Studies in German Idealism Volume 11

Marco Giovanelli

Reality and Negation - Kant's Principle of Anticipations of Perception

An Investigation of its Impact on the Post-Kantian Debate

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VOLUME 11

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by

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ISSN 1571-4764
ISBN 978-94-007-0064-2 e-ISBN 978-94-007-0065-9
DOI 10.1007/978-94-007-0065-9
Springer Dordrecht Heidelberg London New York

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

a mio padre, *in memoriam*

Abbreviations

Original German Texts (abbreviations are in alphabetical order)[1]

AA	Kant, Immanuel. *Kant's gesammelte Schriften*. eds. Preussische Akademie der Wissenschaften, Berlin-Brandenburgische Akademie der Wissenschaften and Akademie der Wissenschaften in Göttingen (Berlin: Reimer, 1900)
CGW	Cassirer, Ernst. *Gesammelte Werke*. ed. Birgit Recki (Hamburg: Meiner, 1998)
CW	Cohen, Hermann. *Werke*. ed. Helmut Holzhey (Hildesheim: Olms, 1977)
FGA	Fichte, Johann Gottlieb. *Gesamtausgabe der Bayerischen Akademie der Wissenschaften*. eds. Reinhard Lauth, Hans Jacob and Hans Gliwitzky (Stuttgart: Frommann-Holzboog, 1962)
FSS	Fries, Jakob Friedrich. *Sämtliche Schriften*. eds. Gert König and Lutz Geldsetzer (Aalen: Scientia Verlag, 1967)
GM	Leibniz, Gottfried Wilhelm. *Leibnizens mathematische Schriften*. ed. Carl Immanuel Gerhardt (Halle: Schmidt, 1850)
GP	Leibniz, Gottfried Wilhelm. *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*. ed. Carl Immanuel Gerhardt (Berlin: Weidmann, 1875)
HKA	Schelling, Friedrich Wilhelm Joseph von. *Historisch-kritische Ausgabe*. eds. Hans Michael Baumgartner, Wilhelm G. Jacobs, Hermann Krings and Hermann Zeltner (Stuttgart: Frommann-Holzboog, 1976)
HSW	Herbart, Johann Friedrich. *Sämtliche Werke*. eds. Karl Kehrbach and Otto Flügel. Neudruck ed (Aalen: Scientia Verlag, 1964)
HW	Hegel, Georg Wilhelm Friedrich. *Werke in zwanzig Bänden*. eds. Eva Moldenhauer and Karl Markus Michel (Frankfurt, Main: Suhrkamp, 1969)
MGW	Maimon, Salomon. *Gesammelte Werke*. ed. Valerio Verra. Reprogr. Nachdr. ed (Hildesheim: Olms, 1965)
SW	Schelling, Friedrich Wilhelm Joseph von. *Sämmtliche Werke*. ed. Karl Friedrich August Schelling (Stuttgart, Augsburg: J.G. Cotta, 1856)

[1]Unless a reference is provided to an existing English translation, all quotes referring to the original German texts are my translation.

Most Frequently Cited English Translation

A	Kant, Immanuel. *Critique of Pure Reason*, tr. Paul Guyer and Allen W. Wood (Cambridge, New York: Cambridge University Press, 1998)
B	Kant, Immanuel. *Critique of Pure Reason*, tr. Paul Guyer and Allen W. Wood (Cambridge, New York: Cambridge University Press, 1998)

Contents

Introduction

Kant's principle of the Anticipations of Perception has been rightly called the "forgotten principle."[2] Critical literature dedicated to the other "principles of pure understanding" in particular to the Analogies of Experience, is nearly endless. In contrast, disregarding allusions to the principle in monographs and commentaries that aim to give an overall picture of the *Critique of Pure Reason (Kritik der reinen Vernunft)*,[3] contributions dedicated specifically to the Anticipations of Perception are relatively few. This disparity in treatment seems completely justified by the fact that in the structure of the *Critique of Pure Reason*, or even within the chapter entitled "System of all Principles of Pure Understanding" alone, the relation between reality and negation that is discussed the Anticipations of Perception seems to be a minor question. The question certainly cannot be placed on the same level as other great topics in Kant's work, in particular that of the cause-effect connection, which seems contain the very meaning of the Copernican revolution.

However, the post-Kantian debate, which extends from the end of the 1700s to the epoch of the so-called "return to Kant," gives a completely different impression. The reformulation of the relation between reality and negation that Kant introduces proves to be an indispensable component of "critical" thought that distinguishes it from every form of "dogmatic" metaphysics, especially that of the Leibnizian-Wolffian tradition. Moreover, the relation was understood by post-Kantian philosophers as a breach, allowing them to penetrate Kant's system in order to reconstruct it from the inside, rescuing it from the seemingly inescapable difficulties that it had fallen into.

The guiding hypothesis of the present work is that if we are to fully understand the Anticipations of Perception's meaning, we need not only to examine Kant 's

[2]See "The Forgotten Principle: Kant's Anticipations of Perception" is the title of Theodore E. Uehling's intervention at the Fifth International Congress on Kant held in Mainz, Germany in 1981: Uehling, Theodore E. "The Forgotten Principle: Kant's Anticipations of Perception." In Akten des 5. Internationalen Kant-Kongresses Mainz. April 4–8, 1981. ed. Gerhard Funke. (Bonn: Bouvier, 1981).

[3]I have provided the original German title following the first reference to texts that have been translated into English. Vice versa, I have provided an approximate English translation of the title following the first reference to any untranslated text.

text closely and attentively (Chapter 1 and 2), as far as possible, but also elucidate the impact that the few pages that Kant dedicates to the second of his synthetic principles in the *Critique of Pure Reason* had on succeeding philosophy, from so-called German "Classical" Idealism (Chapter 3) to Neo-Kantian Idealism (Chapter 4). I certainly do not pretend to cover every thinker who deals with or refers to this theme in the abundant essays and publications that, directly or indirectly, treat Kant's theoretical philosophy. Nor do I intend to retrace every interpretation that has been proposed during the two centuries following the appearance of Kant's *opus maius*. Rather, I aim to identify the essential philosophical nucleus of the Anticipations of Perception in what remains constant despite the various transformations that it undergoes in Post-Kantian philosophy. Even in authors so distinct that any comparison seems impossible at first sight, one discovers an astounding affinity between themes and problems, so much so that the very process of comparison reveals the most appropriate means of approaching, with ever more precision, the fundamental problem raised by the Anticipations of Perception.

Here, one can observe the "intentional tension"[4] that characterizes critical philosophy. The interpreter is obliged to go beyond Kant in order to access what he alludes to only indirectly. On the one hand, such an approach inevitably tends to strain the meaning of Kant's text. On the other hand, I will show that only in this manner is a genuinely "philosophical" understanding of the problem possible, a problem that seems especially hard to grasp on the basis of philological analysis alone.

Acknowledgements There are many people who have contributed to making this book possible. First and foremost, I would like to thank my Ph.D. advisors, Andrea Poma, for giving me both freedom and guidance to conduct my research and Helmut Holzhey for early encouragement to work on this topic as well as for his hospitality at the University of Zürich, where most part of this book was written. Through their own work and through discussions with me, they have been a continuing source of inspiration and advice over the past years. This book owes a great deal to Reinier Munk for his interest in my work and for accepting the present volume in his distinguished series. Anonymous referees' comments helped to improve the manuscript in many respects and are gratefully acknowledged. Lastly, I am happy to have this occasion to express my deep gratitude to my friend Roberto Toledo for his patient and meticulous work in translating the Italian manuscript into English and for his numerous suggestions that significantly improved it.

[4]Mathieu, Vittorio. *La filosofia transcendentale e l' 'Opus postumum' di Kant* (Turin: Filosofia, 1958), 6.

Chapter 1
From the Anticipations of Perception to the Dynamic Conception of Matter

1.1 Anticipation as *A Priori* Knowledge

In the first edition of the *Critique of Pure Reason*, the "Proof" (*Beweis*)[1] of the second of the synthetic principles, the Anticipations of Perception, opens with a definition of the term "anticipation": "All knowledge by means of which I am enabled to know and determine *a priori* what belongs to empirical knowledge may be entitled an anticipation" (B208). Based on this definition, "anticipation" should be understood, firstly, as *a priori* knowledge, thus independent of experience. Kant's claim that "this is undoubtedly the sense in which Epicurus employed the term πρόληψις (B208) is probably the fruit of a misinterpretation, perhaps drawn from Cicero.[2] In Epicurus' gnoseology, the "prolepses" indicate those general concepts (man, horse, etc.) that, in as much as they are derived from the repetition of representations similar to one another, allow one to know in advance that which characterizes certain objects on the basis of previous representations of similar objects.[3] For Kant, in contrast, the "anticipations" should be understood as *a priori* knowledge in a rigorous sense; not as knowledge "independent of this or that experience, but as knowledge absolutely independent of all experience" (B2–).

Moreover, the above definition clarifies that the "anticipations" are not aimed directly at things, but only at that which "belongs to … knowledge" (B208). At first glance, the definition of the term "anticipation" seems to recall that of "transcendental knowledge" from a celebrated passage from the Introduction to the *Critique of Pure Reason* a knowledge that "is occupied not so much with objects as with the

[1] The title "Proof" (*Beweis*) was added in the second edition, as is the case with the proof of the "Analogies of Experience" (see A176 and B218) and of the of the third analogy (see A210 and B256).

[2] See Cicero, *De natura deorum*, 1:16, 43 (255 Us).

[3] See Diog Laertius 9:33 (255 Us).

M. Giovanelli, *Reality and Negation – Kant's Principle of Anticipations of Perception: An Investigation of Its Impact on the Post-Kantian Debate*, Studies in German Idealism 11,
DOI 10.1007/978-94-007-0065-9_1, © Springer Science+Business Media B.V. 2011

mode of our knowledge of objects [unserer Erkenntnißart von Gegenständen][4] in so far as this mode of knowledge is to be possible *a priori*" (B25). However, in respect to this definition of "transcendental knowledge" as *a priori* knowledge that does not concern objects, but rather our manner of cognizing them, Kant's further clarification seems to characterize the "anticipations" as such: the "anticipations" do not determine *a priori* the qualities that belong to all knowledge in general, but only those that belong to "empirical knowledge" (B208).

Therefore, not all "transcendental" knowledge can be defined as "anticipations." Kant's first choices for possible examples of anticipations are the *a priori* determinations of space and time: "we might very well entitle the pure determinations in space and time, in respect of shape as well as of magnitude, anticipations of appearances, since they represent *a priori* that which may always be given *a posteriori* in experience" (B209). Kant's choice of example seems to suggest that to "anticipate" is not only to determine *a priori* something about an object of pure thought (which is generally admitted as possible) but also to determine *a priori* that which can only be "given" and thus seems to exclude by definition any *a priori* determination.

Kant distinguishes "*intuition*, through which [an object] is given, [and] secondly, *concept*, through which an object is thought corresponding to this intuition" (B125)[5]. One of the defining characteristics of critical philosophy is the admission that not only understanding, through which objects in general are thought, but sensibility itself, through which objects are given to us in single concrete examples, also admits *a priori* principles. The first part of the "Doctrine of Elements" (*Elementarlehre*), the "Transcendental Aesthetic" (*Transzendentale Ästhetik*) is precisely "[t]he science of all principles of *a priori* sensibility … in distinction from that part which deals with the principles of pure thought, and which is called transcendental logic" (B35–). Thus, not only are there *pure concepts* of the understanding

[4] Unless specified otherwise, all brackets inserted into quotations are the author's.

[5] The problem of the distinction between concept (*Begriff*) and intuition (*Anschauung*) has been especially discussed in English language literature on Kant. See above all Hintikka, Jakko. "On Kant's notion of intuition (Anschauung)." In *The First Critique: Reflections on Kant's Critique of Pure Reason*. eds. Terence Penelhum and John James Macintosh. (Belmont, California: Wadsworth, 1969); Parsons, Charles. "Kant's Philosophy of Arithmetic." In *Mathematics in Philosophy: Selected Essays*, 110–49. (London: Cornell University Press, 1983), 112; Howell, Robert. "Intuition, Synthesis, and Individuation in the Critique of Pure Reason," *Noûs* 7, no. 3 (1973): 207–32. Houston Smit effectively summarized the major positions in this debate: "Jaakko Hintikka ascribes to Kant the view that an intuition is simply a singular representation, the counterpart of a singular term in the latter's system of representation … and that the immediacy criterion on intuition is merely a logical corollary of the singularity criterion. … Charles Parsons agrees with Hintikka that, in being singular, an intuition is the analogue of a singular term … Parsons maintains, then, that the immediacy criterion is not merely a logical corollary of the singularity criterion, but an independent constraint … Robert Howell takes a middle course between Hintikka and Parsons … Moreover, he suggests, Kant complements his strict definition of intuition's immediacy with a positive conception of this immediacy, analogous to the contemporary notion of the direct reference had by demonstrative terms" (Smit, Huston. "Kant on Marks and the Immediacy of Intuition," *The Philosophical Review* 109, no. 2 (2000): 235–66).

but *pure intuitions* as well, which, "even without any actual object of the senses or of sensation", exist "in the mind *a priori*" (B35).

An apparently insurmountable dilemma arises precisely from this concession on Kant's part. As Kant himself asks in his *Prolegomena to Any Future Metaphysics* (*Prologemena zu einer jeden künftigen Metaphysik*), "how can the intuition of an object precede the object itself?" (AA 4:282). According to Kant one can quite well form *concepts* (a mediate representation through a characteristic common to many objects through which an object can be *thought*), or at least some of them, *a priori*, without an immediate relation to an object. On the contrary it seems impossible at first sight to have an *intuition* (an immediate and individual representation through which the object is *given*) completely *a priori*. In this case, "the intuition would have to occur without an object being present, either previously or now, to which it could refer, and so it could not be an intuition" (AA 4:282).[6] According to Kant, as is well known, there is only one way in which "my intuition" can "precede the actuality of the object and occur as an *a priori* cognition, *namely if it contains nothing else except the form of sensibility, which in me as subject precedes all actual impressions through which I am affected by objects*" (AA 4:282[7]; see also AA 20: 266). Thus, space and time, as the *Critique of Pure Reason* explicitly states, are the formal conditions without which nothing could be given:

> It is evident from the above that the first condition, namely, that under which alone objects can be intuited, does actually lie *a priori* in the mind as the formal ground of the objects. All appearances necessarily agree with this formal condition of sensibility, since only through it can they appear, that is, be empirically intuited and given. (B125)

In this way, the *pure intuitions* (space and time) are the conditions without which objects could not be given because "only by means of such pure forms of sensibility can an object appear to us, and so be an object of empirical intuition" (B121). By virtue of this fact, the formal nature of the pure intuitions can be cognized completely *a priori.* In contrast, all other determinations "belong merely to the subjective constitution of our manner of sensibility, for instance, of sight, hearing, touch, as in the case of the sensations of colours, sounds, and heat, which, since they are mere sensations and not intuitions, do not of themselves yield knowledge of any object, least of all any *a priori* knowledge" (B44).

The general framework of the "Transcendental Aesthetic" is best stated in Kant's own words:

> That in the appearance which corresponds to sensation [was der Empfindung correspondirt], I term its *matter*; but that which so determines the manifold of appearance that it allows of

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[6] English translation from Kant, Immanuel. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, tr. Curtis Bowman, Paul Guyer, and Frederick Rauscher. ed. Paul Guyer, The Cambridge Edition of the Works of Immanuel Kant (Cambridge UK; New York: Cambridge University Press, 2005), 78.

[7] Kant, Immanuel. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." In *Theoretical Philosophy after 1781*, tr. Gary Hatfield. eds. Henry E. Allison, Peter Lauchlan Heath, and Gary C. Hatfield. (Cambridge, UK; New York: Cambridge University Press, 2002), 78.

> being ordered in certain relations, I term the *form* of appearance … [W]hile the matter of all appearance is given to us *a posteriori* only, its form must lie ready for the sensations *a priori* in the mind, and so must allow of being considered apart from all sensation. (B34)

Thus, one can affirm, as Kant does in a "note" (*Reflexion*) from the 1790s, that the "only thing that is cognized *a posteriori* in the appearances is the matter or what is real in it, namely that which corresponds to sensation. What is formal in appearances [Das Formale der Erscheinungen], space and time, is cognized *a priori* and is pure intuition" (AA 18:374; Refl. 5875).[8]

From the preceding definitions, such knowledge of the *a priori* forms of sensibility could also be called "anticipation" in as much as it concerns *a priori* knowledge of that which "belongs to empirical knowledge." That is to say, "anticipation" is *a priori* knowledge of that which can only be "given" in intuition in as much as it represents the latter's formal conditions. The Anticipations of Perception involve a further step, however. Not only can one establish something *a priori*, that is, independent of all experience, concerning the "forms" of "empirical knowledge", space and time. Even the "matter" itself of such knowledge can be determined *a priori*, despite the fact that "the matter of appearances, by which *things* are given us in space and time, can only be represented in perception, and therefore *a posteriori*" (B748).

Here, Kant is making the paradoxical attempt to affirm something *a priori* concerning that which is considered to be inherently *a posteriori*, namely, perception or sensation, the very criterion that is used to distinguish *a posteriori* from *a priori* knowledge. In the "Proof" (*Beweis*) of the Anticipations of Perception, Kant suggests that such *a priori* knowledge seems to be impossible at first sight:

> [T]here is an element in the appearances (namely, sensation, the matter of perception) which can never be known *a priori*, and which therefore constitutes the distinctive difference between empirical and *a priori* knowledge, it follows that sensation is just that element which cannot be anticipated (B209).

Nevertheless, through the Anticipations of Perception, Kant aims to demonstrate the possibility of precisely this paradox: "If, however, there is in every sensation, as sensation in general (that is, without a particular sensation having to be given), something that can be known *a priori*, this will, in a quite especial sense, deserve to be named anticipation" (B209). Therefore, the second synthetic principle of pure understanding is the "principle which anticipates all perceptions, as such" (A166) in as much as it can affirm something *a priori*, not in respect to this or that particular perception, but something that characterizes perception as such.

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[8] English translation from Kant, Immanuel. "Metaphysical Foundations of Natural Science." In *Theoretical Philosophy after 1781*, tr. Michael Friedman. eds. Henry E. Allison, Peter Lauchlan Heath, and Gary C. Hatfield. (Cambridge, UK; New York: Cambridge University Press, 2002), 214.

1.2 Perception and Sensation

The above section has clarified that anticipations are *a priori* knowledge in the strict sense of the term, that is, independent of all experience. Perception, however, the object of anticipations, requires a more precise definition. In a passage of the *Critique of Pure Reason*, Kant defines perceptio (the Latin term usually translated in German as Wahrnehmung) as "representation with consciousness [Vorstellung mit Bewußtsein]," distinguishing it as a species of the "genus [Gattung] … representation in general (*repraesentatio*)" (B376). Proceeding in a dichotomous fashion, Kant introduces a further subdivision within the species "representations with consciousness": "A perception which relates solely to the subject as the modification of its state is sensation (sensatio), an objective perception is knowledge (cognitio)" (B376). "[T]o perceive [wahrnehmen] (percipere)" in this context is thus meant firstly as perceptio generaliter, that is, "to present something to oneself in consciousness [sich mit Bewußtsein etwas vorstellen]" (AA 9:64).[9]

From this definition, the *perceptio sensitiva* should be understood as a particular class of perceptions in which an empirical intuition or sensation is accompanied by consciousness. Kant confirms this in a passage of the *Prolegomena*: "At bottom [of experience] lies the intuition of which I am conscious, i.e., perception (*perceptio*), which belongs solely to the senses" (AA 4:300).[10] The restriction, "which solely belongs to the senses," seems to be the distinctive feature of the species "sensible perception" (sensation accompanied by consciousness) within the genus "perception" in general (representation accompanied by consciousness).

In the section entitled Anticipations of Perception, Kant seems to use the term "*Wahrnehmung*" without further specification, in the most restricted sense of the term; as "sensation with consciousness [Empfindung mit Bewustseyn]"[11] (AA 16:494, Refl. 2740). "Perception [Wahrnehmung]," writes Kant in the "Proof" of the second principle of pure understanding, "is empirical consciousness, that is, a consciousness in which sensation [Empfindung] is to be found" (B207). This definition returns in various forms throughout Kant's writings. In a letter to Jacob Sigismund Beck from the 20th of January, 1792, Kant provides a rather incisive formulation of the definition, speaking of empirical knowledge "as "senses' representation," which "includes both sensation and sensation bound up with consciousness, i.e., perception" (AA 11:315).[12]

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[9] English translation from Kant, Immanuel. *Logic*. eds. Robert Hartman and Wolfgang Schwarz (New York: Dover Publications, Inc., 1974), 71.

[10] English translation Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." 91.

[11] Particularly in regard to Kant's "notes" (*Reflexionen*), the reader should be aware that the German terms placed between brackets are provided as they appear in the reading of the Akademie-Ausgabe even when they differ from current orthography.

[12] English translation from Kant, Immanuel. Correspondence. ed. Arnulf Zweig, The Cambridge Edition of the Works of Immanuel Kant (Cambridge: Cambridge University Press, 1999), 400.

The reference to consciousness suggests firstly that, although sensation (*Empfindung*) belongs to sensible perception (*Wahrnehmung*), distinguishing the latter from perception in general, sensation alone does not exhaust the essence of sensible perception. In the "Proof" of the second principle, Kant clarifies that sensation is only the "matter of perception" (B209). If "sensation," according to the definition in the "*Transcendental Aesthetic*," is "[t]he effect of an object upon the faculty of representation, so far as we are affected by it," then "perception" is distinguished from mere "sensation," as immediate sensible impression, by the fact that the former presupposes awareness of the relationship to the object that modifies the senses. Kant explains that "sensation as merely subjective representation" is that "which gives us only the consciousness [man sich nur bewußt werden kann] that the subject is affected, and which we relate to an object in general" (B207–8).

"Consciousness" thus indicates the awareness of the presence of an objective element that transcends sensation and to which the latter refers. In Kant's words, "*perception of an object* is a consciousness of the object through sensation" (AA 30:999).[13] One can distinguish two aspects of sensible perception: (1) sensation, understood as immediate impression, as the mere modification of the state of the subject (in whom sensation and sensed are, in a manner of speaking, one) and (2) the awareness of the presence of something that causes the modification. Of these two aspects, "the first is the subjective part of 'sensation' (internal modification), and the second is the objective part (the relation to the object [Gegenstand]) of consciousness" (AA 15:688; Refl. 1482). Thus, perception is the awareness of the relationship between sensation and that which is sensed, namely matter: "The influence [Einflus] of this matter on the subject's faculty or representation is the efficient cause of its representation (which, combined with consciousness [mit Bewustseyn verbunden], is called *perception* [Wahrnehmung]" (AA 21:602.5).[14]

Kant insists on distinguishing perception from sensation. What should be emphasized here is that Kant's distinction between "sensation" and "perception" seems to allude to the presence of an objective element beyond sensation itself, determining it and modifying the state of the sentient subject. Kant, as I will discuss, calls this objective element "reality" (*Realität*) or "real" (*Reale*): "The awareness of the presence of an object is perception. The subjective part of perception is sensation, whereas the objective part, that is, the concept of the sensed [der Begrif des Empfundenen], is reality [Realitaet]" (AA 18:654; Refl. 6333).

What is anticipated and known *a priori* is thus something articulated, namely, "perception (sensation and with it reality) [Wahrnehmung (Empfindung und mithin

[13] English translation from Kant, Immanuel. *Lectures on Metaphysics*. eds. Karl Ameriks and Steve Naragon, The Cambridge Edition of the Works of Immanuel Kant (Cambridge, UK; New York: Cambridge University Press, 1997), 467.

[14] English translation from Kant, Immanuel. *Opus postumum*. eds. Eckart Förster and Michael Rosen, The Cambridge Edition of the Works of Immanuel Kant (Cambridge, UK; New York: Cambridge University Press, 1993), 95.

Realität)]" (B212). As the following sections will show, one of the most obscure aspects of the second synthetic principle is the difficulty in unambiguously establishing the relationship between sensation (the subjective aspect of perception) and that which is sensed (the objective component). Kant does not always seem to rigorously maintain this distinction. However, this "wavering" from one aspect of the problem to the other , as I will be able to clarify more precisely further below, is precisely where one should look for the key to understand the meaning of the principle.

1.3 The Strangeness of the 'Anticipations'

As discussed above, "in so far as … perception contains sensation" (B208), it is by definition something empirical (that is to say, dependent on sensible impressions) and can thus only be given *a posteriori*. The possibility of establishing something *a priori* about that which is *a posteriori* is something that Kant himself, in the "Proof" of the Anticipations of Perception, does not hesitate to call "strange [befremdlich]" (B209). "[For] anyone trained in transcendental reflection," Kant writes further below, "so asserting the possibility of an internal distinction in sensation itself (abstraction being made of its empirical quality), awakens doubts and difficulties" about "how the understanding can thus in *a priori* fashion pronounce synthetically upon appearances, and can indeed anticipate in that which in itself is merely empirical and concerns only sensation" (B217).

This "strangeness" is reaffirmed, almost in the same words, in the *Opus Postumum*, in which Kant increasingly emphasizes the need to anticipate experience as well as *quoad materiale*:

> It is strange [befremdlich] – it even appears to be impossible, to wish to present *a priori* that which depends on perceptions (empirical representations with consciousness of them): E.g. sound, light, heat, etc., which, all together, amount to the subjective element in perception (empirical representation with consciousness) and hence, carries with it no knowledge of an object. Yet this act of the faculty of representation is necessary. (AA 22:493.11).[15]

The following question arises: what can be established *a priori* about sensation, something that is eminently *a posteriori* and that in fact "constitutes the distinctive difference between empirical and *a priori* knowledge" (B210)?

In the final part of the "Proof" of the second principle, Kant explains with particular clarity the nature of this curious form of anticipation: Concerning *quantity*, only one *quality* can be anticipated *a priori*, that of being continuous. Inversely, when what is given *a priori* of *qualities* is that they have a *quantity*, or more precisely, a particular form of quantity called the intensive magnitude or degree, "[t]he *quality* of sensation is always merely empirical and cannot be represented *a priori* at all (e.g. colors, taste, etc.). … All sensations are thus, as such, given only *a posteriori*, but their property of having a degree can be cognized *a priori*" (B217–).

[15] English translation from Ibid., 141.

Colors, for example, are "merely modifications of the sense of sight that is affected by light in a certain manner" (A28) and are rightly considered, "not as properties of things, but simply as modifications of our subject that can be different for different people" (A28). They "are in no way necessary conditions for things to become objects for our senses," but instead, "are connected with the phenomena only as effects, accidentally added, of our particular constitution. For this reason, they are not *a priori* representations, but rather founded in sensation" (A28). The sensation remains purely empirical, depending on the relationship between the external stimulus (such as a light wave) and the nature of our psychophysical constitution (our eyes). Therefore, "no one can have an *a priori* representation of color or of taste" (A28).

Kant provides a succinct explanation of the subjective nature of sensation in the *Critique of the Power of Judgment* (*Kritik der Urteilskraft*):

> If sensation, as the real in perception, is related to cognition, it is called sensory sensation; and its specific quality can be represented as completely communicable in the same way only if one assumes that everyone has a sense that is the same as our own, but this absolutely cannot be presupposed in the case of a sensory sensation. Thus, to someone who lacks the sense of smell, this kind of sensation cannot be communicated; and, even if he does not lack this sense, one still cannot be sure that he has exactly the same sensation from a flower that we have from it. (AA 5:291)[16]

Therefore, one can affirm that "what concerns the properties of sensuous intuition in regard to its material content, namely sensation – for example, bodies in light as colors, in vibration as sounds, in salts as acids, and so on – remains merely subjective and yields no knowledge of objects," and, consequently, "do not, like space and time, contain data for *a priori* cognitions, and cannot even be counted as knowledge of objects at all" (AA 20:269).[17]

According to the traditional distinction between primary and secondary qualities, sensible qualities are indefinable without going back to experience. They can only be exemplified through sensible intuition and therefore are not susceptible to any *a priori* determination. According to Kant, one can nevertheless establish *a priori* and thus independently of all experience that sensible qualities have a quantity, or more precisely, that particular form of quantity that is called "intensive magnitude" or "degree": "in all quality … we can know *a priori* nothing save [in regard to] their intensive *quantity*, namely that they have degree. Everything else has to be left to experience" (B218).

A reflection in the margin of the first edition of the *Critique of Pure Reason* (A143; B182) emphasizes that sensation is indeed *a posteriori*, but only in regard to its quality, while the fact that sensible qualities have a degree can be affirmed

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[16] English translation from Kant, Immanuel. *Critique of the Power of Judgment*, tr. Eric Matthews. ed. Paul Guyer, The Cambridge Edition of the Works of Immanuel Kant (Cambridge, UK; New York: Cambridge University Press, 2000), 171.

[17] English translation from Kant, Immanuel. "What Real Progress Has Metaphysics Made in Germany since the Time of Leibniz and Wolff?" In *Theoretical Philosophy after 1781*. eds. Henry Allison and Peter Lauchlan Heath. (Cambridge, UK; New York: Cambridge University Press, 2002).

a priori: "Sensation is that which is really empirical in our cognition …. Sensation therefore lies outside all *a priori* cognition. But only therein, how one sensation differs from another with regard to quality, outside of the *a priori* degrees [ausser den Graden *a priori*], and not in relation to its quantity" (AA 23:27; Refl. LX; A143).

The *Prolegomena* equally highlights this distinction: "although sensation, as the *quality* of empirical intuition with respect to that by which a sensation differs specifically from other sensations, can never be cognized *a priori*, it nonetheless can, in a possible experience in general, as the *magnitude* of perception, be distinguished intensively from every other sensation of the *same kind*" (AA 4:309; emphasis mine).[18] Thus, what can be "anticipated" is "the possibility of an *internal distinction* in sensation itself (abstraction being made of its empirical quality)" (B217; emphasis mine), not the possibility of qualitatively distinguishing one sensation from another (e.g., colors from sounds, red from blue, etc.). In other words, for any given sensible quality, one can affirm that, within it, one can distinguish quantitative differences that do not jeopardize the quality's identity.[19]

1.4 The Quantity of Quality: Degree or Intensive Magnitude

"Every magnitude has a quality, i.e., continuity. Every quality has a magnitude, i.e., intensity (degree)" (AA 18:268; Refl. 5636).[20] This note from Kant effectively expresses the conclusion that can be drawn from the preceding sections. The determinateness of sensible qualities is given *a posteriori*. Nevertheless, that such qualities

[18] English translation from Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." 103.

[19] Interestingly, even Hume admits the possibility of 'anticipating' differences in degrees before having direct experience of them: "Now if this be true of different colours, it must be no less so of the different shades of the same colour; and each shade produces a distinct idea, independent of the rest. For if this should be denied, it is possible, by the continual gradation of shades, to run a colour insensibly into what is most remote from it; and if you will not allow any of the *means* to be different, you cannot, without absurdity, deny the extremes to be the same. Suppose, therefore, a person to have enjoyed his sight for 30 years, and to have become perfectly acquainted with colours of all kinds except one particular shade of blue, for instance, which it never has been his fortune to meet with. Let all the different shades of that colour, except that single one, be placed before him, descending gradually from the deepest to the lightest; it is plain that he will perceive a blank, where that shade is wanting, and will be sensible that there is a greater distance in that place between the contiguous colour than in any other. Now I ask, whether it be possible for him, from his own imagination, to supply this deficiency, and raise up to himself the idea of that particular shade, though it had never been conveyed to him by his senses? I believe there are few but will be of opinion that he can: and this may serve as a proof that the simple ideas are not always, in every instance, derived from the correspondent impressions; though this instance is so singular, that it is scarcely worth our observing, and does not merit that for it alone we should alter our general maxim" (Hume, David. *A Treatise of Human Nature*. eds. Peter Nidditch and Lewis Amherst Selby-Bigge (Oxford: Oxford University Press, 1978), 23.).

[20] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 260.

have a quantity can be affirmed *a priori*; or rather, that they have a specific form of quantity called "intensive magnitude" or "degree." The latter should be distinguished from extensive quantity, which pertains to space and time in which qualities are dispersed: «*quantitas qualitatis est gradus* [The magnitude of quality is degree]» (AA 4:309n).[21]

Degree or intensive magnitude involves a different "logic" than that of extensive magnitude. If one takes two volumes of water of the same temperature and adds them together, the volume of water is doubled but the temperature remains the same. From the perspective of temperature, a liter of boiling water is not equal to 4 l of water at 25° or to an ocean of tepid water. Although doubling the quantity of water doubles the "quantity of heat" that it contains, the temperature does not vary: "a drop of boiling water is certainly smaller than a cooking pot, but both are equally hot" (AA 30:999). Temperature is an intensive magnitude whereas volume is an extensive magnitude:

> If I take a kettle and a thimble full of warm water then the former is extensively greater than the latter, but the water in the kettle is only lukewarm and that in the thimble boiling, then the latter is in this case intensively greater than the first (AA 28:425).[22]

Extensive magnitude is an "additive magnitude"[23] that one can measure by juxtaposing a standard unit of measure in order to determine how many units the given magnitude contains: a large quantity may always be formed by the addition of a certain number of smaller quantities of the same kind. In contrast, intensive magnitude is a "magnitude of level"[24] that allows differences in the distribution of a quality (e.g. "heat") to be measured relative to situations in which the quality is uniformly distributed. One does not measure intensive magnitude through the *juxtaposition* of parts (two bodies have the same volume when they contain the same number of a smaller volume chosen as a unit of measure), but rather through *equilibrium* (two bodies have the same temperature if putting them into contact does not lead to mutual variations).[25] The degree of temperature does not determine "how much heat" is contained in bodies, but only the difference in the "level" of its distribution

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[21] English translation from Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." 102.

[22] English translation from Guyer, Paul. *Kant and the Claims of Knowledge* (Cambridge: Cambridge U. Press, 1987), 199–.

[23] See: Hempel, Carl Gustav. *Grundzüge der Begriffsbildung in der empirischen Wissenschaft* (Düsseldorf: Bertelsmann, 1974), 69–.

[24] See: Mach, Ernst. *Die Principien der Wärmelehre historisch-kritisch entwickelt* (Leipzig: Barth, 1900), 57.

[25] "If the element of fire is in a state of equilibrium among the bodies in a certain space, they are, relative to each other, neither hot nor cold" (AA 2:185). English translation from Kant, Immanuel. "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy." In *Theoretical Philosophy, 1755–1770*. ed. David Walford. (Cambridge: Cambridge University Press, 1992), 223–.

relative to their heat capacity. Without this difference in level, the very distinction between hot and cold, and thus the concept of 'degree', is meaningless.[26]

A body is hotter than another if it tends to cede heat to the second, when put in contact with it as long as the temperatures are not equal, and equilibrium, that is, the same level of distribution of the quality "heat" in the two bodies, is not reached,[27] that is, as long as the difference in level does not disappear and the difference in degree is $= 0$. In reality, the temperatures never equalize entirely. Instead, the difference in temperature between the two bodies approaches zero in an asymptotic fashion as long as the difference does not reach a point at which the thermometer is unable to register it. A more sensitive thermometer would show that one of the two bodies remains slightly hotter than the other and will continue to do so to infinity.[28]

These few considerations should help to make sense of the distinction between extensive and intensive magnitude as it is formulated in the *Critique of Pure Reason*: "I call an extensive magnitude that in which the representation of the parts makes possible the representation of the whole (and therefore necessarily precedes the latter)" (B203; emphasis mine). In contrast, "I call that magnitude which can only be apprehended as a unity, and in which multiplicity can only be represented through the approximation to negation = 0, intensive magnitude" (B210).

Thus, the intensive magnitude of a body does not measure the *quantitas molis*, but rather the *quantitas virtutis*, not the "amount" of a phenomenon,[29] but rather the "degree" of efficacy,[30] such as the "capacity to cede heat" that one body possesses in respect to another:

> Thus, for example, one cannot say that heat consists of tepidness [bestehe aus Lauigkeiten]. Its magnitude, consequently, is determined not by the *parts that it contains*, but rather by the *effects that it produces* [nach den Wirkungen, die sie hervorbringt], for example, a body that it dilates. Thus there cannot be attributed to it a magnitude true and proper [man kann ihr daher nicht eine eigentliche Größe beilegen], but rather a degree. (AA 18:322; Refl. 5636 (1788–1790); emphasis mine).

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[26] One cannot affirm that a body is hot or cold "in itself," but only that, if the "equilibrium is removed, then the material into which the elemental fire passes is, relatively to the body which is thus deprived of the elemental fire, *cold*, whereas the latter body, in so far as it yields this heat to that material, is called, relatively to the material receiving the elemental fire, *warm*" (AA 2:185; English translation from Ibid., 224.).

[27] "The state which prevails during this change is called, in the former case, 'growing warm' and in the latter case 'growing cold'; this process of change continues until everything reaches the state of equilibrium again" (AA 2:185;Ibid.).

[28] See Scaravelli, Luigi. *Saggio sulla categoria della realtà* (Florence: Le Monnier, 1947). Reprint, "Kant e la fisica moderna." In *Scritti kantiani*, ed., Mario Corsi (Florence: La Nuova Italia, 1973), 1–189, 167.

[29] Though many English translations of Kant designate "phenomena" and "phenomenal reality" by the terms "appearances" and "reality of appearances," I consistently use "phenomenon" to maintain the opposition with "noumenon".

[30] Daniel Warren, in his valuable explanation of intensive magnitude, discusses this connection between "degree" and the "causal power" that can be evaluated through the consequences it produces Warren, Daniel. *Reality and Impenetrability in Kant's Philosophy of Nature* (New York: Routledge, 2001), 22–30. See also Guyer. *Kant and the Claims of Knowledge*, 200.

Intensive magnitude cannot be measured directly through a unit of measure that is contained a certain number of times in the measured body, but it can be measured indirectly through the consequences that it can produce: "magnitude which cannot be immediately intuited as magnitude is appraised by way of a consequence" (AA 30:834).[31]

Such consequences can receive an initial subjective evaluation through the more or less intense sensation that they produce in the subject. According to Kant's *Metaphysik von Schön* (Schön's Metaphysics), "[t]hat which is the object of sensation, we call degree; for example, the degree of heat, cold, light. Why do we attribute a magnitude to it? Because we represent the magnitude of a ground [Grund] through it,"[32] that, if very effective, is said to "have caused a great sensation" (AA 28:502).

Obviously, this first evaluation is only approximate, merely showing that a certain quality can be perceived in a more or less intense manner without allowing any precise measurement: "heat can nevertheless be distinguished through sensations, but [such an evaluation] is very uncertain" (AA 30:120). In this manner however it is possible to pass from a mere qualitative or classificatory distinction among different sensations (for instance cold, lukewarm, warm, hot, etc.) to a comparative or topological distinction between intensities of the same sensation, that is to say a "minus or plus" distinction, which allows the sensation of warmth to be arranged in an ordered series[33] (*a* is warmer/less warm than *b*). However, it is equally evident that "we also have other perceptions of heat: one can evaluate heat objectively … through its consequences," such as "when the volumes of bodies expand and increase" (AA 30:120) as occurs in common thermometers. In this way one can define quantitative or metrical relations between intensities, that is to say arrange them in an interval scale in which the concept of distance is defined.[34]

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[31] English translation from Kant. *Lectures on Metaphysics*, 192.

[32] Several of Kant's *„Reflections"* affirm that intensive magnitude is the magnitude of a cause or of a ground: "the magnitude of a whole [eines Gantzen] is extensive," whereas "the magnitude of a ground [eines Grundes] is intensive or degree" (AA 17:536; Refl. 4411). A ground or a cause's "capacity to produce effects" cannot be considered more or less extended nor can it contain more or less parts, but rather should be thought as more or less effective: "the magnitude of an (*g*intensive) ground does not demonstrate any composition [keine Zusammensetzung] from smaller [elements]" (AA 17:448; Refl. 4183); "in this, we do not distinguish any multiplicity; only the consequences of the multiplicity can be represented and is thus the magnitude of a ground" (AA 28:507).

[33] See Höffe, Otfried. *Kants Kritik der reinen Vernunft. Die Grundlegung der modernen Philosophie* (Munich: Beck, 2003), 178.

[34] For a more detailed reconstruction see: Stegmüller, Wolfgang "Theorie und Erfahrung (1. Halbband)." In *Probleme und Resultate der Wissenschaftstheorie und der analytischen Philosophie*. (Berlin: Springer, 1973), 2:19–68. Some scholars point out that an extensive magnitude is a magnitude that can be subjected to cardinal measurability, i.e., an additive operation. In contrast, intensive magnitudes allow the ordinal measurement of qualities. See for example: Walker, Ralph Charles Sutherland. *Kant: The Arguments of the Philosophers* (London: Routledge & Kegan, 1978), 96n11. Such an interpretation is also supported by Hegel: "The degree is thus a specific magnitude, a quantum; but at the same time it is not an aggregate or plural within itself … The determinateness of degree must, it is true, be expressed by a number, the completely determined form of quantum, but the number is not an amount but unitary, only a degree. When we speak of 10° or 20°, the quantum that has that number of degrees is the tenth or twentieth degree, not the amount and sum of them – as such, it would be an extensive quantum – but it is only one

From this point of view one can understand Kant's claim that, "every intensive magnitude should, in the end, be brought back [gebracht auf] to the extensive" (AA 18:242; Refl. 5590), making use of the consequences that it is capable of producing.[35]

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degree, the tenth or twentieth." (HW V, 251). English translation from Hegel, Georg Wilhelm Friedrich. *Hegel's Science of Logic*, tr. Arnold V. Miller (Amherst, N.Y.: Humanity Books, 1998), 218. Later, Wilhelm Ostwald developed a similar conception. For example, in the *Vorlesungen über die Naruphilosophie*, he distinguishes between "magnitudes" (*Größen*) and "intensities" (*Intensitäten*): magnitudes are expressed by mean cardinal numbers and intensities by mean ordinal numbers. See: Ostwald, Wilhelm. *Vorlesungen über Naturphilosophie: gehalten im Sommer 1901 an der Univ. Leipzig* (Leipzig: Veit, 1902), 129–.

[35] In numerous instances, Kant seems to use the photometry study of his friend and correspondent Johan Heinrich Lambert . On this subject, see Vuillemin, Jules. *Physique et métaphysique kantiennes* (Paris: Presses Universitaires de France, 1955), 129–32. See also Brittan, Gordon. "Kant's Two Grand Hypotheses." In *Kant's Philosophy of Physical Science: Metaphysische Anfangsgründe der Naturwissenschaft, 1786–1986*. ed. Robert E. Butts. (Dordrecht, Netherlands Kluwer, 1986), 70–72n13. Photometry starts from the hypothesis of the conservation of "quantity of light"; that diffusing light in a larger space produces a gradually decreasing "density" as it moves away from the light source and therefore a weaker degree of illumination. In the *Metaphysical Foundations of Natural Science* (*Metaphysische Anfangsgründe der Naturwissenschaft)*, Kant claims that "[t]hus light, for example, diffuses from an illuminating point in all directions on spherical surfaces, which constantly increase with the squares of the distance, and the quantum of illumination on all of these spherical surfaces, which become greater to infinity, is always the same in total. But it follows from this that a given equal part of one of these spherical surfaces must become ever less illuminated with respect to its degree, as the surface of diffusion of precisely the same light quantum becomes greater." (AA 4:519). From this point of view, it is possible to derive the law by which the degree of illumination should decrease according to the square of the distance from the luminous source. In other words, as Lambert argues, one can double an illumination by doubling the intensity of the luminous source (*I*) at the same distance (*r*), or by diminishing the distance by ¼ without varying the quantity of light: one candle placed at a certain distance from a sheet of paper illuminates it in a determined way; if the distance is doubled, four candles are necessary to create the same degree of sensation, while doubling the distance without augmenting the intensity of the luminous reduces the effect of the illumination by four times. In an example that is reported in the *Metaphysik von Schön*, Kant, in accordance with this point of view, states: "The illuminative power of a wax: candle is greater than that of a tallow candle, for with the first we can read at a distance of 2 feet and with the second only at 1 foot; the former is therefore the ground of a greater effect, and the latter the ground of a lesser" (AA 28:424–; Guyer. *Kant and the Claims of Knowledge*, 199–200). Kant also appeals to photometry in the *Critique of Pure Reason* itself in order to clarify the meaning of the Anticipations of Perception: "I would be able to compose and determine *a priori*, i.e., construct the degree of the sensation of sunlight out of about 200,000 illuminations from the moon. Thus we can call the former principles constitutive" (B221). Here, Kant has not all of a sudden forgotten about the "non-additivity" of intensive magnitude that he demonstrated a few pages before. In this case, measurement does not require the summing of magnitudes, but rather the equalizing of relations. This point of view also sheds light on this puzzling passage of the *Critique of Pure Reason*: "the very same extensive magnitude of intuition (e.g., an illuminated surface) can excite as great a sensation as an aggregate of many other (less illuminated) surfaces taken together. One can therefore abstract entirely from the extensive magnitude of appearance and yet represent in the mere sensation in one moment a synthesis of uniform increase from 0 up to the given empirical consciousness" (B217–8). One can produce the effect of two candles through four candles with half the intensity, but at the same distance. For a different interpretation see Nayak, Abhaya C. and Eric Sotnak. "Kant on the Impossibility of Soft Sciences," *Philosophy and Phenomenological Research* 75, no. 1 (1995): 133–51, 140.

Intuitively, one can say that temperature depends on how much heat is packed into a body of a certain heat capacity implying that more heat is required to bring a larger body to the same temperature than a smaller one of the same matter. In other words, the possibility of a quantitative treatment is actually possible where the intensive magnitude can be interpreted as a *quotient of extensive magnitudes*. The same "degree of sensation" can be namely determined by the relation between different extended magnitudes. Thus, one can disregard the "absolute" value of the extended magnitudes and merely consider the value of their quotient. The inclination of a straight line, for example, depends on the quotient of the difference of their coordinates and not on their absolute value: "the angle is equally large as much as one extends the lines or sides: what counts is the equality of the inclination, and its difference is again only a difference of intensive magnitude" (AA 30:999).[36]

The distinction between intensive and extensive magnitudes, which I have attempted to clarify, was certainly not introduced by Kant, nor does he feel the need to clarify its meaning in depth in his most important works. Rather, Kant goes back, at least indirectly, to a long and consolidated tradition that dates back to the Middle Ages[37]

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[36] On the reduction of extensive quantity to intensive quantity, see Franz Brentano's manuscript "Kants Intensitätslehre" (Kant's Doctrine on Intensity) in Brentano, Franz. *Kategorienlehre*. ed. Alfred Kastil (Hamburg: Meiner, 1993), 95. This point of view offers a possible response to Nayak and Sotnak's thesis "that Kant really does allow for the cardinal measurability of qualities" (Nayak and Sotnak. "Kant on the Impossibility of Soft Sciences," 140). For Nayak and Sotnak,' "Kant takes weight to be an intensive" (Nayak and Sotnak. "Kant on the Impossibility of Soft Sciences," 139.), but it seems clear that Kant rather takes "density" for an intensive magnitude, namely the quotient mass/volume. The "moment of weight" ($\frac{dv}{dt}$ at the beginning of the fall of a body) is for Kant an intensive magnitude, not the weight. Similarly, when they write that "Kant says that velocities (which are intensive magnitudes) can be added (by means of vector addition),", one must not forget the meaning of this addiction. In Kant's own words "If, for example, a speed AC is called doubled, nothing else can be understood by this except that it consists of two simple and equal speeds AB and BC […]. If, however, one explicates a doubled speed by saying that it is a motion through which a doubled space is traversed in the same time, then something is assumed here that is not obvious in itself – namely, that two equal speeds can be combined in precisely the same way as two equal spaces – and it is not clear in itself that a given speed consists of smaller speeds, and a rapidity of slownesses, in precisely the same way that a space consists of smaller spaces." (AA 4:494 -). The addiction of velocities is the geometrical composition of velocities and not the mechanical variation of velocities through a cause.

[37] Fourteenth century scholasticism enters profoundly into the so-called problem of the *intensio* e *remissio formarum*. The Aristotelian conception of the immutability and eternality of substantial forms apparently excludes this ability to undergo an increase and a decrease while conserving the species. Conceiving of the transition from one species to another (e.g. from sensible soul to intellectual soul) within the same genus is not the issue here. Instead, it is a matter of admitting a transformation from white to less white, from hot to less hot, in which the species "whiteness" or "hotness" is conserved. According to Aristotelian logic, since substance does not present a more or a less, the *perfectio specifica*, which distinguishes each species in respect to another, does not admit variations. Intellectual soul cannot become more or less perfect. Even if differences can be admitted in respect to *perfectio individualis* (in as much as each individual "participates" in the same species), the *perfectio specifica* is immutable and does not admit a multiplicity of degrees: it either possesses the essential property that makes it what it is, a determinate species, or else it must belong to another species. On this point, see Maier, Anneliese. *Zwei Grundprobleme der*

and that modern philosophy adopts,[38] specifically, eighteenth century Leibnizian metaphysics.[39] I will show, however, that the Kantian concept of "degree" clearly intends to distance itself from the "metaphysical" concept, which ultimately relies

scholastischen Naturphilosophie: Das Problem der intensiven Grösse. Die Impetustheorie (Rome: Storia e Letteratura, 1951), 3–5. Instead, one should think of a hierarchy in which different entities are articulated according to the *gradus perfectionis*, culminating in God, *ens perfectissimus* to whom *nulla deest perfectio, vel gradus perfectionis*. However, a difficulty arises in the case of sensible qualities (white, hot, etc.) that can present a *uniformitas* or *difformitas* in spatial or temporal distribution. When the quality is diffused in a non-uniform manner, it necessarily admits a particular quantitative variation. The medieval *calculators* attempted to mathematically measure what they termed intension and remission of forms. Depicting extension on a horizontal axis and intension on a vertical axis, they graphically represented variations in these magnitudes. Thus one can capture in an intuitive manner how a quality that is diffused in a uniform manner (*qualitas simpliciter uniformis*), represented by a horizontal line parallel to the axis that represents extension, can be thought as if all of it were placed on the same "level." In contrast, a quality that is diffused in a "uniform" manner (*qualitas uniformiter difformis*) can be represented as a straight inclined line in which the quality is distributed on different increasing and decreasing levels in successive instants or in diverse points in space. Finally, a curved line represents a quality that is distributed in a non-uniform manner (*qualitas difformiter difformis*). See: Clagett, Marshall. *The Science of Mechanics in the Middle Ages* (Madison: University of Wisconsin Press, 1959), 364–406. The use of the concepts *gradus motus* and *gradus velocitatis* to indicate intensity of movement is particularly significant: as an alteration process (e.g. transition from hot to cold) is measurable through variations in the *gradus intensitatis* of a quality acquired in successive instants, in the same manner, something in movement can possess a greater *gradus intensitatis velocitatis* in one instant than in a preceding instant.

[38] Modern philosophy directly appropriates the medieval tradition's concept of velocity as a "quality" that a body can possess to a greater or lesser degree. Galileo makes widespread use of the concept of *gradus velocitatis*, and Leibniz explicitly discusses velocity as *intensio motus* (see below 245; 250-). Concerning the persistence of this terminology in modern science, see: Ranea, Alberto Guillermo. "The *a priori* Method and the actio Concept Revised: Dynamics and Metaphysics in an Unpublished Controversy between Leibniz and Denis Papin," *Studia Leibnitiana* 21 (1989): 42–68. Even modern thought however has not renounced the medieval idea of the *gradus perfectionis*. Leibniz again, explicitly recognizing his indebtedness to scholastic debates, provides the most significant examples of this idea: "it should be noted that there are many different perfections in nature, that God possesses all of them, and that each one belongs to him in the highest degree" (GP 4:427; see also his 1676 essay: *Quod Ens Perfectissimum existit*; GP 7:261–). In his correspondence with Eckhart, Leibniz establishes an explicit parallel between the *perfectio*, understood as *quantitas realitatis seu essentiae*, and the *intensio*, conceived as *gradus qualitatis* (GP 1:266), showing how easily these two concepts can be confused. If Leibniz here still seems to think that a greater degree of velocity is more perfect than a lesser (see below 39), he nevertheless delineates later a difference, which becomes central in Kant, between *gradus perfectionis* that admits a maximum and *gradus velocitatis* that has neither a *maximum* nor a *minimum* (see GP 4:445). About Leibniz's conception of intensive magnitudes see also De Risi, Vincenzo. *Geometry and Monadology: Leibniz's analysis situs and Philosophy of Space* (Basel ; Boston: Birkhäuser, 2007), 266–.

[39] Unable to cover all the possible sources of Kant's thought, I will only note the diffusiveness of scholastic language in the debate of his time. Christian Wolff's *Ontologia*, directly referring to the scholastic doctrine of *remissio e intensio formarum*, defines degree as "*quantitas qualitatum*." See: Wolff, Christian von. "Philosophia prima sive Ontologia." In *Gesammelte Werke*. ed. Jean École, 3 Abt. (Hildesheim: Olms, 1962), §747. In Baumgarten's *Metaphysica*, the text Kant used in his

on the idea of "degree of perfection,"[40] which is still present in the Leibnizian tradition. Kant only considers the purely physical-mathematical sense of "degree of efficacy." Over and against the image of a "great chain of being," of a world hierarchically structured according to different degrees of perfection that merge in God, the "maximum of perfection"[41] that represents their "common measure" (AA 2:396),[42] Kant only admits the idea of a mere relative difference in degrees of sensible qualities in relation to the point in which the difference between them is = 0. "Maximum et minimum," in this context, are reduced to "conceptus deceptores" as Kant states, adopting Leibniz's expression (AA 17:399*;* Refl. 4051; see also AA 2:32; 28:561).

1.5 The Two Formulations of the Principle

I will take the opportunity to summarize what has been said up to now: By the term "Anticipations," Kant means a rigorously *a priori* knowledge that is completely independent of every possible sense experience. The Anticipations of Perception paradoxically attempt to attribute something *a priori* precisely to that which is is *a*

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lectures, the definition "quantitas qualitatis est GRADUS" [the magnitude of quality is degree] appears, which Kant's passage from the *Prolegomena* cites (see below 18), as well as a reference to the doctrine of *intensio* e *remmissio formarum*: "haec si augetur, QUALITAS, cuisus gradus est INTENDITUR, si minuitur, qualitas, cuius gradus est, REMITTITUR [intension is the increasing of quality's degree; remission is its decreasing]" (Baumgarten, Alexander. *Metaphysica*. ed. Herman Carol (Halle, Germany Hemmerde, 1757), §247). See also Moretto, Antonio. *Dottrina delle grandezze e filosofia trascendentale in Kant* (Padua, Italy: Il Poligrafo, 1999), 259. This terminology is used by Kant himself in "Widerlegung des Mendelssohnschen Beweises der Beharrlichkeit der Seele" (Refutation of Mendelssohn's Proof of the Immortality of the Soul). Here Kant considers the possibility of attributing the soul "no manifold [of parts] *outside one another*, and hence no extensive magnitude" but "intensive magnitude, i.e., a degree of reality." Such a degree might diminish through all the infinitely many smaller degrees and … could be transformed into nothing … by a gradual remission (*remissio*) of all its powers (hence, if I may be allowed to use this expression, through elanguescence)" (B414, see also AA 8:165 and 286). For discussion of Kant's refutation, see Powell, Charles Thomas. "Kant, Elanguescence, and Degrees of Reality," *Philosophy and Phenomenological Research* 46, 2, no. 2 (1985): 199–217 and Martinelli, Riccardo. "Kant, Mendelssohn e l'immortalità dell'anima," *Studi kantiani* 15 (2002): 93–126. Moses Mendelssohn and Lambert may be two other equally important sources for Kant's work. See Mendelssohn, Moses. "Morgenstunden oder Vorlesungen über das Daseyn Gottes." In *Gesammelte Schriften. Jubiläumsausgabe*. ed. Eva J. Engel. (Stuttgart-Bad Cannstatt: Fromman, 1974), 82; Lambert, Johann Heinrich. "Anlage zur Architektonik, oder Theorie des Einfachen und Ersten in der philosophischen und mathematischen Erkenntnis." In *Philosophische Schriften*. ed. Hans Werner Arndt. (Hildesheim, Germany: Olms, 1965), 2:359:§21.

[40] A good exposition of this concept that was later repudiated is found in Kant's *Versuch einiger Betrachtungen über den Optimismus* (Reflections on Optimism) from 1759 (AA 2:31–).

[41] See Schoenfield, Martin. *The Philosophy of the Young Kant: The Precritical Project* (Oxford: Oxford University Press, 2000), 109.

[42] English translation is from Kant, Kant, Immanuel. "On the Form and Principles of the Sensible and the Intelligible World." In *Theoretical Philosophy, 1755–1770*. eds. David Walford and Ralf Meerbote. (Cambridge, UK; New York: Cambridge University Press, 1992), 388.

posteriori par excellence, namely, perceptions. That is to say, they attempt to attribute something *a priori* to sensations, the subjective side of perceptions, and its objective corollary, that which causes a sensation, also called by Kant the "real" or "reality." According to Kant, although it is impossible to affirm something about the quality of sensation (hot, white, etc.) without resorting to experience, one can nevertheless "anticipate" the fact of experience that every sensible quality has a quantity, that is, a degree or an intensive magnitude.

These few considerations should help clarify the rather obscure formulation of the Anticipations of Perception in the *Critique of Pure Reason*. In order to do so, one must first take into account that Kant chooses two different formulations in the two editions of his major work. In the second edition, Kant writes:"Its Principle [Prinzip] is: In all appearances the real, which is an object of the sensation, has intensive magnitude, i.e., a degree" (B208). In the first edition, the formulation was slightly but significantly different: "The *principle* [Grundsatz], which anticipates all perceptions, as such, runs thus: In all appearances the sensation, and the *real*, which corresponds to it in the 'object' (*realitas phaenomenon*), has an *intensive magnitude*, i.e., a degree" (A166).

The significance of the reformulation seems clear at first sight. In the first edition, Kant attributes intensive magnitude to the sensation *and* to the real that corresponds to it in the object; in the second edition however, he decides to attribute a degree to the "real, which is an object of the sensation" (B207) instead. Kant apparently wants to avoid the possibility of understanding the principle as a psychological description of behavior of sensations as if a psycho-psychical fact were at issue.[43] From this point of view, as was noted above, the A version gives rise to equivocations. It seems to suggest that "the sensation has a degree and then the real that corresponds to it," that is. to the sensation. However, the principle first of all affirms "that it is actually the real that has the quantity of degree – and consequently 'sensation' as well."[44]

Nevertheless, in "On the schematism of pure concepts of the understanding" (the so-called *Schematismuskapitel*), which does not vary in the two editions, Kant seems equally decided that "every sensation has a degree" (B182). Other passages give the impression that Kant even intends to deduce the property of the real that is the object of sensation from the nature of sensation:[45] "every sensation, *thus* also every reality in appearances … has a degree" (B211, emphasis mine). Analogously, the *Prolegomena* states that, since sensation presents degrees, "(consequently that

[43] See Baumanns, Peter. *Kants Philosophie der Erkenntnis. Durchgehender Kommentar zu den Hauptkapiteln der Kritik der reinen Vernunft* (Würzburg, Germany: Königshausen und Neumann, 1997), 575–.

[44] Heidegger, Martin. "Die Frage nach dem Ding. Zu Kants Lehre von den transzendentalen Grundsätzen." In *Gesamtausgabe*. ed. Friedrich-Wilhelm von Herrmann. (Frankfurt am Main: Klostermann, 1984–), 214–. English translation from Heidegger, Martin. *What is a Thing*, tr. W. B. Barton Jr, and Vera Deutsch (South Bend, IN: Gateway Editions, Ltd., 1967), 215–.

[45] Paul Guyer seems to choose this interpretation: "Because the sensation which is a component of our representation of an empirical object has a degree of intensity, the object itself, or the real thing in the object … must be assigned a degree" (Guyer. *Kant and the Claims of Knowledge*, 199).

what is real in all phenomena has) a degree" (AA 4:307, emphasis mine).[46] In a rather late reflection dating back to the 1890s, Kant still fails to escape this ambiguity: "Each sensation can be thought of as gradually disappearing, i.e., as decreasing from stronger to weaker, thus as [crossed out: disappearing][47] decreasing to nothing or to a part, and in the same way it can also be increased, hence sensation and the reality of the object that corresponds to it has a degree" (AA 18:661; Refl. 6338a; second emphasis mine).[48] Therefore, one can reasonably ask whether "sensation has a degree, as the first edition of the *Critique of Pure Reason* says," or "reality as the second says,"[49] or perhaps both. In the last case, one might ask whether reality has a degree because sensation has a degree or, vice versa, whether sensation can present diverse degrees because the real that corresponds to it in the object presents differences in degrees: "from the citations the argument cannot be resolved."[50]

In working through all these apparent incongruencies, one must first remember that anticipation is *a priori* knowledge, independent of all possible experience. That Kant, who was rather skeptical of the possibility of conferring a scientific status to empirical psychology,[51] wanted to provide a "psychological description" of sensory behavior

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[46] English translation from Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science."

[47] Bowman's brackets.

[48] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 378.

[49] Scaravelli. *Saggio sulla categoria della realtà*, 170.

[50] Ibid. An intresting discussion of this point can be found in Longuenesse, Béatrice. *Kant and the Capacity to Judge Sensibility and Discursivity in the Transcendental Analytic of the Critique of Pure Reason* (Princeton, NJ: Princeton University Press, 1998). Longuenesse correctly observes that Hermann Cohen is the first thinker (see below, 204–) who "charges Kant with psychologistic confusion for attributing to sensation an intensive magnitude that can belong only to the object of sensation, reality … Kant himself acknowledged his mistake and modified his formulation of the principle of the Anticipations of Perception in the second edition of the Critique." However, Martin Heiddeger, whose interpretation of Kant could be considered the opposite of that of Cohen, also does not hesitate to affirm that the formulation of the A edtion of the *Critique of Pure Reason* expresses "almost the opposite of the true meaning of the principle" (Heidegger. "Die Frage nach dem Ding. Zu Kants Lehre von den transzendentalen Grundsätzen." 214. English translation from Heidegger. *What is a Thing*, 215.) Although I sympathize with such an antipsychologistic perspective, I think that one should nevertheless take into account Longuenesse's suggestion not to consider "*sensation*, in the Anticipations of Perception, as a mere empirical and psychological given" (Longuenesse. *Kant and the Capacity to Judge Sensibility and Discursivity in the Transcendental Analytic of the Critique of Pure Reason*, 319). However, in contrast to Longuenesse, I shall argue that this implies taking into account Kant's distinction between *realitas noumenon* and *realitas phaenomenon* (see below §1.7).

[51] Kant does not regard psychology as a science, or as ever likely to become a science. Kant identifies science with the quantitative treatment of phenomena. Every science must therefore be mathematical. However, he did not think that mental reality could ever be measured, implying that psychology could never become mathematical, and therefore a science: "empirical doctrine of the soul must remain even further from the rank of a properly so-called natural science than chemistry. In the first place, because mathematics is not applicable to the phenomena of inner sense and their law" (AA 4:471; see also B876-7). See Nayak and Sotnak. "Kant on the Impossibility of Soft Sciences," 144–.

is thus difficult to admit. Certainly the "subjective/psychological" perspective seems to become inevitably confused with the properly "critical" perspective, but it nevertheless seems implausible that Kant so openly violated his own distinction, between a *quaestio facti* and a *quaestio iuris* (see B116; AA 20:275).

The reference to sensation should be understood from another point of view. One must not forget that "the principle says something about sensations, not on the basis of a psychological empirical description or even a physiological explanation of its formation and origin, but by way of a transcendental consideration."[52] As the distinction between perception and sensation discussed above suggests, the solution to the problem should not be sought in sensation as such, nor in the concept of reality alone, but rather in the inseparable connection that Kant establishes between reality and sensation; a connection whose ambiguity and fleetingness may lie precisely in this inseparability.

1.6 The Category of Reality

To move in this direction, one must first understand the meaning that Kant attributes to the word "reality" in this context. *Realität* in German, in its everyday as well as philosophical sense, can be considered as a synonym of *Wirklichkeit* [actuality], *Existenz* [existence], and *Dasein* [being];[53] terms that indicate the effective existence of something. For example, one can discuss the problem of the "*Realität der Außenwelt*" (reality of the external world), that is, the question of whether the world is only a projection of our consciousness or whether it has an effective existence independent of our consciousness, whether it is purely "imaginary" or rather something "real."[54]

However, Kant's use of the term *Realität,* or of *Reale*, in the Anticipations of Perception, and in his work in general, does not correspond to the common use. Thus, the common sense of these terms should be set aside when considering the meaning of the principle. The Latin term *realitas* is derived from *res*, which can be translated as *Sache* or as *Ding* (thing) in German. Kant's writing sometimes interchanges the term *Realität* with *Sachheit* or *Dingheit* (thingness, thingliness, thinghood), which can be considered literal translations of the Latin term *realitas.* Following the same logic, philosophical jargon contains expressions like *Etwasheit*[55]

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[52] Heidegger. *What is a Thing*, 216.

[53] On the possibility of different nuances in the meanings of *real* and *wirklich* even in daily use, see Holzhey, Helmut. "Das philosophische Realitätsproblem: Zu Kants Unterscheidung von Realität und Wirklichkeit." In *200 Jahre Kritik der reinen Vernunft*. eds. Joachim Kopper and Wilhelm Marx. (Hildesheim, Germany: Olms, 1984), 95.

[54] See Heidegger. *What is a Thing*, 216.

[55] Natorp, Paul. "Quantität und Qualität in Begriff, Urteil und gegenständlicher Erkenntnis," *Philosophische Monatshefte* 27 (1891): 1–32; 129–60, 151.

(somethingness) or *Washeit*[56] (whatness), which indicate something being (*etwas* or *was*) in general; what the scholastics call *quidditas: "quidditas* is reality" *(*AA 17:674, Refl. 4685). To be "real" in this context is synonymous with being "something".

Thus for Kant, *Realität* does not indicate the existence of a thing, but rather the qualitative determination that makes an object, even if only possible, that which it is, defining it as a determinate "*some*thing", that is to say "*aliqvid sive obiectum qvalificatum*" (AA 18:663; Refl. 6338a), in opposition to what lacks determination and is therefore to be considered "*no*thing". To define a "thing" as such means, even etymologically, to circumscribe it in respect to all other "things" that are not it (consider the Spinozian motto that *omnis determinatio est negatio*). Here, the sense of "reality" is akin to "quality"[57] because "the quality of a thing is the determination that represents it as a something or as mere absence, i.e., whose concept contains a being or non-being" (AA 18:662; Refl. 6338a).[58]

Kant does not oppose "reality" to "appearance" ("mere semblance and illusion"[59]), but to "negation" (the absence or lack of determination): "The difference between reality and negation is: reality is that whose concept contains in itself a being; negation, whose concept contains in itself a non-being" (AA 28:552–).[60] In addition to reality and negation, a third concept is needed to complete the "categories of quality," namely, the concept of "limitation." According to the tripartite schema praised by Hegel, the latter "arises from the combination of the first two in its class" (B110) indicating that negation also contains reality (see AA 18:560). As I will show, Kant's interpretation of the category of limitation is perhaps the most effective expression of the innovative way in which Kant conceives the relation between the first two categories.[61]

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[56] Heidegger, Martin. "Die Frage nach der Technik." In *Gesamtausgabe*. ed. Friedrich-Wilhelm von Herrmann. (Franfurt am Main: Klostermann, 1984–), 28.

[57] "Quality is used by Kant almost as a synonym of reality." (Delekat, Friedrich. *Immanuel Kant. Historisch-kritische Interpretation der Hauptschriften*. 2nd ed (Heidelberg,: Quelle & Meyer, 1963), 125).

[58] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 379.

[59] Heidegger. *What is a Thing*, 214.

[60] English translation from Kant. *Lectures on Metaphysics*, 318.

[61] Even if some obscurity remains, this point of view should help clarify the correspondence between the three categories of quality (reality, negation, and limitation) and quality's three forms of judgment (affirmative, negative, and infinite): *Realität* is "that which can be thought only through an affirmative judgment" (A246; e.g., *A* is *B*, body is extended). An affirmative judgment indicates that the sphere of concept A is found within the wider sphere of the predicate B: "in logical subdivisions one limits the *sphaeram*; in real determinations, reality" (AA 17:330; Refl. 3890). In contrast, the category of negation corresponds to the negative form of judgment (e.g., A is not B, soul is not extended) that denies that the concept *A* is found in *B*'s sphere. *B* is thus excluded from the determinations that constitute A's "thinghood in general" or "reality" (AA 16:638; Refl. 3063). English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 62. More complex but with richer consequences, however, is the correspondence between the category of limitation and the infinite form of judgment that does not deny the

Above all, I want to clarify that *Realität* is a category of quality that is opposed to negation and that should not be confused with *Wirklichkeit*, a modal category that is opposed to mere possibility.[62] I cannot linger any longer on the not always so clear relations between the diverse forms of the "reality problem"[63] in Kantian thought. However, the preceding considerations should be sufficient to circumscribe the object of our investigation: reality responds to the question of "what" (*Was*) a certain

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copula, as in the negative judgment, but rather the predicate (e.g., A is not-B; soul is not-extended). In logic in general, which "abstracts from all content of the predicate" (B97), infinite judgments are considered simply as affirmative judgments. According to Kant, in contrast, "in a transcendental logic, *infinite judgments* must also be distinguished from *affirmative* ones" (B97). Infinite judgment does not merely exclude A negatively from B's "sphere" but also positively affirms that A is included among the infinite number of things that are excluded from the latter's sphere: "In *iudicio affirmativo*, the subject is thought under the *sphaera* of a predicate; in the *iudicio negativo*, the subject is posited outside of the *sphaera* of the latter. In the *iudicio infinito*, the subject is thought in the *sphaeram* of a concept that lies outside the sphere of another" (AA 16:640; *Refl.* 3068). English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 62. For example, affirming that the "soul is not extended" does not imply that it possesses certain finite extension or that it has an extension = 0 (as if it were concentrated in one point), but rather that it is absolutely incomparable with extension: "a 'spirit is not extended' does not mean the same as 'its extension is disappearing,' but rather that 'it can absolutely have none.' A point, on the contrary, is not extended, but it is like a disappearing space. Spirits cannot be considered as points. If I affirm the non-being of a predicate, its mere disappearing is not immediately thought, and I cannot consider the subject as belonging to the same species, but that it should be often included among things of different species" (AA 18:363-; *Refl.* 5826). Logically, between two opposite predicates (e.g., extended and non-extended), there is no third (*exclusi tertii*). The infinite judgment, however, transcending formal logic, seems to lead to the idea of a *tertium comparationis* (e.g., extension) in reference to which the opposition of two "disparate" predicates (*per disparate*), such as "extended" and "red," can be distinguished from two "comparable" predicates, such as extended and non-extended. Infinite judgment does not indicate a mere "exclusion" like negative judgment, but should rather be considered a "positive action" (AA 9:104n1). An infinite judgment "delimits" the sphere of the predicate "extension" within which the opposition of extended and non-extended assumes a particular meaning in respect to the opposition between extension and all predicates that are not "extended" (e.g., red, perfumed, etc.): "Infinite judgment does not only show that a subject is not contained in the sphere of a predicate but that it also lies somewhere outside of its [the predicate's] sphere. Thus, the judgment presents the sphere of the predicate as limited [beschränkt] (AA 9:104, Fn. 1). The soul, the *negation* of all spatial determination, has nothing to do with extension. The point, in contrast, is only a disappearing extension. It is the *limitation* of extension and thus remains a spatial determination. For an analysis of post-Kantian philosophy's interpretation of infinite judgment, which emphasizes the different readings of Georg Wilhem Friederich Hegel and Herman Cohen in particular, see Gordin, Jakob. *Untersuchungen zur Theorie des unendlichen Urteils* (Berlin: Akademie Verlag, 1929).

[62] For an analysis of the difficulty of expressing the distinction between *Realität* and *Wirklichkeit*, see Schwarz, Wolfgang. "Kant's Categories of Reality and Existence," *Philosophy and Phenomenological Research* 48 (1987): 343–46.

[63] It should be noted that Kant also uses the expression "empirical reality" (*emprische Realität*) as a synonym of "objective validity" (*objective Gültigkeit*), thus opposing it to mere dreams or illusions. Here, a single category is not at stake, but rather the general problem concerning the possibility of applying the categories to objects. For a discussion of the relations between the three aspects of the "reality problem" in Kant's thought, see Zöller, Günter. *Theoretische Gegenstandsbeziehung bei Kant. Zur systematischen Bedeutung der Termini "objektive Realität"*

thing is, specifying "that internal determination of a thing through which it can be distinguished from another as a unity" (AA 18:663; Refl. 6338a),[64] whether or not "that" (*dass*) the thing exists can be established. To understand the Anticipations of Perception, one must be attentive to the fact that they do not refer to "reality" as existence, but to "reality" as a category of quality, and that they are involved in defining the relations between this category and its opposite, negation.

This meaning of the term "reality," and of the corresponding category, is supported by a long tradition that dates back to the Middle Ages.[65] Again, Kant surely drew it from the philosophical debate of his time, and from the Leibnizian-Wolffian tradition in particular.[66] The *Lectures on Metaphysics (Vorlesungen über die Metaphysik*), where Kant comments on Baumgarten's *Metaphysica* for his students,

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und "objektive Gültigkeit" in der "Kritik der reinen Vernunft", Kant Studien, Ergänzungsheft, 117 (Berlin: Gruyter, 1984). in particular 213–29. On the ambiguous relation of *Realität* and *Wirklichkeit*, see Delfour, Jean-Jacques. "Une équivocité énigmatique dans le quatrième paralogisme de la Critique de la raison pure. La labilité de la frontière entre réalité et effectivité," *Kant Studien* 88, no. 3 (1997): 280–310.

[64] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 280.

[65] This use of the term *realitas* dates back to Francisco Saurez's *Disputationes Metaphysicae*, which interprets "reality" as thinkable essence, in as much as this is logically possible, in opposition to pure nothingness, which is logically impossible. The use of the term probably derives from the Scotus tradition that came to identify *realitas*, *aliquitas*, *quidditas*, and *essentia*. In 1692, Stephanus (Étienne) Chauvin effectively expressed this sense of the term: "*realitas* is a *diminutivum* of *res*"; the followers of Scotus, "who first found this word," attributed to each *res* (e.g., human) different *realitates* (living being, sensible being, etc.), ending in ultimate *realitas* (rationality), which distinguishes human from all other *res*. Chauvin, Stephanus. *Lexicon Philosophicum … ita tum recognitum & castigatum; tum varie variis in locis illustratum, tum passim quammultis accessionibus auctum & locupletatum, ut denuo quasi novum opus in lucem prodeat*. 2nd ed (Leeuwarden: F. Halma, 1713), 557–.

[66] In his *Ontologia*, Christian Wolff considers *realitas* synonymous to *quidditas*: "quicquid est vel esse posse concipitur, dicitur res, quatenus est aliquid; ut adeo res definiri possit per id, quod est aliquid. Unde realitas et quidditas apud scholasticos synonyma sunt [everything that is or could be thought is called *res*, thing, which is something, *aliquid*; therefore a *res* could be defined as that which is *aliquid*. For this reason, realitas and quidditas are synonymous for the scholastics]" (Wolff. "Philosophia prima sive Ontologia." §243). This "synonymy between 'being real' (*realitas*) and 'being something,'" as Pietro Kobau notes in respect to Wolff, is also maintained by Baumgarten, who uses the expression in an even more restricted sense, calling "realities" the single properties of the essence of a thing: "quae determinando ponuntur in aliquo (notae et praedicata), sunt DETERMINATIONES, altera positiva, et afferativa, quae si vere sit, est REALITAS, altera negativa, quae si vere sit, est NEGATIO [what is posited in something in order to determine it (marks and predicates) are determinations, some of them are positive, or affirmative, and if they are true, are called reality, Others are negative and if they are true, are called negations]." (Baumgarten. *Metaphysica*, §36). See also Kobau, Pietro. *Essere qualcosa. Ontologia e psicologia in Wolff* (Turin: Trauben, 2004). Reality is thus the affirmative predicate of a thing, that which can truly be attributed to it, whereas negations are something negative: "Hinc negationes et realitates sunt sibi invicem oppositae. Tam realitates ipsae, quam entia, quibus insunt, ENTIA REALIA seu positiva dicuntur. Negationes autem ENTIA NEGATIVA [Therefore negations and realities are opposite to one another. The realities themselves and the entities which they refer to are called real entities; negation, in contrast, negative entities]." (Baumgarten. *Metaphysica*, §135.).

is probably the place where his debt to the terminology of the "School" is most apparent: *"realitas* cannot be adequately translated into German. The correct meaning of the term is *Dingheit, Sachheit*" (AA 18:1146); in fact, the "[r]eality of a thing, according to the sense of the word is the thingliness [Sachheit] of a thing, therefore something positive in itself" (AA 30:998).[67] Something, in as much as it is at least possible, indicates in itself a being something. In this sense, "reality" is opposed to that which is lacking; that which is absent and thus "is not." I will show that Kant, by retrieving this meaning of the terms "reality" and "negation," radically reformed the Leibnizian conception of the relation between positive and negative. Kant's modification, which profoundly influenced the post-Kantian debate, allows the possibility of thinking that even a negative "is," that is, constituted as an equally positive "reality."

1.7 The Distinction Between Realitas Phaenomenon and Realitas Noumenon

To fully grasp the innovativeness of Kant's contribution, one must take into consideration the further specification concerning the above definition of "reality" that the Anticipations of Perception introduce. In the first edition of the *Critique of Pure Reason*, as I noted, the principle attributes a degree to sensation and to the reality which corresponds to it called *realitas phaenomenon*, whereas the second edition speaks of the "real that is the object of sensation." The relative proposition, "that is the object of sensation" or "that corresponds to a sensation," does not seem here to have a simple "apposite" value, assuming a "restrictive" function instead. In other words, speaking of the "real that is the object of sensation," Kant intends to isolate, in the genus "reality," the species "reality that can be the object of sensation" (*realitas phaenomenon*) in order to separate it from the "reality that is the object of understanding" (*realitas noumenon*):[68] "A reality <*realitas*> is either *phenomenon* or *noumenon*. Everything that is exhibited positively to our senses is called: phenomenal reality <*realitas phaenomenon*>; and everything that is exhibited positively to our pure understanding is noumenal reality <*realitas noumenon*>. Phenomenal reality <*realitas phaenomenon*> or reality in appearance (or apparent reality) is that which lies in our senses" (AA 28:560).[69] One should distinguish between "reality" in as much as it is "a phenomenon, that is in as much as it is

[67] English translation from Kant. *Lectures on Metaphysics*, 466 Amerik translates Sachheit with "materiality." In the footnote on the same page, however, he observes that "this might also be rendered as *thingliness*". Especially in this context, in order to avoid misunderstanding, this second translation is preferable.

[68] "The expression in parentheses, *realitas phaenomenon*, from [edition] A is without doubt an addition to clarify that it does not refer to the thing in itself." (Broad, Charlie Dunbar. *Kant: An Introduction*. ed. Casimir Lewy (London: Cambridge University Press, 1978), 242).

[69] English translation from Kant. *Lectures on Metaphysics*, 324.

considered an object of the senses, and noumenon, that is in as much as it is considered through the understanding as such" (AA 28:421).

For the understanding, to be something "positive" means not to present contradictions such that "reality" indicates the possibility of being thought.[70] For sensibility however, "positivity" is found in the capacity to provoke a certain sensation and "reality is therefore that which can be sensed [Empfindbare]" (AA 28:1250). The qualitative determination that characterizes noumenal realities, which are objects of pure intellect, is thus that which permits the distinction of the sphere of one concept from that of all others. For phenomena, it indicates that which characterizes the qualities that are objects of the senses (e.g., hot and cold, clear and obscure, heavy and light, etc.)[71] With this reference to sensation, Kant pinpoints the import of the Anticipations of Perception: reality as such does not have a degree or an intensive magnitude; but only the "reality of the phenomenon (the characteristic of a thing in as much as it is an object of the senses)" (AA 8:154).

Reality, as a pure category, is thus generally defined as that which makes something be what it is and thus different from everything else. This definition is valid just as much for objects of pure thought (*realitas noumenon*) as for sensible qualities (*realitas phaenomenon*). The Anticipations of Perception restrict the meaning of the category of reality to phenomenal reality alone, that is, to the reality that corresponds to sensation. The anticipations attribute an intensive magnitude to this reality, and to this reality alone: "that in the empirical intuition which corresponds to the sensation is reality (*realitas phaenomenon*); that which corresponds to its absence is negation $= 0$ … [B]etween reality in appearance and negation there is a continuous nexus of many possible intermediate sensations, whose difference from one another is always smaller than the difference between the given one and zero, or complete negation" (B209). Reality can be distinguished from negation not as one distinguishes a concept from its opposite but as one distinguishes a greater or lesser difference in degree (reality) from the vanishing of this difference, that is, from the difference $= 0$ (negation). "The 0 [is] a *realitas evanescens*, that is, a vanishing reality … and the negation can be understood as a disappearing reality of this sort"(AA 28:426) as the limit to which a difference becoming ever smaller tends.

If reality is cognized through the understanding, the difference between reality and negation is rigid like the one that separates A and not−A; that is, reality is distinguished as a specific concept among all others that does not admit mediation between it and the other (negation) because, as Lambert explains, for example, "the concepts of being and non-being do not have any degree of intensity. Rather, c is b or

[70] Consider Leibniz's words: "Nihil aliud enim realitas quam cogitabilitas[reality is nothing other than thinkability]" (GP 1:272).

[71] See for instance this passage: "Differentiating logically means recognizing that a thing *A* is not *B*; it is always a negative judgment. In contrast, physically differentiating [physisch unterscheiden]" means "different things cause different sensations" (AA 2:60; English translation is from Kant, Immanuel. "The False Subtlety of the Four Syllogistic Figures." In *Theoretical Philosophy, 1755–1770*. eds. David Walford and Ralf Meerbote. (Cambridge, UK; New York: Cambridge University Press, 1992), 104).

it is not b. Here there is no third."[72] Phenomena, however, can allow that hot and cold, full and empty, light and darkness, and so forth are all united through the same quality (heat, density, and light respectively), but are nevertheless capable of being distinguished in terms of quantity; that is, in terms of intermediate degrees of intensity between the one and the other: "everything real has for the *same quality* its degree … which … can become infinitely smaller until it is transformed into emptiness and disappears" (B216).

The particular qualitative nature of phenomenal reality (colors, sounds, heat, etc.) can be provided through sensation alone, thus only *a posteriori*, in any given example furnished by experience. Nevertheless, one can establish completely *a priori*, that is, independently of all experience, that quantitative differences, specifically differences in degrees,[73] can be distinguished within each sensible quality without compromising the quality's identity. The quality positively represents a sort of *tertium comparationis* between reality and its respective negation. Negation is therefore distinguished from reality only by a difference in quantity and thus represents "simply a limit and not the contradictory opposite of reality" (AA 18:362; Refl. 5816),[74] that is to say, negation indicates a mere difference in degree that can diminish until vanishing.[75]

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[72] Lambert. "Anlage zur Architektonik, oder Theorie des Einfachen und Ersten in der philosophischen und mathematischen Erkenntnis." 359, §21. Also See Guyer. *Kant and the Claims of Knowledge*, 198.

[73] From this point of view, Ralph Charles S. Walker's objection to Kant does not seem convincing. Walker argues that it is possible to "imagine an experience rather unlike ours … in which sensible qualities are not naturally thought of as coming in degrees ; things might feel either hot or cold , without gradations between them" (Walker. *Kant: The Arguments of the Philosophers*, 95). For Kant, such a world is certainly imaginable (hence not contradictory), but could not be subjected to scientific knowledge, that is, to measurement. As noted above, the principles, which include the Anticipations of Perception, are conditions of possibility of objective knowledge and thus refer to the object of such knowledge as well.

[74] On the schematization of the category of reality in particular, see Haas, Bruno. "Kants Qualitätsschematismus." In *Analysen – Probleme – Kritik 1*. ed. Hariolf Oberer. (Würzburg, Germany: Königshausen und Neumann, 1988), 133–74.

[75] It is also no accident that, in the *Schematismuskapitel*, Kant abandons the opposition between reality and negation understood as an opposition between two contradictory concepts: "[t]he opposition of the two thus takes place in the distinction of one and the same time as either a filled or an empty time" (B182). This is the very chapter where the mediation between "category" and "principle" (*Grundsatz*) happens, and thus between "reality" understood as a pure category, that, even "without any conditions of sensibility, should hold for things in general, as they are." (B186) and phenomenal reality reduced to being "that to which a sensation in general corresponds" (B182) ("*sensatio realitas pheanomenon*" [B186]). In contrast to what occurs in pure understanding between a concept and its opposite, the full and the empty in intuition are only distinguished by their quantity and can be more or less full or more or less empty the same time: "every sensation has a degree or magnitude which it can *more or less* fill the same time … until it ceases into nothingness (= 0 = *negatio*)" (B183, emphasis mine). What permits the same "reality" to be thought as a *quantum* is thus the possibility of admitting "a relation and connection between, or rather a *transition* from reality to negation"; and "the schema of a reality, *as the quantity* of something" is therefore a "continuous and uniform *generation* of that quantity in time, as one descends in time from the sensation that has a certain degree to its disappearance or gradually ascends from negation to its magnitude" (B183, emphasis mine).

1.8 The Critique of the Concept of Gradus Perfectionis

Notes in the margin of the very section that addresses the Anticipations of Perception (A169; B210) in Kant's copy of the first edition seem to confirm what I have attempted to demonstrate: "I do not claim that every reality has a degree [Ich sage nicht, alle Realität hat einen Grad], let alone that everything has an extensive magnitude" (AA 23:29; Refl. 72). Thus, Kant's problem in the Anticipations of Perception does not concern reality as such, but only phenomenal reality. Only the reality that "corresponds to a sensation", *realitas phaenomenon*, can present a greater or lesser intensive magnitude, whereas *realitas noumenon*, that of the things-in-themselves that populate the intelligible world and can only be known through the pure understanding, cannot present any difference in "degree": "all properties of things have a degree, but not the thing itself (substance)" (AA 18:241s.; Refl. 5590). Instead, *realitas noumenon* admits a form of quantity by which "all things as objects of pure understanding also have a magnitude, namely a metaphysical one" (AA 18:241, Refl. 5589),[76] but only in the sense that every entity possesses a degree in relation to God, the realest entity: "in *noumeno*, I can think quantity [quantitaet] only through the relation to the *illimitato*, that is, to that which contains *omnitudinem* as a unity measure [Maasstabe]" (AA 18:380, Refl. 5905). Quantity in respect to noumenal reality involves comparing limited creatural being with supreme infinite being.

Here, perhaps for the first time, the inextricable connection between *realitas* and *perfectio*[77] in traditional Metaphysics is broken. Traditional metaphysics identifies reality and positivity with perfection (*perfectio est gradus realitatis positivae*), and being with goodness (*omne ens est perfectum et bonum trascendentaliter*). According to the traditional conception, "[m]etaphysical perfection consists in reality. – Reality or thingness is that something is perfect as a thing" (AA 28:211). The greater degree

[76] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 249.

[77] Concerning this proposition, Wolffian scholasticism again merely recuperates a long and consolidated tradition of medieval origin that has many resonances in modern thought. Descartes approaches *realitas* and *perfectio* in the *Secundae Responsiones* of the *Meditationes*. See Descartes, René. *Oeuvres de Descartes*. eds. Charles Adam and Paul Tannery (Paris: Vrin, 1964–1974), Vol. 7, 165. Spinoza also explicitly affirms that: "per realitatem et perfectionem idem intelligo." Spinoza, *Etica*, Pars II, def 6 in Spinoza, Benedictus de. *Opera*. ed. Carl Immanuel Gerhardt (Heidelberg,: C. Winter, 1925). See also Totaro, Giuseppina. "Perfectio e realitas nell'opera di Spinoza." In *Lexicon Philosophicum. Quaderni di terminologia filosofica e storia delle idée*. eds. Antonio Lamarra and Lidia Procesi, 71–113. (Rome: Edizioni dell'ateneo, 1988). The same identification occurs in Leibniz's *Monadology* (§41)*:* "perfection is nothing but an amount of positive reality, in the strict sense, leaving out of account the limits or bounds in things which are limited. And where there are no bounds, that is to say in God, perfection is absolutely infinite" (GP 6:613). Leibniz, Gottfried Wilhelm. *The Monadology*, tr. Robert Latta (Oxford: Oxford University Press, 1925), 9. On the relation between Kant and this tradition of thought, see Sala, Giovanni B. *Kant und die Frage nach Gott. Gottesbeweise und Gottesbeweiskritik in den Schriften Kants* (Berlin: Gruyter, 1997), 137–.

of positivity of being, the more perfect it is because "[a] real thing is something positive, where negations are as well. Metaphysical perfection thus consists in the degree of reality" (AA 28:211).[78] One can thus construct a metaphysics on the idea of a hierarchy of infinite degrees of creatures that culminates in the *Ens illimitatum*. From this point of view, different noumenal realities can be considered more or less perfect (that is, containing a greater or lesser *gradus realitatis*) when compared to the being that contains all the perfections and that represents the unity of all positivities. In relation to this being, finite things are distinguished by the fact that they possess some predicates while they are denied others: "[S]ince each thing must have reality, we can represent every possible thing either as an *ens realissimum* or as an *ens partim reale*, *partim negativum*" (AA 28:1013).[79]

This assumption leads to the conclusion that, in *noumeno*, "[a]ll true negations are nothing but boundaries [Schranken],[80] which they could not be called unless they were grounded in the unlimited (the All)" (B604). The "*negations* are *limitations* from a metaphysical point of view" (AA 28:635). "Negations are not specific concepts. Rather, they are only limitations [Einschränkungen] of the concept of reality" (AA 28:1156). Negations have no ontological consistency and no autonomous being; they are nothing more than a "boundary" (*Schrank*) in respect to the entirety of possible reality. When one speaks of "*mixed realities*, [one] is using an improper expression" because "a mixture of a reality and a negation, of something and nothing, cannot be thought" (AA 28:1015–).[81] Rather, every entity, in as much as it is something, contains a certain degree of positive reality, and negation pertains to it only extrinsically, that is, when it is compared with infinite being.

In the phenomenal world, in contrast, each reality possesses a degree, not compared to *different* realities with greater or lesser degrees of perfection in respect to the sum of all perfections, but simply in relation to the vanishing of the difference in degree of the *same* reality or quality, in respect to the difference $=0$: "all magnitudes in phenomena are only comparative; in things in themselves, absolute" (AA 18:705; Refl. 6398). From the point of view of phenomena, negation assumes a positive meaning as the "limit" (*Grenze*) that tends toward the vanishing of the difference in degree; a limit that indicates the point of *indifference* between reality and negation that is neither one nor the other and that is, nevertheless, something perfectly definable (AA 28:570): "The concept of limit belongs only to phenomena <*phenomenis*>, but that of boundaries to noumena <*noumenis*>" (AA 28:570).

In noumena, "[l]imit <*limes*> is the negation, so that the thing may not be the greatest <*ut ens non sit maximum*>" (AA 28:570)[82] and does not express anything

[78] English translations from Kant. *Lectures on Metaphysics*, 33.

[79] English translation from Kant, Immanuel. *Lectures on Philosophical Theology*. eds. Allen W. Wood and Gertrude M. Clark (Ithaca: Cornell University Press, 1978), 44.

[80] I prefer the translation "boundary" for "*Schranke*" instead of "limit." I will use "limit" for "*Grenze*."

[81] English translation from Kant. *Lectures on Philosophical Theology*, 445.

[82] English translation from Kant. *Lectures on Metaphysics*, 334.

determinate, but only the fact that, in relation to God, a finite creatural entity necessarily lacks "something." Limitation in this context assumes a mere negative meaning. Limitation in phenomena, in contrast, is the tending toward the negation $= 0$ of a certain difference in finite degree and should thus be thought as a disappearing reality and, as such, something positive and determinate. One cannot simply say that stillness is not a movement. Rather, it represents an infinitely slow movement.

1.9 From the Ontological Meaning to the Physical Meaning of the Concept of Degree

Kant clearly separates the ontological meaning of *gradus realitatis*, as "degree of perfection," from the physical-mathematical one that is reduced to a simple "degree of efficacy"[83] that is subjectively manifested in sensation: "mathematics teaches how to know and determine *quantitas phaenomenon*, only philosophy *quanta noumena*" (AA 17:679s; Refl. 4698). "There is with the noumena <*noumenis*> a greatest <*maximum*>, but with the phenomena <*phaenomenis*> there occurs neither a greatest <*maximum*> nor a smallest <*minumum*>" (AA 30:835–)[84] because, from

[83] See Delekat. *Immanuel Kant. Historisch-kritische Interpretation der Hauptschriften*, 126. Understanding "intension" and "remission" as "perfection" and "imperfection" was common in medieval debates. Intension was often understood as degree of perfection: the perfection or intension of things is measured by their nearness (*propinquitas*) to the most perfect being, the highest intensity (*gradus summus*) and their imperfection or remission is measured by their distance (*distantia*) from that being. This conception, however, was already criticized by the so-called *Calculatores*, who regarded qualitative variation as a purely relative distinction between great and small: 'Whiteness A is more intense than whiteness B,' or 'whiteness B is more remiss than whiteness A.'. The theory of the *Calculatores* was later attacked in turn by those who, like Pietro Pomponazzi, still made constant use of the scheme of God as the measure of all things in the metaphysical hierarchy of being as they approach toward him or recede from him as a pole measuring the various degrees of perfection. See: Wilson, Curtis. "Pomponazzi's Criticism of Calculator," *Isis* 44, no. 4 (1953): 355–62, 361. See also Thorndike, Lynn. "Calculator," *Speculum* 7, no. 2 (1932): 221–30. The modern era never really abandoned this last conception. In his correspondence with De Volder, even Leibniz seems to confuse these two concepts of degree when he writes: "in ipso motu promtiore plus est *realitatis et perfectionis*" (GP 2:185; emphasis mine; see also De Volder's response in GP 2:188): a faster motor is more perfect than a slower one. However, Leibniz himself later proceeds to clearly distinguish the two concepts of degree, using the same language that Kant resorts to. The concept of maximum velocity is illegitimate and meaningless, while a maximum degree of perfection is admissible: "We must also know what a perfection is. A fairly sure test for being a perfection is that forms or natures that are not capable of a highest degree are not perfections, as for example, the nature of number or figure. For the greatest of all numbers (or even the number of all numbers), as well as the greatest of all figures, imply a contradiction, but the greatest knowledge and omnipotence do not involve any impossibility. Consequently, power and knowledge are perfections, and, insofar as they belong to God, they do not have limits" (GP 4:427). English translation from Leibniz, Gottfried Wilhelm. *Philosophical Essays*. eds. Roger Ariew and Daniel Garber (Indianapolis, IN: Hackett Pub. Co., 1989), 35–.

[84] English translation from Kant. *Lectures on Metaphysics*, 193.

the phenomenal point of view, "how large something is can only be known relatively [relativisch]. Absolute unity does not exist outside the *ens realissimum*" (AA 18:338; Refl. 5729).

Therefore, knowledge of degree in phenomena is legitimate because, among the degrees that are given in experience, one can always choose a certain arbitrary degree to serve as a point of indifference in relation to which one can evaluate that which is greater or lesser than the point. Concerning noumena however, "metaphysical perfection is the degree of reality and we cannot appraise that because we have no concept of the highest degree of reality" (AA 30:766–).[85] The concept of *ens realissimus* cannot be given *in concreto* in any possible experience.

Consequently, the Anticipations of Perception presuppose the passage from "reality," in the metaphysical sense, to phenomenal reality. The concept of *quidditas entis* is abandoned and the concept of reality is limited to *virtutes* that can be measured, admitting a gradual transition between reality and negation, and vice versa, something that is inconceivable through the "binary" logic that the understanding is founded on.[86] The Anticipations of Perception, together with the "Axioms of Intuition" (*Axiomen der Anschauung)*, lead to the conclusion that "[e]verything in appearance is quantum insofar as it contains time or space (*extensive tale*) or fills it, i.e., is contained in time or space (*intensive tale*: reality in sensation)" (AA 18:409; Refl. 5970).[87] If differences of extension (e.g., of volume or duration), as I have shown, can only be determined in space and time, one must assert the opposite that "the matter (the physical), or the content, which signifies a something that is encountered in space and time … and corresponds to sensation" (B751) admits differences in degree: "all objects in space in time have an extensive magnitude; all the objects of sensible intuition have an intensive magnitude" (AA 28:704).

In the *Critique of Pure Reason*, Kant explains the connection between the principles of the *Axioms of Intuition* and the Anticipations of Perception and mathematics:

> The … two principles, which I named the mathematical ones, in consideration of the fact that they justified applying mathematics to appearances … and taught how both their intuition and the real in their perception could be generated in accordance with rules of a mathematical synthesis, hence how in both cases numerical magnitudes and, with them, the determination of the appearance as magnitude, could be used (B221).

If the *Axiomen der Anschauung* make possible the "mathematics of extension" (B204), then the Anticipations of Perception are the condition of possibility for "the second application of mathematics (*mathesis intensorum*) to natural science" (AA 4:307).[88] The first regulates pure mathematical knowledge, the second the application of mathematics to physics.[89]

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[85] Ibid., 176.

[86] Ibid., 128.

[87] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 260.

[88] English translation from Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." 100.

[89] See Guyer. *Kant and the Claims of Knowledge*, 200.

Extensive magnitude allows differences in place and time to be distinguished. In contrast, intensive magnitude allows differences of "state" to be determined. "Geometrical" variations of space and time (which depend on the point of view of the observer) as well as true and proper physical changes, like variations in pressure, temperature, and velocity (all requiring a cause), can be determined mathematically. The first refer to differences in extension; the second to differences in degree. In the *pure undifferentiated form* of space and time, the distinctions between "here" and "there" or "now" and "then," between high and low, right and left, in front and behind, as well as between past and future, are not the consequence of determinate "properties" of points and instants, but depend only on their reciprocal relation. In the same way, even for the *qualitatively determined content* diffused in space and time, the pairs of opposite terms (e.g., hot-cold, fast-slow, heavy-light, etc) are not related as "qualities" that correspond to completely different sensations. Instead, they can be reduced to mere relative determinations of "position" within an ordered series of degrees. Here, "reality" and "negation" are distinguished only comparatively, that is, by their relative position, in respect to an intermediate point of indifference that is neither the one nor the other, but rather separates them as their reciprocal "limitation."

Only quantitative *differences* (of extension or of degree) can be known: "Everything that is represented in space and in time has extensive magnitude. All reality in space and time has a degree" (AA 28:562).[90] Absolute values elude our knowledge and are irrelevant to it. The issue is not to affirm that "things-in-themselves" have a greater or lesser quantity of perfection in respect to God but only that phenomenal realities fill space and time to a greater or lesser degree: "For all things as appearances have a magnitude: extensive and intensive. Through this mathematics acquires objective reality. It does not pertain to *entia rationis*" (AA 18:242; Refl. 5589).[91]

1.10 Force and Sensation

The goal of the Anticipations of Perception is not to establish the distinction between intensive and extensive magnitude, which Kant could have assumed to have been common knowledge, but rather to attribute a "transcendental" sense to it. In other words, Kant "is not too interested in the formal and mathematical distinction of extensive and intensive magnitudes. Rather, the two magnitudes are distinguished by their differing systematic positions."[92] Extensive and intensive magnitudes serve

[90] English translation from Kant. *Lectures on Metaphysics*, 326.

[91] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 247.

[92] Gernot Böhme, Böhme, Gernot. "Über Kants Unterscheidung von extensiven und intensiven Größen," *Kant Studien* 65 (1974): 239–58, 239.

to distinguish in a given phenomenon that which pertains to the "pure (merely *formal*) intuitions" (B207, emphasis mine), that is to say, space and time and that which should be attributed to "the *materials* for some object in general […] i.e. the real of sensation" (B207; emphasis mine).

"Space and time do not have degree" (AA 17:54; Refl. 3557) because all the regions of space and intervals of time are placed, in a manner of speaking, on the same level (no space is "more space" than another and no time is "more time" than another). In contrast, "The combination of reality with the concept of magnitude (is intensive)" (AA 18:664, Refl. 6338a)[93] because only intensive magnitude allows one to distinguish, within the perfect uniformity of space and time, differences, not only like larger and smaller, longer or shorter, but also like more or less effective. Whereas space and time "cannot be perceived in themselves" (B207) because of their perfect uniformity, "all objects of perception, insofar as they contain sensation, must be ascribed an intensive magnitude, i.e., a degree of influence on sense" (B208).

Therefore, phenomena "can be evaluated according to their spatial relations (extensively) or according to the degrees of their effects (intensively)" (AA 22:169.8). Sensation is the merely subjective manifestation of the efficacy of such effects: "all objects as objects of intuition have an extensive magnitude, intensive [magnitude] as the grounds of sensation [Gründe der Empfindung]" (AA 18:369; Refl. 5853). If degree indicates the "capacity to produce effects," such efficacy can be measured objectively by regarding "reality as cause … of another reality in appearance, e.g., an alteration)." In this case, "one calls the degree of reality as cause a 'moment,' e.g., the moment of gravity." Such efficacy can be also measured in a merely subjective manner through the influence that it exercises on the senses, that is as the cause … of the sensation" (B210): "In the same way that different moments produce diverse degrees of velocity, diverse degrees of pressure produce different degrees of sensation" (AA 18:239; Refl. 5582).

I will return to the concept of "moment" further below. Here, it is above all important to emphasize that phenomenal reality (that is, reality that fills space and time) is characterized by the capacity to exert a greater or lesser degree of influence on the senses: "Reality is that through which something is an object of perception" (AA 17:668; Refl. 4682).[94] In other words, all phenomena "as objects of possible experience contain in themselves time, space, and what is capable of being sensed in general [das Empfindbare überhaupt]" (AA 18:648; Refl. 6324). What can exert a greater or lesser degree of influence on the senses, "modifying" our sensibility? The *Critique of Pure Reason*, forced to remain on a general level, does not speak about what is "concretely" intended by the objective correlate of sensation beyond characterizing it as that which possesses a greater or lesser "degree of efficacy."

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[93] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 247.

[94] English translation from Ibid., 175.

Other texts by Kant help characterize the nature of the objective correlate of sensation more precisely. A reflection from the end of the 1890s states:

> [I]ntuition and sensation are found in all phenomena. The first contains the form, the second the matter of phenomena. The form of external intuition is space; that of apperception, and thus of every intuition in general, time. Space and time are conditions (*g* of principles) of all *a priori* knowledge of nature. The *principium* concerning the matter of all phenomena is force ('cause of' the production of sensation). Force as the ground [Grund] of spatial relations is moving force (*g* or, that which is the same [welches einerley ist], the force that resists all movement (the external cause of sensation). These are the ground [Grund] of all phenomena (space, time, and force) (AA 14:119; Refl. 40).[95]

On the basis of a "mechanistic" model (in the broad sense of the term), which seems to form the background of Kantian thought, different sensible qualities (colors, sounds, heat, etc.) can be referred to the effect that moving forces (that is, causes of the emerging or vanishing of a movement) provoke on our sensory organs. Sensible qualities are thus revealed only as an accidental difference of the effect that a movement produces by acting on sensory organs: "[T]he principle characteristic of corporal substance is moving force. Movement is the only condition through which something can become an object of the external senses" (AA 30:75). Only movement can affect our senses because "only through movement can something touch our senses. In short, moving force is the origin of the possibility for external phenomena" (AA 30:75). Sounds and colors are wave-like movements (*pulsus*) of air and ether; heat is a *motus tremulus* of ether: "[T]he sensations of our senses all come from movement. These act in a chemical manner or, as in the case of taste, in a mechanical manner. Thus, they are subject to mathematical laws" (AA 30:120).

Motion is that "by which the *subject*, as an object of sense, is affected. For without this motion, that is, without the stimulation of the sense organs [ohne Erregung der Sinnenorgane], which is its effect, no perception of any object of the senses, and hence no experience, takes place" (AA 22:551.12).[96] Movement "modifies" our senses and our senses can only be affected by the action that movement produces on these such that "external sensations must result in movements" (AA 21:452.4). The *Opus postumum*, from which the last quotes are drawn, seems to confirm that

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[95] The relation between force and sensation is also affirmed by Lambert: subjectively, "we begin to be aware with sensation of different degrees of something" (Lambert. "Anlage zur Architektonik, oder Theorie des Einfachen und Ersten in der philosophischen und mathematischen Erkenntnis." 4:398); but objectively "we feel different levels or degree of force" (Lambert. "Anlage zur Architektonik, oder Theorie des Einfachen und Ersten in der philosophischen und mathematischen Erkenntnis." 4:3). See also Abicht, Johann Heinrich. "Kurze Darstellung der kantischen System." In *Neues philosophisches Magazin: Erläuterungen und Anwendungen des Kantischen Systems bestimmt* (Leipzig: Haug, 1790), 1:3:98: "if external forces are the cause of our consciousness, then one says that they act on us; in the face of this action, we *undergo* … such a consciousness stimulated by the action of external force is called a sensation." On the relation between force and sensation, Schelling's position is also important (see below 105); and in a completely different context, Stadler, August. Kants Theorie der Materie (Leipzig: Hirzel, 1883), 207.

[96] English translation from Kant. *Opus postumum*, 87.

this is the model that most likely underlies Kant's arguments in the Anticipations of Perception. In another fragment that was written between February and September 1799, Kant writes: "Space and Time cannot be sensed. The form of intuition of objects in space and time is not empirical, but rather is given *a priori*. The moving forces of matter contain the material contents [das Materiale] of the sensation, which … presents [darbietet] the real [das Reale] of sensible representation, by means of which the object becomes perceivable" (AA 21:202.12n). This and numerous other passages lead to the conclusion that "all perceptions can only be brought about through the influence of matter's moving forces on the subject and on the senses. These forces necessarily result in apprehension as a reaction to the moveable object in space (the matter) … and its movement" (AA 22:389.10).[97]

Emil Arnoldt summarizes what has been said in a simple and effective manner: "on the basis of this principle [of the Anticipations of Perception], I can suppose *a priori* that all sensation corresponds to the intensity [Stärke] with which the real, the sensed [das Empfundene], modifies [affiziert] the senses"; in this way, "every real exerts an influence on my sense or on another real in space and time … [O]nly through the employment of such an influence do I begin to construct a physical object, a body. As simple extensive magnitudes, objects are merely mathematical objects. These become physical bodies only in as much as I attribute certain determinations to them through which they can exercise a greater or lesser influence on something else."[98]

That the "anticipations" say something *a priori* regarding "perception" and not simply regarding "sensation" is based on a conscious choice of terms that prevents the question from being reduced to a simple description of a psychological fact. What can be anticipated is a complex with a certain unity, namely, the "real that is the object of sensation," (*realitas phaenomenon*). Perception, as I have shown, is distinct from sensation because it is "the awareness of an object through sensation" (AA 30:999), that is, the consciousness of the subject's being influenced by something (the real) that cannot be reduced to the sensation of which it is the cause. "Perceptions [Wahrnehmungen]" should thus be understood "as the awareness of the effects [Einwirkungen] on the sensory capacity of the subject, and hence of the sensations, which depend on the influence of external things on it [the subject]" (AA 22:348.10). The translation of the subjective and fluctuating aspect of sensation into objective relations between phenomenal realities is that which permits the connection of perceptions in a unitary and coherent experience, making scientific knowledge of nature possible: "Physics is the principle [das Princip] which contains both the subjective aspect of the perception of moving forces and the objective aspect of their connections as the foundation of experience" (AA 22:297.10).

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[97] On the relation between force and sensation, see Falkenstein, Lorne. *Kant's Intuitionism: A Commentary on the Transcendental Aesthetic*, Toronto Studies in Philosophy (Toronto; Buffalo: University of Toronto Press, 1995). See in particular, the discussion of this subject in the appendix, *"Sensations as Effects of the Intensity of Force,"* 133–.

[98] Arnoldt, Emil, *Gesammelte Schriften* , ed. Otto Schöndörffer (Berlin: Cassirer, 1906–11), 2:80–.

One can speak of perception precisely because of this possibility of considering the sensible element, not as isolated, but as inserted in the net of relations that Kant calls "experience." In the *Postulate des empirischen Denkens überhaupt* (Postulates of Empirical Thought in General), the reference to consciousness assumes a more specific meaning:

> perception, thus sensation of which one is conscious – not immediate perception of the object itself the existence of which is to be cognized, but still its connection with some actual perception in accordance with the analogies of experience, which exhibit all real connection in an experience in general. (B272)

The awareness of the presence of an objective element that is beyond sensation, implicit in the definition of sensible perception as sensation of which we have consciousness, can be further clarified as the possibility of passing from the purely subjective aspect of sensation to the objective dimension, according to which sensation can be part of the whole of experience.

Appealing to sensation seems to inextricably confound the "transcendental" and "psychological" aspects of the problem. However, Kant's characterization of phenomenal reality as "sensible" in contrast to noumenal reality as "intelligible" is an important move, allowing him to define phenomenal reality as that which possesses a greater or lesser degree of efficacy. This is the necessary first step in Kant's substitution of relations between "concepts," characterizing *realitas noumenon* and cognized through the pure understanding, with the conflict between "forces" (with all the ambiguity this term implies) that is the distinctive feature of *realitas phaenomenon*: "logical relation is the principle of contradiction; the relations of space and time *phaenomenorum* and of things [Sachen] and of their real relationships are forces" (AA 17:597; Refl. 4570).

1.11 The Dynamic Conception of Matter

In the "Proof" of the Anticipations of Perception, Kant refers specifically to an example from the natural sciences:

> Nearly all the natural philosophers, since they perceive a great difference in the quantity of matter of different sorts in the same volumes (partly through the moment of gravity, or weight, partly through the moment of resistance against other, moved matter) unanimously infer from this that this volume (extensive magnitude of the appearance) must be empty in all matter, although to be sure in different amounts. (B215)

Kant's polemical objection is directed above all at the atomist conception of matter that reduces the greater or lesser density of bodies to a relation of full and empty. From this perspective, matter is composed of corpuscles of equal density that fill the same space in a greater or lesser number depending on the empty intervals that separate them from one another, and the processes of rarefaction and condensation are explained as a reduction or an increase in the number of corpuscles and a corresponding variation in the extension of the empty space between them: "they assume that the real in space (I cannot call it here impenetrability or weight, since

these are empirical concepts), is everywhere one and the same, and can be differentiated only according to its extensive magnitude, i.e., amount" (B215).

In contrast, Kant maintains the possibility of an alternative solution that will at least "entirely obviate that alleged necessity of the presupposition that the difference in question cannot be explained except by the assumption of empty spaces" (B215). Rather than a variation in the number of particles in respect to the empty extension in which they are distributed, one can think of a decrease in the degree by which matter fills space: "it is false to assume that the real in appearance is always equal in degree and differs only in aggregation and its extensive magnitude" (B216). It is equally plausible to sustain that "everything real has … its degree (of resistance or of weight) which, without diminution of the extensive magnitude or amount, can become infinitely smaller until it is transformed into emptiness and disappears" (B216). In asserting that not only the quantity of corpuscles in respect to empty space can vary but also the intensity with which matter fills space, it is not necessary to claim that "one matter [is] denser than another when it contains less emptiness" (AA 4:525).[99] Kant suggests the following alternative: "space, if it should be necessary, can be assumed to be completely filled, and in different degrees, even without dispersing empty interstices within matter" (AA 4:523).[100]

Absolute fullness and emptiness, as intrinsic properties of atoms, are replaced by a relative fullness and emptiness: "a space should only be considered empty in relation to another space" (AA 18:364; Refl. 5826) because "all experience yields only comparatively empty spaces for our cognition" (AA 4:535;[101] see also AA 21:588). In the "Proof" of the Anticipations of Perception, Kant resolutely affirms that "no perception, and hence no experience, is possible that, whether immediately or mediately (through whatever detour in inference one might want), would prove an entire absence of everything real in appearance" (B214) because every phenomenon involves a mere "difference in the degree of its reality" (B214) that can be as small as one likes. Consequently, every reality "must yield infinitely different degrees with which space or time is filled" (B214).

In the *Metaphysical Foundations of the Natural Sciences* (Metaphysische Anfangsgründe der Naturwissenschaft) and in particular the second section entitled "Metaphysical foundations of dynamics" (Metaphysische Anfangsgründe der Dynamik (AA 4:496–534),[102] Kant attempts to give a concrete physical expression

[99] English translation from Kant. "Metaphysical Foundations of Natural Science." 253.

[100] Ibid.

[101] Ibid., 244.

[102] Ibid., 209–44. For a detailed commentary, see Pollok, Konstantin. *Kants "Metaphysische Anfangsgründe der Naturwissenschaft". Ein kritischer Kommentar* (Hamburg: Meiner, 2001), 222–341; with specific references to Anticipations of Perception on 225–, 233, 247, 281–2, and 345–6.

for this philosophical hypothesis. Within this context, Kant challenges those thinkers who, like Lambert, hold a purely logical conception of the impenetrability of matter, opposing full and empty as one opposes A and not−A: "Lambert and others[103] called the property of matter by which it fills a space solidity (a rather ambiguous expression) …. According to their ideas the presence of something real in space must already, through its concept, and thus in accordance with the principle of noncontradiction, imply this resistance …. But the principle of noncontradiction does not repel a matter advancing to penetrate into a space where another is found" (AA 4:497–).[104] Kant substitutes the relation between the concepts A and not−A, contradictory opposites that cannot coexist (the one "rejects" the other, excluding the other's presence in the same position), with the relation between two opposing forces that establish an equilibrium, implying that "Matter fills a space, not through its mere existence, but through a particular moving force." AA 4:497).[105] This force's resistance to penetration impedes the co-presence of matter in the same position: "everything real in the objects of the outer senses, which is not merely a determination of space (place, extension, and figure), must be viewed as moving force." (AA 4:523);[106] that is, a force that causes a movement or that resists a movement: "the existence of substances in space is not [determined] by the principle of contradiction, but rather by means of a force, which causes resistance" (AA 14:113; Refl. 36).

What is perceivable; that is, what can exercise an influence on the senses; is precisely this repulsive force or tendency to expand: "Impenetrability, as the fundamental property of matter, whereby it first manifests itself to our outer senses, as something real in space, is nothing but the expansive power of matter" (AA 4:508).[107] Matter "discloses its existence to us in no other way than through that sense whereby we perceive its impenetrability, namely, sense of touch [Gefühl],[108] and thus only in relation to contact" (AA 4:510).[109] The greater or lesser resistance that touch encounters in trying to penetrate matter is the subjective valuation of different degrees with which repulsive force resists attempts to be compressed because "the body that I touch resists my hand that seeks to penetrate the space that the body occupies."[110] Here, unlike in the *Critique of Pure Reason,* the subjective

[103] For whom Kant means by "other authors," see Pollok. *Kants "Metaphysische Anfangsgründe der Naturwissenschaft". Ein kritischer Kommentar*, 229–.

[104] English translation from Kant. "Metaphysical Foundations of Natural Science." 210.

[105] Ibid., 210.

[106] Ibid., 233.

[107] Ibid., 220. English translation slightly modified by the author.

[108] On *Gefühl* [feeling] as the sense of "touch," see Satura, Vladimir. *Kants Erkenntnispsychologie in den Nachschriften seiner Vorlesungen über empirische Psychologie* (Bouvier: Bonn, 1971), 90.

[109] English translation from Kant. "Metaphysical Foundations of Natural Science." 221.

[110] Beck, Jakob Sigismund. *Erläuternder Auszug aus den critischen Schriften des Herrn Prof. Kant auf Anrathen derselben* (Riga, Germany: Hartknoch, 1796), 3:145.

aspect of sensation is subordinated to Kant's primary interest in the "objective correlate," namely, repulsive force.[111]

However, if only repulsive force were admitted, that is, only the tendency for matter to expand, this force "would disperse itself to infinity, and no assignable quantity of matter would be found in any assignable space," and, consequently, "all spaces would be empty, and thus, properly speaking, no matter would exist at all" (AA 4:508).[112] Attractive force "acting in the opposite direction to the repulsive force" (AA 4:509)[113] must also be supposed. While repulsive force, through its tendency to expand, is responsible for the *density* of matter; attractive force, through which the parts of matter attract one another by resisting tensile stress, makes the *cohesion* of matter possible. Not even this force by itself, however, is sufficient to explain the being of matter since the "parts of any body which cohere together press against each other with true forces (of *attraction*) and the effect of these strivings would be a reduction in spatial volume, were it not for the fact that equally true activities operated in the same degree against them, the operation of the repulsion being the ground of impenetrability" (AA 2:199).[114] The greater or lesser filling of space is thus due to two forces that limit each other.

The relation between reality and negation, between being and non-being, is not thought in terms of the model of non-contradiction between A and not–A, but instead on the model of the equilibrium between forces through which "attraction and repulsion (+*a* and −*a*)" are opposed to each other "really (not logically like *a* and not−*a*)" (AA 21:287.3):

> First, the *real* in space (otherwise called the solid), in the filling of space through repulsive force; second, that which in relation to the first, as the proper object of our outer perception, is *negative*, namely, attractive force … third, the *limitation* of the first force by the second, and the determination of the degree of filling of a space that rests on this. Hence, the quality of matter, under the headings of *reality, negation, and limitation*, has been treated completely, so far as pertains to a metaphysical dynamics (AA 4:523; emphasis mine).[115]

Here, Kant brings to fruition the rethinking of the relations between reality and negation for which he already provided the bases in the *Critique of Pure Reason*: reality and negation do not contradict each other, but rather limit each other, determining differences in degree, that is, differences in quantity, in the filling of space.

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[111] This interpretation is put forth by both Schelling (see below 105) and Hegel: "Kant from the start one-sidedly attributes to the concept of matter only the determination of impenetrability, which we are supposed to perceive by the sense of touch [Gefühl]" (HW 5:201; English translation from Hegel. *Hegel's Science of Logic*, 180). According to Hegel Kant presents matter as impenetrable" "since it presents itself under this category to the sense of touch [Gefühl] by which it manifests itself to us" (HW 5:201; Hegel. Hegel's Science of Logic, 180).

[112] English translation from Kant. "Metaphysical Foundations of Natural Science." 220.

[113] Ibid.

[114] English translation from Kant. "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy." 236.

[115] English translation from Kant. "Metaphysical Foundations of Natural Science." 76.

Atomism, with its *mechanical* conception of matter, is constrained to start from a rigid and insuperable opposition between the absolutely full and the absolutely empty. Instead of explaining the phenomena of compression and rarefaction, atomism merely rescues its explanation by ultimately resorting to something that cannot be compressed or rarefied. In contrast, a *dynamic* conception allows the possibility that matter "fills its space through the repulsive forces of all of its parts, that is, through an expansive force of its own, having a determinate degree, such that smaller or larger degrees can be thought to infinity" (AA 4:499).[116] It is not necessary to affirm that matter "is not capable of compression except insofar as it contains empty spaces within itself" (AA 4:502)[117] because one can think of an increase in degree by which matter fills space as when it is reduced to a more restricted space.

For example, "[w]hen, in the barrel of an air pump filled with air, the piston is driven closer and closer to the bottom," so that "the air-matter is compressed" (AA 4:500),[118] the gradual transition from a certain density of air to a greater one can be easily understood by resorting to the idea of cramming corpuscles into a smaller space. However, an inconsistency is revealed in this explanation when the corpuscles themselves are considered, which must admit a sudden transition from absolute emptiness to absolute density. Kant argues that instead of an "*absolute* impenetrability," which "rests on the assumption that matter, as such, is capable of no compression at all," one can admit a "*relative impenetrability*" of matter "that rests on resistance increasing in proportion to the degree of compression" (AA 4:501–).[119]

The atomist theory commits the error of assuming "absolute solidity" as a *qualitas occulta* in the analogous explanation of the cohesion of bodies, that is, of their resistance to tensile stress. The theory argues that absolutely indivisible particles connected through hooks and fasteners are at the root of the cohesion of matter. Such solutions only defer the explanation, attributing to atoms absolutely that which bodies only possess relatively. In fact, the hooks and fasteners that should make matter cohere presuppose the firmness that they are meant to explain.

In respect to the "atomistic" conception, the "dynamic" conception of matter is able to replace clear distinctions with gradual transitions and absolute properties with relative differences. Instead of "qualitative" differences between corpuscles with various forms and natures, one can conceive of the mere "quantitative" difference in the degree of filling of space; that is to say, a "difference which is not a difference," reduced to a distinction between more or less with no privileged point of transition. The dynamic conception of matter thus allows all the qualitative differences between matter to be reduced to mere differences in the degree of the filling of space due to various reciprocal relations between attractive and repulsive force. These differences do not involve differences in the "original configurations"

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[116] Ibid., 211.

[117] Ibid., 214.

[118] Ibid., 214.

[119] Ibid.

of the corpuscles "and its interspersing of empty spaces" (AA 4:525).[120] Even though Kant cautiously avoids entering into the details of this type of explanation, it nevertheless had a notable influence on later philosophy of nature. An example is Carl August Eschenmayer's *Säze aus der Natur-Metaphysik auf chemische und medicinische Gegenstände angewandt* [Passages from the Metaphysics of Nature Applied to Chemical and Medical Objects], a work that resonated widely in its day (just consider the importance that Schelling attributes it). Eschenmayer, explicitly referring to Kant, aims precisely to reduce all qualitative differences of matter to differences in degree, thus admitting the possibility of "considering this diversity [of matter] also as degrees" where "a degree of matter is a relation involving attractive or repulsive force."[121]

Further below, I will discuss how Eschenmayer's conception succeeds in grasping one of the most important aspects of the dynamic conception of matter even if in a simplistic manner. For now, I will merely take the opportunity to focus on what should be considered the fundamental advantage of Kantian dynamism: that the concept of matter is effectively reduced to that of movement. As I have shown, movement is that which can exercise the action on our senses that is the cause of sensation: "The basic determination of something that is to be an object of the outer senses had to be motion, because only thereby can these senses be affected. The understanding traces back all other predicates of matter belonging to its nature to this, and so natural science, therefore, is either a pure or applied *doctrine of motion*." (AA 4:476–).[122] According to Kant, this implies not only that the multiplicity of phenomena should be understood in terms of movement between bodies that hit and attract one another (i.e., what occurs in the *Mechanik*), but also that the very "concept of matter," the subject of the *Dynamik*, "is reduced to nothing but moving forces, and one could not expect anything else, since no activity or change can be thought in space except mere motion" (AA 4:524).[123]

"Penetration into a space … is a motion," and "the resistance that a matter offers in the space that it fills to every penetration by other matters is a cause of the motion of the latter in the opposite direction" (AA 4:497).[124] This explanation provides "a concept of an acting cause, together with its laws, whereby the action, namely, the resistance in the filled space, can be estimated in regard to its degrees" (AA 4:502).[125] The forces of attraction and repulsion are the "[o]nly … two moving forces of matter

[120] Ibid., 235.

[121] Eschenmayer, Carl August. *Säze aus der Natur-Metaphysik auf chemische und medicinische Gegenstände angewandt*, Propositions from the Metaphysics of Nature Applied to Chemical and Medical Objects (Tübingen: Heerbrandt, 1797), 5.

[122] English translation from Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." 191.

[123] Ibid., 234.

[124] Ibid., 210.

[125] Ibid., 214.

[that] can be thought." All movement that a material can impress on another can be conveyed as a straight line, "[b]ut in this straight line there are only two possible motions: the one through which the two points *remove* themselves from one another, the second through which they *approach* one another. …. But the force causing the first motion is called *repulsive force*, whereas the second is called *attractive force*" (AA 4:498–).[126]

Undeniably, this dynamic conception of matter presents various difficulties and incongruencies when considered from a physical[127] point of view. The philosophical import of its reducing matter to relations between forces, however, is that Kant is able to rethink the relationship between "substance" and its "properties." Rather than matter *having* forces, matter *is* the conflict between moving forces; rather than the concept of matter being presupposed in the concept of movement as its "subject," matter can be defined starting from movement and through movement. Kant thus transforms the matter of a "thing" that can entertain relationships with other "things" into a mere "relation." Moreover, this is the ideal that Kant already announced in the *Critique of Pure Reason* itself: "[E]verything in our cognition that belongs to intuition … contains nothing but mere relations of places in an intuition (extension), alteration of places (motion), and laws in accordance with which this alteration is determined (moving forces). But what is present in the place, or what it produces in the things themselves besides the alteration of place, is not given through these relations" because "through mere relations no thing in itself is cognized" (B66–7)

[126] Ibid., 211.

[127] See Vuillemin. *Physique et métaphysique kantiennes*, 168 On the confusion between force and pressure, see Ibid., 90–94.

Chapter 2
From Real Opposition to the Problem of Change[1]

2.1 Logical Opposition and Real Opposition

As the preceding chapter has shown, the conception of phenomenal reality that Kant defends in the Anticipations of Perception finds its empirical application in the dynamism of the *Metaphysical Foundations of Natural Science*. In Kant's dynamism, agreement between "realities" is not constituted on the lines of a relation between non-contradictory concepts, but rather on the model of a relation between opposed forces that establish an equilibrium. The meaning of this concession and its philosophical implications cannot be understood without considering the central function that the evolution of Kantian thought assigned to the distinction between two forms of opposition: logical opposition (between concepts) and real opposition (between forces).[2]

Since the pre-critical period, and particularly since his 1763 essay, "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy" (*Versuch, den Begriff der negativen Größen in eine Weltweisheit einzuführen*), Kant consistently insisted on the importance of this distinction: "Two things are opposed to each other if one thing cancels that which is posited by the other. This opposition [Entgegensetzung] is twofold: it is either *logical* through contradiction or it is *real*, that is to say, without contradiction" (AA 2:171).[3] The first form of opposition generally occurs when two predicates are related as *A* and not-*A*, in which case the

[1] Kant's term, *Veränderung* is normally translated either as "change" or as "alteration." I have used both terms interchangeably throughout the text.

[2] Paul Guyer affirms the relation between the Anticipations of Perception and the distinction between logical opposition and real opposition: "Indeed, the whole argument of anticipations might be viewed as an illustration of a distinction between logical and real opposition with which Kant had been concerned since his 1763 essay, *Negative Quantities*. Precisely because reality and negation in objects, are not themselves logical contradictions but rather real states which may be in physical opposition, the differences between them may admit degrees" (Guyer. *Kant and the Claims of Knowledge*, 199).

[3] English translation from Kant. "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy." 211.

M. Giovanelli, *Reality and Negation – Kant's Principle of Anticipations of Perception: An Investigation of Its Impact on the Post-Kantian Debate*, Studies in German Idealism 11,
DOI 10.1007/978-94-007-0065-9_2, © Springer Science+Business Media B.V. 2011

position of the one necessarily entails the logical cancellation of the other: "A body which is in motion is something; a body which is not in motion is also something (*cogitabile*); but a body which is both in motion and also, in the very same sense, not in motion, is nothing at all" (AA 2:171).[4] Attempting to think a body as moving and not moving at the same time results in a vacuous and impossible thought; that is, a contradiction.

The second form of opposition, real opposition, is characterized by the opposition of two determinations (i.e., two predicates) of a thing, but not through the principal of non-contradiction: "Here, too, one thing cancels that which is posited by the other; but the consequence is *something* (*cogitabile*). The motive force [Bewegkraft] of a body in one direction and an equal tendency [Bestrebung] of the same body in the opposite direction do not contradict each other; as predicates, they are simultaneously possible in one body. The consequence of such an opposition is rest, which is something (*rapraesentabile*)" (AA 2:171).[5] Whereas the result of a logical contradiction is a concept that negates itself, destroying precisely that which makes it a concept, real opposition leads to a clearly determined physical state called "rest" or "equilibrium": "rest is, indubitably, possible. From this it is also apparent that real opposition [Realrepugnanz] is something quite different from logical opposition or contradiction, for the result of the latter is absolutely impossible" (AA 2:86).[6]

Logical opposition is expressed by the conflict between *A* and not-*A*. In contrast, "[m]athematicians make use of the concepts of this real opposition in the case of mathematical magnitudes. In order to indicate them, the mathematicians designate them by means of the signs '+' and '−'" (AA 2:172).[7] These signs can represent how "one of these magnitudes cancels an amount which is equal to that which is posited by the other, and the consequence is zero" (AA 2:174).[8]

In the first form of contrariety, the attempt to establish a logical connection between *A* and not-*A* leads to a nonsensical result, *nihil negativum, irrepraesentabile*. In the case of real opposition, the result of the conflict between A and $-A$ is a perfectly determined magnitude, 0, which is no less definite than a positive or negative number; it is a *nihil privativum, repraesentabile*:

> Suppose that there are +8 units of capital and −8 units of passive debt; no contradiction is involved in attributing them to the same person. However, one of these magnitudes cancels an amount which is equal to that which is posited by the other, and the consequence is zero (AA 2:174).[9]

In this case, the opposites are not set against each other according to the rules of formal logic, which does not allow them to coexist in a single subject, but rather on

[4] Ibid.

[5] Ibid.

[6] English translation is from Ibid.

[7] English translation from Ibid., 212.

[8] Ibid., 214.

[9] Ibid.

the basis of an opposition between magnitudes: "for example, falling is not to be distinguished from rising merely in the way in which 'not a' is distinguished from 'a'. It is rather the case that falling is just as positive as rising. It is only when the former is combined with the latter that it contains the ground of a negation" (AA 2:175).[10]

Attempting to clarify the meaning of this form of opposition, Kant refers to various physical phenomena, the importance of which was already discussed above, introducing the difference between extensive and intensive magnitudes:

> Now, my contention is this: whenever the temperature is raised or lowered, in other words, whenever the degree of heat or coldness is changed … [t]here are always two poles, so to speak, of warmth to be found: one of them is positive, that is to say, its temperature is higher than the previous temperature of the body in question, while the other pole is negative, its temperature, namely, being lower than the previous temperature of the body, in other words, it is cold (AA 2:186).[11]

The difference between the two poles of heat, as I have shown, tends to diminish and cease when the heat is uniformly distributed and the difference in temperature vanishes and is $=0$. The concept of "temperature" reduces the difference between "heat" and "cold," which appear to intuition as two different sensible "qualities," to a mere difference in degree that is distinguished in respect to an intermediate point of indifference. Relative to this point of indifference, "positive heat" and "negative heat" can be defined as the tendency to cede or acquire heat, that is to say only by the opposite "senses,"[12] $+$ and $-$: "Absolute coldness is unknown in nature, and if it is discussed, then it is understood only in a comparative sense" (AA 2:185).[13]

Even though the expressions "positive heat" and "negative heat" certainly have not found their place in the terminology of scientific knowledge, the opposition between positive magnitude and negative magnitude has acquired an essential function in other areas:

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[10] Ibid., 215. On the concept of real opposition and on the mathematical debate in Kant's time on negative magnitudes, see Wolff, Michael. *Der Begriff des Widerspruchs eine Studie zur Dialektik Kants und Hegels* (Königstein: Hain, 1981), 62–82.

[11] Kant. "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy." 224.

[12] In mathematics it is usual to distinguish between "magnitude" (or length), "direction," and "sense" (i.e., orientation along a given direction) of a vector. Kant normally uses the term *Richtung* to indicate both direction and sense. He recognizes that this could be confusing, however. In the *Metaphysical Foundations of Natural Science*, he observes: "A body moving in a circle changes its *direction* continuously, … yet one says that it moves always in the same *direction*" (AA 4:483; my emphasis; English translation from Kant. "Metaphysical Foundations of Natural Science." 196.). The first use of the term "direction" is in accordance with the modern one. In the second case, Kant asks instead "what is … the *side* towards which the motion is directed" (AA 4:483; English translation from Kant. "Metaphysical Foundations of Natural Science." 196.), that is, in what *sense* the body is moving (dextrorotatory or levorotatory). In a passage of the *Danziger Physik*, Kant distinguishes more clearly between *Direktion* and *Gegend* (see AA 29: 113), that is to say, between "direction" and "sense." The usual translation of "Gegend" by "regions," especially in the title of Kant's pre-critical writing, *Von dem ersten Grunde des Unterschiedes der Gegenden im Raume* [Concerning the Ultimate Foundation of the Differentiation of Regions in Space], is completely misleading.

[13] Kant. "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy." 223.

> It has long been known that magnetic bodies have two extremities which are opposed to each other and which are called 'poles'. Of these two poles, the one repels the like-named pole in another such body, and attracts the other. However, the celebrated Professor Aepinus showed in his treatise on the similarity between electrical and magnetic energy that electrified bodies, when treated in a certain way, likewise display two poles, of which he called one the *positive* pole and the other the *negative*. (AA 2:185)[14]

In this case as well, a difference in degree is present and needs to be filled as in the phenomenon of heat. As when a hot and a cold body are put in contact, the first is hot compared to the other because it tends to cede heat; when a conductor connects a plate of copper with a plate of zinc, the potential of the copper is higher than that of the zinc in the sense that $+2$ is greater than -2, and the two charge each other, the one positively and the other negatively. Two different electricities, such as vitreous electricity opposed to resinous electricity, are not involved, but only one varying distribution of the quantity of electricity, a different degree of concentration of them. When a metallic wire joins the two conductors, the difference is rapidly nullified, according to a procedure similar to that of the passage of "heat" between two bodies with different temperatures, until the difference in potential is $=0$. The "absolute quantity" of electricity or heat (that is, heat and electricity as substance) is meaningless here since their quantity is only determined through the difference in temperature or potential, the difference between the $+$ and the $-$ and the conservation of their algebraic sum.

Further below, I will address the extraordinary influence of this conception on Romantic *Naturphilosophie*. Here, it is important to emphasis that the dualism between logical opposition and real opposition is reaffirmed in the *Critique of Pure Reason* itself. In Kant's appendix "On the Amphiboly of Concepts of Reflection" [*Von der Amphibolie der Reflexionsbegriffe*],[15] the two types of opposition become the distinguishing mark between *realitas phaenomenon* and *realitas noumenon*, the focus of the previous chapter:

> If reality is represented only through the pure understanding (*realitas noumenon*), then no opposition between realities can be thought … Realties in appearance [das Reale in der Erscheinung], on the contrary, can certainly be in opposition with each other (B320–21).

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[14] Ibid., 224. On Aepinus's doctrine in a philosophical context, see Moiso, Francesco. "Magnetismus, Elektrizität, Galvanismus" in Schelling, HKA 1 (Ergänzungsband zu Werke 5 bis 9. Wissenschaftshistorischer Bericht zu Schellings naturphilosophischen Schriften 1797–1800), 4:254-. For more about Aepinus in general, see: Home, Roderick Weir. "Aepinus, the Tourmaline Crystal, and the Theory of Electricity and Magnetism," *Isis* 67, no. 1 (1976): 21–30.

[15] Among contributions dedicated specifically to the *Amphiboliekapitel*, see Broeken, Renate. *Das Amphiboliekapitel der "Kritik der reinen Vernunft": Der Übergang der Reflexion von der Ontologie zur Transzendentalphilosophie* (Köln: Mosebach, 1970); Parkinson, George Henry R. "Kant as a Critic of Leibniz: the Amphiboly of Concepts of Reflection," *Revue Internationale de Philosophie* 35 (1981): 302–14; Hessbrüggen-Walter, Stefan. "Topik, Reflexion und Vorurteilskritik: Kants 'Amphibolie der Reflexionsbegriffe' im Kontext," *Archiv für Geschichte der Philosophie* 86, no. 2 (2004): 146–75; Hess, Heinz-Jürgen. "Zu Kants Leibniz-Kritik in der 'Amphibolie der Reflexionsbegriffe'." In *Beiträge zur Kritik der reinen Vernunft: 1781–1981*. eds. Ingeborg Heidemann and Wolfgang Ritzel, 201–32. (Berlin: de Gruyter, 1981); Funke, Gerhard. "Systematische Voraussetzungen der Leibniz-Kritik Kants im 'Amphibolie-Kapitel." In *Akten des 4. Internationalen Kant-Kongresses*. ed. Gerhard Funke. (Berlin: de Gruyter, 1974).

The understanding only knows conflict as the conflict between concepts and is thus constrained by the form of contradiction. If reality, as I have shown, defines something by distinguishing it from something else (*A* is *A* and thus is not not-*A*), then the negation that sets them against each other (*A* is not-*A*) is a meaningless assertion, signifying "the removal [Aufhebung] of everything" (B603). The assertion does not simply mean that a certain thing is not, but that it is not a thing at all because "the contradiction entirely annihilates and cancels them" (B190): "That of which the thought contradicts itself is absolutely impossible, that is the negative nothing<*nihil negativum*>. *Reality* is something; *negation* is nothing" (AA 28:543).[16] Logical nothingness does not only indicate the absence of something. It annihilates all content of thought. Consequently, "the impossible is not a negative concept. It is not even a concept" (AA 17:532; Refl. 4399):

> Contradiction [Wiederspruch] is the connection of two opposed predicates in a contradictory manner [*contradictorie*], that is to say, logically opposed [*logice*] in a subject. Contradiction results in a *nihil negativum*: *a* and not-*a* together. Real opposition [der reale Widerstreit] is the uniting of two real elements in a subject [die Verbindung zweyer realgründe in einem Subjekt]. One element removes [aufhebt] the consequence of the other, and the result, which is not a *nihil negativum*. (AA 17:267; Refl. 3720)

In this way, the difference between *realitas phaenomenon* and *realitas noumenon* can be established not only, as in the Anticipations of Perception, from a "subjective" point of view, that is to say, referring to sensation, to the action that phenomenal reality can exert on the subject (the effect of which is precisely sensation [see B34]). Instead, the difference can be established "objectively," in a manner of speaking, by considering the difference between the reciprocal relation that exists between phenomenal realities and the relations that can be established between noumenal realities. The latter, which can be thought through pure understanding, do not admit any opposition since logical opposition equals annihilation of the thought itself and of its contents: "to no subject does there belong a predicate opposed to it<*nulli subjecto competit praedicatum ipsi oppositum*>. The negative thing<*nihil negativum*>is that which cannot even be thought" (AA 28:543).[17] In conflict, *realitates phenomena* admit a conflict that produces something perfectly determinate $= 0$: "Only the reality found in phenomena can be opposed to another reality and the negation can concord with a reality" (AA 18:63; Refl. 5823). While the first case involves an opposition between concepts, here the conflict is constituted on the model of an opposition between forces:

> the relation of opposition between these forces is, like $+a$ in confrontation with $-a$, a relation of real opposition [der realen Entgegensetzung], not one of logical opposition [nicht der logischen Opposition] like that between *A* and not-*A*. Otherwise it would not be a relationship between forces (AA 21:190.2).

Consequently, "logic" does not offer any sufficient instrument for mastering phenomenal reality, or, as one could say, "physical" reality. The principle that "realities (as mere affirmations) never logically oppose each other is an entirely true

[16] English translation from Kant. Lectures on Metaphysics, 310.

[17] Ibid.

proposition about the relations of concepts, but signifies nothing at all … in regard to nature" (B328-9). In physical reality, the agreement between realities is not based on non-contradiction (the possibility of placing two concepts together without their contradicting each other), but on the model of equilibrium (the possibility of placing two forces together without one prevailing over the other):

> Real opposition always obtains $A - B$, i.e., where one reality, if combined in one subject with another, cancels out the effect of the latter, which is unceasingly placed before our eyes by all hindrances and countereffects in nature, which, since they rest on forces, must be called *realitates phaenomena* (B329).

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A passage from the *Lectures on Metaphysics* (*Vorlesungen über die Methaphysik*) effectively summarizes what has been said until now:

> In all that of which one is conscious, one distinguishes something real and something negative. Negation is opposed to reality. An opposite is either logical or real. When someone denies something, then this is a logical opposite<*oppositum*>. Reality and negation cannot be posited in one and the very same thing. *Real* opposition consists in the connection of two real grounds, of which one ground cancels the consequence of the other. Among realities there can be an opposition. A reality is opposed not only to negation, but rather also to another reality that cancels the consequence of the other. (AA 28:559–)[18]

The importance of this formulation of the problem is most evident in Kant's own recognition of his radical departure from the Leibnizian-Wolffian tradition: "Leibniz took the appearances for things in themselves, thus for *intelligibilia*, i.e., objects of the pure understanding" (B320). Consequently, he "compared the objects of the senses with each other as things in general, merely in the understanding" (B327), that is, only by means of concepts. As is typical of dogmatic philosophy, Leibniz presupposes that this connection can be translated into objective reality such that "if a certain distinction is not to be found in the concept of a thing in general, then it is also not to be found in the things themselves" (B337).

Considering the relation between reality and negation from this point of view, "merely affirmative concepts cannot, in combination, effect any cancellation" (B338). Two concepts in which one is the negation of the other cannot coexist. In the unity of a concept, that which makes it something, a "reality", rather than nothing, a "negation," is its non-contradictoriness. According to the metaphysical tradition, a "negative thing," implying that it should be and not-be at the same time, is a simple contradiction since "an object *must* have *something* positive, and *can* have many positives or perfections" thus a "merely negative thing<*ens mere negativum*>, i.e., something which would have nothing positive at all, is a direct contradiction, for even the being of the thing already involves something positive" (AA 29:1001).[19]

The possibility of no longer conceiving the relation between reality and negation on the model of the relation between *A* and not-*A*, but on that of $+A$ and $-A$, radically

[18] Ibid., 324.

[19] Ibid., 469.

changes this perspective. "Things" are no longer at stake, but rather "the relationship between certain things" (AA 2:175). From this point of view, a "negative relation" is simply the consequence of the notion itself of "relation," which implies the ability of following two opposite directions: "What is pain? Reality or negation? It is just as real as pleasure … The two realities are in a relation of simple opposition [Widerspiel]" (AA 18:502). For this reason, one can say that "pain is not … a negation, but rather a reality that is opposed as the contrary to another reality" (AA 28:420–).

A "negative reality" is a logical contradiction if reality is conceived through pure understanding because "a concept that contains only affirmations [lauter Bejahungen] does not contain anything negative [nichts Verneinendes]: a proposition that we have never doubted" (B338n). However, "[w]e do not always require true reality [wahre realitaet], in which no negation (*non esse*) can be thought; but from the beginning we are dealing with *realitatibus phaenomenis*" (AA 18:361; Refl. 5814); that is to say, with those realities characterized by real opposition. "Reality is not merely opposed to negation, which is logically impossible, but also to another reality" (AA 28:421). They are considered positive or negative in respect to each other, but both should be subsumed under the same title as "realities":

> in the sensible *intuition* in which reality (e.g., motion) is given, there are conditions (opposed directions), from which one had abstracted in the *concept* of motion in general, that make possible a conflict [Widerstreit], which is certainly not a logical one, that produces a zero $= 0$ out of that which is entirely positive; and one could not say that all reality is in agreement just because no conflict is to be found among its concepts. (B338, emphasis mine)

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The idea that an entity is something "whose concept involves something positive or that which can be conceived by us provided what we conceive is possible and involves no contradiction" (GP 7:319),[20] represents at least one aspect of the Leibnizian tradition. The question whether this point of view can exhaust Leibniz's conception must of course be left aside here. One can anyway surely affirm that Kant recognizes in this tendency to eliminate all oppositions the characteristic trait of Leibniz's thought. In Leibnizian metaphysics "being is apparently thought as constituted by infinite 'perfections' or 'realities' that do not imply anything negative and that are composed without creating any contradiction. No entity, as such, can admit any negation without annihilating its own possibility based on non-contradiction. As I have shown, from the perspective of dogmatic metaphysics, 'reality' should be understood as "that which is simply positive in things" (AA 20:415), as a certain degree of being and perfection. From critical philosophy's point of view, however, "it is always a serious mistake to conflate the sum of reality with the magnitude of perfection … [D]ispleasure is just as positive as pleasure, but who would call it a perfection?" (AA 2:198).[21]

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[20] English translation from Leibniz, Gottfried Wilhelm. *Philosophical Papers and Letters*. ed. Leroy E. Loemker (Chicago: University of Chicago Press, 1956), 2:602.

[21] English translation from Kant. "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy." 2366.

The opposition between positive and negative magnitudes unequivocally distinguishes transcendental idealism from Leibnizian-style metaphysics.[22] According to Kant, from Leibniz's point of view:

> all things metaphysically considered, would be compounded of reality and negation, of being and nonbeing, as in Democritus everything in the universe is made up of atoms and void; … and thus out of all so-called metaphysical evil, in combination with good of that kind, he created a world of mere light and shadows, without considering that, in order to put a space in shadow, a body must be present, and hence something real that prevents the light from penetrating into the space. According to him, pain would be grounded merely on lack of pleasure, vice merely on the want of virtuous motives, and the rest of a moving body merely on the absence of moving force, since by mere concepts reality = a can be contrasted, not to reality = b, but only to privation = 0 – there being no consideration of the fact that in intuition, e.g. of the outer, *a priori*, namely in space, an opposition of the real (the moving force) to another real, namely a moving force in the opposite direction, can be combined in one subject, … and that the *a priori* knowable result of this conflict of realities might be negation. But for this purpose he would assuredly have had to assume mutually opposing directions, which can be represented only in *intuition* and not in mere *concepts*. (AA 20:282–, emphasis mine)[23]

This long passage from "What Real Progress Has Metaphysics Made in Germany since the Time of Leibniz and Wolff" effectively summarizes Kant's image of Leibniz's philosophy[24] and how the relation between reality and negation can be considered the distinctive feature of critical thought as opposed to dogmatic metaphysics.

The allusion to problems related with theodicy, which cannot be exhaustively treated here, highlights the novelty of Kantian philosophy in respect to a long tradition that dates back to Augustine[25] at least and that stretches to Leibniz.[26] Evils for

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[22] See Gueroult, Martial. *Leibniz. Dynamique et Metaphysique* (Paris: Aubier-Montaigne, 1967), 169n.

[23] English translation from Kant. "What Real Progress Has Metaphysics Made in Germany since the Time of Leibniz and Wolff?", 373.

[24] This also shows that Kant's thought is based on a simplification. Kant's depiction of Leibniz seems to be employed as a typical-ideal model considering that in the *Critique of Pure Reason* itself, Kant cautiously notes, "Herr von Leibniz did not exactly announce this proposition [the reduction of the principle of sufficient reason to that of identity] with the pomp of a new principle," but rather, "his successors expressly incorporated it into their Leibnizian-Wolffian doctrine" (B329). In fact, Leibniz is far from wanting to reduce the principle of sufficient reason to that of identity, a position that should be attributed to Christian Wolff and his successors instead.

[25] "[W]hatever is, is good. … [E]vil … is not any substance." Augustine. *The Confessions of Saint Augustine*, tr. Edward Bouverie Pusey (New York, NY: P. F. Coillier and Son Company, 1909), 115.

[26] Leibniz himself admits the idea of "real opposition," giving evil its own reality, in an early letter to Arnold Eckhard from 1677: "In our discussion, when you seemed to have said that what is perfect is that which is purely positive, I countered with the example of pain, which is no more the privation of pleasure than pleasure is the privation of pain" (GP I, 221); on the importance of this point, see Poma, Andrea. *Impossibilità e necessità della teodicea: gli "Essais" di Leibniz* (Milan: Mursia, 1995), 183. However, Leibniz seems to abandon this framework in later years, affirming the position well expressed by the ancient motto *bonum ex causa integra, malum ex quolibet defectu* that he cites in his *Essais de Théodicée* (see GP 6:122). Thus, one can accurately say that, for

Kant are not just "consequences of the limits of creatured beings" (B329) that merely result from the absence of positivity. Rather, evil is just as "positive" reality as the good and is actively opposed to the latter: "Evil can have originated only from moral evil (not just from the limitations of our nature)" (AA 6:43).[27]

Kant breaks the traditional connection between *ens* and *bonum* (*ens et bonum convertuntur*). "Being" is not the same as the "good," because even evils have, in a manner of speaking, their ontological consistency, and "the not good can also be called positive evil" (AA 6:23n).[28] Schelling's *Of Human Freedom* (*Untersuchungen über das Wesen der menschlichen Freiheit*) is a particularly significant example of this change of perspective in post-Kantian philosophy. Schelling simply adopts and radicalizes this fundamental achievement of Kant, criticizing the Leibnizian idea (see SW 6:369) that evil consists of "limitation, lack, privation," that is, that it can be reduced to "a *malum metaphysicum* or the negative concept of creaturely imperfection" (SW 6:367). Kant replaces Leibniz's conception of evil with a "real opposition [reellen Gegensatz]" (SW 6:370) to the good that is founded "on a positive inversion or overturning of the principles" (SW 6:366).[29]

These few comments clearly show how the relation between reality and negation represents a central aspect of Kantian-inspired philosophy in respect to Leibnizian metaphysics. For Kant, and the idealist tradition indebted to him, the conflict between opposed forces that establish an equilibrium, and are thus both real, is the adequate model for representing the agreement between phenomenal realities in contrast to simple non-contradictoriness, which excludes every conflict and opposition. Opposites do not simply exclude each other, avoiding any reciprocal contamination that would introduce contradiction in them and threaten their very being. Rather, opposites can cohabit, like two weights that, despite moving the arms of a balance in opposite directions, create an equilibrium at the same point. Post-Kantian philosophy, as I will show in more detail, insistently resorts to precisely this metaphor of equilibrium, and to the lever in particular, to indicate the unity of opposed elements that cohabit, reciprocally limiting each other.

Leibniz, "*bonum metaphysicum* always consists of a positive, however limited this may be, whilst *malum metaphysicum* can consist of a lack, a deprivation, a limitation" (Martin, Gottfried. *Leibniz: Logik und Metaphysik* (Köln: Kölner Universitätsverlag, 1960), 135). Martin, Gottfried. *Leibniz, Logic and Metaphysics*, The Philosophy of Leibniz (New York: Garland, 1985), 108. For more details on this point, see Heinekamp, Albert. "Zu den Begriffen realitas, perfectio und bonum metaphysicum bei Leibniz." In *Akten des ersten Internationalen Leibniz-Kongresses: Hannover, 14–19. November 1966*. (Wiesbaden: Steiner, 1968), 207–22.

[27] English translation from Kant, Immanuel. *Religion within the Boundaries of Mere Reason and Other Writings* (Cambridge: Cambridge U. Press, 1998), 48.

[28] Ibid., 64.

[29] For more details on the evolution of Schelling's thought concerning the problem of evil, see Riconda, Giuseppe. "Filosofia moderna e problematica del male nelle Untersuchungen über das Wesen der menschlichen Freiheit di Schelling," *Paradosso* (1993): 9–28.

2.2 Quantitative and Qualitative Opposition

One of Kant's reflections serves as a good summary of what has been said to this point: "The reality in phenomena (experience) can conflict with each other, but not *in noumenis* because in these the *oppositum* of reality [realitaet] must be thought *a priori*. For this reason, the opposition can only be logical, that is, negation" (AA 18:238; Refl. 5578). If realities are distinguished as *A* and not-*A*, thinking of something that is both *A* and not-*A* is impossible. For any two contradictorily opposed predicates in a given concept, "only one can apply to it" (B599). That something is neither *A* nor not–*A* is also inadmissible since "among all possible predicates of things, insofar as they are compared with their opposites, one must apply to it" (B599–600). In real opposition, however, a third that is indifferent to the opposition can be thought; a third that is neither $+A$ nor $-A$, or equally, both $+A$ and $-A$: "Between two logical opposites <*logice opposites*> there is no third (<*tertium non datur*; G: *giebt's kein Drittes*>); but between two real opposites <*realiter opposites*> there is a third (<*tertium datur*; G: *giebt es ein Drittes*>) (AA 28:549).[30] Whereas from a logical point of view, "[r]eality and negation cannot be posited in one and the very same thing" (AA 28:559–);[31] in real opposition, reality and negation, positive and negative, can be thought together: "there is no third between two *logice opposites*, but between *realiter opposites* … there is the neutral point $=0$" (AA 18:105; Refl. 5164).

The sharp alternative, *A* or not-*A*, is valid for *noumena*. In phenomena, however, one can think of the difference between two opposites as that between $-A, 0, +A$. Between $+A$ and $-A$, one can always think of a difference that, no matter how small, allows one to choose a point to signify zero: "*inter realitates phaenomena datur tertium*" (AA 17:447; Refl. 4182). *Noumenal* reality, that is to say, the logical possibility of something, its non-contradictoriness, "has no degree, for we can cognize it only according to the principle of contradiction <*principio contradictionis*>" (AA 28:562).[32] As I have shown, no mediation between reality and negation is possible from a logical point of view. Either something is possible, that is, free of contradiction and negativity; or it is impossible, contradictory, negative: *tertium non datur*. The Anticipations of Perception intend to demonstrate that phenomena, in contrast, can admit different degrees between reality and negation that can "increase or decrease to infinity or also disappear through $a - a$" (AA 22:533.11).[33] Reality and negation, conceived through pure understanding, "are distinguished

[30] Schelling, Friedrich Wilhelm Joseph von. *Philosophical Investigations into the Essence of Human Freedom*, SUNY Series in Contemporary Continental Philosophy (Albany: SUNY, 2006), 315–6.

[31] Ibid., 324.

[32] Ibid., 327.

[33] "In as much as the real is present as intensive, a continuous connection between reality and negation necessarily exists, such that negation is not opposed logically but really" (Haas. "Kants Qualitätsschematismus." 163).

from each other in terms of quality or they are *disparate*" (AA 17:630s.; Refl. 4666), excluding each other. In contrast, reality and negation given in intuition limit each other reciprocally, which is only possible because negation is only distinguished from reality by degree: "limitation [limitation] has degrees up to zero, thus reality as well" (AA 18:363; Refl. 5821).

Consequently, logical opposition is *qualitate* whereas real opposition is simply *quantitate*: "*oppositum* can be a qualitative or a quantitative *oppositum*. The first is contradiction; the second is the *quantum* $= 0$ or the limitation [Einshrankung]. Thus, in the formula, rest can be regarded as a movement $= 0$; pure<extension in space>as an extension $= 0$, unchanging duration [Unveranderte Dauer] as an alteration $= 0$" (AA 18:365; Refl. 5831). Negation in phenomena, far from contradicting the concept of reality, should only be thought as a reality that disappears, as the limit of a process of diminution: "negation … is in respect to quantity or in respect to quality. In the first case, it is a disappearing *quantum* and nothing other than a mere limitation that is not opposed to reality in a contradictory fashion … [I]n the second case, it is *negatio oppositionis*" (AA 18:362s; Refl. 5816). Distinguished through quantity alone, the opposites conserve their reciprocal affinity as opposed within a greater qualitative unity that contains both of them: "a point is the limit [Grenze] of a line, yet is nonetheless a locus in space" (AA 4:354)[34] because in the point "one finds the same quality of presence [Gegenwart], and the point is a disappearing space" (AA 18:362; Refl. 5816). Therefore, the vanishing of the quantity does not threaten the unity of the quality, "every *negatio* is merely *limitatio* – that is, quantitative *oppositum* or rather *negatio repugnantiae* – or qualitative *oppositum*" (AA 18:360; Refl. 5815).

In contrast to the absolute opposition of contradictory concepts in which they have nothing to do with each other, real opposition always presupposes a *tertium comparationis*, something that associates the opposites and in respect to which they can be compared in terms of more or less: "Now nothing can be combined with a motion, which diminishes it or destroys it, except another motion of precisely the same movable in the opposite direction" (AA 4:497).[35] A real opposition between movement and heat, for example, is meaningless because "the difference is not yet an opposition if not in a subject. Two different things are not necessarily opposed, but two things are opposed if they make a unity out of that which makes them different. For example, two bodies that move towards each other" (AA 15:189; Refl. 458).

A body launched upwards that is moving at a uniformly slow rate, upon reaching the vertex of its course, will move at a uniformly fast rate, and vice versa. The two motions are identical (a typical example of a reversible phenomenon), that is, they are described according to the same law. The only difference is that which

[34] English translation from Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." 144.

[35] English translation from Kant. "Metaphysical Foundations of Natural Science." 209.

is expressed through a difference in sign: "$+a$ and $-a$ are not qualitatively opposed to each other, but only in terms of their sense [Richtung]" (AA 22:177.8). Only a point of inversion separates the two opposites, the point where an ascending motion passes into its opposite. The third, which is neither an ascending nor a descending movement, is the point where the difference between the two motions is $=0$, and thus at rest since "a point does not move immediately from one direction into another without an intermediate rest" (AA 28:203–).[36] The third between two opposites has an eminently paradoxical nature. It is the point where contraries determine each other as such since "the cessation of positive magnitudes marks the start of negative magnitudes" (AA 2:169).[37] At the same time, this point where they remove each other (because the point $=0$ is neither positive nor negative) is "the middle between two (opposed in terms of degree)" (AA 18:625; Refl. 6317). One of the fundamental problems in post-Kantian philosophy is precisely how to grasp the ambiguous nature of this intermediate point in respect to which the opposites are defined as such while also removing each other, the one passing into the other.

2.3 The Problem of Change

Understanding the meaning of the distinction in Kantian thought between logical and real opposition, and thus between qualitative and quantitative opposition, requires the further step of showing how this distinction is connected to the broader problem of the possibility of change and becoming. The latter problem is not only one of the most persistent questions in the history of Western thought, but also one of the central problems of critical philosophy, perhaps even the most fundamental.

The simple logical connection between concepts merely confirms their identity or ratifies their irreducible difference: movement is movement, rest is rest, and the two determinations remain completely external to one another. The transition from rest to movement cannot be comprehended by through the logic of pure understanding. Trying to establish the instant in which a process undergoes a qualitative transformation, such as the transition from rest to movement, leads to an insurmountable difficulty. The last moment in which a body is *still* at rest is also the first moment that it is *already* moving, forcing one to regard the body as at rest and in motion at the same time and thus to attribute contradictory determinations to it. Holding to a firm logical opposition between being and not-being, reality and negation, does not allow a third where both opposites can exist together and where the one can pass into the other (*principium exclusi medii inter duo contradictoria*).

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[36] English translation from Kant. *Lectures on Metaphysics*, 26.

[37] English translation from Kant. "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy." 209.

Nevertheless, change involves precisely this contradictory mixture of irreconcilable predicates, or in Kant's words, "the combination of contradictorily opposed determinations in the existence of one and the same thing" (B291).[38] For Kant, change is:

> the succession of opposite determinations of the same thing <*successio determinationum oppositarum in eodem ente*>, E.g., a body is altered externally if it is set out of rest into motion … What matters most here is, how is alteration possible? I.e., how can opposed determinations be in one thing? One must *not* at all times believe that one comprehends what one understands; for comprehending is: cognizing something *a priori* through reason (AA 28:558–).[39]

According to the logic of the *concept*, a subject can only contend for one of two opposed predicates, and two contradictory predicates cannot coexist simultaneously in the same subject without the two competing: "Now how it is possible that from a given state an opposed state of the same thing should follow not only cannot be made understandable by reason without an example, but cannot even be made understandable without *intuition*" (B291–92, emphasis mine).

The possibility of becoming is incomprehensible to the understanding, which cannot master its intrinsic contradictoriness, is unable to think two opposed states together in the moment of transition. Only through intuition's testimony of the transition of one thing from one state to another can it be admitted that a substance that is *first* in one state A can *then* transmute into a completely different state B. Temporal intuition defuses the logical contradictoriness of becoming that the concept cannot grasp in any manner, "the concept of motion … is only possible through and in the representation of time – that if this presentation were not *a priori* … *intuition*, then no *concept*, whatever it might be, could make comprehensible the possibility of an alteration, i.e., of a combination of contradictory opposed predicates … in one and the same object" (B48, emphasis mine).

However, time is merely the expression of separation, that an object was first in one state and then was taken over by an opposite state. The two states always remain separate whereas to "conceive" the transition from *A* to not-*A* means that contradictory determinations do not occur in different instants, but meet in a single point. Such a transition could not occur in a determinate time because in no temporal point can one "think" the simultaneous presence of contradictory conditions. However, if the two opposed states did not meet, remaining separate for a period of time no matter how small, no alteration could be verified, but only a succession of states that have nothing to do with each other.

An unusual but effective example to help understand this difficulty is the transition of an object's ownership from *Ego* to *Alter*, which Kant describes as occurring through a business contract in the *Metaphysic of Morals* [*Methaphysik der Sitten*]. Trying to follow the transition step by step, it is difficult to establish when the

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[38] Kant writes: "Now how in general anything can be altered, how it is possible that upon a state in one point of time an opposite one could follow in the next – of these we have *a priori* not the least concept. For this acquaintance with actual forces is required, which can only be given empirically, e.g., acquaintance with moving forces, or, what comes to the same thing, with certain successive appearances (as motions) which indicate such forces" (B252).

[39] English translation from Kant. *Lectures on Metaphysics*, 323.

promittens ceases to be the possessor of the object in question and begins to be the *acceptans*: "what belongs to the promisor does not pass to the promisee (as acceptant) by the separate will of either but only by the *united will* of both, and consequently only insofar as both wills are declared *simultaneously*" (AA 6:272).[40] To understand the not simply theoretical nature of this difficulty, one should consider "external formalities (*solemnia*) in concluding a contract (shaking hands, or breaking a straw, *stipula*, held by both persons)," gestures that "manifest the perplexity [Verlegenheit] of the contracting parties as to how and in what way they are going to represent their declarations as existing *simultaneously*, at the same moment, although they can only be successive" (AA 6:272).[41] However, the transaction (*translatio*) cannot admit a continuous solution either and the possessor of the object should never be "interrupted for a moment during this act; for otherwise I acquire, in this condition, an object as something that has no possessor (*res vacua*)" (AA 6:274).[42]

Thus, one is confronted with two demands, both of which seem impossible to satisfy. On the one hand, one affirms that becoming is possible only if the two states, the point of departure and the point of arrival of the process, remain separate no matter how much they approximate each other because only in this manner can the logical contradiction of the concept be avoided. On the other hand, this concession by no means guarantees the transition of ownership, because "if I have promised and the other now wants to accept, I can still during the interval (however short it may be) regret having promised, since I am still free before he accepts; and because of this, the one who accepts it, for his part, can consider himself as not bound to his counter-declaration after the promise" (AA 6:272).[43] Either, the object never passes from the possessor to the acquirer because the two acts always remain separate. Or, the process could be interrupted at some point and the latter could acquire a *res nullius*, lacking a guarantee for the upholding of the terms of the contract, if one maintains that the object is first the property of one contractor and then that of the counterparty. In order for such a transition to be possible, one must admit that a "[t]ransfer is therefore an act in which an object belongs, for a moment, to both together" (AA 6:274).[44] This is something contradictory for the understanding, but at the same time necessary to make transfer of property possible.

2.4 Change and Real Opposition

The example of the contract demonstrates that, from a "logical" point of view, a thing would destroy its identity if it alternated between being and non-being because, during the intervals of time in which it was not, it would lose its connection with

[40] English translation from Kant, Immanuel. *The Metaphysics of Morals*, tr. Mary Gregor (Cambridge: Cambridge University Press, 1991), 91.

[41] Ibid., 92. Brackets mine.

[42] Ibid., 93.

[43] Ibid., 91–.

[44] Ibid., 93.

itself. The second period of its existence would be completely alien to the first and it could not be distinguished from an entirely new object in respect to which it could be very similar, but not identical. However, if the temporal distance that separates them were removed, then being and non-being, no longer separated by any period of time, would contradictorily coincide in the same instant.

The possibility of change therefore requires the possibility of a form of opposition in which two opposites can coexist without creating a contradiction, and in which reality and negation limit each other in a point of indifference. Such an opposition is not logical, but real: "the entire series of alterations seems to arise [herzurühren] from real oppositions" (AA 17:502; Refl. 4309). Kant thus explicitly states that "[T]he opposition of real grounds makes all alteration possible." (AA 28:560).[45]

The process of change presupposes the possibility of thinking the co-presence of two contradictory opposites, "just as when a stone that has been thrown reaches the apex of its parabolic path is to be regarded as, for just a moment, simultaneously rising and falling, and so first passing from its rising motion to its falling" (AA 6:274).[46] In the process of changing from one state to another, there is always a point in which the opposites must be able to coexist, such as in the point of inversion, which is neither an ascending nor a descending movement and in which velocity has neither a positive nor a negative sign. "There should be a *Punctum flexus contrarii* in the progression, there where direction ends and the other begins" (AA 16:767; Refl. 3305), where the acceleration is neither positive nor negative. The transition from one opposite to another occurs at a point in which the two opposites coexist: "*lex continuiti* means that two states that follow each other always have something in common [etwas gemeinschaftliches], that is, they share a limit [Grentze]" (AA 17:631; Refl. 4666). What the two opposites have in common is the point of indifference $=0$ where they disappear as such since that which defined them as opposites was precisely their quantitative difference: "the law of continuity: between a and $-a$ (for example, attraction and repulsion in a bar magnet), there is a point where the predicate of the thing disappears, becoming $=0$" (AA 18:624; Refl. 6317).[47]

The indifferent third between the opposites is $=0$ and perfectly comprehensible as such: "*negatio* cannot be precisely distinguished from reality [von realitaet nicht specifisch unterschieden]. This is the middle between two opposed realities < and

[45] English translation from Kant. *Lectures on Metaphysics*, 324. The same idea underlies the text about negative magnitude: "All change consists in this: either something positive, which was not, is posited; or something positive, which was, is cancelled … I maintain, however, that if A arises, then, in a natural change occurring in the world, –A must also arise" (AA II, 194). English translation from Kant. "Attempt to Introduce the Concept of Negative Magnitudes into Philosophy." 232.

[46] English translation from Kant. *The Metaphysics of Morals*, 93.

[47] The Dutch physicist Anton Brugmans arrives at this conclusion, regarding the magnet in particular, in his *Tentamina Philosophica de materia magnetica* (1765). In showing how a magnetic bar must pass through an intermediate point of indifference in the transition from one pole to the other, Brugmans claims to have deduced "this proposition *a priori* applying the law of continuity." The citation is drawn from the German edition of Brugmans, Antonius. *Philosophische Versuche über die magnetische Materie, und deren Wirkung in Eisen und Magnet*, tr. Christian Gotthold Eschenbach (Leipzig: S. L. Crusius, 1784), 76.

the transition[48] >, which is related to both" (AA 18:363; Refl. 5824). The point $=0$ is neither positive nor negative, but rather the point in which the difference between the two opposites, distinguished only through their sign, vanishes. "Princip: continui transßc: Non datur progreßus a ratione data ad realiter oppositam secundum regulam nisi per intermedium determinationis quae aeqvivalet ziphoni s. *nullitati vtriusqve h.e. indifferens*, e.g. in oscillatione – in magnete – (in transitu a vitio ad virtutem –) in transitu a voluptate ad taedium (Transcendental principle of continuity: There is no progress from a given state to a real opposite one according to a rule if not per intermediate determinations, that which is equivalent to a siphon, i.e. to the nothing of both or indifference, for example – in oscillation – in a magnet [in the transition between vice and virtue], in the transition from pleasure to tedium)" (AA 21:461.4; emphasis mine). Thus, real opposition eliminates the contradictoriness of becoming by admitting a point where the two states can coexist.

In the case of real opposition, negation and reality are not distinguished by some "quality," but only by "degree" or "quantity"; that is, only by the fact of occupying a determinate "position" within an ordered series. They are not opposed like two contradictory concepts, but only because one can establish that the *n*th "degree" is found between the $(n-1)$ th and $(n+1)$ th degree and that the distance between these is as small as one likes. For example: "if virtue and vice were distinct only in terms of degree, then a certain degree at the limit of the two would be equally virtue and vice" (AA 17:630; Refl. 4666). This limit would be the "state of indifference" in which the opposites can simultaneously exist because it is neither of the two while, at the same time, the two together.

For pure understanding, there is no mediation between reality and negation, which cannot be thought together because "[t]wo opposites <*opposita*> cannot be in one concept" (AA 28:552)[49] without annihilating its identity. In phenomena, however, reality and negation can coexist in the point of indifference, which is neither positive nor negative: "rest is not a *nihil negativum*, but only a *privativum*. The state of indifference is zero" (AA 28:635). The quantitative and logical opposition that characterizes noumenal realities is substituted with a real and quantitative opposition. The latter is characterized by a simple difference in degree to which the "being" of phenomenal realities is reduced, such that a third $=0$ can always be found between two opposed states of change without implying any contradiction. The opposition between "things in themselves," which can be thought by pure understanding, does not make any process of transformation possible: "Alteration is not an intellectual predicate at all. Hence it is not the things but their *phenomena* that are altered" (AA 17:425; Refl. 4122).[50] Only in phenomena can one think another form of opposition that makes becoming possible: "the possibility of alteration is based on the contrariety of certain *realitatum phaenomenorum*" (AA 18:363; Refl. 5825).

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[48] This is a likely interpretation of the insertion "und der Ub" that appears in the original German.

[49] English translation from Kant. *Lectures on Metaphysics*, 318.

[50] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 115.

2.5 Change as Quantitative Variation

A tight connection exists between the new conceptions that the Anticipations of Perception attribute to reality and negation in phenomena, in which a certain finite difference in degree is distinguished from the indifference $=0$, and the problem of change and becoming. In order for alteration to be possible, a quantitative consideration of variation must occur. The alteration must be understood as the transition in time from a greater to a lesser degree, the first representing positive reality and the second its extreme opposite. In the proof of the second of the "Analogies of experience" in the *Critique of Pure Reason*, Kant is able to describe the process of change in the following manner:

> If a substance passes out of a state *a* into another state *b*, then the point in time of the latter is different from the point in time of the first state and follows it. Likewise the second state as a *reality (in the appearance)* is also distinguished from the first, in which it did not yet exist, as *b* is distinguished from zero; i.e., if the state *b* differs from the state *a* even *only in magnitude* [nur der Größe nach], then the alteration would be an arising of $b-a$, which did not exist in the prior state, and with regard to which the latter $=0$. (B253, emphasis mine)

If the two states are only distinguished by degree of reality, becoming is the emerging of a difference in degree $b-a$. What is essential is not the *absolute values* of *a* and *b*, but only their *relative difference* to which their opposition should be reduced.

However, the problem does not seem to be resolved in this manner at all, but simply deferred: "The question therefore arises, how a thing passes from one state $=a$ into another one $=b$. Between two instants there is always a difference that has a magnitude" (B253). No matter how small the difference in degree that separates two opposed states and the difference in duration that separates two instants, the states and instants remain separated. An alteration, as much as it approaches the temporal points that can be thought as *terminus a quo* and *ad quem*, "takes place continuously throughout a time, and thus equally through an infinite series of moments" (AA 4:531).[51] A quantitative value corresponds to each of these instants, that is, a determinate "degree of reality" that should be effectively passed through at least once in the course of the process. But this representation of the alteration "is to be attributed to the inconceivability of dividing any such continuum in general to infinity" (AA 4:531),[52] the impossibility of thinking of finite change as passing through infinite variations.

Kant seems to note this difficulty in the proof of the Anticipations of Perception. In fact, he defines the continuity that pertains to magnitude, extensive as well as intensive, as that "quality of magnitudes" according to which "no part of them is the smallest" (B211). Just as space and time are *quanta continua* because each of their parts is always a space and a time in turn, one can also affirm that "every reality in appearance, however small it may be, has a degree, i.e., an intensive

[51] English translation from Kant. "Metaphysical Foundations of Natural Science." 240.

[52] Ibid.

magnitude, which can always be diminished, and between reality and negation there is a continuous nexus of *possible* realities" (B211, emphasis mine). Thus, one can infer that "all appearances whatsoever are … continuous magnitudes, either in their intuition, as extensive magnitudes, or in their mere perception … as intensive ones" (B212).

An apparently cogent consequence can be drawn from the above considerations: "Now if all appearances, considered extensively as well as intensively, are continuous magnitudes, then the proposition that all alteration (transition of a thing from one state into another) is also continuous could be proved here easily and with mathematical self-evidence" (B212–13). However, Kant refuses to draw this conclusion in order to not threaten the unity of his system and to not "anticipate general natural science" (B213). No reference to the concept of change in general is possible, a concept that is "entirely beyond the boundaries of a transcendental philosophy," presupposing "empirical principles" (B213): "the understanding gives us no inkling *a priori* that a cause is possible which alters the state of things, i.e., determines them to the opposite of a given state" because alteration is a concept "about which experience alone can teach us" (B213).

Consequently, a smaller difference can be thought in any difference in degree without implying that the variation in degree is continuous, that it passes from one degree to another through all of the infinite intermediate degrees. However, Kant does not seem to remain faithful to this simply negative definition of continuity. In the proof of the second of the "Analogies," he writes, "No difference of the real in appearances is the *smallest*, just as no difference in the magnitude of times is, and thus the new state of reality grows out of the first, in which it did not exist, *through all the infinite degrees of reality*, the differences between which are all smaller than that between zero and *a*" (B254, emphasis mine). In reality, it is false to maintain that, just because it is *possible* to subdivide alteration into ever-smaller portions, this should pass de facto through all of its infinite degrees, in the same way that a totality is not constituted of infinite parts just because it is infinitely divisible. Explicitly alluding to the Anticipations of Perception, Kant writes, "[w]e *anticipate* only our own apprehension, the formal condition of which, since it is present in us prior to all given appearance, must surely be able to be cognized *a priori*" (B256, emphasis mine).

Here the connection, as well as the difficult conciliation, between the problem of change and that of Anticipations of Perception[53] is most evident. The Anticipations of Perception make change possible by reducing the opposition between reality and negation to a quantitative opposition, in which the state a and the state b are only distinguished in degree, by the difference $b - a$. However, the transformation contains no privileged point in which the alteration can be verified. A time always

[53] A reflection from the late 1870s also testifies to this connection: "*Principium* of the mathematical cognition of appearances: All appearance has as intuition its extensive magnitude and as sensation its degree. For (as far as the latter is concerned) every sensation arises from non-being, since it is a modification. Thus through alteration. All alteration, however, proceeds from 0 to a through infinitely small steps" (AA 18:241; Refl. 5585) English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 247.

exists between the two instants and a difference in degree between the two states that, no matter how small, are never the smallest possible. Thus, they always remain separate and never pass into one another whereas the transition can only occur where they can coexist together, that is to say, where the quantitative difference between the two opposed states has vanished and is $=0$.

The point where the two opposed states should pass into one another is, at the same time, the point where their difference vanishes, and thus that which defines them as opposites as well. In other words, precisely where change should take over the rigid immobility of being, in a moment without duration, no difference in degree can be admitted, and thus no change can occur. In the same way that Xenon's arrow is at rest in every instant, paradoxically, in each instant of a change, where the difference between the opposites is $=0$, no change occurs. Again, the possibility of alteration cannot be "comprehended" by the concept; only intuition can guarantee such a possibility by furnishing a concrete example, given directly through experience.

2.6 Maimon's Theory of Differentials

Modern thought, however, discovered a *concept* capable of determining the unity of opposed predicates that, for Kant, can only be given in intuition.[54] This difference is thought as neither finite nor equal to zero but as an infinitely small difference like a "differential." In the "haze of the infinitely small" [Nebel des Unendlichkleinen]

[54] Leibniz's philosophy offers the most philosophically pregnant expression of this solution to the problem. Leibniz himself defines alteration (*mutatio*) as "aggregatum duorum statuum contradictoriorum," something that is impossible at first sight "quia non datur tertium inter contradictoria" (Leibniz, Gottfried Wilhem. *Textes inédits d'après les manuscrits de la Bibliothèque provincale de Hanovre*. ed. Gaston Grua (Paris: Presses universitaires de France, 1948), 1:323). If no mediation is possible between two contradictory states, then a quantitative consideration of the variation is presupposed, something whose course of change can be thought as a simple "more or less", thus allowing one to conserve the relation between that which is found at the beginning of the process of transformation and that which is found at its conclusion: "If a thing alters so much that it exhausts itself (i.e., becomes nothing), and if that which is produced during the change is always *alike*, that which comes before will *bigger* and that which comes after will be *smaller* if it returns from nothing through the same alteration. If the result is always *alike*, that which comes first will be *smaller*, and that which comes after will be *bigger*. This is clear from what was stated before. And this can continue to infinity since, because of likeness, there is always the same relation" (Leibniz, Gottfried Wilhelm. *Die Leibniz-Handschriften der Königlichen Öffentlichen Bibliothek zu Hannover*. ed. Eduard Bodemann (Hildesheim: Olms, 1966). Reprint, Hannover, 1989, 35 I 12, n11). To master becoming means to find a conceptual tool with which it is possible to "think" that restricted zone where opposite states seem to be able to coexist precisely where they should annihilate each other. On Leibniz's framing of the problem, see Pasini, Enrico. *Il reale e l'immaginario: la fondazione del calcolo infinitesimale nel pensiero di Leibniz* (Turin: Sonda, 1993), 24–28. Thus, if one admits that opposites "only differ in terms of more and less" (GM 2:119), then one can conceive of the possibility that "they always pass from the small to the big and vice versa through the middle, in degrees as in parts" (GM 5:30).

(HW 5:319),[55] to use Hegel's expression, opposites confuse themselves in the contradictory concept of a "momentary" change. Kant himself actually cleared the way for this type of solution, for instance in this Note: "the meaning of the principle of continuity is simply this: all *different* things are *remota*, that is they are connected only *per intermedia*, between which the difference [Unterschied] can be even smaller. That is to say, no difference is the smallest because no transition is elementary and is the smallest, but it always has a magnitude […] the smallest difference would be called a *differential*" (AA 18:167; Refl. 5382).

The possibility of proceeding in this direction should not be understood as an abstract theoretical alternative. The writings of Salomon Maimon (1753–1800) provide an historical example of the possibility of following this line of thought. In his 1790 *Versuch über die Tranzendentalphilosophie* (Essay on Transcendental Philosophy, which Kant himself, receiving the work from his friend Marcus Herz,[56] recognized as excellent) as well as in some later publications, in particular his 1797 *Kritische Untersuchungen über den menschlichen Geist oder das höhere Erkenntniss- und Willensvermögen* (Critical Investigation of Human Spirit or Higher Faculties of Knowledge and of Will). Maimon clearly poses the problem of becoming, as it emerged from the folds of Kant's thought, as well as a possible solution through infinitesimal calculus.

Maimon conceives change, according to the traditional definition, as a "succession [Wechsel] of modifications in one and the same subject" (MGW 2:308). For this reason, "a determinable (substance) can have two determinations in a succession of time that exclude each other (predicates), one of which is reality [Realität] and the other its negation [Negation]" (MGW 2:142):

> [A] determination should be something positive (if it is to be perceived in intuition because a negative determination is only logical) and the successive determination should be opposed [entgegengesetzt] to the preceding one; but that which is opposed to something positive cannot be negative, rather both opposed qualities are necessary for experience. To resolve [heben] this contradiction, and thus make experience possible, these [the opposed determinations] should be unified in the object such that they conflict with each other as little as possible [am wenigsten Abbruch thun]; that is to say, their opposition [Gegensetzung] should be a *minimum*. (MGW 2:139)

As I have shown, in order for becoming to maintain its unity and to be referred to a single subject, succession cannot be interrupted at any point: "if something

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[55] English translation from Hegel. *Hegel's Science of Logic*, 273. Translation slightly altered by the author.

[56] On April 7, 1789, Marcus Herz notified Kant of receiving by mail a "manuscript" from "Herr *Salomon Maimon* … containing penetrating reflections on the Kantian system" (AA 11:14). Kant, overburdened with the writing of the *Critique of Judgment*, wrote a letter on May 24, 1789 to Herz in which he states, "I had half decided to send the manuscript back immediately, with the aforementioned, totally adequate apology. But one glance at the work made me realize its excellence and that not only had none of my critics understood me and the main questions as well as Herr Maimon does but also very few men possess so much acumen for such deep investigations as he" (AA 11:49; see AA 11:48 for Kant's letter to Maimon himself on May 26, 1789) English translation from Kant. *Correspondence*, 291, 311–12.

suddenly emerges (without continuity) … then we will not be able to believe that we are dealing with the same thing that alters, but rather that we are dealing with two different things" (MGW 2:139). In contrast, becoming should be able to unfold in the variation of states of the same substance:

> [I] n this case we have experience, that is, perception [Wahrnehmung], of the same permanent something [Beharrlichen] connected with different determinations that succeed each other in time [mit verschiedenen in der Zeit wechselnden Bestimmungen verknüpft]. These determinations are also at the same time positive because the opposition that is noticed [die darin bemerkte Gegensetzung] (that is necessary for experience) is *the smallest possible*. And this is the so-called law of continuity (MGW 2:139-, emphasis mine).

Thus, for Maimon as well, opposition should be conceived as a simple difference in degree, not of nature, because "one cannot say that cold water has become sweet, but that it has become hot" (MGW 2:137). Only if opposites are distinguished in terms of more or less can the difference between these be thought as "the smallest possible":

> the perception of an alteration, again requires unity in multiplicity; that is to say, the reciprocal relation of two states in one thing. If these were completely different, … only a simple manifold would be possible. If, on the contrary, they were completely identical, there would not be any manifold; that is, there would no longer be two states, but one and the same state. (MGW 2:216).

In order that an alteration can be thought as a unitary process, "the states should be partly identical and partly different … such a difference should thus be an *infinitely small difference* through which the thing obtains only a *differential* of a state that is different from the preceding one" (MGW 2:216–, emphasis mine). The transition of a determination into an opposite one (e.g., a movement in one direction into a movement in a different direction) can occur, not because they coexist at the same time, but because the difference in time is assumed to be infinitely small (see MGW 4:555-).[57]

As I have shown, the difference between two successive states of a change is, in every instant, $=0$. However, the vanishing of the quantitative difference should not compromise the unitary meaning of the process. Maimon states, "the differential of every object in itself, in respect to intuition, is $=0$, $dx=0$, $dy=0$, etc.; but their relations are not $=0$" (MGW 2:32). In fact, "*dx* and *dy*, considered in themselves as magnitudes, are $=0$. However it is possible that $dx=2dy$" (MGW 7:211); that is, it is possible that their relation has a determinate value.

Maimon discusses the philosophical significance of this resorting to infinitesimal analysis in particular detail in the remarks and clarifications appended to the *Versuch über die Transzendentalphilosophie*. He distinguishes between the "symbolic" and the "metaphysical" meaning of the infinitely small:

> If one affirms that $dx:dy=a:b$, this does not signify that abstract *x* of every magnitude is related to abstract *y* of every magnitude, etc., because nothingness cannot have any

[57] On the difference between Kant and Maimon on this point, see Freudenthal, Gideon. "Maimon's Subversion of Kant's Critique of Pure Reason. There are non Synthetic *a priori* Judgements in Physics." In *Salomon Maimon: rational dogmatist, empirical skeptic critical assessments*. ed. Gideon Freudenthal, 144–75. (Dordrecht: Kluwer Academic, 2003).

> quantitative relation to nothingness. Rather, the meaning is the following: for any *x*, no matter how large or small … it always follows from the equation between these two magnitudes that *x:y* [is = *a:b*] (MGW 2:355).

The "symbolic infinitely small," according to Maimon, "is only an invention [Erfindung] by mathematicians that gives generality to their own affirmations" (MGW 2:355). Alongside this strictly mathematical sense of the infinitely small, another sense can be found that is loaded with philosophical implications. For example, the extensive magnitudes of the sides of a triangle can be thought as vanishing completely in respect to their extensions while, nevertheless, conserving their reciprocal relations: "The extensive magnitude of the sides is completely annihilated [hört alsdann gänzlich auf] and is reduced to its differential. In contrast, the relation between the sides always remains the same (MGW 2:395–). Here, the issue is no longer quantity, but the "quality of the quantum" (MGW 2:395–): "the metaphysical infinitely small is real because the quality can be considered as abstracted from all quantity" (MGW 2:395).[58]

This is precisely the point of support that can be used to move beyond what Kant seems to be willing to admit. For Kant, as I have shown, the unity of quality ("if I say for example: red is different than green" [MGW 2:32]) is always a posteriori and cannot receive any rigorous determination. In contrast, Maimon claims that, "in differential calculus, space is considered a concept abstracted from all quantity in intuition and determined nevertheless by different types of qualities" (MGW 2:22–). This is the achievement of the "great Leibniz" through his "discovery of infinitesimal calculus": that a "magnitude (*quantitas*) is not regarded as something large (*quantum*), or rather, it is a quality abstracted from quantity" (MGW 2:28n).

Maimon's conception is nearly incomprehensible according to today's standard conception of analysis. However, the idea that the "character" of the magnitude is conserved in the disappearing of the quantitative difference is not only explicitly present in Leibniz,[59] as Maimon correctly observes, but Kant himself also refers to this conception. In a very significant "note," Kant writes: "that which holds for a *quantum*, also holds for the *limite quanti*, because the quality remains" (AA 18:360;

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[58] On this point, see Thielke, Peter. "Intuition and Diversity: Kant and Maimon on Space and Time." In *Salomon Maimon: Rational Dogmatist, Empirical Skeptic: Critical Assessments*. ed. Gideon Freudenthal. (Dordrecht: Kluwer Academic, 2003), 312–; Atlas, Samuel H. *From Critical to Speculative Idealism; the Philosophy of Solomon Maimon* (The Hague: Nijhoff, 1964), 109–23; Bergman, Samuel Hugo. *The Philosophy of Solomon Maimon*, tr. Noah J. Jacobs (Jerusalem: Magnes Press, Hebrew University, 1967), 262–3.

[59] Leibniz himself, in a letter to Guido Grandi (1713), indicates that this is the characteristic feature of infinitely small magnitudes: "interea infinite parva concipimus non ut nihila simpliciter et absolute, se ut nihila respectiva … id est ut evanescentia quidem in nihilum, retinentia tamen characterem ejus quod evanescit [We consider infinitely small quantities not as an absolute nothing, but as respective nothing: the quantities that vanish into nothing maintain the character of what is vanishing]" (GM 4:128).

Refl. 5815). The becoming $=0$ of a quantitative difference is not absolute nothingness: "the general mathematical law of continuity," according to Kant, only affirms that "what can be the predicate of the relationship between two unequal magnitudes, also holds if these are equal, that is, in as much as their inequality has vanished" (AA 15:243; Refl. 560), that is to say, has become $=0$. Only from this point of view is change comprehensible. It should not be thought as a metábasiV eÎV Állo gænoV (a crossing over to another genus), but rather as a purely quantitative transformation of the same quality: "Every difference (in appearance) is a *quantum* …; thus 0 must be regarded as homogenous with *A*, only as vanishingly or infinitely small; thus there is no *progressus* in the determination of a thing to another state except by means of an increase of the *same quality* from the infinitely small" (AA 18:410; Refl. 5973; last emphasis mine).[60]

This sort of interpretation of infinitesimal analysis was probably common in Kant's time. Lazarus Bendavid's 1789 *Versuch einer logischen Auseinandersetzung des mathematischen Unendlichen* [Essay on a Logical Confrontation of the Mathematical Infinite][61] is another significant example. Bendavid was not only, together with Herz and Maimon, one of the Jewish philosophers who helped spread Kantian philosophy at the end of the eighteenth century,[62] but he also had a good knowledge of mathematics (his earliest published work was on a geometrical subject).[63]

According to Bendavid, infinitesimal calculus offers the possibility of thinking zero in a different way than the zero that emerges "from the opposition of two equal magnitudes."[64] For infinitesimal calculus namely treats that which in general cannot have a "more or less," that which makes the very concept of magnitude meaningless, that is to say, not quantity but quality: "A magnitude is no longer measurable when its value is eliminated [aufgehoben] through an opposite one and becomes 0. In contrast, a magnitude is infinite if one considers it as a simple quality"[65] For instance, to be tangent of a circle is a quality. It makes no sense to speak of something that is more or less "tangent" to a "circle." On the contrary, the tangent of a circle, "when

[60] English translation from Kant. *Notes and Fragments: Logic, Metaphysics, Moral Philosophy, Aesthetics*, 319.

[61] Bendavid, Lazarus. *Versuch einer logischen Auseinandersetzung des mathematischen Unendlichen* (Berlin: Petit und Schöne, 1789).

[62] On Bendavid, see Rosenkranz, Karl. "Geschichte der Kantischen Philosophie." In *Kants Werke*. eds. Karl Rosenkranz and Friedrich Wilhelm Schubert. (Leipzig: Voss, 1842), 315. According to Rosenkranz (see Rosenkranz. "Geschichte der Kantischen Philosophie." 315), Bendavid tried to disseminate Kantian Philosophy in Vienna (without much success) where, in 1795, he had already published his Bendavid, Lazarus. *Vorlesungen über die Critik der reinen Vernunft*. Fotomechan. Nachdr. ed (Brussels: Culture et Civilisation, 1968). Many other commentaries on Kant's books followed, such as Bendavid, Lazarus. *Vorlesungen über die Critik der Urtheilskraft*. 2nd ed (Brussels: Culture et Civilisation, 1968).

[63] Bendavid, Lazarus. *Über die Parallellinien. In einem Schreiben an Herrn Hofrath Karsten* (Berlin: Voss, 1786).

[64] Bendavid. *Versuch einer logischen Auseinandersetzung des mathematischen Unendlichen*, 39.

[65] Ibid., 44.

it ceases to be a magnitude, nevertheless remains the tangent of a circle and conserves the property [Eigenschaft] of having a determinate position in respect to the circle."[66] One can take a secant that cuts the curve in two points and make the two points coincide so that the difference between them vanishes. The relation between the tangent and the circle remains perfectly determined nevertheless. Such a relation thus has no magnitude. It pertains to a completely different domain: it is "a simple quality" by which "the concept 'magnitude' has no meaning."[67]

Significantly Bendavid tries to explain this transition from quantity to quality by referring to the concept of intensive magnitude (a fact that Hermann Cohen did not fail to notice; see CW 5:1, 112–), emphasizing its character as a non-additive magnitude. As one cannot make lemon more sour by putting many lemons together, similarly, "as soon as a tangent has become infinite, it is simply a contact line," and "it has no more contact because one adds a magnitude to it."[68] Considering the infinitely small quantity dx as the expression of the transition from quantity to quality, the equation $x + dx = x$, according to Bendavid, loses its paradoxical character. For, in reality, it only shows how adding a "property" to a "magnitude" is senseless.[69]

Referring to Kant's concept of "alteration" in his *Vorlesungen über die Critik der reinen Vernunft*,[70] Bendavid explicitly resorts to this solution to the problem. Bendavid shows that "the transition from the state a to the state x does not happen suddenly [plötzlich], but in such a way [dergestalt] that, in the state that flows between these two, no part is the smallest, but rather should always [stets] be preceded by a smaller one."[71] Thus, "first the substance was in the state $= x$ and then in the state $x + dx = x$" in such a way that it is clear, however, that, "the newly arisen state $= dx$ … as a magnitude, is to be considered $= 0$."[72]

Maimon seems to to respond rather polemically to Bedavid's interpretation of the concept, "differential":

> Mr. Bendavid says then that $dx + a = a$ because a simple property cannot be added to a magnitude … The real reason is not the one provided by Mr. Bendavid … [the real reason is] because different types of magnitude cannot be added. One can say that a dx cannot be added to a, just as a pound cannot be added to a cubit [Elle]. (MGW 2:290)

Maimon thus does not seem to agree with Bendavid's complete identification of the infinitely small with zero, understood as the quality abstracted of all quantity. This gives Maimon the occasion to further clarify his conception of the relations between quantity and quality. "Is a certain velocity the simple quality of velocity in general?" (MGW 2:291), Maimon asks rhetorically against Bendavid. On the contrary, the concept of "infinitely small movement" serves precisely to

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[66] Ibid., 41.

[67] Ibid., 77.

[68] Ibid., 48.

[69] Ibid.

[70] Bendavid. *Vorlesungen über die Critik der reinen Vernunft*, 48.

[71] Ibid., 93.

[72] Ibid.

compare different velocities in every instant, that is to say, where apparently there are no quantities that could be compared. Therefore, according to Maimon, "the relations of these differentials are the relations of these velocities to one another" (MGW 2:291).

For this reason too, Maimon resorts to the idea of the differential as something with a zero for extension, but not for intension.[73] However, he emphasizes, as opposed to Bendavid, that the intensive magnitude is also a magnitude, even if of a different kind. Through the notion of "degree," "the velocity of a movement in a point can be compared with the velocity of the same in another point and in this way can be determined as a magnitude" (MGW 2:290). Thus, in every instant, one can think that the moving thing has a determined "degree of velocity," even if in fact the movement in the instant has vanished: "the velocity in every temporal point is a real object [ein reelles Objekt] (a determinate intensive magnitude), a how much of a determined quantity" (MGW 2:290–).

However, Maimon also seems to emphasize that the transition from quantity to quality should be understood as a transition from *magnitude* to the *relation* between magnitudes:

> this determined quantity cannot be recognized through this velocity in itself, but only by means of its effects; that is, by means of the space that the body with this velocity (if it remains unvaried) travels. However, the duration of the movement and the space that is traveled does not belong to the essence of velocity. The latter should be thought as abstracted from these; that is, it should be reduced to an infinitely small space and an infinitely small time, which are no less real as a result (MGW 2:291–).

The degree of velocity can be expressed by a relation between finite extensive magnitudes if the velocity remains constant in time. If it changes from instant to instant, such a relation must also be represented as variable and should ultimately be defined as a relation between infinitely small differences: "when an extensive magnitude is reduced to its differential, this can be expressed, as a result of its intensive magnitude, as a relation between two differentials" (MGW 2:395).

However, one should be careful not to be misled by Maimon's attempt to distance himself from Bendavid's work. Both seem to refer to a common Kantian framework and attempt to solve the problems that Kantian philosophy raised by means of similar conceptual tools. Reading the work of Bendavid and Maimon clearly shows that philosophy seems to have found in infinitesimal calculus the possibility of determining the "quality" of magnitude independently of its intuitive being. This is a conception that, as I will show in the next chapters, continually reappears in different forms in post-Kantian debate. According to Maimon, infinitesimal calculus shows that "fluctions and differentials," despite having no finite extension, "are … distinct through their manner of emerging [Entstehungsart]" (MGW 7:210).

Moreover, and particularly significant for the present work, Maimon explicitly relates this proposal to the Anticipations of Perception (see. MGW 7:210–11, 214–15). "[A]ll objects presentable *a priori*," Maimon writes, reformulating Kant's

[73] "Intensive magnitude is the differential of extensive magnitude and this in turn is the integral of the first" (MGW 2:122).

principle, "[that is] all *quanta* (since we have no other *a priori* objects)," are, "according to their manner of emerging, intensive magnitudes" and "can be regarded as velocities in a certain point or as first and last relations" (MGW 7:215). In the same way, all "*a posteriori* appearances … are in their emerging and vanishing intensive magnitudes" because sensation, "which corresponds to the matter of sensible perception … does not have any extensive magnitude," but rather, allowing different degrees, "has an intensive magnitude" (MGW 7:215–).

The possibility of determining the quality of magnitude through infinitesimal calculus allows mediation between these two points of views, between sensible qualities and intelligible qualities. Here, Maimon seems to offer a solution to the fundamental problem that critical philosophy had left unresolved: "[H]ow can the agreement between *a priori* forms and *a posteriori* things be conceived? … How can the understanding actually submit to its power (the rules) something that is not in its power (the given objects)? According to Kant's system, for which sensibility and understanding are two completely different sources of knowledge, this question, as I have shown, is irresolvable" (MGW 2:32). Only if "empirical" multiplicity is substituted with a "rational" multiplicity; only if the different qualities (heat, color, sound, etc.), which according to Kant[74] are simply "given" intuitively, can themselves be determined conceptually; does it seem possible to heal the apparently overwhelming fracture between the two sources of knowledge: "this manner of consideration also serves in the solution to the problem: *Quid juris*? Since the concepts of the understanding or the categories never refer directly to intuitions … but rather to the way of emerging [Entstehungsart] of intuitions" (MGW 2:355).

2.7 Change and Synthetic Unity

My aim is not to reconstruct Maimon's views in detail or to enter into the specific philosophical questions that they raise. Rather, the above section aims to highlight several important results for the present investigation. Up to this point, I have attempted to demonstrate that the meaning of the Anticipations of Perception ultimately consists in the idea that the opposition between reality and negation (in phenomena) is not logical and qualitative, but should be thought as real and quantitative instead. Whereas no third exists between two contradictory opposites, between two real opposites, as close as they may be to one another, there is always an intermediate point, a point of indifference in which their reciprocal quantitative difference is $= 0$. As Kant notes in a reflection that seems to adequately summarize what has been said until now, "the direct connection of opposites in the same subject is only possible in as much as [ausser so fern] I take zero to be the infinitely small of the property of *A*. For example, the emergence of pain from indifference [Gleichgültigkeit] is possible only if the latter is considered an infinitely small

[74] See above 14.

pain" (AA 18:410; Refl 5971.). When the quantitative difference between the two opposites diminishes to the point that they coincide, vanishing into each other, the absolute nothingness of contradiction is not found since "the negation of the lack can be considered as the infinitely small, but not as the negation of the contradiction of the concepts (qualitative opposition)" (AA 18:378; Refl. 5894).

To grasp the theoretical significance of this solution to the problem and the way in which it can overcome the conflict between intuition and concepts, one first needs to acknowledge that the problem of change and becoming is not merely a marginal problem in Kant's philosophy. The characteristic of alterations consists in the fact that by means of them, completely new contents are constituted, contents that emerge from the unknown depths of "being" and oppose themselves to "knowledge" as something autonomous and independent. Becoming is the irruption of the absolutely unexpected, of that which confronts the subject as something entirely strange in respect to the knowledge it possesses. In every change, the new state opposes the preceding one as something that cannot be taken apart analytically. The concept of change reveals, in the most intuitive form, the fundamental problem of critical philosophy: How is it possible to "go beyond the concept *A* in order to cognize another *B* as combined with it" (A9)? How is it possible to admit that "if A is posited … something altogether different from it, B, must necessarily also exist" (AA 5:51)?

Kant solves this problem, as is well-known, through the concept of synthetic unity and *a priori* synthesis. Overcoming mere analytic identity between concepts, he shows how the possibility of knowledge is not based on the homogeneity of the identical, but on the necessary connection of the different. If the concept *B* that should be connected with *A* cannot be considered immediately or mediately as identical to *A*, this signifies that it is not-*A*, that it is radically opposed to *A* as its contradictory opposite. Thus, no mediation is possible between the concept *A* and the concept *B* because everything that begins to be, before beginning, is not something, but "nothing," and as "nothing," *B* can be anything provided that it is not *A*.

However, if one concedes with Kant the possibility of "anticipating" not only the form of the connection but also its very content, one admits the possibility of establishing completely *a priori* that the concepts *A* and *B* to be connected are not distinguished *qualitate*, but only *quantitate*. *B* is certainly different from *A* and is not contained in *A*, but *B* is not simply the logical negation of *A* (i.e., not-*A*) since *alterum contradictoriorum dicitur de quolibet* [the contradictory other can be said of everything]. One must affirm that concept *A* is distinguished from *B* not by "nature," but only by "degree".

The Anticipations of Perception seem to provide the basis for such a solution to the problem. The theoretical import of the principle is its establishing *a priori* that the opposition between reality and negation in phenomena only involves quantity. It is the difference between $+A$ and $-A$ in respect to the point of indifference $=0$, rather than the qualitative opposition of A and its contradictory opposite not-*A*. However, as I will attempt to demonstrate in the following chapters, this theoretical import is only fully clarified in post-Kantian philosophy's appropriation and development of the principle. Only by moving beyond Kant can one appreciate the

fruitfulness of this specific form of opposition between reality and negation that Kant indicates as characteristic of *realitas phaenomenon*.

"That all opposition is only quantitative was for some time a cardinal thesis of recent philosophy" (HW 5:269).[75] Hegel's words effectively summarize what can be considered to be one of the fundamental postulates of post-Kantian philosophy; an idea that, as I have shown, flows from the heart of critical philosophy itself and, as I will discuss in further detail, seems to have irresistibly spread throughout later philosophy. As Fichte writes in "Foundations of the Entire Science of Knowledge" (*Grundlage der gesamten Wissenschaftslehre*) (1794–95), "Just as, previously, a not-I was opposed to the not-I in general, as an opposite *quality*, so here, an objective is opposed to the subjective […] simply by and by means of *quantity* […] and this procedure is a quantitative antithesis, just as the earlier procedure was a qualitative one" (FGA 1:2:351).[76] In *Darstellung meines Systems der Philosophie* (Presentation of my System of Philosophy), Schelling explicitly reaffirms this conception, providing a particularly effective formulation: "between the subject and object a difference is not possible unless it is quantitative … that is, a difference that involves quantity of being" (SW 3:19).

The logical opposition between reality and negation, being and non-being, seems somehow to bring with it all the others. Dogmatic metaphysic merely translates the opposition between "concepts" into an objective opposition between "things" such that an unbridgeable ravine seems to prevent any mediation between knowing and being, between the subject and the object, between the known and the unknown. This *impasse* can only be overcome by thinking these oppositions in such a way that the two opposites are from the beginning thought within a common horizon, being distinguished only in terms of more and less. Logical opposition between concepts must be substituted with real opposition: "the opposition of subject and object is a real opposition [eine reelle Entgegensetzung]"(HW 2:97)[77] and "if the opposition is real it is merely quantitative"(HW 2:99).[78]

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[75] English translation from Hegel. *Hegel's Science of Logic*, 233.

[76] English translation from Fichte, Johann Gottlieb. "Foundations of the Entire Science of Knowledge." In *Science of Knowledge*. eds. Peter Lauchlan Heath and John Lachs. (Cambridge: Cambridge University Press, 1982), 186.

[77] English translation from Hegel, Georg Wilhelm Friedrich. *The Difference between Fichte's and Schelling's System of Philosophy*, tr. Henry S. Harris and Walter Cerf (Albany: State University of New York Press, 1977), 157.

[78] English translation from Ibid.

This, as Hegel attempts to demonstrate in *The Difference Between Fichte's and Schelling's System of Philosophy* (the so-called *Differenzschrift*), is where Fichte's and Schelling's philosophies converge. Referring particularly to Fichte in *Faith and Knowledge* (*Glauben und Wissen*), Hegel explicitly characterizes the two opposites as a positive and negative magnitude like $+1$ and -1, affirming that the limit that separates the one from the other and the point in which they touch is $=0$:"empty thought [das leere Denken], 0, which is the middle between $+1$ and -1 wherein $+1$ and -1 disappear" (HW 2:2:413).[79] The reality of this empty thought "consists in the $+1-1$, and the standing of this antithesis provides the content of this idealism […] But at the same time these opposites are ideal ($=0$) and their true truth is […] in their being nothing" (HW 3:413).[80]

Finding a *concept* that is capable of establishing the third that, for Kant, is only given in *intuition*, a third in which the opposites (+ and −) can be distinguished from each other while coexisting at the same time, would also resolve the problem concerning the opposition of the two sources of knowledge. In as much as post-Kantian philosophy, up to neo-Kantianism, has attempted to overcome this conflict, it has necessarily resorted to such a conceptual tool. Thus, Maimon's thesis has not remained an isolated hypothesis. The infinitesimal method, or rather the speculative and philosophical interpretation of it, seems to be precisely one such conceptual tool for thinking the "affinity" of elements that, despite being quantitatively different, can be considered under a common conceptual point of view.

If a firm logical opposition between *A* and not-*A* is maintained, then any "third" between the opposites is excluded and knowledge of one's own ignorance is merely contradictory since "[t]he ignorant person has no concept of his ignorance, because he has none of science" (B603). Only by admitting another form of opposition, not logical but real, can one think something intermediate between knowledge and ignorance. Only "real" negation can represent the positive affirmation of something new opposed to that which is already known, and that which nevertheless has an intimate correspondence to the latter: "if the subject and object are completely identical, then there is no knowledge at all; if they are completely heterogeneous, then there is no a *priori* knowledge. Only the intermediate route between the two extremes is accessible and only by means of it can one resolve the problem."

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[79] English translation from Hegel, Georg Wilhelm Friedrich. *Faith and Knowledge: An English Translation of G. W. F. Hegel's Glauben und Wissen*, tr. Henry S. Harris (Albany, NY: SUNY Press, 1996), 172.

[80] Ibid.

Chapter 3
The Anticipations of Perception in Post-Kantian Idealism

3.1 Fichte and the Opposition Between the I and the Not–I

The preceding chapter suggests that the significance of the Anticipations of Perception can be ultimately located in the substitution of logical and qualitative opposition between reality and negation, which for Kant characterizes *realitas noumenon*, the reality that is object of the pure intellect, with the real and quantitative opposition that is the characteristic feature of *realitas phaenomenon*, that is, the "reality that corresponds to sensation." If this distinction still appears to be a secondary aspect of critical thought, a look at the history of post-Kantian Idealism immediately shows that this conviction is unfounded.

I certainly do not intend to give an exhaustive account of one of the most extraordinarily creative periods in the entire history of Western thought.[1] Instead, my aim is to highlight the profound influence that Kant's redefinition of the relations between reality and negation had on what is conventionally called "German Idealism." I start from the premise that merely outlining the history of this problem cannot illuminate its authentically theoretical aspect. The very possibility of recognizing the identity of a problem in the variety of its different formulations, without dissolving it in the series of particular forms in which it is presented, is what permits its philosophical meaning to emerge.

While the principle of the Anticipations of Perception seems to have a marginal role in Kant's text, running the risk of becoming a "forgotten principle,"[2] as was stated above, its full importance is revealed when one considers how the form of opposition between negation and reality that it introduces becomes, in post-Kantian

[1] The literature on this subject is nearly endless. The classic Kroner, Richard. *Von Kant bis Hegel* (Tübingen: Mohr, 1921). Reprint, Tübingen, Mohr 2006 is still worth to read. Among more recent literature see Ameriks, Karl. *The Cambridge Companion to German Idealism* (Cambridge: Cambridge University Press, 2000); Beiser, Frederick C. *German Idealism: The Struggle against Subjectivism, 1781–1801* (Cambridge, MA: Harvard University Press, 2002); Beiser, Frederick C. *The Cambridge Companion to Hegel and Nineteenth-Century philosophy* (New York: Cambridge University Press, 2008).

[2] See Introduction, footnote 1.

M. Giovanelli, *Reality and Negation – Kant's Principle of Anticipations of Perception: An Investigation of Its Impact on the Post-Kantian Debate*, Studies in German Idealism 11,
DOI 10.1007/978-94-007-0065-9_3, © Springer Science+Business Media B.V. 2011

thought, the distinguishing feature of "critical" thought itself in respect to "dogmatic" metaphysics. The absurd problem of how "absolute being" can be transformed into "absolute knowledge", or, in the language characteristic of Fichte, how the "being" of the I can encounter the "being" of the not–I, is fundamentally erroneous in its very formulation. On the one hand, if one recognizes "absolute certainty of the proposition '*A* is not equal to *A*,'" one must admit that "*so surely is a not–I opposed absolutely to the I*" (FGA 1:2:267).[3] On the other hand, if this opposition is maintained as such, leaving the sphere of the I to access that which the I is not would be impossible, precisely because that which the I is not, the not–I, logically contradicts the I and cannot be posited together with it: "As opposed to the absolute I … the not–I is *absolutely nothing* [schlechtin nichts]" (FGA 1:2:271).[4]

The problem of finding a form of opposition between reality and negation that allows mediation between absolutely irreducible opposed elements now becomes the fundamental problem of transcendental philosophy itself: "how can *A*and– *A*, being and nonbeing, reality [Realität] and negation, be thought together without mutual elimination [vernichten] and destruction [aufheben]?" (FGA 1:2:269)[5] as Fichte formulates the problem in the *Foundations of the Entire Science of Knowledge.* According to Fichte only one possible solution exists: I and not–I, reality and negation, must not be logically opposed (as two contradictory concepts), but rather in a real manner (as two opposed magnitudes). "[A]s opposed to the limitable I [dem Einschränkbaren Ich Entgegensetzt] it [the not–I] is a *negative quantity*" (FGA 1:2:271; emphasis mine).[6] The opposition between I and not–I should no longer be considered on the basis of that between *A* and not–*A*, but rather on the model of that between + *A* and −*A*; no longer as a qualitative opposition, but simply as a quantitative opposition: "This is actually the case in mathematics, which disregards quality entirely and looks only to quantity. Whether I choose to count backward or forward

[3] English translation from Fichte. "Foundations of the Entire Science of Knowledge" 104. For reasons of uniformity, the author preferred to translate the celebrated Fichtean expressions "Ich" and "Nicht-Ich" as "I" and "non–I," following Daniel Breazeale in his edition of Fichte's *Early Philosophical Writings*, instead of using Self and not-Self, like Peter Heath and John Lachs in their translation of Fichtean writings, *The Science of Knowledge*. All Fichte's quotes from the latter translation have thus been modified by the author correspondingly.. As Breazeale observes, Fichte did not choose the expression "das Ich," which even in German sounds odd, "because of any lack of more natural-sounding German alternatives (e.g., *das Selbst*)" (Fichte, Johann Gottlieb. *Fichte: Early Philosophical Writings*. ed. Daniel Breazeale (Ithaca, NY: Cornell University Press, 1988), xiv). Therefore, a literal translation is probably more appropriate even if it might be less "readable" and "natural-sounding" in English (Fichte. *Fichte: Early Philosophical Writings*, xiv).

[4] Fichte. "Foundations of the Entire Science of Knowledge" 109. See Seidel, George Joseph. *Fichte's Wissenschaftslehre of 1794: a Commentary on Part 1* (West Lafayette, IN: Purdue University Press, 1993), 66. As Seidel observes, "the word 'quantity' here is *Größe*, which is likely a reference to Kant's discussion of intensive and extensive magnitude in the 'Anticipations of Perception' in the *Crìtique of Pure Reason*" (Seidel. *Fichte's Wissenschaftslehre of 1794: a Commentary on Part 1*, 66).

[5] Fichte. "Foundations of the Entire Science of Knowledge" 108.

[6] Ibid., 109.

steps as positive quantities is in itself a matter of complete indifference … So too in the *Science of Knowledge*. Whatever is negation in the I is reality in the not–I, and vice versa" (FGA 1:2:292).[7] Thus, central to Fichte's reformulation of Kant's philosophy is exactly that "quantification of the category of reality"[8] which I have taken to be the fundamental nucleus of the Kantian conception of the relations between reality and negation: "the Not–I," writes Fichte, "was posited as a quantum, but every quantum is *something*, and hence also *reality* [Realität]. Thus the Not–I must be negation, and in some sense a real negation [reale Negation], or negative quantity" (FGA 2:292).[9]

The importance of this rethinking of the relations between reality and negation for Fichtean thought is clearly seen in the fact that, in contrast to Kant who begins from the category of quantity, Fichte begins from the category of quality, deducing quantity from quality. As commentators have noted, the category of quality, in particular the category of reality (*Realität*), is transformed into a true and proper *Ur-Kategorie*[10] (originary category) from which all the others can be obtained. In this way, the tripartite schema reality-negation-limitation becomes the supporting structure of the *Science of Knowledge*. Reality and negation do not simply exclude each other, but rather reciprocally limit each other, being distinguished only in terms of more or less: "Thus, apart from reality and negation, the notion of a limit also contains that of divisibility (the capacity for quantity in general, not any determinate quantity)" (FGA 1:2:270),[11] what Fichte also calls "the category of determination [Bestimmung] (bounding [Begrenzung], or as Kant calls it, limitation [Limitation]). For a positing of quantity in general, whether it be quantity of reality or of negativity, is called determination" (FGA 1:2:282).[12]

The reality of the I and the not–I, which in the complex constitutes the "absolute totality of the real," should be considered "capable of quantity", that is, "capable of increase or diminuition" (GA 1:2:291).[13] Therefore, "in the I I oppose a divisible not–I to the divisible I" (GA 1:2272)[14] What should be known and, as such, is not yet known is actually always thought through principles of knowledge itself; for if it were simply other than knowledge, it could not even be recognized as something unknown. A real and quantitative opposition between I and not–I only has sense

[7] Ibid., 128–.

[8] Ibid., 128. See Baumanns, Peter. *Fichtes ursprüngliches System: Sein Standort zwischen Kant und Hegel* (Stuttgart: Frommann-Holzboog, 1972), 101.

[9] English translation from Fichte. "Foundations of the Entire Science of Knowledge" 128. On the difference between Kant and Fichte concerning their use of negative and positive magnitude, particularly in respect to Kant's conception in which the difference between positive and negative can only be presented in intuition, see Philonenko, Alexis. *La liberté humaine dans la Philosophie de Fichte* (Paris: Vrin, 1980), 166.

[10] Baumanns. *Fichtes ursprüngliches System: Sein Standort zwischen Kant und Hegel*, 99.

[11] English translation from Fichte. "Foundations of the Entire Science of Knowledge" 108.

[12] Ibid., 119.

[13] Ibid., 128.

[14] Ibid., 110.

within the I itself, within a common horizon in respect to which the two opposites are posited in such a way that the I "apportions [verteilt] the totality of posited being in general to the I and the not–I" (GA 1:2:254).[15]

Thus, the opposition between I and not–I should not be compared to that which occurs between two completely heterogeneous elements, but rather to that which is established between two qualitatively identical factors that are only distinguished in terms of more or less:[16] "I and not–I, as equated and opposed through the concept of their capacity of mutual limitation, are themselves both something (namely accidents) in the I as divisible substance" (FGA 1:2:279).[17] They are not simply "different" from one another such that their opposition cannot admit a nuanced intermediary ($A = A$ or $A \neq -A$). Rather, their contrast should allow solutions of "compromise" such that "A [is] in part = $-A$ and vice versa" (FGAGA 1:2:272).[18] Fichte tries to clarify the nature of the problem in an intuitive manner: "[P]ut light at a point *m*, and at the point *n*, darkness [Finsterniss], then, given that the space is continuous, and there is no hiatus between *m* and *n*, there must necessarily be a point *o* somewhere between the two, which is both light and darkness; a contradiction." Such a contradiction can only be avoided by admitting that reality and negation, light and darkness "are not opposed [entgegengesetzt] in principle, but differ only in degree [nur den Graden nach]. Darkness is simply a very minute amount of light. – That is precisely how things stand between the I and the not–I" (GA 1:2:301).[19]

Only if the I and the not–I are distinguished by quantity can a common line be traced that confines both of the opposites, allowing them to pass into one another: "this boundary is the actual point of union between the I and the not–I. They have no other point in common, nor could they, since they are supposed to be completely opposed to each other" (GA 1:3:156).[20] However, if the I and the not–I are only distinguished in terms of more or less, then the limit that separates them, and thus defines them as such, is at the same time the point in which their quantitative difference has disappeared. The limit that should permit reality and negation to exist together is simultaneously the suppression of both. If one attempts to determine the transition from negation to reality, or vice versa, one is once more confronted with the ineludible difficulty of fixing the point of transition where the second begins and the first ends and where they paradoxically seem to need to exist together:

> [P]osit light at instant A, and darkness at the immediately subsequent instant B: light and darkness are thereby sharply distinguished, as they should be. But instants A and B

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[15] Ibid., 225.

[16] See Philonenko. *La liberté humaine dans la Philosophie de Fichte*, 160.

[17] English translation from Fichte. "Foundations of the Entire Science of Knowledge" 116–.

[18] Ibid., 110.

[19] Ibid., 138. On this point, see Grant, Iain Hamilton. *Philosophies of Nature after Schelling* (London: Continuum, 2006), 87–.

[20] English translation from Fichte, Johann Gottlieb. "Outline of the Distinctive Character of the Wissenschaftslehre with respect to the Theoretical Faculty." In *Fichte, Early Philosophical Writings*. ed. Daniel Breazeale. (Ithaca, NY: Cornell University Press, 1988), 257.

> immediately bound one another, and there is no interval between them. Picture to yourself the sharp boundary between the two instants = Z. What is there at Z? Not light, for that is at instant A, and Z is not identical with A; and not darkness either, for that is at instant B. So it is neither of the two. – But I might equally well say that both are present, for if there is no interval between A and B, there is none between light and darkness either, and both are in immediate contact at Z. (FGA 1:2:352)[21]

As is well-known, Fichte discovered the solution to this problem in one of the I's faculties, the productive imagination, "the most wondrous of its powers" (FGA 1:2:350),[22] which he calls "the faculty of what is opposed" (FGA 1:2:375–);[23] the faculty "which intervenes between elements that would mutually abolish [die sich gegenseitig aufheben müssten] each other, and thereby preserves them both [und dadurch beide erhält]" (FGA 1:2:350).[24] For knowledge to be possible, "the subjective is neither to be destroyed by the objective, nor the objective by the subjective, any more than the I in general was formerly to be annulled by the not–I, or vice versa; both, on the contrary, are to subsist alongside each other. Hence they must be synthetically united, and are so by the third thing, in respect of which they are both alike"; that is, in which their difference disappears. At the same time, however, "they are fixed and held fast by that power of the I (imagination) which is active in the synthesis" (FGA 1:2:351).[25] The result of this wavering is "no sort of fixed boundary [keine feste Grenze]" or "an indeterminate … limit [unbestimmte Grenze]" (FGA 1:2:358),[26] an intermediary "between determination [Bestimmung] and non determination [nicht Bestimmung]" (FGA 1:2:360).[27]

Clarifying in an intuitive manner the meaning of this overlap of contraries at the limit that separates them, Reinhard Lauth speaks of a "ubiquitarian differential [übiquitäres Differential]" in which the opposites coexist precisely where they annihilate each other; something indeterminate in which the difference between the opposites is = 0, but that nevertheless contains the possibility of determination. Fichte calls this something "mere [blosse] determinability" (FGA 1:2:359):[28] something that has no determinate quantity, but that is "capable of quantity" (*Quantitätsfähig*). Fichte, in contrast to Maimon (who is known to have influenced Fichte tremendously), does not explicitly employ conceptual baggage from mathematics. However, Lauth's use of the "differential" metaphor highlights how, here in Fichte's work, all the problems raised by the relation between reality and negation in Kant's work return.

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[21] English translation from Fichte. "Foundations of the Entire Science of Knowledge" 186.

[22] Ibid., 185.

[23] Ibid., 209.

[24] Ibid., 185.

[25] Ibid., 186.

[26] Ibid., 194.

[27] Ibid., 187.

[28] Ibid., 186.

Despite the level of abstraction that Fichte's argumentation demands, the idea of the real opposition between self and not–I merely introduces the fundamental question of Kantian philosophy in a new form: "The celebrated question which Kant placed at the head of the *Critique of Pure Reason*: How are synthetic judgments *a priori* possible? is now answered in the most universal and satisfactory manner. In the third principle we have established a synthesis between the two opposites, I and not–I, by postulating them each to be divisible" (FGA 1:2:275).[29] In *a priori* synthesis, as I have discussed, *B* must be able to be connected to *A* while being different than *A*, forcing *B* to go beyond the concept *A*. Fichte appears to clarify how *A* is no longer opposed to *B* as if *B* were simply not–*A*. *A* and *B* are distinguished only in terms of "quantity." Only in this way can they be thought together as a synthetic unity, rather than a simple analytic unity, in a third where the one passes into the other without losing itself as such:

> *A* and *B* are opposed, and if the one is posited, the other cannot be: and yet they have to stand together … without mutually abolishing each other. But they can be thought of together in no sort of fashion, and under no possible predicate, save merely insofar as they mutually destroy each other. We are not to think of *A*, and not to think of *B*; but the clash – the incursion [Eingreifen] of each upon the other is what we are to think of, and this alone is the point of union between them. (FGA 1:2:352)[30]

The opposition between reality and negation, or rather the possibility of finding the manner in which these can coexist without reciprocally annihilating each other, can be ultimately considered the fundamental question of transcendental philosophy: "how can the opposites be united in one?" (GA 2:3:31). If they were opposed like two contradictory concepts, no point of union between the two could be found. There would be nothing that is neither I nor not–I but at the same time could be thought of as in common with both, a restricted zone in which the one passes into the other. Thus, according to Fichte, "only one thing can avoid contradiction: quantity" (GA 2:3:93).[31] Only by transforming the qualitative opposition between the I and the not–I into a quantitative opposition, only by substituting logical opposition with a real opposition, can one conceive the possibility of choosing an intermediate point C of indifference: "Suppose that C has a degree of reality. In that way, it can have a certain amount of the reality of A and a certain amount of −A; the reality of C has less reality than A and more reality than −A. Such a thing is called bounded [eingeschränkt] (limited [limitiert]) C is limited by A and −A" (GA 2:3:53). What is other in respect to the I is actually the same as the I itself; not in the sense of the "identity" that characterizes two concepts that are ultimately the same concept, but in the sense of the "homogeneity" that is presupposed by two "different" elements that are only distinguished in terms of quantity.

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[29] Ibid., 112.

[30] Ibid., 187.

[31] See Baumanns, Peter. *Fichtes Wissenschaftslehre. Probleme ihres Anfangs: mit einem Kommentar zu § 1 der "Grundlagen der gesamten Wissenschaftslehre"* (Bonn: Bouvier, 1974), 85–.

Once again, and more clearly than before, the dogmatic rationalism is overcome, specifically the Leibnizian-Wolffian type founded on the mere principle of identity and thus the complete "neutralization of the negative"; that is, on the removal of all contradictions that threaten the unity of the concept. Through the apparently simple idea of a divisible I and not–I, Fichte expresses the very heart of critical philosophy. He substitutes "analytic unity," the absence of contradiction, with a "synthetic unity" that does not reject contrast and opposition, but rather, in a manner of speaking, conserves it in itself, within the pure I.

Therefore, on the basis of what I will show to be a locus communis in post-Kantian thought, Fichte substitutes the model of non-contradiction between concepts with an equilibrium of opposed forces, now completely liberated from all physical meaning: "in striving of the I there is simultaneously posited a counterstriving of the not–I, which holds the former in equilibrium" (FGA 1:2:438).[32] The I and not–I cannot exist separately and "hence the forces of both must maintain an equilibrium" (FGA 1:2:439).[33] While metaphysical and dogmatic thought searches for a "thing in itself," something absolutely positive that contains nothing negative, the defining feature of critical philosophy is the possibility of thinking positive and negative in a reciprocal equilibrium within the very self. What is a mere contradiction from the point of view of absolute "being" becomes possible as soon as one adopts the point of view of "knowledge."

> What the I has to do is to posit this conflict of opposing directions or (which here amounts to the same thing) opposing forces. It must not posit either one of them alone, but must posit them both and must posit them in conflict, that is, in opposed but perfectly balanced activity. But perfectly balanced opposing activities cancel each other out and nothing remains. (GA 1:3:147–)[34]

This nothing seems to be the point in which theoretical philosophy is caught. In his writings on negative magnitude, Kant seems to have glimpsed this fundamental difficulty: "the totality of the world in itself is nothing and is something through the will of an other" (AA 2:97). Only an act of will, a free act, can posit the existence of opposites that, annulling each other, are nothing "in themselves." In *Faith and Knowledge* (*Glauben und Wissen*), Hegel not coincidentally suggests that precisely at this point it becomes clear that, in Fichte's philosophy, what eludes theoretical reason in Fichte's concept can only be justified through practical reason: "Thus both are constituted as realities, and this constitutive act is called the pure act of will. This is the act that decrees that the nothing – the nothing of +1 and −1 – to be an absolute something" (HW 2:414).[35] Thus, the relation between I and not–I cannot be conceived as a static relation, but as dynamic. This implies the idea of a quantitative difference that becomes ever smaller but can never be filled; that is, the idea of a *Streben*, a striving, toward an

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[32] English translation from Fichte. "Foundations of the Entire Science of Knowledge" 251.

[33] Ibid., 254.

[34] English translation from Fichte. "Outline of the Distinctive Character of the Wissenschaftslehre with respect to the Theoretical Faculty."

[35] English translation from Hegel, Georg Wilhelm Friedrich. *Faith and Knowledge* (Albany: State University of New York Press, 1977), 173.

infinite task. Only through such a striving can the unity of the I and not–I be thought, a unity that is not given (*gegeben*), but entrusted as a task (*aufgegeben*). This task cannot be grasped from a theoretical point of view. To use Hegel's words, "in the practical sphere this ideality is to be suspended. +1 and −1 shall not equal 0" (HW 2414).[36]

The whole reality in which the opposites are located is not based on the "being" (*Sein*) of an ontological foundation, but exclusively on the "should-being" (*Sollen*) of a task; not on a "something" that should be sought outside of consciousness and to which the latter should conform, but on a "nothing" that only has meaning thanks to the power of the autonomy of knowledge itself, which should not seek any point of support outside of itself: "Now the essence of the critical philosophy," Fichte writes, "consists in this, that an absolute self is postulated as wholly unconditioned and incapable of determination by any higher thing; … Any philosophy is, on the other hand, dogmatic when it equates or opposes anything to the I as such; and this it does in appealing to the supposedly higher concept of the thing (*ens*), which is thus quite arbitrarily set up as the absolutely highest conception" (FGA 1:2:279).[37]

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How the above relates to what has been said up to this point on the Anticipations of Perception can be better understood by taking into account that, according to Fichte, what separates the I from the not–I, the point in which the two opposite actions counter-balance each other, corresponds to *Gefühl* (feeling); for example, "the feeling of sweetness, of red, of cold, etc." (GA 1:5:243); that is to say, to what Kant calls *Empfindung* (sensation, see GA 1:5:243). Sensation – as one can read in the in "Outline of the Distinctive Character of the Wissenschaftslehre with respect to the Theoretical Faculty" (*Grundriss des Eigenthümlichen der Wissenschaftslehre in Rücksicht auf das theoretische Vermögen*) (1795) – reveals the presence of something different than the I. It is something that the I finds in itself (Empfindung [sensation] is equal to *Insichfindung* [finding-within-oneself]; see: GA 1:3:150–[38]; 251), but that is at the same time foreign to the I because "only what is foreign [Fremdartige] is ever found [gefunden]; what was originally posited in the I is always present" (GA 1:3:150–).[39] Any given sensation is qualitatively determined and is thus such by being opposed to a different sensation: "why is sweet something other than bitter? Both, in general, are determinates. But apart from this common characteristic, what is their ground of distinction?" (FGA 1:2:442).[40] For Fichte, distinctions between sensible qualities are merely subjective. *What* each sensible quality is cannot be explained in any way, but only accepted as a simple given fact:[41]

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[36] Ibid., 172.

[37] English translation from Fichte, "Foundations of the Entire Science of Knowledge," 117.

[38] Ibid.

[39] Ibid.

[40] English translation from Ibid., 278.

[41] See Mues, Albert. "Fichtes Kritik an Kants Verständnis der Physik." In *Tranzendentalphilosophie als System. Die Auseinandersetzung zwischen 1794 und 1806*. ed. Albert Mues, 68–80. (Hamburg: Meiner, 1989), 78.

> Let a thing, for example, be sweet, sour, red, yellow, or the like. Such a determination is manifestly something purely subjective … Anything sweet or sour, or red or yellow, is absolutely incapable of being described, and can only be felt, nor can it be communicated by any description to someone else … All that can be said is that the sensation of bitter, sweet, etc., is in me, and nothing more. (FGA 1:2:439)[42]

In sensation, the I only perceives *that* there is something other than itself, something that does not depend on it and thus seems to limit it from without: "the interaction between the I and some other thing outside it" (FGA I, 2, 411)[43] For Fichte, as for Kant before him, the other in respect to the I is revealed particularly in "the sense of touch [Gefühl] (*tactus*)," that is, it "evinces itself only through the sensation of a resistance, an inability [Nicht-könnens], which is subjective" (FGA 1:2:440).[44]

Therefore, "[the I] limits itself in sensation. It excludes something from itself as foreign to the I, and in doing this it posits itself within certain boundaries [Schranken], on the other side of which is supposed to lie not the I, but something opposed to the I" (GA 1:3:157).[45] In as much as it is limited, "it [the I] extends only up to a boundary" (GA 1:3:157),[46] beyond which something that is not I is located. However, as soon as the I recognizes itself as limited, it necessarily goes beyond the limit. The I would be unable to think of itself as "limited" if it did not surpass the limit, if it did not become aware that there is something that is beyond the limit in respect to which the limit is defined as such since "a boundary is nothing apart from two opposing things" (GA 1:3:157).[47] In this going beyond the limit toward that which is not I, the I recognizes the other than itself, which thus cannot be thought as the contradictory opposite of the I since in that case the latter could only be absolutely nothing for the I: "The I is to encounter in itself something heterogeneous [Fremdartiges], alien, and to be distinguished from itself … For all that, this alien element is to be encountered in the I … If it lay outside the I, it would be nothing for the I, and nothing would follow for the I from this. Hence, in a certain respect, it must also be cognate to the I; it must be capable of ascription thereto." (FGA 1:2:405).[48]

Thus, the I can recognize itself as limited only if that which is beyond the limit, that which is not I, is not absolutely other than the I. "Thought" cannot even think of itself as limited by something that simply "is not thought" since what is not thought, precisely cannot be thought and thus thought cannot even recognize it as "other than itself." If, as was discussed above, the I and not–I first appear as opposed

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[42] English translation from Fichte. "Foundations of the Entire Science of Knowledge." 274.

[43] Ibid., 246. See Lauth, Reinhard. "Kants Kritik der Vernunft und Fichtes ursprüngliche Einsicht." In *Transzendentale Entwicklungslinien von Descartes bis zu Marx und Dostojewski*. (Hamburg: Meiner, 1989), 145.

[44] Fichte. "Foundations of the Entire Science of Knowledge." 275.

[45] English translation from Fichte. "Outline of the Distinctive Character of the Wissenschaftslehre with respect to the Theoretical Faculty." 257.

[46] Ibid., 258.

[47] Ibid.

[48] English translation from Fichte. "Foundations of the Entire Science of Knowledge." 240.

like red and not-red,[49] that is, in terms of *quality*, such an opposition leads to a dead-end because the I would not even be able to understand itself as such. For the I, to affirm its essence, must be able to recognize itself as limited and, in becoming aware of its being limited, go beyond the limit and affirm itself as independent of the limit.

To look beyond the limit means recognizing that such a limit is only the vanishing of a difference in *quantity*. In the same way that going beyond a line, that is, the limit of a surface, leads to a new surface, that which is located beyond the I should be again regarded as identical to it. Here, what is involved is not a "limitation of space," but rather "we are speaking of a limitation of the intensity [Begrenzung des Intensiven], i.e. of that which distinguishes the sweet from the bitter, and so forth" (FGA 1:2:441).[50] The not–I should be distinguished from the I only by "degree," not by "essence," such that, to use the example cited above, darkness is only a minute degree of light.[51] When the I encounters the other than itself, recognizing it as such, the I projects onto it that "intensive matter [intensiver Stoff.]" (FGA 1:2:438)[52] of which the I itself is made:[53] "in striving of the I there is simultaneously posited a counter-striving of the not–I, which holds the former in equilibrium [Gleichgewicht]" (FGA 1:2:438),[54] that is, "an opposing force [which] is in fact only felt to be such" (FGA 1:2:411).[55] This force is only given in sensation or feeling while "*that which resists*," that which opposes itself to the I, "is not felt, but only inferred" (FGA 1:2:440).[56]

Kant seems to simply assert *that* qualitatively distinct sensations exist within which different degrees of intensity can be distinguished and then seems to project, in a purely *theoretical* manner, such distinctions on the reality itself that corresponds to a sensation. In contrast, Fichte attempts a sort of *practical* deduction of the concept

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[49] See Fichte, Johann Gottlieb. *Nachgelassene Schriften*. 2 vols (Berlin: Junker & Dünnhaupt, 1937), 2:107–9.

[50] Fichte. "Foundations of the Entire Science of Knowledge." 277. Translation was slightly changed by the author. "As finite surfaces in space are measured in respect to their difference, analogously, degrees of reality, in as much as they are different degrees, are measured in respect to their difference" (Lauth, Reinhard. *Die transzendentale Naturlehre Fichtes nach den Prinzipien der Wissenschaftslehre* (Hamburg: Meiner, 1984), 35). Also see Grant. *Philosophies of Nature after Schelling*, 90.

[51] George Joseph Seidel explicitly emphasizes the connection with the Anticipations of Perception: "The background is, of course, Kantian; and Fichte refers to Kant in this connection. Quantity means determination, reality or negativity posited. In his first critique, under the rubric of 'Anticipations of Perception,' Kant notes that every sensation has a certain degree of quantity, an intensive magnitude, which can always be diminished down to zero. Fichte takes essentially the same meaning for the word quantity in the context of his discussion of the relative activity of self and non-self." Seidel, George Joseph, *Activity and Ground: Fichte, Schelling, and Hegel* (Hildesheim: Olms, 1976), 54.

[52] Fichte. "Foundations of the Entire Science of Knowledge." 273.

[53] See: Lohmann, Petra. *Der Begriff des Gefühls in der Philosophie Johann Gottlieb Fichtes* (Amsterdam: Rodopi, 2004), 85.

[54] English translation from Fichte, "Foundations of the Entire Science of Knowledge," 251.

[55] Fichte. "Foundations of the Entire Science of Knowledge." 246.

[56] Ibid., 275–6.

of degree.[57] If the I must affirm itself as free, in other words, if it must continuously surpass its own limit, then what is beyond such a limit, what is negation in respect to reality, not–I in respect to the I, must be distinguished only in terms of quantity from what is on this side. Only in this manner can that which is beyond the limit, that which presents itself as a negation in respect to reality, be admitted. This beyond the limit is not an absolute non-being (that could not, as such, even be thought), but instead is a simple negative magnitude that is homogenous[58] with the I and is only distinguished from the I in virtue of a "limit which is common to both [beiden gemeinschaftlichen Grenze]" (GA 1:3:156).[59]

In "Outline of the Distinctive Character of the Wissenschaftslehre with respect to the Theoretical Faculty," Fichte writes: "Kant starts by presupposing the existence of a manifold, which may be absorbed into the unity of consciousness" (GA 1:3:145).[60] Without justifying this assumption, Kant thus reduces *a priori* synthesis to a mere "collective universal" (GA 1:3:145),[61] a union of elements that are assumed to be absolutely separate: "It must prove that a manifold is given for possible experience. This proof will go as follows: Whatever is must be something, but it is something only insofar as there is something else – which is also something, though a different something" (GA 1:3:145).[62] Multiplicity is the first condition for the determinateness of the content, the condition through which a certain "reality" is distinguished from something other that it is not, from its "negation." However, as I have shown, in order to be able to recognize the "other" as such, it should once again be posited as the same. That is, it should be considered the "quantitative" limitation of the single "reality" of the absolute I. "Knowledge," as Fichte states in the "New Exposition of the Science of Knowledge" (*Darstellung der Wissenschaftslehre*)

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[57] See Mues. "Fichtes Kritik an Kants Verständnis der Physik." 69 and 72–76. Comparing Kant and Fichte, Lauth writes: "In reality, Kant arrives at the *theoretical* anticipability of perception as an intensive magnitude only by presupposing the diversity given through sensation. We would not know that every sensation possesses a varying intensity if we were not empirically given different and intensively diverse sensations" (Lauth, Reinhard. "Kants Lehre von den 'Grundsätzen des reinen Verstandes' und Fichtes grundsätzliche Kritik derselben." In *Transzendentale Entwicklungslinien von Descartes bis zu Marx und Dostojewski*, 111–24. (Meiner: Hamburg, 1982), 114, emphasis mine). In contrast, for Fichte, "intensity does not originate, as Kant believes, from categorical quality, but from a constitutive *practice*, from a projection of an intensive force into the object" (Lauth. *Die transzendentale Naturlehre Fichtes nach den Prinzipien der Wissenschaftslehre*, 52; emphasis mine).

[58] See Philonenko. *La liberté humaine dans la Philosophie de Fichte*, 280.

[59] Fichte. "Outline of the Distinctive Character of the Wissenschaftslehre with respect to the Theoretical Faculty." 257. Translation slightly altered by the author. Not coincidentally, Philonenko wonders whether the principle of the Anticipations of Perception is the point in which, for Kant as well, the transition from intuition to intellectual intuition occurs, the point in which the object itself is seen as continuously emerging from negation; see Philonenko. *La liberté humaine dans la Philosophie de Fichte*, 287.

[60] English translation from Fichte. "Outline of the Distinctive Character of the Wissenschaftslehre with respect to the Theoretical Faculty." 245.

[61] Ibid.

[62] Ibid.

from 1801–2, "is not based in any way on uniting, nor on disintegrating, but rather is based completely on the melting of these two moments … since there is no unity there other than of that which is separate and there is nothing separate except that which is united … for this reason, your knowledge fluctuates in between the two and it is canceled if it does not oscillate between the two" (GA 2:6:152).

For Kant, the "systematic unity of experience" and the corresponding "uniformity of nature" remain a mere regulative ideal. For Fichte, in contrast, this unity assumes a constitutive[63] meaning. Thus, even though this unity clearly cannot be founded on any "logical necessity," it can nevertheless be based on a "practical necessity"; not on any metaphysical foundation, but on a free act. The illusion that something can be given from outside of the "I" as if a "thing" were involved that as such would not be the I is erased by the idea of an opposition between I and not–I that from the beginning should be thought within the unity of the absolute I. In respect to this unity, the I and the not–I can be distinguished only in terms of more of less since, from the beginning, they are only "subdivisions" of the same I. "Knowledge" does not depart from a simple logical negation, from absolute not-knowledge. What knowledge finds in front of itself as unknown is not simply "other" than knowledge itself because otherwise it could not even be recognized as "unknown." If the subject and object, considered from the point of view of "absolute being," cannot be conceived as separate and irreducible "entities," they can be reconciled by being defined from the point of view of knowledge and on the basis of the latter.

3.2 Schelling and the Philosophy of Nature

The preceding reconstruction of Fichte's philosophy, though inevitably superficial and partial, should nevertheless suffice to highlight the multiple consequences that seem to be entailed by the substitution of the qualitative opposition between reality and negation with quantitative opposition, of the opposition between contradictory concepts with the conflict between opposed magnitudes; a series of consequences that Kant's text only vaguely suggests. The precise connection between the themes raised in the above analysis of Fichte's thought and the Anticipations of Perception remains unclear and it is left to the interpreter to gather the brief hints that emerge in Fichte's text.

The guiding hypothesis of the present work can be more clearly defined, however, by following the development of Fichte's formulation of the problem in the philosophy of his time. As is well known, Schelling's early publications attempt to provide an "objective" meaning, in a manner of speaking, to the relation between I and not–I that Fichte conceives on the model of opposed forces. In so doing, Schelling explicitly connects the relation to Kant's conception of matter: "Matter," writes Schelling

[63] See Mues. "Fichtes Kritik an Kants Verständnis der Physik." 77.

in an article in *Allgemeine Übersicht der neuesten Philosophischen Literatur* (General Outline of the Newest Philosophical Literature), "is nothing other than the spirit intuited in the equilibrium of its activity" (HKA 1:4:108).[64] By following the development of this conception within Schelling's philosophy of nature in as much detail as possible, one can confirm the historical and systematic relation of the concept of degree and intensive magnitude with the idea of the opposition between positive and negative magnitudes; that is, with the two distinctive characteristics of *realitas phaenomenon* for Kant.

In his 1797 essay, "Ideas for a Philosophy of Nature" (*Ideen zu einer Philosophie der Natur*), Schelling seems to incorporate every detail of Kant's formulation of the principle of the Anticipations of Perception: "[R]eality [Realität] is only *felt* [gefühlt], is only present in sensation. Yet what is felt [empfunden] is called quality. Thus, only in that it starts from the generality of the *concept* does the object first acquire *quality*, and cease to be mere *quantity*." (HKA I:5:249).[65] Only through sensation is the transition from the perfect uniformity of "form" to the qualitative differentiation of its "content" possible: "Only now does the mind [Gemüth] relate [bezieht] the real [Reale] in sensation (as the contingent) to an object as such (as the necessary), and vice versa" (HKA 1:5:249).[66] Thus, the particular property of an object is provided through the specific character of the sensation. Sensation is what permits the transition from that which characterizes objects in general to that which distinguishes a particular object.

The contingency that Schelling attributes to the "real of sensation" is based on the fact that (as in Kant), while things in general necessarily have a quality, establishing *a priori* that they have a determinate quality is nevertheless impossible: "What you sense of matter you call quality, and only insofar as it has a determinate quality is it said to be real for you. That it has quality at all is necessary, but that it has this determinate quality appears to you as contingent" (HKA 1:5:81).[67] Once again adopting Kant's formulation, Schelling highlights the fact that any pretence of conceptually securing the unity of sensible qualities, that is, every attempt to refer them to the sphere of the understanding, merely results in the negation of the quality itself that one aims to determine:

> But what the original real [Reale] in the object is, what corresponds to the passivity in myself, is a contingent (accident) in regard to this sphere. So we attempt in vain to derive it *a priori*, or to reduce it to concepts. For the real itself exists only insofar as I am affected. Yet for me there is absolutely no concept of an object, but only a consciousness of the state

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[64] See Bonsiepen, Wolfgang. *Die Begründung einer Naturphilosophie bei Kant, Schelling, Fries und Hegel: mathematische versus spekulative Naturphilosophie* (Frankfurt: Klostermann, 1997), 211. On the relation between "forces" in matter and the "activity" of spirit, see Rudolphi, Michael. *Produktion und Konstruktion: zur Genese der Naturphilosophie in Schellings Frühwerk* (Stuttgart: Frommann-Holzboog, 2001), 98–108.

[65] English translation from Schelling, Friedrich Wilhelm Joseph von. *Ideas for a Philosophy of Nature as Introduction to the Study of this Science, 1797*, tr. Errol E. Harris and Peter Lauchlan Heath (Cambridge, UK; New York: Cambridge University Press, 1988), 215.

[66] Ibid.

[67] Ibid.

> of passivity that I am in [leidende Zustand] … But to transform what is actually sensed into concepts is to rob it of its reality. For it has reality only at the moment of its effect upon myself. (HKA 1:5:249–)[68]

To the aposteriority of sensible qualities regarding what each can present as specifically different in regard to the other, Schelling seems to oppose the apriority of degree. If determining "what" a certain quality is in itself is impossible, it is nevertheless possible to establish that they present distinctions in terms of "more or less": "And that is how it is. We feel merely the more or less of elasticity, heat, brightness and so on, not elasticity, heat, etc., themselves" (HKA 1:5:249).[69] From this first subjective evaluation, one can pass to the "objective" one; to that which Kant indicates as the *a priori* property of the real that is the object of sensation, the *a priori* property of any sensible quality: "the real in sensation must be able to increase, or diminish, indefinitely; it must, that is, have a specific degree, though one that can equally well be thought of as infinitely greater, or as infinitely smaller; or, to put it otherwise, between which and the negation of all degree (= 0) an infinite sequence of intermediate grades can be imagined" (HKA 1:5:249).[70]

The relation between the subjective evaluation of the more or less of sensible qualities and the objective affirmation of their degree occurs – exactly as in Kant once again – through the mediation of the concept of "force": "force as such, not specific force. Force is simply that which *affects* us [was uns afficiert]. What affects us we call real [real], and what is real exists only in sensation; force is therefore that which alone corresponds to our concept of quality. But every quality, insofar as it is to affect us, must have a degree, and that a specific degree" (HKA 1:5:250).[71] On the one hand, sensation is only the subjective evaluation of the intensity of a force such that "force as such makes itself known only to your feeling [sense of touch] [Gefühl]. Yet feeling [Gefühl] alone gives you no objective concepts" (HKA 1:5:79).[72] On the other hand, the degree of force should be admitted as the very condition of the type of modification that is called sensation: "Thus force as such can affect us only insofar as it has a particular degree." (HKA 1:5:250).[73] The subjectivity of the more or less in sensible perceptions thus finds its objectification in the fact that the multiplicity of degrees of matter, the more and less with which matter is presented in sensation, is the consequence of the very constitution of matter, which occurs through a "synthesis (of opposite forces)" (HKA 1:5:226):[74] "matter is already allowed to arise only through the interaction of forces, so that (in accordance with the natural law of continuity) between every possible degree of these forces, down to the total disappearance

[68] Ibid.

[69] Ibid.

[70] Ibid.

[71] Ibid.

[72] Ibid., 18.

[73] Ibid., 216.

[74] Ibid., 189.

of all intensity (= 0), it is possible to have an infinite number of intermediate degrees" (HKA I:5:226).[75]

All variety of matter is merely the consequence of the reciprocal delimitation of opposed forces: "*[a]ll quality of matter rests wholly and solely on the intensity of its basic forces*" (HKA 1:5:251).[76] Thus, for Schelling, the apparently irreducible difference between qualities is purely contingent and, as solely due to the qualitative difference between different sensations, purely subjective. The difference between what appear to be distinct qualities is actually only a difference in quantity: "If all difference of matter rests merely on the differing relationships of its basic forces, we shall have as many different matters as we are acquainted with qualities. But *quality* is valid as such only in respect of sensation. So differing sensations also entitle us to assume different qualities, and thus different matters" (HKA 1:5:273).[77]

Positing a different "substance" (the caloric [*Wärmestoff*], luminous matter [*Lichtstoff*], resinous and vitreous electricity, two magnetic fluids, etc.) that corresponds objectively to every quality is not in the least necessary. All these elements, which are intended to explain the corresponding phenomena, are nothing more than the result of hypothesizing certain sensible qualities in substantially independent entities. Addressing the scientists of his time, Schelling writes, "you heap elementary stuff on elementary stuff, but these are nothing else than just so many refuges of your ignorance" (HKA 1:5:83).[78] If, instead, all differences are conceived as produced through the relation between fundamental forces, "[w]ith this there is an end to all those absolute *qualitative* differences of matter which a false physics fixes and makes permanent in the so-called basic substance [Grundstoffe]: All matter is intrinsically one, by nature pure identity; all difference comes solely from the form and is therefore merely ideal and *quantitative*" (SW 2:175; emphasis mine).[79]

One must certainly admit that the "endeavour [das Bestreben] of ordinary chemistry, to reduce substances [Stoffe] as much as possible to *basic substances* [Grundstoffe], already betrays that (in idea at least) it has a principle of *unity* in view, which it constantly seeks to approach, so far as it may. But if there is such a principle, it provides no reason for halting anywhere in the effort to unify our knowledge" (HKA 1:5:265–).[80] There is no reason to fix a certain number of given fundamental materials. Doing so does not in any way solve the problem, but rather makes it even more obscure and incomprehensible: "So the regulative principle of a scientifically progressive chemistry will always be the idea of regarding all qualities as merely different modifications and relationships of the basic forces" (HKA 1:5:265–).[81]

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[75] Ibid.

[76] Ibid.

[77] Ibid., 239.

[78] Ibid., 21.

[79] Ibid., 137. This observation is added in the second edition of the *Ideen* which has not yet appeared in the Academia edition.

[80] Ibid., 232.

[81] Ibid., 233.

The different qualities of bodies, as has been shown, "can only be called qualities, after all, with respect to your sensation" and "to transfer something that is merely valid for your sensation to the objects themselves" is unjustified (HKA 1:5:266).[82] If, instead, one accepts that qualities as such have an objective value, "one may unhesitatingly postulate as many different qualities of matter and hence as many basic substances as are needed for purposes of empirical scientific research." However, to accumulate different fundamental materials "all differing from one another by special qualities" means to erect "so many barriers to further research" (HKA 1:5:266).[83] Only by maintaining "the idea which, in regulative fashion, must underlie all inquiries as to the different qualities of matter, are we obliged to assume that the whole distinction among these basic substances will rest merely on differences of degree" (HKA 1:5:268).[84]

The problem of reducing every qualitative difference to a quantitative difference seems to be nothing other than the expression of the fundamental postulate of the unity of nature, of the need not to remain fixed fixed to a distinction between different areas of the real, juxtaposing them to each other, but rather to think of them as part of a single possible experience. The recognition of the purely relative opposition between reality (a certain difference in degree) and negation (the difference = 0), the dynamic conception of matter, and the considerations in the "Appendix to the transcendental dialectic" and in the two "Introductions" to the *Critique of the Power of Judgment* on the "Principle of the formal finality of Nature" thus seem to stem from a single fundamental demand: to "anticipate" *a priori* the same content that is "given" *a posteriori*, thus guaranteeing what Kant has called the "lawfulness of the contingent" (20:217).[85]

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Certainly Schelling, like Kant himself (see AA 4:530), does not believe it is possible to deduce *a priori* the multiple qualities of matter. If one can affirm that every difference between qualities in general is reducible to a relation between fundamental forces, the task of determining this particular relation of magnitude only concerns experience. Philosophy cannot explain the specific quality of our sensation. Instead, it claims that this particularity should be reduced to a quantitative relation between fundamental forces.

This demand to reduce differences in quality to mere differences in degree is central to the post-Kantian debate. In the *Versuch die Gesetze magnetischer Erscheinungen aus Säzen der Naturmethaphysik a priori zu entiwickeln* (1798) [Attempt to Develop *a priori* the Laws of Magnetic Phenomenon from the principles of the Metaphysics of Nature] already cited, Carl August Eschenmayer explicitly refers to the dynamic conception of matter that Kant develops: "the dynamic that teaches us [belehrt uns] that the being of matter can be conceived only as the conflict of originally opposed forces. These

[82] Ibid.

[83] Ibid.

[84] Ibid.

[85] Kant. *Critique of the Power of Judgment*, 20.

forces are the force of attraction and of repulsion."[86] The force of attraction tends toward the infinitely small, the force of repulsion toward the infinitely large. A determinate finite product is constituted only in the equilibrium of these two opposite tendencies:[87] "qualities are degrees and a degree of matter is a relation of magnitude in which forces of attraction and repulsion exist."[88] According to Eschenmayer, one can use the model of a numerical series to establish a "series of degrees [Gradreihe]" in which the qualitative differences of materials are distributed, changing with the variations in degrees of density of matter.

$$\overset{+\infty}{A\cdot B},\ldots,\ \overset{+n}{A\cdot B},\ldots,\ \overset{+2}{A\cdot B},\ldots,\ \overset{+1}{A\cdot B},\ \overset{0}{M}\ \overset{-1}{A\cdot B},\ \overset{-2}{A\cdot B},\ldots,\ \overset{-n}{A\cdot B},\ldots,\ \overset{-\infty}{A\cdot B}$$

The prevalence of the repulsive force is represented by the positive part of the series, which gradually passes into the negative part in which the attractive force prevails. On the basis of the relations that exist between positive and negative magnitudes, a point of indifference must necessarily be crossed: "Since the negative gradation diminishes as much as the positive one augments, there must be a point in which both are equal; and in this point, since it is here that the degree must be removed, no quality for our intuition can be represented."[89] If one indicates attractive force with the letter A and repulsive force with the letter B, "$A = \frac{1}{\infty}$ and $B = \infty$. Thus, as $\frac{1}{\infty} \cdot \infty = 1$, $A \cdot B$ gives something finite as well."[90] While exponents proceed through an arithmetic progression, bases follow a geometric progression. While the product $A \cdot B$ is always = 1, the transition from the positive series of the exponents to the negative one occurs through the point of power = 0, in which the difference itself between qualities should disappear, reduced to a mere difference in degree.[91] Through this schema, Eschenmayer believed he could deduce all the different qualities of matter from differences in quantity distributed on the line of gradation, passing from the most dense (e.g., metals) to the least dense (e.g., gases). All differences in material contents can be reduced to differences of + and −, which are constituted through the opposed tendency toward the infinitely large (repulsive force) and the infinitely small (attractive force).

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[86] Eschenmayer. *Säze aus der Natur-Metaphysik auf chemische und medicinische Gegenstände angewandt*, 12.

[87] Marks, Ralph. *Differenz der Konzeption einer dynamischen Naturphilosophie bei Schelling und Eschenmayer* (Munich: Dissertation, 1984), 15.

[88] Eschenmayer. *Säze aus der Natur-Metaphysik auf chemische und medicinische Gegenstände angewandt*, 5.

[89] Ibid., 40.

[90] Eschenmayer, Carl August. *Principia quaedam disciplinae naturali, inprimis Chemiae, ex Metaphysica naturae substernenda* (Tübingen: Heerbrandt, 1796), 8.

[91] See Marks. *Differenz der Konzeption einer dynamischen Naturphilosophie bei Schelling und Eschenmayer*, 18-. Eschenmayer's conceptions certainly leave much to be desired in terms of mathematical precision and their meaning should not be judged from such a point of view. Obviously, this was one of the motives for discrediting *Naturphilosophie* in the second half of the 1800s, within the scientific community as much as in philosophical debates.

Schelling, at least in this early work, adopts Eschenmayer's model in substance, citing among other things a long piece from his *Principia quaedam disciplinae naturali, in primis Chemiae, ex Metaphysica naturae substernenda*. Though Schelling later accuses Eschenmayer of being too simplistic, at this point, Schelling agrees with him at least on the fundamental postulate of the dynamic conception of matter. In Eschenmayer's words, "qualitas materiae sequitur ratione mutuam virium attrattivorum repulsivorum [the quality of matter derived from the mutual relation of attractive and repulsive forces]":[92] Matter is nothing but the relation between forces and each of its particularities can only be founded in the particularity of this relation in which the two infinite forces, limiting each other, determine a finite difference in degree.

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"All quality in bodies" writes Schelling, successfully summarizing the connection between all the questions raised above, "rests on the quantitative (gradual) relationship of their basic forces. For quality exists only in relation to sensation. But only what has a degree can be sensed: Now in matter no degree is conceivable save that of the forces, and even of these only in their relation to one another. So all quality rests on forces insofar as they have a specific quantity (degree), and, since matter presupposes for its possibility opposing forces, on the relationship of these forces according to their degree" (HKA 1:5:287).[93] All the necessary elements for a correct understanding of the Anticipations of Perception (the givenness of the sensible, the relation between the concept of degree and the real opposition of forces, the connection between force and sensation) are here placed side by side, effectively showing their reciprocal relations.

Above all, Schelling deserves credit for elucidating that, already in Kant's text, a difference in "degree" expresses a difference in "level," which is defined by the tendency to return to indifference, that is, to the state in which such a difference vanishes and in which the equilibrium that had been destroyed is restored.

Beginning in the *Ideen*, this formulation of the problem becomes a characteristic trait, if not the very supporting structure, of Schelling's philosophy of nature: "Those substances are said to be homogeneous," writes Schelling, referring to chemical phenomena, "in which the quantitative relationship of the basic forces is the same" (HKA 1:5:287).[94] Homogeneity indicates the presence of the same quality. If qualitative differences can be referred back to quantitative relations between fundamental forces, the identity of the quality is reduced to the uniform distribution of such relations. In contrast, "[t]wo substances are said to be heterogeneous if the quantitative relationship of the basic forces in one is the inverse of that relationship in the other" (HKA 1:5:287).[95] Heterogeneous and homogeneous matter are only distinguished from each other through the fact that, in the first case,

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[92] Eschenmayer. *Principia quaedam disciplinae naturali, inprimis Chemiae, ex Metaphysica naturae substernenda*, 12.

[93] English translation from Schelling. *Ideas for a Philosophy of Nature as Introduction to the Study of this Science, 1797*, 252.

[94] Ibid.

[95] Ibid., 253.

the "quantitative relationship of the basic forces is the same" (HKA 1:5:288),[96] while in the second the fundamental forces' quantitative relation is the same.

Studying chemical phenomena shows that "[o]nly if the quantitative relationship of basic forces in one body is the inverse of the same relationship in the other, is a chemical process possible between two bodies" (HKA 1:5:288),[97] in particular, "only if mass and elasticity in one body have an inverse relation to mass and elasticity in the other, does a chemical process take place" (HKA 1:5:288).[98] When two qualitatively distinct materials (e.g., a solid and a liquid body) are put in reciprocal contact, a difference in the level of concentration of the two substances is manifested; in Schelling's terms, a difference in the relation between mass and elasticity. The tension that this difference produces determines the chemical process, which is none other than the tendency to annul such a difference and to return to a state of equilibrium: "Every chemical motion is an endeavour towards equilibrium" (HKA 1:5:289),[99] and "a chemical process is nothing else but a restoration of the disturbed equilibrium" (HKA 1:5:289).[100]

Therefore, in the chemical process, "the basic forces reciprocally confine one another until an identity of degree is present. The product of an elastically fluid and a solid body, for example, may be expressed by the median relationship between the mass of the solid and the elasticity of the fluid, and vice versa" (HKA 1:5:290).[101] This average relation always remains constant throughout the process considering that the second matter acquires as much density as the first matter loses. In this way, starting from the initial difference, two forces in opposite directions are created, a tendency of the degree of concentration to augment and a tendency for it to diminish: "Every chemical motion is merely a change of degree-relationships. It consists in mere changes of degree, where one body loses by degrees what the other gains, and vice versa" (HKA 1:5:296).[102] The increasing of the density of one of the reactants is thus maintained in a "relative equilibrium" in respect to the diminishing density of the other. "Relative equilibrium" becomes "absolute equilibrium"[103] in which "the density of the fluids in the mixture is equal to the median relationship between the densities of both before the mixture" (HKA 1:5:296)[104] at the moment that the difference in degree vanishes.

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[96] Ibid., 252.

[97] Ibid.

[98] Ibid., 255.

[99] Ibid.

[100] Ibid.

[101] Ibid.

[102] Ibid., 265.

[103] For the difference between relative and absolute equilibrium, see Eschenmayer, Carl August. *Versuch die Geseze magnetischer Erscheinungen aus Säzen der Naturmetaphysik mithin a priori zu entwickeln* (Tübingen: Heerbrandt, 1798), 74 and 80.

[104] English translation from Schelling. *Ideas for a Philosophy of Nature as Introduction to the Study of this Science, 1797*, 260.

As I have shown, the concept of degree allows one to distinguish, in the uniformity of space and time, differences in level by reducing the differences between two opposed qualities (of which one would be a "reality" and the other the negation of that "reality") to a mere relative difference in intensity. The latter difference (reality) is defined as such in respect to a state of equilibrium in which the difference is = 0 (negation). Different qualities "are" only in the relative difference in degree. They do not have an absolute significance, but rather appear as such only to sensation, which is nothing but the merely subjective manifestation of a conflict that unfolds in the phenomenal world. Only the relation between the difference and the tendency to return to equilibrium reveals that "elective affinity" (*Wahlverwandtschaft*)[105] between opposed elements, to which, in chemistry, the concept of quality should be reduced.

Schelling, through the example of chemical phenomena, seems effectively to incorporate and develop in all its consequences the essential nucleus of the Anticipations of Perception: "Chemical motion, as such, can therefore be constructed only as intensive magnitude, according to laws of continuity" (HKA 1:5:303),[106] that is, according to "a continuous approximation of degree, from both sides, towards the common product." The very concept of intensive magnitude or degree, though perceived in the moment as something simple and unitary, only has sense in respect to a difference that separates it from this state of equilibrium = 0 and has no significance outside of this difference: "everything that corresponds to sensation," writes Schelling, almost citing the *Anticipations of Perception* word for word, "is apprehended only as a unity; the whole does not arise through composition of the parts, but on the contrary, parts, or better, multiplicity, can be conceived in it only by approximation to zero" (HKA 1:5:305).[107]

However, even regarding the particular scientific questions concerning chemistry,, Schelling merely adopts and develops what Kant himself observes concerning the intussusception (chemical penetration opposed to mechanical penetration) in the "Metaphysical Foundations of the Natural Sciences": when two materials of different density, a solvent and a solute, enter into contact, "the matters together occupy a space, which accords with the sum of their densities, not outside, but inside one an other, that is, through intussusception (as it is customarily called)" (AA 4:531).[108] The neutral state (complete chemical penetration) is conceived as the return to a state of equilibrium that is produced from the conflict between the force of the

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[105] To highlight the importance of the problem of the *Wahlverwandtschaften,* on which I cannot linger any longer here, I will merely refer to Johann Wolfgang Goethe's novel with the same title, which should be placed in context precisely with the chemical doctrine of his time to be fully understood. The connection between the characters Ottilie and Charlotte, the captain and Eduard, is based on the idea that kindred natures are those that possess qualities that are not simply different, but opposed, such as those seen in magnetism, in positive and negative electricity, and in chemical reactions. See, for example, Adler, Jeremy. *'Eine fast magische Anziehungskraft'. Goethes Wahlverwandtschaften und die Chemie seiner Zeit* (Munich: Beck, 1987).

[106] English translation from Schelling. *Ideas for a Philosophy of Nature as Introduction to the Study of this Science, 1797*, 265.

[107] Ibid., 267.

[108] English translation from Kant. "Metaphysical Foundations of Natural Science." 240.

solvent (attractive) and that of the solute (repulsive), which resists the first: "A dissolution of specifically different matters by one another, in which no part of the one is found that would not be united with a specifically different part of the other, in the same proportion as the whole, is *absolute dissolution*, which can also be called chemical *penetration*" (AA 4:530).[109]

Even if Kant is skeptical about the scientific nature of chemistry, the dynamic conception of matter as a conflict between forces had an extraordinary influence on German chemistry of his time.[110] According to Schelling, "Kant (in the work cited) has nowhere expressly declared himself concerning his conception of chemistry, but this utterance (as to the necessity of assuming a chemical penetration) obviously presupposes the notion that chemical operations are possible only through dynamic forces, insofar as they are thought of in motion." (HKA 1:5:223).[111]

Alexander Nicolaus Sherer's 1796 work, *Nachträge zu den Grundzügen der neuern chemischen Theorie* [Supplements to the Main Features of the New Theories of Chemistry],[112] is a good example of how the Kantian conception resonated even among those occupied with empirical research. All the themes that characterize Kant's dynamism, as I have shown, return in Scherer's work: the impossibility of knowing the essence of matter and the corresponding idea that the object of research is always only constituted through its exterior effects, through the modification of our senses which present different sensible qualities (color, smell, heat, etc.).[113] All attempts to make a different imponderable substance correspond to each quality are shown to be empty explanations that merely defer the explanation of an immediate "fact" of sensation to a "substrate" beyond sensation. In so doing, however, the properties are merely reproduced without furnishing an explanation, delegating them instead to something that is inexplicable in turn.[114] The qualitative diversity of matter should only be considered an accidental effect of the "reciprocal influence of fundamental forces and of their respective intensities."[115]

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In *Von der Weltseele: eine Hypothese der höhern Physik zur Erklärung des allgemeinen Organismus* [On the World Soul: A Hypothesis of Higher Physics to

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[109] Ibid., 239.

[110] On this topic, see Carrier, Martin. "Kants Theorie der Materie und ihre Wirkung auf die zeitgenössische Chemie," *Kant Studien* 81 (1990): 170–210. See also Carrier, Martin. "Kant's Theory of Matter and his Views on Chemistry." In *Kant and the Sciences*. ed. Eric Watkins. (Oxford: Oxford University Press, 2001).

[111] English translation from Schelling. *Ideas for a Philosophy of Nature as Introduction to the Study of this Science, 1797*, 257.

[112] Scherer, Alexander Nicolaus. *Nachträge zu den Grundzügen der neuern chemischen Theorie. Nebst einigen Nachrichten von Lavoisier's Leben und einer tabellarischen Uebersicht der neuern chemischen Zeichen* (Jena: Göpferdt, 1796), 166.

[113] Ibid., 84.

[114] Ibid., 23–.

[115] See Ibid., 164.

explain the General Organism], Schelling effectively summarizes the perspective that emerges from this point of view: "since nature is a general tendency toward equilibrium, every activated cause in action necessarily incites, according to a general law, an *opposite* cause with which it is in equilibrium" (HKA 1:6:125). The *Weltseele* clarifies the relation between the concept of degree and the difference in level that this implies with the idea of opposition between positive and negative magnitudes. While the previous chapter aimed to connect these problems by comparing Kant's different definitions of *realitas phaenomenon* (in particular that of the Anticipations of Perception and that of the Amphiboly of Concepts of Reflection), Schelling seems to explicitly establish these connections, making them the structural element of his philosophy of nature:

> We affirm that matter itself is only a product of opposed forces. When these reach an equilibrium in matter, each movement is either positive (repulsion) or negative (attraction); but when this equilibrium is disturbed [gestört], the movement is both positive and negative and a reciprocal action of the two original forces occurs. Such a disturbance [Störung] of the original equilibrium occurs in chemical reactions, and, for this reason, every chemical process is a coming into being of new matter. Thus, that which philosophy maintains *a priori*, that matter is the product of opposed forces, is intuitive in every chemical process. (HKA 1:6:187-)

Consequently, "one should assume that every chemical process is dominated by a dualism of opposed forces which incite each other [wechselseitig-erregter]" (HKA 1:6:126).

The same fundamental schema exemplified by chemical phenomena can also be observed in thermal phenomena. According to Schelling, the research that Adair Crawford (1749–1795) summarizes in *Experiments and Observations on Animal Heat and the Inflammation of Combustible Bodies*[116] demonstrates that "heat is an altogether relative concept, that different bodies are quite differently warmed by equal amounts of heat. For this varying constitution of bodies, Crawford has invented the term capacity [Kapacität], which was very well chosen, since it designated the phenomenon completely – but also not more than that." (HKA 1:5:259).[117] It is well known that an equal quantity of heat produces different changes in temperature in bodies that have different thermal capacities: the lesser the thermal capacity, the easier it is to arrive at a greater concentration of heat and vice versa: "There is thus no *absolute* heat, and heat in general is merely the phenomenon of a *state* in which the body happens to be" (HKA 1:5:259).[118] "To postulate a heat-matter as the cause of heat," Schelling continues, "is not to explain the situation, but to pay oneself with words." (HKA 1:5:259).[119] Everything that is observable is a relative difference in temperature indicating the inverse relation of quantity of heat to thermal capacity:

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[116] Crawford, Adair. *Experiments and Observations on Animal Heat, and the Inflammation of Combustible Bodies; Being an Attempt to Resolve these Phenomena into a General Law of Nature*. 2nd ed (London,: Printed for J. Johnson, 1788).

[117] English translation from Schelling. *Ideas for a Philosophy of Nature as Introduction to the Study of this Science, 1797*, 252.

[118] Ibid., 227.

[119] Ibid., 228.

"But a heat-stuff does not exist, for heat is a quality that can pertain to all matter, is contingent and relative, and has to do only with the state of body: by its presence or absence the body neither gains nor loses a single absolute quality." (HKA 1:5:268).[120] The same quantity of heat provokes a greater increase in temperature in a body with less thermal capacity and "different bodies are heated in an equal fashion through different quantities of heat: this equilibrium is called equilibrium of temperature." Therefore, "the degree by which each body is heated" should be considered "separate[ly] from the heat-stuff [Wärmematerie] that was necessary to impart [erteilen] this temperature to it" (HKA 1:6:101–). The problem of measuring the "absolute" quantity of heat is simply shown to be a wrongly formulated problem. Only relative differences in temperature can be determined.

Relying on this understanding of the problem, Schelling seeks to explain the process of combustion as well. Bodies with less thermal capacity are more inflammable for having less capacity to "store" heat. In the words of Crawford's theory, they possess a greater quantity of phlogiston. If an oxidizing agent, the combustive agent (oxygen), acts on a combustible in the process of combustion, "this oxygen clearly *does not exist at all in itself* and is thus not presentable in intuition. It *exists as such only in the moment of reciprocal relation* between it and the negative cause of the combustible body" (HKA 1:6:125; emphasis mine). This negative principle is the phlogiston, which equally "cannot be presupposed as a constitutive part of bodies (as the defenders of phlogiston had done), because it does not exist at all in itself. It only exists in opposition with oxygen" and "represents nothing other than a reciprocal concept [Wechselbegiff]" (HKA 1:6:99). Phlogiston and oxygen are not two separate "substances." Each one of them "does not exist as such, but only in the moment of conflict" (HKA 1:6:99), precisely as occurs in the distinction between positive and negative magnitudes which are such that they remove each other in the point = 0: "In contrast with phlogiston … oxygen acquires a positive quality. Thus, phlogiston is nothing more and nothing less than the negative of oxygen. It is therefore clear that phlogiston conceived in an absolute manner and in itself is nothing" (HKA 1:6:99). Both of these principles "are only positive and negative reciprocally, in reference to one another, that is, they enter into this relation (real opposition) only in the moment of the phlogistic process" (HKA 1:6:103).

Positing the existence of absolutely different qualities opposed to one another is not in the least necessary. Rather, "all heterogeneity of matter is lost in the idea of the original homogeneity of every positive beginning in the world. The fundamental opposition itself … disappears in this idea. No basic natural phenomenon can be explained without this opposition" (HKA 1:6:103). At the same time, however, "this conflict is only in the moment of the phenomenon itself. Every natural force of nature calls forth [weckt] its opposite. The latter *does not exist in itself*, *but only in this conflict*, which momentarily gives it a distinct existence. As soon as the conflict ends, it disappears, returning to the general sphere of identity" (HKA 1:6:103; emphasis mine). Qualities are constituted as such only in the "relative equilibrium" between

[120] Ibid., 234.

opposed tendencies, which aim to eliminate a difference in degree. A different "substance" does not correspond to each quality as its substrate because substance is reduced in Kantian fashion to a mere "relation": "material contents are thus reduced to different degrees and are not distinct in terms of obscure and absolute properties, but only in terms of relations between degrees" (HKA 1:6:102–).

Electrical phenomena possibly provide the best example of this conception. Two conductors, such as two spheres of differing magnitude, can have the same quantity of electrical charge while having a differing respective potential; larger for the smaller sphere, and smaller for the larger sphere: "In this way, a law for the relation between both electricities can be established *a priori* (without investigating their specific nature more closely)" (HKA 1:6:127). This law is based on the inverse relation of the quantity of charge to electrical capacity such that Eschenmayer's[121] formula, "2EM=2ME expresses the equilibrium of both electricities" (HKA 1:6:127). Qualitative distinctions between two types of positive and negative electricity, thought as two distinct fluids, are thus replaced with a merely relative distinction between opposite signs according to the model of real opposition: "That two opposed magnitudes can be reciprocally related to each other as negative and positive follows immediately from the concept of real opposition. The signs ± do not express a determinate (specific) quality [Beschaffenheit] of the two electricities, but only the relation of opposition in which they find themselves" (HKA 1:6:127).

The bottle of Leida (the prototype of a condenser), invented by the celebrated Dutch physicist Pieter van Musschenbroeck, offers insight into this characteristic of electrical phenomena. When the bottle is held so that the external frame (normally a sheet of tin attached to the outside of the bottle) is "grounded," and when the internal frame (an analogous metallic frame connected to a small metallic bar ending in a bead) is charged with electricity of a determinate sign, the external frame is charged with the contrary sign such that the two opposed electricities create a reciprocal equilibrium:

> many phenomena [Erscheinungen], in particular the phenomena [Phänomene] of the bottle of Leida, demonstrate that electrical phenomena involve movement in opposite directions. Thus, $+E$ and $-E$ are grounds that are really and positively opposed [reell- und positiv-entgegengesetze] … that the two electricities must have something in common follows *a priori* from this concept because only magnitudes of the same species can be really opposed [reell-entegegengesetz] (HKA 1:6:129)

As is the case for every real opposition in general, the distinction between the two electricities is only based on a relative difference in quantity (of potential). Speaking of electricity as different in itself is meaningless because it is only "when electrical stimulation causes two heterogeneous bodies to approach each other that the positive and negative electricity is distributed between both" (HKA 1:6:167).

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[121] In a chemical reaction between distinct materials, according to Eschenmayer, "elasticity and density are in reciprocal equilibrium such that a material of single density and double elasticity is in equilibrium with one of double density and single elasticity, or rather $D \cdot 2E = 2D \cdot E$" (Eschenmayer. *Versuch die Geseze magnetischer Erscheinungen aus Säzen der Naturmetaphysik mithin a priori zu entwickeln*, 59). Eschenmayer compares this equilibrium to a lever in which the length of the arms or the velocities are in an opposed relation to the masses; an indicative example as I will show.

There is no reason to admit the existence of two opposed electric fluids that are distinguished by the *property* of being "in themselves" positive or negative because the distinction between positive and negative electricity only arises when a certain relative *quantitative* difference is created and only has sense in reference to the latter. Only a naïve realism would postulate a *quid* for every different quality. This separation of the "substrate" from its "properties" is the error of every dogmatism which attempts to explain experience by something located beyond all possible experience. In a magnet, for example, to indicate a peculiar qualitative determination that characterizes the positive pole and the negative pole as they are "in themselves". These "are" only in their reciprocal opposition because they have equal but opposed charges. It is impossible to distinguish a magnet's south pole and north pole outside of their reciprocal relation.

In his 1801 *Beyträge zur innern Naturgeschichte der Erde* [Contributions to the Internal Natural History of the Earth], Norwegian physicist Henrik Steffens indicates that this characteristic of magnetism offers a model to express the structure of matter without resorting to some substrate beyond it: "the magnet is the only true *compass* [Compass], by means of which we can orient ourselves in the confusion of a sea of forms that intersect each other in thousands of ways."[122] The theory of magnetism can be modeled on that of electricity: "once it is established that magnetic polarity is instigated according to the same laws of electric polarity, there is no doubt that the former arises in the same manner and according to the same mechanism."[123] If the positive pole of a magnet approaches the negative end of another magnet, a reciprocal attraction is observed. If the pole of the first magnet is run alongside the other magnet, one observes that the attraction gradually diminishes and, after passing through an intermediate in which no force is manifested, transforms into repulsion. The two poles are distributed such that one extremity exhibits attraction, the other repulsion, and there is equilibrium in the center where the forces are equal and opposed, creating the equilibrium: "magnetism itself exhibits nothing other than the phenomenon of an opposition in general, nothing other than the pure conditions of a completely ideal opposition, a *plus* and a *minus* that repel and attract each other and a point of indifference in which the opposed activities are removed [sich … aufheben]."[124] To explain this phenomenon, one should not recur to two substances that maintain reciprocal relations since the positive and negative pole are only constituted as such in the opposition:

> [I]t is excusable, and to a certain extent justifiable, if the scientist [Naturforscher] seeks a corporal substrate [ein körperliches Substrat] for every phenomenon; but he should not forget that every substrate is simply a fiction [Fiction] … for example, the scientist assumes two opposed magnetic fluids … [but] if we want to represent the phenomenon of magnetism without hypothetical fictions, no other representation remains than that of … polarity. [125]

[122] Steffens, Henrik. *Beyträge zur innern Naturgeschichte der Erde* (Freyberg, Germany: Craz, 1801), 254.

[123] Ibid., 354.

[124] Ibid., 210.

[125] Ibid.

For Steffens, the magnet represents the general schema of the manner in which qualitative differences that are found in nature can be traced back. The complete decomposition of metals, for example, leads to the claim that these are composed of carbon and nitrogen, which according to Steffens should not be thought as elements, but as forces: "repulsive force is conveyed where it is manifested in the purest manner, as in nitrogen [Stickstoff], attractive force where it is manifested with greater intensity, as in carbon [Kohlensstoff]."[126] The various materials are nothing more than limits in which these opposed forces find equilibrium. They are "expressions of conflicts between two types of forces, that is, nothing other than the expression of certain gradations of density and coherence" that result from the "contention between difference and indifference"[127] that characterizes nature. The qualitative differences between metals are configured as a continuous series of different degrees of coherence and density that spans from mercury (lack of all coherence and strong density) to steel (maximum of coherence and decreasing density).

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The schema offered by chemical, thermal, electrical, and magnetic phenomena represents the schema for the procedure of nature in general for Schelling, and, as the above examples seem to testify, for the entirety of Romantic philosophy of nature:[128] "it is a principle of natural philosophy," writes Schelling in the *Weltseele*, "that all of nature moves through polarity and dualism" (HKA 1:6:151) and "that in all of nature there are separate causes, really opposed and of course *a priori*. When these opposed causes are unified in a body, they give [ertheilen] it polarity" (HKA 1:6:167). The influence of Kant's conception of matter is particularly evident here insofar as Schelling resorts to the schema of the opposition between positive and negative in order to represent the universal structure of the phenomenal world: "In nature, everything tends to continue forward. From this fact [dass dies so ist], we should look for the fundamental cause by which an inexhaustible source of positive force entertains movement interruptedly and initiates it ever again. This positive cause is the first force of nature" (HKA 1:6:77). "[T]he fundamental positive force," continues Schelling, "would completely fall out of all the limits of possible perception if it were infinite. Limited by the opposed [force], it becomes a finite magnitude – begins to be an object of perception, or, in other words, appears as a phenomenon" (HKA 1:6:77).

Thus, the finite phenomenal world arises through the conflict and the equilibrium of opposite tendencies that are in themselves infinite: "natural evolution, <in as much as> it <happens> at a finite speed, presupposes … an accelerating and a slowing force as ultimate factors, each one infinite in itself and reciprocally limited only through the other" (SW 2:262). The variety and the becoming of nature are only

[126] Ibid., 260.

[127] Ibid.

[128] For a detailed reconstruction of German science in the Romantic period, see Poggi, Stefano. *Il genio e l'unità della natura* (Bologna: Il Mulino, 2000). The book is also useful for its rich bibliography.

possible on the basis of an underlying dualism, that is, on the basis of the real opposition between positive and negative: "no vital movement is possible without opposed forces. Real opposition, however," continues Schelling, "is only conceivable at the point where the opposites are placed together in a subject" (SW 2:390), that is, "it is only possible for *magnitudes of the same species*" (SW 2:390n1;[129] emphasis mine). The two forces should be considered as if "they were of *one and the same nature*, which merely act in *opposite directions*" (SW 2:390; emphasis mine).

All the traits that Kant indicates as characteristic of real opposition are thus explicitly adopted by Schelling in the *Weltseele* and brought to their extreme consequence in a "speculative" conception of nature that certainly goes well beyond Kant's intentions: "where there are phenomena, there are also opposed forces. The doctrine of nature presupposes as a basic principle a general duplicity and, in order to conceive this, a general identity of matter. Neither the principle of absolute difference nor that of absolute identity is the true principle. The truth is found in the union of both" (HKA 1:6:86[130]).

In an essay appended to the 1806 edition[131] of the *Weltseele*, the problem of finding a third between two opposites in which they can coexist, which I have suggested more than once to be the fundamental question that continually returns in post-Kantian philosophy, is explicitly indicated as the eternal problem of philosophy: "The ancient philosophers already partly foretold and partly recognized the opposition, the duplicity, at the foundation of matter. That this opposition is removed through a third present in matter, and that matter thus exhibits a closed and identical triplicity, is on everyone's tip of the tongue" (SW 2:359). Philosophy of nature merely restates the problem of locating the transition from the infinite to the finite, from the indeterminate to the determinate, from the unlimited to the limited and vice versa, of locating the point in which these contradictory and irreducible elements can transform into one another: "now the infinite can not reach the finite because it would have to go out of itself, that is, it would therefore not be infinite. In the same way, however, it is inconceivable that the finite can reach the infinite" because, in the infinite, every finite determination vanishes. Thus a "third" that unites both is required, "the absolute bond [Band] or the *copula*" (SW 2:360), the unity itself of the bond (das Band) and of that which is connected to it (das Verbundene): "the first glance at nature already shows us what the latter teaches us … since matter exhibits nothing less than the bond [Band], which in reason is the eternal unity of the infinite with the finite" (SW 2:360).

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[129] The expression appears in the first edition of the work.

[130] In the first edition, Schelling prefers the terms "heterogeneity" and "homogeneity" in place of the terms "identity" and "difference" (see SW 2:390n).

[131] The reference is to *Über das Verhältnis des Idealen und in der Natur oder Entwicklung der Ersten Grundsätze der Naturphilosophie an den Principien der Schwere und des Lichts* [On the Behaviour of Ideals and of Nature or the Development of the First Foundations of Natural Philosophy on the Principles of Weight and Light] that has not yet been published in the Academia edition.

At this point, the Anticipations of Perception seem to be relegated to the background, translated into a "metaphysical" conception that has little to do with the original critical problem. Nevertheless, already in an early commentary on Plato's *Timaeus* from 1794,[132] Schelling explicitly relates Kant's principle to the Platonic term "bond" (δεσμός; in German, *Band*) that is posited as the third between the finite and the infinite (see *Timaeus*, 31c). Present-day critical literature seems to agree that this work represents a fundamental step in the evolution of Schelling's thought in which he highlights some themes that he insistently returns to in subsequent years, particularly in the *Weltseele.*

On the basis of a Kantian interpretation of Plato and an equally Platonic reading of Kant, Schelling asks why the conception of matter outlined in the *Timaeus* is traced back to the concept of the unlimited (ἄπειρον). As is well known, Plato systematically treats the concepts of unlimited and limited, in the *Philebus*. In this dialogue, the ἄπειρον is indicated as that which is capable of proceeding in the two opposite directions of more and less (μᾶλλον καὶ ἧττον), hotter and colder, stronger and weaker (24a–d), to which are added moister and dryer, scarcer and more abundant, faster and slower, larger and smaller (26a). It is in this characteristic of the unlimited, argues Schelling that one can glimpse "the traces of the Kantian principle of quality very clearly."[133] According to Schelling, this is demonstrated by the fact that, among "the phenomena that should be assumed under the concept of ἄπειρον (unlimited), he [Plato] cites the sensations in particular (according to the principle of quality)."[134]

The fact that Plato asks "whether the 'more and the less'" should be considered an ineliminable feature of all sensations (see 24 a–b),[135] shows, according to Schelling "fairly clearly the necessity by which every reality [Realität] in sensation can, in a continuous manner, increase or decrease to infinity. This continuous increasing and decreasing is the necessary form of every sensation such that if the infinite is not found in their continuous increasing and decreasing, … the sensation itself could not be present either."[136] According to Schelling, Kant has demonstrated that "this continuous nexus of sensation is a necessary form of every sensation and thus every degree of sensation encloses the concept of sensation within contradictory limits."[137]

In Schelling's view, when Plato writes that "pleasure, since it is under the category of ἄπειρον (illimited), has no beginning nor middle nor end for itself

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[132] Schelling, Friedrich Wilhelm Joseph von. *Timaeus*. ed. Hartmut Buchner (Stuttgart: Frommann-Holzboog, 1794). On the relation of this work with Schelling's philosophy of nature, see Harmut Buchner's introduction to the German edition as well as Herman Krings's contribution "Genesis und Materie – zur Bedeutung der *Timaeus* – Handschrift für Schellings Naturphilosophie" that accompanies the German edition. On this subject, also see Distaso, Leonardo V. *The Paradox of Existence. Philosophy and Aesthetics in the Young Schelling* (Dordrecht: Kluwer, 2004), 37–.

[133] Schelling. *Timaeus*, 60.

[134] Ibid.

[135] English translation from Plato. "Philebus." In Plato. *Complete Works*, ed. John Cooper (Indianapolis, IN: Hackett Publishing Company, Inc., 1997), 412.

[136] Schelling. *Timaeus*, 60.

[137] Ibid., 61.

(ἀφ' ἑαυτοῦ)," he merely indicates what is the specific characteristic of every sensation. In Schelling's own words:

> [There is] no beginning because if it [pleasure] increases from weaker degrees to stronger degrees in a continuous fashion like sensations, one can never find the weakest, but rather every degree is made to begin again from a weaker degree. For this reason, there is no middle, no intermediate degree, because every chosen point is divisible again into smaller parts since each intensive magnitude is continuous and thus another point is always thinkable between two points. There is no end because each degree that is taken as the last is further divisible since there is an infinity of possible degrees of sensation between reality and zero.[138]

By means of this analogy, Schelling concludes "that Plato includes nothing more and nothing less than the category of reality [Realität] under (ἄπειρον) the unlimited and subsumes under this category all the objects that occur in sensation and in as much as they occur there."[139] In opposition to the unlimited multiplicity of that which is still quantitatively indeterminate, the limit is thought of as a being of such and such size (posón). Thus, for Plato, "every object is rather a ἄπειρον, connected to the πέρας, that is, the determinate reality of quantity"[140] and "the entire world," continues Schelling, is nothing but "quality (reality) determined by quantity."[141]

Schelling seems to maintain that the unlimited, though undetermined from the point of view of quantity, is nevertheless determined in terms of quality such that, while temperature in general is unlimited and quantitatively indeterminate (susceptible to being infinitely augmented and diminished), 20° can be considered a πέρας, a certain finite difference. Since, according to Schelling, "Plato understands πέρας, limit, as the very opposite of the unlimited (ἄπειρον),"[142] it is necessary to clarify how one opposite can pass into the other, how the more or less unlimited can enter into the limit. It is precisely in the transition from the ἄπειρον to πέρας that one finds the γένεσις εἰς οὐσίαν, [143] the "going toward being" that Plato calls the third genus, the mixed genus. According to Plato, the two opposed elements cannot unite without the intervention of a "third"; between the two, a bond (δεσμός) is established, specifically a *Band* (the term used by Schelling in the *Weltseele*) that is capable of maintaining the opposites united in something common: "quantity (πέρας), quality (ἄπειρον), and that which emerges from the union of both (κοινόν)."[144]

In his philosophy of nature, Schelling simply reintroduces the same "polar"[145] structure that, in this early commentary, he already found hints of in Greek thought for which "substance and existing things are composed of contraries": "some as Odd and

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[138] Ibid., 60.

[139] Ibid.

[140] Ibid.

[141] Ibid., 62.

[142] Ibid., 61.

[143] See Ibid.

[144] Ibid., 50.

[145] On this theme, see Philippson, Paula. *Untersuchungen über den griechischen Mythos* (Zürich: Rhein-Verl., 1994), 65.

Even, some as Hot and Cold, some as Limit and Unlimited, some as Love and Strife."[146] In the list of contraries, "the second column of contraries is privative,"[147] lack of determinateness, mere indeterminate plurality of possibilities always open to the double direction of "more or less"; thus, something "unstable, shapeless, indefinite and non-being as the negation of being."[148] In contrast, the other column indicates the determinacy posited by the limit, signaling the transition to the dimension of being "something." In the transition from ἄπειρον to πέρας, however, Schelling finds himself again in the same quandary that, in Kant's philosophy, is involved in the transition from negation = 0 (which has no determinate difference) to a certain difference of finite degree. Therefore, the Anticipations of Perception is the place where Kant reintroduces the eternal problem of overcoming the opposition between finite and infinite.

Plato's *Philebus,* as Hegel notes in §95 of the *Encyclopaedia*, is the first text to grasp the problem in all its weight, the problem that, as Hegel already attempted to demonstrate in *The Difference Between Fichte's and Schelling's System of Philosophy*, constitutes the core of Fichte and Schelling's philosophy: "there is no transition from the infinite to the finite, from the indeterminate to the determinate"; reflection that "separates absolutely," cannot grasp such a transition, such a "synthesis of the finite and the infinite, of the determinate and the indeterminate" (HW 2:97).[149] Only reason has the capacity to determine the opposites in the form of the identity of identity and of non-identity, in the form of a real opposition of the opposites, which only occurs because of the identity of both (see HW 2:97).[150]

Not by chance, Hegel, in a footnote, cites the exact passage from the *Timaeus* that Schelling refers to: "Plato expresses real opposition through absolute identity thus: 'the truly beautiful bond [das wahrhaft schöne Band] is that which makes itself and what it binds one' [Timaeus 31c]" (HW 2:97n).[151]

3.3 The Problem of Quality and the Opposition between Positive and Negative

Up to this point, my goal has been to demonstrate that the model of the relations between reality and negation that is outlined in the Anticipations of Perception seems to have played a fundamental role in the post-Kantian philosophical debate. The apparently

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[146] English translation from Aristotle. *Metaphysics, vol. 4*, tr. Hugh Tredennick (Cambridge, MA Harvard University Press, 1933), 1004b2705a2.

[147] Ibid.

[148] Simplicius, *In Arist. Phys*., 248, 13–16 Diels (Gaiser, Test. Plat. 31 = Krämer 13).

[149] Hegel. *The Difference between Fichte's and Schelling's System of Philosophy*, 158.

[150] Ibid.

[151] Ibid., 158n. This is the English version of Hegel's translation of Plato into German: "Das wahrhaft schöne Band ist das, welches sich selbst und die Verbundenen eins macht" (HW 2:97n). The original Greek is: "δεσμῶν δὲ κάλλιστος ὃς ἂν αὑτὸν καὶ τὰ συνδούμενα ὅτι μάλιστα ἓν ποιῇ" [Timaeus 31c. Benjamin Jowett translates Plato in the following manner: [T]he fairest bond is that which makes the most complete fusion of itself and the things which it combines" (*Timaeus*, Charleston, SC: BiblioBazar, 2007).

simple idea that reality and negation are only distinguished in terms of degree; only by a relative difference in level, a difference that can be expressed through the simple opposition between + and − in respect to a point of indifference = 0; seems to have been transformed into one of the necessary presuppositions for speculative philosophy as much as for scientific research: every qualitative difference between specifically distinct materials should be dissolved into a mere quantitative difference. As Schelling comments:

> I speak of the modes of representation which have been put into philosophic heads by Kant, and which may be mainly reduced to this: that we see in matter nothing but the occupation of space in definite degrees, and in all variety of matter, therefore, only mere difference of occupation of space (i.e., density), in all dynamic (qualitative) changes only mere changes in the relation of the repulsive and attractive forces. (SW 3:281)[152]

However, attempting to trace back all qualitative differences in matter to a mere difference in density seems inevitably to lead to the pretense of deducing *a priori* all the differences between materials. The biologist Karl Friedrich Kielmeyer (1763–1844) seems to be particularly perceptive of this danger in a pamphlet entitled *Über Kant und die deutsche Naturphilosophie* (On Kant and German Philosophy of Nature, 1807):

> [I]f the essence of matter is located in general in the being together [Zusammensein] of these two forces, then one should draw the consequence that the difference between materials only depends on the different quantitative relations of these two forces; gold and limestone would thus only differ in respect to the quantitative relation of the repulsive and attractive force. Since only quantitative relations are determinable *a priori*, the entire variety of matter on our earth should be thought as being capable of being easily derived from that concept of matter in general that is determined *a priori*.[153]

As has already been discussed, Schelling recognizes that Kant's position, as put forth in the *Metaphysical Foundations of the Natural Sciences*, is decisively more nuanced: "Kant has nowhere genuinely ventured to construct the specific (qualitative) diversity of matter out of his two basic forces. A few who wished to apply his dynamic principles have gone further" (HKA 1:7:279; see also HKA 1:5:228).[154] Eschenmayer, whose model Schelling initially adopted as a point of reference to be developed, now becomes the polemical target. Eschenmayer is guilty of the "ill-conceived attempt to construct the qualities and series of degrees of qualities according to Kantian principles" (HKA 1:7:279),[155] attempting to "reduce qualities to analytical formulas, and to express them by means of the variable relations of repulsive and attractive force" (HKA 1:7:279).[156]

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[152] English translation from Schelling. *Ideas for a Philosophy of Nature as Introduction to the Study of this Science, 1797*.

[153] Kielmeyer, Karl Friedrich. *Gesammelte Schriften. Natur und Kraft*. ed. Fritz-Hein Holler (Berlin: Keiper, 1938), 245. See also Bach, Thomas. "Kielmeyer als 'Vater der Naturphilosophie'? Anmerkungen zu seiner Rezeption in deutschen Idealismus." In *Naturphilosophie nach Schelling* eds. Thomas Bach and Olaf Breidbach, 232–51. (Stuttgart-Bad Cannstatt: Frommann-Holzboog, 2005), 232–51.

[154] English translation from Schelling, Friedrich Wilhelm Joseph von. *First Outline of a System of the Philosophy of Nature*. ed. Keith R. Peterson (Albany: State University of New York Press, 2004), 200n (first edition).

[155] Ibid., 22n.

[156] Ibid.

This transition from quality to quantity, as I have shown, is one of the essential characteristics of the dynamic conception of matter on the basis of which "electrical [phenomena], are not appearances or effects of determinate individual materials," but only "alterations … in repulsive and attractive force" (HKA 1:7:278).[157] However, the pretense of deducing all qualitative differences in matter from mere differences in density falls outside the authentic spirit of Kantian dynamism. As Schelling explains in his 1801 *Über den wahren Begriff der Naturphilosophie und die richtige Art ihre Probleme aufzulösen* (On the True Concept of the Philosophy of Nature and the Right Way to Solve its Problems):

> The difference between me and Mr. E[schenmayer] does not lie in these propositions [the reduction of matter to a relation between forces], but rather in the fact that he, in the relation of opposed forces, retained the possibility of a mere quantitative difference, determinable through the relative more or less of the one or the other force … and that he, through these different quantitative forces and the formulas that these express, believed he had deduced the entire specific difference of matter.… [such that] the properties of bodies are in direct relation to the degrees with which they fill space [Graden ihrer Raumerfüllung]. (SW 4:94–5)

Eschenmayer himself is aware of Kant's caution in face of the possibility of "going beyond what makes the universal concept of matter in general possible and against wanting to explain *a priori* the particular or specific determination and variety of matter"[158] (see AA, 4:524). Nevertheless, he does not hesitate to admit "that the analysis [Zergliederung] of the general concept" of matter should simultaneously provide "the principles for the construction of its specific determinations" (AA 4:524).[159] The same concept of matter as a conflict between attractive and repulsive forces leads to the possibility of constructing a "scale" on which the different qualities of materials are positioned, reduced to mere differences in degree.

In the *First Outline of a System of the Philosophy of Nature* (Erster Entwurf eines Systems der Naturphilosophie) and in the *Introduction to the Outline of a System of the Philosophy of Nature* (Einleitung zu dem Entwurf eines Systems der Naturphilosophie), both from 1799, Schelling emphasizes that this simplistic model must be abandoned without losing, however, what should be considered the true significance of Kant's conception: the "being" of different qualities cannot be thought outside of their opposition, as if they could by hypothesized in some entity located beyond all consciousness. In Schelling's words, "[t]he condition of all formation is duality. (This is the more profound signification that lies in Kant's construction of matter from opposite forces)" (SW 3:299).[160] Precisely for this reason, and in contrast to what Eschenmayer thought, one should affirm that "[q]uality considered as

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[157] Ibid. The preceding citations are explanatory notes from Schelling regarding his model in the *First Outline* (*Erster Entwurf*).

[158] Eschenmayer. *Versuch die Geseze magnetischer Erscheinungen aus Säzen der Naturmetaphysik mithin a priori zu entwickeln*, 69–.

[159] Ibid.

[160] English translation from Schelling. *First Outline of a System of the Philosophy of Nature*, 212.

absolute is inconstructible, because quality generally is not anything absolute, and there is no other quality at all except that which bodies show mutually in relation to each other, and all quality is something by virtue of which the body is, so to speak, raised above itself" (SW 3:295).[161] The schema of the relation between positive and negative magnitudes demonstrates how a given quality is only manifested through the presence of a certain state of "tension," which is created every time that an equilibrium is disturbed. The difference between opposed qualities (hot and cold, oxygen and phlogiston, positive and negative electricity, etc.) is not "absolute"; it is possible only in the "moment of the conflict": "Two specifically different bodies will relate positively and negatively to one another reciprocally, and their qualitative difference can be expressed through this positive or negative mutual relation" (HKA 1:7:165).[162]

Again, the model of this tension that Schelling refers to is above all that of electrical phenomena:[163] "The absolute relativity of all quality may be shown from the electric relation of bodies, inasmuch as the same body that is positive with one is negative with another, and conversely" (SW 3:294–).[164] Therefore, "*all quality is electricity, and conversely, the electricity of a body is also its quality* (for all difference of quality is equal to difference of electricity, and all quality is reducible to electricity). – Everything that is sensible for us (sensible in the narrower sense of the term, like colors, taste, etc.), is doubtless sensible to us only through electricity, and the only immediately sensible factor would then be electricity, a conclusion to which the universal duality of every sense leads us independently, since in Nature there is properly only one duality" (SW 3:295).[165] Every quality is only constituted in the "difference" between + and − and, in this sense, one can affirm that "phenomena of electricity show the scheme of nature" and that "[t]his condition of oscillation [schweben] or change [Wechsels], attractive and repulsive force, is the real condition of formation [Bildens]" (SW 3:308n).[166] Without opposition and outside of opposition, no quality exists because "quality … is determined by its opposite, with which it is placed in conflict, and this antithesis is itself again determined by a higher antithesis, so back into infinity" (SW 3:294).[167] Only in the conflict of two opposites that tend toward equilibrium are different qualities defined as such: "all dynamical phenomena are phenomena of transition from difference to indifference, but it is in this very transition that matter is primarily constructed" (SW 3:321n).[168]

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[161] Ibid.

[162] Ibid.

[163] On this point, see Moiso, Francesco. "Schellings Elektrizitätslehre 1797–1799." In *Natur und Subjektivität. Zur Auseinandersetzung mit der Naturphilosophie des jungen Schelling*. eds. Reinhard Heckmann, Hermann Krings, and Rudolf W. Meyer, 39–91. (Stuttgart: Frommann-Holzboog, 1985), 39–91.

[164] English translation from Schelling. *First Outline of a System of the Philosophy of Nature*, 209–10.

[165] Ibid., 210.

[166] Ibid.

[167] Ibid., 209.

[168] Ibid., 228n.

Therefore, what should be emphasized in Kant's conception is the polar form of thought that conceives and organizes the world as a unity of contrary pairs. Not only are the contraries of a pair indissolubly connected to each other, but also, in their more intimate logical existence, are conditioned by their opposition; without their opposite pole, they would lose their very sense. Their sense consists in the fact that, as contraries, they are part of a larger unity that is not definable exclusively on the basis of them, but at the same time only "is" through them and in their relation. Schelling seems to delineate the general meaning of the concept of quality through this unity that transcends the opposites and that is only manifested in their reciprocal conflict. Qualities exist as such only where the "tendency" emerges to suppress the difference itself to which the qualities should be referred back. They are unthinkable outside of such a tendency.

As discussed above, electricity only occurs where there is a difference in potential and the concept itself of electricity has no determinate meaning outside of this difference: "now what, then, is it that becomes an object to us through sensation? Nothing else but quality. But all quality is simply electricity, a proposition that is demonstrated in natural philosophy." (HKA 9:9:137)[169] because, exactly like electricity, qualities only exist in the moment of the conflict between + and –.[170] In regard to an electrically neutral body, in which all difference in potential has disappeared, talking about "electricity" is meaningless. Analogously, if all difference in "degree" in general disappeared, "all matter would likewise sink back into dynamical inactivity, that is, into absolute absence of quality" (SW 3:294).[171] To ask "what" is electricity "in itself," outside of the difference in potential that reveals it, is to pose a meaningless question. For this reason, "qualities generally are just the absolutely empirical factors in our knowledge of Nature, of which no construction is possible" (SW 3:294).[172]

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At this point, however, the same ineludible difficulty in Kant's text and in the concept of real opposition arises again: "as soon as one undertakes to carry out [zu Stande zu bringen] the construction of a finite product from these opposed tendencies, one encounters an irresolvable difficulty. For if we let both coincide at one and the same point, then their effects toward one another will reciprocally be cancelled, and the product will be = to 0" (HKA 1:7:82).[173] Analogously, Schelling's *Ideas for a Philosophy of Nature* states: "In order to arrive at a product, these opposite tendencies must encounter one another. But since they are supposed equal …, wherever they meet they will annihilate each other; the product is therefore = to 0, and once

[169] English translation from Schelling, Friedrich Wilhelm Joseph von. *System of Transcendental Idealism (1800)*, tr. Peter Lauchlan Heath (Charlottesville: University Press of Virginia, 1978), 91.

[170] See Moiso. "Schellings Elektrizitätslehre 1797–1799." 39–91.

[171] English translation from Schelling. *First Outline of a System of the Philosophy of Nature*, 209.

[172] Ibid.

[173] Ibid., 17.

more no product is reached" (SW 3:289).[174] As discussed above, the being of qualities only emerges in the moment of the conflict between opposed tendencies, only in the moment in which + and −, encountering each other, limit each other and define one another, but also, and precisely for this reason, annihilate each other. One can compare nature's process "to a series in which positive and negative magnitudes constantly succeed each other. But within this series the product cannot be inhibited, for it would be either[175] =1−1+1−1," that is, = 0, "or some positive actant would have to gain preponderance. [übergewicht]," that is to say, = 1, but "[n]either of these alternatives can come to pass." (HKA I:7:98–).[176]

According to Schelling, "[t]his inevitable, though hitherto not very closely remarked contradiction (namely, that a product can arise only through the concurrence of opposite tendencies, while at the same time these opposite tendencies mutually annihilate each other)" can be resolved "only in the following manner. Absolutely no subsistence [Bestehen] of a product is thinkable without a continual process of being reproduced. The product must be thought as annihilated at every step, and at every step reproduced anew. We do not really see the subsisting of a product, but only the continual process of being reproduced" (SW 3:288).[177] 1+1−1+1−1 and so forth annul each other when taken as pairs, such that all of them should be reduced to a series of 0s. The same series thought to infinity,[178] however, is = ½: It is of course quite conceivable how the series 1−1+1−1 … on to infinity is thought as equal neither to 1 nor to 0. The reason why this series is thought as = ½ lies deeper. There is one absolute magnitude (= 1) which, though continually annihilated in this series, continually recurs, and by this recurrence produces, not itself, but the mean between itself and nothing" (SW 3:289).[179] Nature is nothing but a tending toward indifference, and "[i]t is precisely zero to which Nature continually strives to revert, and to which it would revert if the antithesis were ever canceled. Let us suppose the original condition of Nature = 0 (lack of reality). Now zero can certainly be thought as dividing

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[174] Ibid., 205.

[175] The edition of Schelling's work that SW refers to contains the formulation 1−1+−1. The critical edition (HKA) introduces the correction 1−1+1−1.

[176] English translation from Schelling. *First Outline of a System of the Philosophy of Nature*, 33.

[177] Ibid.

[178] Guido Grandi discovered the sum of this infinite series, moving from the presupposition that the series $1 + x^2 + x^3 + x^4 + \ldots$ has $\frac{1}{1-x}$ as its sum. For $x = -1$, precisely Schelling's result is obtained. Associating two successive terms (first and second, third and fourth), Grandi arrives at the even more paradoxical conclusion that a sum of 0s should have $\frac{1}{2}$ as a result, leading him to see a "model" for creation from nothing in the series. However, Grandi's arguments (as well as those of Leibniz, who accepted the result) rely on the false presupposition that every infinite series must have a sum (a generally-accepted conviction in eighteenth-century mathematics). Today, in contrast, we know that there is no sense in affirming that Grandi's series has a sum, not being convergent. On this theme, see: Moiso, Francesco. "Identità, differenza, indifferenza in Schelling." In *La differenza e l'origine*. ed. Virglilio Melchiorre, 97–132. (Milan: Vita e Pensiero, 1987), 112. Also see Moiso, Francesco. *Vita natura liberta: Schelling, 1795–1809* (Milan: Mursia, 1990), 210–12.

[179] English translation from Schelling. *First Outline of a System of the Philosophy of Nature*, 205.

itself into 1−1 (for this = 0); but if we posit that this division is not infinite (as it is in infinite series 1−1+1−1…), then Nature will, as it were, oscillate continually between zero and unity – and this is precisely its condition." (SW 3:313n).[180]

In order for the conflict between two opposed tendencies to result in a finite product, they must "offset" each other in a dynamic equilibrium; that is, it must be shown that the "indifference is canceled at every step, and at every step it is restored…. The universal restoration of duality, and its recanceling at every step, <that is> can appear only as a *nisus* toward a third factor. This third factor is therefore the pure zero [Null]> abstracted from tendency is nothing <= 0>" (SW 3:313).[181] The possibility of nature is founded on the continual oscillation between permanence and becoming, the tendency toward conservation and the tendency toward transformation, such that the "absolute rest in the world is a non-thing [Unding], all rest in the world is only apparent. It is in actuality only a *minus*, but in no way a complete lack of movement (= 0)" (SW 2:383). In every point in which opposites encounter each other, their difference necessarily vanishes, but this does imply a return to absolute nothingness because "the permanence, the resting of the products of Nature …, is not to be viewed as an absolute resting, but only as an evolution proceeding with infinitely small rapidity or with infinite tardiness" (SW 3:287)[182]

These references to infinitesimal analysis are certainly very significant and confirm the general tendency of post-Kantian philosophy to look to the mathematics of infinity for the solution to the fundamental problem that Kant leaves unresolved (reality and negation are defined as positive and negative, but precisely by being defined as such, annul each other as well). Nevertheless, Schelling apparently does not intend to develop a true and proper philosophy of mathematics (as I will show Hegel does).[183] The example of the series serves rather to show that nature is a continuous oscillation between permanence and change; that all that endures in nature, all finite products, only make sense through the general concession that every element cannot be constituted as such if considered separately, but *is* only in respect to every other and in respect to the totality of all the others:

> The originally *infinite series*, of which every individual series in mathematics is an imitation, does not arise through *aggregation* [Zusammensetzung], but through *evolution*, through evolution of a *magnitude* already *infinite* in its *point of origination* which runs through the entire series. The whole infinity is originally concentrated in this one magnitude. The succession in the series signifies only, as it were, the individual inhibitions [Hemmungen] which continually set bounds to the expansion of that magnitude into an infinite series …, and which moreover happens with an infinite velocity and would permit no *real intuition.* (HKA 1:7:80)[184]

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[180] Ibid., 222n.

[181] Ibid., 223.

[182] Ibid., 204. On Schelling's use of mathematical language of the infinite, see Moiso, Francesco. "La Naturphilosophie e i paradossi dell'infinito." In *Romanticismo e modernità*. eds. Claudio Ciancio and Federico Vercellone, 143–205. (Turin: Zamorano, 1997).; in which the particular importance of the Anticipations of Perception is highlighted.

[183] See below 156–.

[184] English translation from Schelling. *First Outline of a System of the Philosophy of Nature*, 16.

Thus, in its continual changing, "Nature EXISTS nowhere as product; all individual productions in Nature are merely apparent products [Scheinprodukte], not the absolute product, that always BECOMES and never IS" (HKA 1:7:80).[185] Each product is constituted through the determination of limitations or "inhibitions" (Hemmungen) of a single productive activity; that is, through the emergence of finite differences. For example, electricity only "exists" through a difference in "potential": "Electricity exists only at that point at which limits are given, and it is only a poverty of conception that would look for anything else in its phenomena beyond the phenomena of (limited) productivity" (SW 3:298-).[186] Different qualities are nothing other than "points of inhibition" (Hemmungspunkte) in which a single productive activity is limited: "This limitation [Begrenzung] of the dynamical process, that is, the proper determination of quality [Qualitätsbestimmung], takes place by means of no other force than that by which the evolution is universally and absolutely limited" (SW 3:294).[187]

If qualities are nothing outside of those "states of tension" that emerge every time a difference (of temperature, of potential, etc.) is constituted, they do not need any material "substrate" to guarantee their being, but should be thought as" *pure intensity* [reine Intensität], pure action [reine Aktion]" (SW 3:294; my emphasis).[188] Schelling, resorting to a Leibnizian expression, emphasizes that qualities are located "*imo extensione.*" They are indivisible, not because they interrupt the process of subdivision at a certain point or exhaust themselves in the infinitely small, but because they are beyond all subdivision and pertain to a different dimension than that of extension. On the basis of what Schelling calls "dynamic atomism" (see HKA 1:7:278n),[189] as opposed to traditional atomism, qualities are "beyond space" (SW 3:292).[190] Every part of matter is always matter again just as every part of a warm body is warm again. "[T]his pure intensity is what, even in infinite divisibility, sustains the substrate." (SW 3:293)[191] such that "[a] body divided to infinity still occupies space to the same degree as its smallest part" (SW 3:292).[192]

The productivity of nature is only one. However, it does not manifest itself without duality and difference: "that which is purely productive without being a product is but the ultimate ground of quality. But every quality is a determinate one, whereas productivity is originally indeterminate. In the qualities, therefore, productivity appears as already inhibited [gehemmt]" (SW 3:292).[193] That which is called

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[185] Ibid.

[186] Ibid., 212.

[187] Ibid., 209.

[188] Ibid.

[189] Ibid., 21n. On this point, see Rudolphi. *Produktion und Konstruktion: zur Genese der Naturphilosophie in Schellings Frühwerk*, 146–53 and Lauth, Reinhard. "Die Genese von Schellings Konzeption einer rein aprioristischen spekulativen Physik und Metaphysik aus der Auseinandersetzung mit Le Sages spekulativer Mechanik," *Kant Studien* 65 (1974): 397–435.

[190] Schelling. *First Outline of a System of the Philosophy of Nature*, 208.

[191] Ibid.

[192] Ibid.

[193] Ibid.

quality is only manifested when the electric or magnetic potential, or the chemical concentration, etc., is larger or smaller in respect to that which surrounds it:[194] "all the particular determinations of matter that we conceive under the name quality … are based in the variety of relations between bodies in respect to those three functions [electricity, magnetism, chemical processes] and, from this proposition, one arrives above all at the general principle [Prinzip] of a construction of the difference of quality" (SW 4:51). All these phenomena are characterized by a difference in "intensity" in which the infinite productivity of nature is manifested as limited, and outside of this difference, they are nothing.

On the one hand, the productivity is unconditioned, or more precisely, it is the "*natura naturans*" (SW 3:284)[195] such that "the unconditioned cannot be sought in any individual "thing" nor in anything of which one can say it 'is'." (HKA 1:7:80)[196] On the other hand, "wherever the productivity is *limited*, the productivity manifests itself" (SW 3:298),[197] where a finite difference emerges: the productivity appears only as a "*natura naturata*" (SW 3:284).[198] The unconditioned productivity cannot be grasped as such because beyond limited "productivity there is [only] pure identity, the limitation cannot be established by a difference already existing, and therefore must be furnished by an opposition arising in productivity itself" (SW 3:308).[199] For this reason, the productivity itself can only be given a "negative presentation" (HKA 1:7:84).[200]

If the productivity were not limited, it would not even be productivity. The oppositions in which the unique and identical infinite productivity is manifested as inhibited and limited are nothing other than the tendency to return to indifference, but if difference did not exist, the tendency to suppress it could not even be manifested:

> Opposition [Gegensatz] is the removal [Aufhebung] of identity. But nature is originarily identity – there is thus *in* this opposition a tendency [Streben] toward identity again. This tendency is immediately conditioned by the opposition because if there were no opposition, there would be identity, absolute rest and also no *tending* toward identity. If, on the other hand, there were again no identity in indifference, the opposition itself would not be able to persist. (HKA 1:8:63)[201]

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[194] See Heuser-Kessler, Marie-Luise. *Die Produktivität der Natur: Schellings Naturphilosophie und das neue Paradigma der Selbstorganisation in den Naturwissenschaften* (Berlin: Duncker & Humblot, 1986), 104–09. See in particular Heuser-Kessler. *Die Produktivität der Natur: Schellings Naturphilosophie und das neue Paradigma der Selbstorganisation in den Naturwissenschaften*, 107, where the comparison with modern science is more explicit.

[195] English translation from Schelling. *First Outline of a System of the Philosophy of Nature.*

[196] Ibid.

[197] Ibid.

[198] Ibid.

[199] Ibid. See Schmied-Kowarzik, Wolfdietrich. *'Von der wirklichen, von der seyenden Natur'. Schellings Ringen um eine Naturphilosophie in Auseinandersetzung mit Kant, Fichte und Hegel* (Stuttgart-Bad Cannstatt: Frommann-Holzboog, 1996), 72–82.

[200] English translation from Schelling. *First Outline of a System of the Philosophy of Nature*, 19.

[201] See Rudolphi. *Produktion und Konstruktion: zur Genese der Naturphilosophie in Schellings Frühwerk*, 140–46.

The relation between productivity and product simply raises the fundamental problem that emerged in the commentary on the *Timaeus*: how finite determination can spring from absolute indeterminateness. However, Schelling now expresses it in the language characteristic of Kant's real opposition, of the opposition between indeterminate zero and finite difference, between positive and negative on the one hand and the point of indifference that separates them on the other hand. In the *Allgemeine Deduktion des dynamischen Processes oder der Kategorien der Physik* (General Deduction of Dynamic Processes or of the Categories of Physics), Schelling discusses this form of opposition:

> [W]hen speculation reaches that absolute unification of opposed activities, which we conceive in the concept of nature, then we have no other object besides absolute identity, which for intuition is indicated by a mere zero, or rather, with absolute lack of reality … Now how something finite, that is, something real, can emerge from this infinite which is = zero for phenomena can only be understood by the fact that we separate [trennen] that zero into its factors (1−1) and that we assume this separation [Trennung] to be infinite (HKA 1:8:300).

On the one hand, "this difference, thought in a pure manner [rein gedacht], is the first condition of all of activity in nature" (SW 3:308). On the other hand, "nature in all phenomena is the tendency [Bestreben] to return to this 0, even if it never successfully reaches absolute identity since everything it reaches is only a relative identity" (HKA 1:8:300): "if we thus assume the fundamental opposition [Gegensatz] of two activities, we must also assume a third which represents nothing other than the infinite tendency of nature to return to that absolute identity" (HKA 1:8:300). Thus, the eternal problem of philosophy is to present the union of the unlimited and the limited, to show how finite difference can arise from the indifference = 0 and then resolve itself again in infinite indifference: "the greatest problem of every science," that is to say, the "possibility of the exhibition of the infinite in the finite" (HKA 1:7:79),[202] is the problem of conceiving a third between contraries, in which the finite product appears only as the "unstable equilibrium"[203] of two opposed tendencies that determine each other precisely while annihilating each other.

3.4 Speculative Philosophy of Nature and Mathematical Philosophy of Nature

Schelling's philosophy of nature, as well as the philosophical controversies that accompany the scientific debate of his time, involve the fundamental problem raised by the Anticipations of Perception. Though the problem assumes a form that is increasingly farther from its formulation in the *Critique of Pure Reason*, its general philosophical meaning seems to be delineated ever more precisely: reality and

[202] Schelling. *First Outline of a System of the Philosophy of Nature*, 1.

[203] Ibid., 147.

negation are distinguished only by degree and their difference is only constituted paradoxically in the opposite aspirations of the positive and the negative to fill this difference; each "reality" is determined only in relation to an opposed "reality," in the "moment" of the conflict, and has no sense outside of it. No substance, no some "thing," exists beyond the conflict that can guarantee their merely relative existence. However, if the point in which positive and negative meet is= 0, precisely where the opposites are constituted as such, everything should at the same time be resolved again into nothingness.

This formulation of the problem has become a sort of stereotype for Romantic philosophy of nature: "nature is the conflict of a positive force and a negative force," writes Stephen August Winkelmann, for example, in his 1803 *Einleitung in die dynamische Physiologie* (Introduction to Dynamic Physiology). This conflict "is only manifested as a succession at the center of which is the equilibrium of two forces that appear as positive in one direction and as negative in the opposite one."[204] "The schema of physics" is nothing other than the theory of these relations between opposed magnitudes such that "magnetism" can be seen as "the single manifestation of the negative and the positive in equilibrium within the same single sphere"[205]: "if a pole manifests the appearing negative force, then the other expresses the positive one, and thus, between both, the point of indifference and the equilibrium of both forces remains."[206]

This simple schema insistently returns again and again in the debate of the time, manifesting itself in increasingly diverse forms, from those more connected to empirical research to more speculative conceptions. Lorenz Oken, in his *Lehrbuch der Naturphilosophie* (Textbook on Philosophy of Nature), explicitly applies this conceptual tool: "the highest mathematical idea and the fundamental principle of all mathematics is the zero = 0."[207] "[F]rom the fact that the fundamental of mathematics is 0, it directly follows that + and − also exist"[208] All forces of nature arise according to the model of this duality since "there is no positive number without a negative one, there is thus no simple force, but rather each one is posited in terms of + and −."[209] "When this positing is repeated, movement arises, that is, when many + − + − are placed one after another." [210] All movement in nature is founded on a "polar tension,"[211] which is manifested in thermal, chemical, electric, magnetic, and other similar phenomena.

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[204] Winkelmann, Stephen August. *Einleitung in die dynamische Physiologie* (Göttingen: Dieterich, 1803), 13.

[205] Ibid., 23.

[206] Ibid.

[207] Oken, Lorenz. *Lehrbuch der Naturphilosophie* (Hildesheim-Zürich-New York: Olms, 1991), 4.

[208] Ibid., 14.

[209] Ibid., 17.

[210] Ibid.

[211] Ibid.

At this point, however, the same problem arises that Schelling elucidated, a problem that cannot be resolved without resorting to a conceptual tool capable of conserving the meaning of the opposition, even when the quantitative difference between the opposites has disappeared. Joseph Görres, commenting on the work of Oken, highlights that the relation between positive and negative, and their meeting in a point of equilibrium, should be effectively considered as the fundamental schema of natural phenomena, which electrical phenomena particularly demonstrate: "it is required that − and + are annulled [zernichtet] in their transition and this actually occurs in electricity, for example, in the bottle of Leida. $-E$ and $+E$ become = 0 in their transition."[212] Precisely here, however, the same difficulty is manifested that Oken, according to Görres, did not know how to adequately confront:

> [T]he fundamental error of the author [Oken], which brings with it all the others, is to want to hold firm to absolute zero since zero means nothing other than the point of transition from the positive to a negative and indicates a position [Stelle] in this transition in which a null value should exist. In the absolute, certainly everything relative has vanished [verschwunden], but not as in a zero, but rather as a differential vanishes in a finite magnitude, and the finite magnitude is an infinite one.[213]

Görres's use of the term "differential" illustrates once more how the recourse to a speculative conception from infinitesimal calculus seems to have represented the only way of resolving the problem of the vanishing into indeterminate zero of every difference between two opposites, and thus of their "reality," during the moment of transition between them.

Similarly, Johann Wilhelm Ritter, in *Fragmente aus dem Nachlasse eines jungen Physikers: ein Taschenbuch für Freunde der Natur* (Fragments from the Unpublished Writings of a Young Physicist: A Pocketbook for Friends of Nature), argues that "nature is action [Handeln] and only as such is nature. Action requires the manifold, because it is through the latter that the action is possible; without the manifold the action disappears [fällt … weg]. Thus, every action presupposes a difference. This, however, is opposition [Gegensatz], polarity. As action is only found in nature, it is there where polarity should be everywhere."[214] In another fragment, Ritter emphasizes how the polarity's intensity is born from the difference that is established in respect to a state of equilibrium such that "the natural state of an opposition is neither indifference (= 0), nor even difference (= + e − completely separated), but only relative difference."[215] This intermediate state between finite difference and difference = 0, which is neither one or the other, is what permits nature to be grasped in its becoming: "absolute indifference would be exclusion of time, absolute difference would be

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[212] Görres, Joseph. *Ausgewählte Werke in zwei Bänden*. ed. Wolgang Frühwald. 2 vols (Freiburg im Breisgau: Herder, 1978), 2:197.

[213] Ibid.

[214] Ritter, Johann Wilhelm. *Fragmente aus dem Nachlasse eines jungen Physikers: ein Taschenbuch für Freunde der Natur* (Heidelberg: Schneider, 1969; anastatic reprint of 1810 edition), 1:18–, fr[agment. 26.

[215] Ibid., 1:105, fr. 61.

exclusion of the present." The difference between two things should be such that they are different, but still assumed as identical, as the relation between positive and negative electricity shows and as "the voltaic battery most beautifully confirms":[216] "The entire present should be considered the copula of the before and the after … but since it connects the before and the after, it cannot be absolutely before or absolutely after."[217] Rather, it should be understood as an intermediate state between finite difference and complete indifference = 0, that is, as a differential: "Every polarity, every difference, should be thought as temporal, as history and as *differential* of history. All equality [Gleichung] is historical. And, in order for two things to be equal, they must be unequal and in this inequality, are equal."[218]

The connection between the problem of opposition between positive and negative magnitudes and infinitesimal calculus is even more evident in Novalis's numerous entries on physical-mathematical arguments in which infinitesimal calculus is explicitly defined as "polar calculus."[219] Integration is "a positive and negative process that remove the given errors (in reference to the finite magnitudes) through an opposite process."[220] It develops in two directions: on the one hand, it "annuls the differentials." On the other hand, it adds "their apparent differences," that is, their disappearing differences, "until arriving at finite magnitude."[221] The latter is nothing other than the limit towards which the sum of such differences tend as they become ever smaller, while their number increases beyond every limit. Thus, according to Novalis, the formula that expresses the complex sense of infinitesimal calculus is $\frac{a}{\infty} \cdot \infty = a$ "[222] The formula not coincidentally, recalls the formula that Eschenmayer argues is the basis for the relation between attractive and repulsive forces in matter.[223] In fact, it is known that if one subdivides a (e.g., the area of a curvilinear figure) in a *n* number of parallelograms of diminishing size, and if one increases their number infinitely, the final sum of the disappearing parallelograms coincides in every of its parts with the rectilinear figure:[224] "the differential of the infinitely large is related to the infinitely small as its integral."[225]

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[216] Ibid., 2:181, fr. 597.

[217] Ibid., 2:105, fr. 597.

[218] Ibid. On Ritter and the concept of the differential, see Dietzsch, Steffen. "Marginalien zur Leibniz-Rezeption der Jenaer Romantik." In *Beiträge zur Wirkungs- und Rezeptionsgeschichte von Gottfried Wilhelm Leibniz*. ed. Albert Heinekamp. (Wiesbaden-Stuttgart: Steiner, 1986).

[219] Novalis. *Das philosophische Werk II*. eds. Richard Samuel and Hans J. Mähl (Stuttgart: Kohlhammer, 1983), 66.

[220] Ibid.

[221] Ibid., 291.

[222] Ibid., 66.

[223] See above 109.

[224] See Newton, Isaac. *Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World*, tr. Andrew Motte (Berkeley, CA: University of California Press, 1960), 20–30.

[225] Novalis. *Das philosophische Werk II*, 291.

According to Novalis, the totality of nature is a continuous process, based on the same fundamental polarity for which a determinate finite "degree" is actually only a precarious point of equilibrium between a process that augments and a process that diminishes. Thus, for him as well, the very concept of "degree" depends on that of "polarity" because "polarity arises through the decomposition of a degree into its elements.… the properties of a degree confront one another like positive and negative."[226] Such a conflict between a tendency to increase and a tendency to decrease would produce nothing more than the annihilation of both: "*status naturalis polaris est bellum omnium contra omnes* [the polar natural state is the war of everyone against everyone]. Here, the null arises – 0."[227] Yet this null is not absolute nothing. Infinitesimal calculus (a problem I will return to) allows the conserving of the "quality" of the magnitude in the vanishing of its intuitive dimension. The quality is conserved "because one magnitude can be more evanescent than another [verschwindender, als die andere seyn]" such that "nothingness has degrees and, in the reciprocal relations between different 0s, each one of them obtains a relative content" since "the relative something is 0 in respect to an absolute something."[228]

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The above examples should suffice to demonstrate the extensive consequences for *Naturphilosophie* of the idea of the conflict between positive and negative magnitudes, which for Kant represented the schema for modeling the conflict between *realitas* and *negatio phaenomenon*. These consequences were, not coincidentally, noted by another thinker, Jakob Friedrich Fries (1773–1843), who began to appear on the German philosophy scene during those years. Fries, as I will show, also elaborated a different conception of philosophy of nature than that of the Romantics. Fries correctly explains that *Naturphilosophie* seeks to reduce all phenomena to "a dualism of nature … that should be explained through the conflict between negative and positive principles" (FSS 5:139). This conception, as he observes in his early work *Reinhold, Fichte und Schelling* from 1803, "was explicitly elaborated by Schelling in his book, the *Weltseele*, but was also expressed in the same period by two bright [Hellsehende] and promising youths, Mr. Görres from Koblenz and Mr. Winkelmann from Göttingen" (FSS 24:514), who, as discussed above, based their speculative conception of nature precisely on the "conflict of the positive and the negative, of the mathematical + and –" (FSS 24:513).

Fries thus clearly recognizes what seems to be the fundamental principle at the basis of Romantic philosophy of nature. At the same time, however, he denounces what he finds to be a substantial unsustainability in it: "if I should limit opposed activities in such a way that through their limitation a product arises, then these

[226] Ibid., 342–.

[227] Ibid.

[228] Ibid., 291. On Novalis and mathematics, see Hamburger, Käte. "Novalis und die Mathematik. Eine Studie zur Erkennistheorie der Romantik." In *Philosophie der Dichter: Novalis, Schiller, Rilke*. (Stuttgart: Kohlhammer, 1966). See ibid, 13- for a particularly interesting comparison with the Marburg school.

cannot be considered a pure mathematical + and − because their product would not be a continual coming and vanishing, but rather a permanent = 0, and thus, in this manner, nothing is explained" (FSS 24:192). Even the solution to this problem that the philosophers of nature tried to provide, maintaining the possibility of thinking this 0 as an infinitely small difference, is devoid of any foundation, according to Fries. If one holds the latter point of view, the finite, which should, for Schelling, manifest itself as the "inhibition [Hemmung] of infinite productivity" and which is thus "explained through opposed tendencies" (FSS 24:206), would be reduced to being "infinitely small" and thus would not be a finite product. From whichever angle one considers the problem, "through a mere opposition of forces (pure + and −), nothing is constituted" (FSS 24:206).

Fries is certainly aware of the magnitude of the project that forms the background of *Naturphilosophie* and of Schelling's work. He even recognizes that "Schelling's philosophy of nature or speculative physics is the only original and great idea that, since the publication of Kant's primary writings, has appeared in Germany in the field of free speculation. Here, for the first time since the new formation [Ausbildung] of natural science, the complexity of physics has been grasped in a glance" (FSS 24:179). At the same time however, he realizes that as ambitious as the project is, its supporting foundations are fairly uncertain. A few years later in his most important work, *Neue oder Anthropologische Kritik der reinen Vernunft* (New or Anthropological Critique of Pure Reason), first published in 1809, Fries affirms that Schelling's philosophy of nature is founded on the idea that "all external phenomena are the product of forces in opposition that limit [beschränken] each other." Such an opposition is expressed by means of "mathematically opposed magnitudes" that "are the magnitudes of the product of the one and the other force, of which the one is measured through the negative numbers of the other" (FSS 5:140). This conception is undermined not simply by the fact that such a conflict could never allow anything but an empty zero to emerge (see FSS V, 124). Even if the problem could be somehow eluded, the project of *Naturphilosophie* would only have sense "if the forces were opposed as an increase or decrease in homogeneous" (FSS 5:140–); that is, only "where all qualitative differences are referred back to purely quantitative differences" (FSS 5:141). While philosophy of nature certainly recognizes this requirement, it does not possess adequate tools to arrive at this result: the reduction of qualitative differences to quantitative differences, according to Fries, is "only possible through mathematical physics" (FSS 5:141).

Fries harshly condemns philosophy of nature's "fantastic conception and its indifference towards mathematics." Even if a few of his manuscripts note the "talent and the skilled presentation of the author" in Schelling's work, Fries does not hesitate to openly affirm that Schelling "possesses no mathematical sensibility and no awareness of the mathematical doctrine of nature" (cited in FSS 13:21*). This lack completely compromises his project of constructing a "philosophy of nature": "we must pass through the mathematical route a lot or a little we are going to be able to arrive through it" (cited in FSS 13:21*).

Fries credits Schelling for attempting to construct a "metaphysics of nature" capable of illuminating the fact that the fundamental laws of nature "are of philosophical and

metaphysical origin" (FSS 15:22). Or rather, Fries admits that "in science's actual state of development," its own conception "finds itself among the many explanations of so-called *dynamic philosophy* or *philosophy of nature*, such as that which is developed on the basis of Kant's doctrine by Schelling and by Schelling's school" (FSS 15:22). However, Schelling and his followers were completely devoid of the tools necessary to realize such a project because they were incapable of mastering the requisite mathematical knowledge: "for him and for his school, a few mathematical signs and a few technical expressions became a toy" that seemed to work openly "against mathematical precision and, in particular, against the Newtonian school" (FSS 13:508).

Backed by a solid scientific awareness that was recognized and appreciated by his contemporaries (e.g., Georg Friedrich Gauss; see FSS 13:39n62-*), Fries was able to distance himself from the speculations of *Naturphilosophie* and to insist on "calling science which my attention is turned to, *mathematical* philosophy of nature" (FSS 13:V).[229] From this point of view, the fundamental philosophical motive of Romantic philosophy of nature; that is, the reduction of qualitative differences to quantitative differences; should be realized through a conception of natural science in which the speculative elements give way to the tools offered by mathematics. Referring to the Newtonian concept of nature, Fries certainly highlights the fundamental concept of equilibrium between opposed forces. But he maintains that these should be expressed above all through the principle of action and reaction, which is at the basis of static laws (such as in the lever) and of mechanics (such as in collisions);[230] that is, the laws by which parts of matter communicate movement with each other, not by which matter itself is constructed by means of attractive and repulsive forces. Fries argues that "Schelling," in contrast, "commits the error of eliminating material substance, mass, from Kant's construction as the fundamental concept and of wanting to complete such a construction through opposed forces alone" (FSS 13:508). Thus, Schelling resorts to "a useless mathematical thought," the idea of an "attracting and repelling, without anything that is repelled or attracted" (FSS 13:508).

The spirit of Kant's project of referring all qualities back to local movement and moving forces that determine the variations in it is grasped once again with precision: "the being of things in space and their reciprocal relations is referred back to the entirely explicable relation of friction and collision such that every empirical quality that is initially manifested to external sensation is reduced to movement" (FSS 5:175). From this point of view it is correct to maintain that "all real multiplicity in space should be conceived through a mere quantitative difference," as Schelling and philosophy of nature do. This implies, however that "all qualities of the external senses should be resolvable by merely referring them back to simple quantitative

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[229] See Bonsiepen. *Die Begründung einer Naturphilosophie bei Kant, Schelling, Fries und Hegel: mathematische versus spekulative Naturphilosophie*, 320. For Fries's critique of Schelling, see ibid., 45-, and for the foundations of a mathematical philosophy of nature, see ibid., 45–. See also, Kay, Hermann. *Mathematische Naturphilosophie in der Grundlagendiskussion. Jakob Friedrich Fries und die Wissenschaften* (Göttingen: Vandenhoeck und Ruprecht, 2000).

[230] See Apelt, Ernst Friedrich. *Metaphysik.* [Metaphysics] (Halle: Hendel, 1910), 281.

relations of the filling of space and of movement in space" (FSS 5:175). If "matter appears to me immediately as hot or cold, as sound, color; I objectively recognize, in variations in of temperature, oscillating and radiant movement. I refer it back physically to a quantitative difference" (FSS 5:175).

Such a "resolution of external qualities" does not in any way signify "an explanation of them." It only implies "[that] the qualitatively diverse [is replaced] with the quantitatively different, without the first being renounced [aufzugeben] as a result" (FSS 5:175). The constructions of matter proposed in the *Metaphysical Foundations of the Natural Sciences*, which still remains Fries's principal source of inspiration, does not in any manner signify "an explanation of the qualitative differences in matter because this would be an impossible request, but rather a representation [eine Aufstellung] of the actually present differences under the conditions for a Kantian construction" (FSS 24:178). The possibility of reducing differences in matters to "the conflict between carbon and nitrogen" and to establish "a connection between these two substances and the poles of a magnet, unfortunately Steffens's preferred philosophical idea, has not yet been proved anywhere" (FSS 24:243).

One should not attempt to deduce the qualitative differences in matter from a single series of quantitative differences, according to the model that Eschenmayer's work exemplifies and that Schelling argues is inconsistent. One should limit oneself to recognizing that "we arrive at a complete scientific understanding only when, in the understanding of things, we attain the characters and the states of form and of movement and when we completely do without the sensible qualities of colors and sounds" (FSS 13:10). One should not attempt an impossible deduction of all sensible qualities *a priori* from some unifying principle. Rather, "independently of sensible qualities," one should "develop a mathematical understanding of the things outside of us" (FSS 14:282). The "senses only show us the different qualities of the objects outside of us," whereas the objects outside of us, or "the bodies, we represent through a pure mathematical intuition as mobile masses in space with form" (FSS 14:277).

As the pages dedicated to this theme in *System der Metaphysik* (1824) demonstrate, Fries intends to recuperate the authentic relation between quality and quantity that Kant had established in the Anticipations of Perception: "we only know the modes [Arten] of quality empirically," while we regard "as an *a priori* characteristic, the fact that every mode [Art] still admits a multiplicity in the homogenous, a determination of magnitude of degrees" (FSS 8, 289). The above distinction is particularly important for Fries as well as for his most famous student, Ernst Friedrich Apelt,[231] who insists on this point with equal resolve in his *Metaphysik*. They both distinguish "the quality of things recognized through sensible intuition in sensation"[232] from "the degree," which "is the only mathematical [determination] of quality" while "all the rest is empirical."[233]

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[231] For a brief portrait of Apelt, see Groß, Stefan. "Ernst Friederich Apelt." In *Naturphilosophie nach Schelling*. eds. Thomas Bach and Olaf Breidbach, 1–18, 2005). See also Bonsiepen. *Die Begründung einer Naturphilosophie bei Kant, Schelling, Fries und Hegel: mathematische versus spekulative Naturphilosophie*, 400.

[232] Apelt, Ernst Friedrich. *Metaphysik* (Halle: Hendel, 1910), 178.

[233] Ibid.

The Anticipations of Perception establish something *a priori* concerning quality: that it has a quantity, a degree. However, the principle does not claim to dissolve the identity of quality into a series of quantitative differences that should allow the one to pass into the other without interruption. The principle merely affirms that, to use Fries's words again, while "we know the qualities of things only through sensible intuitions, in internal perceptions," we know *a priori* that "the phenomenal increasing and disappearing of qualities, for example of air, of sound, of luminosity, requires their degrees to be passed through in a continuous manner, from zero onward, or backwards until zero" (FSS 8:289).

For example, "the Anticipations of Perception or the law that every real has a degree" can be applied "to the degrees of density with which all matter occupies a space such that beyond the zero of empty space, infinitely greater degrees are possible" (FSS 13:449). If "we posit *M* and *m* as the masses, *V* and *v* as the corresponding volumes, and *D* and *d* as the densities of the two bodies, then we have $D : d = \frac{m}{v} : \frac{M}{V}$" (FSS 13:449). Thus, the degree is determinable through the relation of two factors: mass and volume. This relation is constant when the mass is distributed in a constant fashion in a given volume and inconstant when the mass is distributed in an uneven manner. Analogously, one can affirm that "velocity is … an intensive magnitude of a determinate degree below which there are lesser ones and above which there are greater ones" (FSS 13:417–). If the degree of velocity remains constant, then "for diverse uniform movements," one can posit "the spaces S and s and the corresponding times *T* and *t* and the velocities *V* and *v*, thus giving $V : v = \frac{s}{t} : \frac{S}{T}$" (FSS 13:418).

The above example clearly shows that the relation between finite differences is sufficient to compare uniform movements, measuring the average velocity that a moving object maintains during a finite period of time. In contrast, for non-uniform movements, in which the degree of velocity can continuously vary from instant to instant, one "can assume, for a differential of the movement, the movement as rectilinear and uniform" (FSS 13:292). Moreover, "since every non-uniform change in its infinitely small parts can be judged as uniform, we obtain for every movement the differential equation $v = \frac{ds}{dt}$; the velocity in every moment of its movement is equal to the ratio of the differential of space to that of time" (FSS 13:418).

The simple notations cited above only offer a peak at the relation between the *Anticpationen der Wahrnehmung* and infinitesimal calculus, a theme that I will discuss further. More interesting is how Fries furnishes an interpretation of "higher analysis" free of speculative pretensions; such as those of Maimon and Novalis as well as the even more articulated speculations of Hegel, whom I will return to further below. In contrast, Fries offers a detailed "technical" presentation of infinitesimal calculus, demonstrating his mastery of the mathematical literature of his time. Here, he enters into a very interesting discussion on the classical problem of the *status* that should be attributed to infinitely small magnitudes.[234]

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[234] See Hogrebe, Wolfram and Hermann Kay, eds., *Jakob Friedrich Fries Philosoph, Naturwissenschaftler und Mathematiker. Verhandlungen des Symposions Probleme und Perspektiven von Jakob Friedrich Fries Erkenntnislehre und Naturphilosophie vom 9.–11. Oktober 1997 an der Friedrich-Schiller-Universitat Jena* (Frankfurt am Main: Klostermann, 1997).

Fries relies on the idea that the continuum does not consist in the *de facto* sum of existing parts, but rather in the *possibility* of its subdivision: "the infinite is the inexhaustible, and an infinite largeness and an infinite smallness can never be considered a completely given whole" (FSS 13:258). Thus, "an infinitely large is just as impossible as an infinitely small" (FSS 13:258). This is true as much for space and time as for differences in degree: at the basis "of our conception of the real in time according to degrees," writes Fries, is "the form of continuous intensive magnitude, of which there is no largest or no smallest one." In the same way, it is the foundation of the "law of change in nature, that every change of a determinate degree … must have passed through all the smallest degrees" (FSS 13:272).

However, the question remains, as Fries argues quoting Adolf Kästner: "how does the final state arise from the first? – through an intermediary. And this from the first? Again through an intermediary. And this from the first? Again through an intermediary."[235] Admitting that something new arises from a given state thus seems impossible. According to Fries, however, "the discrete steps [Auffassungen] from which the questions follow one after the other" (FSS 13:274) do not pertain to the law of continuity as if change is composed of an infinite number of smaller changes through which it must necessarily pass. Rather, finite change, which determines a determinate difference in degree, should be conceived as a "connected totality" given in intuition while "the discrete conception of separate intermediate states, which always admit an intermediary" is an operation that one performs on this totality. Thus, one should not begin with a series "of entirely separate particles" (FSS 13:273–) that are given as existents, and then successively compose a totality from them. On the contrary, one should start from "a continuous whole" within which possible subdivisions can be thought: "the continuity of magnitude" signifies nothing other than "its infinite divisibility" (FSS 13:274).

If "the scientifically determined concept of an infinitely small magnitude is thus called *differential*," then "the correct use of this concept depends on the fact that the infinitely small" exactly like the infinitely large, "*cannot be considered a given whole*" (FSS 13:276). If I regard the "differential as an infinitely small part of a magnitude, I contradict myself because a given part that is smaller than any part that can be given is a contradiction" (FSS 13:276). In the same way, the "representation of the sum of infinite parts that are infinitely small in a finite magnitude, where the infinitely small is presupposed as the first simple part," is entirely unacceptable (FSS 13:276). Rather, in any continuous finite change, "the increments can be thought as small as one likes, as smaller than a determinate magnitude" (FSS 13:292). Neither a maximum nor a minimum exists neither in space, nor in time, nor in degrees; but rather everything is resolved into relative differences in respect to which one can think of differences as large or as small as one likes.

The above considerations are surprising for their rigor and their modernity,[236] and they seem to interpret correctly a notion of continuity that is already found in

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[235] Kästner, Abraham Gotthelf. *Anfangsgründe der höhern Mechanik: welche von der Bewegung fester Körper besonders die praktischen Lehren enthalten* (Göttingen: Vandenhoek, 1766).

[236] See: FSS 13:36*.

Leibniz and that returns in Kant. Here, I will merely emphasize that the speculative conceptions of infinitesimal calculus, which Maimon provided the first examples of and which Hegel later reintroduces during the epoch of the "return to Kant", seem to be preoccupied with another problem that Fries leaves in the shadows, the problem of rigorously fixing the *qualitative* unity of the given "whole" whose parts should only be considered possible subdivisions. Infinitesimal calculus seems to have been the apt tool to determine conceptually the very "qualities" that Kant, as well as his faithful interpreter Fries, considered to be irreducible empirical facts.

3.5 The Absolute as Quantitative Indifference

"Every quality should be overcome by the omnipotence of quantity" (FSS 24:490) because "every relative dynamic quality is only a quantitative difference" (FSS 24:490). With these words written in 1803, Fries, though polemically simplifying the positions of his opponents, effectively grasps the principle hidden behind the doctrines of the philosophers of nature. Considered from this point of view, no matter how arbitrary and inconsistent it may seem compared to the "standard" of Newtonian science, Kant's conception of the relation between reality and negation is brought to its extreme consequences in the *Naturphilosophie*; the conception that Kant conceived in terms of the model of the relation between opposed magnitudes and for which philosophy of nature believed to have found a sensible and intuitive representation in the opposition between the poles of a magnet.

As Fries himself, a member of Schelling's generation, exemplifies, these elements of Kant's thought could have been developed in a completely different direction, in which their "critical" sense, respectful of the autonomy of particular sciences, was not irremediably lost in the attempt to erect an ambitious metaphysical construction in which it was believed that even particular scientific questions could be mastered. Nevertheless, in order not to lose sight of the general "philosophical" meaning in the background of these attempts to develop Kant's conception of the reality-negation connection, one should not become distracted by the multiplicity and the strangeness of the doctrines that *Naturphilosophie* claims to deduce.

In *Darstellung des philosophischen Empirismus* (Presentation of Philosophical Empiricism), one of Schelling's last writings, posthumously published in 1861, Schelling explains the core of the question with particular clarity:

> [D]uring the first spark of joy felt in the moment of the first discovery of this general unity of the subject and the object, this relation was compared to the general polarity that is found in nature. Since the powers that are united to each other in a magnet are in such a relation that one flees that which is homologous and equal and avidly attracts that which is opposed to it. Such a retention of its own contrary is the first sign of life in nature. Thus it was admitted that the concrete, the real, exists and is affirmed only in as much it is neither pure subjectivity nor pure objectivity, but rather an objectivity which, so to speak, attracts subjectivity to itself" (SW 10:286).

Thus, the metaphor of the magnet; that is, the distinction between +, −, and an intermediate point = 0 that separates them; is the model for the very relation between the subject and the object:

> Since every magnetic line (as big or small as it may be) does not contain in its entirety more than three points [the two poles and a central point called the point of indifference] …. Since in the entire magnetic line there is therefore only these three points but each part of the line considered by itself contains these three points again and every point, depending on how it is considered, is the north pole, the south pole or the point of indifference, and since the determination of each point is thus purely relative, in the totality of the universe there is nothing that is absolutely objective in respect to another thing (SW 10:286).

The relation between knowing and the being that is opposed to it, between the known and the unknown, can be thought precisely through this new conception of opposition: "[W]ithout doubt," continues Schelling, "philosophy of nature has perceived this fact more profoundly than any preceding philosophy … That which is posited outside of consciousness is in essence the same thing that is posited in consciousness" (SW 10:285). The subject and the object are not distinguished as two distinct "things." They are in their "essence" one and the same and can only be distinguished reciprocally such that "every point of the line is a point of indifference, or a positive or negative pole, depending on how it is related to the other points [je nachdem er bezogen wird]" (SW 7:185n). Only because the object is fundamentally identical to the subject can a mediation between the subjective sphere and the objective sphere, and a transition from one to the other, be thought: "starting from that which is only cognizable … to that which contains a greater number of determinations and finally to the cognized itself; the substance, id quod substat, always remains the same" (SW 10:286).

If a magnet is split at a point along its side, the parts obtained do not possess opposite charges. Rather, each exhibits poles of contrary sign at their new extremities. No matter how many times a magnet is split, each piece is always a complete magnet: "If I break [zerstückle] a magnet, which conserves this schema in itself in a purer manner, into small pieces, then each part, "writes "Schelling, "event the littlest part, is a new magnet. That is, it has two new poles and a point of indifference" (SW 6:291). There is no "internal" property that allows the positive pole to be distinguished from the negative pole (as if the north pole possessed certain characteristics that the south pole lacked and vice versa). They "are" only in their reciprocal opposition. Thus, Schelling does not hesitate to affirm that "all matter should be considered to be an infinite magnet" (SW 4:153). In the same way that no point on a magnetic bar can "accurately be called positive or negative, or the point of indifference" (SW 7:185). Every difference in matter in general should only be considered a respective and relative difference.

Here, the problem already raised by Fichte seems to emerge again in a new form, the problem of substituting the logical opposition between the subject and object with a real opposition: "If we conceive the objective self (the thesis) as absolute reality," – writes Schelling, "its opposite will have to be absolute negation." (HKA 9:9:86).[237] This form of opposition does not allow any conciliation between the subject and the

[237] English translation from Schelling. *First Outline of a System of the Philosophy of Nature*, 157.

object. The opposition must be such that "[t]his conflict is a conflict of activities originally opposed, not so much in subject as in direction, for both are activities of *one and the same self*" (HKA 9:9:86; emphasis mine).[238] The subject certainly negates the object, not in the way that not–*A* is the negation of *A*, but "only through real opposition 1 + 0 = 1, 1 − 1 = 0" (HKA 9:9:72).[239] In the same way that, in this form of opposition, + only has sense in relation to − and vice versa, "[t]he subject asserts itself only in opposition to the object, and the object only in opposition to the subject; neither, that is, can become real without destroying the other, but the point of destruction of one by the other can never be reached, precisely because each is what it is only in opposition to the other. Both have therefore to be united, for neither can destroy the other, and yet nor can they subsist together" (HKA 9:9:85).[240]

The model that Schelling chooses to clarify the opposition between subject and object is not that of "non-contradiction," but once again that of "equilibrium": the opposition "[therefore] has to be thought of, not as an annihilation of the two activities by each other, but rather as an equilibrium to which they reduce one another, and whose continuance is conditioned by the persistent rivalry" (HKA 9:9:92).[241] To clarify this point, Schelling resorts to the example of the lever in which "the two weights merely act upon the fulcrum, which is thus the common substrate of their activity" (HKA 9:9:93):[242]

> In order for it [the lever] to remain in balance, equal weights must bear upon it at both ends, at equal distances from the fulcrum. Each individual weight acts, but cannot achieve its effect (it does not appear as active); both are confined to the common effect. So in intuition. The two activities that preserve equilibrium do not thereby cease to be activities, for the equilibrium only exists insofar as both are actively opposed to one another, only the product is static (HKA 9:9:135).[243]

The possibly of thinking the subject and the object together is not based on the suppression of every opposition between concepts, but on the equilibrium between opposed forces: "Thus if the I could reflect at this present stage upon its construction, it would find the latter to be a composite of two forces maintaining an equilibrium, of which one on its own would produce the infinitely large, while the other in its unrestricted form would reduce the product to the infinitely small" (HKA 9:9:135).[244] The two activities that are maintained in "equilibrium in the product … can appear only as fixed, static activities, that is as forces." (HKA 9:9:137).[245] Consequently, "[t]he

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[238] Ibid.

[239] Ibid., 36.

[240] Ibid., 46.

[241] Ibid., 51.

[242] Ibid.

[243] Ibid. For a detailed analysis of the importance of the "model" of the lever in Schelling's philosophy and its relation to that of the magnet, see Ziche, Paul. *Mathematische und naturwissenschaftliche Modelle in der Philosophie Schellings und Hegels* (Stuttgart-Bad Cannstatt: Frommann-Holzboog, 1996), 210–12.

[244] Schelling. *First Outline of a System of the Philosophy of Nature*, 83. Translation slighty modified by the author.

[245] Ibid.

first of these forces will be by nature positive, so that if unrestricted by any opposing force it would expand out to infinity" (HKA 9:9:135).[246] However, "[t]hat it is actually retained in a finite product, is explicable only through an opposing, negative, restraining force" (SW 3:441).[247] The identity between subject and object is not based on the absence of conflict and opposition, as in the model of the unity of a logically possible (i.e., non-contradictory) concept, but on the equilibrium of two activities equal in "nature" but opposed in "direction." The latter model is exemplified by the lever or the magnet in which + and − limit each other in a point of indifference = 0.

So-called "philosophy of identity" reaffirms this conception, developing it in a direction that is significantly distant from the original "Fichtean" formulation that characterizes the *System*. Particularly in his *Darstellung meines Systems der Philosophie* [Presentation of my Philosophical System] (1801), Schelling seems to arrive at the extreme speculative consequences of Kant's principle, by which reality and negation are only distinguished in terms of "degree" or "quantity," and not by "quality" or "essence:"

> Since it is the same equally absolute identity that is posited as subject and object, the difference is consequently not qualitative. Therefore it remains true that, since no differentiation of the two is possible in respect to being itself (in fact, absolute identity is equally unconditioned as subject and object, and is thus also the same in respect to essence), only a *quantitative difference* can be admitted, that is, a difference that occurs in terms of the *magnitude* of being and specifically in such a way that the one and the identical is posited, but with a preponderance of subjectivity [cognition] or of objectivity [being]."(SW 3:123)

In his 1804 *System der gesammten Philosophie und der Naturphilosophie insbesondere* (System of the Whole of Philosophy and the Philosophy of Nature in Particular), Schelling reaffirms this point with equal clarity:

> *In the whole* [All]*, no essential or qualitative difference is thinkable*. A qualitative difference would exist only if the subjective and the objective could be different in terms of *essence*, in terms of substance. But the subjective and the objective are *one and the same* and *are* nothing in themselves, if not the infinite identity of both. Thus, in the infinite, the subjective as subjective and the objective as objective can never be posited, but rather everything that is in the whole, in as much as it is, it is necessarily universal, eternally equal, indivisible essence of all being. Therefore, no difference in terms of essence can be thought in the whole (SW 6:179).

Here, it is interesting to highlight how the model of real and quantitative opposition permits Schelling to conceive the absolute, the infinite, not as "a pure identity in which nothing is distinguishable," and in which there is no opposition because the identity itself would be annihilated if it was conceived purely logically, but rather as "a perfect quantitative equilibrium of subjectivity and objectivity <of the real and the ideal>" (SW 4:127). Again, Schelling emphasizes that this is precisely what occurs "in the lever whose fulcrum [Ruhepunkt] … represents the equilibrium of the two opposed tendencies. This is what unites [das Vereiningende] both, but

[246] Ibid.

[247] Ibid.

which is not in any way absolute identity and it is, what it is (that is, the fulcrum [Ruhepunkt]) not in itself, but only relatively to both the opposed activities. These are reciprocally reduced to zero in it, but not the fulcrum itself: as it is the positive zero of both" (SW 7:155-). Thus, the formula $A = A$ that Schelling uses to express the nature of the absolute does not indicate logical identity, but rather quantitative equivalence. It seems to be nothing other than a reformulation of Kant's expression $A - A = 0$ through which the absolute essence is not reduced to a merely analytic identity (founded on non-contradiction), but rather to a synthetic unity (in which differences are not annihilated but are necessarily connected together).[248]

In his 1795 *Philosophische Briefe über Dogmatismus und Kriticismus* (Philosophical Letters on Dogmatism and Criticism), Schelling explains with exceptional clarity how this represents the core of Kant's philosophy:

> The *Critique of Pure Reason* began its struggle with this point: How in general do we manage to judge in a synthetic matter? Kant himself asks this at the beginning of his work. And this phrase is at the basis of his entire philosophy as the true point in common at the base of all philosophy. Expressed in other words, the question seems to be: How in general can I succeed in going out of the absolute to one of its opposites? (HKA 1:3:60)

How can "knowledge" leave itself to access that which is opposed to it as other than itself, as "being"? How can one pass from the known to the unknown if the second is completely irreducible to the first? Philosophy of identity seems to ultimately provide a response to these questions by demonstrating "that a going out of the absolute from itself, in any determinate manner, is absolutely unthinkable. This is, like the unity and the intimate affinity of all things with each other and with the divine essence, a further axiom of true philosophy (SW 4:390).

If *A* and *B*, which are irreducible to each other, should be synthetically connected according to Kant's conception, then *B* is not simply not–*A*, "since being *A* is negation for *B*. In the being A, B perishes." However, B "is not at any point purely" *B*, but always only something intermediate between *B* and *A"* (SW 10:173). "[T]his is something that always oscillates between being and non-being, between positing and negation" (SW 10:173). Thus, *A* and *B* "cannot be posited in themselves, but only the 'one' and the 'same', the preponderant objectivity and subjectivity together with the quantitative indifference of both" (SW 4:136). They are not distinguished "according to their nature," but only "in terms of opposite directions" (SW 4:137) in the same way as the poles of a magnet, which are "identical" in terms of essence even though they are "different" from one another. The fundamental question of all philosophy, the problem posed in *Philebus*, is how the indeterminate can go beyond itself and become determinate, how the undifferentiated infinite can become finite difference. Here, it is expressed in terms of the problem of establishing how, from

[248] On the significance of Schelling's use of the metaphor of the magnet and the lever in his philosophy of identity, see Rang, Bernhard. *Identität und Indifferenz: eine Untersuchung zu Schellings Identitätsphilosophie* (Frankfurt am Main: Klostermann, 2000), 116–41 in particular. The book provides evidence of the influence of Kant and Eschenmayer's dynamic conception of matter, and the concept of real opposition, on philosophy of identity. On the formula $A = A$ as a reformulation of $A - A = 0$, see 136.

negation = 0, one can arrive at finite difference: "the quantitative difference between the subjective and the objective is the basis of everything finite, and, vice versa, the quantitative indifference between both is the infinite"(SW 4:131).

3.6 Hegel and the "Mathematics of the Infinite"

The reduction of quantitative difference to qualitative difference is, as Fries already recognizes, the common characteristic of post-Kantian philosophy. Despite scorning the "superficiality" of Fries's philosophy (HW 7:18n),[249] Hegel – who was nevertheless well repayed by Fries's writings with evocative titles like the 1815 *Nichtigkeit der Hegelschen Dialektik* (The Nothingness of Hegelian Dialectic) – seems to present a very similar opinion about the essence of *Naturphilosophie*. In the *Phenomenology of Spirit* (Phenomenologie des Geistes), Hegel offers a particularly lucid reconstruction of this aspect of the problem. Philosophy of nature, according to Hegel, sought to substitute differences in quality for which "one existence is distinguished from another … is for itself, or … subsists through this simple oneness with itself" (HW 3:52)[250] with simple differences in quantity: "immediate sensuous being is immediately one with the determinateness as such, and therefore expresses a qualitative difference in that being, e.g. blue against red, acid against alkaloid, etc." When one considers it in a scientific manner, in contrast, these differences between sensible qualities are reduced to mere differences in magnitude: "the way in which difference, qua inert, expresses itself is just this, that it is an indifferent difference, i.e. difference as magnitude" such that, "the qualitative element falls solely in the magnitude" (HW 3:212).[251]

The different sensible qualities that the senses communicate (wet and dry, light and dark, hot and cold, dense and thin, etc.) are regarded at first as fundamental properties of things themselves. The qualities of sensation are converted into objective principles that exist in themselves and that can act on their own, and an independent substantial being is attributed to each of these properties. A historical example of this process is found in Bernardino Telesio's philosophy of nature, in which hot and cold are elevated to supreme principles from whose combination all natural phenomena arise: "the sun, emanating only heat" is opposed to the earth that "emanates cold, produces effects opposed to those of the heat of the sun." Thus, it "is equally evident that both – the hot and the cold – are states endowed with the quality of rejecting the other and its faculties and conditions from itself."[252] The introduction of the concept of temperature completely alters this perspective.

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[249] English translation from Hegel, Georg Wilhelm Friedrich. *Elements of the Philosophy of Right*. eds. Allen W. Wood and Hugh Barr Nisbet (Cambridge: Cambridge Univerity Press, 1991), 15n.

[250] English translation from Hegel, Georg Wilhelm Friedrich. *Phenomenology of Spirit*, tr. Arnold V. Miller and John Niemeyer Findlay (Oxford: Clarendon Press, 1977), 33.

[251] Ibid., 168.

[252] Telesio, Bernardino. *De rerum natura iuxta propria principia libri IX*. ed. Cesare Vasoli (Hildesheim: Olms, 1971). Reprint, of the 1586 Naples edition, 1:1–6.

Through it, differences between hot and cold can be reduced to a difference of more or less in which no privileged point that separates the one from the other can be found. Thus, in Hegel's words, their reciprocal difference is "a difference which is no difference, or only a difference of what is selfsame [des Gleichnamigen], and its essence is unity" (HW 3:130).[253] A quantitative difference that is defined in respect to an identical "substrate" is involved, "a difference belonging to the thing itself" (HW 3:125).[254] In this case, the substrate is temperature, in respect to which the opposition between hot and cold is reduced to a difference "which is not only not a difference for us, but one which the movement itself cancels as a difference" (HW 3:125).[255]

According to Hegel, the history of scientific thought offers a variety of examples of this conception:

> For example, negative electricity, which at first comes to be known, say, as resin-electricity, and positive electricity as glass-electricity, these, as a result of experiments, lose altogether such a significance and become purely positive and negative electricity, neither of which is any longer attached to a particular kind of thing; and we can no longer say that there are bodies which are positively electrical and others which are negatively electrical (HW 3:193).[256]

The two electricities, that were first regarded as two opposed substances, two different imponderable fluids opposed to each other, can now be defined as such in relation to each other: "Electricity itself is not difference per se, or is not in its essence the dual essence of positive and negative electricity" (HW 3:193).[257] Rather, "electricity is indifferent to positive and negative electricity" (HW 3:193)[258] and the difference between the two electricities is created in an entirely relative manner when a difference in potential is constituted. Thus, such a difference can be represented as a difference of + and − that expresses the twofold and opposed tendency to reconstitute the equilibrium = 0. The same applies to the differences between acids and bases in chemistry, which do not involve two different "substances" or determinate properties that can "remain apart on their own and as such be pointed out." Acids and bases "are only this relation" and the "essential nature of theirs" is "to pass over immediately into a neutral product…. Just as glass and resin can just as well be positively as negatively electrical, in the same way acid and base are not tied as properties to this or that actuality; each thing is only relatively acid or base" (HW 3:194).[259]

Studies of organic nature, according to Hegel, offer equally appropriate examples of such a process, in particular in investigations of the relations between sensibility

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[253] Hegel. *Phenomenology of Spirit*, 99.

[254] Ibid., 94.

[255] Ibid., 95.

[256] Ibid., 153.

[257] Ibid., 92.

[258] Ibid., 99.

[259] Ibid., 153.

and irritability. This doctrine, formulated by Albrecht von Haller and reaffirmed in Schelling and in Romantic philosophy in general, exhibits once again the same schema that prevails in inorganic phenomena:

> If they are distinguished, as they necessarily are, this is in accordance with the Notion, and their opposition is qualitative. But when, apart from this true difference, they are also posited as they immediately are, and for ordinary thought, as they might be as aspects of the law, then they appear as *quantitatively* distinct. Their peculiar *qualitative* antithesis thus becomes one of magnitude, and there arise laws of this kind, for example, that sensibility and irritability stand in an inverse ratio of their magnitude, so that as the one increases the other decreases; or better, taking directly the magnitude itself as the content, as its smallness decreases. (HW 3:205–; emphasis mine)[260]

It is a matter of indifference whether an organic phenomenon is regarded as irritability or as sensibility and "this is so in general and equally when its magnitude is under discussion. Similarly, it is a matter of indifference whether we speak of the increase of a hole as an increase of the hole qua emptiness, or as an increase of the filling removed from it" (HW 3:206).[261] What appears to sensible intuition as a qualitative opposition is reduced to the quantitative opposition that exists between positive and negative magnitudes: "Or again, a number, e.g. three, remains the same quantity whether it is taken positively or negatively … just as the south pole of a magnet is exactly as strong as its north pole, or a positive electricity, or an acid, is exactly as strong as its negative, or the base on which it acts" (HW 3:206–).[262]

The philosophical significance of the transition from quality to quantity can be appreciated by taking into account that, for Hegel, this transition represents the characteristic feature of post-Kantian thought: "In the more abstract exposition of the Kantian philosophy, or at least of its principles, namely in Fichte's *Wissenschafstlehre*," writes Hegel in the *Science of Logic* (Wissenschaft der Logik), "the first axiom, I = I, is followed by a second, independent of it, the opposition of the non–I; the relation between the two is also directly assumed as a quantitative difference, that is, the non–I is partly determined by the I, and partly not" (HW 5:268-).[263] Fichtean philosophy, according to Hegel, "which is powerless to overcome the *qualitative opposition* between the finite and infinite …, has recourse to *quantity* in order to use it as a mediator, because *it is sublated quality, the difference which has become indifferent*. But since both members of the antithesis remain implied as *qualitatively* distinct, the fact is rather that each is straightway made indifferent to this alteration, because it is as *quanta* that they are related to each other" (HW 5:268; emphasis mine).[264]

Schelling's philosophy of identity reintroduces this merely quantitative opposition in another form:

> Because the quantitative is determinateness posited as sublated it was thought that much, or rather everything, had been gained for the unity of the absolute, for the one substantiality,

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[260] Ibid.

[261] Ibid., 163.

[262] Ibid., 164.

[263] English translation from Hegel. *Hegel's Science of Logic*, 232-. Translation slightly altered by the author.

[264] Ibid., 232.

> when opposition generally had been reduced to a *merely quantitative difference* … [T]he opposed determinations *have the same nature*, the same content; they are real sides of the opposition in so far as each of them has within it both determinations, both factors of the opposition, only that on one side one of the factors preponderates, on the other side the other, that is, one of the factors, a material substance or activity, is present in a *greater quantity* or in an *intenser degree* in one side than in the other" (HW 5:269; emphasis mine).[265]

As I have shown, this general schema, which Schelling retrieves from the very heart of philosophy of nature, becomes the structure of the absolute itself: "The difference of the absolute unity is supposed to be only quantitative …. When being and thought are represented as quantitative determinations of absolute substance they too, as quanta, become completely external to each other and unrelated as, in a subordinate sphere, do carbon, nitrogen, etc." (HW 5:269).[266] In other words, the difference "is merely quantitative external difference; there are two distinct quanta of one and the same substrate" (HW 5:446).[267] "Their quantitative difference is that indifference in accordance with which they continue themselves into each other and this continuation as the self-sameness of the qualities is in each of the two unities" (HW 5:447).[268] Here, "[t]he substrate itself" should be understood "as an indifference" (HW 5:446)[269] in respect to which "each side is in its own self an inverted relation." From this, "it follows that they [the two qualities] are in equilibrium; that by as much as the one increases or decreases, the other likewise would increase or decrease and in the same proportion" (HW 5:449).[270]

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Hegel effectively summarizes what has been discussed up to this point:

> Something is what it is through quality. Altering the quality does not only alter the determination of something or of the finite, but also the finite itself. Quantity is, in contrast, the determination that does not decide [ausmacht] more of the nature of the thing itself, but rather is an indifferent difference [gleichgültiger Differenz]. When quantity alters, the thing remains that which it is (HW 4:168).

Thus, quality is "determinacy as being in itself [als an sich seiend], the alteration [Veränderung] of which leads to the alteration of that which[271] it is the determinacy." Whereas a determinate quality, when it changes, becomes something different, another quality, "quantity is determinacy in its exterior clothing for which the alteration [Veränderung] of the object to which such determinacy belongs remains an indifferent alteration, or, in other words, quantity is the determinacy that is not determinacy at the same time [die zugleich keine ist]" (HW 4:86).

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[265] Ibid., 233.

[266] Ibid.

[267] Ibid., 376.

[268] Ibid., 377.

[269] Ibid., 376.

[270] Ibid., 378.

[271] The first version of this fragment states: "with whose vanishing it stops being that" [mit deren Verschwinden auch dasjenige zu sein aufhört]» (HW 4:86).

"If, however, by limit we mean quantitative limit, writes Hegel in the *Science of Logic*, "then when, for example, a field alters its limit it still remains what it was before, a field" (HW 5:209).[272] In contrast, when "its qualitative limit is altered, then since this is the determinateness which makes it a field, it becomes a meadow, wood, and so on. A red, whether brighter or paler, is still red; but if it altered its quality it would cease to be red, would become blue or some other colour" (HW 5:209).[273]

On the one hand, one is faced with "[d]eterminateness thus isolated by itself in the form of being [als seiende Bestimmtheit]" that is to say "quality" (HW 5:117),[274] for which "the real [das Reelle] or something [Etwas] is different from another real" (HW 4:13): "Quality, taken in the distinct character of being, is reality [Realität]; as burdened with a negative, it is negation in general, likewise a quality but one which counts as a deficiency, and which further on is determined as limit, limitation" (HW 5:117).[275] "Limit is the middle between the two of them in which they cease" (HW 5:136),[276] the point where something (reality) ends and that which it is not (negation) begins. In other words, "through the limit something is what it is, and in the limit it has its quality" (HW 5:136).[277] By means of such a limit, it is distinguished from what it is not, from the other than itself.

On the other hand, when the difference between reality and negation is reduced to a merely quantitative difference, the limit is posited "as the limit which is no limit" (HW 4:14). A change in terms of more or less always has to do with a change in terms of more or less of the same quality, and the limit that separates one quantitative determination from another that is not the first is purely accidental and external. For, "[q]uantum alters and becomes another quantum … Quantum becomes an other; but it continues itself into its otherness" (HW 5:260).[278] If reality and negation are only distinguished in terms of quantity, then their "determinateness … has become indifferent to being, a limit which is just as much no limit, being-for-self which is absolutely identical with being-for-other" (HW 5:208).[279]

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The general meaning of the distinction between qualitative and quantitative opposition, the fundamental distinction for Fichte as well as Schelling, is thus brought to its highest level of abstraction in Hegel. However, the connection between his formulation of the problem and Kant's philosophy becomes evident as soon as one considers Hegel's chapter dedicated to "Being-for-self" (*Fürsichsein*). Here, the dynamic conception of matter (as the product of the conflict between attraction and repulsion),

[272] English translation from , Hegel. *Hegel's Science of Logic*, 186–.

[273] Ibid., 232–.

[274] Ibid., 111.

[275] Ibid.

[276] Ibid., 127.

[277] Ibid., 126.

[278] Ibid., 225.

[279] Ibid., 185.

which Kant, as discussed above, elaborates in the *Metaphysical Foundations of the Natural Sciences* as an alternative to the atomistic conception, becomes the model for the transition from quality to quantity. For atomism, "The one and the void is being-for-self, the highest qualitative being-within-self, sunk back into complete externality" (HW 5:185).[280] The full is absolutely distinguished from the empty such that "the first is being-in-itself, which distinguishes it completely from the second" (HW 4:167). However, as soon as one substitutes a conflict between repulsion and attraction for the absolute difference between full and empty; that is, "as soon as this difference, repulsion, is removed through attraction, the difference is posited as removed [ist … als aufgehobener gesetzt]. With that, it has passed into another determination, quantity" (HW 4:167).[281] While the limit that separates the full and the empty is rigid (i.e., qualitative), in the dynamic conception of matter, it is only the relative point of transition in which attraction is converted into repulsion and vice versa:[282] "Quality is the first, immediate determinateness [Bestimmtheit], quantity is the determinateness which has become indifferent to being, *a limit which is just as much no limit*, being-for-self which is absolutely identical with being-for-other – a repulsion of the many ones which is directly the non-repulsion," that is, attraction, "the continuity of them" (HW 5:208; emphasis mine).[283]

In a qualitative determination, as discussed above, "something has no meaning without its limit. If I alter the limit of something, it does not remain what it is," but rather becomes something different. As a result, "the something disappears as such with its alteration" (HW 4:198). When one passes from quality to quantity, in contrast, the limit becomes indifferent because "if I alter the limit of a field [Acker], the field remains what it is and merely becomes bigger. In this case, I have not altered its limit as a field, but only as a *quantum*. Its quality remains unchanged and the field has not become something else; a forest, for example" (HW 4:167). Thus, in general, in the transition from quality to quantity, as the *Phenomenology of Spirit* explains, differences given in intuition are shown to be "differences … that are no differences." At first "that what is *selfsame* repels [abstößt] itself from itself," positing the other than itself. Since, however, "the differences are only such as are in reality no differences and which cancel themselves; in other words, what is not *selfsame* attracts itself [anzieht]" and that which appeared as other is posited again as the same (HW 3:126).[284]

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[280] Ibid., 166.

[281] Here, I draw from Hegel's simpler exposition of the problem that is found in a few fragments (1801–2) shortly before the publication of the first edition of the *Science of Logic* [Wissenschaft der Logik] in 1812. For a recent and more detailed presentation on the logic of *Fürsichsein* (Being-for-self), see Schick, Friedrike. "Absolutes und gleichgültiges Bestimmtsein – Das Fürsichsein in Hegels Logik." In *Hegels Seinslogik. Interpretation und Perspektiven*. eds. Andreas Arndt and Christian Iber, 235–51. (Berlin: Akad.-Verl, 2000).

[282] See Lefèvre, Wolfgang. "Repulsion und Attraktion: Der Exkurs 'Die Kantische Konstruktion der Materie aus der Attraktiv- und Repulsionskraft in Hegels Wissenschaft der Logik." Ibid. eds. Andreas Arndt and Christian Iber, 252–70. (Berlin: Akademie Verlag, 2000).

[283] English translation from Hegel. *Hegel's Science of Logic*, 185.

[284] English translation from Hegel. *Phenomenology of Spirit*.

Considered from the point of view of the "logic of essence," Hegel once again seems to come face to face with Kant's distinction between logical opposition and real opposition. At first, *A* "is immediate determinacy whose alteration is a passing into its opposite," not–*A* (HW 4:165). Each change in a certain concept *A* makes it a different concept, which should be understood as not–*A*. Between *A* and not–*A*, no intermediary exists; either a concept is *A* or, if there is something in it that is different than *A*, then it is not–*A tout court*: "'Of two opposed predicates, only one belongs to something' and 'there is no third'" (HW 8:242; §119).[285]

This first form of opposition is distinguished from that for which the changing of *A* into its opposite does not change the nature of *A*. In this latter form of opposition, *A* remains as the indifferent third in respect to the opposites:

> A must be either +*A* or −*A*; thus the third [term], the *A* which is neither + nor − and which is posited also equally as +*A* and as −*A*, is already expressed. If + *W*means 6 miles in the westerly direction, but − *W* 6 miles in the easterly direction, and + and − sublate each other, then 6 miles of road or of space remain what they were, with or without the antithesis. Even the mere plus and minus of number or of abstract direction have, if one pleases, zero for their third [term]. (HW 8:242; §119)[286]

That Hegel was familiar with Kant's early writing on negative quantity is difficult to establish,[287] but he certainly had access to an entire tradition of thought whose essential content is precisely Kant's conception: "The notion of polarity, which is so generally current in physics," writes Hegel in the *Encyclopaedia*, "contains within itself a more correct determination of opposition; but if physics holds onto ordinary logic as far as its thoughts are concerned, it would easily get scared, if it were to develop polarity for itself, and would thus come to the thoughts that are implied in it" (HW 8:242; §119).[288] In philosophy of nature, as I have shown in detail, "the category of polarity …, which is the determination of a difference in which the different terms are inseparably conjoined, has played the leading part although it has been used inordinately in connection with all phenomena, even with light" (HW 5:21).[289] It becomes evident in this conception that the positive and the negative "are implicitly the same, and therefore we could call the positive 'the negative' if we liked, and conversely we could call the negative 'the positive' as well…. There cannot be

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[285] English translation from Hegel, Georg Wilhelm Friedrich. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*. eds. Théodore F. Geraets, Wallis Arthur Suchting, and Henry S. Harris (Indianapolis: Hackett, 1991), 185.

[286] Ibid.

[287] See Wolff. *Der Begriff des Widerspruchs eine Studie zur Dialektik Kants und Hegels*, 112.

[288] English translation from Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 186.

[289] English translation from Hegel. *Hegel's Science of Logic*, 32. Here, according to Sergio Landucci, Hegel has "siphoned all the material from the Kantian tradition of negative magnitudes" (Landucci, Sergio. "Opposizione e contraddizione nella logica de Hegel," *Verifiche* 1–3 (1981): 89–105, 101). Landucci continues, "the closest example is physical polarity … as Hegel does not tire to repeat … one merely needs to consider Hegel's celebration of the category of polarity, even at the end of his life," in the second edition of the *Science of Logic* (Landucci. "Opposizione e contraddizione nella logica de Hegel," 103).

the north pole of a magnet without the south pole nor the south pole without the north pole. If we cut a magnet in two we do not have one piece and the south pole in the other. And in the same way, positive and negative electricity are not two diverse, independently subsisting fluids. [zwei verschiedene, für sich bestehende Fluida]" (HW 8:245; §119).[290] Thus, the character of the opposition between positive and negative magnitudes does not depend on a certain qualitative or "essential" distinction by which opposites can be distinguished from one another, but rather "each is only the opposite of the other, the one is not as yet positive, and the other is not as yet negative, but both are negative to one another" (HW 6:56).[291] The positive and the negative are such only in respect to "a third point of view outside them that makes one positive and the other negative" (HW 6:60),[292] so that "[t]he opposites are cancelled in their combination":

> An hour's journey to the east and the same distance travelled back to the west, cancels the first journey; an amount of liabilities reduces the assets by a similar amount, and an amount of assets reduces the liabilities by the same amount. At the same time, the hour's journey to the east is not in itself the positive direction, nor is the journey west the negative direction; on the contrary, these directions are indifferent to this determinateness of the opposition (HW 6:60).[293]

In this reciprocal removal of the opposites, the identity of the "substrate" is presupposed ("money" in the case of debtors and creditors, the "path" in the difference in direction toward the east and the west, etc.). For, "opposition through which what is distinct does not have an other in general, but its *own other* facing it" (HW 8:245; §119; emphasis mine).[294] "The opposites certainly cancel one another in their relation, so that the result is zero; but there is also present in them their identical relation, which is indifferent to the opposition itself; in this manner they constitute a one" (HW 6:60).[295] The qualitative difference between A and that which is not A is dissolved into an indifferent difference, the simple + and − of the same A in respect to which these are defined as such. These are, in Hegel's words, "different in one identity [in einer Identität verschiedene]" (HW 6:55).[296] However, such an identical determination, "overarching" the opposites, can only exist in the distinction between + or −. What is indifferent in respect to + and − is simple = 0.With the disappearance of + and −, the substrate in respect to which they were + and − seems to disappear as well. With the disappearance of the difference in quantity, to which

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[290] English translation from Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 186–.

[291] English translation from Hegel. *Hegel's Science of Logic*, 425.

[292] Ibid., 428.

[293] Ibid.

[294] English translation from Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 185.

[295] English translation from Hegel. *Hegel's Science of Logic*, 428–.

[296] Ibid., 424.

the difference between the opposites was reduced, the quality is also dissolved and nothing remains.[297]

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The connection between the problems that I have addressed here in a simplified form and the fundamental philosophical question that insistently returns in Kant and post-Kantian philosophy emerges clearly in the preparatory fragments of the *Science of Logic*. These fragments, the so-called *Jena System* (1804–5) (*Jenaer Systementwürfe*), were probably already edited between 1803 and 1804 during the Jena period. In the section "Simple Relation" (Einfache Beziehung) from the chapter of these drafts entitled "Logic" (Logik), Hegel provides a clear historical and systematic exposition of the problem's development, a problem that the German idealist tradition delineates the most precisely. The bad synthesis of reality and negation, according to Hegel, leads to "the concept of limit [Gränzte]"[298] where the opposites subsist "on their own account."[299] The philosophies of Kant, Fichte, and Schelling are most likely Hegel's critical point of reference: "Thus the so-called construction of the idea out of the opposed activities, of the ideal; and real, ones, as unity of both, has produced nothing but the limit."[300] "The same occurs with the construction of matter out of opposed forces, the forces of attraction and repulsion."[301]

Post-Kantian philosophy, as discussed above, affirms the possibility that the difference between opposed determinations is "a more of the one and a less of the other; but they have significance simply as connected with each other, as opposed

[297] For a more detailed analysis of Hegel's conception of the relations between positive and negative magnitudes and for a comparison with Kant's conception, see Wolff, Michael. "Hegel und Cauchy: Eine Untersuchung zur Philosophie und Geschichte der Mathematik." In *Hegel und die Naturwissenschaften*. eds. Rolf-Peter Horstmann and Michael John Petry, 197–263. (Stuttgart: Klett-Cotta, 1986), 110–68. See also the chapter "Opposizione reale" in Landucci, Sergio. *La contraddizione in Hegel* (Florence: La Nuova Italia, 1978). Despite highlighting the difference between Hegel's and Kant's formulations and arguing against those "who simply associate Hegel's unity of opposites with Kant's real opposition," Landucci recognizes that "in any case, what Hegel (and Fichte before him) inherit from Kant is the idea of a negation that has no affinity with the traditional contradiction: in this sense, Hegel's not–*A* and Fichte's not–I descend from Kant's – *a*." (Landucci. *La contraddizione in Hegel*, 7n). This point is especially significant for the present work. See also note 116 above.

[298] Hegel, Georg Wilhelm Friedrich. "Jenaer Systementwürfe II." In *Gesammelte Werke*. eds. Rolf-Peter Horstmann and Johann Heinrich Trede. (Hamburg: Meiner, 1972-), 3. English translation from Hegel, Georg Wilhelm Friedrich. *The Jena System, 1804–5: Logic and Metaphysics*. eds. John W. Burbidge, George Di Giovanni, and Henry S. Harris (Kingston: McGill-Queen's University Press, 1986).

[299] Hegel. "Jenaer Systementwürfe II." 5. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 6.

[300] Hegel. "Jenaer Systementwürfe II." 3. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 5.

[301] Hegel. "Jenaer Systementwürfe II." 4. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 5.

directions to the extent that one went beyond the other [über die andere hinausgehe], to that extent it would itself cease to be. In their equilibrium, however, both are just as sublated within their distinction."[302] Thus, this "equilibrium is not itself the true unity because a oneness of those things which have being essentially, on their own account, is just their nothingness." What is at stake is therefore not "a true reality that has been posited, but only a limit, the nothingness of the opposites and their being."[303] For, "that through which the directions are truly distinguished – a posited point – would already be the oneness of them both, in which all opposition and the directions themselves are dissolved; apart from this their being-dissolved they are nothing."[304] In other words: "In the limit, the nothingness of reality and negation is posited."[305]

If the difference only involves quantity and if this disappears, the opposites also lose their meaning as such: "the sublating of magnitude would indeed sublate that to which it is ascribed if [magnitude] were essential to it; but it is of the nature of magnitude to be accidental, an excluding which in truth however does not exclude, a limit which in truth however is no limit."[306] Thus, the quantitative difference remains a mere accidental difference that cannot fix the "being" of the two opposites: "The determinacy of quantum as a limit of the many is no determinacy whatever of the Thing itself; its concept is not affected thereby [afficirt]."[307] If the difference between the opposites is thus reduced to being "quantitative," it is only an apparent difference, a difference within that which is identical. However, this identical element itself is only manifested in the difference since, when the difference is removed, only a nothing = 0 remains.

The solution to this problem, as I have suggested more than once, is found in infinitesimal calculus, according to Hegel, "is at the same time the true meaning of the *vanishing magnitudes of analysis*."[308] In infinitesimal calculus, the removing of the quantitative difference leaves the reciprocal relations between finite magnitudes intact: "when one lets a posited magnitude within a system of magnitudes vanish absolutely, just for that reason the concept of what is to be determined comes forth purely as an absolute ratio, which is all we want to know, not the determinate magnitudes."[309] In the disappearance of the difference, "two diverse magnitudes are

[302] Hegel. "Jenaer Systementwürfe II." 3. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 6.
[303] Hegel. "Jenaer Systementwürfe II." 6. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 6.
[304] Hegel. "Jenaer Systementwürfe II." 5. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 6.
[305] Hegel. "Jenaer Systementwürfe II." 5. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 6.
[306] Hegel. "Jenaer Systementwürfe II." 18. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 20.
[307] Hegel. "Jenaer Systementwürfe II." 21. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 19.
[308] Hegel. "Jenaer Systementwürfe II." 18. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 20.
[309] Hegel. "Jenaer Systementwürfe II." 18. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 20.

completely the same, and since the diversity has been presented as a differential, nothing occurs but the elimination of the diverse magnitudes and the establishment of the concept."[310] In the transition from the relation between finite differences (e.g., between the successive values of the *y*-axis and the successive values of the *x*-axis) to the differential relation (that is, to the relation between the differential of the *y*-axis and the differential of the *x*-axis), the differences have vanished, becoming "in themselves" = 0. However, their relation does not lose its meaning as a result.[311] The difference between the successive values of the *x*-axis vanishes, "while the abscissa as such simply remains, so it is in general the case that the internal ratio and its moments remain simply as such."[312] Specifically, one observes that "wherever the ordinate remains as ordinate, the abscissa remains abscissa; and their determinate ratio to each other remains the same in their decrease *ad infinitum*, by which it is not at all affected."[313] Arriving at their "last ratios," that is, passing to the limit, "the diversity of magnitude, the duality of ordinate and abscissa, is totally nullified, and hence the determination is a pure connection through its ratio, not through its magnitudes as such or through them as concepts."[314] This is the sense of the concept of "differential": it is neither a "difference" so small that a smaller one cannot be found nor a relatively small difference (like a grain of sand in respect to a mountain, as in Wolff's[315] effective image). Rather, the differential no longer has anything to do with a magnitude because "the disappearance does not become intelligible by increase or decrease":[316] "The disappearing of the differential, alternatively, is a disappearing of a magnitude; but this differentiation [Differenz] is just as much only a semblance of a diversity [Verschiedenheit] … for in the ratio it is only the concept that is involved."[317]

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The first edition of the *Science of Logic* in 1812, and the 1831 edition in an even more articulated manner, develops this interpretation of infinitesimal calculus in

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[310] Hegel. "Jenaer Systementwürfe II." 21. Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 21.

[311] Hegel. "Jenaer Systementwürfe II." 21. On this point, see Ziche. *Mathematische und naturwissenschaftliche Modelle in der Philosophie Schellings und Hegels*, 212-. See also Moretto, Antonio. "Matematica e contraddizione nella 'Logica di Jena'," *Verifiche* 1–3 (1981): 291–301.

[312] Hegel. "Jenaer Systementwürfe II." 22. English translation from Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 21.

[313] Hegel. "Jenaer Systementwürfe II." 21; Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 21.

[314] Hegel. "Jenaer Systementwürfe II." 19; Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 21.

[315] See Hegel. "Jenaer Systementwürfe II." 18; Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 20.

[316] Hegel. "Jenaer Systementwürfe II." 17; Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 19.

[317] Hegel. "Jenaer Systementwürfe II." 20; Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 21.

detail through a profound analysis of the epoch's scientific literature.[318] The interpretation thus becomes a supporting column in Hegel's logic and the model on which the philosophical concept of the infinite itself is constructed: "the character of the mathematical infinite and the way it is used in higher analysis corresponds to the Notion of the genuine infinite" (HW 5:283).[319] Only through infinitesimal calculus can one pass from "bad infinity," which characterizes quantity as such, to "a quantitative determinateness in qualitative form", to the true infinity that "[consists] in its being a qualitative determinateness" (HW 5:284).[320]

As it was observed, "to grasp the sense of Hegel's discourse, one must refer to Kant. Behind the discussion of infinitesimal calculus lies the critique of the Anticipations of Perception."[321] The return from quantity into quality does not assume the form of a transition from extensive magnitude to intensive magnitude as in Kant. Through infinitesimal calculus, "the sublated quantum has returned into a simple unity and self-relation; but *not merely like the extensive quantum which, in passing into intensive quantum*, has its determinateness only in itself [or implicitly] in an external plurality, towards which, however, it is indifferent and from which it is supposed to be distinct" (HW 5:284; emphasis mine).[322] For Kant, and for Schelling as well, the transition from quantity to quality ultimately corresponds to the transition from extensive magnitude to intensive magnitude, from the magnitude of the extension to the magnitude of that which is extended. For Hegel, in contrast, this "conversion of the one-sided form of extensive magnitude into its other, intensive form, makes no difference to the nature of the fundamental determination,

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[318] An articulated analysis of this aspect can be found in the classic Moretto, Antonio. *Hegel e la "matematica dell'infinito"* (Trento, Italy: Verifiche, 1984), from which the title of the present section is drawn. Moretto's work addresses the particular technical issues involved, while also providing a general historical framework. For a closer focus on the differences between the two editions of the *Science of Logic,* see another fundamental essay, Wolff. "Hegel und Cauchy: Eine Untersuchung zur Philosophie und Geschichte der Mathematik". For a recent panoramic view, see Bonsiepen, Wolfgang. "Hegels Theorie des qualitativen Quantitätsverhältnisses." In *Konzepte des mathematischen Unendlichen im 19. Jahrhundert*. eds. Gert König and Detlef Laugwitz, 101–29. (Göttingen: Vandenhoeck & Ruprecht, 1990). See also the pages dedicated to Hegel in Bell, John L. *The Continuous and the Infinitesimal in Mathematics and Philosophy* (Milan: Polimetrica, 2005), 130ff.

[319] English translation from Hegel. *Hegel's Science of Logic*, 244.

[320] Ibid., 241.

[321] Ibid., 246.

[322] English translation from Ibid., 244. Simon Duffy in his Duffy, Simon. *Quality, Quantity, and Intensity in Spinoza, Hegel and Deleuze* (Aldershot: Ashgate Publishing, 2006) holds an entirely different point of view. He explicitly notes that Hegel took the differential "as an intensive magnitude" Duffy. *Quality, Quantity, and Intensity in Spinoza, Hegel and Deleuze*, 62. Even if for Hegel, as I have shown, differential calculus has to do with "qualities," I think that he rejects Maimon's and Bendavid's identification of differential and intensive magnitude (an identification that Hermann Cohen later made explicit ; see below §4.4). The intensive magnitude, even if it is the "quantity of quality," remains a "quantity" and is therefore incapable of expressing the passage from quantity to quality, which Hegel is interested in.

which is one and the same quantum" (HW 5:421).[323] Extensive magnitude and intensive magnitude "are only distinguished by the one having amount within itself and the other having amount outside itself" (HW 5:253).[324] The essential form of quantity, its indifference in face of the more and the less, does not change in the two cases: "When considering *intensive*, and *extensive* quantum … we found that it is the *same* quantum which is present, once in the form of intensity and again in the form of extension. In this difference the quantum lying at the base suffers no alteration, the difference being only an outer form" (HW 5:399).[325]

Kant deserves credit for showing the possibility of distinguishing not only differences in number but differences in degree as well: "Kant has already opposed *intensity* to the quantitative determination of *amount*, and instead of explaining the different densities of bodies which occupy the same volume by assuming that the heavier body contains more particles, he has assumed that in the heavier body the same number of particles fill space to a greater degree: in this way, he founded a so-called dynamic physics" (HW 9:159; §293).[326] However, the transition from the quantity of the extension to the quantity of the quality that is extended, from the extensive to the intensive, does not imply an overcoming of the logic of quantity, but rather an affirmation and furthering of it. The transition reveals that quality itself can demonstrate a specific form of quantity:

> Dynamic physics, alternatively, wants to cognize this diversity not as something external but as something in and of itself in matter.… [H]owever, the degree is so far from sublating multitude and externality that that is rather what it essentially is. A larger or smaller multitude of mass = heat = etc. particles transformed into a higher or lower intensity of mass or heat, etc., sublates, to be sure, the semblance of atomic multiplicity in what appears to be mass or heat; but if this [multiplicity] now has actually to be expressed as a determinate magnitude, then this can only happen through connection with numbers. Admittedly the fortieth, the hundredth, etc., degree still does not express a multiplicity with respect to the degree itself but with its simplicity; however, this diversity has significance simply and solely in relation to another.[327]

On first consideration, a degree seems to be an entirely simple determination, located beyond every subdivision, beyond every quantity, such that it can even be determined in a point or an instant: "So, a certain degree of temperature, for instance, is an intensive magnitude, to which, as such, there corresponds a wholly simple sensation" (HW 8:217; §103).[328] However, the "determination of *intensive* magnitude" is a determination that "only appears as a quantum by way of *comparison*"

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[323] Hegel. *Hegel's Science of Logic*, 356.

[324] Ibid., 220.

[325] Ibid., 237.

[326] English translation from Hegel, Georg Wilhelm Friedrich. *Hegel's Philosophy of Nature: Being Part Two of the Encyclopedia of the Philosophical Sciences (1830)* tr. Arnold V. Miller (Oxford; New York: Clarendon Press, Oxford University Press, 2004), 127.

[327] English translation from Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 15.

[328] English translation from Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 164.

(HW 9:159; §293).[329] One can correctly say that, "[w]hen we speak of ten or twenty degrees, the quantum that has that number of degrees is the tenth or twentieth degree, not the amount and sum of them" (HW 5:251).[330] A degree can be defined as tenth or twentieth only in respect to another degree that is posited as a point of reference. Velocity, density, and temperature are determined in each point and in each instant as something simple, but such a determination only has sense in relation to another point and another instant in respect to which it is possible to establish a greater-lesser relation. According to Hegel, the use of ordinal numbers shows how a degree is simple in itself and that its quantum is determined only from its position in an ordered series. In contrast, extensive quantity, expressed through cardinal numbers, results from the aggregation of a determinate number of parts.

As close as it may be to a qualitative determination, difference in degree is not exempted from the logic of "quantum": "This determinate intensity is not this at all, if it is not this for something external; and it is simply not at all what it would be for itself."[331] The intensive magnitude is determined in respect to the difference that it (the intensive magnitude) presents in respect to other intensive quantities, "so that its determinateness consists in this relation to its otherness" (HW 5:253).[332] As discussed above, the opposed tendencies to reconstruct an equilibrium in which the degree itself is manifested as such can only emerge on the basis of such a difference: "The form of intensity, the so-called dynamic factor, does not help, because this too has its determinateness in quantum and consequently can express only as much force (which is the measure of its existence) as is opposed to it by the opposite force" (HW 5:453).[333] As I have emphasized several times, only through a difference in degree (e.g., a difference in temperature) can something like heat be constituted, something that *is* nothing outside of this difference: "Understanding makes itself substrates which are not recognized through the Notion [Begriff]." For example, "heat do[es] not exist on [its] own account like heavy matter." "[S]o called … heat-matter," as well as other analogous substances that are introduced to explain determinate phenomena, "are merely physical fictions of the metaphysics of the understanding." In actuality, "they are no more than moments," and heat is "not capable of qualitative dimensions within itself, but only of the abstract opposition of positive and negative." Only in respect to this opposition can one speak of "quantum and degree, in the form of an abstract equilibrium, an equalization of temperature in the bodies among which varying degrees of temperature are distributed [verteilt]" (HW 9:189; §305).[334] With the disappearence of the difference

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[329] English translation from Hegel. *Hegel's Philosophy of Nature: Being Part Two of the Encyclopedia of the Philosophical Sciences (1830)* 127.

[330] English translation from Hegel. *Hegel's Science of Logic*, 118.

[331] English translation from Hegel. *The Jena System, 1804–5: Logic and Metaphysics*.

[332] Ibid., 220. See Wladika, Michael. *Kant in Hegels "Wissenschaft der Logik"* (Frankfurt am Main; New York: P. Lang, 1995), 91–8.

[333] Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 220.

[334] English translation from Hegel. *Hegel's Philosophy of Nature: Being Part Two of the Encyclopedia of the Philosophical Sciences (1830)*.

in degree, as Schelling clearly understood, nothing more remains of quality either since it is nothing outside of such a quantitative difference.

In the *Jena System* (1804–5), Hegel notes not coincidentally how Kant seems to avoid precisely this aspect of the nature of intensive magnitude. As is well-known, Kant's "Widerlegung des Mendelssohnschen Beweises der Beharrlichkeit der Seele" (Refutation of Mendelssohn's Proof of the Persistence of Soul) attributes an intensive magnitude to the soul which little by little can disappear into nothingness, reducing the "being" of soul to an external difference in degree and thus introducing "into the essence of spirit the determination of magnitude."[335] According to Hegel, however, the true nature of the soul should be revealed precisely in the disappearance of the difference in degree. The disappearance of this relative and accidental difference does not imply that what it was a magnitude of "disappears" as well. On the contrary, here, consciousness "comes forth purely as what it truly is in itself. *Only consciousness having no degree is true consciousness*"[336] free of all accidental and comparative elements. This is precisely the meaning of infinitesimal calculus: quantity, the pure such-and-such size of something, disappears, but the nature of that something of which the quantum was the magnitude emerges precisely in this disappearing. Such a nature is therefore independent of the relative difference of more or less, of extensive and of intensive magnitude.

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All these elements, which are present in Maimon only in a confused manner, are now clearly distinguished. The return of quantity into quality is not posited as a transition from extensive quantity to intensive quantity, as it is in Kant and in Kantian-influenced philosophy of nature. The problem is not the transition from one quantity to another, but rather the transition from quantity as such to that which does not depend on quantity, from the magnitude to the relation between magnitudes that remains as such even if the magnitudes that are related to each other are conceived as variable.[337] In the *Science of Logic*, Hegel writes: "Let us then first take quantum in the *relation* where it is a *fractional number.* Such fraction, $\frac{2}{7}$ for example, is not a quantum like 1, 2, 3. Although it is an ordinary finite number it is not an immediate one like the whole numbers" (HW 5:284–).[338] It can be easily observed that, "[I]f 2 and 7 are counted as mere quanta, then 2 is just 2 and nothing more, and 7 is simply 7; 4, 14, 6, 21 etc., are completely different from them and, as only immediate quanta, cannot be substituted for them" (HW 5:285).[339] Here, in contrast, one can say $\frac{2}{7} = \frac{4}{14} = \frac{6}{21}$, giving 2 and 7 a particular qualitative determination in respect to which the quantity becomes indifferent.

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[335] Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 19.

[336] Ibid.

[337] On this point, see Bonsiepen. "Hegels Theorie des qualitativen Quantitätsverhältnisses".

[338] English translation from Hegel. *Hegel's Science of Logic*, 245.

[339] Ibid., 242.

In this example, 2 and 7 are still determinate quantities that have a sense outside of the relation as well. However, they also acquire another sense because the relation that exists between them remains the same even if their absolute value changes. The number of times that 2 is in 7 is the same as the number of times that 4 is in 14 and so forth: "But now in the function," $y = px^2$, "for example, this is essentially changed":" here, it is true that x and y can stand for definite quanta," but, in reality, only y e x^2 "have a determinate quotient" (HW 5:293).[340] In this case, "these sides of the ratio x and y, not any determinate quanta but, secondly, their ratio is not a fixed quantum (nor is such a quantum meant as in the case of a and b), not a fixed quotient, but this quotient is, *as a quantum*, absolutely *variable*" (HW 5:293).[341] $y = ax$ is the equation of a line. The relation $\frac{y}{x}$ remains constant and expresses the line's inclination: $\frac{y}{x} = a$; no matter how large or small is the difference between the two successive values of $y$and that between the two successive values of x, the inclination can still be determined with equal precision since the relation $\frac{\Delta y}{\Delta x}$ is always = a: "there is no point in differentiating for their own sakes," Hegel continues, "the equations $y = ax + b$ (of the straight line) or $s = ct$ (of the plain uniform velocity)" (HW 5:397).[342] In fact in the function of the straight line $y = ax$, the quotient "$\frac{y}{x}$ is an ordinary fraction and quotient; consequently this function is only formally function of variable magnitudes, or x and y here are what a and b are in $\frac{a}{b}$" (HW 5:327).[343] In contrast, in the "relation between powers" (*Potenzenverhältnis*), $y = px^2$, if the relation $\frac{y}{x^2}$ is constant, resulting in = p, the relation $\frac{y}{x}$ is variable. Thus, in the case of a motion with a uniform acceleration $s = at^2$, the relation $\frac{s}{t^2}$ is constant according to "Galileo's law, which states that the traversed spaces are the square of traversed times" (HV 9:77; §267), corresponding to the uniform acceleration. In contrast, the relation "$\frac{s}{t}$, that is, a purely uniform velocity" (HV 9:77; §267) has a different value in different periods of time and ultimately in each instant. In these cases, considering that the relation between finite differences only results in an approximation of its value. If one attempts to determine the precise value that the relations possesses in each point of the curve or in each instant of the motion, the difference between the successive values of the variables would be = 0, leading to the impossible attempt to attribute a determinate value to the relation $\frac{0}{0}$.

As Schelling observes, "no degree is possible except through the inverse ratio of opposed factors, e.g., a determinate degree of velocity through the inverse ratio of

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[340] [Ibid., 252.] To simplify subsequent comparisons with the themes discussed in the next chapter, I have indicated the independent variable by the letter x, as is customary, writing $y = px^2$ in place of Hegel's example $\frac{y^2}{x} = p$.

[341] Ibid.

[342] Ibid., 279.

[343] Ibid., 252.

the space that is traversed and the amount of time that is required [to traverse it]" (SW 4:94). Thus in the variation of the degree of velocity, what is involved is not so much the transition from a purely geometrical problem (change of position) to a physical-dynamic problem (change of velocity). On the contrary, for Hegel, the transition "from said simply uniform velocity to a uniformly accelerated velocity" (HW V, 281)[344] is only a particular case of the purely mathematical problem raised by the "ratio of powers." One can thus disregard "those determinations which belong to the idea of motion and velocity … because in them the thought does not appear in its proper abstraction but as concrete and mixed with non-essential forms" (HW 5:281).[345]

As the *Jena System* (1804–5) already recognizes, the significance of infinitesimal calculus consists in the possibility of admitting that, in the transition from the "difference quotient" $\frac{\Delta y}{\Delta x}$ to the "differential quotient" $\frac{dy}{dx}$, the finite differences have indeed disappeared and have become = 0, but their relation still has a determinate value:

> *dx* and *dy* are no longer quanta, nor are they supposed to signify quanta; it is solely in their relation to each other that they have any meaning, a meaning merely as *moments*. They are no longer something (something taken as a quantum), not finite differences; but neither are they *nothing*; not empty *nullities*. Apart from their relation they are pure nullities, but they are intended to be taken only as moments of the relation, as *determinations* of the differential coefficient" $\frac{dy}{dx}$. (HW 5:296)[346]

Therefore, the differentials quantities "*dx* and *dy*are supposed to be taken simply and solely as *moments* of" the differential coefficient $\frac{dy}{dx}$, which is thus not a fraction, but should be considered "a single indivisible symbol" (HW 5:295–).[347]

According to Hegel, "the quantum has no limit that is determinate in itself" [hat keine an sich selbst bestimmte Grenze]. Thus, there is no quantum "beyond which a bigger or smaller one could not exist" (HW 4:169). On the one hand, a quantum consists only in its limit (a quantum is such only because it is a certain finite difference). On the other hand, it goes beyond this limit because an amount is such only if compared with another. For example, 1 m can be defined as 100 cm or as a thousandth of a kilometer, but once this definition is accepted, one must then ask what a centimeter and a kilometer are equivalent to, and so on to infinity. As soon as one sets the limit that defines a quantum as such, the quantum already overcomes the limit and that which is beyond the limit is a quantum again.

[344] Ibid., 272.

[345] Ibid., 255.

[346] Ibid., 253.

[347] Ibid., 266.

If one disregards this purely relative and indifferent determination, which characterizes an amount as such, "this stops being a quantum as well and, for this reason, is = 0" (HW 4:169). However, in this vanishing of the finite quantitative difference, which defines a quantum as such, the quantum is not necessarily reduced to an absolute nothing. The quantum "still has meaning only as a determination of a relation, where, in-itself, there is no longer a magnitude, but only a determination in relation to another. This is the more precise concept of mathematical infinity" (HW 4:169). If, "in arithmetic and geometry, the *quanta* are compared in such a way that, as arbitrary and general that their magnitudes can be …, they completely retain the value in themselves as determinate *quanta*, as finite magnitudes," that is, as determinate differences. In "infinite analysis," in contrast, "specifically in differential and integral calculus," one considers "infinite magnitudes, that is say, those which no longer have the sense of finite magnitudes, of magnitudes in themselves and of completely determinate magnitudes; but rather of disappearing magnitudes that have their value only in their last relations or in their limits, that is, *purely in their relation*" (HW 4:35; emphasis mine).

The true significance of infinitesimal analysis should be sought in the notion of "the preservation of the ratio in the *vanishing* of the quanta" (HW 5:298; emphasis mine):[348] "the expression that by virtue of the *law of continuity* the disappearing magnitudes still retain the ratio from which they come, before they disappear … *expresses* the true nature of the matter" and "[t]his purification of the quantitative ratio is thus analogous to grasping an empirical reality in terms of its Notion. The empirical reality is thereby raised above itself in such a way that its Notion contains the same characteristic features as it has itself, but these are grasped in their essentiality and are taken into the unity of the Notion in which they have lost their indifferent, Notion-less existence" (HW 5:299).[349] The bad quantitative infinite in virtue of which, for any given difference, a smaller one can always be thought, is thus replaced by the true qualitative infinity: "In this concept of the infinite, the quantum is genuinely completed into a qualitative reality; it is posited as actually infinite; it is sublated not merely as this or that quantum but as quantum generally" (HW 5:295).[350]

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As discussed above, thought first proceeds by substituting rigid qualitative distinctions with simple quantitative differences:

> the progression of thought here is from the specific quality of the thing [*Sache*] to a stage at which this determinate quality is indifferent, i.e. that of quantity…. If, for example, I define a circle, an ellipse, or a parabola, it can be seen that they are specifically different. Nevertheless, the distinction between their different curves is defined purely quantitatively, that is, in such a way that the only relevant factor is a quantitative distinction which relates to their coefficients alone, to their purely empirical determinations (HW 7:136).[351]

[348] Ibid., 256.

[349] Ibid., 257.

[350] Ibid., 253.

[351] Hegel. *Elements of the Philosophy of Right*, 93.

For the logic of the understanding, on the one hand, two contradictory marks cannot coexist in the same concept. On the other hand, even if a "polygonal circle or a rectilinear arc contradicts this principle […] geometers do not hesitate to regard and to treat the circle as a polygon with rectilinear sides" (HW 8:244; §119),[352] as a polygon with an infinite number of sides. What are only contradictory and reciprocally irreducible determinations for the understanding are reduced to mere quantitative differences. That which is "other" is thought again as the "same". In the same way, on the basis of the "Principle of identity: motion is motion, and rest is rest" and "the two determinations are external to each other." (HW 9:66; §265).[353] Rest, however, can be thought as an infinitely small movement. The logical opposition between movement understood as "change" and rest understood as "immobility" is dissolved into an accidental and external difference. However, this very idea of rest as movement = 0 is only made possible through a radical rethinking of the concepts of rest and movement, which can now be "compared" in respect to the common definition of "state of motion" that embraces both. Thus, in the disappearence of the magnitude, a dimension that is not reducible to magnitude is revealed: the same "qualitative unity" that remains the same in the transition from rest to motion, a transition that is reduced to a variation in terms of more or less of the same quality.

If "[e]ven the first sublation, the negation of quality as such whereby quantum is posited, is in principle [an sich] the sublating of the negation" (HW 5:277),[354] then the rigid limit that distinguishes a quality from another and negates it is transformed into a purely accidental and exterior limit: "in the quantitative, the limit through which something is a determinate something is an indifferent limit" [eine gleichgültige Grenze], which immediately aims beyond itself [auf ihr Jenseits]" (HW 4:91). The establishing of a limit and the defining of a determinate amount already posits that which is beyond the limit and in respect to which the limit itself assumes a meaning, and so forth to infinity: "the quantum [das Große] is in this way determined as something finite [ein Endliche] beyond which it is necessary to go. Every expansion or diminution leads to another finite something in front of which infinity is posited. It only tends [strebt] toward the infinite without every reaching it" (HW 4:91).

Hegel conceives of the infinity of the quantum as a "bad infinity," as an "infinite progression." It is "only the empty repeating of the removing of the limit and the positing of a new one" such that "the infinite is, in part merely the empty negation of the finite. In part, however, it itself is finite (HW 4:92). The finite is removed in order to be posited again as finite and the infinite is reduced to a vacuous repetition (see HW 5:265).[355] However, the indifference itself of the quantum vis-à-vis its

[352] English translation from Hegel Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 186.

[353] English translation from Hegel. *Hegel's Philosophy of Nature: Being Part Two of the Encyclopedia of the Philosophical Sciences (1830)*.

[354] English translation from Hegel. *Hegel's Science of Logic*.

[355] Ibid.

limit already indicates the presence of a different dimension to which the quantum is not reducible; a larger or smaller field, as we said, is still a field, a more or less intense red is still a red, and so forth. In this mutual dissolving of qualitative oppositions (hot and cold) into purely quantitative differences (greater or lesser temperature), the quantum returns again into the simplicity of its quality (temperature), which remains identical in the purely external variations of the quantity: "it [the quantum] is qualitatively that which it is … that is, qualitatively determined" (HW 5:277).[356] In this identity of the quality that remains identical, the true "infinite" emerges, the infinite that only mathematical analysis succeeds in rigorously fixing: "The infinite quantum, on the contrary, contains within itself first externality and secondly the negation of it; it is thus no longer any finite quantum, not a quantitative determinateness which would have a determinate being as quantum (HW 5:284).[357] This "has meaning solely with reference to that which stands in *relation* to it. *Apart from this relation it is a nullity*" (HW 5:284).[358] In the differential quotient, as I have shown, the quantity returns into the quality and the quantum becomes "a quantitative determinateness in qualitative form; its infinity consists in its being a *qualitative determinateness*" (HW 5:284).[359]

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[356] Ibid.

[357] Ibid.

[358] Ibid.

[359] Ibid. Hegel's conception of infinitesimal calculus is undeniably far from the rigorous standard that was imposed in the second half of the eighteenth century. However, his conception seems to effectively recuperate an important aspect of Leibniz's philosophy. The infinite, the infinitely large as much as the infinitely small, should not be thought as a magnitude beyond which a bigger or smaller one cannot be thought, but rather as a conservation of the identity of a relation despite varying magnitudes that are put in relation to each other. In the *New Essays* [*Nouveaux essais*], Leibniz writes, "Let us take a straight line, and extend it to double its original length. It is clear that the second line, being perfectly similar to the first, can be doubled in its turn to yield a third line which is also similar to the preceding ones; and since the same principle is always applicable, it is impossible that we should ever be brought to a halt; and so the line can be lengthened to infinity." Thus, it is a mistake to try and imagine "an infinite whole made up of parts. … [T]hese infinite wholes, and their opposites the infinitesimals have no place." Rather, according to Leibniz, the idea of the infinite "comes from the thought of likeness, or of the same principle" (GP 5:145) Leibniz, Gottfried Wilhelm. *New Essays on Human Understanding*. eds. Peter Remnant and Jonathan Francis Bennett (Cambridge, UK; New York: Cambridge University Press, 1996), 158. The idea of infinity results from the repetition of a constantly uniform operation that remains the same during an infinite increase as well as during an infinite decrease. For a comparison of Hegel and Leibniz, particularly regarding the "qualitative" meaning of the concept of "similarity," see Wolff. "Hegel und Cauchy: Eine Untersuchung zur Philosophie und Geschichte der Mathematik". Leibniz himself, particularly in *De Analysi Situs* [On Analysis Situs], considers similarity the "qualitative" aspect of a figure as opposed to its quantitative aspect: "Besides quantity, figure in general includes also quality or form. And as those figures are *equal* whose magnitude is the same, so those are *similar* whose form is the same" (GM 5:178) Leibniz. *Philosophical Papers and Letters*, 391. On the relation between Leibniz and Hegel in general, which I cannot address further here, see Zingari, Guido. *Leibniz, Hegel e l'idealismo tedesco*, Mursia (Milan, 1991).

In particular, the so-called "characteristic triangle" (for its ability to determine the "character," the "quality" of a curve in a given point) is similar in its infinitesimal figure to the ordinary triangle, the length of whose catheti correspond to the difference of the *x*-axes and to the difference of the *y*-axes: thus in the final ratio, for example, as Hegel explains, "the quanta of abscissa and ordinate vanish; but the sides of this ratio essentially remain, the one an element of the ordinate, the other an element of the abscissa" (HW 5:315).[360] The unity of quality, which, as Fries keenly observes, Kant relegates to the mere givenness of sensible impressions (through which heat is distinguished from red, for example), can now be rigorously defined in the unity of the law that remains the same in face of the variations of the finite quantitative differences: "The alteration of variables is in this way qualitatively determined, and hence continuous." This continuity, Hegel continues, is nothing other than the "identity, of a determinateness which is preserved and remains self-same in the alteration" (HW 5:327).[361]

3.7 Being, Nothingness, and Becoming

To fully understand the philosophical significance of Hegel's conception of infinitesimal calculus, one must take into consideration that Hegel's notion of the conserving of quality in the vanishing of quantity is meant as a response to what the preceding chapter argues is the fundamental problem of transcendental philosophy, the problem of "thinking" the contradictory concept of becoming and change. The solution, which Hegel's philosophy most clearly outlines, clears the path for the overcoming of the conflict between concept and intuition that Kant at first declares to be irreducible before claiming to "unit[e] what, a moment earlier, was declared to be independent and therefore not unifiable" (HW 8:145; §60).[362]

Becoming, for Hegel and according to Kant's earlier definition, is the union of two contradictory and opposed determinations, the unity of being and nothingness. For "ordinary dialectic", that is, for the "understanding" or for "reflection", becoming is "unthinkable" because it is contradictory: "common sense [der gesunde Menschenverstand] … rejects the union of being and nothing: either something is being or it is not; there is no third; that which is does not begin, that which is nor that which is not. Common sense thus affirms the impossibility of the beginning" (HW 4:166). The problem, the eternal problem of philosophy whose role in Kant's thought I have investigated, is proposed again in exactly the same terms in post-Kantian philosophy. Schelling, for example, in the *System of Transcendental Idealism* (*System des transzendentalen Idealismus*), summarizes the status quaestionis particularly

[360] English translation from Hegel. *Hegel's Science of Logic*, 269.

[361] Ibid., 278.

[362] English translation from Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 105. Translation slightly modified by the author.

well: "In every change [Veränderung], there is a transition from one state into its contradictory opposite, as, for example, when a body switches from movement in direction *A* to movement in direction– *A*" (HKA 9:9:216).[363] Difficulties arise when one seeks to "mediate the contradiction between the opposites," as Schelling observes:

> There is a well-known sophism whereby the ancient sophists contest the possibility of communicating motion. Take, they say, the last instant at which a body is at rest, and the first at which it moves; there is no intermediate between the two…. Hence, if a body is set in motion, this happens either at the last instant of its rest, or the first instant of its motion; but the former is impossible, because it is still at rest, and the latter impossible, because it is already in motion (SW 3:589).[364]

Schelling's example intuitively clarifies the difficulty in which the understanding is caught when it seeks to grasp the transformations and changes that occur in the phenomenal world: "With the absolute separateness [Geschiedenheit] of being from nothing," writes Hegel in a classic page of the Science of Logic, "beginning or becoming is something incomprehensible [Unbegreifliches]" (HW 5:109):[365]

> It is impossible for anything to begin, either in so far as it is, or in so far as it is not; for in so far as it is, it is not just beginning, and in so far as it is not, then also it does not begin…. For the same reason, too, something cannot cease to be for then being would have to contain nothing, but being is only being, not the contrary of itself" (HW 5:109).[366]

The concept of becoming requires the co-presence of two contradictory opposites and, in consequence, annihilates itself as a "concept." The understanding cannot in any way think becoming: "becoming is the vanishing of being in nothing and of nothing in being and the vanishing of being and nothing generally; but at the same time it rests on the distinction between them. It is therefore inherently self-contradictory, because the determinations it unites within itself are opposed to each other; but such a union destroys itself" (HW 5:112).[367] As I have shown, the problem of becoming is central to Kant's own philosophy. It is none other than the problem of establishing how the concepts A and B, despite being different and the one being the contradictory opposite of the other, can nevertheless be connected in a necessary fashion: "Kant's notion of synthetic *a priori* judgments," writes Hegel, that is to say, "the notion of something differentiated which equally is inseparable, of an identity which is in its own self an inseparable difference, belongs to what is great and imperishable in his philosophy" (HW 5:239).[368] On the one hand, this problem ultimately seems to represent the unified point of view from which the diverse aspects of Kant's thought can be understood. On the other hand, the problem seems

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[363] English translation from Schelling. *System of Transcendental Idealism (1800)*, 380.

[364] Ibid., 144.

[365] English translation from Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 104.

[366] Ibid.

[367] Ibid., 106.

[368] Ibid., 209.

to be the clearest expression of the splitting of Kant's thought into a series of opposed elements (understanding and sensibility, concept and intuition, etc.), which seem unsusceptible to all mediation once separated. If becoming cannot be "thought" by the intellect, it is nevertheless "given" in intuition; if contradictory concepts cannot exist together, they can nevertheless coexist by being placed one-outside-the-other. Thus, on the one hand, "In the *a priori* synthesis of the Notion, Kant possessed a higher principle in which a duality in a unity … the material of sense, the manifold of intuition, was too strong for him and he was unable to get away from it" (HW 6:266).[369] "The very expression synthesis," Hegel writes, "easily recalls the conception of an external unity and a mere combination of entities that are intrinsically separate" (HW 6:260).[370]

As discussed above, the only way to overcome the conflict between *understanding* and *sensibility*, between the synthetic unity and the given manifold, is to find a "concept" that allows "thought" to master becoming itself, a concept that permits the fixing of the *third* in which the two opposites can coexist, the one passing into the other. Such a third, incomprehensible for "reflective understanding" and apparently only graspable for intuition, is "[t]he unity, whose moments, being and nothing, are inseparable, is at the same time different from them and is thus a third to them; this third in its own most characteristic form is *becoming*" (HW 5:96).[371]

The unity of that which is different, the problem that change and becoming bring to the fore, now assumes a new form: "The synthesis, which is the point of interest, must not be taken as a connection of determinations already externally there." Rather, what is at issue is "immanent synthesis, synthesis *a priori*, a self-subsistent, self-determined unity of distinct moments. Becoming is this immanent synthesis of being and nothing; but because synthesis suggests more than anything else the sense of an external bringing together of mutually external things already there [äußerlich gegeneinander Vorhandener], the name synthesis, synthetic unity, has rightly been dropped" (HW 5:99).[372] While this is the fundamental novelty that Kant introduces, that *A* and *B* should be thought as "different" but should nevertheless be connected, it now becomes clear that *A* and *B* cannot be simply "given" and remain incomparable as such, at least in principle. In order to ovecome Kant's difficulty, Hegel develops a new dialectical concept of synthesis: "we call dialectic the higher movement of reason in which such seemingly utterly separate terms pass over into each other spontaneously, through that which they are, a movement in which the presupposition sublates itself. It is the dialectical immanent nature of being and nothing themselves to manifest their unity, that is, becoming, as their truth" (HW 5:110).[373]

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[369] Ibid., 594.

[370] Ibid., 598.

[371] Ibid., 93.

[372] Ibid., 96.

[373] Ibid., 105.

Reflective understanding is "understanding as abstracting, and hence as separating and remaining fixed in its separations" (HW 5:37),[374] that is, the understanding that "determines, and holds the determinations fixed" ($A = A$ and $\neq$ not–A). In contrast, "*reason* is" above all "negative and *dialectical*, because it resolves the determinations of the understanding into nothing" (HW 5:15).[375] Speculative thought, however, does "not stay in the nothing of this result but in the result is no less positive" (HW 5:15).[376] Only dialectical thought is capable of grasping "the vanishing of being in nothing and of nothing in being … but at the same time it rests on the distinction between them" (HW 5:112),[377] which is called becoming. This becoming is ungraspable through abstracting understanding. Only speculative reason can determine it, fixing the third between the opposites in which they pass into each other: "It is in this dialectic [in diesem Dialektischen] as it is here understood, that is, in the grasping of opposites in their unity or of the positive in the negative, that speculative thought [das Speculative] consists" (HW 5:51).[378]

For Hegel, there is nothing mystical about this conception. Even though the union of being and nothingness, the union of opposites that is realized in becoming, appears nonsensical to abstract logical thought as much as for common sense, "[I] t is to the adoption of the said determination, which understanding opposes, that mathematics owes its most brilliant successes" (HW 5:110):[379]

> The foregoing dialectic is the same, too, as that which understanding employs the notion of *infinitesimal magnitudes*, given by higher analysis … These magnitudes have been defined as such that they are *in their vanishing*, not before their vanishing, for then they are finite magnitudes, or after their vanishing, for then they are nothing. (HW 5:110)[380]

The ratio between vanishing magnitudes is not the ratio "before which and after which they vanish, but with which they vanish (*quacum evanescunt*[381])" (HW 5:297).[382] This relation, as discussed above, is not a relation between finite differences, nor is it a relation between null differences: "Against this pure notion," Hegel continues "it is objected and reiterated that such magnitudes are either something or nothing; that there is no intermediate state between being and non-being"

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[374] Ibid., 45.

[375] Ibid., 28.

[376] Ibid.

[377] Ibid., 106.

[378] Ibid., 105.

[379] Ibid., 93.

[380] Ibid.

[381] The allusion is to Newton's famous statement: "per ultimam rationem quantitatum evanescentium, intellegendam esse rationem quantitatum, non antequam evanescunt, non postea, sed quacum evanescunt" ("And in like manner, by the ultimate ratio of evanescent quantities is to be understood the ratio of the quantities not before they vanish, nor afterwards, but with which they vanish"; Newton, *Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World*, 39).

[382] Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 255.

(HW 5:110).[383] On the one hand, the problem of fixing an "intermediate state" between finite magnitude and zero is nothing other than a particular case of the problem of becoming in general, the problem of determining the unity of being and nothingness. On the other hand, the problem assumes an entirely new form.

The vanishing of magnitude is not in the least a "state" of magnitude (a term that Hegel calls "unsuitable, barbarous" [HW 5:110][384]) and one cannot resort to the same "logic" that governs the "quantum" to grasp it. As I have shown, vanishing magnitudes are = 0 from the point of view of quantity; their meaning can only be found in their reciprocal relation, which exists independently of the quantity: "that which has being *solely* in the ratio," writes Hegel, "is not a quantum; the nature of quantum is such that it is supposed to have a completely indifferent existence apart from its ratio" (HW 5:296).[385] Infinitely small magnitudes, in contrast, "are" only in their relation, outside of which they are pure zeros. They are only "moments of … the ratio" (HW 5:296).[386]

The "infinitude = 0 of the indeterminate from which each and everything determinate has yet to proceed" (HW 5:95)[387] is therefore not "pure nothing, but a nothing from which something is to proceed … the unity of being and nothing; or is non-being which is at the same time being, and being which is at the same time non-being" (HW 5:72).[388] This null represents the absolute beginning [Anfang]; that which is *no longer* only nothing, but which is *not yet* something: "it [the beginning] is their undifferentiated unity" (HW 5:73),[389] the indeterminate from which determinacy should spring and in which determinacy must already be enclosed: "the beginning itself is also becoming" and in it "being and nothing show themselves to be undivided within a unity" (HW 8:190: §88).[390]

Only through this concept of the infinite, which is found in higher analysis, can one truly move beyond Schelling's idea of the absolute as a purely quantitative indifference.[391] As in the "vanishing" of magnitude, the negation that the quantum represents in respect to quality is negated again. In this double negation, the "bad" infinity of quantity is removed in the "true" and "qualitative" infinite. In the same way, finite

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[383] Ibid., 104. Hegel alludes to d'Alembert who, arguing against Newton, writes: "A quantity is something or nothing: if it is something, it has not yet vanished; if it is nothing, it has literally vanished. The supposition that there is an intermediate state between these two is a chimera" (quoted in Boyer, Carl Benjamin. *The History of the Calculus and its Conceptual Development* (New York: Dover, 1959), 521). Boyer draws the citation from *Mèlanges de litèrature, d'histoire et de philosophie* (Amsterdam: Zacharie Chatelain & Fils 1766–1770).

[384] English translation from Hegel. *Hegel's Science of Logic*, 104.

[385] Ibid., 254.

[386] Ibid., 255.

[387] Ibid., 86.

[388] Ibid., 73.

[389] Ibid., 74.

[390] English translation from Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 185.

[391] See Bodei, Remo. *Sistema ed epoca in Hegel* (Bologna: Il Mulino, 1975), 210.

things in general "are, but the truth of this being is their end. The finite not only alters, like something in general, but it ceases to be [vergeht]; and its ceasing to be is not merely a possibility, so that it could be without ceasing to be, but the being as such of finite things is to have the germ of decease as their being-within-self: the hour of their birth is the hour of their death" (HW 5:139).[392] However, if their "end is to be grasped as the nothing, then we should be back again at that first, abstract nothing which itself has long since passed away" (HW 5:132).[393] The "one-sidedness and restrictedness of the determinations of the understanding" (HW 7:90)[394] are negated, but this negation is not an "empty abstract nothing, but the negation of certain determinations, which are contained in the result precisely because it is not an immediate nothing, but a result." It is not a "simple formal unity, but a unity of distinctive determinations" (HW 7:90).[395] The finite does not simply negate itself in the impenetrable night of absolute indifference because "[i]t is not in the sublating of finitude in general that infinity in general comes to be; the truth is rather that the finite is only this, through its own nature to become itself the infinite" (HW 5:149).[396]

In the "double negation" through which, according to Hegel, the finite returns into the infinite, the authentic "spirit" of *critical* thought seems to have been completely lost. The need of *dogmatic* metaphysics to find an "in itself" beyond all the conflicts and "relative" oppositions that characterize phenomenal reality, which can reveal that these are mere appearances, seems to resurface as the ultimate end of speculative philosophy: "The idealism of philosophy consists in nothing else than in recognizing that the finite has no veritable being" (HW 5:169).[397] Diverting one's attention from this simple result, however, one immediately notes how, in Hegel's concept of the infinite, the fundamental problem of metaphysical thought, while indeed revalidated, has at the same time adopted an entirely new form. The absolutely "positive," which is arrived at here through the suppression of all "negativity," brings with it "the labour of the negative" (HW 3:12).[398] The negative is indeed "removed," but at the same time "conserved":

> Something is sublated only in so far as it has entered into unity with its opposite; in this more particular signification as something reflected, it may fittingly be called a moment. In the case of the lever, weight and distance from a point are called its mechanical moments on account of the sameness of their effect, in spite of the contrast otherwise between something real, such as a weight, and something ideal, such as a mere spatial determination, a line (HW 5:114).[399]

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[392] English translation from Hegel. *The Encyclopaedia Logic, with the Zusätze. Part I of the Encyclopaedia of Philosophical Sciences with the Zusätze*, 129.

[393] Ibid., 131.

[394] English translation from Ibid., 128.

[395] Ibid., 131.

[396] English translation from Hegel. *Hegel's Science of Logic*, 138. On the connection between infinitesimal calculus and Hegel's conception of the relations between the finite and the infinite, see Bodei. *Sistema ed epoca in Hegel*, 200–10.

[397] Hegel. *Hegel's Science of Logic*, 154–.

[398] English translation from Hegel. *Phenomenology of Spirit*, 10.

[399] English translation from Hegel. *Hegel's Science of Logic*, 107.

Once again, the metaphor of the lever shows itself to be the adequate expression for the unity of opposites in which these are reciprocally reduced to *nothing* in order to affirm their *being* precisely in this reciprocal negation; that is to say, the adequate expression for the notion of "the unity in which they are preserved" (HW 5:114).[400] The unity of being and nothing, that is, becoming, was "unthinkable" from the point of view of "logical opposition" because the disappearing of being into nothingness and of nothingness into being is nothing more than *contradiction*. In contrast, this unity is perfectly definable from the point of view of "real opposition" because becoming is nothing more than "equilibrium [Gleichgewicht] of coming-to-be and ceasing-to-be" (HW 5:112; emphasis mine).[401]

Thus, it is no coincidence that, in the *Jena System* (1804–5), Hegel posits $a - A = 0$ as the "formula" of the "true infinite."[402] In contrast to Kant's formulation $A - A = 0$, the lowercase letter seems to indicate that an absolute "identity" is no longer involved, but rather a mere "equivalence" of different elements:[403] "In connection with the lever, for instance," writes Hegel in the *Encyclopaedia*, "distance can take the place of mass, and vice versa, and a quantum of ideal moment produces the same effect as the corresponding real amount." Similarly, "in connection with the magnitude of motion, … the real effect is the same if the mass is increased and the velocity proportionately decreased" (HW 9:60).[404] For example, "[I]f the mass is six pounds and the velocity four units, then the force is twenty-four; but it is the same, too, if eight pounds moves with a velocity of three units" because if one attributes contrary signs to the quantity of movement that results from contrary velocities, the sum is nothing. The contrary velocities taken alone would reciprocally create the equilibrium ($mV - Mv = 0$), just as "the length of the arm on the one side of the Hypomochlion [fulcrum] where the (motive) weight hangs has its counterpoise on the other side where the weight (to be moved) hangs" (HW 9:67)[405] ($mL - Ml = 0$).

3.8 Herbart and Being as Pure Affirmation

The examples chosen by Hegel further confirm that post-Kantian philosophy, in as much as it is no longer commensurable with the original critical formulation, merely reaffirms and explores the extreme speculative consequences of what

[400] Ibid.

[401] Ibid., 106. On the importance of the metaphor of the lever in Hegel's philosophy, see Ziche. *Mathematische und naturwissenschaftliche Modelle in der Philosophie Schellings und Hegels*, 222.

[402] English translation from Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 18. For a discussion of this formula, see Ziche. *Mathematische und naturwissenschaftliche Modelle in der Philosophie Schellings und Hegels*, 222.

[403] For a more detailed treatment of this point, see Hegel. *The Jena System, 1804–5: Logic and Metaphysics*, 35–.

[404] English translation from Hegel. Hegel's Philosophy of Nature: Being Part Two of the Encyclopedia of the Philosophical Sciences (1830) 42.

[405] Ibid., 51.

appears to be the characteristic manner in which Kant addresses one of the classical themes in the philosophy of the "School": "in the questions concerning the possibility of a conflict of realities, of intensive magnitude, and so on" (AA 4:478).[406] As I have shown, Leibniz-inspired metaphysics thinks that realities should be considered "positive affirmations" (AA 20:302),[407] endowed with a certain positive degree of "perfection." From this point of view, the "concept *de ente omni modo negativo*" is reduced to the "concept of a *non entis*" (AA 28:1013)[408] to which nothing can be made to correspond, without falling into a gross contradiction. According to Kant, in contrast, in the phenomenal world where only simple relations are given, "reality" no longer appears as absolutely positive being (*positivo sive reale*), which excludes all negativity from itself, and "negation" is not reduced to being a simple removal (*remotio*), the mere absence of being (see AA 20:302).[409] Instead, reality and negation behave like an equally positive something in respect to one another such that negation is itself a reality that is actively opposed to another reality.

Therefore, the conflict and agreement between reality and negation, as Kant notes in numerous passages, should not be "thought of as *logical* (as between *A* and non–*A*)," and can thus be compared to the case of "forces effective in space which (like attraction and repulsion) affect one another by opposite direction of motion" (AA 21:311.5).[410] In contrast to conceptual opposition, "the opposition in the moving forces of matter is like + *a* and − *a*, that is, not merely logical, but real," as in the way that "the lever, which is moved only through its own weight, is held in equilibrium by arms of equal length" (AA 21:530–.5). That the image of the "lever," and of the "equilibrium" between opposed forces in general, returns so insistently in the great thinkers of German idealism can be considered a consequence of the prevalence of Kant's conception of the relation between reality and negation over the one imposed by the Leibnizian-Wolffian tradition.

The manner in which Kant defines the relation between reality and negation offers an effective tool for furnishing an intuitive representation of the coexistence of elements that, even though different and opposed, can be thought, even in their opposition, as manifestations of a single principle when considered in relation to each other. Behind the metaphysical speculations that emerge in the peak years of *Naturphilosophie* and of the great idealist systems, the following fundamental logical and ontological conception is concealed: the idea that, in the words of Kuno Fischer, "positive and negative, being opposites, remove each other in the indifferent unity … they become equal to each other and appear in this relation as negative

[406] English translation from Fichte. "Foundations of the Entire Science of Knowledge" 92.

[407] English translation from Kant. "What Real Progress Has Metaphysics Made in Germany since the Time of Leibniz and Wolff?", 390.

[408] English translation from Kant. *Lectures on Philosophical Theology*, 44.

[409] English translation from Kant. "What Real Progress Has Metaphysics Made in Germany since the Time of Leibniz and Wolff?", 390.

[410] English translation from Kant. *Opus postumum*, 25–6.

magnitudes."[411] This conception, though translated into the terms of a doctrine that seems far from the letter as well as the spirit of Kant's thought, appears to be the direct consequence of the conception of the relations that exist between reality and negation that Kant himself defends against Leibniz-oriented metaphysics.

Particularly significant is the attempt of Johann Friedrich Herbart (1776–1841), one of the most important non-idealist thinkers during the epoch of idealism, to construct his metaphysics on the explicit and systematic confutation of such a conception by resorting to certain assumptions from Wolffian scholastics. In his 1828 *Allgemeine Metaphysik* (General Metaphysics), he seeks to demonstrate the impossibility of attributing any form of "being" to negations: "according to the terminology of the old school, realities are those determinations of the quality of a thing, which are only thought by means of an affirmation," while "positing a negation precisely means removing a what from its place. At least this is what the old school meant when it observed that a thing could not be totally negated" (HSW 8:62). Without succeeding in developing this point in all its rigor, Wolffian metaphysics intuited that, according to Herbart, "the quality of an existent is entirely positive, or rather affirmative, without the intrusion of negations" (HSW 8:63). The quality cannot "be determined in any way through internal antitheses" (HSW 8:63).

What distinguishes an "existent as such" can only be found in the "complete absence of all negation in the being as a pure positive" (HSW 8:63) because "a positive in which a negative is concealed would be a deceit" (HSW 8:63). One cannot even maintain, as the "School" does, that a "thing" is defined by its negations, that the quality of things "is partly positive and partly negative" (HSW 8:63); because that would involve the unacceptable concession that negations in some way "are," which is manifestly a contradiction: "the negative part of the quality would not itself be posited, but rather removed" (HSW 7:63) and thus would not be at all because "the one would be the positive in this position and the other the negative and consequently would not be an entity, or rather both would only be reciprocal negations and thus neither would be originally positive, something that would be even more senseless to attribute to real entities" (HSW 8:63). Since "not–A is not in the least an object with an absolute position" (HSW 7:63), "the positive part" is "sufficient to furnish the quality" (HSW 8:63).

Though Herbart's philosophy is certainly not a mere recuperation of the legacy of the Wolffian School in the face of Kantian idealism,[412] he undeniably credits the "metaphysics of the School," as opposed to all post-Kantian metaphysics, for valorizing the principle, "*omni enti quaedam inest realitas*" (HSW 8:62); the principle by which every entity in as much as it is, must necessarily contain something positive and affirmative. The "old school," identifying "the good with the real, and the bad with negation" (HSW 7:49), as Herbart explains, certainly "admitted a *malum*

[411] Fischer, Kuno. *Logik und Metaphysik oder Wissenschaftslehre*. ed. Hans-Georg Gadamer (Heidelberg: Mauntius, 1998), 106.

[412] On Herbart's relationship to Kant and to idealist philosophy in general, see Pettoello, Renato. *Idealismo e realismo: La formazione filosofica di J. F. Herbart* (Florence: La Nuova Italia, 1986).

metaphysicum in things, which would consist in the negations that intervene in them. However, the traces of a better spirit are also indicated, which are already found in the paradoxical principal, *omne ens unum verum bonum*. Here is an effort, worthy of noting despite being in vain and far from firm, to approach the absolute position and the simplicity of the quality" (HSW 8:69).

Since being is absolute positivity and perfection, traditional metaphysics can maintain that "every thing is good: *omne ens est perfectum et bonum transcendentaliter*" (HSW 8:69) and that the bad comes from non-being and from nothingness. Herbart argues that this conception is still inadequate, despite providing a glimpse of the correct solution to the problem of "being." Herbart's position thus helps to identify through contrast, so to say, the peculiarity of Kant's conception that later found fertile terrain for further development in idealistic philosophy, a conception that is based on the attempt to recognze the positive "reality" of the negative. From this point of view, as Søren Kierkegaard perceptively notes, Herbart can be considered "the only modern philosopher who has pronounced in favor of the positive,"[413] committing himself to negating all ontological consistency of the "negative" for the sake of liberating experience from its contradictions.

Herbart's treatment of magnetism is interesting in this context. As discussed above, the image of the magnet is a sort of *locus communis* in Romantic idealism's philosophy of nature for its ability to represent the opposition of elements that, despite presenting no essential difference, are manifested as opposites. Herbart, in contrast, emphasizes the intrinsic contradictoriness of the phenomenon of magnetism as it appears to experience:

> [W]hat is magnetism for example? Unity of opposed polarities. We can simply think of a south pole that repels other south poles and attracts north poles. Frequent attempts have been made to break magnets in order to obtain simple north poles and simple south poles. What is opposed is found here as distinctly separate, at the extremities of a line that is as long as the entire magnet. Who would ever believe that these opposed elements are only one thing? Who would not prefer to subdivide magnetism into two types according to south and the north? But experience is stubborn (HSW 8:38).

Experience tells us that we should not "distinguish … two opposed types." Instead, if we find "one, this should serve as a signal to us that the other is in the vicinity. Neither of the two is something in itself. Magnetism is precisely the opposition of the two" (HSW 7:38).

However, logic and the principle of identity necessarily repel the unity of opposed terms, forcing one to put into question what experience reveals: "the contradictory opposite of unity is non-unity; and elementary logic tells us that non-unity is attributed to opposed elements." Even if "separation conflicts here with experience" (HSW 8:38), experience must be overcome in virtue of the fact that one and the same element "cannot carry opposed elements in itself" (HSW 8:39). Thus, the contradiction present in experience should be eliminated, proceeding "to the

[413] Kierkegaard, Søren. *The Concept of Dread*, tr. Walter Lowrie (Princeton, NJ: Princeton University Press, 1957), 130.

separation of the unity that must connect that which is opposed and cannot. The fault of the contradiction is found in this unity. If the unity is eliminated, the given oppositions remain, as should be the case, and the contradiction is overcome" (HSW 8:38). "Thought" must not welcome "negation" and make "negation" its very dynamic element, as idealism pretends to do by resorting to a different form of the notion of a *coincidentia oppositorum*. Rather, "negation" should be removed and eliminated: "Through a double negation an affirmation is derived and only the latter provides the concept of being" (HSW 8:38).

The purpose of the recourse to *duplex negatio* here is not to clarify the negativity revealed by the experience of a changing and varying reality reveals, but rather to eliminate the "appearance" of negativity in order to reconstruct the absolute positivity and immutability of being. Certainly, "no one doubts that alterations are given in the experience" (HSW 8:92). However, if one considers closely the "alteration" as it is "given" through experience, as the transition of the same substance from one state into its opposite, one immediately realizes that change is presented in the form of a unity of two contradictory predicates, a unity that elementary logic categorically rejects.

If an alteration in a thing means that the thing should "alternatively be and not be, the intervals of time in which it was not would destroy its existence such that it would lose its connection with itself" (HSW 8:104). If "the temporal determination is removed such that the temporal points coincide, then being and nonbeing, no longer separated through the diversity of durations, will also come to coincide *in a single concept*" (HSW 8:104). Consequently, "in every instant … substance is posited at the same time as existing and as not existing, which is evidently a contradiction" (HSW 8:104). Contrary to idealism, Herbart refutes the idea that one should search for tools capable of mastering the contradiction sparked by becoming. He recognizes rather openly that imagining "that changes occur in things, such as when they pass from rest into motion and vice versa" (HSW 7:04), involves a clearly contradictory appearance, and, as such, cannot be considered real. Instead, it should be reduced to the merely relative point of view of the observer. Such is the case in mechanics, where during the "decomposition of forces," one can "eliminate those parts which completely oppose each other and eliminate each other" (HSW 8:102), leaving everything unaltered beyond the appearance of change.

For Herbart, the absolute coincidence of an entity with itself is the only guarantee of its being. Thus, no entity can endure any action inflicted by another entity because nothing can "disturb" its rigid identity with itself, an identity which excludes and repels all negation from itself: "real occurrence is nothing other than persistence against a negation, an auto-conservation". Given that $+\beta$ and $-\beta$ remove each other because "contraries can turn into each other and disappear" (HSW 8:102), if "$A = \alpha + \beta + \gamma$ together with $C = p + q - \beta$" (HSW 8:101) are posited, then A and C are conserved as identical to themselves. Consequently, nothing occurs except from the contingent point of view of the observer: if something "seems" to occur, "in reality," everything remains as it is. The apparent existence of negations, as experience presents them, can be eliminated by showing how these

are ultimately only "contingent points of view" (*zufällige Ansichten)* that possess no autonomous being.

Herbart uses the opposition between positive and negative magnitudes for an entirely different end than that of the idealist tradition:

> [T]he signs + γ and − γ here do not necessarily indicate two concepts such that one of them is to be understood as *only* positive and the other as *only* negative. It suffices that we represent all this in the same manner as the relation that exists between opposite directions. If one must distinguish positive and negative *x*-axes and *y*-axes, the *x*-axes that head to the right must be considered positive as well as the *y*-axes that extend upwards; the negative *x*-axes are oriented towards the left and the negative *y*-axes descend from the horizontal of the axes. But everyone knows that this construction is arbitrary. We could easily consider the *y*-axes that descend and the *x*-axes that head left as positive. The opposition is entirely relative (HSW 8: 101).

Thus, one cannot attribute any ontological consistency to such contingent points of view such that "one would be positive … and the other negative and consequently not an entity … or rather both would be nothing more than reciprocal negations and thus neither of the two originally positive" (HSW 8:103).

The "concept of negative magnitude does not belong to the concept of number as one of its properties" (HSW 8:133). It assumes a determinate meaning only relative to the position that a certain element occupies in an ordered *series* (as occurs in space, in time, and in degrees) and from the possibility of following such a series in two opposite directions: "[O]ne can choose: to go from *C sharp* to *G sharp* through all the intermediate tones: then the opposition to *C sharp* arises. But if one goes from *G sharp* to *C sharp*, again through the intermediate tones, then the opposition also arises, that is, the opposition to *C sharp*" (HSW 8:101). No "true" opposition within the absolute positivity and simplicity of the single elements is involved here: "[P]erhaps one believes that in the notes *C sharp* and *G sharp* something negative in itself is hidden, in which neither of the two would ever hide? One can choose: either one can just as easily be considered that which is farther from the other by a certain degree" (HSW 8:101). Thus, according to Herbart, none of these elements are "constituted of parts, but each one, in comparison to the other, permits one to distinguish (not to separate, however) that which is equal or opposite in respect to the other" (HSW 8:100). Thus, it can be justly affirmed that "these oppositions can be different as magnitudes" (HSW 8:100).

The consequence of this formulation is the exclusion of "every concept of quantity" from the quality of any given existent (HSW 8:66). Thus, Kant's attempt to "attribute to the existing a real multiplicity of degree, which could effectively increase or decrease," is misguided for Herbart. Being can possess "neither magnitude, nor degree" (HSW 2:68) because no intermediary can be thought between being and nothingness. One can certainly say that, "blue is opposed to red more than to purple. And equally that *C sharp* is less opposed to *G sharp* than to *A* and more than to *G*" (HSW 8:101). However, this does not imply that each simple element possesses a quantity because "if one listens to a pure single note, whether it is an eighth or a seventh is not heard" (HSW 8:73). Each element in the series remains simple in itself without any internal multiplicity. One can certainly affirm that "the

seventh degree … is located rectilinearly … between the eighth and the sixth" (HSW 8:73); that is to say, a certain degree is found between the $(n - 1)$th and $(n + 1)$th position, but each one of them remains in itself devoided of any form of quantity.

The concept of "degree" is legitimately used only when one speaks of physical appearances, like velocity, or even psychic appearances, like representations; but not in regard to being-in-itself. The "Herbartian" Bernhard Riemann, in a few posthumously published fragments, may have most effectively summarized the complex sense of this conception: "What permits us to know our conception of things are … quantitative relations, spatial and temporal relations of sensible things, and relations of intensity between notes and their qualitative differences: *there are no degrees of being however*: a difference in degree is only thinkable for states and relations." [414]

As I have shown above, Kant does not intend to use the principle of the Anticipations of Perception to attribute "degrees" to "being"-in-itself. Rather, moving beyond the traditional conception of *gradus perfectionis*, he limits the concept of "degree" to "phenomenal reality," in which only simply relations can be thought, relations that appear positive or negative according to the "direction" in which they are followed. Herbart's critique of the idea that "being" can present negations and degrees, though only partially relevant to Kant's conception, effectively highlights the particular point of view that prevails in post-Kantian idealism, a point of view that Herbart animatedly opposes. "Idealism" seeks to introduce an opposition between reality and negation into the sphere of "knowledge" in order to permit the thinking of the unity of opposed elements. Herbart's "realism," in contrast, aims to highlight the contradictoriness that one falls into every time that a negation is introduced into the sphere of absolute positivity that characterizes being.

[414] Riemann, Bernhard. *Gesammelte mathematische Werke und wissenschaftlicher Nachlass*. ed. Richard Dedekind. 2nd ed (Leipzig: Teubner, 1892), 550. On this subject, see Banks, Erik C. "Kant, Herbart and Riemann," *Kant Studien* 96, no. 2 (2005): 208–34. This essay discusses many points addressed in this section, but often from a different point of view. In particular, I reject the thesis that Herbart's conception "would certainly have fit with Kant's view of the world (*qua realitatis phaenomena*) as consisting of an equilibrium of opposing forces" (Banks. "Kant, Herbart and Riemann," 219). See, for example, Trendelenburg's critique of Herbart, which the next chapter addresses.

Chapter 4
The Anticipations of Perception in Neo-Kantian Idealism

4.1 From Classical Idealism to the Return to Kant

Friedrich Aldolf Trendelenburg (1802–1872), an often neglected yet fundamental thinker for understanding the evolution of philosophy in Germany following the great idealist systems, notes that, with Herbart, one has the impression of breathing "another air."[1] Compared to the speculative arbitrariness of the "poetic philosophers" (HSW 8:73), Herbart is more attentive to the data of experience, data that metaphysics should not "reconstruct" on the basis of *a priori* principles, but simply "liberate" from internal contradictions. One can argue that the publication of Trendelenburg's *Logische Untersuchungen*[2] (Logical Investigations) in 1840 initiated a new phase in German philosophy, culminating in the affirmation of the great neo-Kantian schools[3] that dominated the philosophical scene in Germany. Trendelenburg was the first to conceive philosophy as "theory of science" with the end of discovering the presuppositions behind what he calls the "facts of science,"[4] an expression destined for success. In respect to the present work, however, Trendelenburg deserves credit above all for demonstrating how essential the opposition

[1] Trendelenburg, Adolf. "Geschichte der Kategorienlehre: zwei Abhandlungen." In *Historische Beiträge zur Philosophie* (Bethge: Berlin, 1846–1867), 1:361.

[2] Trendelenburg, Adolf. *Logische Untersuchungen*. 3rd ed. 2 vols (Leipzig: Hirzel, 1870).

[3] On the importance of Trendelenburg for the origins of the neo-Kantian movement, see Köhnke, Klaus Christian. *Entstehung und Aufstieg des Neukantianismus: Die deutsche Universitätsphilosophie zwischen Idealismus und Positivismus* (Frankfurt am Main: Suhrkamp, 1986), 23–; Poggi, Stefano. *I sistemi dell'esperienza: psicologia, logica e teoria della scienza da Kant a Wundt* (Bologna: Il Mulino, 1977), 315–48.

[4] Trendelenburg. *Logische Untersuchungen*, 1:11, 130 and 306. See also, Ferrari, Massimo. *Categorie e a priori* (Il Mulino: Bologna, 2003), 95–100.

M. Giovanelli, *Reality and Negation – Kant's Principle of Anticipations of Perception: An Investigation of Its Impact on the Post-Kantian Debate*, Studies in German Idealism 11,
DOI 10.1007/978-94-007-0065-9_4, © Springer Science+Business Media B.V. 2011

between "reality" and "negation" was to the great idealist systems, and in particular to Hegelian dialectic.[5]

Trendelenburg's observations effectively bring together the results of the preceding chapters:

> What is … the essence of dialectical negation? Negation can have a double nature: either it is conceived in a *purely logical manner*, such that it simply negates what the first concept affirms without putting anything new in its place; or it can be understood *in a real manner*, and thus the affirmative concept is negated by a new affirmative concept in as much as both should be referred to each other. We call the first logical negation, and the second real opposition.[6]

According to Trendelenburg, "[t]he concept, which, like an innate impulse, sets the dialectic in motion degree by degree, is negation."[7] Thus, negation is the indispensable instrument for Hegelian philosophy and at the same time the fundamental difficulty from which it seems unable to escape: "[A]ll of this results in an ineludible dilemma for the dialectic of thought. Either, the negation," which permits the dialectic to pass from one determination to another, is "the pure logical negation (*a*, not-*a*)."[8] In this case, how it could produce a new concept from the given one would be incomprehensible since "negation (in as much as it is not-*a*), understood logically … does not produce anything, but only excludes."[9] Or, the negation is "real conflict" and thus adds something new and determinate beyond the given concept, something that cannot be obtained "in a logical way … and the dialectic is thus not the dialectic of pure thought,"[10] as it nevertheless claims to be:

> The applied negation is not a logical negation that behaves like *A* and not-*A*, but a real opposition that produces the contrary [Contrarium] of the contrast [Gegensatz]. The contrary does not mean the undetermined contradiction of the contrast, but rather another positive determination, …, in this manner it is clear that the negation of dialectic cannot be reached in a logical way.[11]

If the problem of knowledge, as Kant formulates it, consists in moving beyond concepts already possessed of objects in order to access different concepts of them, which are still unknown, then logical negation is insufficient. The latter does not provide any exit from a given concept. Instead, logic merely decomposes it into its

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[5] For a detailed treatment of this point, of which I can only provide a simplified reconstruction, see Schmidt, Josef. *Hegels Wissenschaft der Logik und ihre Kritik durch Adolf Trendelenburg*, Pullacher philosophische Forschungen (Munich: Berchman, 1977), 93–. See also Köhnke. *Entstehung und Aufstieg des Neukantianismus: Die deutsche Universitätsphilosophie zwischen Idealismus und Positivismus*, 49.

[6] Trendelenburg. *Logische Untersuchungen*, 1:43 (emphasis mine).

[7] Ibid.

[8] Ibid., 1:56.

[9] Trendelenburg. "Geschichte der Kategorienlehre: zwei Abhandlungen." 1:361.

[10] Trendelenburg. *Logische Untersuchungen*, 1:56.

[11] Trendelenburg, Adolf. *Die logische Frage in Hegel's System: Zwei Streitschriften* (Leipzig: Brockhaus 1843), 15.

internal articulations. A different form of opposition is required that is not simple *contradictoriness*, but that is configured as a *contrariety*; that is, as an opposition between two determinations that are not simply "disparate," simply excluding each other, but that are only distinguished in terms of "a difference in degree":[12] "[A]ffirmation and negation of the same concept exclude each other without any prospect, without any hope [Aussicht] of an agreement. In contrast, contraries also have something essentially in common,"[13] whereas "concepts that do not share anything cannot even enter into conflict."[14]

Here, Trendelenburg seems to project Aristotle's distinction between contrariety and contradiction onto Kant's distinction between logical opposition and real opposition.[15] Even if the correctness of such a formulation is doubtful, the meaning of his objection to classical idealism, and Hegelian dialectic in particular, is nevertheless evident:[16] "pure negation is found nowhere except in thought" and, "in nature, nothing can be conceived as simple negation."[17] Thus, pure thought cannot go outside of itself because it is unable to "elevate pure negation to real and autonomous factors, as if non-being could act."[18] Dialectical negation can only go beyond a given concept and produce something new and positive if it is a "real opposition; but, as such, it cannot be obtained through pure thought."[19]

This feature of dialectical negation is already demonstrated in the first passages of Hegel's logic: in the Hegelian doctrine of being, "the Kantian concepts of quality, reality, negation, and limitation are transformed into the kindred concepts of pure being, nothingness, and becoming."[20] In order to complete this transformation, however, Hegel is forced surreptitiously to introduce an element that was not contained in the premises: "[P]ure being as equal to itself is at rest; nothingness, as equal to itself, is equally at rest. In what manner does becoming arise from the union of two static representations?"[21] Becoming, far from being the fruit of the union of being and non-being, should rather be presupposed from the beginning: becoming "could not in any way spring from being and from non-being if the representation

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[12] Trendelenburg. "Geschichte der Kategorienlehre: zwei Abhandlungen." 102.

[13] Trendelenburg. *Logische Untersuchungen*, 1:170–.

[14] Ibid., 1:171–.

[15] See Mangiagalli, Maurizio. *Logica e metafisica nel pensiero di F. A. Trendelenburg* (Milan: CUSL, 1983).

[16] See Rossitto, Cristina. "Negazione logica e negazione reale in F.A. Trendelenburg: significato della distinzione e suoi precedenti storici," *Verifiche*, no. 1–3 (1981): 304–22. See also Rossitto, Cristina. "Aristotelian Theory of Opposites in the Criticism of F.A. Trendelenburg of Hegel's Dialectic." In *Proceedings of the World Congress on Aristotle*, 37–42. (Athens: Publication of the Ministry of Culture and Science, 1982).

[17] Trendelenburg. *Logische Untersuchungen*, 1:169.

[18] Ibid.

[19] Trendelenburg. "Geschichte der Kategorienlehre: zwei Abhandlungen." 361.

[20] Ibid., 164.

[21] Ibid., 39.

of becoming did not precede it."[22] Becoming could not emerge if, in the words of Hegel, "this movement of the immediate vanishing of the one in the other: becoming" was not already admitted (HW V, 82).[23] Hegel himself is forced to admit, at least implicitly, the necessity of a non-logical element, namely, "movement," to provide a positive meaning for dialectical negation.

According to Trendelenburg, "movement traverses the entirety of Hegelian logic"; it is the "vehicle and the presupposition of dialectically productive thought."[24] *Pure thought*, through the principle of non-contradiction, "does not produce, but only defends and conserves; it does not acquire, but only affirms the acquired … only protects the known."[25] Consequently, knowledge, "in as much as something new is placed in it," must always resort to an *intuition* that introduces an element that cannot be deduced from pure thought: "thought goes outside of itself in intuition and this occurs with movement."[26] Thus, according to Trendelenburg, "if thought does not want to entangle itself in its own images, if it wants to have access to being in general, it must have in itself the possibility of a community of things … Such activity, which uniformly determines thought and being, was individuated in the constructive movement."[27] Only *intuition of the movement* allows us to go beyond the given concept in order to add something new to it, something not-given, which nevertheless has an indissoluble connection with that which is already known: "movement is … common to thought and to things."[28] Movement represents a determination of thought but at the same time an "anticipation"[29] of the experience of nature because, "in the external world, every activity is connected with movement."[30]

I cannot elaborate further on the meaning of "constructive movement" here.[31] More interesting for the purposes of the present work is Trendelenburg's observation that even Herbart,[32] despite seeming to move in the opposite direction to Hegel, falls into the same error of claiming that pure thought, through mere "logical opposition" between concepts, can grasp being. Herbart's "initial realism"[33] turns into an

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[22] Ibid.

[23] English translation from Hegel. *Hegel's Science of Logic*, 83.

[24] Trendelenburg, Adolf. "Über Herbart's Metaphysik und eine neue Auffassung derselben (I)." In *Historische Beiträge zur Philosophie* (Bethge: Berlin, 1846–1867).

[25] Ibid., 2:231.

[26] Trendelenburg. *Logische Untersuchungen*, 1:143.

[27] Trendelenburg. "Geschichte der Kategorienlehre: zwei Abhandlungen." 368.

[28] Ibid., 365.

[29] Trendelenburg. *Logische Untersuchungen*, 1:323.

[30] Ibid., 1:14.

[31] Above all, see Fugali, Edoardo. *Dialettica e trascendentale nella teoria del movimento di F. A. Trendelenburg* (Trieste: E.U.T, 1999).

[32] For Trendelenburg's criticisms of Herbart, see Pettoello, Renato. *Introduzione a Herbart* (Roma/Bari: Laterza, 1988), 135–.

[33] Trendelenburg. *Logische Untersuchungen*, 2:516.

attempt to grasp concrete reality through the most abstract of logical principles, namely, the principle of identity, which exerts its "tyranny"[34] on the entirety of Herbart's metaphysics, transforming it into "a form of idealism."[35]

If the principle of identity and non-contradiction is elevated to the point of becoming the unique principle, "it is certain that the mere negative, since it is not and has its essence precisely in unconditioned removal [Aufhebung], cannot be thought as existing."[36] Consequently, "being can in no way be grasped as negative,"[37] and, as I have shown is the case for Herbart, the "quality of the real" can be posited only "as absolutely positive and affirmative."[38] In this manner, however, the possibility of grasping reality in its concrete multiplicity and changeability is precluded for Herbart: if "everything is measured through the principle of identity," writes Trendelenburg, then "unity and multiplicity cannot be brought together in a concept because unity is unity and not non-unity (because *A* is *A* and not not-*A*). In this principle, only being is grasped; how it remains equal to itself."[39]

Herbart's strategy for avoiding this difficulty, as I have shown, consists in substituting the opposition between concepts with that between positive and negative magnitudes: "since opposed qualities encounter each other as positive and negative magnitudes, as + and −, each one of these," writes Trendelenburg, "conserves its essence by the fact that they remove each other. Thus, the existent remains equal to itself, even if change appears to the external observer."[40] The appearance of becoming in the face of the immobility of the real is thus explained "through a mathematical analogy, that is to say, according to the conception of calculus with opposed magnitudes: in as much as positive and negative encounter each other, they remove each other and everything remains equal to itself; if the phenomenon changes, the existent is conserved and remains equal to itself."[41]

If, in this manner, Herbart succeeds in explaining how "the reals remain equal to themselves and are conserved – and nevertheless movement appears,"[42] he does not in any way clarify "from where the concept of negation emerges," or on what "the analogy of positive and negative magnitudes"[43] is founded. According to Trendelenburg, Herbart does not explain how one passes from the level of logical opposition to the level of real opposition without unjustifiably resorting once again to a non-logical

[34] Ibid., 1:182.

[35] Ibid., 2:516.

[36] Ibid., 1:176.

[37] Ibid.

[38] Ibid.

[39] Ibid., 1:182.

[40] Trendelenburg, Adolf. "Über Herbart's Metaphysik und eine neue Auffassung derselben (II)." In *Historische Beiträge zur Philosophie* (Bethge: Berlin, 1846–1867), 3:82.

[41] Trendelenburg. *Logische Untersuchungen*, 1:188n .

[42] Trendelenburg. "Über Herbart's Metaphysik und eine neue Auffassung derselben (I)." 2:334.

[43] Ibid., 2:337.

element, namely, movement: "the analogy of positive and negative magnitudes … leads … to movement in space and time"[44] and Herbart does nothing more than tacitly transport "relations of movement into reals assumed as simple."[45] Thus, paradoxically, "movement, which according to Herbart should fall under the concept of apparent occurrence, insinuates itself [schiebt sich] into real occurrence instead."[46]

Herbart's error is the exact mirror image of the error of Romantic idealism. While the latter deludes itself in transforming real opposition into a product of pure thought, tacitly passing from the opposition between contradictories to that between contraries, Herbart, in contrast, attempts to "refer contraries back to contradictories" through the deceptive method based on the "translation of a determinate concept into an indeterminate one."[47] Herbart does not realize that, in this manner, he shuts himself within pure thought without any possible exit, just like the idealists. However, Trendelenburg also emphasizes that his own *Logische Untersuchungen* "had a very particular destiny. The Hegelians interpreted what was directed against Herbart, and the Herbartians, as being directed against Hegel. However, the underlying reason for the critique of both systems, to a certain extent at least, derives from the same positive motive." Particularly significant for the present work is that the common difficulty which Trendelenburg discovers in Herbert as well as Hegel consists precisely in the ambiguous relation that they institute between "reality" and "negation," between logical opposition and real opposition: a theme that is once again revealed to be fundamental for understanding the post-Kantian debate.

4.2 From Psychophysics to Infinitesimal Calculus

One could certainly object that the principle of the Anticipations of Perception is not considered here specifically from a philological or from a theoretical point of view. Nevertheless, little by little, from this point to the zenith of the movement known as the "return to Kant," this principle finally assumes a role as the protagonist. Hermann Cohen (1842–1918), who in his beginnings was profoundly influenced by the "Herbartian school"[48] as much as by Trendelenburg,[49] turns the Anticipations of

[44] Trendelenburg. "Über Herbart's Metaphysik und eine neue Auffassung derselben (II)." 83.

[45] Trendelenburg. *Logische Untersuchungen*, 266.

[46] Trendelenburg. "Über Herbart's Metaphysik und eine neue Auffassung derselben (I)." 339.

[47] Ibid., 333.

[48] Cohen's first writings appear in the journal *Zeitschrift für Völkerpsychologie und Sprachwissenschaft* [Journal for Folk Psychology and Theory of Language] founded by Heymann Steinthal and Moritz Lazurus who adopt and develop elements of Herbart's psychology.

[49] Cohen was a student of Trendelenburg in Berlin. Moreover, he sided with Trendelenburg in a famous controversy that set him at odds with Kuno Fischer. See Cohen, Hermann, "Zur Controverse zwischen Trendelenburg und Kuno Fischer," in *Zeitschrift für. Völkerpsychologie* (1871), 7:249–296, now in CW 13:100–. On the controversy between Trendelenburg and Fischer, see also Poma, Andrea. *The Critical Philosophy of Hermann Cohen* (Albany, NY: State University of New York Press, 1997), 4–.

Perception into the fundamental nucleus of not only his interpretation of Kant, one of the most influential in the history of Kantian criticism, but also of his own philosophy. Indirectly, the same principle also becomes fundamental to a whole series of thinkers connected to him within the so-called "Marburg School."

No philosophical current, that has directly or indirectly addressed Kant's philosophy, has attributed so profound a theoretical significance to the Anticipations of Perception. The evolution of Marburg neo-Kantianism, from Cohen's first *Kant-Bücher* to the works of his most famous students Paul Natorp and Ernst Cassirer, hinges in many respects on the different valuations of the function of this principle in the structure of critical philosophy. As Cassirer observes, "the principle of the Anticipations of Perception … contains the path that led" Cohen's philosophy "to its own systematicity in the *Logik der reinen Erkenntnis* [Logic of Pure Knowledge]," published in 1902 (CGW 9:126). Precisely through this redefinition of the relation between reality and negation, Cohen transformed Kant's "transcendental logic" into a "logic of pure knowledge," replacing Kant's concept of "synthesis" with that of "origin," the idea of a connection of external contents of thought with the idea of the production of the contents by thought itself. The major works of the other members of the school particularly Natorp in *Die logischen Grundlagen der exakten Wissenschaften* (The Logical Foundations of the Exact Sciences) and Cassirer in *Substanzbegriff und Funktionsbegriff* (Substance and Function),[50] both published in 1910, were in many respects the attempt to take a position on Cohen's systematic completion of Kantian philosophy, leading to a debate that, as recent scholarship has shown, was far less conciliatory as previously thought.

Following Cohen's interpretation of the Anticipations of Perception in detail, until the elaboration of his own original theory and the lively discussions that ensued, leads us into an entirely new territory than that of the preceding chapter. After Hegel's death in 1831, the face of German philosophy changed radically. Philosophers seemed to put aside the themes and problems that had been the object of passionate discussions during the first decades of the 1800s. Idealist philosophy was now accused of being lost in abstruse speculations, constructing, in the words of Friedrich Eduard Beneke, "castles in the air …. thrones erected among the clouds"[51] and abandoning "the fertile *bathos* of experience" (AA 4:373n)[52] which Kant, in contrast, had claimed to be his firm basis. Overwhelmed by the forceful development of the empirical sciences, Romantic idealism was no longer even considered worthy as a polemical object of debate.

Surprisingly, the internal debates of the Marburg school, despite starting from the completely different premises that were typical of the new philosophical temperament

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[50] English translation from Cassirer, Ernst. *Substance and Function, and Einstein's Theory of Relativity*, tr. William Curtis Swabey and Marie Taylor Swabey (Chicago, London: Open Court, 1923), 75.

[51] Beneke, Friedrich Eduard. *Kant und die philosophische Aufgabe unserer Zeit. Eine Jubeldenkschrift auf die Kritik der reinen Vernunft* (Culture et Civilisation: Brussels, 1869; reprint of the Mittler, Berlin 1832 edition), 10.

[52] English translation from Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." 164.

and despite following an entirely autonomous course, seem to have ultimately lead neo-Kantianism to the same theoretical alternative that emerged in the history of Romantic idealism discussed above. Once again, the Marburg school resorted to a speculative interpretation of infinitesimal calculus as the adequate instrument for rigorously fixing the particularity of the qualitatively differentiated content of experience in the face of the empty abstractness of its *a priori* forms. In this way, the school bridges the purely "logical" moment and the purely "sensible" moment of knowledge.

While the relation between these themes and the Anticipations of Perception is in all respects secondary in "classical" idealism, except for a few allusions perhaps, it becomes the very core of neo-Kantianism, at least in its Marburg School version. What I sought to develop from a "systematic" point of view in the previous chapter can now be confirmed "historically," even in its smallest details. Moreover, precisely this possibility of clearly individuating the unitary problem that both of these forms of idealism are faced with will illuminate the different objectives that each of them seeks to achieve. While the resolution of the conflict between concept and intuition in Romantic idealism leads to the determination of a "metaphysical unity" in which spirit and nature, subject and object, are all one; now, in contrast, only the "logical unity" of scientific knowledge seems capable of overcoming this conflict.

On the one hand, the analysis of the relations that exist between reality and negation is once again the adequate tool for clarifying the profound rupture that runs through the entirety of Kantian thought, and infinitesimal calculus once again allows the unification of that which remains irremediably separated in Kant. On the other hand, the different ways of applying these conceptual tools, reveal an attempt to save the "critical" aspect of Kantian idealism; an attempt to save it from what now appears as its "degeneration" into an absolutely "metaphysical" form. These tools seek to hold firm to the "reflexive" nature of Kant's philosophy in the face of efforts to mold the contents of scientific knowledge itself on the basis of principles deduced in a purely speculative fashion.

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The attempt to compare the ideas of the Marburg School and the results of the previous chapter may seem forced if one considers the beginnings of this neo-Kantian movement's reflection. In the first edition of *Kants Theorie der Erfahrung* (Kant's Theory of Experience), published in 1871, Cohen outlines a preliminary interpretation of the principle of the Anticipations of Perception, an endeavor that he himself puts into question in the second version of the work. Nevertheless, it cleared the ground for a whole series of studies and investigations conducted by his students. Their research profoundly influenced Cohen's own work, leading him to elaborate a more articulate and complex interpretation. The latter interpretation begins to show itself in Cohen's 1883 work *Das Prinzip der Infinitesimal-Methode und seine Geschichte* (The Principle of the Infinitesimal Method and its History) and in the second edition of *Kants Theorie der Erfahrung*, published in 1885.

"What is the real as an object of simple sensation as an intensive quantity in antithesis to an extensive?" asks Cohen. "[T]he real of sensation is that quantity

which 'can only be apprehended as a unity' [B 209]: it is the unity of the stimulus in which we objectify sensation" (CW 1:3:146). Cohen is convinced that the variation of the principle in the second edition of the *Critique of Pure Reason* is due to a justified need to find an objective correlate of sensation: "the intention to represent, as far as it is possible, in such a harsh way, this dependence of the real as object of sensation on the latter, and to clarify the real as a simple unity of the objectified stimulus, seems to me to be the motive for modifying this affirmation in the second edition. We do not want to expand on this point, but it cannot avoid a thorough comparison" (CW 1:3:146). Thus, Cohen opposes sensation, as the subjective and changing element of consciousness, to its objectification, represented by the external stimulus. Cohen's opposition seems to correspond effectively to Kant's distinction between sensation as "merely subjective representation" (B207) and reality, defined as that which is capable of exerting a "degree of influence on sense" (B208).

These few, apparently rather simple, observations echoed loudly among thinkers close to Cohen. August Stadler writes, "Cohen very happily defines the objective correlate of the intensive magnitude of the stimulus. The magnitude that corresponds to the stimulus of the intensive magnitude would thus be called the magnitude of the stimulus."[53] Stadler develops this Cohenian intuition in a detailed manner. As I discussed above, Kant uses the expression "moment" to indicate "reality as cause" and, in particular, as "cause of sensation." According to Stadler, "the term moment immediately leads one to understand the "influence," the "stimulus," as representing nothing other than the relation that pure theory of knowledge defines by the word force."[54] The "moment is the magnitude of the force that corresponds to the intensive magnitude of the sensation," which only represents the "subjective evaluation"[55] of it. Thus, the "affection" that Kant speaks of "signifies nothing other than this mediated dependence of the subject on the external object";[56] that is to say, that dependence which psychology objectifies in the relation between the stimulus and the sensation. Thinking the "objects that stimulate our senses" (B1) as they are "in themselves," independently of the conditions that make any experience possible, is not at issue here. In the same way, one can only metaphorically call "a rainbow a mere appearance in a sun-shower, but one would call this rain the thing in itself" (A45): "this level," according to Stadler, "has absolutely nothing to do with the noumenal" since the relation between the stimulus and the sensation are comprised in the phenomenal realm. The fact that "we recognize a force only through its effect does not make it transcendental for this reason. It is nevertheless in the phenomenon and not behind it. The movements of luminous ether are not noumenal, even if we do not perceive them, nor are atoms."[57]

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[53] Stadler. *Kants Theorie der Materie*, 60. On the relation between Sadler and Cohen, see: Giovanelli, Marco. *August Stadler interprete di Kant* (Napoli: Guida, 2003).

[54] Stadler. *Kants Theorie der Materie*, 60.

[55] Ibid., 61.

[56] Ibid., 11.

[57] Ibid., 9.

As I have shown, the relation between sensation and force, in which force is understood as a stimulus of sensation, represents an important aspect of the Kantian conception that Romantic idealism returns to so insistently. However, this same problem is now plunged into a radically different context. The reference to "this dependence – what today would be called psychophysical"[58] – between the stimulus and the sensation, to the idea of the stimulus as an objective sensation, presupposes empirical psychological studies that were at the center of a significant debate within German philosophy at the time.[59] The debate specifically concerns studies that submit psychical magnitudes to mathematical measurement, establishing a functional relation between stimulus and sensation: "this principle," notes Cohen, referring to the Anticipations of Perception, "is of great importance for that discipline whose influence on Kant is less discussed: physiology of the senses" (CW 1: 3:146).

Stadler intervened in this debate in 1877 with an article published by *Philosophische Monatshefte* entitled "Über die Ableitung des Psychophysichen Gesetzes" (On the Deduction of Psychophysical Law).[60] The article, apparently free of philosophical implications, thoroughly investigates the so-called Weber–Fechner law that regulates the relations between the stimulus and the sensation. The physicist and physiologist Ernst Heinrich Weber discovered through a long series of experiments that the intensity of the variation of the stimulus that one can perceive ($\Delta\beta$) is proportional to the magnitude of the intensity of the stimulus (β); in order for the stimulus to be able to be perceived, the relation $\frac{\Delta\beta}{\beta} = c$ should hold. The constant c is different for different sensations (for the intensity or pitch of sound it is 1/11, for the volume of sound, 1/22, for luminosity, 1/62, etc.). In lifting a 10 kg weight, in order to be able to perceive a difference in respect to the initial weight, given that the constant for this type of sensation is 1/53, one must add 10/53 kg to the 10 kg that one is sustaining; that is, about 188 g: "According to this law," Stadler comments, "hardly observable variations in different degrees of sensation are not produced each time through an equal increase of the stimulus, but only when the equation $\frac{\Delta\beta}{\beta} = c$, or rather the equation $\Delta\beta = c\beta$ is satisfied."[61] If one imagines gradually increasing the weight from 10 kg until doubling it, not all of the infinite possible values between 10 and 20 kg can be perceived, but only those for which the equation $\Delta\beta = c\beta$ holds: "the essence of the relation between $\Delta\beta$ and $\Delta\gamma$,"

[58] Ibid., 58.

[59] For a detailed reconstruction of the significance of psychophysics for the eighteenth-century philosophical debate, see Heidelberger, Michael. *Nature from Within: Gustav Theodor Fechner and his Psychophysical Worldview* (Pittsburgh, PA: University of Pittsburgh Press, 2004) and Martinelli, Riccardo. *Misurare l'anima. Filosofia e psicofisica da Kant a Carnap* (Macerata, Italy: Quodlibet, 1999).

[60] Stadler, August. "Über die Ableitung des Psychophysischen Gesetzes," *Philosophische Monatshefte* 14 (1877): 215–23.

[61] Ibid., 219.

writes Stadler, "consists in the fact that the variation of the sensation does not correspond to any $\Delta\beta$, but rather $\Delta\gamma$ remains void for all values $\Delta\beta < c\beta$."[62]

Weber did not intend to found a true and proper mathematical psychology capable of measuring psychical phenomena in the way that physical phenomena are measured. In what would later be called the Weber law, the "magnitude" of sensation still does not appear. Rather, Weber sought to determine the precise moment in which an increase in excitation provokes a variation of the sensation, calculating the value of this threshold for different types of sensation. Gustav Theodor Fechner's *Elemente der Psychophysik*[63] takes the further step of attempting to measure sensation true and proper as a psychical phenomenon. His work is crucial in the history of empirical psychology for its ambitious aim to formulate an "exact doctrine of the functional relations between body and soul."[64]

Fechner, departing from Weber's law, sought to establish a general relation between the stimulus and the sensation, on the basis of which a variation of the stimulus $\Delta\beta$ leads to a variation of the sensation $\Delta\gamma$ that becomes perceivable in the moment that the relation results in c. One can thus assert that c is equivalent to the smallest observable difference, what Fechner calls *eben merklicher Unterschied* (e. m. U). A difference in the stimulus is thus connected to a difference in the sensation according to the law: $\Delta\gamma = K\frac{\Delta\beta}{\beta}$, where K signifies a new constant. This relation holds for every possible variation of the stimulus that, as small as it may be, is connected to a corresponding variation of the sensation through the same functional equation. Since, as discussed above, a difference in the stimulus inferior to the threshold does not actually determine any sensation, Fechner postulates the existence of unconscious sensations: "a stimulus or a difference in stimulus [Reizunterschied] has a physical value in general, which is connected to a value of the sensation. In order for the sensation to become observable, it must reach a certain intensity or magnitude; as long as this value of magnitude is still not reached, the sensation remains – as we are accustomed to express it – unconscious."[65]

Presupposing the continuous variation of sensation, Fechner can substitute the finite difference $\Delta\beta$ and $\Delta\gamma$ with infinitely small differences $d\gamma$ and $d\beta$ such that "the just noticeable difference becomes an infinitesimal magnitude":[66] $dy = K\frac{d\beta}{\beta}$

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[62] Ibid. For further discussion on this point, see: Heidelberger. *Nature from Within: Gustav Theodor Fechner and his Psychophysical Worldview* 213–.

[63] Fechner, Gustav Theodor. *Elemente der Psychophysik*. 2 vols (Leipzig: Breitkopf & Härtel, 1860).

[64] Ibid., 1:8.

[65] Fechner, Gustav Theodor. *Revision der Hauptpuncte der Psychophysik* (Leipzig: Breitkopf & Härtel, 1882), 177.

[66] Paulsen, Johannes. *Das Problem der Empfindung*. eds. Hermann Cohen and Paul Natorp, Philosophische Arbeiten (Töpelmann: Giessen, Germany, 1907), 282.

(where K is a new constant). In this way, the value of a determinate sensation can be obtained through a summation of infinitely small elements: $\int_{\gamma_0}^{\gamma} d\gamma = K\int_{\beta_0}^{\beta} \frac{d\beta}{\beta}$. By choosing a determinate value b, above the threshold, as the initial stimulus (the stimulus = 0 would not correspond to any sensation), one obtains $\int_{0}^{\gamma} d\gamma = K\int_{b}^{\beta} \frac{d\beta}{\beta}$ from which the logarithmical formula that regulates the relation between stimulus and sensation is derived: $\gamma = K \log_e \frac{\beta}{b}$. On the basis of this law, and assuming that the sensation corresponds to the logarithm of the stimulus, a variation of sensation corresponds to every "infinitely small" variation of the stimulus in such a way that the functional relation between stimulus and sensation can be represented as a continuous curve: "If we let the intensity of a sound or of an illumination [Licht] rise increasingly higher than the threshold," writes Fechner, "then we sense [spüren] the continuous escalation through all intermediate degrees, from the smallest to the highest. Every smaller increment of the stimulus necessarily causes an increase of the sensation since only in this way can the sensation rise by a lesser or greater value"[67] and be measured through integration.

According to Stadler, however, Fechner's formula directly contradicts Weber's law from which Fechner claims to have deduced it. Fechner's formula establishes that a variation of the sensation corresponds to every possible variation of the stimulus. A tiny variation of the weight being lifted would result in a sensation as well. The sensation occurs even if it is unobservable, that is, unconscious. The difference in sensations, which continuously rises in relation to the stimulus, is only observable when the stimulus has passed the threshold. In contrast, as Stadler observes, "Weber's law is derived from experience and only holds for real sensations; that is, empirically given; not for so-called unconscious sensations."[68] Thus, it is not legitimate to "represent the reciprocal process of the stimulus and of the sensation through a continuous function or a curve. We conceive an increase of the stimulus on the *x*-axis and the corresponding variation of the sensation as depicted on the *y*-axis. The values of the latter would remain equal each time for values of $\Delta\beta$ below $c\beta$. The varying of the sensation thus obtains the form of a scale with rising degrees of magnitude. If the escalations $\Delta\gamma$ were infinitely small, the scale would not separate itself by a finite distance from the *x*-axis."[69] Weber's law establishes that the relation between two successive stimuli cannot go further than a certain limit value in approaching the unity (that is to say the value 1), or else the difference could not be observed: "the essence of the relation between $\Delta\beta$ and $\Delta\gamma$," concludes Stadler, "is discontinuity. The logarithmic curves, with which one attempts to represent the psychophysical law, lack empirical truth."[70]

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[67] Fechner. *Elemente der Psychophysik*, 1:155.

[68] Stadler. "Über die Ableitung des Psychophysischen Gesetzes," 220.

[69] Ibid.

[70] Ibid., 223.

Almost 30 years later, Paul Natorp effectively summarized these results in his 1910 *Allgemeine Psychologie* (General Psychology):

> It is from the beginning erroneous to think of sensation as a continuous variable magnitude in a mathematical relation to the physical magnitude of the stimulus (as a function of the latter). Fechner's psychophysics is based on the presupposition that a correspondence can be established between the sensation and the stimulus in conformity with the law and, through it, between the psychical and the physical. He thus affirms that the sensation is proportional to the logarithm of the stimulus. For this reason, [the law] is already founded on an unsustainable presupposition since it assumes that, in general, the sensation is a continuous function of the stimulus (that is, variable in a continuous manner with the continuous variation of the stimulus). In reality, precisely the facts that should support it (the facts of the "law of Weber") lead to the consequence that the number of perceivable differences between two given limits of the stimulus is always finite [beschränkt]; the continuous variation of the sensation is a presupposition introduced *a priori* in an unacceptable manner and is not supported by the facts.[71]

Interesting for the purposes of the present work, however, is the fact that such an *a priori* presupposition could seem to be a scientifically "acceptable" reformulation of the Anticipations of Perception. In other words, Fechner had "anticipated" sensations, relying on an analogous presupposition to that which Kant seems to allude to in a few passages; the presupposition that sensations "can proceed in a certain time to vanish through infinite intermediate degrees, or to grow from nothing to a determinate sensation through infinite moments of accretion" (AA 4:309n6).[72] By means of a law that establishes a functional relation between the sensation and the stimulus, the still "philosophical" formulation of the problem of the degree of sensation, as it appears in the *Critique of Pure Reason*, seems to have finally assumed a scientific form.

Moreover, Stadler himself, in 1876, already intervened in the discussion on the relation between the problem of the continuity of sensation and on the demonstration of it that Kant claims to give in the Anticipations of Perception. In his 1876 *Die Grundsätze der reinen Erkenntnisstheorie in der kantischen Philosophie* (The Foundations of Pure Theory of Knowledge in Kantian Philosophy), where the philosophical intents of his debate with Fechner are most explicit, Stadler states that "pure theory of knowledge could have and should have described an *a priori* property of everything empirical: intensive magnitude."[73] One can establish *a priori* that "all sensations have an intensive magnitude."[74] If sensations did not have a magnitude, they would not be sensations and the succession of sensations would be interrupted at some point, making the constitution of the "unity of consciousness"

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[71] Natorp, Paul. *Allgemeine Psychologie. In Leitsätzen zu akademischen Vorlesungen* (Marburg: Elwert, 1910), 135.

[72] Kant. "Prolegomena to Any Future Metaphysics that Will be Able to Come Forward as Science." 102n.

[73] Stadler. *Kants Theorie der Materie*, 58.

[74] Stadler, August. *Die Grundsätze der reinen Erkenntnisstheorie in der Kantischen Philosophie: Kritische Darstellung* (Leipzig: Hirzel, 1876), 65.

impossible: "the continuity of the connection and the condition for the identity of consciousness or for the unity of representations is possible only through a continuous synthesis of the sensations."[75] Thus, one must presuppose *a priori* that "from the point in time when the connection begins to the point in which it concludes, these simple elements succeed each other in such a way that an uninterrupted consciousness arises."[76] This presupposition must be valid independently of all experience since nothing could ever arise as a unitary experience without this continuity.

According to Stadler, however, Kant "moves two steps further beyond this point, which I cannot follow"[77] First, he seeks to demonstrate *a priori* that every difference between sensations must always be reduced to a difference in degree such that one must posit the further problem of "intermediate sensations between light and heat, pressure and sound, etc."[78] Second, he "attributes continuity to this quantity as well,"[79] allowing that sensations arise and disappear in consciousness by passing through all the infinite degrees in between.

In contrast, Stadler asserts that one can certainly prove *a priori* that, in Kant's own words, "the succession of many sensations" (B209) must be continuous because, "if the synthesis of the manifold of appearance is interrupted," this would not be a phenomenon, but rather "an aggregate of many appearances" (B212). To affirm the contrary, that each sensation arises in a continuous manner through all infinite intermediate degrees, means adopting an entirely different assumption that should not be confused with the first[80] (as the Garve-Feder[81] review of the *Critique of Pure Reason* had done) and that, above all, cannot be demonstrated *a priori*.[82] The continuous succession of sensations is an *a priori* condition. It is the condition

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[75] Ibid.

[76] Ibid.

[77] Ibid., 145n76.

[78] Stadler. *Kants Theorie der Materie*, 198.

[79] Stadler. *Die Grundsätze der reinen Erkenntnisstheorie in der Kantischen Philosophie: Kritische Darstellung*, 145n76.

[80] For example, see Bernhard Riemann's comment: "Observation only attests to the transition of a thing from one state into another, or more generally, from one mode of determination to another, without any jump being perceived. In the process of integration of perceptions, one can hypothesize that the transition occurs through a very high, but always finite, number of states that are not perceivable by the senses. Or rather, that a thing passes from one state into another through all intermediate degrees." (Riemann. *Gesammelte mathematische Werke und wissenschaftlicher Nachlass*, 554).

[81] Landau, Albert, ed., *Rezensionen zur Kantischen Philosophie* (Bebra, Germany: Landau, 1991), 10–17.

[82] Luigi Scaravelli most effectively clarifies the difference between these two perspectives: "[T]here is no doubt that the connection of degrees between themselves is truly a continuum. But this is not the problem." If one can affirm that the "succession of the modifications (equipped with degrees) is continuous (in time) one cannot say, in contrast, whether the variation of the 'magnitude or quantity" of these degrees is a continuous variation or rather a variation with clearly discontinuous jumps" (Scaravelli. *Saggio sulla categoria della realtà*, 108). One cannot establish *a priori* whether natural forces act without interruption or whether their successive actions are simply separated by intervals of time whose duration is not sensible.

for the unity of consciousness. In contrast, the continuous arising of sensation from its absence to a certain finite degree can, at most, have the status of an empirical claim and cannot be established independently of experience.

However, in his article "Gesetz der Stetigkeit bei Kant"[83] (The Law of Continuity according to Kant) published a few years later, however, Stadler argues that Kant does not maintain this distinction at all. On the one hand, as I have shown, Kant declares in the Anticipations of Perception that the continuity of the variation of intensive magnitude is indemonstrable. On the other hand, he attempts to demonstrate in the second Analogy of Experience the continuity of change from the continuity in the arising of sensation in consciousness. In other words, he tries to demonstrate "the possibility of cognizing *a priori* a law concerning the form of alterations," relying on the fact that "[w]e anticipate only … our own apprehension" (B256) of these appearances. Therefore, we *anticipate* the fact that "the generation of a perception as a magnitude" as Kant writes, explicitly alluding to the Anticipations of Perception, passes "through all degrees, of which none is the smallest, from zero to its determinate degree". (B255; emphasis mine).

Kant's error, according to Stadler, consists in "exchanging the continuity of intensive magnitude with the continuity of its becoming conscious, of its production."[84] In so doing, Kant relies on the idea that "the new state of reality grows out of the first, in which it did not exist, through all the infinite degrees of reality" (B254). He argues that Kant "confused the 'continuous connection of possible realities, and of the smallest possible perceptions' with a connection of effective realities. He affirms that the magnitude of reality 'is produced,' 'increases'" [B 254].[85] The intensive magnitude of sensation is certainly continuous, but not in the sense that it arises in a continuous manner through all intermediate degrees. It is continuous simply in the sense that "between every degree and nothing, one can always think another smaller degree."[86]

Stadler further observes that research using the methods of empirical psychology does not permit any *a posteriori* demonstration of what the Anticipations of Perception attempt in vain to demonstrate *a priori*:

> In parentheses, I would like to note that modern psychology has not offered any reason to revise Kant's concept of the degree of sensation. In fact, it is known that one of the principle progresses in psychology consists in its pretence of analyzing the unconscious as well. From this point of view, perhaps one could affirm that the transition from one sensation to another occurs in a continuous manner, but in such a short period of time that attention is not capable of holding firm to any intermediate state.[87]

In contrast, "Fechner … decided to proceed to the end of this path of attributing an intensive magnitude to sensation, an intensive magnitude that is not only thinkable as a progressive diminution until zero, but, on the contrary, is scientifically

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[83] Stadler, August. "Das Gesetz der Stetigkeit bei Kant," *Philosophische Monatshefte* 16 (1880): 577–97, later published in Stadler. *Kants Theorie der Materie*, 187–204, from which the citations are drawn.

[84] Stadler. *Kants Theorie der Materie*, 196.

[85] Ibid.

[86] Ibid., 197.

[87] Ibid., 198.

described as effectively and really consisting in an intensive variation from zero to the degree reached from time to time, and in as much as it is measurable through integration."[88] According to Stadler, however, psychophysics is unable to demonstrate anything in respect to this question: "as far as intensity is concerned, in my opinion, psychophysical research has rather shown the discontinuity in psychical transition in relation to the continuous growth of the stimulus."[89]

If a certain difference in weight or temperature is perceivable, one can certainly affirm the possibility of observing smaller differences with a scale or a thermometer and of distinguishing even smaller differences with ever more precise instruments. Moreover, even between two extremely close values, one can still think an infinite number of possible intermediate values. Fechner's law, in contrast, not only presupposes that the thinking of this ever-smaller difference is *possible*, but that sensation also passes *de facto* through all these infinite differences even if, before reaching the threshold, such sensations are unconscious. It seems illegitimate to postulate *a priori* the existence of all these intermediate values on the basis of a particular interpretation of the Anticipations of Perception (and, in so doing, to move away from Kant's concept of *a priori* in general). It is therefore equally illegitimate to claim to "reform" Kant's principle by resorting to psychophysics.

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To fully comprehend the importance of these studies, one should take into account the extent of the debate over psychophysics in Marburg during this period. In 1880, Cohen banned the following *philosophische Preisaufgabe* (philosophical essay prize): "[E]xplain Kant's mathematical principles and clarify their relation to the fundamental tenets of his Metaphysical Foundations of the Natural Sciences. In particular, Kant's first principle [Axioms of Intuition] should be evaluated according to its significance for new gnoseological research into the science of space, and the *second principle* [Anticipations of Perception] *in respect to psychophysics.*"[90] The recipient of the prize was Adolf Elsas, assistant at the Institute of Physics and Mathematics[91] who, several years later, in 1886, published the essay *Über die Psychophysik. physikalische und erkenntsstheoretische Betrachtungen*[92] (On Psychophysics: Physical and Epistemological Considerations). In 1882, August Müller published *Das Axiom der Psychophysik und die physiologische Bedeutung der Weber'schen Versuche. Eine Untersuchung auf kantischer Grundlage*[93] (The Axiom of Psychophysics and the Physiological Meaning of Weber's

[88] Martinelli. *Misurare l'anima. Filosofia e psicofisica da Kant a Carnap*, 46 (emphasis mine).

[89] Stadler. *Kants Theorie der Materie*, 198.

[90] Citation from Holzhey, Helmut. *Cohen und Natorp*. 2 vols (Basel: Schwabe, 1986), 1:381 (emphasis mine).

[91] Ibid.

[92] Elsas, Adolf. *Über die Psychophysik. Physikalische und erkenntnisstheoretische Betrachtungen* (Marburg: Elwert, 1886).

[93] Müller, Ferdinand August. *Das Axiom der Psychophysik und die psychologische Bedeutung der Weber'schen Versuche* (Marburg: Elwert, 1882).

Experiments: A Study on the Basis of Kantian Principles) under the guidance of Cohen. An example of a work on the psychology of sensation specifically is Johannes Paulsen's *Das Problem der Empfindung*[94] (The Problem of Sensation), published in 1907 in the series *Philosophische Arbeiten* directed by Cohen and Natorp.[95]

For Hermann Cohen, the results of these studies, and particularly those by Stadler, represent a fundamental terrain of comparison that would lead him in the end to a complete rethinking of his own interpretation of the Anticipations of Perception: "to understand, on the one hand, the new systemic value of Kant's conception of intensive magnitude and, on the other hand, the lack in its foundation and in its exposition, which Stadler was the first to draw attention to; … it is first of all necessary to take into account how tight the systematic kinship relationship is between the infinitely small and sensation and how, for both problems, the concept of intensive magnitude seems to assume a role and become useful" (CW 5:1:105). According to Cohen, Stadler rightly demonstrates that the continuity of the intensive magnitude of sensation cannot be demonstrated *a priori*, as Kant claims to do in the Anticipations of Perception, and studies on the psychology of sensation are even less capable of furnishing a solution to the problem. In Cohen's words, "Stadler rightly opposes the continuity of sensation as an *a priori* determination" (CW 1:1:558). However, he did not realize that "another foundation for continuity"[96] could be found: "the 'pure transcendental foundation' that Stadler lacked, he too was only looking for it *in* sensation … however, it is found *through* sensation … in the new and independent mode of magnitude that, in the infinitesimal method, is effective in constituting the object in its mechanical significance" (CW 1:1:558).

In other words, Stadler still moves on terrain chosen by his adversary in his debate with Fechner and psychophysics. He remains in the terrain of sensation, attributing only to the latter an intensive magnitude: "intensive magnitude," affirms Stadler, "cannot be immediately attributed to an object since it can be defined as a phenomenon of consciousness and internal states cannot as such be transferred to an object. One can attribute consciousness an extensive magnitude just as little as one can think the magnitude of sensation as a determination" of an object itself.[97] According to Cohen, however, this restricted point of view is inadequate for understanding Kant's concept of "intensive": "the so-called intensity of sensation must absolutely be distinct from the intensive magnitude or reality of sensation" (CW 5:1:156): "intensity … is not a mode of excitation of consciousness, but rather

[94] Paulsen. *Das Problem der Empfindung*.

[95] A detailed reconstruction of this debate on psychophysics among the "Cohenian school" can be found in Heidelberger. *Nature from Within: Gustav Theodor Fechner and his Psychophysical Worldview*, 215–.

[96] Cohen, Hermann. "August Stadler: Ein Nachruf," *Kant Studien* 15 (1910): 403–. Reprinted in Cohen, Hermann. *Schriften zur Philosophie und Zeitgeschichte*. eds. Albert Gorland and Ernst Cassirer. 2 vols (Berlin: B. Cassirer, 1928), 2: 440–58.

[97] Stadler. *Kants Theorie der Materie*, 59.

the designation of the origin [Ursprungsbezeichnung] of the objectivity [Objectivität] of the object [Gegenstand]" (CW 1:1:558).

Thus, if it is recognized that sensation is nothing other than "the effect of an object on the capacity for representation, insofar as we are affected by it" (B34), the relation between the external stimulus and the sensation seems to raise a new problem, Already in this first formulation, even if it is "still too psychological" (CW 1:1:556), subjective sensations (seeing color, hearing sound, etc.) are clearly referred back to an objective element from which sensation is determined. However, now the problem is clearly no longer in the reciprocal relation of dependence between the sensation and that which corresponds to it in the object. Rather, the problem is in the conditions by virtue of which something can "claim to be" an object of sensation. Cassirer, in *Hermann Cohen und die Erneuerung der kantischen Philosophie* (Hermann Cohen and the Renewal of Kantian Philosophy), insists on the importance of this transition in the evolution of Cohenian philosophy: "It is completely inaccurate to affirm that Cohen's critique of knowledge is unilaterally directed at mathematical theory of nature." On the contrary, "even the genesis of this fundamental idea" shows how it was directed at "critique of physiology no less than at critique of physics." However, "the concept of object that physiology itself presupposes cannot be fixed in a clear and secure fashion in terms of its most general meaning if not through the language of mathematical physics. The concept of sensation leads to that of 'stimulus,' and the latter in turn to the general concept of movement. Thus, nature, for knowledge, must be understood as a system of processes of movement connected to each other in a law-bound fashion" (CGW 9:123).

This further step clarifies why the problem of the reality of sensation should be considered from an entirely new point of view. Stadler himself explicitly refers the concept of movement back to that of stimulus in *Kants Theorie der Materie*. Commenting on the passage in the *Metaphysical Foundations of the Natural Sciences* where Kant asserts that "[t]he basic determination of something that is to be an object of the outer senses had to be motion, because only thereby can these senses be affected" (AA 4: 476–),[98] Stadler writes: "that only through movement can the senses be affected signifies: in as much as psychology is forced – if it wants to become a science – to think internal phenomena in an intimate connection with external phenomena, it must attribute this movement to every alteration in sensation: movement must be thought as a stimulus of sensation."[99] While recognizing the importance of such a "psychological" deduction in his review of Stadler's book in the *Philosophische Monatshefte*, Adolf Elsas nevertheless emphasizes that the problem should be understood in a fundamentally different manner, in an eminently "logical" instead of a "psychological" manner: the "being" of movement itself

[98] Kant. "Metaphysical Foundations of Natural Science." 191.

[99] Stadler. *Kants Theorie der Materie*, 11.

should become the fundamental problem of critical philosophy because natural science is essentially a science of movement.[100]

This same demand, already stated in Cohen's first interpretation of the principle in 1871, to find something objective in sensation and to identify "reality" with the "stimulus" that exerts a certain "degree of influence on the senses," requires a complete rethinking of the problem: "instead of the unity of the stimulus," Cohen writes in the second edition of *Kants Theorie der Erfahrung*, "it is rather the unity of the movement" (CW 1:1556) that should be rigorously defined. The issue is no longer one of establishing a relation between "movement as a stimulus" (CW 7:129) and "sensation" as its reaction, that is, thinking movement "*as a source originating from the outside*" (CW 7:129), but rather one of determining movement itself as an object of scientific knowledge.

This point of view becomes ever clearer in Cohen's work from the following years:

> [W]hen modern psychology and modern physiology speak of a motory sensation [Bewegungsempfindung] and of motory representation [Bewegungsvorstellung], they mean a sensation or a representation of – and thus also after – movement. Therefore, this movement must already have occurred if it is to allow a sensation and a representation of itself … [I]s not the real problem rather how the first movement is born? Is it born only beyond consciousness, or instead in consciousness itself and not in such a way that sensation is only its echo? (CW 7:129).

The error of psycho-physiology is that it considers sensation a consequence of movement. It relies on the assumption that "sensation already presupposes the external world" and "thus also presupposes movement, material movement, which reaches sensation as a *representation* [Abbild]. However, the question that remains unconsidered [ausser Betracht] is the following: how does material movement itself arise? Or in a more logically precise form: *how do the problem and the concept of movement arise in pure knowledge*?" (CW 8:128).

Psychophysics, as I have shown, searched in vain to "make sensation arise and grow from the infinitely small until entering into consciousness and making itself integral, and thus finite" (CW 6:493). It attempted to determine "the emergence of consciousness as a consequence of movement" (CW 7:126); the emergence of the intensive magnitude of sensation in connection with the variation of the external stimulus. The attempt to do so was in vain, however, because "there is no intensive magnitude. There cannot be any because sensation, as a mode of consciousness, does not have any type of magnitude" (CW 6:493). The problem that Kant brings to the fore with his concept of intensive magnitude cannot be considered from this point of view. The origin of sensation is not the problem raised by the Anticipations of Perception, but rather that of the origin of movement: "the supreme point of this development, however," writes Cassirer in regard to Cohen's interpretation, "is only achieved if we return to the fundamental mathematical motive that is at the basis [vorausliegt] of each particular scientific formulation of the concepts [Begriffsbildungen].

[100] Elsas, Adolf. "Review of Kants Theorie der Materie," in *Philosophische Monatshefte* (1884), 20:147.

We find this motive in the conceptual methodology [in der gedanklichen Methodik] of the 'infinitesimal'. Without this, it would not even be possible to characterize rigorously, let alone master conceptually, the concept of movement as it is presupposed in mathematical natural science" (CGW 9:127).

Cohen arrived at this new formulation of the problem, as it has been observed, "through the problem of psychophysics … and not by applying the infinitesimal method directly."[101] Psychophysics offered hints to Cohen on how to connect intensive magnitude with infinitesimal magnitude. Cohen could be "pushed also from this side in the direction of the concept of the differential."[102] However, the systematic refutation of psychophysics by Cohen's students demonstrated how this connection should now be shown at a completely different level. The problem of the intensive magnitude of sensation, as well as the problem of how to measure it, should be completely abandoned in favor of the problem of movement as an authentic object of mathematical science of nature. It is certainly correct to recognize that "intensive magnitude as degree emerged with the sensation of heat. Even Ernst Heinrich Weber, in his fundamental research that was unintentionally responsible for the field of psychophysics, starts from the sensation of temperature [Temperaturempfindung] (CW 6:494).

However the qualitative uniqueness of heat in respect to other phenomena is only apparent: "Heat, not only as a sensation, but in general, does not present a particular problem. It is rendered as the general problem of movement and of energy … This is the methodological [methodische] meaning of thermodynamics [Wärmelehre] … [Thermodynamics] reduced the most fundamental sensations to general movement" (CW 6:494). The particular qualitative determinateness of the sensation of heat is shown to be completely irrelevant in respect to the general scientific problem to which thermal phenomena should be referred back: "the [mechanical] equivalent of heat" reduced thermal phenomena "to mechanical movement" (CW 6:493–). Moreover, the subdivision of physics that is based on the distinctions between different sensible qualities (optics, acoustics, etc.) is shown to be completely insufficient and no longer tenable: "[T]he defect in the ostensive capacity [Anzeigekraft] of sensation goes even deeper. Electricity and magnetism cannot be represented as specific sensations. One should think of this [the fact that electricity and magnetism cannot be represented as specific sensations] as an *instantia crucis* against sensation … it is shown to be unscientific to remain fixed to five senses when one needs to add another pair" (CW 6:465–).

Sensation should now become a mere "question mark" (CW V, 2, 65), the position of a problem that sensation cannot solve, since the solution pertains to "mathematical natural science with its laws of motion" (CW 1:1:83). While Kant himself, as I have

[101] Gigliotti, Gianna. *Avventure e disavventure del trascendentale: studio su Cohen e Natorp* (Napoli: Guida, 1989), 92n.

[102] Schultess, Peter, "Introduction" to Cohen, Hermann, *Das Prinzip der Infinitesimal-Methode und seine Geschichte*. In CW V:1:12*.

shown, says sensations must ultimately be referred back to movement and the motive forces that are the cause of it, Cohen transforms movement into the central problem of critical philosophy: "according to Kant, substance belongs to movement; only movement … unfolds [entrollt] the multiplicity of that which, in the common sense of the term, presents itself as being. So-called being is actually the being of movement" (CW 1:1793). "Movement" is the "fundamental problem of mathematical natural science" (CW 6:225). In the concept of movement and its laws, knowledge clarifies itself and its proper object for the first time since nature is entirely known only when it is completely reduced to "an ensemble of movements" (CW 5:2:65).

According to Cohen, only such an overall systematic view allows the relation between sensation, intensive magnitude, and the real to be defined with precision; the relation that Kant established, even if ambiguously and imprecisely, in his two different formulations of the principle of the Anticipations of Perception. Only this point of view can clarify how the second "synthetic principle" does not raise the "psychological" question of the nature of sensation, but rather the "gnoseological–critical" [erkenntniskritisch] problem of the constitution of movement as an authentic "object" of scientific knowledge: "That which sensation can signify as magnitude, unlike the concept of 'just noticeable difference' [Ebenmerklichkeit]," writes Johannes Paulsen, effectively summarizing Cohen's development beginning from the first edition of *Kants Theorie der Erfahrung*, "is the objectified content of sensation, that is, the stimulus." But the stimulus

> is referred … to an object of nature [Naturgegenstand] as an object [Gegenstand] of knowledge … sensation according to its concept must be determined in the system of pure knowledge … thus it is in the infinitesimal reality of pure natural science that one must define the 'psychic meter' that measures being in its continuous change and becoming … the material world must be grasped and determined as movement … this movement is the most fundamental hypothesis of consciousness.[103]

Cohen's historical–systematic investigation, *Das Prinzip der Infinitesimal-Methode und seine Geschichte* [The Principle of the Infinitesimal Method and its History], aims to demonstrate that movement was able to become an authentic object of scientific knowledge only through the introduction of infinitely small magnitudes. By following the history of the infinitesimal method, one can determine the nature of the problem that such magnitudes attempted to resolve; the same problem which, according to Cohen, the Anticipations of Perception had furnished a philosophical formulation of: "every objectification of physics," states the second edition of *Kants Theorie der Erfahrung* in which the results of Cohen's research are explicitly inserted into the structure of Kantian philosophy, "presupposes … that of mathematics, and not only in the sense of extensive intuition, but also in the sense of that enlargement of modern mathematics, which is founded on the concept of the differential." The significance of the Anticipations of Perception should be understood

[103] Paulsen. *Das Problem der Empfindung*, 11.

from this point of view, from the idea that "intensive magnitude signifies nothing more than that differential magnitude" (CW I,1, 544).[104]

4.3 Cohen and the History of the Infinitesimal Method

Das Prinzip der Infinitesimal-Methode und seine Geschichte has been defined, not without a certain irony, as "one of the most difficult books in the history of German philosophy."[105] Its theoretical conclusions were severely criticized (not only by Bertrand Russell,[106] Gottlob Frege,[107] and Georg Cantor,[108] but by Cohen's friends and students as well, such as August Stadler and Ferdinand August Müller). At the same time, however, some of the most important members of the Marburg school passionately defended the book (as in the heated debate between Cassirer and Leonard Nelson[109]). The overall content of the objections to Cohen's conception of

[104] For more details, see: Giovanelli, Marco. "Grandezza intensiva e grandezza infinitesimale Hermann Cohen e il principio kantiano delle Anticipazioni della percezione," *Annuario filosofico* 19 (2003): 275–318.

[105] Kuntze, Friedrich. *Die kritische Lehre von der Objektivität. Versuch einer weiterführenden Darstellung des Zentralproblems der Kantischen Erkenntniskritik* (Heidelberg: Winter, 1906), 249. Here, Kuntze sarcastically notes that for most readers the only comprehensible parts of Cohen's book are the citations from other authors. On the context in which the book appeared, see also Moynahan, Gregory B. "Hermann Cohen's Das Prinzip der Infinitesimalmethode, Ernst Cassirer, and the Politics of Science in Wilhelmine Germany," *Perspectives on Science* 11, no. 1 (2003): 35–75.

[106] Russell, Bertrand. *The Principles of Mathematics* (Cambridge: Cambridge University Press, 1903).

[107] Frege, Gottlob. Review of Hermann Cohen, Das Prinzip der Infinitesimal-Methode und seine Geschichte, *Zeitschrift für Philosophie und philosophische Kritik* 87 (1885): 324-9; now in Frege, Gottlob. 'Rezension von Das Prinzip der Infinitesimal-Methode und seine Geschichte." In *Kleine Schriften*. ed. Ignacio Angelelli, 99-102. (Darmstadt: Wissenschaftliche Buchgesellschaft, 1967), from which the citations are drawn. English translation Frege, Gottlob. *Collected Papers on Mathematics, Logic, and Philosophy*. ed. Brian McGuinness (Oxford, UK; New York, NY, USA: B. Blackwell, 1984), 108–11.

[108] Cantor, Georg. Review of Hermann Cohen, Das Prinzip der Infinitesimal-Methode und seine Geschichte, *Deutsche Literaturzeitung* 5 (1884): 266–8. An overview of the criticisms of Cohen's interpretation of the calculus is found in Bell. *The Continuous and the Infinitesimal in Mathematics and Philosophy*, 177–.

[109] See Cassirer, Ernst. *Der kritische Idealismus und die Philosophie des 'gesunden Menschenverstandes'*. eds. Hermann Cohen and Paul Natorp, Philosophische Arbeiten (Giessen, Germany: Töpelmann, 1906). Now in CGW 9: 3–36). Here, Cassirer responds to the "neo-Friesian" Leonard Nelson's harsh criticism of the *Logik der reinen Erkenntnis* published in *Göttingische Gelehrte Anzeigen*. See Nelson, Leonard. Review of Hermann Cohen, Logik der reinen Erkenntnis, *Göttingische Gelehrte Anzeigen* 167 (1905): 610–30.; now in Nelson, Leonard. *Gesammelte Schriften in neun Bänden*. ed. Paul Bernays. 9 vols (Hamburg: Meiner, 1970), 2:3–27. Natorp defended Cohen from Russell's criticisms of Cohen's conception of the infinitesimal method.

the infinitesimal method can be easily summarized: Cohen, identifying infinitesimal magnitude with intensive magnitude, resorts to a "realist" concept of infinitesimals, understanding them as "existing" entities from which finite magnitudes emerge through the continuous addition of infinitely small elements. He completely overlooks the fact that "it is the doctrine of limits that underlies the Calculus"[110] and that "there is no allusion to, or implication of, the infinitesimal in any part of this branch of mathematics."[111] The neo-Friesian Leonard Nelson, in his review of Cohen's *Logik der reinen Erkenntnis* [Logic of Pure Knowledge], successfully explains the core of these criticisms: "the works of Cauchy, Weierstrass, and their pupils irrefutably demonstrated that, in every field of analysis, a precisely defined meaning cannot be attributed to the so-called infinitely small from a mathematical point of view and this [analysis] never has anything to do with really existing infinitely small magnitudes, understood in some sort of mystical sense."[112]

Understanding the connection between the history of the infinitesimal method and Cohen's interpretation of Kant's principle of the Anticipations of Perception requires a clarification of Cohen's motivation for highlighting the historical significance and the systematic value of the introduction of infinitely small magnitudes in calculus, despite the fact that analysis had proceeded to completely abandon infinitesimals for a calculus of limits.

Some mathematicians still preferred the "method of infinitesimals" in the 1800s. For example, in an 1892 article, Charles Sander Peirce does not hesitate to assert, "as a mathematician, I prefer the method of infinitesimals to that of limits, as far easier and less infested with snares."[113] In general, however, eighteenth-century mathematics went in the opposite direction to that which Cohen seems to delineate in his book. Already in 1817, an article by Bernard Bolzano insisted that "the concepts of time and of movement (and even more so for the latter) are completely foreign to general mathematics as much as to the concept of space." All demonstrations for the fundamental theorems of analysis should be conducted in a "purely analytical"[114] manner. Bolzano seems to lucidly delineate the rigorous ideal that, during the 1800s, led to a

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See Natorp, Paul. *Die logischen Grundlagen der exakten Wissenschaften* (Leipzig; Berlin: Teubner, 1910), 221n–. Cassirer also proposed to write an article on this conception entitled: "Das Problem der Kontinuität und der Grenzbegriff" [The Problem of Continuity and the Concept of Limit]. See Cassirer's comment in a footnote to Cassirer, "Kant und die moderne Mathematik. Mit Bezug auf Bertrand Russells und Louis Couturats Werke über die Prinzipien der Mathematik," in *Kant Studien* (1907), 12:1–49*;* now in CGW 9:55n.

[110] Russell. *The Principles of Mathematics*, 329.

[111] Ibid.

[112] Nelson. 617.

[113] Peirce, Charles Sander. "The Law of Mind." In *The Essential Writings*. ed. Edward C. Moore. (Amherst, NY: Prometheus Books, 1998), 202.

[114] The passage from Bolzano is cited in Botazzini, Umberto, *Il Calcolo sublime. Storia dell'analisi matematica da Euler a Weierstrass* (Turin: Boringhieri, 1981), 92.

complete "arithmetization of analysis"[115] and a rigorously arithmetic definition of the concept of continuity: "the arithmetization of mathematics," recognizes Cassirer, "made constant progress until the first half of the nineteenth century. It began with Cauchy's *Course d'analyse* in which doubts concerning the infinitely small were eliminated for the first time." Only "Cauchy succeeded in showing the way … to an unobjectionable theory of analysis," which, "excluding the ambiguous concept of the infinitely small," is based exclusively "on the concept of limit" (CGW 5:66–).

Gottlob Frege's critique, published in the *Zeitschrift für Philosophie und philosophische Kritik*, of Cohen's interpretation in *Das Prinzip der Infinitesimal-Methode und seine Geschichte* expresses this fundamental tendency in the history of eighteenth-century mathematics: "The infinitesimal calculus is purely arithmetical in nature, … one must not go back to geometry or mechanics in defining or justifying its fundamental concepts."[116] However, "[a]s far as the foundations of the differential calculus are concerned, we shall, I believe, have to go back for this purpose to the concept of a limit in the sense of algebraic analysis, and though the author [Cohen] belittles this as 'negative', this would seem to be due only to a misunderstanding."[117]

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This misunderstanding should be attributed to the fact that Cohen's conception of the "method of limits" does not take into consideration the formulation it obtained over the course of the 1800s.[118] His essentially historiographical work is primarily modeled on the definition of the concept of limit formulated by Jean le Rond D'Alembert, who was one of the first to identify the method of limits as "the basis of the true metaphysics of differential calculus."[119] Remaining attached to this conception, Cohen understands the *Grenzmethode* (method of limits) in terms of all the foundational attempts of calculus that are concerned with avoiding all recourse to different magnitudes than ordinary finite ones and all recourse to any tool that is not already used in traditional mathematics. That fact that, "beginning with D'Alembert," writes Cohen, "mathematicians are concerned with founding infinitesimal calculus on the method of limits" can be essentially understood as the mere

[115] In particular, see the classic Klein, Felix Christian. "Über Arithmetisierung der Mathematik (1895)." In *Gesammelte mathematische Abhandlungen*, 232–40. (Berlin: Springer, 1973), 2:232–40. For further details, see Kline, Morris. *Mathematical Thought from Ancient to Modern Times* (New York: Oxford University Press, 1990), 972.

[116] Frege. "Rezension von Das Prinzip der Infinitesimal-Methode und seine Geschichte." 101. English translation from Frege. *Collected Papers on Mathematics, Logic, and Philosophy*, 110.

[117] Frege. "Rezension von Das Prinzip der Infinitesimal-Methode und seine Geschichte." 102; Frege. *Collected Papers on Mathematics, Logic, and Philosophy*, 111.

[118] On this subject, see Peter Schultess's considerations in Schultess, "Introduction" to Cohen, Hermann, *Das Prinzip der Infinitesimal-Methode und seine Geschichte*. In CW 5:1:17*–21*.

[119] From D'Alembert, Jean le Rond, the entry "Limit" in the *Encyclopédie, ou Dictionnaire Raisonné des Sciences, des Arts et des Métiers* (Paris, 1751–1780), 9–542. D'Alembert writes: "I have never regarded differential calculus as a calculus of infinitely small quantities, but as the method of first and last reasons; that is, as the method for finding the limits of relations" (Ibid.).

"continuation of the classical method of exhaustion" (CW 6:135). For example, the French mathematician Simon L'Huiller, author of the 1787 essay entitled *Exposition élémentaire des Principes des calculs supérieurs* (well known by Cohen[120]), maintains that "the method of the ancients appropriately extended" was "sufficient to establish with certainty the principles of the new calculus."[121] On the basis of this formulation, the new calculus modified the method of exhaustion, understanding it in terms of limits such that it became nothing more than, as D'Alembert writes, "the classical method of exhaustion reduced to a simple and convenient analysis."[122]

The recourse to the method of exhaustion (that is, to the indirect demonstration of the equality of two magnitudes through *reductio ad absurdum* in "Archimedean style") as much as to the succeeding method of limits (in which, instead, two magnitudes are considered effectively equal if their difference becomes ever smaller) aimed to remove the ambiguous and contradictory notion of the infinitely small. These methods justify the results obtained by calculus without resorting to any resource foreign to traditional algebra. According to Cohen, however, these alternatives obscure the authentic motive that induced the discoverers of the infinitesimal method to introduce a new type of magnitude, unknown to previous mathematics:

> [T]he method of the limit, can also be useful and necessary in the assessment [Kontrolle] of *calculus*, but the *discovery* [Entdeckung] of the [infinitesimal] *method* is not concerned with this, but in its contrary, in its opposite. This opposite consists in the affirmation and the fixation of that which cannot be determined from a finite point of view, and which nonetheless, or rather, precisely for this reason, can represent the foundation of the finite. This is the new idea. And in this, mathematics and natural science are united (CW 6:135, emphasis mine).

This passage from the *Logik der reinen Erkenntnis* seems to effectively summarize the complex formulation of the concept of the infinitesimal in Cohen's studies: on the one hand, he recognizes the legitimacy of the method of limits for infinitesimal *calculus* on the basis of which the relation between finite magnitudes is conserved even when the difference between them has *vanished*, "as if the differential had nothing more determinate to say than being the limit of the differences" (CW 5:178). On the other hand, he affirms that the *discovery* of the infinitesimal *method* occurred through a contrary procedure that understands finite magnitude as the result of a process of *production* relying on an understanding of the infinitesimal as a generative element.

Only by tracing the *history of the discovery* can the philosophical sense of the infinitesimal method be clarified, a method that was only later laid down into a series of "algorithms" rendered rigorous by avoiding any recourse to non-finite quantities: "through the representation of scientific relations that led to the discovery

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[120] See: CW 5:1:96.

[121] L'Huiller, Simon. *Exposition élémentaire des Principes des calculs supérieurs* (Berlin: G. J. Decker, 1786), 6. Quoted in Boyer. *The History of the Calculus and its Conceptual Development*, 439.

[122] From D'Alembert, Jean le Rond, entry on "Exhaustion" in the *Encyclopédie, ou Dictionnaire Raisonné des Sciences, des Arts et des Métiers*, 5:436.

of calculus, the possibility of understanding its significance for theory of knowledge [erkenntniskritisch] is better guaranteed" (CW 5:1:11); that is, for understanding its capacity to unite mathematics and mathematical natural science. Though, "*after … the discovery* was completed and determined, one can also comprehend the attempts of the discoverers themselves to affirm and defend it [the infinitesimal method] through the traditional method of limits" (representing the calculus *stile archimedeo*, in Archimedean style); "what nevertheless remains clear and plausible is that its [calculus'] clarification from the point of view of the critique of knowledge [erkenntniskritische Beleuchtung] belongs to the *discovery of the idea*" (CW 5:1:88; emphasis mine).

Why does Cohen attribute so much relevance to an apparently insignificant question like that of the "discovery"; so much that he sees the true significance, the "philosophical" significance of the infinitesimal method in it? Why should a merely historical or psychological fact serve a purpose for the "critique of knowledge"? This was essentially the same question that Frege, in his review of *Das Prinzip der Infinitesimal-Methode* [The Principle of the Infinitesimal Method] cited above, was unable to find a response for:

> His [Cohen's] opinion that historical insight alone can first disclose what has a claim to being a logical presupposition of the science (p. iv) is an erroneous one. On the contrary, those logical foundations are perhaps always discovered only later on, after a considerable amount of knowledge has been accumulated. From the logical point of view, the historical starting-point appears as something accidental.[123]

Concerning infinitesimal calculus in particular, even though one can certainly affirm that its "historical starting-point lies in geometrical and mechanical problems,"[124] this historical starting-point cannot provide its theoretical justification. Rather, "a logical concept does not develop and it does not have a history, at least not in the currently fashionable sense." To be terminologically precise, one absolutely cannot "speak about the history of the development of a concept"[125] because the genesis of a concept inevitably compromises its identity, making it a different concept.

In contrast, Cohen explicitly insists on the necessity of not reducing the historical investigation to a mere appendix to the true and proper philosophical and theoretical justification. Instead, he makes the "history of scientific reason … the ideal of all knowledge" (CW 1:1:10). In the introductory pages to *Das Prinzip der Infinitesimal-Methode*, Cohen states, that "nothing seems more necessary to me, and nothing more immediately useful than following, simultaneously with the

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[123] Frege. "Rezension von Das Prinzip der Infinitesimal-Methode und seine Geschichte." 103; Frege. *Collected Papers on Mathematics, Logic, and Philosophy*, 109.

[124] Frege. "Rezension von Das Prinzip der Infinitesimal-Methode und seine Geschichte." 103; Frege. *Collected Papers on Mathematics, Logic, and Philosophy*, 110.

[125] Frege, Gottlob. "Über das Trägheitgesetz." In *Kleine Schriften*. ed. Ignacio Angelelli, 99–102. (Darmstadt: Wissenschaftliche Buchgesellschaft, 1967), 122. English translation from Frege. *Collected Papers on Mathematics, Logic, and Philosophy*, 133.

unfolding of a decisive systematic idea, its historical development" (CW 5:1:3). Thus, understanding the reasons for which Cohen wrote a history of the infinitesimal method, and did not limit himself to determining abstractly its logical fundamentals, is essential to understanding the significance of his work. The clear distinction that Frege proposes between what today is called the "context of the justification" and the "context of the discovery" is certainly applicable to Marburg neo-Kantian philosophy itself considering its anti-psychologism; that is, its clear separation of "physiological [or psychological] deduction" from "transcendental" deduction, psychology from theory of knowledge. Not coincidentally, this is precisely the only point in which Frege, in the same review, claims to agree with Cohen's formulation: "I agree with Cohen that knowledge as a psychic process does not form the object of the theory of knowledge, and hence, that psychology is to be sharply distinguished from the theory of knowledge."[126]

However, the model of philosophical reflection on presuppositions of knowledge that the Marburg school[127] explicitly defends aims to connect "the history of philosophical theory of knowledge with the history of scientific knowledge."[128] This project cannot be completely encaged within the rigid dichotomy between "justification" and "discovery." A 1931 book by Boris Pasternak, who studied at Marburg under Cohen and Natorp, contains an effective description of the Marburg style of connecting philosophy and history of science:

> That repellent condescension to the past was foreign to the [Marburg] school, and it did not look down on it as on a poorhouse where a handful of old men in chlamyses and sandals or perukes and long jackets utter their lying and obscure lines …. [T]he school did not speak of the stages in the development of the 'Weltgeist', but, say, of the postal correspondence of the Bernoulli family, though it knew that every thought of however distant a time, surprised in its place and at its task, must be laid bare to our logical commentary. Otherwise it loses its immediate interest for us and submits to the guidance of the archeologist.[129]

This passage seems to provide a simple explanation of the complex sense of the transcendental method (which for the Marburg School is the method of scientific philosophy in general) and its relation to the evolution of scientific thought. The *a priori* "is not a system of laws that fell from the sky."[130] It does not possess an intrinsic universality and necessity. The *a priori* must be retrieved, as a transcendental

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[126] Frege. "Rezension von Das Prinzip der Infinitesimal-Methode und seine Geschichte." 102. English translation from Frege. *Collected Papers on Mathematics, Logic, and Philosophy*, 111.

[127] On this point, see Ferrari, Massimo. *Ernst Cassirer. Dalla scuola di Marburgo alla filosofia della cultura* (Florence: Olschki, 1996), 20–26.

[128] Natorp, Paul. *Descartes' Erkenntnisstheorie. Eine Studie zur Vorgeschichte des Kriticismus* (Marburg: Elwert, 1882), 25.

[129] English translation from Pasternak, Boris. *Safe Conduct: An Autobiography and Other Writings* (New York, NY: New Directions Publishing Corporation, 1949), 41–.

[130] Natorp, Paul. "Kant und die Marburger Schule," *Kant Studien* 17 (1912): 193–221, 194. Now in Flach, Werner and Helmut Holzhey, eds., *Erkenntnistheorie und Logik im Neukantianismus* (Hildesheim: Gerstenberg, 1980). This edition preserves the page numbers of the original to which I refer.

condition, in the "fact of science" as it is "effectively realized in the history" that is contained "in published books" (CW 2:33): "criticism discovers the pure in reason in as much as reason discovers the conditions of certainty [Gewissheit] on which knowledge as science is founded" (CW 5:16). The *a priori* principles at the foundation of experience should not for this reason be "found" *a priori* as well. Rather, they can only be determined while examining reason, in a manner of speaking, at work in history.

Scholars associated with the Marburg school produced historical works that were often of value (e.g., Paul Natorp's writings on the "prehistory" of criticism,[131] Kurd Lasswitz's *Geschichte der Atomistik* [History of Atomism],[132] and Cassirer's *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit* [The Problem of Knowledge in Philosophy and Science in the Modern Age][133]). These works, of which *Das Prinzip der Infinitesimal-Methode* provided the first significant (even if not the most successful) example, establish an inseparable connection between the historical and the systematic task. From this point of view, the "discovery" of a concept becomes "the distinctive mark of creative reason, which makes itself independent of all other charms of consciousness, and which produces a pure form."[134] Thus, it should be no surprise that, according to Cohen, the authentic significance of the infinitesimal method can only emerge if one arrives at a *systematic* definition of the question which the discoverers endeavored to solve in a *historical* manner: "we abide by a historical perspective," writes Cohen in the *Logik*, "the really creative elements in scientific thought are manifested in the history of scientific thought" (CW 6:50).

The history of the formulation of the laws of collision of bodies, that is to say, the attempts to determine exactly what magnitude is conserved in the collision (energy, momentum), seems for example to permit a precise definition of the most appropriate meaning of Kant's category of substance and of the corresponding synthetic principle that prescribes its permanence. Cohen's theses can be summarized in the same fashion. In the history of science, the infinitesimal method and the concept of the infinitely small was discovered in order to resolve the problem that Kant formulated, even if only imprecisely, "in the category of reality" and "thus in the principle of intensive magnitudes and of anticipations" (CW 5:1:14). Conversely, the meaning of the category of reality and of the Anticipations of Perception can be understood when one sees in them the expression of an ineludible demand in the history of scientific thought: "the lack of a foundation for the concept of differential

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[131] In particular, see Natorp, Paul. "Leibniz und der Materialismus [published posthumously by Holzhey]," *Studia Leibnitiana* 17, no. 1 (1985): 3–14; Natorp, Paul. "Galilei als Philosoph. Eine Skizze," *Philosophische Monatshefte* 18 (1882): 193–229; Natorp. *Descartes' Erkenntnisstheorie. Eine Studie zur Vorgeschichte des Kriticismus*.

[132] Lasswitz, Kurd. *Geschichte der Atomistik* (Hamburg-Leipzig: Voss, 1890).

[133] See volumes 2–5 of Cassirer's *Gessammelte Werke* (CGW).

[134] English translation from Cohen, Hermann. *Religion of Reason: Out of the Sources of Judaism*, tr. Simon Kaplan (New York, NY: Frederick Ungar Publishing Co., 1972), 34.

from the point of view of the theory of knowledge is at the same time the foundation for the lacuna that the fundamental concept of reality represents in the series of categories" (CW 5:1:26).

Thus, the difficulty of *Das Prinzip der Infinitesimal-Methode* is found first of all in the guiding hypothesis and objective of precisely defining this category and the corresponding principle. Firstly, Cohen presupposes the meaning of the term *Realität* as it is found in the *Critique of Pure Reason*; or rather, one of Cohen's greatest interpretive achievements is his understanding of the importance of the clear and rigorous distinction between *Realität* and *Wirklichkeit*, between the qualitative concept of reality (opposed to that of negation) and the modal concept (opposed to that of mere possibility): "[T]he existing [das Wirkliche] and the real [das Reale]," writes Cohen in *Kants Theorie der Erfahrung*, "should be considered distinct as principles. The real in itself is not effectively existent [wirklich]" (CW 1:1:620). Such a distinction was anything but obvious in the *Kantforschung* in those years. Friederich Paulsen understands "reality" and "negation" in terms of *Wirklichkeit* and *Unwirklichkeit*,[135] while Alois Riehl, despite seeking to reveal the different nuances between the two terms, nevertheless maintains that both involve the problem of *Dasein*, of concrete existence.[136]

According to Cohen, however, the meaning of the category of reality assumed in the Anticipations of Perception, namely, the reality that is the object of sensation, can only be fully comprehended by reconstructing the history of the infinitesimal method, and through this, the history of the problem that calculus sought to solve. What Kant seeks to express by recurring to the *psychological* fact of sensation should be defined from the point of view of *critique of knowledge*. The merely "subjective" contrast between pure intuition, space and time (that cannot be perceived in themselves), and reality (that can exercise an influence on the senses) should he replaced with the "objective" conflict between the logical demand for *discrete* numbers and the demands that arise when *continuous* processes of change that unfold in space and time are considered: "Here as well, one can recognize the idealizing meaning of mathematical natural science. Things [Dinge] are not given to it, but rather movements represent its problem" (CW 6:224). If mathematical science was "limited to finite numbers, as the ancients were … then movement could not be determined as the movement that is presented in natural processes, as real movement" (CW 6:134).

The number is only capable of expressing finite extended differences. In contrast, the mathematical definition of physical movement, even in instants and points without extension, can determine the characteristic "property" that distinguishes a particular movement from another: "[I]n space and time, so-called things are in most cases given only as mathematical bodies. Thus, the number that counts them would be a

[135] Paulsen, Friederich. *Immanuel Kant: sein Leben und seine Lehre. Mit Bildnis und einem Briefe Kants aus dem Jahre 1792* (Stuttgart: Frommann, 1898), 68–, 180–.

[136] Riehl, Alois. *Der philosophische Kritizismus. Geschichte und System*. 2 vols (Leipzig: Kröner, 1924–1926), 1:534n.

unity of measure developed for these ideal forms; but just as those mathematical forms would not be given as physical objects, in the same way, the number would only be a fictive [fictive] scientific magnitude correlative to the ideal things that are measured as magnitudes of pure intuition." If, however, "geometric bodies become physical," then "this last meaning does not seem capable of being guaranteed by the number of the ancients [antike Zahl]" (CW 5:1:22). Thus, the problem of reality does not consist in the filling of pure intuition through sensible qualities given *a posteriori*, but in the transition from geometry and mathematics to that "science which is concerned with the last instruction for this material meaning of things [welche sich mit der letzten Instruction für diese materielle Bedeutung der Dinge beschäftigt], that science which considers geometrical bodies as physical objects"; in sum, the transition from geometry and mathematics to "mechanics" (CW 5:1:22).

In other words, the transition from the purely geometrical meaning of movement, as simple succession of positions in time, to its physical meaning, as variation in velocity, finds its expression in the category of reality. While geometric movement is a mere changing of place in a certain period of time, such that one and only one position corresponds to each instant, the mechanical conception of motion involves different motions in each instant and in each point, even if there is no change of position and thus no movement strictly speaking in these points and instants: "the difficulties that surround [umgeben] the concepts of mechanics" involve nothing other than the "difficulties that the concept of differential is characterized [behaftet] by" (CW 5:2:87).

If "finite" magnitudes are no longer sufficient to define the physical being of movement, a new type of magnitude must be introduced, one that is not reducible to the first ones and that is not a descendent of traditional mathematics: "to guarantee things as physical bodies, as real objects, infinitesimal numeration is required [infinitesimaler Zählung]" (CW 5:1:22). The Kantian problem of passing from the extensive magnitude of "space" and "time" to the intensive magnitude of the "real" that is extended in these corresponds to the transition from the geometric conception of movement to the physical one, or, as Cohen sometimes describes it, from the kinematic to the kinetic.[137] Since only the infinitely small made such a transition possible, it "corresponds to a fundamental concept of pure thought, to the category of reality" (CW 5:1:23): "this presupposition" is at the same time "the meaning of reality and the secret of the concept of differential" (CW 5:1:28).

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If one considers, for example, what can be called the first law of physics in the new science; namely Galileo's law of gravity, "the prototype of all natural forces" (CW 5:1:47), the *status quaestionis* can be easily comprehended. According to the law, bodies fall with a uniform acceleration. Given that the "intensity of velocity" of a body that is dropped from a certain height increases according to the "extension of

[137] On this distinction, see Maxwell, James Clerk. *Matter and Motion* (New York, NY: Dover, 1952), 26–.

time,"[138] the body will acquire a certain *gradus velocitatis* in respect to its initial state of rest and over the course of its fall. Galileo himself realized that this process leads to a crucial question: how is it possible to pass from rest (or rather, the absence of motion) to movement, that is, from nothing to something?

One could postulate a continuous increase in velocity from the initial state of rest, admitting that the body passes through all the degrees of velocity and of slowness, as Galileo says. However, how is it possible to conceive infinite degrees of velocity in a finite period of time? How can a finite change pass through infinite variations? To avoid this difficulty, one could admit a "minimum amount" of movement to which the moving body directly jumps without mediations. In this way, the total quantity of motion is obtained by the successive addition of a finite number of these elementary movements. How much should this minimum amount of movement be? What motive can be adduced for interrupting the resolution of the motion into smaller parts in a certain point? Both solutions present insurmountable difficulties when considered from the point of view of ordinary magnitudes: if velocity is conceived as the space that a body travels through even in a very small, but finite, period of time, such an element could not be represented as ultimately indivisible, as the final element. However, if one attempts to define the movement of a body in the instant, reconstituting the motion of the fall from a so-to-speak "immobile" movement seems impossible.

The only way that has been devised to exit from this "labyrinth," as Giulio Vivanti (mathematician and historian of mathematics who also contributed to Moritz Cantor's *Vorlesungen über Geschichte der Mathematik* [Lectures on History of Mathemathics][139]) explains in *Il concetto di infinitesimo e la sua applicazione alla matematica* (The Concept of Infinitesimal and its Application to Mathematics),[140] was that of considering un-extended points as possessing a property that makes them capable of generating extension. This property expresses "the law of generation of a phenomenon, the manner in which it tends to continue, or more precisely, the fact that the element considered is able to generate a determinate motion or a determinate line."[141] The infinitesimal would be introduced with the purpose of serving as a generating element (*elemento generatore*) of finite magnitudes: it "is an entity whose magnitude is *null*," but that nevertheless possesses, in a *greater or lesser degree*, "the aptitude, the tendency to generate magnitudes."[142] "It is what the

[138] «Et sic a recta ratione absonum nequaquam esse videtur, si accipiamus, intentionem velocitatis fieri iuxta temporis extensionem» (Galilei, Galileo. "Discorsi e dimostrazioni matematiche intorno a due nuove scienze." In *Le opere di Galileo Galilei*. (Florence: Barbera, 1890–1909), 8:198; see CW 5:1:49.

[139] See Cantor, Moritz. *Vorlesungen über Geschichte der Mathematik*. 4 vols (Leipzig: Teubner, 1907–1910). Vivanti contributed to the fourth volume with an essay on mathematical analysis in the second half of the seventeenth century.

[140] Vivanti, Giulio. *Il concetto d'infinitesimo e la sua applicazione alla matematica: saggio storico* (Mantua, Italy: G. Mondovi, 1894).

[141] Ibid., 9.

[142] Ibid.

philosophers refer to," writes Vivanti, proceeding to explicitly cite Cohen and Lasswitz,[143] "saying that the infinitesimal is an intensive magnitude."[144]

This formulation of the problem most effectively expresses the idea that seems to be in the background of Cohen's work. As is well known, the movement of a body can be arbitrarily subdivided into extended parts (that is, into stretches of time) during which bodies traverse a determinate space, or in other words, the body possesses a certain velocity. If a body moves in a rectilinear and uniform fashion, its velocity in a determinate stretch of time is identical to its velocity in each moment without duration. As soon as one considers a more complex case, however, such as the motion of a falling body, which in the same period of time moves "not with uniform motion but slowly at the beginning and with a continuously accelerated motion";[145] this conception of velocity no longer seems sufficient. A rectilinear and uniform movement and a movement accelerated with the same average velocity would be completely indistinguishable.

In contrast, in a varying motion, velocity should be thought as variable from moment to moment. Therefore, it should be possible to determine the velocity of a moving body in every indivisible instant in respect to the velocity that the body assumes has in preceding or subsequent instants: "Velocity," writes Cohen, "is simply proportional to time, being the uniform [gleichartige], *extensive* application of time to space [der Zeit auf dem Raum] … velocity can thus be initially represented, still in its sensible primitiveness [in sinnlicher Naivität], as an attribute of time in space [der Zeit am Raume] … for acceleration, in contrast, one cannot avoid assuming infinitesimals from the beginning" (CW 5:1:49). In the case of varying motion it is no longer sufficient to assume finite extended differences (the spatial extension that a body travels in a period of time of a certain duration). Instead, one must admit that in each instant in which there is no genuine change of position and thus no movement, the body still possesses a well-determined "degree of velocity" in respect to the body's velocity in another instant. Since one cannot avoid introducing the concept of the infinitely small to resolve this problem, this concept should be considered a necessary condition of possibility for mathematical natural science.

Thus, it is "from the source and from the beginning of mechanical problems" that "the concept of differential ultimately emerged" (CW 5:1:23): "when we … show that in the laws of gravity the infinitesimal principle is revealed as creative, then we regard it [gewähren wir denselben] as a fundamental mechanical concept"

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[143] See Ibid., 7–9. Vivanti demonstrates his knowledge of the works of Cohen, Lasswitz, and Müller (see Vivanti. *Il concetto d'infinitesimo e la sua applicazione alla matematica: saggio storico*, 916, 66, 72, 91, 94, 109, 15) and that he in turn is an author well-versed in German neo-Kantianism. (e.g., see CGW I, 172, 180, 185). See also Cohn, Jonas. *Geschichte des Unendlichkeitsproblems im abendländischen Denken bis Kant* (Leipzig: W. Engelmann, 1896), 129–31, 37.

[144] Vivanti. *Il concetto d'infinitesimo e la sua applicazione alla matematica: saggio storico*, 9.

[145] Galilei. "Discorsi e dimostrazioni matematiche intorno a due nuove scienze." 8:200. English translation from Galilei, Galileo. *Dialogues Concerning Two New Sciences*, tr. Henry Crew and Alfonso de Salvio (New York: The Macmillan Co., 1914), 164.

(CW 5:1:47). The new analysis of the infinite, from a "historical" as much as from a "systematic" point of view, refers to the fundamental problem of dynamics: "the power of this thought is demonstrated in the principle of conservation [Prinzip der Beharrung]" (CW 5:1:51; see CW 1:1:33). In his attempt to define the moment in which there is no movement, the tendency of a mobile to continue with the same velocity and direction, Galileo discovered the new conceptual tool that contains, in an embryonic form, the later analysis of the infinite. Through terms like "*l'impeto, il talento, l'energia, il momento del discendere* [impetus, faculty, energy, moment of descent],"[146] this concept was "described only … in a sensible manner" and "not fixed conceptually"; but here one can see "the *desideratum* [Desiderat] of this epoch-making thought" (CW 5:1:51), namely, the definition of the problem that analysis should resolve to make mathematical science of movement possible: "Galileo's innovation is the presupposition, the anticipation [Vorwegnahme], of this concept, which only later will arrive at its fulfillment: in Galileo's conception, the concept of differential is already contained in its creative efficacy" (CW 5:2:70).

Only now can one comprehend how "rest, which does not have any velocity," can enter into movement; how "emerging movement," which is not actually movement, can be distinguished "from finite [vom fertigen] movement" (CW 5:1:50). Rest, which to sensible consideration appears to be the opposite of motion, can now be thought as an infinitely slow movement that encloses in itself an infinitely small tendency toward movement. "[T]hus, when" one observes "that a rock that descends from a certain height, starting from rest, acquires new increments of velocity along the way,"[147] each one of these increments are assumed to be infinitely small in respect to an ordinary movement. In this way, one can describe how velocity is "produced" from rest through all the intermediate degrees of velocity: "in this final meaning,"…" writes Cohen, "the work of the infinitely small is already found in the man who established and secured the first great foundation for their valorization. Galileo presupposes the infinitely small in the sense of productive reality in his definition and demonstration of the concept of accelerative force" (CW 5:1:45).

Galileo's idea of rest as "infinite slowness of motion" is still far from the mathematical formulation of "infinitesimal" adopted by Newton in his concept of "fluxion" and by Leibniz in his concept of "differential." Nevertheless, according to Cohen, in these first intuitions of Galileo, one is able to see the work of scientific reason in the process of discovering a new concept: since the concept "only arrived at its development through its mechanical efficacy …. already in Galileo, even before it was fixed in its conceptual rigor … it performed its creative work" (CW 1:1:33). "Even before it was successfully defined," it nevertheless demonstrated "a latent and not yet clearly [frei] recognized productivity" (CW 5:2:70). The successive introduction of an adequate mathematical tool can thus be conceived as the attempt to satisfy what was revealed as a necessary condition of possibility for the science of motion: "for no other motive," affirms Cohen, "did Leibniz introduce his *dx* and Newton his $\dot{x}o$ if not for the purpose

[146] The terms are in Italian in Cohens's original text.

[147] English translation from Galilei. *Dialogues Concerning Two New Sciences*, 164.

of amplifying and securing that experience, which Kant, turned into a problem for the critique of reason, understanding it as mathematical natural science" (CW 5:1:11). Even in these thinkers, as Cohen seeks to demonstrate, the questions of a more properly algebraic character (the study of numerical series) and geometric character (the problem of the tangent and of squaring) are only the necessary premises for the solution to the fundamental problem: "these three relations worked together in mechanics in determining the discovery of the differential" (CW 5:1:23).[148]

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According to Cohen, in order fully to understand the philosophical significance of the connection between the problem of the reality of movement and the concept of differential, it is necessary to reflect on the manner in which Leibniz in particular arrived at the discovery of the infinitesimal method. This example is particularly significant not only because Leibnizian thought represents one of the fundamental points of reference for Marburg neo-Kantianism, but above all because Cohen insist on the "mechanical" meaning that the concept of differential assumes in Leibniz, which can be considered the key to Cohen's interpretation: "If we want to comprehend the principal meaning that the concept of differential had for its discoverer," writes Cohen, "then we should focus on the interest from which it arose" (CW 5:1:54). Consequently, "the fact that it was discovered in connection with a philosophical system," should be considered "an eloquent testimony to the gnoseological–critical [erkenntniskritisch] foundation of the concept of differential" (CW 5:1:52).

According to Cohen, Leibniz did not introduce the concept of infinitesimal magnitude "simply in the interest of solving and amplifying pure mathematical problems, but also in connection with those considerations that in his thought are dedicated to the securing of the reality of things" (CW 5:1:52). These same considerations ultimately led him to introduce "the relations and the fictions of the concepts of space and number," namely, the monad as the "solid basis of the real" (CW 5:1:51). However, Leibniz's monadology is not comprehensible "without [its] profound connection with his dynamics" (CW 5:1:53). Cohen realizes that Leibniz arrives at calculus from the study of numerical series, and by passing from these to the consideration of series on the *x*-axis and the *y*-axis in his studies of curves.[149]

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[148] See Flach, Werner. "Einleitung." In *Das Prinzip der Infinitesimal-Methode und seine Geschichte*. (Frankfurt am Main: Suhrkamp, 1968), 16.

[149] On this argument, see the classic: Bos, Henk J. "Differentials, higher-order differentials, and the derivative in the Leibnizian calculus," *Archive for History of Exact Sciences* 14 (1974): 1–90 and Bos, "Fundamental concepts of the Leibnizian calculus," in *Studia Leibnitiana*, special edition 14. See also Bos, Henk J. "Fundamental concepts of the Leibnizian calculus." In *300 Jahre "Nova Methodus" von G.W. Leibniz (1684–1984)*. ed. Albert Heinekamp, 103–18. (Wiesbaden-Stuttgart: Steiner, 1986).: Bos writes, "The Leibnizian calculus has its origins in the theory of number sequences and the difference sequences and sum sequences of such sequences. … He applied it to the study of curves by considering sequences of ordinates, abscissas etc., and supposing the differences between the terms of these sequences infinitely small" (Bos. "Differentials, higher-order differentials, and the derivative in the Leibnizian calculus," 13.).

However, Cohen maintains that, even in Leibniz's case, the first formulation of the concept of differential through algebra and geometry is only the premise for the solution of the authentic problem that drives Leibniz's entire philosophical system, namely, the problem of overcoming "comparative magnitudes of extension with their arbitrary unity that, like the origin of the coordinates, are only founded on conventions and sensible relativity" (CW 5:1:70). This "third motive [the mechanical motive after the geometrical and arithmetic ones] is that which we esteem to be the decisive motive itself for the *discovery* of the concept of differential" (CW 5:1:47; emphasis mine).

According to Leibniz, to overcome the relativity that characterizes mere geometrical considerations of motion, one must admit that motion, beyond being a change in position in a determinate time, possesses a certain "capacity for action" that confers "physical reality" on its imaginary phoronomic nature: "For, strictly speaking, [ad Âkríbeian]" – writes Leibniz in the *Specimen Dynamicum*, "motion (and likewise time) never really exists, since the whole never exists, inasmuch as it lacks coexistent parts. And furthermore, there is nothing real in motion but a momentary something [momentaneum illud] which must consist in a force [in vi] striving [nitente] toward change" (GM 6:235).[150] Thus, one must attribute to movement, beyond dislocation in a certain period of time, the force, which, present in movement even in the moment without duration, confers on motion the unity that alone is capable of making motion a proper object: "that force is something absolutely real …, while space, time, and motion are, to a certain extent, beings of reason" (GM 6:247).[151] Removing all metaphysical implications that the meaning of the word "real" has in Leibniz, Cohen is able to regard the problem of fixing the *momentaneum illud* that defines motion, *praeter extensionem, imo extensione prius*, beyond the dispersion of the parts in time, as the problem which infinitely small magnitudes were introduced to resolve. Thus, Leibniz did not introduce the concept of "differential" only to resolve purely mathematical problems. Rather, Cohen argues, "the mechanical motive of the differential must have its roots in the critique of knowledge" (CW 5:1:52), as the expression of the problem of fixing the reality of things beyond the imaginary dimension of space and time. Mechanics, the mathematical science of motion (what Leibniz actually calls "dynamics"), is thus the true *focus immaginarius* that Leibniz's mathematical research tends towards. Cohen speaks of a "prevalent [vornehmliche] tendency towards mechanics," of a "converging [einmünden] in this of all operations with the differential" (CW 5:1:63). The significance of the infinitesimal "was first appreciated in the purest form through the discovery of geometry. But the roots of this valorization are nevertheless in its realizing value, thus in mechanics" (CW 5:1:63).

Only by referring to the problem of movement can one comprehend the connection between differential, the intensive, and the problem of reality. This connection,

[150] English translation from Leibniz. *Philosophical Essays*, 118.

[151] English translation Ibid., 130.

according to Cohen, contains the expression of the problem that the discovery of infinitesimal calculus was intended to resolve. At the same time, this problem expresses "the meaning of the category of reality" and "the secret of the concept of differential" (CW 5:1:28). By grasping the connection between intensive and infinitesimal in "mechanics," that is to say, the idea of the differential of velocity as "*intensio motus*" (GM 6:399; see CW 5:1:71),[152] one can clarify the significance of the infinitely small for the "critique of knowledge." One can grasp how, as Leibniz himself affirms, "the infinitesimal calculus has given us the means for allying geometry with natural science [Physique]" (GP 5:370)[153] in order to connect mathematics and mathematical natural science: "for this reason, even in Leibniz, mechanical thought, if it has not produced the intensive magnitude of the infinitely small, it has even less brought it to its completion" (CW 5:1:45).

Thus, the characteristics that Kant attributes to the "real of sensation" express an essential problem in the history of scientific thought, a problem which allows us to define more precisely the authentic meaning of the Anticipations of Perception: on the one hand, sensation is the "instance to which we appeal when magnitude of intuition is not sufficient and it … seems to me to be only a comparative magnitude, which lacks a foundation. Sensation seems to contain this foundation" (CW 1:1:540). On the other hand, "if there were not a reality founded in the infinitely small intensive, then sensation would remain merely subjective, and there would be no mean [kein Mittel] in it, no *a priori*, substantial, real element to objectify" (CW 5:1:77). In the "historical connection" (CW 5:1:77) between the discovery of the differential and the problem of the reality of movement, "both *desiderata* are combined: the necessary overcoming [Hinausgehen] of intuition and the purity of sensation" (CW 5:1:109). Now thought itself can determine the "real" in such a way that, "at the same time, it satisfies the demands of sensation" (CW 5:1:77): "What is new in Kant's teachings consists in this: reality is not in the raw material [in dem Rohen] of sensible perception [sinnlichen Empfindung], nor even in the pure material [in dem Reinen] of sensible intuition, but rather it should be esteemed a particular presupposition of thought … a condition of experience" (CW 5:1:14).

Kant's conception of reality establishes the question that infinitesimal calculus tried to respond to. Cohen writes in the *Logik der reinen Erkenntnis*, where this aspect seems to emerge more clearly, that "one can recognize the profound connection between Galileo and Leibniz. Galileo sought the right expression [ringt mit dem Ausdruck] to determine the internal tendency [Aufstreben] of movement from its very source as force. He prefers the term *impetus* to make this effort of movement intuitable; this springing [Entspringen] of force." Consequently, "we also recognize the connection between Galileo's principle of force and the category of reality as we have recognized it in the infinitesimal number" (CW 6:262): "only at

[152] The passage is drawn from the text *Dynamica de Potentia et Legibus Naturae corporeae*. On the significance of the intensio-extensio opposition in *Dynamica de Potentia,* see Duchesneau, Francois. *La dynamique de Leibniz* (Paris: Vrin, 1994), 180–95. On the importance of the notion of the intensive in Leibniz's correspondence with De Volder, see ibid., esp. 231.

[153] English translation from Leibniz. *New Essays on Human Understanding*, 389.

this point can one legitimately affirm that the term 'reality' is completely justified for the differential because one sees here how reality refers to movement and is demanded by the latter" (CW 6:295).

Cohen maintains that an "accumulative" model of the continuous was of great importance for Leibniz as for Galileo before him.[154] On the basis of this model, the finite arises through a sum of infinitely small elements. A body suspended from a certain height does not instantaneously assume a determinate movement when it is allowed to fall freely. Nevertheless, in the moment that it is released, one cannot expect it to remain still. The body must possess some sort of tendency to move, which is still not movement, but should be considered an infinitely small movement, "an infinity of which are needed to compose an ordinary movement [dont il en faut une infinité pour composer un mouvement ordinaire]" (GM 6:234), that is, a finite movement. Thus, according to Cohen, "if it seems appropriate to define everything that is the object of natural science as finite, then [scientific] investigation is ultimately founded on this: that the finite can be thought as a sum of these intensive infinitesimal realities, as a definite integral" such that the "finite" can be conceived as "an infinite or as an infinite sum" of infinitely small elements (CW 5:1:144).

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Here I must disregard questions about the plausibility of Cohen's historical reconstruction. Instead, my aim is to highlight how, according to Cohen, the problem that only the introduction of infinitesimal magnitudes could resolve, that of conceptually determining the physical "being" of movement, corresponds to the problem of determining "reality" beyond its extension in space and time, a problem in which mathematics and mathematical science of nature are inextricably tied together: "[I]n order to logically legitimate the concept of differential," writes Cohen, "I have referred to the fundamental concept of reality … reality is the problem that, through the concepts of mass, force, and energy, raises the difficulty that the infinitesimal method overcame" (CW 5:2:88). Conversely, one could say that this is the correct formulation that Kant should have given for the pure concept of reality in the principle of the Anticipations of Perception. Cohen does not always rigorously hold to the connection between mechanics, intensive magnitude, and infinitesimal magnitude. Instead, he moves freely, and often seamlessly, between mathematical, geometrical, and physical considerations. In the concept of the infinitesimal as an intensive magnitude, these three fields seem at times to be inextricably connected: "geometry in the problem of the tangent, analysis in the infinite series, and mechanics in the problem of acceleration" (CW 1:1:547). *Das Prinzip der Infinitesimal-Methode* delineates a true and proper "equivalence between the differential and the intensive" (CW 5:1:15) such that, for Cohen, "the infinitely small as an intensive magnitude means reality" (CW 5:1:133).

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[154] Here, Cohen fails to grasp the sense of Leibniz's concept of the continuous, which means the possibility of subdividing parts and not of composing on the basis of already given, even if vanishing, parts. On this subject, see the classic Breger, Herbert. "Leibniz, Weyl und das Kontinuum." In *Beiträge zur Wirkungs- und Rezeptionsgeschichte von Leibniz.* ed. Albert Heinekamp, 316–30, 1986).

However, a noteworthy development occurs in the *Wirkungsgeschichte* of the book among the thinkers associated with the Marburg school[155] who, as opposed to Müller or Stadler,[156] follow in Cohen's tracks. Increasingly more importance is given to the fact that the connection between infinitesimal magnitude and intensive magnitude in the concept of reality can only be established by taking into account the transition from the geometric conception of motion as translation, in which rest and movement are equivalent, to the mechanical conception which attributes to movement in each instant a peculiar "capacity for action" that makes it a physical object. In the end, and in light of the above discussion, one can recognize the systematic core of Cohen's historical research here.

The pages that Kurd Lasswitz dedicates to this subject, in his *Geschichte der Atomistik* (1894),[157] have become classic in historiography of science in the neo-Kantian vein:

> The difference between the modern concept of movement and the ancient one … consists in the fact that antiquity did not possess the concept of energy … the modern expression "energy," which is mathematically expressed through the product of the mass multiplied by the square of the velocity, is only one possible form of conceptually expressing the intensive, the true and proper reality of movement.[158]

Resorting to Cohen's conception, Lasswitz considers more explicitly the "intensive" as the expression of the physical being of movement in the instant, that distinguishes

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[155] On the role of this book in the evolution of Cohen's thought, see the introductions to the two most recent editions of *Das Prinzip der Infinitesimal-Methode und seine Geschichte*: Flach. "Einleitung." and Schultess, Peter, "Introduction to Cohen, *Das Prinzip der Infinitesimal-Methode und seine Geschichte*." In CW 5:5:7*–46*. See also, Holzhey, Helmut. Review of Hermann Cohen's Das Prinzip der Infinitesimal-Methode und seine Geschichte. Einleitung von W. Flach, *Archiv für Geschichte der Philosophie* 56 (1974): 225–9.

[156] *Das Prinzip der Infinitesimal-Methode und seine Geschichte* also provoked lively reactions among thinkers initially in line with Cohen's thought. For example, Ferdinand August Müller, in a letter to Lasswitz, calls the book "one of the most monstrous parts of all the history of philosophy" (cited in CW 5:1:25*). In *Das Problem der Continuität in Mathematik*, Müller, despite crediting Cohen for being clear on the distinction between *Realität* and *Wirklichkeit*, claims that he makes the mistake of "attributing a magnitude, specifically, an intensive magnitude, to something that, like the differential, is not a real object, but only the indicator of the position of content without extent [Umfang]." Though reality certainly does not signify existence for him, "Cohen nevertheless makes the error" of identifying "reality as such" with "spatial differential" and of attributing intensive magnitude and reality to the differential of space. Cohen's identification is based on "a complete misunderstanding of the method of limits" (Müller, Ferdinand August. *Das Problem der Continuität im Mathematik und Mechanik: historische und systematische Beiträge* (Elwert: Marburg, 1886), 96n). In a few unpublished manuscripts, August Stadler explicitly refuses to call "*dx* a new type of magnitude true and proper," understanding it instead as the mere "indication of a limit," that is, to use Kant's term, as an "idea" at the most. Stadler cut off a decade-long collaboration with the future founder of the Marburg school probably for this very reason. In particular, see folder 6 of Stadler's *Nachlass* conserved by the *Zentralbibliothek* in Zurich.

[157] For a direct comparison of Lasswitz, Kurd. "Das Problem der Continuität," *Philosophische Monatshefte* 24 (1888): 9–36.

[158] Lasswitz. *Geschichte der Atomistik*, 2:4.

phoronomic movement from dynamic movement, geometric movement from real movement:

> As long as movement is purely phoronomic, then abstracting from the course [Verlauf] of time must … destroy the concept of movement. Phoronomic movement signifies that the position [Lage] of a body changes relative to the position of another body. Phoronomic movement thus necessarily implies the course of time. Removing the course of time and fixing a determinate temporal point removes the concept of movement itself as well. It seems therefore that such an abstraction is inadmissible [nicht statthaft], and yet is inevitable.[159]

Therefore, the "being" of movement does not consist in change of position, which is eliminated from movement in every single instant where there is no change of position and therefore no movement. Nevertheless, according to Lasswitz, in the temporal moment, "something still remains, a reality [Realität] that is independent of time."[160] As Lasswitz observes, "[t]his reality is not present in phoronomic movement, it consists in the dynamic capacity for action of a body in movement."[161] Such a "capacity for action" cannot be "expressed by any extensive magnitude that is eliminated through abstraction, but rather by an intensive magnitude."[162]

In this transition from the extensive to the intensive, "the 'quality' of the velocity is not removed along with the 'quantity' of the time" because "thought assumes and firmly holds to a velocity that exists only in a temporal point."[163] Lasswitz may express the fundamental "Cohenian" point more effectively than Cohen himself:

> [I]t was in dynamics that, in order to master the problem of the continuous, the necessity of a concept was revealed that, in the moment, still contained the tendency to continue … Such an element is a quality which, determined as a magnitude, is not a finite magnitude, but rather the beginning of a finite magnitude and signifies the limit of the magnitude or an infinitely small magnitude; that is, as a differential.[164]

Galileo was the first to understand the importance of this problem in fixing the concept of "moment," as the measure of the capacity of effect of a body in motion, which should be fixed in a durationless temporal point. According to Lasswitz, "resorting to the infinitely small temporal instant allowed Galilei to grasp conceptually the intensive of the movement and overcome the Eleatic objection to the possibility of alteration; still in the moment of time the tendency remains."[165] In this way Galilei could bridge "the gap between the not-being and the being of movement," regarding "the movement as becoming, as arising from infinitely small

[159] Ibid., 2:5.

[160] Ibid., 2:7.

[161] Ibid., 2:5.

[162] Ibid., 2:4.

[163] Ibid., 2:7.

[164] Ibid.

[165] Ibid., 2:30.

increments,"[166] resolving the increasing of velocity "in infinitely many moments."[167]

For abstract logical thought, founded on the "conceptual tool of substantiality [Denkmittel der Substantialität] … the thing remains unchanged or is no longer the same thing."[168] This precludes the possibility of solving the conflict between the requirements of "logic" and those of sensible experience, which, in contrast, reveals a world dominated by becoming an alteration: "The essence of change [Veränderlichkeit] is the true and proper reality of things, the rational element through which it is possible to think change [Wandel] in a phenomenon."[169] "Reality," that which defines "something" in respect to "something else," should not find its expression in the rigidity of the category of "substance," but in the "capacity of change [Veränderungsfähigkeit]"[170] through which every reality can be connected with others by cause-effect relations: "In all phenomena, "writes Lasswitz, reformulating the Anticipations of Perception, "the real consists in the tendency to continue in time."[171] Only in this way can one mediate "between the Kantian principle of intensive magnitude and the Analogies of Experience founded on the category of relation."[172] For this reason, Lasswitz prefers the term "variability" [Variabilität] to Kant's term "reality" since the former expresses the "possibility of continuation" that defines each "state as a condition of another in the moment of transition …. Only mechanical science of nature has made variability a concept capable of fixing mathematical movement."[173]

Cohen does not fully agree with Lasswitz's choice of terminology, as he makes apparent in a letter from October, 1896: "I know that your 'variability' wants to and can lead to an effective improvement, but I could not truly convince myself of the accuracy of its formulation."[174] Despite the marked differences between Lasswitz's positions and the those of the Marburg School, the young Cassirer also confirms that the essential core of Cohen's formulation should be sought in this direction. It is no coincidence that Cassirer, in his 1902 *Leibniz' System in seinen wissenschaftlichen Grundlagen* (Leibniz's System in its Scientific Foundations), never uses the expression "intensive magnitude" in the chapter dedicated to the foundations of infinitesimal calculus[175] (which I will discuss further below). Only in the chapter that discusses Leibniz's "dynamic" is there a reappearance of the idea of the "unity of quality and of intension," which is distinct "from the extension that

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[166] Ibid., 2:31.

[167] Ibid.

[168] Ibid., 1:270.

[169] Ibid.

[170] Ibid.

[171] Ibid., 1:271.

[172] Ibid.

[173] Ibid., 2:273.

[174] Cited in Holzhey, "Einleitung zu *Einleitung mit kritischem Nachtrag zur 'Geschichte des Materialismus' von F. A. Lange.*" In CW 5:2:13*n13.

[175] See Ferrari, Massimo. *Il giovane Cassirer e la scuola di Marburgo* (Milan: Angeli, 1988), 214.

instead signifies the externality [Auseinander] and a 'dispersion' of parts" (CGW 1:259). As Cohen's work also seems to affirm at times, only the union of these elements, and not the mere "differential" as such, can confront the problem of the reality of motion: "movement," writes Cassirer, "is a continuous arising and disappearing of determinations in succession. Therefore, movement also lacks the consistency of identity that is demanded as a logical presupposition of reality" (CGW 1:260). However, "the reality that we have denied to movement, in as much as it is thought only as a multiplicity [Vielheit] of different spatial locations in continuous succession, is obtained again by fixing the overall process in the single temporal element [im einheitlichen Zeitmoment]" (CGW 1:261). The unity of movement that, understood as "a totality of partial states of being, would be truly inconceivable," can thus be fixed "in the differential of motion" (CGW 1:261). The determinacy of reality in the instant "cannot signify a single sensible presence, but rather a conceptual fixation … this demand is satisfied through the Leibnizian concept of force" (CGW 1:257).

In this sense, one can affirm "that the thought that produced the concept of force signified nothing other to us than that logical function that showed itself to be effective [wirksam erwies] in the fundamentals of infinitesimal calculus" (CGW 1:266). Thus, according to Cassirer, the "fundamental idea of the infinitesimal produced, in its application to the ideal systems of spatial and temporal positions, the content as intensive magnitude" (CGW 1:271). The "differential" as such is not "reality," but rather "the single elements contained in the concept of differential. Quality and intension are finally brought together in the definition of the concept of force," which "for Leibniz … is synonymous with reality [Realität]" (CGW 1:258). Cassirer certainly no longer sees the "foundation" of infinitesimal calculus in the connection between infinitesimal and intensive, but rather, isolating that which seems to be the essence of Cohen's formulation, he highlights how "this function of infinitesimals and their value for the problem of reality is only completely developed in the logic of the concept of force" (CGW 1:258).

In his later writing on the theory of relativity, Cassirer seems to express this position even more clearly, apparently reconstructing the history of mechanics according to a decidedly "Cohenian" point of view, but purified from every attempt to simply identify intensive and infinitely small magnitude:

> The velocity of a material system is more than a mere factor for calculation; it not only really belongs to the system but defines its reality since it determines its *vis viva*, i.e., the measure of its dynamic effectiveness. In the differential quotient of the space by the time, Galileo's physics claims to have reached the kernel of all physical being, to have defined the *intensive reality* of motion. By this reality, the dynamic consideration is distinguished from the merely phoronomic. … Leibniz, too, in his foundation of dynamics, stands throughout at this standpoint … Motion conceived as a mere change of place in the purely phoronomical sense, he explains, remains always something purely relative; it only becomes an expression of a true physical and metaphysical reality when we add to it an inner dynamic principle, a *force* (CGW 10:15; emphasis mine)[176]

[176] English translation from Cassirer. *Substance and Function, and Einstein's Theory of Relativity*, 336.

The use of the expression "differential quotient", as I shall explain, suggests that Cassirer's position is actually rather different from that of Cohen. However, in this passage, the connection between the infinitesimal, intensive, and the category of reality is ultimately referred back to the problem of fixing the "being" of movement in the instant. As I tried to demonstrate, this is the fundamental idea that remains in the background of Cohen's historical-systematical considerations of the "infinitesimal."

Natorp, in *Die logischen Grundlagen der exakten Wissenschaften* (1910), summarizes in a similar way that which seems to be presupposed by Cohen's research: "the reality of movement is … to be defined in a given something, which, despite undergoing a continuous change and consisting only in this variability itself, nevertheless, precisely in the substance of this variability, is conserved as constant."[177] Thus, what is real in movement does not consist in the "variation of location in a given period of time," but rather "in the state of motion (including rest understood as movement $= 0$) that tends to conserve itself."[178] Consequently, one must admit "that a variation in the state of a real should be thought only as a variation of velocity and not as a change in position."[179] The authentic reality of movement is that which, despite tending to remain identical, can undergo variation: "the state of a moving object in a given period of time can be represented as a sum of given tendencies to movement, each of which, taken in itself without regard to the conditions that could modify it from some other part, would be determined as a rectilinear and uniform movement. In respect to a given point of space and time, such a tendency to movement should be thought as an infinitesimal."[180] In this way, the "real" of movement can be conceptually fixed, determining in the infinitely small that which defines it as such even when there is no movement and no change in position, strictly speaking. In this sense, one can affirm that infinitesimal calculus represents the necessary condition for defining movement as an authentic physical object. As this necessary condition, calculus provides the transition "from simple mathematics to mathematical natural science, above all mechanics."[181]

4.4 Intensive Magnitude and Infinitesimal Magnitude

The problem raised in the previous section, far from grafting foreign ideas onto Kant's philosophy, seems to bring out successfully an essential aspect of it, the importance of which has not always been recognized. While the previous chapters have started from the general concept of change, here, for the first time, this concept seems to assume precise connotations. In a note to the "Proof" of the second Analogy of Experience, Kant writes: "Note well that I am not talking about the alteration of certain

[177] Natorp. *Die logischen Grundlagen der exakten Wissenschaften*, 356.
[178] Ibid., 357.
[179] Ibid., 359.
[180] Ibid., 367.
[181] Ibid., 224.

relations in general, but rather of the alteration of the state. Hence, if a body is moved uniformly, then it does not alter its state (of motion at all, although it does if its motion increases or diminishes" (B252n). In every day experience, motion appears as a sort of becoming, as a process of transformation that is often opposed to rest as that which indicates stability, permanence, in the same condition. In contrast, scientific knowledge, as I have shown, paradoxically transforms movement into a "state" equivalent to rest. If a body moves with the same direction and velocity, its "state of motion" remains identical in time, exactly as when it is at rest. Change of position in the perfectly uniform space of physics does not actually change anything because only the subjective point of view of the observer is changed.

Phoronomy regards motion only as a relative change of position, as a change of "external relations." For phoronomy, a body in motion occupies a determinate position in every instant, but there is no "intrinsic" property that, in the perfectly uniform extension of space and time, distinguishes a given instant from the succeeding one. All the instants are placed, in a manner of speaking, on the same level and the state of motion is uniformly "distributed" in each of these. By contrast, according to the mechanical conception of movement, in the case of movement that "accelerates or decelerates," it becomes clear that to be able to determine a variation in velocity (that is, in order to compare the velocity in one instant in respect to the preceding or succeeding instant), it becomes clear that "the instant of rest must be distinct from the instant of motion" (AA 14:151; Refl. 40).

If the extensive magnitude of space and time cannot establish any difference between points and instants besides their reciprocal position, then another form of magnitude must be introduced; one that can be different in the same points or in the same instants: "space and time are extensive, velocity intensive" (AA, 18:241; Refl. 5587). It is not the case that a given velocity is composed of smaller velocities, "that a given speed consists of smaller speeds, and a rapidity of slownesses, in precisely the same way that a space consists of smaller spaces" (AA 4:493).[182] Velocity does not have parts that are located outside each other, but rather it is completely concentrated in a point or in an instant, despite being capable of being precisely determined as a magnitude in respect to another velocity. As Kant explains in the *Metaphysical Foundations of Natural Science*: "the parts of the speed are not external to one another like the parts of the space, and if the former is to be considered as a quantity, then the concept of its quantity, since this is intensive, must be constructed in a different way from that of the extensive quantity of space" (AA 4:494).[183] Just as the density of matter can be greater or lesser in a body of the same volume, similarly, a motion can be more or less fast during periods of time of the same duration.

In this way, Cohen is able to confirm the fact that in Kant, "the production of velocity is presupposed and understood as intensive magnitude [Grössenart] and with this the infinitely small magnitude is thought and indicated as an intensive magnitude" (CW 5:1:110), exactly as "Galileo and Leibniz spoke of the infinitely small as an

[182] English translation from Kant. "Metaphysical Foundations of Natural Science." 206.

[183] Ibid.

intensive magnitude" (CW 1:1:545). Kant's concept of "degree," which initially seemed to be concerned with sensible qualities, should be thought instead as the heir to Galileo's idea of *intensio* or of *gradus velocitatis* as it is proposed in the *Discourses*, or as the heir to Leibniz's idea of velocity as *intensio motus*. In the *New Essays*, Leibniz, who, like Galileo, adopts the medieval *calculatores*' formulation of the problem, also argues that just as a sensible quality (e.g., whiteness) can present differences in degree, in the same way, in "fundamental or distinctly knowable qualities," one can conceive "*intensity* or *degrees*, e.g. with regard to speed" (GP 5:145–).[184]

Cohen can thus draw the conclusion that "Kant, in the spirit of the Leibnizian era, thought of intensive magnitude as a differential" (CW 1:1:790). Or rather, given that "the identity of intensive magnitude and the magnitude of the infinitely small was a general assumption in Kant's time" (CW 5:1:14; see CW 1:1:548), he did not need to insist further on this connection, but only had to demonstrate "that the function [Leistung] of intensive magnitude corresponded to the presupposition of reality" (CW 5:1:14).

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In order to demonstrate that this is effectively the problem at basis of the Anticipations of Perception, Cohen's position can be supported by several passages in which Kant seems to move in this direction, including one that I have already addressed more than once: "If one regards this reality as cause (whether of the sensation or of another reality in appearance, e.g., an alteration), then one calls the *degree of reality* as cause a '*moment*,' e.g., the moment of gravity" (B210, emphasis mine). As discussed above, Kant claims to be incapable of further clarifying the question because, for now, he is not concerned with causality.

Specifically in the second Analogy of Experience, Kant returns to the concept of "moment," affirming that change is "possible only through a continuous action of causality, which, *insofar as it is uniform*, is called a *moment*. The alteration does not consist of these moments, but it is generated through them as their effect" (B254; emphasis mine). In this passage, as Cohen notes, Kant seems to allude to the fact that, in order to be able to determine a variation in velocity, one must presuppose that, in a sufficiently small duration of time, motion tends to conserve its own state. In this way, if a new urging does not occur in the succeeding instant, the body continues with the velocity and direction that it obtained.[185] Even though "Kant does

[184] English translation from Leibniz. *New Essays on Human Understanding*, 159.

[185] See also Pollok. *Kants"Metaphysische Anfangsgründe der Naturwissenschaft". Ein kritischer Kommentar*, 463-. On the connection between the static and the dynamic meaning of moment, see Moiso. "La Naturphilosophie e i paradossi dell'infinito." 152–6. See also p. 181 on the relation between the mathematical and the mechanical meaning of "moment." An unambiguous interpretation of the term is made difficult by Kant's imprecise terminology, which, according to Adickes, involves seven distinct definitions of the term "moment" in the entirety of his work. See Adickes, Erich. *Kant als Naturforscher* (Berlin: de Gruyter, 1924), 25–. See also Adickes comments in a long note on Reflection 40 in AA 14:122–9. For a history of the philosophical use of the concept of "moment", see Ziche. *Mathematische und naturwissenschaftliche Modelle in der Philosophie Schellings und Hegels*, 309–21.

not refer to inertia here" (CW 1:1:592), according to Cohen, Kant probably uses the term "moment" to indicate this tendency to continue in rectilinear and uniform motion. This tendency represents the "state" of movement in an instant, even where "*de facto*" there is no movement, and it thus defines the true and proper "reality" of movement independent of the "comparative magnitude" (CW 1:1:539) of intuition.

Moreover, Cohen's position is supported by several of Kant's "Reflections" in which "moment," that is, "the degree of reality as cause," is understood as nothing other than an infinitely small magnitude, thus making plausible the "identification" between reality, intensive magnitude, and differential.[186] In the "Reflection" entitled "Über das Moment der Geschwindigkeit im Anfangsaugenblick des Falls" [On the Moment of Velocity in the Initial Instant of Fall] in particular, one finds several lines that Cohen could only understand as confirming his interpretation:

> the moment of velocity should not be called velocity for this reason, but only the tendency to communicate a certain velocity to a body; not as an extensive magnitude, but as an intensive magnitude that nevertheless contains the basis of extensive magnitude. One cannot say that the moment of velocity is nothing, however. Otherwise, no finite magnitude could emerge from their sum. (AA 14:495; Refl. 67).

In the first instant of the motion of falling, the moving object, even though it effectively does not travel through any space, nevertheless possesses a "moment of acceleration":[187] "the beginning of every velocity is called the moment of acceleration. The moment has no velocity, but rather produces a certain velocity in a certain amount of time" (AA 14:150 Refl. 40). This is not simply the absence of velocity, but rather an infinitely small velocity in respect to the velocity that the motion acquires in a certain period of time (see also AA 4:551).

Therefore, if in the Anticipations of Perception, the "degree of reality as cause" (of sensation or of another alteration) is called "moment" (e.g., the moment of weight), one can suppose that Kant uses the term "moment" for the infinitely small variation of velocity as the effect of the force of gravity at the beginning of the motion of the fall, as in each subsequent durationless instant. In the "moment" understood as that which exercises a determinate "degree of influence on sense," one can catch a glimpse of "the mechanical motive which intensive magnitude as differential ultimately produced" (CW 1:1:556). If Kant understands the schema of reality as the "continuous and uniform generation [Erzeugung] of that quantity in time" (B183), then one can affirm that this schema corresponds to the schema of the emergence or the disappearance of velocity in a given acceleration: "degree of reality. Production of a reality by $= 0$, that is, the moment [Moment] of the production.

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[186] On the relation between the concept of intensive magnitude and that of "moment," see Jonas Cohn's reference to the passage from Kant cited above, which Cohn relates to Cohen's interpretation. Cohn. *Geschichte des Unendlichkeitsproblems im abendländischen Denken bis Kant*, 241–.

[187] The fact that Kant uses the expressions "moment of velocity" and "moment of acceleration" somewhat loosely seems to indicate that he does not clearly see the possibility of distinguishing higher order differentials. The question of higher order differentials, which certainly deserves greater attention, is particularly important for Cohen. In particular, see CW 5:1:55–.

The magnitude of movement depends on time and on space. The magnitude of a velocity in an instant [Augenblick] is the magnitude of the moment of the motion through which a space is actually traversed in a certain amount of time" (AA, 17:536; Refl. 4411). Thus, the degree of velocity is the tool that serves to distinguish, in a given accelerated motion, the "moment in which it goes out of a preceding state" from the "moment in which it comes into the following state" (AA 28:203)[188] and to determine in this way a true and proper physical change as opposed to a mere relative change of position.

Therefore, according to Cohen, Kant's concept of "moment," understood as "degree of reality," contains nothing other than the fundamental problem of infinitesimal calculus. More precisely, Kant simply reproduces the Galilean way of conceiving motion, which regarded velocity as an "intensive magnitude" increasing by successive additions of degrees, a conception which, according to Cohen, is the basis of the "discovery" of calculus.

Cohen can therefore affirm that Kant obtains the "response to the question" concerning "the form of change … through a reference [Hinweis] to the anticipation of the intensive magnitude of reality [der intensiven Realitäts-Grössen]" (CW 1:1:593). In this way "the principle of intensive magnitude, that serves at the same time as principle of the Anticipations of Perception," should be thought as derived, "ultimately from a mechanical motive": in this principle, "the real of phenomena is anticipated and, in this anticipation, is made an object" (CW 1:1:593). This is the sense of Kant's concept of intensive magnitude: "it represents the *foundation and the origin* of the extensive, it makes the latter arise from itself. In this way, in the intensive magnitude, the real can be incorporated into that process through which physics becomes a science [in derjenigen wissenschaftlichen Zurüstung der Physik]," in which, in intimate connection with the presuppositions of mechanics, the infinitesimal calculus was discovered." In the "laws of motion of point particles [materieller Punkte]," that "which is revealed in sensation becomes accessible to scientific determination" (CW 1:1:758).

4.5 From Synthesis *A Priori* to the Logic of the Origin

In the preceding pages, what has been repeatedly called the fundamental problem of Kantian thought seems to have returned again: Kant's philosophy ultimately starts from the problem of "change," as the transition from one state to its opposite through an infinite series of intermediate degrees. That such change for scientific knowledge is nothing other than "movement," or better, variation of velocity, is now clear. Otto Buek, one of the many "minor" figures with a significant connection with the Marburg school, states: "[S]cience aims to explain the becoming of things;

[188] English translation from Kant. *Lectures on Metaphysics*, 26.

that is, to learn to understand the existence of things through their becoming. However, in the system of physics, becoming is determined from a closer angle as movement and movement is thus understood as fundamental to the knowledge of bodies in their being."[189]

If this is the general presupposition in the background of critical philosophy, then the meaning of the Anticipations of Perception should be understood from this point of view: "we have only three *quanta*," writes Kant in a "Reflection", "space, time, and sensation (movement, reality [realitaet])" (AA 18:240; Refl. 5582). Thus, the problem of movement is the problem that the entire structure of transcendental idealism ultimately leads to. At the same time, however, Kant insists that the "[m]otion of an object in space does not belong in a pure science … for, that something is movable cannot be cognized *a priori*, but only through experience" (B154n; see also B58; AA 4:482; 195).

The concept of movement, as well as that of change in general (see B58), remains an "empirical" concept for Kant: "motion of a point in space," that is, the being of a body "in different places (as a sequence of opposed determinations)" (B292), just as much as the transition of movement from one state into its opposite, that is, the change in the degree of velocity, cannot be conceived in any way through mere "concepts." For, as I have shown, one would "never be able to imagine *a priori* how such an uninterrupted sequence of being and not-being is possible" (B479) without appealing to the testimony of "intuition."

According to Cohen, however, the history of science provided the adequate tools for overcoming this conflict between concept and intuition. On the one hand, the empirical concept of motion was transformed to the point of assuming the rigorous form of a "state" equivalent to rest. On the other hand, mathematics itself was liberated from the rigid opposition between "immutable forms," which alone could become its object, and the world of change and becoming, which had to remain in the indeterminacy of sensory perception. Incorporating the concept of movement, and transforming it from an empirical concept into a true and proper ideal form, was one of the great accomplishments of modern mathematics: "Physics," writes Cohen, "handles motion in general. And so in mathematics as well, all is becoming" (CW 6:121).

In modern thought, mathematics and mathematical natural science are united as a single whole for the first time: "we already know what significance we should attribute to mathematics: mathematics is the mathematics of mathematical natural science, we see its purity not in its isolation, but precisely in its applicability" (CW 6:121). In turning to the problems of physics, mathematics did not merely transfer its methods and results to the empirical concept of movement. Rather, in the very attempt of determining such a concept in a rigorous manner, mathematics itself, as I have shown, assumed a new form.

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[189] Buek, Otto. *Michael Faradays System der Natur und seine begrifflichen Grundlagen*, Philosophische Abhandlungen: Hermann Cohen zum 70sten Geburtstag (4. Juli 1912) dargebracht (Berlin: B. Cassirer, 1912), 109.

In movement, as in the concept of change in general, Kant only saw the passing of opposed and irreducible states located one-outside-the-other or one-after-the-other in intuition. For him, the problem was how these states could be connected synthetically in the unity of the concept: "*A* and *B*," states Cohen, "must be rigorously thought as different from each other. Because this is the problem: that, despite being different, their unification is possible and necessary … And the idea that these are different is in fact the point of departure for the characterization of causality in Kant" (CW 6:274). Here, as I have shown, is the fundamental problem of the *Critique of Pure Reason*; the problem of the possibility of *a priori* synthetic judgments; the problem of establishing how "a *B* has to be added to an *A*, if a content shall arise. But how can a *B* arise, if it's not going to be only another *A*? This other *A* would be distinct from the first *A* only as a psychological process, as a representation … If, on the contrary, *B* really means *B* as opposed to *A*, then a difficult question arises: through what methodological tool can *B* be discovered?" (CW 6:146).

Kant, "using intuition alone [der Anschauung allein gebraucht], did not bring the arising of the first original element of the synthesis as purely to production as pure thought can require and perform [wie das reine Denken zu fordern und zu leisten vermag]." Only the "logic of pure knowledge," rethinking the very core of the concept of *a priori* synthesis, could free thought from its subjection to intuition: "we direct the question to the production of *B* itself, not to the connection with *A*. The production of *B* as different, as true and proper content, is the problem" (CW 6:276). According to Cohen, "Kant used the problem of diversity for the possibility of synthesis, as the connection of *A* and *B*" (CW 6:276) understood as elements "given" to thought. Now, however, it seems clear that "*B* different from *A* should have been produced" (CW 6:269). Instead of mere "dispersion [Auflösung] of being together with time … movement should become production" (CW 6:237). "*B* should be produced and should become produced as an object through movement" (CW 6:269).

Establishing a necessary connection between two contradictory and reciprocally irreducible elements, between rest and movement, is not at issue, but rather showing how movement can arise from rest, where there is no movement, that is, how movement itself can be "produced": "infinitesimal analysis is the legitimate tool of natural science … this mathematical production of movement, and through it, of nature, is the triumph of pure thought" (CW 6:33). If nothing can emerge from nothing, the infinitesimal method nevertheless shows how rest, where there is no movement but from which movement must necessarily arise, contains in itself a tendency to movement, an infinitely small movement: "the tendency corresponds to Galileo's *impetus*, to *impulsus*, or even more to his *propensione*. It expresses the origin of movement" (CW 7:122).

If the "infinitesimal principle had obtained the place it deserved in criticism, then sensibility should have no longer preceded thought; then pure thought would not have been overshadowed [geschwächt] in its autonomy" (CW 6:35). In Kant's philosophy, "the synthesis of thought presupposes the manifold of intuition" (CW 6:27). In contrast, the "logic of the principle of infinitesimal calculus" (CW 6:34) replaces the question concerning the *connection* of *A* and *B* as given elements with

the problem of the *origin* of *B* from that which it *is not*: "it is the problem of the origin that the new calculus raised and brings the thought as production to clarity and precision" (CW 6:35). The issue is not to retrace in Hegelian fashion, through a double negation, the "metaphysical" unity of being and nothingness, which is capable of expressing the absolute *beginning* (*Anfang*). Instead, the aim is to demonstrate, through the negation itself of the infinite judgment, how the *origin* (*Ursprung*) of being can only be sought in that which is not being but which, precisely for this reason, indicates that from which being comes from.[190]

Nothingness is no longer something contradictory, which cannot be thought, but rather the very condition of being, the principle from which being should be produced. The young Nicolai Hartmann, in his 1909 *Platos Logik des Seins* (Plato's Logic of Being), published in Cohen's and Natorp's series *Philosophische Arbeiten* while he was still a faithful member of the Marburg school, offers a good overview of this problem. According to Hartmann, "with this conception of existing non-being as the synthetic principle, we … have arrived at the exact meaning of Cohen's judgment."[191] Through the "infinite judgment, the thought of non-being," developing that which was "also latent in Kant's conception of 'limitation'" is no longer a mere contradictory concept that cancels itself. Instead, it "becomes origin."[192] Cohen's "origin" of "nothingness" refers back to "Kant's 'synthetic unity', to Plato's μὴ ὄν."[193] It is the relative "nothingness" from which "something" emerges, as something "other" than that which is intimately connected to it, thus revealing "the most intimate synthetic essence of thought."[194]

The problem of *a priori* synthesis, of how an already known *A* can be connected to a completely different *B* that, as such, is first of all not-*A*, receives a new and more fertile formulation in the idea of "production" from nothing. The negation of *A* does not represent a mere indeterminate exclusion of everything that *A* is not, but rather a positive methodological principle: "Negation," writes Hartmann, "has become the conceptual tool in another and a more profound sense, which a mere removal [Aufhebung] does not possess. This is, in Kant's language, the synthetic: this consists in nothing other than 'going out'"[195] from the already possessed concept toward a new concept that *is not* the first. In this way, the negation of the already known at the same time implies being directed "toward the unknown, the undiscovered."[196] If the unknown were opposed to knowing in the form of a logical negation, as a not-knowing, then knowledge would be impossible. According to the

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[190] For greater clarification of this point, see Poma. *The Critical Philosophy of Hermann Cohen*, 90–.

[191] Hartmann, Nicolai. *Platos Logik des Seins*. eds. Hermann Cohen and Paul Natorp. Vol. 3, Philosophische Arbeiten (Gieszen: Töpelmann, 1909). Reprint, Berlin: de Gruyter, 1965, 160.

[192] Ibid.

[193] Hartmann. *Platos Logik des Seins*, 160. On Cohen's interpretation of the concept of μὴ ὄν in Plato and in ancient philosophy in general, see Poma. *The Critical Philosophy of Hermann Cohen*, 96–.

[194] Ibid.

[195] Ibid., 154.

[196] Ibid., 160.

eristic argument that Plato recounts in the *Meno*,[197] one cannot know that which one does not know. From this point of view, "Socratic not-knowing does not imply skepticism. There is no doubt concerning knowledge. Rather … it represents a step toward knowing … because it contains the posing of the question concerning knowing."[198] The not-known becomes the paradoxical object of knowledge in the form of the "problem."[199] While it is true that "the problem concerning being represents a non-being,"[200] this does not imply an unthinkable nothing, but rather the search for that from which being emerges and that is not being for this reason. It represents the problem of "from where" being originated.

The value of these pages is not only in Hartmann's effective summarization of some of the great themes of Marburg neo-Kantianism, and of Cohen's thought in particular, but also in his clarification of the intimate connection and, at the same time, the profound difference of these themes in respect to all post-Kantian philosophy: "[T]here is nothing more instructive for this historical situation than the vital need for a completion of Kant, which is already manifested in the first works of post-Kantian philosophy. Fichte and Schelling, each in their own manner, attempted to find the principle for the recently discovered problem of the synthetic." According to Hartmann, it is worthwhile to note that, "after all, both confront the fundamental systematic question through the methodological tool of being and non-being."[201] The idea of the real opposition between reality and negation is the point of departure for the rethinking of the concept of "*a priori* synthesis." As I have shown, this opposition, guaranteeing the legitimacy of the thought of being and non-being, allows the not-I in face of the I, the object in face of the subject, to be thought as "negative magnitudes." However, Hartmann argues that "the most extensive attempt of this type is found in Hegel's philosophy."[202] "The concept of 'non-being' or of nothingness" represents, "in the synthesis with that of being," the problem that Hegel places at the peak of his system, fixing it in the concept of the "absolute 'beginning'."[203] In Hegel's "beginning," being and nothingness "pass into" each other and become confused such that "the absolute beginning coincides … in fact with the concept of transition."[204] Precisely for this reason, however, "origin means something different than 'beginning'[205] This does not imply any "metaphysical"

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[197] "[A] man cannot search either for what he knows or for what he does not know[.] He cannot search for what he knows – since he knows it, there is no need to search – nor for what he does not know, for he does not know what to look for" (*Meno*, 80e). Plato. *Complete Works*. ed. John Cooper (Indianapolis, IN: Hackett, 1997), 880.

[198] Hartmann. *Platos Logik des Seins*, 160.

[199] Hartmann, Nicolai. *Der Aufbau der realen Welt. Grundriss der allgemeinen Kategorienlehre* (Berlin: de Gruyter, 1964), 328.

[200] Hartmann. *Platos Logik des Seins*, 160.

[201] Ibid., 161.

[202] Ibid.

[203] Ibid., 162.

[204] Ibid., 163.

[205] Ibid., 15.

confusion between being and non-being, but rather represents the "critical" guarantee of their separation in as much as "nothingness" is understood as that from which "something" comes and that consequently cannot be confused with it. Finding a third in which the opposites are confused is not at issue. Rather, the negation itself of the infinite judgment evokes the *tertium comparationis* from which the opposites can be defined as such. This negation does not imply the annihilation of their reciprocal difference, but instead merely indicates a direction for investigation instead.

4.6 From the Logic of the Origin to the Logic of the Concept of Relation

The present analysis of the concept of "origin," though unable to do justice to all its richness, should be sufficient to demonstrate how the rethinking of the meaning of the Anticipations of Perception, that is, the possibility of establishing a connection between the "degree of reality" and the concept of the "infinitely small," is Cohen's point of departure for a radical revival of Kantian philosophy. One certainly cannot deny that, in "identifying" intensive magnitude and differential magnitude, Cohen left himself open to completely justified objections. Can one speak of infinitesimals as true and proper magnitudes? Can one isolate the differential from the differential quotient and consider it an entity with an independent meaning? "[W]hen we turn to such works as Cohen's," writes Bertrand Russell in the *Principles of Mathematics*, "we find the *dx* and the *dy* treated as separate entities, as real infinitesimals, as the intensively real elements of which the continuum is composed."[206] In reality, according to Russell, "The *dx* and *dy* of a differential are nothing in themselves."[207] Russell's objections reveal Cohen's vagueness concerning the connection between the history of the discovery of the differential and the birth of mathematical natural science, that is, concerning what seems to be the true core of his work. In his detailed examination of *Das Prinzip der Infinitesimal-Methode*, Russell writes, "[s]ince the analysis of space and motion is still to come, I shall confine myself for the present to such arguments as can be derived from purely numerical instances. For the sake of definiteness, I shall as far as possible extract the opinions to be controverted from Cohen."[208] Nevertheless, it is precisely this more properly mathematical aspect that does not interest Cohen in his history of the infinitesimal method (and not of infinitesimal calculus itself), whose "excellence"[209] is recognized by Russell himself.[210] Instead, *Das Prinzip der Infinitesimal-Methode*

[206] Russell. *The Principles of Mathematics*, 339.

[207] Ibid., 338.

[208] Ibid., 339.

[209] Ibid., 338.

[210] "The historical part of this work, it should be said is admirable." (Ibid., 327n.)

is based on the hypothesis that, "historically," the discovery of the concept of the "infinitely small" occurred in the attempt to determine the possibility of a scientific definition of movement and, with it, the transition from mathematics to mathematical natural science of nature.[211]

Leibniz's dx, isolated from the differential relation, and Newton's $\dot{x}$ are transformed in Cohen's work, though unfortunately not always explicitly, into something that should not be identified with the symbols used in true and proper algorithms; they should rather be considered "parts and examples of that problem which Kant put forward [aufgestellt] with his new concept of experience and of mathematical natural science" (CW 5:1:8). The "differential" and "fluxion" represent the point of arrival for a long tradition composed of all the attempts at fixing that what, using finite magnitudes alone, seemed impossible to define: Galileo's moment, faculty, energy, and tendency to movement; Hobbes's *conatus*;[212] and so forth. Precisely in these attempts to define a body in motion's "tendency to continue" in the instant, one sees the need to conceptually clarify that "something" which, despite not being a determinate extended magnitude, nevertheless possesses to a greater or lesser degree the "capacity" to produce magnitudes, thus representing the "origin" of magnitude.[213]

This problem, which scientific reason could not evade in its attempt to objectify the concept of movement, is the same problem that Kant sought to establish in the connection between reality and intensive magnitude as it is formulated in the principle of the Anticipations of Perception. In this way, the infinitesimal lost its

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[211] Russell, in Russell, Bertrand. *A Critical Exposition of the Philosophy of Leibniz: With an Appendix of Leading Passages* (London: Routledge, 1992)., seems closer to Cohen's project in a reference to *Das Prinzip der Infinitesimal-Methode* in a note. He argues that, since "[m]otion is change of position; but at any one instant the position is one and only one," Leibniz was convinced that, "as the Calculus was likely to suggest … the momentary increment was real" (Russell. *A Critical Exposition of the Philosophy of Leibniz: With an Appendix of Leading Passages*, 103–04).

[212] See Boyer. *The History of the Calculus and its Conceptual Development*, 43.

[213] A recent reprise of this interpretation of the Anticipations of Perception can be found in (Kauark-Leite, Patricia. "The Transcendental Role of the Principle of Anticipations of Perception in Quantum Mechanics." In *Constituting Objectivity: Transcendental Perspectives on Modern Physics*. eds. Michel Bitbol, Pierre Kerszberg, and Jean Petitot, 203–12. (Berlin: Springer, 2009)). According to Kauark-Leite, "Hermann Cohen draws our attention to the fact that the essence of the principle of intensive magnitude lies in the differential concept. The importance of the principle of anticipations actually derives from the fact that it contains the transcendental foundation of differential calculus" (Kauark-Leite. "The Transcendental Role of the Principle of Anticipations of Perception in Quantum Mechanics." 207). In the essay, Kauark-Leite notes that "[a]lthough Kant does not explicitly mention the predictive role of differential equations in the process of Anticipations of Perception, the notion of differentiability introduced by infinitesimal calculus permeates his text.". Kauark-Leite seems to almost literally adopt Cohen's interpretation when she argues that "unlike extensive magnitudes, intensive magnitudes are infinitesimal magnitudes" (Kauark-Leite. "The Transcendental Role of the Principle of Anticipations of Perception in Quantum Mechanics." 207). However, such a formulation of the problem was rather controversial, also for the Marburg School itself.

authentically mathematical character to be transformed into a sort of philosophical "category." The destiny of the concept of "infinitesimal" was certainly paradoxical, but essentially similar to what Cassirer's concept of "function" underwent 30 years later. In this case as well, a mathematical concept was shown to be capable of expressing an overall tendency in the history of science, while being, at the same time, a condition of its possibility. Cassirer himself provides a brief history of analysis of the infinite that, even though written many decades later, effectively summarizes what seems to be the overall sense of Cohen's formulation:

> Modern science … started out from the basic phenomenon of motion, and by the analysis of this phenomenon gained certain fundamental concepts, which on being pursued further led to an entirely new structure of mathematics. The new analysis of the infinite can be traced back, historically and systematically, to the problems of dynamics. In his endeavor to define the concepts of velocity and acceleration Galileo discovered those new modes of thought which already contained, in embryo, the later methods of analysis of the infinite. …. This development attained its logical consummation when Leibniz introduced the general concept of function and formulated precisely the concept of the continuity of a function. (CGW 19:158)[214]

This resorting to the concept of function and its opposition to the concept of substance represents, as is well-known, the *Leitmotiv* of Ernst Cassirer's work. In this way, he seems to continue in Cohen's footsteps in the attempt to free knowledge from all reference to an irreducible "given." At the same time, however, the concept of function transformed Cohen's idea of the infinitesimal understood as the "dynamic" or "intensive" element into something radically different.

In *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit* (The Problem of Knowledge in Philosophy and Science in the Modern Age) Cassirer agrees on the one hand with Cohen on the fact that Galileo's concept of "moment" shows that "the concept of infinity, which did not manage to escape internal difficulties when it was attributed to spatial continuity, was clarified and fixed for the first time through the concept of velocity" (CGW 2:357). Nevertheless, on the other hand, he draws a completely different conclusion than Cohen. Cassirer's formulation of the problem clarifies that the "differential of space, like that of time," cannot indicate the way. Cassirer (essentially in the same way as Russell) argues instead that, "historically and logically, the concept of the *differential quotient* was the point of departure" (CGW 2:357). Thus, not *dy* and *dx* taken alone, but rather the "functional equation" within which these assume their significance "offers the most secure and 'substantial' base that scientific thought can give for the generation of magnitude" (CGW 2:357). A body's velocity at a determinate point of its course in a given period of time "can only be conceived and represented by the comparison and reciprocal relation of a series of space values and time values. Logically considered, velocity is no absolute property of the moving thing but

[214] English translation from Cassirer, Ernst. *Determinism and Indeterminism in Modern Physics: Historical and Systematic Studies of the Problem of Causality* (New Haven: Yale University Press, 1956), 170–.

merely an expression of this *reciprocal relation of dependence*" (CGW 6:135; emphasis mine).[215]

Cohen's idea that the "differential" intensively contains in itself the capacity to generate magnitudes is replaced with the idea of the prevalence of the *relation* over the terms that it connects: "Cohen's *Logik der reinen Erkenntnis* developed its fundamental thought of the origin in connection with the principles of the infinitesimal calculus" (CGW 6:106).[216] If, on the one hand, this is the first significant example of the transition from "quantity" to "quality," on the other hand, "[i]n spite of the protests of idealistic logic, the concept of the 'infinitely small' has continually led to the misunderstanding that here magnitudes are not understood from their conceptual principle, but rather compounded from their vanishing parts" (CGW 6:107).[217] The transition from "quantity" to "quality" thus seems to be reduced to the attempt at finding the "ultimate substantial constitution [den letzten substanziellen Bestand] of magnitudes" (CGW 6:107).[218]

According to Cassirer, however, the true innovation of infinitesimal calculus can only be comprehended when it is introduced, together with other types of "calculus," into "the more inclusive system of 'analysis of relations'" (CGW 6:107).[219] The passing of a *magnitude* into another (e.g., from extensive to intensive magnitude, from finite to infinitely small magnitudes) is not at stake here, but rather the passing from magnitude as such to the *functional relation* between magnitudes:

> The analysis of the infinite is logically the first and complete expression of this intellectual tendency. For even here mathematical investigation advances beyond the mere consideration of magnitudes and turns to a general theory of functions. The 'elements' here joined into new unities are themselves not *extensive magnitudes* which are combined as 'parts' of a whole, but are forms of function which reciprocally determine each other and unite into a *system of dependencies*" (CGW 6:80; emphasis mine).[220]

The concept of function is characterized by its connecting *two series of values*. It departs "from a fundamental series $x_1 x_2 \ldots x_n$ which is coordinated, according to a definite rule, with another series of values $y_1 y_2 \ldots y_n$" (CGW 6:78).[221]

It is particularly evident in analysis of the infinite that the meaning of such a relation does not depend on the value of the magnitudes between which a relation of reciprocal dependence is established. Such a relation conserves its value even if these finite quantitative differences have vanished: "if we think of two series of variable magnitudes that are opposed to each other and reciprocally bound through a solid law of correspondence, we will see that the meaning and validity of this law

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[215] English translation from Cassirer. *Substance and Function, and Einstein's Theory of Relativity*, 75.

[216] Ibid., 99. Translation slightly altered by the author.

[217] Ibid., 100.

[218] Ibid.

[219] Ibid.

[220] Ibid., 76.

[221] Ibid., 73–.

will remain completely intact when the absolute quantitative values that we compare are decreased as much as one likes" (CGW 3:125).

This interpretation of infinitesimal calculus was already clearly delineated in *Leibniz's System*, although Cassirer seems at first sight closer to Cohen's original formulation. On the one hand, Cassirer insistently emphasized in this earlier work that "the historical origin of infinitesimal calculus in Leibniz goes back to the founding philosophical idea: either in the mediation through algebraic problems or in the preparation," that is, with the idea that the extensive is "created through an originary law-bound determinacy [ursprünglichen Gesetzesbestimmtheit] that precedes it as a logical *prius*" (CGW, 1:157). On the other hand, he quickly recognized that Leibniz, alongside this "synthetic" formulation, provides an opposite one as well: "continuity, which at first signified the "unfolding [Entfaltung] of the conceptual germ [des Begriffskeimes] of magnitude until arriving at its intuitive form," is made ever clearer through the idea of the "conservation of the logical value of the magnitude even in the elimination [Aufhebung] and the canceling of its intuitive being" (CGW 1:157). This formulation prevails in the "*purely mathematical* deduction of differentials, whereas the first, philosophically prior, direction found its definitive expression in Leibnizian *mechanics*" (CGW 1:157; emphasis mine).

The *production* of finite magnitude from the infinitesimal through "the continuous 'summing' of infinitesimal moments" (CGW 1:154) finds its place where what is at stake is the "creation of 'quantities of finite motion' from the 'elementary impulse' such as the development of the element of velocity from the continuous repetition of accelerations (as elements of second degree)" (CGW 1:154). In contrast, when higher analysis is structured and organized according to a strictly mathematical point of view, the idea of the *vanishing* of the magnitude and the conserving of the relation prevails, in conformity with the principle that "the special case of a vanishing magnitude" should be thought "as falling under the general rule" that is valid for finite differences (CGW 1:158): "the magnitude must first vanish [verschwinden] for sensible apprehension in order for us to become aware of its determinacy in the pure concept" (CGW 1:157).

Cassirer can thus insist on the fact that, in the classical Leibnizian formulation of the law of continuity, *datis ordinatis etiam quaesita sunt ordinata* [as the data are ordered, the unknowns are also ordered],[222] "the concept of function [is already] implicitly presupposed." According to Cassirer, from a mathematical point of view, the conceptual relationship between "*data*" and "*quaesita*" is entirely represented by the fact that "if the difference in the 'datum' falls as far as one likes below a given value, then it should be possible to go below any given magnitude no matter how small the difference in the 'result' as well." In this way, "the definition of function is immediately implied in the general expression of the idea of continuity" (CGW 1:216).

Now completely putting aside Cohen's critique of the *Grenzmethode* (method of limits), Cassirer sees in this concession the expression of "the ordinary analytic

[222] English translation from Leibniz. *Philosophical Papers and Letters*, 447.

definition of continuity" given by Weierstrass, "according to which, for $|x' - x < \varepsilon|, |f(x') - f(x)|$ becomes $< \delta$" (CGW 1:216).

The "productivity" of thought is revealed precisely in this process that, for Cohen, is purely negative. Here, it becomes clear that, "if the transition to quantitative zero does not remove the lawfulness of the magnitude, then this demonstrates that the magnitude has its foundation in a different principle than the quantitative one" (CGW 1:157). Infinitesimal calculus succeeds in fully expressing that which already seemed to be presupposed in algebra: the idea that "calculus refers solely to the ordering and the reciprocal conditioning of qualitative relations, and in no way to quantitative relations" (CGW 3:125). Concerning the problem of the "composition" of the continuous, one can observe that the "true 'element' of the continuous [Stetigen] is thus not … a partial magnitude [Teilgröße], but the quale of the law [das Quale des Gesetzes]…, in as much as it can be understood independently of any extensive givenness" (CGW 1:247). The "characteristic triangle," as I have already discussed, conserves its "similitude" with the "ordinary triangle" in such a way that the relation between the catheti is conserved even when these become infinitely small: "[T]hat the 'proportion' is part of quality and thus part of pure thought is implied in the difference between quantity and quantity" (CGW 1:160).

The problem is no longer the transition from the extensive to the intensive, but rather the transition from magnitude to the relation between magnitudes: thus, not the differential, but rather "the *differential quotient* is the mathematical expression for the independence and originality of the relation in respect to the individual elements [Einzelgebilde] from which it is obtained" (CGW 1:161, emphasis mine). Only here does it become clear that the essence of continuity should not be sought in the idea of the production of finite quantity from the infinitely small, but rather in the idea "of the conservation of the qualitative unity of the law of the relation in which magnitudes subsist, even in the removal of their extensive value."[223]

In this way, infinitesimal calculus succeeds in showing the "affinity [Verwandschaft] between elements – that are quantitatively heterogenous – but which are nevertheless included within a determinate and common conceptual point of view" (CGW 1:166). At first, "the concepts stay rigid and deprived of relations next to each other. Since the difference is conceived as a contraposition [Gegensatz], the passage of the one concept into the other must be considered a contradiction" (CGW 1:199). Now, however, one can demonstrate that "the opposed [Das Entgegengesetzte] do not have necessarily the status … of a contradictory determination [Widerspruch]" (CGW 1:200); an opposition of "quantity" also exists, an opposition of greater and lesser between elements that participate in the same qualitative unity, a unity that is fixed in the functional relation between magnitudes.

Already in his 1891 essay "Quantität und Qualität" published in *Philosophische Monatshefte*, Natorp seems to move in this very direction: "in fact, one never proceeds

[223] Paulsen. *Das Problem der Empfindung*, 289.

from an *isolated magnitude*, but from *two magnitudes* that are in a constant reciprocal relation."[224] The legitimacy of the transition from finite differences to infinitely small ones is justified by the fact that "the relation between the variations of both of the magnitudes (the function) remains unaltered in terms of its substance, merely assuming another expression, when both of the differences approach zero. Thus, one always proceeds from the differential quotient whose determinate value is not in the least hocus-pocus [Hexerei], so to speak. The 'differential' does not have any need of a particular foundation."[225]

The differential quotient, which is most extensively treated in *Die logischen Grundlagen der exakten Wissenchaften* (The Logical Foundations of the Exact Sciences), shows how, "from the law valid for finite variations of compared magnitudes, according to which the varying of one depends on a change in the other, one can obtain a new form of the law of change for any and every point in the alteration," in which the finite differences have vanished and are $=0$: in the transition at the limit, "the *magnitudes* of the differences become infinitely small or 'vanish'," while the "*relation* between both of the differences is conserved, if both approach zero at the same time."[226] This relation "does not indicate a mere *quantitative* variation of the relation between given finite values, but rather signifies something *qualitatively* different. The term 'quotient' … is only the reminder of the path of deduction through which the new expression was obtained." The "differential quotient" is not actually a fraction, but rather "a new function that is something effectively different from that which expresses the given function."[227]

Even if the differential quotient can be simplified as if it were a fraction, "the form of the quotient is not essential in this respect. Rather, it is even misleading precisely because it leads one to understand *dy* and *dx* as new finite magnitudes, which are just extremely small, as small as one likes." In actuality, "the rigorously infinitely small would have no determinate relation [Wertverhältnis], in as much as its numerical value would be $\frac{0}{0}$, that is, there would be no expression of a relation." Thus, "the infinitesimal" is not "absolute," as Cohen affirms, "but the procedure [Verfahren] that uses the infinitesimal effectively expresses the sovereign power of the thought of being."[228]

Once again, the essential aspect seems to be the "vanishing," not the "arising," of extensive magnitude. According to Natorp, "the productive force of the infinitesimal method consists precisely in the transition at the limit,"[229] that is to say, "the persisting of the legality of a relation … even in the transition at zero, in virtue of

[224] Natorp. "Quantität und Qualität in Begriff, Urteil und gegenständlicher Erkenntnis," 153 (emphasis mine).

[225] Ibid.

[226] Natorp. *Die logischen Grundlagen der exakten Wissenschaften*, 211 (emphasis mine).

[227] Ibid., 216.

[228] Ibid., 219.

[229] Ibid., 220.

the participation of both of the magnitudes in the same law."[230] Thus, according to Natorp, "the true producer of finite magnitude is not the infinitely small magnitude (instead, the infinitesimal would be 0 from a quantitative point of view), but rather it is the law of magnitude (as variable)";[231] that law which does not lose its significance in the transition at quantitative zero, "after all magnitude is removed [Aufgehoben]":[232] "magnitude, that is, the being such and such size has vanished and has become quantitatively null, but with this the law of magnitude is not qualitatively annulled."[233]

Philosophy's interest in infinitesimal calculus, despite the different forms that this interest has assumed, seems to reflect a shared point of view: "the important general sense of the procedure consists in this, that through it, the conceptual limits can be overcome … in this way, both cases, separated by the concept, fall qualitatively under the consideration of a common law."[234] The speculative sense of infinitesimal calculus consists in nothing other than the constitution of a continuity of thought in which the cases *A* and not-*A*, at first distinct, are unified again underneath a higher consideration."[235] They are merged in a qualitative unity that comprises both of them.

Kant "posed the question in a penetrating manner: how is it possible that through something (*A*), something else (*B*) can be posited, which logically, that is, according to the principle of contradiction, is in no way contained in the first? 'Something else,' that is to say, clearly a not-*A*!" such that *A* should become not-*A* and at the same time *B* should arise from not-*B*. In face of this problem, if one does not want to move from *A* to *B* as ultimately unjustifiable given elements, "the only thinkable solution is to discover a conceptual tool capable of overcoming the absolute contrast between position and negation, a tool that is directed from the beginning and principally to the reunification of that which was only separated."[236] Infinitesimal calculus, reducing the difference between *A* and *B* to a difference in quantity, thus seemed to provide the adequate tool for transforming Kant's "*a priori* synthesis" into the concept of "origin": "in the becoming zero of the finite quantity, the qualitative unity of the law remains, as the qualitative characteristic of 'magnitude' … the third must be the quality, it is the overcoming … of the continuous proceeding of every discrete through the return to the *origin* of every discrete in the continuum that is posited as fundamental."[237]

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[230] Paulsen. *Das Problem der Empfindung*, 293.

[231] Natorp. *Die logischen Grundlagen der exakten Wissenschaften*, 215.

[232] Natorp. "Quantität und Qualität in Begriff, Urteil und gegenständlicher Erkenntnis," 155.

[233] Natorp. *Die logischen Grundlagen der exakten Wissenschaften*, 217.

[234] Ibid.

[235] Ibid., 218.

[236] Natorp. "Quantität und Qualität in Begriff, Urteil und gegenständlicher Erkenntnis," 138.

[237] Natorp, Paul. "Zu Cohens Logik." In *Cohen und Natorp*. ed. Helmut Holzhey, 2:6–40. (Basel: Schwabe, 1986), 53.

With these words from a celebrated review of Cohen's *Logic*, which remained unpublished because Cohen claimed he was misconstrued, Natorp puts forward a conception of the "infinitesimal method" that is actually very far from Cohen's. The problem that Cohen poses in his interpretation of the Anticipations of Perception; the problem of passing from the purely comparative nature of extended magnitude to its foundation, from extension to the qualitatively determinate "what" that is extended, seems to assume a completely new form. Thus, *dx* is no longer "a new type of magnitude" that does not have extension, but rather "intensively" possesses the capacity to generate a finite magnitude *x*. The significance of "differential" should now be sought in the fact that "*dx*, which is zero in respect to its *extensive quantity*, is perfectly determined, in respect to the concept, by all the *relations* that define the *x*" because the "removal [Aufheben] of the *quantity* allows the totality of *qualitative* relations to subsist" (CGW 1:158; emphasis mine). The problem of the transition from *dx* to *x*, which Cohen insists on in the *Logik*, as Natorp notes, always presupposes "the functional connection with a possible *y*."[238] The idea of *differential* understood as "intensive reality" is replaced by the concept of *differential quotient* as the typical example of a relation that has a meaning independent form the terms it connects.

The reference to a "Kantian" critique of the "substantialism" of traditional metaphysics seems to appear in two different forms: "continuity," Cassirer already observes in *Leibniz' System*, "which used to signify the *origin* [Ursprung] of a variable magnitude considered principally in methodological isolation, now refers explicitly to the *reciprocal dependence* of variable magnitudes" (CGW 1:215, emphasis mine). The "logic of the origin" is transformed into "a logic of concepts of relation," the problem of the intensive, of the concentration in one point of the *tendency to produce* magnitudes, is replaced by the problem of "law" or of "function,"[239] implying the idea of the *reciprocal relation* between magnitudes that is conserved even if the magnitude has *vanished*.

Dimitri Gawronsky's *Das Urteil der Realität und seine mathematischen Voraussetzungen* (The Judgment of Reality and its Mathematical Premises), intended as a sort of update of the *Logik der reinen Erkenntnis* in light of developments in eighteenth-century mathematics, may be an effective example of this difference in perspective: "even the concept of function is not sufficiently primordial," writes Gawronsky, interpreting Cohen's thought, "even it presupposes magnitudes that are connected in a functional way, without being able to bejustified [legitimieren] through the demonstration of their legitimate [rechtmäßigen] origin."[240] Through the formula *datis ordinatis, etiam quaesita sunt ordinata*, Leibniz aims to show how, in the passing of both of the successions at the limit, their reciprocal relation

[238] Natorp. "Quantität und Qualität in Begriff, Urteil und gegenständlicher Erkenntnis," 138.

[239] Ferrari. *Il giovane Cassirer e la scuola di Marburgo*, 215.

[240] Gawronsky, Dimitri. *Das Urteil der Realität und seine mathematischen Voraussetzungen* (Marburg: Dissertation, 1910).

is conserved,[241] which, however, necessarily presupposes the continuity of the magnitudes that are put in relation. In contrast, Newton, resorting to the idea of magnitudes for continuous motion, "still does not mention the concept of function," writes Gawronsky. For Newton, the problem is not primarily "the connection between magnitudes … but rather the connection of the magnitude with its productive source [Erzeugungsquelle]."[242] In this way, according to Gawronsky, Newton "separated the problem of the differential from that of the differential quotient"[243]. Leibniz, in contrast, relying on "Pascal's concept of the characteristic triangle … bases his deduction of differential calculus on the concept of function and of differential quotient and, for this reason, the concept of differential is not brought to full clarification."[244]

4.7 Concept of Thing and Concept of Function

The preceding reconstruction of the Marburg school's internal debate is obviously too simplistic to account for the subtle nuances that distinguish the positions of the members of the school. My sole aim was to emphasize the themes and problems that recur in the detailed Marbug discussion concerning the Anticipations of Perception and that, in a radically different context, have played a central role in the entire history of German idealism. In particular, what materializes again and again is the demand to fix through "thought" the determinacy of "quality" that Kant had left to the givenness of sensible "intuition."

In identifying intensive magnitude and differential, Cohen sought to bridge the gap between reality (whose qualitative unity that distinguishes it from another reality could only be given *a posteriori*) and degree (which is the authentic *a priori* determination); between quality and its quantity. Using the concept of differential as intensive magnitude, he "connects the *a priori* of the pure *quale* with that of intensity in a [unique] conceptual function."[245] From this point of view, reality does not only *have* an intensive magnitude, but also *is* an intensive magnitude, is an "intensive infinitesimal reality" (see CW 5:1:144).[246]

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[241] Ibid., 49.

[242] Ibid., 82.

[243] Ibid., 83.

[244] Ibid., 91.

[245] Holzhey. "Das philosophische Realitätsproblem: Zu Kants Unterscheidung von Realität und Wirklichkeit." 98. According to Holzhey, this forms the basis for establishing an antithesis between the two (see note 282).

[246] The "Axioms of Intuition" (*Axiomen der Anschauung*) establish that "[a]ll intuitions *are* extensive magnitudes" (B202*;* while the Anticipations of Perception make the more limited claim that "[i]n all appearances the real, which is an object of the sensation, *has* intensive magnitude" (B207; emphasis mine), indicating that the quality of the real and the degree that it possesses are two distinct things, the first only determinable *a posteriori* and the second *a priori*.

In Cassirer and Natorp's thought, the same objective of arriving at an objective "conceptual" definition of the "qualitative" determinations that Kant leaves to "intuition" seems to be achieved via a different route. The concepts of "relation," of "law," or of "function" are shown to be capable of precisely expressing and defining the connection between quantity and quality that Kant did not fully grasp: "the fundamental and truly ultimate concept of mathematics and of all rigorous thought in general," writes Natorp, "is relation … the *relata* are placed for the first time in the relation alone."[247] Only by adopting this premise can one aspire "to a mathematics of quality, as Leibniz demanded … [W]hen mathematics reflects on its own logical foundation, which does not reside solely in quantity, the logic of quality necessarily receives a mathematical form as well."[248] Analogously, Cassirer argues that, in this prevailing of the relation over the terms that it connects, mathematics is revealed not as "the general science of magnitude but of form, not the science of quantity but of quality" (CGW 6:98).[249]

Infinitesimal calculus seems to be the first and most fundamental historical expression of this conception. It demonstrates how magnitude itself can be annihilated, allowing the relation as such to emerge in its purity and independence; that is, the "qualitative" dimension of magnitude. In this way, the "infinitesimal procedure not only contains a method of calculus, but also the general method … for expressing "true qualities" in a rigorous and lawful manner,"[250] the way to mediate between "subjective and objective quality" and thus to determine, as demanded by the Anticipations of Perception, "the quality that 'corresponds to sensation in the object.'"[251]

However, the diverse interpretations of infinitesimal calculus that have emerged in the preceding pages seem to lead to a profoundly different gnoseological perspective. While Cohen's thought is, not coincidentally, centered on the rethinking of Kantianism itself concerning the category of "quality," Cassirer and Natorp rely on the category of "relation" instead. For them, the need to reform critical philosophy should not be directed at the Anticipations of Perception, but rather at the Analogies of Experience. While for Cohen, "infinitesimal reality is the presupposition of the concept of law, which is indicated by the function as a new term" (CW 6:277), the concepts of law or function now seem to serve as the premise for the definition of "that which Kant refers to by the term reality … that which distinguishes a thing from a thing, something from something; its 'quality.'"[252]

The common goal is still the overcoming of Kant's opposition between "concept" and "intuition." However, this objective seems to be arrived at via a substantially

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[247] Natorp, Paul. "Die erkenntniskritischen Grundlagen der Mathematik," *Unterrichtsblätter für Mathematik und Naturwissenschaft* 8 (1902): 2–3, 2.

[248] Natorp, Paul. *Platos Ideenlehre. Eine Einführung in den Idealismus* (Leipzig: Meiner, 1921), 439.

[249] English translation from Cassirer. *Substance and Function, and Einstein's Theory of Relativity*, 90.

[250] Natorp. *Die logischen Grundlagen der exakten Wissenschaften*, 21.

[251] Natorp. "Quantität und Qualität in Begriff, Urteil und gegenständlicher Erkenntnis," 152.

[252] Ibid., 151.

different route than Cohen's. The fundamental problem is no longer that of making "reality" emerge from its "negation," understood as its *origin*, but rather to arrive at the extreme consequences of the *relational* nature that the categories of "reality" and "negation" come to assume in Kant's thought. Cohen insists on the idea that "negation" is not merely an infinitely small "reality," a theme that Kant at least alludes to. For Natorp and Cassirer, in contrast, the relation between positive and negative magnitudes, which is elaborated in so many directions in the history of post-Kantian philosophy, seems to assume an exemplary value again.

Already in *Quantität und Qualität*, Natorp emphasizes that, while traditional logic relied on the radical opposition between the identical and the different, this opposition "was to be relativized in an opposition of directions (a relation of more or less)."[253] The relation between reality and negation should not be thought on the model of an opposition between *concepts* (*A* and not-*A*), between which there is no third, but rather on the basis of the opposition between the *directions* $+A$ and $-A$ that are defined as such in respect to an intermediate point $=0$: "Kant," writes Natorp, "in his valuable pre-critical writing *Versuch, die negativen Größen in eine Weltweisheit einzuführen* [sic], used the expression 'real opposition' [Realopposition] to this end. Characteristic of opposition between 'contraries' [Conträren Gegensatz] is not the exclusion of a third or of a middle, but rather its requirement, such is the contrariety of the opposition of directions. The middle between + and − is zero; just as the middle between movement in one direction and that in the opposite direction is rest."[254]

Precisely this traditional doctrine, founded on the relation between the "concept" and its "marks," that should be replaced by a new logical conception,based on the relation between the "series" and its "members." While each concept is distinct from others on the basis of determinate *characteristics*, each member of a series is defined by the *position* that it assumes in respect to an initial term:[255] "in the opposition of more or less" one should no longer see an "opposition between things" whose *identity* or *difference* can be established, but rather "an opposition of mere relations"[256] from which a *difference in direction* can be defined. "Things" are

[253] Natorp. *Die logischen Grundlagen der exakten Wissenschaften*, 245.

[254] Natorp. "Quantität und Qualität in Begriff, Urteil und gegenständlicher Erkenntnis," 29. Like Trendelenburg, Natorp also seems at first sight to superimpose Aristotle's concept of "contrariety" (ἐνατίωσις) onto Kant's concept of "real opposition." However, he proceeds to demonstrate the profound difference between the Aristotelian and the Kantian point of view. Certainly, Aristotle's work already makes an attempt to admit a "quantification" of qualitative differences, to admit that "things which differ can differ from one another in a greater or lesser degree [πλεῖον καὶ ἔλαττον]" Aristotle. *Metaphysics, vol. 4*, 10: 1055a5. However, Aristotle understands contrariety as "maximum difference [μεγίστε διαφορά]» Aristotle. *Metaphysics, vol. 4*, 10: 1055a5. See Natorp. "Quantität und Qualität in Begriff, Urteil und gegenständlicher Erkenntnis," 152. In contrast, Kant's use to positive and negative magnitudes seems to concede the possibility of going to infinity in both directions, without there being a maximum or a minimum.

[255] See also Natorp. "Die erkenntniskritischen Grundlagen der Mathematik."

[256] Natorp. *Die logischen Grundlagen der exakten Wissenschaften*, 245.

identical and are thus the same thing or they are diverse and incomparable. In contrast, the same identical relation can be followed in two different directions, "from 0 to 1 or from 1 to 0."[257]

This conception seems to overturn traditional logic completely. Whereas the "third" between two contradictories is the destruction of any determinate logical content, "the arithmetical concept of zero [Null] cannot be that of logical nothing … The difference $1-1=0$ certainly is not 'null and senseless' [null und nichtig] in the sense of logical non-existence, but rather it exists with just as much certainty [so sicher] as the difference $<>$, for which it represents the limit."[258] Clearly, "zero represents the common point of departure of the plus series and the minus series [Plus- und Minusreihe]. It represents the common term of comparison [Vergleichungsgrundlage] for the more and the less" from which "one posits the definition (totally arbitrary!) that one is >0 and the other <0, so that these are positive and negative numbers."[259]

The distinction between positive and negative numbers is thus an effective example of how "mathematical concepts are function concepts and not thing concepts."[260] If, as Cassirer notes, "a *negative substance*, which would be at once being and not-being, would be a *contradictio in adjecto*; a *negative relation* is only the necessary logical correlate of the concept of relation in general, for every relation of *A* to *B* can also be represented and expressed as a relation of *B* to *A* … The positive and negative numbers ($+a$, $-a$) now appear merely as expressions of progress [Fortgang] in these two directions [Richtungen] of the relation" (CGW 6:58; emphasis mine).[261]

The very possibility of thinking "negative numbers" is thus the simplest expression of the insufficiency of traditional logic founded on *generic concepts* (*Gattungsbegriffe*) and of the need to construct a logic founded on *concepts of relation* (*Relationsbegriffe*); the need to substitute the procedure of "abstraction" with that of the "formation of series." Each element is indeed perfectly defined in respect to all the others, but not through any *absolute property* that makes it "different" from these, but only through its *relative position*: "relativity is the fundamental characteristic of number … the absolute position is provisional, the relative position is definitive."[262] The elements in the series are completely *indiscernible* as far as their "internal determinations" are concerned, but at the same time characterized as

[257] Ibid., 248.

[258] Ibid., 121.

[259] Ibid., 141.

[260] Ibid., 144.

[261] English translation from Cassirer. *Substance and Function, and Einstein's Theory of Relativity*, 56.

[262] Natorp, Paul. "Nombres, temps et espace dans leurs rapports avec les fonctions primitives de la pensée." In *Bibliothèque du Congres international de la Philosophie, vol. 1, Philosophie générale et Métaphysique* (Paris: Colin, 1900), 360.

different through their "reciprocal relations." They possess "no specific character [besitzt keinerlei spezifisch-inhaltliche Merkmale]" (CGW 6:63)[263] because their essence "is completely expressed in their positions [Stellenwert]" (CGW 6:40).[264]

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Kant's distinction between logical and real opposition seems to call for a reformation of traditional logic's "theory of the formation of concepts" itself in the direction of a *logic of relations*: "Thought" is no longer distinct from "intuition" in the same way that the opposition between "concepts" is irreducible to the opposition between "opposite directions." Rather, thought itself is nothing other than "an establishing of relations,"[265] which no longer proceeds on the "logic of the generic concept [Gattungsbegriffs]," but on the "logic of the mathematical concept of function [Funktionsbegriffs]" (CGW 6:20).[266]

As far as this position may be from Romantic idealism and its use of the opposition between positive and negative magnitudes, it is interesting to note that, in Natorp's later works, precisely these more ardently speculative elements return; those elements which I have shown to be linked to the distinction between real and logical opposition: "Kant," writes Natorp in the *Philosophische Systematik*, "was driven" to the concept of synthesis, "as it seems, firstly through the observation that there is a form of opposition that has nothing to do with logical contradiction. He calls it real opposition [Realopposition]; it is properly the opposition [Gegensatz] of directions, of the + and of the −, as well as the plus and minus direction [Plus und Minusrichtung] of movement: the movements from *A* to *B* and from *B* to *A* do not exclude each other in every sense, but rather they reciprocally need each other." Clearly, it is not "a contradiction the fact that, in the zero-point [Nullpunkt] (and, in general, in every point considered as a transition [Durchgangspunkt]), the plus and minus exist together [zusammenbestehen]."[267] Here, more is involved than the reciprocal exclusion of two opposites because "in the plus and minus, the opposition [Gegensatz] is" indeed "conflictuality [Gegenseitigkeit]," but "in the conflictuality … the most intimate community"[268] is expressed.

As soon as the opposition between reality and negation is reduced to a merely relative difference, Kant's concept itself of a synthesis of a given multiplicity also seems to be overcome: "for this purpose," Natorp proceeds, "'synthesis' is not an adequate expression. It seems to assume duality and being-outside-the-other, mere being-together even when simultaneous. However, it is something much more than a mere being-together [zusammenstehen] and existing-together [zusammenbestehen]; it is a true coincidence [Koinzidenz], a coinciding-together-in-one [In-eins-zusammenfallen]; not

[263] Ibid., 63.

[264] Ibid., 39.

[265] Natorp. *Die logischen Grundlagen der exakten Wissenschaften*, 99.

[266] English translation from Cassirer. *Substance and Function, and Einstein's Theory of Relativity*, 21.

[267] Natorp, Paul. *Philosophische Systematik*. ed. Hinrich Knittermayer (Hamburg: Meiner, 1958), 309.

[268] Ibid., 310.

merely an artificial counterbalancing." Rather "it is a return into the continuity of everything [Allkontinuität]":[269] "in fact, Kant's synthetic unity (which can be noted in many points even if it is never said openly) is complete [durchaus] unity of continuity."[270]

The concept of origin itself acquires a meaning that is now very far from that of Cohen's. The origin is no longer the "nothing" from which "something" must be *produced* as motion is generated from rest. Rather, it appears as the point where the difference between "nothing" and "something" has *vanished*, just as the quantitative difference between rest and motion vanishes in the point of transition from one to the other. In the *Philosophische Systematik*, the origin is thus transformed into that indeterminate element, which Natorp defines with very significant metaphors, as the "point of indifference,"[271] the "hovering [Schwebe]" between being and nothingness.[272] Cohen tried to grasp this tension between nothing and something through the metaphor of the "production" of something from nothing. Natorp, in contrast, seems to try and annihilate this tension in the "third" that overarches both in the "point in which these," namely, nothing and something, "encounter each other [aufeinandertreffen]."[273] Natorp's late work seems to consciously mingle Cohen's "origin" (*Ursprung*) with Hegel's "beginning," (*Anfang*) that is, with the idea of a nothing that is no longer only nothing, but not yet something either. Not coincidentally, Natorp refers to this "origin" by the expression, $A - A = 0$, which has been used frequently in the present work.[274]

4.8 Ontological Interpretations of Kant and the Crisis of the Neo-Kantian Hermeneutic Paradigm

Natorp's last writings,[275] which were edited around the middle of the 1920s, are an effective testimony to how the philosophical climate was completely different from that in which the neo-Kantian movement took form.[276] For the late Natorp, who was also one of the most authoritative protagonists in the neo-Kantian movement, the

[269] Ibid., 311.

[270] Ibid., 108.

[271] See Ibid., 31.

[272] Ibid. See Wolzogen, Christof, von. *Die autonome Relation. Zum Problem der Beziehung im Spätwerk Paul Natorps* (Würzburg: Königshausen und Neumann, 1984), 121–.

[273] Wolzogen. *Die autonome Relation. Zum Problem der Beziehung im Spätwerk Paul Natorps*, 73.

[274] Ibid., 66.

[275] See Gigliotti. *Avventure e disavventure del trascendentale: studio su Cohen e Natorp*, 270- and Stoltzenberg, Jürgen. *Ursprung und System. Probleme der Begründung systematischer Philosophie im Werk Hermann Cohens, Paul Natorps und beim frühen Martin Heidegger* (Göttingen: Vandenhoeck und Ruprecht, 1995), 188–255.

[276] See Wetz, Franz-Josef. "Die Überwindung des Marburger Neukantianismus in der Spätphilosophie Paul Natorps," *Zeitschrift für philosophische Forschung* 47 (1993): 75–92.

"fact of science" no longer represents the most fundamental problem of philosophy,[277] but rather the much more primordial "fact" that something *is* in general.[278] Moreover, this change of perspective is not only manifested in the area of true and proper theoretical debate (for instance, the work of Heidegger, who was deeply connected to Natorp for more than academic reasons),[279] but also in the more restricted field of Kantian interpretation where the neo-Kantian hermeneutic paradigm, essentially based on the reduction of critical philosophy to "theory of knowledge," seems ever more exhausted and incapable of grasping all the facets of Kant's philosophy.

From this point of view, 1924 is traditionally considered an "epochal year"[280] in the history of Kantian interpretation. In issue 29 of *Kant Studien*, Heinz Heimsoeth and Nicolai Hartmann, both from the Marburg school,[281] published two fundamental articles, respectively: "Metaphysische Motive in der Ausbildung des kritischen Idealismus" (Metaphysical Motives in the Construction of Critical Idealism)[282] and "Diesseits von Idealismus und Realismus. Ein Beitrag zur Scheidung des Geschichtlichen und Übergeschichtlichen in der Kantischen Philosophie" (From this Side of Idealism and Realism: An Article on the Separation between the Historical and the Trans-Historical in Kantian Philosophy).[283] Both writers attempt to clarify, even if in profoundly different ways, a variety of themes that undeniably form a relevant part of Kant's thought, but that are badly adapted to the idea of "*Kant als Erkenntnistheoretiker*" (Kant as Theorist of Knowledge).[284] Instead, these themes lead to the image of "*Kant als Metaphysiker*" (Kant as Metaphysician), not coincidentally the title of a celebrated monograph by Max Wundt from the same year.[285]

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[277] Natorp, Paul. *Vorlesungen über praktische Philosophie* (Erlangen: Verlag der philosophischen Akademie, 1925), 5.

[278] See Ferrari. *Ernst Cassirer. Dalla scuola di Marburgo alla filosofia della cultura*, 274.

[279] Natorp himself invited Heidegger to be a professor in Marburg in 1923. See Sheehan, Thomas, "Heidegger e il suo corso sulla 'Fenomenologia della religione' (1920–1921)," in *Filosofia* (1980), 3:432. On the relationship between Natorp and Heidegger from a philosophical point of view, see Ibid.

[280] Wagner, Hans. "Zur Kantinterpretation der Gegenwart. Rudolph Zocher und Heinz Heimsoeth," *Kant Studien*, no. 53 (1961/2): 235–54, 246.

[281] See Hartmann, Frida and Renate Heimsoeth, eds., *Nicolai Hartmann und Heinz Heimsoeth im Briefwechsel* (Bonn: Bouvier, 1978).

[282] Heimsoeth, Heinz. "Metaphysische Motive in der Ausbildung des kritischen Idealismus," *Kant Studien* 29 (1924): 121–59.

[283] Hartmann, Nicolai. "Diesseits von Idealismus und Realismus: Ein Beitrag zur Scheidung des Geschichtlichen und Übergeschichtlichen in der Kantischen Philosophie," Ibid.29 (1924): 160–206.

[284] Holzhey, Helmut. "Zu Natorps Kantauffassung." In *Materialien zur Neukantianismus-Diskussion*. ed. Hans Ludwig Ollig, 134–49. (Darmstadt: Wiss. Buchges., 1987), 135–.

[285] Wundt, Max. *Kant als Metaphysiker. Ein Beitrag zur Geschichte der deutschen Philosophie im 18 Jahrhundert* (Stuttgart: Enke, 1924).

As Gottfried Martin writes, in one of the most authoritative and balanced ontological interpretations of Kant, "it is the lasting merit of the Neo-Kantians to have shown the importance of the natural sciences and especially of physics for Kant. The *Critique of Pure Reason* is certainly a theory of physical science; it is certainly, as is shown at length in Cohen's presentation of it, a theory of experience"[286] According to Martin, however, one must equally admit that Kant's philosophy "is at least to the same degree an ontology."[287] Even if "the Marburg school's interpretation demonstrated once and for all the significance of mathematical and scientific thought in understanding Kant,"[288] the credit goes to the "ontological interpretation of Kant" for having shown that "Kant's philosophy is not exhausted in theory of knowledge."[289]

Regarding the subject of my investigation, the principle of the Anticipations of Perception, completely different research directions begin to emerge in this area as well. Cohen's writings certainly had a profound influence on the history of interpretation of the Anticipations of Perception, highlighting the importance of a subject for the first time that, despite attracting the attention of all the great thinkers of the post-Kantian era, was never made the object of a systematic analysis. From this point of view, Cohen's interpretation represents an unavoidable point of reference for anyone today who wants to confront the subject of the Anticipations of Perception.[290]

Even Heimsoeth, who had distanced himself from his roots in Marburg school neo-Kantianism, still openly recognized the fundamental contribution of "Hermann Cohen's interpretive interest," which sought "to clarify Kant's category of reality (placed under the heading of quality), and his schematization in the direction of the foundation for mathematical natural science."[291] However, Heimsoeth also claims that this interpretive approach, as effective as it may be for comprehending individual aspects of Kant's text, completely leaves in the dark and at least equally important dimension of the problem: if one approaches the problem "from the point of view of mathematical natural science," he writes, "and of philosophy of nature in general," perhaps one can successfully grasp the meaning "of the concepts of quantity, causality, and reciprocal action." Nevertheless, any comprehension of the category of "'reality' (to the extent that one truly understands what is meant by this category)" is still precluded. The same applies to the categories "of negation and of

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[286] English translation from Martin, Gottfried. *Kant's Metaphysics and Theory of Science*, tr. Peter Gray Lucas (Manchester: Manchester University Press, 1955), 42.

[287] Ibid.

[288] Martin, Gottfried. "Die deutsche ontologische Kantinterpretation." In *Gesammelte Abhandlungen*. (Cologne: Kölner Universitätsverlag, 1961), 1:108.

[289] Ibid.

[290] See Moretto. *Dottrina delle grandezze e filosofia trascendentale in Kant*, 350n–.

[291] Heimsoeth, Heinz. *Studien zur Philosophie Immanuel Kants*. 2 vols (Bonn: Bouvier, 1970), 2:50.

limitation" as well.[292] According to Heimsoeth, "of all the groups of categories in Kant's table …. those that fall under the heading of quality" refer back, "in the most immediate manner, to their origin in the ontological tradition."[293]

Precisely this particular "ontological" and "metaphysical" point of view, which is most clearly manifested in the concept of reality, negation, and limitation, was intentionally left in the dark in neo-Kantian interpretations. According to Heimsoeth, however, "alongside and before the gnoseological foundation [erkenntnistheoretische Begründung]," favored by neo-Kantianism, Kant's philosophy exhibits "compelling metaphysical motives."[294] Concerning the category of quality in particular, Heimsoeth argues that studying the relation between Kant and "Leibnizian–Wolffian metaphysics" is indispensable for understanding Kant's discourse: "this situation," writes Heimsoeth, "was clarified above all in Anneliese Maier's excellent monograph, which is highly instructive precisely in respect to Kant's relationships or conflicts [Zusammenhänge oder Auseinandersetzungen] with the scholastic tradition of his time."[295] The principle value of *Kants Qualitätskategorien*, the study by Anneliese Maier that Heimsoeth alludes to, is its clarification of the significance of "Baumgarten and Meier's books, which constitute the basis of Kant's lectures,"[296] for understanding the category of reality and the Anticipations of Perception. Maier studies the sources of Baumgarten and Meier's books in detail and successfully addresses an aspect of the problem that, as obvious as it may appear today, was never analyzed by Cohen and the Marburg school who concentrated on its physical-mathematical implications instead.

In contrast, the "possible positive-scientific consequences of the principle" are excluded from the work of Anneliese Maier who explicitly concentrates on "its logical-categorical presuppositions."[297] Thus, the principle value of her research, as Heimsoeth rightly notes, is her recuperation, through her study of manuals inspired by the dictates of Wolffian metaphysics, the of the "ontological" meaning of the term *realitas*. This ontological meaning has nothing to do with the idea of "reality" as existence that predominated in English empiricism, but rather indicates "a part of the essence or the essence itself."[298] From this point of view, *Kants Qualitätskategorien* can be considered the most important contribution to nineteenth-century criticism on the category of quality. The text places Kant's few pages on the Anticipations of Perception in a completely different perspective than that which emerged from Cohen's *Kant-Bücher* and even more from the discussion

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[292] Ibid., 1:51.

[293] Ibid. See Knittermayer, Hinrich. "Zu Heinz Heimsoeths Kantdeutung," *Kant Studien* 49, no. 2 (1957–1958): 293–311.

[294] Heimsoeth. "Metaphysische Motive in der Ausbildung des kritischen Idealismus," 136.

[295] Heimsoeth. *Studien zur Philosophie Immanuel Kants*, 1:52n118.

[296] Ibid.

[297] Maier, Anneliese. *Kants Qualitätskategorien*, Kant Studien. Ergänzungshefte (Berlin: Metzner, 1930), 59.

[298] Ibid., 11.

that ensued. Cohen was the first to insist on the difference between *Realität* and *Wirklichkeit* and to draw all the theoretical consequences from it. However, one must turn to Kant's historical precedents to throw new light on this distinction.

The recuperation of this dimension of the problem is certainly Maier's fundamental achievement, which all succeeding studies could not ignore. As I have shown, "reality" for Wolffian scholastics indicates "the true positive property as such,"[299] which is expressed through an "affirmative predicate."[300] Kant reappropriates precisely this meaning of the term, recuperating the "ontological" meaning that the word *realitas* and the corresponding terms, *negatio* and *limitatio*, had for Wolff and for the Wolffian school. Anneliese Maier successfully highlights Kant's indebtedness to these traditions of thought and also the radical innovation that Kant brought to this debate.

Kant projects "the distinction between phenomenon and noumenon on to the concept of reality as well":[301] "*realitates noumena* refer back to the *ens originarium*, the substrate of all *a priori* possibility and thinkability,"[302] while "*realitas phaenomenon*, the reality that 'is only in our senses'," is … defined as that 'which in appearance corresponds to a perception (sensation)'."[303] According to Kant, for *realitates noumena*, understood as "the truly positive property of things themselves, the principle of old ontology holds; the principle that no contradiction or reciprocal cancellation [Aufhebung] can exist between affirmative predicates."[304] In contrast, "*realitates phaenomena*, phenomenal realities," can "reciprocally cancel each other through their effects."[305] Thus, as I have shown, "for a *realitas noumenon*, there is only a logical opposition"[306] and "any other form of opposition" is inconceivable, whereas "between *realitates phaenomena*, there is a real conflict."[307]

According to Maier, "*realitas noumenon* remained [for Kant] the positive predicate that is truly inherent in things themselves … and that thus represented the true and proper primordial ontological concept of reality,"[308] which Kant opposed to negation understood as non-being or removal. In contrast, "*realitas phaenomenon* becomes the sensible given," the "correlate of sensation," "the object of sensation *in abstracto*, while negation becomes the lack of the same."[309] From this point of view, "negation in appearance," far from being understood as the contradictory of the pure

[299] Ibid.

[300] Ibid.

[301] Ibid., 33.

[302] Ibid., 34.

[303] Maier. *Zwei Grundprobleme der scholastischen Naturphilosophie: Das Problem der intensiven Grösse. Die Impetustheorie*, 33.

[304] Maier. *Kants Qualitätskategorien*, 33.

[305] Ibid.

[306] Ibid.

[307] Ibid., 63.

[308] Ibid., 39.

[309] Ibid., 33.

positivity of being and, as such, as something unthinkable, could itself be considered a reality, a "*realitas evanescens* or a reality with a vanishing degree."[310]

Maier highlights the difference between the old concept of "reality," as that which is positively present to the "understanding" (in as much as it does not contradict itself), and reality understood as that which appears positively to the "senses" (in as much as it exerts a certain influence on it). Maier clarifies the simultaneous indebtedness and novelty of Kant's concept of reality in respect to the traditional one that converged in Wolffian metaphysics. In this way, Maier successfully recuperates an aspect of the problem that Kant explicitly insisted on: distinguishing "the form of intensity from the form of quality. The first, for Kant, is guaranteed a categorical nature; that is, an *a priori* rational nature; and the second, a form of intuition"[311] given *a posteriori.*[312] With the notable exception of Fries, post-Kantian philosophy, and above all Cohen, who more or less consciously adopts a tradition that dates back to Maimon, attempted to bridge this divide and to connect "quality" and "degree" in a single graspable element of "thought" through a speculative interpretation of infinitesimal calculus. Not without ambiguity, "Quality" and "degree" are united in the concept of "differential." Cassirer summarizes the Marburg school point of view in a fair manner: "This analysis is the true *mathesis intensorum*, the mathematics of intensive magnitudes … [the] quality which has its subjective, psychological expression in sensation, is grasped by a pure concept and … the 'real' in appearance achieves its first scientific designation and objectification" (CGW 8:174–).[313]

Kant, in contrast, according to Anneliese Maier, maintained these two aspects as clearly distinct. Kant understood intensity "as a noetic category, as an intellectual function [Verständismässig] of the manifold," affirming that "the primordial representation of quality has the form of an apprehension rather than a conceptual nature."[314] Thus, in sensation, "one can … distinguish a formal-apprehensive grasping of a given quality" on the one hand, and "the subsumption of it [the quality] under the category of intensity" on the other hand.[315] The *Anticipations* "explicitly refer to 'sensation in general'; that is, they express the application of the *category of intensity* [Intensitätskategorie] to quality in general given in a possible sensation in general, without considering their particular empirical character … What remains is the general representation of the mere *quale*, which is not thought as a red or as a weight, for example, but as a *qualitative-apprehensive* moment in general."[316]

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[310] Ibid., 61.

[311] Ibid., 70–.

[312] On the relationship between the two, see Holzhey. "Das philosophische Realitätsproblem: Zu Kants Unterscheidung von Realität und Wirklichkeit." 96–8.

[313] English translation from Cassirer, Ernst. *Kant's Life and Thought* (New Haven, CT: Yale University Press, 1981), 180.

[314] Maier. *Kants Qualitätskategorien*, 61.

[315] Ibid., 67.

[316] Ibid., 63 (emphasis mine).

For this reason, Maier proposed the introduction of a "form of apprehension of quality,"[317] as an *a priori* condition for the givenness of "qualities" in general; a new form to be posited alongside the *a priori* forms of space and time to clearly distinguish it from the *category* of intensity, which represents true and proper conceptual determination.

Though this solution is certainly disputable, Maier's interpretation of the relation between "quality" and "degree," which moves in the opposite direction of most post-Kantian philosophy, deserves credit for returning an aspect of the problem that Kant himself emphasizes, as I have shown, back to the center of attention: what can be "anticipated" *a priori* is not the "quality" of sensation, but only the fact that it has a "degree." *Kants Qualitätskategorien* thus succeeds in clarifying the function that the use of "sensation" performs in the Anticipations of Perception, without reducing this use to a mere "psychological" element. Instead, it projects "sensation" on to the distinction, which assumes an "ontological" connotation, between *realitas phaenomenon* as sensible givenness and *realitas noumenon* as the object of pure intellect.

However, the book does not seem to explain the theoretical implications of the diverse interpretations that the concept of "degree" assumes in the field of the sensible and the intelligible. The use of the concept of *gradus realitas*, which Kant defends in the pages of the "Transcendental Dialectic" dedicated to the subject of *ens realissimus*, which the critical turn hardly touched, seems unchanged to Maier in respect to the manner of understanding "*realitas noumenon* … in the 1970s." Kant does nothing more than posit "alongside it, *realitas phaenomenon*,"[318] without successfully reconciling these two conceptions of *reality*.

Though the investigation of the historical roots of the category of quality in the metaphysics of the Wolffian school is certainly fruitful, it seems to have reached a point where, to use the words of Gerhard Lehmann, "the critical Kant is under threat of disappearing behind the metaphysical Kant."[319] Explicitly arguing against Maier on this point, Heimsoeth says that one should not forget "that the critical revolution … did not merely change the sensible reference [Sinnbezug] of the category" of reality, "but that, on the contrary, it separated its cognitive use, properly speaking, in the field of phenomena, from its intellectual reference [Denkbezug] to the intelligible."[320] In contrast to dogmatic metaphysics, "for the critical point of view," according to Heimsoeth, "it is no longer legitimate to speak of intensive magnitude in the sense of measurability in a dimension of continuity of noumena. The use of the category of quality, restricted to that which can be the object of a possible experience, does not permit any metaphysics based on an infinity of degrees [Stufe] that culminate in the *ens illimitatum*, as dogmatism and especially

[317] Ibid.

[318] Ibid., 42.

[319] Lehmann, Gerhard. "Kritizismus und kritisches Motiv in der Entwicklung der kantischen Philosophie," *Kant Studien* 68 (1956–1957): 29, 29.

[320] Heimsoeth. *Studien zur Philosophie Immanuel Kants*, 1:10.

the Leibnizians understood it."[321] Thus, the traditional concept of *gradus realitatis* is consciously abandoned and not merely juxtaposed to the concept that is referred to in the case of the "schematized" category: the principle of the Anticipations of Perception is construed "as the transcendental principle of quality for possible experience (and not beyond it)"[322] such that, against any confusion with the idea of degree of perfection, "the principle of the Anticipations of Perception … only concerns degrees of intensity of physical *realitates*."[323]

While, for the Leibnizians, "realities in general are, as such, *perfections* and the degrees of reality are *eo ipso* degrees of perfection,[324] in contrast, "the identification of *realitas* and *perfectio*, which Spinoza, for example, explicitly formulates … is disputable for Kant."[325] For traditional metaphysics, only "the essential positive determinations of things have the characteristic of *realitates*,"[326] such that "the *negationes metaphysicae* are merely reduced to limitations,"[327] to "removals,"[328] to something that is "lacking" (and thus is not) in respect to the fullness of the "*ens realissimus*." Reducing the legitimate use of the concept of reality and of degree to the phenomenal realm, Kant can regard the difference between positive and negative, between reality and negation, as a merely "comparative" difference instead, in the same way that the background only appears in respect to that which is placed in the foreground. Thus, *realitates phaenomena* inhabit a world where not only that which is "positive" possesses reality, but "where negative magnitudes are realities in turn,"[329] just as, for example, "force and counterforce (attraction and repulsion) are, in opposition with each other, both *realitates*."[330]

Thus, only "the difference between phenomena and noumena … makes real repugnance in the realm of phenomena possible,"[331] and only from this point of view can one admit the paradox that "the null belongs to the positive realm [zum positiven Sachbestand]."[332] This fundamental achievement of critical philosophy, according to Heimsoeth, shows that, "even if Kant did not put forth an ontology proper, this *terminus ontologicus* of the null must have occupied him extensively."[333] This ontological status of the null even led Kant to admit that "negation can be

[321] Ibid., 2:55.
[322] Ibid., 2:56.
[323] Ibid., 2:128n46.
[324] Ibid., 1:61.
[325] Ibid., 1:62.
[326] Ibid., 1:51.
[327] Ibid., 1:56–.
[328] Ibid., 1:57–.
[329] Ibid., 1:58–.
[330] Ibid., 2:118–.
[331] Knittermayer. "Zu Heinz Heimsoeths Kantdeutung," 300.
[332] Heimsoeth. *Studien zur Philosophie Immanuel Kants*, 1:10.
[333] Ibid.

indicated as a reality,"[334] something that could have appeared nothing less than absurd to pre-critical metaphysics. To this exemplary reconstruction of Heimsoeth, I can only add the confirmation that this rethinking of negation received in the "history of the effects" that this conception had on post-Kantian philosophy. Only in this history does it seem possible to delineate the theoretical core of the reality–negation relation with ever more clarity beyond its particular historical realization.

[334] Knittermayer. "Zu Heinz Heimsoeths Kantdeutung," 300.

Conclusion

These few comments on the ontological interpretations of Kant, besides offering at least a glimpse at twentieth-century interpretations of the Anticipations of Perception, help to retrace the basic themes that were the guiding thread of the present investigation. In the end, the fundamental result is found in the connection between the problem of the degree, which the Anticipations of Perception attribute to the "reality in appearance," and the problem of the opposition between positive and negative magnitude, which Kant focuses on starting from his pre-critical essay *Versuch, den Begriff der negativen Größen in eine Weltweisheit einzuführen* and which, in the "amphiboly" chapter, he identifies as the distinguishing feature of *realitas phaenomenon* in respect to *realitas noumenon*. By clarifying this connection, I have demonstrated how, within transcendental philosophy, the conception of the relation between reality and negation, or rather between being and non-being, which characterized traditional metaphysics, was completely revolutionized.

As I have shown, in Kant's use of the metaphysical concept of reality, he mostly follows the text-book of Baumgarten in which the tradition of medieval and modern metaphysics are reprised and ultimately summarized through the principle, *omnes realitates sunt vere* positiva [all realities are truly positive], from which its correlate, *nulla negatio est realitas* [no negation is reality], immediately follows. Everything that *is*, any being something in general, possesses, as such, "reality": *omni enti quaedam insit realitas* [in any being whatsoever there is reality] , that is, *omne ens est reale* [every being is real], while negation, as absence or lack of reality, is deprived of any ontological consistency; *ens mere negativum, cui nulla inesset realitas, est non ens* [a mere negative being, to which no reality can be attributed, is a not-being].

Kantian philosophy reappropriates the concept of reality and negation from this tradition of thought, but "Kant gives a new critical interpretation to *realitas*, as he does to all the fundamental concepts he takes from traditional metaphysics."[1] Negation, according to Kant, is opposed to reality merely as a negative magnitude is opposed to a positive one: a line drawn toward the west "is" just as much as one drawn toward the east, and one is positive or negative only in respect to the other

[1] Ibid., 214.

even if both should be considered real: "thus $-a$ is something real in itself, but … in connection [Verbindung] with a it is $a - a$, a negation $= 0$" (AA 18:647; Refl. 6324).

Kant moves beyond the absolute and unbridgeable opposition between a and not-a, which admits neither mediations nor compromises and which leads to an unthinkable and inconceivable *nihil negativum* because *praedicatorum contradictoriorum nullum est subiectum, seu, nihil est, et non est (0 = A + non-A)*. Kant replaces this absolute opposition with a mere relative quantitative difference for which it is always possible to think, between $+ a$ and $- a$, an intermediate term which is indifferent in respect to the opposites, a mere *nihil privativum* that does not imply any contradiction: "between two *logice oppositis*, there is no third, but between *realiter oppositis* … there is a third that is indifferent $= 0$" (AA 18:105; Refl. 5164.) that is neither the one nor the other, neither positive nor negative. Negation in phenomena is not the contradictory opposite of reality, but rather is itself a reality, a *realitas evanescens*; it is the vanishing of a mere difference in quantity.

In this way, the traditional idea of *gradus perfectionis*, hierarchically arranging creatures according to their *gradus entis* within "a great chain" that culminates in the *ens realissimus*, *gradum realitatis maximum habens*, is relegated to a realm that is indeed intelligible, but for which no example can be given in any possible experience. In contrast, the legitimate use of the concept of "degree" is limited to the *virtutes* that can be measured (velocity, temperature, etc.) and thus to the sensible qualities of the phenomenal world where "*omne minimum vel maximum est tantum relative tale*" [every minimum or maximum is only relative] (AA 17:290; Refl. 3775), that is, where absolute "maximum" and "minimum" are nothing but *notions deceptrices* since the series of degrees proceeds to infinity in both directions.

The philosophical meaning of this apparently simple conception seems to remain in the dark if one merely considers the few pages in the *Critique of Pure Reason* dedicated to the Anticipations of Perception or, more generally, to the subject of the relation between reality and negation, as in the "Amphiboly" chapter, for example. In order to fully comprehend its theoretical import, Kant's lectures and reflections, despite their indispensability for situating the categories of quality within their historical context, are not as important as the history of post-Kantian philosophy, where Kant's formulation is revealed to be extraordinarily fruitful. Fichte's point of departure is the relation between reality and negation, leading him to compare the not-I to a negative magnitude that is not distinguished from the I by anything essential, but only by "quantity." Schelling, in turn, along with a large part of Romantic *Naturphilosophie*, often resorts to the metaphor of the "magnet," which is nothing other than the speculative expression of the new point of view that Kant introduced: the positive and negative pole, the north and south pole, do not exhibit any intrinsic quality that distinguishes the one from the other. Rather, they are defined only in respect to a point of indifference $= 0$. Schelling's thesis that the "absolute" should be conceived on the basis of this model could have appeared to be nothing more than an abstruse speculation that unacceptably passes the limits with which Kant had circumscribed the realm of the legitimate use of *a priori* concepts. However, that this conception contains in itself the "distinctive feature"

of Kantian-inspired idealism cannot be ignored. It is no coincidence that the "realism" of Herbart, who more than any other thinker opposed the idealistic degeneration of Kantianism during the very years that idealism flourished, is based instead on the explicit and conscious recuperation of the thesis, typical of Leibnizian–Wolffian metaphysics, of the absolute "positivity" of being, which in itself excludes all negation.

On the one hand, the concept of intensive magnitude, which Kant uses to in order to characterize the "real in appearance," acquired an important role in the history of modern natural science, particularly in the form of *gradus velocitatis*. Having its beginning in scholastic investigations on *intensio* and *remissio qualitatum*, and passing through Galileo and Leibniz, the concept seems to have finally found its expression and scientific development in analysis of the infinite. The credit goes above all to the Marburg school of neo-Kantianism, and to Cohen in particular, for having clarified, through a historical investigation of the scientific presuppositions of the Anticipations of Perception, the connection that was completely lost in Romantic idealism between Kant's philosophy and the history of mathematical natural science.

On the other hand, that the concept of "degree," in the form of *gradus realitatis sive perfectionis*, simultaneously represents "an ancient difficulty of ontology"[2] should not be forgotten. From this point of view, Kant's philosophy also enters into and renews a long and well-consolidated tradition from the inside, a tradition that dates back to the Middle Ages and that traverses the entirety of modern philosophy. The great figures of modern thought, Descartes, Spinoza, and above all Leibniz, did not renounce the typical medieval concept of *gradus entis*, but rather made it their own. Nothing negative or imperfect can be, but rather everything, in as much as it is something and not nothing, participates to a varying degree in that positivity and perfection that belongs to every being as such. Taking a position against this tradition of thought, at least in the form that it took in Leibnizian–Wolffian ontology, Kant aims instead to show that negation should not necessarily be understood as removal or absence of being, which excludes and repels reality, but rather as an equally positive reality. For, in phenomena, reality and negation are distinguished only through a merely relative difference in position, such as that which exists between + and −. Clearly and definitively separating these two conceptions of the concept of "degree," Kant shattered the "metaphysical" unity of *ens* and *bonum* (*ens et bonum convertuntur*) that characterized a large part of the history of Western thought; not in order to destroy it, but in order to reconstruct it in an entirely new and much solider form in the "transcendental" unity between theoretical and practical reason.

[2] Martin, Gottfried. "Die metaphysische Probleme der Kritik der reinen Vernunft." In *Gesammelte Abhandlungen*. (Cologne: Kölner Universitätsverlag, 1961), 1:157.

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