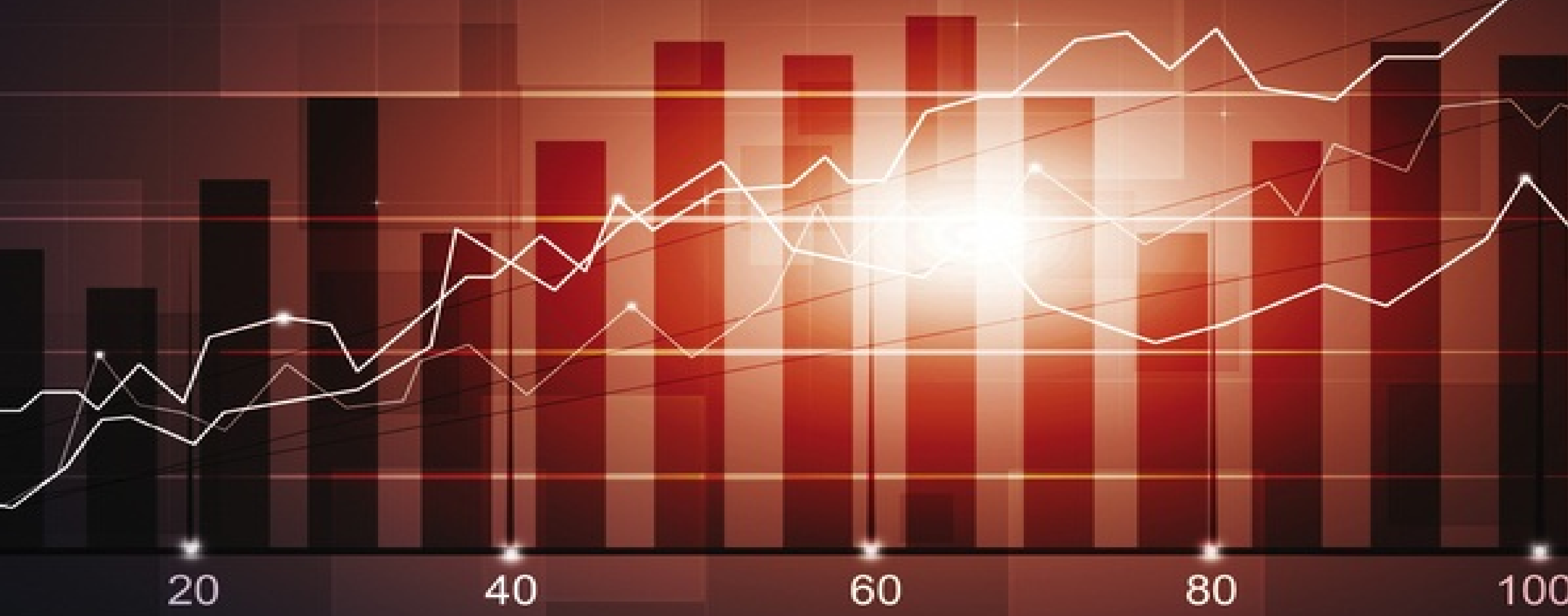
Microeconometrics in Business Management

Jerzy Witold Wiśniewski



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Preface

Decision-making requires having access to various increasingly effective tools serving preparation of such processes. *Econometrics* provides economists with efficient decision-making instruments.[1] The world economy crisis between 1929 and 1933 has triggered necessity of continual analysis and prediction of the business cycle. In answer to the demand, first econometric tools, the so-called *economic barometers*, emerged.

Econometrics is a branch of economic studies. As Oskar Lange[2] wrote, "econometrics is a science concerned with assessment of specific quantitative regularities occurring in the economy, through the use of statistical methods. (…) it combines economic theory with statistics, and seeks to use mathematical-statistical methods for assigning a specific quantitative expression to general diagrammatic regularities determined by economic theories."

On one side, econometrics creates research tools for economics and management, called the models, on the other, it uses those instruments to study economic phenomena and processes. As such, two branches can be distinguished: *the theory of econometrics*, which provides such models along with the methods for their construction and operation, and *applied econometrics*, which uses theoretical models for specific applications in a given economy.

Affiliation of econometrics usage with macroeconomic research falls within the realm of *macroeconometrics*. In contrast, the use of economic models for a single business entity, called *microeconometrics*, is a subject matter within a part of applied econometrics.[3] *Microeconometrics***,** therefore, deals with creation and customization of the tools dedicated to mathematical statistics, which are applicable to business entities, including companies of all sizes, as well as with construction of empirical models designed to support decision-making processes of those economic entities.

Efficiency of microeconometrics is conditioned by a respectively proper background knowledge of microeconomics and of enterprise theory as well as by knowledge of basic economic entity management on the part of the constructor of *empirical microeconometric models*. One important prerequisite, on which the design of such models is based, is correct measurement of the phenomena and processes occurring in an enterprise and within its environment. Mass processes, during which statistical regularities are being observed, can be the subject to such modeling.

The purpose of this book is to present econometric tools, which are practical for management of variably sized enterprises. Familiarity with business processes constitutes the basis for such structures. It is necessary to have knowledge about the possibilities of measuring the characteristics, which are expressed directly using numbers on a relativity scale, as well as about the quality characteristics, also called the descriptive ones.

Economists often divide statistical characteristics into quantitative (measurable) and qualitative (immeasurable) ones. In the eyes of measurement theory, all mass processes are measurable; they can, therefore, be reflected by numbers. The numbers, however, have various contents, depending on their affiliation with a corresponding measurement scale, often also called a measurement level.[4]

The magnitudes of measurement scales stem from the sense and meaning of the numbers, which result as a consequence of a given measurement. The following measurement scales can be distinguished[5]:

- nominal
- ordinal (ranked)
- range (interval)
- ratio (quotient).

On the nominal scale, numbers are used for labeling, identification, or classification of disjoint categories. Resultant numbers play the role of symbols, which usually substitute names or verbal descriptions. On this scale, the only acceptable relations between the numbers are (i) equality of elements within distinguished category frames or (ii) variety of disjoint categories. Summing the numbers up is the only acceptable arithmetic procedure. Out of the available statistical techniques, only those based on counting are allowed. Some examples of nominal scale measurements can be, for instance, citizen's Social Security Numbers, tax identification numbers, postal codes, phone numbers, and so on.

Within the nominal scale, attention is drawn onto a special case – the dichotomous scale, which is commonly used in statistics to extract disjoint category pairs. A simultaneous definition of an A variant of a given phenomenon allows classification of events in a variant form: A or $\bar{A}$ (not A). Assignment of the number 1 to each observation A, while the number 0 is assigned to observation $\bar{A}$, forms the so-called dummy variable.

On the ordinal scale, numbers are the ranks indicating the order of elements or characteristics of a given phenomenon. Not only the ranks reflect the elements' irregularities, but also their arrangement in terms of a considered ownership. The categories of the phenomenon being considered, in this case, are disjoint. The numbers on this scale are comparable on a modular basis. However, they are of a relative (not absolute) importance, since the distances between the ranks are not known. What is more, the distances between the adjacent ranks are not equal. As such, comparison of the ranks can be done by finding equality and majority relationships, and thus, minority relationships as well. Determining the distances between the ranks, that is, determining how they differ, is not possible.

The range scale, also called the *interval scale,* has the ordinal scale's characteristics, but the distances between the numbers are known. What is more, the distances between each pair of the adjacent figures are equal. Zero natural, as a zero on the interval scale, is contractual in nature. An example of such scales application can be temperature measurement in Celsius degrees; zero, in this case, is the temperature of water's change of state from liquid to solid and the other way around. Therefore, the numbers on this scale[6] are the distances from the contractual zero, which prevents the use of the relationship aspect ratio (their division).

The numbers belonging to the *ratio scale* are the distances from zero. The *quotient scale* has properties of all weaker scales and of a natural zero point. All arithmetic operations, including multiplication and division, are thus allowed. Using statistical techniques is also possible. For instance, production capacity of natural or valuable units, employment size, wages, demand, prices, and so on, can be examples of a ratio measurement.

Conversion of the numbers from a stronger scale into the numbers belonging to a weaker one is possible; however, it involves partial loss of numerical information. Occasionally, when the numbers on the stronger scale carry excess information and therefore create the so-called information noise, such operations are necessary.

During measurement, possible errors have to be taken into account. Errors can be divided into two categories: *random errors*, that is, *accidental ones* and *systematic (tendentious) ones*. Random errors are an inherent feature of measurement. They result from imperfections of measuring tools as well as from imperfections of the person performing the measurement. Random measurement errors are characterized by a nominal distribution with a zero mathematical expectation.[7] This means that positive and negative errors, during a long-term measurement, compensate each other.

Systematic errors result in a faulty measurement result, which signifies excess or insufficiency. This type of an error is caused by human interest in falsifying measurement results, usually done by, for example, providing a taxable income lower than the actual one or a company profit write-up, in hope for a higher reward.

It is also necessary to draw attention to two types of measurement: direct and indirect measurements. Direct measurement involves using a suitable measuring device to determine the measurements of features or things. For example, placing a few slices of cheese on a weight scale allows measurement of its weight. Using a graduated vessel filled with liquid allows measurement of its volume. Indirect measurement occurs in at least two stages. In the first, a physical measurement of features or things is conducted; then in the second, an appropriate system of weights is used to determine those measures in other units. For example, in the first stage, we determine the weight of the cheese. In the second, we use a system of weights, in this case the prices. This allows us to determine the value of that cheese in monetary units; in other words, it is transition from natural units onto economic ones.

A significant part of economic measurements is done indirectly, which is more risky than direct measurement. With indirect measurement, there is, at least, some accumulation of random errors. It is much worse when systematic errors accumulate during both stages. The value as well as suitability of the obtained statistical material may then be scant.

Notes

1 In the global school of economics, authorship of the word *econometrics* is attributed to a Norwegian economist and a professor at the University of Oslo, the co-founder of

Econometric Society and a 1969 Nobel Prize winner – Ragnar Frisch (1895–1973). The first textbook in which the term *econometrics* has ever been used is "Econometrics Outline and Bookkeeping Theory," written by Paweł Ciompa (1867–1913) a professor at the Higher School of Economics in Cracow. See: Universal Encyclopedia PWN, 3rd ed., Vol. 1, Warsaw 1983.

2 See: Lange, O. (1967) Introduction to Econometrics, PWN, 4th ed. Warsaw, pp. 11 and up.

3 In this work, issues of microeconometrics will be presented as a research area of econometrics, though it will not treat microeconometrics as an area of microdata analysis. Microdata may come from various noneconomic areas. In microeconometrics, the following subfields can be distinguished: enterprise microeconomics, household microeconomics, and institutional microeconomics.

4 Foundations for the measurement theory were developed by S.S. Stevens in his work titled *On the Theory of Scales Measurement*, "Science" (1964) Vol. 103, No. 2684. See: Wiśniewski J.W. (1986), *Econometric Study of Qualitative Phenomena*, "Methodological Study", Chapter 1, UMK, Toruń.

5 The scales are listed from the weakest to the strongest, according to information content, generated figures, as well as to application of analytical tools.

6 In statistics, transformations of the random variable, called *normalization* or *standardization*, are often used. They consist in such transformation of the variable, which generates a new zero on the original mean level. Therefore, the standardized variable as well as the normalized one both belong to the results of a range measurement (the interval scale).

7 A German mathematician, astronomer, physicist, geodesist, and a professor at the University of Göttingen, Carl Friedrich Gauss (b. April, 30th 1777, d. February, 23rd 1855), during his tests on the random measurement errors, discovered the nominal distribution curve (the Gaussian curve) and described it with an appropriate density function.

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1
A single-equation econometric model

1.1 The essence of an econometric model

An econometric model, in the form of a single stochastic equation, is a primary tool in econometrics. The subject of its description consists of a dependent variable Y with y_t observations, where t is the statistical observation's number ($t = 1, \ldots, n$) and n is the sample size. The dependent variable is economic in character and represents a specific economic category [1].[1]

Explanatory variables marked as $X_1, \ldots, X_j, \ldots, X_k$, essentially, represent the factors causing variations of the dependent variable Y. Also, some statistical observations are assigned to each dependent variables: x_{t1}, representing the variable $X_1, \ldots, x_{tj}$, representing the variable $X_j, \ldots$ as well as x_{tk} for the variable X_k.

The most general form of a model with a single stochastic equation can be written as follows:

$$y_t = f\left(x_{t1}, \ldots, x_{tj}, \ldots x_{tk}, \eta_t\right), \tag{1.1}$$

with one more variable η_t, the random component. This random component gives the model its stochastic character and results from the following:

- The random nature of economic phenomena and processes.
- A conscious and purposeful resignation from complying with less important and statistically insignificant factors.
- Inaccuracies during observation and measurement of economic phenomena and processes.
- A lack of full precision in determining the equation's analytical form.
- Round-ups in the course of numerical calculations during application of the procedures used to estimate the model's parameters.

The most frequently used analytical form of the model is linear in character

$$y_t = \alpha_0 + \alpha_1 x_{t1} + \cdots + \alpha_j x_{tj} + \cdots + \alpha_k x_{tk} + \eta_t. \tag{1.2}$$

A shorter version of this model can be written as follows:

$$y_t = \sum_{j=0}^{k} \alpha_j x_{tj} + \eta_t. \tag{1.3}$$

In Equations 1.2 and 1.3, some structural parameters ($\alpha_1, \ldots, \alpha_j, \ldots, \alpha_k$) appear, which are the measures of each explanatory variable's impact on the dependent variable.[2] The parameter α_0 is called a constant term of the model and cannot always be interpreted in economic terms.

Product models, in literature also called multiplicative models, are the most commonly used ones among the nonlinear analytical forms of an econometric model. The first variant of this model is the power-law model, in the form

$$y_t = \alpha_0 x_{t1}^{\alpha_1} \cdots x_{tj}^{\alpha_j} \cdots x_{tk}^{\alpha_k} e^{\eta_t}, \tag{1.4}$$

which concisely can be written as

$$y_t = \alpha_0 \prod_{j=1}^{k} x_{tj}^{\alpha_j} e^{\eta_t}. \tag{1.5}$$

The second multiplicative model is exponential in the form

$$y_t = \alpha_0 \alpha_1^{x_{t1}} \cdots \alpha_j^{x_{tj}} \cdots \alpha_k^{x_{tk}} e^{\eta_t}. \tag{1.6}$$

Briefly, this model can be written as follows:

$$y_t = \prod_{j=0}^{k} \alpha_j^{x_{tj}} e^{\eta_t}. \tag{1.7}$$

Finally, it is possible to use a type of a mixed multiplicative model, that is, a model of a power-exponential character. An exemplary power-exponential[3] model can be written as follows:

$$y_t = \alpha_0 x_{t1}^{\alpha_1} x_{t2}^{\alpha_2} \alpha_3^{x_{t3}} \alpha_4^{x_{t4}} e^{\eta_t}. \tag{1.8}$$

Construction of an econometric model occurs in the following five subsequent stages:

1. specification of the model,
2. identification of the model,
3. estimation of the model's parameters,
4. verification of the model,
5. application of the model

During the specification stage, the purpose and the scope of the test are established as well as a set of the model's variables: a dependent variable and explanatory variables. A measurement method for those variables is then indicated. All statistical data necessary for the test, such as the time series or cross-sectional data,[4] have to be collected. Finally, it is necessary to formulate the model's hypothesis in an adequate analytical form of an equation(s).[5] Such a specification results in a hypothetical (theoretical) econometric model.

Model identification becomes necessary in case of a model composed of many stochastic equations. Mathematical accuracy of the model's structure, which is discussed in Chapter 2, needs to be settled.

Estimation of the model's parameters involves a selection of an estimator, which is appropriate for the hypothetical econometric model.[6] An estimator is used for numerical calculations. Using all available statistical information for calculations, the model's structural parameters and its stochastic structure's parameters are estimated.

Model verification involves checking its statistical quality and examining the empirical model's economic logic. Analysis of statistical quality requires using specialized goodness measures[7] and various statistical tests.

Model application is using the empirical model in accordance with its purpose and its constructional aim. This can serve as a support in economic decision-making. Another line of the model's use is simulation. The models based on time series are most frequently used in forecasting of economic phenomena and processes.

1.2 Specification of an econometric model

From an economic viewpoint, econometric model's specification is the key to its proper construction. Any deficiencies in specification can result innumerous defects of that model.

Defining an economic system and its components is fundamental in specification. The components of an economic system are represented by variables: the dependent and the explanatory ones. In the model represented by a dependent variable, many of those components can be defined and measured in several ways. The dependent variable should be defined and explained in such way that it is *equivalent*[8] to the economic object or its feature.

For instance, an economic category such as production can be represented by a number of variables. These could, for example, be the value of ready-made production accordingly to its manufacturing costs, the value of a finished production in sale prices, net sales income, gross sales income,[9] or the amount of the cash inflows from the sales of goods and services. In a given model, depending on the study's aim, production can be represented by a variable appropriate for a particular case.

Consideration of factors that may influence a given dependent variable (either by stimulating or by impeding it) leads to specification of potential explanatory variables in the model. Influence of these explanatory variables on the dependent variable will be a subject to verification through the empirical model.

After both types of variables, the dependent and the explanatory ones, are determined, it is imperative to collect all statistical data necessary for their analysis. The number of statistical observations done on each potential variable of the model is important and must visibly exceed the number of explanatory variables. A condition of the so-called large statistical sample ought to be met. Lack of essential statistical information, its poor quality, or a significant number of gaps in the statistical material all can preclude the conduction of an econometric investigation based on the model. Slight gaps in statistical data can be complemented by using some statistical techniques (i.e., interpolation and extrapolation). Collection of valid statistical data on the model's potential defined variables completes the *variable specification* stage, which allows transition onto the equation specification stage.

Specification of equations consists in determining the number of equations within the model and a choice of an analytical form for each of those equations. Econometrics offers a vast arsenal of possible analytical forms. However, type 1.2 linear equations as well as type 1.4 and 1.6 product equations are most frequently used. The model's specification stage ends with a choice of the equation's (equation) analytical form. A hypothetical (theoretical) econometric model is its result.

1.3 Estimation of an econometric model's parameters

Estimation of the model's structural parameters and its stochastic structure parameters requires having a theoretical model as well as all necessary data collected on each variable of that model. First, an estimator, that is, a function estimating the model's parameters, must be selected. An estimator holds the following properties:

1. *Unbiasedness* – let $\hat{\theta}$ be an estimator of the parameter θ, based on the set of observations $\{y_i\}$ as $\hat{\theta} = h(y_1, y_2, \ldots, y_n)$. If equality $E(\hat{\theta}) = \theta$ occurs, then $\hat{\theta}$ is called the unbiased estimator of the parameter θ. When $E(\hat{\theta}) < \theta$, the estimator is negatively biased; but if $E(\hat{\theta}) = \theta$, the estimator is positively biased.

2. *Consistency* – if the estimator $\hat{\theta} = h(y_1, y_2, \ldots, y_n)$ converges in probability to θ, and when $n \to \infty$, then $\hat{\theta}$ is the consistent estimator of the parameter θ; $\hat{\theta}$, according to probability, seeks to be θ, when

$$\lim_{n\to\infty} P\left[\left(\hat{\theta} - \theta\right) = 0\right] = 1. \tag{1.9}$$

   It is worth noting that for appropriately large n-values, the consistent estimator is always unbiased. However, the opposite is not always true, since an unbiased estimator does not have to be consistent.

3. *Efficiency* – let $\hat{\theta}_j = h_j\left(y_1, y_2, \ldots, y_n\right)$, $j = 1, 2$ be the two estimators of the parameter $\hat{\theta}$ that are based on the observation set $\{y_{es}\}$. Efficiency of the estimator $\hat{\theta}_2$, in relation to the estimator $\hat{\theta}_1$, can be defined as the quotient of

$$\lambda = \frac{E(\theta - \hat{\theta}_1)^2}{E(\theta - \hat{\theta}_2)^2} = \frac{H_1}{H_2}. \quad (1.10)$$

In this case, we did not limit ourselves to a class of unbiased estimators; thereby H_1 and H_2 do not need to be the error variances. If we limit ourselves to the unbiased estimators $\hat{\Theta}_1$ and $\hat{\Theta}_2$, then H_1 and H_2 are the variances. The estimator $\hat{\Theta}_1$ is called an effective estimator of the parameter θ, if $H_2 \geq H_1$ occurs for each of the other unbiased $\hat{\Theta}_2$. In other words, no other unbiased estimator has a variation lower than $E[(\Theta - \hat{\Theta}_1)^2]$. It is worth to mention a volatility characteristic that is alternative to a statistical deviation – the estimator's precision (κ), defined as $\kappa = \frac{1}{\sigma}$, where σ is the standard deviation. An estimator of a lower variance, that is, of a lower standard deviation, 87 + 910 is characterized by a higher precision. Thus, a statement can be made that *an estimator of a higher efficiency* is a *more precise estimator.*

4. *Sufficiency* – an estimator is sufficient when it contains all information included in an observation set of the parameter being assessed. Let us suppose that $y_1, y_2, \ldots, y_n$ is an observation sequence in a sample randomly selected from a population having the density function $f(y, \theta)$. If $\hat{\Theta} = h(y_1, y_2, \ldots, y_n)$ is such an estimator of the parameter θ that the conditional expected value $E[\hat{\Theta} \mid (y_1, y_2, \cdots y_n)]$ does not depend on θ, then $\hat{\Theta}$ is a sufficient estimator.

The ordinary least squares method (OLS), developed by Carl F. Gauss, is the first general estimation method having many mutations. It consists in such selection of an estimator, $\hat{\Theta} = h(y_1, y_2, \ldots, y_n)$, so that the sum of squared differences between the observations y_i and their corresponding values of the function $f(y_i \hat{\Theta})$ is minimal

$$S = \sum_{i=1}^{n} \left[y_i - f\left(y_i, \hat{\theta}\right) \right]^2 = \min. \quad (1.11)$$

The OLS method is widely applied in practice, although it requires meeting such criteria, which give the OLS estimator some essential statistical characteristics. Let us consider a linear model

$$y = \mathbf{X}\alpha + \eta, \quad (1.12)$$

where

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & \dots & x_{1j} & \dots & x_{1k} \\ 1 & x_{21} & \dots & x_{2j} & \dots & x_{2k} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_{t1} & \dots & x_{tj} & \dots & x_{tk} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_{n1} & \dots & x_{nj} & \dots & x_{nk} \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_t \\ \dots \\ y_n \end{bmatrix}, \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \dots \\ \eta_t \\ \dots \\ \eta_n \end{bmatrix}, \alpha = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \dots \\ \alpha_j \\ \dots \\ \alpha_k \end{bmatrix},$$

and

$$y = \mathbf{X}\hat{\alpha} + \mathbf{u}, \tag{1.13}$$

where

$$\hat{\alpha} = \begin{bmatrix} \hat{\alpha}_0 \\ \hat{\alpha}_1 \\ \dots \\ \hat{\alpha}_j \\ \dots \\ \hat{\alpha}_k \end{bmatrix} \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_t \\ \dots \\ u_n \end{bmatrix}.$$

In model 1.12, **X** is the matrix of observations on the model's explanatory variables, **Y** is the vector of an observation on the dependent variable, **η** is the vector of the random components, **α** is the vector of the structural parameters, *n* is the number of statistical observations, and *k* is the number of explanatory variables. Hypothetical model 1.12 was assigned an empirical model (1.13), in which there are two new vectors: $\hat{\alpha}$ – an estimator of the vector **α,** and **u** – a residue vector. Having the estimator $\hat{\alpha}$, theoretical values of the dependent variable can be assigned

$$\hat{y} = \mathbf{X}\hat{\alpha}, \tag{1.14}$$

where a vector of the dependent variable's theoretical values, which were calculated using the model 1.14, emerges:

$$\hat{\mathbf{Y}} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \dots \\ \hat{y}_t \\ \dots \\ \hat{y}_n \end{bmatrix}.$$

The conditions of the OLS method application can be described as follows:

1. An econometric model must be linear in form, such as Equation 1.2. If a nonlinear model can be transformed into a linear one, then the OLS method is allowed. For example, the power-law model 1.4 and the exponential model 1.6 both can be transformed into linear forms using a logarithm of both sides.
2. Mathematical expectation of the random component should be equal to zero:

$$E(\eta_t) = 0. \tag{1.15}$$

3. The random component's variation (σ^2) should be constant and finite, such as

$$\sigma_1^2 = \cdots = \sigma_t^2 = \cdots = \sigma_n^2 = \sigma^2 < \infty. \tag{1.16}$$

4. The sequence of matrices of observations on explanatory variables, represented by **X,** is equal to the number of the model's structural parameters ($k + 1$):

$$r(X) = k + 1 < n. \tag{1.17}$$

This means that the n number of statistical observations is higher than the number of the model's structural parameters. In other words, the model has a positive degree of freedom. Moreover, none of the explanatory variables is a linear combination of another variable of the same type.

5. Explanatory variables should be correlated with the random component. This can be written as

$$E(X^T \eta) = 0. \tag{1.18}$$

6. The random component should be devoid of autocorrelation, such as

$$E(\eta\eta^T) = \sigma^2 I = \begin{bmatrix} \sigma_1^2 & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \sigma_t^2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & \sigma_n^2 \end{bmatrix}, \tag{1.19}$$

where $E(\eta\eta^T)$ is the matrix of random components' variances and covariances. Zero elements outside the main diagonal indicate that the covariances of the random components for their various pairs are equal to zero, that is

$$\operatorname{cov}(\eta_t \eta_{t'}) = 0, (t, t' = 1, \ldots, n; t \neq t'). \tag{1.20}$$

The second group of econometric model's parameters is formed by the stochastic structure's parameters. They describe a distribution of the random component **η.** It is usually assumed that distribution of the econometric model's random component is normal [$N(0, \sigma^2)$]. The assumption of a normal distribution of the random component **η,** which has a zero expected value, can be interpreted as follows:

1. positive and negative random deviations compensate each other;
2. the number of positive random deviations is close to the number of negative ones;
3. it can be expected that most random deviations will slightly differ from zero, and over 99.7% of all random deviations should fall within the spectrum of ±3SD. A standard deviation of the random component (σ) provides information on how much, in plus or in minus, the standard observations of a dependent variable (y_t) deviate from the function $E(Y) = X\hat{\alpha}$. The lower the σ value, the smaller the random part of the explanatory variable.

Using the criterion written as Equation 1.11, the OLS estimator $\hat{\alpha}$ for vector **α** can be written as follows:

$$\hat{\alpha} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Y}, \quad (1.21)$$

where $\mathbf{X}^T$ is a transposition of matrix of observations on the explanatory variables **X.** The random component's variance (σ^2) needs to be estimated as well. It can be shown that a residue variance (Su^2) is the unbiased estimator of a variance of the model's random component, and it can be calculated using the following equation:

$$Su^2 = \frac{1}{n-k-1}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2 = \frac{1}{n-k-1}\sum_{t=1}^{n}u_t^2, \quad (1.22)$$

where $\hat{y}_t$ represents the theoretical values of the dependable variable, which are calculated using the empirical model, while u_t represents the model's residuals. Alternatively, Equation 1.22 can be written as a matrix

$$\mathbf{S}\mathbf{u}^2 = \frac{1}{n-k-1}\mathbf{u}^T\mathbf{u}, \quad (1.23)$$

where **u** is the residual vector, defined in relation to Equation 1.13.

The Aitken's method, which is a generalized OLS method, also called a generalized method of the least squares,[10] is another method used for estimation of the model's parameters.

An Aitken estimator ($\overset{*}{\alpha}$) can have the following form:

$$\overset{*}{\alpha} = \left(X^T\Omega^{-1}X\right)^{-1}X^T\Omega^{-1}Y, \quad (1.24)$$

where a weight matrix **Ω** appears in the form of

$$\Omega = \begin{bmatrix} \omega_1 & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \omega_t & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & \omega_n \end{bmatrix}, \tag{1.25}$$

where $\omega_1, \ldots, \omega_t, \ldots, \omega_n$ are the weights, which include the volatility of the random component's variance for various observations. What is more, when $\omega_1 = \cdots = \omega_t = \cdots = \omega_n = 1$, matrix $\mathbf{\Omega} = \mathbf{I}$; in other words, it becomes a unit matrix of the n degree. As such, the Aitken estimator becomes equipotent to the OLS estimator.

In the Aitken's method, an estimator of the random component's variation is given by the following formula:

$$Su^2 = \frac{1}{n-k-1} u^T \Omega^{-1} u. \tag{1.26}$$

1.4 Verification of the model

Statistical verification of the model involves using multiple measures which, first and foremost, characterize the random component's role in that model. The first of such measures is a residual variance, discussed in the previous subsection. It does not have any economic interpretation. The square root of the residual variance

$$Su = \sqrt{Su^2} \tag{1.27}$$

is called standard residual error. *Su* is expressed in the same measurement unit as the dependent variable y_t. It provides information on how much, on average, during an n number of statistical observations, the theoretical values of the dependent variable $\hat{y}_t$, which are calculated using an empirical model, are different from the actual (observed) values of that variable (y_t).

The second-general measure of a model's accuracy provides information on the relative role of the random component. The convergence factor ϕ^2 is calculated using the following formula:

$$\varphi^2 = \frac{\sum_{t-1}^{n} u_t^2}{\sum_{t=1}^{n} \left(y_t - \bar{y}\right)^2}, \tag{1.28}$$

where $\overline{y} = \frac{\sum_{t=1}^{n} y_t}{n}$ represents the average arithmetic value of an observation on the dependable variable. It measures the relative share of the model's random fluctuations $\left(\sum_{t=1}^{n} u_t^2\right)$ in the total variability of the dependent variable $\sum_{t=1}^{n} (y_t - \overline{y})^2$. The smaller the share of random variability in the total volatility of the dependable variable, the better the empirical model. The convergence factor is a normalized number and meets the condition of $0 \leq \varphi^2 \leq 1$. By this criterion, the closer to zero the ϕ^2 is, the better the empirical model. Expression 100 ϕ^2 (%) provides information on what percentage of the total variation of the dependable variable y_t is random.

A square root of a multiple-correlation coefficient, also called a coefficient of determination, represented by R^2, is an alternative measure of the model's accuracy. This measure indicates which part of the dependable variable's total fluctuation is generated by explanatory variables of the empirical model.

A coefficient of determination is calculated as follows:

$$R^2 = 1 - \varphi^2. \tag{1.29}$$

The expression 100 R^2 provides information on what percentage of the dependable variable's total fluctuation results from the impact of the set of the empirical model's explanatory variables. Therefore, the closer to unity the coefficient R^2 is, the better the model.

The next issue is to examine the random component's autocorrelation.[11] Lack of this autocorrelation means that we are dealing with the so-called pure random component.

The presence of the random component's autocorrelation means that the random component creates an autoregressive process, in the form of

$$\eta_t = f\left(\eta_{t-1}, \eta_{t-2}, \ldots, \eta_{t-n+1}, \varepsilon_t\right), \tag{1.30}$$

where ε_t is the pure random component. The autocorrelation coefficients of the random component $\rho_1, \rho_2, \ldots, \rho_{n-1}$ with a value different from zero, indicate this autocorrelation's occurrence. The Durbin–Watson test is a tool used to test the random component's autocorrelation of the first order.[12] It verifies the null hypothesis, which assumes ρ_1 equal to zero and is written as $H_0{:}\rho_1 = 0$. An alternative hypothesis assumes that ρ_1 is positive, and so $H_1{:}\rho_1 > 0$.

The Durbin–Watson (DW) statistic tests the null hypothesis and is calculated by the following formula:

$$\mathrm{DW} = \frac{\sum_{t=2}^{n}(u_t - u_{t-1})^2}{\sum_{t=1}^{n} u_t^2}, \tag{1.31}$$

where u_t marks the residuals from the t-period ($t = 1, \ldots, n$), while u_{t-1} represents residuals delayed by 1 period. In case of a large sample, the DW statistic falls within $0 \le \mathrm{DW} \le 2$, where ρ_1 is positive. Thus, if the DW statistic > 2, the alternative hypothesis ought to be changed to assume occurrence of the random component's negative autocorrelation; in other words, H_1: $\rho_1 < 0$. In such cases, a corrected Durbin–Watson statistic should be calculated using the following formula:

$$\mathrm{DW}^* = 4 - \mathrm{DW}. \tag{1.32}$$

The estimated DW (or DW*) values are compared with the test's critical values: a lower d_l value and an upper d_u value, both taken from Durbin–Watson[13] tables, on an appropriate γ level of significance.

If, based on the above introduced tools of statistical verification, the empirical model can be considered acceptable, the statistical significance of explanatory variables should be studied as next. Having the empirical econometric model

$$y_t = a_0 + a_1 x_{t1} + \cdots + a_j x_{tj} + \cdots + a_k x_{tk} + u_t, \tag{1.33}$$

in which a_j ($j = 0, 1, \ldots, k$) represents assessments of the structural parameters and u_t represents the residuals. The empirical model, alternatively, can be written as follows:

$$\hat{y}_t = a_0 + a_1 x_{t1} + \cdots + a_j x_{tj} + \cdots + a_k x_{tk}, \tag{1.34}$$

where $\hat{y}_t$ represents a theoretical value of the dependable variable in a t ($t = 1, \ldots, n$) period.

Equations 1.33 and 1.34 differ by their residuals ($y_t - \hat{y}_t = u_t$). Each structural parameter's estimate (a_j) is characterized by its corresponding average estimation error S_{a_j} ($j = 0,1, \ldots, k$), which is a square root of the jth variance of the structural parameter's estimate $\left(S_{a_j}^2\right)$ and provides information on that estimate's accuracy. It is, therefore, necessary to assign estimation variances of the model's structural parameters. The matrix of the structural parameters' variations and their covariations [$D^2(a)$] should be assessed, using the following formula:

$$D^2(a) = Su^2\left(\mathbf{X}^T\mathbf{X}\right)^{-1} \tag{1.35}$$

where Su^2 is the residual variance provided by Equation 1.22 and $(\mathbf{X}^\mathrm{T}\mathbf{X})^{-1}$ is an inverse of the so-called Hess matrix occurring in Equation 1.21. Diagonal elements of the matrix $D^2(\mathbf{a})$ are the variances of the structural parameters' respective estimates, that is

$$S_{a_j}^2 = \mathrm{diag}\left[D^2(a)\right]. \tag{1.36}$$

Using the average errors of the structural parameters' estimations, the empirical model can be written as follows:

$$y_t = \underset{(S_{a_0})}{a_0} + \underset{(S_{a_1})}{a_1} x_{t1} + \cdots + \underset{(S_{a_j})}{a_j} x_{tj} + \cdots + \underset{(S_{a_k})}{a_k} x_{tk} + u_t, \tag{1.37}$$

where the average estimation errors are written in parenthesis under the structural parameters' estimates.

Having the model 1.37, a test on explanatory variables' statistical significance can be conducted. We pose the null hypothesis $H_0{:}\alpha_j = 0$ ($j = 1, \ldots, k$), which means that the jth structural parameter equals zero. In an economic sense, this is a hypothesis about insignificance of the model's jth explanatory variable. An alternative hypothesis $H_1{:}\alpha_j \neq 0$ assumes that the jth structural parameter is different from zero, which marks statistical significance of the jth explanatory variable.

An empirical statistic of a t-Student serves as a null hypothesis test, provided by the equation[14]

$$t_j = \frac{|a_j|}{S_{a_j}}, (j = 1, \ldots, k), \tag{1.38}$$

where the absolute value of the jth structural parameter's estimate is in the numerator, while its average estimation error is in the denominator.

The critical value $t_{\gamma;n-k-1}$ should be read from the tables of a t-Student distribution's critical values. A reasonable level of significance[15] γ is selected arbitrarily. The reading is done while having an $-k-1$ number of the degrees of freedom. Comparing the empirical statistic t_j ($j = 1, \ldots, k$) with the critical value $t_{\gamma;n-k-1}$, we infer the significance of the jth variable. If there is an inequality $t_j \le t_{\gamma;n\text{-}k1}$, then there is no reason to reject the null hypothesis. Basically, this forces a removal of that variable from the empirical model, then re-estimation of its parameters and verification of the respecified model. When $t_j > t_{\gamma;n-k-1}$, the hull hypothesis is rejected in favor of an alternative hypothesis and we infer a statistically significant impact of the jth explanatory variable on the dependent variable. Statistically insignificant variables are eliminated from the model. In a given iteration, only one irrelevant variable – the one for which the statistic t_j is the smallest – ought to be eliminated. Such recalculation and reverification of the model is done until all empirical variables of the model are statistically relevant on a reasonable level of significance. By doing so, we get an acceptable empirical econometric model.[16] This empirical model is often written as follows:

$$y_t = \underset{(t_0)}{a_0} + \underset{(t_1)}{a_1} x_{t1} + \cdots + \underset{(t_j)}{a_j} x_{tj} + \cdots + \underset{(t_k)}{a_k} x_{tk} + u_t, \tag{1.39}$$

where the empirical *t*-Student statistics are under the structural parameters' estimations. With such a model, we make its economic evaluation, which involves assessing the compatibility of modeling results with economic theory and the logic of economic practice.

1.5 Multiplicative econometric models

Multiplicative models[17] – after linear models – belong to a category of nonlinear models most frequently used in economic research. Both groups of multiplicative models can be converted into linear ones. Let us consider a power-law model 1.4

$$y_t = \alpha_0 x_{t1}^{\alpha_1} \cdots x_{tj}^{\alpha_j} \cdots x_{tk}^{\alpha_k} e^{\eta_t}.$$

A logarithm on both sides of the above equation is obtained as follows:

$$\ln y_t = \ln \alpha_0 + \alpha_1 \ln x_{t1} + \cdots + \alpha_j \ln x_{tj} + \alpha_k \ln x_{tk} + \eta_t. \tag{1.40}$$

Substituting $y_t^* = \ln y_t$, $\alpha_0^* = \ln \alpha_0$, $x_{tj}^* = \ln x_{tj}$, for $j = 1, \ldots, k$; model 1.40 can be written as

$$y_t^* = \alpha_0^* + \alpha_1 x_{t1}^* + \cdots + \alpha_j x_{tj}^* + \cdots + \alpha_k x_{tk}^* + \eta_t. \tag{1.41}$$

Equation 1.41 is linear in character due to its parameters. The structural parameters $\alpha_0^*, \alpha_1, \ldots, \alpha_j, \ldots, \alpha_k$ thus can be assessed using the OLS method.

A similar transformation can be done using the exponential model 1.6

$$y_t = \alpha_0 \alpha_1^{x_{t1}} \cdots \alpha_j^{x_{tj}} \cdots \alpha_k^{x_{tk}} e^{\eta_t}.$$

Applying a logarithm on both sides to the above equation, we get its following converted form:

$$\ln y_t = \ln \alpha_0 + x_{t1} \ln \alpha_1 + \cdots + x_{tj} \ln \alpha_j + x_{tk} \ln \alpha_k + \eta_t. \tag{1.42}$$

Consecutively substituting $y_t^* = \ln y_t$, $\alpha_j^* = \ln \alpha_j$, $j = 0,1, \ldots, k$, the model 1.41 can be written in a linear version

$$y_t^* = \alpha_0^* + \alpha_1^* x_{t1} + \cdots + \alpha_j^* x_{tj} + \cdots + \alpha_k^* x_{tk} + \eta_t. \tag{1.43}$$

The parameters of Equation 1.43 can be assessed using the OLS method. Using the formula 1.21, we get the following matrices of observations on the dependable variables

$$X^* = \begin{bmatrix} 1 & \ln x_{11} & \cdots & \ln x_{1j} & \cdots & \ln x_{1k} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \ln x_{t1} & \cdots & \ln x_{tj} & \cdots & \ln x_{tk} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \ln x_{n1} & \cdots & \ln x_{nj} & \cdots & \ln x_{nk} \end{bmatrix}, \ Y^* = \begin{bmatrix} \ln y_1 \\ \cdots \\ \ln y_t \\ \cdots \\ \ln y_n \end{bmatrix}.$$

Equation 1.21 will take the form

$$\hat{\alpha}^* = \left(X^{*T} X^*\right)^{-1} X^{*T} Y^*, \tag{1.44}$$

where

$$\hat{\alpha}^* = \begin{bmatrix} \hat{\alpha}_0^* \\ \hat{\alpha}_1 \\ \cdots \\ \hat{\alpha}_j \\ \cdots \\ \hat{\alpha}_k \end{bmatrix}.$$

Parameters of the equation type 1.43 can also be assessed using the OLS method, while the estimator takes on the following form:

$$\hat{\alpha}^* = \left(X^T X\right)^{-1} X^T \mathbf{Y}^*, \tag{1.45}$$

where $\mathbf{Y}^*$ has the same form as in the previous case 1.41 and the vector $\hat{\alpha}^*$ has the form

$$\hat{\alpha}^* = \begin{bmatrix} \hat{\alpha}_0^* \\ \hat{\alpha}_1^* \\ \cdots \\ \hat{\alpha}_j^* \\ \cdots \\ \hat{\alpha}_k^* \end{bmatrix}.$$

Let us suppose that the parameters of the power-exponential model in the form of Equation 1.8 were assessed:

$$y_t = \alpha_0 x_{t1}^{\alpha_1} x_{t2}^{\alpha_2} \alpha_3^{x_{t3}} \alpha_4^{x_{t4}} e^{\eta_t}.$$

The vector of the structural parameters' estimates was obtained in the following form:

$$a^* = \begin{bmatrix} 0.944 \\ 0.657 \\ 0.446 \\ 0.057 \\ -0.034 \end{bmatrix}.$$

Thus, the estimates of the structural parameters are known

$$a_0^* = 0.944, a_1 = 0.657, a_2 = 0.446, a_3^* = 0.057, a_4^* = -0.034.$$

We note that the estimates of parameters a_0^*, a_3^*, a_4^* are given in the form of logarithms. It is therefore necessary to perform the following calculations:

$$a_0 = \exp\left(a_0^*\right) = \exp\left(0.944\right) = 2.570,$$

$$a_3 = \exp\left(0.057\right) = 1.059 \text{ and}$$

$$a_4 = \exp\left(-0.034\right) = 0.967.$$

The power-exponential empirical model, thus, has the form

$$\hat{y}_t = 2.570 x_{t1}^{0.657} x_{t2}^{0.446} 1.059^{x_{t3}} 0.967^{x_{t4}}. \quad (1.46)$$

In the power-law model, all estimates of the structural parameters are obtained directly (except the a_0 estimate), whereas in the exponential model, additional calculations are always necessary to obtain the estimates a_j ($j = 0,1, \ldots, k$).

1.6 The limited endogenous variables

The dependent variable of an econometric model should be characterized by a relatively large area of volatility. It should also not be limited. What it means is that it should have neither a lower nor an upper limit. Meanwhile, sometimes the model has variables performing the role of dependable ones with observations $y_t^{(o)}$, which can have bilateral restrictions. Their specificity is that of having a lower and an upper limit, namely as in

$$y_{\min} \leq y_t^{(o)} \leq y_{\max}, \quad (1.47)$$

where $y_{\min}$ means the lowest possible observation value of the considered variable, while $y_{\max}$ is the highest possible observation value for this dependable variable.

Let us suppose that the limited variable $y_t^{(o)}$ will be described using a linear model

$$y_t = \alpha_0 + \alpha_1 x_t + \eta_t.$$

Figure 1.1 shows a linear econometric model for the limited dependable variable. The consequences of possible extrapolation beyond the statistical observation area are noted. Such an attempt of extrapolation may lead the extrapolated values to fall outside the area of limited variable's volatility, which is contrary with logic. For instance, the structure indicator, which satisfies the inequality $0 \le y_t^{(o)} \le 100$, can be the limited variable. An attempt to extrapolate the variable in the form of a structure indicator can lead the extrapolated values to be less than 0%, or greater than 100%.

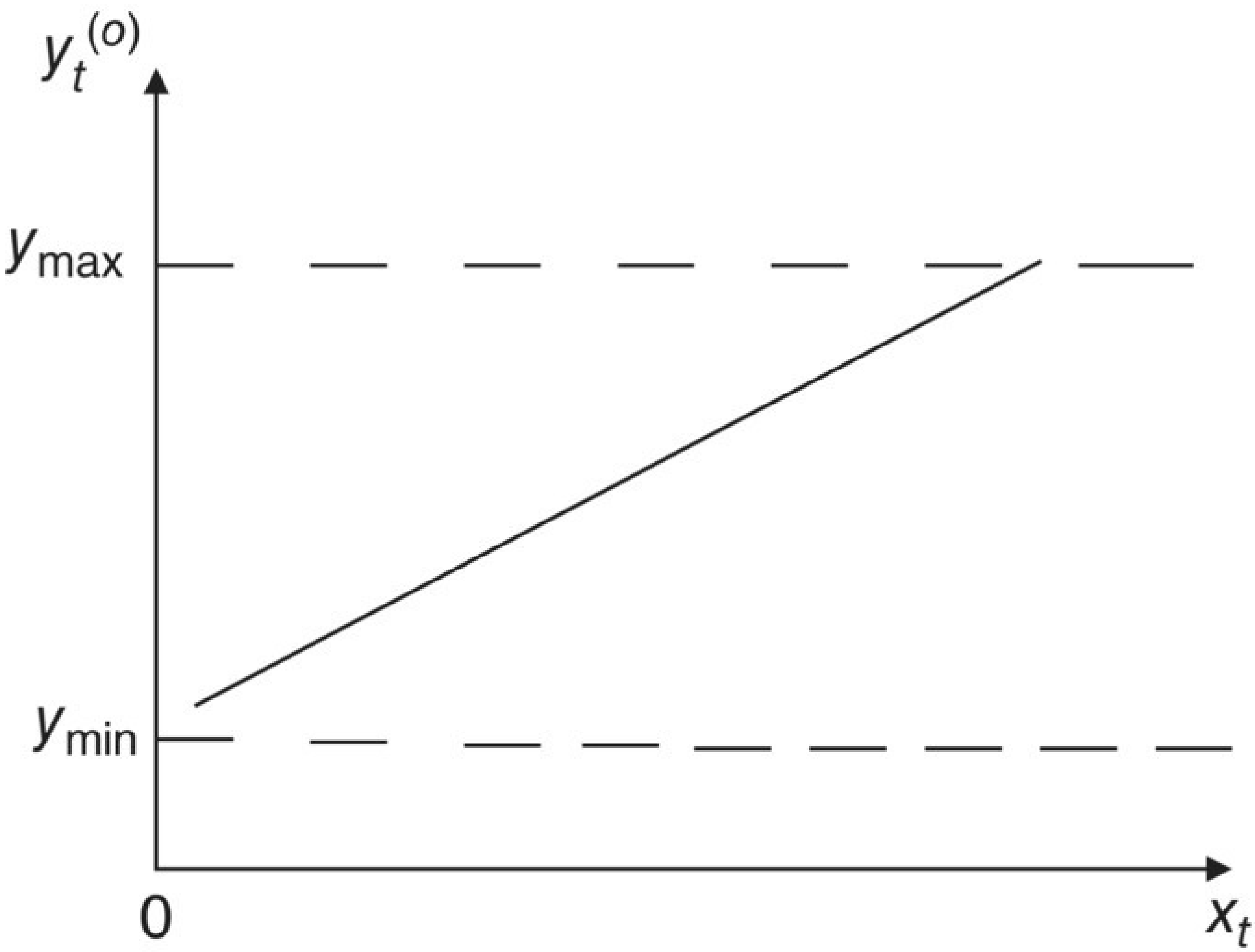


Figure 1.1 A linear model of the limited dependent variable.

Application of one out of many possible transformations of the limited dependent variable could be a possible solution here. The first of such transformations is a basic transformation of the limited variable, given by the following formula:

$$y_t^{(p)} = \frac{y_t^{(o)} - y_{min}}{y_{max} - y_t^{(o)}}, \tag{1.48}$$

where the symbols are identical to those in formula 1.46, while on the contrary, $y_t^{(p)}$ represents a *basic transformation* of the both-sides limited variable $y_t^{(o)}$. A basic transformation of the limited dependent variable converts it into a variable, which takes its value from the range $0 \le y_t^{(p)} \le \infty$. A variable in the form of $y_t^{(p)}$ is unlimited in its non-negative values. Still, its lower limit is on the 0 level; thus it has the characteristics of many economic variables that reach non-negative values.

Figure 1.2 shows a basic transformation of the variable $y_t^{(o)}$, wherein the minimum value of the dependent variable is 0, that is, $y_{min} = 0$. It does not, however, change the generality of the idea shown in this figure.

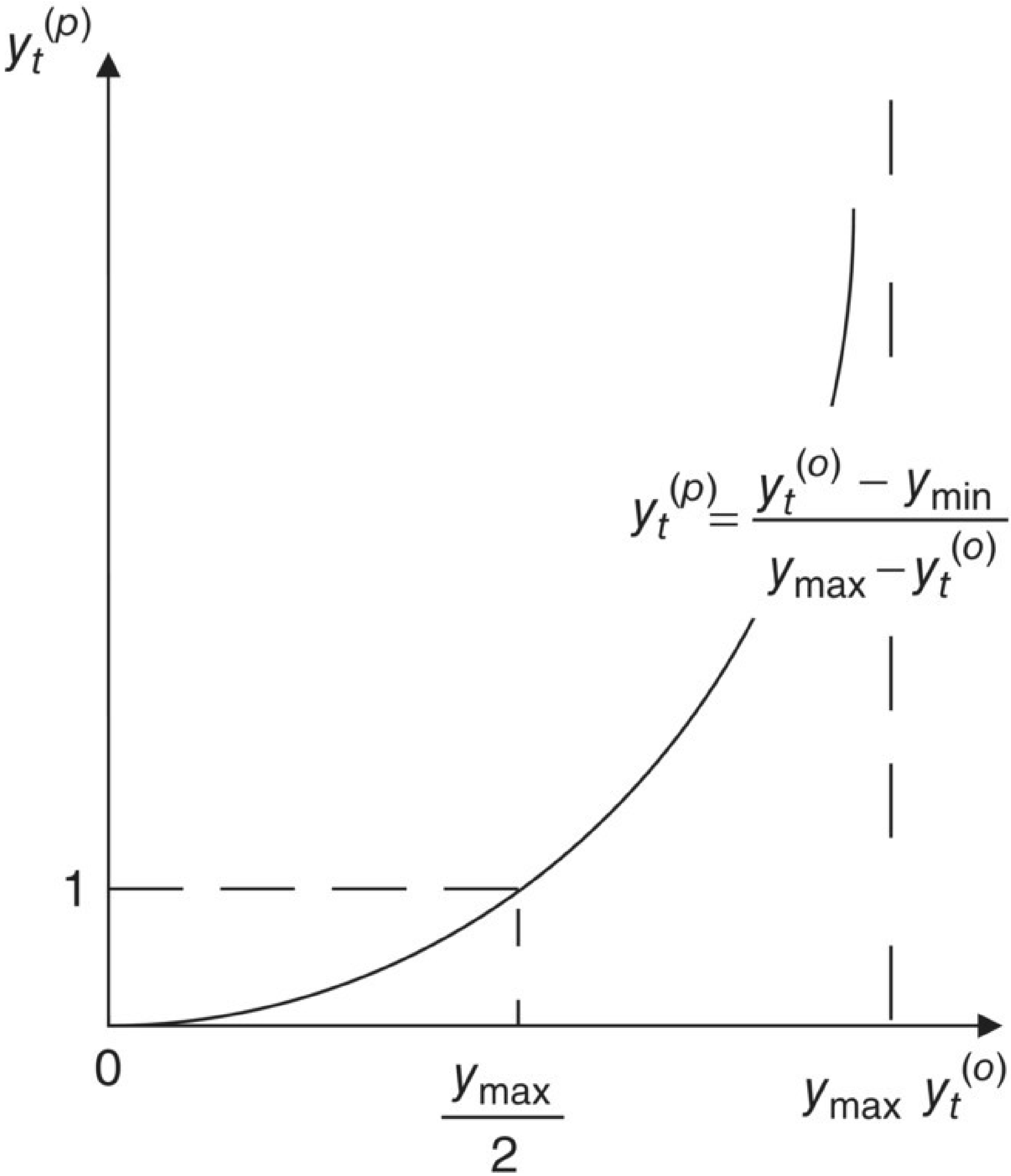
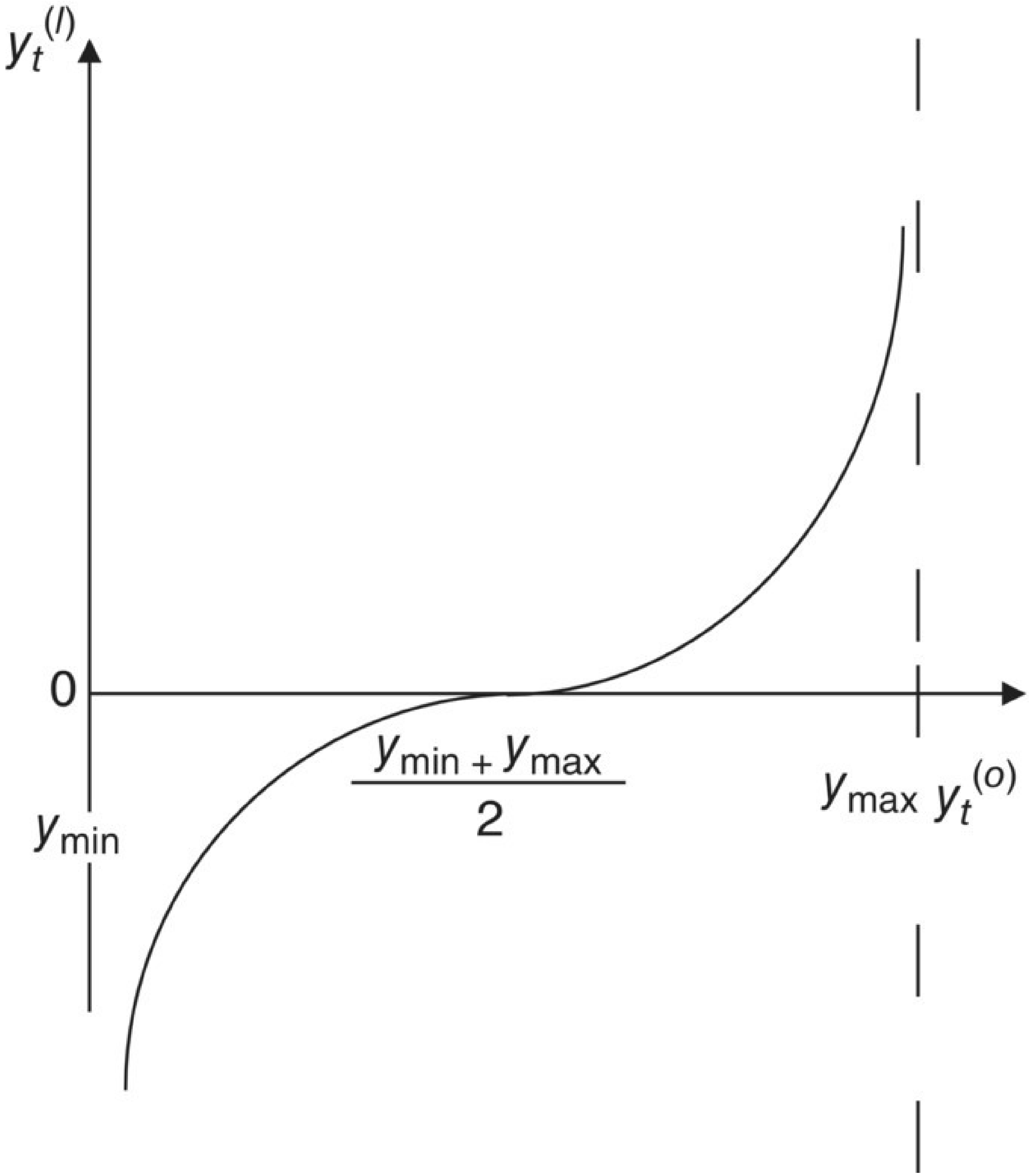


Figure 1.2 A basic transformation of the limited dependent variable.

Next important transformation of a both-sides limited variable is the logit transformation, the idea of which is shown in Figure 1.3 and is given by the following formula:

$$y_t^{(l)} = \ln y_t^{(p)} = \ln \frac{y_t^{(o)} - y_{min}}{y_{max} - y_t^{(o)}}. \tag{1.49}$$



[Figure 1.3](#) A logit transformation of the both-sides limited dependent variable.

A logit transformation of the limited variable is thus a logarithm of the basic transformation. It converts the variable into a both-sides limited variable. In fact, we can see that the variable in a logit form fulfills the inequality $-\infty \leq y_t^{(l)} \leq +\infty$. Thus, application of linear models, such as

$$y_t^{(p)} = \sum_{j=0}^{k} \alpha_j x_{tj} + \eta_t \tag{1.50}$$

or

$$y_t^{(l)} = \sum_{j=0}^{k} \alpha_j x_{tj} + \eta_t \tag{1.51}$$

eliminates the risk associated with extrapolation of the dependent variable beyond the statistical observation area.[18]

Estimation of the parameters from a model with a limited dependable variable can be done using the classic least squares method, with application of the procedure provided by Equations 1.21–1.23. Goldberger suggests[19] that in such a case, the Aitken estimator provided by Equation 1.24 is more accurate. As such, a question emerges, how to estimate the components of the matrix Ω provided by Equation 1.25.

In this case, a double-step procedure is required. In the first step, the OLS method should be used to estimate the parameters of the model with an endogenous dummy variable. After theoretical values from a 1.34-type empirical equation are calculated, weights for each observation can be assigned, using the following calculation:

$$w_t = \hat{y}_t\left(1-\hat{y}_t\right), \qquad \left(t=1,\ldots,n\right) \tag{1.52}$$

A result, an empirical matrix $\hat{\Omega}$ can be constructed in the following form:

$$\hat{\Omega} = \begin{bmatrix} w_1 & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & w_t & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & w_n \end{bmatrix}. \tag{1.53}$$

In practice, negative values of the weights w_t can appear. Therefore, it is better to use weight modules calculated using formula 1.52. Matrix $\hat{\Omega}$ then will take the following form:

$$\hat{\Omega} = \begin{bmatrix} |w_1| & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & |w_t| & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & |w_n| \end{bmatrix}. \tag{1.54}$$

Aitken estimator for the dummy explanatory variable will then have the following form:

$$\overset{*}{\alpha} = \left(X^T \hat{\Omega}^{-1} X\right)^{-1} X^T \hat{\Omega}^{-1} Y. \tag{1.55}$$

Matrix $\hat{\Omega}^{-1}$ will have the following structure:

$$\hat{\Omega}^{-1} = \begin{bmatrix} \left|\frac{1}{w_1}\right| & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \left|\frac{1}{w_t}\right| & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & \left|\frac{1}{w_n}\right| \end{bmatrix},$$

or

$$\hat{\Omega}^{-1} = \begin{bmatrix} \left|\frac{1}{\hat{y}_1(1-\hat{y}_1)}\right| & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \left|\frac{1}{\hat{y}_t(1-\hat{y}_t)}\right| & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & \left|\frac{1}{\hat{y}_n(1-\hat{y}_n)}\right| \end{bmatrix}. \quad (1.56)$$

Estimator 1.55 provides more effective (precise) parameter assessments of the model with a dummy explanatory variable, in comparison with the OLS estimator.

1.7 Econometric forecasting

1.7.1 The concept of econometric forecasting

Forecast estimation is one of the possible courses of econometric model's application. By *econometric prediction* we mean inference into the future, using an econometric model. Prediction, therefore, involves a set of research-procedure activities. Econometric forecast is a result of an econometric prediction.

> The process of predicting and assessing the future, which is based on theoretical studies, analytical considerations, logical presumptions, as well as on practical experience, is an essential basis for the currently rapidly growing statistical (probabilistic) forecasting theory.[20]

Various quantitative methods, especially the mathematical–statistical ones, as well as analytical concepts and instruments of probability calculus are used in such a process of inference about the future. Moreover, econometric models constructed especially for that purpose and based on observed regularities in the past economy are also used.

Application of mainly mathematical and statistical tools of inference into the future allows forecast estimation that is based on a relatively objective method. Objectivity of an econometric forecast primarily results from the fact that if a prediction rule is selected, the manner of forecast construction is defined explicitly. Application of econometric methods prevents forecast "corrections," depending on subjective feelings or suggestions of prediction participants.[21]

Economic forecast, as a result of an econometric prediction, is understood as such numerical evaluation of the considered reality fragment, during formulation of which the knowledge about past regularities or tendencies is used. Appropriate empirical econometric models, which describe the economic systems and its elements, are the starting point of econometric forecasting. A rational steering of the economic system requires recognition of its future behavior and the changes therein.

Econometric forecasts are important in rational programming of economic processes. They provide relatively objective information about the future, and therefore provide additional presumptions in decision-making. However, business practice in many countries, scarcely and not often enough, uses such statistical and econometric tools while estimating the forecasts. Making decisions about future solutions is too excessively dominated by autopsy and faith in decision-makers' intuition. A seldom use of scientific forecasting methods often hinders the effects of that decision-making.

The purpose of a forecast is to create new presumptions by providing new information for the decision-making process. Forecasting allows accounting for anticipated trends and the dynamics of an economic system. It also allows early intervention – by recognition of important elements of the economic system's behavior – to actively influence such processes. In this sense, forecasting may be of a *warning* character, because it indicates, early enough, the negative economic and social consequences of current trends and regularities of economic system's behavior.

Forecasting often has research characteristics. *A research forecast* is a numerical assessment of a future condition of an economic object or a system, based on permanent cause-and-effect relationships, which characterize the subsequent changes. The future condition of an economic object or a system is regarded as a consequence of a previous state combined with a set of hypotheses concerning both the general conditions as well as specific factors of economic development. *Normative forecasts* are often used as well. Their specification is concerned with estimation of the results, which should be achieved in the future (especially over longer periods). Thus, development objectives are formulated to some extent. Cause-and-effect relationships, however, are being considered from future to present. Therefore, the sequence of events to occur as well as the tasks, which ought to take place to achieve a given final result in the form of a forecast, are considered.

Generally, we can distinguish *quantitative* and *qualitative* forecasts. *Quantitative forecast* refers to a numerical value of a specific random variable. In contrast, *qualitative forecast* indicates whether a certain random event is realized a certain number of times within a forecasted period. Qualitative forecasts can be spot or range implemented. A spot conceptualization of forecasting consists in choosing one number, which, under certain conditions resulting from a prediction theory, can be considered as the best assessment of the forecasted variable in the forecasted period. Interval forecasting is characterized by specification of a numerical range, corresponding with an appropriately high (close to unity) probability for a true value of the forecasted variable to be included within the T-period.

A possibility of major qualitative changes, resulting from general socioeconomic politics, should also be taken into consideration. In such transient states, an analysis using econometric tools meets significant difficulties; sometimes it is even impossible.

Accuracy of econometric forecasting is, first and foremost, conditioned by precision with which an empirical model describes the economic system. Estimation of a forecast, based on an econometric model, is more justified when[22]

1. a prediction horizon is shorter, that is, the time interval $(t_0; t_0 + \tau)$, where t_0 is the current period and $t_0 = n$ often is the last observation period, τ is the length of prediction horizon; prediction horizon marks the point for which the constructed forecasts are acceptable (reasonable, sensible);
2. the period, on the basis of which the empirical forecasting model is constructed, is longer;
3. the changes (evolutionary, not revolutionary) of forecasted variables are slower;
4. the nature of forecasted variables is more autonomous, that is, less dependent on strategic decisions.

1.7.2 The conditions of econometric forecast estimation

Performing an econometric prediction of an economic system or its component's is justified, if the basic and thus necessary conditions, called *the* basic assumptions of econometric forecast theory, are met. Such assumptions[23] are as follows:

1. If the prediction is for one economic variable, then the empirical model, which describes formation of that variable, must be known. In case of an economic system's prediction, the empirical econometric model of that system, whose objects are the interdependent variables described by individual equations of that model, must also be known. Knowledge of the model's structural parameter estimations and estimations of stochastic structure parameters is also necessary.
2. The mechanism linking endogenous variables with explanatory variables is stable over the whole time period, beginning with the period from which the sample forming the basis for estimation of the model's parameters originates, up to the forecasted period (including the forecasted period). When dealing with changes of the structure, they can be slow and

regular. Such structure changes within the model can be captured by varying the structural parameters of its equation(s).

3. Distribution of the random components is stationary, both in the period from which the sample was taken as well as in the forecasted period. The changes may cover the type of distribution or modification of the parameters. If there are changes in the random component's distribution, they ought to be regular enough to enable their detection and extrapolation into the forecasted period.
4. The values of explanatory variables of the model's equations in the forecasted period ought to be known. To meet this requirement, first and foremost, the variables, which play a crucial role in achieving the tested regularities as well as those, for which the values of a forecasted period can be predicted with a sufficient accuracy, should be inserted into the econometric model that is used for prediction purposes. The values of explanatory variables in the forecasted period T ($T = t_0 + 1, t_0 + 2,\ldots, t_0 + \tau$) can be predicted:
   a. on a planned level, which allows conclusions about the effects of realization of those plans;
   b. using the already existing forecasts of those variables;
   c. by designation of trend models, and then extrapolation of the trends for the values of those variables. The values of those trends for the forecasted period T are supposed to be the estimates of explanatory variables in the forecasted period;
   d. through construction of a new model, in which exogenous variables will function as endogenous ones. A new empirical model will be used to estimate the values of exogenous variables in the forecasted period, and then to estimate the forecasts of endogenous variables representing the elements of an economic system. This method allows positive results at a small number of exogenous variables. On the other hand, it fails at a greater number of those variables, because it requires collection of a bigger statistical material that is not always available.[24] In practice, the values of explanatory variables in the forecasted periods are not known. In a classic prediction theory, econometric forecasting is conditional in character and depends on achievement of specific values by the explanatory variables. The values of explanatory variables in a forecasted period, in fact, may shape themselves at a level different than the one assumed while estimating the forecast. In such cases, a significant discrepancy between implementation of the forecasted variable and the estimated forecast should be accounted for.
5. In terms of content, it is allowed to extrapolate the model beyond the variables' volatility area observed in the statistical sample, which had served to estimate the model's parameters. This assumption is intended to protect against an automatic generalization of the regularities observed in the sampling. Caution is required when extrapolating the model, especially when the number of sample observations was small or when the area of explanatory variables' volatility was scant. In such cases, there is a risk of selecting a faulty analytical form for one of the equations, which, outside the tested area of volatility,

can result in a different form of the endogenous variable's dependency on the explanatory variables.

Basic assumptions of econometric prediction theory usually are supplemented by two praxeological postulates.[25] The first states that prediction effects should include both an adequate forecast as well as an evaluation of its accuracy rank, provided in a suitable measure. The second indicates that, when several manners of forecast construction are possible, the best method according to a chosen criterion (forecast accuracy rank meter) should be selected.

1.7.3 Forecasts based on single-equation models

Let us suppose that the following single-equation linear econometric model is used in a prediction

$$y_t = \alpha_0 + \alpha_1 x_{t1} + \cdots + \alpha_j x_{tj} + \cdots + \alpha_k x_{tk} + \eta_t,$$

where η_t is the pure random component of a zero expected value. Depending on the applied estimator of the structural parameters' vector, we can get various predictors.[26] If the parameters of the above model were estimated using the least squares method (OLS), the predictor used in the forecast will be according to the OLS method and will have the following form:

$$y_{Tp} = \hat{\alpha}_0 + \hat{\alpha}_1 x_{T1} + \cdots + \hat{\alpha}_j x_{Tj} + \cdots + \hat{\alpha}_k x_{Tk}, \tag{1.57}$$

where $\hat{\alpha}_0, \hat{\alpha}_1, \ldots, \hat{\alpha}_j, \ldots, \hat{\alpha}_k$ are the estimates of parameters $\alpha_0, \alpha_1, \ldots, \alpha_j, \ldots, \alpha_k$, calculated using the OLS method. The symbol T represents the forecasted period, wherein $T = n + 1, n + 2, \ldots, n + \tau$. For example, using an estimator of a generalized OLS method (Aitken's method), we will get a predictor according with the Aitken's method, and so on. The predictor equation 1.57 can be written in a matrix form as

$$y_{Tp} = \mathrm{X}_T \hat{\alpha}, \tag{1.58}$$

where $X_T = [1 x_{T1} \ldots x_{Tj} \ldots x_{Tk}]$ is the vector of explanatory variables in the forecasted period T, while the transposed vector of the structural parameters' estimations has the following form:

$$\hat{\alpha}^T = \left[\hat{\alpha}_0, \hat{\alpha}_1, \ldots, \hat{\alpha}_j, \ldots, \hat{\alpha}_k\right].$$

A prediction variation for the predictor equation 1.57, thereby for Equation 1.58, is assigned using the formula

$$V_T^2 = \sigma^2 \left[1 + \mathbf{X_T} \left(\mathrm{X}^T \mathrm{X}\right)^{-1} \mathrm{X}_T^T\right], \tag{1.59}$$

where V_T^2 represents a prediction variation of the forecasted variables in the T period, $\mathbf{X_T}$ is the vector of explanatory variables' values in the forecasted period T, σ^2 – a variation of the model's random component.[27] Equation 1.59 alternatively can be written as

$$V_T^2 = \sigma^2 + X_T D^2(\hat{\alpha}) X_T^T, \tag{1.60}$$

where the matrix of the model's structural parameters' variance and covariance $D^2(\hat{\alpha})$, calculated using the OLS method, appears.

It can easily be shown that the following inequality occurs:

$$V_T^2 \geq \sigma^2. \tag{1.61}$$

This means that prediction accuracy cannot be greater than the accuracy of the model used in the empirical model's prediction.

Square root of the prediction variance is the average prediction error, that is

$$V_T = \sqrt{V_T^2}. \tag{1.62}$$

The average prediction error is expressed in the units of the forecasted variable Y_T. It allows assessment of prediction accuracy in a period *T*. The requirement appropriate forecast accuracy is defined by its user, setting the prediction's limiting error V_G. If the following inequality occurs:

$$V_T \leq V_G, \tag{1.63}$$

then the forecast is admissible, since it fulfills the requirement of the precision desired by the user. In case of the following:

$$V_T > V_G, \tag{1.64}$$

the forecast is inadmissible, since it is not accurate enough for the user's needs.

Often, it is difficult for the user to determine the value of the average prediction error V_G for each of the forecasted variables. It is easier to determine the relative limiting error of prediction V_G^* expressed as a percentage of the forecast's value. In that case, a prediction accuracy measure is used, such as the *relative limiting error* calculated using the following formula:

$$V_T^* = \frac{V_T}{y_{TP}} 100(\%). \tag{1.65}$$

Comparison of the relative average error of prediction with the relative limiting error of prediction allows appropriate decision-making. In case of the following:

$$V_T^* \leq V_G^*, \tag{1.66}$$

the forecast is deemed admissible; in terms of the user's needs, it is sufficiently accurate. However, in case the following inequality occurs:

$$V_T^* > V_G^*, \tag{1.67}$$

the forecast is inadmissible, since from the user's perspective, it is not precise enough.[28]

1.7.4 Analysis of econometric forecasts' precision

Besides using the measures of prediction accuracy, which allow its *ex ante* type of estimation, it is necessary to observe and to register the realizations of the forecasted variable y_T. Knowledge of the forecasted variable's realization allows its comparison with the forecast. This enables testing the *expired forecast*,[29] using forecast accuracy measures of the *ex post* type.

The difference between the forecasted variable's realization in the T (y_T) period and the (y_{Tp}) forecast, marked as ω_T, will be called the forecast error; in other words

$$\omega_T = y_T - y_{Tp}. \tag{1.68}$$

Even one forecast error observation ω_T can cause a necessity of interference into the forecast results. A grossly inaccurate forecast may emerge. This happens when a forecast error exceeds an average forecast error ($|\omega_T| > V_T$). Such a case may signify a future set of inaccurate forecasts, often with homonymous signs of forecast errors.

Emergence of a set of forecast errors with the same sign means that a set of underestimated or overestimated forecasts has formed. A sequence of *overestimated forecasts* appears, when $\omega_T < 0$, in few consecutive forecasted periods. A sequence of *underestimated forecasts* appears, when inequality $\omega_T > 0$ occurs in at least three periods. A reaction to such an occurrence should consist in predictor's correction, which involves a change of the set of explanatory variables in the empirical model, a change of the equation's analytical form, and supplementation of modeling information with the data resultant from realization of a forecasted variable.[30]

Valuable information on the accuracy of the forecasted prognoses against the forecasted variable's realization is provided by the *average forecast error* δ_υ, which can be calculated using the following formula:

$$\delta_\upsilon = \sqrt{\frac{1}{\upsilon}\sum_{T=t_0+1}^{t_0+\upsilon}\left(y_T - y_{Tp}\right)^2}, \tag{1.69}$$

where T ($T = t_0 + 1, \ldots, t_0 + \upsilon$) denotes the forecasted period's number and υ represents the amount of expired forecasts. The average forecast error provides information on how much (on average) the forecasted variable's realizations differ from the earlier estimated forecasts. The meter 1.69, of course, can be calculated only after the forecasted variable's realization is obtained, that is, while having the υ string of expired variables. The smaller the value of δ_υ, the more accurate the expired forecasts.

As proposed by A. Gadd and H. Wold, Janus coefficient J is an interesting measure of forecast accuracy. It is calculated using the following formula:

$$J = \frac{\frac{1}{\upsilon}\sum_{T=n+1}^{n+\upsilon}\left(y_T - y_{Tp}\right)^2}{\frac{1}{n}\sum_{t=1}^{n}\left(y_t - \hat{y}_t\right)^2}, \tag{1.70}$$

where $\{\hat{y}_t\}$ is the sequence of endogenous variable's theoretical values in the sample, on the basis of which empirical model's parameter estimation was conducted, while n is the number of observations in the statistical sample. All other signs are the same as in formula 1.69. The Janus coefficient, thus, is the quotient of an average squared forecasting error and the average squared equation residuals in the sample. An econometric model can be used as a predictor in the prediction process, as long as the J coefficient is equal to unity or only slightly exceeds 1. If, however, J significantly exceeds 1, predictor correction should be applied, using the newest statistical data.

Notes

1 The nature of the category represented by a dependent variable assigns the model to a specific discipline. For example, the dependent variable representing a demographic category means that the model is demometric; if the dependent variable is sociological, the model is sociometric; when the dependent variable represents a psychological category, the model is psychometric.

2 The structural parameter α_j ($j = 1, \ldots, k$) indicates that an increase in the value of observation x_{tj} by one unit changes the size of y_t by α_j units, while assuming immutability of other explanatory variables (the *ceteris paribus* principle).

3 Explanatory variables of a discrete character (discrete variables) should be included in the model only exponentially, because, in terms of power series, it is difficult to give the structural parameter an economic interpretation.

4 While collecting statistical data, one should remember about its appropriate quality. They should be comparable, with no gaps in the statistical rank, in their interior as well as on the edges. Any minor deficiencies can be supplemented using statistical techniques (interpolation or extrapolation). Statistical material should not contain any statistical biases.

5 In case of a model consisting of multiple equations.

6 The estimator should be chosen so as to have all the necessary statistical properties, that is, compliance, unbiasedness, effectiveness, and adequacy.

7 They belong to a group of global and specific quality measures.

8 See: Wiśniewski J. W. (1986) "An econometric study of qualitative phenomena." *Methodological Study*, UMK, Toruń, Section 1.5. This concept is understood as follows: an economic variable, which, from the research point of view, best reflects an economic category being the subject of an empirical verification, will be called an *equivalent variable*. See also: Wiśniewski J. W. (2013). "Correlation and regression of economic qualitative features." Lap Lambert Academic Publishing, Saarbrucken, subchapter 1.3 and Wiśniewski, J. W. (2012). *Dilemmas of Economic Measurements in Weak Scales*, Folia Oeconomica Stietinensia, No. 10 (18) 2011/2, University of Szczecin Press, Szczecin 2012, pp. 50–59.

9 Including the amount of tax on goods and services.

10 The Aitken's method is recommended when random component variations for various statistical observations are not equal, that is, when Equation 1.16 does not occur.

11 Random component's autocorrelation is an error of the model's specification. It can result from: (i) omission of an important, statistically significant explanatory variable in the empirical model, which results in positive autocorrelation; (ii) a defective analytical form of the empirical model, causing a positive autocorrelation of the random component; (iii) an excess of statistically insignificant variables in the empirical model, resulting in negative autocorrelation of the random component.

12 It can be shown that if autocorrelation of the first order occurs in the model then there is no autocorrelation of higher rows. Appearance of first-order autocorrelation signifies an error in the model's specification, which causes the necessity of its respecification. Respecification should be continued until the moment, when the empirical model will lack a first-order autocorrelation of the random component.

13 If $DW(DW^*) > d_u$, then there are no grounds for rejection of the null hypothesis (H_0), which means, that along with a risk of a first-type error (based on the significance γ), we infer that there is no random component's autocorrelation. In case of $DW(DW^*) < d_l$, we reject the hypothesis $H_0$for an alternative one, by which we infer an autocorrelation of the random component of the first order. Finally, when $d_l \leq DW(DW^*) \leq d_u$, a test does not settle the question of whether there is autocorrelation or not. This means, that the DW statistic hits the test's insensitivity area. In such case, it is necessary to apply another test to autocorrelation of the random component, for example the *i*-Student test to the autocorrelation coefficient test.

14 From the alternative hypothesis, it is inferred that it is a statistical test with the so-called two-sided rejection region.

15 Most frequently the level of significance $\gamma = 0.01$ or $\gamma = 0.05$, which means that there is an agreement to risk of a first-type error on the 1% or 5% level.

16 Provided that all previous measures of the model's accuracy are on a satisfactory level.

17 Product models, in literature, are also called multiplicative models.

18 Extensive discussion on construction of econometric models for transformation of limited dependable variables can be found in the work of J.W. Wiśniewski: *Econometric Research on Qualitative Occurrences: A Methodological Study*. UMK, Toruń, 1986.

19 See the work of: A.S. Goldberger (1972), p. 321

20 See: A. Zeliaś*: Forecasting Theory*, Warsaw 1979, p.15.

21 See: Z. Pawłowski: *Econometric Forecasting*, Warsaw 1973, p.15.

22 See: A. Zaliaś, op. cit, p.16.

23 See: Z. Pawłowski: *Econometric* …, pp. 38–45.

24 See: A. Zeliaś, op. cit, pp. 129–130.

25 See: Z. Pawłowski: *Econometric* …, p. 45.

26 "A predictor" means an empirical function, which serves as a tool for estimation of forecasts.

27 In practice, the residual variance $Su2$ is used as the estimator $\sigma 2$.

28 In admissible forecast is not always a useless forecast. If its accuracy only slightly deviates from the user's expectations, then the forecast can be used as an "indicator" for a given forecasted variable. It may enable the user to prepare for the anticipated direction of the forecasted variable's formation.

29 "An expired forecast" means such a forecast, for which realization of the dependable variable y_T is known.

30 A string of expired forecasts should be characterized by a variety of forecasting error signs (ω_T) and by the values of those errors' modules ($|\omega_T|$) that are smaller than the average forecast errors (V_T).

2
Multiple-equation econometric models

2.1 Classification of multiple-equation models

A multiple-equation model is a system of equations consisting of many (at least two) equations, which describe a given economic system or its part called a subsystem. The model contains a G of endogenous variables: $Y_1, \ldots, Y_g, \ldots, Y_G$ with statistical observations $y_{1t}, \ldots, y_{gt}, \ldots, y_{Gt}$. *The endogenous variable* is characterized by the fact that in one of the model's equations it acts as a dependable variable; however, it can also act as an explanatory variable. This type of a model also contains exogenous variables $X_1, \ldots, X_j, \ldots, X_k$ with observations $x_{t1}, \ldots, x_{tj}, \ldots, x_{tk}$. *Exogenous variables* in the equations act solely as explanatory variables. Endogenous variables without anytime-delays will be called the model's *total interdependent variables*. An alternative group of *predetermined variables* $Z_1, \ldots, Z_j, \ldots, Z_K$ (with observations $z_{t1}, \ldots, z_{tj}, \ldots, z_{tK}$) is formed by exogenous variables and delayed endogenous variables, which in the model's equations appear as explanatory variables.

A system of G equations in a multiple-equation model in a structural form[1] can be written as follows:

$$y_{1t} = \alpha_{10} + \sum_{g=2}^{G} \beta_{1g} y_{gt} + \sum_{j=1}^{K} \alpha_{1j} z_{tj} + \eta_{1t},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$y_{gt} = \alpha_{g0} + \sum_{g''=1;\ g\neq g'}^{G} \beta_{gg'} y_{g''t} + \sum_{j=1}^{K} \alpha_{gj} z_{tj} + \eta_{gt},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$y_{Gt} = \alpha_{G0} + \sum_{g=1}^{G-1} \beta_{Gg} y_{gt} + \sum_{j=1}^{K} \alpha_{Gj} z_{tj} + \eta_{Gt}. \tag{2.1}$$

In the above G equations, some random components as well as structural parameters associated with the total interdependent variables have appeared. Additionally, the parameters α_{gj} ($g = 1, \ldots, G$; $j = 0, 1, \ldots, K$), associated with the predetermined variables, have appeared. In practice, it is natural for only some of the total interdependent variables and the predetermined variables acting as explanatory variables to occur in individual equations. This means that a significant part of parameters β_{gg} and α_{gj} ($g, g' = 1, \ldots, G$; $j = 0, 1, \ldots, K$) takes zero values. What is more, the parameters $\beta_{gg} = 1$ indicate the explanatory variable of the gth equation.

A multiple-equation model can also be written in as a matrix

$$\mathbf{BY} + \mathbf{AZ} = \eta, \tag{2.2}$$

where

$$\mathbf{B} = \begin{bmatrix} 1 & \cdots & -\beta_{1g} & \cdots & -\beta_{1G} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\beta_{g1} & \cdots & 1 & \cdots & -\beta_{gG} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\beta_{G1} & \cdots & -\beta_{Gg} & \cdots & 1 \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} y_{1t} \\ \cdots \\ y_{gt} \\ \cdots \\ y_{Gt} \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} z_{t0} \\ z_{t1} \\ \cdots \\ z_{tj} \\ \cdots \\ z_{tK} \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} -\alpha_{10} & -\alpha_{11} & \cdots & -\alpha_{1j} & \cdots & -\alpha_{1K} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\alpha_{g0} & -\alpha_{g1} & \cdots & -\alpha_{gj} & \cdots & -\alpha_{gK} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\alpha_{G0} & -\alpha_{G1} & \cdots & -\alpha_{Gj} & \cdots & -\alpha_{GK} \end{bmatrix}, \eta = \begin{bmatrix} \eta_{1t} \\ \cdots \\ \eta_{gt} \\ \cdots \\ \eta_{Gt} \end{bmatrix}.$$

Matrix **B** contains the model's structural parameters along its total interdependent variables. Matrix **A** contains the model's structural parameters occurring along the predetermined variables. Vector **Y** contains the model's total interdependent variables, vector **Z** contains the model's predetermined variables, and vector **η** holds the random components of the equations of the structural-form's model.

If we consider the mechanism of interrelations between the total interdependent variables, a multiple-equation model can belong to one of three classes. The manner of those interrelations between the total interdependent variables allows distinction of the following:

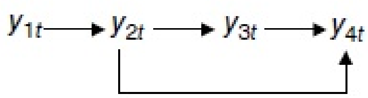
- simple models,
- recursive models,
- systems of interdependent equations.

In *simple models,* there are no direct relations between the total interdependent variables. This means that none of the total interdependent variables act as explanatory variables in any of the equations. The following system of equations can be an example of a simple model:

$$\begin{aligned} y_{1t} &= \alpha_{10} + \alpha_{11}x_{t1} + \alpha_{14}t + \alpha_{16}y_{3t-1} + \eta_{1t}, \\ y_{2t} &= \alpha_{20} + \alpha_{22}x_{t2} + \alpha_{24}t + \alpha_{25}y_{2t-1} + \eta_{2t}, \\ y_{3t} &= \alpha_{30} + \alpha_{33}x_{t3} + \alpha_{36}y_{3t-1} + \eta_{3t}. \end{aligned} \tag{2.3}$$

In the model 2.3, total interdependent variables (y_{1t}, y_{2t}, y_{3t}) are not related to each other. None of them acts as an explanatory variable. Only the time-delayed endogenous variables y_{2t-1} and y_{3t-1}, which belong in the group of predetermined variables, act as explanatory variables.

A recursive model is characterized by a chain (recursive) nature of relations between the total interdependent variables. This chain character of those relations signifies their one-directionality, with a possibility of indicating the chain's beginning and its end. The following can be an example of such a chain denoting the model's recursivity:



The beginning of the chain is formed by the variable y_{1t}, while the variable y_{4t} is its end. A recursive model of the relations between total interdependent variables, which are presented in the above diagram, can have the following form:

$$
\begin{aligned}
y_{1t} &= \alpha_{10} + \alpha_{11} x_{t1} + \alpha_{14} t + \alpha_{16} y_{3t-1} + \eta_{1t}, \\
y_{2t} &= \alpha_{20} + \alpha_{22} x_{t2} + \beta_{21} y_{1t} + \alpha_{25} y_{2t-1} + \eta_{2t}, \\
y_{3t} &= \alpha_{30} + \alpha_{34} t + \beta_{31} y_{1t} + \beta_{32} y_{2t} + \alpha_{35} y_{2t-1} + \eta_{3t}, \\
y_{4t} &= \alpha_{40} + \alpha_{43} x_{t3} + \beta_{43} y_{3t} + \alpha_{46} y_{3t-1} + \eta_{4t}.
\end{aligned}
\tag{2.4}
$$

A system of interdependent equations is characterized by mutual multilateral relations between the total interdependent variables. There may be two kinds of such relations: direct feedback or indirect feedback, also called a closed cycle of relations between the total interdependent variables. Feedback is based on a simultaneous self-impact of such a pair of variables. For example, the variables y_{gt} and $y_{g't}$ (for g, $g' = 1, \ldots, G$; $g \neq g'$) are linked by feedback when

$$y_{gt} \underset{\rightarrow}{\overset{\leftarrow}{\ }} y_{g't}.$$

For example, indirect feedback (a closed cycle of relations) of the total interdependent variables occurs in the following situation:

$$
\begin{array}{ccc}
y_{1t} & \rightarrow & y_{2t} \\
\uparrow & & \downarrow \\
y_{4t} & \leftarrow & y_{3t}
\end{array}
$$

Both kinds of relations can occur in a model simultaneously. This often occurs in large multiple-equation models. However, appearance of one of the above indicated mechanisms is enough for the model to form a system of interdependent equations. The following can be an example of such a model with direct feedback:

$$
\begin{aligned}
y_{1t} &= \alpha_{10} + \alpha_{11} x_{t1} + \alpha_{14} t + \alpha_{15} y_{3t-1} + \eta_{1t}, \\
y_{2t} &= \alpha_{20} + \alpha_{22} x_{t2} + \alpha_{24} t + \beta_{23} y_{3t} + \eta_{2t}, \\
y_{3t} &= \alpha_{30} + \alpha_{33} x_{t3} + \beta_{32} y_{2t} + \eta_{3t}.
\end{aligned}
\tag{2.5}
$$

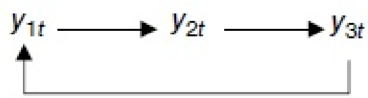
Model 2.5 contains feedback between the variables y_{2t} and y_{3t}. The variable y_{3t} influences the variable y_{2t}, while acting as an explanatory variable in the second equation. What is more, the variable y_{2t} influences the variable y_{3t}, acting as an explanatory variable in the third equation. As such, the requirement of a direct feedback is met.

In the model 2.5, the first equation draws attention. In that equation's set of explanatory variables, only predetermined variables occur. Thereby, the equation has the nature of a simple model. Such an equation in a system of interdependent equations, in which only predetermined variables are explanatory variables, is called a *detached equation*.

Let us consider the following model:

$$\begin{aligned} y_{1t} &= \alpha_{10} + \alpha_{11}x_{t1} + \beta_{14}y_{3t} + \alpha_{15}y_{3t-1} + \eta_{1t}, \\ y_{2t} &= \alpha_{20} + \alpha_{22}x_{t2} + \alpha_{24}t + \beta_{21}y_{1t} + \eta_{2t}, \\ y_{3t} &= \alpha_{30} + \alpha_{33}x_{t3} + \beta_{32}y_{2t} + \eta_{3t}. \end{aligned} \tag{2.6}$$

In the above model, the total interdependent variables form a closed cycle of relations, which has the form



Thereby, model 2.6 is a system of interdependent equations and can be classified as one of the most complicated econometric models.

2.2 A reduced form of the model

Having a model in a structural form, written in a matrix form as Equation 2.2, its left-side multiplication by a matrix $\mathbf{B}^{-1}$ can be performed. As a result we get the following:

$$\mathbf{B}^{-1}\mathbf{B}\mathbf{Y} + \mathbf{B}^{-1}\mathbf{A}\mathbf{Z} = \mathbf{B}^{-1}\eta.$$

The ratio of matrices $\mathbf{B}^{-1}\mathbf{B} = \mathbf{I}$, where $\mathbf{I}$ is the unit matrix of a G degree. By moving the expression $\mathbf{B}^{-1}\ \mathbf{AZ}$ to the right side, we obtain

$$\mathbf{Y} = -\mathbf{B}^{-1}\mathbf{A}\mathbf{Z} + \mathbf{B}^{-1}\eta.$$

By substituting $\mathbf{C} = -\mathbf{B}^{-1}\ \mathbf{A}$ and $\boldsymbol{\varepsilon} = \mathbf{B}^{-1}\boldsymbol{\eta}$ we arrive at a reduced form of the model

$$\mathbf{Y} = \mathbf{C}\mathbf{Z} + \varepsilon, \tag{2.7}$$

where[2]

$$C = \begin{bmatrix} c_{10} & c_{11} & \cdots & c_{1j} & \cdots & c_{1K} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{g0} & c_{g1} & \cdots & c_{gj} & \cdots & c_{gK} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{G0} & c_{G1} & \cdots & c_{Gj} & \cdots & c_{GK} \end{bmatrix}, \ \varepsilon = \begin{bmatrix} \varepsilon_{1t} \\ \cdots \\ \varepsilon_{gt} \\ \cdots \\ \varepsilon_{Gt} \end{bmatrix}.$$

Matrix ***C*** contains the structural parameters of the equations of the reduced-form's model, while **ε** is the vector of the random components of the reduced-form's equations.

Reduced-form's equations (in the number of *G*) can be written as follows:

$$\begin{aligned} y_{1t} &= c_{10} + c_{11}z_{t1} + \cdots + c_{1j}z_{tj} + \cdots + c_{1K}z_{tK} + \varepsilon_{1t}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_{gt} &= c_{g0} + c_{g1}z_{t1} + \cdots + c_{gj}z_{tj} + \cdots + c_{gK}z_{tK} + \varepsilon_{gt}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_{Gt} &= c_{G0} + c_{G1}z_{t1} + \cdots + c_{Gj}z_{tj} + \cdots + c_{GK}z_{tK} + \varepsilon_{Gt}. \end{aligned} \tag{2.8}$$

Based on the above, it can be inferred that each of the reduced-form's equations contains an identical set of explanatory variables. At the same time, all predetermined variables of the entire multiple-equation model make up a set of explanatory variables of each reduced-form equation. For instance, a system of reduced-form's equations for the model 2.6 form will be written as follows[3]:

$$\begin{aligned} y_{1t} &= c_{10} + c_{11}x_{t1} + c_{12}x_{t2} + c_{13}x_{t3} + c_{14}t + c_{15}y_{3t-1} + \varepsilon_{1t}, \\ y_{2t} &= c_{20} + c_{21}x_{t1} + c_{22}x_{t2} + c_{23}x_{t3} + c_{24}t + c_{25}y_{3t-1} + \varepsilon_{2t}, \\ y_{3t} &= c_{30} + c_{31}x_{t1} + c_{32}x_{t2} + c_{33}x_{t3} + c_{34}t + c_{35}y_{3t-1} + \varepsilon_{3t}. \end{aligned} \tag{2.9}$$

2.3 Identification of the model

Application of a multiple-equation model requires determining whether it has a correct form, as far as the relationships between its reduced- and structural-forms are concerned. Let us consider the equation

$$\mathbf{C} = -\mathbf{B}^{-1}\mathbf{A}, \tag{2.10}$$

which combines the structural-form with the reduced one. Left-side multiplication of both sides of Equation 2.10 by matrix **B** results in an *identification equation*:

$$\mathbf{B}\,\mathbf{C} = -\mathbf{A}. \tag{2.11}$$

The model is identifiable when, based on the components of the matrix ***C,*** it is possible to solve the system of linear equations, with regard to the components of matrices **B** and **A.** This means that it is necessary to solve the $G(K + 1)$ system of linear equations.[4]

While attempting to solve the $G(K + 1)$ system of linear equations, three options can be encountered:

1. There is only one solution of the system of equations. We, then, can speak of an explicit solution. In such cases, the multiple-equation model is *identifiable explicitly;*
2. There are many solutions of the $G(K + 1)$ system of equations. The system's solution is ambiguous. This means that the model is *identifiable ambiguously.* Such model has a correct form. It is also called an *overidentified* model.
3. There is no solution of the system. In this case, the multiple-equation model is *unidentifiable*, which signifies its faulty construction. Such a model needs to be reconstructed (respecified) in such a way that it can be identifiable at least ambiguously.

In the empirical model's identification test, two identifiability conditions, which arise from the need to impose the so-called zero-limits onto some of the structural parameters, must be met. This means that some of each equation's structural parameters must take zero-values. Thus, practically speaking, some of the total interdependent variables and some of the predetermined ones should not occur in the set of explanatory variables of specific equations of that form. The identification test is done for each equation separately. The *necessary condition* for identifiability of the gth equation ($g = 1, \ldots, G$) is for the number of the entire model's variables, which are not present in that equation (L_g), to be at least equal to the number $G - 1$, that is:

$$L_g \geq G - 1. \tag{2.12}$$

The second condition, which is a necessary and a sufficient requirement, is for the matrix sequence W_g ($g = 1, \ldots, G$) to be equal to $G - 1$, that is[5]:

$$rz\left(W_g\right) = G - 1. \tag{2.13}$$

If the condition 2.13 is met, then the gth equation is identifiable explicitly, when $L_g = G - 1$. However, if the condition 2.13 is met, the gth equation is identifiable ambiguously (overidentified), when $L_g > G - 1$.

The gth equation is not identifiable if $L_g < G - 1$, or $rz(W_g) < G - 1$. This means that the entire model is not identifiable and requires reconstruction. When all model equations are identifiable, it is identifiable explicitly, provided that each of its equations is identifiable explicitly. A multiple-equation model is identifiable ambiguously, if all of its equations are identifiable, or at least one of them is overidentified. Let us consider the model 2.6

$$y_{1t} = \alpha_{10} + \alpha_{11}x_{t1} + \beta_{14}y_{3t} + \alpha_{15}y_{3t-1} + \eta_{1t},$$
$$y_{2t} = \alpha_{20} + \alpha_{22}x_{t2} + \alpha_{24}t + \beta_{21}y_{1t} + \eta_{2t},$$
$$y_{3t} = \alpha_{30} + \alpha_{33}x_{t3} + \beta_{32}y_{2t} + \eta_{3t}.$$

First, all variables of the model, that is, y_{1t}, y_{2t}, y_{3t}, x_{t0}, x_{t1}, x_{t2}, x_{t3}, t, $y_{3t-1,}$ should be specified. It can be seen that in the first equation the variables y_{2t}, x_{t2}, x_{t3}, and t are absent. This means that $L_1 = 4$ is the number of variables (L_1), which are not present in the first equation. As such, the necessary condition is met, since $L_1 = 4 > G - 1 = 2$. Similarly, in the second equation, the variables y, y_{3t}, x_{t1}, x_{t3}, or y_{3t-1}, are absent, which means that $L_2 = 4$. It means that the second equation can be identifiable. In the third equation, the variables y_{1t}, x_{t1}, x_{t2}, t, and y_{3t-1} are absent, as a result of which $L_3 = 5$. As such, the second and the third equations can be identifiable.

It is therefore necessary to construct matrices W_1, W_2, and W_3, which will contain the coefficients of the variables that are absent in a given equation. Matrix W_1 will thus contain the structural parameters of the variables y_{2t}, x_{t2}, x_{t3}, and t from the second and the third equation, that is

$$W_1 = \begin{bmatrix} 1 & -\alpha_{22} & 0 & -\alpha_{24} \\ -\beta_{32} & 0 & -\alpha_{33} & 0 \end{bmatrix}. \tag{2.14}$$

As long as the parameters α_{22}, α_{24}, α_{32}, α_{33}, and β_{32} are different from zero, the sequence of that matrix $r(W_1) = 2 = G - 1$. Thus, the condition 2.13 is met, as a result of which the first equation is identifiable. Since the inequality $L_1 = 4 > G - 1 = 2$ occurs, the first equation is identifiable ambiguously.

Analogically, matrices W_2 and W_3 are as follows:

$$W_2 = \begin{bmatrix} -\beta_{14} & -\alpha_{11} & 0 & -\alpha_{15} \\ 1 & 0 & -\alpha_{33} & 0 \end{bmatrix}$$

and

$$W_2 = \begin{bmatrix} -\beta_{14} & -\alpha_{11} & 0 & -\alpha_{15} \\ 1 & 0 & -\alpha_{33} & 0 \end{bmatrix}$$

It can be demonstrated that $r(W_2) = r(W_3) = 2 = G - 1$. What is more, $L_2 = 4 > G - 1$ and $L_3 = 5 > G - 1$. Equations: the second and the third are thus identifiable ambiguously. Model 2.6 is therefore identifiable ambiguously (overidentified), which signifies its correct construction and enables further work on it in the subsequent stages.

2.4 Estimation of the parameters of a multiple-equation econometric model

The methods for estimation of the parameters of a multiple-equation model, essentially, are divided into two groups. The first group entails assessment methods for each equation separately, same as in a single-equation model. The second group entails estimation methods for the parameters of all equations simultaneously, called the total estimation methods.

In econometric literature, the prevailing viewpoint is that the parameters of equations in simple and recursive models can be assessed using the ordinary least squares method (OLS). This means that during estimation, each equation in those models can be treated as a single-equation model.

The OLS method does not provide consistent estimators of the parameters of the structural-form's equations of the systems of interdependent equations. This inconsistency results from the correlation of the total interdependent variables, which in the equations are explanatory, with the parallel random components. In this case, the condition of the OLS applicability, written as Equation 1.18, is not met. When this occurs, it is necessary to seek other methods of structural parameters' estimation in the models with interdependent equations. At the same time, it should be remembered that transition to the estimation stage is only possible when the model is identifiable explicitly or ambiguously.

It is noticeable that the reduced-form of the system of interdependent equations has characteristics of a simple model. Therefore, if there are no particularly unfavorable conditions, the parameters of the equations of the reduced-form's model can be assessed using the OLS method, for each equation separately.

Let us consider a case, when the system of interdependent equations is identifiable explicitly. This means that the identification equation 2.11 $\mathbf{BC} = -\mathbf{A}$ has an explicit solution. Let us consider the model

$$\begin{aligned} y_{1t} &= \alpha_{10} + \beta_{12} y_{2t} + \alpha_{11} x_{t1} + \eta_{1t}, \\ y_{2t} &= \alpha_{20} + \beta_{21} y_{1t} + \alpha_{22} x_{t2} + \eta_{2t}. \end{aligned} \tag{2.15}$$

The above model is a system of interdependent equations, in which each equation is explicitly identifiable. A reduced form of that model has the following form:

$$\begin{aligned} y_{1t} &= c_{10} + c_{11} x_{t1} + c_{12} x_{t2} + \varepsilon_{1t}, \\ y_{2t} &= c_{20} + c_{21} x_{t1} + c_{22} x_{t2} + \varepsilon_{2t}. \end{aligned} \tag{2.16}$$

An identification equation for the above structural- and reduced-forms of the model has the following form:

$$\begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix} \begin{bmatrix} c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} \alpha_{10} & \alpha_{11} & 0 \\ \alpha_{20} & 0 & \alpha_{22} \end{bmatrix}. \tag{2.17}$$

Let us suppose that using the OLS method and applying all available statistical data, the parameters of each equation of the reduced-form's model were assessed, the following empirical equations were obtained:

$$y_{1t} = 3.2 + 0.8x_{t1} + 2.4x_{t2} + e_{1t},$$
$$y_{2t} = 0.4 - 1.6x_{t1} + 6.2x_{t2} + e_{2t}. \quad (2.18)$$

In Equation 2.18, some residuals are present and they are respectively marked by the symbols e_{1t} and e_{2t}. The empirical identification equation can be written as

$$\begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} 3.2 & 0.8 & 2.4 \\ 0.4 & -1.6 & 6.2 \end{bmatrix} = \begin{bmatrix} a_{10} & a_{11} & 0 \\ a_{20} & 0 & a_{22} \end{bmatrix}. \quad (2.19)$$

In the matrices of the system 2.19, the symbols b_{12} and b_{21} represent estimations of the parameters β_{12} and β_{21}, while a_{10}, a_{11}, a_{20}, a_{22} represent estimations of the parameters α_{10}, α_{11}, α_{20}, α_{22} in the system of structural-form 2.15.

A system of six linear equations with six unknowns results from the matrix 2.19:

$$a_{10} = 3.2 - 0.4b_{12},$$
$$a_{11} = 0.8 + 1.6b_{12},$$
$$0 = 2.4 - 6.2b_{12},$$
$$a_{20} = 0.4 - 3.2b_{21},$$
$$0 = -1.6 - 0.8b_{21},$$
$$a_{22} = 6.2 - 2.4b_{21}. \quad (2.20)$$

The solution of the system of Equation 2.20 reveals estimations of the structural parameters in the system of Equation 2.15. They reach the following numerical values:

$$b_{12} = 0.387, b_{21} = -2.0, a_{10} = 3.045, a_{11} = 2.787, a_{20} = 6.8, a_{22} = 11.0.$$

The above estimations of the parameters of the structural-form's equations were assessed using the *indirect least squares method* (ILS). As a result, empirical equations of the model 2.15 can be written as follows:

$$\hat{y}_{1t} = 3.045 + 0.387y_{2t} + 2.787x_{t1},$$
$$\hat{y}_{2t} = 6.8 - 2.0y_{1t} + 11.0x_{t2}. \quad (2.21)$$

Bases on the above, the ILS method is easy in application. It takes place in two stages: in the first, the OLS method is used to estimate the parameters of equations of the reduced-form's model, while in the second, the system of linear equations, obtained from the matrix identification equation, is solved. This drawback of this method is the lack of a matrix of variances and covariances of the structural parameters' estimations for the empirical structural-form's equations. This prevents assignation of the average parameter assessment errors for the equations of such a model. As a result, there is no opportunity to test the significance of the explanatory variables in each of the structural-form's empirical equations.

If the system of interdependent equations is identifiable ambiguously, then the ILS method cannot be applied. In that case, the most commonly used estimation procedure is the *double least squares method* (2LS). It involves a twofold application of the least squares method. In the first step, the OLS method is used to estimate the parameters of the equations in the reduced-form's model. Based on the empirical reduced-form equations, the theoretical values of the total interdependent variables, which at the same time are devoid of their random part, are assigned. Next, in the structural-form's equations, the total interdependent explanatory variables are substituted with their theoretical values obtained from the reduced-form's empirical equations. The parameters of such modified structural-form's equations can be assessed using the ordinary least squares method.

As an example, we are going to follow the 2OLS method applied to the system of interdependent equation 2.5:

$$
\begin{aligned}
y_{1t} &= \alpha_{10} + \alpha_{11}x_{t1} + \alpha_{14}t + \alpha_{15}y_{3t-1} + \eta_{1t},\\
y_{2t} &= \alpha_{20} + \alpha_{22}x_{t2} + \alpha_{24}t + \beta_{23}y_{3t} + \eta_{2t},\\
y_{3t} &= \alpha_{30} + \alpha_{33}x_{t3} + \beta_{32}y_{2t} + \eta_{3t}.
\end{aligned}
$$

A system of the reduced-form's equations of this model is as follows:

$$
\begin{aligned}
y_{1t} &= c_{10} + c_{11}x_{t1} + c_{12}x_{t2} + c_{13}x_{t3} + c_{14}t + c_{15}y_{2t-1} + \varepsilon_{1t},\\
y_{2t} &= c_{20} + c_{21}x_{t1} + c_{22}x_{t2} + c_{23}x_{t3} + c_{24}t + c_{25}y_{2t-1} + \varepsilon_{2t},\\
y_{3t} &= c_{30} + c_{31}x_{t1} + c_{32}x_{t2} + c_{33}x_{t3} + c_{34}t + c_{35}y_{2t-1} + \varepsilon_{3t}.
\end{aligned}
$$

After assessing the parameters of the above equations using the OLS method, we will get the following reduced-form's empirical equations:

$$
\begin{aligned}
y^*_{1t} &= \hat{c}_{10} + \hat{c}_{11}x_{t1} + \hat{c}_{12}x_{t2} + \hat{c}_{13}x_{t3} + \hat{c}_{14}t + \hat{c}_{15}y_{2t-1},\\
y^*_{2t} &= \hat{c}_{20} + \hat{c}_{21}x_{t1} + \hat{c}_{22}x_{t2} + \hat{c}_{23}x_{t3} + \hat{c}_{24}t + \hat{c}_{25}y_{2t-1},\\
y^*_{3t} &= \hat{c}_{30} + \hat{c}_{31}x_{t1} + \hat{c}_{32}x_{t2} + \hat{c}_{33}x_{t3} + \hat{c}_{34}t + \hat{c}_{35}y_{2t-1}.
\end{aligned}
$$

In the above system of empirical equations of the reduced-form's model 2.3, the symbols y^*_{1t}, y^*_{2t}, y^*_{3t} represent the theoretical values of each equation's total interdependent variables, which resulted from calculations after application of the OLS method.

Now, the second step of this estimation procedure can be performed. Where the actual amounts of the total interdependent variables in structural-form's equations act as the explanatory variables, they are replaced with their theoretical amounts obtained from the reduced-form's empirical equations. As such, let us consider a new system of structural-form's equations:

$$y_{1t} = \alpha_{10} + \alpha_{11}x_{t1} + \alpha_{14}t + \alpha_{15}y_{3t-1} + \eta_{1t},$$
$$y_{2t} = \alpha_{20} + \alpha_{22}x_{t2} + \alpha_{24}t + \beta_{23}y^*_{3t} + \eta_{2t},$$
$$y_{3t} = \alpha_{30} + \alpha_{33}x_{t3} + \beta_{32}y^*_{2t} + \eta_{3t}. \quad (2.22)$$

Parameters of each equation in the system of Equation 2.22 can be assessed using the OLS method. Attention draws the first equation, whose set of explanatory variables has no total interdependent variables; meaning that the parameters of the detached equation in the system of interdependent equations can be directly estimated using the ordinary least square method.

In the second equation, the explanatory variable y_{3t} has been substituted with a variable in the form of theoretical values calculated from the reduced-form $\left(y^*_{3t}\right)$. At the same time, in the second equation, there is no correlation of the total interdependent explanatory variable that is nonrandom, with the random component. Thus, estimation of the parameters in the second equation, which is modified by the OLS method, is permitted. A similar change has occurred in the third equation, in which y_{2t} was substituted by y^*_{2t}, thus enabling estimation of the equation's parameters using the OLS method.

2.5 Forecasts estimation based on multiple-equation models

A model composed of many equations, for which fundamental assumptions of econometric prediction theory are met, can become a predictor. A forecast estimation procedure based on a multiple-equation model, ultimately, can be conducted very similarly to the one used during a construction of the forecasts that are based on a single-equation model. Predictive techniques differ for each class of multiple-equation models. However, similarities in the way the forecasts are achieved can be found, regardless of the multiple-equation model's class.

Regardless of the multiple-equation model's class, the key issue in econometric prediction is to determine the values of exogenous variables for each forecasted period. This requirement does not concern the time-delayed endogenous variables. The values of the endogenous variables, which are delayed by 1 period, are known for the first forecasted period $T = t_{00} + 1$ as the y_{en} $(g = 1, G)$ amounts for $n = t_0$. In the subsequent forecasted periods $T = t_0 + 2$, $T = t_0 + 3$, $T = t_0 + \tau$ the values are obtained by a sequential reference from the already estimated forecasts of endogenous variables. This procedure is called a *sequential prediction*.

Another common feature of the predictions that are based on various classes of multiple-equation models is the possibility of forecast estimation that is based on each of the equations separately. The procedure, thus, can be reduced to the same one, which takes place during a simultaneous prediction based on the G single-equation models.

Positive results of a simple model's verification allow extrapolation outside the statistical sample, on the status quo principle. Then, a *vector forecasts* for the vector of the total interdependent variables is reached:

$$Y_{Tp} = \left[y_{1T}^{(p)} \dots y_{gT}^{(p)} \dots y_{GT}^{(p)} \right], \tag{2.23}$$

where the vector's components are made up by the forecasts of each of the forecasted variables.

A forecast in the form of the above vector is obtained by substituting the values of explanatory variables for the forecasted period T in the empirical multiple-equation model. Thus, vector $\mathbf{Y}_{Tp}$ emerges by combining the G forecasts, which arose independently based on each equation separately. Forecasting based on each equation is identical to that in a single-equation model. *A prediction from a simple model* is thus a G-fold prediction based on a single-equation model.

In a recursive model, each of the equations can be considered separately, identically as in a single-equation model. It is necessary to forecast each equation in the correct order. Such procedure is called a *chain prediction*. In a recursive model, each endogenous variable is numbered accordingly to its causal order. In such ordering, the variable y_{it} depends on the predetermined variables and solely on those total interdependent variables y_{lt}, for which indicators 1 and i fulfill inequality $l < i$. Chain prediction, in this case, involves construction of forecasts for each individual component of the forecasted vector $\mathbf{Y}_{Tp}$ in a recursive manner, according to the order of the variables, which is reflected by the model. A specific feature of a chain forecast is that: if the model indicates that the forecasted variable $\mathbf{Y}_{iT}$ depends on any other simultaneous variable $\mathbf{Y}_{1T}$, $\mathbf{Y}_{1T}$, …, $\mathbf{Y}_{lT}$, while $l < i$, then during the forecasting $y_{iT}^{(p)}$, the prognoses referring to the total interdependent variables $y_{1T}^{(p)}, y_{2T}^{(p)}, \dots, y_{lT}^{(p)}$ occurring earlier in the chain are used.

If the forecasted period does not occur directly after the period t_0 and is in the $h > 1$ time-unit distance from it, then the chain prediction involves an h-fold repetition of the above procedure. As such, we get the h number of forecasted vectors for the subsequent periods. The $\left\{Y_T^p\right\}$ sequence of those vectors marks the expected paths of individual forecasted variables. Thus, it provides information on the expected manner of achieving the amounts forecasted for the last period. Therefore, it can be stated that when $h > 1$, the chain prediction, in relation to each forecasted variable, generates a sequence of forecasts for the subsequent periods, which signifies a sequential prediction. By combining a chain prediction with a sequential prediction, we get multiple forecast vectors, which can be written in a form of a suitable forecast matrix.

During a chain prediction, it is worth to determine the matrix of the correlation coefficients of the random components from each equation of the model:

$$\rho = \begin{bmatrix} 1 & \cdots & \rho_{1g} & \cdots & \rho_{1G} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{g1} & \cdots & 1 & \cdots & \rho_{gG} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{G1} & \cdots & \rho_{Gg} & \cdots & 1 \end{bmatrix}.$$

Matrix ρ of a $G \times G$ size contains the $\rho_{gg'}$ elements, which are the coefficients of a linear correlation between the random components of the g-th and g'-th equations, while g, $g' = 1, \ldots, G$ and $g \neq g'$. In practice, based on the residuals of the model's equations, the coefficients of the correlation $\rho_{gg'}$ are estimated and their assessments $\rho_{gg'}$ are obtained.

Very small correlation coefficients – as far as the module is concerned – suggest that individual equations are independent of each other. When $\hat{\rho}_{gg'}$ are close to +1, it can be inferred that the residuals of the gth and g'th equations, simultaneously, took the values of the same sign. However, when the $\hat{\rho}_{gg'}$ is close to −1, it can be assumed that the signs of the examined residuals of the equations numbered g and g' generally were different.

Let us consider the prediction technique from the following recursive model 2.4:

$$\begin{aligned} y_{1t} &= \alpha_{10} + \alpha_{11} x_{t1} + \alpha_{14} t + \alpha_{16} y_{3t-1} + \eta_{1t}, \\ y_{2t} &= \alpha_{20} + \alpha_{22} x_{t2} + \beta_{21} y_{1t} + \alpha_{25} y_{2t-1} + \eta_{2t}, \\ y_{3t} &= \alpha_{30} + \alpha_{34} t + \beta_{31} y_{1t} + \beta_{32} y_{2t} + \alpha_{35} y_{2t-1} + \eta_{3t}, \\ y_{4t} &= \alpha_{40} + \alpha_{43} x_{t3} + \beta_{43} y_{3t} + \alpha_{46} y_{3t-1} + \eta_{4t}. \end{aligned}$$

A predictor according the OLS method for the model 2.4 will be written as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -b_{21} & 1 & 0 & 0 \\ -b_{31} & -b_{32} & 1 & 0 \\ 0 & 0 & -b_{43} & 1 \end{bmatrix} \begin{bmatrix} y_{1T}^{(p)} \\ y_{2T}^{(p)} \\ y_{3T}^{(p)} \\ y_{4T}^{(p)} \end{bmatrix} + \begin{bmatrix} -a_{10} & -a_{11} & 0 & 0 & -a_{14} & 0 & -a_{16} \\ -a_{20} & 0 & -a_{22} & 0 & 0 & -a_{25} & 0 \\ -a_{30} & 0 & 0 & 0 & -a_{34} & -a_{35} & 0 \\ -a_{40} & 0 & 0 & -a_{43} & 0 & 0 & -a_{46} \end{bmatrix} \begin{bmatrix} x_{T0} \\ x_{T1} \\ x_{T2} \\ x_{T3} \\ T \\ y_{2T-1} \\ y_{3T-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{2.24}$$

Constructing forecasts for forecasted variables should begin with the so-called initial equation, which is the first equation in the model 2.4. Each equation, in which only the predetermined variables[6] are the explanatory variables, is *an initial equation* of the recursive model. Estimation of the forecast $y_{1T}^{(p)}$ will require determining the value of the exogenous variable x_{T1} in the forecasted period T. The time variable will reach a T value in the forecasted period. In contrast, the time-delayed variable y_{3T-1} will be known from the observation set for the $T = n + 1$ as y_{3n}, or when $T > n + 1$, we will use the forecast $y_{3T}^{(p)}$ estimated earlier on. When estimating forecasts $y_{2T}^{(p)}$, we will apply a procedure analogical to that in the first equation, with one difference – it will be necessary to use the predetermined forecasts $y_{1T}^{(p)}$.

Continuing the chain proceeding, we eventually arrive at the fourth equation, after obtaining a prediction from the third equation, where the forecast $y_{3T}^{(p)}$ was estimated. The forecast $y_{4T}^{(p)}$ is going to be obtained using the forecasts $y_{2T}^{(p)}$ and $y_{3T}^{(p)}$ estimated earlier on. As a result, we have a vector forecast of type 2.23 in the following form:

$$Y_{Tp} = \left[y_{1T}^{(p)} \quad y_{2T}^{(p)} \quad y_{3T}^{(p)} \quad y_{4T}^{(p)} \right], \tag{2.25}$$

where each of the components of the vector $\mathbf{Y}_{Tp}$ were calculated in the same way as the forecast from a single-equation model. Each time, using a formula 2. 25, we also calculate average prediction errors

$$V_T = \left[V_{1T} \quad V_{2T} \quad V_{3T} \quad V_{4T} \right]. \tag{2.26}$$

Prediction from a system of interdependent equations can be done in two ways. In the first method, equations of the structural-form's model are used, while in the second, inference into the future is based on equations of the reduced-form. These methods do not replace each other, and their applicability depends on the type of questions, which are posed and need to be answered by performing such inference into the future.[7]

Structural equations can be used when existence of causal interrelations in the stochastic total interdependent variables is omitted in considerations and when the aim is to estimate the effect of one-side interdependence of those variables. In such cases, the procedure is close to that which is applied in case of simple equations. At the same time, the values of those endogenous variables, which in equations act as explanatory variables, are assessed for the forecasted period T, using the same methods as for exogenous variables.

A prediction based on structural-form's equations, respecting only one side of the multiple-sided interrelations between the total interdependent variables, has the nature of inference into the future only for very short periods. Only during a very short time-period, it is allowed to abstract from the other aspects of interdependence between the total interdependent variables. In longer periods, interdependencies between endogenous variables play an important role and their omission can distort the sense as well as the results of a predictive testing.

With this in view, the second way of inference into the future – based on equations of a reduced-form's model – has greater practical importance. In this method, forecasting can be regarded as a conditional mathematical expectation, where in the condition some predetermined variables occur. Forecasting is based on each of the reduced-form's equations separately. The procedure is the same as in the case of a simple model, because the reduced-form has characteristics of a simple model.

If parameters of the reduced-form's equations were estimated directly, then the variances and covariances of structural parameters' estimations for each equation of that form are known. It is easy to determine the prediction variances for each equation; however, it is more difficult when the reduced-form was determined from an empirical structural-form. It is worth noting that the reduced-form's equations, each of which contains all the predetermined variables, usually are characterized by a presence of statistically insignificant explanatory variables. Therefore, it is worthwhile to determine the average prediction errors for the forecasts from the systems of interdependent equations that were obtained from the reduced-form's equations, from the matrix of variances and covariance's of structural parameters' estimations that are obtained from the structural-form's equations.

Prediction based on equations of the reduced-form of the model, in certain sense, has optimal properties, provided that an appropriate method was used to estimate the parameters. Prediction based on equations of the reduced-form is optimal, in such sense that it provides smaller average prediction errors than other methods using the same information resources.[8]

Let us consider a prediction from the system of interdependent equations, based on the model 2.5:

$$\begin{aligned} y_{1t} &= \alpha_{10} + \alpha_{11} x_{t1} + \alpha_{14} t + \alpha_{15} y_{3t-1} + \eta_{1t}, \\ y_{2t} &= \alpha_{20} + \alpha_{22} x_{t2} + \alpha_{24} t + \beta_{23} y_{3t} + \eta_{2t}, \\ y_{3t} &= \alpha_{30} + \alpha_{33} x_{t3} + \beta_{32} y_{2t} + \eta_{3t}. \end{aligned}$$

A prediction from the first equation of the above system for the $T = n + 1$ period can be performed independently of other equations, since it is a detached equation. The forecasts $y_{2T}^{(p)}$ and $y_{3T}^{(p)}$ ought to be estimated on the basis of a predictor from the reduced-form

$$\begin{aligned} y_{2T}^{(p)} &= \hat{c}_{20} + \hat{c}_{21} x_{T1} + \hat{c}_{22} x_{T2} + \hat{c}_{23} x_{T3} + \hat{c}_{24} T + \hat{c}_{25} y_{3T-1}, \\ y_{3T}^{(p)} &= \hat{c}_{30} + \hat{c}_{31} x_{T1} + \hat{c}_{32} x_{T2} + \hat{c}_{33} x_{T3} + \hat{c}_{34} T + \hat{c}_{35} y_{3T-1}, \end{aligned}$$

where the symbols $\hat{c}_{gj}$ ($g = 2, 3; j = 0, 1, \ldots, 5$) represent the estimations of the parameters of the second and third equation from the reduced-form that were obtained using the OLS method. The following will be the predictor for the first interdependent variable:

$$y_{1T}^{(p)} = a_{10} + a_{11} x_{T1} + a_{14} T + a_{15} y_{3T-1},$$

in which the symbols a_{10}, a_{11}, a_{14}, a_{15} represent the structural parameters' estimations of the equation that were obtained using the OLS method.

It can be noticed that in the subsequent forecasted periods ($T = n + 2, n + 3, \ldots, n + \tau$) it becomes necessary to apply sequential forecasting. The delayed variable y_{3T-1}, which appears in each equation of the considered predictor, forces the forecast estimation of the third total interdependent variable to be done in the first instance. This will allow using the forecast $y_{3T-1}^{(p)}$ as each equation's explanatory variable – in each equation of the predictor $\mathbf{Y}_{\mathrm{Tp}}$ – in subsequent periods.

Forecasts from a system of interdependent equations can also be partially estimated from the reduced-form's equations as well as from the structural-form's equations. Let us consider the following system of equations:

$$\begin{aligned} y_{1t} &= \alpha_{10} + \alpha_{11}x_{t1} + \alpha_{14}t + \alpha_{16}y_{3t-1} + \beta_{14}y_{4t} + \eta_{1t}, \\ y_{2t} &= \alpha_{20} + \alpha_{22}x_{t2} + \beta_{21}y_{1t} + \alpha_{25}y_{2t-1} + \eta_{2t}, \\ y_{3t} &= \alpha_{30} + \alpha_{34}t + \beta_{31}y_{1t} + \beta_{32}y_{2t} + \alpha_{35}y_{2t-1} + \eta_{3t}, \\ y_{4t} &= \alpha_{40} + \alpha_{43}x_{t3} + \beta_{43}y_{3t} + \alpha_{46}y_{3t-1} + \eta_{4t}. \end{aligned} \tag{2.27}$$

A closed cycle of relations between the total interdependent variables can be noticed:

$$\begin{array}{ccc} y_{1t} & \rightarrow & y_{2t} \\ \uparrow & & \downarrow \\ y_{4t} & \leftarrow & y_{3t}, \end{array} \tag{2.28}$$

which signifies a system of interdependent equations. Forecasting of the above model can be done using a mixed technique: partially from the reduced-form and partially from the structural one, applying a technique of chain prediction, which is specific for a recursive model. Applying in the following structural-form's predictor:

$$\begin{aligned} y_{1T}^{(p)} &= a_{10} + a_{11}x_{T1} + a_{14}T + a_{16}y_{3T-1} + b_{14}y_{4T}^{(p)}, \\ y_{2T}^{(p)} &= a_{20} + a_{22}x_{T2} + b_{21}y_{1T}^{(p)} + a_{25}y_{2T-1}, \\ y_{3T}^{(p)} &= a_{30} + a_{34}T + b_{31}y_{1T}^{(p)} + b_{32}y_{2T}^{(p)} + a_{35}y_{2T-1}, \\ y_{4T}^{(p)} &= a_{40} + a_{43}x_{T3} + b_{43}y_{3T}^{(p)} + a_{46}y_{3T-1}, \end{aligned} \tag{2.29}$$

in the forecasting is not possible immediately. Lack of the initial equation forming the "loop" 2.29 is an obstacle resultant from the closed cycle of relations. The "loop" can be eliminated using a reduced-form's equation for forecast estimation $y_{1T}^{(p)}$, that is

$$y_{1T}^{(p)} = \hat{c}_{10} + \hat{c}_{11}x_{T1} + \hat{c}_{12}x_{T2} + \hat{c}_{13}x_{T3} + \hat{c}_{14}T + \hat{c}_{15}y_{3T-1} + c_{16}\hat{y}_{3T-1}. \tag{2.30}$$

Knowledge of the forecast $y_{1T}^{(p)}$ allows application of a chain prediction technique to the subsequent equations of the structural-form's predictor. We can thus estimate the forecast $y_{2T}^{(p)}$ from the following equation:

$$y_{2T}^{(p)} = a_{20} + a_{22}x_{T2} + b_{21}y_{1T}^{(p)} + a_{25}y_{2T-1}.$$

Having the forecasts $y_{1T}^{(p)}$ and $y_{2T}^{(p)}$ allows estimation of the forecast $y_{3T}^{(p)}$ on the basis of the following equation:

$$y_{3T}^{(p)} = a_{30} + a_{34}T + b_{31}y_{1T}^{(p)} + b_{32}y_{2T}^{(p)} + a_{35}y_{2T-1}.$$

Having the forecast $y_{3T}^{(p)}$, it is possible to estimate the forecast $y_{4T}^{(p)}$ on the basis of the following equation:

$$y_{4T}^{(p)} = a_{40} + a_{43}x_{T3} + b_{43}y_{3T}^{(p)} + a_{46}y_{3T-1}.$$

The technique of predicting subsequent forecasted *T* periods should take into account the necessity for sequential proceedings resultant from the occurrence of the delayed endogenous variables $y_{2T-1}^{(p)}$ and $y_{3T-1}^{(p)}$. Ultimately, a prediction from a system of interdependent equations can connect a prediction from reduced-form's equations with a sequential and chain prediction.

Notes

1 In a structural form, the multiple-equation model reflects a full structure of interdependencies between the total interdependent variables as well as a direct effect of the predetermined variables on each total interdependent variable.

2 The symbols **Y** and **Z** were explained in relation with entry 2.2.

3 It is possible to use the original markings of the variables from the structural-form or to introduce new ones by giving a new symbol z_{tj} for each of the delayed exogenous and endogenous variables.

4 The number of $G(K + 1)$ results from the matrix **A**, which contains that amount of elements.

5 The matrix W_g is composed of the model's parameters, which occur with the variables that are absent in the *g*th equation ($g = 1, \ldots, G$).

6 This means that there can be more than one initial equation in a recursive model. Initial equation has the nature of an equation from a simple model, same as a detached equation in a system of interdependent equations.

7 See: Z. Pawłowski: *Forecasts*…, pp. 259–265.

8 See: Z. Pawłowski: *Forecasts*…, p. 254.

3
Econometric modeling of a large- and medium-sized enterprise’s economic system

3.1 Specification of a large- and medium-sized enterprise’s econometric model

Study of company’s production process, in large part, is falls within a realm of econometrics applicability. Productive process is a structure created by a person equipped with means of production. In market economy, it adapts the production profile to services and customer’s expectations. Requirement of rationality in all decision-making and actions means that this person must be aware of cause and effect relationships occurring between various relevant economic variables in a given enterprise.

In company-management practice, strong multilateral relationships between various economic processes exist. Such links occur only within a company. They are subject to influence of various external factors. A mechanism of economic links that occur in large- and medium-sized enterprise is presented in Figure 3.1.[1]

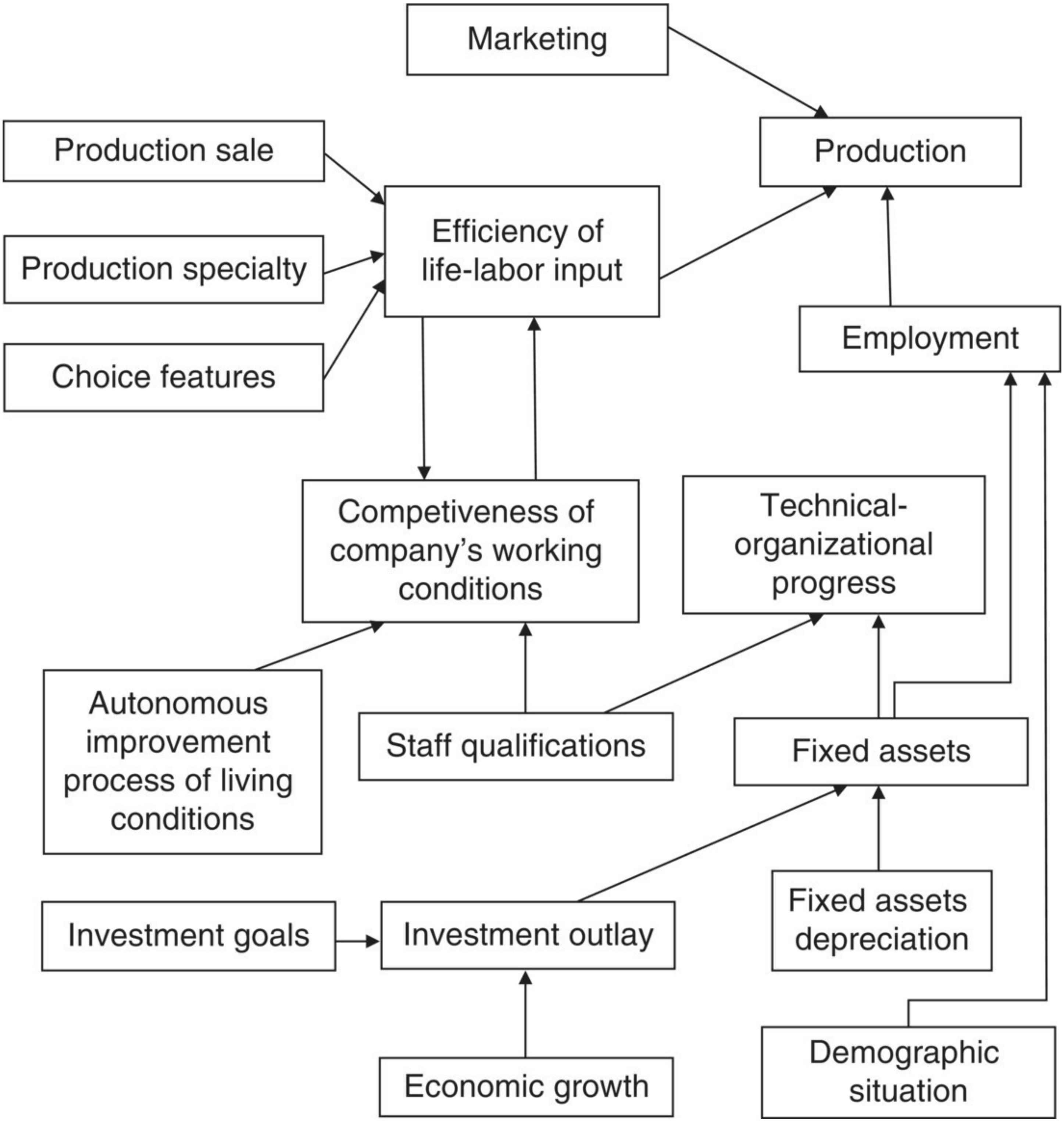


Figure 3.1 Economic interrelations in a large-sized (medium-sized) enterprise.

Source: Wiśniewski, J. W. Econometric Model of a Small Enterprise, chapter two[3] [2003].

Production is the final process described in a mechanism of inter-relations in a large (medium)[2] enterprise. This type of a company has a complex structure containing specialized services, which deal with individual elements occurring in Figure 3.1. The concept of *production* here will be understood as the sum of net[4] sales income. Production of finished goods is subordinate to a generated demand for company's goods and services, as observed by marketing services. Execution of liabilities for sold goods (services) is dealt with by debt recovery specialists. Therefore, a time interval between manufacture of goods (production of a service), its invoicing, and receiving a payment from a client is relatively short. It can be assumed that it is a period negligible from the perspective of a large enterprise's operation time.

Production volume results from the impact of human labor, presented in the model by *employment* magnitude and efficiency of *live-labor inputs*, as well as from activity of services concentrated within the field specified as *marketing*. *Marketing* can be regarded as an instrument created within an enterprise, where it can be applied as a management tool used, among other things, for production volume formation.

Employment is a variable inside a company, which influences its final result. It is also a subject to influence of factors, some of which are located within the enterprise and other outside it. The second of important production factors – *efficiency of labor input (labor productivity)* – is similar in character. *Efficiency* depends on conditions inside the company as well as outside it. What draws attention is feedback between *efficiency of live-labor input* and *competitiveness of working-conditions*[5] of an enterprise. This signifies simultaneous reciprocal influence of the pair of variables mentioned here. The level of technology and production organization, located within the term technological-organizational progress, is an important agent in *efficacy of a labor factor.*

Widely interpreted *competitiveness of working conditions* of an enterprise is shaped by many factors. Two particularly important ones are shown here: *staff qualifications* as well as the *autonomous process of living-conditions improvement* ongoing outside the enterprise,[6] independently of its inside processes.

Employment volume is shaped, above all, by the available *fixed assets*. Complementarity and substitutability relations occur between *employment* and *fixed assets*. This means that increment of company's fixed assets can cause *employment* increase (complementarity) or decrease (substitution). Generally, in an enterprise, complementarity and substitution occur simultaneously. Changes in *employment* volume are the outcome of these processes.

Enterprise's fixed assets undergo physical and moral (economic) *consumption*; thus, it must be regenerated. It follows that *assets consumption* causes quantitative and qualitative changes in the mass of those assets. On the other hand, enterprise's development requires *investment outlays*, which increase and modify the company's existent *fixed assets*.

Investment outlays result from a general atmosphere in the economy and within the country, as those can cause inclinations to invest or to refrain from such projects. Economic growth, being a most synthetic conceptualization of given area's economic conditions in a given time period, is an expression of that economic and national climate. The size and the structure of *investment outlays* are determined by needs and opportunities arising from a *progressive consumption of fixed assets* as well as by defined within company goals, which are later on translated into the language of company's *investment objectives*.

3.2 Structural form of an econometric model of a large- and medium-sized enterprise

Econometric model's endogenous variables describing a large-sized (medium-sized) enterprise can be as follows[7]:

Y_1 – net sales income of an enterprise (in millions PLN)[8] for a t ($t = 1, \ldots, n$) time-period

Y_2 – the average annual[9] number of employees, in full-time equivalent,

Y_3 – labor productivity calculated as the ratio[10] of $\frac{Y_1}{Y_2}$,

Y_4 – average monthly wage, in PLN monthly[11] for one employee,

Y_5 – initial value of active fixed assets, in millions PLN,

Y_6 – technical labor devices measured by the initial value of the fixed assets, for one employee (in thousands PLN/1 employee), adjusted by a shift coefficient,

Y_7 – value of investment outlays in a t period, that is, in thousands PLN.

The following can be exogenous variables of a model for a large (medium) enterprise:

X_1 – cost of marketing activity,[12]

X_2 – production volume in a natural measure unit,[13] that is, in thousands of tons,

X_3 – number of manufactured product range; production entropy H_t in a t ($t = 1, \ldots, n$) period can be an alternative and is calculated using the following formula:

$$H_t = -\sum_{i=1}^{m_t} p_{ti} \log_2 p_{ti}, \tag{3.1}$$

where p_{ti} represents the share of ith product range[14] in production value, whereas m_t is the number of manufactured assortments in a t period.

X_4 – value of special-order production[15] in millions PLN,

X_5 – number of employees with higher education,[16]

X_6 – annual depreciation of fixed assets[17] in millions PLN,

X_7 – GDP growth rate,[18]

X_8 – number of the unemployed having qualifications necessary in the company, who are on the market on which the company functions,[19]

X_9 – value of sales on new markets,[20] in millions PLN,

X_{10} – time variable[21] t ($t = 1, \ldots, n$).

Using Figure 3.1, hypothetical equations of structural-form econometric model for a large-sized enterprise can be written as follows[22]:

$$Y_1 = f(Y_2, Y_3, X_1, \eta_1), \tag{3.2}$$

$$Y_2 = f(Y_5, X_8, \eta_2), \tag{3.3}$$

$$Y_3 = f(Y_4, Y_6, X_2, X_3, X_4, \eta_3), \tag{3.4}$$

$$Y_4 = f(Y_3, X_5, t, \eta_4), \tag{3.5}$$

$$Y_5 = f(Y_7, X_6, \eta_5), \tag{3.6}$$

$$Y_6 = f(Y_5, X_5, \eta_6), \tag{3.7}$$

$$Y_7 = f(X_6, X_7, X_9, \eta_7). \tag{3.8}$$

In the system of theoretical structural-form equations,[23] Equations 3.2–3.8, symbols $\eta_1, \eta_2, \ldots, \eta_7$ represent random components of each equation.

It can be noticed that the equation describing variable Y_7 has characteristics of a detached equation, since only explanatory variables having characteristics of predetermined ones can occur in that equation. A hypothetical system of reduced-form equations of the above model can be written as follows[24]:

$$Y_1 = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, t, \varepsilon_1), \tag{3.9}$$

$$Y_2 = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, t, \varepsilon_2), \tag{3.10}$$

$$Y_3 = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, t, \varepsilon_3), \tag{3.11}$$

$$Y_4 = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, t, \varepsilon_4), \tag{3.12}$$

$$Y_5 = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, t, \varepsilon_5), \tag{3.13}$$

$$Y_6 = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, t, \varepsilon_6), \tag{3.14}$$

$$Y_7 = f\left(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, t, \varepsilon_7\right). \tag{3.15}$$

It is noticeable that in the system of Equations 3.9–3.15, each equation is characterized by an identical set of explanatory variables.[25]

3.3 Empirical econometric model of a medium-sized enterprise

3.3.1 Assumptions for an econometric empirical model

This subsection is going to present an empirical econometric model, which describes an enterprise of medium-size category (according to the European Union classification). The company code-named ENERGY[26] carries a production business activity of a trade and service character. The statistical data was obtained on a monthly basis for years 2008–2012. Information was aggregated, which resulted in quarterly time series containing 20 statistical observations each (see Table 3.1).

Table 3.1 Basic statistical data of the ENERGY enterprise.

Source: Own elaboration on the basic of the ENERGY company's documentation.

Period	Net sales income (in thousands PLN)	Employment, in full-time job positions (number of job-positions)	Fixed assets (in millions PLN)	Payroll (in thousands PLN)
2008:1	10 302	195	10.6	2 209
2008:2	15 120	191	11.1	2 160
2008:3	15 618	188	11.4	2 056
2008:4	16 526	182	11.4	2 230
2009:1	12 209	178	12.4	2 016
2009:2	17 212	176	12.0	2 152
2009:3	16 488	174	11.7	1 862
2009:4	17 512	171	11.4	1 894
2010:1	8 730	167	11.1	1 568
2010:2	14 174	163	10.7	1 606
2010:3	16 518	159	10.3	1 765
2010:4	18 729	157	10.1	2 283
2011:1	10 805	154	10.2	2 362
2011:2	13 090	146	9.6	1 864
2011:3	19 165	128	9.4	1 783
2011:4	17 016	110	9.3	1 733
2012:1	9 172	95	8.3	968
2012:2	11 678	90	8.2	898
2012:3	11 709	85	8.1	1 032
2012:4	12 984	83	8.0	1 065
∑	284 757	2992	205.3	35 506

The econometric model is composed of six stochastic equations. Compared to the hypothetical one presented in Section 3.2, equation describing company's investments is missing. This results from the fact that in the years 2008–2012, the company suffered meager investment. The investment equation, thus, does not bring any significant systemic information to the area of company's management, because most statistical information in time series takes zero values. Therefore, the following endogenous variables will be described in the model:

SNET – quarterly net sales income (in millions PLN),

EMP – average quarterly employment, in full-time job positions (number of job positions), EFEMP – labor productivity, per 1 quarterly full-time employee (in thousands PLN/1 employee),

APAY – gross average quarterly wage, per one employee (in thousands PLN),

FIXAS – company's average quarterly fixed assets value (in millions PLN),

TAL – technical labor equipment measured in company's fixed assets, per one full-time employee (in thousands PLN/one employee).

The variables presented in Section 3.2 were considered, which belong to a set of exogenous variables of the model. Not all of them can be used in an econometric model of the company ENERGY. For example, company's specification prevents the use of a specialization measure. Multispecificity of business activity results in the lack of company's specialization. In empirical model's equations, many exogenous variables that proved to be statistically insignificant were eliminated. Simultaneous endogenous variables were also eliminated. As a result, the hypothesis about the system of interdependent equations failed. The model of the company ENERGY is recursive in character.

3.3.2 Equation of the sales income

A hypothetical equation describing a mechanism of the net sales income (SNET) took into account autoregression up to and including the fourth order. None of the autoregressive variants proved to be statistically significant. Labor productivity (EFEMP) in the form of an unbiased index of dynamics as well as the impact of an EFEMP variable delayed by 1, 2, 3, and 4 quarters was entered into the hypothetical equation. The delayed variables proved to be statistically insignificant. Similarly, variables representing employment (EMP) delayed by 1–4 quarters were insignificant. Equation of sales income also contained three variants of variables describing the company's marketing activity. These were advertisement expenses, representative expenses, and the sum of advertisement and representative expenses. Simultaneously, those variables' values as well as delays from 1 to 4 quarters were considered. Finally, the variable MARK(−1) representing advertisement expenses delayed by 1 quarter proved to be statistically significant. The results of parameters' estimation of an empirical equation of net sales income are shown in Table 3.2. Graphically, estimation results are shown in Figure 3.2.

Table 3.2 An empirical equation of the sales income.

Source: Own calculations using the EViews 4 package.

Dependent variable: SNET				
Method: least squares				
Date: 04/01/2014, time: 15:09				
Sample (adjusted): 2008:2 2012:4				
Included observations: 19 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	−17114.14	1621.608	−10.55381	0.0000
EFEMP	71.74060	4.018011	17.85476	0.0000
EMP	123.2627	7.381952	16.69784	0.0000
MARK(−1)	−9.898086	3.333035	−2.969692	0.0095
R-squared	0.962390	Mean dependent var		14445.00
Adjusted *R*-squared	0.954868	S.D. dependent var		3134.381
S.E. of regression	665.8806	Akaike info criterion		16.02476
Sum squared resid	665.0955	Schwarz criterion		16.22359
Log likelihood	−148.2352	*F*-statistic		127.9419
Durbin–Watson statistic	1.821667	Prob (*F*-statistic)		0.000000

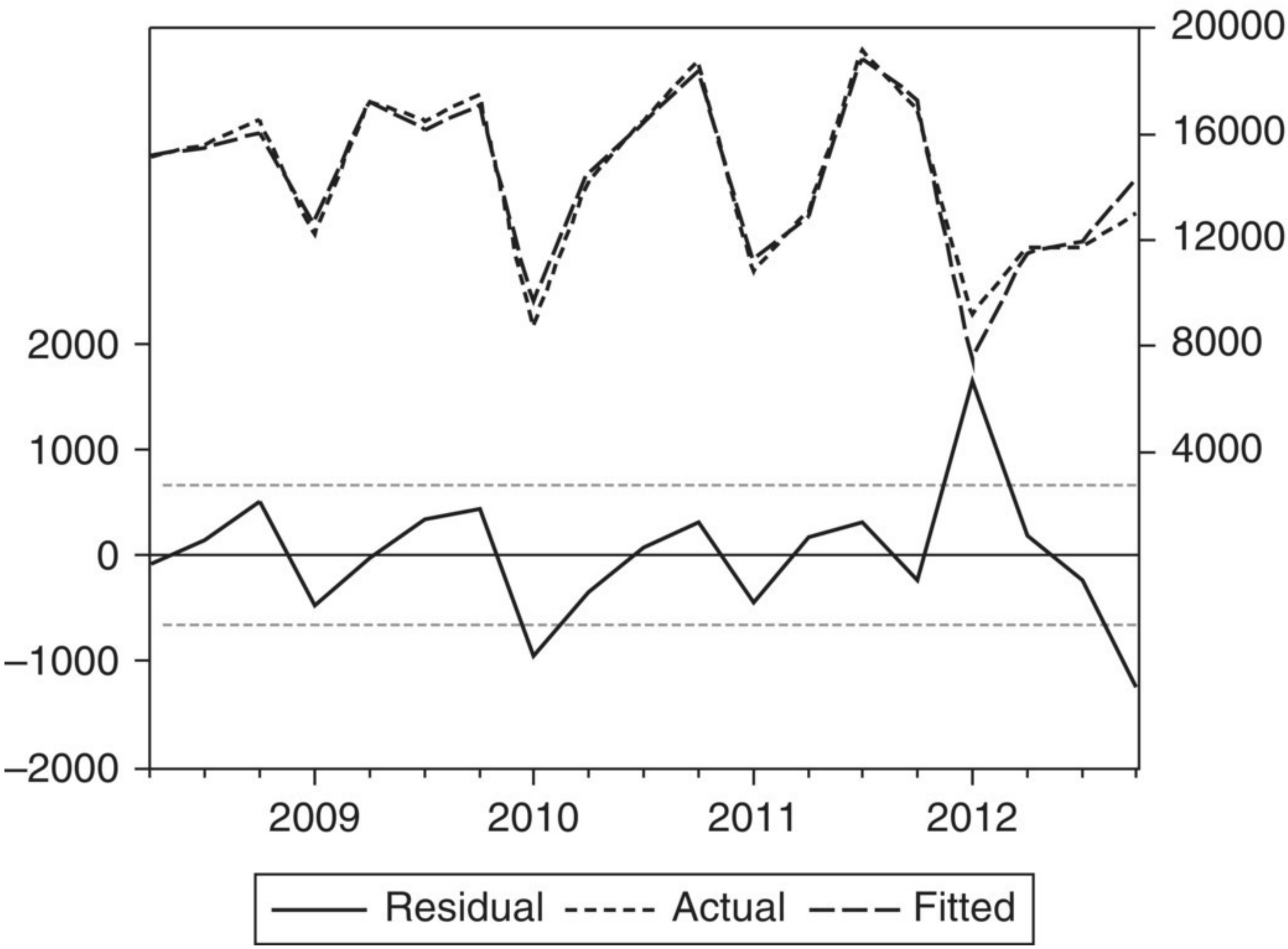


Figure 3.2 The actual monthly net sales income, the theoretical values, and the residuals calculated based on the equation from Table 3.2.

Source: Table 3.2.

As expected, labor productivity and employment volume have turned out to be simulators of the amount of net sales income. In contrast, advertisement expenses delayed by 1 quarter had negative influence on sales income. This probably resulted from a high intensity of advertisement expenses in the beginning of the study period, before 2008. Since 2010, advertisement expenses decreased significantly. A decrease in those expenses was accompanied by an increase in sales income, which probably resulted from previously established business relations.

3.3.3 Equation of employment

The company was characterized by high labor costs before 2010. Savings achieved by reduction in employment level were necessary. As a consequence of a downward trend in the number of employees – along with an established customer range – there was no impact of the fixed assets as well as of the demographic situation on the company's area. Employment was characterized only by autoregressive processes of the first, second, and fourth order. Figure 3.3

presented actual quarterly employment, theoretical employment, and residuals calculated based on the equation from Table 3.3.

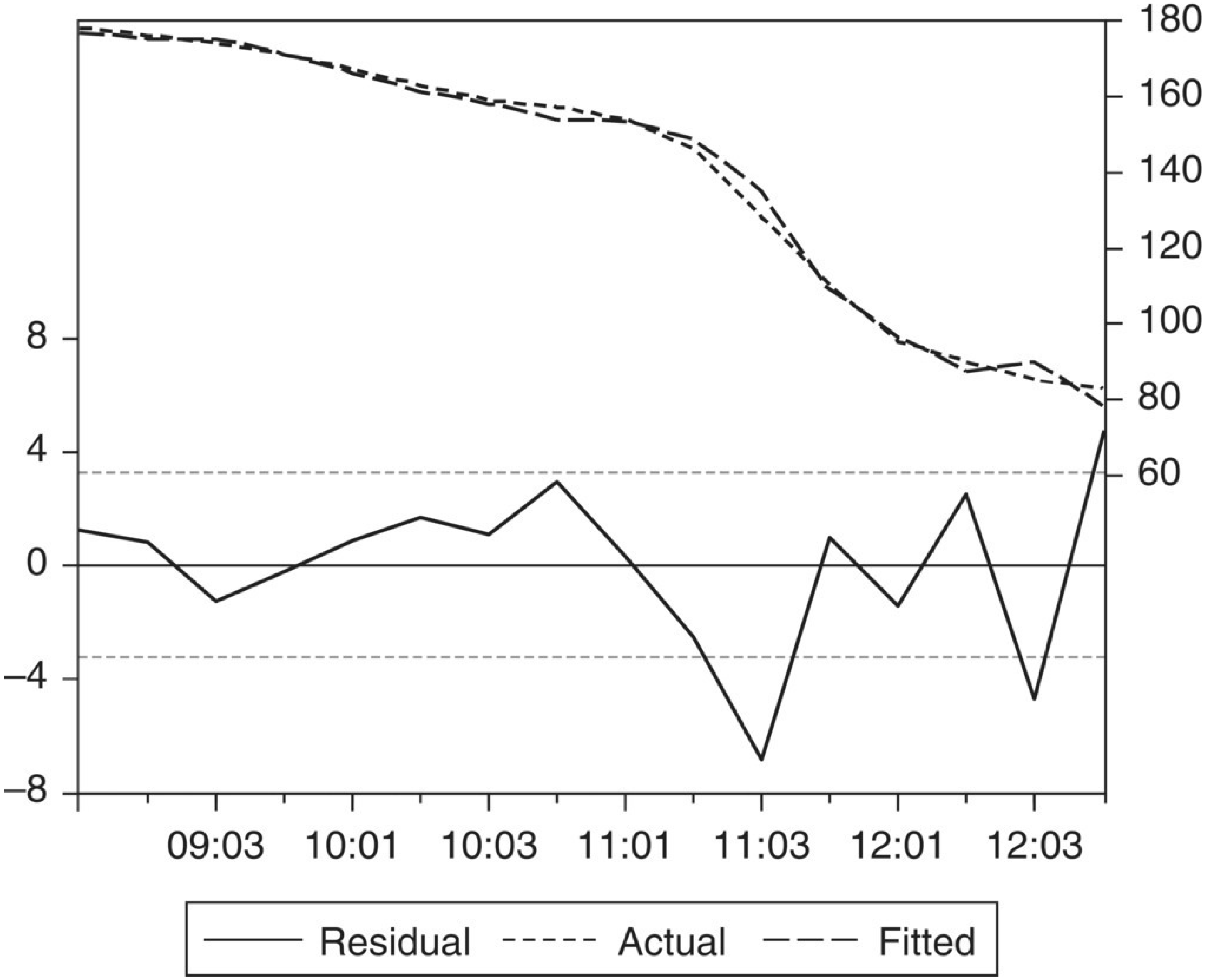


Figure 3.3 Actual quarterly employment, theoretical employment, and residuals calculated based on the equation from Table 3.3.

Source: Table 3.3.

Table 3.3 An empirical equation of employment.

Source: Own calculations using the EViews 4 package.

Dependent variable: EMP				
Method: least squares				
Date: 04/01/2014, time: 17:34				
Sample(adjusted): 2009:1 2012:4				
Included observations: 16 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	−19.07222	9.272138	−2.056938	0.0621
EMP(−1)	1.994066	0.179455	11.11179	0.0000
EMP(−2)	−1.372568	0.258369	−5.312427	0.0002
EMP(−4)	0.466590	0.159079	2.933073	0.0125
R-squared	0.993331	Mean dependent var		139.7500
Adjusted *R*-squared	0.991664	S.D. dependent var		35.43915
S.E. of regression	3.235600	Akaike info criterion		5.398624
Sum squared resid	125.6293	Schwarz criterion		5.591771
Log likelihood	−39.18899	*F*-statistic		595.8284
Durbin–Watson statistic	2.153871	Prob (*F*-statistic)		0.000000

Employment volatility in any enterprise is specified by inertia, which is manifested by autoregressive dependencies of various orders. Most frequently, there is a positive autoregression of the first order, which can be seen in the case of the analyzed company ENERGY. Negative autoregression of the second order primarily results from the mode of employment contracts' termination, which is regulated by Labor Law. Employment equation was dropped from the chain that composes a recursive mechanism of the model. It became the so-called detached equation, meaning a specific type of an equation for the simple model.

3.3.4 Equation of labor productivity

Team-labor productivity in the company ENERGY is affected by a significant number of explanatory variables. There are autoregressive mechanisms of the first, second, and fourth order. A significant level of autoregressive dependency of the fourth order draws attention. There was no feedback between the average wage and labor productivity.[27] Only delayed impact of the average wage on productivity occurs. At the same time, the impact of average wage, delayed by 1 and 3 quarters, is negative. In contrast, impact of the average wage delayed by 2 quarters on labor productivity is positive. What draws attention is the bigger amount of negative coefficients than that of the positive one. This may signify defectiveness of a motivational mechanism dominating in the company. Lack of a simultaneous impact of the average wage on employees' efficiency also confirms such thesis.

Negative impact of the technical equipment delayed by 4 quarters on current labor productivity is significant. This may result from the aimed at reduction of business operating costs reduction of fixed assets and employment. A positive trend in labor productivity is a favorable development. It turns out, that labor productivity has increased, on average, by 3000 PLN per one employee in 1 quarter.

An equation of labor productivity does not co-create the system of interdependent equations with an average wage equation. It remains, however, in an interdependency relationship of chain character, typical for a recursive model. Empirical results of modeling are provided in Table 3.4 and in Figure 3.4.

Table 3.4 An empirical equation of labor productivity.

Source: Own calculations using the EViews 4 package.

Dependent variable: EFEMP				
Method: least squares				
Date: 04/04/2014, time: 15:57				
Sample (adjusted): 2009:1 2012:4				
Included observations: 16 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	140.8303	54.57531	2.580477	0.0364
EFEMP(−1)	0.622322	0.133667	4.655762	0.0023
EFEMP(−2)	−0.481733	0.114267	−4.215850	0.0040
EFEMP(−4)	1.041329	0.120529	8.639625	0.0001
APAY(−1)	−5.644315	1.601560	−3.524261	0.0097
APAY(−2)	11.88797	2.266970	5.243989	0.0012
APAY(−3)	−8.988088	1.827948	−4.917036	0.0017
TAL(−4)	−2.241678	0.683094	−3.281654	0.0135
TIME	2.795396	1.125612	2.483446	0.0420
R-squared	0.977291	Mean dependent var		106.9183
Adjusted *R*-squared	0.951337	S.D. dependent var		31.68854
S.E. of regression	6.990365	Akaike info criterion		7.025264
Sum squared resid	342.0564	Schwarz criterion		7.459845
Log likelihood	−47.20211	*F*-statistic		37.65562
Durbin–Watson statistic	2.099497	Prob (*F*-statistic)		0.000045

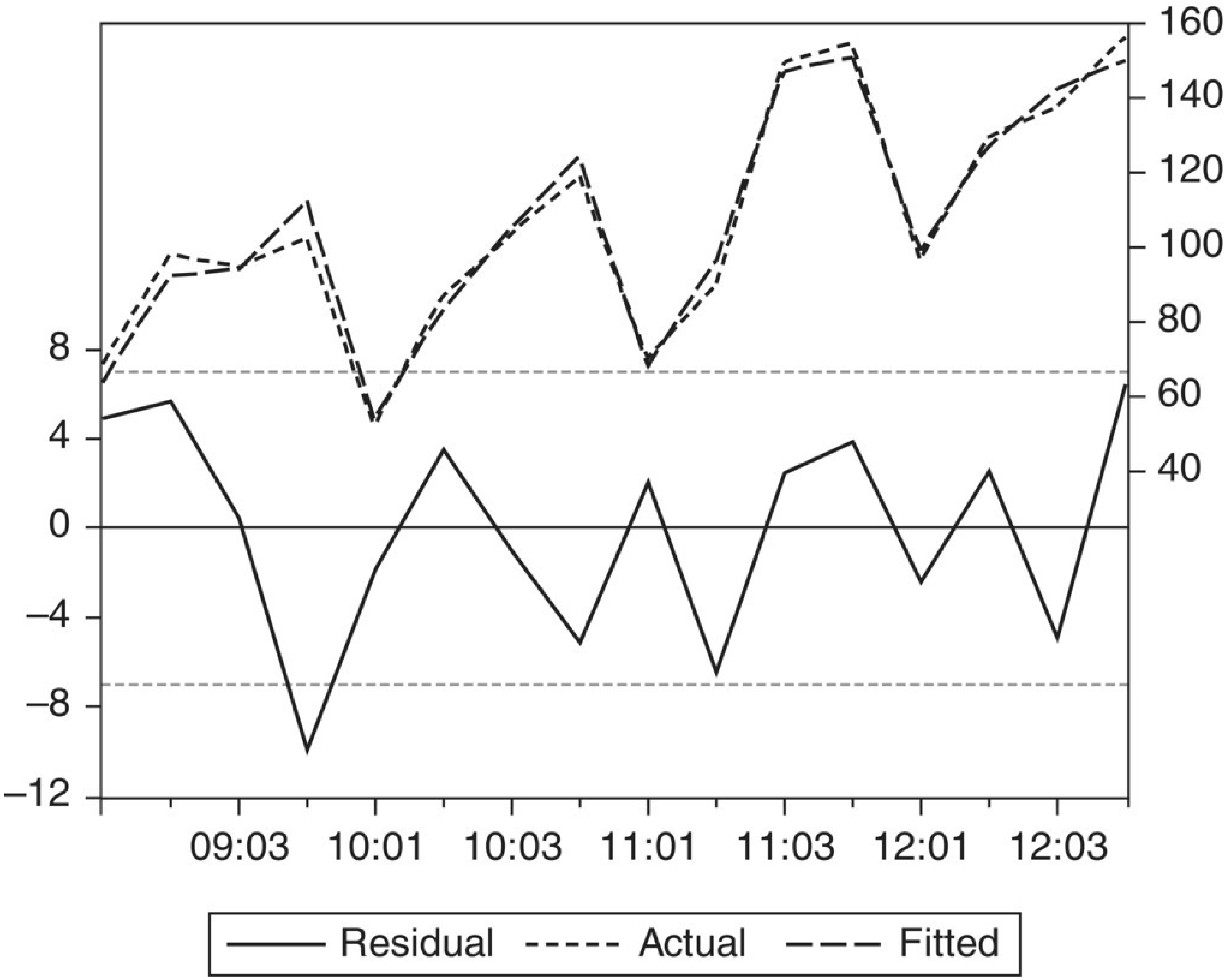


Figure 3.4 Actual quarterly team-labor productivity, theoretical volumes, and the residuals calculated based on the equation from Table 3.4.

Source: Table 3.4.

3.3.5 Equation of the average wage

In an equation describing a mechanism of the average wage formation, autoregression of the first, second, and fourth orders as well as impact of labor productivity have occurred. Autoregression of the first order is positive, while autoregressions of the second and fourth order are negative in character. A tendency to lower the wages in the company by elimination of inertia in wages is seen. Appearance of a positive impact of labor productivity informs about untaken actions aimed at activation of "healthy" motivational role of wages, according to the principle higher wages for more productive work.

Accuracy of wage mechanism's description in the equation is significantly lower $\left(R^2 = 0.769\right)$, compared to the earlier presented empirical equations, in which R^2 significantly exceeded the level of 0.95. This may suggest a significant impact of other factors on the average wage level in the analyzed company.

Empirical results of the average wage modeling are presented in Table 3.5 and in Figure 3.5. Seasonal wage fluctuations in each quarter, with a tendency to increase, can be noticed.

Table 3.5 An empirical equation of the average wage.

Source: Own calculations using the EViews 4 package.

Dependent variable: APAY				
Method: least squares				
Date: 04/01/2014, time: 17:46				
Sample (adjusted): 2009:1 2012:4				
Included observations: 16 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-statistic	Probability
C	11.23787	2.765680	4.063329	0.0019
APAY(−1)	0.700296	0.163949	4.271424	0.0013
APAY(−2)	−0.477972	0.166112	−2.877415	0.0150
APAY(−4)	−0.678725	0.190013	−3.571994	0.0044
EFEMP	0.059336	0.011705	5.069207	0.0004
R-squared	0.768637	Mean dependent var		12.07144
Adjusted *R*-squared	0.684505	S.D. dependent var		1.988078
S.E. of regression	1.116682	Akaike info criterion		3.308907
Sum squared resid	13.71677	Schwarz criterion		3.550341
Log likelihood	−21.47126	F-statistic		9.136088
Durbin–Watson statistic	2.732821	Prob (F-statistic)		0.001667

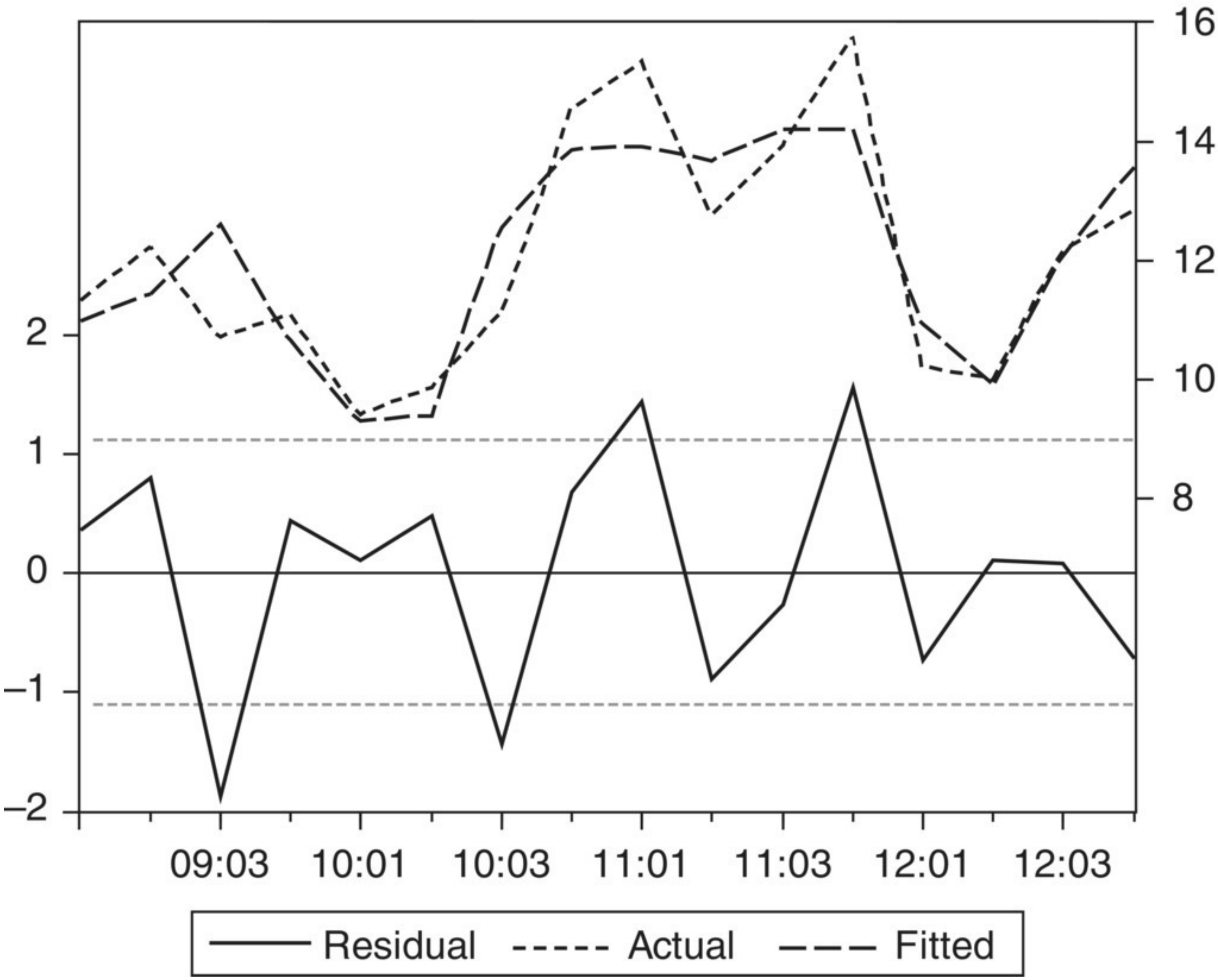


Figure 3.5 The actual average quarterly net wages, their theoretical values, and the residuals calculated on the basis of the equation from Table 3.5.

Source: Table 3.5.

3.3.6 Equation of the fixed assets

The company ENERGY was equipped with the fixed assets (FIXAS) through the deed of foundation of a holding to which it belongs. In years 2008–2012, a systematic and significant decrease in the value of that variable is observed. Equation describing the variable FIXAS has characteristics of a detached one, although, the model hypothesis assumed recursiveness of that endogenous variable's contribution.

Table 3.6 and Figure 3.6 present a mechanism of the volatility of company's fixed assets. In the equation, negative autoregression of the fourth order occurred, which resulted in systematic decrease in the variable FIXAS's value. A negative tendency informing about an average quarterly decrease of fixed assets, by around 320 thousands PLN, occurs as well. At the same time, the value of fixed assets' depreciation has a positive influence on the growth of tangible assets. This means that attention to replacement of fully exploited fixed assets occurs.

Table 3.6 An empirical equation of the fixed assets.

Source: Own calculations using the EViews 4 package.

Dependent variable: FIXAS				
Method: least squares				
Date: 04/04/2014, time: 15:56				
Sample (adjusted): 2009:1 2012:4				
Included observations: 16 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	15.81238	1.011460	15.63324	0.0000
FIXAS(−4)	−0.249517	0.103405	−2.413016	0.0327
DEPR(−4)	0.003374	0.001431	2.358760	0.0361
TIME	−0.320004	0.014186	−22.55784	0.0000
R-squared	0.989627	Mean dependent var		10.05000
Adjusted *R*-squared	0.987034	S.D. dependent var		1.441296
S.E. of regression	0.164117	Akaike info criterion		−0.564158
Sum squared resid	0.323212	Schwarz criterion		−0.371011
Log likelihood	8.513264	*F*-statistic		381.6291
Durbin–Watson statistic	2.141813	Prob (*F*-statistic)		0.000000

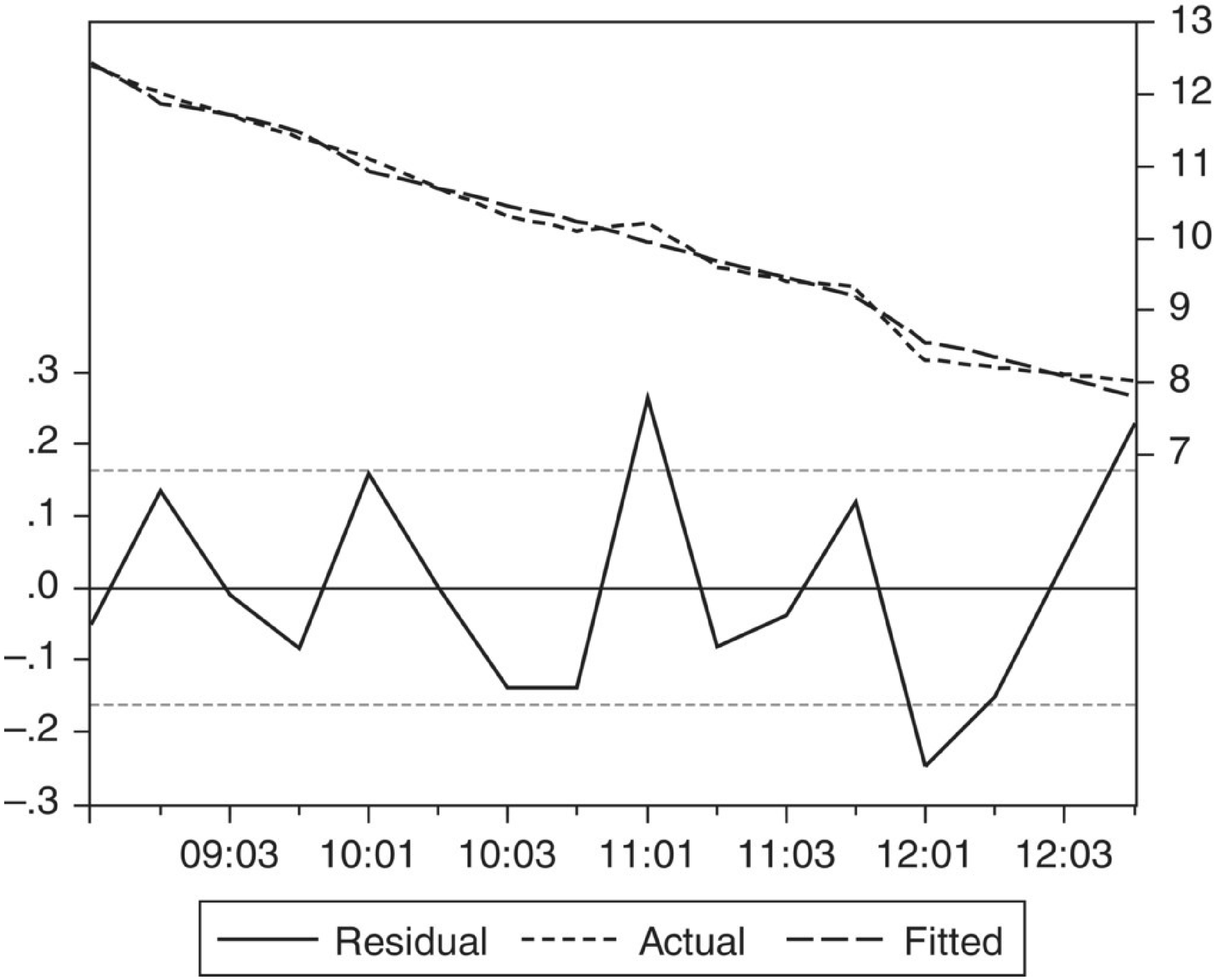


Figure 3.6 The actual quarterly value of the fixed assets, the theoretical values, and the residuals calculated using the equation from Table 3.6.

Source: Table 3.6.

3.3.7 Equation of the technical labor equipment

Technical labor equipment belongs to characteristics of technical development in an enterprise. Empirical equation describing the volatility mechanism of labor technical equipment (TAL) is presented in Table 3.7 and in Figure 3.7. From Figure 3.7 results a dynamic growth of a variable TAL after 2010.

Table 3.7 An empirical equation of the technical labor equipment.

Source: Own calculations using the EViews 4 package.

Dependent variable: TAL				
Method: Least squares				
Date: 04/01/14 time: 18:10				
Sample (adjusted): 2009:1 2012:4				
Included observations: 16 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	−38.19772	19.68906	−1.940049	0.0935
TAL(−1)	3.219578	0.368494	8.737126	0.0001
TAL(−2)	−2.731791	0.407198	−6.708751	0.0003
TAL(−4)	0.876021	0.186679	4.692666	0.0022
FIXAS	11.97294	1.892871	6.325280	0.0004
FIXAS(−1)	−20.33832	2.661017	−7.643062	0.0001
FIXAS(−2)	14.41811	2.286484	6.305803	0.0004
FIXAS (−4)	−7.373084	1.407030	−5.240176	0.0012
DEPREMP(−3)	0.018297	0.003847	4.756120	0.0021
R-squared	0.996096	Mean dependent var		74.56903
Adjusted *R*-squared	0.991634	S.D. dependent var		11.88248
S.E. of regression	1.086854	Akaike info criterion		3.302773
Sum squared resid	8.268763	Schwarz criterion		3.737354
Log likelihood	−17.42219	*F*-statistic		223.2409
Durbin–Watson statistic	1.991177	Probability (*F*-statistic)		0.000000

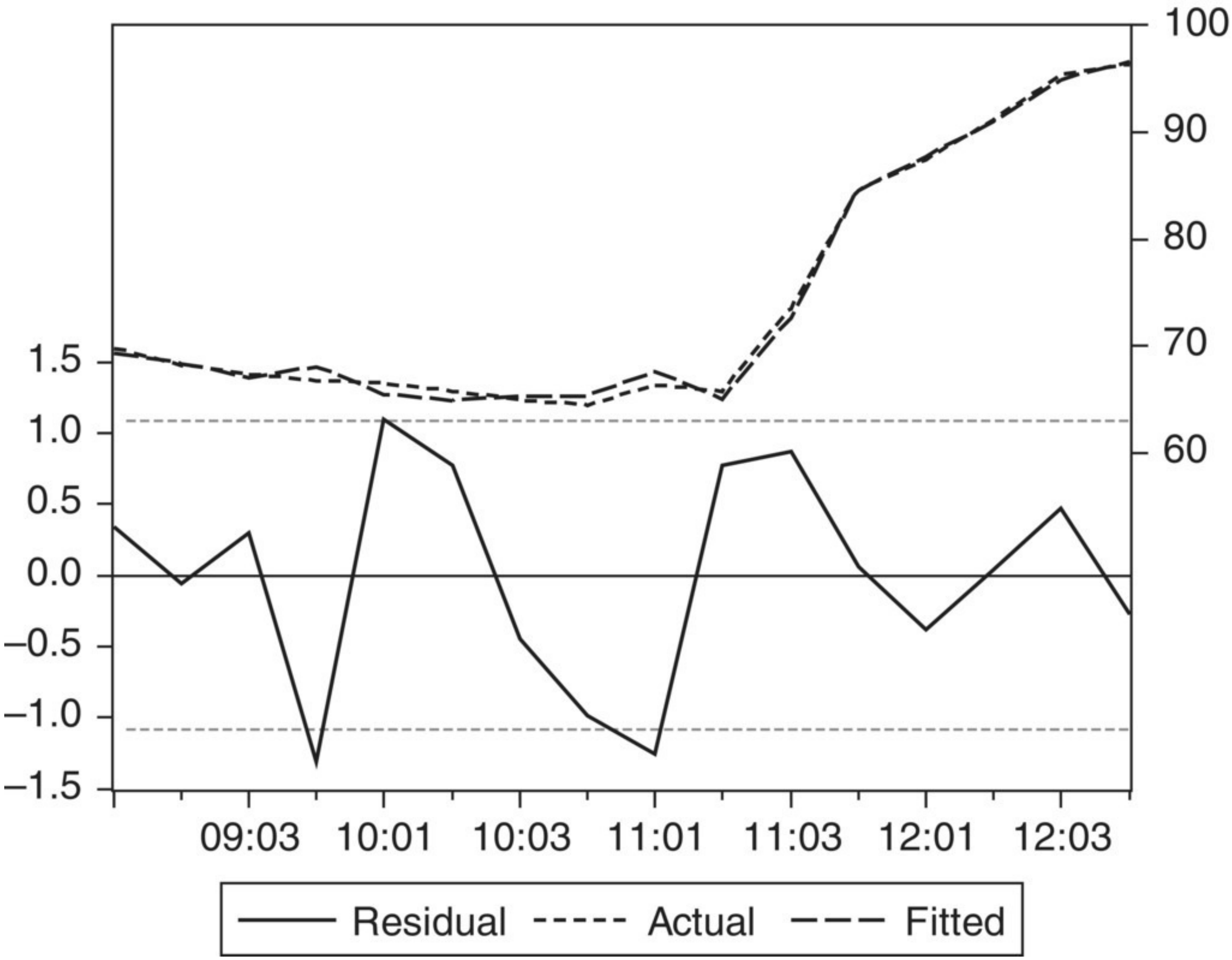


Figure 3.7 Actual quarterly labor technical equipment, its theoretical volumes, and the residuals calculated based on equation from Table 3.7.

Source: Table 3.7.

Table 3.7 indicates that the variable TAL is characterized by autoregression of the first, second, and fourth order. Autoregression of the first and fourth order is positive, while autoregression of the second order is negative. The sum of coefficients for the variables delayed by 1 and 4 quarters significantly exceeds the negative value of a coefficient for the variable TAL delayed by 2 quarters. This results in an increase in the level of technical labor equipment.

The value of tangible assets' depreciation, delayed by 3 quarters exerted a positive impact on the company's labor technical equipment. Consumption of fixed assets forced positive adjustments of tangible assets. It resulted in an increase in the value of labor technical equipment.

The equation in Table 3.7 is characterized by very good stochastic properties. The value of coefficient R^2 is very high and the value of Durbin–Watson's statistic is close to 2. Such values of stochastic characteristics often occur in case of equations of econometric micromodels.

The above presented empirical multiple-equation micromodel of an average enterprise can be a useful tool in estimation of forecasts of economic variables that are important for the company. It can be used for current decision-making. Decisions prepared in such a way will bear a relatively low risk of inaccuracies.

Collection of statistical information about the variables presented in Section 3.2 will increase security of empirical model's application. Information enrichment of the company's model should bring significant benefits for the managing business entity.[28]

3.4 Application of the company's model during a decision-making process

A multiple-equation model of an enterprise can be applied multidirectionally in that company's decision-making processes. Most commonly, the model or its equations are useful in forecast estimation of economic variables. It is also possible to conduct a simulation of the results of various decisions, both in a given equation as well as in the enterprise's entire system described in Figure 3.1.

Cases when an empirical equation is autoregressive in character with a trend are relatively uncomplicated. It signifies inertia of the endogenous variable. In such a case, it is easy to estimate a forecast for the period following the last observation or for few successive periods. Let us consider the empirical equation of employment presented in Table 3.3.

Predictor will have the following form:

$$\mathrm{EMP}_{Tp} = -19.07 + 1.994\mathrm{EMP}_{T-1} - 1.373\mathrm{EMP}_{T-2} + 0.467\mathrm{EMP}_{T-4}. \qquad (3.16)$$

Forecast of employment volumes for the first quarter of 2013 will be calculated as follows:

$$\mathrm{EMP}_{2013.1\mathrm{p}} = -19.07 + 1.994 \cdot 83 - 1.373 \cdot 85 + 0.467 \cdot 95 \approx 74 \text{persons}.$$

The average prediction error, calculated using formula 1.55, here is equal to $V_{\mathrm{EMP13.1}} = \pm 3.236$ persons. The relative prediction error, calculated using formula 1.58, is equal to $V^{*}_{\mathrm{EMP13.1}} = 4.37\%$. Typical value of the relative limiting prediction error is often equal to $V^{*}_{G} = 5\%$. Assuming such a limiting value, employment forecast for the first quarter of 2013 can be deemed as admissible, because the following inequality occurs:

$$V^{*}_{\mathrm{EMP13.1}} = 4.37\% < V^{*}_{G} = 5\%.$$

The results of employment forecast estimation for the first and the second quarter of 2013 in the company ENERGY are presented in Figure 3.8.

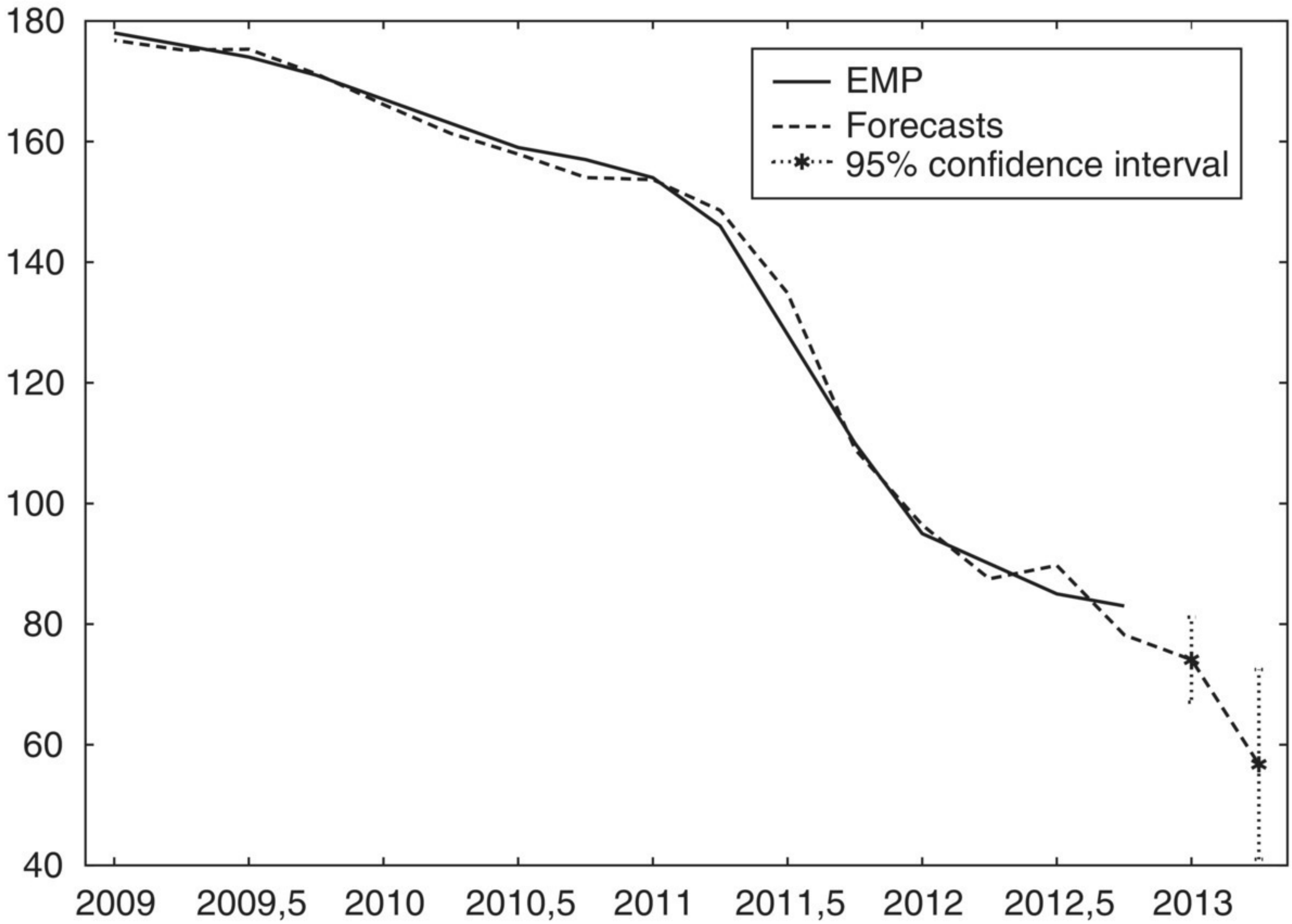


Figure 3.8 *Forecasts of EMP.*

Source: Own calculations using the GRETL package.

Employment forecast for the second quarter of 2013 will be as follows:

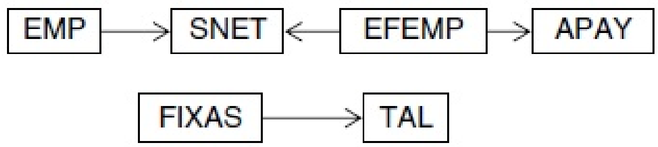
$$\text{EMP}_{2013.2p} = -19.07 + 1.994 \cdot 74 - 1.373 \cdot 83 + 0.467 \cdot 90 \approx 57 \text{persons.}$$

The average prediction error, calculated using formula 1.55, here is equal to $V_{\text{EMP13.2}} = \pm 7.218$ persons. The relative prediction error, calculated using formula 1.58, is equal to $V^*_{\text{EMP13.2}} = 12.66\%$. Forecast for the second quarter of 2013 is characterized by relatively small precision. The following inequality occurs:

$$V^*_{\text{EMP13.2}} = 12.66\% > V^*_G = 5\%.$$

This signifies an inadmissible forecast. However, further declines in employment volumes can be expected. After the first quarter of 2013, evaluation of the prognosis' relevance will be necessary. It may turn out that respecification of the employment equation, done through reconsideration of the hypothetical mechanism from Figure 3.1, will also is required. In case such an attempt fails, it is worth to consider changing the analytical form of the employment equation. Application of one of the nonlinear forms of the employment equation can prove to be an effective way of establishing its good predictive qualities.

The empirical model of a medium-sized enterprise presented in this chapter is recursive in character. Hypothetical feedback was not sustained; it was “broken.” The following chains of links between the total interdependent variables occur in this model:



(3.17)

Consequently, it is necessary to use chain prediction, interspersed with sequential prediction resulting from the delays of the endogenous variables. Possibilities of estimating the forecasts are dependent on the following necessities:

1. $SNET_{Tp}$ is having forecasts of EMP_{Tp} and $EFEMP_{Tp}$,
2. $APAY_{Tp}$ requires having the forecast of $EFEMP_{Tp}$,
3. TAL_{Tp} can be obtained while knowing the forecast of $FIXAS_{Tp}$.

The entire predictive proceeding requires a lot of attention to take into account both the delays of the system’s endogenous variables as well as the existing chains of links forming a recursive mechanism.

Let us take a look at the prediction process in the following chain of links:

We have at our disposal previously estimated forecasts of $EMP_{2013.1p}$ and $EMP_{2013.1p}$, with an indication that only the forecast of $EMP_{2013.1p}$ is admissible. Possibility of estimating the forecasts of $SNET_{Tp}$ will only appear when forecasts of $EFEMP_{Tp}$ are estimated in before. As such, it is necessary to estimate the forecasts of $EFEMP_{Tp}$ using the predictor from the equation in Table 3.4:

$$\begin{aligned} EFEMP_{TP} = {} & 140.83 + 0.622EFEMP_{T-1} - 0.482EFEMP_{T-2} \\ & + 1.041EFEMP_{T-4} - 5.644APAY_{T-1} \\ & + 11.888APAY_{T-2} - 8.988APAY_{T-3} + 2.795T. \end{aligned} \tag{3.18}$$

The values of the variable APAY delayed by 1, 2, and 3 quarters are provided for the first forecasted period. Successive delays can be estimated from a predictor equation for the equation in Table 3.5, while having forecasts of $EFEMP_{Tp}$. Figure 3.9 presented forecasts of labor efficiency ($EFEMP_{Tp}$) in the company ENERGY obtained for three quarters of 2013.

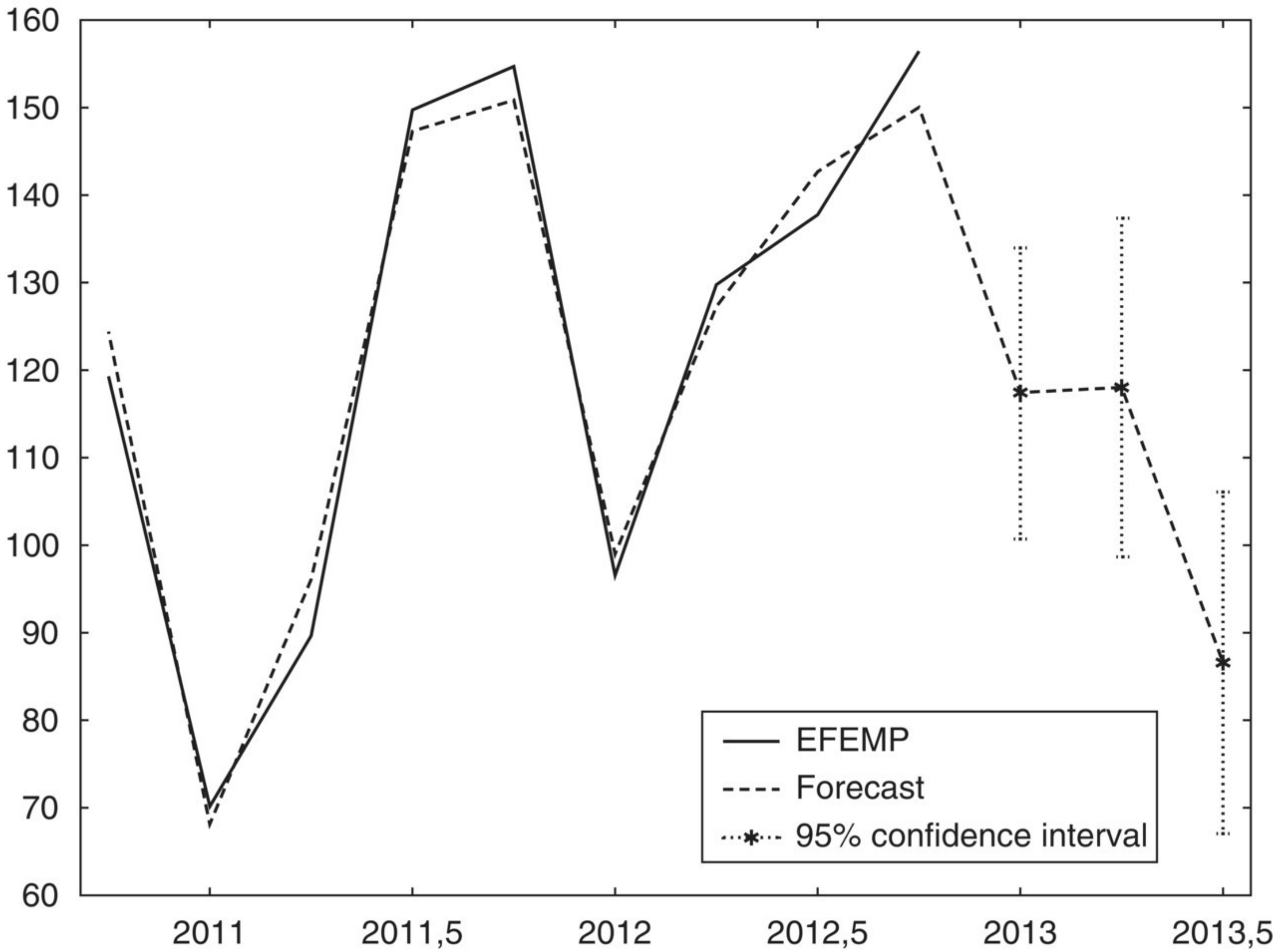


Figure 3.9 Forecasts of labor efficiency (EFEMP$_{Tp}$) in the company ENERGY obtained for three quarters of 2013.

Source: Table 3.8 (designed using the GRETL package).

Table 3.8 Forecasts of labor efficiency in the company ENERGY obtained for three quarters of 2013.

Source: Own calculations using the GRETL package.

Forecasted period	**Forecast of EFEMP$_{Tp}$**	**Average prediction error**	**95% Confidence interval**
2013:1	117.44	6.99	100.91 ÷ 133.97
2013:2	118.03	8.23	98.56 ÷ 137.50
2013:3	86.59	8.26	67.05 ÷ 106.12

Relative accuracy of the forecasts of EFEMP$_{Tp}$ can be noticed. As a result, only forecast estimation for the first forecasted period, that is for the first quarter of 2013, will be relatively safe. The predictor of SNET$_{Tp}$ will be formed on the basis of the equation in Table 3.2:

$$SNET_{Tp} = -17114 + 71.74EFEMP_{Tp} + 126.26EMP_{Tp} - 9.898MARK_{T-1}. \quad (3.19)$$

Figure 3.10 illustrates a forecast compared with the actual values of the sales income in the past.

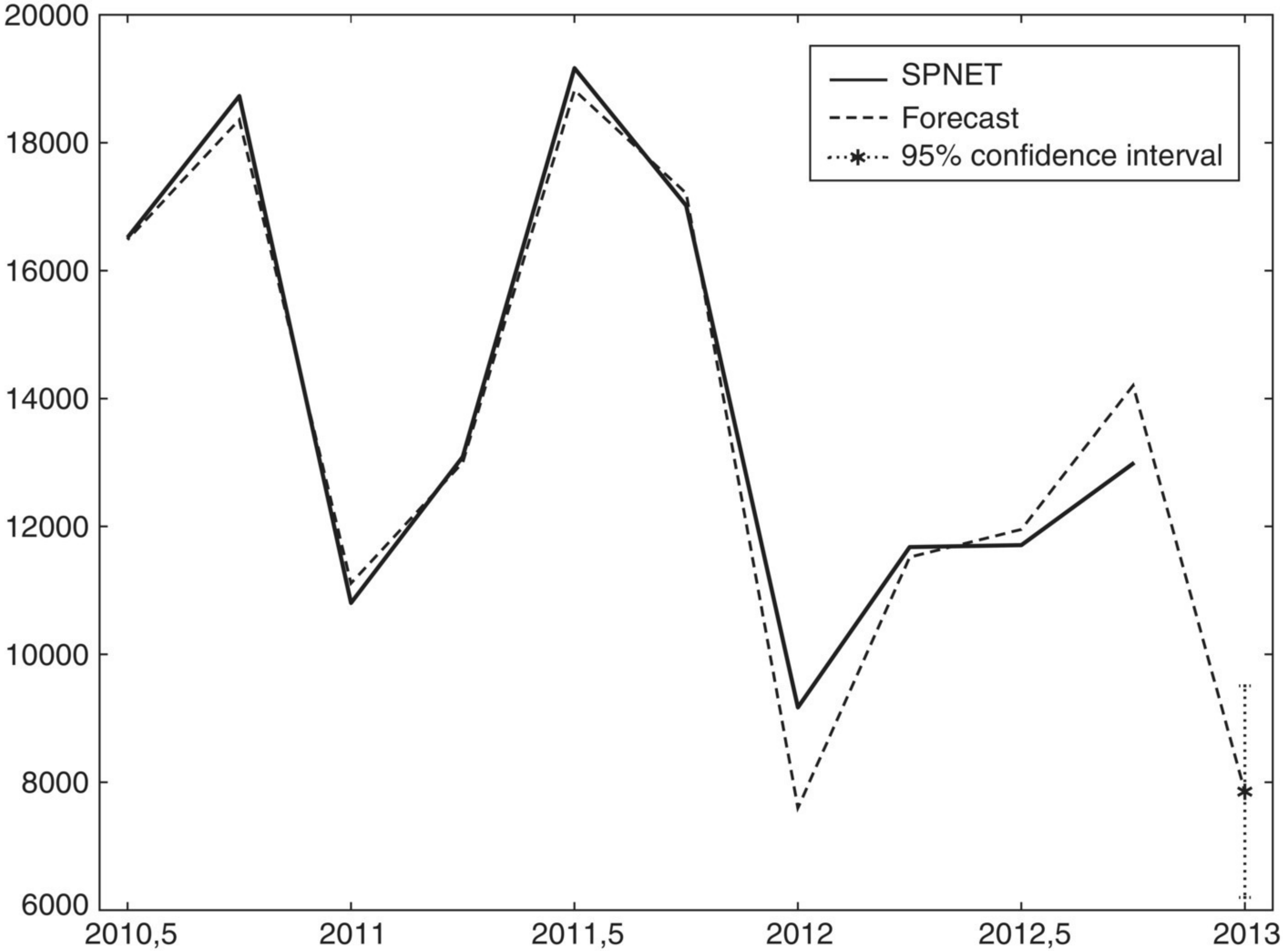


Figure 3.10 Forecast of the sales income ($SNET_{Tp}$) in the company ENERGY obtained for the first quarter of 2013.

Source: Table 3.9 (constructed using the GRETL package).

Table 3.9 Forecast of the sales income ($SNET_{Tp}$) in the company ENERGY obtained for the first quarter of 2013.

Source: Own calculations using the GRETL package.

Forecasted period	**Forecast of $SNET_{Tp}$**	**Average prediction error**	**95% Confidence interval**
2013:1	7852.13	776.197	6197.71 ÷ 9506

Forecast of the sales income is characterized by a moderately high prediction error, which is equal to

$$V^*_{\text{SNET}2013.1} = \frac{100 \cdot 776.197}{7852.13} = 9.885\%.$$

The estimated forecast is characterized by higher relative prediction error $V^*_G = 5\%$ than the previously established limiting error. However, it is worth to consider large volatility amplitude of the variables of a streaming character in each of the company's models that are based on quarterly or monthly data. Therefore, the relative limiting prediction error in such cases is often established at the level of $V^*_G = 10\%$. With such an assumption, forecast $\text{SPNET}_{2013.1p} = 7852.13$ thousands PLN becomes admissible,[29] because $V^*_{\text{SNET}2013.1} = 9.885\% < V^*_G = 10\%$. In such a case, management of the company ENERGY obtains information that with such formulated assumptions, a decline in the company's net sales income in the first quarter of 2013, even down to the level of 8 million PLN, should be expected. Having this information allows the company to adequately prepare for anticipated characteristics of the forecasted variables. These forecasts can be regarded as a warning, if they are contrary to the company's strategy. Some actions can, therefore, be undertaken for the forecasted variables' realizations to form at more optimistic levels. This challenge can only be met while having an empirical econometric model of the company.

Notes

1 A prototype of the presented here mechanism of interconnections in an enterprise was used in the work of J.W. Wiśniewski and Z. Zieliński. *Econometrics*, Part II, UMK, Toruń, 1989. p. 44.

2 We skip here definitions relating to isolation of a term "large (medium) enterprise." Legal definitions can be found in adequate legal acts (bills). Economic definitions can be found in enterprise literature (i.e., in S. Sudoł, 2002). A significant amount of statistical information characterizing activity of a large-sized enterprise must come from account books, which are obligatorily kept by such companies.

3 The mechanism of interdependencies between economic variables in a large- and medium-sized enterprise can differ in its details, depending on business type, the country in which it conducts its business, and many more other characteristics. Figure 3.1 presents universal, general dependencies in this category of companies, especially of industrial character.

4 Net sales income comprises the enterprise's actual property. Gross sales income includes the tax on goods and services, which belongs to the state.

5 *Working conditions competitiveness* in an enterprise is a complex characteristic (multidimensional). It can be comprised of such simple features as salary amount, company's prestige, market position which is decisive for a low lay-off risk, possibility of the so-called self-realization of an employee, worker benefits, possibility of traveling to interesting places, and so on.

6 Manifestation of this can be minimum wage increase, high dynamics of economy which increases employment demand. As a result, its price goes up (the wage).

7 Differences in the purposes of econometric model construction can influence the constructional variety of relationship patterns between important economic categories. The degree of detail in the approach to description of a large-sized enterprise can vary. Thus, it is necessary to consider a variety of sets of endogenous and exogenous variables in such econometric model.

8 Depending on the type of time series, it can be the annual, quarterly, or monthly net sales income. The time variable t can signify a number of the year, the quarter, or the month.

9 Depending on the type of data, it can be the average quarterly or average monthly number of employees, measured in full-time job positions.

10 In the denominator, the variable Y_1 – for the purpose of measuring labor efficiency – should be defined as the value of production performed in t period.

11 Competitiveness of work conditions is described by an average monthly wage, because it is the most understandable in the market category for potential candidates for employment in an enterprise, considered for annual, quarterly as well as monthly data.

12 The category of *marketing* allows us to generate a large variety of endogenous and exogenous variables in an econometric model of an enterprise. The role of a variable representing *marketing* can be fulfilled by, that is, television commercial expenses, amount of taxes paid for participation in fairs, and so on.

13 The variable expressed in a natural measure unit can be easily used in the case of production as a homogenous one, that is, coal mining volume, beer production in million hectoliters, tonnage of manufactured ships. In case of diverse manufacture, *production scale* can be represented by, that is, the volume of basic raw materials consumption, expressed in physical units.

14 The more ranges of products, the lower the level of production specialization. The variable X_3 can be defined as, that is, $X_3 = H_t$.

15 The category of *product features* can be represented by many other exogenous variables, that is, the value of special equipment in non-standard products, and so on.

16 The variables representing *staff qualifications* can be defined in different ways. It is important for such a variable to reflect those qualifications of employees which are most important for the company's efficiency.

17 A variable such defined, representing *fixed assets' consumption*, exhibits streaming of its consumption for a given time period. However, it can turn out, that the value of fixed assets' redemption in a t period, a percentage of the net assets' value in the initial value, or the

average age of the fixed assets – all can be better representatives of such consumption.

18 GDP growth rate in *t* period contains information about the economic situation, which can increase the tendency to invest or limit it.

19 Obviously, there are other variables representing a *demographic situation*, that is, number of working-age population in the company's employment area.

20 This variable represents the category of *investment policy aims*. Expansion onto new markets requires appropriate investment decisions, which result in an investment outlay increase. *Investment policy aims* can be represented by various variables, that is, the value of new product ranges, various dummy variables representing the goals that were verbally defined.

21 The time variable, occurring in the equation describing wages, is a carrier of an autonomous wage increase. Time variable can also occur in other equations of the model, if there is a trend in the dependent variable.

22 Many other explanatory variables should be considered in the set of explanatory variables of each equation, that is the time-delayed endogenous variables placed in a given equation, time-delayed exogenous variables occurring in a given equations, or a time-delayed variable as well as variable describing periodic fluctuations. Empirical verification of the model will determine which delayed variables will occur in the final empirical equation.

23 The presented general formula of an econometric model of a large- or medium-sized enterprise does not have to contain all the variables presented in the hypothetical formula. A detailed model solution may depend on many circumstances. Differentiation may result from the industry, specificity of the company's activity, the time-period from which statistical data was obtained, and observation period (annual, quarterly, monthly). Different solutions may involve enterprise's autonomy or the fact of its belonging to a particular capital group.

24 It should be remembered that in empirical equations of a reduced-form, endogenous time-delayed variables as well as exogenous time-delayed ones may occur.

25 Detailed explanation of this issue can be found in the second chapter of this book.

26 The company's managing director did not agree to use its name, industry, or localization. Therefore, it occurs here anonymously. The company emerged out of a large holding (in the 1990s of the twentieth century) to conduct an independent, economically efficient business. All information has been transformed using a single multiplier to prevent identification of the economic entity. Proportions in relations between variable of the econometric model, however, are authentic. The empirical model presented in this subchapter is poorer – in comparison to the theoretical version. The reasons for this are the company's specification and a limited, only partial access to statistical information.

27 The presence of feedback of the average wage and labor productivity is a proof of the company's motivational system's accuracy. Such type of necessity only occurs in the modeling based on data with observation time longer than a month.

28 The condition for an effective use of an econometric micromodel in an enterprise is to have a specialist, who can effectively construct such a tool and indicate the possibilities of its application in management decision-making.

29 Manipulation of a limiting prediction error V_G^* and its adaptation to the obtained results is not allowed. However, experience in company management and knowledge of specificity of the statistical data obtained from an enterprise allow the manager to determine forecast accuracy limits early enough for them to be useful during decision-making. It is worth noticing that the value of one limiting error, which will be the same for all forecasted variables of the model, does not have to be established. Different value V_G^* can be estimated for each forecasted variable. Typically, this value should fall within the range $5\% \le V_G^* \le 10\%$.

4
An empirical econometric model of a small-sized enterprise[1]

4.1 Specification of a small-sized enterprise's econometric model

The mechanism linking important economic variables of a small-sized enterprise has many features that are common with a structure of a large- or medium-sized company. There are, however, more differences than similarities, which can be noticed while analyzing Figure 4.1.

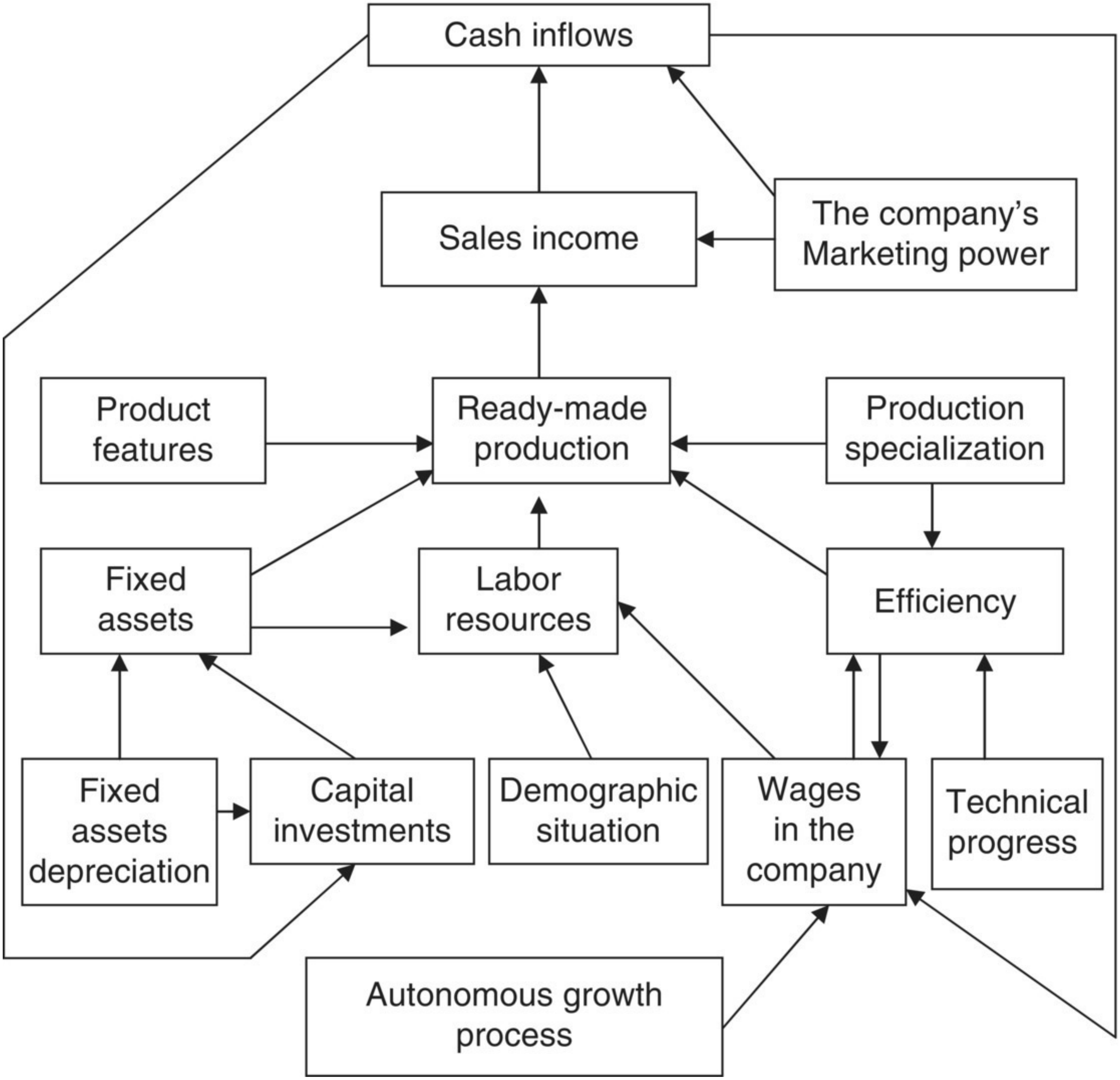


Figure 4.1 Economic interdependencies in a small-sized enterprise.

Source: Wiśniewski, J. W. (2003): An econometric model of a small-sized enterprise, *Chapter 2*.

There is a significant difference in the perception of reality within a small-sized company, in comparison to a large-sized one – the domination of a short time-horizon. In a small business entity, monthly vision prevails over the annual one. A quarter, in a small-sized company, is a period of a longer perspective, than a year in a large-sized company. Therefore, for example, perception of a small-sized company's production is divided into three parts. Specificity of a small-sized manufacturing enterprise requires distinction of three concepts: *manufacture of ready-made production*, the amount of the *sales income*, and the *cash inflows* obtained from the sales. Between those concepts, each of which represents a category of economic *production,* there is a time interval which is significant for a small-sized business entity. The company can manufacture and store in confined spaces the goods that were not sold. Even if formally there is a sale expressed in the sales income resultant from invoicing the goods, still, it can indicate a beginning of hardships for the company. As a result of manufacturing the goods and post-delivery invoicing the recipients, numerous liabilities on the part of the company follow: to the suppliers of raw materials and substances, to the workers, public and legal obligations, and many others. If manufacturing the goods and their delivery to customers do not result in an income of respective amounts of money, and if the company cannot – using other sources of financing – pay off its obligations, in an extreme case, it can lead to bankruptcy of a small business entity.

Figure 4.1, thus, distinguishes the *cash inflows* as a result of delivering the goods, formally reflected by the *sales income* being the sum of the amounts on the invoices issued during a given period of time. *Sales income* can be generated if the finished goods, in the diagram represented by the *ready-made production,* were manufactured in advance. The company's *marketing potential*[2] is an important factor impacting *the sales income*. *Production size,* understood as the volume of the ready-made goods prepared for sale, is conditioned by some conventional factors, among which *labor resources* and *fixed assets* play a dominant role. It may happen that *production* is subject to influence of other conditions, some of which may be *production specialization*, *product properties,* and others. *Labor efficiency* plays a significant role in shaping the size of *production.*

Effectiveness of major tangible factors determines the enterprise's competitive strength. A company characterized by high *labor productivity,* or by higher *productivity of the noncurrent assets,* has a chance for lower production costs, which makes that company stronger in the area of price competition. Labor efficiency, here understood as team efficiency,[3] is in a feedback relation with the *wage,* provided that the motivational system is properly constructed. *Efficiency* stays under the influence of various numerous factors, among which *technical progress* and *production specialization* were indicated in the mechanism of linkage.

Wages – being subject to influence of *labor efficiency* – depend on many different factors, among which the following deserve to be highlighted: the *autonomous process of a wage increase*[4] as well as the company's financial situation, largely determined by the *cash inflows*. The *wages* in the company, in turn, affect the *labor resources,* both, in their quantitative as well as qualitative characteristics.

The volume and the quality of *labor resources* depend on the characteristics of the *noncurrent assets*.[5] The labor supply resultant from a *demographic situation* also plays an important role.

Fixed assets, on one side, result from the *capital investments* realized by the company and, on the other, result from *consumption of the assets' components,* making it necessary to carry out replacement investments. Investment opportunities, to a large extent, are conditioned by the company's financial situation, which largely depends on the sizes of the *cash inflows*.

4.2 The structural form of a small-sized enterprise's econometric model

4.2.1 The model's total interdependent variables

The multiple-equation econometric model will describe economic interdependencies of a small manufacturing enterprise belonging to a publishing and printing sector.[6] It belongs to a business category, which allows simplified accounting in the form of the so-called revenue and expense ledger. At the same time, the company has been liable for the tax on goods and services (VAT), ever since its introduction in Poland. Since 1991, the company has been operating on a highly competitive market. The study covers the years 1996–2000. This period, in which the company was operating in its own facility connected with a property constituting its mortgage ownership,[7] can be described as relatively stabilized. As such, it is a time interval in which stability of the company's headquarters was guaranteed, thus the owners had favorable conditions for development through appropriate investments.

Figure 4.1 constitutes the basis for defining the variables of a small-sized enterprise's econometric model. Equivalent variables discussed in Section 4.1, which represent the categories under the influence of at least one other element, will form a set of endogenous variables of an econometric model composed of many stochastic equations.

This means that the following will be the total interdependent variables of the econometric model:

CASH – the amount of the cash inflows during a period *t*, in thousands PLN[8];

SBRUT – gross sales income during a period *t*, in thousands PLN;

PROD – the value of ready-made production (in sale prices) during a period *t*, in thousands PLN;

EMP – the number of employees calculated in full-time employment during a period *t* (number of people)[9];

FIXAS – the initial value of the tangible fixed assets in use during a period *t*, excluding buildings and constructions, in thousands PLN;

EEFEMP – labor efficiency as a quotient of the ready-made production's value and the number of employees (PROD/EMP), in thousands PLN per 1 employee, during a period *t*;

SAL – net salary paid to the company's employees for their work during a period *t*, in thousands PLN;

APAY – the average monthly wage paid to the employees during a period *t* (in PLN);

INV – the value of capital expenditures during a period *t* (in net purchasing prices[10]), in thousands PLN.

Some of the elements described in Figure 4.1 have been additionally expressed by other variables, which will belong to the group of the total interdependent variables. These are as follows:

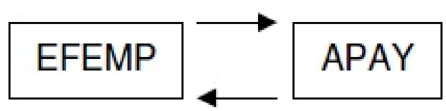
SALPR – effectiveness of the net salary expressed by a quotient of the ready-made production and the net salary (PROD/SAL), in production PLN per 1 PLN of the net salary;

EFSAL – effectiveness of the net salary calculated as a ratio of the sales income to the sum of net salaries (SNET/SAL), in PLN income per 1 PLN of the net salary;

ESC – effectiveness of the net salary as a relation of the cash inflows to the sum of the net salaries (CASH/SAL), in PLN inflows per 1 PLN of the net salary[11];

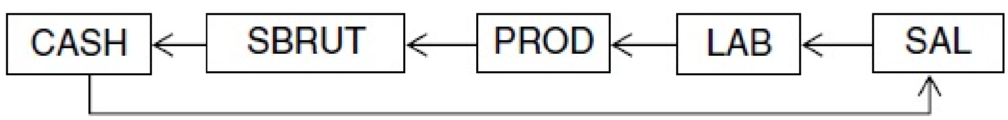
MACH – the value of equipment and machinery (its initial value) in thousands PLN.[12]

Figure 4.1 shows direct feedback between the *wage* and the *labor efficiency*. In the econometric model, this feedback will be manifested by the following hypothetical linkage:



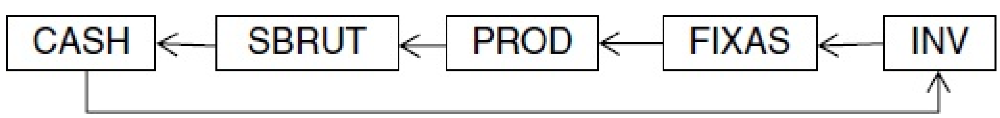
(4.1)

What is more, at least three closed cycles of links between the total interdependent variables[13] occur in the econometric model. The first one has the following form:



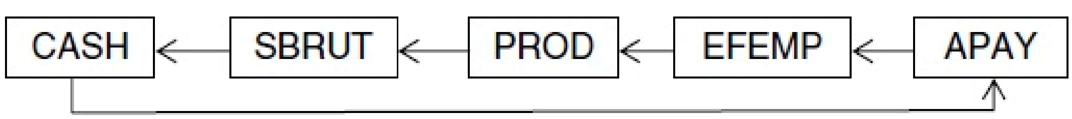
(4.2)

The second of those closed cycles linking the total interdependent variables is as follows:



(4.3)

Finally, the third indirect feedback takes on the following form:



(4.4)

Indirect and direct feedback in the hypothetical model, both determine its affiliation with a class of interdependent systems. In this model, it is also possible to predict occurrence of the so-called detached equations, which are characterized by the fact that only the predetermined variables are those equations' explanatory variables.

4.2.2 The model's predetermined variables

The first group of the variables forming a set of econometric model's predetermined variables is composed of exogenous variables. They represent the categories in Figure 4.1, which have impact on other elements of the system, but are not dependent on the rest of its categories. The second group, constituting a fraction of the set of predetermined variables, is composed of time-delayed endogenous variables.[14]

In the econometric modeling, the so-called methodology of dynamic consistency models[15] will be used. Accordingly, application of autoregressive solutions in various stochastic equations will be attempted. This means that each of the endogenous variables potentially will appear in the group of predetermined variables.

The following will be the model's exogenous variables:

DEPR – the value of the tangible fixed assets' depreciation during a period *t*, in thousands PLN;

AMPL – the quotient of the net payroll during a period *t* and the amount of the fixed assets' depreciation (SAL/DEPR), in PLN (of the wages) per 1 PLN of depreciation;

RAN – the number of the product assortments for sale (the mean during a period *t*), as occurring on the price list;

MARK – the dummy variable, distinguishing the periods in which the company's representatives actively attended fairs,[16] whereas RAN = 1 in quarters (in months) of fair participation and RAN = 0 in the remaining periods;

TAM – technical devices as a ratio of the initial value of machinery and equipment to the number of employees (MACH/LAB), in thousands PLN of machinery and equipment's value per 1 employee;

TAL – technical devices as a ratio of the initial value of the tangible fixed assets in use to the number of employees (FIXAS/LAB), in thousands PLN per 1 employee;

FIXD – initial value of tangible fixed assets in use per one unit of depreciation costs during a period *t* (FIXAS/DEPR),[17] in PLN per 1 PLN of depreciation costs;

TIME – the time variable, assuming the values 1, …, 84 in case of monthly data and 1, …, 28 in quarterly time series.

In the econometric modeling, exogenous variables with their adequate time delays will also occur. In the model, they will also belong to a category of predetermined variables.

4.2.3 Structural-form's equations of a small-sized enterprise's econometric model

In our considerations, the system of interdependent stochastic equations is formed by[18]

$$\mathrm{CASH} = f(\mathrm{CASH}_{-1}, \ldots, \mathrm{CASH}_{-m}, \mathrm{SBRUT}, \mathrm{SBRUT}_{-1}, \ldots, \mathrm{SBRUT}_{-m}, \mathrm{TIME}, \eta_1), \tag{4.5}$$

$$\mathrm{SBRUT} = f(\mathrm{SBRUT}_{-1}, \ldots, \mathrm{SBRUT}_{-m}, \mathrm{PROD}, \mathrm{PROD}_{-1}, \ldots, \mathrm{PROD}_{-m}, \mathrm{RAN}, \mathrm{MARK}, \mathrm{TIME}, \eta_2), \tag{4.6}$$

$$\text{PROD} = f(\text{PROD}, \text{PROD}_{-1}, \ldots, \text{PROD}_{-m}, \text{FIXAS}, \text{FIXAS}_{-1}, \text{FIXAS}_{-m}, \text{EMP}, \text{EMP}_{-1}, \ldots, \text{EMP}_{-m}, \text{FIXD}, \text{FIXD}_{-1}, \ldots, FIXD_{-m}, \text{RAN}, \text{TIME}, \eta_3), \tag{4.7}$$

$$\text{EMP} = f\left(\text{EMP}_{-1}, \ldots, \text{EMP}_{-m}, \text{MACH}, \text{MACH}_{-1}, \ldots, \text{MACH}_{-m}, \text{TIME}, \eta_4\right), \tag{4.8}$$

$$\text{EFEMP} = f(\text{EFEMP}_{-1}, \ldots, \text{EFEMP}_{-m}, \text{APAY}, \text{APAY}_{-1}, \ldots, \text{APAY}_{-m}, \ldots, \text{TAL}, \text{TAL}_{-1}, \ldots, \text{TAL}_{-m}, \text{TIME}, \eta_5), \tag{4.9}$$

$$\text{APAY} = f(\text{APAY}_{-1}, \ldots, \text{APAY}_{-m}, \text{EFEMP}, \text{EFEMP}_{-1}, \ldots, \text{EFEMP}_{-m}, \text{TIME}, \eta_6), \tag{4.10}$$

$$\text{SAL} = f\left(\text{SAL}_{-1}, \ldots, \text{SAL}_{-m}, \text{CASH}, \text{PCASH}_{-1}, \ldots, \text{CASH}_{-m}, \text{TIME}, \eta_7\right). \tag{4.11}$$

Occurrence of many time-delayed variables in the model causes a modeling impediment for two reasons. First, the delays decrease the number of statistical observations – in this case, by a maximum of *m* number of series components[19] (in quarters or months). What is more, taking those delays into account increases the number of structural parameters in the structural-form's equations, especially in the equations of a reduced form. In extreme cases, this leads to a lack of a positive number of the degrees of freedom, which prevents estimation of their parameters.

The study examined some delays, in monthly data by $m = 12$ periods and delays for quarterly series by $m = 4$. Consequently, in the reduced-form's equations, which take into account all the delayed variables, after reducing the number of observations by *m* (12 or 14), the difference between the number of statistical observations ($n = 84$ monthly or $n = 28$ quarterly) and the number of structural parameters became negative. Such situation prevents using the ordinary least square (OLS) method to estimate the parameters of the structural-form's equations. Therefore, it was necessary to experiment on the structural-form's equations using the OLS method. The purpose of this procedure was to eliminate from the equations all the variables that are statistically insignificant, that is, the variables for which empirical statistics of a *t*-Student have turned out to be particularly small,[20] at a particularly high risk of a type I error. With these preparatory proceedings, it was possible to use the 2LS method for each of the structural-form's interdependent equations. In the subsequent iterations, statistically insignificant variables were eliminated from each empirical equation. As a result of this procedure, the reduced-form of the model was also modified. The empirical equations presented below result from the examination proceedings described here. This book is going to present empirical results of econometric modeling, which are based on monthly data. This type of results is used for current management decision-making in small enterprises.

4.3 Equation of the cash inflows

An empirical equation, constructed on the basis of monthly data, has the following form:

$$\underset{(2.472)}{\text{CASH} = 23.387} + \underset{(1.435)}{0.104\text{CASH}_{-11}} + \underset{(9.726)}{0.522\text{SBRUT}} + \underset{(2.853)}{0.135\text{SBRUT}_{-2}} \quad (4.12)$$

$$+ \underset{(3.899)}{0.174\text{SBRUT}_{-4}} - \underset{(5.103)}{0.243\text{SBRUT}_{-10}} + \underset{(1.447)}{0.171\text{TIME}} + u_1,$$

$$R_1^2 = 0.834, S_{1u} = 17.897 \text{ thousands PLN}, \text{DW}_1 = 2.185.$$

In Equation 4.12, there are two explanatory variables with relatively small empirical values of the t-Student statistics.[21] They allow an inference that the variables CASH_{-11} and TIME can be deemed significant at a significance level of $\gamma \approx 0.15$. These variables, however, were left in the equation due to their epistemic qualities. Figure 4.2 showed the actual and the theoretical monthly amounts of the cash inflows as well as the residuals calculated on the basis of Equation 4.12, while Figure 4.3 presented distribution of the residuals of Equation 4.12.

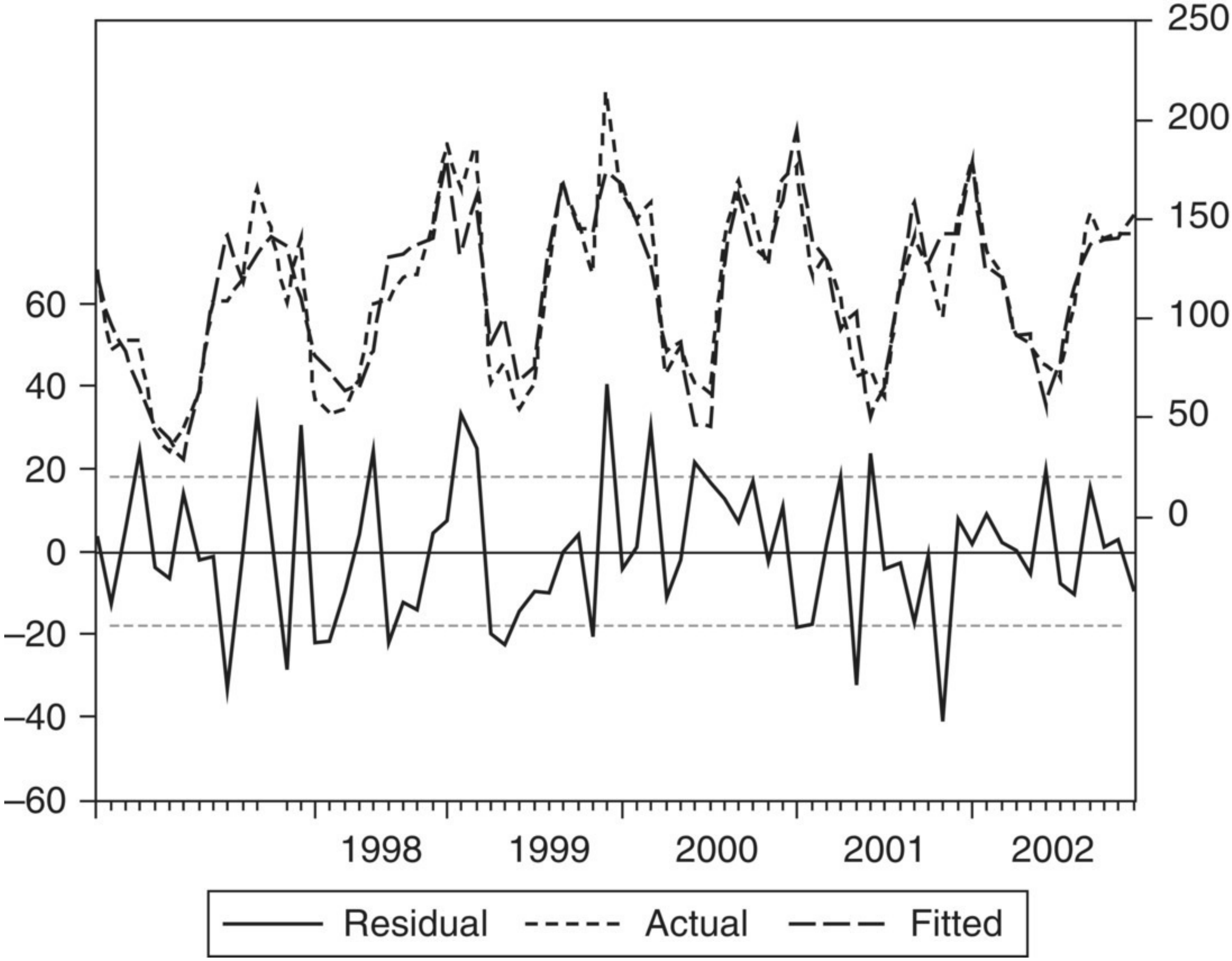


Figure 4.2 The actual and the theoretical monthly amounts of the cash inflows as well as the residuals calculated on the basis of Equation 4.12.

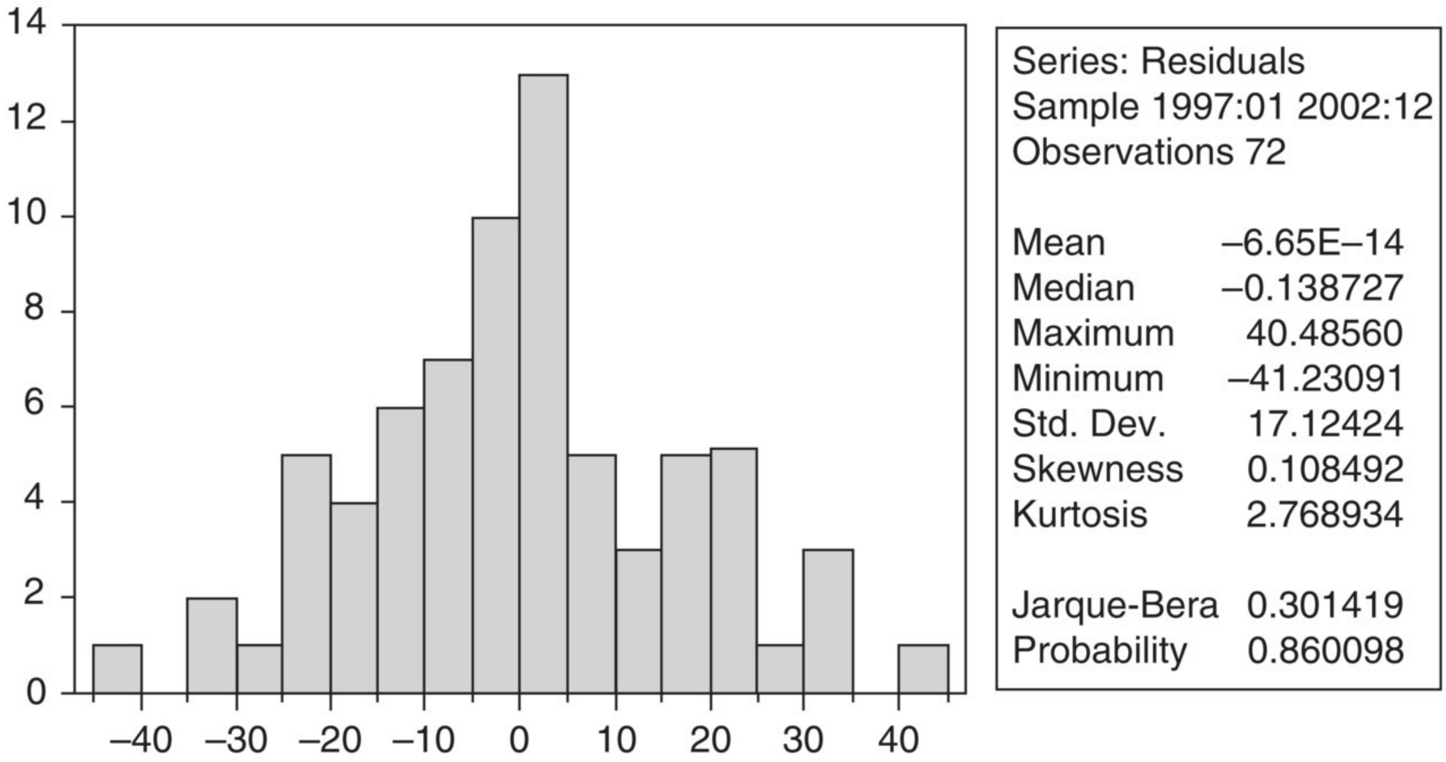


Figure 4.3 Distribution of the residuals of Equation 4.12.

It can be assumed that Equation 4.12 quite accurately describes the principles of monthly cash inflows, since around 83.4% of their volatility results from the impact of the variables $CASH_{-11}$, SBRUT, $SBRUT_{-2}$, $SBRUT_{-4}$, $SBRUT_{-10}$, and TIME. The actual monthly amounts of cash inflows differ from those calculated on the basis of Equation 4.12, on average, by 17.9 thousands PLN, which represents 16.16% of the average monthly value of the variable CASH during the period of 1996–2002. At the same time, an empirical size of the Durbin–Watson (DW) statistic = 2.185 indicates[22] that there are no grounds for rejection of the null hypothesis about the lack of the first-order autocorrelation of the random component.

Autoregression of the 11th order, in the statistical sense, is weak. The revenues obtained 11 months earlier, in the amount of 1000 PLN, entail an increase in the current inflows by around 104 PLN.[23] Also, the time variable in Equation 4.12 belongs to those relatively weak statistically. The upward linear trend indicates an increase in the cash inflows, independently of the other explanatory variables. It can be inferred that the enterprise's receivables collection system has been improving regularly. This can result, inter alia, from elimination of unreliable trading partners as well as from systematic contacts and work with the clients. It is facilitated through the use of the so-called direct marketing combined with rewarding the sales representatives solely on the basis of the amounts of money obtained from the supported sales network.

Sales income, that is the sales validated by invoices, are major source of cash inflows. For 1 thousand PLN invoiced in a current month,[24] the company gains, on average, around 522 PLN in cash. Invoicing a 1000 PLN prior to 2 months results in current inflows bringing, on average, 135 PLN. Lastly, a 1000 PLN invoiced prior to 4 months results in current cash inflows in the amount of 174 PLN.

The variable $SBRUT_{-10}$ has a negative impact on the current cash inflows. This dependence confirms that customers often rely on their guaranteed right to return a product by issuing a corrective invoice, which causes a decrease in the accounts receivable, and consequently a lack of adequate cash inflows.

A linear equation describing the mechanism of cash inflows' formation has the following empirical form (in case of quarterly observations)[25]:

$$\text{CASH} = \underset{(1.973)}{66.48} - \underset{(3.737)}{0.298}\text{CASH}_{-3} + \underset{(10.200)}{0.658}\text{SPBRUT} + \underset{(7.026)}{0.391}\text{SBRUT}_{-1} + u_2, \quad (4.13)$$
$$R_2^2 = 0.893, S_{2u} = 36.8 \text{ thousands PLN}, \text{DW}_2 = 2.054^{26}$$

Figure 4.4 presents the company's actual quarterly cash inflows, the theoretical values calculated using Equation 4.13, and the rests, that is, the differences between empirical and theoretical values.

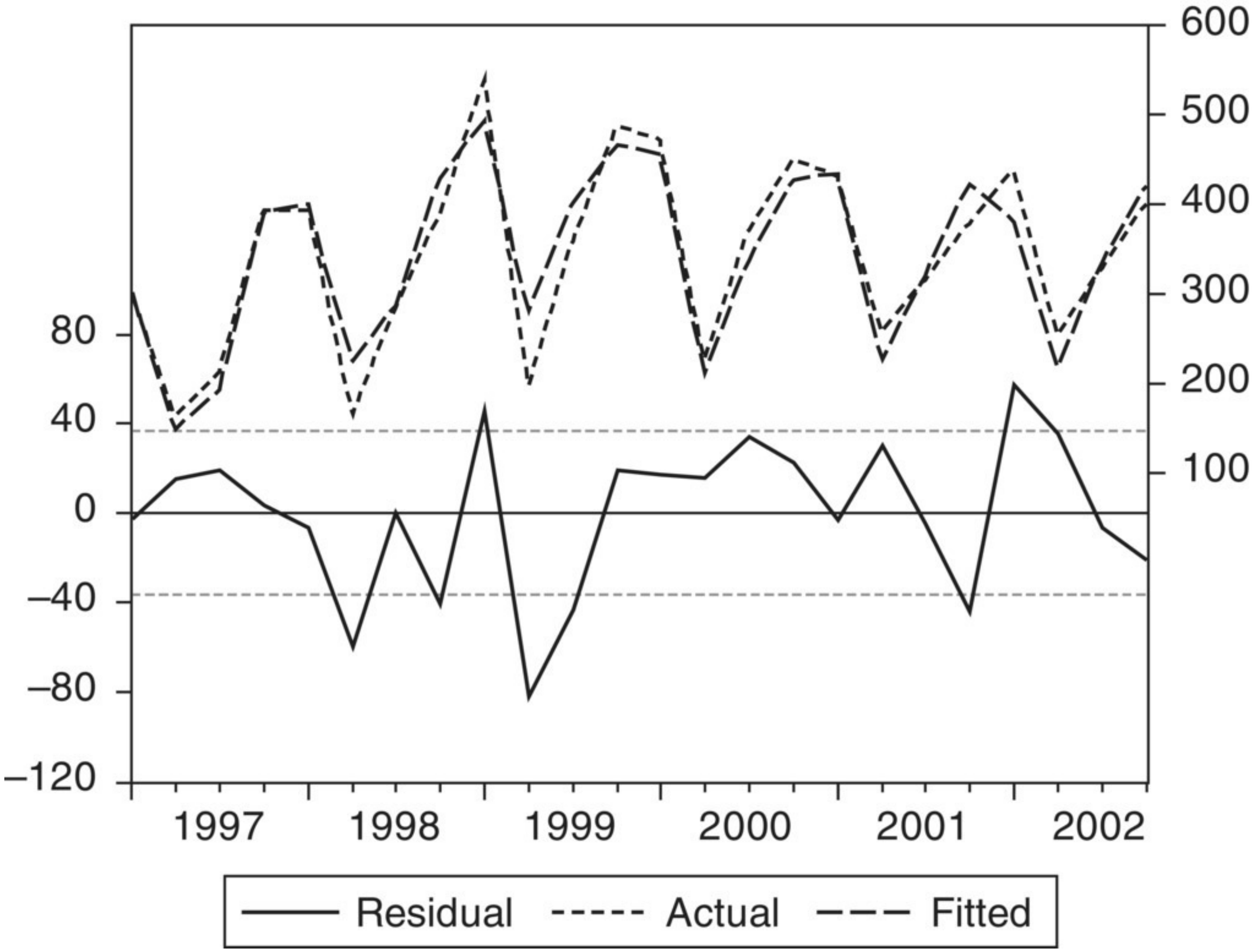


Figure 4.4 The actual and theoretical[27] *quarterly values of the cash inflows and the residuals of Equation 4.13*.

Equation 4.13 indicates that the quarterly amounts of cash inflows are characterized by a third-order autoregression. This autoregression shows that the company's cash inflows from 3 quarters ago shape their current amount, wherein the relationship is negative. It allows a conclusion that the sums previously acquired from the receivers reduce current liabilities, and thereby reduce the current inflows.

The impact of the sales income's size on the cash inflows, in quarterly terms, is both current as well as delayed by 1 quarter. Impact of the concurrent amounts resultant from invoicing customers on the variable CASH, expressed by a parameter estimation equal to 0.658, means that for every thousand of the sales income achieved in the quarter for which the invoices were issued, a sum of 658 PLN is obtained in the form of cash and/or a bank transfer to the company's bank account. For 1 thousand PLN invoiced in a previous quarter, the company obtained, on average, 391 PLN of cash inflows. Each of the explanatory variables occurring in the empirical equation 4.13 is statistically significant at a low significance level of $\gamma < 0.01$. Figure 4.5 presented distribution of the residuals of Equation 4.13.

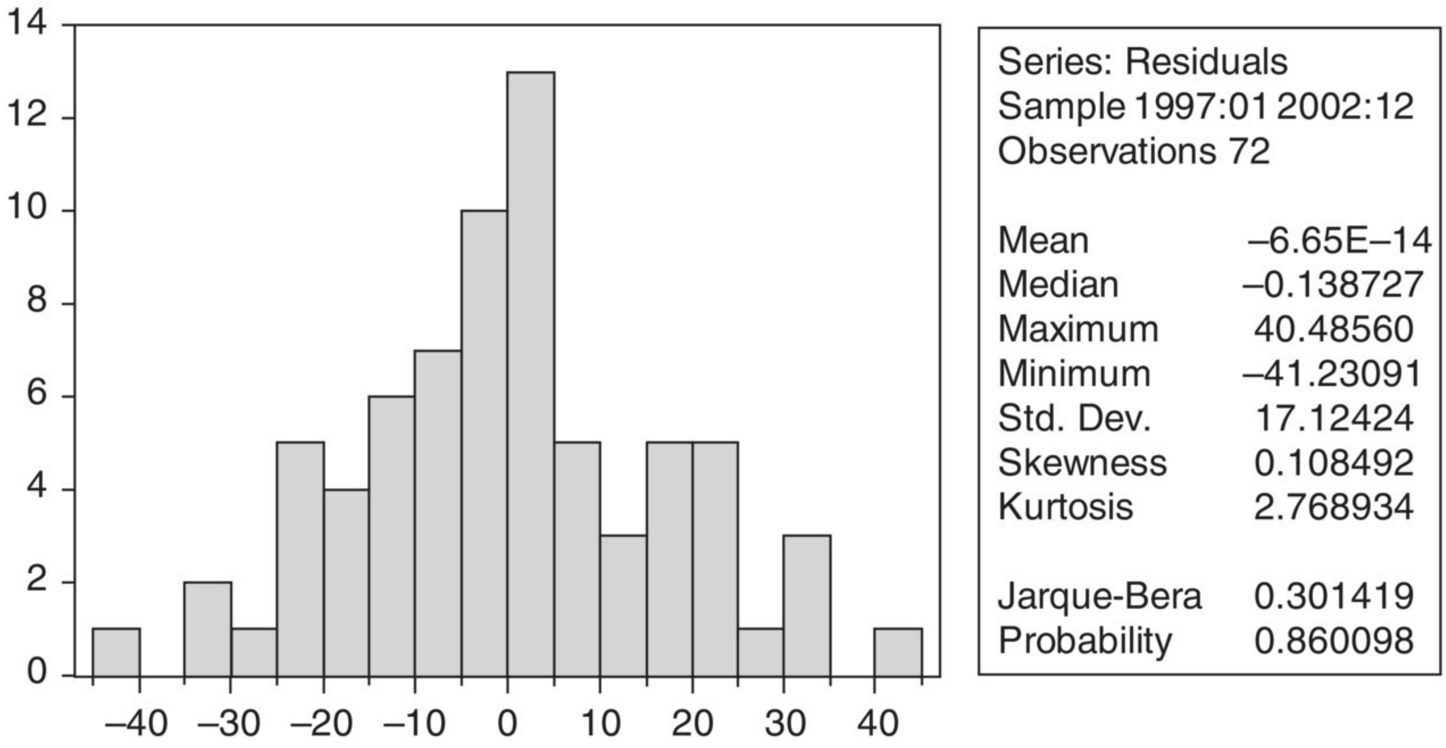


Figure 4.5 Distribution of the residuals of Equation 4.13.

The multiple-correlation coefficient squared indicates that 89.3% of the total volatility of the cash inflows is explained by the variables $CASH_{-3}$, SBRUT i, $SBRUT_{-1}$. Such result marks high description accuracy of the variable CASH for quarterly time series. Random fluctuations of the variable CASH are relatively small, since an estimated value of the standard residual deviation, equal to 36.8 thousands PLN, can be considered as small. It indicates that the theoretical values of the cash inflows, calculated using Equation 4.13, differ from the actual ones, on average, by 36.8 thousands PLN. The value of the DW_2 statistic equal to almost 2 indicates that there is no first-order autocorrelation of the random component.

4.4 Equation of the sales income

An empirical equation describing monthly gross sales income has the following empirical form:

$$\begin{aligned} SBRUT = \underset{(0.898)}{14.188} + \underset{(8.247)}{0.574} SBRUT_{-12} + \underset{(4.578)}{0.481} PROD \\ + \underset{(1.485)}{0.135} PROD_{-4} - \underset{(3.081)}{0.232} PROD_{-7} + u_3, \\ R_3^2 = 0.831, S_{3u} = 24.763 \text{ thousands PLN}, DW_3 = 1.919. \end{aligned} \tag{4.14}$$ [28]

Equation 4.14 is characterized by good description accuracy of monthly gross sales income. Around 83.1% of SBRUT's volatility is explained by an autoregression of the 12th order, the executed current value of ready-made production, and by ready-made production delayed by 4 and 7 months. Figure 4.6 shows this equation's high fitting accuracy, when compared to the equations based on microeconomic monthly data. Figure 4.7 presented distribution of the residuals of Equation 4.14.

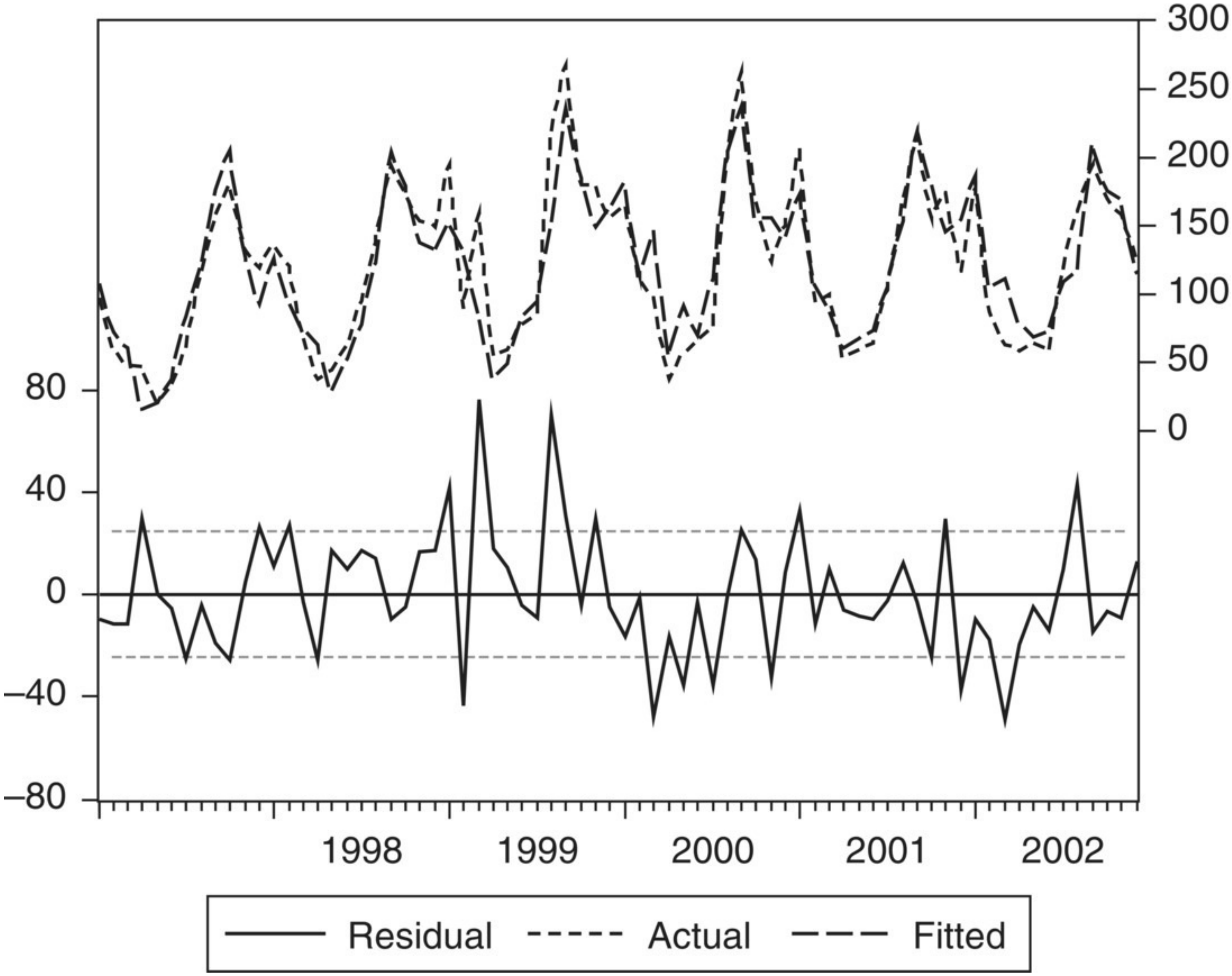


Figure 4.6 The actual monthly gross sales income, the theoretical values, and the residuals calculated on the basis of Equation 4.14.

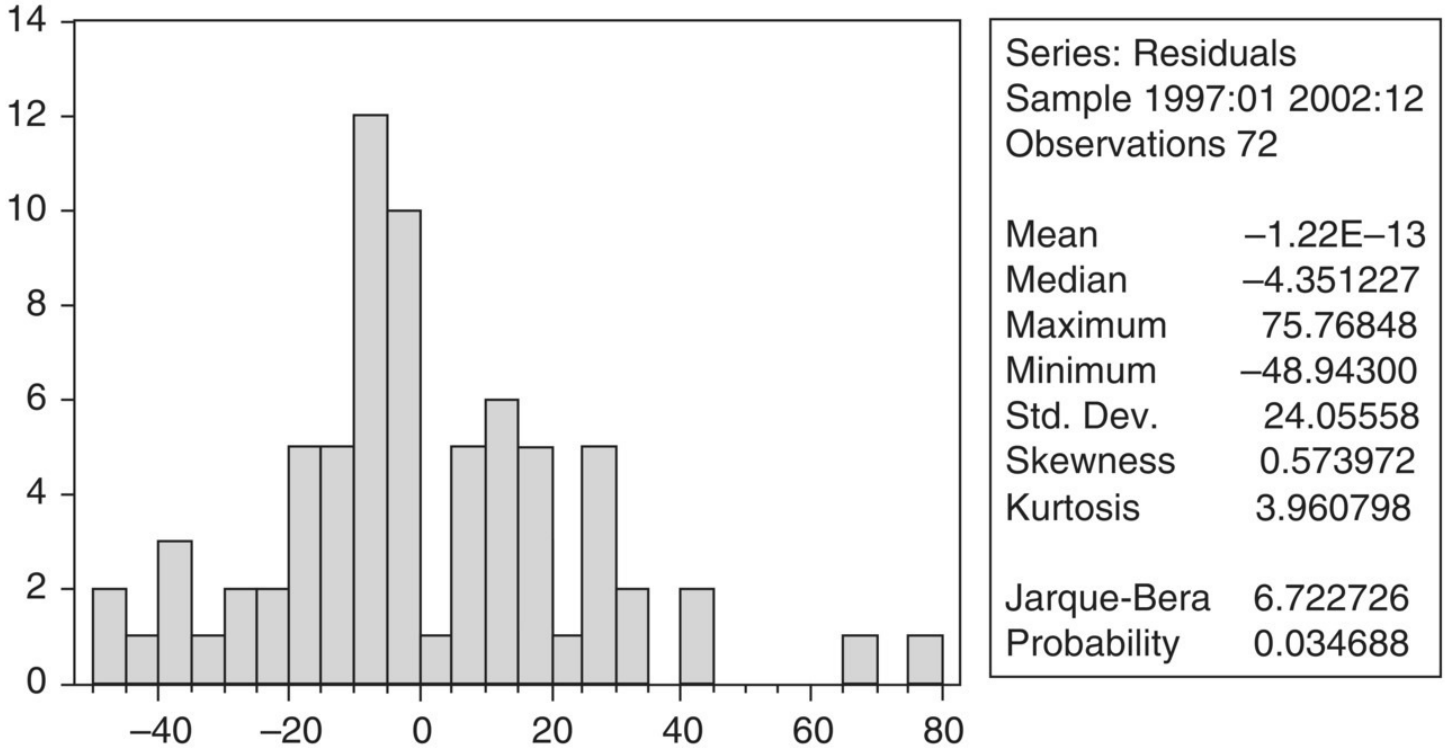


Figure 4.7 Distribution of the residuals of Equation 4.14.

Equation 4.14 reveals repetitiveness of income every 12 months. This means that a sales income of a 1000 PLN achieved prior to 12 months entails current amounts of that income in about 574 PLN. A current value of ready-made production, in the amount of 1000 PLN, results in the sales income of 481 PLN. Ready-made production, in the amount of 1000 PLN prior to 4 months, has a current effect in the form of the sales income amounting to 135 PLN. The variable $PROD_{-4}$ can be regarded as statistically significant,[29] with a 6.95% risk of a type I error. Finally, negative autoregression of the seventh order confirms elimination of some of the expired ready-made goods from the sales, while the waiting period for their designation as waste paper, on average, lasts 7 months.

An empirical equation of the gross sales income, based on quarterly data, has the following form:

$$\begin{aligned} SBRUT = \underset{(0.291)}{-10.31} - \underset{(3.015)}{0.208}SRUT_{-3} + \underset{(5.469)}{0.523}SRUT_{-4} \\ + \underset{(4.899)}{0.843}PROD + u_4, \\ R_4^2 = 0.921, S_{3u}, S_{4u} = 42.739 \text{ thousands PLN}, DW_4 = 1.586. \end{aligned} \tag{4.15}$$

Explanatory variables of the equation are statistically significant at a significance level γ below 0.01. The empirical value of the Durbin–Watson statistic $DW_4 = 1.586 > d_u = 1.30$ at a significance level of $\gamma = 0.01$, which allows a conclusion that in Equation 4.15 there is no first-order autocorrelation of the random component. Figure 4.8 illustrates the volatility of the actual and the theoretical values of the sales income as well as the residuals of Equation 2.3, while Figure 4.9 shows distribution of this equation's residuals.

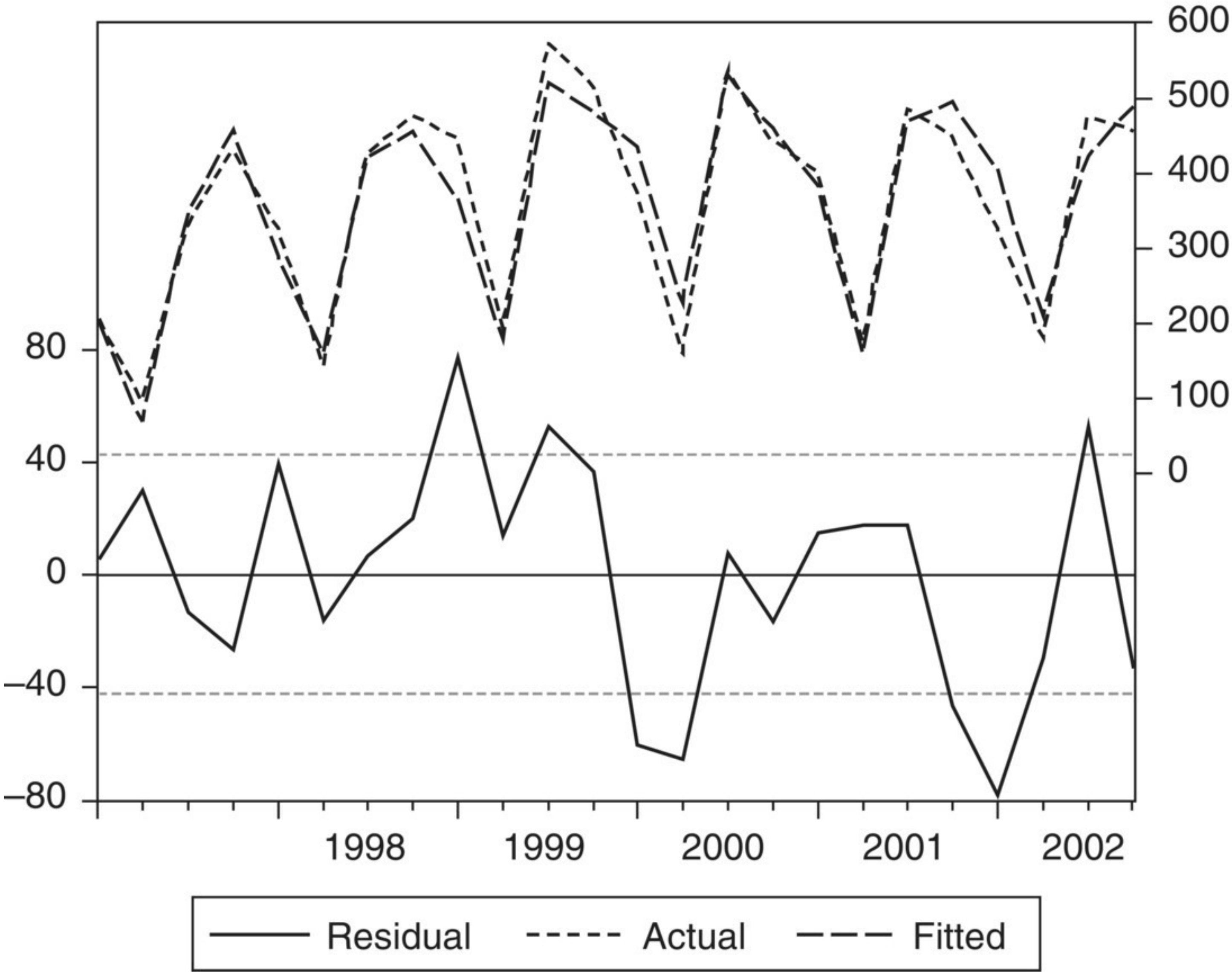


Figure 4.8 The actual quarterly values of the gross sales income, the theoretical values and the residuals calculated on the basis of Equation 4.15.

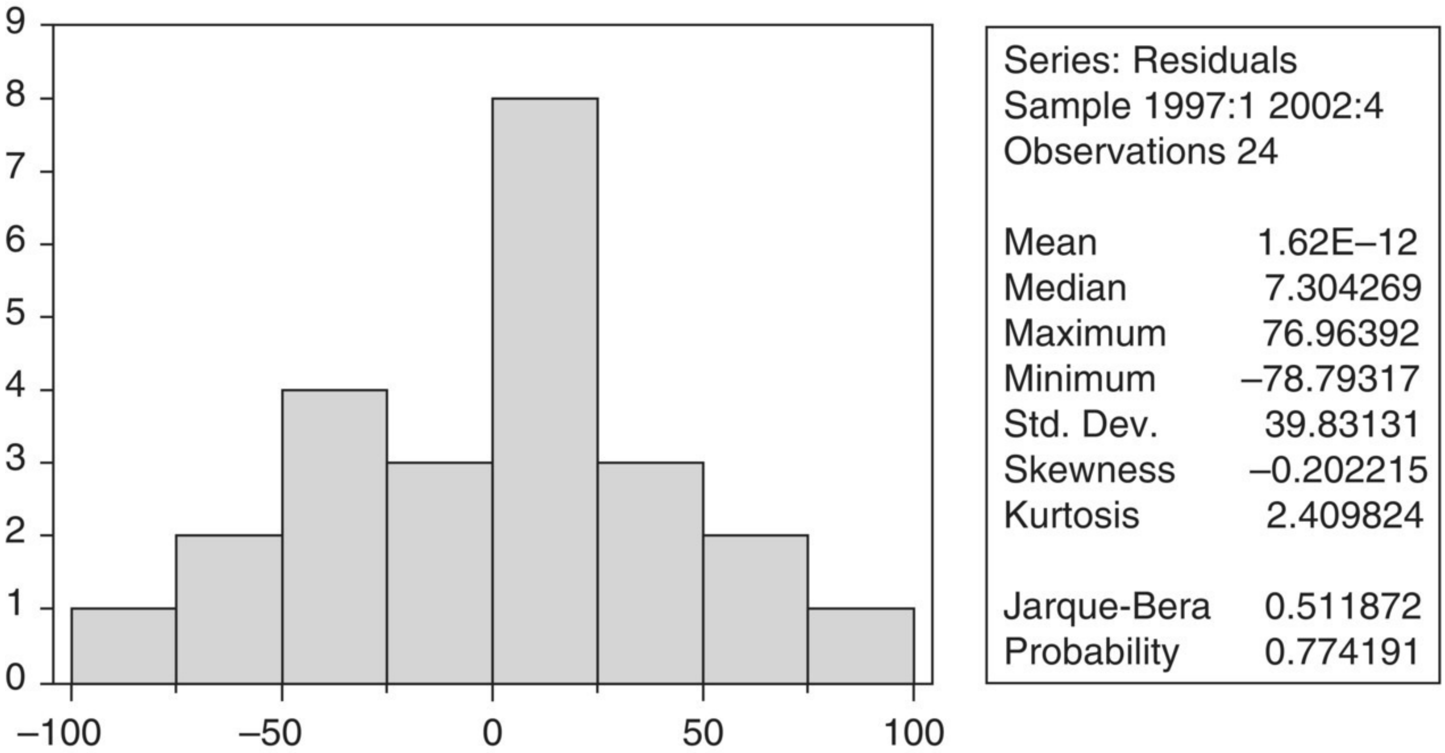


Figure 4.9 Distribution of the residuals of the empirical equation of the gross sales 4.15.

It is somewhat surprising that Equation 4.15 contains only the volume of the current ready-made production PROD as well as third-order and fourth-order autoregressions. In the empirical equation there was no RAN or MARK variables characterizing, on one hand, the variety and versatility of the trade offers, and on the other, participation in trade fairs, as a way of influencing the customers.

For each 1000 PLN of quarterly ready-made production, there is a gross sales income of about 843 PLN. This confirms the initial hypothesis about a necessary production in advance, where, in some cases, the goods await delivery to the client for a period longer than 3 months. Such situation results from a highly competitive market with a large amplitude of seasonal sale fluctuations.

Negative impact of the delayed by 3 quarters sales income on the current amounts obtained as a result of deliveries can, on one hand, result from the returns of some seasonal goods,[30] while on the other, the goods lingering at the recipient’s cause his/her natural aversion for additional purchases. This mechanism forces the manufacturer to more accurately and precisely account for the size of the finished production to avoid returns or not to run out of the goods during a period of intensive purchases, as not to cause the supplier to be displaced by a competitor who is able to satisfy the demand on the market.

Positive interaction of the variable $SBRUT_{-4}$ indicates repeatability of the sales every 4 quarters. This means that a sales income achieved by the end of 4 quarters, in the amount of 1000 PLN, generates its current amount of about 523 PLN.

4.5 Equations of ready-made production

An empirical equation describing the volatility mechanism of a monthly ready-made production has the following from:

$$\underset{(1.747)}{PROD = -83.333} - \underset{(2.053)}{0.175PROD_{-4}} + \underset{(2.750)}{0.422PROD_{-12}} + \underset{(2.026)}{2.894EMP}$$
$$+ \underset{(1.806)}{2.918EMP_{-8}} + \underset{(1.891)}{0.616FIXD_{-2}} - \underset{(2.161)}{0.505FIXD_{-12}} + \underset{(2.370)}{2.162RAN}$$
$$- \underset{(1.916)}{0.716TIME} + u_5,$$
$$R_5^2 = 0.650, S_{5u} = 26.21 \text{ thousands PLN}, DW_5 = 1.862.$$ [31] (4.16)

Description accuracy of the formation mechanism of the monthly ready-made production values is significantly worse in comparison with Equations 4.12 and 4.14. This is due to the specificity of manufacturing for store, which entails production of semifinished goods that were not included in the ready-made production account. This principle is visible in Figure 4.10, which shows the seasonality of the ready-made production's dynamics, on a monthly basis.[32] Therefore, at a large amplitude of the variable PROD's seasonal fluctuations, the degree of explanation of the ready-made production's volatility at a level of 65% can be considered as satisfactory. Figure 4.11 presented empirical values of monthly ready-made production, the theoretical values, and the residuals, calculated on the basis of Equation 4.16.

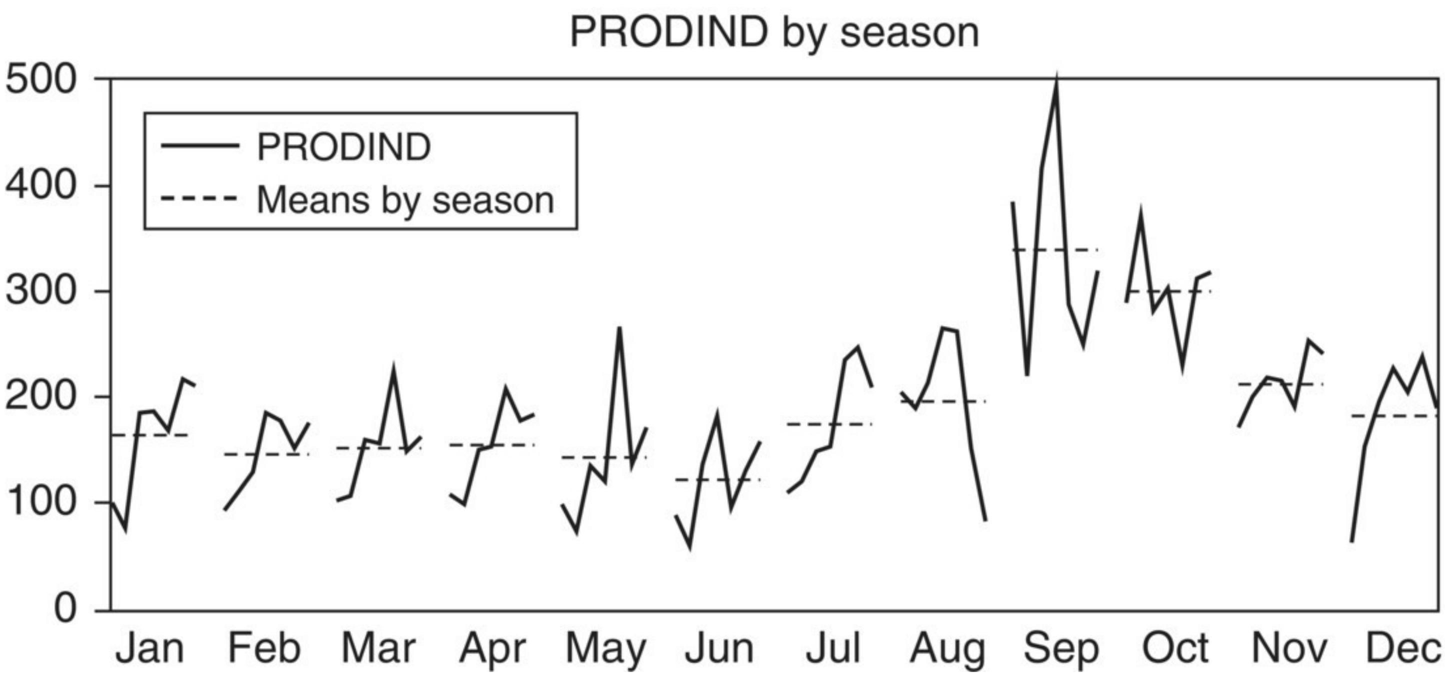


Figure 4.10 Monthly seasonal fluctuations of the ready-made production's dynamics index, in the years 1996–2002.

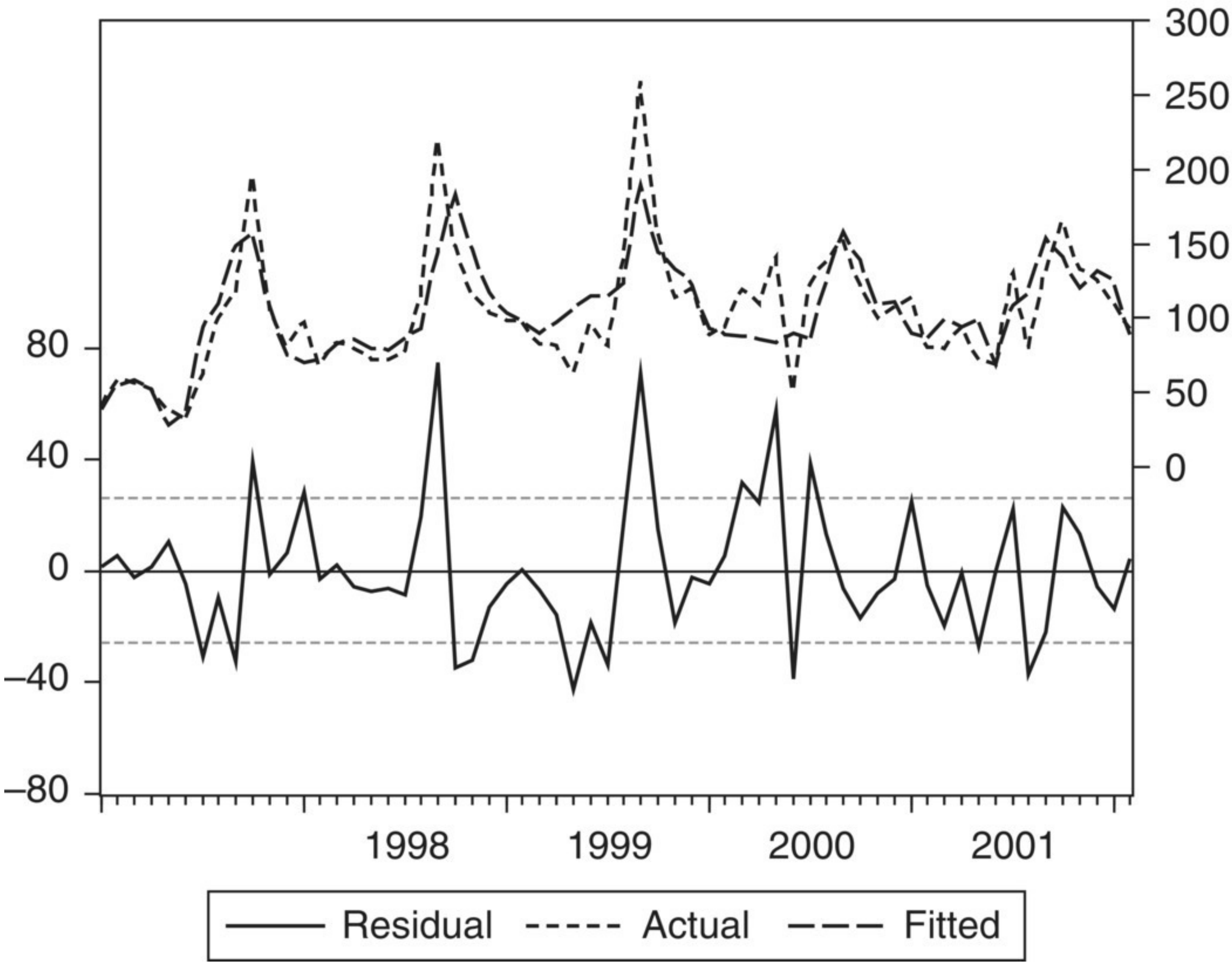


Figure 4.11 Empirical values of monthly ready-made production, the theoretical values and the residuals, calculated on the basis of Equation 4.16.

The explanatory variables appearing in Equation 4.16 are statistically significant on a significance level ranging from γ = 0.0766 (for EMP_{-8}) to less than γ = 0.001 (for PROD). Autoregressions of the 4th and the 12th order signify repetitiveness of the production scale every 12 months[33] as well as its correction every 4 months[34] by the amount of about 175 PLN.

The current and the delayed by 8 months employment, both are stimulators of the ready-made production's size. An increase in employment by one person allows a simultaneous increase in the ready-made production's value, on average, by 2894 PLN. Employment growth by one person, prior to 8 months, generates a current production-effect in the amount of about 2918 PLN, which means that the period of employee's adaptation to his/her work position needs to pass, ultimately resulting in his/her improved efficiency.

In the equation discussed here, the variable FIXAS has been supplanted by a type of its mutation in the form of FIXAS delayed by 2 and 12 months. An increase in the initial value of the tangible fixed assets in use by 1 PLN per 1 PLN of depreciation costs, prior to 2 months, causes an increase in the current ready-made production's value, on average, by about 616 PLN. Along with an increase in the variable FIXAS's value, prior to 12 months, by 1 PLN, there is a current decline in the ready-made production's value, on average, by 506 PLN. The downward trend of the ready-made production, as evidenced by a negative assessment of the parameter along the variable TIME, draws attention. The company follows a deceleration of its production activity, due to an unfavorable entrepreneurial situation, especially owning to the law regulations and law enforcement in the past period. Finally, the variable RAN positively influences the value of the ready-made production. An increase in the number of manufactured assortments by 1 allows an increase in the ready-made production, on average, by 2162 PLN.

An empirical equation of the finished production's value constructed using quarterly data has the following form:

$$\begin{aligned} PROD = \underset{(2.687)}{1103.8} - \underset{(1.965)}{0.403}PROD_{-2} - \underset{(3.017)}{0.399}PROD_{-3} + \underset{(2.584)}{0.599}PROD_{-4} \\ - \underset{(1.886)}{1.709}FIXAS + \underset{(2.473)}{13.791}EMP_{-3} - \underset{(2.614)}{23.032}EMP_{-4} - \underset{(3.074)}{17.084}FIXD \\ + \underset{(1.976)}{7.782}FIXD_{-2} - \underset{(2.734)}{10.696}FIXD_{-3} + \underset{(2.269)}{30.227}TIME + u_6, \\ R_6^2 = 0.876, S_{6u} = 40.680 \text{ thousands PLN}, DW_6 = 2.163.^{35} \end{aligned} \quad (4.17)$$

Among the explanatory variables, the variable FIXAS is statistically significant at a significance level of $\gamma = 0.0818$. All other explanatory variables are statistically significant at $0.01 < \gamma < 0.07$. Explanatory variables included in Equation 4.17 explain 87.6% of the finished production's quarterly volatility. The actual values of the finished production, calculated on the basis of Equation 4.17, differ from its theoretical ones, on average, by 40.680 thousands PLN.

Commutativity of the signs of the coefficients by the variables forming autoregressive dependencies ($PROD_{-2}$, $PROD_{-3}$, $PROD_{-4}$) result from the process of creating inventories of finished goods. During the periods of low-intensity sales, semifinished goods, which can be quickly converted into finished goods and delivered to clients, are manufactured and stored. This way, the company's manufacturing potential is used more rhythmically.

The negative sign of the assessment of the structural parameter by the variable FIXAS can be surprising. Based on that, it cannot be inferred that an increase in the value of the company's noncurrent assets causes a decrease in the ready-made production. However, it follows that the accumulated noncurrent assets are not fully utilized. The resources gathered depend on the demand during the periods of the so-called production peak. This allows avoidance of outsourcing some of the tasks to outside subcontractors, who limit the company's independence and generate higher costs.

The variable FIXD, which expresses a general relation of the noncurrent assets' initial value to the depreciation costs in a given period, contains complementary information about the impact of the noncurrent assets on production performance. A decrease in the FIXAS rate indicates a renewal of the noncurrent assets, while an increase in this variable's value indicates aging of the manufacturing potential. Negative signs of the parameters at FIXAS and $FIXAS_{-3}$ confirm a positive impact of new[36] machinery and equipment on the size of the manufactured production. A positive sign of the assessment of the variable $FIXD_{-2}$ can be interpreted as information about a production increase influenced by an increasing value of the considered variable, with a half-year delay. This may be a consequence of a second quarter period, which is necessary for the employees to adapt to new machinery (equipment), after expiration of which they acquire proper equipment operating efficiency.

Finally, the time variable provides information about a positive trend of manufactured production, followed by an independent increase in the finished production's quarterly value, on average, by 30.227 thousands PLN. It can be interpreted as a result of a neutral technological and organizational progress, which occurred in the company (Figure 4.12).

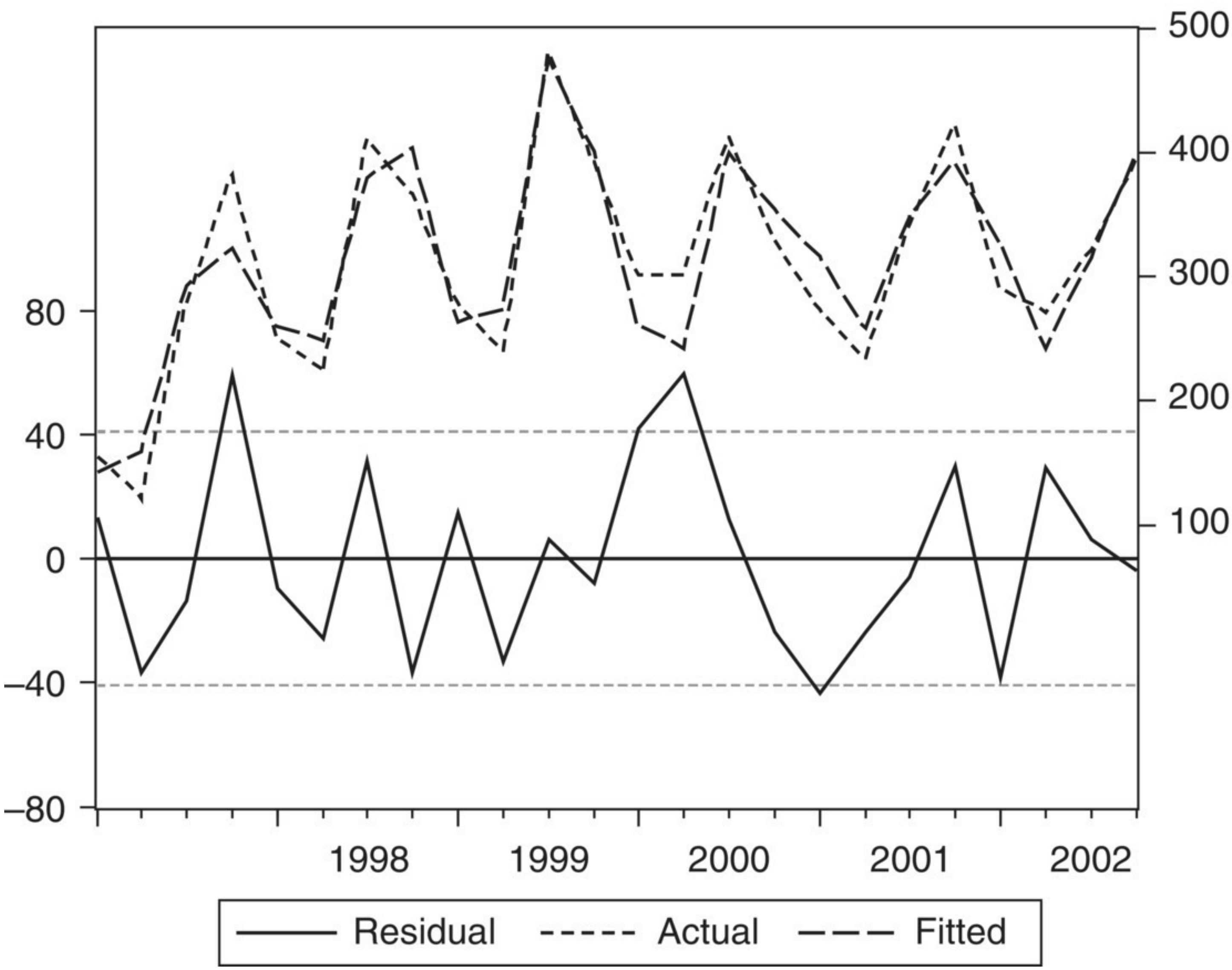


Figure 4.12 The actual quarterly values of finished production, the theoretical values and the residuals, calculated on the basis of Equation 4.17.

4.6 Equation of labor efficiency

The formation mechanism of monthly team-labor efficiency is much more complicated. An empirical equation describing a monthly volatility of the variable WP has the following form:

$$
\begin{aligned}
\text{EFEMP} = & \underset{(1.658)}{1.773} - \underset{(3.138)}{0.262}\text{EFEMP}_{-4} - \underset{(3.531)}{0.320}\text{EFEMP}_{-6} - \underset{(2.573)}{0.225}\text{EFEMP}_{-8} + \\
& - \underset{(2.857)}{0.264}\text{EFEMP}_{-10} + \underset{(2.595)}{0.248}\text{EFEMP}_{-11} + \underset{(5.007)}{0.00877}\text{APAY} - \underset{(2.789)}{0.00648}\text{APAY}_{-5} \\
& + \underset{(3.193)}{0.00836}\text{APAY}_{-6} + \underset{(1.682)}{0.0403}\text{APAY}_{-10} - \underset{(2.355)}{0.0059}\text{APAY}_{-11} + \underset{(3.176)}{0.394}\text{TAL}_{-2} + \\
& - \underset{(2.521)}{0.398}\text{TAL}_{-3} + \underset{(2.173)}{0.259}\text{TAL}_{-4} - \underset{(2.541)}{0.096}\text{TIME} + u_7, \\
& R_7^2 = 0.677, S_{7u} = 1.268 \text{ thousands PLN / 1 employee}, \text{DW}_7 = 2.024.
\end{aligned} \tag{4.18}
$$

[37]

In the empirical equation 4.18, autoregressive interdependencies of the 4th, 6th, 8th, 10th, and 11th order are significant. Negative signs of the assessments of the structural parameters by the efficiency volumes that are delayed by 4, 6, 8, and 10 months indicate the mechanism's commutativity. On the other hand, an increase in labor productivity by 1000 PLN per 1 employee, prior to 11 months, results in an increase in the variable EFEMP's current value, on average, by 248 PLN per 1 employee. Figure 4.13 presented the actual monthly values of team-labor efficiency, the theoretical values, and the residuals calculated on the basis of Equation 4.18.

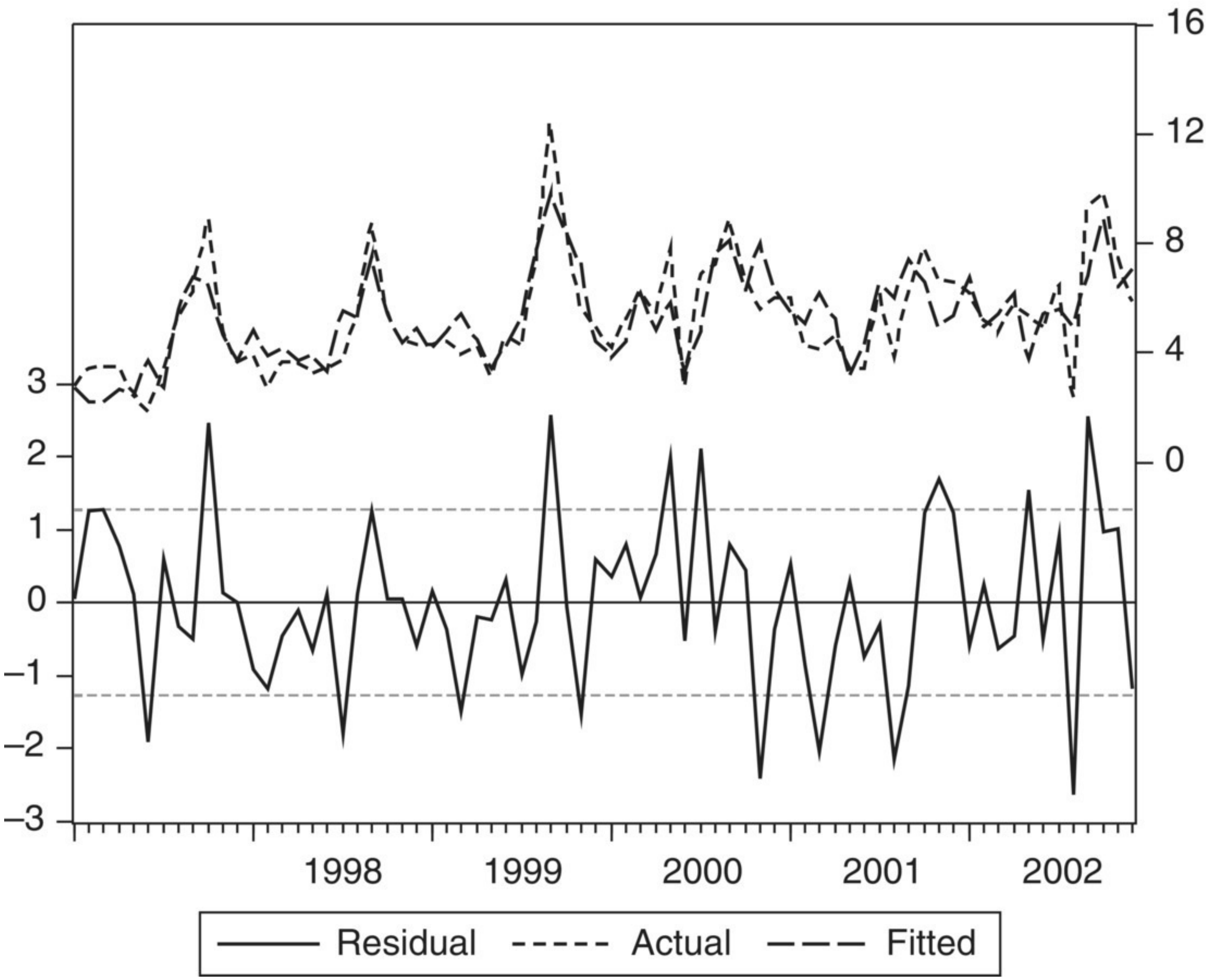


Figure 4.13 The actual monthly values of team-labor efficiency, the theoretical values, and the residuals calculated on the basis of Equation 4.18.

The average monthly net pay is an important efficiency stimulator.[38] An increase in the average net pay by 1 PLN per 1 employee results in an increase in labor efficiency, in this respect, on average, by 8.77 PLN per 1 employee. The average monthly net wages delayed by 6 and 10 months positively influence labor efficiency as well. The average monthly net wages delayed by 5 and 11 months inhibit labor efficiency. However, the sum of the parameter estimations along the current SRPL and the delayed SRPLs is distinctly positive, which indicates a positive influence of the average wages on labor efficiency. This confirms the thesis about the company's properly constructed motivational wage-system.

Technical devices (TAL) affect labor efficiency with the delays of 2, 3, and 4 months. The delays in TAL's impact on the labor efficiency result from an imperative period of human adaptation to new technology. The sum of the coefficients for the variables TAL_{-2}, TAL_{-3}, and TAL_{-4} is positive, which signifies a final positive impact of the technical devices' increase on labor efficiency. At the same time, the negative linear trend of labor efficiency suggests negative technical-organizational changes, detrimental to human-labor effectiveness.

Next empirical equation belongs to the pair expressing a feedback between the team efficiency and the average pay. It describes the quarterly formation mechanism of labor efficiency in the following form:

$$\mathrm{EFEMP} = \underset{(3.916)}{10.224} - \underset{(4.309)}{0.518}\mathrm{EFEMP}_{-2} + \underset{(6.197)}{0.0056}\mathrm{APAY} + u_8, \quad (4.19)$$

$$R_8^2 = 0.694,\ S_{8u} = 2.506 \text{ thousands PLN / 1 employee},\ \mathrm{DW}_8 = 2.023.$$ [39]

Equation 4.19 indicates that an increase in the average net pay by 1 PLN net per 1 employee is followed by an increase in the team-labor efficiency, on average, by 5.6 PLN per 1 employee. This means that the net pay is a significant stimulator of labor efficiency. It allows a conclusion about a properly constructed motivational system within the company. Simultaneously, there is a second-order autoregression of the team-labor efficiency. A negative sign of the coefficient by the variable EFEMP_{-2} indicates alternation of increases and decreases of the EFEMP's value every two quarters. An increase in the variable EFEMP by 1 thousand PLN per 1 employee, prior to 2 quarters, is followed by a current decrease in efficiency, on average, by 518 PLN per 1 employee. An average increase by 518 PLN per 1 employee is a response to a 1 thousand PLN per 1 employee decline in the team-labor efficiency prior to 2 quarters. There is an effect of producing semifinished goods during the periods of sale declines to prepare a necessary inventory that would allow satisfaction of the demand on the market during a period of the so-called peak season.

Description accuracy of the quarterly mechanism of team-labor efficiency is significantly worse than in case of other interdependent variables, which is illustrated in Figure 4.14. Only 69.4% of the labor efficiency's volatility is explained by autoregression of the second order as well as by an average monthly pay in a given quarter.

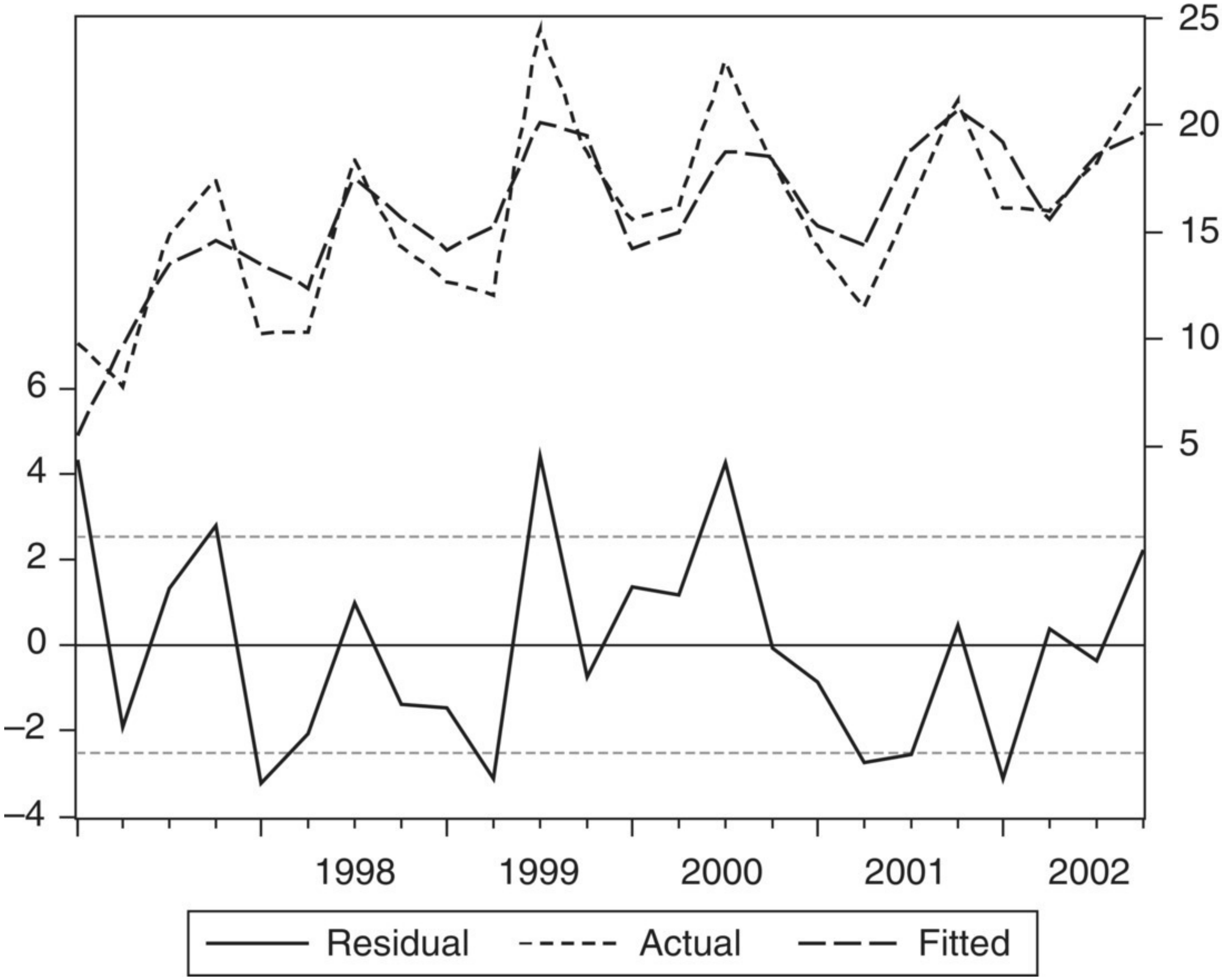


Figure 4.14 The actual values of quarterly team-labor efficiency, its theoretical values, and the residuals calculated on the basis of Equation 4.19.

4.7 Equations of the average wage

An empirical equation describing formation of the average monthly net wage, based on monthly time series, has the following form:

$$\begin{aligned} \mathrm{APAY} = &\underset{(3.232)}{122.17} + \underset{(5.129)}{0.4528}\mathrm{APAY}_{-1} + \underset{(2.361)}{0.2128}\mathrm{APAY}_{-6} + \underset{(2.383)}{0.2313}\mathrm{APAY}_{-12} \\ &+ \underset{(3.243)}{20.701}\mathrm{EFEMP} - \underset{(2.020)}{7.893}\mathrm{EFEMP}_{-7} - \underset{(2.724)}{13.197}\mathrm{EFEMP}_{-12} + u_9, \\ &R_9^2 = 0.906, S_{9u} = 62.21\,\mathrm{PLN\ per\ 1\ employee}, \mathrm{DW}_9 = 2.232.^{40} \end{aligned} \quad (4.20)$$

Figure 4.15 presented the actual average monthly net wages, their theoretical values, and the residuals, calculated on the basis of Equation 4.20. Likewise, in the equation describing APAY, feedback between the average net wage and the team-labor efficiency is confirmed, based on monthly time series. The variable EFEMP is statistically significant in Equation 4.20. An increase in monthly labor efficiency by 1 thousand PLN per 1 employee results in a simultaneous increase in the average net wage, roughly, by net 20.7 PLN per 1 employee. At the same time, there are negative impacts of the labor efficiency amounts delayed by 2 and 12 months on the current average net pay. This means that due to seasonal declines in labor efficiency, caused by production of semifinished inventories, employees are paid accordingly to their labor input. Thus, a decrease in labor efficiency, prior to 7 and 12 months, results in a current increase in the average net wage, due to the employees' significant input into creation of grounds for future ready-made production.

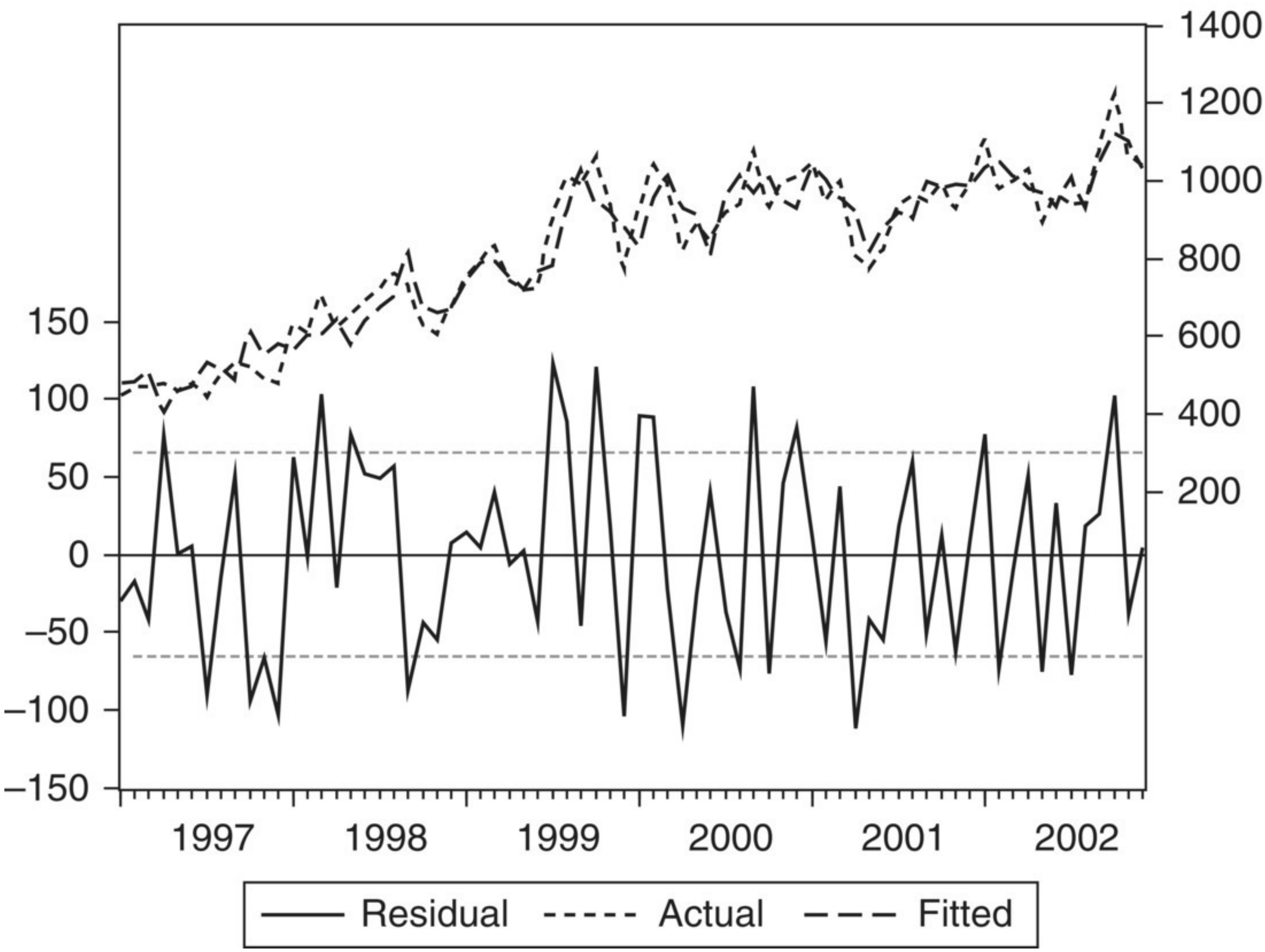


Figure 4.15 The actual average monthly net wages, their theoretical values and the residuals, calculated on the basis of Equation 4.20.

The variable APAY, in a system of monthly observations, is characterized by autoregressions of the 1st, 6th, and 12th order. This signifies stabilization of a part of the standard net wage.[41] An increase in the average net wage by 100 PLN per 1 employee in a previous month causes its current rise by about 45.28% PLN per 1 employee. The effects of similar wage raises prior to 7 and 12 months are lower and, respectively, equal to net 21.28 PLN and 23.13 PLN per 1 employee.

Description accuracy of the average net wage's mechanism, based on monthly time series, is relatively high, since the explanatory variables of Equation 2.10 explain 90.6% of the variable APAY's volatility. Also, S_{5u} = 62.21 PLN per 1 employee confirms the above observation, because the theoretical values of the average net wage, calculated on the basis of Equation 4.20, are different from the actual ones, on average, by ±62.21 PLN per 1 employee, which represents 8.2% of the variable APAY's average value in the years 1996–2002.

An empirical equation describing formation of the average monthly net wage, based on quarterly time series, has the following form:

$$\text{APAY} = \underset{(5.120)}{690.46} + \underset{(12.425)}{0.734}\text{APAY}_{-4} + \underset{(5.009)}{56.8}\text{EFEMP} - \underset{(4.987)}{46.461}\text{EFEMP}_{-4} + u_{10}, \tag{4.21}$$

$$R_{10}^2 = 0.940,\ S_{10u} = 156.00 \text{ PLN per 1 employee},\ \text{DW}_{10} = 2.242.$$ [42]

The mechanism of the average monthly net pay is characterized by high description accuracy, since the explanatory variables of Equation 4.21 explain 94.0% of its volatility. High adherence precision of the theoretical values of the average pay in subsequent quarters in the years 1996–2002 is confirmed by Figure 4.16.

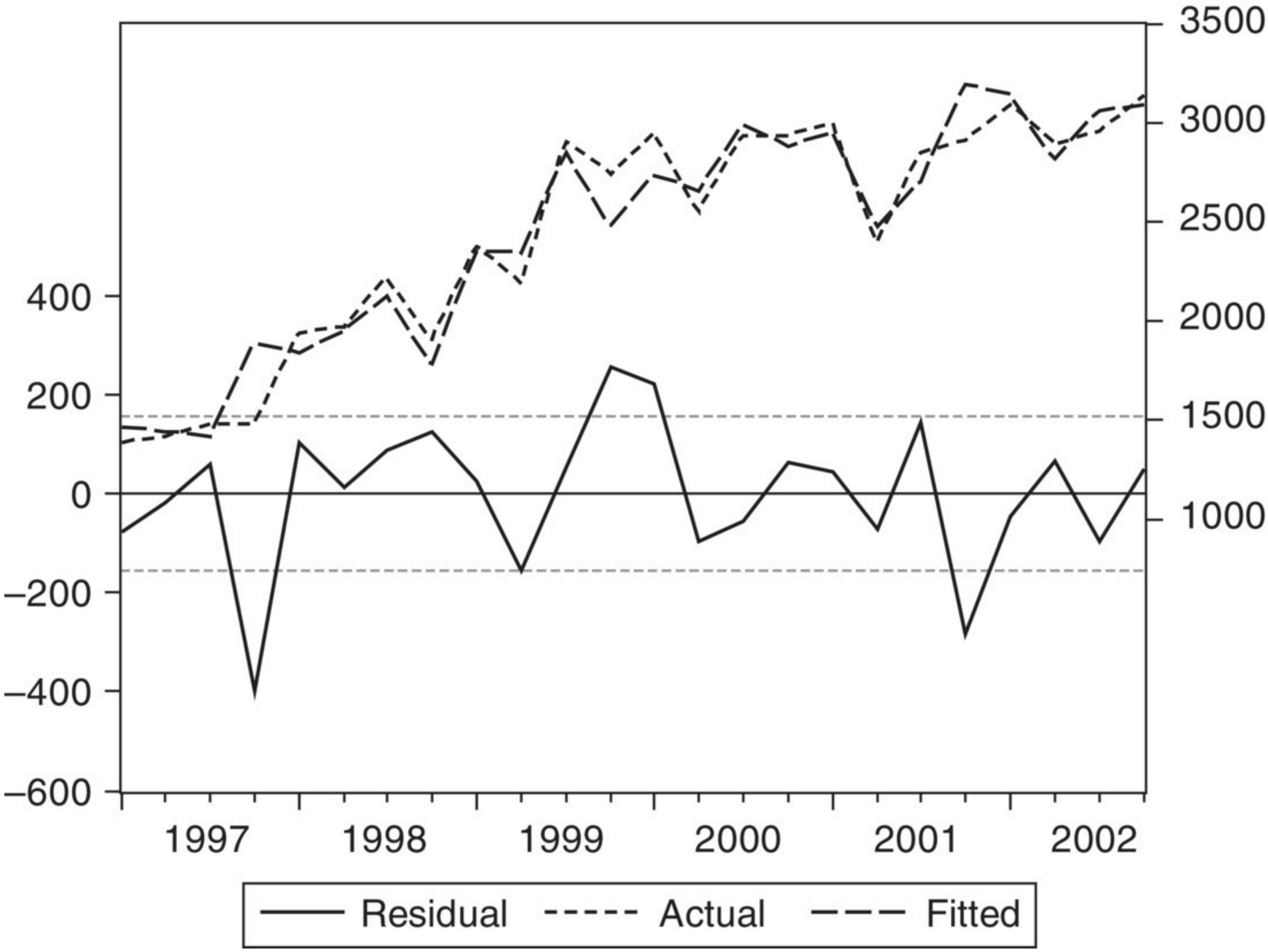


Figure 4.16 The actual average monthly net pay, the theoretical values and the residuals calculated on the basis of Equation 4.21.

Confirmation of a feedback between the average wage and the team-labor efficiency is economically important information contained within Equation 4.21. An increase in the average monthly net wage, on average, by 56.8 PLN per 1 employee, in a simultaneous quarter, occurs along with an increase in EFEMP by 1 thousand PLN per 1 employee. A negative assessment of the structural parameter at the variable $EFEMP_{-4}$ indicates that every 4 quarters, correction of the average monthly pay occurs as a result of a significant arrhythmicity, especially in labor efficiency.

Autoregression of the fourth order is an important principle of the average monthly net pay's quarterly volatility. This indicates consolidation of the wage level, as a company's rule in force. An increase in the average monthly net pay by 100 PLN per 1 employee prior to 4 quarters results in an increase in the current monthly net wage by 73.4 PLN per 1 employee. Despite this fixed mechanism of the average wage increase, the motivational principle of determining a significant part of the wage still functions.

4.8 Equations of the net payroll

An empirical equation of monthly net payroll has the following form:

$$\begin{aligned} SAL = & \underset{(3.554)}{2.373} + \underset{(10.656)}{0.737} SAL_{-1} + \underset{(2.658)}{0.133} SAL_{-11} + \\ & + \underset{(2.971)}{0.01226} CASH - \underset{(3.028)}{0.01181} CASH_{-2} + u_{11}, \\ & R_{11}^2 = 0.895, S_{11u} = 1.246 \text{ thousands PLN}, DW_{11} = 2.235.^{43} \end{aligned} \quad (4.22)$$

Formation mechanism of the enterprise's monthly net payroll has many analogies compared to the equation describing a quarterly wage mechanism. Explanatory variables included in the empirical Equation 4.22 explain 89.5% of the payroll's total volatility. The theoretical values of the variable SAL, calculated on the basis of Equation 4.22, are different from those observed, on average, by 1246 PLN, which represents 8.55% of the average monthly payroll in the years 1996–2002. It is illustrated by Figure 4.17.

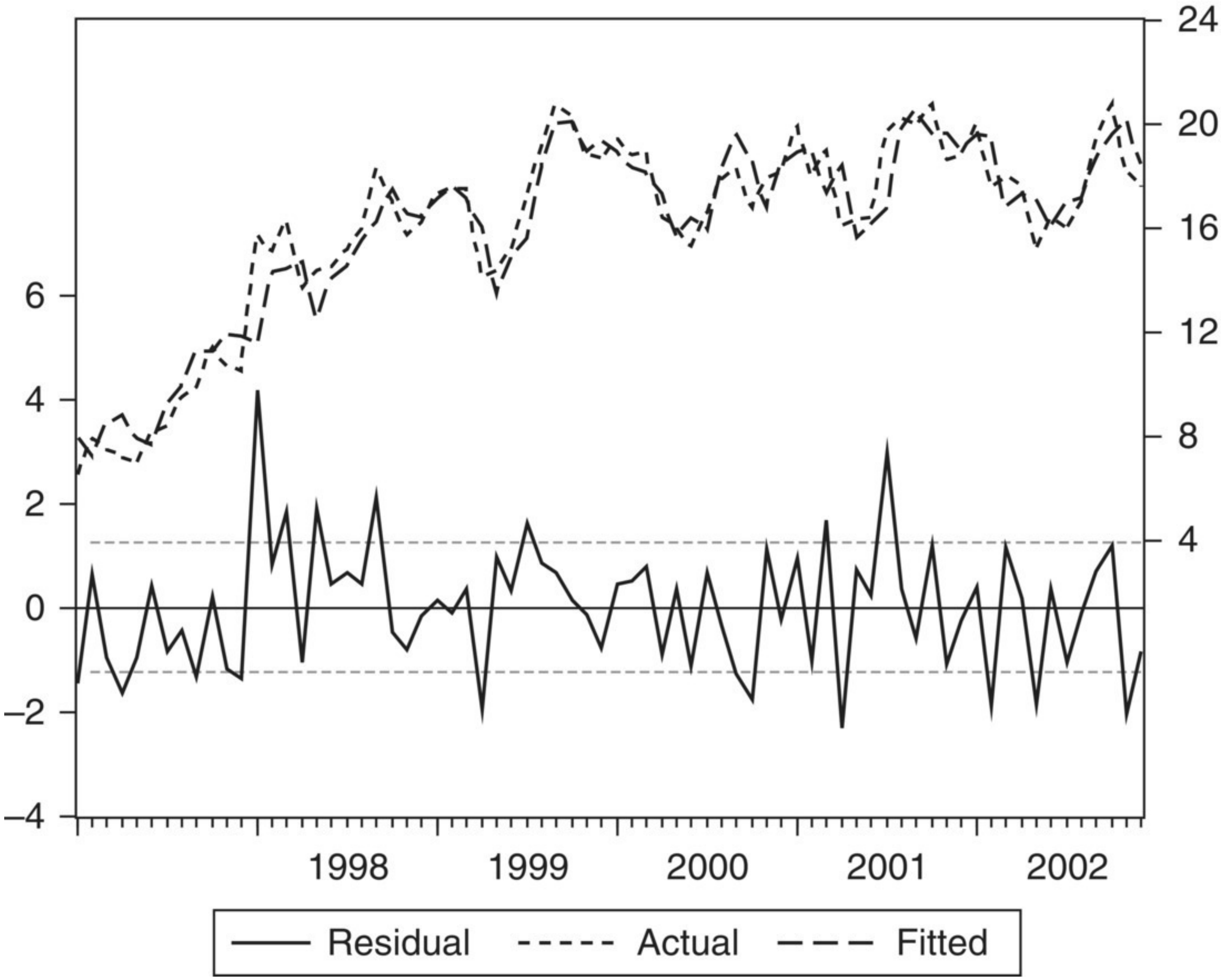


Figure 4.17 The actual monthly values of the net payroll, the theoretical values and the residuals calculated on the basis of Equation 4.22.

Autoregressions of the 1st and the 11th order delimit an important part of the monthly payroll's amount. An increase in the amount of this payroll by 1000 PLN in a previous month causes an increase in the variable SAL's current value, on average, by 737 PLN. In contrast, the same increase in the discussed here payroll, prior to 11 months, results in a current increase in the net payroll in the amount of 133 PLN. Current cash inflows increased by a 1000 PLN cause a simultaneous increase in the net payroll, on average, by 12.26 PLN. The impact of the cash inflow amounts delayed by 2 months on the net payroll is negative, which signifies adequate fluctuations of the variable SAL.

Apart from regular payroll fluctuations, in each month of the year, its upward trend, in the nature of a fading curve, can be generally observed. Subsequent payroll increases are thus slower and become stabilized during the final observation periods.

The company's net payroll depends, among other things, on its current financial situation conditioned by obtaining payments for the goods delivered to the customers. An empirical equation of the net payroll has the following from:

$$\begin{aligned}\mathrm{SAL} = \underset{(0.322)}{1.859} + \underset{(7.842)}{0.707}\mathrm{SAL}_{-1} + \underset{(4.081)}{0.0426}\mathrm{CASH} - \underset{(2.725)}{0.0241}\mathrm{CASH}_{-1} \\ + \underset{(2.131)}{0.0204}\mathrm{CASH}_{-2} + u_{12}, \\ R_{12}^2 = 0.898, S_{12u} = 3.896 \text{ thousands PLN}, \mathrm{DW}_{12} = 2.121.\end{aligned} \quad (4.23)$$

[44]

Equation 4.23 highly accurately explains the mechanism of quarterly fluctuations of the net payroll. Explanatory variables in this equation explain 89.8% of the variable SAL's total volatility. At the same time, the theoretical values of the net payroll, calculated on the basis of the empirical Equation 4.23, differ from its actual ones in the previous quarters, on average, by 3896 PLN, which constitutes 8.9% of the company's average quarterly net payroll amount. It is illustrated by Figure 4.18, which reveals a small discrepancy between the lines labeled as *fitted* and *actual*.

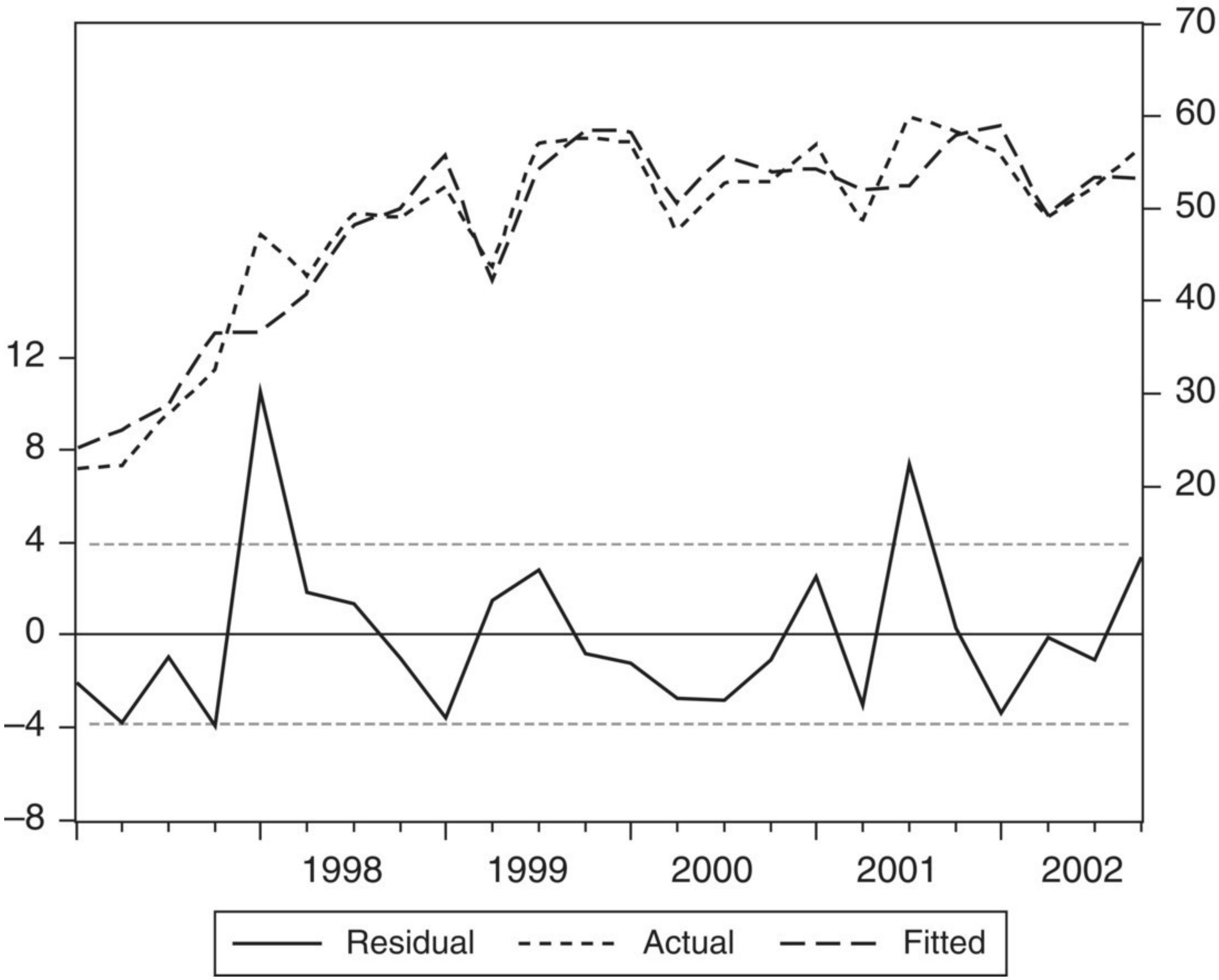


Figure 4.18 The actual quarterly values of the net payroll, the theoretical values, and the residuals calculated on the basis of Equation 4.23.

Autoregression of the first order is the first characteristic of the net payroll's volatility. It shows that 1000 PLN increase in payroll in a previous quarter results in its current increase by around 707 PLN.

The volume of the cash inflows has a current and delayed by 1 and 12 quarters impact on the net payroll. This means that the financial situation, in terms of financial liquidity during the last half of the year, influences the size of the variable CASH. Generally, the company's improved financial situation allows payout of higher net wages.

4.9 The employment equation[45]

An empirical equation describing the principles of formation of a monthly employment has the following from:

$$\underset{(3.092)}{\text{EMP} = 4.443} + \underset{(7.872)}{0.860\text{EMP}_{-1}} - \underset{(2.106)}{0.303\text{EMP}_{-2}} + \underset{(3.341)}{0.368\text{EMP}_{-3}} + \tag{4.24}$$

$$- \underset{(3.133)}{0.292\text{EMP}_{-5}} + \underset{(1.845)}{0.134\text{EMP}_{-8}} + \underset{(2.493)}{0.164\text{MACH}_{-12}} - \underset{(2.841)}{0.087\text{TIME}} + u_{13},$$

$$R_{13}^2 = 0.803, S_{13u} = 1.2 \text{ persons}, \text{DW}_{13} = 1.941.$$[46]

Figure 4.19 presented the actual monthly employment, theoretical employment, and the residuals calculated on the basis of Equation 4.24. The employment equation based on monthly data reveals the variable EMP's strong autoregressive relations. Conclusion of an employment contract, especially for an indefinite time-period, makes the current status of employment a consequence of its level in previous months. Apparently, current employment status depends on the situation in a preceding month. Not all autoregressive relations are equally clear. The variable EMP_{-8} can be considered statistically significant when $\gamma = 0.0696$, while the variable EMP_{-2} – at a significance level of $\gamma = 0.0391$. These are not – as in the microeconomic equation based on monthly time series – very high risk-levels of a type I error.

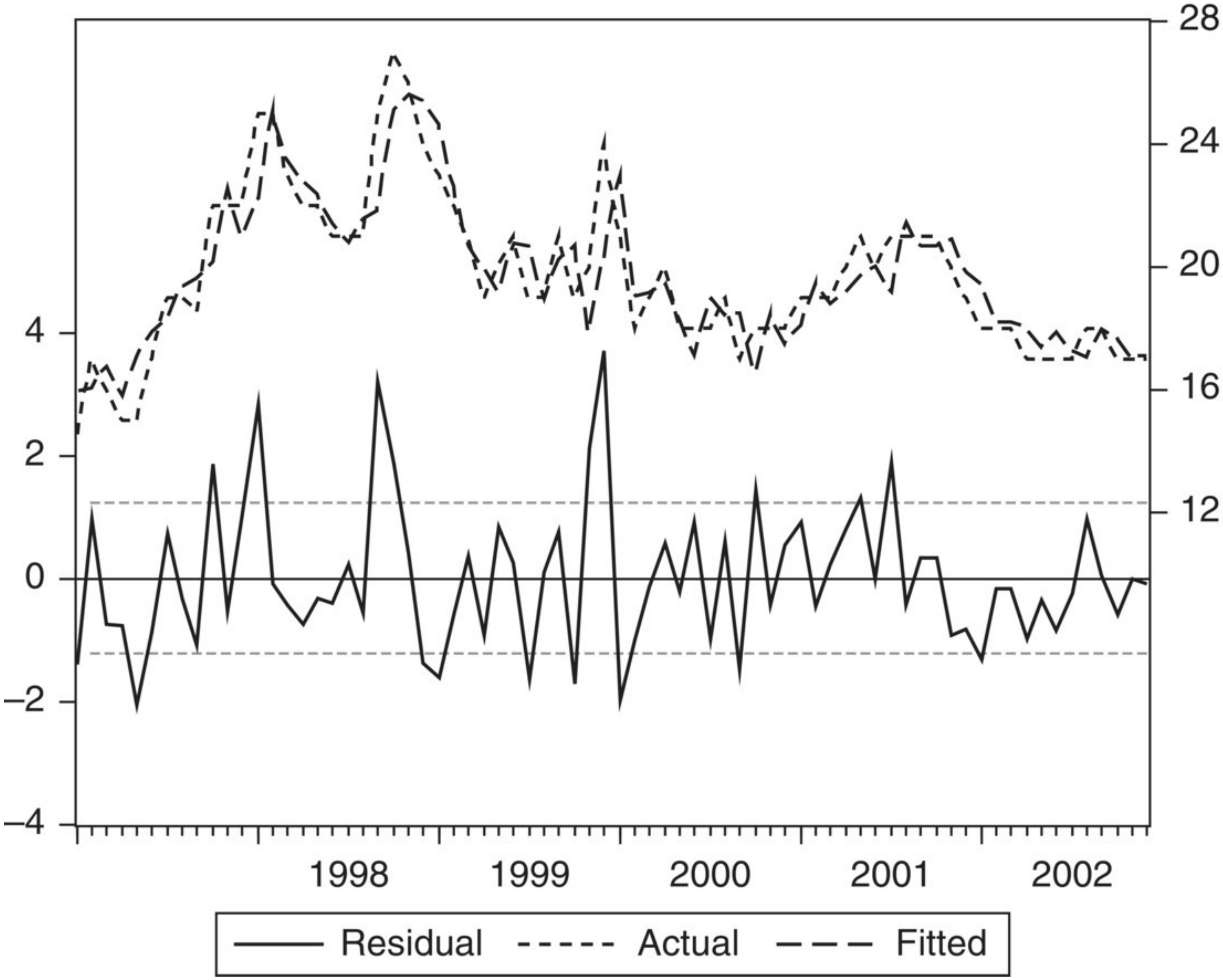


Figure 4.19 The actual monthly employment, theoretical employment, and the residuals calculated on the basis of Equation 4.24.

The initial value of machinery and equipment, prior to 12 months, turns out to be employment stimulus. An increase in employment by 1 person in a current month results from an increase in machinery resources, prior to 12 months, by the amount of 61 thousands PLN, according to its net purchasing value. As such, complementary relations of employment and of the value of machinery and equipment are observed in the company.

The study has shown that the initial value of the tangible fixed assets in use[47] FIXAS did not prove to be statistically significant, both in the employment equations describing a quarterly mechanism as well as monthly regularities. In practice, only ownership changes in terms of the variable MACH (with an adequate delay) cause necessary employment adjustments in the group of workers. Transportation means, in fact, are related to the sales representatives supporting the sales network. During the discussed period, the networks itself as well as the number of the sales representatives were stable. As a result, only the change of the so-called productive apparatus caused an adequate reaction of the variable EMP. Negative employment trends, both the quarterly as well as the annual ones, result from the process of substituting labor with capital, which occurred highly intensively in the period passed.

An equation describing employment formation, based on quarterly time series, has the following empirical from:

$$\begin{aligned} \mathrm{EMP} = {} & \underset{(2.715)}{7.668} + \underset{(4.195)}{0.801}\mathrm{EMP}_{-1} - \underset{(2.423)}{0.604}\mathrm{EMP}_{-2} + \underset{(1.898)}{0.359}\mathrm{EMP}_{-3} \\ & + \underset{(3.068)}{0.0441}\mathrm{MACH}_{-4} - \underset{(3.273)}{0.651}\mathrm{TIME} + u_{14}, \\ & R^2_{14} = 0.734, S_{14u} = 1.4 \text{ persons}, \mathrm{DW}_{14} = 2.257. \end{aligned} \tag{4.25}$$

A dominant characteristic of the company's employment volume is expressed by autoregressive relations up to and including the third order. There is a mechanism correcting employment, characterized by a negative assessment of the structural parameter by the variable EMP_{-2}. It means that employment adjustment is made averagely every semester, involving reduction of the number of employees, on average, by one person annually.

Increasing the machinery resources results in an increase in employment, whiles the initial value of the machinery and equipment, prior to 4 quarters, is a statistically significant explanatory variable. Increasing the company's machinery resources by around 23 thousands PLN triggers a response in the form of an employment increase by one person.

The quarterly volumes of employment are characterized by a linear trend. On average, a decrease in the number of employees, equal to a reduction of 1 employee each half of the year, occurs quarterly. This trend had become stronger at the end of the 1990s and lasted until the end of the observation period, as a reaction to inflexible employment opportunities in Poland and to rapidly rising costs of labor resultant from the actions undertaken by successive governments.

Description accuracy of the quarterly employment mechanism is moderate, since the explanatory variables included in Equation 4.25 explain 73.4% of the EMP's overall volatility. There is no first-order autocorrelation of the random component in Equation 4.25, since $\mathrm{DW}_{14}^* = 1.743 > d_u = 1.53$, at a significance level of $\gamma = 0.01$. Figure 4.20 illustrates the employment's quarterly volatility and the results obtained from the empirical equation 4.25.

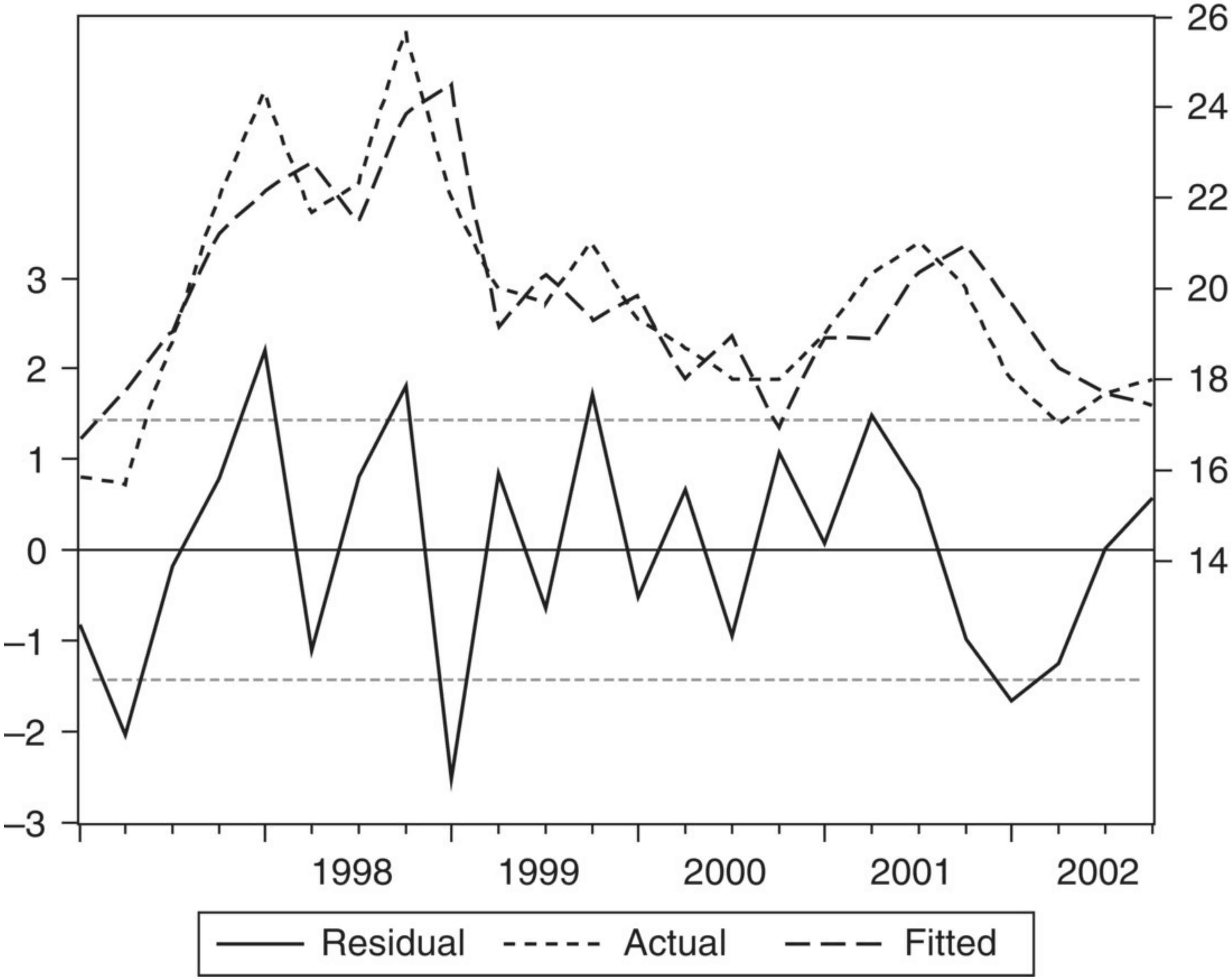


Figure 4.20 The actual employment, the theoretical employment volumes, and the residuals calculated on the basis of Equation 4.25 (quarterly data).

4.10 Equations of the fixed assets

An empirical equation describing the variable FIXAS, based on monthly data, has the following form:

$$\begin{aligned} \mathrm{FIXAS} = \underset{(1.575)}{17.911} + \underset{(10.042)}{0.755}\mathrm{FIXAS}_{-1} + \underset{(3.819)}{0.422}\mathrm{FIXAS}_{-7} - \underset{(2.602)}{0.292}\mathrm{FIXAS}_{-8} + \\ - \underset{(1.421)}{0.064}\mathrm{CASH}_{-2} + \underset{(2.633)}{0.116}\mathrm{CASH}_{-4} - \underset{(2.013)}{0.1004}\mathrm{CASH}_{-7} + \underset{(1.682)}{0.803}\mathrm{CASH}_{-8} \\ + \underset{(1.710)}{0.0796}\mathrm{CASH}_{-12} - \underset{(1.935)}{2.133}\mathrm{SAL}_{-6} + \underset{(1.841)}{2.225}\mathrm{SAL}_{-7} + \underset{(1.845)}{1.549}\mathrm{SAL}_{-12} + u_{15}, \\ R_{15}^2 = 0.991, S_{15u} = 10.472 \text{ thousands PLN}, \mathrm{DW}_{15} = 1.835. \end{aligned} \tag{4.26}$$

Monthly fluctuations of the initial value of the fixed assets are caused by numerous explanatory variables. Autoregressive relations of the first, the seventh, and the eighth order play a significant role. The amounts of the money obtained earlier from the customers play an important role in shaping the noncurrent assets. The impact of the variables $CASH_{-2}$ (while $\gamma = 0.16$), $CASH_{-8}$ (for $\gamma = 0.098$), and $CASH_{-12}$ (while $\gamma = 0.0925$) is relatively weak. Impact of the variable CASH with various delay periods confirms the earlier hypotheses as well as the calculation results obtained earlier on.

The impact of the variable SAL delayed by 6, 7, and 12 months on the size of the noncurrent assets is quite interesting. The sum of the positive parameter assessments by the variables SAL_{-7} and SAL_{-12} exceeds the negative value by the variable SAL_{-6}. On the balance sheet, this means that an increase in the net payroll is followed by an increase in the initial value of the fixed assets. This can be interpreted as a reaction to the rising labor costs by further expansion of the noncurrent assets.

Equation 4.26 highly accurately explains the volatility of the variable FIXAS, which is illustrated by Figure 4.21. Explanatory variables of Equation 4.19 explain up to 99.1% of the overall volatility of the variable FIXAS. The theoretical values of the fixed assets, calculated on the basis of Equation 4.26, are different from their empirical values, on average, by 10472 PLN, which is 2.8% of their average monthly value in the years 1996–2002.

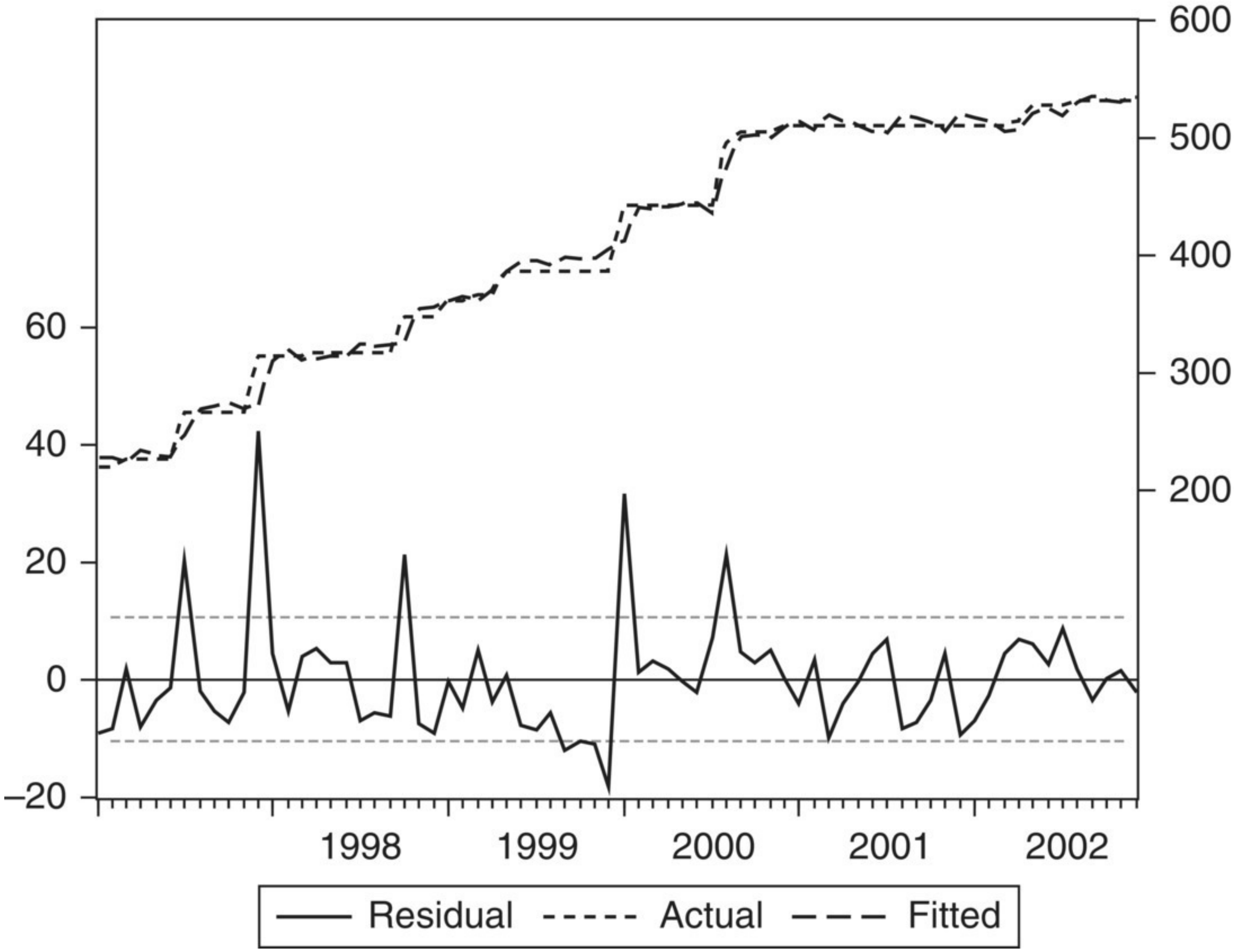


Figure 4.21 The actual initial values of the tangible fixed assets in use, the theoretical values, and the residuals calculated on the basis of Equation 4.26 (monthly).

A quarterly mechanism describing the initial value of the fixed assets is expressed by the following empirical equation:

$$\underset{}{\mathrm{FIXAS}} = \underset{(2.128)}{24.853} + \underset{(32.413)}{0.919}\mathrm{FIXAS}_{-1} + \underset{(2.374)}{0.0645}\mathrm{CASH}_{-4} + u_{16}, \tag{4.27}$$

$$R_{16}^2 = 0.984, S_{16u} = 13.716 \text{ thousands PLN}, \mathrm{DW}_{16} = 2.590.$$

Figure 4.22 presented the actual initial values of the fixed assets, the theoretical values, and the residuals calculated on the basis of Equation 4.27 (quarterly). The company's quarterly fixed assets on the quarterly basis are characterized by a very strong autoregression of the first order. Additionally, the cash inflows from four quarters back positively influence its financial resources. This is due to application of the company's principle of investing only own resources in the fixed assets. In fact, only a large inflow of cash prior to 4 quarters allows an increase in the noncurrent assets. An inflow of 100 thousands PLN prior to 4 quarters increased the current value of the variable FIXAS, on average, by 6540 PLN.

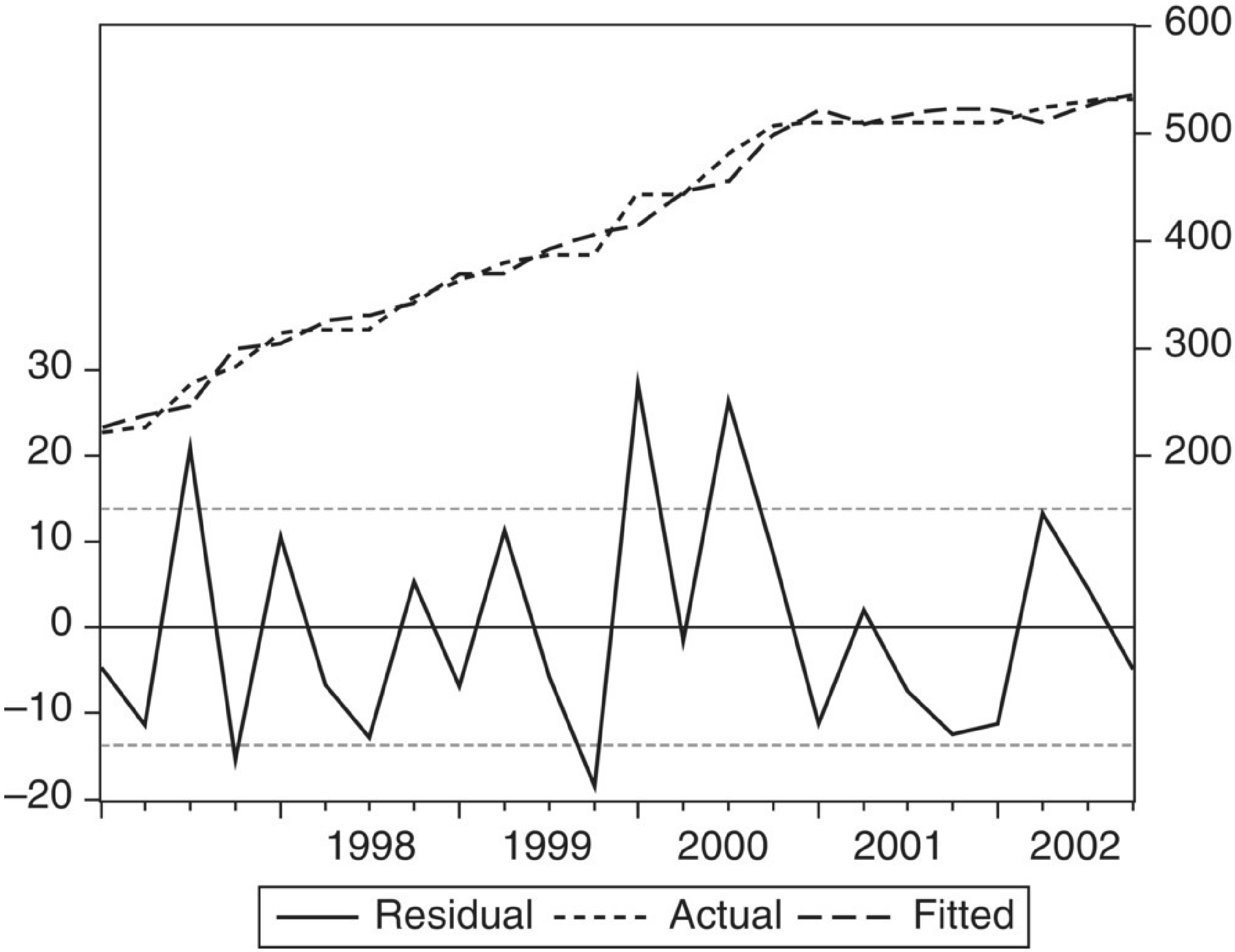


Figure 4.22 The actual initial values of the fixed assets, the theoretical values, and the residuals calculated on the basis of Equation 4.27 (quarterly).

Bank credit is too expensive for the company. Moreover, the procedures involving obtaining external sources of financing the business activity and development, for a small-sized company, are too complicated and require excessive security measures. The whole complexity of using a credit deters the company's owner from applying to any of the financial institutions granting credits. As such, development opportunities of a small-sized company are significantly limited.

Equation 4.27 highly accurately describes volatility of the fixed assets, since autoregression and the cash inflows obtained prior to 4 quarters explain 98.4% of the total volatility of FIXAS. The theoretical values of the fixed assets, calculated on the basis of Equation 4.27, differ from their actual values, on average, by 13.716 thousands PLN, which constitutes 3.69% of the average value of FIXAS in the period between 1996 and 2002. In the equation, there is also a first-order autocorrelation of the random component, since $DW_{16}^* = 1.410 > d_u = 1.20$, at a significance level of $\gamma = 0.01$.

Next equation explains a quarterly mechanism[48] of the machinery and equipment's volatility. It has the following form:

$$\text{MACH} = \underset{(4.128)}{45.886} + \underset{(23.79)}{0.884}\text{MACH}_{-1} + u_{17}, \quad (4.28)$$

$$R_{17}^2 = 0.958, S_{17u} = 18.979 \text{ thousands PLN}, \text{DW}_{17} = 1.642.^{49}$$

Figure 4.23 presented the actual initial values of machinery and equipment, the theoretical values, and the residuals calculated on the basis of Equation 4.28 (quarterly). The value of machinery and equipment is subject only to an autoregressive dependence of the first order, which explains up to 95.6% of its total quarterly volatility. There was no impact of the variable CASH with any delay.

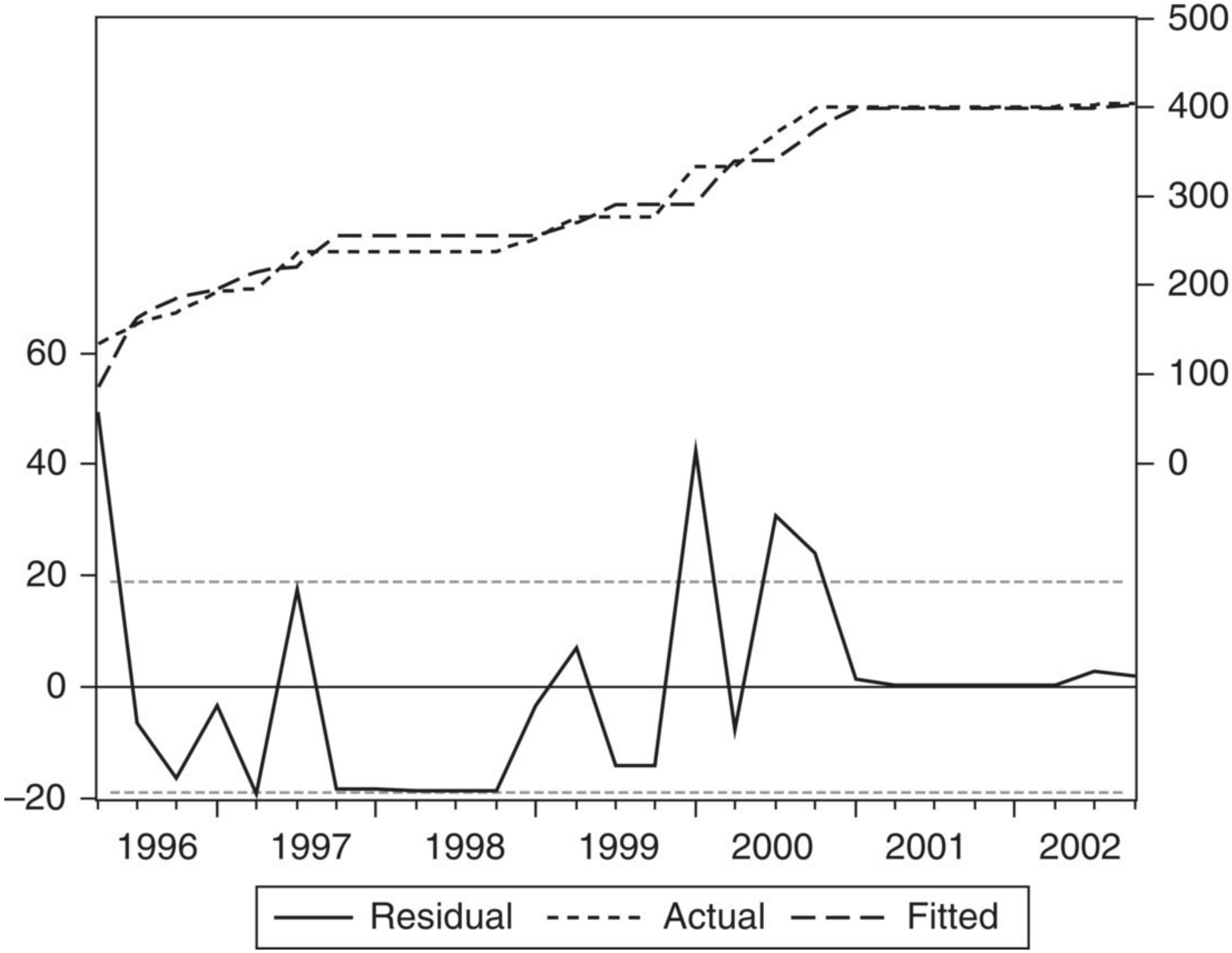


Figure 4.23 The actual initial values of machinery and equipment, the theoretical values and the residuals calculated on the basis of Equation 4.28 (quarterly).

An empirical equation explaining the mechanism of machinery and equipment's volatility has the following form:

$$\underset{(0.564)}{\text{MACH} = 4.419} + \underset{(13.394)}{0.756\text{MACH}_{-1}} + \underset{(4.960)}{0.463\text{MACH}_{-7}} - \underset{(3.531)}{0.320\text{MACH}_{-8}} \quad (4.29)$$

$$+ \underset{(3.325)}{0.0995\text{CASH}_{-4}} + \underset{(3.117)}{0.0973\text{CASH}_{-8}} + \underset{(3.052)}{0.1061\text{CASH}_{-12}} - \underset{(3.267)}{1.607\text{SAL}_{-1}}$$

$$+ \underset{(3.212)}{1.667\text{SAL}_{-12}} + u_{18},$$

$$R^2_{18} = 0.991, S_{18u} = 8.091 \text{ thousands PLN}, \text{DW}_{18} = 2.129.$$

Figure 4.24 presented the actual initial values of machinery and equipment, the theoretical vales, and the residuals calculated on the basis of Equation 4.29 (monthly). In Equation 4.29, some analogies to the volatility mechanism of the total value of the fixed assets can be seen. It results from the fact that the variable MACH is a dominant component of the variable FIXAS, which, in the following years, is about 77%. All observations and explanations included in the characteristic of Equation 4.19 can be referred to in this case as well. A significant role of the cash inflows (CASH) with various delay periods in the formation of the machinery and equipment's volatility (MACH) can also be recognized.

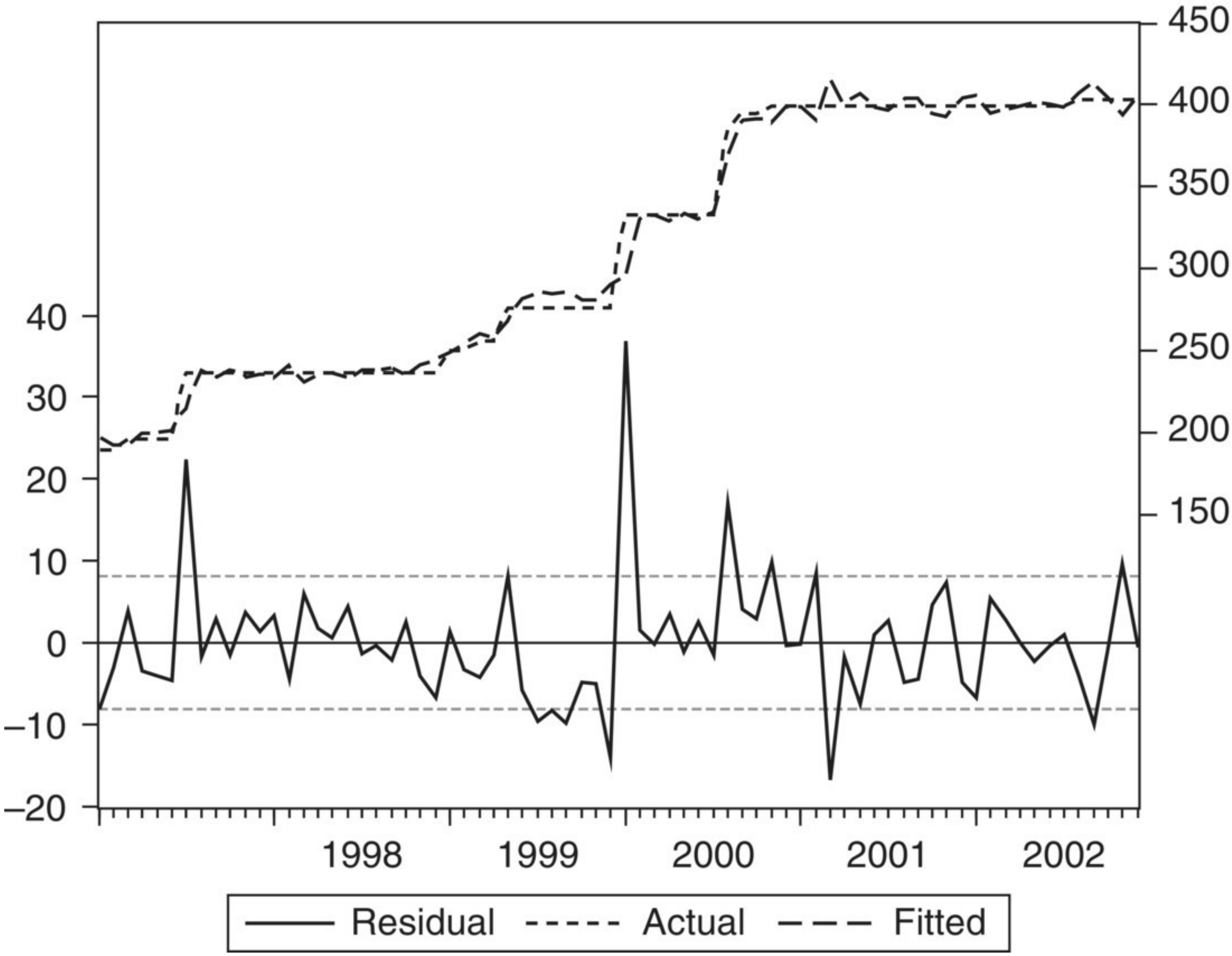


Figure 4.24 The actual initial values of machinery and equipment, the theoretical vales, and the residuals calculated on the basis of Equation 4.29 (monthly).

Next four empirical equations describe the mechanisms of the investments aimed at expansion of the total amount of fixed assets as well as on expansion of the machinery and equipment, on a quarterly and monthly basis. Figure 4.25 presented the actual quarterly investment outlays, the theoretical values of investments, and the residuals calculated on the basis of Equation 4.30. Respectively, these are as follows:

- An empirical equation of the investments expanding the value of the total value of fixed assets, on a quarterly basis:

$$\begin{aligned} INV = &\underset{(0.529)}{-10.058} - \underset{(1.874)}{0.237} INV_{-4} + \underset{(1.965)}{0.084} CASH_{-1} + \underset{(2.455)}{0.063} CASH_{-4} \\ &+ \underset{(2.133)}{20.518} S3 - \underset{(3.994)}{1.851} TIME + u_{19}, \\ &R_{19}^2 = 0.522,\ S_{19u} = 12.48 \text{ tys.zł},\ DW_{19} = 2.321; \end{aligned} \tag{4.30}$$

- An empirical equation of the investments expanding the value of the fixed assets, on a monthly basis:

$$\begin{aligned} \mathrm{INV} = & \underset{(1.305)}{-13.97} - \underset{(1.666)}{0.188}\mathrm{INV}_{-1} - \underset{(1.982)}{0.224}\mathrm{INV}_{-2} + \underset{(2.889)}{0.313}\mathrm{INV}_{-7} \\ & + \underset{(1.766)}{0.087}\mathrm{CASH}_{-4} + \underset{(2.130)}{0.098}\mathrm{CASH}_{-8} + \underset{(1.938)}{0.085}\mathrm{CASH}_{-10} + \underset{(2.398)}{0.106}\mathrm{CASH}_{-12} \\ & + \underset{(2.511)}{3.108}\mathrm{DEPR} - \underset{(3.207)}{4.008}\mathrm{DEPR}_{-3} - \underset{(1.995)}{0.467}\mathrm{RAN} + u_{20}, \\ & R_{20}^2 = 0.298, S_{20u} = 10.93 \text{ thousands PLN}, \mathrm{DW}_{20} = 2.013; \end{aligned} \quad (4.31)$$

- An empirical equation of investments expanding the value of machinery and equipment, quarterly:

$$\begin{aligned} \mathrm{INVMACH} = & -\underset{(2.006)}{59.71} - \underset{(1.412)}{0.293}\mathrm{INVMACH}_{-1} + \underset{(1.879)}{0.075}\mathrm{CASH}_{-1} \\ & + \underset{(2.106)}{0.082}\mathrm{CASH}_{-2} + \underset{(1.543)}{0.0602}\mathrm{CASH}_{-3} + \underset{(2.658)}{0.1027}\mathrm{CASH}_{-4} - \underset{(2.942)}{2.126}\mathrm{TIME} + u_{21}, \\ & R_{21}^2 = 0.381, S_{21u} = 14.929 \text{ thousands PLN}, \mathrm{DW}_{21} = 1.835. \end{aligned} \quad (4.32)$$

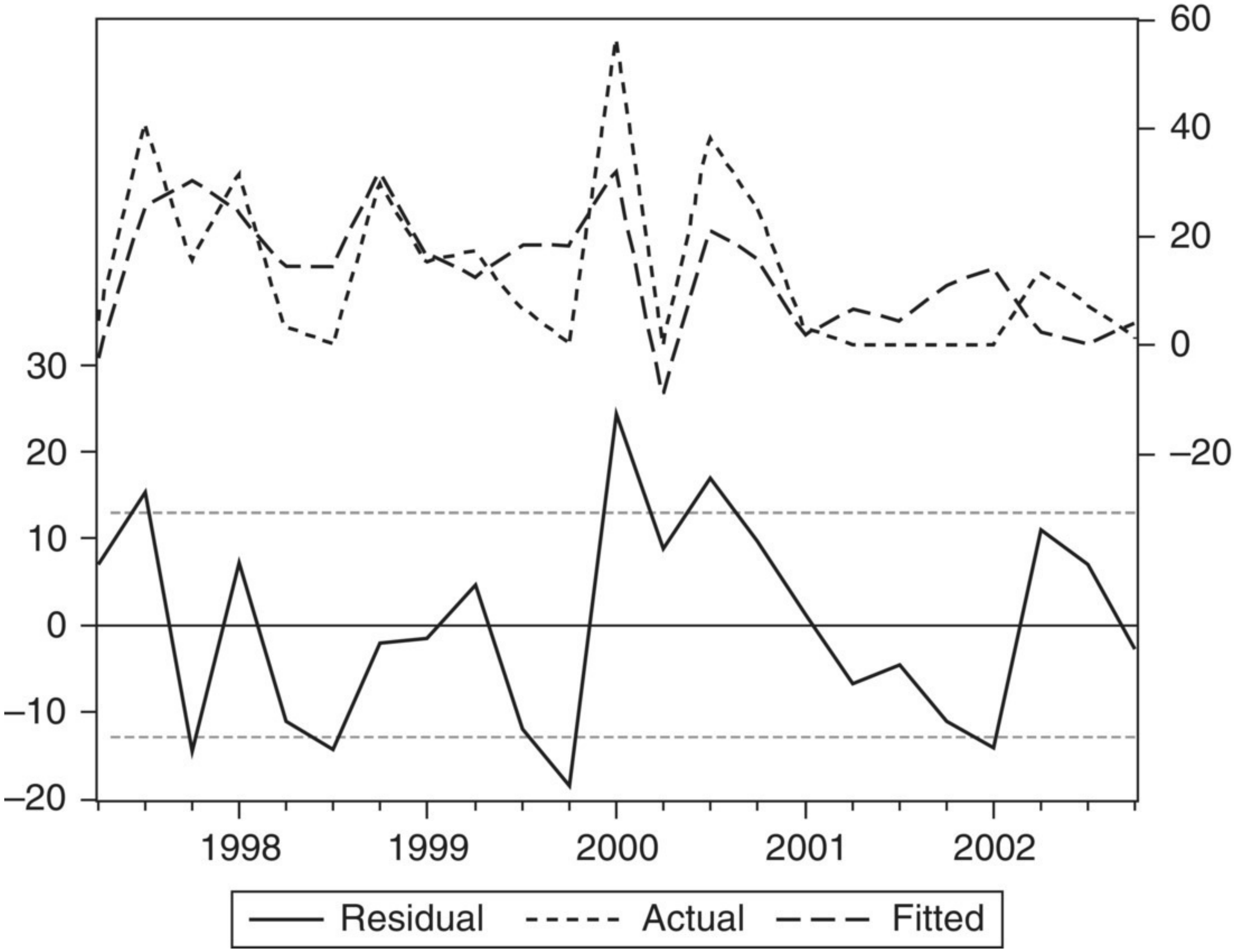


Figure 4.25 The actual quarterly investment outlays, the theoretical values of investments, and the residuals calculated on the basis of Equation 4.30.

Figure 4.26 displayed the actual monthly investments, the theoretical values, and the residuals calculated on the basis of Equation 4.31. Figure 4.27 analyzed the actual quarterly investment outlays for machinery and equipment, its theoretical values, and the residuals calculated on the basis of Equation 4.32. All Equations 4.30–4.32 are characterized by low accuracy in explaining the enterprise’s investment principles. A downward trend is a common feature of the overall investment or the investments in machinery and equipment. It indicates that the strategy of a dynamic development was abandoned in favor of the care for the company’s survival. The investments implemented after the 2000, essentially, were “forced.” Thus, a necessary replacement of old delivery vehicles by new ones followed, due to their excessive exploitation expenses and a risk of malfunction, since using the hitherto vehicles threatened regularity of the sales network servicing.

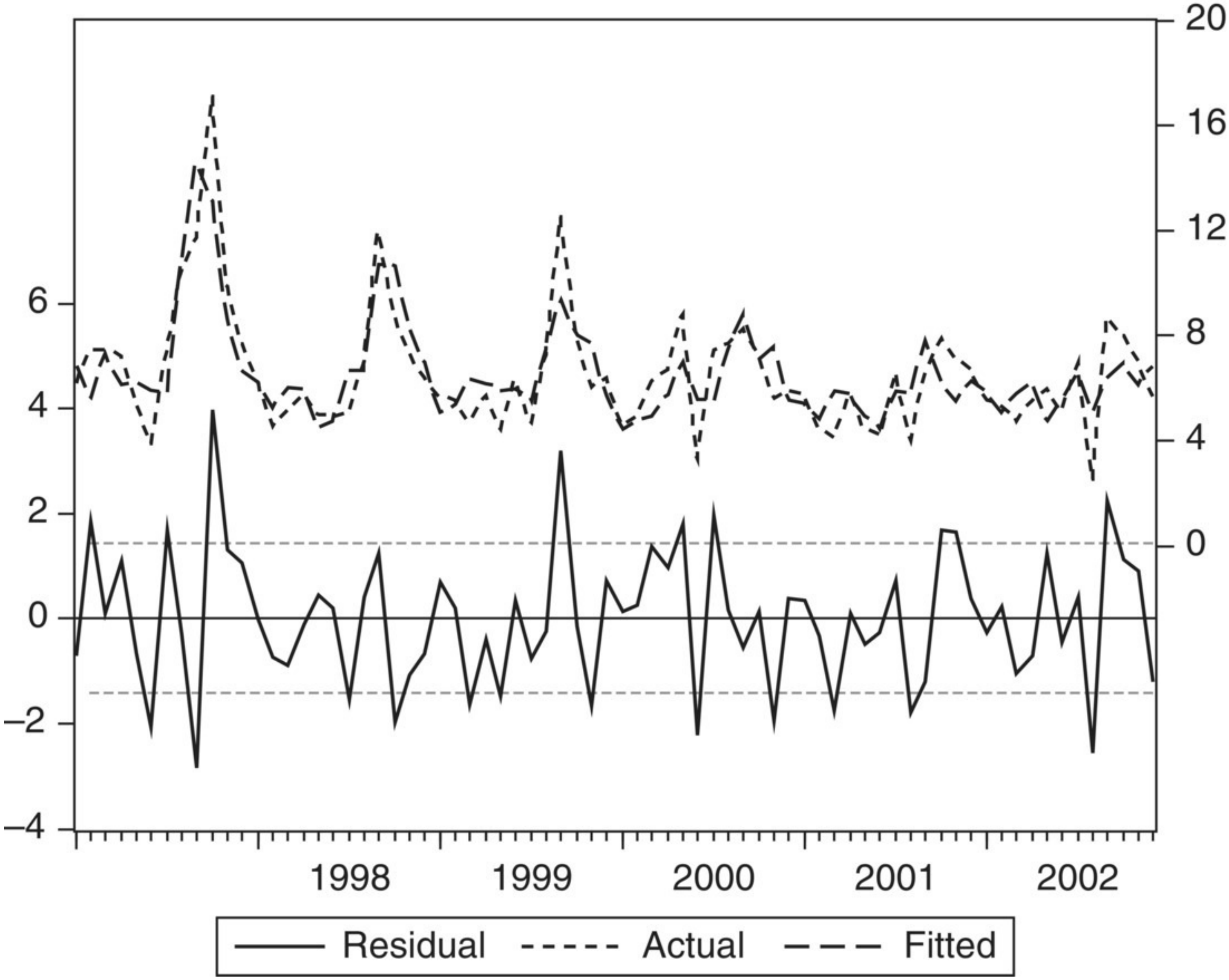


Figure 4.26 The actual monthly investments, the theoretical values, and the residuals calculated on the basis of Equation 4.31.

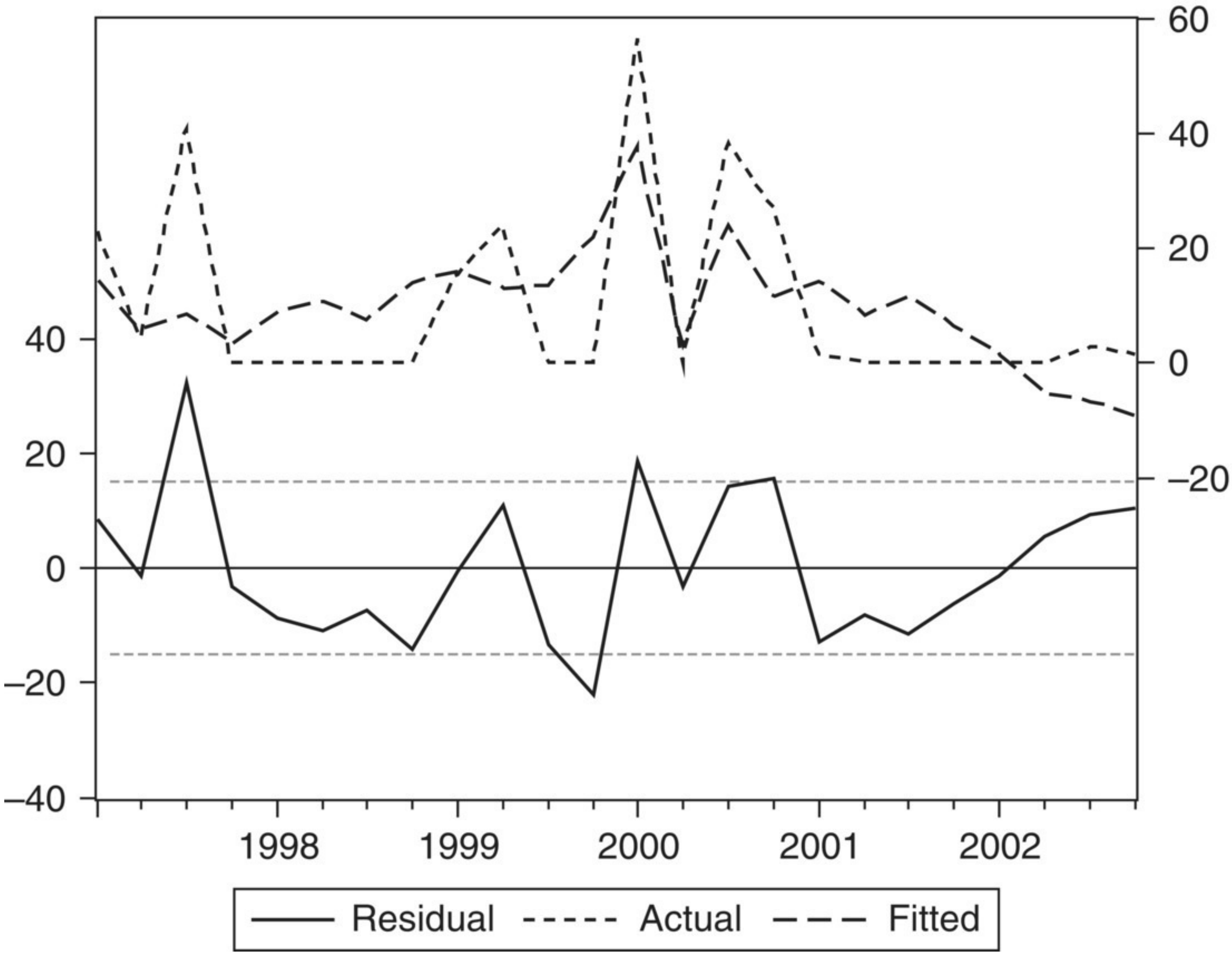


Figure 4.27 The actual quarterly investment outlays for machinery and equipment, its theoretical values and the residuals calculated on the basis of Equation 4.32.

4.11 Equations of wage effectiveness

Effectiveness of human-labor resources was examined in Chapter 4 using stochastic equations, which describe *labor efficiency*. Labor efficiency, analyzed earlier on, measured the ratio of the production volume to the resource meter of human-labor resources (the number of employees). Currently, the denominator in the formula used to calculate labor efficiency is going to be expressed by a variable of a streaming character, in the form of the enterprise's payroll size. Thus, we will consider the following equations of wage effectiveness, where the efficiency meter is in the formula's ratio

$$WP = \frac{\text{production}}{\text{labor}} \tag{4.33}$$

is going to successively contain each of the variables representing production.[50]

An empirical equation describing productive efficiency of the net payroll (ESAL), based on monthly time series, is as follows:

$$\begin{aligned} \mathrm{ESAL} = \underset{(3.121)}{6.461} + \underset{(1.677)}{0.143}\mathrm{ESAL}_{-2} - \underset{(3.081)}{0.158}\mathrm{ESAL}_{-4} - \underset{(2.194)}{0.125}\mathrm{ESAL}_{-7} + \\ - \underset{(2.729)}{0.184}\mathrm{ESAL}_{-10} + \underset{(2.585)}{0.216}\mathrm{ESAL}_{-11} + \underset{(4.392)}{0.297}\mathrm{ESAL}_{-12} - \underset{(2.266)}{5.423}\mathrm{APAY}_{-5} \\ + \underset{(1.606)}{3.820}\mathrm{APAY}_{-6} + \underset{(1.976)}{0.150}\mathrm{EMP}_{-2} - \underset{(2.342)}{0.165}\mathrm{EMP}_{-7} + u_{22}, \\ R_{22}^2 = 0.670,\ S22_u = 1.420\,\mathrm{PLN}/1\,\mathrm{PLN},\ \mathrm{DW}_{22} = 2.282. \end{aligned} \quad (4.34)$$

[51]

The set of explanatory variables in Equation 4.26 (outside the autoregressive dependencies) clearly differs from the set making up the equation describing a quarterly volatility mechanism of payroll's productive efficiency. Autoregressions of the 2nd, the 11th, and the 12th order show a sequential reference of the variable ESAL's value to its current level. On the other hand, autoregressions of the 4th, the 7th, and the 10th order indicate commutativity of the variable ESAL's volatility during the indicated time intervals.

The monthly time series revealed an impact of the average monthly pay (with a delay of 5 and 6 months) and of the technical devices (with a delay of 2 and 7 months). As such, an impact of conventional factors of human-labor effectiveness occurs and it is characterized by a variety of time-delays.

Large time fluctuations of the variable ESAL cause the explanatory variables of Equation 4.34 to explain only 67.0% of its overall volatility. All stochastic characteristics of the considered equations are correct. Also, Figure 4.28 does not arouse a feeling that the empirical equation was wrongly fit to the actual data.

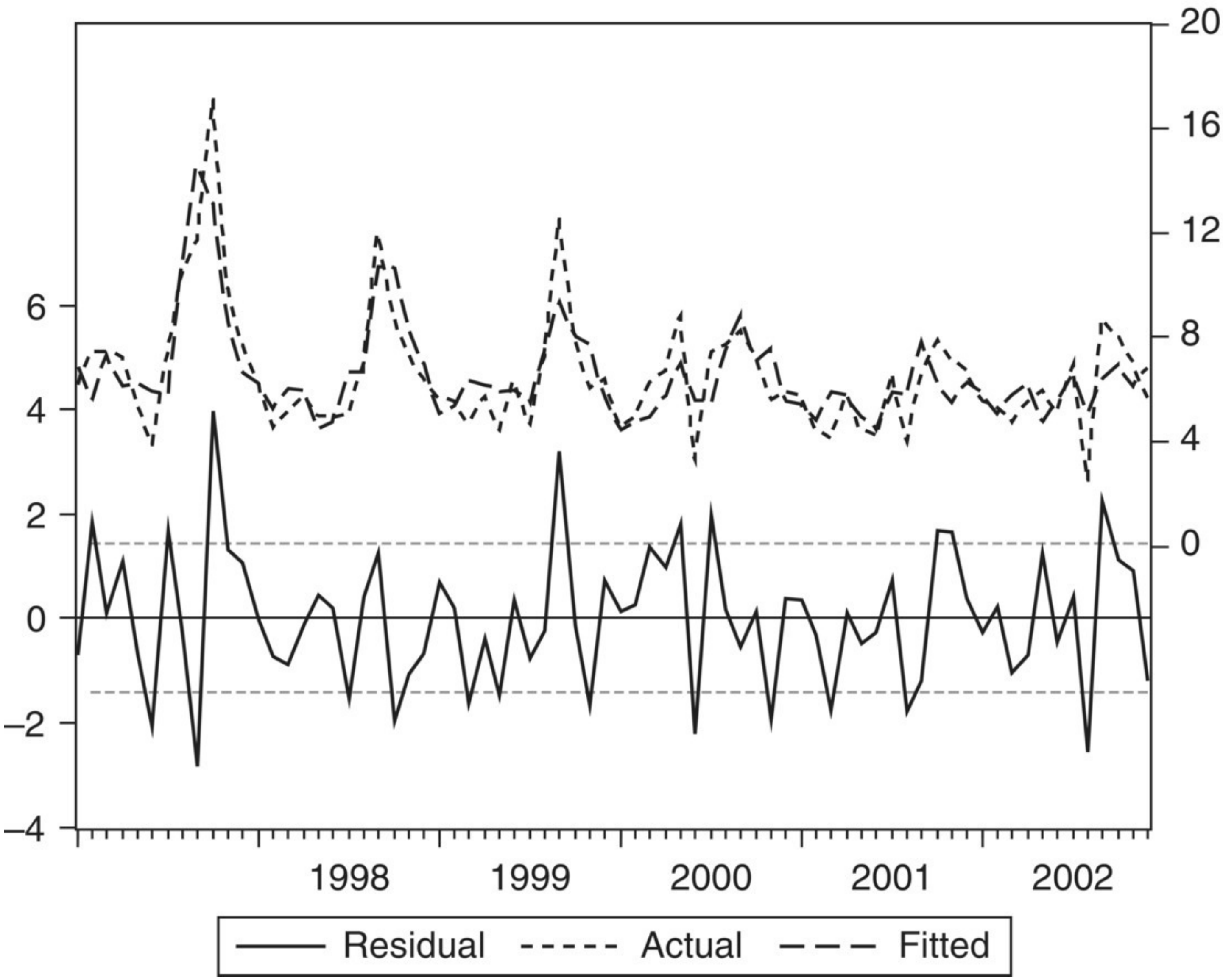


Figure 4.28 The actual monthly ESAL values, the theoretical values, and the residuals calculated on the basis of Equation 4.34.

The first empirical equation of the considered block of equations describes the variable SALPR, which expresses the relation of the ready-made production to the net payroll. An equation based on quarterly time series has the following form:

$$\begin{gathered} SALPR = \underset{(5.731)}{4.372} - \underset{(4.643)}{0.271} SALPR_{-2} + \underset{(8.946)}{0.518} SALPR_{-4} - \underset{(3.135)}{0.227} FIXD \\ + \underset{(3.956)}{0.240} FIXD_{-1} + u_{23}, \\ R_{23}^2 = 0.831, S_{23u} = 0.775 \text{ PLN / 1 PLN}, DW_{23} = 2.038. \end{gathered} \tag{4.35}$$

[52]

Figure 4.29 presented the actual quarterly EPL values, the theoretical values, and the residuals calculated on the basis of Equation 4.35. Economic efficiency of the net payroll is characterized by autoregressions of the second and the fourth order. This dependency, resultant from a delay by 2 quarters, signifies commutativity of the variable SALPR's fluctuations every

2 periods, that is, an efficiency decline occurs after its increase and an increase in efficiency measured in this way is a consequence of this decline.

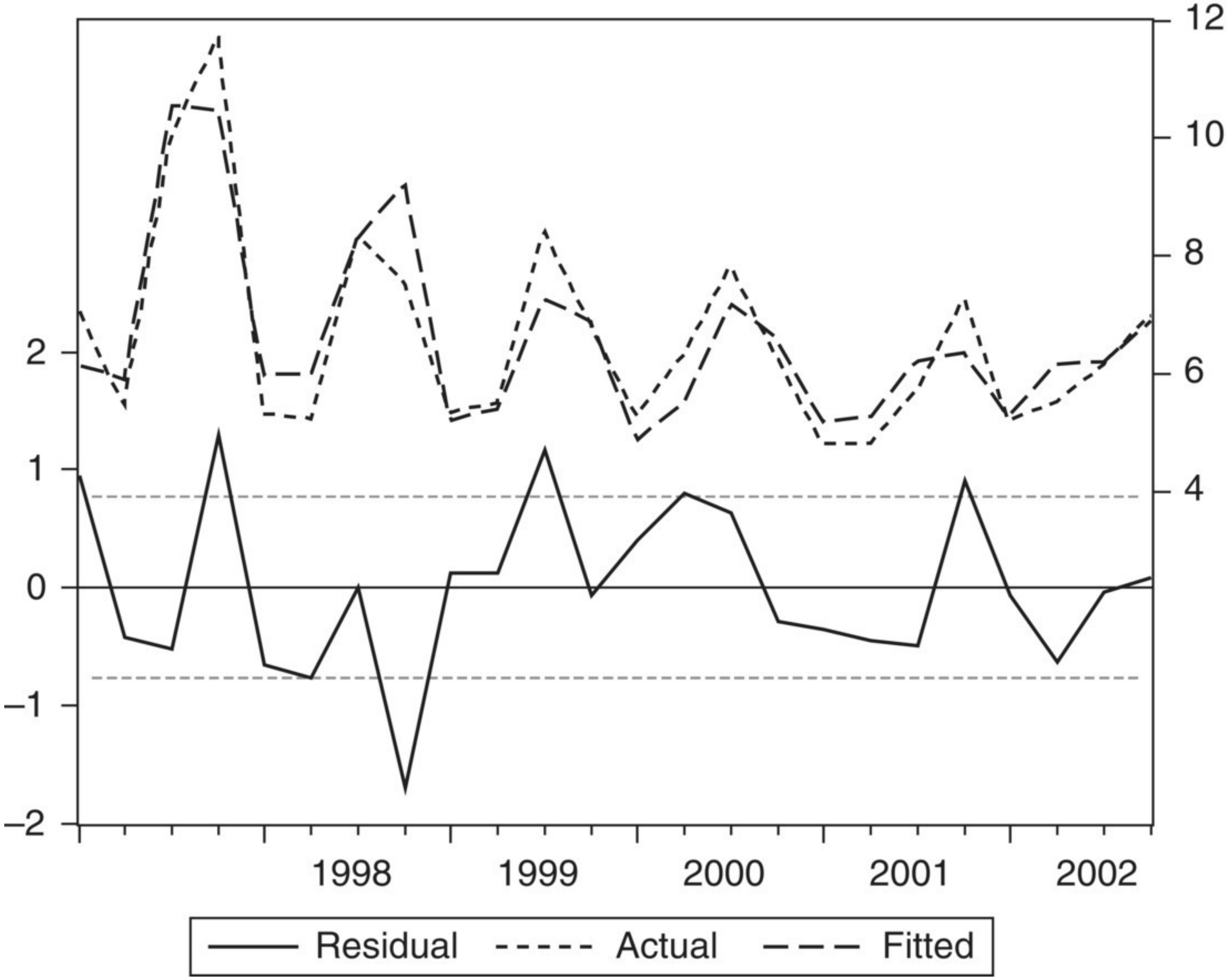


Figure 4.29 The actual quarterly EPL values, the theoretical values, and the residuals calculated on the basis of Equation 4.35.

Indicator of the noncurrent assets' renewal is a statistically significant factor of the wage expenses' efficiency, meaning both its current level as well as its size achieved in a previous quarter. Modernization of the assets[53] fosters a simultaneous increase in the variable SALPR's value, as evidenced by the parameter assessment by the variable FIXD, equal to −0.227. In contrast, improvement of the FIXD index in a previous quarter worsens the current wage efficiency, while deterioration of this characteristic fosters wage efficiency.

Equation 4.35 fairly accurately describes the mechanism of the quarterly net payroll's efficiency, since explanatory variables explain around 83.1% of the SALPR's volatility. Additionally, the theoretical values of the variable SALPR, calculated on the basis of Equation 4.35, differ from the empirical values of this variable, on average, by 77.5 cents (PL: grosz) of the ready-made production's value per 1 PLN of net wages.

Next equation is going to describe a formation mechanism of the variable ESBRSL, which measures the relation of the sales income (SBRUT) to the net payroll (SAL). An empirical equation describing the variable ESBSL on a monthly basis has the following form[54]:

$$\begin{aligned}
\mathrm{ESBSL} = \underset{(2.685)}{3.717} - \underset{(2.870)}{0.134}\mathrm{ESBL}_{-2} - \underset{(8.886)}{0.304}\mathrm{ESBSL}_{-6} + \underset{(10.120)}{0.354}\mathrm{ESBSL}_{-12} \\
+ \underset{(2.359)}{0.979}\mathrm{SALDE}_{-2} + \underset{(4.675)}{1.890}\mathrm{SALDE}_{-4} - \underset{(2.732)}{0.820}\mathrm{SALDE}_{-8} - \underset{(2.153)}{0.769}\mathrm{SALDE}_{-9} \\
+ \underset{(4.998)}{1.505}\mathrm{SALDE}_{-10} - \underset{(4.111)}{0.301}\mathrm{EMP} + \underset{(4.546)}{0.467}\mathrm{EMP}_{-4} - \underset{(2.472)}{0.228}\mathrm{EMP}_{-6} \\
+ \underset{(3.145)}{0.325}\mathrm{EMP}_{-7} - \underset{(2.006)}{0.181}\mathrm{EMP}_{-8} - \underset{(7.607)}{0.646}\mathrm{EMP}_{-12} - \underset{(3.010)}{5.604}\mathrm{APAY}_{-4} + \\
- \underset{(2.893)}{5.564}\mathrm{APAY}_{-5} + \underset{(3.311)}{6.347}\mathrm{APAY}_{-6} + \underset{(8.686)}{17.585}\mathrm{APAY}_{-12} + u_{24}, \\
R^2_{24} = 0.943, S_{24u} = 0.927\ \mathrm{PLN}/1\ \mathrm{PLN}, \mathrm{DW}_{24} = 2.415.
\end{aligned} \qquad (4.36)$$

The structure of Equation 4.36 is considerably more complicated compared to all previous similar constructions. Besides autoregressive dependencies of the 2nd, the 6th, and the 12th order, there are the so-called classic impacts, including an impact of the average net wage with various delays as well as an impact of the technical devices also taking into account various delay periods.

Appearance of a significant impact of the variable SALDE on the sale effectiveness of the net wages – compared to the existing empirical results – is a novelty. Monthly dynamics of the variable SALDE in the years 1996–2002 are illustrated by Figure 4.30. The discussed variable turns out to be statistically significant, with the delays of 2, 4, 8, 9, and 10 months. An increase in the value of the SALDE index more often causes an increase in the sale effectiveness of the wages, rather than its decline. Three parameter assessments for the variable SALDE delayed by 2, 4, and 10 months, in fact, are positive. It can, therefore, be concluded that better financing of new techniques results in a higher, on average, wage effectiveness.

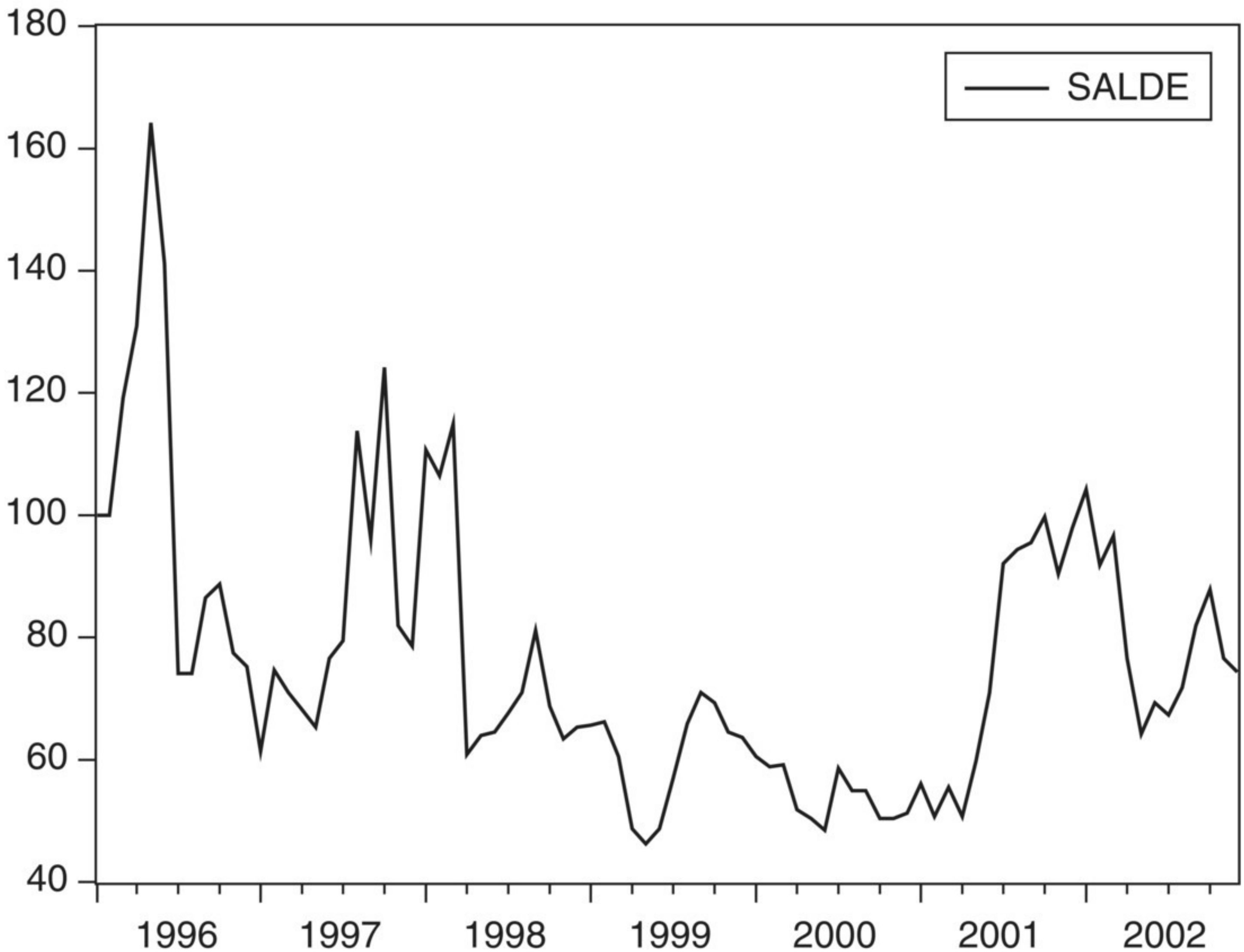


Figure 4.30 A single-base dynamics indexes of the variable SALDE in the years 1996–2002 (monthly, 1996, I = 100).

Empirical equation 4.36 highly accurately explains the mechanism of the variable ESBRSL's volatility, since as much as 94.3% of its volatility results from the impact of a vast set of explanatory variables. Figure 4.31 confirms that opinion.

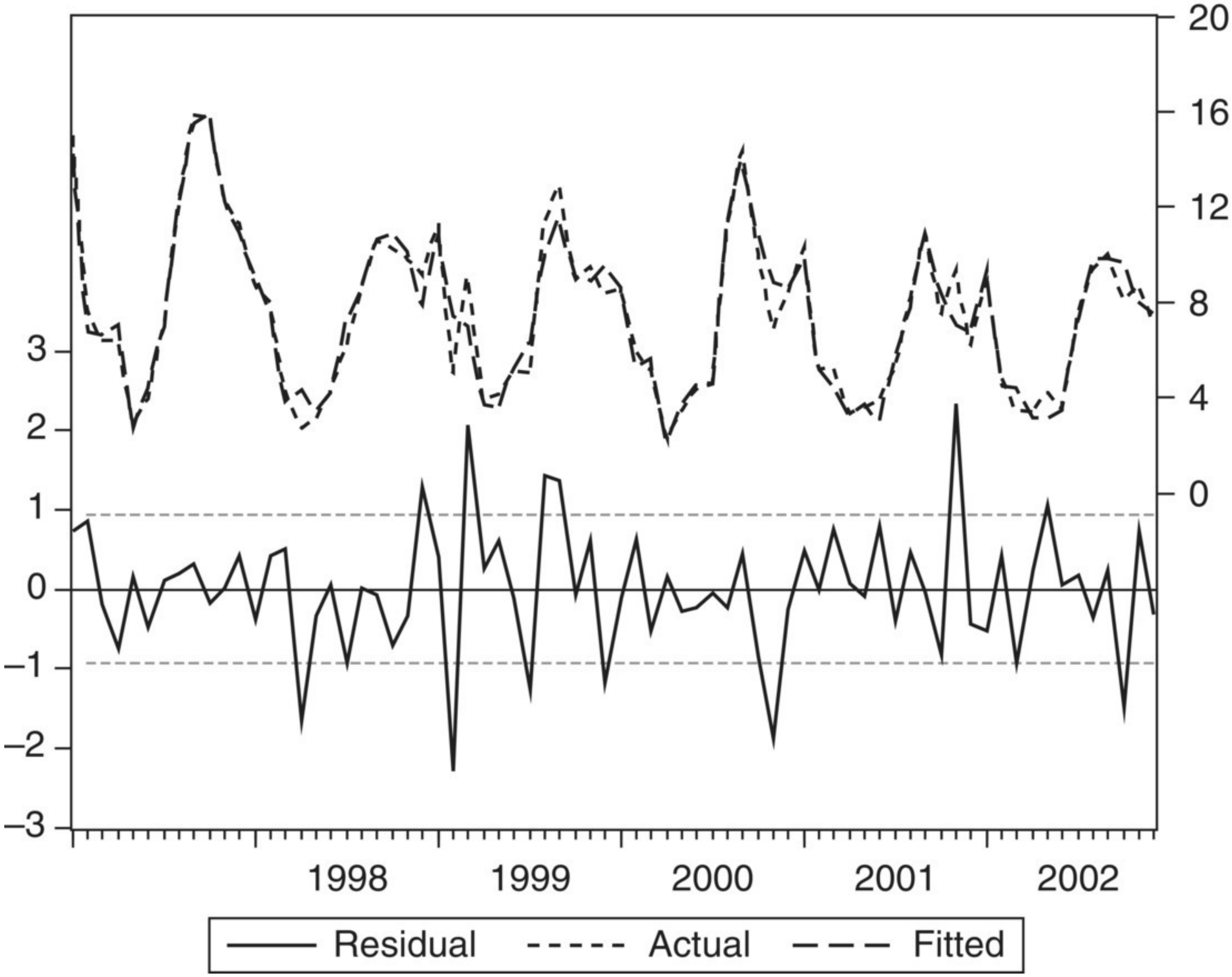


Figure 4.31 The actual monthly ESBSL values, the theoretical values, and the residuals calculated on the basis of Equation 4.36.

Next equations will describe a formation mechanism of the variable EFSAL, which measures the relation between the sales income and the net payroll. An empirical equation of the income efficiency of wages, based on quarterly data, has the following form:

$$\mathrm{EFSAL} = \underset{(1.451)}{2.055} + \underset{(10.176)}{0.631}\mathrm{EFSAL}_{-4} - \underset{(3.196)}{2.579}\mathrm{APAY}_{-1} + \underset{(2.960)}{1.691}\mathrm{APAY}_{-4} + \tag{4.37}$$

$$- \underset{(2.915)}{0.130}\mathrm{FIXD} + \underset{(3.024)}{0.129}\mathrm{RAN} + u_{25},$$

$$R_{25}^2 = 0.918, S_{25u} = 0.880\ \mathrm{PLN} / 1\mathrm{PLN}, \mathrm{DW}_{25} = 2.025.$$

Figure 4.32 illustrates the above equation.

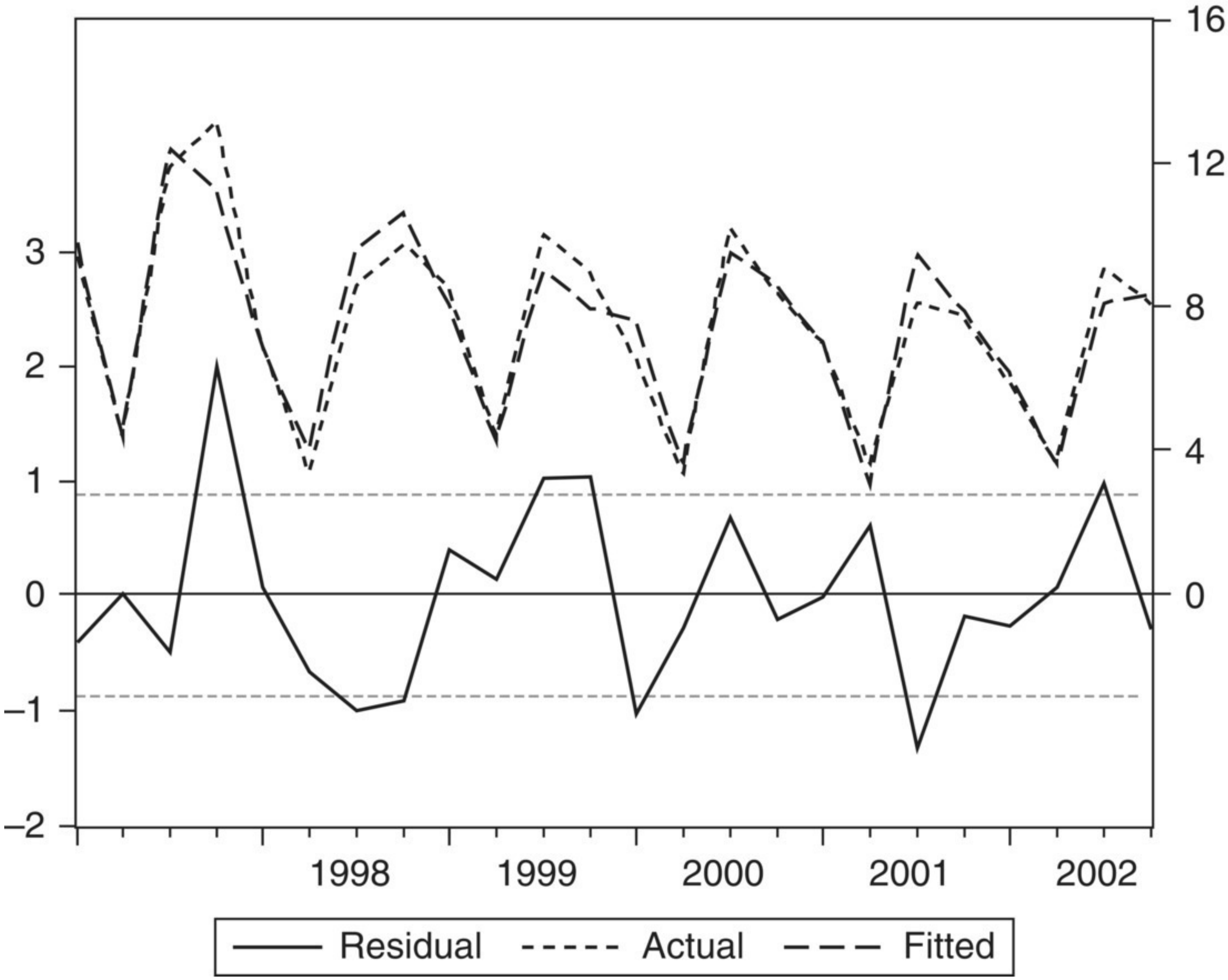


Figure 4.32 The actual quarterly values of EFSAL, the theoretical values, and the residuals calculated on the basis of Equation 4.37.

A fourth-order positive autoregression in Equation 4.37 indicates repetitiveness of the variable EFSAL's value every 4 quarters in a significant part of its volume. An increase in the sales income's value by 1 thousand PLN per 1 thousand PLN of net payroll, prior to 4 quarters, entails a current increase of this variable's value by 631 PLN of the sales per 1 thousand PLN of the net payroll.

Impact of the average net wages delayed by 1 and 4 quarters on the labor efficiency defined as such is significant. A negative sign of the assessment of the structural parameter by the variable $APAY_{-1}$ draws attention. Additionally, a simultaneous impact of the variable FIXD on the variable EFSAL can be considered as a classic situation. Modernization of the noncurrent asset resources fosters sales efficiency of the pay and salary costs.

The positive impact of the variable RAN[55] on the wage efficiency is interesting. It results from the fact that during the periods of increased sales of the assortments, the sales income increases significantly, at a less dynamic increase in the net payroll. The period of purchasing the stationery, related to the beginning of a school year, as well as purchasing of the calendars for the following year are characterized by a higher number of assortments, which appear on the issued invoices.[56] The result is a higher sales income per 1 thousand PLN of the payroll being paid out.

Next equation describes labor effectiveness as a quotient of the cash inflows (CASH) and the net payroll (SAL), expressed by the variable ECS. An empirical equation expressing formation rules of the variable ECS on a monthly basis has the following form:

$$\begin{aligned} ECS = &\underset{(1.677)}{3.004} + \underset{(2.094)}{0.163}ECS_{-3} - \underset{(2.219)}{0.159}ECS_{-4} - \underset{(2.963)}{0.240}ECS_{-6} + \\ &- \underset{(2.310)}{0.162}ECS_{-7} + \underset{(1.976)}{0.135}ECS_{-9} - \underset{(3.137)}{0.257}ECS_{-10} + \underset{(2.845)}{0.185}ECS_{-12} \\ &+ \underset{(1.790)}{3.468}APAY_{-5} + \underset{(5.305)}{11.978}APAY_{-12} + \underset{(3.866)}{0.342}TAL_{-7} - \underset{(2.131)}{0.215}TAL_{-10} + \\ &- \underset{(4.842)}{0.466}TAL_{-12} + \underset{(3.736)}{0.087}FIXD_{-1} + \underset{(3.352)}{0.082}FIXD_{-4} - \underset{(3.305)}{0.09}FIXD_{-6} \\ &+ \underset{(1.998)}{0.036}FIXD_{-7} - \underset{(2.645)}{0.041}FIXD_{-8} + \underset{(2.118)}{0.024}FIXD_{-10} + \underset{(3.920)}{0.041}FIXD_{-12} + \\ &- \underset{(2.408)}{0.107}TIME + u_{26}, \\ &R^2_{26} = 0.901, S_{26u} = 1.062\ PLN / 1\ PLN, DW_{26} = 2.185. \end{aligned} \tag{4.38}$$

The last empirical equation of wage effectiveness contains up to 20 explanatory variables. Autoregressive dependencies (of seven variables) and impacts of the variable FIXD with various delay periods (seven variables) are dominant. Again, a significant role of technical devices in the formation of an economic wage effectiveness (TAL_{-7}, TAL_{-10}, and TAL_{-12}) has revealed itself. The average net pay ($APAY_{-5}$, $APAY_{-12}$) is still important for effectiveness. Compared to previous empirical results, a negative linear trend, which occurred in the discussed meter of net wage effectiveness, is a new element. Confirmation of this trend can be seen in Figure 4.33.

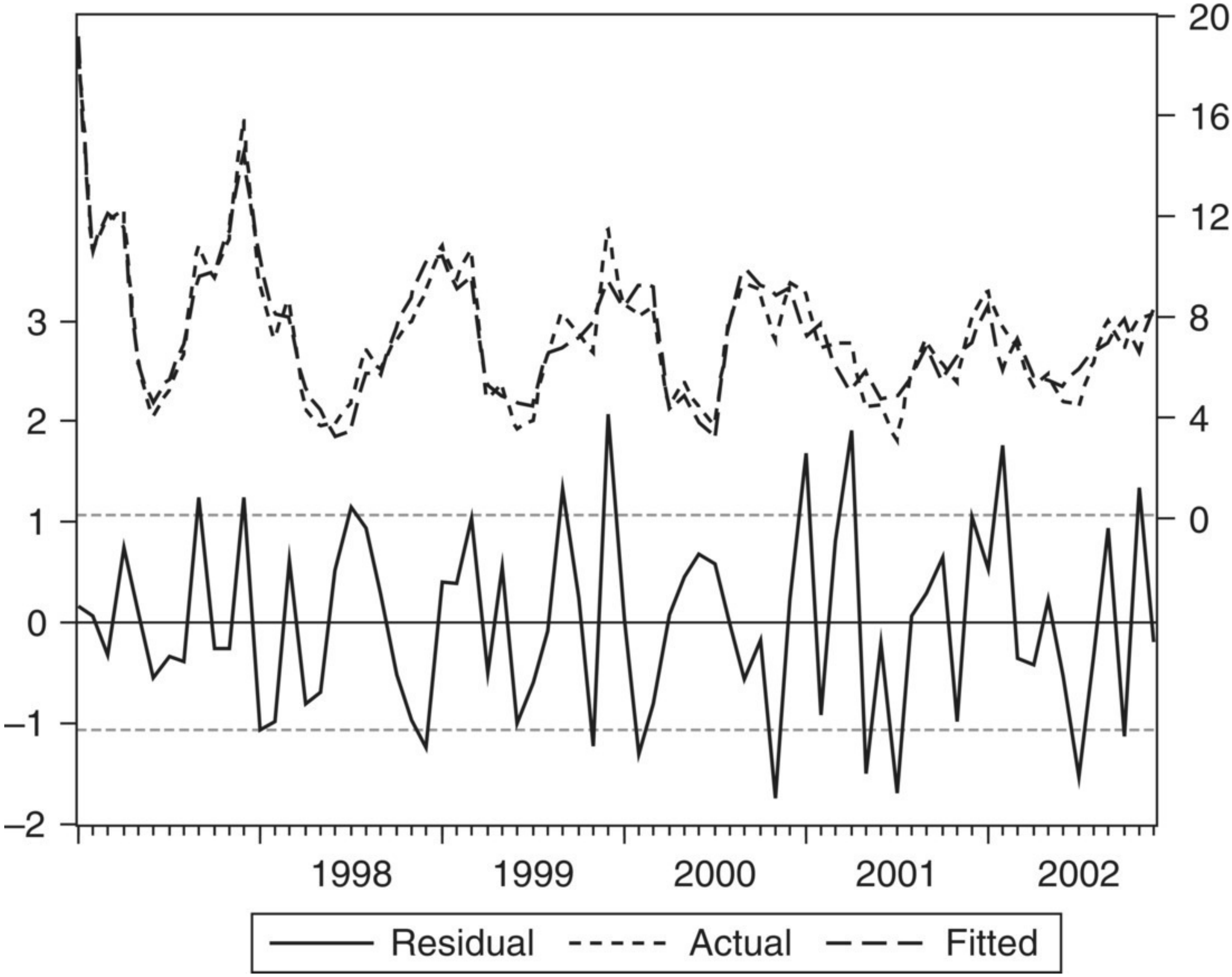


Figure 4.33 The actual monthly EPLW values, the theoretical values, and the residuals calculated on the basis of Equation 4.38.

Compared to the other empirical equations that were constructed based on monthly time series, the volatility of ECS in Equation 4.28 was explained by the equation's variables in 90.1%. The actual and the theoretical values of the variable ECS, in this case, are slightly different, which is shown in Figure 4.33.

Next two equations describe labor effectiveness as the ratio of the amounts of cash inflows and the net payroll, expressed using the variable ECS. An empirical equation based on quarterly data has the following form:

$$\begin{gathered}\mathrm{ECS} = \underset{(1.641)}{2.393} - \underset{(3.452)}{0.229}\mathrm{ECS}_{-2} + \underset{(5.279)}{0.320}\mathrm{ECS}_{-4} + \underset{(5.989)}{2.876}\mathrm{APAY}_{-4} + \\ - \underset{(6.351)}{0.525}\mathrm{TAM}_{-4} + \underset{(3.230)}{0.180}\mathrm{FIXD} + \underset{(3.538)}{0.113}\mathrm{FIXD}_{-4} + u_{27}, \\ R_{27}^2 = 0.937, S_{27u} = 0.668\ \mathrm{PLN}/1\mathrm{PLN}, \mathrm{DW}_{27} = 2.704.\end{gathered} \tag{4.39}$$

The nature of Equation 4.39, to some extent, is similar to Equation 4.37. We are dealing here with autoregressions of the second and fourth order as well as with a significant positive impact of the average net wage delayed by 4 quarters. The technical devices of machinery and equipment (TAM) delayed by 4 quarters turn out to be a brake for the variable ECS. This may result from a physical process of machinery aging, which results in its higher failure rate reflected through cash efficiency of the wage.

A significant impact of the variable FIXD on the variable ECS, both in the simultaneous and the delayed by 4 quarters values, does not require commenting. Figure 4.34 is an illustration of the mechanism described by Equation 4.39.

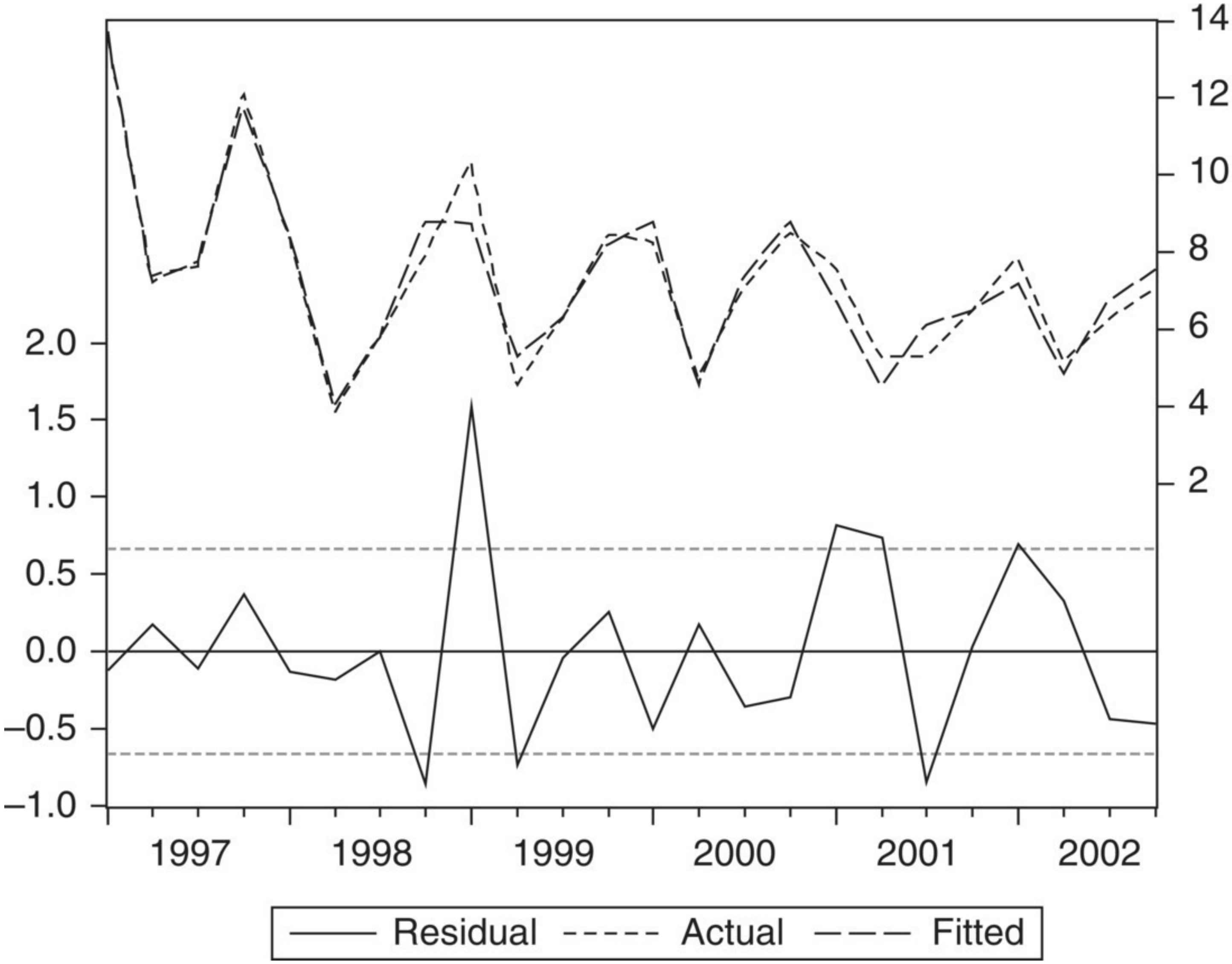


Figure 4.34 The actual quarterly EPLW values, the theoretical values, and the residuals calculated on the basis of Equation 4.39.

4.12 Equations of the efficiency of implementing the fixed assets

The last group of equations detached from the econometric model of a small-sized enterprise is going to characterize the size of the production effects generated by involving the tangible fixed assets in use. Two variables are going to be described:

EFMACH – the ratio of the ready-made production's value (PROD) and the initial value of active machinery and equipment[57] (MACH),

EFBFIX – as the ratio of the sales income to the initial value of the active fixed assets.[58]

All equations of effectiveness of the fixed assets are autoregressive-trended in nature.

The variable EFMACH is described by the following empirical equation obtained on the basis of monthly time series:

$$\begin{aligned} \mathrm{EFMACH} = & \underset{(3.366)}{-0.669} - \underset{(3.990)}{0.267}\mathrm{EFMACH}_{-4} - \underset{(1.986)}{0.132}\mathrm{EFMACH}_{-7} \\ & + \underset{(6.301)}{0.354}\mathrm{EFMACH}_{-12} + \underset{(6.207)}{0.0781}\mathrm{TIME} - \underset{(6.004)}{0.00165}\mathrm{TIME}^2 \\ & + \underset{(5.550)}{0.0000103}\mathrm{TIME}^3 + u_{28,} \\ & R_{28}^2 = 0.599, S_{28u} = 0.102\ \mathrm{PLN}/1\,\mathrm{PLN}, \mathrm{DW}_{28} = 1.840. \end{aligned} \quad (4.40)$$

An important analogy can be seen in a monthly mechanism of the variable EFMACH's volatility when compared to the equation for quarterly data. Description accuracy is slightly smaller than in case of quarterly time series. Only 59.9% of the volatility of the machinery and equipment's monthly productivity is explained by autoregressions of the 4th, the 7th, and the 12th order as well as by a trend in the form of a third-degree polynomial. This observation is confirmed by Figure 4.37. In Equation 4.40, there is no reason to reject the hypothesis about the lack of a first-order autocorrelation of the random component (Figure 4.35).[59]

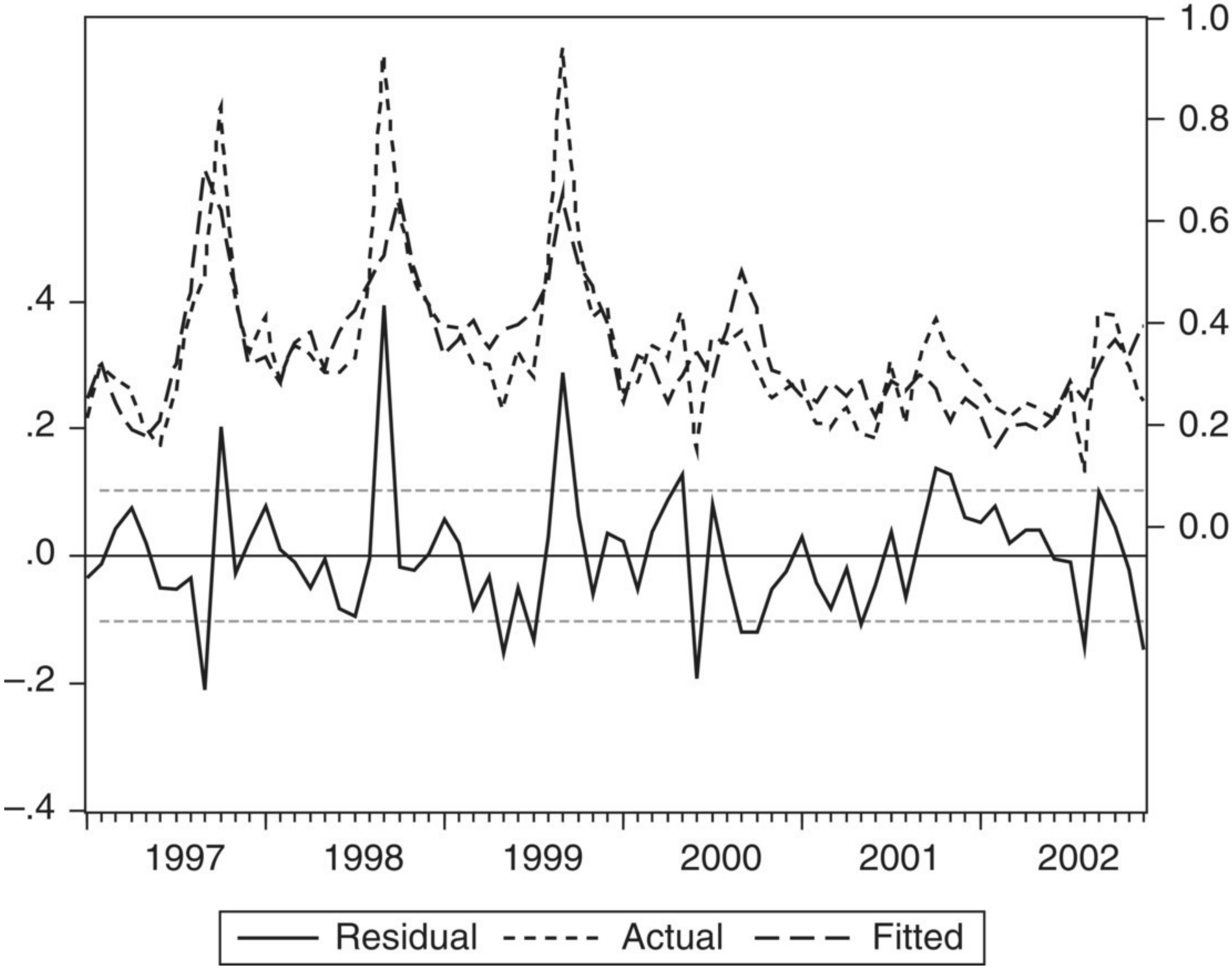


Figure 4.35 The actual monthly EFMACH volumes, the theoretical values, and the residuals calculated on the basis of Equation 4.40.

Autoregression of the 12th order indicates a sequential reference of EFMACH values with accuracy up to 12 months. This means that an increase in the ready-made production by a 1000 PLN per 1 thousand PLN of the machinery and equipment's value, prior to 12 months, results in a current increase in EFMACH's volume by 354 PLN of the ready-made production per 1000 PLN of the initial value of this group of the tangible fixed assets in use. Impacts of $EFMACH_{-4}$ and $EFMACH_{-7}$ indicate a change in the explanatory variable's sign every 4 and 7 months.

An analogical empirical equation constructed on the basis of monthly data has the following form:

$$\underset{}{\text{EFBFIX}} = \underset{(3.339)}{-0.462} + \underset{(3.937)}{0.285}\text{EFBFIX}_{-1} - \underset{(2.223)}{0.126}\text{EFBFIX}_{-4}$$
$$+ \underset{(2.219)}{0.147}\text{EFBFIX}_{-11} + \underset{(5.498)}{0.322}\text{EFBFIX}_{-12} + \underset{(4.129)}{0.0402}\text{TIME} +$$
$$- \underset{(3.939)}{0.00839}\text{TIME}^2 + \underset{(3.701)}{0.0000053}\text{TIME}^3 + u_{29},$$
$$R^2_{29} = 0.782,\ S_{29u} = 0.075\ \text{PLN}/1\ \text{PLN},\ \text{DW}_{29} = 2.250. \quad (4.41)$$

Equation 4.41 relatively accurately describes the variable EFBFIX's monthly volatility, since 78.2% of its volatility is explained by autoregressive relations of the 1st, the 4th, the 11th, and the 12th order as well as by a polynomial trend of the 3rd order. Equation's residuals do not exhibit an autoregressive process of the first order.[60] Figure 4.36 confirms a relatively high explanation precision of the EFBFIX's mechanism. Sequential linkage of the productivity levels of the noncurrent assets occurs every 1, 11, and 12 months. A sign change, as a result of an autoregressive relation, occurs every 4 months. Clearly, it can be seen that the volatility of EFBFIX is oscillatory in character and, in the end, depends on the assortment structure of invoiced deliveries as well as on the demand that is changing seasonally.

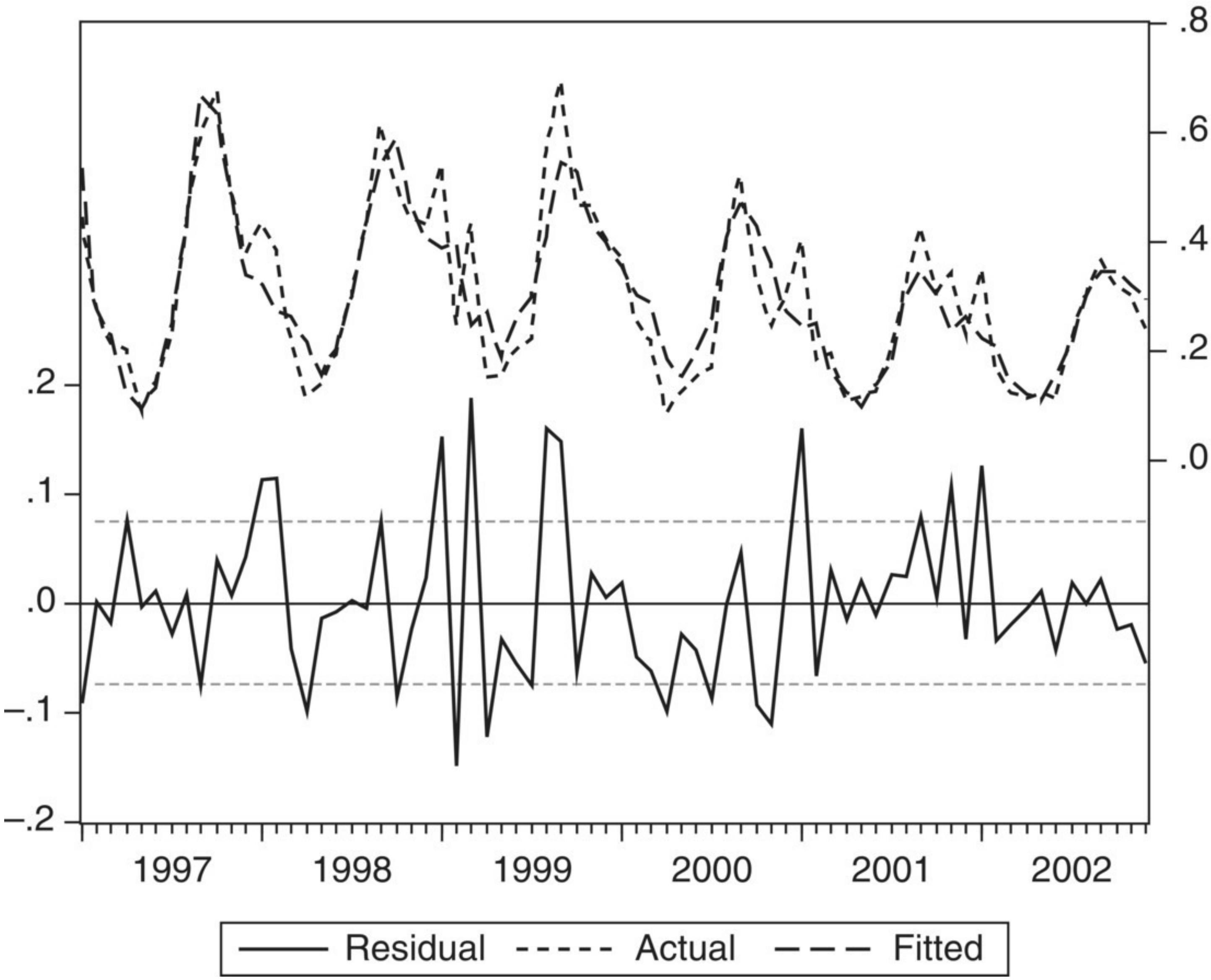


Figure 4.36 The actual monthly EFBFIX volumes, the theoretical values, and the residuals calculated on the basis of Equation 4.41.

Next empirical equation based on quarterly data has the following from:

$$\text{EFBFIX} = \underset{(2.418)}{-0.671} - \underset{(3.778)}{0.378}\text{EFBFIX}_{-2} + \underset{(3.602)}{0.273}\text{EFBFIX}_{-4} + \underset{(5.171)}{0.645}\text{TIME} - \underset{(5.121)}{0.0408}\text{TIME}^2 + \underset{(4.794)}{0.000757}\text{TIME}^3 + u_{30}, \tag{4.42}$$

$$R^2_{30} = 0.795,\ S_{30u} = 0.177\ \text{PLN} / 1\ \text{PLN},\ \text{DW}_{30} = 2.272.$$

Equation 4.42 describes formation of the efficiency of the machinery and equipment's use with moderate accuracy. Autoregressions of the second and the fourth orders as well as a trend in the form of a polynomial of the third order explain 79.5% of the total volatility of the variable EFBFIX. Equation's residuals are a realization of the pure random component, since in the equation there is no first-order autocorrelation[61] of the random component.

It is illustrated by Figure 4.37.

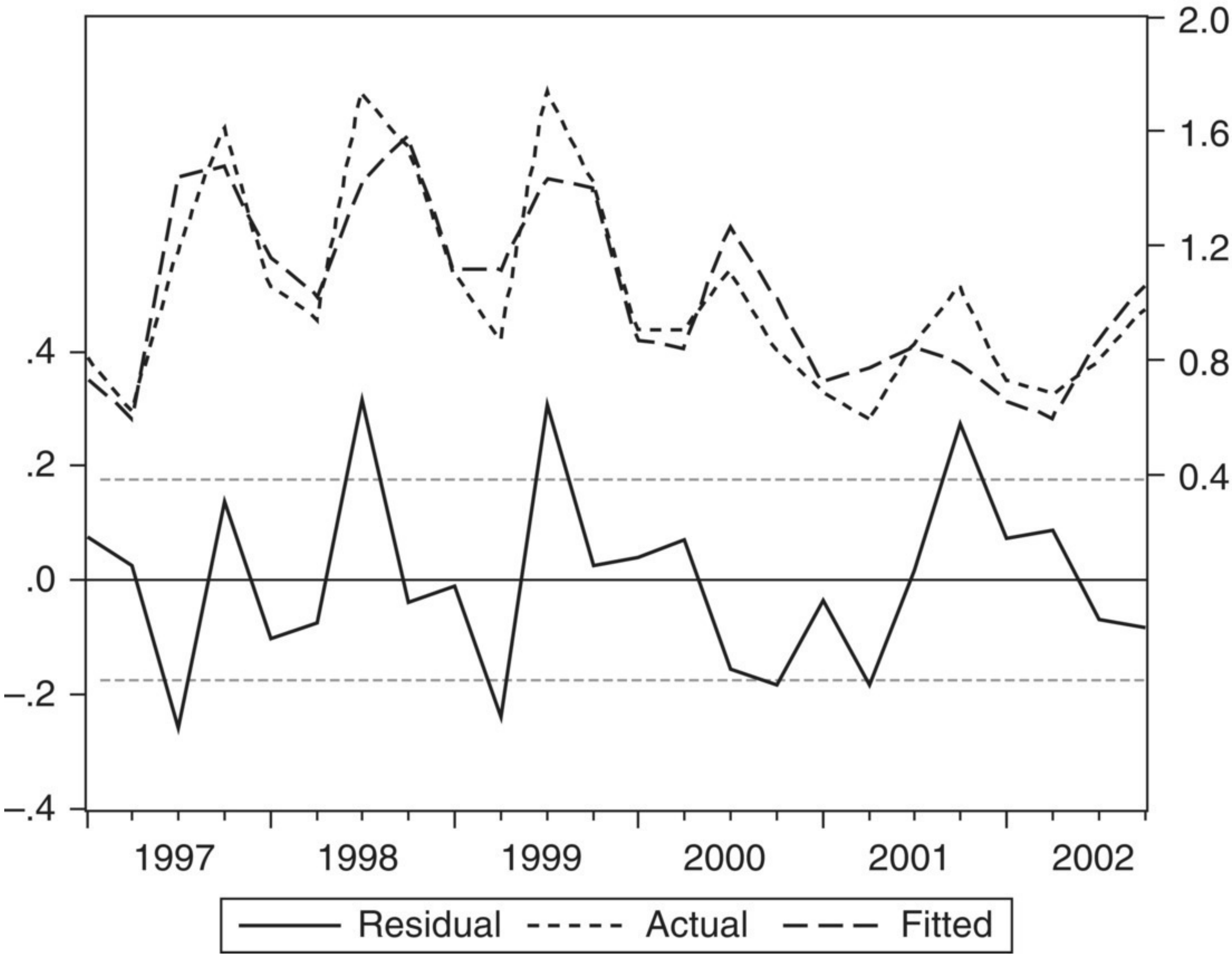


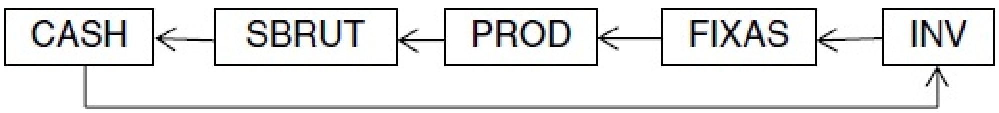
Figure 4.37 The actual quarterly values of EFMACH, the theoretical values, and the residuals calculated on the basis of Equation 4.42.

Equation 4.42 indicates that the fourth-order autoregression in the variable EFBFIX means that an increase in the ready-made production by 1 thousand PLN per 1 thousand of the initial value of machinery and equipment, prior to 4 quarters, entails an increase in the current level of EFBFIX by 273 PLN of production per 1000 PLN of the variable MACH. Autoregression of the second order indicates commutativity of the variable EFBFIX's volatility every 2 quarters. An increase in this variable is followed by a decline in its value after 2 quarters, which in turn causes another increase – after two periods.

4.13 Practical applicability of a small-sized enterprise's model

An econometric model of a small-seized enterprise mainly can be used for forecasts estimation of endogenous variables. It can also serve as an analysis tool allowing assessment of the effects of various possible decisions.[62] For instance, company's marketing strengths influence its production results. Those marketing strengths can be represented by various exogenous variables, characterizing the company's marketing activity. Examples of such variables can be as follows: a variable representing advertisement expenses, a variables characterizing participation in fairs, and many others. This group of variables belongs to a category of decisional variables, called the steering variables. If the model contains statistically significant steering variables, then analysis can be performed using the model – of the effects of various values of particular decisional variables on the entire economic system. It will allow selection of most rational level variants of the steering variables, in terms of the decision maker's needs (the manager's needs).

Let us look at a hypothetical closed cycle of relations of type 4.3. An attempt to present a forecasting mechanism based on quarterly data will be made



(4.43)

From the empirical equation 4.30, it can be inferred that empirical simultaneous impact of the cash inflows on investment value does not exist. As a result, prediction from the sequence of relations 4.3 is going to have the nature of a chain sequence and is going to occur according to the following chain of relations:

$$\text{CASH} \leftarrow \text{SBRUT} \leftarrow \text{PROD} \leftarrow \text{FIXAS} \leftarrow \text{INV}$$

As such, it is going to be necessary to estimate the forecast INV_{Tp} in the first instance, using a predictor based on the empirical equation 4.30, that is:

$$\text{INV}_{Tp} = -10.058 - 0.237\text{INV}_{T-1} + 0.084\text{CASH}_{T-1} + 0.063\text{CASH}_{T-4} + 20.518S3 - 1.851T. \quad (4.44)$$

Quarterly forecasts of investment volumes, calculated using GRETL package, are presented in Table 4.1 and on Figure 4.38.

Table 4.1 Quarterly forecasts of investments for the year 2003 (in thousands PLN).

Forecasted period	**Forecast INV_{Tp}**	**Average prediction error**	**Forecast range (95% trust level)**
2003:1	3.910	12.4795	−22.308 ÷ 30.129
2003:2	−18.017	12.4795	−44.236 ÷ 8.201
2003:3	12.881	12.4795	−13.337 ÷ 39.100
2003:4	−16.410	12.4795	−42.628 ÷ 9.809

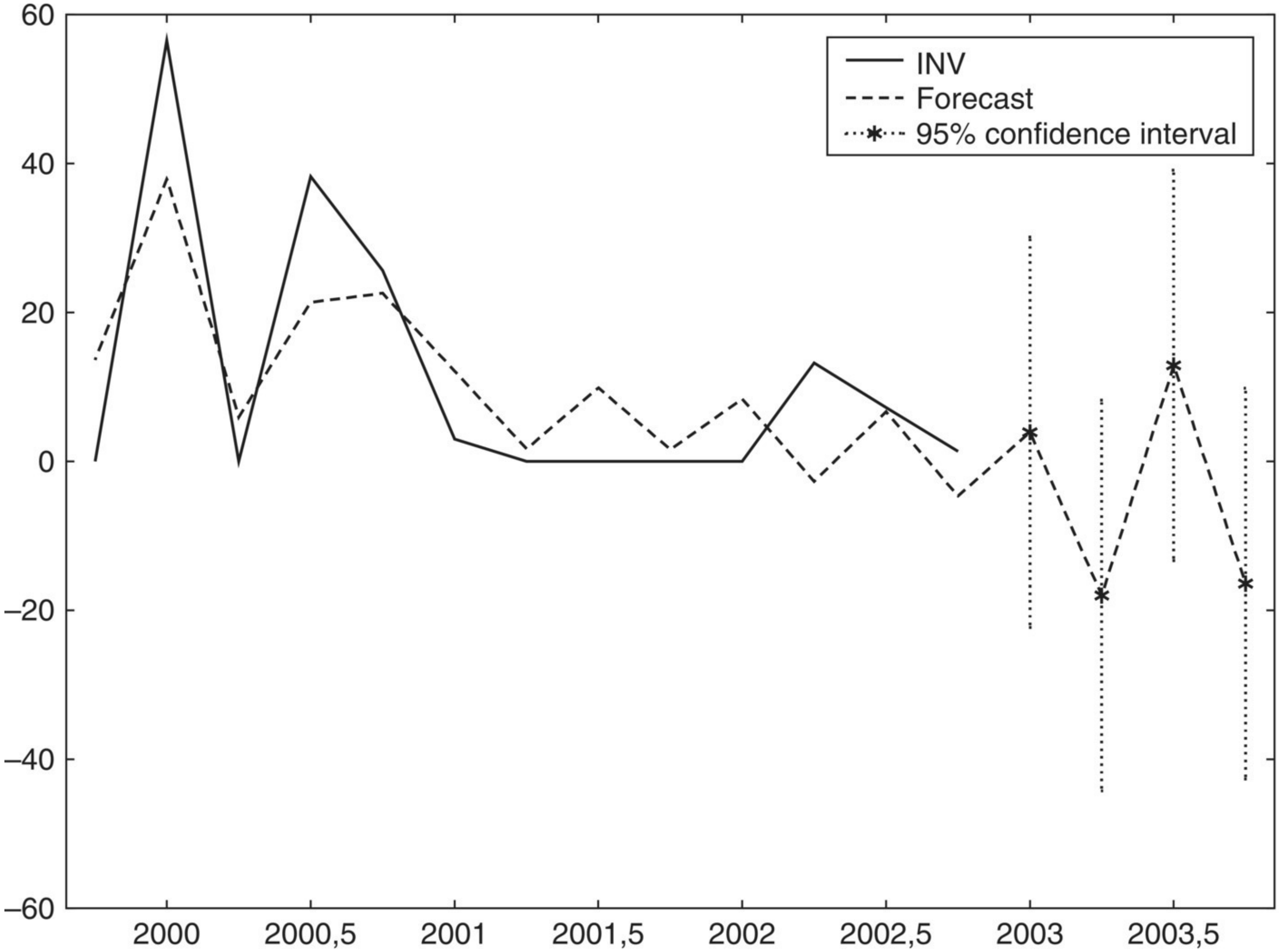


Figure 4.38 Quarterly forecasts of investments for the year 2003 (in thousands PLN).

Negative values of the forecasts of INV_{Tp} in the second and the fourth quarter of 2003 are rarely noticed. These values ought to be regarded as zero values. With the automatism contained within the predictor 4.44 illogical negative investment results appear. Thus it can be inferred that investments in the enterprise can occur in the first quarter at a level of close to 4 thousands PLN and in the third quarter – of a value of close to 13 thousands PLN.

It should be remembered that subjecting investments to a regular mechanism is difficult in a small-sized enterprise; for instance, purchases of machinery in connection with the so-called opportunity, where adequate cash resources must be gathered as a result of the variable CASH's mechanism. Variable INV in a small company is an important decisional variable (steering variable), which stays in relation with the cash inflows. It is highlighted by the mechanism described in Equation 4.30, in which quarterly cash inflows delayed by 1 and 4 quarters are the investment stimulators.

Another variable forecasted for quarterly FIXAS data from the chain 4.43, described by Equation 4.27, is empirically explained by a first-order autoregression and by impact of the delayed variable $CASH_{-4}$. In the empirical equation 4.27, the variable INV occurring in the hypothetical chain 4.43 was eliminated. Statistically insignificant delayed variables INV_{-1}, INV_{-2}, INV_{-3}, and INV_{-4} were also deleted. As a result autonomous predictor, which allows estimation of forecasts $FIXAS_{Tp}$, emerged:

$$FIXAS_{Tp} = 24.853 + 0.919FIXAS_{T-1} + 0.0645CASH_{T-4}. \tag{4.45}$$

Forecasts of the value of the fixed assets $FIXAS_{Tp}$ during subsequent quarters of 2003, estimated using the predictor 4.45, are presented in Table 4.2 and in Figure 4.39.

Table 4.2 The company's quarterly forecasts of the values of the fixed assets, for the year 2003 (in thousands PLN).

Forecasted period	**Forecasts of $FIXAS_{Tp}$**	**Average prediction errors**	**Forecast range (95% trust level)**
2003:1	541.036	13.7160	512.512–69.560
2003:2	538.288	18.6254	499.554–77.022
2003:3	540.582	21.9297	494.976–86.187
2003:4	547.262	24.3724	496.577–597.947

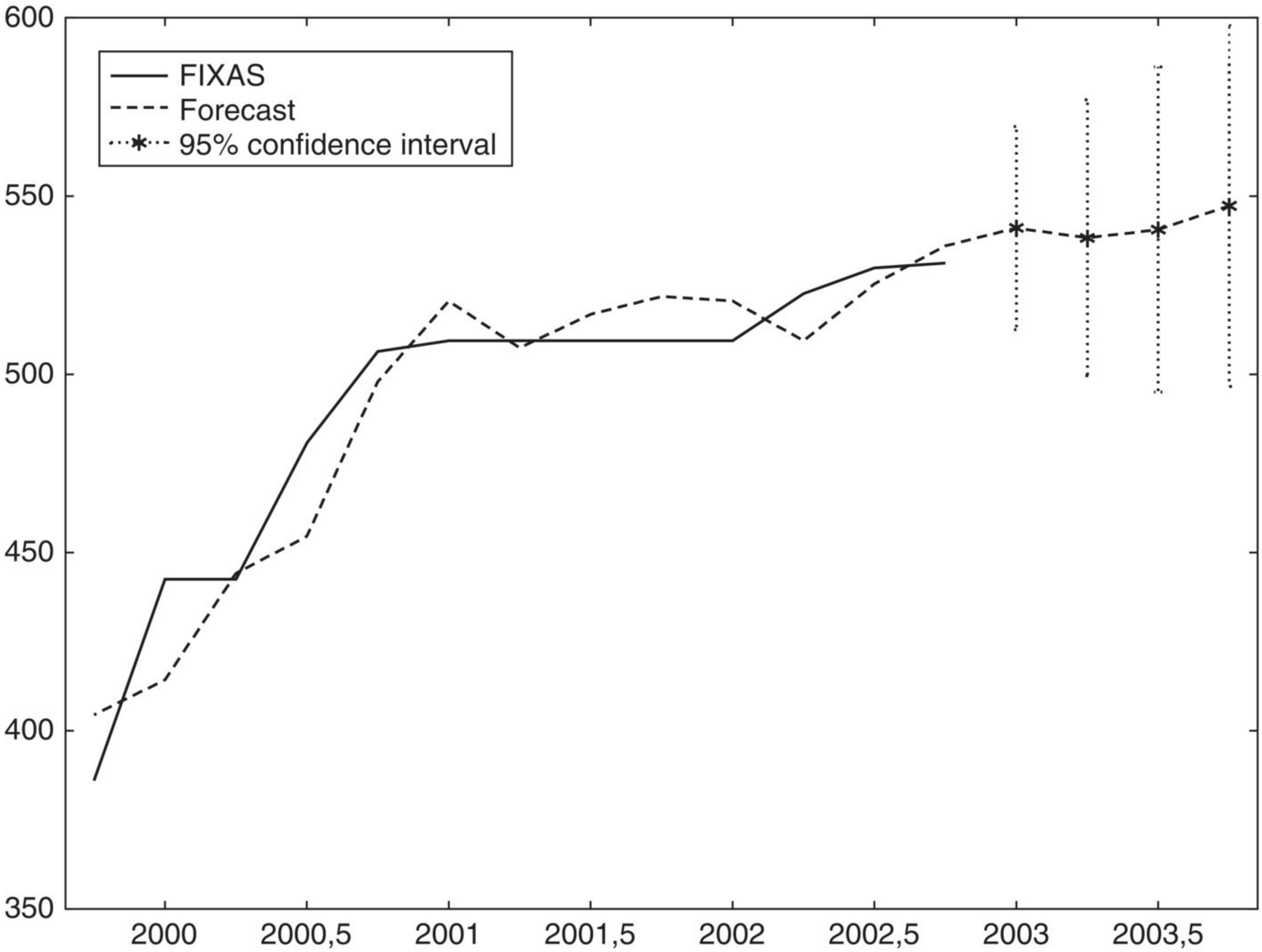


Figure 4.39 The company's quarterly forecasts of the values of the fixed assets in use, for the year 2003 (in thousands PLN).

These forecasts indicate that in the subsequent quarters of 2003 oscillations around the value of about 540 thousands PLN can be expected. Most likely, this will result from small investment outlays. These forecasts are characterized by high precision. Relative prediction errors are at the level from $V^*_{F2003.1} = 2.54\%$ to $V^*_{F2003.4} = 4.45\%$.

Having the forecasts INV_{Tp} and $FIXAS_{Tp}$ allows forecast estimation of the other variables from the chain 4.43, that is, $PROD_{Tp}$, $SBRUT_{Tp}$, and $CASH_{Tp}$. The predictors used for estimation of these forecasts will emerge from the empirical equations 4.17, 4.15, and 4.13. Attention should be paid to the fact that it is necessary to have a significant number of explanatory variables in the predictor $PROD_{Tp}$. Forecast estimation performed using the GRETL package requires much concentration and attention.

Notes

1 The empirical model of a small-sized enterprise presented here comes from the work of

Wiśniewski J.W. (2003), An *Econometric Model of a Small-Sized Enterprise*, Chapter 2. Description and interpretation of the results also come from this work. Each of the total interdependent variables is empirically verified in a given equation of the model, using quarterly and then monthly data.

2 A company's marketing potential is multidimensional and it is composed out of many simple characteristics. See, that is, the works: Grabowski L., K-osiewicz U. (1999), *Marketing Management of a Small-Sized and Medium-Sized Enterprise*, Delfin, Warsaw; Johnston J., Chambers S., Harland Ch., Harrison A., Slack N. (2002), *Operations Management. Case Analysis*, PWN, Warsaw; Lange O. (1967), *Introduction to Econometrics*, PWN, Warsaw (4th ed.); Pawłowski Z. (1976), *Econometric Analysis of the Production Process*, PWN, Warsaw.

3 As opposed to individual labor efficiency, where a particular employee performing his/her duties with a certain efficiency undergoes observation. Speaking of team efficiency, let us consider the so-called statistical employee.

4 The autonomous process of a wage increase mainly results from the country's legal regulations (e.g., the minimum wage increase) and from the overall welfare improvement resultant from an economic growth.

5 The situation here is similar to that of a large-sized enterprise (as an outcome of complementarity and substitution).

6 Before 1996, the company had provided educational services, expert services in economics and in publishing, which constituted a major part of the company's income.

7 Earlier on, the company conducted its business activity in the facilities rented in Torun; that did not foster its development. The company's headquarters had been changed multiple times, due to excessive "appetites" of the real-estate owners, who quite regularly tried to raise the rent, rightly presuming, that, mostly due to marketing reasons, it is difficult for a stable business entity to change its headquarters.

8 All the variables of the econometric model will be presented in two variants: in the form of quarterly data ($n = 28$ observations) or in the form of monthly data ($n = 84$ observations).

9 The average amount of employees (quarterly or monthly).

10 After deduction of VAT.

11 The variables EPL, EPLP, and EPLW are team-labor efficiency's meters, representing a stream variant of the human-labor resources in the denominator of the formula defining labor efficiency.

12 MACH is an alternative for the variable FIXAS, constituting a part of FIXAS less the value of the transportation means.

13 A closed cycle of relations between total interdependent variables, in literature, can be called an indirect feedback.

14 The endogenous variables without time-delays make up a set of total interdependent variables of a multiple-equation econometric model. Random interference should be taken into account.

15 This term was introduced by Z. Zielin&c.acute;ski. See the work: Wiśniewski J.W., Zieliński Z. (2004): *Elements of Econometrics*, UMK, Toruń (5th ed.). The attempt to apply traditional econometric modeling to the statistical material collected, basically, was unsuccessful. Effective solutions emerged after transition to methodology of the dynamic consistency models. This type of a model will be presented further in the book.

16 As exhibitors in an adequately prepared stand.

17 This variable provides information about the level of noncurrent assets' consumption. A high value of the variable FIXD indicates a large value of the noncurrent assets per 1 PLN of the depreciation costs deducted only at a certain time. A decrease in the volume of FIXD means that the tangible fixed assets in use that had been consumed were eliminated and substituted by new ones, allowing depreciation deductions.

18 The stochastic equations were formulated as general functions, which in empirical studies are treated as linear ones. The variables' indices ($-1, \ldots, -m$) represent adequate time-delays, for example, by 1 month (quarter), until a delay by the m number of months (quarters). The symbol η_g ($g = 1, \ldots, G$) represents the random component of a given gth equation, while G represents the number of the model's stochastic equations.

19 Consequently, there is a decrease in the number of the degrees of freedom, as a result of which less predetermined variables can be inserted into the reduced-form's equations.

20 See: the procedures of eliminating the statistically insignificant explanatory variables from the stochastic equations during verification of an econometric model. For example, See the work: Wiśniewski J.W. (2002): *Businessman's Decision-Making Instruments. Applied Econometrics*, IW *GRAVIS*, Toruń (a revised and supplemented 4th ed.), Chapter 6.

21 Empirical values of the *t*-Student statistics are provided in each equation under the structural parameters' assessments.

22 The calculated value of the statistic $DW^* = 4 - DW = 1.815$ exceeds the so-called upper critical value $d_u = 1.62$, at a significance level $\gamma = 0.01$.

23 Difficulties with execution of the receivables are quite common in Poland. This mechanism confirms significant payment delays, even up to 11 months.

24 As evidenced by impact of the variable SBRUT.

25 Empirical statistics of a *t*-Student test were placed under the assessments of the structural

parameters. The symbol u_1 represents the equation's residuals, which are estimations of the random components, obtained using the 2SL. *M*-indices by the explanatory variables indicate the length of the delay (expressed in months or in qs).

26 Further in this work, the following symbols will be used: R^2_g – the coefficient of determination of the *g*th equation ($g = 1, \ldots, G$), being a multiple-correlation coefficient squared; S_{gu} – the random component's standard deviation assessment in the *g*th equation; DW_g – an empirical value of the Durbin–Watson statistic in the *g*th equation, being a double check of the Durbin–Watson test on the random component's first-order autocorrelation.

27 The line marked as *Actual* represents the empirical cash inflow amounts, *Fitted* – the theoretical values, and *Residual* – the residuals. Further in this work such markings will be analogous.

28 At $\gamma = 0.01$ a critical upper value $d_u = 1.55$, which signifies a lack of the first-order autocorrelation of the random component, because $DW_2 > d_u$.

29 The other explanatory variables are statistically significant at the significance level $\gamma < 0.01$.

30 After issuing a corrective VAT invoice that decreases the current sales income, past a prolonged detention of the goods by a wholesaler.

31 The calculated statistic $DW_5 = 1.862 > d_u = 1.62$ signifies a lack of the first-order autocorrelation of the random component at significance level $\gamma = 0.01$.

32 The abscissa axis of Figure 4.10 contains English abbreviations of the months, while the ordinate contains fixed-base dynamics indexes expressed in percentage.

33 An increase in ready-made production by a 1000 PLN results in a current increase in the ready-made production's value, on average, by 422 PLN.

34 An increase in production by a 1000 PLN, prior to 4 months, results in its current decrease by around 175 PLN; while a decrease in the value of ready-made production, prior to 4 months, by around 1000 PLN, causes its current increase by 175 PLN.

35 The calculated statistic $DW_6{}^* = 4–DW_6 = 1.837 > d_u = 1.66$ indicates a lack of the first-order autocorrelation of the random component at a significance level of $\gamma = 0.01$.

36 New noncurrent assets are subject to depreciation deductions, which lower the level of FIXD.

37 The calculated statistic $DW_7{}^* = 1.976 > d_u = 1.62$ signifies a lack of the first-order autocorrelation of the random component at a significance level $\gamma = 0.01$. See footnote 16 in this chapter.

38 APAY is a variable statistically significant in Equation 4.15 at $\gamma < 0.001$.

39 The DW statistic calculated as $DW_8^* = 1.977 > d_u = 1.22$ indicates a lack of the first-order autocorrelation of the random component at a significance level $\gamma = 0.01$.

40 The calculated statistic $DW_5^* = 1.768 > d_u = 1.61$ signifies a lack of the first-order autocorrelation of the random component at a significance level $\gamma = 0.01$.

41 It can be seen that along with a decrease in economic effects in the enterprise, the average net pay increases systematically, provided that existence of periodic fluctuations in the monthly as well as in the quarterly time series is allowed.

42 The calculated statistic $DW_{10}^* = 1.758 > d_u = 1.30$ indicates a lack of the first-order autocorrelation of the random component at a significance level of $\gamma = 0.01$.

43 Comparison of the statistic $DW_{11}* = 1.765$ with the critical value $d_u = 1.56$ (at $\gamma = 0.01$) allows inference, that in Equation 4.22 here is no first-order autocorrelation of the random component.

44 The Durbin–Watson statistic $DW_{12}* = 1.879 > d_u = 1.41$. Thus, there are no grounds for rejection of the null hypothesis about a lack of the first-order autocorrelation of the random component at a significance level $\gamma = 0.01$.

45 The subsequent equations of the small-sized enterprise's empirical model, which are presented in subsequent subchapters of this book, form a block of the so-called detached equations.

46 An upper critical value at $\gamma = 0.01$ is $d_u = 1.62$. Thus, inequality $DW_{13} = 1.941 > d_u = 1.62$ occurs. This means that there are no grounds for rejection of the null hypothesis about a lack of the first-order autocorrelation of the random component.

47 The variable FIXAS contains the value of machinery and equipment as well as the value of the buildings, constructions, and the transportation means.

48 As an exception, we present here the volatility mechanism of machinery and equipment on a quarterly basis as well. It results from a relatively small frequency of the company's investment purchases. Therefore, a quarterly description of the volatility of the fixed assets or its groups, from a perspective of the needs of small business management, is important.

49 A critical upper value $d_u = 1.23$ at $\gamma = 0.01$.

50 These variables are CASH, SBRUT, and PROD.

51 $DW_{14}* = 1.718 > d_u = 1.61$ signifies a lack of the first-order autocorrelation of the random component at $\gamma = 0.01$.

52 There is no first-order autocorrelation of the random component, since $DW_{23}^* = 1.962 > d_u = 1.41$ (at $\gamma = 0.01$).

53 A decrease in the value of the variable FIXD means that for 1 PLN of depreciation, there is a lower initial value of the tangible fixed assets in use. Thus, there are more tangible fixed assets in use that are subject to depreciation deductions, as a result of modernization (purchasing) of the noncurrent assets.

54 In Equation 4.36, there is a variable SALDE expressing the amount of the net pay per 1 PLN of the depreciation costs in a time-period. An increase in the variable SALDE's value signifies an increasing dominance of the labor costs of a manufacturing process over the costs of new technology, expressed by possession of new fixed assets (which are subject to depreciation).

55 The number of price list positions invoiced in a given quarter (month).

56 At a stable sales level of the so-called all-year assortments.

57 Ready-made production is manufactured mainly through the use of machinery and equipment, while the company's transportation means only serve the sale process.

58 The sales income results from the use of machinery and equipment in the production process as well as from the use of vehicles to supply the customers. Therefore, it is logical to relate the SBRUT's volume to FIXAS as an efficiency measure of the entire noncurrent assets in the enterprise.

59 $DW_{17} = 1.840 > d_u = 1.62$ at a significance level $\gamma = 0.01$.

60 $DW_{18}^* = 1.750 > d_u = 1.61$ at $\gamma = 0.01$.

61 $DW_{30}^* = 1.728 > d_u = 1.53$ at $\gamma = 0.01$.

62 It concerns the consequences inside the economic system, resultant from selection of various variants decisional variants on the part of the manager. This group of variables also includes characteristics of the level of production specialization. If they are statistically significant in the equations, various effects of the decisions within those variables can be analyzed.

5
Econometric modeling in management of small-sized enterprise

5.1 The concept of financial liquidity and its measurement in a small-sized enterprise

Financial liquidity of an enterprise is fundamental for its viability and development.[1] Liquidity plays a particular role in a small-sized enterprise, which during the time of transformation in Poland faced the banks and their reluctance to grant credit, which is a liquidity buffering tool. As such, financial liquidity of a small-sized enterprise depends on the company's ability to sell its products and to execute liabilities for the goods and services sold. Possible cash shortages are rarely complimented by bank loans. Typically, such supplementation comes from own funds of the company's owner and of his family, including the amounts accumulated earlier on as a result of having the so-called periodical excess financial liquidity.

Manufacture of the goods for sale requires possession of necessary cash funds,[2] which in turn come from the sale of the goods for which payment has been made, at the same time closing the circular cycle of capital.[4] Production of the goods allows their delivery to the customers, which results in issuance of adequate invoices creating the receivables. After the time specified in an agreement, payment for the goods sold follows. This enables initiation of the next production cycle. The mechanism described here is presented in Figure 5.1.

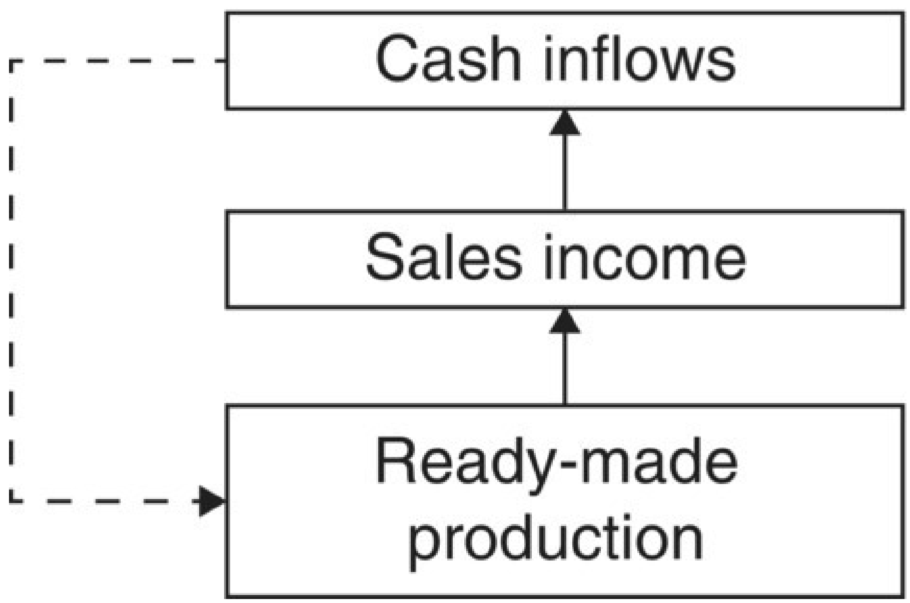


Figure 5.1 The links between cash inflows, gross sales income, and ready-made production in a small-sized enterprise.[3]

Source: Own studies.

Completion of production allows a possibility of generating the sales income, which results in adequate cash inflows. Between:

- the ready-made production and the sales income,
- the sales income and the cash inflows, and
- the cash inflows and completion of ready-made production.

various time intervals occur.

In a small-sized enterprise, monthly and quarterly data are the most important time series related to cash settlements. These intervals are most crucial due to the frequency of implementing the company's various commitments (from a time interval of one month to around three months).

Study of an enterprise's financial liquidity uses a variety of tools of an indicating character. Typically, measurement is done during a predetermined time-period or – for comparison – over two time-periods. Information that is obtained essentially is statistical in character, as a result of which it is very poor and of a little use in financial management.

In a small-sized enterprise, access to information is much more difficult. Owners of this type of companies do consider the data from the past as important. They only collect information which they are obliged by law to keep on their records. Meanwhile, possession of some information from the past, in the form of sufficiently long time series, especially monthly ones, can simplify management of the company in various areas. Past financial characteristics as well as those reflecting intensity of the sales and of the manufacturing process are particularly important.

One of the most significant problems of a small-sized enterprise is having the cash necessary for timely payments of liabilities. In a small-sized company, scarcity of statistical data causes liquidity to play a particular role. Financing business activity in a small business entity, usually, is done by own funds. Very rarely such company uses a bank loan. During the entire time after 1990, the company encountered reluctance of Polish banks to grant credit loans constituting buffering tools for companies' liquidity.

Multitude of the measures of financial liquidity[5] does not mean it is always possible to use them, especially in a microenterprise. Lack of adequate statistical information is the main difficulty. Keeping simplified accounting in a small-sized enterprise is uncomplicated. The price behind this simplicity is unavailability of important information that would enable a precise diagnosis of the situation and evaluation of the past and the future.

Information collected about the company's cash inflows and the value of the finished ready-made production[6] is beneficial to its owners. It allows, inter alia, an approximate account of the company's financial liquidity. Comparison of the cash amounts resultant from realization of receivables with the value of ready-made production[7] provides a relatively precise picture of the company's liquidity. The symbol $\mathbf{cash}_t$ denotes the value of cash inflows, while $\mathbf{prod}_t$ denotes the value of finished production (in net sales prices). Comparison of those variables' amounts in a given period, allows assessment of the company's current financial liquidity. Only the manner of those variables' comparison (the variable $\mathbf{cash}_t$ with the variable $\mathbf{prod}_t$) requires consideration. The first option entails comparison of the values of simultaneous cash inflows with the value of finished ready-made production.[8] If there is inequality: $\mathbf{cash}_t \geq \mathbf{prod}_t$ ($t = 1, \ldots, n$), then the enterprise has the cash funds necessary to cover the liabilities during a t period. When $\mathbf{cash} < \mathbf{prod}_t$ it can signify deficiency of cash funds. It is worth noting though, that an entrepreneur, who must rely primarily on his/her own foresight, can accumulate cash funds during the periods of its surplus over liabilities, and can use it during a current shortage. As such, a better analytical solution involves examining the cumulated value of the cash funds in subsequent periods of a given year and its comparison with the cumulated value of ready-made production.

As a result, we are going to use three measures of a small-sized company's liquidity in this work. The first of these measures will be the difference between the cumulated monthly cash inflows and the cumulate of the finished ready-made production,[9] that is:

$$\text{liq}_t = \text{cum.cash}_t - \text{cum.prod}_t, \tag{5.1}$$

where:

$$\text{cum.cash}_t = \text{cum.cash}_{t-1} + \text{cash}_t, \text{in year } t^*,$$

$$\text{cum.prod}_t = \text{cum.prod}_{t-1} + \text{prod}_t, \text{in year } t^*,$$

$$\left(t^* = 1, \ldots, 11; t = 2, \ldots, 12\right) \text{and}$$

$$\text{cum.cash}_1 = \text{cash}_1, \text{cum.prod}_1 = \text{prod}_1.$$

An alternative measure of the cumulated monthly financial liquidity is going to be the relative measure of this liquidity for the current production. It is calculated using the following formula:

$$\text{liqproc}_t = 100 \cdot \left(\text{liq}_t / \text{prod}_t\right). \tag{5.2}$$

Variable liqproc_t is expressed in percentage points. It provides information on what percentage of the finished production's value in a t month constitutes the value of the enterprise's cumulated monthly financial liquidity.

Next measure of a small-sized company's financial liquidity can be the ratio of cumulated cash inflows and cumulated finished production's value, that is, an indicator of relative liquidity for the cumulated production:

$$\text{liqrel}_t = \left[\frac{\text{cum.cash}_t}{\text{cum.prod}_t} - 1\right] 100. \tag{5.3}$$

Measure 5.3 contains information similar to that in measure 5.2. It is also expressed in percentage. It provides information on whether in a given month cumulated cash inflows were higher than cumulated finished production’s value and if so, by what percentage. Positive value of an observation on $liqrel_t$ indicates by what percentage the cumulated cash inflows were higher than the cumulated finished production’s value, in a given month. Negative value of $liqrel_t$ informs about the risk of a lack of liquidity, although not always.[10]

5.2 econometric modeling of monthly financial liquidity

A study of a small-sized manufacturing enterprise’s monthly financial liquidity was done from January 1996 to December 2006. Time series of the variables **liq** and **liqproc** are presented, respectively, in Figures 5.2 and 5.3. Significant seasonal fluctuations of cumulated liquidity expressed in monetary units as well as fluctuations of relative liquidity expressed in percentage were noticed. However, downward amplitude of seasonal fluctuations, indicating progressive stabilization of liquidity understood as such, can be seen. What is more, both liquidity measures are positive during most periods, while during many periods they are higher than zero. This signifies a generally good or a very good financial condition in the company.

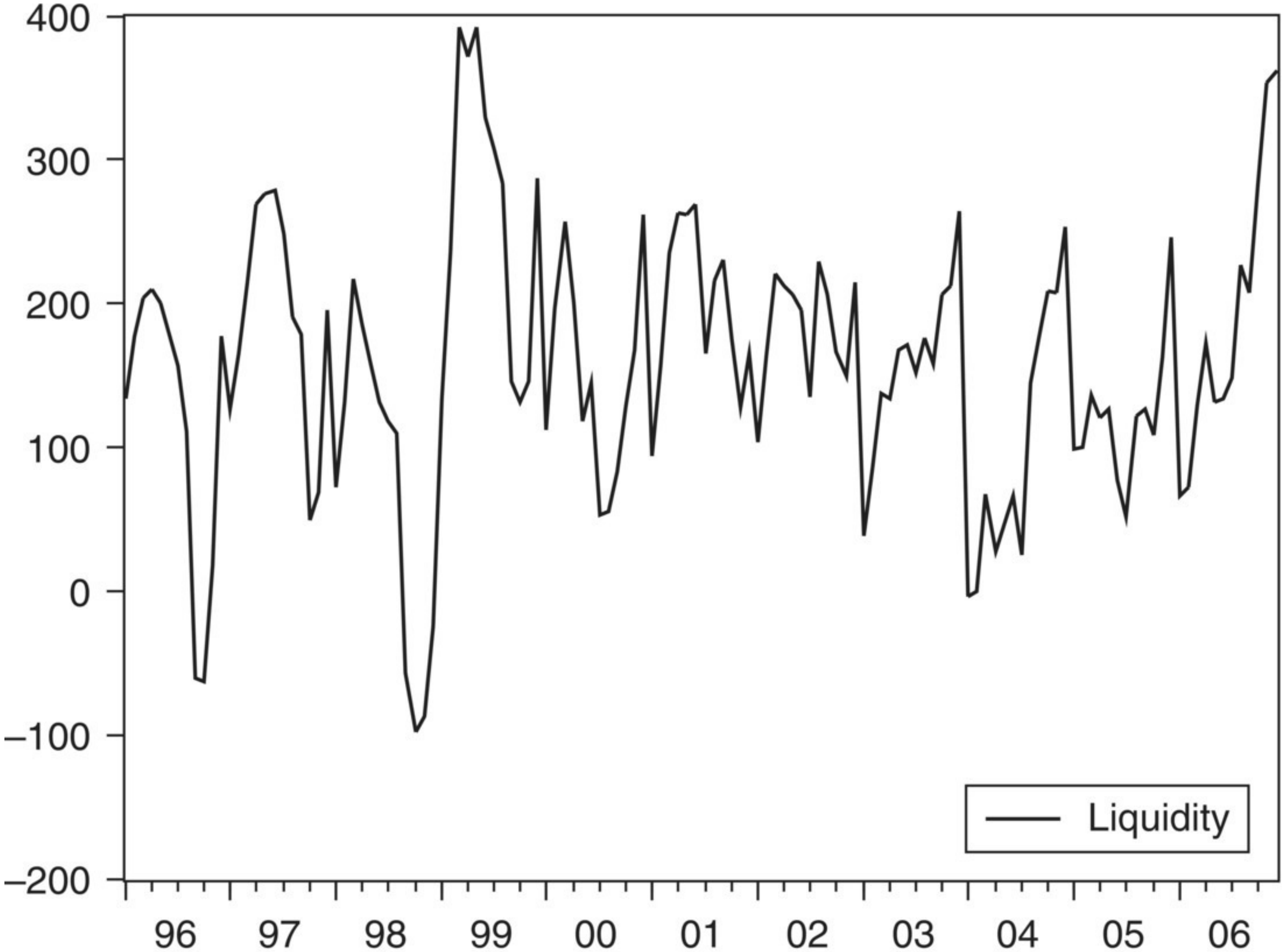


Figure 5.2 Monthly financial liquidity of the company REX during the years 1996–2006 (in thousands PLN).

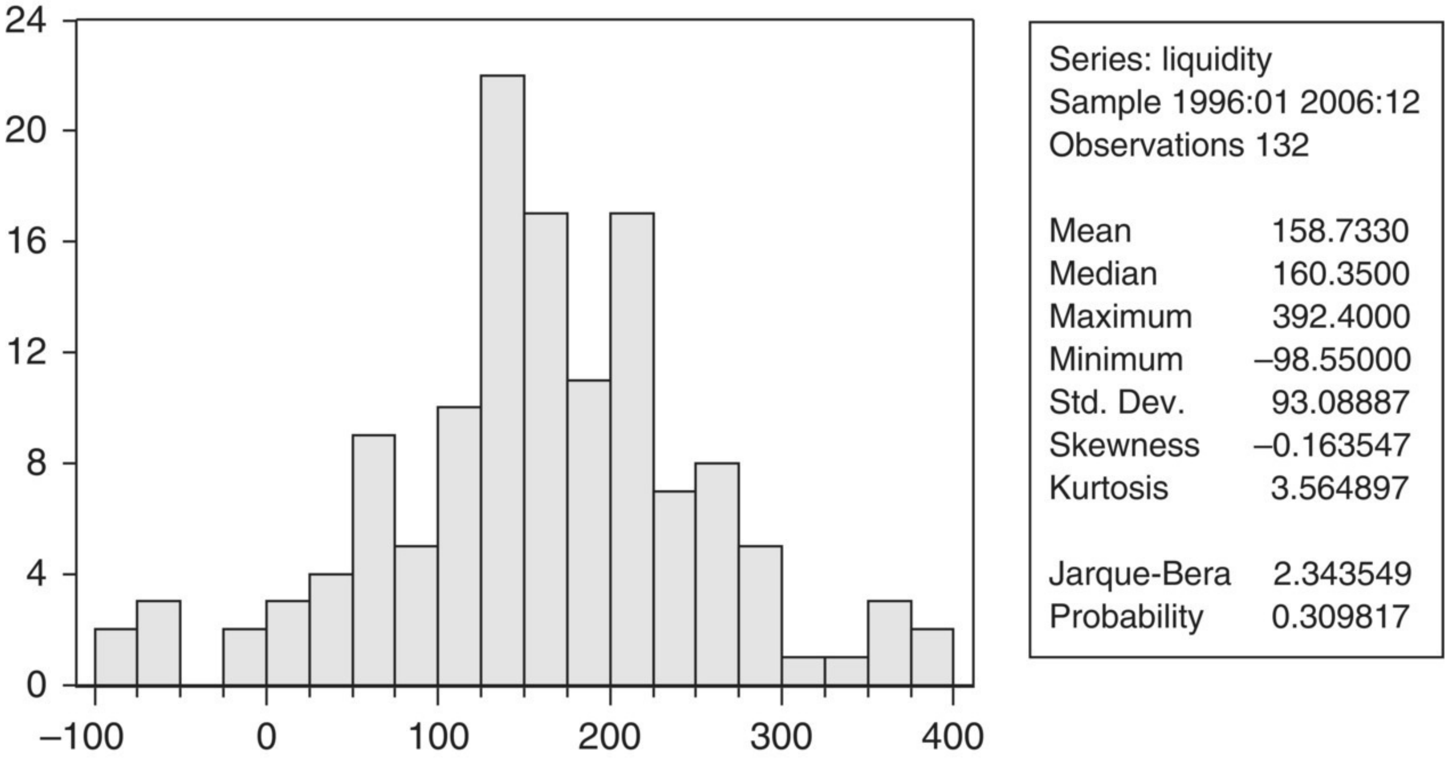


Figure 5.3 The structure of monthly cumulated financial liquidity of the company REX during the years 1996–2006 (in thousands PLN).

Figures 5.3 and 5.4 show the structure of the enterprise's valuable financial liquidity and its relative financial liquidity during the years 1996–2006. While distribution of liquidity, expressed in its value, can be visually evaluated as close to normal, distribution of relative liquidity is visibly right-skewed. The average size of liquidity expressed in value was close to 106 thousands PLN, while the volume of relative liquidity – over 31%. Still, the company's averagely very good financial condition cannot lead to the owners' self-reassurance that everything is under control. All it takes is lack of financial liquidity (negative values) during few subsequent periods in order to cause significant difficulties in running the company. When the values are less than zero, the company faces the risk of bankruptcy.

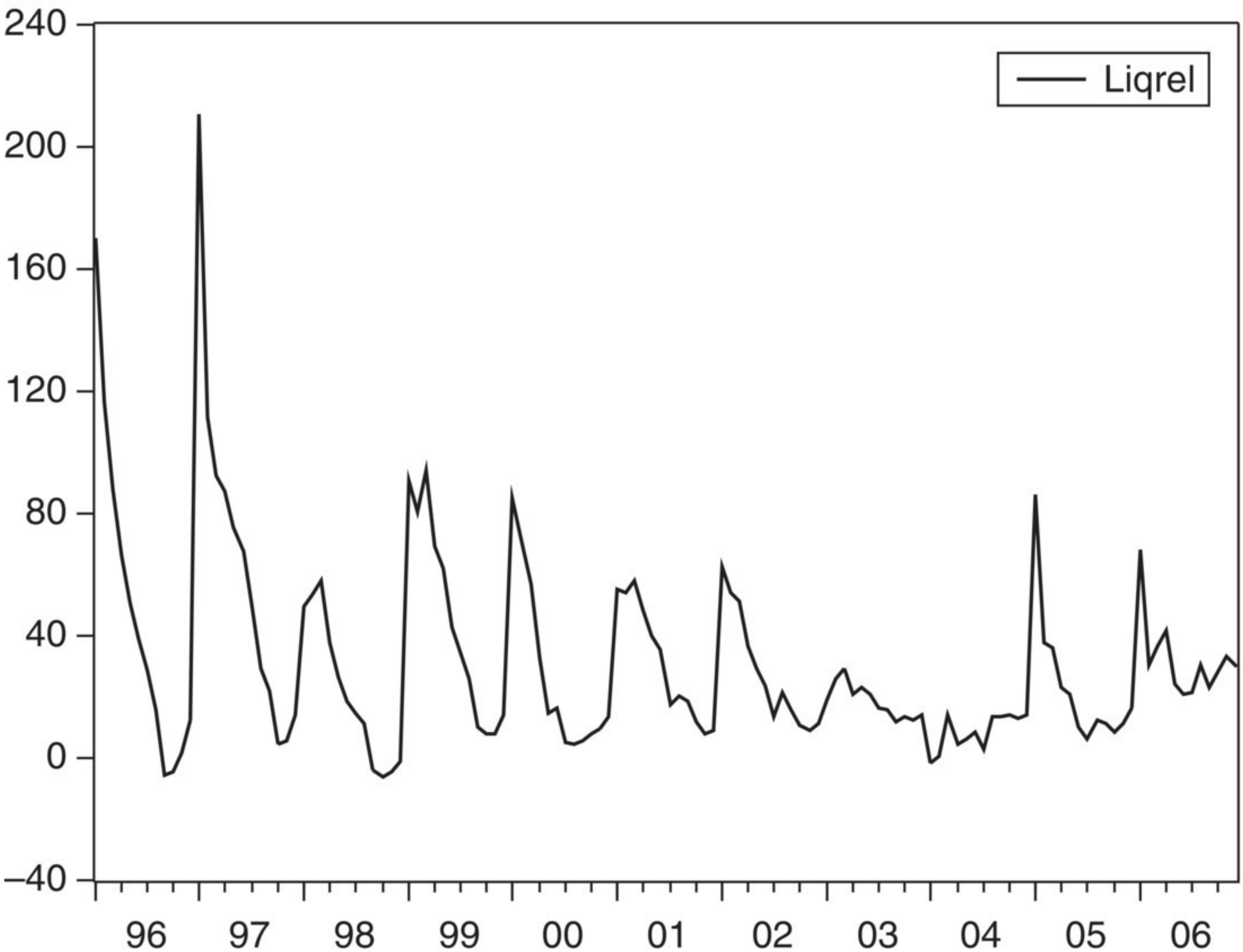


Figure 5.4 Cumulated monthly financial liquidity of the company REX during the years 1996–2006 (in% %).

A detailed analysis of liquidity time series allows inference, that during 7 out of the 132 months liquidity measures were negative. The worst situation occurred during the period from September to December 1998, since financial liquidity measures were negative during four subsequent months. However, these values were below zero, which did not pose any threat for the company's viability. During the worst financial month, the cumulated value of cash inflows was lower than the value of the finished ready-made production,[11] by a mere 6.37%. In turn, during a subperiod between the years 1999–2006, the company's financial liquidity measure was negative only during 1 month (January 2004).

At the same time, during many periods the company was characterized by an excess financial liquidity. During 29 out of the 132 months the company had an excess of cumulated cash inflows over the cumulated value of the finished ready-made production by 210%. Therefore, during the months of significant excess liquidity it was possible to accumulate cash funds. It is quite interesting, that excess liquidity usually occurred in the first three to five months of calendar year. As an exception, in 2003 and 2004, liquidity during the first months was grossly higher compared to the months of the second half-year.

Finally, large dispersion of the financial liquidity's measures is noteworthy. Standard deviations of both variables reached relatively high values. However, it was mainly a differentiation between normal liquidity and excess financial liquidity (Figure 5.5).

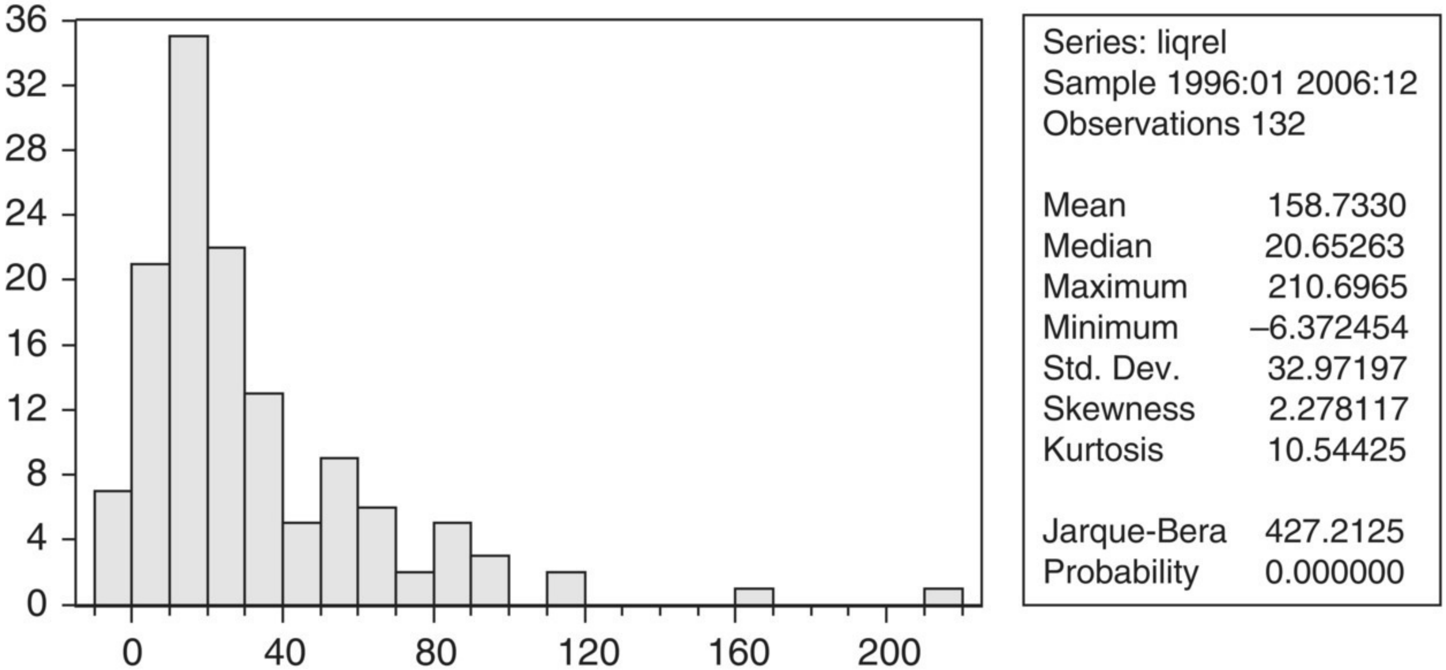


Figure 5.5 The structure of cumulated relative monthly financial liquidity in the company REX during the years 1996–2006 (in%%).

The mechanism of the company's monthly financial liquidity expressed by the variable liq is described by the empirical dynamic econometric model, written in Table 5.1.

Table 5.1 An empirical dynamic econometric model of the company's financial liquidity liq, based on the monthly data for the years 1996–2006.

Dependent variable: liq				
Method: least squares				
Date: 10/31/2008; time: 19:17				
Sample(adjusted): 1997:01 2006:12				
Included observations: 120 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	48.21616	14.12626	3.413230	0.0009
liq(−1)	0.797203	0.047525	16.77429	0.0000
liq(−5)	−0.161817	0.047803	−3.385088	0.0010
liq(−12)	−0.146115	0.052655	−2.774944	0.0066
sbrut(−2)	0.209423	0.073306	2.856831	0.0052
sbrut(−10)	−0.211553	0.073948	−2.860841	0.0051
sbrut(−12)	0.322061	0.082158	3.920044	0.0002
jan	−54.74449	19.66362	−2.784050	0.0064
sep	−110.3969	24.51906	−4.502495	0.0000
oct	−150.1994	24.64126	−6.095444	0.0000
nov	−98.59027	26.32944	−3.744488	0.0003
*t**jan	−1.211317	0.262139	−4.620888	0.0000
*t**feb	−0.344235	0.134695	−2.555660	0.0121
*t**jul	−0.349798	0.135516	−2.581222	0.0113
*t**sep	0.565260	0.260045	2.173703	0.0320
*t**oct	1.117471	0.266116	4.199190	0.0001
*t**nov	0.776393	0.273300	2.840812	0.0054
R-squared	0.828659	Mean dependent var		108.3792
Adjusted *R*-squared	0.802043	S.D. dependent var		61.30909
S.E. of regression	27.27783	Akaike info criterion		9.580596
Sum squared resid	76640.24	Schwarz criterion		9.975491
Log likelihood	−557.8358	*F*-statistic		31.13388
Durbin–Watson statistic	2.079405	Prob (*F*-statistic)		0.000000

In the model presented in Table 5.1, autoregressive dependencies as well as the following variables occur: t^* – the time variable representing the number of the month ($t^* = 1, \ldots, 132$);

$sbrut_{-2}$, $sbrut_{-10}$ *i* $sbrut_{-12}$ – the value of the gross sales income, appropriately delayed by 2 months and by 10 and 12 months. A separate group of exogenous variables consists of the dummy variables, taking the value 1 during the indicated month and the value of 0 during the remaining periods. The following symbols represent the dummy variables, distinguishing by number 1: jan – January, feb – February, mar – March, apr – April, may – May, jun – June, jul – July, aug – August, sep – September, oct – October, nov – November.[12]

The empirical equation demonstrated in Table 5.1 indicates that first-order autoregression forming the current sequential liquidity is crucial in the formation of the variable liq_t. Commutativity of this variable's volatility is formed by autoregressive dependencies of the 5th and 12th order. Gross sales income constituting a gross sum of the invoices issued during a given month, converted into the cash inflows, play a significant role in formation of the company's financial liquidity. Time-constant seasonal fluctuations are adjusted by adequately variable seasonality, which ultimately results in a decrease of the amplitude of seasonal fluctuations during the following years. The actual and the theoretical values of monthly financial liquidity in the company REX as well as the residuals, calculated on the basis of the equation considered here, are presented in Figure 5.6.

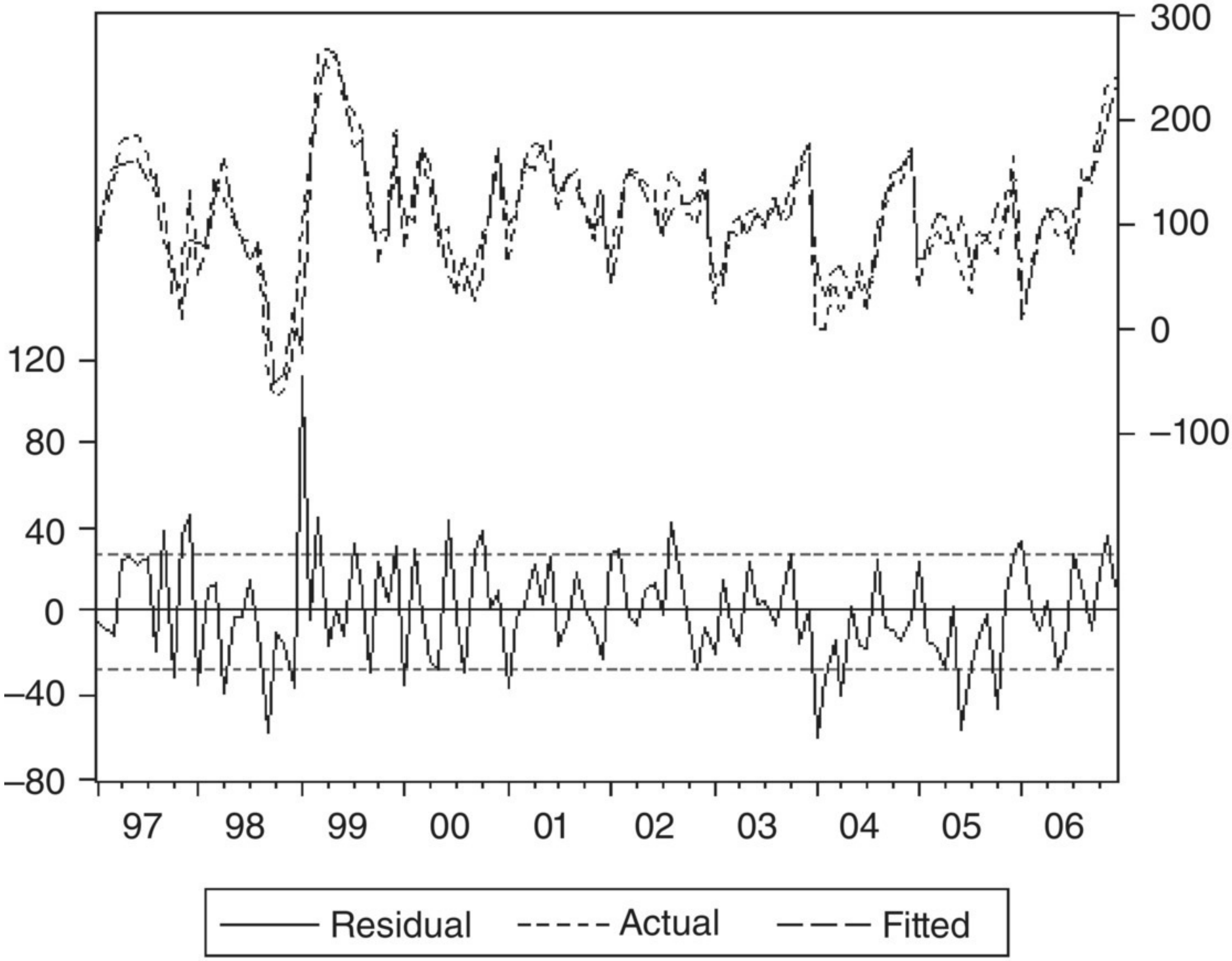


Figure 5.6 Empirical and theoretical values of monthly financial liquidity liq in the company REX[13] *as well as the residuals during the years 1996–2006 (in thousands PLN), calculated on the basis of the equation given in Table 5.1.*

An alternative dynamic empirical model describes the variable liqproc. It has the empirical form[14] shown in Table 5.2. This model describes a volatility mechanism of the company's relative financial liquidity in a very simplified way. There is a variable describing only the first-order autoregression, which indicates sequencing in the variable liqproc's formation. What is more, very large positive constant seasonality in January as well as a significant positive seasonality in March appears. January's constant seasonality is systematically decreased by a time variable correcting its seasonality. The actual and the theoretical values of cumulated relative monthly financial liquidity in the company REX as well as the residuals, calculated on the basis of the equation from Table 5.2, are shown in Figure 5.7.

Table 5.2 An empirical dynamic econometric model of the company's financial liquidity, based on monthly data from the years 1996–2006.

Dependent variable: liqproc				
Method: least squares				
Date: 12/17/2007; time: 10:48				
Sample (adjusted): 1996:02 2006:12				
Included observations: 131 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	3.748842	1.906265	1.966591	0.0514
liqproc(−1)	0.661443	0.041386	15.98223	0.0000
jan	120.0150	10.48327	11.44824	0.0000
MAR	13.71131	4.863916	2.818987	0.0056
*t**jan	−0.881299	0.137312	−6.418243	0.0000
R-squared	0.770874	Mean dependent var		30.30581
Adjusted R-squared	0.763600	S.D. dependent var		30.77835
S.E. of regression	14.96474	Akaike info criterion		8.286691
Sum squared residual	28216.86	Schwarz criterion		8.396431
Log likelihood	−537.7782	*F*-statistic		105.9789
Durbin–Watson statistic	2.307237	Prob (*F*-statistic)		0.000000

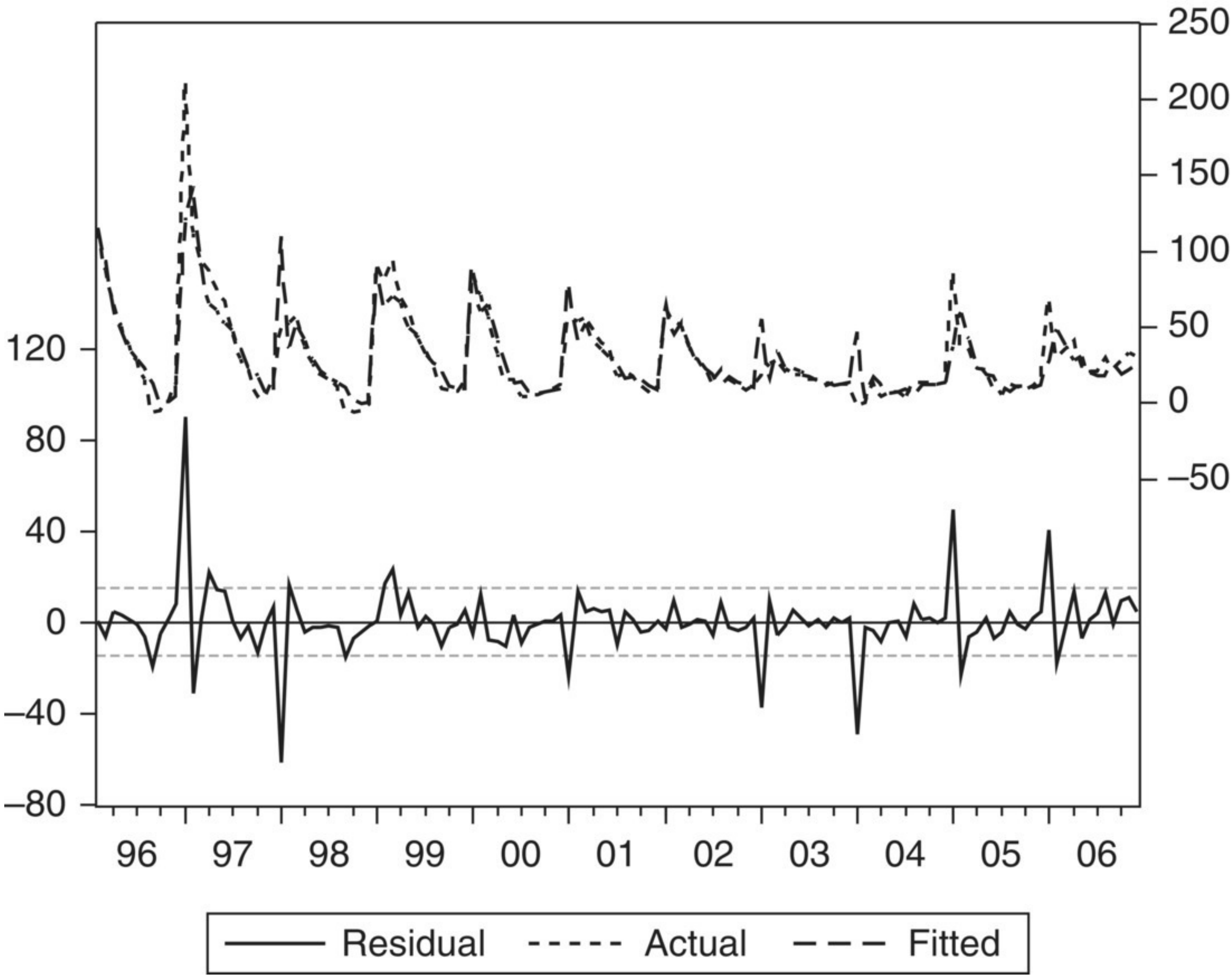


Figure 5.7 Empirical and theoretical values of monthly financial liquidity in the company REX (calculated on the basis of the equation from Table 5.2) as well as the residuals, during the years 1996–2006 (in%%).

The presented here simplified method of calculating and modeling a small-sized enterprise's financial liquidity is characterized by accuracy sufficient for the company's current management purposes. Using only adequately prepared graphs allows visual evaluation of the liquidity measures applied and of the scales of seasonal fluctuations. Using adequate and not very complicated econometric models will allow the owner to predict the scale of future financial liquidity. At the same time, it will be possible to adequately prepare for the expected liquidity levels, which may require appropriate accumulation of cash funds, in order to preserve security of the production process. It also becomes possible to indicate the periods, in which it will be relatively easy to finance investment purchases from the owner's own funds.

5.3 econometric modeling of quarterly financial liquidity

The quarterly financial liquidity accounting is connected to the payment due dates (almost three-months-long) specified in the settlements for the suppliers of basic raw materials needed for production. Liabilities to the supplier of electricity are settled every two months. Sufficiently precise information about quarterly liquidity can refer to statutory liabilities[15] such as: income tax, the tax on goods and services, and property tax. Analysis of liquidity during period of three months seems to be appropriately accurate and significant for most payments arising in connection with small-sized enterprise's business activity.

The mechanism of quarterly financial liquidity in the company REX was described using two stochastic equations. The first empirical equation is specified in Table 5.3. In the equation from Table 5.3, the symbols: sbrut(_1) represent gross sales income delayed by 1 quarterly, snet and snet(_3) – current and delayed by 3 quarters net sales income, prod, prod(_1) – current and delayed by 1 quarter volumes of the company's financial liquidity (Figure 5.8).

Table 5.3 An empirical dynamic econometric model of the company's financial liquidity (liq), based on quarterly data from the years 1996–2006.

Dependent variable: liq				
Method: least squares				
Date: 01/19/2008; time: 10:31				
Sample(adjusted): 1996:4 2006:4				
Included observations: 41 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	111.3354	36.81731	3.023995	0.0047
sbrut(−1)	0.637303	0.093277	6.832402	0.0000
liq(−1)	0.207248	0.102916	2.013761	0.0520
snet	0.674523	0.113202	5.958589	0.0000
snet(−3)	0.151101	0.065464	2.308146	0.0272
prod	−0.924006	0.164212	−5.626896	0.0000
prod(−1)	−0.653855	0.160410	−4.076157	0.0003
R-squared	0.708044	Mean dependent var		124.9390
Adjusted *R*-squared	0.656522	S.D. dependent var		61.49152
S.E. of regression	36.03833	Akaike info criterion		10.16130
Sum squared residual	44157.89	Schwarz criterion		10.45386
Log likelihood	−201.3066	*F*-statistic		13.74263
Durbin–Watson statistic	2.009039	Prob (*F*-statistic)		0.000000

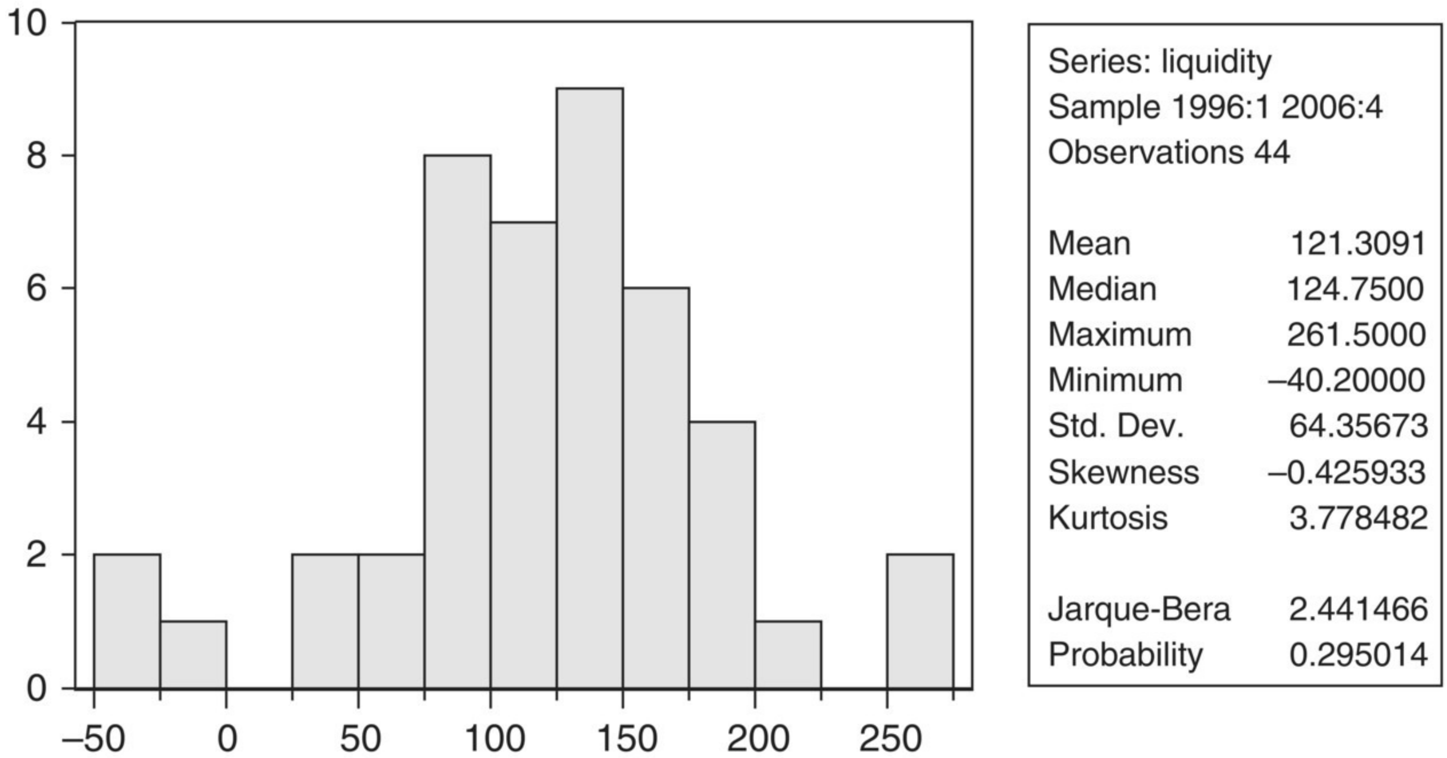


Figure 5.8 The structure of quarterly financial liquidity in the company REX during the years 1996–2006 (in%%).

The equation demonstrated in Table 5.3 explains 70.8% of the volatility of quarterly financial liquidity in the company REX. Liquidity measured in such way is characterized by an autoregression of the first order. There is a positive impact of the simultaneous net sales income and of the gross sales income delayed by 1 quarter. This means, that higher frequency of visiting the customers fosters improvement of the company's financial liquidity. There is also a slight positive impact on improvement liquidity of the net sales income[16] delayed by three quarters. Moreover, there is a negative impact on liquidity of the current and the delayed by 1 quarter value of the finished ready-made production. Manufacturing of ready-made production requires having the cash funds for its financing. Therefore, an increase of ready-made production's value results in a decrease of quarterly liquidity. Figure 5.9 illustrates the actual and the theoretical quarterly values of the variable liq in company REX as well as the equation's residuals.

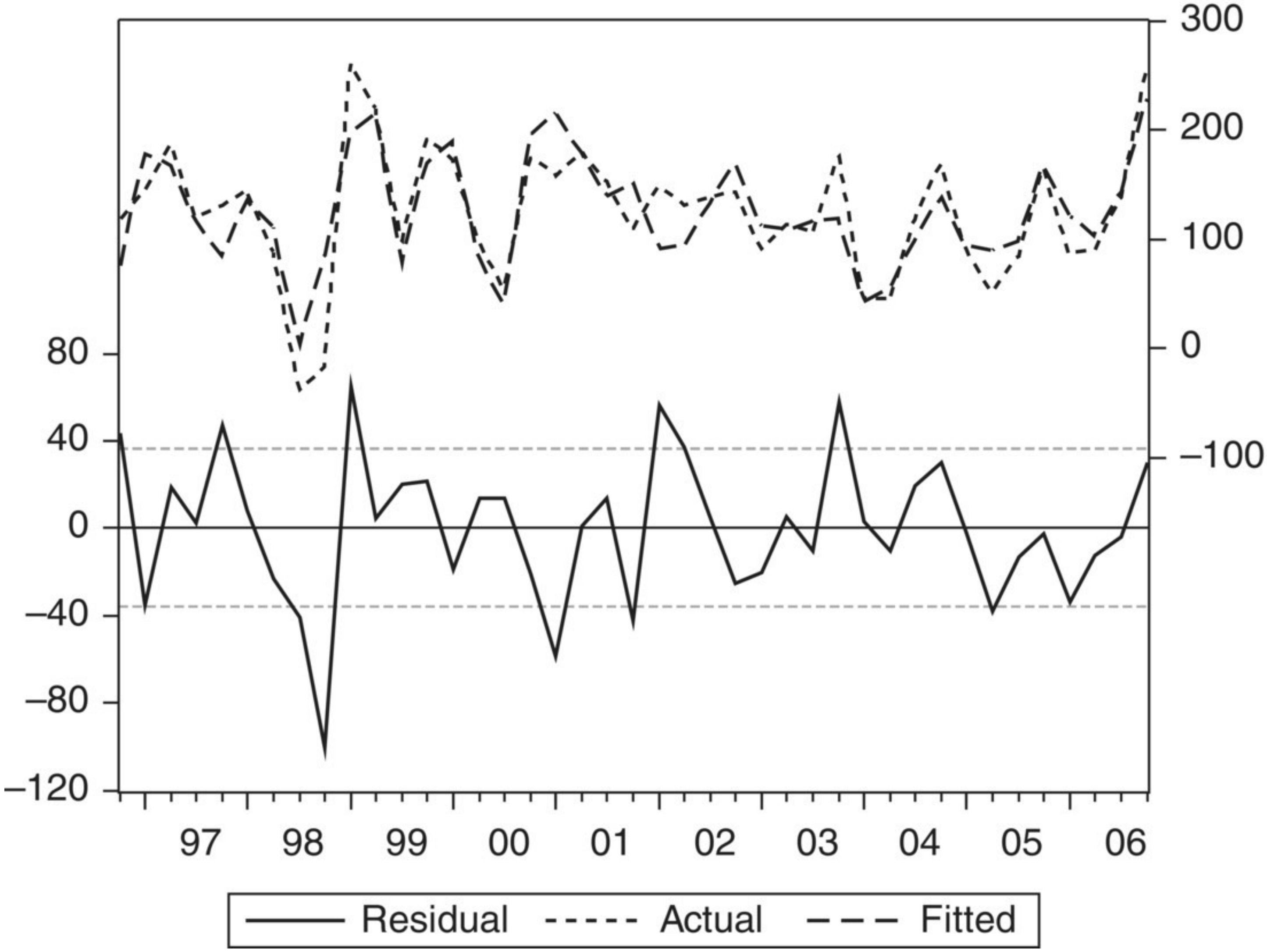


Figure 5.9 Empirical and theoretical values of relative monthly financial liquidity in the company REX as well as the residuals (calculated on the basis of the equation from Table 5.3), during the years 1996–2006 (in%%).

An empirical equation describing a quarterly volatility mechanism of financial liquidity ($liqrel_t$) has the form presented in Table 5.4. In the equation from Table 5.4, the symbols S1, S2, and S3 represent the dummy seasonal variables, taking the values of 1, respectively, in the following quarters: the first, the second, and the third as well as the values of zero in the remaining quarters.

Table 5.4 An empirical dynamic econometric model of the company's financial liquidity, based on quarterly data from the years 1996–2006.

Dependent variable: liqrel				
Method: least squares				
Date: 03/10/2014; time: 18:28				
Sample(adjusted): 1996:2 2006:4				
Included observations: 43 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	7.195236	3.246103	2.216576	0.0329
liqrel(−1)	0.396522	0.071658	5.533516	0.0000
S1	82.08507	7.862608	10.43993	0.0000
S3	−23.95108	6.922035	−3.460122	0.0014
*t**S1	−1.801646	0.285289	−6.315153	0.0000
*t**S3	0.726515	0.254730	2.852095	0.0071
R-squared	0.817315	Mean dependent variable		25.36152
Adjusted *R*-squared	0.792628	S.D. dependent variable		22.74613
S.E. of regression	10.35816	Akaike info criterion		7.642214
Sum squared residual	3969.784	Schwarz criterion		7.887962
Log likelihood	−158.3076	*F*-statistic		33.10691
Durbin–Watson statistic	1.690178	Prob (*F*-statistic)		0.000000

Description accuracy of the variable $liqrel_t$ in the equation demonstrated in Table 5.4 is slightly higher than for the variable liq_t in the equation demonstrated in Table 5.3. In the considered empirical equation, up to 81.7% of the variable liqrel's total volatility is explained by first-order autoregression as well as by constant and time-variable seasonality. Thus, relative cumulated liquidity is characterized by constant seasonal fluctuations, adjusted by variable seasonality during the first and the third quarters. Autoregression of the first order signifies transfer of about 39.7% of the relative liquidity from the previous quarter into the current quarter. The sizes of seasonal fluctuations for each quarter are as follows: $Q_1 = 82.1\%$, $Q_2 = -43.6\%$, $Q_3 = -24.0\%$, and $Q_4 = -14.5\%$. Suppression of negative seasonal fluctuations of the variable $liqrel_t$ in the first quarter, on average, by 1.80% annually and suppression of its positive seasonal deviations in the third quarter, on average, by 0.73% annually occur (Figure 5.10).

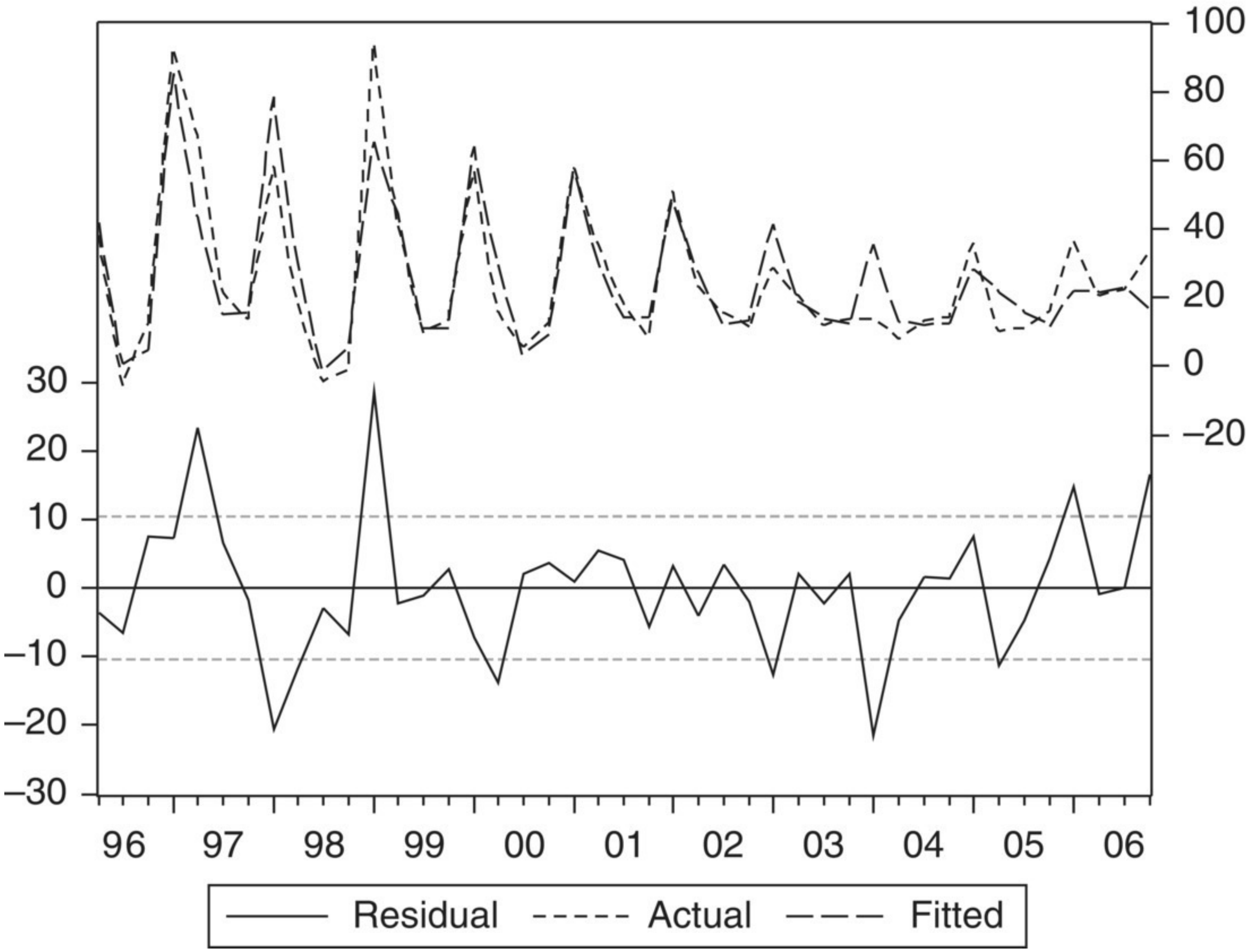


Figure 5.10 Empirical and theoretical values of relative cumulated quarterly financial liquidity in the company REX[17] *as well as the residuals (calculated on the basis of the equation from Table 5.4) during the years 1996–2006 (in%%).*

Financial liquidity of a small-sized enterprise plays cardinal role in its operational efficiency. Liquidity research methods and its measures known in economic and financial analysis, in a microenterprise can be used within a very limited scope. A simple statistical method of analyzing financial liquidity presented in this work can be widely and easily used in a small-sized manufacturing enterprise.

5.4 Econometric modeling of debt recovery efficacy

5.4.1 measuring the effectiveness of debt recovery in an enterprise

Production cycle in an enterprise, encompassing a given batch of products, ends with obtaining payment for the goods and services sold. Common practice of granting a trade credit means, that after delivery of the merchandise and its invoicing, there is a waiting period, which lasts – depending on the type of industry – from few days to few months. The law forces business entities to execute noncash payments, which makes it easier to control the company's turnovers within the banking system.

When a producer grants trade credits to customers, it means that between the date of invoicing a delivery and the actual payment for that delivery there is a time gap of approximately one month. Expiration of the agreed-on time should result in payment for the goods sold. In the enterprise considered here, dominant payment due dates ranged from 21 to 30 days. This means that part of the payment for invoiced deliveries was executed in current month, part in next month, and in case of slight payment delays some of the invoices were settled in 2 months period.[18]

The above facts necessitate searching for differences between the income amounts for the goods sold ($cash_t$) and the value of simultaneous gross sales income ($sbrut_t$) and the gross sales income delayed by one month ($sbrut_{t-1}$) as well as by two months ($sbrut_{t-2}$). It is though necessary to consider the following differences:

$$\text{vind0}_t = \text{cash}_t - \text{sbrut}_t, \tag{5.4}$$

$$\text{vind1}_t = \text{cash}_t - \text{sbrut}_{t-1}, \tag{5.5}$$

$$\text{vind2}_t = \text{cash}_t - \text{sbrut}_{t-2}. \tag{5.6}$$

Fully effective debt recovery ought to be manifested by almost zero values of the measure wind 0_t during all t periods ($t = 1, \ldots, n$). The sum of the values of the measure $\sum_{t=1}^{12} \text{vind}t0_t$ in the year t^* ($t^* = 1, \ldots, n^*$)[19] should be close to 0. This means that the amounts due for the goods and services sold have been converted into cash funds. The value of $\sum_{t=1}^{12} \text{vind}t0_t$ cannot be expected to be positive. However, if $\sum_{t=1}^{12} \text{vind}t0_t$ is significantly less than zero, it signifies lack of successful debt recovery in the enterprise, which threatens its viability.

The measure of debt recovery efficacy ($evind_t$) will be an arithmetic mean of detailed efficacy measures of debt recovery:

$$\text{evind}_t = \left(\text{vind0}_t + \text{vind1}_t + \text{vind2}_t\right) / 3. \tag{5.7}$$

Variable $evind_t$, having characteristics of a moving average, will be characterized by a much lower dispersion – in comparison with detailed measures of debt recovery efficacy.

5.4.2 Statistical analysis of debt recovery efficacy in enterprise

A detailed approach to analysis of payment for the goods and services sold is dominant in small-sized enterprises. In general, daily, weekly, or even monthly registers of the volumes of cash inflows are not kept. This prevents using a debt recovery efficacy's measure of Equations 5.4–5.6 type. It is though difficult to find a small-sized enterprise having statistical information in the form of time series relating debt recovery efficacy. Meanwhile, such time series allows precise diagnostics of the past and the present situation. They also allow forecasting of the company's debt recovery efficacy and of its financial liquidity, thus making its management easier.

Having time series of debt recovery efficacy allows the simplest analysis of the visual assessment of volume volatility. It allows the person managing the company to become oriented in periodicity sizes of the changes. Figure 5.7 demonstrates fluctuations of debt recovery efficacy meters in a small-sized manufacturing company called REX. Appropriate graduation of each volatility graph representing any meter allows quite precise evaluation of efficacy of the activities that are aimed at converting the amounts due into cash funds.

Another analytical possibility constitutes statistical characteristics of debt recovery efficacy measures. Table 5.5 shows the average measures, the dispersion measures, the measures of skewness, and of concentration for the variables $\mathrm{vind0}_t$, $\mathrm{vind1}_t$, $\mathrm{vind2}_t$, and evind_t. Each of those variables is oscillatory in character. By their nature they should oscillate around the number 0. Hence, their arithmetic mean that ought to be close to zero, although, in practice it is negative, slightly less than zero (Figure 5.11).

Table 5.5 Statistical characteristics of the efficacy measures of monthly debt recovery in the company REX, during the years 1996–2006.

Measure	**evind**	**vind0**	**vind1**	**vind2**
Mean	−1.669	−1.111	−1.673	−1.742
Median	1.900	2.650	2.900	1.950
Maximum	42.63	84.00	97.90	94.60
Minimum	−85.27	−110.8	−118.1	−144.2
Standard deviation	25.33	40.14	37.62	39.24
Skewness	−0.899	−0.345	−0.534	−0.916
Kurtosis	3.925	2.851	3.409	4.741
The Sum	−216.93	−146.6	−219.1	−226.4
Observations (*n*)	130	132	131	130

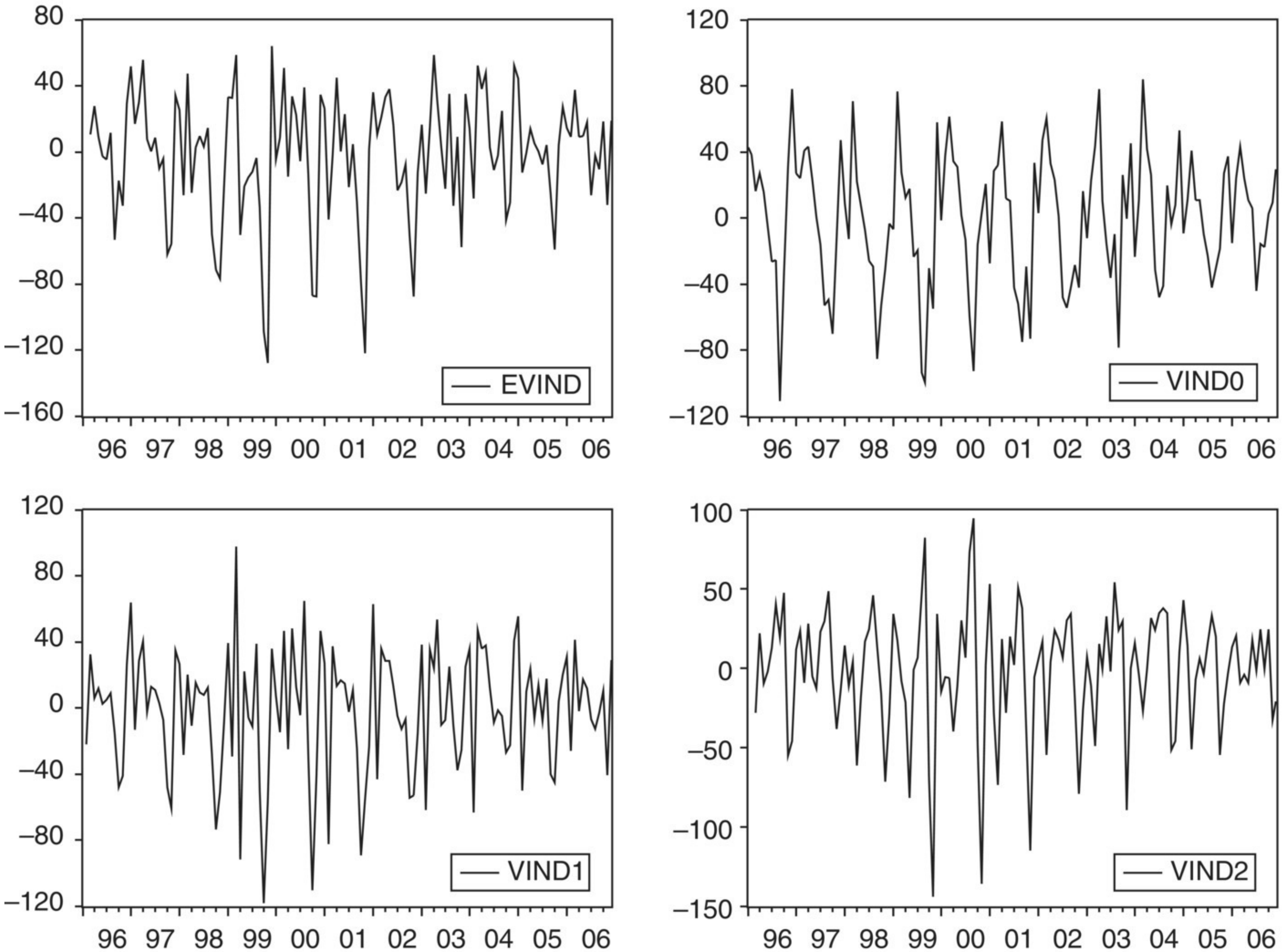


Figure 5.11 Measures of debt recovery efficacy in the company REX, during the years 1996–2006 (in thousands PLN).

The average value of efficacy measures of debt recovery are slightly less than zero. They range from 1111 to 1742 PLN. This allows an inference that conversion of the receivables into cash funds was appropriately effective. These figures should be compared with the average monthly gross sales income and with the cash inflows, which respectively amounted to 109.814 thousands PLN and 108.703 thousands PLN. Average observation of the variable $vind0_t$, in the amount of 111 PLN, represents the size of the gross sales income which was converted into cash funds. This marks the scale of the losses caused by dishonest debtors. In the statistical sense, this amount is considered small. In contrast, the sum of observations of the variable $wind0_t$, which is equal to 1446.6 thousands PLN, provides information about the total amount of the receivables lost by the company during the years 1996–2006. Figure 5.12 shows the structure of the measure $evind_t$. Its left-side skewness is an interesting characteristic of this distribution.

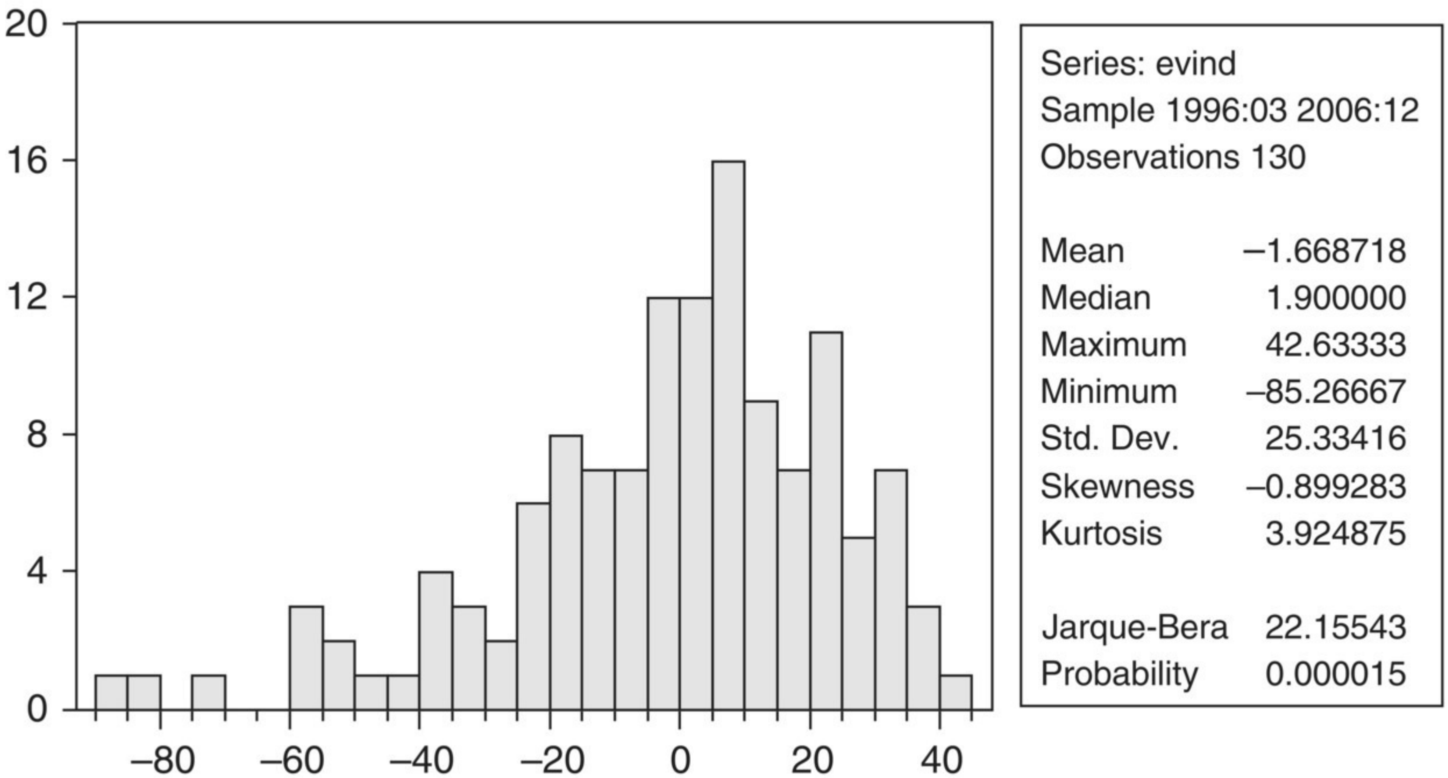


Figure 5.12 The structure of the efficacy measure of monthly debt recovery $evind_t$ *in the company REX, during the years 1996–2006.*

Successful debt recovery, on the one side, settles the level of the company's financial liquidity, and on the other side, it determines feasibility of the production process. The cash funds obtained from successful debt recovery can be used for the successive manufacture of goods and services during next production cycles. Therefore, the efficacy measure of debt recovery should be compared with the value of the ready-made production in a given time-period ($evind_t/prod_t$), where $prod_t$ represents the value of the ready-made production in a t period. A non-negative value of the quotient $evind_t/prod_t$ signifies the company's ability to realize the manufacturing process. Figures 5.13 and 5.14 demonstrate monthly fluctuations of the ratio $evind_t/prod_t$ in the company REX during the years 1996–2006, as well as the structure of monthly debt recovery's efficacy measure $evind_t$ and its share in the value of that company's ready-made production.

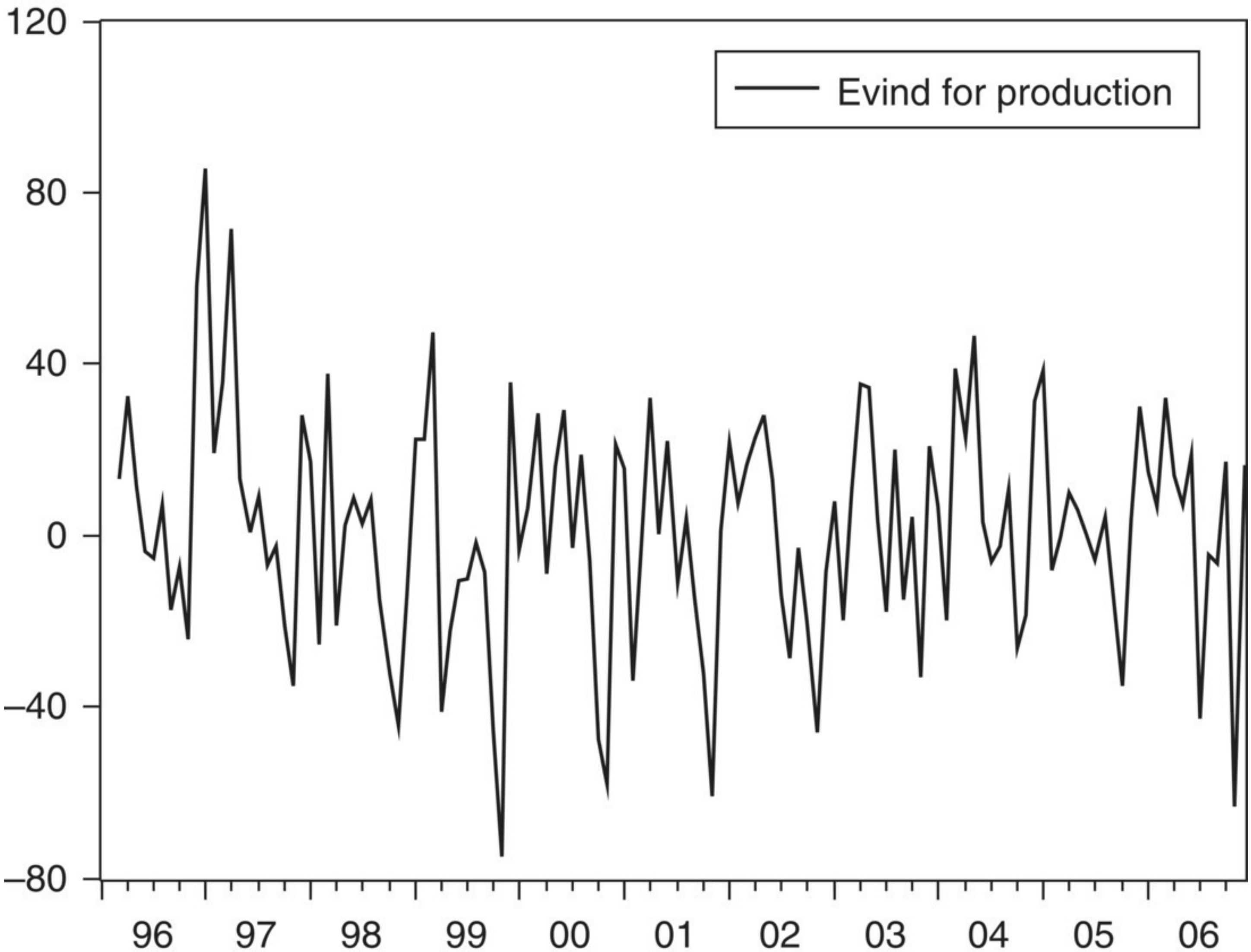


Figure 5.13 The share of the measure evind$_t$ in the value of monthly ready-made production in the company REX, during the years 1996–2006 (%).

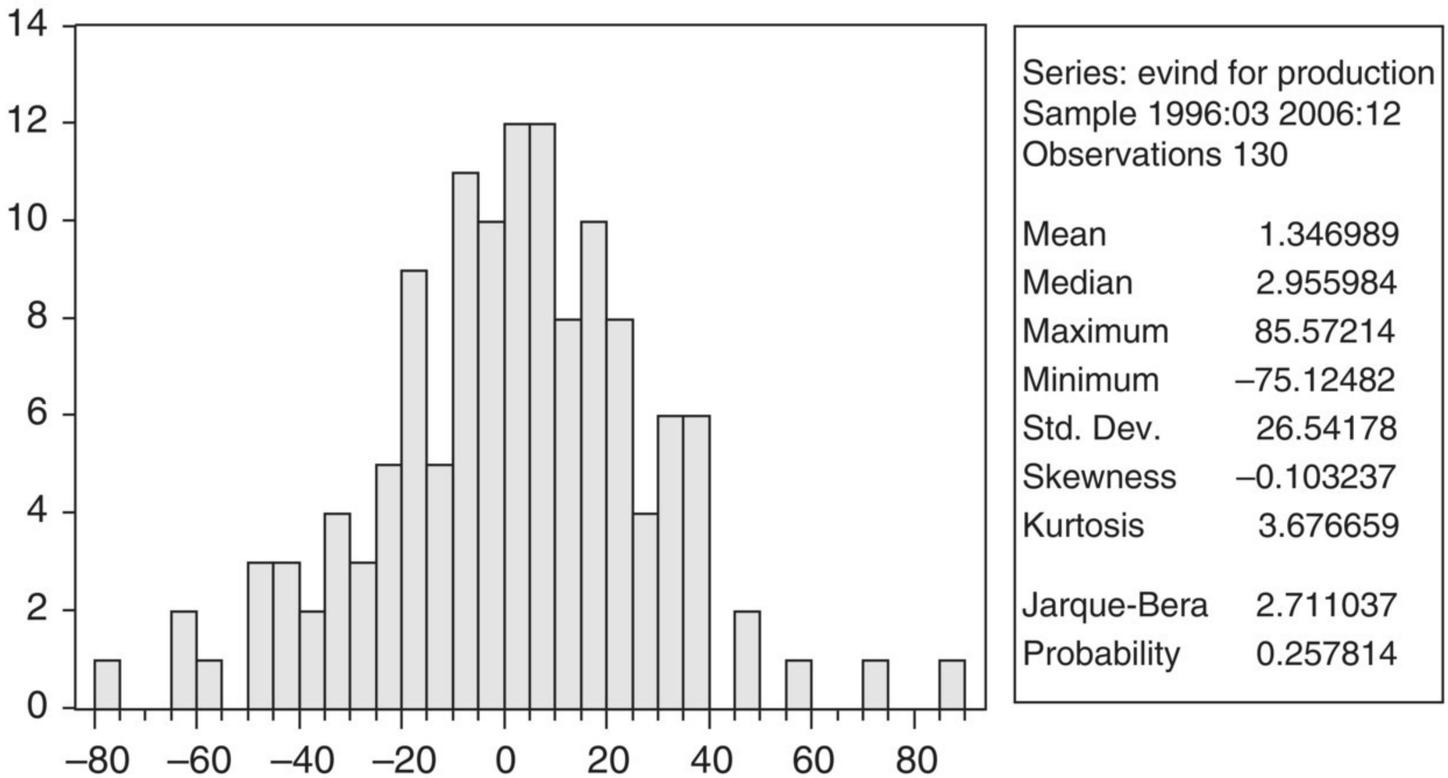


Figure 5.14 The structure of the share of monthly debt recovery's efficacy measure ewind$_t$ in the value of ready-made production in the company REX, during the years 1996–2006.

Annual settlement of the differences between the cash inflows and the gross sales income should be complementary in debt recovery analysis. Obtained differences vind = cash – sbrut provide the company's owner with information whether the company had successfully solicited the payments for the goods and services during a given year or not. Table 5.6 as well as Figures 5.14 and 5.15 present such information about debt recovery efficacy in the company REX.

Table 5.6 Annual results of debt recovery in the company REX during the years 1996–2006.

Years	VIND (tys. PLN)	PRVIND (%)
1996	47.1	4.59
1997	3.6	0.34
1998	−138.9	−10.13
1999	−136.4	−7.93
2000	5.8	0.38
2001	−124.7	−8.28
2002	−36.9	−2.54
2003	73.2	5.35
2004	97.4	7.61
2005	2.8	0.23
2006	56.2	5.65

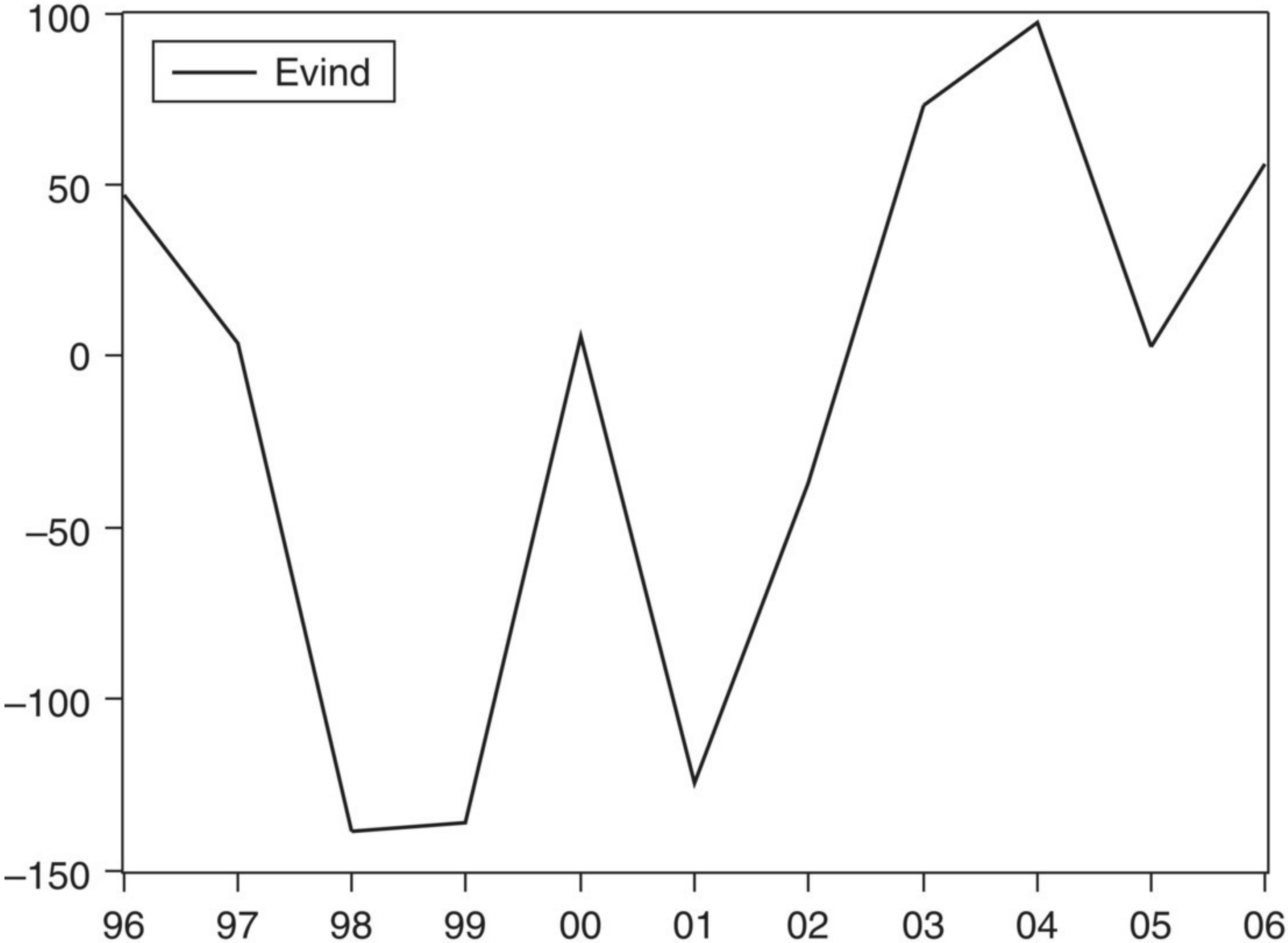


Figure 5.15 Annual results of debt recovery in the company REX during the years 1996–2006 (in thousands PLN).

Source: Table 5.6.

5.5 Econometric model describing interdependencies between the financial liquidity and the debt recovery efficacy in an enterprise

The practice of short-term financial management in a small-sized enterprise forces simultaneous control of its financial liquidity as well as of its debt recovery efficacy. A low level of financial liquidity can result from a small debt recovery activity. Improvement of the debt recovery efficacy causes the company's better financial liquidity. Decisions regarding this matter, in the company are made as they arise. Therefore, we can assume that the variables liq_t and $evind_t$ form direct feedback, that is:

$$\mathrm{liq}_t \underset{\leftarrow}{\overset{\rightarrow}{\ }} \mathrm{evind}_t \tag{5.8}$$

A hypothetical system of two interdependent equations with endogenous variables liq_t and evind_t will be identifiable ambiguously.[20]

The equation describing financial liquidity, contains autoregressions up to the 12th order among its explanatory variables and as well as the interdependent variable ewind_t. Moreover, the following predetermined variables: evind_{t-1}, evind_{t-2}, …, evind_{t-12} – the volumes of the debt recovery efficiency delayed by 1, 2, …, 12 months occur in the equation; the dummy variables used to extract monthly periodical fluctuations, taking the value of 1 for the month indicated and the value of 0 in the remaining periods, while: jan – singles out January, feb – February, mar – March, apr – April, may – May, jun – June, jul – July, aug – August, sep – September, oct – October, nov – November. Moreover, a time variable *t* was taken into the account in order to consider a possible linear and square trend.

In the equation describing debt recovery efficacy (evind_t), explanatory variable liq_t and delayed endogenous variables liq_{t-1}, liq_{t-2}, …, liq_{t-12} are going to occur naturally. Additionally, autoregressions up to the 12th order as well as the dummy variables describing monthly fluctuations will be included. Moreover, a variable representing business activity within the sales network snet_t – net sales income (in thousands PLN) along with its delays ranging from 1 to 12 months (snet_t, snet_{t-2}, …, snet_{t-12}) will also occur. The variable snet_t provides information on the intensity of servicing the sales network, which is always connected with a simultaneous debt recovery.

Parameters of both structural-form's equations were assessed using the double least squares method (2LS). At the same time, it is worth noting, that the empirical reduced-form's equations were characterized by a matching accuracy typical for this type of data. The coefficients of determination in each of these equations, respectively, were: $R_{lz}^2 = 0.784$ and $R_{evz}^2 = 0.731$. Figures 5.16 and 5.17 demonstrate the actual and the theoretical values of the variables liq and evind along with the residuals obtained from the empirical reduced-form's equations, providing a view on their matching accuracy.

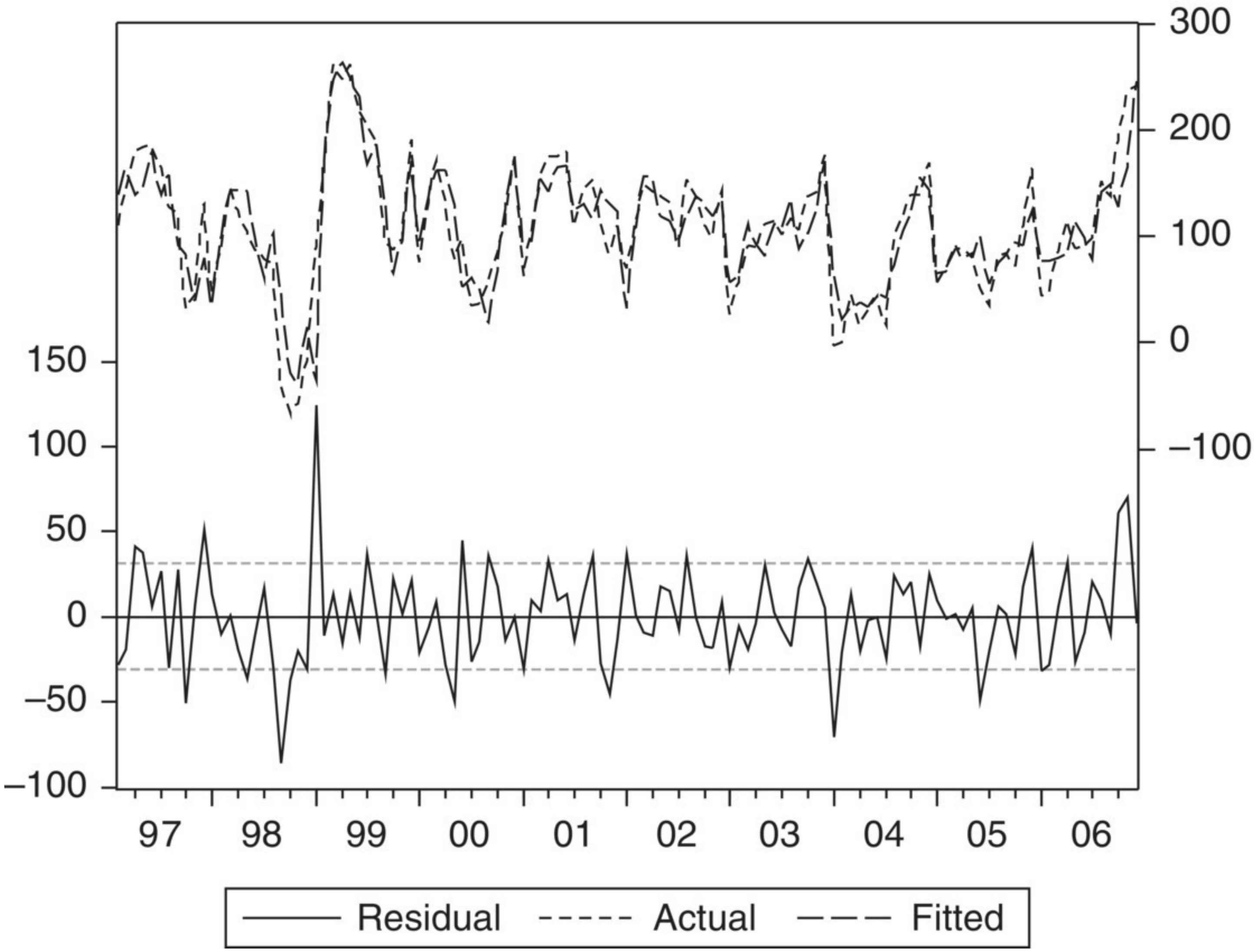


Figure 5.16 The actual and the theoretical values of the variable liq and the residuals from the empirical reduced-form's equation ($R^2_{1z} = 0.784$*).*

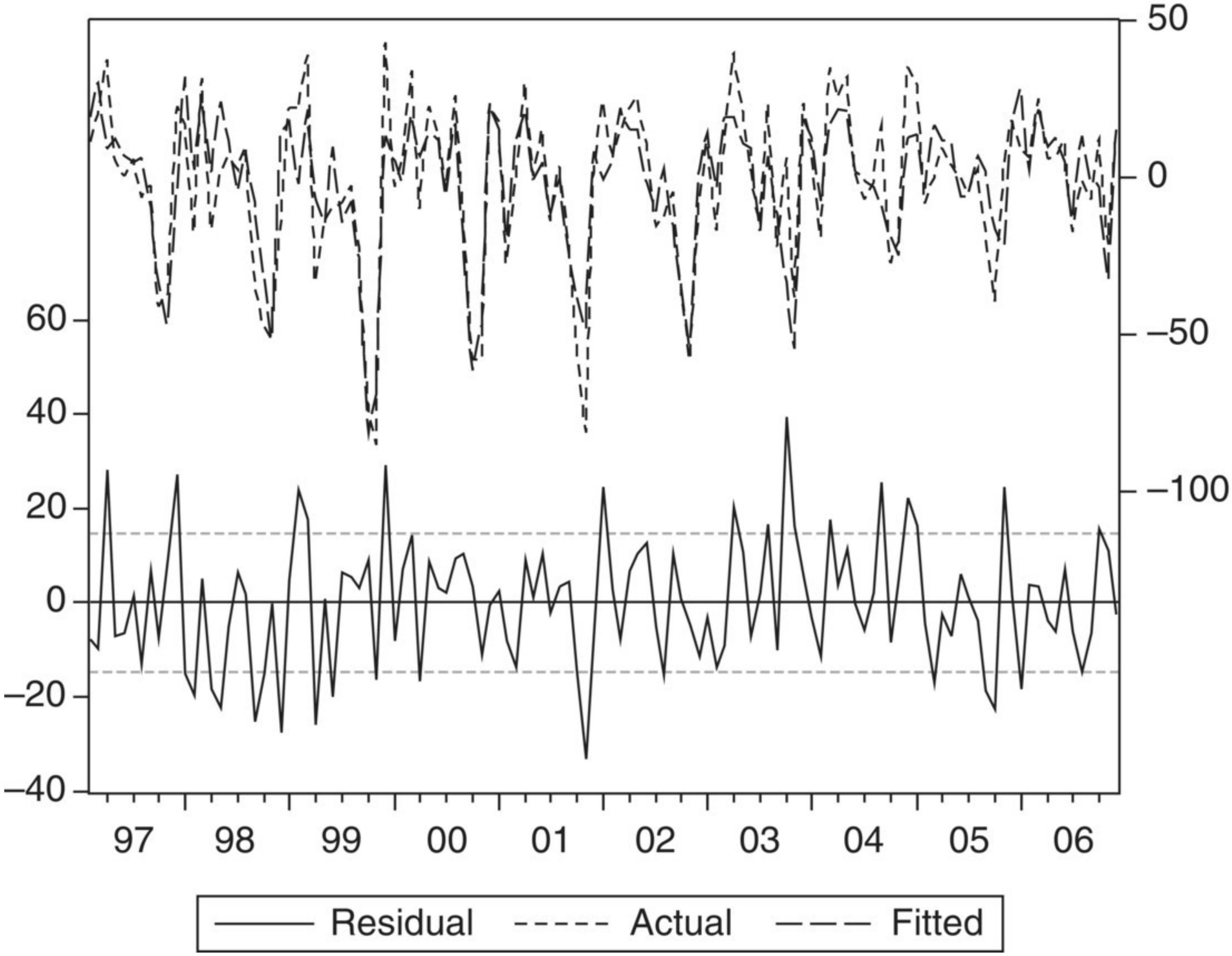


Figure 5.17 The actual and the theoretical values of the variable ewind and the residuals from the empirical reduced-form's equation ($R^2_{evz} = 0.731$*).*

An empirical structural-form's equation describing the mechanism of financial liquidity in a small-sized enterprise has the following form[21]:

$$\begin{aligned} liq_t = {} & \underset{(5.733)}{59.900} + \underset{(3.468)}{0.642} evind + \underset{(18.351)}{0.936} liq_{-1} - \underset{(4.802)}{0.254} liq_{-4} + \underset{(1.998)}{0.146} liq_{-11} + \\ & - \underset{(2.094)}{0.170} liq_{-12} - \underset{(4.130)}{0.514} evind_{-1} - \underset{(2.505)}{0.307} evind_{-3} - \underset{(2.310)}{0.384} evind_{-11} - \underset{(9.522)}{132.036} jan + \\ & - \underset{(3.640)}{38.531} may - \underset{(2.108)}{22.512} jun - \underset{(3.028)}{31.364} jul - \underset{(3.208)}{39.782} sep - \underset{(2.399)}{32.929} oct + u_{lq}, \end{aligned} \tag{5.9}$$

$$R_l^2 = 0.808, Su_1 = 28.721, DW_1 = 1.999.$$

Equation 5.9 confirms the economic logic, which shows, that the simultaneous financial liquidity increases along with improvement of the company's debt recovery efficacy. An increase of the debt recovery measure by 1 thousand PLN increases the company's simultaneous financial liquidity, on average, by about 642 PLN. Other impacts are relatively easily explained, which we leave to the inventiveness of a PT reader. Significant periodic fluctuations during many of the months, the sizes of which are easy to determine, can be noticed. Description accuracy of the company's financial liquidity is relatively high, since the coefficient $R_l^2 = 0.808 > 0.8$.

At the same time, using the 2LS method, parameters of the equation describing debt recovery efficacy were assessed. Its empirical form is demonstrated in Table 5.7. Matching accuracy of the equation demonstrated in Table 5.7 is significantly worse, compared to Equation 5.9. Moreover, the variable liq_t turns out to be statistically insignificant (the *t*-Student statistic $t_{liq} = 0.233$). As such, it is necessary for the insignificant variables to be reduced in the subsequent iterations. On the one hand, this results from a greater role of random fluctuations in the process of evind. On the other hand, the result obtained is contrary to the feedback hypothesis formulated in Equation 5.8. Financial management of a small-sized company in practice demonstrates appearance of that feedback in monthly periods; since its lack signifies faulty management of the cash flows. As such, the question arises: Does the lack of feedback result from errors in financial management or does it result from a faulty research procedure? Further in this book we will conduct an experiment, in which an assumption about an apparent insignificance of the variable liq in the empirical equation presented in Table 5.7 is going to emerge. This statistical insignificance can result from deterioration of the 2LS estimator's efficiency, resultant from a too small description accuracy of the variable liq in the reduced-form's equation. Indeed, the theoretical values of the variable liq in this reduced-form's equation differ from the actual values of the company's financial liquidity by 22.6%. In this particular case, it could have been an excessive difference between the original and a kind of a "copy."

Table 5.7 An empirical econometric equation describing the company's debt recovery efficacy measure (evind), based on monthly data from the years 1996–2006.

Dependent variable: evind				
Method: two-stage least squares				
Date: 04/26/2011; time: 12:16				
Sample(adjusted): 1997:02 2006:12				
Included observations: 119 after adjusting endpoints				
Instrument list: C liq(−1) liq(−3) liq(−4) liq(−11) liq(−12)				
evind(−1) evind(−3) evind(−11) snet(−1) snet(−2) snet(−3)				
jan mar apr may jun jul sep oct nov				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	23.06067	5.918440	3.896410	0.0002
evind(−10)	−0.282791	0.092047	−3.072259	0.0027
liq	−0.007159	0.030786	−0.232528	0.8166
liq(−3)	−0.083816	0.026763	−3.131843	0.0022
snet(−1)	−0.205731	0.048978	−4.200456	0.0001
snet(−2)	−0.288518	0.065474	−4.406576	0.0000
snet(−3)	0.324142	0.050556	6.411493	0.0000
jan	16.33361	6.193039	2.637415	0.0096
mar	37.17356	6.412103	5.797406	0.0000
apr	−12.10009	5.754871	−2.102581	0.0378
jul	−11.35580	5.419012	−2.095549	0.0385
nov	−23.58830	6.277700	−3.757475	0.0003
R-squared	0.701331	Mean dependent variable		−1.987955
Adjusted *R*-squared	0.670626	S.D. dependent variable		25.83556
S.E. of regression	14.82730	Sum squared residual		23523.83
F-statistic	22.89048	Durbin–Watson statistic		2.033665
Prob (*F*-statistic)	0.000000			

This, ultimately, leads to application of the least squares method. As a result, we get the following empirical equation describing the company's debt recovery efficacy:

$$\underset{(4.104)}{\text{evind} = 22.593} - \underset{(3.112)}{0.279\text{evind}_{-10}} - \underset{(3.426)}{0.086\text{liq}_{-3}} - \underset{(4.386)}{0.202\text{snet}_{-1}} + \\ - \underset{(4.519)}{0.290\text{snet}_{-2}} + \underset{(6.677)}{0.321\text{snet}_{-3}} + \underset{(2.908)}{16.809\text{jan}} - \underset{(5.870)}{37.209\text{mar}} - \underset{(2.131)}{12.126\text{apr}} + \\ - \underset{(2.106)}{11.228\text{jul}} - \underset{(3.808)}{23.367\text{nov}} + u_{ev2}, \quad (5.10)$$

$$R_{e2}^2 = 0.705, Su_{e2} = 14.663, \text{DW}_{e2} = 2.039.$$

Equations 5.9 and 5.10, thus, form a recursive set of equations. Feedback 5.8 had collapsed. Only one-way impact occurred $\text{liq}_t \leftarrow \text{evind}_t$.

Let us assume that management of the company’s liquidity and its debt recovery was correct. Will a change of an estimation procedure reveal feedback between the financial liquidity and the debt recovery efficacy? We will apply the ordinary least squares method to estimate the parameters of the equation describing the variable evind, risking a lack of this estimator’s compliance. An empirical equation describing the volatility mechanism of the variable evind has the following form:

$$\underset{(3.297)}{\text{evind} = 15.232} + \underset{(5.704)}{0.202\text{liq}} - \underset{(3.250)}{0.179\text{evind}_{-10}} - \underset{(6.094)}{0.210\text{liq}_{-1}} - \underset{(4.475)}{0.189\text{snet}_{-1}} + \\ - \underset{(4.636)}{0.267\text{snet}_{-2}} + \underset{(6.244)}{0.250\text{snet}_{-3}} + \underset{(6.411)}{37.575\text{jan}} + \underset{(4.499)}{25.874\text{mar}} - \underset{(4.531)}{24.720\text{jul}} + u_{e3}, \quad (5.11)$$

$$R_{e3}^2 = 0.753, \text{Su}_{e3} = 13.409, \text{DW}_{e3} = 1.944.$$

Variable liq_t which closed feedback 5.8 appeared among the explanatory variables. In Equation 5.11, it had turned out to be a strong explanatory variable, thus it confirmed the results of financial management practice in a small-sized enterprise. Characteristics of the matching level to actual data ($R_{e3}^2 = 0.753$, $Su_{e3} = 13.409$, $\text{DW}_{e3} = 1.944$) are significantly better, compared to Equations 5.9 and 5.10. The system of interdependent equations: from Tables 5.7 and Equation 5.11 satisfies expectations of the decision maker managing the company’s finances.

What also requires settlement is the question whether it is more important during econometric modeling of micromodels to obtain a compatible 2LS estimator or to care for effectiveness (precision) that is guaranteed by the LS method[22]? In practical activity of a small-sized enterprise during making short-term decisions, low variances of parameter assessments are more important than asymptomatic properties of estimations.

The discrepancies in theoretical values of the total interdependent variables, obtained from empirical reduced-form’s equations, compared to their actual values, can cause a decrease in effectiveness of parameter estimates of the structural-form’s equations, as a result of application of the 2LSs method. A possible consequence of that can involve “breaking” of feedback, leading to a set of recursive equations, and sometimes even to a simple model. It is worthwhile to find the answer to the question: Why does this happen in some of the cases and in other situations this weakness of empirical reduced-form’s equations has no significance?

It seems that the source of this phenomenon should be sought in various configurations of explanatory variables of the reduced-form's equations, especially all resultant effects of a stochastic interdependency. Additionally, interconnections of the interdependent variables, which remain in feedback relations, in the area of the so-called white-noise dependencies, can play a significant role.

5.6 Econometric forecasting of financial liquidity

In this book, empirical linear econometric model is going to be the tool used for forecast estimation of a small-sized enterprise's financial liquidity during a *T* period. A relative measure of liquidity (liqproc), which is described by the following econometric model 5.12, will be the forecasted variable:

$$\begin{aligned} \mathrm{liqproc} = \underset{(7.631)}{212.59} + \underset{(7.104)}{0.414}\mathrm{liqproc}_{-1} + \underset{(2.984)}{0.19}\mathrm{liqproc}_{-3} - \underset{(11.264)}{1.97}\mathrm{prod} + \\ + \underset{(2.884)}{0.649}\mathrm{prod}_{-9} - \underset{(3.037)}{73.13}\mathrm{jan} - \underset{(2.242)}{66.99}\mathrm{may} - \underset{(2.984)}{83.73}\mathrm{jun} - \underset{(4.009)}{96.55}\mathrm{jul} + u_{\mathrm{lpr}}, \end{aligned} \tag{5.12}$$

$$Su_{\mathrm{lpr}} = \pm 119.71[\%], V_{\mathrm{lpr}} = 87.65\%, R^2_{\mathrm{lpr}} = 0.696, \mathrm{DW}_{\mathrm{lpr}} = 1.987.$$

In model 5.12, there are[23]: liqproc – observations of the explanatory variable, which represent a relative measure of the financial liquidity in the company REX, calculated using Equation 5.2; $\mathrm{liqproc}_{-1}$, $\mathrm{liqproc}_{-3}$ – volumes of the variable liqproc delayed by 1 and 3 months; prod – the value of ready-made production in a *t* month; prod_{-8} – the value of ready-made production delayed by 8 months; jan, may, jun, jul – the dummy variables, taking the value of 1 in the indicated month and the value of 0 in the remaining months – where the variables for January (jan), May (may), June (jun), and July (jul) turned out to be statistically significant; u_{lpr} – represents equation's residuals; *t* – the number of statistical observations (*t* = 1, …, 132).

Model 5.12 shows that the relative liquidity described here is characterized by autoregressions of the first and the third order that are sequential in nature. Impact of the current and the delayed by eight months values of the finished ready-made production is negative, thus it causes reduction of the relative liquidity's level. The dummy variables appearing in the equation indicate that in January, May, June, and in July, the value of the liquidity measure was less than the value of corresponding parameter estimations, in comparison with the so-called systematic component of the model.

Considering the type of statistical data, the stochastic characteristics indicate good description accuracy of the variable liqproc described by model 5.12. It is illustrated by Figure 5.18. All explanatory variables can be considered statistically significant on the significance level $\gamma \leq 0.047$. In the model, there is no autocorrelation of the random component, which is evidenced by the empirical Durbin–Watson statistic ($\mathrm{DW}_5 = 1.987$). As such, this model can be considered a good analytical instrument and a forecasting tool of the company's liquidity.

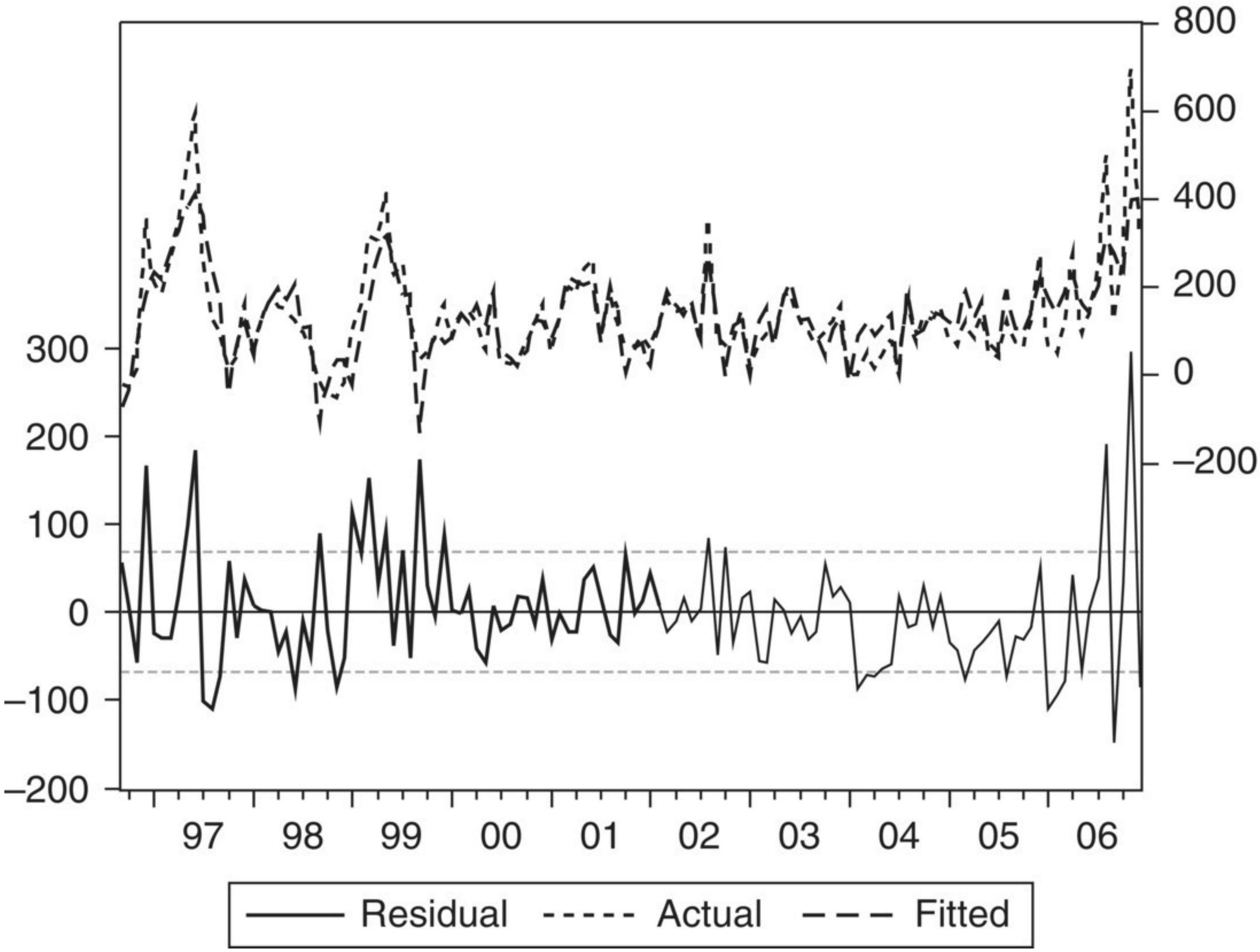


Figure 5.18 The actual and the theoretical monthly values of the variable liqproc (calculated on the basis of model 5.12) during the years 1996–2006.

Further in this work, forecasts of the considered variable characterizing financial liquidity in the company REX were estimated. These forecasts will be short termed in nature and will involve the 12 months of the year 2007.

Forecasts of relative liquidity ($liqproc_{Tp}$) were estimated, assuming limitations of the ready-made production volumes planned for 2007, implemented during successive months of that year. Based on this assumption, the following forecasts are generated automatically. The principle of sequential forecasting is applied here, due to the delayed variables $liqproc_{T-1}$, $liqproc_{T-3}$, and $prod_{T-8}$. The volumes of the dummy seasonal variables are known automatically. Results of the calculations done using the ***GRETL*** package are presented in Table 5.8 and in Figure 5.19.

Table 5.8 Monthly forecasts of ready-made production for the year 2007, in thousands PLN.

Forecasted period	**Forecast ($prod_{Tp}$)**	**Average prediction error**	**Forecast range at a confidence level of 95%**
2007:01	61.2	26.03	9.6 ÷ 112.7
2007:02	65.1	26.03	13.5 ÷ 116.6
2007:03	53.6	26.03	2.0 ÷ 105.1
2007:04	45.8	26.41	−6.5 ÷ 98.1
2007:05	63.6	26.41	11.3 ÷ 115.9
2007:06	41.3	26.41	−11.0 ÷ 93.6
2007:07	25.7	26.42	−26.7 ÷ 78.0
2007:08	44.3	26.42	−8.0 ÷ 96.6
2007:09	70.3	26.42	18.0 ÷ 122.6
2007:10	38.1	26.42	−14.2 ÷ 90.5
2007:11	33.2	26.42	−19.1 ÷ 85.6
2007:12	55.0	27.37	0.8 ÷ 109.2

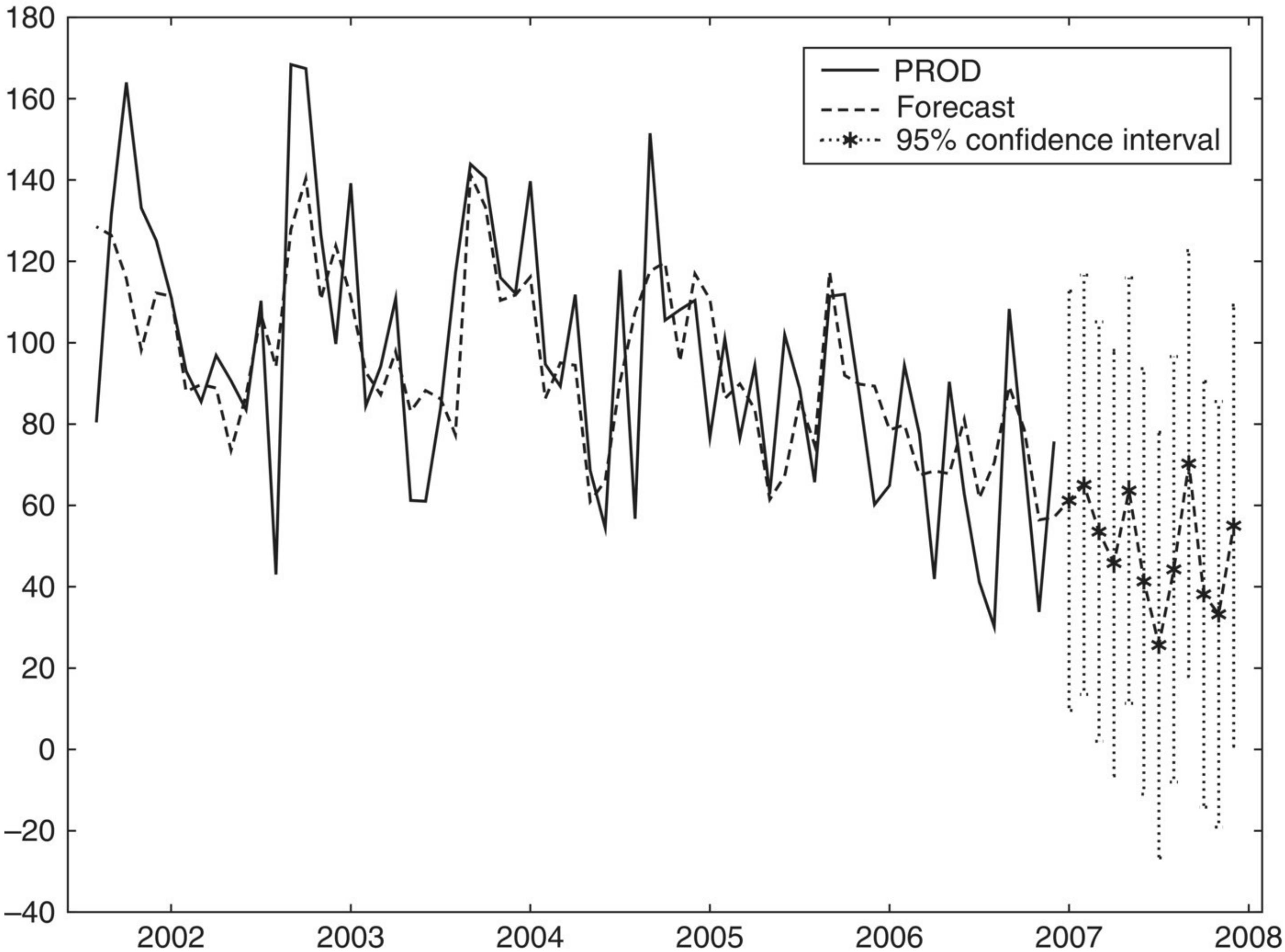


Figure 5.19 Monthly forecasts of ready-made production for the year 2007, in thousands PLN.

Source: Table 5.8 (generated using the GRETL package).

The estimated forecasts are characterized by a precision sufficient for the purposes of current management of a small-sized company. They allow appropriate preparation of the projects assumed during the forecasted period, which comprise obligations to various contracting partners.

It is the responsibility of modern economics to offer simple tools supporting decision-making to business practices, including the persons managing variously-sized enterprises. The simple statistic method of financial liquidity analysis, which is presented in this book, can be widely and easily used in every manufacturing company. It requires, however, creation of an appropriate system for collecting relevant data about the results of the company's past activity. Collection of such important statistical data small-sized enterprises can occur in only when their owners (managers) will recognize and appreciate all significant benefits of possessing such data, especially economic and management benefits.

We are going to estimate the prognoses of $liqproc_{Tp}$ for 12 successive months of 2007. To do so, a predictor emerging the empirical Equation 5.12 is going to be used:

$$\text{liqproc}_{Tp} = 212.59 + 0.414\text{liqproc}_{T-1} + 0.19\text{liqproc}_{T-3} - 1.97\text{prod}_T + \\ +0.649\text{prod}_{T-8} - 73.13\text{jan} - 66.99\text{may} - 83.73\text{jun} - 96.55\text{jul}, \quad (5.13)$$

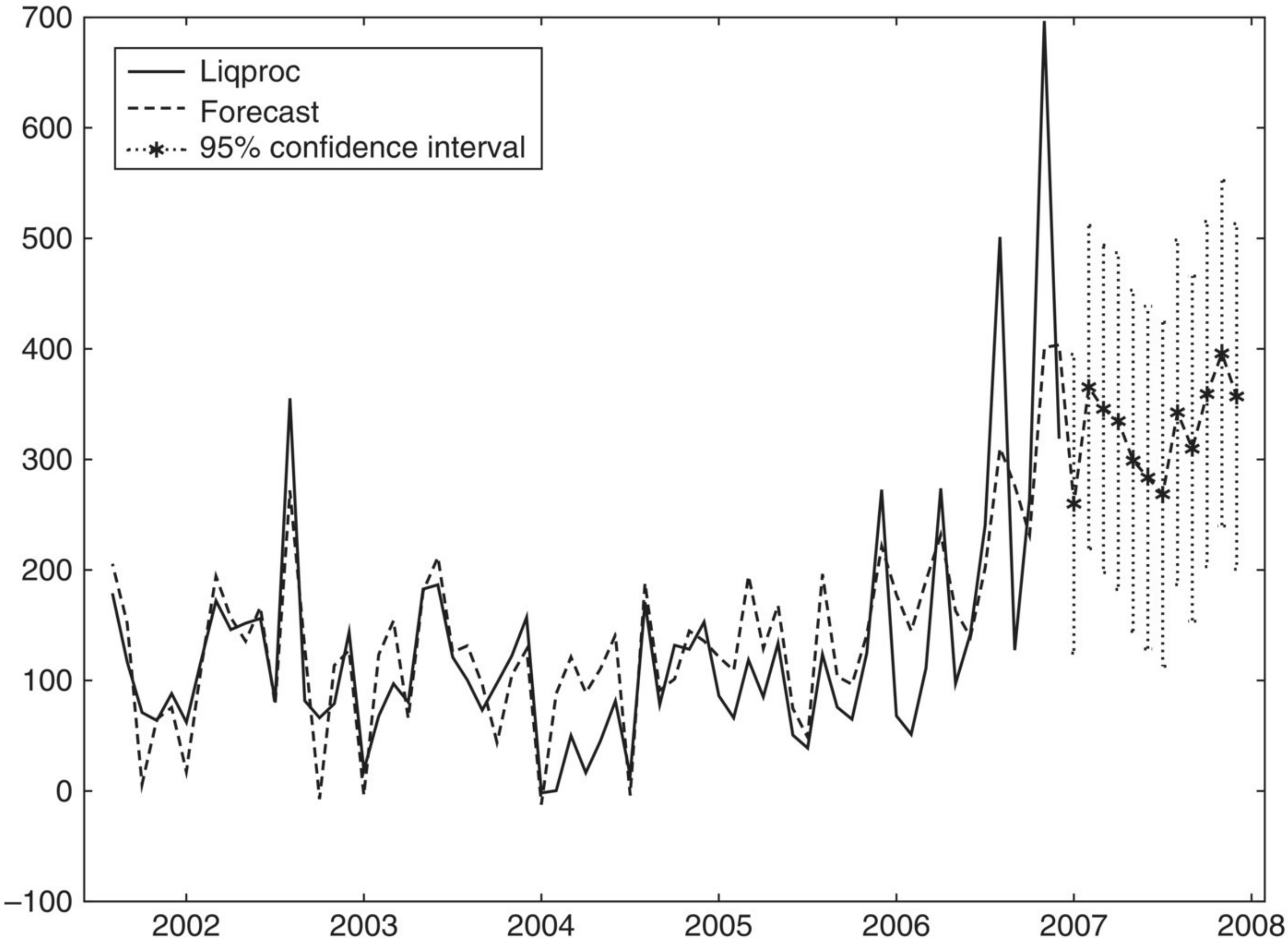
The condition for a forecast estimation of liqproc_{Tp} for successive future time-periods is possession of information about future values of ready-made production (prod_T). Thus, it will be necessary to have a predictor, which allows forecast estimation (prod_{Tp}). As such, an autoregressive-trendy equation describing the volatility mechanism of ready-made production was constructed. It enabled construction of a predictor in the following form:

$$\text{prod}_{Tp} = 18.396 - 0.216T + 0.171\text{prod}_{T-3} + 0.275\text{prod}_{T-11} + 0.514\text{prod}_{T-12}. \quad (5.14)$$

The prognoses of ready-made production for successive 12 months of 1007 were estimated. Estimation results are presented in Table 5.8.

Forecast estimations of ready-made production for successive months of 2007 (prod_{Tp}) are illustrated in Figure 5.19. Large average prediction errors for each of the prognoses (V_T) as well as high relative prediction errors (V_T^*) draw attention. They result from specificity of small-sized enterprise's statistical data. Large amplitude of fluctuations in the time-series of small-sized enterprises results in a significant share of the random variable. Consequently, it is difficult to obtain empirical equations with a high value of R^2. In our case, forecast values of prod_{Tp} play an important role as the expected values at the autoregressive values of explanatory variables and of the time variable T in the predictor equation 5.14. The obtained forecasts of ready-made production allow estimation of a small-sized enterprise's financial liquidity forecasts, expressed by the variable liqproc_{Tp}, in the form of a forecast.

Forecasts of financial liquidity liqproc_{Tp} were estimated using predictor 5.13 (Figure 5.20). Forecast estimations are presented in Table 5.9.



[Figure 5.20] *Forecast estimations of relative liquidity (liqproc$_{Tp}$) and forecast rages at a confidence level of 95%* t *(115, 0.025) = 1.981.*

Source: [Table 5.9] (generated using the GRETL package).

Table 5.9 Monthly forecast estimations of relative liquidity ($liqproc_{Tp}$), of average prediction errors (V_T) and of relative prediction errors (V_T^*) as well as forecast ranges at a confidence level of 95% t (115, 0.025) = 1.981.

Forecasted period	**Forecast ($liqproc_{Tp}$)**	**Average prediction error (V_T)**	**Relative prediction error (V_T^*)**	**Forecast range at a confidence level of 95%**
2007:01	259.91	68.2	26.2	124.75 ÷ 395.07
2007:02	365.37	73.9	20.2	219.07 ÷ 511.67
2007:03	345.77	74.8	21.6	197.64 ÷ 493.90
2007:04	334.68	76.9	23.0	182.39 ÷ 486.96
2007:05	298.80	77.9	26.1	144.44 ÷ 453.17
2007:06	283.51	78.3	27.6	128.42 ÷ 438.59
2007:07	268.49	78.6	29.3	112.87 ÷ 424.11
2007:08	342.56	78.7	23.0	186.61 ÷ 498.51
2007:09	309.70	78.8	25.4	153.59 ÷ 465.81
2007:10	359.20	78.9	22.0	202.00 ÷ 515.40
2007:11	395.98	78.9	19.9	239.73 ÷ 552.24
2007:12	356.96	78.9	22.1	200.67 ÷ 513.25

In case of forecast estimation of a small-sized enterprise's financial liquidity, a relative limiting error at a level 15–20% is admissible. The estimated forecasts of $liqproc_{Tp}$ are characterized by adequate precision – in terms of decisional needs in a small-sized enterprise. They indicate that during the entire 2007 high level of the company's financial liquidity can be expected. It does not mean that this liquidity will emerge spontaneously, or almost automatically. It means that the system of product manufacturing quality as well as the principles of debt recovery, both recently used in the company, should be maintained. It is also going to be necessary to monitor the company's actual financial liquidity during successive months of 2007. Possible excessive negative derogation of the actual liquidity, compared to the forecasted liquidity, will require an adequate reaction on the part of the company's management.

An approach enterprise's financial liquidity[24] – different from those so far presented in literature – makes it possible to obtain more precise results. It allows a detailed insight into the present and into the history of the changes in liquidity. It increases security during a possible credit granting for the company. Particularly, simultaneous examination of the liquidity, using the three research tools proposed here, can effectively increase the description precision of liquidity, and thus enhance the company's competitive strength. Having the time series indicated in this work, allows construction of econometric models describing financial liquidity. Besides its diagnostic properties, such a model can also be used, to estimate shortly-timed liquidity forecasts. Widespread availability of ***GRETL*** (http://gretl.sourceforge.net/) provides an opportunity of sequential correcting the forecasts, performed along with flow of information, due to lapsing of time.

Notes

1 By financial liquidity of an enterprise, we understand its ability to realize the payments resultant for liabilities in a given period of time.

2 We are omitting here the case of the so-called initial production, implemented immediately after founding of the company. In such a case, the funds involved in a small-sized enterprise are almost exclusively those of the owners, sometimes supplemented by the money from his/her family. Only in exceptional cases, cash funds of a small-sized enterprise come from external sources, for example, a bank credit.

3 Compare the work of Wiśniewski J.W. (2003), *Econometric model of a small-sized enterprise*, Section 2.2. Impact of cash inflows on ready-made production is not simultaneous; it occurs with various delays in time. Therefore, the arrow connecting those two variables is marked by a dotted line.

4 Commercial credit is quite common in business activity. Small manufacturing companies must grant it in order to stay in the market game. Being solid payers of liabilities, they also benefit from such credits.

5 Systematic record-keeping of liabilities, with distinction of payment due dates, is very rarely done in small-sized enterprises. Much more attention is paid to the records of claims, since they play a decisive role in accumulation of the cash funds necessary to conduct business in small-sized enterprises. Compare the works of: Wiśniewski J.W. (2003), *Econometric Model of a Small-Sized Company*; Wiśniewska E., Wiśniewski J.W. (2005), *Econometric Analysis of the Impact of the Tax on Goods and Services on the Services in a Small-Sized Enterprise*.

6 There is no legal obligation to accumulate information about the value of ready-made production or about the cash inflows in the companies keeping accounting in the form of a revenue and expenses ledger.

7 The value of ready-made production embodies all the company's liabilities: to the raw material suppliers, to energy suppliers, to employees, statutory obligations, and others. It entails a full mass of liabilities, including the elements not comprising any liabilities, that is, the costs of amortizations and the profit.

8 Compare the work: Wiśniewska E., Wiśniewski J.W (2007), *Econometric Modeling of a Small-Sized Enterprise's Financial Liquidity*.

9 Application of accumulated values results from assumption of the owner's adequate forethought. He/she accumulates funds during the periods of a financial surplus for a period of lower cash inflows. An owner, who does not have accumulative skills, generally, is not able to maintain his/her company on a highly competitive market. The symbol t^* represents the number of the year, while t represents the number of the month in the year t^*.

10 A small observation value of liqrel_t (when it is negative), of about a small percentage, can signify the company has financial liquidity. Only when $\text{liqrel}_t \ll -10\%$, it can threat the ability to pay current liabilities.

11 With such a little deficiency of cash funds, there was no threat to settlement of liabilities, especially those most urgent, that is, statutory obligations, obligations to employees and to the partners with a monopolistic position on the market.

12 In each empirical equations 5.4 and 5.5, there are adequate residuals (u_1, u_2). Moreover, adequate equation accuracy measures were indicated: R_1^2, R_2^2, – the squares of multiple correlation coefficients, Su_1, Su_2, – standard residual errors, DW_1, DW_2 – empirical Durbin–Watson statistics. In each Equations 5.4 and 5.5, corresponding statistics of a t-Student appear under the structural parameters' assessments. The calculations were performed and the graphs were constructed using the EViews 4 package. Both models describe volatility of the company's financial liquidity, with an accuracy sufficient for management practices. None of the equations contain autocorrelations of the random component.

13 Empirical and theoretical accumulated values of the company's financial liquidity are marked on the right ordinate axis. The left axis is used to read the residual values u_1.

14 The designations here are identical to those in the model demonstrated in Table 5.1.

15 Liability for income tax is due on the 20th of each month, that is 50 days from the beginning of its formation. Payment of VAT falls on the 25th day of a month, that is after 55 days from the beginning of its formation. Quarterly payments are valid only for property tax, while payment for consumed electric energy occurs every 2 months.

16 Specificity of conducting a business includes emergence of debtors. An execution payment for the goods and services from those debtors is difficult. It may happen that payments can be executed only after 6 months from issuance of an invoice, as a result of visiting the sales network.

17 The equation's residuals are on the left ordinate axis, while the actual and the theoretical values of a quarterly relative liquidity are on the right axis.

18 For instance, when billing a delivery on the 30th of January – payment at the beginning of March – was considered as paid within the agreed time. Meanwhile, in the statistical sense, payment in March for a delivery in January is characterized by an interval of 2 months.

19 Symbol *t** represents the number of the year, while *n** represents the amount of considered years.

20 The parameters the structural-form's equations in this model should be assessed using the double least squares method (2LS). The values of the empirical *t*-Student statistics will be under the assessments of this equation's structural parameters. Additionally, the values of the determination coefficient (R^2), the standard error of the residual (*Su*), and the value of the Durbin–Watson statistic (DW) are going to be provided.

21 This equation's parameters were assessed using the double least squares method (2LS).

22 A. S. Goldberger (1972) in his work *The Theory of Econometrics*, PWE Warsaw, writes: "(…) despite their noncompliance, the estimators obtained using the ordinary least squares method retain the property of a minimal variance" (p. 454). Further the author writes: "The above analysis suggests, that for small samples, second moments of the estimators obtained by the ordinary least squares method (with respect to the actual value of the parameter) can be smaller than the corresponding moments of the estimators obtained by the 2LS. The variances of the estimators obtained using the ordinary least squares method can be sufficiently small, in order to compensate those estimator's load." (pp. 455–456).

23 The classic least squares method was used for estimation of the model's parameters. The calculations were conducted and the graphs constructed using the EViews 4 package. Empirical values of the *t*-Student statistics are under the structural parameter's assessments. Each of the explanatory variables is statistically significant on the significance level of $\gamma \leq 0.05$. The description accuracy of the variable based on a small-sized enterprise's monthly data, at the R^2 level of about 0.7, should be considered as good. Such accuracy – in case of annual macroeconomic data – is regarded as weak.

24 Traditional approach in corporate finance literature favors statistical and snapshot analysis of enterprises' financial liquidity. This generates a high risk of random results, which would not be relevant to a current situation, that is, during a period of several trimesters. Analysis based on time series eliminates the risk of random results.

6
Econometric model in the analysis of an enterprise's labor resources

6.1 A study of a mechanism of the demand for labor

In an enterprise, the mechanism of the demand for labor can be described using equations, in which employment functions as the explanatory variable. Familiarity with the mechanism of the employment's volatility in an enterprise allows recognition of employment stimulators, inhibitors, as well as some neutral[1] factors. Small enterprises play a particular role in the study on labor demand, since in those companies lays the biggest potential to generate job positions and consequently, the growth of the domestic product. Also, knowledge about labor demand in large- and medium-sized enterprises can foster creation of the regulators of the processes, both within the company as well as in the so-called outside world. In this chapter, various approaches to econometric modeling of employment in as small-sized enterprise will be presented. Similar procedures can be used to analyze employment in medium- and large-sized enterprises.

A relative ease of establishing and developing small-sized enterprises – within certain limits – provides an opportunity of fast economic growth, thereby reducing the number of the unemployed. Therefore, this group of companies constitutes the biggest opportunity for every economic system. Development of small businesses, thus, should belong to the strategic goals defining our future. Therefore, it is necessary to quickly eliminate the barriers hindering both the creation as well as operation of this group of companies. It is urgently necessary to remove the causes of the decreasing labor demand of small companies. In turn, in small-sized companies, it is necessary to implement new technologies of diagnosing the situation on important fields of their activity, which will contribute to rationalization of the decisions.

In this chapter, we will present the consequences of the changes in various external and internal factors, including an increase in a small-sized enterprise's tax burdens, on shaping the size of employment. Internal factors result from the actions of the company's owner, his/her autonomous decisions about the production and its structure, the markets, investments, and the company's organizational structure. External conditions result from actions of the state, the local government, or the competitive environment, as well as from the condition and the dynamics of the job market.

Development of a small-sized enterprise, in the first phase, generally entails an increase in employment volumes. An increase in production requires greater production potential, embodied in its factors. Job resources of a company result from a dynamic economic calculation. This calculation ultimately settles the company's employment trends. The components being considered are both the company's internal elements and external ones. The external and the internal factors sometimes penetrate each other, forming various hybrids.

The products as well as the technology used for its manufacture in reference to the entrepreneur's resources, both decide about the proportions of the production factors implemented by him/her. The considered costs of each of those factors are generally formed outside the enterprise, only partially being formed directly in the company. A group of institutional factors, which are regulated by the state, plays a large part in creating the demand for labor. These include various solutions contained in law, especially in the labor law, such as, for example:

- working time,
- the minimum wage,
- the burdens of various costs connected with hiring and employee,
- the length and variety of vacation time,
- the conditions related to a conclusion and termination of employment contracts, and
- the benefits associated with absenteeism at work and many others.

Institutional solutions form many of the variables in the enterprise, according to a principle of savings. Thus, the impacts of external factors on the volumes of employment in reality are transferred onto the variables, on which it is possible to conduct statistical observations inside the enterprise. The results of this impact, ultimately, are embodied in the company's labor resources.

Figures 6.1 and 6.2 demonstrate the changes in the employment volume in the company REX,[2] respectively, monthly and quarterly in the years 1996–2006. Both graphs reveal a significant increase in employment since 1998.[3] The process of reductions in the company's labor resources started in 1999. Increases in the sales income were obtained through intensification of business activity.

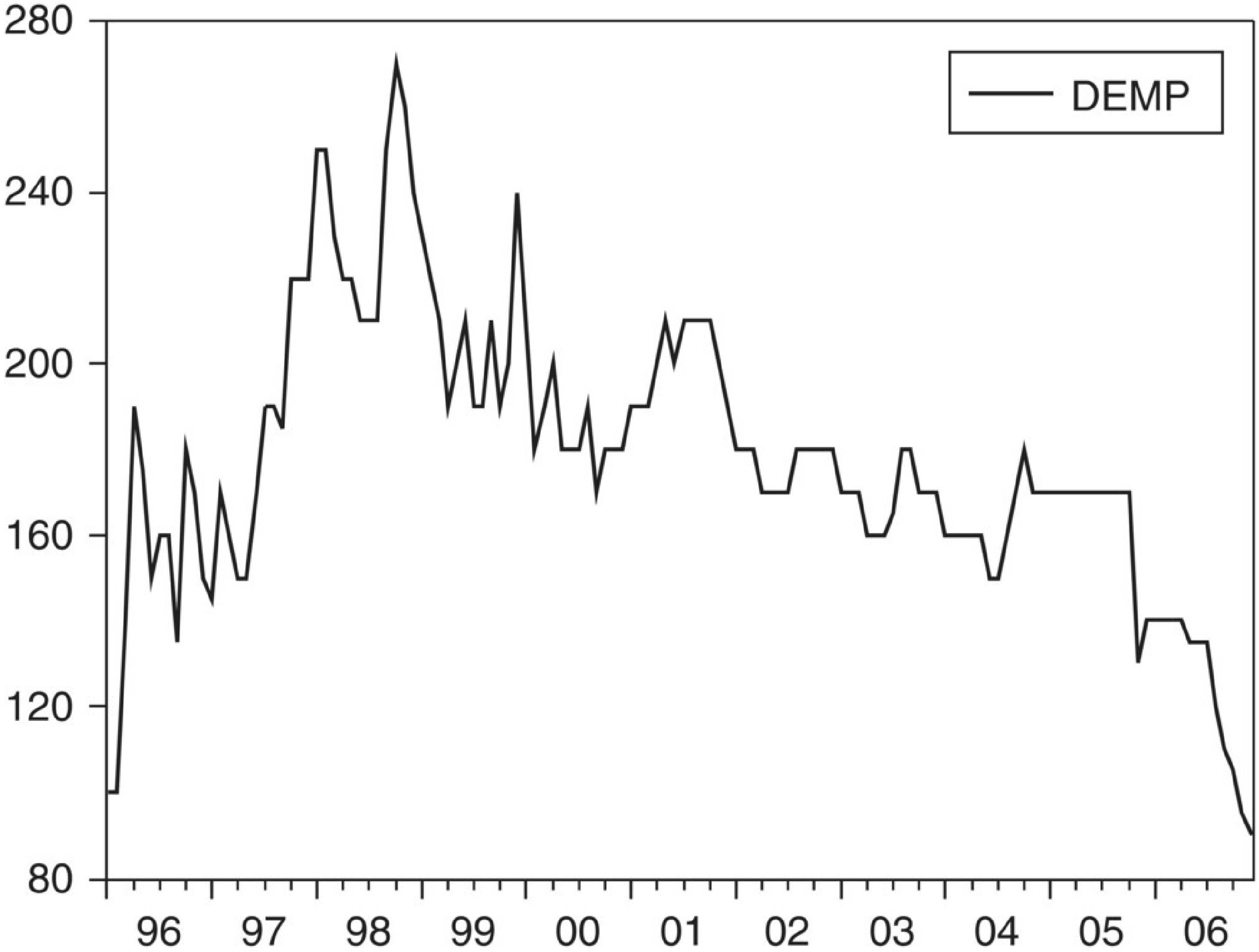


Figure 6.1 The monthly dynamics of employment in the company REX, in the period from January 1996 to December 2006.

Figure 6.2 The quarterly dynamics of the company REX's employment in the period from the first trimester of 1996 to the fourth trimester of 2006.

During the period being considered, the cost of labor in the enterprise had significantly increased. The average net wages increased both monthly as well as quarterly, which is illustrated by Figures 6.3 and 6.4. Similar trends occurred in the cost of labor per one employee, which is demonstrated by Figure 6.5. The requirements of the competition on the market, especially of the price competition, forced the company to adjust the costs to the standards of their competitors. It was necessary to take the measures increasing labor efficiency. This resulted in a decrease in the unit cost of the products, or at least in stopping its increase.

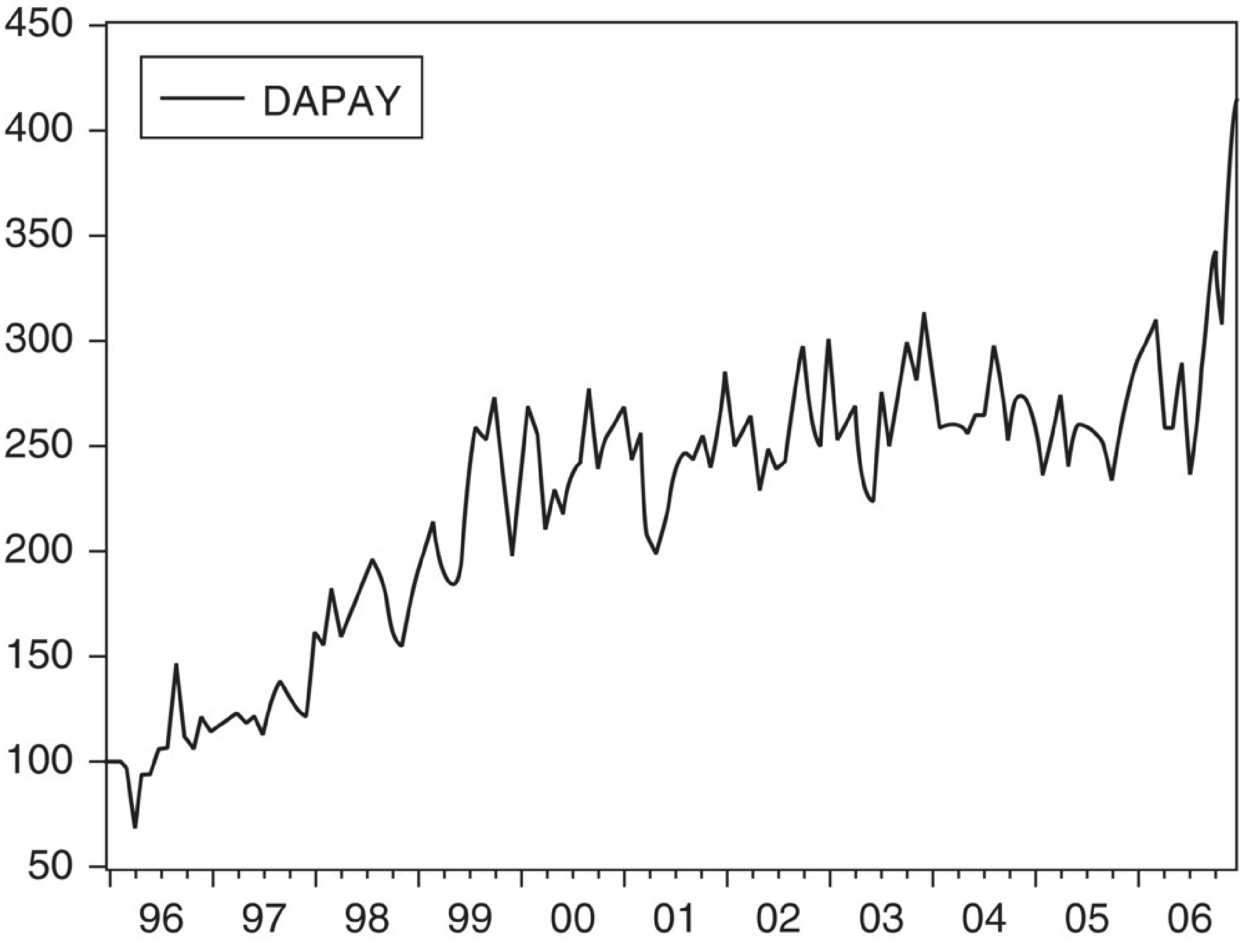


Figure 6.3 The monthly changes of the average net wage in the company REX, in the years 1996–2006.

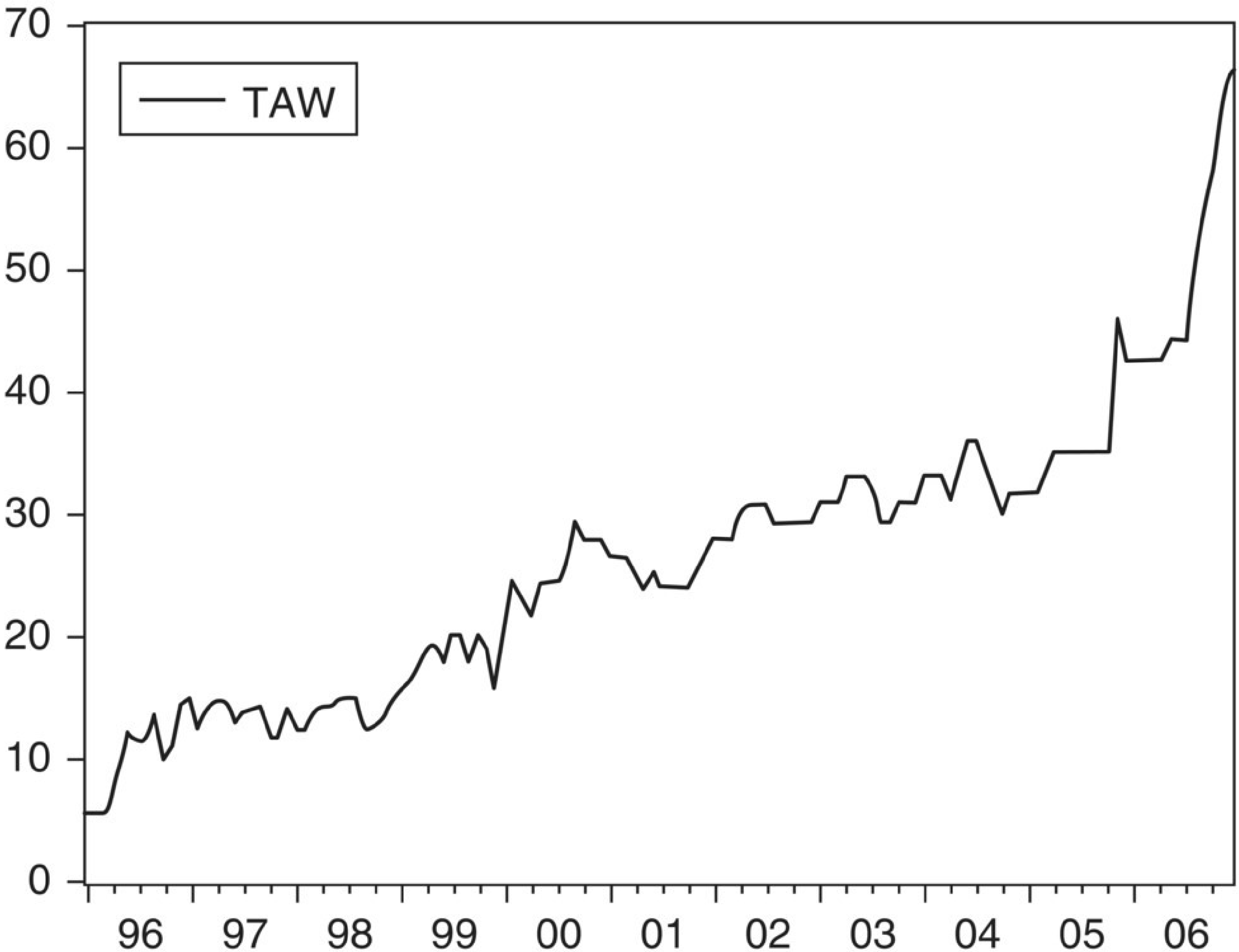


Figure 6.4 The monthly changes in the technical devices into machinery and equipment, in the years 1996–2006.

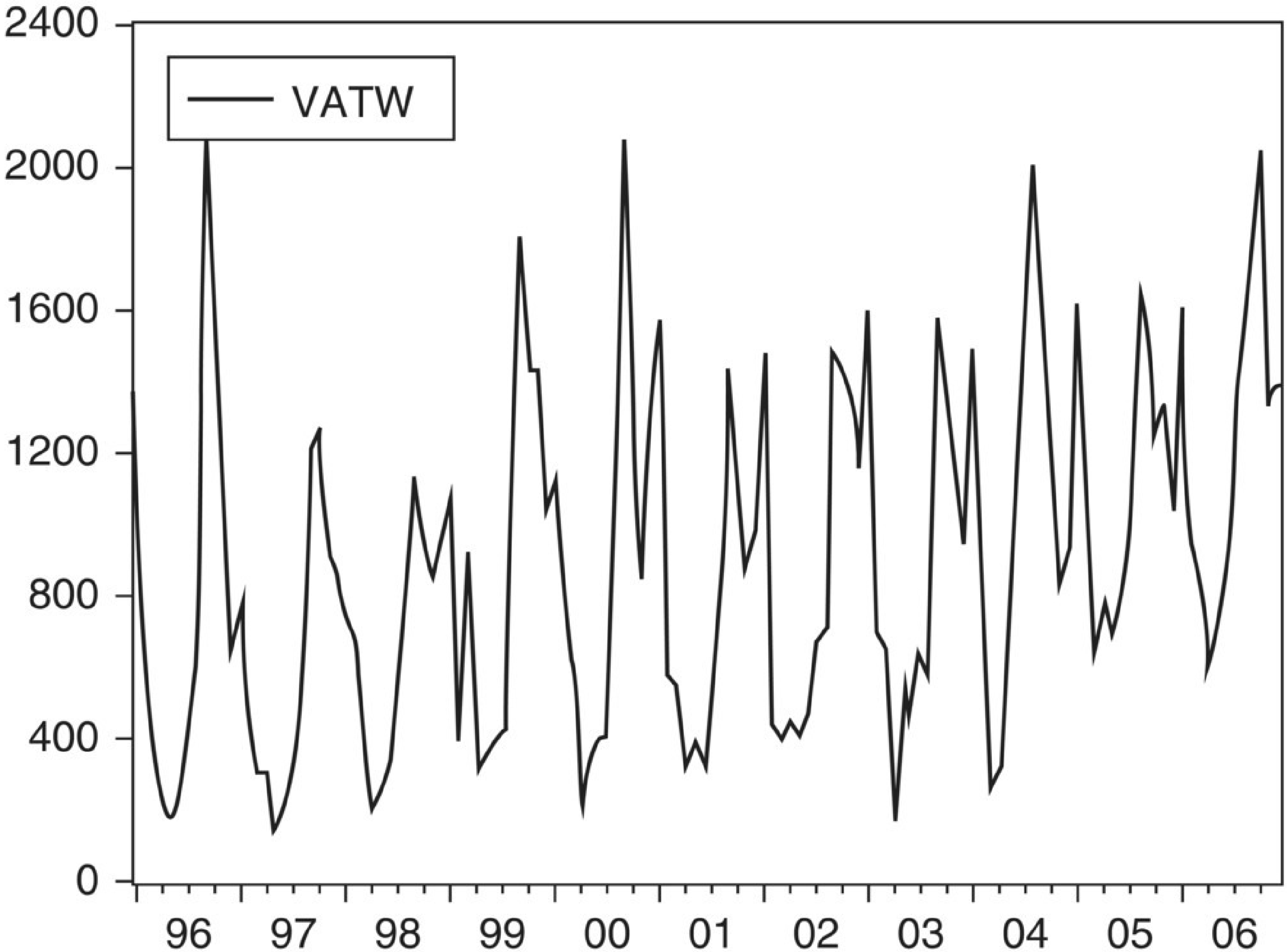


Figure 6.5 The monthly changes of the VAT tax, in PLN per one employee, in the years 1996–2006.

Consequently, some investments[4] appeared which substituted human labor with technology, embodied by machinery and equipment. An increase in the technical devices' level through machinery and equipment (Figure 6.4) caused an increase in the labor efficiency. Simultaneously, during the period being considered, a process of a decline in an economic efficiency of the wages occurred. Changes in the value of the wage efficiency of the net sales meant that out of each thousand PLN of wages in the enterprise, lower and lower net sales income was obtained. In annual terms, during the initial period, out of a 1000 PLN of the net wages, over 12,000 PLN of the sales income was obtained.

Since 1997, the labor efficiency measured as such, which in 2006 had reached the level of just above 4700 PLN of net sales income per 1000 PLN of the net wages, was systematically declining. The curve of this variable's trend is slowly decreasing in character. Maintenance of the market position required keeping up with the competitors' prices. After 2000 begun the process of decreasing prices in this industry, which was forced by a major import of cheap finished products from Asia and especially from China.

In the past period, the burden of the business' fiscal expenses had been increasing. In addition to the part contained within the labor costs, the burden of the tax on goods and services (Figure 6.5) had been increasing as well. In the initial period (1996–1998), the annual value of VAT, calculated per one employee, oscillated around 8000 PLN. In the subsequent years, it had increased systematically, exceeding the amount of 14000 per one employee in 2006.

As a result, a further decline in the profitability of the enterprises forced them to seek savings. Significant savings of expenses were possible in their largest areas. The costs of labor were in this group. Stopping an excessive increase in the labor costs had induced a process of employment volume reduction in the enterprise.

The volume of the enterprise's employment resulted from an impact of various internal and external factors. At the same time, a view can be expressed that the internal and the external factors penetrated each other. As a result of that, hybrid variables formed, which embodied the forces inside the company as well as outside it.

A modeling study based on monthly data, from January 1996 until December 2006, was conducted. The dependable variable is expressed by a dynamics index of a constant base.[5] The variable DEMP here represents the dynamics index of employment (in percentage points). Autoregressions of the 1st and the 12th order[6] were considered in the model. The following explanatory simultaneous variables along with their delays were also considered:

VATW – the monthly value of the tax on goods and services in PLN per one employee,

TAM – the technical devices in machinery and equipment, monthly in thousands PLN per one employee.

Each of the explanatory variables was considered with an adequate time-delay from 1 to 12 months.[7] In the initial hypothetical version of the model, the dummy seasonal variables and the time variable were also considered. They turned out to be statistically insignificant, similarly to the time-delayed variables. As a result, the following empirical[8] model was obtained, which is presented in Table 6.1.

Table 6.1 An empirical monthly econometric model describing the changes in the company REX's employment dynamics.

Dependent variable: DEMP				
Method: least squares				
Date: 03/25/2014, time: 12:34				
Sample(adjusted): 1996:12 2006:12				
Included observations: 121 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	40.64787	8.664094	4.691531	0.0000
DEMP(−1)	0.953657	0.052992	17.99614	0.0000
DEMP(−2)	−0.223040	0.064047	−3.482414	0.0007
DEMP(−3)	0.127650	0.048642	2.624245	0.0099
DEMP(−10)	−0.101014	0.043173	−2.339768	0.0211
DEMP(−11)	0.090973	0.040679	2.236343	0.0273
VATW(−6)	−0.004254	0.001277	−3.332787	0.0012
TAM	−7.230007	0.438921	−16.47223	0.0000
TAM(−1)	6.833970	0.478718	14.27555	0.0000
R-squared	0.964636	Mean dependent var		180.1653
Adjusted *R*-squared	0.962110	S.D. dependent var		32.14507
S.E. of regression	6.257127	Akaike info criterion		6.576788
Sum squared resid	4384.984	Schwarz criterion		6.784739
Log likelihood	−388.8957	*F*-statistic		381.8860
Durbin–Watson stat	1.793400	Prob (*F*-statistic)		0.000000

The model in Table 6.1 describes the mechanism of the employment's[9] volatility with high accuracy. The explanatory variables explain 96.8% of the total employment volatility in the enterprise. The actual values of the dynamics indexes differ from the theoretical values calculated on the basis of the model 6.1, on average, by 6.26% points, which is 3.47% of this index's average monthly value in the years 1996–2006. In the model, there is no autocorrelation of the random component (Figure 6.6).

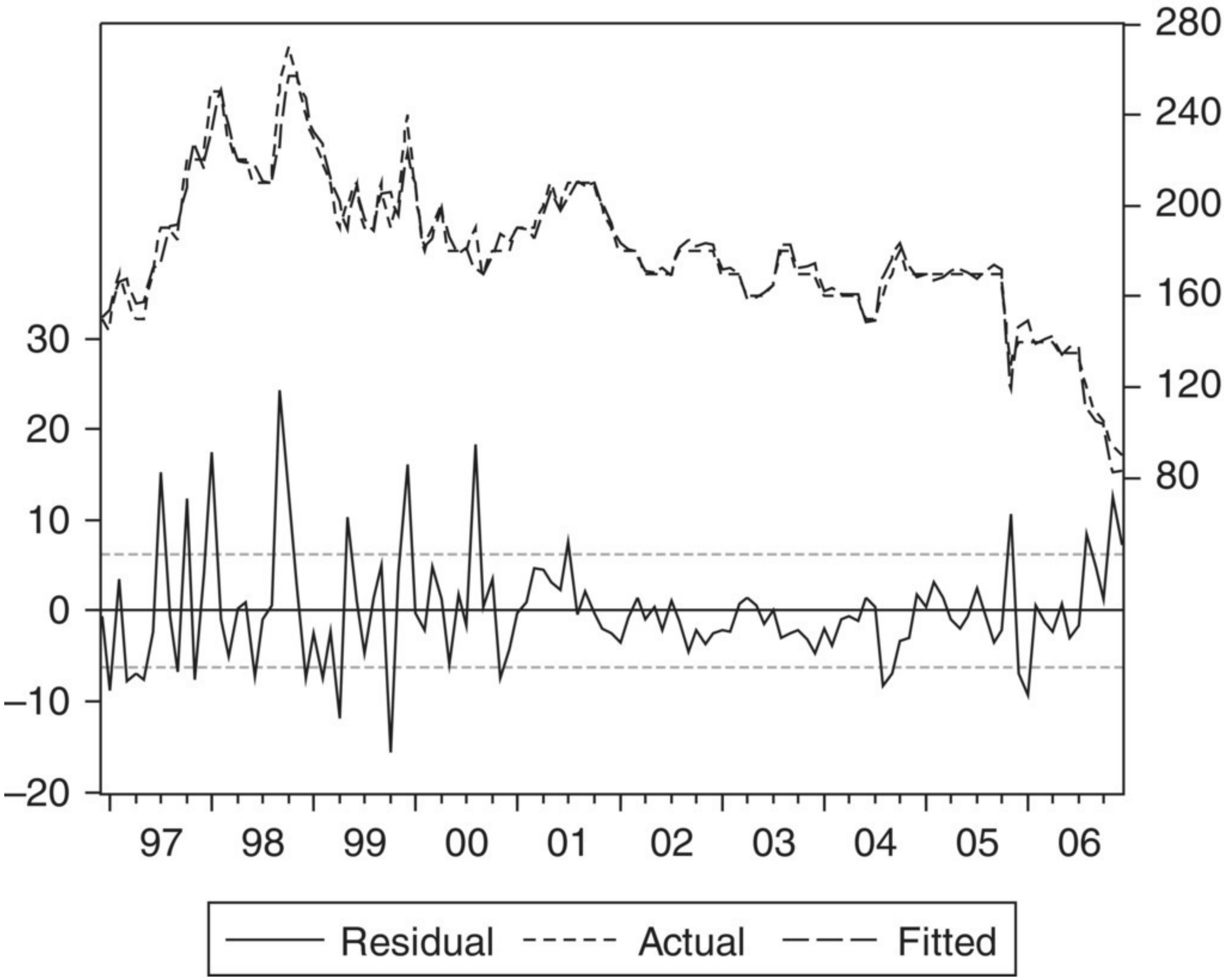


Figure 6.6 The actual and the theoretical volumes of the labor dynamics indexes (of the model from Table 6.1) and the model's[11] *residuals.*

Autoregressive[10] dependencies play the most important role in description of the employment's volatility mechanism. Autoregressions of the 1st, 2nd, 3rd, 10th, and 11th order turn out to be statistically significant. At the same time, an impact of the employment delayed by 2 and 10 months signifies the corrections of the signs in the employment changes with those period lengths.

The amount of the tax on goods and services per one employee plays an important role in the formation of the employment volume. The variable VATW(−6) significantly influences the dependable variable. The sum of the signs along those variables is negative (it is equal to −0.003). This means that an increase in the burden of the tax on goods and services, calculated per one employee, results in a decline in employment after the period of 12 months. An increase of that tax has price-making significance; it results in an increase in the prices. After 1 May 2004, a significant increase in the VAT burden is observed, since over 99% of the goods were covered by the 22% tax rate, while earlier on only about half of the company's goods were under this tax rate.

The current and the delayed by 1 month technical devices in machinery and equipment have significant influence on the size of employment. A negative value of the structural coefficient's assessment is higher, as far as the module and the openness of the positive parameter's assessment are concerned. This means that on the balance sheet, expansion of the technical devices into machinery and equipment induces a decline in the company's employment sizes.

Extracting "clean" internal and external factors, which form the enterprise's employment, is possible only on a sufficiently high level of abstraction. Empirical assignment of adequate variables to one of the abovementioned groups of employment factors does not seem to be possible.

The variables that possibly can be included in the study, stimulating or hindering the size and the structure of employment, essentially have a hybrid nature. They form a kind of a "mix" of various types of factors. Thus, an empirical definition of the variables representing a particular external or internal factor of employment's volatility seems to be an illusion.

The econometric model of employment in a small-sized enterprise presented in this work contains explanatory variables of varied nature. Each of those statistically significant variables is a condensate of many employment factors. Only those possibilities – in fact, limited – of empirical studies on employment factors in a small-sized enterprise exist.

The next econometric model of employment in the enterprise is designed to assess the impact that the burden of the tax on goods and services has on companies. This is due to permanent changes within the taxation legislature. Instability of the institutional system is most burdensome for companies. As a result of this situation, it is difficult to conduct long-term policies within the company. This results in an intensified caution when deciding about increasing resources, especially of one of the production factors, which is the labor potential. This factor, on one hand, can guarantee the company's stability and on the other hand it can potentially be a factor of the biggest risk.[12]

The enterprise of a printing and publishing industry suffered severely from introduction of the standard VAT rate on almost all of its products.[13] This caused significant increases in retail prices for the products manufactured in the company GR. The need had emerged to search for savings, which resulted in a price increase to a lesser extent than the amount of the increase in the tax on goods and services.

The changes in the average rate of the tax on goods and services as well as in the value of this tax[14] calculated per one employee, who followed in the company REX in the years 1996–2006, are demonstrated in Figure 6.7. It shows that in the years 1996–2006, the average VAT rate was stabilized at the level of 14–15%, while from 2004 there was a sharp increase in the rate up to almost 22% in 2005 (Figure 6.8).

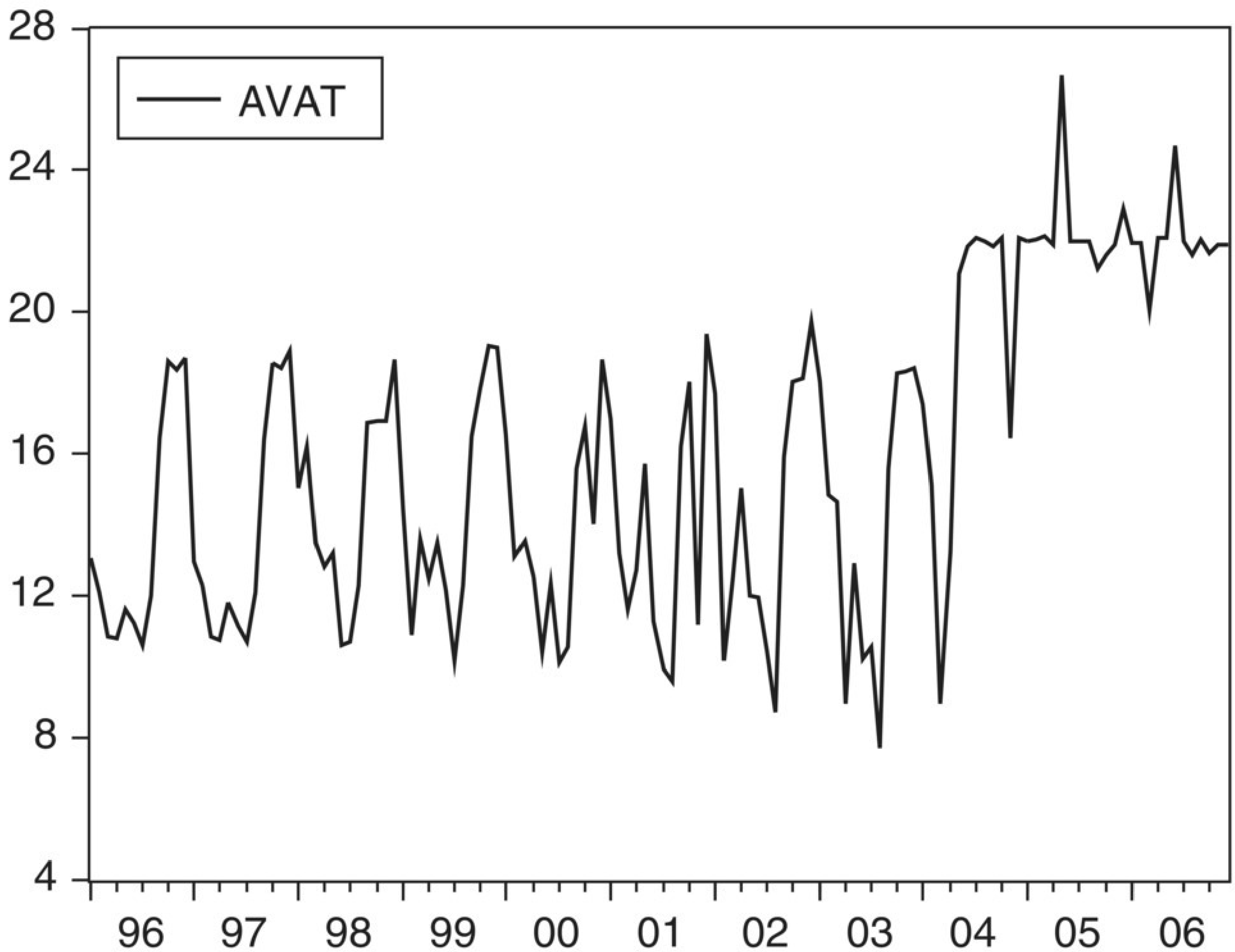


Figure 6.7 Formation of the average VAT rate for the company REX's sales income, in the years 1996–2006 (%).

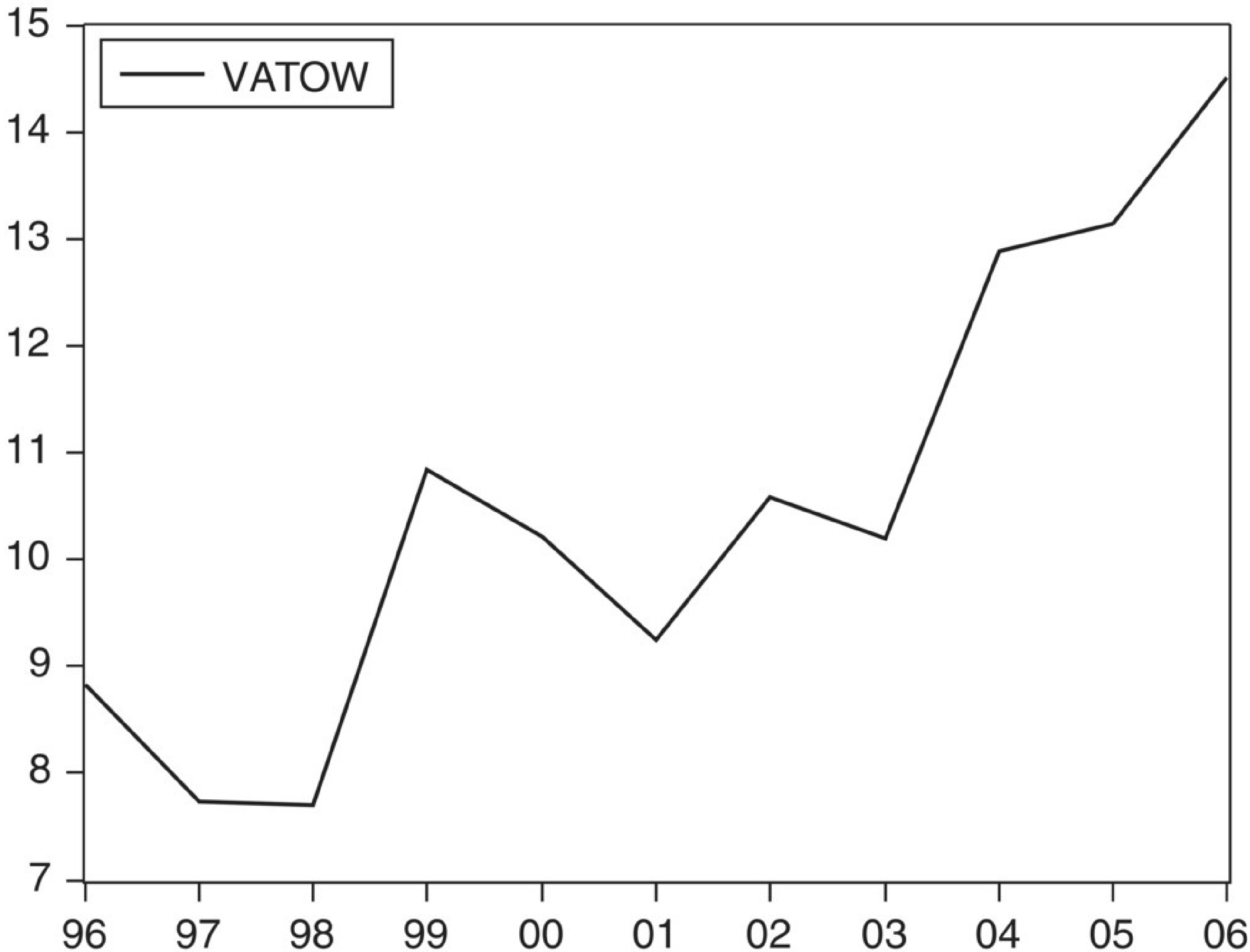


Figure 6.8 The value of the input VAT tax per one employee in the company REX, in the years 1996–2006 (in thousands PLN).

The graph demonstrates the changes in the enterprise's burdens related to the tax on goods and services calculated per one employee. There is an evident upward trend in the value of the input VAT tax attributable to one employee. Table 6.2 demonstrates the empirical autoregressive-trendy equation of the variable AVAT (Figure 6.9).

Table 6.2 An empirical monthly econometric model describing the changes in the VAT tax per one employee, in the years 1996–2006, in the company REX.

Dependent variable: VATOW				
Method: least squares				
Date: 03/25/14, time: 12:47				
Sample (adjusted): 1997:01 2006:12				
Included observations: 120 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	209.8777	81.65109	2.570421	0.0114
VATOW(−1)	0.294970	0.060779	4.853156	0.0000
VATOW(−5)	−0.201949	0.058704	−3.440129	0.0008
VATOW(−12)	0.490617	0.064510	7.605293	0.0000
TIME	2.529976	0.804658	3.144165	0.0021
R-squared	0.725615	Mean dependent var		890.5900
Adjusted *R*-squared	0.716072	S.D. dependent var		479.4315
S.E. of regression	255.4647	Akaike info criterion		13.96482
Sum squared resid	7,505,157	Schwarz criterion		14.08097
Log likelihood	−832.8892	*F*-statistic		76.02994
Durbin–Watson stat	1.720102	Prob (*F*-statistic)		0.000000

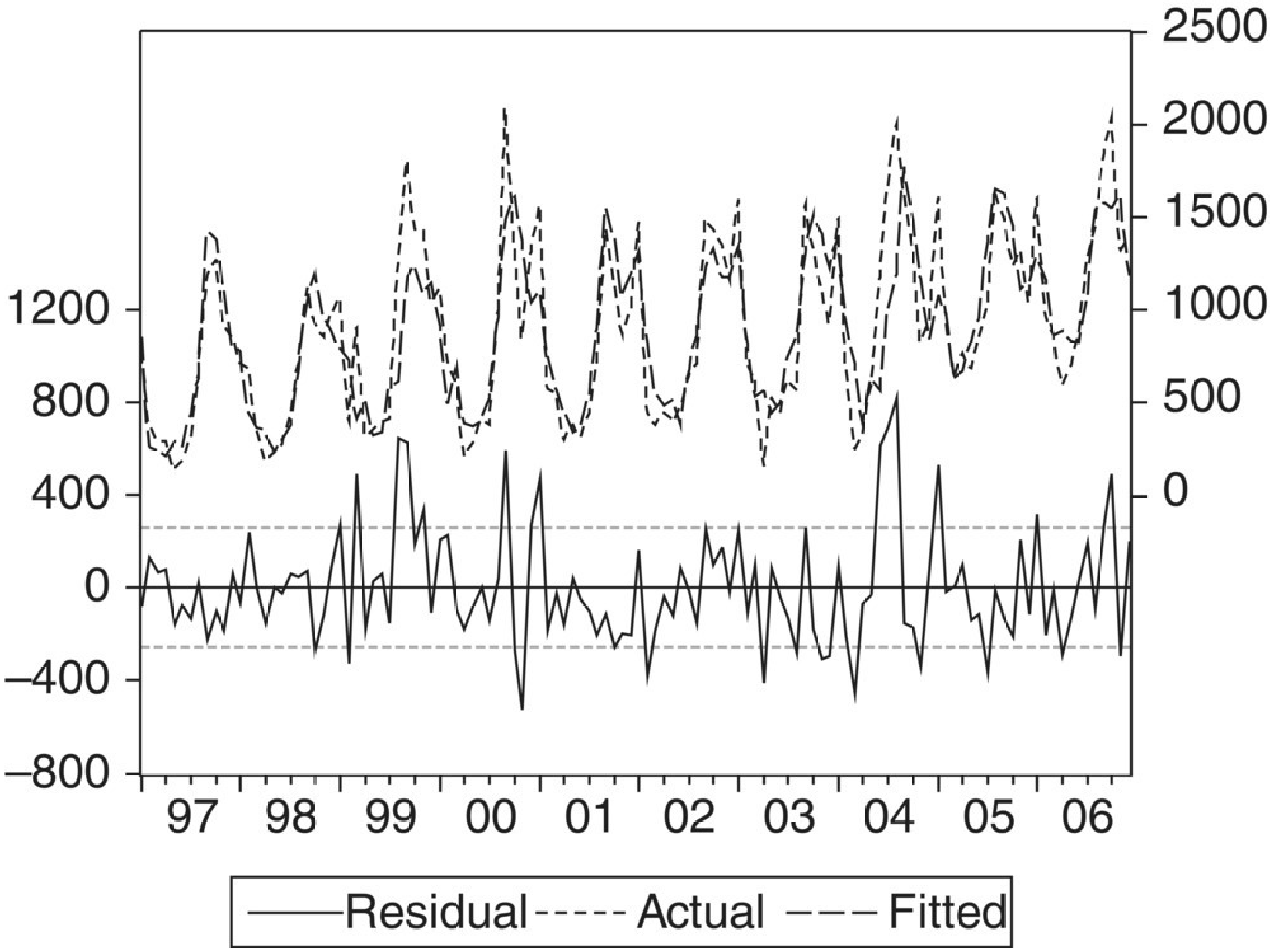


Figure 6.9 The actual and the theoretical values of VAT (of the model from Table 6.2) per one employee as well as the residuals.

An increase in the tax burden, combined with other employment[15] destimulants, causes reactions (especially in microenterprises) resultant in a macroeconomic effect, that is, high unemployment in Poland. The below econometric linear model, describing the company REX's employment mechanism, confirms that.

$$\underset{(0.427)}{\mathrm{EMPL} = 2.496} - \underset{(4.124)}{1.659\mathrm{VATOW}} + \underset{(2.758)}{0.937\mathrm{AVAT}} + \underset{(5.232)}{0.015\mathrm{PROD}} + u, \quad (6.1)$$
$$R^2 = 0.863, Su = 1.31, \mathrm{DW} = 1.509.$$

In the model 6.1, EMPL represents the enterprise's average annual employment volume, VATOW the annual value of the input tax on goods and services attributed to one employee, AVAT the average annual VAT rate on the company's goods and services, and *u* the residuals. The actual and the theoretical employment volumes as well as the residuals, calculated on the basis of the model 6.1, are demonstrated in Figure 6.10.

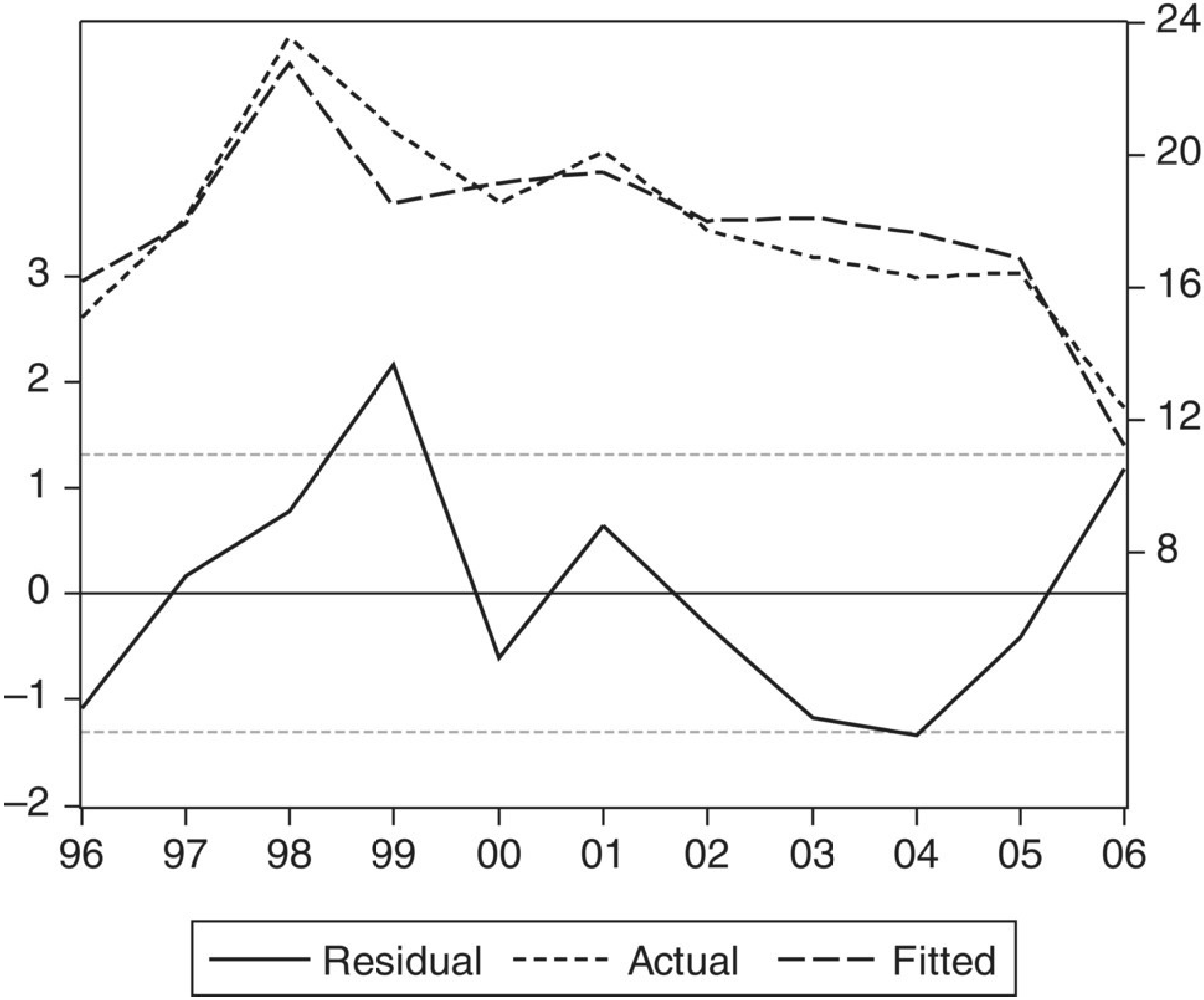


Figure 6.10 The actual and the theoretical employment volumes as well as the residuals, calculated on the basis of the model 6.1.

An alternative variant of an econometric model, describing the employment volatility in the company REX, has the following power-law[17] form:

$$\mathrm{EMPL} = \underset{(2.313)}{0.0279}\,\underset{(2.520)}{\mathrm{AVAT}}^{0.674}\,\underset{(4.365)}{\mathrm{VATOW}}^{-0.815}\,\underset{(5.340)}{\mathrm{PROD}}^{0.919}\,e^{u}, \quad (6.2)$$

$$R^2 = 0.892, V = 2.3\%, \mathrm{DW} = 1.727.$$

Both the above employment equations 6.1 and 6.2 describe the volatility mechanism with similar accuracy. The power-law model is characterized by a slightly better value of the Durbin–Watson statistics (Figure 6.11).

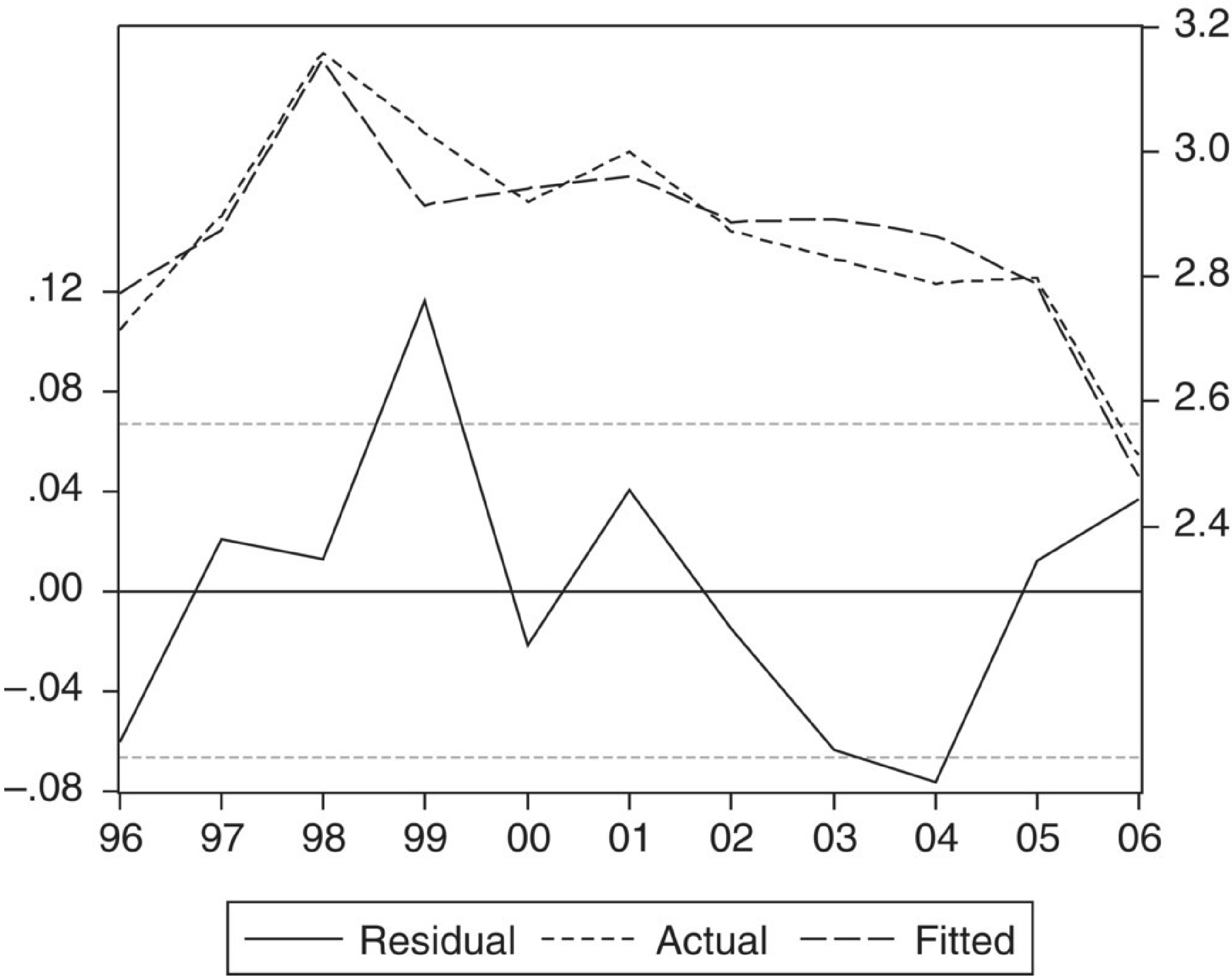


Figure 6.11 The actual and the theoretical employment volumes as well as the residuals, calculated on the basis of the model 6.2.[16]

The empirical models presented clearly indicate the positive role of production and of the annual VAT rate in shaping the employment. The burden of the VAT tax attributed to one employee had a negative impact on the size of employment in the company. Because the VAT rate in the years 2004–2006 had reached its maximum[18] level (22%), an increase in employment resultant from the impact of this variable cannot be expect in the future. At the same time, the ready-made production (PROD) was the employment's stimulator. An increase in the VAT rates on the company REX's products, after all, caused an increase in the retail prices of those products. It resulted in a decrease in the demand for those products, and consequently, a decrease in the net[19] sales income's value. A decrease in the net sales income entails a decrease in employment in the company.

A decline in productive efficiency of the wages and growth of the tax burdens both increase the risk-related economic activity. It is the statutory liabilities as well as payroll liabilities which belong to the category of the so-called rigid ones, that is, the ones absolutely required in a given time. They can significantly decrease and endanger the company's financial liquidity, thus making the entrepreneurship more expensive and risky.

The growth of the company REX's burdens associated with the tax on goods and services attributed to one employee (VATOW) entails a significant decrease in employment. Both models show this. The employment decline observed in the microenterprise REX after 1998 depends on a number of conditions. The causes of employment limitations prior to 2004 were enforced by an additional fiscal factor, which is the growth of the burdens of the VAT tax.

Another model describes the formation mechanism of employment in the enterprise, depending on the fiscal factors. Fiscal operating conditions played a significant role in shaping the demand for labor in small-sized enterprises. During the period passed, the charges to the wages[20] of various contributions increase. The costs of labor exhibited the highest growth rate among all groups of costs. The charges of the benefits for Social Security, personal taxes, and the rising cost of the health insurance significantly contributed to this. The growing charges of fiscal labor[21] costs are illustrated by Figures 6.12–6.14. Evident upward trends of the fiscal labor costs are noticeable.

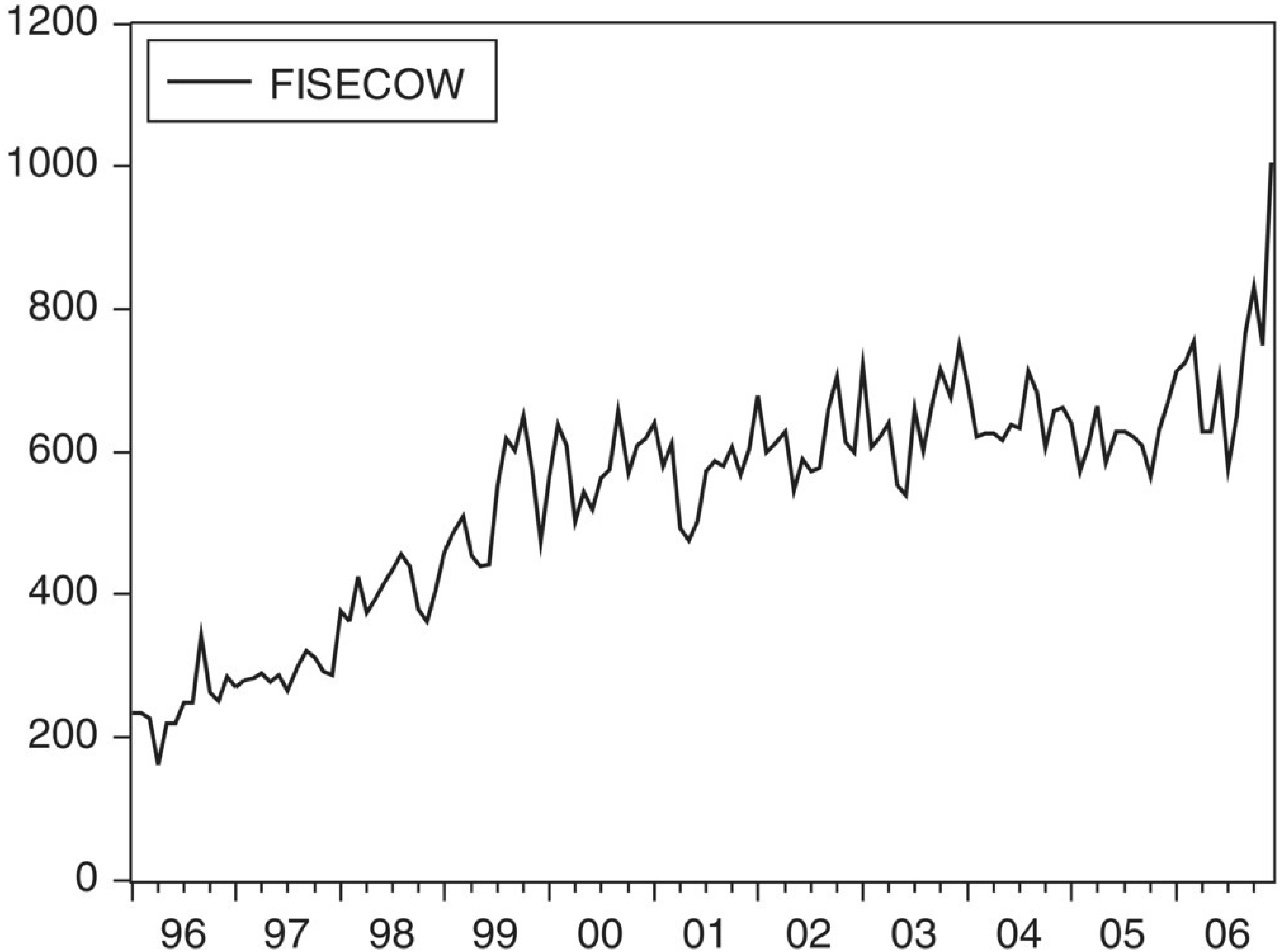


Figure 6.12 The monthly values of the fiscal labor costs in the company REX, in the years 1996–2006 (in PLN per one employee[22]*).*

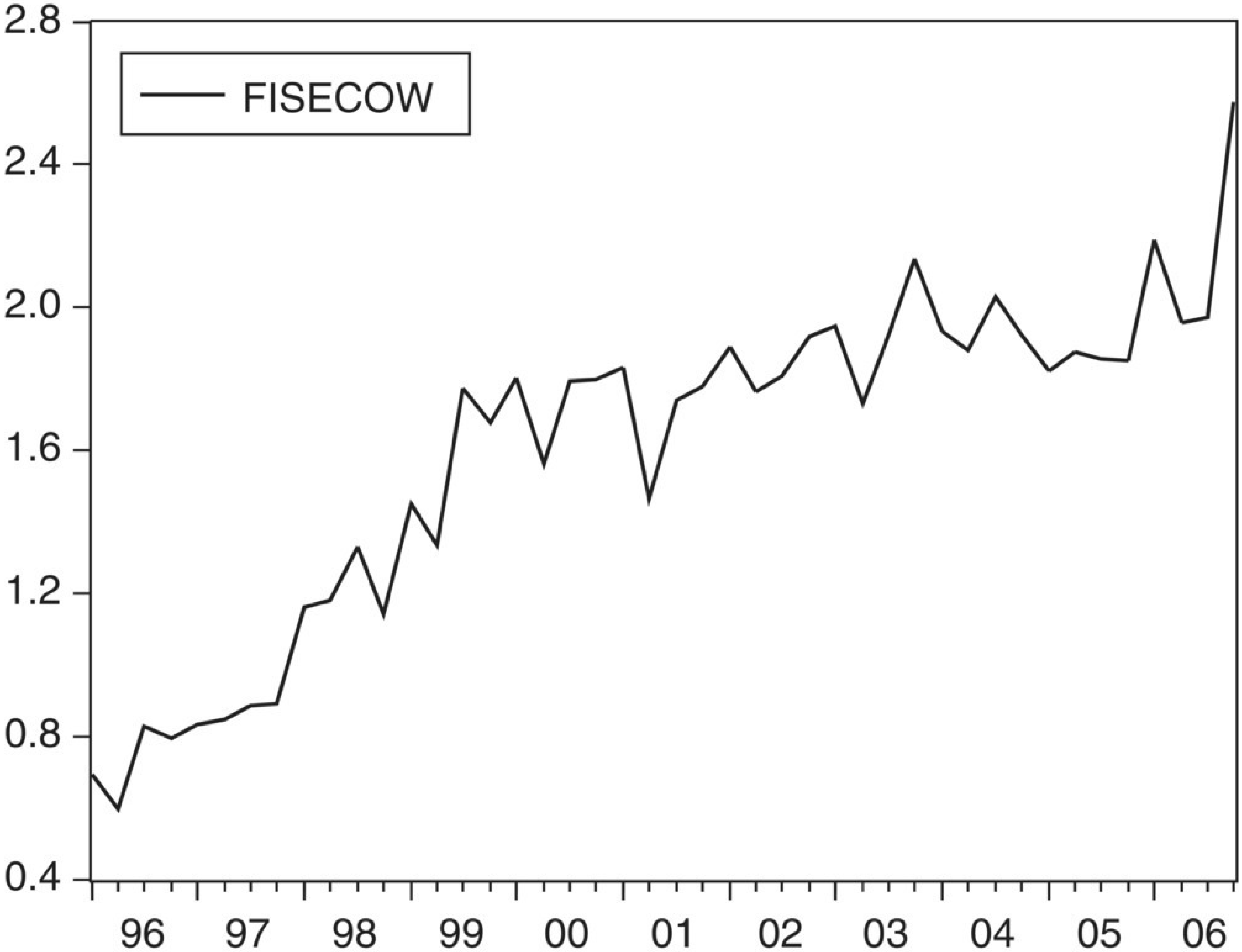


Figure 6.13 The quarterly values of the company REX's fiscal labor costs, in the years 1996–2006 (in PLN per one employee).

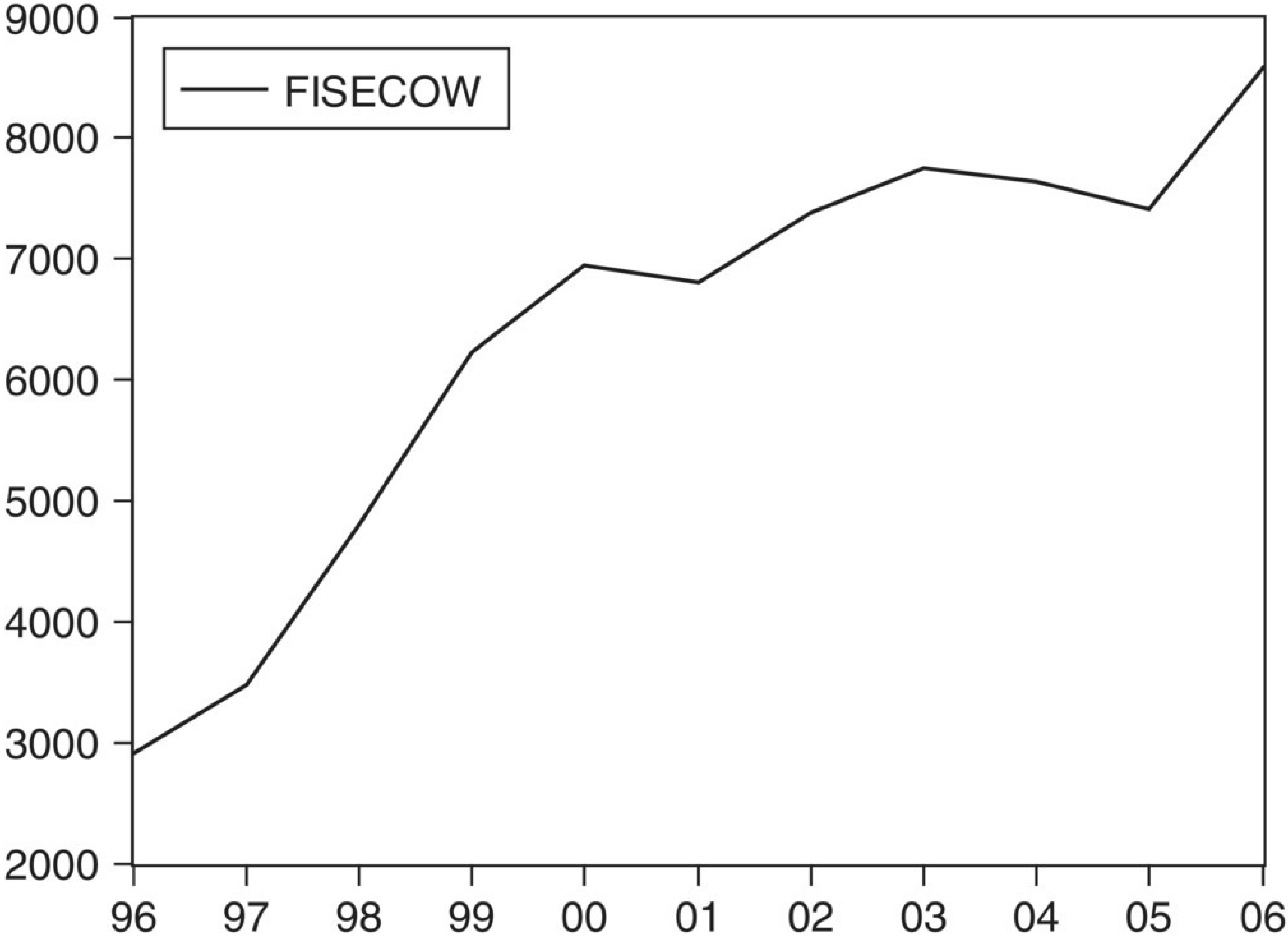


Figure 6.14 The annual fiscal labor costs in the company REX, in PLN per one employee.

Significant seasonal fluctuations of the tax burdens, resultant from the seasonality of the net sales income, are quite natural here. The demand's specification occurring in the industry is the root cause of the seasonality of both, the fiscal charges to the labor costs per one employee as well as the burden of the VAT tax counted per one employee.

A study of the impact of the tax burdens on the changes of the company REX's employment volume was conducted. The possibility of autoregressive dependencies of a trend occurrence and of seasonal fluctuations were also taken into consideration. An econometric linear model based on monthly data, covering 132 statistical observations from January 1996 to December 2006, was constructed. In the model the following variables occurred:

EMPL – the average monthly number of employees,

$EMPL_{-1}$, …, $EMPL_{-12}$ – the variable EMPL delayed adequately from 1 to 12 months,[23]

AVAT, $AVAT_{-1}$, …, $AVAT_{-12}$ – the value of the VAT tax in PLN per one employee monthly in a current month and the delayed values adequately from 1 to 12 months,

FISECOW, $FISECOW_{-1}$, …, $FISECOW_{-12}$ – the value of the fiscal labor costs in PLN counted per one employee monthly in a current year and the delayed value adequately from 1 to 12 months.[24]

The empirical model demonstrated in Table 6.3 was obtained as a result of numerical calculations.

Table 6.3 The empirical monthly econometric model describing the changes in the company REX's employment dynamics, depending on the fiscal factors.

Dependent variable: EMPL				
Method: least squares				
Date: 03/25/2014, time: 14:52				
Sample(adjusted): 1996:12 2006:12				
Included observations: 121 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	2.059000	1.028113	2.002698	0.0476
EMPL(−1)	0.940299	0.082076	11.45641	0.0000
EMPL(−2)	−0.326948	0.108716	−3.007367	0.0032
EMPL(−3)	0.331766	0.080076	4.143147	0.0001
VATOW(−11)	0.000680	0.000221	3.076332	0.0026
FISECOW	−0.007940	0.001700	−4.669978	0.0000
FISECOW(−1)	0.004921	0.001725	2.853569	0.0051
R-squared	0.895239	Mean dependent var		18.01653
Adjusted *R*-squared	0.889725	S.D. dependent var		3.214507
S.E. of regression	1.067462	Akaike info criterion		3.024556
Sum squared resid	129.9002	Schwarz criterion		3.186296
Log likelihood	−175.9856	*F*-statistic		162.3651
Durbin–Watson stat	1.933110	Prob (*F*-statistic)		0.000000

The model is characterized by good stochastic qualities. The explanatory variables considered in the equation explain 89.5% of the total volatility of the single-base employment index. Empirical values of the employment index differ from their corresponding theoretical values, which were calculated on the basis of the equation from Table 6.3, by approximately one employee. The value of the Durbin–Watson statistic indicates that in the model, there is no autocorrelation of the random component (Figure 6.15).

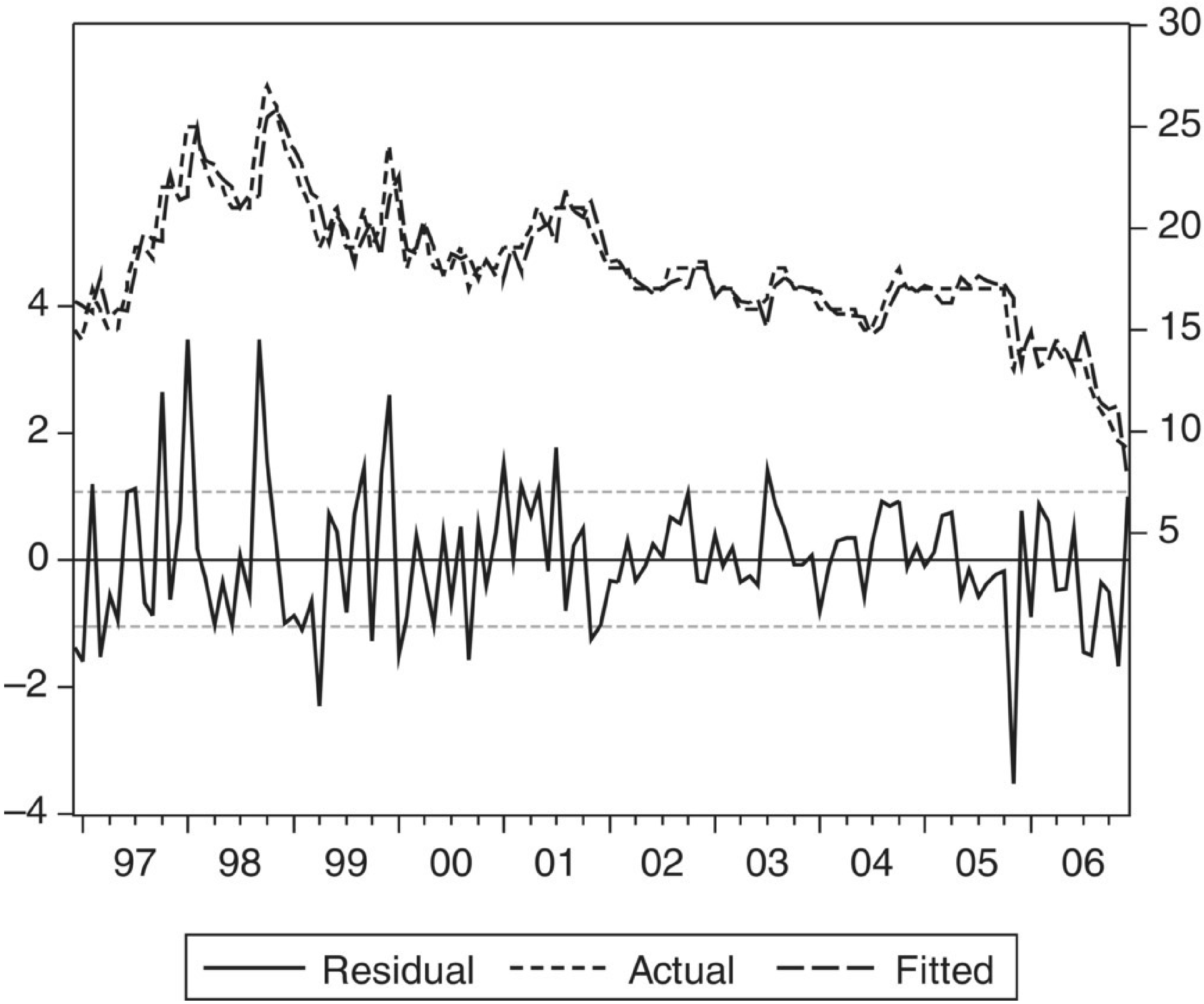


Figure 6.15 The empirical and the theoretical (calculated on the basis of the model from Table 6.3) employment volumes (EMPL) as well as the residuals.

The biggest role in the formation of employment's volatility in the enterprise plays autoregression, wherein autoregressive dependencies of the first, the second, and the third order have turned out to be statistically significant. This signifies employment inertia, wherein the impact of the employment level delayed by 2 months is characterized by a negative sign of the structural parameter's assessment, which signifies commutativity of the fluctuations in a 2-month cycle. Autoregressive dependency of the first order is the strongest and it indicates that over 94% of the number of the employed in the previous month makes up current month's employment.

The value of the tax on goods and services calculated per one employee delayed by 11 months positively influences the volume of employment. An increase in the burden of the VAT tax attributed to one employee entailed an employment increase after 11 months. The burdens of the fiscal costs of employment calculated per one employee negatively influence the current level of employment and positively influence it with a delay by 1 month. However, the sum of the parameter estimates along the variables FISECOW and $FISECOW_{-1}$ is negative and equals to around −0.0012. This means that the overall relative growth of the burdens of the fiscal labor costs results in a decrease in employment volumes, in the considered here company REX.

In the above models, it has been shown that an excessive growth of the burdens of the fiscal costs to the wages in an enterprise produces consequences in the form of a decline in the demand for labor. Also, the excessive amounts of the tax on goods and services, in a small-sized enterprise, result in a decrease in the level of employment. An increase in the VAT rates causes a decrease in the demand, which results from risen prices of the commodity. Smaller production requires less employment, which ultimately was revealed by the above presented econometric models.

6.2 Econometric modeling of labor intensity of production

Labor-intensity of production as well as labor efficiency belong to a common family of economic categories; one is an inverse of the other.[25] Each of those concepts is adequately relevant in an enterprise, providing it with information of a diverse, but a mutually complementary purpose. Ability to measure efficiency and labor intensity is a prerequisite for an accurate diagnosis of the situation and an appropriate support of the ownership decisions in a small company.

A known weakness of Polish small-sized enterprises is their informational negligence. The essence of this neglect lies in the lack of the time-series important for decision-making, which ought to be accumulated to improve decision-making processes. Even the mandatory information, including all the data related to statutory accounts, is not joined together into appropriate series of statistical data. All the more, we cannot speak about its appropriate processing, to prepare the foundations for at least rational decisions. Diagnosis of the situation as well as analysis and prediction of important changes in the enterprise is extremely difficult.

In this part of the book, the tendencies of the changes in the labor intensity of production in a small-sized enterprise will be presented. Depending on the manner of defining the numerator and the denominator of the labor-intensity formula ($LC = L/P$), various volumes of different economic sense can be obtained. Let us consider two concepts of the labor input (L) in a small company and four conceptual formulas of the production (P).

In Table 6.4 and Figure 6.16, the time series of labor intensity of production in a small-sized enterprise,[26] expressed in monetary units, were demonstrated. Labor input, appearing in the numerator, represents the annual, quarterly, or monthly labor costs. The production volume, appearing in the denominator, stands for the net sales income, the cash inflows, the gross sales income, and the values of the finished ready-made production in the sales prices. As such, the numbers in Table 6.4 represent the annual labor costs (in PLN) attributed to one thousand of the production value in one of the meters listed here.

Table 6.4 The company REX's labor intensity of production in monetary[27] units (annually).

Source: The company's documentation.

Year	LABCN	LABCC	LABCB	LABCP
1996	122.8571	102.2092	106.9026	114.8168
1997	169.5511	146.7834	147.2799	167.2178
1998	237.0013	228.9064	205.7237	225.8646
1999	211.2210	199.5265	183.7073	237.7567
2000	238.3379	207.7849	208.5811	234.7235
2001	254.1660	242.9409	222.8236	263.8387
2002	253.2822	226.4211	220.6642	250.9600
2003	267.2298	221.7364	233.6039	252.7890
2004	282.0118	219.2015	235.8800	249.8138
2005	298.8716	244.5517	245.1226	283.1824
2006	313.7652	243.3860	257.1385	315.9358

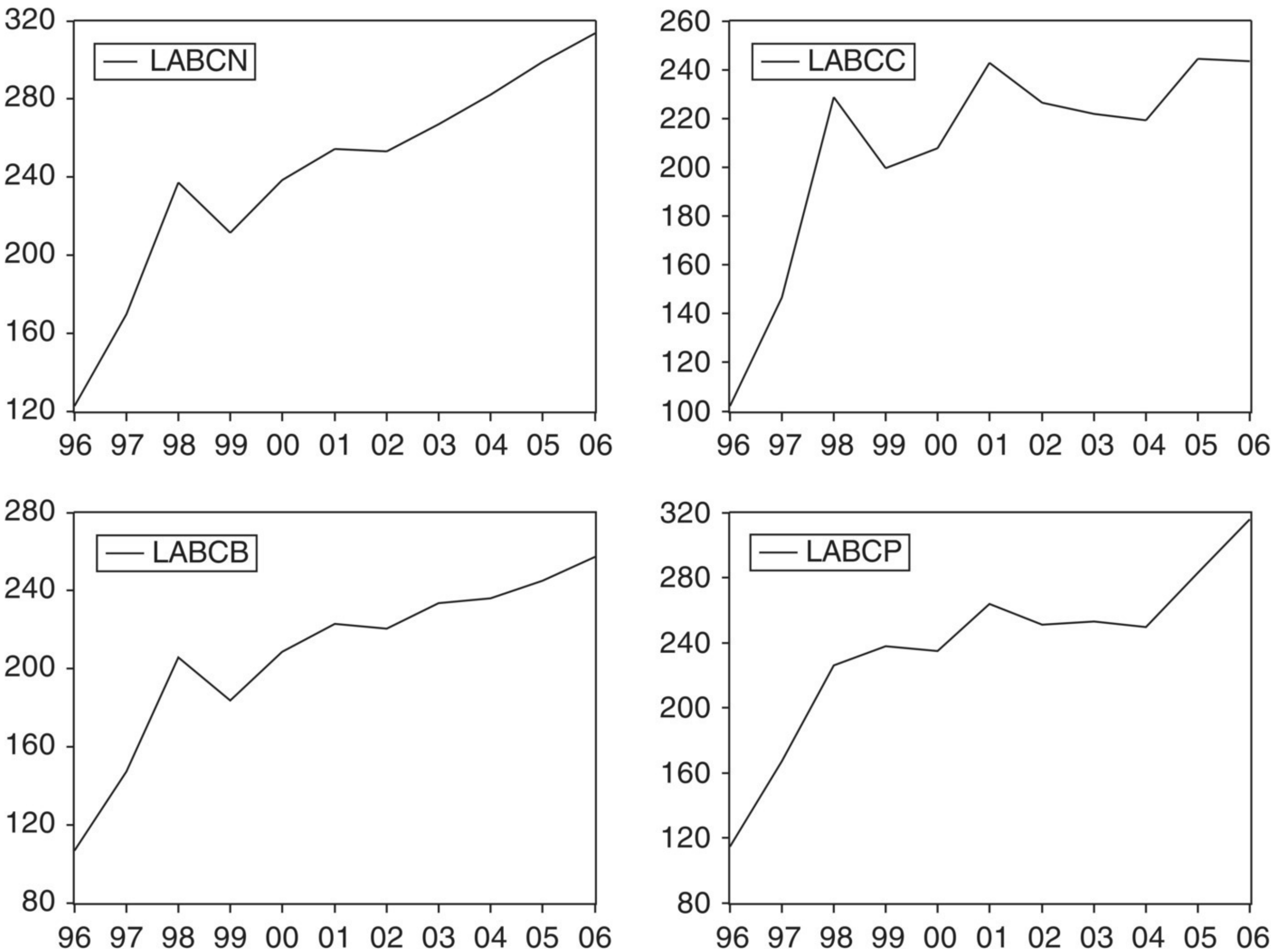


Figure 6.16 The company REX's labor intensity of production in monetary units.

Source: Table 6.4.

Symbols in Table 6.4 are:

LABCN – the cost of labor in PLN attributed to 1000 PLN of the company's net sales income,

LABCC – the cost of labor in PLN attributed to 1000 PLN of the cash inflows,

LABCB – the cost of labor in PLN attributed to 1000 PLN of the gross sales income,

LABCP – the cost of labor in PLN attributed to 1000 PLN of the finished ready-made production's value.

In the years 1996–2006, a systematic increase in the labor costs attributed to 1000 PLN of the production value can be seen in each of the labor-intensity's financial measures that were used. Especially in the 1990s, the labor intensities of a thousand of the production value, of the sales, and of the realization of the company's[28] receivables grew rapidly. Since 2000 the dynamics in each of the time-series considered here decreased significantly, approaching the saturation level. The best situation, from the perspective of the company's interests, is observed in the labor intensity of the ready-made production, where after 2001 stabilization under this year’s level occurred.

Considering the financial measures of the labor involvement into manufacture and sale of a small-sized enterprise’s production, various upward tendencies of the above presented measures of the labor intensity can be noticed.

The labor intensity of production (of the manufacture and the sales), expressed in a natural measure of labor involvement, has been defined using four measures, which demonstrate the changes in the demand for labor, measured by the average annual number of job positions. These are as follows:

LABCD – the average annual number of employees attributed to one manufacture of the ready-made production worth 1000,000 PLN,

LABCB – the average annual number of employees attributed to 1000,000 PLN of the gross sales income,

LABCN – the average annual number of employees attributed to 1000,000 PLN of the net sales income,

LABCC – the average annual number of employees attributed to 1000,000 PLN of the cash inflows.

Statistical information about the labor intensity of production, based on the average annual employment, is presented in Table 6.5 and Figure 6.17. Since 1998, there is a marked drop in the demand for workers in the small enterprise. The emerged phenomenon of a decreasing demand for labor is a reaction to the effect of a dynamic increase in the labor cost during that period. Investments in the fixed assets of a substitution character, in terms of the labor factor, had the desired effect of reductions in the job positions needed in the company.

Table 6.5 The company's labor intensity of production in natural units (annually).

Source: The company's documentation.

Year	LABCP	LABCB	LABCN	LABCC
1996	15.79058	14.70215	16.89636	14.05667
1997	19.28739	16.98769	19.55652	16.93042
1998	18.81204	17.13452	19.73961	19.06539
1999	15.54954	12.01465	13.81407	13.04924
2000	13.75260	12.22090	13.96437	12.17426
2001	15.78865	13.33422	15.20982	14.53808
2002	13.84688	12.17529	13.97501	12.49293
2003	13.35549	12.34189	14.11843	11.71490
2004	13.44420	12.69432	15.17699	11.79673
2005	15.81584	13.69018	16.69208	13.65829
2006	15.28721	12.44219	15.18219	11.77674

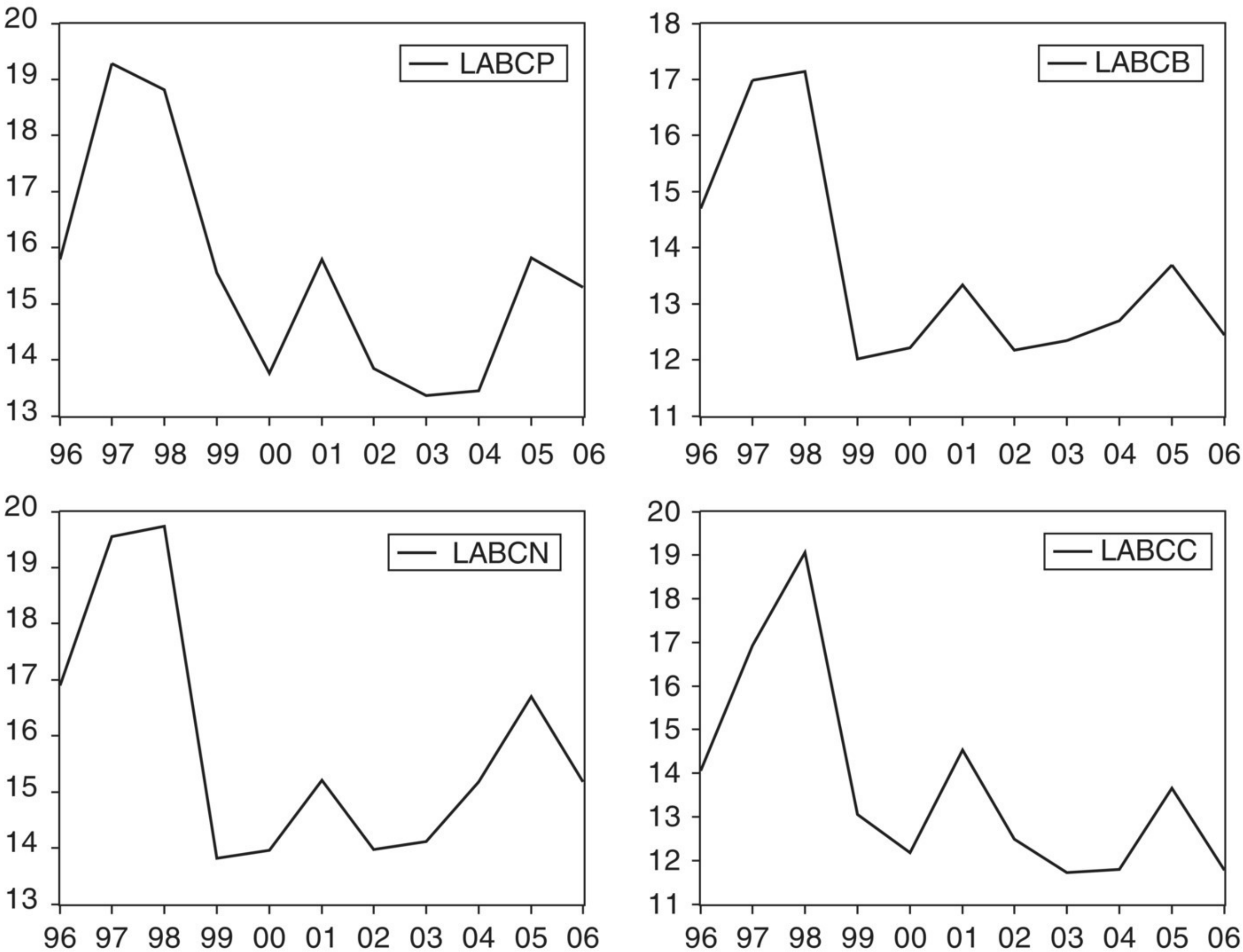


Figure 6.17 The company REX's labor intensity of production in natural measures.

Source: Table 6.5.

The econometric model of the labor intensity of production, based on the monthly data, describes the formation mechanism of the variable LABCD, which represents the value of the labor costs (in thousands PLN), attributed to 100,000 PLN of the ready-made production's value (in the net sale prices) is presented in Table 6.6. The empirical econometric model, resultant from estimation of the parameters of the finished production workload model (LABCD, in monetary units), is presented in Table 6.6. In this model the following occur:

LABCD(−11) – the monthly labor intensity of the ready-made production in monetary units delayed by 11 months;

TAM(−5), TAM(−11) – the technical devices in machinery and equipment per one employee (in thousands PLN), delayed adequately by 5 and 11 months;

MAR – the dummy variable taking the value of 1 in March of every year and the value of 0 in the remaining months;

JUN – the dummy variable taking the value of 1 in June of every year and the value of 0 in the remaining months;

AUG – the dummy variable taking the value of 1 in June of every year and the value of 0 in the remaining months;

SEP – the dummy variable taking the value of 1 in June of every year and the value of 0 in the remaining months;

OCT – the dummy variable taking the value of 1 in June of every year and the value of 0 in the remaining months.

Table 6.6 The parameter estimation results of the model for the labor intensity of the ready-made production, in monetary units (LABCD), based on monthly data.

Dependent variable: LABCD				
Method: least squares				
Date: 03/25/2014, time: 15:49				
Sample(adjusted): 1996:12 2006:12				
Included observations: 121 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	110.8799	28.88057	3.839 256	0.0002
LABCD(−11)	0.362905	0.096687	3.753 395	0.0003
TAM(−5)	8.714928	2.741211	3.179 226	0.0019
TAM(−11)	−5.194010	2.736619	−1.897 966	0.0603
MAR	50.52375	25.28412	1.998 240	0.0481
JUN	89.00362	24.98374	3.562 461	0.0005
AUG	119.2980	27.58651	4.324 506	0.0000
SEP	−75.15040	26.94066	−2.789 479	0.0062
OCT	−59.48777	25.76085	−2.309 232	0.0228
R-squared	0.472216	Mean dependent var		292.3565
Adjusted *R*-squared	0.434517	S.D. dependent var		96.92015
S.E. of regression	72.88257	Akaike info criterion		11.48704
Sum squared resid.	594929.3	Schwarz criterion		11.69500
Log likelihood	−685.9662	*F*-statistic		12.52602
Durbin–Watson stat	2.054223	Prob (*F*-statistic)		0.000000

In the model from Table 6.6, a positive autoregression of the 11th order occurs. Additionally, the technical devices in machinery and equipment delayed by 5 and 11 months (TAM)[29] are the explanatory variables. Employment increased along with the growth of the variable TAM delayed by 5 months, and it decreased after 11 months from the increase in the technical devices in machinery and equipment. Based on that, a conclusion can be made that newer machinery and equipment encouraged a decrease in the company REX's labor intensity after 11 months. The dummy variables also appeared, revealing periodic positive fluctuations: in March, in June, and in August as well as negative ones in September and in November. The positive periodic fluctuations result from the industry's specificity, in which, especially from June till August, the demand for labor grows due to an increased demand for its products in autumn and winter time. It requires manufacture for store. Since September, the demand for work drops, as a result of storing the ready-made products manufactured in advance (Figure 6.18).

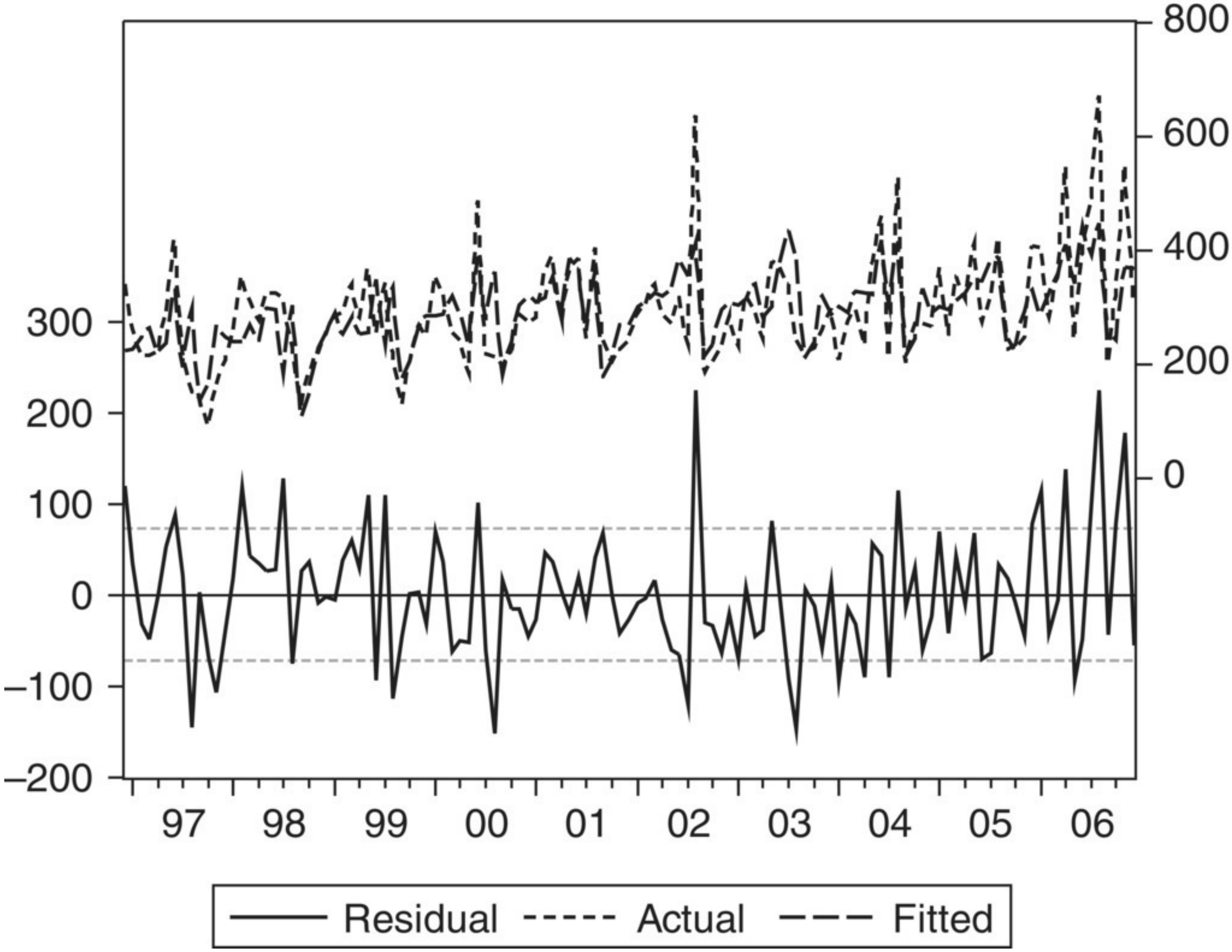


Figure 6.18 The actual and the theoretical monthly values of the company's labor intensity of the ready-made production (in thousands PLN) as well as the residuals.

The fitting accuracy of the model can be considered as moderate.[30] The coefficient of determination ($R^2 = 0.472$) indicates that about 47.2% of the monthly volatility of the labor intensity of production is explained by the equation from Table 6.6. Also, the standard residual error ($Su = 72.9$) confirms that the random fluctuations of the labor intensity can be considered as moderate. The Durbin–Watson statistic (DW = 2.054) indicates that in the equation from

Table 6.6, there is not any statistically important autocorrelation of the random component. These conclusions can be confirmed during analysis of Figure 6.19, which demonstrates the actual and the theoretical volumes of the ready-made production workload and the equation's residuals.

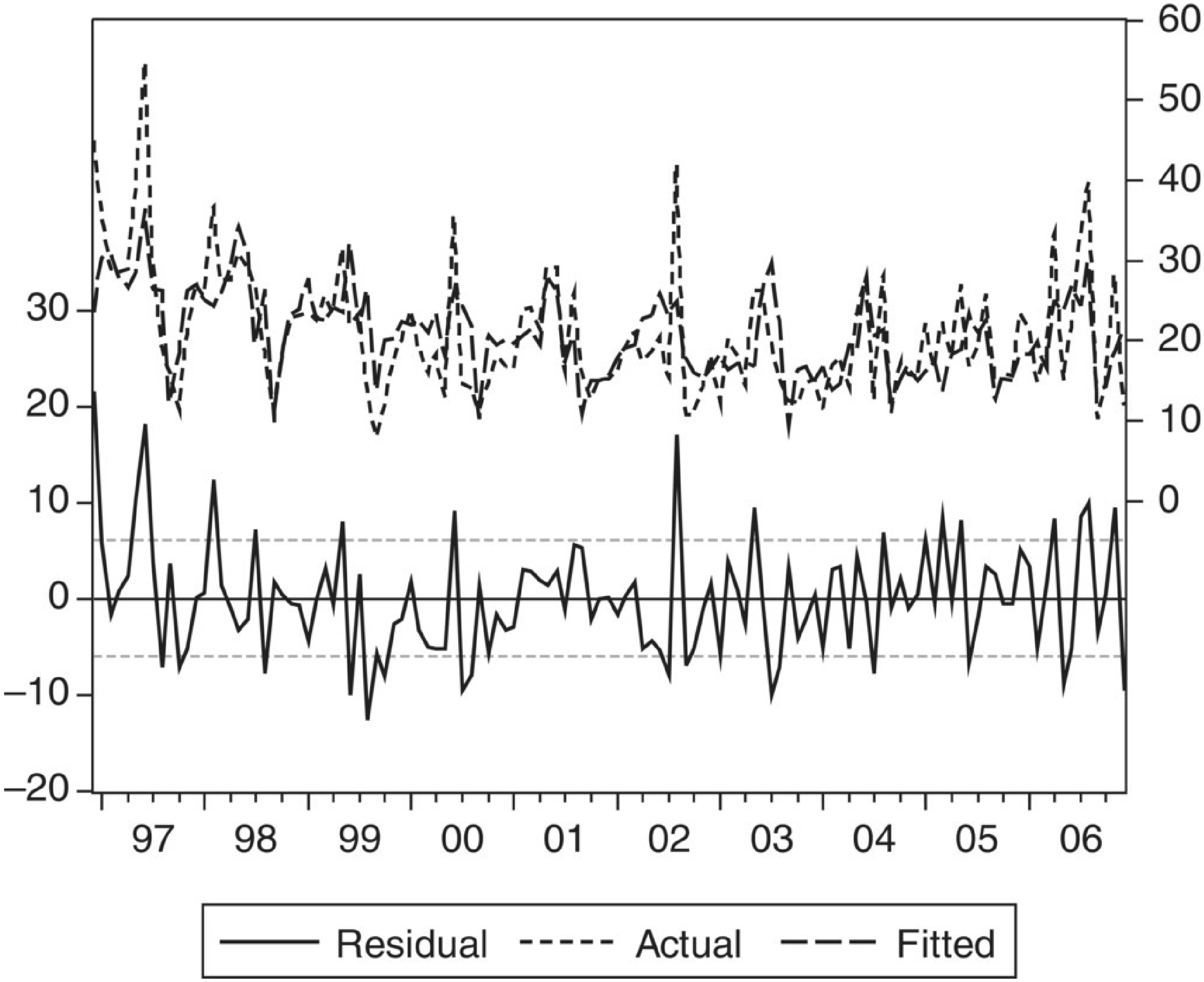


Figure 6.19 The actual and the theoretical monthly values of the company's labor intensity of the ready-made production (in persons/100 thousand PLN) as well as the residuals.

Table 6.7 The parameter estimation results of the model for the labor intensity of the ready-made production in natural units (LABCD), based on the monthly data.

Dependent variable: LABCD				
Method: least squares				
Date: 03/25/2014, time: 16:08				
Sample (adjusted): 1996:12 2006:12				
Included observations: 121 after adjusting endpoints				
Variable	Coefficient	Standard error	*t*-Statistic	Probability
C	7.722308	3.808304	2.027755	0.0449
LABCD(−1)	0.260855	0.075126	3.472237	0.0007
LABCD(−11)	0.390441	0.089307	4.371898	0.0000
TAM(−5)	0.581510	0.244232	2.380976	0.0189
TAM(−10)	−0.670380	0.231987	−2.889729	0.0046
JUN	6.772573	2.092653	3.236358	0.0016
AUG	6.680682	2.261812	2.953686	0.0038
SEP	−6.278568	2.219735	−2.828522	0.0055
R-squared	0.451951	Mean dependent var		20.88974
Adjusted R-squared	0.418001	S.D. dependent var		8.029835
S.E. of regression	6.125871	Akaike info criterion		6.526748
Sum squared resid.	4240.472	Schwarz criterion		6.711593
Log likelihood	−386.8682	*F*-statistic		13.31225
Durbin–Watson stat	1.906838	Prob (*F*-statistic)		0.000000

Similarly to the above, we present an econometric model for the monthly labor intensity of production in natural units (the group of Figures 6.17). Downward trends of that labor intensity in natural units are noticed. In both groups of the monthly labor intensity measures some periodical fluctuations of large amplitude occur. The empirical econometric model of the monthly labor intensity of the ready-made productions in natural units is presented in Table 6.7.

In the equation from Table 6.7, the dependent variable (LABCD) was explained with a moderate accuracy by configuration of the considered explanatory variables. The coefficient $R^2 = 0.452$ indicates that the set of explanatory variables of the considered equation explains 45.2% of the total volatility mass of the labor intensity. What is more, the actual values of the variable LABCD differ from its theoretical values that were calculated on the basis of this equation, on average, by slightly more than 6 persons per 100,000 PLN of the ready-made production's value ($Su = 6.1$). The empirical Durbin–Watson statistic (DW = 1.907) indicates that in the considered equation, there is no autocorrelation of the random component. The description accuracy of the variable LABCD along with the sizes of the fluctuations (in the form of the residuals) is illustrated in Figure 6.19.

The model from Table 6.7 includes autocorrelation of the variable LABCD. The autoregressive dependencies of the 1st and the 11th order turned out to be statistically significant. Additionally, the technical devices in machinery and equipment with the delays of 5 and 10 months also played an important role in this equation. Both the character of the autoregression as well as the impact of the technical devices on the labor intensity are analogical to the results observed in the model presented previously in Table 6.6.

The monthly periodic fluctuations of the labor intensity of the ready-made production, expressed in a natural measure, are specific here. Labor-intensity's positive deviations from the systematic component occurred in January and in August, while the negative ones in September. The positive as well as the negative deviations of the labor intensity were formed on the level slightly over six employees employed monthly per 100,000 PLN of the ready-made production.

The labor intensity of production in a small-sized enterprise has much significance. Its observation and analysis lead to various conclusions and decisions. The changes in the labor intensity are resultant from an impact of many factors, part of which is outside the enterprise and part within it.

Very strong impulses for the entrepreneur's actions aimed at suppressing the growth of labor costs, attributed to one production unit; in Poland they appear through political decisions. They can include the changes in the Labor Code,[31] regular minimum wage raises as well as increases in the social benefits, or an increase in the burden of the Social Security. Entrepreneurs react to those changes by initiating investment[32] processes, which are aimed at substitution of the human labor with machinery and equipment. Thus, investments occur, which allow saving the human labor.

The result of the investments substituting human labor is a decline in the labor intensity of production, expressed in the number of employees attributed to a variously measured unit of production.[33] Thus, by keeping the level of production, even by increasing it, the volume of employment in the enterprise can be limited. This process can be ascribed to the rapid growth of the unemployment rate in Poland, especially since the first half of the 1990s in the twentieth century.

6.3 Econometric model in selection of an efficient worker

Under the conditions of unemployment, attracting the right employee available on the labor market is not easy. Big corporations often engage specialized companies[34] to search for suitable workers. Finding a candidate suitable for the job, even for a worker position, can pose difficulties. Choosing one from a group of many candidates requires experience, intuition, and having an adequate set of criteria necessary to make a rational decision.

Requirements for a candidate for a given job position can be defined in various ways. Some entrepreneurs rely on intuition, which is mostly based on his/her own experience. The below method of indicating the necessary qualities of a worker is based on statistical regularities observed in the activities of the entrepreneur himself/herself or in similar enterprises with the same type of job positions. Efficiency of the worker's labor[35] should be the criterion for assessing the worker's suitability, measured by his/her individual labor efficiency. Having a homogenous data on individual labor efficiency of each employee and on their personal characteristics allows construction of an econometric model. Such model can serve as a good tool for selection of workers for a given type of job positions. An econometric model of *individual labor efficiency* can have the following from:

$$y_i = \alpha_0 + \alpha_1 x_{i1} + \cdots + \alpha_j x_{ij} + \cdots + \alpha_k x_{ik} + \eta_i, \tag{6.3}$$

where

y_i	Individual labor efficiency of the ith employee ($i = 1, \ldots, n$)
$x_{i1}, \ldots, x_{ij}, \ldots, x_{ik}$	The explanatory variables representing personal characteristics of the ith employee
$\alpha_0, \alpha_1, \ldots, \alpha_j, \ldots, \alpha_k$	The model's structural parameters
η_i	The random component
N	The number of employees, whose individual labor efficiency was measured

Out of worker's personal characteristics, the ones which differentiate him/her from other workers can be listed, for example:

- gender
- age
- profession
- education
- marital status
- family status
- place of residence
- owned assets.

Other personal characteristics can also be considered if there are indications pointing to the importance of those characteristics in crating the workers' labor efficiency.

Measuring both the workers' labor efficiency as well as their personal characteristics is essential for application of the decision-making tool indicated here. Measuring the individual labor efficiency can be troublesome, when some difficulties with establishment of a uniform and comparable efficiency measure of each employee[36] occur as a result of work division. An experienced entrepreneur, however, can overcome this difficulty. Measuring employee's personal characteristics is not easy though.

Quantification of measurable characteristics, using the numbers belonging to the relative scale, seems obvious. It is thus easy to measure the characteristics such as age, the number of the dependents, the number of children, the distance from the place of residence to the place of employment, the commute time, the number of years of education, seniority, and the value of his/her assets. However, using measurement in the relative scale is not always possible. Using a relative measurement is not always rational from the perspective of the study. In fact, measurement of the characteristics such as gender, marital status, profession, or education can be uncomplicated. Using the dummy variables makes it easy to measure each of the so-called qualitative characteristics.

Numerical expression of a personal quality, which is characterized by having only two variants, is least complicated. An example of such characteristic is the worker's gender. Existence of two variants provides a possibility of using only one dummy variable, defined, for example, as follows:

$$x_{i1} = \begin{cases} 1, \text{ woman}, \\ 0, \text{ man}. \end{cases}$$

The parameter α_1 then will provide information on by how much, on average, does labor efficiency of a woman differs compared to that of a man.[37] A zero value of the parameter α_1 indicates that labor efficiency of a woman does not differ – in the statistical sense – from that of a man.

Measuring the properties of the so-called immeasurable properties, when it is possible to distinguish many variants of a given characteristic, is much more difficult. A good example of that can be employee's education. Various possibilities can be taken into account, such as, for example, elementary education, vocational education, medium school education, or higher degree education (bachelor and masters). In such case, there is no need of a simultaneous extraction of the dummy variables for each of the education variants specified, which can differentiate individual labor efficiency for a given job position. As such, we can define the following dummy variables representing a considered characteristic:

$$x_{i2} = \begin{cases} 1, \text{ when an employee has elementary education}, \\ 0, \text{ in other cases}; \end{cases}$$

$$x_{i3} = \begin{cases} 1, \text{ when a worker has medium-school eductaion}, \\ 0, \text{ in other cases}; \end{cases}$$

$$x_{i4} = \begin{cases} 1, \text{ when a worker has at least medium-school education,} \\ 0, \text{ in other cases.} \end{cases}$$

Defining many other dummy variables, representing employees' education, is possible. Their application should arise from the needs of a particular study.

Let us suppose that the entrepreneur has in his disposition an empirical econometric model of individual labor efficiency, for the TXP in FP ALFA. This model has the following form[38]:

$$\hat{y}_i = 238 + 2.8x_{i1} - 3.6x_{i2} + 1.4x_{i3} - 0.8x_{i4} - 4.89x_{i5} + 6.2x_{i6} + 2.1x_{i7} - 3.4x_{i8}, \quad (6.4)$$

where[39]

$\hat{y}_i$ – the amount of details XC, manufactured by the ith worker during a work shift,

x_{i1} – the dummy variable representing the worker's age, defined as follows:

$$x_{i1} = \begin{cases} 1, \text{ for the workers younger than } 40, \\ 0, \text{ the remaining workers;} \end{cases}$$

x_{i2} – the dummy variable describing gender:

$$x_{i2} = \begin{cases} 1, \text{ a woman,} \\ 0, \text{ a man;} \end{cases}$$

x_{i3} – the variable characterizing the worker's family situation, expressing the number of the dependents,

x_{i4} – the dummy variable indicating the worker's marital status:

$$x_{i4} = \begin{cases} 1, \text{ when the worker is single,} \\ 0, \text{ the others;} \end{cases}$$

x_{i5} – the dummy variable representing education:

$$x_{i5} = \begin{cases} 1, \text{ when the worker has at least a bachelor,} \\ 0, \text{ the others;} \end{cases}$$

x_{i6} – the dummy variable describing vocational training, defines as follows:

$$x_{i6} = \begin{cases} 1, \text{ for the workers, who underwent a specialization course,} \\ 0, \text{ the others;} \end{cases}$$

x_{i7} – the dummy variable expressing the worker's place of residence:

$$x_{i7} = \begin{cases} 1, \text{when the worker lives in another city / town,} \\ 0, \text{others;} \end{cases}$$

x_{i8} – the dummy variable, differentiating the workers' financial status:

$$x_{i8} = \begin{cases} 1, \text{ when the worker owns a house,} \\ 0, \text{ others.} \end{cases}$$

The empirical model 6.4 can be an effective tool for selecting workers for the TXP division in the company FP ALFA. It allows a preference of those people, among the many candidates for work, who possess personal characteristics fostering high individual labor efficiency. The model 6.4 suggests that among the candidates for work in the TXP division, those should be preferred who are characterized by the following personal characteristics:

- age up to 40 years;
- men;
- the candidates having a significant number of dependents;
- not having higher education;
- those who have completed a specialized vocational course;
- persons commuting to work from nearby cities/towns;
- who do not have any property.

The model 6.9 indicates that a worker with the abovementioned characteristics guarantees – in the statistical sense – the highest expected individual labor efficiency for the TXP division. However, some cases when the above diagnosis will not be fully accurate should be accounted for. At the same time, it is possible to define personal characteristics of a candidate for work, which are far less favorable for high labor efficiency.

Summing up, the empirical econometric model of individual labor efficiency for a given group of work positions can be an effective tool for selecting the candidates for employment in that place. By inserting the values of statistically significant personal characteristics of a candidate for work into the model, his/her potential individual labor efficiency ($\hat{y}_{ip}$) can be obtained, similar to that obtained for the workers with comparable personal characteristics, employed in the same company in the past. Thus, it is possible to sort the candidates for the vacant job positions accordingly to their potential labor efficiency level. Ultimately, the one should be chosen whose potential efficiency is the highest, that is

$$\hat{y}_{ipw} = \max_{i=1,\ldots,v} \hat{y}_{ip}, \tag{6.5}$$

where v is the number of the candidates, who applied for the job during recruitment, and $\hat{y}_{ipw}$ is the potential efficiency of the best candidate, with a given criterion of the choice.

A situation may arise when the employer establishes a minimal norm of potential labor efficiency ($\hat{y}_{pN}$). In such cases, only those candidatures will be considered who fulfill the condition $\hat{y}_{ip} \geq \hat{y}_{pN}$ ($i = 1, \ldots, v$). Then, it is possible that none of the candidates meets the employer's requirement, that is, in all cases $\hat{y}_{ip} < \hat{y}_{pN}$ ($i = 1, \ldots, v$). In such a state of things, none of the candidates will be employed, which results in repetition of the recruitment.

The structure of individual labor efficiency usually is characterized by a right-sided skewness. Therefore, approximately one-third of workers achieve the average arithmetic labor efficiency. At the same time, employment of a new worker with potential labor efficiency $\hat{y}_{ipw} < \bar{y}$ (where $\bar{y}$ represents the average arithmetic labor efficiency in the considered group of workers) will cause a decrease in its averaged value in the considered group of workers. In practice, however, it is difficult to find a person with labor efficiency $\hat{y}_{ipw} \geq \bar{y}$, who would guarantee maintenance or growth of the average team efficiency. Thus, it is worthwhile to make sure that the number $\hat{y}_{pN}$ is at the level above the team efficiency's median.

Selection of a candidate for a vacant job position can be done using a recursive model composed of two equations. The first equation will describe a formation mechanism of individual labor efficiency in a group of workers, according to their personal characteristics. The second equation will describe the principles of workmanship volatility of the products manufactured by this group of workers, according to their personal characteristics and their individual labor efficiency. A hypothesis can be posed that excessive emphasis on individual team-labor efficiency can result in a decrease in workmanship quality.

More precise information on the relations between labor efficiency and workmanship quality of the production manufactured by the workers and their personal characteristics is to be supplied by regression models. In our case, an econometric model composed of the following two equations will be considered:

$$
\begin{aligned}
y_{1i} &= \alpha_{10} + \beta_{12} y_{2i} + \alpha_{11} x_{i1} + \alpha_{12} x_{i2} + \alpha_{13} x_{i3} + \alpha_{14} x_{i4} + \alpha_{15} x_{i5} + \eta_{1i}, \\
y_{2i} &= \alpha_{20} + \alpha_{21} x_{i1} + \alpha_{22} x_{i2} + \alpha_{23} x_{23} + \alpha_{24} x_{i4} + \alpha_{25} x_{i5} + \eta_{2i},
\end{aligned} \tag{6.6}
$$

where

α_{1i}, α_{2i}	Structural parameters at the model's predetermined variables (i = 0, 1, 2, 3, 4)
β_{12}	the structural parameter at the model's interdependent variable
η_{1i}, η_{2i}	random components of the model's equations

Statistical data presented in Table 6.8 provides information about the quality of manufactured production (produced by each one of the 30 workers) during a given month (y_{1i}), about individual labor efficiency (y_{2i}) and about selected personal characteristics.[40] The variable y_{1i} ($i = 1, \ldots, 30$) has characteristics of a dummy variable. It takes the value of 1 if the worker falls within the accepted standards of production shortages, and the value of 0 when the worker goes over the standard limit of the shortages predetermined for a given job position during a given time-period. Variable x_{i1} takes the value of 1 when the worker has vocational education and the value of 0 in the remaining cases. Variable x_{i2} represents gender; it takes the value of 1 for females and the value of 0 for males. Variable x_{i3} provides information about the workers' age, expressed in the number of completed years of age. Variable x_{i4} provides information about work seniority, expressed in the number of full working years. The last variable x_{i5} provides information about the time of commuting to work from the place of residence, calculated in minutes.

Table 6.8 The worker's efficiency and labor quality, as well as some of their personal characteristics.

Source: Contractual data corresponding with actual data.

The worker's id number (*i*)	**Quality of manufactured production (y_{1i})**	**Labor-efficiency (in items) (y_{2i})**	**Education (x_{i1})**	**Gender (x_{i2})**	**Age (years) (x_{i3})**	**Seniority at work (x_{i4})**	**Commuting time (x_{i5})**
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
1	0	92	1	0	23	3	20
2	1	93	1	0	25	4	25
3	1	95	1	0	27	7	30
4	1	96	1	0	26	8	15
5	1	98	1	0	28	5	15
6	1	99	1	0	27	7	10
7	1	102	1	0	29	6	15
8	1	103	1	1	30	11	25
9	1	104	1	0	30	10	20
10	1	105	1	0	29	7	15
11	0	107	0	0	31	12	10
12	1	108	0	1	32	9	15
13	0	110	0	0	33	13	20
14	0	115	0	0	34	12	20
15	1	118	0	0	33	14	25
16	1	119	1	0	35	16	30
17	1	120	1	0	36	15	35
18	1	120	1	1	36	16	40
19	1	122	1	0	34	14	20

20	1	123	1	0	36	17	15
21	1	124	1	0	37	16	20
22	1	125	1	0	38	19	10
23	1	122	1	0	39	16	20
24	1	122	1	0	40	14	15
25	1	121	1	0	40	19	30
26	0	119	0	1	41	23	10
27	1	120	0	0	40	22	15
28	0	118	0	0	39	19	20
29	0	117	0	0	24	4	20
30	1	116	0	0	25	5	15
∑	23	3353	20	4	977	363	595

The first empirical equation describing the mechanism from the system of a 6.6-type, based on the statistical data from Table 6.8, has the following form:

$$\hat{y}_{1i} = \underset{(0.749)}{-0.865} + \underset{(1.020)}{0.012} y_{2i} + \underset{(3.656)}{0.594} x_{i1} + \underset{(0.586)}{0.124} x_{i2} + \underset{(0.073)}{0.003} x_{i3} - \underset{(0.379)}{0.015} x_{i4} - \underset{(0.054)}{0.0005} x_{i5},$$
$$R_1^2 = 0.417,\ S_{1u} = 0.369. \quad (6.7)$$

Most of the explanatory variables from Equation 6.7 are characterized by statistical insignificance. After elimination of insignificant variables, we will get the following empirical equation describing volatility of workmanship quality in the considered group of workers[41]:

$$\hat{y}_{1i} = \underset{(0.529)}{-0.380} + \underset{(1.097)}{0.0068} y_{2i} + \underset{(4.258)}{0.581} x_{i1},$$
$$R_1^2 = 0.402,\ S_{1u} = 0.345. \quad (6.8)$$

In Equation 6.8, the variable y_{2i} is kept, although its significance can be considered at a mere significance level of $\gamma = 0.28$. However, variable y_{2i} in Equation 6.8 provides important information, from the perspective of the company's management. First of all, it does not confirm the hypothesis that an increase in labor efficiency decreases the products' workmanship quality. On the contrary, an increase in labor efficiency improves the products' workmanship quality, as a result of the workers' increasing experience. Individual efficiency did not reach the critical level, beyond which labor quality deteriorates. What is more, Equation 6.8 also indicates that workers with vocational education produce better labor quality – compared to the workers with different education. Workers with vocational education realize defect-free production, on average by 58%, than the other education groups. This suggests that during selection of the worker for a vacant job position, those candidates should be preferred, who have vocational education, which will contribute to the quality of manufactured goods.

The random component of Equation 6.7 (and thus of Equation 6.8) is characterized by heteroscedasticity. Therefore, the Aitken's estimator provided by Equation 1.55 will be more precise (effective) in this case. As such, it will be necessary to use the theoretical values of explanatory variable $\hat{y}_{1i}$ from Equation 6.7 to assign the weights w_i calculated according to the formula 1.52. After estimating the parameters of the equation describing $\hat{y}_{1i}$, using the Aitken's estimator according to Equation 1.55, and with subsequent reduction in statistically insignificant variables, we get an empirical equation in the following form:

$$\overset{*}{y}_{1i} = \underset{(0.167)}{-0.072} + \underset{(1.123)}{0.00410}\, y_{2i} + \underset{(4.800)}{0.589}\, x_{i1}, \qquad R_1^{*2} = 0.482. \tag{6.9}$$

In the empirical equation 6.9, the value of the coefficient $R_1^{*2} = 0.482$ was higher in comparison with $R_1^2 = 0.402$. Moreover, empirical values of *t*-Student statistics at explanatory variables increased as well. Thus, variable y_{2i} can be considered as statistically significant, with a risk of type I error that is slightly higher than before, that is, at the significance level of $\gamma = 0.27$. Decisional conclusions, resulting from Equation 6.9, are identical to those which emerged in reference to Equation 6.8.

An equation describing a volatility mechanism of individual labor efficiency affected by the considered personal characteristics has the following empirical form:

$$\hat{y}_{2i} = \underset{(4.164)}{65.32} - \underset{(1.452)}{4.031}\, x_{i1} - \underset{(1.198)}{4.397}\, x_{i2} + \underset{(1.915)}{1.365}\, x_{i3} + \underset{(0.258)}{0.180}\, x_{i4} + \underset{(0.897)}{0.156}\, x_{i5}, \qquad R_2^2 = 0.680,\ S_{2u} = 6.568. \tag{6.10}$$

After elimination of the explanatory variables which are statistically insignificant, we get the following equation of individual labor efficiency:

$$\hat{y}_{2i} = \underset{(8.410)}{61.521} + \underset{(6.961)}{1.543}\, x_{i3}, \qquad R_2^2 = 0.634,\ S_{2u} = 6.505. \tag{6.11}$$

It turns out that the worker's age (x_{i3}) is the only statistically significant variable impacting his/her individual labor efficiency. Over 63% of individual efficiency fluctuations result from variability of the workers' age. Therefore, similarities between the distribution of individual labor efficiency and the structure of workers, in terms of their age, can be expected. Average labor efficiency at $\bar{y}_2 = 111.8$ items is smaller than the medians of $y_{2\mathrm{Me}} = 116.5$ items. This signifies left-sided skewness of labor efficiency's distribution, which is confirmed by the coefficient of skewness equal to −0.548. Distribution of the workers according to their age is slightly left-skewed, since the average arithmetic age at $\bar{x}_3 = 32.6$ years is slightly smaller than the median, which is equal to $x_{3\mathrm{Me}} = 33$ years. The skewness coefficient of the age structure is equal to −0.11. Staff is characterized by significant professional experience. The result of the workers' age impacting their individual labor efficiency can be referred to the age range observed, that is, from 23 to 41 years of age. Thus, the group's age range is 18 years. It is easy to calculate that the expected efficiency of a 23-year-old worker is $\hat{y}_{2i} = 61.521 + 1.543 \cdot 23 \approx 97$ items monthly; while the expected labor efficiency of a 41-year-old worker will be about $\hat{y}_{2i} = 61.521 + 1.543 \cdot 41 \approx 124$ items monthly.

In the case considered here, a vacant job position should be filled by a candidate in the age of 35–40 years, so he/she can achieve high labor efficiency. Moreover, it ought to be a candidate with vocational education, which increases the chance of better production quality.

It is worth keeping in mind that the result obtained in the above case is not universal. Comparable statistical information about labor efficiency and labor quality in given group of worker positions should be collected for every case. Knowledge of important personal characteristics which shape production quality and individual labor efficiency – for every case – allows reduction in the risk associated with erroneous employment decisions. The procedure offered here is meant to reduce the risk of faulty decisions. Each wrong personnel decision may result in additional costs for the company to incur. Other negative consequences, which companies want to avoid, may also emerge.

6.4 Econometric model in the selection of an efficient white-collar worker

The above proposed employee recruitment procedure can be used for every job position, for which the results of activity can be measured. In the enterprise, such position can be that of a salesman (sales person, sales representative, etc.), for whom the generated sales income will be the measure of his/her effectiveness. It is worthwhile to pay a bit of attention to the traders. A good trader activates the capital frozen in the merchandise. This provides an opportunity to recover the amounts engaged into production, thus increasing the capital by the values of income. An appropriately trained, efficient, and loyal sales representative can play a decisive role in the economic condition of the enterprise.

The entrepreneur should be aware of the role a trader plays in the company. Because of that, it is important to have the knowledge about the selection methods and the methods of assessing the salesperson's work. The art of conversation combined with client psychology, negotiation techniques, the knowledge about trade agreements, are the principal issues, which the sales representative must be familiar with. Every salesperson should be skillful in those areas, regardless the sphere in which he/she sells the goods (services). A perfect knowledge of the subject for sale is also necessary.

Some personal characteristics foster salesperson's work efficiency, others are obstacles in practicing this type of activity. An entrepreneur must be familiar with this dichotomy of traits. Knowledge of this can be acquired from an econometric model structured analogously to the model 6.4. The natural assessment measures of a salesperson's labor efficiency can be the number of new clients acquired and the value of the sales or the profit earned by him/her from all transactions in a given period. Those measures can be used separately or jointly. In the econometric model, they will function as the dependable variable (y_{1i}, y_{2i}, or y_{3i}), forming the criterion for assessment and for the choice of the trader.

The effects of a salesperson's work must be associated with his/her personal characteristics. The characteristics indicated in Section 6.3 and those analyzed during the assessment of a worker's job performance as well as other characteristics, which can affect the salesperson's labor efficiency, should be considered. These will include both, objectively defined characteristics as well as subjective ones. Among the subjective ones, the following assessment types can be distinguished: a "handsome" salesman, a "good looking" sales woman, a representative "having good taste," an "inventive" salesperson. The use of subjective characteristics, however, requires having a relatively objective system of testing those types of characteristics.

The objective characteristics of a salesperson, different from the ones previously considered in the models 6.3 and 6.4, can be as follows:

- possession of a driver's license
- possession of a car
- possession of a phone
- Internet access and so on.

The entrepreneur must be able to identify the characteristics favoring or hindering the salesperson's work efficiency. Some of those characteristics, which are considered to be important, in practice can turn out to be statistically insignificant. Information about this can be obtained from the empirical econometric model, which should be an incentive for its use.

Having an empirical econometric model of the economic work efficiency of a salesperson, describing the variable $y_{pi}(p = 1, \ldots, P; i = 1, \ldots, n)$, allows the use of a selection procedure for a given job position. An approach similar to the one described in Section 6.3 can be effective during a selection using the formula 6.5.

Labor-efficiency assessment in an office is one of those difficult issues, which always arouse all sorts of controversy. A good functioning of an enterprise, however, relies heavily on the efficiency of its office work. Therefore, selection of workers, who guarantees maximum work effectiveness on the job positions connected with the company's organization and management, keeping the financial-accounting and technical documentation, supply, and so on, is of high priority.

Evaluation of intellectual work can be done intuitively. It is a most commonly used method. Often, however, this method is largely unreliable. Therefore, it is worth to use a score method, in which an adequate number of points for particular effects of the worker's actions are determined. Using the score method for office work allows a discourse about the office worker's efficiency. The number of the points scored (y_{ib}) is the measure of the office worker’s assessment; thus it can be the dependable variable in an econometric model of the worker’s effectiveness in this group. The set of dependable variables of such a model ought to be complementarily specified. These variables will represent the worker’s personal characteristics. Having an empirical econometric model of the office worker’s efficiency facilitates making right decisions regarding employing new people to replace the ones leaving or employing workers for newly opened job positions in a given group. The procedure of econometric modeling is then analogous to the proceedings relating to the model 6.4 and to a decision-making according to the formula 6.5.

In some cases, it is possible to apply worker’s assessment using the “beam” criteria. It is expected that the worker ought to be creative, be a good manager, be fair in evaluating subordinates, have good contacts with his/her coworkers, and so on. Each of these assessment components can be verbally defined so that the description is precise. The dummy variable (y_{iz}) can then be the measure of such worker’s assessment, having the value of 1, when he/she fulfills all the requirements for each assessment criterion, and the value of 0, when he/she does not meet any of the conditions. The tool of choice may then be an econometric model with a dummy dependable variable, which allows estimation of the probability of meeting all the criteria of a worker’s assessment. Having such empirical model allows making a rational choice out of the candidates for the job, according to the formula 6.5. The selection, in this situation, is made according to the principle of the highest respect of the expected probability of meeting the criteria verbally defined.

Let us consider an example of using an econometric model for selecting a good candidate for the position of a trader in the enterprise. Table 6.9 contains statistical information about effectiveness of the traders’ work in the company MAX and information about some of evaluations of their personal characteristics. Work effectiveness of this group of workers, on one side, is measured by the value of the sales income obtained by issuing appropriate invoices in a given period and by timely payments for the sales resultant from those invoices, which makes up the company’s receivables. The measurement of the claims recovery was done using the dummy variables, defined as follows[43]:

y_{1i} – the dummy variable taking the value of 1 when in the sales network of the *i*th sales representative overdue[44] receivables had emerged and the value of 0 in the opposite case.

Table 6.9 The effectiveness of claim recovery, the net sales income as well as the selected personal characteristics of the traders in the company MAX, annually for the years 2010–2013 (in thousands PLN annually)[42].

Source: The company MAX’s data analogous to the actual data.

Trader’s # (*i*)	y_{1i}	y_{2i}	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}	x_{i6}	x_{i7}
1	0	872	1	0	3	0	0	24	0
2	0	880	0	0	3	0	0	25	0
3	0	900	0	0	2	0	0	23	0

4	0	910	0	0	4	0	2	25	0
5	0	912	0	0	3	0	1	27	1
6	0	930	1	1	5	0	1	26	1
7	1	933	0	0	5	0	1	28	1
8	0	940	0	0	5	1	3	27	0
9	0	945	1	1	3	0	0	29	0
10	0	950	1	0	4	0	0	30	0
11	0	952	1	0	7	0	1	30	0
12	0	955	0	0	6	0	1	29	0
13	0	960	0	1	5	0	1	31	1
14	0	966	0	0	8	0	1	32	0
15	0	967	0	0	6	0	2	33	0
16	0	968	1	0	8	1	1	34	0
17	0	970	0	0	8	1	2	33	0
18	1	985	0	1	7	0	1	35	1
19	0	990	0	0	9	0	1	36	1
20	0	992	1	0	8	0	2	36	0
21	0	998	1	0	9	0	2	34	1
22	0	1000	0	0	9	0	3	36	1
23	1	1020	0	1	7	0	1	37	1
24	0	1025	0	0	8	1	2	38	0
25	0	1030	0	0	9	1	2	39	1
26	0	1060	1	0	10	0	3	40	1
27	0	1100	1	1	10	1	3	40	0
28	0	1160	0	1	11	1	4	41	0
29	1	1204	0	0	12	0	4	40	0
30	0	1260	0	1	11	1	2	39	1

The remaining symbols representing the variables listed in Table 6.9 are as follows:

y_{2i} – the annual net sales income obtained by this *i*th trader (in thousands PLN),

x_{i1} – the dummy variable, representing the trader's gender, taking the value of 1 for women and of 0 for men,

x_{i2} – the dummy variable, providing information on competitive sports practiced by the trader, taking the value of 1, when he/she had practiced sports professionally and the value of 0, in the opposite case,

x_{i3} – seniority in the trader position, expressed by the number of working years,

x_{i4} – the dummy variable, informing about the trader's economic education, taking the value of 1, when the trader has economic education and the value of 0, when he/she does not have such education,

x_{i5} – the number of the trader's dependents,

x_{i6} – the trader's age, expressed by the number of completed years of life,

x_{i7} – the dummy variable, providing information about an educational status, taking the value of 1, when the trader has higher education and the value of 0, when he/she does not have such education.

More precise information about labor-efficiency relations and about the quality of production done by the workers along with their personal characteristics are provided by the regression models. In our case, a recursive econometric model composed of two equations is the subject of discussion:

$$y_{1i} = \alpha_{10} + \beta_{12} y_{2i} + \alpha_{11} x_{i1} + \alpha_{12} x_{i2} + \alpha_{13} x_{i3} + \alpha_{14} x_{i4} + \alpha_{15} x_{i5} + \alpha_{16} x_{i6} + \alpha_{17} x_{i7} + \eta_{1i}, \quad (6.12)$$

$$y_{2i} = \alpha_{20} + \alpha_{21} x_{i1} + \alpha_{22} x_{i2} + \alpha_{23} x_{i3} + \alpha_{24} x_{i4} + \alpha_{25} x_{i5} + \alpha_{26} x_{i6} + \alpha_{27} x_{i7} + \eta_{2i} \quad (6.13)$$

where

α_{1i}, α_{2i}	The structural parameters along the model's predetermined variables ($i = 0, 1, \ldots, 7$)
β_{12}	The structural parameter along the model's total interdependent variable
η_{1i}, η_{2i}	The model's random components

Recursiveness of the hypothetical econometric model results from the assumption that targeting high amounts of the sales income obtained by the trader lowers his/her attention to the timeliness of settling by the clients their liabilities. In commercial practice, it happens that over-intensification of the sales causes delays in settling the payments for the invoiced goods and services.

Using the data presented in Table 6.9, an empirical probability function was obtained, which describes the mechanism of an occurrence frequency of the trader's abnormal overdue claims, depending on his/her personal characteristics and the obtained sales income. It has the following form[45]:

$$\hat{y}_{1i} = \underset{(0.308)}{-0.268} - \underset{(1.775)}{0.219x_{i1}} + \underset{(0.892)}{0.126x_{i2}} - \underset{(0.004)}{0.0003x_{i3}} - \underset{(1.952)}{0.287x_{i4}} - \underset{(0.400)}{0.032x_{i5}} + \underset{(0.887)}{0.245x_{i6}} + \underset{(0.216)}{0.029x_{i7}} - \underset{(0.364)}{0.0002y_{i2}}, \quad R_1^2 = 0.278. \tag{6.14}$$

Most of the dependable variables of Equation 6.14 are the variables statistically insignificant. Therefore, in the following estimations, reduction was performed. As a result, an empirical model with acceptable decision-making qualities has emerged:

$$\hat{y}_{1i} = \underset{(0.933)}{-0.318} - \underset{(1.963)}{0.214x_{i1}} - \underset{(2.404)}{0.308x_{i4}} + \underset{(1.723)}{0.018x_{i6}}, \quad R_1^2 = 0.230. \tag{6.15}$$

The empirical Goldberger's equation explains only 23% of the volatility of the dependable variable characterizing the formation of the overdue[46] claims. Three (out of eight) explanatory variables differentiate the frequency of formation of the overdue claims in the traders' activity. The results obtained can serve in the decision-making process during recruitment for a vacant job position in the sales department as well as in implementation of staff training. This means that effectiveness of the trader's claim recovery does not depend on the amount of the sales income generated by him/her.

Similarly to the case of Equations 6.7 and 6.8, heteroscedasticity of the random components also occurs in Equations 6.14 and 6.15. This signifies a possibility of improving the estimative precision through application of the Aitken's method for assessment of the parameters in Equation 6.12. As in the previous case, the weights w_i were assigned on the basis of the theoretical values of $\hat{y}_{1i}$, determined from Equation 6.14, using the formula 1.52. As a result of the calculations estimating the parameters in Equation 6.14 and of elimination of statistically insignificant variables, the following empirical equation was obtained:

$$\overset{*}{y}_{1i} = \underset{(1.799)}{-1.015} - \underset{(1.854)}{0.285x_{i1}} - \underset{(2.916)}{0.593x_{i4}} + \underset{(2.343)}{0.039x_{i6}}, \quad R_1^{*2} = 0.381. \tag{6.16}$$

The results obtained are more accurate, compared to Equation 6.15, as evidenced by empirical values of t-Student statistics and by a better value of $R_1^{*2} = 0.381$. The conclusions emerging from the empirical equation 6.16 are the same as those emerging from Equation 6.15.

An equation describing the impact of the traders' personal characteristics on the generated sales income by them is as follows[47]:

$$\hat{y}_{2i} = \underset{(6.096)}{961.2} + \underset{(2.354)}{83.18x_{i2}} + \underset{(5.241)}{65.0x_{i3}} - \underset{(1.775)}{12.77x_{i6}}, \tag{6.17}$$

$S_u = 88.88\,tys.zł$, $V = 8.49\%$, $R^2 = 0.778$

Equation 6.10 shows that the sales income generated by a trader depends on x_{i2} (the dummy variable providing information about the fact of practicing competitive sports by a trader), on x_{i3} (the seniority in the trader's profession expressed by the number of working years), as well as on x_{i6} (the trader's age).

The system of Equations 6.12 and 6.13 can be used in employment decision-making, in case of a vacant trader job position in the enterprise. Having information about the candidates, it should be noticed that the most important trader characteristics influencing the sales income generated by a given trader are as follows: the fact of practicing competitive sports, seniority in trading, and the age. This allows using Equation 6.17, assessment of potential efficiency for each of the candidates for the job. Employment decision can be then made accordingly with the formula 6.5. In case of similar values[48] of $\hat{y}_{2\text{ipw}}$ for two or more candidates, Equation 6.16 should be additionally used, which will indicate a candidate of a smaller risk of creating overdue claims, that is, of the smallest forecast value of the frequency of exceeding the allowed payment due dates $\hat{y}_{1\text{ipw}}$. A candidate selected in this way is laden with the smallest risk of a faulty decision about the staff. There is no guarantee that ultimately, the candidate will satisfy the expectations during the tasks entrusted to him/her. Selection of each of the remaining candidates will be even more risky, compared to the case indicated using the empirical set of equations.

Notes

1 Some of the employment factors in a particular enterprise can be inactive, that is, statistically insignificant. The knowledge of such a state of things can influence the possibilities of their activation.

2 The enterprise was founded in 1991. The process of a systematic collection of statistically important data had begun only in the beginning of 1996, which results in the data from the level of weekly series for a period of 11 years.

3 The year 1991–1998 was a period of learning the entrepreneurship and of extensive production growth. In 1998, intensive investments begun in the machinery, which was installed in the production facility owned ever since it was bought in 1996.

4 Possibilities of such investments in machinery and equipment arose after the purchase of the property used in the business for production purposes and for office space.

5 The assumed number of the employed in January 1996 was 100.

6 Employment belongs to typical variables containing autoregression. It is not a subject for rapid changes. The current level of employment largely depends on the previous values of that variable.

7 Attention for compliance of the structures of the explained process with the explanatory

processes has been maintained, which is the constructional condition of the so-called compatible model. See: Z. Zieliński, in the work of Wiśniewski J.W., Zieliński Z. (2004), Chapter V.

8 The ordinary least squares method was used to estimate the model's parameters. Calculations were done and the graphs were constructed using the EViews 4 package.

9 It is worth underlining, that in practice, accuracy of the models describing micro-variables on the basis of monthly data, in which the value of the coefficient $R^2 > 0.9$ is obtained, does not belong to common cases.

10 Employment is such an economic variable, which is usually characterized by a strong inertia. In an econometric model, inertia is manifested by a presence of autoregression.

11 The empirical and the theoretical values of the employment dynamics indexes are marked on the right ordinate axis (DEMP), while the left axis is used to read the residuals u.

12 The labor factor's risk is due to many reasons. Reduction in working time, increasing of the cost burdens, extension of vacation time, numerous opportunities of excused absences, the necessity of a long-term service for ex-employees, and so on have the most negative impact on a tendency to create job positions in a small-sized enterprises.

13 Until 30 April 2004, paper products intended for school use were covered by the VAT rate on the level of 7%, while since 1 May of the same year all products were covered by a 22% rate.

14 The value of the annual VAT tax in the company REX.

15 See the footnote 12.

16 The actual and the theoretical volumes of the variable EMPL, expressed by singe-base logarithms of the dynamics indexes (1996 = 100%) are on the right ordinate axis, while the equation's residuals are on the left axis.

17 V represents the coefficient of random volatility, which is the percentage share of the standard error of the residuals Su in the arithmetic dependable variable.

18 In the period of the company REX's operation, the maximum obligatory VAT rate was 22%. Currently the VAT rate in Poland is 23%.

19 The effects of the VAT rate increase, resulting in the retail price increase and in a decrease of the demand, have appeared in 2004.

20 In 2006, the average monthly net wage was approximately 280% of its amount in 1996, which signifies a nominal increase of 180%. Meanwhile, the prices of most of the company's products in this period were stabilized, some systematically got cheaper, few slowly got more expensive over the next years. On the average, during the examined 11

years, the prices of the products were stable.

21 The fiscal costs of labor include contributions for pension and disability insurance, sickness/accident insurance, health insurance, employee find of guaranteed benefits, jointly covered by the employee and by the employer, as well as an advance on the income tax.

22 Calculated in full-time employment.

23 In the primary version of the model, all the delayed variables, of the variable EMPL as well as of the variables AVAT and FISCOW, were included. As a result of elimination procedures, only explanatory variables statistically significant on the significance level of $\gamma < 0.05$ were left in the empirical model.

24 In the primary version of the model, a time variable t, which was supposed to extract a linear trend and the dummy variables for the remaining months, had also appeared. However, it was eliminated from the model, because it had turned out to be statistically insignificant.

25 Labor-efficiency (LPROD) – according to a classic definition – is the relation of the production (P) to the labor input used (L), that is $\text{LPROD} = P/L$, while labor intensity of production (LC) is an inverse of the labor efficiency, that is $LC = 1/\text{LPROD} = L/P$.

26 We are not going to provide the enterprise’s name, using only a code name: the company **G**.

27 The measures of labor intensity of production presented in Table 6.1 are characterized by a construction of the formula’s numerator in the form of the company’s labor costs (compare the footnote 26).

28 See the work of Wiśniewski J.W. (2003). *Econometric Model of a Small-Sized Enterprise*, Subchapter 2.2. Toruń.

29 TAM represents the labor devices in machinery and equipment, expressed in the initial value of the machinery and equipment (in thousands PLN) per one employee, on average, monthly.

30 As for microeconometric series of quarterly data.

31 For example, shortening of the working time, extending vacation time, and many employee benefits.

32 Investments occur, provided that the entrepreneur optimistically perceives his/her the future of his business activity, as a result of which he/she plans his/her own company’s development. However, entrepreneurs’ optimism does not always outweigh their pessimism.

33 At the same time, the average labor efficiency increases, thus improving the entrepreneur’s competitive position on the widely interpreted market.

34 Such companies, which specialize in finding appropriate workers, colloquially are called "head hunters."

35 The worker's individual labor efficiency is the most important characteristics of a worker. The more a worker is efficient, at an adequate work quality, the more desirable he/she is in the enterprise.

36 The added value, which the worker performs during a given time unit, is the best measure of a worker's individual labor efficiency. Application of that measure, however, is very difficult.

37 This difference will be expressed in the same units, in which the individual labor efficiency is measured.

38 The results presented here are of a contractual nature, that is, fictional. They are used only to illustrate the way of perceiving the contraction of an econometric model of individual labor, for interpretation of its results as well as for practical application of this decision-making tool.

39 We are assuming that in the empirical model of individual labor efficiency, the only variables present are the explanatory variables that are statistically significant.

40 Typically, quality and efficiency of the workers' labor is affected by a small number of personal traits. Personal characteristics play the greater role, the less mechanized the work on a given job position is.

41 Generally, in equations describing individual efficiency or labor quality cannot be expected to have high values of determination coefficients R^2. These values usually do not exceed 0.5 ($R^2 \leq 0.5$). Personal characteristics usually shape a little part of those variables' volatility.

42 See the work: Wiśniewski J.W. (2013), *Forecasting staffing decisions, ECONOMETRICS* 1(39) 2013, Publishing House of Wroclaw, University of Economics Wroclaw, pp. 22–29.

43 The trader should achieve high sales incomes and strive for effectiveness of claim recovery in the sales network serviced by him/her.

44 We mean the overdue receivables that are over the norm specified by the company.

45 Empirical values of the *t*-Student statistics are in the parenthesis under the assessments of the structural parameters.

46 In econometric micromodels, the obtained values of the coefficient R^2 are inherently lower compared to macromodels. The R^2 of a value 0.23 can be considered sufficient for decision-making purposes in the enterprise, in case of an empirical linear probability function.

47 Each of the explanatory variables of Equation 6.1 is statistically significant on the

significance level of $\gamma \leq 0.086$.

48 The symbol $\hat{y}_{2ipw}$ represents the forecasted (potential) labor efficiency of a candidate for a trader position.

Conclusion

In this book, basic econometric models were presented, which can be used in an enterprise. Collectively, they can be defined as *microeconometric models*. The main incentives for their application are the possible benefits to be obtained from their use. This book presented only some potential effects of using the presented models in enterprise management.

Out of the empirical models presented most come from economic practice. Only few of them are in the form of a hypothesis. Others are based on the data analogous to actual information. There are some difficulties in constructing such models, resulting from some banal reasons – absence of necessary statistical information. The purpose of this book is to invoke awareness for the need of collecting statistical data. Having adequate statistical material at one's disposition allows application of statistical and econometric tools for improving the decision-making processes and for increasing their effectiveness in an enterprise. Free software designed for the purposes of dealing with those issues is currently available on the Internet.

An entrepreneur does not have to be a constructor of the analytical and decision-making instruments presented in this book. However, he or she should be aware of their existence and availability as well as should know where and how to become a disposer of the decision-making tools offered in this book. A modern *economist is a specialist, who must be able to prepare, to interpret, and to indicate application of econometric and statistical decision-making tools that discussed in this work.* Awareness of this fact – in terms of possible benefits to be obtained – is one of the author's main intentions.

The decision-supportive instruments presented in this book can be used in every enterprise, regardless its size. Larger scope of their application, obviously, exists in a large and medium-sized enterprise having trained economists on staff. In a small-sized enterprise, possibilities of applying the tools presented here only emerge when the company's owner has adequate qualifications. Rarely a small-sized enterprise hires a specialist with the skills indicated above.

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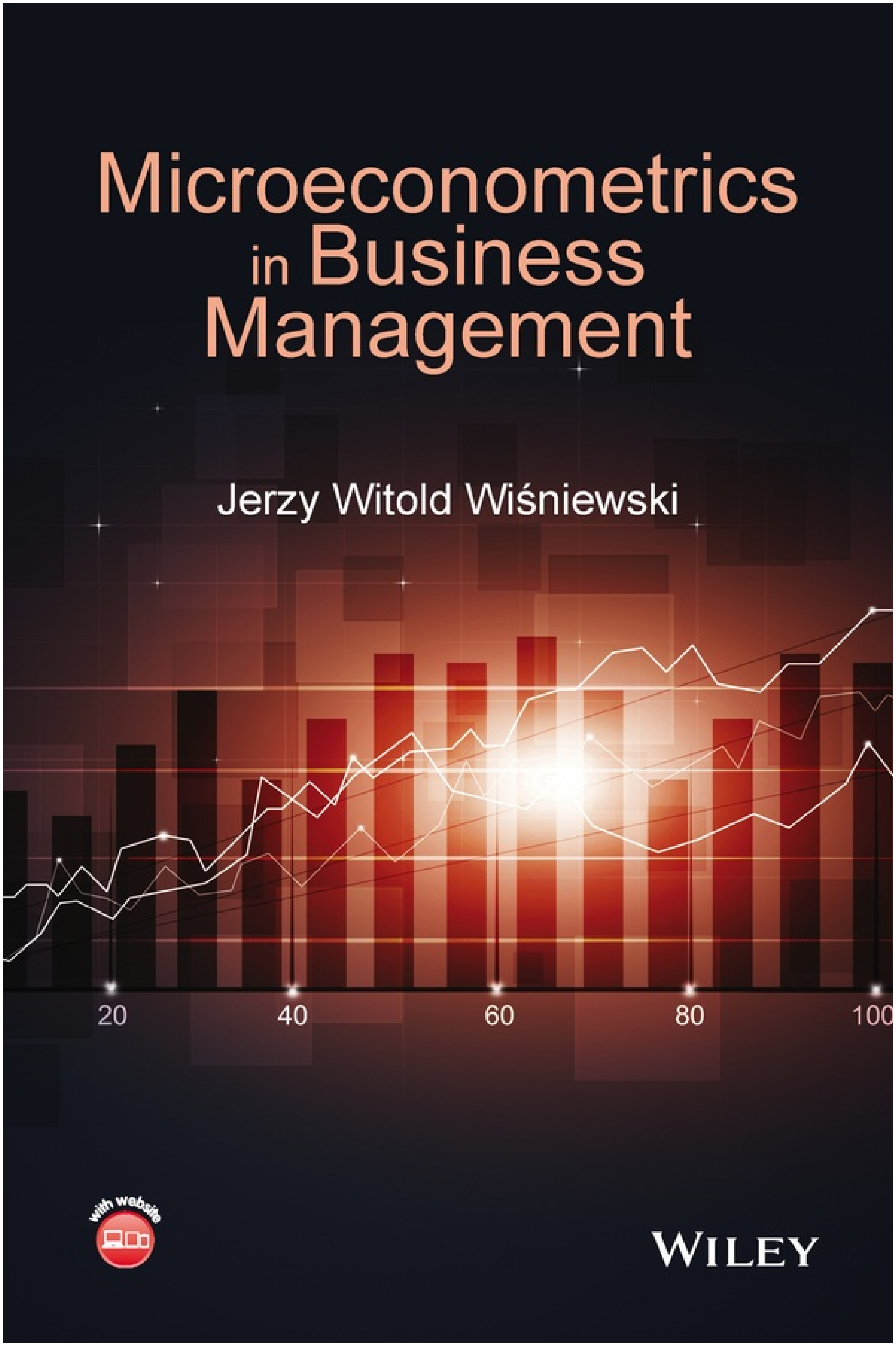
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