DEVELOPMENTS IN WATER SCIENCE

19

DAVID STEPHENSON

PIPEFLOW ANALYSIS

ELSEVIER

PIPEFLOW ANALYSIS

DEVELOPMENTS IN WATER SCIENCE, 19

OTHER TITLES IN THIS SERIES

1 G. BUGLIARELLO AND F. GUNTER
COMPUTER SYSTEMS AND WATER RESOURCES

2 H.L. GOLTERMAN
PHYSIOLOGICAL LIMNOLOGY

3 Y.Y. HAIMES, W.A.HALL AND H.T. FREEDMAN
MULTIOBJECTIVE OPTIMIZATION IN WATER RESOURCES SYSTEMS:
THE SURROGATE WORTH TRADE-OFF-METHOD

4 J.J. FRIED
GROUNDWATER POLLUTION

5 N. RAJARATNAM
TURBULENT JETS

6 D. STEPHENSON
PIPELINE DESIGN FOR WATER ENGINEERS

7 V. HÁLEK AND J. ŠVEC
GROUNDWATER HYDRAULICS

8 J. BALEK
HYDROLOGY AND WATER RESOURCES IN TROPICAL AFRICA

9 T.A. McMAHON AND R.G. MEIN
RESERVOIR CAPACITY AND YIELD

10 G. KOVÁCS
SEEPAGE HYDRAULICS

11 W.H. GRAF AND C.H. MORTIMER (EDITORS)
HYDRODYNAMICS OF LAKES: PROCEEDINGS OF A SYMPOSIUM
12–13 OCTOBER 1978, LAUSANNE, SWITZERLAND

12 W. BACK AND D.A. STEPHENSON (EDITORS)
CONTEMPORARY HYDROGEOLOGY: THE GEORGE BURKE MAXEY MEMORIAL VOLUME

13 M.A. MARIÑO AND J.N. LUTHIN
SEEPAGE AND GROUNDWATER

14 D. STEPHENSON
STORMWATER HYDROLOGY AND DRAINAGE

15 D. STEPHENSON
PIPLELINE DESIGN FOR WATER ENGINEERS
(completely revised edition of Vol. 6 in the series)

16 W. BACK AND R. LÉTOLLE (EDITORS)
SYMPOSIUM ON GEOCHEMISTRY OF GROUNDWATER

17 A.H. EL-SHAARAWI (EDITOR) IN COLLABORATION WITH S.R. ESTERBY
TIME SERIES METHODS IN HYDROSCIENCES

18 J. BALEK
HYDROLOGY AND WATER RESOURCES IN TROPICAL REGIONS

PIPEFLOW ANALYSIS

DAVID STEPHENSON

Civil Engineering Department, University of the Witwatersrand,
1 Jan Smuts Avenue, 2001 Johannesburg, South Africa

ELSEVIER

Amsterdam – Oxford – New York – Tokyo 1984

ELSEVIER SCIENCE PUBLISHERS B.V.
Molenwerf 1
P.O. Box 211, 1000 AE Amsterdam, The Netherlands

Distributors for the United States and Canada:

ELSEVIER SCIENCE PUBLISHING COMPANY INC.
52, Vanderbilt Avenue
New York, NY 10017

ISBN 0-444-42283-8 (Vol. 19)
ISBN 0-444-41669-2 (Series)

Printed in The Netherlands

PREFACE

The advent of economical and simple micro computers has left no excuse for engineers to avoid computer solutions to problems. Hydraulic engineers now have tools for modelling steady and unsteady flows in complex pipe networks. Tedious graphical and analogue simulations can be discarded with relief. This book is aimed at the water engineer who has to design water reticulation pipe networks, trunk mains, pumping lines and storage reservoirs. The practising engineer often tends to negelect the theoretical side, but when the occasion arises he requires a rapid, simple answer to problems of head loss, discharge capacity and pressures. The various computational methods available to him are summarized in this book, starting with simple steady flow problems and advancing through slow motion to water hammer in complex networks. The subject matter will also be of use to students of hydraulic engineering and those contemplating research in this field.

The iterative techniques for flow analysis of pipe networks such as the Hardy Cross method are known to most water engineers and have been applied extensively without the aid of computers. Some lesser known techniques are in fact simpler to apply on computers, e.g. the linear method. When it comes to unsteady flow, e.g. water hammer, computer analysis is much more rapid than the older graphical method and can account for many more factors such as column separation, changes in section and branch pipes. Numerical methods for computers are easy and accurate provided simple rules are followed. This book condenses and compares various methods for analysing flows and pressure variations in pipelines, whether they be pumping systems or multiple reservoir gravity systems. Simple BASIC computer programs are given in many chapters and these will serve as a basis for more comprehensive programs which the reader should be able to write after reading this book.

In addition to those on flow analysis, sections are given on design of pipe systems using optimization programs, and operation using computer simulation programs. An introduction to computer graphics is given but the book does not cover structural design of pipes. That is covered in another book by the same author, 'Pipeline Design for Water Engineers' (Elsevier, 1981).

This Page Intentionally Left Blank

CONTENTS

PAGE

CONTENTS (cont.) **PAGE**

CONTENTS (cont.) **PAGE**

This Page Intentionally Left Blank

CHAPTER 1

HYDRAULICS AND HEAD LOSS EQUATIONS

BASIC EQUATIONS

Most hydraulic problems in pipe systems can be solved starting with one or more of the basic equations described below, or an adaptation of them. It is the methods of solution of the equations, be they analytical, graphical or numerical, which this book is about.

The three equations which will appear in various forms throughout the book are the continuity equation, the momentum equation and the energy equation. For steady, incompressible one-dimensional flow the continuity equation is simply obtained by equating the flow rate at any section to the flow rate at another section along the stream tube. By 'steady flow' it is implied that there is no variation in velocity at any point. As far as unsteady flow is concerned the continuity equation has an additional term, namely the change in storage between the sections.

The momentum equation stems from Newton's basic law of motion and states that the change in momentum flux between two sections equals the sum of the forces on the fluid causing the change. For steady, one- dimensional flow this is

$$\Delta F_x = \rho Q \ \Delta V_x \qquad (1.1)$$

where F is the force, ρ is the fluid mass density, Q is the volumetric flow rate, V is velocity and subscript x refers to the 'x' direction.

The basic energy equation is derived by equating the work done on an element of fluid by gravitational and pressure forces to the change in kinetic energy. Mechanical and heat energy transfer are excluded from the equation. In most systems there is a loss of energy due to friction and turbulence and a term is included in the equation to account for this. The resulting equation for steady flow of incompressible fluids is termed the Bernoulli equation and is conveniently written as:

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{p_1}{\gamma} + Z_2 + h_\ell \qquad (1.2)$$

where V = mean velocity at a section
$V^2/2g$ = velocity head (units of length)
g = gravitational acceleration
p = pressure
p/γ = pressure head (units of length)
γ = unit weight of fluid
Z = elevation above an arbitrary datum
h_ℓ = head loss due to friction or turbulence between sections 1 and 2.

The sum of the velocity head plus pressure head plus elevation is termed the total head. Strictly the velocity head should be multiplied by a coefficient to account for the variation in velocity across the section of the conduit. The average value of the coefficient for turbulent flow is 1.06 and for laminar flow it is 2.0. Flow through a conduit is termed either uniform or non-uniform depending on whether or not there is a variation in the cross-sectional velocity distribution along the conduit.

For the Bernoulli equation to apply the flow should be steady, i.e. there should be no change in velocity at any point with time. The flow is assumed to be one-dimensional and irrotational. The fluid should be incompressible, or else a term for strain energy has to be introduced.

The respective heads are illustrated in Figure 1.1. For most practical cases the velocity head is small compared with the other heads, and it may be neglected. In fact it is often the case that minor head losses due to bends, expansions, etc. can also be neglected and friction need be the only method whereby head is lost.

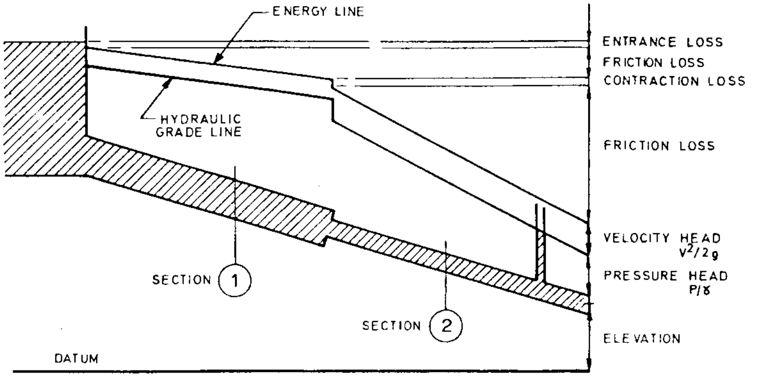


Fig. 1.1 Energy heads along a pipeline

FLOW-HEAD LOSS RELATIONSHIPS

Empirical Flow Formulae

The throughput or capacity of a pipe of fixed dimensions depends on the total head difference between the ends. This head is consumed by friction and other losses.

The first friction head loss/flow relationships were derived from field observations. These empirical relationships are still popular in waterworks practice although more rational formulae have been developed. The head loss/flow formulae thus established are termed conventional formulae and are usually given in an exponential form of the type

$$V = KD^{x}S^{y} \tag{1.3}$$

or $$S = K'Q^{n}/D^{m} \tag{1.4}$$

where V is the mean flow velocity, m, n, x, y, K and K' are constants, D is the inside diameter of the circular pipe and S is the head loss gradient (in m head loss per m length of pipe). Some of the equations more frequently applied are listed below:

	Basic Equations	S.I. units	ft-sec	
Hazen-Williams	$S=K_1(V/C_w)^{1.85}/D^{1.167}$	$K_1=6.84$	$K_1=3.03$	(1.5)
Manning	$S=K_2(nV)^2/D^{1.33}$	$K_2=6.32$	$K_2=2.86$	(1.6)
Chezy	$S=K_3(V/C_z)^2/D$	$K_3=13.13$	$K_3=4.00$	(1.7)
Darcy	$S=\lambda V^2/2gD$	Dimensionless		(1.8)

Except for the Darcy formula the above equations are not universal and the 'constant' in the equation depends on the units. It should be borne in mind that some of the formulae were intended purely for standard water engineering practice and take no account of variations in gravity, temperature or type of liquid. The friction coefficients vary with pipe diameter, surface configuration and age of pipe.

The conventional formulae are comparatively simple to use as they do not involve fluid viscosity. They may be solved directly as they do not require an initial estimate of Reynolds number to determine the friction factor (see next section). On the other hand, the more modern equations cannot be solved directly for rate of flow. Solution of the formulae for velocity, diameter or friction head gradient is simple with the aid of a slide rule, calculator, computer, nomograph or graphs plotted on log-log paper. The equations are of particular use for analysing flows in pipe networks where the flow/head loss equations have to be iteratively solved many times.

A popular flow formula in waterworks practice is the Hazen-Williams formula. Friction coefficients for use in this equation are tabulated in Table 1.1. If the formula is to be used frequently, solution with the aid of a chart is the most efficient way. Many waterworks organizations use graphs of head loss gradient plotted against flow for various pipe diameters, and various C values. As the value of C decreases with age, type of pipe and properties of water, field tests are desirable for an accurate assessment of C.

TABLE 1.1 Hazen-Williams friction coefficients C

Type of Pipe	Condition			
	New	25 years old	50 years old	Badly Corroded
PVC:	150	140	130	130
Smooth concrete, AC:	150	130	120	100
Steel, bitumen lined, galvanized:	150	130	100	60
Cast iron:	130	110	90	50
Riveted steel, vitrified, woodstave	120	100	80	45

Rational flow formulae

Although the conventional flow formulae are likely to remain in use for many years, more rational formulae are gradually gaining acceptance amongst engineers. The new formulae have a sound scientific basis backed by numerous measurements and they are universally applicable. Any consistent units of measurements may be used and liquids of various viscosities and temperatures conform to the proposed formulae.

The rational flow formulae for flow in pipes are similar to those for flow past bodies or over flat plates (Schlichting, 1960). The original research was on small-bore pipes with artificial roughness. Lack of data on roughness for large pipes has been one deterrent to the use of the relationships in waterworks practice.

The velocity in a full pipe varies from zero on the boundary to a maximum in the centre. Shear forces on the walls oppose the flow and a boundary layer is established with each annulus of fluid imparting a shear force onto an inner neighbouring concentric annulus. The resistance to relative motion of the fluid is termed kinematic viscosity, and in turbulent flow it is imparted by turbulent mixing with transfer of particles of different momentum between one layer and the next.

A boundary layer is established at the entrance to a conduit and this layer gradually expands until it reaches the centre. Beyond this point the flow becomes uniform. The length of pipe required for fully established flow is given by

$$\frac{x}{D}=0.7\ Re^{1/4} \text{ for turbulent flow} \qquad (1.9)$$

The Reynolds number $Re = VD/\nu$ is a dimensionless number incorporating the fluid viscosity ν which is absent in the conventional flow formulae. Flow in a pipe is laminar for low Re (<2000) and becomes turbulent for higher Re (normally the case in practice). The basic head loss equation is derived by setting the boundary shear force over a length of pipe equal to the loss in pressure multiplied by the area:

$$\tau\pi DL = \gamma h_f\ \pi D^2/4 \qquad (1.10)$$

$$\therefore h_f = \frac{4\tau/\gamma}{V^2/2g}\frac{L}{D}\frac{V^2}{2g} \qquad (1.11)$$

$$= \lambda\frac{L}{D}\frac{V^2}{2g} \qquad (1.12)$$

where $\lambda = (4\tau/\gamma)/(V^2/2g)$ is the Darcy friction factor, τ is the shear stress, D is the pipe diameter and h_f is the friction head loss over a length L. λ is a function of Re and the relative roughness k/D. For laminar flow, Poiseuille found that $\lambda = 64/Re$ i.e. λ is independent of the relative roughness. Laminar flow will not occur in normal engineering practice. The transition zone between laminar and turbulent flow is complex and undefined but is also of little interest in practice.

Turbulent flow conditions may occur with either a smooth or a rough boundary. The equations for the friction factor for both conditions are derived from the general equation for the velocity distribution in a turbulent layer, which is derived from mixing length theory:

$$\tau = \rho k^2 \ell^2 \left(\frac{dv}{dy}\right)^2 \qquad (1.13)$$

Integrating with the constant k found to be 0.4 and converting to $\log_{10}$:

$$\frac{v}{\sqrt{\tau/\rho}} = 5.75 \log \frac{y}{y'} \qquad (1.14)$$

where v is the velocity at a distance y from the boundary. For a hydrodynamically smooth boundary there is a laminar sub-layer, and Nikuradse found that $y' \propto \nu / \sqrt{\tau/\rho}$ where y' is the boundary layer thickness, so

$$\frac{v}{\sqrt{\tau/\rho}} = 5.75 \log y \frac{\sqrt{\tau/\rho}}{\nu} + 5.5 \qquad (1.15)$$

The constant 5.5 was found experimentally.

Where the boundary is rough the laminar sub-layer is affected and Nikuradse found that $y' = k/30$ where k is the boundary roughness.

Thus $$\frac{v}{\sqrt{\tau/\rho}} = 5.75 \log \frac{y}{k} + 8.5 \qquad (1.16)$$

Rearranging equations 1.15 and 1.16 and expressing v in terms of the average velocity V by means of the equation $Q = \int v dA$ results in

$$\frac{1}{\sqrt{\lambda}} = 2 \log Re \sqrt{\lambda} - 0.8 \qquad (1.17)$$

(turbulent boundary layer, smooth boundary) and

$$\frac{1}{\sqrt{\lambda}} = 2 \log \frac{D}{k} + 1.14 \qquad (1.18)$$

(turbulent boundary layer, rough boundary).

Notice that for a smooth boundary, λ is independent of the relative roughness k/D and for a very rough boundary it is independent of the Reynolds number Re for all practical purposes.

Colebrook and White combined Equations 1.17 and 1.18 to produce an equation covering both smooth and rough boundaries as well as the transition zone:

$$\frac{1}{\sqrt{\lambda}} = 1.14 - 2 \log \left(\frac{k}{D} + \frac{9.35}{Re\sqrt{\lambda}}\right) \qquad (1.19)$$

Their equation reduces to Equ. 1.17 for smooth pipes, and to Equ. 1.18 for rough pipes. This semi-empirical equation yields satisfactory results for various commercially available pipes. Nikuradse's original experiments used sand as artificial boundary roughness. Natural roughness is evaluated according to the equivalent sand roughness. Table 1.2 gives values of k for various surfaces.

TABLE 1.2 Roughness of pipe materials (Hydraulics Research Station, 1969). Value of k in mm

Finish	Smooth	Average	Rough
Glass, drawn metals	0	0.003	0.006
Steel, PVC or AC	0.015	0.03	0.06
Coated steel	0.03	0.06	0.15
Galvanized, vitrified clay	0.06	0.15	0.3
Cast iron or cement lined	0.15	0.3	0.6
Spun concrete or wood stave	0.3	0.6	1.5
Riveted steel	1.5	3	6
Foul sewers, tuberculated water mains	6	15	30
Unlined rock, earth	60	150	300

Fortunately λ is not very sensitive to the value of k assumed. k increases linearly with age for water pipes. The various rational formulae for λ were plotted on a single graph by Moody and this graph is presented as Figure 1.2.

A close approximation to λ is often given by the following equation:

$$\lambda = 0.0055 \{ 1+(20000\,k/D + 10^6/Re)^{1/3} \} \qquad (1.20)$$

This equation avoids an implicit situation but is only a first approximation which should be substituted in the r.h.s. of (1.19) to obtain a better value.

Unfortunately the Moody diagram is not very amenable to direct solution for any variable for given values of the dependent variables, and a trial and error analysis may be necessary to get the velocity for the Reynolds number if reasonable accuracy is required. The Hydraulics Research Station at Wallingford (1969) re-arranged the variables in the Colebrook- White equation to produce simple explicit flow/head loss graphs. Thus equation 1.19 may be arranged in the form

$$V = -2\sqrt{2gDS}\ \log\left(\frac{k}{3.7D} + \frac{2.51\nu}{D\sqrt{2gDS}}\right) \qquad (1.21)$$

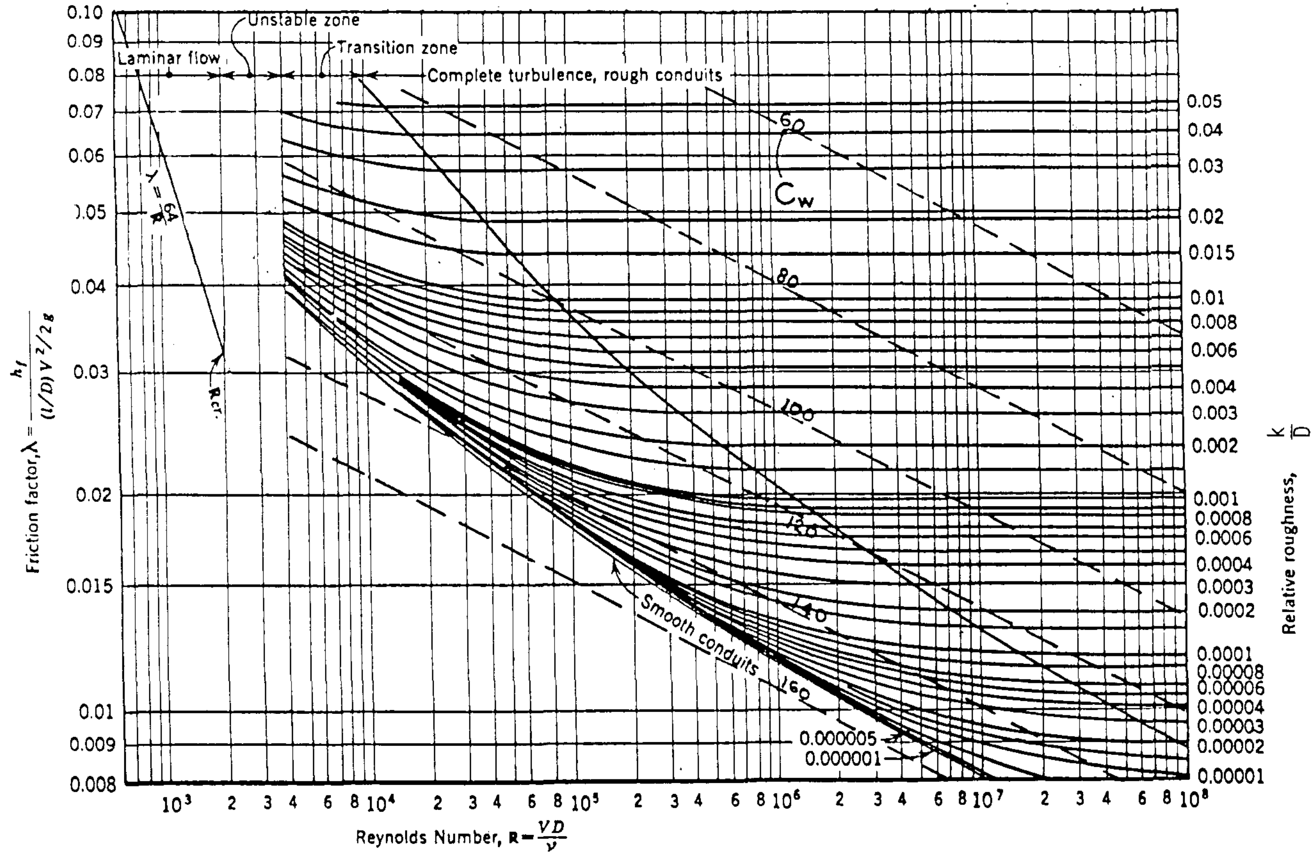


Fig. 1.2 Moody resistance diagram for uniform flow in pipes

Thus for any fluid at a certain temperature and defined roughness k, a graph may be plotted in terms of V, D and S. Figures 1.3 to 1.7 are such graphs for water at 15°C and for various roughnesses K.

In summary the Darcy-Weisbach equation for head loss in circular pipes

$$h_f = \frac{\lambda L V^2}{2gD} \qquad (1.22)$$

where h is the head loss (or the energy loss per unit weight), λ is the Darcy-Weisbach friction factor, L is the pipe length, D is the pipe diameter, g is the gravitational acceleration and V is the average fluid velocity. It may be noted that in North America λ is replaced by f, whereas British practice is to use another f′ in the equation

$$h_f = f'LV^2/2gR \qquad (1.23)$$

where R is the hydraulic radius D/4 for a circular pipe. Thus $f' = \lambda/4$.

$$\text{Since } V = 4Q/\pi D^2 \qquad (1.24)$$

we can also rewrite equation 1.22 as

$$\frac{h_f}{L} = \frac{8\lambda Q^2}{\pi^2 gD} \qquad (1.25)$$

Solution of the basic equations

The Colebrook-White formula (1.19) which forms the basis of the Moody diagram may also be written as

$$\frac{1}{\sqrt{\lambda}} = -0.8686 \ \ell n \left(\frac{k}{3.7D} + \frac{2.51}{Re\sqrt{\lambda}}\right) \qquad (1.26)$$

$$\text{Also } Re = VD/\nu = 4Q/\pi D\nu \qquad (1.27)$$

The equation must be solved by iteration for h. This can be conveniently done by letting $x = \lambda^{-\frac{1}{2}}$ and using the Newton Raphson iterative scheme,

$$x^+ = x - \frac{g(x)}{g'(x)} \qquad (1.28)$$

where in this case we have from equation 1.26

$$g(x) = x + 0.8686 \ \ell n \left(\frac{k}{3.7D} + \frac{2.51x}{Re}\right) \qquad (1.29)$$

and g'(x) is the derivative of g(x). We then obtain

$$x^{+} = x - \left\{ \frac{x + 0.8686\ \ell n\left(\frac{k}{3.7D} + \frac{2.51x}{Re}\right)}{1 + 0.8686\ \left[\frac{2.51/Re}{\left(\frac{k}{3.7D} + \frac{2.51x}{Re}\right)}\right]} \right\} \tag{1.30}$$

where x^{+} is a successive approximation to the solution of $g(x) = 0$ using the prior approximation x. Setting $x = x^{+}$ and solving for x^{+} several times, the solution can be obtained to any desired accuracy.

This equation converges rapidly for almost any starting value of x and can be easily solved on a computer or on a programmable calculator. Using this equation in combination with equations (1.22) and (1.27) one can solve for head loss.

Note that since $\lambda = \frac{h_f 2gD}{LV^2}$ (1.31)

$$Re\ \sqrt{\lambda} = \sqrt{2gh_f/L}\ D^{3/2}/\nu \tag{1.32}$$

so that (1.26) can be solved directly for V given h_f, L, g, D, ν and k:

$$V = \sqrt{2gDh_f/L}\ \ -0.8686\ \ell n\ \left(\frac{k}{3.7D} + \frac{2.51\nu}{\sqrt{2gD^3 h_f/L}}\right) \tag{1.33}$$

If however, V is given then either λ or h_f must be obtained employing an iterative procedure.

Comparison of Friction Formulae

The Darcy equation may be written as

$$V = \sqrt{2g/\lambda}\ \sqrt{SD} \tag{1.35}$$

$$\text{or } V = C_z\ \sqrt{SR} \tag{1.36}$$

which is termed the Chezy equation and the Chezy coefficient is

$$C_z = \sqrt{8g/\lambda} \tag{1.37}$$

The Hazen-Williams equation may be rewritten for all practical purposes in the following dimensionless form:

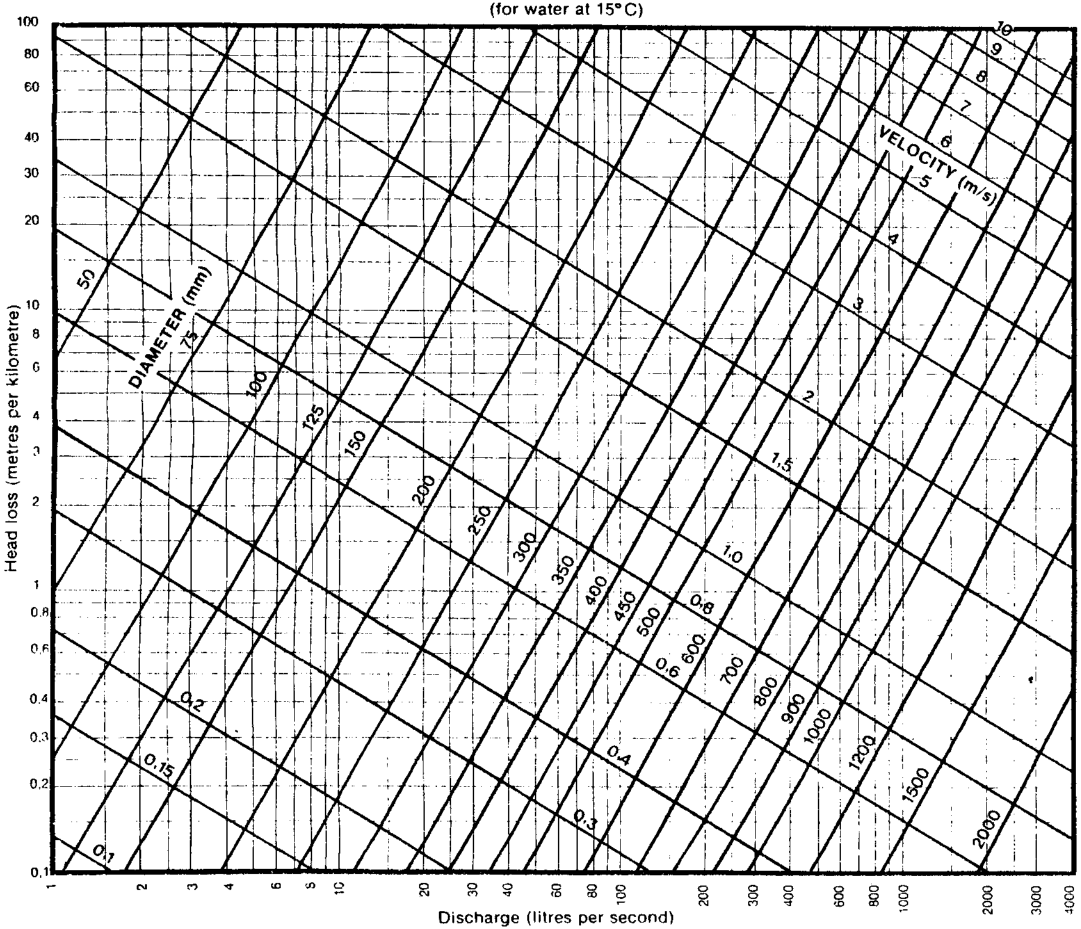
$$S = 515(V/C_w)^2\ \ (C_w/Re)^{0.15}/gD \tag{1.38}$$

By comparing this with the Darcy-Weisbach equation (1.25) it may be deduced that

$$C_w = 42.4\ \ (\lambda^{0.54} Re^{0.08}) \tag{1.39}$$

FIG. 1.3 FRICTION LOSS CHART FOR PIPES FLOWING FULL

(for water at 15°C)



k = 0.015 mm

Typical examples:

Uncoated steel
PVC or AC

This chart is derived from the Darcy-Weisbach equation:

$$\frac{h}{L} = \frac{8\lambda Q^2}{\pi^2 g D^5}$$

where

- h = head loss due to friction
- L = length of pipe
- λ = friction factor
- Q = discharge
- g = acceleration due to gravity (9.81 m/s²)
- D = internal diameter of pipe

The friction factor, λ, has been determined using the Colebrook-White equation:

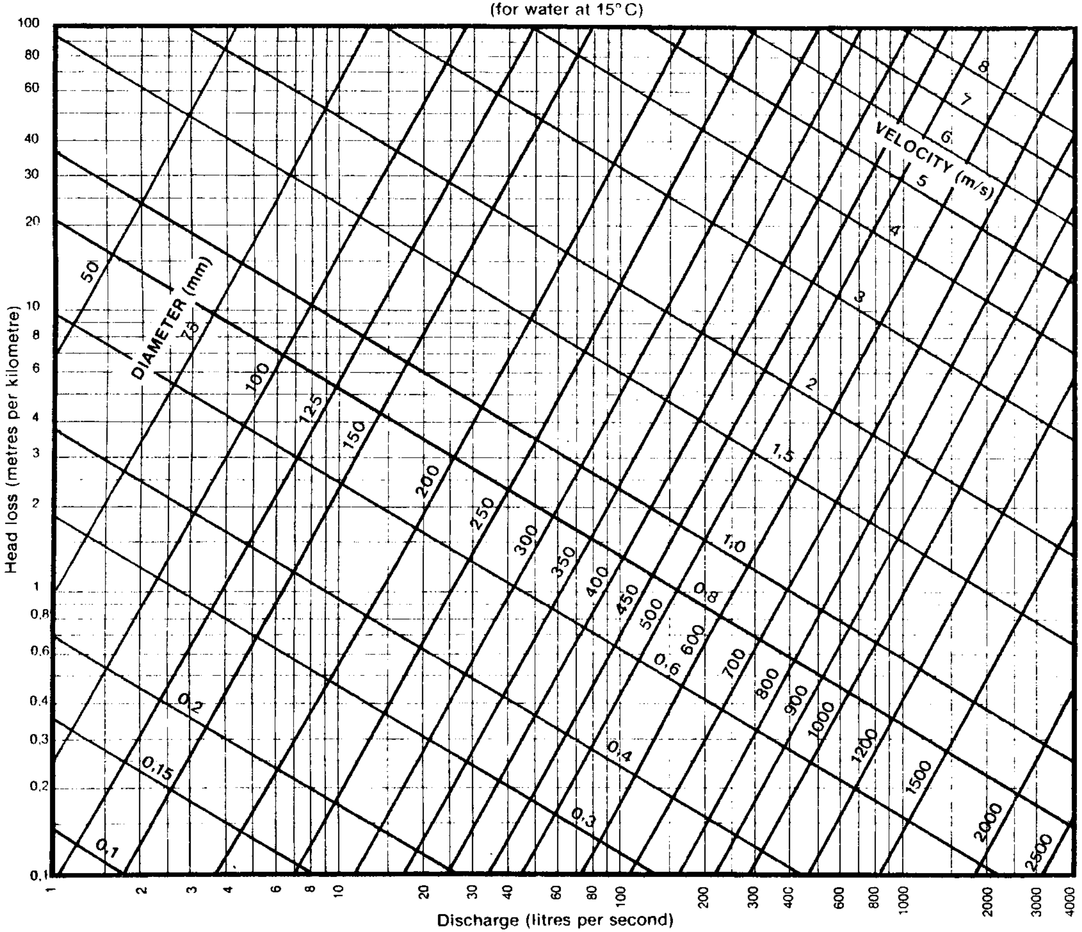
$$\frac{1}{\sqrt{\lambda}} = -2 \log_{10}\left(\frac{k}{3.7D} + \frac{2.51}{R_e\sqrt{\lambda}}\right)$$

where

- k = boundary roughness
- R_e = Reynolds number $= \frac{4Q}{\pi D \nu}$
- ν = kinematic viscosity = 1.15×10^{-6} m²/s for water at 15°C

FIG. 1.4 FRICTION LOSS CHART FOR PIPES FLOWING FULL

(for water at 15° C)



k = 0.03 mm

Typical examples:

Coated steel
Wrought iron

This chart is derived from the Darcy-Weisbach equation:

$$\frac{h}{L} = \frac{8\lambda Q^2}{\pi^2 g D^5}$$

where

h = head loss due to friction
L = length of pipe
λ = friction factor
Q = discharge
g = acceleration due to gravity (9,81 m/s²)
D = internal diameter of pipe

The friction factor, λ, has been determined using the Colebrook-White equation:

$$\frac{1}{\sqrt{\lambda}} = -2 \log_{10}\left(\frac{k}{3.7D} + \frac{2.51}{R_e\sqrt{\lambda}}\right)$$

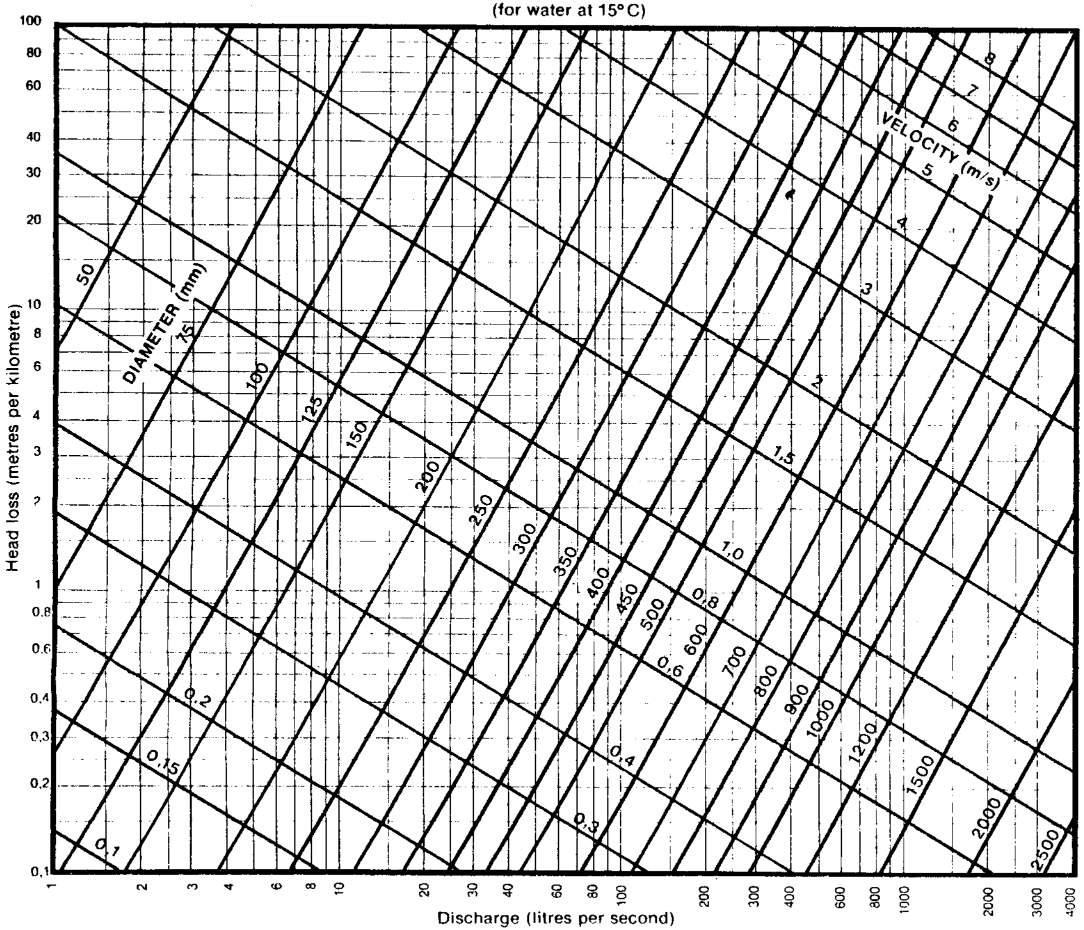
where

k = boundary roughness
R_e = Reynolds number
$= \frac{4Q}{\pi D \nu}$

ν = kinematic viscosity
$= 1.15 \times 10^{-6}$ m²/s for water at 15°C

FIG. 1.5 FRICTION LOSS CHART FOR PIPES FLOWING FULL

(for water at 15°C)



k = 0.06 mm

Typical examples:

Galvanised iron
Coated cast iron
Glazed vitrified clay

This chart is derived from the Darcy-Weisbach equation:

$$\frac{h}{L} = \frac{8\lambda Q^2}{\pi^2 g D^5}$$

where:

- h = head loss due to friction
- L = length of pipe
- λ = friction factor
- Q = discharge
- g = acceleration due to gravity (9.81 m/s²)
- D = internal diameter of pipe

The friction factor, λ, has been determined using the Colebrook-White equation:

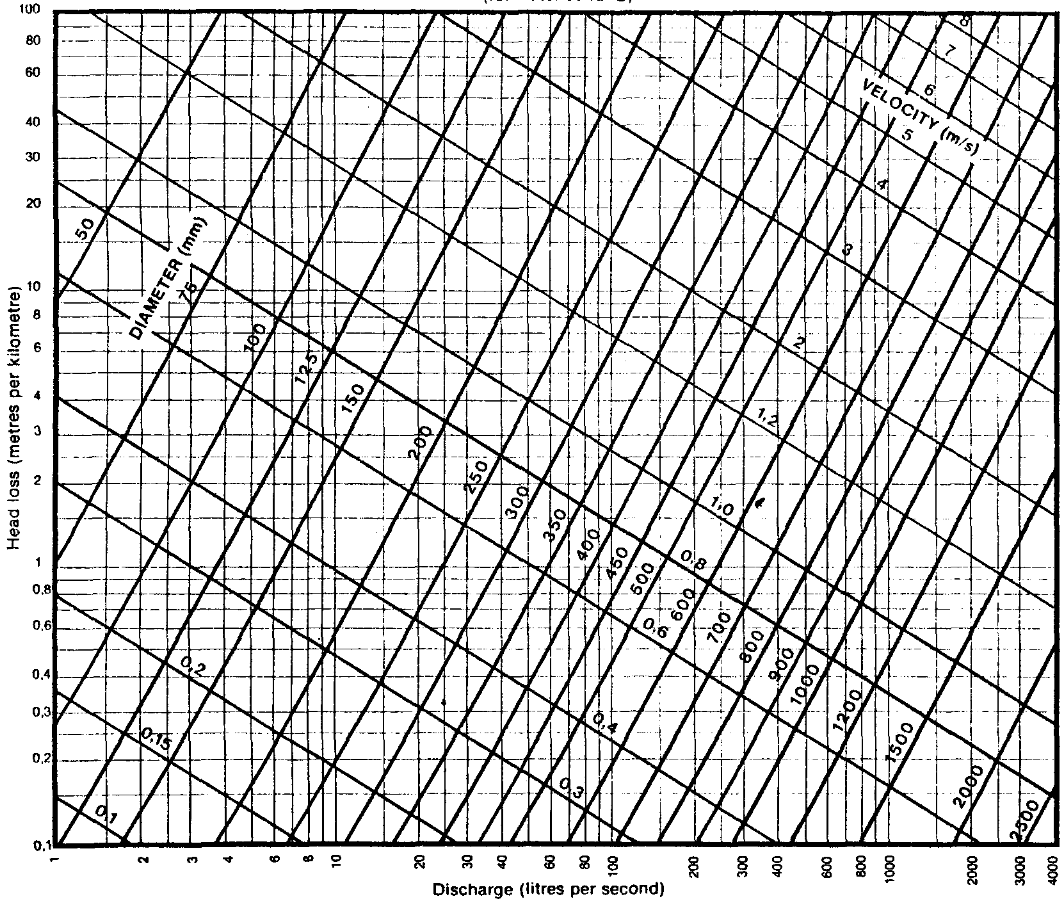
$$\frac{1}{\sqrt{\lambda}} = -2 \log_{10}\left(\frac{k}{3.7D} + \frac{2.51}{R_e\sqrt{\lambda}}\right)$$

where

- k = boundary roughness
- R_e = Reynolds number $= \frac{4Q}{\pi D \nu}$
- ν = kinematic viscosity = 1.15×10^{-6} m²/s for water at 15°C

FIG. 1.6 FRICTION LOSS CHART FOR PIPES FLOWING FULL

(for water at 15°C)



k = 0.15 mm

Typical examples:

Rusty wrought iron
Uncoated cast iron

This chart is derived from the Darcy-Weisbach equation:

$$\frac{h}{L} = \frac{8\lambda Q^2}{\pi^2 g D^5}$$

where

- h = head loss due to friction
- L = length of pipe
- λ = friction factor
- Q = discharge
- g = acceleration due to gravity (9,81 m/s²)
- D = internal diameter of pipe

The friction factor, λ, has been determined using the Colebrook-White equation:

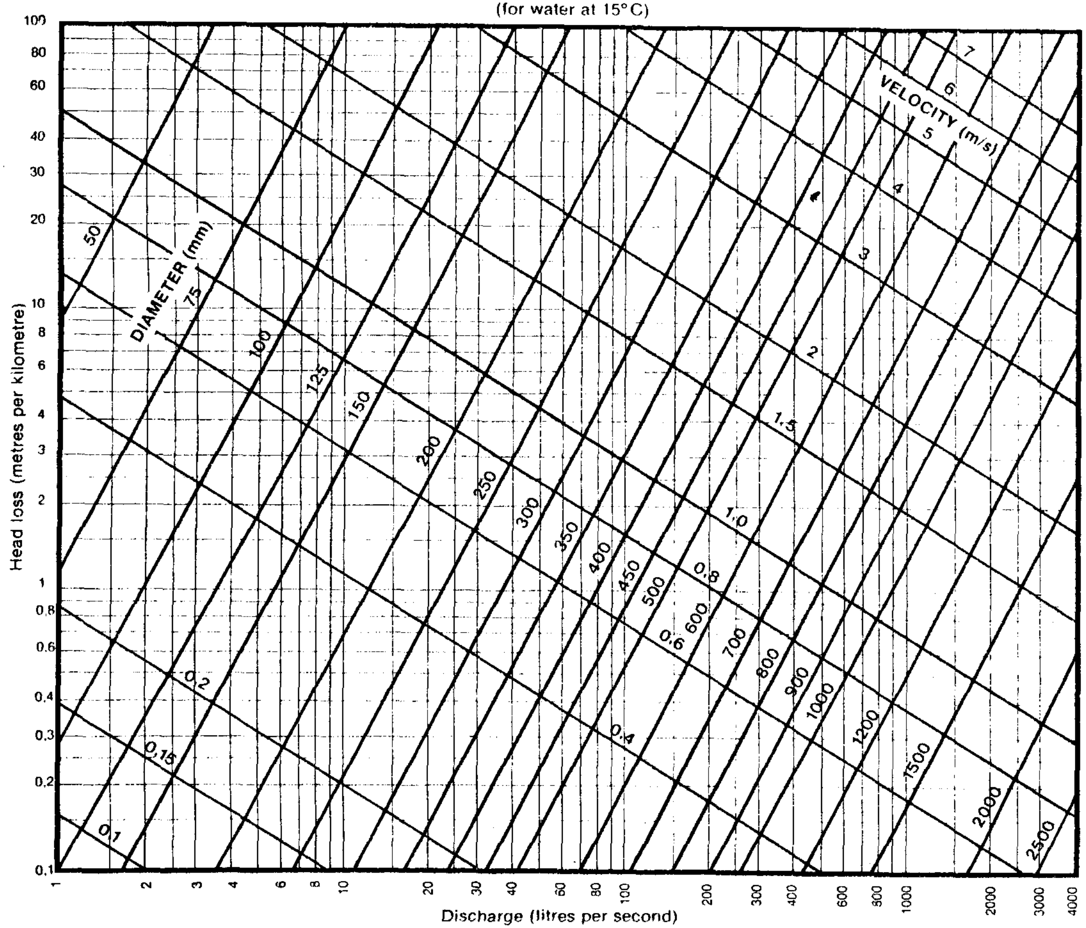
$$\frac{1}{\sqrt{\lambda}} = -2 \log_{10}\left(\frac{k}{3{,}7D} + \frac{2{,}51}{R_e\sqrt{\lambda}}\right)$$

where

- k = boundary roughness
- R_e = Reynolds number $= \frac{4Q}{\pi D \nu}$
- ν = kinematic viscosity = 1,15 x 10^{-6} m²/s for water at 15°C.

FIG. 1.7 FRICTION LOSS CHART FOR PIPES FLOWING FULL

(for water at 15°C)



k = 0.3 mm

Typical examples:

Wood stave pipes
Spun concrete

This chart is derived from the Darcy-Weisbach equation:

$$\frac{h}{L} = \frac{8\lambda Q^2}{\pi^2 g D^5}$$

where

- h = head loss due to friction
- L = length of pipe
- λ = friction factor
- Q = discharge
- g = acceleration due to gravity (9,81 m/s²)
- D = internal diameter of pipe

The friction factor, λ, has been determined using the Colebrook-White equation

$$\frac{1}{\sqrt{\lambda}} = -2 \log_{10}\left(\frac{k}{3,7D} + \frac{2,51}{R_e\sqrt{\lambda}}\right)$$

where

- *k* = *boundary roughness*
- R_e = Reynolds number $= \frac{4Q}{\pi D \nu}$
- ν = kinematic viscosity $\approx 1,15 \times 10^{-6}$ m²/s for water at 15°C

The Hazen-Williams coefficient C_w is therefore a function of λ and Re and values may be plotted on a Moody diagram (see Figure 1.2). It will be observed from Figure 1.2 that lines for constant Hazen-Williams coefficient coincide with the Colebrook-White lines only in the transition zone. In the completely turbulent zone for non-smooth pipes the coefficient will actually reduce the greater the Reynolds number, i.e. one cannot associate a certain Hazen-Williams coefficient with a particular pipe as it varies with the flow rate.

The Manning equation is widely used for open channel flow and part full pipes. The equation is

$$V = \frac{K}{n} R^{2/3} S^{\frac{1}{2}} \tag{1.40}$$

where K is 1.000 in SI units and 1.486 in ft lb units, and R is the hydraulic radius A/P where A is the cross sectional area of flow and P the wetted perimeter. R is D/4 for a circular pipe, and in general for non-circular sections, 4R may be substituted for D.

TABLE 1.3 Values of Manning's 'n'

Smooth glass, plastic	0.010
Concrete, steel (bitumen lined), galvanized	0.011
Cast Iron	0.012
Slimy or greasy sewers	0.013
Rivetted steel, vitrified, wood-stave	0.015
Rough concrete	0.017

MINOR LOSSES

One method of expressing head loss through fittings and changes in section is the equivalent length method, often used when the conventional friction loss formulae are used. Modern practice is to express losses through fittings in terms of the velocity head, i.e. $h_\ell = KV^2/2g$ where K is the loss coefficient. Table 1.4 gives typical loss coefficients although valve manufacturers may also provide supplementary data and loss coefficients K which will vary with gate opening. The velocity V to use is normally the mean through the full bore of the pipe or fitting.

TABLE 1.4 Loss coefficients for pipe fittings.

Bends $h_B = K_B V^2/2g$

Bend angle	Sharp	r/D=1	2	6
30^o	0.16	0.07	0.07	0.06
45^o	0.32	0.13	0.10	0.08
60^o	0.68	0.18	0.12	0.08
90^o	1.27	0.22	0.13	0.08
180^o	2.2	0.40	0.25	0.15

Valves $h_v = K_v V^2/2g$

Opening:	1/4	1/2	3/4	Full
Sluice	24	5.6	1.0	0.2
Butterfly	120	7.5	1.2	0.3
Globe	160	40	20	10
Needle	4	1	0.6	0.5
Reflux				1-2.5

Contractions and expansions in cross section

Contractions: $h_c = K_c V_2^2/2g$ Expansions: $h_c = K_c V_1^2/2g$

Wall angle	A_2/A_1 0	0.2	0.4	0.6	0.8	1.0	A_1/A_2 0	0.2	0.4	0.6	0.8	1.0
7.5^o							.13	.08	.05	.02	.0	0
15^o							.32	.24	.15	.08	.02	0
30^o							.78	.45	.27	.13	.03	0
180^o	.5	.37	.25	.15	.07	0	1.0	.64	.36	.17	.04	0

Entrance and exit losses: $h_e = K_e V^2/2g$

	Entrance	Exit
Protruding	0.8	1.0
Sharp	0.5	1.0
Bevelled	0.25	0.5
Rounded	0.05	0.2

It is frequently unnecessary to calculate the losses in all pipe fittings if in fact they need to be considered at all. The full velocity head $V^2/2g$ is typically less than 0.2m so even the full velocity head is small in comparison with many line friction head losses. The losses due to fittings rarely exceeds 5% of the friction loss except for very high design pressure heads when it is economical to reduce the diameter of fittings such as valves.

On the other hand, the presence of air in pipelines can add to the friction loss. Although air will seldom build up to create a large pocket in comparison with the cross sectional area there is frequently air in free bubble form in transport. The presence of 1% of free air is not impossible and since the head loss is nearly proportional to V^2 the corresponding head loss increases 2%.

Fittings can be accounted for by adding the losses to the friction loss:

$$\text{Thus} \quad h = h_f + h_\ell \tag{1.41}$$

$$= \frac{\lambda L}{D}\frac{V^2}{2g} + \Sigma K\frac{V^2}{2g} \tag{1.42}$$

$$= \left(\frac{\lambda L}{D} + \Sigma K\right)\frac{V^2}{2g} \tag{1.43}$$

One method is to add on an equivalent length of pipe to give the same head loss as the fitting. The equivalent length is from (1.43)

$$L' = (D/\lambda)\Sigma K \tag{1.44}$$

Obviously the variation of λ with Re is not strictly accounted for this way and neither is the variation of K or discharge coefficient of valves with opening.

USE OF HEAD LOSS CHARTS FOR SOLUTION OF SIMPLE PIPE SYSTEMS

Many of the following chapters describe methods of calculating head losses in complex pipe networks. Those problems have a number of unknowns. For instance if a pipe with a known head changes diameter along its length, the head loss for each section, as well as the total flow rate may be unknown. Similarly if the pipe branches, the flows as well as the head losses become unknowns. Some additional facts are needed to solve for all the unknowns. The continuity equation is invariably utilized in such cases. In general, however, it is necessary to simplify the flow-head loss relationships somewhat where simultaneous solutions of a number of equations are involved. Thus if the Darcy head loss equation is utilized the friction factor λ could be assumed a constant. Alternatively an equation of the following form is used

$$h = KLQ^n/D^m \tag{1.45}$$

where K is assumed constant for all pipes. In many of the numerical and iterative methods explained later these simplifications are not essential and friction factors varying with pipe and flow can be accounted for. If equivalent pipe or analytical methods are employed the simplifications are necessary, or at least the assumption that friction factor is independent of flow rate.

One method of accounting for simple pipes in series or in parallel is the equivalent pipe method. The diameter of an equivalent pipe to replace a compound pipe may be derived analytically (see Chapter 2) or more simply, if computations are to be performed by hand, obtained from a head loss chart such as Figure 1.5

Consider as an example a compound pipe comprising a 300 mm bore 1000 m long pipe in series with (leading into) a 400 mm bore pipe 2000 m long. A simple relationship between head loss and flow rate is not possible unless the pipe is replaced by an equivalent uniform bore pipe, say 3000 m long. A method of obtaining the diameter of such a pipe is to assume a reasonable flow rate (e.g. 100 ℓ/s) and read off the total head loss from Figure 1.5: h = 5.4 + 1.3 X 2 km = 8 m, so that the mean head loss gradient is 8/3 = 2.67 m/km and the equivalent pipe diameter to discharge 100 ℓ/s is 345 mm.

As another example consider a 500 mm bore pipe, 3000 m long laid parallel to a 700 mm bore pipe 4000 m long and both pipes are connected at both ends. In this case a total head loss is assumed, say 10 m. The head gradient along the 500 mm pipe is 3.33 m/km and the flow rate from Figure 1.5 is 300. The head gradient along the 700 mm bore pipe is 2.5 m/km and the flow rate 620 ℓ/s. An equivalent pipe with a length of, say, 4000 m to discharge 920 ℓ/s at a gradient of 10 m/4 = 2.5 m/km would require a bore of 820 mm. It should be noted that Figure 1.5 is for a particular roughness and slightly different results would be obtained if alternative head loss charts were employed or even if different flow or head loss assumptions were made to start with. That is because the effect of varying Reynolds number was ignored. A more accurate solution could have been obtained by using the Moody diagram or the Colebrook-White equation and iterating. The additional effort is, however, seldom justified in real systems as the losses due to many factors (including roughness) are only estimates anyway.

REFERENCES

Albertson, M.L., Barton, J.R. and Simon, D.B., 1960. Fluid Mechanics for Engineers, Prentice Hall, N.J.

Diskin, M.H., Nov. 1960. The limits of applicability of the Hazen-Williams formulae, La Houille Blanche, 6.

Hydraulics Research Station, 1969. Charts for the Hydraulic Design of Channels and Pipes, 3rd Edn., H.M.S.O., London.

Schlichting, H., 1960. Boundary Layer Theory, 4th Edn., McGraw Hill, N.Y.

Watson, M.D., July, 1979. A simplified approach to the solution of pipe flow problems using the Colebrook-White method. Civil Eng. in S.A., 21(7), pp. 169-171.

CHAPTER 2

ALTERNATIVE METHODS OF PIPE NETWORK FLOW ANALYSIS

TYPES OF PIPEFLOW PROBLEMS

The hydraulic engineer is confronted with many problems in the planning, design and operation of piped water supply systems. The problems can be divided into analysis and design types, both for steady flow and unsteady flow.

The analysis of steady flows in simple pipes may be for flow rate (given the head loss) or for head loss (given the flow rate). The same calculations apply to compound pipes (in parallel or series) although solution of more than one equation is then involved. When it comes to branched or looped networks more sophisticated methods become necessary.

The design problem is usually treated as a steady state problem. That is for known heads and drawoffs, the engineer has to select the pipe layout and diameter and reservoir location and size. The latter aspect, namely reservoir sizing is really an unsteady flow problem which may often be solved using steady-state equations. Net outflow over the peak drawoff period may be assessed by multiplying drawoff rate by time.

For more rapid variations in flow 'rigid column' surge theory or even elastic water hammer theory is necessary to determine heads and transient flows. Computer analysis is practically essential. Once suitable programs are available they could, however, even be used to determine steady-state flows or heads.

The design problem associated with unsteady flows is the determination of pipe wall thicknesses, and the operating rules for valves, pumps etc.

METHODS OF SOLUTION

Where complex pipe networks are utilized for water distribution it is not easy to calculate the flow in each pipe or the head at each point. Even if the flow-head loss equation assumed is explicit for each given pipe length, diameter and roughness, the non-linear relationship between head loss and flow makes calculation difficult. In unlooped tree-like networks the flows will be defined by the drawoffs but if the pipe network incorporates closed loops flows are unknown as well as heads at the various nodes.

The complexity of the pipe network, as well as the facilities available for computation, will dictate which method of analysis is to be utilized. Many of the following methods can be performed manually whereas computers are required for the more complex methods, particularly where unsteady flow is involved:

1. Equivalent pipes for compound pipes in series.
2. Equivalent pipes for complex pipes in parallel.
3. Trial and error methods for multiple reservoir problems.
4. Analytical solution of flow-head loss equations for compound pipes.
5. Analytical solution of flow-head loss equations for pseudo-steady flow.
6. Iterative node head correction for predominantly branched networks (by hand or computer).
7. Iterative loop flow correction for looped networks (by hand or computer).
8. Simultaneous solution of the head loss equations for all pipes by matrix or iterative methods (easiest for laminar flow when head loss is linearly proportional to flow).
9. Linearization of head loss equation and iterative solution for heads at nodes.
10. Linearization of head loss equation and iterative solution for flows in pipes.
11. Analytical solution of rigid column unsteady flow equation.
12. Numerical solution of finite difference form of rigid column acceleration equation, head loss equation and continuity equation.
13. Graphical analysis for unsteady rigid column flow.
14. Graphical analysis for unsteady elastic water hammer.
15. Finite difference and characteristic solution of differential water hammer equations using computers. Valves, pumps, vaporization, release systems and branches may be considered.

Generally the equations used are:

(i) Continuity of flow at junctions (net inflow less drawoff must be zero).

(ii) Head differences between nodes equal friction head loss in the pipes linking them. Minor losses and velocity head are generally neglected or included in the friction term, or an equivalent length of pipe is added to the pipeline to allow for minor losses.

(iii) Dynamic equations of motion - only where acceleration or deceleration of water is significant.

SIMPLE PIPE PROBLEMS

Calculation of Head Difference, Given Flow Rate

For the case of a known flow rate in a pipe of known length, diameter and roughness, the calculation of head loss is simple and direct, using for example the Hazen-Williams equation,

$$h_f = 6.84L(V/C)^{1.85}/D^{1.167} \text{ (m-s units)} \tag{2.1}$$

where C is the Hazen-Williams coefficient e.g. 140 for smooth pipes reducing for rough pipes, or the Darcy-Weisbach equation,

$$h_f = \frac{\lambda L}{D}\frac{V^2}{2g} \tag{2.2}$$

where λ is a function of the pipe relative roughness and the Reynolds number, $Re = VD/\nu$, ν is the kinematic viscosity of the fluid, V is the mean velocity, D the pipe bore, L the pipe length and g is gravitational acceleration.

Calculation of Flow Rate, Given Head Difference

The exponential type of equation, e.g. Hazen-Williams, can be solved directly for flow velocity and hence discharge rate. This is one reason why this type of equation remains popular despite its empiricism. The more scientific formula of Darcy has to be solved iteratively for this case as the Reynolds number is a function of flow velocity which is an unknown. The procedure is to guess a V, calculate the corresponding Re, read λ from a Moody diagram (or calculate it from the Colebrook- White equation) and solve for a new flow velocity from the head loss equation. It is generally not necessary to repeat the process with the new velocity more than twice as the flow converges rapidly. If the procedure is to be utilized in a computer a Newton Raphson convergence method could be programmed (see Chapter 1). If the computations are performed manually a head loss chart based on the equations can be used to simplify the calculations.

INTER-CONNECTED RESERVOIRS

A more complicated problem involves the calculation of both flow rate and head drop along compound pipes inter-connecting a number of reservoirs. For example a drawoff may occur from a point in a pipe network which is fed from a number of reservoirs - the problem is to estimate the flow rate in each pipe and the head at the drawoff point.

It is assumed that the friction factors, diameters and lengths of all pipes are known. Also, the level in each reservoir remains constant and the pressure head is nowhere negative in the pipe network. A simple such supply system is depicted in Figure 2.1

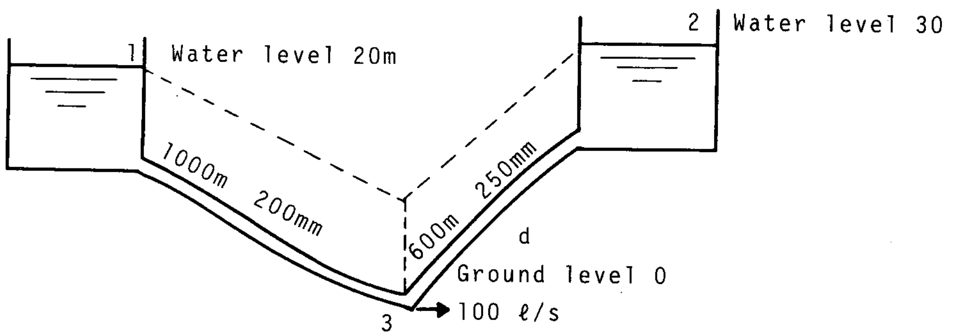


Fig. 2.1 Inter-connected Reservoirs

The problem can be reduced to the estimation of the residual head at node 3. Then the corresponding flow in each of pipes 1-3 and 2-3 can be calculated. If the net flow to node 3 does not equal the drawoff, 100ℓ/s, the assumed head at node 3 is incorrect and should be revised. A trial and error method could be employed to obtain a more accurate head at 3. Thus if the net flow towards 3 in both pipes exceeded 100ℓ/s the head should be increased, and if the flow was too low the head should be decreased. The final head at 3 should be above ground level or else the flow of 100ℓ/s would be unobtainable as air would be drawn into the pipelines through air valves and fittings.

If k = 0.01mm, $\lambda \doteq 0.014$. Try H_3 = 25m and solve for Q_{31} = 35ℓ/s, Q_{23} = +80ℓ/s, so H_3 is too high. Repeating with H_3 = 21m, it will be found Q_{23} = 110ℓ/s, Q_{31} = 10 ℓ/s.

A methodical and converging method for improving the estimation of the head at 3 at each iteration is demonstrated later (the node head correction method). The methods (trial and error, or iterative relaxation) could be employed for more complex situations, e.g. 3 or more reservoirs, more than one drawoff and compound pipes with changes in diameter.

Alternatively an analytical solution could be obtained.

Thus if $H_1 - H_3 = k_1 Q_1^2$ (2.3)

and $H_2 - H_3 = k_2 Q_2^2$ (2.4)

Then using $Q_1 + Q_2 = Q_3$ to eliminate Q_2 and H_3 (2.5)

$$Q_1 = \frac{-2k_2Q_3 \pm \sqrt{(2k_2Q_3)^2 - 4(k_1-k_2)(H_2-H_1-k_2Q_3^2)}}{2(k_1-k_2)} \quad (2.6)$$

For λ = 0.014 then k_1 = 3622 and k_2 = 712 so Q_1 = -0.014 or -0.035 m^3/s. The latter solution is found on inspection to be incorrect and so Q_2 = 0.1 + 0.014 = 0.114 m^3/s.

PSEUDO STEADY FLOW

If the water levels in two inter-connected reservoirs change with time, it may still be possible to obtain an analytical solution to the flow rate and water levels at any time. Acceleration is neglected in the calculation for the example in Figure 2.2.

Assuming a head loss equation of the form

$$h = H_1 - H_2 = K Q^n \quad (2.7)$$

(e.g. $K = \lambda L/2gD^5(\pi/4)^2$ and n = 2 for the Darcy equation). (2.8)

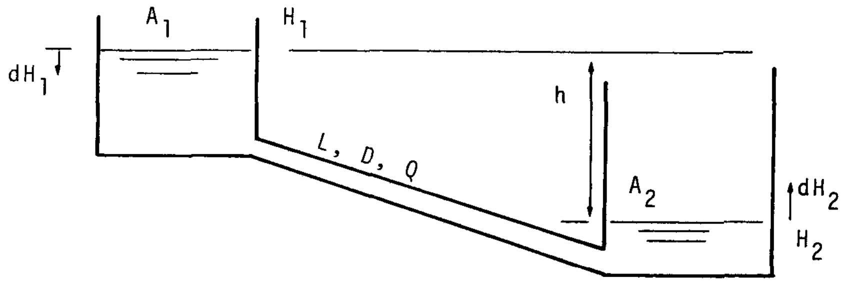


Fig. 2.2 Inter-connected reservoirs with heads converging

then

$$Q = (h/K)^{1/n} \tag{2.9}$$

For continuity

$$dH_1A_1 = dH_2A_2 = Qdt \tag{2.10}$$

$$\therefore -dh = dH_1+dH_2 = Qdt(1/A_1+1/A_2) \tag{2.11}$$

$$= (h/K)^{1/n}(1/A_1+1/A_2)dt$$

$$\frac{-dh}{h^{1/n}} = \frac{(1/A_1+1/A_2)dt}{K^{1/n}} \tag{2.12}$$

Integrating, $$\frac{K^{1/n}}{(1-1/n)(1/A_1+1/A_2)}(h_t^{1-1/n}-h_o^{1-1/n}) = \Delta t \tag{2.13}$$

i.e. starting with a known head difference h_o, the relationship between h_t and time t may be determined from this equation.

The problem could also be solved numerically and such methods are discussed in chapters 7 and 8.

COMPOUND PIPES

Equivalent Pipes for Pipes in Series

It is often useful to know the equivalent pipe which would give the same head loss and flow as a number of inter-connected pipes in series or parallel. The equivalent pipe may be used in place of the compound pipes to perform further flow calculations.

The equivalent diameter of a compound pipe composed of sections of different diameters and lengths in series may be calculated by equating the total head loss for any flow to the head loss through the equivalent pipe of length equal to the length of compound pipe:-

$$K(\Sigma\ell)Q^n/D_e{}^m = \Sigma K\ell Q^n/D^m \qquad (2.14)$$

$$\therefore D_e = \left\{\frac{\Sigma\ell}{\Sigma(\ell/D^m)}\right\}^{1/m} \qquad (2.15)$$

(m is 5 in the Darcy formula and 4.85 in the Hazen-Williams formula).

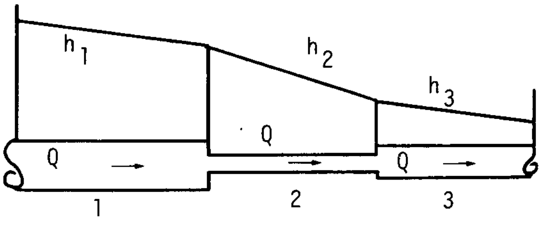


Fig. 2.3 Pipes in Series

Complex Pipes in Parallel

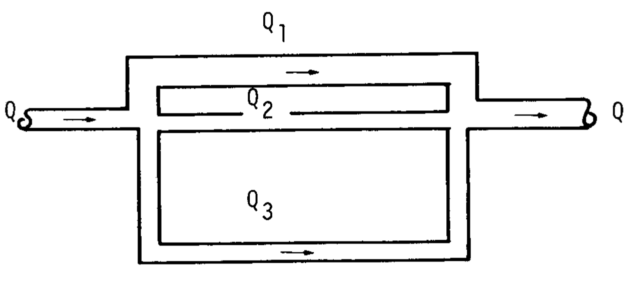


Fig. 2.4 Pipes in Parallel

Similarly, the equivalent diameter of a system of pipes in parallel is derived by equating the total flow through the equivalent pipe 'e' to the sum of the flows through the individual pipes 'i' in parallel:

Now $h = h_i \qquad (2.16)$

i.e. $K\ell_e Q^n D_e^m = K\ell_i Q_i^{\,n}/D_i^m$ (2.17)

So $Q_i = (\ell_e/\ell_i)^{1/n}(D_i/D_e)^{m/n}Q$ (2.18)

and $Q = \Sigma Q_i = \Sigma[(\ell_e/\ell_i)^{1/n}(D_i/D_e)^{m/n}Q]$ (2.19)

Cancelling out Q, and bringing D_e and ℓ_e to the left hand side,

$(D_e^{\,m}/\ell_e)^{1/n} = \Sigma(D_i^{\,m}/\ell_i)^{1/n}$ (2.20)

and if each ℓ is the same,

$D_e = (\Sigma D_i^{\,m/n})^{n/m}$ (2.21)

The equivalent diameter could also be derived using a flow/head loss chart. For pipes in parallel, assume a reasonable head loss and read off the flow through each pipe from the chart. Read off the equivalent diameter which would give the total flow at the same head loss. For pipes in series, assume a reasonable flow and calculate the total head loss with the assistance of the chart. Read off the equivalent pipe diameter which would discharge the assumed flow with the total head loss across its length.

It often speeds network analyses to simplify pipe networks as much as possible using equivalent diameters for minor pipes in series or parallel. Of course the methods of network analysis described below could always be used to analyse flows through compound pipes and this is in fact the preferred method for more complex systems than those discussed above.

NODE HEAD CORRECTION METHOD

A converging iterative method of obtaining the correct heads at nodes and the corresponding pipe flows is often used, especially for complex branched networks with multiple reservoirs. This method is termed the node head correction method. It is necessary to assume initial heads at each node. Heads at nodes are then corrected by successive approximation. The steps in an analysis are as follows:-

(1) Draw the pipe network schematically to a clear scale. Indicate all inputs, drawoffs, fixed heads and booster pumps.

(2) Assume initial arbitrary but reasonable heads at each node (except if the head at that node is fixed). The more accurate the initial assignments, the speedier will be the convergence of

the solution.

(3) Calculate the flow in each pipe to any node with a variable head using a formula of the form $Q = (hD^m/K\ell)^{1/n}$ or using a flow/head loss chart.

(4) Calculate the net inflow to the specific node and if this is not zero, correct the head by adding the amount

$$\Delta H = \frac{\Sigma Q}{\Sigma (Q/nh)} \qquad (2.22)$$

This equation is derived as follows:-

$$\text{Since} \quad Q = (hD^m/K\ell)^{1/n} \qquad (2.23)$$

$$dQ = Qdh/nh \qquad (2.24)$$

We require $\Sigma (Q + dQ) = 0$

$$\text{i.e.} \quad \Sigma Q + \Sigma \frac{Qdh}{nh} = 0 \qquad (2.25)$$

But $dH = -dh$ where h is head loss and

H is head at node (2.26)

$$\text{So} \quad \Delta H = \frac{\Sigma Q}{\Sigma (Q/nh)} \qquad (2.27)$$

Flow Q and head loss are considered positive if towards the node. H is the head at the node. Inputs (positive) and drawoffs (negative) at the node should be included in Q.

(5) Correct the head at each variable-head node in similar manner, i.e. repeat steps 3 and 4 for each node.

(6) Repeat the procedure (steps 3 to 5) until all flows balance to a sufficient degree of accuracy. If the head difference between the ends of a pipe is zero at any stage, omit the pipe from the particular balancing operation.

It should be noted that the node head correction method in some cases converges slowly if at all or it can be unstable. This can be the case if the system is unbalanced, i.e. one pipe with a high head loss is adjacent to another pipe with a low head loss. A small head correction at the common node could cause a large change in flow, even a flow reversal in the low head loss pipe. In fact a head correction should be less than the head loss in any pipe connecting to the node in question. It may therefore take many iterations to work a correction out of the system. Alternatively the flow rate could oscillate wildly and no longer comply with the linearized relationship between a small change in head loss and change in flow.

On the other hand, data input is very simple for the node head correction method. No loops have to be assumed and initial flows need not be estimated. Heads can be taken as ground levels for each node to start with so sometimes no initial heads have to be estimated. It is easier to identify pipes by the nodes they connect rather than the loops they separate. It is easier to interpret the results. Final heads are given for each node instead of just line head losses as for the loop method. If changes are required once a preliminary analysis is done it is easy to add, subtract or alter pipes with the node method. Many of these advantages are retained in the linear method described in Chapter 4.

COMPUTER PROGRAM BASED ON NODE HEAD CORRECTION METHOD

The method is very easy to use and simple to program (see computer program appended to this chapter). Data input is also simple, and easy to change after a run. The method is, however, slow to converge in cases of unbalanced layouts or low and high head loss combinations. An example is depicted after the program listing, together with input listing and output. The same example is analysed in chapter 3 using a different method.

Program Input

The program is run interactively and will prompt for the following information:

Line 1: Network name

Line 2: No. of pumps; No. of nodes (total); No. of fixed head nodes (reservoirs); permissible final maximum change in head per iteration in m (the 'error'); Darcy factor (assumed constant for all pipes); Head at node 1 (which must be a reservoir) in metres. Note that in numbering nodes, the fixed heads should be numbered first.

Line 3 and subsequent lines (one for each pipe): Top node No.; Bottom node (the correct order is not important at this stage); Length in metres; Diameter in metres; First estimate of head at bottom node in metres; Drawoff at bottom node in m^3/s.

Pipe data can be put in any order but every node in the system should have its head and drawoff defined in the data input, i.e.

each node should be a bottom node in at least one pipe. If data (head and drawoff) for any node is supplied twice thus it is only the last data which is recorded. Drawoffs are ignored at fixed head nodes.

When the answers are printed, the top head node and bottom node will be in the correct order thus defining flow direction. The head at the bottom node is also given.

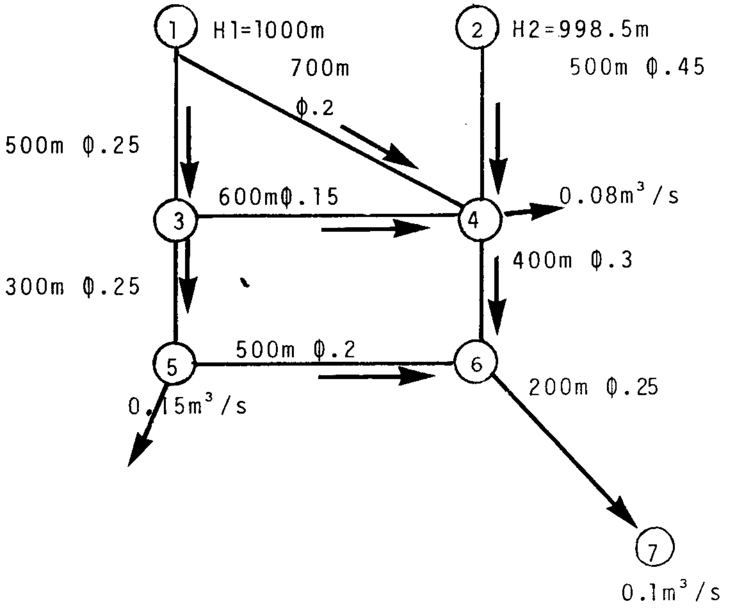


Fig. 2.5 Network analysed by program

NEWTON-RAPHSON METHOD

Most methods of network analysis described here are based on a linearization of the head loss equations. Even the Hardy Cross methods are based on a linear increment in flow being proportional to a linear increment in head loss. The method hereunder does not require such a linearization by the analyst but a mathematical approximation to the non-linear head loss term is improved in steps using linear increments. The Newton- Raphson technique is used to solve the non-linear head-flow equations simultaneously for each pipe in the network. The method is said by Jeppson (1976) to have rapid convergence, but requires a close initial estimate of flows if instability is to be avoided. It also requires relatively little computer storage.

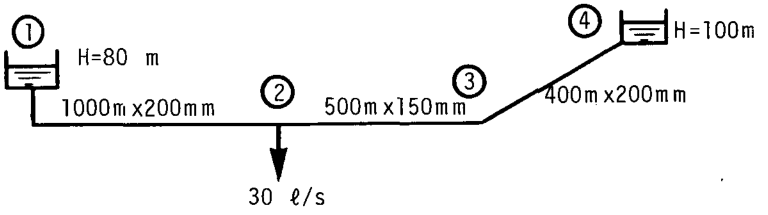
The method used in Chapter 1 to solve the Colebrook-White equation for λ, is based on the approximation

$$x_{m+1} = x_m - \frac{F(x_m)}{F'(x_m)} \tag{2.28}$$

where F (x) is the function of x to be solved (F(x) = 0) and F′ is the differential of F with respect to x. Subscript m refers to the m-th iteration. This equation comes directly from the first order approximation

$$F'(x_m) = \frac{F(x_{m+1}) - F(x_m)}{x_{m+1} - x_m} \quad \text{where } F(x_{m+1}) = 0 \tag{2.29}$$

The convergence is rapid because the error in the m+1-th iteration is proportional to the error in the m-th iteration squared. This is termed quadratic convergence. The method applied to the simultaneous solution of a number of non-linear equations for heads at nodes is demonstrated with an example below



$$\text{Head losses } H = \frac{0.02LQ^2\ \ell/s}{D^5 m 1000^2 \times (\pi/4)^2}$$

$$= 0.0325\times10^{-6}\ LQ^2/D^5 \text{ (D in m, Q in } \ell/s\text{, L in m)}$$

$$\therefore Q = 5547h^{\frac{1}{2}}D^{5/2}/L^{\frac{1}{2}}$$

$$Q12 = 3.14\ H12^{\frac{1}{2}}$$

$$Q23 = 2.16\ H23^{\frac{1}{2}}$$

$$Q34 = 4.96\ H34^{\frac{1}{2}}$$

$$\text{Set } F2 = Q12-Q23-30 = 0 \quad \text{for flow balance at node 2}$$
$$= 3.14(80-H2)^{\frac{1}{2}} - 2.16(H2-H3)^{\frac{1}{2}} - 30$$
$$F3 = Q23-Q34 = 0 \quad \text{for flow balance at node 3}$$
$$= 2.16(H2-H3)^{\frac{1}{2}} - 4.96(H3-100)^{\frac{1}{2}}$$
$$\frac{dF2}{dH2} = 1.57/(80-H2)^{\frac{1}{2}} - 1.08/(H2-H3)^{\frac{1}{2}}$$
$$\frac{dF3}{dH3} = -1.08/(H2-H3)^{\frac{1}{2}} - 2.48/(H3-100)^{\frac{1}{2}}$$
$$\text{Try } H2 = 55\text{m}, \; H3 = 90\text{m}$$
$$\text{Then} \quad H2_2 = 55 - \frac{3.14(80-55)^{\frac{1}{2}} - 2.16(55-90)^{\frac{1}{2}} - 30}{1.57/(80-55)^{\frac{1}{2}} - 1.08/(55-90)^{\frac{1}{2}}} = 56.06$$
$$H3_2 = 90 - \frac{-2.16(55-90)^{\frac{1}{2}} - 4.96(90-100)^{\frac{1}{2}}}{-1.08/(55-90)^{\frac{1}{2}} - 2.48/(90-100)^{\frac{1}{2}}} = 87.0$$

Note $(55-90)^{\frac{1}{2}}$ is assumed to be $-(90-55)^{\frac{1}{2}}$ etc. to account for flow direction. The new heads can be re-substituted into the equations for the next iteration and so on.

It will be observed that the resulting equations are in effect the same as those for the node head correction method. If the basic equations were written in terms of unknown flows instead of heads, the resulting equations would be the same as the loop flow correction equation (see Ch. 3). The equations may, however, be solved simultaneously for all new flows instead of for one loop at a time (Ch. 3).

REFERENCES

Davis, C.V., 1952. Handbook of Applied Hydraulics, McGraw Hill, 2nd Edn. N.Y.

Jeppson, R.W., 1976. Analysis of Flow in Pipe Networks. Ann Arbor Science. 161pp.

Stephenson, D., 1981. Pipeline Design for Water Engineers, 2nd Edn. Elsevier, Amsterdam. 234pp.

Webber, N.B., 1971. Fluid Mechanics for Civil Engineers, Chapman and Hall, London.

Network analysis by Node Head Correction-Program Output and Input

```
10 PRINT "PIPE NET ANAL BY NODE
    HEAD CORR"
20 DIM K(50),L(50),X(50),D(50),
   Q(50),H(50),P(50)
30 DISP "NETWORK NAME";
40 INPUT L$
50 G=9.8 ! 32 IF FT-S UNITS
60 DISP "NPIPES,NNODES,NRESS,ER
   Rm,DARCYf,TOPHm";
70 INPUT N1,N2,N3,H3,F,H(1)
80 FOR J=1 TO N1 ! FIXED HEADS
   NUMBERED FIRST
90 DISP "NODE1+,NODE2-,Lm,Dm,IN
   ITLH2,Q2m3/s";
100 INPUT K(J),L(J),X(J),D(J),H2
   ,Q2
110 H(L(J))=H2
120 P(L(J))=Q2
130 F(J)=F
140 NEXT J
150 FOR I1=1 TO 50 ! MAX ITNS
160 V=0
170 FOR J=1 TO N1
172 IF H(K(J))<>H(L(J)) THEN 180
174 H(K(J))=H(L(J))+.01
180 IF H(K(J))>=H(L(J)) THEN 220
190 U=L(J)
200 L(J)=K(J)
210 K(J)=U
220 H1=H(K(J))-H(L(J))
230 Q(J)=SQR(H1*D(J)^5*.785^2*2*
   G/F/X(J))
240 NEXT J
250 FOR I=N3+1 TO N2
260 R=-P(I)
270 S=0
280 FOR J=1 TO N1
290 IF K(J)<>I THEN 330
300 R=R-Q(J)
310 S=S+Q(J)/(H(K(J))-H(L(J)))
320 GOTO 360
330 IF L(J)<>I THEN 360
340 R=R+Q(J)
350 S=S+Q(J)/(H(K(J))-H(L(J)))
360 NEXT J
370 H(I)=H(I)+2*R/S
380 IF ABS(2*R/S)<=V THEN 400
390 V=ABS(2*R/S)
400 NEXT I
410 IF V<H3 THEN 430
420 NEXT I1
430 PRINT L$
440 PRINT " N+ N-   X(M) D(M) QM
   3/S H2M"
450 FOR J=1 TO N1
460 PRINT USING 470 ; K(J),L(J),
   X(J),D(J),Q(J),H(L(J))
470 IMAGE DDD,DDD,DDDDD,DD.DDD,D
   D.DDD,DDDDD.D
480 NEXT J
490 END
```

```
PIPE NET ANAL BY NODE HEAD CORR
TESTRUN
 N+ N-    X(M)  D(M)  QM3/S  H2M
  1  4    700   .200  .028   997.9
  2  4    500   .450  .133   997.9
  1  3    500   .250  .091   994.8
  4  3    600   .150  .018   994.8
  3  5    300   .250  .108   990.3
  6  5    500   .200  .041   990.3
  4  6    400   .300  .144   993.8
  6  7    200   .250  .099   991.1

NETWORK NAME?
TESTRUN
NPIPES,NNODES,NRESS,ERRm,DARCYf,
TOPHm?
8,7,2,.1,.015,1000
NODE1+,NODE2-,Lm,Dm,INITLH2,Q2m3
/s?
1,4,700,.2,997,.08
NODE1+,NODE2-,Lm,Dm,INITLH2,Q2m3
/s?
4,2,500,.45,998.5,0
NODE1+,NODE2-,Lm,Dm,INITLH2,Q2m3
/s?
1,3,500,.25,995,0
NODE1+,NODE2-,Lm,Dm,INITLH2,Q2m3
/s?
3,4,600,.15,997,0
NODE1+,NODE2-,Lm,Dm,INITLH2,Q2m3
/s?
3,5,300,.25,992,.15
NODE1+,NODE2-,Lm,Dm,INITLH2,Q2m3
/s?
5,6,500,.2,990,0
NODE1+,NODE2-,Lm,Dm,INITLH2,Q2m3
/s?
4,6,400,.3,990,0
NODE1+,NODE2-,Lm,Dm,INITLH2,Q2m3
/s?
6,7,200,.25,987,.1
```

CHAPTER 3

LOOP FLOW CORRECTION METHOD OF NETWORK ANALYSIS

INTRODUCTION

An efficient method of analysing the flows in pipe networks was developed by Hardy Cross (1936). The numerical method can be performed manually or by computer and is one of the most rapid methods of manually estimating flows and head losses in a network with closed loops. By closed loops it is implied that there are alternative routes supplying any of the (known) drawoffs from the system. Provided the loops are selected judiciously the number of iterations may be considerably less than the number of individual pipes in the network and convergence is rapid.

The method suffers the disadvantages that the network has to be drawn in the form of a number of closed loops or routes and initial flows have to be estimated such that flows balance at each node or intersection. There are further complications when more than one reservoir or fixed head is included or when branch pipes feature.

Pipes join nodes or junctions and drawoffs should be taken from the nodes. The number of nodes should be minimized to minimize the number of pipes in the network. This may be done by lumping a lot of drawoffs, e.g. from a block of houses, together and taking them off a central point. Where local head losses are important the number of nodes may not be able to be reduced much. On the other hand, the longer the lengths of individual pipes the more rapidly data assembly and analysis can be done.

METHOD OF FLOW CORRECTION

The method requires an initial realistic estimate of flows which are successively corrected. Corrections are made in steps to the flows in closed loops, using a relaxation or converging iterative method. The basis of the correction is derived as follows. A first order approximation to the differential of the head loss equation is made:

Assuming $h = KLQ^n/D^m$ (3.1)

Then $dh = KLnQ^{n-1}dQ/D^m = (nh/Q)dQ$ (3.2)

The total net head loss around any closed loop should be zero and if it is not, the head losses are incorrect and should be adjusted by dh:

Then $\Sigma(h+dh) = 0$ (3.3)

$\Sigma h + \Sigma dh = 0$ (3.4)

$\Sigma h + \Sigma(nh/Q)dQ = 0$ (3.5)

$$\Delta Q = \frac{-\Sigma h}{n\Sigma(h/Q)} \quad (3.6)$$

If the Darcy-Weisbach equation is used for head losses, n=2 and m=5, whereas the Hazen-Williams equation would give n=1.85 and m=4.85. Some degree of freedom is available in selecting n. It may also be noted that Q and h may be in any units, e.g. ℓ/s and m respectively.

Barlow and Markland (1969) showed how a second order approximation to the differential produced a more rapid convergence.

Steps in analysis may be set out as follows (Instn. of Water Engineers, 1969);

1. Draw the network plan to a clear scale and set all data such as pipe lengths and diameters, reservoir heads and drawoffs on the plan. In fact it is often convenient to set the calculations out on such a plan.
2. Number the closed loops consecutively and mark positive directions, e.g. clockwise.
3. Starting with any pipe in any loop, assign an arbitrary but sensible flow and flow direction to this pipe. Repeat for an adjacent pipe taking care that the flow balances at the intermediate node, i.e. net inflow to less drawoff from each node must be zero. Proceed thus through the network allocating flows to each pipe. The number of assumptions necessary will equal the number of inner loops, i.e. a new flow will have to be assumed at each new loop. The more accurate the initial estimates of flows the speedier will be the solution.
4. Calculate the head loss in each pipe using a formula of the form $h = KLQ^n/D^m$. Also calculate the term h/Q for each pipe retaining the positive value of h and Q in this case.

5. Select a loop such as the most prominent one if performing the analysis manually, or proceed in numerical order starting at loop 1 if using a computer. Calculate the net head loss around the loop, adding head drops, or subtracting head increases if the flow is in the adverse direction.
6. If the net head loss around the loop is not zero, correct the flows in each pipe in the loop using the formula

   $$\Delta Q = -\Sigma h/n\Sigma(h/Q). \qquad (3.6)$$

7. Repeat steps 5 and 6 for each loop.
8. Repeat steps 4 to 7 until the head losses around each loop balance to a satisfactory degree or until the flow corrections are negligible.
9. Calculate the head at each node by starting at a known head point and going from pipe to pipe. Compare the heads at each node with ground levels to determine the residual pressure head.

It is easier to estimate head losses from a head loss chart than to use an equation if the procedure is done manually. The value of n can also be set at 2 for ease of manual computations.

If h (and consequently Q) works out at zero for any pipe during the computations, h/Q should be assumed to be zero in calculating $\Sigma h/Q$ for the corresponding loop or loops.

LOOP SELECTION

It is most convenient to select and number consecutively loops adjacent to each other when proceeding across the network. This is, however, not necessarily the method which will result in most rapid numerical convergence when correcting flows. By carefully designating loops the convergence can be speeded up. Remember a correction can only proceed at the rate of one loop at a time through the system.

It is often convenient to have some of the loops embracing the larger pipes, i.e. cutting across small minor loops. Alternatively the outer pipes can be imagined as a loop turned inside out. The space outside the network thus has a loop number. The method will work whether or not such additional loops are included. If a pipe is identified by two loop numbers then it is necessary to number the outer loop.

The procedure of identifying pipes by the loops they separate may also be used to indicate the direction of flow. If all loops obey a sign convention such as clockwise positive, then the loop in which the flow is in the positive direction can always be given first when identifying a pipe and its flow direction.

When computing the flow correction for successive loops, the head loss calculation and flow corrections may be made for one loop at a time, or else all the flow corrections may be computed before correcting all the loop flows together. The former procedure is easiest to computerize whereas the latter procedure is often the easiest when doing the calculations manually. Careful attention has to be paid to signs and directions, however, or else the flow balance can be lost for some of the nodes.

MULTIPLE RESERVOIRS

The loop network method becomes tricky when more than one fixed head reservoir is incorporated in the pipe network. The simplest method of coping with multiple reservoirs is to insert a dummy pipe connecting the reservoirs (one dummy pipe per additional reservoir above one). One additional loop is thereby created per dummy pipe. The length and diameter of dummy pipes may be selected arbitrarily and the estimated flow in a dummy pipe should correspond to the head difference between the reservoirs it connects.

The flow and head loss in the dummy pipe are taken into account in calculating Σh and $\Sigma h/Q$ around the respective loops, but the flow is not corrected in dummy pipes when the flow correction is made around the relevant loop.

BRANCH PIPES

Although predominantly tree-like networks without closed loops can often be handled most efficiently using the node head correction method, it is possible to include branch pipes in the loop method. The flow in an isolated branch pipe should be pre-defined (unless the branch connects a reservoir when it will be incorporated in a loop with a dummy pipe). Therefore the head loss can be estimated without resorting to iterations except if it is a compound pipe made up of a number of diameters in series. Even then, equivalent pipe methods are possible.

In general either the head at a node, or the drawoff, can be defined. In either case the other variable has to be calculated in the network analysis.

The method of including branch pipes with defined flows in a network programme is to ascribe arbitrary loop numbers greater than the number of real loops to the branch pipe, and not correct the flows in those loops.

PUMPS AND PRESSURE REDUCING VALVES

Pumps may be assumed to increase or boost the head in a pipe in one direction by a specified amount. Pressure reducing valves, on the other hand, may reduce the head by a specified head and can be treated as a negative pumping head. (If either operates to give a fixed total head at a point that point may be treated like a fixed-head reservoir).

A pumping head would be included in an analysis by subtracting head generated from the friction head drop if proceeding around the loop in the direction of positive flow in the pipe with pump. If the flow is against the pumping head add the friction head and pumping head. This pumping head is included in Σh but not in $\Sigma h/Q$ when proceeding around the loop. The computation for Q is then as before.

Where a preset valve or fitting in a pipe causes a head loss in the direction of flow the fitting can generally be converted to an equivalent length of pipe and added to the real pipe length. Thus if the head loss through the fitting is $KV^2/2g$ and the friction head loss in the pipe is $(\lambda L/D)V^2/2g$, then the equivalent additional length of pipe is

$$\Delta L = KD/\lambda \qquad (3.7)$$

If the pumping head is a function of the flow rate, e.g.

$$h_p = a_0 + a_1 Q + a_2 Q^2 \qquad (3.8)$$

where a_0 = the pump shutoff head and a_1 and a_2 are pump constants,

then equation (3.6) will be replaced as follows:

The revised $Q_1 = (Q + dQ)$ (3.9)

For any pipe head drop $h = KLQ^n/D^m - a_0 - a_1Q - a_2Q^2$ (3.10)

or if h is the uncorrected head drop,

$$h+dh = KL(Q^n + n\,Q^{n-1}dQ + \ldots)/D^m - a_0 - a_1(Q+dQ) - a_2(Q^2 + 2QdQ + \ldots) \quad (3.11)$$

The total head drop around a loop must be zero:

$$\Sigma(h+dh) = 0 \quad (3.3)$$

Hence
$$dQ = -\frac{\Sigma(KLQ^n/D^m - a_0 - a_1Q - a_2Q^2)}{n\,\Sigma\,KLQ^{n-1}/D^m - a_1 - 2Qa_2} \quad (3.12)$$

or
$$\Delta Q = \frac{-\Sigma(h_f - h_p)}{n\,\Sigma\,(h_f/Q) - \Sigma(a_1 + 2Qa_2)} \quad (3.13)$$

PRACTICAL DESIGN

The water engineer may be required to analyse a pipe network to check flows or heads. He may have to check pressure heads for aged networks to see if they are sufficient, or to re-analyse a network for a revised demand pattern. Alternatively the system may have been designed to cope with a certain demand pattern and abnormal conditions have to be checked. For example, design of pipe sizes may have been based on peak period drawoffs, but an abnormal load may come on the system when a fire hydrant is operated. Then lower pressures generally may be tolerable provided the fire demand is met. For example spread over a township, peak house demands may be assumed to be 0.1 ℓ/s whereas individual fire hydrants may have to deliver more than 10 ℓ/s.

Pipe sizes have to be selected by trial and error. The network will be analysed for each assumed pipe network layout and pipe diameter until a satisfactory arrangement is at hand. Generally a tree-like network with a radiating distribution of pipes becoming smaller in diameter towards the tips is the most economic, but closed loops are required for security in case pipes are damaged or taken out of commission. The tree - like network with consequent known flows can be optimized using linear programming methods but trial and error design is necessary for looped systems. As a guide the head loss gradients should be lesser the larger the pipe diameter or flow, but care is necessary to ensure pressures are not too high in valleys(e.g. 100m upper limit for household fittings) or too low at high points (e.g. 15m lower limit for residential areas). In undulating areas, pressure reducing valves may be required to meet these pressure limits, or many reservoirs could be used to supply separate zones.

COMPUTER PROGRAM FOR LOOP FLOW CORRECTION METHOD

The accompanying computer program in BASIC language is based on the loop flow correction method. It is adapted to suit a HP-85 computer and as the program stands it can handle 50 pipes. No pumps are permitted. It is in SI units (metres and seconds) but by adjusting G from 9.8 to 32 it may be run in ft-sec units. Also where m is indicated in the printout, ft should be assumed if calculations are to be in ft-sec units.

The Darcy head loss equation is used with a constant friction factor λ (equivalent to f in North America). The symbol F in the program represents λ and is assumed the same for each pipe. λ and pumping heads could in fact be read for each pipe but this would increase data input.

Input requested interactively is:

<u>Line 1</u>; Name of system

<u>Line 2</u>; No. of pipes; No. of loops; No. of reservoirs; Error permitted in m^3/s; Darcy friction factor.

<u>Line 3 etc.</u> (One for each pipe starting with dummy pipes); Positive loop; Negative loop; Length (m); Diameter (m); Initial estimate of flow (m^3/s).

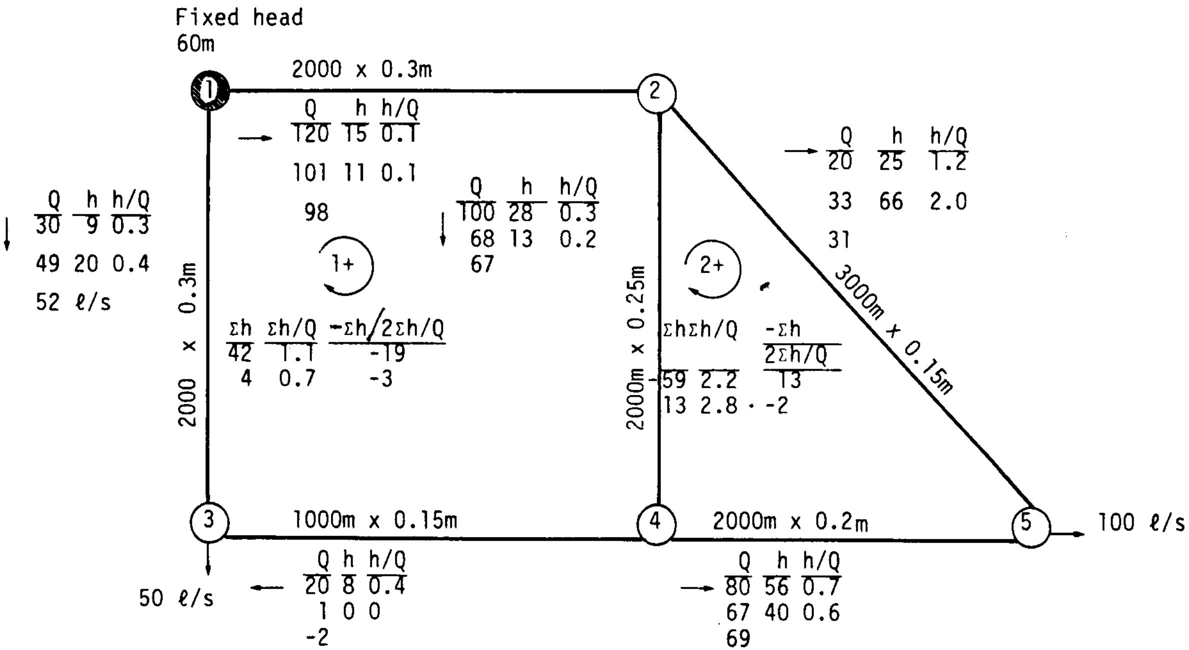
Running time is roughly 1s per 3 pipes per iteration.

Fig. 3.1 - Loop flow correction network analysis

$$h = 0.015L\ Q^2/2gD^5 \left(\frac{\pi}{4}\right)^2 1000^2$$

(h in m, L in m, Q in ℓ/s, D in m

$$\therefore\ h = 1.24 \times 10^9 LQ^2/D^5$$



Computer program for Hardy Cross Loop Method with Output and Input

```
10 PRINT "PIPE NET ANAL USNG H-
   C LOOP MTD"
20 DIM K(50),L(50),X(50),D(50),
   Q(50),H(50),F(50)
30 DISP "NETWORK NAME";
40 INPUT L$
50 G=9.8 ! 32 IF FT-S UNITS
60 DISP "NPIPES,NLOOPS,NRESS,ER
   m3/s,DARCYf";
70 INPUT N1,N2,N3,Q1,F
80 FOR J=1 TO N1 ! DUMMY PIPES
   JOINING RESS FIRST
90 DISP "LOOP1+,LOOP2-,Lm,Dm,IQ
   m3/s";
100 INPUT K(J),L(J),X(J),D(J),Q(
   J)
110 IF Q(J)>0 THEN 130
120 Q(J)=Q1/2
130 F(J)=F
140 H(J)=F(J)*X(J)*Q(J)^2/2/G/.7
   85^2/D(J)^5
150 NEXT J
160 FOR I1=1 TO 50 ! MAX ITNS
170 V=0
180 FOR I=1 TO N2
190 R=0
200 S=0
210 FOR J=1 TO N1
220 IF K(J)<>I THEN 270
230 H(J)=F(J)*X(J)*Q(J)^2/2/G/.7
   85^2/D(J)^5
240 R=R+H(J)
250 S=S+H(J)/Q(J)
260 GOTO 310
270 IF L(J)<>I THEN 310
280 H(J)=F(J)*X(J)*Q(J)^2/2/G/.7
   85^2/D(J)^5
290 R=R-H(J)
300 S=S+H(J)/Q(J)
310 NEXT J
320 T=-(R/S/2)
330 FOR J=N3 TO N1
340 IF K(J)<>I THEN 370
350 Q(J)=Q(J)+T
360 GOTO 390
370 IF L(J)<>I THEN 390
380 Q(J)=Q(J)-T
390 IF Q(J)>=0 THEN 440
400 Q(J)=-Q(J)
410 U=L(J)
420 L(J)=K(J)
430 K(J)=U
440 IF ABS(T)<=V THEN 460
450 V=ABS(T)
460 NEXT J
470 NEXT I
480 IF V<Q1 THEN 500
490 NEXT I1
500 PRINT L$
510 PRINT " L+ L-    X(m) D(m) Qm
   3/s H(m)"
520 FOR J=1 TO N1
530 PRINT USING 540 ; K(J),L(J),
   X(J),D(J),Q(J),H(J)
540 IMAGE DDD,DDD,DDDDD,DD.DDD,D
   D.DDD,DDD.D
550 NEXT J
560 END
```

```
PIPE NET ANAL USNG H-C LOOP MTD
TESTLOOP
 L+ L-   X(m)  D(m)  Qm3/s  H(m)
  1  4   100   .100   .010   1.2
  2  1   700   .200   .032   2.8
  4  2   500   .250   .092   5.4
  2  3   600   .150   .016   2.6
  4  3   300   .250   .109   4.6
  3  4   500   .200   .041   3.2
  1  4   500   .450   .206   1.4
  3  4   400   .300   .141   4.0
  5  5   200   .250   .100   2.5
```

```
NETWORK NAME?
TESTLOOP
NPIPES,NLOOPS,NRESS,ERm3/s,DARCY
f?
9,4,2,.001,.015
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
1,4,100,.1,.01
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
2,1,700,.2,0
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
4,2,500,.25,.18
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
3,2,600,.15,0
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
4,3,300,.25,.18
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
4,3,500,.2,.03
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
1,4,500,.45,.15
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
3,4,400,.3,.07
LOOP1+,LOOP2-,Lm,Dm,IQm3/s?
5,5,200,.25,.1
```

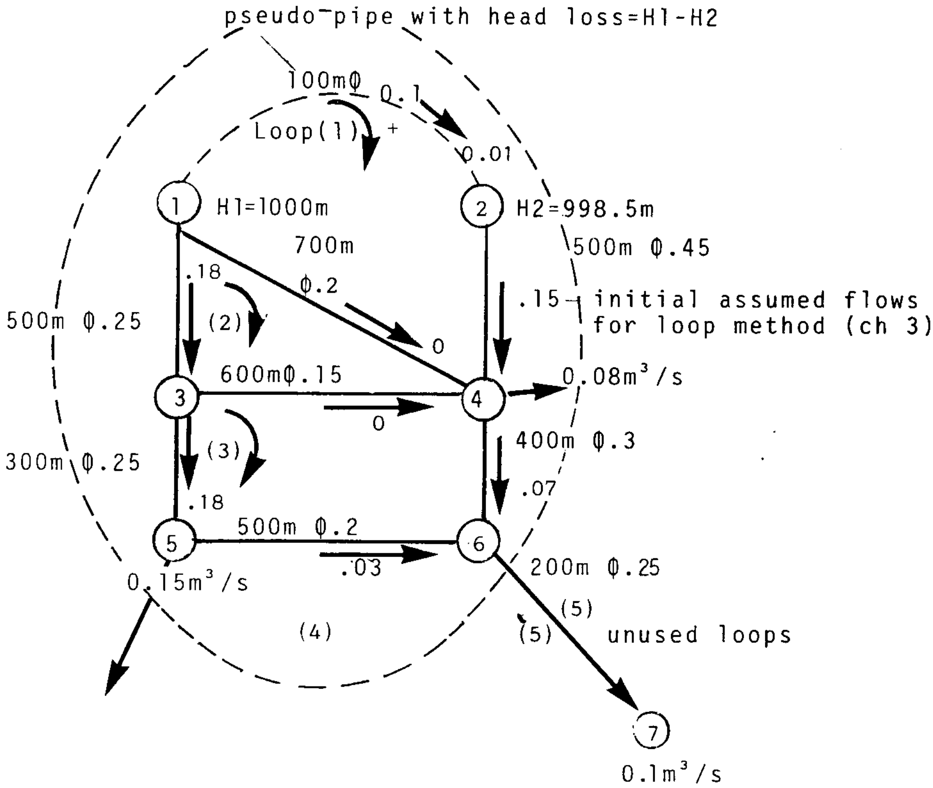


Fig. 3.2 Network analysed by program using loop method

REFERENCES

Barlow, J.F. and Markland, E., June 1969. Computer analysis of pipe networks, Proc. Instn. Civil Engrs., 48, paper 7187, p249.

Cross, H., 1936. Analysis of flow in networks of conduits or conductors. University of Illinois, Bulletin 286.

Institution of Water Engineers, 1969. Manual of Water Engineering Practice, 4th Edn., London.

CHAPTER 4

LINEAR METHOD

INTRODUCTION

The trial and error and relaxation methods of pipe network analysis were originally developed for manual solution. Calculations proceed from one pipe to the next in a routine manner and large matrices or solution of simultaneous equations are not required. The methods suffer disadvantages such as poor convergence for some systems unless improved solution methods are used (e.g. Barlow and Markland, 1969). With the development of digital computers came methods of inversion of matrices and methods of solution of sets of linear equations, e.g. Gauss' method of elimination. Rapid convergence of non-linear equations, employing numerical method such as that of Newton-Raphson (e.g. Martin and Peters 1963) is also possible.

The possibility of solution of a matrix of equations for pipe networks relating flows to head losses and including flow balance at each node thus arises. Except for laminar flow which is rarely encountered in waterworks practice, the flow-head loss relationship for pipes is non-linear. This makes the establishment and solution of linear simultaneous equations difficult. A method of converting the equations to a linear form was proposed by Wood and Charles (1972). The linearization makes matrix solution of a set of simultaneous equations relatively easy. The linear form of head loss-flow equation is in fact an approximation to the true equation and the approximation is improved iteratively by substituting revised flows into a 'constant' in the head loss equation. The linearization techniques can be applied in solving for flows around loops or heads at nodes (Isaacs and Mills, 1980). A comparison of the various methods was made by Wood and Rayes (1981). Although the linear method reduces computational effort, it can result in oscillating solutions and averaging methods may be required to speed convergence (Wood and Charles, 1972). There are also problems of convergence for the node method where some pipes have low head losses, as for the node head correction method of Hardy Cross.

In general a set of equations is established as follows:
At each node, for continuity

$$\Sigma Q_{in} = \text{drawoff} \qquad (4.1)$$

There are j equations for j 'closed' nodes or junctions. That is only nodes which can vary in head are considered thus, and junctions where the head is fixed have to have a variable volume. For such nodes the continuity equation becomes

$$\Delta t \Sigma Q_{in} = \text{increase in volume stored} \qquad (4.2)$$

If the relationship between volume and head is known this could be utilized but it is common practice to assume reservoir levels are fixed, that is for steady state conditions volumes are assumed constant. Therefore the mass balance equations at reservoirs or fixed head nodes are not required as the number of variables in the networks is reduced by the number of fixed head reservoirs. Alternatively the above equation is replaced by one of the form for each known head junction

$H = \text{constant}$

There are also i equations (one for each closed loop) of the following form

$$\Sigma h = 0 \qquad (4.3)$$

where h is the head loss in a selected direction in one pipe forming the loop, i.e. the net head loss around each closed loop must be zero. Now each head loss h is related to the flow rate in the pipe, Q, by an equation of the form

$$h = KQ^n \qquad (4.4)$$

so that heads can be replaced by the term involving Q for each pipe. It can be shown that in a network with i pipes, j nodes and m loops, (only counting internal loops)

$$i = j + m - 1 \qquad (4.5)$$

This holds for all networks with closed loops as well as open tree-like branches. There are j − 1 continuity or fixed head equations (the additional one is redundant) and m loop equations, thus giving a total of i equations for i unknowns if flows are used as the unknowns. The j_f equations for known heads can be omitted where j_f is the number of fixed head nodes.

omitted where j_f is the number of fixed head nodes.

Alternatively, if heads at nodes are regarded as the unknowns there are j equations for nodes, where each flow Q in m pipes is replaced by $(h/K)^{1/n}$.

There are m equations relating head loss to flow in the pipes, so that once the Q's in the head loss equations are replaced by $H_1 - H_2$ the number of equations is still j for solving for j unknown heads.

Unfortunately, except in laminar flow situations the equations relating h and Q are non-linear so direct simultaneous solution of both the node equations and the loop flow equations is difficult. There is no easy method of solution of non-linear simultaneous equations and trial and error or numerical methods usually have to be employed. The method used here, the linear method, is to approximate the head loss equations by linear equations and then they can be solved simultaneously by various methods. Where computer storage is no problem, Gauss elimination is an efficient method, and if storage is limited but computing time is readily available, successive approximation, e.g. successive over-relaxation and Newton Raphson methods can be employed.

LINEAR METHOD APPLIED TO LOOP FLOWS

If the head loss in any pipe can be expressed by an equation such as

$$h_i = K_i Q_i^{\,n} \tag{4.6}$$

it can be rewritten approximately as follows

$$h_i \simeq K_i Q_{io}^{\,n-1} Q_i = K_i' Q_i \tag{4.7}$$

where K_i is a function of the length of pipe i, its diameter and roughness, Q_i is the flow rate and Q_{io} is the flow rate indicated by the previous iteration. Both Q_i and h_i are unknowns for each iteration and a set of equations relating flow and head loss is established:

Around each loop, $\Sigma h_\ell = 0$ (4.8) i.e. $\Sigma K_i' \; Q_{im} = 0$ (4.9)

and for each node j: $\Sigma Q_{ij} = q_j$ (4.10)

where Q_{ij} is the flow in pipe i to node j, Q_{im} is the flow in pipe i in loop m and q_j is the drawoff at node j.

If there are i pipes, j nodes and m loops, then it was stated that

$i = j + m - 1$ (4.11)

There are j node equations and m loop equations so there are i+1 equations in total of which one is redundant so one can solve for i flows. The resulting set of linear equations has to be solved a number of times. Each time, new Q_i's emerge and the K_i''s are revised (improved) before re-solving the equations. Initial flow rates have to be estimated prior to the first solution of the linear equations.

Although the above method converges fairly well the following method is easier to visualize and is explained in more detail.

LINEAR METHOD FOR NODE HEADS

In this case the Darcy head loss equation will be used to simplify the calculations. The friction loss equation for a pipe is then written as $Q_{ij}|Q_{ij}| = h_{ij}/K_{ij}$ where Q_{ij} is the flow from node i to node j, h_{ij} is the head difference between i and j and $K_{ij} = 8\lambda_{ij}\ell_{ij}/gd_{ij}^5\pi^2$. If Q_{ijo} is an approximate solution to Q_{ij} (obtained from a previous iteration or from an initial estimate) and if one substitutes $C_{ij} = 1/(K_{ij}|Q_{ijo}|)$, then a 'linear' equation results;

$\Sigma Q_{ij} = C_{ij}h_{ij}$ (4.12)

Substituting into the continuity equation at each node j,

$\Sigma Q_{ij} = q_j$ (4.13)

$\Sigma C_{ij}h_{ij} = q_j$ (4.14)

Replacing h_{ij} by $H_i - H_j$

$\Sigma(C_{ij}H_i - C_{ij}H_j) = q_j$ (4.15)

Hence $H_j = \dfrac{\Sigma C_{ij}H_i - q_j}{\Sigma C_{ij}}$ (4.16)

One has J linear equations (equal to the number of variable head nodes) for J unknowns (the heads H at each node). The equations can be solved by various techniques, e.g. Gauss elimination method, or the Gauss Siedel iterative method. A successive over-relaxation method is employed in the accompanying computer program as it requires little memory whereas a matrix would require a large computer storage capacity.

To avoid overshoot an averaging procedure can be introduced after each step,

$$H_j' = wH_{j1}+(1-w)H_{jo} \tag{4.17}$$

where $0 \leq w \leq 1$ and subscript o refers to the previous H_j and 1 to the recent H_j.

After solution of the equations for each H_j, flows Q_{ij} in each pipe are calculated and then each C_{ij}. The linearization procedure is then repeated and a new set of equations solved for the heads at each node H_j. The procedure is repeated until convergence is satisfactory.

Pumps

If a pump in a line generates a specific head, h_p, then the head loss is $H_i-H_j+h_p$. Equation (4.10) is therefore replaced by

$$\Sigma C_{ij}(H_i-H_j+h_p) = q_j \tag{4.18}$$

$$\therefore H_j = \frac{\Sigma C_{ij}(H_i+h_p)-q_j}{\Sigma C_{ij}} \tag{4.19}$$

If the pumping head is a function of the flow rate, the convergence can be slow.

COMPUTER PROGRAM FOR LINEAR METHOD

A BASIC program is presented. The program is written to minimize data input. Thus no pumps or pressure reducing valves are possible as the program stands. Also the Darcy friction factor is assumed constant. In fact it could be varied from pipe to pipe and with Reynolds number and pipe roughness with small modifications, but the former would increase input and the latter would increase computational time.

A great advantage is that no initial flows or heads need be assumed. Flows are initially set to correspond to unit velocity and the corresponding heads are calculated at successive nodes. This procedure can lead to slow convergence for some cases but is in fact one of the main advantages of this method.

Input and output are in metres and cubic metres per second. The input is prompted at each line. Typical running time on an HP85 is 5s per pipe and the number of pipes could be increased above 30 by altering the dimension statement.

The appended program follows the procedure described previously. The variable names used follow the nomenclature used in the above section in general. Although data is in S.I. units here it will also work in ft- s units if G is altered to 32.2 (line 69 of program). It should be noted that the speed of the program could be improved for large networks if the iterations between lines 360 and 410 were limited to the pipes connecting to that node. This would require a new dimensioned variable for each node and a connectivity search.

Description of Variables in Program

C(K)	SD/FX
C2	Sums H(I) for each S.O.R. iteration
C3	Sums ΔF
D(K)	Pipe diameter (in m)
D2	Holds the old value of H(I) to compute ΔH
F(K)	Darcy friction factor F of pipe K
F1	Darcy friction factor for all pipes
H(I)	Head at junction
I	Node counter
J	Number of nodes in system
J1(I)	Joint beginning number
J2(I)	Joint end number
J3	Number of fixed head nodes (Numbered first)
K	Iteration counter
L	Node counter
M	Pipe counter
NO	Maximum number of main iterations e.g. J+5
N1	Maximum number of S.O.R. iterations within each main iteration e.g. J+10
N2	Counts the number of main iterations
N3	Counts the total number of S.O.R. iterations

P	Number of pipes in system
Q(K)	Flow in pipe m^3/s
Q1	Drawoff in m^3/s
Q2(I)	Drawoff from junction (+ve out of junction)
S	$g\pi^2/8$
S2(I)	ΣK_{ij}
S3	Counts the number of H(I)'s
S4(I)	$\Sigma K_{ij}H_j$
S5	Holds old Q(K) value for calculation of average of old value and continuity value
TO	Tolerance on head calculation in F e.g. 0.0001
T1	Tolerance on S.O.R. loop in metres on heads e.g. 0.01
W	S.O.R. factor w e.g. 1.3m. Must be between 1 and 2.
X(K)	Pipe length (in m)

Data Required

Line 1:	Heading. This can be any alphanumeric expression up to 18 characters long. Will be printed out at head of results.
Line 2:	P : Number of pipes J : Total number of junctions or nodes J3 : Number of junctions with fixed heads F1 : Darcy friction factor, assumed the same for all pipes e.g. 0.015
Line 3 to J3+3: (J3 lines)	J(I): Head at successive fixed-head nodes (which must be numbered first)
Line 4+J3 to 4+J3+P:	(P lines, in order such that any node except fixed head is referred to first as a J2 then as a J1) J1(K): Joint begin no. J2(K): Joint end no. Make sure each node is an end number at least once to allocate a drawoff. X(K): Pipe length (in m) D(K): Pipe diameter (in m) Q2(J2(K)): Drawoff from end node (in m^3/s); if this information is read twice the last value is retained.

General Comments

Generally the linear method converges in far less iterations than the Hardy Cross method. Between 4 and 10 successive approximations to the flow is all that is required even for networks involving up to 100 pipes. The snag is the solution of large numbers of simultaneous equations. If successive over-relaxation is used at least that number of substitutions may be required.

The initial flow assumption may be critical in some cases. The program assumes a velocity of 1m/s and calculates the corresponding flow rate in m^3/s for each pipe. The initial heads at each node are also based on this assumption and these values are used to start the SOR improvement of head at each node. When data for each pipe are read into the computer the head at the assumed downstream node is calculated on this basis provided the upstream head is known. Hence the order in which data are supplied should be such that the assumed upstream node head has been calculated, or supplied in the case of reservoirs. Wood and Charles (1972) use a unit flow rate in each pipe for the first estimate but this can lead to large errors especially if the pipe diameters vary widely. Alternatively it could be assumed to start that flows are laminar.

Although the convergence is fairly rapid, it has been observed that successive trials oscillate about the final value. To overcome this the next approximation can be taken as the mean of the previous two values (whether flows or heads are the unknowns).

Computer Program for Linear Method Network Analysis with Output and Input (same example as in Chapters 2 and 3).

```
10 ! NETLIN LINEAR NETWORK ANAL
20 DIM C(50).Q(50),H(50),Q2(50)
   .S2(50),S4(50),F(50),J1(50),
   J2(50),D(50),X(50)
30 DISP "NAME"
40 INPUT N$
50 DISP "NPIPES,NODES.FIXH,DARC
   Yf"
60 INPUT P,J,J3,F1
62 DISP "FIXED HEADS,m"
64 FOR L=1 TO J3
65 DISP L;
67 INPUT H(L)
68 NEXT L
69 G=9.8
70 S=3.14159^2*G/8
75 DISP "TOPN,BOTN.Lm,Dm,QBOTm3
   /s"
80 FOR K=1 TO P ! PIPE DATA
85 DISP K;
90 INPUT J1(K),J2(K),X(K),D(K),
   Q1
100 Q2(J2(K))=Q1
110 F(K)=F1
120 Q(K)=3.14159*D(K)^2/4
125 C(K)=S*D(K)^5/F(K)/X(K)
127 IF J2(K)<=J3 THEN 140
130 H(J2(K))=H(J1(K))-1/C(K)*Q(K
    )^2
140 NEXT K
150 W=1.3
160 T0=.0001
165 T1=.01
170 N0=SQR(J)+5
180 N1=10
190 N2=0
200 N3=0
210 FOR I=1 TO N0
220 N2=N2+1
230 FOR K=1 TO J ! NEW H BY SOR
240 S2(K)=0
```

```
250 NEXT K
260 FOR K=1 TO P
270 S2(J1(K))=S2(J1(K))-C(K)/ABS
    (Q(K))
280 S2(J2(K))=S2(J2(K))-C(K)/ABS
    (Q(K))
290 NEXT K
300 FOR K=1 TO N1
310 C2=0
320 S3=0
330 N3=N3+1
335 IF J3+1>J THEN 480
340 FOR L=J3+1 TO J
350 S4(L)=0
360 FOR M=1 TO P
370 IF J1(M)<>L THEN 390
380 S4(J1(M))=S4(J1(M))+C(M)/ABS
    (Q(M))*H(J2(M))
390 IF J2(M)<>L THEN 410
400 S4(J2(M))=S4(J2(M))+C(M)/ABS
    (Q(M))*H(J1(M))
410 NEXT M
420 D2=H(L)
430 H(L)=H(L)*(1-W)+W*(Q2(L)-S4(
    L))/S2(L)
440 C2=C2+ABS(H(L)-D2)
450 S3=S3+1
460 NEXT L
470 IF C2/S3<=T1 THEN 500
480 NEXT K
500 FOR K=1 TO P ! NEW FLOWS
510 S5=Q(K)
520 Q(K)=C(K)/ABS(Q(K))*(H(J1(K)
    )-H(J2(K)))
530 IF I=1 THEN 550
540 Q(K)=.5*(Q(K)+S5)
550 NEXT K
560 C3=0 ! TOLERANCE CHECK
570 FOR K=1 TO P
580 C3=C3+ABS(F(K)-ABS(H(J1(K))-
    H(J2(K)))/Q(K)^2*C(K)*F(K))
590 NEXT K
600 IF C3/P<=T0 THEN 620
610 NEXT I
620 PRINT "PIPENET",N$
630 PRINT "TOPN BOTN Xm    Dm   Qm
    3/s HBOTm"
640 FOR K=1 TO P
650 PRINT USING 670 ; J1(K),J2(K
    ),X(K),D(K),Q(K),H(J2(K))
660 NEXT K
670 IMAGE DDD,DDDD,DDDDD,DD.DDD,
    DD.DDD,DDDDD.D
680 STOP
690 END
```

```
PIPENET                     TESTLIN
TOPN BOTN Xm     Dm    Qm3/s  HBOTm
  1    3   500   .250   .093   994.5
  1    4   700   .200   .033   997.1
  3    4   600   .150  -.016   997.1
  2    4   500   .450   .204   997.1
  3    5   300   .250   .110   989.9
  5    6   500   .200  -.040   993.1
  4    6   400   .300   .140   993.1
  6    7   200   .250   .100   990.5
NAME
?
TESTLIN
NPIPES,NODES,FIXH,DARCYf
?
8,7,2,.015
FIXED HEADS,m
 1 ?
1000
 2 ?
998.5
TOPN,BOTN,Lm,Dm,QBOTm3/s
 1 ?
1,3,500,.25,0
 2 ?
1,4,700,.2,.08
 3 ?
3,4,600,.15,.08
 4 ?
2,4,500,.45,.08
 5 ?
3,5,300,.25,.15
 6 ?
5,6,500,.2,0
 7 ?
4,6,400,.3,0
 8 ?
6,7,200,.25,.1
```

REFERENCES

Barlow,J.F. and Markland, E., June, 1969. Computer Analysis of pipe networks. Proc. ICE, 43 p 249-259.

Campbell, G.V., 1981. Pipe network analysis program (linear method). Project, University of the Witwatersrand.

Isaacs, L.J. and Mills, K.G., July, 1980. Linear theory methods for pipe network analysis. Proc. ASCE 106 (HY7). p1191-1201

Martin, D.W., and Peters, G., 1963. The application of Newton's method to network analysis by digital computer. J. Instn. Water Engrs. 17, p115-129

Wood, D.J. and Charles, O.A., July, 1972. Network analysis using linear theory. Proc. ASCE, 98 (HY7) p1157-1170.

Wood, D.J. and Rayes, A.G., Oct. 1981. Reliability of algorithms for pipe network analysis. Proc. ASCE, 107 (HY10) p 1145-1161.

CHAPTER 5

OPTIMUM DESIGN OF BRANCHED PIPE NETWORKS BY LINEAR PROGRAMMING

INTRODUCTION

In the previous chapters the pipe diameters, lengths and layouts were assumed known and the corresponding flows and heads were computed. Design of pipe networks could only be undertaken by trial. The design problem is not as easy as the analysis problem. In the next two chapters approximate methods for direct design of pipe networks are given. Economics dictates the most practical design in each case.

Linear programming is a powerful optimization technique, but it may only be used if the relationship between variables is linear. Linear programming cannot be used for optimizing the design of pipe networks with closed loops without resort to successive approximations. It can be used to design trunk mains or tree-like networks where the flow in each branch is known. Since the relationships between flow, head loss, diameter and cost are nonlinear, the following technique is used to render the system linear: For each branch or main pipe, a number of pre-selected diameters is allowed and the length of each pipe of different diameter is treated as the variable. The head losses and costs are linearly proportional to the respective pipe lengths. Any other type of linear constraint can be treated in the analysis. It may be required to maintain the pressure at certain points in the network above a fixed minimum (a linear inequality of the greater-than-or-equal-to-type) or within a certain range. The total length of pipe of a certain diameter may be restricted because there is insufficient pipe available, etc.

SIMPLEX METHOD FOR TRUNK MAIN DIAMETERS

The following example concerns a trunk main with two drawoff points. The permissible diameters of the first leg are 250 and 200 mm, and of the second leg, 200 and 150 mm. There are thus four variables, X_1, X_2, X_3 and X_4 which are the lengths of pipe of different diameters. This simple example could be optimized by manual comparison of the cost of all alternatives giving the correct head loss, but linear programming is used here to demonstrate the technique.

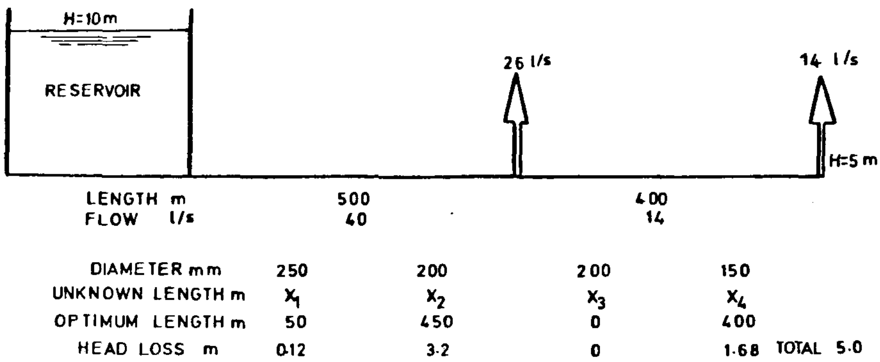


Fig. 5.1 Least-cost trunk main by linear programming.

The head losses per 100 mm of pipe and costs per m for the various pipes are indicated below:-

TABLE 5.1 Pipe Diameters and Costs

Diameter mm	Head loss @ 40 ℓ/s m/100 m	@ 14 ℓ/s m/100 m	Cost $100/100m
250	0.25		5
200	0.71	0.1	4
150		0.42	3

The linear constraints on the system are expressed in equation form below and the coefficients of the equations are tabulated in Table 5.3 (I). Lengths are expressed in hundred metres.

TABLE 5.2 Constraint Equations

Lengths	X_1 +	X_2			= 5
			X_3 +	X_4	= 4
Head Loss	$0.25X_1$ +	$0.71X_2$ +	$0.1X_3$ +	$0.42X_4$	= 5
Objective Function:	$5X_1$ +	$4X_2$ +	$4X_3$	$3X_4$	= min.

The computations proceed by setting all real variables to zero, so it is necessary to introduce artificial slack variables into each equation to satisfy the equality. The slack variables are designated a, b and c in Table 5.3(I), and their cost coefficients are set at very high values designated m. To initiate the solution, the slack variables a, b and c are assigned the values 5, 4 and 5 respectively (see the third column of Table 5.3(I).

The numbers in any particular line of the main body of the table indicate the amount of the program variable which would be displaced by introducing one unit of the column variable. Thus one unit of X_1 would displace 1 unit of a and 0.25 units of c.

To determine whether it is worthwhile replacing any variable in the program by any other variable, a number known as the opportunity number is calculated for each column. If one unit of X_1 was introduced, then the cost would increase by (5 - (1 x m) - (0 + m) - (0.25 + m)), which is designated the opportunity value, i.e. the opportunity value for each column is calculated by multiplying the entries in that column by the corresponding cost coefficients of the program variable in the second column and subtracting the total thus formed from the cost coefficient of the column variable. The most profitable variable to introduce would be X_2, since it shows the greatest cost reduction per unit (or negative opportunity value). The X_2 column is now designated the key column. The key column is that which shows the lowest opportunity value (in the cost minimization case). Only one variable may be introduced at a time.

To determine the maximum amount of the key column variable which may be introduced, calculate the replacement ratios for each row as follows:-

Divide the amount of the program variable for each row by the corresponding number in the key column. The lowest positive replacement ratio is selected as that is the maximum amount which could be introduced without violating any of the constraints. The row with the lowest positive replacement ratio is designated the key row and the number at the intersection of the key column and key row, the key number.

After introducing a new variable, the matrix is rearranged (Table 5.3(II)) so that the replacement ratios remain correct. The program variable and its cost coefficient in the key row are replaced

TABLE 5.3 Linear programming solution of pipe problem

I			Variable Cost coef.							
Prog. Var.	Cost Coef.	Amt.	X_1	X_2	X_3	X_4	a	b	c	Repl. ratio
			5	4	4	3	m	m	m	
a	m	5	1	1			1			5/1*
b	m	4			1	1		1		∝
c	m	5	0.25	0.71	0.10	0.42			1	5/.71
OPPORTUNITY VALUE:			5-1.25m	4-1.71m KEY COLUMN	4-1.1m	3-1.42m	0	0	0	*key row
II										
			X_1	X_2	X_3	X_4	a	b	c	
			5	4	4	3	m	m	m	
X_2	4	5	1	1			1			∝
b	m	4			1	1		1		4
c	m	1.45	-0.46		0.1	0.42	-0.71		1	3.45*
			1+0.46m	0	4-1.1m	3-1.42m	1.71m-4	0	0	
III										
			X_1	X_2	X_3	X_4	a	b	c	
			5	4	4	3	m	m	m	
X_2	4	5	1	1			1			5
b	m	0.55	1.1		0.76		1.69		-2.38	0.5*
X_4	3	3.45	-1.1		0.24	1	-1.69		2.38	-
			1.1-1.1m	0	3.28-0.76m	0	1.1-0.69m	3.38-8.2		
IV										
			X_1	X_2	X_3	X_4	a	b	c	
			5	4	4	3	m	m	m	
X_2	4	4.5		1	-0.69		1		2.16	
X_1	5	0.5	1		0.69		1.52		-2.16	
X_4	3	4			1	1				
			0	0	0.31	0	m-	m-	m-	

(NO FURTHER IMPROVEMENT POSSIBLE)

by the new variable and its cost coefficient. The amount column as well as the body of the table are revised as follows:-

Each number in the key row is divided by the key number.

From each number in a non-key row, subtract the corresponding number in the key row multiplied by the ratio of the old row number in the key column divided by the key number. The new tableau is given as Table 5.3(II).

The procedure of studying opportunity values and replacement ratios and revising the table is repeated until there is no further negative opportunity value. In the example, Table 5.3 (IV) shows all positive opportunity values so the least-cost solution is at hand (indicated by the current program variables and their corresponding values).

The reader should refer to a standard textbook on linear programming (e.g. Dantzig, 1963) for a full description of the technique. There are many other cases a few of which only can be mentioned below:-

(1) If the constraints are of the < = (less-than-or-equal- to) type and not just equations, slack variables with zero cost coefficients are introduced into the l.h.s. of each constraint to make them equations. The artificial slack variables with high cost coefficients are then omitted.

(2) If the constraints are of the > = (greater-than-or- equal-to) type, introduce artificial slack variables with high cost coefficients into the l.h.s. of the constraint and subtract slack variables with zero cost coefficients from each inequality to make them equations.

(3) If the objective function is to be minimized, the opportunity value with the highest negative value is selected, but if the function is to be maximized, the opportunity value with the highest positive value is selected.

Note all variables are assumed to be positive.

(4) The opportunity values represent shadow values of the corresponding variables i.e. they indicate the value of introducing one unit of that variable into the program.

(5) If two replacement ratios are equal, whichever row is selected, the amount of program variable in the other row will be zero when the matrix is rearranged. Merely assume it to have a

very small value and proceed as before.

NETWORK DESIGN

Most networks can be simplified to a tree-like network with known design flows. The most economic network is in fact a tree-like network and loops are purely for backup. In tree-like networks the pipe legs can be made up of lengths of commercially available diameters with costs and head losses per metre or kilometre inserted as a function of flows beforehand. The range of diameters can be limited by experience.

Example - Determine the least-cost pipe diameters for the network illustrated below

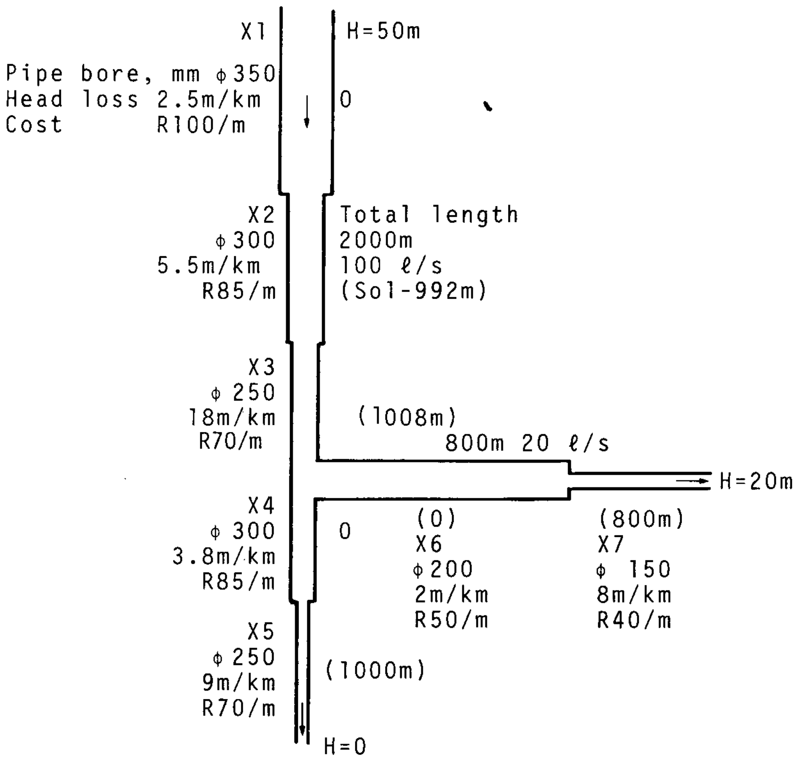


Fig. 5.2 Plan of branch network for example

Denote the unknown lengths of individual sections as X.

In algebraic form the constraints are:

Head losses along each route:

$$0.0025X1+0.0055X2+0.018X3+0.0038X4+0.009X5 \leq 50$$

$$0.0025X1+0.0055X2+0.018X3+0.002X6+0.008X7 \leq 30$$

Lengths:

$$X1+X2+X3 = 2000$$

$$X4+X5 = 1000$$

$$X6+X7 = 800$$

Objective function:

Minimize 100X1+85X2+70X3+85X4+70X5+50X6+40X7

Solution using an LP computer package on an HP85 micro computer :

```
            PIPE

VARIABLE    #   1 = X1
VARIABLE    #   2 = X2
VARIABLE    #   3 = X3
VARIABLE    #   4 = X4
VARIABLE    #   5 = X5
VARIABLE    #   6 = X6
VARIABLE    #   7 = X7
CONSTRAINT  #   1 = H1
CONSTRAINT  #   2 = H2
CONSTRAINT  #   3 = L1
CONSTRAINT  #   4 = L2
CONSTRAINT  #   5 = L3
CONSTRAINT H1
     +.00 X1       +.01 X2
     +.02 X3       +.00 X4
     +.01 X5  <=       50.00
CONSTRAINT H2
     +.00 X1       +.01 X2
     +.02 X3       +.00 X6
     +.01 X7  <=       30.00
CONSTRAINT L1
    +1.00 X1     +1.00 X2
    +1.00 X3  =      2000.00
CONSTRAINT L2
    +1.00 X4     +1.00 X5
  =     1000.00
CONSTRAINT L3
    +1.00 X6     +1.00 X7
  =      800.00
OBJECTIVE FUNCTION
MINIMIZE
  +100.00 X1        +85.00 X2
   +70.00 X3        +85.00 X4
   +70.00 X5        +50.00 X6
   +40.00 X7
```

```
              PIPE

VARIABLES     FROM   THROUGH

PROBLEM        1        7
SLACK          8        9
ARTIFICIAL    10       12

TABLEAU AFTER   0 ITERATIONS
           .00        .01        .02
           .00        .01       0.00
          0.00       1.00       0.00
          0.00       0.00       0.00
         50.00

           .00        .01        .02
          0.00       0.00        .00
           .01       0.00       1.00
          0.00       0.00       0.00
         30.00

          1.00       1.00       1.00
          0.00       0.00       0.00
          0.00       0.00       0.00
          1.00       0.00       0.00
       2000.00

          0.00       0.00       0.00
          1.00       1.00       0.00
          0.00       0.00       0.00
          0.00       1.00       0.00
       1000.00

          0.00       0.00       0.00
          0.00       0.00       1.00
          1.00       0.00       0.00
          0.00       0.00       1.00
        800.00
```

100.00	85.00	70.00
85.00	70.00	50.00
40.00	0.00	0.00
0.00	0.00	0.00
0.00		

OPTIMAL SOLUTION: PIPE

BASIS AFTER 5 ITERATIONS

VARIABLE	VALUE
SLACK 1	17.400
X3	1008.000
X2	992.000
X5	1000.000
X7	800.000

OBJ FUNC VALUE = 256880.000

DUAL VARIABLES

COLUMN	CONSTRAINT	VALUE
8	H1	0.000
9	H2	1200.000
10	L1	-91.600
11	L2	-70.000
12	L3	-49.600

TABLEAU AFTER 5 ITERATIONS

0.00	0.00	0.00
-.01	0.00	.01
0.00	1.00	-1.00
0.00	-.01	.01
17.40		
-.24	0.00	1.00
0.00	0.00	-.48
0.00	0.00	80.00
-.44	0.00	-.64
1008.00		
1.24	1.00	0.00
0.00	0.00	.48
0.00	0.00	-80.00
1.44	0.00	.64
992.00		
0.00	0.00	0.00
1.00	1.00	0.00
0.00	0.00	0.00
0.00	1.00	0.00
1000.00		
0.00	0.00	0.00
0.00	0.00	1.00
1.00	0.00	0.00
0.00	0.00	1.00
800.00		
11.40	0.00	0.00
15.00	0.00	2.80
0.00	0.00	1200.00
-91.60	-70.00	-49.60
-256880.00		

SURPLUS VARIABLES ARE SUBTRACTED FROM THE LEFT SIDE OF >= INEQUALITIES

SLACK VARIABLES ARE ADDED TO THE LEFT SIDE OF <= INEQUALITIES

ARTIFICIAL VARIABLES ARE ADDED TO THE LEFT SIDE OF EQUALITIES & >= INEQUALITIES TO GENERATE AN INITIAL BASIC FEASIBLE SOLUTION

THE FINAL BASIS CONTAINS ALL VARIABLES IN THE SOLUTION

THE DUAL VARIABLE VALUE (SHADOW PRICE) IS THE AMOUNT OF CHANGE IN THE VALUE OF THE OBJECTIVE FUNCTION FOR EACH UNIT BY WHICH THE CONSTRAINT RIGHT-HAND-SIDE (RHS) VALUE IS CHANGED

SENSITIVITY ANALYSIS

CONSTRAINT RHS VALUE RANGING

CON	LOWER LIMIT	RHS VALUE	UPPE LIMI
H1	32.60	50.00	UNBN
H2	17.40	30.00	42.4
L1	1311.11	2000.00	4290.9
L2	.00	1000.00	2933.3
L3	.00	800.00	2375.0

OBJ FUNC COEFF RANGING
BASIS VARIABLES

VAR	LOWER LIMIT	OBJ FNC VALUE	UPPER LIMIT
X3	64.17	70.00	85.0
X2	70.00	85.00	90.8
X5	.00	70.00	85.0
X7	UNBND	40.00	42.8

OBJ FUNC COEFF RANGING
NON-BASIS VARIABLES

VAR	LOWER LIMIT	OBJ FNC VALUE	UPPER LIMIT
X1	88.60	100.00	UNBND
X4	70.00	85.00	UNBND
X6	47.20	50.00	UNBND

WITHIN THE LIMITS YOU MAY CHANGE THE VALUE OF ANY ONE CONSTRAINT RHS OR OBJ FUNC COEFF WITHOUT CAUSING VARIABLES TO ENTER OR LEAVE THE SOLUTION

BUT, VALUES FOR THE OBJ FUNC AND SOLUTION VARIABLES MAY CHANGE WHEN THIS IS DONE

LOOPED NETWORKS - LP OPTIMIZATION

When pipe loops are created by connecting pipes into the system at more than one place, the problem description becomes non- linear and cannot be solved directly by linear programming methods. That is largely because the flows are unknown. If, however, the network can be reduced to a tree-like network the problem is again simplified. The least-cost network is in fact invariably a tree-like network - the problem being to identify the tree. Because of the economy of scale in pipe transport, the most economic layout is with only one pipe supplying to any point. If this is accepted, then a close approximation to the best (least cost) tree-like network can be obtained by linear programming as follows.

Starting with a looped network number each node j, and each pipe i, define arbitrarily the positive flow directions, then set up the following (linear) constraints in terms of the unknown, Q, in each pipe i:

For flow balance at each node j,

ΣQ_i into node j = q_j (drawoff from node j)

Objective function: Minimize $\Sigma Q_i L_i$

where L_i is the known length of pipe i.

This will minimize the bulk transport, i.e. the litres per second times metres. Strictly this will not be the optimum for non-linear flow rate-cost relationships since economy of scale is not introduced. Each pipe cost is more likely to be proportional to Q^m $(m<1)$, so that a more accurate but non-linear objective function would be Min $\Sigma L_i Q_i^m$. Separable programming methods (Hadley, 1964) could be employed to optimize such a problem but the approach here plus engineering judgement should generally suffice. Bhave (1978), presented a manual method of obtaining a similar optimum network to that proposed here, and Powell and Barnes (1982) proposed an alternative hierarchical method.

LINEAR PROGRAMMING PROGRAM

The appended program is suitable for optimizing both stages of a network, the network 'layout' and the pipe diameters. The program minimizes the objective function, and supplies the coefficients for all dummy variables and artificial variables required for the simplex method.

The program is followed by an example. A network is reduced to a branched system and then the pipe diameters are selected. The input for each section follows and the optimal solution, namely optimum program variables, their magnitudes and costs.

Symbols in linear programming optimization program by minimization

A(I)	Objective coefficient of variable in program
B(J)	Objective coefficient of variable in program
B2	Net cost
E(J)	B - ΣX/A
E2	Min E
I	Row no.
I2	Key I
J	Column no.
J2	Key J
N	Total no. of variables plus dummys
N1	No. variables
M	Total columns
M1	No. of <= constraints
M2	No. of = constraints
M3	No. of >= constraints
R(I)	Replacement ratio
R2	Min replacement ratio
V(I)	Variable no. in program
X(J,I)	Matrix coefficient
X1(J,I)	Matrix coefficient
Z(I)	Magnitude of variable in program
Z1(I)	Magnitude of variable in program

Note input numbers should be between 0.001 and 1000

First Problem: Network Layout

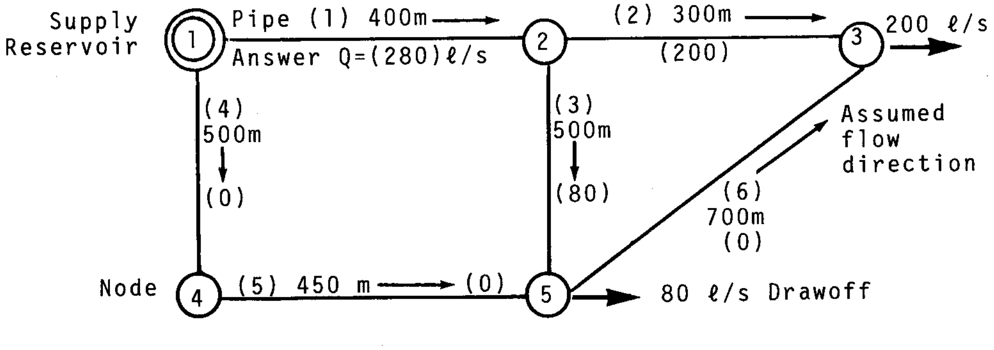


Fig. 5.3

Constraints: node 2: $Q_1 - Q_2 - Q_3 = 0$

node 4: $Q_4 - Q_5 = 0$

node 3: $Q_2 + Q_6 = 200$

node 5: $Q_3 + Q_5 - Q_6 = 80$

Minimize $\Sigma\, QL = 400Q_1 + 300Q_2 + 500Q_3 + 500Q_4 + 450Q_5 + 700Q_6$

```
NAME?
NETLAYOUT
NO.VARBLS,NO.CONSTRAINT<=,=,>=?
6,0,4,0
INPUT COEFS IN CONSTRAINTS IN OR
DER <=,=,>=
COEFS IN CONSTRAINT 1
X 1 ?
1
X 2 ?
-1
X 3 ?
-1
X 4 ?
0
X 5 ?
0
X 6 ?
0
COEFS IN CONSTRAINT 2
X 1 ?
0
X 2 ?
0
X 3 ?
0
X 4 ?
1
X 5 ?
-1
X 6 ?

0
COEFS IN CONSTRAINT 3
X 1 ?
0
X 2 ?
1
X 3 ?
0
X 4 ?
0
X 5 ?
0
X 6 ?
1
```

```
COEFS IN CONSTRAINT 4
X 1 ?
0
X 2 ?
0
X 3 ?
1
X 4 ?
0
X 5 ?
1
X 6 ?
-1
RHS 1 ?
0
RHS 2 ?
0
RHS 3 ?
200
RHS 4 ?
80
OBJ FN COEFS FOR MINIM,1- 6
OBJ.COEF.X 1 ?
400
OBJ.COEF.X 2 ?
300
OBJ.COEF.X 3 ?
500
OBJ.COEF.X 4 ?
500
OBJ.COEF.X 5 ?
450
OBJ.COEF.X 6 ?
700

LP OPTIMZN NETLAYOUT
VARIABLE, MAGNITUDE, COST COEF
     1     280.000   400.000
     2     200.000   300.000
     5        .000   450.000
     3      80.000   500.000
```

Second Problem: Pipe Sizing

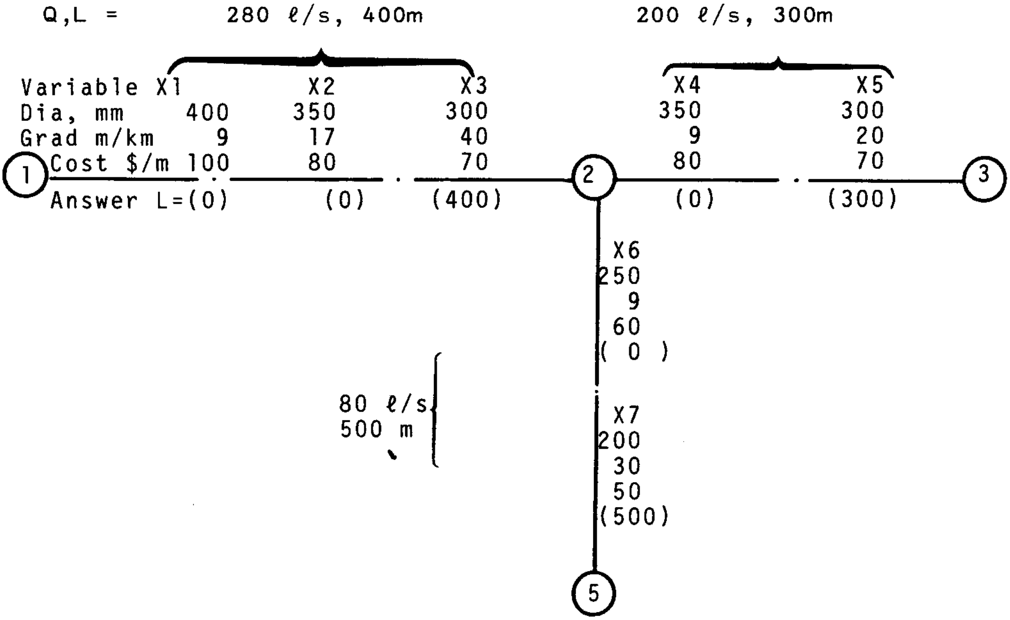


Fig. 5.4

Constraints; Head loss to 3: $.009X1+.017X2+.04X3+.009X4+.02X5 \leq 50$

5: $.009X1+.017X2+.04X3+.009X6+.03X7 \leq 70$

Length X1-X3: $X1+X2+X3 = 400$

X4-X5: $X4+X5 = 300$

X6-X7: $X6+X7 = 500$

```
NAME?
PIPE DIAS
NO.VARBLS,NO.CONSTRAINT<=,=,>=?
7,2,3,0
INPUT COEFS IN CONSTRAINTS IN OR
DER <=,=,>=
COEFS IN CONSTRAINT 1
X 1 ?
.009
X 2 ?
.017
X 3 ?
.04
X 4 ?
.009
X 5 ?
.02
X 6 ?
0
X 7 ?
0
COEFS IN CONSTRAINT 2
X 1 ?
.009
X 2 ?
.017
X 3 ?
.04
X 4 ?
0
X 5 ?
0
X 6 ?
.009
X 7 ?
.03
COEFS IN CONSTRAINT 3
X 1 ?
1
X 2 ?
1
X 3 ?
1
X 4 ?
0
X 5 ?
0
X 6 ?
0
COEFS IN CONSTRAINT 4
X 1 ?
0
X 2 ?
0
X 3 ?
0
X 4 ?
1
X 5 ?
1
X 6 ?
0
X 7 ?
0
COEFS IN CONSTRAINT 5
X 1 ?
0
X 2 ?
0
X 3 ?
0
X 4 ?
0
X 5 ?
0
X 6 ?
1
X 7 ?
1
RHS 1 ?
50
RHS 2 ?
70
RHS 3 ?
400
RHS 4 ?
300
RHS 5 ?
500
OBJ FN COEFS FOR MINIM,1- 7
OBJ.COEF.X 1 ?
100
OBJ.COEF.X 2 ?
80
OBJ.COEF.X 3 ?
70
COST=       74000
OBJ.COEF.X 4 ?
80
OBJ.COEF.X 5 ?
70
OBJ.COEF.X 6 ?
60
OBJ.COEF.X 7 ?
50
LP OPTIMZN PIPE DIAS
VARIABLE, MAGNITUDE, COST COEF
     8       28.000     0.000
     9       39.000     0.000
     3      400.000    70.000
     5      300.000    70.000
     7      500.000    50.000
```

Linear programming program

```
10 ! LINEAR PROGRAMMIING MINIMI
   ZN
20 DIM X(30,20),X1(30,20),E(30)
   ,B(30),V(20),A(20),Z(20),Z1(
   20),R(20)
30 DISP "NAME";
40 INPUT N$
50 DISP "NO.VARBLS,NO.CONSTRAIN
   T<=,=,>=";
60 INPUT N1,M1,M2,M3
70 M=M1+M2+M3
80 N=N1+M1+M2+M3*2
90 FOR I=1 TO M
100 FOR J=1 TO N
110 B(J)=0
120 X(J,I)=0
130 NEXT J
140 NEXT I
150 DISP "INPUT COEFS IN CONSTRA
    INTS IN ORDER <=,=,>="
160 FOR I=1 TO M
170 DISP "COEFS IN CONSTRAINT";I
180 FOR J=1 TO N1
190 DISP "X";J;
200 INPUT X(J,I)
210 NEXT J
220 J=N1
230 IF I>M1 THEN 280
240 X(J+I,I)=1
250 V(I)=J+I
260 A(I)=B(J+I)
270 GOTO 390
280 IF I>M2+M1 THEN 340
290 X(J+I,I)=1
300 B(J+I)=99999999
310 V(I)=J+I
320 A(I)=B(J+I)
330 GOTO 390
340 X(J+I,I)=-1
350 X(J+M3+I,I)=1
360 B(J+M3+I)=99999999
370 V(I)=J+M3+I
380 A(I)=B(J+M3+I)
390 NEXT I
400 FOR I=1 TO M
410 DISP "RHS";I;
420 INPUT Z(I)
430 NEXT I
440 DISP "OBJ FN COEFS FOR MINIM
    ,1-";N1
450 FOR J=1 TO N1
460 DISP "OBJ.COEF.X";J;
470 INPUT B(J)
480 NEXT J
490 FOR J=1 TO N
500 E(J)=B(J)
510 FOR I=1 TO M
520 E(J)=E(J)-X(J,I)*A(I)
530 NEXT I
540 NEXT J
550 E2=0
560 FOR J=1 TO N
570 IF E(J)>=E2 THEN 600
580 E2=E(J)
590 J2=J
600 NEXT J
610 IF E2>=0 THEN 930
620 FOR I=1 TO M
624 IF Z(I)>0 THEN 630
626 Z(I)=.00001
630 IF X(J2,I)<>0 THEN 650
640 X(J2,I)=.0000001
650 R(I)=Z(I)/X(J2,I)
660 NEXT I
670 R2=999999999999
680 FOR I=1 TO M
690 IF R(I)<=0 THEN 730
700 IF R(I)>=R2 THEN 730
710 R2=R(I)
720 I2=I
730 NEXT I
740 FOR I=1 TO M
750 Z1(I)=Z(I)
760 FOR J=1 TO N
770 X1(J,I)=X(J,I)
780 NEXT J
790 NEXT I
800 FOR I=1 TO M
810 Z(I)=Z1(I)-Z1(I2)*X1(J2,I)/X
    1(J2,I2)
820 FOR J=1 TO N
830 X(J,I)=X1(J,I)-X1(J,I2)*X1(J
    2,I)/X1(J2,I2)
840 NEXT J
850 NEXT I
860 Z(I2)=Z1(I2)/X1(J2,I2)
870 FOR J=1 TO N
880 X(J,I2)=X1(J,I2)/X1(J2,I2)
890 NEXT J
900 A(I2)=B(J2)
910 V(I2)=J2
920 GOTO 490
930 B2=0
940 FOR I=1 TO M
950 B2=B2+A(I)*Z(I)
960 NEXT I
970 PRINT "LP OPTIMZN ";N$
980 PRINT "VARIABLE, MAGNITUDE,
    COST COEF"
990 FOR I=1 TO M
1000 PRINT USING 1010 ; V(I),Z(I
     ),A(I)
1010 IMAGE DDDDDD,DDDDDDD.DDD,DD
     DDDD.DDD
1020 NEXT I
1030 PRINT USING 1035 ; B2
1035 IMAGE "COST=",DDDDDDDDDDD
1040 STOP
1050 END
```

REFERENCES

Bhave, P.R., Aug. 1978. Optimization of single source networks, Proc. ASCE, 104 (EE4) p799-814.

Dantzig, G.B., 1963. Linear Programming and Extensions. Princeton Univ. Press, Princeton.

Hadley, G., 1964. Nonlinear and Dynamic Programming, Addison-Wesley, Reading.

Powell, W.F. and Barnes, J.W., Jan., 1982. Obtaining layout of water distribution systems. Proc. ASCE., 108 (HY1).

CHAPTER 6

DYNAMIC AND NON-LINEAR PROGRAMMING FOR LOOPED NETWORKS

Chapters 2 to 4 described methods for calculating the flows in pipe networks with or without closed loops. For any particular pipe network layout and diameters, the flow pattern corresponding to fixed drawoffs or inputs at various nodes could be calculated. To design a new network to meet certain drawoffs, it would be necessary to compare a number of possibilities. A proposed layout would be analysed and if corresponding flows were just sufficient to meet demands and pressures were satisfactory, the layout would be acceptable. If not, it would be necessary to try alternative diameters for pipe sizes and analysis of flows is repeated until a satisfactory solution is at hand. This trial and error process would then be repeated for another possible layout. Each of the final networks so derived would then have to be costed and that network with least cost selected.

A technique of determining the least-cost network directly, without recourse to trial and error, would be desirable. No direct and positive technique is possible for general optimization of networks with closed loops. The problem is that the relationship between pipe diameters, flows, head losses and costs is not linear and most routine mathematical optimization techniques require linear relationships. There are a number of situations where mathematical optimization techniques can be used to optimize layouts and these cases are discussed and described below. The cases are normally confined to single mains or tree-like networks for which the flow in each branch is known. To optimize a network with closed loops, random search techniques or successive approximation techniques are needed.

Mathematical optimization techniques are also known as systems analysis techniques (which is an incorrect nomenclature as they are design techniques not analysis techniques), or operations research techniques (again a name not really descriptive). The name mathematical optimization techniques will be retained here. Such tecniques include simulation (or mathematical modelling) coupled with a selection technique such as steepest path ascent or random searching.

The direct optimization methods include dynamic programming, which is useful for optimizing a series of events or things, transportation progamming, which is useful for allocating sources to demands, and linear programming, for inequalities, (van der Veen, 1967 and Dantzig, 1963.) Linear programming usually requires the use of a computer, but there are standard optimization programs available.

DYNAMIC PROGRAMMING FOR OPTIMIZING COMPOUND PIPES

One of the simplest optimization techniques, and indeed one which can normally be used without recourse to computers, is dynamic programming. The technique is in fact only a systematic way of selecting an optimum program from a series of events and does not involve any mathematics. The technique may be used to select the most economic diameters of a compound pipe which may vary in diameter along its length depending on pressures and flows. For instance, consider a trunk main supplying a number of consumers from a reservoir. The diameters of the trunk main may be reduced as drawoff takes place along the line. The problem is to select the most economic diameter for each section of pipe.

A simple example demonstrates the use of the technique. Consider the pipeline in Figure 6.1. Two consumers draw water from the pipeline, and the head at each drawoff point is not to drop below 5 m, neither should the hydraulic grade line drop below the pipe profile at any point. The elevations of each point and the lengths of each section of pipe are indicated. The cost of pipe is $0.1 per mm diameter per m of pipe. (In this case the cost is assumed to be independent of the pressure head, although it is simple to take account of such a variation). The analysis will be started at the downstream end of the pipe (point A). The most economic arrangement will be with minimum residual head i.e. 5 m, at point A. The head, H, at point B may be anything between 13 m and 31 m above the datum, but to simplify the analysis, we will only consider three possible heads with 5 m increments between them at points B and C.

The diameter D of the pipe between A and B, corresponding to each of the three allowed heads may be determined from a head loss chart and is indicated in Table 6.1 (1) along with the corresponding

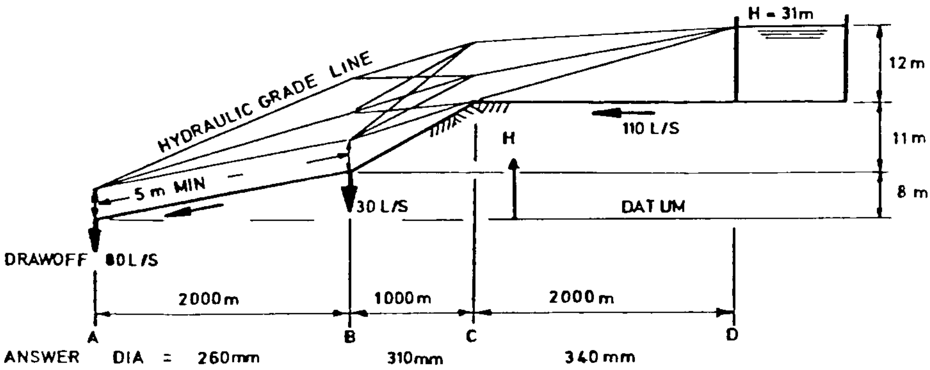


Fig. 6.1 Profile of pipeline optimized by dynamic programming.

cost.

We will also consider only three possible heads at point C. The number of possible hydraulic grade lines between B and C is 3 x 3 = 9, but one of these is at an adverse gradient so may be disregarded. In Table 6.1 (II) a set of figures is presented for each possible hydraulic grade line between B and C. Thus if H_B = 13 and H_{CE} = 19 then the hydraulic gradient from C to B is 0.006 and the diameter required for a flow of 110 ℓ/s is 310 mm (from Figure 1.3). The cost of this pipeline would be 0.1 x 310 x 1 000 = $31 000. Now to this cost must be added the cost of the pipe between A and B, in this case $60 000 (from Table 6.1(I)). For each possible head H_C there is one minimum total cost of pipe between A and C, marked with an asterisk. It is this cost and the corresponding diameters only which need be recalled when proceeding to the next section of pipe. In this example, the next section between C and D is the last and there is only one possible head at D, namely the reservoir level.

In Table 6.1 (III) the hydraulic gradients and corresponding diameters and costs for Section C - D are indicated. To the costs of pipe for this section are added the costs of the optimum pipe arrangement up to C. This is done for each possible head at C, and the least total cost selected from Table 6.1 (III). Thus the minimum possible total cost is $151 000 and the most economic diameters are 260, 310 and 340 mm for Sections A - B, B - C and C - D

TABLE 6.1 Dynamic programming optimization of a compound pipe

I

HEAD AT B	HYDR. GRAD.	DIA. mm	
H_B	h_{B-A}	D_{B-A}	COST $
13	.004	300	60000
18	.0065	260	52000
23	.009	250	50000

II

H_C =	19			24			29		
H_B	h_{C-B}	D_{C-B}	COST $	h_{C-B}	D_{C-B}	COST $	h_{C-B}	D_{C-B}	COST $
13	.006	310	31000 60000 91000*	.011	270	27000 60000 87000	.016	250	25000 60000 85000
18	.001	430	43000 52000 95000	.006	310	31000 52000 83000*	.011	270	27000 52000 79000*
23				.001	430	43000 50000 93000	.006	310	31000 50000 81000

III

H_C	h_{D-C}	D_{D-C}	COST $
19	.006	310	62000 91000 153000
24	.0035	340	68000 83000 151000*
29	.001	430	86000 79000 165000

respectively. It may be desirable to keep pipes to standard diameters in which case the nearest standard diameter could be selected for each section as the calculations proceed or each length could be made up of two sections; one with the next larger standard diameter and one with the next smaller standard diameter, but with the same total head loss as the theoretical result.

Of course many more sections of pipe could be considered and the accuracy would be increased by considering more possible heads at each section. The cost of the pipes could be varied with pressures. A booster pump station could be considered at any point, in which case its cost and capitalized power cost should be added in the tables. A computer may prove useful if many possibilities are to be considered, and there are standard dynamic programming programs available.

It will be seen that the technique of dynamic programming reduces the number of possibilities to be considered by selecting the least-cost arrangement at each step. Kally, (1969) and Buras and Schweig, (1969) describe applications of the technique to similar and other problems.

TRANSPORTATION PROGRAMMING FOR LEAST-COST ALLOCATION OF RESOURCES

Transportation programming is another technique which normally does not require the use of a computer. The technique is of use primarily for allocating the yield of a number of sources to a number of consumers such that a least-cost system is achieved. The cost of delivering the resource along each route should be linearly proportional to the throughput along that route and for this reason the technique is probably of no use in selecting the optimum pipe sizes. It is of use, however, in selecting a least-cost pumping pattern through an existing pipe distribution system, provided the friction head is small in comparison with static head, or for obtaining a planning guide before demands are accurately known.

An example serves to illustrate the technique. In this example, there are two sources of water, A and B, and two consumers, M and N. A and B could deliver 12 and 20 ℓ/s respectively and M and N require 10 and 15 ℓ/s respectively. Thus there is a surplus of water. The cost of pumping along routes A - M, A - N, B - M and B - N are 5,7, 6 and 9 c/1 000 litres respectively.

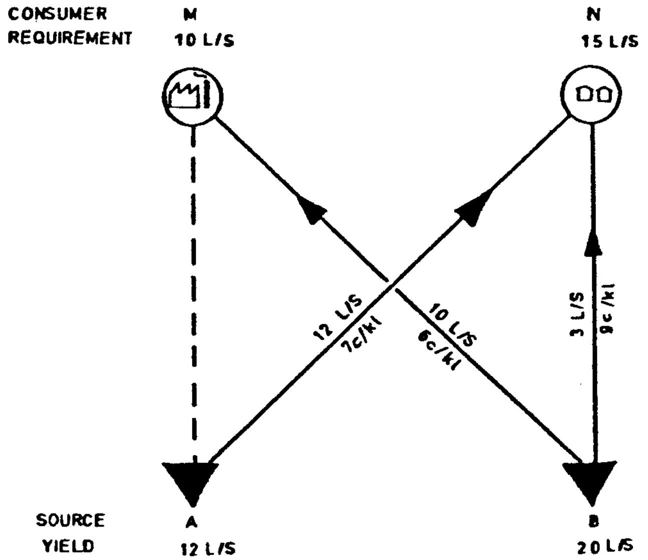


Fig. 6.2 Least-cost allocation pattern for transportation programming example.

TABLE 6.2 Transportation programming - optimization of an allocation system

(I)

SOURCE	YIELD	CONSUMER: REQUIREMENTS:	M 10	N 15	SURPLUS 7	EVALUATION NUMBER:
A	12		5 10	7 2	0 -2	0
B	20		6 7	9 13	0 7	2
EVALUATION NUMBER:			5	7	-2	

(II)

		10	15	7	
A	12	5 4	7 12	0 -2	0
B	20	6 10	9 3	0 7	2
		4	7	-2	

The data are set out in tabular form for solution in Table 6.2(I). Each row represents a source and each column a demand. The unit cost of delivery along each route is indicated in the top right corner of the corresponding block in the table. The first step is to make an arbitrary initial assignment of resources in such a manner that each yield and demand is satisfied. Starting with the top left block of the table, the maximum possible allocation is 10. This satisfies the demand of column M and the amount is written in the bottom left corner of block AM. Proceeding to the next column, since the first column is completed, the maximum possible allocation in the first row is 2, which satisfies the yield of row A. So the next block to be considered is in row B, namely column N. Proceed through the table making the maximum possible assignment at each stage until all resources are allocated (even if to the slack column). Thus the next allocation is the 13 in the second row, then the 7 in the third column.

Once an initial allocation is made the figures are rearranged methodically until a least-cost distribution emerges. To decide which would be the most profitable arrangement, assign a relative evaluation number to each row and column as follows:-

Assign the value 0 to row 1 and work out the other evaluation numbers such that the sum of the row evaluation number and column evaluation number is equal to the cost coefficient for any occupied block. The value for column M is 5, for column N is 7, for row B is 2, and so on. Now write the sum of the row and column evaluation numbers beneath the cost coefficient of each unoccupied block. If this sum is bigger than the cost coefficient of the block, it would pay to introduce a resource allocation into the block. This is not easy to see immediately, but stems from the method of determining each evaluation sum from the cost coefficients of occupied blocks. The biggest possible rate of improvement is indicated by the biggest difference between the evaluation sum and the cost coefficient. The biggest and in fact in our case the only, improvement would be to introduce an amount into Block BM. The maximum amount which can be put in block BM is determined by drawing a closed loop using occupied blocks as corners (see the dotted circuit in Table 6.2(I)).

Now for each unit which is added to block BM, one unit would have to be subtracted from block BN, added to block AN and subtracted from block AM to keep the yields and requirements consistent. In this case the maximum allocation to BM is 10, since this would evacuate block AM. The maximum re-distribution i.e. 10 is made, and the amount in the block at each corner of the closed loop adjusted by 10 to satisfy yields and requirements. Only one re- distribution of resources should be done at a time.

After making the best new allocation, re-calculate the evaluation number and evaluation sums as in Table 6.2 (II). Allocate resource to the most profitable block and repeat the re-distribution procedure until there is no further possible cost improvement, indicated by the fact that there is no evaluation sum greater than the cost coefficient in any block. In our example we arrived at the optimum distribution in two steps, but more complicated patterns involving more sources and consumers may need many more attempts.

The example can only serve to introduce the subject of transportation programming. There are many other conditions which are dealt with in textbooks on the subject of mathematical optimization techniques such as van der Veen (1967) and Dantzig (1963) and this example only serves as an introduction. For instance, if two blocks in the table happened to be evacuated simultaneously, one of the blocks could be allocated a very small quantity denoted by 'e' say. Computations then proceed as before and the quantity 'e' disregarded at the end.

STEEPEST PATH ASCENT TECHNIQUE FOR EXTENDING NETWORKS

A 'steepest path ascent technique' can be used for extending pipeline networks at minimum cost (Stephenson, 1970). The technique is primarily for adding new pipes to existing networks when demands exceed the capacity of the existing pipe networks.

It is usually possible to supply points in a system along various routes or from alternative sources. It may not be necessary to lay a new pipeline all the way from the source to the demand, or alternatively the diameter of the new pipe may vary from one section to another. The fact that elevation, and hence pressure head, varies along a pipeline route causes different pipe diameters to be optimal at different sections.

On account of the complexity of the optimization technique, use of a computer is essential. Alternative routes along which water could be supplied to the node in question are pre-selected manually. The computer program is used to determine the optimum pipelines and corresponding diameters to meet the specific demands.

An informal demonstration that the method yields an optimum design is given with the aid of diagrams.

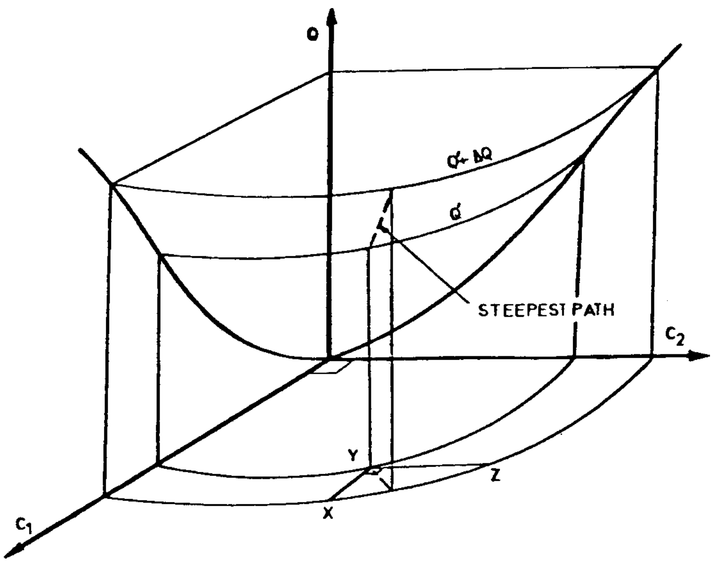


Fig. 6.3 Relationship between discharge and cost of two pipes.

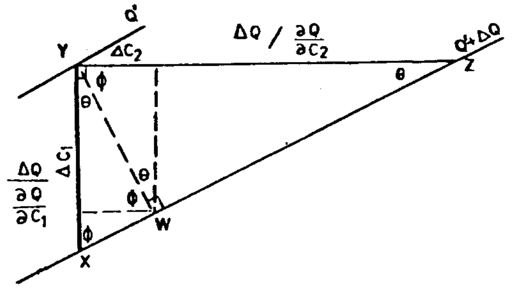


Fig. 6.4 Steepest path projected onto C_1-C_2 plane

Pipeline costs increase with increasing diameter and wall thickness. Wall thickness usually increases in proportion to the diameter, so the cost per metre will be a function of the square of the diameter.

Now for a given head gradient, the discharge is proportional to the pipe diameter to a power of approximately 2.5. However, if the pipe is not laid along the entire route from the source to the demand, but merely reinforces part of the network, the capacity will be limited by the capacity of the remainder of the network. Hence it may be deduced that discharge varies with cost of a new pipe section to a power greater than unity, but is limited by the capacity of the remainder of the pipe network.

If more than one proposed new pipe is involved the relationship between discharge and pipe costs is multi-dimensional. Figure 6.3 illustrates the relationship between discharge at a particular node and cost of two possible pipes in the network.

The curves on the $C_1 - C_2$ plane in Figure 6.3 are lines of constant discharge Q. The shortest path between two Q lines spaced a small distance apart is a line perpendicular to the Q lines. This is the path with the steepest discharge/cost gradient, and is the one sought. The procedure is therefore to start at the origin and proceed in increments on the $C_1 - C_2$ plane, each increment being perpendicular to the next Q line, until the desired discharge Q is attained.

To determine the increments in diameter corresponding to increments in cost, the actual increment in cost of each pipe for a step on the $C_1 - C_2$ plane has to be calculated. Consider the triangle XYZ, enlarged in Figure 6.4. C_1 and C_2 corresponding to increment YW in Q are to be determined.

$$\text{Now } \cos\Theta = \frac{\Delta Q}{\frac{\partial Q}{\partial C_2}} \Bigg/ \sqrt{\left(\Delta Q/\frac{\partial Q}{\partial C_2}\right)^2 + \left(\Delta Q/\frac{\partial Q}{\partial C_1}\right)^2}$$

$$= \frac{\partial Q}{\partial C_1} \Bigg/ \sqrt{\left(\frac{\partial Q}{\partial C_1}\right)^2 + \left(\frac{\partial Q}{\partial C_2}\right)^2}$$

$$YW = \Delta Q \Big/ \frac{\partial Q}{\partial C_1} \cos\Theta$$

$$\Delta C_1 = YW \cos \Theta = \Delta Q / \frac{\partial Q}{\partial C_1} \cos^2 \Theta$$

$$= \frac{\Delta Q \dfrac{\partial Q}{\partial C_1}}{(\dfrac{\partial Q}{\partial C_1})^2 + (\dfrac{\partial Q}{\partial C_2})^2}$$

Similarly $\Delta C_2 = YW \cos \phi$

$$= \Delta Q \frac{\partial Q}{\partial C_2} \cos^2 \phi$$

$$= \frac{\Delta Q \dfrac{\partial Q}{\partial C_2}}{(\dfrac{\partial Q}{\partial C_1})^2 + (\dfrac{\partial Q}{\partial C_2})^2}$$

In a similar manner, for n possible pipes, it may be proved that

$$\Delta C_i = \frac{\Delta Q \dfrac{\partial Q}{\partial C_i}}{\sum_{i=1,n} (\dfrac{\partial Q}{\partial C_i})^2}$$

The rate of increase of discharge with respect to the cost C_i of any pipe, $\partial Q / \partial C$ is determined by analyzing the network with and without a small increment in diameter D_i.The increase in discharge, divided by the increase in cost of pipe i associated with the increase in diameter, gives the required relationship. The increment in discharge per step, ΔQ, is pre-selected so that the increase in cost of each pipe is yielded by the above equation at each step. The corresponding increases in diameter are then calculated from the known diameter/ cost relationships. The diameters of the proposed pipes are increased in steps until the discharge at the specified node is sufficient. A network analysis should be performed after each step to re-balance the system.

It will be observed from Figure 6.3 that it is unlikely that any local maxima will be reached with the technique as the discharge/cost curves are generally concave upwards, and have few points of inflection.

So far the technique has been used to supplement the supply to only one node at a time. It has been found that the pipeline system should be well conditioned for satisfactory convergence. It is usually necessary to initialize the diameters of proposed pipes at a value greater than zero (say of the order of 1/4 of the anticipated final diameter). Otherwise the linear approximation to the differential equations is unrealistic for the initial steps and false results are yielded.

DESIGN OF LOOPED NETWORKS

It was explained previously that it is not possible to design a pipe reticulation network with closed loops without recourse to trial and error or successive approximations. The non-linear flow/head loss relationship, the fact that flow magnitudes and directions are initially unknown, that pipe diameters should conform to standard sizes and be larger than specified minimum sizes, and that certain minimum pressures are required, all pose problems. There are many approaches to the solution for the least-cost looped network, none of which, it should be noted, overcome all the problems and ensure that a true least-cost solution, and not a local peak in the cost function, is at hand. The solutions are nevertheless invariably more economic than a network which is designed by standard methods, and offer a starting system for manipulation by the design engineer.

Some techniques proposed for achieving least-cost solutions, together with their limitations, are outlined below.

(i) Loop/node correction method

A method of least-cost design, which does not depart radically from the familiar methods of Hardy Cross analysis, was developed by the author. The optimization procedure is not based on linear or non-linear programming techniques which are unfamiliar to most engineers. Instead successive cost revisions are performed for each node and for each loop in the network using a correction based on the differential of the cost function determined as follows:-

Assume any pipe cost $C = a\,D^b \ell$ (6.2)

where a and b are constants.

Now the diameter, D, can be expressed in terms of flow Q and head loss h for any pipe: $D=(K\ell Q^n/h)^{1/m}$

where K, m and n are constants

So $C = a(K\ell Q^n/h)^{b/m}\ell$

Differentiating, $dC=(nb/m)(C/Q)dQ-(b/m)(C/h)dh$ (6.3)

i.e. the cost of any pipe can be varied in two ways: by varying flow, Q, keeping the head loss, h, constant, and by varying h while maintaining Q constant. Actually, both factors must be considered in designing a least-cost network. The fact that the diameters and corresponding costs are functions of the two independent variables is often overlooked in mathematical optimization models.

The complete optimization procedure for a network is therefore as follows:

(1) Assume a pipe layout and assume any reasonable initial diameter for each pipe.

(2) Analyse the network using, say, the Hardy Cross method, to determine flows in each pipe and heads at each node. Any number of constant head reservoirs and drawoffs is permitted.

(3) For each loop in turn, calculate the sum of $dC/dQ = (nb/m)\,C/Q$ for each pipe in the loop. If Q is in the assumed positive direction around the loop take the positive value of dC/dQ, otherwise the negative value. Now if $\Sigma dC/dQ$ is positive, i.e. cost increases if flow increases, it would pay to reduce the flow in the positive direction around the loop. Conversely if $\Sigma dC/dQ$ is negative, it would pay to increase the flow around the loop. Subtract or add an increment in flow around the loop depending on the sign of $\Sigma dC/dQ$, and decrease or increase the diameter of each pipe in the loop respectively to keep the head losses constant. The maximum size of increment is that which would reduce any flow to zero, or reduce any pipe diameter to a specified minimum size. An increment slightly smaller than this, say half this value, is preferable. Proceed from loop to loop, repeating this analysis. It is preferable to proceed in the order of decreasing absolute value of $\Sigma dC/dQ$, which means the loops should be ranked in order before making the flow corrections.

(4) For each node in turn other than fixed-head reservoirs, calculate the sum of $dC/h = (b/m)(C/h)$ for each pipe connecting the node. Take the positive value if the head drops towards the node in question and the negative value if the head drops away

from the node. If $\Sigma dC/dh$ is positive it pays to reduce the head, H, at the node, and if it is negative, it pays to increase the head. By increasing the head at the node, pipes leading to the node will have to be increased in diameter and the pipes leading away reduced in diameter to maintain the flows. Conversely a decrease in head will decrease diameters of pipe leading to the node and increase the diameters of pipes leading away. Determine the maximum change in head permissible to produce a decrease in cost without altering any flow directions or reducing pipe diameters to less than specified minimums. The head of the node should also be maintained above the specified minimum. Vary the head correspondingly, or preferably limit the head change to, say, half the maximum permissible and calculate the new pipe diameters connecting the node.

(5) Repeat steps 3 and 4 until no further improvement in cost is discernable. No further network analyses are necessary as once the initial flow balance of step 2 has been achieved it is not unbalanced. Notice however that once the flow directions have been established they cannot be altered. It is therefore important that the initially assumed diameters are realistic and that the corresponding flow pattern is generally correct.

The technique will yield non-standard pipe diameters and these will have to be corrected by assuming the nearest standard pipe size or by letting each pipe comprise two sections, one the next standard size greater and the other the next standard size less than the diameter yielded by the analysis. The corresponding length of each section is calculated from the fact that the total head loss must equal that indicated by the analysis. The calculations should be performed by computer as they are lengthy and definitely not as simple as those for a Hardy Cross network analysis.

(ii) Flow correction by linear programming

In the previous section it was demonstrated that for each pipe dC is linearly proportional to dQ and dH. (This is provided the increments in Q and H are small). If the objective function is taken as the minimization of ΣdC for each pipe, the problem may be set up as a linear programming optimization problem. The linear constraints would be:

For each node, ΣdQ in = 0 where dQ could be positive or negative, and $H \geq$ a specified minimum.

The objective function is ΣdC = minimum, where dC is a linear function of increments in flows in each pipe and heads at each node. A standard linear programming program could be used to select the changes in flow along each pipe and head at each node once an initial network is assumed and analysed. The corresponding diameters could then be calculated. Unless the increments in flow and head are confined to very small values, the head losses will be unbalanced after the linear programming optimization and a network analysis will be required. The linear programming optimization and network analysis should be iterated until there is no further reduction in the total network cost. The subprogramme for setting up the linear programming tableau is complicated and a large computer core storage is required for reasonably large networks.

The core storage required is proportional to the square of the number of pipes, whereas it is proportional to the linear number of pipes for the Loop/Node Correction Method.

Non-standard diameters are yielded, and the flow directions are not altered once an initial assumption is made.

(iii) Non-linear programming

As the problem of design of a pipe network is non-linear a standard or "canned" non-linear programming computer program could be used. Many of these programs are based on the steepest path ascent technique. A sub-program would be required to formulate the constraints and the objective function. The constraints could be expressed as linear functions but the objective function is nonlinear. The constraints are:-

For each node ΣQ in = 0

For fixed-head nodes H = a specified value.

For variable head nodes, $H \geq$ a specified minimum.

For each loop $\Sigma h = 0$.

The objective function is $\Sigma C = \Sigma a\ (K\ell\ Q^n/h)^{(1/m)}$

$= \text{minimum}$

Non-standard diameters are yielded and flow directions must be assumed beforehand. As the technique of nonlinear programming is fairly complicated, it may be difficult to debug the program if errors occur.

(iv) Optimum length method

Kally (1971) proposed a method very similar to the linear programming method of optimization of tree-like networks with known flows.

Since the flows are likely to re-distribute after optimization of a looped network by this method, Hardy Cross analysis is necessary to balance flows at each node, after which a further optimization is performed, and so on.

An initial estimation of diameter is fed into the program, which calculates flows by Hardy Cross analysis. For a small change in diameter along a portion of any pipe, the corresponding head changes at various nodes are calculated. The relationships between head change and length of enlarged (or reduced) pipes is assumed linear, and the optimum lengths of each section of new diameter calculated by linear programming. Diameters may be confined to standard sizes.

(v) Equivalent pipe method

Deb, (1973) replaces all pipes in a layout by pipes with a common predefined length and equivalent diameters (i.e. such that head losses remain unaffected).

An initial flow pattern is assumed and corrected in steps by adjusting pipe sizes for successive loops. The total pipe cost for any loop is a minimum at some flow extreme i.e. with the flow in some pipes in the loop equal to the specified minimum.

A constraint limiting the minimum rate of flow through each pipe may be imposed (for reliability and continuity of supply in case of bursts or blockage closed loops and specified minimum flows in pipes are usually required).

A minimum cost if assumed for each loop and a flow correction which will bring the loop cost to the assumed figure is calculated.

If the minimum flow constraint is violated, a new minimum cost figure is assumed and then the flows corrected accordingly. If the flows are within permissible limits, a slightly lower loop cost is assumed, and flows corrected again. This is repeated until the cost cannot be reduced any more without violating the constraints.

The technique yields non-standard diameters, and is highly dependent on the initial flow assumptions. The equations involved tend to obscure the initial assumptions.

REFERENCES

Appleyard, J.R., Aug. 1975. Discussion on least cost design of branched pipe network systems. Proc. Am. Soc. Civil Engs., 101, (EE4).

Buras, N. and Schweig, Z., Sept. 1969. Aqueduct route optimization by dynamic programming, Proc. Am. Soc. Civil Engs., 95 (HY5).

Dantzig, G.B., 1963. Linear Programming and Extensions, Princeton University Press, Princeton.

Deb, A.K., Jan. 1973. Least-cost pipe network derivation, Water and Water Engg., 77 (928).

Kally, E., April 1971. Automatic planning of the leastcost water distribution network, Water and Water Engg., 75 (8902).

Kally, E., March 1969, Pipeline planning by dynamic computer programming, J. Am. Water Works Assn., (3).

Lam, C.F., June, 1973. Discrete gradient optimization of water systems, Proc., Am., Soc. Civil Engrs., 99 (HY6).

Stephenson, D., 1970. Operation research techniques for planning and operation of pipeline systems, Proc. Pipeline Engg. Convn., Brintex Exhibs., London.

Van der Veen, B., 1967. Introduction to the Theory of Operational Research, Cleaver Hume, London.

CHAPTER 7

CONTINUOUS SIMULATION

The previous chapters have been confined to steady flow in pipe networks with constant reservoir heads. In subsequent chapters unsteady flow is considered. As a first approximation the water column in a pipe is assumed to be incompressible and the accelerations and decelerations are estimated on that basis. In subsequent chapters the compressibility of the water and the pipe is considered. The latter is termed water hammer.

There are many unsteady flow conditions, however, in which neither the compressibility nor accelerations are significant. An example is the slow depletion of storage in a reservoir over a number of hours, for instance during peak flows in a reticulation system. The reservoir will subsequently be refilled by pumping when the demand is lower, for example during the nights and weekends. This is an unsteady flow situation to which the steady flow equations can be applied without appreciable error. That is, pipe friction is the major head loss term but continuity is also applied in order to observe the variations in storage levels. Corresponding to changes in stored volume the water level in the reservoirs will change and consequently the heads on the reticulation system will change. These in turn will affect the discharges.

It is frequently necessary to analyse reticulation systems on this basis in order to determine reservoir capacities and pumping rates for a water reticulation system. The omission of the acceleration and elasticity terms makes the simulation much easier and the time interval can be hours instead of seconds which reduces computational time considerably. Although the so-called global computer program based on the water hammer equations could be employed it would be inefficient and unjustified. In this chapter the coninuity equation and pipe friction equations are used to develop a general purpose simulation program for studying fluctuations in storage in a complex pipe water supply system.

Most water supply systems can be assumed to comprise two major components, one a link or pipe and the other a node which could be a pipe junction or a reservoir. The handling of the node would depend on the description of its operation. A constant head reservoir would be regarded as a node of infinite surface area whereas at the other extremity the outflow could be a function of the storage volume which in turn controls the head on the discharge orifice.

The simulation approach can frequently be used to optimize the design of a water reticulation system where direct optimization techniques would be too complicated. Another advantage of simulation is that it can incorporate non-linear equations and specified functions and does not have to be based on average or assumed steady state conditions.

SYSTEMS ANALYSIS TECHNIQUES AND THE USE OF SIMULATION MODELS

Developments in operation research have led to numerous extremely powerful systems analysis techniques. These techniques can be broadly classified into two main categories:

1) Direct optimization techniques
2) Simulation techniques.

Direct optimization techniques can be used to find the optimum solution to certain problems. Grosman (1981) describes the application of transportation programming, extended transportation programming, linear programming and separable programming. Costs are estimated for raw water, conveyancing and desalination. The techniques are used to calculate the average flows from each source to each demand point. The flows satisfy minimum water quality and quantity constraints and result in the minimum total cost solution.

The techniques used in many studies assume steady state conditions. Average flows and constant water quality are assumed at all the sources and demand points. The average flows are calculated which result in the minimum cost. Real water systems are never in a steady state. Water required at demand points generally varies between zero and several hundred litres a second. During peak demand periods it may not be possible to draw water from the different sources in the optimum ratio determined. The concentration of pollutants in some sources varies throughout the day and week depending on what source was used and the proportion of clean water. It can be concluded that optimum solutions derived from deterministic models which assume steady flows and constant water quality may not be realistic. The 'optimum solution' cannot guarantee that the constraints will be satisfied at all times, that the solution is practical or, indeed, that the solution is an optimal one at all.

Simulation provides a means of observing the behaviour of the components of a system under varying conditions. No 'solution' in the mathematical sense is sought. The objective is to gain an understanding of the relationships among components of the system and to find ways to make them work together in the best possible way. Simulation does not yield an optimal solution directly and it is thus necessary to simulate iteratively in order to achieve an optimum. Even when combined with efficient techniques for selecting the values of each decision variable, an enormous computational effort may lead to a solution which is still far from the best possible.

To its credit, simulation can be used to solve models with highly non-linear relationships and constraints. The direct optimization techniques are seldom able to deal with all the complexities and non-linearities which are easily incorporated into a simulation model. Simulation can be used to experiment with alternative 'optimum solutions' and together with direct optimization techniques it may be possible to narrow the search for a real global optimum. Little or no cost, time or risk is involved with simulation. The time scale can be controlled and long and short term effects of quantity and quality can be determined and used as an aid to decision making and planning. More important, variables and parameters can be identified by changing their values and studying the effects. Certain parameters and relationships can be determined from simulations. A simulation model can be visualised by most people and the results are generally more convincing than those obtained from deterministic approaches.

The rate of change of water quantity and quality, with respect to time can always be described by a set of first order ordinary differential equations. These equations can be solved simultaneously at each iteration in a simulation using powerful numerical methods. The solution to the set of equations yields the volume of water in each storage component of the system at the end of each simulation time-step. These values can be used in conjunction with the operating rules and various relationships to determine pump on/off settings, make-up flows, demands, overflows etc. The degree to which the model represents the real system and the accuracy of the results depends on the validity of the model and the accuracy of the solution of the set of equations.

A general simulation program has been written which can be used to simulate water models. The model must be described by a system of first-order ordinary differential equations. Such a model, consisting of j equations and involving q variables, can be written in the general form:

$$\frac{dx_i}{dt} = \phi(t, x_1, x_2, --- x_q, \frac{dx_1}{dt}, \frac{dx_2}{dt} --- \frac{dx_j}{dt}) \qquad (7.1)$$

where i = 1,2 --- j

According to James (1978), the orderly procedure for constructing simulation models is:

1) Systems Analysis: the salient components, inter-actions, relationships and dynamic mechanism of a system are identified.
2) Systems Synthesis: the model is constructed and coded in accordance with Step 1).
3) Verification : the model's responses are compared with those which would be expected if the model's structure was prepared as intended.
4) Validation: the responses from the verified model are compared to corresponding observations of, and measurements from the actual system.
5) Inference: experiments with, and comparisons of responses from, the verified and validated model - this is the design stage.

MATHEMATICAL MODELLING OF WATER QUALITY

Modelling Concepts

A field to which many of the present concepts can be applied is that of water quality deterioration in industrial systems. Cooling and washing systems are examples where quality will deteriorate in time.

It is not easy to predict the rate of build-up of dissolved salts or the equilibrium concentrations in water reticulation systems, even with an understanding of the origins and methods of concentration of salts. This is because of the complex nature of industrial water recirculation systems. One way of accounting for all these effects in a real system appears to be by modelling the system on a computer.

Once a model is produced and validated, it may be used to improve the operation of existing service water reticulation systems and for optimizing the design of new systems. It is one of the objectives of a research programme to produce such a mathematical model which will be formulated in general terms for adaptation to any particular system.

The build-up of impurities in water can be simulated mathematically together with the water recirculation cycle. The flows of water in conduits or in vapour form in the air in and out of the system can be calculated. The processes of evaporation, condensation, pollution and make-up can all be modelled.

Mass Balances

For the purposes of mathematical simulation of water systems, the system must be described in terms of equations. One-stage systems can be described in terms of a mass balance equation which can be solved analytically. In other more complex situations it is necessary to express the equations in finite difference form and solve them numerically. Different types of models and the assumptions therein are described below.

Parameters whereby pollution is measured may either be conservative or non-conservative. In a conservative system input to any part of the system equals outflow. Thus, if the parameter studied is water flow then evaporation will be neglected in a conservative model. Similarly if the parameter is a chemical compound it is assumed there is no reaction, deposition or solution in a conservative model.

The model may be steady-state or time-varying. During the start-up period of a mine as concentrations build up the system is said to be unsteady. After a while the system may reach equilibrium. That is, in the case of salts in solution, the increases in mass of dissolved solids in the system due to leaching or evaporation equals the loss by pumping or deposition.

Mixed and Plug Flow Systems

In a plug-flow system, the water is assumed to travel through the pipes and drains at a certain rate, conveying impurities at that rate. The salts content at any point can therefore be affected in a series of steps as water with different concentrations arrives at that point. In a completely mixed system, the concentration of salts will be the same at every point. An input is assumed to spread instantaneously through the system so that the concentration increases by the mass of salt input divided by the total volume of water in the system. This simplified mechanism is often satisfactory to describe systems which exhibit gradual rates of change in concentrations. Real systems will probably be between plug flow and completely mixed, as there will be diffusion and mixing due to turbulence and cross connections. In general salts are conveyed by advection (lateral transport) and dispersion.

Examples

The simplest illustration of the use of the mass balance equations is for a steady-state system. Q is flow rate in ℓ/s or Mℓ/d, C is the concentration in mg/ℓ. Inflow of water and of salts per unit time equals outflow rate:

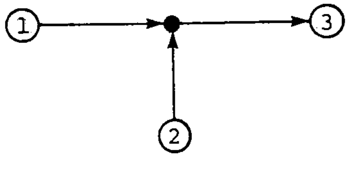


Fig. 7.1 Point Node

$$Q_1 + Q_2 = Q_3 \quad (7.2)$$

$$Q_1C_1 + Q_2C_2 = Q_3C_3 \quad (7.3)$$

$$\therefore C_3 = \frac{Q_1C_1+Q_2C_2}{Q_1+Q_2} \quad (7.4)$$

e.g. if Q_1 = 5 Mℓ/d, Q_2 = 10 Mℓ/d (water flow rate)

C_1 = 400 mg/ℓ, C_2 = 100 mg/ℓ (salt concentration)

then C_3 = 200 mg/ℓ

and the total mass of salt discharged per day

$$= Q_3C_3 = 15 \times 200 = 3000 \text{ kg/d.} \qquad (7.5)$$

A completely mixed system can be described by differential equations: Subscript i refers to inflow, e to exit, s to initial conditions.

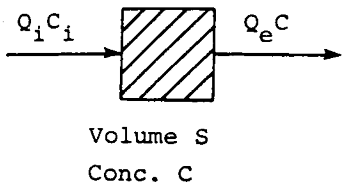


Fig. 7.2 Mixed flow node

$$Q_iC_i = Q_eC + \frac{d(SC)}{dt} \qquad (7.6)$$

$$= Q_eC + S\frac{dC}{dt} \text{ for constant S} \qquad (7.7)$$

$$\therefore dt = \frac{SdC}{Q_iC_i - Q_eC} \qquad (7.8)$$

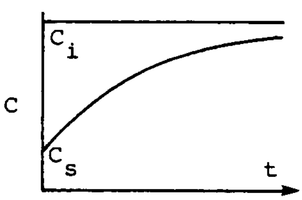


Fig. 7.3 Diffuse node

Integrating and evaluating the constant of integration from the fact that $C = C_s$ at $t = 0$:

$$t = \frac{S}{Q_e} \ell n \left(\frac{Q_iC_i - Q_eC_s}{Q_iC_i - Q_eC}\right) \qquad (7.9)$$

$$\text{or } C = \frac{Q_iC_i}{Q_e} - \frac{Q_iC_i/Q_e - C_s}{e^{Q_et/S}} \qquad (7.10)$$

e.g. at $t = 0$, $C = C_s$, and at $t = \infty$, or $Q_e = \infty$ or $S = 0$,

$C = (Q_i/Q_e)C_i$

Observe that if Q_i does not equal Q_e, there must be internal gains or losses, e.g. due to evaporation.

The previous example could be studied numerically. Although this requires specific numbers, it is often the only practical way of solving more complex problems.

Assume $S = 1000\ m^3$, $Q_i = 1\ m^3/s = Q_e$, $C_s = 0$, $C_i = 500$ mg/ℓ.

Choose $\Delta t = 100$ s. The choice of Δt can affect the speed of solution, the accuracy of results and the numerical stability of the computations. It must be determined by trial, from experience or from theoretical considerations.

$$\text{Now } Q_iC_i - Q_eC = S\,\frac{C_2-C_1}{\Delta t} \qquad (7.11)$$

$$\therefore C_2 = C_1 + \frac{\Delta t}{S}Q_i(C_i-C_1) = C_1 + 0.1(500-C_1) \qquad (7.12)$$

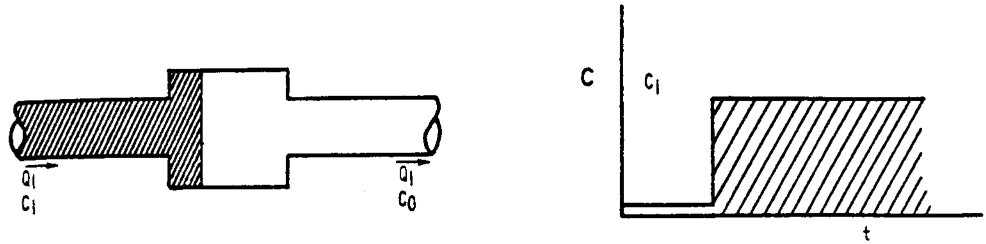
The computations can be set out in tabular form as follows:

t	C_1	$500-C_1$	×0.1	C_2
0	0	500	50	50
100	50	450	45	95
200	95	405	40	135
300	135	365	37	172
.				
.				
.				
1000	326	174	17	343 mg/ℓ

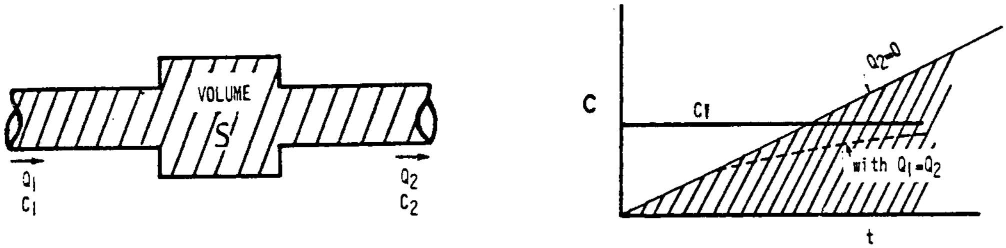
Equation (7.10) would indicate C = 316 mg/ℓ at t = 1000s, which is comparable with the result indicated by the numerical solution of 343 mg/ℓ.

Systems Analysis

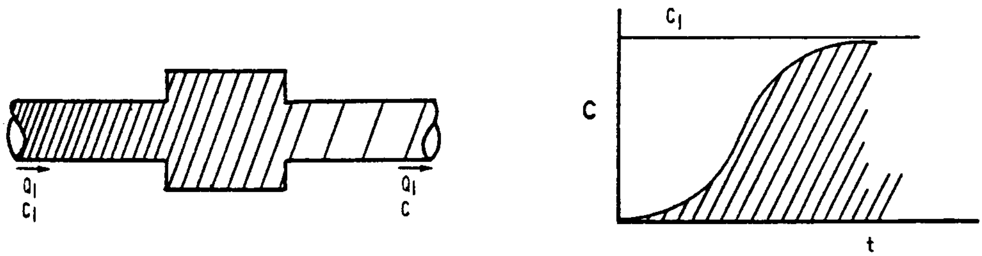
A more sophisticated approach than the simulation method described above is the use of systems analysis and optimization techniques, with the assistance of computers if necessary. The methods allow an optimum design to be selected from numerous alternatives.



a. Plug flow



b. Completely mixed system



c. Diffuse system

Fig. 7.4 Comparison of plug flow and mixed systems

The alternative standard engineering approach is to select the best option from a few selected designs. The latter approach is tedious where there are many alternatives.

The design optimization approach involves the creation of a general configuration in which the numerical value of independent variables has not been fixed. An overall economic objective is defined and the system is described in terms of equations or constraints.

NUMERICAL METHODS FOR THE SOLUTION OF SINGLE DIFFERENTIAL EQUATIONS

Numerical solutions appear in the form of a tabulation of the values of the functions at various values of the independent time variable and not as a functional relationship. Numerical methods have the ability to solve practically any equation but they have the disadvantage that the entire table must be recomputed if the initial conditions are changed.

If a function f(t) can be represented by a power series in a certain interval then it can be represented by the Taylor series expanded about a point $t = t_0$, i.e. about the initial value:

$$y(t)=y(t_0)+y^{I}(t_0)(t-t_0)+\frac{y^{II}(t_0)(t-t_0)^2}{2!}+\frac{y^{II}(t_0)(t-t_0)^3}{3!}+\ldots \quad (7.13)$$

Letting n represent the previous step at time t_0 and n+1 represent the next step at t_0+h, the series can be written as:

$$y_{n+1}=y_n+hy_n^{I}+\frac{h^2}{2}y_n^{II}+\frac{h^3}{6}y_n^{III}+\ldots \quad (7.14)$$

Consider the example problem

$$y' = \frac{dy}{dt} = y+t \quad (7.15)$$

with initial conditions

$$y(0) = 1 \quad (7.16)$$

This is a linear time variant 1st order differential equation. The analytical solution to the problem, $y = 2e^t-t-1$ will be used to compare the numerical results of some of the methods and to illustrate the error at any step.

The Euler Method

The Euler method is the simplest but least accurate of all the methods discussed. To obtain an exact numerical solution to the example problem (7.15), all the derivatives y^{II}, y^{III}, y^{IV} ... must be evaluated and substituted into the Taylor series (7.14). Knowing the initial values of y_n, y_n^I, y_n^{II} ..., y_{n+1} could be evaluated after a time increment h. The values of all the derivatives could then be calculated at n+1, and y_{n+2} could be evaluated after the next time increment and so on. Derivatives of arbitrary functions cannot easily be formulated in computer programs. The derivatives y^{II}, Y^{III}, etc. are easy to evaluate for the example (7.15) but this is not generally the case. The Euler method truncates the Taylor series by excluding the terms after the first derivative and eliminates the problem of having to evaluate the second and subsequent derivatives. Then

$$y_{n+1}=y_n+hy_n^I+0(h^2) \text{ error} \tag{7.17}$$

Neglecting $h^2y_n^{II}/2$ and the subsequent terms in (7.14) results in a truncation error of order h^2 which is denoted $0(h^2)$. This is the local error and results from one step only, i.e. from n to n+1. It can be shown that the global error accumulated over many steps becomes 0(h), i.e. an error of order h.

Substituting the example (7.15) into the Euler algorithm (7.17) gives:

$$y_{n+1}=y_n+h.(y_n+t_n) \tag{7.18}$$

The initial condition y(0)=1 means that y=0 at t=0. Choosing the *time increment h=0.02 and letting the step number n=0 at t=0,* the values for y can be evaluated at successive time increments as follows:

$$y_1=y_0+h(y_0+t_0) = 1+0.02(1+0) = 1.0200 \tag{7.19}$$
$$y_2=y_1+h(y_1+t_1) = 1.0200+0.02(1.0200+0.02) = 1.0408 \tag{7.20}$$
$$y_3=y_2+h(y_2+t_2)= 1.0408+0.02(1.040+0.04) = 1.0624 \tag{7.21}$$
$$y_4= \quad = 1.0848 \tag{7.22}$$
$$y_5= \quad = 1.1081 \tag{7.23}$$

etc.

The numerical solution after 5 steps is y(0.10)=1.1081 whereas $y=2e^{t}-t-1$ gives the exact analytical solution as y(0.10)=1.1103. Hence the absolute global error is 0.0022, i.e. two-decimal-place accuracy. Since the global error of the Euler method is proportional to h, i.e. 0(h), the step size h must be reduced at least 22- fold to gain four-decimal accuracy, i.e. h 0.004. This would increase the computational effort 22-fold. Fig. 7.4 shows how the slope at the beginning of the interval y_n' is used to determine the function value at the end of the iteration in the Euler method.

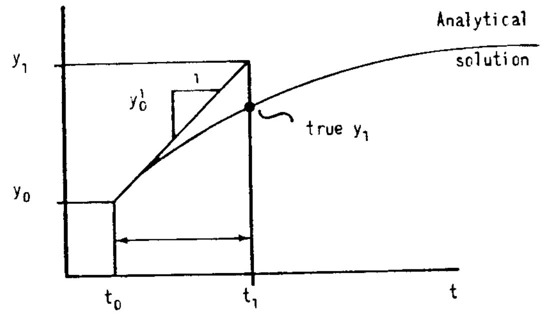


Fig. 7.4 The Euler method

The slope at the beginning of the interval is always wrong unless the solution is a straight line. Thus the simple Euler method suffers from the disadvantage of lack of accuracy, requiring an extremely small step size.

The Modified Euler Method

Fig. 7.4 and the subsequent discussion suggest how the Euler method can be improved with little additional computational effort. The arithmetic average of the slopes at the beginning and the end of the interval is used (only the slope at the beginning is used in the Euler method).

$$y_{n+1} = y_n + h\frac{y_n' + y_{n+1}'}{2} \tag{7.24}$$

The Euler algorithm must first be used to predict y_{n+1} so that y'_{n+1} can be estimated. Applying the same example (7.15) as before and substituting $y^1 = x+t$ into (7.24) gives

$$y_{n+1} = y_n + h\frac{(y_n+t_n) + (y_{n+1}+t_{n+1})}{2} \tag{7.25}$$

Substituting the Euler equation (7.18) for y_{n+1} gives

$$y_{n+1} = y_n + h\frac{(y_n+t_n) + (y_n+h(y_n+t_n) + t_{n+1})}{2} \tag{7.26}$$

Using h=0.02 and the initial conditions: $y_0=1, t_0=0$

$$y_1 = y_0 + h\frac{(y_0+t_0) + (y_0+h(y_0+t_0)+t_1)}{2} \tag{7.27}$$

$$=1+0.02\frac{(1+0) + (1+0.02(1+0)+0.02)}{2} \tag{7.28}$$

$$=1.0204 \tag{7.29}$$

$$y_2 = 1.0204+0.02\frac{(1.0204+0.02)+(1.0204+0.02(1.0204+0.02)+0.04)}{2} \tag{7.30}$$

$$=1.0416 \tag{7.31}$$

.

.

.

$y_5 = 1.1104$ cf analytical solution 1.1103

The answer agrees to within 1 in the fourth decimal place. Nearly twice as much work was done as in the Euler method but certainly not the 22 times more that would have been needed with that method to attain four decimal place accuracy. It can be shown that the local and global errors of the Modified Euler method are

$0(h^3)$ and $0(h^2)$ respectively. The Modified Euler and the simple Euler methods are often referred to as second and first order methods respectively.

Runge-Kutta Methods

The Fourth-Order Runge-Kutta methods are amongst those which provide the greatest accuracy per unit of computational effort. The development of the method is algebraically complicated and is given completely in Stummel and Hainer (1978) while Gerald (1980) derives the Second-Order Runge-Kutta algorithm and explains the principles behind the methods. All the Runge-Kutta methods use the simple Euler method as a first estimate. Improved estimates are then made using previous estimates and different time- values within the time interval h. A weighted average of all the estimates is used to calculate y_{n+1}. The Fourth-Order Runge-Kutta methods are the most widely used because of their power and simplicity. The following is a particular Fourth-Order method which is commonly used and which is included in the simulation program:

$$y_{n+1}=y_n+\frac{1}{6}(k_1+2k_2+2k_3+k_4) \qquad (7.32)$$

$$k_1 = hf(t_n,y_n) \qquad (7.33)$$

$$k_2 = hf(t_n+\tfrac{1}{2}h,y_n+\tfrac{1}{2}k_1) \qquad (7.34)$$

$$k_3 = hf(t_n+\tfrac{1}{2}h,y_n+\tfrac{1}{2}k_2) \qquad (7.35)$$

$$k_4 = hf(t_{n+1},y_n+k_3) \qquad (7.36)$$

Again the problem given in (7.15) above is solved as an example: $dy/dt=f(t,y)=t+y, y(0)=1$. This time $y(0.1)$ is calculated in one step ($h=0.1$) whereas $y(0.1)$ was calculated in five time increments ($h=0.02$) using the simple and modified Euler methods above.

$$k_1=h(t_n+y_n)$$

$$=0.1(0+1) = 0.10000 \qquad (7.37)$$

$$k_2=0.1(0.05+1.05) = 0.11000 \qquad (7.38)$$

$$k_3=0.1(0.05+1.055) = 0.11050 \qquad (7.39)$$

$$k_4=0.1(0.10+1.1105) \quad = 0.12105 \tag{7.40}$$

$$y(0.1)=1.0000+\frac{1}{6}(0.10000+2\times0.11000+2\times0.11050+0.12105) \tag{7.41}$$

$$=1.11034 \tag{7.42}$$

This agrees to five decimals with the analytical result and illustrates a further gain in accuracy with less effort than required by the previous Euler methods. It is computationally more efficient than the modified Euler method because, while four evaluations of the function are required for each step rather than two, the steps can be many-fold larger for the same accuracy. The simple Euler method would have required of the order of 220 steps to achieve five-decimal accuracy in y(0.1) but each step involves only one evaluation of the function. The efficiency of the Euler and Runge-Kutta methods can be roughly compared by calculating the number of function evaluations required for the same order of accuracy. In this particular example the Runge-Kutta method is approximately 50 times more efficient than the simple Euler method (220/4). The local error term for the Fourth-Order Runge- Kutta algorithm (7.36) is $0(h^5)$ and the global error would be about $0(h^4)$.

Multistep Methods

The simple Euler, Modified Euler and Runge-Kutta methods are called single step methods because they use only the information from the last step computed. In this they have the ability to perform the next step with a different step size and are ideal for beginning the solution where only the initial conditions are available. The principle behind a multistep method is to utilize the past values of y and/or y^{I} to construct a polynomial that approximates the derivative function and to extrapolate this into the next time interval. Most multistep methods have the disadvantage that they use a constant step size h to make the construction of the polynomial easier. Another disadvantage of multistep methods is that several past points are required whereas only the initial conditions are available at the start. The starting values are generally calculated from the initial conditions using a single-step method such as a Runge-Kutta method.

REAL-TIME OPERATION OF WATER SUPPLY SYSTEMS

Although numerical models are of direct use for planning and eventually design of the components in a water supply system they can equally well be applied on-line for the operation of the system. Shamir (1981) described a number of applications of the connection between data loggers and mini computers for optimizing the operation of systems of reservoirs and water supply pipes. Cost minimization of these operations can be performed on a continuous basis (example Sterling and Coulbeck, 1975). On the other hand, when applied off-line the methods can be used to identify shortfalls in the system (Rao and Bree, 1977).

The telecommunication of reservoir levels, pipe pressures and flow rates is relatively simple whether in analogue or digital form. Although there are problems with the measurement of discharge rate the accuracy of simple methods is still probably more than adequate to cope with the predicted future demands. Forecasting in fact is the most difficult aspect of the real time simulation. Demand patterns may be approximated by Fourier methods. It is generally possible to prepare a daily and weekly demand pattern on a deterministic basis but the introduction of probability makes the calculations more cumbersome. Growth in demand can also be included in the simulation.

The simulation program can be used to investigate alternative operating methods. Constraints on resource availability such as water, power or manpower can be built into the program. Costs can be minimized by including energy tariffs and income from water sales. Quality constraints can also be included.

COMPUTER PROGRAM TO SIMULATE RESERVOIR LEVEL VARIATIONS IN A PIPE NETWORK

The accompanying computer program will simulate the variations in water level in reservoirs in addition to performing a network flow balance.

The program is based on the node head correction program in Chapter 2 with an additional variable, area of reservoir for each 'fixed head' or, in this case, 'reservoir type' node. If the simulation duration T1 in hours and time increment T2 are input, for example 24 and 1, then the heads at each node and water level in each reservoir will be printed out for every hour. The actual network iterations each time interval after the first should be minimal since the network flows are balanced in the first iteration and only unbalance due to reservoir level changes which will have to be corrected at subsequent time intervals. Although drawoffs are time-fixed in the present program, they could be altered at pauses in the running or inserted in equation form.

The output, namely level variations, could be used to estimate required reservoir depths (using trial reservoir surface areas) and in fact to see at which reservoir locations the storage is most

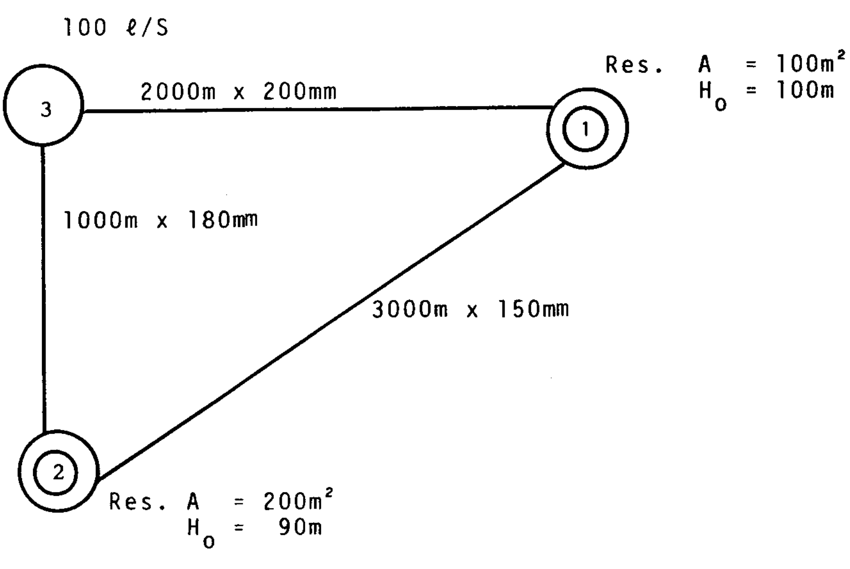


Fig. 7.5 System for continuous simulation example

required. Data requirements are similar to the analysis program with the following additions.

In the first data line after the name, the simulation duration and increment in hours is added at the end of the line. In the pipe data, the first pipes should be from the various reservoirs with the surface areas of the up-stream reservoirs in square metres given at the end of the pipe data lines. In order to display the reservoir levels in the biggest reservoir it is necessary to have a supply pipe from a pseudo fixed head, very large, reservoir to represent a pumped supply feeding into the actual biggest level reservoir in the distribution system.

REFERENCES

Gerald, C.F., 1980. Applied Numerical Analysis. 2nd Ed., Addison Wesley.

Grosman, D.D., 1981. Optimum Allocation of Mine Service Water Subject to Quality Constraints. MSc(Eng) Dissertation, University of the Witwatersrand.

Holton, M.C., 1982. A Computer Programme for the Simulation of Water Reticulation Systems in Gold Mines. MSc (Eng) Dissertation University of the Witwatersrand.

James, W., 1978. Developing and Using Computer Simulation Models of Hydrological Systems. Computational Hydraulics Inc. Hamilton.

Rao, H.S. and Bree, D.W., 1977. Extended period simulation of water systems, Proc. ASCE, 103 (HY2) p97-108.

Shamir, U., June 1981. Real-time control of water supply systems. Proc. Intl. Symp. on Real-time Operation of Hydro-systems. Univ. of Waterloo, p555-562.

Sterling, M.J.H. and Coulbeck, B., 1975. Optimization of water pumping costs by hierarchical methods. Proc. ICE, 59, p789-797.

Stummel,F. and Hainer, K., 1980. Introduction to Numerical Analysis. Scottish Academic Press.

Computer program for continuous simulation of pipe network with reservoirs

```
 10 PRINT "PIPE NET SIMUL BY NOD
    E HEAD CORR"
 20 DIM K(50),L(50),X(50),D(50),
    Q(50),H(50),P(50),A(50)
 30 DISP "NETWORK NAME";
 40 INPUT L$
 50 G=9.8 ! 32 IF FT-S UNITS
 60 DISP "NPIPES,NNODES,NRES,ERm
    ,DARCYf,TOPHm,SIMLNh,DTh";
 70 INPUT N1,N2,N3,H3,F,H(1),T1,
    T2
 80 FOR J=1 TO N1 ! FIXED HEADS
    NUMBERED FIRST
 90 DISP "NODE1+,NODE2-,Lm,Dm,IN
    TLH2,Q2m3s,Am2FORFHUS";
 95 IF J<=N3 THEN 108
100 INPUT K(J),L(J),X(J),D(J),H2
    ,Q2
103 GOTO 110
108 INPUT K(J),L(J),X(J),D(J),H2
    ,Q2,A(K(J))
110 H(L(J))=H2
120 P(L(J))=Q2
130 F(J)=F
140 NEXT J
142 PRINT L$
145 FOR T3=0 TO T1 STEP T2
150 FOR I1=1 TO 50 ! MAX ITNS
160 V=0
170 FOR J=1 TO N1
180 IF H(K(J))>=H(L(J)) THEN 220
190 U=L(J)
200 L(J)=K(J)
210 K(J)=U
220 H1=H(K(J))-H(L(J))
230 Q(J)=SQR(H1*D(J)^5*.785^2*2*
    G/F/X(J))
232 NEXT J
233 IF I1>1 THEN 250
234 FOR I=1 TO N3
235 H(I)=H(I)-P(I)/A(I)*T2*3600
236 FOR J=1 TO N1
238 IF K(J)=I THEN 246
240 IF L(J)=I THEN 242
241 GOTO 249
242 H(I)=H(I)+Q(J)/A(I)*T2*3600
243 GOTO 249
246 H(I)=H(I)-Q(J)/A(I)*T2*3600
247 GOTO 249
248 NEXT J
249 NEXT I
250 FOR I=N3+1 TO N2
260 R=-P(I)
270 S=0
280 FOR J=1 TO N1
290 IF K(J)<>I THEN 330
300 R=R-Q(J)
310 S=S+Q(J)/(H(K(J))-H(L(J)))
320 GOTO 360
330 IF L(J)<>I THEN 360
340 R=R+Q(J)
350 S=S+Q(J)/(H(K(J))-H(L(J)))
360 NEXT J
370 H(I)=H(I)+2*R/S
380 IF ABS(2*R/S)<=V THEN 400
390 V=ABS(2*R/S)
400 NEXT I
410 IF V<H3 THEN 440
420 NEXT I1
440 PRINT " N+ N-    X(M) D(M) QM
    3/S H2M";T3
450 FOR J=1 TO N1
460 PRINT USING 470 ; K(J),L(J),
    X(J),D(J),Q(J),H(L(J))
470 IMAGE DDD,DDD,DDDDD,DD.DDD,D
    D.DDD,DDDDD.D
480 NEXT J
485 NEXT T3
490 END
```

```
NETWORK NAME?
TESTRES
NPIPES,NNODES,NRES,ERm,DARCYf,TO
PHm,SIMLNh,DTh?
3,3,2,.1,.02,100,3,1
NODE1+,NODE2-,Lm,Dm,INTLH2,Q2m3s
,Am2FORFHUS?
1,2,3000,.15,90,.05,100
NODE1+,NODE2-,Lm,Dm,INTLH2,Q2m3s
,Am2FORFHUS?
2,3,1800,.18,70,.1,200
NODE1+,NODE2-,Lm,Dm,INTLH2,Q2m3s
,Am2FORFHUS?
1,3,2000,.2,70,.1
```

```
PIPE NET ANAL BY NODE HEAD CORR

TESTRES
 N+ N-    X(M)  D(M)  QM3/S  H2M 0
  1  2 3000   .150   013    89.3
  2  3 1800   .180  .041    63.2
  1  3 2000   .200  .059    63.2
 N+ N-    X(M)  D(M)  QM3/S  H2M 1
  1  2 3000   .150  .013    88.6
  2  3 1800   .180  .041    63.2
  1  3 2000   .200  .059    63.2
 N+ N-    X(M)  D(M)  QM3/S  H2M 2
  1  2 3000   .150  .013    88.0
  2  3 1800   .180  .041    62.1
  1  3 2000   .200  .059    62.1
 N+ N-    X(M)  D(M)  QM3/S  H2M 3
  1  2 3000   .150   013    87.3
  2  3 1800   .180   041    62.1
  1  3 2000   .200  .059    62.1
```

CHAPTER 8

UNSTEADY FLOW ANALYSIS BY RIGID COLUMN METHOD

RIGID WATER COLUMN SURGE THEORY

Transients in closed conduits are normally classed into two categories: slow motion mass oscillation of the fluid which is referred to as surge, and rapid change in flow accompanied by elastic strain of the fluid and conduit which is referred to as water hammer. For slow or small changes in flow rate or pressure the two theories yield similar results.

It is normally easier to analyse a system by rigid column theory than by elastic theory. On the other hand there are many situations where it is inaccurate or even dangerous to apply this simplified theory, and water hammer theory must be applied. With rigid column theory the water in the conduit is treated as an incompressible mass, although the water column is free to move around bends and through expansions etc. A pressure difference applied across the ends of the column produces an instantaneous acceleration throughout its length. The basic equation relating the head difference between the ends of the water column in a uniform bore conduit to the rate of change in velocity is derived from Newton's basic law of motion, and is

$$h = \frac{-L}{g}\frac{dv}{dt} \tag{8.1}$$

where h is the difference in head between the two ends, L is the conduit length, v is the flow velocity, g is gravitational acceleration and t is time.

The equation is useful for calculating the head rise associated with slow deceleration of a water column. It may be used for calculating the water level variations in a surge shaft following power trip or starting up in a pumping line, or power load changes in a hydroelectric installation fed by a pressure pipeline. The equation may be solved in steps of Δt by computer, in tabular form or graphically.

Example 1

Numerical Analysis of Surge Shaft

A 100 m long penstock with a cross-sectional area, A_1, of 1 m^2 is protected against water hammer by a surge shaft at the turbine, with a cross-sectional area, A_2, of 2 m^2 and an unrestricted orifice. The initial velocity in the conduit is 1 m/s and there is a sudden complete load rejection at the turbine. Calculate the maximum rise in water level in the surge shaft neglecting friction.

Take Δt = 1 sec. Then from Equ. 8.1, $\Delta v = -gh\Delta t/L = -9.8h/100 = -0.098h$. By continuity, $\Delta h = A_1 v \Delta t/A_2 = 1v/2 = 0.5v$.

t	Δh=0.5v	h	Δv= -0.098h	v
0-1	0.5	0.5	-0.049	0.951
1-2	0.476	0.976	-0.096	0.855
2-3	0.428	1.404	-0.138	0.717
3-4	0.359	1.763	-0.173	0.544
4-5	0.272	2.035	-0.199	0.345
5-6	0.172	2.207	-0.216	0.129
6-7	0.064	2.271*	-0.223	-0.094

The maximum rise is 2.27 m, which may be compared with the analytical solution of 2.26 m. The accuracy of the numerical method could be improved by taking smaller time intervals or taking the mean v and h over the time intervals to calculate Δh and Δv respectively. The method can readily be extended to include head losses, and is calculator-orientated.

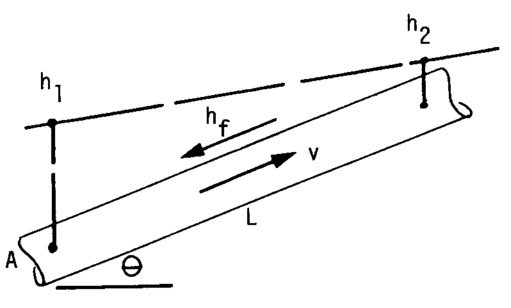


Fig. 8.1 Derivation of rigid column equation of motion

DERIVATION OF BASIC EQUATION

Net body force along pipeline $= wAL \sin \theta$

$$= wA(h_2-h_1) \quad (8.2)$$

Force = mass x acceleration

$$w(h_2-h_1-h_f)A = -(w/g)AL\ dv/dt \quad (8.3)$$

$$\therefore\ (h_2-h_1) = h_f - (L/g)dv/dt \quad (8.4)$$

SOLUTION OF EQUATION OF MOTION

The equation can be solved analytically in some cases, or by graphical means (Jaeger, 1956) or numerically (manually or by computer).

Only the simplest of surge systems (constant conduit cross-section and no friction) can be studied analytically. That is the relationships between velocity and amplitude and time can be derived in algebraic form. Consider as an example the simple U-tube in Fig. 8.2 which is disturbed by forcing the liquid up one leg to start. The equation of motion from (8.1) is

$$\frac{dv}{dt} = \frac{-2g}{L}y \quad (8.5)$$

and since $v = \frac{dy}{dt}, \frac{dv}{dt} = \frac{d^2y}{dt^2}$

Integrating twice with respect to t gives $y = y_{max}\cos(t\sqrt{2g/L})$

(8.6)

where the constants of integration are from $y=y_{max}$ at t=0, and $dy/dt = 0$ when t=0.

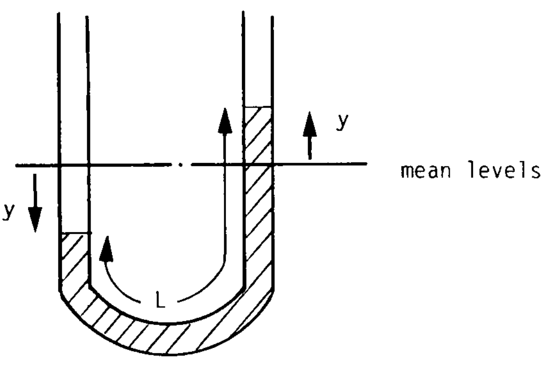


Fig. 8.2 U-tube

The oscillations obviously repeat every $2\pi\sqrt{L/2g}$ which is the period. (8.7)

The velocity is $dy/dt = -y_{max}\sqrt{2g/L}\,\sin\,(t\sqrt{2g/L})$ (8.8)

and $v_{max} = -y_{max}\sqrt{2g/L}$ (8.9)

For the case of a conduit of area A_t leading from a reservoir with constant level to a surge shaft with cross-sectional area A_s, the relationship is slightly different.

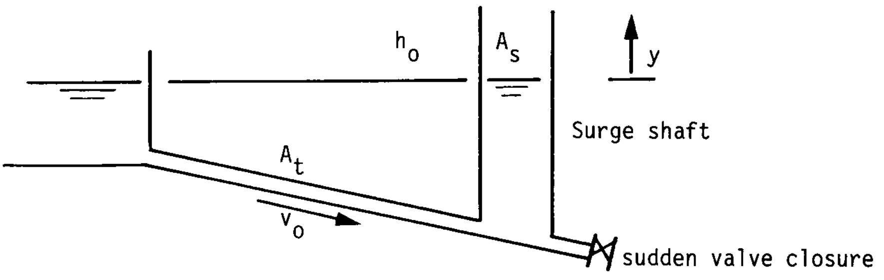


Fig. 8.3 Simple surge shaft

It is customary to neglect the inertia of the water in the surge shaft. Then for the flow in the tunnel,

Dynamics: $\frac{L}{g}\frac{dv}{dt} + y = 0$ (8.10)

Continuity: $vA_t = A_s dy/dt$ (8.11)

$$\therefore \frac{dv}{dt} = \frac{A_s}{A_t}\frac{d^2y}{d_t^{\ 2}} \quad (8.12)$$

$$\therefore \frac{L}{g}\frac{A_s}{A_t}\frac{d^2y}{dt^2} + y = 0 \quad (8.13)$$

The general solution to this is $y=a\cos 2\pi t/T+b\sin 2\pi t/T$ (8.14)

where $T = 2\pi\sqrt{LA_s/gA_t}$ (8.15)

If at t=0, y=0 then a=0

Also $\frac{dy}{dt} = \frac{A_t v}{A_s} = \frac{2\pi}{T}b\cos\frac{2\pi t}{T}$

If at t=0, $v=v_o$ then $b=v_o\frac{A_t}{A_s}\frac{T}{2\pi} = v_o\sqrt{\frac{L}{g}\frac{A_t}{A_s}}$ (8.16)

Then $y= v_o\sqrt{\frac{L}{g}\frac{A_t}{A_s}}\,\sin\,t\sqrt{\frac{g}{L}\frac{A_t}{A_s}}$ (8.17)

A More Precise Method for Manual Numerical Analysis

The simple numerical method illustrated should really only be applied with very small time increments. A large number of steps may therefore be involved and the method is more appropriate to digital computer solution. When manual methods are employed it is suggested the finite difference equations be written in implicit or time averaged form. Thus the average h over a time interval is used, namely $(h_2 + h_1)/2$ to calculate Δv, and the average velocity in the conduit or tunnel $(v_2 + v_1)/2$ to calculate Δh over each time interval. It is then necessary to solve the two equations (dynamic and continuity) simultaneously for h_2 and v_2. Alternatively one may work in terms of flow rate Q instead of v. The following equations are solved for h_2 and Q_2. A head loss term, h_ℓ is also introduced for throttling at the surge shaft inlet. The inlet area is A_i, and the head loss term is expressed in terms of the previous flow rate as an implicit solution would be more complicated for the quadratic term. Thus head losses must be small relative to surge rises.

Dynamic: $Q_2 = Q_1-(gA_t\Delta t/L)\{(h_2+h_1)/2 + h_f\}$ (8.18)

where $h_\ell = KQ_t|Q_t|/(2gA_i^2)$ (8.19)

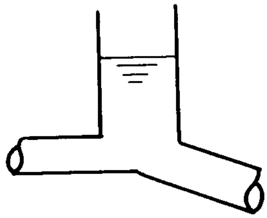
Continuity: $h_2-h_1=(Q_2+Q_1)\Delta t/2A_s$ (8.20)

Solving for Q_2 and h_2:

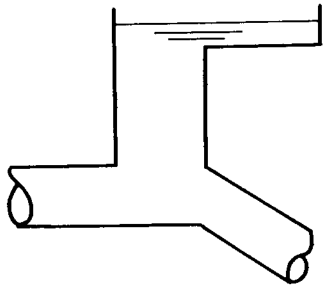
$$Q_2 = \frac{Q_1-(gA_t\Delta t/L)\{h_1+(Q_1\Delta t/4A_s)+KQ_1|Q_1|/(2gA_i^2)\}}{1+gA_t\Delta t^2/4LA_s} \qquad (8.21)$$

and $h_2 = h_1+(Q_2+Q_1)\Delta t/(2A_s)$ (8.22)

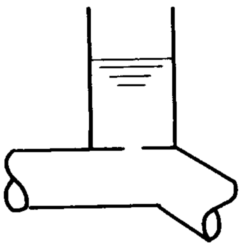
Another useful application of the rigid water column equation is with water column separation. Following the stopping of a pump at the upstream end of a pumping line, the pressure frequently drops sufficiently to cause vaporization at peaks along the line. In such cases the water column beyond the vapour pocket will decelerate slowly and rigid column theory is sufficiently accurate for analysis. Equ. 8.1 may be integrated twice with respect to time t to determine the distance the water column will travel before stopping.



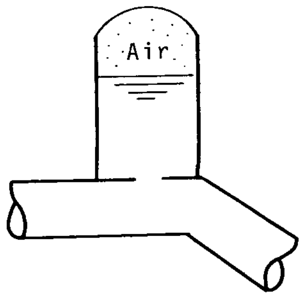
Simple open surge tank



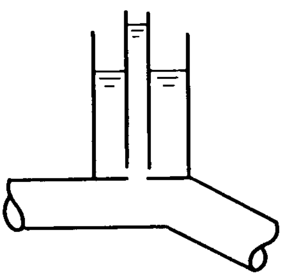
Variable area surge tank



Throttled inlet/outlet



Closed surge tank



Differential area surge tank

Fig. 8.4 Types of surge tanks

If the pumps stop instantaneously the volume of the vapour pocket behind the water column of length ℓ will be $Q=A\ell v_o^2/2gh$ where v_o is the initial flow velocity.

SURGE TANKS

By breaking a length of closed conduit with a free water surface, the water hammer pressure and surge amplitude can be reduced considerably. The use of surge tanks for this purpose is common in hydroelectric installations with tunnels but not encountered regularly in pumped or pipe systems where the pressures are greater and consequently surge tank heights would be excessive. Some shapes of surge tanks are indicated in Figure 8.4. Apart from the throttled and closed surge tanks, the hydraulic calculations, namely water level variations and pressures, can be performed analytically or numerically employing two equations. One is the dynamic equation (8.1) and the other is the continuity equation,

$$v_1A_1 = v_3A_3 = \dot{v}_2A_2 = A_2dy/dt \qquad (8.23)$$

where v is velocity, A is cross-sectional area, y is water depth, t is time, subscript 1 refers to the conduit and 2 to the open surge shaft and 3 to the inlet.

In the case of throttles, the head in the conduit could rise higher than the water level in the surge shaft by the head loss through the throttle. The loss through the restriction could be represented by $h_\ell = K_{i/o}\, v_3|v_3|/2g$ where $K_{i/o}$ could vary depending on whether the flow is into or out of the opening. Generally the throttle serves to reduce the water level fluctuations in the surge shaft and since the head loss is out of phase with the level variations (see Figure 8.5) it will not increase the maximum head in the conduit if not excessive.

Approximations to the damped surges can be obtained by analytical methods (Pickford, 1969).

Alternative methods of analysis of surge shafts are graphically (Jaeger, 1956 and 1977) and with the aid of charts (Rich, 1963).

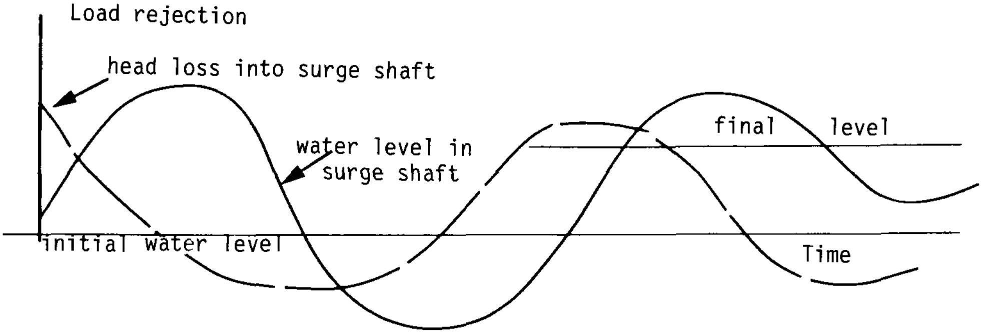


Fig. 8.5 Surge in throttled surge shaft

Example 2

Numerical Analysis of Penstock Protected with an Air Chamber

From Boyles law $H_oS_o = HS$ (8.24)

assuming isothermal expansion

where head H is absolute i.e. + atmospheric head

Continuity: $\Delta S = vA\Delta t$ (8.25)

Dynamics: $\Delta v = \frac{-g}{\ell} H\Delta t$ (rigid column equ.) (8.26)

$L = 1000m$, $H_o = 30m = 40m$ absolute, $v_o = 1.5m/s$, $A = 0.2m^2$,

$S_o = 1m^3$, $\Delta t = .5s$

t	$H = \frac{40}{S}$	$\Delta v = -.005H$	v	$\Delta S = -0.1v$	S
0-.5	30+10=40	-.2	1.3	-.13	.87
.5-1	46.0	-.23	1.07	-.11	.76
-1.5	52.6	-.26	.81	-.08	.68
-2	58.8	-.29	.52	-.05	.63
-2.5	63.5	-.31	.21	-.02	.61
-3	65.6*	-.33	-.12	.01	.62
-3.5	64.5	-.32	-.44	.04	.68
-4					

Max H = 65.6 - 10 = 55.6m.

This analysis is not particularly accurate as accelerations were too great to permit accuracy with rigid column theory. It nevertheless demonstrates use of the technique.

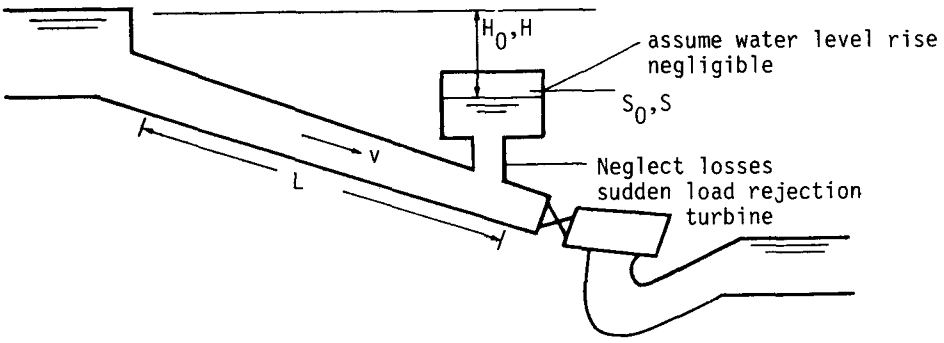


Fig. 8.6 Numerical analysis of penstock protected with an air chamber

REFERENCES

Jaeger, C., 1956. Engineering Fluid Mechanics. Blackie & Son Ltd. London. 529 pp.
Jaeger, C., 1977. Fluid Transients in Hydro-Electric Engineering Practice. Blackie & Son Ltd. London. 413 pp.
Pickford, J., 1969. Analysis of Surge, Macmillan. 203 pp.
Rich, G.R., 1963. Hydraulic Transients, Dover, N.Y. 409 pp.

CHAPTER 9

WATER HAMMER THEORY

BASIC WATER HAMMER EQUATIONS

The fundamental wave equations describing the phenomenon of water hammer may be arrived at from consideration of conservation of momentum and of mass. The following derivation is for the general case of a pipe inclined at any angle to the horizontal and friction head loss varying with the square of the velocity. The notation used is given in Figure 9.1.

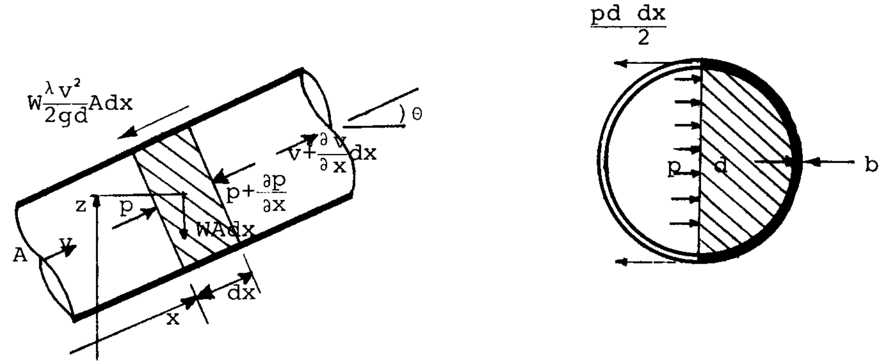


Fig. 9.1 Longitudinal and cross sections through pipe

Conservation of Momentum

The following equation is derived from a force-momentum balance, but it is possible to derive it starting from Bernoulli's energy equation. Newton's Second Law of motion is applied to an element of fluid Adx moving in the x- direction (see Figure 9.1). The resultant force in the x-direction equals the rate of change of momentum in that direction:

$$pA - (p+\frac{\partial p}{\partial x}dx)A - WA\,\frac{\lambda v^2 dx}{2gd} - WAdx\ \sin\theta = \frac{W}{g}Adx\frac{dv}{dt} \qquad (9.1)$$

and since $\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}v$, (9.2) the equation becomes

$$\frac{\partial p}{\partial x} + \frac{W}{g} v \frac{\partial v}{\partial x} + \frac{W}{g} \frac{\partial v}{\partial t} + W \sin \theta + W\frac{\lambda v^2}{2gd} = 0. \qquad (9.3)$$

$$\text{Put} \quad h = \frac{p}{W} + z + \frac{v^2}{2g} \qquad (9.4)$$

$$\text{then} \quad W\frac{\partial h}{\partial x} = \frac{\partial p}{\partial x} + W\frac{\partial z}{\partial x} + \frac{Wv}{g} \frac{\partial v}{\partial x} \qquad (9.5)$$

$$\text{where} \quad \frac{\partial z}{\partial x} = \sin \theta. \qquad (9.6)$$

Hence the momentum equation becomes

$$\frac{\partial h}{\partial x} + \frac{1}{g} \frac{\partial v}{\partial t} + \frac{\lambda v^2}{2gd} = 0 \qquad (9.7)$$

To account for the directional change in head loss with velocity one can write $v|v|$ instead of v^2:

$$\frac{\partial h}{\partial x} + \frac{1}{g} \frac{\partial v}{\partial t} + \frac{\lambda v|v|}{2gd} = 0 \qquad (9.8)$$

In this equation the velocity head $v^2/2g$ has been included in the total head h. One can take h as the piezometric head $p/W + z$ and write the momentum equation thus:

$$\frac{\partial h}{\partial x} + \frac{1}{g} \left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) + \frac{\lambda v^2}{2gd} = 0 \qquad (9.9)$$

Generally the term $v\partial v / \partial x$ is small compared with $\partial v/\partial t$ and can be neglected, but it can be accounted for in numerical solutions if necessary, e.g. in flexible plastic piping.

Conservation of Mass

The second differential equation arises from continuity. The difference between the rate of mass inflow to and outflow from an elemental length of pipe is equated to the rate of increase of storage caused by elastic expansion of the pipe and elastic compression of the water.

The rate of stress increase in the pipe wall is $(d/2b)\partial p/\partial t$
The corresponding rate of strain is $(d/2bE)\partial p/\partial t$
and the rate of increase in diameter is $(d^2/2bE)\partial p/\partial t$

$$\text{Now} \quad A = \pi d^2/4 \qquad (9.10)$$

$$\therefore \quad \frac{\partial A}{\partial t} = \frac{\pi d}{2} \frac{\partial d}{\partial t} \qquad (9.11)$$

$$= \frac{\pi d}{2} \frac{d^2}{2bE} \frac{\partial p}{\partial t} \tag{9.12}$$

The rate of increase in volume over a length dx is given by

$$\frac{\partial s_1}{\partial t} = \frac{\partial A}{\partial t} dx \tag{9.13}$$

$$= \frac{\pi d^3 dx}{4bE} \frac{\partial p}{\partial t} \tag{9.14}$$

Longitudinal strains have been neglected here. It has also been assumed that longitudinal stress which would affect the lateral expansion due to the Poisson's ratio effect is insignificant although they can be included if desired.

The rate of increase in storage caused by elastic compression of water is

$$\frac{\partial s_2}{\partial t} = \frac{\pi d^2}{4K} dx \frac{\partial p}{\partial t} \tag{9.15}$$

Equating inflow minus outflow to rate of change of storage,

$$vA - (v + \frac{\partial v}{\partial x}dx)A = (\frac{\pi d^2 dx}{4K} + \frac{\pi d^3 dx}{4bE}) \frac{\partial p}{\partial t} \tag{9.16}$$

i.e. $$\frac{\partial v}{\partial x} + (\frac{1}{K} + \frac{d}{bE}) \frac{\partial p}{\partial t} = 0 \tag{9.17}$$

Again, if we put $h = p/W$ and recall $\partial z/\partial t = 0$, we get

$$\frac{\partial v}{\partial x} + \frac{g}{c^2} \frac{\partial h}{\partial t} = 0 \tag{9.18}$$

where $c = 1/\sqrt{\rho(\frac{1}{K} + \frac{d}{bE})}$

The basic differential water hammer equations, including a friction term, thus become

$$\frac{\partial h}{\partial x} + \frac{1}{g} \frac{\partial v}{\partial t} + \frac{\lambda v|v|}{2gd} = 0 \tag{9.19}$$

and

$$\frac{\partial h}{\partial t} + \frac{c^2}{g} \frac{\partial v}{\partial x} = 0 \tag{9.20}$$

Omitting the friction term the equations become

$$\frac{\partial h}{\partial x} + \frac{1}{g}\frac{\partial v}{\partial t} = 0 \tag{9.21}$$

$$\frac{\partial h}{\partial t} + \frac{c^2}{g} \frac{\partial v}{\partial x} = 0 \tag{9.22}$$

The general solution to these equations is

$$h = h_o + f_1(t - \frac{x}{c}) + f_2(t + \frac{x}{c}) \quad (9.23)$$

$$v = v_o + \frac{g}{c}f_1(t - \frac{x}{c}) + \frac{g}{c}f_2(t + \frac{x}{c}) \quad (9.24)$$

which indicates that pressure and velocity changes are propagated at speed $\pm c$ along the pipe. c is referred to as the water hammer wave celerity.

Where longitudinal expansion is allowed for a more accurate expression for c is

$$c = 1/\sqrt{(\frac{1}{K} + \frac{kd}{bE})} \quad (9.25)$$

where $k = \frac{5}{4} - \mu$ for pipe supported at one end only

$= 1 - \mu^2$ for both ends fixed

$= 1$ for a pipe with expansion joints

μ is the Poisson ratio which for steel is 0.3

E is the elastic modulus, (210 000 N/mm^2 for steel),

K is the bulk modulus of fluid, (2100 N/mm^2 for water),

ρ is the fluid mass density, (1000 kg/m^3 for water),

d and b are the pipe diameter and wall thickness respectively, h is total head, x is distance along pipe, g is gravitational acceleration (9.81 m/s^2), t is time, v is flow velocity, and λ is the Darcy-Weisbach friction coefficient (λ in British practice and f in U.S. practice). λ is actually a function of the flow velocity as indicated by the Colebrook-White equation or a Moody diagram. It is also affected by unsteady motion but no quantitative assessment of this is available

EFFECT OF AIR

The presence of free air in pipelines can reduce the severity of water hammer considerably. Fox (1977) indicates that the celerity (speed) of an elastic wave with free air is

$$c = \frac{1}{\sqrt{\rho\,(\frac{1}{K} + \frac{d}{bE} + \frac{f}{p})}} \quad (9.26)$$

For large air contents this reduces to $c = \sqrt{gh/f}$ (9.27)

where p is the absolute pressure and f is the free gas fraction by volume.

c is reduced remarkably for even relatively low gas contents. Thus 2% of air at a pressure head of 50 m of water reduces the celerity from about 1100 m/s for a typical steel pipeline to 160 m/s. The Joukowsky water hammer head is

$$\Delta h = \frac{-c}{g} \Delta v \qquad (9.28)$$

where Δv is the change in velocity of flow. There is thus a large reduction in water hammer head h for a relatively small fraction of air. If the air collects at the top of the pipe there is no reason to see why the same equation cannot apply. Stephenson (1967) on the other hand derived an equation for the celerity of a bore in a partly full pipe. The celerity derived from momentum principles is for small air proportions

$$c = \sqrt{g\Delta h/f} \qquad (9.29)$$

where Δh is the head rise behind the bore. This indicates a celerity of 158 m/s for f = 0.02 and h = 50 m.

There is a school of thought which favours the installation of air valves in pipelines as a means of reducing water hammer overpressures. The intention is primarily to cushion the impact of approaching columns. Calculations will, however, indicate that an excessively large volume of air is required to produce any significant reduction in head. The idea stems from the use of air vessels to alleviate water hammer in pipelines. It will be realized that air in air vessels is under high pressure initially and therefore occupies a relatively small volume. Upon pressure reduction following a pump trip, the air from an air vessel expands according to the equation pU^k = constant where U is the volume of air. The size of air valves to draw in the necessary volume of air at low (vacuum) pressures will be found on analysis to be excessive for large diameter pipelines.

METHODS OF ANALYSIS

A common method of analysis of pipe systems for water hammer pressures used to be graphically (Lupton,1953). Friction was assumed to be concentrated at one end of the pipe or at a few points along the line, and the water hammer equations were solved simultaneously with the valve or pump characteristics on a graph of h plotted against v, for successive time intervals. This method is now largely replaced by computers e.g. Chaudhry, 1979.

The most economical method of solution of the water hammer equations for particular systems is by digital computer. Solution is usually by the method of characteristics (Streeter and Wylie, 1967) which differs little in principle from the old graphical method or by finite differences (Stephenson, 1966). The differential water hammer equations are expressed in finite difference form and solved for successive time intervals. The conduit is divided into a number of intervals and Δt is set equal to $\Delta x/c$. The $x - t$ grid on which solution takes place is depicted in Fig. 9.2. Starting from known conditions along the pipeline at time t, one proceeds to calculate the head and velocity at each point along the line at time $t + \Delta t$.

By adding equations (9.19) and (9.20) divided by c, the partial differential terms can be replaced by total differentials and one obtains

$$\text{for } \frac{dx}{dt} = +c : \quad dh + \frac{c}{g}dv + \frac{c\,\lambda v|v|dt}{2gd} = 0 \tag{9.30}$$

$$\text{for } \frac{dx}{dt} = -c : \quad dh - \frac{c}{g}dv - \frac{c\lambda v|v|dt}{2gd} = 0 \tag{9.31}$$

Equs.(9.30) and (9.31) may be solved for h'_p and v'_p at point p at time $t + \Delta t$ in terms of known h and v at two other points q and r at time t. Thus for qp, $dh=h'_p-h_q$, $dv=v'_p-v_q$ and for rp, $dh=h'_p-h_r$ and $dv=v'_p-v_r$. The resulting equations, termed the characteristic equations, are

$$h'_p = \frac{h_q+h_r}{2} + \frac{c}{g}\,\frac{v_q-v_r}{2} + \frac{cdt\lambda}{2gd}\,\frac{v_r|v_r|-v_q|v_q|}{2} \tag{9.32}$$

$$v'_p = \frac{v_q+v_r}{2} + \frac{g}{c}\,\frac{h_q-h_r}{2} - \frac{\lambda dt}{2d}\,\frac{v_r|v_r|+v_q|v_q|}{2} \tag{9.33}$$

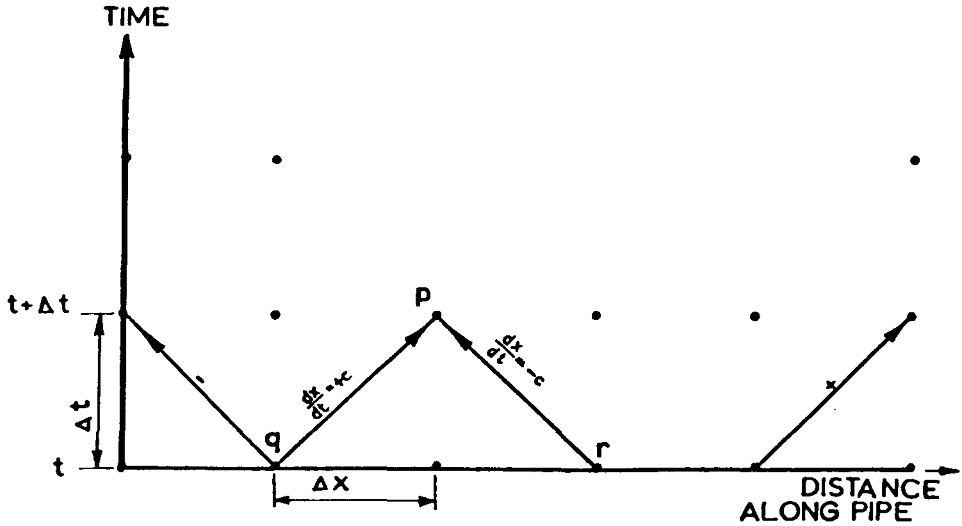


Fig. 9.2 x - t Grid for water hammer analysis by characteristics method.

At the terminal points, an additional condition is usually imposed; either h is fixed, or v is a function of a gate opening or pump speed. The correct Equ. (9.30) or (9.31) is solved simultaneously with the known condition to evaluate the new h and v at time t + Δt. The computations commence at known conditions and are terminated when the pressure fluctuations are sufficiently damped by friction.

Where a branch pipe s occurs (flow out of p to s) or there is a change in diameter, then Equs. (9.32) to (9.33) should be replaced by Equs. (9.34) to (9.37):

$$h'_p = [h_qA_q+h_rA_r+h_sA_s+(c/g)(Q_q-Q_r-Q_s)-(c\lambda dt/2g).$$
$$(Q_q|Q_q|/d_qA_q-Q_r|Q_r|/d_rA_r-Q_s|Q_s|/d_sA_s)]/(A_q+A_r+A_s) \quad (9.34)$$

then $$Q'_{qp} = Q_q+(A_qg/c)(h_q-h'_p)-\lambda Q_q|Q_q|dt/2d_q\ A_q \quad (9.35)$$

$$Q'_{pr} = Q_r+(A_rg/c)(h'_p-h_r)-\lambda Q_r|Q_r|dt/2d_r\ A_r \quad (9.36)$$

$$Q'_{ps} = Q_s+(A_sg/c)(h'_p-h_s)-\lambda Q_s|Q_s|dt/2d_s\ A_s \quad (9.37)$$

Where Q_{pr} is flow out of p to r etc. and Q_q is the flow out of q to p etc.

It should be noted that the finite difference form of the equations above is termed explicit since head losses are expressed in terms of the velocities at the previous time interval. Where head losses are significant compared with the water hammer heads, an implicit solution may be necessary (Chaudhry and Yevjevich, 1981). The latter is, however, more complicated as the equations involve more unknowns and simultaneous solutions of equations for every point in space are necessary. The equations also become non-linear. A method of overcoming these problems is explained in Chapter 12.

VALVES

At a valve or other constriction it is necessary to solve one of the characteristic equations and the valve discharge equation simultaneously. A valve acts in effect like a constriction which increases head losses. One may therefore enquire why the head loss cannot be treated as for friction head loss. It will be recalled that friction head loss was assumed to be a function of velocity at the previous point and previous time interval. Unfortunately this method becomes increasingly inaccurate (and unstable) for increasing head losses and an explicit (new time) function is generally required for the head loss at a constriction. (It is however exceedingly laborious to account for a new velocity head loss at a pipe branch, so in the case of network programs a weighted pseudo-implicit method has been employed).

The discharge characteristic of a valve can be expressed in the following way:

$$Q = C_d FA\sqrt{H} \tag{9.38}$$

where Q is the flow in the valve, H is the head loss across the valve and C_d is a discharge coefficient. F is the fractional opening of the valve (0 is closed and 1 is fully open). The discharge coefficient is often a function of F unfortunately but this will be accounted for here by assuming the only variable is F.

The degree of valve opening as a function of time in the case of uniform stem travel or proportional turns of a handwheel is generally non-linear. For most valves the hole closes more rapidly near the end of the travel of the gate. The following figure illustrates the proportional area open for different valves as a function of time assuming steady turning of the handwheel or actuator, (the actuator can be controlled to close non-linearly if desired and this will be discussed under chapter 12.

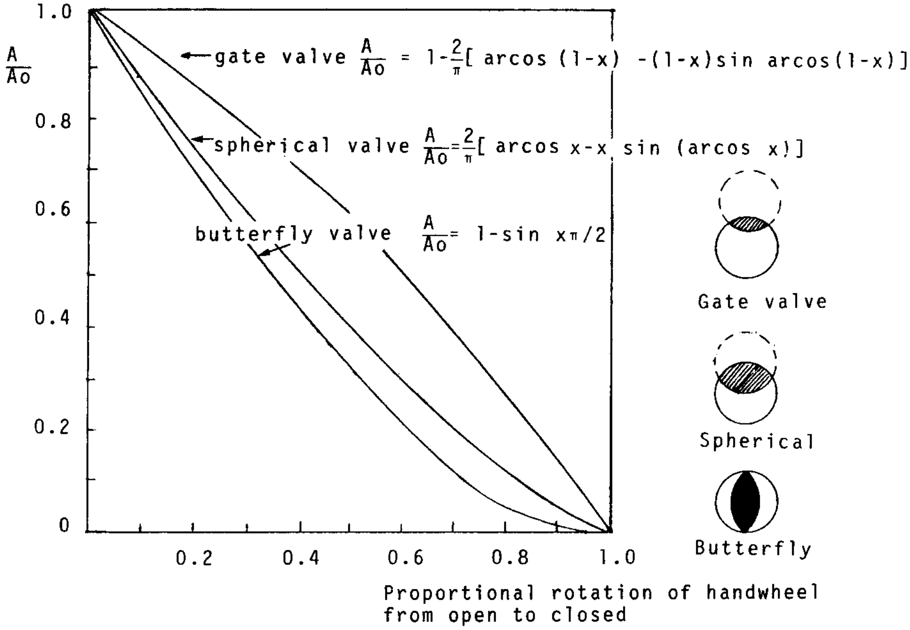


Fig. 9.3 Proportional areas for some valves

For gate valves the F - T relationship can be approximated by a parabolic function:

$$F = 1 - (t/T)^2 \qquad (9.39)$$

In general the head loss function can be written

$$Q = \{1-(t/T)^N\} A_p \sqrt{(H/Ho)} \qquad (9.40)$$

where Ho = initial H/V_o^2 i.e. initial head loss through fully open valve divided by initial velocity squared. A_p is the pipe area. Solving this and the characteristic equation at the downstream end of a pipe

$$dh + \frac{c}{g}dv + \frac{\lambda v|v|dx}{2gd} = 0 \qquad (9.30)$$

yields

$$Q = \{\sqrt{\left(\frac{c}{gA_p}\right)^2 - 4\,\frac{\left(\frac{\lambda Q_1|Q_1|dx}{2gdA_p^2} - \frac{cQ_1}{gA_p} - h + Z\right)}{(A_p[1-(t/T)^N])^2/Ho}} - \frac{c}{gA_p}\}\,\{\frac{A_p[1-(t/T)^N])^2}{Ho}\}/2 \qquad (9.41)$$

$$\text{and } h = \frac{Q^2}{(A_p[1-(t/T)^N])^2/Ho} + Z \qquad (9.42)$$

where Z is the downstream head, H is the head above Z and h is the total head. Subscript 1 refers to the upstream point at the previous time interval, and p to the pipe upstream of the valve. t is time, T is the closure time of a valve, t/T is the proportional operation of the handwheel assuming a constant rate of operation and A is the open area.

ACCURACY AND STABILITY OF FINITE DIFFERENCE SCHEMES

The water hammer equations are non-linear partial differential equations. As they cannot be solved analytically for most cases, numerical techniques are employed. Most existing methods can be classified as follows:

a) explicit finite difference methods
b) implicit finite difference methods
c) finite element methods.

The implicit method of solution is a method whereby a simultaneous solution of all the flow properties is obtained by solving a matrix; its main advantage is that the ratio of space to time interval, $\Delta x/\Delta t$, is not governed by any stability criteria and the method is considered to be stable for any choice of Δx and Δt. Most previous investigators considered this to be an advantage. Others, however, found that it is not always possible to make practical use of this "advantage" as for high ratios of $\Delta x/\Delta t$ inaccuracy in the results was high. Implicit methods are also not convenient for use as one cannot keep track of results at different time periods.

Finite element methods are usually avoided as they are expensive to run and accuracy and stability criteria can be tedious.

Explicit finite difference schemes have been widely used in the past for the solution of different non-linear partial differential equations. They differ from each other in the way they define the variable gradients, but they all express the flow properties at a certain time as a function of the flow properties at a previous time, thus permitting an explicit solution. They are simple to use as they use a fixed regular grid and it is easier to follow the variation of the flow properties along the catchment as the solution is performed explicitly. They have been found to be accurate and economical when properly used. The problems accompanying the choice and the use of an explicit finite difference scheme are, however, those of accuracy and stability. Choosing the most proper scheme and using it accordingly is, therefore, important in obtaining stable and accurate results.

Basic Terms Related to Accuracy and Stability for Difference Schemes

Many natural systems which are continuous functions can be described by differential equations. If the differential equations cannot be solved mathematically one usually resorts to numerical techniques by approximating the differential equations to a computational algorithm using difference schemes. This procedure raises two basic questions. "How well is the natural system modelled by the differential equations?", and "How well is the solution to the *differential equations represented by the computational algorithm?"* In the analysis to follow more attention is paid to the second question. The first question can only be answered by studying the behaviour of the natural system and comparing it to the equations applied to it. Therefore it will be assumed here that the differential equations approximate the system well. The difference between the differential equations and the difference scheme approximating them is called a Truncation error (Tr),

i.e. Differential equations = Difference scheme + Tr (9.43)

The truncation error can readily be established using Taylor's expansion. There is also a difference in the solutions of the two schemes which one calls the Error (E), i.e. Solution of Differential equations = Solutions of Difference equations + E (9.44)

The exact value of the Error cannot really be obtained in this case as the differential equations cannot be solved analytically. We say that a difference scheme is consistent with a set of Differential equations if the Truncation error tends to zero as the space and time increments tend to zero,

i.e. Consistent if limit Tr = 0

$$\text{as } \Delta x, \Delta t \rightarrow 0 \qquad (9.45)$$

We say that the solution of the difference scheme is convergent with the solution of the differential equations if the Error tends to zero as the space and time increments tend to zero,

i.e. Convergent if limit E = 0

$$\text{as } \Delta x, \Delta t \rightarrow 0 \qquad (9.46)$$

Numerical Diffusion is the process in which the Error (E) is formed. It is the development of the truncation error (Tr) to the error (E) through the numerical technique used. It generally manifests in the form of an attenuation and spreading of wave fronts. If computations at points distance Δx apart are at time intervals Δt then numerical diffusion will proceed through the system at a rate $\Delta x/\Delta t$.

Stability and Accuracy Criteria for an Explicit Finite Difference Scheme

Since one is dealing with non-linear partial differential equations (p.d.e's) there is no rigorous proof specifying stability criteria.

In the solution of a non-linear set of p.d.e's like the Water Hammer equations it was found that both stability and accuracy are influenced by the values chosen for the space increment (Δx) and the time increment (Δt). In particular, a critical ratio of $\Delta x/\Delta t$ i.e. $(\Delta x/\Delta t)_{cr}$., exists for determining whether a scheme will run under stable conditions or not. The effects of Δx and Δt on stability and accuracy are summarized in Figure (9.4).

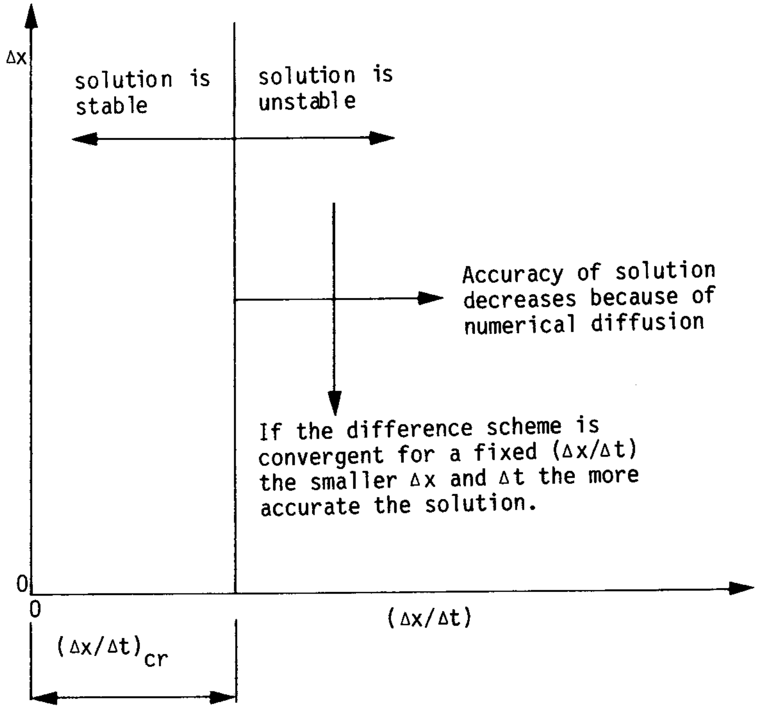


Fig. 9.4 Effect of values on Δx and Δt on stability and accuracy for an explicit finite difference scheme

From Figure (9.4) one can deduce that the main criteria in the selection of Δx and Δt values for an explicit finite difference scheme are:

a) that the scheme shall proceed under stable conditions i.e.

$$\frac{\Delta x}{\Delta t} \geq \left(\frac{\Delta x}{\Delta t}\right)_{cr} \tag{9.47}$$

b) $\frac{\Delta x}{\Delta t}$ shall be close to $\left(\frac{\Delta x}{\Delta t}\right)_{cr}$ to minimize diffusion errors and obtain optimal accuracy.

c) The difference scheme shall be convergent. This could be ascertained by running the scheme with different Δx's and Δt's and comparing with analytical results in a simple case.

Determining $(\Delta x/\Delta t)_{cr}$

$(\Delta x/\Delta t)_{cr}$ has been shown to be the speed of wave as it is propagated. This can be demonstrated by considering the method of characteristics

The method of characteristics describes flow in the form of waves travelling along or against the flow at a specific velocity, dx/dt. The family of curves described by dx/dt in the x - t plane are called the characteristics. The flow properties, velocity and total depth, in the case of the water hammer equations are described by relationships obtained from the wave equations (1 and 2) using the relationships of dx/dt. In other words, the relationships derived describe the flow properties as seen by an observer travelling along the flow at a velocity defined by the characteristics.

In the case of the water hammer equations, a wave is caused by a velocity gradient, i.e. change in velocity anywhere in the flow, e.g. shutting of valves etc. This wave is propagated with or against the flow at a velocity of $\pm$ c as given by the equation

$$dx/dt = \pm c \qquad (9.48)$$

As the wave travels it propagates information about the velocity gradient to different points in the conduit. The concept of information propagation by the wave in time and space can be illustrated by considering the characteristic curves defined by equation (9.48). This is illustrated in Figure (9.5)

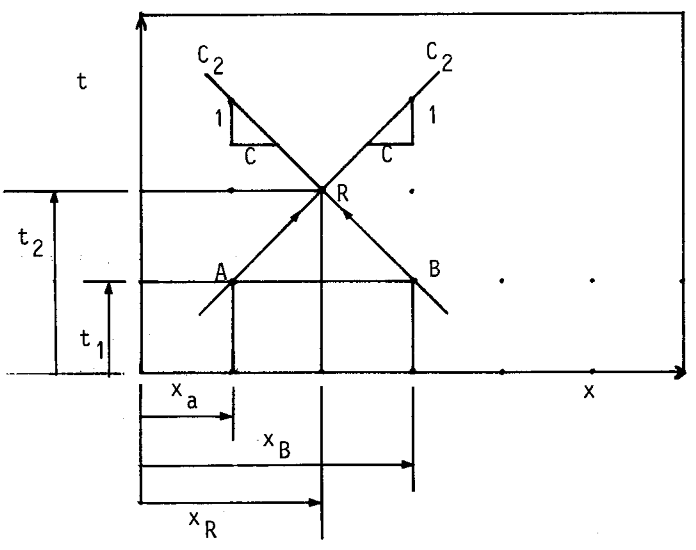


Fig. 9.5 Propagation of information along characteristics of the water hammer equations

In Figure 9.5, C1 and C2 are a set of characteristics described by equation (9.48). Suppose that the flow properties, total head and velocity are known at points A and B, x_A and x_B distances from the origin at time t_1. One can then obtain the flow properties at a point R which lies on the same characteristics as points A and B, x_R distance from the origin at time t_z.

The critical space to time interval ratio, $(\Delta x/\Delta t)_{cr}$, can be shown to be the wave speed by considering a central difference scheme for solving the water hammer equations. The scheme is illustrated in Figure 9.6.

Let i represent a space interval, and k represent a time interval as shown in Figure 9.6. The point in question, i.e. where the flow properties are to be calculated, has the co-ordinates (i, k). Information about the flow properties is sought from the previous time interval. In Figure 9.6(a) the true propagation speed is smaller than the numerical propagation speed while in Figure 9.6(b) the converse is true. Numerical propagation lines are lines that have a slope $\Delta x/\Delta t$ in the x-t plane while true propagation lines have a slope dx/dt in the x-t plane. In Figure 9.6(a) information is obtained within the i-l, i+l range by the true propagation lines. In Figure 9.6(b) information is sought by the true propagation lines outside the i-l, i+l range. Since information outside this range is not propagated by the numerical scheme, it cannot be found and thus instability will result.

For stability of an explicit finite difference scheme the following must, therefore, hold:

$$\frac{\Delta x}{\Delta t} \geq \frac{dx}{dt} \qquad (9.49)$$

This is referred to as the "CFL condition" after Courant, Friedrichs and Lewy (1956), or simply the Courant criterion for stability.

To minimize diffusion errors as mentioned earlier $\Delta x/\Delta t$ must be as close as possible to dx/dt. Using the result in equation (9.48), $\Delta x/\Delta t$ is chosen to be equal to the wave celerity, i.e.

$$\frac{\Delta x}{\Delta t} = c \qquad (9.50)$$

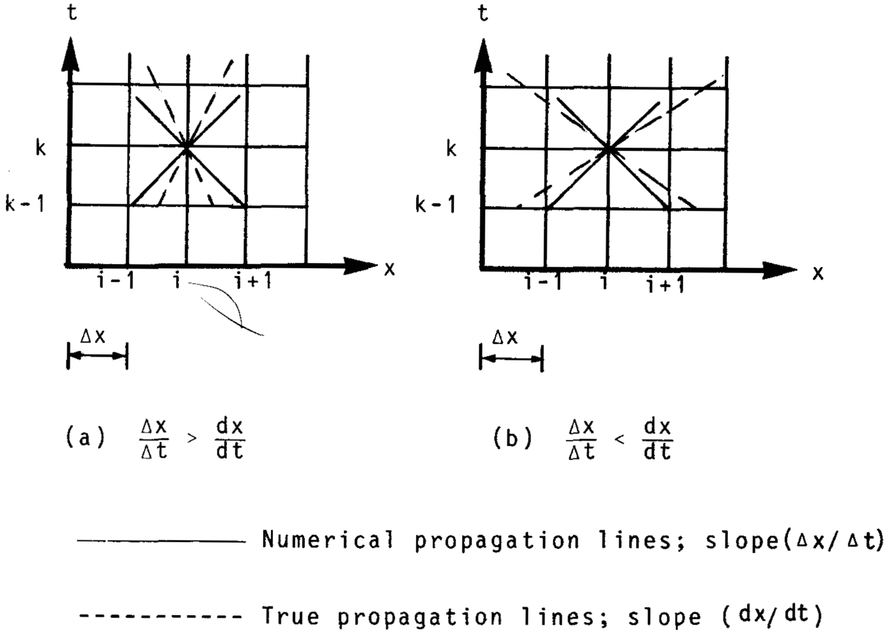


Fig. 9.6 Comparison of numerical and theoretical propagation of information in a central difference scheme

Equation (9.50) is used to define the space and time interval. The relationship ensures that the difference scheme is run under stable conditions with no diffusion errors. The solution of the equation is further made more accurate by the choice of smaller Δx and at Δt intervals as can be seen from Figure (9.4).

BASIC COMPUTER PROGRAM FOR ANALYSING GRAVITY LINE WITH ONE DRAWOFF ALONG THE LINE, VARIABLE PIPE DIAMETER, NUMBER OF INTERVALS AND CLOSURE TIME OF DOWNSTREAM GATE VALVE.

Input is as follows:

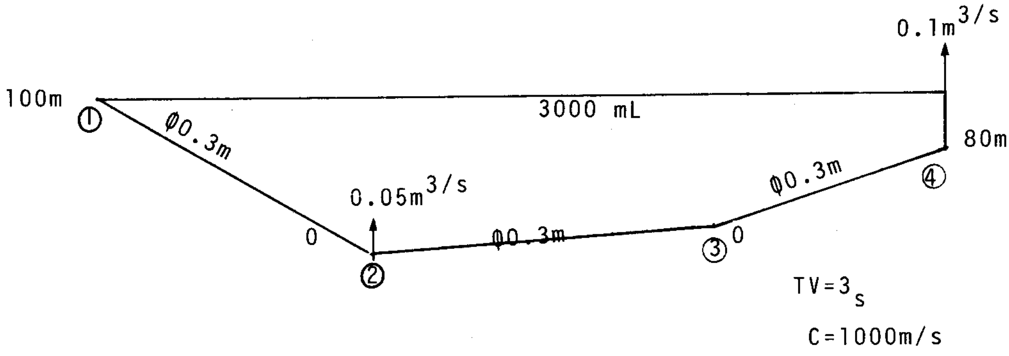
Line 1;	Title
Line 2:	Pipelength, m; Celerity m/s; Valve closure time, s; Number of pipe divisions; Endflow m^3/s; Number of iterations (in 1 iteration wave travels 1 pipe division so $\Delta t= \Delta L/C$); Point at which draw-off occurs.
Line 3;	Drawoff m^3/s (put 0 if none).
Line 4 etc.;	Elevations in m of each point (first one being the fixed reservoir level), (one per line).
Later lines;	Diameters of each pipe division in m (one per line).

```
 10 ! WATER HAMMER GRAVITY LINE
    WITH VALVE CLOSURE AT DS END
 20 DIM Z(21),H(21),D(21),S(21),
    A(21),Q(21),P(21),R(21)
 30 DISP "PIPE NAME";
 40 INPUT N$
 50 G=9.8 ! 32.2 IF FT-S UNITS
 60 DISP "Lm,Cm/s,TVALVEs,NINTS,
    QVm3/s,ITS,DRAWOFFPT";
 70 INPUT X,C,T,N1,Q1,I1,M1
 80 N2=N1+1
 90 DISP "DRAWOFF m^3/s";
100 FOR J=1 TO N2
110 P(J)=0
120 NEXT J
130 INPUT P(M1)
140 DISP "ELEVATIONSm1 BY 1-",N2
    ;
150 FOR J=1 TO N2
160 INPUT Z(J
170 NEXT J
180 H(1)=Z(1)
190 H1=1 ! HL/V0^2 THRU OPN VALV
200 V1=2 ! GATE VALV=2,BV=.5,NV=
    1
210 D1=X/N1
220 D2=D1/C
230 DISP "DIASm1 BY 1-",N1
240 FOR J=1 TO N1
250 INPUT D(J)
260 NEXT J
270 PRINT "WATER HAMMER DT VALVE
     CLOSURE ",N$
280 M2=M1-1
290 FOR J=1 TO M1-1
300 Q(J)=Q1+P(M1)
310 NEXT J
320 FOR J=M1 TO N2
330 Q(J)=Q1
340 NEXT J
350 S1=0
360 FOR J=1 TO N1
370 A(J)=.785*D(J)^2
380 S1=S1+Q(J)^2/D(J)^5
390 NEXT J
400 D(N2)=D(N1)
410 A(N2)=A(N1)
420 H(N2)=Z(N2)+H1*(Q1/A(N1))^2
430 F=(Z(1)-H(N2))*2*G*.785^2/D1
    /S1
440 FOR J2=2 TO N1
450 H(N1+2-J2)=H(N1+3-J2)+F*D1*Q
    (N1+2-J2)^2/2/G/D(N1+2-J2)/A
    (N1+2-J2)^2
460 NEXT J2
470 FOR I=1 TO I1
480 FOR J=1 TO N2
490 R(J)=Q(J)
500 S(J)=H(J)
510 NEXT J
520 FOR J=2 TO N1
530 H2=R(J-1)-R(J+1)-P(J+1)-P(J)
540 H3=-((R(J+1)+P(J+1))*ABS(R(J
    +1)+P(J+1))/A(J)/D(J)*F*D2/2
    )
550 H2=H2-F*D2/2*R(J-1)*ABS(R(J-
    1))/A(J-1)/D(J-1)-H3
560 H2=H2*C/G+(S(J-1)*A(J-1)+S(J
    +1)*A(J))
570 H(J)=H2/(A(J-1)+A(J))
```

```
580 Q2=R(J-1)+G/C*(S(J-1)-H(J))*
    A(J-1)
590 Q(J)=Q2-F*R(J-1)*ABS(R(J-1))
    *D2/2/D(J-1)/A(J-1)-P(J)
600 NEXT J
610 IF I*D2>=T THEN 680
620 T1=(A(N1)*(1-(I*D2/T)^V1))^2
    /H1
630 Q3=C*F*R(N1)*ABS(R(N1))*D2/2
    /G/D(N1)/A(N1)^2
640 Q3=Q3-C*R(N1)/G/A(N1)-S(N1)+
    Z(N2)
650 Q(N2)=(SQR((C/G/A(N1))^2-4*Q
    3/T1)-C/G/A(N1))*T1/2
660 H(N2)=Q(N2)^2/T1+Z(N2)
670 GOTO 700
680 Q(N2)=0
690 H(N2)=(R(N1)-F*D2/2*R(N1)*AB
    S(R(N1))/A(N1)/D(N1))*C/G/A(
    N1)+S(N1)
700 Q(1)=R(2)+P(2)+G/C*A(1)*(H(1
    )-S(2))-F*(R(2)+P(2))*ABS(R(
    2)+P(2))*D2/2/D(1)/A(1)
710 PRINT "NODE FLO.M3^S HEAD"
720 FOR J=1 TO N2
730 PRINT USING 740 ; J,Q(J),H(J
    )
740 IMAGE DDDD,DDD.DDD,DDDD.D
750 NEXT J
760 NEXT I
770 STOP
780 END
```

```
PIPE NAME?
SAMPLE RUN
Lm,Cm/s,TVALVEs,NINTS,QVm3/s,ITS
,DRAWOFFPT?
3000,1000,3,3,.1,20,2
DRAWOFF m^3/s?
.05
ELEVATIONSm1 BY 1-     4 ?
100
?
20
?
30
?
80
DIASm1 BY 1-           3
?
.3
?
.3
?
.3
```

```
WATER HAMMER DT VALVE CLOSURE
SAMPLE RUN
NODE FLO.M3^S HEAD
   1    .150 100.0
   2    .100  90.5
   3    .100  86.2
   4    .100  82.5
NODE FLO.M3^S HEAD
   1    .150 100.0
   2    .100  90.5
   3    .100  86.7
   4    .097  86.1
NODE FLO.M3^S HEAD
   1    .150 100.0
   2    .100  91.0
   3    .097  90.2
   4  0.000 226.5
NODE FLO.M3^S HEAD
   1    .149 100.0
   2    .097  94.4
   3    .001 228.6
   4  0.000 226.7
NODE FLO.M3^S HEAD
   1    .145 100.0
   2    .003 230.3
   3    .001 228.8
   4  0.000 230.7
NODE FLO.M3^S HEAD
   1  -.038 100.0
   2    .000 227.4
   3    .001 232.4
   4  0.000 230.9
NODE FLO.M3^S HEAD
   1  -.038 100.0
   2  -.089 101.8
   3  -.001 229.5
   4  0.000 234.1
NODE FLO.M3^S HEAD
   1  -.040 100.0
   2  -.089 101.9
   3  -.089 105.2
   4  0.000 228.1
NODE FLO.M3^S HEAD
   1  -.040 100.0
   2  -.090 100.7
   3  -.087 102.2
   4  0.000 -20.3
NODE FLO.M3^S HEAD
   1  -.040 100.0
   2  -.088  97.7
   3  -.002 -23.1
   4  0.000 -20.5
NODE FLO.M3^S HEAD
   1  -.036 100.0
   2  -.003 -24.8
   3  -.002 -23.4
   4  0.000 -25.9
NODE FLO.M3^S HEAD
   1    .133 100.0
   2  -.001 -22.1
   3  -.001 -27.6
   4  0.000 -26.3
NODE FLO.M3^S HEAD
   1    .133 100.0
   2    .082  93.0
   3    .001 -25.0
   4  0.000 -29.3
NODE FLO.M3^S HEAD
   1    .132 100.0
   2    .083  92.9
   3    .083  89.9
   4  0.000 -23.7
NODE FLO.M3^S HEAD
   1    .132 100.0
   2    .082  92.3
   3    .081  92.7
   4  0.000 206.1
NODE FLO.M3^S HEAD
   1    .132 100.0
   2    .080  95.2
   3    .001 207.2
   4  0.000 206.4
NODE FLO.M3^S HEAD
   1    .129 100.0
   2    .002 208.9
   3    .001 207.5
   4  0.000 208.3
NODE FLO.M3^S HEAD
   1  -.024 100.0
   2    .000 206.6
   3    .001 209.9
   4  0.000 208.7
NODE FLO.M3^S HEAD
   1  -.024 100.0
   2  -.075 100.6
   3  -.001 207.7
   4  0.000 211.6
NODE FLO.M3^S HEAD
   1  -.025 100.0
   2  -.075 100.7
   3  -.075 103.5
   4  0.000 206.7
```



List of symbols in gravity pipe water hammer analysis program

Symbols

A(J)	Area of pipe
C	Celerity
D(J)	Diameter of next section
D1	Increment in X
D2	Increment in T
F	Darcy friction factor in $H = FXV^2/2GD$
G	Gravitational acceleration, 9.81
H(J)	Head
H2	Dummy variable
H3	Dummy variable
H1	Head loss through valve/V^2
I1	Number of iterations
I	Iteration number
J	Node number
J2	Counter
L	Length
M3	M2-1
NS	Name
N1	Number of sections in main pipe
N2	N1 + 1
M1	J at which draw-off occurs (any node 2 to N1)
P(J)	Draw-off
Q1	Draw-off from main pipe end
Q(J)	Flow
Q3	Draw-off branch
R(J)	Q at previous time interval
S(J)	H at previous time interval
S1	Q^2/D
T	Time in seconds to close main pipe line downstream valve
T1	Dummy variable
V1	Power in valve area-closure equation
X	Length of pipe
Z(J)	Elevation

REFERENCES

Chaudhry, M.H. and Yevjevich, V. (Eds.), 1981. Closed Conduit Flow. Water Resources Publics. Colorado.

Chaudhry, M.H., 1979. Applied Hydraulic Transients, Van Nostrand Reinhold Co. 503 pp.

Courant, R., Friedrichs, K. and Lewy, H., 1956. On the Partial Differential Equations of Mathematical Physics. N.Y. Univ. Inst. Maths.

Fox, J.A., 1977. Hydraulic Analysis of Unsteady Flow in Pipe Networks. Macmillan, London.

Lupton, H.R., 1953. Graphical Analysis of Pressure Surges in Pumping Systems. J. Inst. Water Engs.

Stephenson, D., Oct. 1967. Prevention of vapour pocket collapse in a pumping line. trans., S.A. Instn. Civil Engs. 9(10) p 255-261.

Stephenson, D., Sept., 1966. Water hammer charts including fluid friction. Proc. ASCE., 92 (HY5), p71-94.

Streeter,V.L. and Wylie, E.B., 1967. Hydraulic Transients, McGraw Hill, N.Y.

Wood, D.J. and Jones, S.E., January 1973. Water hammer charts for various types of valves. Proc. ASCE 99, HY paper 9500, p 167-178.

CHAPTER 10

BOUNDARY CONDITIONS IN WATER HAMMER

DESCRIPTION

The finite difference solution of the characteristic equations starts from known conditions along the pipe at a specified time. The conditions may be steady-state flows and heads as defined by the analyst or as determined from a given friction head loss-flow relationship together with given reservoir heads at either end of the line. After the first iteration it is unnecessary to refer to the initial flow conditions at intermediate points but the end conditions should still be known. These conditions may also be specified by the user, but it is these conditions which dictate in what way the flows are to vary.

Thus if valve closure is to be the cause of water hammer, the valve opening has to be specified as well as the relationship between discharge and head loss across the valve. The latter can be given in equation form and the valve opening can also be specified as a function of time, or even controlled by the program (termed valve stroking and can be used to limit head rises) (Wylie and Streeter, 1978).

Where a pipe leads to or from a reservoir, the reservoir head is generally specified. This may be at a fixed level if the water level does not vary much, or the water level may vary as flow occurs into or out of the reservoir.

Other boundary conditions encountered included changes in pipe diameter, closed end pipes, or branches for both of which the relationship between head and flow may be determined by equations such as (9.34) to (9.37) (the cross sectional area of a pipe beyond the closed end is taken as zero).

Conditional Boundary Conditions

During the course of computations the physical limits to an assumption may be encountered. Thus if the head computed using equation (9.34) was to be below vapour head of the liquid, the liquid would vaporize. The head would be unable to drop below the vapour pressure so it should be set equal to vapour pressure and the liquid assumed to vaporize. The extent of the vaporization (the volume) could be computed by calculating the flow rate upstream and downstream of the point using equations (9.35) or (9.36). The increase in vapour volume is the difference in the two flow rates either side of the point, multiplied by the time increment. Chapter 11 elaborates on this type of problem.

It may be that the engineer imposes controls on the system to limit flow rates or pressures. Control valves can be used to limit maximum flow rates, or pressures. The use of control valves and other forms of water hammer protection, e.g. surge tanks, non-return valves and air vessels is described later.

Spring loaded release valves can be set to discharge at a specified rate when the pressure against them exceeds a preset figure. This condition can be programmed as well as the discharge-head relationship through the open valve. A further disadvantage of the rupture disc (Fig.10.1 b) is that the line could drain after it has opened, unless there is also a control valve on the Tee. Both above methods require the head to increase before they operate, and by that time it may be too late to protect other points in the pipe system. There are many control valves which operate on this direct principle e.g. Figure 10.1c. On the other hand, there are pilot operated valves which could commence opening on sensing the first downsurge, e.g. Figure 10.1e. In such cases a sub-program is required to check the operating condition of the valve.

It should be noted that pressure reducing valves, or in fact any constriction is liable to cause cavitation downstream if the cavitation number is too high. The cavitation index is generally of the form

$$K = \frac{P_d - P_v}{P_u - P_d} \qquad (10.1)$$

where P is pressure and subscript d refers to downstream, v to vapour and u to upstream. Depending on the valve design, values as low as 0.3 are possible before the onset of cavitation. If cavitation occurs, vapour bubbles and gas release may occur downstream which actually reduce the water hammer wave celerity.

This can in turn reduce the downsurge downstream if the valve is closing, which in turn provides a stabilizing characteristic to the valve. On the other hand, cavitation erosion of pipework is a troublesome phenomenon to be avoided.

WATER HAMMER PROTECTION OF PUMPING LINES

The pressure transients following power failure to electric motor driven pumps are usually the most extreme that a pumping system will experience. Nevertheless, the over-pressures caused by starting pumps should also be checked. Pumps with steep head/flow characteristics often induce high over-pressures when the power is switched on so a wave with a head equal to the closed valve head is generated. By partly closing the pump delivery valves during starting, the over-pressures can be reduced.

If the pumps supplying an unprotected pipeline are stopped suddenly, the flow will also stop. If the pipeline profile is relatively close to the hydraulic grade line, the sudden deceleration of the water column may cause the pressure to drop to a value less than atmospheric pressure. The lowest value to which pressure could drop is vapour pressure. Vaporization or even water column separation may thus occur at peaks along the pipeline. When the pressure wave is returned as a positive wave the water columns will rejoin giving rise to water hammer over-pressures.

Unless some method of water hammer protection is installed, or friction plays a significant role in reducing water hammer pressures before positive return surge occurs, a pumping pipeline system may have to be designed for maximum water hammer overhead equal to cv_o/g (termed the Joukowsky head). In fact this is often done with high-pressure lines where water hammer heads may be small in comparison with the pumping head. For short lines this may be the most economic solution, and even if water hammer protection is installed it may be prudent to check that the ultimate strength of the pipeline is sufficient should the protective device fail.

The philosophy behind the design of most methods of protection against water hammer is similar. The object in most cases is to reduce the downsurge in the pipeline caused by stopping the

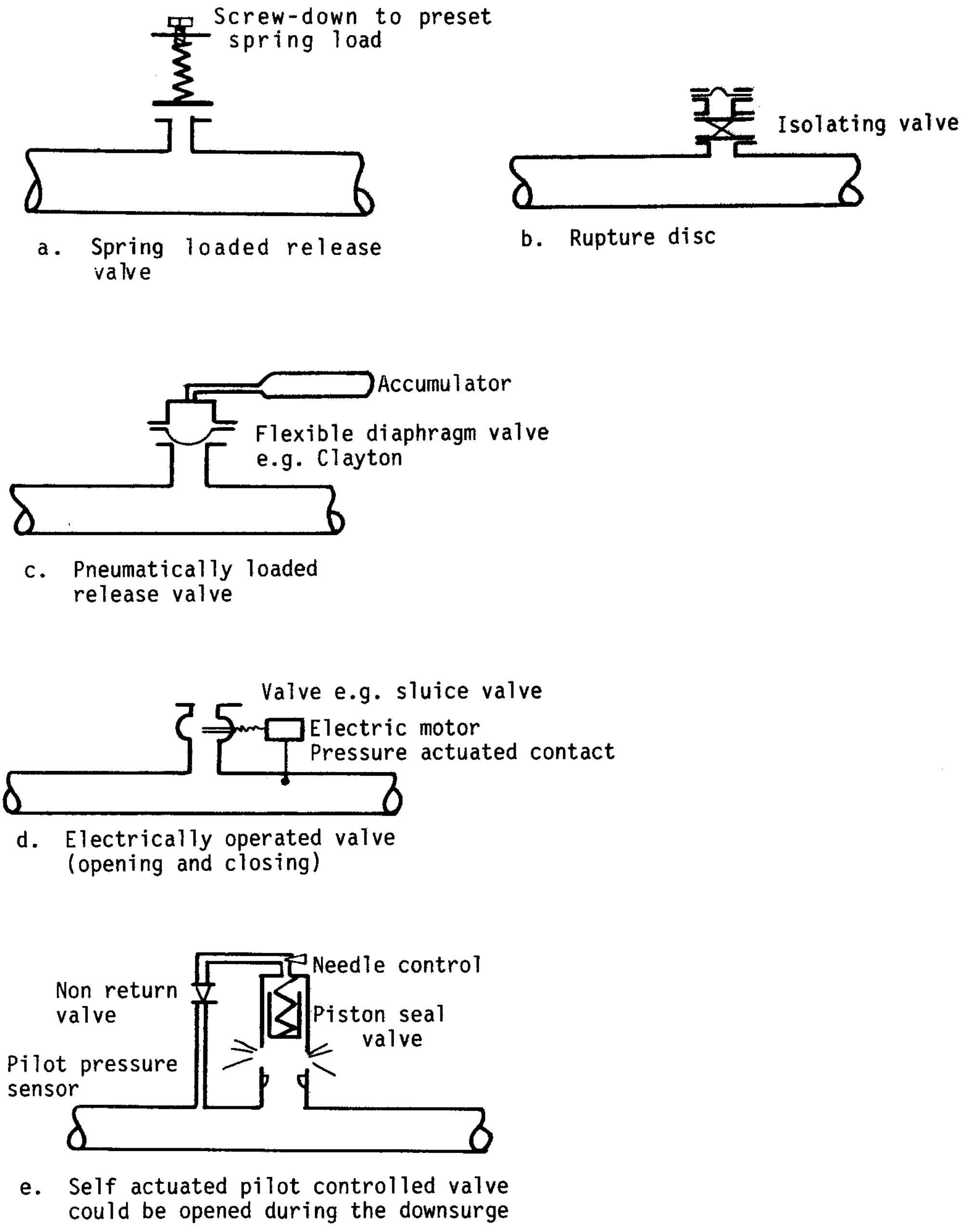


Fig. 10.1 Release valves

pumps. The upsurge will then be correspondingly reduced, or may even be entirely eliminated. The most common method of limiting the downsurge is to feed water into the pipe as soon as the pressure tends to drop.

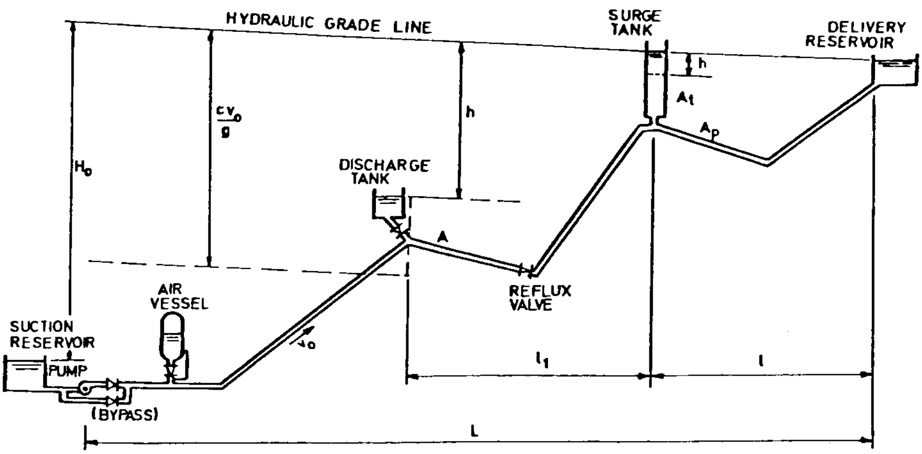


Fig. 10.2 Pipeline profile illustrating suitable locations for various devices for water hammer protection

Suitable locations for various protective devices are illustrated in Figure 10.2. Most of the systems involve feeding water into the pipe. Observe that in all cases the sudden momentum change of the water column beyond the tank is prevented so the elastic water hammer phenomenon is converted to a slow motion surge phenomenon. Part of the original kinetic energy of the water column is converted into potential energy instead of elastic energy. The water column gradually decelerates under the effect of the difference in heads between the ends. If it was allowed to decelerate the water column would gather momentum in the reverse direction and impact against the pump to cause water hammer over-pressures. If, however, the water column is arrested at its point of maximum potential energy, which coincides with the point of minimum kinetic energy, there will be no sudden change in momentum and consequently no water hammer over-pressure. The reverse flow may be stopped by installing a reflux valve or throttling device at the entrance to the discharge tank or air vessel, or in the pipeline. A small orifice bypass to the reflux valve would then allow the pressures on either side to gradually equalize.

Fortunately charts are available for the design of air vessels and for investigation of the pump inertia effects, so that a water hammer analysis is not normally necessary (Stephenson, 1981). Rigid water column theory may be employed for the analysis of surge tank action, and in some cases, of discharge tanks.

If the pipeline system incorporates in-line reflux valves or a pump bypass valve, an elastic water hammer analysis is usually necessary. The analysis may be done graphically or, if a number of solutions of similar systems are envisaged, a computer program could be developed. Normally the location, size and discharge characteristics of a protective device such as a discharge tank have to be determined by trial and error. The location and size of in-line or bypass reflux valves may similarly have to be determined by trial. In these instances a computer program is usually the most economical method of solution, as a general program could be developed, and by varying the design parameters methodically, an optimum solution arrived at.

NON-RETURN VALVES

In some situations the strategic location of a non-return valve, or check valve or reflux valve, is sufficient to prevent or at least reduce water hammer over-pressures. Where water column separation occurs the installation of a non-return valve downstream of the pocket could prevent flow reversal and the subsequent over-pressures.

In another type of application, water could be drawn into the pipeline from the suction reservoir or a tank when the head in the main pipe drops below the head outside. This water would fill the cavity and likewise reduce the return surge.

Non-return valves are included with conditional statements in computer programs. When the flow rate is positive then an in-line valve remains open with minimal head loss. When the flow reverses, or attempts to reverse the valve closes. The flow rate and head are initially calculated using equations (9.32) and (9.33). Then the flow check is made. If negative, the flow rate is reset equal to zero on either side of the valve and the head upstream and downstream re-compiled using equations (9.30) and (9.31) respectively. There may be a further conditional check for head, and if the upstream head is less than vapour head, it is reset and the upstream flow recalculated.

The use of off-line non-return valves with discharge tanks is illustrated in Figure 10.3.

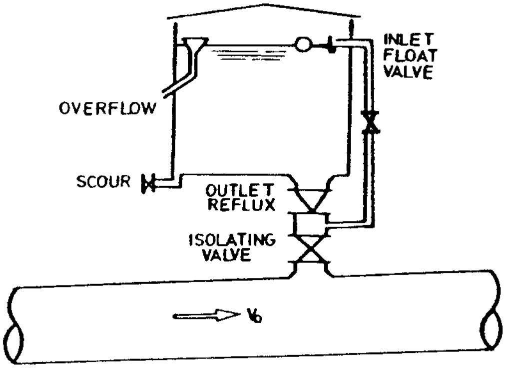


Fig. 10.3 Discharge tank

In principle many forms of water hammer protection for pumping lines operate similarly. That is they discharge liquid into the pipeline when the pressure in the line drops after a pump trip. This fills the potential vacuum or at least reduces the downsurge. When the (positive) water hammer wave then returns from the far end of the line it will be reduced in amplitude correspondingly. The types of protection which operate thus include air vessels, pump bypasses with non-return valves, flywheels on pumps, discharge tanks and in-line non-return valves. Surge tanks operate on a slightly different principle, i.e. they offer a continuous insulation effect protecting the pipeline beyond them against over-pressures. The analyis of various types of surge tank is described by Rich (1963).

AIR VESSELS

Air vessels are often used to cushion water hammer over-pressures or to feed water into the low pressure zone created by stopping pumps. They are also used to balance flow on reciprocating pump systems. Air vessels generally contain air at the operating pressure of the adjacent pipeline. They are connected to the pipeline via a pipe which may have a constriction. This outlet constrition will reduce the volume of water forced from the air vessel into the pipe when a low-pressure zone is created in the pipe. This constriction has the disadvantage that pressures in the pipe are lower after the pump trip than without the constriction.

The air vessel outlet could also have a non-return valve incorporated in the outlet to reduce backflow when the positive surge occurs. This prevents the returning water column gathering momentum and reduces the volume of air needed to cushion the final maximum return flow. The most efficient return flow is usually a restricted bypass pipe around the discharge non-return valve. This lets back some flow but also acts as a constriction or throttle. The optimum combination of air vessel capacity, initial air volume, outlet size and inlet size must be found by trial. There are charts available for preliminary selection of air vessel size (Stephenson, 1981; Thorley and Enever, 1979).

Air Vessel Equations

The air in the air vessel, on release, expands in accordance with the laws of physics. The expansion is usually in-between isothermal (PS = constant, where P is absolute pressure and S is volume) and adiabatic ($PS^{1.4}$ constant). The relationship generally adopted is $PS^{1.2}$ = constant. (10.2)

The increment in volume of air, dS is obtained from the continuity equation

$$dS = (Q_2 - Q_1)dt \tag{10.3}$$

where Q_2 is the discharge rate in the pipeline beyond the air vessel connection and Q_1 before the air vessel, averaged over the time period, dt.

The change in pressure head in the air vessel over the time dt can be calculated from the air expansion. The head in the pipeline adjacent to the air vessel is calculated by subtracting the head loss in the connecting pipe or bypass, (depending on whether flow is out or in). The new discharge rates in the pipeline are then calculated using the respective characteristic equations

$$Q_1'=Q_q+(Ag/c)(h_q-h')-\lambda Q_q|Q_q|dt/2dA \quad (10.4)$$

$$Q_2'=Q_r+(Ag/c)(h'-h_r)-\lambda Q_r|Q_r|dt/2dA \quad (10.5)$$

where q is an upstream point, r is a downstream point and the prime ′ refers to the new values.

If dt is large it may be necessary to use the mean values of Q_1 and Q_2 over the time interval, and not the initial values. This is an implicit solution and would mean the above equations would have to be iterated a few times to obtain the discharge rates at the end of the time interval.

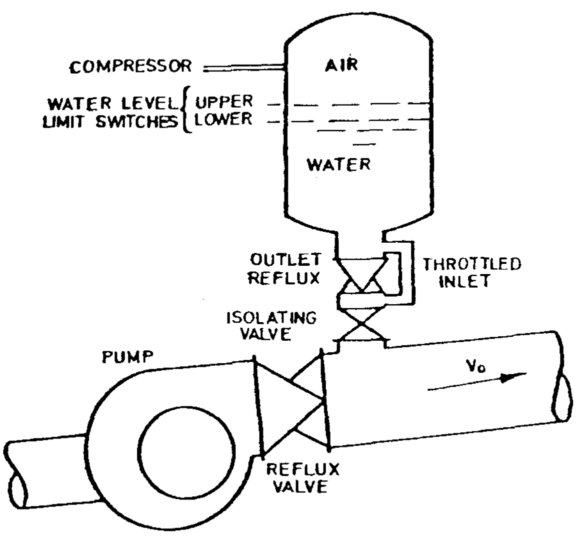


Figure 10.4 Air Vessel

REFERENCES

Rich, G.R., 1963. Hydraulic Transients. Dover Publics. N.Y.
Sharp, B.B., 1981. Waterhammer Problems and solutions. Edward Arnold, London.
Stephenson, D., 1981. Pipeline Design for Water Engineers. Elsevier, Amsterdam.
Thorley, A.R.D. and Enever, K.J., 1979. Control and Suppression of Pressure Surges in Pipelines and Tunnels. Construction Industry Research and Information Association, Report 84. London.
Wylie, E.B. and Streeter, V.L., 1978. Fluid Transients. McGraw Hill.

CHAPTER 11

WATER COLUMN SEPARATION

INTRODUCTION

Water cannot tolerate a pressure less than its vapour pressure, which is a function of the temperature but is only a few metres absolute head. When the pressure in a water column is reduced, for instance by stopping a pump, the pressure may reduce to below atmospheric and to the vapour pressure of the water. The inherent energy then no longer converts to strain energy but kinetic energy. The head may not drop low enough to stop the water column so that downstream of the cavity it proceeds, albeit at a lower velocity, in the same direction as *initially, creating a cavity in the conduit* between the pump and the water column.

The cavity is vacuous but contains water vapour and some air which will come out of solution due to the lowering of the pressure in the water. It is also possible that air will be drawn in through air valves on the line or even through the pumps. This air has a limited elastic effect in cushioning the pressure drop and subsequent rise on return of the water column. It is generally found, however, that the mass of air is too small to have a noticeable effect in reducing the subsequent water hammer pressure rise.

The form of the cavity created by the drop in pressure is time-dependent although the total volume can be computed accurately *using mass balance and even the rigid column equations in some* cases. It is often sufficiently accurate to assume the cavity occupies the full cross sectional area of the pipe and the effective length can thence be determined at any time if the volume is known. Initially the cavity occurs in the form of bubbles dispersed across the section of the conduit. The bubbles rise to the top of the section and may also travel longitudinally before coalescing to create a pocket. In fact it is only in steep pipes that the interface is sharp, and in longitudinal pipes the cavity or vapour pocket spreads longitudinally. A bore may travel up the line to spread the cavity laterally. If the cavity spreads over more than one 'node', water hammer calculations are complicated but often the cavity can be imagined as occuring in separate pockets at nodes (see Martin, 1981).

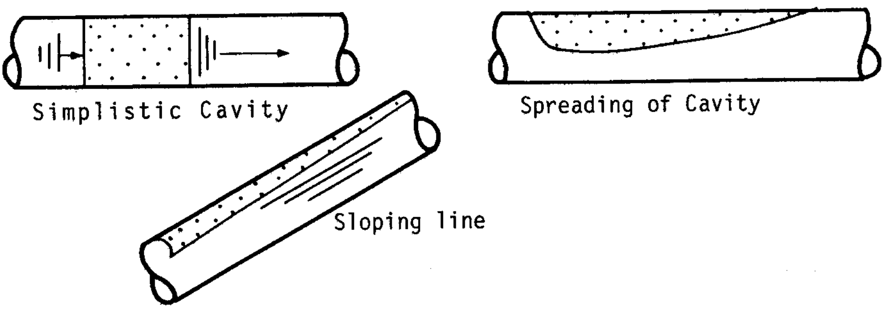


Figure 11.1 Shape of cavity

The mechanics of water hammer accompanied by column separation can most readily be visualized using the graphical method, (see Fig. 11.2).

Whether the graphical method or a computational method is employed, a check is made at each stage to ensure the head does not drop below vapour pressure head. Once it does, a subroutine is entered setting it equal to vapour head and accumulating a volume of vapour pocket by adding $\Delta S = \Delta V \; A \; dt$ to S each time step, where ΔV is the difference in downstream and upstream velocities.

COMPUTATIONAL TECHNIQUE FOR COLUMN SEPARATION

The method of handling water column separation or vaporization numerically is to an extent a process of trial. At any time interval head and flow may first be computed assuming continuity at each point, that is inflow equals outflow. The head is computed without heed to separation. If the resulting head is less than vapour pressure head, it can be set equal to vapour head, e.g. $\pm$ 1 m absolute or -9 m gauge head. If, however, there is an air valve on the line near that point, or even considerable air transported in the flow, the 'separation' head may in fact be higher. The air mass drawn in is generally negligible and will have no cushioning effect when the water columns subsequently rejoin (a comparison with the size of an air receiver to prevent water hammer will reveal this). At the upper limit the head will approach atmospheric, i.e. it will be the elevation of the pipe above datum.

The flow rates upstream and downstream of the pocket are now recomputed using the respective characteristic equations and substituting vapour head for head at the point. Then the increase in vapour pocket volume over the next time interval is computed, $\Delta t(Q_o - Q_i)$ where Q_i is inflow and Q_o outflow at the point.

A tally of the cumulative vapour pocket volume at each such point is maintained during the successive computations. As long as this volume is greater than zero the head must be equated to vapour head and the flows each side of the point computed thus.

When the vapour pocket reverts to zero (or turns negative) then the head is recalculated for a continuous water column. The vapour pocket may collapse between computational time intervals, in which case the head rises to a lesser extent than would be predicted assuming it rises at the end of the time interval (compare point M2 with M1 in Figure 11.2).

If a one-way discharge tank is used to reduce water hammer the same computational procedure is employed. The discharge tank is connected to the pipeline through a non-return valve and discharges only when the head in the pipe drops below the tank head. Thus return flow and vapour pocket collapse are eliminated. The tally on volume of separation is thus maintained as for a vapour pocket but as soon as it attempts to reduce, the non-return valve prevents this and the head rises as for a water column without separation.

SIMPLIFIED RIGID COLUMN ANALYSIS

If the water column can be assumed to separate at a known

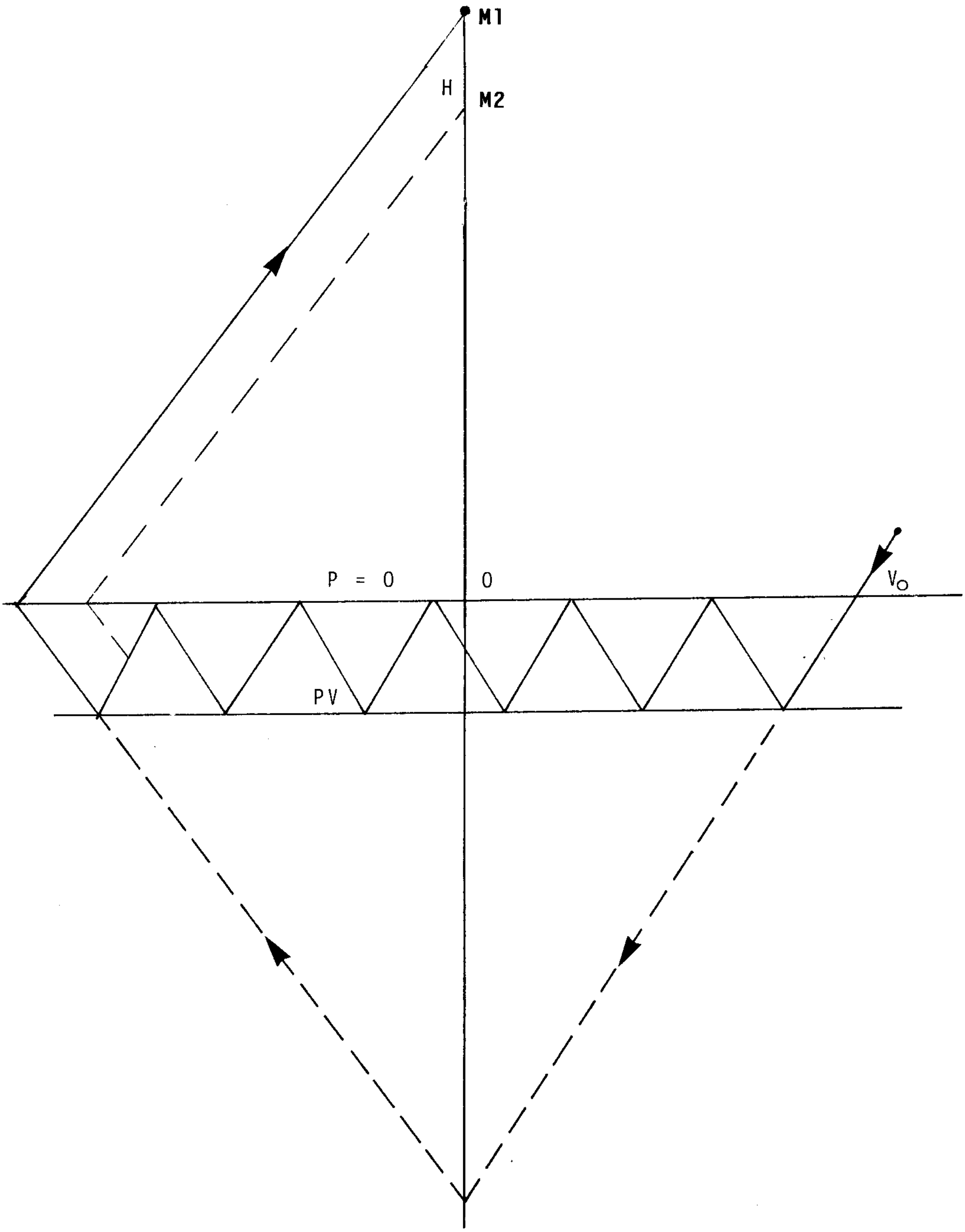


Figure 11.2 Drop in head due to pump trip and subsequent rise in rejoining of water columns

point and the subsequent downstream head changes are small, the following analysis will indicate the extent of column travel before reversal. The downstream column may be assumed to decelerate according to the following equation:

$$\frac{dv}{dt} = -\frac{g}{L}h \qquad (11.1)$$

where h is the decelerating head, which could be the difference between the vapour pressure head in the vapour pocket and the elevation head of the downstream open end. Friction head can also be added to h if it is significant, and it will act in an upstream direction while the water column is decelerating, and subsequently in the opposite, downstream (towards discharge end) direction when the water column reverses. This will cause a reduction in the return velocity of the water column whereas the following analytical solution without friction will indicate a return velocity equal to the original velocity in magnitude:

Integrating the previous equation with time, we get

$$v = v_o - ght/L \qquad (11.2)$$

here v_o is the constant of integration which is the initial flow velocity. Now when the column reaches its extremity before reversing, v = 0 hence $t = v_oL/gh$ (11.3)

Integrating again gives the distance the water column travels

$$x = v_ot - ght^2/2L \qquad (11.4)$$

Substituting for t at v=0 gives

$$x_{max} = v_o^2L/2gh \qquad (11.5)$$

Hence the maximum volume of cavity is $Ax_{max} = ALv_o^2/2gh$ (11.6)

Rising Mains

Along gently rising pumping pipelines the vaporization may occur along a considerable length of pipe. The spreading of the vapour pocket front will occur initially as fast as an elastic wave, but the subsequent refilling will be slower owing to the additional 'elasticity' in the vapour pocket.

In the following analysis the magnitude of the vapour pocket formed in the rising length of pipeline is evaluated in equation form. The cross sectional area occupied by the vapour is shown to be very small in relation to the total pipe cross sectional area. It is proved that the vapour pocket is rapidly filled by surges travelling along the pipeline from both ends, and that no water hammer pressure rise is associated with the filling of the pocket.

To simplify the analysis it is assumed that the pipe beyond point C is inclined at a constant angle θ to the horizontal. Heads are absolute values, that is gauge plus atmospheric.

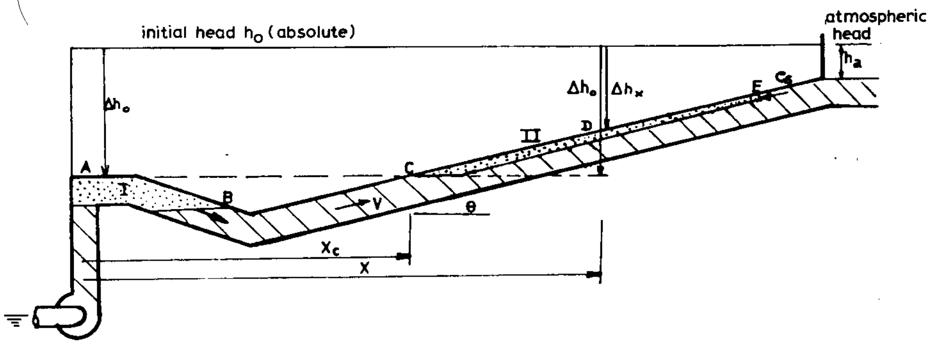


Figure 11.3 Simplified pipe profile for theoretical study

The velocity at any point x between c and the delivery end, (see Figure 11.3) after the initial negative wave has passed, is

$$v_x = v_o - \frac{g}{c}[\Delta h_o - (x - x_c)\sin\theta] - g[t - \frac{x}{c}] \sin\theta \qquad (11.7)$$

The second term on the right hand side is the velocity reduction caused by the initial negative surge, while the last term is the deceleration due to gravity. The continuity equation applicable is

$$\frac{\partial}{\partial x}(av) + \frac{\partial A}{\partial t} = 0 \qquad (11.8)$$

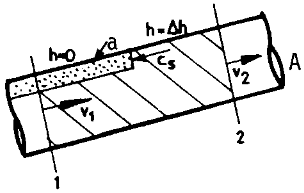


Figure 11.4 Detail at surge front

where A is the cross sectional area of water in the pipe. If the cross sectional area of the vapour 'a' may be assumed to be small in comparison with the total cross section A, the last equation may be simplified to

$$\frac{\partial a}{\partial t} = A\frac{\partial v}{\partial x} \tag{11.9}$$

from (11.7) $\partial v/\partial x = 2g \sin\theta/c$, so that (11.9) may be integrated to yield

$$\frac{a}{A} = \frac{2g}{c} \sin \theta \left(t-\frac{x}{c}\right) \tag{11.10}$$

This equation illustrates that 'a' is small compared with A.

At the open delivery end the head remains at atmospheric head h_a. When the negative wave reaches this point water is forced back into the pipe. A surge travels back along the pipe, filling the vapour pocket. In order to study this surge let the analyst travel with the surge, at celerity c_s. Apply momentum principles to the free body of water shown in Figure 11.4.

$$\rho(v_1+c_s)^2(A-a)+\rho g\bar{y}_1(A-a) = (v_2+c_s)^2 A\rho+\rho g\Delta hA+\rho g\bar{y}_2 A \tag{11.11}$$

Here ρ is the mass density of water and $\bar{y}$ is the depth to the centroid of the cross section of water. The velocity v_2 may be expressed in terms of v_1, c_s, a and A with the aid of the continuity equation:

$$(v_1+c_s)(A-a) = (v_2+c_s)A \tag{11.12}$$

Since (A-a) is very nearly equal to A, y_1(A-a) is nearly equal to y_2A. Solving the last two equations for c_s

$$c_s \approx \sqrt{g\Delta h \frac{A}{a}} \tag{11.13}$$

$$= \sqrt{\frac{c\Delta h}{2 \sin\theta(t-\frac{x}{c})}} \tag{11.14}$$

For many practical situations this equation will yield values of c between 500 and 1000 m/s, illustrating that it is of the same order of magnitude as the wave celerity of a full pipe. The velocity change at the surge front may be expressed in terms of Δh by rewriting (11.12) and (11.13) as follows:-

$$A[(v_1+c_s)-(v_2+c_s)] \approx g\Delta hA/c_s$$

$$\therefore\ (v_1-v_2) = \frac{g}{c_s}\Delta h$$

PROGRAM FOR SIMULATION OF WATER HAMMER IN PUMPLINES FOLLOWING PUMPTRIP

The accompanying basic computer program for an HP-85 calculates water hammer pressures in a pipeline following a pumptrip. The pump is assumed at the upstream end, the pipe diameter is constant, but friction is accounted for. Data is requested interactively as follows;

<u>Line 1</u>: The name of the system

<u>Line 2</u>: The number of sections

<u>Line 3</u>: The number of iterations (e.g. if the pipe is divided into 4 intervals, 4 iterations represent the travel time for the water hammmer wave up the pipe)

<u>Line 4</u>: Length (m)

<u>Line 5</u>: Diameter (m)

<u>Line 6</u>: Wavespeed (m/s)

<u>Line 7</u>: Flow rate (m^3/s)

<u>Line 8</u>: Friction head (m)

Line 9
onwards: Elevation of each successive point (all in m and s units).

The first and last elevations should represent the water level in the suction sump and delivery reservoir (both assumed constant).
The program assumes the pump trips immediately the simulation starts, and prints out heads and flows at each point every interval.

Water column separation is accounted for in a simplistic way. When the head drops to the elevation of the pipe at any point, or tries to drop below the elevation, it is set equal to elevation and the flow rate both before and after the point are re-calculated for the new head. Vapour pocket volume is computed by summing the difference between the two flows and when it turns to zero (or a negative value) the head reverts to that indicated by the finite difference equations for a full pipe. The spreading or longitudinal movement of vapour pockets are neglected, and no reduction in wave speed is made. Due to this simplification and the fact that vapour pocket closure can only occur at the computational time, the program can overestimate water hammer pressure due to pocket collapse.

The maximum heads at each head along the line are plotted above the pipe profile at the end of the run.

List of symbols in program for pumpline water hammer analysis

A	cross sectional area
B	celerity of wave
D	diameter
D1	sum of $1/D^5$
E	mass density of liquid
F	unit friction
F1	total friction head
F2	friction term
F3	friction term
G	gravitational acceleration
H	head
H2	head term
I	iteration number
I9	printout interval (0 = only summary required, 1 = full listing of heads)
J	pipe number

K	number of iterations
L	name
M	number of pipe intervals
M1	M + 1
P	maximum head
P1	head term in plot
Q	flow beyond point
Q2	flow term
R	flow in front of point at previous time interval
S	flow beyond point at previous time interval
T	flow in front of point
T2	time increment
U	head at previous time interval
X	length
X2	length interval
Y	vapour pocket volume
Z	elevation

Computer Program for Analysis of Water Hammer after Pumptrip

```
 10 !  WATER HAMMER IN PIPELINES
     AFTER PUMP TRIP- WAP
 20 DIM A(21),B(21),F(21),H(21),
    P(21),Q(21),R(21),U(21),Z(21
    )
 30 DIM D(20),Y(21),T(21),S(21)
 40 DISP "PUMPSTOP PIPE NAME";
 50 INPUT L$
 52 DISP "NO.SECNS";
 60 INPUT M
 70 M1=M+1
 80 DISP "NO.ITERATIONS ";
 90 INPUT K
100 DISP "PIPE LENGTH,m";
110 INPUT X
120 DISP "DIA.m";
130 INPUT D(1)
140 DISP "WAVESPEED,m/s";
150 INPUT C
160 DISP "FLO RATE,m3/s";
170 INPUT Q(1)
180 DISP "FRICTION LOSS,m";
190 INPUT F1
200 DISP "ELEV(m) POINT 1-",M1
210 FOR J=1 TO M1
220 DISP J;
230 INPUT Z(J)
240 NEXT J
250 I9=1
260 E=1000
270 G=9.8
280 X2=X/M
290 T2=X2/C
300 D(M1)=D(1)
310 D1=0
320 FOR J=1 TO M
330 D(J)=D(1)
340 A(J)=.785*D(J)^2
350 D1=D1+1/D(J)^5
360 NEXT J
370 H(1)=Z(M1)+F1
380 FOR J=1 TO M1 ! PIPE SECNS
390 Q(J)=Q(1)
395 T(J)=Q(J)
398 Y(J)=0
400 F(J)=F1/D1/D(J)^5/Q(1)^2
410 H(J+1)=H(J)-F(J)*Q(1)^2
420 P(J)=H(J)
430 NEXT J
440 PRINT "WATERHAMMER ";L$
450 FOR I=1 TO K
460 IF I9<1 THEN 510
470 PRINT " PNT HEADm FLOms";I
480 FOR J=1 TO M1
490 PRINT USING 750 ; J,H(J),Q(J
    )
500 NEXT J
510 FOR J=1 TO M1 ! PIPE SECNS
520 R(J)=T(J)
530 U(J)=H(J)
532 S(J)=Q(J)
540 NEXT J
550 Q(1)=0
555 H(1)=U(2)+C/G/A(1)*(Q(1)-R(2
    ))+F(1)*R(2)*ABS(R(2))
556 IF Y(1)<=0 THEN 560
558 GOTO 564
560 Y(1)=0
562 IF H(1)>Z(1) THEN 570
564 H(1)=Z(1)
566 Q(1)=R(2)-A(1)*G/C*(U(2)-H(1
    )+F(1)*R(2)*ABS(R(2)))
```

```
568 Y(1)=Y(1)+Q(1)*X/M/C
570 FOR J=2 TO M
580 F2=F(J-1)*S(J-1)*ABS(S(J-1))
590 F3=F(J)*R(J+1)*ABS(R(J+1))
600 Q2=U(J-1)-U(J+1)+C/G*(R(J+1)
    /A(J)+S(J-1)/A(J-1))
610 Q(J)=(Q2-F2-F3)*G/C/(1/A(J)+
    1/A(J-1))
615 T(J)=Q(J)
620 H2=(S(J-1)-R(J+1))*C/G+U(J-1
    )*A(J-1)+U(J+1)*A(J)
621 H(J)=(H2+F3*A(J)-F2*A(J-1))/
    (A(J-1)+A(J))
622 IF Y(J)<=0 THEN 626
624 GOTO 630
626 Y(J)=0
628 IF H(J)>Z(J) THEN 640
630 H(J)=Z(J)
632 T(J)=S(J-1)+A(J-1)*G/C*(U(J-
    1)-H(J)-F(J-1)*S(J-1)*ABS(S(
    J-1)))
634 Q(J)=R(J+1)-A(J)*G/C*(U(J+1)
    -H(J)+F(J)*R(J+1)*ABS(R(J+1)
    ))
636 Y1=(Q(J)-T(J))*T2
637 IF Y(J)+Y1>Y(J)/2 THEN 639
638 Y(J)=0 @ GOTO 610
639 Y(J)=Y(J)+Y1
640 NEXT J
650 Q(M1)=S(M)+A(M)*G/C*(U(M)-H(
    M1)-F(M)*S(M)*ABS(S(M)))
655 T(M1)=Q(M1)
660 H(M1)=Z(M1)
670 FOR J=1 TO M1
680 IF H(J)<=P(J) THEN 700
690 P(J)=H(J)
700 NEXT J
710 NEXT I
720 PRINT " PNT ELV,m FLO,m3s MA
    X H"
730 FOR J=1 TO M1
740 PRINT USING 750 ; J,Z(J),Q(J
    ),P(J)
750 IMAGE DDD,DDDDDD,DDD.DDD,DDD
    DDD
760 NEXT J
765 PRINT
770 PRINT "HEADS @ 100M INCR AB
    VE PUMP "
780 P1=P(1)+200
790 Z1=Z(1)-100
800 GCLEAR
810 SCALE 1,M1,Z1,P1
820 XAXIS Z(1),1
830 YAXIS 1,100
840 MOVE 1,Z(1)
850 FOR J=2 TO M1
860 DRAW J,Z(J)
870 NEXT J
880 MOVE 1,P(1)
890 FOR J=2 TO M1
900 DRAW J,P(J)
910 NEXT J
920 COPY
930 STOP
940 END
```

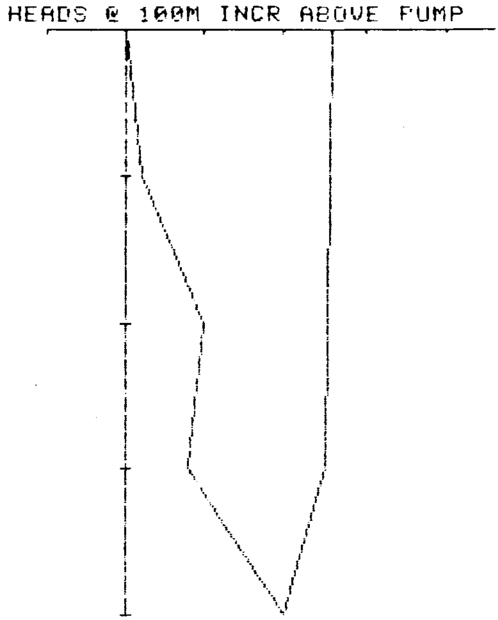
Output

```
WATERHAMMER TESTMAIN
 PNT HEADm FLOms 1
  1   220    .100
  2   215    .100
  3   210    .100
  4   205    .100
  5   200    .100
 PNT HEADm FLOms 2
  1    61  0.000
  2   215    .100
  3   210    .100
  4   205    .100
  5   200    .100
 PNT HEADm FLOms 3
  1    61  0.000
  2    59   .002
  3   210    .100
  4   205    .100
  5   200    .100
 PNT HEADm FLOms 4
  1    56  0.000
  2    59   .002
  3   100   .031
  4   205    .100
  5   200    .100
 PNT HEADm FLOms 5
  1    56  0.000
  2    97  -.026
  3   100   .031
  4    97   .032
  5   200    .100
 PNT HEADm FLOms 6
  1   138  0.000
  2    97  -.026
  3   100   .034
  4    97   .032
  5   200  -.033
 PNT HEADm FLOms 7
  1   138  0.000
  2   141  -.002
  3   100   .034
  4   202  -.031
  5   200  -.033
 PNT HEADm FLOms 8
  1   143  0.000
  2   141  -.002
  3   195  -.036
  4   202  -.031
  5   200  -.029
 PNT HEADm FLOms 9
  1   143  0.000
  2   197  -.034
  3   195  -.036
  4   193  -.034
  5   200  -.029
PNT HEADm FLOms 10
 1    250  0.000
 2    197  -.034
 3    195  -.032
 4    193  -.034
 5    200  -.038
PNT HEADm FLOms 11
 1    250  0.000
 2    248    .001
 3    195  -.032
 4    202  -.036
 5    200  -.038
PNT HEADm FLOms 12
 1    245  0.000
 2    248    .001
 3    255  -.003
 4    202  -.036
 5    200  -.035
PNT HEADm FLOms 13
 1    245  0.000
 2    252  -.004
 3    255  -.003
 4    252  -.001
 5    200  -.035
PNT HEADm FLOms 14
 1    259  0.000
 2    252  -.004
 3    250  -.003
 4    252  -.001
 5    200    .031
PNT HEADm FLOms 15
 1    259  0.000
 2    257    .001
 3    250  -.003
 4    198    .030
 5    200    .031
PNT HEADm FLOms 16
 1    254  0.000
 2    257    .001
 3    205    .034
 4    198    .030
 5    200    .028
PNT HEADm FLOms 17
 1    254  0.000
 2    203    .032
 3    205    .034
 4    207    .032
 5    200    .028
PNT HEADm FLOms 18
 1    152  0.000
 2    203    .032
 3    205    .031
 4    207    .032
 5    200    .037
```

PNT	HEADm	FLOms 19
1	152	0.000
2	154	-.001
3	205	.031
4	198	.035
5	200	.037

PNT	HEADm	FLOms 20
1	157	0.000
2	154	-.001
3	148	.003
4	198	.035
5	200	.033

PNT	ELV,m	FLO,m3s	MAX H
1	0	0.000	259
2	20	.004	257
3	100	.003	255
4	80	.001	252
5	200	.077	200



Input

```
PUMPSTOP PIPE NAME?
TESTMAIN
NO.SECNS?
4
NO.ITERATIONS ?
20
PIPE LENGTH,m?
1000
DIA.m?
.3
WAVESPEED,m/s?
1100
FLO RATE,m3/s?
.1
FRICTION LOSS,m?

20
ELEV(m) POINT 1-       5
 1 ?
0
 2 ?
20
 3 ?
100
 4 ?
80
 5 ?
200
```

REFERENCES

Fox, J.A., 1977. Hydraulic Analysis of Unsteady Flow in Pipe Networks. Macmillan, London.

Martin, C.S., 1981. Two-phase flows. In 'Closed Conduit Flow', Edd by Chaudhry, M.H. and Yevjevich, V., Water Resources Publics., Colorado, p225-251.

Stephenson, D., October 1957. Prevention of vapour pocket collapse in a pumping line. Trans. SAICE, 9(10), p255-261.

CHAPTER 12

WATER HAMMER AND FLOW ANALYSIS IN COMPLEX PIPE SYSTEMS

THE PROBLEM OF FLOW ANALYSIS IN COMPLEX PIPE NETWORKS

In many municipal and industrial water distribution systems the pipework follows no regular geometric layout and is developed as the requirements grow. Although the main supply pipe may be predesigned, extensions are often of variable diameter and lengths and the distribution system at the ends of these pipes is even more complex. Similar problems often arise in pumping columns and in fact the water hammer pressures due to the tripping of a pump can often be the most severe pressure condition in the pipeline.

Where hydraulic pressures can be high, such as down the shaft of a mine, the corresponding pipe velocities are often very high e.g. above 5 metres per second. The high velocity is accepted as it reduces the pipe diameter thus saving in capital cost. In the case of gravity mains the hydraulic gradients can be very high and the corresponding pipe velocities will also be very high. Unfortunately the water hammer heads associated with changes of flow are directly proportional to the flow velocity and consequently an accurate water hammer pressure analysis is necessary or else a sophisticated method of protecting the pipe system against water hammer is required.

There are three main stages in the analysis of a water reticulation system. During the initial planning stages the pipes have to be sized and the corresponding wall thicknesses selected before installation. Pipe bores should reduce the greater the pressure in order to minimize wall thicknesses and costs. Where there are off-takes then the diameter of the pipe may be reduced. The total available head will depend on the elevation of the supply point below the supply reservoir or else on a pumping head. The head-losses along individual sections of the pipe can be distributed between various lengths of pipes depending on flow rates. In fact simple linear programming optimization methods (Stephenson, 1981) can be used to select least cost pipe diameters in such a branch network. Alternatively dynamic programming methods can be used to select successively decreasing diameters down deep mine shafts, (Bernstein, 1982).

Pipe sizes are based on an initial estimate of design flow but in later years the flows may be increased or altered and it is necessary to analyse the system to determine the flows which can be obtained from the system. For example where water is taken from a pipe for supplying cooling coils then it is important that the pressure be known at each branch along the distribution pipe. As the head at each branch point is not known explicitly it must be calculated from headloss equations. This is not an easy matter as there are a number of unknowns and it is often necessary to use successive approximation methods to determine the flows and pressures in the individual pipe lengths. The problem may be aggravated by changing friction factors with time. As the pipes corrode or scale the bores will be affected and the corresponding friction losses change. Although iterative solution of the Darcy-Weisbach flow resistance equation is possible this degree of sophistication is often not warranted as the exact roughness effect on the hydraulic capacity can only be estimated roughly anyway.

The third problem in analysing pipe reticulation systems is the determination of transient water hammer pressures due to closing valves or tripping pumps. The water hammer head can be several hundred metres if the flow in a pipe is reduced rapidly such as due to the closure of a valve. This head should be added onto the static heads so that it may reduce the factor of safety of the system and in fact has been known to burst pipes or fittings. Valve closure times must therefore be selected to prevent water hammer pressures being excessive and similarly precautions have to be taken to minimize water hammer pressures due to pumps tripping. Water hammer pressure waves can also be reflected from the ends of branch pipes and be more severe in the branch pipe than in the main pipe where the flow is altered. Since water reticulation systems can be fairly complex the use of computers is necessary for accurate analysis.

The two problems of analysis of flow in pipe networks and the calculation of water hammer pressures due to changing of the flows can be done using one computational approach. The two problems have been combined in developing a program the basis of which is described here. The effort in feeding data into a computer for the various analyses is thus minimized.

CONVENTIONAL METHODS OF NETWORK ANALYSIS

The standard methods of analysing flows and headlosses in complex piping systems were developed by Hardy Cross in 1936 and are still used by many engineers on account of their simplicity, the ease of visualization of the procedure and the possibility of doing the calculations manually instead of resorting to computers. In earlier chapters these methods and others more computer orientated are described.

Very mathematical methods such as the solution of a set of simultaneous equations describing the flow of headloss relationships in a pipe network have been proposed. An alternative approach is to simulate the system using the differential water hammer equations. Starting with any assumed flows and heads the flows will eventually stabilize at their steady-state values.

Although slightly more complicated if the problem is only the determination of flow rates in closed networks, the following procedure in fact is efficient computationally and is considerably enhanced by the ease of supply of data to the computer. The method relies on the fact that if a valve or pump in a pipe network is operated, then after a length of time friction will damp the elastic waves travelling backwards and forwards in the pipe network and a steady-state will emerge. One therefore has the steady-state flows and pressures throughout the network. The method can be extended to predict pressure and flow histories at every point in the system. These are the pressures and flows which will actually occur at any interval, which is not the case for the network analysis methods described previously. Consequently if transient conditions are to be studied the method will yield them as well.

There have also been many computer programs developed for analysing water hammer pressures in pumping mains and pipe networks. Although the method of finite differences is more rapid for solution of the differential equations of fluid motion, the method of characteristics (Streeter and Wylie, 1967) is generally preferred as the computations are more straightforward and can be applied to varying boundary conditions more easily than the centred finite difference methods. All the methods start with the basic differential equations of continuity and momentum,

$$\frac{\partial h}{\partial t} + \frac{c^2}{g}\frac{\partial v}{\partial x} = 0 \tag{12.1}$$

$$\frac{\partial h}{\partial x} + \frac{1}{g}\frac{\partial v}{\partial t} + \frac{\lambda v|v|}{2gd} = 0 \tag{12.2}$$

The symbol h denotes water head (the sum of pressure head and potential head above a specified datum), v is the water velocity averaged across the cross section of the pipe, d is the pipe diameter, t is time, x is distance along the direction of flow, λ is the Darcy-Weisbach friction coefficient which can vary with Reynolds number, g is gravitational acceleration and the term $v|v|$ is used in place of v^2 as it accounts for changing direction of the friction headloss when the flow changes direction. c is the water hammer wave celerity which may be shown to equal $1/\sqrt{[\rho(1/K + d/Et)]}$ where ρ is the mass density of the fluid, K is its bulk modulus, d is the pipe diameter, t its wall thickness and E its elastic modulus. For steel water pipes c is generally of the order of 1100 m/s but increases for thick-wall pipes. Adopting the time-distance grid indicated in Figure 12.1 then the ensuing characteristic equations give the relationship between head and velocity at known initial or boundary conditions and those at a new point in time. Invariably the friction equation is used in an explicit form, that is v in the friction term is assumed to be the v at the previous time interval and previous point in space.

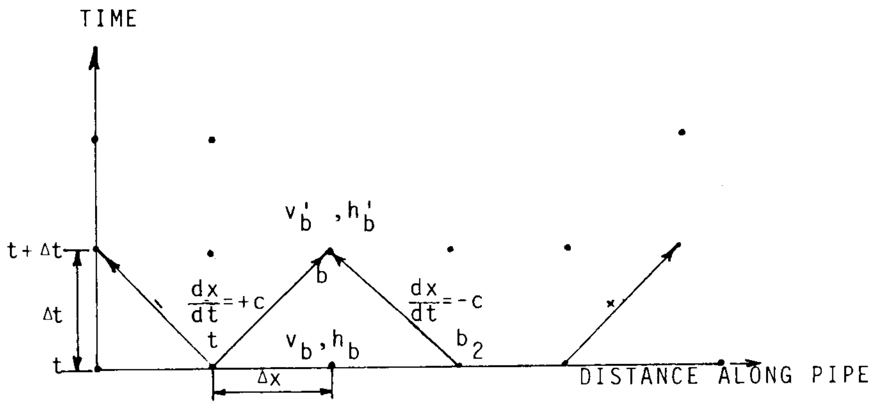


Fig. 12.1 x-t Grid for water hammer analysis by characteristic method.

METHOD OF SOLUTION OF THE EQUATIONS

Starting with the finite difference form of the differential equations and substituting $\frac{dx}{dt} = +c$ yields

$$h_b' - h_t + p + \frac{c}{g}(v_b' - v_t) + fv|v| = 0 \qquad (12.3)$$

where p is pumping head, and the prime ' refers to new values after iteration, and $f = (\lambda L/D + k)/2g$, where losses due to fittings are $kv^2/2g$. Subscript b refers to the bottom (downstream) end and t to the top (upstream) end of a pipe although these are arbitrarily defined as the flow direction can vary.

Generally it is sufficient to use the explicit form of $\lambda Lv^2/2gd$, i.e. the velocity at the previous time interval is used in the finite difference forms of the equations. That is satisfactory provided friction loss is small compared with water hammer head. In some situations this is not the case, for example a pipe burst down a shaft can result in a steady-state friction head loss down the shaft of several hundred metres. The head loss term $kv^2/2g$ is also significant in many situations, e.g. when the head loss is through a valve. It is, nevertheless, much easier from the point of view of computations to lump the two head loss terms together.

It is, on the other hand, difficult to solve the characteristic equations in implicit form, i.e. with the velocity in the head loss term that at the new time. If this is attempted a quadratic expression results and computations are cumbersome. In chapter 9 the head loss term was solved in implicit form for a simple case, i.e. with known head downstream of a valve. In general the head downstream is also an unknown and must be obtained by simultaneous solution of the head loss/discharge equation and the water hammer equation. A further complication arises when there are branches and changes in diameter at the upstream or downstream nodes. The head at such a node is obtained by summating the effects from each branch by employing the continuity equation at the node.

It should also be noted that the answer will depend on whether the head loss is assumed to be at the far end or the near end of the line. An averaging procedure would introduce further complications and, in any case, concentrated head losses such as valves are usually at one end or the other.

In order to avoid many of these difficulties for a branched pipe network some simplification is necessary. A semi-explicit solution is obtained, but a degree of implicitness is introduced to stabilize the solution. This method is sometimes unstable, but, as stated, a complete implicit method is complex. A weighted compromise between a fully explicit form and a semi implicit form is

$$fv|v| = F\ fv_b|v_b| + (1-F)\ fv'|v_b|.$$

Then solving (12.3)
$$v_b' = \frac{v_t + \frac{g}{c}(h_t - h'_b - p - F\ fv_b|v_b|)}{1+(1-F)fg|v_b|/c} \qquad (12.4)$$

Substituting into the continuity equation $\Sigma A_i v_{bi} = q_b$ where q_b is the drawoff at node b connecting i pipes each with area A_i, and solving for h_b'

$$h_b' = \frac{-\frac{q_b}{g} + \Sigma \frac{A(h_t - p + cv_t/g - Ffv_b|v_b|)}{c+(1-F)fg|v_b|}}{\Sigma \frac{A}{c+(1-F)fg|v_b|}} \qquad (12.5)$$

This is then substituted into (12.4) to obtain velocity v_b' in each pipe leading to b. It will be observed that the explicit form of the head loss equation is not the only form used in the above equations. Explicit solution can lead to serious instability in the computations if the time interval selected for analysis is excessive. In fact when rapid valve closure is to be considered the numerical solutions have been known to become unstable and magnify errors. This is because the head loss through the valve or through the pipe is based on the velocity at the previous time interval and not at the new time interval. In fact the most accurate method would be to take the average flow velocity at the beginning and end of the time interval. This would render the equations exceedingly cumbersome and quadratic equations have to be solved on an iterative basis for each node. In order to simplify the procedure and adapt it to microcomputer solution the above simplification was therefore made. The procedure in effect adopted a weighted average of the previously described pseudo- explicit-implicit method and a purely explicit method.

It is also assumed that the relevant velocity for head loss determination is that at the end specified as the 'bottom' or down-stream end of the pipe. Although flow directions can change during flow variations the 'bottom end' is in fact specified as a fixed position by the analyst.

Where water hammer analysis is not important and the program is used purely for the analysis and determination of steady-state flows in a pipe network then damping can be increased by assuming a very low or artificial water hammer wave celerity. The celerity of each pipe in the system to be analysed is defined by the user as being its pipe length divided by the selected time interval between successive analyses. Thus by selecting a long time interval between successive computations the water hammer waves are eliminated and friction controls the equations. The flows will therefore rapidly converge to steady-state flows. Obviously this technique should not be used where true water hammer heads are required and it may be necessary to select a smaller time interval, for example less than one second, for water hammer analyses.

BOUNDARY CONDITIONS

The head at certain input points and discharge points will be fixed by the head in a reservoir and such nodes will not have the head changed in the computations. There is also the problem of very low heads which may cause water column separation. When for instance, a pump is suddenly stopped in a low-head pumping main then the water hammer head initially drops and this may vaporize the water. The program accommodates this effect automatically and the head will not drop below the specified head at any node. A vapour pocket will therefore grow and diminish in size as waves travel up and down the pipes. The corresponding water hammer head when the water columns rejoin will be computed automatically.

Distribution pipes often have pressure reducing valves or surge relief valves installed to ensure no excessive pressure rises. A pressure reducing valve can, in fact, be treated as a reverse pump.

The positioning of control valves in the pipe network is also of importance from the point of view of rapid accessibility and closure in times of emergency or for control purposes. Such valves can be timed to close over a specified period in order to control water hammer pressures. In the case of automatic control valves they can be installed to operate when pressures or flows exceed certain limits. The program can accommodate the opening or closing of valves at any point in time and the combined opening and closing of a valve in the same simulation can be made by imagining two valves in the same position.

An application of the program to steady-state flows is in the analysis of flow in a reticulation system for various conditions. Although pipes are normally designed on a reasonable load factor with selected design flows to meet certain duties, there are often abnormal situations to be considered. Where the water reticulation system is to be used for fire-fighting then high peaks may be required at specified points and at the same time specified minimum pressures will have to be achieved. In such cases it can be assumed that other loads will be reduced or eliminated in order to achieve the necessary peak flows.

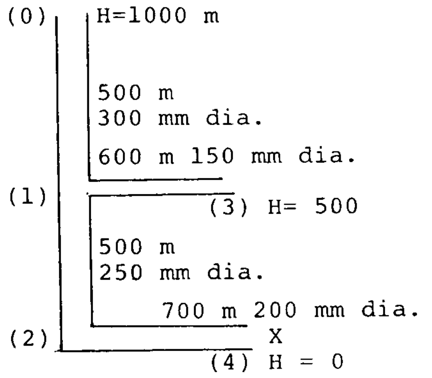


Fig. 12.2 Simple pipe network analysed for flows and heads

The analysis of the system for determining maximum water hammer pressures at various points in the system is also of interest. The water hammer pressures may be caused by the tripping of a pump or the opening or more probably the closing of a valve, or valves, in the system. An example of a case which is likely to be severe is where the pipe supplies water at a high velocity down a shaft at a high pressure. For instance the supply to a hydraulic turbine may have to be rapidly controlled as electrical load is shed. In figure 12.5 the results of such an analysis are plotted for node number 2 in Figure 12.2. The system was initially analysed to determine steady-state heads then the valve in pipe 2-4 was shut at t = 10 sec (at iteration 20 using 0.55 intervals). The valve was shut uniformly in 5 seconds. The heads at each node were tabulated at specified time intervals. The control valves to such turbines can be adjusted to operate over specified times and the closure time can be selected by trial and error or by using an additional algorithm on the program described here which enables the program to recommend the valve closure time.

VALVE STROKING

The reader has been presented with methods of analysing pipe systems for flows and pressures, both under steady flow conditions and for transient conditions. The network analysis for flows gives indirectly the diameters required for the various pipes. Water hammer analysis will then reveal maximum pressures which, in turn, indicate pipe wall thicknesses.

Both types of analysis yield indirect answers, i.e. a trial and error approach is required. In the case of some networks, pipes can be sized directly using optimization methods. When it comes to water hammer analysis the direct design approach is often more difficult when it comes to optimum operation from the water hammer point of view. Thus if the engineer is confronted with a pipe system and is required to specify valve closure times he may resort to a trial and error approach - requiring an analysis for each assumed valve closure rate.

In some cases a direct solution may be obtained for a valve closure relationship where pressures are to be maintained within specified limits. The control of flow in a pre-determined way is complicated by the fact that flow rate depends on pressure which, in turn, is affected by changes in flow. For instance, the closure of a valve gives rise to pressure increases upstream, which in turn can affect the flow rate through the valve.

The procedure of closure of a valve in a pattern which limits pressure rises to set values is referred to as valve stroking (Streeter and Wylie, 1967). A mathematical solution to the valve closure time is possible for single pipes with or without friction. In fact the minimum valve closure time is yielded as well as the rate of closure in various steps.

For a single frictionless pipe the minimum valve closure time (while maintaining the head at some point B less than H_{Bmax}) is obtained with the following operating procedure. At the commencement of operations at the discharge end valve A is closed uniformly, such that the head at point 'B' along the line rises to H_{Bmax} in the time 2L/c (see Figure 12.3). This is assuming H_{Bmax} is less than H_o+cv/g. The actual flow rate through the valve after 2L/c seconds may be solved from the water hammer equation applied to the valve, $\Delta H_A = c\Delta v/g$ and the valve discharge equation $V_A = K\sqrt{H_A}$.

Now the closing of the valve is continued at a uniform rate maintaining the head at B equal to H_{Bmax}.

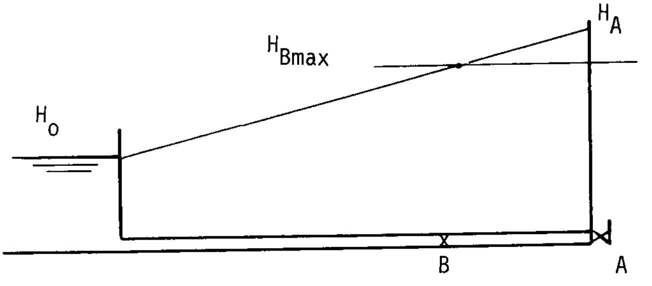


Fig. 12.3 Maximum head along pipe

Complex pipe system

If a valve is to be closed a number of pipe lengths away from a point where the head is to be limited, the mathematical relationship between the cause and effect can become cumbersome. If branch pipes, changes in diameter and friction are involved the relationship becomes even more complicated. For such cases it is often simplest to obtain the relationship between valve closure rate at some point 'A' and head rise at a point 'B' by trial. The relationship between Δh_B and partial valve closure could be computed at each successive time step. If the gradient is too steep or too flat the valve closure over the next time interval could be adjusted geometrically.

Thus assume the line $H_{BO}D$ in Figure 12.4 is a target head rise function at a specified node B. After one iteration with an assumed trial valve closure time T, the head H_B rises to C. This is less than expected so the valve closure could be speeded up by the ratio $S(H_{BO}D)/S(H_{BO}C)$ where S is the slope of the line in Figure 12.4. The correction to the closing speed of the valve could be continued each step.

It will, however, be found that if the time intervals are too large or the point B is too close to the valve A that overshoot or unexpected or even misleading answers can occur. A target closure

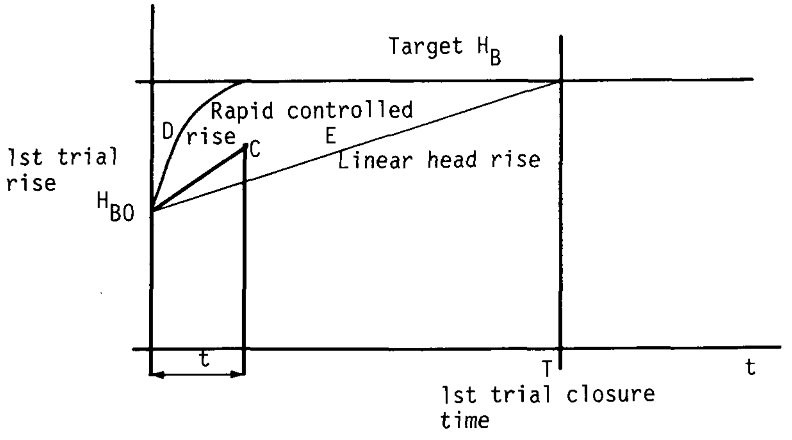


Fig. 12.4 Feedback method of controlling head at B

time and gate closure characteristic must be estimated beforehand and this can only be refined using this technique. If reflections occur somewhere in the system causing the heads to reverse or drop, obviously the proportional correction is unapplicable and the valve is being closed too quickly or the time step is too great.

The accompanying computer printouts and plots of heads versus time are from a program based on the above principle.

The programmer may enquire from the program over what time he should close the valve in order to not exceed a specified head at a specified point in the pipe network. The method is to employ a feedback principle and gradient method for projecting water hammer heads at the specified node. The program will therefore successively correct the closure time of the valve keeping an eye on the head increase at the designated node. In fact the head at the designated node will be plotted on the screen as computations proceed and subsequently be transferred to a graph such as Figure 12.6. The maximum head specified for node 2 was 1100 m above the datum, whereas the maximum which actually occurred was 1121 m c.g. 1167 m for the valve closure in 5 seconds in Figure 12.5

The most severe such conditions are likely to arise where there is a burst in a pipe which is to be followed by a rapid valve closure upstream of the burst. At steady-state runaway conditions the flow velocities can reach very high values. On the other hand the heads at the end of the system are consequently very low as most of the head will be dissipated in friction and through various fittings in the pipe system. The friction is therefore very high and the damping effect of the friction may often result in the water hammer pressure rising not much more than that for valve closure under normal operating conditions. Nevertheless each system should be analysed individually before such conclusions are made.

REFERENCES

Fox,J.A., 1977. Hydraulic Analysis of Unsteady Flow in Pipe Networks. Macmillan, London.

Streeter, V.L. and Wylie, E.B., 1967. Hydraulic Transients. McGraw Hill.

```
PIPE NET WATER HAMMER ANAL PROG

PIPE FLOW L/S DSHEAD SEC= 2.5
0   1    651      882
1   2    429      725
2   4    373        0
1   3    151      500
PIPE FLOW L/S DSHEAD SEC= 5
0   1    572      879
1   2    407      736
2   4    413        0
1   3    173      500
PIPE FLOW L/S DSHEAD SEC= 7.5
0   1    583      886
1   2    411      743
2   4    411        0
1   3    171      500
PIPE FLOW L/S DSHEAD SEC= 10
0   1    582      884
1   2    411      741
2   4    411        0
1   3    172      500
PIPE FLOW L/S DSHEAD SEC= 12.5
0   1    463      988
1   2    253     1022
2   4    236        0
1   3    188      500
PIPE FLOW L/S DSHEAD SEC= 15
0   1    224     1057
1   2      8     1167
2   4      0        0
1   3    204      500
PIPE FLOW L/S DSHEAD SEC= 17.5
0   1    182      948
1   2     -2      924
2   4      0        0
1   3    188      500
PIPE FLOW L/S DSHEAD SEC= 20
0   1    196     1004
1   2            1014
2   4      0        0
1   3    195      500
PIPE     RFLO     MAXH      TMAX
0   1    196     1068      15.50
1   2            1167      15.00
2   4      0        0       0.00
1   3    195      500       0.00
```

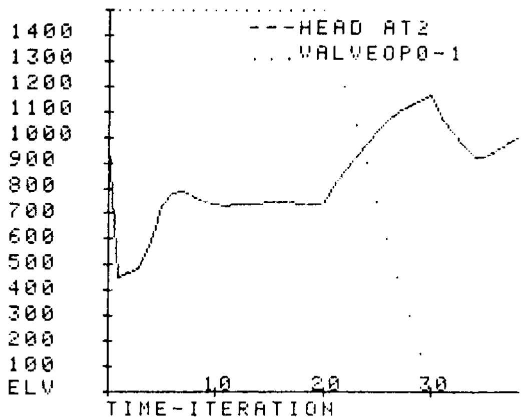
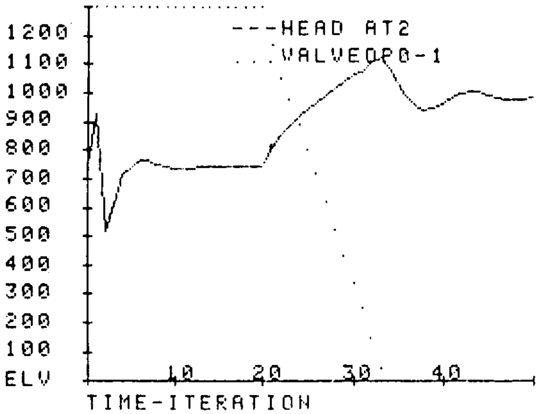


Fig. 12.5 Head variations at node 2 due to valve closure in 5 s after 10 s convergence




```
PIPE NET WATER HAMMER ANAL PROG

PIPE FLOW L/S DSHEAD SEC= 2 5
0   1   616    894
1   2   418    742
2   4   399      0
1   3   163    500
PIPE FLOW L/S DSHEAD SEC= 5
0   1   578    892
1   2   409    738
2   4   411      0
1   3   172    500
PIPE FLOW L/S DSHEAD SEC= 7.5
0   1   583    885
1   2   411    742
2   4   411      0
1   3   172    500
PIPE FLOW L/S DSHEAD SEC= 10
0   1   582    884
1   2   411    741
2   4   411      0
1   3   172    500
PIPE FLOW L/S DSHEAD SEC= 12.5
0   1   485    958
1   2   291    941
2   4   283      0
1   3   185    500
PIPE FLOW L/S DSHEAD SEC= 15
0   1   342   1004
1   2   136   1062
2   4   127      0
1   3   194    500
PIPE FLOW L/S DSHEAD SEC= 17.5
0   1   161   1028
1   2   -17   1036
2   4     0      0
1   3   203    500
PIPE FLOW L/S DSHEAD SEC= 20
0   1   208    972
1   2     7    966
2   4     0      0
1   3   189    500
PIPE FLOW L/S DSHEAD SEC= 22.5
0   1   186    993
1   2    -3    995
2   4     0      0
1   3   195    500
PIPE FLOW L/S DSHEAD SEC= 25
0   1   196    985
1   2     1    984
2   4     0      0
1   3   192    500
PIPE    RFLO    MAXH     TMAX
0   1   196    1048     17.00
1   2     1    1121     16.50
2   4     0       0      0.00
1   3   192     500      0.00
```

Fig. 12.6 Head variations at node 2 and valve closure pattern if maximum head at 2 is limited to 1100m above datum

CHAPTER 13

GRAPHICAL WATER HAMMER ANALYSIS

REASONS FOR GRAPHICAL APPROACH

Although not advocated for general use, the graphical methods of water hammer calculation still retain a place in water hammer analysis. The graphical method proposed by Allievi (1925) and developed by Bergeron (1935) and Schnyder (1937) is well suited for illustrating the mechanics of water hammer wave reflection. A clearer understanding of the wave reflections is possible by students than with the use of a computer program. Variable wave speed, changes in diameter, the effects of valve closure, water column separation and friction can all be portrayed graphically. The technique does become laborious if many steps are required, and the manpower to look at alternative designs often becomes excessive. Computer solution is more economic in the majority of cases, especially if multiple solutions are required.

BASICS OF METHOD

The graphical method is based on the linear relationship $\Delta H = (c/g)\,\Delta V$ (13.1)

If one plots head H against velocity V the line will have a slope of + or $-c/g$. The procedure is therefore to plot lines through the known (starting) points on an H versus V plot. Where the line crosses another known relationship between H and V (e.g. at a valve or another c/g line) the new conditions are obtained. It is in fact a way of solving two equations graphically for 2 unknowns, namely H and V.

The graphical method is in fact identical to the characteristics method, since one starts with the same equations (see chapter 9):

$$\text{for } \frac{dx}{dt} = +c \ ; \quad \Delta H = -\frac{c}{g}\Delta V - \lambda V|V|L/2gd \qquad (13.2)$$

$$\text{for } \frac{dx}{dt} = -c \ ; \quad \Delta H = \frac{c}{g}\Delta V + \lambda V|V|L/2gd \qquad (13.3)$$

where L is the length of pipe over which the flow velocity changes by V and head changes by ΔH.

The procedure employing the equations is to calculate graphically at intervals in time the head and velocity at selected points spaced L apart along the line. If L is not the total pipe length it is necessary that each L is such that waves arrive at the junction from either end at the same time. Thus each L/c should equal the selected time interval between computed conditions. In this way pipes with changes in wave celerity and even changes in diameter can be accounted for. In the latter case, and if branch pipes exist the diagram should be replaced by a H-Q plot (see later).

An explanation of the procedure follows with reference to Figure 13.1. On the H-V graph one marks the known conditions, namely H_o the static head, and V_o the initial line velocity. If line friction is significant the relationship between line velocity and line head at R is plotted as a parabola below the H_o line (the curve H_o-H_f). Note that on the negative V side the parabola curves upwards since $\lambda V|V|L/2gd$ is in the opposite direction.

The valve discharge characteristics are also plotted on the graph since a relationship between discharge through and head across the downstream valve is required to be solved simultaneously with the water hammer characteristic equation. The relationship between flow and head drop through a valve is generally assumed to be of the form $H_L = K\sqrt{V}$ where V is the pipe velocity which is directly proportional to flow rate in m^3/s. The factor K reduces as the valve is closed and it is a function of open area as well as discharge characteristic. If it is reduced in a defined way the factor K is known at each step of L/c seconds. Its initial value is obtained from

$$K_o = H_{vo}/\sqrt{V_o} \qquad (13.4)$$

where H_{vo} is the initial head loss through the valve (H_o-H_f) at a line velocity V_o. A parabola can be drawn through the points (H_o,V_o) with its apex at (0,0). Other parabolae can be drawn at different valve openings provided K is known.

Figure 13.1 illustrates the graphical procedure for the case

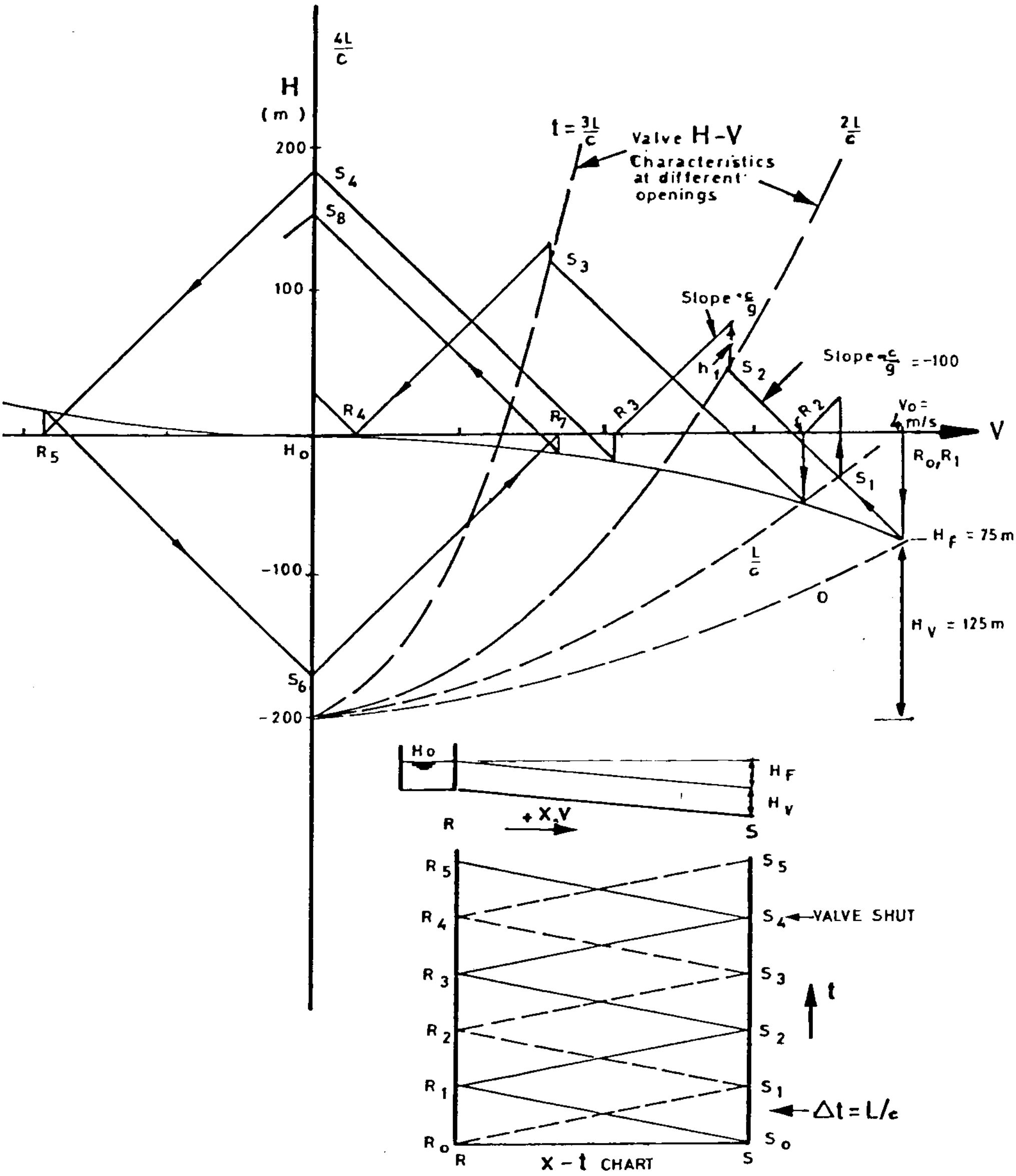


Figure 13.1 Graphical analysis for slow valve closure in gravity line with friction

of slow valve closure at the downstream end (point S) of a gravity line with initial friction head loss H_f.

The waves emanating at the valve proceed up and down the pipeline in time as indicated by the full lines on the x-t chart. The characteristic equations (13.2) and (13.3) are applied across the waves. For instance, in order to compute conditions at point S_1, commence at the defined boundary point R_0 and use the characteristic corresponding to $dx/dt = +c$ to obtain V and H at S_1. On the H-V graph the line starting at R_o (H_o, V_o) drops by $H_f = \lambda V|V|L/2gd$ and then proceeds along a line with slope $-c/g$. The point S_1 is also on the valve characteristic for time L/c.

It may be noted that the line friction is assumed concentrated at the upstream end in this case since it is the velocity at point R_o which was used in establishing the friction head. For characteristics in the opposite direction to compute conditions at point R, the velocity at the downstream end is used to assess friction loss. Thus, starting at point S_1 on the H-V diagram one draws a line up by H_f (the difference between the H_o and H_f lines at VS_1) and then draws a line with slope $+c/g$. Where that line intersects the H_o line is point R_2. One proceeds in this manner to establish points S_3, R_4, S_5, R_6 etc. Similarly, starting at point S_o one can establish successively points R_1, S_2, R_3, S_4 etc. (not done in Figure 13.1)

MID-POINTS AND CHANGE IN DIAMETER

It is frequently necessary to determine heads at intermediate points along the line, for instance in order that the pipe wall thickness can be varied to suit the maximum heads. In such cases the pipe is divided into a number of intervals. The heads at the end points are determined as before commencing from an adjacent known point and using the relevant characteristic equation to establish the boundary condition at the next time interval. The time interval between computations is $\Delta L/c$ where ΔL is the length interval. Each $\Delta L/c$ should be the same no matter how many intervals there are. There is the possibility of c varying from one interval to the other, which can be accommodated in this way. The friction head per interval is $\lambda V|V|\Delta L/2gd$.

The head and flow at intermediate points is established by projecting two characteristic lines from neighbouring points on each side. Where the two lines meet (slope - c/g from the upstream point and + c/g from the downstream point) is the head and line velocity at the mid-point.

The same procedure is applicable if there is a change in diameter. In such cases, however, it is convenient to plot H against flow rate Q not V. Then $\Delta H = \pm[\frac{c}{g}\frac{\Delta Q}{A} + \lambda Q|Q|\Delta L/2gdA^2]$. (13.5)
where A is the pipe cross sectional area so the slope of the lines on the H-Q graph are $\pm$ c/gA, where A (and c) and hence the slopes could be different for each section. The procedure is illustrated in Figure 13.2.

PUMPING LINES

When pumps trip in a pumping line there is first a drop in head downstream (on the delivery side) of the pump. The same graphical procedures can be followed to determine initial head drop and subsequent head rises.

It often occurs that the drop in head is sufficient to cause water vaporization at points along the line. The mechanics of separation and subsequent rejoining of the water columns was described in Chapter 10. A separate tally must be kept of vapour pocket volumes as they expand and later contract in order to identify the time the head rise occurs on rejoining. It may be necessary to divide the pipe into a number of increments in order to pick up the locations of vapour pockets.

A similar procedure can be adopted if water hammer discharge tanks are used to eliminate water hammer overpressures. In such cases the vapour pocket does not need to collapse before the head rises.

Many systems of protecting pumping lines against water hammer overpressures are possible. One method is the use of release valves. A valve immediately downstream of the pump non-return valve is opened after the pump is tripped and subsequently slowly

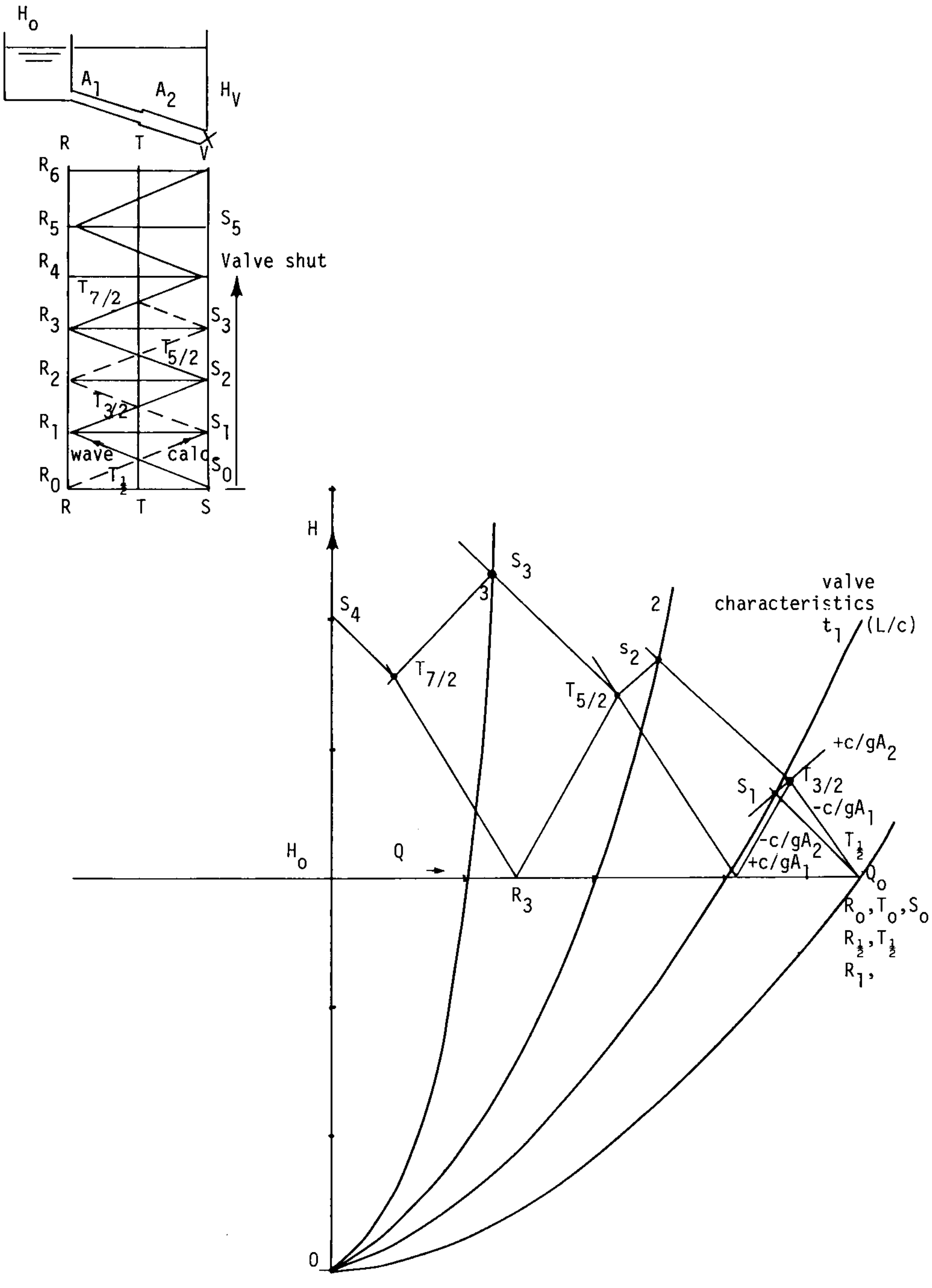


Fig. 13.2 Graphical calculation of water hammer heads at mid-line with change in pipe diameter

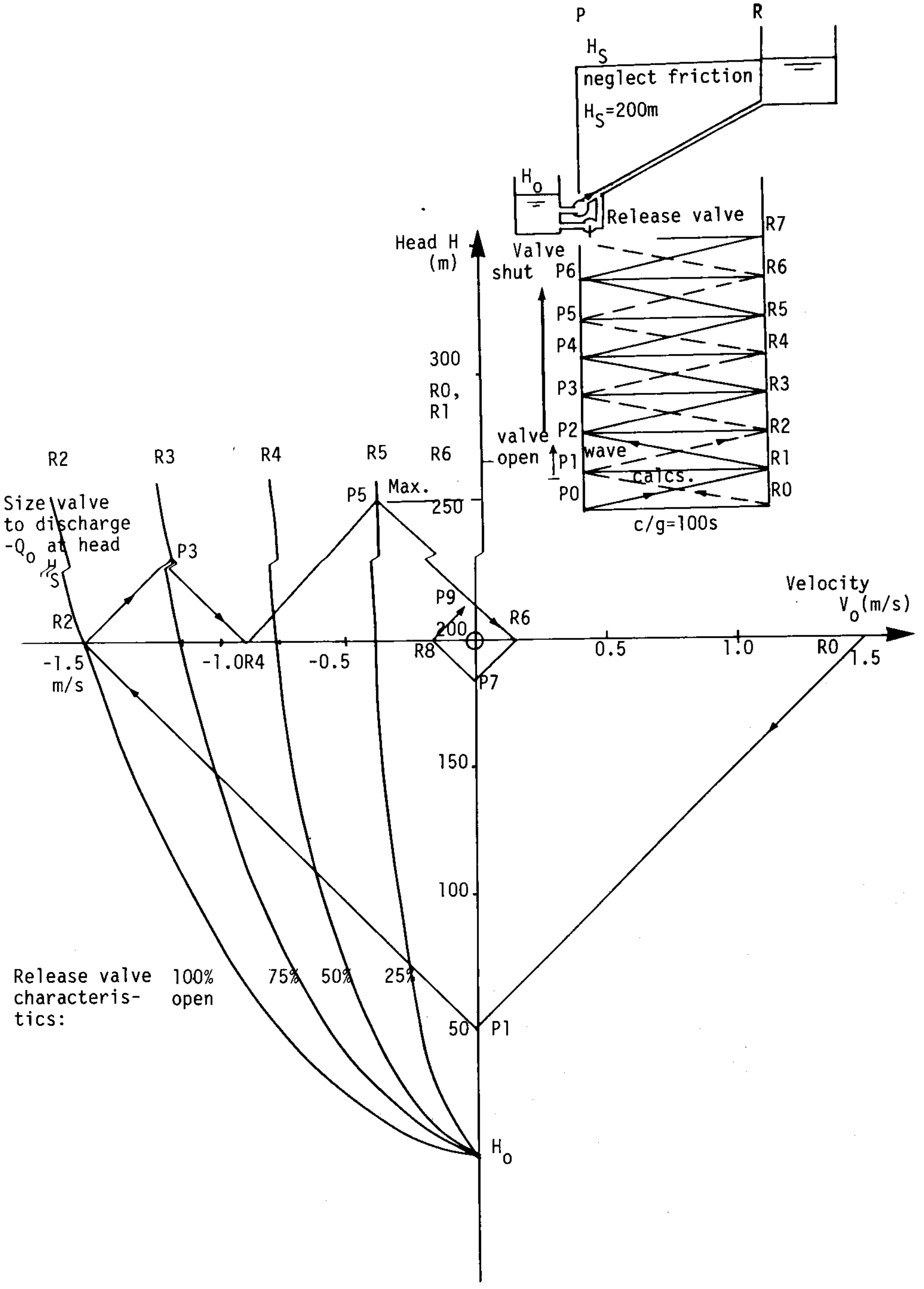


Fig. 13.3 Graphical analysis of pump stop with water hammer release valve

closed. The analysis of such a valve is done graphically in Figure 13.3.

The rotational inertia of the pump can also assist in reducing water hammer. Parmakian (1955) and Pickford (1969) describe methods of graphical analysis of the transients following pumptrip with the pump rotational characteristics included. For graphical analyses of surge shafts and rigid column transients, the reader is also referred to Jaeger (1956).

REFERENCES

Allievi, L., 1925. Theory of Waterhammer, translated by E.E. Halmos, Riccardo Garoni, Rome.

Bergeron, L., 1935. Etude des variations de regime dans les conduites d'eau: Solution graphique generale, Revue Generale de l'hydraulique, Paris. Vol. 1 p12-69.

Jaeger, C., 1956. Engineering Fluid Mechanics, Blackie & Son, London, 529pp.

Parmakian, J, 1955. Waterhammer Analysis, Dover Publics., N.Y., 161pp.

Pickford, J. 1969. Analysis of Surge, Macmillan, 203pp.

Schnyder, O. Nov., 1937. Comparisons between calculated and test results on waterhammer in pumping plant. Trans. ASME, 59, Paper Hyd-59-13, p695-700.

CHAPTER 14

PIPE GRAPHICS

INTRODUCTION

It is frequently informative and useful to draw a pipe network to scale. Sketch plans were utilized in the chapters on pipe network analysis and optimization and pipe profiles were drawn for water hammer analysis. Three dimensional depiction, or viewing from alternative angles would be even more useful. The relationship between elevation of the pipe and the pressure head at all points would be very illustrative when pressure limits and gradients have to be studied. Bottlenecks can frequently be pinpointed in this manner, as steep hydraulic gradients will stand out. High heads likely to cause bursts, and low heads resulting in insufficient pressures, will also be identified. This type of depiction is possible in 3-dimensional graphics.

Industrial water systems can be particularly complicated to visualize unless some form of graphical display is available. Water distribution networks and fire fighting systems supplying factory complexes on a number of levels are typical examples. These systems are frequently extended as the factory grows, and a record of pipe positions would be of use in later extensions. Suitable positions of connections, duplications and additional storage can be visualized on a 3-dimensional drawing.

The computer screen is the ideal place to depict such systems. Interactive construction and amendments to a pipe system can be performed before pipe runs, drawings and orders are finalized. A considerable cost savings is possible if mistakes are ironed out on the screen and not on site.

Computer graphics enables the drawing scale, size and viewing angle to be altered at will, until the best picture for any purpose is obtained. Various pipes, connections or valves may be best viewed from another angle and detail pictures of any particular part of the system can be called on while the engineer sits at the computer.

Graphics are particularly useful in the fabrication of industrial pipework subject to close tolerances. In fact for many plants, factories or refineries a physical model of the plant is made to iron out problems before construction commences. During the modelling problem areas are identified and remedial steps taken. For example pipes which get in the way of each other can be relocated. Support systems can be designed for stacks of pipes to minimize space and maximize the strength of the system. Tight corners, bends and intersections where erection is likely to be difficult will be identified. High head loss areas, e.g. sharp bends one after the other, valves and tees can be ironed out. Locations for measuring orifices in accessible, long, straight sections will be facilitated. It is preferable to sort out such problems at design stage with a physical model or graphical portrayal rather than at erection stage. Erection costs and re-fabrication of incorrect assemblies will be reduced. Erection time will be more rapid and further savings are possible.

It will be found that the cost of a computer graphics model is less than that of a physical model. It can be assembled faster, and can match many of the features of a real model, e.g. colour coding for pipes, standard codes for various devices such as valves, and ease of viewing. In some ways it is superior, e.g. rapid variation of viewing angle, zooming for closeups, linking to a hydraulic analysis model, and rapid alterations to the set-up. In some other features it is inferior. For instance, to view the plant from a slightly different angle may require a coded instruction, a clearing of the screen and re-drawing the system which takes time. The computer is less portable than a physical model and colour screen graphics suffer poor resolution, although this can be overcome by plotting the picture on paper (hard copy).

A computer program for drafting and design of pipe layouts can easily be linked to other programs for analysing flows and pressures, as well as programs for stress analysis. Temperature movements, superimposed loads and supports all add to longitudinal stresses, while internal pressures, supports and external loads cause circumferential stressing. Supports can be repositioned to reduce stress concentration by going back and forth between the layout and the analysis programs.

INTERACTIVE DRAWING

On some of the commercially available computer aided drafting (CAD) systems for piping, editing can be done directly on the drawing or screen. Data can generally be input off a sketch or drawing with the aid of a digitizer. That is, the location of a point is identified with a pointer on a drawing or a cursor or arrow on a screen. The cursor can be moved around by hand controlled buttons until it points to the correct spot on the screen. Then by pushing the correct key, a symbol such as a valve, a flange or reducer is reproduced on the drawing.

In the simple program accompanying data can only be read in or changed in alphanumeric mode, that is by means of co-ordinates and not while the picture is on the screen. This is a limitation of the computer used, but large systems can use more interactive drawing, that is adding to and editing while the picture is in front of one.

In many modern CAD systems the draftsman has a menu in front of him and identifies the correct symbol with an arrow before pushing a button to reproduce the symbol. The size and orientation of the device can also be controlled by keys or buttons suitably designated.

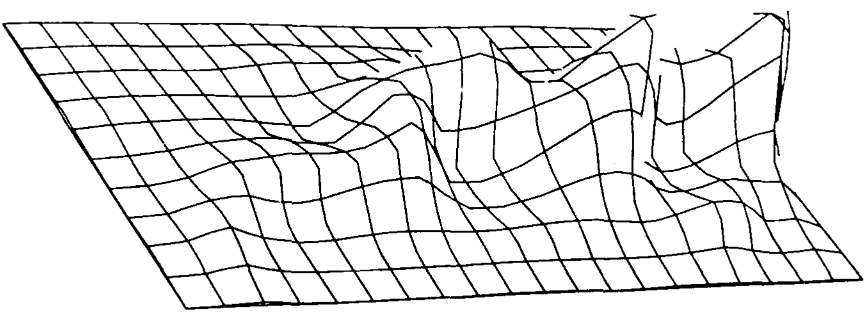


Fig. 14.1 3-dimensional depiction of a catchment

Many of the symbols are diagrammatic, i.e. distinguishing the characteristics of the device rather than its appearance. This assists in distinguishing between, for example, butterfly valves, globe valves, gate valves and control valves.

Figure 14.2 was abstracted by Lamit (1981) from ANSI standards and covers a wide range of symbols.

Annotation and lettering can also be done by locating the label sizing the letters and then typing the labels on a keyboard.

Once the drawing on the screen (cathode ray tube or C.R.T.) is to the satisfaction of the draftsman it may be reproduced on a proper sheet. This can be to a larger scale than on the C.R.T. and, if done on a drafting machine, the quality of lines and resolution will be much better than on the C.R.T. This is particularly so if colour graphics are used, as colour resolution is only 1/3 of that for black and white on a C.R.T. It is still often useful to use different codes to depict different types of line, for example to distinguish between gas, water, products, etc. by means of dashed, dotted or full lines.

COMPUTER PROGRAM FOR PIPE GRAPHICS

The appended computer program was written for a micro computer depiction of piping systems.

Program Description

The program as written can accommodate 100 pipes and 100 devices (flanges, valves or tanks). Unless detail close-up views are used this number of pipes will more than clutter the screen of the HP-85. Alternatively if a colour screen were available or if drawings were reproduced graphically on paper, different colours could be used to depict different pipes, e.g. cold water, hot water, sewers, ventilation ducts or high pressure. Alternative computers, e.g. the HP87, present different types of lines, e.g. dashed or dotted, which are also useful for distinguishing lines.

Input is typed below the program starting from line 700, and

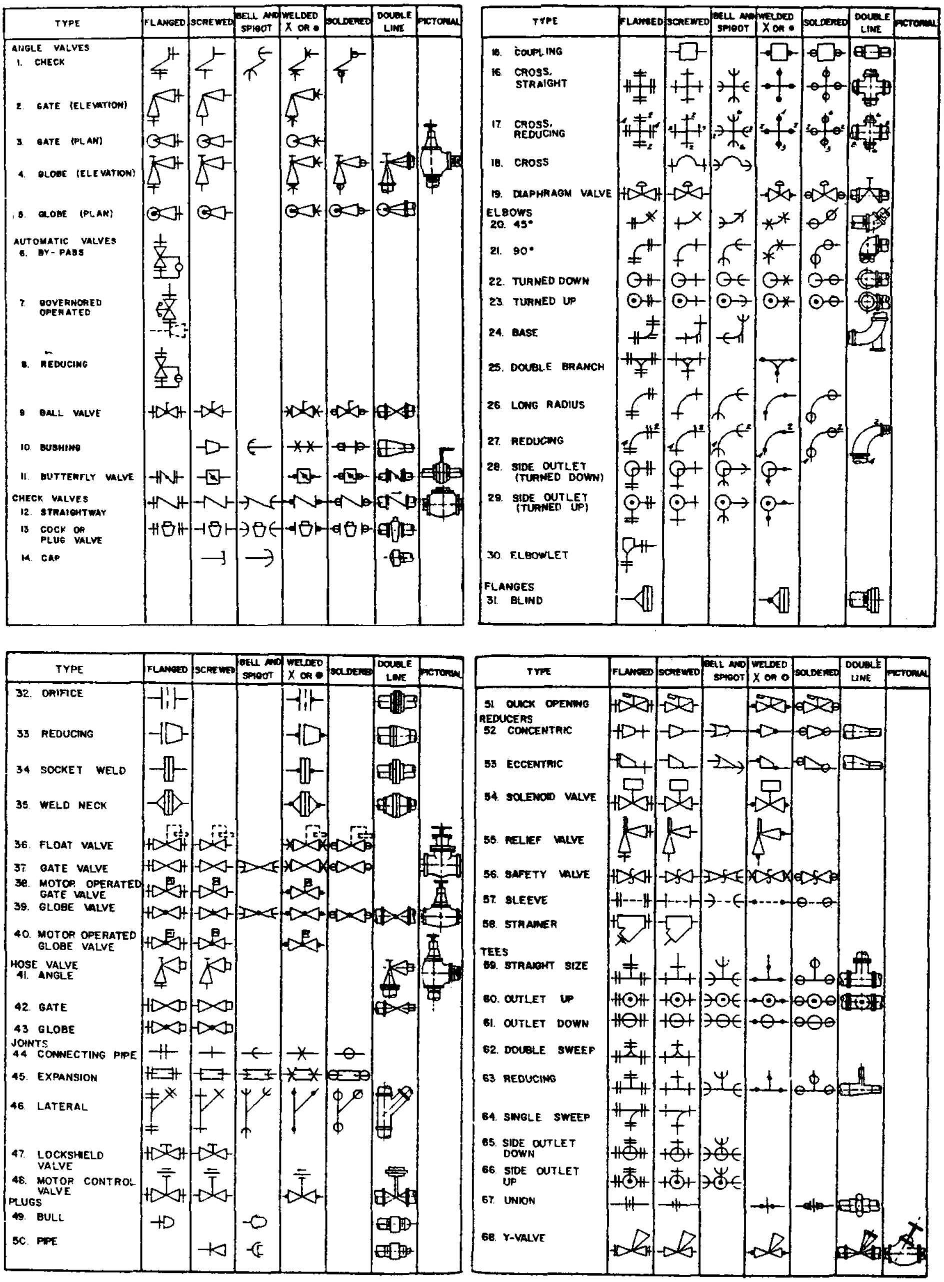


Fig. 14.2 Symbols for pipe fittings

requires line numbers as well as a DATA statement followed by:

Line 1: Left hand limit, Right hand limit, Lower limit, Upper limit of screen display (in metres or ft.), Angle of viewing measured from x to y axis, Angle up of viewing measured from x-z plane. Since the HP-85 screen is 4 x 3, the difference between the left and right hand limits should be 4/3 times the difference between the upper and lower limit for an undistorted scale. A practical angle combination for isometric viewing is 30°, 15°.

Line 2 onwards: Pipe data i.e., X1,Y1,Z1,X2,Y2,Z2 of start and end points on the pipe, Cost per metre of pipe.
After the last real pipe line, insert a data line with 0,0,0,0,0,0,0 to identify the last pipe.

Device lines: The information on devices: Pipe no. (in the order typed in), Distance from start point to end point measured along the pipe, Type of device (1 = flange represented by a vertical line, 2 = valve represented by X, 3 = tank represented by U), Size of device (m or ft.).
After the last device line, insert a data line with 0,0,0,0 to identify end of data. After running the program, it remains activated until PAUSE is pressed, or until K1 is pressed. In the latter case the program is reset to amend any pipe and a screen display requests the pipe number (in order of original pipe input), the new X1,Y1,Z1,X2,Y2,Z2 and C for amendment. Upon typing in this data and pushing END LINE, that pipe on the picture is altered and the cost also corrected. Note that this amend facility is not available for altering screen limits or viewing angles as the whole picture has to be drawn again in these cases.

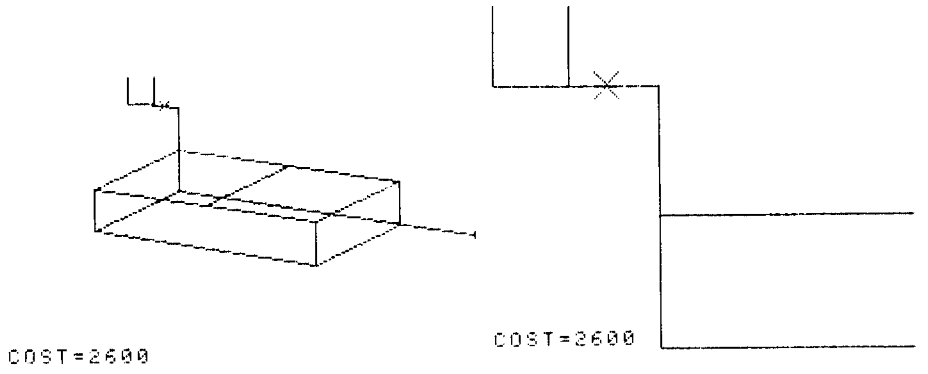


Fig. 14.3 Fire Distribution Piping Depiction

Example

The appended program is used to depict the fire fighting water distribution network to a double-storey building. Water is supplied from a roof tank through a double looped pipe network. An isolating valve is indicated downstream of the tank and a flanged connection extends from the lower floor to a future extension. Data describing the system follows the computer program. The cost of each pipe is estimated to be $10 per metre length.

Upon running the program the general view in Figure 14.3a was depicted and copied from the screen. The viewing angle is 30° from the x-axis and 15° up. A more detailed view (Figure 14.3b) of the supply tank from an (0,0) angle was also obtained. Note that in each case the total cost of the system is also indicated.

Symbols in pipe graphics program

A1	angle of viewing from x-axis measured in y direction
A2	angle of viewing from z-axis measured towards

	x-y plane.
C	cost of pipe per unit length
C2	total cost of pipes
H1	dimension of device
K	0 = original data, 1 = amended pipe data
L(M)	length
L1	distance along pipe from start to device
M	pipe counter
N	device counter
T1	device type. 1 = flange, 2 = valve, 3 = tank
U	x - co-ordinate
U0	left hand limit on screen
U1(M)	start point in screen x-co-ordinates
U2(M)	end point in screen x-co-ordinates
U5	x-co-ordinate of device in plane of screen
U9	right hand limit on screen
V	y-co-ordinate
W	z-co-ordinate
W5	z-co-ordinate of device in plane of screen
W0	lower limit on screen
W1(M)	start point in screen z-co-ordinate
W2(M)	end point in screen z-co-ordinates
W5	z-co-ordinate of device in plane of screen
W9	upper limit on screen
X1	start x of line
X2	end x of line
X3(M)	X1
X4(M)	X2
X5	x-co-ordinate of device
Y1	start y of line
Y2	end y of line
Y3(M)	Y1
Y4(M)	Y2
Y5	y-co-ordinate of device
Z1	start z of line
Z2	end z of line
Z3(M)	Z1
Z4(M)	Z2
Z5	z-co-ordinate of device.

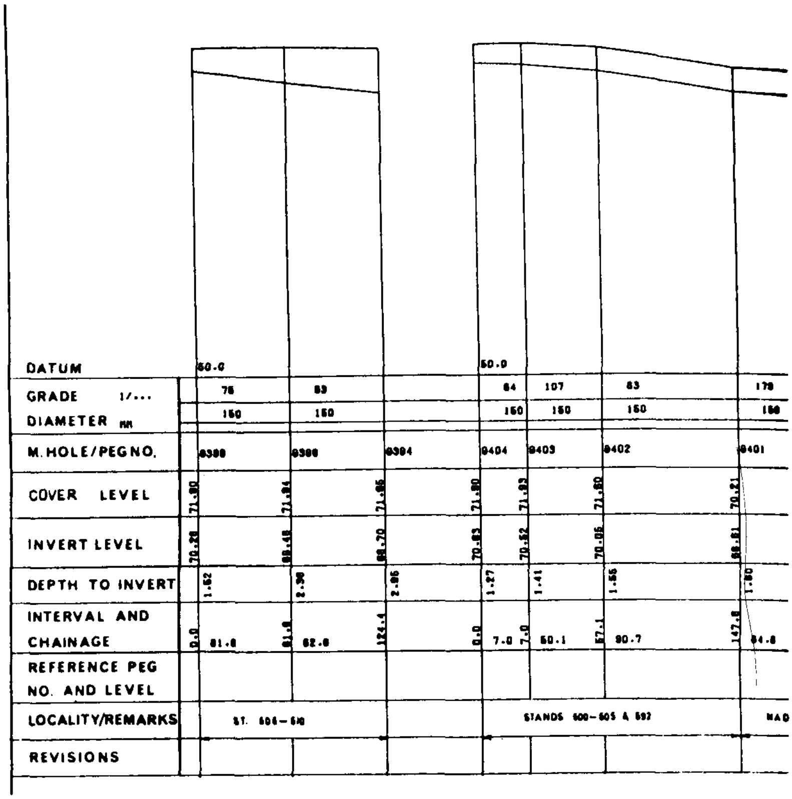


Fig. 14.4 Computer flatbed plot of graded sewer

COMPUTER GRADING

Pipelines are well suited for automatic grading by computer and drawing of sections by a linked plotter. A network is designed largely on empirical bases which can be readily programmed as constraints.

Although it is possible to start from data from a contour plan and develop the plan layout within the computer, it is preferable to lay the network out on a plan, peg and survey it in the field and feed the data from pegging sheets directly to the computer. A suitable program will then select the most economic depths and pipe diameters. The data may be displayed in summary form for visual inspection and adjustment if desired, taking off quantities. The results are then submitted to a separate plotting routine for drawing longitudinal sections. Such a plot is illustrated in Figure 14.14.

REFERENCES

ANSI 232.23. Symbols for pipe fittings

Lamit, L.G., 1981. Piping Systems, Drafting and Design. Prentice Hall, Eaglewood Cliffs. 612 pp.

```
10 ! PIPELINE GRAPHICS
20 ON KEY# 1 GOTO 130
30 DIM U1(99),U2(99),W1(99),W2(
   99),X3(99),X4(99),Y3(99),Y4(
   99),Z3(99),Z4(99),L(99)
40 GCLEAR
50 K=0
60 READ U0,U9,W0,W9,A1,A2
70 SCALE U0,U9,W0,W9
80 C2=0
90 FOR M=1 TO 100
100 READ X1,Y1,Z1,X2,Y2,Z2,C
110 IF C=0 THEN 400
120 GOTO 220
130 ALPHA
140 CLEAR
150 DISP "ENTER NEW M,X1,Y1,Z1,X
    2,Y2,Z2,C"
160 INPUT M,X1,Y1,Z1,X2,Y2,Z2,C
170 K=1
175 PEN -1
180 MOVE U0,W0
185 LABEL "COST="&VAL$(C2)
190 C2=C2-C*L(M)
200 MOVE U1(M),W1(M)
210 DRAW U2(M),W2(M)
220 X3(M)=X1
230 X4(M)=X2
240 Y3(M)=Y1
250 Y4(M)=Y2
260 Z3(M)=Z1
270 Z4(M)=Z2
280 L(M)=SQR((X2-X1)^2+(Y2-Y1)^2
    +(Z2-Z1)^2)
290 C2=C2+C*L(M)
300 DEG
310 U1(M)=X1*COS(A1)+Y1*SIN(A1)
320 U2(M)=X2*COS(A1)+Y2*SIN(A1)
330 W1(M)=Z1*COS(A2)+(Y1*COS(A1)
    -X1*SIN(A1))*SIN(A2)
340 W2(M)=Z2*COS(A2)+(Y2*COS(A1)
    -X2*SIN(A1))*SIN(A2)
350 PEN 1
360 MOVE U1(M),W1(M)
370 DRAW U2(M),W2(M)
380 IF K=1 THEN 510
390 NEXT M
400 FOR N=1 TO 100
410 READ M1,L1,T1,H1
420 IF M1=0 THEN 510
430 X5=X3(M1)+L1/L(M1)*(X4(M1)-X
    3(M1))
440 Y5=Y3(M1)+L1/L(M1)*(Y4(M1)-Y
    3(M1))
450 Z5=Z3(M1)+L1/L(M1)*(Z4(M1)-Z
    3(M1))
460 U5=X5*COS(A1)+Y5*SIN(A1)
470 W5=Z5*COS(A2)+(Y5*COS(A1)-X5
    *SIN(A1))*SIN(A2)
480 ON T1 GOTO 550,580,630
490 ! 1=FLANGE,2=VALVE,3=TANK
500 NEXT N
510 MOVE U0,W0
520 C2=INT(C2)
530 LABEL "COST="&VAL$(C2) ! "PR
    ESS K1 TO ALTER"
540 GOTO 540
550 MOVE U5,W5+H1/2
560 DRAW U5,W5-H1/2
570 GOTO 500
580 MOVE U5-H1/2,W5+H1/2
590 DRAW U5+H1/2,W5-H1/2
600 MOVE U5+H1/2,W5+H1/2
610 DRAW U5-H1/2,W5-H1/2
620 GOTO 500
630 MOVE U5-H1/2,W5+H1
640 DRAW U5-H1/2,W5
650 DRAW U5+H1/2,W5
660 DRAW U5+H1/2,W5+H1
670 GOTO 500
680 END
700 DATA -10,10,0,15,0,0
710 DATA -5,20,10,0,20,10,10
720 DATA 0,20,10,0,20,0,10
730 DATA 0,20,5,0,0,5,10
740 DATA 0,0,5,30,0,5,10
750 DATA 30,0,5,30,20,5,10
760 DATA 30,20,5,0,20,5,10
770 DATA 15,20,5,15,0,5,10
780 DATA 0,0,5,0,0,0,10
790 DATA 30,0,5,30,0,0,10
800 DATA 30,20,5,30,20,0,10
810 DATA 0,20,0,0,0,0,10
820 DATA 0,0,0,30,0,0,10
830 DATA 30,0,0,30,20,0,10
840 DATA 0,20,0,40,20,0,10
850 DATA 0,0,0,0,0,0,0
860 DATA 1,0,3,3
870 DATA 1,3,2,1
880 DATA 14,40,1,1
890 DATA 0,0,0,0
```

CHAPTER 15

COMPUTER PROGRAMMING IN BASIC

DESCRIPTION

BASIC is an elementary computer language orientated to technical problem solving, and somewhat similar to FORTRAN language. BASIC is often used on microcomputers which are designed to streamline the human input effort. Some of the statements in the language are actually commands to the computer as well as program steps.

The programming may be classified into various types, e.g. Statements, Functions, Operators and Commands. Statements are program lines which have to be numbered in the correct order although not necessarily sequentially, e.g.

10
20
21
50
200

Computer Commands

Some statements activate the computer, e.g.

AUTO [begin statement no., [increment]]e.g. AUTO 100,5 makes the computer start at statement 100 and automatically increment each number by 5 during input. Numbers in [] are optional.

CAT — produces a list of everything on the tape.

CONT [statement no.], continues execution of a program at the specified statement no. after a PAUSE.

COPY — reproduces information on the screen to paper.

DELETE first statement no. [last statement no.] removes those statements from the program.

ERASETAPE — initializes a tape

INIT — resets the program to the first line and sets all variables to undefined values.

LIST [begin statement no. [end statement no.]] lists the program on screen.

LOAD "program name" copies the program from a tape or disc to computer memory.

PLIST [begin statement no. [end statement no.]] lists the program on paper.

REN [first statement no.[increment]] renumbers the program lines as specified. Default values are 10,10.

RUN [statement no.] starts execution of the program from the specified statement.

SCRATCH deletes the program from memory.

STORE "program name". Stores the program onto tape or disc.

SECURE "filename", "security code", type. Type 0 secures against LIST, PLIST and EDIT. Type 1 against STORE, LIST, PLIST and EDIT. Type 2 against STORE, PRINT#,STORE BIN and Type 3 against CAT.
e.g. SECURE "PROG1","XY;" 1.

UNSECURE "filename", "security code", type.

ARITHMETIC

The following symbols are used in arithmetic statements

+ add

− subtract

* multiply

÷ divide

^ raise to the power.

The arithmetic functions have certain priorities and the precedence is ^÷,* and , + and −, thus A+B*C^D is A+{B*(C^D)}.

LOGICAL OPERATORS

= equal to

< less than

> greater than

≠ not equal to

VARIABLES

Any letter may be used to represent a variable, as well as any letter followed by a number, e.g. A,B3. Subscript variables are confined to letters, e.g. C(3). Up to two subscripts are possible, e.g. D(4,5). Nested parentheses are evaluated from inwards out.

Arrays should be preceded by a DIM (dimension) statement if subscripts exceed 10, e.g. DIM A(20), B(30,100).

PRECISION

REAL variables are accurate to 12 digits, while SHORT INTEGER variables are truncated to 5 digits. REAL precision is used unless variables are declared SHORT or INTEGER.

FUNCTIONS

ABS(x)	absolute, positive, value of x
ACS(x)	arcos (x)
ASN(x)	arcsin x
ATN(x)	arctan x
CEIL(x)	smallest integer<=x
COS(x)	cosin x
COT(x)	cotangent x
CSC(x)	cosecant x
DTR(x)	degrees to radius
EXP(x)	e^x
FLOOR(x)	same as INT(x)
FP(x)	fractional part of x
INT(x)	largest integer <=x
IP(x)	integer part e.g. IP(-3.276)= -3
LGT(x)	$\log_{10} x$
LOG(x)	natural $\log_e x$
MAX(x,y)	x or y whichever is largest
MIN(x,y)	x or y whichever is smallest
MOD	integer e.g. MOD B = positive integer value of B
PI	3.14159265359
RMD(x,y)	remainder of x/y-y*IP(x/y)
RND	next number x in a random sequence 0<=x<1
RTD(x)	radius to degrees
SEC(x)	secant x
SGN(x)	sign of x, + or -
SIN(x)	sine x
SQR(x)	square root of x

TAB(n)	skips to column n
TAN(x)	tangent of x
VAL(S$)	numerical equivalent of string S$
VAL$(x)	string equivalent of x

SPECIAL CHARACTERS

@	permits multi-statement line, e.g. 100 A=B @ GOTO 200
!	delimits a remark e.g. 100 A=B!A IS A NEW VARIABLE
?	prompts for input
" "	delimits a string of text which is displayed

BASIC PROGRAMMING STATEMENTS

The following statements can be incorporated in programs.

ASSIGN number TO "name", e.g. ASSIGN 2 TO "DATA"

CHAIN "filename". Loads and runs the program.The current program is deleted except for "common" variable and "assigned" buffers.

CLEAR Clears the screen

COM Common variable list. Used with CHAIN

CREATE "filename", number of records [number of bytes per record] Establishes a data file of the specified size.

DATA list. Provides constants and/or strings from which READ statements abstract data.

DEFAULT ON/OFF prevents or cancels some math errors from halting execution

DEF FN name [(parameter)][= expression]. Defines a special function within a program, e.g. DEF FN A(X)=SQR (X↑2+2*X)

DEG Sets degree mode for trig functions

DIM list. Declares the maximum subscripts for arrays. e.g. DIM A(20), B(50,100)

DISP [list] displays the list on the screen. e.g. DISP "A EQUALS", B

DISP USING format line [; list]. Displays on screen according to the format in format line. If a line number is used that line headed IMAGE acts as the format, e.g. DISP USING 50;A,B

END Terminates program execution

FOR counter = first value TO last value [step increment]. The statements up to a line NEXT counter are repeated for the counter proceeding from the first value to the last value. The counter step is one unless otherwise specified.

GOSUB statement no. The next statement executed is the specified statement in a subprogram. Control is returned to the main program with a RETURN statement.

GOTO statement no. Control is transferred to the designated statement

GRAD sets trig operations to grads. (400 grads in 360°)

IF expression THEN statement $number_1$ [ELSE statement $number_2$]. If the expression is true control is transferred to $statement_1$ and if not, to the next statement [or $statement_2$]

IMAGE format. Used with PRINT USING or DISP USING statements to specify the format of the output.

The following can be specified:

n(.....) repeats the operation in parenthesis n times
A string character
Z L.H. digit position or leading zero
D L.H. digit position or leading blank
. decimal point position
S sign (+ or -)
M minus or blank
E exponential form ESDDD
X blank
/ carriage return
" " literal
e.g. IMAGE 2D.DDD,2XZDD

INPUT variable name [, variable name ...] Variables are assigned values on the keyboard when a prompt "?" appears on the screen

INTEGER variable [subscripts] .... Specifies variables as integers and dimensions them

KEY LABEL Displays labels associated with ON KEY statements

LOAD BIN "filename". Loads a binary coded file from tape into memory

NEXT counter. Returns control to a FOR statement

ON ERROR GOTO statement number. Transfers to designated statement if a recoverable execution error is encountered

ON expression GOTO list. Transfers control to the statement no. in the list corresponding to expression being 1, 2, 3 etc.

PAUSE Halts execution until CONT is encountered

PRINT [USING statement number][list]. Prints the list on paper using the format in the specified statement number or in free format. Items in the list must be separated by commas or semicolons, e.g. PRINT USING 100;A,B(5), "IS THE ANSWER"

PRINTER IS code number. Redefines the printer. 1=screen, 2=paper

PURGE file name. Eliminates the designated file from tape

RAD Sets radian mode for trig functions

RANDOMIZE [expression]. Generates a random number seed. By specifying an expression the random number sequence can be repeated

READ [buffer no.] $name_1$[,$name_2$ ...]. Reads a string of variables from DATA statements or from a buffer

REAL variable [(subscripts)] ... Declares variables real and dimensions them

REM [anything] remark statement, not for execution, same as !

RENAME oldname TO newname. File is renamed

RESTORE [statement no,]. Resets data printer to the specified statement or to the beginning of the data file

RETURN Transfers control from the last line of a subroutine back to the original program line following a GOSUB statement

SHORT variable [(subscript)] ... Declares variables short and dimensions them

STOP — Terminates execution and returns pointer to first statement

STORE BIN "name". Stores the file named on tape

TRACE — Used to follow statements executed. NORMAL will cancel the TRACE operation

GRAPHICS STATEMENTS

The HP series 80 microcomputer enables graphs to be plotted on the screen and on paper using the following simple statements.

ALPHA — Sets display to alphanumeric mode

BPLOT string, number of characters per line. Plots a group of dots as specified by the string

DRAW x-co-ordinate, y-co-ordinate. Draws a line from current pen position to specified (x,y)

GCLEAR [y] — clears screen below specified y value

GRAPH — Sets display to graphic mode

IDRAW x-increment, y-increment. Draws a line from current pen position to position determined by the x and y increments

IMOVE x-increment, y-increment. Moves the pen by the specified increments, without drawing a line

LABEL string. Writes the string starting at the current pen position

LDIR angle. Specifies direction of label (angle=0 or 90°)

MOVE x-co-ordinate, y-co-ordinate. Moves pen from current position to (x,y) without drawing a line

PEN number. Specifies whether dots are black (negative number) or white (positive number)

PENUP — Lifts pen up

PLOT x-co-ordinate, y-co-ordinate. Makes a dot at (x,y)

SCALE x-min, x-max, y-min, y-max. Sets the x and y scale on the screen between the limits specified

XAXIS y intercept [,tic spacing [xmin, xmax]]. Draws a horizontal
axis with tic marks and within limits specified.
Negative tics specify the right side as reference

YAXIS x intercept [,tic spacing [ymin, ymax]]. Draws a vertical
axis

(By courtesy of the Hewlett Packard company.)

INDEX