Maarten Hooijberg

Geometrical Geodesy

Using Information and Computer Technology

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With 138 Figures and a CD-ROM

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Preface

Surveying a Century Ago

As it was based on the principles of geometry and trigonometry, surveying may be may be looked upon as a branch of practical mathematics. Hence, it was necessary that land surveyors and hydrographers should have a fair general knowledge, not only of these subjects, but also of all the subjects comprised by the term mathematics. In addition, the knowledge of mathematics required in ordinary chain surveying and levelling was not very extensive but in geodetical work, the highest mathematical ability and great organising power were required for a proper conception and supervision of the operations (Threlfall, 1940).

Only small area of a few hundred square kilometres can be accurately mapped and surveyed without a framework, since no difficulty is encountered because of Earth-curvature. In the past, especially in hydrography due to the type of work, surveying was carried out on the principles of ordinary practice, but in a very rough manner, rapidity of execution being of paramount importance, the permissible error was sometimes large. The relative positions of the main surface features were obtained by aid of portable instruments, such as sextants and lead lines, tide poles, and logships. Sketching, just like military surveying was often filling in the smaller detail. In contrary, survey works done by the national mapping agencies (NMAs) were of a higher-level, and comprised the delimitation of boundaries as well as topographical surveys. However, much of this work was done in wild and unexplored countries devoid of artificial landmarks. It involved a substantial amount of astronomical work. In the late 1940s, the digital computer became the geodesist's most important tool. Indisputably, the information, communication, and computer technology (ICT) has revolutionised the use of different digital (geodetic) instruments to the point where the collection of data is completely automated. A high accuracy framework is imperative for geodetic and hydrographical surveys [16], for example in the southern part of Africa where several disparate datums have been established, between Eastern and Western Europe, and the North Sea with maritime frontiers between several countries: each with a different horizontal and vertical datum, origin, zone and projection system (Bordley, Calvert, 1986; Ihde, 1995; 2002; Laurila, 1976, 1983; Schureman, 1958; Snyder, 1987; Threlfall, 1940; Wylde, 2000).

A more detailed data analysis in the field of Earth sciences that would have taken a lifetime to complete can now be done within minutes by digital computer. Improved means of electronic data technology and developments, provided by vector-computers, high-speed scaleable parallel-processors (SP), fibre-optical cables, and ultra-high-frequency (UHF) radio links offers scientists the ability to store and to retrieve huge amounts of geodetical and geophysical data. Geodesy is certain to redraw its conceptual borders even more. The aspects, influencing the use and development of geodesy, are increasingly requiring land and hydrographic surveys related to recognised 3D-reference systems with a regional, national, or intercontinental scope. In the mean time, developing a better understanding of such reference system matters is imperative (Higgins, 1999; Snyder, 1987).

Physical Earth deals with any of the Geoscience branches [2]. Once considered as a form of art, geodesy is no longer limited to features of the Earth's surface drawn by hand, but also to spatial computing techniques. Technical 3D-drawings or maps of the cylindrical, conical or other type are of special interest to the geodetic design of structures, realised by geometrical computing and measurement techniques. These techniques involve spatial techniques, such as used in medical virtual reality, 3D-seismic surveying, architectural designs such as bridges, church towers, cooling towers, hydro-electric-power dams, parabolic optical mirrors, shape of cars and ships (torus, hyperboloid, paraboloid, onion), represented in conformal, equal-area or equidistant orthogonal or polar projection systems (Grafarend, 1997k, m, 1998a, b).

Devices, such as digital computers, digitisers, scanners, sensors, printers, plotters, have changed the way mapping is done. Various technologies that are transforming traditional geodesy are:

- Electronic Chart Display and Information System (ECDIS)
- electronic total stations (so-called tacheometers)
- geographic information systems (GIS)
- digital ortho-photography by CCD Receivers
- remote sensing
- 3D-positioning.

The principles of digital photogrammetry, image analysis, and remote sensing have become dynamically intertwined (Falkner, 2002; Harper, 1976; Kasser, 2002; Kovalevsky, 2002).

Global Navigation and Positioning Systems

The Time ranging and sequential system (TRANSIT) and the Global positioning system (GPS) are satellite-based navigation technologies funded by the US Department of Defense (DoD) in 1959 and 1973, respectively. While GPS was under development, Russia developed a similar (military) system, called GLONASS.

In the 1960s, the TRANSIT series of satellites, operationally based on observation of the Doppler shift of the satellite's transmitted signal, were launched enabling US Navy submarines to fix their position accurately under all surface weather conditions. By 1967, the Transit Doppler satellite tracking system was revealed for use by the civil (offshore) industry [4.3.1]. Therefore it is, as the late Alwyn R. Robbins, Head and Reader at Oxford University, mentioned in 1978:

> *" the Revolution in Geodesy has only just begun and we are privileged to be taking part in it .... ".*

In the same year, DoD launched the first global positioning system (GPS) satellite. The more accurate GPS [4.3.2] (or NavStar) replaced the Transit navigation system in 1996. Using GPS for instantaneous 3D-positioning in space, the wonder of using digital images for electronic charts, development of completely new tools - such as advanced very high resolution radiometer (AVHRR) imagery, synthetic aperture radar (SAR), inertial surveying system (INS), combined for acoustic hydrographical survey of the oceans and inland waters, digital aerial and terrestrial photogrammetric cameras - will be imperative for the years ahead (Carrara, 1995).

In 1993, many well-known geodesists were completely swept off their feet after the initial GPS-operational network capability was established with more than 24 GPS-satellites in orbit, each circling the Earth once every 24 hours. To fix a position, a GPS-receiver needs to be within range of at least four satellites. From this time forth, many geodesists regarded GPS as the last word in geodesy and the ideal method for solving most existing global problems in geodesy. Actually, GPS is a first rather than a last word. Today, a DGPS World receiver can fix the position to better than a few nanoseconds in time, 0.01 m or 0.01° (degree) in heading from the difference in time between receiving the signals from each satellite (Dixon, 2007).

Because GPS is vulnerable to intentional, unintentional, and natural interference, most governments are placing emphasis on suitable backup systems quoted are triple inertial: VOR/DME and eLoran (FAA, 2004). The eLoran signals are unjammable, and penetrate buildings and foliaged areas because of its high-powered signals. While eLoran uses ground-based transmitters with low frequencies, low-powered GPS signals are often blanked out or disrupted due to satellite-based technology with very high frequencies. It is a Loran system, that incorporates the latest receiver, antenna, and enhanced transmission system technology to serve as a back up to, and complement Global Navigation Satellite Systems (GNSS) for navigation and timing to obtain positioning accuracies of better than 10 metres, with 95% repeatability using eLoran for harbour entrance and approach. Similarly, eLoran is an independent, accurate source of Universal Time Coordinated (UTC), already operational in northwestern Europe. It is expected that the systems will be integrated as hybrid GNSS/eLoran receiver architecture in the near future [4.3.7] (Roth, 1999).

For more than two decades, scientists try to boost system performance without increasing the clock rate by sophisticated packaging micro-electro-mechanical-systems (MEMS or MOEMS) by taking simplification into the extreme. It means simplifying architecture, until a monolithic IC contains an entire computer, including GNSS, gyros, inertial and other sensors to make it possible to build powerful, high quality products at the lowest possible cost. It remains to be seen in which way the GNSS-concept can be successful [4.3.6]. But one thing is in no doubt, it would be worth the time to develop new stimulating methods that replace and disregard several of the fundamental techniques of the classical analogue methods. Currently, it is only possible to perceive dimly the scope and possibilities of GGIS 3D-space technology [4.5]. It has brought the world to an instant in time at which humankind stands on the threshold of extraterrestrial exploration.

Some older methods and tools may surprise the reader. The reader must keep in mind that - for due to political nature or technical grounds in the past - at a certain date several economical hydrocarbon reservoirs and mineral resources are depleted. Unfortunately, less economical oil, gas, and non-ferro mineral resources are in the future for the second time an opportunity provided the original exploration database coordinate listings are obtainable for reconstruction of these resources.

In Europe only, there exist more than fifty-five different national reference systems (NRSs) as combination of different geodetic datums and different map projections. In addition, false eastings, false northings, zone and zone numbering are all handled in a dissimilar way. For that reason, most national mapping agency (NMA) customers (such as civil engineers, cadastral agents) are not interested at all in discarding complete sets of charts or maps, and changing their NRS. Logistical organisations, such as DoD, DoT, FAA (Department of Defense, Department of Transport, Federal Aviation Administration, respectively), demand continental or global mapping systems. Both types of NMA customers' needs may run in parallel during the first part of the third millennium. Thus, in consequence, NMAs try to fulfil most wishes of customers. Consequently, this book is an everlasting treatise. It is devoted to those topics of geodetic surveying expected in the years ahead and to establish a better understanding consistent with the past, the present, and the future.

About This Book

Geometrical Geodesy - Using ICT is an up-to-date publication about geodetic tools with the ability to store and retrieve the geodetical and geophysical data within minutes on a worldwide basis [15, 16]. This treatise has evolved through the author's twenty-five years of advising and writing on geodetic methods and computations, while the author was on the staff of a major Dutch-British company in the field of worldwide exploration and mining of oil, gas and non-ferro mineral resources.

In the field of geodesy, there are many fine textbooks available. Regrettably, practical examples and hints are spread over numerous technical or scientific publications. Computer programs are either hard to find or hidden in software that is not easy accessible to a wider community. Accordingly, this book is launched to satisfy the needs of land and marine surveyors (hydrographers) and engineers of other branches for accurate, ready-to-use tools: formulae, programs, and subroutines, including an abundant demonstration of all tools by selected examples [On_CD].

Regarding the demands, a computer of modest capacity can cope with the algorithms developed for computation for various spherical and ellipsoidal arcs, lines, areas, geodesics, conversion of conformal projections and S-transformations in any hemisphere.

Methods and Algorithms

Most algorithms will be discussed only enough to bring the reader up to where the specialised treatments in the following chapters start. Some older algorithms can be found in (Adams, 1921, 1949, 1990; C&GS SP No. 65-1 Part 49, 1961; C&GS SP No. 8, 1952; Claire, 1968; Clark, 1976; DA, 1958; Jordan, 1959; Pearson II, 1990; Rune, 1954; Thomas, 1952).

The fundamental and most valuable references for geodesy are (Bomford, 1962, 1977; Heiskanen, 1928, 1958, 1960, 1962, 1967; Helmert, 1880, 1884; Hotine, 1969; Jeffreys, 1952; Levallois, 1970; Molodensky, 1960; Moritz, 1962, 1968; Mueller, 1969). Outstanding treatments are (Bjerhammar, 1986; Graaff-Hunter, 1960; Grafarend / Krumm, 2006b; Hirvonen, 1962). For satellite geodesy alone, the works of (Hofmann-Wellenhof, 2001; Kaula, 1962; Leick, 2004; Levallois, 1971; Kovalevsky, 2002; Montenbruck, 2001; Tsui, 2004) are detailed. The papers mentioned in the Bibliography show excellent summaries of the Status in the 2000s.

Equations used in algorithms in this book were obtained from (Boucher, 1979; Burkholder, 1985; Hegemann, 1913; Heuvelink, 1918; Meade, 1987; Moritz, 1992, 2000; NGA, 2000; Redfearn, 1948; Rosenmund, 1903; Schreiber, 1866, 1897; Snyder, 1987; Strang van Hees, 2006; Vincenty, 1984a), used by (Floyd, 1985; Stem, 1989a).
The meridional arc calculations are based on formulae by Klaus (Krack, 1981, 1982), James E. (Stem, 1989a). Isometric latitude calculations are based on formulae by Ralph Moore (Berry, 1970, 1971). Formulae are in radian measure, and optimised by the author.

Contribution of ICT to Geodetic Research

To find a solid understanding of geodesy, the concerned reader must range more widely in the search for full knowledge. He should penetrate to the underlying ideas stripped of sophisticated and pettifogging details; he should learn its objectives, uses, and the genesis of its present concepts and structures.

Geodesy proper is a skeleton. The meaning of geodesy consists in what one can do with it. Therefore, the reader must recognise why a particular result is wanted, and what bearing it has on other results now and in the years ahead. Efficiency of presentation seems to dictate the use of concise English. The omission of some mathematical features of that is vital to its comprehension. In view of multitudinous goals and obligations, it can offer a more enlightening presentation of geodesy and related fields.

The first part of the book describes of various geodetical applications. It is divided into eleven chapters about various geodetical methods of applying the formulae to particular cases, copiously illustrated by examples, and fundamental parameters. The second part covers with eight chapters the information about history, organisations, about the techniques, the influence of the sphericity of the Earth on the field operations. All methods and algorithms are elucidated by selected examples found in the literature, enabling the reader to test subroutines in the programs provided at different stages. [On_CD]: *Related Topics* show additional info in a colour block.

Being separate subjects, within the scope of this book, brief references are made to classical geodesy [6] and techniques of astronomy [11]. Following chapters elucidate technological developments, including opto-electronic, acoustic data and positioning technology [13, 14, 15]. The information, communication and computer technology (ICT) has revolutionised the use of various digital tools to the point where the collection of data is completely automated in (private) communication networks. These networks provide scientists the ability to store, and retrieve huge amounts of data using a worldwide distribution network, including multi-dimensional databases. Using vector- and parallel-computer technology, the compilation of relevant data is entirely automatic. A geodesist identifies the object, and a digital computer, using fibre-optical or UHF-satellite radio links, distribute detailed analyses of data and digital graphics, facts and figures, information, statistics, records, analogue data to one particular ICT-communication centre.

Undeniably, replacing the use of mechanical devices (such as counters used in conventional pulsing marine navigation systems as HIRAN, and in phase comparison electronic distance measurement (EDM) equipment in surveying systems) by a choice of solid-state electronic devices - from transistors to very-large-scale integrated (VLSI) circuits - has revolutionised the development and use of various geodetic and hydrographic instruments.

To satisfy the practical needs of land surveyors or hydrographers, the author tried to illuminate 3D-, and 4D-spatial developments, calculation tools and techniques and to focus on land- and marine applications in any hemisphere [16]. The need to fit the algorithms and results into a logically ordered system has obliged the author to seek out a new structure. The universal subroutines with 2D-, and 3D-formulae listed in this book were developed on an IBM compatible Personal Computer (PC) [17]. The reader is assumed to be acquainted with the general principles of geodesy and associated jargon, which is essential for proper use of the FORTRAN subroutines and related programs. In addition, main programs [18] and subroutines [On_CD] in FORTRAN have replaced the BASICA programs of the edition *Practical Geodesy - Using Computers*. The formulae remained unchanged. This book does not attempt to teach the FORTRAN language.

The author assumes the reader is familiar with ICT and computer programming. All source code is written in the FORTRAN programming language, since 1954 widely used on a variety of different computers and operating systems. The software is built around a library. PC-based geodetic applications are still models of great virtue when it comes to ease of use and flexibility. Using a computer to the fullest extent means that many existing formulae must be rewritten in a form, which fits the computer best (Vincenty, 1971). Many techniques are available for doing coordinate conversions and transformations. A few offer mathematical ease at the sacrifice of accuracy. At the other extreme, some are accurate under all conditions. Those who are conversant with the extended use of physical or mathematical geodesy will find that certain refinements in explanation have been neglected for the sake of simplicity. The essence here is being extremely close to the applications in theory, and to the production of practical data.

Nevertheless, readers are cordially invited to study applicable sources. References to material for further thought as outlined in an abundant Bibliography, with indices to Authors and Subjects. With the aim of making the reference book more comprehensible, and to facilitate reference work, a certain number of equations and subroutines have been repeated in various sections and chapters. Additionally, the reference section includes certain original and review articles dealing with subjects, which are only superficially mentioned or not discussed at all to stress the practical aspects of geodesy and related fields of interests. A series of papers by a variety of authors on a range of subjects is not the substitute for systematic study or technical mastery, but rather a kaleidoscope, whose varicoloured bright flashes of light may illuminate, excite, and inspire. The purpose is not to present additional information that the reader must master, but to give some more insight. They complement the close examination of minute details with broad vistas. This is the primary objective of this reference book.

Every effort has been made to present a practical approach to computing in which the how-to-do-it approach is stressed to provide thoughtful advice, to create inspiration and confidence among professionals with an engagement in the geodetic and hydrographic profession. It is the latter, that the author has had exclusively in view. Furthermore, it should be realised that some electronic processes are presented in a limited context. Corresponding with rapid advancements in electronic data processors, all techniques discussed herein are dynamic. What is in vogue today could be very well obsolete tomorrow. Nevertheless, this book is written for the benefit of those engineers who need to understand the developments, such as global and continental reference systems, computing frames, spheres, ellipsoids, datums, and application of least squares adjustments (LS), multidimensional databases, using parallel computers for striving activities in the years ahead.

Having worked for many years in the field of Geodesy and ICT, the author tried to stay in close touch with investigations, and problems that arose during work. The author was very privileged to receive assistance with algorithms, corrections, criticism, data, ideas, images, and response that contributed significantly to a systematic presentation of the subjects discussed in this book.

Standing on the Shoulders of Giants

Writing this book does not depend on just one individual but on cooperation between the author and more than thirty scientists, colleagues of distinguished agencies, research institutes, and technical universities in the worldwide field of geodesy and geophysics. Every scientist stands on the shoulders of his predecessors.
It should be noted that the remarkable increase in quality and quantity of the results obtained from the geodetical techniques during the last twenty-five years is because they were based on about 250 years of development of theory. Basic concepts, thoughts, and formulae were devised by the pioneers, such as Bessel, Brouwer, Bouguer, Clairaut, Einstein, Gauss, Helmert, Lambert, Laplace, Legendre, Maupertuis, Newton, Poincaré, and Tsiolkovsky [12].

Isaac Newton quoted the saying of a 12th-century French scholar, Bernard of Chartres (Lloyd, 1998):

> *" ... we are as dwarfs mounted on the shoulders of giants, so that we can see more and farther than they: yet not by virtue of the keenness of our eyesight, nor through the tallness of our stature, but because we are raised and borne aloft upon that giant mass ... "*

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This book is dedicated to

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on the occasion of his 65th Birthday

Erik W. Grafarend became Lecturer in 1969, and Associate Professor in 1970 at the Dept. of Theoretical Geodesy, University of Bonn. He joined the FAF University, Munich, Germany as Professor in 1975. From 1980 onward, he was Professor and Head at the Institute of Geodetic Research, The University of Stuttgart, Germany. Erik W. Grafarend's work spans from the rotation of a gyroscope to the rotation of the Earth's axis described by a deformable, viscoelastic body (Krumm, 1999). He is the principal author of more than three hundred papers, and various books in the field of geodesy, mathematics and physics, such as *Map Projections - Cartographic Information Systems* (2006). The author expresses his great appreciation for reading through the manuscript, for the dialogues in Stuttgart and in Continental Europe, for providing valuable comments and substantial corrections, which have greatly enhanced the value of the book.

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To avoid international differences no personal or academic titles are mentioned in this publication. All views expressed are those of the author.

It is the author's sincere wish that the Handbook will perhaps encourage and help those already practising geodesy to achieve an even higher standard of efficiency, to raise the expectation level, and arouse the interest of others, resulting often in some new idea, or at least material for further thought.

Land van Heusden en Altena, Spring 2007
Maarten Hooijberg

Contents

Index of Authors

List of Figures

Related Topics - On_CD contains some corresponding figures in a colour block. In the list mentioned above are these figures indicated in **bold**

1. Time and Reference Systems

According to August de Morgan, Laplace's very last words were:

"Man follows only phantoms".

While time is an absolute quantity in the framework of Newtonian physics, which does not depend on the movement and the position of a clock, the same is no longer true in a general relativistic framework. Different proper times apply for clocks that are related to each other by a 4D-space-time transformation. Transformations require knowledge of the space-time metric, which itself depends on the movement and position of the gravitating masses (Montenbruck, 2001).

1.1 Earth in Space-Time Metric

Generalised Theory of Gravitation

Space is a boundless, three-dimensional extent in which objects and events occur and have relative position and direction. The space beyond the Earth and its atmosphere is described as the cosmos (Britannica, 1999; Dirac, 1963).

In Einstein's picture, 4D-space of physics itself becomes curved and the planets move along the straightest (geodesic) lines in that curved space.

Here he was speaking of geodesics in the 4D-space-time continuum. Using these laws means bringing in something additional to the four-dimensional symmetric dimension is a description of our consciousness of the universe at a certain moment in time. A four-dimensional path (spiral) of a planet is portrayed by showing space in two dimensions and a measured time (Figure 1). If Einstein's picture of a four-dimensional curved space of physics is correct, the spiral becomes a geodetic line! Even Einstein described this article as *not quite easy to grasp* (Einstein, 1950; Gamow, 1961)

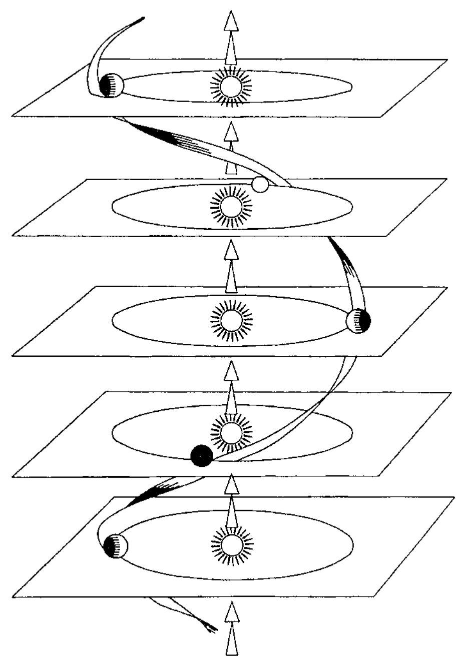

Figure 1: A four-dimensional path (spiral) of a planet in space-time is a geodesic

Principle of General Relativity

The physical foundation of Einstein's view of gravitation, the *principle of general relativity*, lies on two empirical findings that he elevated to the status of basic postulates. The first postulate is the relativity principle, i.e. local physics is governed by the theory of special relativity. The second postulate is the equivalence principle, i.e. there is no way for an observer to distinguish locally between gravity and acceleration.

The principle of general relativity is that the equations expressing the laws of nature must be covariant with respect to all continuous transformations of the coordinates. According to this principle, the concept of space detached from any physical content does not exist. A field whose components are continuous functions of four independent variables - the coordinates of space and time - represents the physical reality of space (Einstein, 1950).

Einstein's Unified Field Theory

Albert Einstein, assisted by Max Planck and Niels Bohr, supplied at several decisive times the background of the quantum theory that ensured the dominant role it played in the 21st century. Launching a revolution in physical thought, Einstein was primarily identified with papers about the theory of relativity. Another paper set forth his celebrated deduction of the equivalence of mass and energy: $E = mc^2$.

Some years later, Einstein had built upon his *special theory of relativity* the great edifice of his *general theory* that subsumes the large-scale mechanics of the universe into comprehensive space-time geometry. From that time on, he set himself the towering target of bringing into this grand generalisation the electromagnetic laws that govern the small-scale territory of atomic particles. Principle of equivalence enunciated by Einstein states that accelerated motion produces effects indistinguishable from those of a gravitational field. The equivalence of the two points of view is the foundation of Einstein's relativistic theory of gravity. This so-called principle of equivalence between observations carried out in an accelerated chamber and in a *real gravitational field* would be trivial, however, if it applies merely to mechanical phenomena. Einstein's deep insight was that the principle is quite general and holds for optical and other electromagnetic phenomena.
Einstein undertook a speculative attack aimed at bringing the electromagnetic field into line with the gravitational field. Having reduced gravity to the geometrical properties of a space-time continuum, he became convinced that the electromagnetic field must also have some purely geometrical interpretation. Then again, the *unified field theory*, which grew out of this conviction, had hard going, and Einstein died without producing anything so simple, elegant and convincing as his earlier work (Dirac, 1963; Gamow, 1961).

1.2 Frequency and Time

"The difficult is what takes little time; the impossible is what takes a little longer"

Fridtjof Hansen, Scientist, and Nobel Laureate

Seizing the Day

Since the time of Aristotle - who believed in *absolute time* (time as a fixed entity which does not vary, in direct contradiction with the theory of relativity) - there have been theories to describe the behaviour of the universe. If interpretations of time change, so do the methods for recording it.

The sundial, which uses the Sun's shadow to show the passing of time, was used from at least 3500 BC when the first obelisks were built in Egypt. While sundials were used until the 1850s, they were obviously limited to daylight. Around 1415-1380 BC the first Egyptian clepsydra (water-clocks) were constructed. The invention was adopted by the Greeks about 325 BC. For a long period, the middle ages seem to have been quite indifferent to time. As they had measured fragments of time, such instruments were unable to account for its continuity. Towards the end of the 13th century, the invention of the mechanical clock replaced the uncertain clerical time by a time that was secular, rationalised, and widely used in Europe until the 18th-century (Lehni, 2004) (Figure 2). The ancient Greek Antikythera Mechanism - a 85 BC version of an astronomical clockwork - acts as a magnet for decoding in the National Museum of Archaeology in Athens.

It is not known where the first weight-driven-clocks appeared, or what drove the adoption of a day divided into 24 equal hours. They often did not keep time particularly well, but they were expressive indicators of status in medieval Europe. In the 16th century the midnight changeover was adopted, largely due to the influence of mechanical clocks, which struggled with measuring hours of different length.

Since October 1884, the International Meridian conference (IMC) held in Washington DC, USA, decided that each new day begins at the Prime Meridian passing through the centre of the Airy transit circle telescope at the Greenwich Royal Observatory, Herstmonceux, UK. However, the International Dateline - by coincidence at 180°E / W which principally runs through the Open Ocean - is nevertheless subject to various historical prejudiced changes by Nations that lie on otherwise close to it.

Precise Time Dissemination

Precise time and frequency are broadcast by radio in many countries. In 1904, transmissions of time signals began as an aid to navigation. Since 1955, they are now widely used for many scientific and technical purposes. The seconds' pulses are emitted on coordinated universal time (UTC), and the frequency of the carrier wave is maintained at some known multiple of the caesium frequency.

Strasbourg Cathedral's Astronomical Clock (1352-1571-1838)

Figure 2: Part of the Astronomical Clock at the Cathedral of Strasbourg

In this era of the atomic clock, the Strasbourg clock might no longer be of interest, but it continues to draw the crowds. It represents a reflection on Time, together with an entertainment by the play of its automata. In 1571, the maker's aims were to describe Time by every means but would also ornament the cathedral and add to the town's prestige. It constitutes a total work of art, a masterpiece unique in the world offered by our ancestors to the enigma of Time (Lehni, 2004; Ungerer, 1922).

Mechanical Clocks

All first mechanical clocks were weight-driven machines, fitted into towers as turret clocks. These mechanical clocks appeared 1300-1400 AD with bells but without hands in the towers of several large Italian cities. The first public clock that struck the hours was made and built in Milan in 1335. Spring-powered clocks were developed 1500-1510 AD in Nuremberg. In 1859, Edmund Beckett invented the double gravity escapement. He used it for the clock known as the Westminster *Big-Ben*. A modern Italian city clock is *Verdi's theatre clock* in Busseto (Figure 3).

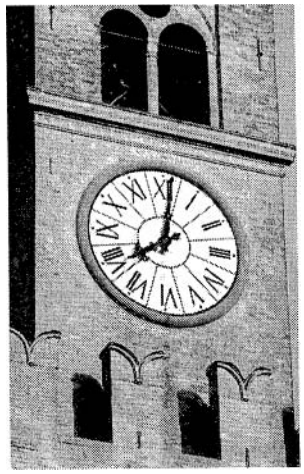

Figure 3: Time - Verdi's clock

Caspar Werner made the earliest known watch in 1548. Galileo's attempts to measure right ascensions directly by means of the moments the stars passed through the meridian were mystified by the irregular running of the clocks. In 1656, Christiaan Huygens created a structure in which the pendulum regulates the turning of the toothed wheels. A short impulse from the escapement at every oscillation keeps the pendulum swinging at constant width. The length of a pendulum with a period of one second was about 0.99 metre. As a result, the pendulum clock became an accurate astronomical instrument for measuring time, an essential aid in all future astronomical measurements.

Both Christiaan Huygens and John Harrison introduced the spring balance and escapement as a regulator. Independently, Pierre Leroy, Ferdinand Berthoud of France, and John Harrison of England made so-called marine chronometers, providing a new means of calculating longitude at sea. Such a chronometer was indispensable on a rolling vessel. It could keep time to about one-fifth of a second during the journey.

Electrical Clocks

In 1918, the timekeeping part of electric clocks with a synchronous electric motor was running in a tune with the frequency of 50 Hz, coupled to a reduction gearing driving the clock hands. The most accurate electrical-mechanical timekeepers were the Shortt pendulum clocks in observatories. Shortt clocks could maintain the time to within $0^s.002$ per day.

In 1929, the quartz crystal, oscillating at frequencies from about 10 kHz to more than 200 MHz, was first applied to timekeeping. The maximum error of an observatory quartz-crystal clock was within $0^s.0002$ per day. By the end of the 1960s, the Bulova watch company invented an electronic tuning fork, controlled by a quartz crystal, reduced by a process known as frequency de-multiplication (Mühe, 1992).

Caesium Atomic Clocks

Atomic clocks are based on the periodic oscillation of a microwave signal that is in resonance with a low-energy state transition of a specific atom or molecule. In 1948, the first microwave amplification by stimulated emission of radiation (Maser) clock built at the National Bureau of Standards used an ammonium gas (NH_3) absorption line to control the frequency generation. In the mid-1960s, Alwyn R. Robbins of Oxford conceived and developed the *Chronochord*, a printing quartz crystal clock for use in the astro-geodetic field. At present atomic clocks are generally based on caesium (^{133}Cs), hydrogen (^{1}H), or rubidium (^{87}Rb). Caesium clocks provide the best long-term stability. So, they are used as primary standards in the practical realisation of atomic time scales (Montenbruck, 2001; Kovalevsky, 2002; Mühe, 1992).

Few laboratories built large caesium-beam oscillators and clocks to serve as primary standards of frequency. In 1942, the Greenwich Observatory adopted quartz clocks as their standard timekeepers. Vibrations of quartz crystals are more regular than those of the pendulums are for measuring time.

At the Bureau International de l'Heure, Paris (BIH), atomic clocks were used as early as 1955 besides traditional astronomical time keeping procedures. Another caesium-beam atomic clock was placed at the National Physical Laboratory (NPL). Currently 65 laboratories worldwide that use 230 atomic clocks maintain the world's official global time. All clocks are matched up to global positioning system time (GPST). Each satellite with several atomic clocks on board uses the vibrations from a *"caesium fountain"* to measure time within a nanosecond per day. Now, five atomic clocks at the National Physical Laboratory (NPL) at Teddington, UK ensure that Britain's time scale agrees with the internationally agreed standard coordinated universal time (UTC) - David Thompson, (1999).

> *"To deal with the uncertainties of life we have invented something like time. Though time is generally linear, nature is cyclical."*

Kristen Lippincott (1999), Greenwich Observatory

In the late 1950s, NPL and USNO (United States naval observatory) conducted a joint experiment to determine the frequency maintained by the caesium-beam oscillator and clock in terms of the ephemeris second, as established by precise observations of the Moon from USNO. In 1972, the BIH atomic time scale was adopted as a worldwide standard time under the name International atomic time (TAI).
The radiation associated with the particular transition of the Caesium-atom (^{133}Cs) was found to have the fundamental frequency $\nu_0 = 9\ 192\ 631\ 770$ Hz ± 20 Hz of ephemeris time (ET). The merits of the (space) caesium-beam atomic clock are:

- an invariant fundamental frequency
- an extremely small fractional error
- convenient to use.

About Ephemeris Time Scale

In 1950, the lack of uniformity of the Earth's rotation led to the introduction of the ET, which may be regarded as the independent variable of Newtonian mechanics [1.6.1]. Among his other notable contributions, Brouwer formulated the term "*ephemeris time*" to describe time measurement unaffected by deviation in the rate of the Earth's rotation. Since 1960, ET is independent of the Earth's rotation but based on the observed motions of the Solar system bodies (Brouwer, 1951; Montenbruck, 2001; Mühe, 1992).

An ephemeris is a table of calculated positions with the time as argument. In calculating a precise ephemeris, it is desirable to treat the Aberration in the same way as the observations are affected to make the two directly comparable (Brouwer, 1961).

A geometric ephemeris gives the actual position of the body at the times indicated, because it is impossible to observe true geometric positions. It is primarily employed when the only purpose is to find the body.

An apparent ephemeris may be obtained by applying the planetary Aberration to a geometric ephemeris. Positions observed with a transit circle are apparent positions. Hence, ephemerides of the Sun, Moon, and principal planets are usually apparent ephemerides. Phenomena, such as eclipses of the Sun, depend on the apparent positions of the bodies concerned.

An astrometric ephemeris may be obtained by antedating the heliocentric position of the object, which may be substituted for the barycentric position with an error not exceeding 0".01. Alternatively, the apparent ephemeris may first be calculated and the circular portion of the annual Aberration then subtracted.

An astrographic ephemeris is easier to calculate than the astrometric ephemeris. It differs only from the astrometric ephemeris by the elliptic portion of the annual Aberration.

$$\mathrm{ET} \approx \mathrm{UT} \quad \text{on January 1, 1900} \tag{1.01}$$

Note

> *An ephemeris is a table of calculated positions with the time as argument and related data for a celestial body for given epochs. In particular, the astronomical almanac is an example publication containing such data for a number of celestial bodies.*

(Brouwer, 1961; GG, 1986)

The arrival of atomic clocks established an atomic time scale as an improved timing system. In practice, the most accurate control of frequency is achieved by detecting the interaction of radiation with atoms that can undergo some selected transition. The frequency of the caesium clock is:

$$\nu_t = \nu_0 + \Delta\nu_0 \tag{1.02}$$

in which ν_0 is the fundamental frequency, and $\Delta\nu_0$ is the frequency shift. The circuitry of the clock is arranged so that ν_t is corrected to generate an operational frequency $\nu_0 + \varepsilon$, where ε is the error in the correction. The accuracy of the frequency-control system is the fractional error $\gamma = \varepsilon / \nu_0$. A large, laboratory-constructed clock can be varied to attain values near γ of $\pm 5 \times 10^{-14}$.

Since 1962 artificial satellites have been used to synchronise for widely separated caesium clocks worldwide within about 0.5 ms. However, the best optically pumped primary frequency standard of clocks is in operation in NIST (National Institute for Science and Technology), USA. It has an accuracy of 5×10^{-15}. The uncertainty in the fundamental frequency of rubidium atomic-beam clocks is greater than the absolute precision of the caesium-beam clocks (Kovalevsky, 2002).

Theory of Time

Stephen Hawking's research showed that Einstein's general theory of relativity implied the universe must have a beginning, and possibly an end. He spoke of space and time being dynamic qualities that could combine to form a fourth dimension called space-time. Used to fix "*events*" in space, space-time is necessary when talking of distant stars or galaxies where the light source from an event such as an explosion may take many years to reach us - Hawking (2000).

1.3 Principal Time Scales in Brief

The SI day is the astronomical unit of time containing 86 400 SI seconds, in which two succeeding meridian transits of the Sun define the length of the Solar day. The period of the Earth's rotation, the time interval between two subsequent meridian passages of the vernal equinox, is known as a sidereal day.
It is equal to $23^h\,56^m\,4^s.091 \pm 0^s\,005$ in Solar time (McCarthy, 1996; Montenbruck, 2001).

The time epoch denoted by the Julian date (JD) is expressed by a certain number of days and fraction of a day after a fundamental epoch sufficiently in the past to precede the historical record, chosen to be on:

January 1, 4713 BC at $12^h.00$ UT.

The Julian century contains 36 525 SI days. The Julian day number denotes a day in this continuous count, or the length of time that has elapsed at $12^h.00$ UT on the day designated since this epoch. The JD of the standard epoch of UT is called J2000.0, in which:

$$\text{J2000.0} = \text{JD } 2\,451\,545.0 = \text{January 1, 2000 at } 12^h.00 \text{ UT)} \tag{1.03}$$

$$\text{J2000} = \text{MJD } 51\,544.5 \tag{1.04}$$

$$\text{MJD} = \text{JD} - 2\,400\,000.5 \tag{1.05}$$

Time arguments denoted by T are measured in Julian centuries relative to the epoch J2000.0 (Bock, 1998; McCarthy, 1996):

$$T = (\text{JD} - 2\,451\,545.0) / 36\,525 \tag{1.06}$$

Time Correction for Height above the Geoid

According to Einstein's theory of general relativity [1.1], the primary standards used to form the frequency of TAI are corrected for height above the geoid or $\approx$ mean sea level. In the 1960s, Robert V. Pound and Glen A. Rebka confirmed that it agreed very closely with the predictions. The mutual relation of the above time scales and their historical evolution are outlined in (Bock, 1998; McCarthy, 1996).

Numerous principal time scales have been formed. Distinction is made between:

- mean Solar time, tied to the motion of the Sun and the rotation of the Earth
- atomic time scale that provides the practical realisation of a uniform clock
- dynamical time scale that serve as independent argument in the equations of motion.

(Montenbruck, 2001)

Mean Solar Time

In 1925, Greenwich mean time (GMT) or universal time (UT) was established as an international time scale for astronomical and civil purposes. Following Kepler's second law, the UT time scale was non-uniform due to irregularities and secular variations in the Earth's rotation. A fictitious mean Sun was defined moving along the Equator with uniform velocity. The hour angle of this fictitious mean Sun was called UT.

Universal time (UT1), corrected for polar motion (PM) [2.2.2], represents the true angular rotation of the Earth. UT1, the mean Solar time of the Greenwich Meridian, UTC (coordinated universal time), the basis of civil time, runs at the same rate as TT (terrestrial time), and TAI (international atomic time). Nevertheless, UTC is incremented by integer leap seconds to follow UT1 within $0^s.9$ periodically (McCarthy, 1996; Montenbruck, 2001).

Atomic Time Scale

TAI is kept by many atomic clocks operated by various national agencies, international earth rotation service (IERS), and Bureau International des Poids et Mésures (BIPM), as the continuous fundamental time scale on Earth, related by definition to TDT by

$$\mathrm{TDT} = \mathrm{ET} = \mathrm{TAI} + 32^s.184 \tag{1.07}$$

Its point of origin was established to agree on January 1, 1958 with $0^h.00$ UT. At this date, universal and sidereal times ceased effectively to function as time systems (Bock, 1998; Kovalevsky, 2002).

Because TAI is the continuous fundamental time scale on Earth, synchronisation with the Solar day (UT1) is not maintained due to the Earth's rotation rate retardation. Atomic time (UTC) is made to keep rough track of the Sun's motion for civil convenience. GPS-receivers label phase- and pseudorange measurements in UTC (or GPS-time GPST), the basis of current civilian timekeeping (Menge, 2001).

Time signals broadcast by the GPS-satellites are synchronised with the GPS Master Control Station's atomic clock in Colorado, USA. Global positioning system time (GPST) is defined as JD 2 444 244.5 and was set to:

$$\mathrm{GPST} = 0^h.00 \text{ UTC on January 6, 1980} \tag{1.08}$$

UTC is incremented by integer leap seconds, initiated by IERS, so that UTC does not vary from UT1 by more than 0.9s:

- first preference is given to the end of June and December
- second preference to the end of March and September

$$\mathrm{DUT1} = \mathrm{UT1} - \mathrm{UTC} \quad \text{broadcast with time signals to a precision of} \pm 0^s.1 \tag{1.09}$$

There is an offset of 19s between GPST and TAI so that (Bock, 1998; Hofmann-Wellenhof, 2001):

$$\mathrm{GPST} = \mathrm{TAI} - 19^s \tag{1.10}$$

Travel times of extraterrestrial signals are measured in space-geodesy. Fundamentally, it is the precise definition of time. Hence, the epoch and the interval are aspects of time (Montenbruck, 2001):

- an epoch is a particular point in units of a time scale to which events are referred
- an interval is the time elapsed between two epochs.

Dynamical Time Scale

In 1960, a new dynamical time scale was established, considering time as a continuously and a uniformly passing physical quantity in the theory of motion.

SI Units - Système International

In 1967, the Conférence Générale des Poids et Mésures (CGPM), Sèvres, France, redefined the unit of time is expressed as the SI second (s) as measured on the geoid, the fundamental interval unit of TAI, as follows:

> *"the second (s) as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-atom (^{133}cs)".*

It is the transition selected for control of the Caesium-beam clock developed at the NPL. The definition implies that the atom should be in the unperturbed state at the geoid or mean sea level. It makes the SI second equal to the ET second.

ET, based on the orbital motion of the Sun, the planets, and the Earth's Moon with data predicted from analytical or numerical theories of motion, defined time as the independent argument of precomputed planetary and Lunar ephemeredes (Cheng, 1992).

In 1976, the IAU (International Astronomical Union) introduced the TDB (barycentric dynamical time) and TDT (terrestrial dynamical time) time scales for dynamical theories and ephemerides to be used in Almanacs beginning in 1984. Responsible for the dissemination of standard time and BIH's earth orientation parameters (EOP) are IERS and the Bureau International des Poids et Mésures (BIPM).

Since 1992, the difference between proper time and coordinate time has led the IAU to adopt two different time scales for use, which are named as terrestrial time (TT), formerly TDT, and geocentric coordinate time (TCG). TT describes the equations of motion in the Earth's gravity field of an Earth (GPS) satellite. The constant difference between TT and TAI makes TT continuous with ET for periods before TT was defined. For practical purposes, TT is actually derived from the atomic TAI time scale (Bock, 1998; McCarthy, 1996; T, 1993).

TDB is the independent variable in the equations, including terms for relativity, of motion of the celestial bodies, relative to the SSB (barycentre of the Solar system). The barycentre is placed to a point near its surface in the direction of Jupiter. According to the theory of general relativity, TDB is the independent variable in the equations of motion of bodies in a gravitational field of the SSB IRF (inertial reference frame). Due to the motion of the Earth in the Sun's gravitational field, an Earth-based clock will display periodical variations as large as 1.6 ms with respect to TDB. Measuring and recording in TDB of extragalactic radio signals by Earth observatories is imperative for very long baseline interferometry (VLBI) (Felli, 1989) [4.2.2].

Barycentric coordinate time (TCB) supersedes a time scale known as TDB, which was defined to differ TT by periodic terms. Accordingly, TDB and TCB are related (Audoin, 2001; Bock, 1998; Kovalevsky, 2002; McCarthy, 1996; Montenbruck, 2001):

$$\mathrm{ET} = \mathrm{TDT} = \mathrm{TT} = \mathrm{TAI} + 32^{s}.184 \tag{1.11}$$

TCB is the relativistic time coordinate of the 4D-barycentric frame

$$\mathrm{TCB} = \mathrm{TDB} + 46.7\ \mathrm{s/cy} \quad (\mathrm{Year}-1977.0) \tag{1.12}$$

Geocentric coordinate time (TCG) is the relativistic time coordinate of the 4D-geocentric frame

$$\mathrm{TCG} = \mathrm{TT} + 2.2\ \mathrm{s/cy} \quad (\mathrm{Year}-1977.0) \tag{1.13}$$

Sidereal and Universal Time Calendar

Sidereal time (ST) and universal time (UT1) are interchangeably employed as time systems. Earth's diurnal rotation is used to measure angles by transforming the celestial reference frames (CRF) into terrestrial reference frames (TRF) and vice-versa. The primary rotation angle between CRF and TRF is given as a sidereal angle Greenwich apparent sidereal time (GAST). The angle between the observer's local meridian and the point on the celestial sphere after correction for Nutation and Precession, the local hour angle (LHA) of the true vernal Equinox is called the apparent sidereal time (AST). When the hour angle is referred to the mean astronomic meridian of Greenwich, it is called GAST (McCarthy, 1996; Montenbruck, 2001; Wilkins, 1989).

Similarly, MST and GMST Fundamental Katalog 5 (FK5) equinox refer to the mean vernal equinox, corrected only for Precession [2.2]. The equation of the Equinoxes relates AST and MST, due to Nutation [2.2]. Conversion between ST and UT1 is rigorously defined in terms of the IAU-(1976) as given in (McCarthy, 1996).

The relationship between atomic time and the universal (sidereal) angle is given by UT1-UTC. It describes the irregular variations of the Earth's rotation. Variations are determined by analysis of GPS and very long baseline interferometry (VLBI) [4.2.2] data that provides long-term stability, the connection between the celestial reference system (CRS), and terrestrial reference system (TRS) [1.6].

Greenwich Mean Sidereal Time (GMST), also known as Greenwich Hour Angle, denotes the angle between the mean Vernal Equinox of date and the Greenwich Meridian as a direct measure of the Earth's rotation. The Earth's rotation, being used in effect as the regulator of a space clock whose tick defines the length of day (LOD), shows small variations. For that reason sidereal time must be computed from astronomical and geodetic observations (Montenbruck, 2001).

1.4 Definitions of the Figure of the Earth or Geoid

The mathematical Figure of the Earth, a term, which in present usage is usually applied to the classical definition of the geoid, is defined as geop, or the equipotential (level) surface of the Earth's gravitational field. This surface, on average, which best fits, in the least squares sense, mean sea level (MSL) is called the geoid, or the Figure of the Earth. In practice, the average position of sea level and a corresponding average over the time-varying geopotential must be accepted (Bomford, 1977; Clarke, 1880; Fischer, 1845; Graaff-Hunter, 1960; Kuroishi, 2001a, b, 2002; Lisitzen, 1974; Novák, 2003a, b; Torge, 2001).

Original Definition of the Geoid

Various definitions of the equipotential surface of the Earth or geoid were given, such as the original (first) definition as given by (Listing, 1873):

> *"Die Oberfläche des gesammten Meeres in seinem Gleichgewichtszustande, also abgesehen von Fluth und Ebbe und Wellenschlag; unter den Continenten denkt man sich diese Fläche erweitert durch ein Netz von Canälen, welche unter sich und mit dem freien Meer in Verbindung stehen. Diese Fläche, welche, dem hydrostatischen entsprechend, alle Lothlinien (Richtung der Schwerkraft) rechtwinklig durchschneidet, heisst das Geoid"*

or in a few words:

> *"The equipotential surface of the Earth's gravity field which would coincide with the ocean surface if the latter were undisturbed and affected only by the Earth's gravity field".*

Second definition of the Geoid

The second definition as given by (Jensen, 1950):

> *"The equipotential surface, through a given point, chosen near MSL, that would exist if only the rotation of the Earth and the Earth's gravitational field affected the potential as a function of the position of the chosen point".*

Third Definition of the Geoid

The third definition as given by Albert Einstein and later by Arne Bjerhammar conceived in his treatise *Relativistic Geodesy* (Bjerhammar, 1986; Einstein, 1950):

> *"In relativistic geodesy, the geoid is the surface nearest to mean sea level on which precise clocks run with the same speed".*

Fourth Definition of the Geoid

An attractive field for future research will be in line of thought about W_0 - the geopotential value of the equipotential surface, the geoid - in which Grafarend has contributed so substantially (Ardalan, 2004; Grafarend, 1996i, 1997h, 1999b, 2002b, c, 2006b).

The fourth definition. A study about the ellipsoidal "*free air*" potential variation may serve as a guide to solve the problem of W_0 without using the gravity field of an ellipsoidal equipotential surface.

In summary, (Grafarend, 1997h) reports:

> " *... the* Baltic Sea Level Project *situated between 53° N, 6° E and 66° N, 28° E is used as an example. Twenty-five GPS-stations are determined in WGS84 3D-coordinates with its orthometric height at the Finnish Height Datum N60, epoch 1997.4, the geopotential value* $W_0 = 62\ 636\ 855.536 \pm 1.692\ m^2/s^2$ *in the vicinity of* primary mareographic (*tide gauge*) stations. *Matching the 3D-coordinates of the primary mareographic stations with the GPS-station coordinates was carried out using a new ellipsoidal free-floating-variation technique. Transformation spherical harmonic coefficients of the gravity potential results into ellipsoidal harmonic coefficients, computed for all GPS-stations using a global model EGM96 of the gravitational potential field. Using the orthometric heights of the GPS-stations with respect to the geoid reference Datum, the gravity potential with respect to a global ellipsoidal model has been transformed to yield the vertical geodetic* W_0 *value ...".*

Variations of the miscellaneous W_0 data vs. 6 263 686.085 kgal.m (Burša, 1992, 1998; GH, 1992; Groten, 1974, 1994a; Poutanen, 1999).

(Doodson, 1960): |[1]

> *" ... to obtain accurate, comprehensive results in the geodetic calculations, it is necessary to know the form of the geoid. Clarifying that the MSL is not an equipotential surface may be necessary and in its simplest definition it would comprise a mean of sea level surface approximated and observed over 18.67 years ... ".*

Geoid undulation is a word to describe the separation (geoidal height = N) between the two surfaces. In a mathematical sense, the geoid is also defined as so many metres above (+N) or below (–N) the ellipsoid.

The geoid can be depicted as a contour chart, which shows the deviations of the geoid from the ellipsoid selected as the mathematical Figure of the Earth. The equipotential surface of the Earth, global gravimetric detailed geoid NASA / GSFC GEM4 Earth Model (1970) with a $1° \times 1°$ gravity data grid is shown in the Miller Projection. The overall size and shape of the GSFC GEM4 1970 ellipsoid is expressed by two parameters, the major axis (a) = 6 378 142.0, and the reciprocal flattening (f^{-1}) = 298.255 (Figure 4).

As a surface perpendicular to the direction of gravity, the geoid manifests the gravitational forces of the Earth, which vary the irregular intensity and direction from place to place due to the irregular mass distribution in the Earth. These forces also affect the orbits of satellites and the trajectories of missiles. A geoid can never be used as a computational surface (Groten, 1967, 1968; Hirvonen, 1962).

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[1]Rotation of the lunar orbital plane. Although the moon completes one orbital pass in one month, it takes 18.67 years for a complete circuit of the orbital plane around the ecliptic pole.

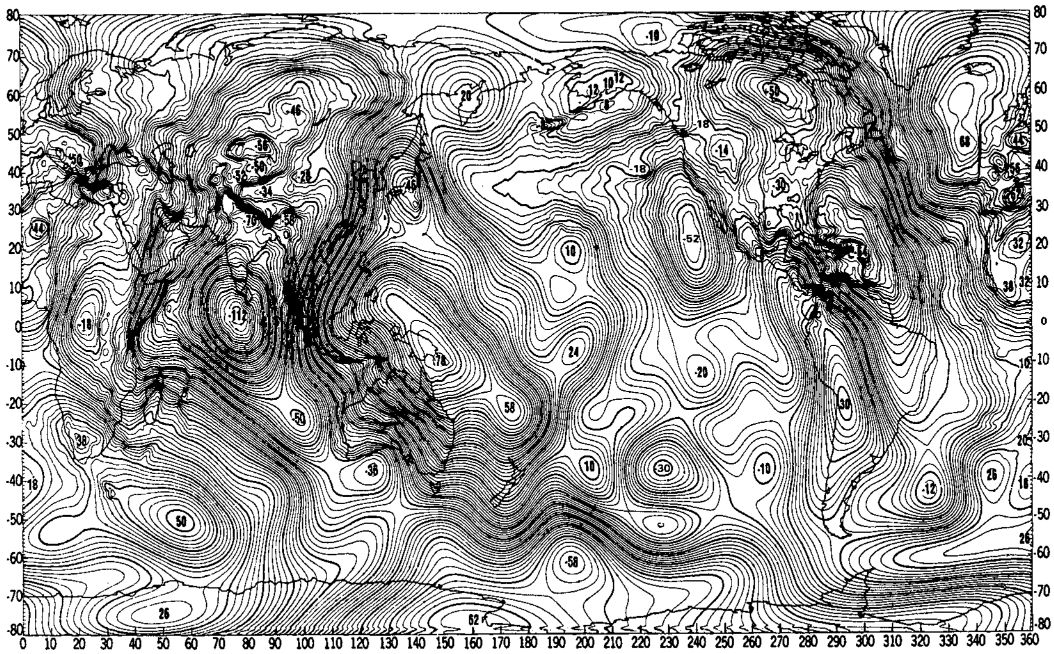

Figure 4: NASA/GSFC GEM4 - Global Detailed Gravimetric Geoid in the Miller projection

The Geoid

About projective heights in geometry and gravity space (Fleming, 2004; Grafarend, 1995d, 1995i, 2001f):

> *With the arrival of artificial satellites, global positioning is carried out in geodetic 3D-reference systems, in Euclidean 3D-space at some reference epoch. In particular, systems are responsible for the materialisation of 3D-geodesy in a Euclidean space.*
>
> *In gravity space, the triplet (x, y, z) is transformed into physical heights with respect to the reference equipotential surface, the geoid, at some reference epoch by a geodesic projection. The geodesic projection is performed by a curved line - the geodesic - as the plumbline / orthogonal trajectory with respect to a family of equipotential surfaces in curved gravity space*

Consequently, the ideal vertical Datum is completely defined by the equipotential surface. The ideal vertical Datum is the geoid. In that case C, the geopotential number, is equal to zero and consequently $H_o = H_n =$ zero.
In practice, the vertical positions are given with respect to the local mean sea level of a determined tide gauge and epoch (Graaff-Hunter, 1960; Groten, 1974, 1981; Kakkuri, 1995).

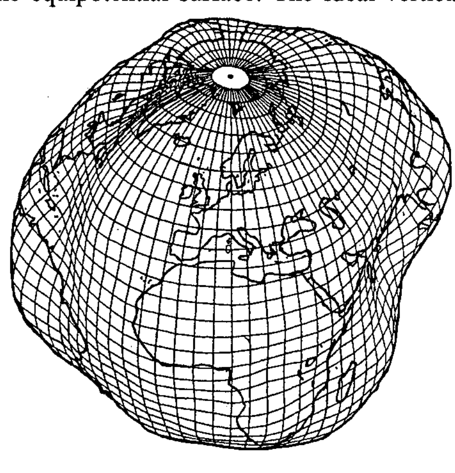

Figure 5: Geoid undulations 15000 × enlarged

Classical Geoid or Quasi Geoid

Defining a geoid requires adopting either a classical geoid or a quasi geoid. Calculating orthometric heights using the actual gravity field will define the classical geoid (Figure 5). Calculating normal heights using the normal gravity field will define a quasi-geoid, which is not exactly an equipotential surface, but an spherop (Graaff-Hunter, 1960).

The difference between the classical geoid and the quasi geoid is approximately proportional to the square of the height above mean sea level (MSL). At MSL, the difference is almost zero. It increases to about 0.1 m for a height of one thousand metres, and extends to about one metre for a height of three thousand metres (Ekman, 1995b; Heiskanen, 1967; Torge, 2001).

Reference Ellipsoids

Using a geometrical figure to describe the geoidal surface of the Earth, a sphere or an oblate ellipsoid is calculated as an approximation for the shape of the geoid [6.2] (Table 18): pp 122. Details of the irregular geoid are described by the separations from the chosen computational reference ellipsoid at specific points (Clarke, 1880; Heiskanen, 1967; Helmert, 1880, 1884; Henriksen, 1977a, b; Hotine, 1969; Molodensky (1960); Torge, 2001).

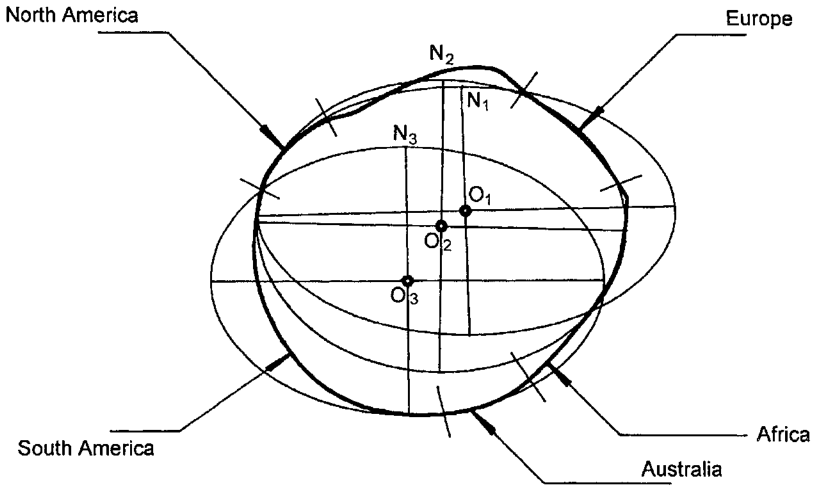

Figure 6: The geoid and three reference ellipsoids

A classical ellipsoid that fits very well, for instance, in Australia, does not necessarily fit in America or Europe (Figure 6). Some of these ellipsoids in use are given in [6.3.1].

Geodetic control data are collected and adjusted on a regional, national, continental or worldwide Datum. Thus, there is little difference between angles and distances measured on the topographic surface of the Earth and their geodetic counterparts represented on the ellipsoid.

As geodetic knowledge of the Earth is refined, and more surface gravity and satellite data are made available, the emphasis in theoretical research will turn to the application of geodetic information to geophysical models, to explore gravitational models, and to revise the global geoid. Nevertheless, research is underway to allow use of the ellipsoid or geoid, as measured by the GPS, as a common worldwide international terrestrial reference. A solution to such an objective is a refined geoid (Ayres, 1995).

1.5 Fundamental Polyhedron

The correction of astronomical observations for the effects of precession, proper motion, nutation, aberration, and parallax belongs to the subject of spherical astronomy, of which only the barest outline can be given here. Valuable treatises on the subject are (Kovalevsky, 2002; Wilkins, 1989).

The fundamental polyhedron (t_0) consists of a set of conventional spatial Cartesian coordinate axes with a particular origin and orientation. The terrestrial system and its frame coincide only at the initial epoch, consequently the polyhedron rotates and moves with the reference system. The system is linked to the initial positioning with a set of changes due to (Bock, 1998):

- Time varying rotation
- micro-plate motion models

- Gravity models
- Nutation models
- Polar motion
- Precession models.

The development of astrometric observations permits to measure any visible distance across the Earth with one cm accuracy. Using a single dual-frequency GPS-receiver, the right software, and the products from the International GPS Service for Geodynamics [19.2], all results are repeatable (Blewitt, 1998; Groten, 1994a, 1999; Kovalevsky, 2002).

Orientation Parameters

The purpose of a reference frame is to provide the means to materialise a reference system, which is in turn defined by station positions (Kovalevsky, 1989). Consequently, space-geodetic positioning is an iterating process by improvements in standard of physical models, reference systems, reference frames, and station positions. Precession [2.2], Nutation, and the BIH earth orientation parameters (EOP) link the fundamental polyhedron to the celestial system. Considering a polyhedron of geodetic stations - with internal motions, deformations, and rotations in space - an angular impetus vector related to the torques and an inertia factor exerted on the Earth can be used to maintain the orientation of this fundamental polyhedron at a later epoch.

IERS Standard of Physical Models

IERS Standards, based on the-state-of-the-art in space-geodetic analysis, are a set of constants and models used by the Analysis Centres. Earth Models may differ from the IAG and IAU adopted standards, such as Precession and Nutation parameters.

The definition of the celestial reference systems and the terrestrial reference systems is complicated by proper motions of stellar objects, and due to an interaction of (Bock, 1998):

- continental drift and crust deformation
- complexity of the Earth's composition
- Earth's atmosphere
- gravitational attraction of Moon and Sun.

The spatial densification, however, requires a processing procedure as outlined by IERS, which can continuously develop the International GNSS Geodynamics Service (IGS) program to densify the international terrestrial reference frame (ITRF) by participation of IGS and scientific agencies. It is based on the principles of hierarchical networks, starting with the IGS global network associate analysis centres (GNAACs), which is then densified by regional network associate analysis centres (RNAACs) to produce the IGS polyhedron. Celestial coordinates of extragalactic radio sources determined from astrometric observations define implicitly spatial coordinate axes (Blewitt, 1998).

About Astrometric Observations

Since the 1970s, astrometry takes part in the general development of astronomy. Proper motions, sizes of stars can only be obtained by astrometric techniques. New techniques such as radio and optical astrometry, CCD-receivers, astrometric satellites, chronometric methods and computers have drastically changed astrometry (Bock, 1998; Grafarend, 1989; Kovalevsky, 2002). Astrometric observations are various space-geodetical operations, such as:

Doppler Orbitography and Radiolocation Integrated by Satellite	DORIS
Global Positioning System	GPS
Lunar laser ranging	LLR
precise range and range rate equipment	PRARE
satellite laser ranging	SLR

very long baseline interferometry ______________________ VLBI

Observed positions obtained by comparing an object with catalogue places of stars in the immediate neighbourhood are an intermediate class that may be termed astrometric positions. They are free from the principal parts of the stellar Aberration, but they are affected by the barycentric motion of the observed object in the time taken for light to travel from it to the observer, and by the elliptic portion of the annual Aberration (Brouwer, 1961).

1.6 Celestial and Terrestrial Reference Systems

Traditionally, celestial reference frames (CRF) have been tied to the Earth's rotation and its annual revolution around the Sun. Two reference planes most commonly used in celestial mechanics are the plane of the ecliptic and the plane of the Equator (Figure 7):

- the global coordinate system defines the positions with reference to the Earth's ecliptic
- the global coordinate system defines the positions with reference to the Earth's equatorial plane

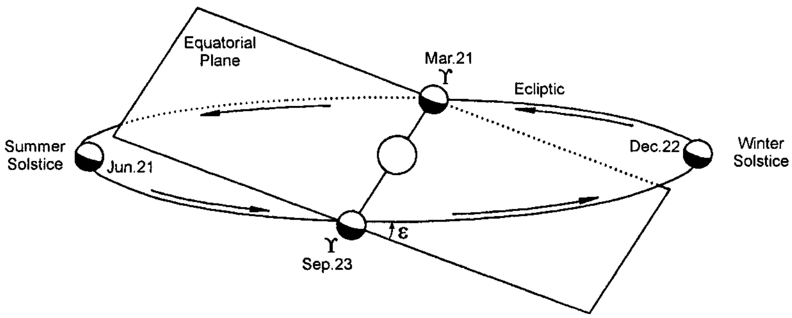

Figure 7: Ecliptic and Equator

The Equinox, or first point of Aries (♈), is the one of the two intersections of the ecliptic and equator that the Sun passes about March 21. The planes of both coordinate systems are inclined at an angle $\varepsilon \approx 23° 27'$. The ecliptic, equator, and equinox are all in uninterrupted motion. Hence, the latitude, longitude, declination, and right ascension of any celestial body all change continuously. When they are developed analytically in the way commonly used, it is found that there are periodic terms, which have in their arguments certain elements of the orbits of the Earth and Moon, called Nutational terms. The secular terms, which contain powers of the time and are independent of the instantaneous positions of the Earth and Moon, are the Precessional terms. These two classes of terms are treated separately. The direction of the Equinox is perpendicular to both the celestial polar axis (z-axis) and the North Polar axis of the ecliptic (z'-axis). The equatorial coordinates r and the ecliptic coordinates r' of a given point are calculated by the rotation (Brouwer, 1961):

$$r' = R_x(\varepsilon)\, r \tag{1.14}$$

Planetary orbits are inclined at small angles to the Earth's orbital plane and are usually calculated in ecliptic coordinates. Equatorial coordinates are calculated in geographical latitudes and longitudes, referring to an earth-centred reference system.

About Frames and Systems

The term reference system means the set of basic concepts and models used to define at any instant the orientation of the reference axes. A reference frame, in contrast, means a specific realisation for observing and computing in agreement with the theory

Currently, scientists calculate with reference frames. A reference frame has an origin, usually fixed to the (earth's) centre, reference axes, such as a Cartesian coordinate system with three orthogonal, straight axes, and a metric scale

Reference frames may have different orientations. At sea, hydrographers on a survey vessel use the electronic centre of vessels' positioning antennae as the origin of a reference frame. They choose Cartesian reference axes with an appropriate scale factor k = 1.0 parallel to the symmetry axes of the vessel as a coordinate system. All transformation activities are related to the Earths true north orientated main reference framework

Relative orientation of frames could also be described by a sequence of rotations (ε) about specific axes. The arrangement for these angles is not connected invariably to any particular sequence of rotations, but the types of rotations (roll, pitch, yaw) executed by a survey vessel.

Satellite observations are usually observed from stations on the surface of the Earth, which is - due to tectonic activities - not at rest with respect to reference systems. Satellite geodesy - dynamical astronomy and astrometry - is concerned with four reference systems (Gaposchkin, 1964; Groten, 1999; Kovalevsky, 2002):

- orbital systems
- inertial systems
- celestial systems
- terrestrial systems.

A concise definition of celestial and terrestrial reference systems [1.6.3] is necessary to compare ground-based measurements with calculated satellite positions.

1.6.1 Orbital Systems

During the 1770s, Lagrange derived analytical methods of differential and integral calculus applied to the observed motions of celestial bodies in the Solar system. During the 1930s, the observed deviations between observations and theory of the Moon's motion revealed the lack of uniformity of the Earth's rotation. In the 1950s, ephemeris time (ET), based on the observed motions of the Moon and the Sun, was introduced as the independent variable of Newtonian mechanics (Brouwer, 1937, 1946, 1958; Cheng, 1992) [1.2].

At present, it is shown by comparisons between precise observations and theory - in which the perturbations are properly taken into account - that the Newtonian laws of motion and law of gravitation approximated the true laws governing motions of celestial bodies as a convincing proof of the theory of relativity. The call for orbital perturbation theories has been replaced by a purely numerical treatment of the equations of motion (2.2.3)

The relations between geodetic type methods and concepts geometric and dynamic satellite geodesy are [4.2]:

- geodesy of Väisälä's type terrestrial methods
- static type satellite geodesy
- dynamic type satellite geodesy

Satellite geodesy is based on practically important differences in similarity underlying these methods. The differences in various methods are discussed in (Brouwer, 1959, 1963; Kaula, 1962; Levallois, 1971; Väisälä, 1947, 1960; Xu, 2003), giving a clear exposition of 3D-geodesy.

1.6.2 Inertial Reference Systems

Since the Earth rotates and is accelerated with respect to the Sun, any coordinate system attached to the Earth is not an inertial reference frame. More exactly, Newton's laws of motion are valid in an inertial reference system with origin at the centre of the Solar system, based on the fixed stars. An inertial system is fundamental to space-dynamics, and the orbit theory is eventually developed in this system. The inertial system is materialised through the celestial system. With respect to distant galaxies, the stars define the celestial system. These extragalactic radio sources, in connection with a limited number of fixed stars, with a combination of astrometric operations (VLBI, GPS, LLR, and SLR global geodetic space-tracking techniques) define a conventional inertial system (CIS). Then, astrometric operations combined with Kalman filtering, are used to arrange and to evaluate an independent BIH earth orientation parameters (EOP) series. These are used at the Jet Propulsion Laboratory in support of tracking and navigation of interplanetary spacecraft (Bozic, 1979; Felli, 1989; Farrell, 1999).

1.6.3 Reference Systems and Frames

A systematic account of systems and their relationships to one another can be found in (Gaposchkin, 1964; McCarthy, 1996; Montenbruck, 2001). In brief:

Individual star catalogues are similar to compilations of geodetic coordinates with relative positions. They can be combined into a uniform system by use of stars common to any catalogue. This procedure was used to compile the Smithsonian astrophysical observatory star catalogue of 1966 in computer-accessible form, covering the whole sky, containing 258 997 stars with their positions and proper motions reduced to *the fourth fundamental catalogue* (FK4, 1963). It was superseded by the stellar FK5 system (Kovalevsky, 1989) |[2].

A small secular variation of the Earth's orbital plane due to the presence of other solar system planets is known as planetary Precession. The torque exerted on the equatorial bulge by the Sun and Moon perturbs the Earth's axis of rotation. This torque tries to align the Equator with the ecliptic and results in a gyroscopic motion of the Earth's rotation axis around the Pole of the ecliptic with a period of about 25 850 years [2.2]. Because of this luni-solar Precession, the vernal equinox (♈) retreats slowly on the ecliptic, while the obliquity of the ecliptic remains essentially constant. Some minor periodic perturbations of the Earth's rotation axis may be observed that are known as Nutation, and reflect variations of the Solar and Lunar torques on time scales larger than a month (Williams, 1994).

Considering the time-dependent orientation of equator and ecliptic, a standard reference frame is usually based on the mean equator, ecliptic, and equinox of a particular epoch. Access to the mean equator and the mean vernal equinox for J2000.0 are realised by the FK5 (1535 stars) and FK5-supplement (3117 stars), which contains precise positions and proper motions of fundamental stars for epoch J 2000 as referred to the given reference frame. Then again, earth-based astrometry can hardly improve ±0.05" accuracy due to refraction qualms (Fricke, 1982, 1988; Kovalevsky, 1989, 2002; Lieske, 1977; McCarthy, 1996; Montenbruck, 2001; Wilkins, 1989).

IUGG-1991 defined by convention the celestial reference frame is as a geocentric, equatorial frame with the Earth mean equator and equinox of epoch J2000.0. IAU-1991 established an extragalactic radio source system as a new international celestial reference system (ICRS) for use from 1998 onward (Arias, 1995. Kovalevsky, 2002).

The origin of the ICRS is defined as the Solar system barycentre (SSB) within a relativistic framework and its axes are fixed with respect to distant extragalactic radio objects, thus ensuring no-net-rotation. For a smooth transition to the new system, the ICRS axes are chosen as to be consistent with the previous FK5 system.

The ICRS would be realised through the international celestial reference frame (ICRF), maintaining the initial definition of the axes to within ±0.0001 arc seconds. The ICRF is largely based on observations of extragalactic radio sources using VLBI for direct access to the fiducial points (radio-emitting stars to nearby quasars), and may be accessed through a catalogue providing source coordinates of 608 objects (equatorial system, epoch J2000.0). Astrometric analyses also provide the monitoring of Precession and Nutation (Groten, 1994a, 1999; Higgins, 1999; Kovalevsky, 1989, 2002; McCarthy, 1996; Monico, 1998; Montenbruck, 2001; Wilkins, 1989).

International Celestial Reference Frame

Recommendations by IAG/IAU specify for the international celestial reference frame (ICRF) epoch J2000.0 that:

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[2] Fifth fundamental catalogue or Fünfte fundamental katalog

- the Origin of the ICRF is at the barycentre of the Solar system (SSB) through appropriate modelling of observations in the framework based on the general theory of relativity
- the x- and y-axes lie on the mean equatorial plane of the SSB at the conventional date of January 1, 2000 (epoch J2000.0)
- the z-axis (polar axis) is orthogonal to the XY-plane.

Hence, the fundamental plane of the ICRS is closely aligned with the mean earth-equator at J2000. At an arbitrary fundamental epoch a catalogue of Cartesian station positions was determined from a variety of extraterrestrial geodetic observations|[3] to conceive a terrestrial reference system (TRS).

IERS Central Bureau distributes information on the Earth's orientation, and maintains the IERS celestial reference frame (ICRF) and the IERS terrestrial reference frame (ITRF) (McCarthy, 1996). The IERS standards are based on the state-of-the-art in space-geodetic analysis and Earth models, and may differ from the IAG and IAU adopted standards, e.g., Precession and Nutation parameters. The ICRF is realised by a catalogue of compact extragalactic radio sources, the ITRF by a catalogue of station coordinates and velocities. The IERS Earth orientation parameters provide the permanent tie of the ICRF to the ITRF. They describe the orientation of the celestial ephemeris Pole (CEP) in the terrestrial system and in the celestial system by polar coordinates, Nutation offsets, and the orientation of the Earth around this axis (UT1-TAI) as a function of time (Bock, 1998).

Conventional Terrestrial Reference System

Complementary to the ICRS, the IERS terrestrial reference system (ITRS) - with consideration of relativistic effects - provides the conceptual definition of an Earth-fixed reference system (Boucher, 1990; McCarthy, 1996):

- the Origin is located at the Earth's centre of mass, including oceans and atmosphere
- the unit of length is the SI metre, consistent with the Geocentric Coordinate Time (TCG) coordinate
- the orientation of the IERS Reference Pole (IRP) and IERS Reference Meridian (IRM) are consistent with the initially BIH orientation at epoch 1984.0, and the Conventional International Origin (CIO)
- the time evolution of the ITRS is such that it exhibits no-net-rotation with respect to the Earth's crust.

In 1898, the 12th General Conference Internationale Erdmessung (IE), now IAG, decided to set up the international polar motion service (IPMS) and the Greenwich Meridian fixed a common worldwide reference X-axis in the equatorial plane. The mean polar axis 1900-05 defined the Z-axis as the CIO [2.2.2] (Grafarend, 1979; Jordan, 1923).

Conventional International Origin

Conventional International Origin (CIO) is approximately the average position, on the Earth, of the Earth's axis of rotation during the period 1900-05. It is used as the Origin for the coordinates of the instantaneous Pole of rotation of the Earth, derived from the following original latitudes of five observatories.

Note that the values of latitude of five ILS stations define the mean Pole of 1903.0, which is identical with the more commonly defined mean Pole of epoch 1900-05 of G. Cecchini.

[3] global geodetic space-tracking network (STN) as GPS, LLR, SLR, VLBI, DORIS, and PRARE

The CIO Pole is the mean direction of the Pole determined by measurements of the five international latitude service (ILS) stations during the period 1900-1905 (Cecchini, 1905). This definition helps to preserve continuity with the long record of optical polar motion determinations, which began formally in 1899 with the establishment of the ILS (Table 1) (Schlesinger, 1900).

Five ILS Stations			
Station	Country		Longitude
Misuzawa	Japan	=	39° 08' 03".602 N
Kitab	Russia	=	39° 08' 01".850 N
Carloforte	Italy	=	39° 08' 08".941 N
Gaithersburg	MD, USA	=	39° 08' 13".2022 N
Ukiah	CA, USA	=	39° 08' 12".096 N

Table 1: Five International Latitude Service stations

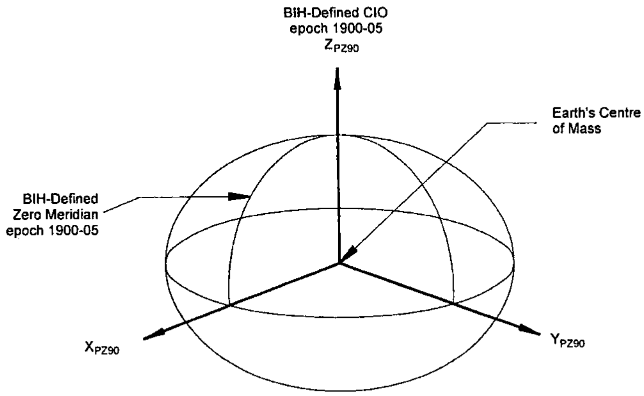

Figure 8: Parametry Zemli System of 1990 definition

IERS Terrestrial Reference System and its Definitions

The IERS terrestrial reference system (ITRS) is defined with Origin at the Earth's geocentre of mass including oceans and atmosphere, and the Pole at the 1903.0 conventional international Origin (CIO) frame as adopted in 1967 (Figure 8). It represents the rectangular coordinate system:

- the Z-axis is the direction of the CIO, directed towards the mean North Pole in the mean epoch 1900-05, as defined in resolutions of the IAU and the IAG
- the X-axis of the system lies in the plane of the terrestrial Equator of the epoch 1900-05. The reference meridian plane XOZ is parallel to the mean Greenwich Meridian. It defines the position of the Origin based on the longitude values adopted for the Bureau International de l'Heure (BIH) stations, and
- the Y-axis supplements the geocentric rectangular coordinate system to a right-handed, earth-centred, earth-fixed (ECEF) orthogonal coordinate system, measured in the plane of the terrestrial Equator of the epoch 1900-05, 90° East of the X-axis.

The scale of the ITRS is defined in a geocentric frame, according to the relativistic theory of gravitation. Its orientation is constrained to have no residual global rotation with respect to the Earth's crust. With respect to the conventional ephemeris Pole (CEP), the effective instantaneous Earth rotation axis direction is in agreement with that of the FK5 within 0".01, as provided by the IAU models for Precession and theory of Nutation. This effect caused by elastic and non-elastic deformations of the Earth, is described by the so-called polar motion

(PM) [2.2.2] whose the ILS historical series is known since 1899 (McCarthy, 1996).

Parametry Zemli System of 1990

The Russian|[4] global orbiting navigation satellite system (GLONASS) has been developed in parallel to GPS. Both satellite navigation systems, GPS and GLONASS, enable users to determine their position and navigation calculations in geocentric global coordinate systems. GPS utilises the WGS84, whereas GLONASS satellite information is given in Parametry Zemli of 1990 (PZ90, formerly SGS90) in 1995. PZ90 Origin and orientation of the x-, y- and z-axes are specified in interface control document GLONASS ICD-2002 (Table 4): pp 21.

Geodetic Reference System of 1980

IUGG Resolution No. 7 specifies that the Geodetic Reference System of 1980 (GRS80) be geocentric. The orientation of the system is specified in the following way (Figure 9):

> *the rotation axis of the reference ellipsoid is to have the direction of the conventional international Origin for polar motion (CIO), the zero-Meridian as defined by the Bureau International de l'Heure (BIH) is used.*

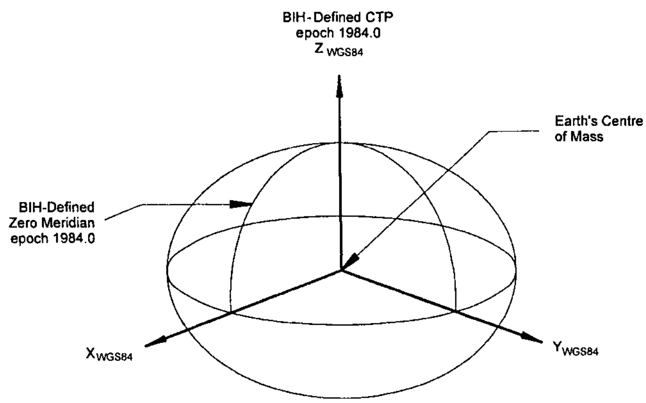

Figure 9: World Geodetic System of 1984 definition

To this definition, there corresponds a rectangular coordinate system XYZ whose Origin is the geocentre, whose Z-axis is the rotation axis of the reference ellipsoid, defined by the direction of CIO, and whose X-axis passes through the zero-Meridian according to the BIH (Guinot, 1969; Heiskanen, 1967; Ihde, 1991; Moritz, 1967-1992).

Overlapping Datums

Some physical points are fundamental to two Datums, e.g. GRS80 and WGS84. Using global positioning system observations in the single point-positioning mode, together with a satellite ephemeris given in the WGS84 coordinate system, these coordinates will be in WGS84 *raw Doppler coordinates*. The user should do something about coordinate misclosures remaining in GRS80: the coordinates receive adjustment corrections from integration with terrestrial observations.

The WGS84 ellipsoid differs very slightly from the GRS80 ellipsoid, which was used for ETRF89 and NAD83. The differences can be seen in (Table 2). ETRS89, NAD83 and WGS84 Datums should be thought of as overlapping Datums. ETRS89 and WGS84 use the same corrections to geodetic coordinates, such as translation, scale, orientation to the celestial Pole, and orientation to the celestial zero-Meridian (Moritz, 1980).

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[4]Formerly CIS (Commonwealth of Independent States), or the USSR

ETRS89, NAD83 and WGS84 Datums should be thought of as overlapping Datums. ETRS89 and WGS84 use the same corrections to geodetic coordinates, such as translation, scale, orientation to the celestial Pole, and orientation to the celestial zero-Meridian (Moritz, 1980).

World Geodetic System of 1984

In 1986, the World Geodetic System of 1984 (WGS84) with a set of quantities based on the geocentric Geodetic Reference System 1980 (GRS80) replaced the WGS72. The ellipsoid parameters for both systems are based on the same equatorial radius and three physical constants for the GRS80. On the other hand, in computing the flattening, NGA did NOT use the defining GRS80 constants as given (Kinoshita, 1994).

GRS80 had used the un-normalised form of the coefficient of the second zonal harmonic of the gravity field as a fundamental geodetic constant, while NGA used the normalised form, obtained by using the mathematical relationship:

$$C_{2.0} = -J_2 / (5)^{1/2} \tag{1.15}$$

and rounding the result to eight significant figures (NGA, 2000). This resulted in two different values for the reciprocal of reciprocal flattening, derived to 16 significant figures:

- GRS80: f^{-1} = 298.25722 21008 827 (Burkholder, 1984, as used by NGA)
- WGS84: f^{-1} = 298.25722 3563

Differences arise for the reason that NGA derived one constant from another one to an insufficient number of significant figures (Vincenty, 1985).

reference ellipsoid: name	GRS80 constants and magnitudes	WGS84 constants and magnitudes
a	6 378 137.0	6 378 137.0
b	6 356 752.31414 035	6 356 752.31424 518
f^{-1}	298.25722 21008 827	298.25722 3563
f	.33528 10681 18363 8E-02	.33528 10664 74748E-02
Rectifying Radius	6 367 449.1458	6 367 449.1458
N (=ν for 40° 40')	6 387 222.3114	6 387 222.3114
M (=ρ for 40° 40')	6 362 551.3828	6 362 551.3828

Table 2: Calculated values for GRS80 and WGS84 for comparison

The coordinates of these points in the two systems may differ in precision to sub-millimetre level. If neither position determination contains a blunder, then the differences of coordinates should be relative small. Measured distance could differ from the value computed from the coordinates by a one metre or more due to differences in adjustment and in survey methodologies (Ashkenazi, 1991; 1993; Hooijberg, 1997; Ollikainen, 1995; Schwarz, 1989a).

Geodetical Constants of Global Reference Systems

Equipotential ellipsoids of revolution are determined by a set of four constants, adopted by the International Union of Geodesy and Geophysics (IUGG), and authorised by the IAG (Table 3).

WGS84, realised through Doppler observations from the TRANSIT satellite system using a worldwide group of TRANET (tracking network) stations, is a conventional terrestrial system (CTS), conceived by modifying the TRANSIT NSWC-9Z-2 Doppler reference frame in origin and scale, by rotating to bring its reference Meridian into coincidence with the BIH terrestrial system (BTS), epoch 1984.0, defined zero-Meridian.

WGS84 provides the means by which about 115 local and regional geodetic horizontal Datums can be referenced to a single geocentric system. WGS84 offers a coherent set of global models and definitions, which form the basis for all current US DoD mapping, charting, navigation and geodesy (GG, 1986; Kouba, 1994; Poder, 1989).

Geodetical constants of World Geodetic System of 1984			
constants designation	notation	ellipsoid WGS84 [5]	
semi-major axis	a	6378137.0	m
reciprocal flattening	f^{-1}	298.25722 3563	
geocentric gravitational constant	GM	$3\,986\,005 \times 10^{8}$	$m^3\,s^{-2}$
angular velocity of the Earth	ω	$7\,292\,115 \times 10^{-11}$	$rad\,s^{-1}$
dynamic form factor, un-normalised form	J_2		(1.15) [6]
dynamic form factor, normalised form	$\bar{C}_{2.0}$	$-484.16685 \times 10^{-6}$	

Table 3: Constants for the WGS84 ellipsoid

Geocentric constants of Parametry Zemli System of 1990			
constants designation	notation	ellipsoid PZ90	
semi-major axis	a	6378136.0	m
reciprocal flattening	f^{-1}	298.25783 9303	
geocentric gravitational constant	GM	$398\,600.44. \times 10^{-9}$	$m^3\,s^{-2}$
angular velocity of the Earth's rotation	ω	$7.29211\,5 \times 10^{-5}$	$rad\,s^{-1}$
dynamic form factor, un-normalised form	J_2	$1\,082\,625.7 \times 10^{-9}$	

Table 4: Constants for PZ90 ellipsoid

WGS84 Definitions and its Revision

The WGS84 terrestrial reference frame is the frame of a standard Earth rotating at a constant rate around a conventional terrestrial Pole (CTP). The WGS84 coordinate System, which is identical to the BIH-defined CTS in its definition, must be related mathematically to an instantaneous terrestrial system (ITS) and to a conventional inertial system (CIS). The relationship between the CIS, ITS, and the WGS84 coordinate System, can be expressed as (Figure 9) (NGA, 2000):

- the Origin is located at the Earth's centre of mass
- the Z-axis is the direction of the conventional terrestrial Pole (CTP) for polar motion, as defined by the Bureau International de l'Heure (BIH) for epoch 1984.0 on the basis of the coordinates adopted for the BIH stations
- the X-axis is the intersection of the WGS84 reference meridian plane and the plane of the CTP's Equator, the reference Meridian being the zero-Meridian defined by the BIH for epoch 1984.0 based on the coordinates adopted for the BIH stations
- the Y-axis completes a right-handed, earth-centred, earth-fixed (ECEF) orthogonal coordinate system, measured in the plane of the CTP Equator, 90° East of the X-axis.

[5] Analogous to the BIH Defined conventional terrestrial system (CTS), or BTS, epoch 1984.0

[6] Calculated J_2=108 262.99890 51944$\times 10^{-8}$

An evaluation of the four WGS84 defining parameters indicated that the WGS84 EGM value is the one that warrants revision to remove a 1.3 m bias in DoD orbit fits (Kouba, 1994; Malys, 1994; Slater, 1998). NGA, 1997 replaced the WGS84 EGM-1984 value by the international earth rotation service (IERS) (McCarthy, 1992), by the standard value of 3 986 004.418 × 10^8 m^3 s^{-2}, mass of the atmosphere included. The change (0.582 × 10^8 m^3 s^{-2}) to the WGS84 EGM is within the original one sigma (1σ) uncertainty for this parameter (0.6 × 10^8 m^3 s^{-2}) as identified in (NGA, 2000).

The Method of Least Squares

In geodesy, certain constants appear that have to be determined by observation. When nothing about these constants is known in advance, as is the case with the elements of the orbit of an artificial satellite, the determination of them may be difficult. When approximate values of the constants are known, however, they may be incorporated into a theory of the motion, which may then be used to calculate theoretical positions of the object. A comparison of the theory with observations will show that the theory does not represent the observations exactly. Each observation furnishes a residual, computed place minus observed place (C-O), which is due to three causes. First, the theory may be inadequate. Second, the observations are affected by errors. Third, the errors of the approximate constants used in determining the computed positions contribute to the residuals. In this chapter are discussed the two classes of errors last mentioned. It is shown how the approximate values of the constants may be improved by an analysis of the discrepancies between theory and observation.

The US Air Force Space Command implemented a refined WGS84 reference frame, designated as WGS84 (G730) referring to GPS week 730, January 2, 1994. the WGS84 (G730) coordinates on June 29, 1994 (Malys, 1994; Swift, 1994), the designation WGS84 (G873) referring to GPS week 873, September 29, 1996 (Slater, 1998), and the designation WGS84 (G1150) refers to GPS week 1150 in the year 2001. Using an improved global Earth gravitational model EGM96 and an associated global geoid, the revisions were essential only for specific high-accuracy DoD applications such as precise orbit determination, and in a more precise ephemeris in the GPS broadcast message.

Muneendra Kumar; James P. Reilly (2006): "... on the other hand, a scientific participation with academia and satellite experts was not followed, and three subsequent updates of WGS84 were carried out without in-depth discussions of satellite geodetic theory, and correct statistical evaluation of the GPS Tracking Network. Imperative is, that the world's eminent geodesists support the "*zero-tide*"of the IAG's Standing Resolution no. 16 of 1983, and the IERS Conventions, Tech Note 21, July 1996."

Note

The European Datum of 1950 (ED50) and its later versions such as ED79, ED87, ETRF89, and PZ90 do not cover the whole world. WGS84 is the only global Datum in use at this time (Poder, 1989).

Those interested in other aspects of the subject will find a wealth of additional information in (Brouwer, 1961a, b; Koch, 1980, 1999, 2000) and other treatises.

Realisation of a Terrestrial Reference System

A conventional terrestrial reference system (TRS) can be realised through a reference frame, such as a set of coordinates for a network of stations. Such a realisation will be specified by Cartesian x, y, and z coordinates, or if geographical coordinates are necessary, the GRS80 ellipsoid is recommended using:

$$a = 6\,378\,137.0 \text{ m}$$
$$e^2 = 0.00669438003$$

The values for a, e^2 and f, calculated by the formulae (Table 5): pp 49 in program [18.8, Geodetic Reference System], are entered in e.g. program Ellipsoid Constants – Arcs [18.4] to derive the associated constants. Derived functions for the reference ellipsoids GRS80 and WGS84 are given in (Table 2).

IUGG-1991 Resolution no. 2 specified the CTRS monitored by an IERS analysis centre as the IERS terrestrial reference system (ITRS). Within IERS, each terrestrial reference frame (TRF) is either directly, or after transformation, expressed as a realisation of the ITRS. The position of a point located on the surface of the solid Earth should be expressed by

$$X(t) \quad = \quad X_0 + V_0\,(\,t - t_0\,) + \Sigma\, \Delta X_i\,(t) \tag{1.16}$$

in which ΔX_i (t) are corrections due to various time changing effects, and - X_0 + V_0 are position and velocity at the Epoch t_0. The corrections to be considered are:

- atmospheric loading
- Coriolis force
- ocean loading
- post-glacial rebound
- solid Earth tide displacement.

Further corrections could be added if they are at mm level or greater, and can be computed by a suitable model.

Realisations of the ITRS are produced by IERS as international terrestrial reference frames (ITRF), which consist of lists of coordinates and velocities for a selection of IERS stations given in a conventional frame where the effects of all tides are removed. Currently, ITRFyy is published annually by the IERS in the Technical Notes (Boucher, 1994; McCarthy, 1992, 1996).

Realisation ITRFyy

IERS produces realisations of the international terrestrial reference system (ITRS) under the name international terrestrial reference frame (ITRFyy), which consist of lists of coordinates and velocities for a selection of IERS tracking stations and international GNSS geodynamics service (IGS) core stations. IGS collects, archives, and exchanges GPS data of geodetic quality, stored in the receiver independent exchange format (RINEX) (Bock, 1998; Boucher, 1998; Gurtner, 1994).

The Cartesian station positions derived by a global network analysis centre (GNAACs) define the fundamental polyhedron at a specified epoch: ITRFyy (or international terrestrial reference frame). The number (yy) following the designation ITRF specifies the year whose data were used in the formation of the frame, such as (ITRF97) [1.5]. Annually, IERS publishes ITRFyy in *Technical Notes*. In addition, some regional network analysis centres (RNAACs) began submitting regional GPS solutions, using the solution independent exchange format (SINEX) for the exchange of geodetic solutions (Boucher, 1994, 1996, 1998; SINEX, 1996).

About the Attribute Fiducial

In photogrammetry: a fiducial marker is one of a set of four small objects rigidly fastened to the interior of a camera's body, also called fiducial mark, though this expression is better applied to the image of the fiducial marker

Tide gauge benchmark: the fiducial mark of the permanent tide scale is established by reference to the zero mark of the permanent tide scale

In surveying: a fiducial mark is a point or line used as a reference or Origin, established by VLBI, SLR, LLR and absolute gravimetry surveys

A fiducial (first order) network, used together with absolute gravity measurements determined by a transportable absolute ballistic laser gravimeter at the fiducial astro-geodetic (FAGN) stations with LS adjusted values of the distances, angles, directions, or heights, determine coordinates of the geodetic control network

A fiducial control network together with the measured or adjusted values of gravity, distances, angles, directions, or heights are used in determining the coordinates of the control network. Three geodetic control networks bear the name fiducial network: IRENET95 (Ireland and Great Britain, using ETRF89), SIRGAS (sistema de referencia geocentrico para America del Sur) of South America, and geocentric Datum of Australia (GDA94). GDA94 is realised through the estimated positions of the Australian fiducial network and the Australian national network of 1995

Essentially two major global networks, the cooperative international GPS network (CIGNET) spearheaded by the US NOAA and fiducial laboratories for an international natural science network (FLINN) led by the US NASA, were merged into the international GNSS geodynamics service (IGS) with several continental-scale networks in North America, Western Europe and Australia. The fiducial concept of the 1980s is now replaced by the Fundamental Polyhedron at a specified epoch.

(Drewes, 1998; Featherstone, 2001; Galera, 1998; Groten, 1967, 1968; Hooijberg, 1997; Hoyer, 1998; Moirano, 1998; Olliver, 1999; Seemüller, 1998).

At the FIG-1990, Resolution 5.3 mentions about a global geocentric geodetic reference system (GRS) as proposed by IAG/IUGG (Higgins, 1999):

" ... that high accuracy geocentric reference networks be established with connections to national or continental networks. These networks will support requirements of the Earth sciences for global measurements. A worldwide geodetic network with GPS-receivers collocated with VLBI, LLR and SLR stations be established to track continuously GPS-satellites signals where the positions of these fiducial stations will be known to 10^{-8} or better for a given epoch relative to the defined ITRS ... ".

Systems for the 2000s

Resolution considering the international terrestrial reference system and frame of 2000:

- the availability of ITRF2000 as an improved and accurate realisation of the ITRS
- the improved determination of the rotation of the Eurasian plate using ITRF2000 site velocities
- the replacement of NNR-NUVEL-1A rotation rate values by the ones derived from ITRF2000 in the transformation formula linking ETRS89 to ITRS.

Resolution considering the European Vertical Reference System and Frame of 2000:

- the significant practical and scientific value of the European Vertical Reference System (EVRS)
- all national systems are accurately related to the European Vertical Reference Frame of 2000 (EVRF2000) realisation
- the levelling data be submitted in the zero tidal system according to the EVRS definition and corresponding IAG-1983 Resolution 16.

Resolution considering EUVN / UELN geoid model (Augath, 1996, 1998):

- the European Vertical GPS Reference Network (EUVN) with its GPS-derived ellipsoidal heights and levelled connections to the unified European levelling network (UELN) (Grafarend, 1988, 2000e)
- the definition of the European Vertical Reference System (EVRS) with its realisation called EVRF2000
- the realisation of a single European geoid consistent with both ETRS89 and EVRS
- the need to generate a European geoid model of decimetre accuracy consistent with ETRS89 and of decimetre accuracy consistent with ETRS89 and EVRS
- the existence of a large number of regional and local geoids in Europe.

Resolution considering unification of EUREF map projections:

- Gauss-Krüger (GK) projection, with UTM grid zone system for maps in a scale larger than 1: 500 000
- Lambert (LCC) conformal projection system for maps in a scale smaller than 1: 500 000

A stable EUREF system. At present, all European terrestrial reference frame (EUREF) stations are found to move with a certain azimuth and velocity, relative and parallel to plate-systems within the Eurasian plate. Accordingly, surveys are anticipated for comparison.

EUREF proposes to adopt the ITRS / ITRF2000 by deriving (Altamimi, 2002):

- the Origin as defined by SLR
- the scale as defined by VLBI
- the orientation as defined by a selected set of about 800 ITRF station sites
- to replace NNR-NUVEL-1A values by new rotation-rate values as derived from ITRF2000 in the ETRS89 - ITRS transformation formulae links.

2. Dealing with Geoscience Branches

The simple conditions of world structure assumed throughout the centuries could not be maintained against modern research, such as the invariable position of the Earth's axis and the invariable period of Earth's rotation. Since the international geophysical year 1957-1958 (IGY), the physical Earth deals with any of the geoscience branches, such as astronomy, geochemistry, geodesy, geology, geophysics, hydrography, meteorology, mineral exploration and exploitation, oceanography, plate tectonics, and physical geography.

2.1 Continental Drift Hypothesis

Clarence E. Dutton, (1892) developed and named the principle of isostasy. He postulated this principle of isostasy in a paper *On some of the greater problems of physical geology*. Richard D. Oldham (1897) hypothesised evidence for the existence of the Earth's core. Rocks can undergo substantial deformations over geologic time scales, particularly at the high temperatures of the interior. In addition, innumerable geologic and geophysical observations at the Earth's surface are found to fit neatly into this picture of global tectonics involving large-scale deformations of the solid interior.

Tectonic Plate Motion

Frank B. Taylor (1908) postulated the equator-ward creep of the continents. Independently, Alfred L. Wegener (1911), a meteorologist, formulated a statement of the continental drift hypothesis: about 250 million years ago, all the present-day continents had formed a single large mass *Pangäea* (or *Pangaea*), which had slowly moved apart over long periods of geological time (Figure 10). He termed this continental drift or displacement *"Die Verschiebung der Kontinente"*. In 1915, he postulated the theory about the origin of continents and oceans in his work: *Die Entstehung der Kontinente und Ozeane*. Alexander du Toit (1937) modified Wegener's hypothesis.

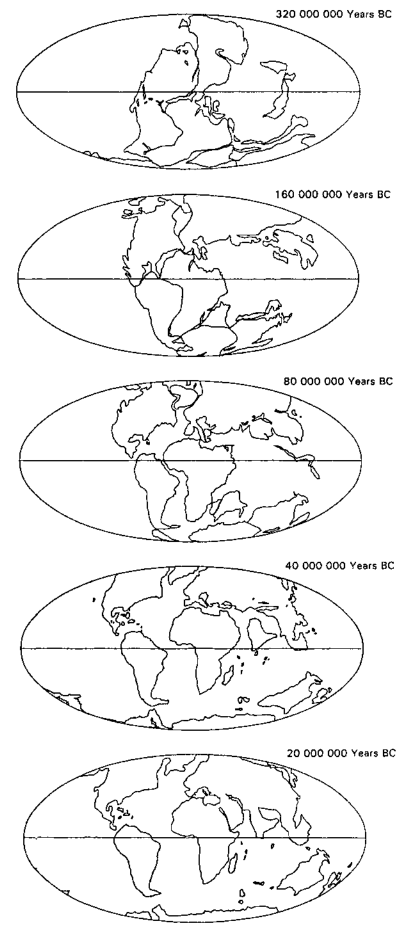


Figure 10: Wegener's Vision of Pangaea

One of the strongest opponents was the British geophysicist Sir Harold Jeffreys, who showed his disbelief so strong that he spent many years to explain that continental drift is impossible. Nevertheless, it took many distinguished scientists more than forty years to prove the massive evidence in favour of plate tectonics. In spite of this, Jeffreys refused to abandon his viewpoint.

In 1929, Felix A. Vening-Meinesz. had shown that some spectacular negative gravity anomalies on Earth were associated with oceanic trenches. Since the 1950s, these regions are called subduction zones, plate margins along which a slab of oceanic lithosphere is forced to descend beneath a continental plate. Therefore, the study of the gravitational field over the oceans is diagnostic for understanding the features of global plate tectonics, such as mid-oceanic ridges and subduction zones. Stanley Keith Runcom's pioneering studies of palaeomagnetism provided early evidence in support of the theory of continental drift. Particularly, the true significance of existence of an ancient land bridge - between India and Africa - did not become apparent until after development of *the theory of plate tectonics* in the 1960s (Vening-Meinesz, 1929).

Since the mid 1950s, the basic structure, constitution, and composition of the Earth's interior have been known based on an array of seismological, experimental, and geochemical observations. Due to inaccurate geodetic computing and measuring techniques then current, polar motion was not accepted as a part of geodesy.

Physical Earth consists of two major regions: a central core, surrounded by a solid shell comprising the mantle and crust together. Observations of earthquake waves had led to a spherical symmetrical crust-mantle-core picture of the Earth. Mohorovicic discovered the Mohorovicic discontinuity, the boundary between the Earth's crust and mantle, called the *Moho*. This crust-mantle interface lies at a depth of 25-40 km (max. 60 km) on continental crust and about 7 km beneath the oceanic crust. The mantle-core boundary is the *Gutenberg-Wiechert discontinuity* at a depth of about 2900 km.

General Scope of the IGY-Enterprise

The international geophysical year 1957-1958 (IGY) was a great research enterprise, shared by many nations, with the aim to increase the knowledge of the planet Earth. Although called the *international geophysical year* - the period was actually a year and a half - and a worldwide program of geophysical research was conducted from the beginning on July 1, 1957, and continuing until the end of 1958. The adjective geophysical was used, because the physical aspect of the Earth was studied. Only an outline of the vast field of geophysical observation and research can be given here (Chapman, 1957):

- the IGY was directed towards a systematic study of Earth and its planetary environment in the fields of astronomy, aurora and airglow, biology, cosmic rays, economy, economy, geochemistry, geodesy, geography, geology, geomagnetism, geophysics, glaciology, gravity, hydrography, ionospheric meteorology, oceanography, physical geography, physics, politicology, seismology, and solar activity
- these fields may be involved in the international geophysical year (IGY), but incidentally. As regards geography, the only large part of the Earth that still offers great scope for exploration is the Antarctic continent. The IGY is an exceptional period in the history of geographic as well as geophysical Antarctic research
- the physical aspects of the Earth concern all its parts, the atmosphere, the oceans and the solid land. They are numerous and interwoven. The atmosphere, the region of weather and climatic change, intimately affects the daily life and the food supply. Though long studied, its changes, their causes and mechanism, still present many baffling mysteries
- it leaded studies of high-altitude and upper atmosphere phenomena using the earliest artificial satellites launched by the (former) Soviet Union and the United States by collecting data. Interest in the concept of continental drift has been strengthened substantially during the 1950s as knowledge of the Earth's magnetic field during the geologic past developed from the studies of Patrick Maynard Stuart Blackett and colleagues. A dynamic picture of the solid interior has been recognised in subsequent years (Figure 11).

The IGY pioneered in the use of rocketry to conduct studies of high-altitude and upper-atmosphere phenomena. Several of the earliest artificial satellites launched by the Soviet Union and the United States in the late 1950s were used to gather data for the IGY.

In retrospect, an important achievement of the IGY was its verification of scientists' suggestion that a continuous system of submarine mid-oceanic ridges exists that encircled the globe.

The discovery of both Van Allen radiation belts, which enclose the Earth at altitudes of hundreds and thousands of kilometres, was another major achievement of IGY. In 1958, instruments aboard the early Explorer satellites first delineated the inner Van Allen belt. Space probes Pioneers III and IV discovered the second Van Allen belt soon afterward. Specific discoveries and findings represented only a part of the technical results of IGY. It gave a comprehensive overview of global physical phenomena.

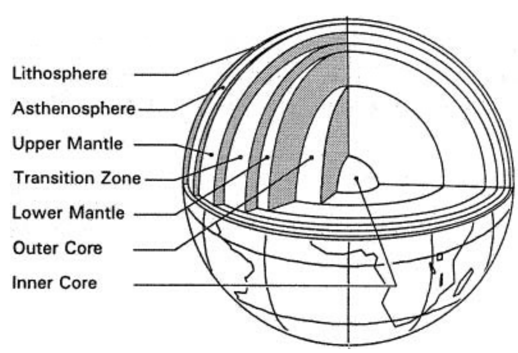


Figure 11: Classification of Earth's depths

Since the early 1960s, an all-encompassing theory grew out of observations about continental drift, and seafloor spreading hypothesis. Advances in deep-water drilling technology tested this theory. The evolution of the Earth's lithosphere is termed plate tectonics, broken into two types of plates, oceanic and continental. It describes the deformation of the Earth's crust for the 3D-Space-Time mechanical behaviour of most outermost rigid plates with lateral movements of the continents at a rate of a few up to several centimetres per year.

John T. Wilson's studies in plate tectonics had an important bearing on the theories of continental drift, seafloor spreading, and convection currents within the Earth. He conceived global patterns of faulting and the structure of the continents at a time when prevailing opinion held that continents were fixed and unmovable. His paper entitled *A new class of faults and their bearing on continental drift* (1965).

In the 1960s, it was only through the insights provided by the theory of plate tectonics that the relationship of geology to global geophysics and geochemistry became thoroughly appreciated. Harry H. Hess, Robert S. Dietz and Xavier Le Pichon, geophysicists, suggested that new ocean crust was formed along mid-oceanic ridges between separating continents. Drummond H. Matthews and Frederick J. Vine proposed that the new oceanic crust acted like a magnetic card recorder as far as magnetic anomaly strips parallel to the ridge had been magnetised alternately in normal and reversed order, reflecting the changes in polarity of the Earth's magnetic field.

Since the mid 1960s and ensuing decades, many of the problems that had been bothering most geodesists, and to whose solution they had been devoting many years of study, had been solved with an accuracy unhoped for in earlier years. At that period, building up a qualitative comprehensive and quantitative models of the Earth was a continuing contribution by satellite geodesy. Crustal motion, Earth tides, and continental drift were all falling within the domain of geodetic analysis. The implications of the largest mountain chain on Earth were understood with the recognition of plate tectonics as a basic phenomenon of the Earth's crust. Objectives of undersea exploration were to describe the ocean waters, the seafloor, and the recognition of global patterns of continental plate motion. Projects, such as:

- Joint Oceanographic Institutions Deep Earth Sampling (JOIDES)
- Deep Sea Drilling Project (DSDP)
- International Phase of Ocean Drilling (IPOD) in 1976

have portrayed a simple picture of the crust beneath the oceans. Geophysical surveys furnish a more comprehensive picture than test boreholes alone, although most boreholes are drilled to verify the geophysical interpretation.

Mechanical Behaviour of Plates

Scientifically understanding the way the Earth works has revolutionised virtually every discipline of the geosciences. In the 1970s, the implications of submarine mid-oceanic ridges that encircled the globe were first understood with the recognition of plate tectonics as a basic phenomenon of the Earth's crust. Recognising the surface of the Earth as an ever-moving mosaic is imperative for establishing geodetic networks.

Interacting plates are classified into three general types:

- divergence - in areas where plates move in opposite directions
- convergence - in areas where plates move towards each other along plate boundaries
- transforming - in areas where the plate boundary slide parallel to one another in opposite directions which is also known as a fault or fracture zone.

By 1997, the sedimentary cycles - dated by the magnetic polarity reversal time scale - were used to calibrate the astronomical time scale, which had been established for the past twelve million years. It was more accurate and has higher resolution than any other time scales.

Complexity of the problem was stated by Milton Keynes:

" ... thus there is still much to be done ... ".

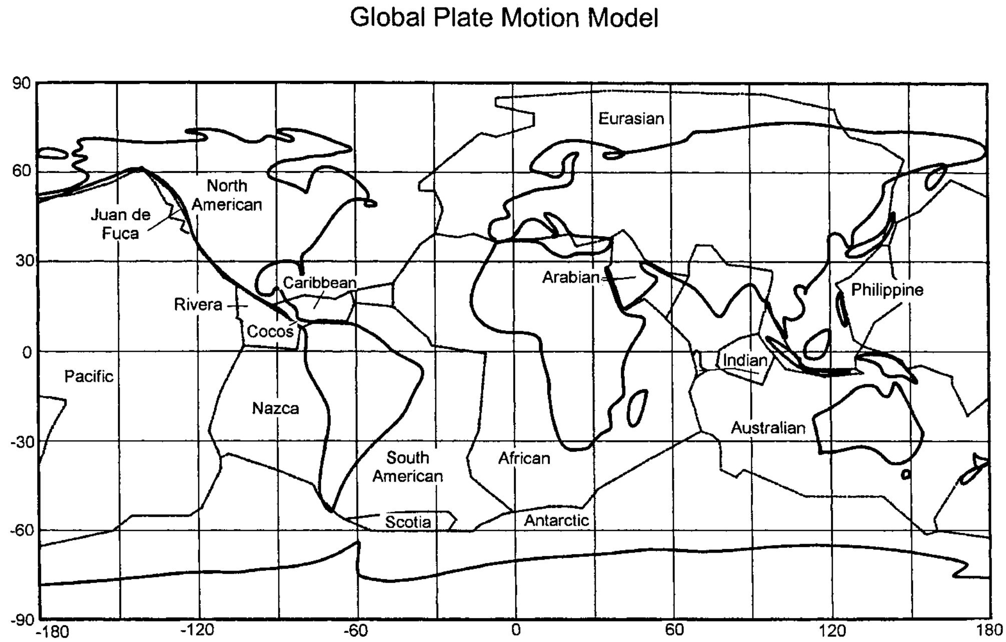


Figure 12: Tectonic plates and its boundaries

New angular rates of the paleomagnetic time scale has led to the models NUVEL-1A and no-net-rotation (NNR)-NUVEL-1A. The NNR NUVEL1 relative plate motion model is derived from paleomagnetic data, transform fault azimuths, and earthquake slip vectors. This plate tectonic model describes the angular velocities of fourteen major tectonic plates - Africa, Antarctica, Arabia, Australia, Caribbean, Cocos, Eurasia, India, Juan de Fuca, Nazca, North America, Pacific, Philippine, Rivera, South America, and Scotia - defined by a NNR constraint which have been adopted by IERS (Figure 12) (McCarthy, 1996).

Deformation of the Earth's crust in 3D-space-time with lateral movements of the continents around a common Pole of the plate rotation axis - not to be confused with the spin-axis of Earth. As the plates move, fixed 3D-coordinates for the observing stations become inconsistent with each other. The computed rates of relative plate motions for observed sites are roughly 0.05 m per year or even larger.

2.2 Concept of Earth's Wobbling-Movements

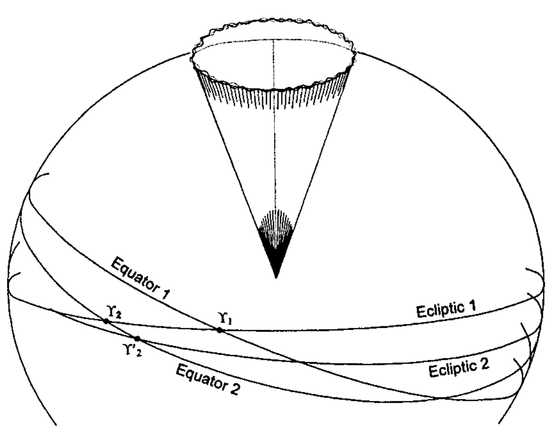


Figure 13: Celestial sphere with intersection of the Equator by ecliptics

In 1852, Léon Foucault, a 19th-century French scientist, invented the gyroscope, a device consisting of a rapidly spinning wheel or rotor, mounted in a framework. Just like the figure of Earth, a gyroscope has certain peculiar properties, such as Precession and Nutation - when its rotor is spinning. The rotation axis of a quickly spinning gyroscope revolves slowly about round a vertical line.

In 1921, M. Schuler developed the first surveying gyroscope. Gyros mounted on theodolites are used for orientation of field artillery, for surveying in underground mine exploitation, since magnetic compasses would be disturbed by mineral deposits (Defraigne, 1996; Dehant, 1997c).

An analytical approach by vector algebra, with properties of gyroscopes mounted on theodolites with the theory-orientated concept of earth's wobbling-movements in the celestial sphere, is discussed in (Grafarend, 1967, 1969, 1974). In these reports, Grafarend explains the gyrocompass consists essentially of a gyroscope.

By the application of suitable controls, use is made of these properties so that the axis of this gyroscope seeks the true North and maintains itself in this direction. If the spin axis of such a gyro is pointing in a certain direction in space, then, when the rotor is spun at a sufficiently high speed, this axis will maintain its direction in space. This motion is a combination of drift and tilts, called apparent motion.

Precession

Precession of the Equinoxes (ϒ), being the intersection of the ecliptic and the equator, moves round the former, as the Earth's axis and the celestial Pole move round the Pole of the ecliptic (Figure 13). Hipparchus (129 BC) discovered Precession. In consequence of the Precession and of the obliquity of the ecliptic, regular Precession of the axis causes ϒ to move regularly round the ecliptic. The rotation axis of the Earth revolves slowly about round the fixed direction in space, making one complete revolution in 25 850 years (Figure 14): line 1, [1.6.3].

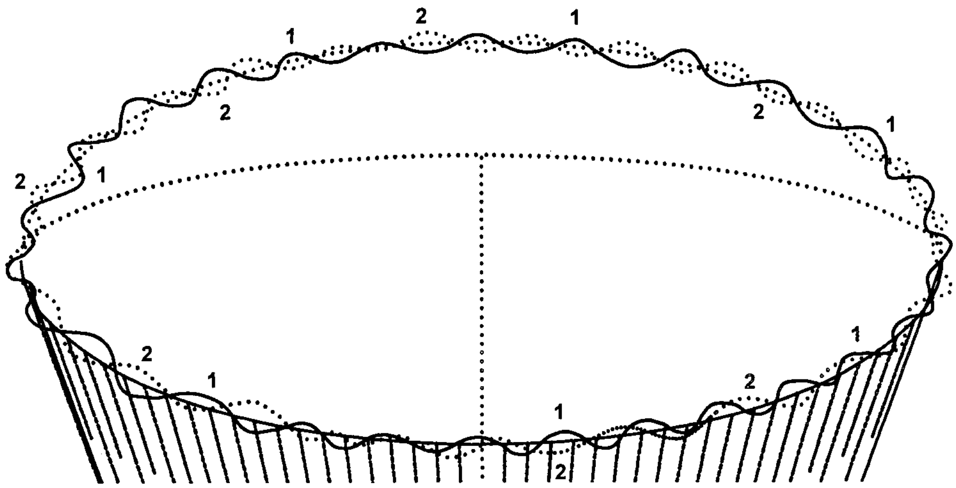


Figure 14: Rotation of axis due to precession (1) and nutation (2) in 25 850 years

The combined attractions of the Moon and Sun on the Earth cause the luni-solar precession (ψ) (Figure 15). The annual motion of ϒ is $\psi \approx 50''.37$.

Planetary Precession (λ') modifies this action. Mutual attraction of the planets and of the Earth tends to draw the latter out of the plane in which its orbit lies, without affecting the position of the Equator. The movement of the ecliptic itself causes a smaller movement of ϒ in the opposite direction, from Υ_2 to $-\Upsilon'_2$. It is given by $\lambda' = 0''.12$.
Consequently, a very small change occurs in the obliquity as the general effect of Precession (p). The equinoxes (ϒ) drift in a retrograde motion, along the ecliptic at the rate being equal to $p = \psi - \lambda' \cos \varepsilon = 50''.26$ of arc per annum as the celestial equator moves with the Earth's Precession (Dehant, 1997a, b).

Nutation

James Bradley, astronomer Royal, found an annual change of declination in his fixed star measurements between 1727-32. He concluded that this was caused by a slight periodic and uneven nodding motion of the Earth's rotation axis that resulted from the changing direction of the gravitational pull of the Moon. Bradley announced Nutation as his discovery in 1747.

The effect of Nutation causes a slight periodic oscillation of the Earth's axis in the form of an ellipse, nearly entirely due to the influence of the Moon, the solar Nutation being almost inappreciable.

The Moon imposes two periodic motions, namely:

- long-period Nutation with an amplitude of about 9".0 and the time taken for the Pole of the Equator to describe this ellipse is 18.67 years
- short-period Nutation with an amplitude of generally less than 0".5 and a period of a fortnight.

If this motion is combined with that of Precession, the true paths of the North and South celestial Poles are of a wavy form (Figure 14): dotted line 2. Apart from Nutation, the movement of the Pole of the ecliptic results in the mean obliquity not being quite constant. Now its value is given by $\varepsilon \approx 23° 27'$.

The IAU 1980 theory of Nutation in longitude $\Delta\psi$ and obliquity $\Delta\varepsilon$, referred to the mean equator and equinox of date, with parameter (t) measured in Julian centuries from epoch J2000.0 [1.3] (1.03). It includes the effects of a solid inner core and a liquid outer core and a distribution of elastic parameters inferred from a large set of seismological data (McCarthy, 1996; Seidelmann, 1980; Wahr, 1981).

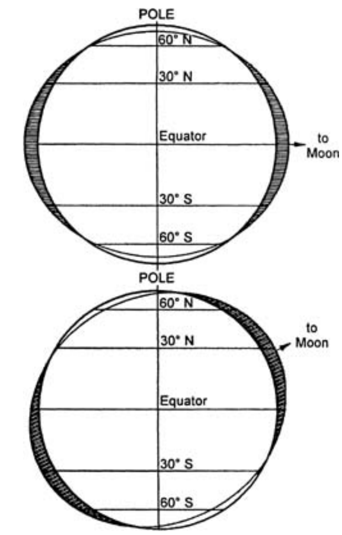


Figure 15: Effect of the Moon on the Earth

Coriolis Force

In 1835, Gaspard Coriolis showed that, if the ordinary Newtonian laws of motion of bodies are to be used in a rotating frame of reference, and an inertial force must be included in the equations of motion. On the Earth an object that moves longitudinally within a rotating coordinate system, will undergo apparent deflection to the right in the Northern Hemisphere and to the left in the Southern Hemisphere due to the effect of the Coriolis force.

The Coriolis effect has great significance in earth sciences, such as astrophysics, electrodynamics, geodesy, meteorology, physical geology, stellar dynamics and oceanography, in that the Earth is a rotating frame of reference. All motions over the surface of the Earth are subject to acceleration from this force as indicated (Bearman, 1989; Ludwig, 1999).

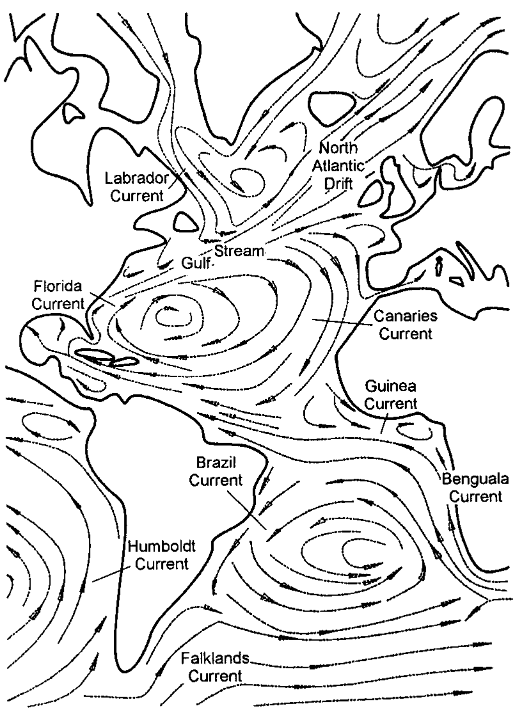


Figure 16: Coriolis Force

2.2.1 Tidal Deformation of the Earth

Solid Earth Tide

The gravitational attractions of the Sun and Moon induce tidal deformations in the solid Earth. The amplitude, period of these variations and the station coordinates will vary periodically. The penalty for ignoring tidal effects will generally be more severe for GPS measurements as baseline length increases. In principle, Earth tides models need to be defined as part of the definition of the terrestrial reference system (TRS). Besides, there are a number of sources of perturbation acting on the satellite, such as the gravitational attraction of the Earth as a rigid body, a fictitious force originating in the deformation of the Earth by the attraction of the Sun and Moon [1.6.3] (Yurkina, 1993).

Love numbers

Site displacements caused by tides of spherical harmonic degree and order (NM) are characterised by the Love number h_{NM} and the Shida number l_{NM}. The effective values of these numbers depend on station latitude and tidal frequency to characterise the changes produced in the free space potential. This dependence is a consequence of the ellipticity and rotation of the Earth, and includes strong frequency dependence within the diurnal band due to the nearly diurnal free wobble (NDFW) resonance. Further frequency dependence in the long period tidal band arises from mantle anelasticity, which leads to corrections to the elastic Earth's Love numbers, named after A.E. Love (Anderle, 1977; Grafarend, 1983; McCarthy, 1996).

The Earth's deformation produces perturbations of satellite orbits. The amount of deformation of the Earth caused by Lunar and Solar tides is a function of a Love number (k). The rigidity of the solid Earth appear as Love number coefficients in the expansion of the exterior tidal potential as spherical harmonies, especially in the long period tidal band. Affects may need to be taken into account when a high accuracy is desired in determining station positions: solid earth tide deformation, atmospheric loading, ocean loading and the Pole tide.

A report by the FGI (Finnish Geodetic Institute) provides the zero-crust hypothesis that corresponds to the theory already accepted in the resolution of IAG in 1983 for gravimetric works which is in tune with up to date practice and technical development in the field of earth-centred, earth-fixed (ECEF) coordinate computation. For further reading, the interested reader may consult (Poutanen, 1996).

Concisely:

> *"Little has been published about the treatment of the tidal deformation of the Earth in GPS [4.3.2] computation. Most ECEF coordinates are reduced to the* nontidal *crust, conventionally defined using physically meaningless parameters. However, the great demand for ever increasing accuracy and the need to combine the GPS based coordinates with other methods requires an agreeable way to handle the tides*
>
> *The tide-generating potential of the Moon and the Sun result in deformation of the shape of Earth. This deformation can be divided into a periodic and a time-independent or permanent part. The periodic part should always be eliminated. Concerning the permanent part, the* nontidal geoid *is the equipotential surface of the gravity field of the Earth when the permanent tidal deformation is eliminated. A* zero geoid *refers to the gravity field of the Earth when the permanent tidal deformation is preserved*
>
> *The gravity field of the Earth with its permanent tidal deformation preserved, plus the time-independent part of the tidal field generated by Sun and Moon refers to a* mean geoid*. Equally, the* nontidal crust *refers to the model of the Earth when the permanent tidal deformation is eliminated and the* zero crust (= mean crust) *refers to the model of the Earth when this deformation is preserved".*

(Kakkuri, 1996).

Error Sources

Main error sources are antenna-phase-centre variations, tropospheric path delay irregularities, vertical station displacement due to ocean tide, and due to atmospheric pressure loading (Menge, 2001).

Atmospheric Loading

IERS-1996 [19.5] conventions recommend considering a time-varying atmospheric pressure distribution loading effect in the analysis of space-geodetic observations. The EUREF permanent network (EPN), extending between northern Scandinavia and the Mediterranean, experienced an atmospheric loading displacement up to 4 centimetres as an elastic response of the Earth's crust (Bock, 1998; Kaniuth, 2002; Xu, 2003).

Ocean Loading

Ocean Loading is the elastic response of the Earth's crust to ocean tides. Displacements can reach centimetres for stations near the continental shelves (Bock, 1998; Xu, 2003).

Pole Tide

The Pole tide is the elastic response of the Earth's crust to shifts in the Pole of rotation. An expression for Pole tide displacement is given in geocentric spherical latitude, longitude and radius, the rotation rate of the Earth, and displacements from the mean Pole. The maximum polar motion displacement is between 0.01-0.02 m (Bock, 1998).

2.2.2 Polar Motion

Observations prove that the Earth's polar motion is essentially a superposition of two components:

- the Chandler period of about 435 days, it agrees not with an expected 305 day period due to a non-rigid Earth model
- an annual motion, induced by seasonal changes of the Earth's mass distribution.

The motion of the rotation axis with respect to the surface of the Earth can be monitored by continuous observations. Consequently, the mean position of the Pole of rotation during the years 1900 to 1905 - the conventional international Origin (CIO) - was usually chosen as the Origin for polar motion measurements.

Historically there are two reference points (estimated difference < 1.0 m) (Groten, 1984):

- the Bureau International de l'Heure (BIH) Pole
- the CIO was defined by the location of five stations of the international latitude service (ILS) that has been involved in polar motion measurements from the beginning of the century.

In 1765, Leonhard Euler proposed that there was a disparity between the earth's axis of rotation, and its axis of figure, the latter being the principal polar axis of inertia. Using dynamical theory and a rigid model of the Earth, he explained that only as a highly exceptional case could the rotation axis keep an invariable position in a rotating body. He computed that such an oscillation period in a rigid Earth is completed in 303 days.

Equatorial coordinates change slowly with time due to a gyrating motion of the axis of the Earth - a deformable, viscoelastic body. The celestial Pole, P moves relative to the fixed stars. The principal change, Precession, carries P roughly in a small circle round the Pole of the ecliptic.

Polar motion is a periodic rotation of the Earth's spin axis about a mean axis, quite like the wobble of a spinning gyroscope. Slight variations in latitude and longitude resulted from this wobble because the Poles are displaced from their mean positions. The North Pole of rotation rotates counter clockwise around its mean position. Seth C. Chandler (1884) analysed a series of latitude determinations in half a year, he found progressive changes of 0".4 of arc. Astronomical observations of the Earth's rotational position in space to determine Universal Time must be corrected for slight variations in longitude caused by polar motion. The period of rotation has turned out to be inconstant, dependent on unknown geophysical events. Chandler's discovery of the Chandler wobble was relieved by his invention of the Almucantar (device for measuring the positions of stars relative to a circle centred at the zenith) (Küstner, 1888).

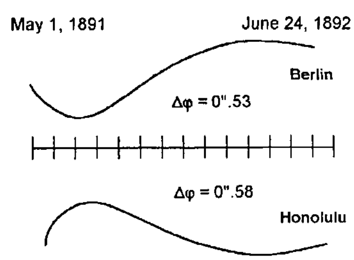


Figure 17: Polar heights as surveyed by Marcuse and Preston

The beginning of the 4D-geodesy may be reckoned from the detection of polar motion. In the 1880s, Küstner published the results of accurate measurements to derive the constant of Aberration Albrecht made in Berlin, Prague, Strassbourg, and Gaillot made in Paris. About this time, Marcuse and Preston went to Waikiki in the Sandwich Isles (Hawaii) on the Western Hemisphere to learn that the combined wobbles period appeared now to be about 437 days (Figure 17)

It causes the Pole to describe a small orbit of a few metres around the mean positions. (Figure 18) shows the path of polar motion between 1909.0 and 1915.0. Simon Newcomb explained the four-month difference between Leonhard Euler's predicted period and the observed duration of the Chandler wobble is due to the elasticity of the Earth's mantle, and yearly displacement - by melting and freezing - of water masses between Arctic and Antarctic regions as a cause of polar variations (Jordan, 1923).

2.2.3 Orbit Perturbations

The equations of motion are easily given in an inertial reference frame. However, in this system, the Earth is moving in an irregular manner, and the gravitational field, assumed static in an Earth-fixed system, has irregular time dependence. This irregular temporal variation will give rise to perturbations [4.2.1].

With the availability of powerful terrestrial and spaceborne computers, in addition to increasing demands for prediction, the need for analytical perturbation theories has been replaced by a purely numerical treatment of the equations of motion. Resonance is induced in the satellite motion when a satellite finds itself at equal intervals of time above the same part of the gravitational field. Even small gravitational forces can then build up into respectable perturbations on the orbit (Brouwer, 1963; Montenbruck, 2001).

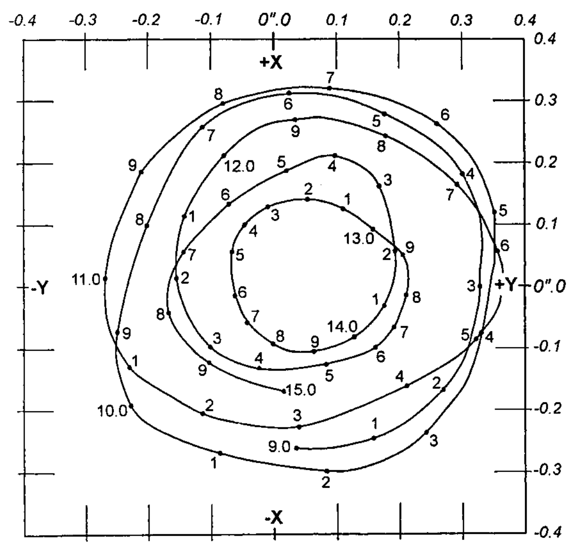


Figure 18: Path of polar motion between 1909.0 and 1915.0

Recent improvements in astrometric surveying techniques have enabled geodesists to detect and measure systematically horizontal and vertical crustal motion to the 1 cm level, and variations in gravity to the 0.1 mgal level. Most topographic features were formed because of vertical tectonic movements, such as uplift and subsidence of the Earth's crust (Even-Tzur, 2005; Mäkinen, 2004).

3. The Figure of Earth

About Mapping the World

> *As early as 1466, Nicholaus Germanus applied the Polyeder projection for a (trapezoidal) Map of the World. Since then, the Polyeder (polyhedral) projection has been used in many countries for mapping purposes. It will be found that these maps cannot be joined properly. Consequently, this projection was never used for nautical charts [10.8]*
>
> *Until 1940, only 10% of the (onshore) world was mapped. Geodesy uses a Euclidean coordinate system, in which the geometric arrangement of the coordinated stations are defined in terms of accessible points, bearings and distances, and measured in terms of the same units*
>
> *Until the completion of Geodetic Reference System of 1967, it was often impossible to compute the transformations between absolute geodetic systems of geographically or politically isolated governments, who set up its own absolute geodetic systems.*

3.1 Astronomic and Geodetic Research

It is not surprising, that the simple conditions of Earth's structure, such as the invariable position of the rotation axis with the invariable period of the Earth's rotation, could not be maintained against present research. The precision of the 19th-century measurements predicted by the theory of Leonhard Euler showed a deviating reality.

Carl F. Gauss, who, with Archimedes of Syracuse and Isaac Newton, ranks as one of the greatest mathematicians of all time, opened the way significantly to pure mathematics. However, he also made practical applications of importance for astronomy, and geodesy. In 1794, he applied a new method, by which the best estimated value was derived from the minimum sums of squared differences - the so-called method of least squares (LS) - in a particular computation. In 1820, Gauss turned his attention to geodesy - the mathematical determination of the shape and size of the Earth's surface, to which he devoted much time in the theoretical studies. To increase the accuracy of surveying, Gauss invented two heliotropes. These instruments use reflected sunlight to get measurements that are more accurate in field surveying (Britannica, 1999, Gauss, 1823).

Gifted in mathematical theory and no less insistent in practical surveying and processing, astronomer Friedrich W. Bessel of Königsberg performed all his work with meticulousness and accuracy far beyond the relatively poor quality of the data. In 1820, he installed Reichenbach's new meridian circle. Moreover, the observations required corrections for changes occurring in the celestial sphere by Precession [2.2], Nutation, and Aberration. In 1830, Bessel published his *Tabulae Regiomontanae* (so-called *Königberg tables*), which became a constituent part of an astronomical almanac.

In 1841 after a full life of astronomical practice, Bessel conceived a greater instrument - made by Repsold - provided with various improvements. Bessel mentioned in a lecture:

> *" ... every instrument is made twice, once in the workshop of the artisan, in brass and steel, and then again by the astronomer on paper, by means of the list of necessary corrections which he derives by his investigations ... ".*

In addition, he made fundamental contributions to accurate positional astronomy, the exact measurement of the positions of celestial bodies; to celestial mechanics, dealing with their movements; and to geodesy. Bessel's contributions include a correction in 1826 to the seconds' pendulum, the length of which is precisely calculated so that it requires exactly one second for a swing. Throughout the period 1839-1841, he deduced a value of 299 for the reciprocal flattening, i.e. ellipticity of the Earth: the amount of elliptical distortion by which the Earth's shape departs from a perfect sphere (Table 17): pp 121 (Strasser, 1957).

George B. (Airy, 1830) mentioned:

> *"... If the Northern and Southern Hemispheres of the Earth were dissimilar, the radii of curvature in corresponding north and south latitudes would be unequal. Therefore, the lengths of degrees in corresponding, north and south latitudes would not be the same. Conversely, if it is found that the length of a degree in any north latitude is different from that in equal south latitude, we must conclude that the Northern and Southern Hemispheres are not similar*
>
> *If the Earth were not a solid of revolution, the different meridians would be different curves, and, therefore, at the same latitude, but in different longitudes, the degrees of meridian would be unequal. The parallels, also, would not be circles, and, consequently, different degrees of longitude on the same parallel would have different lengths. But the earth's form, even though perfectly symmetrical, may be formed by the revolution of some figure differing from the ellipse ... ".*

Establishing Geodesy

Friedrich R. Helmert was well endowed with all those qualities that make a successful geodesist. He was the professor of Geodesy at the Technical University at Aachen, director of the Prussian Geodetic Institute (1892), and of the Internationale Erdmessung (IE) - Central Büreau [19.2]. Through his work, geodesy has experienced discoveries, conceptual inventiveness, and achievement of technical ingenuity, which still have their effect.

The subject was generally treated *as one and indivisible*. Helmert's definition of 1880 was essentially that geodesy was the science concerned with determining the size and shape of the Earth. On the other hand, geodesy was, as in the hands of Helmert, essentially four different disciplines (Helmert, 1880):

- horizontal control
- vertical control
- astro-geodesy
- physical geodesy

which only rarely were coordinated and synthesised. As a consequence, the work went on independently. The development of geodesy reflected this separation (Clarke, 1880; Helmert, 1880, 1884; Torge, 2001).

Geodesy in the Twentieth Century

In practice, the geodetic science was equivalent to locate, in some convenient coordinate system, positions on the Earth's surface, and to determine the Earth's gravity field. In the 20th-century, a large number of different coordinate systems were in use.

Many years, geodesy was subdivided into lower geodesy (concerning techniques, instrumentation), and higher geodesy, (reference ellipsoid, earth's curvature, geoidal heights, and similar concepts). At present, the term geodesy is commonly understood to include seven different geodesy divisions:

- astronomical geodesy is concerned with positional relationships in the Earth's environment obtained by geometric methods
- geometric geodesy, which deals primarily with geometrical relationships
- gravimetric geodesy is concerned with the application of gravimetry to geodesy, such as measurement of the magnitude or direction of the Earth's gravity field
- theoretically, Antonio Marussi formulated intrinsic geodesy, and Martin Hotine extended it considerably. Here, the coordinate system is entirely defined by the Earth's gravity field in the language of tensor analysis.
- marine geodesy is concerned with positioning by geometric means, such as hydrographic surveys of positions at sea surface, or at the (sub) bottom, gravity at sea, and determination of the geoid over the ocean surface
- physical geodesy is concerned with the principles and methods of physics, e.g. the gravity field

- satellite geodesy, using both natural and artificial satellites, deals with the use of targets principally in their greater height above the Earth's surface,

in which satellite- and marine geodesy refer to special geodetic data sources. Because geodetic satellites are constantly in motion, nearly simultaneously observations must be made on targets from particular locations to compensate for their motion (Groten, 1967, 1968).

The Earth - An Imperfect Ellipsoid

Until the early part of the 20^{th}-century, each country, due to due to political nature or financial precautions, used its own coordinate system. Hence, whenever such problems as the calculation of a line between two countries or the determination of the location of a point in one country with respect to the reference system of another country were undertaken, technical complications arose because there was no recognised relationship between the Datums used by the countries involved.

In geodesy exists the problem of using classical 2D+1D-geodesy [5, 6], with horizontal- and vertical coordinates, respectively, and spatial 3D-geodesy [7].

Locally Defined Geodetic Datums

Until 1967, a locally defined geodetic Datum was usual developed using *classical 2D+1D-geodesy*, with independent horizontal and vertical coordinated points. These points on the Earth's surface occurred in sets, and each set belonged to a specific local Datum. Every satellite-tracking instrument, located in some reference system for which a Datum exists, was used to gather data for the static satellite positioning [4.2]. Regrettably, the majority of the satellite-tracking instruments were frequently not in the same reference system or Datum.

Geocentric Reference Systems

Using all available geodetical data, along with the reciprocal flattening (298.3) from Sputnik and Vanguard observations, NGA and NASA developed the astro-geodetic Mercury Datum System of 1960 to position the Mercury project tracking stations (Seppelin, 1974).

Positioning by geodetic satellites is performed in a 3D-earth-centred, earth-fixed (ECEF) geodetic reference system (GRS), utilising a rectangular Cartesian coordinate system, having three mutually perpendicular x-, y-, z-axes with its origin located at Earth's centre of mass. This was applied to the TRANSIT system [4.3.1] as realised in the 1960s, and the GPS or NavStar [4.3.2], as realised in the 1980s. Geocentric Datums were established by space-geodetic measurements, and by calculations that considered the Earth's gravitational field as determined from measurements on the Earth's surface or from analyses of satellite orbits (Austen, 2002; Grafarend, 1995f, 1996e, 1997n, 1999a, 2002o, 2003h, n; Ilk, 2004; King-Hele, 1958; Kopejkin, 1991; Nerem, 1993; Newton, 1961; O'Keefe, 1960; Parkinson, 1996; Reubelt, 2003).

3.2 Horizontal Control Datum

Efforts of mathematicians to establish beyond question the physical soundness of a Euclidean geometry culminated in the creation of non-Euclidean geometries [12.3], which proved as useful as Euclidean geometry for representing the properties of physical space. This unexpected fact gave rise to the question: Since these geometries differ from each other what are we really sure is true about physical space?

Using applied mathematics and computer science, geodesy is the science of the determination of the shape and size of the Earth. This means positioning of points on its ellipsoidal surface, with description of variations of its gravity field. Long before the arrival of artificial satellites, it was supposed by expeditions that the Earth's form was a perfect ellipsoid.

The diversity of local Datums was caused by the political nature of survey activities, or it had an excellent technical justification (Hooijberg, 1997). Closely connected with local Datums are sets of geodetic controlled networks that form the geometric framework of a country or state.

Indicators of Accuracy

Although Gauss introduced the method of least squares (LS) in the late 18th-century, applied it to the adjustment of geodetic network data, and explained its statistical treatment, the results of geodetic computations consisted only of angles, distances, and coordinates without any information on the reliabilities of these data [3.1]. Even today, there exist many large geodetic networks in which the coordinates of the control points are known, but without indicators of accuracy (standard deviation, covariances, et cetera) (Dalton, 1953; Grafarend, 2006a; Koch, 1980, 1999, 2000; Olliver, 1999).

A Datum designed particularly for a specific region can usually provide a better local fit of the ellipsoid to the geoid than does a Datum specifying a single global ellipsoid. Coordinates of points in a earth-centred, earth-fixed (ECEF) system must change every time a transformation is made in the theory connecting the ECEF's origin to surface points. To effect a transformation from one Datum to another, there must be accepted (surveyed) relations between the origin and orientation constants of one Datum and those of the other.

Theoretically was the definition of a Datum correct, but the geodesists applying it to the global- or continental-wide Datum found that only a few of these Datums were well defined.

As a rule, precise relations between two Datums did not exist. When sufficient common stations were known, the transformation was customarily accomplished by conversion of geodetic to Cartesian coordinates, and shifting them to a new origin, rotate them into the new Datum, and finally conversion of Cartesian to geodetic coordinates with the new major axis (a) and its reciprocal flattening (f^{-1}).

As a result, calculations made with most existing geodetical Datums were (trivial) erroneous due to distortions in location-dependent differences. This inhomogeneity makes any transformation formulae between any local 2D-Datum and a 3D-Datum inaccurate. If tables of functions, whose development would require great labour, are furnished with a small tabular interval, a double interpolation method (so-called bi-linear interpolation) is rendered very easily.

Nevertheless, such tabulation is not practicable for tables that must be used frequently, and also is not practicable for further use by hand. Then it is advisable to use the method of bi-linear interpolation by computer, applicable when bi-linear interpolation would lead to a simple transformation between any local 2D-Datum and a 3D-Datum, such as ED50/GRS80. Bi-linear interpolation corrects varying differences between any Datums [7.1].

ECEF Geocentric Coordinate System

In 1967, the International Union of Geodesy and Geophysics (IUGG) recommended that the conventional international Origin (CIO) be used in defining the direction of a North Pole for geodetic reference systems. The Origin of a coordinate system, derived from the original latitudes of observatories (GG, 1986).

Based on geometrical and dynamical satellite data and analyses [4.2], the World Geodetic System Committee (WGSC) decided to adhere to the model of the equipotential ellipsoid as conceived by IUGG in establishing the Geodetic Reference System of 1967 (GRS67) [3.2.2]. Determinations of the Earth's figure from the time of GRS67 onwards are made in the truly earth-centred, earth-fixed (ECEF) geocentric coordinate system. For that reason, the GRS67 ellipsoid was taken as the model for the World Geodetic System of 1972 (WGS72) ellipsoid (Malys, 1994; Slater, 1998).

Ongoing research in improved computational procedures and statistical analyses resulted in new methods for handling and combining enormous quantities of data, such as:

- Baker-Nunn camera data of the SAO (Smithsonian Astrophysical Observatory)
- Doppler data available from Geoceivers (geodetic receivers)
- Doppler data provided by Doppler tracking station network (TRANET) [4.3.1]
- electronic satellite data provided by SECOR (sequential collation of ranges) network
- optical satellite data provided by the Wild BC-4 camera system.

In 1978, with the results of precision gravity-, triangulation-, trilateration- and astronomical surveys, WGSC started the development of the global Datum World Geodetic System of 1984 (WGS84) (Groten, 1981; Ihde, 1981; Seppelin, 1974).

3.2.1 Specification of Size and Shape

The set of constants that defines the relationship between a coordinate system and the Earth is called a Datum. The number of constants in this set depends on the kind of coordinate system chosen. A Datum for a Cartesian orthogonal coordinate system is specified by six constants:

- three linear quantities that give the translation of a point in a body
- three angular quantities that give the rotation of the body about a point.

Time is not relevant, but if a geodetic coordinate system is to be used, the Datum must also specify the size and shape of the ellipsoid. This means:

- if the ellipsoid is a sphere ________ one additional constant
- if the ellipsoid is bi-axial ________ two additional constants
- if the ellipsoid is tri-axial ________ three additional constants [6.2].

Since all reference systems in use specify an oblate bi-axial ellipsoid as a reference surface, all corresponding geodetic Datums should contain eight constants.

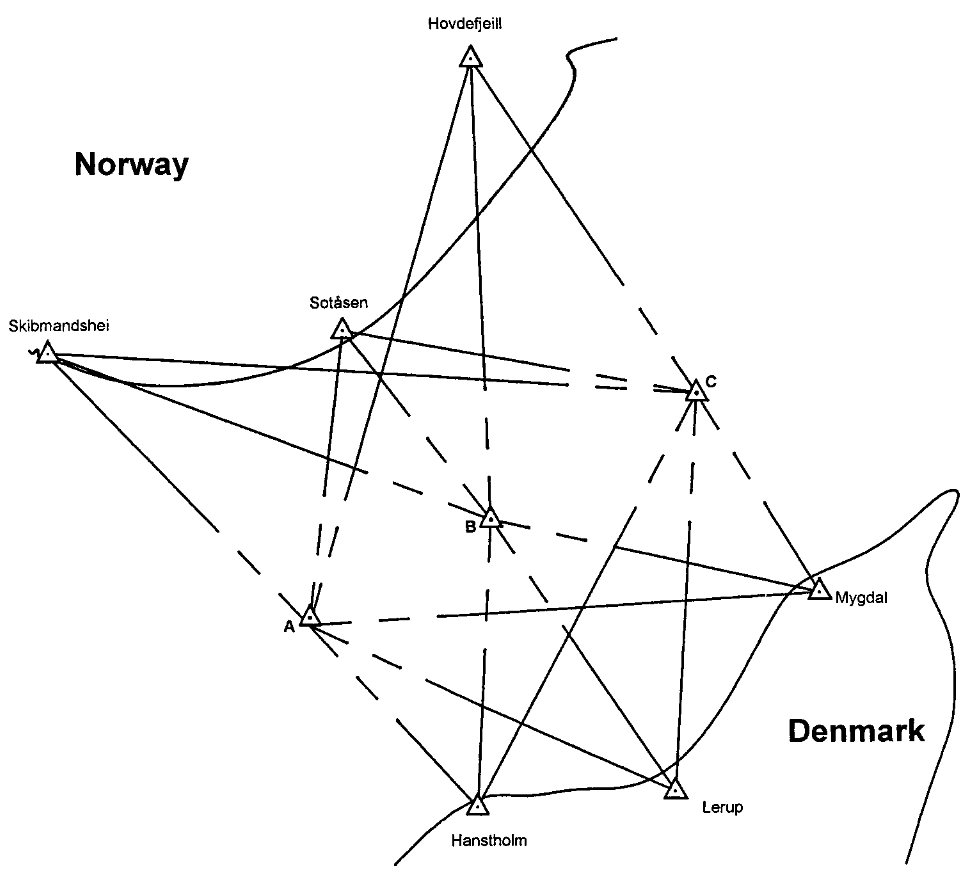


Figure 19: Flare triangulation of the connection Denmark and Norway

Datums were related to one another by the following methods:

- when short stretches of water occurred, the Datums were connected by flare triangulation, as in the connection of Danish and Norwegian Datums across the Kattegat (Simonsen, 1949), in the USA, and for primary triangulation in Finland (Schmid, 1977; Väisälä, 1947, 1960), but not elsewhere
- very long distances over water could not be bridged with high accuracy. Methods used Solar eclipses or occulations by the Moon to get data that would permit connecting NAD27 to ED50, and to Tokyo Datum. Few results were published due to military restrictions, difficulty of measurement and the length of time needed for precise reduction (Kukkamäki, 1954)
- connection of coordinate systems to each other was accomplished by triangulation, trilateration and, sometimes, by levelling. A field party in the Sudan completed a missing link in the triangulation along the EurAfrican Meridian 30° E through Africa of the Meridian 30° E through Africa, connecting European to Cape Datums (Figure 20). The geoidal profile 30° E is on various Datums: ED50 (International Ellipsoid), WGS84, Arc Datum, and Adindan Datum (Clarke 1880 modified ellipsoid) (Fischer, 1959, 1972; Smith, 2005)
- The Inter American Geodetic Survey (IAGS), now a component of NGA, completed a long triangulation arc through Central and South America
- longer distances over water were bridged by use of a high-intensity range navigation system (HIRAN, 1950) or a short range navigation system (SHORAN), such as in the connection of the Hawaiian Islands to each other, and the connection of North American Datum of 1927 (NAD27) to European Datum of 1950 (ED50) via Greenland and Iceland. IGN Institut Géographique National (IGN) bridged connections by IGN from France to Algeria (1962), and from Europe to Azores (1965) (Owen, 1960)
- static and dynamic satellites surveying [4.2] between 1957-1977.

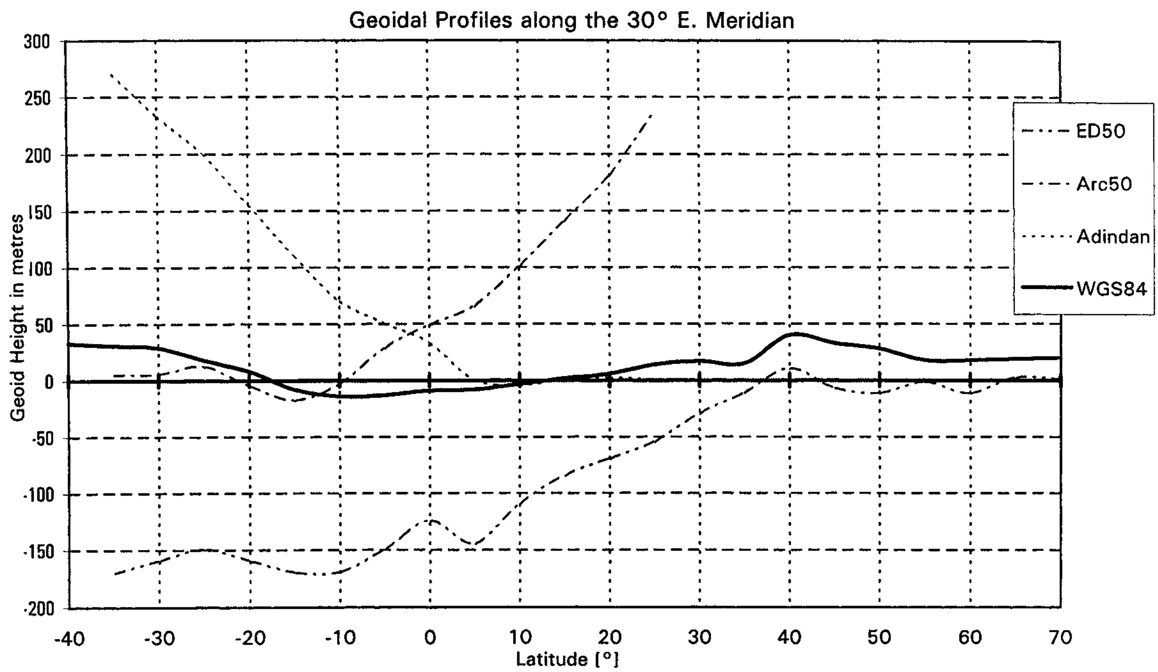


Figure 20: Comparison of various Datums along the 30° meridian from Finland to South Africa

Until the 1900s, each country, due to political nature or financial precautions, used its own coordinate system. Until the early part of the 20th-century, each country, due to political or economic motives, used its own coordinate system. Hence, whenever such problems as the calculation of a line between two countries or the determination of the location of a point in one country with respect to the reference system of another country were undertaken, technical complications arose because there was no recognised relationship between the Datums used by the countries involved.

3.2.2 Combining Local Reference Datums

In the late 1950s, geoidal heights (N) are not among the unknowns present in the observation equations of either static or dynamic satellite geodesy [4.2]. Conversely, they appeared in OSU's work in the condition equations used for determining the size and flattening of the best fitting ellipsoid.

In 1958, the World Geodetic System Committee (WGSC) of DoD generated a World Geodetic System of 1960 (WGS60) to which different local geodetic networks could be referred, using a combination of surface gravity data, astro-geodetic data, triangulation and trilateration network results from high-intensity range navigation system (HIRAN) and short range navigation system surveys (SHORAN).

In 1965, NASA started the US National Geodetic Satellite Program (NGSP) [19.6] by setting up an extensive program of observation and data reduction. The basis for evaluation is comparison of results with values whose accuracies were known (Henriksen, 1977a, b):

- compare gravity computed from observations on satellites with gravity measured on the surface of the Earth
- compare coordinates and/or distances derived by satellite geodesy with corresponding values derived by surveying on the Earth's surface
- accuracies of first-order survey coordinates are known satisfactorily only within the local Datums, but not with respect to a global Datum.

In the 1970s, values of suitable accuracy were known for less than five percent of the Earth's onshore surface. The positions of stations on North American, European, South American, Tokyo, and Australian geodetic datums were determined to reduce the errors in ties between these Datums. But instead of there being one set of coordinates and one gravity field, there were at least seven different major sets of coordinates and five different fields. The existence of different results may indicate just that the NGSP participants envisioned an answer to different questions, all of which contained within the original statement of the purposes of the NGSP program.

Dynamic Datums

Coordinates of satellite-tracking stations were properly classified as unknowns. However, the importance of the surveyed coordinates in fixing the local Datums made it advantageous to use them as data. In the 1960s, the most useful result from dynamic satellite geodesy (DSG) to the geodesist was the Figure of the Earth [1.4]. It is the only connection between heights measured by levelling and geodetic heights found by DSG [4.2.2]. The small variations (to within -0.5 metre) were given e.g. by the Geoid of Vincent and Marsh, which was derived from gravity anomalies to provide the fine detail (Figure 4): pp 11.

Suitably chosen absolute reference systems, such as early World Geodetic Systems of 1960 and 1966 [4.3], were conceived to give a small value for the standard deviation (SD) for most of the points within the region governed by that particular Datum. Obviously, this resulted in larger values for the SD of all other points on the Earth's surface. At that point in time, there was no worldwide geodetic Datum cover!

A large number of "*dynamic*" Datums were established for use in satellite-orbit computation. Their axes were oriented to Bureau International de l'Heure (BIH) meridian of zero Longitude and the other parallel to the mean axis of the 1900-05 conventional Pole (CIO) as recommended by the International Astronomical Union (IAU-1967).

3.3 Vertical Control Datum

Kakkuri mentioned that the perfect vertical geodetic control Datum is completely defined by the equipotential surface. Consequently, the ideal vertical Datum is the geoid. In that case C, the geopotential number, is equal to zero and consequently $H_O = H_N = 0.0$. In practice, the vertical positions are given with respect to local mean sea level (MSL) of a particular tide gauge and epoch (Kakkuri, 1995).

Definition of the Vertical Datum

In the determination of precise geodetic heights, either orthometric or normal heights are employed. Using the geopotential number C, heights are defined as follows (Kakkuri, 1996):

> *"Orthometric Height is the height $H_o = C / \bar{g}$ of a point P, in which $\bar{g}$ is the mean gravity value assigned to the plumbline between the geoid and the point P*
>
> *Normal Height is the height $H_n = C / \bar{\gamma}$ of a point P, in which $\bar{\gamma}$ is the mean normal gravity value assigned to the normal plumbline between the ellipsoid and the point Q, where the normal potential is equal to the actual potential at the point of calculation P". (So-called telluroid mapping $P \rightarrow Q$ (Figure 54): pp 143). The treatment of Telluroid or Terroid is given in (Ardalan, 2002a; Graaff-Hunter, 1960; Grafarend, 1978a).*

System of Sea-Level Heights

An ideal geoid is the mathematical (equipotential) surface that corresponds - in the least squares sense - to a hypothetical Earth in the same way that the geoid corresponds to the real Earth. As defined [1.4], the surface coinciding with mean sea level is usually not the same as the geoid due to the correction of distortions, reduction in centrifugal force, an increase in the force of gravity at higher latitudes, and tidal- and ocean currents [2.2].

Since the 1940s, a major goal of geodesy was the determination of the geoid. Global mapping operations are dependent on an accurate geoid. A chosen ellipsoid, which uses the geoid as a reference surface for elevations, together with its uniquely chosen location (a Datum point) and an orientation with respect to the geoid represents a geodetic 2D+1D-reference Datum.

Worldwide, there are many vertical geodetic Datums, such as:

- continental North American Vertical Datum of 1988 (NAVD88), formerly the US Sea Level Datum of 1929, after May 10, 1973 identified as the National Geodetic Vertical Datum of 1929 (NGVD29) (Zilkoski, 1992)
- continental Baltic system of heights with the Kronstadt Datum being near Saint Petersburg (Bogdanov, 2000)
- normaal Amsterdams peil (NAP)-Datum in the Netherlands and neighbouring European countries.

Still, small differences between various local vertical Datums exist in most countries. Some heights are given in feet, but most recent heights are given in international metres. During the 20^{th}-century, most national mapping agencies (NMAs) increased the density and accuracy of vertical control.

In classical geodetic practice, the geoid was generally a close fit to the reference ellipsoid in the particular area of interest (Figure 6): pp 12. At present, there is no vertical worldwide geodetic Datum cover, with a geoid defined to unify and tie together all local vertical Datum systems (Rummel, 1991; Torge, 2001).

Mean-Sea-Level

Another element is the information that the purely geometric heights coming from traditional local systems regularly need to be related to the local vertical Datums, which in turn are related to a physical surface such as one of the many realisations of mean sea level (MSL). Since 1921, National Hydrographic Offices (NHOs) increased the accuracy of vertical control by means of mareographs (tide gauges). The ellipsoidal height of tidal stations is surveyed as a network or by GPS (Adams, 2004, 2006; Chance, 2003; Higgins, 1998).

Chart Datum is the vertical reference surface used in offshore charts. Moreover, it is necessary to refer all soundings to Chart Datum. In hydrography, the geodetic position and height of the survey vessel's centre are continuously fixed in ellipsoidal terms at the same time with depth soundings. Using the hydrographic combination of electronic positioning with acoustic depth measurements, all data are automatically corrected and related to the Datum, an ellipsoidal surface in use.

Tides

The range of tides varies from mareograph to mareograph due to the hydrodynamic effects of the currents and tides. The lowest astronomical tide (LAT) is the lowest level of the range of the tides, and highest astronomical tide (HAT) is the highest extreme of the range of the tides, that can be predicted to occur at the tidal station. Consequently, the heights of HAT and LAT are relative to MSL and changes from chart to chart. Formerly, mean sea level (MSL), taken as zero elevation for the vertical reference Datum, was usually determined by a series of observations at various mareographs along the seashore taken continuously for a short period of 29 days and a long period of 18.67 years or more (HI, 2005g).

Chart Datum

Most seafaring countries use a (vertical) Chart Datum related to MSL for waterdepths, depth contours, and elevations of onshore and offshore features, such as the Tsing-Tao-Datum, of the Yellow Sea. Any improvement will have a beneficial impact on charting, navigation and in hydrographical positioning. EUREF-1999 defined the policies concerning the European cooperation in the field of science and technology (COST), and initiated the use of the European Sea-Level Observing System (EOSS), including the European Vertical System (EVS) database activities to support the development of:

- GLOSS (Global Sea-Level Observing System)
- ESEAS (European Sea-Level Observing System Service), a regional densification of GLOSS.

ESEAS is a EUREF program to provide long-term monitoring activities and sea-level related information along the entire European coastline. Worldwide, more than fifty National Hydrographic Offices (NHOs) are involved in the operation of over 450 mareographs (tide gauges) with data being stored in data-warehouses. The International Hydrographic Organisation (IHO) [19.4] has consultative activities of technical character. It coordinates the activities of most NHOs (Lurton, 2002).

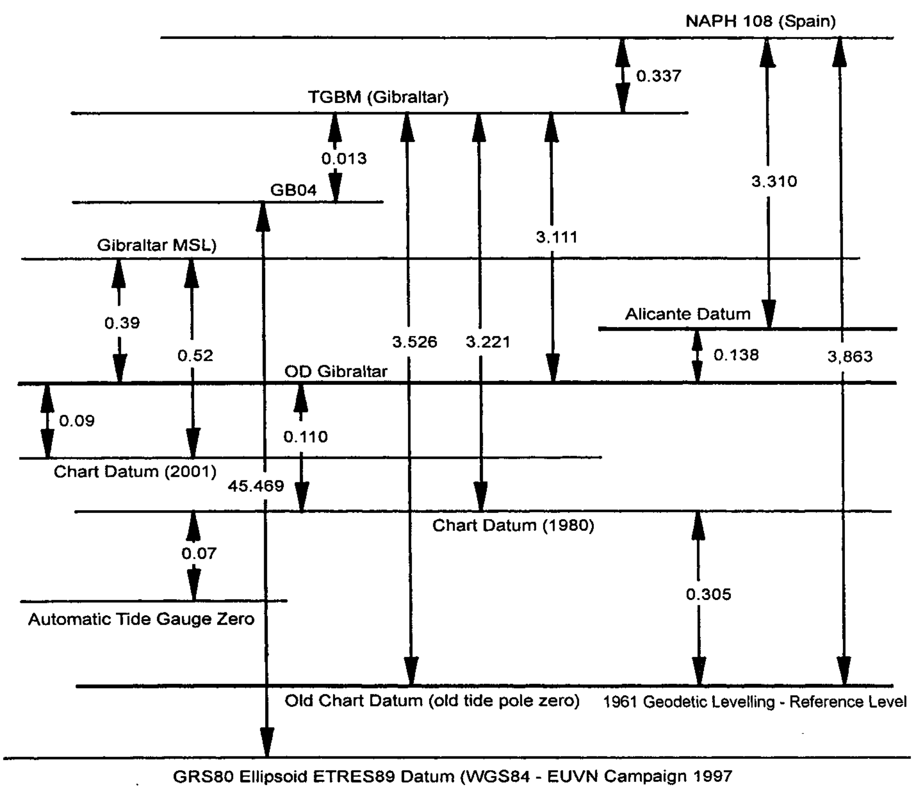


Figure 21: One example of various reference levels and vertical datums in use - Spain / Gibraltar

Consequently, one genuine aim of geodesy is to unify all local national vertical Datums into a single worldwide vertical geodetic Datum with its height in international metres (Figure 21). At present, the global geoid is expected to have a vertical accuracy of better than two metres worldwide. As ongoing geodetic research provides increasingly better estimates of geoidal heights, absolute geoidal height accuracies are being improved as more data are collected (Adams, 2004, 2006; André, 2002; Ayres, 1995).

Reducing Horizontal Distances

The necessity of reducing horizontal distances to the grid distance on the surface of the ellipsoid implies a mathematical operation with formulae. Consequently, a basic requirement for mapping of a given area is an adequate model of vertical control points using the geodetic vertical control Datum. An initial structure of the national levelling network was carried out according to traditional spirit levelling techniques between benchmarks using a least squares (LS) adjustment with a hierarchical structure (1^{st}, 2^{nd}, 3^{rd}, ... order). Currently, the availability of GPS allows the high precision altimetry.

Sea Level Observations

The differences in heights between existing surface features above sea level are surveyed by using:

- space- or airborne profile recorders
- altimetric GPS high precision satellite techniques
- aneroid-barometric measurements
- horizontal spirit levelling
- hydrostatic levelling
- measuring vertical angles and distances.

Benchmarks, connected by precise levelling, constitute the vertical controls of surveying. Elevations of benchmarks are given in feet or metres above or below a height reference surface.

European Vertical Reference Systems

Most countries of Europe were participating in the United European Levelling Network (UELN) project, including the vertical networks of Belarus, Bulgaria, Greece, Russia, Turkey, and Ukraine. The national levelling network of Russia leads to the closing of the UELN around the Baltic Sea with a connection to the fiducial benchmarks of Amsterdam (NAP) and Kronstadt (Alberda, 1960, 1963; Bogdanov, 1995; Bordley; Calvert, 1985; Ekman, 1995a; Grafarend, 1997h, 1999b, 2002c; Ihde, 2002; Kakkuri, 1995; Metzner, 1993a, b; Poutanen, 1999; Sjöberg, 1993; Waalewijn, 1986, 1987).

The IAG Sub Commission for Europe (EUREF) realised the continental-wide height system a European Vertical Reference System (EVRS), for expressing gravity-related heights. It is based on the European Vertical System (EVS). The adoption of the European Vertical Reference Frame of 2000 (EVRF2000) should greatly improve the geodetical vertical links with continental Europe, using the European United Vertical Reference Network (EUVN) and the UELN initiatives.

After the definition of the EVRS, further action has been recognised in connection with the International Gravity and Geoid Commission (IGGC) to arrive at a permanent continental or global height reference surface for scientific and practical purposes according the requirements of the navigation community.

EUREF advises UELN/EUVN levelling data providers to submit all levelling data in the zero tidal system according to the EVRS definition. The European height system shall be defined by the projection with its realisation EVRF2000.

3.4 Stellar Triangulation-Network

Already in 1615, Willebrord Snell van Royen laid an arc of triangulation-network between Alckmaar and Bergen-op-Zoom, the Netherlands, using the first geodetic unit: the Rhineland Rod [12.1].

Guy (Bomford, 1977):

> *" ... the word ellipsoid can be used either with reference to the axes and flattening of the figure or to the three arbitrary constants at the Origin as well. The word Datum can also be used to refer only to the latter, or to the dimensions of the ellipsoid as well. For example, the International Ellipsoid* (a and f), *located by the Potsdam 1950 Origin* (ξ_o, η_o, N_o) *constitutes the European Datum ... "*

Before World War I, the categories were primary, secondary, and tertiary triangulation. Primary triangulation was the category containing triangulation with the highest accuracy. The term was abandoned after WWI, when the terms geodetic or first order, second order, third order and fourth order triangulation were introduced. A first-order triangulation network is a survey network, which consists of first order control stations. The category first-order of the 1990-classification does not correspond exactly to any category of earlier classifications.

For successful application of geodetic positioning techniques, the availability of an accurate network was essential for the maintenance of a Datum, such as:

- a triangulation network is a survey network in which the survey stations are triangulation stations and the lines represent adjusted distances or baselines and directions
- a trilateration network is a survey network in which the survey stations are positioned by trilateration. The lines connecting survey stations represent adjusted distances
- a geodetic control network is constructed partly by trilateration, partly by triangulation, by levelling and/or by gravimetry
- a flare triangulation network is a method of triangulation in which observations of a flare at very high altitude were made simultaneously from a number of stations. The flare is usually carried high in the air by a rocket or airplane and ejected, to drift downward under a parachute. This method was used for extending triangulation over lines so long that the ends are not intervisible (Schmid, 1977; Väisälä, 1947, 1960; Wilcox, 1963) (Figure 22)
- a stellar triangulation network is a survey network method of determining directions between points on the Earth's surface by simultaneously photographing a beacon or lighted object against a stellar background. The derived position is an astrometric position. If three or more points are involved simultaneously, a network of directions is established that determines, except for scale, the relative positions of the points. Flares were used in a stellar triangulation network connecting Bermuda to the USA. Light-carrying or Sun-illuminated artificial satellites were used to establish vertical and horizontal control across continents or between continents (Figure 19; Figure 22)
- a satellite triangulation network is any method of determining the coordinates of points on the Earth by measuring directions from these points to one or more artificial satellites. Generally, the phrase implies that observations were made simultaneously from two or more station points on the ground. One method has been to photograph the satellite against a stellar background, so digitised star catalogues were used to obtain the bearings (Bomford, 1962, 1977).

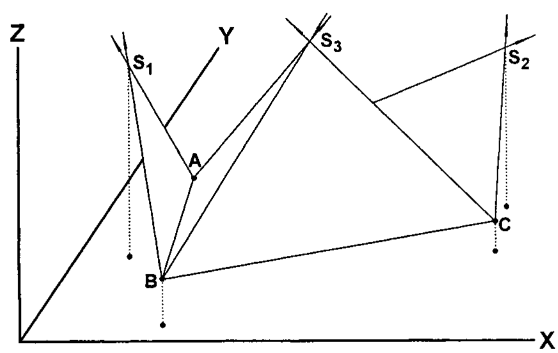


Figure 22: Väisälä's flare-triangulation application

Nevertheless, in the late 1960s, the traditional major fieldwork activities ceased with the arrival of geometric satellite triangulation. At that time a purely geometrically defined, 3D-GRS was desired to go beyond the labour intensive, traditional geodetic triangulation networks.

Since 1994, International GNSS Geodynamics Service

(IGS) / IERS coordinates all operations and analysis of a global Polyhedron-network, using the IGS global network associate analysis centres (GNAACs) instead of the fiducial astrometric concept existing in the 1980s. A reliable update of a global Polyhedron-network is currently available by using regional network associate analysis centres (RNAACs) (Blewitt, 1998).

3D-Reference Systems

Prior to the advent of specifically geodetic satellites, geodesists from NGA developed an astro-geodetic Mercury Datum System of 1960, using available geodetical data, along with an early determination of reciprocal flattening from observations of Sputnik Zemli-I, Zemli-II and the Vanguards. NASA selected the Mercury Datum System of 1960 with an early determination of the reciprocal flattening (f^{-1} = 298.3) to position the Mercury project tracking stations (Seppelin, 1974).

Positioning by geodetic satellites is carried out in a geocentric, geodetic reference system (GRS). This so-called earth-centred, earth-fixed (ECEF) system utilises a rectangular 3D-Cartesian coordinate system having three mutually perpendicular x-, y-, z-axes, with its Origin located at Earth's centre of mass. This applies to the TRANSIT system [4.3.1], as realised in the late 1950s, and GPS-NavStar [4.3.2], or GLONASS [4.3.4] as realised in the late 1970s. Geocentric Datums are located by space-geodetic measurements and calculations that consider the Earth's gravitational field as determined from measurements on the Earth's surface or from analyses of satellite orbits (Austen, 2002; Grafarend, 1995f, 1996e, 1997n, 1999a, 2002o, 2003h, n; King-Hele, 1958; Kopejkin, 1991; Nerem, 1993; Newton, 1961; O'Keefe, 1960; Parkinson, 1996; Reubelt, 2003).

Continental-wide Geodetic Datums

A global, continental, or national triangulation network involves a properly defined ellipsoidal geodetic Datum. New reference systems, such as GRS80, WGS84, and PZ90 are terrestrial continental-wide geocentric 3D-geodetic reference systems for continental mapping, navigation and positioning applications. WGS84, in addition, is currently established to facilitate a worldwide use of efficient and precise geodetic satellite techniques in surveying (Hager, 1990).

A Datum may be defined as a set of specifications for a 3D-coordinate system for a collection of positions on Earth's surface:

- a 2D-reference surface to which the latitude and longitude coordinates are referred as well as the quantities which determine the Origin used and orientation of the ellipsoid with respect to the Earth
- a 3D-Cartesian coordinate system, the Origin, orientation, and scale of which must fit the coordinates of physical points in the system. It is associated with every geodetic Datum
- at least eight constants are needed to form a complete geocentric reference Datum in 3D-satellite geodesy:
    - three constants to specify the location of the Origin of the coordinate system
    - three constants to specify the orientation of the coordinate system, and
    - two constants to specify the dimensions of the Reference Ellipsoid (GG, 1986).

For both the reference surface and the coordinate system, we must consider:

- issues of philosophy or principle
- issues of materialisation, how these attributes are achieved.

Datum as Coordinate System

Describing the relationship between one system and another coordinate system can also specify a Datum. Attempting to match the Bureau International de l'Heure (BIH) Terrestrial System (BTS), NAD83, OSGRS80, ETRS89 and WGS84 coordinate systems were related to the TRANSIT NSWC 9Z-2 coordinate system. These coordinate systems are almost identical because NMAs were coordinating their efforts.

Datum as Coordinates

The Datum is defined as an abstract, equipotential reference surface, usually followed by a definition which states that a horizontal geodetic Datum is composed of adopted horizontal coordinates of a set of physical points. The adopted coordinates actually determine the Origin and orientation of a Datum, such as Tokyo Datum [6.3.1].

Datum Blunder Detection

Families of survey stations with coordinates in two or more Datums (or epochs) do exist in most countries. An oversight will take place when the unaware user is contradicted with sets of coordinates for survey points. However, the differences of position coordinates could be relatively small.

The lack of awareness by the general user of the necessity to link coordinate values with a specific Datum is the main problem associated with geodetic Datums (Chovitz, 1989; Schwarz, 1989a; Sillard, 2001; Wolf, 1963, 1987).

3.5 Toward a Worldwide 3D-Geodetic Reference System

Geodetic Reference Systems (GRS) are based on the theory of the geocentric equipotential ellipsoid, defined by constants. The International Union of Geodesy and Geophysics (IUGG) has chosen a set of conventional ellipsoid constants (Moritz, 1992).

Formulae for Geocentric Ellipsoids

a	in m	semi-major axis or equatorial radius of the Earth
GM	value × $10^{8} m^3 s^{-2}$	geocentric gravitational constant
J_2	value × 10^{-8}	dynamic form factor, un-normalised form
$C_{2.0}$	value × 10^{-6}	dynamic form factor, normalised form
ω	value × $10^{-11} rad\ s^{-1}$	angular velocity of the Earth.

Following relation exists between the zonal sphere function J_2 and the eccentricity e:

e^2	=	$3 J_2 + (4/15)(\omega^2 \times a^3 / GM)(e^3 / 2q_0)$, in which		(3.01)
$2q_0$	=	$(1 + 3/e'^2))\arctan(e') - (3/e')$ and		(3.02)
e'	=	$e(1 - e^2)^{-1/2}$	2^{nd} eccentricity	(3.03)
			equipotential ellipsoid parameters:	
e^2	=	$a^2 - b^2 / a^2$	square of the 1^{st} eccentricity	(3.04)
f	=	$1 - (1 - e^2)^{1/2}$	flattening	(3.05)

Table 5: Closed formulae to calculate the square of the first eccentricity $e^2$and reciprocal flattening

Using basic equation (3.04), and taking (3.02) and (3.03) into account, (3.01) calculates iteratively e^2, and (3.05) furnishes the flattening (f). See program A_08GEOR.FOR [18.8] (Moritz, 1967-1992).

3.6 Fundamental Parameters in Astronomy-Geodesy-Geodynamics

BASED ON A REPORT OF THE IAG-SPECIAL COMMISSION-3 IN CHARGE OF THE WORK RELATED TO FUNDAMENTAL CONSTANTS BY ERWIN GROTEN, PRESIDENT

3.6.1 Estimated Parameters for the 2000s

Current (2003) best estimates of the parameters of common relevance to astronomy, geodesy, and geodynamics, in which SI units are used throughout (Smith, 1993; Williams, 1993).

- velocity of light in vacuum

  $c = 299\,792\,458 \text{ m s}^{-1}$ metric <u>and</u> gravity scale ! (SC-3.01)

- Newtonian gravitational constant:

  $G = (6\,672.59 \pm 0.30) \times 10^{-14} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$ (SC-3.02)

- Geocentric gravitational constant (including the mass of the Earth's atmosphere) (Ries, 1998):

  $GM = (398\,600\,441.8 \pm 0.8\,) \times 10^6 \text{ m}^3 \text{ s}^{-2} \quad \pm 2.10^{-9}$ (SC-3.03)

For the new EGM 96 global gravity model was adopted:

$GM_{EGM\,96} = 398\,600\,441.5 \times 10^6 \text{ m}^3 \text{ s}^{-2}$

In TT (Terrestrial Time) units the value is:

$GM_{TT} = (398\,600\,441.5 \pm 0.8) \times 10^6 \text{ m}^3 \text{ s}^{-2}$ (SC-3.04)

If expressed in old TDB units (Barycentric Dynamical Time of the Solar system), the value is:

$GM_{TDB} = 398\,600\,435.6 \times 10^6 \text{ m}^3 \text{ s}^{-2}$

Based on well-known transformation formulae we may relate GM in SI-units to TT / TCG / TCB (IERS-Convention 1996: pp 85). The well-known secular term was not originally included in the GM(E)-analysis, therefore it was related to TT, neither to SI nor (TCG, TCB). As long as satellite analysis occurs without the secular term, GM(E) in TT is still of geodetic interest. GM(E) = GM of the Earth.

- Mean angular velocity of the Earth's rotation:

  $\omega = 7\,292\,115 \times 10^{-11} \text{ rad s}^{-1}$ truncated! (SC-3.05)

Year	ω 10^{-11} rad s^{-1}	Year	ω 10^{-11} rad s^{-1}	Mean LOD (ms/day)
min: 1978	7 292 114.903	1995	7 292 114.952	-
max: 1999	7 292 115.063	1996	7 292 114.992	-
		1997	7 292 114.991	-
		1998	7 292 115.031	1.37
		1999	7 292 115.063	0.99

Table 6: Mean angular velocity of the Earth's rotation 1978-1999

- Long-term variation in ω:

$$\frac{d\omega}{dt} = (\,-4.5 \pm 0.1\,) \times 10^{-22} \text{ rad s}^{-2} \qquad \text{(SC-3.06)}$$

This observed average is based on two actual components:

a) due to tidal dissipation:

$$\frac{d\omega}{dt}_{(tidal)} = (-6.1 \pm 0.4) \times 10^{-22} \text{ rad s}^{-2} \tag{SC-3.07}$$

This value is commensurate with a tidal deceleration in the mean motion of the Moon n:

$$\frac{dn}{dt} = (-25.88 \pm 0.5) \text{ arcsec cy}^{-2} \tag{SC-3.08}$$

b) non-tidal in Origin:

$$\frac{d\omega}{dt}_{(non\text{-}tidal)} = (+1.6 \pm 0.4) \times 10^{-22} \text{ rad s}^{-2} \tag{SC-3.09}$$

- Second-degree zonal geopotential (Stokes) parameter (tide-free, fully normalised, Love number $k_2 = 0.3$ adopted), in agreement with EGM 96:

$$\bar{J}_2 = 4.84165\,37173\,6 \times 10^{-4} \pm 3.56 \times 10^{-11} \tag{SC-3.10}$$

To be consistent with the IAG General Assembly Resolution 16, Hamburg-1983, the indirect tidal effect on J_2 should be included: then in the zero-frequency tide system (JGM-3):

$$J_2 = (1\,082\,635.9 \pm 0.1) \times 10^{-9} \tag{SC-3.11}$$

geopotential model	zero-frequency tide system		tide-free	
–	$\bar{J}_2$	J_2	$\bar{J}_2$	J_2
–	$[10^{-6}]$	$[10^{-6}]$	$[10^{-6}]$	$[10^{-6}]$
JGM-3	484.16951	1082.6359	484.16537	1082.6267
EGM 96			484.16537	

Table 7: Stokes second-degree zonal parameter; marked with a bar: fully normalised; $k_2 = 0.3$ adopted for the tide-free system

- Long-term variation in J_2:

$$\frac{dJ_2}{dt} = -(2.6 \pm 0.3) \times 10^{-9}\, cy^{-1} \tag{SC-3.12}$$

- second-degree sectorial geopotential (Stokes) parameters (conventional, not normalised, geopotential model JGM-3)

$$J_2^2 = (1574.5 \pm 0.7) \times 10^{-9} \tag{SC-3.13}$$

$$S_2^2 = -(903.9 \pm 0.7) \times 10^{-9} \tag{SC-3.14}$$

$$J_{2,2} = [(J_2^2)^2 + (S_2^2)^2]^{1/2} = (1815.5 \pm 0.9) \times 10^{-9} \tag{SC-3.15}$$

geopotential model	$\bar{C}_2^2$ $[10^{-6}]$	$\bar{S}_2^2$ $[10^{-6}]$
JGM-3	2.43926	-1.40027
EGM 96	2.43914	-1.40017

Table 8: Stokes second-degree sectorial parameters; marked with a bar: fully normalised

Only the last decimal is affected by the standard deviation. For EGM96 more detailed values are found (Marchenko, 1999):

harmonic coefficient			value of coefficient $\times 10^6$	temporal variation $\times 10^{11}$ [yr^{-1}]
$\bar{C}_{20}$	=	$-\bar{J}_2$	–484.16537 1736	1.16275 534
$\bar{C}_{21}$	=		–0.00018698764	–0.32
$\bar{S}_{21}$	=		0.01195 28012	1.62
$\bar{C}_{22}$	=	$-\bar{J}^2_2$	2.43914 35239 8	–0.49473 1439
$\bar{S}_{22}$	=		–1.40016 683654	–0.20338 5232

Table 9: Parameters of the linear model of the potential of 2nd degree

- Coefficient H associated with the Precession constant as derived (Mathews, 2000):

$$H = \frac{C - \frac{1}{2}(A+B)}{C} = 3.27378\,75 \times 10^{-3} \qquad \text{(SC-3.16)}$$

(with an uncertainty better than 0.2 ppm) with Fricke's corrected Precession constant we have

$$H = (3\,27376\,3 \pm 20) \times 10^{-9} \qquad \text{(SC-3.16a)}$$

The value of H as derived by (Mathews, 2000) contains the full permanent tide (direct and indirect effects). In principle, this fact depends on the VLBI-data, on which the semi-empirical solution is based: if the permanent tide is not fully included there, a different tidal reference is being used.

- The geoidal potential W_0 and the geopotential scale factor $R_0 = GM / W_0$ recently derived (Burša, 1998):

$$W_0 = (\,62\,636\,855.611 \pm 0.5\,)\ m^2\,s^{-2}$$

$$R_0 = (\,6\,363\,672.58 \pm 0.05\,)\ m \qquad \text{(SC-3.17)}$$

$$W_0 = (\,62\,636\,856.4 \pm 0.5\,)\ m^2\,s^{-2}$$ (Ries, 1998) conceived rounded!

If W_0 is preserved as a primary constant, the discussion of the ellipsoidal parameters could become obsolete; as the Earth's ellipsoid is basically an artefact. Modelling of the altimeter bias and various other error influences affect the validity of W_0-determination. The variability of W_0 and R_0 was studied by (Burša, 1998) recently. They detected interannual variations of W_0 and R_o amounting to 2 cm.

The relativistic corrections to W_0 were discussed by (Kopejkin, 1991; formulae 67; 77) where tidal corrections were included. Whereas he proposes average time values, Grafarend insists in corrections related to specific epochs to illustrate the time-dependence of such parameters as W_0, GM, and J_n, which are usually, in view of present accuracies, still treated as constants in current literature.

Based on recent GPS data (Ardalan; Grafarend, 1997h) found locally in the Finnish Datum for Fennoscandia:

$$W_0 = (\,6\,263\,685.58 \pm 0.36\,)\ \text{kgal m}$$

The temporal variations in general terms, as discussed by (Wang; Kakkuri, 1998):

- Mean equatorial gravity in the zero-frequency tide system:

$$g_e = (978\,032.78 \pm 0.2) \times 10^{-5}\ m\,s^{-2} \qquad \text{(SC-3.18)}$$

- Equatorial radius of the Reference Ellipsoid (mean equatorial radius of the Earth) in the zero-frequency tide system (Burša, 1998):

  a = (6 378 136.62 ± 0.10) m (SC-3.19)

- the corresponding value in the mean tide system (the zero-frequency direct and indirect tidal distortion included) comes out as:

  a = (6 378 136.72 ± 0.10) m (SC-3.20)

  and the tide-free value:

  a = (6 378 136.59 ± 0.10) m (SC-3.21)

The tide free-value adopted for the new EGM96 gravity model reads:

a = 6 378 136.3 m

- polar flattening computed in the zero-frequency tide system, (adopted GM, ω, and J_2 in zero-frequency tide system):

  f^{-1} = 298.25642 ± 0.00001 (SC-3.22)

The corresponding value in the mean tide system comes out as:

f^{-1} = 298.25231 ± 0.00001 (SC-3.23)

and the tide-free system comes out as:

f^{-1} = 298.25765 ± 0.00001 (SC-3.24)

- equatorial flattening of JGM-3 geopotential model:

  α_1^{-1} = 91 026 ± 10 (SC-3.25)

- longitude of major axis of equatorial ellipse, geopotential model JGM-3:

  Λ_a = (14°.9291 ± 0°.0010) W (SC-3.26)

In view of the small changes (Table 8) of the second-degree tesserals, it is close to the value of EGM96. We may raise the question whether we should keep the reference ellipsoid in terms of GRS80 (or an alternative) fixed and focus on W_0 as a parameter to be essentially better determined by satellite altimetry, where however the underlying concept (inverted barometer, altimeter bias, and so on) has to be clarified (Nesvorni, 1994).

geopotential model	$\frac{1}{\alpha_1}$	Λ_a [deg]
JGM-3	91026	14°.9291 W

Table 10: Equatorial flattening α_1, and Λ_a of the major-axis of equatorial ellipse

- coefficient in potential of centrifugal force:

$$q = \frac{\omega^2 a^3}{GM} = (3\,461\,391 \pm 2) \times 10^{-9} \qquad \text{(SC-3.27)}$$

Computed by using values (SC-3.03), (SC-3.05) and a = 6 378 136.6

- principal moments of inertia (zero-frequency tide system), computed using values (SC-3.11), (SC-3.15), (SC-3.03), (SC-3.02) and (SC-3.16)

$$\frac{C-A}{M a_0^2} = J_2 + 2J_{2,2} = (1\,086.267 \pm 0.001) \times 10^{-6} \qquad \text{(SC-3.28)}$$

$$\frac{C-B}{M a_0^2} = J_2 - 2J_{2,2} = (1\,079.005 \pm 0.001) \times 10^{-6}$$

$$\frac{B-A}{M a_0^2} = 4\, J_{2,2} = (7.262 \pm 0.004) \times 10^{-6}$$

$$M a_0^2 = \frac{GM}{G} a_0^2 = (2.43014 \pm 0.00005) \times 10^{38}\ \text{kg m}^2 \qquad \text{(SC-3.29)}$$

$$(\,a_0 = 6\,378\,137\ \text{m}\,)$$

$$\begin{aligned} C-A &= (2.6398 \pm 0.0001) \times 10^{35}\ \text{kg m}^2 \\ C-B &= (2.6221 \pm 0.0001) \times 10^{35}\ \text{kg m}^2 \\ B-A &= (1.7650 \pm 0.0010) \times 10^{35}\ \text{kg m}^2 \end{aligned} \qquad \text{(SC-3.30)}$$

$$\frac{C}{M a_0^2} = \frac{J_2}{H} = (330\,701 \pm 2) \times 10^{-6} \qquad \text{(SC-3.31)}$$

$$\frac{A}{M a_0^2} = (329\,615 \pm 2) \times 10^{-6}$$

$$\frac{B}{M a_0^2} = (329\,622 \pm 2) \times 10^{-6} \qquad \text{(SC-3.32)}$$

$$\begin{aligned} A &= (8.0101 \pm 0.0002) \times 10^{37}\ \text{kg m}^2 \\ B &= (8.0103 \pm 0.0002) \times 10^{37}\ \text{kg m}^2 \\ C &= (8.0365 \pm 0.0002) \times 10^{37}\ \text{kg m}^2 \end{aligned} \qquad \text{(SC-3.33)}$$

$$\alpha = \frac{C-B}{A} = (327\,353 \pm 6) \times 10^{-8}$$

$$\gamma = \frac{B-A}{C} = (2\,196 \pm 6) \times 10^{-8}$$

$$\beta = \frac{C-A}{B} = (329\,549 \pm 6) \times 10^{-8}$$

3.6.2 Primary Geodetic Parameters and Discussion

It should be noted that parameters a, f, J_2, g_e, depend on the tidal system adopted. They have different values in tide-free, mean or zero-frequency tidal systems. However, W_0 and/or R_0 are independent of the tidal system (Burša, 1995).

The following relations can be used:

$$a_{\text{mean}} = a_{\text{tide-free}} + \frac{1}{2}(1 + k_s) \quad R_0 \ \frac{\delta J_2}{k_s} \qquad \text{(SC-3.34)}$$

$$\alpha_{\text{mean}} = \alpha_{\text{tide-free}} + \frac{3}{2}(1 + k_s) \quad \frac{\delta J_2}{k_s}$$

$$a_{\text{zero-frequency}} = a_{\text{tide-free}} + \frac{1}{2} R_0\, \delta J_2 \qquad \text{(SC-3.35)}$$

$$\alpha_{\text{zero-frequency}} = \alpha_{\text{tide-free}} + \frac{3}{2} R_0\, \delta J_2$$

k_s = 0.9383 is the secular Love number, δJ_2 is the zero-frequency tidal distortion in J_2.

First, the internal consistency of parameters a, W_0, (R_0) and g_e should be examined:

(i) If:

a = 6 378 136.7 m

is adopted as primary, the derived values are:

W_0 = 62 636 856.88 $m^2 s^{-2}$, (SC-3.36)
(R_0 = 6 363 672.46 m) (SC-3.37)
g_e = 978 032.714 × 10^{-5} m s^{-2} (SC-3.38)

(ii) If:

W_0 = (62 636 855.8 ± 0.5) $m^2 s^{-2}$
R_0 = (6 363 672.6 ± 0.05) m

is adopted as primary, the derived values are (mean system):

a = 6 378 136.62 m (SC-3.39)
g_e = 978 032.705 × 10^{-5} m s^{-2} (SC-3.40)

(iii) If (SC-3.18):

g_e = (978 032.78 ± 0.2) × 10^{-5} m s^{-2}

is adopted as primary, the derived values are:

a = 6 378 136.38 m, (SC-3.41)
W_0 = 62 636 858.8 $m^2 s^{-2}$ (SC-3.42)
(R_0 = 6 363 672.26 m) (SC-3.43)

There are no significant discrepancies, the differences are about the standard errors. However, the inaccuracy in (iii) is much higher than in (i) and/or (ii). That is why solution (iii) is irrelevant at present.

If the rounded value:

W_0 = (62 636 856.0 ± 0.5) $m^2 s^{-2}$ (SC-3.44)
R_0 = (6 363 672.6 ± 0.1) [m] (SC-3.45)

is adopted as primary, then the derived length of the semi-major axis in the mean tide system comes out as:

a = (6 378 136.7 ± 0.1) m, (for zero-tide: 6 378 136.6) (SC-3.46)

which is just the rounded value (SC-3.20), and (in the zero frequency tide system):

g_e = (978 032.7 ± 0.1) × 10^{-5} m s^{-2} (SC-3.47)

However, the Special Commission-3 (IAG-SC-3) recommends that GRS80 should be retained as the standard.

3.6.3 Consistent Set of Fundamental Constants (1997)

It is imperative to recognise the consistency problem: In "*current best estimates*", the best available numerical values are given. In sets of fundamental constants - such as the Geodetic Reference System 1980 (GRS80) - consistent sets are demanded. When fundamental parameters are derived (incl. time variations) from one data set, as is often the case with satellite derived data, then this principle is often violated. See e.g. the dependence of GM and a. Similarly, when data derived from systems with different "*defining constants*", as is often the case for time systems, similar inconsistency problems arise. The typical case of an inconsistent system is the WGS84 global systems, which, contrary to GRS80, is inconsistent but being widely, used.

- Geocentric gravitational constant (including the mass of the Earth's atmosphere):

  GM = $(398\,600\,441.8 \pm 0.8) \times 10^6\ \mathrm{m^3\,s^{-2}}$ [value (SC-3.03)]

- Mean angular velocity of the Earth's rotation

  ω = $(7\,292\,115) \times 10^{-11}\ \mathrm{rad\,s^{-1}}$ [value (SC-3.05)]

- second-degree zonal geopotential (Stokes) parameter (in the zero-frequency tide system, epoch 1994):

  J_2 = $(1\,082\,635.9 \pm 0.1) \times 10^{-9}$ [value (SC-3.11)]

- Geoidal potential:

  W_0 = $(62\,636\,856.0 \pm 0.5)\ \mathrm{m^2\,s^{-2}}$ [value (SC-3.44)]

- Geopotential scale factor:

  R_0 = GM / W_0 = $(6\,363\,672.6 \pm 0.05)$ m [value (SC-3.45)]

- Mean equatorial radius (mean tide system):

  a = $(6\,378\,136.7 \pm 0.1)$ m [value (SC-3.46)]

- Mean polar flattening (mean tide system):

  f^{-1} = 298.25231 ± 0.00001 [value (SC-3.23)]

- Mean equatorial gravity:

  g_e = $(978\,032.78 \pm 0.1) \times 10^{-5}\ \mathrm{m\,s^{-2}}$ [value (SC-3.18)]

(Grafarend; Ardalan, 1999d) have evaluated a (consistent) normal field based on a unique set of current best values of four parameters (W_0, ω, $\bar{J}_2$ and GM) as a preliminary "*follow-up*" to the Geodetic Reference System GRS80. It can lead to a level-ellipsoidal normal gravity field with an ellipsoidal external field in the Somigliana-Pizetti sense. By comparing the consequent values for the semi-major and semi-minor axes of the related equipotential ellipsoid with the corresponding GRS80 axes (based on the same theory) the authors end up with axes which deviate by –0.40 m and –0.45 m, respectively from GRS80 axes, and within standard deviations from the current values such as in (SC-3.21); but no g-values are given up till now.

3.6.4 SC-3 Appendices

Appendix I - Zero-frequency tidal distortion in J_2

$$(J_2 = -C_{20})$$

$$\delta J_2 = k_s \frac{GM_L}{GM} \left| \frac{\bar{R}}{\Delta_{\oplus L}} \right|^3 \left| \frac{\bar{R}}{a_0} \right|^2 (E_2 + \delta_{2L}) + k_s \frac{GM_L}{GM} \left| \frac{\bar{R}}{\Delta_{\oplus S}} \right|^3 \left| \frac{\bar{R}}{a_0} \right|^2 (E_2 + \delta_{2S})$$

$$E_2 = -\frac{1}{2} + \frac{3}{4} \sin^2 \varepsilon_0$$

$$\delta_{2L} = +\frac{3}{4} (\sin^2 i_L - e^2_L) + \frac{9}{8} e^2_L (\sin^2 \varepsilon_0 - \sin^2 i_L)$$

$$\delta_{2S} = -\frac{3}{4} e^2_S (1 - \frac{3}{2} (\sin^2 \varepsilon_0)$$

$$\bar{R} = R_0 \left| 1 + \frac{25}{21} v^3 q - \frac{10}{7} v^2 J_2 \right|^{1/5}$$

GM_L	=	$4\,902.799 \times 10^9\ m^3s^{-2}$		(selenocentric grav. const.)
GM_S	=	$13\,271\,244.0 \times 10^{13}\ m^3s^{-2}$		
$\Delta_{\oplus L}$	=	384 400 km		(mean geocentric distance to the Moon)
$\Delta_{\oplus S}$	=	1 AU	=	1.4959787×10^{11} m
a_0	=	6 378 137 m		(scaling parameter associated with J_2)
ε_0	=	23° 26' 21''.4		obliquity of the ecliptic)
e_L	=	0.05490		(eccentricity of the orbit of the Moon)
i_L	=	5° 0'.9		(inclination of Moon's orbit to the ecliptic)
e_S	=	0.01671		(eccentricity of the heliocentric orbit of the Earth-Moon barycentre)
v	=	a_0 / R_0	=	1.00227 29
k_s	=	0.9383		(secular-fluid Love number associated with the zero-frequency second zonal tidal term)
δJ_2	=	$-\delta\, C_{20}$	=	$(3.07531 \times 10^{-8})\, k_s$ (conventional)
$\delta \bar{J}_2$	=	$-\delta\, \bar{C}_{20}$	=	$(1.37532 \times 10^{-8})\, k_s$ (fully normalised)
L	=	Lunar		
S	=	Solar		

Appendix II - Definition

Because of tidal effects on various quantities, the tide-free, zero-frequency and mean values should be distinguished as follows:

- a tide-free value is the quantity from which all tidal effects have been removed
- a zero-frequency value includes the indirect tidal distortion, but not the direct distortion
- a mean tide value includes both direct and indirect permanent tidal distortions.

Acknowledgement

E. Groten's Special Commission-3 (IAG-SC-3) Report is basically an updated version of M. Burša's IAG-SC-3-The Geodesist's Handbook (GH) report presented in 1995, with new material added.

(Groten, 2001, 2005).

4. 3D-Positioning and Mapping

The sequence of events that occurred as the Earth crust cooled is difficult to reconstruct. It is possible, however, that the Earth's surface could still have been dominated by a salty ocean: a hydrosphere.

A major technological achievement has been the navigation over large distances by using electromagnetic waves, such as radar systems, Loran-C, eLoran, Chayka of Russia, and the global navigation satellite systems (GNSS) [4.3].

Land, hydrographic, oceanographic and seismological survey systems call for varying degrees of positional accuracy that is a function of the scale of the survey and the actual requirements of the survey task. Electromagnetic and light waves can propagate both in a vacuum and in the atmosphere. The underwater domain, however, allows propagation of hydro-acoustic waves. Thus, the hydrosphere remains inaccessible to electromagnetic waves [4.4, 16.2.3] (Figure 23) (Lurton, 2002).

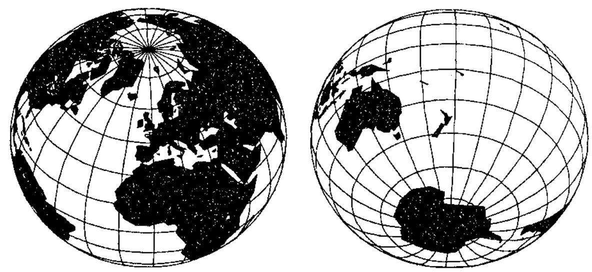


Figure 23: View on the Earthsphere and the Hydrosphere

4.1 At the Dawn of the Space Age

Actually, the theory of satellite surveying started about a century ago. The principle of rocket propulsion was well known earlier, and its possibilities used for achieving speeds sufficient to escape from the Earth's gravitational pull had been published by serious work of talented scientists:

- *Dreams of Earth and Sky* (1895)
- *Exploration of Cosmic Space by Means of Reaction Devices* (1896)
- *The Rocket into Interplanetary Space* (1923)
- *The Possibility of Reaching Celestial Bodies* (1925)
- *Ways to Space flight* (1929).

Simultaneously, a group of German and Romanian pioneers was working along the same and given the resources to develop a rocket capable of delivering a warhead hundreds of miles away. Even so, technology in the early 20th-century was a long way from the level required for rocket-powered space flight. In the 1930s, the use of star images to orient photogrammetric cameras and the corresponding triangulation of additionally photographed target points was used successfully in tracking of missiles at the Peenemünde base on the Island of Usedom in the Baltic (Hohmann, 1925; Hopmann, 1943).

4.2 Geometric and Dynamic Satellite Geodesy

The geometric or dynamic method of analysis and the data have been selected to optimise the results for a global network of reference points. The satellite methods separate nicely into two distinct types of analysis:

- geometrical or static satellite geodesy (GSG)
- dynamical satellite geodesy (DSG).

Classical Treatment of Geometrical Geodesy

In the classical treatment of geometrical geodesy, difficulties arise, as the measured quantities cannot be rigorously related to the geometric model. Physical influences are responsible for this dilemma. Reduction of baseline measurements, measurements of horizontal and vertical angles are (vitiated) to an unknown extent an effect on systematic influences, such as:

- anomalies in the gravity field and refraction
- a classical method of triangulation is forced to adopt a number of complex postulates whose geometric con-

tent is based on the assumption of homogeneity and hydrostatic equilibrium of the masses within the Earth's crust (Khan, 1967)

- an unavoidable characteristic of classical geodetic triangulation consists of the practical limitation of sight length between points on or near the surface of the Earth
- such geodetic triangulations are incapable of making intercontinental connections
- all first-order networks must be pieced together with an excessive number of individual arcs
- the disadvantage of this method arises from the fact that accuracy is impaired, especially in large number of stations involved in an extensive network, as by error propagation.

Principle of Satellite Geodesy

Geodesists have designed methods of adjustment by iteration, permitting the results of geometric adjustment operations to interact until all results become internally consistent. Although they are attractive from a theoretical standpoint, such methods have practical limitations.

For this reason a purely geometrically defined, a 3D-worldwide geodetic reference system was desired to transcend the shortcomings of the classical geodetic triangulation method. Moreover, such a global geometric solution is superior to a meagre connection of the various local geodetic Datums, which has at times been called the purpose of satellite geodesy. The geometric solution was not new. In fact, there was little room for originality in the field of the application of photogrammetry to ballistic and related problems, and the use of star photography for the calibration of photogrammetric cameras was a proven method.

Emphasised repeatedly in the recent history of geodesy, the significance of a 3D-method of triangulation became especially apparent in connection with the branch of satellite geodesy, which, because of its geometric and geophysical aspects, demanded a 3D-solution. The 3D-triangulation method was designed to determine the coordinates of nonintervisible triangulation stations, based on direction measurement.
Then again, the greatest significance of geometric satellite triangulation was the creation of a worldwide 3D-reference system. It was supported by a minimum of a priori hypotheses in particular without reference to either the direction or the magnitude of the force of gravity.

Spatial Triangulation

For the first time in the history of geodesy, selected nonintervisible points of the physical surface of the Earth can be accomplished with spatial triangulation by means of auxiliary targets elevated sufficiently high above the Earth's surface. Generation of targets, such as light signals visible over large distances, was possible by means of flares or artificial satellites. Due to the high velocity of such targets, observation of directions to targets could be made only with photogrammetric precision cameras. Absolute accuracy in addition to reproducibility of the observation conditions in this method was limited. Owing to the physical and chemical properties of the photogrammetric measurement components, adjustment of the photogrammetric measurements had to be based on a method of interpolation to obtain observational results with maximum absolute accuracy.

A suitable reference system into which an elevated target can be intercalated is a right ascension-declination system of metric astronomy. The system was attractive from the geodetic point of view because one of its axes is parallel to the Earth's axis of rotation. A large number of fixed stars were available whose coordinates were tabulated in catalogues. These control points being practically at an infinite distance, it follows that their direction coordinates are insensitive to a parallel displacement of the observer. Hence, they cannot be used for scale determination. It is necessary to determine the scale of the satellite triangulation independently by measuring the distance between two adjacent stations. Accordingly, it is necessary to carry out scale determinations in several portions of the triangulation network.

Differences between GSG and DSG

Dichotomy between GSG and DSG is based on differences, which are less significant from a mathematical point of view. The differences in methods are found almost entirely in the way the observations are expressed as a function of the unknowns:

- geometrical analysis hinges on making simultaneous observations of a satellite from two or more points on the Earth's surface. When these are camera observations, the vector connecting the two stations must lie in the plane defined by the two observed directions. A number of independent simultaneous observations will define the direction between the two stations. The Smithsonian Astrophysical Observatory (SAO) has obtained a sufficient number of simultaneous observations to determine a network for its stations. The National Ocean Survey (NOS) of the National Oceanic and Atmospheric Administration (NOAA) carried out a BC-4 camera program to establish a global geometrical network (Schmidt, 1971a)
- in practice, the orbit was determined from the same observations, but the dynamical analysis assumed that the satellite's orbit was known, and computed the location of the observing station from individual observations. Orbital mode was used by SAO to analyse the tracks on low ellipsoid orbits (LEO) satellites and by the Jet Propulsion Laboratory (JPL) to analyse the tracks on deep-space-probes.

Rocket trajectories measurement techniques had an early influence on the development of photogrammetric data acquisition and its evaluation. It was necessary to adapt the instruments to the geometry requirements:

- a great number of observations in a single photogram
- each individual observation is registered against a time standard
- great stability over extended periods of observation.
- improved accuracy of the direction determination
- arbitrarily orientation of the camera
- using objectives with long focal lengths.

Note

> *The instruments directly involved in measuring the distance, direction, or velocity of a satellite are called in this book "(satellite) tracking systems (STS)" to adhere to common usage.*

The experience gained created a demand for the development of a series of special cameras, such as the BC-4 phototheodolite system of Leica Geosystems AG (previously Wild-Heerbrugg) (Schmid, 1962):

- with different angles of view
- variation in picture sequence over a wide range
- accuracy in exposure synchronisation.

A decrease in viewing angle was inevitable because of limitations on the size of the photographic plate.

4.2.1 Geometrical Satellite Surveying Systems

In 1865, the French author Jules Verne gave a picture of space travel in his work of fiction *From the Earth to the Moon*. Nikolai Kibaltchich of Russia was held under arrest and he died in prison (1881), but his technical papers about cosmic space exploration remained unseen until the 1917s. In fact, there were many space flight writers, such as Hermann Ganswindt of Germany with his lectures about space (1890), Walter Hohmann's *Die Erreichbarkeit der Himmelskörper* (1925), Robert Esnault-Pelterie about research in the upper atmosphere (1928), and Eugen Sängers *Raketenflugtechnik* (1933).

The principle of rocket propulsion was well known earlier. In the late 17th-century, Newton formulated the laws of universal gravitation and motion. He established the basic principles governing the propulsion and orbital motion of spacecraft. Rocket propulsion and its possibilities as a means of achieving speeds sufficient to escape from the Earth's gravitational pull had been pointed out by recognised space flight pioneers: Konstantin Tsiolkovsky, Robert Goddard, and Hermann J. Oberth.

Tsiolkovsky was the first scientist to conceive rationally space flight notes in his book *Gryozy o Zemle i Nebe; Dreams of Earth and Sky* (1895), and his most serious work on astronautics *Exploration of Cosmic Space by Means of Reaction Devices* (1896). Nevertheless, technology in the early 20th-century was a long way from the level required for rocket-powered spaceflight.

Note

Tsiolkovsky pointed out the possibilities of rocket flight in the 1890s, but he too received little support and recognition for several decades. After 1917, he was increasingly recognised with Friedrich A. Tsander and M.K. Tikhonravov by the 1930s. Until 1922, Oberth was unfamiliar with the work of Goddard in the United States and, until 1925, with that of Tsiolkovsky. Oberth published Die Rakete zu den Planetenräumen (Rockets into Interplanetary Space), 1923 and Wege zur Raumschiffahrt (Ways to Space flight), 1929.

In 1926, Robert Goddard built experimental liquid-fuelled rockets. The *"Verein für Raumschiffahrt"* (VfR or *German Space Flight Society)* was formed in 1927. Hence, the theory and dynamics of such flights were rigorously studied.

Hermann J. Oberth was primary a theoretician, but he was the only great pioneer who survived to see the first artificial satellite launched in 1957. After launching into flight by a brief burst of power, the technical problem of German and Romanian pioneers was achieving maximum range of ballistic missiles (rocket-propelled weapons). As the rocket used up its fuel, its weight and velocity would change (Noordung (Potočnik) 1929, 1995).

Techniques and tools, such as accelerometers and radio Doppler systems, used in the measurement of rocket trajectories had an early influence on the development of photogrammetric instrumentation and data acquisition. In the 1940s, instrumentation evaluation reflected these concepts and reached a high point with the Askania phototheodolite, used at the Peenemünde base on the Island of Usedom in the Baltic for measuring the V-2 trajectory up to the point of engine cut-off.

Development of Space Flight

Using the method for measuring the *Vengeance Weapon* (V-2) trajectory up to the point of an engine cut-off, Wernher von Braun and his team found the principle of rocket propulsion. The theory and dynamics of such flights were rigorously studied. By the end of World War II, the German development of rocket propulsion for military aircraft and guided missiles had reached a high level. Von Braun solved the problems of propulsion and guidance that have continued ever since to shape ballistic missile development. It marked the beginning of the 3D-space age.

Britain, France, Russia, and the United States continued the technical programs of rocket power developed by the Germans. The groundwork for the era of geometrical satellite geodesy was formulated greatly in the work of Dirk Brouwer, Yrjö Väisälä, and other geodesists. Exact elements of exterior orientation were obtained with the use of elements from classical theodolites.

The computer, and especially the digital computer, was a crucial piece of equipment, the theory of which was envisioned by Charles Babbage in the 1830s. The essence of this machine is the use of the binary system. In 1935, the first digital computer was at work in Germany. Using a swift computer for orbit computations, Dirk Brouwer laid the basis of dynamic satellite geodesy (DSG) in his papers (Brouwer, 1937, 1946, 1958, 1959, 1961a; b, 1963). It foreshadowed a new world of technology. Nonetheless, the results of great geodetic importance did not start to come in until the International Geophysical Year 1957-1958 (IGY) [2.1].

Observe

Satellite geodesy is carried out in two different kinds of coordinate systems:

- *the system defined by reference to fixed points on the Earth*
- *the system defined by fixed points (stars) in the celestial sphere.*

As the principal objective of Smithsonian Astrophysical Observatory - Sao participation in the International Geophysical Year 1957-1958 (IGY), Fred L. Whipple (1952) and his colleagues conceived the fundamental program to observe positions of artificial satellites, and to derive geophysical information from these observations. The objectives of SAO activities were:

- to tie together observing stations and the centre of the geoid
- to learn about the density distribution in the crustal volumes of the Earth
- to provide the value of the atmospheric density a few kilometres above the initial perigee distance
- to predict any cyclic effects that may occur in the earth's high atmosphere.

The first two objectives evolved into more demanding ones for subsequent programs, such as the National Geodetic Satellite Program (NGSP) [19.6] (Henriksen, 1977a, b).

In 1955, President Eisenhower announced that the United States would launch a scientific satellite during the International Geophysical Year 1957-1958 (IGY). Consequently, results of great geodetic importance came in the IGY, when Explorer-One attained its orbit on January 30, 1958. The SAO observation network and analytical devices were all set with partial operational status.

Historically, IGY is considered as the dividing line between conventional 2D-geodesy and Euclidean 3D-geodesy in space-time. New geodetic information was added by the Army Map Service (AMS, now NGA) in 1958, at the time when artificial Earth satellites - the Sputniks by Russia (formerly USSR), the Explorer and Vanguards by the USA - had been launched in the schedules of the IGY.

The Event of the Twentieth Century

On October 4, 1957, on the Sands of Baikonur (Kazakhstan), through planning and perseverance, the Soviets achieved the milestone of launching the first artificial satellite Sputnik Zemli-I into space flight. The rocket found an even more constructive significance in the US and Russian space programs to provide launch vehicles for satellites and lunar probes (Guier, 1958; King-Hele, 1958).

Between the International Geophysical Year 1957-58 and 1977, the idea of applying the photogrammetric technique for geometric satellite triangulations to the establishment of a continental net was seriously considered. Satellite technology brought improvements through optical and Doppler satellite positioning methods. A worldwide geodetic control network was achieved by corresponding efforts of particular scientific geodetic agencies, such as Centre National d' Études Spatiales (CNES), National Aeronautics and Space Administration (NASA), National Geospatial Intelligence Agency (NGA, formerly NIMA, and DMA), and Ohio State University (OSU).

Far after the launching of the artificial satellites by the Soviet Union and by the United States, the artificial satellites were introduced as a tool to replace all local geodetic networks by a global satellite network. The period after the IGY saw the growth of geodetic satellite networks, the start of work on artificial satellites for use as a geodetic tool and an increasing amount of satellite-tracking data that could be used for determining a more precise shape of the Figure of Earth.

From a mathematical point of view, the significance of an artificial satellite for geodesy consists of the ability to express the position curve of the orbit satellite in terms of function of certain parameters, which give in turn information concerning geometric and geophysical properties of the Earth, and surrounding space. The geophysical forces affecting the orbit were treated as perturbation sources. Hence, orbits of satellites could be determined accurately due to the theory of third zonal harmonics by O'Keefe, Eckels, and Squires, (1959), the theory of motion of artificial satellite by Dirk Brouwer, and an improved theory for satellite orbits (Brouwer, 1959; Kaula, 1962).

Post-Sputnik Era of Geodesy

In 1957 for all practical purposes the geodetic situation was therefore still one in which Datums were local or continental in scope. Satellite technology brought improvements through optical and Doppler satellite positioning methods. A worldwide geodetic control network was achieved by corresponding efforts of scientific geodetic agencies. The US government had provided funds for independent networks of electronic and optical satellite-tracking stations (STS). Another system favoured the use of electronic distance measuring (EDM) equipment to track a reflecting satellite (Henriksen, 1977a, b).

If the orbits of satellites could be determined so accurately, then distance measurements at stations from any other known position could be used to fix unknown positions on the Earth's surface. Frank Telford McClure found that the roles could be inverted. Discussions have led to the insight that the reciprocal value of ellipticity - obtained only a year or so after the launching of the first artificial Earth's satellite (Sputnik Zemli-I) - was fixed at 298.3 by O'Keefe, Marchant, Herz, Buchar, Desmond King-Hele, and the Army Map Service (AMS, now NGA). This is practically the same as had been put forward by Friedrich R. Helmert in 1906 and Fedodosiy N. Krassovsky in 1940. The centre of the Reference Ellipsoid is made to coincide with the geoid (the Earth's gravitational centre) (O'Keefe, 1958: Buchar, 1958; Fischer, 1959; King-Hele, 1958). It was concluded (Airy, 1830):

> *" ... the surveys of the Earth, the observations of pendulums, and the Lunar inequalities, agree in showing that the Earth's form does not differ much from that of an ellipsoid of revolution whose reciprocal ellipticity is greater than 300, whose semi major-axis is about 20 923 700 English feet ... .".*

Worldwide Geodetic Control Network

In the 1960s, the idea of applying static satellite geodesy (GSG) to the establishment of a global geodetic control network (GGCN) was considered by photographing large balloon satellites, such as Echo I, II, passive geodetic Earth orbiting satellite (PAGEOS) against the background of stars by astrophysic- and ballistic-cameras. The brainwave of applying the photogrammetric technique for geometric satellite triangulation to the establishment of a continental net started. In essence, the aim of the reductions was to refer all the photographically registered directions to stars to a consistent stationary coordinated network system.

The task of providing a GGCN was achieved by corresponding efforts of geodetic agencies, such as Aeronautical Chart and Information Centre (ACIC), Applied Physics Laboratory (APL), Jet Propulsion Laboratory (JPL), NGS (formerly part of C&GS), Centre National d'Études Spatiales (CNES), National Aeronautics and Space Administration (NASA), NASA/Goddard Space Flight Centre (GSFC), National Geospatial Intelligence Agency (NGA), formerly DMA, Topographic Centre (NGA/TC). NGA/Aerospace Centre (NGA/AC), Naval Research Laboratories (NRL), Naval Surface Weapons Centre (NSWL), Ohio State University (OSU), Smithsonian Astrophysical Observatory (SAO) (Buchar, 1962; Kaula, 1962; Schmid, 1966; Williams, 1966).

If the direction of the Earth's instantaneous axis of rotation remains invariant in space, the computations produce a rigorous geometric solution. However, the Poles describe more or less irregular loops in a period of approximately 430 days about a mean position. It is immaterial whether this so-called polar motion [2.2.2] is treated as Precession and Nutation relative to the astronomic reference or whether the direction of the rotation axis is accepted as a displacement of the crust [2.1]. Additionally, the influence of polar motion is coupled with the problem of Time determination [1.2].

Time correlation in star observations is of a purely geometric nature, because the spatial position of the Earth with its observation stations changes with time, relative to the celestial reference system (CRS) [1.6]. The measurement of time serves to refer the spatial orientation of the Earth at the instant of observation back to an orientation assumed as a normal position and corresponding at a specified epoch.

The significance of time determination for the GSG is twofold:

- determining the instant of the photographic exposure of the star image within an interval based on astronomic observations
- the instants of observing the pass of the satellite in motion at all stations, correlated with respect to an otherwise arbitrary measuring frequency, which amounted to a relative time determination.

Registering the pass of a satellite whose track is marked by short-duration light flashes, observed from different stations and at different times, will produce uniquely defined points on the orbit in space. The origins of the flash fulfil automatically the geometric condition of simultaneity. The US government provided funds for two independent networks of satellite-tracking stations (STS), the optical STS and the opto-electronic STS.

Optical Satellite Tracking Cameras

Askania phototheodolites had a focal length of 370 mm, aperture f / 5.5, 13×18-cm plate format, and a synchronous drive for the rotary shutters, producing 1.5, 3, 6, and 12 exposures per second with a synchronisation accuracy of 10^{-3} sec. The horizontal and vertical circles could be set to within 3^{cc}. In addition, a louver shutter was available to block out certain exposures in the sequence or to generate time-related star trails (Lacman, 1950).

Another camera was the Baker-Nunn (SAO) named after its optical and mechanical designers, James G. Baker, and Joseph Nunn, respectively. Their intention was to get a wide-angle camera that could be constructed easily. By using a short-focal-length reflector as the principal element, the "*super-Schmidt*" camera obtained a wide angle and large aperture. This camera then became the starting point for design of cameras for satellite photography (Schmidt, 1932). Other cameras have followed the same line of development:

- AFU-75 (Russia)
- Antares (France)
- GSFC MOTS
- Hewitt (UK)
- Maksutov camera (Russia; Schmidt camera derivative)
- MOTS-40
- NAFA (Russia)
- PC-1000
- SBG Zeiss Jena
- Wild BC-4 series (USA)

All data from camera-type STS in the GSG were reduced by using stellar positions taken from Fundamental Katalog 4 (FK4) used by European STS or the Smithsonian Astrophysical Observatory (SAO) star catalogue. In spite of this, dynamical satellite geodesy (DSG) was another concept.

4.2.2 Dynamical Satellite Surveying Systems

In the 1960s, the TRANSIT series of satellites, operationally based on observation of the Doppler shift of the satellite's transmitted signal, were launched enabling US Navy submarines to fix their position accurately under all surface weather conditions. By 1967, the Transit Doppler satellite tracking system was revealed for use by the civil (offshore) industry. The more accurate Global Positioning System (GPS) or Navigation Satellite Timing and Ranging (NAVSTAR) replaced the Transit navigation system in 1996. Determinations of the Earth's figure from the time of Geodetic Reference System of 1967 (GRS67) onwards are actually made in the earth-centred, earth-fixed (ECEF) geocentric coordinate system, and based upon the theory of an equipotential ellipsoid. By 1973, the BC-4 data source - actually called the BC-4 triangulation - was replaced by an even more accurate survey, based on the methods of 3D-dynamical satellite geodesy (DSG) tracking systems (Bomford, 1977). Consequently, the trend was to describe new figures initially in geophysical terms (Arnold, 1970; Maling, 1973, 1992; Richardus, 1972; Seeber, 1993).

Note

> *Since 1997, NGA stopped considering spheroids and ellipsoids as equivalent due to the fact that a spheroid is very complex surface while an ellipsoid is a simple two-degree surface.*

At that moment, it was only possible to perceive dimly the scope and possibilities of space technology. It has brought the world to an instant at which the civilisation stands on the doorstep of extraterrestrial exploration.

USAs National Geodetic Satellite Program

Although the growth of satellites-in-space-time knowledge was thrilling, there was little evidence of it has having direction ever since. Geodesists began to worry about the consequences. In 1971, a US-committee devised the US National Geodetic Satellite Program (NGSP) [19.6] to help the NASA (National Aeronautics and Space Administration) to plan the future. The essence of the committee's recommendations reflected the desire to see the program's results not only presented but also widely used.

Søren (Henriksen, 1977a, b):

> " ... *As every major science, geodesy has its development marked by epochs during which the creation of new ideas and the influx of new data increase beyond the ability of the science to absorb them. The science loses for a time its sense of direction and purpose. Spectacular results were often inconsistent and appear unrelated to each other and to the science as a whole* .... ".

While each geodesist revelled in the abundance of data and rejoiced in his ability to mould them into new results, the ultimate users of geodetic information had no reliable means of choosing among the results or judging their suitability for anyone application. And of course, there was absolutely no way of evaluating the value of the results as a function of the cost of the NGSP.

Global Satellite Tracking Control Network

Using radio frequencies, Doppler observations were made from satellite tracking stations (STS) towards the laser satellites Diadème I (DI-C), Diadème II (DI-D), Explorer 22, and the laser flashlight satellites such as Anna 1B, AJISAI, Etalon, GEOS A, GEOS B, and Stella. Another system favoured the use of EDM equipment, such as C-Band Radar, CNES (laser), Goddard range and range rate (GRARR) system, GSFC DME (laser), MINITRACK (interferometer), to track a comparatively small reflecting satellite (Henriksen, 1977a, b). The distribution locations of STS with respect to one another were known partly by connections through classical triangulation networks [3.2] and partly by connections through the orbits of artificial satellites as computed from the tracking data of these STS.

Since SAO satellites would be detectable only during twilight hours at any particular station, these STS were uniformly spaced around the globe longitudinally. This was exemplified by sequential collation of ranges (SECOR) a type of microwave EDM system, which was used to establish a global equatorial control network around the world.

Very Long Baseline Interferometry

Radio telescopes are used to study naturally occurring radio emissions from stars, galaxies, quasars, and other astronomical objects between wavelengths of about 30 MHz and 300 GHz. Essentially, the radio telescopes have two basic components:

- a large radio antenna
- a radio receiver.

The sensitivity of a radio telescope depends on the receiver bandwidth, the surface area and efficiency of the antenna, the sensitivity of the radio receiver used to amplify and detect the signals, and the duration of the observation. In an antenna, radiation is accepted from a very narrow beam in the sky; the width of the beam depends on the ratio of the wavelength of operation to the diameter of the antenna (Pellinen, 1978).

Signal-Processing by Radio Telescopes

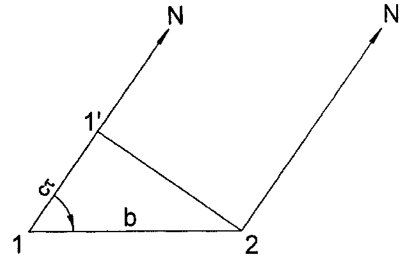


Figure 24: Principle of VLBI with a short base

The high angular resolution of radio telescopes is achieved by using the principles of interferometry. In the simplest form, a receiver is placed directly at the focal point of the parabolic reflector of the radio telescope, and the detected signal is carried by cable to a point where it can be recorded. All tasks as signal processing and analysis are usually executed by a digital vector-computer to obtain images from the interferometry data. The laborious computational task of doing Fourier transforms to translate the raw data into a form useful for analyses is achieved with so-called FFT (fast Fourier transform) techniques. This requires an application of algorithms specially suited for computing discrete Fourier transforms.

Principles of Very Long Baseline (Radio) Interferometry Operation

In 1965, the primary conditions of the very long baseline (radio) interferometry (VLBI) procedures were envisioned by the Russian radio astronomers Matveenko, Kardašev and Šolomickij. Using the geodynamical techniques of VLBI can form interferometry systems of essentially unlimited element separation (Felli, 1989; Pellinen, 1978).

In a VLBI system, the signals received at individual Antennae 1 and 2 are sampled, digitised, and recorded by broad-bandwidth videodisk (or videotape) recorder. The intention is to compare received signals from each individual antenna (Figure 24, Figure 25). Hence, it is correlated with the signals from other antennae at a joint laboratory.

b	=	length of VLBI-base 1-2
c	=	velocity of light in vacuo [12.2]
τ	=	time delay
$c\tau$	=	difference of signals in time
β	=	angle of base
$\cos\beta$	=	$c\tau / b$
t_0	=	time correlation shift

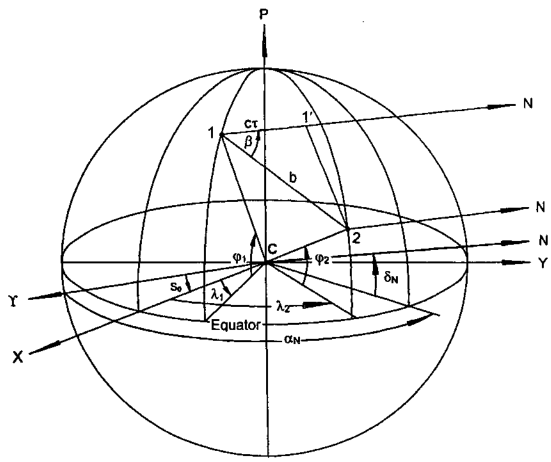


Figure 25: Principle of VLBI with a line base

For that reason, the videotapes of a VLBI station are replayed at a shared laboratory for comparison. The (almost) identical recorded signals $Z_1(T)$ and $Z_2(T)$ are compared, show the time shift (= t_0) and the interference borders (Figure 26) (Pellinen, 1978).

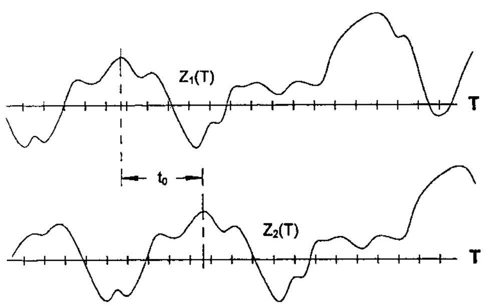


Figure 26: Time-shift of signals

The successful operation of a VLBI system requires that distinguishable signals are received and recorded, the disk or tape recordings are synchronised within a few millionths of a second records, and then sent to a central computer, which performs the computation of the correlations. The local oscillator reference signal uses hydrogen-maser-frequency standards to give the required frequency stability and timing accuracy. For sensitive VLBI applications, magnetic videodisk (or videotape) recorders provide sufficient performance, and are capable of recording for several hours to obtain high-resolution data. The costs of VLBI devices have allowed many radio telescopes to be used simultaneously (Emardson, 1999; Pellinen, 1978; LaRousse, 1999; Kovalevsky, 2002).

4.3 Global Navigation Satellite System

The generic name given to worldwide navigation satellite systems is global navigation satellite systems (GNSS). A full treatment of the GNSS developments is beyond the scope of this book. The present chapter does not aim at presenting details of signal transmission and reception, and the theory of signal processing will not be presented here either, as these techniques are based on fast developing digital technologies. To take full advantage of the information potential of GNSS, the reader may find an introduction to technical details in (Grewal, 2001; Misra, 2001; Tsui, 2004)

Historical GNSS Developments in Brief

- since Apr. 13, 1960, an US DoD Navy navigation satellite system (Transit 1B with the two stages Thor-Able-Star rocket) has been developed, based on the systems of the WGS60, WGS66, WGS72 and WGS84 ellipsoids
- since Feb. 21, 1978, a NAVSTAR/GPS (US DoD global positioning satellite system) has been developed, based on the World Geodetic System of 1984 (WGS84) ellipsoid
- since Oct. 10, 1982, a GLONASS (Russian global orbiting navigation satellite system or Cosmos 1413-1415) has been developed in parallel to GPS. GLONASS was initially based on the Pulkovo system 1942 of survey coordinates (Krassovsky), the WGS84 ellipsoid, and the PZ90 ellipsoid.
- GALILEO is Europe's initiative to the development of an independent worldwide satellite navigation system, providing a precise, worldwide position determination service under civilian control [4.3.5].

This chapter discusses the basic concepts of the global navigation system developments:

- TRANSIT USA between 1959 and 1996.

Available global navigation satellite systems (GNSS) for user position and the foundation of contemporary mapmaking are:

- GPS USA from 1996
- GLONASS Russia from 1995
- GALILEO EU between 2007 and 2010.

4.3.1 TRANSIT System

TRANSIT or navy navigation satellite system (NNSS) Doppler satellite system traces its origin back to 1957. The initial idea was due to G.C. Weiffenbach and W.H. Guier at the Applied Physics Laboratory (APL) of John Hopkins University. They made measurements of the Doppler shift exhibited on signals received from the artificial satellites, such as the Sputniks, Explorers and Vanguards, showing that satellite orbits could be deduced from them to a reasonable degree of accuracy.

By 1959, the first experimental satellite, TRANSIT 1A, had been designed and constructed. NNSS became operational in 1964. It was commonly named the TRANSIT system (Eschelbach, 1979; Newton, 1962).

Determinations of the Earth's figure from the time of Geodetic Reference System of 1967 (GRS67) onwards are made in the earth-centred, earth-fixed (ECEF) geocentric 3D-coordinate system [3.5].

TRANET Network

In the early 1960s, the US Navy established the Doppler satellite tracking system (STS) network TRANET. It was a crucial part of the TRANSIT development effort. The US Navy, however, intended primarily the TRANSIT system for military use. TRANET observations of the Doppler shift were applied to other problems: determination of geocentric coordinates of Earth-fixed points, establishment of ties between the several continental Datums, determination of crustal tides, and refinements to the gravitational model.

The main refraction error is due to ionospheric refraction and tropospheric propagation effects. Signals' passing through the ionosphere stretches the wavelength in the received signal, causing greater apparent orbit curvature. A characteristic of a dispersive medium as the ionosphere is that the wavelength stretch is inversely proportional to the square of transmitted frequency. It was discovered by Guier and Weiffenbach that a refraction correction could be obtained by combining the measurements made at two different frequencies, and when satellites transmit both signals. In 1967, TRANSIT was released to civilian users (Guier, 1958; McClure, 1965).

Using the Translocation Solution

Using the TRANSIT system in the 1970s, a translocation solution consisted of the difference between a users station and another coordinated station. The station-point position solutions were done under special circumstances. Using more than 20 passes but fewer than 35 passes resulted in an exact solution (Arnold, 1970; Bomford, 1977; Maling, 1992; Schwarz, 1989b; Seeber, 1993).

Satellite tracking had yielded much information about the gravity potential of the Earth, and led eventually to mapping of the geoid throughout the world. Consequently, the development was to describe new figures initially in geophysical terms.

4.3.2 GPS System

The GPS is part of a satellite-based navigation system developed by the US DoD under its NAVSTAR satellite program.

In recent years, the GNSS (global navigation satellite system), such as the global positioning system (GPS) of the USA, or global orbiting navigation satellite system (GLONASS) of Russia, have become indispensable as a tool for Inertial Navigation System (INS). In principle, both systems rely on a task force of 24 (excl. about six spare satellites) that can be accessed globally around the clock in conjunction with geodetic receivers on Earth. They provide ability of passive global position fixing for strategic and tactical military forces with accuracy equivalent to or better than conventional geodetic surveys whatever weather conditions (Figure 27) (Torge, 1988).

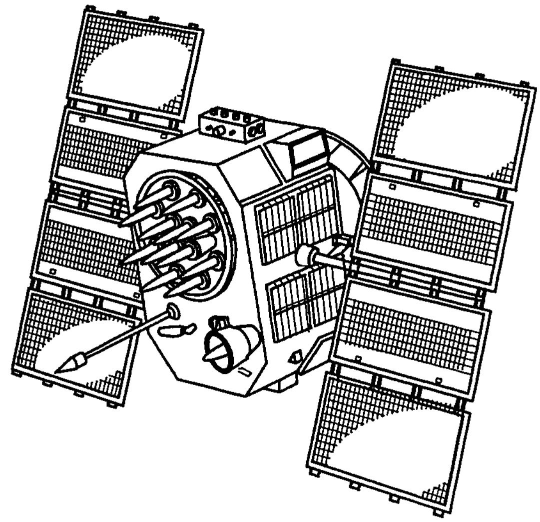


Figure 27: One of the first GPS-satellites

GPS and Inertial Navigation

A key function performed by the Kalman filter is the statistical combination of GPS and INS information to track drifting parameters of the sensors in the INS. As a result, the INS can provide enhanced inertial navigation accuracy during periods when GPS-signals may be lost, and the improved position and velocity estimates from the INS can then be used to make GPS-signal reacquisition happen much faster when the GPS-signal becomes available again. Uses of partial results are not ordinarily accessible (Bozic, 1979; Grewal, 2001).

The DoD has built in the capability to control the accuracy of the Standard Positioning Service (SPS) signals available to civilian and military users by a combination of dithering the satellite clock and manipulating the ephemeris data. SPS provides GPS single-receiver (stand-alone) positioning service to any user on a continuous, worldwide basis. SPS is intended to provide access only to the C/A-code and the L_1 carrier. A great deal of information about GPS was published on the techniques used to provide such precise positioning (DoD, 1995; Grewal, 2001; Parkinson, 1996).

Benefits and Objectives of GPS

An objective of the DoD was to provide continuously an unlimited number of users accurate Time, and worldwide positioning. Additionally, US and allied military forces require a system useable on dynamic platforms, with signal resistance to jamming and interference. The view was that civilian users of GPS would be provided with accuracy consistent with the US national security considerations:

- standard positioning service (SPS) for civilian users (SPS, 1995)
- precise positioning service (PPS) for the DoD-authorised users.

PPS is the full-accuracy, single-receiver GPS positioning service provided to the United States and its allied military organisations and other selected agencies. This service includes access to the unencrypted P-code and the removal of any SA effects. Non-DoD authorised users have restricted access to the full capability of PPS due to the cryptographic features such as Anti-Spoofing (AS).

Function	United States GPS
operational since	March 27, 1996
S/C number in the fully deployed system	24 plus max. 6 spare satellites standby
number of orbit planes	6
circular orbits with four or more satellites	4
orbit inclination relative to the Equator	55°
orbits separation	by multiples of 60° Right Ascension
orbit altitude	20 180 km
revolution period orbit	approx. 12 hours
in each orbit	four primary satellite slots distributed
orbital Parameters updated	every hour
ephemeris data representation	Kepler-like elements of S/C orbit parameters and extrapolating coefficients
geodetic coordinate system	WGS84 ECEF Frame
phase-lock ranging signals	to GPS synchroniser
method of S/C signal division	code division multiplexing (CDMA)
almanac content	152 bit
duration Almanac transmitting time	12.5 min
frequencies in the L_1 range	1575.42 ±1.0 MHz
frequencies in the L_2 range	1227.6 MHz
number of code elements	1023
c/a-code frequency	1.023 MHz chipping rate
P-code frequency	10.23 MHz chipping rate
cross talk level on two adjacent channels	- 21.6 dB
synchro code repetition period	6 s
bit number in synchronic code	8
type of ranging code	C/A and P(Y) Codes
availability	SA, removed since May 1, 2000
time synchronisation	GPS-time with UTC (USNO)
tracking network	worldwide
ground track repeat period	1 sidereal day

Table 11: Original GPS characteristics

Pratap Misra of the Lincoln Laboratory MIT (Misra, 2001):

> *"... the main advantage of PPS over SPS is robustness: higher resistance to jamming. PPS also offers improved positioning performance due to dual-frequency measurements which allow compensation for ionospheric propagation effects faster codes which lead to higher precision of range measurements and lower error due to multipath ... ."*

DoD introduced SPS controlled degraded signals called SA (Selective Availability) to reduce precision, but DoD-authorised users could remove such cryptographic features. In 1985, DoD announced the policy for civilian use of GPS to the ICAO (International Civil Aviation Organisation) to make GPS-SPS available for the foreseeable future on a continuous, worldwide basis, and free of charge (Grewal, 2001; Misra, 2001; Tsui, 2004).

Selective Availability

> *Selective availability (SA) is a combination of methods used by the US Department of Defense for deliberately derating the accuracy of GPS for non-US military users. In accordance with the US*

Presidential Order SA was deactivated on May 1, 2000.

GPS Architecture

(GPS ICD, 2000) is the interface between the GPS space segment, control segment and user segment of the NavStar GPS program.

GPS I space segment, including the satellites
GPS II control segment dealing with the operational use of satellites
GPS III user segment covering civilian and military user receiving equipment.

GPS I Space Segment

The baseline constellation comprises:

- stationary ground tracks
- in each orbit are four primary satellite slots distributed
- satellites are identified by an alphanumeric code
- a spare satellite slot is available in each orbital plane
- supported is the constellation of maximum thirty satellites.

US Department of Defense's global positioning system (GPS) refers to the World Geodetic System of 1984 (WGS84) ellipsoid. In 1978, the feasibility of GPS was demonstrated by the first GPS-satellite prototype. In April 1995, DoD declared the global positioning system operational.

The turning point came as the GPS sparked a Klondikian rush for various navigational solutions that attracted everyone from the inventive genius to the best minds of the age. In position fixing mode, broadcasted ranging signals and navigation data allows the GPS-receiver units to measure pseudoranges of satellites in passive mode.

GPS navigation signals include navigational information on the ephemeris of the satellite, an almanac, corrections for ionospheric signal propagation delays, an offset time between satellite clock time, and true GPS time at a rate of 50 baud. It is possible that most GPS-receiver units with a clear view of the sky would see up to ten GPS-satellites in view, but an official DoD policy concerning a GPS constellation status and scheduled outage was never announced. Hence, the number of working GPS-satellites can vary due to failure and maintenance. So far, GPS-satellites have exceeded their design life significantly.

ITRF97 coordinates in latitude, longitude, ellipsoid height, and velocities were obtained from the globally distributed set of International GNSS Geodynamic Service (IGS) stations. WGS84 precise orbits and satellite clock corrections were held fixed during the computations. The station velocities were used to propagate the ITRF97 coordinates from epoch 1993.0 to epoch 1997.0.

GPS II Control Segment

Master control station (MCS) Schriever AFB (Air Force Base), at Colorado Springs in Colorado, USA, sends data to each satellite. Ephemeris data consist of satellite positions. Almanac data consist of the projected orbits of each satellite and information about their overall status. It is the centre of the control segment - also referred to as the operational control system (OCS). In brief, functions of the OCS are:

- checking satellite orbits
- controlling manoeuvres of satellites to maintain orbit
- examining satellite health
- maintaining GPST
- predicting satellite ephemeris data and clock corrections
- satellite relocations to prevent failures
- updating navigation messages.

Satellite signals are tracked continuously from US Air Force (USAF) and National Geospatial Intelligence

Agency (NGA) monitor stations distributed worldwide. Clock corrections are transmitted if necessary. In turn, the satellites routinely send signals to five monitor stations so that their positions can be tracked. The estimation and prediction of satellite orbits and clock biases are based on data from the monitor stations. These stations also downlink the Almanac information sent to the satellites from MCS. GPS-receiver-units with atomic clocks and meteo-devices at the monitor stations are operated remotely from the MCS, uplink commands and uploads data to the MSC to restore the navigation messages. In addition, measurements are post-processed from the OCS to compute precise ephemerides and clock parameters. The code division multiple access (CDMA) signal is a spread-spectrum system in which all signals are modulated by a set of orthogonal codes, using the same centre frequency (Grewal, 2001; Misra, 2001; Tsui, 2004).

Although the fact is often overlooked, satellites are an indispensable part of any satellite tracking system (STS). Not only is the STS inoperative without the satellite, but also the measurements made by the STS depend in type and value and in precision and accuracy on the satellite. Exploring gravitational models, adding more stations to the STS network resulted in an improved version of the WGS72, GRS80, and later the WGS84 Reference Frame.

GPS III User Segment

In GPS-user segment receivers, digital signal processing is used to track the GPS-signal, determine pseudoranges, Doppler measurements, and demodulate the data stream. For this purpose, the signal is sampled and digitised by an analogue-to-digital-converter (ADC). In most budget GPS-receivers, the final intermediate frequency (IF) signal is sampled, but in geodetic GPS-receivers, the final IF signal is converted down to an analogue baseband-signal prior to sampling (Grewal, 2001).

Hence, GPS-user segment receivers became an essential element of the military and civilian infrastructure. There are various military and civilian GPS user segment receivers. Civilian budget GPS-receivers are low-cost instruments with some advantages:

- automatic gain control (AGC) is not indispensable
- its performance is insensitive to small variations in voltage levels
- severe signal distortion is averaged, resulting in a linear signal component
- using digitised samples with a simple method of ADC.

However, civilian budget GPS-receivers have also some disadvantages (Grewal, 2001):

- no possibility of adaptation (hardwired) to changing circumstances
- perform a capture effect in the interfering signals
- signals are vulnerable to jamming.

Geodetical civilian GPS-receivers are expensive instruments with many advantages:

- eliminates the signal-to-noise-ratio (SNR) degradation
- extend the dynamic range
- high-speed real-time baseband-signal-algorithms are in firmware, are controlled by software to track the GPS-signal, extract navigation data, and measures code, pseudoranges, and Doppler
- improves performance in the presence of jamming signals
- jamming signals are less likely to saturate the analogue-to-digital converter (ADC)
- processing to elevate the signal above the noise will be discussed subsequently
- receiver has automatic-gain-control (AGC) to keep the ADC input level constant.

Receiving Signals

Now, all GPS-satellites transmit two carrier signals in the UHF (ultra-high-frequency) L-band denoted by L_1 and L_2. Two signals are transmitted on L_1, one for civilian users, and the other for DoD-authorised users. L_1 carries position and time data for civilian users. A second, lower-frequency signal L_2 can be used to correct for the effects of the ionosphere for more precise positioning by authorised users, including allied military, government agencies, and approved civilian users.

Antennae, RF chain, and ADC are hardwired used in GPS-receivers without a possibility of adaptation to changing circumstances. Assigned to each satellite is a unique ranging code, a binary pseudo-random noise (PRN) sequence, pseudo-random code (PRC) or pseudo-random number (PRN) a binary sequence with random-noise like properties. It is a very long, complicated but repeatable pattern of binary data. It allows precise range measurements and mitigates multipath effects of scattered, reflected or interfering signals received by GPS-antennae.

As shown in [1.3], GPS-time (GPST) is the Time-scale to which GPS-signals are referenced. GPST is defined by a set of atomic clocks - based on caesium (^{133}Cs), or rubidium (^{87}Rb) - in satellites and monitor stations. Estimating the bias, such as Time offset, drifts, and drift rate relative to GPST, accomplish synchronisation of the satellite clocks.

Basic Perception of Positioning

A GPS-receiver unit determines the arrangement of the satellite constellation, searches for the positions of the satellites and GPST by means of radio signals transmission by the GPS-satellites. Then, the GPS-receiver unit fixes its position in φ, λ, and h, which are: latitude, longitude, and altitude, respectively on or above the Earth's surface.

Because of the advance of integrated circuit technology, the GPS-receivers are getting smaller, so it is desirable to have a small size antenna. The GPS-antenna pattern also contributes to the power level of the receiver with a higher gain in the zenith direction. This includes the ability of attenuating multipath but it loses gain to signals from lower elevation angles. Various types of GPS-receiver antennae are used, such as a choke-ring, dipole, helix, micro-strip or patch, monopole, spiral-helix antennae. The minimum required beam width of the transmitting antenna to cover the Earth is 13°.87, but the beam width of the antenna is 7° wider than needed to cover the Earth. An antenna with two narrow bands L_1 and L_2 can also avoid interference from the signals in between the two bands. It should also minimize multipath effects (Tsui, 2004).

Multipath Effects

> *Since the 1940s, the multipath effect is an acoustic or electronic distance measuring signal reflection from some objects that reach the antenna indirectly. It is not easy to suppress multipath in any system because it can come from any direction. Thus, the multipath requirement complicates the antenna design. Multipath effects can cause severe errors in the user position computation.*

For precise GPS measurements, the calibration of the phase centre variations (PCV) of all satellite- and receiver-antennae cannot be neglected. Hence, important error sources are the effects of PCV, in the mm to cm range (Tsui, 2004).

GPS performance requirements are:

- applicable to real-time positioning or navigation for dynamic users
- no allowance for jamming, intentional or unintentional interference
- using a short period to fix a user's position after switching on.

GPS Receiver Units

Transit time of the signal gives the distance between the electronic centres of the GPS-satellite-antenna and the GPS-receiver-antennae:

- the GPS-receiver unit receives signals transmitted from the GPS-satellites. These signals include the position of the satellite and GPST at which the signal was sent
- the radio frequency (RF) chain amplifies the input signal
- amplitude and frequency are converted to an output signal

- an analogue-to-digital converter (ADC) digitises the output signal
- the signal is processed, and the signal of a specific satellite is found by acquisition
- a tracking program obtains the phase-transition of the navigation data, sub frames and pseudoranges
- ephemeris data are used to obtain the satellite positions
- the user position is calculated for the satellite positions from the pseudoranges.

A GPS electronic receiver unit measures simultaneously the transit time between the ground station and four to ten GPS-satellites. To determine the user position in a 3D-situation, at least four satellites and distances from the user receiver to the satellites are required. When more than four satellites are available, a LS (least squares) calculation shows the quality of the user position fix (Dalton, 1953; Farrell, 1999).

Position Fixing and Accuracy in ECEF Coordinate System

In a GPS operation, the positions of the satellites are known from the ephemeris data transmitted by the satellite with a GPST reference:

- each satellite sends a signal at the true time of transmission from satellite t_{si}
- arrival of a signal at an unknown position (r_i) of a receiver at a receiver time t_{ri}.

By measuring the time difference ($t_{ri} - t_{si}$) of the signal travelling from the satellites to the receiver, the required pseudoranges can be calculated, remembering (Higgins, 1999):

- distances must be measured simultaneously
- all distances, so-called pseudoranges, have an unknown bias (Figure 28).

Initialisation acquisition. Non-linear simultaneous equations solve the user position in a 3D-Cartesian coordinate frame through a linearisation and an iteration method.
In brief:

- initial values are set to zero
- intersection of the divergent rays is established
- the spatial coordinates of the users station choose a position x_i, y_i, z_i, and user clock bias b_i to represent the initial conditions calculate new values using the measured values of a position x_{ui}, y_{ui}, z_{ui}, and a user clock bias b_{ui}
- compare values with an arbitrarily chosen threshold. Previous step will be iterated, until values are equal or less than the threshold.

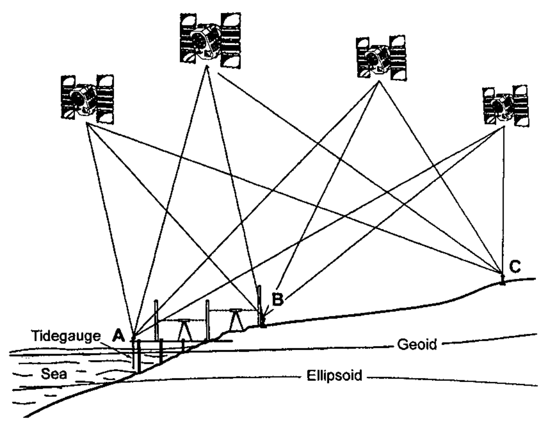


Figure 28: Measuring pseudo-ranges

By measuring the time differences of the signal travelling from the satellites to the receiver, the required pseudoranges can be calculated using:

- true time of a signal transmission from each satellite is t_{si}
- arrival of a signal at the receiver time t_{ri}, true time of reception is t_0
- using the transit time of the satellite radio signal from the satellites to the receiver (t_{ri} - t_{si}) times c_0 (speed of light) gives the calculated pseudoranges, recalling:

as predicted by Einstein's theory of general relativity, time passes more quickly for the satellites than on Earth [1.1]. For this reason relativistic corrections to the transit time must be made (Tsui, 2004, Xu, 2003).

Factors affecting the pseudo-range measurement are:

- user clock bias error
- satellite position error

- delay error due to periodic changes in the troposphere
- delay error due to periodic changes in the ionosphere
- receiver measurement noise error
- relativistic time correction
- multipath effects
- Sagnac effects.

In general, the values calculated in the acquisition by iteration method mentioned above will keep decreasing rapidly. When this value is less than a certain predetermined threshold, the iteration will stop. The required user position and clock bias are expressed in final values of x_u, y_u, z_u, and b_u. The final position is usually transformed into a 3D-ellipsoidal earth-centred, earth-fixed (ECEF) coordinate system (such as WGS84) to user position fixing in terms of latitude, longitude, altitude, and the user clock bias. If accurate data for processing is required, the tracking data and additional broadcast information can be made available to the user.

Note

> *Four or more GPS-satellites can be used to determine an observer's position anywhere on the earth's surface 24 h per day. Theoretically, three or more GPS-satellites will always be visible from most points on the Earth's surface* (Grafarend, 2003g).

Differential GPS

Differential GPS (DGPS) is a technique for reducing the error in standalone GPS single point positioning by using additional data from a reference GPS-receiver at a known coordinated position. DGPS offers several significant advantages for precise determination of new geodetic reference station coordinates.

Typically, a DGPS reference system consists of a reference system at a selected reference location and a mobile system. Such conventional systems operate on the principle that the main error sources due to relativistic time delay, satellite ephemerides, the ionospheric, and tropospheric distortions or atmospheric delays, are closely correlated. Pseudo-range measurements made at precisely located reference stations are compared with corresponding ranges computed from coordinates, and the errors derived are transmitted as differential corrections for application by DGPS users within a specified range. The estimated positions are corrected, and deviations are used for quality control. If this error exceeds a given threshold, various methods are used to correct the reference station messages (DGPS'91, 1991).

The most common form of DGPS involves determining and transmitting pseudorange corrections in real time to a user's receiver, which applies the corrections in the process of determining its position. The consequences are that error sources are cancelled entirely, such as:

- selective availability (SA)
- satellite ephemeris and clock errors.

Cancellation degrades with distance with other error sources:

- ionospheric delay error
- tropospheric delay error.

Some error sources are not cancelled:

- multipath errors
- receiver errors.

The main drawback of DGPS systems is the limited range over which the differential corrections are valid. DGPS systems have been largely limited to areas of commercial interests, such as hydrographical activities and busy sea ways (Grewal, 2001; Johnston, 1993).

Wide-Area Differential GPS

Using GPS observations in the single point method, together with the satellite precise ephemeris in the WGS84 coordinate system will result in a horizontal accuracy of one metre and a vertical accuracy of two metres. However, several techniques are available for accomplishing a Datum transformation.

A wide-area-DGPS (WADGPS) is a system designed to enhance the GPS. It is available over a wide area, in which the user's GPS-receiver collects corrections determined from a network of reference stations distributed over a wide continental area. Any proposed WADGPS must maintain the advantages offered by a conventional DGPS system. The corrections are applied in the user's receiver in computing the receiver's coordinates. The corrections are typically supplied in real time through a network of ground-based transmitters, at a later date by post processing collected data, or by using geostationary communications satellites, such as satellite-based augmentation systems (SBAS), under development at the beginning of the third millennium.

Eliminating the correlation between achievable accuracy and reference-station-to-user distance is reached by separating the combined differential corrections into their component parts, dealing with those which are dependent on the distance separately from those which are not. Separate corrections are usually determined for specific error sources, such as satellite clock, ionospheric propagation delay, and ephemeris.

Indeed, a number of autonomous DGPS networks on a regional scale - local area augmentation systems (LAAS) - are deployed by the private industry to enhance GNSS. Three global DGPS reference networks are designated as SBAS. While these systems are compatible, an international coverage is guaranteed. The correctional signals are sending at the same frequencies as GNSS. Since a modern 12-channel GNSS-receiver has maximally signals from 10 satellites, one channel remains free for a correctional signal:

- European geostationary navigation overlay service (EGNOS)
- Japanese multi-functional transport satellite-based augmentation systems (MSAS)
- US wide area augmentation system (WAAS).

EGNOS is a satellite navigation system under development by the European Space Agency (ESA). The European Commission and EUROCONTROL (European Organisation for the Security of Space Flight) to supplement the GPS and GLONASS systems. It will consist of a precursor to the Galileo positioning system, with three geostationary satellites and a network of ground-based augmentation stations (GBAS), operational in 2006. In fact, the horizontal position accuracy should be better than one meter. EGNOS will be integrated into GALILEO, the European satellite navigation system that is expected to go into operation in 2008. GALILEO is Europe's initiative to the development of a modern global position determination service under civilian control.

US Federal Aviation Administration (FAA) operates several DGPS airport reference stations for wide area augmentation system (WAAS).

The International Association of Lighthouse Authorities (IALA) operates some DGPS reference stations. The coverage area of the local area augmentation system (LAAS) is limited, but each reference station is a valuable source for the maritime community.

4.3.3 Global Differential GPS

US National Aeronautics and Space Administration (NASA) is reportedly implementing a new global differential GPS (GDGPS) developed by the Jet Propulsion Laboratory (JPL) that provides seamless global real-time positioning at 0.2 m vertical accuracy and 0.1 m horizontal accuracy for dual frequency GPS-receivers. Eight GPS Analysis Centres compute the Earth's rotation parameters independently (NASA, 2006).

Using GDGPS, C&C Technologies Inc. began establishing the C-Nav Technologies' RTG Network programme to provide worldwide positioning services for the hydrographic, offshore oil & gas field exploration, salvage- and construction industries. *NavCom's SF-2000R World GPS receivers'* position fixing accuracy - with exceptional accuracy and reliability - is no longer a function of the distance from any reference station (Figure 29). Because the differential GPS corrections are broadcast via Inmarsat geostationary satellites, the users need no local reference stations. The C-Nav RTG data links from the reference stations, using Internet as primary data link. They are backed up by private communications lines, using duplicate receivers, processors, and switching communication-interfaces for reference stations. Due to the worldwide coverage of the geostationary satellites, the same high accuracy is available between 72°N to 72°S latitude (Fortenberry, 2006; Hudson, 2001; JPL, 2006).

Figure 29: C-Nav Starfire GcDGPS World Receiver in the foreground on a barge lifting a Hurricane Katrina damaged platform

The National Geospatial Intelligence Agency (NGA) examines the World Geodetic System 1984 (WGS84) definition and interaction with various local geodetic systems (NGA, 2000). Hence, a relative precision of better than 0.1 m is obtained for the position of the fundamental GPS-stations of ITRF (IERS, 2002). The distribution of frequency bands for all separate satellite systems depends on international agreements. It is the direct responsibility of the International Telecommunication Union (ITU) (Hofmann-Wellenhof, 2001).

Finally, the GNSS technique is an important factor in the study about geophysical models used to describe the dynamics of the Earth. The theory of plate tectonics, crustal deformation of the Earth's surface, and identification of earthquake hazard zones have been in focus at a local scale. Sophisticated DGNSS techniques can give position differences with an accuracy of a few centimetres (Grewal, 2001).

3D-Correction Grid

Marine- and land surveyors need to understand the geodetic reference system aspects in the field of earth-centred, earth-fixed (ECEF) spatial coordinate computation, which is in tune with the technical developments. Such reference systems include the possibility of incompatibilities between spaceborne 3D-systems and combined local 2D-horizontal- and 1D-vertical Datums. Currently, all spaceborne survey systems are performed largely using pseudo-ranging GPS techniques. To apply latitude-, longitude-, and height corrections to the calculation, $\delta\varphi$, $\delta\lambda$, δh, respectively, local correction grids could be applied using e.g. the method of bi-linear interpolation [7.1]. Figure 30 shows a distorted old grid (RD1918), an old corrected grid, and an ideal, true grid (RD2000) due to the application of the transformation, combined with a bi-linear interpolation.

The grids should ideally provide an absolute accuracy of a few millimetres uncertainty in any 10 km square grid area. Yet again, if not, an upgrade of the size of the square grid is highly advisable. Remaining errors are mainly due to the bias in the errors of the pseudo-ranging GPS techniques.

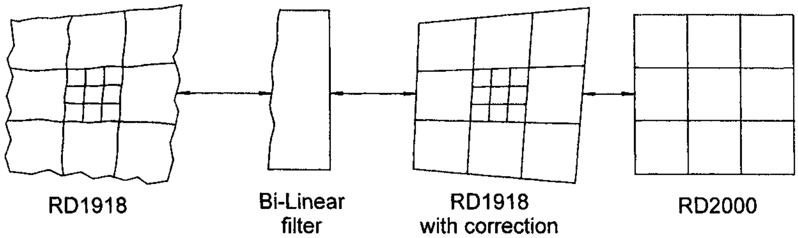


Figure 30: Distorted – filtered - transformed - corrected grids

Dilution of Precision

The dilution of precision (DOP) is used to evaluate quality control of a user position. The smallest DOP value means the best spatial position of the satellites for determining a user position. Subsequently, following definitions of DOPs are used as a function of satellite geometry only:

- geometrical dilution of precision ______ GDOP
- position dilution of precision ______ PDOP
- horizontal dilution of precision ______ HDOP
- vertical dilution of precision ______ VDOP
- time dilution of precision ______ TDOP

However, the solutions obtained through this approach must be modified to reflect the ellipsoidal shape of the Earth.

Broadcast and Precise Ephemeris

> *A broadcast ephemeris is the ephemeris of a satellite broadcast from it, from which Earth-fixed satellite positions can be computed. In the 1980s, the term applied particularly to the Navy Navigation Satellite System and the Global Positioning System. The broadcast ephemeris is designed to provide orbital elements quickly, and is not as accurate as a corresponding precise ephemeris for the same satellite*
>
> *A precise ephemeris is the ephemeris of a satellite computed by adjustment of observations obtained from a worldwide tracking network in order to obtain maximum accuracy (GG, 1986).*

Benefits

A consideration about the practical use of global horizontal and vertical reference systems is imperative, because much of the geospatial data information in the world is still referenced to local Datums. Tectonic plate motion [2.1] is being accounted for in WGS84 geodetic positioning by using a plate motion model (Grewal, 2001; Misra, 2001; Slater, 1998; Tsui, 2004). Few GPS-receivers are intended for geodetic positioning and navigation. Accuracies of centimetre are possible using the static positioning system or kinematic positioning system if at least two (geodetic) receivers are used together (Falkner, 2002).

Remembering that in the mid 1960s the subject of polar motion was not falling within the domain of geodesy, since it dealt with phenomena of magnitude less than could be calculated or measured by the geodetic techniques then current. Now, atmospheric loading, continental drift, Coriolis force, ocean loading, polar motion, Pole tide, resonance, solid earth tides and tectonic (micro)-plate motion, are now all falling within the domain of geodetic analysis. Right now, a continuing contribution by satellite geodesy is building up a qualitative comprehensive and quantitative models of the Earth. The tracking data, such as DORIS of the TOPEX/POSEIDON oceanographic satellite, were used to calculate the coordinates of station positions, velocities and Earth rotation parameters (Costes, 1998; IERS, 1995; Nerem, 1994a, b; Rapp, 1994).

For that reason, many geodesists regard GPS (or GNSS) as the last word for solving existing problems in geodesy. Actually, the GPS concept can be successful, provided time is available to develop new stimulating methods, if possible, combined with the information, communication, and computer technology (ICT) [15], including the acoustic digital subsea positioning systems [16], that replaced several of the analogue techniques of classical methods. It is also recognised as an excellent tool to be of benefit to branches, such as agriculture, astronomy, aviation, charting or mapping, civil navigation, construction, electric power-supply systems, fisheries, geochemistry, geodesy, geology, geophysics, hydrography, logistics, meteorology, mineral exploration and mining exploitation, missile guidance, oceanography, physical geography, recreation, telecommunications, and warship navigation.

4.3.4 GLONASS System

GLONASS System and Signals

Another configuration for continental-wide positioning is the GLONASS (global orbiting navigation satellite system). It is managed and owned by Russia (formerly Russian Federation (RF), or USSR), including Belarus, Kazakhstan, Russia, and the Ukraine. The authorisation to use the space system rests with the RIRT (Russian Institute of Radio-navigation and Time) and the Russian Ministry of Defense (Gouzhva, 1997).

In 1982, the first GLONASS Satellite was launched. GLONASS is similar to the GPS in that it is a space-based navigation system providing global, twenty-four hour-a-day, all weather access to precise position, velocity and time information anywhere in the world or near-earth space to a properly equipped user. GLONASS satellites are launched three at a time from the Tyuratam Space Centre into near-circular orbits (ICD, 2002).

Each GLONASS satellite continuously broadcasts its own precise position as well as less precise position information for the entire constellation. For a given number of satellites in the final operational system, the choice of orbital planes and phases within the plane is controlled to ensure visibility of four well-located satellites on a worldwide basis. GLONASS reached this configuration in January 1996 when there were 24 operating satellites plus a spare one (Walsh, 1998). At present, there are insufficient operating satellites available.

TDMA is a time division multiple access, requiring precise Time synchronisation in a communication method to divide Time into slices to facilitate service to several users. The methods for receiving and analysing GLONASS signals are similar to the methods used for GPS-signals. SA is not appropriate to GLONASS. Frequency division multiplexing access (FDMA) is, in general, a GLONASS signal. The GLONASS system uses two independent satellite signals, corresponding to L_1 and L_2. The frequencies are $f = (1.602 + 9k/16)$ GHz and $f_2 = (1.246 + 7k/16)$ GHz, in which $k = 0, 1, 2, \ldots, 23$ is a satellite number. Every satellite transmits the same two carriers to avoid interferences, which are modulated by a set of codes assigned to each satellite.
The transmitted GLONASS ephemeris message, the satellite position, its velocity and its acceleration are given in Cartesian earth-centred, earth-fixed (ECEF) coordinates and extrapolation terms in Euclidean space and time (Daly, 1992-1994; Hofmann-Wellenhof, 2001; Kazantsev, 1992; Tsui, 2004).

Currently, GLONASS is based on the Parametry Zemli System of 1990 (PZ90), formerly SGS90 (Soviet Geocentric System of 1990) (Moskvin, 1990). The majority of the effort put into GLONASS carrier phase positioning has been with the intention of combining GPS and GLONASS. Possible advantages of combining GPS and GLONASS carrier phase positioning alone are that:

- the availability of the system is improved in areas where there is a great deal of satellite masking
- the integrity of the solution is improved
- the ambiguities can be resolved faster.

Function	GLONASS Space Segment
operational since	December 14, 1995
S/C number in the fully deployed system	24 plus max. 6 spare satellites standby
number of orbit planes	3
near-circular orbits with four or more satellites	8
orbit inclination	64°.8
orbits separation	by multiples of 120° Right Ascension of the Ascending Node
orbit altitude	19 100 km
Dragonian period orbit	11 hours, 15 min, 44 sec.
orbital parameters updated	Satellite broadcast ECEF geocentric position, velocity, and acceleration updated every 30 minutes
ephemeris data representation	9 parameters of S/C motion in the geocentric Cartesian coordinate system
geodetic coordinate system	PZ90 ECEF Frame (1995)
phase-lock ranging signals	to GLONASS synchroniser
method of S/C signal division	frequency division multiplexing (FDMA)
almanac content	120
duration Almanac transmitting time	2.5 min
frequencies in the L_1 range	1602.5625 - 1615.5 ±0.5 MHz
frequencies in the L_2 range	1246.4375 - 1256.5 MHz
number of code elements	511
C/A-code frequency	0.511 MHz
P-code frequency	5.11 MHz
cross talk level on two adjacent channels	- 48 dB
synchro code repetition period	2 s
bit number in synchro code	30
type of ranging code	C/A and P Codes
availability	Non Selective
time synchronisation	GLONASS-time, with UTC(SU)+ 03:00
tracking network	continental-wide
laser tracking	network
ground track repeat period	8 sidereal days

Table 12: Original GLONASS characteristics

Dual GPS/GLONASS receivers exist, and both systems are jointly used for the various scientific objectives. In the frame of IERS, position determinations are made from observations collected by the IGS. About thirty of the IGS sites are collocated with VLBI or satellite ranging facilities so that the link with the ITRF (International Terrestrial Reference Frame) is very strong. The analysis includes the computation of precise ephemeredes, corrections to station coordinates and the Earth's rotation matrix between the reference frames.

4.3.5 GALILEO System

In 1999, the European Union (EU) and the European Space Agency (ESA) made a decision to proceed with definition of the architecture of another passive, one-way ranging satellite system expected to become operational between 2008-2010: Galileo, an European equivalent to GPS and GLONASS (Kannemans, 2005).

Geodesists, land surveyors, software developers, and users of geographic information will use the schema to provide data with consistently defined reference systems with the EU motivations:

- EU has a technical and a commercial interest associated with GNSS.

- EU has no control over GPS or GLONASS|[7]
- under civilian control, the EU must own an independent GNSS for its own safety

4.3.6 Combined GALILEO, GPS and GLONASS Systems in Differential Mode

Any GNSS needs a unique conceptual schema for the geodetic reference system. Hydrographers and land surveyors need to understand the geodetic reference system aspects of GNSS constellation of satellite measurements to apply them to surveying problems. Moving to information based global economy, the major tendency is the realisation that the spatial data which land surveyors produce needs to be seen as infrastructure, facilitated by GIS (geographic information system) technologies [4.5]. Such a reference system matters include the possibility of incompatibilities between the space based systems and the local horizontal working Datum.

Another element is the information that the purely geometric heights coming from these local systems regularly need to be related to the local vertical Datum which in turn are related to a physical surface such as mean sea level.

For a position fix, using a combination of GPS and GLONASS satellite navigation systems, a uniform coordinate system is necessary. Some questions remain concerning the intention of combining GPST, GLONASS-time and reference ellipsoids used by GPS and GLONASS. Although the almanac parameters used for the position determination of GLONASS satellites are similar to those of the GPS, the more precise ephemeris data of GLONASS and GPS differ completely in contents and timely updates. The values of the defining parameters of the PZ90 EGM gravity model and ellipsoid are slightly different from the WGS84 EGM.

As a matter of fact, the appearance of GPS and GLONASS as an element of national security was strongly stimulated by political and technical competition in the years between 1970 and 1980. In the mid-1990s, the economic aspect emerged when both GNSS systems became operational, and their civil segments were declared available free of charge to the world community. To achieve electromagnetic compatibility with satellite navigation and communication systems, GLONASS frequencies are being shifted to provide within the frame of allocated frequency bands non-conflicting satellite navigation and communication systems.

Functional improvements in GLONASS are based on its space segment, ground segment, augmentation systems user segment modernisation and development. Since the year 2003, modernised GLONASS Spacecraft (SC) are launched with a 7-year active life cycle, using two civil signals. The GLONASS Development program implies a reconstruction of a constellation of 18 orbital SC in 3-4 years. Hereafter, the space segment will be renewed by the next generation of SC, at this time in the design phase, with a 10-12 year life cycle, reduced mass to enable simultaneous launch of six SC and three civil signals available to users.

After that, within ten years, about thirty GLONASS/GPS floating reference stations (buoys) will cover the coastal regions. Another twenty-five stations shall operate on the inland waters. An autonomous identification system (AIS) ground stations will cover main ports. The existing systems, developments in orbital constellation along with the terrestrial control segment are continuously improved to provide one metre accuracy worldwide.

With regard to the differences between the ECEF reference frames of WGS84 and Parametry Zemli of 1990 (PZ90), GLONASS/GPS user equipment will operate in both geodetic reference systems without introduction of noticeable errors in the S-transformations. Meanwhile, modernised SC will transmit messages containing the GLONASS/GPS time reference discrepancy (Benhallam, 1996; Shebshaevich, 2006).

Differential mode increases considerably the accuracy of navigation and position fixing by GLONASS- and GPS-signals. Currently, GLONASS' accuracy of position and time determination depends on caesium beam atomic frequency standards, in addition to hydrogen masers, which are used as spaceborne- and terrestrial-GLONASS Frequency and Time Support (FTS). Position fixing by using combined GLONASS/GPS-signals in differential mode permits to increase the accuracy of positioning considerable. Still, the accuracy of position and time determination depends on (Mitrikas, 1998):

[7] due to political nature and financial precautions

- the accuracy of execution depends on the performance of each system
- the method of mutual system time synchronisation of GLONASS (UTC(SU) and GPS (UTC(USNO) United States Naval Observatory system times during its joint use
- providing a possibility of those systems' joint use in differential mode
- optimisation of algorithms for processing of results and routines
- technique enhancement of FTS.

to obtain the accuracy of mutual synchronisation in case of FTS uploading two times in a day (Gouzhva, 1997).

The mutual synchronisation of the GLONASS/GPS system times is in actual fact based on:

- measurements at the master monitoring station of System I, which generates a system time by SV-signals of System II
- simultaneous measurements at the monitoring stations of both systems by SV-signals of any system.

The control centres of systems determine:

- the parameters of mutual system time synchronisation
- computed correction factors for uploading into the spaceborne equipment for next transmission to users.

4.3.7 eLORAN Systems

In the period 1997-2005, the FAA is assessing the Loran-C modernisation program toward an enhanced Loran (eLORAN) system in the following areas (http://www.locusinc.com):

- development of Loran H-field antennae suitable for aircraft installation
- development of an RTCA compliant digital signal processing (DSP) Loran receiver
- development of enhanced Loran communications capability for GPS integrity and potentially for GPS correction data
- development of hybrid GPS/Loran receiver architecture and high performance antennae technology system.

Basic GPS service fails to meet the *accuracy*, *availability*, and *integrity* requirements critical to safety of navigation. An eLoran system provides performance comparable to GPS for numerous time and frequency control equipment (TFE), hence eLoran can act as a traceable, infinite, and independent backup to GPS in these applications. The eLoran system will use time-of-transmission (ToT) control similar to GPS, and adds a Loran data channel (LDC) that will distribute UTC, leap seconds, individual station identification, differential Loran corrections, and other information. Testing indoor antennae, investigations have demonstrated that small eLoran H-antennae offer the opportunity of indoor reception. Time and Frequency control Equipment (TFE) has the capability to use three caesium clocks in the Loran infrastructure to compute a single timescale from a single Loran signal, because TFE steer all transmitters to within 15 ns of UTC(USNO). Technical evaluation confirmed that an eLoran system can meet specific criteria for aviation and marine navigation, so it is sensible to provide a backup to the global differential navigation satellite systems (GDNSS) for critical infrastructure TFE applications, with the recognition that an eLoran system can offer unique capabilities to fulfil this role (Roth, 1999, 2006.

Terrestrial-based, long-range navigation systems like enhanced LORAN are among potential pretenders. Russia has its own system of this type: Chayka. A universal international satellite navigation system will integrate both global navigation and telecommunication functions in one. An enhanced Chayka long-range navigation system will be modernised to provide transmission of differential corrections. This approach provides about one metre accuracy, at the same time with integrity monitoring, notification for safe navigation, and accurate offshore positioning (Shebshaevich, 2006).

4.3.8 Uncorrected Errors due to System Time Drift

The GALILEO-, GLONASS- and GPS-signals mode are different in a variety of ways. On the other hand, using these combined signals in differential mode may increase the accuracy of position fixing and navigation significant. In case of mutual synchronisation of reference times, the accuracy will be reduced due to uncorrected errors for drift of GALILEO, GPS-UTC(USNO) and GLONASS-UTC(SU) system times (Gouzhva, 1997).

Future and Developments of IAG's Global Geodetic Observing System

In a dynamic electronic environment, the developments are continuous. As mentioned before: what is in vogue today could be obsolete tomorrow. The current state of GLONASS, the characteristic combination of GALILEO, GPS, and GLONASS, carrier phase positioning is explained with reference to currently available receivers (Beser, 1992; Gouzhva, 1991, 1995; Ivanov, 1992; Kannemans, 2005; Kazantsev, 1991; Seeber, 1993).

GGOS is the Global Geodetic Observing System of the IAG to provide observations of the three fundamental geodetic observables and their variations:

- the Earth's shape
- the Earth's gravity field
- the Earth's rotational motion.

GGOS integrates geodetic techniques, models, and approaches to ensure a long-term, precise monitoring of the geodetic observables in agreement with the integrated global observing strategy (IGOS). GGOS provides the crucial observational basis to maintain a stable, accurate and global reference frame.

GGOS contributes to the emerging global earth observing system of systems (GEOSS):

- the accurate reference frame required for many components of GEOSS
- the observations related to the global hydrological cycle
- the dynamics of atmosphere and oceans
- the natural hazards and disasters.

GGOS acts as the interface between the geodetic services and external users to ensure the interoperability of the services and GEOSS (Grafarend, 2006a; IAG, 2006}.

4.4 Acoustic 3D-positioning

Using electromagnetic waves, a major technological achievement has been the devise and operation of communication, transmission of information and navigation over large distances, such as telephone, radio, radar systems, remote sensing satellites, and global navigation satellite systems in the atmosphere and in space.

The Earth is the only planet known to have liquid water and ice. The underwater domain allows for propagation of hydro-acoustic waves. As a result, a large portion (70.8 percent) of the Earth's surface, the Hydrosphere, remains inaccessible to electromagnetic and light waves (Lurton, 2002) [6.2].

Underwater Acoustics

Sound waves are an integral part of our natural or artificial aerial environment. Acoustic waves are today the only practical way to carry information underwater. Acoustic waves, consisting in mechanical vibrations of their propagation medium, propagate very easily in seawater over very large distances. Its use - as sound navigation and ranging (SONAR) developed during World War I by the French physicist Langevin - proved to be the practical way to carry information underwater. In water, acoustic waves have different transmission characteristics than in air. The atmosphere propagation velocity is four to five times higher than in water. Sonar signal-processing functions are detection and identification (target recognition). Currently, underwater acoustics plays in the hydrosphere an essential role in naval and civilian techniques [16.2] (Lurton, 2002).

Naval Applications

One of the main differences between using electromagnetic waves in the air (300 000 km/s for radar waves in space) and using acoustic waves (1 500 m/s in water) resides in the constraints brought about by the propagation medium. The environment air-water boundary plays a very complex role in the transmission of signals. Seawater is relatively favourable to the propagation of acoustic waves. For naval sonar applications, passive and active systems are a primary function in anti-submarine warfare, long distance messages between submarines and surface vessels, mine hunting, and target detection [16.2].

Civilian Applications

The hydrographic environment plays a very complex role in civilian sonar applications. There are shipborne and submersible systems for detecting and locating obstacles, for mapping the seafloor, observing tides and tidal currents, and sounding geophysical structures.

Perturbations of propagation by the variations in sound speed and reflections on seafloor and sea surface interfaces, resulting in:

- inhomogeneous insonification of the propagation medium
- multiple paths, generating echoes and interference
- deformation of acoustic signals
- ambient noise, such as self-noise, from current, swell of the sea surface, volcanic or seismological activities, shipping, living organisms, wind, rain, et cetera.

The problem encountered in the use of underwater acoustics are the varying characteristics of a propagation medium due to variations in barometric pressure, currents, salinity, swell, temperature, and tides. Signal processing directly affects the performance of underwater acoustics systems. Hence, during the last half-century, the underwater acoustic propagation became one of the most active branches of hydrography.

4.5 Global Geographic Information Systems

It is imperative for geodesists and hydrographers to understand the various coordinate systems, implications of data transportation and storage, statistics, including the methods of mapping-data compilation.

GIS Applications

Using information, communication and computer technology (ICT), digital data, derived from aerial-, terrestrial photography, and hydrographic systems will find in time its way into georeferenced data-warehouses [15.2]. Data from these and remote sensing devices can be stored, sorted, and incorporated into geospatial referenced information systems (GIS) for detailed analysis and results (Kovalevsky, 2002).

The ability of GIS to overlay new information on top of an existing database, to print or display it in colour is serving users to manage analyses (Figure 118, pp 294). GIS comprise software and hardware systems that relate and display collected data in terms of geographic or spatial location (Falkner, 2002; Kasser, 2002).

GIS in Hydrography - the Marine and Coastal Environment

A swift development in ICT has made many hydrographers increasingly aware of the widespread application of the networking technology, including hardware- and software-technologies ranging from GIS to environmental problems. ICT [16.1] provides hydrographers more insight into development and successful implementation of spatial information data. Using GIS requires a condition to monitor, to map, and to model both the coastal zones, offshore and the coastal areas:

- management of cables and pipelines
- environmental data management
- exploration and production of oil and gas

- mapping fishing resources
- lakes, rivers, ports and coastal zone management
- offshore energy.

Historically, exploration of the Earth's interior was restricted close to the onshore surface. In the last half-century, it was largely a matter of continuing downward all those discoveries made at the sea surface. Electronic- and acoustic-positioning with a wide choice of functions and potentials, with satellite based marine environmental information services provide policy makers with up-to-date information in the environment of water frameworks, and coastal zone management (Green, 2004).

Electronic Total Stations

Electronic Total Stations (electronic tacheometers; ETS) connected to an electronic field book collect digital survey information directly to perform topographic mapping by computer.

Using Analogue and Digital Cameras for Orthophotography

There are several types of aerial- and terrestrial photogrammetric systems available:

- terrestrial photogrammetric tools, such as Leica terrestrial photogrammetric cameras and CycloMedia, the latest terrestrial photogrammetric system for 355° horizontal and 15° vertical scanning and digital image recording, processing, and mapping (Beers, 1995)
- airborne photogrammetric analogue- and digital cameras (Falkner, 2002; Kasser, 2002; Kovalevsky, 2002).

Digital camera systems are relatively similar to analogue systems. A significant difference between analogue and digital camera systems are the charged-coupled device (CCD) to record an image as a matrix of pixels along with a computer data storage device to record a group of image data sets (images). Analogue camera systems do utilise photographic film to record an image. Consequently, digital imagery can be loaded into a workstation immediately, because there are no steps, such as chemical film processing, printing required prior to data manipulation and analysis. Digital imagery systems can collect black and white, natural colour, and colour infrared imagery. Digital data derived from aerial photography or remote-sensing devices will eventually find its way into georeferenced databases for detailed analysis and solutions (Kasser, 2002).

Spaceborne Remote Sensing Systems

By the beginning of the 20^{th}-century most of the Earth's surface has been explored superficially, except for the Arctic and Antarctic regions. USA and Russia have used spaceborne imagery for various purposes, along with GPS-controlled satellite positioning. Heavy clouds, steady rain, and dense jungle vegetation made the exploration of the rainforest difficult. Fortunately, remote sensing systems (RSS) are able to penetrate the cloud cover to produce reliable, detailed maps of the area. Today radar and photographic mapping from aircraft and satellites have filled in the last of the unmarked areas on land maps. An overview of the most used systems in the 1990s (Carrara, 1995):

RSS - Airborne Systems:

- aerial camera photo image scanning
- airborne visible infrared image spectrometer (AVIRIS)
- digital multispectral videography scanner (DMVS)
- light detection and ranging (LIDAR)
- thermal infrared multispectral scanner (TIMS).

RSS - Spaceborne Systems:

- advanced very high-resolution radiometer (AVHRR)
- earth remote sensing (ERS)
- IKONOS-2 -space imaging EOSAT

- LANDSAT enhanced thematic mapper (ETM)
- LANDSAT multispectral scanner systems (MSS)
- LANDSAT thematic mapper (TM)
- optical RADARSAT
- synthetic aperture radar (SAR)
- système pour l'observation de la terre (SPOT)
- thermal infrared multispectral scanners (TIMS)
- video camera satellite imagery.

5. Plane and Spherical Earth Systems

Earth as a Flat Disk

The history of mathematics gives ample evidence that the subject is indelibly stamped by the outlooks and personal qualities of men. Not being able to go to the Moon to look at the true shape Earth, all early civilisations imagined the Earth as a flat disk, with peaks heaped upon it like food on the king's table. Throughout the past five centuries, the mathematicians not only extended the subject vastly but, as they made successive extensions, as they recognised new entities, new phenomena, and as their own understanding improved, they recasted their concepts, processes, and proofs.

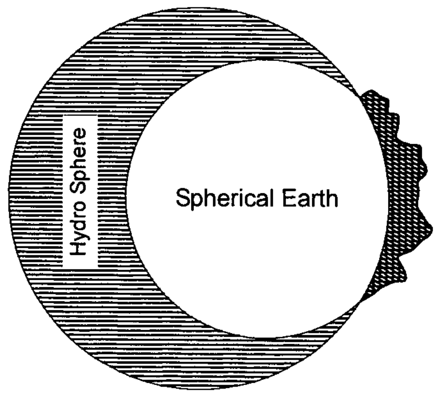


Figure 31: Spherical Earth

In land and hydrographic surveys covering a large extent of country or continent, many mathematical refinements have to be introduced to allow for the curvature of the Earth's surface. To this most accurate mode of surveying the term geodesy or geodetic surveys is applied.

The knowledge of practical geodesy or land- and hydrographic surveying required by civil and mining engineers is, as a rule, not very extensive. A good working knowledge of electronic surveying, setting out whether on the straight or the curve, on the surface or underground, and levelling will satisfy their requirements.
The refinements introduced in geodetic surveying are not necessary in surveys for ordinary engineering purposes, as the corrections introduced by the sphericity of the Earth are negligible in this type of work. Consequently, for all ordinary purposes the operations of the surveyor are carried out as though the Earth's surface were plane and not ellipsoidal (Stevenson, 2004; Threlfall, 1940).

5.1 Plane Trigonometry

Algebraic or analytic geometry of R. Descartes had eclipsed geometry for many years. For two centuries, the pure geometers remained in the shadows. In the 19^{th}-century, Gaspard Monge, a leading French mathematician and adviser to Napoleon, launched the revival of geometry. Monge was such an inspiring teacher that he grouped about him at the Ecole Polytechnique a congregation of scholars, among them L. Carnot, C.J. Brianchon and J.V. Poncelet. These men, sought to explain that geometrical reasoning could accomplish more. The aim became to free geometry from the hieroglyphics of analysis (Crombie, 1959).

Familiar elements of geometry: Parallel lines never meet, and a straight line is the shortest way between two points. The sum of the three angles of a triangle equals two right angles, or 180°. Consider three points on the surface of the Earth. Taking the straight-line segment between any two points, that joins them through the Earth, results in a triangle that has all the properties of a Euclidean triangle. An Euclidean postulate assumes *the flatness of space in its smallest parts*, i.e. taking three points in space very close to one another and joining them by the shortest possible lines, the triangular figure so formed will lie very nearly in a plane. Such a small triangle on the Earth's spherical surface is indistinguishable from a flat triangle. Conversely, the curvature of larger triangles will become increasingly significant. It will show up in more precise methods of surveying in the excess of the sum of their angles over 180°.

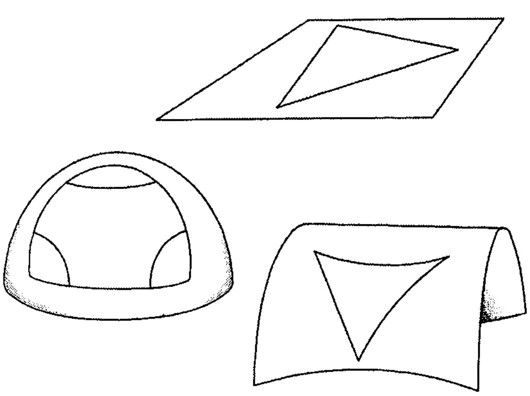


Figure 32: Space curvature illustrated in 2D

In surveying, angles and distances are measured. From these measured values, the coordinates of the surveyed points are calculated. Most examples refer to surveys measured in a sexagesimal system (° = degrees) and metres. The reader can gain valuable checks by recomposing the examples by converting the units to those with which he is familiar, such as a centesimal system (^{g} = gon or so-called grades), and metres or feet [13.1].

Goniometrical Conversion of Angles - the Formulae

It has been thought desirable to give the more important formulae of plane trigonometry to meet the needs of those whose operations are of a simple character. Plane trigonometry deals with the properties of figures drawn wholly in one plane, such as the Flat Earth. Formulae in plane trigonometry are given owing to their general utility to the land surveyor. Let A, B, and C be the angles of a triangle, and let a, b, and c be the sides of the triangle opposite to the angles A, B, and C, respectively.

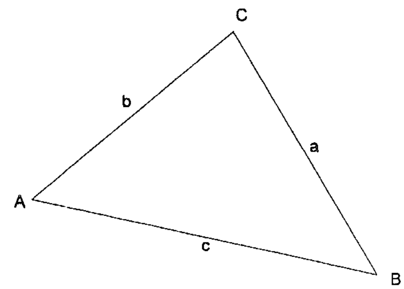


Figure 33: Plane triangle

The following formulae in plane trigonometry are given because of their general utility to hydrographers or land surveyors, but no attempt is made to prove them. Otherwise, any good textbook on plane trigonometry must be referred to. Other more important formulae of spherical trigonometry will be given as the need arises in [5.4].

Sine Formulae

$$2\,r = \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \tag{5.01}$$

$$a = \frac{c\,\sin\alpha}{\sin\gamma} \tag{5.02}$$

$$b = \frac{c\,\sin\beta}{\sin\gamma} \tag{5.03}$$

$$c = \frac{a\,\sin\gamma}{\sin\alpha} \tag{5.04}$$

Cosine Formulae

$$a^2 = b^2 + c^2 - 2\,b\,c\,\cos\alpha \tag{5.05}$$

$$b^2 = a^2 + c^2 - 2\,a\,c\,\cos\beta \tag{5.06}$$

$$c^2 = a^2 + b^2 - 2\,a\,b\,\cos\gamma \tag{5.07}$$

Projection Formulae

$$a = b\,\cos\gamma + c\,\cos\beta \tag{5.08}$$

$$b = c\,\cos\alpha + a\,\cos\gamma \tag{5.09}$$

$$c = a\,\cos\beta + b\,\cos\alpha \tag{5.10}$$

Area Formula

$$\text{area} = \tfrac{1}{2}\,a^2\,\frac{\sin\beta\,\sin\gamma}{\sin\alpha} = \tfrac{1}{2}\,b^2\,\frac{\sin\alpha\,\sin\gamma}{\sin\beta} = \tfrac{1}{2}\,c^2\,\frac{\sin\alpha\,\sin\beta}{\sin\gamma} \tag{5.11}$$

Spherical Access

$$\varepsilon'' = \text{area}\;/\;(\,R^2_{\text{mean}}\ \text{arcsec}\,) \tag{5.12}$$

Addition Formulae

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \tag{5.13}$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \tag{5.14}$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \tag{5.15}$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \tag{5.16}$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \tag{5.17}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} \tag{5.18}$$
$$\sin 2\alpha = 2\sin\alpha\cos\alpha \tag{5.19}$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha \tag{5.20}$$
$$= 2\cos^2\alpha - 1 \tag{5.21}$$
$$= 1 - 2\sin^2\alpha \tag{5.22}$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha^2} \tag{5.23}$$
$$\cos\alpha = 2\cos^2 ½\,\alpha^2 - 1 \tag{5.24}$$
$$= 1 - 2\sin^2 ½\,\alpha \tag{5.25}$$

Sums and Differences of Functions

$$\sin\alpha + \sin\beta = +2\sin ½(\alpha + \beta)\cos ½(\alpha - \beta) \tag{5.26}$$
$$\sin\alpha - \sin\beta = +2\cos ½(\alpha + \beta)\sin ½(\alpha - \beta) \tag{5.27}$$
$$\cos\alpha + \cos\beta = +2\cos ½(\alpha + \beta)\cos ½(\alpha - \beta) \tag{5.28}$$
$$\cos\alpha - \cos\beta = -2\sin ½(\alpha + \beta)\sin ½(\alpha - \beta) \tag{5.29}$$
$$\sin\alpha\cos\beta = ½(\sin(\alpha + \beta) + \sin(\alpha - \beta)) \ldots \quad \ldots \text{if } \alpha > \beta \tag{5.30}$$
$$½(\sin(\beta + \alpha) + \sin(\beta - \alpha)) \ldots \quad \ldots \text{if } \beta > \alpha \tag{5.31}$$
$$\cos\alpha\cos\beta = ½(\cos(\alpha - \beta) + \cos(\alpha + \beta)) \tag{5.32}$$
$$\sin\alpha\sin\beta = ½(\cos(\alpha - \beta) - \cos(\alpha + \beta)) \tag{5.33}$$
$$\tan ½\,\alpha = \frac{\sin\alpha}{1 + \cos\alpha} = \frac{1 - \cos\alpha}{\sin\alpha} \tag{5.34}$$

Calculations in Radian Measure

Personal computers (PC) calculate in radian measure. In addition, PC programs must master the relationships between coordinates, bearings and distances, including observing the sign rules for coordinate differences and angular functions applicable to the four quadrants.

5.2 Plane Coordinate System

In this chapter, a system of plane rectangular coordinates is used in which the position of a station point P_i is fixed uniquely by giving its perpendicular distance from two coordinate axes - in the form of a pair of coordinates.

Position of Origin

Since the 1900s, all rectangular coordinates are reckoned positive from west to east and from south to north in surveying. The position of the point of origin of the grid system should be placed to the extreme southwest of the area to be surveyed so that all coordinated points are positive. The scale along both axes is the same (Figure 34).

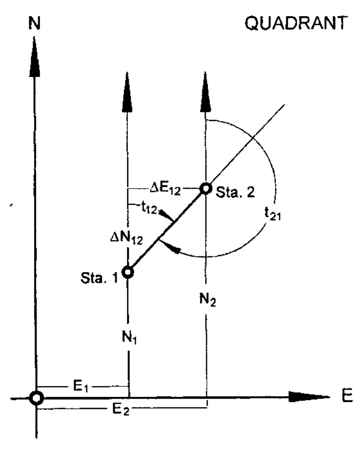


Figure 34: Bearing and distance

The horizontal axis is known as the E-axis (east to west axis) and the vertical axis as the N-axis (north to south axis). Coordinates are often identified as eastings and northings (Allan, 1975).

Relative positions of the stations of a survey line, the points, P_1 and P_2, can be given either by their coordinate difference ΔE_{2-1}, and equally ΔN_{2-1} or by bearing, t_{1-2} (i.e. directional angle or azimuth) and distance D_{1-2} (Figure 34).

$$\Delta E_{2-1} = (E_2 - E_1) \quad \text{- called the departure of the line -} \tag{5.35}$$
$$\Delta N_{2-1} = (N_2 - N_1) \quad \text{- called the latitude of the line -} \tag{5.36}$$

Note

Convention is to quote eastings before northings and it is usually written as (E, N). Other versions are (X, Y)-axes [10.5; 10.7] or (Y, X)-axes [10.6], since there is no internationally accepted convention concerning the use of X and Y.

Bearings reckoned round all four quadrants are called whole circle bearings in this book to distinguish them from other ways of considering bearings, such as used in the USA. In computer programs of this book [18] are only whole circle bearings considered.

Formulae to Calculate Coordinates from Bearing and Distance and Vice-versa

Clockwise Rotation

Furthermore, the direction of a 2D-vector from the point of origin, or its grid bearing, t_{1-2}, is reckoned clockwise from the North, even in the Southern Hemisphere. Latter method is in the polar coordinate system. Every line between the survey stations has two bearings (Figure 34):

forward brg = grid bearing t_{1-2} from P_1 to P_2
inverse brg = grid bearing t_{2-1} from P_2 to P_1

Following relationship exist between both bearings:

$$\text{bearing } t_{2-1} = \text{grid bearing } t_{1-2} \pm 180° \tag{5.37}$$

Following relationships exist between coordinates

$$\tan t_{1-2} = \frac{E_2 - E_1}{N_2 - N_1} = \frac{\Delta E_{2-1}}{\Delta N_{2-1}} \tag{5.38}$$

Computation of Coordinates from Bearing and Distance and Vice-versa

$$\Delta E_{2-1} = D_{1-2} \sin(t_{1-2}) \tag{5.39}$$
$$\Delta N_{2-1} = D_{1-2} \cos(t_{1-2}) \tag{5.40}$$

Using either (5.41; 5.42), a vector between the points P_1, P_2, represented by the bearing, t_{1-2}, and the distance, D_{1-2}, may be calculated as

$$D_{1-2} = \frac{\Delta E_{2-1}}{\sin(t_{1-2})} = \frac{\Delta N_{2-1}}{\cos(t_{1-2})} \tag{5.41}$$

or by using Pythagoras, the differences between eastings and northings of survey points, P_1, P_2

$$D_{1-2} = \sqrt{\frac{(\Delta E_{2-1})^2}{(\Delta N_{2-1})^2}} \tag{5.42}$$

and computation of bearing, t_{1-2}, from coordinates

$$t_{1\text{-}2} \quad = \quad \arctan\frac{E_2 - E_1}{N_2 - N_1} \quad = \quad \arctan\frac{\Delta E_{2\text{-}1}}{\Delta N_{2\text{-}1}} \tag{5.43}$$

Formulae to Calculate a Plane Area

Programs [18.1] calculate a cartographic object in digital storage:

A_01POLA.FOR ________ from polar coordinates to polygonal area
A_01BRDI.FOR ________ from coordinates to bearing, distance
A_01BDAR.FOR ________ from bearing and distance to coordinates and polygonal area

In analytical geometry, an area enclosed by a polygon having n 2D-vectors can be calculated either as the sum of its triangular (5.44) or trapezoidal (5.45) elements, using the coordinates X_1, Y_1, ..., X_n, Y_n in an n-sided figure (Figure 35).

Input: values for stations P_i, in which the subscript i = 1 to n:

easting - P_i	:	E_i
northing - P_i	:	N_i

Output value:

polygonal area = PA

Computation of Models

Using 2D-vectors is reckoned in a clockwise direction resulting in a positive area, however entering 2D-vectors in an anticlockwise direction results in a negative area. The formulae accept a clockwise 2D-vector rotation of input values of eastings and northings, E_i and N_i, respectively for any number of coordinated endpoints P_i, in which subscript i = 1 to n (Bannister, 1994; Borsuk, 1960; Coxeter, 1961; Cromley, 1992; Harvey, 1969)

Model I - Triangular

$$PA \quad = \quad -(½) \sum_{i=1}^{n} (x_i\, y_{i+1} - x_{i+1}\, y_i) \tag{5.44}$$

in which (x_i, y_i) are the coordinated endpoints P_i of the n line segments.

Model II - Trapezoidal

$$PA \quad = \quad -(½) \sum_{i=1}^{n} (x_{i+1} - x_i)(y_{i+1} + y_i) \tag{5.45}$$

in which (x_i, y_i) are the coordinated endpoints P_i of the n line segments.

Application I

Calculation from coordinates to polygonal plane area, using programs: A_01BDAR.FOR and A_01POLA.FOR

An example uses coordinates in a seven-sided figure, in which the successive coordinates are given:

easting-1	:	65 178.3000	northing-1	:	28 483.1800
easting-2	:	65 288.7700	northing-2	:	28 443.7200
easting-3	:	65 304.3100	northing-3	:	28 336.5800
easting-4	:	65 292.5300	northing-4	:	28 244.1400
easting-5	:	65 280.8000	northing-5	:	28 142.2400

easting-6	:	65 258.3100	northing-6	:	28 124.2100
easting-7	:	65 223.3100	northing-7	:	28 098.4100

Enclosed polygonal area is calculated as = 31 592.9298 m^2.

Quadrants

[18.1, A_01BRDI.FOR].

A bearing, $t_{i\text{-}i+1}$ - i.e. the azimuth or directional angle measured clockwise from North - can have any value on the full circle between 0° and 360°. The coordinate axes divide the circle (2π) into four quadrants (Figure 35).

quadrant I	=	between	0°	and	90°
quadrant II	=	between	90°	and	180°
quadrant III	=	between	180°	and	270°
quadrant IV	=	between	270°	and	360°

The functions, sin, cos, tan of equally sized angles in the four quadrants, are distinguished by plus (+) or minus (–) signs.

In quadrants II and IV the so-called co-functions replace the corresponding functions in quadrants I and III, as can be observed (Figure 35), in which angle $t_I = t_{II} - 90° = t_{III} - 180° = t_{IV} - 270°$, respectively.

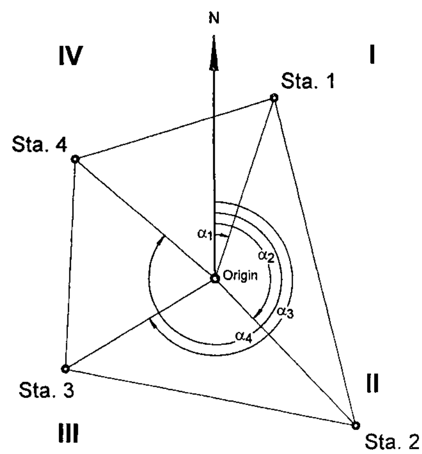


Figure 35: Area calculation in four quadrants

Application II

[18.1, A_01POLA.FOR].

By considering a four-sided figure and using a local framework, the successive bearings and distances (in m) are surveyed in four different quadrants, respectively, as follows (Figure 35):

Bearings and distance are entered – thus coordinates are calculated in quadrants

						Quadrant
bearing-1	:	37° 20' 30".00	distance-1	:	15.1230 m	I
bearing-2	:	135° 41' 23".00	distance-2	:	20.4560 m	II
bearing-3	:	232° 45' 21".00	distance-3	:	10.7890 m	III
bearing-4	:	310° 12' 25".00	distance-4	:	12.1250 m	IV

As a result, all coordinates (in m) are calculated in four different quadrants

						east	north
easting-1	=	12.0233 m	northing-1	=	9.1731 m	+	+
easting-2	=	–14.6376 m	northing-2	=	14.2894 m	–	+
easting-3	=	–6.5296 m	northing-3	=	–8.5887 m	–	–
easting-4	=	7.8273 m	northing-4	=	–9.2601 m	+	–

Finally, an enclosed polygonal area is calculated as = 417.9652 m^2

5.3 Distance Measurement Techniques

This section presents a curve fitting formula intended to perform data analyses. Here, it is especially useful for estimating the relationship between elapsed time and recorded readings of electronic distance measurement (EDM) equipment. The LS curve fitting subroutine provided does not require an iterative solution.

Observations

In refined work, positions that are observed are never used directly, but instead the differences between the observed positions and positions calculated from the least squares theory, in the sense calculated position minus observed position (C-O) are discussed.

Actual data, consisting of time and distances for long baselines, would be regressed to develop a best fit through all the points. The most common meaning of best fit is the line that minimises the square of the vertical distances from each data point to the line of regression. The term least squares (LS) is used to describe this fitting technique (Baarda, 1999).

Available calculation - [18.2]____ Baseline Crossing Application:

A_02BASX.FOR: _________ baseline crossing technique with least squares (LS) curve fit

Baseline-Crossing Geometry Techniques

For trilateration network control point extension the so-called baseline-crossing technique was a special method of long line measurement widely used between 1940 and 1990. In this procedure, the recording master-station is either shipborne or airborne. Two slave-instruments stations are positioned ashore at Sta. P_1 and Sta. P_2, respectively (Figure 36).

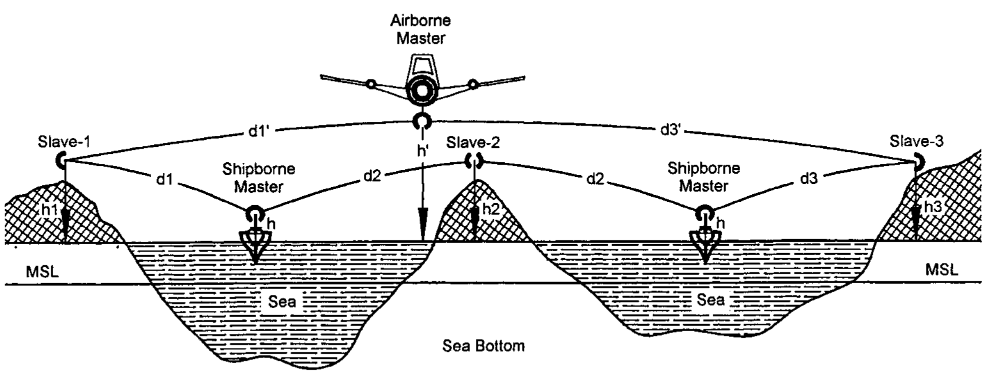


Figure 36: Airborne and shipborne baseline crossing

Continuously, the distances D_1 and D_2 are measured between the master and both slaves at short time intervals. The survey vessel crosses the baseline many times.

Note

Required is a regression technique that minimises the sum of squared deviations between the regression line and the actual data. The minimum sum distance calculated by means of a least squares curve fitting procedure is required, not the shortest observed sum distance. The uninitiated user of statistical models should review this section carefully before drawing any conclusion.

All sets recorded contain elapsed time, X, and summations of distances, Y. The summations of these distances

D_1 and D_2 can be assumed to reach the minimum point of a parabola (Y_{MIN}) when the vessel is exactly at the baseline being measured. Consequently, angle θ between the master and two slave stations is exactly π (180°). The outline of this method is illustrated in (Figure 37).

A second-degree polynomial will estimate the coefficients of such a parabolic curve when four or more points are given:

$$Y = +a + b\,X + c\,X^2 \tag{5.46}$$

The more data points n in a calculation - preferable between 25 and 40 observations - the closer the estimate of a, b and c will be to the expected values (Laurila, 1976, 1983).

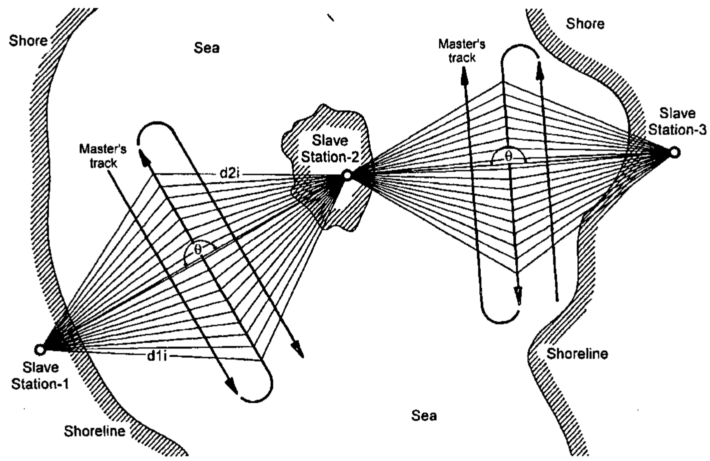


Figure 37: Shipborne baseline crossing technique

Scaling

Most computers are capable of storing fifteen digits in memory. This limitation presents a problem if the less significant digits are lost during calculation. The problem can be particularly acute when one variable is much larger than another one, or when one variable changes much more rapidly than another. A special technique is often used in such cases to maintain maximum accuracy. It consists of subtracting a common value $\bar{Y}$ from each Y_i value before entering the data in such a way that the values of ΔY are all small but positive or equal to zero:

$$\Delta Y_i = +Y_i - \bar{Y} \tag{5.47}$$

To find the values of a, b, and c the general equation of observations (5.46) can be rewritten. After the coefficients have been determined, the value ΔY is then added to the general equation for Y.
X designates the time at which measurements are recorded (Table 13):

$$\Delta Y = +a + b\,X + c\,X^2 \tag{5.48}$$

in which a, b, and c are constants that can be determined by a LS curve-fitting adjustment.

The value of X that causes the function ΔY (5.48) to be a minimum is found by forming the first derivative of this function by setting it equal to zero:

$$\frac{\delta \Delta Y}{\delta x} = +2\,c\,X + b = 0 \tag{5.49}$$

When solved for X, (5.49) gives:

$$X = +\frac{-b}{2\,c} \tag{5.50}$$

The minimum value of ΔY is now found by substituting the value X of (5.50) into (5.48):

$$\Delta Y_{min} = +a + \frac{-b^2}{4c} \tag{5.51}$$

A general equation represents a parabolic curve that fits the data and gives the minimum baseline length. After the coefficients a, b and c are computed, a best-fit curve for the expression is available (Laurila, 1976).

Using a Second Degree Polynomial

The following formulae will estimate the coefficients of such a curve when four or more points are given. X and Y may be positive, or equal to zero:

$$w01 = \Sigma X_i \qquad w04 = \Sigma Y_i^2 \qquad w07 = \Sigma X_i^2 Y_i$$
$$w02 = \Sigma X_i^2 \qquad w05 = \Sigma X_i Y_i \qquad w08 = \Sigma X_i^3$$
$$w03 = \Sigma Y_i \qquad w06 = n \qquad w09 = \Sigma X_i^4$$

in which X, Y values are associated with each data point, and n is the total number of points (Kolb, 1983). The following terms must now be computed to obtain the coefficients a, b and c of the equation:

$$w10 = w02 \times w06 - w01 \times w01 \tag{5.52}$$
$$w11 = w06 \times w07 - w02 \times w03 \tag{5.53}$$
$$w12 = w06 \times w08 - w01 \times w02 \tag{5.54}$$
$$w13 = w05 \times w06 - w01 \times w03 \tag{5.55}$$
$$w14 = w06 \times w09 - w02 \times w02 \tag{5.56}$$

The coefficients a, b and c of the best fit curve are:

$$c = w15 = (w10 \times w11 - w12 \times w13) / (w10 \times w14 - w12^2) \tag{5.57}$$
$$b = w16 = (w13 - w12 \times w15) / w10 \tag{5.58}$$
$$a = w17 = (w03 - w16 \times w01 - w15 \times w02) / w06 \tag{5.59}$$

There are a number of ways to evaluate how good the fit is, each with its own advantages and limitations. In science, most commonly used is to measure the goodness of fit (GOF), or the coefficient of determination. The square root of GOF is called the correlation coefficient (CC).

GOF is calculated from the formula

$$GOF = \frac{w17 \times w03 + w16 \times w05 + w15 \times w07 - w03^2 / w06}{w04 - (w03 \times w03) / w06} \tag{5.60}$$

$$w18 = -w16 / (2 \times w15) = \text{position in time at minimum sum distance} \tag{5.61}$$
$$w19 = +w17 - (w16)^2 / (4 \times w15) = \text{minimum sum distance} \tag{5.62}$$

Following expression gives the error equation

$$v = +a + b X + c X^2 - \Delta Y \tag{5.63}$$

in which calculated minus observed values (C–O) are v

$$w20 = \Sigma v \tag{5.64}$$
$$w21 = \Sigma v v \tag{5.65}$$

are used to find the standard deviation (SD).

$$SD = \sqrt{\frac{n \Sigma v^2 - (\Sigma v)^2}{n(n-1)}} \tag{5.66}$$

5.3.1 Baseline-Crossing

Baseline-crossing data are given in (Table 13). The calculated coefficients of the equations and the goodness of fit are given in (Table 14).

Coefficients of the Parabola

a	=	+.6430157327505234E+00	from ... (5.59)
b	=	−.5089120971709459E−02	from ... (5.58)
c	=	+.1365305814433093E−02	from ... (5.57)

Substituting these coefficients into the general equation (5.48) gives the best fit. Hereafter, the final minimum sum distance is calculated using (5.51).

Computed Vertex

position in time	w18	=	X	=	1.86 s	from ... (5.61)
min. sum distance	w19	=	ΔY_{min}	=	638.3 m	from ... (5.62)

time of position	observed distance	time of position	observed distance
0.1	643.2	2.1	639.8
0.2	643.4	2.2	640.5
0.3	641.6	2.3	639.3
0.4	640.7	2.4	637.9
0.5	639.0	2.5	638.3
0.6	639.1	2.6	639.6
0.7	640.5	2.7	640.1
0.8	640.6	2.8	640.0
0.9	639.7	2.9	639.7
1.0	638.9	3.0	638.5
1.1	639.7	3.1	638.9
1.2	640.1	3.2	641.9
1.3	639.1	3.3	642.5
1.4	637.7	3.4	641.4
1.5	637.3	3.5	640.7
1.6	638.9	3.6	642.7
1.7	638.9	3.7	644.4
1.8	637.0	3.8	643.8
1.9	636.4	3.9	642.8
2.0	637.1	4.0	643.9

Table 13: Baseline-crossing data

calculation of coefficients and the goodness of fit					
w01	=	+ .8200E+02	w10	=	+ .2132E+04
w02	=	+ .2214E+03	w11	=	+ .7543E+01
w03	=	+ .2561E+02	w12	=	+ .8741E+04
w04	=	+ .1639E+02	w13	=	+ .1084E+01
w05	=	+ .5252E+02	w14	=	+ .3811E+05
w06	=	+ .4000E+02	w15	=	+ .1365E−02
w07	=	+ .1419E+03	w16	=	− .5089E−02
w08	=	+ .6724E+03	w17	=	+ .6430E+00
w09	=	+ .2178E+04	GOF	=	+ .7362E+00

Table 14: Calculation of coefficients and the GOF

Baseline-Crossing Statistics

coefficient of determination	GOF	=	0.74 ________________	from ... (5.60).
standard deviation	SD	=	1.03 ________________	from ... (5.66).

in which the SD is a measure of how widely values are dispersed from the average value, the mean.

$$Y_{min} = \bar{Y} + \Delta Y_{min} \tag{5.67}$$

Consequently:

$$Y_{min} = 21\,000.0 + 638.2734\text{ m} = 21\,638.3\text{ m at } 1.86\text{ s}$$ _____ from ... (5.67).

Geometric and Instrumental Corrections

Apply separately geometric and instrumental corrections due to the refraction of atmosphere to find the ellipsoidal distances $S_{1\text{-}2}$ between the stations, such as:

- cyclic zero error in EDM equipment
- geometrical considerations
- ground conductivity
- ground reflection errors
- ground wave propagation
- influence by the signal-to-noise ratio
- instrumentation errors in transmitting equipment
- instrumentation errors in user receiving equipment
- microwave-velocity - index of refraction
- refractive index n as a function of temperature, pressure, and humidity along the ray path.

Some assurance is required that the parabolic line resulting from our regression analysis is a reasonable approximation or good fit (Kolb, 1983.Laurila, 1976, 1983).

In general, a model is only valid over the range of input data. The curve with a small value of the standard deviation (SD) is purely a function of the original values of time and observed baseline. Moreover, a low SD does not prove causality and does not guarantee getting a good fit. Therefore, the curve is plotted along with X-Y data as a visual check of the calculations (Figure 38; Figure 39).

Conventional pulsing marine navigation systems, such as HIRAN, SHORAN, and compact, lightweight in-phase-comparison electronic distance surveying equipment were deployed to measure distances, using rotary, mechanical counters. Jack Kilby and Jean Hoern are usually credited with having developed the concept of integrating device and solid-state electronic circuit elements onto a single silicon chip.

Development of Solid-State Distance Counters

Because the more reliable, very-large-scale-integrated (VLSI) circuits did not burn out in service, all mechanical counters were replaced by electronic VLSI circuits. This fact has revolutionised the development, and the need for the integrated circuit (IC) as used in various geodetic and hydrographic instruments. Robert Noyce is given credit for having conceived the integration of the separate elements on a single IC.

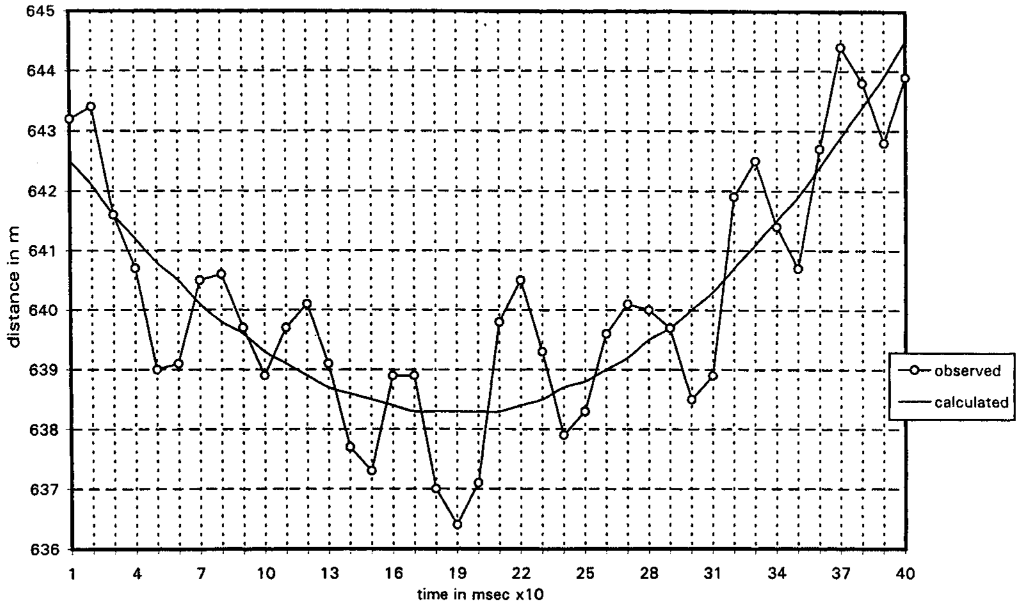


Figure 38: Determination of the shortest LS sum distance of Baseline A1

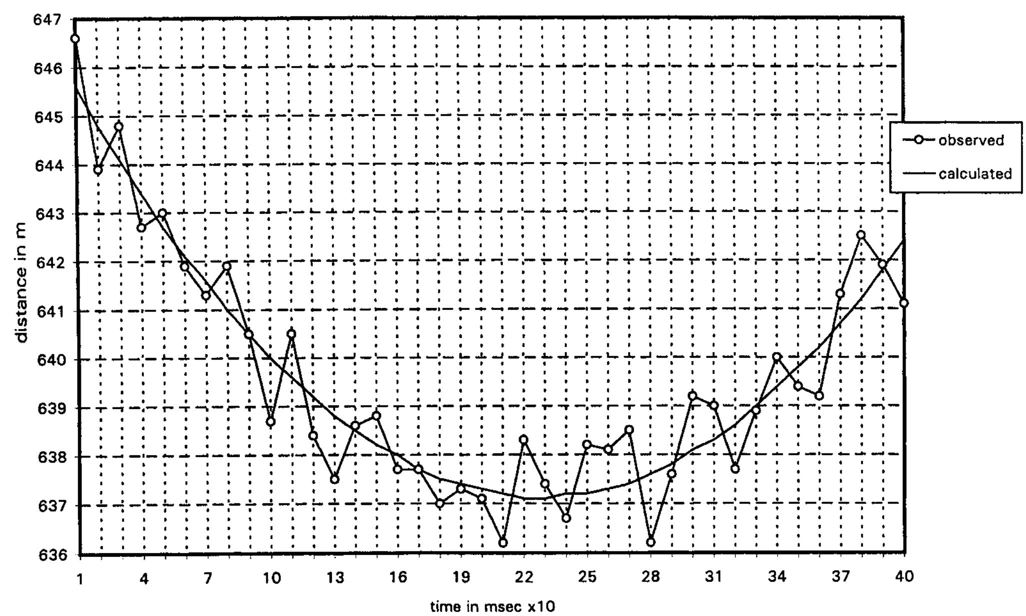


Figure 39: Determination of the shortest LS sum distance of Baseline A2

5.4 Spherical Trigonometry

No attempt is here made to deal fully with the subject of spherical trigonometry. It is attempted only to portray the matter dealing with the geodetic work of Gauss-Schreiber.

Plane trigonometry [5.1] deals with the properties of figures situated wholly in one plane. Spherical trigonometry, on the other hand, deals with the properties of figures drawn on the surface of a sphere. Sides of the figures being situated in three or more planes pass through one point, thus enclosing a pyramidal space. If the base of the pyramid is a polygon, the enclosed pyramidal space may be divided into several pyramids, each of which bounded by three planes containing one common point. Consequently, a triangular pyramid of this kind is called a spherical triangle (Figure 40; Figure 41).

As spherical trigonometry is not abundantly illustrated in most textbooks, it has been thought desirable to introduce a brief description of the subject and to give the important formulae.

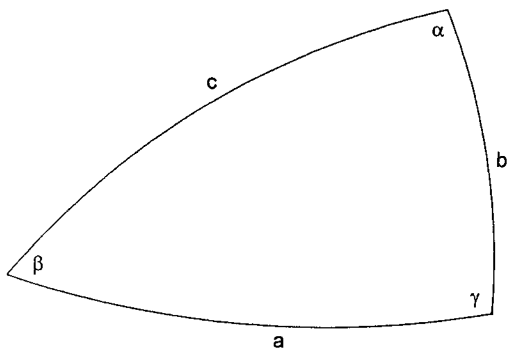


Figure 40: General spherical triangle

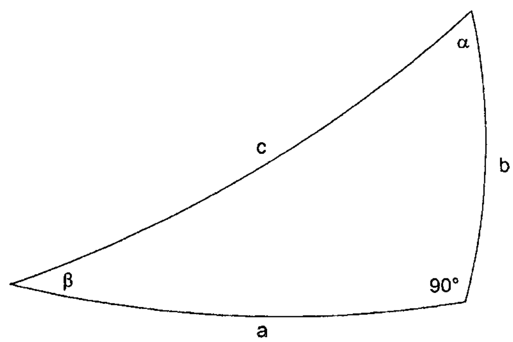


Figure 41: Rectangular spherical triangle

Some formulae are given, pp 100.

Gaussian Sphere

The Gaussian sphere is called a surface of constant curvature. At this point, the abbreviation GS is used for so-called Gauss-Schreiber type (GS) conformal double projection. The GS projection from an ellipsoid to the Gaussian sphere, and the conformal projection from the Gaussian sphere to the plane, and vice-versa, was conceived by Gauss, and described by Schreiber as a conformal projection [10.2, 18.16] (Schreiber, 1897).

Using the GS type conformal projection, Rosenmund's oblique Mercator (ROM) projection of Switzerland [10.6, 18.21] was described by Max (Rosenmund, 1903), and the Oblique stereographic conformal projection (OSC) of the Netherlands [10.7, 18.22] was devised by Hendrik J. Heuvelink (Heuvelink, 1918).

Aposphere

Using an aposphere for geodetic purposes, Brigadier Martin Hotine developed the Rectified Skew Orthomorphic (RSO) projection for Borneo, Malaysia, and the Hotine oblique Mercator (HOM) projection for the US SPCS Alaska zone 10 on the ellipsoid [10.5, 18.20] (Hotine, 1946-1947).

5.4.1 Formulae in Spherical Trigonometry

If α, β and γ are the three angles and a, b, and c the opposite sides:

Delambre Formulae

$$\cos \tfrac{1}{2} c \sin \tfrac{1}{2} (\alpha + \beta) = \cos \tfrac{1}{2} \gamma \cos \tfrac{1}{2} (a - b) \tag{5.68}$$
$$\sin \tfrac{1}{2} c \sin \tfrac{1}{2} (\alpha - \beta) = \cos \tfrac{1}{2} \gamma \sin \tfrac{1}{2} (a - b) \tag{5.69}$$
$$\cos \tfrac{1}{2} c \cos \tfrac{1}{2} (\alpha + \beta) = \sin \tfrac{1}{2} \gamma \cos \tfrac{1}{2} (a + b) \tag{5.70}$$
$$\sin \tfrac{1}{2} c \cos \tfrac{1}{2} (\alpha - \beta) = \sin \tfrac{1}{2} \gamma \sin \tfrac{1}{2} (a + b) \tag{5.71}$$

Neper Formulae

$$\tan \tfrac{1}{2} (\beta + \gamma) = \cot \tfrac{1}{2} \alpha \frac{\cos \tfrac{1}{2} (b - c)}{\cos \tfrac{1}{2} (b + c)} \tag{5.72}$$
$$\tan \tfrac{1}{2} (\beta - \gamma) = \cot \tfrac{1}{2} \alpha \frac{\sin \tfrac{1}{2} (b - c)7}{\sin \tfrac{1}{2} (b + c)} \tag{5.73}$$
$$\tan \tfrac{1}{2} (b + c) = \tan \tfrac{1}{2} a \frac{\cos \tfrac{1}{2} (\beta - \gamma)}{\cos \tfrac{1}{2} (\beta + \gamma)} \tag{5.74}$$
$$\tan \tfrac{1}{2} (b - c) = \tan \tfrac{1}{2} a \frac{\sin \tfrac{1}{2} (\beta - \gamma)}{\sin \tfrac{1}{2} (\beta + \gamma)} \tag{5.75}$$

Formulae for Rectangular Triangles

In case of a right-angled triangle, let α, β, and γ be the angles of a triangle, right-angled at γ, and let a, b, and c be the sides of the triangle opposite to the angles α, β, and γ, respectively.

$$\sin a = \sin c \sin \alpha \tag{5.76}$$
$$\sin b = \tan a \cot \alpha \tag{5.77}$$
$$\sin b = \sin c \sin \beta \tag{5.78}$$
$$\cos c = \cos a \cos b \tag{5.79}$$
$$\cos \alpha = \cos a \sin \beta \tag{5.80}$$
$$\cos \alpha = \cot c \tan b \tag{5.81}$$
$$\cos \beta = \cos b \sin \alpha \tag{5.82}$$
$$\cos \beta = \cot c \tan a \tag{5.83}$$
$$\tan a = \tan \alpha \sin b \tag{5.84}$$
$$\tan b = \tan \beta \sin a \tag{5.85}$$
$$\tan a = \tan c \cos \beta \tag{5.86}$$
$$\tan b = \tan c \cos \alpha \tag{5.87}$$
$$\cot \beta = \cot b \sin a \tag{5.88}$$
$$\sin \alpha = \frac{\sin a}{\sin c} \tag{5.89}$$
$$\cos \alpha = \frac{\tan b}{\tan c} \tag{5.90}$$
$$\tan \alpha = \frac{\tan a}{\sin b} \tag{5.91}$$
$$\sin \beta = \frac{\sin b}{\sin c} \tag{5.92}$$
$$\cos \beta = \frac{\tan a}{\tan c} \tag{5.93}$$
$$\tan \beta = \frac{\tan b}{\sin a} \tag{5.94}$$

Formulae for General Triangles

$$\sin\alpha = \frac{\sin a}{\sin c} \tag{5.95}$$

$$\cos\alpha = \frac{\tan b}{\tan c} \tag{5.96}$$

$$\tan\alpha = \frac{\tan a}{\sin b} \tag{5.97}$$

$$\sin\beta = \frac{\sin b}{\sin c} \tag{5.98}$$

$$\cos\beta = \frac{\tan a}{\tan c} \tag{5.99}$$

$$\tan\beta = \frac{\tan b}{\sin a} \tag{5.100}$$

$$\sin a \sin\gamma = \sin\alpha \sin c \tag{5.101}$$

$$\sin b \sin\gamma = \sin\beta \sin c \tag{5.102}$$

$$\cos\gamma + \cos\alpha\cos\beta = \cos c \sin\alpha \sin\beta \tag{5.103}$$

$$\cos\alpha + \cos\beta\cos\gamma = \cos a \sin\beta \sin\gamma \tag{5.104}$$

$$\cos\beta + \cos\alpha\cos\gamma = \cos b \sin\alpha \sin\gamma \tag{5.105}$$

Sine Formulae

$$\frac{\sin a}{\sin\alpha} = \frac{\sin b}{\sin\beta} = \frac{\sin c}{\sin\gamma} \tag{5.106}$$

Cosine Formulae

$$\cos a = \cos b \cos c + \sin b \sin c \cos\alpha \tag{5.107}$$

$$\cos b = \cos c \cos a + \sin c \sin a \cos\beta \tag{5.108}$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos\gamma \tag{5.109}$$

$$\cos\alpha = -\cos\beta\cos\gamma + \sin\beta\sin\gamma\cos a \tag{5.110}$$

$$\cos\beta = -\cos\gamma\cos\alpha + \sin\gamma\sin\alpha\cos b \tag{5.111}$$

$$\cos\gamma = -\cos\alpha\cos\beta + \sin\alpha\sin\beta\cos c \tag{5.112}$$

$$\cos\alpha = \frac{\cos a - \cos b \cos c}{\sin b \sin c} \tag{5.113}$$

$$\cos\beta = \frac{\cos b - \cos c \cos a}{\sin c \sin a} \tag{5.114}$$

$$\cos\gamma = \frac{\cos c - \cos a \cos b}{\sin a \sin b} \tag{5.115}$$

$$\cos b = \frac{\cos\beta + \cos\gamma\cos\alpha}{\sin\gamma\sin\alpha} \tag{5.116}$$

Tangent formulae

$$\frac{\tan \frac{1}{2}(b+c)}{\tan \frac{1}{2}(b-c)} = \frac{\tan \frac{1}{2}(\beta+\gamma)}{\tan \frac{1}{2}(\beta-\gamma)} \tag{5.117}$$

Cotangent Formulae

$$\cot\alpha = \frac{\sin c / \tan a - \cos c \cos\beta}{\sin\beta} \tag{5.118}$$

$$\cot\beta = \frac{\sin c / \tan b - \cos c \cos\alpha}{\sin\alpha} \tag{5.119}$$

$$\cot\gamma = \frac{\sin a / \tan c - \cos a \cos\beta}{\sin\beta} \tag{5.120}$$

$$\cot a \sin b = \cos b \cos\gamma + \sin\gamma \cot\alpha \tag{5.121}$$

$$\cot a \sin c = \cos c \cos \beta + \sin \beta \cot \alpha \qquad (5.122)$$
$$\cot b \sin c = \cos c \cos \alpha + \sin \alpha \cot \beta \qquad (5.123)$$
$$\cot b \sin a = \cos a \cos \gamma + \sin \gamma \cot \beta \qquad (5.124)$$
$$\cot c \sin a = \cos a \cos \beta + \sin \beta \cot \gamma \qquad (5.125)$$
$$\cot c \sin b = \cos b \cos \alpha + \sin \alpha \cot \gamma \qquad (5.126)$$

Five Element Formulae

$$\sin a \cos \beta = \cos b \sin c - \sin b \cos c \cos \alpha \qquad (5.127)$$
$$\sin a \cos \gamma = \cos c \sin b - \sin c \cos b \cos \alpha \qquad (5.128)$$
$$\sin b \cos \gamma = \cos c \sin a - \sin c \cos a \cos \beta \qquad (5.129)$$
$$\sin b \cos \alpha = \cos a \sin c - \sin a \cos c \cos \beta \qquad (5.130)$$
$$\sin c \cos \alpha = \cos a \sin b - \sin a \cos b \cos \gamma \qquad (5.131)$$
$$\sin c \cos \beta = \cos b \sin a - \sin b \cos a \cos \gamma \qquad (5.132)$$

$$\sin \alpha \cos b = \cos \beta \sin \gamma + \sin \beta \cos \gamma \cos a \qquad (5.133)$$
$$\sin \alpha \cos \beta = \cos b \sin \gamma - \sin \beta \cos \alpha \cos c \qquad (5.134)$$
$$\sin \beta \cos c = \cos \gamma \sin \alpha + \sin \gamma \cos \alpha \cos b \qquad (5.135)$$
$$\sin \beta \cos \gamma = \cos c \sin \alpha - \sin \gamma \cos \beta \cos a \qquad (5.136)$$
$$\sin \gamma \cos a = \cos \alpha \sin \beta + \sin \alpha \cos \beta \cos c \qquad (5.137)$$
$$\sin \gamma \cos \alpha = \cos a \sin \beta - \sin \alpha \cos \gamma \cos b \qquad (5.138)$$

$$\sin \alpha \cos c = \cos \gamma \sin \beta + \sin \gamma \cos \beta \cos a \qquad (5.139)$$
$$\sin \beta \cos a = \cos \alpha \sin \gamma + \sin \alpha \cos \gamma \cos b \qquad (5.140)$$
$$\sin \gamma \cos b = \cos \beta \sin \alpha + \sin \beta \cos \alpha \cos c \qquad (5.141)$$

Six Element Formula

$$\sin b \sin c + \cos b \cos c \cos \alpha = \sin \beta \sin \gamma - \cos \beta \cos \gamma \cos a \qquad (5.142)$$

In any spherical triangle having sides a, b, and c, and angles α, β, and γ.

- $\alpha + \beta + \gamma$ is less than 3 π and greater than π.
- any angle α is less than π, and any side a is less than π r.
- a + b - c is less than 2 π r, and a + b + c is less than 2 π
- a + b is greater than c, and a – b is less than c.
- if a + b = π, then $\alpha + \beta = \pi$.
- if a is $\geq$ or $<$ b, then will α be or $<$ β.

If a general equation be established between the sides and angles of a spherical triangle, a true result is obtained if in the equation the supplements of the sides and of the angles respectively be written for the angles and sides, which enter into the equation.

Goniometrical Conversions

$$\sin^2 \alpha = \tfrac{1}{2} (1 - \cos 2\alpha) \qquad (5.143)$$
$$\sin^3 \alpha = \tfrac{1}{4} (3 \sin \alpha - \sin 3\alpha) \qquad (5.144)$$
$$\sin^4 \alpha = {}^1/_8 (3 - 4 \cos 2\alpha + \cos 4\alpha) \qquad (5.145)$$
$$\sin^5 \alpha = {}^1/_{16} (10 \sin \alpha - 5 \sin 3\alpha + \sin 5\alpha) \qquad (5.146)$$
$$\sin^6 \alpha = {}^1/_{32} (10 - 15 \cos 2\alpha) +6 \cos 4\alpha - \cos 6\alpha) \qquad (5.147)$$
$$\sin^7 \alpha = {}^1/_{64} (35 \sin \alpha - 21 \sin 3\alpha) + 7 \sin 5\alpha - \sin 7\alpha) \qquad (5.148)$$

$$\cos^2 \alpha = \tfrac{1}{2}.(1 + \cos 2\alpha) \qquad (5.149)$$
$$\cos^3 \alpha = \tfrac{1}{4} (3 \cos \alpha + \cos 3\alpha) \qquad (5.150)$$
$$\cos^4 \alpha = {}^1/_8 (3 + 4 \cos 2\alpha + \cos 4\alpha) \qquad (5.151)$$
$$\cos^5 \alpha = {}^1/_{16} (10 \cos \alpha + 5 \cos 3\alpha + \cos 5\alpha) \qquad (5.152)$$
$$\cos^6 \alpha = {}^1/_{32} (10 + 15 \cos 2\alpha + 6 \cos 4\alpha + \cos 6\alpha) \qquad (5.153)$$
$$\cos^7 \alpha = {}^1/_{64} (35 \cos \alpha + 21 \cos 3\alpha + 7 \cos 5\alpha) + \cos 7\alpha) \qquad (5.154)$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha \tag{5.155}$$
$$\sin 3\alpha = 3 \sin \alpha \cos^2 \alpha - \sin^3 \alpha \tag{5.156}$$
$$\sin 4\alpha = 4 \sin \alpha \cos^3 \alpha - 4 \sin^3 \alpha \cos \alpha \tag{5.157}$$
$$\sin 5\alpha = 5 \sin \alpha \cos^4 \alpha - 10 \sin^3 \alpha \cos^2 \alpha + \sin^5 \alpha \tag{5.158}$$
$$\sin 6\alpha = 6 \sin \alpha \cos^5 \alpha - 20 \sin^3 \alpha \cos^3 \alpha + 6 \sin^5 \alpha \cos \alpha \tag{5.159}$$
$$\sin 7\alpha = 7 \sin \alpha \cos^6 \alpha - 35 \sin^3 \alpha \cos^4 \alpha + 21 \sin^5 \alpha \cos^2 \alpha - \sin^7 \alpha \tag{5.160}$$
$$\sin 8\alpha = 8 \sin \alpha \cos^7 \alpha - 56 \sin^3 \alpha \cos^5 \alpha + 56 \sin^5 \alpha \cos^3 \alpha - 8 \sin^7 \alpha \cos \alpha \tag{5.161}$$
$$\sin 9\alpha = 9 \sin \alpha \cos^8 \alpha - 84 \sin^3 \alpha \cos^6 \alpha + 126 \sin^5 \alpha \cos^4 \alpha - 36 \sin^7 \alpha \cos^2 \alpha + \sin^9 \alpha \tag{5.162}$$
$$\sin 10\alpha = 10 \sin \alpha \cos^9 \alpha - 120 \sin^3 \alpha \cos^7 \alpha + 252 \sin^5 \alpha \cos^5 \alpha - 120 \sin^7 \alpha \cos^3 \alpha + 10 \sin^9 \alpha \cos \alpha \tag{5.163}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \tag{5.164}$$
$$\cos 3\alpha = \cos^3 \alpha - 3 \cos \alpha \sin^2 \alpha \tag{5.165}$$
$$\cos 4\alpha = \cos^4 \alpha - 6 \cos^2 \alpha \sin^2 \alpha + \sin^4 \alpha^4 \tag{5.166}$$
$$\cos 5\alpha = \cos^5 \alpha - 10 \cos^3 \alpha \sin^2 \alpha + 5 \cos \alpha \sin^4 \alpha \tag{5.167}$$
$$\cos 6\alpha = \cos^6 \alpha - 15 \cos^4 \alpha \sin^2 \alpha + 15 \cos^2 \alpha \sin^4 \alpha - \sin^6 \alpha \tag{5.168}$$
$$\cos 7\alpha = \cos^7 \alpha - 21 \cos^5 \alpha \sin^2 \alpha + 35 \cos^3 \alpha \sin^4 \alpha - 7 \cos \alpha \sin^6 \alpha \tag{5.169}$$
$$\cos 8\alpha = \cos^8 \alpha - 28 \cos^6 \alpha \sin^2 \alpha + 70 \cos^4 \alpha \sin^4 \alpha - 28 \cos^2 \alpha \sin^6 \alpha + \sin(\alpha)^8 \tag{5.170}$$
$$\cos 9\alpha = \cos^9 \alpha - 36 \cos^7 \alpha \sin^2 \alpha + 126 \cos^5 \alpha \sin^4 \alpha - 84 \cos^3 \alpha \sin^6 \alpha + 9 \cos \alpha \sin^8 \alpha \tag{5.171}$$
$$\cos 10\alpha = \cos^{10} \alpha - 45 \cos^8 \alpha \sin^2 \alpha + 210 \cos^6 \alpha \sin^4 \alpha - 210 \cos^4 \alpha \sin^6 \alpha + 45 \cos^2 \alpha \sin^8 \alpha - \sin^{10} \alpha \tag{5.172}$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1 \tag{5.173}$$
$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha \tag{5.174}$$
$$\cos 4\alpha = 8 \cos^4 \alpha - 8 \cos^2 \alpha + 1 \tag{5.175}$$
$$\cos 5\alpha = 16 \cos^5 \alpha - 20 \cos^3 \alpha + 5 \cos \alpha \tag{5.176}$$
$$\cos 6\alpha = 32 \cos^6 \alpha - 48 \cos^4 \alpha + 18 \cos^2 \alpha - 1 \tag{5.177}$$
$$\cos 7\alpha = 64 \cos^7 \alpha - 112 \cos^5 \alpha + 56 \cos^3 \alpha - 7 \cos \alpha \tag{5.178}$$
$$\cos 8\alpha = 128 \cos^8 \alpha - 256 \cos^6 \alpha + 160 \cos^4 \alpha - 32 \cos^2 \alpha + 1 \tag{5.179}$$

Series Expansions

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \tag{5.180}$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^3}{4!} - \frac{x^5}{6!} + \ldots \tag{5.181}$$
$$\tan x = x + \frac{x^3}{3} + \frac{2\,x^5}{15} + \frac{17\,x^7}{315} + \frac{62\,x^9}{2835} + \frac{1382\,x^{11}}{155925} + \ldots \tag{5.182}$$
$$\operatorname{cotan} x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2\,x^5}{945} \ldots \tag{5.183}$$
$$\sec x = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \ldots \tag{5.184}$$
$$\operatorname{cosec} x = \frac{1}{x} + \frac{x}{6} - \frac{7\,x^3}{360} + \ldots \tag{5.185}$$

(Baeyer, 1862a)

5.5 Area Calculation

Area Calculation using Graphics

A frequently occurring problem in cadastral or hydrographic surveys is an area determined by given points on its boundary. Judicial area determinations are also an integral part of concession surveys related to oil, gas, and mineral resources. Expressing areas is usual in square metres, or square kilometres. In determining large areas by graphical methods, an equal area map projection is necessary, using the true scale of the map, scale of projection, reduction to mean sea level (MSL), and corrections due to shrinkage of the map (Bannister, 1994; Bjørke, 2006; Kimerling, 1984).

Various area determinations by graphical methods are:

- dividing the figure into triangles equalising curvilinear boundaries
- using squared tracing paper: counting the squares and partial squares will give the area directly
- the parallelogram method by dividing a figure by a series of equidistant lines. The length of each line intercepted between the boundary is scaled, the sum of the lengths multiplied by the invariable width will give the area
- the trapezoidal method by breaking the area into a number of trapezoids
- guiding a planimeter-pointer is around the area enables the measurement of areas by a counting mechanism.

Computing the horizontal areas of land parcels, concession areas, et cetera, is a subject of concern to land- and marine surveyors, geologists, and estate planners.

In surveying, two methods are commonly used to compute small areas:

- when bearings and distances of lines have been measured, the plane coordinates of each boundary corner and its area are calculated
- if the plane coordinates of all boundary corner are known, the coordinate method of area calculation is employed.

Geodetic Coordinates Method

Computing surface area from geodetic (latitude and longitude) coordinates is similar to area computation from rectangular (easting and northing) coordinates in that the concept of adding and subtracting geometric shapes underlies both. Trapezoids are summed for rectangular coordinates, whereas spherical or ellipsoidal triangles are created and summed from geodetic coordinates (Kimerling, 1984). A direct method by computer for calculating the areas from geodetic coordinates along its boundary is described in the next section. The accuracy at various locations between 0° and 84°N or S on the ellipsoid are better than 99.999998 percent in any case.

Area Calculation Using Computers

Area determination using coordinated points on the concession boundary. Planar or curvilinear coordinates may be used on the sphere or ellipsoid. Various methods of determining areas from coordinates are, e.g.:

- graphical computer aided design (CAD) programs
- special formulae using planar or curvilinear coordinates, including Gauss-Schreiber (GS) type projection system.

In this section special formulae are given. The programs use formulae, which by virtue of its inherent simplicity are strongly recommended for computing of spherical and ellipsoidal areas. Personal computer (PC) methods are described for calculating the areas of any surface feature on Earth directly from geodetic coordinates along its boundary.

The procedure on a computer is elucidated by different methods of using programs:

A_05ARSQ.FOR [18.5]

Input: geodetic latitude and longitude of two stations.
Output: the ellipsoidal area in sq. m or sq. km

A_06ARPY.FOR [18.6]

Input: geocentric latitude, longitude and distances, and closing station.
Output: spherical area in sq. m or sq. km. To compute an ellipsoidal area in sq. metres or sq. km: first convert all input data, using program A_16GAUS.FOR [18.16]

Area Computation

Symbols and definitions. All angles are expressed in radians.

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
f	flattening of the ellipsoid
B_m	mean parallel of latitude of the sphere
L_m	mean meridian of the sphere
Δb	$b_2 - b_1$ - difference in lat. of a Gaussian sphere
Δl	$l_2 - l_1$ - difference in lon. of a Gaussian sphere
φ_U	upper parallel of geodetic lat. of N/E point, positive north
λ_U	meridian of geodetic lon. of N/E point, positive east
φ_L	lower parallel of geodetic lat. of S/W point, positive north
λ_L	meridian of geodetic lon. of S/W point, positive east
$\Delta\varphi$	$\varphi_2 - \varphi_1$ - difference in latitude
$\Delta\lambda$	$\lambda_2 - \lambda_1$ - difference in longitude
e^2	first eccentricity squared
e'^2	second eccentricity squared
R_{00}	radius of curvature in the Meridian
R_{90}	radius of curvature in the Prime Vertical
R_m	mean geometric radius of curvature
R_A	radius of the Gaussian sphere
r_0	mean geometric radius of curvature scaled to the grid
E_E	ellipsoidal area of the reference ellipsoid of the entire figure of the Earth
E_U	area on the reference ellipsoid, bounded by the Equator and an upper latitude
E_{SQ}	area on the reference ellipsoid, bounded by two latitudinal and longitudinal coordinates

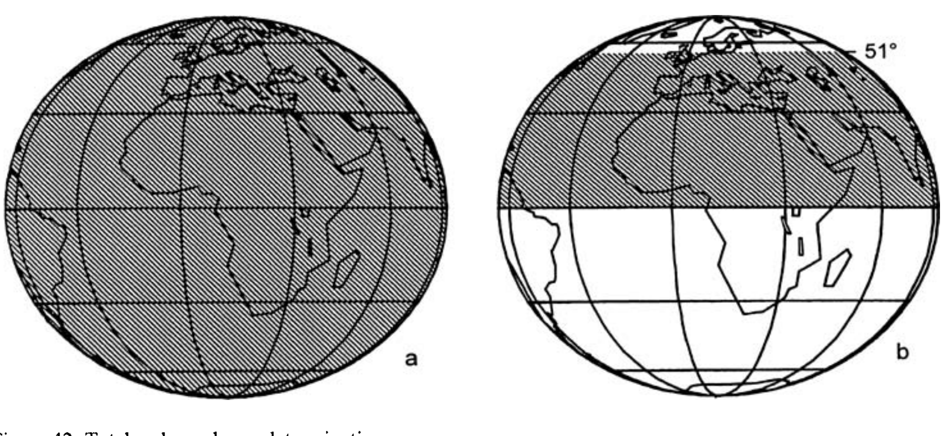


Figure 42: Total and zonal area determination

Ellipsoidal Area of the Earth

Input: basic ellipsoid data a and f^{-1} (IN BOLD).

Output: calculated ellipsoid parameters (Table 19): pp 123.

Calculating Ellipsoidal Constants

$$t_1 = 1 - f$$
$$b = a (1 - f) \quad (5.186)$$
$$e^2 = f (2 - f) \quad (5.187)$$
$$R_{00} = a (1 - e^2) / (1 + e^2 \sin^2 B_m)^{½} \quad (5.188)$$
$$R_{90} = a / (1 - e^2 \sin^2 B_m)^{½} \quad (5.189)$$
$$R_A = (R_{00} R_{90})^{½} = (t_1 / (1 + t_1^2 - 1) \sin^2 B_m) \quad (5.190)$$

Formulae to Calculate the Area of Earth

Equations to compute the ellipsoidal area of the entire figure of the Earth in sq. km and its mean radius:

$$E_E = 10^{-6}\, 4\, \varphi^2 \pi (1 + e^2 (2 / 3 + e^2 (3 / 5 + e^2 (4 / 7 + e^2 (5 / 9 + 6 / 11\, e^2))))) \quad (5.191)$$
$$R_m = 10^{-6} (E_E / (4 \pi))^{½} \quad (5.192)$$

Example I - using program A_05ARSQ.FOR

total area and mean radius of ellipsoid						
parameters for:		NE Hemisphere				
reference ellipsoid	:	**GRS80**	E_E	=	5100 65621.718466	sq. m
semi-major axis a	:	**6 378 137.0**	R_m	=	6371.007181	sq.km
recipr. flattening f^1	:	**298.25722 21008 827**				
upper latitude φ_U	:	**51° 00' 00" N**				(Figure 42a)

Ellipsoidal Zonal Area

Input: basic ellipsoid data a and f^{-1} (in bold).
the latitudinal geodetic coordinates of the Equator
an upper boundary North or South

Output: the ellipsoidal area E_U as defined by the Equator and φ_U - an upper boundary

Equations to compute the ellipsoidal zonal area:

$$\psi_s = \sin B_U \quad (5.193)$$
$$E_U = 10^{-6}\, 2\, b^2 \pi (\psi_s + 2 / 3\, e^2 \psi^3_s + 3 / 5\, e^4 \psi^5_s\, 4 / 7\, e^6 \psi^7_s + 5 / 9\, e^8 \psi^9_s + 6 / 11\, e^{10} \psi^{11}_s) \quad (5.194)$$

Equations to compute the ellipsoidal zonal area between the Equator and an upper-latitude (φ_U) boundary in sq. km, see shaded area (Figure 42b).

Example II - using program A_05ARSQ.FOR

Ellipsoidal zonal area bounded by max. latitude - output: zonal area				
parameters for:	NE Hemisphere			
reference ellipsoid:	**GRS80**	E_U =	1978 45570.95611 1	sq. km
semi-major axis a :	**6 378 137.0**			
recipr. flattening f^1 :	**298.25722 21008 827**			
upper latitude φ_U :	**51° 00' 00" N**			(Figure 42b)

Calculating an Ellipsoidal quadrilateral area

Input: basic ellipsoid data a and f^{-1} (in bold).
a boundary bounded between two latitudes and two longitudes through P_1 and P_2

Output: the ellipsoidal area is defined between two points P_1 (B_U, L_U) and P_2 (B_L, L_L)

Formulae for the Ellipsoidal Quadrilateral Area

Equations to compute the ellipsoidal quadrilateral area ABGH in sq. m or sq. km (Figure 43).
Two sets of geodetic coordinates are entered into the program, which returned the quadrilateral area of the reference ellipsoid in sq. km bounded between two latitudes and two longitudes, using the intersecting points P_1 (B_U, L_U)and P_2 (B_L, L_L):

$$\Delta\psi = .5\,(B_U - B_L) \tag{5.195}$$

$$\phi = .5\,(B_U + B_L) \tag{5.196}$$

$$\Delta L = 2\,b^2\,(\,(L_U - L_L)\,) \tag{5.197}$$

$$t_2 = \cos\phi \sin\Delta\psi\,[\,1 + e^2/2\,(\,1 + e^2/4\,(\,3 + e^2/2\,(\,5 + 7/8\,e^2\,(\,5 + 9/2\,e^2\,)\,)\,)\,)\,]$$

$$t_3 = \cos 3\phi \sin 3\Delta\psi\,[\,-e^2/2\,(\,1/3 + e^2/8\,(\,3 + e^2\,(\,3 + 5/4\,e^2\,(\,7/3 + 9/4\,e^2\,)\,)\,)\,)\,]$$

$$t_4 = \cos 5\phi \sin 5\Delta\psi\,[\,e^4/16\,(\,3/5 + e^2\,(\,1 + 5/4\,e^2\,(\,1 + 9/8\,e^2\,)\,)\,)\,]$$

$$t_5 = \cos 7\phi \sin 7\Delta\psi\,[\,-e^6/16\,(\,1/7 + 5/16\,e^2\,(\,1 + 3/2\,e^2\,)\,)\,]$$

$$t_6 = \cos 9\phi \sin 9\Delta\psi\,[\,e^8/256\,(\,5/9 + 3/2\,e^2\,)\,]$$

$$t_7 = \cos 11\phi \sin 11\Delta\psi\,[\,-3\,e^{10}/5632\,]$$

$$E_E = 10^{-6}\,\Delta L\,(\,t_2 + t_3 + t_4 + t_5 + t_6 + t_7\,) \tag{5.198}$$

Quadrilateral Area ABGH

The quadrilateral area ABGH can be solved by the formulae to find the ellipsoidal area of the whole concession. Given a list of coordinates, suitably reduced, the method of square blocks is convenient. Using program A_05ARSQ.FOR, two latitudinal and longitudinal geodetic coordinates (one NE boundary point and one SW boundary point) may give the quadrilateral area ABGH (Figure 43).

Example III - using program A_05ARSQ.FOR

ellipsoidal zonal rectangular area bounded by two latitudes and longitudes - output: zonal rect. area						
parameters for	:	NE Hemisphere				
reference ellipsoid:		GRS80	E_{SQ}	=	4 63493 08694.4222	sq. m
semi-major axis a	:	**6 378 137.0**	E_{SQ}	=	46349.3087	sq. km
recipr. flattening f^1	:	**298.25722 21008 827**				
latitude φ_{NE}	:	**53° 00' 00" N**				
longitude λ_{NE}	:	**3° 00' 00" E**				
latitude φ_{SW}	:	**50° 00' 00" N**				
longitude λ_{SW}	:	**1° 00' 00" E**				(Figure 43)

Area of a Spherical Irregular Area

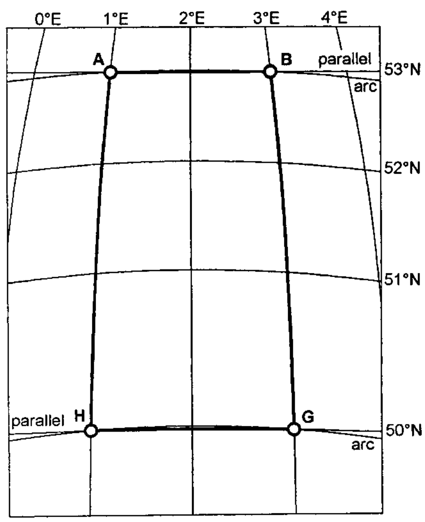


Figure 43: Ellipsoidal quadrilateral area

A method is described for calculating the area of any irregular (polygonal) surface on Earth directly from geodetic coordinates along its boundary. The polygonal method is preferred when more than four points are involved. Consider a figure whose vertices have geodetic coordinates of the spherical or ellipsoidal concession area of ABCDEF (in bold lines) as shown. This form of treatment holds well with any spherical polygon. The essential points to note are that the initial point A is repeated in the layout. The area is given in square kilometres. Consider also the area FEDCGH (II) with coordinates of the vertices. The program gives the *spherical coordinated area* of the polygon I and II in sq. km.

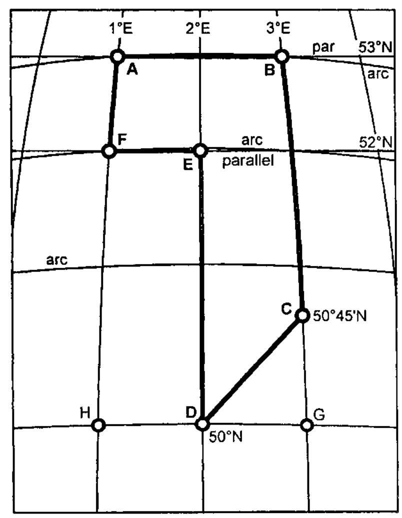


Figure 44: Spherical polygonal area

Formulae for the Spherical Polygonal Area

Input:

- basic ellipsoid data a and f^{-1} (in bold)
- choice of mean latitude in centre of polygon is imperative
- a boundary bounded by polygon points P_i (b, l), in which i is the number of points, i.e. latitudes and longitudes in general through stations P_1, …, and P_N ($=P_1$).

Output:

- the spherical area in sq. m is defined through stations P_1 (B_1, L_1), …, and P_N (B_N, L_N).

Equations to compute the spherical (or ellipsoidal) area of any irregular (polygonal) surface in sq. km. See the figure of Earth ABCDEF (Figure 44), bounded by polygon program [18.6] - A_06ARPY.FOR.

Compute constants and spherical area as given below:

$$R_A = (a (1 / f) / (1 + ((1 / f)^2 - 1) \sin^2 B_M) \quad (5.199)$$

$$E_S = 4 \pi R_A^2 \quad (5.200)$$

Compute closed polygonal area bounded by P_n (b, l) points on the Gaussian sphere (Application IV)

b_1 = first point of latitude of sphere
l_1 = first point of longitude of sphere
b_i = following point of latitude
l_i = following point of longitude

$$\Delta l_i = (l_{i+1} - l_i) \quad (5.201)$$

$$t_1 = \sin (l_i) \quad (5.202)$$

(a) repeat, until exhausted for any line along the great arc, else (b)

$$t_2 = t_2 + 2 \arctan (\tan (\Delta l / 2) \sin ((b_{i+1} + b_i) / 2) / \cos ((b_{i+1} - b_i) / 2)) \quad (5.203)$$

(b) correction for any line along the parallel instead of a great arc

$$t_2 = t_2 - (2 \arctan (t_1 \tan (\Delta l / 2)) - t_1 \Delta l) \quad (5.204)$$

$$E_S = \text{abs} (t_2) R_A^2 \text{ (in sq.m)} \quad (5.205)$$

Using geodetic coordinates of the polygon points P_i (b, l) is an efficient way to compute a closed polygonal area bounded by N {P_n (b, l)} points - for any line along the great arc - on the Gaussian sphere.

A convenient variant of this treatment is the use of the A_16GAUSS.FOR [18.16] - Gauss-Schreiber (GS) type projection formulae - to calculate the area of an ellipsoidal area.

Area of an Ellipsoidal Irregular Area

An ellipsoidal area is bounded by a polygon. First, it is necessary to run the Gaussian program [18.16] to convert all ellipsoidal latitudes, longitudes (φ, λ) into spherical latitudes, longitudes (b, l) to project an adjacent pair of coordinates of a geodetic line algebraically. In the same way, this is done for each succeeding geodetic line. Read also [10.2] about GS (Gauss-Schreiber Conformal Double Projection).

With a little experience, the whole computation of an area can be carried out with only a list of converted ellipsoid-sphere coordinates to compute an ellipsoidal area of any irregular (polygonal) surface on Figure of Earth ABCDEF of the reference ellipsoid in sq. km (Figure 44).

Hereafter, the equations to compute an ellipsoidal area of any irregular (polygonal) surface in sq. km are given in (5.199, ... , 5.205). See Polygon Program [18.6] - A_06ARPY.FOR. The projected spherical coordinates of the vertices A, B, C, D, E, F and A (start/end point) are entered.

Finally, the ellipsoidal coordinated area of the whole concession will be given in sq. km or sq. m.

Area Corrections due to a Line along a Parallel or a Great Arc

A correction to application IV and V is required for any line along a parallel instead of a great arc, area calculations) in [18.6], spherical or ellipsoidal area bounded by polygon. If the computer program detects in two different stations exactly the same latitude, the parallel correction is used and shown in the output. This is the case with line AB and line EF (Figure 44).

Example IV - using program A_06ARPY.FOR

Spherical area bounded by polygon							
input	latitudes and longitudes of sphere			output:	polygonal area (sphere)		
parameters for		:	NE Hemisphere				
reference ellipsoid		:	GRS80				
semi-major axis	a	:	**6 378 137.0**				
recipr. flattening	f^{1}	:	**298.25722 21008 827**				
latitude mean	[MN]	:	**51° 30' 00" N**				
mean radius	[RM]	=	63 82923.21805 2				
area of sphere	[sq.m]	=	51197 54123 37874.1000				
input number of sta.:			6				
latitude	B_1	:	**53° 00' 00" N**	latitude	B_4	:	**50° 00' 00" N**
longitude	L_1	:	**1° 00' 00" E**	longitude	L_4	:	**2° 00' 00" N**
latitude	B_2	:	**53° 00' 00" N**	latitude	B_5	:	**52° 00' 00" N**
longitude	L_2	:	**3° 00' 00" E**	longitude	L_5	:	**2° 00' 00" N**
latitude	B_3	:	**50° 45' 00" N**	latitude	B_6	:	**52° 00' 00" N**
longitude	L_3	:	**3° 00' 00" E**	longitude	L_6	:	**1° 00' 00" N**
E_P	=		2 77325 53215.70609 sq.m	Gaussian sphere			
			or				
E_P	=		27732.55322 sq.km	Gaussian sphere			

Note

The difference between a spherical and an ellipsoidal quantity depends on size and its use. Be aware that the lines AB and EF are often calculated as parallels and not as a part of a great circle or ellipse. The corrections applied may influence the area of the whole polygon dramatically.

When a large area - exceeding more than one hundred sq. km - is involved, the polygonal method mentioned above may be preferred, which will depend on the purpose of the computation. In most cases, the spherical polygonal area calculation is sufficient accurate for most surveys (Example IV). Nevertheless, in case the ellipsoidal polygonal area calculation method is chosen, (Example V), is the correct choice of a mean latitude crucial.

Computer area calculation methods produce satisfactory results primarily because they are modified to the ways boundary lines are usually defined and stored for reference. Technical developments have added a new dimension to boundary data definition, and new storage-boundaries are defined only as strings of geodetic coordinates (φ, λ) for future calculations.

Example V - using program A_06ARPY.FOR

Ellipsoidal area bounded by a polygon

input: converted latitudes and longitudes of ellipsoid — output: polygonal area ellipsoid

parameters for		:	NE Hemisphere
reference ellipsoid		:	GRS80
semi-major axis	a	:	**6 378 137.0**
recipr. flattening	f^1	:	**298.25722 21008 827**
mean latitude	[MN]	:	**51° 28' 00" N**
mean radius	[RM]	=	63 82898.89315 9
area of sphere	[sq.m]	=	51197 15101 37323.20

number of stations: 6 (example [18.6])

latitude	B_1	:	**52° 57' 42".5799 N**	latitude	B_4	:	**49° 57' 56".6610 N**
longitude	L_1	:	**0° 59' 58".1787 E**	longitude	L_4	:	**2° 00' 00".0000 E**
latitude	B_2	:	**52° 57' 42".5799 N**	latitude	B_5	:	**51° 57' 46".6582 N**
longitude	L_2	:	**3° 00' 01".8213 E**	longitude	L_5	:	**2° 00' 00".0000 E**
latitude	B_3	:	**50° 42' 52".6198 N**	latitude	B_6	:	**51° 57' 46".6582 N**
longitude	L_3	:	**3° 00' 01".8213 E**	longitude	L_6	:	**0° 59' 58".1787 E**

E_S = 2 77347 78356.04446 sq.m ellipsoid

or

E_S = 27734.77836 sq.km ellipsoid

6. Classical Datums and Reference Systems

6.1 Standard Units of Linear Measure

Since Eratosthenes, an essential aspect of geodetic measurement is the need to define the standard to which the measured lengths and the dimensions of the reference ellipsoid are related. These are the standard Bar O_1, the legal metre, the foot, the Russian double-toise, the Belgian toise, the Prussian standard toise, the British toise, the Indian- or Australian ten-foot standard bars, the Gunter's chain, Wiener klafter, and finally, the international metre.

The Toise

According to a Royal Act of May 16, 1766, the Peru Toise (Toise du Pérou) made by Langlois in 1735 was declared prototype of the French Toise, the standard temperature being:

$$t_0 \quad = \quad +13°R \quad = \quad +16.25°C \quad = \quad +61.25°F$$

The toise of Peru is the length of an iron bar used as standard in measuring the base lines that controlled lengths in the Peruvian Arc of triangulation surveyed in 1736-1743, to determine the figure and size of the Earth. It became the legal standard of length in France in 1776.

In March 19, 1791, the Assemble Constituante of France sanctioned a project of CGPM (Commission Générale des Poids et Mésures) that one ten-millionth part of the Earth's meridian quadrant with an ellipticity of 1/304 passing through the Observatory of Paris should be adopted as the national standard of length, to be called the *mètre*. Thus, from the toise, the French legal meter was derived as follows: the toise was divided into 6 *pieds* (feet), each pied was divided into 12 pouces (inches), and each pouce was divided into 12 lignes (lines). The *mètre* was defined as equal to exactly 443.296 lignes (lines) of the toise of Peru at the temperature 13°R (Reaumur), equal to $61^1/_4$°F (Fahrenheit).

Note

> *The original "toise of Paris" conversion is at 0°R: 1 metre = 5 130 740 × 864 ÷ 10 000 000 = 443.295 936 Paris' lines. The conversion factor used: 443.296÷864* (Table 15) (Jordan, 1959).
>
> *The notation "legal metre" varies with the country and is therefore only of local national significance.*

In the first decades of the 1800s, direct copies of the Toise of Peru were made and used by various states as base line apparatus for their triangulation. Carelessness in handling the original bar representing the Toise of Peru resulted in its loss. The primary and secondary copies of the bar were then used as reference for the true length of the Toise (Admiralty, 1965, Airy, 1830; Britannica, 1999; LaRousse, 1999; Strasser, 1957).

National Units

Colonel Alexander R. Clarke recognised also that the relation between the various national units has to be determined first when various national arcs are combined in deducing the elements of an ellipsoid. Therefore, he compared several European standards with the British Imperial Yard in 1863-1870. He considered these relations in his computations of elements from 1866 onwards. The unit of his elements are unique, contrary to those computed before (Strasser, 1957).

Legal Metres or International Metres

The conversion from toise to legal (geodetic) metres using the appropriate conversion factor gives the original defining parameters of the Bessel ellipsoid as 52° N arc stations adjusted in Bessel and Clarke Ellipsoids, and (Table 15; Table 17): pp 121, (Table 20): pp 126 (Baeyer, 1862a; König, 1951; Krack, 1995; Torge, 1991).

International System of Units

In 1967, the Conférence Générale des Poids et Mésures (CGPM) redefined the unit of time is expressed as the SI second (s), the fundamental interval unit of TAI [1.3], as follows:

> *"the second (s) as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium atom (^{133}Cs)".*

At its meeting in Paris of October 20, 1983 the Conférence Générale des Poids et Mésures (CGPM) redefined the metre as follows (Price, 1986; Rotter, 1984):

> *"the metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second".*

In other words this definition gives c, a fixed value, to the speed of light of 299 792 458 m s^{-1} exactly.

The units of length, mass, and time are in the International System of Units or Le Système International d'Unités (SI), as expressed by the metre (m), kilogram (kg) and second (s).

Conversion Units

unit of length	equal in international metre	equal to one international metre
international inch	0.0254 (exactly)	39.37
international foot	0.3048 (exactly)	3.28083 98950 13123
international yard	0.9144 (exactly)	1.09361 3298
British Imperial yard	0.91439 84155	1.09361 5193
British Imperial foot	0.30479 94718	3.28084 558
foot of bar O_1	0.30480 07491	3.28083 18318
(South) African geodetic foot	0.30479 72654	3.28086 9335
Indian foot	0.30479 84100 66	3.28085 70097
Indian foot 1956 (Bhattacharji, 1962)	0.3048 (exactly)	3.28083 98950 13123
US survey yard	0.91440 18288 03658	1.09361 11111 11111
US survey foot	0.30480 06096 01219	3.28083 33333 33333
US survey foot	12.00 / 39.37 (exactly)	39.37 / 12.00 (exactly)
toise of Paris	1.94903 63098 24586 73212	0.51307 40740 74074 07407
klafter 1871	1.89648 4	0.52729 156
legal metre (Zeger, 1991)	1.00001 33554 1	0.99998 66447 70

Table 15: Conversion of linear units

Temperature Scales

name	temperature in degrees freezing point	temperature in degrees boiling point	barometric pressure
Reaumur	0°R	80°R	760 mm
Celsius	0°C	100°C	760 mm
Fahrenheit	32°F	212°F	760 mm

Temperature Conversion

Temperatures (in degrees = °) Fahrenheit - Centigrade (or Celcius) - Reaumur are related by the equations:

$$°F = 32° + {}^{9}/_{5}\,°C \quad \text{or} \quad °C = {}^{5}/_{9}\,(°F - 32°) \tag{6.01}$$
$$°F = 32° + {}^{9}/_{4}\,°R \quad \text{or} \quad °R = {}^{4}/_{9}\,(°F - 32°) \tag{6.02}$$
$$°R = {}^{4}/_{5}\,°C \quad \text{or} \quad °C = {}^{5}/_{4}\,°R \tag{6.03}$$

Radio Frequencies

band	frequency	wavelength
very low frequency (VLF)	< 30 kHz	> 10 km
low frequency (LF)	30-300 kHz	1-10 km
medium frequency (MF)	300 kHz-3 MHz	100 m-1 km
high frequency (HF)	3-30 MHz	10-100 m
very high frequency (VHF)	30-300 MHz	1-10 m
ultra high frequency (UHF)	300 MHz-3 GHz	10
super high frequency (SHF)	3-30 GHz	1-10 cm

Table 16: Classification of radio frequencies

6.2 Spheres and Ellipsoids

About the Hydrosphere

The history of mathematics gives ample evidence that the subject is as early as the third century BC. Eratosthenes of Alexandria, believing the Earth to be a globe, first measured its size. After the fall of Greece and Rome came the so-called Dark and Middle Ages. Between 1200 and 1500AD, learning and science revived again in Europe, especially among the seafaring nations by a large-scale study of the globe. Broadly speaking, it was thought the Earth was made-up by a combination of two regions: a Hydrosphere and an Earthsphere. The Earth is the only planet known to have liquid water and ice. It covers 70.8 percent of the Earth's surface. The total mass of the Hydrosphere consists of seawater (98 percent) and fresh water, occurring principally in lakes, rivers, and glaciers [16].

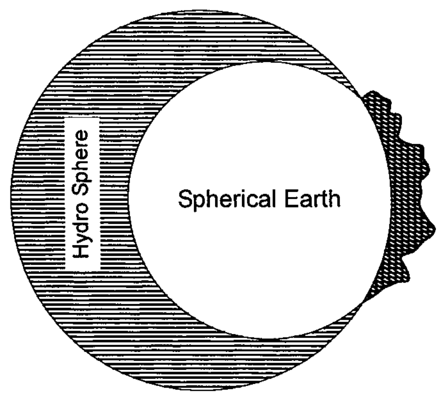


Figure 45: Hydrosphere between 1200-1500 AD

Celestial and Terrestrial Spheres

The celestial sphere [1.6] is an apparent surface of the heavens, on which the stars seem to be fixed. Stars are sometimes conceived as bright points dotted about on the inside of a large hollow spherical dome called the celestial sphere. For establishing coordinate systems to mark the positions of heavenly bodies, it can be considered a real sphere at an infinite distance from the Earth. The Earth's axis, extended to infinity, touches this sphere at the north and south celestial Poles, around which the heavens seem to turn. The plane of the Earth's Equator, extended to infinity, marks the celestial Equator.

The Equator, a great circle around the Earth that is everywhere equidistant from the geographic Poles, lies in a plane perpendicular to the Earth's axis. This geographic, or terrestrial Equator divides the Earth into the Northern and Southern Hemispheres and forms the imaginary reference line on the Earth's surface from which latitude is reckoned; in other words, it is the line with 0° latitude.

In astronomy, the celestial equator is the great circle in which the plane of the terrestrial equator intersects the celestial sphere; it consequently is equidistant from the celestial Poles. When the Sun lies in its plane, day and night are everywhere of equal length. The twice-per-year occurrence is known as an equinox. The bodies of giant ellipticals may not be figures of revolution (e.g., oblate ellipsoids) but may possess three axes of unequal lengths. In the models, the tri-axial shape arises because the random velocities of the stars are not equal in all directions.

An Equipotential Surface

The Figure of the Earth that coincides with mean-sea-level over the oceans and continues in continental areas as an imaginary sea level surface is by definition everywhere perpendicular to the pull of gravity and approximates the shape of a regular oblate ellipsoid. Mathematically speaking, the geoid [1.4] is an equipotential surface that is characterised by the fact that over its entire extent the potential function is constant. This potential function describes the combined effects of the gravitational attraction of the Earth's mass and the centrifugal repulsion caused by the rotation of the Earth about its polar axis.

Basic Surveys for a single World Geodetic Datum

All geodetic surveys are classified according to its accuracy, and must have the smallest permissible error. Geodetic surveys involve such extensive areas that allowance must be made for the Earth's curvature. During the International Geophysical Year 1957-1958 (IGY) [4.2], artificial satellite observations were introduced, because there are many other important characteristics of our planet, such as biological, geographical, geological, and economical aspects.

International collaborations have led to the possibility of adjusting existing primary geodetic surveys and astronomical observations to:

- a single 3D-global Datum
- an accurate determination of the shape of Earth.

Measurements for triangulation, corrections for spherical excess in the angular determinations, trilateration by using electronic distance measuring (EDM) equipment, least square (LS) adjustment and reduction to mean-sea-level (MSL), have facilitated the specifications (Dalton, 1953).

Sphere or an Ellipsoid of Revolution

The problem of finding an exact expression for the perimeter of an ellipse led to the development of elliptic functions, an important topic in mathematics and physics. A special case of elliptic functions arises when the surface is a sphere:

the axes $a_1 = a_2 = b$

For many purposes in land surveying, it is possible to neglect the real Earth's curvature, and to treat its surface as planar area, or to consider the Earth as a sphere and work using the spherical trigonometry. To simplify the fieldwork, it is often sufficiently accurate to consider the surface of a sphere. A rectifying sphere closely approximates the ellipsoid at the specific place in question, and whose radius equals the radius of curvature of the ellipsoid at that point. Such an approximate treatment enable land surveyors to calculate the results of surveys covering lines of up to say 20 km long, although the permissible length varies with latitude and precision required. The ellipsoid is an oblate bi-axial ellipsoid:

if the axes $a_1 = a_2 > b$

The ellipsoid is a tri-axial ellipsoid or spheroid |[8]:

if the axes $a_1 > a_2 > b$

[8] NGA stopped considering spheroids and ellipsoids as equivalent because a spheroid is very complex surface while an ellipsoid is a simple two-degree surface. To avoid confusion, the author used the word ellipsoid throughout.

The Reference Ellipsoid

Many years, an ellipsoid of revolution - the so-called the reference ellipsoid - was used to represent the Earth in geodetic calculations, because such calculations are simpler than those with mathematical models are. A surface generated by rotating an ellipse 360° about its minor axis forms the surface called the ellipsoid of revolution. This ellipsoid is the shape most often used to represent a simple geometric reference surface. An ellipsoid that is used in geodetic calculations to represent the Earth is called a reference ellipsoid, or a spheroid (Helmert, 1880; 1884).

For an ellipse, the centre of which is at the origin and the axes of which are coincident with the x and y-axes. If a, and b are the semi-axes, the general equation of such an ellipsoid is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{6.04}$$

When a more accurate reference figure is required, a bi-axial ellipsoid of revolution is used as a representation of the Earth's shape and size. It is specified by two parameters:

- semi-major axis (a) or equatorial radius
- semi-minor axis (b) or polar radius, or
- flattening (f)

The values of (a), (b), and (f) are related by formulae: (Table 19): pp 123

$$f = (a - b) / a \tag{6.05}$$

Bi-Axial Ellipsoid

The relation between radius r of a sphere, containing a volume equal to an reference ellipsoid with semi-axes (a) and (b) is :

$$4/3\, r^3 \pi = 4/3\, a^2 b\, \pi \tag{6.06}$$

$$r = \sqrt[3]{a^2 b} = a \sqrt[3]{(1 - f^{-1})} \tag{6.07}$$

Instead of a bi-axial ellipsoid, (Ritter, 1861) dreamed up a tri-axial ellipsoid in his paper *Recherches sur la Figure de la Terre* as a conversion of a bi-axial ellipsoid with following (unexcepted) formula:

$$a^2 y^2 + b^2 x^2 = a^2 b^2 + c\, x^2 y^2, \tag{6.08}$$

in which a and b are the semi-axes and c (= numerical constant) was very small value (Günther, 1897).

Tri-Axial Ellipsoid

A tri-axial ellipsoid is an ellipsoid with a different length for each of its three axes. The difference between the two semi-axes of the equatorial ellipse in the case of a tri-axial ellipsoid fitting the Earth is only about 80 metres (Hammer, 1896). In the 1860s, the problem of tri-axiality of the Earth was geophysically very interesting. Most observations were done in America, Europe, and India, which are about 90° apart from one another. Now it happens to be so that the gravity anomalies in North America are slightly negative, in Europe as well as in the eastern part of the North Atlantic they are strongly positive, and in India strongly negative again (Heiskanen, 1928). A tri-axial ellipsoid is specified by three parameters (Baeyer, 1862a; GG, 1986); Pellinen, 1982):

- semi-major axis (a_1) or equatorial radius
- semi-major axis (a_2) or equatorial radius
- semi-minor axis (b) or polar radius, or
- reciprocal flattening (f^{-1}).

An ellipsoid is symmetrical about three mutually perpendicular axes that intersect at the centre.

If a, b, and c are the principal semi-axes, the general equation of such an ellipsoid is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{6.09}$$

G.Th. Schubert supplied accurate values in: *Essai d'une Détermination de la véritable Figure de la Terre, Mémoires de l'Académie de Paris* for the tri-axial ellipsoid:

$a_1 = 6\,378\,556$ m $a_2 = 6\,377\,837$ m $b = 6\,356\,719$ m

in which:

a_1	>	a_2 > b
a_1 and a_2	=	two semi- major axes in the equatorial plane
b	=	the semi-minor or polar axis

The flattening is not a simple value. The eccentricity of the Equator, as well as the maximum and minimum eccentricity of the Meridians were given as 1:326.95 - 1:285.97 - 1:313.38 (Günther, 1897).

Ellipsoid-axes of Clarke 1866 in the equatorial plane are situated as follows:

longest radius	a_1	is situated at	15° 34'	East of Greenwich
smallest radius	a_2	is situated at	105° 34'	East of Greenwich

Helmert's gravity formula (1915) is a development by Helmert's gravity formula, based on a tri-axial ellipsoid, and including a term containing the longitude. Jacobi's ellipsoid is one of a series of tri-axial ellipsoids describing the figure of a homogenous, rotating fluid body in hydrostatic equilibrium (Khan, 1967).

Example Tri-Axial Ellipsoid

Following constants characterise a tri-axial ellipsoid by (GG, 1986; Pellinen, 1982):

reference ellipsoid		-	Krassovsky Ellipsoid of 1940	
geodetic datum		-	system 1942	
semi-major axis	a_1	=	6 378 245.0	m
	a_2	=	6 378 033.0	m
semi-minor axis	b	=	6 356 863.019	m
reciprocal flattening	f^{-1}	=	1 / 298.0	
eccentricity of	a_1 and a_2	=	1 / 30 086	
longest radius is situated at	a_1	=	15° 34' 00".0	E
latitude	φ_0	=	59° 46' 18".55	N
longitude	λ_0	=	30° 19' 42".09	N
deflection of the vertical meridian		=	0° 00' 00".16	N
prime vertical		=	0° 00' 01".78	N

The problem of tri-axiality was not essential, because the undulations (= N) of the geoid were already gravimetrically computed for the provisional geoid of the Northern Hemisphere. The undulations of the geoid (Figure 5): pp 11 seem not to exceed ± 80 m. The surveys of variations of the orbital path of Sputnik (1957α), Vanguard-I (1959α), and Vanguard-II (1959η) revealed that the oblate shape of the Earth was slightly pear-shaped. Distances measured from the Earth's centre to the Northern Hemisphere are greater than those to the Southern Hemisphere. Further observations of anomalies in the orbits of both satellites had confirmed that the Equator is elliptical.

(Heiskanen, Uotila, 1958):

> *"The problem of tri-axiality is no more actual, because we can now gravimetrically compute the undulations N of the geoid-in fact, the tentative geoid of the Northern Hemisphere has already been computed in Columbus, Ohio. Therefore, the tri-axial formulas have been given only from the historical point of view. After some few years, a more detailed geoid will be available.... ".*

Subsequently, the tri-axial formulae have been given only from the historical point of view (Benioff, 1958; Heiskanen, 1928, 1958).

6.3 Basic Control Surveys for Reference Systems

Historically, as countries around the world began to map and survey the nearby land and sea. They developed a local reference frame (geodetic Datum) to provide a point of origin and orientation for the specific area or region. Local geodetic Datums generally employed non-geocentric ellipsoids as the best-fitting figure of the Earth in the region being surveyed or mapped. A Datum is usually given a distinctive name. It is concerned with the realisation of that reference system, through a series of conventionally agreed measures, such as the John F. Hayford Reference Ellipsoid of 1909 or the so-called International Reference Ellipsoid of 1924 (Hristov, 1968; GG, 1986).

The Principle of Least Squares

The mathematical reference surface, where the reductions take place, is called an oblate ellipsoid [12.1] or reference ellipsoid: an ellipse that rotates around its polar, semi-minor axis. International Mapping Agencies, such as NGA, stopped considering spheroids and ellipsoids as equivalent due to the fact that a spheroid is very complex surface while an ellipsoid is a simple two-degree surface. Throughout this book, the word ellipsoid is used to avoid confusion (Ayres, 1995; Bigourdan, 1912; Tardi, 1934).

Computer scientists had to simply judge as they best they could how to use supernumerary angles. A system of corrections ought to be applied to the reduction of the horizontal angles and calculation of the triangulations. The reduction of the supernumerous observations, in the past a very laborious process, led to complex calculations. The first grand development of the principle of least squares (LS) [12.3] is contained in the book entitled *Gradmessung in Ostpreußen und ihre Verbindung* by Friedrich W. Bessel, published in 1838. It marks an era in the science of geodesy.

Defining Ellipsoids and Parameters

The defining fundamental parameters of the Ellipsoid by Bessel were generally considered as one of the most elegant geometrical investigations that were ever published in: Schum. Astr. Nachr. No. 438, and as well presented by Encke in his tables of his astronomical work *Astronomischen Jahrbuch für 1852* (Baeyer, 1862a; Günther, 1897):

log a	=	6.51482 35337	logarithm semi-major axis
log b	=	6.51336 93539	logarithm semi-minor axis
a	=	$3272077^{T}.1399$ |[9]	semi-major axis or the equatorial radius of the Earth
b	=	$3261139^{T}.3284$	(= a / 1.00335 39847), thus
f^{-1}	=	299.15281 8	reciprocal flattening
arc = rad	=	57°.29577 95129	

According to Bessel was the unit of length *legal meter* (mètre) defined by 443.296 lines of the geodetic unit of length *Toise du Peru* at the temperature 13° of Reaumur (Baeyer, 1862a; Günther, 1897).

[9]Toise of Paris

Defining fundamental parameters of the Bessel 1841 Ellipsoid were expressed in legal metres:

a	=	6 377 397.155 m_{LEG}	semi-major axis of the Earth
b	=	6 356 078.963 m_{LEG}	semi-minor axis
f^{-1}	=	299.15281 285	reciprocal flattening

After IAG Metre Convention of 1875, these parameters are used in many countries in Continental Europe and Asia as international metres (m_{INT}).

Nevertheless, in southern Africa (Namibia) the defining parameters of the Bessel Ellipsoid are given in really international metres as (Rens, 1990):

a	=	6 377 483.865 m_{INT}	semi-major axis of the Earth
b	=	6 356 165.38297 m_{INT}	semi-minor axis
f^{-1}	=	299.15281 285	reciprocal flattening

6.3.1 Classical Geodesy

For everyday surveying, using conventional techniques based on angular and distance measurement, it has often been sufficient to use a local and even an arbitrary Datum. Before the 1958s, defining a complete geodetic reference Datum in traditional 2D-geodesy by five quantities was customary:

- latitude of an initial point
- longitude of an initial point
- azimuth of a line from this point
- first unique parameter semi-major axis (a) of a reference ellipsoid, and
- second unique parameter, e.g. reciprocal flattening (f^{-1}) of a reference ellipsoid.

Example of a Bi-Axial Ellipsoid

name of datum horizontal control datum			Tokyo datum Bessel ellipsoid of 1841
origin		=	centre of the transit circle at the old Tokyo Observatory
latitude	φ_0	=	35° 39' 17".51 N
longitude	λ_0	=	139° 44' 40".50 E
semi-major axis	a	=	6 377 397.155
flattening	f^{-1}	=	299.15281 285
azimuth to Kano-Yama	α_0	=	156° 25' 28".44 (clockwise from North)

In addition, specification of the components of the deflection of the vertical at the initial point, or the condition that the minor axis of the ellipsoid be parallel to the Earth's axis of rotation granted one more quantity. The Datum was *not complete* because the origin of the coordinate system remained free to shift in one dimension. The definition does not correspond to present usage (GG, 1986). In geodesy, several Reference Ellipsoids are used (Strasser, 1957).

Meridian Programs

Program A_03MARC.FOR [18.3], A_04ELLI.FOR [18.4] use the parameters a, f, and scale factor k_0 for any reference ellipsoid as input, and calculates the parameters and associated constants according to (Table 19): pp 123 in the meridional plane. Note that the IERS Reference zero-Meridian (IRM), measured in the plane of the geodetic equator, is positive from 0° to 180°E, although negative from 0° to 180°W.

Constants for Bessel 1841

The rotational ellipsoid is created by rotating the meridian ellipse about its minor axis. Using two geometrical parameters, the shape of the ellipsoid is described. Generally, b is replaced by one of a number of quantities, which is more suitable for series expansions.

the semi-major axis	:	a	and	the semi-minor axis	:	b
the semi-major axis	:	a	and	the geometrical flattening	:	f
the semi-major axis	:	a	and	the first eccentricity	:	e
the semi-major axis	:	a	and	the second eccentricity	:	e'

log a	=	6.80464 34637 (unique (Jordan, 1959))
log b	=	6.80318 92839 (unique)
a	=	6 377 397.15507 604|96 89970 31762 |[10]
b	=	6 356 078.96289 778|47 19996 70996
a / b	=	1.00335 39847 9|1992 86789
1 / f	=	299.15281 28533 4|0587 66096
f	=	0.00334 27731 81578 77|108
e^2	=	0.00667 43722 306|14 05990
e'^2	=	0.00671 92187 979|70 65522
c	=	6 398 786.84814 66|733 41962 56064

Table 17: Derived ellipsoidal parameters of Bessel 1841 according to Helmert

Geodetic Latitude and Longitude

A geodetic coordinate system is a 2D-system used to define the unique location of a Datum point on a mathematical model of the Earth, the ellipsoid [6.3.4]. Latitude represents a distance north or south of the Equator while longitude represents a distance east or west from an arbitrarily accepted reference, the Meridian, such as Berne, Ferro, Genoa, Greenwich, Oslo and Paris. Latitude and longitude are both given in sexagesimal or centesimal units.

If 3D-positions are required, heights above or below the ellipsoid at an arbitrary location are given in units of length. Elevation above or below mean sea level can and has been used instead of the ellipsoidal height where accuracy is not critical, or where geoidal heights are not available. The geodetic position of a point on Earth is defined by the ellipsoidal coordinates of the projection of the 2D-point on to the surface of a geodetic reference ellipsoid, along the normal to that ellipsoid (Ashkenazi, 1991).

Geodetic latitude φ_G of a point is the angle φ^P. The geodetic latitude (φ_G) is defined as the inclination of the normal to the ellipsoidal equatorial plane, and geodetic meridian as the plane through the normal and the minor axis of the reference ellipsoid, the angle PME (Figure 46): pp 123 (Maling, 1992).

It follows that geodetic longitude (λ_G) of a point is the angle t, the geodetic longitude angle between its geodetic.

Note

No two points on the same reference Ellipsoid / Datum / Epoch can have identical geodetic coordinates.

[10]Calculated using program "BIGCALC" to 50 decimal places

Reference Ellipsoids - Primary Constants

reference ellipsoid	semi-major axis	unit	reciprocal flattening
Airy 1830	6 377 563.3964	m	299.32496 459
Airy modified 1965	6 377 340.189	m	299.32496 459
Australian National (IAU)	6 378 160	m	298.25
Bessel 1841 original	3 272 077	toise	299.15282
Bessel (Namibia)	6 377 483.865	m	299.15281 285
Bessel 1841	6 377 397.155	m	299.15281 285
Bessel NGO1948	6 377 492.0176	m	299.1528
Clarke 1866	20 925 832.16	US survey feet	294.97869 82
Clarke 1866	6 378 206.4	m	294.97869 82
Clarke 1880 Arc modified	6 378 249.145326	m	293.46630 76
Clarke 1880DoD	6 378 249.145	m	293.465
Clarke 1880G	6 378 249.14533	m	293.465
Clarke 1880IGN	6 378 249.2	m	293.46602 13
Everest (Brunei, East Malaysia)	6 377 298.556	m	300.8017
Everest (Pakistan)	6 377 309.613	m	300.8017
Everest 1830 (India)	6 377 276.3458	m	300.8017
Everest 1830	20 922 931.8	Indian feet	300.8017
Everest 1948 (West Malaysia, Singapore)	6 377 304.063	m	300.8017
Everest 1956 (India)	6 377 301.243	m	300.8017
Everest 1969 (West Malaysia)	6 377 295.664	m	300.8017
Fischer 1960 modified	6 378 155	m	298.3
GRS67	6 378 160	m	298.24716 75
GRS80 (Burkholder, 1984)	6 378 137	m	298.25722 21008 827
IAG 1975	6 378 140	m	298.25722 2
International 1924	6 378 388	m	297
Krassovsky 1940	6 378 245	m	298.3
PZ90 (RU)	6 378 136	m	298.25783 9303
South American 1969 (GRS67)	6 378 160	m	298.25
WGS72	6 378 135	m	298.26
WGS84	6 378 137	m	298.25722 3563

Table 18: Reference ellipsoids

(Table 19): pp 123 lists the Earth's defining parameters and associated constants for that particular ellipsoid for use in geodetic applications, and calculates the meridional arc distance (Figure 46) and [8.4].

Earth's Defining Parameters

notation		a | b	e^2	e'^2	f	n
a						$\frac{c\,(1-n)}{1+n}$
b	=		$a\,(1-e^2)^{1/2}$	$\frac{a}{(1+e'^2)^{1/2}}$	$a\,(1-f)$	
$\frac{a}{b}$	=				$\frac{1}{1-f}$	
$\frac{b}{a}$	=		$(1-e^2)^{1/2}$	$\frac{1}{(1+e'^2)^{1/2}}$	$1-f$	$\frac{1-n}{1+n}$
c	=	$\frac{a^2}{b}$	$\frac{a}{(1-e^2)^{1/2}}$	$a\,(1+e'^2)^{1/2}$	$\frac{a}{1-f}$	
e^2	=	$\frac{a^2-b^2}{a^2}$		$\frac{e'^2}{1+e'^2}$	$f\,(2-f)$	$\frac{4n}{(1+n)^2}$
e'^2	=	$\frac{a^2-b^2}{b^2}$	$\frac{e^2}{(1-f)^2} = \frac{e^2}{1-e^2}$		$\frac{f\,(2-f)}{(1-f)^2}$	$\frac{4n}{(1-n)^2}$
f	=	$\frac{a-b}{a}$	$1-(1-e^2)^{1/2}$	$1-(1+e'^2)^{-1/2}$		$\frac{2n}{1+n}$
n	=	$\frac{a-b}{a+b}$	$\frac{1-(1-e^2)^{1/2}}{1+(1-e^2)^{1/2}}$	$\frac{(1+e'^2)^{1/2}-1}{(1+e'^2)^{1/2}+1}$	$\frac{f}{2-f}$	

Table 19: Earth's defining parameters and associated constants

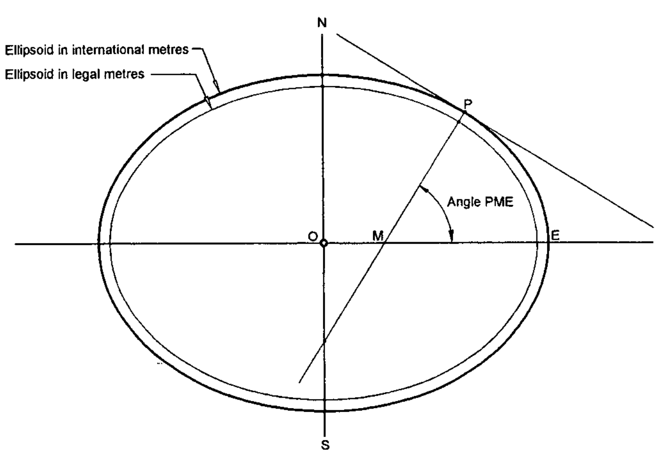


Figure 46: Bessel ellipsoids in legal metres and international metres

Defining Parameters

All angles are expressed in radians (Krack, 1983) [18.3, 18.4].
For geometrical parameters, (Table 19).

Classical Ellipsoid Constants

a	semi-major axis of the bi-axial ellipsoid
b	semi-minor axis
f^{-1}	reciprocal flattening
f	flattening of the ellipsoid
k_0	scale factor assigned to the meridian
φ_i	parallel of geodetic latitude (=B_i^R in radian measure), positive north
G_m	meridional distance (from the equator φ_o to $\varphi_i = S_o$) multiplied by the scale factor
e^2	first eccentricity squared
e'^2	second eccentricity squared
n	second flattening
W	first auxiliary quantity
V	second auxiliary quantity
M	or (ρ) = radius of curvature in the meridian ________________ (Figure 48)
N	or (ν) = radius of curvature in the prime vertical _____________ (Figure 48)
c	polar radius of curvature
r	radius of the rectifying sphere, having the same meridional length as that of the ellipsoid
R_M	arithmetic mean radius of a and b
R_G	geometric mean radius of a and b
R_α	radius of curvature in azimuth α
R_Q	sphere of equal volume as the ellipsoid

Derived Constants

Compute constants for ellipsoid as given below:

$$r = a(1 + n^2/4)/(1+n) \tag{6.10}$$
$$f = 1/f^{-1} \tag{6.11}$$
$$c = a/(1-f) \tag{6.12}$$
$$a/b = 1/(1-f) \tag{6.13}$$
$$n = f/(2-f) \tag{6.14}$$
$$e^2 = f(2-f) \tag{6.15}$$
$$e'^2 = e^2/(1-f)^2 \tag{6.16}$$
$$b = a(1-f) \tag{6.17}$$

Input: geodetic coordinate of a point P_i (B_i^R)

$$W = (1 - e^2 \sin^2 B_i^R)^{1/2} \tag{6.18}$$
$$V = (1 + e'^2 \cos^2 B_i^R)^{1/2} \tag{6.19}$$
$$N = a/W \qquad = c/V \tag{6.20}$$
$$M = c/V^3 \tag{6.21}$$
$$R_M = (M\,N)^{1/2} \qquad = c/V^2 \tag{6.22}$$
$$c = a^2/b \tag{6.23}$$
$$R_\alpha = c/(V + (V^3 - V)\cos^2\alpha) \quad = (M\,N)/(M\sin^2\alpha + N\cos^2\alpha) \tag{6.24}$$
$$R_{MAB} = \frac{(a+b)}{2} \tag{6.25}$$
$$R_{GAB} = \sqrt{(a\,b)} \tag{6.26}$$

$$R_Q = a\left(1 + \frac{-f}{3} + \frac{-f^2}{9}\right) \tag{6.27}$$

Meridional Arc formula

Forward constants:

$$\begin{aligned} E_1 &= -n(36 + n(45 + 39n)) \\ E_2 &= n^2(90 + 280n) \\ E_3 &= -280n^3 \end{aligned} \tag{6.28}$$

Inverse constants:

$$\begin{aligned} F_1= &= n(36 + n(93n - 63)) \\ F_2= &= n^2(126 - 604n) \\ F_3= &= 604n^3 \end{aligned} \tag{6.29}$$

Direct computation

Input: geodetic coordinate of a point P_i (B_i^R)
Output: G_m of a point P_i

$$G_m = k_o r(B_i^R + \sin B_i^R \cos B_i^R / 12(E_1 + \cos^2 B_i^R(E_2 + \cos^2 B_i^R E_3))) \tag{6.30}$$

Inverse computation

Input: G_m of a point P_i
Output: geodetic coordinate of a point P_i (B_i^R)

$$\omega = G / (r \times k_o) \tag{6.31}$$

$$B_i^R = (\omega + \sin\omega \cos\omega / 12(F_1 + \cos^2\omega(F_2 + \cos^2\omega F_3))) \tag{6.32}$$

Derived functions for the WGS84 ellipsoid are given in (Table 22): pp 128. For further reading (Agajelu, 1987; Baeschlin, 1948; Jordan, 1959).

The least squares adjustment (LS) of the European 52°N arc of parallel between stations Feaghmain and Warsaw: for Bessel's Ellipsoid is $\Sigma\xi = 339.9$, $\Sigma\lambda = 320.7$; for the Clarke's ellipsoid is $\Sigma\xi = 346.2$ and $\Sigma\lambda = 530.6$ (Figure 47, Table 20) (Helmert, 1893; Börsch, 1896) [On_CD; 12.2].

The radii N and M (Figure 48) have the following features (Jordan, 1959):

- $N = \nu$; (4.11) $\geq$ M in any latitude
- $M = \rho$; (4.12) $=$ N at the poles (= c), and reach their maximum values
- N $=$ a on the Equator, and M and N have their minimum values
- M $=$ a in latitude 54°.8, approximately
- M $=$ b in latitude 35°.3, approximately.

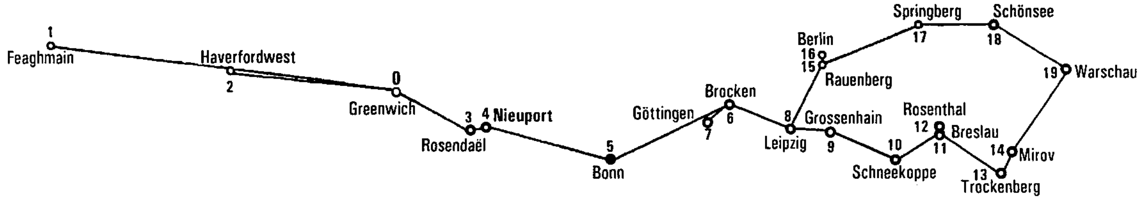


Figure 47: European 52° arc of parallel between Feaghmain and Warsaw

		Bessel's ellipsoid		Clarke's ellipsoid	
no	station	ζ	λ	ζ	λ
1	Feaghmain	-2".93	+2".29	-4".15	-10".66
2	Haverfordwest	-5".62	+8".31	-6".38	-1".37
0	Greenwich	-6".94	+6".40	-6".80	-0".20
3	Rosendaël	-4".46	+0".63	-5".09	-4".44
4	Nieuport	-3".93	+0".45	-4".51	-4".41
5	Bonn	+0".26	-5".29	-0".37	-7".48
6	Brocken	+9".27	+4".21	+9".15	+4".16
7	Göttingen	+1".06	-3".15	+0".82	-3".61
8	Leipzig	+0".65	+4".13	+0".32	+5".16
9	Grossenhain	-1".70	-5".13	-2".06	-3".37
10	Schneekoppe	+7".93	-1".11	+7".27	+1".96
11	Breslau	+2".35	+3".70	+1".83	+7".58
12	Rosenthal	+1".58	+3".96	+1".07	+7".84
13	Trockenberg	+1".62	-3".14	+0".72	+1".79
14	Mirow	+5".35	+0".99	+4".60	+6".15
15	Rauenberg	+0".71	+2".89	+0".86	+4".50
16	Berlin	+0".34	+1".69	+0".52	+3".40
17	Springberg	-5".60	-0".40	-5".18	+3".40
18	Schönsee	-0".36	-7".31	-0".03	-2".06
19	Warschau	+0".17	-0".13	-0".04	+6".34

Table 20: 52° N arc stations adjusted in Bessel and Clarke Ellipsoids

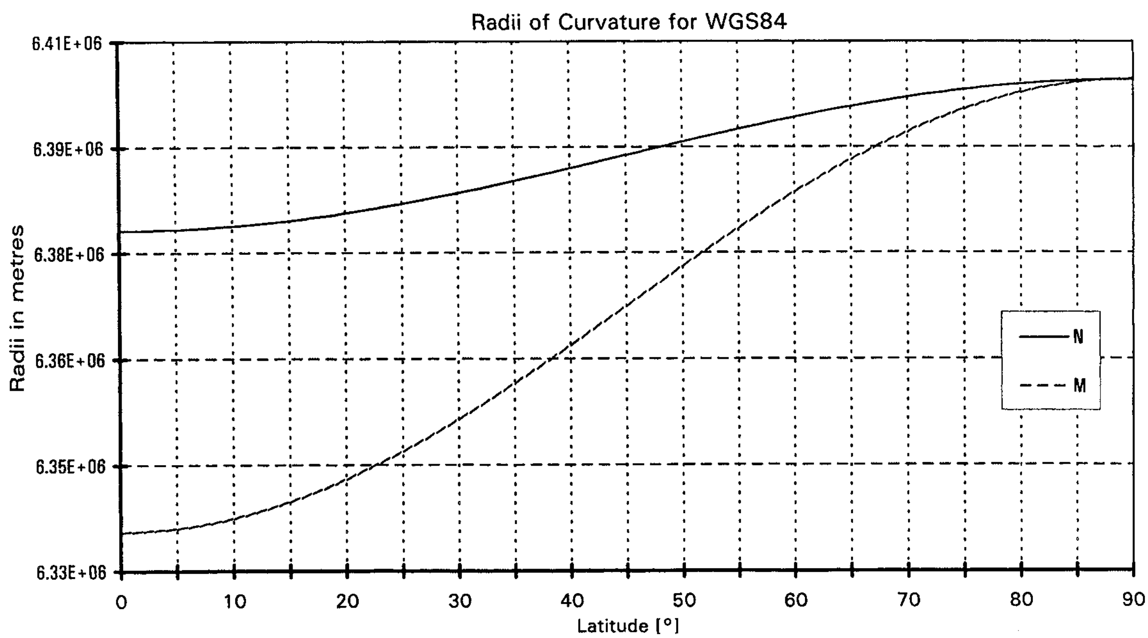


Figure 48: Radii of curvature in Prime Vertical and in the Reference Meridian

6.3.2 Satellite Geodesy

After the 1957s, the introduction of Earth orbiting satellites allowed the development of global geodetic reference frames for practical use. The first of these global reference frames was the US-Department of Defense (DoD) World Geodetic System of 1960 (WGS60), followed by WGS66. At least eight constants are needed to form a complete geodetic reference Datum in 3D-satellite geodesy: WGS72, GRS80 and WGS84 (GG, 1986). A geocentric geodetic Datum specifies the centre of the reference ellipsoid at the Earth's centre of mass. 3D-geodesy is the complete determination of positions in 3D-space by a unified consistent theory.

Boundary Value Problem

In the classical problem, the Earth's surface is the geoid, and the basic solution is Stokes' formula. In the modern formulation, the surface is the physical surface of the Earth. The boundary-value problem of physical geodesy is the problem of determining the shape of the Earth's surface, given values of gravity or gravity potential on that surface. Boundary-value problem in which a partial differential equation is to be solved for the dependent function, with the values of the function or its derivatives given on the boundary of the region over which the function is to be determined (Grafarend, 1984, 1991, 1995g, 1997c, 2001c, g).

3D-Geodesy

Three-dimensional geodesy differs from the so-called classical methods in that 3D-methods are sufficiently flexible to use both traditional and satellite types of data, an international origin, a system of gravity with a set of adjusted values for the acceleration of gravity, velocity of light, and a refractive index of atmosphere, such as:

a	semi-major axis or equatorial radius of the Earth	
GM	value × 10^8 m^3 s^{-2}	G×M product, geocentric gravitational constant
J_2	value × 10^{-8}	dynamic form factor, un-normalised form
$C_{2.0}$	value × 10^{-6}	dynamic form factor, normalised form
ω	value × 10^{-11} rad s^{-1}	angular velocity of the Earth

Parameters and
Associated Constants for WGS84

name		constants and magnitudes	explanation
π / 4	=	0.78539 81633 97448 3	
RD	=	0.01745 32925 19943 3	conversion factor degrees into radians
Reference Ellipsoid: WGS84, a geocentric equipotential ellipsoid (NGA, 2000)			
a	=	6 378 137.0 m	semi-major axis or equatorial radius of the Earth
GM	=	3 986 005 × 10^8 m^3 s^{-2}	geocentric gravitational constant
J_2	=	calculated value (1.15), p. 20, 21:	dynamic form factor, un-normalised form
$C_{2.0}$	=	−484.166 85 × 10^{-6}	dynamic form factor, normalised form
ω	=	7 292 115 × 10^{-11} rad s^{-1}	angular velocity of the Earth

Table 21: WGS84 constants and parameters

name		derived geometric constants and magnitudes for WGS84	explanation
b	=	6 356 752.31424 518	semi-minor axis
f^{-1}	=	298.25722 3563	reciprocal flattening
k_0	=	1.0000	scale factor on central meridian
f	=	3.35281 06647 47481E-03	flattening of meridional ellipse
c	=	6 399 593.62575 8493	polar radius of curvature
a / b	=	1.00336 40898 20976	
n	=	1.67922 03863 83705E-03	ratio of length
e^2	=	6.69437 99901 41317E-03	first eccentricity squared
e	=	8.18191 90842 62149E-02	first eccentricity of meridional ellipse
$1 - e^2$	=	0.99330 56200 098587	
$\sqrt{(1 - e^2)}$	=	0.99664 71893 352525	
e'^2	=	6.73949 67422 76435E-03	second eccentricity squared
$1 + e'^2$	=	1.00673 94967 42276	
$\sqrt{(1 + e'^2)}$	=	1.00336 40898 20976	
r	=	6 367 449.14582 2625	radius of the rectifying sphere
E_1	=	-6.057900872590829E-02	coefficient of meridional arc - direct
E_2	=	2.551061090413531E-04	coefficient of meridional arc - direct
E_3	=	-1.325809497155670E-06	coefficient of meridional arc - direct
F_1	=	6.027472805828684E-02	coefficient of meridional arc - inverse
F_2	=	3.524324588751549E-04	coefficient of meridional arc - inverse
F_3	=	2.859960486721516E-06	coefficient of meridional arc - inverse
R_{MAB}=	=	6367444.657122590	arithmetic mean radius
R_{GAB}	=	6367435.679716192	geometric mean radius
R_Q	=	6371000.804881453	radius of sphere of same volume
$\varphi°$	:	45°	*input:* arbitrary latitude (B_i°)
N	=	6388838.290121148 = ν	radius of curvature in the prime vertical
M	=	6367381.815619549 = ρ	radius of curvature in the meridian
R_M	=	6378101.030201018	mean radius of c and V
R_α	=	6378092.007544452	radius of curvature in azimuth (α)
G_m	=	4984944.377977213	*output* = length of meridional arc
$\varphi°$	=	45°.00000 00000 2307	round-trip latitude (B_i°)
$\varphi°$	:	60°	*input:* arbitrary latitude (B_i°)
N	=	6394209.173847894 = ν	radius of curvature in the prime vertical
M	=	6383453.857229078 = ρ	radius of curvature in the meridian
R_M	=	6388829.252275326	mean radius of c and V
R_α	=	6391516.948369661	radius of curvature in azimuth (α)
G_m	=	6654072.819442124	*output* = length of meridional arc
$\varphi°$	=	59°.99999 99980 4641	round-trip latitude (B_i°)

Table 22: Derived WGS84 constants and parameters

6.3.3 Information in European Standards

geographic information are given in European Standards, CEN/TC 287 (Harsson, 1996)

Astronomic Coordinates (Φ, Λ, H) or (Φ, Λ)

Astronomic latitude and astronomic longitude of a given point, with or without orthometric height.

Astronomic Latitude (Φ)

Angle from the equatorial plane to the direction of gravity through the given point, northwards treated as positive.

Astronomic Longitude (Λ)

Angle from the zero-Meridian to the celestial meridian plane of the given point, an astronomic concept beyond the scope of this standard.

Geodetic Ellipsoid

Flattened ellipsoid of rotation, usually chosen to fit the geoid as closely as possible, either locally or globally.

Geodetic Latitude (φ)

Angle from the equatorial plane to the direction of the perpendicular to the ellipsoid through the given point, northwards treated as positive.

Geodetic Longitude (λ)

Angle from the zero-Meridian plane to the meridian plane of the given point, eastwards treated as positive.

Geographic Coordinates

Geographic latitude and geographic longitude, with or without height.

Geographic Latitude

A generic term for geodetic or astronomic latitude, but more often geodetic.

Geographic Longitude

Generic term for geodetic or astronomic longitude, but more often geodetic.

Geoid

Equipotential surface of the Earth's gravity field, which most closely approximates mean sea level [1.4].

Greenwich Meridian Plane

Meridian plane passing through Greenwich, England, widely used as the *zero-Meridian plane.*

Note - this is actually only a half-plane, on the European side of the polar axis.

Latitude (Φ, φ)

Angle from the equatorial plane to the considered direction at the given point, northwards treated as positive. The considered direction is the perpendicular to the reference surface through the given point; see astronomic latitude, geodetic latitude and geographic latitude.

Longitude (Λ, λ)

Angle from the *zero-Meridian plane* to the meridian plane of the given point, eastwards treated as positive. See *astronomic longitude, geodetic longitude* and *geographic longitude.*

Map Projection

Mathematical mapping of a geodetic ellipsoid, or part of a geodetic ellipsoid, to a plane.

6.3.4 On the Meaning of Geodetic Orientation

The astronomic latitude, (φ_A or Φ) of a point is given by the angle between the vertical at that point and the equatorial plane. The astronomical meridian is defined as the plane passing through the vertical at that point and the spin axis of the Earth. The astronomic longitude (λ_A or Λ) of a point is the angle between two planes, one of which is the local meridian and the other the zero-Meridian at Greenwich. Two points on the same reference ellipsoid may have identical astronomic coordinates. Therefore, astronomic coordinates do not therefore constitute a coordinate system in the geometrical sense. Laplace equations are used to convert astronomic coordinates into geodetic coordinates.

In geometric geodesy, the Laplace equation:

$$\alpha_A - \alpha_G = (\lambda_A - \lambda_G) \sin \varphi_G \tag{6.33}$$

expresses the relationship between astronomic azimuth (α_A) in terms of astronomic longitude (α_G), geodetic longitude (λ_A), and geodetic latitude (φ_G).

Vincenty (1985c) mentioned in his meaning of geodetic orientation, that any systematic errors in astronomic longitude, in astronomic azimuth, or in both may occur due to a misorientation of a geodetic network at the Datum origin, or adjustments of the geodetic network. Hence, it is possible to form a judgment about the orientation of geodetic networks and related rotation angles with regard to data in:

- selected coordinates of two or more systems, compared to determine shared relationships
- coordinates of an existing geodetic or satellite network, re-orientated by corrections in rotations
- computing adjusted azimuths between transformed points
- adjusting a geodetic network with an initial involvement of a spaceborne system.

Whether on a local or on a geocentric Datum, the orientation can be accepted as correct. A distinction is made between two kinds of reference systems:

- a fundamental astronomic reference system with respect to which the numerical values of astronomic latitude, longitude, and azimuth (φ, λ, α, respectively) are given
- any Cartesian XYZ system with respect to which geodetic coordinates are computed and defined by BIH longitude origin and CIO mean Pole.

Adjustments of networks can be conducted with participation of calibrated satellite data, i.e. geocentric positions corrected a priori for known errors in scale and orientation. Overall, orientation of this network, biases in astronomic orientation can be modelled in the adjustment. Two approaches will produce practically the same final geodetic coordinates, although they are based on different assumptions (Vincenty, 1985c; Yeremeyev, 1963).

6.4 Formulae for Various Types of Latitude

In the applications, it is generally desirable to have the differences between the various latitudes, expressed in terms of the sines of the different arcs. The problem is then to determine these series in the most convenient manner (Adams, 1949). In most cases, this can be accomplished by the use of the principles of the functions of a complex variable.

$$m = \frac{e^2}{2 - e^2} \tag{6.34}$$

$$e^2 = \frac{a^2 - b^2}{a^2} \tag{6.35}$$

$$\tan \psi = (1 - e^2) \tan \varphi \tag{6.36}$$

$$\tan\varphi = \frac{1}{1-e^2}\tan\psi \tag{6.37}$$

Geodetic or Astronomic Latitude = φ

$$\tan\varphi = \frac{1}{1-e^2}\tan\psi \tag{6.38}$$

$$\varphi-\psi = m\sin 2\varphi - \frac{m^2}{2}\sin 4\varphi + \frac{m^3}{3}\sin 6\varphi - \ldots \tag{6.39}$$

$$\varphi-\psi = m\sin 2\psi + \frac{m^2}{2}\sin 4\psi + \frac{m^3}{3}\sin 6\psi + \ldots \tag{6.40}$$

Reduced or Parametric Latitude = θ

$$\tan\theta = (1-e^2)^{1/2}\tan\varphi \tag{6.41}$$

$$\varphi-\theta = n\sin 2\varphi - \frac{n^2}{2}\sin 4\varphi + \frac{n^3}{3}\sin 6\varphi - \ldots \tag{6.42}$$

$$\varphi-\theta = n\sin 2\theta + \frac{n^2}{2}\sin 4\theta + \frac{n^3}{3}\sin 6\theta + \ldots \tag{6.43}$$

Geocentric Latitude = ψ

$$\tan\psi = (1-e^2)\tan\varphi \tag{6.44}$$

$$\theta-\psi = n\sin 2\theta - \frac{n^2}{2}\sin 4\theta + \frac{n^3}{3}\sin 6\theta - \ldots \tag{6.45}$$

$$\theta-\psi = n\sin 2\psi + \frac{n^2}{2}\sin 4\psi + \frac{n^3}{3}\sin 6\psi + \ldots \tag{6.46}$$

Isometric Latitude = χ

$$\tan\left[\frac{\pi}{4}+\frac{\chi}{2}\right] = \tan\left[\frac{\pi}{4}+\frac{\varphi}{2}\right]\left[\frac{1-e\sin\varphi}{1+e\sin\varphi}\right]^{e/2} \tag{6.47}$$

$$\begin{aligned}\varphi-\chi = {} & \left[\frac{e^2}{2}+\frac{5e^4}{24}+\frac{3e^6}{32}+\frac{281e^8}{5760}+\ldots\right]\sin 2\varphi \\ & - \left[\frac{5e^4}{48}+\frac{7e^6}{80}+\frac{3e^8}{11520}+\ldots\right]\sin 4\varphi \\ & + \left[\frac{13e^6}{480}+\frac{461e^8}{13440}+\ldots\right]\sin 6\varphi \\ & - \left[\frac{1237e^8}{1612802}+\ldots\right]\sin 8\varphi + \ldots\end{aligned} \tag{6.48}$$

$$\varphi-\chi = \left[\frac{e^2}{2}+\frac{5e^4}{24}+\frac{3e^6}{12}+\frac{13e^8}{360}+\ldots\right]\sin 2\chi$$

$$+ \left[\frac{7e^4}{48} + \frac{29e^6}{240} + \frac{811e^8}{11520} + \ldots \right] \sin 4\chi$$

$$+ \left[\frac{7e^6}{120} + \frac{81e^8}{1120} + \ldots \right] \sin 6\chi$$

$$+ \left[\frac{4279e^8}{1612802} + \ldots \right] \sin 8\chi + \ldots \tag{6.49}$$

Authalic Latitude = β

$$\sin\beta = \sin\varphi \left[\frac{1 + \frac{2e^2}{3}\sin^2\varphi + \frac{3e^2}{5}\sin^4\varphi + \frac{4e^6}{7}\sin^6\varphi + \ldots}{1 + \frac{5e^2}{3} + \frac{3e^4}{5} + \frac{4e^6}{7} + \ldots} \right] \tag{6.50}$$

$$\varphi - \beta = \left[\frac{e^2}{3} + \frac{31e^4}{180} + \frac{59e^6}{560} + \ldots \right] \sin 2\varphi \; -$$

$$\left[\frac{17e^4}{360} + \frac{61e^6}{1260} + \ldots \right] \sin 4\varphi \; +$$

$$\left[\frac{383e^6}{45360} + \ldots \right] \sin 6\varphi \; - \ldots \tag{6.51}$$

$$\varphi - \beta = \left[\frac{e^2}{3} + \frac{31e^4}{180} + \frac{517e^6}{5040} + \ldots \right] \sin 2\beta \; +$$

$$\left[\frac{23e^4}{360} + \frac{251e^6}{3780} + \ldots \right] \sin 4\beta \; +$$

$$\left[\frac{761e^6}{45360} + \ldots \right] \sin 6\beta \; + \ldots \tag{6.52}$$

Rectifying Latitude = ω

Rectifying latitude is the latitude such that a length along the Meridian from the Equator to that latitude on the sphere is exactly equal to the corresponding length on the ellipsoid (8.4.1). For mathematical definition, see (Adams, 1921). Following methods calculate the radius of the rectifying sphere for the International Ellipsoid, using scale factor $k_0 = 1.0$ (Figure 68, pp 166).

Method I (Helmert, 1880, Jordan, 1959):

```
r = a ( 1 - n ) ( 1 - n² ) ( 1 + n² / 4 ( 9 + 25 / 16 n² ( 9 + n² / 4 ( 49 + 81 / 16 n² ))))     (6.53)
Calculation time:                    0.97 ms
Result:                       6 367 654.50005 76 m
```

Method II (Krack, 1982):

```
r = a ( 1 - n ) ( 1 - n² ) ( 1 + 9 / 4 n² ( 1 + 25 / 16 n² ))     (6.54)
Calculation time:                    0.67 ms
Result:                       6 367 654.50005 76 m
```

Method III (Krack, 1983), in which a significant aspect of this method is its speed combined with accuracy:

```
r = a ( 1 + n² / 4 ) / ( 1 + n ) used by programs of this publication     (6.55)
Calculation time:                    0.30 ms
Result:                       6 367 654.50005 67 m
```

7. Spatial Coordinate Calculations

Summary

In the classical treatment of geometric geodesy, difficulties arise that the surveyed quantities cannot be rigorously related to the geometric model that had to be established. These surveys, such as reduction of baselines, measurements of horizontal and vertical angles, are affected to an unknown extent by systematic influences such as refraction and gravity field anomalies. Accordingly, geodetic theory has developed least-squares (LS)-methods of adjustment to eliminate the contradictions in data sets, with instable results near the coastal margins. Aside from the assumptions of homogeneity and hydrostatic equilibrium of the masses, an unavoidable limitation of line-of-sight exists between the stations on the Earth's crust. Hence, 2D-reference Datums may be considered as inhomogeneous due to adjustments with a continuous varying difference in scale (Grafarend, 1978b).

The transformation method can be divided up into two parts:

- polynomial transformation or a S-transformation between two local 2+1D-, or 3D-Datums
- bi-linear interpolation for random Datum differences.

7.1 Using Bi-linear Interpolation

Transformation of two Datums result in different coordinate values. In essence, there are two sources for the differences, the first being the conformal projection between differing ellipsoids with some distortion, the second source is commonly known as the differing between levels with some distortion. Both sources result in 3D-differences, which are smoothly varying over the country, or continent. The curvature of the differences being continuous, where the distortions may result in irregular location-dependent differences. This inhomogeneity makes a simple transformation between any local 2+1D-Datum and a 3D-Datum, such as a WGS84, impossible or inaccurate for further use.

Hydrographers and land surveyors need to understand both the horizontal reference system aspects of coordinate computation, which is in tune with the technical developments [4.3]. Such reference systems include the possibility of incompatibilities between spaceborne or airborne surveying techniques, existing local horizontal Datums, and the vertical reference system aspects in the field of earth-centred-earth, fixed (ECEF) height coordinate computation.

2D-Horizontal Correction Grid

To apply horizontal techniques to field surveying, a geoid correction grid should be applied twice by means of the method of bi-linear interpolation. 2D-horizontal correction grids should ideally provide an accuracy of a few millimetres uncertainty in any 10 km square grid area. If not, an upgrade of the size of the square grid is highly advisable. Correction errors depend on the bias in the errors of the LS (least squares) adjustment techniques and height determination (Figure 30): pp 78.

1D-Vertical Correction Grid

The uses of reference systems include the possibility of incompatibilities between spaceborne or airborne altimetric techniques and local vertical Datums. To apply altimetric correction techniques to vertical surveying, a geoid undulation correction should be applied by means of e.g. the method of bi-linear interpolation. The 1D-vertical correction grid should ideally provide an accuracy of a few millimetres uncertainty in any 10 km square grid area. Once more, if not, an upgrade of the size of the square grid is highly advisable. The correction error depends on the bias in the errors of the vertical component of GPS height determination.

Adjusting a Transformation by Bi-linear Interpolation of Datums

This section deals with aspects of bi-linear interpolation, which minimises geometrical distortions between the curved grid surfaces of transformed projections. One suitable solution for modelling random Datum differences is the bi-linear interpolation using a (polynomial) transformation by computer.

A double interpolation method corrects the varying differences between any Datums as separately operations on latitude, longitude, and height of the geoid, or in the example easting, northing with a look-up computer tables for each component: one for the shifts in eastings and another for the shifts in northings. The shifts between the grid points are carried out using a bi-linear polynomial.

Example: in the past, private companies have developed different methodologies for the transformation of data between the Datum Bessel-Amersfoort (RD1918) and the European Datum of 1950 (ED50). RD1918 is used for all national surveys within the Netherlands and ED50 was used for all surveys on the Dutch continental shelf and coastal margins. The independently derived methods optimise transformations within a particular area of the North Sea. In the polynomial transformation, the Datum shifts were often the result of fitting very high-order polynomials to a limited number of accurate coordinated data points, calculated between ED50-Datum coordinates, using the Gauss-Krüger (GK) type conformal (transverse Mercator) projection [10.4] and RD1918-Datum coordinates, using an GS type oblique stereographic conformal (OSC) double projection [10.7].

Observe

The shifts between two Datums, such as Bessel-Amersfoort and ED50, arise from a difference in the reference and coordinate systems as well as small differences or local distortions arising due to differences in moving tectonic micro-plates, least squares adjustment and survey methodologies.

Unfortunately, an accurate Dutch national ED50 to RD1918 transformation cannot be achieved using a simple polynomial transformation model. A polynomial transformation model approaches the required accuracy to about 1.0 m accuracy at the one ρ level.

Bi-linear interpolation comprises transfer of the weighted mean of the digital number obtained for the four nearest coordinated points of any square: ED50 corrections for ΔE (Figure 51).

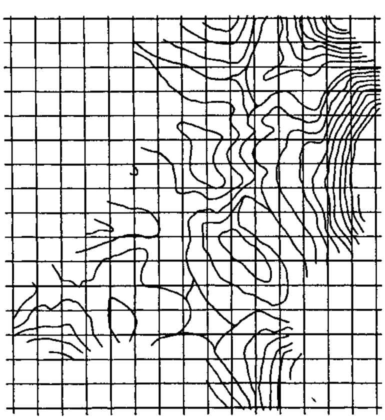


Figure 49: ED50 corrections for ΔE

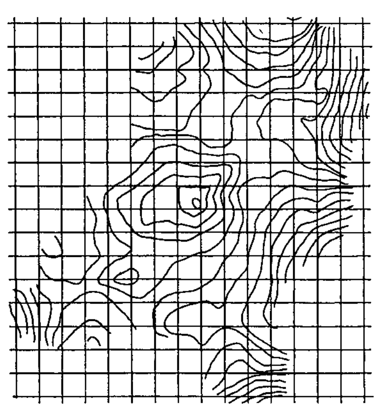


Figure 50: ED50 corrections for ΔN

Here it is used to correct a transformation from the oblique stereographic projection of The Netherlands, reference ellipsoid Bessel-Amersfoort (RD1918), into Gauss-Krüger (GK) projection type - UTM grid, European Datum of 1950 (ED50), reference ellipsoid International 1924 and vice versa. RD1918 and ED50 do not exactly map into one another.

The graphs Figure 49 and Figure 50 - inserted for demonstration purposes only (original size is A4, showing corrective data) - show the ΔE, ΔN correction curves for Eastings and Northings, respectively. The graph values at the grid-intersections were transferred to matrices for application of bi-linear interpolation model in program A_09BILI.FOR [18.9]. An appropriate test case is given On_CD, using A_09ZO31.EXE
The approach of the program relies on a two-step process for UTM Zone 31:

- transformation by a 2D-polynomial from RD1918 to ED50, and vice versa
- using bi-linear interpolations, based on the correction grids, which effectively remove inhomogeneous (distorted) data.

Two tabulated data sets are required for the complete computation of a Datum shift (transformation); one for easting-shifts and another for northing-shifts. Thus, two mathematical correction grids (surfaces) must be prepared for a region, country or continent.

Observe

Please note that, if needed, bi-linear interpolation can be used for many purposes, such as contour charts of geoidal heights or other physical sizes.

A 2D-polynomial, prepared by the Topografische Dienst of the Netherlands (TDN), Department of Defence, was fitted to the actual (observed) Datum shifts between RD1918 and ED50, using first order stations. This method often provides results with an accuracy of better than 1 m. Local distortions, implicit within ED50, are evident and require a correction in eastings and northings, ΔE_i and ΔN_i, respectively (Linden, 1985).

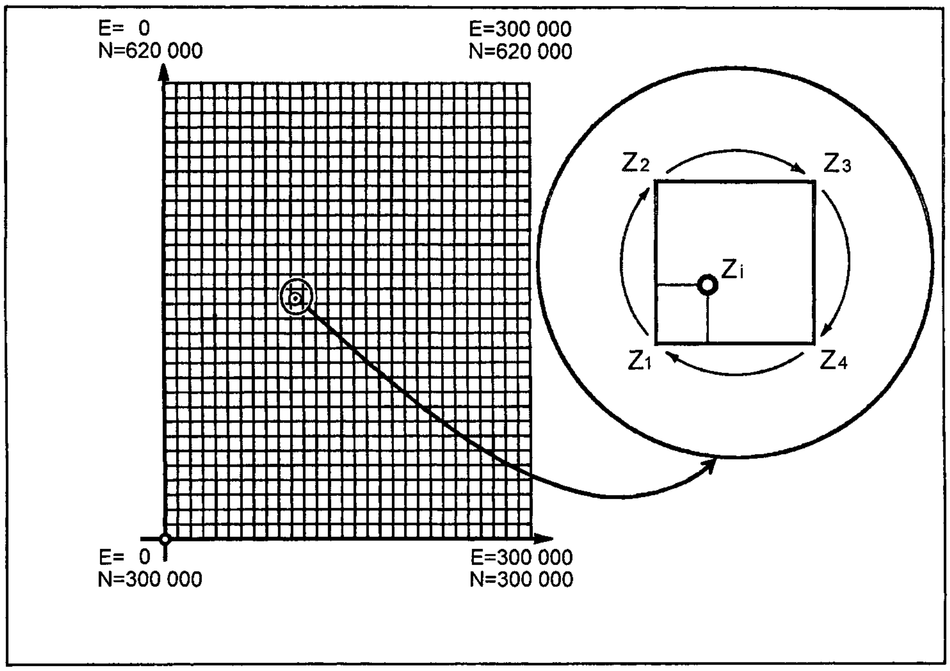


Figure 51: Location of a grid square for interpolation

Contour Charts of Shifts

The corrections ΔE_i, ΔN_i can be depicted as contour charts which show the corrections *in mm* to the transformed data points of RD1918. These contour correction charts, prepared by the TDN, developed from coordinate differences by subtracting the transformed Bessel 1841 numerical grid data (oblique stereographic projection) from the ED50 Gauss-Krüger - UTM numerical grid data. TDN has prepared the difficult aspect of data selection, including two contour charts of shifts, and the computation of two tabulated digital data sets of ΔE_i, ΔN_i-shifts.

Here, the application of bi-linear interpolation is described as it applies to the correction of transformed positional data between the local geodetic Datums RD1918 and ED50. The author has implemented this approach and technique in a computer program, known as A_09ZO31.EXE an acronym standing for the RD1918 to ED50 Conversion, UTM (3°E) Zone 31. A_09ZO31.EXE *was developed to provide a uniform methodology in which a simple interpolation routine automatically provides estimates of ΔE_i, ΔN_i values at random non-nodal points within the area of the first order stations mentioned above.*

Actual application of the processed data was designed in a simple application program for a Hewlett-Packard Desktop 256 KB computer (Bjork, 1974; Hooijberg, 1984; Olson, 1977).
For further reading, see (Alberda, 1978; Heuvelink, 1918; Linden, 1985; Luymes, 1924; Stem, 1989b; Strang van Hees, 2006).

Practical Aspects of Bi-linear Interpolation

Consider a (demo) matrix of grid locations shown in Figure 49 and Figure 50. Note that an estimate at an unknown grid location can be found for each point within this matrix grid.
Tabulated data grids require bi-linear interpolation to be useful for application by computer programs. For any data set of a square grid, a minimum of four values is required. The nodal points Z_1, Z_2, Z_3, Z_4 are known corrections (ΔE_i, ΔN_i, respectively) at grid intersecting points, and the given grid interval (such as Δ=10 km) is imperative (Figure 51). The method will determine a smooth surface.

Horizontal Correction Grids

In general, Geodetic Reference System (GRS) aspects in horizontal coordinate computation include the possibility of incompatibilities between spaceborne 3D-systems and local 2D-horizontal Datums. All spaceborne survey-systems are performed largely using pseudo-ranging DGPS and / or DGLONASS techniques.

To apply latitude- and longitude corrections, $\delta\varphi$, $\delta\lambda$, respectively, to the calculation, a local horizontal correction grid should be applied using e.g. the method of bi-linear interpolation. This correction grid should ideally provide an absolute accuracy of a few millimetres uncertainty in any 10 km square grid area. If not, an upgrade of the size of the square grid is advisable. Remaining errors are mainly due to the bias in errors of the pseudo-ranging GPS techniques.

Bi-linear Interpolation Scheme

The polynomial used for the bi-linear interpolation is simple and accurate (Dewhurst, 1990; OS, 1995b). I and J are calculated, positional indices. The *sign convention* is ED50 minus RD1918 (local geodetic Datum).

Clockwise from the southwest corner of the cell, the PE-indices (and similar for PN-indices) are:

Z_1	=	PE (I, J)	the (easting) index of the *lower left corner* of the cell in which the unknown point resides.
Z_2	=	PE (I, J + 1)	
Z_3	=	PE (I+1, J + 1)	
Z_4	=	PE (I+1, J)	

The following A, B, C, and D are coefficients of the following polynomial (7.05a, 7.05) and are all functions of the shift values of the surrounding nodal points (Figure 51).

$$A = Z_1 \tag{7.01}$$

$$B = (Z_4 - Z_1) / \Delta \tag{7.02}$$

$$C = (Z_2 - Z_1) / \Delta \tag{7.03}$$

$$D = (Z_1 - Z_2 + Z_3 - Z_4) / \Delta^2 \tag{7.04}$$

in which Z_1, Z_2, Z_3, Z_4 are known corrections ΔE_i, ΔN_i, respectively at grid points used in the interpolation process.

The origin is located in the lower-left corner of a grid square of dimension Δ on a side. The sub-gridding technique is known as bi-linear interpolation (Bjork, 1974; Olson, 1977) and is defined as:

$$Z = A + B \times X + C \times Y + D \times X \times Y \qquad (7.05a)$$

The polynomial surface, fit to the four surrounding nodal points Z_1, Z_2, Z_3 and Z_4, is recasted as:

$$Z = A + C \times Y + (B + D \times Y) \times X \qquad (7.05)$$

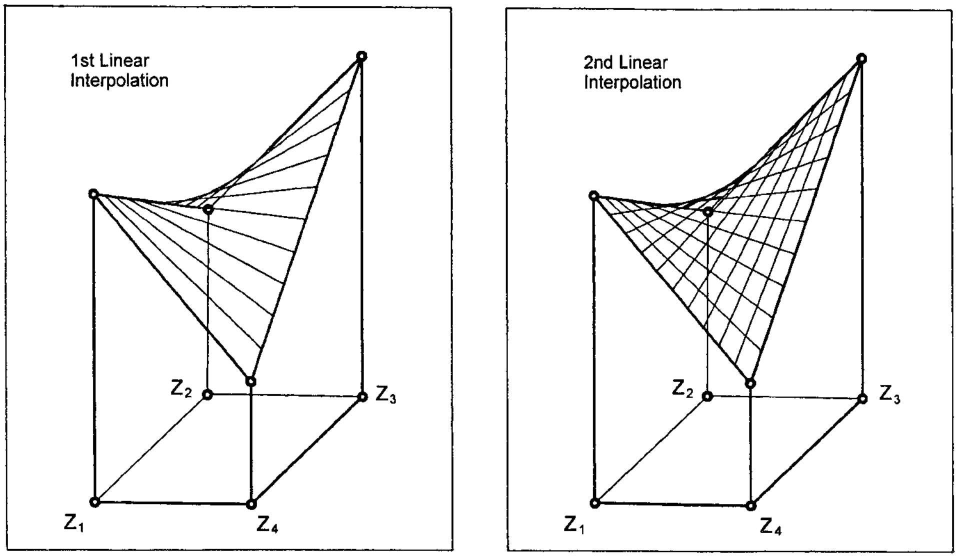


Figure 52: Cross sections of the bi-linear interpolation surface are defined by straight-line elements

and interpolated for point P_i at which place Z is the calculated correction at the unknown point (Figure 51, Figure 52). It is applied *twice*: one for eastings and one for northings.

This equation (7.05) is solved using index location based on the row and column organisation of the grid, as where X and Y represent the coordinates of the unknown point.

The grids are organised as matrix rows, minimum easting to maximum easting, and matrix columns, minimum northings to maximum northings. Thus in this case (Figure 51):

• matrix rows:	eastings run from	0 m to 300 000 m in 10 km (Δ) steps
• matrix columns:	northings run from	300 000 m to 620 000 m in 10 km (Δ) steps

Interpolated results are calculated to nearest millimetre, using an integer precision matrix.

A_09ZO31.EXE program [On_CD] provides for the application of the interpolation method outlined above on all of the tabulated 10 km gridded data sets. The program user merely has to ensure that the proper gridded data set forever resides in the program or storage area. Eastings and Northings provided by the user will be converted between Datums via the program. A_09ZO31.EXE allows for the conversion of individual points in UTM Zone 31, CM 3°E (Hooijberg, 1997).

Bi-Linear Interpolation from Bessel to ED50 and vice versa

ellipsoid old:		Bessel-Amersfoort	ellipsoid new:		ED50
semi-major axis	a_o :	**6 377 397.155**	semi-major axis	a_n :	**6 378 388.0**
recipr. flattening	f^{-1}_o :	**299.15281 285**	recipr. Flattening	f^{-1}_n :	**297.000**

Input coordinates of a point

RD1918-X	[xi]	:	189121.4430
RD1918-Y	[yi]	:	510886.0950

Compute position of Z1 in matrix:

we1		=	180000.0000
wn1		=	510000.0000
xd		=	9121.4430
yd		=	886.0950

Input the values of the corner points Z1, Z2, Z3, Z4, in matrix:

I .... J		=	19, ... , 22
IZ1 [east]		=	-20
IZ2 [east]		=	10
IZ3 [east]		=	-30
IZ4 [east]		=	-70
easting correction	[δe]	=	−0.0621
IZ1 [north]		=	79
IZ2 [north]		=	107
IZ3 [north]		=	83
IZ4 [north]		=	69
northing correction	[δn]	=	+0.0712

from RD1918 with the correction (δ) to ED50 UTM grid zone 31						
RD1918-X :	189 121.4430	ED50-E =	695 920.6339	−0.0621 (δe)	=	695 920.5718
RD1918-Y :	510 886.0950	ED50-N =	5 830 185.0216	+0.0712 (δn)	=	5 830 185.0928

The Bi-linear interpolation method is documented in:

- (Bjork, 1974)
- (Dewhurst, 1990) for transformation of NAD27 to NAD83
- (NGA, 2000) TR 8350.2 for transformation of local geodetic Datum to WGS84
- (Hooijberg, 1984) for transformation of ED50 grid to RD1918 grid
- (Olson, 1977)
- (OS, 1995b) for transformation of OSGRS80 grid to the National grid
- (Hooijberg, 1997).

7.2 S-transformation

Changing from one Datum to another implies a mathematical operation with Datum transformation formulae that fit the geoid as closely as possible. Thus, the geoidal heights will be small.

Employing analytical and exact techniques for the transformation of data from one Datum to another are widely accepted in geodetic and surveying organisations.
A transformation can be accomplished in curvilinear and in rectangular space. The procedure is only possible if the Datum shifts, e.g. $d\varphi$, $d\lambda$ and dh, are measured or calculated. NGA has prepared methods for the transformation of data from various local geodetic Datums into the WGS84, the only global and geocentric Datum right now (Ayres, 1995; NGA, 2000).

Several types of Datum transformation formulae have been developed, such as:

- the standard Molodensky formulae or the abridged Molodensky formulae
- multiple regression equation (MRE) technique

- 2D high-order polynomials fitting Datum shifts
- affine transformation that relies upon actual and observed coordinates in both Datums
- the 3D-transformation parameters for local regions.

Eliminating the conversion from geodetic to rectangular coordinates, the 3-parameter rectangular case is embedded mathematically in the Molodensky formulae. Depending on the availability of accurate transformation parameters, the 3D-similarity transformation is the most commonly used technique in the industry.

Representing the same point on the Earth's surface, data with two different, distinct coordinate values is all that is required to convert data between two Datums. The defining parameters provide the basis for computation of all other positions in the geodetic network. S-Transformation, [18.10] is such a program designed to transform or convert data. Readers requiring more details are requested to consult the references (Brouwer, 1989; Floyd, 1985; Ihde, 1995; Paggi, 1994a, 1994b; Rapp, 1981).

Earth's Average Spin Axis

Datum differences attributed to differences in the ellipsoid can be more than 200 m (e.g. in SE Asia) in amplitude due to differences in adjustment and survey methodologies implicit within classical Datums.
Redefining the deflection of the vertical at the origin and the orientation of the reference ellipsoid requires determination of the geodetic and astronomic positions of many monumented points throughout the geodetic network, making the geodetic Datum a close fit to the Earth.

A perfect geodetic Datum is not necessarily geocentric. If a Datum is created with care, the equatorial plane of the ellipsoid is nearly coincident with the equatorial plane of the Earth, and the minor axis of the ellipsoid is nearly parallel with the average spin axis of the Earth at a designated epoch.

Space Coordinates

In real time positioning, the user is fixed in inertial space. Coordinates are expressed in the Earth-Centred-Inertial (ECI) coordinate system and the True Time. Assuming an exact user's clock, the satellite and user coordinates are both expressed in an ECI coordinate system - which is nonrotating with the X-axis aligned with a vector from the Sun's centre to the Earth's position at the Vernal Equinox, with the Origin at the Earth's centre (Parkinson, 1996).

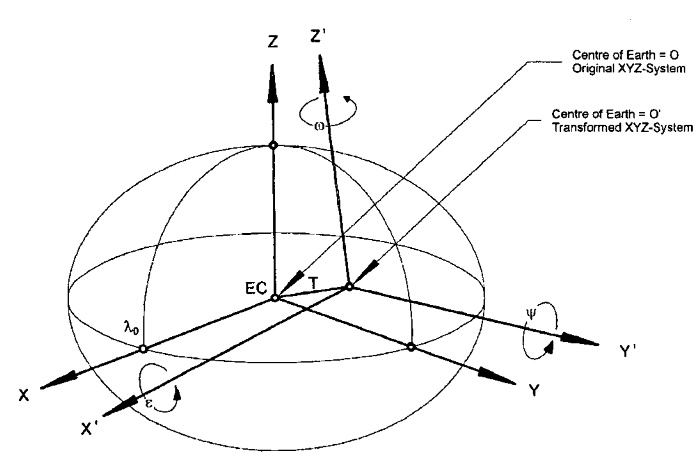


Figure 53: Translated and rotated 3D-coordinate system

An XYZ-space coordinate system is simply a 3D-system of Cartesian coordinates in a Euclidean-space-and-time. ECEF (earth-centred, earth-fixed) coordinates rotate with the Earth, and the XZ-plane contains the meridian of zero longitude - the Greenwich Meridian. When used for satellite surveying, the origin of the coordinate system is at the Earth's centre of mass, providing a reference ellipsoid upon which a geodetic Datum is based with truly geocentric coordinates. The XY-plane of the system is coincident with the equatorial plane of the ellipsoid, the Z-axis is initially coincident with the minor axis of the ellipsoid (Figure 53).

Curvilinear Geodetic Datum Transformation

Methods for transforming Cartesian coordinates into 3D-data are widely accepted in geodetic and surveying organisations. One technique used here is to carry out a similarity or S-transformation:

- geodetic coordinates in the old Datum (φ_o, λ_o, h_o) are converted to old XYZ-coordinates: X_o ,Y_o ,Z_o (X_o)
- old XYZ-coordinates (X_o) are transformed to new XYZ-coordinates, X_n ,Y_n ,Z_n (X_n), by translation (T) of the origin O, rotation of the axes Z, X, Y, (R, and a change of the scale (1+k)
- new XYZ-coordinates (X_n) are converted to new geodetic coordinates in the new Datum: φ_n, λ_n, h_n of the origin O'.

Conversion of geodetic coordinates of a point P (φ_p, λ_p, h_p) to XYZ-coordinates X_p ,Y_p ,Z_p initially requires knowledge of the height h_p of that point P above the ellipsoid, which is the sum of orthometric height (H = elevation above MSL) and geoidal separation (N_s).

Symbols and Definitions of S-transformation Algorithms

For computation of the ellipsoid, constants (Table 19): pp 123, and expressions may contain subscripts "$_o$" and "$_N$", which denote old and new, respectively. All angles are expressed in radians (Figure 54, Figure 55), program A_10STRM.FOR, [18.10].

Definitions

a	semi-major axis of the ellipsoid	
b	semi-minor axis of the ellipsoid	
f	flattening	
e^2	first eccentricity of ellipsoid squared	
e'^2	second eccentricity of ellipsoid squared	
H	geoidal orthometric height	
h	elevation of the Datum	
NS	geoidal separation of the Datum (N)	
RN	radius of curvature in the meridian	
φ	geodetic latitude	
λ	geodetic longitude	
X_o	old 3D Cartesian coordinate system	
X_n	new 3D Cartesian coordinate system	
O	origin of the old 3D Cartesian system	
O'	origin of the new 3D Cartesian system	
ΔX	component of origin translation X	direction new minus old Datum
ΔY	components of translation Y	direction new minus old Datum or (Y_n - Y_o)
ΔZ	components of translation Z	direction new minus old Datum or (Z_n - Z_o)
T	O-O' translation vector (ΔZ, ΔX, ΔY)	
R	rotation matrix of XYZ system	about ω, ε, ψ-angles (rad)
ω (γ)	omega	rotation of XYZ system about Z-axis
ε (α)	epsilon	rotation of XYZ system about X-axis
ψ (β)	psi	rotation of XYZ system about Y-axis
Δk	change in scale of the Datum	from old to new Datum in ppm (10^{-6})
$\Delta k'$	change in scale Δk_z, Δk_x, Δk_y	for 9-parameter transformation

The similarity transformation in 3D provides an accurate methodology for the transformation of original XYZ-coordinates X_o ,Y_o ,Z_o to new XYZ-coordinates X_n ,Y_n ,Z_n.

Here, the ω, ε, ψ Eulerian angles for reference frame orientation, are rotated successively about the Z, X, Y axes respectively from the old Datum to the new Datum. The specific sequence of individual rotations is here as follows:

1. ω-rotation of XYZ system about Z-axis |[11] rotation, in counter-clockwise direction

[11]Rotation is counter-clockwise direction as seen from the positive axes toward the origin

2. ε-rotation of XYZ system about X-axis after application of ω, rotation in same sense as ω
3. ψ-rotation of XYZ system about Y-axis after application of ω and ε.

6-Parameter S-transformation

$$X_n = X_\Delta + Rx_o \tag{7.06}$$

or:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{new} = \begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix} + \begin{vmatrix} 1 & \omega & -\psi \\ -\omega & 1 & \varepsilon \\ \psi & -\varepsilon & 1 \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{old} \tag{7.07}$$

7-Parameter S-transformation

$$X_n = X_\Delta + RX_o\ (1+k) = X_\Delta + R_k X_o \tag{7.08}$$

or:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{new} = \begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix} + \begin{vmatrix} (1+\Delta k) & \omega & -\psi \\ -\omega & (1+\Delta k) & \varepsilon \\ \psi & -\varepsilon & (1+\Delta k) \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{old} \tag{7.09}$$

See also: General 7-parameters transformation algorithm, (7.12).

9-Parameter S-transformation

$$X_n = X_\Delta + RX_o(1+k') = X_\Delta + R'_k X_o \tag{7.10}$$

or:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{new} = \begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix} + \begin{vmatrix} (1+\Delta k_x) & \omega & -\psi \\ -\omega & (1+\Delta k_y) & \varepsilon \\ \psi & -\varepsilon & (1+\Delta k_z) \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{old} \tag{7.11}$$

Observe

Specifying the orientation of the three axes, the definition follows astronomical usage, and in theoretical mechanics according to US usage. In principle, the formulae are different for any other sequence of Eulerian angles, but in practice, there will be no dissimilarity in accuracy (Brouwer, 1961; Jekeli, 2001). Assuming a transformation for nearly aligned coordinate systems, the Eulerian angles ω, ε, and ψ are very small. Hence, the sines of the angles are approximately equal to the angles themselves. Consequently, the sines are approximately equal to zero, and the cosines are approximately equal to one. Therefore, it is allowable to simplify the matrix of rotation (R).

General 7-Parameters Transformation Algorithm

The Cartesian coordinates $(X_i, Y_i, Z_i)_o$ of a point P on the old Datum are to be transformed into its coordinates $(X_i, Y_i, Z_i)_n$ on the new Datum. X_o, Y_o, Z_o are Cartesian coordinates of a defining, initial point on the old Datum, about which the scale, shift and rotation changes are applied.

In the 7-parameter transformation algorithm, the Cartesian coordinates X_O, Y_O and Z_O of the Initial Point on old Datum may be set in such a way that the initial point is:

- at the centre of Earth - identified as R (the regular case) or
- at the Origin (φ_o, λ_o) of the old Datum - identified as the G (general case)

G is exemplified in case IIb, used *here for RD1918 only.*

$$\begin{vmatrix} X_i \\ Y_i \\ Z_i \end{vmatrix}_{new} = \begin{vmatrix} X_i \\ Y_i \\ Z_i \end{vmatrix}_{old} + \begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix} + \begin{vmatrix} \Delta k & \omega & -\psi \\ -\omega & \Delta k & \varepsilon \\ \psi & -\varepsilon & \Delta k \end{vmatrix} \begin{vmatrix} X_i - X_0 \\ Y_i - Y_0 \\ Z_i - Z_0 \end{vmatrix}_{old} \quad (7.12)$$

in which the Cartesian coordinates of P_i on the old Datum $(X_i, Y_i, Z_i)_o$ are to be transformed into its coordinates $(X_i, Y_i, Z_i)_n$ on the new Datum and the parameters are, using program S-transformation, [18.10].

S-transformation Equations

S-transformation: menu: R (regular) or G (general).

Input and output data

Input:	geodetic coordinates of a point P $(\varphi_o, \lambda_o, H, N_o)$.
Output:	geodetic coordinates of a point P $(\varphi_n, \lambda_n, h_n, N_n)$.

Compute Cartesian coordinates in the Old System

$$h_o = H + Ns_0 \quad (7.13)$$
$$RN = a / (1 - e^2 \sin^2 \varphi)^{½} \quad (7.14)$$
$$X_0 = (RN_0 + h_0) \cos \varphi_0 \cos \lambda_0 \quad (7.15)$$
$$Y_0 = (RN_0 + h_0) \cos \varphi_0 \sin \lambda_0 \quad (7.16)$$
$$Z_0 = (RN_0 (1 - e_0^2) + h_0) \sin \varphi_0 \quad (7.17)$$

Compute Cartesian coordinates in the New System

Using the following equations, the S-transformation can be performed in spatial coordinates:

$$\begin{aligned} X_n &= \Delta X + (X_o \ +\omega Y_o \ -\psi Z_o) \ (1+\Delta k) \\ Y_n &= \Delta Y + (Y_o \ -\omega X_o \ +\varepsilon Z_o) \ (1+\Delta k) \\ Z_n &= \Delta Z + (Z_o \ +\psi X_o \ -\varepsilon Y_o) \ (1+\Delta k), \end{aligned} \quad (7.18)$$

in which subscripts "$_O$" and "$_N$" denote OLD and NEW, respectively.

ΔX, ΔY, ΔZ, ω, ε, ψ, and Δk are the seven transformation parameters needed to compute X_n's, Y_n's, Z_n's coordinates. In a well-designed geodetic Datum, Eulerian angles ω, and especially ε and ψ are (almost) equal to zero, the reference ellipsoid is properly oriented to Physical Earth. Then the Datum transformation can be used for geodetic- and precise positioning applications.

Figure 54 and Figure 55 illustrate the relations between h, Ns, and H.

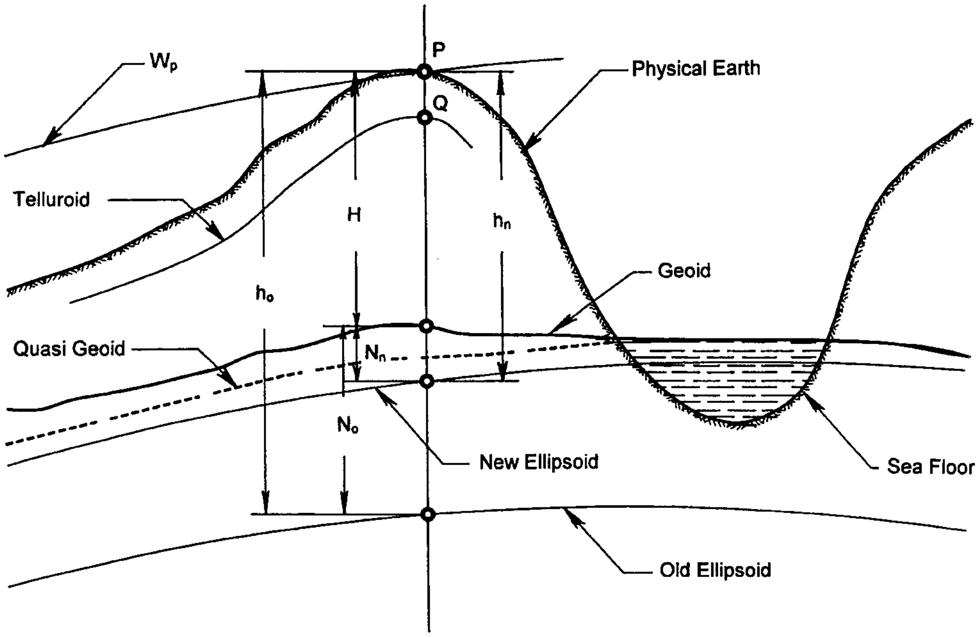


Figure 54: Geoid separation of the old and the new Datum

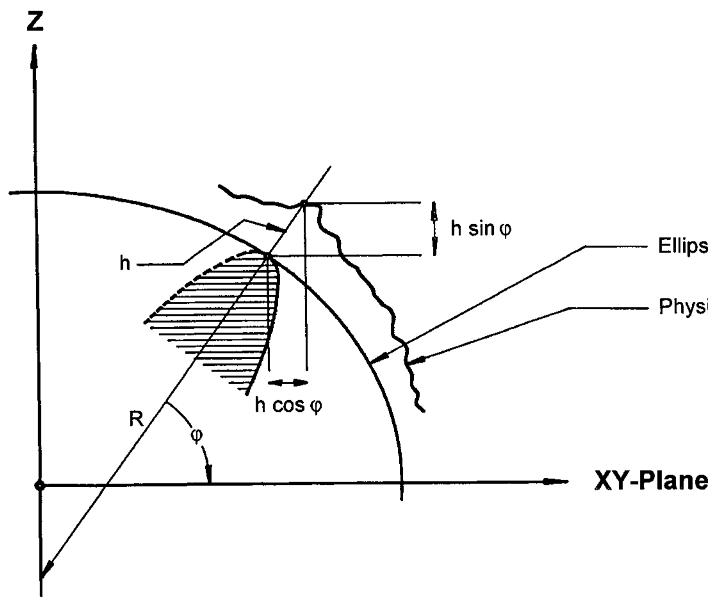


Figure 55: Ellipsoidal and physical Earth

Computation of Geodetic Coordinates:

Computation of latitude is obtained without iteration (Borkowski, 1989; Bowring, 1976):

$$p = (X_n^2 + Y_n^2)^{½} \tag{7.19}$$
$$\theta = \tan^{-1} [a_n Z_n / (b_n p) \tag{7.20}$$
$$\varphi_n = \tan^{-1} [(Z_n + e_n'^2 b_n \sin^3\theta) / (p - e_n^2 a_n \cos^3\theta)] \tag{7.21}$$

- As mentioned above: in the computation is usually no iteration required. Only in special cases is the use of a corrected value of θ imperative, and iterate if $[(\varphi_n' - \varphi_n) > 1.10^{-11}]$ before obtaining φ_n (Schuhr, 1996):

$$\varphi_n' = \varphi_n$$
$$\theta = \tan^{-1} [b_n / a_n . \tan \varphi_n') \tag{7.22}$$
$$\varphi_n = \tan^{-1} [(Z_n + e_n'^2 b_n \sin^3\theta) / (p - e_n^2 a_n \cos^3\theta)] \tag{7.23}$$

$$\lambda_n = \tan^{-1} (Y_n / X_n) \tag{7.24}$$
$$Rn_n = a_n / (1 - e^2_n \sin^2\varphi_n)^{½} \tag{7.25}$$
$$h_n = p / \cos \varphi_n - RN_n \tag{7.26}$$
$$Ns_n = h_n - H \tag{7.27}$$

Notes

For more features on Datum transformation in a Euclidean 3D-space, the reader may consult the report about: Conformal group $C_7^{(3)}$ - Curvilinear Geodetic Datum transformations (Grafarend, 1995e):

> *"Positioning by global satellite positioning systems results in the problem of curvilinear geodetic Datum transformations between stations in a local (2+1)D-geodetic reference system and a global 3D-geodetic reference system.*
>
> *In the process of (2+1)D towards 3D-geodesy the Datum parameters of the seven parameter global conformal group $C_7^{(3)}$ with translation, rotation, and scale observational equations are solved by a least squares (LS) adjustment".*

Or a report about the ten parameter conformal group $C_{10}^{(3)}$ - curvilinear geodetic datum transformations (Grafarend, 1996b):

> *"3D- geodetic Datum transformations with two data sets of 3D-Cartesian coordinates which leave angles and distance ratios equivariant (covariant, form invariant) are generated by the ten parameter conformal transformation group $C_{10}^{(3)}$ in a Euclidean three-dimensional space*
>
> *A geodetic Datum transformation whose ten parameters are determined by effective adjustment techniques will play a central role in contemporary Euclidean point positioning".*

or consult (Baarda, 1981; Borkowski, 1989; Bowring, 1976; Burša, 1966; Leick, 1975; Molenaar, 1981; Paggi, 1994; Rapp, 1981; Schuhr, 1996) for more information about mathematical developments and derivations.

Converting geodetic coordinates of a national or continental terrestrial Datum into the WGS84 Datum and vice versa requires specific transformation formulae. An S-transformation produces an average result due to local differences in adjustment and distortions. Using a least squares adjustment (LS) method, the relationship between WGS84 and a national terrestrial Datum may remove significant distortions. Many transformation parameters between WGS84 and national Datums have been computed by the US Army using the Doppler and the global positioning systems using the values of primary stations worldwide (Grafarend, 1995e, 1996b, 2002g, 2003c, g; Awange, 2003d, 2004; Rens, 1990).

Application I - Similarity Transformations of Italy

S-transformation from WGS84 to IGM1940
Datum: Roma, M. Mario, Sistema Geodetico Nationale

ellipsoid old:			WGS84	ellipsoid new:			IGM1940
datum $_0$			WGS84	datum $_n$			Roma, M.Mario
semi-major axis	a_0	:	**6 378 137**	semi-major axis	a_n	:	**6 378 388**
recipr. flattening	f^{-1}_0	:	**298.25722 3563**	recipr. flattening	f^{-1}_n	:	**297**
see parameter specifications below.							

Cartesian coordinates of P_i $(X_i, Y_i, Z_i)_o$ on the old Datum are to be transformed into its coordinates $(X_i, Y_i, Z_i)_n$ on the new Datum. Using program S-transformation [18.10], and using the 6-, 7- or 9-parameter data mentioned below in the S-transformation algorithm, result in calculated differences as shown in Figure 56, Figure 57, and Figure 58.

S-transformation Parameters of Italy

In Italy, relationships between the *national Datums:* IGM1940 and WGS84 were developed and transformation parameters between the Datums were derived (Surace, 1995). Examples are [On_CD]:

6-Parameters Transformation WGS84 → IGM1940 (Figure 56)

ΔX	=	+265.358 m
ΔY	=	+177.209 m
ΔZ	=	−86.405 m
ε	=	−1.7199E-05 rad
ψ	=	+1.2662E-05 rad
ω	=	+8.4928E-06 rad

7-Parameters Transformation WGS84 → IGM1940 (Figure 57)

ΔX	=	−139.052 m
ΔY	=	+90.470 m
ΔZ	=	−471.337 m
ε	=	−1.7199E-05 rad
ψ	=	+1.2662E-05 rad
ω	=	+8.4928E-06 rad
Δk	=	+8.8720E-05

9-Parameters Transformation WGS84 → IGM1940 (Figure 58)

ΔX	=	+655.941 m
ΔY	=	+98.669 m
ΔZ	=	−1232.988 m
ε	=	−2.8062E-05 rad
ψ	=	+9.7721E-05 rad
ω	=	−1.7656E-06 rad
Δk_x	=	−2.5246E-06
Δk_y	=	+8.0713E-05
Δk_z	=	+1.7246E-04

(Achilli, 1994; Anzidei, M.; Paggi, 1994)

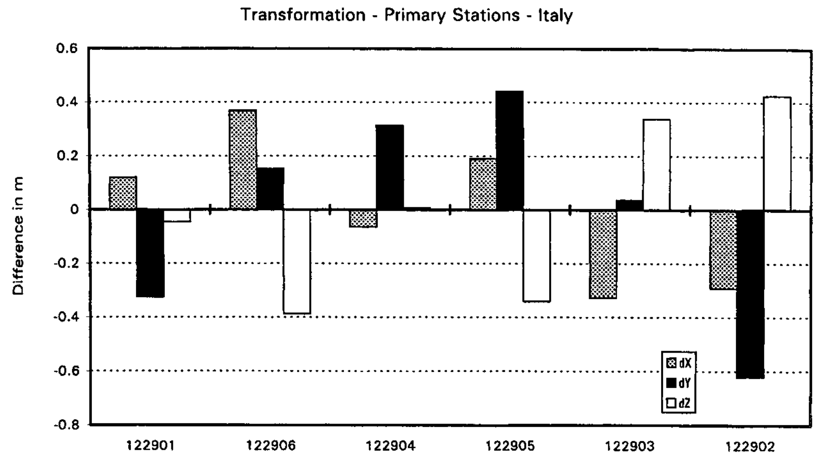


Figure 56: Calculated differences for the 6-parameter transformation solution

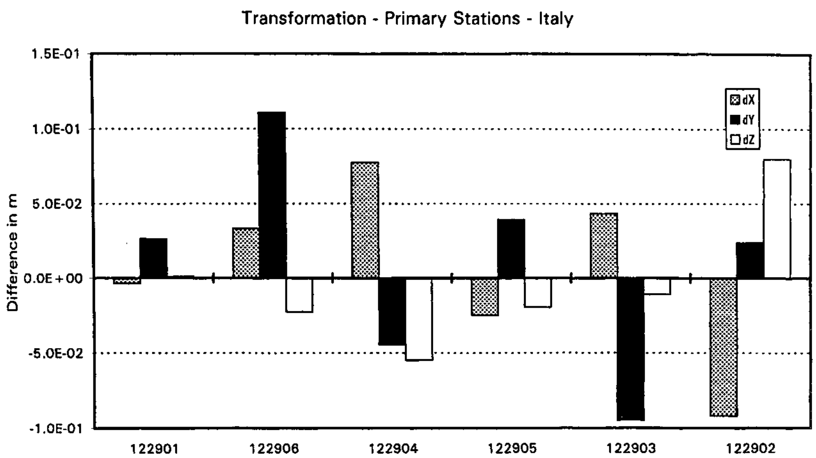


Figure 57: Calculated differences for the 7-parameter transformation solution

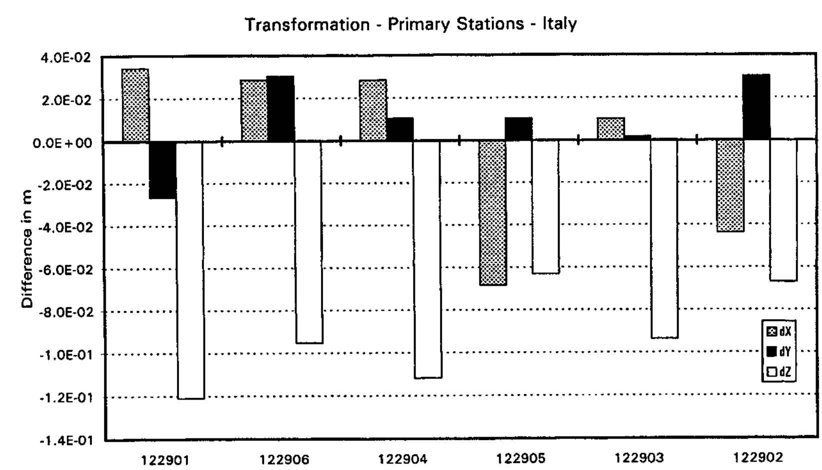


Figure 58: Calculated differences for the 9-parameter transformation solution

Application II - Similarity Transformations of the Netherlands

Converting geodetic coordinates of a national or continental terrestrial Datum into the ETRS89 Datum and vice versa requires S-transformation formulae. (NGA, 2000, and later) gives such formulae for most Datums, but an S-transformation produces only an average result due to limited numbers of stations, differences in adjustment and distortions. Using a least squares adjustment (LS) method, a relationship between ETRS89 and Bessel-Amersfoort, RD1918, was developed. Nevertheless, significant local distortions are removed by other (bi-linear interpolation) formulae. Transformation parameters between these Datums have been computed by Schut (1991) using the Doppler system derived values of 13 primary stations in the Netherlands. Geodetic coordinates of Sta. Amersfoort "*Lieve Vrouwe*" Tower (RD-origin) are in RD1918 and ETRS89, respectively:

φ RD.$_0$	:	52° 09' 22".1780 N	λ RD.$_0$	:	5° 23' 15".5000 E	h RD.$_0$	:	0.000 m
φ ETRS.$_0$	:	52° 09' 18".6200 N	λ ETRS.$_0$	:	5° 23' 13".9327 E	h ETRS.$_0$	:	43.348 m

S-transformation Parameters

The seven parameter transformation algorithm, (7.12 ... 7.27) is used here. The Cartesian coordinates (X, Y, Z)$_o$ of a point P on the old Datum are to be transformed into its coordinates (X, Y, Z)$_n$ on the new Datum. X_o, Y_o, Z_o are Cartesian coordinates of a defining, initial point on the old Datum, about which the scale, shift and rotation changes are applied (Bruijne, 2004; Min, 1996; Schut, 1991; Strang van Hees, 2006).

Regular Application case IIa

The geoid is not used. In the 7-parameter transformation algorithm, the Cartesian coordinates X_o, Y_o, Z_o of the initial point on old Datum may be set so that the Initial Point is at the centre of Earth: $X_o = Y_o = Z_o = 0$.

Replacing geoid height values by zero will result in latitude (φ), longitude (λ) and no height.

Regular case IIa - the following ETRS89 to RD1918 S-transformation parameters are given:

$X_{0\,RD}$	=	0.0	initial point (earth centred)
$Y_{0\,RD}$	=	0.0	initial point (earth centred)
$Z_{0\,RD}$	=	0.0	initial point (earth centred)
ΔX	=	–565.040 m	from ETRS89 to Bessel-Amersfoort
ΔY	=	–49.910 m	
ΔZ	=	–465.840 m	
ε	=	$-1.9848\ 10^{-6}$ rad	
ψ	=	$+1.7439\ 10^{-6}$ rad	
ω	=	$-9.0587\ 10^{-6}$ rad	
Δk	=	–4.0772 ppm	

Using $C_7^{(3)}$ transformation Regular algorithm (7.09) in [18.10], A_10STRM.FOR.

Bessel-Amersfoort to ETRS89 Transformation - Regular case IIa

S-transformation from Bessel-Amersfoort (RD1918) to ETRS89 - (Strang van Hees, 2006).

ellipsoid old:			Bessel1841	ellipsoid new:			GRS80
semi-major axis	a_o	:	**6 377 397.155**	semi-major axis	a_n	:	**6 378 137.0**
recipr.flattening	$f^{-1}{}_o$	:	**299.15281 285**	recipr.flattening	$f^{-1}{}_n$	:	**298.25722 21008827**
delta	X_m	:	**565.040**	omega	rad	:	**+9.0587 E-06**
delta	Y_m	:	**49.910**	epsilon	rad	:	**+1.9848 E-06**
delta	Z_m	:	**465.840**	psi	rad	:	**–1.7439 E-06**
				delta	k	:	**4.0772 E-06**

$C_7^{(3)}$ Curvilinear Geodetic Datum Transformation

Transformation of Bessel-Amersfoort geodetic coordinates into ETRS89 geodetic coordinates:

input:	latitude	longitude	H	N_0	output:	latitude	longitude	h_n	N_n

given: initial point

$X_{RD.O}$:	**0**	$Y_{RD.O}$:	**0**	$Z_{RD.O}$:	**0**

Sta. DUT		RD1918	ETRS89		
latitude-old	:	**51° 59' 13".3938 N**	latitude-new I	=	51° 59' 09".9144 N
longitude-old	:	**4° 23' 16".9953 E**	longitude-new I	=	4° 23' 15".9531 E
H-old	:	**30.809 m**	h-new I	=	74.3125m
Nsep	:	**−0.113 m**	N-new I	=	43.5035 m
h-old	=	30.696 m			

calculated Cartesian coordinates		
$X_{RD\text{-}I}$ =3 924 096.8506	$Y_{RD\text{-}I}$ =301 119.8207	$Z_{RD\text{-}I}$ =5 001 429.8963
$X_{ETRS\text{-}I}$=3 924 689.3397	$Y_{ETRS\text{-}I}$ =301 145.3379	$Z_{ETRS\text{-}I}$ =5 001 908.6872

General case IIB - for RD1918 only

The geoid NLGEO2004 is used. In the general 7-parameter S-transformation algorithm, the Cartesian coordinates X_0, Y_0 and Z_0 of the Initial Point on old Datum may be set in such a way that the Initial Point is at Origin *"Amersfoort"* of the old Datum Bessel-Amersfoort.

Geodetic coordinates of the RD-Origin (φ RD_o, λ RD_o) are converted into X_0, Y_0 and Z_0.

General case IIb - the following ETRS89 to RD1918 S-transformation parameters are given:			
$X_{0\,RD}$	=	3 903 453.1482	initial point (Datum Amersfoort)
$Y_{0\,RD}$	=	368 135.3134	initial point (Datum Amersfoort)
$Z_{0\,RD}$	=	5 012 970.3051	initial point (Datum Amersfoort)
ΔX	=	−593.032 m	ETRS89 to Bessel-Amersfoort
ΔY	=	−26.000 m	
ΔZ	=	−478.741 m	
ε	=	$-1.9848\ 10^{-6}$ rad	
ψ	=	$+1.7439\ 10^{-6}$ rad	
ω	=	$-9.0587\ 10^{-6}$ rad	
Δk	=	−4.0772 ppm	

Case IIb is exemplified using the general $C_7^{(3)}$ S-transformation algorithm (7.12) A_10STRM.FOR [18.10].

Bessel-Amersfoort to ETRS89 Transformation - General case IIb

S-transformation from Bessel-Amersfoort (RD1918) to ETRS89

ellipsoid old:	Bessel1841	ellipsoid new:	GRS80
semi-major axis a_o :	**6 377 397.155**	semi-major axis a_n :	**6 378 137.0**
recipr. flattening $f^{-1}{}_o$:	**299.15281 285**	recipr. flattening $f^{-1}{}_n$:	**298.25722 21008827**
delta X_m :	**593.032**	omega $_{rad}$:	**+9.0587 E-06**
delta Y_m :	**26.000**	epsilon $_{rad}$:	**+1.9848 E-06**
delta Z_m :	**478.741**	psi $_{rad}$:	**−1.7439 E-06**
		delta k :	**4.0772 E-06**

Given: initial point:

$X_{RD.O}$:	**3 903 453.1482**	$Y_{RD.O}$:	**368 135.3134**	$Z_{RD.O}$:	**5 012 970.3051**
$X_{ETRS.O}$=	3 904 046.1802	$Y_{ETRS.O}$=	368 161.3134	$Z_{ETRS.O}$=	5 013 449.0461

$C_7^{(3)}$ Curvilinear Geodetic Datum Transformation

Transformation of Bessel-Amersfoort geodetic coordinates into ETRS89 geodetic coordinates:

input:	latitude	longitude	H	N_0	output:	latitude	longitude	h_n	N_n

Given: initial point

$X_{RD.O}$:	**3 903 453.1482**	$Y_{RD.O}$:	**368 135.3134**	$Z_{RD.O}$:	**5 012 970.3051**

Sta. DUT	RD1918	ETRS89	
latitude-old :	**51° 59' 13".3938 N**	latitude-new II =	51° 59' 09".9144 N
longitude-old :	**4° 23' 16".9953 E**	longitude-new II =	4° 23' 15".9530 E
H :	**30.809 m**	h-new II =	74.312 m
Nsep :	**- 0.113 m**	N-new II =	43.503 m
h-old =	30.696 m		

calculated Cartesian coordinates

$X_{RD.i}$ =	3 924 096.8506	$Y_{RD.i}$ =	301 119.8207	$Z_{RD.i}$ =	5 001 429.8963
$X_{RD.i-RD.o}$ =	20 643.7024	$Y_{RD.i-RD.o}$ =	−67 015.4927	$Z_{RD.i-RD.o}$ =	−11 540.4088
$X_{ETRS.i}$ =	3 924 689.3396	$Y_{ETRS.i}$ =	301 145.3376	$Z_{ETRS.i}$ =	5 001 908.6872

Example from (Strang van Hees, 2006).

Note

Formulae used are those given in S-transformation (7.06 … 7.12). *Interested readers may also consult the* Cadastre, Apeldoorn-NL *for the latest information about a new map (NLGEO2004) of geoid heights in the Netherlands for the 2000s.*

Accuracy

The term accuracy refers to the closeness between calculations and their correct or true values.

Transformation Round-trip data Errors

The "*round-trip data errors*" are the differences of latitude and longitude in degrees or height in metres between the starting and ending coordinates, illustrated in Figure 59, Figure 60, and Figure 61 for a latitudinal distance 0° N to 84° N, a longitudinal distance 0° E to 180° E, and are calculated as follows:

Latitudes, longitudes and heights are converted to the X, Y and Z coordinates, which are converted back to latitudes, longitudes and heights (Floyd, 1985).

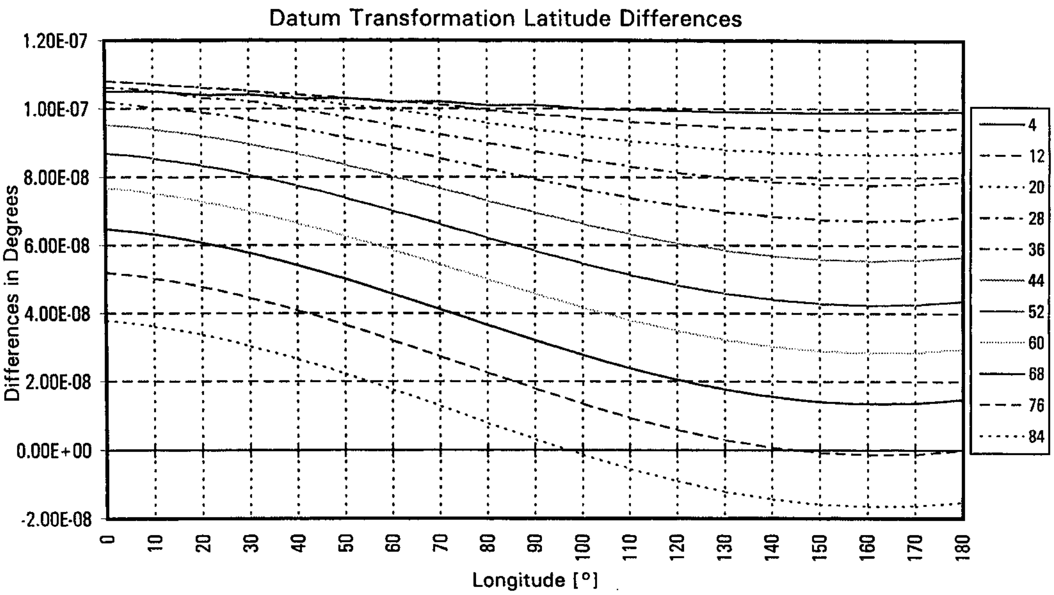


Figure 59: Datum transformation round-trip error of latitude

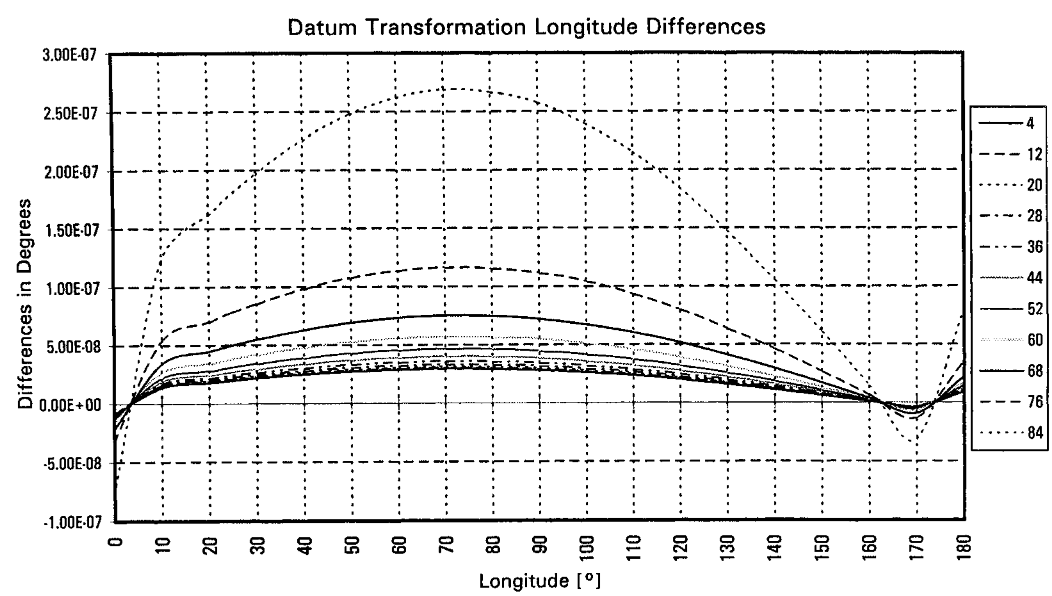


Figure 60: Datum transformation round-trip error of longitude

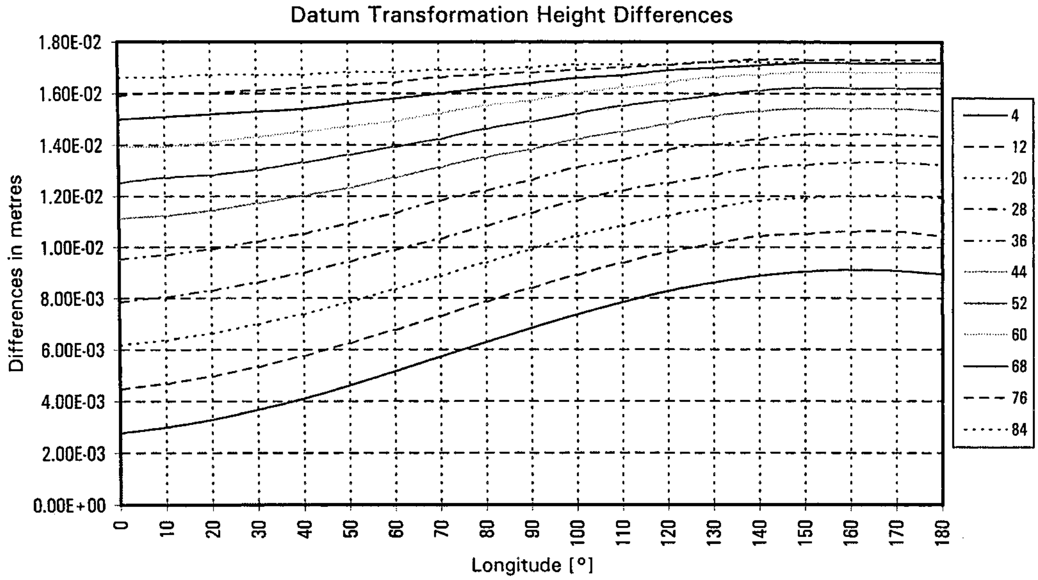


Figure 61: Datum transformation round-trip error of height

Transformation round-trip data

Input data:

latitude	=	0° N to 84° N
longitude	=	0° E to 180° E
δX	=	60 m
δY	=	- 75 m
δZ	=	- 375 m
ω	=	2".1
ε	=	0".35
ψ	=	- 0".3
δk	=	- 0.3E-04
H	=	5000 m
Nsep	=	80 m.

Note

Interested readers may also consult (Bruijne, 2004; Min, 1996) *for more information about a map of geoid heights in the Netherlands.*

7.3 Cartesian Coordinates

Application III - Cartesian X Y Z- coordinates

3D-Turtmann terrestrial network 1985-1986 - Datum: Berne geodetic, CH-1903

ellipsoid old:	Bessel 1841	ellipsoid new:	Bessel 1841
semi-major axis a_o :	**6 377 397.155**	semi-major axis a_n :	**6 377 397.155**
recipr. flattening f^{-1}_o :	**299.15281 285**	recipr. flattening f^{-1}_n :	**299.15281 285**

Conversion of geodetic coordinates into X, Y, Z coordinates and vice versa

input:	latitude	longitude	H	N_0	output:	X	Y	Z	latitude	longitude	h_n

Sta. 7.Turt

latitude-old :	**51^g 45 35 20036 N**	latitude-new	=	51^g 45 35 20036 N
longitude-old :	**8^g 55 72 49986 E**	longitude-new	=	8^g 55 72 49986 E
h :	**622.4474**			
Nsep :	**1.3098**			
h-old =	623.7572	h-new	=	623.7572

X= 4373700.0722 Y= 591465.9898 Z= 4588963.1846

Sta. 1.Brun

latitude-old :	**51^g 46 04 18828N**	latitude-new	=	51^g 46 04 18828 N
longitude-old :	**8^g 51 89 93719 E**	longitude-new	=	8^g 51 89 93719 E
H :	**1008.8876**			
Nsep :	**1.2478**			
h-old =	1010.1354	h-new	=	1010.1354

X= 4373824.6037 Y= 588806.6270 Z= 4589719.3096

Sta. 2.Brae

latitude-old :	**51^g 47 52 29006 N**	latitude-new	=	51^g 47 52 29006N
longitude-old :	**8^g 54 84 62246 E**	longitude-new	=	8^g 54 84 62246 E
H :	**1506.8276**			
Nsep :	**1.3621**			
h-old =	1508.1897	h-new	=	1508.1897

X= 4372830.4541 Y= 590733.7474 Z= 4591102.8446

8. Geodetic Arc Calculations

In the field of geodesy, there are at least three distinct geodetic curves. Each curve gives different distances and different azimuths for anyone reference ellipsoid:

- a great elliptic arc
- normal sections
- a geodesic

Whatever type of curve is involved, calculations of distance and azimuth are directly related to:

- formulae utilised
- quality of stations P_1, P_2
- reference Datum
- parameters of reference ellipsoid

Arc distance P_1 P_2 and also the size of angles P_n P_1 P_2, P_n P_2 P_1 (Figure 62) depend upon the characteristics and behaviour of the surface curve joining the end points.

8.1 Great Elliptic Arc

The great Elliptic arc is not the shortest possible connection between two points, but it may be of more importance than the geodesic for inertial or ballistic missile computations.

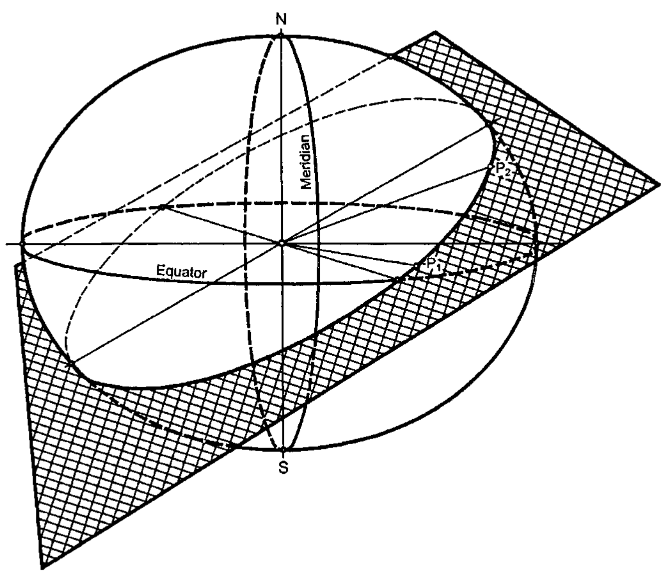


Figure 62: The great elliptic arc and the reference ellipsoid

The great Elliptic arc is a curved line in which a plane through the centre of the reference ellipsoid cuts the surface of the reference ellipsoid:

- it is comparable to a geodesic except that it is always a plane curve
- it is comparable to a normal section except that only one great elliptic exists between any two points.

The equator and meridians are the special cases, which may be considered as great elliptic arcs in addition to geodesics or normal sections.

Figure 62 illustrates the geometrical relations that exist between a great elliptic and a reference ellipsoid. In this drawing, any solid curved line is a great elliptic. The solid arc between P_1 and P_2 is an elliptic section of the great elliptic arc in a plane containing the centre of the ellipsoid, and the two points P_1, P_2.

8.2 Normal Section

The normal section is significant for EDM instrument distances. It is of more importance than the geodesic for inertial or ballistic missile computations (ACIC, 1957).

The Normal is a straight line perpendicular to the surface of a reference ellipsoid. A "*normal section*" is a curved line on the surface of the reference ellipsoid, which connects two points on that surface. It lies in a plane, which contains the normal at the one point and passes through the other point.

- it is similar to a geodesic except that it is always a plane curve
- it is different from a geodesic in that two normal sections exist between any two points except in the cases of the meridians and the equator (Bowring, 1971).

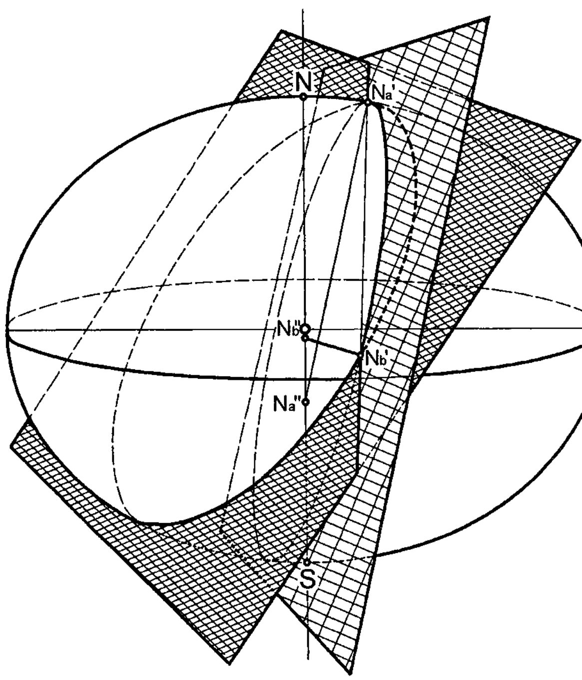


Figure 63: Normal sections and the reference ellipsoid

Figure 63 illustrates the geometrical relations that exist between the normal sections and the reference ellipsoid. Here, the solid elliptic arc connecting the points N_a' and N_b', is a normal section because it lies in the plane of the normal N_a'-N_a'' and a point N_b' of the ellipsoid.
Equally, the dashed elliptic arc between the two points N_a' and N_b' is a second normal section. It lies in a plane of the normal N_b'-N_b'' and the point N_a' of the ellipsoid.

8.3 Geodesics up to 20 000 km

In (Figure 64), the geodesic is considered the principal path in that it represents the shortest possible distance in a unique way between any two given points on the ellipsoidal Earth (Leick, 2004).

Clairaut found that for any point along a geodesic on a surface of revolution the distance from the axis of rotation times the sine of the azimuth (= α) remains constant.

The concern is to obtain accurate values of the shortest, smooth but mathematically complicated geodesic line S between two given points and of the azimuths of the geodesic. Two fundamental geodetic calculations on a reference ellipsoid are generally known as the direct and inverse problems (Bomford, 1977; Hopfner, 1949; Leick, 2004; Pfeifer, 1984; Sodano, 1958; Urmajew, 1955).

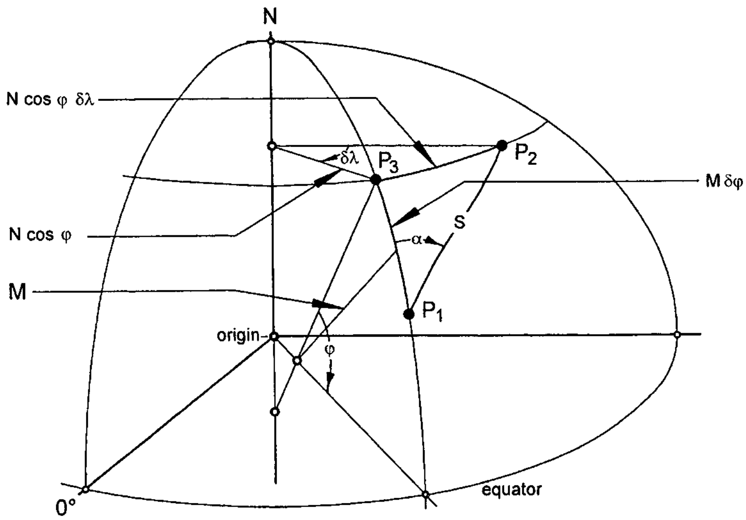


Figure 64: Geodetic triangle

The azimuths are conventionally calculated clockwise from True North. Generally, it is not a plane curve except when it follows a meridian or the equatorial circle. Any segment of a meridian or the equator is a geodesic.

Likewise, the S-shaped curved line between points P_1 (φ, λ) and P_2 (φ, λ) is a geodesic. On the mathematically defined reference ellipsoid, a unique line representing the shortest possible distance between P_1 and P_2. It contains the Normal at each point.

There are several sets of formulae to solve the direct and the inverse problem on the surface of a reference ellipsoid. If 0.01 m or 0".0001 accuracy is desired most of these methods can be used for distances not greater than 150 km, beyond which results might be erroneous. Some can be used up to 400 km but only a very few from 400 km to 20 000 km, the latter being the maximum possible computable distance between two points on the Earth's surface.

8.3.1 Using Kivioja's Method

In theory, the computer offers a choice of integration methods in the solution of the direct and inverse problems, which has been extensively documented (Kivioja, 1971; Jank, 1980; Murphy, 1981; Robbins, 1950, 1962).

Point-by-Point Method

Integrating the differential equations directly, the practical implementation of such a method on a computer works briefly as follows:

If a geodesic, length S, is divided into n equal elements length δS, then, providing δS is small, the elemental triangle $P_1P_2P_3$ (Figure 64) may be considered plane. Also, the sides P_1P_2, P_2P_3, P_1P_3 can be determined. Therefore, the problem can be reliably solved with excellent accuracy by adding together the lengths of small line elements, δS, of the length of the geodetic line, S.

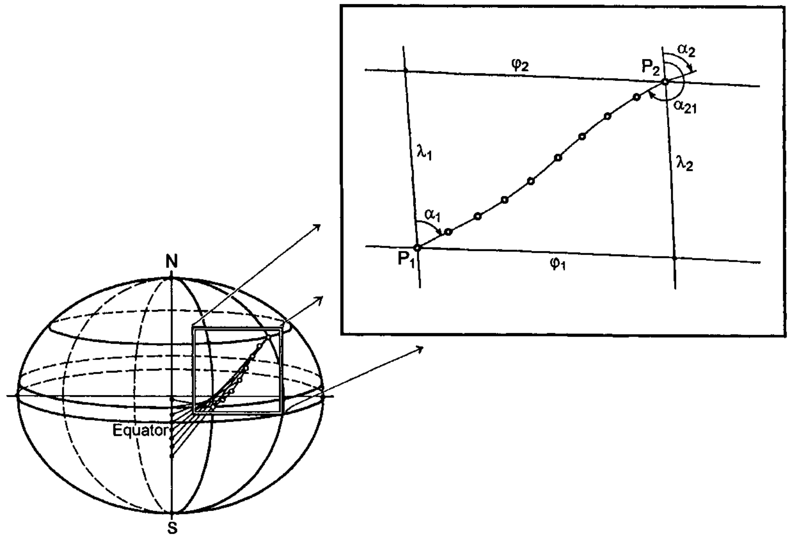


Figure 65: Geodesic divided into dS line elements

To solve the direct problem by the algorithm, distance S is divided into n line elements, each of length δS, so that (Figure 65):

$$S = n \times \delta S$$

Equal line elements are kept in correct azimuths by estimating and correcting Clairaut's Equation:

$$C = N_1 \cos \varphi_1 \sin \alpha_1 = N_2 \cos \varphi_2 \sin \alpha_2, \text{ and so forth} \quad (8.01)$$

for the geodetic line to force each line element to lie on the geodetic line, in which:

φ = geodetic latitude
N = radius of curvature in prime vertical
α = bearing of the geodetic line element

The inverse problem calculates between points P_1 and P_2 approximations for the distance S, an azimuth α. Starting from one initial point P_1, computes the coordinates of the second position P_2 by the same algorithm used in the direct problem. The computed coordinates will not generally coincide with the given values, but the differences between the computed and the known points are used to correct the initial approximations for the distance and azimuth. The program computes the latitude ($\delta\varphi_i$) and longitude ($\delta\lambda_i$) increments for each equal line element and cumulatively adds them to the latitude and longitude of the previous point. Kivioja's solution by integration is called the "*point-by-point method*".

As in numerical integrations in general, the accuracy of the result can be increased by decreasing the length of the geodetic line elements δS. Computing time of the program depends upon the number of integrations of the geodetic line. If millimetre accuracy is desired, δS should be between 100 to 200 m, and for centimetre accuracy, δS should be between 1 000 or 2 000 m but should not exceed 4 000 m (Kivioja, 1971).

Obviously, this method can be used to check any other classical method. The point-by-point method has an additional advantage of calculating intermediate geodetical positions (Figure 65). Unfortunately, the mathematical developments of the geodesic on the ellipsoid and its use on the computer are complex. Because the solutions are fully documented in literature and the companion handbook (Hooijberg, 1997), this section contains only the basic concept and computer algorithms A_11BDGK.FOR; A_12GBDK.FOR as found in [18.11; 18.12], respectively.

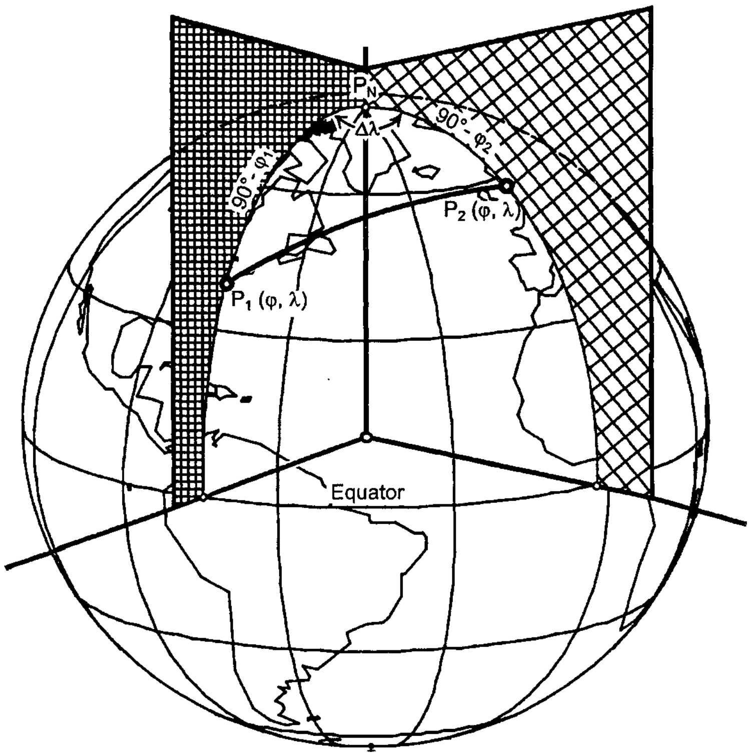


Figure 66: The geodesic and the reference ellipsoid

Solution of the Direct Problem

The direct problem involves computation of the latitude, longitude, and the back-azimuth of a point P_2 whose distance and azimuth from given initial point P_1 are known.

The geodetic line (geodesic) runs through points P_1 and P_2. The length of the geodetic line between points P_1 and P_2 is S. Therefore S runs in azimuth α_1 at P_1 and in azimuth α_2 at P_2. The back azimuth at point P_2 is obtained by adding 180° to α_2, or generally $\alpha_{2\text{-}1} = \alpha_2 \pm 180°$. (Figure 65, Figure 66) shows the meridians λ_1 and λ_2 through P_1 and P_2, and φ_1 and φ_2 are parallels of latitude through P_1 and P_2.

The program computes the latitude and longitude increments $\delta\varphi$ and $\delta\lambda$ for each geodetic line element δS in a repetitive manner. Over the first δS, the increments are $\delta\varphi_1$ and $\delta\lambda_1$ over the second δS, the increments are $\delta\varphi_2$ and $\delta\lambda_2$; and so on, until over the last δs_n, the increments are $\delta\varphi_n$ and $\delta\lambda_n$.

Starting with latitude φ_1, and longitude λ_1, at point P_1 after the first δS, the latitude will be $\varphi_1+\delta\varphi_1$, and the longitude will be $\lambda_1+\delta\lambda_1$; after the second δS, the latitude will be $\varphi_1+\delta\varphi_1+\delta\varphi_2$, and the longitude will be $\lambda_1+\delta\lambda_1+\delta\lambda_2$; and so on. The computation proceeds until all n elements δS are covered. To obtain these increments, the radii of curvature are needed for the reference ellipsoid (Kivioja, 1971; Jank, 1980).

Kivioja's Equations

The equation of the reference ellipsoid with semi-axes a and b is:

$X^2 / a^2 + Y^2 / a^2 + Z^2 / b^2$	=	1	(8.02)
M	=	c / V^3, radius of curvature in the Meridian,	(8.03)

in which:

c	=	a^2 / b	
V	=	$(1+e'^2 \cos^2 \varphi)^{\frac{1}{2}}$ (φ=geodetic latitude)	
e'^2	=	$(a^2 - b^2) / b^2$	
N	=	c / V, the radius of curvature in the Prime Vertical	(8.04)

Figure 64 shows:

$\delta S \cos \alpha$	=	$M\, \delta\varphi$, or	$\delta\varphi$	=	$\delta S \cos \alpha / M$	(8.05)
$\delta S \sin \alpha$	=	$N \cos \varphi\, \delta\lambda$, or	$\delta\lambda$	=	$\delta S \sin \alpha / (N \cos \varphi)$	(8.06)

in which δS is any curve element in azimuth α on the surface of the ellipsoid, $M\, \delta\varphi$ is the corresponding element of meridian between parallels φ and $\varphi+\delta\varphi$, and $N \cos \varphi\, \delta\lambda$ is the corresponding element of the parallel circle between meridians λ and $\lambda+\delta\lambda$. Clairaut's equation (8.01) for a geodetic line is used to control the azimuth of δs:

$$N \cos \varphi \sin \alpha \quad = \quad C \text{ (Clairaut's constant)} \qquad (8.07)$$

C is constant for one geodetic line. It can have any value between zero and the Equatorial radius a.

The equations (8.05, 8.06, 8.07) are required to compute a geodetic line. The shorter δS, the more accurate are the results (Kivioja, 1971).

Examples are calculated by the programs A_11BDGK.FOR [18.11] and A_12GBDK.FOR [18.12] according to Kivioja's method.

Important:

To solve the problem it is advisable that the position of Station P_1 is situated south of Station P_2. Furthermore, the Line P_1-P_2 should not run exactly east-west or exactly north-south. In case this may occur during data processing, the computer will issue an error message. See the FORTRAN-Handbook, which explains the various error conditions. Some calculations in the Western or Southern Hemisphere can be mirrored and calculated in the N/E Hemisphere (Figure 67): pp 164.

Using equations (8.05, 8.06, 8.07), it is possible to estimate an approximate azimuth and an increment at P_1. Thus, the distance S and the azimuth α between points P_1 (φ_1, λ_1) and P_2 (φ_2, λ_2) are calculated.
Use $\delta\varphi = (\varphi_2 - \varphi_1) / n$ if the $\alpha < 45°$ or use $\delta\lambda = (\lambda_2 - \lambda_1) / n$ if the azimuth $\alpha > 45°$, or use both.
If $\delta\varphi$ is used, compute M, N, $\cos \varphi$, and α for latitude $\varphi + \frac{1}{2}\, \delta\varphi$, then δs from (8.05), and then $\delta\lambda$ from (8.06), and the azimuth α from (8.07). Do this *n* times accumulating the increments of distance and longitude.

A computed longitude difference will of course not correspond with $\lambda_2 - \lambda_1$, because none of the approximate azimuths will guide the geodesic exactly through P_2, but this geodetic line will cut the parallel of φ_2 either East or West from P_2. Using these disagreements, the estimated azimuth can be substantially improved. If desired, the computer can show the latitudes and longitudes at the last set of increments.

8.3.2 Using Vincenty's Method

Using Vincenty's method, the direct and inverse solutions of geodesics on the ellipsoid with application of nested equations gives compact formulae for the direct and inverse solutions of lines of any length, from a few centimetres to nearly 20 000 km. The feature of the formulae is the use of nested equations for elliptic terms, which reduces the length of the program and the time of execution, and minimises the possibility of overflow or underflow. Both Vincenty's solutions are iterative.

Kivioja's (noniterative) solutions provide very accurate points by the step-by-step-method.

Vincenty's Equations

Symbols and definitions

The equations for the ellipsoid constants are given in (Table 19): pp 123 (Vincenty, 1975).
All angles in are expressed in radians, A_13BDGV.FOR [18.13], A_14GBDV.FOR [18.14]

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
f^{-1}	reciprocal flattening of the ellipsoid
φ_1	parallel of geodetic latitude, positive north
λ_1	meridian of geodetic longitude, positive east
φ_2	parallel of geodetic latitude, positive north
λ_2	meridian of geodetic longitude, positive east
L	difference in longitude, positive east
S	length of the geodesic
α_1	azimuth of the geodesic, clockwise from north in the direction P_1 - P_2
α_2	azimuth of the geodesic, clockwise from north in the direction P_2 - P_1
α	azimuth of the geodesic at the equator
U	reduced latitude
λ	difference in longitude on an auxiliary sphere
σ	angular distance P_1 - P_2 on the sphere
σ_1	angular distance on the sphere from the equator to P_1
σ_m	angular distance on the sphere from the equator to the midpoint of the line P_2 - P_1
e^2	first eccentricity squared
e'^2	second eccentricity squared
k_1	Helmert's quantity

Calculate Ellipsoid Parameters

$$fl = (a - b) / a \tag{8.08}$$
$$f = 1 / fl \tag{8.09}$$
$$t_1 = 1 - 1 / fl \tag{8.10}$$
$$b = a - a / fl \tag{8.11}$$
$$e^2 = (2 - f) / fl \tag{8.12}$$
$$e'^2 = e^2 / t_1^2 \tag{8.13}$$

Direct Computation

Given: geodetic coordinates of a point P_1 (φ_1, λ_1)
true bearing $\alpha_{1\text{-}2}$
distance $S_{1\text{-}2}$

Output: geodetic coordinates of a point P_2 (φ_2, λ_2)
true bearing $\alpha_{2\text{-}1} \pm 180°$

Equations

$$u^2 = \cos^2\alpha\,(a^2 - b^2) / b^2 \tag{8.14}$$
$$\tan U = (1 - f) \tan\varphi \tag{8.15}$$
$$\tan\sigma_1 = \tan U_1 / \cos\alpha_1 \tag{8.16}$$
$$\sin\alpha = \cos U_1 \sin\alpha_1 \tag{8.17}$$

k_1	=	$((1+u^2)^{½} - 1) / ((1+u^2)^{½} + 1)$	(8.18)
A	=	$(1 + k_1^2 / 4) / (1 - k_1)$	(8.19)
B	=	$k_1 (1 - 3/8\, k_1^2)$	(8.20)
$2\sigma_m$	=	$2\sigma_1 + \sigma$	(8.21)
E	=	$2 \cos^2 2\sigma_m - 1$	(8.22)
E1	=	$2\,E - 1$	(8.23)
$\Delta\sigma$	=	$B \sin \sigma (\cos^2 2\sigma_m + B / 4 (\cos \sigma\, E - B / 6 \cos^2 2\sigma_m\, E1 (4 \sin^2 \sigma - 3)))$	(8.24)
σ	=	$S / (b\, A) + \Delta\sigma$	(8.25)

Iteration: the first approximation of σ is $S / (b\, A)$. Henceforth, the equations (8.21, ……, 8.25) are iterated until there is an insignificant change in σ (Vincenty, 1975): pp 88-93, 294.

t_2	=	$\sin U_1 \sin \sigma - \cos U_1 \cos \sigma \cos \alpha_1$	
$\tan \varphi_2$	=	$((\sin U_1 \cos \sigma + \cos U_1 \sin \sigma \cos \alpha_1) / (t_2 (\sin^2 \alpha + t_2^2)^{½}$	(8.26)
$\tan \lambda$	=	$\sin \sigma \sin \alpha_1 / (\cos U_1 \cos \sigma - \sin U_1 \sin \sigma \cos \alpha_1)$	(8.27)
C	=	$f / 16 \cos^2 \alpha (4 + f (4 - 3 \cos^2 \alpha))$	(8.28)
L	=	$\lambda - \sin \alpha (\sigma + C \sin \sigma (\cos 2\sigma_m + C \cos \sigma\, E)) (1 - C) f$	(8.29)
$\tan \alpha_2$	=	$\sin \alpha / (\cos U_1 \cos \sigma \cos \alpha_1 - \sin U_1 \sin \sigma))$	(8.30)

Inverse Computation

Given: geodetic coordinates of a point $P_1 (\varphi_1, \lambda_1)$
$P_2 (\varphi_2, \lambda_2)$

Output: true bearing $\alpha_{1\text{-}2}$
true bearing $\alpha_{2\text{-}1}$
distance $S_{1\text{-}2}$

Equations

u^2	=	$\cos^2 \alpha (a^2 - b^2) / b^2$	(8.31)
$\tan u$	=	$(1 - f) \tan \varphi$	(8.32)
λ	=	L (= first approximation)	(8.33)
$\sin^2 \sigma$	=	$(\cos U_2 \sin \lambda)^2 + (\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos \lambda)^2$	(8.34)
$\cos \sigma$	=	$\sin U_1 \sin U_2 + \cos U_1 \cos U_2 \cos \lambda$	(8.35)
$\tan \sigma$	=	$\sin \sigma / \cos \sigma$	(8.36)
$\sin \alpha$	=	$\cos U_1 \cos U_2 \sin \lambda / \sin \sigma$	(8.37)
$\cos 2\sigma_m$	=	$\cos \sigma - 2 \sin U_1 \sin U_2 / \cos \alpha$	(8.38)

by iteration: the first approximation of λ is obtained by equations (8.28, 8.29). Henceforth, the procedure is iterated, starting with $\Delta\sigma$ and the equations (8.34, ……, 8.40) until there is an insignificant change in λ.

A	=	$(1 + k_1^2 / 4) / (1 - k_1)$	(8.39)
B	=	$k_1 (1 - 3 / 8\, k_1^2)$	(8.40)
E	=	$2 \cos^2 2\sigma_m - 1$	(8.41)
E1	=	$2\,E - 1$	(8.42)
$\Delta\sigma$	=	$B \sin \sigma (\cos^2 2\sigma_m + B / 4 (\cos \sigma\, E - B / 6 \cos^2 2\sigma_m\, E1 (4 \sin^2 \sigma - 3)))$	(8.43)
$S_{1\text{-}2}$	=	$b\, A (\sigma - \Delta\sigma)$	(8.44)

$\Delta\sigma$ comes from equations (8.19, 8.20, and 8.24)

$\tan \alpha_{1\text{-}2}$	=	$\cos U_2 \sin \lambda / (\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos \lambda)$	(8.45)
$\tan \alpha_{2\text{-}1}$	=	$\cos U_1 \sin \lambda / (\cos U_1 \sin U_2 \cos \lambda - \sin U_1 \cos U_2)$	(8.46)

The inverse formula may give no solution over a line between two nearly antipodal points. This will occur when λ, as computed by equation (8.29), is greater than π in absolute value (Vincenty, 1975)

Accuracy Checks on Both Methods

Elliptic terms have their maximum effect on angular and geodesic distances over in north-south lines. Independent checks on distances between 2 000 km up to 18 000 km, which gave a maximum disagreement of 0.01 mm.

The direct and inverse solutions duplicate each other perfectly if the values obtained from the previous computation are used without rounding, since one formula was obtained by reversing the other. They were tested independently on the examples given by (Jank; Kivioja, 1980), using direct and inverse subroutines prepared in FORTRAN. The programs iterate until the change in σ in the direct or λ in the inverse computation diminishes in absolute value to 10^{-14} radians or less.

Errors in computed azimuths due to rounding of coordinates are large for short lines and decrease progressively up to about 10 000 km, after which they start increasing. Very large errors can be expected over lines between nearly antipodal points.

The direct subroutines demand two to four iterations in most cases, but up to 26 iterations in the inverse subroutine solution. Lines connecting nearly antipodal points may require considerably more repetitions.

Applications of a Direct Problem

Given: geodetic coordinates of a point P_1 (φ_1, λ_1)
true bearing $= \alpha_{1\text{-}2}$
distance $S_{1\text{-}2} = S$

Output: geodetic coordinates of a point P_2 (φ_2, λ_2)
true bearing $\alpha_{2\text{-}1} = \alpha_{1\text{-}2} \pm 180°$

Using Clairaut's equation for the geodetic line, the direct problems can be solved. The point-by-point method "*stakes out*" the geodetic line. It computes the coordinates for each intermediate geodetical position (Kivioja, 1971).

Kivioja's paper gives some hints, e.g. integration could also utilise the differential elements $\delta\varphi$, $\delta\lambda$, or $\delta\alpha$, rather than δS, using rearrangements of equations (Kivioja, 1971; Jank, 1980).

Using a varying number n of δs equal line elements, the following examples are calculated by programs according to Kivioja's method. The examples are calculated using varying number n of δS line elements.

Solution Kivioja

Application Geodetic Line - Direct Problem I

Given:		:	Bessel (Jordan, 1959)			
semi-major axis	a	:	**6 377 397.155**	recipr. flattening	f^{-1} :	**299.15281 285**
latitude	φ_1	:	**45° 00' 00" N**	true bearing	$\alpha_{1\text{-}2}$:	**29° 03' 15".458713**
longitude	λ_1	:	**10° 00' 00" E**	true distance	S :	**1 320 284.368 37**
equal lines	n	:	**100 … 100 000**			

Output:

n	δS_m	latitude - φ_2	longitude - λ_2	true bearing - $\alpha_{2\text{-}1}$
100	13 202.843683	55° 00' 00".00615	19° 59' 59".97525	216° 45' 07".40575
1 000	1 320.284368	55° 00' 00".00016	19° 59' 59".99952	216° 45' 07".39937
10 000	132.028437	55° 00' 00".00010	19° 59' 59".99976	216° 45' 07".39931
100 000	13.202844	55° 00' 00".00010	19° 59' 59".99976	216° 45' 07".39931

Application Geodetic Line - Direct Problem II

Given:			ellipsoid international 1924			
semi-major axis	a	:	**6 378 388**	recipr. flattening	f^{-1} :	**297.0**
latitude	φ_1	:	**46° 55' 09".9100 N**	true bearing	$\alpha_{1\text{-}2}$:	**119° 17' 21".9200**
longitude	λ_1	:	**7° 26' 40".4700 E**	true distance	S :	**64 865.007**
equal lines	n	:	**10 000**			

Output:

n	latitude - φ_2	longitude - λ_2	true bearing - $\alpha_{2\text{-}1}$
10 000	46° 37' 53".68297 N	8° 10' 59".93073 E	299° 49' 39".82352

if required, the point-by-point method computes the coordinates for intermediate geodetic positions.

decr.- φ n	latitude - φ_i	decr.- λ n	longitude - λ_i
φ_{5001}	46° 46' 33".9488 N	λ_{5001}	7° 48' 53".7430 E
φ_{5000}	46° 46' 33".8452 N	λ_{5000}	7° 48' 54".0090 E
φ_{4999}	46° 46' 33".7416 N	λ_{4999}	7° 48' 54".2749 E

Application Geodetic Line - Direct Problem III

Given:			ellipsoid international 1924 (Jordan, 1959)			
semi-major axis	a	:	**6 378 388**	recipr. flattening	f^{-1} :	**297.0**
latitude	φ_1	:	**50° 00' 00" N**	true bearing	$\alpha_{1\text{-}2}$:	**140° 00' 00".0000**
longitude	λ_1	:	**10° 00' 00" E**	true distance	S :	**15 000 000.000**
equal lines	n	:	**10 000 … 750 000**			

Output:

n	latitude - φ_2	longitude - λ_2	true bearing - $\alpha_{2\text{-}1}$
10 000	62° 57' 03".203645 S	105° 05' 38".300738 E	294° 46' 41".484846
100 000	62° 57' 03".203865 S	105° 05' 38".299675 E	294° 46' 41".483913
750 000	62° 57' 03".203867 S	105° 05' 38".299664 E	294° 46' 41".483904

Applications of an Inverse Problem

An inverse problem calculates the distance and azimuths between two geodetic positions, P_1 and P_2.

Given:	geodetic coordinates of a point P_1 (φ_1, λ_1)	
	geodetic coordinates of a point P_2 (φ_2, λ_2)	
Output:	true bearing	$\alpha_{1\text{-}2}$
	true bearing	$\alpha_{2\text{-}1} \pm 180°$
	distance	$S_{1\text{-}2} = S$

Using Clairaut's equation for the *geodetic line*, the inverse problems can be solved. The point-by-point method "*stakes out*" the geodetic line. It computes the coordinates for each intermediate geodetical position (Kivioja, 1971).

Application Geodetic Line - Inverse Problem I

Given: ellipsoid international 1924

semi-major axis a :	**6 378 388**	recipr. flattening f^{-1} :	**297.0**
latitude φ_1 :	**46° 37' 53".6830 N**	latitude φ_2 :	**46° 55' 09".9100 N**
longitude λ_1 :	**8° 10' 59".9308 El**	longitude λ_2 :	**7° 26' 40".4700 E**
equal lines n :	**10 000**		

Output:

n	geodesic - S_m	true bearing - $\alpha_{1\text{-}2}$	true bearing - $\alpha_{2\text{-}1}$
10 000	64 865.00777	299° 49' 39".81880	119° 17' 21".91522

Application Geodetic Line - Inverse Problem II

Given: ellipsoid Bessel 1841

semi-major axis a :	**6 377 397.155**	recipr. flattening f^{-1} :	**299.15281 285**
latitude φ_1 :	**45° 00' 00" N**	latitude φ_2 :	**55° 00' 00" N**
longitude λ_1 :	**10° 00' 00" E**	longitude λ_2 :	**20° 00' 00" E**
equal lines n :	**5 000 ... 25 000**		

Output:

n	δS_m	geodesic - S_m	true bearing - $\alpha_{1\text{-}2}$	true bearing - $\alpha_{2\text{-}1}$
5 000	264.056874	1 320 284.368415	29° 03' 15".45874	216° 45' 07".39924
10 000	132.028437	1 320 284.368381	29° 03' 15".45872	216° 45' 07".39921
25 000	52.811375	1 320 284.368372	29° 03' 15".45871	216° 45' 07".39920

Programming Suggestions

A useful FORTRAN function to evaluate arc tangent in double precision is DATAN 2(E, N) which accepts the numerator and the denominator as arguments and gives the result between $-\pi$ and $+\pi$.
Exemplified (program A_01BRDI.FOR)

Some results become indeterminate over equatorial lines but this will not cause trouble, if division by zero is excluded. In this case, it is unimportant what values are computed by these equations, so that all values will be computed correctly (Vincenty, 1975).

8.4 The Meridional Arc

The arc of the meridian is complicated to evaluate because the meridional radius of curvature varies continuously with latitude. To solve the direct question on the arc of the meridian G_m of a reference ellipsoid, the method starts with the differential formulae and proceeds by series development.

Determining the arc G_m measured from the equator to geodetic latitude B_i. Latitude B_i is the angle PME $=\varphi^P$ between the major axis of the ellipsoid and the normal to the tangent plane at an arbitrary point i on the surface of the ellipsoid measured at the point of intersection of the normal with the equatorial plane (Figure 46): pp 123.

Considering any point along the arc as an infinitely small part of a curve, corresponding to an infinitely small change in latitude, the length of the whole curve G_m is defined from the equator B_o to a point i in latitude B_i.

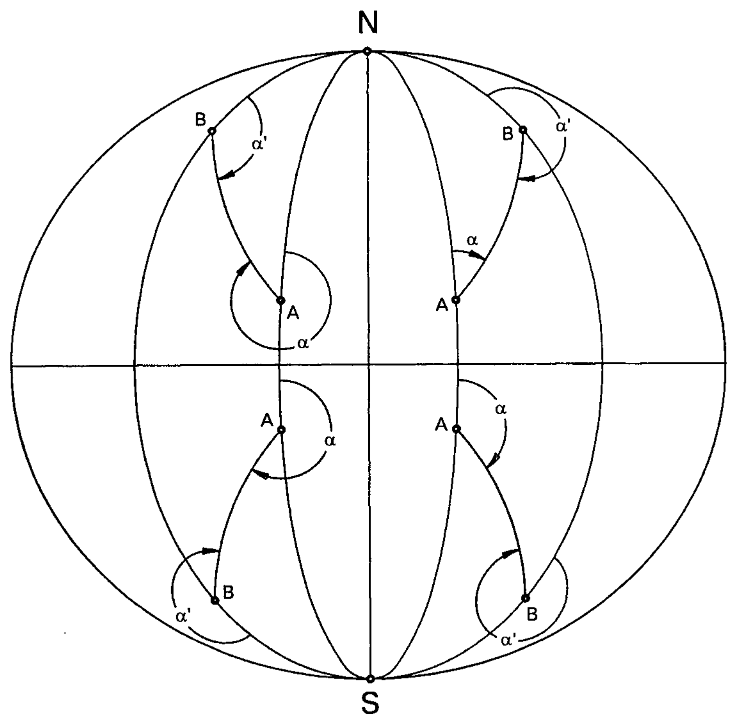


Figure 67: Different positions of the geodesic A - B

It is now necessary to determine, first, the length of a very short arc at the equator, and then accumulate the lengths of all the small elements, which form the whole arc G_m. Since the limits of the arc have already been specified, the arc distance on the ellipsoid, G_m, may be written as an integral

$$\delta G_m = M\,\Delta B = c\,(1+e'^2 \cos^2 B)^{-3/2}\,\delta B \text{ (Jordan, 1959), in which:} \tag{8.47}$$

c	=	polar radius of curvature		
e'^2	=	second eccentricity squared		
M	=	ρ =	radius of curvature in the meridian ___________	(8.03)

Obviously, these parameters are closely related.

After integration of this expression, the equation is simplified to a form suitable for calculation as shown by (Helmert, 1880).

Direct Equations

$$G_m = c\left[E_0 B + \frac{E_2}{2}\sin 2B + \frac{E_4}{4}\sin 4B + \frac{E_6}{6}\sin 6B + \frac{E_8}{8}\sin 8B\right] \tag{8.48}$$

in which the coefficients E_0, E_2, E_4, E_6, E_8 are expressed in radians in terms of e' as given by (Jordan, 1959):

$$\begin{aligned}
E_0 &= 1 - \frac{3}{4}e'^2 + \frac{45}{64}e'^4 - \frac{175}{256}e'^6 + \frac{11025}{16384}e'^8 - \frac{43659}{65536}e'^{10}\\
E_2 &= -\frac{3}{4}e'^2 + \frac{15}{16}e'^4 - \frac{525}{512}e'^6 + \frac{2205}{2048}e'^8 - \frac{72765}{65536}e'^{10}\\
E_4 &= +\frac{15}{64}e'^4 - \frac{105}{256}e'^6 + \frac{2205}{4096}e'^8 - \frac{10395}{16384}e'^{10}\\
E_6 &= -\frac{35}{512}e'^6 + \frac{315}{2048}e'^8 - \frac{31185}{131072}e'^{10}\\
E_8 &= +\frac{315}{16384}e'^8 - \frac{3465}{65536}e'^{10}
\end{aligned} \tag{8.49}$$

See (Table 19): pp 123, in which a and b are the semi-major axis and semi-minor axis of the ellipsoid used.

Inverse Equations

The arc of the meridian from the equator to an arbitrary point P_i is G_m as before. Most computing algorithms calculate the inverse solution by a time-consuming iterative process.

Rather than use the coefficients E_2, E_4, E_6, E_8 as derived for this equation, better consistency is obtained by reversing the equation used in the direct computation. This was done by Klaus Krack in 1982.
An explanation of the inversion can be found in a report described by (Krack, 1982). He explains in the paper a way for an accurate solution.

In principle, formulae (8.48) are divided by c E_0. This results in:

$$\frac{G_m}{c\,E_0} = B + \frac{E_2}{E_0}\sin 2B + \frac{E_4}{E_0}\sin 4B + \frac{E_6}{E_0}\sin 6B + \frac{E_8}{E_0}\sin 8B \tag{8.50}$$

or respectively the abbreviated formulae for g and e_2, e_4, e_6, e_8 in:

$$g = B + e_2 \sin 2B + e_4 \sin 4B + e_6 \sin 6B + e_8 \sin 8B \tag{8.50a}$$

By reversing (8.50a) Krack finally gets the equation:

$$B_i^R = g + f_2 \sin 2g + f_4 \sin 4g + f_6 \sin 6g + f_8 \sin 8g \tag{8.51}$$

in which the coefficients f_2, f_4, f_6, f_8 are expressed in terms of e' as:

$$\begin{aligned}
f_2 &= +\frac{3}{8}e'^2 - \frac{3}{16}e'^4 + \frac{213}{2048}e'^6 - \frac{255}{4096}e'^8 + \frac{166479}{655360}e'^{10}\\
f_4 &= +\frac{21}{256}e'^4 - \frac{21}{256}e'^6 + \frac{533}{8192}e'^8 - \frac{152083}{327680}e'^{10}\\
f_6 &= +\frac{151}{6144}e'^6 - \frac{3171}{86016}e'^8 + \frac{2767911}{9175040}e'^{10}
\end{aligned} \tag{8.52}$$

$$f_8 = \qquad + \frac{38395}{4587520} e'^8 - \frac{427277}{4587520} e'^{10} \tag{8.52}$$

The coefficients calculated in (8.49) and (8.52) are substituted in formulae (8.48) and (8.51), respectively.

Direct and inverse problems can be solved with good accuracy using the formulae mentioned above. It must be stressed that computing the various trigonometric functions, expressed in radians, for 2B, 4B, 6B, 8B and 2g, 4g, 6g, 8g are time consuming. All angles are expressed in radians.

$\sin 2\alpha$	=				$2 \sin\alpha \cos\alpha$
$\sin 4\alpha$	=			$8 \sin\alpha \cos^3\alpha$	$- 4 \sin\alpha \cos\alpha$
$\sin 6\alpha$	=		$32 \sin\alpha \cos^5\alpha$	$- 32 \sin\alpha \cos^3\alpha$	$+6 \sin\alpha \cos\alpha$
$\sin 8\alpha$	=	$128 \sin\alpha \cos^7\alpha$	$- 192 \sin\alpha \cos^5\alpha$	$+80 \sin\alpha \cos^3\alpha$	$- 8 \sin\alpha \cos\alpha$

Table 23: Reduction table

A higher computational speed may be achieved by the equations converted to a nested form and the use of a single trigonometric function in both direct and inverse computations. The equations (8.48, 8.51) are modified so that the only trigonometric function appearing in them is a cosine (x). This is done by substituting the appropriate trigonometric identities from reduction (Table 23), as given by (Adams, 1949), in the formulae. New constants for the calculation of direct and inverse constants are obtained. These constants in terms of e' and one cosine function allow calculation of the arc G_m and latitude B_i directly [GK Mapping Equations] (10.04.04, … , 10.04.40).

A higher computational speed may be achieved by the use n in substitution for e'^2, see the equations described by (Krack, 1983; Stem, 1989a, modified), (8.65, …, 8.72).

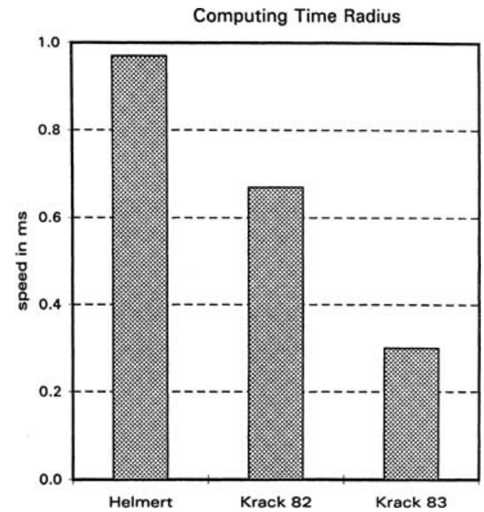


Figure 68: Time to calculate a rectifying sphere

8.4.1 Recasting Algorithms

The following examples 1, 2, 3, 4 and 5 with formulae (8.53, … , 8.72) are incomplete FORTRAN programs [18.3, On_CD] and refer to the calculation of the meridional arc (G_m) according to the formula:

$$\delta G_m = \rho_m \delta B \text{ (Helmert, 1880).}$$

Example 1 (Jordan, 1959). See the calculation of the meridional arc, direct; inverse by iteration:

```
E2 = C/2.D0*( -3.D0/4.D0*EC3+15.D0/16.D0*EC3**2 -525.D0/512.D0*EC3**3+22 05.D0/2048.D0*
EC3**4-72765.D0/65536.D0*EC3**5)                                                    (8.53)
G1 = R0*LT1+E2*DSIN(2.D0*B)+E4*DSIN(4.D0*B)+E6*DSIN(6.D0*B)+E8*DSIN(8.D0*B)          (8.54)
F2 = 3.D0/8.D0*EC3 -3.D0/16.D0*EC3**2+213.D0/2048.D0*EC3**3 -255.D0/4046.D0*EC3**4+166479.D0/655360.D0*EC3**5
                                                                                     (8.55)
L1 = G+F2*DSIN(2.D0*G)+F4*DSIN(4.D0*G)+F6*DSIN(6.D0*G)+F8*DSIN(8.D0*G)               (8.56)
Calculation time: 11.0 ms (direct)
Calculation time: 16.7 ms (inverse by iteration)   (Figure 69)
```

Example 2 (Krack, 1982), solving direct and inverse without iteration; equations in brackets:

```
E2 = C/2.D0*EC3/4.D0*(-3.D0+EC3/4*(15.D0+EC3/32.D0*(-525.D0+EC3/4.D0*(2205.D0-72765.D0/32.D0*EC3))))   (8.57)
G2 =  R0*LT2+E2*DSIN(2.D0*B)+E4*DSIN(4.D0*B)+E6*DSIN(6.D0*B)+E8*DSIN(8.D0*B)          (8.58)
F2 = 3.D0/2.D0*EC3*(1.D0/4.D0 -EC3*(1.D0/8.D0 -EC3*(71.D0/1024.D0 -EC3*(5.D0/119.D0 -55493.D0/327680.D0*EC3))))
                                                                                     (8.59)
```

```
L2 = G+F2*DSIN(2.D0*G)+F4*DSIN(4.D0*G)+F6*DSIN(6.D0*G)+F8*DSIN(8.D0*G)                    (8.60)
Calculation time: 10.4 ms (direct)
Calculation time: 10.4 ms (inverse)
```

Example 3 (Hooijberg, 1996), solving direct and inverse without iteration, with only sin(x)= f(C):

```
E2 = C*((((( -55.D0/8.D0*EC3+7.D0)*9.D0/64.D0*EC3 -1.D0)*35.D0/4.D0*EC3+9.D0)*5.D0/16.D0
*EC3-3.D0)/4.D0*EC3)                                                                       (8.61)
G3 = (((E8*B2+E6)*B2+E4)*B2+E2)*B1*DSQRT(1.D0-B2)+R0*LT3                                   (8.62)
F2 = ((((917504.D0/280.D0*EC3-2205.D0)/64.D0*EC3+35.D0)/4.D0*EC3-9.D0)*5.D0/16.D0*EC3 +3.D0)/4.D0*EC3   (8.63)
L3 = (((F8*B2+F6)*B2+F4)*B2+F2)*B1*DSQRT(1.D0-B2)+TL                                       (8.64)
Calculation time: 6.0 ms (direct)
Calculation time: 6.0 ms (inverse)
```

Example 4 (Krack, 1983), solving direct and inverse without fractions/iteration, with only one sin(x)= f(C):

```
E2 = -N*(36.D0+N*(45.D0+39.D0*N))                                                          (8.65)
G4 = R0*(LT4+DSQRT(1.D0-B2)*B1/12.D0*(E2+B2*(E4+B2*E6)))                                   (8.66)
F2 = N*(36.D0+N*(-63.D0+93.D0*N))                                                          (8.67)
L4 = TL+DSQRT(1.D0-B2)*B1/12.D0*(F2+B2*(F4+B2*F6))                                         (8.68)
Calculation time: 3.2 ms (direct)
Calculation time: 3.2 ms (inverse)
```

Example 5. NGS5 (Stem, 1989a), modified equations by Hooijberg (1998), solving direct and inverse with fractions without iteration, with only one sin(x)= f(C):

```
E2 = -N*(3.D0+N/4.D0*(15.D0+13.D0*N*(1.D0+15.D0/16.D0*N)))                                 (8.69)
G5 = (TL+(DSQRT(1.D0-B2)*B1)*(E2+B2*(E4+B2*(E6+E8*B2))))*R0                                (8.70)
F2 = N*(3.D0-N/4.D0*(21.D0-N*(31.D0-657.D0/16.D0*N)))                                      (8.71)
L5 = TL+(DSQRT(1.D0-B2)*B1)*(F2+B2*(F4+B2*(F6+F8*B2)))                                     (8.72)
Calculation time: 9.0 ms (direct)
Calculation time: 9.0 ms (inverse)
```

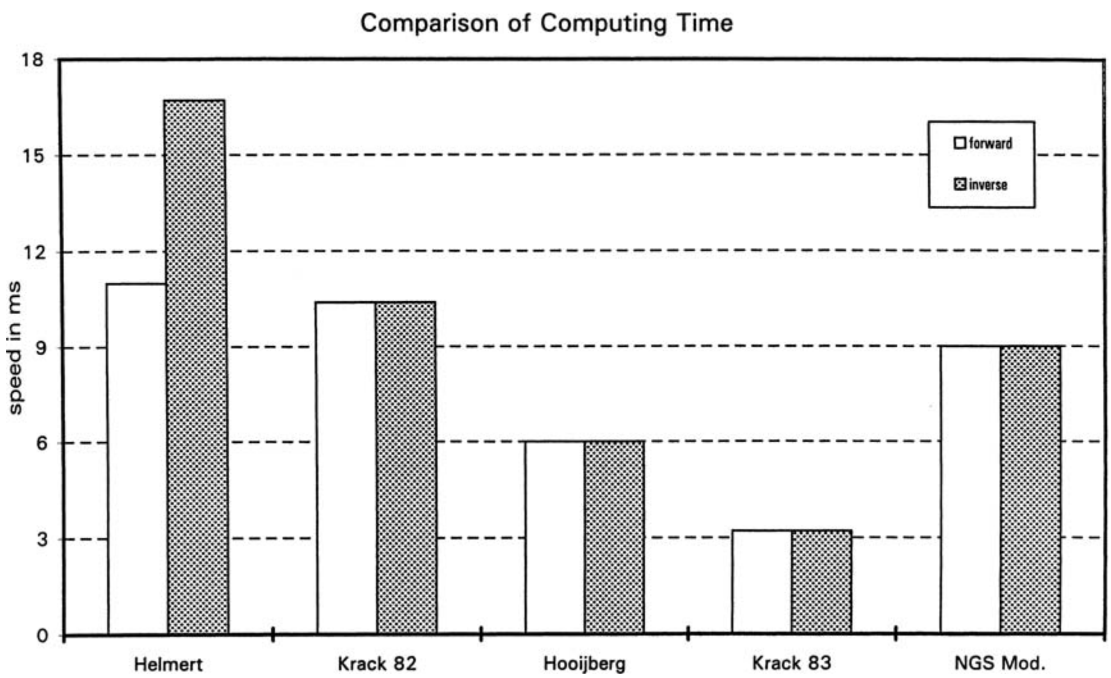


Figure 69: Comparison of computing time to calculate the Meridional arc

8.4.2 Accuracy and Precision

The term accuracy refers to the closeness between calculated values and their correct or true values. The further a calculated value is from its true value, the less accurate it is. Conversely, calculated values may be accurate but not precise if the calculated values are well distributed about the true value, but are significantly different from each other.

Calculated values will be both precise and accurate if the values are very closely grouped around the true value. Precision is expressed in terms of the mean of the squares of the errors (Bomford, 1977).

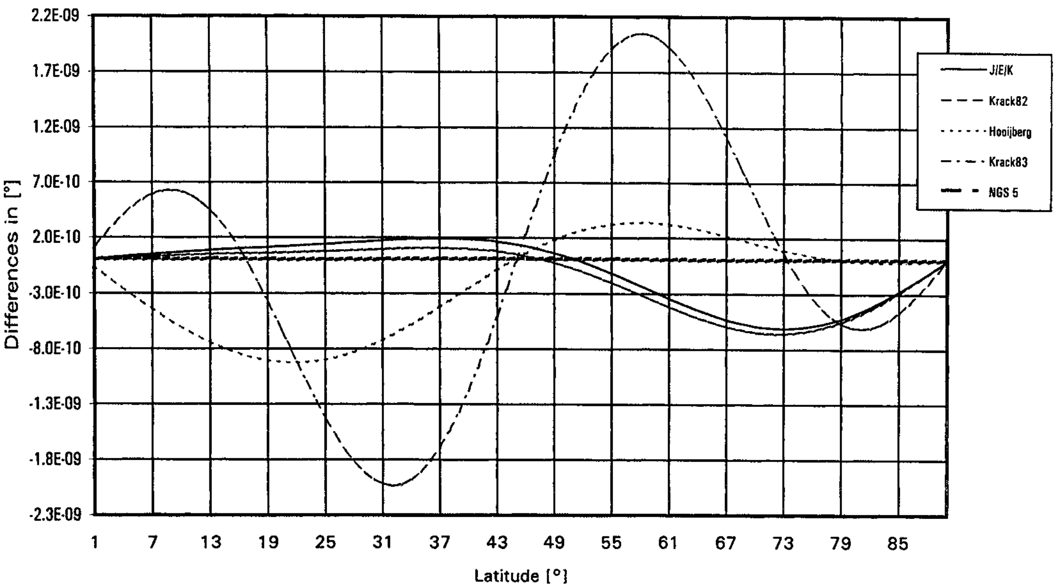


Figure 70: Round-trip error of the meridional arc in degrees

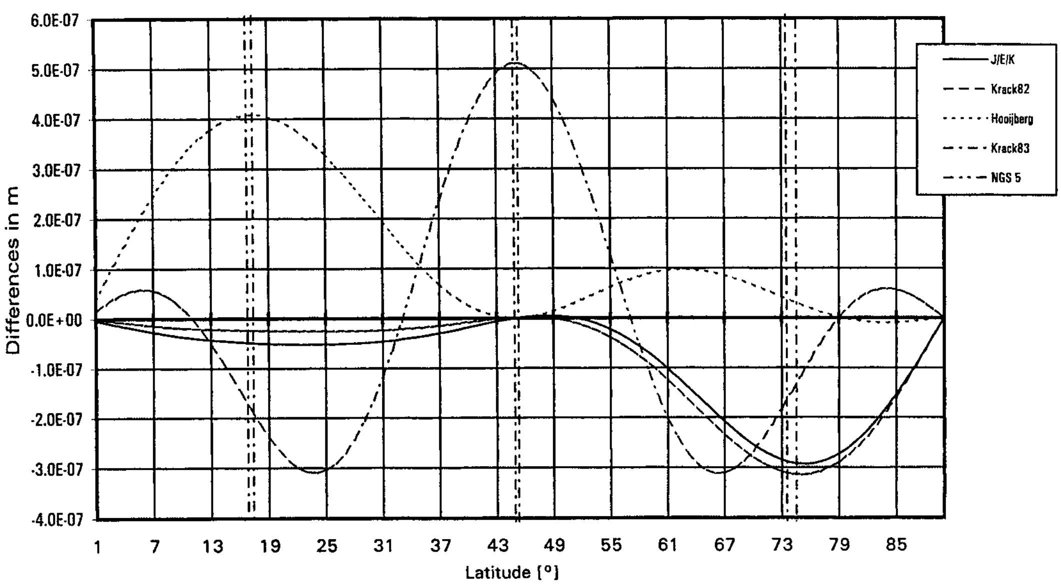


Figure 71: Round-trip error of the meridional arc in m

Errors

The term error can be considered as referring to the difference between a given calculation and a true value of the calculated quantity.

The *round-trip errors* are the differences in degrees or in metres between the starting and ending coordinates, illustrated in (Figure 70; Figure 71), and are calculated as follows:

- latitudes (B) are converted to the meridional arc distances (G), which are converted back to latitudes
- meridional arcs are converted to latitudes, which are converted back to meridional arcs.

round-trip error of (Jordan, 1959) is:	3×10^{-7} m for G = f(B), and	$6 \times 10^{-10}\,°$ for B = f(G)
round-trip error of (Krack, 1982) is:	3×10^{-7} m for G = f(B), and	$6 \times 10^{-10}\,°$ for B = f(G)
round-trip error of (Hooijberg, 1996) is:	4×10^{-7} m for G = f(B), and	$9 \times 10^{-10}\,°$ for B = f(G)
round-trip error of (Krack, 1983) is:	2×10^{-4} m for G = f(B), and	$2 \times 10^{-9}\,°$ for B = f(G)
round-trip error of NGS5 modified is: {optimised by the author}	3×10^{-7} m for G = f(B), and	$3 \times 10^{-12}\,°$ for B = f(G)

Caution

Users of some rotary calculators can monitor all calculations step-by-step, so unreasonable intermediate results caused by algorithm inadequacies are recognised when they appear. Nevertheless, when using an electronic computer, most users are often aware of only the input and the final answer. Errors can be divided into two types, avoidable and unavoidable.

Avoidable errors are errors caused by inadequacies in the algorithms. An example of this type of error is shown in calculating 2^3.
The electronic calculator subroutines calculate y^x by the formula $y^x = e^{x \text{ LN } y}$. Using the sequence x⇔y, LN, x, e^x, the formula is evaluated by subroutine calls to LN, multiplication, and exponential. The answer 2^3 = 8.000000003 has been subject to three intermediate roundings, but an improvement was made by allowing the basic subroutines for carrying intermediate results to extra digits.

Unavoidable errors are errors caused by using finite computers to approximate non-finite processes. An example of this type of error is shown in the repeating non-finite decimal representation of $^1/_7$ = 0.142857 142857 142857 ... . Computing $1/(^1/_7)$ = 1/(0.142857 1429) = 6.99999 99979 00000 00062 99... , which rounds to 6.99999 9998 due to an unavoidable consequence of rounding to 10 decimal places, instead of 7 (Cochran, 1972; Egbert, 1977a, b; Harms, 1976; Whitney, 1972); Kahan, (1974).

About Nonlinear Equations and Failure of Iterative Methods

Most geodetic models necessitate the solution of nonlinear equations. In a few cases is it possible to solve such equations explicitly, but equations involving x^{10} or higher powers cannot be solved explicitly. In that case, repetition, so-called iteration, is an efficient way to solve large systems. An iterative method can be described as one in which the process can be repeated: an accurate answer about each succeeding answer is used to compute a more accurate answer. Consequently, an initial guess is called $x^{(0)}$, and each new approximation is named $x^{(i)}$, in which i = 1, 2, ..., n. The values $x^{(0)}$, $x^{(1)}$, $x^{(2)}$, . . . form an infinite sequence. Consequently, $x^{(n)}$ converges to the correct solution, as n gets larger.

Usually, it is thought that as much accuracy as is needed can be obtained by performing enough iterations. It one of the fastest methods for finding the guess fairly close to the answer, but $x^{(n)}$ may not converge to lack of computer accuracy. This problem represents cumulative errors, loss of accuracy in floating point numbers (round off error, overflow error) and range [17.1].

Nonlinear equations can be very difficult to solve computationally, in part because it is not possible to know where a solution may exist from local knowledge of the functions. Hence, it is necessary to plot a graphic overview. Using an iterative method, two nonlinear equations to compute geodetic- and conformal latitudes failed to converge above 68° (Figure 72). Here, plotted graphic results show as positive-, and negative nearby values of the expected solution. Simultaneously, varying values ($^+/_-$) prove the algorithms are correct.

Note

About iterating formulae, (Vincenty, 1975) mentioned: " ... the recommended direct and reverse solutions duplicate each other perfectly if the values obtained from the previous computation are used without rounding. This was to be expected, since one formula was obtained by reversing the other ".

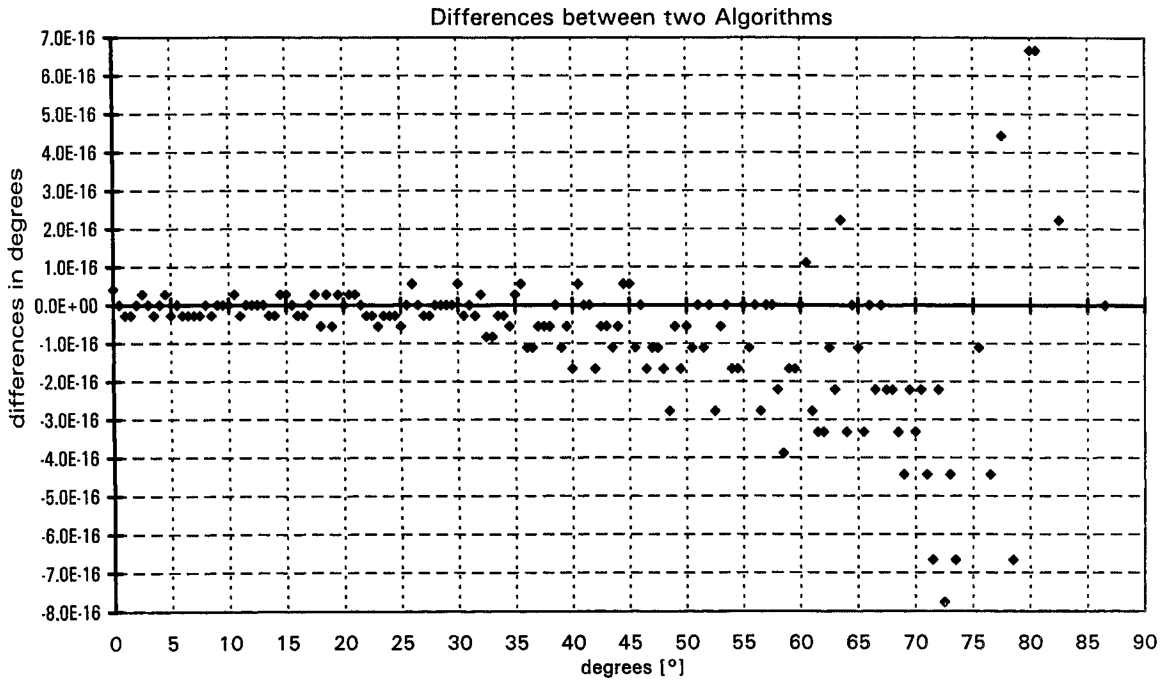


Figure 72: An LCC algorithm shows that iterating is not always successful

8.5 Arc of Parallel

Compared with the meridional arc, the arc of parallel of a reference ellipsoid is simple to evaluate. To solve the arc of parallel $\Delta\lambda$, the method starts with entering latitude φ. Furthermore, the calculated length of the whole curve parallel $\Delta\lambda$ is defined from the point of longitude, λ_1, and to a point of longitude, λ_2.

Arc Equations

All angles are expressed in radians. See program [18.7], Length of Parallel - A_07PARA.FOR

The equations for the ellipsoid constants are also given (Table 19): pp 123.
All angles are expressed in radians.

a	semi-major axis of the ellipsoid	
b	semi-minor axis of the ellipsoid	
c	polar radius of curvature	
f^1	reciprocal flattening of the ellipsoid	
f	flattening of the ellipsoid	
e^2	first eccentricity squared	
e'^2	second eccentricity squared	
M	or ($\rho = R_{00}$) = radius of curvature in the meridian ________	(Figure 48): pp 126
N	or ($\nu = R_{90}$) = radius of curvature in the prime vertical ____________	(Figure 48)

φ_i	parallel of geodetic latitude (at P_i), positive north
$\Delta\lambda_i$	length of the parallel at P_i

Derived Ellipsoid Constants

Compute constants for an ellipsoid as given below:

$$f = 1 / fl$$
$$e^2 = f(2 - f) \tag{8.73}$$
$$e'^2 = e^2 / (1 - f)^2 \tag{8.74}$$
$$t_1 = 1 - f$$
$$c = a / t_1$$

Length of a Parallel

Compute the length of the parallel at $P(\varphi)_i$ for an ellipsoid as given below:

$$V = (1 + e'^2 \sin \varphi^2)^{½} \tag{8.75}$$
$$W = (1 - e^2 \cos \varphi^2)^{½} \tag{8.76}$$
$$M = c / V^3 \tag{8.77}$$
$$M = R_{00} = a(1 - e^2) / (1 + e^2 \sin^2 \varphi)^{½} \tag{8.78}$$
$$N = a / W \tag{8.79}$$
$$N = R_{90} = a / (1 - e^2 \sin^2 B_m)^{½} \tag{8.80}$$
$$\Delta\lambda_1 = N \cos \varphi \,.\, \lambda \tag{8.81}$$

Length of a Parallel - Application

Given:

ellipsoid		:	Bessel 1841			
semi-major axis	a	:	**6 377 397.155**	recipr. flattening	f^{-1} :	**299.15281 285**
latitude	φ_1	:	**45° 00' 00" N**	Latitude	φ_2 :	**55° 00' 00" N**
longitude	$\Delta\lambda$	:	**1°** -	**1'** -	**1"**	

φ	$\Delta\lambda = 1°$ m	$\Delta\lambda = 1'$ m	$\Delta\lambda = 1''$ m
45°	78837.29341	1313.954890	21.89924817
46°	77453.91115	1290.898519	21.51497532
47°	76046.76765	1267.446128	21.12410212
48°	74616.28344	1243.604724	20.72674540
49°	73162.88715	1219.381452	20.32302421
50°	71687.01462	1194.783577	19.91305962
51°	70189.10917	1169.818486	19.49697477
52°	68669.62128	1144.493688	19.07489480
53°	67129.00870	1118.816812	18.64694685
54°	65567.73593	1092.795599	18.21325998
55°	63986.27472	1066.437912	17.77396520

(Jordan, 1959)

9. Conversions and Zone Systems

At the outbreak of World War I in August 1914, the Artillery of the Allied Forces in France used French Survey System with 2D-coordinates, continuous from sheet to sheet, for calculating distance and direction (Seymour, 1980).

> *A projection is a systematic representation of a portion of the Earth's curved surface upon a plane. The projections have been devised with a geodetical curvilinear lattice of latitude and longitude. The coordinate systems are based on projections and are determined by the orientation of a particular area of the Earth's curved surface. A grid is a system of coordinates on the earth's surface expressed in linear units: Eastings, Northings, or X, Y. There is no internationally accepted convention concerning the use of X and Y.* (Maling, 1992).

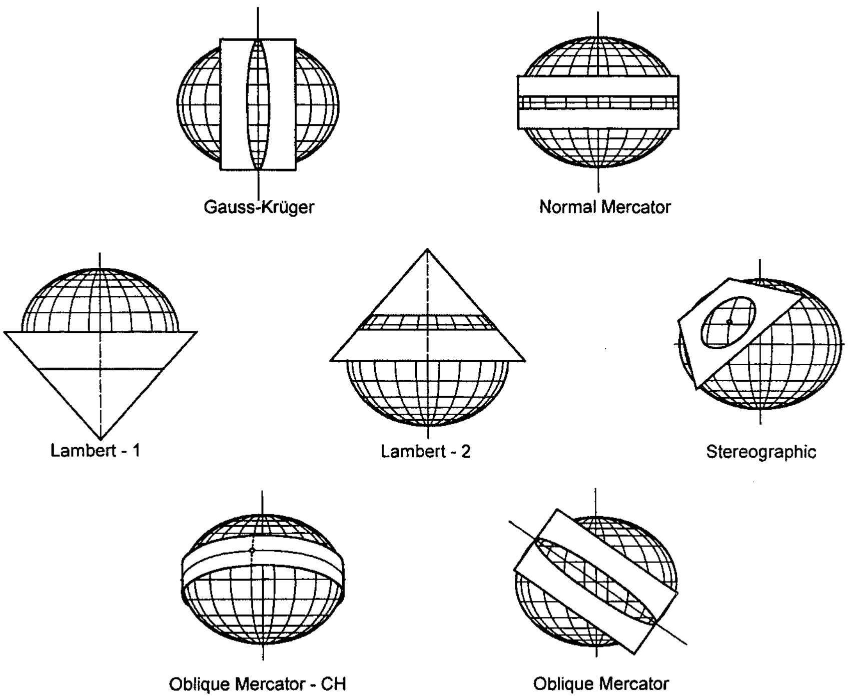


Figure 73: Tangency or secancy of various map projection surfaces

The mathematical function defines a relationship of coordinates between the ellipsoid and a Gaussian sphere or aposphere, between that sphere and a plane, or both, between the ellipsoid and a plane.

Conformal or Orthomorphic

Arthur R. Hinks, in (Adams, 1921), defines orthomorphic, which is another term for conformal, as follows:

> *"If at any point the scale along the meridian and the parallel is the same in the two directions and the parallels and meridians of the map are at right angles to one another, then the shape of any very small area on the map is the same as the shape of the corresponding small area upon the Earth. The projection is then called orthomorphic".*

A full treatment of map projections is beyond the scope of the book. For more information, the interested reader should refer to (Sacks, 1950; Hotine, 1946-1947) - Nos. 65, 66 being regarded as "*classics*" - (Hotine, 1969; Engels, 1995f; Grafarend, 1995f, h, k, 1996h; Steers, 1948).

9.1 Scope and Terminology

Normal Mercator projection

For mapping an area of considerable extent in longitude along the equator (or parallel), the normal Mercator (NM) projection based on the idea of an east-west centre line - can be used. The continuously increasing scale of the Mercator projection becomes more pronounced as the spacing of meridional parts north and south of the Equator increase rather rapidly. Consequently, the Mercator projection is clearly unsuitable for high latitudes. Especially, the Poles are infinitely distant from the Equator and cannot be shown on the projection. The Mercator projection provides a convenient working base for the mariner, excepting the high latitudes beyond 72° N and S due to distortion. Owing to its unique properties, the projection has attained a status that is well established and is used widely for naval navigational charts.

The central line can just as well be any geodesic with an orientation at some azimuth intermediate between 0° and 90° which becomes the axis of an oblique projection. Otherwise, a meridian at an azimuth of 90° is used which then becomes the axis of the transverse Mercator projection (Wilford, 1981).

For mapping an area of considerable extent in longitude along the equator or a parallel, a geometrical system - the normal Mercator (NM) projection [10.3] based on the idea of an east-west centre line - can be used. In such a system the differences of longitude to be covered will be comparatively small, excepting the high latitudes. It is used widely for navigational charts (Grafarend, 1996h, 1997f).

Gauss-Krüger or Transverse Mercator projection

The name Gauss-Krüger (GK) type projection or transverse Mercator projection [10.4], derives from the fact that this projection is the transverse case of the NM type projection (Grafarend, 1996h, 1997e). GK coordinates are best suited to an area or country, which has a large extent in a north-south direction and a relatively small extent in an east-west direction. It is the most commonly used conformal projection system in the world.

Lambert conformal conical projection

For mapping an area of considerable extent in longitude, a geometrical system - the Lambert conformal conical (LCC) projection [10.1] based on the idea of an east-west centre line - can be used. In such a system, the differences of latitude to be covered will be comparatively small.

Stereographic conformal projection

The oblique Stereographic conformal (OSC) projection [10.7] coordinates are best suited to a "*square*" state or country that has a medium extent in a north-south direction and a medium extent in an east-west direction. The stereographic projection it is employed in geodesy and cartography in three forms:

- polar stereographic projection system
- oblique stereographic projection system
- horizontal stereographic projection system.

The universal polar stereographic (UPS) grid system is extended for the regions south of 79° 30' S and north of 83° 30' N, hence providing a 30' overlap with the UTM grid. For UPS, the meridians are straight lines radiating from a central point - the North or South Pole of the ellipsoid. The parallels are concentric circles about this central point. Noticeably, the UPS is a special case of the LCC projection in which the standard parallel is taken to be the Pole (Ayres, 1996).

Proposals for Unifying European 3D-coordinate Systems

A Map Projection Workshop (CPW, 2000) Paris, recommends the EC/EuroGeographics proposals to:

- adopt vector data for the ellipsoidal coordinates of ETRS89 for expressing positions
- to adopt ellipsoidal coordinates (geodetic latitude, geodetic longitude, and (if appropriate) ellipsoidal height) for expressing positions of vector data
- the European height system shall be defined by the EVRS and shall be realised by the height system European Vertical Reference Frame of 2000 (EVRF2000), and the national systems accurately related to this common reference.
- adopts ellipsoidal coordinates (geodetic latitude, geodetic longitude, and if appropriate ellipsoidal height) for expressing positions of vector data;
- adopts the GK projection system with UTM grid-zone system for topographic maps with scales larger than 1 : 500.000
- adopts the Lambert conformal projection (LCC) with two parallels for cartographic maps with scales equal 1 : 500 000 or less
- adopts the Lambert equal area azimuthal projection for statistical analysis and display
- includes the above coordinate systems in the future specifications of the products to be delivered to the EC, within projects, contracts, et cetera to the EC
- to promote wider use of the coordinate systems mentioned above within all member states, by any means, such as recommendations, official statements)
- support the Working Groups of EUREF / EuroGeographics in collecting and publication of various coordinate reference system definitions and definitive transformation parameters between the national and the pan-European CRS (Ihde, 2000).

9.2 Projection Zone Systems

Projection systems are often limited to a small region. An example of such a region is the Great Britain with a longitudinal difference of 11° width. In order to keep all coordinates within the map system positive, 400 000 m are added to all Eastings since these would otherwise be negative for points west of 2° W, also, 100 000 m are subtracted from all Northings. In the United Kingdom is 11° width approximately the longitudinal difference. It follows that there are some linear distortions present e.g. in the Western Isles (Maling, 1992).

Fitting a single projection system to a country or state may give rise to problems. Hence, a zone system with distinctive "*Baumgart*" numbers, combined with one or more projection systems, may propose a convenient solution. The origin of each zone-grid is generally located close to the centre of the grid. The unit of measure is either metres, or feet (yards). It may bear no relation to the origins of adjacent grid zones. Please note there is a change of zone hazard!

A projection system should be limited to a region bounded by a latitudinal- and longitudinal distance, the central line, which will depend on the purpose of the projection. Still, when used in a zone system, either the Lambert conformal conic (LCC) type systems [10.1], normal Mercator (NM) type systems [10.3], or the Gauss-Krüger (GK) type systems [10.4] are suitable.

Tangency or Secancy

The local tangent plane is a plane that has been developed from another regular mathematical surface, such as the cylinder of the Mercator projection or the cone of the Lambert projection. The plotting surface may be either tangent or secant to the reference ellipsoid (Figure 73). If a secant surface is used, one true length line is defined on the map at the line of secancy. For the conical and cylindrical projections, the line of tangency is a single true length line. In the secant case, two true length lines occur at the two lines of secancy. Note that secancy is not a geometrical property, but only a conceptual one in a projection such as the Lambert conformal conical with two standard parallels (Maling, 1992).

False Eastings and Northings

Easting is the grid coordinate of a point eastward (positive) or westward (negative) from a reference meridian. Northing is the linear distance of a point northwards (positive) or southwards (negative) from a (false) origin. In the Southern Hemisphere, Northings are defined by adding 10 000 000 metres to the negative value of N.

Having grid quantities changing from positive to negative is inconvenient as the grid zero lines are crossed. To avoid this, UTM has a special standard for recording false values added to prevent negative grid coordinates within a grid zone.

Non-Standard Projection and Grid Systems

- Norwegian specifications for the National GK Grid

The National projection and grid system of Norway is based on the GK grid system. It has a latitude of true origin 58° N and eight longitudinal zones.

- US State Plane Coordinate System

The US State Plane Coordinate System (SPCS27/SPCS83), with reference ellipsoids Clarke 1866 / GRS80, respectively has following projections: transverse Mercator, Lambert conformal conic, and Hotine oblique Mercator. Units: US States use international feet, US survey feet, and metres.

The coordinate system in each zone is based on a map projection, which is determined by the orientation of the zone relative to the Earth's meridian. For instance, the US State of Alaska is divided into 10 zones for establishing a SPCS. In Zone 1, the curve in question is a geodesic passing through an arbitrary central point of the area and roughly bisecting the strip. The resulting projection is an oblique Mercator projection of the ellipsoid [10.5]. In Zone 10 the centre line is a parallel of the ellipsoid near the centre of the zone, and the resulting projection is the well-known LCC. Alaska Zones 2-9 all have the same set of plane coordinates based on the GK projection in which the strips extend to both sides of a central meridian. In each of Zones 1-9, the central axis, instead of being mapped true to scale, has been reduced by a scale factor 0.9999 to reduce the maximum error in any portion of the zone, because any curve on any portion of the Earth's curved surface can be mapped in its true length. A narrow strip on either side of this curve will be sufficiently undistorted to serve as a map on which distances can be measured to satisfy the demands of surveying accuracy. State boundaries are the latitude limits of SPCS (Dracup, 1994; Hooijberg, 1997; Mitchell, 1945; Osterhold, 1993; Schwarz, 1989a; Stem, 1989a).

The coordinate system in a zone was usually based on a logarithmic method of computation; not strictly exact, but the differences between stations in any particular area were sufficiently correct for all practical purposes (Krüger, 1919). In the USA, a rotary-machine method was introduced, now based on a method to arrive at the same results as the logarithmic method of computation.

- system of oblique stereographic conformal projections

A system of oblique stereographic conformal (OSC) projections [10.7] applied to an area of considerable extent (Western- and Central-Europe and the Middle East) was proposed in (Roussilhe, 1922).

- non-standard GK-projection and zone-systems

A GK projection system will easily accommodate wide zones with submillimetre accuracy. Hence, GK is a universal projection in that the same set of coordinates can be used for any narrow strip following any arbitrary meridian, designated the central meridian. The origin of a GK-zone system usually refers either to the 0° E (Greenwich) meridian or the Ferro meridian.

Four continental-wide standard Gauss-Krüger (GK) type projection and grid systems in current use are available with specifications for a uniform cover in zones which correspond to the 6° (and 3°) longitudinal units, starting at a specified meridian, such as:

- CK-42 grid system of Russian Federation (formerly Warsaw pact states, or CIS)
- Xi'an-grid system of China (Xiang, 1988)
- GK-grid system of Germany (Krüger, 1919)
- AMG-system of Australia
- GK type system as a basis for a worldwide standard universe transverse Mercator (UTM) grid system, the US Army standard.

- Russian specifications for the Gauss-Krüger grid system

The specifications for the CK42 GK/transverse Mercator is a uniform cover of the Eurasian continent in zones which correspond to the 6° (and 3°) longitudinal units, starting at the Greenwich Meridian on the zone 1 (0°E to 6°E), and increasing eastwards to prefixed number 60 on the zone from 6°W to 0°E. Zone numbers are prefixed to the false easting in most cases, i.e. the false easting for the CK Zone 7 is 7 500 000 m.

- China - Xi'an grid system specifications for the Gauss-Krüger grid

GK type specifications for the Xi'an state reference grid system is a uniform cover of the Asian continent in zones which correspond to the 6° (and 3°) longitudinal units, starting at the Greenwich Meridian on the zone number 1 (0°E to 6°E), and increasing eastwards to prefixed number 60 on the zone from 6°W to 0°E.
The zone number is prefixed to the false easting in most cases, i.e. the false easting for the GK type, zone 20 is 20 500 000 meters (Zhu, 1986).

- German specifications for the Gauss-Krüger grid system

The northern axes of the FRG-West Coordinate System of 1922 coincide with the meridians at 3°, 6°, 9°, 12°, 15°, et cetera. The northern axes of the FRG-East coordinate system 1942 / 83) coincide with the meridians at 3°, 9°, 15°, 21°, et cetera. Eastings receive a prefixed "*Baumgart*" number. Numbering of UTM-zones and GK-zones differ by 30.

- Australia's AMG system

In Australia, the National Mapping Council (NMC-1965) adopted the ANS (Australian national spheroid) and recommended the introduction of the AGD66 (Australian Geodetic Datum of 1966) for all geodetic surveys in Australia and Papua New Guinea. During a period of change to the AMG (Australian map grid), the opportunity was taken to change the unit from the imperial yard to the metric system. AMG covers the Australian mainland, Tasmania and features close to their shores.

Standard GK-projection and zone-system for the UTM-grid system

In 1947, the US Army Map Service (AMS), now the National Geospatial Intelligence Agency (NGA), devised the universal transverse Mercator (UTM) grid system as a mapping base. It was adopted by the US Joint Chiefs of Staff as the official US Army UTM grid system. It is defined for military mapping by a transverse Mercator projection, GK type, for areas between latitudes 80° S and 84° N. Since that time, the UTM grid system is a uniform cover of the whole world in 6° wide zones. The GK-zone system is numbered from 1 through 60 beginning at 180° W and proceeding eastwards to 180° E. The UTM grid system extends to 84° 30' N and 80° 30' S, providing a 30' overlap with the UPS grid system. Consequently, the concept of the coordinated grid had to receive its proper place. Coordinates were to be redefined in an operational sense as a function of a worldwide transverse Mercator grid system (Ayres, 1995). For authorative guidance for the use of grids and reference systems, see (DA, 1958; Fister, 1980) and NGA (DMA) TM 8358 series. The UTM grid system specifications, i.e. scale factor, central meridian, defining constants and ellipsoid parameters appear in many manuals of the US DoD NGA Topographic Centre (Ayres, 1995; Burton, 1996).

9.3 Computation Zone Systems

In this handbook, all computer calculations refer to the northeastern Hemisphere by means of internal so-called Flags (18.23) in programs. Flags keep a close watch on correct signs and coordinates in any Hemisphere.

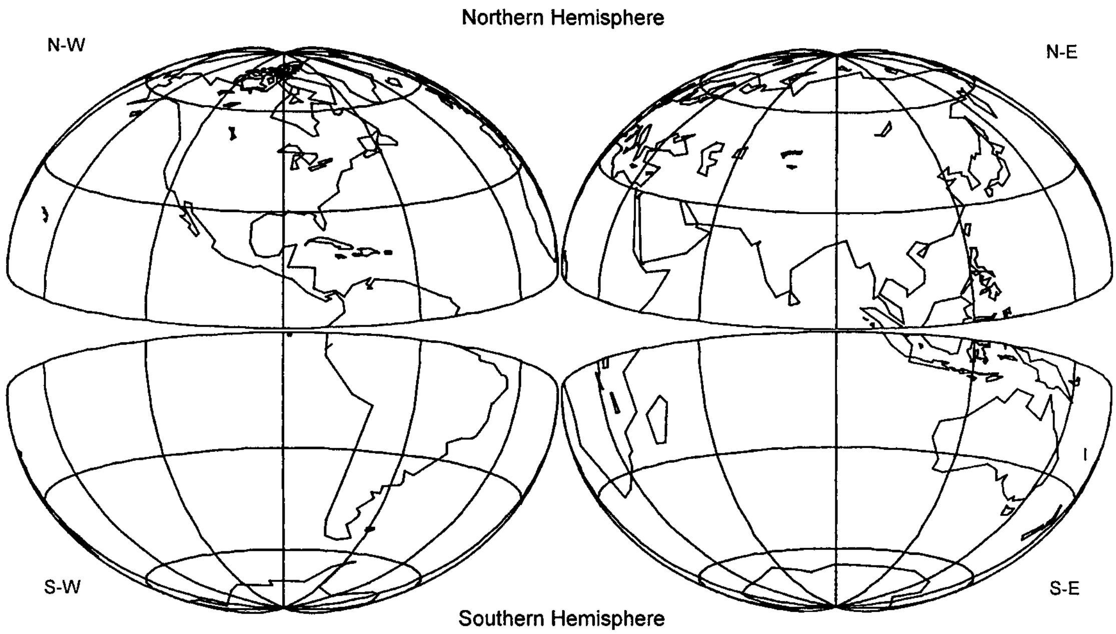


Figure 74: Outline shows the FOUR QUADRANTS as used for computing correct signs using Flags (i_1; ... ..., i_n)

9.4 Conversions and Transformations

In converting coordinates from one positional reference system to the coordinates of that position represented in any other reference system, the coordinate conversions involve coordinate calculations and sometimes Datum transformations (Floyd, 1985).

Reference systems can be categorised into three broad groups:

- geodetic coordinates in terms of a curvilinear lattice of latitudes and longitudes
- plane coordinates in terms of a rectilinear lattice of X and Y, or eastings and northings
- latitudes, longitudes, and height (space coordinates) in terms of a Cartesian system of X, Y, and Z.

Algorithms are provided for any geodetic Datum for the following projections:

- 10.1. Lambert's Conformal Conical Projection ______ A_19LC00.FOR
- 10.2. Gauss-Schreiber Conformal Double Projection ______ A_16GAUS.FOR
- 10.3. Normal Mercator Projection ______ A_17NM00.FOR
- 10.4 Gauss-Krüger Conformal Projection ______ A_18GK00.FOR
- 10.5. Hotine's Oblique Mercator Projection ______ A_20HM00.FOR

Special algorithms are provided for a geodetic Datum for the Gauss-Schreiber (GS) type double projections:

- 10.6. Rosenmund's Oblique Mercator Projection (Switzerland) ______ A_21RM00.FOR
- 10.7. Stereographic Conformal Projection ______ A_22ST00.FOR

Calculations apply specifically to the process of coordinate conversions and Datum transformation (Maling, 1992). Changes can be listed as follows:

- change in Datum ______ A_10TRMO.FOR
- change in epoch (Ollikainen, 1995) ______ A_10TRMO.FOR
- change in zone ______ see zone conversion example
- change in projection ______ see projection conversion example.

Coordinate Conversions

- converting geodetic coordinates to plane coordinates referenced to the same ellipsoid, Datum and epoch or vice versa
- plane coordinates on one projection to plane coordinates on another projection, both projections referenced to the same ellipsoid, Datum and epoch, for instance due to change in scale factor, origin, orientation of grid axes.

Datum Transformation

- transformation of geodetic coordinates based on one ellipsoid, Datum and epoch to geodetic or plane coordinates based on another ellipsoid, Datum and epoch
- transformation of plane coordinates on one projection based on one ellipsoid, Datum and epoch to plane coordinates on the same type of projection referenced to another ellipsoid, Datum and epoch
- transformation of plane coordinates on one projection based on one ellipsoid, Datum and epoch to plane coordinates on a different type of projection referenced to another ellipsoid, Datum and epoch.

Plane Coordinates and Origins

Two types of plane coordinates and three plane coordinate origins have the following significance:

- true origin is the fundamental origin of the projection that defines and orients the projection surface
- true coordinates are those reckoned from the true origin of the projection at a scale given by the projection parameters
- normally the grid origin is located by design to the west and south of the region of interest; resulting grid coordinates are always positive-valued
- grid coordinates are at the same scale and referenced to an origin situated more conveniently for a particular area of interest
- false eastings and false northings are X, Y constants used in the translation of coordinates from a true origin to a false origin assigned to the true origin of the grid system.

α	= geodetic azimuth ______	reckoned from north
T	= projected geodetic azimuth	
t	= grid azimuth ______	reckoned from north
γ	= convergence angle	
δ	= (t - T) ______	arc-to-chord correction

Symbology

Convergence is an angular difference between geodetic north and grid north. Geodesic azimuths are referred to meridians: the projected geodetic azimuth or geodetic North. Grid azimuths are referred to north-south grid lines. Defined another way, the convergence angle is the angle between a north-south grid line and the meridian as represented on the plane grid. A geodetic azimuth is symbolised as "α". A convergence angle is symbolised as "γ " (Bomford, 1977; Stem, 1989a).

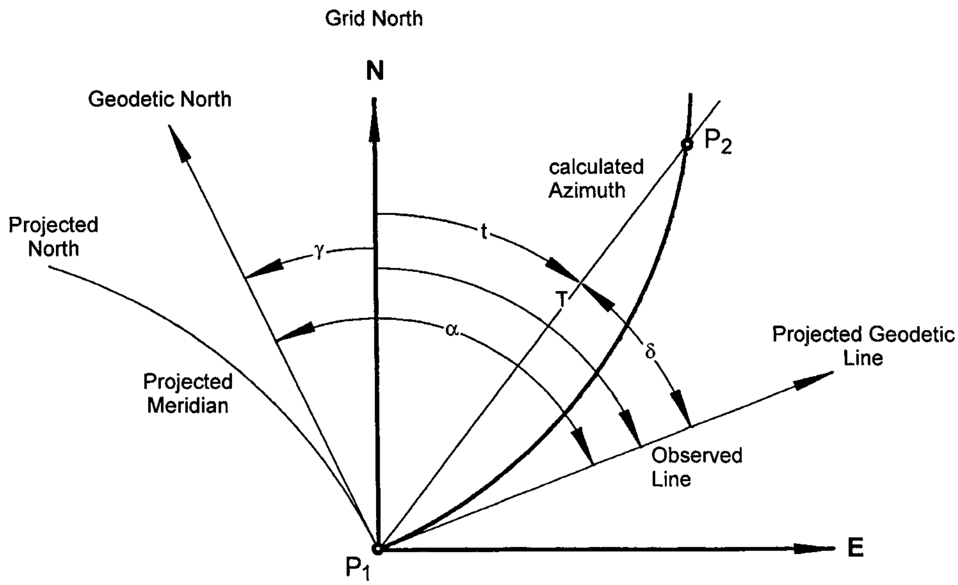


Figure 75: Azimuth and convergence

In the Northern Hemisphere, when a point is east of the central Meridian (CM), grid north is east of true north: γ is positive. When a point is west of the CM, grid north is west of true north: γ is negative. In the Southern Hemisphere the reverse condition is true.

Grid Azimuth and Projected Geodetic Azimuth

The projection of the geodetic azimuth α onto the grid is not the grid azimuth, but the *projected geodetic azimuth* symbolised as "T". Convergence γ is defined as the difference between a geodetic and a projected geodetic azimuth. Hence by definition, $\alpha = T + \gamma$, and the sign of γ should be applied accordingly. The value of γ varies with latitude according to $\gamma = \delta\lambda \sin \varphi$. The angle obtained from two projected geodetic azimuths is a true representation of an observed angle.

When an azimuth is computed from two plane coordinate pairs, the resulting quantity is the grid azimuth symbolised as "t". The angular difference between the projected geodetic azimuth T and grid azimuth t is a calculable quantity symbolised as "δ", or more often identified as the (t - T) correction. The relationship between these azimuths is shown in Figure 75. For sign convention it is defined as $\delta = (t - T)$ and is also identified as the *arc-to-chord* correction, which should always be considered (Stem, 1989a).

Figure 76 and Figure 77 illustrate the projected geodetic lines T, the bold lines in both figures - which always bow away from the central line - and the grid lines t. Both figures illustrate the (t - T) correction in the projections of a traverse. Given the definition of α and δ, we obtain:

$$t = \alpha - \gamma + \delta \tag{9.01}$$

The arc-to-chord correction δ or (t – T) is normally determined for a pair of points on the ellipsoid, such as P_1 and P_2, whose grid coordinates are known. There are two corrections: one to be applied at P_1 for the line $P_{1\text{-}2}$ and another one to be applied at P_2 for the line $P_{2\text{-}1}$ (Stem, 1989a).

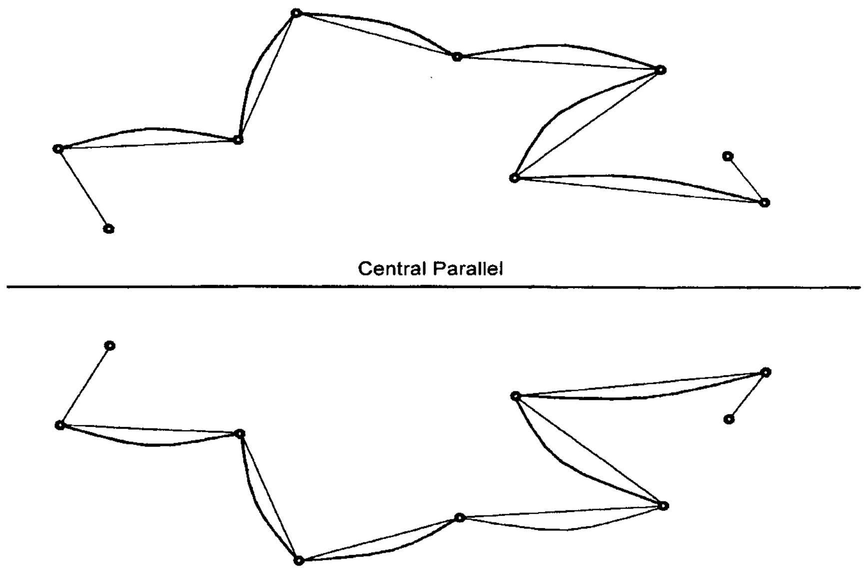


Figure 76: Normal Mercator and Lambert projection - projected geodetic vs. grid angles

The Elementary Concept of Scale and True Distances

The scale factor is the ratio of infinitesimal linear distance in any direction at a point on the plane grid to the corresponding true distance on the ellipsoid. Thus, the grid scale factor is the measure of the linear distortion that has been mathematically imposed on ellipsoid distances so they may be projected onto a plane. The grid scale factor - symbolised by k - is constant at a point, regardless of the azimuth, when conformal projections are used.

In order to obtain the true distance, S, from the grid distance, D, derived from grid coordinates, or alternatively, to convert a true distance to a grid distance (for plotting on the map or projection) it is necessary to calculate the scale factor and apply it in the correct sense:

$$D = k \times S, \text{ or}$$
$$S = D / k \qquad (9.02)$$

The scale factor changes along a line so slowly that for most purposes it may be taken as constant within any 10 km square and equal to the mean of the value at the centre at the square considered.

To apply a correction to an ellipsoidal distance or plane grid distance, it is necessary to use the values of all points making up the line. A more efficient method is to apply Simpson's Rule of numerical analysis to compute a line scale-factor:

$$k = {}^1\!/_6\,(k_1 + 4\,k_m + k_2) \qquad (9.03)$$

See the examples, in which k_1 and k_2 are the scale factors at the end of each line and k_m is the scale factor at the midpoint of the line (Bomford, 1977; Maling, 1992; Stem, 1989a). Interesting illustrations on the subject can be found in (Arrighi, 1994).

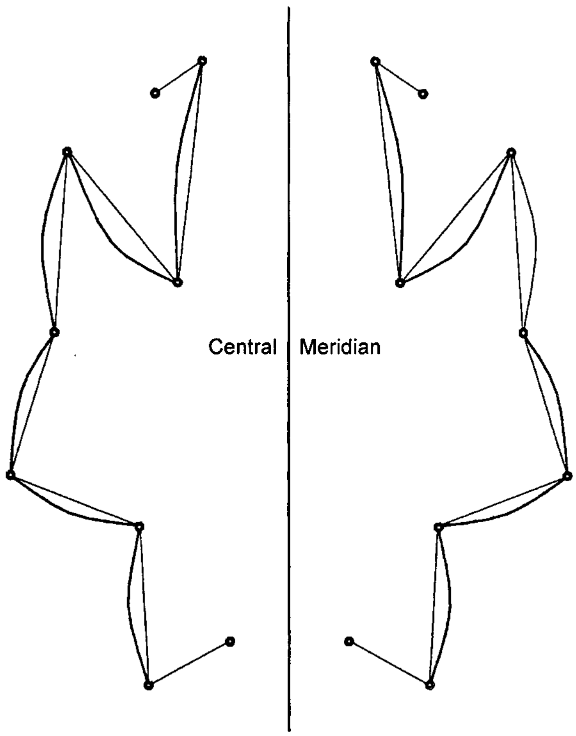


Figure 77: Transverse Mercator vs. grid angles

Practice

At least 12 significant figures are adequate for the desired accuracy in projection zone widths that may be encountered. Therefore, double precision variables will be required on most computers. This book contains very efficient algorithms and program listings for coordinate conversion (0.1 mm accuracy or better) and coordinate transformations (Floyd, 1985; Stem, 1989a; Vincenty, 1985a, b, c, 1986a, b).

10. Conformal Projections - Using Reference Ellipsoids

10.1 Lambert's Conformal Conical Projection

For mapping an area of considerable extent in longitude, a geometrical system based on the notion of an east-west centre line can be used.

Johann Heinrich Lambert devised and published such a projection in *Beiträge zum Gebrauche der Mathematik und deren Anwendung* in the year 1772. The Lambert projection remained almost unknown until the beginning of the World War I. The system was introduced and has been brought to conspicuous attention by the French Military Survey (*"Le Service Géographique de l'Armée"*) under the name *"Quadrillage kilomètrique système Lambert"* (Tardi, 1934).

The grid of the military mapping system presented various properties useful for the rapid computation of distances and azimuths in military operations. These properties are (Adams, 1921):

- uniquely defined by the property of conformality
- an unique reference Datum
- using the centesimal system instead of the sexagesimal system
- grid system of rectangular coordinates.

Quadrillage Kilomètrique Système Lambert

A quadrillage is a grid system of squares determined by the rectangular coordinates of the projection. Referring to one origin, the grid system is projected over the whole area of each zone of the original projection. Therefore, every point on that map is fixed both regarding its position in a given quadrilateral as well as to its position in curvi-linear coordinates. Quick computations of distances are possible between points whose grid coordinates are given. Determinations of the azimuth of lines joining any two points within artillery range are of great value to military operations.

The French Institut Géographique National (IGN) divides the circumference of the circle into the centesimal system (400^g grades) instead of the sexagesimal system ($360°$ degrees). During World War I and II, the essential tables were produced for the conversion of degrees, minutes, and seconds into grades, the conversion of miles, feet, and inches into the metric system, and vice versa for the allied military forces (AMF). In the Lambert projection for the map of France employed by the AMF in their military operations, the maximum scale errors are practically negligible, while the angles measured on the map are practically equal to those in the field.

Since 1947, the system has been superseded by the Universal Grid with a transverse Mercator conformal projection system (UTM) for worldwide military mapping. UTM was developed by the US DoD National Geospatial Intelligence Agency (NGA) Topographic Centre (SDHG), Washington DC [10.4].

Lambert's conformal conical projection

The Lambert conformal conical (LCC) projection is often used where the area of concern lies with its predominating, longer dimension in an east-west direction. The LCC projection is not suited for mapping areas of very wide latitudes.

Using the defining parameters of an ellipsoid, the fundamental parameters are the central parallel (CP), Central or Reference Meridian (RM) and the mapping radius of the Equator. In the LCC projection, all meridians are represented by straight lines that meet a common centre S outside the limits of the map projection.

All parallels are sections of concentric circles centred at the point S of intersection of the meridians. Accordingly, their radii are functions of latitude. All elements retain their authentic forms and meridians and parallels intersect at right angles. With it, the angles formed by any two lines on the Earth's surface are correctly represented on this projection.

The algorithm handles three cases for defining a projection:

- secant projection with upper and lower standard parallels known
- secant projection with standard parallel and its imposed scale factor known
- tangent projection with one standard parallel and its scale factor of unity known (Figure 78).

The Reference Meridian (RM) orients the cone with respect to the ellipsoid selected. The latitude of the standard parallel and the longitude of the RM would be chosen to run through the centre of the zone to be computed. Consequently, the geometry of a conical projection is described by the value of the constant of the cone which is defined either by a standard parallel with a desired scale factor or two (upper and lower) standard parallels.

Usually, a scale factor of unity can be applied to make the scale true along two selected parallels. For the equal distribution of the scale factors, the standard parallels are placed at distances from its northern and southern limits of the projection, each equal to $^1/_6$ of the total meridional distance to be portrayed. The scale will be less than the nominal value between these parallels.

Moving these parallels close together results in the LCC with one standard parallel. In that case, a reference meridian and a standard parallel are assumed with a cone tangent along the standard parallel.

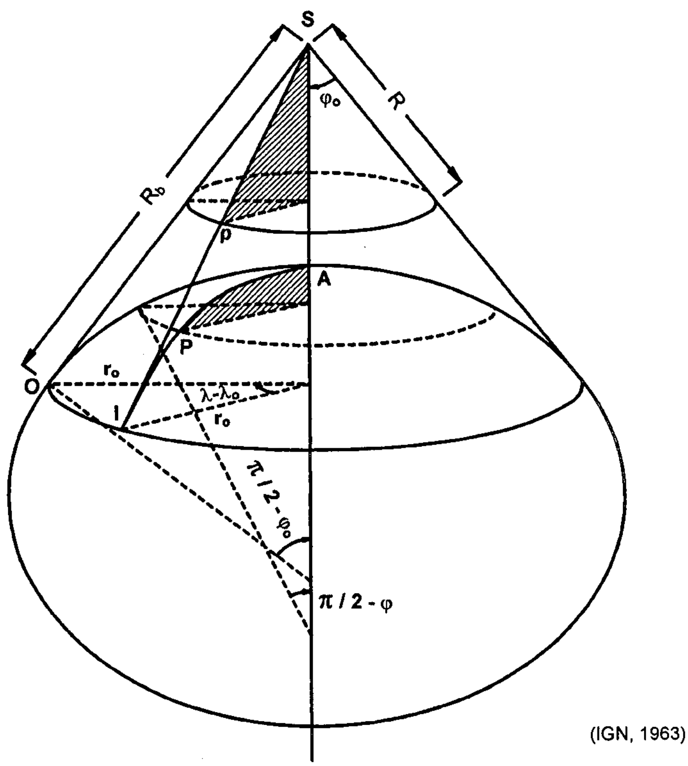


Figure 78: Lambert's conformal conical projection

Secant Conical Projection

The LCC projection employs a cone intersecting the ellipsoid at two parallels known as the upper and lower standard lines or parallels - automecoic - for the section to be represented. Bringing them closer together in order to have greater accuracies in the centre of the map at the expense of the upper and lower border areas may be advisable in some localities, or for special reasons. Instead of two standard parallels - the lines of secancy - some computer programs require the input of a scale factor (k_0 <1) and one standard parallel, which lies close to the midpoint between these standard parallels (IGN, 1994). For further reading (Otero, 1990).

Tangent Conical Projection

This conical projection employs a standard parallel that represents a line of tangency between the cone and the surface of the ellipsoid. The projection parameters required are the reference meridian, a standard parallel, a scale factor of unity, and defining parameters of the ellipsoid selected (Pearson, 1990).

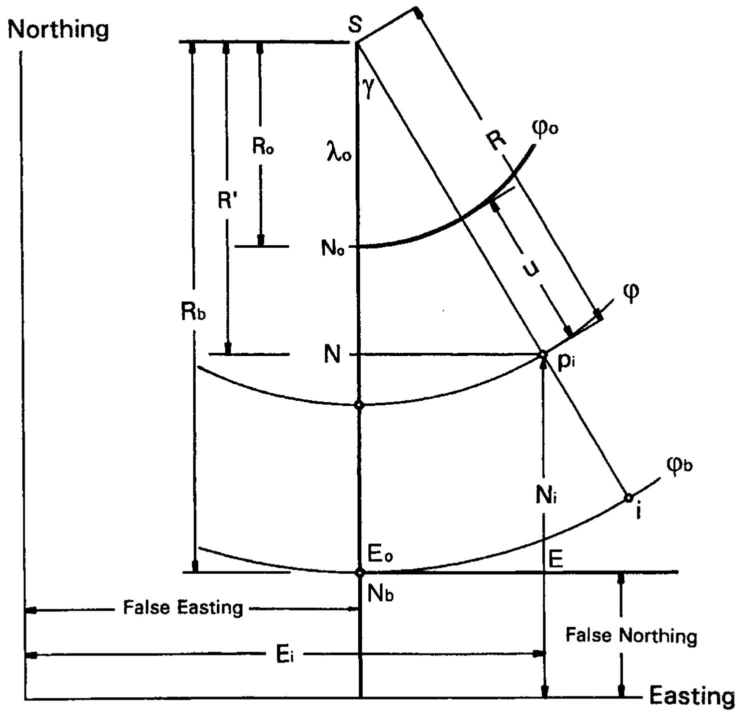


Figure 79: Lambert's grid system

Scale Factor

A reference meridian and a standard parallel are assumed with a cone either tangent along the standard parallel ($k_o = 1$) or secants along two standard parallels - with an imposed scale factor, $k_o < 1$. Nevertheless, on two selected parallels, arcs of longitude are usually represented to a scale factor $k_o = 1$. Between these parallels the scale factor k is <1.0000 and outside them the scale factor k is >1.0000.
Remember that in conical projections, the scale errors vary increasingly with the range of latitude north or south of the standard parallels.

Origins

The true origin is at the intersection of the Reference Meridian (RM) with the apex of the cone.

The equations for the ellipsoid constants are given in (Table 19): pp 123.
The false grid origin is specified by the intersection of the RM longitude, λ_o, and the latitude of the standard parallel, φ_b. Adding the false northing to the false grid origin furnishes the true grid origin, which lies on the RM. A false easting and a false northing may have been assigned to the false grid origin allowing the zone to be extended farther southwest to maintain positive-valued coordinates (Adams, 1921).

(t - T) Correction

The arc-to-chord or (t - T) correction requires the input of two sets of grid coordinates (E_1, N_1, E_2, N_2) and the northing constant, N_0 of the true projection origin. Using the latitude of CP (φ_b) and the longitude of RM (λ_0), program A_19LC00.FOR furnishes that value N_0.

The Lambert conformal conical (LCC) projection has conclusively superior merits for areas of extended longitudes. Consequently, many LCC grids exist, e.g. in Central and North America, Europe, Asia, and North Africa. Each is designed for a particular state or country of the world with its own set of constants. A few Lambert conformal cases follow this introduction (Adams, 1921; Bomford, 1971; Claire, 1968; GG, 1986; IGN, 1994; Otero, 1990; Tardi, 1934).

LCC Mapping Equations

Symbols and definitions

The equations for the ellipsoid constants are given (Table 19): pp 123.
All angles are expressed in radians [18.19], A_19LC00.FOR.

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
f	flattening of the ellipsoid
e^2	first eccentricity squared
φ_u	upper parallel
φ_l	lower parallel
φ_o	central parallel (CP), latitude of projection origin
φ_b	latitude of (false) grid origin, in case of 2 parallels
φ_b	latitude of standard parallel, in case of 1 parallel
k_o	point scale factor at CP
λ_o	central reference meridian (RM, λ_o), longitude of true and grid origin (Figure 79)
E_o	false easting constant at grid and projection origin, (RM, λ_o)
N_b	false northing constant for φ_b at the RM, λ_o
N_o	northing constant at intersection of RM, λ_o, with CP, φ_o
M_o	scaled radius of curvature in the meridian at φ_o
R	mapping radius at latitude, φ
R_b	mapping radius at latitude, φ_b
R_o	mapping radius at latitude, φ_o
K	mapping radius at the equator
Q	isometric latitude
φ	parallel of geodetic latitude, positive north
λ	meridian of geodetic longitude, positive east
E	easting coordinate
N	northing coordinate
γ	convergence angle
δ_{12}	(t - T), arc-to-chord correction for a line from p_1 to p_2
k	grid scale factor at a general point
k_{12}	line scale factor of a line from p_1 to p_2

Computation of Zone and Ellipsoid Constants

Constants and expressions within Lambert's conical mapping equations are ellipsoid and zone specific.

$$Q_l = \tfrac{1}{2}\,[\,(\ln(\,(1+\sin\varphi_l)\,/\,(1-\sin\varphi_l)\,) - e\ln(\,(1+e\sin\varphi_l)\,(1-e\sin\varphi_l)\,)\,] \qquad (10.01.01)$$
$$W_l = (\,1 - e^2\sin^2\varphi_l\,)^{\frac{1}{2}} \qquad (10.01.02)$$

Similarly for Q_u, W_u, Q_b, Q_o, and W_o upon substitution of the appropriate latitude.

$$\sin\varphi_o = \ln[\,W_u\cos\varphi_l\,/\,(\,W_l\cos\varphi_u\,)\,]\,/\,(\,Q_u - Q_l\,) \qquad (10.01.03)$$
$$K = (\,a\cos\varphi_l\exp(\,Q_l\sin\varphi_o\,)\,)\,/\,(\,W_l\sin\varphi_o\,) \qquad (10.01.04)$$
$$= (\,a\cos\varphi_u\exp(\,Q_u\sin\varphi_o\,)\,)\,/\,(\,W_u\sin\varphi_o\,) \qquad (10.01.05)$$

$$R_b = K\,/\exp(\,Q_b\sin\varphi_o\,) \qquad (10.01.06)$$
$$R_o = K\,/\exp(\,Q_o\sin\varphi_o\,) \qquad (10.01.07)$$
$$k_o = (\,W_o\tan\varphi_o\,R_o\,)\,/\,a \qquad (10.01.08)$$
$$N_o = R_b + N_b - R_o \qquad (10.01.09)$$

Note: $\exp(x) = \varepsilon^x$, in which ε= 2.71828 18284 59045 23536 02875 (base of natural logarithms)

Recognise in (10.01.01, 10.01.11): $\ln(\,\tan((\,\pi/4\,)+(\,\varphi/2\,))) = \tfrac{1}{2}\ln((\,1+\sin(\,\varphi\,))\,/\,(\,1-\sin(\,\varphi\,)))$ (10.01.10) (Gretschel, 1873; Le Pape, 1994).

Direct Conversion Computation

Input: geodetic coordinates of point P (φ, λ) on meridian API, projected on Spi.
Output: grid coordinates of point p (E, N), convergence angle (γ), grid scale factor (k).

$$Q = \tfrac{1}{2}\,[\,\ln(\,(1+\sin\varphi)\,/\,(1-\sin\varphi)\,) - e\ln(\,(1+e\sin\varphi)\,/\,(1-e\sin\varphi)\,)\,] \qquad (10.01.11)$$
$$R = K\,/\exp(\,Q\sin\varphi_o\,) \qquad (10.01.12)$$
$$E = E_o + R\sin\gamma \qquad (10.01.13)$$
$$N = R_b + N_b - R\cos\gamma \qquad (10.01.14)$$
$$\gamma = (\,\lambda_o - \lambda\,)\sin\varphi_o \qquad (10.01.15)$$
$$k = (\,1-e^2\sin^2\varphi\,)^{\frac{1}{2}}(\,R\sin\varphi_o\,)\,/\,(\,a\cos\varphi\,) \qquad (10.01.16)$$

Inverse Conversion Computation

Input: grid coordinates of point p (E, N) on a line Spi (Figure 78; Figure 79).
Output: geodetic coordinates of point P (φ, λ), convergence (γ), grid scale factor (k).

$$R' = R_b - N + N_b \qquad (10.01.17)$$
$$E' = E - E_o \qquad (10.01.18)$$
$$\gamma = \tan^{-1}(\,E'/R'\,) \qquad (10.01.19)$$
$$\lambda = \lambda_o - \gamma\,/\sin\varphi_o \qquad (10.01.20)$$
$$R = (\,R'^2 + E'^2\,)^{\frac{1}{2}} \qquad (10.01.21)$$
$$Q = [\,\ln(\,K/R\,)\,]\,/\sin\varphi_o \qquad (10.01.22)$$

Use an approximation for φ as follows:

$$\sin\varphi = (\,\exp(\,2Q\,) - 1\,)\,/\,(\,\exp(\,2Q\,)+1\,) \text{ and iterate } \sin\varphi \text{ as follows:} \qquad (10.01.23a)$$
$$f_1 = \tfrac{1}{2}\,[\,\ln(\,(1+\sin\varphi)\,/\,(1-\sin\varphi)\,) - e\ln(\,(1+e\sin\varphi)\,/\,(1-e\sin\varphi)\,)\,] - Q \qquad (10.01.24)$$
$$f_2 = 1\,/\,(\,1-\sin^2\varphi\,) - (\,e^2\,/\,(1-e^2\sin^2\varphi\,)\,) \qquad (10.01.25)$$
$$\sin\varphi = \sin\varphi - (\,f_1/f_2\,) \text{ and iterate to obtain } \varphi \text{ with sufficient accuracy} \qquad (10.01.23b)$$

In the LCC program, latitude is obtained without iteration using the five constants F_0, F_2, F_4, F_6, and F_8 as it is computed in the HOM program, oblique Mercator section [10.5]:

$$k = (1 - e^2 \sin^2 \varphi)^{1/2} (R \sin \varphi_o) / (a \cos \varphi) \qquad (10.01.26)$$

Arc-to-Chord Correction δ = (t - T)

Grid azimuth (t), geodetic azimuth (α), convergence angle (γ), and arc-to-chord correction (δ) at any given point are related as follows: $t = \alpha - \gamma + \delta$ (9.01).

Input: P_1 (E_1, N_1), and P_2 (E_2, N_2)
Output: δ_{12}

$$p_1 = N_1 - N_o \qquad (10.01.27)$$
$$p_2 = N_2 - N_o \qquad (10.01.28)$$
$$q_1 = E_1 - E_o \qquad (10.01.29)$$
$$q_2 = E_2 - E_o \qquad (10.01.30)$$
$$R'_1 = R_o - p_1 \qquad (10.01.31)$$
$$R'_2 = R_o - p_2 \qquad (10.01.32)$$
$$\Delta N = N_2 - N_1 \qquad (10.01.33)$$
$$M_o = k_o\, a (1 - e^2) / (1 - e^2 \sin^2 \varphi)^{3/2} \qquad (10.01.34)$$

$$u_1 = p_1 - q_1^2 / 2 R'_1 \qquad (10.01.35)$$
$$\varphi_3 = \varphi_o + (u_1 + \Delta N / 3) / M_o \qquad (10.01.36)$$
$$\delta_{12} = (\sin \varphi_3 / \sin \varphi_o - 1) (q_2 / R'_2 - q_1 / R'_1) / 2 \qquad (10.01.37)$$

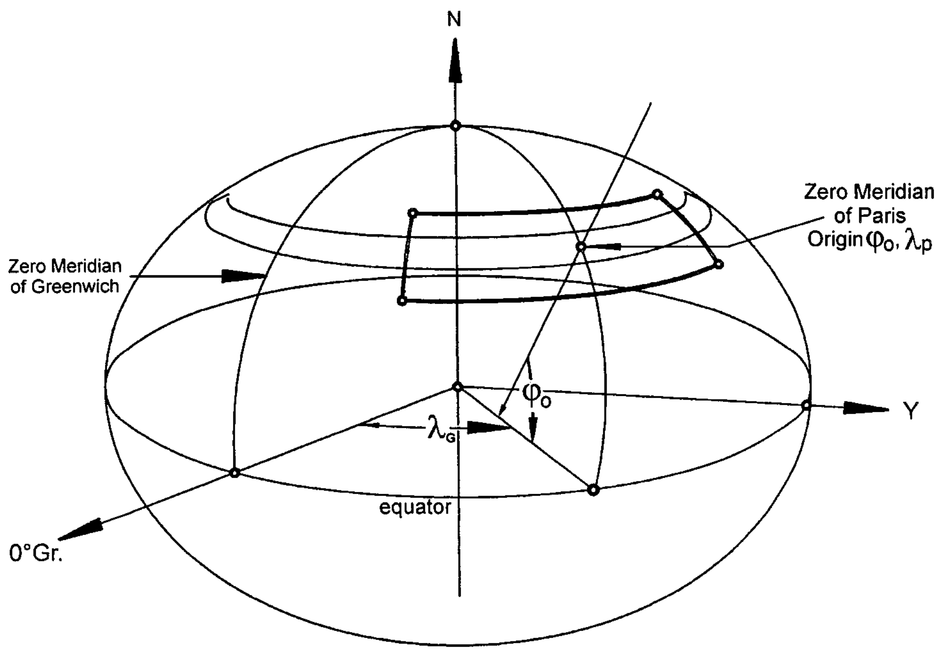


Figure 80: Lambert's conical IGN zone

The size of δ varies linearly with the length of the ΔE (Δλ) component of the line and with the distance of the point from the central parallel.

Note

The lengths of the lines are 20 km, the orientation of the lines in azimuths of 90°, 135°, and 180°. Dividing the line into several traverse legs results in a proportional decrease in the required correction to a direction. However, it does nothing to diminish the closure error in azimuth because errors due to omission of δ are cumulative.

(Vincenty, 1986a).

The examples below (Table 24; Table 25), referring to example LCC SPCS83 Texas Central, are computed for a hypothetical zone with a central parallel of approximately φ_o= 32° on the GRS80 ellipsoid.

values of (t - T) and computational errors in δ determination for φ_o = 32° (in seconds of arc)					
$\varphi_1 - \varphi_o$		1°	2°	1°	2°
$\lambda_1 - \lambda_o$		5°	5°	10°	10°
true δ, azimuth =	90°	5.67	11.44	5.67	11.44
error		0.00	0.01	0.01	0.06

Table 24: Values of computational errors in δ for φ_o = 32° from the LCC equations

The examples are computed for a hypothetical zone with a central parallel of approximately 42° (standard parallels 41° and 43°), on the GRS80 ellipsoid (Stem, 1989a).

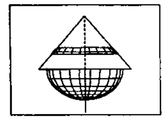
values of (t - T) and computational errors in δ determination for φ_o = 42° (in seconds of arc)					
$\varphi_1 - \varphi_o$		1°	2°	1°	2°
$\lambda_1 - \lambda_o$		5°	5°	10°	10°
True δ, azimuth =	90°	5.67	11.44	5.67	11.44
Error		0.00	0.02	0.03	0.11
True δ, azimuth =	135°	3.83	7.91	3.83	7.91
Error		0.00	0.01	0.02	0.08
True δ, azimuth =	180°	0.00	0.00	0.00	0.00
Error		0.00	0.00	0.00	0.00

Table 25: Values of computational errors in δ for φ_o = 42° from the LCC equations

The equations in this section are found in (Berry, 1970; Burkholder, 1985; Boucher, 1979; IGN, 1963, 1994)|[12] and (Floyd, 1985; Stem, 1989a), based on (Vincenty, 1985a, b, 1986a, b).

[12]Explanations in full, see (Boucher, 1979)

LCC - Applications



Countries like France with a predominantly east-west extent employ the Lambert conformal conical projection (LCC). Actually, the IGN (Institut Géographique National)-grid with two standard parallels is a Lambert grid with one standard parallel to which a grid scale constant has been applied (IGN, 1994). In 1993, IGN conceived a new RGF1993 permanent geodetic reference system, using the RBF network, connected to a GPS-station RGP-network, and to the old NTF triangulation network. It is intended for future use in the 2010s (Kasser, 2002).

Centesimal Units. The longitude of the Meridian of Paris, $2^g.59\ 68\ 98$ E of Greenwich, is used as the Datum for National surveys and maps. Here, geodetic coordinates - expressed in centesimal units - are used by LCC's IGN-grid for the civilian surveys in France since 1920 (Figure 80). IGN defined the Clarke 1880IGN ellipsoid uniquely (IGN, 1994):

Clarke 1880IGN			
a	semi-major axis	=	6 378 249.2000 m (unique)
b	semi-minor axis	=	6 356 515.0000 m (unique)
a / b		=	1.00341 92006 1|5431 56902 79972
1 / f		=	293.46602 12936 2|9395 14681 92986
f		=	0.00340 75495 20015 61|807 90176
e^2		=	0.00680 34876 46299 87|748 88800
e'^2		=	0.00685 00921 637|11 70567 63915
c		=	6 400 057.71359 001|59 12807 56829 80375

The adapted and validated algorithms are given in program listing [18.19], A_19LC00.FOR.

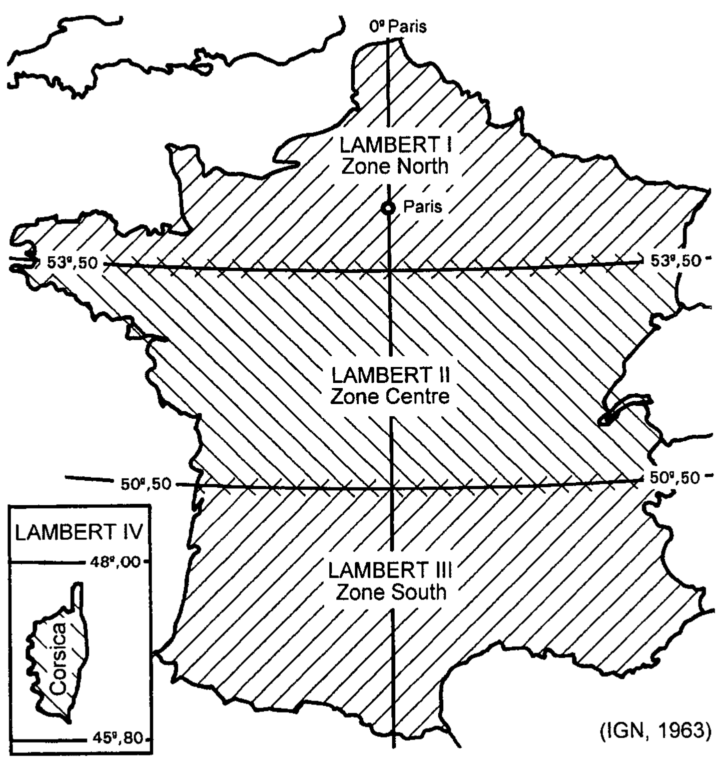


Figure 81: Lambert's conformal conical projection of France

LCC Application I

Lambert II - zone centre – France - two standard parallels

zone parameters		:	Lambert II, (Figure 81) (IGN, 1994)
reference ellipsoid		:	Clarke 1880IGN
semi-major axis	a	:	**6 378 249.2**
recipr. flattening	f^1	:	**293.46602 12936 294**
lower parallel	φ_l	:	**50^g.99 87 98849 354 N**
upper parallel	φ_u	:	**52^g.99 55 71668 931 N**
lat. grid origin	φ_b	:	**52^g.0 N**
central parallel	φ_0	=	52^g.00 00 00000 00353 N
lon. grid origin	λ_p	:	**0^g E of Paris**
scale factor	k_0	=	0.999 877 420 000
false easting	E_0	:	**600 000**
false northing	N_b	:	**200 000**

Conversion of IGN1922 Geographicals to Lambert Grid

Direct calculation is to convert geodetic coordinates into LCC-planar coordinates:

input: latitude longitude output: easting northing convergence scale factor

latitude	φ_1	:	**51^g.80 72 3130 N**	easting	E_1	=	632 542.0576
longitude	λ_1	:	**0^g.47 21 6690 E**	northing	N_1	=	180 804.1446
convergence	γ_1	=	+ 0^g.34 41 9486	scale factor	k_1	=	0.999 881 984 229

Conversion of Lambert Grid to IGN1922 Geographicals

Inverse process is to convert LCC-planar coordinates into geodetic coordinates:

input: easting northing output: latitude longitude convergence scale factor

easting	E_1	:	**632 542.0576**	latitude	φ_1	=	51^g.80 72 3130 N
northing	N_1	:	**180 804.1446**	longitude	λ_1	=	0^g.47 21 6690 E
convergence	γ_1	=	+ 0^g.34 41 9486	scale factor	k_1	=	0.999 881 984 229

Note

The N_b (false northing, or Y_o values) of the Lambert projection in France receive the prefixed numbers 1, 2, 3, and 4 for the Lambert Zones 1, 2, 3, and 4, respectively.

Thus N_b = 4 185 861.369 for Lambert IV, Corsica (IGN, 1986).

The "round-trip errors" are calculated for the area between latitude 49^g N to 55^g N and longitude 9^g W to 9^g E of Paris in (Hooijberg, 1997).

LCC Application II

Lambert SPCS83 - State Texas central – USA - two standard parallels

zone parameters		:	SPCS Texas, zone central
reference ellipsoid		:	GRS80
semi-major axis	a	:	**6 378 137**
recipr. flattening	f^{1}	:	**298.25722 21008 827** (Burkholder, 1984)
lower parallel	φ_l	:	**30° 07' 00" N**
upper parallel	φ_u	:	**31° 53' 00" N**
lat. grid origin	φ_b	:	**29° 40' 00" N**
central parallel	φ_0	=	31° 00' 05".00701 57360 N
lon. grid origin	λ_0	:	**100° 20' 00" W of Greenwich**
scale factor	k_0	=	0.999 881 743 629 290
false easting	E_0	:	**700 000**
false northing	N_b	:	**3 000 000**
unit		:	metre

Conversion of NAD83 geographicals to Lambert grid

Direct calculation is to convert geodetic coordinates into LCC-planar coordinates:

input: latitude longitude				output: easting northing convergence scale factor			
latitude	φ_1	:	**32° 00' 00" N**	easting	E_1	=	117 571.2278
longitude	λ_1	:	**106° 30' 00" W**	northing	N_1	=	3 274 824.8169
convergence	γ_1	=	- 3° 10' 34".3072	scale factor	k_1	=	1.000 033 424 208
latitude	φ_2	:	**31° 54' 15" N**	easting	E_2	=	127 581.7269
longitude	λ_2	:	**106° 23' 16" W**	northing	N_2	=	3 263 631.5509
convergence	γ_2	=	- 3° 07' 06".2234	scale factor	k_2	=	1.000 005 662 807

Conversion of Lambert grid to NAD83 geographicals

Inverse process is to convert LCC-planar coordinates into geodetic coordinates:

input: easting northing				output: latitude longitude convergence scale factor			
easting	E_1	:	**117 571.2278**	latitude	φ_1	=	32° 00' 00" N
northing	N_1	:	**3 274 824.8169**	longitude	λ_1	=	106° 30' 00" W
convergence	γ_1	=	- 3° 10' 34".3072	scale factor	k_1	=	1.000 033 424 208
easting	E_2	:	**127 581.7269**	latitude	φ_2	=	31° 54' 15" N
northing	N_2	:	**3 263 631.5509**	longitude	λ_2	=	106° 23' 16" W
convergence	γ_2	=	- 3° 07' 06".2234	scale factor	k_2	=	1.000 005 662 807

Lambert SPCS83 - State Texas central – USA - two standard parallels (cont'd)

Input:	easting	northing	Output:	arc-to-chord or (t - T) correction	
N_0: 3 147 960.7784 (calc.)					
E_1:	**117 571.2278**	E_2: **127 581.7269**	(t - T)	$\delta_{1\text{-}2}$ =	+2".9100
N_1:	**3 274 824.8169**	N_2: **3 263 631.5509**	(t - T)	$\delta_{2\text{-}1}$ =	- 2".8243

ellipsoidal and grid calculation (manual)

Using [18.12, 18.14] gives:		grid bearing	$t_{1\text{-}2}$	138° 11' 33".7429
		+ convergence	γ_1	- 3° 10' 34".3072
true distance $S_{1\text{-}2}$:	15 016.3439	- (t - T)	$\delta_{1\text{-}2}$	+0° 00' 02".9100
true azimuth $\alpha_{1\text{-}2}$:	135° 00' 56".5292	true azimuth	$\alpha_{1\text{-}2}$ =	135° 00' 56".5257
true azimuth $\alpha_{2\text{-}1}$:	315° 04' 30".3301			
		grid bearing	$t_{2\text{-}1}$	318° 11' 33".7429
grid distance $D_{1\text{-}2}$:	15 016.6340	+ convergence	γ_2	- 3° 07' 06".2234
true distance $S_{calc\ 1\text{-}2}$:	15 016.3437	- (t - T)	$\delta_{2\text{-}1}$	- 0° 00' 02".8243
See (9.03), using		true azimuth	$\alpha_{2\text{-}1}$ =	315° 04' 30".3438
scale factor $k_{1\text{-}2}$:	1.000 019 329 958			

Note

Documentation may be found on the Lambert conformal conical projection for the scale factor, the use of polynomial coefficients, and the arc-to-chord correction using this example (Vincenty, 1985b, 1986a, b).

Use of Polynomials

Conversions of coordinates with hand calculators from geodetic coordinates to Lambert grid coordinates and vice versa can be greatly simplified by using computed constants for a specific area. With 10 significant figures and some care in handling large numbers of the input, the method gives results correctly to the millimetre. The polynomial approach is simple in application, efficient in calculation, and sufficiently precise in zones of wide north-south extent.

Use of polynomial coefficients in conversions of coordinates on the LCC projection method, and the constants for the ConUS (Contiguous United States and Alaska) can be found in (Stem, 1989a). Formulae are given in (Pearson, 1990).

Observe

Coordinates of historical data points must be taken at face value, with the realisation that such co-ordinates could be significantly in error. In practice, exact coordinates in any reference system are not obtained (Floyd, 1985).

10.2 Gauss-Schreiber Conformal Double Projection

The Gauss conformal double projection is referred to as the Gauss-Schreiber (GS) type projection (Schreiber, 1866, 1897). The Gauss-Schreiber double map-projection- first employed by the geodesists Gauss and Schreiber - is identical to the transverse-Mercator map-projection. It is derived by a conformal transformation from the rotational ellipsoid to an auxiliary Gaussian sphere and then by another conformal transformation to the plane.

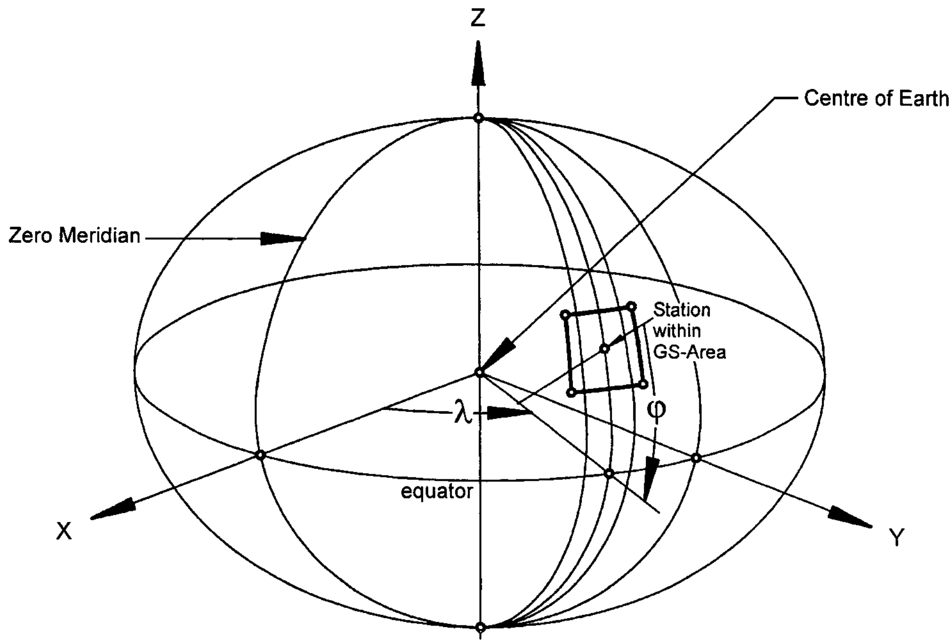


Figure 82: Gaussian sphere / ellipsoid mapping system

It is also called the Gauss-Laborde map projection, or the Gauss-Schreiber map projection. Using elementary considerations, the *Projection Cylindrique Orthomorphe* was invented by Lambert, though Carl F. Gauss gave the analytic derivation later in a paper presented to the Academy of Sciences of Copenhagen in 1822, and published by Schumacher, (1825). Gauss demonstrated the conformal representation of one surface upon another as a particular case of his general theory. This conformal map projection has become of the greatest importance to pure mathematics.

In 1866, eleven years after the death of Gauss, Oscar Schreiber published an account by Gauss of the projection in the Survey of Hannover, *Theorie der Projektionsmethode der Hannover'schen Landesvermessung.*

Schreiber's Projection Story

Theoretical speculation about the Figure of the Earth gave way to serious attempts to determine it by measurement. Gauss's name is most popularly associated with astronomy and mathematics, but his contribution to the discipline was far greater in the application of Geodesy, and led into territory where few could follow.

> *When Gauss died, the scientific world had not yet caught up with his ideas. Even today, the scientific high society has not fully explored the Kingdom created by his thoughts.*

Gauss himself was very much interested in the practical side of his profession. In 1816, it was a very hazy area when he began. Gauss was closely involved in the work of the Survey, particularly with the observations for determining the shape of the Earth. Using Walbeck's Figure of Earth, he measured an arc between the observatories of Altona and Göttingen in Germany. It was accurate enough to connect it to the Dutch triangulation network in 1824 (Hooijberg, 1997).

Although by 1827 the primary triangulation of Gauss covered the Kingdom of Hannover, the old projection would have been unsatisfactory if it was used to cover the whole Kingdom. Therefore the Survey Committee's Recommendations made it necessary to adopt a new projection, a fact recognised at that time by Gauss. It was decided to extend the survey across the whole Kingdom of Hannover. The survey was continued until 1844.

In spite of this, a representation of Gauss' theory of this projection was not possible. Gauss himself intended but did not publish a textbook due to political reasons. Wittstein (1866) concluded the Survey of Hannover provided at least an impression of the framework of Gauss' intentions.

The arithmetic involved was so laborious that errors were almost unavoidable. To finish the work of Gauss demanded a certain level of mathematical ability. Army commander Oscar Schreiber was a perceptive mathematician and very gifted engineer, who had many of the qualities needed. Schreiber was perfectly equipped to lead the Topographical Services working on the frontiers of technology. He combined vision and inventiveness, and he completed the work - begun by Gauss - between 1860 and 1866. This, he achieved in remarkable measure. Not only did he remodel the calculating procedure, but he also studied ways of perfecting the methods used and directing the computations.

Conversion from Ellipsoid to Gaussian Sphere

The Mercator projection uses the equator as a straight line. In the projection of Gauss, the Earth is established as an ellipsoid of revolution. On the contrary - Gauss projected the central meridian projected as a straight line. The main property, viz., the conformality in the smallest parts with the projected lines has both projections in common.

The original Gauss-Schreiber (GS) type projection system devised by Gauss projects the ellipsoid conformally upon the plane by the series of conformal projections:

1. conformal conversion from an ellipsoid to the Gaussian sphere and vice-versa [18.16]
2. conformal projection from the Gaussian sphere to the plane (not used in this book)

Oscar Schreiber recovered with great pains the original Altona-Göttingen documents, and hints carefully at various fragmentary documents, bringing the analytical developments again completely into live, using e.g. Gauss' authoritative works, such as:

- Allgemeine Auflösung der Aufgabe: *Die Theile einer gegebenen Fläche auf einer andern gegebenen Fläche so abzubilden, dass die Abbildung dem Abgebildeten in den kleinsten Theilen ähnlich wird* (Gauß, 1822)
- Correspondence between C.F. Gauss and H.C. Schumacher, published by C.A.F. Peters, Altona, 1860
- *Untersuchungen über Gegenstände der höhern Geodäsie, Erste Abhandlung Gauß' Werke*, Band IV (Gauß, 1843) and:
- *Geodätische Tafeln, et cetera,* by Taaks, Aurich (1865), (Marek, 1875).

Sexagesimal Formulae in Radian Measure

Oscar Schreiber completed his sexagesimal formulae in seconds. In this book, all formulae are in radian measure, ready for use by digital computers. A large part of Schreiber's volume *Theorie der Projektionsmethode der Hannover'schen Landesvermessung* deals with work and recommendations which was of the greatest international importance in geodesy, because the character of the projection changed to meet the varying demands made on it (Schreiber, 1897).

Observations

Originally, the equations for the Gauss- Schreiber projection should be limited to a region bounded by latitude between 47° 26' and 55° 24' N, and longitude between 23° 30' and 40° 33' E of Ferro, using the central meridian (CM) 31° E of Ferro. The Gauss- Schreiber and the Gauss-Krüger projection limit the latitudinal and longitudinal distances from the CM, but will depend on the purpose of the projection (Figure 82).

The Gauss-Schreiber Grid of 1866 uses the origin of the Observatory of Göttingen ("Göttinger Sternwarte"), and a different quadrant convention: latitude and longitude are reckoned to be positive south and west of the origin, respectively (Figure 83).

Due to different quadrant convention, all signs should be reversed to compare results computed by [18.18], Gauss-Krüger projection, with the original calculation (Hooijberg, 1997, Schreiber, 1866)

The Gauss-Krüger conformal projection system [10.4] converts the ellipsoid conformally upon the plane and vice-versa.

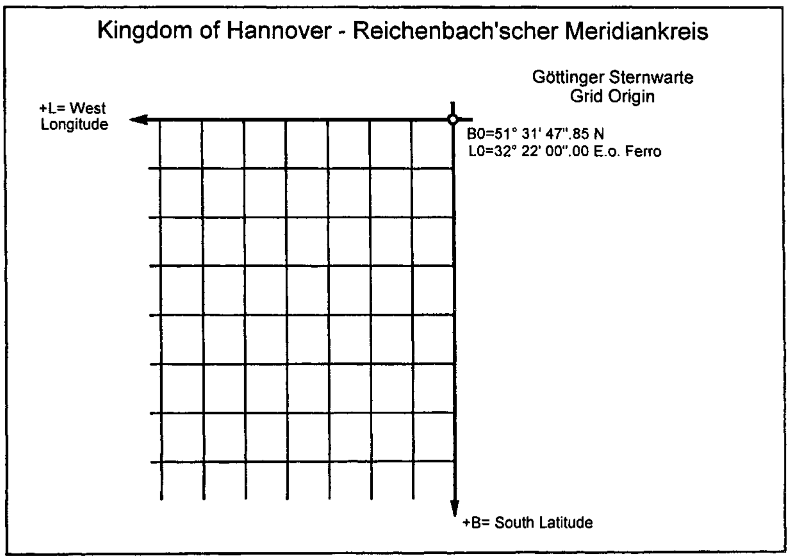


Figure 83: Gauss-Schreiber grid of the Hannover'schen Landesvermessung 1866

Meridian of Ferro

The Meridian of Ferro (Canary Islands, now Hierro) was established 20° West of Paris by Royal Act of King Louis the XIII of France in the year 1630. Germany and Austria accepted the Meridian of Ferro as 17° 39' 59".41, rounded to 17° 40' W of Greenwich (Gretschel, 1873).

In 1878, the use of Gauss-Schreiber's (GS) type of double projection system was changed according to international grid representation, essentially in use today.

Using the GS type projection presented here, the Gauss-Schreiber's formulae are used to calculate the ellipsoid-to-sphere projection and vice-versa. Use of the formulae are shown in area calculation [5.5] equations to compute the ellipsoidal area of any irregular (polygonal) surface in sq. km. See the figure ABCDEF (Figure 44): pp 108 bounded by polygon. GS type projections are applied in [10.6] Rosenmund's oblique Mercator projection of Switzerland (1903), and in [10.7], Heuvelink's oblique stereographic projection of the Netherlands (Heuvelink, 1918; Schreiber, 1897).

Surfaces of Constant Curvature

The Gaussian sphere and the Aposphere are called surfaces of constant curvature.

Gauss conceived the so-called Gauss-Schreiber (GS) type conformal double projection: for conversion of coordinates from the ellipsoid to the Gaussian sphere, and then from the sphere to the plane

Martin Hotine mentioned the aposphere, a sophisticated surface whose total curvature, R, is the same at all points, attributing the name to C. J. Sisson of London University. The ellipsoid is projected conformally upon the plane by conformal triple projection: from the ellipsoid to the aposphere, then from the aposphere to the sphere, and then finally from the sphere to the plane.

Gauss-Schreiber Mapping Equations

Symbols and definitions

The equations for the ellipsoid constants are given (Table 19): pp 123.
All angles are expressed in radians [18.16], A_16GAUS.FOR

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
f	flattening of the ellipsoid
B_0	φ_0 - latitude origin of ellipsoid, positive north
L_0	λ_0 - longitude origin of ellipsoid, positive east
b_0	latitude of the sphere
l_0	longitude of the sphere
B	latitude of the ellipsoid
L	longitude of the ellipsoid
b	latitude grid of the sphere
l	longitude grid of the sphere
Δb	b - b_0 - difference in latitude of the sphere
e	first eccentricity
e^2	first eccentricity squared
e'^2	second eccentricity squared
α	Gauss' dilatation constant
M	modulus of common logarithm
R_{00}	radius of curvature in the Meridian
R_{90}	radius of curvature in the prime vertical
R_A	radius of the Gaussian sphere

Ellipsoid Parameters

$$e^2 = f(2 - f) \quad (10.02.01)$$
$$e'^2 = e^2 / (1 - e^2) \quad (10.02.02)$$

Origin Constants for Gauss-Schreiber Projection

Compute constants of the origin for ellipsoid to sphere:

$$\alpha = (1 - e^2 \sin^2 \varphi_0)^{½} / (1 - e^2)^{½} \cos \varphi_0) / \cos B_0 \quad (10.02.03)$$
$$\alpha = (1 + e^2 / (1 - e^2)(1 - \alpha^2 \sin^2 (b_0))^2)^{½} \quad (10.02.04)$$
$$r = a(1 - e^2)^{½} / (1 - e^2 \sin^2 B_0) \quad (10.02.05)$$

Input: latitude and longitude of ellipsoid of origin (B_0, L_0)
Output: latitude and longitude of sphere of origin (b_0, b_0)

s_0	$= \sin B_0$	(10.02.06)
s_2	$= \sin^2 B_0$	(10.02.07)
c_0	$= \cos B_0$	(10.02.08)
c_2	$= \cos^2 B_0$	(10.02.09)
t_4	$= c_2^2 e'^2$	(10.02.10)
uu	$= t_4 / 2 (1 + t_4 (- 1/2 + t_4 (1/3 + t_4 (- 1/4 + t_4 (+ 1/5 + t_4)))))$	(10.02.11)
vv	$= s_0 / c_0\, uu (1 - 1 / (2 c_2)\, uu + (1 + 2 s_2) / (6 c_2^2)\, uu^2 - (1 + 10 s_2 + 4 s_2^2) / (24 c_2^3)\, uu^3 + (1 + 36 s_2 + 60 s_2^2 + 8 s_2^3) / (120 c_2^4)\, uu^4 - (1 + 116 s_2 + 516 s_2^2 + 296 s_2^3 + 16 s_2^4) / (720 c_2^5)\, uu^5)$	(10.02.12)
b_0	$= \arcsin (\sin B_0 / \exp (uu))$	(10.02.13)
l_0	$= L_0$	(10.02.14)

Compute constants of the origin for sphere to ellipsoid:

Input: latitude and longitude of origin - sphere (b_0, l_0)
Output: latitude and longitude of origin - ellipsoid (B_0, L_0)

s_1	$= \sin b_0$	(10.02.15)
s_2	$= \sin^2 b_0$	(10.02.16)
c_1	$= \cos b_0$	(10.02.17)
c_2	$= \cos^2 b_0$	(10.02.18)
t_4	$= e'^2 c_2 s_2$	(10.02.19)
u	$= t_4 / 2 (1 + t_4 (-3 / 2 + t_4 (10 / 3 + t_4 (- 35 / 4 + t_4 (126 / 5 + t_4 (- 77 + 1716\, t_4 / 7))))))$	(10.02.20)
v	$= c_1 / s_1\, u (1 + u / (2 s_2) (- 1 + (1 + 2 c_2) / (3 s_2)\, u) - (1 + c_2 (10 + 4 c_2)) / (24 s_2^3)\, u^3 + (1 + c_2 (36 + c_2 (60 + 8 c_2))) / (120 s_2^4)\, u^4 - (1 + c_2 (116 + c_2 (516 + c_2 (296 + 16 c_2)))) / (720 s_2^5)\, u^5)$	(10.02.21)
B_0	$= \arccos (\cos b_0 - \cos b_0\, u (1 + u / 2 (-1 + u / 3 (1 + u / 4 (- 1 + u / 5)))))$	(10.02.22)
L_0	$= l_0$ (= longitude of origin)	

Common Constants:

α	$= (1 + e'^2 \cos^4 B_0)^{1/2}$	(10.02.23)
s_0	$= \sin B_0$	(10.02.24)
s_2	$= \sin^2 B_0$	(10.02.25)
c_0	$= \cos B_0$	(10.02.26)
c_2	$= \cos^2 B_0$	(10.02.27)
M	$= 1 / \ln (10)$	(10.02.28)
R_A	$= a (1 - e^2)^{1/2} / (1 - e^2 \sin^2 B_0)$	(10.02.29)

Compute constants of the origin for sphere to ellipsoid:

Input: latitude and longitude - sphere (b, l)
Output: latitude and longitude - ellipsoid (B, L)

Δb	$= b - b_0$	(10.02.30)
w	$= (1 + e'^2 c_2)^{1/2}$	(10.02.31)
A_1	$= (w - 1)\, \Delta b$	(10.02.32)
A_2	$= - 3 e'^2 s_0 c_0 / 2\, \Delta b^2$	(10.02.33)
A_3	$= e'^2 / (6 w)\, \Delta b^3 (3 - 6 c_2 + e'^2 3 c_2 (5 - 6 c_2))$	(10.02.34)
T_4	$= 16 + e'^2 ((- 45 + 118 c_2) + e'^2 3 c_2 (- 35 + 54 c_2))$	(10.02.35)
A_4	$= T_4 e'^2 s_0 c_0 / (24 w^2)\, \Delta b^4$	(10.02.36)
A_5	$= e'^2 / (120 w^3)\, \Delta b^5 (4 ((- 3 + 7 c_2) + e'^2 (45 + c_2 (- 150 + 161 c_2))) + e'^4 10 c_2 (63 + 10 c_2 (- 27 + 22 c_2)) + e'^6 3 c_2^2 (315 + 8 c_2 (- 118 + 81 c_2)))$	(10.02.37)

A_6	$= e'^2 s_0 / (720 w^4 c_0) \Delta b^6 (4 ((3 - 17 c_2) + e'^2 c_2 (339 - 755 c_2)) + e'^4 c_2 (- 1575 + 4 c_2 (4215 - 5296 c_2)) + e'^6 2 c_2^2 (- 4725 + 2 c_2 (11823 - 11218 c_2)))$	(10.02.38)
A_7	$= e'^2 / (5040 w^5 c_2) \Delta b^7 (4 ((15 - 29 c_2^2) + e'^2 c_2 (- 357 + c_2 (3403 - 3476 c_2))) + e'^4 c_2 (1575 + 2 c_2 (- 34335 + 2 c_2 (58100 + c_2 - 43217 c2))) + e'^6 c_2^2 (42525 + c_2 (-486570 + c_2 (1107988 - 676872 c_2)))))$	(10.02.39)
A_8	$= e'^2 s_0 / (40320 w^6 c_0^3) \Delta b^8 (12 (30 + c_2 (-21 + 23 c_2)) + 24 (e'^2 c_2 (-18 + c_2 (-1147 + c_2 (2481)))) + e'^4 (4 c_2^2 (38430 + c_2 (- 287367 + 324841 c_2)))))$	(10.02.40)
A_9	$= e'^2 (362880 w^7 c_2^2) \Delta b^9 (2520 + c_2 (- 3420 + c_2 (816 + 468 c_2)) + e'^2 c_2 (-3240 + c_2 (40212 + c_2 (- 262980 + 257892 c_2))) + e'^4 c_2^2 (- 149040 + c_2 (4271364 + c_2 (- 12998488 + 9211028 c_2)))))$	(10.02.41)
B	$= b + (B_0 - b_0) + A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9$	(10.02.42)
L	$= l - (\alpha - 1) / \alpha (l - l_0)$	(10.02.43)

Compute constants of the origin for ellipsoid to sphere:

Input: latitude and longitude of origin - ellipsoid (B, L)
Output: latitude and longitude of origin - sphere (b, l)

s_0	$= \sin B$	(10.02.44)
s_2	$= \sin^2 B$	(10.02.45)
c_0	$= \cos B$	(10.02.46)
c_2	$= \cos^2 B$	(10.02.47)
k_1	$= - (\alpha - 1) M \ln (\tan (\pi /4 + B_0 / 2)) - M / c_0 vv (1 + vv (- s_0 / (2 c_0) + vv ((1 + s_2) / (6 c_2) - (5 s_0 + s_0^3) / (24 c_0^3) vv)) + (5 + s_2 (18 + s_2)) / (120 c_2^2) vv^4 - s_0 (61 + s_2 (58 + s_2)) / (720 c_0^5) vv^5)$	(10.02.48)
k_2	$= M \alpha e^2 s_0 (1 + s_2 e^2 (1/3 + s_2 e^2 (1/5 + s_2 e^2 (1/7 + e^2 / 9 s_2))) + e^{10} s_2^5 (1 / 11 + e^2 / 13 s_2))$	(10.02.49)
k_l	$= k_1 + k_2$	(10.02.50)
mz	$= M (\alpha e^2 s_0 (1 + s_2 e^2 (1/3 + e^2 / 5 s_2) + s_2^3 e^6 (1/7 + 1/9 e^2 s_2)) - (\alpha - 1) \ln (\tan (\pi /4 + B_i / 2))) - k_l$	(10.02.51)
z	$= mz / M$	(10.02.52)
nz	$= z c_0 (1 + z s_0 / 2 + z^2 / 6 (1 - 2 c_2) + z^3 / 24 (1 - 6 c_2) s_0)$	(10.02.53)
b	$= B - nz$	(10.02.54)
l	$= \alpha (L - L_0) + L_0$	(10.02.55)

GS - Application

Gauss-Schreiber's ellipsoid to sphere conversion and vice-versa

parameters for:			Germany-Göttingen				
reference ellipsoid		:	Bessel 1841	latitude	b_0 :	**52° 40' N**	of Equator
semi-major axis	a	:	**6 377 397.155**	longitude	l_0 :	**31° 00' E**	of Ferro
recipr. flattening	f^1	:	**299.15281 285**				

Conversion of Origin Geographicals

input:	latitude and longitude of sphere	output:	latitude and longitude of ellipsoid

latitude	b_0 :	**52° 40' 00" N**	latitude	B_0 =	52° 42' 02".53251 56394 53976 N
longitude	l_0 :	**31° 00' 00" E**	longitude	L_0 =	31° 00' 00" E

or

input:	latitude and longitude of ellipsoid		output:	latitude and longitude of sphere	
latitude	B_0 :	**52° 42' 02".53251 56394 53976 N**	latitude	b_0 =	52° 40' 00" N
longitude	L_0 :	**31° 00' 00" E**	longitude	l_0 =	31° 00' 00" E
radius	R=	6 383 037.5644			
dilatation	α=	1.00045 29181 18262			

Conversion of Ellipsoid to Sphere Geographicals

forward process is to convert ellipsoid geographicals into sphere geographicals:

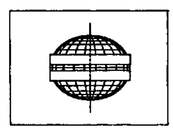
input:	latitude and longitude of ellipsoid		output:	latitude and longitude of sphere	
latitude	B_1:	**45° 24' 51".14736 N**	latitude	b_1=	45° 23' 37".31312 N
longitude	L_1:	**37° 12' 12".12357 E**	longitude	l_1=	37° 12' 22".23819 E
latitude	B_2:	**60° 59' 10".75429 N**	latitude	b_2=	60° 56' 51".63873 N
longitude	L_2:	**37° 00' 00".00000 E**	longitude	l_2=	37° 00' 09".78303 E

Conversion of Sphere to Ellipsoid Geographicals

inverse process is to convert sphere geographicals into ellipsoid geographicals:

Input:	latitude and longitude of sphere		output	latitude and longitude of ellipsoid	
latitude	b_1:	**45° 23' 37".31346 N**	latitude	B_1=	45° 24' 51".14736 N
longitude	l_1:	**37° 12' 22".23820 E**	longitude	L_1=	37° 12' 12".12358 E
latitude	b_2:	**60° 56' 51".63807 N**	latitude	B_2=	60° 59' 10".75429 N
longitude	l_2:	**37° 00' 09".78303 E**	longitude	L_2=	37° 00' 00".00000 E

10.3 Normal Mercator Projection



For mapping an area of considerable extent in longitude along the equator or parallel near the equator, a Meridional map projection system based on an east-west centre line - can be used.

Equatorial Projections

Some map projections are centred at a certain point on the Equator, and are called equatorial projections and serve as a basis for dialogue in this chapter:

- normal Mercator projection
- polycylindrical map projection

Owing to its unique properties the normal Mercator projection has reached a status that is well established and is used widely for naval navigational charts. The projection was devised by Gerardus Cremer (Latin surname: Mercator), and described in 1569. Cremer was born in Rupelmonde, Flanders (now Belgium). Mercator, graduated at Louvain university, was a proficient copperplate engraver, who made in his workshop scientific tools, such as astrolabes and globes. Particularly, Mercator's aim was to convert the curving rhumb line to a line of constant compass bearing - a straight line - enabling mariners to sail by following a fixed rule to their destinations (Wilford, 1981).

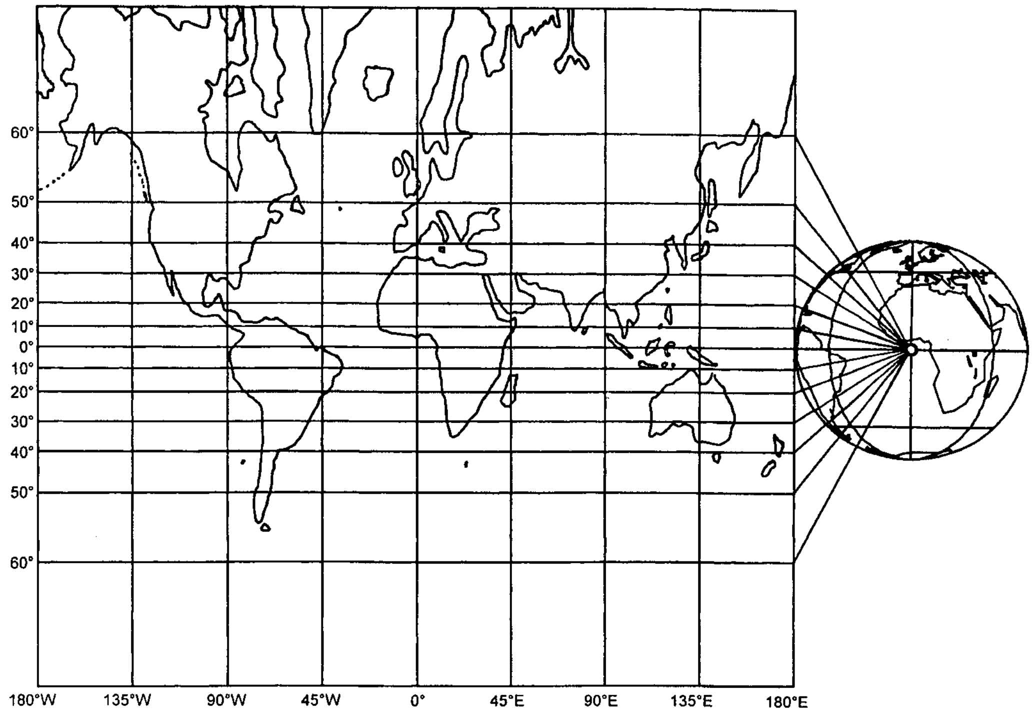


Figure 84: Perspective projection of the sphere

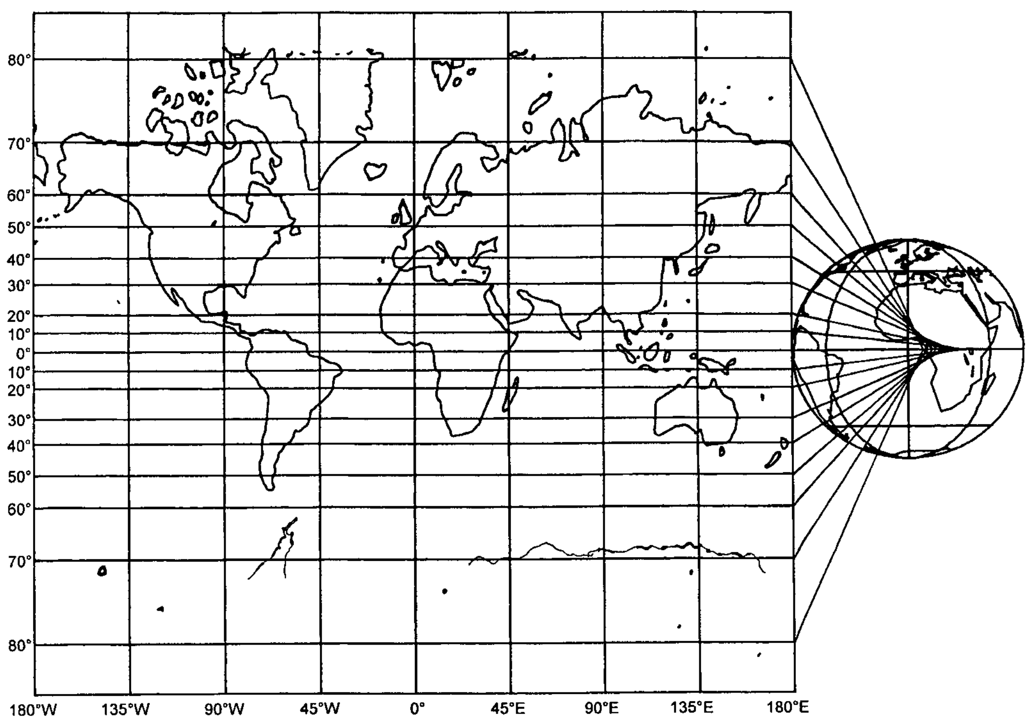


Figure 85: Mercator projection of the sphere

By the age of fifty-seven, Mercator published *the Map of the World*, the greatest achievement in cartographic history. Known as the Mercator projection, the map revealed the world, projecting the globe onto a flat plane. Nevertheless, it must be stated that his results were derived by inexact "*secant latitude*" graphical methods. Accurate values did not become available until logarithmic tables were developed by Edward Wright of Cambridge in 1590. In 1599, it was published in his treatise entitled "*Certaine Errors in Navigation*" (Adams, 1921; Verstelle, 1951; Wilford, 1981).

The projection provides a convenient working base for the mariner, excepting the high latitudes beyond 72° N and 72° S due to distortion. This is the case of the Mercator projection, belonging to a class of maps called conformal projections. The continuously increasing scale of the Mercator projection becomes pronounced as the spacing of meridional parts north and south of the Equator increase rather fast. Consequently, the Mercator projection is clearly unsuitable for high latitudes. Especially, the Poles are infinitely distant from the Equator and cannot be shown on the projection. A line of tangency (or lines of secancy) runs along the Equator.

In practice, projections are reduced to the plane surface with a minimum distortion by use of mathematical formulae. An advantage of the Mercator projection, with the polar or minor-axis of the sphere or ellipsoid in coincidence with the cylinder axis, is its simplicity. All meridians are represented by straight lines perpendicular to the straight line representing the Equator. Accordingly, the lines of latitude and the parallels of latitude would intersect each other at right angles.

In latitudes above 72° N or S, where the meridional parts of Mercator projection increase quite rapidly, charts covering considerable area may be constructed advantageously either on a LCC projection, if the locality has a predominating east-and-west extent, or on a polar stereographic (UPS) projection (Adams, 1921).

Misleading Statements

In literature, it is often stated that a projection can be visualised as a globe projected onto a cylinder (or cone) with tangency (or secancy) established at the Equator (Figure 84). For that reason, (Adams, 1921) said:

> "*It has frequently been stated in textbooks on geography that any geometric or perspective projection is projected upon a cylinder or cone with all the projecting lines radiating from the centre of the sphere or ellipsoid. It is misleading to speak of the tangent or secant cylinder and cone in connection with a geometric projection, to think of it as a perspective or geometric projection upon the cylinder with all the projecting lines radiating from a centre. Values required are not obtained by a simple formula*".

For that reason, it is better to deal with it as a projection upon a plane, which must be derived by mathematical analysis (Figure 85).

NM Mapping Equations

Symbols and definitions

The equations for the ellipsoid constants are given in (Table 19): pp 123.
All angles are expressed in radian measure [18.17], A_17NM00.FOR.

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
f	flattening of the ellipsoid
k_0	scale factor at the latitude of reference
k_e	scale factor at the Equator
φ_r	latitude of reference
φ_0	latitude of true origin, the Equator
λ_0	longitude of reference or true origin
E_0	false easting
N_0	false northing

Q	isometric latitude φ: parallel of geodetic latitude, positive north
λ	meridian of geodetic longitude, positive east
E	easting coordinate
N	northing coordinate
k	scale factor
e	first eccentricity
e^2	first eccentricity squared
e'^2	second eccentricity squared

Zone Constant

Constant and expression within the normal Mercator mapping equations is ellipsoid and zone specific:

$$k_i = k_e (1 - e^2 \sin^2 \varphi_r)^{1/2} / \cos \varphi_r \quad (10.03.01)$$

Direct Conversion Computation

Input: geodetic coordinates of a point P (φ, λ)
Output: grid coordinates of a point P_i (E, N), the scale factor (k)

$$Q = .5 [\ln \{ (1 + \sin \varphi) / (1 - \sin \varphi) \} - e \ln \{ (1 + e \sin \varphi) / (1 - e \sin \varphi) \}] \text{ or} \quad (10.03.02)$$
$$Q = \ln [\{ (1 + \sin \varphi) / (1 - \sin \varphi) ((1 - e \sin \varphi) / (1 + e \sin \varphi))^e \}^{1/2}] \quad (10.03.03)$$
$$L = (\lambda - \lambda_0)$$
$$E = a L k_e + E_0 \quad (10.03.05)$$
$$N = a Q k_e + N_0 \quad (10.03.06)$$
$$k = k_e (1 - e^2 \sin^2 \varphi_i)^{1/2} / \cos \varphi_i \quad (10.03.07)$$

Recognise in (10.03.02, 10.03.03): $\ln (\tan ((\pi / 4) + (\varphi / 2))) = ½ \ln ((1 + \sin (\varphi)) / (1 - \sin (\varphi)))$ (Gretschel, 1873: pp 117; Le Pape, 1994) ... (10.01.10).

Inverse Conversion Computation

Input: grid coordinates of a point P (E, N)
Output: geodetic coordinates of a point P (φ, λ), the scale factor (k)

$$\Delta E = (E - E_0) \quad (10.03.08)$$
$$\Delta N = (N - N_0) \quad (10.03.09)$$
$$X = 1 / \exp [\Delta N / (a k_e)] \quad (10.03.10)$$

Computation of latitude φ is iterative with the approximation:

$$\varphi_t = (.5 \pi - 2 \arctan (X) \quad (10.03.11)$$
$$\varphi = (.5 \pi - 2 \arctan [X (\{ (1 - e \sin \varphi_t) / (1 + e \sin \varphi_t) \}^e)^{1/2}] \quad (10.03.12)$$

Replace φ_t by the corrected value of φ, and iterate if [abs $(\varphi_t - \varphi) > 1.10^{-15}$] before obtaining φ

$$\lambda = \lambda_0 + \Delta E / (a k_e) \quad (10.03.13)$$
$$k = k_e (1 - e^2 \sin^2 \varphi_i)^{1/2} / \cos \varphi_i \quad (10.03.14)$$

Arc-to chord correction $\delta = (t - T)''$

Input: geodetic coordinates of points P_1 (φ_1, λ_1) and P_2 (φ_2, λ_2)
Output: correction δ_{1-2} (t - T) for the line from point P_1 to P_2

$$\Delta\varphi = .5 (\varphi_2 - \varphi_1) \quad (10.03.15)$$
$$\Delta\lambda = .5 (\lambda_2 - \lambda_1) \quad (10.03.16)$$
$$\delta_{1-2} = \arctan (\sin ((2 \varphi_1 + \varphi_2) / 3)) \tan \Delta\lambda / \cos \Delta\varphi) \quad (10.03.17)$$

The equations in this section are found in (Adams, 1921; Floyd, 1985; Snyder, 1987; Thomas, 1952).

NM - Application

Bessel's Figure of the Earth for Indonesia (Hager, 1990):

Indonesian normal Mercator equatorial zone grid has the following specifications:

- projection : normal Mercator
- reference ellipsoid : Bessel 1841
- zone : between 0° S and 8° N
- latitude of origin φ_0 : 0° N ______ Equator
- longitude of origin λ_0 : 110° E
- unit : metre
- false northing N_0 : 900 000 m ______ south of true origin
- false easting E_0 : 3 900 000 m ______ west of true origin
    - latitude of false origin 8° 08' S ______ (approx. value)
    - longitude of false origin 74° 51' E ______ (approx. value)
- scale factor k_e : 0.997 ______ at latitude of reference

Equatorial zone grid of the Republic of Indonesia					
zone parameters		:	Equatorial	e	= 8.169683121571159E-02
reference ellipsoid		:	Bessel 1841	e^2	= 6.674372230688467E-03
semi-major axis	a	:	**6 377 397.155**	e'^2	= 6.719218798046066E-03
recipr. flattening	f^1	:	**299.15281 285**	k_e	= 0.997 ____ at the Equator
scale factor	k_0	:	**0.997**	Q	= 0.0 ___ isometric latitude
latitude of reference	φ	:	**0° 00' N**		
longitude centre	λ_c	:	**110° 00' E**	at station P_0:	
false easting,	E_0	:	**3 900 000**	Q	= - 5.05878 19452 45878E-02
false northing	N_0	:	**900 000**	X	= 1.05188 92356 34597
input unit		:	m		

Conversion of Bukit Rimpah geographicals to normal Mercator grid

direct calculation is to convert geodetic coordinates into NM-planar coordinates:

input:		latitude	longitude	output:	easting	northing	scale factor
latitude	φ_1	:	**2° 55' 00".0000 S**	easting	E_1	=	3 669 432.4160
longitude	λ_1	:	**107° 55' 20".2900 E**	northing	N_1	=	578 349.2400
				scale factor	k_1	=	0.998 284 565 271

Conversion of normal Mercator grid to Bukit Rimpah geographicals

inverse process is to convert NM-planar coordinates into geodetic coordinates:

input:		easting	northing	output:	latitude	longitude	scale factor
easting	E_1	:	**3 669 432.4160**	latitude	φ_1	=	2° 55' 00".0000 S
northing	N_1	:	**578 349.2400**	longitude	λ_1	=	107° 55' 20".2900 E
				scale factor	k_1	=	0.998 284 565 271

The scale factor is shown as a function of latitudinal zone widths between 0° and 8° N in (Figure 86), and between 0° and 75° N in (Figure 87).

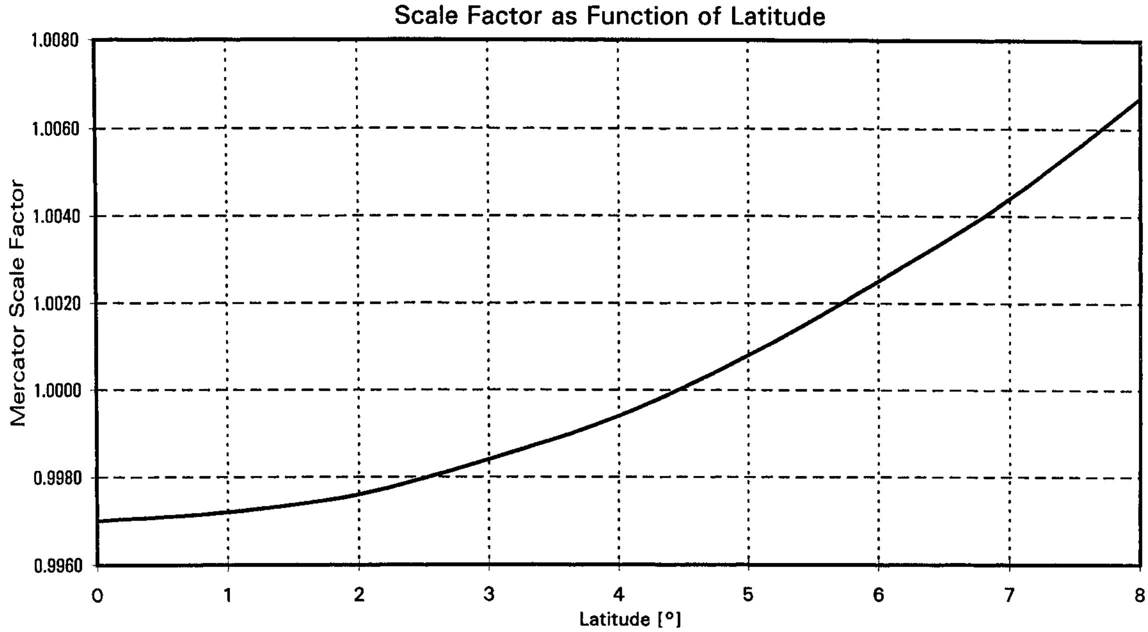


Figure 86: Scale factor as a function of 8° latitudinal zone width

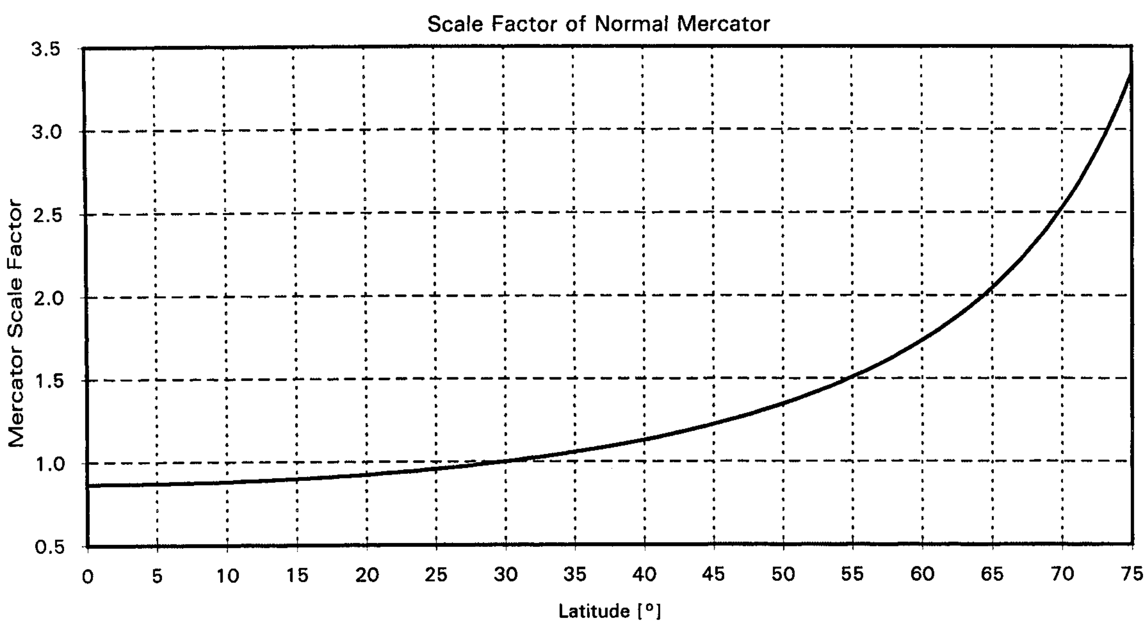


Figure 87: Scale factor as a function of 75° latitudinal zone width

Characteristics of the normal Mercator projection

- tangency — one standard parallel at the Equator or:
- secancy — two standard parallels equidistant from the Equator
- meridians — equally spaced parallel straight lines
- parallels — are unequally spaced parallel straight lines, closest near the Equator
- scale factor — tangency: ________________________ increases away from the Equator or:
- scale factor — secancy: _______ increases outward from secancy, decreases toward Equator
- rhumb line — represented by a straight line on the sphere - imperative for naval navigation
- great circle — represented by a curved line on the sphere - imperative for naval navigation
- convergence — does not exist

The scale factor is shown as a function of latitudinal zone widths between 0° and 8° N in (Figure 86), and between 0° and 75° N in (Figure 87).

Optimising Mercator Projections

Of great value is the report of Grafarend about optimising Mercator projections in which the Airy optimality criterion of a least average distortion is discussed (Grafarend, 1998c).

Proposed Polycylindric Conformal Mapping

Latitude and longitude of the Origin or Datum must be known in geodetic terms for the derivation of formulae. The polycylindric mapping projection subject follows here.

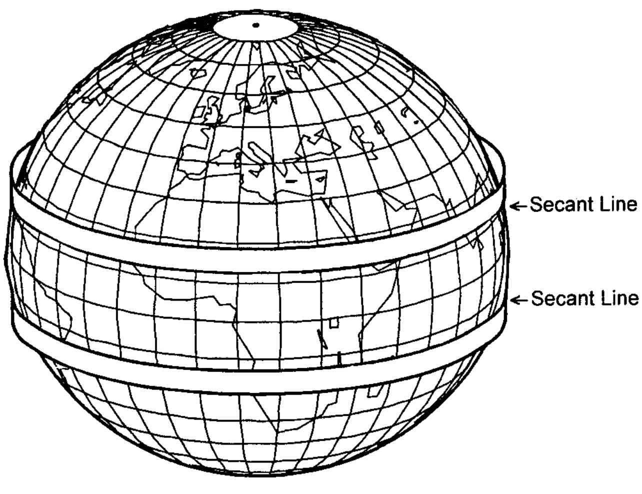


Figure 88: Normal Mercator with two secant lines

An academic view about a solution may be found by application of the normal Mercator (NM) projection: the polycylindrical projection with a family of secant equidistant reference lines - parallels of latitude - running parallel to the Equator (Figure 88).
The true origin is defined at the intersection of the longitude of the reference Meridian and the latitude of the secant line of reference. As mentioned before, the scale along the latitude of the line of reference is constant.
The polycylindrical projection is conformal, so point grid scale factors are calculated by the program.
True eastings and northings are reckoned from the true origin. Grid coordinates are reckoned from the grid Origin, which is defined by a false easting and a false northing (GG, 1986).

In the report both the normal Mercator (NM) and the polycylindric Mercator (PM) projection are outlined as the optimum universal Mercator (UM) projection systems. The conformal NM projection maps the equator equidistantly. So, it is used for areas where the area of interest lies with its longer dimension in an east-west direction along the equator.

A basis for discussion serves the equatorial territory of the Republic of Indonesia between 95°E and 142°E. Using WGS84, the report (Grafarend, 1998c) focuses on a case study - with a family of secant latitudinal zones between 8°N and 12°S - in which dissimilar optimised scale factors are found for both the Mercator and the Airy polycylindric projection of the conformal type. UM reveals its merits for those regions, which extend the latitude of parallel circles.

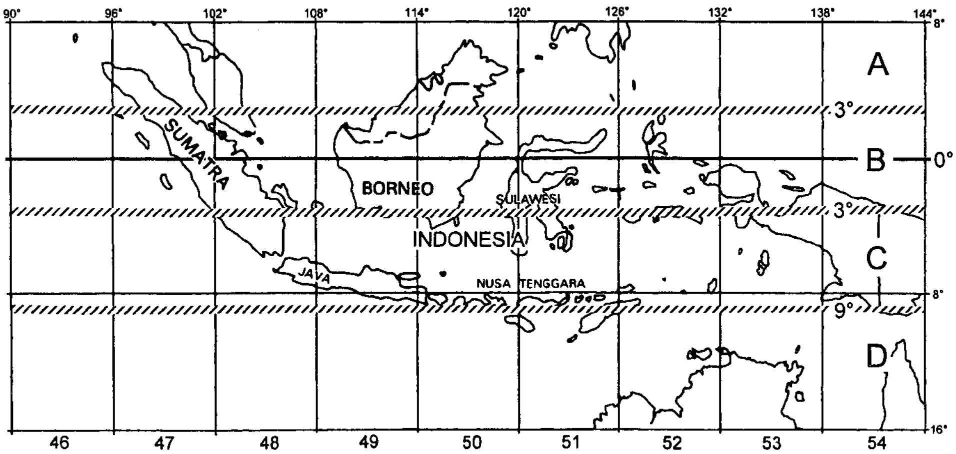


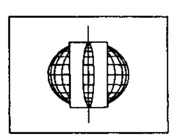
Figure 89: UM and UTM grid zones for the equatorial territory of Indonesia

Determining an optimisation criterion for the UM projection for various zone widths results in an optimum scale factor. An Airy optimum scale factor depends solely on the latitude of the reference line and on latitudinal distances of a finite zone bounded by parallel circles near or along the equator. It is independent of the longitudinal extension of the area. Furthermore, an optimum scale factor is a valuable tool of quality control. It sets the average areal distortion over the zone to zero, which is quite a desirable result of optimisation a map projection. For the equatorial territory of Indonesia (Figure 89), the scale factors of the zones A, B, C and D are calculated (Table 26):

zone	scale factor	central parallel	zone parallels
A	0.999536	6°N	3°N - 9°N
B	0.999546	0°	3°N - 3°S
C	0.999536	6°S	3°S - 9°S
D	0.999504	12°S	9°S - 15°S

Table 26: Proposed zones for the Indonesian Archipelago

10.4 Gauss-Krüger Conformal Projection



The transverse Mercator projection of the sphere was described by Lambert in 1772, and is occasionally known as the Lambert I projection (Lee, 1962). The first known appearance of the name "*transverse Mercator*" is found in Germain's *Traité des Projections*, Paris 1865. Gauss-Krüger (GK) is identical to the transverse Mercator conformal map projection. It is derived by mapping directly from the ellipsoid onto the plane.

Gauss-Krüger or Transverse Mercator Conformal Projection System

The GK-type projection, also called the Gauss conformal map projection in continental Europe - first employed by the geodesist Gauss - is used because of its longer dimension in a north-south direction. The abbreviation TM is used for Thematic Mapper (Landsat), and universal transverse Mercator (UTM) grid is a derivative of the general GK projection (Burton, 1996; Clarke, 1973; DA, 1958; Field, 1980; Grafarend, 1995h, 1998d; Greenfield, 1992; Hooijberg, 1997; Krüger, 1912, 1919; Lee, 1962; Meade, 1987; OS, 1950; 1995a; Rune, 1954; Vincenty, 1984a).

Origin and Central Meridian

The true origin of this projection is at the intersection of the Central Meridian (CM) with the Equator, and a grid origin may be established to the south and west of that area.

False Origin

The definition of the north-south location of the grid origin: if a false northing (Y) is specified rather than a latitude of false origin, the false origin is located at the intersection of the central meridian with the Equator. Then the false northing, or Y constant of the false origin, would be negative-valued in the Northern Hemisphere, moving the grid origin northwards.
In the Southern Hemisphere, the false northing would be positive-valued. The false easting is the constant assigned to the false origin. It would be positive in either hemisphere.

Region

The equations for the GK projection become unstable near the Poles. The GK projection should be limited to a region bounded by maximum latitude and a longitudinal distance from the central meridian, which will depend on the purpose of the projection.

GK Mapping Equations

Symbols and definitions

The equations for the ellipsoid constants are given in (Table 19): pp 123.
All angles are expressed in radians [18.18], A_18GK00.FOR.

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
f	flattening of the ellipsoid
k_o	grid scale factor assigned to the central meridian
φ_o	parallel of geodetic latitude grid origin
λ_o	central meridian (CM)
E_o	false easting (constant assigned to the CM)
N_o	false northing (constant assigned to the latitude of grid origin)
φ	parallel of geodetic latitude, positive north
λ	meridian of geodetic longitude, positive east
E	easting coordinate on the projection
N	northing coordinate on the projection
γ	meridian convergence
$\delta_{1\text{-}2}$	arc-to-chord correction (t - T), for a line from P_1 to P_2
k	point grid scale factor
$k_{1\text{-}2}$	line grid scale factor (P_1–P_2)
ω	rectifying latitude
S	meridional distance

S_o	meridional distance from the equator to φ_o, multiplied by the CM scale factor	
ΔN	$N_2 - N_1$ - difference in northings	(10.04.01)
ΔE	$E_2 - E_1$ - difference in eastings	(10.04.02)
E'	$E - E_o$	(10.04.03)
e^2	first eccentricity squared	
e'^2	second eccentricity squared	
n	second flattening	
R	radius of curvature in the prime vertical	
r_o	geometric mean radius of curvature scaled to the grid	
r	radius of the rectifying sphere	
t	tan φ, see formulae direct and inverse computation	
t	grid azimuth, see formulae (t - T)"	
η^2	$e'^2 \cos^2 \varphi$	

Constants for Meridional Arc

Compute constants for meridional arc as given below

$$c = a / (1 - e^2)^{1/2} \quad (10.04.04)$$

$$r = a (1 + n^2 / 4) / (1 + n) \quad \ldots (6.55)\ (\text{Krack, 1983}) \quad (10.04.05)$$

$$\begin{aligned} U_0 &= c [((((- 86625 / 8 e'^2 + 11025) / 64 e'^2 - 175) / 4 e'^2 + 45) / 16 e'^2 - 3) / 4 e'^2] \\ U_2 &= c [(((- 17325 / 4 e'^2 + 3675) / 256 e'^2 - 175 / 12) e'^2 + 15) / 32 e'^4] \\ U_4 &= c [- 1493 / 2 + 735 e'^2] / 2048 e'^6 \\ U_6 &= c [(- 3465 / 4 e'^2 + 315) / 1024 e'^8] \end{aligned} \quad (10.04.06)$$

$$\begin{aligned} V_0 &= ((((16384 e'^2 - 11025) / 64 e'^2 + 175) / 4 e'^2 - 45) / 16 e'^2 + 3) / 4 e'^2 \\ V_2 &= (((- 20464721 / 120 e'^2 + 19413) / 8 e'^2 - 1477) / 32 e'^2 + 21) / 32 e'^4 \\ V_4 &= ((4737141 / 28 e'^2 - 17121) / 32 e'^2 + 151) / 192 e'^6 \\ V_6 &= (- 427277 / 35 e'^2 + 1097) / 1024 e'^8 \end{aligned} \quad (10.04.07)$$

Meridional Arc formula

$$\omega_o = \varphi_o{}^{\circ} + \sin \varphi_o \cos \varphi_o (U_0 + U_2 \cos^2 \varphi_o + U_4 \cos^4 \varphi_o + U_6 \cos^6 \varphi_o) / r \quad (10.04.08)$$

$$S_o = k_o\, \omega_o\, r \quad (10.04.09)$$

Direct Computation

Input: geodetic coordinates of a point P (φ, λ)
Output: grid coordinates of a point P (E, N), convergence angle (γ), scale factor (k)

$$L = (\lambda - \lambda_0) \cos \varphi \quad (10.04.10)$$

$$\omega = \varphi^{\circ} + \sin \varphi \cos \varphi (U_0 + U_2 \cos^2 \varphi + U_4 \cos^4 \varphi + U_6 \cos^6 \varphi) / r \quad (10.04.11)$$

$$S = k_0\, \omega\, r \quad (10.04.12)$$

$$R = k_o\, a / (1 - e^2 \sin^2 \varphi)^{1/2} \quad (10.04.13)$$

$$\begin{aligned} A_1 &= - R \\ A_3 &= 1 / 6 (1 - t^2 + \eta^2) \\ A_5 &= 1 / 120 (5 - 18 t^2 + t^4 + \eta^2 (14 - 58 t^2)] \\ A_7 &= 1 / 5040 (61 - 479 t^2 + 179 t^4 - t^6) \end{aligned} \quad (10.04.14)$$

$$\begin{aligned} A_2 &= \tfrac{1}{2} R\, t \\ A_4 &= 1 / 12 [5 - t^2 + \eta^2 (9 + 4 \eta^2)] \\ A_6 &= 1 / 360 [61 - 58 t^2 + t^4 + \eta^2 (270 - 330 t^2)] \end{aligned} \quad (10.04.15)$$

$$E = E_o + A_1 L [1 + L^2 (A_3 + L^2 (A_5 + A_7 L^2))] \quad (10.04.16)$$

N	$= S - S_o + N_o + A_2 L^2 (1 + L^2 (A_4 + A_6 L^2)]$	(10.04.17)
C_1	$= - t$	
C_3	$= {}^1/_3 (1+3 \eta^2 +2 \eta^4)$	(10.04.18)
C_5	$= 1 / 15 (2 - t^2)$	
F_2	$= ½ (1 + \eta^2)$	
F_4	$= 1 / 12 [5 - 4 t^2 + \eta^2 (9 - 24 t^2)]$	(10.04.19)
γ	$= C_1 L [1 + L^2 (C_3 + C_5 L^2)]$	(10.04.20)
k	$= k_o [1 + F_2 L^2 (1 + F_4 L^2)]$	(10.04.21)

Inverse Computation

Input: grid coordinates of a point P (E, N)
Output: geodetic coordinates P (φ, λ) convergence angle γ, grid scale factor k

ω	$= (N - N_o + S_o) / (k_o r)$	(10.04.22)
φ_f	$= \omega + (\sin \omega \cos \omega) (V_0 + V_2 \cos^2 \omega + V_4 \cos^4 \omega + V_6 \cos^6 \omega)$	(10.04.23)
R_f	$= k_o a / (1 - e^2 \sin^2 \varphi_f)^{½}$	(10.04.24)
Q	$= E' / R_f$, in which $E' = E - E_o$	(10.04.25)
B_2	$= - ½ t_f (1+ \eta_f^2)$	
B_4	$= - 1 / 12 [5 + 3 t_f^2 + \eta_f^2 (1 - 9 t_f^2) - 4 \eta_f^4]$	(10.04.26)
B_6	$= 1 / 360 [61 + 90 t_f^2 + 45 t_f^4 + \eta_f^2 (46 - 252 t_f^2 - 90 t_f^4)]$	
B_3	$= - 1 / 6 (1 + 2 t_f^2 + \eta_f^2)$	
B_5	$= 1 / 120 [5 + 28 t_f^2 + 24 t_f^4 + \eta_f^2 (6 + 8 t_f^2)]$	(10.04.27)
B_7	$= - 1 / 5040 (61 + 662 t_f^2 + 1320 t_f^4 + 720 t_f^6)$	
φ	$= \varphi_f + B_2 Q^2 [1 + Q^2 (B_4 + B_6 Q^2)]$	(10.04.28)
L	$= Q [1 + Q^2 (B_3 + Q^2 (B_5 + B_7 Q^2))]$	(10.04.29)
λ	$= \lambda_o - L / \cos \varphi_f$	(10.04.30)
D_1	$= t_f$	
D_3	$= - {}^1/_3 (1+ t_f^2 - \eta_f^2 - 2 \eta_f^4)$	(10.04.31)
D_5	$= 1 / 15 (2+5 t_f^2 + 3 t_f^4)$	
G_2	$= ½ (1+ \eta_f^2)$	
G_4	$= 1 / 12 (1+5 \eta_f^2)$	(10.04.32)
γ	$= D_1 Q (1+Q^2 (D_3 +D_5 Q^2))$	(10.04.33)
k	$= k_o (1+G_2 Q^2 (1+G_4 Q^2))$	(10.04.34)

Arc-to-Chord Correction δ = (t - T)

Grid azimuth (t), geodetic azimuth (α), convergence angle (γ), and arc-to-chord correction (δ) at any given point are related as follows: $t = \alpha - \gamma + \delta$ ... (9.01).

Input: $P_1 (E_1, N_1)$, and $P_2 (E_2, N_2)$
Output: $\delta_{1\text{-}2}$

N_m	$= ½ (N_1 + N_2)$	(10.04.35)
ω	$= (N_m - N_o + S_o) / (k_o r)$	(10.04.36)
φ_f	$= \omega + V_0 \sin \omega \cos \omega$	(10.04.37)
F	$= (1 - e^2 \sin^2 \varphi_f) (1 + \eta_f^2) / (k_o a)^2$	(10.04.38)

$E_3 \quad = 2\,E'_1 + E'_2$ (10.04.39)

$\delta_{1\text{-}2} \quad = -\,1\,/\,6\,\Delta N\,E_3\,F\,(\,1 - (\,1\,/\,27\,)\,E_3^2\,F\,)$ (10.04.40)

Note

The computer subroutines were written in the most suitably economical form. Therefore, these basic equations may deviate from the subroutine listings in this book.

The equations in this section are found in (Krack, 1982; Hooijberg, 1996, 1997), and (Floyd, 1985; Stem, 1989a) based on (Meade, 1987; Vincenty, 1984a).

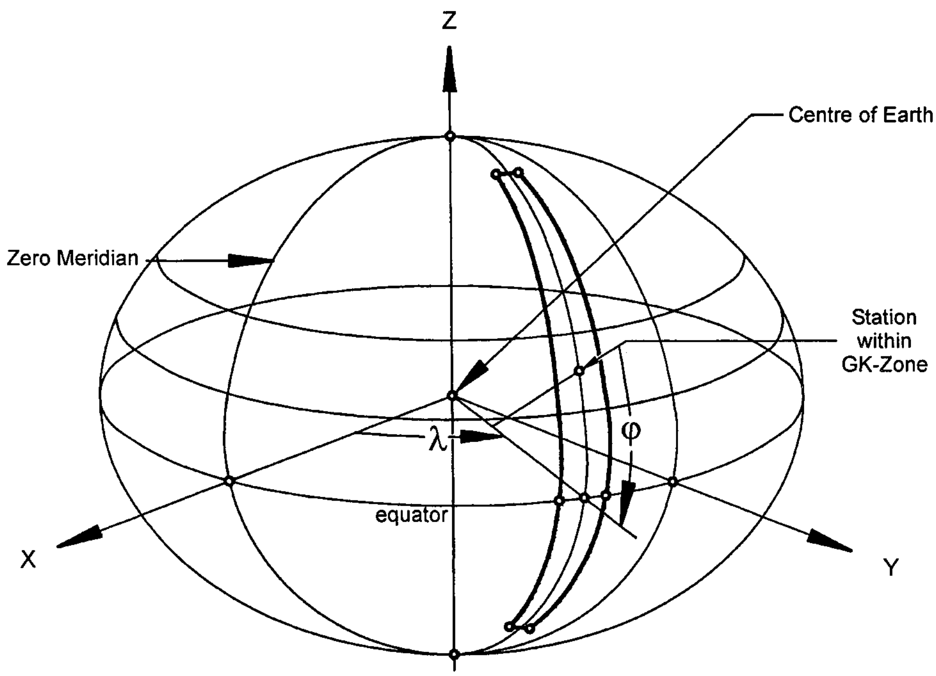


Figure 90: Transverse Mercator grid zone system

Boundaries of the GK-projection Systems

The Gauss-Schreiber (GS) projection system [10.2] should be limited to a region bounded by maximum latitude and a longitudinal distance from the central meridian (CM), which will depend on the purpose of the projection (Figure 82, pp 194).

The Gauss-Krüger (GK) conformal projection system should be limited to a region bounded by a longitudinal distance from the CM, which will depend on the purpose of the projection, zone system and its grid system as discussed in [9] (Figure 90).

GK - Application

OSGB36 is used for mapping in Great Britain. The re-triangulation of Great Britain has been computed on the same Airy ellipsoid, resulting in the transverse Mercator (GK) of Gt Britain in 1963 (MacDonald, 1991).

This application does not use OSGRS80, associated with GRS80

GK projection and grid system of Gt. Britain, excl. N. Ireland, has the following specifications:

projection		:	Transverse Mercator of *Great Britain,* (OS, 1950, 1995a)
reference ellipsoid		:	Airy (National projection grid only)
semi-major axis	a	:	**6 377 340.189000**
reciprocal flattening	f^{1}	:	**299.32496 459**
latitude of true origin	φ_0	:	**49° 00' 00"N** of the Equator
longitude of true origin	λ_0	:	**2° 00' 00"W** of Greenwich
unit		:	metre
false northing	N_0	:	**100 000.0** m north of true origin
false easting	E_0	:	**400 000.0** m west of true origin
scale factor at CM	k_0	:	**0.99960 12717**
the false origin, used by formulae			
latitude of false origin		=	49° 46' N of the Equator (approx.)
longitude of false origin		=	7° 33' W of Greenwich (approx.)

Conversion of OSGB36 geographicals to transverse Mercator grid

Direct calculation is to convert geodetic coordinates into GK-planar coordinates:

input: latitude longitude output: easting northing convergence scale factor

Sta. Framingham (OS, 1950, 1995a)

Latitude	φ_1:	**52° 34' 26".8915 N**	Easting	E_1=	626 238.2477
Longitude	λ_1:	**1° 20' 21".1080 E**	Northing	N_1=	302 646.4119
Convergence	γ_1=	+ 2° 39' 10".4691	Scale Factor	k_1=	1.000 229 694 988

Sta. Caister Water Tower

latitude	φ_2 :	**52° 39' 27".2531 N**	easting	E_2 =	651 409.9029
longitude	λ_2 :	**1° 43' 04".5177 E**	northing	N_2 =	313 177.2703
convergence	γ_2 =	+ 2° 57' 26".5561	scale factor	k_2 =	1.000 377 316 229

intermediate point

latitude	φ_m :	**52° 36' 57".6300 N**	easting	E_m =	638 824.0761
longitude	λ_m :	**1° 31' 42".1863 E**	northing	N_m =	307 911.8536
			scale factor	k_m =	1.000 301 560 388

Boundaries of the Transverse Mercator Projection

The transverse Mercator projection grid is derived by mapping directly from the ellipsoid onto the plane. Consequently, the algorithm used in the program will easily accommodate wide zones with submillimetre accuracy. An excellent example of such a region is the United Kingdom, see Great Britain, with a longitudinal difference of 11° width, approximately. Such accuracy is unimportant, but it can serve as a standard by which approximations may be judged (Bomford, 1977).

Conversion of Transverse Mercator Grid to OSGB36 Geographicals

inverse process is to convert GK-planar coordinates into geodetic coordinates:

input:	easting	northing	output:	latitude	longitude	convergence	scale factor

easting	E_1 :	**626 238.2477**	latitude	φ_1 =	52° 34' 26".8915 N
northing	N_1 :	**302 646.4119**	longitude	λ_1 =	1° 20' 21".1080 E
convergence	γ_1 =	+ 2° 39' 10".4692	scale factor	k_1 =	1.000 229 694 581
easting	E_2 :	**651 409.9029**	latitude	φ_2 =	52° 39' 27".2531 N
northing	N_2 :	**313 177.2703**	longitude	λ_2 =	1° 43' 04".5177 E
convergence	γ_2 =	+ 2° 57' 26".5561	scale factor	k_2 =	1.000 377 315 457
easting	E_m :	**638 824.0761**	latitude	φ_m =	52° 36' 57".6300 N
northing	N_m :	**307 911.8536**	longitude	λ_m =	1° 31' 42".1863 E
			scale factor	k_m =	1.000 301 559 823

ellipsoidal and grid calculation (manual)

Using [18.12, 18.14] gives:

true distance	$S_{1\text{-}2}$:	27 277.4892	grid bearing	$t_{1\text{-}2}$	67° 17' 50".7461
			+ convergence	γ_1	+ 2° 39' 10".4691
			- (t - T)	$\delta_{1\text{-}2}$	- 6".2563
true azimuth	$\alpha_{1\text{-}2}$:	69° 57' 07".4747	true azimuth	$\alpha_{1\text{-}2}$ =	69° 57' 07".4715
true azimuth	$\alpha_{2\text{-}1}$:	250° 15' 10".8182			
			grid bearing	$t_{2\text{-}1}$	247° 17' 50".7461
grid distance	$D_{1\text{-}2}$:	27 285.7326	+ convergence	γ_2	+2° 57' 26".5561
true distance	$S_{calc\ 1\text{-}2}$:	27 277.4891	- (t - T)	$\delta_{2\text{-}1}$	+6".4798
See (9.03), using			true azimuth	$\alpha_{2\text{-}1}$ =	250° 15' 10".8224
scale factor	$k_{1\text{-}2}$:	1.000 302 208 795			

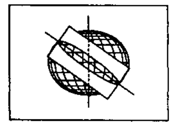
Airy 1830 reference ellipsoid

a	=	20 923 713 (*unique* in feet of Bar O_1)
b	=	20 853 810 (*unique* in feet of Bar O_1)
a - b	=	69 903
a	=	6 377 563.396|35 34083 m
b	=	6 356 256.909|58 9071 m
1 / f	=	299.32496|45937 94257 75717
f	=	0.00334 08506 41566 34|149
e^2	=	0.00667 05400 00123 42|876
n	=	0.00167 32203 10356 83|948
feet of Bar O_1	=	0.30480 07491

Note

Verification of historical data and information is not a function of a data-processing system. Coordinates of historical data points must be taken at face value, with the realisation that such coordinates could be significantly in error. In practice, exact coordinates in any reference system are not obtained. Starting with coordinates known to be correct in the reference system is the crux of the problem (Floyd, 1985).

10.5 Hotine's Oblique Mercator Projection



An Aposphere

For many purposes, an adequate geometric representation of the Earth is a sphere, for which only the radius (r) of the sphere must be given. The aposphere is called a surface of constant curvature. It is a surface of rotation whose meridional section is defined by the equation:

$$r = A \operatorname{sech} [B (\tau + C)], \tag{10.05.01}$$

in which A, B, and C are constants, τ is the isometric latitude, and r is the perpendicular distance from the axis of rotation to the surface. The constants are chosen in such a way that the aposphere touches the ellipsoid with which it has a common axis of rotation along some parallel. That parallel passes through the centre of the area for which the transformation is required.

The Hotine's oblique Mercator (HOM) or rectified skew orthomorphic (RSO) grid projections for which the basic formulae were originally developed by Martin Hotine is used where the area of interest is oblong, and the longer axis through the territory is skewed with respect to the meridians (Hotine, 1946-1947).

An Introduction to the Triple Projection System

The fact that the RSO projection is skewed introduces complications, but in certain areas, it has advantages over the Gauss-Krüger (GK) type and the Lambert (LCC) type of projection. Its errors are very similar to those of the Gauss-Krüger projection.

The projection is actually made by mapping the ellipsoid onto an auxiliary surface of an aposphere, the general surface of constant Gaussian curvature, by equating longitude λ and isometric latitude μ for corresponding points on the two surfaces in order to establish conformality. The parameters of the aposphere:

$$r = A \operatorname{sech} (B\mu + C) \tag{10.05.02}$$

are selected so as to make it fit the ellipsoid as closely as possible at the centre of the map. Hereafter, the aposphere is mapped on a sphere with radius A / B, in which:

$$r_s = (A / B) \operatorname{sech} \mu_s, \tag{10.05.03}$$

by means of the correspondence

$$\mu_s = B\mu + C \tag{10.05.04}$$

$$\lambda_s = B\lambda \tag{10.05.05}$$

Two surfaces being applicable, this sphere is mapped conformably on a plane as in the ordinary oblique Mercator projection of the sphere. The central line - called the initial line - of the projection runs in the direction of the longer dimension of the area of interest. The projection parameters for the oblique Mercator projection are provided by latitude and longitude of a centre point, the azimuth of the Initial line, and the scale factor at the centre point of the skewed Initial line, which is in this projection a geodesic, positive between 0° and 180°.

Two grids constructed on the skew orthomorphic projection exist for:

- Borneo (Brunei Darussalam, East-Malaysia), and West-Malaysia - the rectified skew orthomorphic (RSO) projection
- US State Alaska, - the Hotine oblique Mercator (HOM) projection (Bomford, 1977).

Case State Alaska Zone 1. The axis of the strip - the geodesic through the point $\varphi = 57° 00'$ N, $\lambda = 133° 40'$ W, azimuth of arctan (-¾) - is mapped in this triple projection into a geodesic on the aposphere, a great circle on the sphere, and finally the u-axis to the mapping plane. A linear transformation gives the plane coordinates x, and y. This process is complicated, but conformality is preserved throughout.

HOM (or RSO) Mapping Equations

Symbols and definitions

The equations for the ellipsoid constants are given in (Table 19): pp 123.
Original Borneo formulae use the constants: x, y, ω_o, E and N
Original Alaska formulae use instead the constants: u, v, λ_o, x and y

All angles are expressed in radians [18.20], A_20OM00.FOR.

a	equatorial radius of the ellipsoid
b	semi-minor axis of the ellipsoid
f	flattening of the ellipsoid
k_c	grid scale factor at the local origin
φ_c	latitude of local origin
λ_c	longitude of local origin
α_c	azimuth of positive skew axis (u-axis) at local origin, Initial line
α_o	azimuth of positive skew axis at Equator, line of projection
E_o	false easting
N_o	false northing
M_o	conversion factor
Q	isometric latitude
X	conformal latitude
λ_o (ω_o)	basic longitude of the true origin
φ	parallel of geodetic latitude, positive north
λ	meridian of geodetic longitude, positive east
E (x)	easting rectified coordinate |[13]
N (y)	northing rectified coordinate
γ	convergence
k	point grid scale factor
k_{1-2}	line scale factor between P_1 and P_2
δ_{1-2}	(t - T)", arc-to-chord correction from P_1 to P_2
e^2	first eccentricity squared
e'^2	second eccentricity squared

Computation of the Constants

$$\begin{aligned}
c_2 &= e^2/2 + 5e^4/24 + e^6/12 + 13e^8/360 + 3e^{10}/160 \\
c_4 &= 7e^4/48 + 29e^6/240 + 811e^8/11520 + 81e^{10}/2240 \\
c_6 &= 7e^6/120 + 81e^8/1120 + 3029e^{10}/53760 \\
c_8 &= 4279e^8/161280 + 883e^{10}/20160 \\
c_{10} &= 2087e^{10}/161280
\end{aligned} \tag{10.05.06}$$

$$\begin{aligned}
F_0 &= 2(c_2 - 2c_4 + 3c_6 - 4c_8 + 5c_{10}) \\
F_2 &= 8(c_4 - 4c_6 + 10c_8 - 20c_{10}) \\
F_4 &= 32(c_6 - 6c_8 + 21c_{10}) \\
F_6 &= 128(c_8 + 8c_{10}) \\
F_8 &= 512c_{10}
\end{aligned} \tag{10.05.07}$$

[13]Original Borneo formulae use the constants: x, y, ω_o, E and N; for Alaska instead: u, v, λ_o, x and y

Computation of Projection Zone Constants

Constants and expressions within the oblique Mercator mapping equations are ellipsoid and zone specific.

$$B = (1 + e'^2 \cos^4 \varphi_c)^{1/2} \qquad (10.05.08)$$
$$W_c = (1 - e^2 \sin^2 \varphi_c)^{1/2} \qquad (10.05.09)$$
$$A = a\, B\, (1 - e^2)^{1/2} / W_c^2 \qquad (10.05.10)$$
$$Q_c = \tfrac{1}{2} [\ln \{ (1 + \sin \varphi_c) / (1 - \sin \varphi_c) \} - e \ln \{ (1 + e \sin \varphi_c) / (1 - \sin \varphi_c) \}] \qquad (10.05.11)$$
$$C = \cosh^{-1} [B (1 - e^2)^{1/2} / (W_c \cos \varphi_c)] - B\, Q_c \qquad (10.05.12)$$
$$D = k_c\, A / B \qquad (10.05.13)$$
$$\sin \alpha_o = (a \sin \alpha_c \cos \varphi_c) / (A\, W_c) \qquad (10.05.14)$$
$$\lambda_o\ (\omega_o) = \lambda_c + (\sin^{-1} [\sin \alpha_o \sinh (B\, Q_c + C) / \cos \alpha_o]) / B \qquad (10.05.15)$$
$$F = \sin \alpha_o \qquad (10.05.16)$$
$$G = \cos \alpha_o \qquad (10.05.17)$$
$$I = k_c\, A / a \qquad (10.05.18)$$

Direct Conversion Computation

Input: geodetic coordinates of a point P (φ, λ).
Output: grid coordinates of a point P (E, N), convergence angle (γ), the grid scale factor (k).

$$L = B (\lambda - \lambda_o) \qquad (10.05.19)$$
$$Q = \tfrac{1}{2} [\ln \{ (1 + \sin \varphi) / (1 - \sin \varphi) \} - e \ln \{ (1 + e \sin \varphi) / (1 - e \sin \varphi) \}] \qquad (10.05.20)$$
$$J = \sinh (B\, Q + C) \qquad (10.05.21)$$
$$K = \cosh (B\, Q + C) \qquad (10.05.22)$$
$$u\ (x) = D \tan^{-1} [(J\, G - F \sin L) / \cos L] \quad \text{(skew coordinates along axis of proj.)} \qquad (10.05.23)$$
$$v\ (y) = \tfrac{1}{2} D \ln [(K - F\, J - G \sin L) / (K + F\, J + G \sin L)]$$
$$\text{(skew coordinates perpendicular to axis of projection)} \qquad (10.05.24)$$

$$E\ (X) = u \sin \alpha_c + v \cos \alpha_c + E_o \qquad (10.05.25)$$
$$N\ (Y) = u \cos \alpha_c - v \sin \alpha_c + N_o \qquad (10.05.26)$$
$$\gamma = \tan^{-1} ((F - J\, G \sin L) / (K\, G \cos L)) - \alpha_c \qquad (10.05.27)$$
$$k = I (1 - e^2 \sin^2 \varphi)^{1/2} \cos (u / D) / (\cos \varphi \cos L) \qquad (10.05.28)$$

> Recognise in (10.05.11, 10.05.20): $\ln (\tan ((\pi / 4)+(\varphi / 2))) = \tfrac{1}{2} \ln ((1+\sin (\varphi)) / (1 - \sin (\varphi)))$ (Gretschel, 1873; Le Pape, 1994).

Inverse Conversion Computation

Input: grid coordinates of a point P (E, N).
Output: geodetic coordinates of a point P (φ, λ).

To compute the convergence angle (γ), the grid scale factor (k), see the equations of the direct conversion computation.

$$u\ (x) = (E - E_o) \sin \alpha_c + (N - N_o) \cos \alpha_c \quad \text{(skew coordinates along axis of projection)} \qquad (10.05.29)$$
$$v\ (y) = (E - E_o) \cos \alpha_c - (N - N_o) \sin \alpha_c \quad \text{(skew coordinates perpendicular to axis of projection)} \qquad (10.05.30)$$
$$R = \sinh (v / D) \qquad (10.05.31)$$
$$S = \cosh (v / D) \qquad (10.05.32)$$
$$T = \sin (u / D) \qquad (10.05.33)$$
$$Q = [\tfrac{1}{2} \ln \{ (S - R\, F + G\, T) / (S + R\, F - G\, T) \} - C] / B \qquad (10.05.34)$$
$$X = 2 \tan^{-1} [(\exp (Q) - 1) / (\exp (Q) + 1)] \qquad (10.05.35)$$

$$\varphi = X + (\sin X \cos X)(F_0 + F_2 \cos^2 X + F_4 \cos^4 X + F_6 \cos^6 X + F_8 \cos^8 X) \quad (10.05.36)$$
$$\lambda = \lambda_o + (1 / B) \tan^{-1} [(R G + T F) / (\cos (u / D))] \quad (10.05.37)$$

Note

cosh x	=	$(\varepsilon^x + \varepsilon^{-x}) / 2$
$\cosh^{-1} x$	=	$\ln [x + (x^2 - 1)^{1/2}]$
sinh x	=	$(\varepsilon^x - \varepsilon^{-x}) / 2$
$\sinh^{-1} x$	=	$\ln [x / (x^2 + 1)^{1/2}]$
exp (Q)	=	ε^Q
in which ε	=	2.71828 18284 59045 23536 02875 (base of natural logarithms)

Arc-to-Chord Correction δ = (t - T)"

Grid azimuth (t), geodetic azimuth (α), convergence angle (γ), and arc-to-chord correction (δ) at any given point are related as follows: $t = \alpha - \gamma + \delta$ ... (9.01).

Input: P_1 (u_1, v_1), and P_2 (u_2, v_2).
Output: (t - T) correction (δ_{12}) for the line from P_1 to P_2, and line scale factor (k_{12})

$$\varphi_m = (\varphi_1 + \varphi_2) / 2 \quad (10.05.38)$$
$$Q = \tfrac{1}{2} [\ln \{ (1 + \sin \varphi_m / (1 - \sin \varphi_m) \} - e \ln \{ (1 + e \sin \varphi_m) / (1 - e \sin \varphi_m) \}] \quad (10.05.39)$$
$$\delta_{12} = (u_1 - u_2) (2 v_1 + v_2) / (6 D^2) \quad (10.05.40)$$
$$k_{12} = [k_c \{ 1 + (v_1^2 + v_1 v_2 + v_2^2) / (6 D^2)\} (1 - e^2 \sin^2 \varphi_m)^{1/2}] / [\cos \varphi_m \cosh (B Q + C)] \quad (10.05.41)$$

Observations on Oblique Mercator

Equations for the ellipsoid constants are given (Table 19): pp 123.

The equations in this section are found in (Brazier, 1947, 1950; Admiralty, 1965; NOAA-C&GS, 1961) and (Floyd, 1985; Stem, 1989a), based on (Vincenty, 1984b). The equations for the isometric latitude constants are found in (Berry, 1970; Burkholder, 1985). The equations were further manipulated by the author.

The algorithms [18.20] for conversion of coordinates, local scale factor and the arc-to-chord correction for any point on the grid are based on Hotine's rectified skew orthomorphic (RSO) projection. Some grids may include a false easting and / or a false northing constant in the formulae.

The oblique Mercator projection and its conversion equations should not be used under certain circumstances:

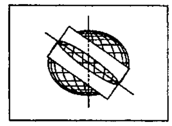
- if the centre point of the area of interest lies near either Pole: the stereographic projection may be used
- if a point of the central line defining the projection lies at either Pole, use equations for the GK type projection instead
- if the two points of the central line both lie on or close to the Equator, the normal Mercator projection (NM) type may be used
- in general, if the two points of the central line lie on the same parallel of latitude other than the Equator, use equations for the Lambert conformal conical (LCC) type projection instead.

The program is using the following constants:

- for the Borneo formulae use the constants: x, y, ω_o, E and N
- for the Alaska formulae use the constants: u, v, λ_o, x and y.

RSO - Application I

Everest's Figure of the Earth for Borneo



zone parameters		:	Borneo RSO
reference ellipsoid		:	Everest 1830
semi-major axis	a	:	**6 974 310.600** Indian yards or
	a	:	**6 377 276.3458** m
reciprocal flattening	f^{1}	:	**300.8017**
scale factor at origin	k_c	:	**0.99984**
defined true origin	φ_c	:	**4° 00' 00" N**
longitude centre	λ_c	:	**115° 00' 00" E**
skew angle $_{\text{True Origin}}$		:	**+ 53° 18' 56".95370 00000 4016**
skew angle $_{\text{False Origin}}$		:	**+ 53° 07' 48".36847 49615 2333**
atan $_{\text{True Origin}}$		=	1.342376 31592 6919
sin $_{\text{false Origin}}$		=	0.8
cos $_{\text{False Origin}}$		=	0.6
false easting	E_f	:	**0**
false northing	N_f	:	**0**
coordinates at origin	E_0	=	29 352.4763 chains west from true origin
	N_0	=	22 014.3572 chains south from true origin
input unit:	m	=	output unit in Indian chains (i.c.)
basic longitude	ω_o (λ_o)	=	109°.6855202029758 E
conversion factor	m	:	**0.04970 99546 8**

The skew coordinates x, y, parallel to the initial line, and at right angles to the initial line, respectively, and the map coordinates E, N are interrelated by the formulae:

$$E = 0.8x + 0.6y, \text{ and}$$
$$N = 0.6x - 0.8y, \text{ accordingly it follows that:}$$

$$x = 0.8E + 0.6N, \text{ and}$$
$$y = 0.6E - 0.8N$$

The x, y terms are referred to as skew coordinates and the E, N terms as rectified, planar mapping coordinates.

The semi-major axis, a, is entered in metres, thus all calculations are made in metres. The resulting plane coordinates are then converted into the unit required, for Borneo into Indian chains (Brazier, 1947, 1950).

Figure 91 shows a part of the Borneo rectified skew orthomorphic (RSO) grid. The initial line O'OQ makes an angle θ'=53° 07'48".3685 with the true meridian S'O'N' at the false point of origin O' (Figure 92).

At O, the true origin, the initial line of the projection makes an angle θ = 53° 18' 56".9537 E of true North with the true meridian SON. The difference between the angles θ and θ' is the convergence of these meridians.

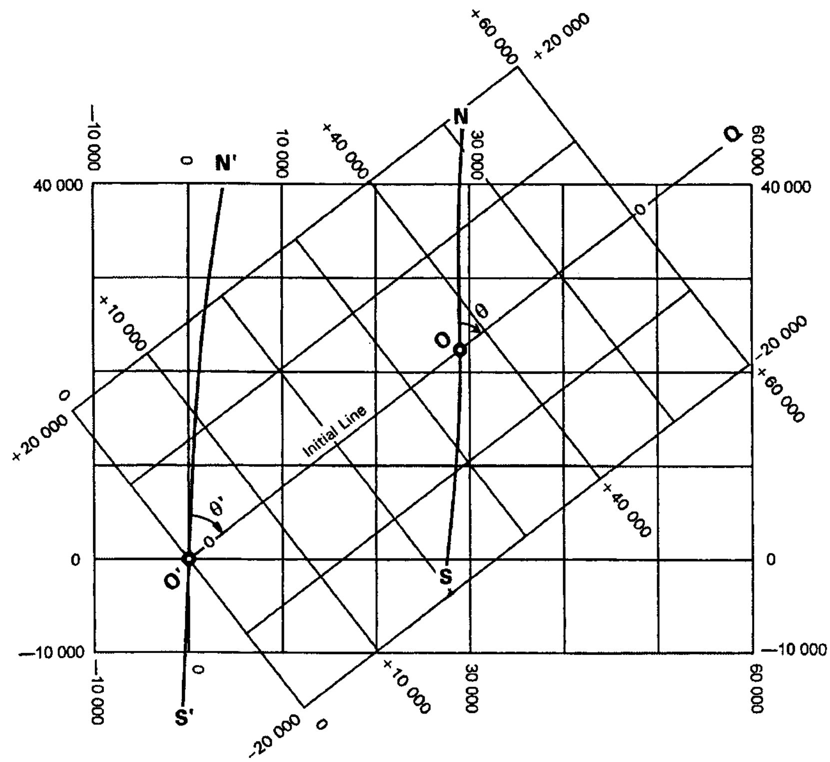


Figure 91: Borneo Rectified Skew Orthomorphic grid

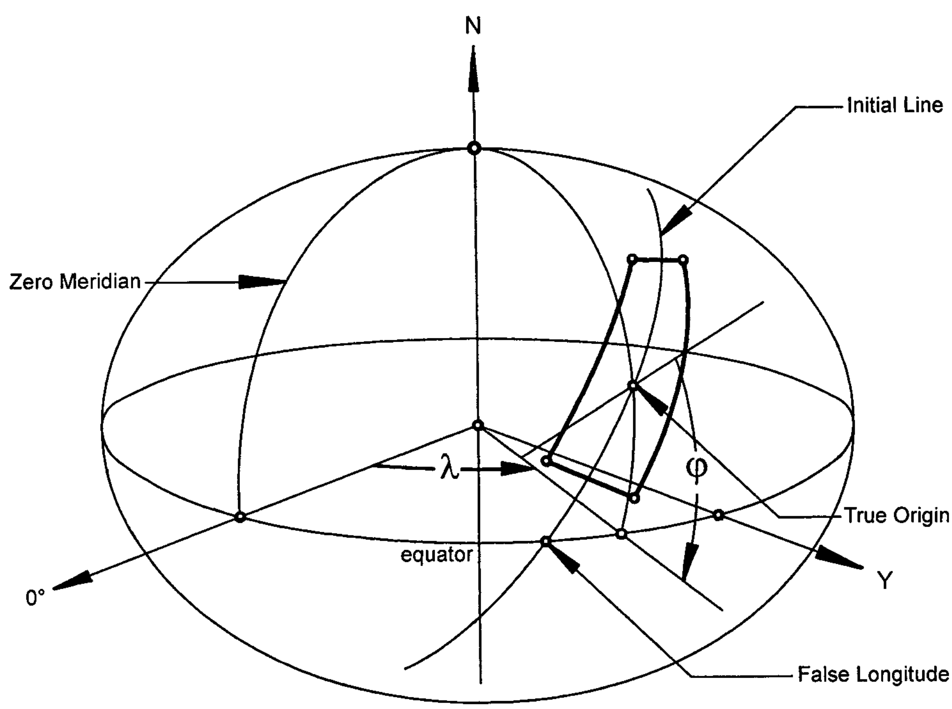


Figure 92: Initial Line of the Borneo RSO grid

Conversion of Timbalei1948 geographicals to oblique Mercator grid

Direct calculation is to convert geodetic coordinates into OM-planar coordinates:

input:	latitude	longitude	output:	easting	northing	convergence	scale factor
Sta.: Ulu Membakut							
latitude	φ_1 :	**5° 23' 14".1129 N**	easting	E_1 =			33 765.15703 i.c
longitude	λ_1 :	**115° 48' 19".8196 E**	northing	N_1 =			29 655.00567
convergence	γ_1 =	+ 0° 14' 36".8379	scale factor	k_1 =			0.999 900 131 343
Sta.:Kinandukan							
latitude	φ_2 :	**5° 43' 08".3444 N**	easting	E_2 =			34 253.65351
longitude	λ_2 :	**115° 53' 44".2988 E**	northing	N_2 =			31 480.41516
convergence	γ_2 =	+ 0° 14' 55".8046	scale factor	k_2 =			0.999 947 468 405

Conversion of oblique Mercator grid to Timbalei1948 geographicals

Inverse process is to convert OM-planar coordinates into geodetic coordinates:

input:	easting	northing	output:	latitude	longitude
easting	E_1 :	**33 765.15703** i.c	latitude	φ_1 =	5° 23' 14".1129 N
northing	N_1 :	**29 655.00567**	longitude	λ_1 =	115° 48' 19".8196 E
easting	E_2 :	**34 253.65351**	latitude	φ_2 =	5° 43' 08".3444 N
northing	N_2 :	**31 480.41516**	longitude	λ_2 =	115° 53' 44".2988 E

(t - T) corrections

input:	easting		northing	output:	arc-to-chord or (t - T) correction	
E_1:	**33 765.15703**	E_2:	**34 253.65351**	(t - T)	$\delta_{1\text{-}2}$ =	+5".9181
N_1:	**29 655.00567**	N_2:	**31 480.41516**	(t - T)	$\delta_{2\text{-}1}$ =	- 6".5156

Borneo RSO - ellipsoidal and grid calculation

ellipsoidal and grid calculation (manual)

Using [18.12, 18.14] gives:				grid bearing	$t_{1\text{-}2}$	14° 58' 54".5749
				+ convergence	γ_1	+ 0° 14' 36".8379
true distance $_{(m)}$	$S_{1\text{-}2}$	:	38016.3011	(t - T)	$\delta_{1\text{-}2}$	+ 0° 00' 05".9181
true distance	$S_{1\text{-}2}$	:	1 889.78860	true azimuth	$\alpha_{1\text{-}2}$ =	15° 13' 25".4947
true azimuth	$\alpha_{1\text{-}2}$	:	15° 13' 25".4956			
true azimuth	$\alpha_{2\text{-}1}$	:	195° 13' 56".8951	grid bearing	$t_{2\text{-}1}$	194° 58' 54".5749
grid distance	$D_{1\text{-}2}$	:	1 889.64246	+ convergence	γ_2	+ 0° 14' 55".8046
distance	$S_{calc\ 1\text{-}2}$	:	1 889.78855	- (t - T)	$\delta_{2\text{-}1}$	- 0° 00' 06".5156
See (9.03), using				true azimuth	$\alpha_{2\text{-}1}$ =	195° 13' 56".8951
scale factor	$k_{1\text{-}2}$	:	0.999 922 694 259			

(For Borneo: all values are expressed in i.c.{Indian chains})

HOM Projection SPCS27 of the USA

The defining elements of the HOM projection are:

- parameters of the Clarke ellipsoid of 1866 for SPCS27 (old Datum) and parameters of the Geodetic Reference system of 1980 for SPCS83 (new Datum)
- the geodesic passing through the point φ_c=57° 00' N, λ_c=133° 40' W, designated the centre of the projection, at an azimuth α_c=arctan (-¾, by definition). This skew angle is 360° – 36° 52' 11".63152503847826
- a grid scale factor k_c = 0.9999 (exactly)
- Alaska formulae use the constants: u, v, λ_o, X and Y, for Borneo instead: x, y, ω_o, E and N. A rotation of the skew plane coordinate system u, v to minimise the map convergence of meridians (γ_c=0) and a shift of origin, as defined by the equations:
  - X (easting)__________________________ = −0.6u + 0.8v + 5 000 000
  - Y (northing)_________________________ = +0.8u + 0.6v − 5 000 000
- because all computations were made in meters, the resulting X (easting) and Y (northing) coordinates are then converted as a final step, into *US survey feet* by means of the relation 1.00 m= 39.37 inches (on NAD27, old Datum only)
- for Alaska zone 1 the true origin is in the vicinity of the 0° 15' N latitude, 101° 31' W longitude. The grid origin lies at a specified point near the area of interest, for Alaska zone 1, that point is 5 000 000 metres to the north and 5 000 000 metres to the west of the true Origin.

Example State Alaska Zone 1, USA - SPCS27 - SPCS83

The calculation for SPCS27, State Alaska zone 1, United States of America, are exemplified [On_CD], and in (C&GS SP No. 62-4, 1968; C&GS SP No. 65-1 Part 49, 1961).

Note

> *Coordinates of historical data points must be taken at face value, with the realisation that such coordinates could be significantly in error. In practice, exact coordinates in any reference system are not obtained*
>
> (Floyd, 1985).

Observations

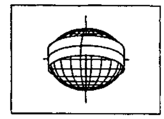
It will be observed that the initial line - a geodesic - of the projection has a scale factor that is nearly constant throughout its length.

The interested reader may consult a report about development of algorithms to calculate Hotine's Oblique Mercator projection (HOM) of the ellipsoid of revolution by Grafarend:

> *"An investigation of the conformal projection is the basis of a paper about the ellipsoidal oblique Mercator projection as devised by Hotine 50 years ago. Research in the Oblique Mercator was raised by Hotine's cryptic procedure to derive the mapping equations which should be based on similar concepts known for the normal Mercator and the transverse Mercator projection, thus extending the exercises of Hotine. Enhanced up to date mapping equations of the conformal oblique Mercator projection are given in the publication"*

(Grafarend; Engels, 19951).

10.6 Rosenmund's Oblique Mercator Projection



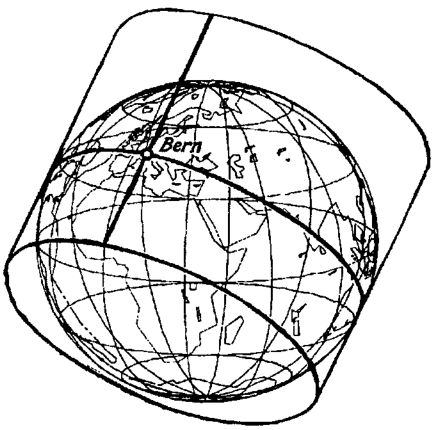


Figure 93: Oblique Mercator Grid of Switzerland

Rosenmund's oblique Mercator (ROM) projection of the ellipsoid was portrayed by Max (Rosenmund, 1903). The Gauss-Schreiber conformal double projection (GS) [10.2] is intertwined with the computation of the projection from the ellipsoid to the sphere and vice versa (Figure 93).

Oblique Mercator Projection System

The oblique Mercator conformal projection is based on the Gauss-Schreiber (GS) type projection, devised by the geodesists Gauss and Schreiber. This projection is used because of its longer dimension in an east-west direction (Grob, 1941; Signer, 2002; Wicki, 2002).

True Origin

The true origin of the CH1903 projection was chosen at the Observatory of Bern, Capital of Switzerland. A grid origin was established to the southwest of that observatory (Figure 94).

Grid Origin

The definition of the north-south location of the grid origin is specified by a false easting = y_0 constant, and a false northing = x_0 constant.

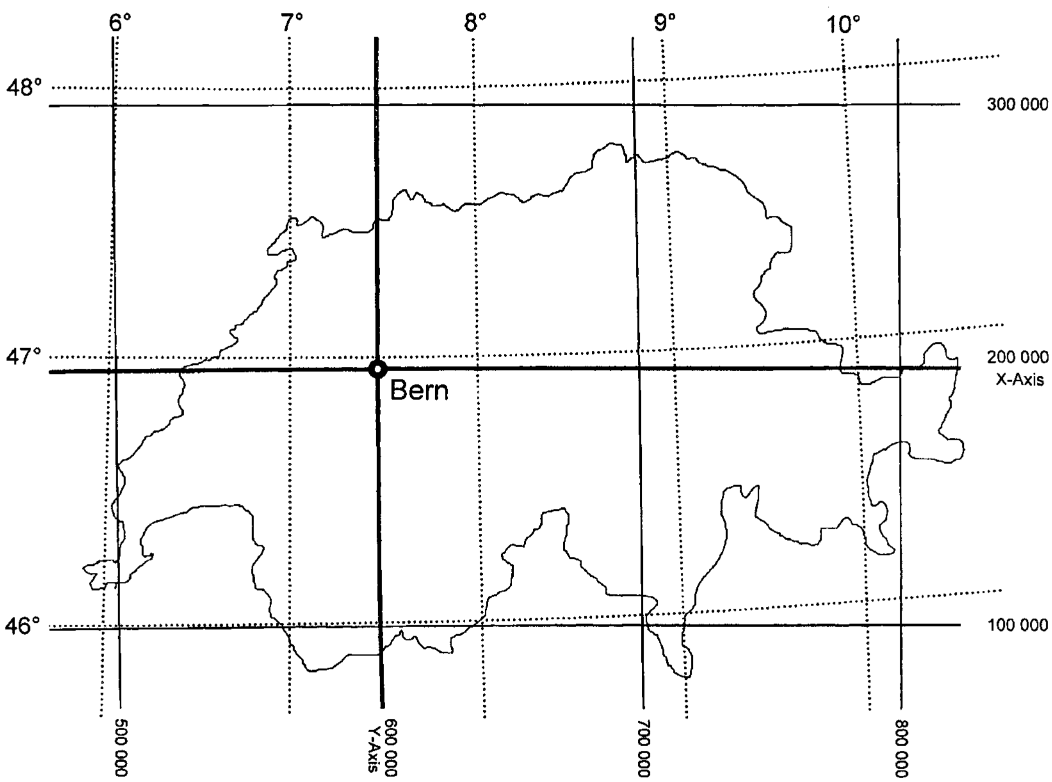


Figure 94: Rosenmund's Oblique Mercator system

Note

Using a set of Y-, X-coordinates, the old local reference system is designated as CH1903. Its reference frame is LV1903. Using a set of E-, N-coordinates, the state-wide GPS-Network uses a landesvermessung 1995 (LV1995) reference frame.

the oblique Mercator grid system of Switzerland has the following specifications:					
projection			CH-1903 oblique Mercator		
reference ellipsoid			Bessel 1841		
latitude of true origin Bern	B_0	:	46° 57' 08".66 N		
longitude of true origin Bern	L_0	:	7° 26' 22".5 E		
area bounded by eastings	y	:	5 000	...	280 000
area bounded by northings	x	:	300 000	...	624 000

Stable Region

To prevent that equations for the Rosenmund's oblique Mercator (ROM) projection become unstable, the ROM projection should be limited to a region bounded by latitudes and longitudinal distances from the origin (Bern). These distances depend on the purpose of the projection (Grob, 1941).

ROM Mapping Equations

Symbols and definitions

Equations for the Ellipsoid constants are given in (Table 19): pp 123.
All angles are expressed in radians. See program [18.21], A_21RM00.FOR.

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
f	flattening of the ellipsoid
k_0	grid scale factor assigned to the central line
B_0	geodetic latitude of the origin
L_0	geodetic longitude of the origin
x_0	false easting - constant assigned to the longitude of the grid origin
y_0	false northing - constant assigned to the latitude of the grid origin
B	parallel of geodetic latitude, positive north
L	meridian of geodetic longitude, positive east
Y	easting coordinate
X	northing coordinate
γ	meridian convergence
k	scale factor
e^2	first eccentricity squared
e'^2	second eccentricity squared
n	second flattening
R_{00}	radius of curvature in the meridian
R_{90}	radius of curvature in the prime vertical
R_A	radius of the Gaussian sphere
α	Gaussian dilatation constant

Constants for Zone

Compute ellipsoid constants as given below:

e^2	$= f (2 - f)$	(10.06.01)
e	$= (e^2)^{½}$	(10.06.02)
e'^2	$= e^2 / (1 - e^2)$	(10.06.03)
n	$= f / (2 - f)$	(10.06.04)
R_{00}	$= a / (1 - e^2 \sin^2 l_0)^{½}$	(10.06.05)
R_{90}	$= (1 - e^2) n_r / (1 - e^2 \sin^2 l_0)$	(10.06.06)
R_A	$= (R_{00} R_{90})^{½}$ or:	(10.06.07)
R_A	$= a (1 - e^2)^{½} / (1 - e^2 \sin^2 B_0)$	(10.06.08)

Compute CH1903-zone parameters of the double projection.

α	$= (1 + e'^2 \cos^4 B_0)^{½}$	(10.06.09)
$\sin b_0$	$= \sin B_0 / \alpha$	(10.06.10)
l_0	$= L_0$	(10.06.11)
k	$= \log (\tan (\pi / 4 + b_0 / 2)) - \alpha \log (\tan (\pi / 4 + B_0 / 2)) + \alpha e / 2 \log (1 + e \sin B_0) / (1 - e \sin B_0)$	(10.06.12)

Calculate oblique Mercator CH1903 projection

Direct conversion computation

Input: geodetic coordinates of a point P (B, L or φ, λ, resp.)
Output: grid coordinates of a point P (y, x or E, N, resp.), convergence (γ) and scale factor (k).

l	$= \alpha (L - L_0)$	(10.06.13)
t_1	$= e \sin B$	
t_2	$= \alpha (\log ((1 + \sin B) / (1 - \sin B)) / 2 - e / 2 \log ((1 + t_1) / (1 - t_1))) + k$	(10.06.14)
$\sin b'$	$= \cos b_0 \sin b - \sin b_0 \cos b \cos l$	(10.06.15)
$\cos b' \sin l'$	$= \cos b \sin l$	(10.06.16)
b	$= 2 (\arctan (\exp t_2) - \pi / 4)$	(10.06.17)
b'	$= \arcsin (\cos b_0 \sin b) - \sin b_0 \cos b \cos l)$	(10.06.18)
l'	$= \arcsin (\cos b \sin l / \cos b')$	(10.06.19)
t_1	$= \sin b'$	(10.06.20)
x	$= x_0 + ½ R_A \log (((1 + t_1) / (1 - t_1)))$	(10.06.21)
y	$= y_0 + R_A l'$	(10.06.22)
$\cos b' \sin \gamma$	$= \sin b_0 \sin l$	(10.06.23)
$\cos b' \cos \gamma$	$= \cos b_0 \cos b + \sin b_0 \sin b \cos l$	(10.06.24)
γ	$= \arcsin (\sin b_0 \sin l / \cos b')$	(10.06.25)
k	$= 1 + (x - x_0)^2 / (2 R_A^2)$	(10.06.26)

Inverse conversion computation

Input: grid coordinates of a point P (y, x or E, N, resp.)
Output: geodetic coordinates of a point P (B, L or φ, λ, resp.).

Δx	$= x - x_0$	(10.06.27)
Δy	$= y - y_0$	(10.06.28)
x'	$= 1 / \exp (\Delta x / R_A)$ or:	(10.06.29)
x'	$= \tan (\pi / 4 + b' / 2)$	(10.06.30)
b'	$= 2 (\arctan x' - \pi / 2)$	(10.06.31)
l'	$= \Delta y / R_A$	(10.06.32)
$\sin b$	$= \cos b_0 \sin b' - \sin b_0 \cos b' \cos l'$	(10.06.33)
$\cos b \sin l$	$= \cos b' \sin l'$	(10.06.34)
$\cos b \cos l$	$= \sin b_0 \sin b' + \cos b_0 \cos b' \cos l'$	(10.06.35)

B	$= B_0$	(10.06.36)

iterate for b:

t_1	$= e \sin B$	(10.06.37)
t_2	$= (\log ((1 + \sin b) / (1 - \sin b)) / 2 - k) / \alpha + e / 2 \log ((1 + t_1) / (1 - t_1))$	(10.06.38)
B	$= 2 (\arctan (\exp t_2) - \pi / 4)$	(10.06.39)

continue

L	$= L_0 + 1 / \alpha$	(10.06.40)

Calculate (t – T)" corrections

δy	$= y_2 - y_1$	(10.06.41)
δx	$= x_2 - x_1$	(10.06.42)
δx_2	$= x_2 + x_1 - 2 x_0)$	(10.06.43)
δ_{12}	$= -\delta y / (2 R_A)^2 (-\delta x_2 + (\delta x + \delta x_2^3 / (2 R_A)^2) /3)$	(10.06.44)
δ_{21}	$= -\delta y / (2 R_A)^2 (+\delta x_2 + (\delta x - \delta x_2^3 / (2 R_A)^2) /3)$	(10.06.45)

ROM – Application

Rosenmund's oblique Mercator conformal projection and grid system of Switzerland

the oblique Mercator grid system has the following specifications:

projection		:	Oblique Mercator projection
reference ellipsoid		:	Bessel 1841
semi-major axis	a	:	**6 377 397.155**
reciprocal flattening	f^1	:	**299.15281 285**
latitude of true origin	B_0	:	**46° 57' 08".66 N**
longitude of true origin	L_0	:	**7° 26' 22".5 E**
lat. of true origin sphere	b_0	:	**46° 54' 27".83324 84575 N**
lon. of true origin sphere	l_0	:	**7° 26' 22".5000 E**
false easting	y_0	:	**600 000.**
false northing	x_0	:	**200 000.**
scale factor at the central line	k_0	:	**1.0000**
unit		:	metre
Gaussian dilatation constant	α	=	1.00072 91384 30386
dilatation factor	m	=	3.06673 23773 07039E-03
radius of the Gaussian sphere	R_A	=	6 378 815.9036

conversion of CH1903 Geographicals to oblique Mercator Grid

direct calculation is to convert geodetic coordinates into OM-planar coordinates:

input: latitude longitude output: easting northing convergence scale factor

latitude	φ_1	:	**46° 07' 28".4697 N**	easting	y_1	=	693 479.8247
longitude	λ_1	:	**8° 38' 56".8777 E**	northing	x_1	=	108 705.0672
convergence	γ_1	=	+ 0° 53' 02".3377	scale factor	k_1	=	1.000 102 419 633
latitude	φ_2	:	**46° 07' 29".1992 N**	easting	y_2	=	731 863.7519
longitude	λ_2	:	**9° 08' 45".1043 E**	northing	x_2	=	109 441.4681
convergence	γ_2	=	+ 1° 14' 49".0772	scale factor	k_2	=	1.000 100 774 027

latitude	φ_3	:	**46° 06' 18".3197 N**	easting	y_3	=	710 364.0174
longitude	λ_3	:	**8° 52' 01".6034 E**	northing	x_3	=	106 822.8217
convergence	γ_3	=	+ 1° 02' 35".8086	scale factor	k_3	=	1.000 106 686 379

conversion of oblique Mercator grid to CH1903 geographicals

inverse process is to convert Rosenmund's oblique Mercator grid into geodetic coordinates:

input: easting northing		output: latitude longitude		(t - T) correction	
easting y_1:	**693 479.8247**	latitude φ_1=	46° 07' 28".4697 N	$\delta_{1\text{-}2}$ =	−8".8575
northing x_1:	**108 705.0672**	longitude λ_1=	8° 38' 56".8777 E	$\delta_{2\text{-}1}$ =	+8".8336
easting y_2:	**731 863.7519**	latitude φ_2=	46° 07' 29".1992 N	$\delta_{1\text{-}3}$ =	−3".9336
northing x_2:	**109 441.4681**	longitude λ_2=	9° 08' 45".1043 E	$\delta_{3\text{-}1}$ =	+3".9604
easting y_3:	**710 364.0174**	latitude φ_3=	46° 06' 18".3197 N	$\delta_{2\text{-}3}$ =	+4".9821
northing x_3:	**106 822.8217**	longitude λ_3=	8° 52' 01".6034 E	$\delta_{3\text{-}2}$ =	−5".0297

ellipsoidal and grid calculation (manual)

grid bearing	$t_{1\text{-}2}$	88° 54' 03".2668	grid bearing	$t_{1\text{-}3}$	96° 21' 39".7962
+ convergence	γ_1	+0° 53' 02".3377	+ convergence	γ_1	+ 0° 53' 02".3377
− (t - T)	$\delta_{1\text{-}2}$	-0° 00' 08".8575	−(t - T)	$\delta_{1\text{-}3}$	− 0° 00' 03".9336
true azimuth	$\alpha_{1\text{-}2}$ =	+ 89° 47' 14".4620	true azimuth	$\alpha_{1\text{-}3}$ =	+ 97° 14' 46".0677
grid bearing	$t_{2\text{-}1}$	268° 54' 03".2668	grid bearing	$t_{2\text{-}3}$	263° 03' 20".2888
+ convergence	γ_2	+ 1° 14' 49".0772	+ convergence	γ_2	+ 1° 14' 49".0772
− (t - T)	$\delta_{2\text{-}1}$	+ 0° 00' 08".8336	− (t - T)	$\delta_{2\text{-}3}$	+ 0° 00' 04".9821
true azimuth	$\alpha_{2\text{-}1}$ =	+ 270° 08' 43".5104	true azimuth	$\alpha_{2\text{-}3}$ =	+ 264° 18' 04".3840
grid bearing	$t_{3\text{-}1}$	276° 21' 39".7962	grid bearing	$t_{3\text{-}2}$	83° 03' 20".2889
+ convergence	γ_3	+ 1° 02' 35".8086	+ convergence	γ_3	+ 1° 02' 35".8086
− (t - T)	$\delta_{3\text{-}1}$	+ 0° 00' 03".9604	− (t - T)	$\delta_{3\text{-}2}$	− 0° 00' 05".0297
true azimuth	$\alpha_{3\text{-}1}$ =	+ 277° 24' 11".6444	true azimuth	$\alpha_{3\text{-}2}$ =	+ 84° 06' 01".1272

Using [18.12, 18.14], and formula (9.03), gives:

scale factor=	k_m	1.000 101 596 830	true distance =	$S_{1\text{-}2}$ (calc.)	38 387.0905
grid distance:	$D_{1\text{-}2}$	38 390.9905	true dist 1-2 =	$S_{1\text{-}2}$	38 387.0902
scale factor=	k_m	1.000 104 553 006	true distance =	$S_{1\text{-}3}$ (calc.)	16 987.0089
grid distance:	$D_{1\text{-}3}$	16 988.7849	true dist 1-3 =	$S_{1\text{-}3}$	16 987.0088
scale factor	k_m	1.000 103 730 203	true distance =	$S_{2\text{-}3}$ (calc.)	21 656.3753
grid distance:	$D_{2\text{-}3}$	21 658.6217	true dist 2-3 =	$S_{2\text{-}3}$	21 656.3754

10.7 Oblique Stereographic Conformal Projection

Hipparchus (about 150 BC) is also credited with the invention of the stereographic projection. Besides its use for the solution of problems in mining, seismology, astronomy, navigation, and hydrodynamics, it is employed in geodesy and in cartography, such as the universal polar stereographic (UPS) projection grid system of the ellipsoid over 80°S and 84°N.
For UPS, the meridians are straight lines radiating from a central point - the North or South Pole of the ellipsoid - and the parallels are concentric circles about this central point. Clearly, the UPS of the ellipsoid is a special case of the Lambert projection where the standard parallel is taken to be the Pole.

The oblique stereographic conformal (OSC) projection of the ellipsoid is used e.g. in Canada, the Netherlands, Poland and Syria.

Gauss-Schreiber's double Projection

As with all other conformal projections discussed, the stereographic projection of the ellipsoid is complicated compared with the sphere. A way of avoiding the difficulty is to employ the Gaussian method, which has been explained in [10.2], namely that of projecting the ellipsoid conformal to the Gaussian sphere. Hereafter, the sphere is projected to the plane. The abbreviation GS type is used, the so-called Gauss-Schreiber double projection (Schreiber, 1897). Using the GS type projection, the OSC projection of the Netherlands was devised because of its circular dimension by (Heuvelink, 1918).

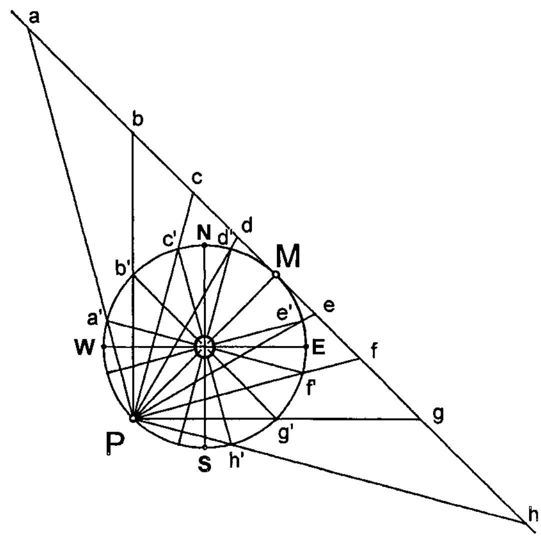


Figure 95: Construction of the stereographic projection

Zone System

Oblique stereographic projections applied to an area of considerable extent are given in (Roussilhe, 1922). On the other hand, when used in a zone system, either the Lambert conformal conic or the transverse Mercator is more suitable, and can be computed with greater ease.

For a small country whose boundaries are contained in a small circle of a radius between three and six degrees, the stereographic projection is effective using the conformal Gaussian sphere. The scale is about unity at a distance of 170 kilometres from the central point.

The sphere is projected from any point of the sphere upon any plane perpendicular to the diameter passing through the given point of perspectivity means stereographic projection. Using the perspective projection of the sphere, the only difference for different choices of planes is the one of scale.

For the geometric demonstration in Figure 95, P - the centre of projection - is on the circumference of the sphere, a diametrically opposite the plane of a projection tangent or secant - passing through - to the sphere.
A diametrical plane of the sphere is no longer regarded as making contact along the equator, but at any parallel north or south of the equator. Origin or central point M is perpendicular projected from P upon the diametrical plane a-h. In the same way is the sphere projected from P through a'-b'-c'-d'-e'-f'-g'-h' upon the diametrical plane a-b-c-d-e-f-g-h.

OSC Mapping Equations

Symbols and definitions

The equations for the ellipsoid constants are given in (Table 19): pp 123.
All angles are expressed in radians [18.22], A_22ST00.FOR.

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
f	flattening of the ellipsoid
k_0	scale factor assigned to the grid origin
B_0	φ_0 - latitude origin of the ellipsoid
L_0	λ_0 - longitude origin of the ellipsoid
b_0	latitude origin of the sphere
l_0	L_0 - longitude origin of the sphere
x_0	false easting (constant assigned to the longitude of the origin)
y_0	false northing (constant assigned to the latitude of the origin)
B	φ - geodetic latitude of the ellipsoid, positive north
L	λ - geodetic longitude of the ellipsoid, positive east
x	easting coordinate of the grid
y	northing coordinate of the grid
γ	convergence of the meridian
k	grid scale factor
e	first eccentricity
e^2	first eccentricity squared
e'^2	eccentricity squared
r	radius of circle centred at the origin
R_{00}	radius of curvature in the meridian
R_{90}	radius of curvature in the prime vertical
R_A	radius of the Gaussian sphere
α	Gaussian dilatation constant
q	isometric latitude of the ellipsoid
w	isometric latitude of the Gaussian sphere

Constants for Ellipsoid

Compute constants for the ellipsoid as given in (Table 19): pp 123:

$$f = (a - b) / a \qquad (10.07.01)$$
$$e^2 = f (2 - f) \qquad (10.07.02)$$
$$e = (e^2)^{½} \qquad (10.07.03)$$
$$e'^2 = e^2 / (1 - e^2) \qquad (10.07.04)$$
$$n = f / (2 - f) \qquad (10.07.05)$$

Compute parameters of the projection (Strang van Hees, 2006), and (Heuvelink, 1918; Schreiber, 1897):

$$\alpha = (1 + e'^2 \cos^4 B_0)^{½} \qquad (10.07.06)$$
$$b_0 = \arcsin (\sin B_0 / \alpha) \qquad (10.07.07)$$
$$l_0 = L_0 \qquad (10.07.08)$$
$$q_0 = .5 (\log ((1 + \sin B_0) / (1 - \sin B_0)) - e \log ((1 + e \sin B_0) / (1 - e \sin B_0))) \qquad (10.07.09)$$
$$w_0 = .5 \log ((1 + \sin b_0) / (1 - \sin b_0)) \qquad (10.07.10)$$
$$m = w_0 - \alpha\, q_0 \qquad (10.07.11)$$
$$R_A = a (1 - e^2)^{½} / (1 - e^2 \sin^2 B_0) \qquad (10.07.12)$$

Direct Computation

Input: geodetic coordinates of a point P (B, L or φ, λ), resp.
Output: grid coordinates of a point P (x, y), convergence angle (γ), scale factor (k).

$$q = .5 (\log ((1 + \sin B) / (1 - \sin B)) - e \log ((1 + e \sin B) / (1 - e \sin B))) \qquad (10.07.13)$$
$$w = \alpha\, q + m \qquad (10.07.14)$$
$$b_i = 2 \arctan (\exp w) - \pi / 2 \qquad (10.07.15)$$

ΔL	$= \alpha (L - L_0)$	(10.07.16)
ψ'	$= \sin^2 ((b_i - b_0) / 2) + \sin^2 (\Delta L / 2) \cos b_i \cos b_0$	(10.07.17)
ψ	$= 2 \arcsin^{1/2} \psi'$	(10.07.18)
$\sin \alpha$	$= \sin \Delta L \cos b_i / \sin \psi$	(10.07.19)
$\cos \alpha$	$= (\sin b_i - \sin b_0 \cos \psi) / (\cos b_0 \sin \psi)$	(10.07.20)
r	$= 2\, k_0\, R_A \tan (\psi / 2)$	(10.07.21)
x	$= r \sin \alpha + x_0$	(10.07.22)
y	$= r \cos \alpha + y_0$	(10.07.23)

Inverse Computation

Input: grid coordinates of a point P (x, y), convergence angle (γ), scale factor (k).
Output: geodetic coordinates of a point P (B, L or φ, λ), resp.

r	$= ((x - x_0)^2 + (y - y_0)^2)^{1/2}$	(10.07.24)
$\sin \alpha$	$= (x - x_0) / r$	(10.07.25)
$\cos \alpha$	$= (y - y_0) / r$	(10.07.26)
ψ	$= 2 \arctan (r / (2\, k_0\, R_A))$	(10.07.27)
$\sin b_i$	$= \cos \alpha \cos b_0 \sin \psi + \sin b_0 \cos \psi$	(10.07.28)
ΔL	$= \arcsin (\sin \alpha \sin \psi / \cos b_i)$	(10.07.29)
L	$= \Delta L / \alpha + L_0$	(10.07.30)
w	$= .5 \log ((1 + \sin b_i) / (1 - \sin b_i))$	(10.07.31)
q	$= (w - m) / \alpha$	(10.07.32)
B'	$= 2 \arctan (\exp q) - \pi / 2$	(10.07.33)

iterate for B = B'

Δq	$= .5\, e \log ((1 + e \sin B') / (1 - e \sin B'))$	(10.07.34)
B'	$= 2 \arctan (\exp (q + \Delta q)) - \pi / 2$	(10.07.35)

continue:

B	$= B'$	(10.07.36)

Computation (t – T)" correction

δx_1	$= (x_2 - x_1)$	(10.07.37)
δx_2	$= (x_1 - x_2)$	(10.07.38)
δy_1	$= (y_2 - y_1)$	(10.07.39)
δy_2	$= (y_1 - y_2)$	(10.07.40)
r_1	$= ((x_1 - x_0)^2 + (y_1 - y_0)^2)^{1/2}$	(10.07.41)
r_2	$= ((x_2 - x_0)^2 + (y_2 - y_0)^2)^{1/2}$	(10.07.42)
k_2	$= k_0 (1 + r_1^2 / (4\, R_A^2))$	(10.07.43)
k_3	$= k_0 (1 + r_2^2 / (4\, R_A^2))$	(10.07.44)
c_2	$= 1 / 4 / r_0^2$	(10.07.45)
p_1	$= +(y_1 - y_0) / k_2\, c_2$	(10.07.46)
q_1	$= -(x_1 - x_0) / k_2\, c_2$	(10.07.47)
p_2	$= +(y_2 - y_0) / k_3\, c_2$	(10.07.48)
q_2	$= -(x_2 - x_0) / k_3\, c_2$	(10.07.49)
$\delta_{1\text{-}2}$	$= p_1\, \delta x_1 + q_1\, \delta y_1$	(10.07.50)
$\delta_{2\text{-}1}$	$= p_2\, \delta x_2 + q_2\, \delta y_2$	(10.07.51)

True Origin

The true origin of this OSC projection is at the point of reference, O.L. Vrouwe toren, Amersfoort. A grid origin is established to the south and west of that point of reference.

False Origin

Before 1960, the definition of the location of the grid origin was at the true origin. Hereafter, a false easting or x constant = x_0 and a false northing or y constant = y_0 is specified, moving the grid origin south-westwards.

Region

The equations for the OSC projection system are applied to the area inside the Netherlands (Figure 96).

The continental shelf-region bounded by a small belt along the coast uses the GK type projection system with the WGS84 (and previously ED50) Datum. Using this belt for hydrographic surveys requires a transformation as recommended in [18.9].

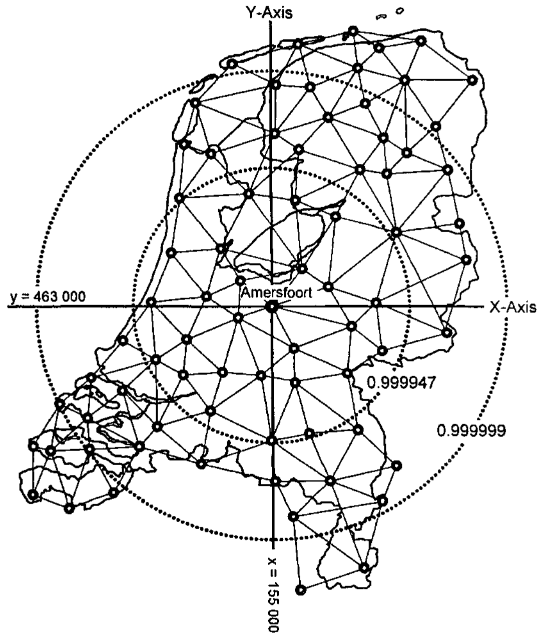


Figure 96: Stereographic grid of the Netherlands

OSC RD1918-Application

oblique stereographic projection and grid system of Netherlands has the following specifications:

projection		:	oblique stereographic conformal (OSC) projection
reference ellipsoid		:	Bessel 1841 (RD1918)
semi-major axis	a	:	**6 377 397.155**
reciprocal flattening	f^{1}	:	**299.15281 285**
latitude of true origin	B_0	:	**52° 09' 22".178 N**
longitude of true origin	L_0	:	**5° 23' 15".5 E**
latitude of sphere - origin	b_0	=	52° 07' 15".95009 59008 N
longitude of sphere - origin	l_0	=	5° 23' 15".5 E
false easting	x_0	:	**155 000 m**
false northing	y_0	:	**463 000 m**
scale factor at origin	k_0	:	**0.99990 79**
unit		:	metre
Gaussian dilatation constant	α	=	1.00047 58566 84165
dilatation factor	m	=	3.77395 38313 04509E-03
radius of the Gaussian sphere	R_A	=	6 382 644.5710
area bounded by eastings		:	5 000 E ... 280 000 E
area bounded by northings		:	300 000 N ... 624 000 N

Direct calculation to convert RD1918 geographicals into stereographic planar coordinates, and vice-versa by the inverse process, including ellipsoidal and grid calculations, is exemplified [10.9], ap. III and [On_CD].

OSC Arab Republic of Syria - Application

Oblique stereographic conformal (OSC) projection system has the following specifications:

projection		:	oblique stereographic conformal projection (GS type)
reference ellipsoid		:	Clarke 1880S
semi-major axis	a	:	**6 378 247.842**
reciprocal flattening	f^{1}	:	**293.46630 766**
latitude of true origin	B_0	:	**34° 12' 00" N**
longitude of true origin	L_0	:	**39° 09' 00" E**
latitude of sphere - origin	b_0	=	34° 08' 15".95636 75073 N
longitude of sphere - origin	l_0	=	39° 09' 00" sec E
false easting	E_0	:	**0 m**
false northing	N_0	:	**0 m**
scale factor at origin	k_0	:	**0.99953 4317**
unit			metre
Gaussian dilatation constant	α	=	1.00160 14362 95382
dilatation factor	m	=	1.50188 17298 24720E-03
radius of the Gaussian sphere	R_A	=	6 370 206.2753

Conversion of Clarke 1880S geographicals to stereographic grid

Direct calculation is to convert geodetic coordinates into stereographic planar coordinates:

input: latitude longitude output: easting northing convergence scale factor

latitude	φ	:	**37° 00' 00" N**	easting	E	=	267 078.4121
longitude	λ	:	**42° 09' 00" E**	northing	N	=	314 642.5483
convergence	γ	=	+ 1° 44' 49".7551	scale factor	k	=	1.000 583 192 502

Conversion of stereographic grid to Clarke 1880S geographicals

Inverse process is to convert stereographic planar coordinates into geodetic coordinates:

input: easting northing output: latitude longitude convergence scale factor

easting	E	:	**267 078.4121**	latitude	φ	=	37° 00' 00" N
northing	N	:	**314 642.5483**	longitude	λ	=	42° 09' 00" E
convergence	γ	=	+ 1° 44' 49".7551	scale factor	k	=	1.000 583 192 502

10.8 Polyeder Mapping

A trapezoidal map is the map of an ellipsoid on an enclosing polyhedron. Equidistant straight lines characterise the Polyhedral or Polyeder projection representing parallels of latitude and convergent straight lines representing meridians (Figure 97). It is the earliest form of a polyhedric map, having been used for star charts as early as 1426. Nicholaus Germanus applied it for a Map of the World in 1466.

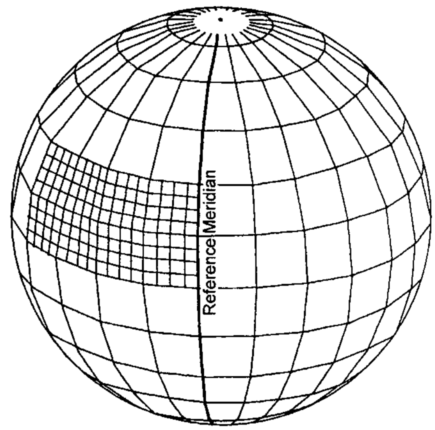


Figure 97: Polyeder mapping system

Prussian Polyhedric Mapping

Initially, the Topographic Services of the Royal Kingdom systematically produced polyhedric maps of Prussia. Therefore, geodesists also title it the Prussian polyhedral projection, intending to project the ellipsoid orthogonally onto each map face. The polyhedral projection has been used in various countries for mapping purposes. When many of these maps have been made, it will be found they cannot be joined properly. Consequently, this projection was never used for nautical charts (GG, 1986).

Indonesian Polyhedric Mapping

Before 1950, the topographical services (TS) of Indonesia have used this type of projection system for topographical maps between 6° N and 10° S. In Indonesia, the geodesists identify it as a polyeder projection. The TS projects the surface under the ellipsoid on the developable cones concerned according to an uncomplicated method so that the representation is conformal. It is a conformal projection using a series of cutting cones. Polyeder maps have an approximate square (trapezoidal) form AB (Figure 98). This zone is projected on the developed surface of the cone ABS_1, also zone BC on the developed surface of the cone BCS_2, and so on. All other succeeding cones can be developed to a flat plane. Consequently, in fact two non-parallel converging meridians and two circular parallels limit every map.

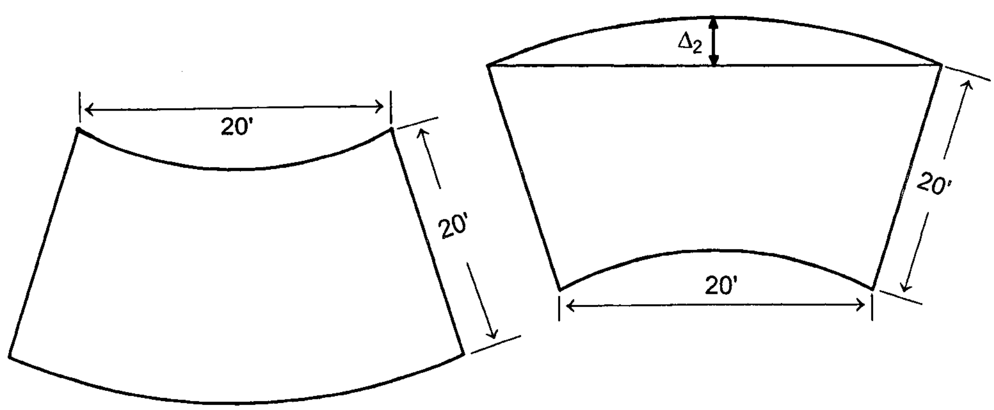


Figure 98: Polyeder conical construction AB

Beside conformality, this technique - just as any other method - does not satisfy the both other principles of equidistance and equivalence. Because topographical maps are often used to measure distances, bearings, angles and areas, retaining conformality is essential and to satisfy the requirements of equidistance and equivalence as close as possible. Choosing only a small square portion of a country is one method to reduce distance and area distortion. Therefore, the TS of Indonesia have limited the arc distance between both meridians and parallels to a section of 20' square.

Figure 99 and Figure 100 give a much-exaggerated representation of the situation. That the distortions are negligible can be concluded from (Figure 98), in which the meridional arc-to-chord distance, Δ_1, is 26.9 m only. The difference in the length between that latitudinal arc S and its chord D is 0.05 m approximately. The convergence factor, γ, is 3' 37" between both meridians.

At the 9° 50' S parallel the distance, Δ_2, between the longitudinal arc and its chord is only 5.0 m in the worst case. For the 9° 50' S parallel, the difference in length between the lower and the upper parallel is 37.0 m, i.e. 0.05 millimetres at a map scale of 1:100 000.

Matching adjacent sheets using this projection without gaps is not possible. Furthermore, in a strict sense of geometric projections cannot be spoken of a proper, conventional geodetic method. In modern times, a better solution is imperative for an equatorial territory of the Republic of Indonesia between 95° and 142° E.

Adopting LCC [10.1], normal Mercator [10.3], or Gauss-Krüger [10.4] projection maps instead of local polyeder maps would be a solution. Consequently, an inverse conversion is essential as discussed in [conversion of polyeder coordinates]. In the USA, a polyconic map projection was adopted instead (Adams, 1921).

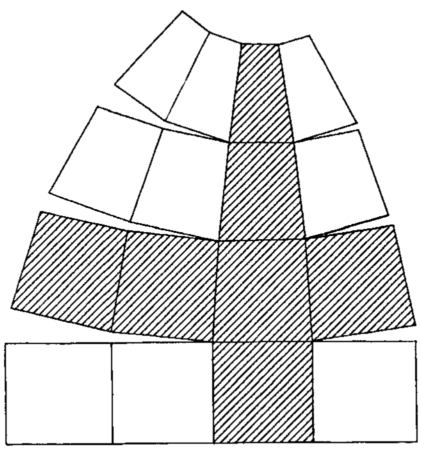


Figure 99: Parallel bursts

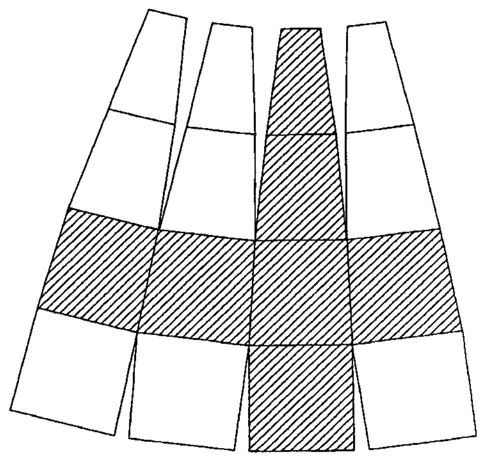


Figure 100: Meridian bursts

Polyeder Mapping Equations

Symbols and definitions

The equations for the ellipsoid constants are given in (Table 19): pp 123.
All angles are expressed in radians [18.15], A_15POLY.FOR

a	semi-major axis of the ellipsoid
b	semi-minor axis of the ellipsoid
fl	flattening of the ellipsoid
f	inverse flattening of the ellipsoid
φ_r	latitude of reference (usually the equator)
λ_r	longitude of reference, positive east
φ_0	local latitude of a polyeder central point (CP), positive north
λ_0	local longitude of a polyeder CP, positive east
φ	parallel of geodetic latitude of ellipsoid, positive north
λ	meridian of geodetic longitude of ellipsoid, positive east

x	easting polyeder grid coordinate
y	northing polyeder grid coordinate
B_p	polyeder latitude, referring to φ_r
L_p	polyeder longitude, referring to λ_r
e	first eccentricity
e^2	first eccentricity squared

Computation of Zone Constants

Constants and expressions within the polyeder mapping equations is ellipsoid and zone specific.

$$N_0 = a / (1 - e^2 \sin \varphi_0^2)^{½} \tag{10.08.01}$$

$$R_0 = a (1 - e^2) / (1 - e^2 \sin \varphi_0^2)^{3/2} \tag{10.08.02}$$

$$A' = 1 / (N_0 \cos \varphi_0 \sin 1'') \tag{10.08.03}$$

$$B' = 1 / (R_0 \sin 1'') \tag{10.08.04}$$

$$C' = \tan \varphi_0 / (N_0^2 \cos \varphi_0 \sin 1'') \tag{10.08.05}$$

$$D' = \tan \varphi_0 / (2 R_0 N_0 \sin 1'') \tag{10.08.06}$$

Inverse Conversion Computation

(Direct conversion computation is not required)

Input: grid coordinates of a polyeder point P (X, Y)
Output: geodetic coordinates of a point P (φ, λ), E or W of Greenwich

formulae hold for a central point in the Northern Hemisphere:

$$B_p = + B' Y - D' X^2 \tag{10.08.07}$$

$$L_p = + A' X + C' X Y \tag{10.08.08}$$

formulae hold for a central point in the Southern Hemisphere:

$$B_p = - B' Y - D' X^2 \tag{10.08.09}$$

$$L_p = + A' X - C' X Y \tag{10.08.10}$$

$$\varphi = \varphi_r + \varphi_0 + B_p \tag{10.08.11}$$

$$\lambda = \lambda_r + \lambda_0 + L_p \tag{10.08.12}$$

Organisation of the Indonesian Polyeder mapping system

Reference Meridian (RM) of Jakarta1910 or RM Jakarta1924 is the zero-Meridian of the polyeder mapping system in Indonesia. The polyeder projection is defined as a cluster of noncontiguous blocks for the area between 10°S and 6°N, and 12°E and 12°W, and shown in (Table 27).

map nr		1	2	3	....
I	φ =	5° 50' N	5° 50' N	5° 50' N	....
	λ =	11° 50' W	11° 30' W	11° 30' W	....
II	φ =	5° 30' N	5° 30' N	....	
	λ =	11° 50' W	11° 30' W	....	

Table 27: Organisation of polyeder mapping

The Indonesian grid system is defined by latitudinal and longitudinal squares (20' × 20') with the following properties:

- centre of each square is defined by the latitude [20' (m – 1)+10'] north or south of the equator, see note
- centre of each square is defined by the longitude [20' (n – 1)+10'] east or west of the RM
- all maps have a central point (CP) and a local, rectangular grid system with its own origin
- origin is the centre (CP) of each square, thus X (easting) and Y (northing) are always zero
- Y-axes of each block coincide with the map meridian referring to the RM

Quadrant	NW	NE	SW	SE
x' =	−18 461.12	+18 461.12	−18 471.40	+18 471.40
y' =	+18 431.63	+18 431.63	−18 426.38	−18 426.36

Table 28: Polyeder corner coordinates

The corner points of map (1-II) have the following coordinates (Table 28):
All maps of a zone, such as 1-II, 2-II, 3-II, and so on, have identical coordinates as map 1-II (Figure 101)

Conversion of Polyeder Coordinates

Adopting universal Mercator (UM) or transverse Mercator (GK) projection maps instead of local polyeder maps requires the conversion from eastings, northings (x, y) into geodetic coordinates (φ, λ_p) related to the zero-Meridian of Greenwich.

For the Indonesian territory, the conversion from local eastings and northing into geodetic coordinates requires some uncomplicated formulae and the use of a reference meridian (RM):

- Jakarta1910 106° 48' 37".05 E of Greenwich
- Jakarta1924 106° 48' 27".79 E of Greenwich

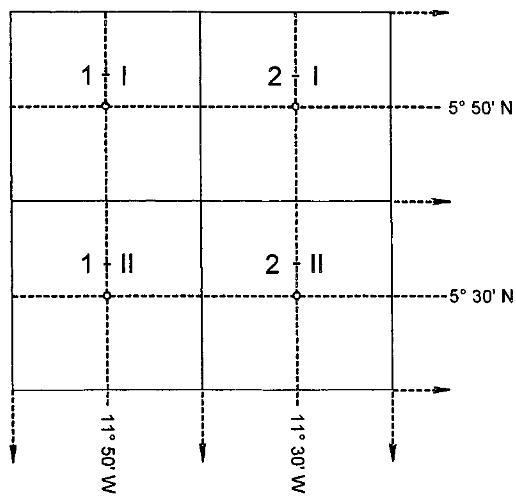


Figure 101: Part of the Indonesian Polyeder mapping system

Polyeder mapping system - survey grid of Indonesia					
Zone		: polyeder zone 1-I			
reference ellipsoid		: Bessel 1841	lat. of CP-polyeder	φ_0 :	**5° 50' 00" N**
semi-major axis	a	: **6 377 397.155**	lon. of CP-polyeder	λ_0 :	**11° 50' 00" W of J.**
recipr. flattening	f^1	: **299.15281 285**	parallel of origin,	φ_r :	**0° 00' 00" N**
A'	=	3.251033366934337E-02	reference meridian	λ_r :	**106° 48' 27".79 E**
B'	=	3.255705509093393E-02			
C'	=	5.207884527277476E-10			
D'	=	2.607684454830447E-10			See (Anon, 1950), Tafel XXII

Conversion of Polyeder-grid to Jakarta1924 geographicals

Converting polyeder coordinates into geodetic coordinates:

input:	easting	northing		output:			latitude	longitude
x	easting	:	**-18450.24**	latitude	φ =	B_p	6° 00' 00" N	
y	northing	:	**18431.91**	longitude	λ_p =	L_p	12° 00' 00" W. of Jak	
				longitude	λ =		94° 48' 27".79 E	

input:	easting	northing		output:			latitude	longitude
x	easting	:	**18461.12**	latitude	φ =	B_p	5° 40' 00" N	
y	northing	:	**-18426.46**	longitude	λ_p =	L_p	11° 40' 00" W. of Jak.	
				longitude	λ =		95° 08' 27".79 E	

Note

In Indonesia, the central point of each square is defined by the latitude [20' (m - 1)+ 10'] north or south of the Equator and by the longitude [20' (n - 1)+ 10'] east or west of the RM, in which m and n are the number of maps counted from the Equator and the RM, respectively.

10.9 Conversions between Grid Systems

Conversion of coordinates between projections and zone systems

Each time a survey line crosses from one zone to another, a conversion is required. Conversion of coordinates from one grid system into a different grid system is done, using the preferred procedure as outlined in [9.4]. To convert plane coordinates in one system to plane coordinates in another system or zone, it is paramount to calculate the geodetic coordinates from the plane coordinates of the first grid system, and next to convert those geodetic coordinates positions to the plane coordinates of the adjacent zone or the second grid system (Figure 102) (Field, 1980; Clarke, 1973).

LCC and GK Zones - Applications I - IV

Coordinates may be converted from one grid system to another. Exemplified are conversions between:

- conversion from LCC-SPCS27 Texas north central to UTM grid zone 14 and 15 (DA, 1958)
- conversion from CK42 grid zone 4 (CM 21° zone 6°) to (CM 24° zone 3°) and to zone 5 (CM 27° zone 6°)
- conversion from GK grid (CM 24° zone 3°) to CK42 grid zone 5 (CM 27° zone 6°)
- conversion from OSC RD1918 the Netherlands to OSC grid
- conversion from OSC RD2000 the Netherlands to OSC grid and to GK grid CM 5° zone 6°
- conversion from ANS zone 54 (CM 141° zone 6°) to ANS zone 55 (CM 147° zone 6°) and as a test
- geodesic from ANS zone 54 (CM 141° zone 6°) to ANS zone 55 (CM 147° zone 6°).

Application Ia - LCC-SPCS27 Texas zone north central in the Northwestern Hemisphere			
zone parameters		:	*SPCS Texas*, zone north central
reference ellipsoid		:	Clarke 1866 (NAD27)
semi-major axis	a	:	**20 925 832.16 ft**
recipr. flattening	f^{-1}	:	**294.97869 82**
lower parallel	φ_l	:	**32° 08' 00" N**
upper parallel	φ_u	:	**33° 58' 00" N**
lat. grid origin	φ_b	:	**31° 40' 00" N**
central parallel	φ_0	=	**33° 03' 05".8352 N**
lon. grid origin	λ_p	:	**97° 30' 00" W**
scale factor	k_0	=	**0.999 872 629 332**
false easting	E_0	:	**2 000 000 ft**
false northing	N_b	:	**0 ft** (Units in US survey feet)

Conversion of Lambert grid to NAD27 geographicals

inverse calculation is to convert LCC- planar coordinates into geodetic coordinates

input: easting northing				output: latitude longitude convergence scale factor			
easting	E_i	:	**2 439 603.2585**	latitude	φ_i	=	34° 15' 34".7420 N
northing	N_i	:	**946 451.2622**	longitude	λ_i	=	96° 02' 43".1579 W
convergence	γ_i	=	+0° 47' 36".1444	scale factor	γ_i	=	1.000 094 894 466

Conversion from LCC-SPCS27 to UTM grid

zone parameters UTM zone 14 and zone 15 - NW Hemisphere							
zone parameters:			UTM, zone 14 and zone 15				
reference ellipsoid		:	Clarke 1866	parallel of origin	$\varphi_{0\,Z14}$	:	**0° 00' 00" N**
semi-major axis	a	:	**6 378 206.4 m**	CM zone 14	λ_0	:	**99° 00' 00" W**
recipr. flattening	f^{1}	:	**294.97869 82**	parallel of origin	$\varphi_{0\,Z15}$	:	**0° 00' 00" N**
scale factor	k_0	:	**0.9996**	CM zone 15	λ_0	:	**93° 00' 00" W**
false easting	E_0	:	**500 000 m**	false northing	N_0	:	**0 m**

Application Ib - conversion of NAD27 geographicals to GK-UTM grid zone 14

input: latitude longitude				output: easting northing convergence scale factor			
zone 14:			CM 99°W				
latitude	φ_i	:	**34° 15' 34".7420 N**	easting	E_i	=	772 075.8149
longitude	λ_i	:	**96° 02' 43".1579 W**	northing	N_i	=	3 794 702.1725
convergence	γ_i	=	+ 1° 39' 51".6267	scale factor	k_i	=	1.000 512 588 175

Application Ic - conversion of NAD27 geographicals to GK-UTM grid zone 15

input:	latitude	longitude	output:	easting	northing	convergence	scale factor

zone 15: CM 93°W

latitude	φ_i	:	**34° 15' 34".7420 N**	easting	E_i	=	219 574.6186
longitude	λ_i	:	**96° 02' 43".1579 W**	northing	N_i	=	3 794 948.5090
convergence	γ_i	=	- 1° 42' 55".6734	scale factor	k_i	=	1.000 569 468 231

For further reading: (Schödlbauer, 1982).

Observe

For the military maps, the worldwide grid system adopted by the US DoD NGA makes use of the universal transverse Mercator UTM-grid and the universal polar stereographic UPS-grid superimposed on the respective projections.

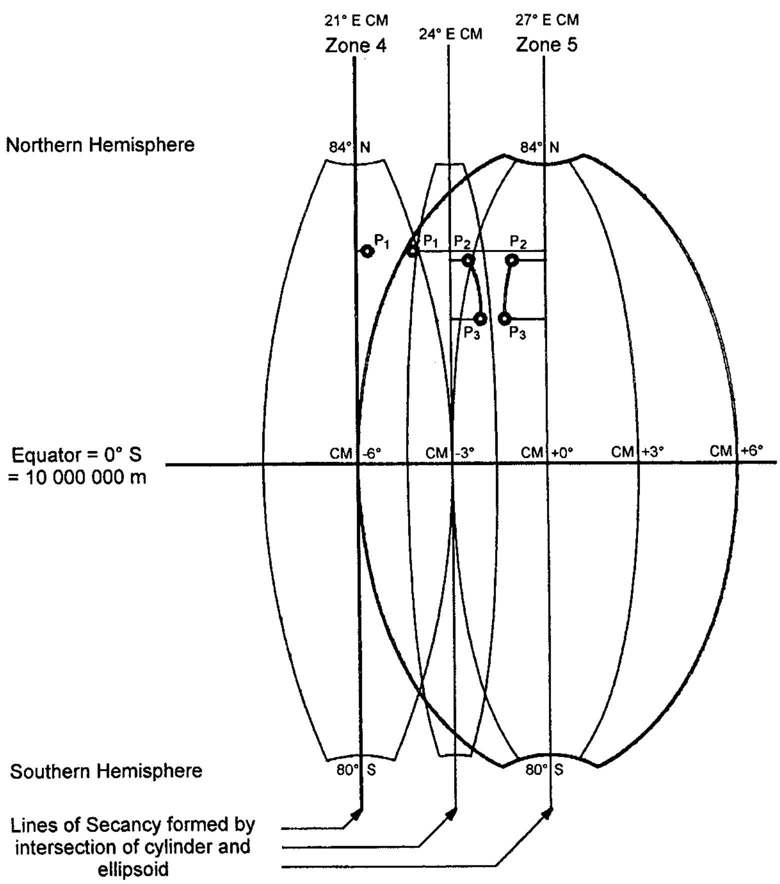


Figure 102: Conversion between 3° - 6° zone systems and 6° - 6° zone systems

Application IIa - GK-zone conversion 6° GK-zone into 3° and 6° GK-zones

zone parameters		:	CK42, Zone 4	(Tarczy-Hornoch, 1959)			
reference ellipsoid		:	Krassovsky 1940	false easting	E_0	:	**500 000 m**
semi-major axis	a	:	**6 378 245**	false northing	N_0	:	**0 m**
recipr. flattening	f^1	:	**298.3**	parallel of origin	φ_0	:	**0° 00' 00" N**
scale factor	k_0	:	**1.0000**	CM of zone	λ_0	:	**21° 00' 00" E**

conversion from CM 21°E (6° zone) zone 4

CM	λ_0	:	21° E				
easting	E_1	:	**501 500.000**	latitude	φ_1	=	47° 50' 00".335083 N
northing	N_1	:	**5 300 000.000**	longitude	λ_1	=	21° 01' 12".128378 E

conversion from 6° GK-zone into 3° GK-zone of CM 24°

CM	λ_0	:	24° E				
latitude	φ_1	:	**47° 50' 00".33508 N**	easting	E_1	=	276 910.3034
longitude	λ_1	:	**21° 01' 12".12838 E**	northing	N_1	=	5 304 301.6859

conversion from 6° GK-zone into 6° GK-zone of CM 27° zone 5

CM	λ_0	:	27° E				
latitude	φ_1	:	**47° 50' 00".33508 N**	easting	E_1	=	52 381.4871
longitude	λ_1	:	**21° 01' 12".12838 E**	northing	N_1	=	5 317 343.4834

Application IIb - GK-zone conversion 3° Gk-zone into 6° GK-zone

Zone Parameters			CK42	(Tarczy-Hornoch, 1959)			
reference ellipsoid		:	Krassovsky 1940	false easting	E_0	:	**500 000 m**
semi-major axis	a	:	**6 378 245**	false northing	N_0	:	**0 m**
recipr. flattening	f^1	:	**298.3**	parallel of origin	φ_0	:	**0° 00' 00" N**
scale factor	k_0	:	**1.0000**	CM of zone	λ_0	:	**24° 00' 00" E**

conversion from CM 24°E (3° zone)

CM	λ_0	:	24° E				
easting	E_2	:	**537 127.9991**	latitude	φ_2	=	42° 24' 17".6282 N
northing	N_2	:	**4 696 793.1726**	longitude	λ_2	=	24° 27' 03".5764 E

conversion from 3° zone into (6° zone) zone 5

CM	λ_0	:	27° E				
latitude	φ_2	:	**42° 24' 17".6282 N**	easting	E_2	=	290 147.0589
longitude	λ_2	:	**24° 27' 03".5764 E**	northing	N_2	=	4 699 843.6807

Note

This procedure does not change the Datum. See [7, Spatial Coordinate Calculations], for the procedure to use when changing from one Datum to another Datum.

Application IIIa - conversion of OSC-Zone and GK-Zone

Original Hendrik J. Heuvelink's (RD1918)
Stereographic projection and grid system of the Netherlands has the following specifications:

projection		:	Stereographic Conformal - Gauss-Schreiber (GS) type
Datum		:	RD1918
reference ellipsoid		:	Bessel 1841
semi-major axis	a	:	**6 377 397.155**
reciprocal flattening	f^{1}	:	**299.15281 285**
latitude of true origin	B_0	:	**52° 09' 22".1780 N (Amersfoort)**
longitude of true origin	L_0	:	**5° 23' 15".5000 E**
latitude of sphere - origin	b_0	:	52° 07' 15".9501 N
longitude of sphere - origin	l_0	:	5° 23' 15".5000 E
false easting	x_0	:	**155 000 m**
false northing	y_0	:	**463 000 m**
scale factor at origin	k_0	:	**0.99990 79**
unit		:	metre
Gaussian dilatation constant	α	=	1.00047 58566 84165
dilatation factor	m	=	3.773953831304509E-03
radius of the Gaussian sphere	R_A	=	6 382 644.5710
area bounded by eastings	x	:	5 000 ... 280 000
area bounded by northings	y	:	300 000 ... 620 000

Application IIIa - conversion of RD1918 geographicals to stereographic grid

converting geodetic coordinates into stereographic planar coordinates:

input: latitude longitude output: easting northing convergence scale factor

latitude	φ_i	:	**51° 59' 13".3938 N**	easting	x_i	=	86 346.7840
longitude	λ_i	:	**4° 23' 16".9953 E**	northing	y_i	=	444 659.9720
convergence	γ_i	=	- 0° 47' 18".4573	scale factor	k_i	=	0.99993 88854

Conversion of stereographic grid to RD1918 geographicals

converting stereographic planar coordinates into geodetic coordinates:

input: easting northing output: latitude longitude convergence scale factor

easting	x_i	:	**86 346.7840**	latitude	φ_i	=	51° 59' 13".3938 N
northing	y_i	:	**444 659.9720**	longitude	λ_i	=	4° 23' 16".9953 E
convergence	γ_i	=	- 0° 47' 18".4573	scale factor	k_i	=	0.99993 88854

(Figure 96): pp 230; (Heuvelink, 1918; Schreiber, 1897; Schut, 1992; Strang van Hees, 2006).

Application IIIb - conversion of OSC-zone

using oblique stereographic projection and grid system with following specifications:

projection:			Heuvelink's stereographic conformal of the Netherlands - GS type
hypothetical datum		:	RD2000 modified
reference ellipsoid		:	GRS80 (new international)
semi-major axis	a	:	**6 378 137.**
reciprocal flattening	f^{1}	:	**298.25722 21008 827**
latitude of true origin	B_0	:	**52° 09' 18".6200 N (Amersfoort)**
longitude of true origin	L_0	:	**5° 23' 13".9327 E**
latitude of sphere - origin	b_0	:	**52° 07' 12".0048 N**
longitude of sphere - origin	l_0	:	**5° 23' 13".9327 E**
false easting	E_0	:	**1 155 000 m**
false northing	N_0	:	**2 463 000 m**
scale factor at origin	k_0	:	**0.99990 79**
unit		:	metre
Gaussian dilatation constant	α	=	1.00047 7334813218
dilatation factor	m	=	3.785137356788674E-03
radius of the Gaussian sphere	R_A	=	6 383 399.99756 33720
area bounded by eastings	E	:	1 005 000 ... 1 280 000
area bounded by northings	N	:	2 300 000 ... 2 620 000

Conversion of RD2000 geographicals to stereographic grid

input: latitude longitude output: easting northing convergence scale factor

latitude	φ_i	:	**51° 59' 09".9145 N**	easting	E_i	=	1 086 346.9323
longitude	λ_i	:	**4° 23' 15".9531 E**	northing	N_i	=	2 444 660.1749
convergence	γ_i	=	- 0° 47' 18".0054	scale factor	k_i	=	0.99993 88779

Application IIIc - conversion GK-Zone ETRS89

using transverse Mercator projection and grid system with following specifications

projection		:	*hypothetical* transverse Mercator - Gauss-Krüger - GK type
datum		:	ETRS89
reference ellipsoid		:	GRS80 (new international)
semi-major axis	a	:	**6 378 137.0**
reciprocal flattening	f^{1}	:	**298.25722 21008 827**
latitude of true origin	φ_0	:	**0° 00' 00".00 N**
cm - longitude of true origin	λ_0	:	**5° 00' 00".00 E**
false easting	E_0	:	**500 000 m**
false northing	N_0	:	**0 m**
scale factor at origin	k_0	:	**0.9996**
unit		:	metre
6° zone bounded by longitudes		:	2° 00' 00".00 E ... 8° 00' 00".00 E

Conversion of RD2000 geographicals to transverse Mercator grid

input: latitude longitude output: easting northing convergence scale factor

latitude	φ_i	:	**51° 59' 09".9145 N**	easting	E_i	=	457 956.8844
longitude	λ_i	:	**4° 23' 15".9533 E**	northing	N_i	=	5 759 667.7883
convergence	γ_i	=	– 0° 28' 56".5082	scale factor	k_i	=	0.99962 16994

Application IVa - conversion GK-zone

UTM zone 54 and 55 - Southern Hemisphere (NMC, 1986)						
zone parameters		: zone 54 / 55				
reference ellipsoid		: ANS1966	false easting	E_0	:	**500 000 m**
semi-major axis	a	: **6 378 160**	false northing	N_0	:	**10 000 000 m**
recipr. flattening	f^1	: **298.25**	parallel of origin	φ_0	:	**0° 00' 00" S**
scale factor	k_0	: **0.9996**	CM of zone 54	λ_0	:	**141° 00' 00" E**
			CM of zone 55	λ_0	:	**147° 00' 00" E**

Conversion of AGD66 geographicals to UTM grid - zone 54

direct calculation is to convert geodetic coordinates into GK-planar coordinates

input: latitude longitude			output: easting northing convergence scale factor			
Sta. Flinders Peak						
Latitude	φ_1 :	**37° 57' 09".1288 S**	easting	E_1	=	800 817.4065
longitude	λ_1 :	**144° 25' 24".7866 E**	northing	N_1	=	5 793 905.6504
convergence	γ_1 =	- 2° 06' 25".5312	scale factor	k_1	=	1.000 714 682 440
Sta. Buninyong (Australia)			(NMC, 1986)			
Latitude	φ_2 :	**37° 39' 15".5571 S**	easting	E_2	=	758 053.0896
longitude	λ_2 :	**143° 55' 30".6330 E**	northing	N_2	=	5 828 496.9735
convergence	γ_2 =	- 1° 47' 16".6717	scale factor	k_2	=	1.000 420 299 351

Conversion of AGD66 geographicals to UTM grid - zone 55

Direct calculation is to convert geodetic coordinates into GK-planar coordinates

input: latitude longitude			output: easting northing convergence scale factor			
Sta. Flinders Peak						
latitude	φ_1 :	**37° 57' 09".1288 S**	easting	E_1	=	273 629.4358
longitude	λ_1 :	**144° 25' 24".7866 E**	northing	N_1	=	5 796 305.2356
convergence	γ_1 =	+1° 35' 06".7564	scale factor	k_1	=	1.000 231 177 620
Sta. Buninyong						
latitude	φ_2 :	**37° 39' 15".5571 S**	easting	E_2	=	228 742.0764
longitude	λ_2 :	**143° 55' 30".6330 E**	northing	N_2	=	5 828 074.2081
convergence	γ_2 =	+1° 52' 46".3554	scale factor	k_2	=	1.000 506 410 833

AMG test line connects Flinders Peak and Buninyong between AMG Zones 54 and 55.

Application IVb - AMG geodetic test-line - inverse problem

Given	:	ellipsoid ANS			
semi-major axis a	:	**6 378 160**	recipr. flattening f^{-1}	:	**298.25**
latitude φ_1	:	**37° 57' 09".1288 S**	latitude φ_2	:	**37° 39' 15".5571 S**
longitude λ_1	:	**144° 25' 24".7866 E**	longitude λ_2	:	**143° 55' 30".6330 E**

Output:

n	geodesic - $S_{1\text{-}2}$	true bearing - $\alpha_{1\text{-}2}$	true bearing - $\alpha_{2\text{-}1}$
1 000	54 972.1599	306° 52' 07".3416	127° 10' 27".0839

Zone 54

ellipsoidal and grid calculation (manual calculation)

using [18.12, 18.14] gives:				grid bearing	$t_{1\text{-}2}$	128° 58' 07".6874
				+ convergence	γ_1	−1° 47' 16".6717
true distance	$S_{1\text{-}2}$	:	54 972.1599	- (t - T)	$\delta_{1\text{-}2}$	+23".9243
true azimuth	$\alpha_{1\text{-}2}$	:	127° 10' 27".0839	true azimuth	$\alpha_{1\text{-}2}=$	127° 10' 27".0914
true azimuth	$\alpha_{2\text{-}1}$	:	306° 52' 07".3416			
				grid bearing	$t_{2\text{-}1}$	308° 58' 07".6874
grid distance	$D_{1\text{-}2}$	:	55 003.1493	+ convergence	γ_2	−2° 06' 25".5312
true distance	$S_{calc\ 1\text{-}2}$	:	54 972.1704	- (t - T)	$\delta_{2\text{-}1}$	−25".1751
See (9.03), using				true azimuth	$\alpha_{2\text{-}1}=$	306° 52' 07".3313
scale factor	$k_{1\text{-}2}$	:	1.000 563 537 799			

Zone 55

ellipsoidal and grid calculation (manual calculation)

using [18.12, 18.14] gives:				grid bearing	$t_{1\text{-}2}$	125° 17' 20".0532
				+ convergence	γ_1	+1° 52' 46".3554
true distance	$S_{1\text{-}2}$	:	54 972.1599	- (t - T)	$\delta_{1\text{-}2}$	−20".6818
true azimuth	$\alpha_{1\text{-}2}$	:	127° 10' 27".0839	true azimuth	$\alpha_{1\text{-}2}=$	127° 10' 27".0904
true azimuth	$\alpha_{2\text{-}1}$	:	306° 52' 07".3416			
				grid bearing	$t_{2\text{-}1}$	305° 17' 20".0532
grid distance	$D_{1\text{-}2}$	:	54 992.2053	+ convergence	γ_2	+ 1° 35' 06".7564
true distance	$S_{calc\ 1\text{-}2}$	:	54 972.1497	- (t - T)	$\delta_{2\text{-}1}$	+19".4756
See (9.03), using				true azimuth	$\alpha_{2\text{-}1}=$	306° 52' 07".3340
scale factor	$k_{1\text{-}2}$	:	1.000 364 832 614			

Observe

The conversion procedure does not change the Datum. For this reason, it is not allowed to use two different Datums! See [7, Spatial Coordinate Calculations], for the procedure to use when changing from one Datum to another.

11. Astrolabe Observations

Prismatic Astrolabe

An astrolabe is used to determine the latitude and longitude of the geodesist, assuming the star positions are known from an Almanac. The aperture of a prismatic astrolabe is small, a refracting prism and some mercury make up the most important parts of the astrolabe. A direct image is observed along with an image reflected off the pool of mercury to provide the position data. An example of this type is the French Danjon astrolabe. A special prepared Carl Zeiss Ni2 automatic level instrument offers an alternative.

11.1 Reduction of Astrolabe Observations

Author: L.A. Kivioja, J.A. Mihalko (Kivioja, 1985).
New Method for Reduction of Astrolabe Observations Using Rectangular Coordinates on the Celestial Sphere

In Brief

A new reduction method using x, y, z-coordinates is derived for astrolabe observations. By this method, the latitude and the longitude of the station are computed without the need of a priori knowledge of the station position. This method is a significant development in data reduction of astrolabe and other almucantar observations due to its mathematical exactness, simplicity, and the ease of handling the associated statistics.

Introduction

By definition, an almucantar is a small circle on the celestial sphere of constant zenith distance, or of constant altitude. Three stars (points) on one almucantar define the plane of the almucantar and its position on the celestial sphere.

The meridian of a station by definition is the great circle on the celestial sphere, which is cut by the plane passing through the spin axis of the Earth and the zenith of the station.

The angle between the Greenwich and the local meridian is the longitude of the station, and the angle in the meridian plane from the zenith to the nearest celestial Pole is the co-latitude of the station, and the declination of the zenith is also equal to the latitude of the station.

In astrolabe observations, stars are timed at the instant, they cross the almucantar of the astrolabe. This new reduction method computes the location of the observer's zenith on the celestial sphere, and then the astro latitude and astro longitude of the station.

The method does not require a priori approximation of the latitude, nor the longitude of the station to be known - except for the creation of the star list for the observations.

Knowledge of the refraction is not needed as such, only the variations in the refraction for the stars within a set, because the variable refraction lowers or elevates the stars affecting the times when the stars cross the almucantar of the astrolabe. Thermal effects may also slightly alter the angle between the two reflecting surfaces of the astrolabe with similar effects.

11.2 Derivation of the Method

Select a rectangular coordinate system (x, y, z) so that its origin is at the centre of the infinitely large celestial sphere placing the origin at the station; select the positive z-axis to go through the north celestial Pole; select the positive x-axis to be the intersection of the planes of the celestial equator and the Greenwich Meridian and then the positive y-axis is in the meridian plane of 90° E longitude in the equatorial plane.

The x- and y-axes are thus in the plane of the celestial equator, and they do rotate together with the Earth. The astronomical zenith with its constant coordinates, and the almucantar around it will also rotate together with the Earth.

The Greenwich sidereal time GST is given by:

$$GST = GHA + RA \tag{11.01}$$

in which GHA is the Greenwich hour angle of any star, and RA is the right ascension of the same star. The Greenwich hour angle of a star is obtained from Equation (11.01) as:

$$GHA = GST - RA \tag{11.02}$$

The GST is computed from the recorded UTC (Universal Coordinated Time) for the instant the star crosses the almucantar, and the declinations and right ascensions (δ, RA) are computed (updated) for the same instant.

The rectangular Cartesian coordinates x, y, z in the chosen coordinate system can be obtained for any star on the sphere with radius r from:

$$\begin{aligned} x &= r \cos\delta \cos(360° - GHA) \\ y &= r \cos\delta \sin(360° - GHA) \\ z &= r \sin\delta \end{aligned} \tag{11.03}$$

in which δ is the declination of the star, and GHA is the Greenwich hour angle of the same star.

For simplicity, select the celestial sphere to be a unit-sphere, and then $r = 1$. Equation (11.03) will become equation (4) giving x, y, and z-coordinates for all stars in the set when the corresponding δ's and GHA's are used for each star:

$$\begin{aligned} x &= +\cos\delta \cos(GHA) \\ y &= -\cos\delta \sin(GHA) \\ z &= +\sin\delta \end{aligned} \tag{11.04}$$

The rectangular coordinates of each star in the set, at the instant the star crosses the almucantar of the astrolabe will be obtained from equation (11.04):

x_1, y_1, z_1 for star S_1,
x_2, y_2, z_2 for star S_2,
x_3, y_3, z_3 for star S_3,

| | | |

x_n, y_n, z_n for the last star in the set

A common practice is to have approximately 30 stars in a set, which takes approximately two hours to observe, so n is approximately 30.

From analytical geometry, recall that the angle ξ = angle S_1OS_2 at the origin O between lines

$OS_1 = l_1 = (x_1^2 + y_1^2 + z_1^2)^{½}$, and
$OS_2 = l_2 = (x_2^2 + y_2^2 + z_2^2)^{½}$ is obtained from

$$\cos\xi = (x_1 x_2 + y_1 y_2 + z_1 z_2) / (l_1 l_2) \tag{11.05}$$

Because the unit sphere was chosen for equation (11.04), and because the stars are on the surface of this chosen unit sphere, then $l_1 = l_2 = 1$, and equation (11.05) reduces to

$$\cos\xi = x_1 x_2 + y_1 y_2 + z_1 z_2 \tag{11.06}$$

Apply equation (11.06) to the zenith having coordinates x'_0, y'_0, z'_0, and to the stars S_1, S_2, S_3, ... S_n when they are on the almucantar at zenith distance ξ and obtain:

$$\begin{array}{llllll} x_1\, x'_0 & + y_1\, y'_0 & + z_1\, z'_0 & = & \cos \xi & \\ x_2\, x'_0 & + y_2\, y'_0 & + z_2\, z'_0 & = & \cos \xi & \\ x_3\, x'_0 & + y_3\, y'_0 & + z_3\, z'_0 & = & \cos \xi & \quad (11.07) \\ \vdots & \vdots & \vdots & & \vdots & \\ x_n\, x'_0 & + y_n\, y'_0 & + z_n\, z'_0 & = & \cos \xi & \end{array}$$

If the set has 30 stars, there are 30 equations in (11.07) with three unknowns. The coordinates x'_0, y'_0, z'_0 of the zenith are the unknowns to be solved from equation (11.07). The coordinates x'_0, y'_0, z'_0 of the zenith as solved from equation (11.07), must place the zenith on the chosen unit sphere. Mainly due to small observational errors, the computed zenith coordinates may not perfectly fulfil the condition:

$$(x'^2_0 + y'^2_0 + z'^2_0)^{½} = 1 \qquad (11.08)$$

as they should. The coordinates of the zenith fulfilling equation (11.08) on the unit sphere are obtained from:

$$\begin{array}{lll} x_0 & = & x_0' / (x_0'^2 + y_0'^2 + z_0'^2)^{½} \\ y_0 & = & y_0' / (x_0'^2 + y_0'^2 + z_0'^2)^{½} \\ z_0 & = & z_0' / (x_0'^2 + y_0'^2 + z_0'^2)^{½} \end{array} \qquad (11.09)$$

Because the declination of the zenith is φ (= latitude of the station), and the angle between the Greenwich and local meridian is λ (= longitude of the station, 0° to 360°, positive to the east), the rectangular coordinates x_0, y_0, z_0 of the zenith are:

$$\begin{array}{lll} x_0 & = & \cos \varphi \cos \lambda \\ y_0 & = & \cos \varphi \sin \lambda \\ z_0 & = & \sin \varphi \end{array} \qquad (11.10)$$

When equations (11.07) and (11.09) are solved using adjustment computation, x_0, y_0, z_0, are easily obtained with their standard deviations.

After x_0, y_0, z_0 are known, the latitude and the longitude of the station can be obtained from equation (11.10):

$$\begin{array}{lll} \varphi & = & \arctan z_0 / (x_0^2 + y_0^2)^{½} \\ \lambda & = & \arctan y_0 / x_0 \end{array} \qquad (11.11)$$

Equation (11.11) gives the desired astro-latitude φ, and astro-longitude λ of the station with their standard deviations in a very simple, and in a beautiful way. As can be seen, the mathematics of the step-by-step derivation of this new method shown by Equations (11.01, ..., 11.11), is quite easy, and straightforward, without any need for approximations of the station position. It can be expected that this method of reduction of astrolabe observations will be preferred by many over other methods so far published for all almucantar observations by astrolabes and theodolites.

A sample computation of this x-, y-, z-method is shown in (Table 29) for three stars to illustrate how easy this x-, y-, z-method is. The same zenith coordinates x_0, y_0, z_0 are obtained whether or not the constant refraction for the almucantar is included in ξ.

The actual computation, of course, is best done on a large computer with all the standard corrections (UTC-UT1, refraction, polar motion, diurnal Aberration, height of the station, ...) included, as described in astronomy textbooks (Mueller, 1977).

Numerical Example, Equations (11.01, ... , 11.11)

x, y, z - method, Stars 3367, 847, 667 on
$\zeta = 29° 59' 50''$ almucantar on 1984 Sep 20.

STAR No.	GST 1984 Sep 20	δ*	RA*	GHA*
3367	$01^h 40^m 41^s.901$	52° 25' 53".774	$17^h 10^m 08^s.5778$	127°.6380125
847	$01^h 43^m 31^s.796$	58° 20' 14".941	$22^h 28^m 37^s.5094$	48°.7279251
667	$01^h 46^m 09^s.730$	27° 43' 55".781	$17^h 45^m 50^s.5848$	120°.0786750

$$(11.07) \quad \begin{aligned} -0.372330823 \quad x_0' -0.482818460 \quad y_0' +0.792626074 \quad z_0' -0.866049644 &= 0 \\ +0.346252486 \quad x_0' -0.394518563 \quad y_0' +0.851154697 \quad z_0' -0.866049644 &= 0 \\ -0.443618438 \quad x_0' -0.765938856 \quad y_0' +0.465338964 \quad z_0' -0.866049644 &= 0 \end{aligned}$$

$$\begin{aligned} x_0' &= +0.040565383 \\ y_0' &= -0.760094999 \\ z_0' &= +0.648686065 \end{aligned}$$

$$(11.09) \quad \begin{aligned} x_0 &= x_0' / (x_0'^2 + y_0'^2 + z_0'^2)^{½} &= +0.040561661 \\ y_0 &= y_0' / (x_0'^2 + y_0'^2 + z_0'^2)^{½} &= -0.760025244 \\ z_0 &= z_0' / (x_0'^2 + y_0'^2 + z_0'^2)^{½} &= +0.648626534 \end{aligned}$$

$$(11.11) \quad \begin{aligned} \varphi &= \arctan z_0 / (x_0^2 + y_0^2)^{½} &= 40° 26' 17''.262 \text{ N} \\ \lambda &= \arctan y_0 / x_0^2 &= 86° 56' 42''.321 \text{ W} \end{aligned}$$

Means for the whole set of 30 stars:

$$\begin{aligned} \varphi &= 40° 26' 17''.355 \quad \pm 0''.215 \text{ N} \\ \lambda &= 86° 56' 42''.367 \quad \pm 0''.152 \text{ W} \end{aligned}$$

Table 29: Numerical example astrolabe observations

(War Office, 1958)

Acknowledgement

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12. About History - A Bird's-Eye View

12.1 In Antiquity

"What we know is very slight; what we don't know is immense" - Laplace (1827)

The history of astronomy is the growth of man's concept of his world, the source and essence of his life. In the ancient world and in the following centuries astronomy was already a highly developed science.

The other realms of knowledge, such as physics and chemistry, developed into sciences only in later centuries. Astronomy had already manifested itself in the ancient world as a system of theoretical knowledge that enabled man to prophesy even the terrifying eclipses and had become a factor in his spiritual strife.

In early times, when physical theory was but abstract speculation, astronomy was already an ordered system of knowledge giving practical orientation in 3D-space-and-time. In later centuries, astronomical research was directed ever more towards theoretical knowledge of the structure of the universe, far beyond any practical application, to satisfy the craving for truth.

Mathematicians gradually found the types of numbers and the operations with these numbers that are now comprised in the complex number system. The rigorous, axiomatic, deductive style of geometry yielded to inductive, intuitive insights, and purely geometric notions gave way to concepts of number and algebraic operations embodied in analytic geometry, the calculus and mechanics. It was the small intellectual aristocracy of the new mathematics that now spearheaded the forward thrust of science (Courant, 1964).

The complexity of a civilisation is mirrored in the complexity of its numbers. Number systems employed in mathematics are divided into five principal stages:

- system consisting of the positive integers only
- positive and negative integers and zero
- rational numbers, which include fractions as well as the integer
- real numbers, including the irrational numbers, such as π
- complex numbers, which introduce the "*imaginary*" number $\sqrt{-1}$

Positive integers are usually written 1, 2, 3, 4, .. However, they can and have been written in many other ways. The Romans wrote them I, II, III, IV, ... ; the Greeks wrote them α, β, γ, δ ... . All variations are equivalent. They use different symbols for entities whose meaning and order are uniformly understood. Negative numbers were not fully incorporated into mathematics until the publication of Girolamo Cardano's *Ars Magna* in 1545. Fractions are more ancient than the negative numbers, and were discussed at some length as early as 1550 BC in the Rhind Papyrus of Egypt. The present way of writing fractions (such as 1/4, 1/5, 8/13), and also the present way of doing arithmetic with them dates from the 15^{th}- and 16^{th}-centuries (Davis, 1964).

Observation of Earth's Shadow

By observation of Earth's shadow, it was probably the reason the Figure of the Earth was first found out to be nearly spherical. However, astronomers of antiquity had some incorrect considerations about its size, but sufficient for their purposes. Assuming was necessary as a preliminary step to establish a hypothesis on the Figure of the Earth. The method pursued in the earliest measure is that of Eratosthenes.

Eratosthenes, who flourished about 230 BC, is considered one of the founders of geodesy because he was the first to describe and apply a scientific measuring technique for determining the size of the Earth. He used a simple principle of estimating the size of a great circle passing through the North and South Poles. Eratosthenes observed that at Syene, in Upper Egypt, the Sun was exactly vertical at noon on the day of the summer solstice. At Alexandria the Sun's zenith distance was at the same day 7°12'. According to Cleomedes was the first observation made by remarking that the edges of a deep well threw no shadow on the bottom, the latter by using a σκαφη (a hemispherical bowl) with a vertical style. Eratosthenes supposed Alexandria and Syene to lie in the same meridian. He estimated their distance at five thousand stadia, which gave the Earth's circumference is

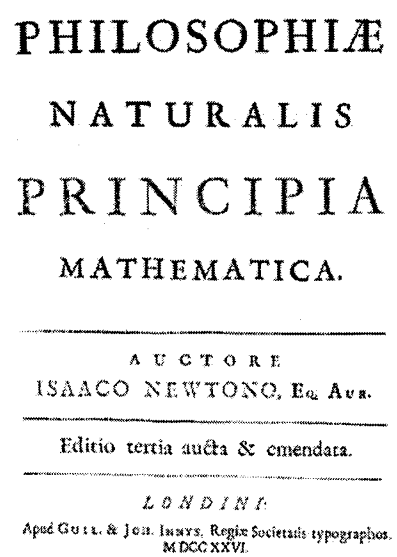
50 × 5000 = 250 000 stadia or 46250 km. The length of the stadium used here, is unknown. Pointing out the sources of error in this survey is unnecessary.

Reason for believing the Earth to be spherical was that eclipses of the Moon take place at different times about the noon of the places of observation. In the astronomical works of Ptolemy (about AD 137), the Earth's dimensions were not revealed.

An estimate was also attempted by Posidonius. It appears to have been intended only as rough guess. He observed that the star Canopus was seen at Rhodes just to touch the horizon. At Alexandria, he estimated its meridian altitude at 7½°. In this measure, it is merely necessary to observe that refraction was neglected, and that no accurate estimate could be formed of the distance of two places separated by the Mediterranean.

The Dark Age that followed the overthrow of the Alexandrian school put a stop to all speculations on the Figure of the Earth. The empires of the caliphs had extended over the greatest part of the civilised world. Works of the Greek astronomers had been translated and studied by the Arabs. Abdullah Al-Mamûn, who began his reign at Baghdad AD 814, measured an arc of the meridian in the plains of Mesopotamia. However, the Cubit, the unit of length is unknown (Airy, 1830).

Towards the end of the seventeenth century, hypothetical consideration about the shape of the Earth had given way to the first serious attempt to determine it by measurement. In 1528, the scene was shifted to Western Europe. Jean Fernel, a Parisian, observed the Sun's meridian altitude close to Paris. His observations were made with a triangle. The distance was calculated from the number of turns made by the wheel of his carriage.



PHILOSOPHIÆ

NATURALIS

PRINCIPIA

MATHEMATICA.

AUCTORE

ISAACO NEWTONO, Eq. Aur.

Editio tertia aucta & emendata.

LONDINI:

Apud Guil. & Joh. Innys, Regiæ Societatis typographos.
MDCCXXVI.

Figure 103: Cover 3rd amended edition of the Principia

12.1.1 Trigonometrical Surveys

Willebrord Snel van Royen (Snellius) was professor of mathematics at the University of Leyden, the Netherlands. In 1617, van Royen's survey Alcmar - Bergen-op-Zoom (1615) was published under the quaint title *Eratosthenes Batavus de Terræ ambitûs verâ quantitate, a Willebrordio Snello suscitatus*. Calculating the length of the base from an auxiliary base, this survey is distinguished by substitution of trigonometrical surveys for the meridional arc measurement of the whole distance by rods (Kreffer, 2005).
In 1637, Richard Norwood, Reader of the Mathematics, published *The Seaman's Practice, contayning a Fundamentall Probleme in Navigation experimentally verified, namely touching the Compasse of the Earth and Sea, and the Quantity of a Degree in our English Measures*.
Between 1669-1671, the trigonometrical survey of Jean Picard was commenced between Paris and Amiens in France, which was not free from errors, but the errors almost balanced each other (Airy, 1830).

Theory of Gravitation

An arc of latitude of the astronomers Picard and Richer enabled Huygens and Newton to confirm their hypothesis of gravitation. In 1673 appeared the work of the Dutch mathematician and scientist Christiaan Huygens, entitled *De Horologio Oscillatorio* with correct notions on the subject of the pendulum and centrifugal force. These were not applied to the theoretical investigation of the Earth's form before the publication of Newton's *Principia*. Huygens published *De Causâ Gravitatis* in 1688, an investigation of the Figure of the Earth.

Discovering the law of gravitation was due to Robert Hooke, an English physicist in 1678. He found the inverse law to describe the planetary motions and concluded that the force of gravity could be measured by using the motion of the pendulum. Remarkably, Augustin-Louis Cauchy and Siméon-Denis Poisson extended Hooke's formulation. Isaac Newton formulated a special case of gravitation and proofs of his (imperfect) theory in the *Philosophiae Naturalis Principia Mathematica* (Figure 103) known as the *Principia.* Unfortunately, Hooke laid his claim to the inverse-square law of gravitation. So, Newton threatened to withdraw some parts of his work. The Royal Society urged the astronomer Edmund Halley to get Newton to write the Principia, and to see it undiminished through the press. Halley's credit in this enterprise is enormous. He even paid the costs of publication in 1686 (Airy, 1830; LaRousse, 1999; Britannica, 1999).

12.1.2 Prolate or Oblate Ellipsoid

A new ellipsoidal era was begun. The axis of revolution of the Earth must be shorter that its equatorial diameter. In addition, Newton proofed, that gravity must be less at the Equator than near the Poles.

In this magnificent work, a prodigious step was made towards the theory of the Earth's form. On the supposition that the Earth had been in the state of a homogeneous fluid and combined with the theory of centrifugal forces, Newton calculated the Earth must be ellipsoidal (Airy, 1830; Cohen, 1955; Grafarend, 1999a; Torge, 1989).

The Cassini's seem to have been firmly convinced that the Figure of the Earth was a prolate ellipsoid. It appeared then that the degrees shortened in going from the south to the north, a conclusion in which direct opposition to Newton's theory (Figure 104) excited a great sensation among the mathematicians of Europe.

To settle the controversy, the origin of the celebrated expeditions of the French academicians to Peru and Lapland was to escape from a state of doubt. An arc of a meridian should be measured near the equator for comparison with an arc measured in northern Europe.

Meridional Arcs

In May 1735, Bouguer, Godin, La Condamine, Juan and Ulloa, sailed to Peru (now Equador), South America. The great valley between Tarqui and Colchesqui in the Andes was found favourable. They affected the survey of the meridional arc of more than 3° at an elevation of two kilometres above sea level.

Camus, Celcius, Clairaut, Le Monnier, Maupertuis, and Outhier reached the Gulf of Bothnia in July 1736. Survey stations were on the hills of each side of the river Tornea. A pendulum showed that gravity increased in going from the equator towards the Pole. All attempts to determine the shape of the Earth by measurement by scientists in Peru (1735-1744) and by scientists in Lapland (1736-1737) had given proof of the Earth's oblateness from arcs measured (Kakkuri, 1986; Maupertuis, 1753, 1768; Outhier, 1744).

Latitudo loci.	*Longitudo penduli.*		*Menſura gradus unius in meridiano.*
grad.	*ped.*	*lin.*	*hexapedæ.*
0	3	7,468	56637
5	3	7,482	56642
10	3	7,526	56659
15	3	7,596	56687
20	3	7,692	56724
25	3	7,812	56769
30	3	7,948	56823
35	3	8,099	56882
40	3	8,261	56945
1	3	8,294	56958
2	3	8,327	56971
3	3	8,361	56984
4	3	8,394	56997
45	3	8,428	57010
6	3	8,461	57022
7	3	8,494	57035
8	3	8,528	57048
9	3	8,561	57061
50	3	8,594	57074
55	3	8,756	57137
60	3	8,907	57196
65	3	9,044	57250
70	3	9,162	57295
75	3	9,258	57332
80	3	9,329	57360
85	3	9,372	57377
90	3	9,387	57382

Figure 104: Pendulum observations in Newton's *De Mundi Systemate*

Remeasuring the French meridional arc by Jacques Cassini, his son César F. Cassini de Thury, and LaCaille (1740) resulted as the subject of *la vérification de la méridienne de France.* The comparison of the three arcs in Peru, in France, and in Lapland showed that the degrees increased in going towards the Pole. Consequently, an inescapable conclusion was that the Earth's Figure was flattened at the Poles.

In the history of several important surveys, many others attempted to measure a meridional arc: Edward Wright, London-York between 1633-1635, Grimaldi and Riccioli, Roma, in 1645, Jean Richer, Cayenne, between 1671-1673, Jacques Cassini, France, between 1684-1718, Boscovitch and Le Maire, Vatican-Rimini, in 1752, LaCaille, Cape of Good Hope, between 1750-1754, Liesganig, Hungary, between 1762-1769, Mason and Dixon, Pennsylvania, North America, between 1764-1768, Becaria, Turin, in 1768, Roy, Hounslow Heath, between 1783-1794, Delambre and Méchain, from Dunkerque to Barcelona, Biot and Arago, from Dunkerque to Formentera between 1792-1808, Burrow, Dalby and Reuben, East-Indies, in 1790, Lambton, East-Indies, 1802, Svanberg, Lapland, between 1801-1803, Lambton and Everest, East-Indies, between 1805-1825, Plana and Carlini, Piédmont, 1829, MacLear, Cape of Good Hope, between 1836-1848, and Gauss, Altona-Göttingen, between 1821-1825 (Airy, 1830; LaRousse, 1999; Britannica, 1999).

By the discoveries of Newton, the Figure of the Earth was shown to depend on an astronomical theory, which explained with such an extraordinary accuracy the motions of the planets and their satellites.

Differences of Longitude

Measuring an arc of longitude was done by Jacques D. Cassini de Thury and Nicolas L. de LaCaille measured an arc across the mouth of the Rhône between 1739-1740, by Jacques D. Cassini de Thury for connecting the Observatories of Greenwich and Paris in 1785, by General Roy near Dunnose, and by Tiarks between Dover-Falmouth, in 1823.

As the Earth's rotatory motion was expected to be uniform, the difference of solar or sidereal time serves to measure the angle. His method was to observe from two places some instantaneous signal, and to note the time indicated by both clocks at the time of observation. It was only necessary that the rates of the clocks be known pretty accurately. It was found in practice extremely difficult to establish the system of co-operation necessary for this method.

The difference of longitudes was determined by observing the explosion of ten pounds of gunpowder at one intermediate point. The duration of the flash was estimated at less than half a second. Between 1821-1824, signals were made by the explosion of gunpowder on the summit of Monte Baldo, near the Lago di Garda. Each evening ten signals were observed by several astronomers at the observatories of Padua, Milan, Bologna, Modena, and Verona (Airy, 1830; Britannica, 1999; LaRousse, 1999).

12.2 A Quarter of a Millennium Ago

An Equilibrium

As early as 356 BC, Eudoxas conceived the principle of analysis and prediction of tides.

The Academy of Paris proposed *The Tides* as a subject for a prize essay in the year 1740. An admirable essay to the theory about the Figure of the Earth was sent by Maclaurin. It was proved in this most elegant geometrical investigation for the first time, that the oblate ellipsoid is a form of equilibrium. His transcendental equations were given by which the ellipticity can be found when the proportion of the centrifugal force at the equator to gravity is known. The results are the same as those of the theory of Newton.

Clairaut's *Theorie de la Figure de la Terre* (Figure 105) was the most valuable work written upon the subject, in which he has shown upon what the possibility of equilibrium depends. Principles to the discovery of an equilibrium on the suppositions had engaged the attention of the most distinguished mathematicians (Clairaut, 1743).

The application of Hooke's theory of gravitation to the behaviour of the planets and their satellites entailed fearful difficulties. At the conclusion of the 18th-century, Laplace showed that the complex problem of perturbations of the planets was not cumulative but periodic. The changes of the entire Solar system repeat themselves at regular intervals by the stellar clockwork of the universe. These never exceed a certain moderate amount as a great pendulum of eternity that beats ages.

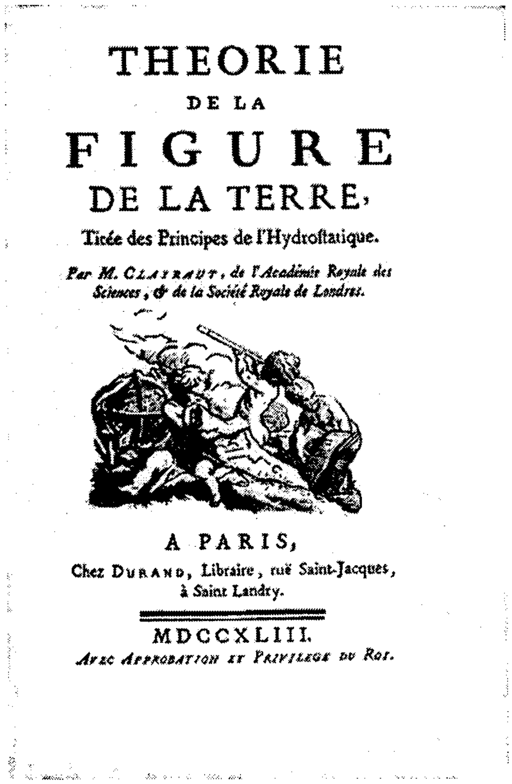
In the early part of the 19th-century Laplace wrote in the Mécanique Céleste:

> *" ... The irregularities of the two planets appeared formerly to be inexplicable by the law of universal gravitation ... ".*

His work appeared in five immense volumes between 1798 and 1825. In his books he described the general principles of the equilibrium, the principal equations of the motions of bodies and planets with the oscillations of the fluids which cover them, the law of universal attraction, the force of gravity at the surface of the Earth, the known phenomena of the flow and ebb of the tides, the Precession of the Equinoxes and the figure and rotation of Saturn's rings remaining permanently in the plane of its equator. Laplace recognised the existence of partial tides and applied the essential principles of the harmonic analysis to the reduction of high and low waters. At about the same time an important part in laying the foundation for the harmonic analysis of the tides devised George B. Airy.

However, the credit for having devised the method of reduction of tides by harmonic analysis on a practical basis is given to Sir William Thomson (Lord Kelvin). His paper was published in the Report of the British Association for the Advancement of Science in 1868.

At about the same time, *The Tidal Researches* and additional articles on the Harmonic Analysis were published by William Ferrel and Rollin A. Harris (*Manual of Tides*) of the NOAA, previously C&GS; (Airy, 1830; Laplace, 1795, 1798a, 1798b, 1802; Newman, 1954; Schureman, 1958).



THEORIE
DE LA
FIGURE
DE LA TERRE,
Tirée des Principes de l'Hydrostatique.
Par M. Clairaut, de l'Académie Royale des Sciences, & de la Société Royale de Londres.

A PARIS,
Chez Durand, Libraire, ruë Saint-Jacques, à Saint Landry.

MDCCXLIII.
Avec Approbation et Privilege du Roi.

Figure 105: Cover of the Theorie de la Figure de la Terre

Standard of Linear Measure

In the true spirit of the revolutionary philosophy, the expanded ideas of the French legislators, the *Convention Nationale* of France (1795) fixed a new worldwide standard of linear measure. Therefore, the length of the quadrant of meridian passing through the Observatory of Paris was undertaken by Delambre and Méchain. Adopting an ellipticity of 1/304, one ten-millionth part of the quadrant of the Parisian meridian was to be called the mètre, as described in Delambre's *Base du Système Métrique.* The *mètre* was defined by 443.296 lines of the Toise of Peru at the temperature 13°R (Reaumur) or 61½°F (Fahrenheit) (Airy, 1830; Britannica, 1999; LaRousse, 1999).

About Projections

Johann Heinrich Lambert had the facility for applying mathematics to practical questions that are not only interesting, but are still in use among geodesists.

> *"... It is no more than just, therefore, to date the beginning of a new epoch in the science of map projection from the appearance of Lambert's work ... " (Craig, 1882).*

The purpose of the zenith sector was to determine zenith-distances or astronomical latitude from stars near the zenith with great accuracy, where errors due to atmospheric refraction are minimised. The telescope of this superb instrument was 2.5 m long. It was moved by an external micrometer. The extend of the Indian Arc was found by numerous observations of twenty-four stars with the zenith sector used by Colonel Lambton.

12.2.1 Principle of Gauss' Least Squares adjustment

Until the time of Carl F. Gauss and Friedrich W. Bessel, computer scientists had to simply judge as best as they could how and when to use supernumerary angles. Using the zenith-sector, the difference of latitudes between the observatories at Göttingen and Altona was observed by Gauss himself. The details are found in the *Bestimmung des Breitenunterscheides zwischen den Sternwärten von Göttingen und Altona* by Gauss. It was stated by Gauss, the results were, undoubtedly, one of the most accurate determinations ever made at that time. Gauss developed a new technique for calculating orbital components *Corporum Coelestium*.
The discovery of the method of least squares (LS) by Gauss was published in Art. 186 of *Theoria Motus* in 1795. He wrote " *... principium nostrum, quo jam inde ab anno 1795 usi sumus ...* "(Airy, 1830).

The principle of LS |[14] showed that a system of corrections ought to be applied to the reduction of the horizontal angles and calculation of the triangulations. The reduction of the supernumerous observations, in the past a very laborious process, led to complex calculations. The first grand development of this principle is contained in the book entitled *Gradmessung in Ostpreußen und ihre Verbindung* by Friedrich W. Bessel, published in 1838. It marks an era in the science of geodesy. The defining fundamental parameters of the Bessel 1841 ellipsoid are the length of the equatorial radius of the Earth:

$a = 3272077^{T}.14$ |[15], $b = 3261139^{T}.33$ ($= a / 1.00335\ 39847$), thus reciprocal flattening $f^{1} = 299.152\ 82$.

Furthermore, two different heliotropes were invented by Gauss in 1821. Using a dilatation factor α [10.2] (10.02.03), Gauss masterminded the conformal projection from the plane via the sphere to the ellipsoid. His concepts had been applied in the Ordnance Survey of Mecklenburg (1853-1873) by Friedrich Paschen, and Oscar Schreiber changed the character of the arc measurement into a projection and grid system of a network in 1866, Hannover (Schreiber, 1866, 1897; Vogeler, 1895).

An Electric Telegraph

One of the main problems of determining *the Figure of the Earth* was the lack of knowledge about a very accurate determination of longitude. Gauss and Wilhelm Weber devised an electric telegraph to measure differences of longitude (1830-1832). In 1857 Friedrich G.W. von Struve, Director of the Observatory at Pulkova in Russia, invited Germany, France, Great Britain, and the Flandern to collaborate in the survey of an arc along the 52° N parallel, covering more than 68° 31' of longitude. Between 1857-1896, the project connected the countries from the Ural to Ireland (Figure 47: pp 125). In 1861, as part of the same project Airy was closely involved in remeasuring longitudes between Valencia and Greenwich Observatory using the electric telegraph (Helmert, 1895; Börsch, 1896).

In 1862 Austria, Prussia, and Sachsen founded an intergovernmental organisation the Mitteleuropäische Gradmessung (MG) in Berlin. Baeyer was perfectly equipped to lead an organisation as MG (now IAG) working on the frontiers of geodetic technology, combining vision and practicality. MG changed its name to Europäische Gradmessung (EG) in 1867. After the death of Baeyer in 1885, the states decided to rename the EG organisation to the Internationale Erdmessung (IE). Helmert took over the presidency of the IE in 1886.

In 1886, the physicist Heinrich Hertz at Bonn made his first experiments with electrical oscillations propagated as radio waves. He used semiconductors as early as 1889. For the centuries-old problem of the longitude at sea, wireless time signals gave meanwhile a complete different solution. Hence, in 1895, Marconi introduced wireless telegraphy by means of these radio waves. Subsequently, Greenwich time signals were sending out every exact hour by international collaboration as from 1913. Ships at any point of the ocean could observe these time signals. The precise standard time gives, compared with the local time, the longitude. The problem of the longitude at sea was an episode in the history of astronomy, but highly important for the progress of science now closed (Airy, 1830, Helmert, 1884; Britannica, 1999; Brockhaus, 2000; Seymour, 1980).

---

[14]Methods of H.J. Walbeck, later on C.F. Gauss and E.J.C. Schmidt

[15]Toise of Paris

An essential but sometimes confusing aspect of geodetic measurement is the need to define the standard to which the measured lengths and the dimensions of the reference ellipsoid are related. Both Airy and Clarke called the shape and dimensions of the oblate reference ellipsoid the Figure of the Earth, a term which in current usage is usually applied to the geoid.

Johann B. Listing renamed the equipotential surface of the Earth at sea level into *the geoid.* Excellent treatment of the theory of the geoid and equipotential surfaces (geops) are those of (Bjerhammar, 1986; Bruns, 1878; Grafarend, 1983, 1997c; Hirvonen, 1962; Graaff-Hunter, 1960; Listing 1873; Sludskii, 1888). Two principal types of geoids were derived according to different definitions, the astro-geodetic geoid and the gravimetric geoid (Fischer, 1968). One of the first gravimetric geoid of global extent was Columbus geoid of the Ohio State University (OSU), 1957.

12.2.2 Frameworks for Mapping

The triangulation of Europe, carried out in the first part of the nineteenth century, furnished the frameworks for the survey and mapping of the various countries and states. The changing nature from the surveys of scientists through to the cadastre infrastructures addressed the future requirements of a taxation cadastre. As a basis for taxation and juridical purposes, maps incorporating the information of location, quantity, value, and describing ownership of real estate, were required to anticipate the (fiscal) future.

Different projections preserve different geometric properties, but no one projection can preserve all geometric properties simultaneously. Unfortunately, the Gauss map projection - the product of two sequential map projections - was found quite unsuitable by the tax collectors and cadastre-administrations due to *unacceptable* distortions. Consequently, the Gaussian projection was treated as the caboose.

Cadastre and Taxation Committee's recommendation was the geometric property most needed was a *congruent* or equal-area projection. It preserves a constant ratio between the area of the region on the ellipsoid, and the area of the corresponding region on the map.

The surveyors of Dépôt Général de la Guerre (France), Deposito della Guerra (Italy), Liesganig in Austria, Clarke, Colby, Mudge, General Roy, and Yolland in England, General von Müffling, J.G. Bohnenberger, and General Baeyer in Germany played a key role in a range of events to prepare the maps required, such as Bonne (modified Flamsteed), Cassini, Cassini-Soldner, normal Flamsteed, and Polyeder. Although these congruent maps were used without any systematic connection to adjacent maps, no apparent need pursued any other course (Seymour, 1980).

In 1895, the farsighted Jordan predicted following outlook:

> *" ... In one country (Germany) are more than forty different non-orthomorphic mapping systems with only one exception: the Gauss-Paschen conformal conical projection of Mecklenburg. Within twenty years, and between 1916-1920, all existent mappings will be recasted on one national conformal projection and grid system ... "*
>
> *(Jordan, 1895).*

Until 1915, all land information systems (LIS) were focused on an administration of land information. A cadastre addressed the requirements of valuation of real estate - centred on payment of taxes (Rosenmund, 1903).

Changing Requirements

The Internationale Erdmessung (IE) was discontinued in 1916 due to World War I (1914-1918) and the death of Friedrich R. Helmert in 1917. Until 1924, Geodesy is represented by the l'Association de Géodésique (AG) and since 1932 by the International Association of Geodesy (IAG). Objectives of the IAG are specified in [19.2]. In 1924, the Baltic Geodetic Commission was founded.

The requirements of the administration and development of geodesy changed from rural to industrial societies in developed and densely populated countries. In the first part of the twentieth century new mapping and grid systems were established, such as in Switzerland (1903), The Netherlands (1918), France (1922), Germany (1927), USSR, now CIS, (1932/1942), USA (1933), Great Britain (1936), and a worldwide transverse Mercator (UTM) grid system in 1947 (Izotov, 1959; Rosenmund, 1903; Roussilhe, 1922.

12.2.3 Radar and Velocity of Light in Vacuo

In 1864, the physicist James Clerk Maxwell devised the general equations of the electromagnetic field, determining that both light and radio waves are examples of electromagnetic waves governed by the same fundamental laws. His work led to the conclusion that radio waves can be reflected from metallic substances and refracted just like light waves [12.3.2]. Basic research of the propagation velocity of light goes back to 1676: Römer measured c = 215 000 km/s. In 1839, the French physicist Fizeau found a first sensibly value. With a range of methods, Weber and Kohlrausch measured c = 310 800 km/s (1857), Maxwell c = 284 300 km/s (1860), and Albert A. Michelson c = 299 910 km/s (1879). In addition, Michelson was awarded the Nobel Prize for his pioneering work in Interferometry techniques in 1907. At this time, c = 299 792 458 m/s.

In 1886, the experimental work of the physicist Heinrich Hertz set out to verify the existence of radio waves and demonstrated that radio waves perform much like lightwave properties with widely different frequencies. The basic development of radar had its origins in the experiments on electromagnetic radiation at a frequency of about 455 MHz conducted during the late 1880s. The potential benefit of Hertz's work as the basis did was recognised. Christian Hülsmeyer was one of the first to apply Hertz's conclusions on radar. He developed "*das Telemobiloskop*", a radio echo device for the detection of targets of practical interest and for safety in navigation. A patent was issued to Hülsmeyer on April 30, 1904. His primitive radar-like system aroused an interest at the First Nautical Conference in Rotterdam. Hülsmeyer built his invention and he was invited to demonstrate it to the Holland-America-Line (HAL) company. However, despite improvements and another patent, he failed to attract interest due to its technical limitations at the Second Nautical Conference in Rotterdam, Fall 1905.

Invar, a 36%-nickel / 64%-steel alloy, was an important discovery by Ch. Guillaume, BIPM in 1896. This metal could be rolled into tapes for measurements. In the 1920s, Heinrich Wild of Switzerland was making great improvements in the design of theodolites. This resulted in increased accuracy and speed of observations, combined with rugged instrument construction (Deumlich, 1982; Seymour, 1980; Strasser, 1966; Völter, 1963). After the end of World War I, the Väisälä Light Interference Comparator was designed and developed by Yrjö Väisälä. Using the Comparator for baseline measurements of up to 200 metres resulted in an accuracy of one part in 10^7. It was put into practical use by the Finnish Geodetic Institute (FGI) in 1929 (Kukkamäki, 1954, 1978; Logsdon, 1992; Price, 1986; Rotter, 1984; Väisälä, 1923, 1930).

In 1922, the possibility of using the radio reflection phenomenon for detection purposes was further explored after the engineer Guglielmo Marconi elaborated its principles. The US Naval Research Laboratory tested Marconi's proposal, employing continuous-wave (CW) radiation to detect a ship passing between a radio transmitter and receiver. In 1924, physicist Edward Victor Appleton measured the height of the Earth's ionosphere. One year later, Gregory Breit and Merle Antony Tuve developed the operating principle of pulse ranging, while engaged in ionospheric research by bouncing radio pulses off the ionised layer of air and determining the time taken by the echoes to return. In 1929 improved Hidechugu Yagi the principle of Marconi's directional radar antennae of 1896. The width of the beam produced by the antenna is directly proportional to the wavelength of the radiation and inversely proportional to the width of the antenna.

Until the early 1930s, there was simply no economic, or military need for radar. Several countries initiated research to look for a means with which to detect the approach of hostile long-range military bomber aircraft and surface vessels under conditions of poor visibility. The countries that developed radar prior to World War II first experimented with:

- listening for the acoustic noise of aircraft engines
- detecting the electrical noise from their ignition
- experiments with infrared sensors.

Still, none of these proved effective.

12.2.4 Electronic Surveying Systems

Historically, World War II is the dividing line between conventional and electronic surveying systems. Radio Detection and Ranging (RADAR), as conceived by Robert Watson-Watt in 1935, was used by Allied forces since 1938 and by the German forces (FREYA-system) since 1939, and during the World War II for a variety of devices concerned with radio detection, positioning and navigation.

Using electronic positioning was born in the Channel on (D-Day) June 6, 1944. The classified two-range hyperbolic Decca Project QM, devised by W.J. O'Brien and Harvey Schwarz, was activated for minesweepers. Such devices not only indicate the presence and range of a distant object, called the target, but also to:

- locate objects beyond the range of vision
- determine their distance
- determine position in space
- determine its size
- determine its shape
- determine its velocity
- determine its direction of motion

One of the first electronic-optical distance measuring (EDM) instruments to be produced for geodetic purposes was the Geodimeter (using the *Kerr* cell), a bulky instrument and laborious to use. It was developed by Ragnar Schöldström, and patented by Erik Bergstrand, geodesist of the Swedish National Survey Agency in 1947 (Ritchie, 1992).

Using the propagation velocity of light in vacuo, 299 793.1 km/s, the Geodimeter - based on the principle of *Fizeau* - was used by NGA to test it over an accurately measured base of length in 1953. The Ridge Way (RW) base was established as probably the most accurate and longest baseline in the world, giving an accuracy of two ppm. Measuring the RW-base in the UK, 11 260.1887 m, the base differed by only +26 mm (2.3 ppm) from the taped length. This result of the RW-base gave a velocity of light of 299 792.4 km/s. A few years later, it was followed by an electronic-magnetic distance-measuring (EDM) instrument, the Tellurometer (using a *Klystron*), a system developed by T.L. Wadley of the South African Council for Scientific Research. IUGG-1983 (International Union of Geodesy and Geophysics) adopted the velocity of light value of c = 299 792.458 km/s (Price, 1986; Rotter, 1984; Seymour, 1980; Wadley, 1957, 1958).

Since the 1950s, the efficiency and reliability of radar equipment has included significant improvements of components and circuitry, with introduction of scanning methods and implementation of high-speed digital computers for signal processing with increasing use of solid-state electronic devices from transistors to very-large-scale integrated (VLSI) circuits.

Group of Radar Devices

- a primary radar system operates on the principle of a passive echo from the target
- a secondary radar system depends on a active response from the target as used in navigation and in communication
- continuous-wave (CW) radar broadcasts a continuous signal rather than pulses
- frequency-modulated (FM) radar broadcasts a continuous signal of uniformly changing frequency. During the time it takes a signal to be transmitted, reflected, and received, the transmitting frequency changes

Radar systems operate by transmitting electromagnetic waves of microwave frequency, toward an object and receiving the waves reflected from it. The properties of the received radio waves, or echoes, are amplified and analysed by a signal processor. The processed signals are then converted into a form usable by a human operator or by a device controlled by the radar unit. Information about the target object (distance, direction, or altitude) is typically displayed on the screen of a flat panel or cathode-ray tube (CRT). It provides a map like image of the area scanned by the radar beam.

There are several types of radar, each of which involve different kinds of signals from the radar transmitter and makes use of different properties of the received echo:

- the most widely used type of radar is pulse radar
- a general type of radar is continuous wave (CW) radar. This type cannot measure distances. Frequency-modulated (FM) radar is able measure distance
- another form of radar is microwave radar (LIDAR) in which very narrow laser light beams are transmitted. LIDAR operates with high frequencies ranging between 10 MHz and 40 GHz (Table 16): pp 115.

Radar serves as a valuable tool in civilian applications:

- an navigational aid for commercial airplanes and marine vessels
- to direct the movements of approaching and departing aircraft when visibility is poor at major airports
- to map planetary and satellite surface features, the surfaces of the Moon, Mars, and Venus in detail.
- to aid weather forecasters in making short-range predictions

Doppler Shifts

Because a continuous echo cannot be associated with a specific part of the transmitted wave, it is not possible to derive range information from continuous-wave radar, but it is used to determine the speed of the target by measuring Doppler shifts - a change in observed frequency produced by motion.

World's First Digital Computers

Independent developments associated with digital-relay computers began in 1935. Trying to automate large-scale calculations, the German scientist Konrad Zuse masterminded the Z1 in the living room of his parent's home. Z1 was the world's first entirely mechanical binary digital computer (Zuse, 1993).

Separately, the American scientist George Stibitz developed the electro-mechanical complex number calculator, the first American digital computer assembled at Bell Laboratories in 1939. The digital computer is distinguished by the fact that it does not measure. It counts (Davis, 1949).

Trilateration Networks

Initially, progress using traditional triangulation techniques was slow, but the arrival of EDM instruments caused a revolution in surveying methods. A great variety of electronic surveying systems and methods were developed, depending on the mission to be accomplished: circular systems, hyperbolic systems, baseline measuring systems [5.3], radar altitude measuring systems [12.2.3], and sound navigation and ranging (Sonar). These tools were applied to geodetical- and hydrographical surveys, oceanographic research, navigation of ships and aeroplanes, including microwave instrument landing systems (ILS). Except for small tasks associated with plane surveying, the popularity of measuring distances using tapes, stadia, subtense bars, and similar equipment is eroding as more convenient tools and techniques pass into common use (Höpke, 1959; Laurila, 1976; Ledersteger, 1954; Leick, 2004; Luse, 1985).

Triangulation procedures were abandoned in favour of trilateration networks, i.e. geodetic control by EDM instruments and first order traversing by theodolites, which were applied more and more after 1960. However, these methods do not measure positions, which is a crucial limitation. With the introduction of the artificial satellite as a geodetic tool, not only did new results appear but also new theories had to be provided to explain these results (Bomford, 1977; Meade, 1967, 1968).

12.3 About Mathematics

Experience alone can decide on truth

(Einstein, 1950)

Two-Dimensional Symmetry

In 1604, the diagnostician Johannes Kepler abstracted from the wealth of Tycho Brahe's observations that planetary orbits are ellipses. Before Newton, people looked on the world as being essentially two-dimensional, with an up-and-down dimension (Figure 45), pp 115.

By bringing in gravitational forces and the differential equations of mechanics, Newton brought in a main step in the process of evolution. Combining the law of gravitation with his laws of motion, Newton was able to derive mathematically the rules governing planetary motion down to the minutest details. Newton's laws:

- first law is known as the law of inertia.
- second law states that the time rate of change of the velocity, or acceleration, is directly proportional to the force $F = m \times a$. The only force acting on the body produces a downward acceleration equal to the acceleration of gravity, symbolised as g (approx. 9.8 m /s /s) near the surface of the Earth.
- third law states that the actions of two bodies upon each other are always equal. However, it may not hold for electromagnetic forces when the bodies are far apart (Britannica, 1999)

Newton's mathematical analysis showed that the force of gravity decreases as the square of the distances between the attracting bodies. He could now write the formula for gravitational force: $F = G\ (M_1\ M_2)\ /d^2$, in which G is the constant of proportionality, or the gravitational constant. It enabled physicists to modify a picture with 2D-symmetry into a picture with 3D-symmetry. In the unforgettable era that followed, the nature of gravitational interaction, and in particular the reason for the mysterious proportionality between gravitational mass and inertial mass, remained completely hidden. Half a century after Newton's death, Henry Cavendish demonstrated the existence of universal gravitation experimentally (Courant, 1964; Dyson, 1964; Gamow, 1961).

Innovators, such as Archimedes, Euclid, Leonhard Euler, Pierre de Fermat, Carl F. Gauss, Hamilton, Jacobi, Lagrange, Laplace, Maxwell, Newton, Poincaré, and Pythagoras, furthered mathematics most. The right type of mathematical curiosity is a precious possession that usually belongs only to professionals of the highest rank, who are distinguished by the power of intuition rather than by the capacity to make rigorous proofs. Sense and purpose of all of mathematics do not lie in the series of logically related collections of symbols but in taking its driving force from concrete specific substance and aiming again at some layer of reality, such as geodesy. Comparison with previous stages in its development appraises the current role of geodesy best.

Revolution in Euclidean and non-Euclidean geometry

Euclid's celebrated geometric proofs, inherited from the ancients and only insufficiently augmented during the intervening two millenniums, were found to contain severe breaks. What the mathematicians thought to be the reality of nature, believing that their intelligence gave unfailing support for this cognition, proved unreliable sense data. Then began a radical and rapid transformation of mathematics (Halmos, 1958; Kline, 1964, 1968).

The truth that destroyed truth was seen clearly by the greatest of all 19th-century mathematicians, Carl F. Gauss. He affected the revolution in his work on non-Euclidean geometry. Gauss began his work in mathematics in the 1790s when Immanuel Kant's thinking epitomised the intellectual scene (Figure 106). Gauss' attitude forced succeeding generations of mathematicians to abandon the privilege that mathematics offers truths. The new ideas about algebra that emerged at the beginning of the 19th-century grew naturally out of the older algebra, such as the concept of the square root of minus one, customarily denoted by i, which was used to solve a wide range of problems in the 17th- and 18th-centuries, but without satisfactorily explanation (Kline, 1964, 1968; Quine, 1964).

A Memoir of Carl F. Gauss

In 1827, Gauss had published nothing but a cryptic, though yet not widely appreciated, memoir on curved surfaces. Gauss' insight was that on a surface of any curvature the distance between point P_1 and P_2 cannot be determined by the Pythagorean equation $a^2 + b^2 = c^2$. Gauss defined it in a famous mathematical equation:

$$ds^2 = E\,dp^2 + 2F\,dp\,dq + G\,dq^2,$$

in which the square of the distance ds is given by the sum of three terms, in which ds is arbitrarily small. He introduced the entities E, F and G, which are a function of the two arbitrary small quantities p and q on a curved surface. The equation is a function of the intersecting coordinates locating points and curvature varying on the surface from point to point. Gauss called his new geometry non-Euclidean geometry. It could be used to represent physical space just as well as Euclidean geometry does (LeCorbeiller, 1954; Kline, 1964).

A straight line is the shortest way between two points; parallel lines never meet. Consider connecting all three points on the surface of the Earth, the length of the ordinary straight-line segments that joins any two points through the Earth forms a triangle that has all the properties of a Euclidean triangle. In particular, the sum of this triangle is 180°. The shortest route between two points on a sphere is called a great circle. However, connecting all three points, the line elements along the great circle determine a so-called a spherical triangle. Such triangles possess quite different properties. The sum of the angles in them can be any number between 180° and 540°. When segments of three great circles, such as one quarter of the Earth's equator and the northern halves of two meridians, are taken to form a spherical triangle on the surface of a sphere, the three angles of 90° add up to 270°, or three right angles. The difference between a spherical triangle and one in a plane derives from the fact that the sides of the former are drawn on a curved surface instead of on a flat one, thus the elements of a plane and of a sphere obey different rules (LeCorbeiller, 1954; Kline, 1964).


Figure 106: Carl F. Gauss

Riemannian Geometry

In 1854, Georg F.B. Riemann gave a lecture before the Faculty of Philosophy at Göttingen. The subject Carl F. Gauss - the dean of German mathematicians - had chosen for Riemann's lecture was entitled *Über die Hypothesen, welche der Geometrie zu Grunde liegen (On the Hypotheses which Underlie Geometry)*. Riemann had given much thought to the 1827 memoir of Gauss. Actually he needed to generalise is Gauss' equation, which works for any curved surface whatsoever, including a plane as a very special case. Riemann launched an even deeper investigation of possible spaces, utilising only the surest facts about physical space. He constructed an elliptic geometry - now so-called Riemannian geometry - that opened up the variety of mathematical spaces a thousand fold. Riemann had reached into empire of thought so innovative that few scientists then could follow him.

Riemann created Riemannian geometry for the purely abstract purpose of unifying, clarifying and deepening the non-Euclidean geometry of Lobachevsky, Bolyai and Gauss. This geometry turned out to be the crucial implement for Einstein's world-shattering elucidation of the gravitational force.

This new geometry, called hyperbolic geometry, permitted more than one line passing through a point to be parallel to a line not containing the point. Soon other geometries were discovered and a hierarchy of geometries was formulated (Figure 107).

12.3.1 Topology

Riemann was also the founder of topology, another branch of geometry in which research is most active today. He worked with functions of a complex variable, and he introduced Riemann surfaces, to represent such functions, in which the precise shape of the surface was not critical. Hence, he proved that the topological facts concerning the properties of so-called Riemann surfaces that does not change under arbitrary continuous deformation (Cohen, 1967; Courant, 1964; Davis, 1964; Halmos, 1958; Kline, 1955, 1964; LeCorbeiller, 1954).

Cauchy's contributions to mathematics include principles of calculus and his development of the theory of functions of a complex variable (a variable involving a multiple of the square root of minus one), today indispensable in applied mathematics from to geodesy to physics.

Leaving these remarks together, Riemann coined the name *continuum* (no breaks) for any geometrical object of any number of dimensions upon which a point can continuously roam about. A straight line is a continuum in

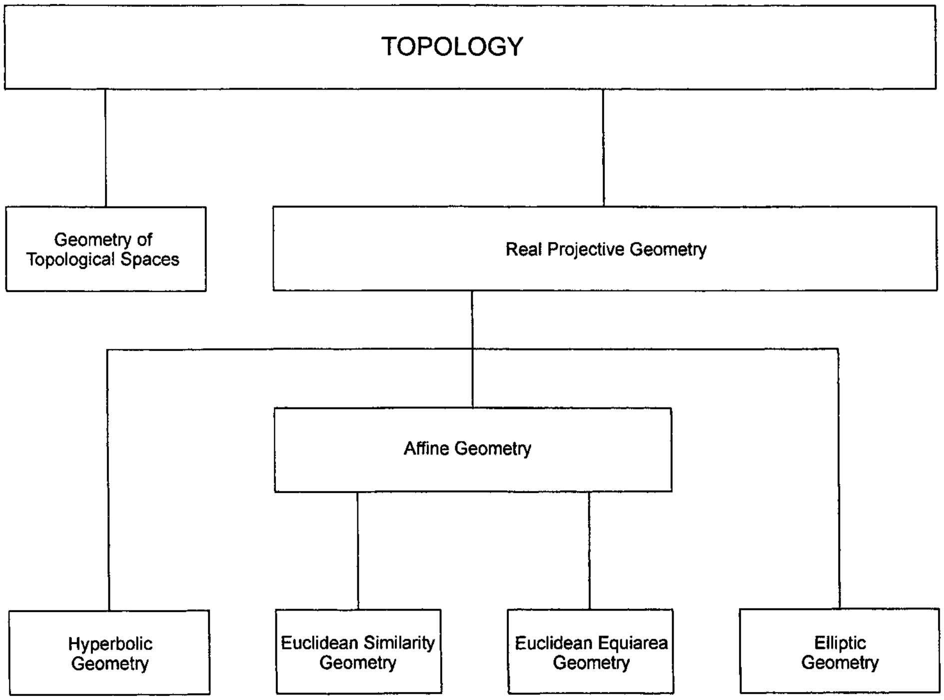


Figure 107: Formulating a hierarchy of geometries

one dimension. A two-dimensional continuum is a surface, and a single number - positive on an egg-shaped surface, negative on a saddle-shaped surface - defines its curvature, for a small region surrounding any point of the surface. A set of three numbers will be needed to define the curvature of a continuum of three dimensions and a set of six numbers for one of four dimensions. Riemann showed that the mathematical concept of curvature could be generalised for the case of a continuum to manifolds of an arbitrary number of dimensions (Le-Corbeiller, 1954; Kline, 1964; Nagel, 1956; Whittaker, 1954).

Vector in a Plane

A vector is a line segment with a specified direction. A vector in a plane could be represented by a complex number; that is, a number formed of both real and imaginary numbers, or $x + y\sqrt{-1}$. The square root of -1, an imaginary number, is usually written i, so that the expression becomes $x + yi$.

Hamilton's Quaternions and Matrices

William Rowan Hamilton's fame has had some curious vicissitudes of life. During his lifetime, he was celebrated but not understood. In the 20th-century, he has become the subject of an extraordinary revival of interest and appreciation.

Quaternions were William R. Hamilton's (1843) great creation. In quaternion algebra two new symbols, i and j, were introduced, with the rules $i^2 = -1$, $j^2 = -1$. When quaternions are reversed, the product may be changed such as: $i \times j = k$, but $j \times i = -k$. Hamilton devised a multiplication table for quaternions (Figure 108), e.g. the row quantity j multiplied by the column quantity k produces I, but row k times column j produces -1 instead.

Each of the quantities multiplied by itself is equal to –1. Arthur Cayley and James J. Sylvester founded the theory of algebraic invariants, algebraic-equation coefficients that are unaltered when the coordinate axes are translated or rotated (Davis, 1964).

Quaternions are given in the general form of a quaternion: a + bi + cj + dk, in which a, b, c and d denote real numbers. However, the commutative law of multiplication, embodied in the algebraic formula $a \times b = b \times a$, does not hold for quaternions, because the latter describes geometrical operations such as rotations.
His discovery was fifteen years later followed by other new algebras, such as the theory of matrices, which is likewise non-commutative (Britannica, 1999, Sawyer, 1964; Whittaker, 1954).

Non-Commutative Algebra

Showing that the ordinary Hamiltonian equations of dynamics are valid in quantum theory, Werner Heisenberg, Max Born and Pascual Jordan (1925) brought non-commutative algebra into quantum theory. Working from different points of view, Heisenberg and Schrödinger's vision led to another picture of the (atomic) world. Heisenberg brought his scheme by matrix mechanics, the Principle of Uncertainty; Schrödinger worked from a mathematical point of view (Figure 109) (Dirac, 1963; Ludwig, 1999).

Topology is one of the most fundamental branches of mathematics that deals with properties of position that are unaffected by changes in size or shape. Topological properties are geometric surfaces, which stay the same in spite of stretching or bending. Topology is full of apparent paradoxes and impossibilities.

On intuitive basis, Henri Poincaré and others built a fascinating edifice of topological theory. In the 1910s, Luitzen Brouwer formulated single-handed topological products on the solid ground of logically impeccable reasoning (Courant, 1964; Harvey, 1969. Kline, 1964; Newman, 1948; Tucker, 1950).

	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

Figure 108: Hamilton's multiplication table for quaternions

Betti number

Further development of Riemannian geometry became the task of other scientists, such as Eugenio Beltrami, Rudolf Lipschitz (theoretical mechanics), and Elwin B. Christoffel, (tensor analysis). Gregorio Ricci introduced the tensor calculus; Tullio Levi-Civita brought a concept of the Euclidean notion of parallelism for more spaces that are general to Riemannian geometry. Hermann Weyl introduced affinely connected spaces, a concept that uses Levi-Civita's notion of parallelism. Gustav R. Kirchhoff, James C. Maxwell had used the idea of the Betti number, and Henri Poincaré established it in 1895. Poincaré, father of modern topology, named the Betti number after Enrico Betti, who in 1871 had generalised the connectivity numbers of Riemann (Cohen, 1967).

Heady with success, the mathematicians rushed into the calculus, differential equations, partial differential equations, differential geometry, the calculus of variations, and functions of a complex variable, and other branches of analysis while constantly using intuitive and physical arguments to support these additions without the capability to make painstaking verifications. Fortunately, through the rigorisation of analysis of Bernhard Bolzano, Augustin L. Cauchy, and Karl Weierstrass, the superstructure was rebuilt. In addition, the great mathematician David Hilbert finally plugged the holes in his reasoning at the end of the 20th-century (LeCorbeiller, 1954; Kline, 1968).

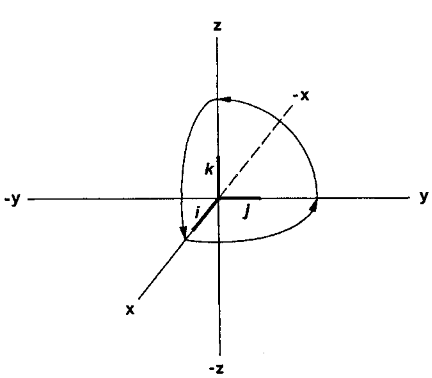


Figure 109: Non-Commutative Algebra

12.3.2 Maxwell's Electromagnetic Wave

For the point of departure, we must go back to earlier work by Faraday discovered the law of electromagnetic induction. He established a body of experimental findings linked by his own ingenious interpretations. He abstracted from mathematical qualitative some laws of electromagnetism. Faraday's diary of 1849 stated: "*Gravity must be capable of an experimental relation to electricity, magnetism and other forces, so as to build it up with them in reciprocal action and equivalent effect*". The results of numerous experiments were negative (Gamow, 1961).

Wilhelm Weber and Friedrich Kohlrausch found - on the relationship between electrostatic and electrodynamical forces - that the velocity of propagation of an electric disturbance along a perfectly conducting wire was close to 3×10^{-11} centimetres per second. Similarly, Maxwell illuminated the wave nature of electromagnetic phenomena by a general quantitative law that combines a unified system of differential equations. Maxwell's equations appear in *A Dynamical Theory of the Electromagnetic Field*, (1864) and his *Treatise on Electricity and Magnetism* (1873). These equations inspired Heinrich Hertz's experiments. Starting an entire new technology led him into demonstrating the existence and propagation of light and radio waves. However, the electrodynamic laws could not be satisfactorily incorporated into the Newtonian system.

Considering Maxwell's electrodynamics as a complete theory is not possible, because it does not provide a theory of the behaviour of electrical densities. Maxwell's equations for empty space remain unchanged if spatial coordinates and time are subjected to the Lorentz group. Covariance holds for the Lorentz group, which is composed of two or more Lorentz transformations. Thus Maxwell's equations imply the group property of Lorentz transformations but the Lorentz group does not imply Maxwell's equations (Courant, 1964; Dirac, 1963; Dyson, 1964; Einstein, 1950; Newman, 1955).

Lorentz Transformation

> *The discovery of special relativity brought quaternions to the fore, because Arthur Cayley (1854) revealed that quaternions could be applied to the representation of rotations in four-dimensional space. His result yielded a particularly elegant expression for the most general Lorentz transformation. Under the Lorentz transformations, the velocity of a body with respect to one inertial frame of reference is related to its velocity with respect to another frame. The rule takes into account the changes in scale length, in clock time, and in simultaneity. Moreover, the quaternions emphasised the importance of action, which preserves its form in different reference systems and is therefore fundamental in relativity physics. The Lorentz group may indeed be defined independently of Maxwell's equations as a group of linear transformations, which leave a particular value of the velocity of light invariant. In 1896, at the suggestion of Hendrik A. Lorentz, Pieter Zeeman investigated the Zeeman effect of magnetic fields on a source of light and found that each of the lines in the spectrum of emitted light split into several lines The Lorentz transformations are rotations of space-time axes. Since the classic interpretation of Einstein's special theory of relativity by Hermann Minkowski, it was shown that physics deals with a unitary entity space-time, in which space like and timeline directions can be distinguished*
>
> *(Einstein, 1950; Whittaker, 1954).*

4D-Space-Time

> *Hermann Minkowski (1907) pointed out that the invariant interval between two events has some of the properties of the distance in Euclidean geometry. Based on Euclidean geometry, the Cartesian coordinate system identifies any point in space by its reference to three mutually perpendicular axes meeting at an arbitrary origin. His brainwave of a four-dimensional space combining the three dimensions of physical space with that of time (arc length of a curve in three-dimensional space), laid the mathematical foundation of Einstein's general theory of relativity. It is refers to as the Minkowski universe*
>
> *(Minkovski, 1907).*

12.3.3 Albert Einstein's Vision

Albert Einstein one of the most engaging and successful intellects in human history proposed entirely new ways of thinking about space, time, and gravitation. His theories were a thoughtful advance over the old Newtonian physics and revolutionised scientific and philosophic investigation. Space-time, in physical science, a single concept that recognises the union of space and time, portrayed in the theories of relativity by Albert Einstein (1905, 1915).

Einstein took the revolutionary step of identifying our physical space-time with a curved non-Euclidean space, so that the laws of physics became propositions in geometry radically different from the classical flat-space geometry. In actuality, physical space-time is four-dimensional.

The most remarkable thing about non-Euclidean geometry is that it turned out to be an indispensable tool for Einstein's revolutionary reinterpretation of the gravitational force and an essential prerequisite for Einstein's general theory of relativity, which today controls our view of the universe (Cohen, 1967; Dyson, 1964; Kline, 1964).

> *The efforts of mathematicians to establish beyond question the physical soundness of Euclidean geometry culminated in the creation of non-Euclidean geometries, which proved as useful as Euclidean geometry for representing the properties of physical space. This unexpected fact gave rise to the question: Since these geometries differ from each other what are we really sure is true about physical space?*

Einstein's special theory of relativity (1905) first printed in *Zur Elektrodynamik bewegter Körper* (*On the Electrodynamics of Moving Bodies*), was his initial paper on special relativity. An addendum to the special theory of relativity established the equivalence of mass and energy expressed in the form $E = mc^2$, in which E is energy, m is quantity of matter, and c is the square of the velocity of light. The influence on his work by his friends Paul and Tatiana Ehrenfest and Lorentz of Leiden on Einstein's special theory is still contentious. For all frames of reference, the theory held that if speed of light is constant, and if all natural laws are the equivalent, then both time and motion are relative to the observer.

Then, Albert Einstein (1914) lifted the veil. Newton's law of gravitational interaction between masses is quite similar to the law of electrostatic interaction between charges, and Einstein's theory of the gravitational field has many common elements with James Clerk Maxwell's theory of the electromagnetic field. Therefore, it is natural to expect that an oscillating mass should give rise to gravitational waves just as an oscillating electric charge produces electromagnetic waves.

Einstein (1916) was primarily engaged in perfecting his general theory of relativity, which he published in Annalen der Physik as *Die Grundlagen der allgemeinen Relativitätstheorie* (*The Foundation of the General Theory of Relativity*). In a well-known article published Einstein (1918) indeed obtained solutions of his basic equation of general relativity that represent such gravitational disturbances propagating through space with the velocity of light. In Einstein's model (1917) of the universe, the curvature occurs only in space. There is no need to distinguish locally between acceleration and gravity--the two are in some sense equivalent. Nevertheless, if that is the case, then it must be true that gravity - *real gravity* - can actually bend light. Hence, concluded Einstein, if the principle of equivalence holds in all of physics, light rays from distant stars that pass close to the Sun on their way to the Earth should bend toward the Sun. Moreover, indeed it can, his prediction was brilliantly confirmed by a party of British astronomers (1919) observing a total Solar eclipse in Africa (Dirac, 1963; Dyson, 1964; Gamow, 1961).

For more than 20 years, Einstein sought to relate the universal properties of matter and energy in a single equation or formula, in what came to be called a *unified field theory*. This turned out to be a fruitless quest that occupied the rest of his life. In 1953 came his new version of the unified field, *The Meaning of Relativity* a most meticulous mathematical essay, which includes the generalised theory of gravitation, the first edition of Einstein's unified-field theory (Einstein, 1953).

13. Tools and Topics

13.1 History of Tables

Europe in the middle Ages had inherited the clumsy Roman numerals. Then gradually the Indian system of numerals penetrated from the Arabic world into Europe.

In the beginning of the 13^{th}-century a manual, *Liber Abaci,* appeared, written by *Fibonacci* (Leonardo of Pisa) that gave instructions for computing with Arabic numerals. Gradually the Arabic numerals spread over Europe, drawn in different shapes before they got their present forms.

Most important were the complicated formulae for spherical triangles to derive angles and distances at the celestial sphere. Georg von Pürbach began with the computation of a more extensive and accurate table of sines for angles increasing by 10-minute sines are given in seven figures.

In 1464, *Regiomontanus* (Johann Müller) worked on five books on triangles of all classes *de Triangulis Omnimodis Libri Quinque*, a advanced dissertation of plane and spherical trigonometry. It uses algebraic techniques to simplify solutions. He computed a table of tangents. Hence, he was accountable for the promotion of trigonometry in Europe.

In 1540, *Rheticus* (Georg Joachim de Porris) conceived an important palatine work on triangles *Opus Palatinum de Triangulis.* Rheticus' treatise also contains tables of values for the trigonometric functions of sines, tangents and secants computed in intervals of 10-seconds, calculated to 10 decimal places. At his death, the tables were not yet ready; his scholar Valentin Otto published them in 1596. They served as the basis for many later tables; other computers later on corrected some remaining errors. The system of relations between the goniometric functions and of the formulae for computation of all the elements of triangles was built up into a complete trigonometry.

At that time, in 1579, it was perfected by François Viète. Using six trigonometric functions for calculating plane and spherical triangles, he invented mathematical laws applied to equations in his treatise *Canon Mathematicus Seu ad Triangula.* Concerning the recognition and improvement of equations, Viète's treatises *In Artem Annalyticem Isagoge* and *De Aequationum Recognitione et Emendatione* conceived methods of solving equations of higher degrees, roots of equations and coefficients of the different powers in 1591. However, the extension and accuracy of these tables could suffice for the astronomical practice of that time. Now the more general formulae were adapted to universal use (Pannekoek, 1951).

13.1.1 Dividing Circumference of the Circle

Military Trigonometric Units. The circumference of the circle ($2\pi \times r$) has been divided into mills ($^0/_{00}$), a military unit ($^1/_{6400}$ = one mill). There are also sexagesimal units ($^1/_{360}$ = one degree) and centesimal units ($^1/_{400}$ = one gon).

Sexagesimal System. The circumference of the circle has usually been divided into 360 equal parts, called degrees (°). Each of these are subdivided into 60 equal parts, called minutes ('). Again, each of these minutes are subdivided into 60 equal parts, called seconds ("). Example: 59° 59' 59".9999.

Centesimal System. A quarter of a millennium ago the centesimal system was introduced in continental Europe. The circumference of the circle has been divided into 400 equal parts, called grade or gon (^{g}). A gon is subdivided into 100 units, called centigons (cgon), or subdivided into 1000 units, called milligons (mgon). Example: $66^g.98\ 769$.

13.2 Trigonometrical Tables

Trigonometric functions are defined as ratios of line segments and are, consequently, dimensionless. They depend on dimensionless quantities (angles). Defining the functions of arbitrary angles - greater than right angles and also negative angles - require consideration of trigonometric functions in a circle, on a sphere or ellipsoid. The sole measure of angles used in geometrical geodesy and higher mathematics is the radian.

Showing some basic techniques of the trigonometrical table usage and its construction is the aim of this section.

On the Use of Subsidiary Angles

In antiquity, the possession of trigonometrical tables enabled to shorten numerical calculations, which have no relation whatever to trigonometry. The angles, which are used in this process, are called subsidiary angles.

Example 1: suppose to calculate the formula $x = \sqrt{(a^2 - b^2)}$. If the logarithms of a and b are already found and if b / a is the sine of angle ϕ, then $x = a \cos \phi$.

Example 2: suppose to calculate the formula $e'' = [1 - \sqrt{(1 - e^2)}] / [1 + \sqrt{(1 - e^2)}]$.
If $e = \sin \phi$, then $\cos \phi = \sqrt{(1 - e^2)}$. Consequently: $e'' = (1 - \cos \phi) / (1 + \cos \phi) = \tan^2 (\phi / 2)$. Revealing what is gained by this arrangement is easy to understand (Airy, 1830).

The relation between goniometric functions and the formulae for computation of all the elements of triangles was built up into a complete trigonometric system. Georg von Pürbach computed an extensive, accurate table of sines for angles. Regiomontanus expanded it by means of interpolation into a table with the angles increasing by 1-minute, Rheticus began with the computation of sines, tangents and secants in 15 figures and it was perfected by Vièta [13.1]. It served as the basis for many later tables (Pannekoek, 1951).

Using Goniometric Functions

Trigonometry is the science of triangles, the branch of mathematics, which treats the application of arithmetic to geometry. In the application of mathematics to geometrical geodesy, no branch is so important than trigonometry. The practical exactness of arithmetic calculations with the hypothetical accuracy of geometrical constructions is the connecting link (Airy, 1830; Steinhauser, 1880).

Trigonometrical tables have generally sines, cosines, tangents, et cetera, up to 45°, the cotangent of an arc being the tangent of its complement. Until 1850 the following principal tables were commonly in use:

- Vega's, containing the logarithms of numbers, log. sines, et cetera, for every ten seconds to ten decimals. One of the best tables (Vega, 1794)
- Callet's logarithms of numbers, trigonometrical functions, et cetera, for every ten seconds to seven decimals with tables for the decimal division of the circle
- *Tables du Cadastre* by Borda, revised by Delambre, a useful collection for the decimal division
- Hutton's and Sherwin's, containing logarithms of numbers, sines, cosines, tangents, et cetera, natural and logarithmic, for every minute, to seven decimals
- Gardiner's, with log-sines, et cetera, for every ten second to seven decimals
- Taylor's, with log-sines, et cetera, for every second to seven decimals.

The trigonometric functions depend on dimensionless quantities. They are defined as ratios of line segments and are, consequently, dimensionless.

About the Construction of Tables

The aim of this section is concerned with the construction aspects of a trigonometrical table. It may serve as a guide to check the internal working of an arithmetic (electronic) processor.

Reading the natural arrangement of sines, tangents, and so forth, up to 90° in the reverse order would give the cosines, cotangents, and so on. Nevertheless, there was frequently some confusion about an arc greater than 45°, or greater than 90° (Airy, 1830).

In the past, the construction of a table naturally divided itself into two parts:

- determination of values of the function to be tabulated for certain values of the arc, at large intervals
- filling up of the tables by inserting the values included between these using interpolation methods.

In this order the formation of tables of the values of trigonometrical functions were considered. The method for the determination of natural sines, was to take some arcs whose sine and cosine are known, such as 30°, 45°, 18°, 54°, and so forth. According to:

$$\sin x = x(1 - x^2/\pi^2)(1 - x^2/4\pi^2)(1 - x^2/9\pi^2)\ldots \tag{13.01}$$

tables of their logarithms might be formed by taking from logarithmic tables the logarithms of these numbers. It was considered this as being upon the whole the easiest way. As they may be found independently, and therefore free from all errors of previous computations:

$$\log \sin x = \log x + \log(1 - x^2/\pi^2) + \log(1 - x^2/4\pi^2) + \log(1 - x^2/9\pi^2)\ldots \tag{13.02}$$

The cosine of half the arc was determined by the formula:

$$\cos\alpha = \sqrt{[\tfrac{1}{2}(1 + \cos 2\alpha)]} \tag{13.03}$$

and the sine was determined by the formula:

$$\sin\alpha = \sqrt{[\tfrac{1}{2}(1 - \cos 2\alpha)]} \tag{13.04}$$

or the sine and cosine may be determined by the formulae:

$$\sin\alpha = \tfrac{1}{2}[\sqrt{(1 + \sin 2\alpha)} - \sqrt{(1 - \sin 2\alpha)}] \tag{13.05}$$

$$\cos\alpha = \tfrac{1}{2}[\sqrt{(1 + \sin 2\alpha)} + \sqrt{(1 - \sin 2\alpha)}] \tag{13.06}$$

This method, when 2α is small, is more accurate than the former, with fewer, more laborious operations. It was found that:

$$\sin 5\alpha = 5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha \tag{13.07}$$

Conversely, the solution of the equation

$$\sin 5\alpha = 5x - 20x^3 + 16x^5 \tag{13.08}$$

will give the value of sin a. Again:

$$\sin 2\alpha = 2\sin\alpha\cos\alpha\text{, from here on:} \tag{13.09}$$

$$\sin 20° = 2\sin 10°\cos 10°$$

was found. From sin 15° (found by bisection), sin 3° was found:

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha \tag{13.10}$$

Solving this equation, the value of $\sin\alpha$ was calculated from $\sin 3\alpha$. Similarly, from sin 3° the value of sin 1° was found. Repeating this method, from now on the application descended to sin 30', sin 15', sin 3', sin 1'. The same method could be applied to the sine of 18°, or any arc whose sine is known.

For instance, from sin 30° by trisection sin 10° is found. From this cos 10° or sin 80° was found. Since:

$$\sin\alpha = \sin(60° + \alpha) - \sin(60° - \alpha)\text{, then} \tag{13.11}$$

$$\sin 20° = \sin 80° - \sin 40°$$

whence sin 40° was found. So sin 50° or cos 40° was found, and:

$$\sin 10° = \sin 70° - \sin 50°$$

The sines for every 10° of the quadrant being found, those of every degree should then be calculated as verifications for those of every minute. All sines for every calculated degree should be verified.

All calculations must be observed with regard to the number of decimals. Beginning with the sines, these were taken to two or three figures more than it was intended to preserve in the tables to calculate with great accuracy to prevent any unnecessary labour. The calculation of the differences was rather tedious, but the tables were formed then with great ease. The certainty that any error will be discovered at the next place of verification makes this method superior to any other.

From the relation among the differences of natural sines, it is not necessary to calculate more than the first of them. If the sines up to 60° were calculated, those above 60° were found by a simple addition, from the convenient formula:

$$\begin{aligned} \sin(60° + \alpha) &= \sin(60° - \alpha) + \sin\alpha \\ \sin 70° &= \sin 50° + \sin 10° \end{aligned} \tag{13.12}$$

Thus if the sines were found for the quadrant, consequently, all the cosines were known. Tangents were found by dividing the sines by the cosines.

After 45° they were found by the formula:

$$\begin{aligned} \tan(45° + \alpha) &= \tan(45° - \alpha) + 2\tan 2\alpha, \text{ so} \\ \tan 60° &= \tan 30° + 2\tan 30° \end{aligned} \tag{13.13}$$

Tangents may also be found independently, but this process was too laborious to be applied to all small divisions. It cannot be extended with ease farther than to every degree.

Interpolating

This chapter is concerned with neither a theory of finite differences used for the investigation of interpolations, nor a means of verification of the computations, but rather with how trigonometric tables were constructed to solve arithmetic problems. Having an interpolating formula was favourable to interpolate for finding the differences for smaller divisions, such as seconds, by means of differences for the larger ones. Interpolating formulae are discussed at length in (Airy, 1830).

Verifying Results

However, one error in the calculation of one number would affect all the following ones. Calculating independently some numbers in the same series at certain intervals to verify for the rest was clearly necessary. Verification the accuracy of the numbers for the small divisions of the arc was unnecessary. Therefore, the agreement of the last of a series of numbers computed by differences with the ones, which have previously been calculated by an independent process, gave a better verification.

13.3 Trigonometric Approximation Techniques

The aim of this section is to show some basic techniques of trigonometric approximation.

Computers provide transcendental instructions for logarithmic calculations, exponentiation, and trigonometric calculations. The development of the (sub) routines requires consideration of both range and accuracy. The inverse trigonometric functions require less programming, because the forward trigonometric functions are periodic, where the inverse trigonometric functions are not.

Having defined derivatives, generally, any real continuous function f(x) can be expressed as a polynomial expansion. Entering function values from high-accuracy function tables can generate the high-accuracy polynomial expansions. In summary, trigonometric functions can be approximated to high accuracy with a short and simple polynomial expansions series, or using approximation techniques such as least squares fitting by iterations and recursion. Further, the major concerns in using these routines are round off errors and a potential problem is the possibility of an underflow or overflow condition (Abramowitz, 1970; Ruckdeschel, 1981a).

In this chapter, the method considered is the coordinate rotational digital computer (CORDIC) algorithm, described in short notes and extensively.

Volder devised a CORDIC technique. In essence, it is based on vector rotations that can be calculated using the fast register shifting operations available in the hardware or machine language of binary-based computers. The

CORDIC method is very effectively used in calculators such as the Hewlett-Packard series because it provides excellent approximations to the trigonometric functions and their inverses (Volder, 1959). A CORDIC algorithm aims the following section specifically at computing in Reverse Polish Notation (RPN). CORDIC algorithms for the BASIC language are given in (Ruckdeschel, 1981b).

David S. Cochran and William M. Kahan of Berkeley, whose state-of-the-art development of the algorithms for the mathematical functions methods have served the HP electronic calculator family well for many years, are given below.

13.3.1 CORDIC Trigonometric Functions

A description of the forward trigonometric algorithms to compute sine, cosine, and tangent - as used in HP hand-held calculators in the 1970s - is given here. The RPN pocket calculators have used essentially the same algorithms for computing complex mathematical functions in their Binary-Coded Decimal (BCD) microprocessors. Although tailored for efficiency within the environment of a special-purpose BCD microprocessor, the basic mathematical equations and the techniques used to transform and implement them are applicable to a wide range of computing problems and devices (Egbert, 1977a).

Computations of forward trigonometric functions are devised in radians (^{R}). An angle in degrees ($\theta°$) is converted to radians using the scaling factor by: $\theta^R = \theta° \, \pi \,/\, 180°$.

Prescaling

- scaling the input angle to a number between zero and 2π is imperative.

In computers, numbers can be expressed only to a limited number of digits. Producing the exact answer to all trigonometric problems in radian measure, the use of π, an irrational number, with 13 digits is required. Scaling the input angle to a number between zero and 2π is imperative. Every angular argument is reduced to an angle between zero and 2π by subtracting 2π from $|\theta|$ repeatedly.

A negative argument is treated in a similar way. Then 2π is added, giving a number between 0 and 2π, causing a significant digit to be lost and an error creeps in. Calculators obviate this problem by scaling to a number between zero and ¼ π, which prevents an asymmetry such as $\sin(.15) \neq \sin(-.15)$ (Egbert, 1977a).

CORDIC Forward Algorithm

- using the pseudo-division process, the scaled number is divided into groups of selected smaller angles.

$\mathrm{Tan}^{-1}(10^{-j})$ needs to be expressed to at least ten digits. The tangent of θ is found as follows. In the pseudo-division process θ is divided into a sum of smaller angles whose tangents are powers of 10, such as:

$$\tan^{-1}(1) = 45°,\ \tan^{-1}(.1) \approx 5.7°,\ \tan^{-1}(.01) \approx .57°,\ \tan^{-1}(.001) \approx .057°,\ \tan^{-1}(.0001) \approx .0057°, \dots .$$

Pseudo-Division

First, 1 (= 45°) is subtracted from θ until overdraft, keeping track of the number q_0 of subtractions. Then 0.1(≈5.7°) is repeatedly subtracted from the remainder, again keeping track of the number q_1 of subtractions. The process is repeated with smaller and smaller angles. So far, a pseudo-quotient has been generated that represents the division of the given angle θ into a sum of smaller and smaller angles. As a result:

$$\theta = q_0 \tan^{-1}(1) + q_1 \tan^{-1}(.1) + q_2 \tan^{-1}(.01) + \dots + r \tag{13.14}$$

in which r is the residual angle remaining after the previous subtractions. The coefficient q_i refers to the count in a particular pseudo-quotient digit. Each q_i is equal to or less than 10, so it can be stored in a single four-bit digit.

In many calculators, the pseudo-quotient is five hexadecimal digits long. Each digit represents one series of subtractions and is a number from 0 to 10. An example is given here. Again, the tangents are powers of 10.

$$\theta = 7\tan^{-1}(1) + 7\tan^{-1}(.1) + 8\tan^{-1}(.01) + 7\tan^{-1}(.001) + 7\tan^{-1}(.0001) + r$$

Therefore, the pseudo-quotient would be 77877 for $\theta \approx 359°.9999$

Forward Vector Rotation

Tan θ can be found using the vector rotation process. In vector geometry, an angle can be expressed as a vector R having x and y components (Figure 110). If R is the unit vector, then $\sin\theta = y$, $\cos\theta = x$. $\tan\theta = y / x$ and $\cot\theta = x / y$. This holds true for all values of θ between zero and 2π. Generating x_i and y_i for a given θ_i results in its trigonometric function.

Rotating a vector R anti-clockwise through an angle θ_1 gives the components x_1 and y_1 (Figure 111). Rotating the vector anti-clockwise through an additional angle θ_2 results in the components x_2 and y_2 given by the formula:

$$x_2 = x_1\cos\theta_2 - y_1\sin\theta_2 \qquad (13.15)$$
$$y_2 = y_1\cos\theta_2 + x_1\sin\theta_2 \qquad (13.16)$$

Dividing both sides by $\cos\theta_2$ results in:

$$x_2 / \cos\theta_2 = x_1 - y_1\tan\theta_2 = x_2' \qquad (13.17)$$
$$y_2 / \cos\theta_2 = y_1 + x_1\tan\theta_2 = y_2' \qquad (13.18)$$

Note that x_2' and y_2', while not the true values of x_2 and y_2, both differ by the same factor, $\cos\theta_2$, thus $y_2' / x_2' = y_2 / x_2$. (Figure 111) shows that the quotient y_2' / x_2' is equal to $\tan(\theta_1 + \theta_2)$.

- the tangent of a large angle can be found by manipulating smaller angles whose sum equals the large one.

Using (13.17; 13.18) to generate x_2' and y_2', x_1, and y_1 need to be multiplied by $\tan\theta_2$ and mathematically added. θ_2 is chosen in such a way that $\tan\theta_2$ is a power of 10. The multiplications simply amount to shifting x_1, and y_1. Consequently, to generate x_2' and y_2', only a shift and an add (or subtract) are needed (Egbert, 1977a).

Pseudo-Multiplication

- with the pseudo-multiplication process - applied once for each angle resulting from the division of the input argument - generate an x and a y that are proportional to the sine and cosine of the input angle.

Using (13.17; 13.18) requires an initial x_1 and y_1, corresponding to the x and y of the residual angle r in radian measure. Since this angle r is very small (< 0.001°), $\sin\theta \approx \theta$ and the initial $y_1 = r$, and the initial $x_1 = 1$. Applying the formulae repeatedly, in which $\tan\theta_2 = 10^{-j}$, a new x_1, y_1 are produced, i.e., x_2', y_2'. The use of the formulae is counted in the pseudo-quotient digit for that angle θ. If the angle θ had the pseudo-quotient digit counter of $\tan^{-1}(.1) \approx 5.7°$ is $q = 3$, the formulae would be used three times with x_1 and y_1 being shifted one place right for tan $(\tan^{-1}(.1))$ before the addition (or subtraction). In this way, a new x_1 and y_1 are formed as the vector is rotated the amount equal to the count in the pseudo-quotient digits which sum to the original angle θ.

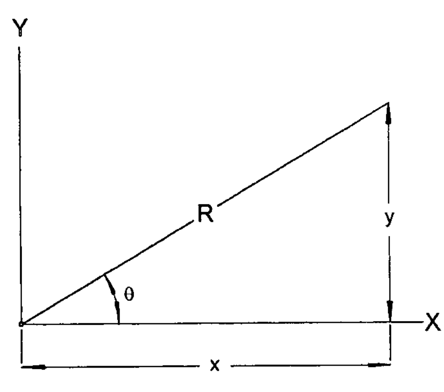


Figure 110: Vector rotation

Using (13.17; 13.18) to generate x_2 requires a shift of y_1 and a subtraction from x_1. Similarly, y_2 requires a shift of x_1 and an addition to y_1. To implement this would require either two extra registers to hold the shifted values of x_1 and y_1, or else shifting one register twice and the other once. However, it is possible to shift only one register once. If for example y = 123, x = 456 in the formula y + .01x. Keeping the decimal points in the same places and shifting y two places left - y is multiplied by 100 - results in: 12300 + 456 = 12756. Therefore, to avoid shifting x, y must be multiplied by 10^j, in which j is the decade digit.

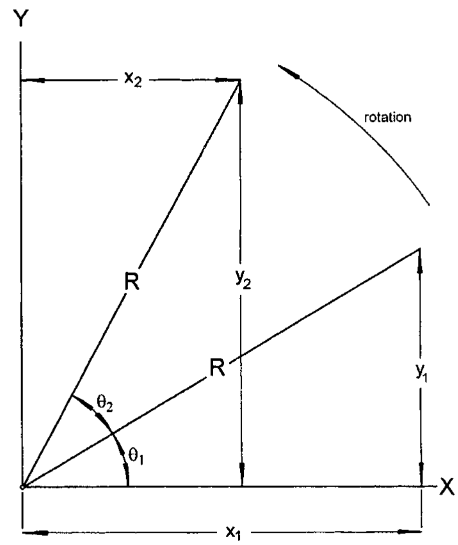


Figure 111: Forward vector rotation

The problem of accuracy. During pseudo-division, the angle θ is resolved until a small angle residual (r) is left as the original y value. Since this is carried out in fixed-point arithmetic, zero digits are produced following the decimal point (e.g. .00123). The zero digits indicate the position of the decimal point. The remainder is shifted one place to the left (i.e. multiplied by 10) during each decade of pseudo-division. This preserves an extra digit of accuracy with each decade. If the pseudo-quotient is five digits long, the final remainder is equal to $r \times 10^4$.

The implementation requires only a single register shift, see formulae (13.17; 13.18), and replace $\tan\theta_2$ by 10^{-j}, in which j is the decade digit. This substitution is correct because $\theta_2 = \tan^{-1} 10^{-j}$:

$$x_2' = x_1 - y_1\, 10^{-j} \tag{13.19}$$
$$y_2' = y_1 + x_1\, 10^{-j} \tag{13.20}$$

If $z = y_1 \times 10^j$, or $y_1 = z \times 10^{-j}$ then substituting in (13.19; 13.20) gives:

$$x_1' = x_1 - z\, 10^{-2j} \tag{13.21}$$
$$y_2' = z \times 10^{-j} + x_1 \times 10^{-j} \tag{13.22}$$

Multiplying the equation (13.22) by 10^j gives:

$$y_2' \times 10^j = z + x_1 \tag{13.23}$$

The left-hand side of (13.23) is in the correct form to be the new z for the next iteration. Each iteration gives within a decade:

$$x_2' = x_1 - z \times 10^{-2j} \tag{13.24}$$
$$y_2' \times 10^j = z + x_1 \tag{13.25}$$

in which x_2' becomes the new x_1, and $y_2' \times 10^j$ becomes the new z.

The shifted remainder ($r \times 10^4$) is z for the first iteration, and j = 4. Using (13.24; 13.25), x_1 and z are stored in two registers. Then $z \times 10^{-2j}$ is formed and stored in a third register. Then x_1 is added to z to form a new z. This leaves x_1 undisturbed so that $z \times 10^{-2j}$ can be subtracted from it to form the new x_2'. This execution saves extra shifts and increases accuracy by removing leading zeros in z. The only register shifted is z.

Applying (13.24; 13.25) a number of times as indicated by a pseudo-quotient digit, z is shifted one place to the right, and a new pseudo-quotient digit is fetched. This creates $y_1 \times 10^j$, in which j is one less than before. Now, the formulae (13.24; 13.25) are applied repeatedly until all five pseudo-quotient digits have been exhausted. The result is an x and y that are proportional to the cosine and sine of the angle θ. Because the final j is zero, the final y = z is correctly normalised with respect to x (Egbert, 1977a).

As soon as an x, y pair has been generated by a pseudo-multiply operation, consisting of shifts and additions, the sine and cosine functions are available.

- computing the required function using the elementary operations with x and y.

To minimise program length a single function, tan θ, is generated first using y / x. Hereafter, sin θ is found by the formula:

$$\sin\theta = \pm \tan\theta / \sqrt{(1 + \tan^2\theta)} \quad (13.26)$$

as shown cot θ can be found while preparing the function tan θ. Then cos θ is calculated using the formula:

$$\cos\theta = \pm \cot\theta / \sqrt{(1 + \cot^2\theta)} \quad (13.27)$$

The difference between the computation for sin θ and that for cos θ is whether x and y are exchanged (Egbert, 1977a).

Inverse CORDIC Trigonometric Functions

- calculate $A / \sqrt{(1 - A^2)}$ if the desired function is $\sin^{-1} A$ or $\cos^{-1} A$.

A description of the inverse trigonometric algorithms used in hand-held calculators to compute $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ is given here (Egbert, 1977b). To minimise program length, the function $\tan^{-1} A$ is always computed, regardless of the inverse trigonometric function required. If the desired function is $\sin^{-1} A$ or $\cos^{-1} A$, then:

$$\sin^{-1} A = \tan^{-1} A / \sqrt{(1 - A^2)} \quad (13.28)$$

If $\cos^{-1} A$ is required, then:

$$\cos^{-1} A = \tfrac{1}{2}\pi - \sin^{-1} A \quad (13.29)$$

Cos^{-1} is found in the range $0 \le \theta \le \pi$. Sin^{-1} and $\tan^{-1}$ are computed for the range $-\frac{1}{2}\pi \le \theta \le \frac{1}{2}\pi$. The $\tan^{-1}$ routine solves only for angles between zero and ½ π, since −tan A = +tan (-A). Consequently, A is assumed positive, and the sign of the input argument becomes the sign of the result in radians. All angles are calculated in radians and converted to degrees or gons (grads) if necessary.

Inverse Vector Rotation

A vector rotation process similar to that used in the forward routine is applied in the inverse process.
A vector, expressed in its x and y components, can be rotated through certain specific angles using nothing more than shifts and adds of integers. In the algorithm for $\tan^{-1}|A|$, the input argument is |A|, or |tan θ|, in which θ is the unknown value.
Assuming $\tan\theta = y_1 / x_1$, then |A| can be expressed as |A| / 1, in which $y_1 = |A|$ and $x_1 = 1$.

The vector rotation process (Figure 112) is used repeatedly to rotate the vector in the clockwise direction through a series of successively smaller angles θ_i, counting the number of rotations (q_i) for each angle, until $y_2 \approx 0$. If q_i denotes the number of rotations for θ_i then:

$$|\theta| = q_0 + q_1\theta_1 + q_2\theta_2 + \ldots + q_i\theta_i + \ldots \quad (13.30)$$

The number of rotations and the amount of each rotation is stored as a pseudo-quotient.

- place |A| and 1 in fixed-point format into appropriate registers, while preserving the sign of A.

Basically, store |A| and 1 in fixed-point format into appropriate registers corresponding to y_1 and x_1, preserving as many digits of A as possible when the exponent of A differs from zero. Then $y_1 = |A|$, and the sign of A is saved (Egbert, 1977b).

If the vector R (Figure 112) is rotated clockwise to find θ, y_2 becomes smaller and smaller, until $y_2 \approx 0$. The amount of rotation of R is stored and is equal to the angle $\theta = \tan^{-1}|A|$.

Rotate R according to the formulae:

$$x_2 / \cos\theta_2 = x_1 + y_1 \times \tan\theta_2 = x_2' \quad (13.31)$$

$$y_2 / \cos\theta_2 = y_1 - x_1 \times \tan\theta_2 = y_2' \quad (13.32)$$

In comparison to (13.17; 13.18), the plus and minus signs are now exchanged because R is now rotated clockwise.

- repeatedly rotate the vector with A = y and 1 = x clockwise using (13.31; 13.32) until y approaches zero. The number of rotations and the amount of each rotation is stored as a pseudo-quotient along the way.

As before, $\tan\theta_2$ is chosen in such a way that execution requires a shift and an add: $\tan\theta_2 = 10^{-j}$. To find θ, R is initially rotated with $\tan\theta_2 = 1$ ($\theta_2 = 45°$). Y_2' soon becomes close to zero and the number of rotations is stored as the first digit so-called of the pseudo-quotient. Y_2' is restored to the last positive value it had and R is rotated again.
Now, the vector R is rotated through a smaller angle, i.e., $\tan\theta_2 = 0.1$ ($\theta_2 = 5°.7$). This process is repeated with the angle of rotation becoming smaller and smaller until five pseudo-quotient digits have been generated. At the end of each series of clockwise rotations, y_2 is multiplied by 10 to preserve accuracy.

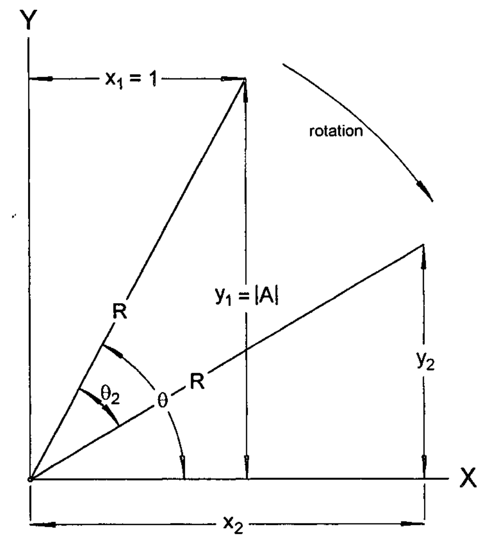


Figure 112: Inverse vector rotation

Pseudo-Multiplication

- using the pseudo-multiplication process, sum all the angles represented by the pseudo-quotient digits to form |θ|.

There remains a residual angle, r, represented by the (final) x_2' and y_2'. The angle $r \approx \sin(r) = y_2'$. This is true if $x_2' = 1$, but x_2' in this case is the product of all $1/\cos\theta$ terms resulting from all applications of using (13.31; 13.32). However, y_2' is the same product times y_2, in other words, $y_2' / x_2' = y_2 / 1$. The last y_2' is divided by the last x_2', and the result is $\sin r \approx r$.

Hence, θ is generated by adding the angles represented by the digits of the pseudo-quotient (the reverse of the pseudo-division operation discussed in the forward trigonometric routine) to using the residual angle r as the first partial sum:

$$\theta = q_0 \tan^{-1}(1) + q_1 \tan^{-1}(.1) + q_2 \tan^{-1}(.01) + \ldots + r \quad (13.33)$$

in which r is the residual angle. The coefficient q_i refers to the count in a particular pseudo-quotient digit. The result of this pseudo-multiplication process is the angle θ, that is equal to $\tan^{-1}|A|$, in which |A| is the input argument with the proper sign of A appended to θ of the $\tan^{-1}$ routine.

For $\tan^{-1}$, the angle is normalised. Recall that for $\sin^{-1}$, $A / \sqrt{(1 - A^2)}$ was first generated. Thus for $\sin^{-1}$, the result of the $\tan^{-1}$ routine is again normalised. For $\cos^{-1}$, the $\tan^{-1}$ routine returns $\sin^{-1}$. Append the proper sign to the answer and calculate $\cos^{-1} A = \frac{1}{2}\pi - \sin^{-1} A$ (Egbert, 1977a, b).

14. Computing Techniques

Abacus

The earliest known analogue computing device was the abacus. Having originated in Asia more than 5000 years ago, the abacus is still employed in some parts of the world. It is not a true computer, because calculation is carried out by the user who must remember the rules for arithmetic operations. The abacus was perhaps the earliest mathematical machine. The abacus is lost in antiquity, and computers of some kind were evidently built by the ancient Greeks. Considering the abacus, numbers are represented by physical objects in a way that offers wider scope for number representation and calculation in commercial applications. Theoretically, the number of physical objects is limiting man's advancement.

14.1 Logarithms and Slide Rules

John Napier of Edinburgh is also recalled as the inventor of logarithms. However, Jobst (Joost) Bürki conceived independently a system of logarithms in 1588, published in 1620. Napier (or Latin: Naper) originated the concept of logarithms in two treatises: *Mirifici Logarithmorum Canonis Descriptio* (1614) and *Mirifici Logarithmorum Canonis Constructio* (1615).

In 1617, Henry Briggs invented the common logarithms. Most readers have never used logarithms, or the knowledge of logs is shrouded in the dark mists of time. Logarithm tables were considered as one of the simplest and most widely used of all aids to calculation. A calculation procedure performs a process of simple inspection, consultation of tables and addition or subtraction, such as in multiplication and division.

The more common tables of logarithms and of trigonometric functions are provided with such a small tabular interval that interpolation is made easy, the process being known as linear interpolation. Theoretically, the availability of eight-place logarithm and interpolation tables for the Gauss-Krüger transverse Mercator projection system made the use of 3° wide zones in Germany essential (Brouwer, 1961).

Slide Rules

An alternative method of logarithmic calculation is the method illustrated by the slide rule (Figure 113). William Oughtred invented the circular and the rectilinear slide rule. Robert Bissaker (1654) conceived the first known slide rule, and Peter M. Roget (1814) invented the log-log-slide rule. The slide rule uses the basic characteristics of the logarithm tables, which is a radical change in the principle. A slide rule does not handle numbers in digital form. It measures.

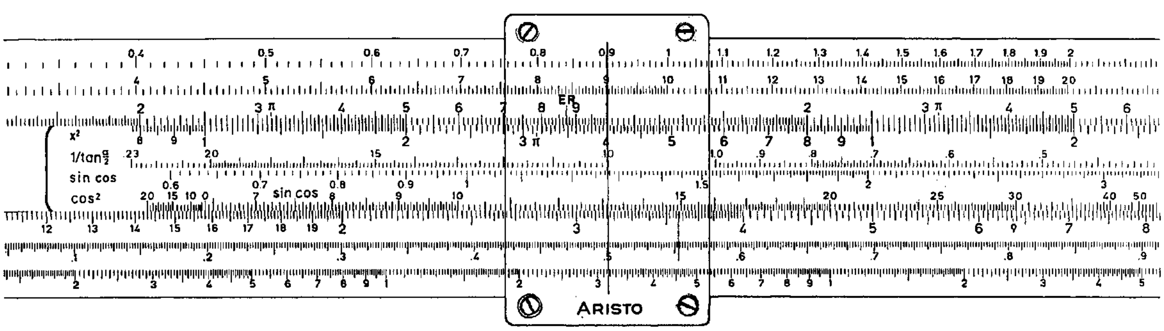


Figure 113: Part of the "Aristo-Geodät" slide rule in Gon

Number-length representation is by graduations along the scales showing lengths proportional to the logarithm of a number. Since multiplication can be done by the addition of logarithms, the slide rule combines two scales for multiplication, the resultant length being proportional to the logarithm of the product. This method to solve arithmetic problems was cumbersome and impractical in the field of land surveying and hydrography.

14.2 Mechanical Calculators

First Mechanisms

In 1594, John Napier conceived methods of multiplying and dividing to mechanise multiplication. Napier's square rods - in fact a slide rule - were never made to work satisfactorily.

In the 17th-century, Wilhelm Schickard conceived the first mechanical calculator. Blaise Pascal constructed working mechanisms to do arithmetical operations. Gottfried Wilhelm von Leibniz created an advanced calculator to multiply, divide, and calculate square roots. Leibniz never lost sight of the fact that everything interlocks. Even though he did not succeed in writing history, his effort was influential because he devised new combinations of old ideas and invented very new ones. He outlined a program for what would now be called automatised thinking (LaRousse, 1999).

Difference Engine

The first advanced mathematical machine to compute and to print mathematical tables was conceived by Charles Babbage. His first brainchild was labelled *the Difference Engine*. It was to be based on the principle of constant differences. Babbage's great conception was commenced in 1823, and abandoned in 1842. In 1991, UK Scientists managed to construct a Difference Engine, a copy of Babbage's design.

Analytical Engine

Later, a second mathematical machine was designed to carry out any mathematical operation, which performs the operation of multiplying or dividing a number by any power of ten. Using his fortune, Babbage aimed with another brilliant idea at *an Analytical Engine*; something higher than a desktop calculator did to compute lengthy mathematical tables. Henceforth, he worked for several years on an improved and simplified second difference engine (LaRousse, 1999; Morrison, 1952).

It was the work of a designer for which his epoch was not ready, because his vision was greater than the mechanical means then available for achieving it. Artisans of that time could just not make sufficiently accurate parts. The conception of the engines was genius, but his great engines never cranked out the answers. Charles Babbage of England clearly visualised a general-purpose computer, complete with a flexible programming scheme and memory units. Early electronic computers contained what Babbage was trying to carry it out mechanically. He proposed to do it all with punched cards modelled on those used in the Jacquard's textile machines in 1804 (Morrison, 1952).

Rotary Calculating Machine

A jump may be made to the rotary calculating machine. An addition can be achieved by a simple gear method, and with further mechanical ingenuity, the wheels may be connected so that subtraction and multiplication can be done. Charles Xavier Thomas de Colmar (1820) constructed the commercially available arithmometer. William Seward Burroughs (1875) patented the first recording adding machine (1892), and Frank Stephen Baldwin the *Arithmometer* in1890.

In the 20th-century, several rotary calculating machines were used for geodetical engineering purposes, such as Brunsviga, Facit, Ohdner and the electric Friden, Monroe and Olivetti with a built-in printer mechanism. Curt Herzstark's Curta-I and Curta II rotary pocket machines were in use by geodetic engineers in land surveying.

The manual positioning of the decimal point was imperative. Again, the method of doing calculations that are more complex was awkward.

14.3 Mathematical Functions for Use in Subroutines

Forward and Inverse Functions can be calculated

forward function		equivalent
sec (x)	=	1 / cos (x)
cosec (x)	=	1 / sin (x)
cot (x)	=	1 / tan (x)
sinh (x)	=	[exp (x) – exp (– x)] / 2
cosh (x)	=	[exp (x) + exp (– x)] / 2
tanh (x)	=	[exp (x) – exp (– x)] / [exp (x) + exp (– x)]
sech (x)	=	2 / [exp (x) + exp (– x)]
csch (x)	=	2 / [exp (x) – exp (– x)]
coth (x)	=	exp (– x) / [exp (x) – exp (– x)] 2 + 1

inverse function		equivalent
arcsin (x)	=	arctan [x / sqr (1 – x^2)]
arccos (x)	=	π / 2 –arctan [x / sqr (1 – x^2)]
arcsec (x)	=	arctan [x / sqr (x^2 – 1)] + sgn (sgn (x) –1) π / 2
arccsc (x)	=	arctan [x / sqr (x^2 – 1)] + sgn (x) –1) π / 2
arccot (x)	=	arctan (x) + π / 2
arcsinh (x)	=	log [x / sqr (x^2 + 1)]
arccosh (x)	=	log [x + sqr (x^2 – 1)]
arctanh (x)	=	log [(1 + x) / (1 – x)] / 2
arccsch (x)	=	log [sgn (x) sqr (x^2 + 1) + 1] / x
arcsech (x)	=	log [sqr (1 – x^2) + 1] / x
arccoth (x)	=	log ((x + 1) / (x – 1)) / 2

14.4 Electronic Computers

The technical history of the section electronic computers, such as basic hardware, firmware and software is concerned with a big leap from the rotary calculating machines to electronic digital computers that have only come into prominence since the late 1930s.

In the realm of science, computers have - as a tool - not only replaced the tedious work of calculation, but techniques have been developed to review and enhance many calculation methods, leavening the geoscientist free to concentrate on those areas in which is no substitution for human endeavour, intuition and creative development. They were developed almost exclusively to solve mathematical problems without continual calls to outside sources such as tables. Digital computers are based on the binary system (London, 1968).

Electric-Digital-Relay

Independent developments associated with digital-relay computers began in 1935. Trying to automate large-scale calculations, Konrad Zuse masterminded the Z1 in the living room of his parent's home. It was the world's first entirely mechanical binary computer (Zuse, 1993).

Independently, George Stibitz developed the electro-mechanical Complex Number calculator, the first American relay machine assembled at Bell Laboratories in 1939. In these computers, binary numbers were represented by flip-flops (on/off positions) of the electric-digital-relay-switches denoting the binary bits zero or one. The digital computer is distinguished by the fact that it does not measure. It counts (Davis, 1949).

At the same time, Howard Aiken of Harvard University began designing the Automatic Sequence Controlled Calculator, the Harvard Mark I, an electro-mechanical machine, approximately 15.5 m long and 2.5 m high. It was put into operation use in 1944. All input data were entered on punched cards. Operations were controlled by a sequence of instructions on punched paper tapes, and output either was recorded on punched cards or printed by an electric IBM-typewriter.

Vacuum Tubes

Using vacuum tubes, the ENIAC (Electronic Numerical Integrator And Calculator) built at the University of Pennsylvania was devised by J.P. Eckert and J.W. Mauchly in 1945. The vacuum tube assembly at top was used in first generation of computers built by the International Business Corporation (IBM), starting in 1946.

It had been thought for three decades that the Colossus (England, 1943) and the ENIAC were the first electronic digital computers. Nevertheless, the first electronic digital ABC computer was actually built by John V. Atanasoff and C.E. Berry. The recognition of Atanasoff's achievement resulted from a lawsuit filed in 1967 by the Sperry-Rand Corp. against Honeywell Corp. to protect the patent on the ENIAC held at that time by Sperry. Portions of the patent covering essentially all aspects of electronic digital computers were shown to be derived from the ABC (Atanasoff-Berry-Computer), from information granted to J.W. Mauchly by Atanasoff in the early 1940s. In 1973, the ENIAC patent was ruled to be invalid by the US Federal Court.

Transistorised Circuits

Although it was invented in 1947, an improved transistor - superior to the vacuum tube - became commercially available as an alternative to the vacuum tube. In 1956 began a new computer generation when Konrad Zuse of Zuse KG (since 1967 Siemens AG) introduced the Z33 with micro-miniaturised-transistorised circuits. Control Data Corp. and IBM followed in 1960. By using long-life transistors in electronic circuits, along with an improved magnetic-core memory, computer manufacturers could produce smaller, more reliable digital computers (Strachey, 1966).

Integrated and Very-Large-Scale-Integrated Circuits

During the late 1960s, transistorised circuits were replaced by integrated circuits (ICs), which characterised another generation. This permitted the construction of computers with high speeds and reliability at low cost. Using IC fabrication technology showed a continuous progress of the computer generation. The integration size of an IC chip was followed by large-scale-integrated (LSI) circuits, with thousands of transistors and other arithmetic, logic, and control circuitry components packed onto a single chip.

In 1971, Intel Corporation introduced the first successful microprocessor (4004). In 1974, Rockwell Corp. introduced the first microcomputer PPS/four. Manufacturing of random-access-memory (RAM) chips, and semiconductor memories replaced the huge magnetic core memories in new computers. The microcomputers, including Cathode-Ray-Tube (CRT), keyboard, and I/O units - appeared on the market as a forerunner of the IBM Personal Computer (PC).

Packing related electronic components onto a single chip, the integration size had advanced to very-large-scale-integrated (VLSI) circuits in the 1980s. The progress of computer technology had permitted computer producers to reduce the size of chips used in their machines. Moreover, focusing on logic programming languages had a more gradual implication (LaRousse, 1999; Pool, 1999).

Generally, mainframes, such as the International Business Corporation (IBM) ES/9000 family and minicomputers, such as IBM AS/400 and DEC VAX family, were used by agencies and companies for corporate-wide business data processing in the 1990s, where a myriad of transactions must be processed by the computer within a short time. Beyond that, they performed complex calculations for scientific and geodetic problems as well (IBM, 2001).

14.5 SuperComputers

Electromagnetic signals cannot travel faster than the speed of light, which constitutes a fundamental speed limit for signal transmission and circuit switching. The current determination of the maximum velocity of magnetic radiation is based on the standard frequency of magnetic radiation emitted by the caesium atom (^{133}Cs) [6.1]. This limit has almost been reached due to decreasing the length of wires connecting electronic circuits, and the miniaturization of circuit components. To bypass physical limits of the circuits, computer manufacturers developed systems with more than one central processor, whereby it was possible to share a task using separate components that can be handled by each processor simultaneously. Supercomputers have a very large shared memory, in addition to a very fast input/output capability.

The late Seymour R. Cray was an unsurpassed designer of the high-speed super-computers that have achieved the maximum performance with most advanced hardware and software technology for large-scale scientific applications. The Cray Y-MP (1988) could do 2.7×10^9 flops. In 1992, Steve Scott was the pre-eminent architect of the Cray X1 scalable vector supercomputer. The Cray machines use vector arithmetic hardware, able to operate on long strings of numbers in complex scientific computing problems. Scott holds many patents in scalable parallel architecture, and high performance (HPC) interconnection networks. Burton J. Smith, an inventive scientist, began software development for the multithreaded architecture (MTA) to enhance parallelism to make effective use of multi-processors resources. Margaret Williams is responsible for research regarding Cray X1E, XT3 and XT1 parallel-vector-processor computers.

Ever-Faster Computers in the 2000's

Cray Inc. combines a diverse portfolio of purpose built supercomputers that combine fast processors with high-bandwidth, and low-latency interconnect technologies. This interconnect directly connects all processing elements in a 3D-torus topology, eliminating the complexity of external switches of all I/O traffic. Very fast large-scale research techniques call for capacity and balance:

- CrayT3E uses an optimised massively parallel processing (MPP) operating system with AMD Opteron processors and memory
- Cray X1E combines the processor performance of traditional vector systems with the scalability of micro-processor-based architectures (from 16 to 8192 processors), delivering up to 147 TFlops/s in a single system image
- Cray XT3 operating system UNICOS is designed to run large complex applications and scale efficiently to 30 000 processors.

Higher-performance supercomputers were developed through increased use of massively parallel processing (MPP) - using systems that connect together thousands of individual processors to share the workload - by Cray Inc., International Business Corporation (IBM), Silicon Graphics (SG), and Sun computers. These computers rely on microprocessors tied together to make ever-faster machines. Meanwhile, however, most MPP computers have proven being inefficient in some programming circumstances.

14.6 Scaleable Parallelism

IBM's computer architecture resulted in state-of the-art supercomputers, such as four innovations:

- IBM's reduced-instruction-set-computing (RISC) in the 1980s. RISC caused a giant leap in computing power by reducing the size of the instruction set on a chip
- boosting IBM's complementary metal-oxide semiconductor (CMOS) circuitry
- IBM's Deep Blue, the chess-playing supercomputer, defeated the chess champion Garry Kasparov in May 3-11, 1993 in New York City
- in the 1990s, scaleable parallelism has made it possible to build computers with anywhere from a few POWER (performance-optimisation-with-enhanced-RISC) processors to many thousands.

- since 1993, every half year a list of the TOP500 computer systems is published. In 2006, IBM's BlueGene/L supercomputer achieved a Linpack performance record of 280.6 TFlops/s (10^{12} Flops/s). No. 2, an IBM Server Blue achieved 91.2 TFlops/s Linpack performance. French Atomic Energy Authority's Tera 10 Supercomputer is confirmed No. 1 in Europe and No. 5 in the world. It achieved a performance of 42.9 TFlops/s Linpack performance.

About MEMS - boosting CMOS Circuitry - and Moore's Law

In 1972, IBM derived so-called key scaling factors for CMOS transistor circuits driving improvements. Scaling down of transistors are based on:

- thinner Silicon-On-Insulator (SOI) layers, reducing dimensions, and voltage levels.
- shortening the channel length (distance source - drain)
- reducing voltage to a critical value between 1.0 and 1.5 V
- superior electrical conductors - such as copper - for circuitry on silicon (SiO_2) wafers
- keeping clock rates advancing up to 10 GHz.

For more than two decades, computer designers have relied on an outstanding materialisation of progress known as Moore's law. This is a tendency for doubling the number of transistors on a chip every 18 to 24 months. It occurs in part because the transistors can be made smaller. In the past, shrinkage of components has meant greater reliability and speed plus substantial savings in construction and operation of computer systems. Still, at some point, smaller transistors may not perform faster (Lerner, 1999; Stone, 1993); Moore (1965).

Since the 1990s, scientists try to boost system performance without increasing the clock rate by packaging micro-electro-mechanical-systems (MEMS) by taking simplification of semiconductors into the extreme. It means simplifying architecture, until a monolithic IC contains an entire computer, including GNSS, gyros, inertial and other sensors to make it possible to build powerful, high quality products at the lowest possible cost.

14.6.1 SP Hardware

In the year 2000, the IBM RS/6000 SP linked IBM's POWER (performance-optimisation-with-enhanced-RISC) microprocessors in so-called scaleable-parallel (SP) Nodes (Figure 114). RS/6000-SP configuration range in size up to 128 symmetric multi-processing (SMP) Nodes per RS/6000 SP system - 512 Nodes as special bid - controlled by one Central Operator Workstation.

Nodes are controlled by a SP-Switch (Figure 115). Within a SP Server Frame drawer, each Node is a complete standalone RS/6000 processor. Each of these Nodes is equipped with matched components: two to eight PowerPC or POWER microprocessors lashed together, memory system, expansion slots, and internal (or external) disk storage devices.

Choice of Nodes

- SMP thin-Node, max. sixteen inside a tall or eight inside a short SP-system frame
- SMP wide-Node, max. eight inside a tall or four inside a short SP-system frame
- SMP high-Node, max. four inside a tall SP-system frame.

SP Facts

- execution of any large diversity of jobs at the same time
- continuous processing as obligatory in (geo)sciences
- adding capacity, improving performance without suspending operations
- flexible system management, servers controlled by one central operator workstation
- reducing complexity by using PSSP management software
- system scaleability configured to run a single MP, multiple symmetric-multi-processor (SMP) applications, or both Node modes

- upgradability of system partitioning and through individual Node upgrades.

IBM RISC System SP models provide the capabilities and solutions required to manage virtually every aspect of an open SP system environment at any level concerning application development, such as automated input/output (I/O) management (Figure 116).

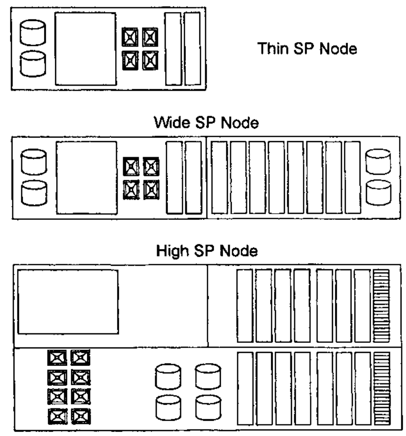


Figure 114: Overview IBM RS/6000 SP Nodes

SP Switch

SP Switch controls inter-node high-speed communications inside the SP-Frame:

- SP Switch board in a Frame interconnects up to 16 Nodes
- 16 paths for other Switches in other SP Frames
- peak throughput of each switch board - 4800 MB/s
- controls performance between Node pairs
- bi-directional peak of 300 MB/s
- bi-directional TCP/IP up to >160 MB/s - depending on Node.

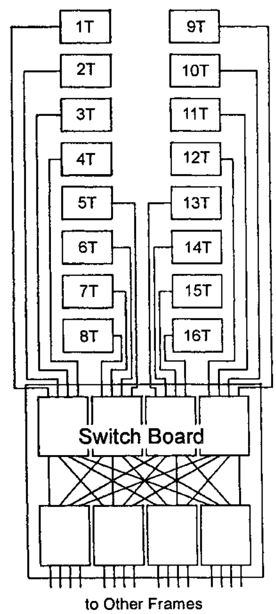


Figure 115: SP-Switch Circuitry

Control Workstation

RS/6000 SP Control Workstation commands:

- Console
- Frame Management
- HPGN Switch Router
- Network Clients and Servers to Nodes
- Power Control
- System Management.

Node Hardware Specs

processors:	2, 4, 6, or 8-way >1 GHz POWER or PowerPC processors
L1 cache:	for data and for instructions
L2 cache:	up to 256 KB
RAM memory:	up to one GB
memory bus width:	128-bits
disk bays:	up to four (26 disk bays, with SP Exp. I/O Unit)
PCI expansion slots	up to 10 (53 expansion slots, with SP Exp. I/O Unit)
bus speed I/O Adapter	up to 264 MB/s (cont'd)

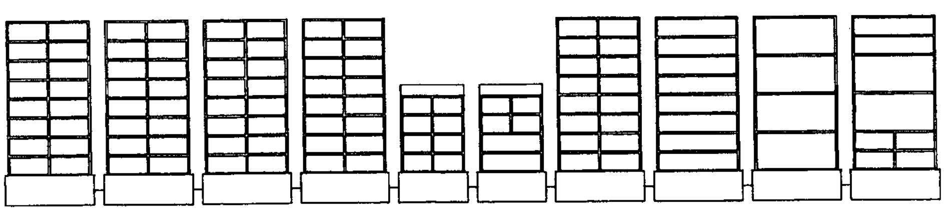


Figure 116: IBM RISC System / 6000 SP Massively Parallel Processors

(cont'd)

bus speed Switch adapters	400 MB/s
adapters:	Ultra SCSI and Ethernet (10/100 Mbps)

System Expansion

maximum RAM:	up to 16 GB
maximum internal storage:	up to 110 GB (946 GB, with SP Exp. I/O Unit)
SP Switch and adapter:	300 MB/s bi-directional
system dimensions	Tall Frame: .... 1.9 x 0.9 x 1.3 m Short Frame: .... 1.3 x 0.7 x 1.0 m

14.6.2 SP Software

SP Software Overview:

- Operating System AIX
- General Parallel File System (GPFS)
- Parallel Environment (PE)
- Parallel System Support Program (PSSP)
- Non-Uniform-Memory-Access (NUMA)
- IBM FORTRAN for AIX.

System software can be mixed and matched to support a broad range of applications common in scientific and technical computing.

Operating System AIX

IBM's UNIX Operating System (OS), AIX, is an integrated OS environment that supports 64-bit RS/6000 SP systems in their full range of scaleability connectivity, performance, interoperability, sharing access to files, memory and other system services, and usability with Internet technology.

Java development kit (JDK), just-in-time compiler (JIT), OpenGL and graPHIGS API's are included as standard components. While maintaining binary compatibility, AIX offers 64-bit scaleability for Web serving. It includes the next generation of Internet protocol (IPV6) and a lightweight directory access protocol (LDAP) compliant directory.

AIX maintains conformity to the X/Open XPG4 UNIX 95 branding and conforms to Open Group UNIX 98 specifications. It complies with the portable operating system interface for computer environments (POSIX) IEEE1003/1996. It conforms to single UNIX Specification (SPEC 1170).

General Parallel File System

IBM's General Parallel File System (GPFS) for AIX was designed expressly to deliver scaleable high performance across multiple disk-, and multiple file system Nodes, and to comply with UNIX file standards. GPFS is a logging file system that allows access to files within an RS/6000 SP system from any GPFS Node in the system. It can be exploited by parallel jobs running on multiple SP Nodes. A single GPFS multi-Node command can perform a file system function across the entire SP system. In addition, most existing UNIX utilities will also run unchanged. GPFS supports the file system standards of X/Open. It allows a configuration with an optimised RS/6000 SP Switch, component fail-over and disk use. Consequently, it recovers from most node, disk and adapter failures.

GPFS provides the scaleability needed as RS/6000 SP systems grow with additional processors and disks while maintaining standard UNIX file interfaces. GPFS scales beyond single-server (Node) performance limits, which is achieved by using IBM Virtual Shared Disk (VSD), client-side data caching, large file block support, and the ability to perform read-ahead and write-behind file functions.

Parallel Environment

Parallel environment (PE) for AIX on the RS/6000 platform is a complete development and execution environment for parallel applications on the AIX parallel-operating-environment (POE) platform. PE has been enhanced to exploit SMP Nodes, and also threaded applications and the thread-safe message-passing-interface (MPI) API for communications between tasks in a Fortran, C, C^{++} parallel program. It supports low-level-application-programming-interface (LAPI) programs. Visualisation tools (VT) show improved performance and communication characteristics of an application.

Parallel System Support Program

Through its use of the RS/6000 Cluster technology capabilities of IBM's parallel system support program (PSSP) for AIX in combination with IBM recoverable-virtual-shared-disk (RVSD), GPFS continues to operate in case of disk connection failures. GPFS allows data replication to reduce the chances of losing data if storage media fail.

Non-Uniform-Memory-Access

A non-uniform-memory-access (NUMA) program solves the problem of insufficient memory by combining memories of other nodes into one large single memory block, available for use by processors of these Nodes to run a single job. SP-Switch sees the memory of e.g. four individual Nodes as one large single memory block. All processors within the four Nodes may search in the combined memory block to complete the single job.

14.6.3 Parallel Applications for SP Platforms

XL-Fortran Compiler for AIX

IBM's XL-FORTRAN compiler (XLF) for AIX (exploits the RS/6000 SP-Family architecture with POWER and PowerPC microprocessor architectures) is the first XLF compiler to provide support for the full OpenMP Fortran application-program-interface (API). API is a new industry-standard for multi-platform symmetric multi-processing (SMP) programming on UNIX-based platforms. XLF for AIX was created in response to the needs of AIX users who must handle large applications in a multiple-workstation environment (IBM, 2000a, b, c, d).

XLF uses the graphical user interface (GUI)-based common-desktop-environment (CDE). CDE integration consists of an XLF application folder integrated within the CDE application manager. The XL-FORTRAN application folder contains icons representing the XLF tools and applications.

OpenMP is a portable, scaleable programming model designed to give SMP programmers a simple and flexible interface for developing parallel applications for platforms ranging from the desktop to the SP.

XL-FORTRAN Compiler Tools:

- supports 64-bit pointer and 64-bit addressability support for SMP and serial codes
- direct manipulation of the floating-point status, control register and asynchronous I/O
- to explicitly parallelise Fortran codes through the use of supported SMP directives for 64-bit codes
- to automatically parallelise DO loops
- supports automatic parallelisation of a Fortran program as well as explicit parallelisation of selected program sections
- support for the OpenMP Fortran API
- IBM's XL-Fortran run-time environment (RTE) for AIX environment to support (math)-library modules, asynchronous I/O, threadsafe I/O, and I/O services and utilities.
- GUI-based command-line builder
- supports FORTRAN-77 (F77) standards, Fortran90 (F90) and Fortran95 (F95) language extensions
- live parsing extensible (LPEX) editor is a language-sensitive and fully programmable editor that supports full Fortran95 function.

Asynchronous I/O may make it possible for other program statements to be executed while massive amounts of data transfer is taking place.

SMP applications may be run on a single SMP node on a SP system across multiple SMP nodes. User may manually distribute the code across SP SMP nodes using MPI, because automatic parallelisation is not supported

The XLF for AIX, and XLF RTE component for AIX have different software and hardware prerequisites depending on the support that is wanted. The Debugger is being provided as a technology preview with the XLF product. It provides support for C, C^{++}, Fortran, and Interpreted JAVA languages. Due to its common use as a powerful language for scientific research, its ease of use and intuitive structure, Fortran remains the language of choice for scientists and engineers around the world (IBM, 2000d).

Use of equipment, such as SP equipment, is imperative for the industry such as indicated [15.1, 15.2]. Already at the time of writing (2001), the IBM SP/2 equipment was replaced by the *Genesis parallel computing system*, using a similar architecture, the Linux operating system and IBM-Pentium servers. The scaleable parallel computing system is a combination of 32 racks with 32 servers each. Again, it is a system with separate nodes, in which every Pentium IBM-server is equipped with two SCSI hot-swappable hard disks, connected to a Cisco 2 Gigabit optical-fibre connection network.

Loading the software is accomplished with a special automated disk robot with 200 I/O units within one hour. The Linux RedHat operating system (OS) is installed on the *Genesis System* as is as non-propriety software, because Linux is an Open Source Architecture System, maintained, expanded, and extended by thousands of Linux users, and the Company's ICT Dept.

14.7 Operating Systems

Types of Software

- Operating Systems (OS) - support for direct running of application programs
- compilers such as FORTRAN
- application programs that direct the processing for an application [18; On_CD]. FORTRAN examples are produced by application programs are [On_CD].

Using a specific OS, such as the IBM-AIX, IBM-MVS, IBM-DOS/VSE, IBM-VM, IBM-OS/2, Linux, Microsoft PC-DOS, UNIX (several manufacturers), will result in features that are unique to the system.

FORTRAN and other programming languages [14.8] can create executable files, despite which OS is used.

In early computers, the user typed programs onto punched tape or cards, from which they were read into the computer for assembling or compiling and executing them. The results were then transmitted to a printer.

Evidently, much valuable computer time was wasted between users and while programs to be executed (jobs) were being read or while the results were being printed. Executive programs take care of housekeeping and allocate priorities, providing essential facilities that cannot economically be provided by hardware. It is also called an Operating System (OS). The earliest OS, developed by Johann (John) von Neumann, consisted of first generation machine language software resident in the computer that handled batches of user jobs without intervention by user or operator. An OS is a master control program held in memory throughout the time that the computer is in use. Most computers have such a control program and, indeed, many computers cannot operate without one. It bridges the gap between hardware and user's programs (software).

Thus, an operating system (OS) consists of a remarkably complex set of software instructions that schedules different operations of a computer, directs and coordinates program-processing. It controls the user applications to be performed, and allocates them to the computer's hardware, such as the central processing unit (CPU), main memory, and all peripheral systems. The central processor and OS directs the loading, storage, and execution of programs, such as tasks of accessing files, the OS applications controlling the memory storage devices, and interpreting in- and output commands. Usually, a computer executes several jobs simultaneously, and the OS proceeds to allocate the computer's time and resources in an efficient manner, prioritising waiting jobs over others in a process called time-sharing. An OS governs all computers' interactions within a computer-network.

The purpose of an OS is to make programming and machine operations more efficient and simpler. The functions of the OS depend on the size and complexity of the machine. Basic functions include in short:

- loading the required programs
- communications via monitor messages doing general housekeeping functions, such as initiating and controlling peripheral transfers, checking parity errors, and so forth.
- opening and closing files.

The CPU will be controlled by an OS that will monitor interruption and ensure that the appropriate results are output on the screen. OS ensures that each program is executed according to established priorities. Each program is executed according to the state of processing and the availability of the appropriate machine facilities at anyone time.

Personal Computer (PC) users have been choosing different approaches to solve the incompatibility problems, i.e., specifying standard OS. PC's use a simple OS, usually named some variant of the disk operating system (DOS), with the main jobs of managing the user's files, providing access to other software and supporting keyboard input and screen display. OS are becoming increasingly machine-independent. Thus, a user of an OS need not be concerned about the particular hardware platform on which it is running.

Linux Operating System

In 1991, Linus B. Torvalds conceived the kernel of the Linux Operating System, and placed it on the Internet /World Wide Web. Accordingly, the Linux system is free available from distributors, e.g. GNU/Linux or RedHat. Many hardware manufacturers, such as Compaq, Dell, Hewlett Packard, and IBM, support it. Linux is an Open Source Architecture System, maintained, expanded, and extended by thousands of enthusiastic global Linux users. Linux is as non-propriety software mainly used as a web server, without marketing departments or licences. Now, it runs on much architecture, including 334 of the 500 fastest supercomputers.

14.8 Programming Languages

Many third generation programming languages exist for different purposes, such as APL, C^{++}, and FORTRAN.

Assembly Language

Although computers from one manufacturer tend to have the same machine language, those from different manufacturers do not. Accordingly, different computers have different assemblers and assembly languages. Besides the conversion of the mnemonic operation code and decimal operand in each instruction into machine code, most assembly languages have functions to simplify programming, such as combining a sequence of several instructions into one pseudo-instruction. Machine code instructions are then generated from this pseudo-instruction. Programming in assembly (second generation) languages requires a solid knowledge of computer architecture. In addition, it is more time-consuming than programming in high-level languages.

4GLs

Fourth-generation languages (4GLs) are closer to human language than other high-level languages. 4GLs are intended to be easier for users than machine languages (the first generation), assembly languages, and high-level languages. Many 4GLs actually incorporate third-generation software as well. 4GLs are used primarily for database management. Applications are becoming increasingly complex as new features are continually added. Thus, the development of new application programs and the improvement of old ones are becoming extremely time-consuming.

BASIC Languages

A third generation beginner's all-purpose symbolic instruction code (BASIC) is a general-purpose third generation programming language developed by John G. Kemeny and Thomas E. Kurtz at Dartmouth College in the mid-1960s. It is the simplest high-level languages. Since about 1980, all BASIC languages have become popular for use on personal computers. In the 2000s, the Hewlett Packard pocket calculator HP-48 and its predecessor HP-41CV were replaced by the Ti-89 Titanium calculator, running a Ti-BASIC language, with a view-screen monitor, or linked to a Ti-Presentator, using a *Ti-Data Projector* (Purnell, 2005).

C^{++} Language

C^{++}, an object-oriented programming language, was developed by Bjarne Stroustrup of AT&T Bell Labs in the early 1980s, has become extremely popular because of its high programming productivity. Object-oriented programming is a programming technique in which a program is written with discrete objects that are self-contained collections of computational procedures and data structures. New programs can be written by assembling a set of these predefined, self-contained objects in far shorter time than by writing complete programs from scratch (LaRousse, 1999).

FORTRAN Languages

In 1954, the third generation FORmula TRANSlation (FORTRAN) was developed as a language for numerical analysis computation by John W. Backus and colleagues at IBM. It was announced by IBM in 1957 [17.1].

Within half a century, FORTRAN compilers were developed by software companies, such as Absoft, Cray, DEC, GNU, HP, IBM, Intel, Lahey, Microsoft, NAG, OpenWatcom, Ryan-McFarland, Salford, and SUN in various versions, such as Fortran II, ANSI Fortran 77, ANSI Fortran 90, ANSI Fortran 95, ANSI Fortran 2003.

Various host environments (operating systems (OS) under which Fortran run), such as IBM-AIX, Cray operating system (COS), Linux, Mac OS-X, SPARK Solaris, UNIX, MS-DOS, PC-DOS, including OS/2, accept the implementation of a FORTRAN compiler versions.

14.9 Timeline of Calculating

inventors	year	name of invention
Pürbach, Georg von	1450	table of sines and tangents
Regiomontanus	1464	expended table of sines and tangents
Rheticus	1540	table of sines, secants and tangents, publ. 1596
Viète, François	1579	math. laws applied to triangles
Bürki, Joost	1588	published table of logarithms in Prague (1620)
Napier. John	1594	multiplying and dividing using Napier's bones
Briggs, Henry	1617	invented the common logarithms - a base of 10
Gunter, Edmund	1620	invented forerunner of the slide rule
Schickard, Wilhelm	1623	first mechanical calculator
Oughtred, William	1632	conceived the circular / rectilinear slide rule
Pascal, Blaise	1642	an adding machine
Bissaker, Robert	1654	first known slide rule
Leibniz, Gottfried Wilhelm	1673	advanced calculator
Pascal, Blaise	1673	mechanical calculator
Roget, Peter M.	1814	conceived log-log-slide rule
Colmar, Charles Xavier Thomas de	1820	commercially available Arithmometer
Babbage, Charles	1822	Differential Engine – it was never completed
Babbage, Charles	1834	Analytical Engine - never completed
Mannheim, Amédée	1859	conceived first of modern slide rules

cont'd

inventors	year	name of invention
cont'd		
Hollerith, Hermann	1880	electromechanical and sensing punching device
Baldwin, Frank Stephen	1875	patented. Baldwin Computing Engine (1890)
Burroughs, William Seward	1892	first recording Adding Machine
Zuse, Konrad et al	1935	Zuse's Z1 mechanical digital computer
Atanasoff, John V.; Berry, Clifford E.	1939	prototype ABC - Atanasoff-Berry Computer
Atanasoff, John V.; Berry, Clifford E.	1942	ABC - Atanasoff-Berry Computer
British Scientists, Bletchley Park	1943	Colossus Electronic Digital computer
Aiken, Howard Hathaway; Lake, Clair D.;	1944	Harvard Mark-I, using punched paper tape
Neumann, Johann von (John)	1945	conception general internal stored program
Eckert, John Presper, Jr; Mauchly, John W.	1946	ENIAC (Electronic Numerical Integrator And Computer)
Williams, Frederic (Calland)	1946	CRT memory system
Durfee, B.M.; Hamilton, F.E.	1947	Harvard Mark II
Shockley, William Bradford; Bardeen, John;	1947	point-contact germanium-transistor
Forrester, Jay Wright	1949	3D-magnetic core RAM
Eckert, John Presper, Jr; Mauchly, John W. Neumann, Johann von (John)	1950	Computer (EDVAC) Electronic Discrete Variable Automatic computer
IBM	1952	IBM 701 computer
Kilby, Jack, of TI (Texas Instruments Inc.)	1956	Integrated Circuit (IC), a solid-state device
Brattain, Walter H. of Bell Labs	1957	silicon-transistor in operational use
Noyce, Robert Norton; Hoerni, Jean	1959	IC of Fairchild Corp.
Moore, Gordon	1965	Moore's Law: doubling proficiency
Hoff, Ted; Vadasz, Les; Mazor, Stan;	1970	Very-Large-Scale Integration (VLSI)
Faggin, Federico of INTEL	1971	INTEL 4-bit 4004, world's first microprocessor
Hewlett-Packard	1972	HP-35 Pocket Calculator
INTEL	1972	INTEL 8-bit 8008
Xerox	1973	Xerox Alto - Graphical User Interface (GUI)
Hewlett-Packard	1974	HP-65 Programmable Pocket Calculator
Rockwell	1974	world's first microcomputer, 4-bit PPS/4
MITS	1975	Altair 8800 hobbycomputer
Cray , Seymour R. of Cray Research Inc.	1976	Cray-I, high-speed supercomputer (SC)
TI (Texas Instruments Inc.)	1976	16-bit TI 9900 microprocessor
Hewlett-Packard	1977	HP-9845 Desktop Calculator
Jobs, Steve; Wozniak, Stephen	1977	Apple II computer
Gates, Bill; Allen, Paul (Microsoft Corp.)	1981	MS-DOS software v1.00
IBM	1981	Personal Computer (PC) with Intel 8088 (1978)
Jobs, Steve; Wozniak, Stephen	1984	Macintosh Lisa - GUI
INTEL 32-bit	1985	INTEL 32-bit 80386
IBM	1987	PS/2 Computer
Cray , Seymour R. of (Cray Research Inc.)	1988	Cray Y-MP (SC)
Microsoft Corp	1990	Microsoft Windows v3.00
Linus Torvald's system on Internet	1991	Linux Operating System
Pentium 80586	1993	Pentium 80586, microprocessor
IBM	1993	SP (Massively Parallel Processor)
IBM	1997	Deep Blue chess computer
IBM	1999	PowerPC microprocessor - 1 GHz

15. Information and Computer Technology

Acquisition and Recording Analogue Information

To record original text the mechanical typewriter was replaced by superior means of recording and storing data, such as the photographic process. It offered a method of capturing and storing reduced copies from the original analogue information precisely on microfilm strips and on microfiches, a film sheet containing multiple micro images. By the late 1950s, multipurpose electromagnetic media, such as magnetic (video) tape presented possibilities for recording analogue voice and video signals, and direct recording of analogue information, such as alphanumeric characters, bar codes, and special marks for subsequent sensing by magnetic or optical readers, and conversion to a digital form. In the mean time, paper continued to be the leading media for storage of textual information in an analogue form.

Modern information is stored in binary components of digital technology. These systems stems from their ability to represent information, stored in binary components of digital technology, as signals at exceptionally high speeds. Alphanumeric character tables and binary digit strings are called coding systems, such as the American seven- or eight-bit code for information exchange (ASCII), and the corresponding eight-bit extended binary coded decimal interchange code (EBCDIC) for IBM (International Business Machines) computers and compatible systems. In the 1990s, the computer industry devised a new international 16 bit-coding standard.

Recording and Storage Digital Information

Storage media based on electromagnetic and optic-electro technologies with random access or sequential access has superseded punched cards and perforated paper tape. Since the 1960s, the magnetic disk provides random access to data. In the 1970s, the flexible or floppy disk was introduced for use in microcomputer systems. Making use of laser technology, the optical disk became available during the early 1980s. It was a very different kind of digital recording and storage medium, in which the digital signals are converted to analogue information. In the 1990s, magnetic video recording tape, a serial-access medium, provided digital storage. Computers were used to abstract and digitise text to reduce space, cost and time. Advances in hard- and software for computers, automated electronic storage techniques, telecommunications, indexing and searching led to the establishment of electronic digital applications to control "*virtual*" archives, residing in global databases. After the 1990s, the explosive growth of communication networks have led to the introduction of widespread programs, catalogues, and database networks.

Growing portions of the resources, such as scientific data from various disciplines, software archives, environmental reports, plus electronic mail (e-mail) are accessible almost instantaneously via intranet, Internet (WWW), private telephone networks, and satellite communication linking agencies of private companies or National mapping agencies (NMA) and National Hydrographic Office (NHO) organisations.

Retrieval of Information

Inventory of recorded information can only be useful if it is systematically organised and if tools exist for locating, storing and retrieving of required information. Recorded information relies on extended description of its content and of some features of the physical items. The method employs tools of content analysis that consequently facilitate accessing and searching recorded information (RockWare, 2002).

A useful access key to analogue-form items is the subject. Lists of subject headings of library classification schemes provide only a gross access tool to the content of the items. Printed journals provide a means of keeping users informed of primary information sources only. An electronic document generally follows the principles of a digital database catalogue that is expected to be accessed individually. When the database is a part of a global database, computers and servers usually consist of a set connected scaleable parallel-processors (SP), and if several complexities are introduced, physical attributes, such as format and size, are highly variable in electronic documents. An challenging design is the "*living*" document, consisting of segments electronically copied from various documents, interspersed with graphics, voice comments contributed by scientists in various locations, whose dissimilar versions reside on a range of servers.

15.1 Relational Databases

Since the 1960s, retrieving information by computer and communication technology (ICT) employs techniques, using searching by means of logical operators and for traversing the database with links for corporate information retrieval. Image processing is a computational technique for analysing images. Image processing recognition has extensive applications in scientific areas, including astronomy, geology, geophysics, and hydrography. Digital information is stored in complex patterns. Therefore, the principal objective of all storage structures is to process data elements based on their relationships. Since the 1970s, databases containing highly structured information have the ability to handle unanticipated data relationships in relational storage structures, such as large-scale management information applications. Allowing users to access relational databases, a structured sequential query language (SQL) is necessary as an interface for retrieving records that perform calculations before displaying results

Information Searching and Retrieval

At present, a personal computer, computer terminal, or the Internet computer network, can search a number of databases maintained on heterogeneous computer servers. The latter are usually located at different geographic sites, and their databases contain different data types and incompatible data formats. The search is possible because in a standard document, an addressing scheme and a common communications protocol accommodate all the data types and formats used. Information video displays originate increasingly from digitally stored data, with the output media being print, video, beamers, and sound:

- prints and images on paper
- projected on a high-resolution digital television
- projected at a video system.

Information by computer and communication technology (ICT) are, converging in a high-resolution digital television set, capable of receiving alphanumeric, video, and audio signals, using displays, enabling users to scroll, zooming-in for enhancement, dividing the screen into multiple viewing areas. Displays use various media for visual representation of data, such as cathode ray tubes (CRT), flat-panel-displays (light-emitting diodes, liquid crystal, plasma panels), and video beamer projection. The capacity of the video-data or graphics screen depends on its resolution, but a very high resolution is fundamental in workstations (PWs).

In the 1990s, the optical disks (CD, DVD) provided the digital mass production technology for publication in machine-readable form of having large libraries of information available in almost every professional workstation. The combination of digital computers and telecommunications is also changing the modes of information dissemination. Updates of archival databases, catalogues, and computer software, are distributed electronically between remote computers. A network offers instantaneous access to vast resources of scientific information in the field of earth sciences as stored in computers around the world. Various types of communications, such as e-mail, telephone, electronic bulletin boards, and electronic teleconferencing, are aided by digital satellite communications using networks connecting areas of geographically spreaded scientists with common interests (Figure 117): pp 292.

Observe

The reader interested in more ICT theory is invited to read any of the many books on data communication, data transfer, data warehouses, database management, data-mining and processing.

Design of Science-oriented Information Systems

Since the 1960s, the cost performance of computer components in addition to the speed of microprocessors chips, at that time measured in millions of instructions per second (MIPS), increased exponential. The capacity of the basic integrated circuit (IC) has doubled consistently in intervals of eighteen months according to Moore's Law [14.6]. This means that electronic information systems (EIS) are closely tied with advances in digital communications and integration technology. Full exploitation of the developments for the realm of information systems requires corresponding advances in software disciplines.

At present, database management system (DBMS) software requires automatic high-level data handling facilities. The complexity of the data processes that comprise very large information systems has escaped from major breakthroughs. Many databases require frequent updating. In addition, cost-effectiveness of the software development sector improves slowly. Conversely, a wide-area computer communication network (WAN) permits sharing of data, electronic mail, printers, and programs. It uses a variety of media, such as fibre-optical data cable links or ultra-high-frequency (UHF)-radio or satellite links, designed to send data at speeds in the terabit-per-second (TB/s) range, contentedly moving huge volumes of information across a web of digital highways worldwide. Such a network operates according to a network protocol using the open system interconnection (OSI) reference model, formulated by the International Organisation for Standardisation. If all individual protocols conform to the OSI recommendations, computer networks can be interconnected resourcefully through gateways.

Global Database Facilities

Operating companies, functioning in the global hydrocarbon industry (GHI), have access to databases, data warehouses, and electronic teleconferencing facilities (ETF) in different *ICT friendly* locations around the world. One of those GHIs has global master-facilities (GMFs), or so-called master-hubs (MHs) in Amsterdam for the European and African continents, in Kuala Lumpur for the Australian and Asian continents, and in Houston for the American continent between the Hawaiian Isles and Greenland. These three GMFs are linked through a fibre-optical data transmission network system.

Third parties - ICT-Vision (ICTV) estates - care for trouble free GHI-accommodation including various means or devices designed, such as a uninterrupted power supply, maintaining room temperature, various cooling systems, extinguishers, safety, security systems, to guard the GHI (persons and property) against a broad range of hazards.

An GHI is in each master-hub (MH) owner of all available multi-axial copper or fibre-optical data transmission cable links, connectors, computing equipment, and database-facilities. Each MH consists of a so-called Twin-Data-Warehouse, in fact two different ICTV accommodation estates, with forty- and sixty-percent backup capabilities in Data-Warehouse-1, and Data-Warehouse-2, respectively. These data-warehouses are provided with two double sets of interconnecting fibre-optical 30 Gbit/s Cisco data transmission cable links.

Each MH is connected to a variety of slave-hub facilities (SHs). In addition, some hubs are connected to set personal computers (PCs) outside the office hours, personal workstations (PWs), massively scaleable parallel-processors (MPP), peripherals, or a vector supercomputers, such as the Cray|[16]. An automatic robotic data library (RDL) cares for database maintenance under control of the ICT Dept., e.g. in Amsterdam for 40 000 ICT employees in Continental Europe and Africa|[17].

Common information technology service is an internal technology company, and independently it has its own bottom-line worldwide responsibilities. Besides the service provision to sites in Europe, the service is involved in numerous global office projects. Simultaneously, various IT services are supporting the world's largest SAP/R3 implementation projects. Querying and updating by RDL are carried out simultaneously against the composite database centres or slave-facilities to safeguard their interests. As all graphics, sound, text, video, and voice information carriers can be converted and controlled by digital techniques, the scope of functions and potentials of ICT are irregularly expanding. Usually, database maintenance is outsourced to the third party industry [On_CD].

[16] Computer network users prefer software packs that fit their individual requirements using miscellaneous types, versions and languages. For that reason, most computers are replaced by offering standard firmware, hardware and software only, to reduce any maintenance to a minimum.

[17] In additional use outside the office hours, up to 75 percent of PCs are available for massively parallel systems to provide the high processing speeds required for the quick search and retrieval of needed information.

Electronic Teleconferencing Facilities

Since the late 1980s computer simulations and visualisation is a new field of tools that has grown expansively. It deals with the conversion of huge amounts of data originating from instruments, databases, or generation of synthetic environments into a visual 3D-display. An electronic teleconferencing facility (ETF) is the most efficient method of human information reception, analysis, and exchange using 3D-displays, interactive devices, related to virtual reality (Figure 117) [On_CD].

Figure 117: Electronic Teleconferencing Facility Centre fed by 1024 servers - Each server is equipped with two SCSI-discs

Any record-keeping structure may be considered as an electronic information system (EIS). Special is that digital information systems permit extremely fast, automated manipulation of digitally stored data and their transformation to and from analogue representation. Imperative for processing of information are techniques that collect, display, format, organise, process, and store, such as graphics, video, sound, voice, and text. In future, an information system may permit use of knowledge-based models, search and retrieval in 3D- or 4D-virtual worlds.

Geodetical, geological, and geophysical information systems fall within the domain of earth sciences. The main purpose of these systems is to gather, analyse, and integrate internal and external data. Correct understanding of the requirements and preferences of users is crucial to the design and success of an EIS. The development phase consists of writing and testing computer software, hardware, and of developing data input, output, and conventions. EIS in scientific organisations may be further distinguished according to their purpose, and services.

EIS maintenance refers to the structure of a system that results from changing requirements, experience with the system's use. It demands training of database managers, operators and users. Most EIS are implemented with one type of standard hardware, firmware, and software. Management-oriented information and decision-making functions are supported by executive information systems, using computational aids for data classification, and simulation.

Effects of Discarding Data

One important characteristic common to the reduction procedures has been to discard data that differ from their expected values by more than a certain threshold. Discarding in a Gaussian LS distribution is known by various names: filtering, pre-processing, data improvement, and so on (Bozic, 1979). It is contrary to sound statistical principles if applied rigorously to data from a distribution. But if a valid reason cannot be found, the value should be retained regardless of how far it may be from the expected value, because discarding of values farther than a certain amount from the expected value is a direct violation of the assumption that a Gaussian distribution is present. After the discarding, the set of values is no longer Gaussian.

15.2 Spatial 3D- or 4D-Databases

Epstein and Duchesneau discuss a report *The Use and Value of a Geodetic Reference System*, a benefit vs. cost framework for identifying and assessing economic value arising from the use of a geodetic reference system. In the context of economic value, a geodetic reference system (GRS) itself can be viewed as information for the production process. Using a geodetic reference system avoids costs, but does not automatically generate any benefits; these occur only if a demand exists (Epstein, 1984).

Most benefit occurs with the use of 3D- or 4D-information based on a geodetic system by engineers and scientists in the fields of agriculture, astronomy, aviation, charting or mapping, civil navigation, construction, electric power-supply systems, fisheries, geochemistry, geodesy, geography, geology, geophysics, hydrography, logistics, meteorology, mineral exploration and mining exploitation, missile guidance, oceanography, physical geography, recreation, telecommunication, and warship navigation. These users have the need to integrate information produced for geodetic purposes. Activities and decisions, which generate an economic benefit by using universally compatible spatial information, provide an opportunity to measure the benefits obtained. The establishment of universal multi-dimensional databases and EIS-networks is a part of the solution (Gardner, 2005).

It is a key element too (Leick, 2004):

- strengthen the worldwide research infrastructure, because fundamental, technological and geodetic research requires good ICT media to work effectively while geographically dispersed, through better and quicker access to relevant information sources
- improve the process and speed of innovation and stimulate the exploitation of results, through the links provided between universities, research centres, standardisation and specialists
- lead to consistent approach and implementation of worldwide common standards. The objective is the provision of a common integrated computer communication infrastructure and associated services. It will allow exchange of data and results of research on a worldwide basis.

Traditionally, the national geodetic network of every country uses a properly defined Datum and ellipsoid. New reference frames will be created and research is underway to arrive at a refined, optimal, and universal geoid together with a worldwide geodetic reference system and frame. To acquire, integrate, process and disseminate spatial data in a digital dataflow without operational limits, several types of digital instruments are in use, such as:

- multispectral scanning systems (MSS), remote sensing systems (RSS, ERTS, LandSat) [4.5] (Barbor, 2001)
- spaceborne photogrammetric cameras, such as the Russian KFA3000, KWR1000
- airborne photogrammetric CCD cameras (Kasser, 2002)
- terrestrial photogrammetric tools, such as CycloMedia (Beers, 1995)
- total-stations.

Spatial data of such instruments are used for integration to other sets of similar data.

ICT Effects on System Architecture

Concerning progress of computer technology, International Business Corporation (IBM) introduced the successful scaleable parallel-processor (SP) in the 1990s [14.6]. This had a significant impact on the methods applied to the qualitative comprehensive and quantitative models of the Earth. The continuing contributions by ICT and satellite geodesy allow scientists to explore complex applications in the domain of geophysics in 4D-space-and-time (4D-SAT), such as earth-crustal-deformations, plate-tectonic-faults and other geophysical phenomena [2.2].

The methods used in classical algorithms as by their nature are unnecessary slow and cumbersome. Using multi-dimensional calculation techniques, new algorithms support the advanced digital CCD-sensor instruments, such as mentioned above (Grafarend, 1997a, b, 2002f, 2003j, o).

Using the least squares adjustment (LS) of spaceborne-, airborne-, shipborne-, and terrestrial-surveys, the accuracies of the observations exceed those of any existing first-order geodetic network. Because spatial tools and surveying techniques yield three-dimensional vectors, the spatial graphical and alpha-numerically geodetic database is expected to generate benefits.

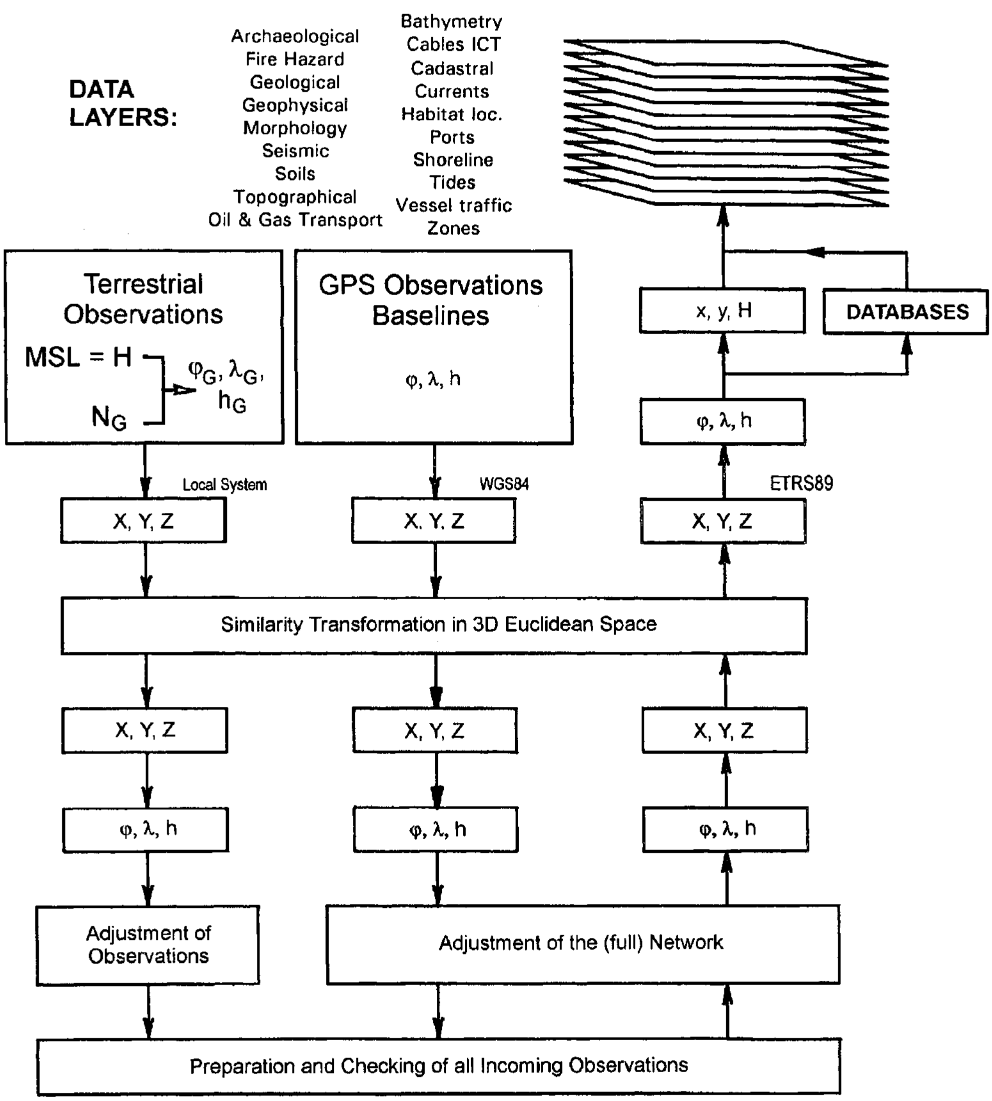


Figure 118: Sketch of spatial dataflow using multi-dimensional Databases and data Overlays in brief

Of course, the best universal multi-dimensional database and information network design - the requirements for the exchange of observation data among co-operating National Mapping Agencies (NMAs) and National Hydrographic Offices (NHOs) - have to be identified. Eventually, spatial measurements, least squares network adjustments and spatial layered database administration will be achieved by using a multi-dimensional database management system (DBMS) to arrive at the expected benefits. In many respects the future of geodetic and associated disciplines, and ICT from spatial data, is already with us.

The outline of the process and dataflow with overlays is given in brief (Figure 118) (Beers, 1995; Burkholder, 1995a, b; Epstein, 1984; Grafarend, 1985a, b, 1997a, b; Jivall, 1995; Leick, 2004, RockWare, 2002).

15.3 ICT-Human Resources

There is a lot more to report about the staff organisation working in the hydrocarbon and mineral resources industry than meets the eye [On_CD]. Even so, most engineers and scientists are not familiar with the core business as exploration and production. They may be surprised by the total scale, and by the opportunities of the operations offered in information technology to engineers with a high degree of personal motivation and drive. Such company depends on most advanced information technology systems, and inevitably, on initiatives, which will set the pace for others to follow. An ingenious, talented, or imaginative team - working at the forefront of technology - has the ability to turn ideas into robust reality, and with a commitment to see long-term projects through from start to finish, to maintain and to develop a strong global market position.

Customarily, the internal information technology services is an internal technology company (ITC), and as such has its own global bottom line responsibilities. Besides the service provision to sites in Europe, ITC is involved in numerous global office projects worldwide, such as upgrading networks and tens of thousands personal computers (PCs) in Europe alone, enabling to run state-of-the-art-tools such as desktop video, electronic documents, groupware, and internet/intranet management. Simultaneously, ITC are strengthening various IT services for some of the world's largest SAP/R3 implementation projects. Broadly speaking, in a dynamic IT environment an organisation requires many types of workforce: system engineers, telecom engineers, IT-project leaders, desktop custodians, and IT-contract analysts. They conceive a wide range of opportunities across a miscellaneous skill base, database systems, and storage management in a dynamic environment with support of a supercomputer with 20 000-40 000 on-line users [On_CD].

The Future of the World-Wide-Web

On the topic of the World-Wide-Web (WWW), one point is in no doubt. WWW will develop into new approaches, that replace and disregard the techniques of present fundamental methods. Currently, it is only possible to perceive dimly the scope and possibilities of the WWW-technology [4.5]. It brings - just like GNSS - the whole world to an instant in time at which humankind stands on the threshold of completely new technologies.

16. ICT Applied to Sea Surveying

Together with ice, the liquid water covers over 70.6 % of the Earth's surface, the hydrosphere. Despite great advances in hydrographic surveying and remote sensing techniques, this area cannot be easily mapped. The world's oceans are the largest not fully explored regions in the world. It remains a major frontier in the third millennium.

In the earth science disciplines, the Earth as a whole is a laboratory. In order to draw valid conclusions from his observations the geodesist amasses vast collections of data. Unique observations have only as much value as the capacity to analyse them keeps pace with the collection. Already in the 1950s, Beno Gutenberg remarked cogently:

> *" ... for the evaluation of observations, it is more important to reduce and analyse well-selected data by the best possible methods than to study all available data by less time-consuming but less accurate methods. Electronic tools should be used whenever practicable ... "*

It is only in recent history that the inner space beneath the sea surface has been subjected to scientific research and industrial exploration. In the late 1950s, the views of industrialists have been turned seaward by the insight that the requirements of the expanding world-population cannot for much longer be entirely satisfied by the onshore resources. The hostile marine environment has long inhibited all but a handful of explorers. To exploit the mineral resources of the marine environment successfully, the hydrographer or marine-surveyor, geologist or geophysicist has to determine the parameters involved. Skills of those familiar with the traditional marine activities as charting, diving, dredging, location, and positioning are in demand. Efforts peculiar to the marine environment have to be resolved, especially in the exposed areas far offshore to which hydrographical activity has spread.

Definition of Hydrography

> *Hydrography is that branch of applied sciences which deals with the measurement and description of the features of the seas and coastal areas for the primary purpose of navigation and all other marine purposes and activities, including offshore activities, research, protection of the environment, and prediction services*
>
> *(Kwok-Chu, 2003).*

Key technology in hydrographic surveying is based on precise navigation, data acquisition, presentation, storage and processing large volumes of data, all in real time to produce final results on the fly for telemetry, and designed according to IHO, IMO and IEC standards.

Hydrography deals with navigation, measurement of position and water depth, and description of physical features of the navigable portion of the hydrosphere, with a particular reference to make use of sub-sea bottom profiling and mapping. A description and the correction of hydrographical observations for the effects of waves, tides, and currents belong to the subject of hydrography (Lurton, 2002). Only the barest outline can be given here.

Historical Highlights

In 1912, SONAR (sound navigation and ranging) was used for the first time. In active SONAR systems, the returned acoustic signal determines the range from the transducer to the detected object, and the object location in the Sonar beam can be determined together with the object's relative location, shape and size. Its technological development has brought enormous improvement in transducer and signal processing technology (Gjerstad, 2002).

In the 1940s, radio positioning evolved spreading rapidly to improve ship navigation and safety, and eventually high frequency radio systems enabled surveyors to locate their work precisely. Surveying could proceed without interruption until the ship ran out of fuel, or food and water. At that moment, hydrographic surveying entered the realms of mass production.

Hydrography began as an activity to collect data for chart production and safe navigation. During the 1950s, with the start of offshore hydrocarbon exploration and exploitation, hydrographic data also came of vital importance for other users. The defining difference between the two applications is the significance of water depth. The requirements for nautical charting are depth dependent. Nevertheless, requirements for hydrocarbon exploration and exploitation, including the development of an extensive transportation system, are independent of depth. Seabed construction activities generally require the same type and quality of measurement (positioning, geological sampling, et cetera), as similar activities onshore (Enabnit, 2001; Gardner, 2005; Kreffer, 2004; Stevenson, 2004).

The story of the offshore survey industry is inevitably intertwined with the story of sea surveying or hydrography. Hydrographers were suddenly in demand until, by the 1960s, their ranks were joined by an influx of graduates in geography or oceanography, electronic technicians, professional mariners, anyone else who dared to read the Hi-Fix-6, HIRAN- or SHORAN-receiver-dials, and to plot a fix on a hyperbolic lattice chart. Experienced land and mining surveyors managed quite well while they could see survey stations ashore, but out of sight of land was a different matter. However it was a time of impending upheaval in government hydrography as political pressure increased to privatise data collection activities while, concurrently, the introduction of digital chart production moved forward swiftly (Haskins, 2001).

In response to the rapid expansion of the world's oil & gas industries, and the inherent need for suitably skilled personnel to supply the new demand, well-established hydrographic technical colleges developed courses in hydrography to teach students the basic principles and practices. These projects proved to be an enormous success. Students from all over the world attended. On the other hand, there remained one problem, the lack of a focal point for the people in the new profession: practitioners, engineers and manufacturers, all felt a need for information, to discuss solving technical problems, and seek to improve the standing of hydrography, its usefulness to the level of research into equipment and methodology.

In the early 1970s, the International Hydrographic Society, Plymouth, was formally established to promote the science of marine surveying and related disciplines in addition to fostering of recognised standards of education and training for those engaged in the hydrographic profession. The first hydrographic courses were organised with the aim of providing practical assistance to those from third world countries and newly emerging coastal states. In the following decades, national hydrographic societies (NHSs) spread around the world in countries, such as Australia, Canada, China, Denmark, France, Germany, Japan, the Netherlands, New Zealand, Russia, South Africa, and the USA. The objectives have largely been realised by regular promotion and organisation of international symposia, seminars, workshops, and publication of literature: *Hydro International* (HI) and *The Hydrographic Journal* (HJ). Individual and corporate members represent the fields of hydrography, oceanography, civil engineering, dredging and salvage contractors, environmental protection organisations, equipment manufacturers and suppliers, oil & gas companies, research and educational institutes at all levels of expertise (Atkinson, 2005; Haskins, 2001).

Hydrographic Markets

- planning of survey projects that support our onshore and offshore 3D- and 4D-seismic, drilling, crude oil and natural gas exploration and production
- naval expenditure
- seaport development and security
- marine equipment
- seaport management
- National Exclusive Economic Zones (EEZ) surveys 200 nmi
- submarine cables for the worldwide telecommunication system, Internet (WWW)
- route surveys of pipelines
- desalination
- ocean survey
- mineral exploration and exploitation

(Westwood, 2004).

16.1 Navigation and Positioning at Sea

During the 1890s, the potential for applying technology to oil field development was recognised. the basics of petroleum engineering were established in the states California and Texas, USA. Geologists were employed to correlate oil-producing zones and water zones from well to well. Between 1900 and the late 1930s, petroleum engineers were busy with drilling problems, designing casing strings, and improving the mechanical operations by using special drilling fluids, and directional drilling. Petroleum engineering focussed on complete systems of oil-gas-water reservoirs, and in petrophysics, electric logging was used to determine fluid and rock characteristics. In the reservoirs, estimates could be made of gas-oil-water saturations.

In the 1950s, the offshore oil industry sector started with the development of navigation and positioning at sea, a technology called hydrography. Hydrographers, land surveyors joined the oil and gas engineers to develop sea surveying standards for navigation, and horizontal and vertical positioning.

Using positioning system principles, measurements are carried out relative to the vessel's reference frame to keep an acceptable, accurate position. This position is affected by heading, heave, ray bending, pitch, and roll. Moreover, all measurements must be compensated by altitude measurements in such a way to avoid x-, y-, z-coordinate errors in conversion and timing. An acoustic geophysical sensor package mounted in a hull unit of a vessel with electronic positioning may be used for hydrographic route surveys, saving the time and cost of setting up an accurate seafloor acoustic array (Grewal, 2001).

Now, the geodetical differential global positioning system solution is integrated with an Inertial Navigation System (INS), using a wide area dual frequency DGNSS service with data telecommunication links. An INS is essentially the proper integration of dead reckoning (DR), using a state-of-the-art ring-laser gyro (RLG), and aided by a Doppler velocity log (DVL). It is capable to autonomous navigation using location and velocity to determine Earth referenced heading and positioning. A higher level of accuracy is only achieved by supporting the system with the measurements from external position sensors. Furthermore, an accuracy of DGNSS is determined by error sources in baseline, clock, clock-error, multipath, orbit, troposphere and ionosphere. Chart Datum - the height above the reference ellipsoid of the GPS-antenna and the movements of the heave-pitch-roll sensors are reported for position-fix corrections. Availability of the precise ellipsoid-height from satellite positioning represents an invaluable improvement in offshore surveying, but primarily for recording, processing, compiling and filing of depth data. MSL is anticipated to remain the final mapping depth Datum, but on a precise basis (Adams, 2006; Chance, 2003).

Survey Platforms

Various hydrographic survey platforms have become commercially feasible for specific applications, where they have delivered major improvements in quality and survey cost reductions, such as:

- inflatable survey boats for extremely shallow waters
- shallow-draught vessel for shallow waters
- survey vessels in deep water (McKay, 2004)
- buoys for real-time measurements of digital waves and location
- spaceborne remotely sensed SPOT imagery (LeGouic, 2004)
- airborne LIDAR bathymetry
- surface current drifters
- current meters
- current profiling devices
- autonomous underwater vehicle (AUVs)
- umbilical tethered (micro and mini)-ROVs.

As an example, recently, SHOM (Service Hydrographique et Oceanographique de la Marine of France) decided to construct a hydrographic vessel with the following technology:

- deep water multibeam hydrographic echo sounder

- shallow to medium water multibeam hydrographic echo sounder
- sub-bottom profiler
- 12 kHz deep-water single beam hydrographic echo sounder
- shallow to medium water single-beam hydrographic echo sounder
- integrated heading, position and attitude sensors
- dynamic positioning system (DPS)
- various sensors, such as (ADCP).

Observe

> *Abrupt termination of surveying operations is not unknown in hydrographic fleets. Unpredictable discontinuities as a result of an electronic breakdown, grounding or another disaster have to be accepted. Other factors can be foreseen and avoided by cautious management measures, in which contractors must demonstrate that they are competent, and completely understand what is required*

(Haskins, 1992)

Portable Acoustic Position System

A basic hydrographic system is a stand-alone portable underwater acoustic position system, based around a portable case, containing all essential electronics for underwater positioning. An electronic position system, together with applicable software and transducers deployed from any hydrographic support vessel or platform, is easy to use by interfacing the transducers for underwater positioning to the portable unit. In addition, the system can be directly interfaced to the reference frame of a differential global navigation satellite system (DGNSS) receiver, making it possible to give positions, using an ultra short baseline (USBL), or a super short baseline (SSBL), or a long baseline (LBL) in a coordinated reference frame (Lurton, 2002).

Observe

> *In the underwater environment, where GNSS is unavailable, aiding by ground referenced Doppler velocity measurements can reduce the long-term error by considerable several orders of magnitude.*

System Sensor Platforms

Bathymetric, side-scan, and sub-bottom profiler system sensors are:

- hull-mounted, and positioned by DGPS
- mounted in a tethered, buoyant towed-Fish
- mounted in a remote operated vehicle (ROV)
- mounted in an autonomous underwater vehicle (AUV).

Acoustic Data Transmission

In the hydrographic domain, underwater acoustic data transmission applications are used mainly between the support vessel and a submersible or an automated system, to get data available at the bottom (recordings, status information) without having to recover a submerged instrument physically. Similarly to other transmission systems, underwater acoustic data transmission is generally performed using digital electromagnetic signals. Data are coded as binary symbols, each type of symbol being transmitted with different acoustic signals. Of course, the design of acoustic digital data transmission systems benefits from the latest telecommunication techniques.

Underwater acoustic data transmission presents a few problems that are difficult to avoid. The first one is the low data rate to obtain acceptable ranges. Available bandwidths are reduced, and therefore the amount of information that can be transmitted. Unpredictable propagation strongly degrades the transmitted signal quality, in particular through interference and scattering. Using directive antennae may decrease the effects due to multiple paths and reverberation. The operator station can log selected sensor data received from the submersible, and store it in a file on hard disk, CD-Rom, or as data file communicated by telemetry up to the office network

(LAN) onshore (Lurton, 2002).

16.1.1 Underwater Acoustic Positioning

Single-function positioning transponder (SPT) and multi-function positioning transponder (MPT) are the sub-sea and seafloor elements of the High Precision Acoustic Positioning or hydro-acoustic position reference, such as the Kongsberg HiPAP and HPR systems, respectively. These transponder models have 56 channels available for use with the systems. The shipborne system comprises:

- shipborne operator station with a framework for acoustic positioning
- vessel positioning with a framework for input from DGPS or WADGPS
- AUV/ROV position input from an acoustic system plus INS (inertial positioning) principles
- computer interfaced to ROV sensors
- software collection.

Underwater ultra-short-baseline (USBL) systems use a SPT with telemetry functionality featuring one small array, and sub-sea super short-baseline (SSBL) systems use more than one positioning transponder with telemetry functionality featuring small arrays. Measuring the phase differences between different points on the array determines the direction of arrival of the acoustic waves from the transmitter placed on the moving object. Depth is measured using a pressure sensor, and transmitted acoustically. USBLs and SSBLs are easier to use than long-baseline (LBL) systems, but with less accuracy.

USBL and SSBL Positioning

The obvious advantage of the underwater USBL principle is that it only requires installation of one hull mounted transducer and one subsea transponder to establish a 3D-position of an underwater transponder (or ROV) in a rectangular coordinate frame (Figure 119).

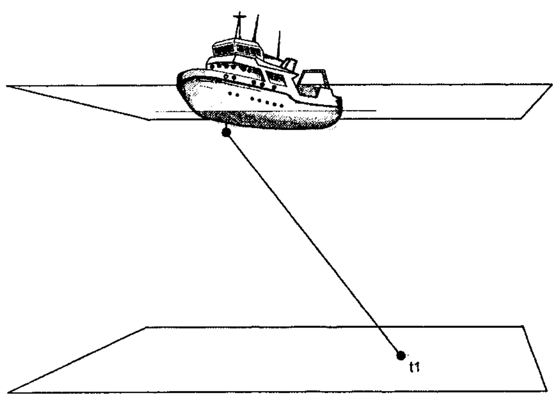


Figure 119: Ultra short baseline (USBL) configuration

Underwater super short baseline (SSBL) configuration uses a single transmitter and an array of transceivers (Figure 120). Positions are calculated by measuring the bearing and distances to the transponder. Accordingly, the positioning error will increase with the range to the transponder. Processing of data is done on a shipborne operator workstation with a computer, a high-resolution graphic monitor, a colour printer, and a plotter.

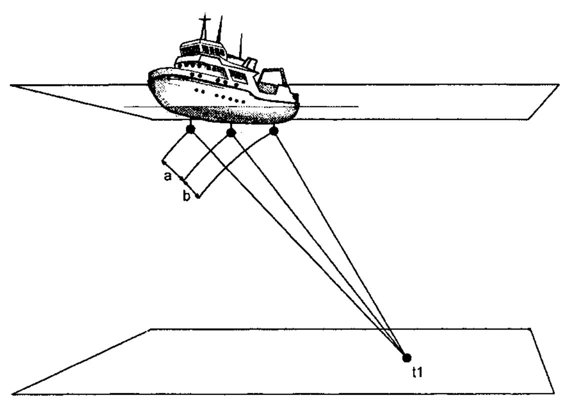


Figure 120: Super short baseline (SSBL) configuration

LBL Positioning

The underwater long base line (LBL) system depends on measuring ranges relative to a seabed basic multi-function positioning transponder (MPT) with build-in depth rated temperature sensors and telemetry functionality, widely spaced over the area to be covered. It can operate in both USBL (Figure 119) and LBL modes (Figure 121). For use in the LBL deep-water positioning is a dual beam option required.

LBL uses slant range-range measurements only from a transceiver mounted on a vessel, towed vehicle or sub-

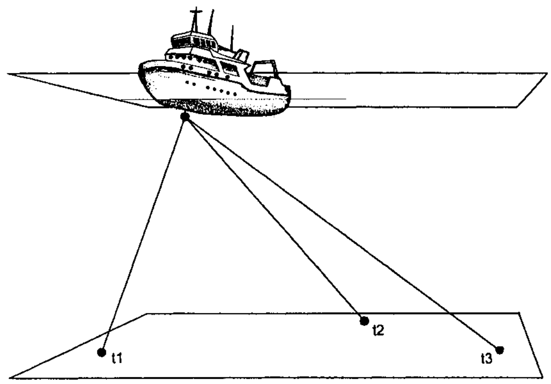


Figure 121: Long baseline (LBL) configuration

mersibles to an array of three or more transponders deployed at known positions on the seabed-framework (Figure 121). Distances are derived by multiplying sound velocity by the elapsed travel time of acoustic wave propagating between the master-transceiver and slave-transponders. The baseline length or distance between the seabed transponders may vary with water depth, bottom topography and the frequency of acoustic waves being used from of less than 50 m to 6000 m.

The main advantage of the LBL positioning technique is the high accuracy over the working area. Relative and absolute calibration of the array of framework-transponders is required to determine the precise geometry and location of the transponder framework. Several methods, including auto-calibration and direct baseline measurements, are available for the calibrating phase. Repeatability and total position accuracy is dependent on array calibration and geometry, correction of sound velocity variations, measurement errors, number of transponders, and reverberation (ray-bending and multipath) (Lurton, 2002).

After calibration, underwater long-baseline (LBL) systems can produce framework-position-fixing accuracies better than one metre. Positional accuracy and stability of a submersible depends on practical and physical factors, but within a few decimetres. An underwater LBL system has the transceiver installed on the ROV together with the transducer. The interface to the surface is via an optical or electrical serial line through the umbilical to the onboard operator workstation to secure full acoustic synchronisation.

The main limitation and drawbacks of the LBL system are the lengthy operations required to lay, deploy, and calibrate the frameworks of the seabed transponders. However, once the framework of the transponder array is geographically orientated and calibrated, a LBL system does not require any surface positioning system (such as GPS), which makes the LBL principle well suited for relative positioning. LBL systems are used worldwide to support a wide variety of sub-sea applications in water depths of less than 50 m down over 6000 metres, with a positional accuracy varying between 0.05 m up to 3-5 m, depending on operating conditions.

MuLBL Positioning

An underwater multi-user long base line system (MuLBL) (Figure 122) system is seen as the multi-purpose underwater acoustic high precision acoustic positioning framework. These systems are being developed that offer the profit of supporting an unrestricted number of subsea and surface users over a wide (underwater) area (up to 100 sq. km).

An additional potential is to transmit data throughout an operational area. Quite a lot of vessels, remotely operated vehicles (ROVs), autonomous underwater vehicles (AUVs), and other submersibles can navigate using the same seabed transponder array framework, because all vessels are in listening mode only. A MuLBL positioning system uses the standard LBL transponders, but the MuLBL avoids transponder frequency collisions when more vessels or ROVs are working in the same framework area (Oshima, 2005).

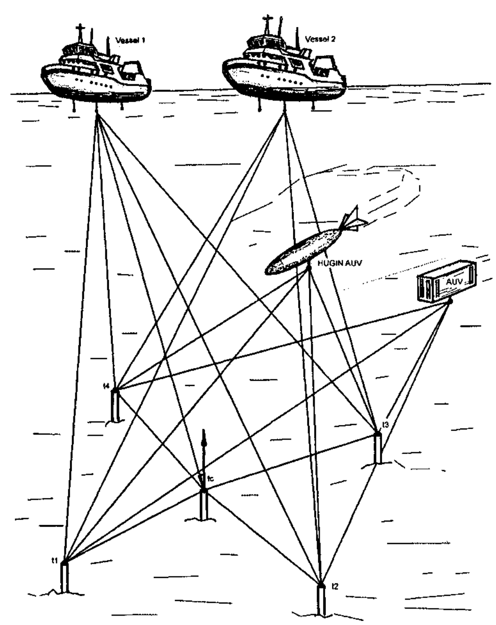


Figure 122: Multi-user Long Baseline (MuLBL) configuration

A variant consists in installing acoustic transponders attached to GPS-tracked travelling surface buoys, in fact an inverted LBL configuration.

Reverberation

The marine environment plays a complex role in the underwater acoustic propagation of signals. A transmitted signal is sometimes scattered back by objects present in the water column, the seabed or the sea surface, masking the authentic signal. This reverberation phenomenon occurs in all applications of current systems. Reverberation is an important process affecting underwater acoustic signals, analogous to electronic distance measurement clutter (multipath) in land surveying. As far as underwater acoustic signal detection is concerned, reverberation acts like noise: it adds an undesirable random component to the expected signal (Lurton, 2002)

Shipborne Operator Station

An underwater acoustic hydrographic inertial navigation computer (INC) onboard the vessel interfaces the inertial measurement unit (IMU) via a serial line (RS-232), and the HiPAP (Figure 123) or HPR system via an *Ethernet* interconnection. The INC gives continuity in position output, and it calculates a new acoustic position every second regardless of water depth. The operation in deeper waters is allowed since both the accuracy and the position update rates are better. Using a seafloor-based long baseline (LBL) system, precise hydrographic positioning of an acoustic geophysical sensor package in the deep-towed submersibles depends on measuring distances relative to a seabed transponder array framework, at the expense of extensive deployment and calibration time. Another support vessel would follow the vehicle and track it using an ultra short baseline (USBL) to supply accurate positioning. USBL positioning is often used for route surveys due to the time and cost of setting up the seafloor acoustic array framework (Grewal, 2001).

While improvements were made to the sensors over the years, the basic towing method remained the same for nearly two decades. In the mean time, the oil industry was slowly moving into deeper water. Because the cable length required for a deep tow is two to three times the water depth, the quantity of cable in the water also increased, at times to almost 10 000 m. The drag on such a cable length requires tow speeds of 2 knots or less. Accordingly, deep-towed positioning costs are dominated by time spent on line, and in line-turns. Hence, with the vehicle as much as 5 000 m behind the tow vessel, the tow-time spent in line turns could exceed four hours per turn, exceeding the time spent for collecting data. However, deep-towed sonar systems have significant limitations:

- a long time is needed to make the large turns required of many survey patterns
- flying at sufficient altitude to avoid crashing into the seabed or some variation in towing movements is necessary.

The tether is attached to a cable, which when dragged along the seafloor allows the suspended vehicle to collect high-quality data at a constant altitude, regardless of water depth or seafloor complexity. Real-time framework-data was transmitted up the tether to the operating station onboard of the towing-support vessel. Acoustic and inertial positioning principles in combination is ideal, since they have complementary qualities. Acoustic positioning is characterised by relatively high and evenly distributed noise and no drift in the position, whilst inertial positioning has very low short-term noise and relatively large drift in the position over time (McKay, 2004).

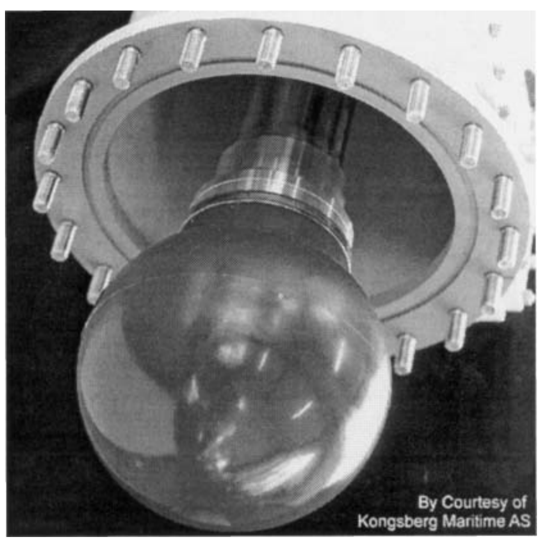


Figure 123: Kongsberg HiPAP using an array of 241 elements assembled into one sphere

Towed ROV Positioning

Since the early 1990s, there is an increasing use for micro and mini-ROVs in port security, port surveillance, docking, ships, and other structures of high-value assets. Important to the civilian and military user community is the ease of deployment and recovery of micro-ROVs in current use, reliability, and performance versus cost. Real-time 3D-sonar has been used by dredging monitoring, drilling platforms, inspection of rigs, pipelines, pipeline touchdown monitoring, and ROV navigation in the hydrocarbon industry.

Deep ROV Positioning

The umbilical remotely operated vehicle (towed ROV) concept is a buoyant tow-body, completely operated under acoustic control from a surface vessel. A manoeuvrable ROV is capable of performing fast surveys of excellent quality, using an acoustic geophysical sensor-framework-package for bathymetric, side-scan, and sub-bottom profiling. It is essentially the same as used in the deep-towed submersible in the deep tow system with a large rectangular grid pattern survey. In case of total system failure, an acoustic emergency data link is physically separated from the rest of the vehicle control system. It has a redundant power system, emergency pingers, light, radio beacons, and a drop weight, that will force the vehicle to the sea surface.

AUV Positioning

The key to autonomous underwater vehicle (AUV) technology is the inertial navigation system (INS) framework. The AUV-INS, using an inertial measurement unit (IMU), allows the vehicle autonomously to maintain a true course speed with only occasional updates from an external navigation position fixing system, such as LBL, SSBL or USBL. The position updates are sent by acoustic telemetry to the AUV. The telemetry link may be used to:

- set the transponder into different modes of operation, including altering transponder channels
- transfer data from the transponder.
- set the transponder receiver sensitivity and transmitter power level
- read battery lifetime.
- acoustic release.

Most autonomous underwater vehicles (AUVs) are used for tasks traditionally performed by ROVs for geophysical-, oceanographic-, or deep-sea-survey operations. An AUV concept is a battery powered, untethered, self-propelled autonomous survey vehicle, operating primarily under acoustic supervision from a surface vessel, or in autonomous modus completely relying on the intelligence and built-in control systems (Larsen, 2002).

Calibrating Underwater Acoustic Systems

After installation of a system, it is necessary to determine offsets between sensor reference points and axes:

- vertical angular offset between transducer axes and roll/pitch sensor axes
- horizontal angular offsets between roll/pitch sensor and heading reference
- horizontal angular offsets between transducer axis and heading reference
- horizontal distance offsets between transducer location and the reference point.

The principles for alignment procedures are based on the combination of geodetical position of the vessel (in latitude and longitude) and on the underwater position of fixed seabed transponders, by fixing the support vessel's position from DGPS along with the coordinated position of a underwater transponders.

An excellent replacement for this method is a calibrating computer program to calculate the alignment parameters. In summary, during the data acquisition, the procedure is to locate the vessel at a number of cardinal points. All alignments are computed and transferred to the alignment parameters, using the integrated transducer alignment utility in the *operator station* [On_CD]. Manual involvement is not required. All alignments are displayed both numerically and graphically on the operator station. The traditional calibration procedure, necessary and adequate for proving and documenting the accuracy of the system prior to any survey task uses the system similar to the base line crossing method [5.3]. Calculated parameters are (Lurton, 2002):

- gyro alignment
- sound velocity scale
- transducer inclinations
- transducer offsets
- transponder position.

Detailed knowledge about the sub-seabed and seabed environments becomes a key factor affecting the exploration, engineering and construction activities. Charting the pipeline route to the most optimal site means significant lower construction costs. The pipeline transport systems bringing the oil and gas ashore often have to cross rough seabeds, steep escarpments, and shallow continental shelves. Production of oil and gas into deep seas is facing a significant challenge of installing subsea production equipment in a complex seabed terrain.

Proficiency in Practice

Global operating C&C Technologies Inc., contractor in the field of earth sciences, uses autonomous underwater vehicles (AUVs), equipped with side-scan sonar, seismic systems, caesium magnetometers, sub-bottom profilers and multi-beam bathymetry systems for collection of high-resolution data from the shore to 4500 m ocean depth, together with DGPS-equipment for surface positioning, and underwater acoustic equipment for positioning surveys in shallow, deep, and ultra-deep waters (Fortenberry, 2006).

Since 1995, the high precision untethered geosurvey and inspection AUV model in the HUGIN-AUV family, manufactured by Kongsberg Maritime AS, is a well proven, third generation of the three preceding HUGIN vehicles, which were developed and operated in partnership with various private companies, such as BP, C&C Technologies, Geoconsult AS, Norsk Hydro, Shell, and Statoil. An HUGIN, rated to 1000 m, 3000 m or 4500 m water depths, is between 5.3 and 6.3 m long [On_CD]. An AUV is deployed from the supporting ship in a free-swimming mode (without umbilical) for up to 50 hours endurance before resurfacing. It is powered by a state-of-the-art *aluminium-oxygen-fuel-cell battery* (Kleiner, 2004). A HUGIN can be equipped with a full range of payload sensors depending on the application and user needs. It will initially be integrated with a variety of geosurvey sensors including:

- multibeam swath echo sounder
- chirp side-scan sonar
- sub-bottom profiler.

Underwater positioning will be performed using:

- HiPAP ultra short base line (USBL) system integrated with:
- Doppler speed log
- inertial navigation system (INS)
- DGPS for surface reference
- acoustic links for control of the vehicle, reading of sensor data and emergency control.

The system also includes a well-proven high-seas launch and retrieval system. It is stored in two containers that can be airfreighted worldwide. Exceptionally accurate data quality, coupled with time and cost savings, has given options that were never feasible with the various deep-tow methods. An HUGIN will be integrated with a variety of survey sensors, including multibeam echo sounder, side-scan sonar and sub-bottom profiler. Underwater positioning will be performed using a HiPAP USBL system integrated with Doppler speed log, an INS for surface reference, DGPS, and underwater acoustic links for control of the vehicle, reading of sensor data and emergency control.

The latest HUGIN utilises a state-of-the-art multibeam bathymetry and imagery system, dual frequency chirp side-scan sonar, and a chirp sub-bottom profiler. This positioning system combination allows an accuracy of 3-

6 m in survey depths up to 4500 metres. A deep-sea AUV-survey job that would have required one week on location was five times more efficient than the traditional deep-towed methods. Moreover, the acoustic sonar sensor package as used in the AUV-4500 was essentially the same as in the deep-tow system.

Using AUVs to survey close to the ocean floor, their hydrographic and geophysical tools are used to inspect historic wrecks, identification of moved pipelines, location of missing assets, plotting ocean-bottom maps to warrant safe installation of underwater pipelines, cables and production equipment for decades ahead. C&C-AUVs have surveyed 102 AUV projects, more than 60 000 km worldwide since 2001.

16.1.2 Chart Datum

In 1926, IHO-member states agreed that Chart Datum should be a plane so that the tide will seldom fall below it. Current practice is to establish the Datum at or near the level of the lowest astronomical tide (LAT). This Chart Datum is dependent on the range of tides at a particular point. Tidal Datums serve as reference surface against which the depths on navigational charts are shown, as the basis for the definition of various coastal jurisdictional boundaries, as reference surfaces for height Datums:

- the Datum may be established by terrestrial levelling, using an established tide gauge as a starting point
- GNSS ellipsoidal height differences can be determined by GNSS methods and the geoidal undulation difference applied to obtain Datum height at the remote site
- a temporary tide gauge (mareograph) can be established at the remote site and the Datum transferred by comparing the tidal record at the permanent gauge with that at the temporary gauge.

A large error stems from the non-uniformity or poorly defined levels to which the depths on the charts have been expressed relative to a range of vertical Datums. Consequently, the International Hydrographic Organisation (IHO) has set standards, which are applied to navigation charts around the world. The standards were first published in 1986, but advances in technology in satellite positioning, Sonar and shipborne computers led to tightening of the standards in 1998. Systematic errors and blunders must have been detected and removed from the survey data prior to evaluating them against IHO standards.

A sounding Datum is the level to which all Sonar soundings are reduced during the course of a hydrographic survey. Chart Datum is the datum of depth finally adopted for the published charts. It is the level above which tidal predictions, and tide levels are given. An onshore land-levelling-map Datum is usually different from a Chart Datum. Ideally, the Chart Datum should be used as the sounding Datum. An invaluable improvement in offshore hydrographic surveying becomes the precise WGS ellipsoid height from 3D-GPS positioning. This height represents the most precise and unambiguous repeatable offshore-Datum of depth, but primarily for recording, processing, compiling, and filing of depth data. An MSL related Chart Datum is anticipated to stay the final map Datum of depth, but in future, it will be computed using precise GPS- and mareograph-based observations, and approved by IAG and IUGG (Adams, 2004, 2006; Chance, 2003).

Chart Compilation

Detailed analysis of any positioning system, including the actual sea conditions experienced during the calibration, provides real insight into the performance of all instruments used. However, using the least squares (LS) solution as the most suitable mathematical method for positioning calculations, the benefits of redundancy in the observations provides the soundness of the method adopted. Any solution provides an optimum result, provided the algorithms and weights are applied correctly to the data. An acceptable standard deviation (SD) of the data set shows that the observations are of a good quality.

During the compilation of charts, simplifications are made to present the data from the *"lead line"* and echo sounder surveys on the chart. For navigation safety, values of depth relative to a reference Datum, such as mean sea level (MSL), are usually rounded downward.

The private sector is equally responsible in relation to inland waterways monitoring, precise navigation monitoring, disasters, and seaport approach. The government-supported surveys are generally multi-vessel programs requiring hydrographers, databases and equipment to produce hydrographic surveys according to IHO S-44 standards. Requirements include bathymetry, magnetometry, side-scan sonar, wreck investigations, tidal- and current observations, inspections of harbours and naval aids. Consequently, hydrographic charting is a large sector (71%), followed by coal, oil and gas (15%), and seaport / harbour surveys (12%).

The legal shoreline, a crossing line between land and water, for the country as shown on nautical charts is difficult to survey in a consistent way along the coast of the country. A shoreline is customarily tied to a particular vertical reference Datum, based on the tide, such as low low water (LLW). Only in an area with vertical artificial constructions, such as seawalls and piers, remains the position of the shoreline fixed as the water level varies with the tide. A very simple concept is the use of airborne photogrammetry to fix the position of the shoreline (Parker, 2001).

16.1.3 Electronic Chart Systems

Since the 1980s, electronic charting systems and their associated data handling aspects are of interest to the hydrographic navigator in his work. The majority of the hydrographic community is interested in an Electronic Chart System (ECS) or an electronic chart display and information system (ECDIS). While a paper chart is very effective for planning and looking-ahead, ECDIS is superior for tactical, real-time navigation and situational awareness. To mention some groups:

- national hydrographic offices (NHOs), which are involved in data production
- private ECS manufacturers
- sea-going surveyors in safely navigating their ships
- private sector marine surveyors who may use the systems as a basis for their own surveying activities.

Electronic digital charting offers many opportunities. Marine surveyors can use in ECS the advantages of the IHO S-57-standard, an accepted and prescribed data format of interest to the field of private sector surveying, with specifications IHO S-52 for chart content and display aspects. One application is the use of data to be displayed on top of navigational data, such as environmental data. ECDIS is a real-time GIS, optimised for maritime navigation. In addition, other hydrographic applications would benefit from an increased GIS focus. This includes coastal zone management, seafloor classification, and marine environmental protection (Alexander, 2004; Maratos, 2004; Yogendran, 2001).

The ENC is integrated with shipborne equipment for position fixing, monitoring speed, direction, meteorological and oceanographic conditions prevailing in the marine environment, and up to date systems, such as:

- GPS-receivers
- gyro compass
- magnetic compass
- echosounder
- log
- radar
- acoustic positioning systems and
- electronic positioning systems.

Developments in the third millennium paved the way for cost-effective solutions. NHOs have a mission to collect and to compile hydrographic data. Private companies are dedicated to an entire business sector to support them with tools and services. Fast-moving technologies, markets, and long production cycles mean that private companies and NHOs need a continuing and prospective overview of marine chart markets (Enabnit, 2001).

A system electronic navigation chart (SENC) is the digital database resulting from the transformation of the ENC by ECDIS for updates to the ENC by appropriate means and other supplementary data by the mariner (Hecht, 2002). Since the 1999s, the purpose of additional digital military data layers (AML) is to provide a unified series of geospatial data products - laid down by a NATO working group concerning WECDIS (Warship ECDIS) - to support safe navigation, while at the same time bundling and providing additional naval information (Freytag, 2004).

Maritime specifications published are:

- contour line bathymetry (CLB): provides simple depth information as points, lines and areas
- environment seabed and beach (ESB): provides features for amphibious operations
- large bottom objects (LBO): provides all known bottom contacts
- marine foundation and facilities: provides information about coastline and boundaries
- routes, areas and limits (RAL): provides, for example, selected aeronautical information, marine management areas, restricted areas.

Primary objectives of ENC and ECDIS are:

- coastal zone management
- development of seaports
- naval applications
- safe navigation
- scientific research
- seaport and harbours vessel traffic management.

Differing requirements for nautical charting and for offshore exploration and exploitation can be most readily appreciated where sub-sea construction activities are involved. Seabed surveying and mapping for such construction activities is based on industry standards of the hydrocarbon industry. Rocky and steep seabed near coastal areas, and significance of seabed topography was recognised as an imperative basis of project design. An optimised design is positively affecting construction safety. High priority within the context of such project activities resulted in detailed mapping of the seabed topography, together with survey operational cost-efficiency (Gardner, 2005).

Electronic nautical charts (ENC) have become commonplace on vessels and allow seamless updating via satellite communication with packages that include *Notice to Mariners* and chart corrections, weather forecasting and routing services. Development of such value added services is one option available to the commercial DGPS service suppliers.

In 2003, the General Bathymetric Chart of the Oceans (GEBCO) - the archetype of ocean charting at a worldwide scale - celebrated its centenary with the publication of a new, digital atlas. The chart contains soundings and contours, the culmination of one hundred years of successful unification of hydrography and oceanography. In addition, it reflects the co-operation of organisations, such as FIG, IHO, IMO, IOC (intergovernmental oceanographic commission (Monahan, 2004).

Exclusive Economic Zone

Even the definition of EEZ (Exclusive Economic Zone) requires the use of hydrographers to determine the location of the outer edge of the continental margin, because the environment of the continental shelf has proven to be significantly more complex than the featureless shallow water coastal margins. For that reason, the hydrographic survey industry plays an important role in full development of the world economy, because the EEZ shall not extend beyond 200 nmi (nautical miles) from baselines drawn by the coastal states, within which certain specific additional national legislation laws may be applied. The intention was (Burlik, 2004; Guy, 2005):

- to ensure that the United Nations convention on the law of the sea (UNCLOS) protects the resources of the sea, the seabed, its subsoil, to the benefit of states and countries
- that the IMO's Safety Of Life At Sea (SOLAS) convention states that the *Contracting Government undertakes to arrange for the collection and compilation of hydrographic data and the publication, dissemination and keeping up to date of all nautical information necessary for safe navigation*
- to take the opportunity to regularise the rights and responsibilities of all users of the sea.

UNCLOS will impact the entire field of Ocean Mapping for the next two decades. Delineating the juridical Continental Shelf requires a large amount of seafloor mapping and cooperation with the Commission on the Limits of the Continental Shelf (CLCS). These CLCS guidelines will influence data collections over extensive areas for the coming decades. Mapping the foot of the slope may refine our ability to discern small features at great depths and accelerate development of mathematical models of the seafloor. Isolated elevations adjacent to continental margins may require additional investigation to determine origin and nature of some ridges. It is imperative to maintain an infrastructure with the supporting database that could be observed by CLCS to consider the proposals (Gardner, 2005).

Accordingly, UNCLOS has forced coastal states to recognise the requirement in both customary and convention law to provide charting and maritime safety information, and determination of the hydrographic responsibility. Coastal States may lay pipelines, submarine cables and the right to authorise construction of installations and artificial islands within its EEZ (Guy, 2005).

16.2 Geo Marine Surveying

Naval applications represent the major part of activities related to underwater acoustics, such as systems of passive SONAR designed for detection, tracking, and identification of submarines (Figure 124).

Civilian underwater acoustics are used for fishery, geology, geophysics, mapping, navigation, physical oceanography and environmental investigations, estimates of attenuation in the sediment, evaluation of sound velocity in the water and in sediments, investigation of building sites (bridges and tunnels), marine archaeological investigations (search for wrecks, historical buildings and settlements), morphologies of the bottom surface or sediment layers, searches for mineral resources, inspection of pipelines, telephone cables, navigation obstacles, and surveying of the crossable water depths. However, the differences between electromagnetic- and underwater acoustic systems result from the physics of the environment. Propagation ranges using electromagnetic waves in the air hardly exceed a few kilometres, while sound propagation in water can currently be observed at ranges of up to thousands of kilometres (Figure 125) (Lurton, 2002; Müller, 1999; Simons, 2004).

Hydrographic bathymetry sounding is subject to very strict accuracy standards, particularly for coastal areas,

Figure 124: Kongsberg HUGIN-1000 - Mine-hunting survey

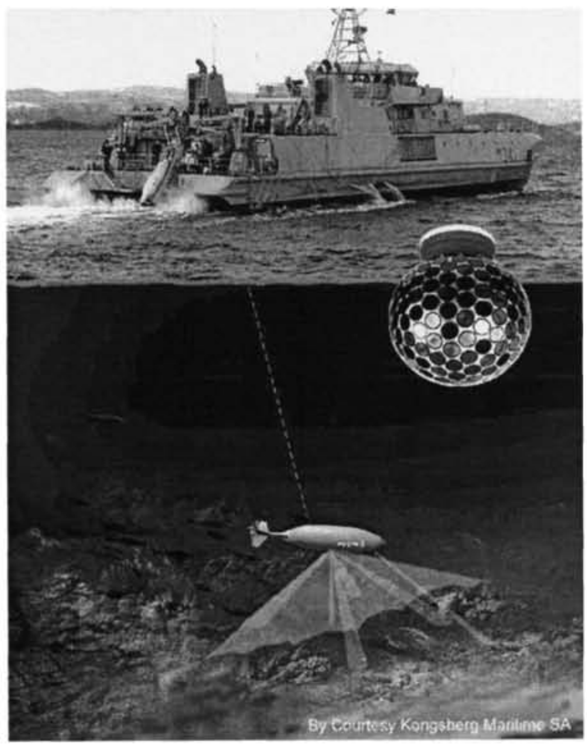


Figure 125: Hugin-3000 AUV using GNSS and Doppler log

harbours, and tidal estuaries. The performance of underwater acoustics mostly depends on the acoustic Signal-to-Noise-Ratio (SNR). One function of the sonar receiver is to improve the SNR.

The hydrocarbon industry conducts engineering- and hazard-surveys using geophysical instruments to assess the proposed pipeline route. Prior to conducting the survey, the proposed centreline for the route is provided by a reservoir engineer. During the survey, a side-scan sonar determines seafloor features, bathymetry data is recorded to examine the seafloor topography, and a sub-bottom profiler provides a cross-sectional view beneath the seafloor within the area of interest. A magnetometer is always used for detection of ferrous materials or objects within the survey lane (MacMillan, 2005).

Underwater Active and Passive Acoustic Systems

Most scientific knowledge of the underwater acoustics was obtained through geophysical research conducted since the International Geophysical Year 1957-1958 (IGY), including the deep Earth. Exploration of lake-, sea- and ocean depths has been made possible by the use of underwater acoustic sensors and related devices, because in the sub-sea environment is an electronic positioning system, such as GNSS, not available. One of the main differences between using electromagnetic waves in the air, and using acoustic waves in water, resides in the constraints brought about by the wave propagation. Target or obstacle detection, positioning using underwater acoustics is performed by using so-called SONAR (sound navigation and ranging) systems. These have been grouped in two categories: *naval sonar*, and *civilian sonar*. Very distinct types of underwater acoustic systems are:

- ASS (active sonar systems) use controlled signals, whose characteristics are imposed during the transmission phase
- PSS (passive sonar systems) intercept, and analyse signals
- seafloor charting systems.

Geo-Imagery

Geo-surveys are conducted over various water depths, each one with a unique geological seabed structure. Mining companies routinely acquire ultra high-resolution geophysical data in support of its offshore exploration and exploitation programs for mineral deposits situated on the continental shelf. Geophysical and geological imaging constitutes the framework upon which the entire exploration and mining operation is based, using some areas of activity (Stevenson, 2004):

- geophysical data collection
- geological mapping
- sub-bottom samples collection
- seabed geotechnical information
- geo-statistical evaluation
- mining area concessions
- geo-technical information
- policy of mining
- mining evaluation
- environmental impact of various activities.

Possible offshore hydrocarbon exploitation techniques have important economic implications. These have led to the development of electromagnetic navigation and underwater acoustic techniques. All activities required by the offshore industry are carried out for the determination of currents, depths, sub-bottom morphology, salinity, sediment transport, tidal currents, tides, waves and winds. Leading and accompanying every hydrographic survey, the hydrographer uses methods varying from mapping the sub-sea bottom and simple depth measurements by using a lead line via single-beam linear equipment to sophisticated digital multibeam echo sounders (MBES).

The expenditure for the gathering of hydrographical data runs into millions of dollars yearly. Even so, many of the phenomena still elude the scientist's perception (Benioff, 1958). The arrival of matured digital electronic instruments using solid-state sensors changed the work of the hydrographical community completely. Hence, a flood of more observed data has started which have to be added to the stockpile. Manpower for the manual evaluation of the data is either not available or too expensive. Even, it is doubtful if currently more than a fraction of the information content of the basic data is exploited. For that reason, electronic systems and underwater acoustics, data acquisition, data reduction processing, and data transmission have come into existence, and are undergoing further development. Using GNSS, and merging different types of acoustic techniques, such as swathe-sounding and MBES , the charts have greatly gained in accuracy and in completeness. The revolutionary impact this has on hydrography justifies a brief review.

Seismic Surveying

Hydrocarbon prospecting and geophysical exploration is characterised by the deployment of towed array Sonar systems (TASS). 4D-seismic surveying application technology is employed in marine seismology, in which several very long sonar systems may be towed at the same time. Modern naval passive sonars are characterised by the deployment of very long TASS, able to efficiently detect and locate low-frequency noise sources (Lurton, 2002).

Pipeline Inspection Survey

One of the primary differences between digital video storage and analogue video storage is its versatility. Analogue video can only be stored on tape. In contrast, digital video files can be stored on any type of digital storage media. Current practice for most forms of offshore inspection is to use an ROV equipped with a digital video camera and other sensors. It is now possible to integrate digital video with database information. During pipeline inspection survey, the side-scan sonar sensor is towed along the length of the pipeline at an optimum position from which to obtain a high-resolution sonar image of the pipeline and adjacent seabed. This data is analysed to identify features along the pipeline, in addition to seabed features such as boulders and sand waves.

16.2.1 Multi-Frequency Signal Processing

The design of SONAR signals and the signal processing by extracting the expected signal directly affects the performance of underwater acoustics (Blackband, 1968):

- detection of a signal and filtering
- evaluation of parameters, propagation time and tracking the angular direction
- recognition and summation to improve SNR.

Sonars, generated electrically by a source, such as the electrodynamic transducer (Figure 126; Figure 127), are propagated through the seawater. The power of the acoustic waves is attenuated as distance from the source increases. On encountering a target, such as the seabed, the waves are reflected or refracted to a particular degree. A quantity of the remaining power returns to the source as an echo. The signal strength of this echo is dependent upon the power of the original transmission, the effects on its propagation of the medium, and any targets on its path.

Figure 126: Single beam ceramic transducer

Underwater acoustic applications use a variety of signals allowing the determination of sediment characteristics. These underwater acoustics use a variety of multi-frequency signals chosen to carry information in applications.

Acoustic Pressure Waves

power - would spread spherically away from the source, are experienced in the far field as plane waves. 90 to 95 percent of the underwater parts are compact ceramic transducer arrays. They can be mounted to the ship's hull, towed body or be installed on a special rack for transportable applications. Composite piezoelectric transducer materials comprising ceramic rods embedded in a polymer matrix, possess an unique property to provide an operational gain bandwidth product in which the element shape is defined solely via the electrode pattern. The plane waves are reflected at the target. The reflected power spreads spherically from it, the returning echo being experienced at the receiving antenna (possibly an array made of several transducers or hydrophones) as a plane wave. The voltage of an alternating current plotted against time results in a sinusoidal waveform: This electrical wave - related to the rotating coil, which produced the current - has a phase (ϕ), which varies with time according to the angular velocity of the coil (ω).

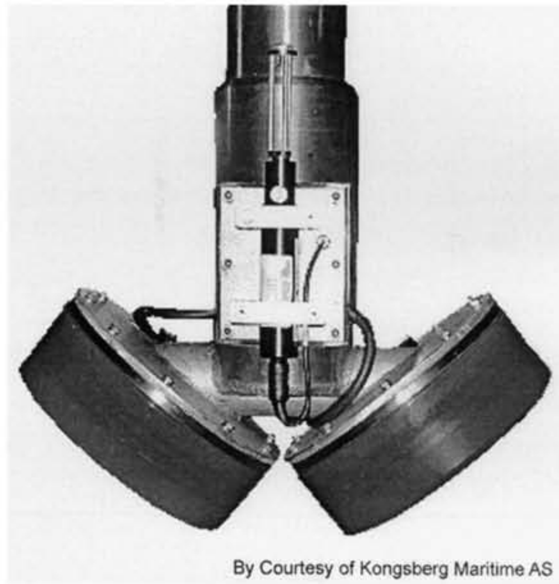


Figure 127: Kongsberg BM-636 dual beam ceramic transducer

Sonar instrumentation resolves target ranges by measuring the two-way travel time of a pulse of acoustic energy between source and target. The propagation velocity of the acoustic waves is clearly a factor in the conversion of time to slant distance or depth.

Sound velocity is proportional to the square root of the reciprocal product of density and elasticity of the medium. Accordingly, the velocity is dependent on temperature, salinity, and pressure (or depth) of the medium. In seawater, the velocity increases approximately 4.5 m s^{-1} per 1°C increase in temperature, 1.3 m s^{-1} per $1^{0}/_{00}$ increase in salinity and 1.7 m s^{-1} per 100 m increase in depth. The value c is usually between 1470 m s^{-1} and 1540 m s^{-1}. Empirical formulae, such as Wilson's equation, have been evolved for the calculation of sound velocity:

$$c\,(\mathrm{m\,s^{-1}}) \quad = \quad 1449.1 + c_P + c_S + c_T + c_{STP}$$

in which c_P, c_S, c_T, and c_{STP} are expressions allowing for changes in pressure, salinity, temperature, and for simultaneous changes in salinity, temperature and pressure, respectively. These are invariably found in *Oceanographical Tables*. For sonar instruments an average sounding velocity value c = 1500 m s^{-1} is often accepted as the sound velocity between source and target.

Taking into account the transmitting and receiving qualities of a sonar system, an echo level is of greatest interest to the hydrographer. It enables to draw an evaluation between various systems given similar operating conditions. Using the instrumental properties, the maximum range capability of selected systems may be compared, and shows how specific values of noise level and detection threshold can be taken into account (Ingham, 1975).

16.2.2 Acoustic Geo-Sensors

Acoustic geophysical sonar systems may be used for the measurement of depth (using echosounders), penetration of seabed sediments to reveal the characteristics of the substrata (using sub-seabed profilers), investigation of the seabed lithology and objects on the seabed (using side-scan sonars) at either short range or long range. Multibeam echo sounders therefore fall into categories, which may be described in general terms as:

- long range at 10 000 m, using 12 kHz
- medium range at 5 000 m, using 30 kHz
- short range at 300 m, using 10-200 kHz.

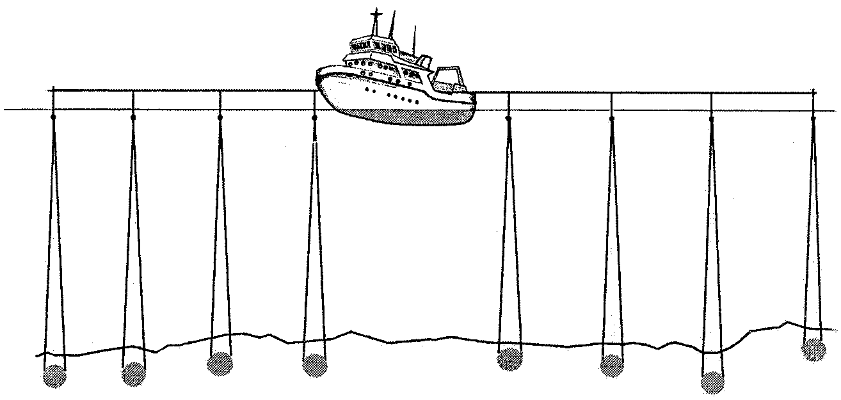


Figure 128: Simultaneous deployment of multi single-beam echosounders

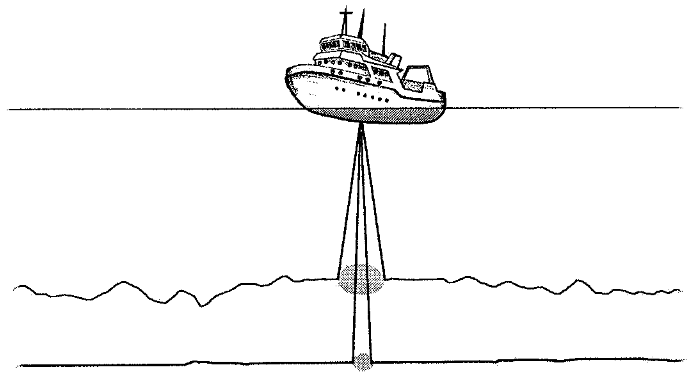


Figure 129: Hull mounted multibeam echosounder geometry

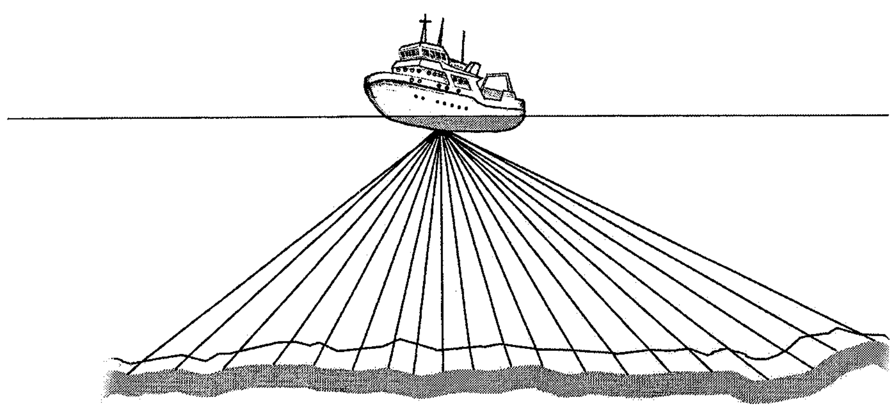


Figure 130: Hull mounted multibeam echosounder geometry

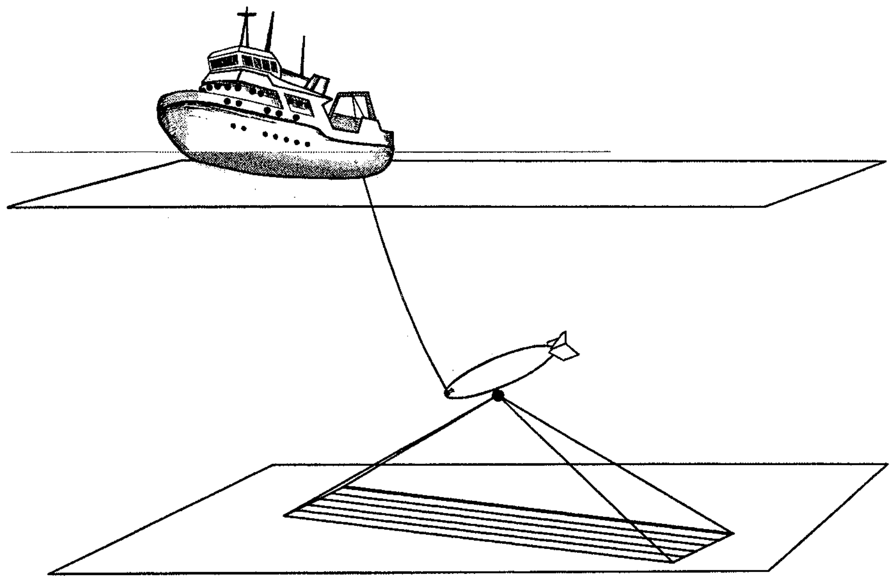


Figure 131: Towed Fish mounted sidescan sonar

The operation bears directly on the efficiency with which the frequency of a sonar system may fulfil the requirement and affects almost all the sonar parameters.

Sonar parameters are:

- low frequencies imply a long pulse and a long range with a good seabed penetration
- high frequencies imply a short pulse, a high attenuation and a short range with little or no penetration of the seabed.

16.2.3 Underwater Acoustic Systems

Underwater acoustic systems use a diversity of signals, chosen for their capacity to carry the information sought by the user in specific applications. Configurations are:

- bathymetric sounders - water depth
- sidescan sonars - acoustic imaging seabed
- multibeam sonars - acoustic seafloor mapping
- sediment profilers - seismological systems
- acoustic positioning - Cartesian x, y, z-systems
- ship ADCP - acoustic Doppler current profilers (HI, 2001a).

Since the 1960s, non-linear acoustics have been used in sediment profilers (Rines, 2001; Lurton, 2002).

Besides a good understanding of hydro-acoustics, development of new products such as wide-band multibeam echo sounders together with new signal processing algorithms, software, motion sensors, positioning instrumentation, have been supported by the hydrographic industry to meet their needs.

Various configurations of bathymetric echosounders are given in Figure 128, as used in estuaries and harbours, Figure 129, Figure 130, Figure 131 as used at sea and inland waters.

LIDAR System

H. Boot and J.T. Randall conceived the LIDAR (Light Detection And Ranging) system. At present, it is used in the field of telecommunication, and in hydrography to measure water depths up to 80m. Using acoustic methods is time-consuming in large, shallow waters. Sea surveying operations in these areas are difficult to survey by boat without extensive downtime, and they can be hazardous. Airborne LIDAR Bathymetry (ALB) is based on the differential timing of laser pulses reflected from water surface and underwater bottom to determine accurate estimates of water depth at the point where the laser pulses strike the water bottom. However, the propagation of light in water is hindered by the absorption of light by the elements in the water. Light is attenuated at increasing depth because of diffraction and absorption. This promising technology can also produce a high-resolution bathymetric digital terrain model (DTM). Regrettably, it is sensitive to environmental conditions, such as water clarity, depth, cloud ceiling and wind. Combined employment of the ALB in conjunction with shipborne multibeam altimetry results in a faster turn-around time for updating the nautical chart, and thus decreasing the survey backlog (Burlik, 2004; Dehling, 2004; Gjerstad, 2004; Millar, 2005; Velthoven, 2004).

16.2.4 Sub-Bottom Profiling

Since the 1960s, hydrography has grown from an administration responsibility into a science indispensable to the offshore mining industry. Using a Decca Sea-Fix 2D-positioning system, Aleva and Bon conceived a simple exploration method by sonar sub-bottom profiling to search for mineral deposits. Using an adapted echosounder with an 850 Hz square pulse transducer, Bon was successful to chart large cassiterite-bearing layers in the South China Sea (Aleva, 1973; Bon, 1979).

Sub-Seabed Logging Profiles

After a seismic survey, the electromagnetic sub-seabed logging (SBL) is an additional operation to show a possibility to separate the hydrocarbon (HC) from the non-hydrocarbon state by data or graphics, prior to drilling a borehole. SBL techniques identify the nature of the reservoir below the seafloor in deep water. Accordingly, it determines whether the reservoir-liquids imaged in the seismic are just water (non-hydrocarbon state), or gas, oil (Eidesmo, 2002; Løseth, 2000).

In the 2000s, the electromagnetic (EM) sounding technology was conceived by Terje Eidesmo, Svein Ellingsrud and Ståle E. Johansen of Statoil's research centre, Trondheim, to increase the success ratio in oil & gas exploration. In essence, the SBL technique uses an active source EM-transmitter towed by a hydrographic support vessel, using an array of seafloor-based EM-field receivers. Sensors are weighed down with a concrete base on the seabed along the predetermined track. Thereafter, towing an EM-transmitter along the track to carry out the EM-survey is quite a laborious process. The track starts by towing the source 30 m above the seabed at 1.5 knots, emitting a square EM-signal pulse with base frequency of 0.25 Hz into the water column, and downwards into the seabed. In a layer like an HC-reservoir, the EM-signal energy is attenuated and efficient propagated in the conductive seafloor sediments along the resistive layer in deep water. Then the energy leaks from the HC-filled layers back to the seafloor receivers. Once finished, and using an acoustical release technology controlled from the support vessel, the seafloor instruments are recovered to readout the recorded information. Thus, SBL's resistivity tools for saturation measurement increases the success ratio in oil & gas drillings massively. It operates on the principle that EM provides a resistivity contrast, in water filled reservoirs 0.5-1.0 Ωm, and in HC-filled reservoirs 50-100 Ωm [On_CD].

16.2.5 Parametric Echosounders

It is fundamental that every phase of an offshore operation is more costly than that which precedes it. The surveyor's work can influence the degree of economic success of any offshore venture. The hydrographic surveyor's skill must embrace a sound understanding of all the processes of organisation, measurement, including positioning and meteorology. In the mean time, it is essential that related 3D-space-time position fixes, are combined with previously established coordinated reference stations within one and the same mathematical framework (Datum), using conversions, transformations and least squares adjustments (LS) (Stevenson, 2004).

An important problem in sediment acoustics is to determine the thickness of sediment layers. If the sound velocity is known, the thickness can be calculated from travel times. Non-linear acoustics combined with principles of refraction seismology allow estimating the sound velocity of selected sediment layers.

Most hydrographic systems require an onboard laptop computer, or a computer designed to withstand the offshore conditions. Furthermore, a straightforward portable hydrographic survey system contains a GPS receiver, a differential signal receiver, a data-logging device, sonar equipment, and a course guidance indicator. An operator station is designed for layer digitising and for comparison with drilling information to cover processing requirements with data quality checks.

Containing all surface electronics, the shipborne portable operator station is connected to a portable personal computer (PC), using an adequate post-processing software package. The submerging transducer, connected by cable directly to the operator station, may be deployed from any vessel, platform, or even from an amphibian car.

Thinking about improved positioning systems, central data collecting and post processing, the parametric echosounder is an essential piece of equipment in many survey applications. When an acoustic wave is transmitted at a very high-pressure level, the regions of high and low pressures in the propagation medium causes non-linearities in sound velocity.

During simultaneous transmission at very high levels of two slightly different frequencies, non-linearity in sound velocity induces the phantom of a secondary wave with a frequency equal to the difference between the two primary frequencies. This secondary wave has the directivity pattern of the two primary waves. Detection of objects embedded in the sediments of rivers, lakes, or a coastal area is imperative for a range of applications. Parametric sub-bottom profilers based on non-linear acoustics offer many advantages due to very narrow beams with a small transducer size without sidelobes at a low frequency (Lurton, 2002).

Portable Compact Systems

Sub-bottom profiler systems deliver a narrow beam for high-resolution monitoring of sedimentation, and siltation processes, in addition to detecting embedded pipelines. Innomar GmbH of Germany conceived the portable PC based SES-parametric echo sounders for survey applications to major dredging and marine surveying companies (Figure 132). For non-linear acoustics, short pulses, narrow beams and the absence of side lobes result in a better signal-to-noise-ratio (SNR). In combination with motion sensors, the Innomar SES-2000 system supports real time beam stabilisation for roll, pitch, and wave movement compensation. It delivers the narrow beam parametric sub-bottom profiler for high-resolution monitoring of sedimentation in addition to detecting embedded pipelines (Lowag, 2002, Wunderlich, 2004).

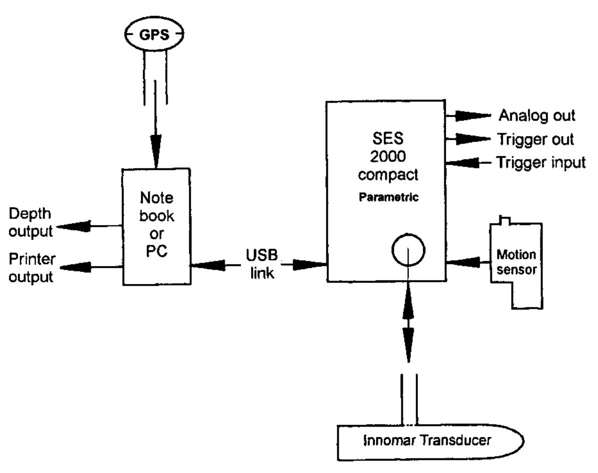


Figure 132: Sketch of DGPS surveying by parametric echosounder

By 2003, an Innomar SES-2000-C (compact system) was introduced, as shown in (Figure 133; Figure 134) and [On_CD]. It is intended as a self-contained stand-alone portable system, directly interfaced to a DGPS-receiver, which allows rack mounting of the transducer for measuring water depth range between 3 m - 1 500 m and an operating range between 5 - 200 m. The shipborne portable operator station, containing all surface electronics, is connected to a portable personal computer (PC), using an adequate post-processing software package, is designed for layer digitising and for comparison with drilling information to cover processing requirements and data quality checks.

The survey system can be readily installed on any boat in a convenient location by fixing the mounting bracket to the side of the boat to attach a transducer

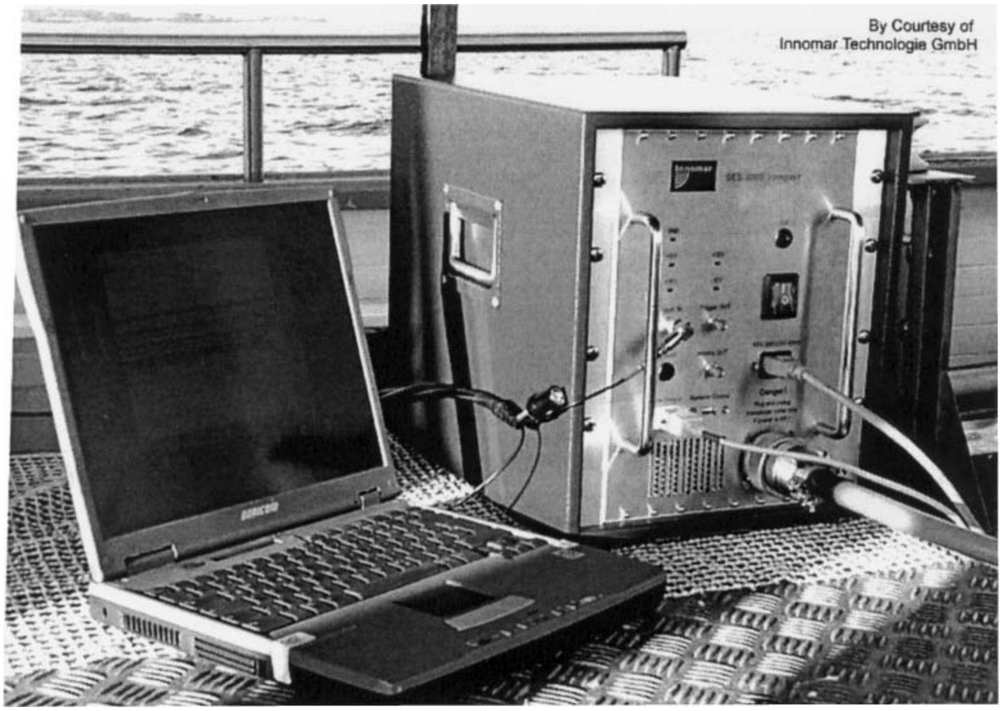


Figure 133: Laptop PC with Innomar SES 2000 Compact - parametric sub-bottom profiler

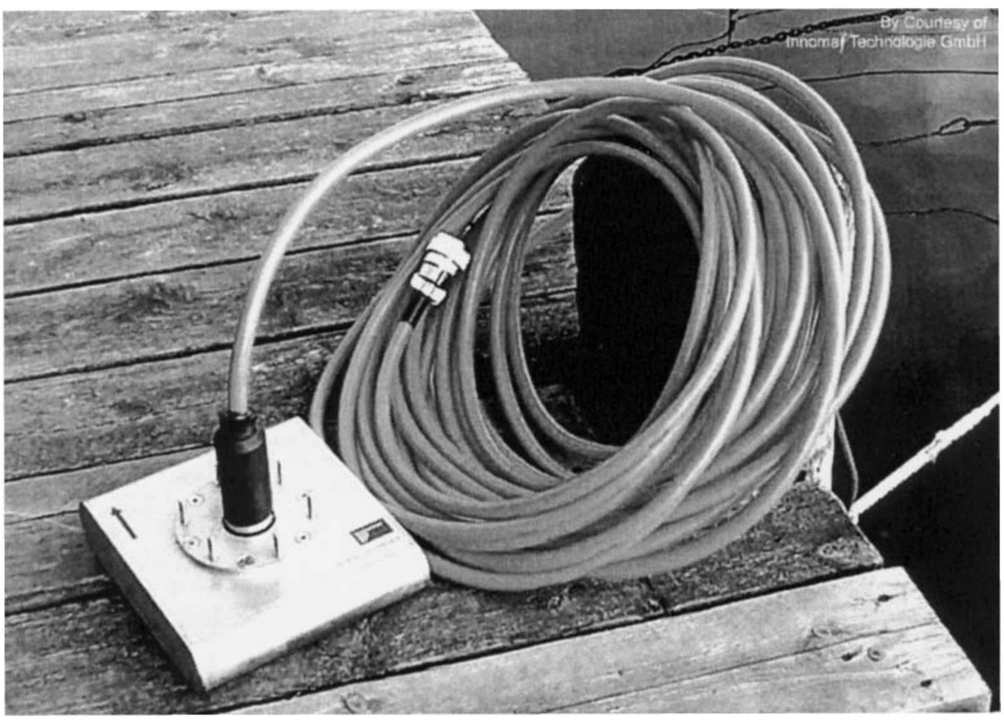


Figure 134: Innomar SES 2000 Compact - parametric transducer with cable

Geographical Surveys

Survey imagery systems are often used on the most remote places on this hydrosphere, and in a harsh environment. In 2001, C&C Technologies, Inc. of Lafayette conducted a deep-water pipeline survey near a reported location of shipwrecks in the Gulf of Mexico. Using the untethered HUGIN-3000 AUV, it can operate at a speed of four knots, using a multibeam bathymetry and an imagery system, a dual frequency chirp side-scan sonar, a chirp sub-bottom profiler, and an INS (inertial navigation system). Analysing the data, the C&C marine archaeologists realised that the debris scatter could be the remains of the U-166, the only German U-boat lost in the Gulf of Mexico during World War II. Both BP and Shell research teams voluntarily sponsored further site-specific investigations of the suspected U-166, using C&C's HUGIN 3000 AUV (Figure 135).

DGPS provides the supporting vessel the geographic positioning, while the position fixing of the AUV is done by means of the combination HiPAP, INS, and a Doppler velocity log. Using an survey grid, the survey was carried out within two hours at a depth of 1500 metres (Figure 136). The efforts enabled the expedition to reveal the final resting-place of the U-166, thus solving one of the great mysteries of World War II in the Gulf of Mexico (Figure 137) (Fortenberry, 2006).

In 2002, C&C Technologies, Inc. surveyed a BBC-sponsored a multibeam search for the aircraft carrier *H.M.S. Ark Royal* southeast of Gibraltar. Analysing the data, the C&C marine archaeologists confirmed that the debris scatter could be the remains of the aircraft carrier (Church, 2002; Ferrante, 2005; Warren, 2004).

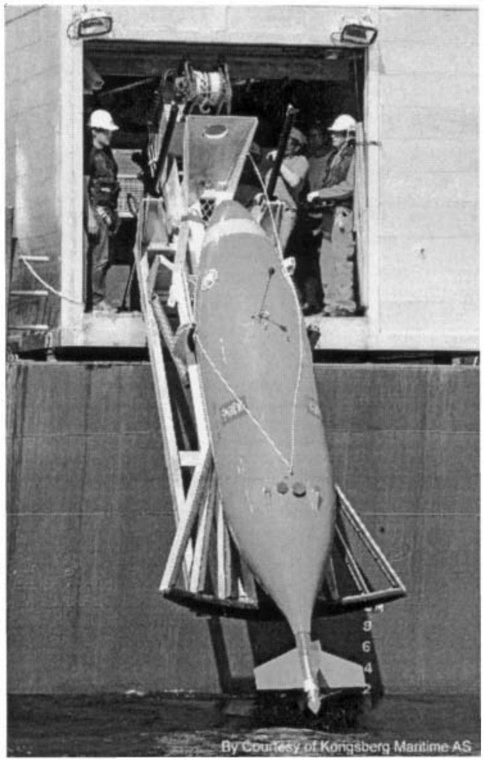


Figure 135: Recovering HUGIN-3000 AUV from deep sea operations

16.2.6 Hydrographical Literature

The literature about the use of electronics available to the hydrographers is limited, most of the published texts being directed at an electronic expert and an equipment designer. Manufacturers' data sheets are instructive, although the user is expected to have only a basic technical knowledge. Explanation on the calculations involved in the calculation of sound velocity is given in *Handbook of Oceanographic Tables*, 1966, USN Oceanographic Office No. SP-68 (Admiralty, 1965).

The International Hydrographic Organisation (IHO) has published the *IHO Manual on Hydrography M-13*.
It features in seven chapters the principles of hydrography, positioning, depth determination, seafloor classification and feature detecting, water levels and flow, topographic surveying, and hydrographic practice.

Hydrographic Journals

Finally, in view of the rapid development of the electromagnetic- and acoustic-sonar-technology, the hydrographic community is obliged to peruse a number of journals, such as the worthy:

Hydro International (HI), Reed Business Publishing Co., PO Box 112, NL-8530 AC Lemmer
International Hydrographic Review, Reed Business Publishing Co., PO Box 112, NL-8530 AC Lemmer

The Hydrographic Journal (HJ), IFHS, PO Box 103, Plymouth, U.K.

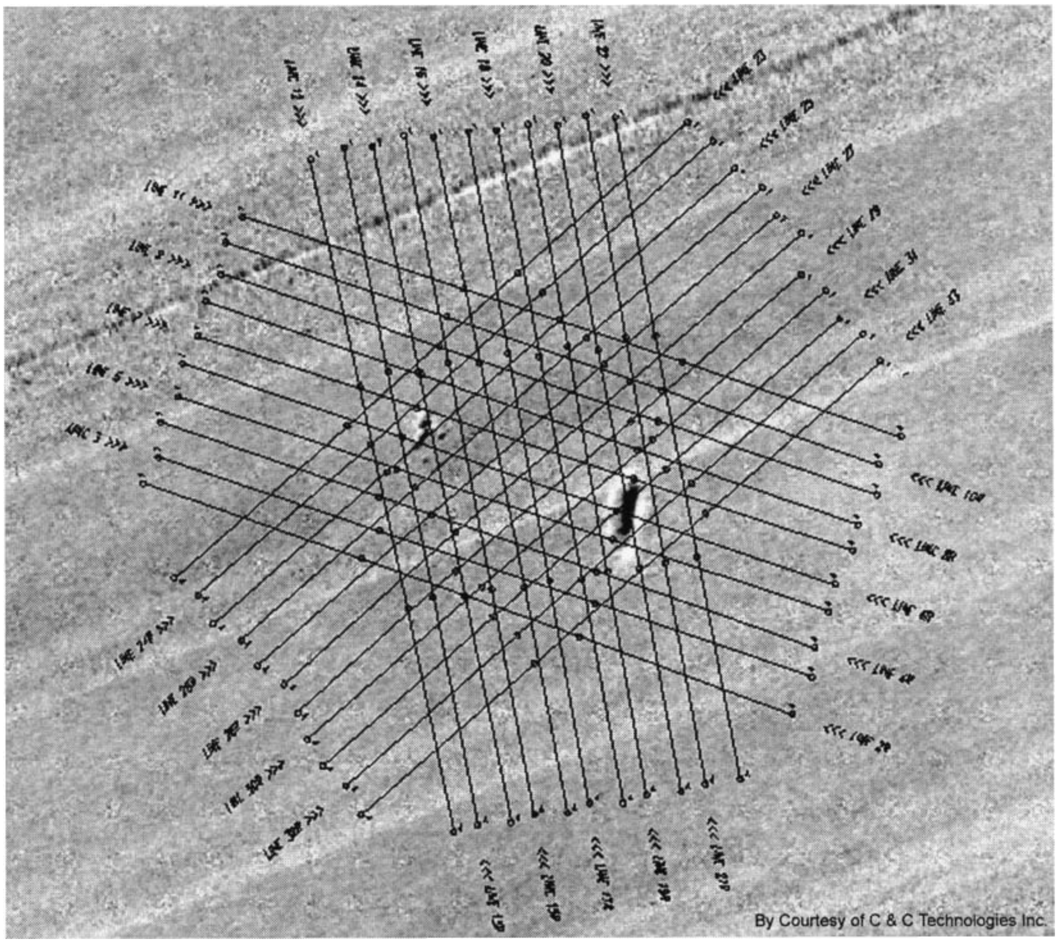


Figure 136: C&C side-scan sonar survey pattern for the German U-166 submarine in the Gulf of Mexico

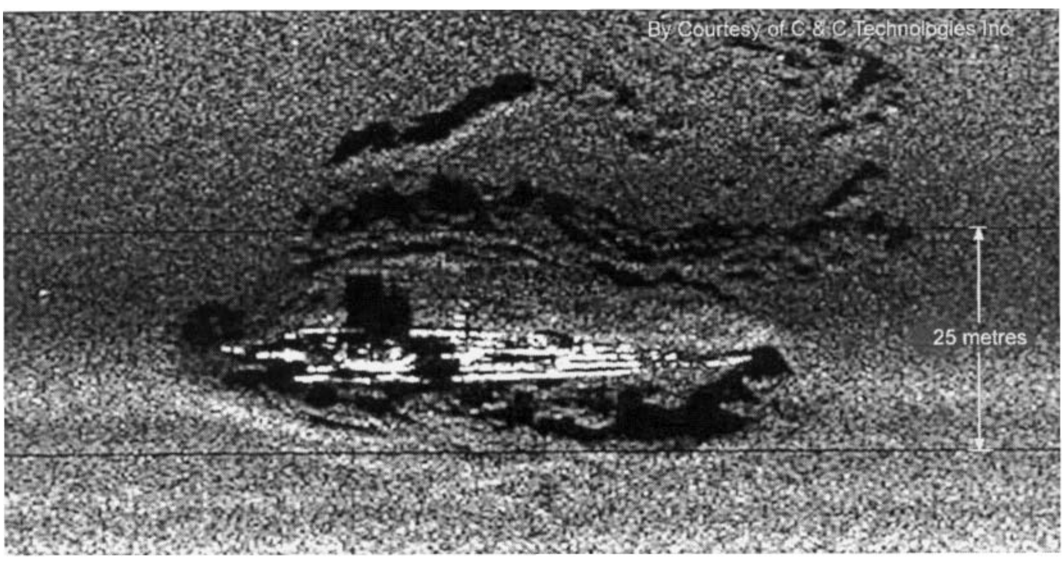


Figure 137: Side-scan sonar 410 kHz image of the German U-166 submarine discovered at 1100 m depth

17. Using Computers

As early as 1671, Leibniz, the German philosopher and mathematician, was writing:

> *... The astronomers surely will not have to continue to exercise the patience, which is required for computation. It is this, which deters them from computing or correcting tables ... working on hypotheses, and from discussions of observations with each other. For losing hours like slaves in the labour of calculation which could be safely relegated to someone else if the machine were used, is unworthy of excellent men ... ".*

Important points are to be made both about the nature of the subject and the content of this chapter. The concept of the process of calculation means many things. The ideas of calculation are based on the formal approach of using the simple rules of arithmetic and the complexities of higher mathematics (Airy, 1830).

Computing Tools

For many years the tools for calculation consisted of the Abacus, logarithm tables, trig. tables, slide-rules, rotary calculating machines. Later, digital computers (1935), pocket calculators (1972), supercomputers (1976), personal computers (PC, 1978), and massively symmetric parallel processors (1990s) were introduced [14.6].

17.1 Using FORTRAN Programs

Compiler Programs

Compiler programs operate on symbolic data in a source program, produce an object program with machine language function codes and absolute addresses. After code writing and entering of a program, commands are needed to compile the code, to link object files, and to continue the build process. Compiler instructions are quite simple under the control of the system. Hence, object files are compiled from the source file, and libraries are indispensable to build the (*.EXE) executable file. Compiler programs, such as FORTRAN, can create executable files, despite which Operating System (OS) is used (London, 1968).

FORTRAN (an acronym for FORmula TRANslator) belongs to a class of high-level languages called scientific or algebraic languages [14.8]. The American National Standards Institute (ANSI) has revised it several times since then. Even though other languages, such as C++, are becoming popular for scientific and engineering computations, FORTRAN is still the language of choice for numerical analysis. Due to its common use as a powerful language for scientific research, its ease of use and intuitive structure, Fortran remains the language of choice for scientists and engineers around the world.

FORTRAN was developed as a third generation language for numerical analysis computation by John W. Backus and colleagues at IBM. It was released to customers of IBM computers in April 1957. In 1966, a voluntary FORTRAN standard, American National Standard (ANS) FORTRAN-66 F66), was adopted. Standard FORTRAN's include IBM's Professional FORTRAN, FORTRAN-F77L by Lahey Computer Systems, Microsoft FORTRAN Professional Developing System and FORTRAN-RM by Ryan-McFarland Corporation.

Revised ANS FORTRAN-77 (F77) conforms to the American National Standard Programming Language F77, as described in the ANSI X3.9-1978 standard. Further Fortran90 (F90) - ANSI X3.198-1992, Fortran95 (F95) language (1997). In the 2000's, three other versions were introduced: IBM XL-Fortran (XLF) for IBM's UNIX, AIX, with support for the IBM RISC System/6000 family [14.6.2]; IBM XL High Performance Fortran with support for multiple workstation environments; IBM VS Fortran with support for OS/390 systems and VM environments (IBM, 2000a, b, c, d).

In order to extend its applicability to scientific computations beyond numerical analysis, facilities for handling structured data, dynamic data allocation, recursive calls, and other features were added in the version released in FORTRAN77 (1978), Fortran90 (1992). Fortran95 (1997) XL-FORTRAN compiler (2003) with support for OpenMP FORTRAN API v1.0 SMP. Hence, FORTRAN remains the language of choice for scientists and engineers around the world (IBM, 2000d).

17.1.1 Installation of FORTRAN

Installing and configuring FORTRAN includes the use of libraries, mathematical (math) options, and some memory models. It is used for a variety of mathematical and quantitative analysis procedures for computers.

Note

FORTRAN may include many optional features that are not part of American National Standards Institute (ANSI) standard FORTRAN. Hence, this book avoids the use of non-standard features for all FORTRAN programmes given in [18].

17.1.2 Compilation

Compiler programs operate on symbolic data in a source program, produce an object program with machine language function codes and absolute addresses. After code writing and entering of a FORTRAN source program, commands are essential to compile the source code, to link the object files, and to continue the build process. The instructions to compile are simple under control of FORTRAN. Consequently, object files are compiled from the source file, and libraries are needed to build the executable file (*.EXE).

FORTRAN (Object) programs also use prewritten program modules from the FORTRAN library of routines. In the compilation process of FORTRAN statements into machine-level, executable instructions are done first, and at that moment, the executable program is stored for execution. Execution may be done immediately after compilation. When compilation and linking are complete, the status, compilation results and error messages are displayed. An unsuccessful compilation demands debugging (London, 1968).

Debugging Programs

If compilation or linking is unsuccessful, the result is displayed with a list of all operations and errors to debug the program, if necessary.

17.1.3 FORTRAN Application Program Modules

One key idea is a modular designed application program, thus as a set of units called modules. A program module is the part of a program that does a distinct function, such as:

- initialisation establishes initial values for some variables, shows headings, messages, et cetera
- input data
- validation of input data to detect errors or omissions
- processing or data manipulation
- output of data
- analysis of error condition and output of error messages
- conclusion of processing - ending program execution

These modules reflect a logical flow for a computer program.

Standard Code

Alphabetic text or numeric digits are not represented as individual characters. Instead, characters are represented by a code consisting of an individual set of seven or eight bits, 0's and 1's. The common binary coding scheme is the standard code called American standard code for information interchange (ASCII). Seven bits can be arranged in 128 different combinations to represent the numeric digits, upper-case letters, lowercase letters, punctuation symbols, and some special control codes. Eight bits provide 256 different extended ASCII codes.

Upper-Case-Only Code

The computer code for an upper-case "A" is different from a lowercase "a". ASCII code for an upper-case "A" is 065, or 01000001. The ASCII code for a lowercase "a" is 097, or 01100001. In this book FORTRAN listings use upper-case throughout.

The OS environment in which programs are created, compiled, and executed is not standardised. This means that all non-standard FORTRAN instructions should be avoided. Nevertheless, straightforward programs, written in FORTRAN, should be transferable without substantial change to another computer having a FORTRAN compiler.

column	character interpretation
1-5	statement label used as an identifier for referencing the statement
6	continuation character for succeeding lines if the FORTRAN statement is too long for one line
7-72	FORTRAN statements.
73 and above	ignored

Table 30: FORTRAN Program Structure

Input Preparation

Source programs are usually entered computer memory using a text editor. Five FORTRAN statements are required to write an application program (Table 30):

input	READ format
output	PRINT format
arithmetic	an assignment statement
STOP	stopping execution
END	last statement

From now on, it is submitted to the FORTRAN system to build a program source file. This file is a text file without connection with the FORTRAN compiler. Word processors are easier to write or correct programs and to prepare data files.

Job control instructions (JCI) will be required in the FORTRAN program. Each computer installation will assign standard device numbers to I/O-devices, such as keyboard, printer, or monitor.

Listing files send listing options to the PC: AUX - an auxiliary device, CON - console, PRN - printer, and NUL means that a null file (a non-existent file) is created. A standard device can be changed by JCI. The completed program is immediately available for compilation. Usually, the FORTRAN system provides job messages and error messages to the user at the computer.

Arithmetic Assignment Statements

General form of an arithmetic statement is v = e, in which:

=	equals sign means assignment, but not as an equality sign in mathematical sense
v	is a variable name, not a constant or arithmetic operation
e	arithmetic expression

FORTRAN uses a precedence rule and a parenthesis rule for handling arithmetic expressions. FORTRAN uses a parenthesis rule for handling arithmetic expressions. Operations inside parentheses are done first. Parentheses must always be used in pairs. Without parentheses, the precedence rule for doing arithmetic operations specifies the order.

Arithmetic operators are recognised by FORTRAN. They appear in order of precedence:

** for exponentiation, * for multiplication, / for division, + for addition, – for subtraction or negation.

Within precedence levels, the operations will be done in order from left to right. The order of operation is not important, if operations are commutative.

CPU Time

Data transfer instructions move data back and forth between the CPU and memory and shuffle data internally among the CPU registers. At the heart of the CPU instruction set are the arithmetic operations for addition, subtraction, multiplication, and division - plus some extras such as square root and absolute value. In addition, the CPU hardware has built-in transcendental capabilities for computing logarithms and trigonometric functions in radian measure, e.g., θ^R.

They compose the process of going from command to answer of three phases:

- translation time is the time the computer takes to figure out what to do. The program compiler makes the same translation as the interpreter, but only once, rather than every time they execute a line. Interpreted programs spend much time in the translation phase while compiled programs spend none at all.
- invocation time is the time it takes the computer to calculate the addresses of the variables and to call the appropriate internal floating-point addition subroutine or a similar routine in the run-time library.
- calculation time is the time the computer spends doing the actual addition. All the direct advantage of the CPU hardware comes from improvement in this phase.

Reduction of translation and invocation time depends on the appropriate choice of a program translator, and compilers produce faster programs than interpreters. There is an important warning about using a CPU version. A CPU with a coprocessor and a CPU without a coprocessor are not fully compatible. They represent floating point numbers differently in their precision (round off error, overflow error) and range. Two irresolvable problems exist, but quite unlikely to be a major concern for most users (Ruckdeschel, 1981a).

Data Type

Numeric constants can be integers, single-precision or double-precision numbers. Integer constants are stored as whole numbers only (Table 31).

A single-precision IEEE real value occupies four bytes of memory. The precision of this data type is between six and seven decimal digits, but six decimal digits are significant, including negative numbers.
The range of single-precision numbers is between:

–3.40282 35E+38	and	–1.17549 44E–38
+1.17549 44E–38	and	+3.40282 35E+38
	0 (zero)	

The problem with single precision is the loss of accuracy from cumulative errors. Doing all the calculations in double precision is almost as good as holding everything in double precision.

A double-precision IEEE real value is usually an approximation, and occupies eight bytes of memory. Precision is between 15 and 16 decimal digits, but 15 decimal digits are significant.

The range of double-precision numbers is between:

–1.79769 31348 62316D+308	and	–2.22507 38585 07201D–308
+2.22507 38585 07201D–308	and	+1.79769 31348 62316D+308
	0 (zero)	

Numerical programming practice dictates using double precision throughout.

The functions affected are: `DABS DACOS DASIN DATAN DATAN2 DCOS DCOSH DCOTAN DEXP DINT DLOG DLOG10 DMAXI DMINI DMOD DSIGN DSIN DSINH DSQRT DTAN DTANH`

17.1.4 Program Execution

The program statements and data are encoded in the computer in a special computer code. Once entered and stored by the computer, the program can be executed. After initialisation, processing goes on logically with input, input validation, various processing modules, and output. In each structure is a single point of entry plus a single point of the exit.

The last output with an end-of-job message will be the successful result from the execution of the program. The output from compilation of a FORTRAN object program will vary depending on whether or not the job ran successfully to completion.

Incorrect Results

Input validation is a process of testing all input data to detect whether they meet the criteria set for them. Input data maybe tested for values within an acceptable range. In reading printer or monitor output, differentiating between numbers and letters that are similarly is difficult. The letter O and 0 (zero) are the biggest problems, but B and 8 (eight), S and 5 (five), Z and 2 (two), and I (i), l (L), and 1 (one) are often doubtful.

A program may produce incorrect results, due either to program errors or to incorrect information (either text or numeric data). The FORTRAN error diagnostics' program gives a listing of error messages. The program should be designed so that errors will, whenever possible, be detected by the program itself during execution.

Therefore distinguishing between errors is imperative - either nonfatal or fatal - detected during compilation and errors detected during execution. Fatal errors prevent compilation or lead to execution errors. Nevertheless, if errors are detected during compilation, the user can correct the source program and resubmit it for compilation.

Note

> *Bug Hazard! Occasionally, errors - such as bugs in computing hardware of numerical data processors (NDP) - affect seriously the results of a project. The calculation, a purely mechanical process, would be pure guesswork, which would be biased by such intangible factors*

17.1.4.1 Error Messages

Error messages that you might encounter while using FORTRAN. If, during the evaluation of an expression a division by zero is encountered, the "*division by zero*" error message appears, machine infinity with the sign of the numerator is supplied as the result of the division and execution continues. If the evaluation of an Exponentiation results in zero being raised to a negative power, the "*division by zero*" error message appears, positive machine infinity is supplied as the result of the exponentiation, and execution continues. If an overflow occurs, the "*overflow*" error message appears, machine infinity with the algebraically correct sign is supplied as the result and execution continues. The errors that occur in overflow and division by zero will not be trapped by the error trapping function.

Errors that might result from using number crunching subroutines can be loosely grouped into four classes:

- programming errors in the subroutines.
- errors in using the subroutines.
- recoverable precision errors.
- non-recoverable precision errors.
- programming errors

Long real (double precision) corresponds to FORTRAN's double precision data type to reduce the effect of round off error in intermediate steps.

data type	bits	significant digits	range		
word integer	16	4	-32768	to	32767
short integer	32	9	$-2\ 10^{9}$	to	$2\ 10^{9}$
long integer	64	15	$-9\ 10^{18}$	to	$9\ 10^{18}$
packed decimal	80	18	18 decimal digits + sign		
short real	32	6 or 7	10^{-17}	to	10^{11}
long real	64	15 or 16	10^{-307}	to	10^{308}
temporary real	80	19	10^{-4932}	to	10^{4932}

Table 31: NDP data types

Note

Please do consider that manufacturers would not disclose any part of their proprietary information technology.

17.1.4.2 Detailed Information

This section is oriented towards the professional FORTRAN user. Detailed description about planning the general flow of program logic, managing program lists, job control instructions (JCI), Fortran intrinsic functions, using number crunching numeric data processors (NDP), mixed languages, general purpose microprocessors, et cetera, would require an user's manual and / or user's reference manual, such as (Holzner, 1987). Moreover, the user - besides being fully conversant within his own field - must be conversant with computer techniques.

18. FORTRAN Application Programs

A further important factor is the design of the equations. Many early computing programs were not written in a suitably economical form. The nested form of an equation e.g. for calculation of the Meridional arc distance is equally valuable. Operations within parentheses are performed first. Inside the parentheses, the usual order of precedence is maintained.

It follows that the programs without parentheses spent a certain amount of computing time without furthering progress in the calculations. Not only are the equations easier to write in an appropriate programming language, they depend upon progressively raising terms to higher powers, and therefore reduce the risk of underflow or overflow conditions (Hooijberg, 1979, 1996; Vincenty, 1971).

Note

The structure of the subroutine listings is not fully utilised to permit wider use of the routines. In addition, all input- and output formats are kept as simple as possible.

Design of the Equations

The design of the equations is imperative. In general, the procedures comprise different cases, each involving one or more conversions and / or transformations, using the properties of a projection, such as:

- arc-to-chord (δ) or (t - T) correction
- bi-linear interpolation
- conversion of coordinates from system to system
- Datum s-transformation
- geodetical to grid coordinate conversion and vice versa
- grid scale factors (k)
- length of an arc of latitude and longitude
- meridian convergence (γ)
- properties of geodetic lines.

Program Execution

The program statements and data are encoded in the computer in a special computer code. Once entered and stored by the computer, the program can be executed. The last output with an end-of-job message will be the successful result from the execution of the program. Error messages that you might encounter while using a compiler are discussed in [17.1.4.1].

Note

In this book is the rounding function – intended for printing only - not applied in the subroutines. As a result, all examples are printed without rounding.

Using FORTRAN Application Programs and Subroutines

The Fortran subroutine listings [On_CD] are followed by examples with an intermediate output, because situations may arise in which a program fails to operate as expected.Vincenty (Vincenty, 1976b):

> *" ... many apparently unrealisable programmes can be written for a computer if more thinking is given to recasting the equations ... "*

18.1 Using Flat Earth Applications

A_01BDAR.FOR Program I - from Bearing and Distance to Coordinates and Polygonal Area

```
*************************************************************************
*     PROGRAM A_01BDAR.FOR - Date 01-06-2006                            *
*     FROM BEARING AND DISTANCE TO POLYGONAL AREA - FORTRAN PROGRAM     *
*     COPYRIGHT SPRINGER-VERLAG, BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 2001-2006                                     *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Main Program, Subroutines, and Examples
C
```

A_01BRDI.FOR Program II - Coordinates to Grid Bearing and Distance

```
*************************************************************************
*     PROGRAM A_01BRDI.FOR - DATE 01-06-2006                            *
*     FROM COORDINATES TO GRID BEARING AND DISTANCE - FORTRAN           *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK              *
*     AUTHOR M. HOOIJBERG 2001-2006                                     *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER VR*1
C
       OPEN(11,FILE= '\FORFILES\BRDIDATA.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
----------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- BRDI     ')
      WRITE(11,3)
    3 FORMAT(10X,'COORDINATES TO BEARING AND DIS-
TANCE       ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK ')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
----------',/)
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
C
      WRITE(11,20)IYR,IMON,IDAY,IHR,IMIN
   20 FORMAT(10X,'DATE AND TIME     [HH:MM] =
',I3,I3,I5,' - ',I3,I3,/)
C
      PI4    = DATAN(1.D0)
      RD     = PI4/45.D0
C
C     -- COMPUTE BEARING AND DISTANCE - FLAT EARTH
--
C
   30 CONTINUE
      WRITE(*,31)
   31 FORMAT(' INPUT EASTING          [STA.1] : X
XXX XXX.XXXX')
      READ(*,32)E1
   32 FORMAT(F22.4)
      WRITE(*,33)
   33 FORMAT(' INPUT NORTHING         [STA.1] : X
XXX XXX.XXXX')
      READ(*,34)N1
   34 FORMAT(F22.4)
      WRITE(*,35)
   35 FORMAT(' INPUT EASTING          [STA.2] : X
XXX XXX.XXXX')
      READ(*,36)E2
   36 FORMAT(F22.4)
      WRITE(*,37)
   37 FORMAT(' INPUT NORTHING         [STA.2] : X
XXX XXX.XXXX')
      READ(*,38)N2
   38 FORMAT(F22.4)
C
      WRITE(11,39)E1
   39 FORMAT(10X,'EASTING            [STA.1] :
',F16.4)
      WRITE(11,40)N1
   40 FORMAT(10X,'NORTHING           [STA.1] :
',F16.4)
      WRITE(11,41)E2
   41 FORMAT(10X,'EASTING            [STA.2] :
',F16.4)
      WRITE(11,42)N2
   42 FORMAT(10X,'NORTHING           [STA.2] :
',F16.4,/)
C
      CALL BRGDI1(E1,E2,N1,N2,DI,BR1,BR2,PI4)
C
      WRITE(11,60)DI
   60 FORMAT(10X,'GRID DISTANCE   [STA.1-2] =
',F16.4)
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
      WRITE(11,61)ID,IM,M3
   61 FORMAT(10X,'GRID BEARING    [STA.1-2] =
',I4,I3,F8.4)
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,62)ID,IM,M3
   62 FORMAT(10X,'GRID BEARING    [STA.2-1] =
',I4,I3,F8.4,/)
C
      CALL BRGDI2(E1,E2,N1,N2,DI,BR1,BR2,PI4)
```

```
C
      WRITE(11,70)DI
   70 FORMAT(10X,'GRID DISTANCE     [STA.1-2] =
',F16.4)
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
      WRITE(11,71)ID,IM,M3
   71 FORMAT(10X,'GRID BEARING      [STA.1-2] =
',I4,I3,F8.4)
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,72)ID,IM,M3
   72 FORMAT(10X,'GRID BEARING      [STA.2-1] =
',I4,I3,F8.4,/)
      CALL BRGDI3(E1,E2,N1,N2,DI,BR1,BR2,PI4)
C
      WRITE(11,80)DI
   80 FORMAT(10X,'GRID DISTANCE     [STA.1-2] =
',F16.4)
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
      WRITE(11,81)ID,IM,M3
   81 FORMAT(10X,'GRID BEARING      [STA.1-2] =
',I4,I3,F8.4)
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,82)ID,IM,M3
   82 FORMAT(10X,'GRID BEARING      [STA.2-1] =
',I4,I3,F8.4,/)
C
C     -- END OF CALCULATION --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c') GOTO 30
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
```

```
*********1*********2*********3*********4*********5*********6*********7**
-- SUBROUTINE BRGDI1 GRID BEARING AND DISTANCE USING ACOSINE --
-- SUBROUTINE BRGDI2 GRID BEARING AND DISTANCE USING DATAN --
-- SUBROUTINE BRGDI3 GRID BEARING AND DISTANCE USING DATAN2 --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC
*********1*********2*********3*********4*********5*********6*********7**
```

A_01POLA.FOR Program III - Calculation from Grid Coordinates to Polygonal Area

```
************************************************************************
*     PROGRAM A_01POLA.FOR - Date 01-06-2006                           *
*     FROM COORDINATES TO POLYGONAL AREA - FORTRAN PROGRAM             *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 2001-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
```

```
C     See [On_CD] for Main Program, Subroutines, and Examples
```

18.2 Baseline Crossing Application

A_02BAXG.FOR - Program - Baseline Crossing Technique with Least Squares Curve Fit

```
************************************************************************
*     PROGRAM A_02BAXG.FOR - DATE 01-06-2006                           *
*     BASELINE CROSSING WITH L.S. PARABOLIC CURVE FIT - FORTRAN        *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1984-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
```

```
C     See [On_CD] for Main Program, Subroutines, and Examples
```

18.3 Bi-Axial Meridional Arcs

A_03MARC.FOR - Five Meridional Arc and Geodetic Latitude Computation

```
************************************************************************
*     PROGRAM A_03MARC.FOR - Date 01-06-2006                           *
*     COMPUTATION OF VARIOUS MERIDIONAL ARCS - FORTRAN PROGRAM         *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1999-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
```

```
C     See [On_CD] for Subroutines, and Examples
```

```
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,VR*1
C
       OPEN(11,FILE= '\FORFILES\A3MARC00.TXT',
STATUS='OLD
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
-------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- MARC        ')
      WRITE(11,3)
    3 FORMAT(10X,'MERIDIONAL ARC <> GEODETIC LATI-
TUDE CONVERSION')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
-------------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
      WRITE(*,10)
   10 FORMAT(' INPUT REFERENCE ELLIPSOID    : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
      WRITE(*,12)
   12 FORMAT(' INPUT SEMI-MAJOR AXIS        : ')
      READ(*,13)A
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT(' INPUT RECIPROCAL FLATTENING  : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
C
C     -- BASIC ELLIPSOID PARAMETERS --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,20)IYR,IMON,IDAY,IHR,IMIN
   20 FORMAT(10X,'DATE AND TIME                =
',I3,I3,I5,' - ',I3,I3)
C
      CALL ELDAT1(A,C,EC2,EC3,FL,N,PI4,RD)
C
      WRITE(11,21)DATUM
   21 FORMAT(10X,'REFERENCE ELLIPSOID          :
',A30)
      WRITE(11,22)A
   22 FORMAT(10X,'SEMI-MAJOR AXIS        [A] :
',F22.6)
      WRITE(11,23)FL
   23 FORMAT(10X,'REC. FLATTENING       [FL] :
',F22.16)
      WRITE(11,24)EC2
   24 FORMAT(10X,'SQ.1ST ECCENTRICITY [EC2] =
',E22.16)
      WRITE(11,25)EC3
   25 FORMAT(10X,'SQ.2ND ECCENTRICITY [EC3] =
',E22.16)
      WRITE(11,26)C
   26 FORMAT(10X,'C                       [C] =
',E22.16)
      WRITE(11,27)N
```

```
   27 FORMAT(10X,'N                       [N] =
',E22.16)
      WRITE(11,28)RD
   28 FORMAT(10X,'RAD-DMS CONV.          [RD] =
',E22.16,/)
C
C     -- INPUT LATITUDE IN DEGREES-MINUTES-SECONDS
--
C
   50 CONTINUE
      I2    = 1
      WRITE(*,51)
   51 FORMAT(10X,'INPUT LATITUDE                :
DD,MM,SS.SSSSSS ')
      READ(*,52)ID,IM,M3
   52 FORMAT(I3,I3,F8.4)
      WRITE(11,53)ID,IM,M3
   53 FORMAT(10X,'LATITUDE                      :
',I3,I3,F12.6,/)
      CALL DMSRAD(LT0,ID,IM,M3,I2,RD)
      CALL ELFOR1(LT0,C,EC3,ARC1)
      WRITE(11,54)ARC1
   54 FORMAT(10X,'ARC OF HELMERT          [1] =
',F22.6,/)
C
      CALL ELFOR2(LT0,C,EC3,ARC2)
C
      WRITE(11,55)ARC2
   55 FORMAT(10X,'ARC OF HELMERT-MOD.     [2] =
',F22.6,/)
C
      CALL ELFOR3(LT0,C,EC3,ARC3)
C
      WRITE(11,56)ARC3
   56 FORMAT(10X,'ARC OF KRACK 82         [3] =
',F22.6,/)
C
      CALL ELFOR4(LT0,A,N,ARC4)
C
      WRITE(11,57)ARC4
   57 FORMAT(10X,'ARC OF KRACK 83         [4] =
',F22.6,/)
C
      CALL ELFOR5(LT0,A,N,ARC5)
C
      WRITE(11,58)ARC5
   58 FORMAT(10X,'ARC OF NGS-MODIFIED     [5] =
',F22.6,/)
C
C     -- INPUT MERIDIONAL ARCS --
C
      WRITE(*,65)
   65 FORMAT(10X,'MERIDIONAL ARC LENGTH         :
XXXXXXX.XXXXXX')
      READ(*,66)ARC0
   66 FORMAT(F22.4)
      WRITE(11,67)ARC0
   67 FORMAT(10X,'LENGTH OF MERIDIONAL ARC  :
',F22.6,/)
      CALL ELINV1(LT1,C,EC3,ARC0)
      CALL RADDMS(LT1,ID,IM,M3,I2,RD)
C
      WRITE(11,68)ID,IM,M3
   68 FORMAT(10X,'LATITUDE HELMERT         [1] =
',I3,I3,F12.8,/)
C
      CALL ELINV2(LT2,C,EC3,ARC0)
      CALL RADDMS(LT2,ID,IM,M3,I2,RD)
```

```
      WRITE(11,69)ID,IM,M3
   69 FORMAT(10X,'LATITUDE HELMERT MOD. [2] =
',I3,I3,F12.8,/)
C
      CALL ELINV3(LT3,C,EC3,ARCO)
      CALL RADDMS(LT3,ID,IM,M3,I2,RD)
C
      WRITE(11,70)ID,IM,M3
   70 FORMAT(10X,'LATITUDE KRACK 82     [3] =
',I3,I3,F12.8,/)
C
      CALL ELINV4(LT4,A,N,ARCO)
      CALL RADDMS(LT4,ID,IM,M3,I2,RD)
C
      WRITE(11,71)ID,IM,M3
   71 FORMAT(10X,'LATITUDE KRACK 83     [4] =
',I3,I3,F12.8,/)
C
      CALL ELINV5(LT5,A,N,ARCO)
      CALL RADDMS(LT5,ID,IM,M3,I2,RD)
C
      WRITE(11,72)ID,IM,M3
   72 FORMAT(10X,'LATITUDE NGS          [5] =
',I3,I3,F12.8,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT LATITUDE - (C)ONT OR (S)TOP
(C/S) ? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'C')GOTO 50
      IF (VR .EQ. 'S' .OR. VR .EQ. 'S')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
```

```
*********1*********2*********3*********4*********5*********6*********7**
-- SUBROUTINE ELDAT1 ELLIPSOID PARAMETERS --
-- SUBROUTINE ELFOR1 TO CALCULATE - VERSION 'HELMERT' FORWARD [1] --
-- SUBROUTINE ELINV1 TO CALCULATE - VERSION 'HELMERT' INVERSE [1] --
-- SUBROUTINE ELFOR2 TO CALCULATE - VERSION 'HELMERT' FORWARD [2] --
-- SUBROUTINE ELINV2 TO CALCULATE - VERSION 'HELMERT' INVERSE [2] --
-- SUBROUTINE ELFOR3 TO CALCULATE - VERSION 'KRACK 82' FORWARD [3] --
-- SUBROUTINE ELINV3 TO CALCULATE - VERSION 'KRACK 82' INVERSE [3] --
-- SUBROUTINE ELFOR4 TO CALCULATE - VERSION 'KRACK 83' FORWARD [4] --
-- SUBROUTINE ELINV4 TO CALCULATE - VERSION 'KRACK 83' INVERSE [4] --
-- SUBROUTINE ELFOR5 TO CALCULATE - VERSION 'NGS MODIFIED' FORWARD [5] --
-- SUBROUTINE ELINV5 TO CALCULATE - VERSION 'NGS MODIFIED' INVERSE [5] --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
*********1*********2*********3*********4*********5*********6*********7**
```

18.4 Ellipsoid Constants – Arcs – Radii

A_04ELLI.FOR Program - Meridional Arc Constants

```
************************************************************************
*     PROGRAM A_04ELLI.FOR - Date 01-06-2006                           *
*     ELLIPSOID CONSTANTS AND MERIDIONAL ARCS - FORTRAN PROGRAM        *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1999-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
```

C See [On_CD] for Subroutines, and Examples

```
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,VR*1
C
       OPEN(11,FILE= '\FORFILES\A4ELLI00.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
----------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- ELLI          ')
      WRITE(11,3)
    3 FORMAT(10X,'ELLIPSOID - MER. ARC CONSTANTS -
VARIOUS RADII  ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
----------------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
      WRITE(*,10)
   10 FORMAT(' INPUT REFERENCE ELLIPSOID     : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
      WRITE(*,12)
   12 FORMAT(' INPUT SEMI-MAJOR AXIS         : ')
      READ(*,13)A
```

```
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT(' INPUT RECIPROCAL FLATTENING  : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
C
C     --  CALCULATE ELLIPSOID PARAMETERS --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,20)IYR,IMON,IDAY,IHR,IMIN
   20 FORMAT(10X,'DATE AND TIME     [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
C
      CALL ELDATA(A,B,C,AB,EC0,EC1,EC2,EC3,F,FL,
AE2, SE2,AE3,SE3,N,PI4,
     + RD,T1)
C
      WRITE(11,21)DATUM
   21 FORMAT(10X,'DATUM OF ELLIPSOID        :
',A30)
      WRITE(11,22)A
   22 FORMAT(10X,'SEMI-MAJOR AXIS       [A] :
',F22.8)
      WRITE(11,23)B
   23 FORMAT(10X,'SEMI-MINOR AXIS       [B] =
',F22.8)
      WRITE(11,24)FL
   24 FORMAT(10X,'REC. FLATTENING      [FL] :
',F22.16)
      WRITE(11,25)F
   25 FORMAT(10X,'FLATTENING            [F] =
',E22.16)
      WRITE(11,26)EC0
   26 FORMAT(10X,'1ST ECCENTRICITY    [EC0] =
',E22.16)
      WRITE(11,27)EC1
   27 FORMAT(10X,'2ND ECCENTRICITY    [EC1] =
',E22.16)
      WRITE(11,28)EC2
   28 FORMAT(10X,'SQ.1ST ECCENTRICITY [EC2] =
',E22.16)
      WRITE(11,29)EC3
   29 FORMAT(10X,'SQ.2ND ECCENTRICITY [EC3] =
',E22.16)
      WRITE(11,30)C
   30 FORMAT(10X,'C                         =
',E22.16)
      WRITE(11,31)RD
   31 FORMAT(10X,'RAD-DMS CONV.             =
',E22.16)
      WRITE(11,32)T1
   32 FORMAT(10X,'1-F                       =
',E22.16)
      WRITE(11,33)N
   33 FORMAT(10X,'N                         =
',E22.16)
      WRITE(11,34)AB
   34 FORMAT(10X,'A/B                       =
',F22.16,/)
      WRITE(11,35)AE2
   35 FORMAT(10X,'(1-EC2)                   =
',E22.16)
      WRITE(11,36)SE2
   36 FORMAT(10X,'SQRT(1-EC2)               =
',E22.16)
      WRITE(11,37)AE3
   37 FORMAT(10X,'(1+EC3)                   =
',F22.16)
      WRITE(11,38)SE3
   38 FORMAT(10X,'SQRT(1+EC3)               =
',F22.16,/)
C
C     -- INPUT LATITUDE IN DEGREES-MINUTES-SECONDS
--
   40 CONTINUE
      I2    = 1
      WRITE(*,41)
   41 FORMAT(10X,'INPUT AZIMUTH             :
DD,MM,SS.SSSSSS ')
      READ(*,42)ID,IM,M3
   42 FORMAT(I3,I3,F8.4)
      WRITE(11,43)ID,IM,M3
   43 FORMAT(10X,'AZIMUTH                   :
',I3,I3,F10.6,/)
      CALL DMSRAD(AZ,ID,IM,M3,I2,RD)
C
C     -- CALCULATE VARIOUS RADII --
C
      CALL RADII(A,AZ,B,C,F,M0,N,N0,EC2,EC3,R0,RA,
RB, RG,RE,RM,RP,V,W)
C
      WRITE(11,50)M0
   50 FORMAT(10X,'M (R00)                [M] =
',F22.4)
      WRITE(11,51)N0
   51 FORMAT(10X,'N (R90)                [N] =
',F22.4)
      WRITE(11,52)RM
   52 FORMAT(10X,'MEAN RADIUS M*N       [RM] =
',F22.4)
      WRITE(11,53)RP
   53 FORMAT(10X,'POLAR RADIUS          [RP] =
',F22.4)
      WRITE(11,54)R0
   54 FORMAT(10X,'RECTIFYING RADIUS     [R0] =
',F22.4)
      WRITE(11,55)RA
   55 FORMAT(10X,'RADIUS OF AZIMUTH     [RA] =
',F22.4)
      WRITE(11,56)RB
   56 FORMAT(10X,'MEAN RADIUS A+B       [RB] =
',F22.4)
      WRITE(11,57)RG
   57 FORMAT(10X,'GEO-MEAN RADIUS       [RG] =
',F22.4)
      WRITE(11,58)RE
   58 FORMAT(10X,'RADIUS EQ.VOL.SPHERE [RE] =
',F22.4,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 40
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
```

```
C                                              *********1*********2*********3*********4*********5
C     -- END OF PROGRAM --                     *********6*********7**
C
C
************************************************************************
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- SUBROUTINE RADII TO CALCULATE VARIOUS RADII --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
************************************************************************
```

18.5 Quadrilateral Ellipsoidal Area

A_05ARSQ.FOR Program - Quadrilateral Ellipsoidal Area Calculation

```
************************************************************************
*     PROGRAM A_05ARSQ.FOR - Date 01-06-2006                           *
*     QUADRILATERAL AREA CALCULATION FOR ELLIPSOID - FORTRAN PROGRAM   *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1998-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
```

```
C     See [On_CD] for Subroutines, and Examples
```

```
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1
C
       OPEN(11,FILE= '\FORFILES\ARSQDAT0.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'--------------------------------
---------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- ARSQ          ')
      WRITE(11,3)
    3 FORMAT(10X,'QUADRILATERAL AREA CALCULATION
')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'--------------------------------
---------------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT(' INPUT REFERENCE ELLIPSOID    : ')
      READ(*,12) DATUM
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT(' INPUT SEMI-MAJOR AXIS    [A] : ')
      READ(*,14)A
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT(' INPUT REC. FLATTENING   [FL] : ')
      READ(*,16)FL
   16 FORMAT(F22.16)
C
C     --  CALCULATE ELLIPSOID PARAMETERS --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME     [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
C
      CALL ELDAT5(A,B,EC2,FL,PI4,RD)
      CALL AREMAX(B,EC2,PI4,R,GASPH)
C
      WRITE(11,33)DATUM
   33 FORMAT(10X,'DATUM OF ELLIPSOID          :
',A30)
      WRITE(11,34)A
   34 FORMAT(10X,'SEMI-MAJOR AXIS        [A] :
',F22.6)
      WRITE(11,35)B
   35 FORMAT(10X,'SEMI-MINOR AXIS        [B] =
',F22.6)
      WRITE(11,36)FL
   36 FORMAT(10X,'REC.FLATTENING        [FL] :
',F22.16)
      WRITE(11,37)EC2
   37 FORMAT(10X,'SQUARE ECC            [EC2] =
',E22.16)
      WRITE(11,38)GASPH
   38 FORMAT(10X,'AREA-TOTAL          [SQ.KM] =
',F22.6)
      WRITE(11,39)R
   39 FORMAT(10X,'RADIUS OF SPHERE       [KM] =
',F22.6)
C
C     -- INPUT UPPER LATITUDE --
C     -- AREA ELLIPSOID ZONE --  EQUATOR-UPPER-
LATITUDE --
C
   40 WRITE(*,41)
   41 FORMAT(' LATITUDE                [UPPER] :
DD,MM,SS.SSSS ')
      READ(*,42)ID,IM,M3
   42 FORMAT(I3,I3,F8.4)
C
      I2    = 1
      NS    = 'N'
      WRITE(11,43)ID,IM,M3,NS
   43 FORMAT(10X,'LATITUDE               [UPPER] :
',I4,I3,F8.4,A2,/)
      CALL DMSRAD(PH,ID,IM,M3,I2,RD)
```

```
      CALL AREZON(B,PH,EC2,PI4,AREAZ)
C
      WRITE(11,44)AREAZ
   44 FORMAT(10X,'ZONE-AREA           [SQ.KM] =
',F22.6,/)
C
C     -- QUADRILATERAL-AREA ELLIPSOID BOUNDED BY
TWO POINTS --
C     -- INPUT POINT-ONE - [N/E point] --
C
   50 CONTINUE
      WRITE(*,51)
   51 FORMAT(' LATITUDE     [STA.NORTH-EAST] :
DD,MM,SS.SSSS ')
      READ(*,52)ID,IM,M3
   52 FORMAT(I3,I3,F8.4)
C
      I2    = 1
      NS    = 'N'
      WRITE(11,53)ID,IM,M3,NS
   53 FORMAT(10X,'LATITUDE [STA.NORTH-EAST] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
      WRITE(*,54)
   54 FORMAT(' LONGITUDE    [STA.NORTH-EAST] :
DDD,MM,SS.SSSS ')
      READ(*,55)ID,IM,M3
   55 FORMAT(I4,I3,F8.4)
C
      I2    = 1
      EW    = 'E'
      WRITE(11,56)ID,IM,M3,EW
   56 FORMAT(10X,'LON.      [STA.NORTH-EAST] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
C
C     -- INPUT POINT-TWO - [S/W point] --
C
      WRITE(*,57)
   57 FORMAT(' LATITUDE     [STA.SOUTH-WEST] :
DD,MM,SS.SSSS ')
      READ(*,58)ID,IM,M3
   58 FORMAT(I3,I3,F8.4)
C
      I2    = 1
      NS    = 'N'
      WRITE(11,59)ID,IM,M3,NS
   59 FORMAT(10X,'LATITUDE [STA.SOUTH-WEST] :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LT2,ID,IM,M3,I2,RD)
C
      WRITE(*,60)
   60 FORMAT(' LONGITUDE    [STA.SOUTH-WEST] :
DDD,MM,SS.SSSS ')
      READ(*,61)ID,IM,M3
   61 FORMAT(I4,I3,F8.4)
C
      I2    = 1
      NS    = 'E'
      WRITE(11,62)ID,IM,M3,EW
   62 FORMAT(10X,'LONG.     [STA.SOUTH-WEST] :
',I4,I3,F8.4,A2,/)
      CALL DMSRAD(LN2,ID,IM,M3,I2,RD)
C
C     -- CALCULATE QUADRILATERAL-ELLIPSOIDAL AREA
C     -- BOUNDED BY ONE N/E POINT AND ONE S/W
POINT IN GEOGRAPHICALS --
C
      CALL ARZON2(B,EC2,LT1,LT2,LN1,LN2,AREAM)
C
      WRITE(11,63)AREAM
   63 FORMAT(10X,'AREA SQ.METRES               =
',F22.6)
C
      AREAK = AREAM/1.D6
      WRITE(11,64)AREAK
   64 FORMAT(10X,'AREA SQ.KILOMETRES           =
',F22.6,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 40
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
*********1*********2*********3*********4*********5*********6*********7**
  -- SUBROUTINE ELDAT5 ELLIPSOID-05 PARAMETERS --
  -- SUBROUTINE AREMAX to CALCULATE RADIUS AND AREA OF ELLIPSOID --
  -- SUBROUTINE AREZON to CALCULATE AREA BETWEEN EQUATOR AND A UPPER LATITUDE --
  -- SUBROUTINE ARZON2 TO CALCULATE QUADRILATERAL-ELLIPSOIDAL AREA
  -- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
*********1*********2*********3*********4*********5*********6*********7**
```

18.6 Polygonal Area on a Sphere or Bi-Axial Ellipsoid

A_06ARPY.FOR Program - Polygonal Ellipsoidal Area Calculation

```
************************************************************************
*     PROGRAM A_06ARPY.FOR - Date 01-06-2006                           *
*     POLYGONAL ELLIPSOIDAL AREA CALCULATION - FORTRAN PROGRAM         *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1997-2006                                    *
```

```
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1
C
       OPEN(11,FILE= '\FORFILES\A6ARPY00.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
---------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- ARPY   ')
      WRITE(11,3)
    3 FORMAT(10X,'POLYGONAL ELLIPSOIDAL AREA CAL-
CULATION   ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
---------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT(' INPUT REFERENCE ELLIPSOID    : ')
      READ(*,12) DATUM
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT(' INPUT SEMI-MAJOR AXIS    [A] : ')
      READ(*,14)A
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT(' INPUT REC. FLATTENING   [FL] : ')
      READ(*,16)FL
   16 FORMAT(F22.16)
C
      WRITE(*,17)
   17 FORMAT(' LATITUDE MEAN           [MN] :
DD,MM,SS.SSSS ')
      READ(*,18)ID,IM,M3
   18 FORMAT(I3,I3,F8.4)
C
C     --  CALCULATE ELLIPSOID PARAMETERS --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME     [HH:MM] =
',I3,I3,I5,' - ',I3,I3,/)
C
      I2     = 1
      NS     = 'N'
C
      CALL ELDAT6(FL,PI4,RD)
      CALL DMSRAD(LTM,ID,IM,M3,I2,RD)
      CALL SPHERA(A,FL,LTM,PI4,RM,AREAS,T1)
C
      WRITE(11,31)DATUM
   31 FORMAT(10X,'DATUM OF ELLIPSOID          :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS        [A] :
',F22.6)
      WRITE(11,33)FL
   33 FORMAT(10X,'REC.FLATTENING        [FL] :
',F22.16)
      WRITE(11,34)T1
   34 FORMAT(10X,'T1                    [T1] =
',F22.16)
      WRITE(11,35)ID,IM,M3,NS
   35 FORMAT(10X,'LATITUDE MEAN         [MN] :
',I4,I3,F8.4,A2)
      WRITE(11,36)LTM
   36 FORMAT(10X,'LAT.MEAN IN RAD      [LTM] =
',F22.16)
      WRITE(11,37)RM
   37 FORMAT(10X,'MEAN RADIUS           [RM] =
',F22.6)
      WRITE(11,38)AREAS
   38 FORMAT(10X,'AREA OF SPHERE      [SQ.M] =
',F22.6,/)
C
C     -- INPUT NUMBER OF STATIONS ON A SPHERE --
C
   50 CONTINUE
      WRITE(*,51)
   51 FORMAT(' INPUT NUMBER OF STA.             :
XXX')
      READ(*,52)J
   52 FORMAT(I4)
C
C     -- INPUT ELLIPSOID TO SPHERE CONVERTED
GEOGRAPHICALS ONLY !! --
C     -- I.E. INPUT SPHERICAL GEOGRAPHICALS OF
STATIONS--
C
   60 CONTINUE
      WRITE(*,61)
   61 FORMAT(' LATITUDE              [STA.1] :
DD,MM,SS.SSSS ')
      READ(*,62)ID,IM,M3
   62 FORMAT(I3,I3,F8.4)
      I2     = 1
      NS     = 'N'
      WRITE(11,63)ID,IM,M3,NS
   63 FORMAT(10X,'LATITUDE              [STA.1] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
      WRITE(*,64)
   64 FORMAT(' LONGITUDE             [STA.1] :
DDD,MM,SS.SSSS ')
      READ(*,65)ID,IM,M3
   65 FORMAT(I4,I3,F8.4)
      I2     = 1
      EW     = 'E'
      WRITE(11,66)ID,IM,M3,EW
   66 FORMAT(10X,'LONGITUDE             [STA.1] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
C
C     -- INPUT GEOGRAPHICALS I=2 TO J+1 --
C
      W6     = 0.D0
      DO 80 I = 2,J+1
      WRITE(*,67)I
```

```
   67 FORMAT(10X,'STATION NUMBER        [I] =
',I4)
C
C     -- INPUT LATITUDE/LONGITUDE STA.I (I=2 TO
J+1) --
C
   70 CONTINUE
      WRITE(*,71)
   71 FORMAT(' LATITUDE              [STA.I] :
DD,MM,SS.SSSS ')
      READ(*,72)ID,IM,M3
   72 FORMAT(I3,I3,F8.4)
      I2    = 1
      NS    = 'N'
      WRITE(11,73)ID,IM,M3,NS
   73 FORMAT(10X,'LATITUDE           [STA.I] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LT,ID,IM,M3,I2,RD)
C
      WRITE(*,74)
   74 FORMAT(' LONGITUDE             [STA.I] :
DDD,MM,SS.SSSS ')
      READ(*,75)ID,IM,M3
   75 FORMAT(I4,I3,F8.4)
      I2    = 1
      EW    = 'E'
      WRITE(11,76)ID,IM,M3,EW
   76 FORMAT(10X,'LONGITUDE          [STA.I] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LN,ID,IM,M3,I2,RD)
C
      CALL
SPHERB(LT,LT1,LN,LN1,PI4,RM,AREAM,AREAK,W6)
C
   80 CONTINUE
C
      WRITE(11,81)AREAM
   81 FORMAT(10X,'AREA SQ.M  (SPHERE)   [M] =
',F22.6)
      WRITE(11,82)AREAK
   82 FORMAT(10X,'AREA SQ.KM (SPHERE)  [KM] =
',F22.6,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 50
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**

C
*****************************************************************************
-- SUBROUTINE ELDAT6 ELLIPSOID-06 PARAMETERS --
-- SUBROUTINE SPHERA SPHERE-AREA PARAMETERS --
-- SUBROUTINE SPHERB AREA OF POLYNOMIAL ON A SPHERE --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
*****************************************************************************
```

18.7 Length of Parallel

A_07PARA.FOR Program - Length of Parallel

```
*****************************************************************************
*     PROGRAM A_07PARA.FOR - Date 01-06-2006                                *
*     LENGTH OF PARALLEL - FORTRAN  PROGRAM                                 *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK                  *
*     AUTHOR M. HOOIJBERG 1999-2006                                         *
*********1*********2*********3*********4*********5*********6*********7**
 C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,EW*1,NS*1,VR*1
C
       OPEN(11,FILE= '\FORFILES\A7PARALO.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
---------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- PARA   ')
      WRITE(11,3)
    3 FORMAT(10X,'COMPUTE LENGTH OF PARALLEL
')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'--------------------------------
---------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
      WRITE(*,10)
   10 FORMAT('  INPUT REFERENCE ELLIPSOID    : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
```

```
      WRITE(*,12)
   12 FORMAT('  INPUT SEMI-MAJOR AXIS       : ')
      READ(*,13)A
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT('  INPUT RECIPROCAL FLATTENING : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,20)IYR,IMON,IDAY,IHR,IMIN
   20 FORMAT(10X,'DATE AND TIME                =
',I3,I3,I5,' - ',I3,I3)
C
C     -- CALCULATE ELLIPSOID PARAMETERS --
C
      CALL ELDATA(A,B,C,EC2,EC3,F,FL,PI4,RD,T1)
      WRITE(11,21)DATUM
   21 FORMAT(10X,'REFERENCE ELLIPSOID          :
',A30)
      WRITE(11,22)A
   22 FORMAT(10X,'SEMI-MAJOR AXIS        [A] :
',F22.6)
      WRITE(11,23)B
   23 FORMAT(10X,'SEMI-MINOR AXIS        [B] =
',F22.6)
      WRITE(11,24)FL
   24 FORMAT(10X,'REC. FLATTENING       [FL] :
',F22.16)
      WRITE(11,25)F
   25 FORMAT(10X,'FLATTENING              [F] =
',E22.16)
      WRITE(11,26)EC2
   26 FORMAT(10X,'SQ.1ST ECCENTRICITY [EC2] =
',E22.16)
      WRITE(11,27)EC3
   27 FORMAT(10X,'SQ.2ND ECCENTRICITY [EC3] =
',E22.16)
      WRITE(11,28)C
   28 FORMAT(10X,'C                            =
',E22.16)
      WRITE(11,29)RD
   29 FORMAT(10X,'RAD-DMS CONV.                =
',E22.16)
      WRITE(11,30)PI4
   30 FORMAT(10X,'PI/4                         =
',E22.16)
C     -- INPUT GEOGRAPHICALS (LAT./ LON.) NORTH-
EAST DATA --
C
   40 CONTINUE
      I2    = 1
      NS    = 'N'
      EW    = 'E'
C
      WRITE(*,41)
   41 FORMAT(' INPUT LATITUDE                  :  DD
MM SS.SSSS ')
      READ(*,42)ID,IM,M3
   42 FORMAT(I3,I3,F8.4)
      WRITE(11,43)ID,IM,M3,NS
   43 FORMAT(10X,'LATITUDE                     :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
   50 CONTINUE
      WRITE(*,51)
   51 FORMAT(' INPUT LONGITUDE                 : DDD
MM SS.SSSS ')
      READ(*,52)ID,IM,M3
   52 FORMAT(I4,I3,F8.4)
      WRITE(11,53)ID,IM,M3,EW
   53 FORMAT(10X,'LONGITUDE                    :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
      CALL PARDAT(A,C,EC2,EC3,LN1,LT1,N,M,PAR,V,W)
C
      WRITE(11,60)N
   60 FORMAT(10X,'N                  [V=R90] =
',F22.4)
      WRITE(11,61)M
   61 FORMAT(10X,'M                  [P=R00] =
',F22.4)
      WRITE(11,63)PAR
   63 FORMAT(10X,'LENGTH OF PARALLEL  [PAR] =
',F22.4,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 50
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
```

```
*************************************************************************
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- SUBROUTINE PARDAT LENGTH OF PARALLEL --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
*************************************************************************
```

18.8 Geodetic Reference System

A_08GEOR.FOR Program - Geodetic Reference System

```
*************************************************************************
*     PROGRAM A_08GEOR.FOR - Date 01-06-2006                            *
```

```
*     GEODETIC REFERENCE SYSTEM - FORTRAN PROGRAM                      *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1996-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
 C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30, VR*1
C
       OPEN(11,FILE= '\FORFILES\GA8GEOR0.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
---------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- GEOR   ')
      WRITE(11,3)
    3 FORMAT(12X,'GEODETIC REFERENCE SYSTEM PA-
RAMETERS     ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(11X,'-------------------------------
---------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT(' INPUT REFERENCE ELLIPSOID   : ')
      READ(*,12) DATUM
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT(' INPUT SEMI-MAJOR AXIS       : ')
      READ(*,14)A
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT(' INPUT GM*10**8 M**3 S**-2   : ')
      READ(*,16)GM
   16 FORMAT(F22.16)
      WRITE(*,17)
   17 FORMAT(' INPUT J2*10**-8             : ')
      READ(*,18)TJ2
   18 FORMAT(F22.16)
      WRITE(*,19)
   19 FORMAT(' INPUT W*10**-11 RAD S**-1   : ')
      READ(*,20)W
   20 FORMAT(F22.14)
      WRITE(*,21)
   21 FORMAT(' INPUT SQUARE ECC [ESTIMATE] : ')
      READ(*,22)T3
   22 FORMAT(F22.16)
C
C     -- CALCULATE ELLIPSOID PARAMETERS --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
C
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME               =
',I3,I3,I5,' - ',I3,I3)
      WRITE(11,31)DATUM
   31 FORMAT(10X,'NAME OF DATUM               :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS         [A] :
',F22.8)
      WRITE(11,33)GM
   33 FORMAT(10X,'GM*10**8 M**3 S**-2         :
',E22.16)
      WRITE(11,34)TJ2
   34 FORMAT(10X,'J2*10**-8                   :
',E22.16)
      WRITE(11,35)W
   35 FORMAT(10X,'W*10**-11 RAD S**-1         :
',E22.16)
      WRITE(11,36)T3
   36 FORMAT(10X,'SQUARE ECC [ESTIMATE]       :
',E22.16)
C
C     -- CALCULATE FLATTENING --
C
      CALL GEDATA(A,E1,EC2,F,FL,GM,T3,TJ2,Q2,W)
C
      WRITE(11,40)F
   40 FORMAT(10X,'F                           =
',E22.16)
      WRITE(11,41)FL
   41 FORMAT(10X,'FL                          =
',F22.12,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP :
(C/S) ? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 10
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
C
**************************************************************************
-- SUBROUTINE GEDATA GEODETIC REFERENCE DATA --
**************************************************************************
```

18.9 Bi-Linear Interpolation

A_09BILI.FOR - Program - Bilinear Interpolation

Bi-linear interpolation TDN-data matrix given is 30×32 (integers). For calculation, one column and one row of zero data (given as: 999) are added to each matrix. Hence, in the program the data-matrix used for calculation is 31×33 (integers).

In (Hooijberg, 1997), RDED00031.FOR and RDED00032.FOR routines are for both UTM zone 31 and UTM zone 32, using the matrix 31×33.

```
*****************************************************************************
*     PROGRAM A_09BILI.FOR - Date 01-06-2006                                *
*     BILINEAR - INTERPOLATION - FORTRAN PROGRAM                            *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK                  *
*     AUTHOR M. HOOIJBERG 1984-2006                                         *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,M-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER XRD*10,YRD*10,VR*1
C
       OPEN(11,FILE= '\FORFILES\A9BILI00.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
---------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- BILI   ')
      WRITE(11,3)
    3 FORMAT(10X,'BI-LINEAR INTERPOLATION
')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
---------',/)
C
C     -- EXAMPLE FOR CORRECTION RD1918 COORDINATES
--
C
      XRD    = 'RD1918-X'
      YRD    = 'RD1918-Y'
C
  100 WRITE(*,101)
  101 FORMAT(' INPUT RD1918-X             [XI] : ')
      READ(*,102)XI
  102 FORMAT(F16.4)
      WRITE(*,103)
  103 FORMAT(' INPUT RD1918-Y             [YI] : ')
      READ(*,104)YI
  104 FORMAT(F16.4)
      WRITE(11,105)XRD,XI
  105 FORMAT(10X,A10,10X,' [XI] : ',F16.4)
      WRITE(11,106)YRD,YI
  106 FORMAT(10X,A10,10X,' [YI] : ',F16.4,/)
C
C     APPLY BI-LINEAR INTERPOLATION ACCORDING TO
C
C     EI     = EI+CE  =  OUTPUT
C     NI     = NI+CN  =  OUTPUT
C
C     -- LOCATING VALUES FOR BI-LINEAR INTERPOLA-
TION --
C
      CONTINUE
      WE     = XI
      WN     = YI
      WRITE(11,300)WE
  300 FORMAT(10X,'WE                               =
',F16.4)
      WRITE(11,301)WN
  301 FORMAT(10X,'WN                               =
',F16.4)
      DI     = 1.D+04
      WE1    = DINT(WE/DI)*DI
      WN1    = DINT(WN/DI)*DI
      WRITE(11,302)WE1
  302 FORMAT(10X,'WE1                              =
',F16.4)
      WRITE(11,303)WN1
  303 FORMAT(10X,'WN1                              =
',F16.4)
      XD     = WE-WE1
      YD     = WN-WN1
      WRITE(11,304)XD
  304 FORMAT(10X,'XD                               =
',F16.4)
      WRITE(11,305)YD
  305 FORMAT(10X,'YD                               =
',F16.4,/)
      I      = DINT(WE1/DI)+1
      J      = DINT((WN1-30.D0*DI)/DI)+1
      IF (I .LT. 1 .OR. I .GT. 30 .OR. J .LT. 1
.OR. J .GT. 32) GOTO 400
C
      WRITE(*,306)I,J
  306 FORMAT(10X,'I .... J                         =
',I5,I5,/)
      WRITE(11,307)I,J
  307 FORMAT(10X,'I .... J                         =
',I5,I5,/)
C
C     -- FOUR CORNER EASTING VALUES FOR 2X-LINEAR
INTERPOLATION --
C
      WRITE(*,308)
  308 FORMAT(' INPUT E-VALUE (I;J)    [IZ1] : ')
      READ(*,309)IZ1
```

```
  309 FORMAT(I5)
      WRITE(*,310)
  310 FORMAT(' INPUT E-VALUE (I;J+1)  [IZ2] : ')
      READ(*,311)IZ2
  311 FORMAT(I5)
      WRITE(*,312)
  312 FORMAT(' INPUT E-VALUE (I+1;J+1)[IZ3] : ')
      READ(*,313)IZ3
  313 FORMAT(I5)
      WRITE(*,314)
  314 FORMAT(' INPUT E-VALUE (I+1;J)  [IZ3] : ')
      READ(*,315)IZ4
  315 FORMAT(I5)
C
      WRITE(11,316)IZ1,IZ2,IZ3,IZ4
  316 FORMAT(10X,'IZ1 ... IZ4        [EAST] =
',I5,I5,I5,I5,/)
      IF (IZ1 .EQ. 999 .OR. IZ2 .EQ. 999 .OR. IZ3
.EQ. 999 .OR. IZ4 .EQ.
     + 999) GOTO 410
C
C     -- EASTING INTERPOLATION --
C
      CALL BILINR(DI,XD,YD,IZ1,IZ2,IZ3,IZ4,CE)
C
      WRITE(11,317)CE
  317 FORMAT(10X,'EASTING CORRECTION   [CE] =
',F16.4,/)
C
C     -- FOUR CORNER NORTHING VALUES FOR 2X-LINEAR
INTERPOLATION --
C
      WRITE(*,318)
  318 FORMAT(' INPUT N-VALUE (I;J)    [IZ1] : ')
      READ(*,319)IZ1
  319 FORMAT(I5)
      WRITE(*,320)
  320 FORMAT(' INPUT N-VALUE (I;J+1)  [IZ2] : ')
      READ(*,321)IZ2
  321 FORMAT(I5)
      WRITE(*,322)
  322 FORMAT(' INPUT N-VALUE (I+1;J+1)[IZ3] : ')
      READ(*,323)IZ3
  323 FORMAT(I5)
      WRITE(*,324)
  324 FORMAT(' INPUT N-VALUE (I+1;J)  [IZ4] : ')
      READ(*,325)IZ4
  325 FORMAT(I5)
C
      WRITE(11,326)IZ1,IZ2,IZ3,IZ4
C
  326 FORMAT(10X,'IZ1 ... IZ4        [NORTH] =
',I5,I5,I5,I5,/)
      IF (IZ1 .EQ. 999 .OR. IZ2 .EQ. 999 .OR. IZ3
.EQ. 999 .OR. IZ4 .EQ.
     + 999) GOTO 410
C
C     -- NORTHING INTERPOLATION --
C
      CALL BILINR(DI,XD,YD,IZ1,IZ2,IZ3,IZ4,CN)
C
      WRITE(11,327)CN
  327 FORMAT(10X,'NORTHING CORRECTION  [CN] =
',F16.4,/)
      GOTO 900
C
  400 CONTINUE
      WRITE(11,401)
  401 FORMAT(10X,'-> OUTSIDE TRANSFORMATION
LIMITS')
      CE    = 0.D0
      CN    = 0.D0
      GOTO 900
C
  410 CONTINUE
      WRITE(11,411)
  411 FORMAT(10X,'-> NO CORRECTION APPLIED
')
      CE    = 0.D0
      CN    = 0.D0
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP :
(C/S) ? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 100
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
C********1*********2*********3*********4*********5
*********6*********7**
*****************************************************************************
-- SUBROUTINE BILINR BI-LINEAR INTERPOLATION --
*****************************************************************************
```

18.10 S-Transformation

A_10STRM.FOR Program - S-Transformation

Transformed curvilinear coordinates can be obtained without iteration. However, situations may arise in which iterative correction is required due to a large ellipsoidal height, e.g. satellite tracking at 20 000 km height + 6 378 km (Earth's radius)=26 400 km height. Schuhr's algorithm can provide corrections to such data (Schuhr, 1996).

```
*****************************************************************************
*     PROGRAM  A_10STRM.FOR - Date 01-06-2006                              *
*     DATUM S-TRANSFORMATION - FORTRAN PROGRAM                             *
```

```
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK          *
*     AUTHOR M. HOOIJBERG 1995-2006                                 *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUMO*30,DATUMN*30,VR*1,NS*1,
W*1,PP*1,GR*1
C
       OPEN(11,FILE= '\FORFILES\A10STR00.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,' ------------------------------
---------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- STRM   ')
      WRITE(11,3)
    3 FORMAT(10X,'DATUM S-TRANSFORMATION
')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,' ------------------------------
---------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA - OLD SYSTEM -
-
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT('  INPUT NAME OF OLD DATUM     : ')
      READ(*,12) DATUMO
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT('  INPUT OLD SEMI-MAJOR AXIS   : ')
      READ(*,14)A0
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT('  INPUT OLD REC.FLATTENING    : ')
      READ(*,16)FLO
   16 FORMAT(F22.16)
C
C     -- INPUT BASIC ELLIPSOID DATA - NEW SYSTEM -
-
C
      WRITE(*,20)
   20 FORMAT('  INPUT NAME OF NEW DATUM     : ')
      READ(*,21) DATUMN
   21 FORMAT(A30)
      WRITE(*,22)
   22 FORMAT('  INPUT NEW SEMI-MAJOR AXIS   : ')
      READ(*,23)AN
   23 FORMAT(F22.8)
      WRITE(*,24)
   24 FORMAT('  INPUT NEW REC.FLATTENING    : ')
      READ(*,25)FLN
   25 FORMAT(F22.16)
C
C     -- COMPUTE ELLIPSOIDAL PARAMETERS --
C
      CALL ELDAT2(AN,BN,EC20,EC2N,EC3N,FLO,FLN,
PI4, RD,R1)
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
C
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME                 =
',I3,I3,I5,' - ',I3,I3)
      WRITE(11,31)DATUMO
   31 FORMAT(10X,'REFERENCE ELLIPSOID [OLD] :
',A30)
      WRITE(11,32)A0
   32 FORMAT(10X,'SEMI-MAJOR AXIS OLD  [A0] :
',F22.6)
      WRITE(11,33)FLO
   33 FORMAT(10X,'REC.FLATTENING  OLD [FLO] :
',F22.16)
      WRITE(11,34)EC20
   34 FORMAT(10X,'SQ.1ST ECC.OLD     [EC20] =
',E22.16,/)
      WRITE(11,35)DATUMN
   35 FORMAT(10X,'REFERENCE ELLIPSOID [NEW] :
',A30)
      WRITE(11,36)AN
   36 FORMAT(10X,'SEMI-MAJOR AXIS NEW  [AN] :
',F22.6)
      WRITE(11,37)BN
   37 FORMAT(10X,'SEMI-MINOR AXIS NEW  [BN] =
',F22.6)
      WRITE(11,38)FLN
   38 FORMAT(10X,'REC.FLATTENING  NEW [FLN] :
',F22.16)
      WRITE(11,39)EC2N
   39 FORMAT(10X,'SQ.1ST ECC.NEW     [EC2N] =
',E22.16)
      WRITE(11,40)EC3N
   40 FORMAT(10X,'SQ.2ND ECC.NEW     [EC3N] =
',E22.16)
      WRITE(11,41)RD
   41 FORMAT(10X,'RAD-DMS CONV.        [RD] =
',E22.16)
      WRITE(11,42)R1
   42 FORMAT(10X,'R1                        =
',F22.12,/)
C
C     -- INPUT ALL TRANSFORMATION PARAMETERS --
C
   50 CONTINUE
      WRITE(*,51)
   51 FORMAT('  INPUT DELTA X          [dX] : ')
      READ(*,52)DX
   52 FORMAT(F22.16)
      WRITE(*,53)
   53 FORMAT('  INPUT DELTA Y          [dY] : ')
      READ(*,54)DY
   54 FORMAT(F22.16)
      WRITE(*,55)
   55 FORMAT('  INPUT DELTA Z          [dZ] : ')
      READ(*,56)DZ
   56 FORMAT(F22.16)
      WRITE(*,57)
   57 FORMAT('  OMEGA ANGLE        [Z-AXIS] : ')
      READ(*,58)OG
   58 FORMAT(F22.16)
      WRITE(*,59)
   59 FORMAT('  EPSILON ANGLE      [X-AXIS] : ')
```

```
      READ(*,60)EP
   60 FORMAT(F22.16)
      WRITE(*,61)
   61 FORMAT('  PSI ANGLE              [Y-AXIS] : ')
      READ(*,62)PS
   62 FORMAT(F22.16)
      WRITE(*,63)
   63 FORMAT('  DELTA K (SCALE)           [dK] : ')
      READ(*,64)DK
   64 FORMAT(F22.16)
C
      WRITE(11,70)DX
   70 FORMAT(10X,'DELTA X                 [dX] :
',F22.4)
      WRITE(11,71)DY
   71 FORMAT(10X,'DELTA Y                 [dY] :
',F22.4)
      WRITE(11,72)DZ
   72 FORMAT(10X,'DELTA Z                 [dZ] :
',F22.4,/)
      WRITE(11,73)OG
   73 FORMAT(10X,'OMEGA               [Z-AXIS] :
',E22.8)
      WRITE(11,74)EP
   74 FORMAT(10X,'EPSILON             [X-AXIS] :
',E22.8)
      WRITE(11,75)PS
   75 FORMAT(10X,'PSI                 [Y-AXIS] :
',E22.8)
      WRITE(11,76)DK
   76 FORMAT(10X,'DELTA K SCALE           [dK] :
',E22.8,/)
C
      PP    =   'P'
      GR    =   'R'
   80 WRITE(*,81)
   81 FORMAT('  (P)PM. OR IN (S)ECONDS        :
(P/S)? ')
      READ(*,82)PP
   82 FORMAT(A1)
      IF (PP .EQ. 'P' .OR. PP .EQ. 'p') GOTO 90
      IF (PP .EQ. 'S' .OR. PP .EQ. 's') GOTO 83
      GOTO 80
C
   83 OG    = OG*R1
      EP    = EP*R1
      PS    = PS*R1
C
      WRITE(11,173)OG
  173 FORMAT(10X,'OMEGA               [Z-AXIS] :
',E22.8)
      WRITE(11,174)EP
  174 FORMAT(10X,'EPSILON             [X-AXIS] :
',E22.8)
      WRITE(11,175)PS
  175 FORMAT(10X,'PSI                 [Y-AXIS] :
',E22.8)
C
   90 CONTINUE
      WRITE(*,91)
   91 FORMAT('  (G)ENERAL OR (R)EGULAR        :
(G/R)? ')
      READ(*,92)GR
   92 FORMAT(A1)
      IF (GR .EQ. 'G' .OR. GR .EQ. 'g') GOTO 100
      IF (GR .EQ. 'R' .OR. GR .EQ. 'r') GOTO 200
      GOTO 90
C
C     -- INPUT OFFSET X-Y-Z ORIGIN - GENERAL MODE
--
C
  100 CONTINUE
      WRITE(*,101)
  101 FORMAT('  INPUT OFFSET X-ORIGIN  [XC] : ')
      READ(*,102)XC
  102 FORMAT(F22.6)
      WRITE(*,103)
  103 FORMAT('  INPUT OFFSET Y-ORIGIN  [YC] : ')
      READ(*,104)YC
  104 FORMAT(F22.6)
      WRITE(*,105)
  105 FORMAT('  INPUT OFFSET Z-ORIGIN  [ZC] : ')
      READ(*,106)ZC
  106 FORMAT(F22.6)
C
      WRITE(11,107)XC
  107 FORMAT(10X,'OFFSET X-ORIGIN      [XC] :
',F22.4)
      WRITE(11,108)YC
  108 FORMAT(10X,'OFFSET Y-ORIGIN      [YC] :
',F22.4)
      WRITE(11,109)ZC
  109 FORMAT(10X,'OFFSET Z-ORIGIN      [ZC] :
',F22.4,/)
C
C     -- REGULAR MODE - NO X-Y-Z ORIGIN OFFSET --
C
  200 CONTINUE
      WRITE(*,201)
  201 FORMAT(' LATITUDE          :  DD,MM,SS.SSSS ')
      READ(*,202)ID,IM,M3
  202 FORMAT(I3,I3,F8.4)
  203 WRITE(*,204)
  204 FORMAT('  NORTH OR SOUTH:          (N/S)? ')
      READ(*,205)NS
  205 FORMAT(A1)
      I2    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 207
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 206
      GOTO 203
  206 I2    = -1
  207 CONTINUE
C
      WRITE(11,208)ID,IM,M3,NS
  208 FORMAT(10X,'LATITUDE            [D,M,S] :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LTO,ID,IM,M3,I2,RD)
C
      WRITE(*,209)LTO
  209 FORMAT(10X,'LATITUDE            [RAD] =
',F22.18)
      WRITE(*,210)
  210 FORMAT(' LONGITUDE                      :
DD,MM,SS.SSSS ')
      READ(*,211)ID,IM,M3
  211 FORMAT(I4,I3,F8.4)
C
  212 WRITE(*,213)
  213 FORMAT(' EAST OR WEST   :   (E/W)? ')
      READ(*,214)EW
  214 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 216
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 215
      GOTO 212
```

```
  215 I2    = -1
  216 WRITE(11,217)ID,IM,M3,EW
  217 FORMAT(10X,'LONGITUDE            [D,M,S) :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LNO,ID,IM,M3,I2,RD)
C
      WRITE(*,218)LNO
  218 FORMAT(10X,'LONGITUDE             [RAD] =
',F22.18)
C
C     -- INPUT HEIGHT VALUES --
C
      WRITE(*,220)
  220 FORMAT('  HEIGHT-OLD [MSL]            : ')
      READ(*,222)H
  222 FORMAT(F22.10)
      WRITE(*,223)
  223 FORMAT('  N-OLD [GEOID SEPARATION]    : ')
      READ(*,224)NSO
  224 FORMAT(F22.10)
C
C     -- COMPUTE HEIGHTS --
C
      CALL GEOXYZ(AO,EC2O,H,HO,LNO,LTO,NSO,RNO,XO,
YO,ZO)
C
      WRITE(11,225)H
  225 FORMAT(10X,'HEIGHT-OLD [MSL]          :
',F22.4)
      WRITE(11,226)NSO
  226 FORMAT(10X,'N-OLD [GEOID SEPARATION]  :
',F22.4)
      WRITE(11,227)HO
  227 FORMAT(10X,'h-OLD [H+N]               =
',F22.4 ,/)
C
C     -- OUTPUT CARTESIAN OLD X-Y-Z COORDIINATES -
-
C
      WRITE(11,230)XO
  230 FORMAT(10X,'X - OLD                   :
',F22.4)
      WRITE(11,231)YO
  231 FORMAT(10X,'Y - OLD                   :
',F22.4)
      WRITE(11,232)ZO
  232 FORMAT(10X,'Z - OLD                   :
',F22.4,/)
C
      IF (GR .EQ. 'R' .OR. GR .EQ. 'r') GOTO 300
C
C     -- COMPUTE NEW GENERAL - XYZ SYSTEM --
C
      CALL GENXYZ(XO,XC,XN,XT,YO,YC,YN,YT,ZO,ZC,
ZN,ZT, DX,DY,DZ,DK,OG,EP,
     + PS)
C
      WRITE(11,233)XT
  233 FORMAT(10X,'X [REDUCED VALUE]         :
',F22.4)
      WRITE(11,234)YT
  234 FORMAT(10X,'Y [REDUCED VALUE]         :
',F22.4)
      WRITE(11,235)ZT
  235 FORMAT(10X,'Z [REDUCED VALUE]         :
',F22.4,/)
      IF (GR .EQ. 'G' .OR. GR .EQ. 'g') GOTO 301
C
C     -- COMPUTE NEW REGULAR - XYZ SYSTEM --
C
  300 CALL REGXYZ(XO,XN,YO,YN,ZO,ZN,DX,DY,DZ, DK,
OG,EP,PS)
C
  301 WRITE(11,302)XN
  302 FORMAT(10X,'X - NEW                   :
',F22.4)
      WRITE(11,303)YN
  303 FORMAT(10X,'Y - NEW                   :
',F22.4)
      WRITE(11,304)ZN
  304 FORMAT(10X,'Z - NEW                   :
',F22.4,/)
C
C     -- ALGORITH BOWRING --
C
      CALL BOWRIN(AN,BN,EC2N,EC3N,LTN,P,XN,YN,ZN)
C
C     -- ALGORITHM SCHUHR - ONLY FOR LARGE ELLIP-
SOIDS REQUIRED ! --
C
      CALL SCHUHR(AN,BN,EC2N,EC3N,LTN,P,ZN)
      CALL RADDMS(LTN,ID,IM,M3,I2,RD)
C
      NS    = 'N'
      IF (I2 .LT. 0) NS = 'S'
      WRITE(11,310)ID,IM,M3,NS
  310 FORMAT(10X,'LATITUDE  - NEW           =
',I4,I3,F8.4,A2)
      LNN   = DATAN2(YN,XN)
      CALL RADDMS(LNN,ID,IM,M3,I2,RD)
      EW    = 'E'
      IF (I2 .LT. 0) EW = 'W'
      WRITE(11,311)ID,IM,M3,EW
  311 FORMAT(10X,'LONGITUDE - NEW           =
',I4,I3,F8.4,A2,/)
C
      CALL HEIGHT(AN,HN,EC2N,LTN,P,RNN)
C
      WRITE(11,320)HN
  320 FORMAT(10X,'h - NEW                   =
',F22.4)
      NSN   = HN-H
      WRITE(11,321)NSN
  321 FORMAT(10X,'N - NEW                   =
',F22.4,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c') GOTO 200
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
```

```
C
******************************************************************************
-- SUBROUTINE ELDAT2TWO ELLIPSOID PARAMETERS --
-- SUBROUTINE GEOXYZ GEO-XYZ TRANSFORMATION --
-- SUBROUTINE GENXYZ GENERAL-XYZ TRANSFORMATION --
-- SUBROUTINE REGXYZ REGULAR-XYZ TRANSFORMATION --
-- SUBROUTINE BOWRIN BOWRING --
-- SUBROUTINE SCHUHR SCHUHR - ALGORITHM FOR LARGE ELLIPSOIDS TO CALCULATE HEIGHT --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
-- SUBROUTINE GRDRAD CONVERTING GRAD TO RADIAN --
-- SUBROUTINE RADGRD CONVERTING RADIAN TO GRAD --
******************************************************************************
```

18.11 Forward Long Line - Kivioja's Method

A_11BDGK.FOR Program - from Bearing and Distance to Geodetic Coordinates - Kivioja's Method

Computing of intermediate points in the direct problem of A_11BDGK.FOR requires to Remind (or C) lines. All intermediate points are computed in lines as indicated. The results (accuracy) depend on the setting of the integrations (number). The program avoids iteration techniques.

```
******************************************************************************
*     PROGRAM A_11BDGK.FOR - Date 01-06-2006                                 *
*     LONG LINE CALCULATION METHOD KIVIOJA FORTRAN PROGRAM                   *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK                   *
*     AUTHOR M. HOOIJBERG 1999-2006                                          *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
```

```
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1
C
       OPEN(11,FILE= '\FORFILES\BDDATK00.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
-----------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- BDGK          ')
      WRITE(11,3)
    3 FORMAT(10X,'FORWARD LONG-LINE CALCULATION -
METHOD KIVIOJA   ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
-----------------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
      WRITE(*,10)
   10 FORMAT('  INPUT NAME OF ELLIPSOID      : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
      WRITE(*,12)
   12 FORMAT('  INPUT SEMI-MAJOR AXIS    [A] : ')
      READ(*,13)A
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT('  INPUT REC. FLATTENING   [FL] : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
C
C     -- COMPUTE BASIC ELLIPSOID PARAMETERS --
C
      CALL ELDATA(A,C,EC2,EC3,F,FL,PI4,RD)
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
C
   30 FORMAT(10X,'DATE AND TIME      [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
      WRITE(11,31)DATUM
   31 FORMAT(10X,'NAME OF ELLIPSOID          :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS        [A] :
',F22.6)
      WRITE(11,33)FL
   33 FORMAT(10X,'REC. FLATTENING       [FL] :
',F22.14)
      WRITE(11,34)EC2
   34 FORMAT(10X,'SQUARE ECC            [EC2] =
',E22.16)
      WRITE(11,35)EC3
   35 FORMAT(10X,'SQUARE 2ND ECC        [EC3] =
',E22.16)
      WRITE(11,36)C
   36 FORMAT(10X,'C                       [C] =
',F22.12,/)
C
C     -- INPUT GEOGRAPHICALS - BEARING - DISTANCE
--
C
   40 CONTINUE
      WRITE(*,41)
```

```
   41 FORMAT(' LATITUDE               [STA.1] :
DD,MM,SS.SSSSS')
      READ(*,42)ID,IM,M3
   42 FORMAT(I3,I3,F9.5)
   43 WRITE(*,44)
   44 FORMAT(' (N)ORTH OR (S)OUTH              :
(N/S)? ')
      READ(*,45)NS
   45 FORMAT(A1)
      I2    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 47
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 46
      GOTO 43
   46 I2    = -1
   47 CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
      WRITE(11,48)ID,IM,M3,NS
   48 FORMAT(10X,'LATITUDE-1          [STA.1] :
',I4,I3,F9.5,A2)
      LT    = LT1
      WRITE(*,49)
   49 FORMAT(' LONGITUDE-1            [STA.1] :
DDD,MM,SS.SSSSS')
      READ(*,50)ID,IM,M3
   50 FORMAT(I4,I3,F9.4)
   51 WRITE(*,52)
   52 FORMAT(' (E)AST OR (W)EST                :
(E/W)? ')
      READ(*,53)EW
   53 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 55
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 54
      GOTO 51
   54 I2    = -1
   55 CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
      WRITE(11,56)ID,IM,M3,EW
   56 FORMAT(10X,'LONGITUDE           [STA.1] :
',I4,I3,F9.5,A2,/)
      LN     = LN1
      WRITE(*,57)
   57 FORMAT(' TRUE BEARING         [STA.1-2] :
DDD,MM,SS.SSSSSS')
      READ(*,58)ID,IM,M3
   58 FORMAT(I4,I3,F10.6)
C
      WRITE(11,59)ID,IM,M3
   59 FORMAT(10X,'TRUE BEARING     [STA.1-2] :
',I4,I3,F12.6)
      I2    = 1
      CALL DMSRAD(BR1,ID,IM,M3,I2,RD)
      BR     = BR1
      WRITE(*,60)
   60 FORMAT(' TRUE DISTANCE       [STA.1-2] :  XX
XXX XXX.XXXXX')
      READ(*,61)DI
   61 FORMAT(F16.6)
C
      WRITE(11,62)DI
   62 FORMAT(10X,'TRUE DISTANCE    [STA.1-2] :
',F16.5,/)
C
C     -- ENTER INTEGRATION/NUMBER --
C
   63 WRITE(*,64)
   64 FORMAT(' NUMBER OF INTEGRATIONS         : XX
XXX XXX.X')
      READ(*,65)CNT
   65 FORMAT(F12.0)
C
      WRITE(11,66)CNT
   66 FORMAT(10X,'NUMBER OF INTEGRATIONS    :
',F12.1,/)
C
      IF (CNT .EQ. 0.D0) GOTO 63
C
C     -- INTEGRATE DIRECT LONG-LINE PROBLEM --
C
      CALL FRWKIV(C,EC3,LT,LT2,LN,LN2,DI,BR,BR2,
PI4,RD,CNT)
C
      CALL RADDMS(LT2,ID,IM,M3,I2,RD)
C
      IF (I2 .EQ.  1) NS = 'N'
      IF (I2 .EQ. -1) NS = 'S'
      WRITE(11,67)ID,IM,M3,NS
   67 FORMAT(10X,'LATITUDE            [STA-2] =
',I4,I3,F9.5,A2)
C
      CALL RADDMS(LN2,ID,IM,M3,I2,RD)
C
      IF (I2 .EQ.  1) EW = 'E'
      IF (I2 .EQ. -1) EW = 'W'
      WRITE(11,68)ID,IM,M3,EW
   68 FORMAT(10X,'LONGITUDE           [STA-2] =
',I4,I3,F9.5,A2,/)
C
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
C
      WRITE(11,69)ID,IM,M3
   69 FORMAT(10X,'TRUE BEARING     [STA.1-2] =
',I4,I3,F9.5,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 40
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
C
**************************************************************************
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- FORWARD FRWKIV SUBROUTINE KIVIOJA  --
-- SUBROUTINE INTSTA DISPLAY AN INTERMEDIATE STATION --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
**************************************************************************
```

18.12 Inverse Long Line - Kivioja's Method

A_12GBDK.FOR Program - from Geodetic Coordinates to Bearing and Distance - Kivioja's Method

Computing intermediate points in the inverse problem of A_12GBDK.FOR. This computation requires the insertion of lines, such as given in program A_11BDGK.FOR. The intermediate points computed depend on the setting of the integrations (number). The program avoids iteration techniques.

In order to solve the inverse problem it is advisable that the position of point P_1 is situated south of point P_2. Furthermore, the line P_1-P_2 should not run exactly east-west or exactly north-south. In case this may occur during data processing, the computer will issue an error message without further notice!

The FORTRAN-Handbook explains the various error conditions. Some calculations in the Western or Southern Hemisphere can be mirrored and calculated in the N/E Hemisphere [Figure 67: pp 164].

```
*****************************************************************************
*     PROGRAM A_12GBDK.FOR - Date 01-06-2006                                *
*     LONG LINE CALCULATION METHOD KIVIOJA FORTRAN PROGRAM                  *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK                  *
*     AUTHOR M. HOOIJBERG 1999-2006                                         *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1
C
       OPEN(11,FILE= '\FORFILES\GBDK0000.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
-----------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- GBDK          ')
      WRITE(11,3)
    3 FORMAT(10X,'INVERSE LONG-LINE CALCULATION -
METHOD KIVIOJA   ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
-----------------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
      WRITE(*,10)
   10 FORMAT('  INPUT NAME OF ELLIPSOID      : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
      WRITE(*,12)
   12 FORMAT('  INPUT SEMI-MAJOR AXIS   [A] : ')
      READ(*,13)A
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT('  INPUT REC. FLATTENING  [FL] : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
C
C     -- COMPUTE BASIC ELLIPSOID PARAMETERS --
C
      CALL ELDATA(A,C,EC2,EC3,F,FL,PI4,RD)
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
C
   30 FORMAT(10X,'DATE AND TIME        [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
      WRITE(11,31)DATUM
   31 FORMAT(10X,'NAME OF ELLIPSOID              :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS            [A] :
',F22.6)
      WRITE(11,33)FL
   33 FORMAT(10X,'REC. FLATTENING           [FL] :
',F22.14)
      WRITE(11,34)EC2
   34 FORMAT(10X,'SQUARE ECC               [EC2] =
',E22.16)
      WRITE(11,35)EC3
   35 FORMAT(10X,'SQUARE 2ND ECC           [EC3] =
',E22.16)
      WRITE(11,36)C
   36 FORMAT(10X,'C                          [C] =
',F22.12,/)
C
C     -- INPUT GEOGRAPHICALS --
C
   40 CONTINUE
      WRITE(*,41)
   41 FORMAT(' LATITUDE                  [STA.1] :
DD,MM,SS.SSSSS ')
      READ(*,42)ID,IM,M3
   42 FORMAT(I3,I3,F9.5)
C
   43 WRITE(*,44)
   44 FORMAT(' (N)ORTH OR (S)OUTH                 :
(N/S)? ')
      READ(*,45)NS
   45 FORMAT(A1)
      I2    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 47
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 46
```

```
      GOTO 43
   46 I2     = -1
   47 WRITE(11,48)ID,IM,M3,NS
   48 FORMAT(10X,'LATITUDE          [STA.1] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
      B      = LT1
      WRITE(*,49)
   49 FORMAT(' LONGITUDE         [STA.1] :
DDD,MM,SS.SSSSS ')
      READ(*,50)ID,IM,M3
   50 FORMAT(I4,I3,F8.5)
C
   51 WRITE(*,52)
   52 FORMAT(' (E)AST OR (W)EST             :
(E/W)? ')
      READ(*,53)EW
   53 FORMAT(A1)
      I2     = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 55
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 54
      GOTO 51
   54 I2     = -1
   55 WRITE(11,56)ID,IM,M3,EW
   56 FORMAT(10X,'LONGITUDE         [STA.1] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
      CT     = LN1
      WRITE(*,57)
   57 FORMAT(' LATITUDE          [STA.2] :
DD,MM,SS.SSSSS ')
      READ(*,58)ID,IM,M3
   58 FORMAT(I3,I3,F9.4)
C
   59 WRITE(*,60)
   60 FORMAT(' (N)ORTH OR (S)OUTH           :
(N/S)? ')
      READ(*,61)NS
   61 FORMAT(A1)
      I2     = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 63
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 62
      GOTO 59
   62 I2     = -1
   63 WRITE(11,64)ID,IM,M3,NS
   64 FORMAT(10X,'LATITUDE          [STA.2] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LT2,ID,IM,M3,I2,RD)
      D      = LT2
      WRITE(*,65)
   65 FORMAT(' LONGITUDE         [STA.2] :
DDD,MM,SS.SSSSS ')
      READ(*,66)ID,IM,M3
   66 FORMAT(I4,I3,F9.4)
C
   67 WRITE(*,68)
   68 FORMAT(' (E)AST OR (W)EST
(E/W)? ')
      READ(*,69)EW
   69 FORMAT(A1)
      I2     = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 71
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 70
      GOTO 67
   70 I2     = -1
   71 WRITE(11,72)ID,IM,M3,EW
   72 FORMAT(10X,'LONGITUDE         [STA.2] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LN2,ID,IM,M3,I2,RD)
      ET     = LN2
C
C     -- ENTER  INTEGRATION/NUMBER --
   73 WRITE(*,74)
   74 FORMAT(' NUMBER OF INTEGRATIONS       : XX
XXX XXX.X')
      READ(*,75)CNT
   75 FORMAT(F12.0)
      WRITE(11,76)CNT
   76 FORMAT(10X,'NUMBER OF INTEGRATIONS   :
',F12.1,/)
      IF (CNT .EQ. 0.D0) GOTO 73
C
C     -- INTEGRATE INVERSE LONG-LINE PROBLEM --
C
      CALL INVKIV(C,EC2,EC3,B,CT,D,ET,DI,BR1,BR2,
PI4,RD,CNT)
C
      WRITE(11,80)DI
   80 FORMAT(10X,'TRUE DISTANCE         [1-2] =
',F16.6)
C
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
C
      WRITE(11,81)ID,IM,M3
   81 FORMAT(10X,'TRUE BEARING          [1-2] =
',I4,I3,F9.5)
C
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
C
      WRITE(11,82)ID,IM,M3
   82 FORMAT(10X,'TRUE BEARING     [STA.2-1] =
',I4,I3,F9.5,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 40
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
**************************************************************************
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- SUBROUTINE INVKIV - REPEAT INTEGRATION --
-- SUBROUTINE INTSTA DISPLAYING INTERMEDIATE STATIONS --
-- CORRECTION SUBROUTINE S2100 ROUTINE ONE --
-- CORRECTION SUBROUTINE S2200 ROUTINE TWO --
```

```
-- CORRECTION SUBROUTINE S2300 ROUTINE THREE --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
****************************************************************************
```

18.13 Forward Long Line - Vincenty's Method

A_13BDGV.FOR Program - from Bearing and Distance to Geodetic Coordinates - Vincenty 's Method

```
****************************************************************************
*     PROGRAM A_13BDGV.FOR - DATE 01-06-2006                               *
*     FORWARD LONG LINE - METHOD VINCENTY - FORTRAN 77 PROGRAM             *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK                 *
*     AUTHOR M. HOOIJBERG 1999-2006                                        *
****************************************************************************
C     See [On_CD] for Subroutines, and Examples
C
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1
C
       OPEN(11,FILE= '\FORFILES\A13DATA0.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'--------------------------------
----------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- BDGV         ')
      WRITE(11,3)
    3 FORMAT(10X,'FORWARD LONG LINE CALCULATION -
METHOD VINCENTY ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'--------------------------------
----------------',/)
C
      WRITE(*,10)
   10 FORMAT('  INPUT NAME OF ELLIPSOID      : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
      WRITE(*,12)
   12 FORMAT('  INPUT SEMI-MAJOR AXIS    [A] : ')
      READ(*,13)A
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT('  INPUT REC. FLATTENING   [FL] : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
C
C     -- COMPUTE BASIC ELLIPSOID PARAMETERS --
C
      CALL ELDATA(A,B,EC2,F,FL,PI4,RD)
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
C
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME      [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
      WRITE(11,31)DATUM
   31 FORMAT(10X,'NAME OF ELLIPSOID          :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS        [A] :
',F22.6)
      WRITE(11,33)B
   33 FORMAT(10X,'SEMI-MINOR AXIS        [B] :
',F22.6)
      WRITE(11,34)FL
   34 FORMAT(10X,'REC. FLATTENING       [FL] :
',F22.14)
      WRITE(11,35)EC2
   35 FORMAT(10X,'SQUARE 2ND ECC.      [EC2] =
',E22.16)
C
C     -- INPUT GEOGRAPHICALS - BEARING - DISTANCE
--
C
   40 CONTINUE
      WRITE(*,41)
   41 FORMAT(' LATITUDE              [STA.1] :  DD
MM SS.SSSS ')
      READ(*,42)ID,IM,M3
   42 FORMAT(I3,I3,F9.5)
C
   43 WRITE(*,44)
   44 FORMAT(' (N)ORTH OR (S)OUTH              :
(N/S)? ')
      READ(*,45)NS
   45 FORMAT(A1)
      I2     = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 47
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 46
      GOTO 43
   46 I2     = -1
   47 WRITE(11,48)ID,IM,M3,NS
   48 FORMAT(10X,'LATITUDE           [STA.1] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
      WRITE(*,49)
   49 FORMAT(' LONGITUDE             [STA.1] :
DDD,MM,SS.SSSSS ')
      READ(*,50)ID,IM,M3
   50 FORMAT(I4,I3,F9.5)
C
   51 WRITE(*,52)
   52 FORMAT(' (E)AST OR (W)EST                :
(E/W)? ')
```

```
      READ(*,53)EW
   53 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 55
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 54
      GOTO 51
   54 I2    = -1
   55 WRITE(11,56)ID,IM,M3,EW
   56 FORMAT(10X,'LONGITUDE           [STA.1] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
C
      WRITE(*,57)
   57 FORMAT(' TRUE BEARING        [STA.1-2] :
DDD,MM,SS.SSSSS')
      READ(*,58)ID,IM,M3
   58 FORMAT(I4,I3,F9.5)
C
      I2    = 1
      WRITE(11,59)ID,IM,M3
   59 FORMAT(10X,'TRUE BEARING     [STA.1-2] :
',I4,I3,F9.5)
      CALL DMSRAD(BR1,ID,IM,M3,I2,RD)
C
      WRITE(*,60)
   60 FORMAT(' TRUE DISTANCE       [STA.1-2] :  XX
XXX XXX.XXX')
      READ(*,61)DI
   61 FORMAT(F16.6)
C
      WRITE(11,62)DI
   62 FORMAT(10X,'TRUE DISTANCE    [STA.1-2] :
',F16.6,/)
C
C     -- ITERATE DIRECT LONG-LINE PROBLEM --
C
      CALL FRDVIN(B,BR1,BR2,DI,EC2,F,LN1,LN2,LT1,
LT2,PI4)
C
      WRITE(11,70)LT2
   70 FORMAT(10X,'LAT IN RADIANS     [STA.2] =
',F22.16)
      WRITE(11,71)LN2
   71 FORMAT(10X,'LON IN RADIANS     [STA.2] =
',F22.16)
      WRITE(11,72)BR2
   72 FORMAT(10X,'BRG IN RADIANS   [STA.1-2] =
',F22.16,/)
C
      CALL RADDMS(LT2,ID,IM,M3,I2,RD)
C
      IF (I2 .EQ.  1) NS = 'N'
      IF (I2 .EQ. -1) NS = 'S'
      WRITE(11,73)ID,IM,M3,NS
   73 FORMAT(10X,'LATITUDE            [STA.2] =
',I4,I3,F9.5,A2)
C
      CALL RADDMS(LN2,ID,IM,M3,I2,RD)
C
      IF (I2 .EQ.  1) EW = 'E'
      IF (I2 .EQ. -1) EW = 'W'
      WRITE(11,74)ID,IM,M3,EW
   74 FORMAT(10X,'LONGITUDE           [STA.2] =
',I4,I3,F9.5,A2,/)
C
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,75)ID,IM,M3
   75 FORMAT(10X,'TRUE BEARING     [STA.1-2] =
',I4,I3,F9.5,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 40
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
****************************************************************************
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- FORWARD FRDVIN SUBROUTINE - METHOD VINCENTY --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
****************************************************************************
```

18.14 Inverse Long Line - Vincenty's Method

A_14GBDV.FOR Program - from Geodetic Coords to Bearing and Distance - Vincenty 's Method

```
****************************************************************************
*     PROGRAM A_14GBDV.FOR - DATE 01-06-2006                               *
*     INVERSE LONG LINE - METHOD VINCENTY - FORTRAN PROGRAM                *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK                 *
*     AUTHOR M. HOOIJBERG 1999-2006                                        *
*********1*********2*********3*********4*********5*********6*********7**
C
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1
C
```

```
      OPEN(11,FILE= '\FORFILES\A14DATA0.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'--------------------------------
----------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- GBDV          ')
      WRITE(11,3)
    3 FORMAT(10X,'INVERSE LONG LINE CALCULATION -
METHOD VINCENTY ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'--------------------------------
----------------',/)
C
      WRITE(*,10)
   10 FORMAT('  INPUT NAME OF ELLIPSOID      : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
      WRITE(*,12)
   12 FORMAT('  INPUT SEMI-MAJOR AXIS    [A] : ')
      READ(*,13)A
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT('  INPUT REC. FLATTENING   [FL] : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
C
C     -- COMPUTE BASIC ELLIPSOID PARAMETERS --
C
      CALL ELDATA(A,B,EC2,F,FL,PI4,RD)
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME      [HH:MM] =
',I3,I3,I5,' - ',I3,I3,/)
C
      WRITE(11,31)DATUM
   31 FORMAT(10X,'NAME OF ELLIPSOID            :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS         [A] :
',F22.6)
      WRITE(11,33)B
   33 FORMAT(10X,'SEMI-MINOR AXIS         [B] :
',F22.6)
      WRITE(11,34)FL
   34 FORMAT(10X,'REC. FLATTENING        [FL] :
',F22.14)
      WRITE(11,35)EC2
   35 FORMAT(10X,'SQUARE 2ND ECC.       [EC2] =
',E22.16,/)
C
C     -- INPUT GEOGRAPHICALS 1ST STATION --
C
   40 CONTINUE
      WRITE(*,41)
   41 FORMAT(' LATITUDE                        :
DD,MM,SS.SSSS ')
      READ(*,42)ID,IM,M3
   42 FORMAT(I3,I3,F9.5)
C
   43 WRITE(*,44)
   44 FORMAT(' (N)ORTH OR (S)OUTH             :
(N/S)? ')
      READ(*,45)NS
   45 FORMAT(A1)
      I2    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 47
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 46
      GOTO 43
   46 I2    = -1
   47 WRITE(11,48)ID,IM,M3,NS
   48 FORMAT(10X,'LATITUDE            [STA.1] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
      WRITE(*,49)
   49 FORMAT(' LONGITUDE           [STA.1] :
DDD,MM,SS.SSSSS ')
      READ(*,50)ID,IM,M3
   50 FORMAT(I4,I3,F9.5)
C
   51 WRITE(*,52)
   52 FORMAT(' (E)AST OR (W)EST               :
(E/W)? ')
      READ(*,53)EW
   53 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 55
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 54
      GOTO 51
   54 I2    = -1
   55 WRITE(11,56)ID,IM,M3,EW
   56 FORMAT(10X,'LONGITUDE           [STA.1] :
',I4,I3,F9.5,A2,/)
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
C
C     -- INPUT GEOGRAPHICALS - 2ND STATION --
C
      WRITE(*,57)
   57 FORMAT(' LATITUDE            [STA.2] :
DD,MM,SS.SSSSS ')
      READ(*,58)ID,IM,M3
   58 FORMAT(I3,I3,F9.5)
C
   59 WRITE(*,60)
   60 FORMAT(' (N)ORTH OR (S)OUTH             :
(N/S)? ')
      READ(*,61)NS
   61 FORMAT(A1)
      I2    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 63
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 62
      GOTO 59
   62 I2    = -1
   63 WRITE(11,64)ID,IM,M3,NS
   64 FORMAT(10X,'LATITUDE            [STA.2] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LT2,ID,IM,M3,I2,RD)
C
      WRITE(*,65)
   65 FORMAT(' LONGITUDE           [STA.2] :
DDD,MM,SS.SSSSS')
      READ(*,66)ID,IM,M3
   66 FORMAT(I4,I3,F9.5)
C
   67 WRITE(*,68)
   68 FORMAT(' (E)AST OR (W)EST               :
(E/W)? ')
      READ(*,69)EW
```

```
   69 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 71
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 70
      GOTO 67
   70 I2    = -1
   71 WRITE(11,72)ID,IM,M3,EW
   72 FORMAT(10X,'LONGITUDE          [STA.2] :
',I4,I3,F9.5,A2,/)
      CALL DMSRAD(LN2,ID,IM,M3,I2,RD)
C
C     -- ITERATION INVERSE PROBLEM --
C
      CALL INVVIN(B,BR1,BR2,DI,EC2,F,LN1,LN2,LT1,
LT2,PI4)
C
      WRITE(11,73)DI
   73 FORMAT(10X,'TRUE DISTANCE    [STA.1-2] =
',F16.5,/)
C
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
      WRITE(11,74)ID,IM,M3
   74 FORMAT(10X,'TRUE BEARING     [STA.1-2] =
',I4,I3,F9.5)
C
C
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,75)ID,IM,M3
   75 FORMAT(10X,'TRUE BEARING     [STA.2-1] =
',I4,I3,F9.5,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 40
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
*************************************************************************
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- INVERSE INVVIN SUBROUTINE - METHOD VINCENTY --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
*************************************************************************
```

18.15 Polyeder Mapping System

A_15POLY.FOR Program - Polyeder Mapping

```
*************************************************************************
*     PROGRAM A_15POLY.FOR - Date 02-09-2006                            *
*     POLYEDER MAPPING - FORTRAN PROGRAM                                *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK              *
*     AUTHOR M. HOOIJBERG 1998-2006                                     *
C********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1
C
       OPEN(11,FILE= '\FORFILES\POLYDATO.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,'------------------------------
----------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- POLY    ')
      WRITE(11,3)
    3 FORMAT(10X,'POLYEDER MAPPING SYSTEM
')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'------------------------------
----------',/)
C
C     -- BASIC ELLIPSOID DATA --
C
      WRITE(*,10)
   10 FORMAT(' INPUT NAME OF ELLIPSOID       : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
      WRITE(*,12)
   12 FORMAT(' INPUT SEMI-MAJOR AXIS         : ')
      READ(*,13)A
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT(' INPUT RECIPROCAL FLATTENING   : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,20)IYR,IMON,IDAY,IHR,IMIN
```

```
   20 FORMAT(10X,'DATE AND TIME      [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
C
C     -- COMPUTE BASIC ELLIPSOID PARAMETERS --
C
      CALL ELDATA(EC2,EC3,F,FL,PI4,RH,RD,T1)
C
      WRITE(11,21)DATUM
   21 FORMAT(10X,'NAME OF ELLIPSOID           :
',A30)
      WRITE(11,22)A
   22 FORMAT(10X,'SEMI-MAJOR AXIS         [A] :
',F22.8)
      WRITE(11,23)FL
   23 FORMAT(10X,'REC. FLATTENING        [FL] :
',F22.16,/)
      WRITE(11,24)F
   24 FORMAT(10X,'FLATTENING              [F] =
',E22.16)
      WRITE(11,25)EC2
   25 FORMAT(10X,'SQ.1ST ECCENTRICITY [EC2] =
',E22.16)
      WRITE(11,26)EC3
   26 FORMAT(10X,'SQ.2ND ECCENTRICITY [EC3] =
',E22.16)
      WRITE(11,27)RD
   27 FORMAT(10X,'RAD-DMS CONV.          [RD] =
',E22.16)
      WRITE(11,28)T1
   28 FORMAT(10X,'1-F                    [T1] =
',E22.16)
      WRITE(11,29)RH
   29 FORMAT(10X,'RHO                    [RH] =
',F22.12,/)
C
C     -- INPUT LONGITUDE OF REFERENCE MERIDIAN --
C
   40 CONTINUE
      WRITE(*,41)
   41 FORMAT(' LONGITUDE REF.MERIDIAN  [RM] :
DDD,MM,SS.SSSS ')
      READ(*,42)ID,IM,M3
   42 FORMAT(I4,I3,F8.4)
   43 WRITE(*,44)
   44 FORMAT(' (E)AST OR (W)EST              :
(E/W)? ')
      READ(*,45)EW
   45 FORMAT(A1)
C
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 47
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 46
      GOTO 43
   46 I2    = -1
   47 WRITE(11,48)ID,IM,M3,EW
   48 FORMAT(10X,'LONGITUDE             [CM] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LN0,ID,IM,M3,I2,RD)
C
C     -- INPUT GEOGRAPHICALS OF CENTRAL POINT OF
POLYEDER TRAPEZOID --
C
   50 CONTINUE
      WRITE(*,51)
   51 FORMAT(' LATITUDE             [STA.CP] :
DD,MM,SS.SSSS ')
      READ(*,52)ID,IM,M3
   52 FORMAT(I3,I3,F8.4)
   53 WRITE(*,54)
   54 FORMAT(' (N)ORTH OR (S)OUTH            :
(N/S)? ')
      READ(*,55)NS
   55 FORMAT(A1)
C
      I2    = 1
      I3    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 57
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 56
      GOTO 43
   56 I2    = -1
      I3    = -1
   57 WRITE(11,58)ID,IM,M3,NS
   58 FORMAT(10X,'LATITUDE          [STA.CP] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
      WRITE(*,59)
   59 FORMAT(' LONGITUDE            [STA.CP] :
DDD,MM,SS.SSSS ')
      READ(*,60)ID,IM,M3
   60 FORMAT(I4,I3,F8.4)
   61 WRITE(*,62)
   62 FORMAT(' (E)AST OR (W)EST              :
(E/W)? ')
      READ(*,63)EW
   63 FORMAT(A1)
C
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 65
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 64
      GOTO 61
   64 I2    = -1
   65 WRITE(11,66)ID,IM,M3,EW
   66 FORMAT(10X,'LONGITUDE         [STA.CP] :
',I4,I3,F8.4,A2,/)
C
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
      CALL POLCON(A,EC2,LT1,RH,A1,B1,C1,D1)
C
C     -- ENTER X (=EASTING) AND Y (=NORTHING) UP-
PER POINT
C
   80 CONTINUE
      WRITE(*,81)
   81 FORMAT(' INPUT EASTING              XP : ')
      READ(*,82)XP
   82 FORMAT(F22.5)
      WRITE(*,83)
   83 FORMAT(' INPUT NORTHING             YP : ')
      READ(*,84)YP
   84 FORMAT(F22.5)
C
      WRITE(11,85)XP
   85 FORMAT(10X,'EASTING              [XP] :
',F16.5)
      WRITE(11,86)YP
   86 FORMAT(10X,'NORTHING             [YP] :
',F16.5,/)
C
      CALL GEOPOL(I3,LN0,LN1,LN2,LN3,LT1,LT2,LT22,
RH,A1,B1,C1,D1,XP,YP)
C
      CALL RADDMS(LT2,ID,IM,M3,I2,RD)
      NS    = 'N'
      IF (LT2 .LT. 0) NS = 'S'
      WRITE(11,88)ID,IM,M3,NS
```

```
   88 FORMAT(10X,'LAT. OF POLYEDER POINT    =
',I4,I3,F8.4,A2)
C
      CALL RADDMS(LN2,ID,IM,M3,I2,RD)
      EW    = 'E'
      IF (LN2 .LT. 0) EW = 'W'
      WRITE(11,89)ID,IM,M3,EW
   89 FORMAT(10X,'LON. OF POLYEDER POINT    =
',I4,I3,F8.4,A2)
C
      CALL RADDMS(LN3,ID,IM,M3,I2,RD)
      EW    = 'E'
      IF (LN3 .LT. 0) EW = 'W'
      WRITE(11,90)ID,IM,M3,EW
   90 FORMAT(10X,'LON. OF ELLIPSOID P.POINT =
',I4,I3,F8.4,A2,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 80
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********1*********2*********3*********4*********5*********6*********7**
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- SUBROUTINE POLCON BASIC POLYEDER CONSTANTS --
-- SUBROUTINE GEOPOL POLYEDER TO GEOGRAPHICALS --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
*********1*********2*********3*********4*********5*********6*********7**
```

18.16 Gaussian Ellipsoid to Sphere

A_16GAUS.FOR Program - Ellipsoid to Sphere and Vice-versa – Using Gauss-Schreiber Method

```
************************************************************************
*     PROGRAM A_16GAUS.FOR - Date 01-06-2006                           *
*     Conversion Sphere to Ellipsoid and Vice-Versa - FORTRAN PROGRAM  *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1998-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1
C
       OPEN(11,FILE= '\FORFILES\A16GAUSO.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,' ------------------------------
------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- GAUS      ')
      WRITE(11,3)
    3 FORMAT(10X,'SPHERE TO ELLIPSOID AND VICE-
VERSA          ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,' ------------------------------
------------',/)
C
C     -- INPUT BASIC ELLIPSOID DATA --
C
   20 CONTINUE
      WRITE(*,21)
   21 FORMAT('  INPUT NAME OF DATUM          : ')
      READ(*,22) DATUM
   22 FORMAT(A30)
      WRITE(*,23)
   23 FORMAT('  INPUT SEMI-MAJOR AXIS        : ')
      READ(*,24)A
   24 FORMAT(F22.8)
      WRITE(*,25)
   25 FORMAT('  INPUT REC. FLATTENING        : ')
      READ(*,26)FL
   26 FORMAT(F22.16)
C
C     -- CALCULATE ELLIPSOID PARAMETERS --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME      [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
C
      CALL ELDA16(EC2,EC3,F,FL,M,PI4,RD)
C
      WRITE(11,31)DATUM
   31 FORMAT(10X,'NAME OF ELLIPSOID             :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS          [A] :
',F22.6)
      WRITE(11,33)FL
```

```
   33 FORMAT(10X,'REC. FLATTENING      [FL] :
',F22.14)
      WRITE(11,34)EC2
   34 FORMAT(10X,'SQUARE 1ST ECC.     [EC2] =
',E22.16)
      WRITE(11,35)EC3
   35 FORMAT(10X,'SQUARE 2ND ECC.     [EC3] =
',E22.16)
      WRITE(11,36)M
   36 FORMAT(10X,'MODULUS OF LOGARITHM  [M] =
',F22.16,/)
C
      NS    = 'N'
      EW    = 'E'
   50 WRITE(*,51)
   51 FORMAT(10X,'NORTHERN AND EASTERN HEMI-
SPHERE',/)
C
C     -- ELLIPSOID(o) TO SPHERE(o) OR VICE VERSA -
- [o = ORIGIN]
C
  100 CONTINUE
      WRITE(*,101)
  101 FORMAT(' ELLIPSOI(D)o / SPHER(E)o / (S)TOP :
(D/E/S) ? ')
      READ(*,102)VR
  102 FORMAT(A1)
      IF (VR .EQ. 'D' .OR. VR .EQ. 'd') GOTO 150
      IF (VR .EQ. 'E' .OR. VR .EQ. 'e') GOTO 200
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 100
C
C     -- INPUT CO-ORDINATES ELLIPSOID OF ORIGIN --
C
  150 CONTINUE
      WRITE(*,151)
  151 FORMAT(' LATITUDE ELLIPSOID      [Bo] :
DD,MM,SS.SSSSS SSSSS SSSS
     + S ')
      READ(*,152)ID,IM,M3
  152 FORMAT(I3,I3,F14.10)
      I2    = 1
C
      WRITE(11,153)ID,IM,M3,NS
  153 FORMAT(10X,'LATITUDE ELLIPSOID   [Bo] :
',I4,I3,F14.10,A2)
      CALL DMSRAD(LT0,ID,IM,M3,I2,RD)
C
      WRITE(*,154)
  154 FORMAT(' LONGITUDE ELLIPSOID     [Lo] :
DDD,MM,SS.SSSSS SSSSS SSS
     + SS')
      READ(*,155)ID,IM,M3
  155 FORMAT(I4,I3,F14.10)
      I2    = 1
C
      WRITE(11,156)ID,IM,M3,EW
  156 FORMAT(10X,'LONGITUDE ELLIPSOID  [Lo] :
',I4,I3,F14.10,A2)
      CALL DMSRAD(LN0,ID,IM,M3,I2,RD)
C
C     -- CONVERT ELLIPSOID ORIGIN TO SPHERE --
C
      CALL SPDSPE(LT0,LN0,LT1,LN1,EC3,UU,VV,C0,C2,
S0,S2)
C
      CALL RADDMS(LT1,ID,IM,M3,I2,RD)
      WRITE(11,157)ID,IM,M3,NS
  157 FORMAT(10X,'LATITUDE  SPHERE      [bo] =
',I4,I3,F14.10,A2)
C
      CALL RADDMS(LN1,ID,IM,M3,I2,RD)
      WRITE(11,158)ID,IM,M3,EW
  158 FORMAT(10X,'LONGITUDE SPHERE      [lo] =
',I4,I3,F14.10,A2,/)
C
      CALL COMMON(LT0,LT1,A,AL,EC2,EC3,MM0,C0,C2,
S0,S2)
      GOTO 210
C
C     -- INPUT CO-ORDINATES SPHERE OF ORIGIN --
C
  200 CONTINUE
      WRITE(*,201)
  201 FORMAT(' LATITUDE SPHERE          [bo] :
DD,MM,SS.SSSSS SSSSS SSSS
     + S')
      READ(*,202)ID,IM,M3
  202 FORMAT(I3,I3,F14.10)
      I2    = 1
C
      WRITE(11,203)ID,IM,M3,NS
  203 FORMAT(10X,'LATITUDE SPHERE      [bo] :
',I4,I3,F14.10,A2)
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
      WRITE(*,204)
  204 FORMAT(' LONGITUDE SPHERE         [lo] :
DDD,MM,SS.SSSSS SSSSS SSS
     + SS')
      READ(*,205)ID,IM,M3
  205 FORMAT(I4,I3,F14.10)
      I2    = 1
C
      WRITE(11,206)ID,IM,M3,EW
  206 FORMAT(10X,'LONGITUDE SPHERE     [lo] :
',I4,I3,F14.10,A2)
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
C
C     -- CONVERT SPHERE ORIGIN TO ELLIPSOID --
C
      CALL SPESPD(LT0,LN0,LT1,LN1,EC3,U,V)
C
      CALL RADDMS(LT0,ID,IM,M3,I2,RD)
      WRITE(11,207)ID,IM,M3,NS
  207 FORMAT(10X,'LATITUDE  ELLIPSOID  [Bo] =
',I4,I3,F14.10,A2)
C
      CALL RADDMS(LN1,ID,IM,M3,I2,RD)
      WRITE(11,208)ID,IM,M3,EW
  208 FORMAT(10X,'LONGITUDE ELLIPSOID  [Lo] =
',I4,I3,F14.10,A2,/)
C
      CALL COMMON(LT0,LT1,A,AL,EC2,EC3,MM0,C0,C2,
S0,S2)
      CALL SPDSPE(LT0,LN0,LT1,LN1,EC3,UU,VV,C0,C2,
S0,S2)
C
  210 WRITE(*,211)
  211 FORMAT(' ELLIPSOI(D)i / SPHER(E)i / (S)TOP :
(D/E/S) ? ')
      READ(*,212)VR
  212 FORMAT(A1)
      IF (VR .EQ. 'D' .OR. VR .EQ. 'd') GOTO 300
      IF (VR .EQ. 'E' .OR. VR .EQ. 'e') GOTO 250
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
```

```
      GOTO 210
C
C     -- CONVERT SPHERE(i) TO ELLIPSOID(i) COORDI-
NATES -- [i = INDIVIDUAL] --
C
  250 CONTINUE
      WRITE(*,251)
  251 FORMAT(' LATITUDE SPHERE          [bi] :
DD,MM,SS.SSSSS')
      READ(*,252)ID,IM,M3
  252 FORMAT(I3,I3,F9.5)
      I2    = 1
C
      WRITE(11,253)ID,IM,M3,NS
  253 FORMAT(10X,'LATITUDE SPHERE      [bi] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LT3,ID,IM,M3,I2,RD)
C
      WRITE(*,254)
  254 FORMAT(' LONGITUDE SPHERE         [li] :
DD,MM,SS.SSSSS')
      READ(*,255)ID,IM,M3
  255 FORMAT(I3,I3,F9.5)
      I2    = 1
C
      WRITE(11,256)ID,IM,M3,EW
  256 FORMAT(10X,'LONGITUDE SPHERE     [li] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LN3,ID,IM,M3,I2,RD)
C
C     -- CONTINUE CONVERSION SPHERE(i) TO ELLIP-
SOID(i) --
C
      CALL SPHIED(LT0,LT1,LT2,LT3,LN1,LN2,LN3,
EC3,C0, C2,S0,AL)
C
      CALL RADDMS(LT2,ID,IM,M3,I2,RD)
      WRITE(11,257)ID,IM,M3,NS
  257 FORMAT(10X,'LATITUDE ELLIPSOID   [Bi] =
',I4,I3,F9.5,A2)
C
      CALL RADDMS(LN2,ID,IM,M3,I2,RD)
      WRITE(11,258)ID,IM,M3,EW
  258 FORMAT(10X,'LONGITUDE ELLIPSOID  [Li] =
',I4,I3,F9.5,A2,/)
      GOTO 210
C
C     -- CONVERT ELLIPSOID(i) TO SPHERE(i) COORDI-
NATES --
C
  300 CONTINUE
      WRITE(*,301)
  301 FORMAT(' LATITUDE ELLIPSOID       [Bi] :
DD,MM,SS.SSSSS')
      READ(*,302)ID,IM,M3
  302 FORMAT(I3,I3,F9.5)
      I2    = 1
C
      WRITE(11,303)ID,IM,M3,NS
  303 FORMAT(10X,'LATITUDE ELLIPSOID   [Bi] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LT2,ID,IM,M3,I2,RD)
C
      WRITE(*,304)
  304 FORMAT(' LONGITUDE ELLIPSOID      [Li] :
DDD,MM,SS.SSSSS')
      READ(*,305)ID,IM,M3
  305 FORMAT(I4,I3,F9.5)
      I2    = 1
C
      WRITE(11,306)ID,IM,M3,EW
  306 FORMAT(10X,'LONGITUDE ELLIPSOID  [Li] :
',I4,I3,F9.5,A2)
      CALL DMSRAD(LN2,ID,IM,M3,I2,RD)
C
C     -- CONTINUE CONVERSION ELLIPSOID(i) TO
SPHERE(i) --
C
      CALL SPHIDE(LT0,LT2,LT3,LN0,LN2,LN3,EC2,C0,
C2, S0,S2,AL,M,VV,PI4)
C
      CALL RADDMS(LT3,ID,IM,M3,I2,RD)
      WRITE(11,307)ID,IM,M3,NS
  307 FORMAT(10X,'LATITUDE SPHERE      [bi] =
',I4,I3,F9.5,A2)
C
      CALL RADDMS(LN3,ID,IM,M3,I2,RD)
      WRITE(11,308)ID,IM,M3,EW
  308 FORMAT(10X,'LONGITUDE SPHERE     [li] =
',I4,I3,F9.5,A2,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 210
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
*********1*********2*********3*********4*********5*********6*********7**
-- SUBROUTINE ELDA16 ELLIPSOID PARAMETERS --
-- SUBROUTINE SPDSPE CONVERTING ELLIPSOID(o) TO SPHERE(o) --
-- SUBROUTINE SPESPD CONVERTING SPHERE(o) TO ELLIPSOID(o) --
-- SUBROUTINE COMMON CONSTANTS --
-- SUBROUTINE SPHIED ELLIPSOID(i) TO SPHERE(i) --
-- SUBROUTINE SPHIDE SPHERE(i) TO ELLIPSOID(i) --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
*********1*********2*********3*********4*********5*********6*********7**
```

```
*************************************************5*********************
**** - Lat.of Ellipsoid   (Bo) = LT0   Lon.of Ellipsoid   (Lo) = LN0  *
**** - Lat.of Sphere      (bo) = LT1   Lon.of Sphere      (lo) = LN1  *
**** - Lat.of Ellipsoid   (Bi) = LT2   Lon.of Ellipsoid   (Li) = LN2  *
**** - Lat.of Sphere      (bi) = LT3   Lon.of Sphere      (li) = LN3  *
***********************************************************************
```

18.17 Normal Mercator Projection

A_17NM00.FOR Program - Normal Mercator Projection

```
***********************************************************************
*     PROGRAM A_17NM00.FOR - Date 01-06-2006                          *
*     NORMAL MERCATOR PROJECTION - FORTRAN PROGRAM                     *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1996-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1,NI*4
C
       OPEN(11,FILE= '\FORFILES\A17DATA0.TXT',
STATUS='NEW')
C
      WRITE(11,1)
    1 FORMAT(10X,'--------------------------------
---------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- NM00   ')
      WRITE(11,3)
    3 FORMAT(10X,'NORMAL MERCATOR CONFORMAL PRO-
JECTION     ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'--------------------------------
---------',/)
C
C     -- INPUT ELLIPSOID DATA --
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT(' INPUT NAME OF ELLIPSOID      : ')
      READ(*,12) DATUM
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT(' INPUT SEMI-MAJOR AXIS        : ')
      READ(*,14)A
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT(' INPUT REC. FLATTENING        : ')
      READ(*,16)FL
   16 FORMAT(F22.16)
      WRITE(*,17)
   17 FORMAT(' INPUT SCALE FACTOR           : ')
      READ(*,18)K0
   18 FORMAT(F22.16)
      WRITE(*,19)
   19 FORMAT(' INPUT FALSE EASTING          : ')
      READ(*,20)E0
   20 FORMAT(F22.16)
      WRITE(*,21)
   21 FORMAT(' INPUT FALSE NORTHING         : ')
      READ(*,22)N0
   22 FORMAT(F22.16)
C
C     -- NORMAL MERCATOR PROJECTION - PART ONE --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME     [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
C
      CALL ELDATA(EC0,EC2,EC3,F,FL,PI4,RD)
C
      WRITE(11,33)DATUM
   33 FORMAT(10X,'DATUM                     :
',A30)
      WRITE(11,34)A
   34 FORMAT(10X,'SEMI-MAJOR AXIS       [A] :
',F22.6)
      WRITE(11,35)FL
   35 FORMAT(10X,'REC.FLATTENING       [FL] :
',F22.14)
      WRITE(11,36)K0
   36 FORMAT(10X,'SCALE FACTOR         [K0] :
',F22.14)
      WRITE(11,37)E0
   37 FORMAT(10X,'FALSE EASTING        [E0] :
',F22.4)
      WRITE(11,38)N0
   38 FORMAT(10X,'FALSE NORTHING       [N0] :
',F22.4)
      WRITE(11,39)EC0
   39 FORMAT(10X,'1ST ECCENTRICITY    [EC0] =
',E22.16)
      WRITE(11,40)EC2
   40 FORMAT(10X,'SQUARE 1ST ECC      [EC2] =
',E22.16)
      WRITE(11,41)EC3
   41 FORMAT(10X,'SQUARE 2ND ECC      [EC3] =
',E22.16,/)
C
C     -- NORMAL MERCATOR PROJECTION - PART ONE --
C
C     -- CALCULATE ORIGIN GEOGR. - ZONE PARAMETERS
--
C
   50 CONTINUE
      WRITE(*,51)
   51 FORMAT(' LATITUDE GRID REFERENCE      :
DD,MM,SS.SSSS ')
```

```
      READ(*,52)ID,IM,M3
   52 FORMAT(I3,I3,F8.4)
   53 WRITE(*,54)
   54 FORMAT(' (N)ORTH OR (S)OUTH:          (N/S)?
')
      READ(*,55)NS
   55 FORMAT(A1)
      I2    = 1
      I3    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 57
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 56
      GOTO 53
   56 I2    = -1
      I3    = -1
   57 WRITE(11,58)ID,IM,M3,NS
   58 FORMAT(10X,'LATITUDE GRID REFERENCE   :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LR,ID,IM,M3,I2,RD)
C
      WRITE(*,59)
   59 FORMAT(' LON.GRID ORIGIN :
DDD,MM,SS.SSSS ')
      READ(*,60)ID,IM,M3
   60 FORMAT(I4,I3,F8.4)
   61 WRITE(*,62)
   62 FORMAT('  (E)AST OR (W)EST:           (E/W)?
')
      READ(*,63)EW
   63 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 65
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 64
      GOTO 61
   64 I2    = -1
   65 CALL DMSRAD(LNO,ID,IM,M3,I2,RD)
      WRITE(11,66)ID,IM,M3,EW
   66 FORMAT(10X,'LONGITUDE GRID ORIGIN     :
',I4,I3,F8.4,A2)
      WRITE(11,67)LNO
   67 FORMAT(10X,'LON IN RADIANS      [LNO] =
',F22.16)
C
      CALL SCAEQR(KE,KO,EC2,LR)
C
      WRITE(11,68)KE
   68 FORMAT(10X,'SCALE FACTOR AT EQUATOR   =
',F22.16)
      WRITE(11,69)EO
   69 FORMAT(10X,'FALSE EASTING        [EO] :
',F22.4)
      WRITE(11,70)NO
   70 FORMAT(10X,'FALSE NORTHING       [NO] :
',F22.4,/)
C
C     -- NORMAL MERCATOR PROJECTION - PART TWO --
C
      WRITE(11,100)
  100 FORMAT(10X,'--------------------------------
---------------------
     +--------')
      WRITE(11,101)DATUM
  101 FORMAT(10X,'NORMAL MERCATOR PROJECTION
',A20)
      WRITE(11,102)
  102 FORMAT(10X,'--------------------------------
---------------------
     +--------')
  103 CONTINUE
      WRITE(*,104)
  104 FORMAT(' (L)AT/LON - (E)AS/NOR - t-(T) -
(B)RG/DIST - (S)TOP :
     * (L/E/T/B/S) ? ')
      READ(*,105)VR
  105 FORMAT(A1)
C
      IF (VR .EQ. 'L' .OR. VR .EQ. 'l') GOTO 200
      IF (VR .EQ. 'E' .OR. VR .EQ. 'e') GOTO 300
      IF (VR .EQ. 'T' .OR. VR .EQ. 't') GOTO 400
      IF (VR .EQ. 'B' .OR. VR .EQ. 'b') GOTO 500
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 103
C
C     -- COMPUTE FORWARD CONVERSION LAT/LON TO
EAS/NOR --
C
  200 CONTINUE
      WRITE(*,201)
  201 FORMAT(' LATITUDE            [STA.1] :
DD,MM,SS.SSSS ')
      READ(*,202)ID,IM,M3
  202 FORMAT(I3,I3,F8.4)
  203 WRITE(*,204)
  204 FORMAT(' (N)ORTH OR (S)OUTH:          (N/S)?
')
      READ(*,205)NS
  205 FORMAT(A1)
      I2    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 207
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 206
      GOTO 203
  206 I2    = -1
  207 WRITE(11,208)ID,IM,M3,NS
  208 FORMAT(10X,'LATITUDE          [STA.1] :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
      CALL ISOCON(LT1,ECO,QW)
C
      WRITE(11,209)QW
  209 FORMAT(10X,'ISOMETRIC LATITUDE   [QW] :
',E22.16)
      WRITE(*,210)
  210 FORMAT(' LONGITUDE           [STA.1] :
DDD,MM,SS.SSSS ')
      READ(*,211)ID,IM,M3
  211 FORMAT(I4,I3,F8.4)
C
  212 WRITE(*,213)
  213 FORMAT(' (E)AST OR (W)EST:            (E/W)?
')
      READ(*,214)EW
  214 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 216
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 215
      GOTO 212
  215 I2    = -1
  216 WRITE(11,217)ID,IM,M3,EW
  217 FORMAT(10X,'LONGITUDE         [STA.1] :
',I4,I3,F8.4,A2,/)
C
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
      CALL FRDNME(A,LNO,EO,E1,KE,LN1,NO,N1,QW)
C
      I1    = 1
```

```
      IF (E0 .GT. E1)I1 = -1
      WRITE(11,219)E1
  219 FORMAT(10X,'EASTING              [STA.I] =
',F16.5)
      WRITE(11,220)N1
  220 FORMAT(10X,'NORTHING             [STA.I] =
',F16.5,/)
C
      CALL SCAEQR(K1,KE,EC2,LT1)
C
      WRITE(11,221)K1
  221 FORMAT(10X,'SCALE FACTOR         [STA.I] =
',F22.16,/)
C
      GOTO 900
C
C     -- COMPUTE INVERSE CONVERSION EAS/NOR TO
LAT/LON --
C
  300 CONTINUE
      WRITE(*,301)
  301 FORMAT(' INPUT EASTING         [STA.1] : ')
      READ(*,302)E1
  302 FORMAT(F22.5)
      WRITE(*,303)
  303 FORMAT(' INPUT NORTHING        [STA.1] : ')
      READ(*,304)N1
  304 FORMAT(F22.5)
      WRITE(11,305)E1
  305 FORMAT(10X,'EASTING              [STA.1] :
',F16.5)
      WRITE(11,306)N1
  306 FORMAT(10X,'NORTHING             [STA.1] :
',F16.5,/)
C
      CALL INVNME(A,LN0,E0,E1,EC0,KE,LN1,LT1,N0,
N1, PI4)
      CALL RADDMS(LT1,ID,IM,M3,I2,RD)
C
      NS    = 'N'
      IF (I2 .EQ. -1) NS = 'S'
      WRITE(11,309)ID,IM,M3,NS
  309 FORMAT(10X,'LATITUDE             [STA.1] =
',I4,I3,F8.4,A2)
      CALL RADDMS(LN1,ID,IM,M3,I2,RD)
C
      EW    = 'E'
      IF (I2 .EQ. -1) EW = 'W'
      WRITE(11,310)ID,IM,M3,EW
  310 FORMAT(10X,'LONGITUDE            [STA.1] =
',I4,I3,F8.4,A2,/)
C
C     -- CALCULATE LOCAL SCALE FACTOR --
C
      CALL SCAEQR(K1,KE,EC2,LT1)
C
      WRITE(11,311)K1
  311 FORMAT(10X,'SCALE FACTOR AT STA.1      =
',F22.16,/)
      GOTO 900
C
C     -- COMPUTE [t-T"] OR ARC-TO-CHORD CORRECTION
--
C
  400 CONTINUE
      WRITE(*,401)
  401 FORMAT(' LATITUDE              [STA.1] :
DD,MM,SS.SSSS ')
      READ(*,402)ID,IM,M3
  402 FORMAT(I3,I3,F8.4)
C
  403 WRITE(*,404)
  404 FORMAT(' (N)ORTH OR (S)OUTH:             (N/S)?
')
      READ(*,405)NS
  405 FORMAT(A1)
      I2    = 1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 407
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 406
      GOTO 403
  406 I2    = -1
  407 CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
      WRITE(11,408)ID,IM,M3,NS
  408 FORMAT(10X,'LATITUDE             [STA.1] :
',I4,I3,F8.4,A2)
C
      WRITE(*,409)
  409 FORMAT(' LONGITUDE             [STA.1] :
DDD,MM,SS.SSSS ')
      READ(*,410)ID,IM,M3
  410 FORMAT(I4,I3,F8.4)
C
  411 WRITE(*,412)
  412 FORMAT(' (E)AST OR (W)EST:               (E/W)?
')
      READ(*,413)EW
  413 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 415
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 414
      GOTO 411
  414 I2    = -1
  415 WRITE(11,416)ID,IM,M3,EW
  416 FORMAT(10X,'LONGITUDE            [STA.1] :
',I4,I3,F8.4,A2,/)
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
      WRITE(*,417)
  417 FORMAT(' LATITUDE              [STA.2] :
DD,MM,SS.SSSS ')
      READ(*,418)ID,IM,M3
  418 FORMAT(I3,I3,F8.4)
C
  419 WRITE(*,420)
  420 FORMAT(' (N)ORTH OR (S)OUTH:             (N/S)?
')
      READ(*,421)NS
  421 FORMAT(A1)
      I2=1
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 423
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 422
      GOTO 419
  422 I2=-1
  423 CALL DMSRAD(LT2,ID,IM,M3,I2,RD)
      WRITE(11,424)ID,IM,M3,NS
  424 FORMAT(10X,'LATITUDE             [STA.2] =
',I4,I3,F8.4,A2)
C
      WRITE(*,425)
  425 FORMAT(' LONGITUDE             [STA.2] :
DDD,MM,SS.SSSS ')
      READ(*,426)ID,IM,M3
  426 FORMAT(I3,I3,F8.4)
C
  427 WRITE(*,428)
```

```
  428 FORMAT(' (E)AST OR (W)EST:            (E/W)?
')
      READ(*,429)EW
  429 FORMAT(A1)
      I2     = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 431
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 430
      GOTO 427
  430 I2     = -1
  431 CALL DMSRAD(LN2,ID,IM,M3,I2,RD)
      WRITE(11,432)ID,IM,M3,EW
  432 FORMAT(10X,'LONGITUDE          [STA.2] =
',I4,I3,F8.4,A2,/)
C
C     -- CALCULATE ARC-TO-CHORD CORR. FORMULA --
C
      CALL ARCCOR(D12,D21,LNO,LN1,LN2,LT1,LT2)
      CALL RADDMS(D12,ID,IM,M3,I2,RD)
C
      NI     =  '(+)'
      IF (D12 .LT. 0.D0) NI = '(-)'
      WRITE(11,433)NI,ID,IM,M3
  433 FORMAT(10X,'ARC-TO-CHORD     [STA.1-2] =
',A4,I3,I3,F8.4)
C
      CALL RADDMS(D21,ID,IM,M3,I2,RD)
C
      NI     =  '(+)'
      IF (D21 .LT. 0.D0) NI = '(-)'
      WRITE(11,434)NI,ID,IM,M3
  434 FORMAT(10X,'ARC-TO-CHORD     [STA.2-1] =
',A4,I3,I3,F8.4,/)
      GOTO 900
C
C     -- COMPUTE BEARING AND DISTANCE --
C
  500 CONTINUE
      WRITE(*,501)
  501 FORMAT(' INPUT EASTING          [STA.1] : ')
      READ(*,502)E1
  502 FORMAT(F22.5)
      WRITE(*,503)
  503 FORMAT(' INPUT NORTHING         [STA.1] : ')
      READ(*,504)N1
  504 FORMAT(F22.5)
      WRITE(*,505)
  505 FORMAT(' INPUT EASTING          [STA.2] : ')
      READ(*,506)E2
  506 FORMAT(F22.5)
      WRITE(*,507)
  507 FORMAT(' INPUT NORTHING         [STA.2] : ')
      READ(*,508)N2
  508 FORMAT(F22.5)
      WRITE(11,509)E1
  509 FORMAT(10X,'EASTING             [STA.1] :
',F16.5)
      WRITE(11,510)N1
  510 FORMAT(10X,'NORTHING            [STA.1] :
',F16.5)
      WRITE(11,511)E2
  511 FORMAT(10X,'EASTING             [STA.2] :
',F16.5)
      WRITE(11,512)N2
  512 FORMAT(10X,'NORTHING            [STA.2] :
',F16.5,/)
C
      CALL BRGDIS(BR1,BR2,DI,E1,E2,N1,N2,PI4)
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
C
      WRITE(11,513)ID,IM,M3
  513 FORMAT(10X,'BEARING          [STA.1-2] =
',I4,I3,F8.4)
C
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
C
      WRITE(11,514)ID,IM,M3
  514 FORMAT(10X,'BEARING          [STA.2-1] =
',I4,I3,F8.4)
      WRITE(11,515)DI
  515 FORMAT(10X,'DISTANCE         [STA.1-2] :
',F16.5,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 103
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
```

```
*********1*********2*********3*********4*********5*********6*********7**
-- SUBROUTINE ISOCON ISOMETRIC LATITUDE --
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- SUBROUTINE SCAEQR TO COMPUTE SCALE FACTOR --
-- SUBROUTINE FRDNME FORWARD NORMAL MERCATOR --
-- SUBROUTINE INVNME INVERSE NORMAL MERCATOR --
-- SUBROUTINE ARCCOR ARC-TO-CHORD CORRECTION --
-- SUBROUTINE BRGDIS FLAT EARTH - BEARING AND DISTANCE --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
*********1*********2*********3*********4*********5*********6*********7**
```

18.18 Gauss-Krüger Projection

A_18GK00.FOR Program - Gauss-Krüger Projection or Transverse Mercator Projection

The program can also be put to use with the Meridional Arc formulae as shown in [8.4], formulae (8.30 - 8.46). It may speed up the computing time. Recasting the original NGS5 formulae into NGS5 Modified by the author may speed up the computing time. As a result, it is slightly slower than the present algorithm (Hooijberg, 1996, 1997; Vincenty, 1984a).

```
************************************************************************
*     PROGRAM M_18GK00.FOR - Date 19-12-2006                          *
*     GAUSS-KRUEGER CONFORMAL CONVERSION - FORTRAN PROGRAM            *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK            *
*     AUTHOR M. HOOIJBERG 1995-2006                                   *
*********1*********2*********3*********4*********5*********6*********7**
```

```
C     See [On_CD] for Subroutines, and Examples
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1,SGN*4
C
      OPEN(11,FILE= '\FORFILES\A18GDATA.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,'-------------------------------
----------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- GK00    ')
      WRITE(11,3)
    3 FORMAT(10X,'GAUSS-KRUEGER CONFORMAL PROJEC-
TION        ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
----------',/)
C
C     INPUT BASIC ELLIPSOID DATA
C
      WRITE(*,10)
   10 FORMAT(' INPUT REFERENCE ELLIPSOID     : ')
      READ(*,11) DATUM
   11 FORMAT(A30)
      WRITE(*,12)
   12 FORMAT(' INPUT SEMI-MAJOR AXIS     [A] : ')
      READ(*,13)A
   13 FORMAT(F22.8)
      WRITE(*,14)
   14 FORMAT(' INPUT REC.FLATTENING     [FL] : ')
      READ(*,15)FL
   15 FORMAT(F22.16)
      WRITE(*,16)
   16 FORMAT(' INPUT SCALE FACTOR       [K0] : ')
      READ(*,17)K0
   17 FORMAT(F22.16)
      WRITE(*,18)
   18 FORMAT(' INPUT FALSE EASTING      [FE] : ')
      READ(*,19)E0
   19 FORMAT(F22.16)
      WRITE(*,20)
   20 FORMAT(' INPUT FALSE NORTHING     [FN] : ')
      READ(*,21)N0
   21 FORMAT(F22.16)
C
      CALL ELDA18(A,C,EC2,EC3,FL,N,PI4,RD)
C
C     -- BASIC ELLIPSOID PARAMETERS --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
C
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME     [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
C
      WRITE(11,31)DATUM
   31 FORMAT(10X,'DATUM                       :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS        [A] :
',F22.6)
      WRITE(11,33)FL
   33 FORMAT(10X,'REC. FLATTENING       [FL] :
',F22.16)
      WRITE(11,34)K0
   34 FORMAT(10X,'SCALE FACTOR          [K0] :
',F22.14)
      WRITE(11,35)E0
   35 FORMAT(10X,'FALSE EASTING         [E0] :
',F22.4)
      WRITE(11,36)N0
   36 FORMAT(10X,'FALSE NORTHING        [N0] :
',F22.4)
      WRITE(11,37)EC2
   37 FORMAT(10X,'SQ.1ST ECCENTRICITY [EC2] =
',E22.16)
      WRITE(11,38)EC3
   38 FORMAT(10X,'SQ.2ND ECCENTRICITY [EC3] =
',E22.16)
      WRITE(11,39)C
   39 FORMAT(10X,'C                      [C] =
',E22.16)
      WRITE(11,40)N
   40 FORMAT(10X,'N                      [N] =
',E22.16)
      WRITE(11,41)RD
   41 FORMAT(10X,'RAD-DMS CONVERSION    [RD] =
',E22.16,/)
C
      CALL ELLCON(A,N,C,EC3,R0,U0,U2,U4,U6,V0,V2,
V4,V6)
```

```
C
C     -- INPUT CO-ORDINATES OF ORIGIN --
C
   50 WRITE(*,51)
   51 FORMAT('  NORTHERN OR SOUTHERN HEMISPHERE:
(N/S)? ')
      READ(*,52)NS
      I2    = 1
      I3    = 1
   52 FORMAT(A1)
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 54
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 53
      GOTO 50
   53 I3    = -1
      I2    = -1
   54 CONTINUE
   55 WRITE(*,56)
   56 FORMAT(' LATITUDE ORIGIN        [LTO] :
DD,MM,SS.SSSS ')
      READ(*,57)ID,IM,M3
   57 FORMAT(I3,I3,F8.4)
      WRITE(11,58)ID,IM,M3,NS
   58 FORMAT(10X,'LATITUDE ORIGIN     [LTO] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LTO,ID,IM,M3,I2,RD)
      WRITE(*,59)
   59 FORMAT(' LONGITUDE               [CM] :
DD,MM,SS.SSSS ')
      READ(*,60)ID,IM,M3
   60 FORMAT(I4,I3,F8.4)
   61 WRITE(*,62)
   62 FORMAT(' (E)AST OR (W)EST       (E/W) :
(E/W)? ')
      READ(*,63)EW
   63 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 65
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 64
      GOTO 61
   64 I2    = -1
   65 WRITE(11,66)ID,IM,M3,EW
   66 FORMAT(10X,'LON.CENTRAL MERIDIAN [CM] :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LNO,ID,IM,M3,I2,RD)
      CALL MERFWD(LTO,CL1,CL2,SL1,SL2,RO,U0,U2,U4,
U6,KO,SO)
C
      WRITE(11,67)
   67 FORMAT(10X,'--------------------------------
---------------------
     +--------')
      WRITE(11,68)DATUM
   68 FORMAT(10X,'GAUSS-KRUEGER PROJECTION
 CONVERSION - ',A30)
      WRITE(11,69)
   69 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,70)
   70 FORMAT(10X,'--------------------------------
---------------------
     +--------')
C
   80 CONTINUE
      WRITE(*,81)
   81 FORMAT(' (L)AT/LON - (E)AS/NOR - t-(T) -
(B)RG/DIS - (S)TOP
     + (L/E/T/B/S) ? ')
      READ(*,82)VR
   82 FORMAT(A1)
      IF (VR .EQ. 'L' .OR. VR .EQ. 'l') GOTO 100
      IF (VR .EQ. 'E' .OR. VR .EQ. 'e') GOTO 200
      IF (VR .EQ. 'T' .OR. VR .EQ. 't') GOTO 300
      IF (VR .EQ. 'B' .OR. VR .EQ. 'b') GOTO 400
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 80
C
C     -- FORWARD GAUSS-KRUEGER CONVERSION -
LAT/LON TO EAS/NOR --
C
  100 CONTINUE
      I2 = I3
      WRITE(*,101)
  101 FORMAT(' LATITUDE                [STA.i] :
DD,MM,SS.SSSS ')
      READ(*,102)ID,IM,M3
  102 FORMAT(I3,I3,F8.4)
      WRITE(11,103)ID,IM,M3,NS
  103 FORMAT(10X,'LATITUDE            [STA.i] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LTI,ID,IM,M3,I2,RD)
C
      LTI   = DABS(LTI)
      WRITE(*,104)
  104 FORMAT(' LONGITUDE               [STA.i] :  DDD
MM SS.SSSS ')
      READ(*,105)ID,IM,M3
  105 FORMAT(I4,I3,F8.4)
  106 WRITE(*,107)
  107 FORMAT(' (E)AST OR (W)EST                  :
(E/W)? ')
      READ(*,108)EW
  108 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 110
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 109
      GOTO 106
  109 I2    = -1
  110 WRITE(11,111)ID,IM,M3,EW
  111 FORMAT(10X,'LONGITUDE           [STA.i] :
',I4,I3,F8.4,A2,/)
      CALL DMSRAD(LNI,ID,IM,M3,I2,RD)
C
C     -- FORWARD MERIDIAN ARC --
C
      CALL MERFWD(LTI,CL1,CL2,SL1,SL2,RO,U0,U2,U4,
U6,KO,S)
      CALL GKRFWD(A,KO,EO,NO,S,SO,EC2,EC3,LNI,LNO,
LTI,CL2, SL2,EI,NI,CVG,
     + KI,I3)
C
      I1    = 1
      IF (EO .GT. EI)I1 = -1
      WRITE(11,112)EI
  112 FORMAT(10X,'EASTING             [STA.i] =
',F16.4)
      WRITE(11,113)NI
  113 FORMAT(10X,'NORTHING            [STA.i] =
',F16.4,/)
C
C     -- CONVERGENCE AND SCALE FACTOR --
C
      CALL RADDMS(CVG,ID,IM,M3,I2,RD)
      SGN   = '(+)'
      IF ((I1*I3) .EQ. -1) SGN = '(-)'
```

```
      WRITE(11,114)SGN,ID,IM,M3
  114 FORMAT(10X,'CONVERGENCE       [STA.i] =
',A4,I4,I3,F8.4)
      WRITE(11,115)KI
  115 FORMAT(10X,'SCALE FACTOR      [STA.i] =
',F16.12,/)
      GOTO 80
C
C     -- INVERSE CONVERSION  EAS/NOR TO LAT/LON -
CONVERGENCE - SCALE FACTOR --
C
  200 CONTINUE
      WRITE(*,220)
  220 FORMAT(' INPUT EASTING        [STA.i] : ')
      READ(*,221)EI
  221 FORMAT(F22.16)
      WRITE(*,222)
  222 FORMAT(' INPUT NORTHING       [STA.i] : ')
      READ(*,223)NI
  223 FORMAT(F22.16)
      WRITE(11,224)EI
  224 FORMAT(10X,'EASTING           [STA.i] :
',F16.4)
      WRITE(11,225)NI
  225 FORMAT(10X,'NORTHING          [STA.i] :
',F16.4,/)
C
C     -- INVERSE MERIDIONAL ARC --
C
      CALL MERINV(LTF,ARC,V0,V2,V4,V6,I3,K0,NI,
N0,S0,R0)
C
      WRITE(11,226)ARC
  226 FORMAT(10X,'                   [ARC3] :
',F22.12)
      WRITE(11,227)LTF
  227 FORMAT(10X,'                    [LT3] :
',F22.12,/)
C
C     -- INVERSE GAUSS-KRUEGER --
C
      CALL GKRINV(A,K0,E0,EI,EC2,EC3,I1,KI,CVG,
LN0,LNI,LTI,LTF)
      CALL RADDMS(LTI,ID,IM,M3,I2,RD)
C
      WRITE(11,228)ID,IM,M3,NS
  228 FORMAT(10X,'LATITUDE          [STA.i] =
',I4,I3,F8.4,A2)
      CALL RADDMS(LNI,ID,IM,M3,I2,RD)
      WRITE(11,229)ID,IM,M3,EW
  229 FORMAT(10X,'LONGITUDE         [STA.i] =
',I4,I3,F8.4,A2,/)
C
C     -- CONVERGENCE AND SCALE FACTOR --
C
      CALL RADDMS(CVG,ID,IM,M3,I2,RD)
      SGN   = '(+)'
      IF ((I1*I3) .EQ. -1) SGN = '(-)'
      WRITE(11,230)SGN,ID,IM,M3
  230 FORMAT(10X,'CONVERGENCE       [STA.i] =
',A4,I4,I3,F8.4)
      WRITE(11,231)KI
  231 FORMAT(10X,'SCALE FACTOR      [STA.i] =
',F16.12,/)
      GOTO 80
C
C     -- COMPUTE ARC-TO-CHORD CORRECTION --
C
```

```
  300 CONTINUE
      WRITE(*,301)
  301 FORMAT('  INPUT EASTING       [STA.1] : ')
      READ(*,302)E1
  302 FORMAT(F22.16)
      WRITE(*,303)
  303 FORMAT('  INPUT NORTHING      [STA.1] : ')
      READ(*,304)N1
  304 FORMAT(F22.16)
      WRITE(*,305)
  305 FORMAT('  INPUT EASTING       [STA.2] : ')
      READ(*,306)E2
  306 FORMAT(F22.16)
      WRITE(*,307)
  307 FORMAT('  INPUT NORTHING      [STA.2] : ')
      READ(*,308)N2
  308 FORMAT(F22.16)
      WRITE(11,309)E1
  309 FORMAT(10X,'EASTING           [STA.1] :
',F16.4)
      WRITE(11,310)N1
  310 FORMAT(10X,'NORTHING          [STA.1] :
',F16.4,/)
      WRITE(11,311)E2
  311 FORMAT(10X,'EASTING           [STA.2] :
',F16.4)
      WRITE(11,312)N2
  312 FORMAT(10X,'NORTHING          [STA.2] :
',F16.4,/)
C
      CALL ATCRAD(A,K0,N,EC2,EC3,D12,D21,E0,E1,E2,
N0,N1,N2,S0,R0)
      CALL RADDMS(D12,ID,IM,M3,I2,RD)
C
      SGN   = '(+)'
      IF (D12 .LT. 0.D0) SGN = '(-)'
      WRITE(11,313)SGN,ID,IM,M3
  313 FORMAT(10X,'ARC-TO-CHORD    [STA.1-2] =
',A4,I3,I3,F8.4)
      CALL RADDMS(D21,ID,IM,M3,I2,RD)
      SGN   = '(+)'
      IF (D21 .LT. 0.D0) SGN = '(-)'
      WRITE(11,314)SGN,ID,IM,M3
  314 FORMAT(10X,'ARC-TO-CHORD    [STA.2-1] =
',A4,I3,I3,F8.4,/)
      WRITE(11,714)D12
  714 FORMAT(10X,'ARC-TO-CHORD          D12] =
',F22.16)
      WRITE(11,715)D21
  715 FORMAT(10X,'ARC-TO-CHORD          D21] =
',F22.16,/)
      GOTO 80
C
C     -- COMPUTE BEARING AND DISTANCE - FLAT EARTH
--
C
  400 CONTINUE
      WRITE(*,401)
  401 FORMAT(' INPUT EASTING        [STA.1] : ')
      READ(*,402)E1
  402 FORMAT(F22.16)
      WRITE(*,403)
  403 FORMAT(' INPUT NORTHING       [STA.1] : ')
      READ(*,404)N1
  404 FORMAT(F22.16)
      WRITE(*,405)
  405 FORMAT(' INPUT EASTING        [STA.2] : ')
      READ(*,406)E2
```

```
  406 FORMAT(F22.16)
      WRITE(*,407)
  407 FORMAT(' INPUT NORTHING           [STA.2] : ')
      READ(*,408)N2
  408 FORMAT(F22.16)
      WRITE(11,409)E1
  409 FORMAT(10X,'EASTING               [STA.1] :
',F16.4)
      WRITE(11,410)N1
  410 FORMAT(10X,'NORTHING              [STA.1] :
',F16.4)
      WRITE(11,411)E2
  411 FORMAT(10X,'EASTING               [STA.2] :
',F16.4)
      WRITE(11,412)N2
  412 FORMAT(10X,'NORTHING              [STA.2] :
',F16.4,/)
C
      CALL BRGDIS(E1,E2,N1,N2,DI,BR1,BR2,PI4)
C
      WRITE(11,420)DI
  420 FORMAT(10X,'GRID DISTANCE     [STA.1-2] =
',F16.4)
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
      WRITE(11,421)ID,IM,M3
  421 FORMAT(10X,'GRID BEARING      [STA.1-2] =
',I4,I3,F8.4)
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,422)ID,IM,M3
  422 FORMAT(10X,'GRID BEARING      [STA.2-1] =
',I4,I3,F8.4,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP (C/S)
? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c')GOTO 80
      IF (VR .EQ. 'S' .OR. VR .EQ. 's')GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
```

```
******************************************************************************
-- SUBROUTINE ELDA18 ELLIPSOID PARAMETERS --
-- SUBROUTINE ELLCON ELLIPSOID CONSTANTS - VERSION 'KRACK 82' [3] --
-- SUBROUTINE MERFWD - VERSION 'KRACK 82' FORWARD --
-- SUBROUTINE GKRFWD GAUSS-KRUEGER - VERSION 'KRACK 82' FORWARD [3] --
-- SUBROUTINE MERINV - VERSION 'KRACK 82' INVERSE --
-- SUBROUTINE GKRINV GAUSS-KRUEGER - VERSION 'KRACK 82' INVERSE [3] --
-- SUBROUTINE ATCRAD TO CALCULATE ARC-TO-CHORD VALUES --
-- SUBROUTINE BRGDIS TO CALCULATE BRG AND DISTANCE - FLAT EARTH --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
******************************************************************************
```

18.19 Lambert's Conical Conformal Projection

A_19LC00.FOR Program - Lambert Conical Conformal Projection

Using the algorithm as given in the calculation of isometric latitude for the Oblique Mercator program A_20OM00.FOR, the latitude can be obtained with or without iteration as shown.

```
******************************************************************************
*     PROGRAM A_19LC00.FOR   DATE 02-09-2006                                 *
*     LAMBERT CONICAL CONFORMAL CONVERSION - FORTRAN PROGRAM                 *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK                   *
*     AUTHOR M. HOOIJBERG 1995-2006                                          *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1,SGN*4
C
       OPEN(11,FILE= '\FORFILES\LCDATA00.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,'------------------------------
----------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- LC00    ')
      WRITE(11,3)
    3 FORMAT(10X,'LAMBERT CONICAL CONFORMAL PRO-
JECTION      ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'------------------------------
----------',/)
C
C     -- BASIC ELLIPSOID DATA --
```

```
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT(' INPUT REFERENCE ELLIPSOID    : ')
      READ(*,12) DATUM
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT(' INPUT SEMI-MAJOR AXIS     [A] : ')
      READ(*,14)A
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT(' INPUT REC. FLATTENING   [FL] : ')
      READ(*,16)FL
   16 FORMAT(F22.16)
      WRITE(*,17)
   17 FORMAT(' INPUT FALSE EASTING     [FE] : ')
      READ(*,18)E0
   18 FORMAT(F22.4)
      WRITE(*,19)
   19 FORMAT(' INPUT FALSE NORTHING    [FN] : ')
      READ(*,20)N0
   20 FORMAT(F22.4)
C
C     -- LISTING ELLIPSOID CONSTANTS 1 --
C
   30 CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,31)IYR,IMON,IDAY,IHR,IMIN
   31 FORMAT(10X,'DATE AND TIME      [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
C
      CALL ELDA19(EC0,EC2,EC3,FL,N,PI4,RD)
C
      WRITE(11,32)DATUM
   32 FORMAT(10X,'DATUM OF ELLIPSOID          :
',A30)
      WRITE(11,33)A
   33 FORMAT(10X,'SEMI-MAJOR AXIS         [A] :
',F22.6)
      WRITE(11,34)FL
   34 FORMAT(10X,'REC.FLATTENING         [FL] :
',F22.14)
      WRITE(11,35)E0
   35 FORMAT(10X,'FALSE EASTING          [FE] :
',F22.4)
      WRITE(11,36)N0
   36 FORMAT(10X,'FALSE NORTHING         [FN] :
',F22.4)
      WRITE(11,37)EC0
   37 FORMAT(10X,'1ST ECCENTRICITY     [EC0] =
',E22.16)
      WRITE(11,38)EC2
   38 FORMAT(10X,'SQUARE 1ST ECC       [EC2] =
',E22.16)
      WRITE(11,39)EC3
   39 FORMAT(10X,'SQUARE 2ND ECC       [EC3] =
',E22.16)
      WRITE(11,40)N
   40 FORMAT(10X,'N                      [N] =
',E22.16,/)
C
      CALL ISOCON(EC2,F0,F2,F4,F6,F8)
C
C     -- LISTING ELLIPSOID CONSTANTS 2 --
C
C     -- LAMBERT CONICAL CONFORMAL PROJECTION -
PART ONE --
C     -- ENTER CO-ORDINATES OF ORIGIN --
C
  100 WRITE(*,101)
  101 FORMAT('  (N)ORTHERN OR (S)OUTHERN HEMI-
SPHERE: (N/S)? ')
      READ(*,102)NS
      I3    = 1
  102 FORMAT(A1)
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 104
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 103
      GOTO 100
  103 I3    = -1
  104 CONTINUE
C
C     -- INPUT ORIGIN CO-ORDINATES FOR ONE OR TWO
PARALLELS --
C
  105 WRITE(*,106)
  106 FORMAT(' (O)NE OR (T)WO STANDARD PARALLELS:
(O/T)? ')
      READ(*,107)VR
  107 FORMAT(A1)
      IF (VR .EQ. 'O' .OR. VR .EQ. 'o') GOTO 110
      IF (VR .EQ. 'T' .OR. VR .EQ. 't') GOTO 130
      GOTO 105
C
C     -- ZONE PARAMETERS FOR ONE STANDARD PARALLEL
--
C
  110 CONTINUE
      WRITE(*,111)
  111 FORMAT(' LAT. STANDARD PAR.      [LTP] :
DD,MM,SS.SSSS ')
      READ(*,112)ID,IM,M3
  112 FORMAT(I3,I3,F8.4)
      WRITE(11,113)ID,IM,M3,NS
  113 FORMAT(10X,'LAT. STANDARD PAR.  [LTP] :
',I4,I3,F8.4,A2)
      I2    = 1
C
      CALL DMSRAD(LTP,ID,IM,M3,I2,RD)
C
      WRITE(*,114)
  114 FORMAT(' LONGITUDE GRID ORIGIN  [LNC] :
DDD,MM,SS.SSSS ')
      READ(*,115)ID,IM,M3
  115 FORMAT(I4,I3,F8.4)
C
  116 WRITE(*,117)
  117 FORMAT(' (E)AST OR (W)EST                :
(E/W)? ')
      READ(*,118)EW
  118 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 120
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 119
      GOTO 116
  119 I2    = -1
  120 WRITE(11,121)ID,IM,M3,EW
  121 FORMAT(10X,'LON. GRID ORIGIN    [LNC] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LNC,ID,IM,M3,I2,RD)
C
      WRITE(*,122)
  122 FORMAT(' INPUT SCALE FACTOR      [K0] : ')
      READ(*,123)K0
  123 FORMAT(F14.12)
      WRITE(11,124)K0
```

```
  124 FORMAT(10X,'SCALE FACTOR         [KO] =
',F16.12)
C
      CALL LCOFWD(A,ECO,EC2,K,KO,LTP,NC,QC,WC,RO,
RB,RC,RP,SS,NO,I3)
C
      WRITE(11,125)NC
  125 FORMAT(10X,'NORTHING ORIGIN      [NC] =
',F22.4,/)
      GOTO 160
C
C     -- ZONE PARAMETERS FOR TWO STANDARD PARAL-
LELS --
C
  130 CONTINUE
      WRITE(*,131)
  131 FORMAT(' LATITUDE LOWER PAR.    [LTL] :
DD,MM,SS.SSSS ')
      READ(*,132)ID,IM,M3
  132 FORMAT(I3,I3,F8.4)
      WRITE(11,133)ID,IM,M3,NS
  133 FORMAT(10X,'LATITUDE LOWER PAR. [LTL] :
',I4,I3,F8.4,A2)
      I2    = 1
      CALL DMSRAD(LTL,ID,IM,M3,I2,RD)
C
      WRITE(*,134)
  134 FORMAT(' LATITUDE UPPER PAR.    [LTU] :
DD,MM,SS.SSSS ')
      READ(*,135)ID,IM,M3
  135 FORMAT(I3,I3,F8.4)
      WRITE(11,136)ID,IM,M3,NS
  136 FORMAT(10X,'LATITUDE UPPER PAR. [LTU] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LTU,ID,IM,M3,I2,RD)
C
      WRITE(*,137)
  137 FORMAT(' LATITUDE GRID ORIGIN   [LTC] :
DD,MM,SS.SSSS ')
      READ(*,138)ID,IM,M3
  138 FORMAT(I3,I3,F8.4)
      WRITE(11,139)ID,IM,M3,NS
  139 FORMAT(10X,'LATITUDE GRID ORIG. [LTC] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LTC,ID,IM,M3,I2,RD)
      WRITE(*,140)
  140 FORMAT(' LONGITUDE GRID ORIGIN  [LNC] :
DDD,MM,SS.SSSS ')
      READ(*,141)ID,IM,M3
  141 FORMAT(I4,I3,F8.4)
C
  142 WRITE(*,143)
  143 FORMAT(' (E)AST OR (W)EST               :
(E/W)? ')
      READ(*,144)EW
  144 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 146
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 145
      GOTO 142
  145 I2    = -1
  146 WRITE(11,147)ID,IM,M3,EW
  147 FORMAT(10X,'LON. GRID ORIGIN    [LNC] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LNC,ID,IM,M3,I2,RD)
C
      CALL LCTFWD(A,LTL,LTU,LTC,ECO,EC2,K,KO,QO,
QC,QL,QU,WC,WL,WU,RO,
     + RB,RC,RP,LTP,SS,NO,NC,I3)
C
      CALL RADDMS(LTP,ID,IM,M3,I2,RD)
      WRITE(11,148)ID,IM,M3,NS
  148 FORMAT(10X,'LATITUDE CENTR.PAR. [LTP] =
',I4,I3,F8.4,A2)
      WRITE(11,149)KO
  149 FORMAT(10X,'SCALE FACTOR         [KO] =
',F16.12)
      WRITE(11,150)NC
  150 FORMAT(10X,'NORTHING ORIGIN [NC]      =
',F22.4,/)
C
C     -- LAMBERT CONICAL CONFORMAL PROJECTION -
PART TWO --
C
  160 CONTINUE
      WRITE(11,161)
  161 FORMAT(10X,'-------------------------------
---------------------
     +-----------')
      WRITE(11,162)DATUM
  162 FORMAT(10X,'LAMBERT CONFORMAL CONIC PROJEC-
TION - ',A30)
      WRITE(11,163)
  163 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,164)
  164 FORMAT(10X,'-------------------------------
---------------------
     +-----------')
C
  170 CONTINUE
      WRITE(*,171)
  171 FORMAT(' (L)AT/LON - (E)AS/NOR - t-(T) -
(B)RG/DIS - (S)TOP:
     + (L/E/T/B/S) ? ')
      READ(*,172)VR
  172 FORMAT(A1)
      IF (VR .EQ. 'L' .OR. VR .EQ. 'l') GOTO 200
      IF (VR .EQ. 'E' .OR. VR .EQ. 'e') GOTO 300
      IF (VR .EQ. 'T' .OR. VR .EQ. 't') GOTO 400
      IF (VR .EQ. 'B' .OR. VR .EQ. 'b') GOTO 500
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 170
C
C     -- FORWARD LCC CONVERSION : LAT/LON TO
EAS/NOR --
C
  200 CONTINUE
      WRITE(*,201)
  201 FORMAT(' LATITUDE              [STA.i] :
DD,MM,SS.SSSS ')
      READ(*,202)ID,IM,M3
  202 FORMAT(I3,I3,F8.4)
      I2    = 1
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
C
      WRITE(11,203)ID,IM,M3,NS
  203 FORMAT(10X,'LATITUDE           [STA.i] :
',I4,I3,F8.4,A2)
C
      CALL ISORAD(QI,WI,LT1,ECO,EC2)
C
      WRITE(*,204)
  204 FORMAT(' LONGITUDE             [STA.i] :
DDD,MM,SS.SSSS ')
      READ(*,205)ID,IM,M3
```

```
  205 FORMAT(I4,I3,F8.4)
C
  206 WRITE(*,207)
  207 FORMAT(' (E)AST OR (W)EST                :
(E/W)? ')
      READ(*,208)EW
  208 FORMAT(A1)
      I2     = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 210
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 209
      GOTO 206
  209 I2     = -1
  210 WRITE(11,211)ID,IM,M3,EW
  211 FORMAT(10X,'LONGITUDE          [STA.i] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
C
      CALL LCCFWD(A,CVG,ECO,EC2,LT1,LN1,LNC,K,KI,
E0, EI,NI,RC,QI,WI,SS)
      CALL ISORAD(QI,WI,LT1,ECO,EC2)
C
      NI     = NI*I3
      WRITE(11,212)EI
  212 FORMAT(10X,'EASTING            [STA.i] =
',F16.4)
      WRITE(11,213)NI
  213 FORMAT(10X,'NORTHING           [STA.i] =
',F16.4,/)
C
C     -- CONVERGENCE AND SCALE FACTOR --
C
      I1     = 1
      IF ((EI-E0) .LT. 0.D0) I1 = -1
      SGN    = '(+)'
      IF ((I1*I3) .EQ. -1) SGN = '(-)'
C
      CALL RADDMS(CVG,ID,IM,M3,I2,RD)
      WRITE(11,214)SGN,ID,IM,M3
  214 FORMAT(10X,'CONVERGENCE        [STA.i] =
',A4,I4,I3,F8.4)
      WRITE(11,215)KI
  215 FORMAT(10X,'SCALE FACTOR       [STA.i] =
',F16.12,/)
      GOTO 170
C
C     -- INVERSE CONVERSION : EAS/NOR TO LAT/LON -
-
C
  300 CONTINUE
      WRITE(*,301)
  301 FORMAT(' INPUT EASTING         [STA.i] :
XXX XXX.XXXX')
      READ(*,302)EI
  302 FORMAT(F16.4)
      WRITE(*,303)
  303 FORMAT(' INPUT NORTHING        [STA.i] : X
XXX XXX.XXXX')
      READ(*,304)NI
  304 FORMAT(F16.4)
C
      WRITE(11,305)EI
  305 FORMAT(10X,'EASTING            [STA.i] :
',F14.4)
      WRITE(11,306)NI
  306 FORMAT(10X,'NORTHING           [STA.i] :
',F14.4,/)
C
C     -- IF SOUTH --
      NI     = NI*I3
C
      CALL LCCINV(A,CVG,ECO,EC2,LTI,LNI,LNC,K,KI,
E0, EI,DE,F0,F2,F4,F6,
     + F8,NI,RC,WK,QD,SS)
C
      CALL RADDMS(LTI,ID,IM,M3,I2,RD)
      WRITE(11,307)ID,IM,M3,NS
  307 FORMAT(10X,'LATITUDE           [STA.I] =
',I4,I3,F8.4,A2)

      CALL RADDMS(LNI,ID,IM,M3,I2,RD)
      EW     = 'E'
      IF (I2 .EQ. -1) EW = 'W'
      WRITE(11,308)ID,IM,M3,EW
  308 FORMAT(10X,'LONGITUDE          [STA.i] =
',I4,I3,F8.4,A2,/)
      I1     = 1
      IF (DE .LT. 0.D0) I1 = -1
      SGN    = '(+)'
      IF ((I1*I3) .EQ. -1) SGN = '(-)'
C
      CALL RADDMS(CVG,ID,IM,M3,I2,RD)
      WRITE(11,309)SGN,ID,IM,M3
  309 FORMAT(10X,'CONVERGENCE        [STA.i] =
',A4,I4,I3,F8.4)
      WRITE(11,310)KI
  310 FORMAT(10X,'SCALE FACTOR       [STA.i] =
',F16.12,/)
      GOTO 170
C
C     -- COMPUTE ARC-TO-CHORD OR (t-T)" CORRECTION
--
C
  400 CONTINUE
      WRITE(*,401)
  401 FORMAT(' INPUT EASTING         [STA.1] :
XXX XXX.XXXX')
      READ(*,402)E1
  402 FORMAT(F16.4)
      WRITE(*,403)
  403 FORMAT(' INPUT NORTHING        [STA.1] : X
XXX XXX.XXXX')
      READ(*,404)N1
  404 FORMAT(F16.4)
      WRITE(*,405)
  405 FORMAT(' INPUT EASTING         [STA.2] :
XXX XXX.XXXX')
      READ(*,406)E2
  406 FORMAT(F16.4)
      WRITE(*,407)
  407 FORMAT(' INPUT NORTHING        [STA.2] : X
XXX XXX.XXXX')
      READ(*,408)N2
  408 FORMAT(F16.4)
C
      WRITE(11,409)E1
  409 FORMAT(10X,'EASTING            [STA.1] :
',F16.4)
      WRITE(11,410)N1
  410 FORMAT(10X,'NORTHING           [STA.1] :
',F16.4,/)
      WRITE(11,411)E2
  411 FORMAT(10X,'EASTING            [STA.2] :
',F16.4)
      WRITE(11,412)N2
  412 FORMAT(10X,'NORTHING           [STA.2] :
',F16.4,/)
```

```
C
      CALL ATCCOR(LTP,E0,E1,E2,D12,D21,DN,N1,N2,
NC,R0,RC,RP,SS)
C
      WRITE(11,413)NC
  413 FORMAT(10X,'NORTHING OF ORIGIN [NC]    :
',F14.4,/)
C
      SGN   = '(+)'
      IF (D12 .LT. 0.D0) SGN = '(-)'
C
      CALL RADDMS(D12,ID,IM,M3,I2,RD)
      WRITE(11,414)SGN,ID,IM,M3
  414 FORMAT(10X,'ARC-TO-CHORD     [STA.1-2] =
',A4,I3,I3,F8.4)
      SGN   = '(+)'
      IF (D21 .LT. 0.D0) SGN = '(-)'
C
      CALL RADDMS(D21,ID,IM,M3,I2,RD)
      WRITE(11,415)SGN,ID,IM,M3
  415 FORMAT(10X,'ARC-TO-CHORD     [STA.2-1] =
',A4,I3,I3,F8.4,/)
      GOTO 170
C
C     -- BEARING AND DISTANCE - FLAT EARTH --
C
  500 CONTINUE
      WRITE(*,501)
  501 FORMAT(' INPUT EASTING           [STA.1] :
XXX XXX.XXXX')
      READ(*,502)E1
  502 FORMAT(F16.4)
      WRITE(*,503)
  503 FORMAT(' INPUT NORTHING          [STA.1] : X
XXX XXX.XXXX')
      READ(*,504)N1
  504 FORMAT(F16.4)
      WRITE(*,505)
  505 FORMAT(' INPUT EASTING           [STA.2] :
XXX XXX. XXXX')
      READ(*,506)E2
  506 FORMAT(F16.4)
      WRITE(*,507)
  507 FORMAT(' INPUT NORTHING          [STA.2] : X
XXX XXX.XXXX')
      READ(*,508)N2
  508 FORMAT(F16.4)
C
      WRITE(11,509)E1
  509 FORMAT(10X,'EASTING              [STA.1] :
',F16.4)
      WRITE(11,510)N1
  510 FORMAT(10X,'NORTHING             [STA.1] :
',F16.4,/)
      WRITE(11,511)E2
  511 FORMAT(10X,'EASTING              [STA.2] :
',F16.4)
      WRITE(11,512)N2
  512 FORMAT(10X,'NORTHING             [STA.2] :
',F16.4,/)
C
      CALL BRGDIS(E1,E2,N1,N2,DI,BR1,BR2,PI4)
C
      WRITE(11,513)DI
  513 FORMAT(10X,'GRID DISTANCE    [STA.1-2] =
',F16.4,/)
C
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
      WRITE(11,514)ID,IM,M3
  514 FORMAT(10X,'GRID BEARING     [STA.1-2] =
',I4,I3,F8.4)
C
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,515)ID,IM,M3
  515 FORMAT(10X,'GRID BEARING     [STA.2-1] =
',I4,I3,F8.4,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP :
(C/S) ? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c') GOTO 170
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
```

```
******************************************************************************
-- SUBROUTINE ELDA19 ELLIPSOID PARAMETERS --
-- SUBROUTINE ISOCON ISOMETRIC LATITUDE CONSTANTS --
-- SUBROUTINE ISORAD ISOMETRIC LATITUDE CONSTANTS --
-- SUBROUTINE LCOFWD LAMBERT ONE STANDARD PARALLEL --
-- SUBROUTINE LCTFWD LAMBERT CONICAL CONFORMAL - TWO PARALLELS --
-- SUBROUTINE LCCFWD FORWARD LAMBERT CONICAL CONFORMAL --
-- SUBROUTINE LCCINV INVERSE LAMBERT CONICAL CONFORMAL --
-- SUBROUTINE ATCCOR LAMBERT ARC-TO-CHORD CORRECTION --
-- SUBROUTINE BRGDIS TO CALCULATE BRG AND DISTANCE - FLAT EARTH --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
-- SUBROUTINE GRDRAD CONVERTING GRAD TO RADIAN --
-- SUBROUTINE RADGRD CONVERTING RADIAN TO GRAD --
******************************************************************************
```

18.20 Hotine's Oblique Mercator Projection

A_20OM00.FOR Program – Hotine's Oblique Mercator Projection

The constants for the formulae in A_20OM00.FOR to calculate the isometric latitude are adapted by Berry and Burkholder. The equations were recasted by the author in 1994. See also A_19LC00.FOR

```
****************************************************************************
*     PROGRAM A_20OM00.FOR - DATE 01-06-2006                               *
*     OBLIQUE CONFORMAL MERCATOR - FORTRAN PROGRAM                         *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK                 *
*     AUTHOR M. HOOIJBERG 1985-2006                                        *
****************************************************************************
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,PN*4,SGN*4,
VR*1,VS*1
C
      OPEN(11,FILE= '\FORFILES\OMDATA00.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,'--------------------------------
---------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- OM00    ')
      WRITE(11,3)
    3 FORMAT(10X,'OBLIQUE MERCATOR CONFORMAL PRO-
JECTION    ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'-------------------------------
---------',/)
C
C     -- BASIC ELLIPSOID DATA --
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT(' INPUT REFERENCE ELLIPSOID    : ')
      READ(*,12) DATUM
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT(' INPUT SEMI-MAJOR AXIS    [A] : ')
      READ(*,14)A
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT(' INPUT REC. FLATTENING   [FL] : ')
      READ(*,16)FL
   16 FORMAT(F22.16)
      WRITE(*,17)
   17 FORMAT(' INPUT SCALE FACTOR      [KO] : ')
      READ(*,18)KO
   18 FORMAT(F22.16)
      WRITE(*,19)
   19 FORMAT(' INPUT FALSE EASTING     [FE] : ')
      READ(*,20)EO
   20 FORMAT(F22.4)
      WRITE(*,21)
   21 FORMAT(' INPUT FALSE NORTHING    [FN] : ')
      READ(*,22)NO
   22 FORMAT(F22.4)
      WRITE(*,23)
   23 FORMAT(' CONVERSION  FACTOR             : ')
      READ(*,24)CF
   24 FORMAT(F22.16)
C
C     -- OBLIQUE MERCATOR PROJECTION - PART ONE --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME     [HH:MM] =
',I3,I3,I5,' - ',I3,I3)
C
      CALL ELDAT3(EC0,EC2,EC3,F,FL,PI4,RD)
      CALL ISOCON(EC2,F0,F2,F4,F6,F8)
C
      WRITE(11,31)DATUM
   31 FORMAT(10X,'DATUM OF ELLIPSOID         :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS       [A] :
',F22.6)
      WRITE(11,33)FL
   33 FORMAT(10X,'REC.FLATTENING       [FL] :
',F22.14)
      WRITE(11,34)KO
   34 FORMAT(10X,'SCALE FACTOR         [KO] :
',F22.14)
      WRITE(11,35)EO
   35 FORMAT(10X,'FALSE EASTING        [EO] :
',F22.4)
      WRITE(11,36)NO
   36 FORMAT(10X,'FALSE NORTHING       [NO] :
',F22.4)
      WRITE(11,37)CF
   37 FORMAT(10X,'CONVERSION FACTOR    [CF] :
',F22.16)
      WRITE(11,38)EC2
   38 FORMAT(10X,'SQUARE ECC          [EC2] =
',E22.16)
      WRITE(11,39)EC3
   39 FORMAT(10X,'SQUARE 2ND ECC      [EC3] =
',E22.16,/)
      WRITE(11,40)F0
   40 FORMAT(10X,'ISOM. LAT. CONSTANT  [F0] =
',E22.16)
      WRITE(11,41)F2
   41 FORMAT(10X,'ISOM. LAT. CONSTANT  [F2] =
',E22.16)
      WRITE(11,42)F4
```

```
   42 FORMAT(10X,'ISOM. LAT. CONSTANT  [F4] =
',E22.16)
      WRITE(11,43)F6
   43 FORMAT(10X,'ISOM. LAT. CONSTANT  [F6] =
',E22.16)
      WRITE(11,44)F8
   44 FORMAT(10X,'ISOM. LAT. CONSTANT  [F8] =
',E22.16,/)
C
C     -- HOTINE OBLIQUE MERCATOR PROJECTION - PART
ONE --
C
  100 WRITE(*,101)
  101 FORMAT('  (N)ORTHERN OR (S)OUTHERN HEMI-
SPHERE: (N/S)? ')
      READ(*,102)NS
      I3    = 1
  102 FORMAT(A1)
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 104
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 103
      GOTO 100
  103 I3    = -1
      NS    = 'S'
  104 WRITE(*,105)
  105 FORMAT(' (E)ASTERN OR (W)ESTERN HEMISPHERE:
(E/W)? ')
      READ(*,106)EW
  106 FORMAT(A1)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 108
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 107
      GOTO 104
  107 EW    = 'W'
  108 I2    = 1
C
C     -- ENTER BASIC RECT.SKEW ORTHOMORPHIC PRO-
JECTION DATA --
C
      WRITE(*,110)
  110 FORMAT(' LATITUDE CENTRE:  DDD,MM,SS.SSSS ')
      READ(*,111)ID,IM,M3
  111 FORMAT(I3,I3,F8.4)
      WRITE(11,112)ID,IM,M3,NS
  112 FORMAT(10X,'LATITUDE CENTRE          :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LTC,ID,IM,M3,I2,RD)
C
      WRITE(*,113)
  113 FORMAT(' LONGITUDE CENTRE: DDD,MM,SS.SSSS ')
      READ(*,114)ID,IM,M3
  114 FORMAT(I4,I3,F8.4)
      WRITE(11,115)ID,IM,M3,EW
  115 FORMAT(10X,'LONGITUDE CENTRE         :
',I4,I3,F8.4,A2)
      I2    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 116
      I2    = -1
C
  116 CALL DMSRAD(LNC,ID,IM,M3,I2,RD)
C
      WRITE(*,117)
  117 FORMAT(' SKEW ANGLE - TRUE ORIGIN:
DDD,MM,SS.SSSSSSSSSSSSSS ')
      READ(*,118)ID,IM,M3
  118 FORMAT(I4,I3,F22.18)
      PN    = '(+)'
      I2    = 1
  119 WRITE(*,120)
  120 FORMAT('  (P)OSITIVE OR (N)EGATIVE ANGLE:
(P/N)? ')
      READ(*,121)VS
  121 FORMAT(A1)
      IF (VS .EQ. 'P' .OR. VS .EQ. 'p')GOTO 123
      IF (VS .EQ. 'N' .OR. VS .EQ. 'n')GOTO 122
      GOTO 119
  122 I2    = -1
      PN    = '(-)'
  123 CONTINUE
      WRITE(11,124)PN,ID,IM,M3
  124 FORMAT(10X,'SKEW ANGLE - TRUE ORIGIN   :
',A4,I4,I3,F22.18)
C
      CALL DMSRAD(ATO,ID,IM,M3,I2,RD)
C
      WRITE(*,125)
  125 FORMAT(' SKEW ANGLE - FALSE ORIGIN:
DDD,MM,SS.SSSSSSSSSSSSSS ')
      READ(*,126)ID,IM,M3
  126 FORMAT(I4,I3,F22.18)
      WRITE(11,127)PN,ID,IM,M3
  127 FORMAT(10X,'SKEW ANGLE TRUE ORIGIN DMS:
',A4,I4,I3,F22.18)
C
      CALL DMSRAD(TTO,ID,IM,M3,I2,RD)
C
      CALL RSOMER(A,KO,ATO,TTO,LNC,LNO,LTC,ECO,
EC2,EC3,TO,SO,CO,QC,OMO,
     + OM1,OM2,OM3,OM4,OM5,OM6,OM7,OM8)
C
C     WRITE(*,130)
C 130 FORMAT(' PRESS ANY KEY TO CONTINUE
')
C     READ(*,131)VR
C 131 FORMAT(A1)
C
      WRITE(11,132)
  132 FORMAT(10X,'--------------------------------
-----------------')
      WRITE(11,133)
  133 FORMAT(10X,' OBLIQUE CONFORMAL MERCATOR PRO-
JECTION CONSTANTS')
      WRITE(11,134)
  134 FORMAT(10X,'--------------------------------
-----------------')

      WRITE(11,135)TO
  135 FORMAT(10X,'ATAN OF TRUE ORIGIN        =
',E22.16)
      WRITE(11,136)SO
  136 FORMAT(10X,'SIN OF FALSE ORIGIN        =
',E22.16)
      WRITE(11,137)CO
  137 FORMAT(10X,'COS OF FALSE ORIGIN        =
',E22.16)
      WRITE(11,138)QC
  138 FORMAT(10X,'CONSTANT QC                =
',E22.16)
      WRITE(11,139)OMO
  139 FORMAT(10X,'CONSTANT OMO               =
',E22.16)
      WRITE(11,140)OM1
  140 FORMAT(10X,'CONSTANT OM1               =
',E22.16)
      WRITE(11,141)OM2
```

```
  141 FORMAT(10X,'CONSTANT OM2                    =
',E22.16)
      WRITE(11,142)OM3
  142 FORMAT(10X,'CONSTANT OM3                    =
',E22.16)
      WRITE(11,143)OM5
  143 FORMAT(10X,'CONSTANT OM5                    =
',E22.16)
      WRITE(11,144)OM6
  144 FORMAT(10X,'CONSTANT OM6                    =
',E22.16)
      WRITE(11,145)OM7
  145 FORMAT(10X,'CONSTANT OM7                    =
',E22.16)
      WRITE(11,146)OM8
  146 FORMAT(10X,'CONSTANT OM8                    =
',E22.16)
      WRITE(11,147)LNO
  147 FORMAT(10X,'CONSTANT LNO                    =
',E22.16)
      SGN   = '(+)'
      IF (LNO .LT. 0.D0) SGN = '(-)'
C
      CALL RADDMS(LNO,ID,IM,M3,I2,RD)
C
      WRITE(11,148)SGN,ID,IM,M3
  148 FORMAT(10X,'ZERO LONGITUDE                  =
',A4,I4,I3,F8.4)
C
C     -- HOTINE OBLIQUE MERCATOR PROJECTION - PART
TWO --
C
  150 CONTINUE
      WRITE(11,151)
  151 FORMAT(10X,'--------------------------------
---------------------
    +--------')
      WRITE(11,152)DATUM
  152 FORMAT(10X,'OBLIQUE CONFORMAL MERCATOR
PROJECTION ',A20)
      WRITE(11,153)
  153 FORMAT(10X,'     COPYRIGHT SPRINGER-VERLAG
BERLIN HEIDELBERG NEW YORK')
      WRITE(11,154)
  154 FORMAT(10X,'--------------------------------
---------------------
    +--------')
C
  155 WRITE(*,156)
  156 FORMAT('  (L)AT/LON - (E)AS/NOR - t-(T) -
(S)TOP:
    + (L/E/T/S) ? ')
      READ(*,157)VR
  157 FORMAT(A1)
      IF (VR .EQ. 'L' .OR. VR .EQ. 'l') GOTO 200
      IF (VR .EQ. 'E' .OR. VR .EQ. 'e') GOTO 300
      IF (VR .EQ. 'T' .OR. VR .EQ. 't') GOTO 400
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 155
C
C     -- COMPUTE FORWARD CONVERSION LAT/LON TO
EAS/NOR --
C
  200 CONTINUE
      WRITE(*,201)
  201 FORMAT(' LATITUDE               [STA.I] :  DD
MM SS.SSSS ')
      READ(*,202)ID,IM,M3
  202 FORMAT(I3,I3,F8.4)
      WRITE(11,203)ID,IM,M3,NS
  203 FORMAT(10X,'LATITUDE            [STA.I] :
',I4,I3,F8.4,A2)
      I2    = 1
      IF (NS .EQ. 'S') I2 = -1
      CALL DMSRAD(LT1,ID,IM,M3,I2,RD)
      WRITE(*,204)
  204 FORMAT(' LONGITUDE              [STA.I] :
DDD,MM,SS.SSSS ')
      READ(*,205)ID,IM,M3
  205 FORMAT(I4,I3,F8.4)
      WRITE(11,206)ID,IM,M3,EW
  206 FORMAT(10X,'LONGITUDE           [STA.I] :
',I4,I3,F8.4,A2,/)
      I2    = 1
      IF (EW .EQ. 'W') I2 = -1
      CALL DMSRAD(LN1,ID,IM,M3,I2,RD)
C
C     -- ROUTINE RADIANS TO RSO --
C
      CALL RADRSO(LN1,LT1,OM1,OM2,OM3,OM5,OM6,OM8,
E0,E1,N0,N1,EC0,EC2,
     + CVG,K,TT0,CF,QL,C0,S0,LNO)
C
      N1    = N1*I3
      WRITE(11,207)E1
  207 FORMAT(10X,'EASTING             [STA.I] =
',F16.5)
      WRITE(11,208)N1
  208 FORMAT(10X,'NORTHING            [STA.I] =
',F16.5,/)
C
C     --  CONVERGENCE-SCALE FACTOR --
C
      CALL RADDMS(CVG,ID,IM,M3,I2,RD)
C
      I4    =    1
      SGN   =    '(+)'
      IF ((LN1-LNC) .LT. 0.D0) I4 = -1
      IF ((I3*I4) .EQ. -1) SGN = '(-)'
      WRITE(11,209)SGN,ID,IM,M3
  209 FORMAT(10X,'CONVERGENCE         [STA.I] =
',A4,I4,I3,F8.4)
      WRITE(11,210)K
  210 FORMAT(10X,'SCALE FACTOR        [STA.I] =
',F22.12,/)
      GOTO 900
C
C     -- COMPUTE INVERSE CONVERSION EAS/NOR TO
LAT/LON --
C
  300 CONTINUE
      WRITE(*,301)
  301 FORMAT(' INPUT EASTING          [STA.I] : ')
      READ(*,302)E1
  302 FORMAT(F22.5)
      WRITE(*,303)
  303 FORMAT(' INPUT NORTHING         [STA.I] : ')
      READ(*,304)N1
  304 FORMAT(F22.5)
      WRITE(11,305)E1
  305 FORMAT(10X,'EASTING             [STA.I] :
',F16.5)
      IF (NS .EQ. 'S') N1 = -N1
      N1    = N1*I3
      WRITE(11,306)N1
```

```
  306 FORMAT(10X,'NORTHING             [STA.I] :
',F16.5,/)
C
C     -- ROUTINES RSO TO RADIANS --
C
      CALL RSORAD(C0,E0,E1,CF,N0,N1,OM1,OM2,OM3,
OM5,OM6,S0,R,CU,SU,X)
      CALL RADISO(F0,F2,F4,F6,F8,LN0,LN1,LT1,OM1,
OM5,OM6,R,CU,SU,X)
C
      CALL RADDMS(LT1,ID,IM,M3,I2,RD)
C
      WRITE(11,307)ID,IM,M3,NS
  307 FORMAT(10X,'LATITUDE             [STA.I] =
',I4,I3,F8.4,A2)
C
      CALL RADDMS(LN1,ID,IM,M3,I2,RD)
C
      WRITE(11,308)ID,IM,M3,EW
  308 FORMAT(10X,'LONGITUDE            [STA.1] =
',I4,I3,F8.4,A2,/)
      GOTO 900
C
C     -- COMPUTE ARC-TO-CHORD OR (t-T)" CORRECTION
--
C
  400 CONTINUE
      WRITE(*,401)
  401 FORMAT(' INPUT EASTING        [STA.1] :
XXX XXX.XXXX')
      READ(*,402)E1
  402 FORMAT(F22.4)
      WRITE(*,403)
  403 FORMAT(' INPUT NORTHING       [STA.1] : X
XXX XXX.XXXX')
      READ(*,404)N1
  404 FORMAT(F22.4)
      WRITE(*,405)
  405 FORMAT(' INPUT EASTING        [STA.2] :
XXX XXX.XXXX')
      READ(*,406)E2
  406 FORMAT(F22.4)
      WRITE(*,407)
  407 FORMAT(' INPUT NORTHING       [STA.2] : X
XXX XXX.XXXX ')
      READ(*,408)N2
  408 FORMAT(F22.4)
  409 CONTINUE
      N1    = N1*I3
      N2    = N2*I3
      WRITE(11,410)E1
  410 FORMAT(10X,'EASTING              [STA.1] :
',F16.4)
      WRITE(11,411)N1
  411 FORMAT(10X,'NORTHING             [STA.1] :
',F16.4,/)
      WRITE(11,412)E2
  412 FORMAT(10X,'EASTING              [STA.2] :
',F16.4)
      WRITE(11,413)N2
  413 FORMAT(10X,'NORTHING             [STA.2] :
',F16.4,/)
C
      CALL BDCORR(E1,E2,N1,N2,DI,BR1,BR2,PI4)
C
      WRITE(11,414)DI
  414 FORMAT(10X,'GRID DISTANCE   [STA.1-2] =
',F16.5)
C
      CALL RADDMS(BR1,ID,IM,M3,I2,RD)
C
      WRITE(11,415)ID,IM,M3
  415 FORMAT(10X,'GRID BEARING    [STA.1-2] =
',I4,I3,F8.4)
C     BR2   = BR1+PI4*4.D0
C
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
C
      WRITE(11,416)ID,IM,M3
  416 FORMAT(10X,'GRID BEARING    [STA.2-1] =
',I4,I3,F8.4,/)
C
      CALL TTCORR(A,K0,RD,E1,E2,N0,N1,N2,OM1,S0,
C0,CF,D12,D21,I3)
C
      WRITE(11,417)D12
  417 FORMAT(10X,'ARC-TO-CHORD IN SEC (1-2) =
',F10.4)
      WRITE(11,418)D21
  418 FORMAT(10X,'ARC-TO-CHORD IN SEC (2-1) =
',F10.4,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP :
(C/S) ? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c') GOTO 155
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 900
C
C     -- NEXT CASE --
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**

*************************************************************************
-- SUBROUTINE ELDAT3 ELLIPSOID PARAMETERS --
-- SUBROUTINE ISOCON ISOMETRIC LATITUDE CONSTANTS
-- SUBROUTINE RSOMER OBLIQUE MERCATOR CONSTANTS --
-- SUBROUTINE RADRSO CONVERTING RADIANS TO RECT. SKEW MERCATOR --
-- SUBROUTINE RSORAD CONVERTING RECT. SKEW MERCATOR TO RADIANS --
-- SUBROUTINE RADISO RADIAN MEASURE - ISOMETRIC LATITUDE --
-- SUBROUTINE ISORAD ISOMETRIC LATITUDE - RADIAN MEASURE --
-- SUBROUTINE BDCORR CALCULATING BEARING AND DISTANCE --
-- SUBROUTINE TTCORR CALCULATING (t-T)" CORRECTIONS --
```

```
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
*************************************************************************
```

18.21 Rosenmund's Oblique Mercator Projection

A_21RM00.FOR Program – Rosenmund's Oblique Mercator Projection

```
*************************************************************************
*     PROGRAM  A_21RM00.FOR- DATE 01-06-2006                            *
*     ROSENMUND OBLIQUE MERCATOR CONFORMAL DOUBLE - FORTRAN PROGRAM     *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK              *
*     AUTHOR M. HOOIJBERG 2001-2006                                     *
*************************************************************************
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,SGN*4,VR*1
C
      OPEN(11,FILE= '\FORFILES\RMDATA00.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,'--------------------------------
-------------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- RM00       ')
      WRITE(11,3)
    3 FORMAT(10X,'OBLIQUE MERCATOR CH-1903 CONFOR-
MAL PROJECTION')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,'--------------------------------
-------------')
C
C     -- BASIC ELLIPSOID DATA --
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT(' INPUT REFERENCE ELLIPSOID     : ')
      READ(*,12) DATUM
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT(' INPUT SEMI-MAJOR AXIS     [A] : ')
      READ(*,14)A
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT(' INPUT REC. FLATTENING    [FL] : ')
      READ(*,16)FL
   16 FORMAT(F22.16)
      WRITE(*,17)
   17 FORMAT(' INPUT SCALE FACTOR       [K0] : ')
      READ(*,18)K0
   18 FORMAT(F22.16)
      WRITE(*,19)
   19 FORMAT(' INPUT FALSE EASTING      [FE] : ')
      READ(*,20)Y0
   20 FORMAT(F22.4)
      WRITE(*,21)
   21 FORMAT(' INPUT FALSE NORTHING     [FN] : ')
      READ(*,22)X0
   22 FORMAT(F22.4)
C
C     -- LISTING ELLIPSOID CONSTANTS 1 --
C
   30 CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,31)IYR,IMON,IDAY,IHR,IMIN
   31 FORMAT(10X,'DATE AND TIME      [HH:MM] =
',I3,I3,I5,' - ',I3,I3,/)
C
      CALL ELLIPS(EC0,EC2,EC3,F,FL,N,PI4,RD)
C
      WRITE(11,32)DATUM
   32 FORMAT(10X,'DATUM OF ELLIPSOID          :
',A30)
      WRITE(11,33)A
   33 FORMAT(10X,'SEMI-MAJOR AXIS         [A] :
',F22.6)
      WRITE(11,34)FL
   34 FORMAT(10X,'REC. FLATTENING        [FL] :
',F22.14)
      WRITE(11,35)K0
   35 FORMAT(10X,'SCALE FACTOR           [K0] :
',F22.14)
      WRITE(11,36)Y0
   36 FORMAT(10X,'FALSE EASTING          [FE] :
',F22.4)
      WRITE(11,37)X0
   37 FORMAT(10X,'FALSE NORTHING         [FN] :
',F22.4)
      WRITE(11,38)EC0
   38 FORMAT(10X,'1ST ECCENTRICITY      [EC0] =
',E22.16)
      WRITE(11,39)EC2
   39 FORMAT(10X,'SQUARE 1ST ECC.       [EC2] =
',E22.16)
      WRITE(11,40)EC3
   40 FORMAT(10X,'SQUARE 2ND ECC.       [EC3] =
',E22.16)
      WRITE(11,41)N
   41 FORMAT(10X,'N                       [N] =
',E22.16,/)
C
C     -- NORTH-EASTERN HEMISPHERE --
C
      EW    = 'E'
      NS    = 'N'
C
C     -- OBLIQUE MERCATOR CONFORMAL DOUBLE PROJEC-
TION -
C      - (CH-1903) - PART ONE --
C
  109 CONTINUE
```

```
      I2    = 1
      WRITE(*,110)
  110 FORMAT(' REFERENCE LATITUDE      [Bo] :
DD,MM,SS.SSSS ')
      READ(*,111)ID,IM,M3
  111 FORMAT(I3,I3,F8.4)
      WRITE(11,112)ID,IM,M3,NS
  112 FORMAT(10X,'REFERENCE LATITUDE   [Bo] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LT0,ID,IM,M3,I2,RD)
      I2    = 1
      WRITE(*,113)
  113 FORMAT(' REFERENCE LONGITUDE     [Lo] :
DD,MM,SS.SSSS ')
      READ(*,114)ID,IM,M3
  114 FORMAT(I4,I3,F8.4)
      WRITE(11,115)ID,IM,M3,EW
  115 FORMAT(10X,'REFERENCE LONGITUDE  [Lo] :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LN0,ID,IM,M3,I2,RD)
      LN1   = LN0
C
      CALL ISORAD(Q0,W0,LT0,EC0,EC2)
      CALL ELZONE(A,A2,EC2,EC3,LT0,LT1,K1,R0,
Q0,W0)
C
      WRITE(11,116)K1
  116 FORMAT(10X,'DILATION FACTOR      [K1] =
',E22.16)
      WRITE(11,117)R0
  117 FORMAT(10X,'RADIUS (M)           [Rm] =
',F22.12)
      WRITE(11,118)A2
  118 FORMAT(10X,'GAUSS FACTOR      [ALPHA] =
',E22.16,/)
C
      CALL RADDMS(LT1,ID,IM,M3,I2,RD)
      WRITE(11,119)ID,IM,M3,NS
  119 FORMAT(10X,'LATITUDE SPHERE      [bo] =
',I4,I3,F8.4,A2)
C
      CALL RADDMS(LN1,ID,IM,M3,I2,RD)
      WRITE(11,120)ID,IM,M3,EW
  120 FORMAT(10X,'LONGITUDE SPHERE     [lo] =
',I4,I3,F8.4,A2,/)
C
C     -- OBLIQUE MERCATOR CONFORMAL DOUBLE PROJEC-
TION -
C      - (CH-1903) - PART TWO --
C
  160 CONTINUE
      WRITE(11,161)
  161 FORMAT(10X,'--------------------------------
----------------------
     +--------')
      WRITE(11,162)DATUM
  162 FORMAT(10X,'OBLIQUE MERCATOR DOUBLE PROJEC-
TION CH-1903 - ',A30)
      WRITE(11,163)
  163 FORMAT(10X,'     COPYRIGHT SPRINGER-VERLAG
BERLIN HEIDELBERG NEW YORK')
      WRITE(11,164)
  164 FORMAT(10X,'--------------------------------
----------------------
     +--------')
C
  165 CONTINUE
      WRITE(*,166)
  166 FORMAT('  (L)AT/LON - (E)AS/NOR - t-(T) -
(S)TOP (L/E/T/S) ? ')
      READ(*,167)VR
  167 FORMAT(A1)
      IF (VR .EQ. 'L' .OR. VR .EQ. 'l') GOTO 200
      IF (VR .EQ. 'E' .OR. VR .EQ. 'e') GOTO 300
      IF (VR .EQ. 'T' .OR. VR .EQ. 't') GOTO 400
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 165
C
C     -- COMPUTE FORWARD CONVERSION : LAT/LON TO
EAS/NOR --
C
  200 CONTINUE
      WRITE(*,201)
  201 FORMAT(' LATITUDE              [STA.I=BI] :
DD,MM,SS.SSSS ')
      READ(*,202)ID,IM,M3
  202 FORMAT(I4,I3,F8.4)
      I2    = 1
      WRITE(11,203)ID,IM,M3,NS
  203 FORMAT(10X,'LATITUDE         [STA.1=Bi] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LT2,ID,IM,M3,I2,RD)
C
      WRITE(*,204)
  204 FORMAT(' LONGITUDE             [STA.1=Li] :
DDD,MM,SS.SSSS ')
      READ(*,205)ID,IM,M3
  205 FORMAT(I4,I3,F8.4)
      I2    = 1
      WRITE(11,206)ID,IM,M3,EW
  206 FORMAT(10X,'LONGITUDE        [STA.1=Li] :
',I4,I3,F8.4,A2,/)
      CALL DMSRAD(LN2,ID,IM,M3,I2,RD)
C
      CALL GEOROM(EC0,X,X0,Y,Y0,R0,PI4,LT1,LT2,
LN0, LN2,A2,K1,KI,CVG)
C
      WRITE(11,207)Y
  207 FORMAT(10X,'EASTING          [STA.I=Yi] =
',F16.4)
      WRITE(11,208)X
  208 FORMAT(10X,'NORTHING         [STA.I=Xi] =
',F16.4,/)
C
C     -- CONVERGENCE-SCALE FACTOR --
C
      IF (CVG .GT. 0.D0) SGN = '(+)'
      IF (CVG .LT. 0.D0) SGN = '(-)'
C
      CALL RADDMS(CVG,ID,IM,M3,I2,RD)
      WRITE(11,209)SGN,ID,IM,M3
  209 FORMAT(10X,'CONVERGENCE      [STA.I=Ci] =
',A4,I4,I3,F8.4)
      WRITE(11,210)KI
  210 FORMAT(10X,'SCALE FACTOR     [STA.I=Ki] =
',F22.16,/)
      GOTO 900
C
C     -- COMPUTE INVERSE CONVERSION : EAS/NOR TO
LAT/LON --
C
  300 CONTINUE
      WRITE(*,301)
  301 FORMAT(' INPUT EASTING     [STA.I=Yi] :
XXX XXX.XXXX')
```

```
      READ(*,302)Y
  302 FORMAT(F16.4)
      WRITE(*,303)
  303 FORMAT(' INPUT NORTHING     [STA.I=Xi] : X
XXX XXX.XXXX')
      READ(*,304)X
  304 FORMAT(F16.4)
      WRITE(11,305)Y
  305 FORMAT(10X,'EASTING         [STA.1=Yi] :
',F16.4)
      WRITE(11,306)X
  306 FORMAT(10X,'NORTHING        [STA.1=Xi] :
',F16.4,/)
C
      CALL RECGOM(A2,K1,ECO,X,X0,Y,Y0,R0,PI4, LT0,
LT1,LT2,LN0,LN2)
C
      CALL RADDMS(LT2,ID,IM,M3,I2,RD)
      WRITE(11,307)ID,IM,M3,NS
  307 FORMAT(10X,'LATITUDE        [STA.I=Bi] =
',I4,I3,F8.4,A2)
C
      CALL RADDMS(LN2,ID,IM,M3,I2,RD)
      EW    = 'E'
      IF (I2 .EQ. -1) EW = 'W'
      WRITE(11,308)ID,IM,M3,EW
  308 FORMAT(10X,'LONGITUDE       [STA.I=Li] =
',I4,I3,F8.4,A2,/)
      GOTO 900
C
C     -- COMPUTE ARC-TO-CHORD OR (t-T)" CORRECTION
--
C
  400 CONTINUE
      WRITE(*,401)
  401 FORMAT(' INPUT EASTING        [STA.1] :
XXX XXX.XXXX')
      READ(*,402)Y1
  402 FORMAT(F16.4)
      WRITE(*,403)
  403 FORMAT(' INPUT NORTHING       [STA.1] : X
XXX XXX.XXXX')
      READ(*,404)X1
  404 FORMAT(F16.4)
      WRITE(*,405)
  405 FORMAT(' INPUT EASTING        [STA.2] :
XXX XXX.XXXX')
      READ(*,406)Y2
  406 FORMAT(F16.4)
      WRITE(*,407)
  407 FORMAT(' INPUT NORTHING       [STA.2] : X
XXX XXX.XXXX')
      READ(*,408)X2
  408 FORMAT(F16.4)
C
      WRITE(11,409)Y1
  409 FORMAT(10X,'EASTING           [STA.1] :
',F16.4)
      WRITE(11,410)X1
  410 FORMAT(10X,'NORTHING          [STA.1] :
',F16.4,/)
      WRITE(11,411)Y2
  411 FORMAT(10X,'EASTING           [STA.2] :
',F16.4)
      WRITE(11,412)X2
  412 FORMAT(10X,'NORTHING          [STA.2] :
',F16.4,/)
C
      CALL TEECOM(X0,X1,X2,Y1,Y2,DX,DY,R0,D12,D21,
BR1,DI)
C
      WRITE(11,413)D12
  413 FORMAT(10X,'D12               [STA.1-2]  =
',E22.16)
      WRITE(11,414)D21
  414 FORMAT(10X,'D21               [STA.2-1]  =
',E22.16,/)
C
      SGN   = '(+)'
      IF (D12 .LT. 0.D0) SGN ='(-)'
C
      CALL RADDMS(D12,ID,IM,M3,I2,RD)
      WRITE(11,415)SGN,ID,IM,M3
  415 FORMAT(10X,'ARC-TO-CHORD    [STA.1-2] =
',A4,I3,I3,F8.4)
C
      SGN   = '(+)'
      IF (D21 .LT. 0.D0) SGN ='(-)'
C
      CALL RADDMS(D21,ID,IM,M3,I2,RD)
      WRITE(11,416)SGN,ID,IM,M3
  416 FORMAT(10X,'ARC-TO-CHORD    [STA.2-1] =
',A4,I3,I3,F8.4,/)
C
      IF (DX .EQ. 0.D0) DX = 1.D-34
      IF (DY .GT. 0.D0 .AND. DX .GT. 0.D0) GOTO
417
      IF (DX .LT. 0.D0)BR1 = 4.D0*PI4+BR1
      IF (DX .GT. 0.D0)BR1 = 8.D0*PI4+BR1
C
  417 CALL RADDMS(BR1,ID,IM,M3,I2,RD)
      WRITE(11,418)ID,IM,M3
  418 FORMAT(10X,'GRID BEARING    [STA.1-2] =
',I4,I3,F8.4)
      BR2   = BR1+4.D0*PI4
      IF (BR2 .GT. (8.D0*PI4)) BR2 = BR2-8.D0*PI4
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,419)ID,IM,M3
  419 FORMAT(10X,'GRID BEARING    [STA.2-1] =
',I4,I3,F8.4)
      WRITE(11,420)DI
  420 FORMAT(10X,'GRID DISTANCE   [STA.1-2] =
',F18.4,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP :
(C/S) ? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c') GOTO 165
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
```

```
*********1*********2*********3*********4*********5*********6*********7**
-- SUBROUTINE ELLIPS ELLIPSOID PARAMETERS --
-- SUBROUTINE ELZONE GAUSS-SCHREIBER - ZONE PARAMETERS --
-- SUBROUTINE ISORAD ISOMETRIC LATITUDE --
-- SUBROUTINE GEOROM GEOGRAPHICALS TO RECTANGULAR COORDS --
-- SUBROUTINE RECGOM ROM COORDS TO GEOGRAPHICALS --
-- SUBROUTINE TEECOM TO COMPUTE (t-T)', BEARING AND DISTANCE --
-- SUBROUTINE TO COMPUTE BEARING AND DISTANCE --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
-- SUBROUTINE GRDRAD - see A_19LC00 --
-- SUBROUTINE RADGRD - see A_19LC00 --
*********1*********2*********3*********4*********5*********6*********7**
```

18.22 Stereographic Conformal Projection

A_22ST00.FOR Program - Heuvelink's Oblique Stereographic Conformal Projection

```
************************************************************************
*     PROGRAM  A_22ST00.FOR - DATE 01-06-2006                          *
*     STEREOGRAPHIC CONFORMAL DOUBLE PROJECTION - FORTRAN PROGRAM      *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1995-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
C     See [On_CD] for Subroutines, and Examples
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER DATUM*30,NS*1,EW*1,VR*1,SGN*4
C
      OPEN(11,FILE= '\FORFILES\STDATA00.TXT',
STATUS='OLD')
C
      WRITE(11,1)
    1 FORMAT(10X,' ------------------------------
-----------')
      WRITE(11,2)
    2 FORMAT(10X,'GEOMETRICAL GEODESY - USING ICT
- ST00     ')
      WRITE(11,3)
    3 FORMAT(10X,'STEREOGRAPHIC CONFORMAL PROJEC-
TION        ')
      WRITE(11,4)
    4 FORMAT(10X,'COPYRIGHT SPRINGER-VERLAG BERLIN
HEIDELBERG NEW YORK')
      WRITE(11,5)
    5 FORMAT(10X,' ------------------------------
-----------')
C
C     -- BASIC ELLIPSOID DATA --
C
   10 CONTINUE
      WRITE(*,11)
   11 FORMAT(' INPUT REFERENCE ELLIPSOID    : ')
      READ(*,12) DATUM
   12 FORMAT(A30)
      WRITE(*,13)
   13 FORMAT(' INPUT SEMI-MAJOR AXIS    [A] : ')
      READ(*,14)A
   14 FORMAT(F22.8)
      WRITE(*,15)
   15 FORMAT(' INPUT REC. FLATTENING   [FL] : ')
      READ(*,16)FL
   16 FORMAT(F22.16)
      WRITE(*,17)
   17 FORMAT(' INPUT FALSE EASTING     [X0] : ')
      READ(*,18)X0
   18 FORMAT(F22.4)
      WRITE(*,19)
   19 FORMAT(' INPUT FALSE NORTHING    [Y0] : ')
      READ(*,20)Y0
   20 FORMAT(F22.4)
      WRITE(*,21)
   21 FORMAT(' INPUT SCALE FACTOR      [K0] : ')
      READ(*,18)K0
   22 FORMAT(F22.16)
C
C     -- LISTING ELLIPSOID CONSTANTS --
C
      CALL GETDAT(IDAY,IMON,IYR)
      CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(11,30)IYR,IMON,IDAY,IHR,IMIN
   30 FORMAT(10X,'DATE AND TIME     [HH:MM] =
',I3,I3,I5,' - ',I3,I3,/)
C
      CALL ELDATA(EC0,EC2,EC3,FL,N,PI4,RD)
C
      WRITE(11,31)DATUM
   31 FORMAT(10X,'DATUM OF ELLIPSOID        :
',A30)
      WRITE(11,32)A
   32 FORMAT(10X,'SEMI-MAJOR AXIS       [A] :
',F22.6)
      WRITE(11,33)FL
   33 FORMAT(10X,'REC.FLATTENING       [FL] :
',F22.14)
      WRITE(11,34)X0
   34 FORMAT(10X,'FALSE EASTING        [FE] :
',F22.4)
      WRITE(11,35)Y0
   35 FORMAT(10X,'FALSE NORTHING       [FN] :
',F22.4)
      WRITE(11,36)K0
   36 FORMAT(10X,'SCALE FACTOR         [K0] :
',F22.14)
      WRITE(11,37)EC0
   37 FORMAT(10X,'1ST ECCENTRICITY    [EC0] =
',E22.16)
```

```
      WRITE(11,38)EC2
   38 FORMAT(10X,'SQUARE 1ST ECC.      [EC2] =
',E22.16)
      WRITE(11,39)EC3
   39 FORMAT(10X,'SQUARE 2ND ECC.      [EC3] =
',E22.16)
      WRITE(11,40)N
   40 FORMAT(10X,'N                      [N] =
',E22.16,/)
C
C     -- STEREOGRAPHIC CONFORMAL DOUBLE PROJECTION
- PART ONE --
C
  100 WRITE(*,101)
  101 FORMAT('  (N)ORTHERN OR (S)OUTHERN HEMI-
SPHERE: (N/S)? ')
      READ(*,102)NS
      I3    = 1
  102 FORMAT(A1)
      IF (NS .EQ. 'N' .OR. NS .EQ. 'n') GOTO 104
      IF (NS .EQ. 'S' .OR. NS .EQ. 's') GOTO 103
      GOTO 100
  103 I3    = -1
  104 WRITE(*,105)
  105 FORMAT('  (E)ASTERN OR (W)ESTERN HEMISPHERE
: (E/W)? ')
      READ(*,106)EW
  106 FORMAT(A1)
      I1    = 1
      IF (EW .EQ. 'E' .OR. EW .EQ. 'e') GOTO 109
      IF (EW .EQ. 'W' .OR. EW .EQ. 'w') GOTO 107
      GOTO 104
  107 I1    = -1
C
C     -- INPUT ZONE PARAMETERS STEREOGRAPHIC DOU-
BLE PROJECTION --
C
  109 CONTINUE
      I2    = 1
      WRITE(*,110)
  110 FORMAT(' REFERENCE LATITUDE       [Bo] :
DD,MM,SS.SSSS ')
      READ(*,111)ID,IM,M3
  111 FORMAT(I3,I3,F8.4)
      WRITE(11,112)ID,IM,M3,NS
  112 FORMAT(10X,'REFERENCE LATITUDE    [Bo] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LT0,ID,IM,M3,I2,RD)
      I2    = 1
      WRITE(*,113)
  113 FORMAT(' REFERENCE LONGITUDE      [Lo] :
DD,MM,SS.SSSS ')
      READ(*,114)ID,IM,M3
  114 FORMAT(I4,I3,F8.4)
      WRITE(11,115)ID,IM,M3,EW
  115 FORMAT(10X,'REFERENCE LONGITUDE  [Lo] :
',I4,I3,F8.4,A2)
C
      CALL DMSRAD(LN0,ID,IM,M3,I2,RD)
      LN1   = LN0
C
      CALL ISORAD(Q0,W0,LT0,EC0,EC2)
      CALL ELZONE(A,A2,EC2,EC3,LT0,LT1,K1,R0,Q0,
W0)
C
      WRITE(11,116)K1
  116 FORMAT(10X,'DILATION FACTOR       [K1] =
',E22.16)
      WRITE(11,117)R0
  117 FORMAT(10X,'RADIUS (M)            [Rm] =
',F22.12)
      WRITE(11,118)A2
  118 FORMAT(10X,'GAUSS FACTOR       [ALPHA] =
',E22.16)
C
      CALL RADDMS(LT1,ID,IM,M3,I2,RD)
      WRITE(11,119)ID,IM,M3,NS
  119 FORMAT(10X,'LATITUDE SPHERE       [bo] =
',I4,I3,F8.4,A2)
C
      CALL RADDMS(LN1,ID,IM,M3,I2,RD)
      WRITE(11,120)ID,IM,M3,NS
  120 FORMAT(10X,'LONGITUDE SPHERE      [lo] =
',I4,I3,F8.4,A2,/)
C
C     -- STEREOGRAPHIC CONFORMAL DOUBLE PROJECTION
- PART TWO --
C
  160 CONTINUE
      WRITE(11,161)
  161 FORMAT(10X,'-------------------------------
---------------------
     +--------')
      WRITE(11,162)DATUM
  162 FORMAT(10X,'STEREOGRAPHIC CONFORMAL DOUBLE
PROJECTION ',A30)
      WRITE(11,163)
  163 FORMAT(10X,'     COPYRIGHT SPRINGER-VERLAG
BERLIN HEIDELBERG NEW YORK')
      WRITE(11,164)
  164 FORMAT(10X,'-------------------------------
---------------------
     +--------')
C
  165 CONTINUE
      WRITE(*,166)
  166 FORMAT('  (L)AT/LON - (E)AS/NOR - t-(T) -
(S)TOP:
     + (L/E/T/S) ? ')
      READ(*,167)VR
  167 FORMAT(A1)
      IF (VR .EQ. 'L' .OR. VR .EQ. 'l') GOTO 200
      IF (VR .EQ. 'E' .OR. VR .EQ. 'e') GOTO 300
      IF (VR .EQ. 'T' .OR. VR .EQ. 't') GOTO 400
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 165
C
C     -- COMPUTE FORWARD CONVERSION : LAT/LON TO
EAS/NOR --
C
  200 CONTINUE
      WRITE(*,201)
  201 FORMAT(' LATITUDE            [STA.I=Bi] :
DD,MM,SS.SSSS ')
      READ(*,202)ID,IM,M3
  202 FORMAT(I3,I3,F8.4)
      I2    = 1
      WRITE(11,203)ID,IM,M3,NS
  203 FORMAT(10X,'LATITUDE         [STA.1=Bi] :
',I4,I3,F8.4,A2)
      CALL DMSRAD(LT2,ID,IM,M3,I2,RD)
C
      WRITE(*,204)
  204 FORMAT(' LONGITUDE           [STA.1=Li] :
DDD,MM,SS.SSSS ')
      READ(*,205)ID,IM,M3
```

```
  205 FORMAT(I4,I3,F8.4)
      I2    = 1
      WRITE(11,206)ID,IM,M3,EW
  206 FORMAT(10X,'LONGITUDE       [STA.1=Li] :
',I4,I3,F8.4,A2,/)
      CALL DMSRAD(LN2,ID,IM,M3,I2,RD)
C
      CALL ISORAD(Q0,W0,LT2,EC0,EC2)
      CALL GEORST(A2,K1,Q0,LN0,LN2,LT0,LT1,LT2,
LT3,R0, X,X0,Y,Y0,KI,CVG,
     + PI4,W0,K0)
C
      WRITE(11,207)X
  207 FORMAT(10X,'EASTING         [STA.I=Xi] =
',F16.4)
      WRITE(11,208)Y
  208 FORMAT(10X,'NORTHING        [STA.I=Yi] =
',F16.4,/)
C
C     -- CONVERGENCE-SCALE FACTOR --
C
      IF (CVG .GT. 0.D0) SGN = '(+)'
      IF (CVG .LT. 0.D0) SGN = '(-)'
C
      CALL RADDMS(CVG,ID,IM,M3,I2,RD)
      WRITE(11,209)SGN,ID,IM,M3
  209 FORMAT(10X,'CONVERGENCE     [STA.I=Ci] =
',A4,I4,I3,F8.4)
      WRITE(11,210)KI
  210 FORMAT(10X,'SCALE FACTOR    [STA.I=Ki] =
',F22.16,/)
      GOTO 900
C
C     -- COMPUTE INVERSE CONVERSION : EAS/NOR TO
LAT/LON --
C
  300 CONTINUE
      WRITE(*,301)
  301 FORMAT(' INPUT EASTING      [STA.I=Xi] :
XXX XXX.XXXX')
      READ(*,302)X
  302 FORMAT(F16.4)
      WRITE(*,303)
  303 FORMAT(' INPUT NORTHING     [STA.I=Yi] : X
XXX XXX.XXXX')
      READ(*,304)Y
  304 FORMAT(F16.4)
      WRITE(11,305)X
  305 FORMAT(10X,'EASTING         [STA.I=Xi] :
',F16.4)
      WRITE(11,306)Y
  306 FORMAT(10X,'NORTHING        [STA.I=Yi] :
',F16.4,/)
C
      CALL RSTGEO(A2,EC0,K0,K1,KI,LN0,LN2,LT1,
LT2,PI4, Q0,R0,R1,X,X0,Y,
     + Y0,LT0,DLN0,CVG)
      CALL RADDMS(LT2,ID,IM,M3,I2,RD)
      NS    = 'N'
      IF (I2 .EQ. -1) NS = 'S'
      WRITE(11,307)ID,IM,M3,NS
  307 FORMAT(10X,'LATITUDE        [STA.I=Bi] =
',I4,I3,F8.4,A2)
C
      CALL RADDMS(LN2,ID,IM,M3,I2,RD)
      EW    = 'E'
      IF (I2 .EQ. -1) EW = 'W'
      WRITE(11,308)ID,IM,M3,EW
  308 FORMAT(10X,'LONGITUDE       [STA.I=Li] =
',I4,I3,F8.4,A2,/)
C
C     -- CONVERGENCE-SCALE FACTOR --
C
      IF (CVG .GT. 0.D0) SGN = '(+)'
      IF (CVG .LT. 0.D0) SGN = '(-)'
C
      CALL RADDMS(CVG,ID,IM,M3,I2,RD)
      WRITE(11,309)SGN,ID,IM,M3
  309 FORMAT(10X,'CONVERGENCE     [STA.I=Ci] =
',A4,I4,I3,F8.4)
C
      WRITE(11,310)KI
  310 FORMAT(10X,'SCALE FACTOR    [STA.I=Ki] =
',F16.12,/)
      GOTO 900
C
C     -- COMPUTE ARC-TO-CHORD OR (t-T)" CORRECTION
--
C
  400 CONTINUE
      WRITE(*,401)
  401 FORMAT(' INPUT EASTING        [STA.1] :
XXX XXX.XXXX')
      READ(*,402)X1
  402 FORMAT(F16.4)
      WRITE(*,403)
  403 FORMAT(' INPUT NORTHING       [STA.1] : X
XXX XXX.XXXX')
      READ(*,404)Y1
  404 FORMAT(F16.4)
      WRITE(*,405)
  405 FORMAT(' INPUT EASTING        [STA.2] :
XXX XXX.XXXX')
      READ(*,406)X2
  406 FORMAT(F16.4)
      WRITE(*,407)
  407 FORMAT(' INPUT NORTHING       [STA.2] : X
XXX XXX.XXXX')
      READ(*,408)Y2
  408 FORMAT(F16.4)
      WRITE(11,409)X1
  409 FORMAT(10X,'EASTING           [STA.1] :
',F16.4)
      WRITE(11,410)Y1
  410 FORMAT(10X,'NORTHING          [STA.1] :
',F16.4,/)
      WRITE(11,411)X2
  411 FORMAT(10X,'EASTING           [STA.2] :
',F16.4)
      WRITE(11,412)Y2
  412 FORMAT(10X,'NORTHING          [STA.2] :
',F16.4,/)
C
      CALL TEECOR (BR1,DI,DX1,DY1,K0,R0,X0,X1,X2,
Y0,Y1,Y2,D12,D21)
C
      SGN   = '(+)'
      IF (D12 .LT. 0.D0) SGN ='(-)'
C
      CALL RADDMS(D12,ID,IM,M3,I2,RD)
      WRITE(11,413)SGN,ID,IM,M3
  413 FORMAT(10X,'ARC-TO-CHORD    [STA.1-2] =
',A4,I3,I3,F8.4)
C
      SGN   = '(+)'
      IF (D21 .LT. 0.D0) SGN ='(-)'
```

```
C
      CALL RADDMS(D21,ID,IM,M3,I2,RD)
      WRITE(11,414)SGN,ID,IM,M3
  414 FORMAT(10X,'ARC-TO-CHORD      [STA.2-1] =
',A4,I3,I3,F8.4,/)
C
      IF (DY1 .EQ. 0.D0) DY1 = 1.D-34
      IF (DX1 .GT. 0.D0 .AND. DY1 .GT. 0.D0)GOTO
415
      IF (DY1 .LT. 0.D0)BR1 = 4.D0*PI4+BR1
      IF (DY1 .GT. 0.D0)BR1 = 8.D0*PI4+BR1
C
  415 CALL RADDMS(BR1,ID,IM,M3,I2,RD)
      WRITE(11,416)ID,IM,M3
  416 FORMAT(10X,'GRID BEARING      [STA.1-2] =
',I4,I3,F8.4)
      BR2    = BR1+4.D0*PI4
      IF (BR2 .GT. (8.D0*PI4)) BR2 = BR2-8.D0*PI4
      CALL RADDMS(BR2,ID,IM,M3,I2,RD)
      WRITE(11,417)ID,IM,M3
  417 FORMAT(10X,'GRID BEARING      [STA.2-1] =
',I4,I3,F8.4)
      WRITE(11,418)DI
  418 FORMAT(10X,'GRID DISTANCE     [STA.1-2] =
',F12.4,/)
C
C     -- NEXT CASE --
C
  900 WRITE(*,901)
  901 FORMAT(' NEXT CASE - (C)ONT OR (S)TOP :
(C/S) ? ')
      READ(*,902)VR
  902 FORMAT(A1)
      IF (VR .EQ. 'C' .OR. VR .EQ. 'c') GOTO 165
      IF (VR .EQ. 'S' .OR. VR .EQ. 's') GOTO 996
      GOTO 900
C
  996 WRITE(*,999)
  998 WRITE(11,999)
  999 FORMAT(10X,'END-OF-JOB',/)
      END
C
C     -- END OF PROGRAM --
C
*********1*********2*********3*********4*********5
*********6*********7**
```

Subroutines used:

```
*********1*********2*********3*********4*********5*********6*********7**
-- SUBROUTINE ELDATA ELLIPSOID PARAMETERS --
-- SUBROUTINE ELZONE GAUSS-SCHREIBER - ZONE PARAMETERS --
-- SUBROUTINE ISORAD ISOMETRIC LATITUDE --
-- SUBROUTINE GEORST GEOGRAPHICALS TO STEREOGRAPHIC RECTANGULAR COORDS --
-- CONVERTING RSTGEO RECTANGULAR STEREOGRAPHIC COORDS TO GEOGRAPHICALS --
-- SUBROUTINE TEECOR TO COMPUTE (t-T)' AND BEARING AND DISTANCE --
-- SUBROUTINE DMSRAD CONVERTING DEG-MIN-SEC TO RADIANS --
-- SUBROUTINE RADDMS CONVERTING RADIANS TO DEG-MIN-SEC --
*********1*********2*********3*********4*********5*********6*********7**
```

18.23 I/O Subroutines

Program Execution

The program statements and data are encoded in the computer in a special computer code. Once entered and stored by the computer, the program can be executed. The last output with an end-of-job message will be the successful result from the execution of the program. Error messages that you might encounter while using a compiler are discussed in [17.1.4.1].

Subroutines in General

The computer subroutines were written in the most suitably economical form. Therefore, some subroutine listings may deviate from the basic equations given in this book. A short description of transformations [7] and conversions [9], together with the full formulae for computing and the fundamental data used are given.

Considerable space is devoted to the use of the routines, with fully worked out sample computations of all sections. Furthermore, each program listing is preceded by worked examples with an intermediate output for algorithm testing, because situations may arise in which a program fails to operate as expected due to incorrect signs, constants or arithmetic operators. Graphics are added describing the "*round-trip errors*" of the computations indicating when and how these reach certain limiting values. (Vincenty, 1994) said:

> " *...numerical and graphical presentations of running times and accuracies of programs should speak for themselves ...* ".

A demonstration of the reliability of the results by calculating the round-trip errors is imperative. In this respect (Figure 67, Figure 75) may be helpful for use of correct signs. Suspicion may be placed on results that have *not* been checked. When a program known to be operating correctly is used for calculating the solution, checking should be concentrated on *all input* to the computer program to eliminate errors. Rounding off data variables is left to the user of the programs.

Be sure about their significance in the program before changing or removing any statement. Please bear in mind, the book is written chiefly for the reader who has studied, but is by no means familiar with the subject of map projections and their use for global and extended surveys.

360° Sexagesimal FORTRAN Subroutines

```
*********1*********2*********3*********4*********5*********6*********7**
*     CONVERTING RADIANS TO DEG-MIN-SEC AND VICE-VERSA FORTRAN PROGRAM *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1995-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER NS*1,VR*1
C
C     -- BASIC DATA --
C
  100 CONTINUE
      PI4   = DATAN(1.D0)
      RD    = PI4/45.D0
      NS    = 'N'
      I2    = 1
C
C     -- ANGLE IN DEG --
C
  110 CONTINUE
      WRITE(*,111)
  111 FORMAT(' ANGLE IN DEG            [360] :  DD,MM,SS.SSSSSS ')
      READ(*,112)ID,IM,M3
  112 FORMAT(I4,I3,F8.4)
      WRITE(*,113)ID,IM,M3,NS
  113 FORMAT(10X,'ANGLE IN DEG            [360] : ',I4,I3,F8.4,A2)
      I2    = 1
C
      CALL DMSRAD(RA1,ID,IM,M3,I2,RD)
C
C     -- SUBROUTINE CONVERTING DEG-MIN-SEC TO RADIANS --
C
      SUBROUTINE DMSRAD(M0,ID,IM,M3,I2,RD)
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
C
      M0    = (ABS(ID)+(ABS(IM)+DABS(M3)/60.D0)/60.D0)*RD
      M0    = M0*I2
      RETURN
      END
C
      CALL RADDMS(RA1,ID,IM,M3,I2,RD)
C
C     -- SUBROUTINE CONVERTING RADIANS TO DEG-MIN-SEC --
C
      SUBROUTINE RADDMS(M0,ID,IM,M3,I2,RD)
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
C
      I2    = 1
      IF (M0 .LT. 0.D0) I2 = -1
      W1    = DABS(M0/RD)
      ID    = DINT(W1)
      W2    = (W1-ID)*60.D0
      IM    = DINT(W2)
      M3    = (W2-IM)*60.D0
      RETURN
```

```
      END
C
      WRITE(*,115)ID,IM,M3,NS
  115 FORMAT(10X,'ANGLE IN DEG       [360] = ',I4,I3,F8.4,A2)
C
      CALL RADDM2(RA1,ID,IM,M3,I2,RD)
C
C     -- SUBROUTINE CONVERTING RADIANS TO DEG-MIN-SEC - ROUNDED --
C
      SUBROUTINE RADDM2(MO,ID,IM,M3,I2,RD)
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
C
      I2    = 1
      IF (MO .LT. 0.DO) I2 = -1
      W1    = DABS(MO/RD)
      ID    = DINT(W1)
      W2    = (W1-ID)*60.DO
      IM    = DINT(W2)
      M3    = (W2-IM)*60.DO
      T3    = M3+0.5D-4
      IF (T3 .LT. 60.DO) GOTO 20
      M3    = 0.DO
      IM    = IM+1
   20 IF (IM .LT. 60) GOTO 21
      IM    = 0
      ID    = ID+1
   21 IF (ID .LT. 360) GOTO 22
      ID    = 0
   22 RETURN
      END
C
      WRITE(*,116)ID,IM,M3,NS
  116 FORMAT(10X,'ANGLE IN DEG  (RND) [360] = ',I4,I3,F8.4,A2)
C
*********1*********2*********3*********4*********5*********6*********7**
```

in which

```
DOUBLE PRECISION         =    (Mx)
INTEGER                  =    (Ix)
I2                       =    FLAG________________ INTEGER true=(+1) or false=(-1)
PI4      (= π/4)         =    arctan 1.0__________ DOUBLE PRECISION
RD       (= deg to rad)  =    PI4 / 45.0__________ DOUBLE PRECISION
DD       (= degrees)     =    ID__________________ INTEGER
MM       (= minutes)     =    IM__________________ INTEGER
SS.SSSS  (= seconds)     =    M3__________________ DOUBLE PRECISION
```

Note

In this book is the rounding function – intended for printing only - not applied in the subroutines. As a result, all examples are printed without rounding.

400[g] Centesimal FORTRAN Subroutines

```
*********1*********2*********3*********4*********5*********6*********7**
*     CONVERTING RADIANS TO GON AND VICE-VERSA FORTRAN PROGRAM         *
*     COPYRIGHT SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK             *
*     AUTHOR M. HOOIJBERG 1995-2006                                    *
*********1*********2*********3*********4*********5*********6*********7**
C
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
      CHARACTER NS*1,VR*1
C
C     -- BASIC DATA --
C
```

```
  100 CONTINUE
      PI4   = DATAN(1.D0)
      RD    = PI4/45.D0
      NS    = 'N'
      I2    = 1
C
      WRITE(*,211)
  211 FORMAT(' ANGLE IN GON          [GON] :  DD.DD DD DDDD ')
      READ(*,212)M1
  212 FORMAT(F22.16)
      WRITE(*,213)M1,NS
  213 FORMAT(10X,'ANGLE IN GON         [GON] :  ',F16.12,A2)
C
      CALL GRDRAD(RA2,M1,I2,RD)
C
C     -- SUBROUTINE CONVERTING GRAD TO RADIAN --
C
      SUBROUTINE GRDRAD(M0,M1,I2,RD)
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
C
      M0    = (DABS(M1)*0.9D0)*RD
      M0    = M0*I2
      RETURN
      END
C
      WRITE(*,214)RA2,NS
  214 FORMAT(10X,'ANGLE IN RADIANS     [RAD] :  ',F16.12,A2)
C
C     -- SUBROUTINE CONVERTING RADIAN TO GRAD --
C
      SUBROUTINE RADGRD(M0,M1,I2,RD)
      IMPLICIT DOUBLE PRECISION (A-H,K-Z)
      IMPLICIT INTEGER (I-J)
C
      I2    = 1
      IF (M0 .LT. 0.D0) I2 = -1
      M1    = DABS(M0/0.9D0/RD)
      RETURN
      END
C
      CALL RADGRD(RA2,M1,I2,RD)
C
      WRITE(*,215)M1,NS
  215 FORMAT(10X,'ANGLE IN GON         [GON] =  ',F16.12,A2)
      END
C
*********1*********2*********3*********4*********5*********6*********7**
```

in which

DOUBLE PRECISION		=	(Mx)	
INTEGER		=	(Ix)	
I2		=	FLAG	INTEGER true=(+1) or false=(-1)
M1	(= gon)	=	Mx	DOUBLE PRECISION
M0	(= radians)	=	Mx	DOUBLE PRECISION

General Remarks about the Routines

Verification of historical data and output of other programs is *not* a function of these (sub)routines. Coordinates of historical data points must be taken at face value, with the realisation that such coordinates could be significantly in error (Floyd, 1985).

About Rounding and Conversion of Radians into Degrees

In the 360° sexagesimal FORTRAN subroutines, converting radians into degrees (deg-min-sec), occurs sometimes a rounding error as shown in following example:

```
π    =   179° 59' 59".99996   =   179° 59' 60".0000
2π   =   359° 59' 59".99996   =   359° 59' 60".0000
```

For example, by adding a small provisional amount of 0".00005, and the final result shows:

```
X    =   179° 59' 59".99995   +   0".00005   =   180° 00' 00".0000
X    =   in radians                           =   3.14159 26533 47

X    =   200G.00000 000                       =   200.00000 000
X    =   in radians                           =   3.14159 26535 90

X    =   359° 59' 59".99993   +   0".00005   =   359° 59' 59".9999
X    =   in radians                           =   6.28318 53068 40

X    =   359° 59' 59".99996   +   0".00005   =   360° 00' 00".0000
X    =   in radians                           =   6.28318 53069 86

X    =   400G.00000 000                       =   400.00000 000
X    =   in radians                           =   6.28318 53071 8
```

Flags

Using the programs, **FLAGS** I or J (I1, ... , I9) with an integer values eliminate the complexity of quadrant determination in all I/O data traffic as shown in Figure 67; Figure 74: pp 164; 178, respectively.

Using positive `DOUBLE PRECISION` values of eastings, northings, bearings, N-(north)-latitude, E-(east)-longitude, **FLAGS** are set **TRUE.** Hence, using negative `DOUBLE PRECISION` values of northings, northings, bearings, S-(south)-latitude, W-(west)-longitude, **FLAGS** are set **FALSE.** Example I2. It is used as an integer value in the previous I/O subroutines:

```
I2 = FLAG = INTEGER true=(+1) or false=(-1)
```

Program Execution

The program statements and data are encoded in the computer in a special computer code. Once entered and stored by the computer, the program can be executed. The last output with an end-of-job message will be the successful result from the execution of the program. Error messages that you might encounter while using a compiler are discussed in [17.1.4.1].

Using FORTRAN Application Programs and Subroutines

The Fortran subroutine listings [On_CD] are followed by examples with an intermediate output, because situations may arise in which a program fails to operate as expected.

Note

In this book is the rounding function – intended for printing only - not applied in the subroutines. As a result, most examples [On_CD] are printed without rounding.

Method by Iteration

Geodesists have designed computing methods by a time-consuming iterative process, until the results of computing operations become internally consistent. Values calculated by an iteration method decrease rapidly, until the value is less than a certain predetermined threshold. The iteration will stop. In general, they seem to be attractive from a theoretical standpoint. On the contrary, such methods have practical limitations.

Some computing algorithms in this book calculate with or without an iterative solution.

19. International Organisations

General Outlook

Many international hydrographic mapping and land surveying companies in the field of geodesy were or are engaged in activities associated with charting and mapping, such as:

Association Internationale de Géodésique Réduite entre États Neutres	AIGREN
Baltic Geodetic Commission	BGC
Bundesamt für Kartographie und Geodäsie	BKG
Comité de Liaison des Geomètres Européenne	CLGE
Comité Européenne des Responsables de la Cartographie Officielle	CERCO
Europäische Gradmessung	EG
European Geo-Information	EGI
European Organisation for Experimental Photogrammetric Research	OEEPE
European Organisation for Geo-Information	EUROGI
Institut Géographique National	IGN
Inter-American Geodetic Survey	IAGS
International Association of Geodesy	IAG
International Astronomical Union	IAU
International Cartographic Association	ICA
International Civil Aviation Organisation	ICAO
International Council of Scientific Unions	ICSU
International Federation of Surveyors	FIG
International Hydrographic Bureau	IHB
International Society for Photogrammetry and Remote Sensing	ISPRS
International Union of Geodesy and Geophysics	IUGG
Meeting of Officials on Land Administration	MOLA
Mitteleuropäische Gradmessung	MG
National Geospatial Intelligence Agency	NGA
National Oceanic and Atmospheric Administration	NOAA
Ordnance Survey	OS
Ordnance Survey of Ireland	OSI
Pan American Institute of Geography and History	PAIGH
UN Office of Cartography	UNOOC
US Army Corps of Engineers	USACE

In general, these organisations were or are concerned with promotion and various topics, such as:

- dissemination of information on progress
- encouraging co-operation, and meetings
- general bathymetric chart of the oceans (GEBCO)
- general bathymetric chart of the world (GBCW)
- geographical information systems (GIS), development of facilities and programs
- international map of the world (IMW)
- progress in technical processes, and papers in journals
- standardisation of conventional signs, maps and charts
- training in general

19.1 International Union of Geodesy and Geophysics

International Council of Scientific Unions

In 1919 conseil internationale des recherches (CIR) founded the International Union of Geodesy and Geophysics (IUGG) or union géodésique et géophysique internationale (UGGI) with following organisations:

International Association of Geodesy, as from 1932 ______ (IAG)
International Association of Geomagnetism and Aeronomy ______ (IAGA)
International Association of Hydrological Sciences, as from 1922 ______ (IAHS)
International Association of Meteorology and Atmospheric Science ______ (IAMAS)
International Association of Seismology and Physics of the Earth's Interior ______ (IASPEI)
International Association of the Physical Sciences of the Ocean ______ (IAPSO)
International Association of Volcanology and Chemistry of the Earth's Interior ______ (IAVCEI)

The CIR changed its name into conseil internationale des unions scientifique (CIUS) or international council of scientific unions (ICSU) in 1931.

19.2 International Association of Geodesy

Mittel-Europäische Gradmessung or Central-European Triangulation

Geodesy has its development marked by epochs of geodetic co-operation on either national or on central-European level. Until 1860, some of the investigations to determine a more precise shape and the Figure of the Earth were initiated or received assistance by the foundation of scientific academies in Bavaria, France, Great Britain, Italy, Prussia and the Netherlands.

In Germany, the Prussian General Johann Jacob Baeyer's ambition was to unify all existing triangulation networks in Central Europe. Baeyer was perfectly equipped to lead an organisation as working on the frontiers of geodetic technology, combining vision and practicality. Researching the history of the Figure of the Earth, Baeyer investigated a program of systematic connection of various triangulations. Designing a map showing the proposal of the Mitteleuropäische Gradmessung (MG) or Central-European Triangulation with an astronomical network divided into nine polygons with the centre Altona, Berlin-1, Berlin-2, Copenhagen, Florence, Milan, Munich, Prague, and Roma (Figure 138).

In 1862 Austria, Prussia, and Sachsen founded the intergovernmental organisation MG in Berlin. The first general conference was held with official representatives of heads of states: Austria, Baden, Hannover, Hessen-Darmstadt, Hessen-Kassel, Italy, Mecklenburg, Norway, Poland, Prussia, Sachsen, Sachsen-Koburg-Gotha, Sweden and Switzerland in 1864. Representatives of Bayern, Belgium, Denmark, the Netherlands and Württemberg were invited, but did not attend the Conference. The Central Büreau (CB) was headquartered between 1866-1917 in Berlin (Baeyer, 1861, 1862a, b; Hunger, 1962; Torge, 1993; Völter, 1963).

As from 1870 president Baeyer coordinated 3-yearly *Allgemeinen Conferenz* (AC) or general conferences, and were used for discussions and geodetic studies about:

- astronomic observations at observatories
- encourage geodetic research
- imaginary mathematical representation of the Figure of the Earth
- least squares adjustments of triangulations
- mean sea level heights by mareographs
- theoretical methods of least squares adjustments
- definition of the units of length

Europäische Gradmessung or European Triangulation

MG changed its name to Europäische Gradmessung (EG) or European triangulation in 1867. The headquarters of the CB and the Royal Prussian Geodetic Institute (PGI) were in Potsdam, Germany. The countries or States Spain, Portugal, Russia (1866), France (1873), Romania (1881), Hamburg (1883), Austria, Belgium, Denmark, France, Greece, Italy, Mexico, Norway, Portugal, Spain, Sweden, Switzerland, the Netherlands, German States Baden, Bayern, Hamburg, Hessen, Sachsen, Württemberg (1886) Argentine, Chile, Greece, Japan, Mexico, Serbia and the USA (1889) joined the EG.

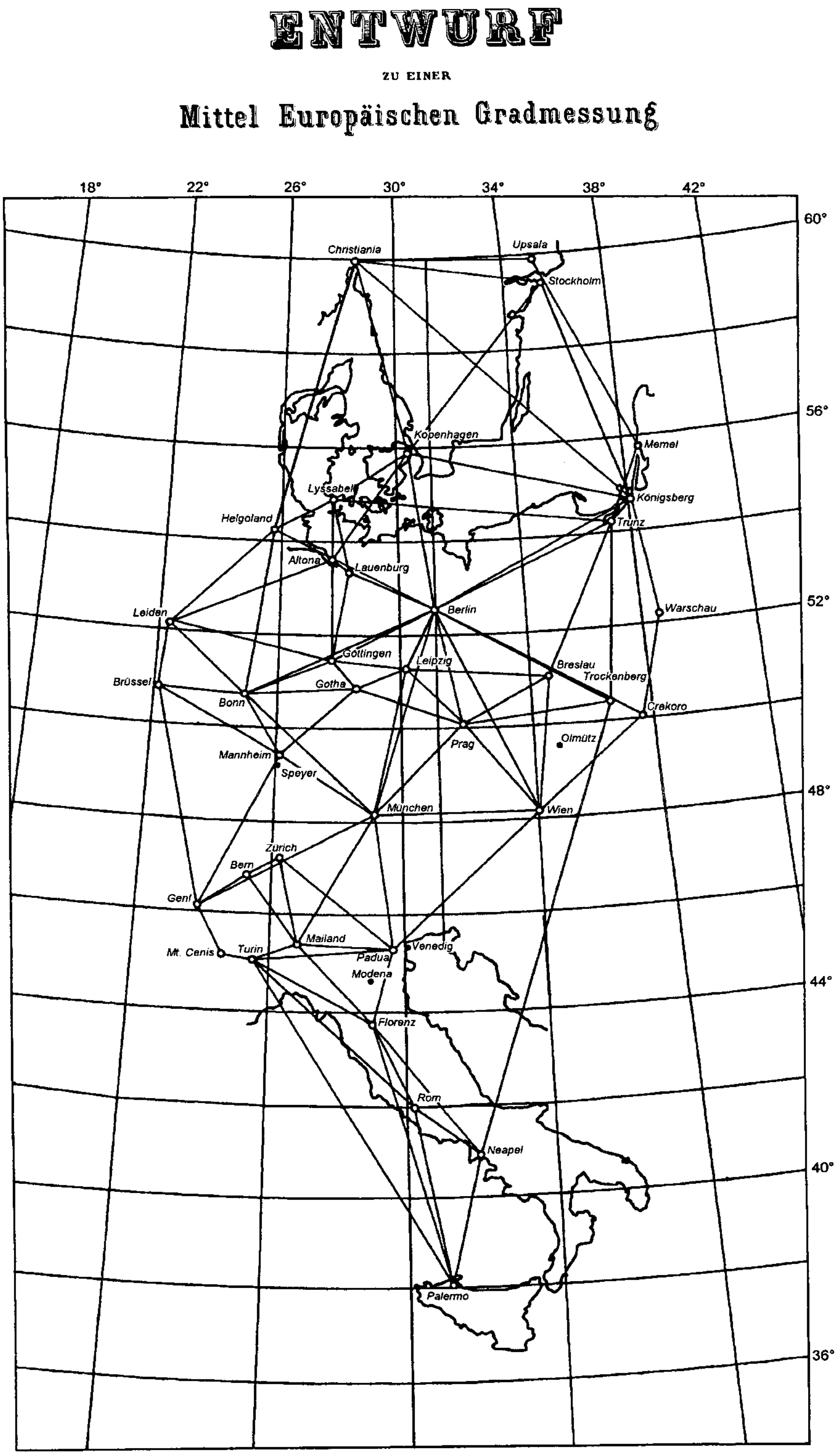


Figure 138: Baeyer's design of the Mittel-Europäischen Gradmessung

In the meantime, the Metre Convention (1875) [6.1], the Longitude and Time Convention, Rome (1883), [1.2] International Meridian Conference, Washington (1884) were held (Baeyer, 1861, 1862a; Hunger, 1962; Torge, 1993; Völter, 1963).

Internationale Erdmessung - IE

After the death of Baeyer in 1885, Helmert took over the presidency of the Internationale Erdmessung in 1886. In the mean time, the EG decided to rename the organisation to Internationale Erdmessung (IE). This led to an increase in international co-operation regarding absolute gravity measurements at Potsdam, astro-geodesy, Earth tide observations, error investigations of instruments, establishment of a world gravity system, gravimetric activities, measurements with Sterneck's pendulum apparatus, absolute gravity measurements at Potsdam, the establishment of a world gravity system (*Vienna* and *Potsdam* system), gravity measurements on the oceans, theoretical error investigations, collection and publication of gravity data, physical geodesy and establishing the true geodetic science. The IE was discontinued in 1916 due to World War I (1914-1918) and the death of Friedrich R. Helmert in 1917.

Association Internationale de Géodésique Réduite entre États Neutres

Denmark, Norway, Spain, Sweden, Switzerland, the Netherlands, and the USA (until 1917) founded the *Association Internationale de Géodésique Réduite entre États Neutres* (AIGREN) - a non-governmental organisation - between 1916-22.

Baltic Geodetic Commission

The Baltic Geodetic Commission was founded in 1924 by its members Estonia, Finland, Germany, Latvia, Lithuania, Polen, Sweden. The USSR (now Russia) joined between 1929 and 1939.

International Association of Geodesy

Until 1924, geodesy is represented by the l'Association de Géodésique (AG) and since 1932 by the International Association of Geodesy (IAG) within the International Union of Geodesy and Geophysics (IUGG), founded in 1919 (Baeyer, 1861, 1862a, b; GH, 1992; Hunger, 1962; Torge, 1990, 1993; Schwarz, 2001; Völter, 1963).

The objectives of the IAG are (GH, 1992):

- to promote the study of all scientific problems of geodesy and encourage geodetic research
- to promote and coordinate international co-operation in this field, and promote geodetic activities in developing countries
- to provide, on an international basis, for discussion and publication of the results of the studies, researches and works indicated above.

IAG promotes geodesy, publication of studies, publication of a magazine (*Bulletin Géodésique*), and research through special commissions (SC), including five technical commissions (TC):

- section I: ____ positioning
- section II: ____ advanced space technology
- section III: ____ determination of the gravity field
- section IV: ____ general theory and methodology
- section V: ____ geodynamics

International Centre for Earth Tides

The International Centre for Earth Tides (ICET) is a scientific organisation in the field of global geodynamics. It is an active member of the IAG, section V, and of IUGG, which it is a member of the ICSU. It has its headquarters in Brussels, due to the generosity of the Royal Observatory of Belgium, Brussels.

Ocean tides are the alternating rise and fall of the surface of the seas and oceans. The tides are due to the gravitational attraction of the Moon and the Sun on the rotating Earth. Gravitational attraction also affects the Figure of the Earth from its centre to its surface, creating deformations and stresses, so-called Earth tides, perturbing all kinds of precise measurements. The regular movements of the oceanic waters associated with the tides cause Load Tides. These are generated by the deformation of the Earth's crust under the pressure of the oceanic tidal load, moreover also the direct attraction and also the redistribution of the water masses. The magnitude of the Earth tides and of the loading effect can reach up to 40 cm and 10 cm for the radial displacement, respectively. Due to the considerable increase of the accuracy of the various observational systems (super-conducting gravimeter, GPS. VLBI) used in geophysical research, these Earth tides and loading effects appear like disturbing effects that must be taken out before doing further studies.

The body Earth tides can be modelled to a precision of few tenth of percent due to observations of the Earth and ocean tide loading from super-conducting and absolute gravimeters and from GPS measurements. It means that the tide displacement can be computed to an accuracy of about 0.2 cm. Nevertheless, the uncertainty in the ocean tide loading computation is still very high between 10 and 20 percent. The main causes are the ocean tide model errors.

A *Bulletin d' Informations Marées Terrestres* is published two or three times a year by ICET. A bibliography is published with more than 6000 references, including translations of Russian and Chinese papers.

The International Centre for Earth Tides (ICET) establishes and maintains a database from about 360 worldwide tidal gravity stations, including data from tiltmeters and extensometers. Hourly values, main tidal waves, residual vectors, oceanic attraction, and loading vectors are obtainable for each station. ICET arranges tidal predictions for any place and time, which are essential for field gravimetry, absolute gravity measurements and for tilt measurements. Predictions can be computed based either on elastic Earth models and oceanic co-tidal maps, or on basis of the results of direct measurements.

Furthermore, ICET is the centre for the archiving, exchanging and processing of the global geodynamics project (GGP) data. It monitors the changes in the Earth's gravity field at periods of seconds and longer by means of a network of super-conducting gravimeters taken over a time span of six years. The ICET provides assistance for the set-up of new stations, calibrations of the instruments, data processing and also for tidal analyses (Francis, 1997).

International GPS Service for Geodynamics

An international GNSS service for geodynamics (IGS) organisation includes an international governing board, a central bureau, three global data centres, five regional data centres, and eight analysis centres with a coordinator. Currently more than fifty institutions and organisations contribute to the IGS.

In 1993, the international association of geodesy (IAG) officially established the IGS to consolidate worldwide permanent GPS tracking networks under a single organisation, including the cooperative international GPS network (CIGNET) and the Fiducial Laboratories for an International Natural science Network (FLINN) [19.2] guided by NOAA and NASA, respectively.

Essentially, since January 1994, the IGS - in partnership with IERS - coordinates all operations and analysis of a global GPS-station-network. Starting global network associate analysis centres (GNAACs) with millimetre precision was possible with a reliable update on a frequent basis.

The primary IGS products include high-quality GPS orbits, satellite clock information, Earth orientation parameters, and ITRF station positions and velocities. The IGS supports worldwide geodetic positioning with respect to the international terrestrial reference frame (ITRF). All data are exchanged and stored in the receiver independent exchange format (RINEX). IGS contributes essential data to the IERS Reference System, including precise geocentric Cartesian station positions and velocities - global polyhedron - and Earth orientation parameters (EOP).

IGS supports worldwide geodetic positioning with respect to primary IGS products:

- high-quality GPS orbits
- GPS-satellite clock information
- Earth orientation parameters, including polar motion, UT1-UTC rate, station coordinates - annually
- ITRF station positions, velocities and EOP information

International Governing Board contributes data to:

- International Earth Rotation Service (IERS) central bureau [19.5]
- IGS Central Bureau Information System (CBIS)
- IGS network operation sub-groups to facilitate GPS extension of the terrestrial reference frame

The IGS sub-bureau for data systems through global, regional, operational data centres provides received/retrieved GPS-station data. It provides on-line archive for analysis centre, and IGS products.
IGS analysis centres produce GPS ephemerides, station coordinates, Earth rotation parameters, special products, official product generation and comparison.
IGS network operation sub-group facilitates station implementation and operation, monitoring and control of stations, status and communications.

IGS central bureau provides services for the IGS user society:

- coordination and management
- compliance to IGS standards
- monitoring all elements
- supporting IERS using GPS
- IGS communications and documentation

The IGS- user society expects high precision, practical geodetical products from IGS analysis centre distributed products. IGS information is provided through the central bureau, located at the Jet Propulsion Laboratory of CIT (JPL) in Pasadena, California, USA.

19.3 Fédération Internationale des Géomètres

Fédération Internationale des Géomètres (FIG) - (or International Federation of Surveyors) organisation was founded in 1878. FIG's work depends on volunteers. Peter Dale: " ... *the strengths of the FIG lie in the enthusiasm and dedication of people who act, not because they are paid, but because they believe in what they do ...* " (Dale, 1998). It has its headquarters - called the FIG permanent office (FIGPO) *The Surveyors House* - in Copenhagen, due to the generosity of Den Danske Landinspektorforening (DDL), which provides it office space.

Representing individual members of member associations in more than 90 countries, FIG is a non-government organisation on surveying activities. An executive committee manages the day-to-day affairs. The FIG is particularly concerned to strengthen professional associations of surveyors, especially in countries in economic transition and in the less developed areas of the world. FIG's objectives are such as:

- to provide an international forum for the exchange of information
- publishing guidelines on continuing professional development, workshops and seminars around the world
- to promote the disciplines of surveying, particularly in developing countries and in countries in economic transition
- to help in the development of national associations of surveyors by promoting professional standards, codes of ethics and the exchange of surveying personnel
- to encourage high standards of education for surveyors by easing continuing professional development, and encouraging the proper use of appropriate technology.

FIG addresses technical and scientific objectives through the work of commissions.

commission 1 - professional standards and practice

commission 2 - professional education
commission 3 - land information systems
commission 4 – hydrography
commission 5 - positioning and measurement
commission 6 - engineering surveys
commission 7 - cadastre and land management
commission 8 - spatial planning and development
commission 9 - valuation and the management of real estate
ad hoc commission (10) on the History of Surveying
ad hoc commission (11) on Construction Economics and Management

19.4 International Hydrographic Organisation

The International Hydrographic Organisation (IHO) is an intergovernmental body. It was formed in 1921. It has its headquarters - called the International Hydrographic Bureau (IHB) - in Monaco, due to the generosity of the government of that country. However, the IHO is not a member of the United Nations group of organisations.

Since 1978, the IHO convention defines its basic objectives in the field of automation, cartography, equipment, geodesy, hydrographic vessels, hydrography, instruments, navigation, oceanography, photogrammetry, radio aids, and on the history and organisation of National Hydrographic Offices (NHOs). IHO cooperates with EU, FAO, FIG, IALA, ICA IMO, IOC, UN, and WMO. The main standards published by the IHO are: S-44, Standard for Hydrographic Surveys, IHO S-52, Standard for Electronic Charts, IHO S-57, Standard for Transfer of Digital Hydrographic Data, and IHO S-100, Standard for Geospatial for Transfer of Digital Hydrographic Data.

IHO objectives are:

- coordination of the NHOs activities
- uniformity in nautical charts, documents, and the development of standards for digital data exchange.
- IHO organisation has consultative activities of technical nature. Its activities shall not include questions of international political affairs.

The *International Hydrographic Review* contains articles on hydrography, oceanography, cartography, and navigation. It is published twice a year in English and French editions.

International Hydrographic Bulletin reports the work undertaken by the IHB and the world hydrographic community, covers recent developments in hydrographic instrumentation and training programmes, describes new survey vessels.

Address:

International Hydrographic Bureau - Directing Committee, BP 445 - 4, Quai Antoine 1^{ER},
MC 98011 Monaco Cedex (Kerr, 1997)

19.5 International Earth Rotation Service

IERS Terms of Reference

The International Earth Rotation Service (IERS) was established in 1987 by IAU and IUGG and it started operation on January 1, 1988. It replaces the international polar motion service (IPMS) and the Earth-rotation section of the Bureau International de l'Heure (BIH); the activities of BIH on time are continued at Bureau International des Poids et Mésures (BIPM). IERS is a member of the Federation of Astronomical and Geophysical data analysis Services (FAGS).

IERS should provide the information necessary to define a conventional Terrestrial Reference System (TRS), a conventional Celestial Reference System (CRS), and relate them as well as their frames to each other and to other reference systems used in determination of the Earth orientation parameters.

IERS is responsible for:

- defining and maintaining a conventional terrestrial reference system based on observing stations that use the high-precision techniques in space geodesy
- defining and maintaining a conventional CRSystem based on extragalactic radio sources, and relating it to other celestial reference systems
- determining the Earth orientation parameters connecting these systems, the terrestrial and celestial coordinates of the Pole and universal time
- organising operational activities for observation and data analysis, collecting and archiving appropriate data and results, and disseminating the results to meet the needs of users.

IERS consists of a Central Bureau (CB) and coordinating centres for each of the principal observing techniques, and is supported by many other organisations that contribute to the tasks of observation and data processing.

The coordinating centres are responsible for developing and organising the activities in each technique to meet the objectives of the service. The CB combines the various types of data collected by the service, and disseminates the appropriate information on Earth-orientation, the terrestrial and celestial reference systems to the user community. It can include sub-bureaux for the accomplishment of specific tasks.

The CB decides and disseminates the announcements of leap seconds in UTC and values of DUT1 to be transmitted with time signals.

IERS Organisation

The principal centres of IERS are as follows:

IERS Central Bureau

Bundesamt für Kartographie und Geodäsie (BKG), Frankfurt am Main, previously Observatoire de Paris

- Terrestrial Frame Section, IGN, St Mandé, France
- Earth Orientation Section, Observatoire de Paris
- Celestial Frame Section, Observatoire de Paris.

IERS Sub-Bureau

- Rapid Service and Predictions, National Earth Orientation, US Naval Observatory (USNO), Washington DC
- Atmospheric Angular Momentum, Climate Analysis Centre, NOAA/National weather service, Washington, DC.

IERS Coordinating Centres

- VLBI Coordinating Centre, NGSG, Astronomy and Satellite Branch, Silver Spring, MD, USA
- LLR Coordinating Centre, OCA / CERGA, Grasse, France
- GPS Coordinating Centre, Jet Propulsion Laboratory, Pasadena, CA, USA
- SLR Coordinating Centre, Centre for Space Research, the University of Texas, Austin, TX, USA.

IERS Reference System

Rotation of the Earth and related space-time References. The IERS Reference System is composed of two parts: IERS Standards and IERS Reference Frames.

The IERS standards used for a Report are the IERS Standards 1993 |[18], published in IERS Technical Note No xx. These are a set of constants and models used by the IERS analysis centres for lunar laser ranging (LLR), satellite laser ranging (SLR), very long baseline interferometry (VLBI), global positioning system (GPS), and by the IERS central bureau in the combination of results (IERS, 1993a, 1993b). The values of the constants are adopted from recent analyses. In some cases, they differ from the current IAU and IAG conventional ones. The models represent, in general, the state of the art in the field concerned.

The IERS reference frames consist of the IERS Celestial Reference Frame (ICRF) and IERS Terrestrial Reference Frame (ITRF). Both frames are realised through lists of coordinates of fiducial stations, terrestrial sites or compact extragalactic stellar or radio sources.

IERS Terrestrial Reference Frame

Origin, reference directions and the scale of ITRF are implicitly defined by the coordinates adopted for the terrestrial sites. The origin of the ITRF is located at the centre of mass of the Earth with an uncertainty of 10 cm. unit of length is the metre (SI). IERS Reference Pole (IRP) and IERS Reference Meridian (IRM) are consistent with the corresponding directions in the BIH Terrestrial System (BTS) within 0".003. The BIH Reference Pole was adjusted to the Conventional International Origin (CIO) in 1967; it was then kept stable independently until 1987. The uncertainty of the tie of the BIH Reference Pole with the CIO was 0".03.

IERS Celestial Reference Frame

The origin of the ICRF is at the barycentre of the Solar system. The direction of the polar axis is the one given for epoch J2000.0 by the IAU 1976 Precession and the IAU 1980 Theory of Nutation. The origin of Right Ascensions is in agreement with that of the FK5 (Fundamental Katalog 5) within 0".01.

IERS Earth Orientation Parameters

The IERS Earth orientation parameters (EOP) are the parameters, which describe the rotation of the ITRF to the ICRF, in conjunction with the conventional Precession-Nutation model. They model the unpredictable part of the Earth's rotation.

IERS Coordinates of the Pole

X, Y are the coordinates of the celestial ephemeris Pole (CEP) relative to the IRP, the IERS Reference Pole. The CEP differs from the instantaneous rotation axis by quasi-diurnal terms with amplitudes under 0".01. The X-axis is in the direction of IRM, the IERS Reference Meridian; the Y-axis is in the direction 90° West longitude.

IERS Celestial Pole offsets

$d\psi$, $d\varepsilon$ are offsets in longitude and in obliquity of the celestial Pole with respect to its position defined by the conventional IAU Precession / Nutation models.

19.6 Participants in National Geodetic Satellite Program

In 1965, NASA started the US national geodetic satellite program (NGSP) by setting up an extensive program of observation and data reduction [3.2]. The major participants in this program were (Henriksen, 1977a, b):

- Air Force Cambridge research laboratories
- Applied Physics Laboratory - the Johns Hopkins University (APL)
- Defense Mapping Agency - aerospace centre
- Defense Mapping Agency - topographic command
- Department of Commerce - National Ocean Survey (NGS)
- Department of Defense (DoD)
- Engineer Topographic Laboratories (ETL)

[18]"IERS Standards" is a yearly report published by IERS with detailed values or constants in IERS "Technical Note"

- Goddard Space Flight Centre (GSFC)
- Jet Propulsion Laboratory - California Institute of Technology (CIT)
- National Aeronautics and Space Administration (NASA)
- Naval Surface Warfare Centre/Dahlgren Laboratory (NSWC)
- Ohio State University (OSU)
- Smithsonian Astrophysical Observatory (SAO)
- University of California (UoC)
- Wallops Flight Centre (WFC).

During the lifetime of the NGSP, some of these investigators left the program and were replaced by others.

The objectives of the NGSP were to get:

- sufficiently improved positions for satellite-tracking stations that errors in connections between major Datums could be materially reduced
- a better determination of the Earth's gravitational field out to the 15th degree and order in the expansion in spherical harmonies
- the quality of the data provided by the various tracking stations participating in the program would have to be determined.

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Index of Subjects

H

M

N

Appendix

Installing and Using Applications

In the period 1957-2006, many Fortran systems were on hand for various hard- and software configurations, from "*simple*" mainframes to Cray-vector-parallel computers, including IBM-PCs. For customising the computer system, see first of all: the Fortran-handbook for the host operating system, specifics for installing FORTRAN, Math options, Memory models, Libraries, and Additional instructions for the CD-ROM and the Mouse-driver (optional).

Using a 100% IBM-compatible computer running disk operating system (DOS) version 7.0, and a Microsoft FORTRAN Professional Development System version 5.1 - using standard options only - this handbook provides the Fortran (F77) geometrical application software with some examples (no utility or library) [On_CD].

System Requirements

FORTRAN geometrical application mainroutines are merged with subroutines, build, and compiled for use. The minimum system requirements are:

- a DOS XT / AT or an IBM-PS/2 or compatible computer of modest capacity
- microprocessor: Intel 8088/8087, or higher
- memory: 640 KB or more is recommended
- operating system: PC-DOS or MS-DOS version 3.3, or IBM OS/2 version 1.1; or higher
- a monochrome display monitor
- a FORTRAN compiler system (not supplied)
- a data printer
- an optional a hard disk drive with 10 MB of free space for using the application programs and data storage
- a CD ROM station is recommended, using the host operating system PC-DOS or MS-DOS version.7.0 or higher

Actual requirements depend on the computer hardware, installed programming tools, and the libraries.

Application Software Requirements

The use and installation requires the installation of the FORTRAN compiler by means of a Fortran SETUP utility. Main FORTRAN routines M_*.FOR|[19] are provided - without subroutines - in print [18.1]; ... ; [18.22] in the book as a general FORTRAN source code. In addition, the book is supplied with the enclosed CD-ROM that contains the required subroutines. These are available as FORTRAN source code (S_*.FOR), with data text examples. The geometrical mainroutines contain a list of arguments that can be retrieved by the subroutines. These subroutines are ready for merging with the mainroutines. At that moment, the merged routines are ready for loading and further use into directory \FORFILES\ or elsewhere. Furthermore, the FORTRAN routines were merged, compiled, and made available as "*ready-to-run*" programs (A_*.EXE), which produce data text files ready for printing. All program applications (A_*.EXE) on CD-ROM are available as "*ready-to-print*" data text files (A*.TXT), allowing for easy software tests and extensive checks by the user.
Hence, FORTRAN geometrical application programs and examples appear as follows:

- as FORTRAN geometrical mainroutines in the book [18] as \M_...... .FOR, and in the On_CD directory
- the subroutines as: ____________\A_......\S_... .FOR
- the examples as: ______________\A_......\A... .TXT
- the ready-to-run-programs as: ___\A_......\A_... .EXE
- three complete FORTRAN application programs A_... .FOR are found in the On_CD directory (see the next page)

---

[19] * = a value between 01name and 22name

Input / Output Data

After loading the program, enter the file name and start the program. For double precision: input all data by using a decimal point (e.g. 1.00). The input data entered in a running program are shown at the monochrome-display only. The input data and the computed output data are in an output ASCII text file, ready for printing, such as the file A18GK000.TXT.

Imperative is the use of Fortran statement: *line OPEN* in the FORTRAN application mainroutines, such as:

- line `OPEN (11,FILE='\FORFILES\A18GK001.TXT', STATUS='NEW'` (produces a new output file, ready for printing) or
- line `OPEN ( *,FILE= \FORFILES\A18GK001.TXT', STATUS='NEW'` (produces a output file for display on screen only)
- line `OPEN (11,FILE= \FORFILES\A18GK001.TXT', STATUS='OLD'` (overrides any previous output file).

A quick overview of the individual routines and programs in the book and on the enclosed CD-ROM are given in (Table 32):

mainroutine book	subroutine CD-ROM	appl. program CD-ROM	book chapter	description of application programs
-	-	A_01BDAR.FOR	[18.1]	Flat Earth - Bearing, Distance and Polygonal Area
M_01BRDI.FOR	S_01BRDI.FOR		[18.1]	Coordinates to Grid Bearing and Distance
-	-	A_01POLA.FOR	[18.1]	from Coordinates to Polygonal Area
-	-	A_02BASX.FOR	[18.2]	Baseline Crossing with LS. Parabolic Curve Fit
M_03MARC.FOR	S_03MARC.FOR	-	[18.3]	Computation of Various Meridional Arcs
M_04ELLI.FOR	S_04ELLI.FOR	-	[18.4]	Ellipsoid Constants, and Meridional Arcs
M_05ARSQ.FOR	S_05ARSQ.FOR	-	[18.5]	Quadrilateral Area Calculation for Ellipsoid
M_06ARPY.FOR	S_06ARPY.FOR	-	[18.6]	Polygonal Ellipsoidal Area Calculation
M_07PARA.FOR	S_07PARA.FOR	-	[18.7]	Length of Parallel
M_08GEOR.FOR	S_08GEOR.FOR	-	[18.8]	Geodetic Reference System
M_09BILI.FOR	S_09BILI.FOR	-	[18.9]	Bi-linear Interpolation
M_10STRM.FOR	S_10STRM.FOR	-	[18.10]	Datum S-Transformation
M_11BDGK.FOR	S_11BDGK.FOR	-	[18.11]	Long Line Calculation - Method Kivioja
M_12GBDK.FOR	S_12GBDK.FOR	-	[18.12]	Long Line Calculation - Method Kivioja
M_13BDGV.FOR	S_13BDGV.FOR	-	[18.13]	Forward Long Line - Method Vincenty
M_14GBDV.FOR	S_14GBDV.FOR	-	[18.14]	Inverse Long Line - Method Vincenty
M_15POLY.FOR	S_15POLY.FOR	-	[18.15]	Polyeder Mapping Projection
M_16GAUS.FOR	S_16GAUS.FOR	-	[18.16]	Conversion Sphere to Ellipsoid and Vice-Versa
M_17NM00.FOR	S_17NM00.FOR	-	[18.17]	Normal Mercator projection
M_18GK00.FOR	S_18GK00.FOR	-	[18.18]	Gauss-Krueger projection
M_19LC00.FOR	S_19LC00.FOR	-	[18.19]	Lambert's Conical projection
M_20OM00.FOR	S_20OM00.FOR	-	[18.20]	Hotine's Oblique Mercator projection
M_21RM00.FOR	S_21RM00.FOR	-	[18.21]	Rosenmund's Oblique Mercator projection
M_22ST00.FOR	S_22ST00.FOR	-	[18.22]	Oblique Stereographic Conformal projection
	S_23IO00.FOR	-	[18.23]	360° sexagesimal / 400g centesimal I/O Subroutines

Table 32: Organisation of Fortran geometrical routines