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Husserlian Phenomenology A Unifying Interpretation



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Husserlian Phenomenology

A Unifying Interpretation



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Preface

What follows is the culmination of my work on Husserl to date. I first became interested in Husserl around 1991, when I was an undergraduate at UC Berkeley, attending Hubert Dreyfus' well-known lectures on Heidegger. Dreyfus would pace back and forth after a student's question, genuinely thinking about it, sometimes saying things like "Well, maybe you're right, and I've been wrong all these years." I took full advantage of Dreyfus' open attitude toward discussion, pressing my doubts about Heidegger in office hours. Dreyfus would patiently entertain my questions, identify errors in my reasoning, and think through the issues on their own terms. At one point he said, "you sound like a Husserlian." It turned out he was right. I went on to study Husserl with David Woodruff Smith at UC Irvine, who introduced me to Husserl's texts, and to a formal, analytic way of reading them.

Since those early days, my larger goal has been to connect phenomenology with the cognitive sciences. But somewhere along the way, I became independently interested in Husserl-interpretation. After years of reading the primary and secondary literature, I came to believe that a certain core aspect of Husserl's theory, an analysis of "world constitution," of the way we develop our sense of reality over time, had not been articulated as clearly as it could be. Moreover, Husserl presents his account of world constitution in many different ways over the course of his career. In a sense, Husserl's corpus is redundant, and at least one aspect of it—a kind of unifying thread—can be compressed into a smaller, more usable format.

In addition to delineating a relatively compact formal theory of world constitution in Husserl, this work also applies several methods to Husserl-interpretation that have not often been used in that context, including computer analysis of texts, and explicit formalization using dynamical systems theory.

I am grateful to many people for helping me either directly or indirectly with the ideas expressed here. More or less in the order I met them, they are: Hubert Dreyfus, John Searle, Bruce Mangan, Russ McBride, David Woodruff Smith, Martin Schwabb, Amie Thomasson, Rolf Johannson, Ron McIntyre, Jeff Barrett, Mark Deering, Wayne Martin, David Kasmier, Walter Hopp, Dallas Willard, Matthew Lloyd, Michael Shim, Charles Siewert, Burt Hopkins, David Pitt, Richard

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Merced, CA, USA June 2015 Jeffrey Yoshimi

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Chapter 1 Introduction

Husserl is well known for his vast literary output. The archives have thus far produced over 60 volumes of edited manuscripts, correspondence, and other materials, spanning 50 years of Husserl's life and 40,000 pages of research manuscripts [49, 52]. Most of the 40,000 pages are from Husserl's daily meditations, which are tentative, fragmentary, and sometimes inconsistent (Husserl seems to change his mind on several key issues over the course of his career).¹ There is also a complex temporal progression in Husserl's work: at different stages of his life, different issues, influences, and metaphor systems are prominent. In his early work of the 1890s he focuses on logic, mathematics, and language. In the 1900s he develops phenomenology in to a distinct discipline, launches a series of "constitutional" investigations, and moves in a more Kantian, idealist direction. In the 1920s he focuses more on history, intersubjectivity, and "genetic phenomenology." These apparent inconsistencies and changes of emphasis give the impression of a disorganized, fragmentary corpus of work.

However, there is more unity to Husserl's corpus than is immediately apparent. This has been increasingly recognized in the secondary literature, from across the spectrum of interpretive tendencies [52, 80, 102]. I continue in this line of unifying interpretations of Husserl, from a formal, mathematical perspective. I will argue that a great deal of the theoretical apparatus he develops can be understood in terms of a supervenience function linking how we see a thing with how we expect it would look relative to different movements, and two dynamical functions or rules: an expectation

¹Prominent examples are his apparent shift from realism to idealism, and his changing attitude towards the transcendental ego. Both changes are believed to have occurred after the publication of the first edition of the *Logical Investigations* (in 1901). The shift to idealism is controversial (see, e.g. [104]), but the changing status of the ego is marked by Husserl himself, who initially says he was "quite unable to find this ego" but later notes (in the second edition of the *Logical Investigations*, published in 1913), "I have since managed to find it."

function and a knowledge update function (the terms "function" and "rule" will be treated as synonyms). Each of these functions corresponds to a kind of transcendental dynamical law, which says how experience must unfold if a certain type of agent (a "stable cognizer") is to experience a stable world.² For example, given what I know about my house, when I walk in to it I must, according to one of these rules, have specific expectations. Moreover, if the house turns out to be different than I expected it to be, I must be surprised. When this happens, I must update my knowledge such that if I were to walk in to the house a second time immediately after, I would no longer be surprised. We will see that these kinds of simple regularities are at the basis of a surprisingly broad range of Husserlian concepts. I focus on intentionality, constitution, horizon, motivation, and genetic phenomenology, all of which can be explained in terms of the same set of dynamical rules. In this way I shed light on a unified structure running through Husserl's work, and draw connections between different Husserlian concepts. Husserl, on my interpretation, had a basic theory that he struggled to express in various ways at different times in his life, but that had a comparatively simple core structure.

This work is intended as a contribution to Husserl interpretation. However, it has significance beyond this. The formalization of Husserl I develop mirrors several parallel formalizations in the cognitive sciences. It is a dynamical systems reading of Husserl (i.e. one which emphasizes spaces of possibilities and rule-governed evolutions in those spaces) [106, 113], which is broadly consistent with dynamical systems approaches to cognitive science [112]. The terms of the formalism can also be associated in a fine-grained way with specific components of theories that treat the brain as a kind of statistical prediction machine. According to these theories, humans and other animals build up a model of their environment over time by generating predictions and comparing those predictions with what is actually observed [9, 27, 91]. The terms of such an account—prediction based on sensory inputs and bodily movements, learning based on comparisons between predictions and observations, etc.-can be associated with specific concepts in Husserl like adumbration, fulfillment, and laws of passive genesis. This treatment of Husserl also highlights relationships between Husserl and various authors, e.g. J.J. Gibson, Gareth Evans, Adrian Cussins, and Alva Noë, who also emphasize the role of bodily movement in perception. In the conclusion I briefly elaborate on these and other directions for future research.

²Most of the substantive claims that follow can be thought of as consequents of an implicit conditional that begins "given a stable cognizer in a stable environment...." This is explicit in the statement of the two main rules in Chap. 3. The point of this condition is to preclude certain logically possible situations in which the relevant dynamical patterns could be violated. If we allow arbitrarily degenerate environments or forms of cognition, then, as Husserl himself notes, various kinds of "chaos" can result, i.e. processes that are subjective but insufficient to present an agent with a coherent sense of an independently existing reality. I use the phrase "stable cognizer in a stable environment" (or just "stable cognizer") to designate an agent whose experiences over time are coherent enough for a stable world to appear. It is interesting to consider what specific constraints are involved in being a stable cognizer: two specific constraints are discussed in Chap. 3, but there are surely others as well. There is additional discussion in [113], but there is further work to be done understanding how the dynamical rules formalized here are related to the Husserlian/Kantian transcendental project.

In Chap. 2, I motivate the project, focusing on the intuitive concept of a world picture or world model. In Chap. 3, I describe the main formalism of the book, which involves two rules (an expectation rule and a learning rule) and some associated constructs. In Chap. 4, I describe the results of keyword searches relating to the concepts I focus on, showing to what extent they were used at different stages of Husserl's career. In the remainder of the book I unify these concepts by showing how they can all be understood in terms of the basic formalism of Chap. 3. I consider intentionality (Chap. 5), constitution (Chap. 6), horizon (Chap. 7), motivation (Chap. 8) and genetic phenomenology (Chap. 9). In Chap. 10, I consider ways this framework could be extended and used to facilitate interdisciplinary collaborations between Husserlian phenomenology, cognitive science, and other disciplines.

I will use "Hua" as shorthand for "*Husserliana*." I also use the following abbreviations: CM (Hua 1), *Ideas 1* (Hua 3; [37]), *Ideas 2* (Hua 4; [38]), *Ideas 3* (Hua 5), *Crisis* (Hua 6; [34]), *First Phil 1* (Hua 7), *First Phil 2* (Hua 8), *Time* (Hua 10; [39]), APS (Hua 11; [41]), PA (Hua 12), *Intersubjectivity 1* (Hua 13), *Intersubjectivity 2* (Hua 14), *Intersubjectivity 3* (Hua 15), TS or *Thing and Space* (Hua 16; [40]), *Prolegomena* (Hua 18; [36]), LI or *Logical Investigations* or *Investigations* (Hua 19; [36]), *Crisis Beilage* (Hua 29), EJ (*Experience and Judgment*; [35]). Page references following these abbreviations are (in cases where there is a suitable English translation) to the translation listed above in the parentheses.

Chapter 2 The Basic Idea and Other Preliminaries

According to Husserl, conscious experience is overwhelmingly outward. We do not live in a private realm of inner thoughts (though our inner life is an important part of our phenomenology), but in a world or reality that we feel to be outside ourselves. Over time, we build up an increasingly detailed sense of this reality, via a kind of "world model" or "world picture" (these are contentious but vivid metaphors). It is instructive to compare this aspect of Husserlian phenomenology with the introspective psychology of Husserl's time. Researchers in Wundt's and Titchener's labs studied experience by reflecting inwards: they were trained to discriminate colors in a changing after-image, tones in a chord, etc. [75]. They would close their eyes and try to look in or "spect intro" on the fluctuating color of an after-image or the components of a sounding chord. Husserl, by contrast, focuses on "external perception" (*äußere Wahrnehmung*).¹ His point of departure is the "natural attitude" of everyday life. In this attitude, we are naïve realists: we take the existence of the world outside of us for granted. In daily life we do not primarily encounter impressions or feelings; things beyond us are the dominating factor: the places we go, the people we interact with-family members, side-walks, bicycles, computers, coffee cups. As Heidegger emphasized, we are In-der-Welt-sein. In contemporary philosophy of mind this is referred to as the "transparency" of perceptual consciousness: we "normally 'see right through' perceptual states to external objects and do not even notice that we are in perceptual states" [53].²

Even though experience is transparent, and we are normally just "in the world," we can, as phenomenologists, take a kind of external perspective on experience itself. I take this to be one of the points of Husserl's method of phenomenological reduction. When we perform the reduction we put a moment or period of everyday life in

¹See, e.g. APS 39.

²Though even in external perception we arguably have some indirect sense of the relationship between what we see and the position of our eyes, body, etc.; for a review of this issue from a phenomenological perspective, see [97].

"parentheses," placing it on a kind of "phenomenological blackboard" (*Ideas* 1, p. 171). We step outside of consciousness, and the naïve realism of everyday life. Within the parentheses, experience stops being transparent; we regard it *as experience*, as an evolving field of consciousness or flux of streaming experiential data. From this standpoint, we can begin to analyze consciousness, identifying mereological parts within it, associating actual experiences with possible experiences (for example, associating what we do see with what we expect we would see if we moved in various ways), and describing phenomenological laws using Husserl's transcendental and "eidetic" methods.

I supplement Husserl's blackboard analogy with an indulgent set of metaphors, that I purposefully mix in order to develop a rich intuition for the formalisms and laws to come. I will argue that we can think of our omnipresent sense of external reality in terms of a kind of internal model. Think of it as an actual physical model, like the clay models used to prototype car designs, or the scale models used in special effects and miniature war-games. This would be a vast model, a model of our sense of the entire physical universe, though we can constrain things a bit by focusing on the surface of Earth. So, think of a huge scale model of the entire surface of the Earth. We can think of the stream of consciousness as unfolding "on top of" this model. As we have an experience in some region of the world, we "see" some part of this model, and we can think of experience as actually changing the model: sculpting and refining it. When experiences frustrate our expectations, the model is altered to reflect what we see. If, driving to work, we see a fumigation tent on a neighborhood building, we update the model to reflect this.

As we move through the world we "move" through this model, updating and refining it in the places we visit. The model is just underneath the surface of consciousness, persisting as experience unfolds. It is like the riverbed below a river, slowly changing and acquiring sediment as the water flows past. Unlike a riverbed, however, which is usually in contact with the water flowing through it, most of this model is dormant; we only "see" one part of it at any time. When we "leave" that part of the model, it persists and slowly changes. Details are washed away, leaving schematic images. When we visit an unfamiliar area, we visit a part of the model that has never been developed at all. Even in these areas the model exists with some kind of structure, based on our general knowledge about that part of the world. Think of the early stages of a clay model, when the colors and shapes are vague, specifying generic things but with none of the details painted in (Diebenkorn paintings come to mind).³ If I were to get off Interstate 40 in Oklahoma City, I'd have general expectations about what a standard middle-American city looks like, but I would not have any specific expectations about the city's layout. If, however, I moved there, then over time that part of the model would be filled in and refined. I would come to have very specific expectations about the layout of the city; that part of the model

³I have heard it said that figurative paintings like Diebenkorn's capture the "essence" of a place without specifying any of its detail.

would become extremely detailed. In other areas the model would remain sketchy and incomplete. Mixing metaphors once again, we can think of the model as varying in pixel density, with high resolution details in some areas (in those places you can "zoom in" on the model and still see details), and lower resolution in other areas.

In what follows I interpret Husserl as describing rules and structures associated with this kind of developing model of reality. What Husserl does, on my account, is allow the phenomenologist to take a God-like view on a person's model of reality, as if it were flattened and laid out on a huge modeling table. "Ah this is how California was constituted for this person," such a phenomenologist might say, contemplating the model, "notice all the detail around Los Angeles and Merced, but Alaska was sketchy for him..."⁴

These ideas and metaphors illuminate an explanatory dimension of Husserl's phenomenology (cf. APS, p. 631ff). The world model and the formal constructs associated with it explain what our expectations are as we move around in the world, and how those expectations change in time. As we will see, these ideas can be made more precise using philosophical tools like supervenience, and mathematical constructs like dynamical systems theory and probability theory. Supervenience formalizes a kind of dependency relation whereby what we immediately or "immanently" see depends on what we would expect relative to different counterfactual sequences of movements (the metaphorical world model is a fusion of expected views of the world relative to different movements). Dependencies between immanent and counterfactual structures will be a running theme in what follows. We also consider mathematical constructs, in particular probability distributions which describe how surprising different possible experiences would be, were they to occur, and dynamical systems which describe updates to the world model based on experience. The image of an evolving world-model thereby connects our understanding of first-person phenomenological structures with a set of mathematically tractable explanatory constructs, which can in turn be linked with parallel formalisms in other disciplines, as we will see in the concluding chapter.

⁴Perhaps the most detailed existing account of what I call a "world model" is Gurwitsch's account of an "order of existence" [30]. The basic idea in Gurwitsch is to start with a particular experience, e.g. of the entrance to a subway station, and then note that one also has a sense (in the "thematic field" of the focal perception; cf. Sect. 7.1) of all the things near the station. These other things are "relevant" to the subway station. We now shift attention to one of these relevant things, e.g. a nearby storefront. That storefront will have its own thematic field. We can imagine repeating the process until we have completely delineated our sense of the physical world, in the form of a network of possible experiences each of which is relevant to its neighbors in the network. Thus an order of existence is "an indefinitely extended thematic field" [30, p. 381]. Gurwitsch goes on to develop a kind of hierarchy of orders of existence and relationships between them. He treats physical reality as the "one all-encompassing order of existence... the life world of all human beings communicating with each other either directly or indirectly" [30, p. 387]. This world has several "sub-orders," including the "spheres" of family-life and work life. Gurwitsch also describes separate orders of existence corresponding to "worlds of the imagination," domains of mathematical objects, and other "eidetic domains" (thus Gurwitsch considers the same kinds of additional "constitutive domains" I do in Sect. 10.3). In each case the order of existence is thought of as a system of potential experiences connected by relevancy relations. I am sympathetic to Gurwitsch's approach, and think it could be cleaned up and fruitfully synthesized with this one (cf. the approach taken to Gurwitsch in [117]).

In order to make this project tractable, I make several simplifications. Like a scientist beginning with a simple *Drosophila* model, or like Husserl beginning *Thing and Space* by focusing on the simple case of stationary objects perceived via visual images and eye movements, we must begin with a simplified framework in order to understand the basic features of the account, which is complicated even on its own.

First, I focus specifically on our model of the physical world insofar as we directly visually perceive it and explore it via bodily movements. This involves several exclusions and abstractions. By focusing on visual perception I exclude non-visual modalities like smell and touch (except insofar as it figures in what Husserl called "kinesthetic experiences" of moving one's body). By focusing on the physical world I exclude non-physical objects, e.g. fictional objects or abstract objects like numbers. By focusing on direct visual perception I exclude indirect forms of visual access to physical objects, e.g. imagining how an object might look from different angles. By focusing on the sensory appearance of a thing I abstract from other strata of meaning Husserl describes, including "active structures" (more on these shortly) as well as implicit social features that are "intersubjectively constituted" (such that, for example, I see the house as something that could be seen by others). So, when I refer without qualification to perceptions, I mean direct visual perception of physical things. Similarly, when I refer without qualification to (for example) my interpretation of Husserl's account of intentionality, I mean Husserl's account as it applies to direct visual perception of physical things.

Second, I focus on *stationary* physical objects, like houses, chairs, and tables (naturally I am setting aside small-scale, unperceived changes which constantly occur in all physical objects). Changing physical objects—e.g. moving cars, rustling trees, and fires—present special issues that I believe are best analyzed on the basis of an initial analysis of stationary objects (in this I follow Husserl, who only considers "the constitution of objective change" at the very end of *Thing and Space*, after having focused almost entirely on unchanging things).

Third, I do not consider what Husserl calls "active processes" where we explicitly think about things using linguistically structured concepts, e.g. looking at a car and thinking "I better get the brakes checked before the family vacation." Rather, I focus on what Husserl calls "passive processes," i.e. our implicit understandings of things independently of any linguistically structured cognitions about them.⁵ In the car example this would correspond to my simply seeing the car, independently of associated thoughts about its brakes. In practice it may be impossible to completely abstract passive perception from active conceptual understanding, which has been

⁵There is some ambiguity in the active/passive distinction. For Husserl active processes are associated with the voluntary direction of attention (i.e. what is today called "endogenous attention"): "the realm of activity is...a realm of free volitional activity" (2001, p. 283); "all genuine activity is carried out in the scope of attentiveness" (2001, p. 276). However, in practice the examples of active processes Husserl uses tend to be linguistically mediated as well. An example would be looking at a house and then saying, "Ah, that's a California Bungalow, probably built in the 1930s." I specifically bracket the linguistic dimension of the active/passive distinction. I am *not* bracketing attention, and in fact focal visual contents will be an emphasis in what follows. For further discussion see [97, 108].

shown to exert a top-down influence on the way we implicitly perceive things ([70], p. 84). Still, we can think of pure passiveness as a useful limit abstraction for the purposes of analysis. Husserl himself takes this approach: "An object... that does not yet bear any traits that stem from active accomplishments is actually a limit-concept for us, an abstraction..." (APS, p. 288).

Fourth, Husserl focuses most of his efforts on analyzing our experiences of *individual objects* over time (as opposed to the entire field of objects given in experience). However, this kind of individual object perception itself assumes an additional set of structures beyond those considered here, e.g. perceptual laws that determine when a perceived thing segregates itself from a visual surround, and dynamical relations such that successive segregated images are felt to present the same thing from different perspectives. Husserl calls this a "synthesis of identity" or "unity of coinciding" (APS 39), as distinguished from other forms of synthesis (e.g. the synthesis of fulfillment) to be discussed below. Given the nature of the cases I focus on—which involve what I will call "object tracking" sequences of movements—I will have to make some use of the concepts of object individuation and identity over time. However, I will not give a detailed analysis, space being limited.⁶

Finally, note that others have formalized Husserlian phenomenology in the same spirit as I have, though with different emphases. Examples include Marbach [58, 59] (who develops a precise "phenomenological notation" for describing, among other things, iterated intentional modifications, e.g. imagining that one is viewing a picture);

Petitot's work formalizing Husserlian concepts using topology and geometry [73], Boi's work on "Husserlian geometry" [6], Miller's work formalizing Husserl's theory of time consciousness [62], Krysztofiak's work on the formal structure of the noema [47], and Smith and McIntyre's work on intentionality and horizon theory [81]. Other examples are reviewed in [106]. These projects generally have the same overarching goal in mind—making Husserl's ideas more precise, using contemporary formal tools.⁷ I will end by making note of Michael Madary's work, which I discovered as

⁶I will note, however, that it is possible in principle to run the entire analysis presented here without assuming any account of object individuation. This can be done by focusing on our visual understanding of the physical world as a whole: our model of our complete physical reality. For example, a sequence of perceptions of a house can be regarded as a sequence of perceptions of (one part of) the entire physical world. So instead of a story about how our model of the house is built up, we have a story about how one part of our model of the *whole physical world* is built up, and thus questions of object individuation are bracketed. Cf. the discussion of outer horizons as maximal inner horizons in Chap. 7, note 2.

⁷Husserl was in some sense opposed to this kind of project: he is famous for his critique of naturalism and for his critical-historical analysis of the "mathematizing" tendencies of Western science. On the other hand, Husserl was a mathematician by training, and he himself formalized certain phenomenological structures, for example the structure of time-consciousness and some features of the constitution of space. He also seems to leave it open that a "*mathesis* of mental processes"

I was making final revisions to this manuscript. He is engaged in a project similar to this one, emphasizing some of the the same core features of Husserl's phenomenology as I do (especially anticipation and fulfillment), and using them to develop new interpretations of an impressive range of problems in philosophy and cognitive science [54, 55, 56].

⁽Footnote 7 continued)

⁽*Ideas* 1, p. 169) might be possible. I have addressed this apparent tension in [106]. Briefly, my analysis is that Husserl rejects any "naïve" use of non-phenomenological structures like mathematics in phenomenology, *before* they have been phenomenologically grounded. To make uncritical use of mathematics in phenomenology would be to violate the principle of the phenomenological reduction, which is that one must initially bracket all non-phenomenological considerations and simply describe objects as they appear to consciousness. However, once the reduction has been carried out and mathematics has itself been properly grounded in phenomenology, its methods can be legitimately used in phenomenological settings.

Chapter 3 The Formalism

3.1 The Dynamical Systems Framework

I interpret Husserl as a proto-dynamical systems theorist, who focused not only on *actual* conscious experiences, but also on structured sets or spaces of *possible* conscious experiences.¹ When a set is a space its members can be called "points" (Husserl himself refers to "manifolds," which are a special kind of space).² Strictly speaking, a space and the points that comprise it are abstract mathematical entities that are used (in this study) to represent actual and possible conscious experiences and their mereological parts. However, it is convenient to conflate the two, and to refer, for example, to "experiences of fulfillment in [0, 1]" rather than "experiences of fulfillment represented by points in [0, 1]".

The rules I describe in this chapter are defined on the basis of several sets of possible experiences. The most fundamental set is what I will call "C", the set of all possible conscious states.³ I take a conscious state to encompass everything a person is aware of at a point in time (or over a brief duration)—thoughts, feelings, the visual field, smells, itches, pains, etc.—what Aron Gurwitsch called a "field of consciousness" or "total field" and what would today be called a "phenomenal conscious state."⁴ A conscious process unfolding over some interval of time can be

¹I have collected evidence that this is a valid way of interpreting Husserl, and addressed some of the many philosophical questions that arise in this connection, elsewhere [106, 110, 113].

²A space is typically taken to be, minimally, a topological space (a space with a concept of neighborhood defined). I assume the spaces I consider have additional structure as well (e.g. a metrical structure that allows distances to be defined between pairs of points).

³Or all possible conscious states for human beings. Some of the issues associated with the "scope" of C are discussed in [110].

⁴Here again many questions arise, e.g. concerning the extent to which the field of consciousness extends beyond the focus of attention, and the coherence of the concept of an instantaneous conscious state. For further discussion see [117].

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represented by a finite sequence of points in C which can be indexed by integers $1, \ldots, n$.⁵

Many of the phenomenological structures Husserl describes can be understood in terms of *subspaces* of *C*, e.g. the space of visual experiences, experiences of bodily movement, thoughts, etc. I take a phenomenological subspace of *C* to be, minimally, a set of mereological parts of states in *C*, subject to the constraint that a person at a time can only instantiate a single state in such a subspace at a time. For example, *V* is a subspace of *C* containing possible "visual images" (roughly, the visual sensory experience at the focus of attention). We assume a person at a time only has one focal visual experience at a time, which corresponds to a unique point in *V*.⁶ Each point in *V* is a mereological part of multiple conscious states in *C*, insofar I can hold my visual experience $v \in V$ constant while varying my thoughts, emotions, etc. The main subspaces considered here are shown schematically in Fig. 3.1, where the dotted lines show superspaces pointing to subspaces. Notice that a subspace can have further subspaces, and that the subspace relation is transitive, so that *V* is a subspace of *C* in Fig. 3.1. Also note that for any subspace we can assume there is a null point corresponding to absence of experience of that type.

A path or other set of points in a space can be *projected* to its subspaces. I take a projection p from space X to subspace Y to be a function that takes a point in Xas input and returns a point in Y as output, where if p(x) = y, then y is the unique point in Y that corresponds to a mereological part of x.⁷ For example a projection from my total field of consciousness right now (a point in C) to V associates my total experience now with the visual image at the focus of attention, which is a point in V(see Figs. 3.1 and 3.2). A path in C representing my evolving field of consciousness can also be projected (by a separate projection function) to a path in B representing my specifically bodily experience during that time. Similarly for any projection from a superspace to a phenomenological subspace.

The spaces I consider are C, V^+ , V, B, F, and K. All of these but K are shown in Fig. 3.1 (K is not a subspace of C). Structures relating these spaces can be used to

⁵This is a convenient approximation of a continuous-time path (a function from an interval of the real numbers, representing an interval of time, to *C*). For similar reasons of convenience variables with integer-indices are used throughout (even if the sets indexed in this way are plausibly thought of as being uncountably infinite).

 $^{^{6}}$ We must either stipulate that there is just one focus of attention, or if we allow split attention, consider the mereological sum of all contents in a split focus of attention to be a point in *V*. Similar qualifications must be in place to ensure uniqueness for any phenomenological subspace.

⁷A projection map is typically many-to-one. Many points in *C* can be projected to the same point in *V*, since many different overall conscious states can involve the same visual experience, as noted above. On the other hand the projection map is *not* one-to-many: each point in a superspace projects to a *unique* point in a given subspace, as already noted. So in some sense *V* is "smaller" than *C*. Similarly for other sensory modalities, cognitions of various kinds, and perhaps emotions and other features of experience. There are many interesting questions here about how many ways there are to parse *C* into smaller subspaces like this, what the smallest subspaces are of *C*, whether subspaces can be meaningfully said to have shapes or volumes, etc. Also note that the projection map is not to be confused with the superspace-to-subspace relationship. One superspace can have many subspaces (the superspace-to-subspace relationship is one-to-many), even if any particular projection map from a superspace to a subspace is *not* one-to-many.



Fig. 3.1 C and some of its subspaces. Dotted arrows point from superspaces to subspaces



Fig. 3.2 A sequence of points in V corresponding to visual images of a house from different angles. The area of the *circle* and the relative positions of the *dots* are not meaningful. Similarly for subsequent figures

formalize a great deal of Husserlian phenomenology, for the special case of visually perceived physical objects.

Let us consider these spaces in turn.

V is a set of possible *visual images* or visual sensory impressions, while V^+ is a set of possible *visual experiences of objects* (in both cases we are talking about visual experiences at the focus of attention). The intuitive contrast is between the immediately present sensory part of a visual experience of an object, and the full visual experience of *that object*. Consider, for example, the contrast between my

seeing a house as a regular house versus seeing it as the front of a movie façade. These correspond to two distinct points in V^+ , two distinct visual experiences of distinct objects. However, they do seem to have a part in common, a raw sensory presentation, which is what I am calling a "visual image."⁸ Visual images are a kind of abstraction that only becomes evident via the kind of comparative process described here. As noted in Chap. 2, it is the experience of external objects that dominates experience. Our phenomenology is "transparent"; we primarily experience what Husserl calls the "meant, purely as meant" (CM, p. 20). We will see that visual experiences of objects in V^+ can be analyzed in various ways, both in terms of our "immanent" sense of the rest of an object beyond what is immediately present, and in terms of our "counterfactual" sense of what we expect we *would* see *were* we to move in various ways with respect to an object.

It will often be useful to consider projections from V^+ to V, and so I denote this projection with its own symbol " π ". For any point $\bar{v} \in V^+$, $v = \pi(\bar{v})$ is the corresponding point in V. The experience of the house and the façade correspond to visual experiences of different objects, and hence different points in V^+ , but they contain the same visual image as a part, and thus correspond to the same point in V: $\pi(\text{façade-percept}) = \pi(\text{house-percept}) = \text{visual image of the front of the house. I$ $adopt the convention of referring to conceptualized images <math>\bar{v} \in V^+$ with an over-bar, and visual images $v \in V$, without one. My use of the phrase "visual experience" is neutral between visual images and visual experiences of objects; I assume context will generally disambiguate the phrase. Husserl himself has a detailed theory of perception, that involves a complex set of overlapping and controversial terms both for visual images (and sensory experiences more generally) and for our sense of a whole object.⁹ To simplify matters, I will prefer my stipulated terms to any of Husserl's own terms.

B is a space of felt bodily movements, something like what Husserl calls "kinesthetic experiences." These are experiences of moving one's body in a certain way.

⁸By "visual image" I do not mean an actual experience of an image *as* an image (e.g. looking at a painting or photograph), which is what Husserl means when he refers to "image consciousness" (*Bildbewusstsein*). I just mean what is the same between, for example, the experience of a house and the same structure when one comes to regard it as a façade.

⁹Husserl refers (roughly) to what I call a "visual image" and its mereological parts as "images" (*Bilder*), "sensations" (*Empfindungen*), "sensation contents" (*Empfindungsinhalte*), "presentative sensations" (*darstellenden Empfindungen*), "hyletic data" (*hyletische Data*), and "sensuous stuff" (*sinnliche Stoffe*). Husserl refers to the part of an experience that corresponds specifically to our sense of the whole object—the part the is different between the house and the façade—in many ways: as an "interpretation" (*Auffassung*), a "noetic moment" (*noetisches Moment*), a set of "intentional characters" (*intentionalen Charaktere*), an "interpretive intention" (*auffassenden Intention*), a "conception" (*Deutung*), and as the "surplus" (*Überschuß*) provided by "apperception" (*Apperzeption*). He also says that the interpretive intentions "animate" or "ensoul" (*beseelt*), "interpret" (*auffassen*) or "bestow sense" (*sinngebende*) on the sense data. As Dallas Willard puts it: "the act character (also called a 'noesis') confers a sense, an ofness, on otherwise dead sensa by forming them, giving them the character of 'pointing beyond themselves'…" ([103], p. 5). See LI 5 Sect. 14, LI VI, Sect. 26, and *Ideas* 1, Sect. 85. The general distinction between sensation and interpretation is also closely related to Husserl's distinction between intuitive or perceptual and empty contents. In the secondary literature see [5, 21, 32, 60].

Since *B* is a subspace of *C*, bodily movements can be taken to be mereological parts of total fields of experience. There is a simplification here, insofar as bodily movements are extended in time, while points in *B* correspond to momentary experiences of bodily movement, e.g. one time-slice of an experience of moving your body. A fuller analysis could define the functions below in terms of short paths in *B* rather than points in *B*. However, this approximation is convenient, and does not seem to compromise the analysis in any essential way.

F is the space of levels or degrees of what Husserl calls "fulfillment" (*Erfüllung*), that is degrees to which what we see confirms or disconfirms prior expectations.¹⁰ I assume that these experiences can be ordered on an interval from the null case of no fulfillment to a maximal case of complete fulfillment. Thus I will represent *F* by the unit interval [0, 1]. 0 represents the null case of minimal fulfillment (and maximal surprise); 1 represents maximal fulfillment. An experience of fulfillment occurs (in the case of visual perception) when a subsequent perception accords with prior expectations. Husserl also calls this an experience of "synthesis" (*Synthesis*), "coincidence" (*Deckung*) or "togetherness" (*Zusammengehörigkeit*) between what we see and what we had anticipated. Fulfillment is inversely related to surprise: fulfilling experiences are unsurprising (things go the way we expect them to) while frustrating experiences involve surprise. In these terms [0, 1] ranges from maximal surprise (0) to minimal surprise (1).¹¹

Husserl sometimes describes levels of fulfillment and frustration, though he often prefers a binary distinction between "consciousness of fulfillment" (*Erfüllungsbewußtsein*) and "frustration" (*Enttäuschung*), which in my terms are just values near 1 and 0 respectively in F. In fact, there is a kind of qualitative difference between the two. When frustration occurs, we experience some level of surprise, which is a distinct phenomenological component of a field of consciousness. We seem to be occurrently, presently aware of surprise. However, fulfillment, when things go as we expect, is not as obviously a component of consciousness. It is more of an implicit background feeling that things are "going as they should." It is almost a non-experience, a default case that does not contribute anything distinctive to the field of consciousness. I will continue to treat "fulfillment" and "surprise" as contrary terms for the same phenomenon, though in an expanded discussion it could be useful to distinguish them.

I also consider a set K of possible states of background knowledge. K is not a subspace of C. States of background knowledge, as I understand them, are not

¹⁰In the discussion of expectation it is important to keep the passive/active distinction in mind. Active expectations, where we dwell on what we think is about to happen, are not what Husserl has in mind. As he says: "Intention is not expectancy, it is not of its essence to be directed to future appearances" (LI VI, Sect. 10). These are not collections of active expectations of further views of an object. Rather what is at work is a whole apparatus of passive expectations, a set of counterfactual patterns whereby if the object turned out to be different than expected I would be surprised, but I otherwise just accept that things are as I expect them to be (cf. [17], p. 93).

 $^{^{11}}$ It may be that the open unit interval (0, 1) would be a better representation of degrees of fulfillment, insofar as the maximal and especially the minimal cases are limit concepts. Such a change would not have any substantive impact on the main theses following.

mereological parts of conscious states. A state of background knowledge corresponds to all of a person's unconscious beliefs, attitudes, tendencies, and understandings at a time. These states persist while we sleep and have an impact on what we do when we are awake.¹² Background knowledge encodes the "world model" sketched metaphorically in Chap. 2, which experiences "moves through." Experience shapes the model and is also influenced by it. Background knowledge does not correspond to any single theoretical construct in Husserl, though he clearly makes use of the idea. Here, for example, Husserl refers to the "*Boden der Welt*" (translated by Churchill and Ameriks as "background of the world") which "has already done its work" with "every act of experience":

every grasping of an individual object, and every subsequent activity of cognition, takes place against the background of the world. For us the world is always a world in which cognition in the most diverse ways has already done its work... every act of experience... [has] a knowledge and a potential knowledge... [a] preknowledge [*Vorwissen*] (EJ, pp. 31–32).

Husserl also refers to a *Hintergrund* which, even if "completely unnoticed, still functions according to its implicit validities" (*Crisis*, p. 149, Hua VI, p. 152).

Whereas a path in *C* represents a changing stream consciousness; a path in *K* represents slower changes in background knowledge due to learning. The dynamics of *K* are intuitively slower than those of *C* (cf. [110]).

Note that background knowledge as I construe it covers all unconscious factors affecting experience, not only deep-seated and slowly changing beliefs (what might be called "deep background"), but also more rapidly changing features of one's unconscious knowledge, e.g. one's sense of where they were a few moments ago, which is not typically present as an explicit content of consciousness, but still affects how consciousness unfolds. This might be called "local background," or "indexical background," since it determines where we assume we are in the world.¹³ The distinction will be important below, since we will sometimes assume that deep background

¹²For more on what I call "background knowledge" see Searle's [76] concept of the "background of intentionality," and Baars' concept of context, "a system that shapes conscious experience without itself being conscious at that time" ([3], p. 138). The Freudian "dynamic unconscious" and more recent theories of the unconscious in cognitive science-sometimes referred to as the "cognitive unconscious" are also relevant (an early discussion is in [18]). Today the concept of unconscious influence is so pervasive in cognitive science that one is hard-pressed to find review articles, except on special topics like unconscious influences on decision making [67]. Neural pathways in the brain are typically taken to be the physical correlate of the cognitive unconscious. James is characteristically evocative on this point (see Chap. 1 of [44]), referring to plaster, India Rubber, and brain tissue as materials that are "weak enough to yield to an influence, but strong enough not to yield all at once." Many of these ideas are brought together in the concept of a "schema," which are "cognitive structures built up in the course of interaction with the environment to organize experience" [2]. The concept of a schema is widely deployed in many disciplines, from neuroscience to cognitive science and even to ethology and kinesiology [2]. Within Husserl scholarship compare Smith and McIntyre [81], who refer to "the subject's background presuppositions about the object, or about objects of its type; we shall call them background beliefs. As beliefs presupposed by the experience, their meanings or Sinne are not explicitly included in the Sinn of the experience but are presupposed by it" [81]. Husserl's notions of *habitus* and habituality are also relevant here (for review, see [66]).

¹³Perhaps retentions in a retentional manifold serve some of the more local functions being attributed to background knowledge here.

has changed, but local or indexical background has stayed the same, for example in cases where we are viewing what we take to be the same thing from the same angle (same indexical background), but where our understanding of the object has changed (different deep background). It may be useful in future work to more explicitly distinguish these different types background knowledge, but for now they are combined into one structure, at least in the formalism.

3.2 Expectation Rule

We are now in a position to describe the first of our two rules. Recall that, relative to Husserl's transcendental project, rules like this can be thought of as the consequents of conditional statements whose antecedent states that an agent is a "stable cognizer," roughly, a conscious being that has a coherent sense of an independently existing world (cf. Chap. 1, note 2). These rules are mathematical functions (again, "rule" and "function" are synonymous for our purposes). This is Husserlian in spirit, insofar as Husserl himself refers to rules and their dependent and independent variables.¹⁴

A function is a rule that associates inputs with unique outputs. For example, $g(x) = x^2$ is a function that associates any real number as input with a unique real number as output; e.g., g(2) = 4. It has the form $g : \mathbb{R} \to \mathbb{R}$ (by the "form of a function" I will mean a description of its domain and range). A function can have multiple arguments or inputs. h(x, y, z) = x + y + z is a function that associates a triple of real numbers with another real number; e.g., h(1, 2, 3) = 6. It has the form $h: \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$. Functions can associate any kind of input and output (not just numbers): we could define a past tense function p that takes the present tense of a verb and produces its past tense, so that p("help") = "helped" and p("run") ="ran". It has the form $p: W^+ \to W^-$, where W^+ is the set of present tense verbs in English and W^- is the set of past tense verbs in English. Functions can even take functions as input and produce functions as output. For example, the derivative from calculus can be associated with a differentiation function D that takes a function as input, and produces another function as output. For example $D(2x^2) = 4x$. It has the form $D: C^1 \to C^1$, where C^1 is the set of differentiable functions. Note that $C^1 \subset \mathbb{R}^{\mathbb{R}}$, which is the set of all real-valued functions (on this notation a set R^D is the set of all functions from domain D to range R). These basic facts about functions are worth bearing in mind as we move forward. The functions of interest here are phenomenological: they describe how (for example) different mereological parts of conscious experiences—i.e. points in the subspaces of C described above—are related to one another in time.

In this section we consider an *expectation function* f. It associates points in K, V, and B with an *expectation gradient*, which is itself a function e from V to F (the interval [0, 1]), that is, from visual experiences to experiences of degrees of fulfill-

¹⁴For example, Husserl refers to an "interplay of independent and dependent variables" relating movements and visual appearances (APS, pp. 51–52); also see [5, 108]. Some of these passages will be discussed in Chap. 8, in the discussion of motivation.

3 The Formalism



Fig. 3.3 A schematic of the expectation function, which takes background knowledge k, an initial visual image v_0 , and a bodily movement b as inputs, and produces an expectation gradient e as output. The expectation gradient is a mapping from visual images in V to degrees of fulfillment between 0 and 1 in F, which says how fulfilling or surprising each visual image would be, should we experience it after movement b. A cross-section through the gradient is shown, which makes its status as a mapping from V to [0, 1] more apparent. For convenience, it is assumed in this figure and some figures below that there is a *most* expected visual image max(e) (3.2). τ is thought of as a threshold fulfillment value above which visual images can be thought of as "expected" (3.3)

ment. An expectation gradient says how fulfilling different subsequent visual images will be.¹⁵ An expectation gradient e is a member of the set of all possible functions from V to [0, 1], which can be written as " $[0, 1]^V$ ". Putting this all together, a person's background knowledge in K, current visual experience in V, and experience of movement in B, determine a unique expectation gradient $e \in [0, 1]^V$.

A stable cognizer's expectations are generated by a rule of this form:

$$f: K \times V \times B \to [0, 1]^{V} \qquad (3.1)$$

Again, this means that any input to f (any triple consisting of a state of background knowledge, a visual image, and a bodily movement) will be associated with an output e which is itself a function—an expectation gradient—from visual images to degrees of fulfillment between 0 and 1. An example of this is shown in Fig. 3.3. A person has just moved to the side of a house, and an expectation gradient has been generated by f. Points in V that the expectation gradient maps to values closer

 $^{^{15}}$ I am assuming that background knowledge together with a visual image determine a sense of location. This is important insofar as a point in *V* is by itself indeterminate with respect to location: an empty blue field could correspond to staring at the sky at any number of places on the surface of the earth, relative to which we have different expectations relative to different movements. But (I claim) given what I know and given that I see a blue sky I will have specific expectations about what I will see when I look back at the ground. This again highlights the fact that I am assuming that background knowledge encompasses more local or indexical forms of background knowledge.

to 1 are associated with darker colors (experiences of fulfillment; we expect these visual images); points in V mapped to values closer to 0 are associated with lighter colors or no color (these images produce experiences of frustration or surprise in varying degrees). We typically have fairly specific expectations; the vast majority of V is comprised of images that would completely surprise us were they to occur—a kangaroo, a galaxy, boiling magma—visual scenes which could in principle suddenly occur in my visual field (were a mad neuroscientist to gain control of my ocular nerves, for example), but that are inconsistent with the expectations generated by the expectation function. Within this already tiny region of V, a 1-dimensional crosssection is shown to illustrate more clearly that the expectation gradient is a function from V to F = [0, 1]. The cross-section is shown as a bell-shaped curve, where the visual image producing maximal fulfillment, max(e) is labelled and shown as a semi-transparent figure (the output of an arg-max function defined in the next section).¹⁶

Here is the basic idea. As a person moves, they have some expectation about what they will see. When I'm at the front of the house and I move to the left of it, I know what to expect. There is some range of subsequent visual images that will not surprise me. These correspond to the dark regions of the expectation gradient generated by f as I start walking. So the idea is that as a person moves around in the world having visual experiences, tracing out a path in V, they are also expecting what will occur next, which we can visualize as an expectation gradient whose "dark region" (containing expected subsequent images) also moves through V, in "front" of the path of actual visual experiences. Usually subsequent visual images "land" in the darker "peaks" of the expectation gradient, and fulfillment occurs (Fig. 3.4), though occasionally they land in the lighter or lower parts, in which case frustration occurs (Fig. 3.8).

Since the output of the function f is itself a function e, we can write things like e(v') or f(k, v, b)(v'), which is a number between 0 and 1 saying how fulfilling a subsequent visual image v' is, given that the person whose background knowledge is k just saw v and made movement b. I will sometimes add a subscript to e, indicating what state of background knowledge was used to generate it, e.g. $e_{k'} = f(k', v, b)$,

An expectation gradient as I have defined it is a mapping from visual images in V (which can plausibly be taken to be a continuum) to experiences of fulfillment in the range [0, 1]. Typically, a non-negative function¹⁷ on a continuum can be divided by a normalizing constant and thereby converted into a probability density function (or, more generally, a "probability distribution"). This makes it possible to interpret an expectation gradient as a probability density function, which says how likely a subsequent visual experience is to fall in any region of V. This framework allows probability theory be applied in a rigorous way to phenomenology (although I only make minimal use of this connection here; there is more work to be done formalizing Husserl's ideas using probability theory).

¹⁶As noted there, a unique max-value will not always exist, e.g. in an unfamiliar area many visual images could be equally fulfilling.

¹⁷A non-negative function is a function whose outputs are 0 or positive.



Fig. 3.4 A case of fulfillment, where a person has an initial visual experience, moves, has an expectation, and then has that expectation confirmed. In the *top row* **1** A house is seen from the front. This corresponds to a visual image $v_0 \in V$. **2** The person moves to the right (a movement $b \in B$) and an expectation gradient e_k is generated by the expectation function f. A most expected visual image $max(e_k)$ is assumed to exist and is displayed as a semi-transparent image. **3** After moving, v' is experienced. What is seen closely matches what was expected, $v' \approx max(e_k)$, and is clearly above threshold ($v' \in \theta_\tau(e_k)$) and so we have an experience of fulfillment, which is near maximal in F. The *bottom row* shows what happens after background knowledge is updated from k to k' by the learning rule (3.10), and the same movements are made. The expectation gradient remains centered on the most expected image, but also "sharpens" around that image; the maximum fulfillment value increases and the gradient narrows. This tracks the intuitive idea that we become incrementally more confident that the house looks a certain way from the side, when we repeatedly see that it that way

The cross-section of the expectation gradient shown in Fig. 3.3 has a bell-shaped, Gaussian form, which seems plausible for the cases we will consider: some further views of the house are most expected, as are nearby views in visual-image space V, and our expectation drops off as visual images become increasingly different from the most expected images.¹⁸

We can intuitively think of expectation gradients as having a "width" or "dispersion," which is something like the length of the dotted line at τ in Fig. 3.3, i.e. a measure of how spread out the gradient is in V (formally specifying dispersion in a

¹⁸I believe a qualified version of the claim that expectation gradients typically have a Gaussian shape (or related shape, e.g. a heavy-tailed distribution like a *t*-distribution) can be defended on empirical grounds (though I will not provide such a defense here), and moreover it provides a useful and intuitive way of thinking about expectation gradients. Still, nothing in what I say here requires that expectation gradients have such a form. Also note that whatever form these gradients have, they will be *n*-dimensional—e.g. *n*-dimensional Gaussians—where *n* is the dimensionality of *V*.

way that generalizes to expectation gradients with arbitrary forms and in arbitrarily many dimensions is more work for a future study). Below I sometimes refer to a "narrowing" expectation gradient, by which I mean one whose dispersion has been reduced (examples of narrowing expectation gradients are shown below in Fig. 3.7 and in Chap. 10, Fig. 10.1). When you know a neighborhood well you have specific expectations about what you will see as you drive around: gradients are "narrow" and "peaked" with relatively low dispersion. On the other hand, when you are unfamiliar with a neighborhood, you have less specific expectations as you drive around: expectation gradients will be "flatter" and "wider" (i.e. more dispersed).¹⁹

The expectation function f has not been specified beyond its form. Equation (3.1) only asserts the existence of a specific type of phenomenological regularity, which involves a specific pattern of inputs and outputs. To go beyond a specification of form would require that we produce a function that allowed us to actually compute a unique expectation function for any input to f. It seems unlikely this could be done using phenomenological resources alone, though we could approximate such a function by drawing on non-phenomenological sources, e.g. in psychology and neuroscience (more in the conclusion). Still, as a general constraint on how experience must unfold, as a statement that some regularity of this type exists for agents who coherently cognize a world, I believe it is plausible, both on its own, and as Husserl interpretation.

3.3 Expected Visual Continuations and Trail Sets

Various derivative constructs can be defined using the expectation function.

First, it will sometimes be useful (as we have already seen) to consider the most *expected* visual image relative to an expectation gradient, assuming such an image exists and is unique. If so, we can define a *maximum argument* or *arg-max* function:

$$max(e)$$
 is the v * such that $e(v*) > e(v), \forall v \in V - \{v*\}$ (3.2)

Since the function *e* is the output of f(k, v, b), we can also write max(f(k, v, b)). An example is shown in Fig. 3.3.

We can also consider the set of all visual experiences above some threshold τ , which can be taken to distinguish subsequent images that are expected (that would not surprise us) from those that would surprise us:

¹⁹There is a subtlety here that should be flagged. When an expectation gradient narrows (again, see Fig. 3.7 and especially Fig. 10.1) its peak increases. We have more specific expectations which will produce *more fulfillment* when they occur. Conversely, a flatter expectation gradient will involve less specific expectations, which will produce *less fulfillment* when they occur. (I am speaking of expectation gradients here, not the probability density functions that result from dividing expectation gradients by a normalizing constant). Husserl suggests that these variations in fulfillment occur (cf. Sect. 8.4), and I find the idea plausible, but am not sure. Does each step through a familiar environment involve more of an experience of fulfillment than a step through an unfamiliar environment? If so, perhaps this is just what familiarity is. This is an area that could benefit from a more precise characterization and perhaps empirical work testing alternative hypotheses.

$$\theta_{\tau}(e) = \{ v \in V \mid e(v) > \tau \}$$

$$(3.3)$$

For example, if $\tau = 0.8$ then $\theta_{\tau}(e)$ is the set of visual experiences mapped to values greater than 0.8 by the expectation gradient *e*, which we take (conveniently, but somewhat artificially) to distinguish expected from unexpected subsequent visual images. Examples which show the threshold τ as a dotted line are Figs. 3.3, 3.4 and 3.8.

We can use θ_{τ} to define *expected visual continuations* (I will also refer to these as "visual continuations" or "trails"), which will be a central construct in my formalization of Husserlian phenomenology. An expected visual continuation of an initial visual image v_0 is a sequence of movements around the object given in v_0 , where each movement is followed by an expected (unsurprising) visual image. An expected visual continuation relative to a starting visual image v_0 is denoted " c^{v_0} " and has this form:

$$c^{v_0} = (v_0, (b_1, v_1), (b_2, v_2), \dots, (b_n, v_n)) \in V \times (B \times V)^n$$
 (3.4)

 c^{v_0} can be read as beginning with visual image v_0 , taking movement b_1 , experiencing an expected visual image v_1 , taking movement b_2 , experiencing an expected visual image v_2 , etc. I refer to v_0 as the "initial" visual image, to each subsequent bodymovement/visual image pair (b_i, v_i) as a "member" of a visual continuation, and to the visual images v_i as "visual members" of a visual continuation. The set $V \times (B \times V)^n$ is the set of all possible n + 1-tuples consisting of an initial v_0 and an *n*-tuple of (b_i, v_i) pairs. Most of these n + 1-tuples will in some sense be incoherent—they are just arbitrary sequences of images and movements. A subset of $V \times (B \times V)^n$ containing "coherent" visual continuations is described below. I leave open what an appropriate value for n is, but assume a collection of visual continuations of length n + 1 is sufficient to determine one's sense of an object (cf. the discussion of supervenience in Sect. 3.4). The superscript to the initial visual image will sometimes be dropped. Note that subscripts are time-indices here, not indices over points in Vor B. Thus all the visual members v_i of an expected continuation might correspond to the same member of V, and similarly for the b_i (e.g. when one stands still before a stationary object for *n* time instants).

To more precisely define expected visual continuations, and to set up the concept of a *trail set*, I introduce an explicit procedure for deriving a visual continuation from an *object tracking sequence of bodily movements*. An object tracking sequence of bodily movements is a finite sequence of bodily movements $(b_1, b_2, ..., b_n)$ that tracks the object given in v_0 (e.g. walking to the right of a house while keeping one's head oriented towards it).²⁰ Given such a sequence of bodily movements (and a person's

²⁰I have left the concept of object tracking intuitive. The basic idea is that such a sequence keeps a person focused on same object as is given in v_0 : examples include walking around the object while fixing one's eyes on it, moving up to it for a closer look, or simply staying still facing towards it. Several questions arise in this connection. First, I have not specified a value for *n*, i.e. a limit on

background knowledge k),²¹ we can construct an expected visual continuation $c^{v_0} = (v_0, (b_1, v_1), (b_2, v_2), \dots, (b_n, v_n))$ using the following procedure, which I will call "procedure P":

Procedure P

For b_1 choose $v_1 \in \theta_{\tau}(f(k, v_0, b_1))$ For b_2 choose $v_2 \in \theta_{\tau}(f(k, v_1, b_2))$: For b_n choose $v_n \in \theta_{\tau}(f(k, v_{n-1}, b_n))$

The idea is to begin with an initial visual image v_0 , then to move through the objecttracking bodily movements $(b_1, b_2, ..., b_n)$. After the first movement b_1 , one of the visual images that would not be surprising is selected and used as v_1 for the next step. The process is repeated relative to v_1 and movement b_2 , and similarly for all bodily movements in $(b_1, b_2, ..., b_n)$. The procedure is illustrated in Fig. 3.5.

Using procedure P, we can define a function T (for "trail set") that associates a state of background knowledge and an initial visual image v_0 with the set of all possible expected visual continuations of that image. I will refer to these as "trail sets" ("continuation tree" might have been better since all trails originate in the same visual image). An example of a small fragment of a trail set is shown in Fig. 3.6. A trail set relative to v_0 is a set of expected continuations of v_0 , which is in turn a member of the set $\mathscr{P}(V \times (B \times V)^n)$ (where " $\mathscr{P}(X)$ " denotes the powerset of a set X, the set of all subsets of X).

The trail set function T has this form:

$$T: K \times V \to \mathscr{P}(V \times (B \times V)^n)$$
(3.5)

That is, *T* associates background knowledge/visual image pairs with sets of expected visual continuations of that visual image (we assume that for any $t = T(k, v_0)$, all $c_i \in t$ have the same v_0 as their initial visual image, and can thus be denoted " $c_i^{v_0}$ "). For

⁽Footnote 20 continued)

how many bodily movements are allowed: it might be reasonable to impose some restriction (e.g. no sequences of more than 100 movements). Second, some sequences of body movements may not be possible relative to some ways the object may turn out to be: we can't just move through solid objects, for example. Third, the issue of object individuation (which I said I would bracket as much as possible; see Chap. 2, note 6) arises here, since we must assume one object is being tracked throughout.

 $^{^{21}}$ I am assuming that in defining trails and trail sets, *k* corresponds to deep background knowledge about an object, and that this knowledge remains constant during procedure P. This makes sense insofar as procedure P is a process of analyzing how we *now assume* (based on our current background knowledge) an object would look, *were* we to move around it. No actual movements take place, and so no actual updates to background knowledge are associated with the steps in procedure P.



Fig. 3.5 Illustration of procedure P for constructing an expected visual continuation (or "trail") relative to an initial visual image $v_0 \in V$. We assume a sequence of object-tracking bodily movements b_1, b_2, \ldots, b_n . We begin with the first movement b_1 in the sequence, and choose some expected visual image v_1 , relative to the expectation gradient generated by k, v_0 , and b_1 . We then consider the second movement b_2 , and do the same thing, this time using v_1 as our visual image. We repeat this produced for each movement in the sequence. Notice that at each stage of the construction procedure there can be multiple visual images to choose from, since θ_{τ} can return multiple expected images

example, given a state of background knowledge k and a visual image of a table from the front, v_{table} , $T(k, v_{table})$ returns a trail set containing all visual continuations of the front of the table $T(k, v_{table}) = \{c_1^{v_{table}}, c_2^{v_{table}}, \ldots\}$ consistent with our knowledge of the table. Note that trail sets can include "branching" trails from the same initial visual image, i.e. trails that involve the same sequence of bodily movements b_1, \ldots, b_m , but a different sequence of visual images v_1, \ldots, v_m . Two trails in the same trail set might have the same visual members following the same bodily movements up to a point, but then have different visual members following the same bodily movements thereafter. Given what I know about this house, I would not be surprised to walk around it and see any of a number of configurations of windows on the side and back. Thus the same body movements of walking around the house could be accompanied by different visual images, given my uncertainty.

We can specify this function more precisely using procedure P. We let $Seq(v_0)$ be the set of all possible sequences of bodily movements of length *n* that track the object given in v_0 . Then we can define the trail set function *T* as:

$$T(k, v_0) = \{c_i^{v_0} \mid c_i^{v_0} \text{ is constructed using procedure P for some } s \in Seq(v_0)\}$$
(3.6)



Fig. 3.6 A trail set $T(k, v_0)$; the set of all possible trails $c_i^{v_0}$ relative to background knowledge k and initial visual image v_0 , which is in turn associated with the way we see the object

We can use *T* to define a set *EC* (for "Expected Continuations") of possible trail sets, that is, *coherent* sequences of expected continuations of an initial image, that present the same thing from different perspectives, consistently with our knowledge about the thing. Thus $EC \subset \mathscr{P}(V \times (B \times V)^n)$ should exclude most of $\mathscr{P}(V \times (B \times V)^n)$, which contains arbitrary sets of arbitrary sequences of body movements and visual images. To a first approximation, *EC* can be defined as the set of trail sets generated by procedure P for some $k \in K$ and $v \in V$. That is, *EC* is the image of *T* in $\mathscr{P}(V \times (B \times V)^n)$

$$EC = T[K, V] \subset \mathscr{P}(V \times (B \times V)^n)$$
(3.7)

3.4 Explanatory Phenomenology and Supervenience

Our next step is to introduce a function that associates trail sets with visual experiences of objects. The function is a "supervenience function," and in developing this idea we also develop the broader idea, alluded to in Chap. 2, that phenomenology has an explanatory dimension. In particular, supervenience functions can be used to associate "immanent" phenomenological structures with "counterfactual" structures.²² Immanent structures are components of lived experience: conscious states or processes and their parts. Counterfactual structures, by contrast, correspond to experiences that *could* occur, *were* certain things to happen. They are the *possibilia* of *C* and its subspaces. The basic idea from Chap. 2 is that actual immanent experience can be thought of as unfolding on top of a model, which is itself a system of counterfactual possibilities, a kind of fusion of the possible experiences we'd expect, were we to move in different ways. Actual experiences are related to these counterfactual possibilities by a kind of dependency or supervenience relation. These kind of immanent / counterfactual relationships are, I think, at work in many of Husserl's discussions, though Husserl's own descriptions are often ambiguous (e.g. Husserl uses "horizon" to refer to both kinds of structure). I take clarification of these matters to be one contribution of this work.

Supervenience is a kind of dependency relation that is well-suited to characterizing the immanent/counterfactual relations I will focus on (for more on supervenience see [107, 111]; for other applications of supervenience to phenomenology see [79, 109]). To simplify matters, we can focus on a functional characterization of supervenience. When one phenomenon Y supervenes on another X, then any complete distribution of X-phenomena (or "X-states" or "base states") will be associated with a unique distribution of Y-phenomena (or "Y-states" or "supervenient states").²³ In that case there exists a function $F : X \to Y$ that associates base states with supervenient states.²⁴

A standard example of supervenience is the relationship between positions and velocities of particles in a region of space, and the temperature in that region. Any configuration of particles in a region (the *X*-state) determines the temperature in that region (the *Y*-state). So temperatures supervene on configurations of particles: we have a function from configurations of particles to temperatures in a region. In a similar way conscious states supervene on brain states, so that there is a function that associates brain states with conscious states.²⁵

²²I am grateful to Philip Walsh for discussion of these matters.

 $^{^{23}}$ I am being somewhat vague about what phrases like "X-state," "X-phenomena," or "distribution of X-phenomena" mean. For our purposes, these phrases refer to members of C and its subspaces, or set-theoretic constructions thereof. The main requirement on a set of states being the domain or range of a supervenience function is that an object or person at a time can only be in one state in such a "state set" at a time (cf. the discussion of phenomenological subspaces above, and the discussion of maximal properties in [107]). For example, a visual image in V is a state, since a person can only instantiate one visual image in V at a time. A trail set in *EC* is a state, since only one trail in *EC* will characterizes a person's expectations about an object at a time.

²⁴Note that this function must be onto (it must be a "surjection") in order for it to properly capture the supervenience relation.

²⁵Both cases require further specification, and the second is controversial (it is for example denied by externalists of various kinds); but they illustrate the concept.

There is a lot of overlapping (and at times confusing) terminology associated with supervenience, so here is a table:

Y supervenes on X
X determines Y
$F: X \to Y$
$x \in X$ determines $F(x) \in Y$

In the examples we consider, the *Y*-phenomena will be immanent (e.g. visual experiences of objects) and the *X*-phenomena will be counterfactual (e.g. trail sets). So the immanent phenomena supervene on counterfactual phenomena, and the counterfactual phenomena determine the immanent phenomena. Notice that the term "determines" is overloaded. It can either apply to a whole set of states (as in "trail sets in general determine visual experiences of objects") or it can apply to individual states (as in, "this particular trail set determines this particular visual experience"). Also note that the term "dependence" does not appear in the table. Roughly speaking, "dependence" is a synonym for "supervenience" (and I will generally treat it that way), so that we could have added a row that says "*Y* depends on *X*." However, it should be noted that "dependence" in Husserl's sense is not equivalent to supervenience.²⁶ Also absent is "realization," which can be thought of as the converse of determination for individual states (as in, this visual experience is realized by this trail set). Thus we could have added yet another row: "*F*(*x*) is realized by *x*", or "y is multiply realized in x_1, \ldots, x_n , where $y = F(x_1) = \ldots = F(x_n)$ ".

When making a first-pass plausibility argument for a supervenience relation, one can consider two questions. First, we can directly ask: is it the case that an X-state determines a unique Y-state? A complete specification of particles in a region of space does determine a unique temperature. A complete specification of a brain state (arguably) determines a unique conscious state. Sometimes it is easier to consider a related question, which also establishes supervenience: does a change in Y entail a change in X? If so, we have evidence that Y-states supervene on X-states. This seems to work in our two examples. A change in the temperature in a region implies a change in the particles in that region. A change in a person's conscious state implies a change in his or her brain state.

Of particular interest here is the idea that *visual experiences of objects supervene on trail sets*. We can formalize this using a function *S* that associates trail sets with visual experiences of objects:

$$S: EC \to V^+ \tag{3.8}$$

²⁶For example, perceived extensions depend (in Husserl's sense) on colors, in the sense that any perceived extension must have some color filling it. If extensions supervened on colors, this would add the requirement that a given color specify a *unique* extension, which is not in general the case. So Husserlian dependence can obtain without supervenience. See [107].

Let us see if a plausibility argument for supervenience works here. First, do trail sets seem to determine visual experiences of objects? Yes: once a complete set of expected continuations of an image is specified, then it seems there is only possible way to experience the given object. Every feature we attribute to the object, and every degree of uncertainty we have with respect to those features, is captured by the trail set. So a trail set does seem to specify a unique way of seeing a thing. Second, do changes in one's visual experience of an object imply a change in expected continuations? Here my intuitions are even stronger. It seems that it must be possible to cash out any change in how an object is seen in terms of some change in how it is expected to look from some angle. If I stop seeing this as an undented car and now see it as a car with a dent on the rear bumper, then there are some continuations of the current experience of the front of the car (but not the previous experience) where I walk to the rear of the car and expect to see a dent.

I will assume that S has an inverse S^{-1} that we can use to reason "backwards" from a visual experience in V^+ to a trail set in EC:

$$S^{-1}: V^+ \to EC \tag{3.9}$$

In this way we can associate our tacit way of conceptualizing a thing with the set of ways we expect that thing to be, relative to different ways of moving with respect to it. If I am now seeing a table via a conceptualized visual experience \bar{v}_{table} , then $S^{-1}(\bar{v}_{table})$ is a set of possible visual continuations of the current table image, which makes more precise what my tacit conceptualization of the table is. We will see that S^{-1} can capture ideas at work in many areas of Husserl's phenomenology. We will also see (in the final chapter) that something like S^{-1} is assumed by many contemporary authors. For example, Glenberg has said that "conceptualization is the encoding of patterns of possible physical interaction with a three-dimensional world" ([26], p. 1).

For *S* to have an inverse it must be a 1-1 function. However, the assumption that *S* is 1-1 is problematic.²⁷ There are at least two ways of dealing with the problem. One is to simply assume we can specify background knowledge, which is something that is assumed throughout most of this study. In that case we can think of S^{-1} as having

 $^{^{27}}$ It seems plausible to assume that the function *S* is many-to-one. Just as different molecular configurations in a region can determine the same temperature (temperature is multiply realized in molecular configurations), it seems likely that slightly different trail sets could determine the same experience of an object (visual experiences are multiply realized in trail-sets). My background knowledge could change slightly, so that I might be surprised in some slightly different way relative to some particular walk around an object, which could in turn change the trail-set. But despite this "counterfactual" difference in the level of surprise/fulfillment that would occur relative to a particular interaction with the object, my overall sense of the object from the front could remain the same.
an implicit second argument k, so that $S^{-1}(\bar{v})$ is shorthand for $T(k, \pi(\bar{v}))$. This is my preferred way of thinking of (3.9). We could also define an inverse without relying on background knowledge, but this approach raises issues of its own.²⁸

3.5 Learning Rule

We now turn to the second rule, a learning rule. As we move around in the world, implicitly having expectations about what we will see, and having those expectations more or less confirmed, our knowledge of the world changes, which in turn changes how we perceive things. Assuming we are stable cognizers, background knowledge must be updated in a coherent way, so that when we see the same things and move in the same ways in the future, our expectations reflect what we previously saw.²⁹ For example, if the right side of a house turns out to be different than I expected it to be—if I discover it to have one window rather than two—then my knowledge must be updated so that the next time I am in front of the house and move to the right, I won't be surprised to see just one window.

As with the expectation rule, we will only specify the form of the rule, i.e. its domain and range:

A stable cognizer's knowledge is updated by a rule of this form:

$$\lambda: K \times V \times B \times V \to K \qquad (3.10)$$

This function takes current knowledge, visual experience, and bodily movement, together with subsequent visual experience, as input (k, v, b, v') and returns an updated state of background knowledge k' as output.

It is helpful to think of this rule as having two main inputs: (1) what you currently know, see and do, which generates an expectation gradient $e_k = f(k, v, b)$, and (2) what you actually see next, v'. At a minimum, λ should update k in such a way that subsequent expectations relative to what was seen and done in (1) reflect what was seen in (2). If v' involves something unexpected or surprising about an object, then we should update k to reflect this new knowledge, so that if we were to go back to

²⁸We could do this by collecting all of the possible trail sets associated with a given $\bar{\nu}$ into one larger trail set. That is: $S^{-1}(\bar{\nu}) = \{c_i^{\nu_0} \mid c_i^{\nu_0}$ is a member of some trail-set *t* such that $S(t) = \bar{\nu}\}$, where $\pi(\bar{\nu}) = \nu_0$. That is, we take the pre-image of $\bar{\nu}$ relative to *S* and collect all the visual continuations of ν_0 in that pre-image into a single "larger" trail set. Compare taking the pre-image of the squaring function, which is also many-to-one, relative to some number: e.g. $sq^{-1}(4) = \{2, -2\}$, or a function from trigonometry, e.g. $\sin^{-1}(0) = \{\dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots\}$. However, there are puzzles about this approach, since it involves aggregating over all possible states of background knowledge that an agent could be in while observing the object, or more accurately, all possible agents that could perceive that object, in all possible states of mind. To me at least, this is hard to think about it, and potentially a problem. For now I simply bracket the issue by assuming background knowledge is available.

²⁹References to the "future" here and below are implicitly references to the "immediate" future.

where we just were and take the same action, v' would no longer be surprising. On the other hand, if v' is what we expected, then k should be left the same, or only incrementally changed, so that if we were to go back to where we just were and take the same action, v' would be at least as fulfilling as it was. Either way, when k is updated to k' by λ , the updated expectation gradient $e_{k'}$ produced relative to seeing and doing the same thing again should make v' as fulfilling or more fulfilling than it was before: that is, $e_{k'}(v') \ge e_k(v')$.³⁰

This rule can be further specified as a form of Bayesian learning (or "Bayesian update"), which is an abstract framework that describes how hypotheses or beliefs should be updated to reflect new data [46]. Bayesian update has been shown to characterize a great deal of human cognitive processing [86]. It is also consistent with Husserl's own *a priori* analysis of knowledge update, as we will see. Though the idea that λ instantiates a form of Bayesian update clarifies its structure, we only sketch the idea qualitatively here; again, formalizing Husserl's ideas explicitly using probability theory is an area for further work.

Two examples of Bayesian update are shown in Fig. 3.7. The figures are adapted from a standard example, in which a coin is flipped repeatedly to determine if it is biased or not (e.g. weighted to unfairly show tails 80% of the time). In both rows of the figure, imagine that the dotted vertical line corresponds to 0.5. In that case, the top row corresponds to an agent believing a coin is fair, and having this belief confirmed by subsequent observations. The distribution at each step narrows around this value, and the height of the probability distribution increases, reflecting the agent's increasing confidence that the coin is fair. In the bottom row the coin is fair, but the agent initially suspects it is unfairly weighted towards tails. After the initial mis-estimate, the distribution moves towards (or is "re-centered" on) the correct value, and then sharpens around it.

The same ideas can be applied to the evolution of expectation gradients relative to knowledge update via the learning rule λ . We can re-interpret the curves in Fig. 3.7 as cross-sections of expectation gradients. The top row then corresponds to a case of fulfillment: when visual images match expectations, knowledge is updated in such a way that expectation gradients "narrow" around them. I expect two windows on the side of the house, and see two windows. Repeatedly seeing this makes me more and more confident that there are two windows on the side of the house: that experience becomes more and more fulfilling, and I come to expect a narrower range of images when moving to the side of the house. The bottom row shows a case of frustration followed by fulfillment. The first visual image is unexpected, and as a result the gradient is re-centered around it. Subsequent experiences are consistent with this new expectation and so dispersion begins to narrow around the expected visual image. These ideas are illustrated for the house example in Figs. 3.4 (fulfillment) and 3.8 (frustration).

³⁰By "seeing and doing the same thing again" I mean immediately going back to the same place one just was, experiencing v again, and taking the same action b. In such a case I assume local or "indexical" background knowledge is the same when we return to where we were, even though deep background knowledge may have changed.



Fig. 3.7 An idealized example of Bayesian update and its impact on expectation gradients. The example can either be understood as two series of coin flips (see main text), or as changes to an expectation gradient. In the latter case, the two rows show four expectation gradients e_1 through e_4 , and the visual experiences v'_1 through v'_4 that occur just after those gradients are generated. The *top row* shows a case of fulfillment. Visual images occur where they are most expected, and as a result the gradient sharpens around that point. The *bottom row* shows a case of frustration followed by fulfillment. The visual image occurred and also sharpen around it. In both cases as fulfillment occurs, the maximum value of the gradient increases

In both cases, our metaphorical world-model is refined; in the first case, details are added to our model of the side of the house; it is rendered in greater detail; "pixel density" increases. In the second case, the model is changed; the previous representation of the side of the house (with two windows) is replaced with a new one (with one window), and in subsequent experiences that change is consolidated as fulfillment occurs.

The learning rule λ can be thought of as inducing a *dynamical system* on *K* (a dynamical system on a space is a rule which can be used to associate any initial state in that space with future states): the application of λ when we see and do things produces incremental changes in background knowledge, so that over time any initial state of knowledge *k* can be associated with a sequence of knowledge states *k*, *k'*, *k''*... in *K*, produced by repeated applications of λ . This is an "orbit" of the dynamical system on *K* induced by λ . Sometimes these orbits will change more rapidly, when expectations are violated and there are more drastic changes to



Fig. 3.8 Application of the learning rule after a case of frustration. In the *top row* **1** A house is seen from the front. **2** The person moves to the right of the house and an expectation gradient is generated. **3** After moving, a new visual image v' of the house is experienced, but it does not match the most expected visual image in step 2 (or any of the above-threshold expected images in $\theta_{\tau}(e_k)$). It turns out the side of the house has one window rather than two. Thus we have an experience of frustration, and background knowledge is updated. The *bottom row* shows the same 3-step process relative to updated background knowledge k'. **1** The same house is seen from the front. **2** After moving to the right, a different expectation gradient is generated based on the change in background knowledge (note that the expectation gradient has shifted to the right). **3** The new visual image of the house matches our (updated) expectations, and so we have an experience of fulfillment

background knowledge, and sometimes more slowly, when expectations are fulfilled and background knowledge is only incrementally refined.

An orbit in K determines a corresponding evolution of expectation gradients. As we learn about a place, expectation gradients will tend to narrow. Expected continuations are thereby changing (trails are being removed from trail sets), and thus, in virtue of the supervenience function, the way we conceive of things changes as well. So, a dynamical system on K induces a cascade of changes in related structures (Husserl will call these "genetic" processes): shifting expectation gradients, expected continuations, and ways of experiencing objects. Metaphorically, the dynamics on K induces a corresponding dynamics on our internal model of the world; describing how new experiences refine and "sculpt" it, filling in details when things go as we expect, and producing more radical changes when expectations are frustrated.

Like f, λ is a dynamical rule of phenomenology. It is a rule that says how background knowledge must be updated for a stable cognizer in a stable environ-

ment. Knowledge update in such cases is not arbitrary: if an agent spends time in an unchanging environment, then fulfillment should occur more frequently and frustration less frequently over time. If I were not a stable cognizer, and this rule were violated, then my knowledge could be updated in a completely arbitrary way, or might never be updated at all. I might walk in to a house, see a typical living room, and then update background knowledge in such a way that on walking back in the house I'd expect to see a canyon or a heap of potatoes. These kinds of changes could occur arbitrarily often, absent any constraint on knowledge update. This kind of haphazard change in background knowledge would clearly undermine a person's sense of a stable reality persisting in time.

Chapter 4 Textual Analysis

I will use the formalism developed in the last chapter to unify five main concepts in Husserlian phenomenology, which are associated with five groups of terms or "term families" in his literary corpus. I label these term families "intentional*", "konsti*", "horizon*", "motiv*", and "genesis*".

The names of the term families correspond roughly to the names of search queries to the Husserl database in Japan, where "*" is intuitively a wild-card (the actual wild-cards used were more complex than this). For example intentional* includes such words as "*intentionalität*" and "*intentionalen*." Each of the term families corresponds to a constellation of ideas in Husserlian phenomenology that are broadly concerned with the kind of developing world picture described in Chap. 2 and formalized in Chap. 3.

The Husserl database in Japan indexes keywords¹ of 23 of the first 29 *Husserliana* volumes,² which covers the series through 1992 (as of May, 2015 there are 42 volumes). These 22 texts cover the entire temporal span of Husserl's career and are to that extent representative (the *Husserliana* volumes consulted and the years they span are shown in the Appendix as Fig. A.1). In some cases I supplemented my searches with searches in Google books.³ For all searches I used my own web-interface to the Japanese database, http://www.husserl.net, and analyzed search results using a computer program. Details on the search method are in the Appendix.

¹For the complete registry of keywords and more information on them, see http://www.let.osakau.ac.jp/~cpshama/HUA/e/register.html.

²Included are Hua 1–8, 10, 11, 13–16, 18, 19, 21–23, 25, 27, 29. Omitted are Hua 9, 12, 17, 20, 24, 26, 28, and all *Husserliana* after 29.

³Those data were not included in the data used to generate Fig. 4.1, but are discussed where relevant below.

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Fig. 4.1 Mean occurrences per page of terms in each term family in 5-year periods between 1885 and 1935. Legend items are ordered from most frequently occurring (konsti*) to least frequently occurring (genesis*) term families in 1920. (Note that the points in the figure correspond the following five-year interval. E.g. the values given for 1925 correspond to mean occurrences per page in the years 1925–1930)

Examples of terms encompassed by the five term families are as follows:

intentional* = {Intentionalität, Intention, Intentionales, intentionale, ...}

konsti* = {Konstitution, konstitutiv, konstitutive, konstitutiven, konstituiere, konstituieren, konstituierend, konstituierendes, konstituiert, konstituierten, mitkonstituierende...}⁴

horizon* = {*Horizont, Horizontes, Horizonten, Horizonte, Innenhorizont, Auβenhorizont...*}

 $motiv^* = \{motivier, motivation...\}$

genesis^{*} = $\{Genesis\}^5$

I observed how often words in each term family were used in the half-decades between 1880 and 1940. I initially considered the raw number of occurrences of these terms. Since this produced inflated values for periods when Husserl wrote more, I shifted my focus to mean occurrences of terms in a term family per page.

⁴konsti* was used instead of konst* to eliminate terms like *konstruktiv* and *konstruktion*.

⁵Only the word "Genesis" occurs in the original Japanese search results, so there was no need for an actual wild-card in this case. I keep the asterisk for consistency with other term-family names.

4 Textual Analysis

The main results are shown in Fig. 4.1. Several clear patterns emerged from the available data, which I suspect would persist in a more detailed analysis.

First, there is a division in this data between two broad periods in Husserl's career: (1) a period before 1915, when terms in intentional* and konsti* occur between 0.2 and 0.6 times per page but terms in horizon* and genesis* are rarely used, and (2) a later period, after 1915, when terms in all five families occur more frequently than they did in the earlier period.⁶ To get a sense of this difference compare the mean occurrences per page of each term family in 1900 and 1920. In 1900 terms in horizon* and motiv* hardly occur at all, whereas by 1920 they are used on average once every 5 pages. The increase around 1915 is consistent with a fairly common division of Husserl's work into early and later periods (or middle and later), where the later works take for granted the overall phenomenological system Husserl had developed by the time he published *Ideas 1* in 1913 [12, 63, 65, 52]. It also tracks Husserl's move from Göttingen to Freiburg in 1916, and the beginning of his work with Heidegger (Heidegger knew of Husserl prior to this, but it seems Husserl only came to know of Heidegger after moving to Freiburg [102]).

Second, terms in intentional* and konsti* are both used regularly (more than 0.2 occurrences per page) as early as 1890. In the case of konsti* this surprised me, since I tended to think of constitutive phenomenology as being associated with Husserl's middle and later works. But constitution of logical structures is a recurring theme in early works. The terms in this family occur frequently in *Studies in Arithmetic and Geometry* (1886–1901) and *Philosophy of Arithmetic* (1890–1901), for example.

Third, konsti* dominates the other four term families throughout Husserl's career. Indeed terms in this family occur in almost all of Husserl's major works. There is a brief period at the end of Husserl's life when (at least in this data) horizon* overtakes konsti*, but otherwise terms in this family occur more frequently than those of any of the other four, in every half-decade considered after 1895.

Fourth, when horizon* terms rise in usage after 1915, they reach the scale of intentional* and eventually konsti*. This late peak is due to *Husserliana* 1 (*Cartesian Meditations*), 14 (*Intersubjectivity* 2), 15 (*Intersubjectivity* 3), and 29 (*Crisis Beilage*).

Finally, "Genesis" is used less frequently than terms in any of the other families considered. It never occurs more than 100 times in any of the searched texts, and only occurs more than 50 times in Hua 11 (*Analyses of Passive Synthesis*), 14, and 15. By contrast, terms in the other families occur hundreds of times in multiple volumes.

⁶Time-averages of mean occurrences of terms in each family, across all half-decades before and after 1915, are: 0.33 versus 0.72 (konsti*), 0.22 versus 0.36 (intentional*), 0.11 versus 0.22 (motiv*), 0.03 versus 0.35 (horizon*), and 0.01 versus 0.06 (genesis*). The year 1915 was chosen based on visual inspection of the data; automated approaches also exist for determining this kind of division, e.g. methods for finding the optimal breakpoint in a segmented linear regression.

Chapter 5 Intentionality

Intentionality corresponds to the outward directed feature of experience emphasized in Chap. 2. In contemporary terms, experience is "transparent"; for the most part, we experience or are intentionally directed towards *things*, not experiences. Intentionality is a dominant feature of Husserl's phenomenology. As he said in 1913, "the problem that encompasses the whole of phenomenology is called 'intentionality'... all phenomenological problems... fall under it" (*Ideas 1*, Sect. 146). Terms in the intentional* term family occur frequently in the *Investigations*—in particular the fifth and sixth investigations—though it continues to be heavily used in middle and late works as well. These terms occur most frequently in: *Logical Investigations* (394 occurrences), *Intersubjectivity 3* (323), *Cartesian meditations/Paris lectures* (288), *Intersubjectivity 2* (280), and *Ideas 1* (269). The term "*intention*" is related, and occurs frequently in the same texts. Intentionality has been a dominant theme in the secondary literature [16, 29, 64, 81].

In this chapter I analyze intentional experience and associated concepts using the formalism developed in Chap. 3. I will focus on Husserl's "dynamical" account of intentionality (as contrasted with his more "static" classificatory analyses),¹ whereby as we perceive a thing via intentional experience over time, our implicit expectations about that thing are either fulfilled (*erfüllt*) or frustrated (*enttäuscht*).

¹The more static analyses describe mereological parts of intentional experiences and classify those experiences in various ways. This work reflects Brentano's influence, and is prominent in the *Investigations*. For example, we can classify intentional experiences according to: (1) their intuitive content (do I see a thing directly or do I "phantasize" it in, for example, memory?); (2) the existential status of the things they posit (do I believe a thing exists, as in perception or memory, or is it a fictional object?); (3) doxic modes (how certain am I about an object's qualities?); and (4) degrees of vivacity and fullness. Because these analyses are focused on individual intentional experiences, this has been described as a form of "static" analysis (see APS, in particular Steinbock's introduction).

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5.1 Intentionality in General

In phenomenology and philosophy of mind, "intentionality" does not correspond to purposiveness (as the term suggests, at least in English), but rather to a kind of directedness. Intentional experiences are directed towards something that is felt to be *objective*, that is felt to exist outside of oneself:

The qualifying adjective intentional names the essence common to the class of experiences we wish to mark off, the peculiarity of intending, of referring to what is objective, in a presentative or other analogous fashion (LI V, Sect. 13).

Intentional experiences present subjects with something beyond them, out there in the world. In the physical case this corresponds to what Husserl calls *"äußere Wahrne-mung."* This is, again, the outward-directed aspect of everyday life I emphasized in the initial motivating discussion of Chap. 2.

Intentional visual experiences correspond to what I called "visual experiences of objects" in Chap. 3:

An intentional visual experience is a visual experience of an object $\bar{v} \in V^+$ (5.1)

More specifically, an intentional visual experience is a mereological part of a field of consciousness that corresponds to a focal visual experience of an object.

It is not clear what exactly our "sense of the whole object" consists in.² In some sense it involves references to further profiles of the object. However, we clearly don't have any *explicit* sense of all the other sides of a thing. In fact explicit imaginations were bracketed in Chap. 2, and even if they weren't, it's impossible to imagine more than a few sides of a thing at a time. Still, we seem to have some tacit sense of the whole object. Husserl puts the point in terms of a kind of "co-consciousness" (*mitbewußt*):

²A related question is whether we can consider this sense of the object to be a separate mereological part of $\bar{\nu}$, a specifically "intentional component" of the experience. If so, we can parse visual experiences of objects \bar{v} into two parts: a visual image, and a part corresponding to our sense of the object itself. The former is the raw sensory impression $\pi(\bar{v})$, which is the same when we compare my seeing a house and my seeing the same house as a façade. The latter is the specifically intentional component of a visual experience of an object. In mereological terms, this specifically intentional part of the experience (this sense of the whole object) can be thought of as the mereological difference or remainder (denoted by "-") between \bar{v} and $\pi(\bar{v})$, i.e. $\bar{v} - \pi(\bar{v})$, which is the mereological sum of all the parts of \bar{v} that do not overlap $\pi(\bar{v})$; see [96]. Anything in an experience of a house that goes beyond its visual image is fused into this intentional component. It can be thought of as a kind of tacit conceptualization or interpretation, whereby we see this as a house rather than as a façade. This component of the experience is specifically about the house in one case, and the facade in the other. There are reasons to be skeptical about these ideas (in what sense exactly can we think of a visual image or intentional component as detached from a visual experience?), and there are alternative accounts which make no use of this kind of mereology. In particular, according to certain kinds of phenomenological holism, e.g. Gurwitsch's [30], it may be impossible to use classical mereology to decompose intentional experiences into parts. Even if this type of holism is accepted, most of the account offered here can be preserved. For example, we can still say that visual experiences of objects (now conceived as non-decomposable wholes) supervene on trail sets.

I say co-conscious, since the non-visible sides are certainly also there somehow [*irgendwie*] for consciousness, "co-meant" as co-present ["*mitgemeint*" als *mitgegenwärtig*] (APS, p. 40).

Notice that Husserl says the non-visible sides are "*somehow*" there in the conscious experience. Husserl insists we have some sense of non-visible sides of the object, but is noncommittal about the details (he does have more to say, however, and there is a literature on the topic; cf. Sect. 7.1).

The nice thing about explanatory phenomenology (Sect. 3.4) is that even if this kind of immanent structure is difficult to characterize, we can still make progress by noting that it depends on a set of counterfactual possibilities. $\bar{\nu}$ presents a *whole object*, which is determined by what we would see were we to move in various ways with respect to it. That is, $\bar{\nu}$ is determined by a trail set $S^{-1}(\bar{\nu})$, as discussed in Sect. 3.4. So the "reference to what is objective" of intentional experience can be understood in terms of all the ways we think an object might look relative to all the ways we could move with respect to it.

5.2 The Dynamics of Intentionality

I will treat dynamical intentionality as an idealized 3-stage process, whereby a person (1) has an initial visual experience of an object, (2) moves with respect to the object and has specific expectations, and (3) those expectations are confirmed or disconfirmed based on what is actually seen. For a graphical sense of what I mean by these three stages, refer back to Fig. 3.5 in Chap. 3. We can take stage 1 to be the person's initial visual experience v_0 , stage 2 to be the period "during" the movement b_1 , and stage 3 to be the subsequent perception v_1 , which in that case is fulfilling.

5.2.1 Partial and Total Intentions

In the first stage of this process, a person simply has an intentional experience \bar{v} . The front of a well-known house is seen *as* the front of that house: it is seen to have a certain number of windows on each side, a specific internal layout, etc. Again, even if it's hard to say in what sense a person is aware of the back-side of a house when it is viewed it from the front, we can sketch out to the best of our ability, what those other sides are implicitly taken to be. That is, we can sketch out some fragment of the trail set $S^{-1}(\bar{v})$. I take this to be what Husserl calls "intentional analysis": "[the] peculiar attainment" of intentional analysis "is an uncovering of the potentialities 'implicit' in actualities of consciousness" (CM, p. 46).

Husserl refers to these implicit references to the other sides of the object as "partial intentions" (*Partialintentionen*), or "adumbrations" (*Abschattungen*), among

other terms.³ Much of this terminology is ambiguous between the "immanent" and "counterfactual" structures I describe here. I will stipulate that a "partial intention" of an object is a counterfactual structure: one way we would expect an object to look, were we to move in a particular way. That is, a partial intention of an object is a visual member of the trail set that visual experience supervenes on:

Partial intentions of the object given in
$$\bar{v}$$
 are (5.2)
visual members of trails $c_i \in S^{-1}(\bar{v})$

A partial intention of an object given in \bar{v} is one expected view of it, relative to a particular sequence of body movements. Partial intentions are directed at the various nonvisible parts of the thing given in \bar{v} : "the correlate of… partial intentions are the thing's parts and aspects" (LI VI 10, p. 701).

Husserl also says a totality of these partial intentions "fuse" into a total intention: "...perceiving and imaging in is, on our view, a web of partial intentions, fused together in the unity of a single total intention" (LI VI 10, p. 701). If we read a "total intention" as an intentional experience $\bar{\nu}$, then "fusion" here corresponds to the relationship between a visual experience $\bar{\nu}$ and the trail set $S^{-1}(\bar{\nu})$ that determines it: each partial intention of an object is a visual member of a trail in the trail set that determines that visual experience.

5.2.2 Specific Adumbrations

In the second stage of an idealized intentional process, a person moves with respect to an object. I move to the right of the house, and now, from among a coalescent mass of partial intentions, I have specific expectations relative to my movements. We can think of these as "activated" or "specific" adumbrations (my terms), or "protentions" (*Protentionen*), our immanent sense of what we are about to see as we take specific movements with respect to an object. In interacting with an object "I am directed towards what is coming…in a protention" (*Time*, p. 91).⁴ In the following passage Husserl considers both the tacit partial intentions of the first stage above (the "also

³Husserl also refers to "expectations" (*Erwartungen*), "anticipations" (*Antizipationen*), "predelineated possibilities" (*vorgezeichnete Möglichkeit*), "motivated possibilities" (*motivierten Möglichkeit*), "systems of rays" (*Strahlensysteme*), and "indicative systems" (*Hinweissysteme*); several of these terms and phrases are embedded in other theoretical contexts and have additional connotations; motivation is discussed further in Chap. 8.

⁴Protentions are the forward-directed counterparts to retentions in Husserl's theory of timeconsciousness. In the discussion of background knowledge in Chap. 3 (in particular, note 13) it was suggested that local or indexical background knowledge may be based on retentions in a retentional manifold. In fact, Husserl's theory of time-consciousness could be fairly tightly integrated with this account, and potentially be enhanced by it, for example insofar as the approach I take suggests some asymmetries between retention (*qua* indexical background knowledge) and protention (*qua* dimly sensed expectation).

meant" sides of an object), and the more explicit adumbrations or protentions of the second stage:

there belongs to every external perception its reference from the "genuinely perceived" sides of the object of perception to the sides "also meant"—not yet perceived, but only anticipated and... as the sides that are "coming" now perceptually... a continuous *protention*, which, with each phase of the perception, has a new sense (CM, p. 44).

I take "protention," "specific adumbration," "anticipatory impression," and "definite intention" to refer to this immanent feature of experience—a vague but occurrent sense of what's coming next (though again, as used by Husserl, these kinds of terms tend to be ambiguous between immanent and counterfactual interpretations).

Protentions have a subtle phenomenology. We do seem to have some kind of anticipatory sense of what's coming when we move with respect to an object, but (as noted above) we certainly *don't* have a distinct sense of every "expected" subsequent image. We just have a vague sense that "one of those things" will occur, where "one of those things" can encompass a greater or lesser range, depending on how familiar we are with our surroundings. When I walk in to an unfamiliar park I have a diffuse protention: a wide range of park-like things would not surprise me. Walking in my front-door I adumbrate something much more specific.

We can capture these features of protentions while leaving open their precise phenomenal character by again drawing on supervenience. Protentions (in a set Pof possible protentions) can plausibly be taken to supervene on the sets of expected visual images generated when we move around an object. Recall that there are two questions we can ask when assessing the plausibility of a supervenience claim; does a base state (in this case a set of expectations generated during movement) determine a unique supervenient state (a protention), and does a change in supervenient state imply a change in base state? Both seem plausible in this case. (1) If two people have the exact same set of counterfactual expectations relative to some movement with respect to an object (the same exact images would or would not surprise them), it seems plausible that they would experience the same specific adumbration or protention while taking that movement. (2) If my immanent protention changes I should have different counterfactual expectations. If we compare my protention walking in to an unfamiliar park with my more specific protention walking in to the same park once it has become familiar, my counterfactual expectations should clearly have changed. I now expect that specific merry-go-round and that specific popcorn stand, whereas I didn't before.

We can formalize these ideas using the expectation function f(3.2) and the threshold function $\theta_{\tau}(3.3)$.⁵ The expectation function is applied *when we move*, generating a new expectation gradient e = f(k, v, b) with each movement. By applying the threshold function θ_{τ} to this gradient, a set of expected image $\theta_{\tau}(e)$ is generated.

⁵What is formalized here is somewhat weaker than the generic claim that protentions supervene on expectations when we move, since f takes background knowledge as an argument. What is formalized here is the claim that protentions supervene on expectations while we move, *relative to a particular state of background knowledge*.

If protentions supervene on expected images when we move, then the following function can be defined:

$$p: K \times V \times B \to P \tag{5.3}$$

Any time you take movement *b* relative to image *v* and knowledge *k*, a set of expectations $\theta_{\tau}(f(k, v, b))$ is generated, which, given the supervenience of protentions on expectations generated during movement, will determine a unique protention in *P*. That is, the set of images that would not surprise you during a movement, were they to occur (a counterfactual structure) determines the immanent phenomenology of your adumbration during that movement.

5.2.3 Fulfillment and Frustration

In the third stage of our idealized intentional process, when we actually have a subsequent visual experience, our specific adumbrations (or "definite intentions") are confirmed or disconfirmed, or, as Husserl says, "fulfilled" or "frustrated." When, for example, "a familiar melody begins, it stirs up definite intentions which find their fulfillment in the melody's gradual unfolding" (LI VI, Sect. 10). Similarly for perceived physical things. Husserl says that in the "transition of appearances, for instance, when approaching or walking around an object, or in eye movement" that our sense of the unity of a thing is based specifically on the relationship between intentions or adumbrations and their fulfillment: "the fundamental relationship in this dynamic transition is that of intention and fulfillment" (APS, p. 48).

We have already seen how this can be formalized in the discussion of fulfillment in Chap. 3 (see in particular Figs. 3.4 and 3.8). After a set of expectations and a corresponding specific adumbration or protention is generated in stage 2, a subsequent visual experience v' either fulfills or frustrates that adumbration, which is to say, it either is or is not a member of $\theta_{\tau}(e)$. That is:

$$v'$$
 fulfills a specific adumbration $: v' \in \theta_{\tau}(e)$ (5.4)
 v' frustrates a specific adumbration $: v' \notin \theta_{\tau}(e)$

where $\theta_{\tau}(e)$ is the set of expectations that determines that specific adumbration. Notice that fulfillment occurs when one of the images in the expectation set that determines the specific adumbration from stage 2 is actualized in stage 3. When frustration occurs, by contrast, this does not happen: an image that did not contribute to the specific adumbration occurs.

I have presented a binary distinction between fulfillment and frustration, but of course the phenomenon occurs in degrees. To capture this we can say that when we know k see v, and do b, then the *degree to which* subsequent experiences fulfill or frustrate our prior adumbrations is just f(k, v, b)(v'). That is:

The degree to which
$$v'$$
 is fulfilling is $e(v')$ (5.5)

where e = f(k, v, b).

Husserl also analyzes changes that occur after frustration and fulfillment. Indeed, the "synthesis of fulfillment" (roughly: the accretions of knowledge that occurs when things are as we expect them to be) is central to his account of fulfillment (see, e.g., LI VI, Chap. 1). When I expect to see a bookshelf in the corner of my office, and that expectation is fulfilled, my sense that there is a bookshelf in that corner is further "synthesized" in to my understanding of the office (it is metaphorically fused into my model of the office, reinforcing this particular part of the model). I will treat these as changes to background knowledge on the basis of the learning rule (3.10). These kinds of changes will be further unpacked in the discussion of motivation in Chap. 8.

Chapter 6 Consitutive Phenomenology

Constitution, on my interpretation, corresponds to the relationship between things as we experience them and all the ways we *could* experience them. Constitutive phenomenology is thus one of Husserl's terms for what I have called "explanatory phenomenology," which studies how immanent phenomenological structures depend on counterfactual phenomenological structures. Husserl also thinks of constitution in a dynamical way which emphasizes how our understandings of things change as we have more experiences with them. We saw in Chap. 3 that konsti* co-occurs with intentional*, and that terms in the two term families occur throughout Husserl's career. We also saw that konsti* dominates the five families. In fact, terms in this family occur over 1000 times in *Intersubjectivity 3* (1113 occurrences), and over 500 times in *Intersubjectivity 2* (786), *Ideas 2* (574), and *Analyses of Passive Synthesis* (518). An important early study of constitution is Sokolowski's 1964 book [82]. Also see [48, 64].

6.1 Constitution in General

In early writings Husserl uses "constitution" to refer to what might be thought of as the cognitive development of conceptual structures, e.g. the "formal constitution of arithmetic" (Hua 21, p. 3), the "constitution of deductive systems" (PA, p. 451), and the constitution of "prime numbers and trapezia" (*Prolegomena*). This usage explains the prevalence of the term in Husserl's early works, until about 1905. Sokolowski defines constitution in this early "technical sense" to be "the subjective process by which objective categories come to be" (p. 35). The concept does not become a central concept in Husserlian phenomenology until the time of *Ideas*, when Husserl increasingly refers to the project of "constitutive phenomenology." The meaning of the term in this context is largely the same, but with an expanded scope: constitution comes to encompass how all types of experienced realities "come to be" via

subjective processes: "Every imaginable sense, every imaginable being... falls within the domain of transcendental subjectivity, as the subjectivity that constitutes sense and being" (CM, p. 84). We can also think of this metaphorically in terms of the gradual "sculpting" of objects in the vast world model described in Chap. 2. As we experience things their current status on the "modeling table" is updated; their "constitution" is refined. In his later, idealistic phase (which coincides with his increased use of konsti* terms), the constitution metaphor arguably becomes literal, since physical objects as constituted in experience are taken to be real objects.¹

Husserl develops his mature theory of constitution in thousands of pages of text, most of which are focused on particular constitutional investigations (e.g., the constitution of time, the self, art objects, "social structures," the "human surrounding world" and "worlds of culture"). However, he does at times describe the general theory of constitution which subsumes these specific studies. In one of the few places where Husserl seems to offers a definition of constitution (more specifically, the "problem of constitution"), he describes a theoretical enterprise where the phenomenologist "surveys" and "seizes upon theoretically" the way appearances "belong together in the unity of what appears":

...the problem of constitution clearly signifies... that the regulated series of appearances necessarily belonging together in the unity of what appears can become intuitively surveyed and seized upon theoretically (*Ideas 1*, p. 362).

On my interpretation, what Husserl calls "regulated series of appearances" are (in the visual case) trails or expected visual continuations c_1, c_2, \ldots, c_m . The "unity of what appears" corresponds to the object as it is given in a visual experience \bar{v}_j . These two structures have a clear relationship in the formalism. Visual experiences of objects are immanent structures, trails are counterfactual structures within trail sets, and the visual experiences supervene on the trail sets. So we have the following formalization of constitution:

The object as given in
$$\bar{v}_i$$
 is constituted in $S^{-1}(\bar{v}_i)$ (6.1)

That is, any object as given in visual experience \bar{v}_j is constituted in a trail set $\{c_1, c_2, \ldots\} = S^{-1}(\bar{v}_j)$.² Each of these trails c_i is itself a sequence of body movement / visual experience pair that correspond to a particular sequence of movements and the visual experiences we would expect were we to move in this way. So, when

¹Subtle issues are raised here. Husserl does not think that objects are "constituted" out of experiences in the same way physical things are made out of their parts, though his idealist tendencies may take him closer to that view than some would like. For a discussion of Husserl's idealism in relation to constitution, with references to the extensive literature on the topic, see [114].

²Note that I refer to the constitution of the object as given in a particular experience from a particular angle. Husserl himself, in part because of his idealist tendencies, tends to be more interested in the constitution of the whole objects which make up our world. However, defining a whole object raises problems of its own, and would involve a more complex set of constructs (e.g. some kind of concatenation of trail sets relative to different angles on an object). I believe the concept of the constitution of an object, *from the standpoint* of one its angles, is phenomenologically coherent, is consistent with Husserl's intentions, and provides a tractable starting point for future work.

Husserl says my sense of the house given in \bar{v}_j has been "constituted" in nexuses of consciousness or regulated series of appearances, I read him as meaning that my visual experience of an object is determined by all the ways I expect the house to look, relative to all the different ways I could move with respect to it. This is another way of stating the supervenience thesis (3.8), according to which visual experiences of objects supervene on trail sets: change the way the object is experienced, and some trail must have changed as well; fix a complete set of visual continuations of a current image, and that determines a unique experience of an object. Metaphorically speaking, an object on the modeling table is determined by all our past interactions with it; \bar{v}_j is constituted in $S^{-1}(\bar{v}_j)$.

Given this interpretation of constitution, several connections can be made between constitution and intentionality, by way of their common formalization (in subsequent chapters I will continue to make these kinds of connection, and in this way show how Husserlian phenomenology is unified by the formalism of Chap. 3).

What is constituted is an object as given in an intentional visual experience $\bar{\nu}$ (5.1). The trails that constitute that object contain "partial intentions" (5.2) as visual members. The other views of a house that constitute my sense of the house as a whole are partially intended relative to a view of the house from the front. When we follow one of the trails that constitutes an object, our specific adumbration or protention (5.3) is determined by our expectations. When I walk to the right of a house, my specific adumbration is determined by all the subsequent visual images which would not surprise me. When subsequent visual experiences match that adumbration, and are in that set of expectations, fulfillment occurs (5.4). When subsequent visual experiences (3.10), and the constitution of the object is altered. This last point highlights the fact that constitution is a dynamical or, as Husserl will also say, "genetic" process, what could also be a called a process of "construction."

6.2 Constitution as Construction

Husserl sometimes refers to constitution in a dynamical way, as a kind of developmental process associated with an intentional experience, a process of "constitutive construction" (*Konstitutive Aufbau*, I3, p. 192) in virtue of which we come to see a thing the way we do. This constructive process involves, metaphorically at least, a kind of weaving together of individual conscious processes or what I have called "trails" into nexuses that constitute the given object:

The thing is constituted in consciousness; there is an intentionality that gives sense to it and to its "true being." This intentionality is one that comes to light, following essential laws, in nexuses of consciousness of a determinate kind... This intentionality is inseparable from such nexuses.... The world is borne, as it were, by consciousness... (TS, p. 34).

The process of constitution shows how a person's sense of an objective thing, e.g. a house, is in some sense built up out of individual subjective experiences of it (this

is related to genetic phenomenology, discussed in Chap. 9). As Sokolowski says, "constitution is thus the process by which subjectivity forms objectivity by virtue of its own activity" ([82], p. 214). Or, as Husserl says in the passage above, "the world is borne, as it were, by consciousness."³ Imagistically, it's as if multiple images have been fused together (recall again the modeling table metaphor from Chap. 2) into a "hanging-together" (*Zusammenhängen*) or nexus in virtue of which we are aware of the thing as a whole.

I take (6.1) to show how these metaphors can be made precise. According to 6.1, a physical thing as given in visual experience is constituted by its expected visual continuations. These expected continuations are determined in part by one's state of background knowledge (recall from Sect. 3.3 that $S^{-1}(\bar{v})$ is shorthand for the trail set function $T(k, \pi(\bar{v}))$). As background knowledge changes via the learning rule (3.10), the visual continuations returned by this function change, and thus the constitution of the thing changes as well. When I learn a poster in the front hall of a house has been removed, I update background knowledge so that I no longer expect to see it when I enter that house. One set of trails is removed from the trail set that constitutes that house from the front, and another set of trails is added.

6.3 Constitutive Phenomenology as a Research Program

Husserl conceived of constitutive phenomenology as an extremely broad enterprise. In late works like *Cartesian Meditations* he says that phenomenology involves a series of constitutional investigations, one for each category of possible experienced objects:

an enormous task is foreshadowed...the task of carrying out all phenomenological investigations within the unity of a systematic and all-embracing order... [which involves] all objects of possible consciousness, including the system of their formal and material categories—the task, I say, of carrying out such investigations as corresponding constitutional investigations, one based upon another, and all of them interconnected, in a strictly systematic fashion (CM, 54).

In his 40,000 pages of manuscripts and notes, Husserl carries out an impressively large part of this task, surveying many areas of experienced reality and in each case giving detailed analyses of the way specific types of thing are constituted in subjective processes (or "regulated series of appearances"), each according to its own specific laws. We already saw that in early works Husserl studies the constitution of formal

³Husserl's idealist tendencies are evident in these and related passages. He at times seems to claim that *all an object is*, is a nexus of consciousness. From that perspective the constitution metaphor become more literal. In fact, the meaning of "constitution" has been a central issue in the debate whether Husserl was a realist (in which case constitution involves a kind of self-manifestation or disclosing in consciousness) or idealist (in which case constitution is closer to a kind of creation). For discussion see [1] (which includes citations to earlier literature), as well as [16, 81, 104, 114]. The issue is important, but orthogonal to this work: Husserl's theory of constitution can be endorsed whether or not one reads Husserl as a realist or idealist.

and mathematical structures like numbers, geometric figures, and deductive systems in general (these studies are taken up again in later works like FTL). *Ideas 2* is organized around the following constitutional investigations:

- (1) Constitution of Material Nature
- (2) Constitution of Animal Nature
- (3) Constitution of Spiritual Nature

The constitution of material nature, which we focus on, is itself a complex, manylayered enterprise. Perhaps the most detailed analysis of this type of constitution is in *Thing and Space*, where Husserl separately considers "The constitution of the temporal and spatial extension of appearance" (TS, Chap. 4), "The constitution of threedimensional, spatial corporeality" (TS, Sect. V) and "The constitution of objective change" (TS, Sect. 6). The intersubjectivity volumes and the fifth Cartesian meditation study the "constitution of social structures," "constitution of the specifically human surrounding world," the constitution of "different surrounding worlds of culture," and the "problem of the transcendental constitution of the world in terms of normality." In other works Husserl considers the constitution of time (Hua 1), the self (APS, CM), and art and aesthetic objects (Hua 23).

I believe all of the classes of constitutional investigation Husserl pursues could be formalized using variants of the apparatus introduced in Chap. 3, in particular the expectation function, supervenience function, and learning rule. However the details are complex (e.g. is there an expectation function involved when one is thinking about mathematics? If so, what is its form?). These and related directions for further research are discussed in the conclusion.

Chapter 7 Horizon Theory

Terms in the horizon* term family begin to appear regularly in Husserl's work around 1905, with peak usage between 1930 and Husserl's death in 1938 (see Fig. 4.1). There are several concepts of horizon at work in Husserl, but in general they correspond to our sense of things beyond what we immediately perceive or intuit. These ideas have their origins in such early writings as the *Investigations*.¹ Husserl sometimes conceives of horizons as structured sets of possible experiences, which he also refers to as "manifolds" (*Mannigfaltigkeiten*), drawing on his early work in logic and mathematics [31]. However, terms in the horizon* family occur most frequently in later works: *Intersubjectivity 3* (423 occurrences), *Crisis Beilage* (256), *Crisis* (172), *First Philosophy 2* (167), and *Intersubjectivity 2* (136). The concept of a horizon has been prominent in the secondary literature on Husserl, e.g. in [30, 84, 102]. One formalization is [81]. There is also a recent book-length study [24].

The horizon metaphor suggests an explorable expanse in the distance, like the literal horizon, where the earth meets the sky. Husserl uses the metaphor in several ways, and distinguishes different types of horizon.² We consider three types of

¹As Hopp says, "Husserl's doctrine of horizons is manifestly a development of his *Investigations* doctrine that every perceptual experience contains both signitive and intuitive components" [33, pp. 21–22].

²Some examples that I will not discuss are the "arithmetical horizon," "value horizon," "horizon of eidetic cognitions," "horizon of reflection," "horizon of the pure Ego," "horizon of inattention [*Unaufmerksamkeit*]," "horizon of Before" or "horizon of the past" and "horizon of After" (these examples are from *Ideas 1* and CM). Most of these are special cases of the horizons discussed here (compare the discussion of specific constitutive domains in 10.2). Husserl also distinguishes between an inner and an outer or "external" horizon. This is an important distinction, since on my reading it allows questions about object-tracking to be bracketed. The inner horizon is what I focus on here: our sense of a whole object relative to a current view of it from one perspective. An outer horizon, by contrast, corresponds to our sense of other things *around* an object: our sense, as it were, of the whole physical environment relative to our current view of it: "everything given in experience has not only an internal horizon but also an infinite, open,

horizon: an immanent horizon, and two kinds of counterfactual horizons: (a horizon of potentialities, and a horizon of continuations).

7.1 Immanent Horizon

In one sense, a "horizon" corresponds to our occurrent sense of the rest of a thing we are looking at, beyond what is immediately present in sensory experience, a "*plus ultra* in the empty horizon" (APS, p. 48). This *plus ultra* corresponds to that aspect of an intentional experience that grasps a whole object, beyond what is immediately given (cf. Sect. 5.1). As Husserl says:

A physical thing is necessarily given in mere "modes of appearance" in which necessarily a core of "what is actually presented" is apprehended as being surrounded by a horizon of co-givenness, which is not givenness proper, and of more or less vague indeterminateness (*Ideas* 1, Sect. 44).

It is unclear what the precise phenomenal character of the immanent horizon is. As noted in Sect. 5.1, Husserl says the non-visible sides of an object are "*somehow*" there in the conscious experience, but how exactly is not clear. There has been some work on this and related problems, both historically and in the more recent psychological literature. One proposal is that our immanent sense of the rest of an object involves "feelings of connection" between the visible and non-visible parts of an object. Husserl (drawing on Herbart, Wundt, and others before him) refers to "apperceptions" of the rest of an object [17]. The Husserlian phenomenologist and psychologist Aron Gurwitsch describes a "thematic field," an experienced context which "tinges" what a person sees at any moment. Both Husserl and Gurwitsch are drawing on James, who refers to "psychic fringes or overtones" which characterize a person's "knowledge about a thing." We are aware of an object "in the penumbral

⁽Footnote 2 continued)

external horizon of objects co-given" (EJ, p. 33). Husserl also refers to a "world horizon," which contextualizes all of our moment to moment experience: "Things... are 'given' as being valid for us in each case... but in principle only in such a way that we are conscious of them as things or objects within the world-horizon. Each one is 'something of' the world of which we are constantly conscious as a horizon" (*Crisis*, p. 143). I will briefly note the following: one can interpret the outer horizon to be the inner horizon of the world as a whole: the set of all ways a very large object—namely, the whole physical world—could appear. All the definitions and concepts above still go through: any sequence of perceptions of a physical thing is, after all, a sequence of perceptions of the one physical world. Focusing on the world horizon as maximal inner horizon has some potential theoretical benefits. For example, it allows us to bracket the questions about object individuation noted at the end of Chap. 2. Rather than considering object tracking bodily movements, for example, we can consider arbitrary sequence of bodily movements (subject, of course, to the constraints of our bodies), since any possible sequence of bodily movements will produce a sequence of experiences of the physical world as a whole. In this way the whole analysis could in principle be re-run without the concept of object individuation.

nascent way of a 'fringe' of unarticulated affinities about it' [43, Chap. 9].³ These ideas have recently been discussed in relation to the empirical literature on inattention, peripheral experience, and object perception [117], and there is clearly more research to be done in this area.⁴

If we accept this account of the immanent horizon as a fringe or penumbra of felt relations, then we can speculate that it is "weighted" towards those parts of the object we expect to see next, i.e. our current protention (5.3). Perceptual processes are, after all, "saturated with anticipation" (EJ, p. 122). Metaphorically, it's as if the immanent horizon were a halo around the current visual experience, with a kind of protrusion or eccentricity in the "direction" of the current protention (compare what James calls the "forward-looking end" of our perception of time, via the "specious present" [43, Chap. XV]).

7.2 Horizon of Potentialities

In a second sense, the "horizon" of an experience corresponds to the set of possible ways the object given in the experience could appear, a "horizon of potentialities" (CM, p. 71). Husserl sometimes refers to a horizon in this sense as a "manifold." I see the house now and have a sense of the rest of it (immanent horizon). My sense of the house in some sense points to these possibilities. Husserl refers to "indicative systems" (*Hinweissysteme*) that function "as systems of rays [*Strahlensysteme*] that point toward corresponding manifold systems of appearance" (APS, p. 42).

Husserl suggests that visual experiences depend on these manifolds.

the individual thing in perception has meaning only through an open horizon of "possible perceptions," insofar as what is actually perceived "points" to a systematic manifold [*mannigfaltikeit*] of all possible perceptual exhibitings belonging to it harmoniously (*Crisis*, p. 162).

The horizon in this second sense is a kind of static collection, which gathers together all the ways we implicitly believe the object could appear relative to all the different ways we could move with respect to it:

Horizon of potentialities associated with
$$\bar{v}$$

{ $v \mid v$ is a visual member of some $c \in S^{-1}(\bar{v})$ } (7.1)

The idea that intentional experiences depend on the horizon in this sense is not quite right, for reasons outlined below in footnote 6. Intentional experiences are

³These different concepts do not perfectly overlap. James' fringes and Gurwitsch's thematic fields encompass more than just our sense of the non-visible parts of a perceived object.

⁴In psychology the phenomenon is referred to as "amodal perception," and is related to "perceptual completion" or "amodal completion" [51], which occurs when, for example, a "pac-man" figure is perceived as an occluded disk in a Kanizsa triangle.

determined by a specific *structure* on this kind of horizon, in particular, a trailset, which is the horizon in a third sense. However, the horizon of potentialities is a useful construct, since it describes in a simple, first-pass way what the counterfactual correlate of a perceived thing is.⁵

The horizon of potentialities is related to constitution, intentionality, and the immanent horizon as follows. We partially intend (5.2) each member of the horizon of potentialities. I see the car and partially intend various features of its unseen sides; these partial intentions collectively comprise the horizon of potentialities associated with my experience of the car. A specific adumbration (5.3) is determined by those members of the horizon of potentialities that we expect will occur based on our current movement. As I get in the car I have a specific adumbration of the car's interior, which is determined by a subset of the horizon of potentialities associated with the car. A "web of partial intentions" (Sect. 5.2.1) just is the horizon in this sense; a totality of ways I expect the car might be, relative to different movements with respect to it. The visual images in a horizon of potentiality are the visual members of the trail set that constitutes an object (6.1). Though the precise status of the immanent horizon is unclear, we can think of it as a dim fringe-like sense of these potentialities—e.g., our "penumbral" sense of the rest of the house when we see it from the front.

7.3 Horizon of Continuations

In a third sense a "horizon" is a system of "continua of possible perceptions" which present the same object from different sides:

every single aspect of the object in itself points to a continuity, to multifarious continua of possible new perceptions, and precisely to those in which the same object would show itself from ever new sides (APS, p. 41).

The horizon in this third sense is another kind of "counterfactual" horizon; not an immanent sense of the rest of a thing, nor a set of static images, but rather a collection of image/movement pairs, connected together as "continua" which present the same object from different sides during an exploration of it. This in turn is the trail set $\{c_1, c_2, \ldots\}$ returned by S^{-1} :

Horizon of continuations of
$$\bar{v}$$
: $S^{-1}(\bar{v})$ (7.2)

At times Husserl personifies the horizon in this sense—this system of possible explorations—describing it as "calling out to us," saying:

⁵In fact, it may be that Husserl is never interested in an unstructured collection of perceptions, since he emphasizes *systems* of rays or indications, or manifolds, which are by their nature structured.

7.3 Horizon of Continuations

There is still more to see here, turn me so you can see all my sides, let your gaze run through me, draw closer to me, open me up, divide me up; keep on looking me over again and again, turning me to see all sides. You will get to know me like this, all that I am, all my surface qualities, all my inner sensible qualities (APS, p. 31).

The horizon of continuations is the trail set that constitutes an object (6.1). The expected continuations that constitute my sense of the car correspond to the horizon of continuations of the current view of the car. Since the horizon in this sense just is the trail set that constitutes an object, the same linkages between constitution and intentionality discussed in Sect. 6.1 apply here. In addition to thinking of the immanent horizon as a dim sense of the horizon of potentialities associated with an object (7.1), we can also think of it as a penumbral sense of the trails in the horizon of continuations, weighted towards the earlier members of those trails, and in particular those trails we are are currently "moving in to" (as noted above, protentions seem to correspond to a forward-directed part of the immanent horizon).

7.4 Horizon Dynamics

Husserl says that the horizon of an experience changes over time: the "horizon is constantly in motion (*Bewegung*); with every new step of intuitive apprehension, new delineations of the object result, more precise determinations and corrections of what was anticipated" (EJ, p. 122). We can understand these changes in terms of the learning rule (3.10) and its implications for the horizon in all three senses.

Changes in background knowledge can have an immediate impact on which expected visual continuations are returned by S^{-1} , since $S^{-1}(\bar{v}) = T(k, \pi(\bar{v}))$ (cf. Sect. 3.3). Thus a change in background knowledge can have an immediate impact on the horizon of continuations, which just is the trail set returned by S^{-1} . If I hike through a particular valley and see, to my surprise, that the wildflowers are suddenly in bloom, then I will update my background knowledge so that I will begin to expect to see those wildflowers on my next hike (assuming it occurs fairly soon after the first one). Some expected visual continuations are added to the horizon of continuations associated with the entrance to that valley, and others are removed. Or again: whereas the horizon of continuations previously contained sequences of visual experiences where I move from the front to the back of the car and expect to see a dent.

Changes in background knowledge that change the horizon of continuations will often (but not always) change the horizon of potentialities. In the dent example the dent image wasn't a part of the horizon of potentialities before the accident, but it is after the accident.⁶ A change in background knowledge can also change the immanent horizon, insofar as the immanent horizon is determined by the horizon of continuations. I come to see the car from the front as a car with a dent on the back.

⁶We can also imagine cases where the horizon of expected continuations of a current visual experience changes but the static horizon of potentialities—of ways the object could appear—does not. For example, a uniform colored sphere with two dots might have a relatively "small" horizon of potentialities, consisting only of images of the ball from angles where no dot is visible and images of the ball from angles where just one dot is visible (assume the dots are placed so that there is no way to see both dots at once). In such a case, if we change where we think one of the dots is relative to the other (but also continue to believe that both dots can't be perceived at once) then the horizon of continuations relative to, say, a perception of the ball from the perspective of the first dot, will change—we will have different expectations about what we will see when the ball is moved in certain ways. Nonetheless, the horizon of potentialities, the static collection of possible ball perceptions, will remain the same.

Chapter 8 Motivation

For Husserl, one experience of an object "motivates" another when, roughly speaking, the two experiences are felt to be naturally connected in some way (since experience is transparent, we can also think of this as a natural connection between two given things or parts of a thing). As with the other concepts we consider in Husserl, motivation is complex, and comes in several varieties. We will focus on a kind of counterfactual motivation, which corresponds to what we would expect to see, were we to move in various ways with respect to an object. These "motivated possibilities" are associated with a strength that can be understood in terms of how fulfilling they would be, were they to occur. As we will see, Husserl gives a particularly detailed account of the learning rule (3.10) when discussing how these motivational strengths change with experience.

The motiv* family includes terms like *motivier* and *motivation*, which occur most frequently in *Ideas 2* (296 occurrences), *Intersubjectivity 1* (292), *First Philosophy 2* (274), *Crisis* (249), *Thing and Space* (182), and *Intersubjectivity 2* (171). However, the term occurs in Husserl's early works as well, e.g. 90 times in the *Logical Investigations*. Motivation has received less scholarly attention than the other concepts discussed above, though it has received some, in particular in [98], who interprets the concept similarly to the way I do (also see [5, 81, 83]).

8.1 Husserl's Concept of Motivation

Husserl develops a fairly complex account of motivation in earlier works like *Logical Investigations* (esp. LI 1) and *Ideas 2* (esp. Sect. 56), and goes on to use the concept frequently in his later writings (see Fig. 4.1). Though it is difficult to unify all of Husserl's examples of motivation under one concept, the general idea seems to be this: x motivates y when x and y are felt to be naturally connected in some way (e.g. they correspond to two parts of the same experienced object) so that when passing

from x to y, y feels like the natural outcome.¹ Husserl describes it as a relation whereby "certain things may or must exist, since other things have been given" (LI, p. 270). Husserl also describes it as a relation between beliefs (though his general account applies equally to perceptions and other types of intentional states):

certain objects or states of affairs of whose reality someone has actual knowledge indicate to him the reality of certain other objects or states of affairs, in the sense that his belief in the reality of the one is experienced... as motivating a belief or surmise in the reality of the other (LI, p. 184).

Husserl also says that we feel a kind of natural connection or unity between two things when they motivate each other. Some examples: my belief in the premises of an argument motivates my belief in its conclusion, and the two are unified in one argument. My perception of smoke motivates my expectation that there is fire at its source, and when I see the fire I see it as unified with the smoke. If I see the front of the house and move to the right, this motivates an experience of two windows, and I see the two sides of the house as unified parts of the one house.

Husserl describes multiple kinds of motivation, each with its own structure. He at times refers to active forms of motivation, e.g. "motivations of reason" where we explicitly think about evidentiary links between propositions. This is contrasted with various types of "passive motivation" (*Ideas 2*, p. 234) (cf. the discussion of the active/passive distinction in Chap. 2). Husserl describes passive motivation as a kind of tendency for one thing to follow another in conscious experience: "What is meant by the universal fact of 'passive motivation'? Once a connection is formed in a stream of consciousness, there then exists in this stream the tendency of a newly emerging connection, similar to a portion of the earlier one, to continue in the direction of the similarity" (*Ideas 2*, p. 234).²

These passive motivations can also be understood in several ways. In one sense, we have an immediate, immanent sense of what will occur next in a stream of consciousness. This seems to be what Husserl has in mind in the passage just quoted, and it corresponds to the forward-directed weighting or "eccentricity" of the immanent horizon *qua* field of relt relations (Sect. 7.1). We will focus on

¹Thus the motivation relation is similar to a kind of conditional probability: if I experience x and take certain actions, then y feels like it should be the natural outcome. However, Husserl is at pains to deny this in one sense: "to talk of an indication is not to presuppose a definite relation to considerations of probability" (LI 1, p. 186). I take him to mean that motivation relations do not imply actual awareness of probabilities, i.e. the kinds of explicit or "active" probability calculations one considers when gambling, solving math problems involving probabilities, or making certain kinds of decision. Motivations are always occurring; we only rarely think explicitly about probabilities.

²There are other distinctions here as well. Husserl distinguishes noetic and noematic motivation: noetically, my perception of the front of the house motivates a particular image of the side of the house; noematically this corresponds to a unity in the house itself, whereby we experience the two sides of the house as "belonging together" as parts of the same object (*Ideas 2*, p. 230). Husserl also considers associative cases where "A thought 'reminds' me of other thoughts and calls back into a memory a past lived experience" and even Freudian cases where the reason for the motivating tendency can only be brought to light by psychoanalysis (*Ideas 2*, p. 234).

passive (i.e. non-explicit) motivations in another, counterfactual sense: possibile visual images that would not surprise us, were they to occur. Husserl sometimes refers to these as "motivated possibilities."

8.2 Motivated Possibilities

I take motivated possibilities (*motivierten Möglichkeit*) to correspond to how we expect an object to look from another perspective when we move around it, while empty possibilities (*leeren Möglichkeiten*) correspond to ways we don't expect it to look. Suppose that you are looking at a desk and can't see all of its legs. That it has 4 legs is a motivated possibility; that it has 10 is an empty possibility:

motivated possibility... is to be sharply distinguished from empty possibility... It is an empty possibility that the now unseen underside of this desk here has ten legs instead of four, which is actually the case. In contrast, the number four is a motivated possibility for the determinate perception which I am in the process of effecting (*Ideas 1*, Sect. 140).

The motivated possibility corresponds to one of the most expected subsequent perceptions in the expectation gradient generated when moving to the underside of the desk, which is the output of a function of our current experience. In fact Husserl sometimes explicitly describes motivated possibilities as "dependent possibilities" (*abhängigen Möglichkeiten*), relative to actual experiences:

the actualization of x first of all makes y a motivated possibility. We have to do everywhere in this domain with these sorts of dependent possibilities, which are... dependent on posited actualities, motivated by them, and which pass over into motivated actualities... (TS, p. 84).

In my terms, motivated possibilities (in the visual case) correspond to expected visual images relative to background knowledge k, current visual experience v, and movement b, while empty possibilities correspond to unexpected visual images relative to k, v, and b:

v' is a motivated possibility relative to k, v, and b if $v' \in \theta_{\tau}(f(k, v, b))$ (8.1) v' is an empty possibility relative to k, v, and b if $v' \notin \theta_{\tau}(f(k, v, b))$ (8.2)

Given a person's knowledge k and visual experience of the top of the desk v, and given that they have moved to view the bottom of it, f will produce an expectation gradient whose motivated possibilities (those producing fulfillment above the threshold τ) include seeing the underside of the desk with four legs, and whose empty possibilities (those below threshold) include seeing the underside of the desk with 10 legs.

A collection of motivated possibilities determines a specific adumbration (5.3). Any motivated possibility is a member of the horizon of potentialities (7.1). When a motivated possibility is actualized, fulfillment (5.4) occurs; when an empty possibility is actualized, frustration occurs. When we follow the current trail in the trail set (or horizon of continuations) that constitutes an object (6.1), the first members of those trails are motivated. Of course, a binary distinction between motivated and empty possibility is a simplification. Motivations come in degrees. Husserl is quite clear about this:

the motivations differ, are more or less rich, are more or less definite or vague in content depending on whether it is a matter of physical things which are already "known" or "completely unknown," "still undiscovered" (*Ideas 1*, Sect. 47).

This is naturally captured by the formalism. Recall that the threshold function θ_{τ} is a convenient approximation, insofar as it somewhat arbitrarily sorts experiences in an expectation gradient in to those that are expected (that produce fulfillment above τ) and those that are not. But the expectation gradient itself maps visual experiences to a continuous range [0, 1] of *degrees* of fulfillment. So we can interpret the strength of a motivated possibility v' as a level of fulfillment returned by an expectation expectation gradient when applied to v':

The strength of motivation of
$$v'$$
 is $e(v')$ (8.3)

where e = f(k, v, b). So, expectation gradients can also be thought of as motivation gradients. The different heights of an expectation gradient correspond to different strengths of motivation.

8.3 Example

In the following passage, Husserl describes relations between series of bodily movements or kinesthetic sensations which are *motivating*, and expected visual images, which are *motivated*. Here refers to

motivating series...systems of kinesthetic sensations, which freely unfold... in such a way that if a free unfolding of one series of this system occurs (e.g., any movement of the eyes or fingers), then the corresponding series must unfold as motivated. In this way, from the ordered system of sensations in eye movement, in head movement freely moved, etc., there unfold...in motivated order, "images" of the thing that was perceptually apprehended... (*Ideas* 2, p. 63).

We can take eye movements $(m_1, m_2, ..., m_n)$ to be bodily movements that motivate visual images $(v_1, v_2, ..., v_n)$. To simplify matters, we can assume that as each eye movement is made, there is a single *most* expected visual image (cf. 3.2). In that case, eye movements $(m_1, m_2, ..., m_n)$ motivate visual images $(v_1, v_2, ..., v_n)$ in the sense that:

$$v_1 = max(f(k, v_0, m_1))$$

 $v_2 = max(f(k, v_1, m_2))$
:

As I move my eye to the right via movements $(m_1, m_2, ..., m_n)$ I expect visual images $(v_1, v_2, ..., v_n)$, where each visual image v_i is the most expected visual image in the expectation gradient produced by f. Something similar could be done for head movements, trunk movements, etc. In each case we tacitly understand how the relevant movements affect what we will see, even in the case where movements don't change what we expect to see (e.g. when I tap my fingers I don't usually expect what I see before me to change in light of those finger taps).

Husserl believes this kind of relation between motivating bodily movements and motivated images to be pervasive in perception:

In the essence of the apprehension itself there resides the possibility of letting the perception disperse into "possible" series of perceptions, all of which are of the following type: if the eye turns in a certain way, then so does the "image"; if it turns differently in some definite fashion, then so does the image alter differently, in correspondence. We constantly find here this two-fold articulation: kinesthetic sensations on the one side, the motivating; and the sensation of features on the other, the motivated. This holds, obviously, for touch, and similarly, everywhere (*Ideas 2*, p. 63).

In the last sentence Husserl suggests that there are (in my terms) more functions in constitutive phenomenology than the visual expectation rule alone: for example, functions from movements to expected auditory experiences or touch experiences. Recall from Sect. 6.3 that Husserl envisioned constitutive phenomenology as a vast enterprise; ways of extending this formalism to encompass more of constitute phenomenology are considered in Sect. 10.2.

8.4 Learning and Motivation

Husserl's discussion of motivation contains what is arguably his most detailed account of knowledge update. Husserl says that motivations are in a certain sense strengthened after experiences of fulfillment, and weakened after experiences of frustration. For example, an increase in "force" (*Kraft*) occurs when similar things repeatedly occur in similar circumstances. In such cases, Husserl says, the force of the prior expectation grows via a kind of inductive process:

the anticipatory belief of expectation has a differentiation of force, that is, a gradation, and this force grows with the number of inductive "instances," that is, with the frequency of what has occurred under similar circumstances (APS, p. 238).

For example, if I repeatedly see that the desk has four legs when I start from the front and move to look under it, then the strength of my expectation of the four legs grows with each inductive instance of looking under the table. In this passage Husserl describes the process using symbols:

If for example earlier in the circumstantial situation C, a b c have occurred, and in the current similar situation C', a' has occurred, then according to what we already said, b' and then c' are naturally motivated as arriving (APS, p. 239).

He goes on to say that motivations will increase in a kind of additive way. With two inductive instances, an expectation becomes "doubly motivated," and presumably also triply motivated with three instances, and so on:

If now, however, b' has actually occurred, then obviously c' is doubly motivated, since here the law gets applied once more. Thus, the occurrence of the expected b' does not only ratify the expectation... it also strengthens this expectation (APS, p. 239).

Husserl also describes conditions in which the strength of motivations decrease. In cases of frustration, where subsequent experiences fail to conform with our expectations, Husserl says there is "a diminution of the force of expectation that has been gathered through repetition" (APS, p. 240). For example, if I walk in to my apartment and find the previously-white walls unexpectedly painted orange, I will stop expecting white in the future. The "force" of the image of a white interior in this situation is diminished.

The passages above can be understood in terms of the learning rule (3.10). Consider the example shown in Fig. 8.1, and assume that what Husserl calls the "situation" C corresponds to seeing the house from the front (in my terms, v_0 together with local or indexical background knowledge k), and a, b, and c correspond to seeing the house from three angles on a walk around it. After walking once around the house (top row), background knowledge k is updated to k'. Now we go back to the front of



the house, so that the situation is the same, but at a later time (bottom row). This is what Husserl calls C' (same v_0 and assuming k' and k overlap in indexical knowledge; all that has changed is our knowledge about the house itself). On the second walk around the house, the relevant motivations are strengthened, in the sense that when the same visual images occur, they are slightly more fulfilling than they were before. The expectation gradients are narrower, and their peaks are higher.

In the case where frustration occurs, the Bayesian learning rule suggests that expectation gradients will move towards the unexpected visual images that are experienced. Recall from Fig. 3.7 how an expectation gradient can "re-center" itself around an initially unexpected state.

Although Husserl's analysis is consistent with Bayesian update in broad outline, there are some differences.

First, the specific way Husserl describes the rule—as involving "doublings" in motivation—is not plausible, insofar as such a rule would lead to unbounded growth in the strength of motivations. This is fixed with Bayesian update, where repeated confirmations produce asymptotic approach towards a maximal value.

Second, Husserl only describes how *specific* experiences like *a*, *b* and *c* become more strongly motivated with experience. However, updates to background knowledge have an impact on whole *ranges* of similar experiences. When I am shocked to discover a dent on my car I immediately come to expect a range of similar but not identical experiences of a dent on the car (I don't have perfect memory, after all). Compare the old psychological concept of stimulus generalization [72], where what an animal learns about one stimulus generalizes to similar stimuli. This suggests that expectations are not updated in an isolated way, but that expectation gradients *as a whole* are altered. This is a natural feature of Bayesian update when applied to probability distributions.

Chapter 9 Genetic Phenomenology

Genetic phenomenology studies *a priori* rules governing changes in phenomenological structures. As we saw in Chap. 4, "*Genesis*" occurs less frequently than the other terms we consider, and there is a rise in usage in Husserl's later works (notice the increase around 1920 in Fig. 4.1). The term occurs most often in *Intersubjectivity 3* (66 occurrences), APS (58), *Intersubjectivity* 2 (53), CM (45), and *Crisis Beilage* (18). Though related terms do occur in earlier works (e.g. *genetisch* occurs 27 times in *Logical Investigations*), as a technical phenomenological concept, genetic phenomenology is a latecomer to Husserl's system. Genetic phenomenology is discussed in the early secondary literature [82, 101],¹ but since the 1990s it has become a more prominent theme [5, 7, 74, 85, 102].

9.1 Husserl's Concept of Genesis

Genetic phenomenology studies the way various types of phenomenological structure change in time. In fact, it could have been called "dynamical phenomenology," and it fits naturally within this kind of dynamical systems framework (cf. Sect. 3.5). In earlier texts Husserl uses the term "genetic" to refer to contingent processes of development, e.g. biological or psychological processes (this usage was current in Germany at the time, and can be found in Dilthey and Brentano, for example [85]). All of the occurrences of "genetisch" in Logical Investigations seem to refer to this type of process, and in most of these cases Husserl is emphasizing their status as empirical and contingent.

Husserl began to use the term "genetic" in its more phenomenological sense in the 1910s, and continued to do so until the end of his life (again, see Fig. 4.1). In this usage, a genetic process is a dynamical process of phenomenology, a form of development that must have occurred for present experience to be the way it is. The

¹For further references to the early literature on genetic phenomenology, see [101, p. 167].

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way I see a house or car reflects my past history with these objects. I see the car as having a dent on its unseen back-side because I have seen that dent in the past. "The phenomenology of genesis follows the history... of the object itself..." (APS, p. 634). The project of genetic phenomenology is to uncover these kinds of historical precursors to present phenomenology, to identify the "antecedent formations" that determine the phenomenal character of current intentional experiences:

[As phenomenologists we] penetrate into the intentional constituents of experiential phenomena... [and] find intentional references leading back to a "history" and accordingly making these phenomena knowable as formations subsequent to other, essentially antecedent formations... [In doing so] we encounter a passive genesis of... manifold apperceptions (CM, p. 79).

For example, the intentional reference to the dent implies a past history where we saw the dent, which is part of the "passive genesis" of the way we see the car now ("passive" because the process concerns a non-linguistic perceptual structure; the contrast is with "active genesis" of judgement formations and other explicitly conceptual or linguistic structures; see note 2). This passive genesis informs the "intentional constituents" or "manifold apperceptions" of the present experience, which encompass our immanent sense of an object as a whole (cf. the discussion of intentionality in Sect. 5.1 and immanent horizons in Sect. 7.1).

In terms of our modeling table metaphor, genetic phenomenology describes laws governing the way experience transforms and "sculpts" the model, altering it when unexpected experiences occur, and refining it when fulfillment occurs. Using these laws we can make certain inferences about what kinds of experience must have occurred in the past in order for the model to be as it is now.

As with "horizon" and "motivation," Husserl uses the term "genesis" (and related terms) in a wide range of ways. Our focus will be on passive genesis of intentional structures, as in the passage above. He refers in this connection to "constitutive genesis" (EJ, p. 274), "genetic intentional analysis" (FTL, p. 316), and "intentional genesis" (FTL, p. 319). He also refers to "universal laws of genesis" (CM, p. 75), "genesis in conformity with eidetic laws" (CM, p. 76), "principles of constitutive genesis" (CM, p. 77), and "phenomenological genesis" (as contrasted with "genesis in the usual sense"; CM, p. 79), as covering terms for phenomenological investigation of these types of processes and the laws governing them.²

²There are additional types of genetic process I will not consider here. First, Husserl sometimes refers to "active genesis" or "genetic logic." In these cases he is emphasizing phenomenological changes brought about by explicit processes of judging, talking, reading, and thinking about objects. For example, as I learn about a painting in an art history class, my understanding of it changes. The way I now see the painting reflects a past history of discussions about the painting, a process in which active cognitions were "sedimented" in to my understanding of it. As noted in Chap. 2, these types of active process are being bracketed. Second, Husserl often refers to genetic processes as they apply to the ego, e.g. the "universal genesis of the ego" (CM, p. 75). These genetic processes refer to changes in a person's habits, personality traits, and explicit self-understandings. These accrue and change according to their own laws, whereby the phenomenologist can work backwards from a person's present self-understanding and personality to past experiences of various kind that have led to this present self-understanding.

9.2 Genetic Phenomenology as Dynamics on *K*

Husserl's many uses of "genesis" and related terms can be placed in a common dynamical framework using the formalism above, and in particular the learning rule (3.10), which, as we saw in Chaps.3 and 8, induces a dynamical system on K, such that any initial state of background knowledge k_1 will give rise to a trajectory k_1, k_2, \ldots , via repeated application of λ . As background knowledge changes in this way, all the phenomenological structures which depend on one's current state of background knowledge change as well. As we get to know how things look in a neighborhood, for example, we align our expectations accordingly, and in time we come to "know our way around," so that what we expect to see generally matches what we do see.

In fact, the learning rule can be used to explain "genetic changes" in almost every phenomenological structure considered in a numbered statement above. To see this, suppose a state of background knowledge *k* has been replaced with *k'* by the learning rule λ (3.10) after taking movement *b* with respect to initial visual image *v*. When *k* changes to *k'*, the following structures may have changed as well:

- 1. The expectation gradient $e_{k'}$ generated by f(k', v, b) (3.1).
- 2. Expected visual experiences $\theta_{\tau}(e_{k'})$ (3.3), which are motivated possibilities (8.1) that determine a specific adumbration (5.3).
- 3. A trail set t = T(k', v) (3.5) of expected visual continuations (3.4), which are constructed using $\theta_{\tau}(e_{k'})$.
- 4. A visual experience of an object $\bar{v} = S(t)$; (3.8), which is an intentional visual experience (5.1).
- 5. The immanent horizon of \bar{v} (Sect. 7.1), our sense of the whole object beyond what is immediately given in sensory experience.
- 6. The horizon $S^{-1}(\bar{v}) = t$ of visual continuations (7.2) which constitute the object given in \bar{v} (6.1).
- 7. The horizon of potentialities associated with $\bar{\nu}$ (7.1), which is a set of partial intentions of the object (5.2).

Notice that each item on the list is determined by some previous item, so that we can read this as a kind of cascade of genetic changes implied by the initial change in background knowledge. On my interpretation, these numbered items can be read as *laws of genesis* of the following form, "If background knowledge *k* is updated to k' by λ , then the impact of this on structure *X* is..."

Genetic phenomenology can also move backwards through the steps above, e.g. beginning with a visual experience \bar{v} of some object, determining what sequences of expectations are implied by this experience, and then determining what prior experiences, what history, must have occurred in order for us to have those expectations. Compare the discussion of "phenomenological archaeology" and of constitutive "building" and "unbuilding" in [100]; also see [7].
9.3 Some Applications

With these observations in place, we can make sense of many of Husserl's statements about genetic phenomenology. For example, when Husserl says that "every shape of apperception is an essential shape and has its genesis in accordance with essential laws" (APS, p. 627), this refers to item 5 in the list above: as background knowledge changes, expectations change and so the way we see a thing—the apperceptions associated with the immanent horizon of the relevant visual image—change as well.

Here is another example:

Genetical intentional analysis... is directed to the whole concrete nexus in which each particular consciousness stands, along with its intentional object as intentional... every single process of consciousness, as occurring temporally, has its own "history"—that is: its *temporal genesis* (FTL, p. 316).

We can read this as referring, at least in part, to a process which begins with reflection on an intentional experience, e.g. a visual experience \bar{v} of an object, and then considers the kinds of past processes that must have occurred in order for the current experience to be the way it is. Why do I see that as a house with two windows on the side or as having a crack that needs fixing above its left awning? Because I've seen the left side and that crack before. When I did, the learning rule was applied, and background knowledge was updated, so that in subsequently seeing the house my expectations relative to various movements, which determine how I see the house, were altered.

There is much more to genetic phenomenology than what has been emphasized here.³ All types of appearing objects—fictional objects, mathematical objects, other persons, etc.—are constituted in subjective processes, and have their horizons, adumbrations, etc., and these formations have their own specific forms of genesis. Genesis is a universal property of intentional experience; history matters in every domain of intentionality and constitutive phenomenology. As this framework is extended to other domains, a similar formalization of genetic phenomenology should become possible (though there may be interesting and subtle differences as well).

³See, for example, APS p. 631, where Husserl catalogues 7 types of "explanatory phenomenology," as well as note 2.

Chapter 10 Conclusion

We have seen that Husserl's account of the world and its constitution can be understood in terms of a kind of model of realty that we maintain and update over time. It can be thought of in terms of a vast collection of organized possibilities, synthesized together into a kind of picture of the world. It can also be thought of, metaphorically, as a kind of scale model of the world, which our experiences "move through." When expectations are fulfilled our experiences incrementally refine the model; when our expectations are frustrated, the model is changed more dramatically. The model is associated with an expectation function that generates more or less specific adumbrations as we move, a supervenience function that associates our expectations about things with how we see them, and a learning rule that updates the model based on the degree to which our expectations are fulfilled.

This apparatus formalizes and unifies a great detail of Husserl's textual corpus, describing in a relatively condensed and precise way how "constitutive phenomenology" works in a specific case (the physical world as visually experienced). But there is more work to be done. Husserl's overall ambition was to develop a comprehensive phenomenological system, which would make sense of how *all* categories of object are constituted, using a hierarchy of phenomenological laws that could ultimately serve as the foundation for the natural and social sciences (which study different kinds of appearing thing) [113]. While I think some of Husserl's foundational ambitions were problematic [116], I also think his work, once it has been cleaned up and revised in light of current evidence, has important contributions to make. Indeed, one of my main reasons for formalizing Husserl has been to make his ideas precise enough to integrate with work in other disciplines, cognitive science in particular. Although efforts to "naturalize phenomenology" are already underway, I believe the project could be further advanced by this formalization of Husserl.

After reviewing just how much of Husserl's work has been unified here, I consider ways this formalization could be extended and applied. I consider how the overall framework of Chap. 3 could be generalized to other categories of object (fictional objects, abstract objects, etc.), and how other concepts in Husserlian phenomenology, beyond those considered here, could be formalized in this framework. Finally, I show how this account of world-constitution can be connected in a fairly precise way

with recent work on predictive models in human perception, action, and cognition. These parallels could be used to visualize neuro-phenomenological dynamics using computer simulations of embodied agents. I end by considering how this framework could be used to integrate Husserlian phenomenology with research in other areas, e.g. anthropology and literary theory.

10.1 The Unifying Interpretation

We have seen how a wide variety of concepts in Husserl—mainly those relating to the five term families (intentional*, konsti*, horizon*, motiv*, and genesis*)— can be understood in terms of the unifying formalism of Chap. 3, which involves a collection of spaces and subspaces, and several rules describing how structures in these spaces are related to each other. Indeed, the formalism developed in just over 20 pages in Chap. 3 can be used to capture ideas spanning literally thousands of pages of Husserl's philosophical work, spread out over a period of more than five decades. Table 10.1 gives a sense of how many ideas in Husserl can be understood in terms of the main features of this formalism.

Expectations generated by movements	"Adumbrations" (<i>Abschattungen</i>), "motivations" (<i>Motivationen</i>), "motivated possibilities" (<i>motivierten Möglichkeiten</i>), "predelineated possibilities" (<i>vorgezeichnete</i> <i>Möglichkeit</i>), "partial intentions" (<i>Partialintentionen</i>), and "expectations" (<i>Erwartungen</i>)
The expected visual continuations or "trail sets" visual experiences of objects supervene on	"Constitutive systems" (Konstituven Systeme), "constitutive constructions" (Konstitutive Aufbau), "horizons of reference" (Horizont der Verweisung), "horizons of possible thing experience" (Horizont möglicher Dingerfahrung), "motivational systems" (Motivationssytemen), a "web of partial intentions" (Gewebe von Partialintentionen), and "indicative systems" (Hinweissysteme)
Updates to background knowledge	The "motion" (Bewegung) of the horizon, the way the horizon is constantly "shaping itself anew" (neu gestaltenden), the "strengthening and inhibiting of expectational belief" (Verstärkung und Hemmung des Erwartungsglaubens), the way the "weight of positings" (Gewicht von setzungen) changes in time, "increases" (Steigerungen) in "motivational force" (motivierende Kraft), and laws of genesis (Gesetzmäßigkeiten der Genesis)

 Table 10.1
 Basic features of the Chap. 3 formalism, and some of the Husserlian ideas they can be applied to

With this unification in hand, it is helpful to read a few long passages, where Husserl draws on all or most of the term families considered above. For example, consider what Husserl calls "intentional analysis" (cf. Chap. 5), which corresponds to the project of explicitly working out or "explicating" how, in my terms, immanent structures are related to counterfactual possibilities:

[intentional analysis] *reaches out beyond the isolated subjective processes* that are to be analyzed. By explicating their correlative horizons, it brings the highly diverse anonymous processes into the field comprising those that function "constitutively" in relation to the objective sense of the cogitatum... not only the actual but also the *potential* subjective processes, which, as such, are "implicit" and "predelineated" in the sense-producing intentionality of the actual ones. Thus alone can the phenomenologist make understandable to himself how... anything like fixed and abiding objective unities can become intended and, in particular, how this marvelous work of "constituting" identical objects is done in the case of each category of objects... The horizon structure belonging to every intentionality thus prescribes for phenomenological analysis and description methods of a totally new kind (CM, p. 48).

Husserl describes how intentional experiences—transparent experiences of things beyond us, out there in the world—involve not just actual subjective processes but also potential ones, horizons of potentialities and continuations, or trail sets. These sets constitute the "objective sense" of the intentional object. They are implicit in and predelineated by what we actually see. Also note that at the end of the passage Husserl suggests that similar constitutive analyses are possible for "each category of objects." We consider such generalizations from perception of physical things to other forms of intentional experience in the next section.

The following passage also emphasizes constitution and intentionality, but with a special focus on the constitution of the ego, the genesis of our sense of ourselves and who we are. Just as we learn more about the physical world over time, so too we learn more about ourselves over time. The passage is subtle in the sense that Husserl emphasizes not only the genesis of the ego, but also the ego's sense of all the objects in its world (which, as a kind of Leibnizian monad, it projects), so that the ego's genesis parallels the world's genesis:

life goes on as a motivated course of particular constitutive performances with a multiplicity of particular motivations and motivation systems, which according to *universal laws of genesis*, produce a unity of *universal genesis of the ego*. The ego constitutes himself for himself in, so to speak, the unity of a 'history'... the *constitutive systems*... by virtue of which such and such objects and categories of objects exist for him, are themselves possible only within the frame of a genesis in conformity with laws. At the same time they are bound, in their constituting, by the universal genetic form that makes the concrete ego (the monad) possible as a unity... (CM, pp. 75–76).

Genetic analysis is similar to intentional analysis. In both cases, we can think of ourselves as phenomenological archaeologists, tracing out a genesis of sense (in part, our sense of *ourselves*), teasing out layers of meaning and how they originated.

10.2 Other Constitutive Domains

Recall from Chap. 6 that Husserl conceived of constitutive phenomenology as a vast project encompassing all categories of experienced object. Husserl pursued constitutional investigations of abstract objects like numbers; social formations like institutions and "worlds of culture"; time; the self; other persons; and art objects. Each category involves a specific type of intentionality with a specific "horizon structure" (thus we have an "arithmetical horizon," "value horizon," "eidetic horizon," "horizon of the pure Ego," etc.)¹ Each is also governed by specific laws of genesis, and should thus be studied in a distinctive way. In discussing intentional analysis, Husserl describes

how this marvelous work of 'constituting' identical objects is done *in the case of each category of objects*—that is to say: how, in the case of each category, the constitutive conscious life looks... The horizon structure belonging to every intentionality thus prescribes for phenomenological analysis and description *methods of a totally new kind* (CM, p. 48, Husserl's emphasis).

Even in the physical case we have maintained a fairly narrow focus. We have considered our model of stationary objects on the surface of the Earth, but of course our internal model of physical reality involves much more than that. We understand that glass shatters when it drops, that flowers bloom in Spring, that certain animals are dangerous, that people are typically not on the roofs of houses, that some people are more prone to anger than others, and so on (cf. what psychologists refer to as "intuitive physics," "folk biology," and "folk psychology"). We maintain internal models of a *dynamic* world, one populated by entities (including biological entities) that change and interact in complex ways, and which can be understood at multiple spatial and temporal scales. Moreover, we did not consider how things appear relative to different sensory modalities, let alone multi-modal forms of perception (recall the simplifying assumptions described in Chap. 2; there is interesting work to be done relaxing each of those assumptions within this framework).

Moving from physical to non-physical cases raises even more difficult questions. When we are aware of objects in any ontological category—numbers, concepts, other people, institutions, etc.—specific variants on the structures described above seem to be at work: special types of intention, horizon, motivation, and genetic law. In each case, we develop and refine a kind of internal model of the domain in question, and that model influences subsequent experience. When reading a book or watching a film, we build up a sense of the characters and places in a fictional world. When learning about some period of history or some new mathematical topic, we build up a sense of the subject matter, and over time begin to feel more comfortable in it; we eventually "know our way around" a difficult text or topic. We can discuss the topic more fluidly, we can field questions with ease, we are better prepared to

¹These quotes are repeated from Chap. 7, and are taken from *Ideas 1* and CM.

engage in debate. In each of these cases, we feel ourselves to be interacting with things that exist outside of us (and that are thus "modelled" by us in some way). As Tragesser says in the context of mathematical phenomenology, "I am aware... that here is something... that I can not arbitrarily ascribe properties to, here is something having a life independent of my whims" ([92], pp. 30–31). In these and other cases something analogous to the world model takes form (perhaps they are in some sense additions to it)—we build up a model of a movie, a person, a period of history, an area of math, a subject matter. It seems the same kinds of metaphors should apply: these models are more sharply tuned in some "areas" than others, they have varying "pixel density," and they are changed as we "move" through them.

In the physical case we can, using the formalism above, make these ideas relatively precise using the concept of an expectation gradient. As I walk through a familiar neighborhood, low-dispersion, "peaked" expectation gradients are produced by the expectation function. I expect specific things as I walk around. But it is not clear how exactly this generalizes to the cognitive case. What is the analogue of bodily movement for someone who "knows her way around" Russian history? What is a "movement" relative to her present thoughts of Pushkin or Stalin? In what sense does she have expectations as she "moves" through a conversation or thought process? If we do have expectations in such cases, can we describe functions analogous to f that generate those expectations? In what ways are expectations confirmed or disconfirmed in these cases? Clearly confirmation and disconfirmation occur: I can check what I say about Russian history or a math problem by consulting a book, googling a query, or observing an interlocutor's nod. In these ways there is a *reality* beyond me that parallels physical reality. Moreover, the relevant feedback helps me build up a map or model of that reality. But the feedback is less continuous, more varied, and more complex, or so it seems. So the questions stand: how are nonphysical entities constituted? What are their horizons? What is their genesis?

I think these are fascinating questions. Developing this interpretation of Husserl further would require developing variants (or perhaps, generalizations) of the expectation rule (3.2), supervenience function (3.8), learning rule (3.10), and associated concepts. It would also draw on Husserl's own extensive discussions of the various regions of constitutive phenomenology (cf. the end of Chap. 6), as well as an expanding body of work in the secondary literature that develops these ideas. Examples include the constitution of abstracta [89, 92, 94], fictional objects and ontologies [30, 87], other persons [83], and social structures [99]. (Also recall that Gurwitsch had the rudiments of an account of these different domains and their relationships with his theory of "orders of existence"; see Chap. 1, note 4). Stein has some particularly interesting examples of fulfillment and frustration with respect to other people, e.g. the "series of corroborating and correcting empathic acts" [83, p. 86] by which we build up a sense of a person's character, whether they are wise, honest, vindictive, etc. Walsh, discussing cases like this, notes that what I call "expectation gradients"

are narrower when interacting with other persons than they are when interacting with inert physical objects:

the range of expectations that characterizes my awareness of a bag blowing around in the wind or a rock rolling down a hill is much narrower than it is when I see a person walking down the street or a student sitting at desk... I expect the student to go about reading or writing, and so on. But I am not surprised in the least when the student suddenly reels backward to stretch her arms, nor am I surprised when the pedestrian changes direction abruptly and stops at a newsstand. I would, however, be quite shocked to see a tumbling rock suddenly cease tumbling, or a blowing bag suddenly begin jerking about at right angles [99, p. 223].

10.3 Other Concepts in Husserlian Phenomenology

There is more work to do applying this framework to Husserl interpretation. Some areas for future research have already been noted above, in particular: further study of the relationship between the rules I describe here and transcendental laws (Chap. 1, note 2), integrating this account of world-constitution with Gurwitsch's account of orders of existence (Chap. 2, note 4), further unpacking the concept of background knowledge in relation to the many other authors who discuss it (Chap. 3, note 12), using this account to refine Husserl's theory of time consciousness (Chap. 5, note 4), and integrating this account with the literature on peripheral experience and inattention (cf. the discussion of immanent horizons in Chap. 7).

One feature of the analysis—dispersion of expectation gradients—is particularly fertile in terms of further work formalizing Husserl's ideas. We have treated expectation gradients as having a kind of width or dispersion, which roughly corresponds to the widths of the curves in Fig. 10.1. Assuming the concept of dispersion were made sufficiently precise, it could be used to explain several additional concepts in Husserlian phenomenology.

What Husserl calls the <u>doxic mode</u> of an experienced object corresponds to our confidence in it being the way it appears. Seeing a familiar house in broad daylight has a doxic mode of "simple belief certainty." An obscured object in the shadows,



Fig. 10.1 An expectation gradient "sharpening" around a point in *V*. Background knowledge has been updated in such a way that expectations about subsequent visual experience have become more specific and fulfilling

which we believe might be a cat but are not sure about, has a doxic mode of uncertainty. Doxic modes could be formalized in terms of mean dispersion of expectation gradients relative to object-tracking body movements: when we are certain about a thing, mean dispersion is low (we have specific expectations about what we will see when we move around it), whereas if we are uncertain about an object mean dispersion will be higher (we are not sure what the shadowy figure is and thus of what we will see as we move toward it). Similarly with familiarity (narrow mean dispersion) and unfamiliarity (wider mean dispersion), which are also Husserlian themes.

When a doxic mode of simple certainty changes to some form of doubt, Husserl says a <u>modalization</u> has occurred. For example, what I thought was a person in the distance might turn out to be a tree rustling in the wind. Here we have a change in background knowledge leading to a widening of expectation gradients: I was relatively sure about what I would see on moving closer to the object, but now I am less sure.²

Husserl contrasts perfect evidence, where we have no doubts about a thing we experience, with imperfect evidence, where there is some uncertainty (CM, p. 12). Perfect evidence is an ideal limiting case in the domain of physical thing perception: we can never see all of a physical thing at once, so we can never be perfectly sure about how it looks an all sides. Imperfect evidence comes in degrees: as we learn more about an object, we "fill in" our knowledge of it. We can interpret this filling in as a progressive reduction in mean dispersion, where expectation gradients become more and more peaked around specific expectations. Perfect evidence then corresponds to the limiting case of an expectation gradient which assigns a maximum value to one point and a 0 everywhere else (something like a Dirac delta function in mathematics): we will not be at all surprised when the thing is as we expect to be, and will be utterly surprised by all else, since we know exactly how the object looks on all sides. Such an expectation gradient is a theoretical limit that expectation gradients tend towards (but never attain) as we learn: consistently with Husserl's doctrine that perfect evidence of external things is impossible for finite beings.

More generally, further developing this formalism using probability theory and information theory (which associates being informative with being surprising) could lead to even more precise formulations of Husserl's theory, and could provide an opportunity to enrich Husserlian concepts using insights from those areas of mathematics.

²There are interesting related cases, for example Husserl's example of a person we find out is a wax dummy, where "two perceptual interpretations... interpenetrate in conflicting fashion" (LI V, 27). As these perceptual interpretations compete, mean dispersion (and various other constructs, e.g. the horizon of potentialities) should fluctuate accordingly: dispersion should remain fairly wide while we are unsure of what we are seeing (we remain open to more possibilities in this period), then "lock down" and become narrower once we have settled on an interpretation (we now have specific expectations about the thing).

10.4 Naturalized Phenomenology

Husserl's theory of the dynamical laws in virtue of which a person's sense of reality is constituted can be linked with a parallel story about the dynamical laws governing the development of an agent's internal neural model of its environment. In some ways this goes against the spirit of Husserl's phenomenology, given his anti-naturalist arguments (cf. Chap. 2, note 7), but the fact remains that there is a remarkable isomorphism between Husserl's constitutive phenomenology and theories of expectation and prediction in the brain, as we will see. The general project of linking phenomenology with empirical work in psychology, neuroscience, and the cognitive sciences broadly has come to be known as "naturalized phenomenology."³ Among those interested in naturalizing phenomenology [88, 95], for example, can be understood as a specific form of naturalized phenomenology that emphasizes links between Husserlian phenomenology and neuroscience by way of dynamical systems theory.

Neurophenomenology is a natural counterpart to this reading of Husserl, which explicitly defines dynamical rules on spaces of possible experiences and their mereological parts. I have argued elsewhere that the kinds of laws described in Chap. 3 can be derived from dynamical laws governing neural activity in an embodied brain [110]. To make this connection more precisely, we can assume that conscious states supervene on brain states, i.e. that there is a mapping from some subset B' of the space of brain states B to the space of conscious states C. We can also assume that subspaces of B' correspond to subspaces of C. For example, activity in the visual cortex may determine visual images in V (a subspace of C). Patterns of synaptic connectivity that shape the evolution of neural activity correspond to states of background knowledge that shape the way conscious processes unfold.

However, the story about world-constitution developed here can be connected in an even more detailed way with cognitive research, in particular theories of predictive coding in the brain. The idea that the brain constructs a model of the environment extends at least as far back as Helmholtz, who famously described perception as a kind of unconscious inference or construction, whereby what we see goes beyond immediate sensation to encompass our implicit understandings of things. In the late 1940s, Tolman [91] showed that behavioral data (concerning rats in various kinds of maze) imply that agents navigate environments using internal "cognitive maps," rather than chains of stimulus-response associations. In recent decades it has become possible to study these kinds of maps using mathematical and computational models. Frameworks for describing unconscious inferences and internal maps in the brain include model-based reinforcement learning ([27, 68]) and Bayesian models of the brain as a prediction machine ([9, 22]). The general idea is that models of the

³The kind of pluralist approach to phenomenology I am defending has gone by a number of names, including "Convergent phenomenology" [57], "Reciprocal constraints" [95] the "Natural Method" [20], and "Mutual Enlightenment" [23]. For more details on these ideas and how they relate to phenomenology see [116].

environment encoded in various parts of the brain (hippocampus, cortex, cerebellum, parts of the basal ganglia) generate predictions and compare those predictions with actual occurrences. The "prediction errors" that result are used to update the brain's internal models so that they can more accurately predict events in the future. Andy Clark, in an influential recent paper that synthesizes much of this literature, has described expectation and prediction as essential hallmarks of brain function:

Brains... are essentially prediction machines. They are bundles of cells that support perception and action by constantly attempting to match incoming sensory inputs with top-down expectations or predictions. This is achieved using a hierarchical generative model that aims to minimize prediction error within a bidirectional cascade of cortical processing. Such accounts offer a unifying model of perception and action ([9], p. 1)

The predictive models Clark describes can be mapped on to the Husserlian formalism in a fairly direct way (similar ideas are developed in [28], and in ongoing work by Michael Madary). When an agent does something, for example move in response to a stimulus, neural models generate predictions about what subsequent sensory states will occur. These predictions are compared with sensory states that actually occur, and prediction error is used to update the parameters of the model, so that it will make more accurate predictions in the future. The predictions made by the neural model are analogous to the adumbrations produced by the phenomenological expectation rule (3.2). Prediction error is analogous to degree of fulfillment (5.5), with low error corresponding to fulfillment and high error corresponding to frustration (5.4). The process by which error is used to update the neural model, so that it is more accurate in the future, is analogous to the way the phenomenological learning rule (3.10) is used to update background knowledge, so that fulfillment tends to occur more often in a stable environment. With these parallels in place, various derived constructs in Husserl can be linked with specific features of a predictive model in the brain; for example, the horizon of potentialities (7.1) can be linked with the set of predictions a neural model would make relative to a range of possible movements with respect to an object.

An advantage of these kinds of neuro-phenomenological connections is that they make it possible to study abstract features of phenomenology in a concrete setting. Recall from Chap. 3 that we could only state the *form* of the expectation rule, trail set function, supervenience function, and learning rule (their domains and ranges). By developing neural network simulations of agents in virtual environments, we can produce approximated but concrete models of these functions, and use them to visualize the relevant neuro-phenomenological dynamics. As an agent moves around in its virtual environment, a trajectory unfolds in its network space N, which approximates the brain space B'. Since brain states in B' can be mapped to conscious states in C, this network space and its subspaces can simultaneously be thought of as an approximation of the phenomenological space C and its subspaces (there are simplifications in this description, but it serves to convey the idea; for a more detailed discussion see [110, 111]). As a trajectory unfolds in this simulated neuro-phenomenological space, shapes or manifolds take form which correspond to different objects in the agent's environment. As the agent learns about its environment these manifolds change. The

agent's predictions about subsequent states can be visualized as a kind of halo "in front of" the unfolding trajectory. As the agent's predictions become more accurate, the width of this halo contracts. In this way a literal picture of an agent's internal model of its environment can be developed and viewed. A simple model along these lines is described in [115], as is the notion that this type of model can serve as a "bridge metaphor," connecting our concrete intuitions about neural dynamics with our more abstract intuitions about phenomenological dynamics.

I have been emphasizing relationships between predictive coding in the brain and the formalism of Chap. 3 in a programatic way. But these topics have also been studied in specific sensory and cognitive domains, which suggests that this neurophenomenological project could be worked out and applied in empirically tractable settings. Expectation has been studied in vision [4, 71], audition [78], touch [93], motor behavior [105], music [45], event perception [118], statistical learning [14], and language [50]. The vast Bayesian literature in cognitive science was already mentioned in Chap. 3 (for review see [86]). A detailed account of synesthesia in terms of predictive models in the brain which are more or less "counterfactually rich" (i.e. models which, in my terms, produce complex trail sets) has recently been developed [77]. In perception, there is evidence that what we hear in speech is based more on expectations than on actual sensory inputs [78]. In the case of language, a prominent theory of syntactic processing emphasizes "incremental probabilistic disambiguation with expectations about upcoming events in a sentence" [50]. In the case of music: "expectations for melodies and harmonies in tonal music are perhaps the most studied aspect of music cognition" [10, p. 33]). Krumhansl (citing Meyer [61]) describes how

expectations play the central psychological role in musical emotions. Some points in the music engender strong expectations for continuation, creating a sense of tension and instability. Other points in the music fulfill expectations, and units are perceived as closed off and completed. Musical meaning and emotion depend on how the actual events in the music play against this background of expectations [45, p. 46].

Studies in this broad literature (or set of literatures) involve detailed experimental and computational work, which emphasize learning and expectation in the same way Husserl does, and thus provide concrete cases and empirical data that could be used to further enrich the kind of dynamical, visualizable neuro-phenomenology described above.

10.5 Other Applications

There are other important strands of philosophy and psychology that cohere well with the story above, and in particular the idea that experiences of objects are dependent on our expectations about how they would appear relative to different sequences of bodily movements. Relevant thinkers include [13, 19, 69]. Evans, for example, says "A perceptual input... cannot have a spatial significance for an organism except in

so far as it has a place in... a complex network of input–output connections." Gibson defines affordances as relations between a perceived object and a perceiver's abilities (see [8]). O' Regan and Noë [69] declare that "seeing is a way of acting" which occurs when an organism masters "governing laws of sensorimotor contingency." Each of these views is itself embedded in a complex theoretical framework, so that working out their connections to this kind of formalized Husserlian phenomenology is a project unto itself (some work along similar lines has begun; see [56]). Benefits could accrue to both sides: these theorists work out details and draw on sources absent from Husserl, and Husserl embeds accounts like this in a comprehensive phenomenological system.

These ideas could also be applied to other areas where phenomenology has previously been applied, e.g. mathematics [89, 92], anthropology and archeology [15, 90], qualitative research in the social sciences [11, 25], and literary theory [42]. In every case I believe a formalized phenomenological apparatus could enhance existing symbioses. To give just one example, it might be possible use the formalism above to characterize in a comparatively precise way how literary works sustains suspense and interest, by, among other things, maintaining the average dispersion of a reader's expectation gradients within a critical range.

Appendix A Search Method

Searches of the Japanese Husserl database return keyword occurrence counts for each *Husserliana* volume in the database's registry. Occurrences in the Editor's prefaces and footnotes were removed from the analysis. Searched *Husserliana* volumes are listed in Fig. A.1, as well as the years these volumes encompass in Husserl's work (year spans for a given *Husserliana* volume were taken from the title page or Editor's introduction).

I only made use of terms which were searched for in all 23 *Husserliana* volumes in the Japanese database. For example, I did not consider *genetisch*, since it was only searched for in 5 volumes. For horizon* I did not use all available prefixes in the search (e.g. *Welthorizont*). Similarly for konsti* (e.g. *Selbstkonstitution*). In each case I considered the omitted terms separately, but did not notice any qualitative difference in the results.

A computer program was written to process the search data from these volumes and produce Fig. 4.1. The program begins with a *data-table* containing 5-year intervals between 1886 and 1935 as rows, and the five term families as columns. Cells of the table were initialized to 0, and a program was written to populate the table with *mean occurrences per 5-year interval*. The structure of the program is shown as pseudocode in Algorithm 1.

For example, since terms in the horizon* term family occur 55 times in *Husserliana* 16 (TS), which is associated with the year 1907, 55 is initially added to the 1905–1910 row/horizon* column of the table. For *Husserliana* volumes associated with multiple 5 year intervals, these occurrence counts are evenly divided across the intervals, with partial overlaps scaled accordingly (e.g. 10 occurrence for a book spanning 1899–1909 would add one occurrences to the row for 1895–1900, 5 to the row for 1900–1905, and 4 to the row for 1905–1910). After total occurrence counts are produced in this way, each 5-year interval is divided by the total number of pages for all *Husserliana* volumes that occur in that year (a number which is produced by scaling and adding page counts for *Husserliana* volumes in a manner similar to the way occurrence counts are computed). For example, in 1925–1930, terms in genesis* occur 88 times in the searched volumes, and 939 pages of searched volumes occur



Fig. A.1 Husserliana volumes that were searched, and what years of Husserl's career they span

```
for term family \in {intentional*, konsti*, horizon*, motiv*, genesis*} do
```



Algorithm 1: Pseudo-code for the program that produced Fig. 4.1, i.e. that determines mean occurrences of terms in each term family in 5-year intervals between 1895 and 1935

in those years, so that the mean occurrences per page of terms in genesis* in that 5-year interval is 8/939 = 0.09.

There are a number of simplifications and potential sources of inaccuracy in this data analysis. First, only half of the currently available *Husserliana* volumes are included in the search results (and moreover, *Husserliana* does not encompass everything Husserl wrote). Some important texts that were excluded are *Philosophy of Arithmetic* (Hua 12) and *Formal and Transcendental Logic* (Hua 16). Both were, however, consulted. Second, texts associated with a single year in this analysis (e.g. *Cartesian Meditations*) actually encompass multiple years of writing. Third, I treat keyword occurrences as being evenly distributed between the years in a span, though they are not. For example, Hua 23 spans the years 1898–1925, but only a handful of passages in that volume are from 1898. Finally, the terms themselves have multiple meanings. E.g. "*Genesis*" sometimes has a technical meaning for phenomenology and sometimes simply pertains to any developmental process. Husserl himself distinguishes five senses of "*intentional*." This issue of multiple meanings of Husserl's terms is addressed in the main text.

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