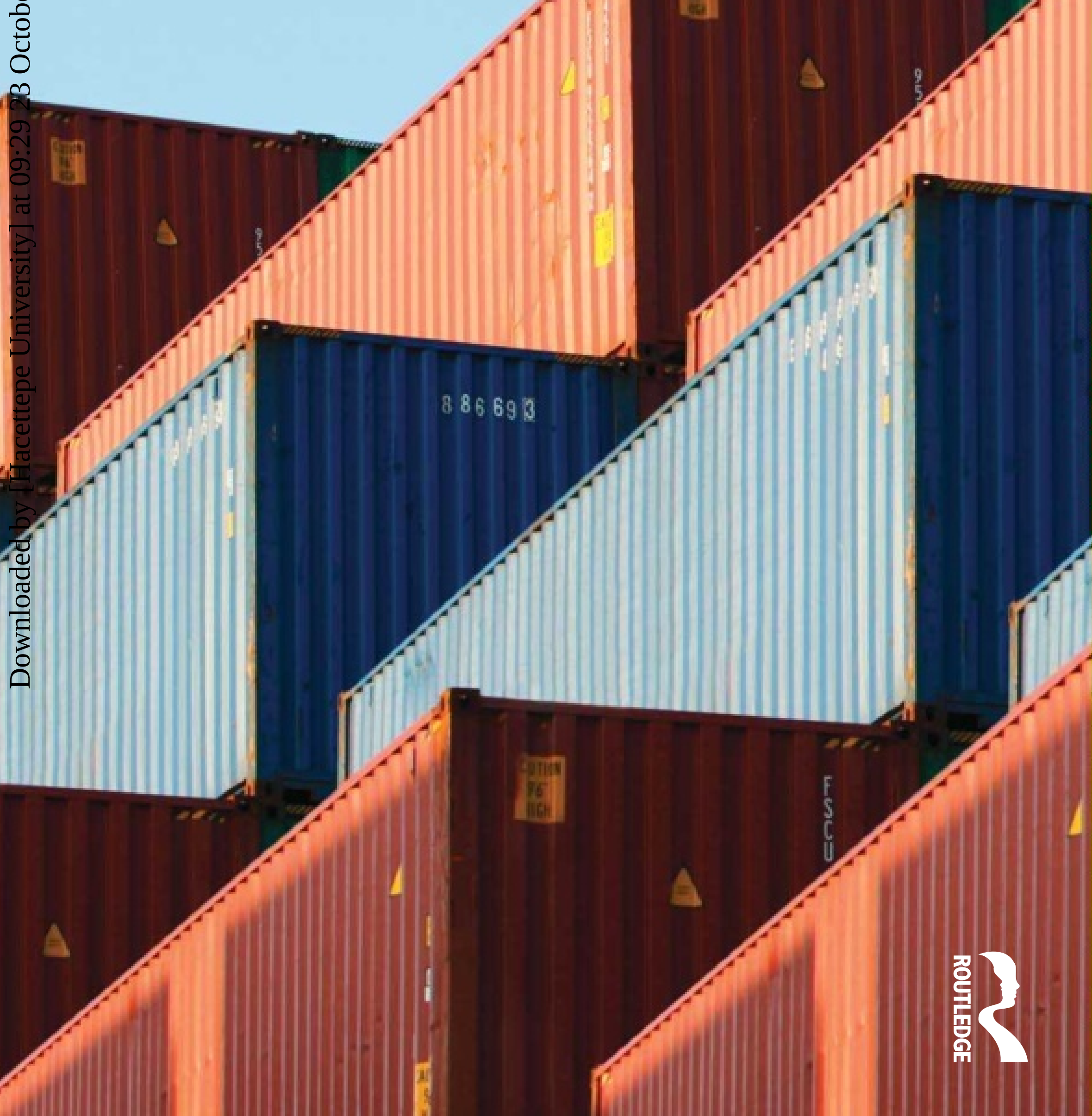
Agricultural Marketing

Structural models for price analysis

James Vercammen



ROUTLEDGE

Agricultural Marketing

The price of food has become very volatile in recent years for a variety of reasons, including a strengthened connection between the prices of agricultural commodities and other commodities such as oil and metals, more volatile production due to more frequent droughts and floods, and a rising demand for biofuels. Understanding the determinants of agricultural commodity prices and the connections between prices has become a high priority for academics and applied economists who are interested in agricultural marketing and trade, policy analysis and international rural development.

This book builds on the various theories of commodity price relationships in competitive markets over space, time and form. It also builds on the various theories of commodity price relationships in markets that are non-competitive because processing firms exploit market power, private information distorts commodity bidding, and bargaining is required to establish prices when the marketing transaction involves a single seller and buyer. Each chapter features a spreadsheet model to analyze a particular real-world case study or plausible scenario, and issues considered include:

- the reasons for commodity price differences across regions
- the connection between the release of information and the rapid adjustment in a network of commodity prices
- the specific linkage between energy and food prices
- bidding strategies by large exporters who compete in import tenders.

The simulation results that are obtained from the spreadsheet models reveal many important features of commodity prices. The models are also well suited for additional "what if" analysis such as examining how the pattern of trade in agricultural commodities may change if shipping becomes more expensive because of a substantial increase in the world price of oil.

Model building and the analysis of the simulation results is a highly effective way to develop critical thinking skills and to view agricultural commodity prices in a rigorous and unique way. This is an ideal resource for economics students looking to develop skills in the areas of Agricultural Marketing, Commodity Price Analysis, Models of Commodity Markets, Quantitative Methods and Commodity Futures Markets.

All the spreadsheets contained in the text book are available for download at www.vercammen.ca

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For Kelleen, Laura and Kelsey

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Preface

At the time of writing this Preface (August 2010) agricultural commodity prices are once again beginning to surge. Prices had surged in the 18 months leading up to the meltdown of world financial markets in the fall of 2008, but then retreated to early 2007 levels as a result of the market meltdown. The 2006–8 price surge has rightfully or wrongfully been attributed to a variety of factors including rapidly rising demand for agricultural commodities in emerging economies such as China and India, rapidly rising demand for corn and soybeans by the biofuels sector and large-scale speculation by hedge funds and other institutional investors. The current surge in commodity prices, including a near doubling in the world price of wheat over the past few months, is being blamed on a severe and widespread drought in Russia, and, more generally, a slowly increasing gap between the demand and supply of agricultural commodities.

In the fall of 2007 I was contacted by Rob Langham (Senior Publisher – Economics and Finance) from Routledge and asked to write a textbook on agricultural marketing. Rob was very concerned about seemingly run-away prices for food and the impact of food price inflation on the world's poor. He felt that a textbook was needed to help students view agricultural markets and commodity prices in an integrated economics framework, and to approach important real-world problems with a solid theoretical foundation and with rigorous quantitative methods. Rob stressed that the textbook should focus on how agricultural markets actually work and how commodity prices are actually determined versus how society would like markets to work and prices to be determined (i.e., positive versus normative economic analysis).

When Rob initially contacted me in 2007 my academic department at the University of British Columbia was in the process of planning a new professional masters program in food and resource economics. The book that Rob envisioned would work well for this program, so I now had additional incentives to launch into a three-year book writing project. When thinking about the style of textbook to write a colleague reminded me about Jon Conrad's 1999 textbook titled *Resource Economics*. Conrad's approach was to simplify relatively complex theoretical models and then present simulation results from the spreadsheet versions of the simplified models. This approach was appealing to me because it would allow students to see the various steps in constructing and solving a model as well as learning the underlying theory and associated issues. I have a long history of using spreadsheets in my teaching and research, so it was natural for me to make spreadsheet analysis a key part of my textbook.

This textbook is designed to equip students with knowledge about arbitrage and the law-of-one-price over space, time and form in competitive markets, and about various other aspects of price determination for agricultural commodities such as imperfect competition, competitive bidding and bargaining. The theory is presented in a "user-friendly" format, and step-by-step instructions are provided to help students master the art of building, calibrating and solving a quantitative model and then performing sensitivity analysis. The students that I teach are typically amazed at the diverse array of tools that are embedded in today's spreadsheet. Array formulas, look-up functions, inverse cumulative probability functions and various optimization tools add considerable power and flexibility to the spreadsheet when solving equilibrium price determination models.

Each chapter of this textbook has a similar format. The chapter begins with a brief description of the issue and then various types of data are presented to add realism to the analysis. A model that uses simple functional forms (e.g., linear, quadratic and constant elasticity) is then constructed, and the conditions that must hold to obtain a pricing equilibrium are specified. In some cases a real-world case study serves to motivate the spreadsheet application of the model. In other cases an artificial example with "realistic" parameter values is used. The formal part of each chapter concludes by using the spreadsheet model to generate base case simulation results and a series of sensitivity results (all spreadsheet models in the text are available for download at www.vercammen.ca). The questions at the end of each chapter are designed to allow students to solve "gentler" versions of the models that were formally presented. An annotated bibliography at the end of the last chapter refers the student to the relevant readings.

I would like to thank Rob Langham at Routledge for his insights and his patience. I would also like to thank three anonymous reviewers for their comments on earlier chapter extracts from this textbook. Their positive assessment allowed Rob to move ahead with the project and gave me confidence that this book could potentially fill an important niche in the agricultural marketing literature. My colleagues at the University of British Columbia also deserve credit for the feedback they provided me on various aspects of this textbook. Finally, I am indebted to Louisa Earls, Donna White, Lucy Spink and the other members of the editorial and production team at Routledge for guiding me through the complex process of preparing the manuscript for submission and creating this final product.

1 Introduction

1.1 Background

This book is about agricultural commodity prices. Commodity prices can be discussed in three dimensions: (1) long-term trends and price volatility over time; (2) pricing relationships at a particular point in time; and (3) the impact of a particular supply or demand shock on the full set of commodity prices (i.e., price integration). Figure 1.1 shows the daily spot price of live steers in the US between June 2004 and June 2009. Notice that steer prices are highly volatile, subject to repeating cycles and do not appear to be trending up or down over time. Figure 1.2 shows the daily spot price of Thai rice over the same June 2004 to June 2009 time period. In contrast to the price of live steers, the price of rice was very stable until early 2008, but then spiked to over US$1,000/tonne by the middle of 2008 and declined substantially along with most other commodity prices with the emergence of the global financial crisis in late 2008. Figure 1.2 reveals a long-term upward trend in the world price of rice.

Long-term price trends and price volatility over time are important from a public policy perspective. The world's poor and foreign aid agencies who distribute food to the poor are very vulnerable to upward trends and fluctuations in the price of stable commodities such as rice, wheat, maize and palm oil. Commodity price fluctuations also result in financial risk and planning uncertainty for farmers, food processors and other agribusiness firms. The sharp increase in prices for a wide array of agricultural commodities in 2007 and the first half of 2008 reignited the public debate regarding long-term affordability of food and the role of non-commercial speculation in agricultural commodity markets. The affordability debate has focused on the sluggish growth in global food supplies due to the on-going loss in arable farmland, climate change, a shrinking supply of fresh water for irrigation, a declining rate of productivity growth for crops and livestock and, more recently, the use of food for fuel. Critics of non-commercial speculation point out that in 2008 the number of agricultural contracts that traded on the Chicago Board of Trade rose by 20 percent to almost one million contracts, and during this same time agricultural commodity prices soared to unprecedented levels.[1]

Despite the public policy importance of price trends and volatility, this topic is too broad in scope to be included in this textbook. This book focuses on the

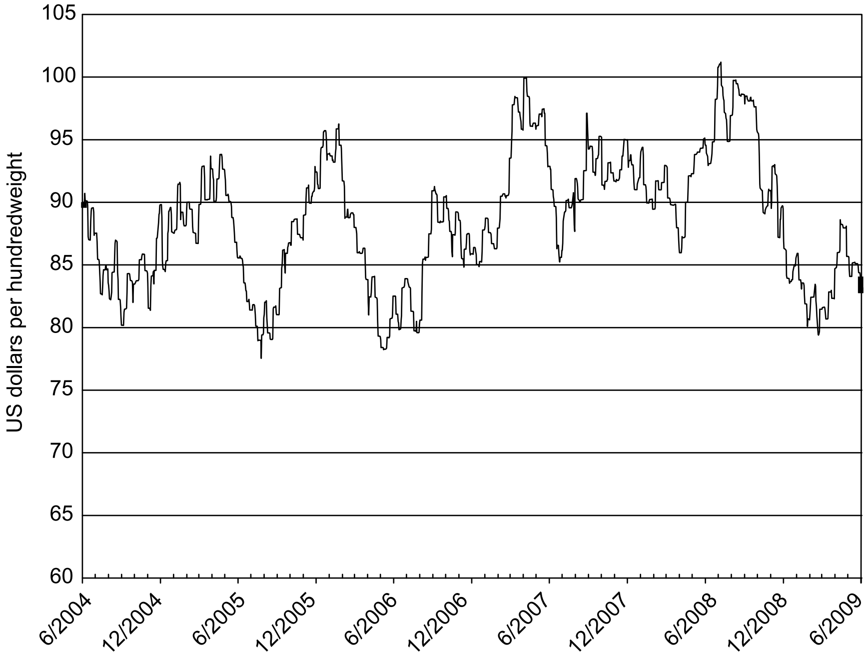


Figure 1.1 Daily live steer spot price, USDA weighted average (five regions): June 2004 to June 2009.

Source: Data from Datastream International Ltd/Datastream database (computer file): USTEERS. London: Datastream International Ltd, retrieved 10 June 2009.

equally important topics of structural pricing relationships at a particular point in time and price integration.[2] Pricing relationships for a particular commodity at a particular point in time have several dimensions. The pricing relationships for the following commodity pairs highlight the different dimensions: (1) a particular type of wheat in two different regions such as France and Saudi Arabia; (2) coffee beans in a Singapore wholesale market and a futures contract for coffee on the Singapore Commodity Exchange; (3) eggs at the farm versus retail level in Australia (i.e., the so-called marketing margin); and (4) a high versus low grade of rice at a Japanese wholesale market.

Price integration is a measure of the extent by which a supply or demand shock in a particular region of a particular market affects the relationship between: (1) the regional spot price and the corresponding futures price; (2) the spot prices in two different regions; and (3) the spot prices of substitute commodities. This textbook emphasizes long-run price integration, which is the change in pricing relationships after the adjustment to the new equilibrium is complete, rather than short-run integration, which is a particular path of price adjustment. As will be shown, a high degree of pricing integration is a standard feature of competitive global commodity markets.

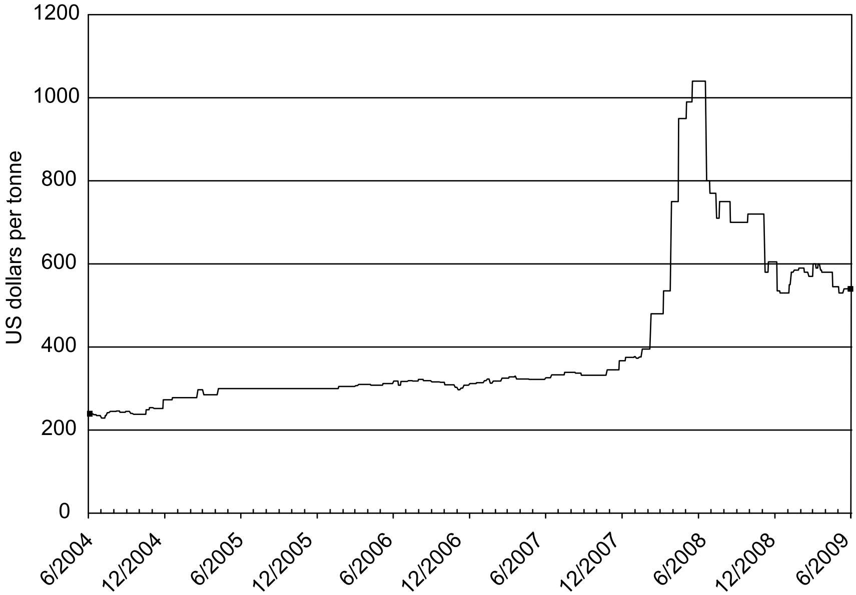


Figure 1.2 Daily rice spot price, Thailand, long grain 100% B grade (FOB): June 2004 to June 2009.

Source: Data from Datastream International Ltd/Datastream database (computer file): RCETILG. London: Datastream International Ltd, retrieved 10 June 2009.

The predominant theme in Chapters 2 through 5 is the law-of-one-price (LOP), which results from the actions of traders seeking arbitrage profits. The LOP gives rise to a specific set of pricing relationships at a particular point in time, and also gives rise to a high degree of price integration over time. In Chapter 6 the focus is on how substitution in supply and/or demand affects the degree of pricing integration for related commodities such as corn and wheat. A high degree of substitution implies that the price response to supply and demand shocks is dampened by substitution and offsetting changes in supply and demand in other markets. In Chapter 7 substitution by consumers of differentiated products determines the level of market power for processing firms, which in turn establishes the marketing margin and the set of equilibrium prices within the food supply chain.

Chapters 8 and 9 focus on two important institutional aspects of commodity price discovery: competitive bidding and bargaining. The assumption of perfect information is maintained for the analysis of competitive bidding, but the presence of private information by participating bidders implies that the LOP no longer holds. Private information induces participating suppliers to submit seemingly random bids that balance the benefit of bidding low, which increases the probability of winning, with the benefit of bidding high, which increases the value of the supply contract when a winning bid is submitted. In Chapter 9 bargaining theory is applied to a situation involving bilateral exchange. In this case the equilibrium price of the commodity depends on the distribution of bargaining power between the two agents, and this distribution in turn depends on the comparative value of the inside and outside options for the two bargaining agents.

1.2 Specific topics

The formal analysis begins with an examination of spatial pricing relationships. These relationships are determined by the particular pattern of excess supply and demand across regions and the matrix of interregional transportation costs. The key result of this analysis is that price relationships across space can be quite unstable in the sense that a comparatively small change in supply or demand in one region can result in a very different pattern of trade and set of prices. For example, a shortage in supply in a distant importing region can change a region from being a commodity importer with a relatively high price to a commodity exporter with a relatively low price. Understanding the reason for this "domino outcome" in spatial price analysis is important from both a business management and a public policy perspective.

Intertemporal price relationships at a particular point in time refer to the relationships between the spot price of a commodity and the set of commodity futures prices. The difference between the spot price and the futures price, which is referred to as the basis, and the price spreads for commodity futures contracts with different expiry dates provide important signals to commodity producers and merchants regarding how much of the commodity to produce and how much of current stocks to store for sale in a subsequent period. For example, news of the worsening of the drought in Australia in 2007 immediately drove up the price of wheat in all of the major spot and futures markets. Price responded rapidly to this news because traders anticipated that more of the current wheat stockpile would be stored to take advantage of the higher prices that would eventually emerge, and the higher volume of storage reduced the short-term supply of wheat to the market.

Substitution is an important determinant of agricultural commodity prices. For example, when prices change, farmers substitute toward the higher-priced set of production activities, feedlots substitute toward the lower-priced set of feed grains and traders change blending practices for commodities with quality variations. In 2009 news of the rapid spread of swine flu across multiple countries caused the price of hogs to tumble and the price of cattle to strengthen in commodity futures markets. The rapid price change occurred because traders anticipated a significant global substitution of beef consumption for pork consumption. Substitution is also a central feature in the food or fuel debate. Farmers have increasingly been shifting land out of crops destined for human food and toward biofuel crops such as corn and soybeans. As well, in response to the higher price of corn and soybeans, feedlots have substituted more non-corn and non-soybean ingredients in their feed mix. The combined effect of substitution by farmers and feedlots is believed to have resulted in a significantly higher price for human food.

Agricultural economists have long worried about excessively high marketing margins because a high margin implies relatively low prices for farmers and relatively high prices for consumers. The model developed in Chapter 7 shows that high marketing margins are the result of high fixed costs and high levels of market power by commodity processors. Market power and high fixed costs normally have a positive association because processing firms achieve market power by differentiating their product, and product differentiation normally raises a firm's fixed costs. For example, marketing margins for fresh fruits and vegetables are comparatively small because of low fixed costs and a reasonably high degree of product substitution. In contrast, processed fruits and vegetables generally have high marketing margins because the products have comparatively high degrees of differentiation, and firms require high margins to cover relatively high fixed operating costs.

As discussed above, the analysis of competitive bidding and bargaining in Chapters 8 and 9 is included in this book to highlight the fact that institutional arrangements can be important for price discovery. In Chapter 8 the theory of competitive bidding is used to analyze import tenders, which are routinely used by countries when importing agricultural commodities such as rice and sugar. Import tenders are an efficient way for the importer to achieve competition amongst potential suppliers, each of whom has private information about their opportunity cost of supplying the commodity. In Chapter 9 the theory of bargaining is used to analyze bilateral exchange between a producer association with single-desk selling privileges and a monopsonistic commodity processor. Understanding the role of inside and outside options in the bargaining process is key for understanding how prices are negotiated in a bilateral exchange environment.

1.3 Motivating data

The purpose of this section is to discuss a series of graphs that highlight the various pricing relationships that have been discussed above.

Spatial relationships

A typical spatial pricing relationship is shown in Figure 1.3. This diagram shows the price of canola in Edmonton (Canada) and the price of soybeans in Norte do Parana (Brazil) over the June 2008 to June 2009 time interval. The Dow Jones Industrial Average over this period of time is included as a pricing benchmark. On most days during the June 2008 to June 2009 period the price of canola is above the price of soybeans. This difference could be unique to this time period because of particular supply and demand conditions, or may reflect a more long-term and fundamental pricing relationship. The fundamental pricing relationship may reflect differences in transportation costs to key import markets, or may reflect differences in the value of the oil and meal that is derived from canola and soybeans.

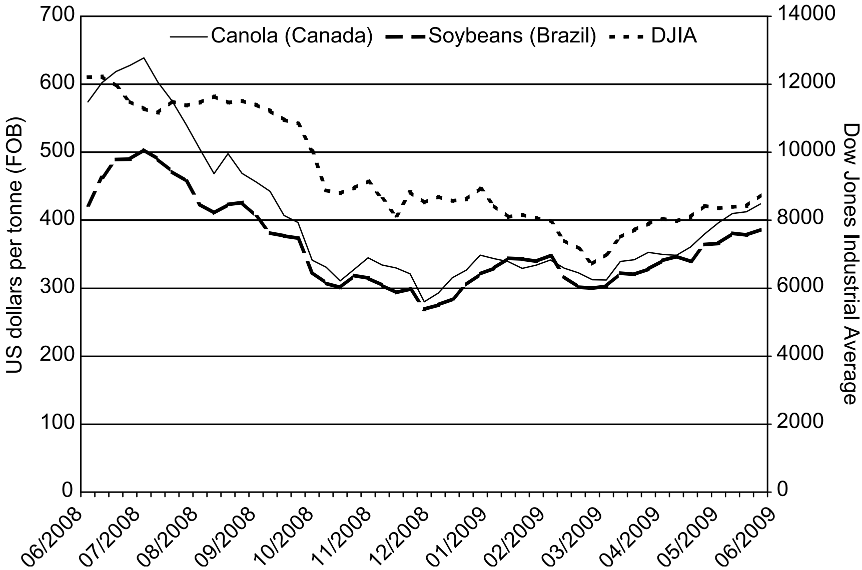


Figure 1.3 Weekly average of Dow Jones Industrial Average, and spot prices for canola (Edmonton, Canada) and soybeans (Norte do Parana, Brazil): June 2008 to June 2009.

Source: Daily commodity data from Bloomberg L.P. (2009). Canola FOB (R) Edmonton, Alberta and Soybean FOB (R) Norte de Parana, Brazil, 1 June 2008 to 1 June 2009. Daily Dow Jones Industrial Average data from Yahoo! Finance (2010) Dow Jones Industrial, 1 June 2008 to 1 June 2009. Data retrieved 10 June 2009 from Bloomberg and 23 August 2010 from Yahoo.

Figure 1.3 reveals that the prices of Canadian canola and Brazilian soybeans are moderately integrated over time. Some of this integration is due to the fact that both prices respond to general conditions of commodity demand, which is reflected by the value of the Dow Jones Industrial Average index. More importantly, however, the prices are integrated because supply and demand shocks in the Canadian canola market are transmitted into the Brazilian soybean market and vice versa. This integration occurs because the spot prices for canola and soybeans are both derived from centralized commodity futures prices. The high degree of substitution between these two commodities implies that traders in the canola and soybean markets, who are continually searching for profitable arbitrage opportunities, can fairly rapidly shift stocks of canola and soybeans across regional markets in response to supply and demand shocks.

Figure 1.4 shows the strong correlation between the price of soft white winter wheat over the June 2008 to June 2009 time interval for two US delivery stations: Bannister, Missouri and Commerce, Colorado. Without knowing the specifics of the winter wheat market it is not possible to explain why the price of winter wheat is higher in Colorado than it is in Missouri, and why the price difference steadily

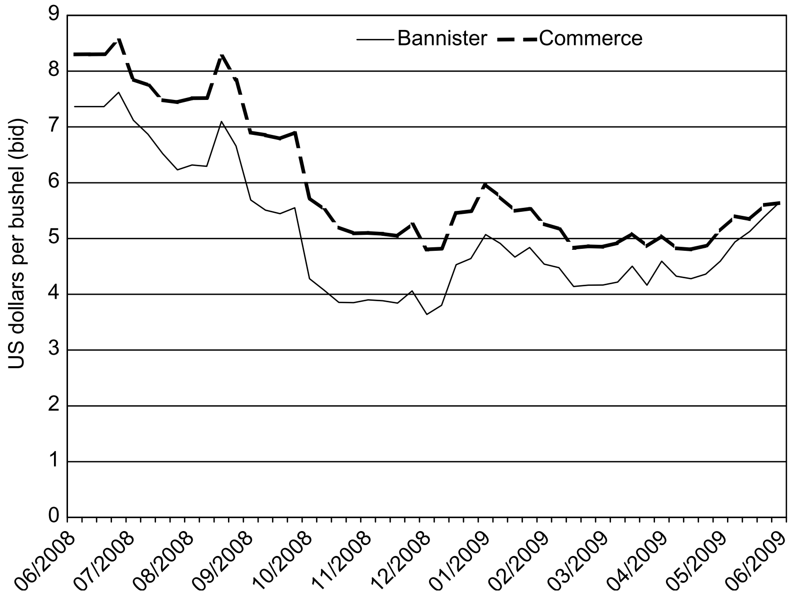


Figure 1.4 Weekly average of spot prices for soft white winter wheat (Bannister, Missouri and Commerce, Colorado): June 2008 to June 2009.

Source: Data from Bloomberg L.P. (2009). Wheat (SWW) bid (R), Bannister, Missouri, SLF Grains, and Commerce, Colorado, Con Agra, 1 June 2008 to 1 June 2009. Retrieved 10 June 2009 from Bloomberg database.

shrank between June 2008 and June 2009. Soft white winter wheat is a relatively minor crop in both regions, so one possible explanation is that the wheat is being processed locally in Colorado whereas it is being exported from Missouri. Local processing typically results in a higher price because the cost of transporting the grain to the export market does not depress the regional selling price.

Substitution relationships

As discussed above, a high degree of crop substitution by farmers and feed grain substitution by feedlot managers implies a strong connection between the price of food and the price of energy via biofuels processing. Figure 1.5 highlights pricing integration for corn and ethanol in the US state of Iowa over the June 2008 to June 2009 time interval. Corn and ethanol prices tend to move in tandem because the price difference between these two commodities is the primary determinant of the profits earned by an ethanol manufacturer. Thus, if the price of ethanol increases, the resulting increase in the production of ethanol will increase the demand for corn, which in turn will bid up the price of corn. A decrease in the price of ethanol will have the opposite effect on the price of corn.

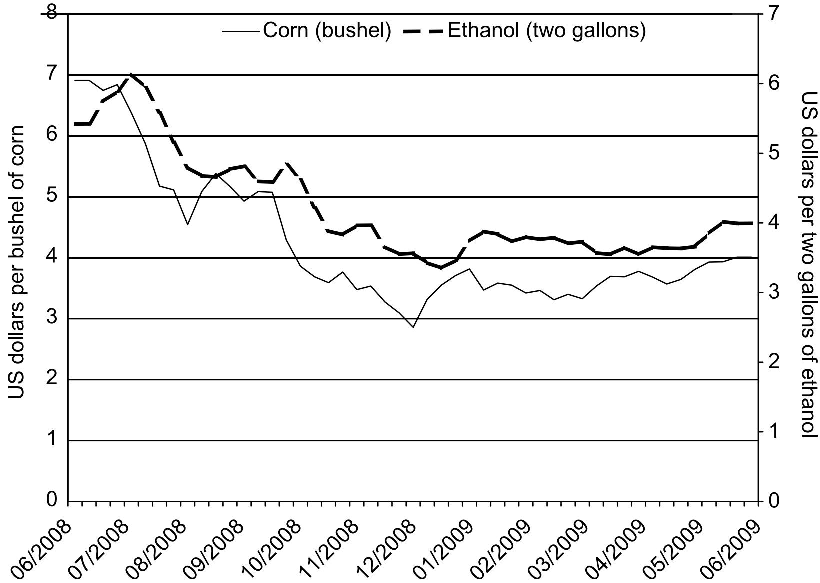


Figure 1.5 Weekly average of spot prices for yellow corn (Iowa) and ethanol (Des Moines, Iowa): June 2008 to June 2009.

Source: Data from Bloomberg L.P. (2009). Ethanol, Des Moines, Iowa FOB, and corn (yellow), Iowa (avg) – bid (R), 1 June 2008 to 1 June 2009. Retrieved 10 June 2009 from Bloomberg database.

The price of ethanol is strongly linked to the price of oil, and the price of corn is strongly linked to the price of other agricultural commodities. Thus, the growing size of the ethanol market implies a strengthening linkage between the price of oil and the price of food. In fact, the relatively strong association between the Dow Jones Industrial Average and the price of soybeans that was shown in Figure 1.3 may be partially the result of biodiesel processing. Critics of biofuels policy argue that mandates for minimum biofuel percentages in gasoline and diesel result in volatile commodity prices because mandates make the demand for biofuel crops by biofuel processors highly inelastic. The inelastic demand for biofuel crops will necessarily exacerbate price spikes in the corn and soybean markets during periods of low stocks.

Many agricultural commodities, particularly crops, differ in quality because of the impacts of weather, disease, insects, etc. Quality differentiated commodities are typically graded, and the price premiums and discounts for the different grades are determined in the market place through conventional market forces and the degree of substitution across different quality versions of the commodity. If a high quality commodity is in short supply, then the grade premium will be relatively large, and the opposite is true if the stocks of high quality commodity are relatively large. Figure 1.6 shows price and the quality premium (expressed as a percent) for grade 1a and 1b cocoa beans in Malaysia for the June 2008 to June 2009 time interval. The premiums are not large, but their variation over time is significant. Speculators actively monitor price premiums and discounts for the different grades of a commodity in an attempt to find arbitrage profits. As well, price premiums and discounts imply that traders have an incentive to blend different quality versions of the commodity in an attempt to raise arbitrage profits.

Intertemporal relationships

For storable commodities, the LOP ensures that intertemporal pricing relationships exist at a particular point in time. Figure 1.7 shows the deviation of the monthly price of wheat in Kansas City (US) from the annual price of wheat, averaged over the years 1970 to 2008. Notice that the June price tends to be 20 cents per bushel below the annual average whereas the February price tends to be about 17 cents per bushel above the annual average. In general, the price of wheat rises between July and February and falls between February and July. This pricing pattern is consistent with the harvesting and storage pattern of wheat in Kansas. Because wheat is harvested in late spring, price is relatively low in the months following harvest and is relatively high in the months leading up to harvest. The higher price in the pre-harvest period compensates traders who choose to store the commodity. The

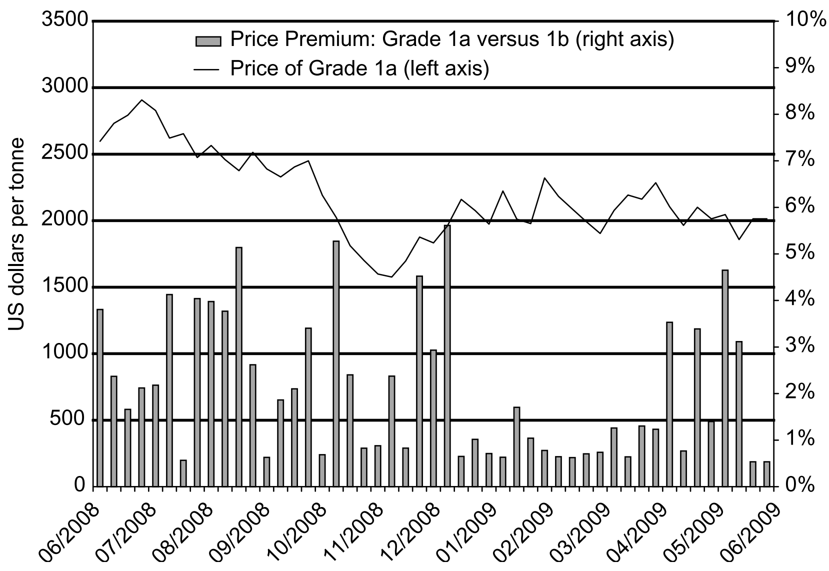


Figure 1.6 Weekly average of spot price and grade premium for dry cocoa beans: Sabah, Malaysia: June 2008 to June 2009.

Source: Data from Bloomberg L.P. (2009) SMC 1a and 1b dry cocoa bean, Malaysia, Sabah, Tawau, 1 June 2008 to 1 June 2009. Retrieved 10 June 2009 from Bloomberg database.

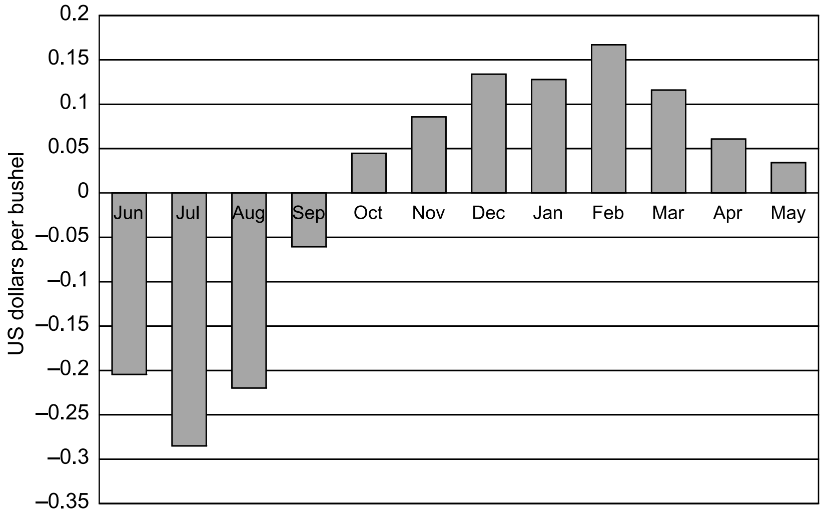


Figure 1.7 Monthly price minus annual price for #1 hard red winter wheat, Kansas City, Missouri, 1970–2008 average.

Source: Table 19 – Wheat: cash prices at principal markets. *Wheat Data: Yearbook Tables* (various years), Economic Research Service, USDA, Agricultural Marketing Service, Grain and Feed Market News.

monthly price differentials that are displayed in Figure 1.7 will be partially reflected in the set of commodity futures prices for wheat.

The top graph in Figure 1.8 is the daily price of a March 2010 soybean futures contract over the August 2009 to early March 2010 time interval. The bottom graph is the price spread for the May 2010 and March 2010 soybean futures contracts over this same time period. The price spread is intended to provide compensation for traders who store soybeans between March 2010 and May 2010. Theory suggests that this spread cannot exceed the unit cost of storage because if it did a trader with the capacity to store soybeans could lock in a profit by simultaneously contracting to accept delivery in March via a long March 2010 futures position and make delivery in May via a short position in a May 2010 futures contract. Theory does not impose a minimum value on the price spread because it is not possible for the trader to borrow stocks from the future and deliver them in the current time period if the price spread becomes excessively narrow or negative. It is for this reason that the price spread for soybeans is able to take on a negative value from late August to early November 2009. A negative price spread is commonly referred to an "inverted" market.

Figure 1.8 shows that the March 2010 to May 2010 price spread is highly volatile over time. It should be obvious that there is no predictable pattern in either the price of soybeans or the price spread. If there was a predictable pattern traders

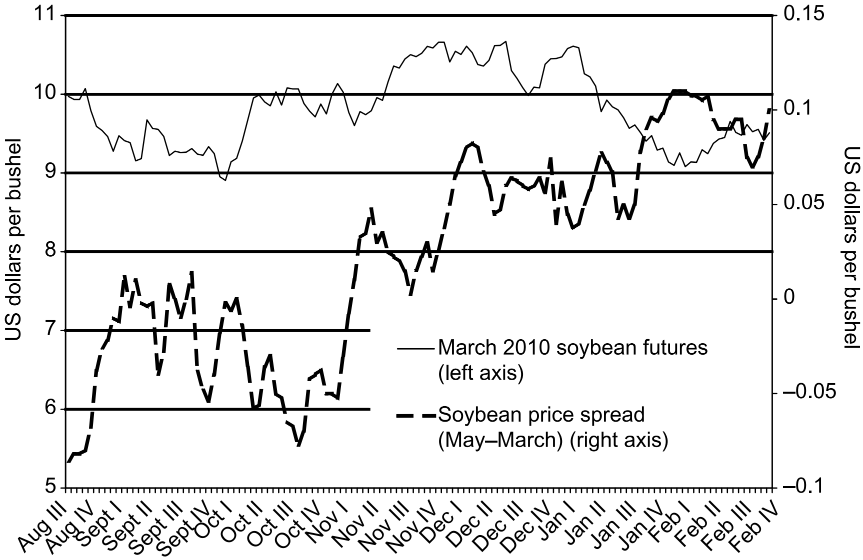


Figure 1.8 March 2010 soybean futures price and price spread for May and March soybean futures: August 2009 to March 2010.

Source: CBOT settlement data was downloaded daily from the CBOT website (http://www.cmegroup.com).

would exploit this through profit seeking trading, and these trades would eliminate all predictable patterns. Is the spread responding in some unpredictable way to fundamental factors of supply and demand in the soybean market and in other commodity markets? Chapter 4 of this text advances two theories concerning why price spreads fluctuate over time, often at a level that fails to provide adequate compensation to traders who choose to store the commodity over time. The theories are useful for explaining long-term patterns in price spreads but have little to say about the causes for short-run price spread volatility, such as the type revealed in Figure 1.8. In general, economists have a long way to go toward advancing a satisfactory explanation of commodity price spreads.

1.4 Outline of this book

Each chapter of this book has the same basic structure. After discussing the underlying theory a simple model is constructed and the conditions that are required to solve for the pricing equilibrium are specified. The model is then entered into a Microsoft Excel workbook, and the parameter values from a case study or hypothetical situation are assigned. Each chapter concludes with a description of how the model is solved, a presentation of the numerical simulation results, and some type of sensitivity analysis.

Chapter 2 begins the analysis of commodity prices by examining the relationship between competitive prices over space. The analysis considers a group of exporting regions and a group of importing regions connected through trade. A spatial equilibrium model is constructed and numerically calibrated for the case of trade in tomatoes. Optimization of the model simultaneously generates the volume of trade and the set of import and export prices within each region. The calibrated model is then used to examine how regional trade and the set of spatially-connected prices respond to exogenous events such as a major supply reduction in one of the regions, or a major increase in the cost of shipping due to a surge in the price of energy.

Chapter 3 continues with the LOP analysis by examining the role of storage as a mechanism for linking commodity prices over time. Dynamic programming techniques are used to obtain the intertemporal LOP for a situation where excess inventory can be stored and later sold in order to maximize the discounted market value of the commodity. A key consideration with intertemporal LOP is that the commodity can be stored forward through time, but borrowing from the futures is not possible. The corner solutions that arise because of this non-negativity constraint imply that price spikes in response to supply and demand shocks are relatively common. A case study of a historical Australian wool reserve scheme is initially solved with non-stochastic supply and demand in order to analytically derive the main LOP results analytically. Numerical dynamic programming procedures are then used to solve for the intertemporal LOP in a scenario where grain is produced with harvest uncertainty. The main result from this analysis is that harvest uncertainty combined with the non-negative storage constraint raises the equilibrium level of storage because traders anticipate future "stock-outs" and the associated price spikes.

Chapter 4 continues with the analysis of prices over time by constructing a simple model of a commodity futures market. In this model speculators trade in a centralized market with a distribution of beliefs about levels of future production. Price discovery in the futures market informs traders in the spot market, who must decide how to allocate their inventory between current sales and storage for future sales. Modeling commodity futures is somewhat complicated because it involves modeling the decisions of forward looking traders who rationally anticipate current and future market outcomes, including the probability that the market will stock out in the future. Chapter 4 concludes with a separate model of convenience yield as a key determinant of price spreads and the market basis. Convenience yield and the potential for a future market stock out are two important reasons why price spreads over time are positively correlated with stock levels.

Chapter 5 considers commodity prices over form by examining quality differentials in a competitive market and the economics of blending and grading. A simple model is constructed where competitive traders seek profits by blending low and high quality versions of a commodity, thereby arbitraging implicit price differences across product form. Grading creates corner solutions during blending arbitrage because the commodity will be blended until the quality is reduced to a minimum acceptable level for a particular grade category. The model assumes that the prices of the various grades of a commodity, and minimum quality standards for each grade are exogenous. The goal is to solve for the equilibrium prices (either implicit or explicit) of the non-blended versions of the commodity. These equilibrium prices are obtained by recovering the set of shadow prices of the resource availability constraints in the linear programming model of value maximizing blending.

Chapter 6 considers how prices are interconnected in a multi-market setting. The goal is to show how a supply or demand shock for one commodity affects the prices of other commodities because of commodity substitution in commodity supply and demand. The model is based on the popular constant elasticity of substitution (CES) function, which is used to represent the production possibility frontier of the farming sector as well as the production isoquant of the livestock sector. Different settings of the substitution parameter for the farming and livestock sectors give rise to different strengths of market connections and thus different pricing impacts that result from a supply or demand shock. A calibrated model of corn and ethanol production in the US Midwest shows how human food markets are connected to energy markets as a result of production substitution by farmers and feed grain substitution by livestock feedlots.

Chapter 7 marks the beginning of the departure from the competitive market assumption. For each of the five pricing scenarios that were highlighted in Chapters 2 through 6 the market equilibrium can be described as a LOP outcome and the LOP outcome can be derived by solving the social planner's problem of maximizing net aggregate welfare. In Chapters 7 through 9 the principles of the LOP, and the equivalence of social welfare maximization and the LOP outcome, no longer hold. Chapter 7 allows for market power by the food processing sector. Chapter 8 focuses on competitive bidding with incomplete information and Chapter 9 examines equilibrium prices in a bilateral monopoly bargaining scenario.

In Chapter 7 a set of monopolistically competitive food processing firms are assumed to each sell a differentiated product to retail customers. The degree of substitution across products implicitly defines the level of market power that each firm possesses. To create the differentiated product the monopolistically competitive firm demands a raw commodity ingredient from the farming sector. In one scenario the food processor has full market power when purchasing the raw ingredient from farmers and in a second scenario the processor is assumed to be a price taker. The focus of the analysis is the degree of product substitution at the retail level as a determinant of the size of the farm to retail marketing margin. The integrated model allows for both processed goods, which have a relatively low degree of product substitution and thus relatively high marketing margins, and semi-processed goods, which have a relatively high degree of product substitution and thus relatively low marketing margins.

Chapter 8 returns to the assumption of a non-differentiated commodity, but it allows for asymmetric information between buyers and sellers. Specifically, sellers of a commodity have different reserve prices, and these prices are not known by a commodity buyer (e.g., a state procurement agency). The agency uses a sealed-bid auction mechanism to minimize the cost of purchasing the commodity subject to the hidden information. If the game is restricted to pure pricing strategies whereby each seller bids a particular price with probability one, then a Nash equilibrium set of prices does not exist. The equilibrium outcome must therefore involve a mixed strategy where for each player there exists an interval from which bid prices are randomly selected according to an endogenously determined probability function. In the mixed strategy equilibrium each seller has expectations of earning positive profits on the transaction.

In Chapter 9 the economics of bargaining is analyzed by examining the strategies of a single buyer and single seller in a game theoretic framework. A necessary condition for a bargaining outcome to emerge is the presence of positive bargaining surplus (i.e., gains from trade). In the non-cooperative approach to solving for the bargaining equilibrium the two players are allowed to make successive offers and counteroffers until an offer is eventually accepted or one of the players decides to permanently terminate the bargaining process. In equilibrium an agreement is reached immediately and the distribution of bargaining surplus depends on the players' degree of patience (i.e., discount rate) while bargaining is underway. In the special case where the time between bargaining rounds is infinitely short, the non-cooperative bargaining equilibrium converges to the well-known Nash bargaining outcome, which relies on axioms rather than game theory to identify the outcome.

2 Prices over space

2.1 Introduction

The purpose of this chapter is to examine how the set of competitive prices for a particular commodity at a particular point in time are connected over space. Prices will be spatially integrated if commodity deficit and commodity surplus regions trade amongst themselves and profit seeking arbitrage results in the LOP. Arbitrage implies that profit-seeking traders will ship the commodity from a low-price exporting region to a high-price importing region if the price difference exceeds the marginal transportation and handling costs. These arbitrage shipments, which serve to raise the price in the exporting region and to lower the price in the importing region, will continue until the price difference is reduced to the marginal transportation cost.[1] The assumption of competitive prices may appear to be unreasonable because agricultural commodity trade is often dominated by large multinational firms and state trading agencies. Nevertheless, easy entry by small traders and heavily traded futures markets is believed to be sufficient to ensure reasonably competitive pricing for the major agricultural commodities.

A simple example will illustrate several important features of prices over space and the spatial version of the LOP. Table 2.1 shows ocean freight rates as of 23 November 2005 for a set of exporting and importing regions. There is considerable variation in shipping rates, varying from a low of $19/tonne when Australia sells to South Korea to a high of $45/tonne when the US sells to South Korea. Figure 2.1 shows a scenario where Australia and the US export hard red winter (HRW) wheat to Egypt and South Korea. The numbers associated with each pair of countries are the transportation cost data from Table 2.1 and the numbers associated with each particular country are the export/import prices. The US price of $165/tonne is exogenous and the remaining prices are endogenous. The $165/tonne value corresponds to the price of HRW #2 11.5 percent protein wheat at the US Gulf Coast on 23 November 2005.[2]

Figure 2.1 implies that a US exporter could profitably purchase Gulf Coast wheat and sell into Egypt provided that the Egyptian import price was 165 + 35 = $200/tonne or better (the freight cost from the US Gulf to Egypt is $35/tonne).[3] Assuming that competition by US exporters bids the price of wheat in Egypt down to $200/tonne, an Australian exporter could profitably sell to Egypt provided that

Table 2.1 Ocean freight rates for grain, select ports, 23 November 2005

US dollars per tonne					
From↓/To→	*Algeria*	*Egypt*	*Iran*	*Korea*	*Morocco*
Australia	na	32	29	19	na
EU	24	26	na	na	22
US Gulf	na	35	Na	45	32

Source: Market Data Center: http://data.hgca.com/demo/archive/physical/xls/Ocean%20Freight%20Rates.xls

Note: "na" indicates that data are not available.

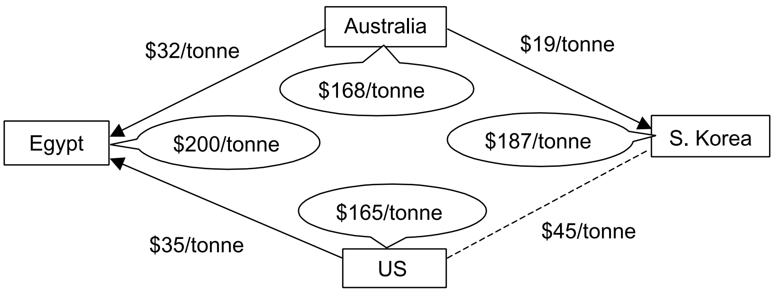


Figure 2.1 Prices and trading partners in a simple spatial price example.

the Australian export price for the same quality wheat was 200 − 32 = $168/tonne or lower (the freight cost from Australia to Egypt is $32/tonne). If there are sizeable stocks of wheat moving from Australia to Egypt, then competition by Australian exporters would bid the Australian export price up to $168/tonne. Based on this price, an Australia exporter could also land wheat in South Korea at cost of 168 + 19 = $187/tonne (i.e., the cost of transporting wheat between Australia and South Korea is $19/tonne). With a landed Australian price in South Korea equal to $187/tonne, a US exporter cannot compete in the South Korean market because a minimum price of 165 + 45 = $210/tonne is required for a US exporter to make a profit (i.e., the transport cost between the US and Korea is $45/tonne).

The data in Table 2.1 reveal several additional relationships between equilibrium prices and transportation costs. For example, if the EU is simultaneously exporting to Egypt and Morocco, then the long-run equilibrium price in Egypt should be higher than the long-run equilibrium price in Morocco by $4/tonne because of the $4/tonne cost advantage that Morocco enjoys when purchasing from the EU. Similarly, if Australia is also exporting to Egypt, then the export price in Australia should be $6/tonne lower than the EU export price because of the $6/tonne cost advantage that the EU enjoys over Australia when exporting to Egypt. A $6/tonne price difference between Australia and the EU implies that these two regions will not trade with each other because the $6/tonne profit that could be made through trade is likely to be insufficient to cover the cost of shipping grain from Australia to the EU.

The purpose of this chapter is to illustrate how interregional trade can be modeled for the case of a perfectly homogenous commodity where price alone fully determines buying and selling patterns. The model consists of a group of exporting and importing regions with unique distances separating the various regions. In equilibrium, a particular exporter will ship to a subset of the nearest importers and a particular importer will purchase from a subset of the nearest exporters.[4] The key feature of the LOP outcome is that: (1) for any pair of trading regions the import and export price difference is equal to the unit cost of transportation; (2) for any pair of exporters selling to the same importer (or for any pair of importers buying from the same exporter), the absolute difference in the pair of export prices (or the pair of import prices) is equal to the absolute difference in the unit cost of transportation; and (3) the absolute price difference between any pair of countries that are not trading with each other will not exceed the unit transportation cost between that pair of countries.

Solving for the equilibrium set of prices by imposing the LOP restriction directly would be both complicated and time consuming because of the potentially large number of different combinations of trading partners. Fortunately, a simple and effective indirect method exists for obtaining the set of equilibrium prices. The method involves constructing a net aggregate welfare function by aggregating consumer and producer surplus across all trading regions and then subtracting from this value the aggregate cost of transportation. The set of shipment quantities that maximize net aggregate welfare subject to a variety of market clearing and non-negativity constraints can be substituted into the set of inverse supply and demand schedules to recover the set of competitive equilibrium prices. This technique of deriving the set of competitive equilibrium prices by maximizing a net aggregate welfare function works because of Adam Smith's "invisible hand" hypothesis. Indeed, a competitive allocation of the commodity across regions by profit-seeking traders leads to maximum net aggregate welfare, so if maximum net aggregate welfare is obtained through optimization, the associated set of prices must correspond to a competitive market outcome.

Maximizing net aggregate welfare to solve for the set of interregional shipments and prices can be complicated when there are many importing and exporting regions because equilibrium trade will be zero for many pairs of countries. These zero-trade outcomes are referred to as "corner solutions" in the language of mathematical programming. If there are a large number of corner solutions then it will be necessary to solve the pricing problem numerically using relatively sophisticated optimization software.[5] Fortunately, Microsoft Excel has a powerful "Solver" tool that can handle small and medium sized numerical optimization problems. The standard version of Solver can also be upgraded if the model is particularly large (i.e., containing dozens of importers and exporters, which results in hundreds of shipment choice variables).

Before proceeding with the construction of the spatial pricing model, it is useful to discuss the specific uses of this type of analysis. First, spatial analysis can be used to predict location-specific price premiums and discounts. As rising world energy prices increase transportation costs, these premiums and discounts will grow in magnitude and become an increasingly important determinant of a region's level of competitiveness. Second, spatial price analysis can be used to illustrate how a change in supply or demand in one region can cause a domino effect that changes the pattern of shipments and prices in a significant and often unpredictable way. Finally, spatial price analysis can be used for strategic decision making by either corporations (e.g., where is the best location for a terminal elevator?) or policy makers (e.g., what are the predicted price impacts of India and Vietnam's February 2008 embargo on rice exports?).

It should be noted that many economists empirically test whether the predictions made by spatial equilibrium models are observed in reality. More specifically, economists are interested in the degree of spatial pricing integration, which is a measure of the time it takes for markets to adjust to the LOP after a regional price shock. Regional markets in developed market economies such as soft white winter wheat in Bannister, Missouri and Commerce, Colorado tend to be well integrated (see Figure 1.4). However, even in emerging markets such as China, prices for agricultural commodities reveal a surprisingly high degree of integration and often follow a well defined "transportation gradient".[6]

In times of rapidly changing transportation costs, prices may appear to have a low degree of spatial integration even though the LOP is hard at work. Consider Table 2.2, which shows how ocean freight rates for grain between select countries have changed between May 2008 and May 2009 due to the world financial crisis. The US regularly ships grain to Japan, so according to the LOP the difference between the Japanese import price and the US export price is equal to the US–Japan freight rate that is displayed in Table 2.2. Thus, the price difference for the same grain in export position at the Gulf Coast and import position in Japan is predicted to have changed by as much as \$81/tonne (\$125 − \$44) between May 2008 and May 2009.

2.2 Basic model

The model consists of *N* regions that competitively produce, consume and trade a homogenous commodity. To keep things simple assume that there is one price for

Table 2.2 Ocean freight rates for grain, select ports, May 2008–May 2009

	US dollars per tonne		
	May 2009	*Nov. 2008*	*May 2008*
US Gulf to EU	30	20	83
US Gulf to Japan	44	28	125
US Gulf to Algeria	31	26	94
Brazil to EU	40	34	96

Source: "Latest Ocean Freight Rates (Weekly)", International Grains Council: http://www.igc.org.uk/en/grainsupdate/igcfreight.aspx each region because production and consumption occurs at the same location. The ith region has an inverse demand schedule, $P_i = a_i + b_i Q_i^D$, and an inverse supply schedule, $P_i = \alpha_i + \beta_i Q_i^S$, where P_i is the price paid by commodity buyers and received by commodity suppliers, and Q_i^D and Q_i^S represent the respective demand and supply quantities. If region i does not produce the commodity in question, then $\alpha_i = \beta_i = 0$. Similarly, if there is no consumption of the commodity then $a_i = b_i = 0$.

Let $T_{ij} \geq 0$ denote the amount of commodity shipped from region i to region j. In equilibrium, total shipments out of region i (including sales to buyers within the region) cannot exceed production, which implies the following supply restriction: $\sum_{j=1}^{N} T_{ij} \leq Q_i^S$. Similarly, total shipments into region j (including purchases from suppliers within the region) must be at least as large as regional demand, which implies the following demand restriction: $\sum_{i=1}^{N} T_{ij} \geq Q_j^D$.

Let C_{ij} denote the cost of shipping a unit of the commodity from region i to region j. Unique values can be assigned to the symmetric parameter pairs C_{ij} and C_{ji}, but for the purpose of this study it is assumed that $C_{ij} = C_{ji}$. The LOP implies that if region i is actively shipping to region j, then it must be the case that $P_j = P_i + C_{ij}$. This condition ensures that a trader cannot profitably purchase a unit of the stock in region i at price P_i, pay for the transportation cost, C_{ij}, and then resell the stock in region j at price P_j. If region i is not exporting to region j, then trading must not be profitable, which implies $P_j < P_i + C_{ij}$.

Welfare measurement on a diagram

Recall that the set of competitive equilibrium prices can be obtained by maximizing net aggregate welfare, which is equal to consumer and producer surplus aggregated across the N regions less aggregate transportation expense. Figure 2.2 illustrates the measurement of net aggregate welfare for the case of two trading regions. The left-hand graph represents the commodity exporter (E) and the right-hand graph represents the commodity importer (I). The middle graph shows the export supply curve of E, which is derived as the horizontal difference between the supply and demand schedules within E. The middle graph also shows the import demand schedule for I, which is derived as the horizontal difference between the demand and supply schedules within I. The export supply schedule for E is shifted up by an amount C_{EI} to account for the cost of transporting the commodity.[7]

The intersection of the raised export supply schedule with the import demand schedule in the middle graph of Figure 2.2 shows the equilibrium level of trade between the two regions, T_{EI}.

Figure 2.2 also shows that the equilibrium price in E is P_E, which leads to consumption at level Q_E^D and production at level Q_E^S, where $Q_E^S - Q_E^D = T_{EI}$. As well, the equilibrium price in I is P_I, which leads to consumption at level Q_I^D and production at level Q_I^S, where $Q_I^D - Q_I^S = T_{EI}$. Figure 2.2 conforms to the LOP, which requires $P_I - P_E = C_{EI}$.

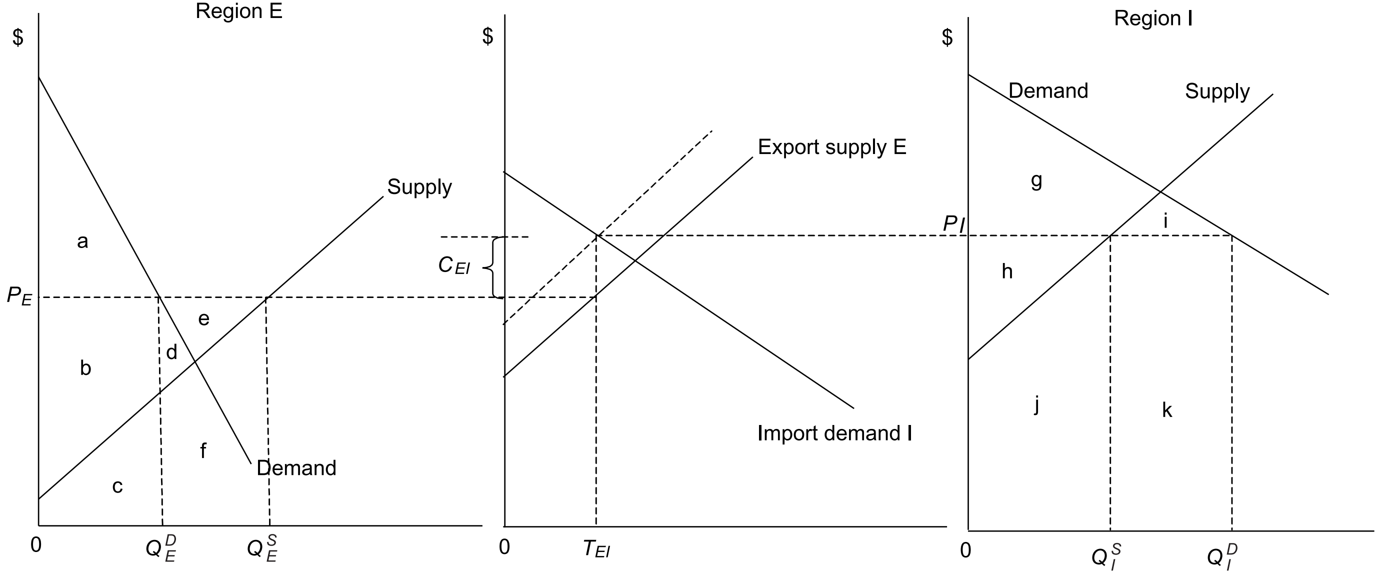


Figure 2.2 Measurement of net aggregate welfare in a spatial equilibrium model.

After trade, consumers in E earn surplus given by area *a* and producers in E earn surplus given by area $b + d + e$. The combined surplus of consumers and producers is therefore equal to area $a + b + d + e$. Similarly, consumers in I earn surplus given by area $g + i$ and producers in I earn surplus given by area *h*. The combined surplus of consumers and producers is therefore equal to area $g + h + i$. Aggregating across both regions implies that net aggregate welfare is given by area $a + b + d + e + g + h + i$.

Net aggregate welfare can also be measured in a way that will prove useful in the optimization model. The first step is to aggregate the areas under the two demand schedules, up to Q_E^D for E and Q_I^D for I. The second step is to aggregate the areas under the two supply schedules, up to Q_E^S for E and Q_I^S for I. To complete the calculation of net aggregate welfare, subtract the latter measure from the former, and then subtract aggregate transportation cost, $C_{EI}T_{EI}$.[8] To establish that this measure is the same as that derived in the previous paragraph, notice from Figure 2.1 that the combined area under the two demand schedules is given by $a + b + c + g + h + i + j + k$ and the combined area under the two supply schedules is given by $c + f + j$. Hence, net aggregate welfare under the proposed scheme is given by area $a + b + g + h + i + k - f - C_{EI}T_{EI}$.

However, as Figure 2.2 shows, area *k* is equal to area $d + e + f + C_{EI}T_{EI}$. After substituting this expression for *k* into the previous equation, the revised area for net aggregate welfare is given by $a + b + d + e + g + h + i$. This outcome agrees with the area for net aggregate welfare, which was derived in the previous paragraph.

Assumptions for mathematical model

To construct a mathematical model it is necessary to assign specific functional forms to the regional supply and demand schedules and then obtain expressions for the aggregate areas under these schedules. Assuming a linear inverse demand schedule for region *i*, $P_i = a_i - b_iQ_i^D$, the formula for the area under the demand schedule is $0.5(a_i - P_i)Q_i^D + P_iQ_i^D$, which becomes $(a_i - 0.5b_iQ_i^D)Q_i^D$ after substituting the demand schedule for P_i. The inverse supply schedule for region *i* given by, $P_i = \alpha_i + \beta_iQ_i^S$. If this schedule intersects the vertical axis at a positive price, then the area under the supply schedule for region *i* is $\alpha_i Q_i^S + 0.5\ (P_i - \alpha_i)Q_i^S$, which becomes $(\alpha_i + 0.5\beta_iQ_i^S)Q_i^S$ after substituting the supply schedule for P_i. Conversely, if the supply schedule intersects the horizontal axis at a positive quantity, then the area under the supply schedule is $0.5(Q_i^S - Q_i^0)P_i$, where $Q_i^0 = -\alpha_i/\beta_i$ is the point of intersection. After making this substitution, along with $P_i = \alpha_i + \beta_iQ_i^S$ for P_i, the formula for the area under the supply schedule for the case of a horizontal axis intersection can be written as $0.5(\alpha_i + \beta_iQ_i^S)\left(Q_i^S + \alpha_i/\beta_i\right)$.

For an arbitrary set of values for Q_i^D, Q_i^S and T_{ij}, and with the supply schedules intersecting the vertical axes at a positive price, the measure of net aggregate welfare (NAW) for all *N* regions can be expressed as:

$$NAW_V = \sum_{i=1}^{n}\left(a_i - 0.5b_iQ_i^D\right)Q_i^D - \sum_{i=1}^{n}\left(\alpha_i + 0.5\beta_iQ_i^S\right)Q_i^S - \sum_{i=1}^{n}\sum_{j=1}^{n}C_{ij}T_{ij} \quad (2.1a)$$

Similarly, if the supply schedules intersect the horizontal axes, the appropriate expression for *NAW* is:

$$NAW_H = \sum_{i=1}^{n}\left(a_i - 0.5b_iQ_i^D\right)Q_i^D - \sum_{i=1}^{n}.5\left(\alpha_i + \beta_iQ_i^S\right)$$

$$\left(Q_i^S + {}^{\alpha_i}\!/_{\beta_i}\right) - \sum_{i=1}^{n}\sum_{j=1}^{n}C_{ij}T_{ij} \quad (2.1b)$$

The spatial pricing equilibrium can be obtained by choosing the set of values for T_{ij}, Q_i^D and Q_i^S that maximize equation (2.1) subject to the import demand restriction $\sum_{j=1}^{N}T_{ji} \geq Q_i^D$ for all i, the export supply restriction $\sum_{j=1}^{N}T_{ij} \leq Q_i^S$ for all i, and the non-negativity restrictions for T_{ij}, Q_i^D and Q_i^S.

Kuhn–Tucker solution

Before describing the numerical optimization procedures Kuhn–Tucker programming is used in this section to derive the LOP relationships. The first step is to construct a Lagrangian function for maximizing net aggregate welfare subject to the various constraints. It is important that the constraints are entered as “less-than-or-equal-to” inequalities, $Q_i^D - \sum_{j=1}^{N}T_{ji} \leq 0$, $\sum_{j=1}^{N}T_{ij} - Q_i^S \leq 0$ and $-T_{ij} \leq 0$, to ensure that the multiplier variables have the correct sign (the constraints are entered by subtracting the terms on the left of the inequality from the terms on the right). If the supply schedule intersects the vertical axis then Lagrangian function can be written as:

$$L = \sum_{i=1}^{n}\left[\left(a_i - 0.5b_iQ_i^D\right)Q_i^D - \left(\alpha_i + 0.5\beta_iQ_i^S\right)Q_i^S\right] - \sum_{i=1}^{n}\sum_{j=1}^{n}C_{ij}T_{ij}$$

$$+\sum_{i=1}^{N}\lambda_i^D\left(\sum_{j=1}^{N}T_{ji} - Q_i^D\right) + \sum_{i=1}^{N}\lambda_i^S\left(Q_i^S - \sum_{j=1}^{N}T_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{ij}^T T_{ij} \quad (2.2)$$

The set of λ^D and λ^S variables are the Lagrangian multipliers associated with the import demand and export supply adding-up restrictions, and the set of λ^T variables are the Lagrangian multipliers associated with the $T_{ij} \geq 0$ restrictions.

The Kuhn–Tucker conditions for the constrained optimization problem are for $i = 1, 2, \ldots, N$:

$$\partial L / \partial Q_i^D = a_i - b_i Q_i^D - \lambda_i^D \leq 0, \quad \left(\partial L / \partial Q_i^D \right) Q_i^D = 0 \quad \text{and}$$

$$\lambda_i^D \left(\sum_{j=1}^{N} T_{ji} - Q_i^D \right) = 0 \tag{2.3a}$$

$$\partial L / \partial Q_i^S = -\left(\alpha_i + \beta_i Q_i^S \right) + \lambda_i^S \leq 0, \quad \left(\partial L / \partial Q_i^S \right) Q_i^S = 0 \quad \text{and}$$

$$\lambda_i^S \left(\sum_{j=1}^{N} T_{ij} - Q_i^S \right) = 0 \tag{2.3b}$$

$$\partial L / \partial T_{ij} = -C_{ij} + \lambda_j^D - \lambda_i^S - \lambda_{ij}^T \leq 0, \quad \left(\partial L / \partial T_{ij} \right) T_{ij} = 0 \quad \text{and}$$

$$\lambda_{ij}^T T_{ij} = 0 \text{ for } j = 1, 2, \ldots, N \tag{2.3c}$$

Additional restrictions include the resource constraints, $\sum_{j=1}^{N} T_{ji} \geq Q_i^D$ and $\sum_{j=1}^{N} T_{ij} \leq Q_i^S$ for $i = 1, 2, \ldots, N$, and non-negative values for all of the multiplier variables.

The consumer and producer price in region i can be expressed as $P_i^D = a_i - b_i Q_i^D$ and $P_i^S = \alpha_i + \beta_i Q_i^S$, respectively. It follows from equations (2.3a) and (2.3b) that $\lambda_i^D = P_i^D$ and $\lambda_i^S = P_i^S$. Producers will sell to domestic consumers before exporting, and consumers will buy from local producers before importing because, by assumption, $C_{ii} = 0$ and $C_{ij} > 0$ for $i \neq j$. Using equation (2.3c), the combination of $T_{ii} > 0$ and $C_{ii} = 0$ implies $\lambda_{ii}^T = 0$ and $\lambda_i^D = \lambda_i^S$. This result, combined with $\lambda_i^D = P_i^D$ and $\lambda_i^S = P_i^S$ implies that $\lambda_i^D = \lambda_i^S = P_i^*$. That is, the price is the same for domestic consumers and producers when stocks are optimally allocated by a social planner.

For interregional shipments there are two possibilities. First, using the last expression in equation (2.3c), if region i ships to region j then $T_{ij} > 0$ and $\lambda_{ij}^T = 0$. Using the second expression in equation (2.3c), the $T_{ij} > 0$ result implies that the first expression in equation (2.3c) must hold as an equality. Thus, if $\lambda_{ij}^T = 0$ along with $\lambda_i^D = \lambda_i^S = P_i^*$ are substituted into the first expression in equation (2.3c) it follows that $P_j^* - P_i^* = C_{ij}$. This result confirms the first property of the intertemporal version of the LOP. The second possibility is that region i does not ship to region j, which implies from the last expression in equation (2.3c) that $T_{ij} = 0$ and $\lambda_{ij}^T \geq 0$. The first two expressions in equation (2.3c), combined with $\lambda_i^D = \lambda_i^S = P_i^*$, therefore imply that $P_j^* - P_i^* \leq C_{ij}$. This result, that shipments are zero when the price difference is less than the unit transportation cost, confirms the second property of the intertemporal version of the LOP.

The Kuhn–Tucker approach to solving for equilibrium prices works well for small problems, but becomes difficult to implement for large problems. Numerical optimization procedures are typically used for spatial price analysis. These procedures are illustrated below in a case study involving the global trade in tomatoes. Before describing the specifics of this case study, a method for scaling the values of price and quantity variables is described.

Scaling procedure

Numerical optimization works best if the variables of the model have values that are roughly the same order of magnitude. To illustrate the scaling procedure consider the generic demand and supply schedules, $P = a - bQ$ and $P = \alpha + \beta Q$, where Q is quantity measured in tons and P is price measured in dollars per ton. Suppose instead quantity is measured in *ktons* and price is measured in *zollars*, where one *kton* is equal to k tons and one *zollar* is equal to z dollars. If one ton is valued at P dollars then one *kton* is valued at kP dollars. Because one dollar is equal to $1/z$ *zollars*, it follows that one *kton* is valued at $(k/z)P$ *zollars*. Letting $\hat{P}$ denote price measured in *zollars* per *kton*, it follows that, $\hat{P} = (k/z)P$, which in turn implies $P = (z/k)\hat{P}$. Similarly, letting $\hat{Q}$ denote quantity measured in *ktons*, it follows that $Q = k\hat{Q}$.

The next step in the scaling procedure is to substitute the expressions for $\hat{P}$ and $\hat{Q}$ into the inverse demand schedule, $P = a - bQ$, and the inverse supply schedule, $P = \alpha + \beta Q$, to obtain scaled demand and supply schedules, $\hat{P} = \hat{a} - \hat{b}\hat{Q}$ and $\hat{P} = \hat{\alpha} - \hat{\beta}\hat{Q}$, where:

$$\hat{a} = {ak}/{z}, \; \hat{\alpha} = {\alpha k}/{z}, \; \hat{b} = {bk^2}/{z} \text{ and } \hat{\beta} = {\beta k^2}/{z} \tag{2.4}$$

The unit transportation cost parameter, C_{ij}, is measured in dollars per ton, so the scaled transportation cost parameter that is measured in *zollars* per *kton* can be expressed as $\hat{C}_{ij} = (k/z)C_{ij}$. After optimization it is often desirable to present the variables in original units rather than scaled units. Reverse scaling is achieved by multiply all scaled quantities by k, all scaled prices by z/k and all scaled surplus measures by z.[9]

To illustrate the scaling technique with a specific example, suppose the demand schedule is given by $P = 9000 - 0.001Q$, which implies $a = 9000$ and $b = 0.001$. Also suppose that the objective of the scaling is to achieve $\hat{a} = 1$ and $\hat{b} = 0.5$. Given that $\hat{a} = {ak}/{z}$ and $a = 9000$ it follows that $z/k = 9000$ to ensure that $\hat{a} = 1$. Similarly, given that $\hat{b} = {bk^2}/{z}$ and $b = 0.001$ it follows that $z/k^2 = 0.002$ to ensure that $\hat{b} = 0.5$. Solving these two equations together implies $k = 4{,}500{,}000$ and $z = 40{,}500{,}000{,}000$. Therefore, to generate the scaled demand schedule, $\hat{P} = 1 - 0.5\hat{Q}$, all price data should be divided by $z/k = 9000$ and all quantity data should be divided by $k = 4{,}500{,}000$. In a more general model with multiple demand and supply schedules, values for the scaling variables k and z should be chosen to ensure that the revised set of intercept and slope parameters have a similar order of magnitude.

2.3 Spatial pricing case study

The following case study focuses on the global trade in tomatoes. Tomatoes are one of the most important commodities that trade in the global vegetable market. Dominant tomato producers include China, the European Union and the United States. With respect to trade, dominant exporters of fresh tomatoes are Spain, Mexico and the Netherlands, and dominant importers are United States, Germany and France. For this particular case study the global market for tomatoes is broken into five regions: Mexico, the US, Canada, the European Union (EU) and Latin America.

Parameter estimates for the regional tomato supply and demand schedules are borrowed from an earlier paper on the global tomato market.[10] Unfortunately, this paper did not report the values that were assumed for regional transportation costs. Consequently, estimates of ocean freight rates for fresh vegetables were obtaining by calculating the nautical mileage between representative ports within each region and then multiplying these mileage values by a fixed price per ton per mile.[11] The parameters values for regional supply/demand and transportation costs are summarized in Table 2.3(a) and (b). Negative values for the intercept

Table 2.3 Pre-scaled parameters for tomato case study: (a) Supply and demand intercept and slope parameters; (b) transportation cost parameters

(a)

	Intercept parameters		*Slope parameters*	
Region	*Supply (α)*	*Demand (a)*	*Supply (β)*	*Demand (b)*
Mexico	−2,532	8,732.3	0.00146	0.00578
US	−1,279	2,217.1	0.00021	0.00011
Canada	−2,128	5,131.1	0.0059	0.00581
EU	−5,337	4,258.7	0.00043	0.00022
L. Amer.	−3,306	2,806.5	0.00059	0.00036

(b)

	US dollars per ton				
Region	*Mexico*	*US*	*Canada*	*EU*	*L. Amer.*
Mexico	0.00	58.50	96.63	155.55	161.88
US	58.50	0.00	42.21	106.98	142.05
Canada	96.63	42.21	0.00	106.47	164.43
EU	155.55	106.98	106.47	0.00	137.37
L. Amer.	161.88	142.05	164.43	137.37	0.00

Source:

(a) See endnote 10.

(b) Representative cities are Veracrux (Mexico), New York (USA), Montreal (Canada), Valencia (Spain/EU) and Rio de Janeiro (Brazil/Latin America). Nautical sea mileage between these port cities was obtained from the Sea Rates.Com website: http://www.searates.com/reference/portdistance
The values in (b) were derived by multiplying the nautical mileage by $0.03 per mile.

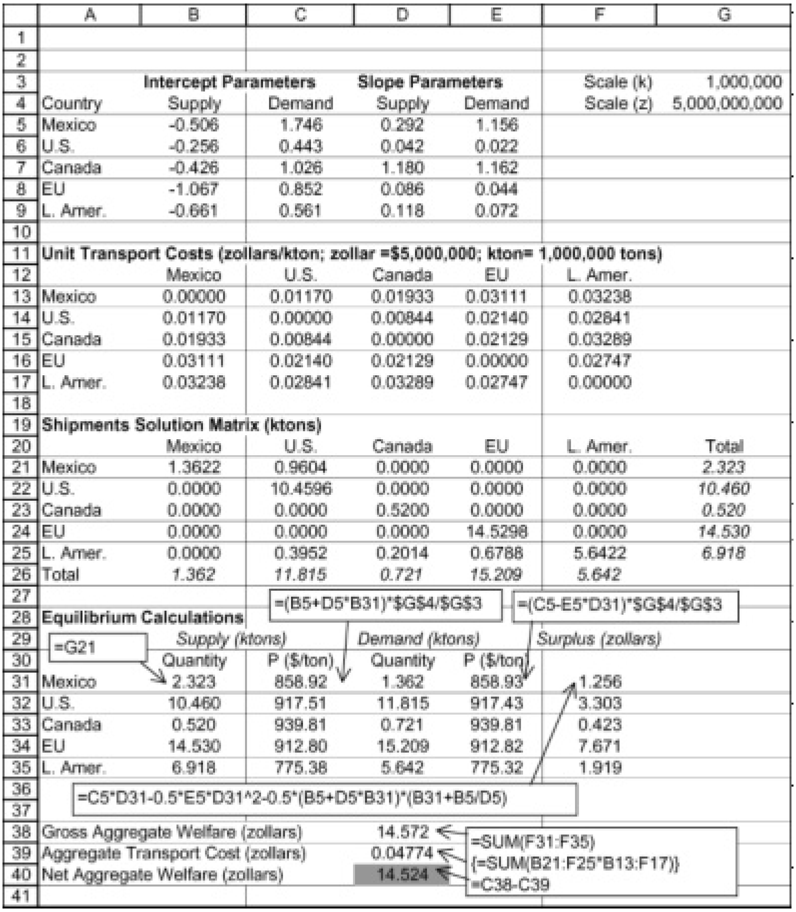
parameters of the supply schedules imply that these schedules intersect the horizontal axis. Equation (2.1b) rather than (2.1a) must therefore be used to calculate net aggregate welfare.

Model setup

Figure 2.3 illustrates both the setup of the spatial model and the post optimization equilibrium outcome. Cells B5:E9 contained scaled parameter values for the regional supply and demand schedules and cells B13:F17 contain the scaled unit transportation costs. Scaling was achieved by using equation (2.4) and $\hat{C}_{ij} = (k/z)C_{ij}$ together with $k = 1{,}000{,}000$ (i.e., one *kton* is equal to a million tons) and $z = 5{,}000{,}000{,}000$ (i.e., one *zollar* is equal to five billion dollars). To properly interpret the transportation cost matrix given by cells B13:F17 note that the cost of shipping from region i to region j is found by looking up region i in cells A13:A17 and looking up region j in cells B12:F12. The unit transportation cost when the product is shipped from region i to region j can now be found where the ith row and jth column intersect. For example, the value in cell E13 represents the cost of shipping a unit of tomatoes from Mexico to the EU.

The T_{ij} shipment variables that are optimally chosen by Excel (more details below) are listed in cells B21:F25 of Figure 2.3. The shipping regions are identified in column A and the receiving regions are identified in row 20. The total shipments leaving each region (including self-shipments) are shown in cells G21:G25 and the total shipments arriving in each region (including self-purchases) are shown in cells B26:F26. The set of shipment outflow values in cells G21:G25 are repeated in cells B31:B35 (labeled "Supply") and the transpose of the set of shipment inflow values in cells B26:F26 are repeated in cells D31:D35 (labeled "Demand"). This procedure implies that the trade constraints, $\sum_{j=1}^{N} T_{ij} \le Q_i^S$ and $\sum_{j=1}^{N} T_{ji} \ge Q_i^D$, automatically hold as equalities. Restricting these constraints to equalities rather than inequalities is acceptable because for the problem at hand there is no economic payoff to selling less than what is produced or consuming less than what is purchased.

The set of $\hat{Q}^S$ values in cells B31:B35 and the set of $\hat{Q}^D$ values in cells D31:D35 of Figure 2.3 can now be inserted into the pair of equations $\hat{P}^S = \hat{\alpha} + \hat{\beta}\hat{Q}^S$ and $\hat{P}^D = \hat{a} - \hat{b}\hat{Q}^D$ in order to generate demand and supply prices for each region. The pre-scaled version of these prices are reported in cells C31:C35 and E31:E35 of Figure 2.3 using the formulas $(\hat{\alpha} + \hat{\beta}\hat{Q}^S)^Z/k$ and $(\hat{a} - \hat{b}\hat{Q}^D)^Z/k$. The combined surplus for consumers and producers that appears in cells F31:F35 of Figure 2.3 was calculated using equation (2.1b). Gross aggregate welfare, which is the sum of the surplus values in cells F31:F35, is shown in cell D38. The aggregate cost of transportation for all shipments, which is shown in cells D39, is subtracted from gross aggregate welfare to give a measure of net aggregate welfare in cell D40.[12]



	A	B	C	D	E	F	G
1							
2							
3		**Intercept Parameters**		**Slope Parameters**		Scale (k)	1,000,000
4	Country	Supply	Demand	Supply	Demand	Scale (z)	5,000,000,000
5	Mexico	-0.506	1.746	0.292	1.156		
6	U.S.	-0.256	0.443	0.042	0.022		
7	Canada	-0.426	1.026	1.180	1.162		
8	EU	-1.067	0.852	0.086	0.044		
9	L. Amer.	-0.661	0.561	0.118	0.072		
10							
11	**Unit Transport Costs (zollars/kton; zollar =$5,000,000; kton= 1,000,000 tons)**						
12		Mexico	U.S.	Canada	EU	L. Amer.	
13	Mexico	0.00000	0.01170	0.01933	0.03111	0.03238	
14	U.S.	0.01170	0.00000	0.00844	0.02140	0.02841	
15	Canada	0.01933	0.00844	0.00000	0.02129	0.03289	
16	EU	0.03111	0.02140	0.02129	0.00000	0.02747	
17	L. Amer.	0.03238	0.02841	0.03289	0.02747	0.00000	
18							
19	**Shipments Solution Matrix (ktons)**						
20		Mexico	U.S.	Canada	EU	L. Amer.	Total
21	Mexico	1.3622	0.9604	0.0000	0.0000	0.0000	*2.323*
22	U.S.	0.0000	10.4596	0.0000	0.0000	0.0000	*10.460*
23	Canada	0.0000	0.0000	0.5200	0.0000	0.0000	*0.520*
24	EU	0.0000	0.0000	0.0000	14.5298	0.0000	*14.530*
25	L. Amer.	0.0000	0.3952	0.2014	0.6788	5.6422	*6.918*
26	Total	*1.362*	*11.815*	*0.721*	*15.209*	*5.642*	
27							
28	**Equilibrium Calculations**						
29		*Supply (ktons)*		*Demand (ktons)*		*Surplus (zollars)*	
30		Quantity	P ($/ton)	Quantity	P ($/ton)		
31	Mexico	2.323	858.92	1.362	858.93	1.256	
32	U.S.	10.460	917.51	11.815	917.43	3.303	
33	Canada	0.520	939.81	0.721	939.81	0.423	
34	EU	14.530	912.80	15.209	912.82	7.671	
35	L. Amer.	6.918	775.38	5.642	775.32	1.919	
36							
37							
38	Gross Aggregate Welfare (zollars)			14.572			
39	Aggregate Transport Cost (zollars)			0.04774			
40	Net Aggregate Welfare (zollars)			14.524			
41							

Figure 2.3 Equilibrium solution for spatial transportation model.

Starting values for T_{ij}

Solver performs best when it is initially supplied with reasonably accurate starting values for the choice variables. Starting values are particularly important when the problem is relatively large and there are many corner solutions. An effective way to generate starting values is to calculate equilibrium values for a scenario where all transportation costs are equal to zero. In this “free flow” equilibrium, there is a common price, P^*, for all five regions. To obtain an expression for P^* solve the scaled inverse demand, $P_i = \hat{a}_i - \hat{b}_i Q_i^D$, for Q_i^D and inverse supply, $P_i = \hat{\alpha}_i + \beta_i \hat{Q}_i^S$, for Q_i^S. Now aggregate across all regions and set aggregate demand equal to aggregate supply to obtain $\sum_{i=1}^{n}\frac{\hat{a}_i}{\hat{b}_i} - P\sum_{i=1}^{n}\frac{1}{\hat{b}_i} = -\sum_{i=1}^{n}\frac{\hat{\alpha}_i}{\hat{\beta}_i} + P\sum_{i=1}^{n}\frac{1}{\hat{\beta}_i}$. This expression can be solved to obtain the free flow equilibrium price:

$$P^* = \frac{\sum_{i=1}^{n}\frac{\hat{a}_i}{\hat{b}_i} + \sum_{i=1}^{n}\frac{\hat{\alpha}_i}{\hat{\beta}_i}}{\sum_{i=1}^{n}\frac{1}{\hat{b}_i} + \sum_{i=1}^{n}\frac{1}{\hat{\beta}_i}} \tag{2.5}$$

Figure 2.4 shows the calculation of P^* for the case study. The array formulas in cells K4:K5 and K7:K8 are expressions for the four terms in the numerator and denominator of equation (2.5). These array formulas are linked to the intercept and slope parameters that reside in cells B5:E9 of Figure 2.3. Equation (2.5) and the four expressions in cells K4:K5 and K7:K8 are used to generate the $P^* = 0.178$ value that appears in cell K10. In cells N5:O9 this free flow equilibrium price is used in conjunction with supply, $Q_i^s = \frac{-\hat{\alpha}_i}{\hat{\beta}_i} + \frac{1}{\hat{\beta}_i}P_i$, and demand, $Q_i^D = {}^{\hat{a}_i}\!/_{\hat{b}_i} - {}^{1}\!/_{\hat{b}_i}P_i$, to generate free flow levels of supply and demand for each region. Free flow exports, which is the difference between free flow supply and demand, are reported in cells P5:P9 of Figure 2.4. The free flow equilibrium is confirmed by noting from cells N10 and O10 that supply and demand aggregated across all regions is equal to 34.788.

The values in bold font in cells N5:O9 of Figure 2.4 provide good starting values for the elements of the principle diagonal of the T_{ij} shipment matrix, which is displayed in Figure 2.5. To see how this works notice from Figure 2.4 that in the free flow equilibrium Mexico will keep 1.357 *ktons* of tomatoes for itself and ship the remaining 0.988 *ktons* to the three importing regions. Similarly, Latin America will keep 5.322 *ktons* for itself and ship the remaining 1.791 *ktons* to the three importing regions. Consequently, 1.357 is a good starting value for the Mexico–Mexico cell in the shipment matrix and 5.322 is a good starting value for the L. Amer–L. Amer cell in the shipment matrix.

	I	J	K	L	M	N	O	P
2			=-B5/D5+(1/D5)*$K$10			=C5/E5-(1/E5)*$K$10		
3	Sum of intercept/slope:							
4		Supply	-26.196		Region	Supply	Demand	Export
5		Demand	49.703		Mexico	2.344	**1.357**	0.988
6	Sum of 1/slope:				U.S.	**10.331**	12.059	-1.728
7		Supply	48.184		Canada	**0.512**	0.730	-0.218
8		Demand	83.796		EU	**14.478**	15.310	-0.832
9					L. Amer.	7.113	**5.322**	1.791
10	Free Flow equil. price:		0.178		Total	34.778	34.778	
11								
12	={SUM(B5:B9/D5:D9)}		=(K5+K4)/		={SUM(1/D5:D9)}			
13	={SUM(C5:C9/E5:E9)}		(K8+K7)		={SUM(1/E5:E9)}			
14								

Figure 2.4 Solving for the free flow equilibrium prices and quantities.

	A	B	C	D	E	F	G
19	**Shipments Solution Matrix with Starting Values (ktons)**						
20		Mexico	U.S.	Canada	EU	L. Amer.	Total
21	Mexico	1.357	0.000	0.000	0.000	0.000	*1.357*
22	U.S.	0.000	10.331	0.000	0.000	0.000	*10.331*
23	Canada	0.000	0.000	0.512	0.000	0.000	*0.512*
24	EU	0.000	0.000	0.000	14.478	0.000	*14.478*
25	L. Amer.	0.000	0.000	0.000	0.000	5.322	*5.322*
26	Total	*1.357*	*10.331*	*0.512*	*14.478*	*5.322*	

Figure 2.5 Initial values for Solver choice variables.

For the three exporting regions, use the free flow production values rather than the demand values as proxies for the self-shipments. Specifically, use 10.311 for the US, 0.512 for Canada and 14.478 for the EU when inserting values on the principle diagonal of the shipment matrix. The initial values for the Solver choice variables are displayed in Figure 2.5. These starting values could be further improved by identifying how aggregate exports from Mexico and Latin America are divided between the three importing regions, but this level of fine tuning is not required to solve the problem.

Solution procedure

The solution procedure begins by transferring the initial "guess" values for the shipment variables from cells B21:F25 of Figure 2.5 to cells B21:F25 of Figure 2.3. Solver must now be instructed to improve upon the 25 values contained in cells B21:F25 of Figure 2.3 so as to maximize the net aggregate welfare expression in cell D40. Begin these Solver instructions by entering "D40" in Solver's "Set Target Cell" slot and entering B21:F25 in Solver's "By Changing Cells" slot. Note that there are no constraints in this problem other than $T_{ij} \geq 0$ because, as discussed above, the $\sum_{j=1}^{N} T_{ij} \leq Q_i^S$ and $\sum_{j=1}^{N} T_{ji} \geq Q_i^D$ constraints have been directly incorporated into the spreadsheet as equalities. The absence of constraints implies that Solver's "Subject to the Constraints" slot can remain empty. The $T_{ij} \geq 0$ constraints can be included by checking the "Assume Non-Negative" box under Solver Options (accessed from Solver's main dialogue sheet). At the same time it is useful to increase solution efficiency by checking the "Quadratic" option.

The programming model can now be solved by clicking Solver's "Solve" button. In the absence of programming errors, Solver will return a message indicating that it has found an optimal solution or that it has converged to the current set of values. If the latter message appears it is a good idea to run Solver again to ensure that the solution has fully converged. Convergence is complete when the demand and supply prices, which are contained in cells C31:C35 and E31:E35, are approximately equal within each region. Cells B21:F25 of Figure 2.3 show the final optimized T_{ij} shipment values. The equilibrium prices and quantities for each region are shown in cells B31:E35.

2.4 Case study results

The equilibrium pricing outcome for the Figure 2.3 base case is illustrated in Figure 2.6. The values in parentheses beside each region are the equilibrium prices. The solid arrows between regions identify the flow of shipments, and the dashed lines between regions indicate zero trade. The values near the lines that connect the regions are the unit transportation costs in dollars per ton.

The pricing and unit transportation cost data allow for easy verification of the LOP. For example, notice that Latin America ships to Canada, the US and the EU. In all cases, the price in the importing region is higher than the Latin American price by (approximately) the corresponding unit transportation cost. A similar relationship holds for Mexico, which exports to the US. For the regions that do not trade (e.g., Mexico and the EU), it is easy to verify in Figure 2.6 that the interregional price difference is less than the corresponding unit transportation cost.

In the base case results that are shown in Figure 2.6, why does Latin America instead of Mexico sell to Canada given that the cost of shipping to Canada is lower for Mexico? The reason is that if Mexico supplies Canada then Latin America must pick up the Mexico–US shortfall. The added cost of shipping from Latin America to the US versus Mexico to the US (142.50 – 58.50 = $83.55/tonne) exceeds the cost savings of shipping from Mexico to Canada versus Latin America to Canada (164.43 − 96.63 = $67.80/tonne). Notice that Canada is approximately indifferent between importing from Mexico and the US because the US price plus the unit cost of shipping between the US and Canada is approximately equal to the Mexican price plus the unit cost of shipping between Mexico and Canada.

The programming model developed in this chapter is very useful for "what if" analysis. For example, policy makers may be interested in knowing how the pattern of trade in tomatoes and the regional prices for tomatoes will change if there is a large supply reduction in the EU, possibly due to a long-term disease problem or a government policy that diverts productive capacity away from tomatoes. To analyze this issue, assume that the intercept term of the EU inverse supply schedule shifts up from $\alpha = -1.067$ to $\alpha = -0.5$. After making this change in cell B8 of Figure 2.3, the model can be resolved to generate a new set of equilibrium prices and trading patterns. The revised results are illustrated in Figure 2.7.

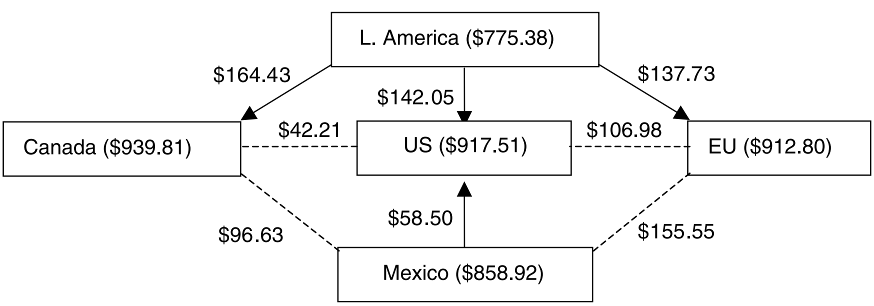


Figure 2.6 Base case results for spatial analysis of global tomato trade.

Notice in Figure 2.7 that the supply reduction in the EU has resulted in the US switching from a net importer with tomatoes flowing in from Latin America and Mexico to a net exporter with supplies tomatoes flowing out to both the EU and Canada. In the base case the EU received tomatoes from Latin America only, but now shipments arrive from Latin American, Mexico and the US. Shipments from Latin America to Canada have been replaced with shipments from the US to Canada. This "what if" scenario nicely illustrates the result that a change in production in one region can have a "domino effect" and result in a very different pattern of trade and set of prices among all trading regions.

Figure 2.7 also reveals that the supply reduction in the EU has resulted in higher importing and exporting prices for all five regions. The price differentials continue to be governed by the spatial version of the LOP, but the absolute price levels have simultaneously risen due to a reduction in production in one region. This result illustrates the extreme case of perfect pricing efficiency and market integration. In particular, with free trade consumers in commodity surplus regions are not immune to large price increases if there are large-scale supply reductions in distant regions. This issue was particularly apparent in the rice market in 2008 when several rice exporting countries imposed export embargoes to halt the rapidly escalating price of rice for domestic consumers.

A second interesting "what if" question concerns the cost of transportation. Suppose a rapid run-up in the price of energy causes a spike in the cost of transporting tomatoes. Specifically, suppose ocean freight rates doubled in value. Will the trade in tomatoes grind to a complete halt with this level of price increase? To investigate this issue, set the EU supply schedule intercept parameter back to its base level of $\alpha = -1.067$ in cell B8 of Figure 2.3, and then multiply the transportation cost parameters contained in cells B13:F17 of Figure 2.3 by two. The pricing and trading pattern results that are generated after resolving the model are illustrated in Figure 2.8. A comparison of Figure 2.6 and Figure 2.8 reveals that the trading pattern with this high transportation cost scenario is the same as the base case. This result implies that the gains from trade must be comparatively strong because in the absence of strong gains from trade, the high cost of transportation would be expected to eliminate much of the global trade in tomatoes.

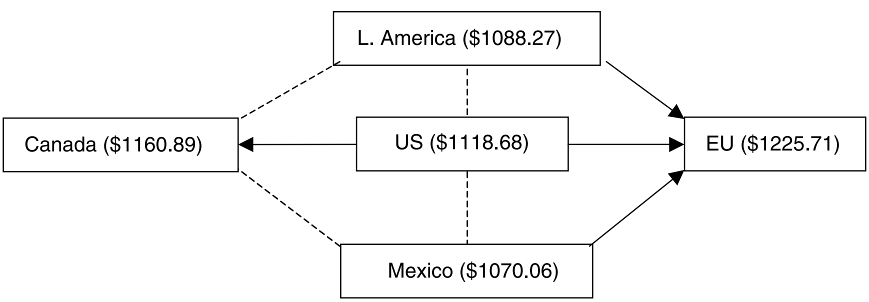


Figure 2.7 Pricing impact from a permanent supply reduction in the EU.

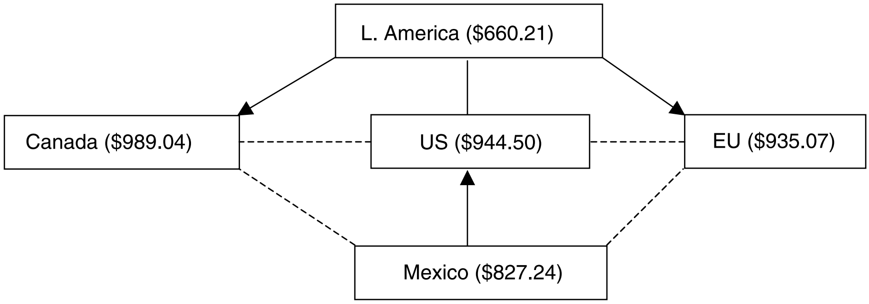


Figure 2.8 Pricing impact from a doubling in transportation costs.

Perhaps the most interesting feature of Figure 2.8 is that the price differences between the importing and exporting regions are now very large. Indeed, the equilibrium price is approximately $990/ton in Canada and is approximately $660/ton in Latin America. The higher transportation cost drives the global trade in tomatoes toward an autarky outcome where regions with excess supply capacity face a relatively low price and regions with excess demand face a relatively high price. The high cost of transportation has significantly reduced the gains from trade.

2.5 Concluding comments

Arbitrage over space is probably the most straightforward application of the LOP to understand. Traders scan various regions for price differences that exceed the unit cost of transportation, and upon finding an arbitrage opportunity arrange for shipments from the low-price to the high-price region. Such shipments, when undertaken by a large number of traders, will continually drive up the price in the low-price region and drive down the price in the high-price region until an equilibrium is reached (i.e., until the price difference is equal to the unit cost of transportation). If the price difference is less than the unit cost of transportation, then that particular pair of regions will not trade with each other in the market equilibrium.

It is important to emphasize that shipping a commodity from one region to another for the purpose of arbitrage can be both time consuming and administratively complicated. Consequently, it is important to view the spatial LOP as a general long-run principle rather than a rule that is expected to hold in all short-run trading situations. However, despite this long-run interpretation of the LOP, the high degree of spatial price integration across markets that exists for most commodities is strong evidence that the LOP is working in both the short run and the long run. Spatial price integration should generally be viewed as a positive feature of markets because demand and supply shocks are much less severe and thus price is much less volatile when markets are well integrated.

The Excel programming procedures used in this chapter are similar to those that would be used in large-scale applications of spatial price analysis that utilize specialized software package (e.g., GAMS) to solve the model. As was hopefully demonstrated in this chapter, programming a spatial pricing problem and performing sensitivity analysis is very straightforward in Excel. Excel is therefore a good choice of software for small to medium spatial pricing problems, especially if Solver's capabilities are extended by scaling the parameters and using reasonably accurate initial values for the model's choice variables. Solver is not particularly good at solving large problems with a large number of corner solutions. It is for this reason that practitioners who frequently work with spatial pricing and similar types of linear programming problems typically choose to use specialized optimization packages rather than Excel.

Questions

1 Countries A and B both produce coffee and export all of it to Country C. The export price is \$0.03/lb higher in A than in B. What can you conclude about the transportation costs that are associated with these three countries?

2 Countries A and B are net exporters of maize and countries C and D are net importers of maize. Assuming that the market consists of these four countries, demonstrate that only in a special case will A export to both C and D and at the same time B will export to both C and D. Assume that the transportation cost between each pair of counties has a unique value.

3 Countries A, B and C produce, consume and trade apples. Last year wholesale prices in Countries A, B and C were \$0.56/kg, \$0.59/kg and \$0.52/kg, respectively. During that period two pairs of countries traded apples with each other (e.g., A exports to B and C). Describe two possible trading scenarios that are consistent with the spatial version of the LOP. For each scenario, what is the set of transportation costs?

4 Referring back to Question 3, suppose that for the current year supply and demand conditions are the same as in the previous year except now production in Country C is significantly lower. As a result of the lower production, Country C is now importing from Country B and Country A is not trading. What can you definitely conclude about the transportation costs between these three countries?

5 The sugar demand and supply parameters for three regions (A, B and C), as well as the grid of transportation costs, are listed in Table 2.Q. Notice that A exports

Table 2.Q Demand, supply and unit transportation cost parameters for Question 2.5

	Demand		*Supply*			*Unit transport cost (\$/tonne)*		
Country	*Intercept*	*Slope*	*Intercept*	*Slope*	*Country*	*A*	*B*	*C*
A	0	0	2	0.5	A	0	2	3
B	100	2	0	0	B	2	0	4
C	75	1	5	0.75	C	3	4	0

everything that it produces because it has no domestic consumers. Conversely, B imports everything that it consumes because it has no domestic producers. Region C, which both produces and consumes, may be a net importer or a net exporter, depending on the particular values assumed for the parameters.

a Solve for the LOP spatial pricing equilibrium using both methods discussed. For the first method, program Solver to choose the three prices in the three regions to ensure that the LOP pricing relationships are not violated and total exports equal total imports. For the second method, program Solver to choose the set of interregional shipments to maximize net aggregate welfare and then recover the set of equilibrium prices. When using this latter method don't allow Solver to choose any shipments for B (because B does not produce) and for A shipping to itself (because there is no domestic market in A).
b Sensitivity analysis: without using your Excel model, predict and *explain* the impact on price in the three countries from the following events.
    i The supply intercept in region A shifts up from 2 to 6 due to disease problems.
    ii The transport cost between A and C increases from 3 to 10 due to the imposition of a tariff by region C.
    iii The demand intercept in region C decreases from 75 to 40 as a result of health concerns about the commodity in region C.

3 Prices over time (storage)

3.1 Introduction

The focus of the previous chapter was the price of a homogeneous commodity selling in different geographical regions at the same point in time (i.e., spatial price analysis). This chapter begins the analysis of the price of a homogenous commodity with periodic production that is sold at the same location but at different points in time (i.e., intertemporal price analysis). This current analysis examines the determinants of the current price and the set of prices that agents expect to observe in the future given the information that is available at the current point in time.[1] In other words, this analysis is about the intertemporal profile of commodity prices. To keep the analysis focused on storage and spot market transactions attention is restricted to commodities that do not trade in a centralized futures market (e.g., potatoes, lentils and apples). Chapter 4 continues the analysis of prices over time by examining the pricing of a storable commodity with periodic production in the presence of a commodity futures market.

A trader has a private incentive to store the commodity if the difference between the discounted expected future selling price and the current price exceeds the marginal physical cost of storage. The marginal physical cost of storage will be treated as a constant for much of the analysis even though in reality this variable may change over time due to a change in aggregate stocks. In a competitive market investment in storage by many traders will raise the current price and lower the expected future price until the marginal profits from investing in commodity storage are driven to zero. If marginal profits from investing in commodity storage are negative for all levels of storage then nothing will be stored by competitive traders and the market will be "stocked out".

The above discussion implies that there are two properties of the intertemporal version of the LOP. First, if a commodity is being stored for speculative reasons (versus pipeline stocks held by commercial firms) then the difference between a commodity's discounted expected future price and its current price should equal the marginal physical cost of storage. Another way of stating this result is that when speculative stocks are being held by competitive traders then the discounted expected price is expected to rise over time at a rate equal to the marginal cost of storage. The second property of the intertemporal LOP is that if traders are not holding any speculative stocks then the difference between the discounted expected future selling price and the current price may be positive or negative but in either case must be less than the marginal physical cost of storage. This situation will be referred to as the stock-out property of the LOP.

Of course the discounted price of a storable commodity cannot be expected to rise indefinitely at a rate equal to the marginal cost of storage. If the commodity is harvested on a periodic basis then price is expected to rise at a rate equal to the marginal cost of storage between two adjacent harvesting periods, but is expected to drop when the next harvest begins to reflect the new availability of the commodity. This repeating "saw-tooth" pattern of prices over time is often difficult to observe in the data because of trade between regions with different harvesting seasons and because prices are continually changing in response to changes in supply and demand conditions. The drop in price at the time of harvest will be particularly small if the volume of stocks carried over from one production period to the next is unusually large.

The "saw-tooth" pricing pattern attributable to storage is readily apparent when long-term averages are considered. Recall Figure 1.7, which was briefly discussed in Chapter 1 and which shows for the case of US winter wheat the set of long-run average deviations between the monthly price and the average annual price. Figure 1.7 clearly demonstrates that on average price is lowest in the months immediately following harvest (June, July and August) and rises steadily over the course of the year until just prior to the next harvesting season (February–March). With the arrival of a new harvesting season the long-run average price drops to previous post-harvest levels in response to the new source of supply, and then the cycle repeats.

Storage has an important price stabilizing role across production periods. Storage creates a pricing buffer by shifting stocks forward through time from periods of above average stocks to periods where stocks are expected to be average or below average. Commercial firms typically require "pipeline" stocks to ensure that processing facilities and feedlots do not run short of key production inputs. Of particular interest in this analysis is speculative storage, which is storage in excess of these pipeline stock requirements. Investment in storage will increase in a high production period and will decrease in a low production period in response to changes in the expected price differential across time. When aggregated across the market as a whole these storage adjustments by individual traders reduce price fluctuations across time. For commodities such as fresh fruit where storage is not possible, price fluctuations across production periods tend to be much more substantial as compared with the storable commodity case.

Figure 3.1 illustrates the basic economics of commodity prices with combined intra-year and inter-year storage for US wheat production for the crops years spanning 1976/7 to 2005/6. The square-shaped markers show annual US wheat production in millions of bushels. The solid line shows quarterly US wheat stocks, also measured in millions of bushels. The dashed line shows the quarterly spot price of wheat ($/bushel), which is averaged over various US locations and over three-month time intervals. Figure 3.1 reveals that within a year stocks are highest

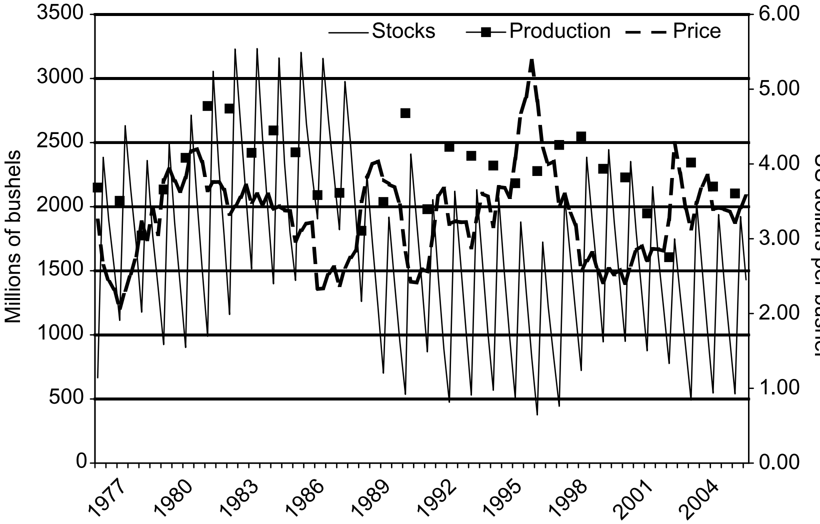


Figure 3.1 Annual US quarterly wheat production, stocks and price: 1976/7–2005/6.

Source: *Wheat Data: Yearbook Tables* (various years), Economic Research Service, USDA. Data downloaded 30 June 2009.

following harvest, are gradually depleted over the course of the year and reach a low just prior to the following year's harvest. Notice that the difference between maximum and minimum annual stocks, which represent wheat usage, is quite stable over the thirty-year period.

Figure 3.1 also reveals that rising wheat production from the late 1970s to the mid 1980s raised stocks to record high levels for much of the 1980s, which in turn caused the price of wheat to decline continually during this decade. The sizeable volumes of inter-year storage throughout the 1980s is consistent with the theory of storage. Specifically, during times of above average production and inventory the market will anticipate an eventual return to normal levels of production and higher prices, which justifies carrying sizeable inventories across years. In contrast, production was significantly below average from 1985 to 1990, which caused wheat inventories to gradually decline and price to gradually rise throughout the 1990s. In this case annual production was significantly higher than peak stocks, which implies that very little wheat was being stored across years during this period. With low levels of production and rising prices, the market anticipates higher future production and lower future prices, so only a minimal amount is stored across years.

Fortunately for government there is little need to actively manage public stockpiles of commodities in order to maximize aggregate social welfare. In the analysis below it is shown that a storage rule that maximizes aggregate welfare also gives rise to the intertemporal version of the LOP. This "invisible hand" result is expected given the analysis of Chapter 2, which showed that a LOP equilibrium is consistent with maximum aggregate welfare. It is important to note that storage by competitive traders does not automatically eliminate the need for government stockpiling. Government intervention may still be socially valuable because the market does not internalize important social externalities such as the impact of high commodity prices on the well-being of the poor and the need for food security during times of political uncertainty.

The specific purpose of this chapter is to construct a model of intertemporal pricing relationships for a storable commodity, initially under the assumption of no production and demand uncertainty and then with the production uncertainty assumption relaxed. In both cases the model involves dynamic programming. This technique is required to solve an intertemporal pricing problem because the amount of commodity the market will choose to store today depends on the price the market expects in the future, but this expected future price depends on how much will be stored in the future, which is not currently known. Dynamic programming solves for the optimal level of stocks and the corresponding set of prices over time by recursively solving the problem backwards through time.

In the absence of production and demand uncertainty, surplus production will be gradually disposed of over time, and the discounted price will rise smoothly over time at a rate equal to the marginal cost of storing the surplus commodity. In this artificial world, price in the future is perfectly predictable once the optimal storage rule has been derived. With production and/or demand uncertainty a significant complication arises. Now the market must anticipate future "stock outs", which occur when production is low and the market has no incentive to store across production periods for speculative reasons. A stock out implies that the linkage between price and storage is broken, and the dynamic programming relationship is severed. It is not possible to obtain an analytical solution to a storage problem when a market is subject to stock outs, and so a numerical-based optimization model must be used.

In the next section a simple two-period model is presented to illustrate the key economic features of the storage decision. Following this, in Section 3.3, the more general dynamic programming model of storage is specified for the case of no uncertainty. In Section 3.4 a case study that involves the gradual sale of a large Australian wool stockpile during the 1990s is calibrated and entered into Excel in order to fully illustrate the solution technique. In Section 3.5 a stochastic version of the storage problem is examined, and features of the solution are discussed. Concluding comments for this chapter are presented in Section 3.6.

3.2 Two-period model of storage

A simple two-period storage model with no uncertainty can be used to illustrate several key principles. Production in periods 1 and 2 are denoted h_1 and h_2, respectively (these h parameters are exogenous, and they can be thought of as harvest levels). Inverse market demand, which is the same in each of the two periods, is given by $P(x)$ where x denotes the quantity consumed and $P'(x) < 0$ because demand slopes down. If S units of period 1 production are stored and sold in period 2, then the pair of equilibrium prices for the two periods will equal $P(h_1 - S)$ and $P(h_2 + S)$, respectively. Let $C(S)$ denote the cost of storing S units of the commodity from period 1 to period 2. It is reasonable to assume that $C'(S) > 0$ and $C''(S) \geq 0$ because the marginal cost of storage is positive and possibly increases as stocks increase.

Surplus for consumers, which is equal to the area below the consumer demand schedule up to the quantity of consumption less expenditures on the commodity, can be expressed as $\int_0^{h_1-S} P(x)dx - P(h_1 - S)(h_1 - S)$ for period 1 and $\int_0^{h_2+S} P(x)dx - P(h_2 + S)(h_2 + S)$ for period 2. Producer surplus, which is equal to the revenues generated by commodity sales, can be expressed as $P(h_1 - S)(h_1 - S)$ for period 1 and $P(h_2 + S)(h_2 + S)$ for period 2. Production costs have no effect on the optimal storage rule and are therefore excluded from the analysis because production is fixed at level h. Let δ denote the discount factor, which is relevant for the two periods.[2]

Using the above expressions, an expression for net aggregate welfare, which is the discounted sum of consumer and producer surplus across the two periods minus the cost of storage, can be expressed as $V(S) = \int_0^{h_1-S} P(x)dx + \delta \int_0^{h_2+S} P(x)dx - C(S)$. This particular specification assumes that $C(S)$ is measured in period 1 and therefore does not need to be discounted. The Kuhn–Tucker conditions for achieving a maximum value for $V(S)$ subject to $S \geq 0$ can be written as $V'(S) + \lambda \leq 0$, $(V'(S) + \lambda)\, S = 0$ and $\lambda S = 0$, where $\lambda \geq 0$ is the Khun–Tucker multiplier variable. Noting that $V'(S) = -[P(h_1 - S) + C'(S)] + \delta P(h_2 + S)$ the Kuhn–Tucker conditions imply that socially optimal stock holding requires[3]

$$S^* = 0 \text{ if } P(h_1 - 0) + C'(0) > \delta P(h_2 + 0) \text{ and}$$
$$S^* > 0 \text{ if } P(h_1 - S^*) + C'(S^*) = \delta P(h_2 + S^*) \quad (3.1)$$

Equation (3.1) reveals that if positive storage is optimal, then storage should continue until the price of the commodity in period 1 plus the marginal cost of storage is equal to the discounted price of the commodity in period 2. Noting that $P'(x) < 0$, equation (3.1) further reveals that a necessary condition for positive storage to be optimal is $h_1 > h_2$. This condition makes sense because it would never be optimal to carry stocks from a relatively low production period into a relatively high production period. If $h_1 \leq h_2$, or if $h_1 - h_2$ is positive but not sufficiently large, then zero stocks is optimal, in which case the discounted price difference across the two periods will be less than the marginal cost of storage and possibly will be negative.

Equation (3.1) can also be interpreted as the pair of arbitrage conditions within a competitive market where all storage decisions are made by profit-seeking agents. Indeed, with $S^* > 0$, equation (3.1) ensures zero profits for an agent who purchases a unit of the commodity in period 1 at price $P(h_1 - S^*)$, pays the marginal cost of storage, $C'(S^*)$, and then sells that unit of the commodity in period 2 at the present value price $\delta P(h_2 + S^*)$.[4] The top line of equation (3.1) shows that $\delta P(h_2 + 0) - P(h_1 - 0) - C'(0) < 0$ when $S^* = 0$, which is also consistent with the no arbitrage outcome. This dual interpretation of equation (3.1) implies that the equilibrium level of storage in a competitive market with profit-seeking traders can be conveniently derived by choosing the level of storage that maximizes aggregate market welfare.

3.3 *T*-period model of storage with no uncertainty

If commodity stocks can be stored over several periods (e.g., multiple months within a growing period or across multiple growing periods), then the storage problem becomes more complicated because optimal storage for period t will depend on the equilibrium price in period $t + 1$, which in turn depends on the optimal level of storage for period $t + 1$, etc. It is therefore necessary to use dynamic programming to solve the storage problem. In general, numerical dynamic programming methods must be used to solve a storage problem. However, in this section a number of simplifying assumptions are made so that an analytical solution is possible. An analytical solution most effectively illustrates the intertemporal properties of commodity prices for a storable commodity.

Similar to the previous section, assume that production is exogenously fixed at level h and is thus independent of price. Also assume that market demand is linear and the marginal physical cost of storage is constant at level m. The LOP equilibrium is solved for by choosing the level of storage that maximizes the sum of producer revenue and consumer surplus minus the cost of storage over a T period time horizon. Once the optimal storage rule has been obtained, the difference between the level of stocks that are brought in and taken out of period t can be added to period t production to obtain total consumption for period t. Total consumption can then be substituted into the market demand schedule in order to recover the period t equilibrium price.

The first step in the analysis is to construct an expression for the present value of net aggregate welfare, which is denoted V. To construct this function, assume that production/harvest occurs at the beginning of each period and consumption occurs immediately after harvest. Let S_{t-1} denote the volume of stocks carried out of period $t - 1$ and into period t. The amount of stocks used for consumption in period t is denoted $x_t = S_{t-1} - S_t$. Total consumption during period t is equal to consumption of stocks, x_t, plus period t harvest, h. For the analysis below it is more convenient to express consumption as $q_t - S_t$, where $q_t = h + S_{t-1}$ is the level of stocks that are available for consumption in period t.

The price of the commodity in period t depends on the level of consumption, $q_t - S_t$, according to the inverse market demand schedule $P_t = a - b(q_t - S_t)$. Because production costs have been excluded from the analysis, the sum of consumer and producer surplus for period t is the area below the consumer demand schedule up to the consumption quantity, $q_t - S_t$. With linear demand, this joint surplus area is equal to $a(q_t - S_t) - 0.5b\,(q_t - S_t)^2$. This joint surplus area must be discounted and summed over time in order to construct the net aggregate welfare function.

Dynamic programming procedure

The equation of motion for the dynamic programming problem can be written as $q_{t+1} = q_t - x_t$.[5] This equation implies that stocks available for consumption, q_t, is the state variable and stock removals, x_t, is the control variable. However, because $S_t = x_t + S_{t-1}$ and S_{t-1} is predetermined in period t, it is acceptable to use S_t rather than x_t as the control variable. In this more convenient specification of the problem the equation of motion for the dynamic programming problem can be written as $q_{t+1} = h + S_t$.

Dynamic programming requires the specification of a Bellman equation. To construct this equation, let $V_t(q)$ denote the present value of net aggregate welfare as of period t assuming that q units of post-harvest stocks are available for consumption at the beginning of period t and storage in the current and future periods is optimally chosen. The $V_t(q)$ expression, which is commonly referred to as the value function, is defined recursively as:

$$V_t(q) = MAX_S \left\{ a(q-S) - 0.5b(q-S)^2 - mS + \delta V_{t+1}(h+S) \right\} \tag{3.2}$$

Equation (3.2) shows that the value function at the beginning of period t is equal to the surplus associated with period t consumption minus the (present value) cost of storage, mS, plus the discounted value function for period $t + 1$.

Equation (3.2) can be solved for the special case where $S_t^*(q_t) > 0$ for $t = 0,1,\ldots,T$. When this assumption holds the general solution procedure for solving the dynamic programming problem is as follows. In period t the optimal level of storage is calculated and written as a function of q_t. This storage function, $S_t^*(q_t)$, is then substituted into the value function to obtain:

$$V_t(q_t) = a\left(q - S_t^*(q_t)\right) - 0.5b\left(q - S_t^*(q_t)\right)^2 - mS_t^*(q_t) + \delta V_{t+1}\left(h + S_t^*(q_t)\right) \tag{3.3}$$

Now substitute the equation of motion, $h + S_{t-1}$, for q_t into equation (3.3) in order to obtain an expression for $V_t(h + S_{t-1})$. This new expression can be substituted into equation (3.2) that has been incremented back in time by one period in order to create the value function for period $t - 1$. The above procedure of optimizing and substitution can now be repeated to obtain expressions for optimal storage and the value functions in periods $t - 2$, $t - 3$, etc. The problem is fully solved when period 0 is reached. At that point the full set of optimized S_t values can be used to recover consumption and prices on a period by period basis.

To put this procedure into practice, note that in the last period $S_T = 0$ (i.e., everything is consumed) and so the value function for period T can be expressed as $V_T(h + S_{T-1}) = a(h + S_{T-1}) - 0.5b(h + S_{T-1})^2$. Therefore, as of the beginning of the second-to-last period $(T - 1)$, with post-harvest stocks at level q_{T-1}, an expression for the value function is:

$$V_{T-1}\left(q_{T-1}\right)=\underset{S_{T-1}}{MAX}\begin{Bmatrix}a\left(q_{T-1}-S_{T-1}\right)-0.5b\left(q_{T-1}-S_{T-1}\right)^2-mS_{T-1}\\+A_{T-1}\left(h+S_{T-1}\right)-0.5B_{T-1}\left(h+S_{T-1}\right)^2\end{Bmatrix} \tag{3.4}$$

where $A_{T-1} = \delta a$ and $B_{T-1} = \delta b$. Next differentiate equation (3.4) with respect to S_{T-1}, set the resulting equation equal to 0 and then rearrange to obtain an expression for optimal level of storage in period $T-1$ as a function of available stocks for that period:

$$S^*_{T-1}\left(q_{T-1}\right)=\frac{A_{T-1}-a-m-B_{T-1}h+bq_{T-1}}{b+B_{T-1}} \tag{3.5}$$

If equation (3.5) is substituted into equation (3.4), the resulting equation with $h + S_{T-2}$ substituting for q_{T-1} can be simplified and rewritten as:

$$V^*_{T-1}\left(h+S_{T-2}\right)=\frac{A_{T-2}}{\delta}\left(h+S_{T-2}\right)-0.5\frac{B_{T-2}}{\delta}\left(h+S_{T-2}\right)^2+\frac{CONSTANT}{\delta} \tag{3.6}$$

where:

$$A_{T-2}=\frac{(a-bh)B_{T-1}+bA_{T-1}-mb}{b+B_{T-1}}\delta \text{ and } B_{T-2}=\frac{bB_{T-1}}{b+B_{T-1}}\delta \tag{3.7}$$

Within equation (3.6), the term *CONSTANT* represents a group of parameter values that are not important for the optimization problem because they are independent of S and q.

Using equation (3.2) as a template, the value function for period $T-2$ can be expressed as:

$$V_{T-2}(q_{T-2})=\underset{S_{T-2}}{MAX}\begin{Bmatrix}a\left(q_{T-2}-S_{T-2}\right)-0.5b\left(q_{T-2}-S_{T-2}\right)^2\\-mS_{T-2}+\delta V^*_{T-1}\left(h+S_{T-2}\right)\end{Bmatrix} \tag{3.8}$$

After substituting equation (3.6) for V^*_{T-1} equation (3.8) can be rewritten as:

$$V_{T-2}\left(q_{T-2}\right)=\underset{S_{T-2}}{MAX}\begin{Bmatrix}a\left(q_{T-2}-S_{T-2}\right)-0.5b\left(q_{T-2}-S_{T-2}\right)^2-mS_{T-2}\\+A_{T-2}\left(h+S_{T-2}\right)-0.5B_{T-1}\left(h+S_{T-2}\right)^2+CONSTANT\end{Bmatrix} \tag{3.9}$$

Notice that equation (3.9) is identical to the value function for period, $T-1$, which is given by equation (3.4), except this new equation contains the *CONSTANT* term and the time subscripts are written as $T-2$ rather than $T-1$. The optimal storage rule for period $T-2$ must therefore be given by equation (3.5) with $T-2$

substituting for $T - 1$. If the above procedure is repeated again, it will be discovered that the optimal storage rule for period $T - 3$ will also be given by equation (3.5) with $T - 3$ substituting for $T - 1$. In general, equation (3.5) is the time-dependent optimal storage rule for all time periods for which $S_t^* > 0$. Once the full set of S_t^* values have been derived, an expression for the equilibrium price in period t can be obtained by substituting $S_{t-1}^* + h - S_t^*$ into the inverse market demand function: $P_t = a - b(S_{t-1}^* + h - S_t^*)$. Using this procedure, the equilibrium price associated with S^* (q_t) can be recovered for each of the T periods.

The remaining complication is that the optimal storage rule given by equation (3.5) has been constructed recursively. Specifically, equation (3.5) shows that optimal storage in period t depends on A_t and B_t, but according to equation (3.7) the variable A_t depends on both A_{t-1} and B_{t-1}, and the variable B_t depends on B_{t-1}. This pair of difference equations for A and B must therefore be solved simultaneously in order to obtain an explicit solution for the optimal level of storage and the corresponding set of prices. Unfortunately, the difference equations are nonlinear, so it is not possible to obtain an analytical solution for this storage problem. A numerical solution technique is described in the next section in the context of a real-world case study.

3.4 Storage problem case study

Australia is a dominant wool producer, accounting for about 25 percent of global wool production. Throughout the 1970s and 1980s Australia operated with a price floor for wool in order to stabilize domestic wool prices. The Australian Wool Commission purchased excess supplies from the market and stored the wool in order to maintain the price floor. However, due to a growing gap between the floor price and the market price, the support scheme had collapsed by 1991 and a large public stockpile remained. Wool International was created by the Australian government to gradually dispose of the stockpile and repay the public loan that had been used to finance the stockpile.

Wool International's specific mandate was to dispose of the stockpile in a way that would minimize market distortions. One way to interpret this mandate is that Wool International should sell the wool similarly to how it would be sold if the stockpile was privately held by a large number of competitive traders. In this section the model developed in Section 3.3 is calibrated with data from the global wool market and used to examine wool disposition and pricing if Wool International had behaved in accordance with the LOP and a competitive market outcome.

In the late 1980s the price of wool was about \$A8.70/kg and annual wool production was about 2.9 million tonnes across all major wool producing countries. Just prior to the creation of Wool International in the early 1990s the Australian wool stockpile was approximately equal to Australia's annual production of about one million tonnes.[6] The price of wool was highly sensitive to residual stocks during this period, so assuming a demand elasticity of −0.1 is not unreasonable. The cost of storing wool is assumed to be \$A0.15/kg per quarter (three months), which is equivalent to assuming an annual storage cost of about 7 percent of the wool's value. The annual discount rate is set equal to 4 percent, which implies a quarterly discount factor of $\delta = (1 + 0.01)^{-1} = 0.9901$.

The first step in the analysis is to use the data to construct a linear market demand schedule. The formula for the elasticity of demand is $\varepsilon = \frac{dQ}{dP}\frac{P}{Q}$, which can be rearranged and written as $\frac{dQ}{dP} = \varepsilon\frac{Q}{P}$. Noting that $\varepsilon = -0.1$, $Q = 2.9$ and $P = 8.7$, it follows that $dQ/dP = -0.03333$. Given the inverse demand schedule $P = a - bQ$, it can be seen that $b = 30$. The inverse demand schedule implies that $8.7 = a - 30(2.9)$, which can be rearranged to give $a = 95.7$. The global inverse demand for wool can therefore be expressed as $P = 95.7 - 30Q$, where P is measured in A\$/kg and Q is measured in millions of tonnes.

Spreadsheet Model I

The parameters of the Australian wool model have been entered and named in cells A2:B7 of Figure 3.2. Optimal storage and prices are displayed in reverse chronological order for 20 quarters (five years). Columns B and C contain the recursive calculations that are required to generate the values for the A_t and B_t variables. Specifically, cell B12 contains $A_{T-1} = delta^* \, a$, cell C12 contains $B_{T-1} = delta^* \, b$. The expressions for the remaining A_t and B_t variables in cells B13:C30 are given by equation (3.7).[7] The storage variable, S_t, is calculated in column C using the Australian stockpile starting value of $S_0 = 1$ in cell D31 and equation (3.5) with $h + S_{t-2}$ substituting for q_{t-1} for the remaining cells. Price is calculated in column E using the demand function $P_t = a - b(h + S_{t-1} - S_t)$.

It is easy to verify in Figure 3.2 that price in column E conforms to the $\delta P_{t+1} = P_t + m$ LOP relationship for all 20 quarters. In other words, the discounted price is rising over time at a rate equal to the marginal cost of storage. With this rate of price increase storage arbitrage by profit-seeking traders is not possible. When examining the set of equilibrium prices in Figure 3.2 it is important to remember that they were derived by maximizing aggregate welfare in the market. In other words, Adam Smith's invisible hand works for competitive prices over time similar to how the invisible hand works for prices over space, as discussed in Chapter 2.

Despite the LOP properties of the set of prices in Figure 3.2, the optimal storage outcomes that reside in column D reveal a serious problem with the results. Specifically, for quarters 14 to 20 optimal storage is negative. This violation of the non-negativity restriction should not be surprising because the chosen time horizon is relatively long and no explicit restrictions were incorporated into the programming model to ensure non-negative values for S_t.[8]

The results in Figure 3.2 can be made to conform to the non-negativity restriction on storage by further exploiting the LOP condition. In this simple model where demand and production is certain and constant over time, the additional

	A	B	C	D	E	F
1	**Parameters**					
2	h	2.9	per-period production			
3	a	95.7	intercept of inverse demand			
4	b	30	slope of inverse demand			
5	m	0.15	marginal cost of storage			
6	delta	0.9901	discount factor			
7	S0	1	Australian stocks into period 1			
8		=delta*a	=delta*b	=a-b*(h+D12-D11)		
9	**Calculations**					
10	Quarter	A	B	S	P	
11	20 (T-0)			0.00	9.38	
12	19 (T-1)	94.753	29.703	-0.023	9.13	
13	18 (T-2)	51.351	14.778	-0.037	8.89	
14	17 (T-3)	36.807	9.803	-0.043	8.65	
15	16 (T-4)	29.477	7.315	-0.042	8.42	
16	15 (T-5)	25.033	5.823	-0.032	8.19	
17	14 (T-6)	22.032	4.828	-0.015	7.95	
18	13 (T-7)	19.856	4.118	0.010	7.73	
19	12 (T-8)	18.196	3.585	0.042	7.50	
20	11(T-9)	16.880	3.171	0.082	7.27	
21	10 (T-10)	15.804	2.839	0.130	7.05	
22	9 (T-11)	14.904	2.568	0.185	6.83	
23	8 (T-12)	14.135	2.342	0.247	6.62	
24	7 (T-13)	13.468	2.151	0.316	6.40	
25	6 (T-14)	12.880	1.987	0.393	6.19	
26	5 (T-15)	12.356	1.845	0.477	5.97	
27	4 (T-16)	11.884	1.721	0.568	5.77	
28	3 (T-17)	11.455	1.612	0.665	5.56	
29	2 (T-18)	11.062	1.514	0.770	5.35	
30	1 (T-19)	10.698	1.427	0.882	5.15	
31	0 (T-20)			1		
32		=delta*b*C29/(b+C29)		=S_0		
33						
34	=delta*((a-b*h)*C29+b*B29-m*b)/(b+C29)					
35						
36		=(B30-a-m-C30*h+b*(h+D31))/(b+C30)				
37						

Figure 3.2 Wool stock disposal model.

LOP restriction is that stocks should decline to zero over the first N periods and then remain equal to zero for all future periods. With this pattern of stock holding, price should rise according to the LOP between periods 1 and N, and then remain constant at the steady-state, zero-storage level, $P = a - bh$, for all remaining periods. The task at hand is to calculate a value for N from the parameters of the model and then incorporate this restriction into the spreadsheet model.

The LOP requires a "smooth" transition from the path of rising prices to the path of steady-state, zero-storage prices. In period N all stocks carried in from period $N - 1$ are consumed, and so $P_N = a - b\,(h + S_{N-1})$. In period $N + 1$, price begins its journey on the steady-state path of zero-storage, and so $P_{N+1} = a - bh$. In order for the LOP to hold during the transition from period N to period $N + 1$ it must the case that $\delta\,(a - bh) = a - b\,(h + S_{N-1}) + m$. Solve this equation to obtain $S_{N-1} = Z$ where

$$Z = \frac{(1-\delta)(a-bh)+m}{b} \tag{3.10}$$

The value of N that solves $S_{N-1} = Z$ is the point in time that the switch to steady-state pricing begins.

It is rather cumbersome to incorporate the $S_{N-1} = Z$ restriction into the spreadsheet model shown in Figure 3.2. An alternative approach is to derive an analytical expression for S_t that conforms to the LOP and then use this analytical expression to solve the $S_{N-1} = Z$ restriction for N. With initial stocks, S_0, and a T period time horizon an analytical expression for S_t that conforms to the LOP is derived in Appendix 3.1 and can be written as:

$$S_t = \left(\frac{\rho^T - \rho^t}{\rho^T - 1}\right)S_0 + \frac{\rho}{\rho-1}\left(\frac{\rho^t - 1}{\rho^T - 1}T - t\right)Z \tag{3.11}$$

Within equation (3.11) the parameter ρ has been substituted for $1/\delta$ to simplify the notation.

When solving for N there is no need to consider time beyond period N because $S = 0$ for all such time periods. Thus, using equation (3.11) the $S_{N-1} = Z$ restriction can be expressed as:

$$S_{N-1} = \left(\frac{\rho^N - \rho^{N-1}}{\rho^N - 1}\right)S_0 + \frac{\rho}{\rho-1}\left(\frac{\rho^{N-1} - 1}{\rho^N - 1}N - (N-1)\right)Z = Z \tag{3.12}$$

It is not possible to obtain an expression for the value of N that solves equation (3.12). A numerical solution value for N is therefore required.

Spreadsheet Model II

Figure 3.3 shows the calculations that are needed to obtain a value for N and to generate a solution for the Australian wool storage problem that does not involve negative storage. The original parameters from Model I (Figure 3.2) have been entered and named in cells A2:B7 of Figure 3.3. An initial guess value for the choice variable, N, resides in cell B10 (Solver or Goal Seek is eventually used to choose the equilibrium value of this variable). The other variables of the model are listed in cells B13:B16. Most important, cell B14 contains the expression for Z, which is given by equation (3.10) and cell B15 contains the expression for S_{N-1}, which is given by the main expression in equation (3.12). The difference in the values of cells B14 and B15 is displayed in cell B16. The solution value for N can be obtained by adjusting the value of N in cell B10 until the value in cell B16 vanishes (i.e., $S_{N-1} = Z$). Excel's Solver or Goal Seek tool can be used for this task.

The optimal level of stocks for each of the 20 quarters are displayed in standard chronological order in cells B21:B41 of Figure 3.3. These values have been calculated using equation (3.11) with $T = N$. An IF statement has been added to the

	A	B	C	D	E	F
1	**Parameters**					
2	h	2.9	per period production			
3	a	95.7	intercept of inverse demand			
4	b	30	slope of inverse demand			
5	m	0.15	marginal cost of storage			
6	delta	0.9901	discount factor			
7	S_0	1	Australian stocks into period 1			
8						
9	**Choice Variable**					
10	N	15.839	Number of quarters until zero-stocks			
11						
12	**Regular Variables**					
13	rho	1.010	← inverse of delta			
14	Z	0.0079	← =(1-delta)*(a-b*h)/b+m/b			
15	S_Tm1	0.0079	← main expression in			
16	Difference	0.0000	equation (3.12)			
17						
18	=IF(A21<=N,S_0*(rho^N-rho^A21)/(rho^N-1)					
19	+Z*(rho/(rho-1))*(N*(rho^A21-1)/(rho^N-1)-A21),0)					
20	Quarter	Stocks	Price			
21	0	1.000		=a-b*(h+B21-B22)		
22	1	0.884	5.22			
23	2	0.775	5.43			
24	3	0.673	5.63			
25	4	0.577	5.84			
26	5	0.489	6.05			
27	6	0.408	6.26			
28	7	0.334	6.48			
29	8	0.267	6.69			
30	9	0.207	6.91			
31	10	0.155	7.13			
32	11	0.110	7.35			
33	12	0.072	7.58			
34	13	0.043	7.81			
35	14	0.020	8.04			
36	15	0.006	8.27			
37	16	0.000	8.52			
38	17	0.000	8.70			
39	18	0.000	8.70			
40	19	0.000	8.70			
41	20	0.000	8.70			

Figure 3.3 Revised wool stock disposal model with non-negative storage.

formula in cells B21:B41 to ensure that stocks take on a value of zero after the current quarter indicator that resides in column A exceeds the value of N that resides in cell B10. Price is calculated in cells C22:C41 using the formula $P_t = a - b(h + S_{t-1} - S_t)$.

Cell B10 in Figure 3.3 reveals that the LOP equilibrium requires $N = 15.839$ quarters, which is approximately four years. This outcome implies that price rises according to the $\delta P_{t+1} = P_t + m$ pricing relationship for approximately the first 16 quarters and then levels off and remains constant at $A8.70/kg for all remaining time periods. By construction the transition between the rising price trajectory and the constant price trajectory satisfies the LOP.[9] The equilibrium prices that are reported Figure 3.3 are somewhat higher than the corresponding prices in Figure 3.2. This outcome is expected because in Figure 3.2 storage is allowed to become negative, which implies higher consumption and lower prices for all time periods.

In Table 3.1 the pricing outcome for the Figure 3.3 base case is compared with the pricing outcomes with alternative parameter settings. The third column shows the results with initial Australian wool stocks set at $S_0 = 1.25$ versus $S_0 = 1$ million tonnes. As expected, these additional stocks result in a longer stock disposal time horizon (18 quarters versus 16 quarters) and lower prices during the stock disposal period. The fourth column of Table 3.1 shows the pricing impact if the marginal physical cost of storage is $A0.3/kg per quarter versus $A0.15/kg per quarter. This higher cost of storage serves to decrease the optimal stock disposal time frame from 16 to 13 quarters, which in turn implies lower prices during the stock disposal time frame.

The impact of a higher discount rate is reported in the fifth column of Table 3.1. With a quarterly discount factor of $\delta = 0.97$ versus $\delta = 0.9901$, storage has a higher implicit cost. Hence, similar to the case of a higher marginal physical cost of storage, a higher discount rate results in a lower value for n (11 versus 16 quarters) and lower prices during the stock disposal time frame. Finally, the last column of Table 3.1 shows the impact on prices if the demand schedule is

Table 3.1 Sensitivity results for price in wool storage problem

Simulated price (A$ per kilogram)					
Quarter	*Base*	$S_0 = 1.25$	$m = 0.3$	$\delta = 0.97$	$\varepsilon = -0.2$
1	5.22	4.82	4.24	3.85	6.23
2	5.43	5.02	4.58	4.21	6.45
3	5.63	5.23	4.93	4.59	6.66
4	5.84	5.43	5.29	4.99	6.88
5	6.05	5.63	5.64	5.41	7.10
6	6.26	5.84	6.00	5.85	7.32
7	6.48	6.05	6.36	6.31	7.55
8	6.69	6.26	6.73	6.81	7.78
9	6.91	6.48	7.10	7.32	8.01
10	7.13	6.70	7.48	7.86	8.24
11	7.35	6.91	7.85	8.51	8.47
12	7.58	7.13	8.23	8.70	8.70
13	7.81	7.36	8.65	8.70	8.70
14	8.04	7.58	8.70	8.70	8.70
15	8.27	7.81	8.70	8.70	8.70
16	8.52	8.04	8.70	8.70	8.70
17	8.70	8.27	8.70	8.70	8.70
18	8.70	8.52	8.70	8.70	8.70
19	8.70	8.70	8.70	8.70	8.70
20	8.70	8.70	8.70	8.70	8.70

assumed to have an elasticity of −0.2 rather than −0.1 (i.e., more elastic).[10] A more elastic demand schedule implies a shorter stock disposal time frame (11 quarters versus 16 quarters) because stocks can be sold more aggressively while maintaining the $\delta P_{t+1} = P_t + m$ pricing relationship. Despite the more aggressive disposal of stocks, prices during the stock disposal period are higher than the base case because of the more elastic demand.

3.5 Storage model with uncertainty

Most commodity storage situations involve uncertainty in production and/or demand. Uncertainty implies that the $S_t \geq 0$ constraint will be binding in some future periods, but the timing of future stock outs is unknown when current storage decisions are being made. The $S_t \geq 0$ constraint will bind if traders have an incentive to "borrow" stocks from the future due to a relatively high current price and/or a relative low expected future price. The current price will be above the expected future price if current stocks are low due to recent production shortfalls and/or if future production is expected to be above average. As discussed earlier in this chapter, in a stock out scenario the $\delta P_{t+1} = P_t + m$ pricing rule for the LOP is replaced with $\delta P_{t+1} < P_t + m$ and $S_t^* = 0$.

With uncertainty added to the problem, an analytical solution for a typical storage problem is no longer possible, so a numerical solution procedure must be used. There are many software packages that are specifically designed to numerically solve dynamic programming problems, and solution algorithms can also be written using a standard programming language such as MATLAB. In this section the basic numerical approach is illustrated in an Excel workbook. The size of the programming problem is purposely kept small by restricting the continuous variables of the model to take on integer values.

Suppose the source of uncertainty is the volume of harvested commodity, h. Assume that h is independently drawn from the set $\{h_1, h_2, \ldots, h_N\}$, and each outcome is equally likely. Let $\{0,1,2, \ldots, K\}$ denote the set of $K + 1$ possible values for the control variable, S_t, which is the amount of commodity stored from period t to period $t + 1$. This assumption implies that storage is subject to a maximum value, K, which may in fact constrain the optimal solution when the inventory level is high. Let $q_t = h + S_{t-1}$ denote the state variable, which is the level of inventory on hand in period t, after harvest takes place. Noting that $S_t = \{0,1,2, \ldots, K\}$, it is appropriate to assume that $q_t \in \{h_1, h_1 + 1, h_1 + 2, \ldots, h_N + K\}$. Price in period t is determined by the inverse demand function, $P_t = a - b\,(q_t - S_t)$.

It is now possible to construct the Bellman equation that is required to solve the dynamic programming problem. This revised Bellman equation is the same as that given by equation (3.2) except now the expected value of V_{t+1} must be utilized. Specifically, instead of using $V_{t+1}\,(h + S)$, it is necessary to use $\frac{1}{N}\sum_{i=1}^{N} V_{t+1}\left(h_i + S\right)$ because each harvest value (h) in the set $\{h_1, h_2, \ldots, h_N\}$ occurs with probability $1/N$. The revised Bellman equation should therefore be written as:

$$V_t\left(q_t\right) = MAX\left\{a\left(q_t - S_t\right) - .5b\left(q_t - S_t\right)^2 - mS_t + \frac{\delta}{N}\sum_{i=1}^{N}\left(V_{t+1}\left(h_i + S_t\right)\right)\right\}$$
$$S_t \in \left\{0,1,...,K\right\} \quad (3.13)$$

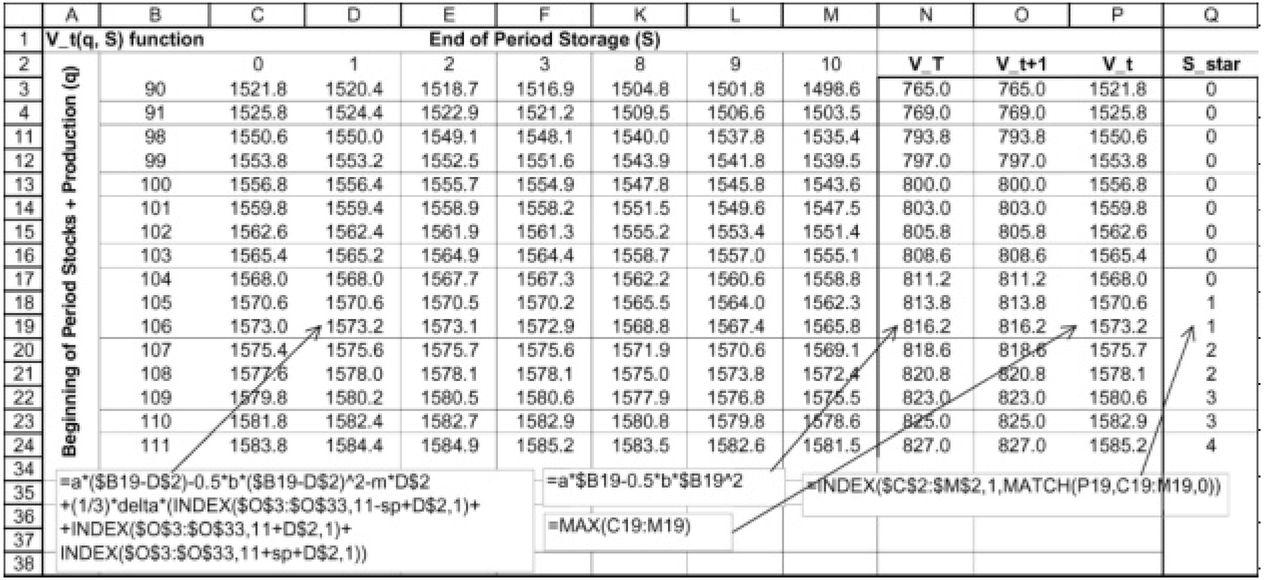
Within equation (3.13) the $S^* = 0$ corner solution must be directly accounted for in the numerical procedure because the $S^* = 0$ outcome will emerge for comparatively small values of q. In period $T - 1$ a numerical value of V_{T-1} will be calculated for each q_{T-1} value and all possible values of S_{T-1}. The value of S_{T-1} that maximizes V_{T-1} (in some cases the optimal S_{T-1} will equal 0) and the value of V_{T-1} that corresponds to the optimal S_{T-1} will both be saved for each q_{T-1} value. The optimized value of V_{T-1} can then be substituted into the Bellman equation. Using the Bellman equation and the relationship $q_{T-1} = h + S_{T-2}$, a value for V_{T-2} can now be calculated for each q_{T-2} value and all possible values of S_{T-2}. This optimization procedure can be repeated indefinitely until period 0 is reached.

Excel application

Suppose random harvest is low ($h_1 = 100 - sp$), average ($h_2 = 100$) or high ($h_3 = 100 + sp$) where $sp \in \{0,10\}$ identifies the degree of production uncertainty ("*sp*" refers to "spread"). With $sp = 0$ there is no uncertainty, in which case the solution to the storage problem will be the same as that described in Section 3.3. With $sp = 10$ production is uncertain because it can occur at level 90, 100 or 110 with equal probability. Further suppose that $T = 6$ (i.e., the market is terminated after five years of operating), the discount rate is $\delta = 0.95$, the unit cost of storage is $m = 0.2$ and the inverse market demand is defined by $a = 13$ and $b = 0.1$, which implies $P_t = 13 - 0.1\ (q_t - S_t)$.[11] These parameter values have been entered and named in an Excel worksheet, which is used below to solve the dynamic programming problem.

The setup of the model is displayed in Figure 3.4. The sheet shows the problem from the perspective of period $T - 1$ with $sp = 10$, but later it is shown that this same sheet can be used to solve the problem for all time periods and for both $sp = 10$ and $sp = 0$. Column B shows the alternative values of post-harvest stocks (q). The values range from a low of $q = 90$ to a high of $q = 120$ (the values from 5 to 10 and from 112 to 120 are hidden from view to conserve on space). The minimum value of $q = 90$ corresponds to a scenario where zero stocks were brought into period $T - 1$ and period $T - 1$ production was low at $h = 90$. The maximum value of $q = 120$ corresponds to a situation where maximum stocks of $S = 10$ units were brought into period $T - 1$ and period $T - 1$ production was high at $h = 110$.

Row 2 of Figure 3.4 shows alternative values for S, which is the amount of commodity that will be stored between periods $T - 1$ and T (the columns corresponding to $S = 4,5,6,7$ are hidden to conserve on space). Columns C through M in the main body of Figure 3.4 show the calculated value of V_{T-1}, as a function of q and S. For example, cell D19 shows that $V_{T-1} = 1573.2$ when $q = 106$ and $S = 1$.



	A	B	C	D	E	F	K	L	M	N	O	P	Q
1	V_t(q, S) function		End of Period Storage (S)										
2	Beginning of Period Stocks + Production (q)		0	1	2	3	8	9	10	V_T	V_t+1	V_t	S_star
3		90	1521.8	1520.4	1518.7	1516.9	1504.8	1501.8	1498.6	765.0	765.0	1521.8	0
4		91	1525.8	1524.4	1522.9	1521.2	1509.5	1506.6	1503.5	769.0	769.0	1525.8	0
11		98	1550.6	1550.0	1549.1	1548.1	1540.0	1537.8	1535.4	793.8	793.8	1550.6	0
12		99	1553.8	1553.2	1552.5	1551.6	1543.9	1541.8	1539.5	797.0	797.0	1553.8	0
13		100	1556.8	1556.4	1555.7	1554.9	1547.8	1545.8	1543.6	800.0	800.0	1556.8	0
14		101	1559.8	1559.4	1558.9	1558.2	1551.5	1549.6	1547.5	803.0	803.0	1559.8	0
15		102	1562.6	1562.4	1561.9	1561.3	1555.2	1553.4	1551.4	805.8	805.8	1562.6	0
16		103	1565.4	1565.2	1564.9	1564.4	1558.7	1557.0	1555.1	808.6	808.6	1565.4	0
17		104	1568.0	1568.0	1567.7	1567.3	1562.2	1560.6	1558.8	811.2	811.2	1568.0	0
18		105	1570.6	1570.6	1570.5	1570.2	1565.5	1564.0	1562.3	813.8	813.8	1570.6	1
19		106	1573.0	1573.2	1573.1	1572.9	1568.8	1567.4	1565.8	816.2	816.2	1573.2	1
20		107	1575.4	1575.6	1575.7	1575.6	1571.9	1570.6	1569.1	818.6	818.6	1575.7	2
21		108	1577.6	1578.0	1578.1	1578.1	1575.0	1573.8	1572.4	820.8	820.8	1578.1	2
22		109	1579.8	1580.2	1580.5	1580.6	1577.9	1576.8	1575.5	823.0	823.0	1580.6	3
23		110	1581.8	1582.4	1582.7	1582.9	1580.8	1579.8	1578.6	825.0	825.0	1582.9	3
24		111	1583.8	1584.4	1584.9	1585.2	1583.5	1582.6	1581.5	827.0	827.0	1585.2	4

Figure 3.4 Period $T-1$ results for optimal storage with uncertainty example.

Column N contains the set of values for V_T corresponding to the different values of q_T that reside in column B. The formula for calculating these V_T values is given by $aq_T - 0.5bq_T^2$. There is no discounted component in the V_T calculation because in period T storage is zero by assumption.

The remainder of the values in Figure 3.4 are calculated as follows. First, the V_T values in column N are manually pasted into the corresponding cells in column O, which is labeled V_t+1 (the reason for doing this is explained below). The formula contained in the main body of the table can now be explained using cell D19 as an example. The formula in cell D19 has two components, corresponding to the non-discounted expression and the discounted expression in equation (3.13). The non-discounted expression

"=a*($B19−D$2)−0.5*b*($B19−D$2)^2−m*D$2"

calculates current market surplus associated with commodity sales and consumption in period $T - 1$ minus the cost of storage. To understand this expression note that the value in cell B19 minus the value in cell D2 is a measure of $q - S$ and is therefore a measure of period $T - 1$ consumption.

The discounted expression in the cell D19 formula can be written as

"=(1/3)*delta*(INDEX($O$3:$O$33,11−sp+D$2,1)
+ INDEX($O$3:$O$33,11+D$2,1)+INDEX($O$3:$O$33,11+sp+D$2,1))"

This expression consists of the weighted sum of three index functions, which return values for V_T corresponding to "low", "average" and "high" levels of period T production.[12] To understand how these index functions work notice that Cell D19 corresponds to the situation where $S = 1$ unit of the commodity is brought into period T. Consequently, $q_T \in \{91,101,111\}$, depending on whether harvest is low, medium or high, respectively. The first value, $q_T = 91$, corresponds to the second position in the V_{t+1} matrix in column O, and so the expression "11 − sp + D$2" is used in the index function to return the second value in the O3:O33 cell vector. The second value, $q_T = 101$, corresponds to the twelfth position in the V_{t+1} matrix in column O, and so the expression "11 + D$2" is used in the index function to return the twelfth value in the O3:O33 cell vector. The index function corresponding to $q_T = 111$ utilizes the expression "11 + sp + D$2" in a way similar to that described above.

The next step in the programming is to instruct Excel to choose the largest value of V_{T-1} and the corresponding value of S for each q value in column B of Figure 3.4. The maximum V_{T-1} values are calculated and reported in column P using a simple "MAX" function. For example, cell P19 in Figure 3.4 reveals that out of the 10 possible V_{T-1} values corresponding to $S = \{0,1,2,\ldots, 10\}$, the maximum occurs at $V_T = 1573.2$ when $S = 1$. Column Q reports the value of S that gives rise to the maximum value of V_T for each value of q in column B. For example, in cell Q19 the formula

"=INDEX($C$2:$M$2,1,MATCH(P19,C19:M19,0))" matches the optimized $V_T = 1573.2$ value contained in cell P19 with the set of V_T values in row 19 and then returns the corresponding value of $S = 1$ from the set of S values in row 2.

The solution procedure for period $T - 1$ is now complete. Column Q of Figure 3.4 shows the optimal storage results for period $T - 1$. The results reveal that nothing should be carried over to period T if post-harvest stocks for period $T - 1$ are 104 units or less. One unit should be carried over if $q = 105$ or $q = 106$, two units should be carried over if $q = 107$ or $q = 108$, etc. These results imply that if the level of stocks carried into period $T - 1$ is four units or less, then storage from period $T - 1$ to period T will be zero unless production is high.

Given the solution procedure for period $T - 1$ displayed in Figure 3.4, solving the storage problem for periods $T - 2$, $T - 3$, etc. is now straightforward. Specifically, the V_{T-1} solution values contained in column P of Figure 3.4 should be manually pasted over the existing V_{t+1} values in column O. After doing this the new set of values that appear for V_t in column P and for S^* in column Q can be interpreted as the optimized period $T - 2$ values for V_t and S_t, respectively.[13] The recursive process of replacing the V_{t+1} values in column O with the optimized V_t values in column P as a method of incrementing the solution back in time by one period can continue indefinitely. For the problem at hand it was assumed that $T = 6$ so five column P to column O paste operations are required to generate the optimal storage results for period 1.[14]

Simulation results

The above procedure for generating optimal levels of storage for period 1 was repeated twice, first for the case of no uncertainty ($sp = 0$) and then for the uncertainty case ($sp = 10$). The results for these two cases are reported in Table 3.2 for the five-year time horizon of the storage problem. Notice that the optimal storage levels quickly converge as the solution moves back through time. Hence, if the solution procedure was continued for a longer time horizon, the values in the last column (period $T - 5$) would simply repeat.

Table 3.2 clearly demonstrates that optimal storage is at least as high and is often higher with uncertainty versus without uncertainty. For example, in period $T - 5$ optimal storage is equal to one unit in the uncertainty case and zero units in the no uncertainty case for $q \in \{103,104\}$. For $q > 104$ optimal storage with uncertainty exceeds optimal storage without uncertainty except for the case of $q = 120$. The remainder of this section is devoted to explaining this important result.

The reason why uncertainty induces the market to store more is because uncertainty raises the expected marginal value of storage. The expected marginal value increases because of the potential for the market to stock out in the future. The potential for a future stock out increases the convexity of the value function because price decreases remain limited by the potential to store in high production years but price increases are not limited because negative storage is not possible in low production years. Increased convexity of the value function raises the marginal value of storage and thus raises the optimal level of storage.

Table 3.2 Simulated optimal storage with and without uncertainty

	T – 1		*T – 2*		*T – 3*		*T – 4*		*T – 5*	
q	*First column (sp = 0)*				*Second column (sp = 10)*					
100	0	0	0	0	0	0	0	0	0	0
101	0	0	0	0	0	0	0	0	0	0
102	0	0	0	0	0	0	0	0	0	0
103	0	0	0	0	0	1	0	1	0	1
104	0	0	0	1	0	1	0	1	0	1
105	1	1	1	1	1	2	1	2	1	2
106	1	1	1	2	1	2	1	2	1	2
107	2	2	2	3	2	3	2	3	2	3
108	2	2	2	3	2	3	2	4	2	4
109	3	3	3	4	3	4	3	4	3	4
110	3	3	3	4	3	5	3	5	3	5
111	4	4	4	5	4	5	4	5	4	6
112	4	4	5	5	5	6	5	6	5	6
113	5	5	5	6	5	7	5	7	5	7
114	5	5	6	7	6	7	6	7	6	7
115	6	6	7	7	7	8	7	8	7	8
116	6	6	7	8	7	8	7	9	7	9
117	7	7	8	9	8	9	8	9	8	9
118	7	7	9	9	9	10	9	10	9	10
119	8	8	9	10	9	10	9	10	9	10
120	8	8	10	10	10	10	10	10	10	10

Note: No uncertainty corresponds to *sp* = 0 and uncertainty corresponds to *sp* = 10.

The intuition concerning storage and production uncertainty can also be described with reference to the expected price. In a market stock out when the $S_t \geq 0$ constraint binds the price will "spike" upward because the market is not able to borrow stocks to buffer the price increase. The potential for a price spike in the future raises the expected value of all future prices above the levels that would be expected for the case of no production uncertainty. In other words, traders should rationally anticipate periodic booms in commodity prices due to unexpected production and demand shocks, similar to the rapid run up in commodity prices that occurred in 2007/8. Having additional stocks on hand to sell during price spikes is a profitable strategy for traders.

3.6 Concluding comments

Storage is a key determinant of price for many of the world's major agricultural commodities. Storage is subject to spatial arbitrage and thus gives rise to an intertemporal version of the LOP. However, unlike spatial arbitrage where, depending on the price difference, the commodity can flow in both directions, the storage function is unidirectional. The inability to "borrow" stocks from the future implies that commodity stock outs are an important component of the intertemporal theory of commodity prices. Commodity suppliers and buyers must anticipate the impact of future stock outs on future prices when making current storage decisions because current storage decisions depend on both current prices and the expected values of the full set of prices in the future.

For the first part of the analysis a series of restrictive assumptions were made about the form of the demand schedule and the nature of storage costs. Under these conditions an analytical solution to the dynamic programming problem is achievable. The main conclusion from this simple analysis is that while carryover stocks remain positive the discounted price of a storable commodity rises over time at a rate equal to the marginal cost of storage. Sensitivity analysis reveals that stocks are depleted faster and the price rises faster if the marginal cost of storage increases or if the market's rate of discount increases. Sensitivity analysis also reveals that stocks are disposed of more quickly the greater the elasticity of market demand.

In the more general model considered in the second part of the analysis, production is assumed to be uncertain and stock out situations are inevitable. Consequently, numerical stochastic dynamic programming techniques must be used to solve for the market equilibrium. In this more general model, production uncertainty increases the optimal amount of storage because uncertainty raises the likelihood of a market stock out and a stock out can result in a price spike. Although not formally discussed, price variability due to the occasional price spike is expected to be higher when stocks are low because with low stock levels a future stock out is more likely.

The stochastic dynamic programming procedures described in this chapter are standard, but using an Excel workbook to solve a numerical dynamic programming problem is not standard practice. Programs such as MATLAB and GAUSS are much better suited for solving these types of programs because the underlying programming language utilizes matrix operations. The Visual Basic programming language, which operates in the background of each Excel worksheet, can be programmed to simplify the solution procedure somewhat, but it is probably wise to abandon the spreadsheet and adopt a more appropriate software package for most real-world applications.

Questions

1. The intertemporal version of the LOP indicates that if storage is positive, then $\delta E\{P_{t+1}\} - P_t = C$ where δ is the discount rate and C is the unit cost of storage between periods t and $t+1$. Suppose time is measured in six-month intervals. Also suppose that C is equal to \$10 per year and the opportunity cost of capital is 6 percent per year. Calculate values for equilibrium prices for the next six time periods assuming $P_0 = 120$ and positive storage.
2. A region will operate for two periods. Production of wheat is 80 units in period 1 and 0 units in period 2. The demand for wheat is $Q^d = 100 - P$ for each of the two periods. The cost of storage is $C = 7.3$ per unit. The region's discount rate is $\delta = 0.9$. Use two methods to calculate the LOP equilibrium

level of storage and the associated commodity prices. For the first method, calculate directly the set of prices that result in the equilibrium condition, $\delta E\ \{P_{t+1}\} - P_t = C$. For the second method, find the level of storage which maximizes market surplus in period 1 plus discounted market surplus in period 2 minus the discounted cost of storage. Ignore period 1 production costs in the calculation of market surplus.

3 A region with a discount rate equal to $\delta = 0.9$ will operate for three years. At the beginning of years 1, 2 and 3, the region will produce either one or two units of the commodity with equal probability. Just prior to production at the beginning of year 1, the region has commodity stocks equal to three units. Assume that production is low in year 1. At the end of year 1 and year 2, the region must decide how much of the available stocks to consume and how much to carry over to the next year (zero stocks should be carried out of year 3). Carryover stocks are restricted to an integer value. Inverse demand for each year is given by $P = 10 - Q$. The unit cost of storage is $m = 0.5$.

a Enter the appropriate formulas in the dark-bordered cells contained in rows 9 to 19 of the worksheet in Figure 3.Q. Specifically, calculate combined producer and consumer surplus for year 3 with different assumptions about the volume of inventory that was carried from year 2 to year 3. Perform these calculations for the case of low production (one unit is produced) and high production (two units are produced).

b Enter the appropriate formulas in the dark-bordered cells contained in rows 23 to 33 of Figure 3.Q. Specifically, calculate combined producer and consumer surplus for year 2 plus the pre-optimized discounted producer and consumer surplus for year 3 (accounting for the fact that the low and high production levels are equally likely in year 3) minus the cost of storage. These calculations must be made for all feasible combinations of stocks carried into year 2, stocks carried out of year 2 and into year 3, and low and high production in year 2.

c Enter the appropriate formulas in the dashed bordered cells residing in cells H23:H27 and H29:H33 of Figure 3.Q. For example, in cell H23 use Excel's MAX function to calculate the maximum of the values in cells C23:D23. In cell H24, calculate the maximum of the values in cells C24:E24, and so forth.

d Insert the appropriate values in the dark bordered cells in row 40 of Figure 3.Q. Specifically, calculate combined producer and consumer surplus for year 1 plus discounted optimized combined producer and consumer surplus for year 2 minus the cost of storage. Be sure to note that production is low in year 1 and that low and high production levels are equally likely in year 2.

e Based on the values calculated in part (d) of this question, what is the optimal amount to store between years 1 and 2? Briefly explain why the optimal amount to store in year 1 rises if the probability of a stock out in year 2 increases because of higher production uncertainty.

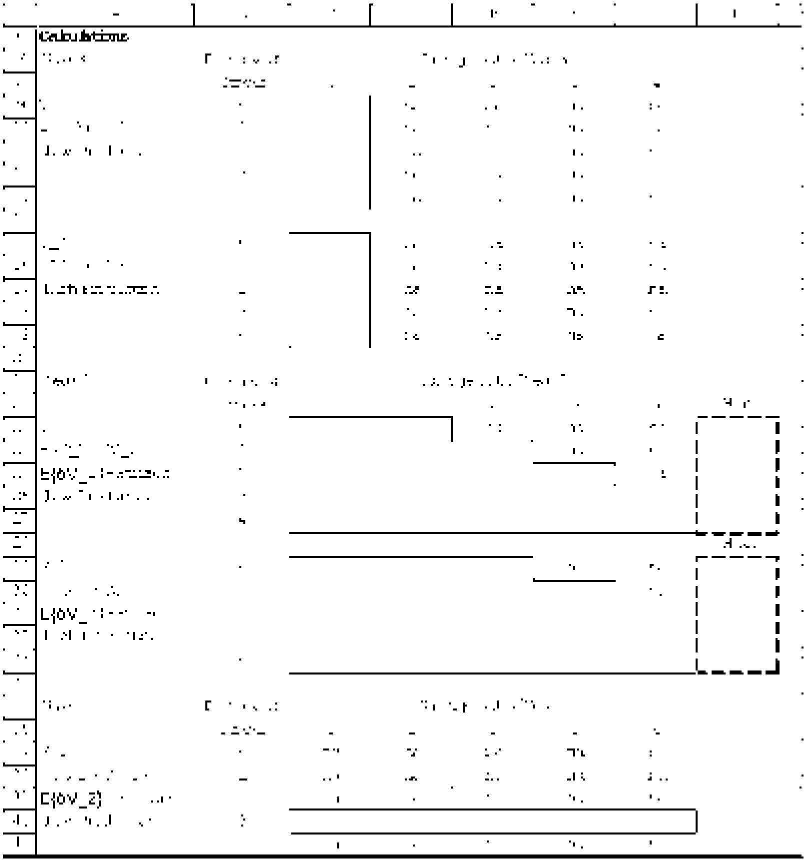


Figure 3.Q Worksheet for completing Question 3.3.

4 Use the Bellman equation approach to analytically solve the storage problem in Question 3 with the assumption that the unit cost of storage is $m = 0$, the discount rate is $\delta = 1$ and annual production is equal to 1.5 units with certainty.

a Derive an expression of V_3, which is combined producer and consumer surplus in year 3 assuming that all post-production inventory is consumed in year 3. This expression should be a function of q_3, which is post-production stocks for period 3. Substitute the equation of motion into the expression that you derived so that V_3 is a function of S_2, where S_2 is the amount stored from year 2 to year 3.

b Construct an expression for V_2 as a function of S_2. In this particular example V_2 is the sum of producer and consumer surplus for year 2 plus V_3. Now maximize the expression with respect to S_2 and assume the parameters are such that $S_2^* \geq 0$. Substitute the optimized expression for S_2 into the V_2 expression to obtain an expression for V_2^*.

c Construct an expression for V_1 as a function of S_1. Now maximize the expression with respect to S_1 and assume the parameters are such that $S_1^* \geq 0$. The derived value of S_1^* is the desired solution to the problem.

Appendix 3.1: Derivation of equation (3.11)

Rather than maximizing an aggregate welfare function to obtain an optimized expression for S_t, the LOP restriction can be imposed directly. Recall that $x_t = S_{t-1} - S_t$ is a measure of removals from the stockpile in period t. Thus, $P_t = a - b(h + x_t)$ and $P_{t+1} = a - b(h + x_{t+1})$. These two equations can be inserted into the LOP condition, $\delta P_{t+1} = P_t + m$, to obtain $x_{t+1} = \rho(x_t - Z)$, where $\rho = 1/\delta$ and Z is given by equation (3.10) in the text.

The equation of motion implies $S_1 = S_0 - x_1$ and $S_2 = S_1 - x_2$. Substituting the former equation into the latter along with $x_2 = \rho(x_1 - Z)$ gives $S_2 = S_0 - (1 + \rho)x_1 + \rho Z$. Now substitute $x_2 = \rho(x_1 - Z)$ into $x_3 = \rho(x_2 - Z)$ to obtain $x_3 = \rho x_3 = \rho^2 x_1 - (1 + \rho)\rho Z$. Substitute this expression along with $S_2 = S_0 - (1 + \rho)x_1 + \rho Z$ into $S_3 = S_2 - x_3$ to obtain $S_3 = S_0 - (1 + \rho + \rho^2)x_1 + (1 + (1 + \rho))\rho Z$. Continue with this substitution and reduction procedure until the following pattern emerges:

$$S_t = S_0 - x_1 \sum_{i=0}^{t-1} \rho^i + \rho Z \sum_{i=1}^{t-1} (t-i)\rho^{i-1} \tag{3.A1}$$

Noting that $\sum_{i=0}^{t-1} \rho^i = \frac{\rho^T - 1}{\rho - 1}$ and $\sum_{i=1}^{t-1} i\rho^{i-1} = \frac{1-\rho^t}{(1-\rho)^2} - \frac{t\rho^{t-1}}{1-\rho}$, equation (3.A1) can be rewritten as:

$$S_t = S_0 - \left(\frac{\rho^t - 1}{\rho - 1}\right)x_1 - \left(\frac{t}{\rho - 1} + \frac{1-\rho^t}{(1-\rho)^2}\right)\rho S_{T-1}^* \tag{3.A2}$$

The boundary condition $S_T = 0$ implies that equation (3.A2) with $t = T$ can be set to zero and the resulting expression solved for x_t in order to obtain a solution value for the variable x_1. After substituting this solution value for x_1 back into equation (3.A2), a revised expression for S_t can be written as equation (3.11) in the text.

4 Prices over time (commodity futures)

4.1 Introduction

The focus of the previous chapter was the intertemporal profile of agricultural commodity prices for a storable commodity at a particular point in time. The analysis was kept simple by restricting attention to prices in a spot market. In reality, commodity futures contracts trade in centralized markets for the major storable agricultural commodities such as maize, wheat, soybeans, sugar, coffee and cotton. Futures markets have existed for hundreds of years, and over the years have grown steadily in terms of volume of trade, breadth of product coverage and number of actively trading contracts for a particular commodity. Some commodity futures markets are very large and geographically diverse, such as the NYSE Euronext Group, which operates agricultural commodity exchanges in Amsterdam, Brussels, Lisbon, London and Paris, and the CME Group, which was formed in Chicago in 2007 with a merger of the Chicago Mercantile Exchange and the Chicago Board of Trade. Other exchanges are relatively small and regionally focused such as the Brazilian Mercantile & Futures Exchange (BM&FE) and the South African Futures Exchange (SAFEX).

Commodity futures markets serve the dual role of efficient price discovery and price risk transfer. Spot prices are normally strongly influenced by commodity futures prices, which in turn are established by competitive traders in an open outcry pit or electronic trading environment. In an efficient market, the futures price is the market's best estimate of the location-adjusted spot price at the time the futures contract expires. Low transaction costs due to market centralization, efficient dissemination of all relevant information and the highly competitive nature of speculation are the key attributes of efficient price discovery. Commodity futures markets also enable commercial firms involved with the purchase and sale of the physical commodity to transfer varying levels of price risk to speculators. This chapter is devoted exclusively to the price discovery role of a commodity futures market.

Traders in a commodity futures market take long or short positions in contracts with a dozen or so expiry dates (e.g., March, May, July, September and December for a 2–3 year horizon). A completed contract requires one long trader and one short trader. A long (short) position is a commitment to purchase (sell) the commodity during the month of contract expiry at the price that prevailed when the position was initially taken. However, rather than actually purchasing or selling the commodity when the contract expires, most traders offset their futures position and thereby eliminate all futures market obligations by taking a short position if they were originally long and vice versa. The difference between the futures market price when the contract is offset versus initiated, multiplied by the number of tonnes under contract, represents the profits or loss for the trader with a long position. The profit or loss earned by a short trader is opposite (i.e., the initial price minus the offset price).

The difference between the futures price and the spot price on a particular trading day is referred to as the basis.[1] The basis is subject to arbitrage, which ensures several key pricing relationships. First, if the futures price rises sufficiently high above the spot price, then a trader can lock in profits by purchasing the commodity, storing it and delivering against a short futures contract. The actions of many traders doing this will drive up the spot price and drive down the futures price until the basis is no larger than the unit cost storing and financing the inventory (i.e., the unit carrying cost). Similarly, if the price spread between two futures contracts with different maturity dates becomes sufficiently large, a trader will simultaneously take a long position in the nearby month and a short position in the more distant month. Because traders can accept delivery with the long contract, store the commodity and then make delivery against the short contract, the price spread is limited by the size of the unit carrying cost. Borrowers cannot borrow inventory from the future so there is no analogous arbitrage that prevents the price differences discussed above from falling below the corresponding carrying cost.

An immediate implication of arbitrage is that as the futures contract nears its expiry date and carrying costs for delivering on that contract disappear, the spot price and futures price will converge. The prices converge because in the absence of any new information the futures price is expected to remain constant over time whereas the spot price is expected to rise. This pricing difference occurs because maintaining a futures position does not result in carrying costs whereas holding inventory in the spot market does. If the spot market location is the same as the delivery location specified in the futures contract, then the spot and futures price should eventually equalize. If there is a difference in location then the price difference should converge to the cost of transporting the commodity from the commodity surplus location to the commodity deficit location. Figure 4.1 shows the gradual full convergence of the daily spot price and the December futures price for white maize, which trades on the SAFEX. The trading dates range from early June 2008 to the middle of December 2008, which is approximately when the December futures contract expires. The average rate of price convergence provides a good estimate of the average change in carrying costs over time.

Another important feature of commodity futures market is the spread between prices for contracts with different maturity dates. Simple economic theory predicts that in normal market conditions the price spread between a pair of adjacent contract months (e.g., January and March) will reflect the cost of carrying the

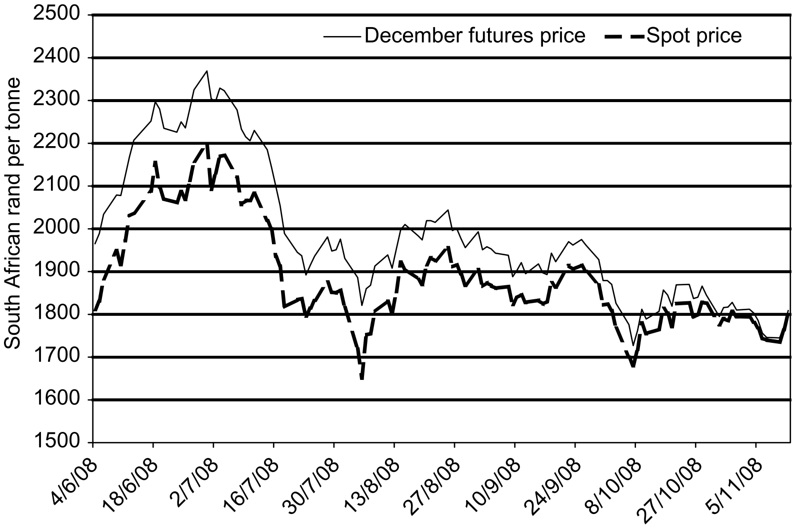


Figure 4.1 Spot price and SAFEX December futures price for white maize: June to November 2008.

Source: South African Futures Exchange (SAFEX). Data from http://www.safex.co.za/ap/market_price_history.asp Downloaded on 23 July 2009.

commodity between January and March. As discussed above, these carrying costs will include the various costs of storage and the opportunity cost of the capital that is tied up in the commodity inventory. Figure 4.2 shows price spreads for corn, wheat and soybean March and May 2010 futures contracts for the period August 2009 to March 2010. The price spread for corn is consistently above the spread for wheat, and both corn and wheat spreads are positive, fairly stable and gradually rising over time. In contrast, the March to May price spread for soybeans starts off negative and rises sharply over time to end up with a positive value. As well, soybean price spreads are highly volatile. Certainly the price spread for soybeans is being determined by factors other than the simple cost of carrying soybeans forward through time.

Figure 4.1 may leave the reader with the impression that the basis is fairly stable and predictable over time. In many cases this is true because the cash price is derived from the futures price so the difference between these two sets of prices (i.e., the basis) is expected to have a predictable pattern over time. However, similar to the volatile price spread for soybeans that was highlighted by Figure 4.2, the basis can also be highly variable and unpredictable over time. A volatile basis is one of the key factors that affect the performance of commodity hedges that are placed by commodity merchants. The economics of hedging are not including in this textbook other than a hedging example that forms the basis of Question 4.2 at the end of this chapter.

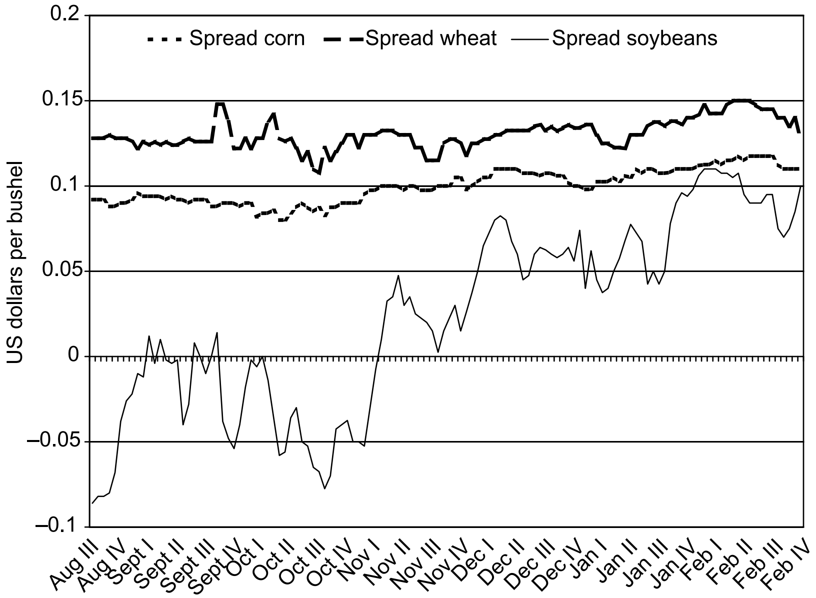


Figure 4.2 CBOT futures price spreads (March to May 2010 contracts) for corn, wheat and soybeans: August 2009 to March 2010.

Source: CBOT settlement price data was downloaded daily from the CBOT website (http://www.cmegroup.com).

The last three columns of Table 4.1 show the difference between the spot price and the November CBOT futures price for soybeans over a two-week period (4 August to 18 August 2010) at three US locations: Chicago, the US Gulf export terminals and Toledo, Ohio export terminals. The CBOT futures price is based on delivery in Chicago, so the Chicago basis that is reported in Table 4.1 excludes any location premium or discount. The basis is largest for the Gulf, presumably because the cost of transporting grain from Chicago to the Gulf is higher than the cost of transporting grain from Chicago to Toledo. Notice that the Chicago basis is quite volatile, ranging from a high of 13.5 cents per bushel on 4 August to a low of negative 15 cents per bushel on 16 August. The basis at Toledo and the Gulf also declined over this period, although in percentage terms the decline in the Gulf basis is comparatively small. The third column of Table 4.1 reveals that the September to November spread in soybean prices was highly volatile over this period, but not subject to the downward trend that characterizes the November basis. The fourth column reveals that the November to January spread is quite stable over this two-week period, despite the high degree of volatility in the September to November spread and the soybean basis.

Undoubtedly one of the weakest areas of commodity price theory involves the economic determinants of commodity price spreads and the basis. The simple

Table 4.1 Soybean futures price spreads and spot market basis (US$ per bushel): 4 August 2010 to 18 August 2010

	Futures spread			*November basis*		
	Nov. futures	*Sept. to Nov.*	*Nov. to Jan.*	*Chicago*	*Gulf*	*Toledo*
4 August	10.240	−0.050	0.058	0.135	1.050	0.375
5 August	10.290	−0.060	0.052	0.080	1.000	0.375
6 August	10.335	−0.055	0.060	0.075	0.965	0.375
9 August	10.350	0.005	0.058	−0.045	0.870	0.300
10 August	10.220	0.008	0.062	−0.040	0.870	0.275
11 August	10.155	−0.001	0.060	0.110	0.880	0.225
12 August	10.300	0.003	0.072	−0.035	0.865	0.210
13 August	10.440	0.005	0.065	−0.075	0.860	0.225
16 August	10.315	−0.025	0.062	−0.150	0.870	0.225
17 August	10.420	−0.032	0.065	−0.150	0.870	0.225
18 August	10.310	−0.045	0.068	−0.150	0.890	0.180

Source: Export spot price data was downloaded on 17 August 17 2010 from USDA Cash Bid Report. CBOT settlement price data was downloaded daily from the CBOT website: http://www.cmegroup.com

Note: Basis calculated as the average of the low and high export price minus the November 2010 futures price. Futures prices are CBOT settle prices.

theoretical prediction that the price spreads and the basis should equal the cost of carrying the commodity forward through time unless the market is stocked out falls well short of explaining the observed spreads and basis, especially for the case of soybeans. The theory of arbitrage allows the price spreads and the basis to fall below the unit carrying cost, but how far below is not well understood. Common explanations for a volatile basis include transportation "bottlenecks" and non-committed supply and demand imbalances in the two distinct markets that cannot be readily arbitraged.[2]

The purpose of this chapter is to construct two simple models of prices for a storable commodity with the assumption that production is uncertain, price is discovered by traders in a futures market, and arbitrage connects the cash price with the futures price. In the first part of the analysis, the LOP from Chapter 3 is imposed on the model. Consequently, in the simulation results the difference between the futures price and the spot price (i.e., the basis) and the difference between a pair of futures prices (i.e., the spread) is equal to the unit carrying cost of storage when storage is positive and is less than the unit carrying cost (and possibly is negative) when the market stocks out. The results reveal that with a low level of production uncertainty, in which case there is minimal chance of a stock out, the basis is expected to remain relatively constant over time. Conversely, with a high level of production uncertainty the basis is expected to fluctuate over time in response to changes in the probability of a stock out. In both cases, information shocks concerning future production and demand have an immediate impact on the interdependent set of futures and spot prices.

In the second part of the analysis a simple two-period model with no production uncertainty is used to describe the convenience yield theory of commodity prices. This theory postulates that commercial firms hold inventories to reduce marketing transaction costs, even if the expected increase in the value of the stored commodity is less than the cost of storing and financing the commodity. The reduction in transaction costs associated with holding inventories is referred to as convenience yield. If convenience yield is included in the LOP equation and treated as a legitimate financial benefit that offsets carrying costs, then the standard properties of an intertemporal market equilibrium continue to hold. If convenience yield is not accounted for, then the measured basis and price spreads will be below the unit carrying cost and will fluctuate over time in response to changes in the actual convenience that is generated by the inventories.

Before beginning the formal analysis, it is useful to comment on the terms "contango" and "backwardation". The term "contango" is commonly used by industry traders to describe a "normal" market situation for a storable commodity where a futures contract with a longer maturity trades at a higher price than the same contract with a shorter maturity. The term "backwardation" is used to describe the opposite situation and is therefore equivalent to an inverted market. These two terms will be avoided in this chapter because in the economics literature on futures markets these terms describe situations where the futures price for a contract with a particular maturity (e.g., December 2011) is expected to rise over time ("contango") or fall over time ("normal backwardation"). Normal backwardation is believed to be a mechanism by which short hedgers of the physical commodity pay a risk premium to long speculators. In the formal analysis below there is no contango or normal backwardation (as defined in the economics literature) because the futures price is assumed to be an unbiased forecaster of the futures cash price. In other words, the futures price is not expected to rise or fall over time.

In the next section a two period model of stochastic production, storage and futures trading without a convenience yield is constructed. Simulation results from this model are presented in Section 4.3. A theoretical examination of convenience yield is presented in Section 4.4. Section 4.5 contains a short summary and concluding comments.

4.2 A model of commodity futures

The commodity futures model has three relevant dates: date 0 (beginning of the first period); date 1 (end of the first period and beginning of the second period); and date 2 (end of the second period). A futures market operates at dates 0 and 1, and a spot market operates at dates 1 and 2. The spot and futures market operate in the same location, so transportation costs are not considered in this analysis.

There are two types of futures contracts that trade at date 0: a date 1 contract that calls for commodity settlement at date 1, and a date 2 contract that calls for commodity settlement at date 2. As of date 1, only date 2 futures contracts trade because date 1 contracts have expired. Risk neutral speculators trade contracts in the futures market, and commodity producers and processors exchange the physical commodity in the spot market. Speculators continually arbitrage the spot and futures markets, and it is this search for arbitrage profits by speculators that links spot and futures prices together.

At date 0 commodity inventories are at level S_0. The commodity is randomly produced at dates 1 and 2, at stochastic levels $\tilde{Q}_1$ and $\tilde{Q}_2$, respectively. These production values are independent across time, which implies that the date 1 expected value of $\tilde{Q}_2$ is independent of the realized value of $\tilde{Q}_1$. There is no supply response in this model, which implies that the values for $\tilde{Q}_1$ and $\tilde{Q}_2$ are not influenced by market prices. Let μ_1 and μ_2 denote the mean values of $\tilde{Q}_1$ and $\tilde{Q}_2$, respectively. Speculators implicitly estimate values for μ_1 and μ_2 as part of the process of trading in the commodity futures market (more on this below).

Inverse demand for the commodity in the spot market is constant in each of the two periods at level $P = a - bQ$ where P is price and Q is the quantity demanded. The spot prices at dates 1 and 2 equal $\tilde{P}_1 = a - b\,(S_0 + \tilde{Q}_1 - S)$ and $\tilde{P}_2 = a - b\,(\tilde{Q}_2 + S)$, respectively, where S is the amount of the commodity that is stored between dates 1 and 2. The level of storage, S, and date 1 production, $\tilde{Q}_1$, are related according to the storage function, $S(\tilde{Q}_1)$, which is derived below. The storage function can be used to rewrite the pricing equations as:

$$\tilde{P}_1 = a - b\,(S_0 + \tilde{Q}_1 - S\,(\tilde{Q}_1)) \text{ and } \tilde{P}_2 = a - b\,(\tilde{Q}_2 + S\,(\tilde{Q}_1)) \tag{4.1}$$

Assume that the unit cost of carrying the commodity forward through time is fixed at level m. As well, the cost of capital associated with storage is assumed to be folded into m. This latter assumption greatly simplifies the analysis because it eliminates the need for discounting across the two time periods.

Price discovery

Speculators invest in information gathering in order to earn profits in the commodity futures market. It is this process of information gathering by profit-seeking speculators that leads to efficient price discovery. Speculators must estimate the production means, μ_1 and μ_2, when trading futures contracts. Information gathering is the process of speculators attempting to improve the accuracy of these estimates.

Consider a speculator who estimates a value for μ_1 that is lower than the average value estimated by all other speculators. This speculator will take a long futures position at date 0 because aggregate market supply is believed to have been over-estimated for date 1. Thus, the speculator believes that the date 2 futures price will rise between date 0 and date 1 (i.e., price will adjust after the estimation error has been discovered). Similarly, a speculator who estimates a relatively high value for μ_1 will take a short futures position at date 0 because this speculator believes that aggregate market supply has been underestimated, which implies a price decrease between date 0 and date 1.

One futures contract transaction requires one long speculator and one short speculator. The futures price will therefore adjust upward if the number of long speculators exceeds the number of short speculators. An increase in the futures price will cause some long speculators to drop out of the market and will cause some short speculators to enter the market. The futures price will continue to be bid up until a futures market equilibrium is achieved, which occurs when the number of long and short speculators are equal. The opposite situation will emerge if the number of short speculators initially exceeds the number of long speculators.

The futures price is said to be *unbiased* and the price discovery is said to be *efficient* if the information brought to the market by speculators is correct on average. An unbiased futures price is, on average, equal to the subsequent spot price. In an unbiased market the futures price is not expected to systematically rise or fall between dates 0 and 1. For the remainder of this chapter, assume that the pair of futures prices are unbiased.

The pair of unbiased futures prices are equal to the expected values of the subsequent spot prices, P_1 and P_2, assuming the values of μ_1 and μ_2 are correctly estimated and the level of commodity storage between dates 1 and 2 has been accurately anticipated. Specifically, $f_0^1 = E_0(\tilde{P}_1)$ and $f_0^2 = E_0(\tilde{P}_2)$ is the pair of unbiased date 0 futures prices for a date 1 and date 2 contract, respectively, and $f_1^2 = E_1(\tilde{P}_2)$ is the unbiased date 1 futures price for a date 2 contract. Expressions for these equilibrium futures prices are presented below.

The unbiased price assumption also implies that the date 1 price of a date 1 futures contract must equal the date 1 spot price, and the date 2 price of a date 2 futures contact must equal the date 2 spot price (i.e., $f_1^1 = P_1$ and $f_2^2 = P_2$). In other words, the price of an expiring futures contract must equal the associated spot price. As indicated earlier, if this was not the case then a speculator could profit by simultaneously taking a spot and futures market position (e.g., if $f_1^1 > P_1$ the speculator could profit by taking a short futures position and then using commodity purchased on the spot market to immediately fulfill the delivery requirements of the futures contract). A summary of the connection between the set of futures prices and spot prices is presented in Table 4.2.

Table 4.2 Notation used in futures price model

Description	*Symbol*	*Formula*
Date 0		
Price of date 1 contract	f_0^1	$f_0^1 = E_0(\tilde{P}_1)$
Price of date 2 contract	f_0^2	$f_0^2 = E_0(\tilde{P}_2)$
Date 1		
Price of expiring date 1 contract	f_1^1	P_1
Price of date 2 contract	f_1^2	$f_1^2 = E_1(\tilde{P}_2)$
Date 2		
Price of expiring date 2 contract	f_2^2	P_2

Optimal storage at date 1

Suppose date 1 harvest has just been completed. Total inventory is S_0 units carried into date 1 plus Q_1 units of date 1 production. The objective of this section is to derive the date 1 storage function, which identifies how much of the current inventory should be carried over to date 2.

It follows from the analysis of Chapter 3 that market arbitrage by traders who are active in both the spot and futures markets will ensure that $E_1(\tilde{P}_2) = P_1 + m$ when $S > 0$ and $E_1(\tilde{P}_2) \leq P_1 + m$ when $S = 0$ (recall that discounting across time periods has been eliminated to simplify the analysis). This pair of arbitrage equations, together with equation (4.1), imply that there exists a critical value of Q_1, call it Q_1^*, for which optimal storage is zero for $Q_1 \leq Q_1^*$ (i.e., a stock out occurs) and optimal storage is positive for $Q_1 > Q_1^*$. As part of the process of price discovery in the futures market speculators must correctly anticipate these storage incentives facing commercial firms in the market.

To derive an expression for Q_1^*, let $S^*(Q_1)$ denote the storage function when $Q_1 > Q_1^*$. Thus, $S(Q_1) = 0$ for $Q_1 \leq Q_1^*$ and $S(Q_1) = S^*(Q_1)$ for $Q_1 > Q_1^*$. To derive an expression for $S^*(Q_1)$, use equation (4.1) to write the $E_1(\tilde{P}_2) = P_1 + m$ pricing equation as:

$$a - b\,(E_1(\tilde{Q}_2) + S^*(Q_1)) = a + m - b\,(S_0 + Q_1 - S^*(Q_1)) \qquad (4.2)$$

Now set $S^*(Q_1) = 0$ and solve for Q_1 to obtain:

$$Q_1^* = \mu_2 - S_0 + m/b \qquad (4.3)$$

For $Q_1 > Q_1^*$ equation (4.2) can be solved for $S^*(Q_1)$ with μ_2 substituting for $E_1(\tilde{Q}_2)$ to obtain:

$$S^*(Q_1) = \frac{b(S_0 + Q_1 - \mu_2) - m}{2b} \qquad (4.4)$$

Equations (4.3) and (4.4) together fully specify the optimal date 1 storage rule.

Date 1 and date 2 spot prices

Now that the optimal date 1 storage function has been derived it is possible to write more complete expressions for the date 1 and date 2 spot prices as a function of Q_1. This can be accomplished by substituting equation (4.4) into equation (4.1) to obtain:

$$P_1(Q_1) = \begin{cases} a - b(S_0 + Q_1) & \text{for } 0 \leq Q_1 \leq Q_1^* \\ a - b(S_0 + Q_1 - S^*(Q_1)) & \text{for } \quad Q_1 > Q_1^* \end{cases} \qquad (4.5a)$$

and

$$P_2(Q_1,\tilde{Q}_2)=\begin{cases} a-b(\tilde{Q}_2) & \text{for } 0\le Q_1\le Q_1^* \\ a-b(S^*(Q_1)+\tilde{Q}_2) & \text{for } \quad Q_1>Q_1^* \end{cases} \tag{4.5b}$$

Of particular interest are the expected dates 1 and 2 spot prices as of date 0. As discussed above, these expected prices are "discovered" in the commodity futures market by profit seeking speculators who are gathering information on μ_1 and μ_2. To obtain expressions for these expected prices, let $f(Q_1)$ denote the probability density function for $\tilde{Q}_1$ and recall that $E_0(\tilde{Q}_1) = \mu_1$. Using equation (4.5a), the expected value of the date 1 price can be expressed as:

$$E_0\left\{P_1(\tilde{Q}_1)\right\}=a-b(S_0+\mu_1)+b\int_{Q_1^*}^{\bar{Q}} S^*(Q_1)f(Q_1)dQ_1 \tag{4.6}$$

To obtain an expression for $E_0\ \{P_2(\tilde{Q}_1,\tilde{Q}_2)\}$ it is necessary to first obtain an expression for $E_0\ \{E_1(\tilde{Q}_2)\}$. The assumption that Q_1, and Q_2 are independent implies that $E_0\ \{E_1(\tilde{Q}_2)\} = \mu_2$, where μ_2 is the mean value for the $\tilde{Q}_2$ random variable. It now follows from equation (4.5b) that:

$$E_0\left\{P_2(\tilde{Q}_1,\tilde{Q}_2)\right\}=a-b\mu_2-b\int_{Q_1^*}^{\bar{Q}} S^*(Q_1)f(Q_1)dQ_1 \tag{4.7}$$

As of date 1, the expected date 2 spot price conditional on date 1 production can also be derived from equation (4.5b) and written as:

$$E_1\left\{P_2(\tilde{Q}_2\,|\,Q_1)\right\}=\begin{cases} a-b(\mu_2) & \text{for } 0<Q_1<Q_1^* \\ a-b(\mu_2+S^*(Q_1)) & \text{for } \quad Q_1\ge Q_1^* \end{cases} \tag{4.8}$$

Equilibrium futures prices

As discussed above, the assumption of efficient price discovery by speculators implies that the futures price must equal the expected spot price while properly accounting for the optimal storage decisions by commercial firms. The efficient price assumption implies that the expected spot prices given by equations (4.6) to (4.8) can also serve as expressions for the equilibrium futures price. Specifically:

$$f_0^1 = E_0\ (P_1(\tilde{Q}_1)),\ f_0^2\ = E_0\ (P_2(\tilde{Q}_1,\tilde{Q}_2)) \text{ and } f_1^2 = E_1\ (P_2(\tilde{Q}_2\,|\,Q_1)) \tag{4.9}$$

The remaining task is to obtain an explicit expression for the integrated term, $\int_{Q_1^*}^{\bar{Q}} S^*(Q_1)f(Q_1)dQ_1$, in equations (4.6) and (4.7). To obtain this expression it is necessary to be specific about the functional form for the probability density function, $f(Q_1)$. When choosing a density function it is desirable to use one that does not allow for negative values of Q_1, and one that is easy to work with in Excel. The normal distribution, truncated from below at 0, satisfies both of these criteria. A truncated normal distribution has two parameters: scale and dispersion.[3] Let $\hat{\mu}_1$ and σ_1 be the scale and dispersion parameters, respectively, for the truncated normal distribution for Q_1.

The value of the dispersion parameter, σ_1, for $f(Q_1)$ will be assigned directly. The value of the scale parameter, $\hat{\mu}_1$, must be calculated to ensure that the mean value of Q_1 is equal to μ_1. It is shown in Appendix 4.1 that the desired value of $\hat{\mu}_1$ is defined implicitly as the solution to:

$$\hat{\mu}_1 + \sigma_1 \frac{\phi\left(\frac{0-\hat{\mu}_1}{\sigma}\right)}{1-\Phi\left(\frac{0-\hat{\mu}_1}{\sigma}\right)} = \mu_1 \qquad (4.10)$$

Within equation (4.10) $\phi()$ is the probability density function and $\Phi()$ is the cumulative distribution function for a standard normal random variable. It is not possible to solve equation (4.10) analytically, and so a numerical solution is presented in Figure 4.4.

Now that a functional form for $f(Q_1)$ and a procedure for calculating $\hat{\mu}_1$ has been specified an explicit expression for $\int_{Q_1^*}^{\infty} S^*(Q_1) f(Q_1) dQ_1$ is derived in Appendix 4.1 using equation (4.4) and presented here as:

$$\int_{Q_1^*}^{\infty} S^*(Q_1) f(Q_1) dQ_1 = \left(\frac{b(S_0-\mu_2)-m}{2b}\right) \int_{Q_1^*}^{\infty} f(Q_1) dQ_1 + 0.5 \int_{Q_1^*}^{\infty} Q_1 f(Q_1) dQ_1 \qquad (4.11)$$

where

$$\int_{Q_1^*}^{\infty} f(Q_1) dQ_1 = \frac{1-\Phi(Z_1)}{1-\Phi(Z_0)} \qquad (4.12)$$

and

$$\int_{Q_1^*}^{\infty} Q_1 f(Q_1) dQ_1 = \frac{\hat{\mu}_1\left[1-\Phi(Z_1)\right]+\sigma_1\phi(Z_1)}{1-\Phi(Z_0)} \qquad (4.13)$$

Equation (4.12) is a measure of the probability that the market will not stock out. Equation (4.13) is a measure of the expected level of date 1 production conditional on a no stock out outcome at date 1. Both of these expressions depend on the scale parameter, $\hat{\mu}_1$, which is implied by equation (4.10), and on the pair of normalizing variables $Z_0 = {(0-\hat{\mu}_1)}/{\sigma_1}$ and $Z_1 = {(Q_1^* - \hat{\mu}_1)}/{\sigma_1}$.

4.3 Commodity futures model application

The parameters of the simulation model are displayed in cells A2:B8 of Figure 4.3. These values do not relate to any specific case study, but they are intended to be representative of a typical production and marketing scenario for a storable commodity with a periodic and uncertain level of production. Mean production (μ_1 and μ_2) is assumed to be equal to 100 units in each of the two periods. The σ_1 parameter is a measure of the dispersion of the Q_1 random variable. For the initial base case there is no production uncertainty and so $\sigma_1 = 0.1$.[4] The intercept and slope for the inverse market demand curve is assumed to take on values $a = 100$ and $b = 0.5$, respectively (prices are measure in dollars/unit).

	A	B	C	D	E	F	G
1	**Parameters**						
2	mew_1	100	mean production in period one				
3	mew_2	100	mean production in period two				
4	sig_1	0.1	dispersion parameter for period 1 production				
5	a	100	intercept for market demand schedule				
6	b	0.5	slope for market demand schedule				
7	m	5	unit cost of storage				
8	S_0	20	stocks carried into period one				
9							
10	**Variables**						
11	Q1_star	90	critical period one production: equation (4.3)				
12	mew_1_hat	100	scale parameter for f(Q1): implied by equation (4.10)				
13	mew_1_hat = 0	0.00	difference between left & right side of eqn (4.10)				
14	Z_0	-1000	first standardized variable				
15	Z_1	-100	second standardized variable				
16	Pr(Q1>Q1_st)	1	probability of positive storage: equation (4.12)				
17	E(Q1|Q1>Q1_st)	100	conditional expected production: equation (4.13)				
18	E(S*|Q1>Q1_st)	5	conditional expected date 1 storage: equation (4.14)				
19							
20	**Results**						
21	*date 0*						
22	f_0_1	42.5	date 0 futures price for contract that expires at date 1				
23	f_0_2	47.5	date 0 futures price for contract that expires at date 2				
24	spread	5	date 0 spread in futures prices				
25	*date 1*						
26	Q1	S(Q1)	P_1	f_1_2	basis		
27	20	0	80	50	-30		
28	60	0	60	50	-10		
29	100	5	42.5	47.5	5		
30	140	25	32.5	37.5	5		
31	180	45	22.5	27.5	5		

Figure 4.3 Model setup and base case results.

These values imply a demand elasticity equal to −1 when 100 units of the commodity are consumed. The unit carrying cost is assumed to equal $m = 5$, which is about 10 percent of the expected selling price. Finally, the level of stocks carried into period 1 is $S_0 = 20$, which is about 20 percent of mean production.

Cells A11:B18 of Figure 4.3 contain the main variables of the model. The formulas corresponding to these variables are displayed in Figure 4.4. Note that the Excel formula for a normal random variable is "=NORMDIST(*x*, *mean*, *standard_dev, cumulative*)" where *cumulative* = *false* to generate the probability density function and *cumulative* = *true* to generate the cumulative distribution function. Thus, "= *normdist* (*x*,0,1,*false*)" can be used to generate $\phi(x)$ and "= *normdist*(*x*,0,1, *true*)" can be used to generate $\Phi(x)$. To simplify the formulation an alternative Excel function, "= *normsdist*(*x*)", is used instead of = *normdist*(*x*,0,1,*true*).

The value for $\hat{\mu}_1$ that is displayed in cell B12 of Figure 4.3 was originally entered as a guess value. The desired equilibrium value of $\hat{\mu}_1$ must solve equation (4.10). The right-hand side of equation (4.10) has been subtracted from the left-hand side and the resulting expression has been entered in cell B13. Excel's Goal Seek or Solver tool will eventually be instructed to adjust the guess value in cell B12 to ensure that the value in cell B13 is equal to zero. Note that a new value for $\hat{\mu}_1$ must be obtained using this procedure each time a change is made to the μ_1 and σ_1 parameters.

Cells B14:B15 hold the two standardizing variables: $Z_0 = \left(0 - \hat{\mu}_1\right) / \sigma_1$ and $Z_1 = \left(Q_1^* - \hat{\mu}_1\right) / \sigma_1$. Cells B16:B17 contain the expressions for the probability of positive date 1 storage and expected date 1 production conditional on positive storage, which are given by equations (4.12) and (4.13), respectively. The value for $\int_{Q_1^*}^{\bar{}} S^*\left(Q_1\right) f\left(Q_1\right) dQ_1$ that is calculated using equation (4.11) is displayed in cell B18.

The simulation results for date 0 are presented in cells A22:B24 of Figures 4.3 and 4.4. Cells B22:B23 contain the date 0 futures prices for the contracts that expire at date 1 and date 2, respectively. Cell B24 contains the spread between these two prices. The pair of futures prices in cells B22:B23 are calculated using equations (4.6), (4.7) and (4.9). These equations are linked to the expression for $\int_{Q_1^*}^{\bar{}} S^*\left(Q_1\right) f\left(Q_1\right) dQ_1$, which resides in cell B18.

The simulation results for date 1 appear in cells A26:E31 of Figures 4.3 and 4.4. Column A shows five alternative assumptions for the realized value of Q_1. Column B shows the amount that will be carried over from period 1 to period 2 for each of the five Q_1 realizations. The formula for making these storage calculations requires inserting equation (4.4) into an IF statement to ensure that storage is zero when $Q_1 \leq Q_1^*$. Cells C27:C31 and D27:D31 show the set of date 1 spot prices and futures prices, respectively, for the assumed set of Q_1 values. These prices are calculated using the pricing formulas, $P_1 = a - b\ (S_0 + Q_1 - S_1\ (Q_1))$ and $f_1^2 = E_1\ (P_2) = a - b$

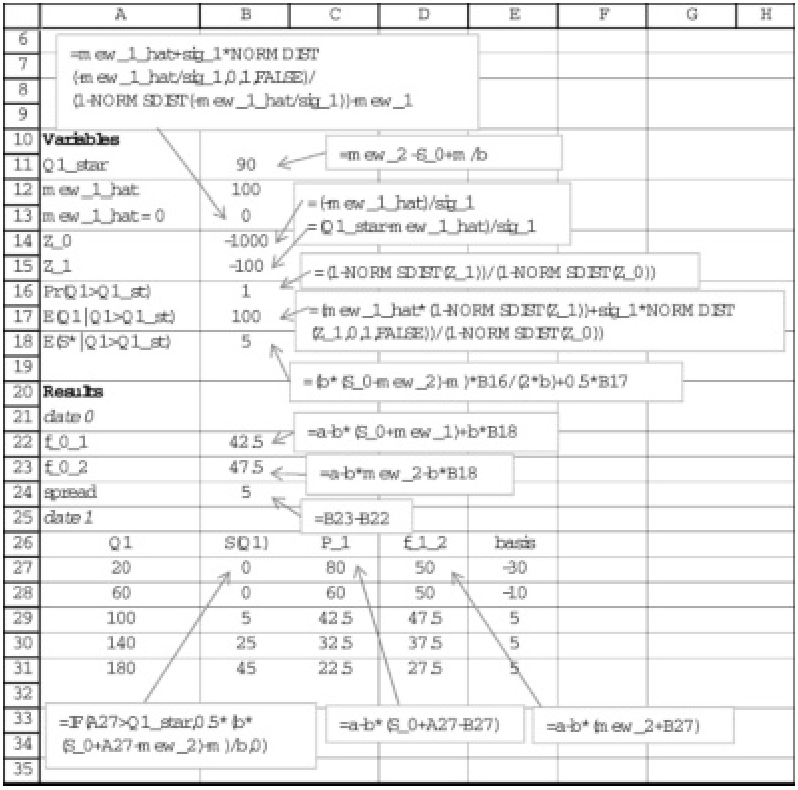


Figure 4.4 Specific equations for model.

$(S_1(Q_1) + \mu_2)$. The basis, which is the difference between the date 1 futures price in column D and the spot price in column C, is reported in column E.

Results with no production uncertainty

The base case results shown in Figure 4.3 correspond to the case of no production uncertainty. The absence of production uncertainty implies that with the initial stocks set at $S_0 = 20$ and equal levels of production in each of the two periods, there is no chance of a stock out. This outcome can be confirmed by noting that the probability of positive storage as displayed in cell B16 takes on a value of 1.0.

Consider first the date 0 results that reside in cells A22:B24. The futures prices for the contracts that expire at dates 1 and 2 are equal to \$42.50 and \$47.50, respectively. The \$5 spread in this pair of date 0 futures prices is reported in cell B24. With $m = 5$ this price spread conforms to the LOP equation, $E_0(P_2) = E_0(P_1 + m$. For the date 1 results in cells A27:E31 only row 29 that corresponds to $Q_1 = 100$ is relevant because by assumption there is no production uncertainty in period 1. With $Q_1 = 100$ the date 1 spot price is equal to \$42.50 and the date 1 price of a futures contract that expires at date 2 is \$47.40. This pair of results implies that the date 1 basis of \$5.00 is equal to the unit carrying cost, which is an outcome that is predicted by the theory of the LOP for the case of no production uncertainty.

Results with production uncertainty

Figure 4.5 displays the simulation results for the case of moderate production uncertainty, which is achieved by setting $\sigma_1 = 30$ in cell B4 of Figure 4.3. Cell B11 of Figure 4.5 shows that Q_1 must take on a value of 90 or larger to avoid a stock out scenario in period 1. Cell B16 shows that this outcome occurs with 63 percent probability.

As discussed earlier in this chapter, a stock out in period 1 implies that the price difference across the two periods will be less than the unit carrying cost and may in fact be negative. For example, cells A28:E28 of Figure 4.5 reveal that if $Q_1 = 60$ then a stock out occurs. In this situation the basis is negative because the date 1 spot price of \$60 is above the \$50 price of a date 1 futures contact with a date 2 expiry. On the other hand, if period 1 production is large (e.g., $Q_1 = 140$), then stocks are carried forward across time and the basis is equal to the $m = 5$ unit carrying cost.

An important feature of production uncertainty is that the potential for a stock out in period 1 affects the spread between the pair of futures prices at date 0. Cells B22:B24 of Figure 4.5 reveal that because of the production uncertainty and the corresponding probability of a period 1 stock out, the date 0 spread in futures

	A	B	C	D	E	F	G
10	**Variables**						
11	Q1_star	90.0					
12	mew_1_hat	99.95					
13	mew_1_hat = 0	0.00					
14	Z_0	-3.3					
15	Z_1	-0.3					
16	Pr(Q1>Q1_st)	0.63					
17	E(Q1|Q1>Q1_st)	74.3					
18	E(S*|Q1>Q1_st)	8.80					
19							
20	**Results**						
21	*date 0*						
22	f_0_1	44.40	date 0 futures price for contract that expires at date 1				
23	f_0_2	45.60	date 0 futures price for contract that expires at date 2				
24	spread	1.20	date 0 spread in futures prices				
25	*date 1*						
26	Q1	S(Q1)	P_1	f_1_2	basis		
27	20	0	80	50	-30		
28	60	0	60	50	-10		
29	100	5	42.5	47.5	5		
30	140	25	32.5	37.5	5		
31	180	45	22.5	27.5	5		

Figure 4.5 Simulation results for the case of production uncertainty ($\sigma_1 = 30$).

prices is \$1.20, which is much less than the \$5 unit carrying cost. The reason for this outcome is that speculators at date 0 account for the probability of a period 1 stock out when formulating price expectations for the two production periods. The higher the probability of a stock out, the smaller the price spread between the pair of futures prices. Moreover, if the probability of a stock out is sufficiently large then the date 0 spread will be negative.

A key variable that affects the probability of a stock out is the level of stocks carried into period 1, which is currently set at $S_0 = 20$. Holding the other parameters constant, suppose $S_0 = 30$. The simulation model displayed in Figure 4.5 can be used to show that these additional stocks cause the pair of date 0 futures prices to decrease from {44.40, 45.60} to {41.14, 43.86}. As well, the extra stocks will cause the date 0 spread in futures prices to increase from \$1.20 when $S_0 = 20$ to \$2.73 when $S_0 = 30$. This last result is expected because the higher volume of stocks carried into period 1 implies a lower probability of a stock out in period 1, and thus a price spread that is closer to that predicted by the LOP relation, $E_0(P_2) = E_0(P_1) + m$.

To conclude this section, two additional comments are in order. First, the results from the simple model presented above suggest that in a model with many periods, the pair of futures prices and the corresponding spread in futures prices are expected to fluctuate over time with negative correlation. Fluctuation with negative correlation is also expected for the futures price and the basis. To best understand this result note that a production shock in a model with many periods is equivalent to shocking S_0 in the current model. Based on the previous results, a shock to S_0 will cause the pair of futures prices and the spread to change in opposite directions (similar results hold for the basis). Fluctuations in production and the corresponding probability of a stock out provides a partial explanation of why the basis and price spreads fluctuate over time.

The second noteworthy point is that the pair of date 0 futures prices and the pair of date 0 spot and futures prices will instantly adjust in response to new information concerning future supply and demand. For example, suppose the market is at date 0 and news arrives that mean production for period 2 is $\mu_2 = 80$ units rather than $\mu_2 = 100$ units. The simulation model (with $\sigma_1 = 30$ and $S_0 = 20$) can be used to show that with μ_2 lowered from 100 to 80 units the date 0 futures market will instantly adjust upward from $f_0^1 = 44.40$ to $f_0^1 = 48.13$ and from $f_0^2 = 45.60$ to $f_0^2 = 51.87$. The spread of \$3.74 when $\mu_2 = 80$ is larger than the spread of \$1.18 when $\mu_2 = 100$ because with an anticipated shortfall in production in period 2, more will be stored from period 1 to period 2 and thus there is a lower likelihood of a period 1 stock out. Similar to the previous example, factors that lower the likelihood of a stock out raise the spread in futures prices toward the level predicted by $E_0(P_2) = E_0(P_1) + m$.

4.4 Convenience yield

In the previous section the potential for a market stock out was the sole reason why the spread in equilibrium futures prices across time was less than the unit carrying cost. The data presented at the beginning of this chapter suggest that basis and spread fluctuations below the unit carrying cost are common and probably depend on factors that extend well beyond the stock out argument. Traditional explanations for the observed properties of the market basis and price spreads include Keynes' theory of normal backwardation, the theory of convenience yield, the theory of the transactions demand for inventories and various ad hoc factors such as transportation bottlenecks and local supply and demand imbalances. The remainder of this chapter is devoted to a detailed examination of convenience yield, which is the most widely accepted theory of basis and spread behavior. The specific objective is to construct a simple model of convenience yield and then use it to explain why the equilibrium spread in futures prices can be lower than the unit carrying cost, even if there is no chance of a market stock out.

The main feature of this model is that commercial firms choose to hold additional stocks to lower marketing transaction costs – a so-called "convenience" yield. The purchase of these additional stocks drives up the spot price of the commodity and the subsequent resale drives down the futures price. This pricing impact lowers the spread in futures prices to a level that is less than the unit carrying cost. In other words, convenience yield can explain why commercial firms choose to hold inventory even though doing so results in an expected monetary loss in the market value of the commodity. The convenience yield is expected to fluctuate over time in response to changes in market inventory, traders' expectations about future supply and demand conditions, and changes in marketing transaction costs. Thus, convenience yield and production uncertainty can help to explain commonly observed random fluctuations in futures price spreads and the market basis at levels below the unit carrying cost.

Model setup

The convenience yield model is similar in structure to the simple two-period model of intertemporal commodity prices that was constructed at the beginning of Chapter 3. In period 1 n identical merchants each own q units of the commodity, which implies that $Q = nq$ units is the inventory level for the industry as a whole. Each merchant must choose the amount to store and subsequently sell in period 2, and the amount to sell immediately in period 1. Let s denote the level of storage chosen by a representative merchant, and let $S = ns$ be the aggregate level of storage across all merchants. Assuming zero production in period 2, the aggregate storage decision determines prices across the two periods according to the following pair of equilibrium conditions: $X_1(P_1) = Q - S$ and $X_2(P_2) = S$, where $X_1(P_1)$ and $X_2(P_2)$ are the market demand schedules for periods 1 and 2 respectively. Assume market demand is linear and the same in both periods:

$$X_1(P_1) = a/b - 1/b\,P_1 \text{ and } X_2(P_2) = a/b - 1/b\,P_2 \tag{4.14}$$

In period 2 each merchant is assumed to sell the commodity to one committed buyer and, if the need arises, to one or more non-committed buyers on an ad hoc basis. Assume there are n committed buyers (one for each merchant) and $k - n$ non-committed buyers, which implies a total of k buyers. If a merchant's inventory is sufficient to meet the amount demanded by his or her committed buyer, then no transaction costs are incurred, and the merchant's residual inventory is sold to either a non-committed buyer or to another merchant. If inventory is insufficient to meet the demand requirements of the committed buyer, then the merchant must purchase stocks from a merchant with non-committed stocks and then resell those stocks to the committed buyer. Purchasing from another merchant is assumed to generate a transaction cost that increases in proportion to the size of the inventory shortfall. Transaction costs can arise for a variety of reasons including the value of time devoted to searching, negotiating and verifying, specific monetary costs such as legal fees and penalties imposed by the buyer, and non-monetary costs such as loss in reputation.

To formally model period 2 transaction costs, assume that period 2 market demand is uniformly distributed across the k buyers, but merchants do know which buyer has which demand prior to the arrival of period 2. Specifically, one of the buyers will have demand $\frac{X_2(P_2)}{0.5k(k+1)}$, a second buyer will have demand $\frac{2X_2(P_2)}{0.5k(k+1)}$, a third buyer will have demand $\frac{3X_2(P_2)}{0.5k(k+1)}$ and so forth. Aggregate demand across all k buyers is equal to $\sum_{i=1}^{k}\frac{iX_2(P_2)}{0.5k(k+1)}$, which reduces to $X_2(P_2)$. Random period 2 demand facing a particular merchant as of period 1 can now be expressed as:

$$q^d(\theta, P_2) = \frac{\theta X_2(P_2)}{0.5k(k+1)} \tag{4.15}$$

where θ is a random variable that takes on values 1,2, . . ., k with equal probability.

The representative merchant will not have sufficient inventory to satisfy the demand requirements for the committed buyer if $q^d(\theta, P_2) > s$. Thus, for $\theta > \theta^*(s)$ where $\theta^*(s) = \frac{0.5k(k+1)s}{X_2(P_2)}$ the merchant will incur a transaction cost because of insufficient inventory. To ensure that some transaction costs are probable, assume that the parameters of the model are such that $\theta^*(s) < k$.

It is now possible to write an expression for $Z(s)$, which is the expected shortfall in period 2 inventories for a merchant that chooses to store s units from period 1 to period 2:[5]

$$Z(s) = \frac{1}{k}\sum_{i=\theta^*(s)}^{k}\left(\frac{iX_2(P_2)}{0.5k(k+1)} - s\right) \tag{4.16}$$

Assume that as of date 0 the expected period 2 transaction costs for the representative merchant are proportional to the expected period 2 shortfall in inventory when supplying the committed buyer. Specifically, assume that $C(s) = \gamma Z(s)$, where $C(s)$ is a measure of the expected period 2 transaction costs as of period 1, expressed as a function of the amount stored by the representative merchant. As will be shown below, the proportionality constant, γ, is a key determinant of equilibrium commodity prices across the two periods.

Equilibrium price spread

As discussed throughout Chapters 3 and 4, the representative merchant will continue to store the commodity from period 1 to period 2 until the expected profits from storing the last unit of the commodity is equal to zero. For the current analysis storing the last unit generates marginal revenues equal to P_2 but also generates three types of marginal cost. The first is P_1, which is the opportunity cost of not selling that unit in period 1. The second is m, which is the unit carrying cost. The third is $C'(s)$, which is the marginal change in the expected transaction cost for the merchant. Consequently, the condition that determines the equilibrium value s of for the representative merchant can be expressed as:

$$P_2 = P_1 + m + C'(s) \tag{4.17}$$

After substituting in the two demand schedules from equation (4.14), this equilibrium condition can be rewritten as:

$$a - bS = a - b\,(Q - S) + m + C'(s) \tag{4.18}$$

To obtain an explicit expression for $C'(s)$ in equation (4.18), recall that $C(s) = \gamma Z(s)$, where $Z(s)$ is the expected shortfall in inventory, as given by equation (4.16). Differentiating equation (4.16) gives $C'(s) = -\gamma\left(1 - \frac{0.5(k+1)}{X_2(P_2)}s\right)$.[6] In equilibrium, $X_2(P_2) = S$ to ensure that market demand is equal to market supply in period 2. Moreover, with identical merchants it follows that $s = S/n$. Substituting these last two expressions into the previous expression for $C'(s)$ allows the expression for marginal transaction costs to be rewritten as:

$$C'(s) = -\gamma\left(1 - \frac{0.5(k+1)}{n}\right) \tag{4.19}$$

Equation (4.19) reveals that the marginal reduction in transaction costs from additional storage is a constant that depends on the scaling parameter, γ, and the approximate ratio of buyers to sellers, $(k+1)/n$. The marginal reduction in transaction costs takes on a negative value because $1 < (k+1)/n < 2$ by assumption.

Now that it has been confirmed that $C'(s)$ takes on a constant negative value, equation (4.18) can be solved for the equilibrium level of storage at the industry level, S^*:

$$S^* = \frac{bQ - m - C'}{2b} \tag{4.20}$$

Having established that $C' < 0$, equation (4.20) reveals that industry storage is higher as a result of the expected transaction costs that are incurred by individual firms.

If equation (4.20) is substituted into the pair of pricing equations given by equation (4.14), then the equilibrium prices for periods 1 and 2 can be expressed as:

$$P_1 = a - \frac{b}{2}\left(Q + \frac{m + C'}{b}\right) \text{ and } P_2 = a - \frac{b}{2}\left(Q - \frac{m + C'}{b}\right) \tag{4.21}$$

Of particular interest is the price spread. Subtracting the two expressions in equation (4.21) gives:

$$P_2 - P_1 = m - (-\gamma C') \tag{4.22}$$

The negative value for C' implies that the price spread is less than the unit carrying cost, m. The more costly the transactions for a firm to address an inventory shortfall, as reflected by a larger value for γ, or the greater the number of merchants relative to the number of buyers, as reflected by a smaller value for $(k+1)/n$, then the smaller the spread in equilibrium prices. For a sufficiently high level of transaction costs, the price spread will be negative.

The term $-\gamma C'$ in equation (4.22) is a measure of the market's convenience yield. If storage is positive, a revised version of the LOP can be expressed as $P_{t+1} = P_t + m - ConvienceYield$. If convenience yield is ignored by the market analyst, it will appear that the LOP is violated because $P_{t+1} < P_t + m$. Convenience yield therefore helps to explain the $P_{t+1} < P_t + m$ outcome that is commonly observed in real-world markets.

In a more general model, the convenience yield is expected to be larger when industry stocks are low and vice versa. This generalization makes sense because when the commodity is scarce it will be more costly to source new supplies when inventories fall short of buyer requirements. Because inventory levels are expected to fluctuate over time in response to stochastic levels of production, it follows from the analysis above that the convenience yield is also expected to fluctuate over time in an inverse relationship to inventory levels. In most real-world markets there is considerable evidence that supports the hypothesis that fluctuations in equilibrium prices and price spreads have an inverse relationship.

4.5 Concluding comments

Commodity futures and spot markets are complex and in many respects are difficult to model. A number of assumptions are built into the pair of models presented in this chapter in order to simplify the analysis. Despite these simplifying assumptions, the models highlight important economic forces that shape the general pattern of commodity futures and spot prices. The first force relates to arbitrage and the intertemporal version of the LOP. Based on the results from Chapter 3, if the likelihood of a stock out is low, then price spreads for futures contracts with different delivery months and the commodity basis should be approximately equal to the unit carrying cost. As a futures contract approaches its expiry date and the cost of carrying the commodity to deliver against the contract vanishes, then the theories of arbitrage and the LOP predict a convergence of the cash and futures prices.

A second economic force that shapes commodity prices is production uncertainty and the corresponding probability of a stock out at some point in the future. During a stock out the cash price will increase relative to the set of futures prices and the set of prices for pre-harvest futures contracts will increase relative to the set of prices for post-harvest futures contracts. If the stock out condition is sufficiently strong, then the market will invert and nearby prices will increase above more distant prices. The greater the potential for a future stock out, the smaller the corresponding price spreads and the basis, and the greater the likelihood of a market inversion.

Real-world markets very seldom complete a crop year with zero stocks, so why has the stock out concept received so much emphasis in this chapter? While it is true that inventories are seldom depleted to zero, it is also true that in some years only minimum amounts of pipeline stocks are carried over from one crop year to the next. If the constraint for minimum pipeline stocks is binding, then the equivalent of a stock out has occurred because if traders could borrow stocks from the future they would. The common observation that markets are more likely to be inverted when stocks are low versus high is strong evidence that stock outs are an important economic determinant of commodity prices over time.

The third economic force that shapes commodity prices is convenience yield. Convenience yield provides a "convenient" explanation as to why price spreads and the basis typically fluctuate at a level that is below the unit carrying cost. It is difficult to empirically test for the presence of convenience yield, so the extent that this theory is useful in explaining real-world commodity prices is largely unknown. As discussed in note 2, some economists believe that a formal theory to explain the existence of spread and basis fluctuations below the unit carrying cost is not required. These economists argue that commodity futures markets are designed to allow commercial firms to shift stocks through time in an efficient matter. Moreover, spread and basis uncertainty is the main reason why multiple contracts for a particular commodity trade on any given day.

The formal analysis in this chapter assumes that futures prices are efficient because they correctly predict future spot prices on average. Whether or not futures prices are efficient is an empirical question. Backwardation is said to occur if the futures price is expected to rise over time. If the futures market rises on average, then traders taking short futures positions will lose profits on average and traders taking long futures positions will gain profits on average. Conclusions from a large volume of empirical research are mixed regarding the overall efficiency of commodity futures markets. In recent years the efficiency of commodity markets has come under attack due to some high-profile price convergence failures in markets such as cotton and rice.

Finally, this chapter has been silent on the hedging aspect of commodity futures. One of the primary roles of a futures market is that it allows agribusiness firms at various positions in the supply chain to reduce price risk through hedging. The effectiveness of the hedge as an instrument for transferring risk depends critically on the level of basis uncertainty and on the overall efficiency of the futures market. Consequently, in thin markets where pricing efficiency is questionable, and in markets with considerable basis risk, the value of hedging is significantly reduced.

Questions

1 A trader has access to grain storage capacity in Chicago. The monthly cost of storage is $0.04/bushel and the trader's cost of capital is 0.5% per month. The trader's main business is to purchase grain in the spot market and deliver it against a short futures contract, or to accept delivery of grain via a long futures contract and then sell the grain in the spot market (assume that these activities can be done at zero cost). In the set of pricing scenarios listed below identify those that can be arbitraged (be sure to describe the specific arbitrage strategy). As well, explain what will happen to market prices if a larger number of traders pursue similar arbitrage activities. To simplify the discussion, assume that the commodity can only be delivered against a futures contract on the last day of the contract's expiry month.

   a On 1 August the Chicago spot price of oats is $2.60/bushel, and the CBOT price for December oat futures is $2.05/bushel.
   b On 1 September the Chicago spot price of corn is $3.80/bushel and the CBOT price for March corn futures is $4.20/bushel.
   c On 1 December wheat futures for the following March and July are trading at $5.50/bushel and $5.80/bushel, respectively.
   d On 1 August local farmers are willing to commit to deliver soybeans to the trader's Chicago warehouse on the last day of December. The contract stipulates that when delivery is made the trader must pay the farmer $9.20/bushel. On 1 August soybean futures for the following January are trading at $9.30/bushel.

2 Hedging converts price risk into basis risk through offsetting positions in the spot and futures markets. A trader who owns the commodity (now or in the future) and plans to later sell the commodity in the spot market will

initiate a hedge by taking a short futures position and will terminate the hedge when the commodity is sold in the spot market by taking an offsetting long futures position. A trader who plans on purchasing the commodity in the spot market at some future date will initiate the hedge with a long futures position and terminate it when the commodity is purchased in the spot market with an offsetting short futures position. If the anticipated size of the spot market transaction is the same as the size of the futures market transaction, then the profit or loss on the hedge will depend only on the change in the basis net of the commodity's carrying cost over the hedging period.

a On 1 September the spot price of wheat is $5.25/bushel and CBOT wheat futures for the following March are trading at $5.55/bushel. A trader owns wheat on 1 September and plans on selling the wheat on 31 January. The trader's cost of storage is $0.04/bushel and cost of capital is 0.05% per month.

   i Explain how the trader can hedge in order to convert price risk into basis risk.
   ii Ignoring trading costs, what is the net loss or gain for the hedger relative to the 1 September spot price if the 31 January spot price of wheat turns out to be $7.15/bushel and the 31 January price of a March CBOT wheat futures contract turns out to be $7.25/bushel?
   iii How would your answer change if the 31 January spot price was $3.90/bushel instead of $7.15/bushel and if the CBOT price for March wheat was $4.00/bushel instead of $7.25/bushel?

b An important reason why farmers are reluctant to ledger is they may not be able to finance large margin calls. With reference to part (a) of this question, calculate the change in the trader's margin account over the 1 September to 31 January period for the scenario where the March futures contract for wheat rose from $5.55/bushel to $7.25/bushel over this period and the trader hedged 10,000 bushels.

c A grain merchant purchases corn from farmers in the local spot market and plans to later resell the corn to a local processor sometime in January or February. Suppose it costs $0.15/bushel to transport the commodity from the local market to the delivery point specified in the futures contract. The trader's cost of storage is $0.04/bushel per month and the trader's cost of capital is 0.5% per month. On 1 October CBOT corn futures for March of the following year are trading at $4.05/bushel.

   i What local spot price will the trader set for 1 October if the trader prices according to an arbitrage strategy? Arbitrage involves delivering the corn that is purchased from the farmer against a March futures contract and just breaking even on the transaction.
   ii Suppose on 1 October the trader takes a short March futures position at $4.05/bushel, pays the spot price calculated in part (i) to acquire

the corn and later resells the corn to the processor on 31 January for \$4.25/bushel. Assuming the trader realizes a net gain of \$0.07/bushel after offsetting the short futures position, calculate the 31 January futures price for the March wheat contract.

d An exporter has committed to sell soybeans at a pre-negotiated price to an overseas buyer with a delivery date set three months in the future. The exporter intends to purchase the beans in the spot market two months in the future and use those purchased beans to fulfill the requirements of the export contract. Describe a possible hedging strategy for the crusher. How do net profits from the hedge depend on the movement in the basis over the three- month period that begins 1 September?

3 The market for a storable commodity consists of two periods. Inverse market demand is constant in each period and is equal to $P = 10 - Q$. Production in period 2 is equal to five units with certainty. Post harvest stocks in period 1 are equal to K units. The cost of storage from period 1 to period 2 is \$1/unit, and the opportunity cost of capital is zero. A zero cost of capital implies that the discount factor is equal to 1.

a Derive the equilibrium price for each period as a function of K accounting for the non-negative storage constraint. Now assume that K is a random variable, in which case equilibrium price in period 2 and the price spread across the two periods are also random. Use your simple model to describe why the price spreads fluctuate over time and occasionally invert in response to changes in the market's expectations about the size of K.

b In this chapter two theories were developed to explain why prices are sometimes inverted in a commodity futures market (i.e., nearby contracts trade at a higher price than more distant contracts). In both the stock out theory and the convenience yield theory there is a connection between the likelihood of price inversion and the level of stocks in the market. For this pair of theories, does the connection between stock levels and inversion work in the same direction or the opposite direction (i.e., are the effects offsetting or reinforcing)?

Appendix 4.1: Derivation of equation (4.10) and the integrated expression in equations (4.11) to (4.13)

An explicit expression is required for $\int_{Q_1^*}^{\infty} S^*(Q_1) f(Q_1) dQ_1$ where $f(Q_1)$ is a $Q_1 \geq 0$ truncated normal density function with scale parameter, $\hat{\mu}_1$, and dispersion parameter σ_1. The S^* (Q_1) function is given by equation (4.4) in the text. Before analyzing the above integral it is useful to establish two important properties of a truncated normal distribution. First, if Q follows a non-truncated normal distribution with mean μ, standard deviation σ and density function $\sigma^{-1}\phi\left({(Q-\mu)}/{\sigma} \right)$, then $Q \geq a$ follows a truncated normal distribution with density function $\sigma^{-1}\phi\left({(Q-\mu)}/{\sigma} \right) \Big/ \left[1-\Phi\left({(a-\mu)}/{\sigma} \right) \right]$ where $\phi()$ and $\Phi()$ are the probability density function and the cumulative distribution function, respectively, for a standard normal random variable.

The second important property of a truncated normal distribution is that: $\int_a^{\infty} \phi\left(\frac{Q-\mu}{\sigma} \right) dQ = \mu\left[1-\Phi\left(\frac{a-\mu}{\sigma} \right) \right] + \sigma\phi\left(\frac{a-\mu}{\sigma} \right)$.[7] This pair of results together imply that the mean of a truncated normal random variable with lower truncation a can be expressed as:

$$\frac{1}{1-\Phi\left(\frac{a-\mu}{\sigma} \right)} \int_a^{\infty} Q\sigma^{-1}\phi\left(\frac{Q-\mu}{\sigma} \right) dQ = \mu + \sigma \frac{\phi\left(\frac{a-\mu}{\sigma} \right)}{1-\Phi\left(\frac{a-\mu}{\sigma} \right)} \tag{4.A1}$$

As discussed in the text, it is necessary to select a value for the scale parameter for $f(Q_1)$ to ensure that the mean of Q_1 is equal to μ_1. Noting that $\hat{\mu}_1$ is the scale parameter for $f(Q_1)$ and further noting that equation (4.A1) with $a = 0$ is the mean of Q_1, it follows that the desired value of $\hat{\mu}_1$ is implied by:

$$\hat{\mu}_1 + \sigma_1 \frac{\phi\left(\frac{0-\hat{\mu}_1}{\sigma} \right)}{1-\Phi\left(\frac{0-\hat{\mu}_1}{\sigma} \right)} = \mu_1 \tag{4.A2}$$

Equation (4.A2) appears as equation (4.10) in the text.

The next step is to construct expressions for $\int_{Q_1^*}^{0} f(Q_1)dQ_1$ and $\int_{Q_1^*}^{0} Q_1 f(Q_1)dQ_1$, which appear as equations (4.12) and (4.13) in the text. Using the previous results it follows that:

$$\int_{Q_1^*}^{\infty} f(Q_1)dQ_1 = \frac{1}{1-\Phi(Z_0)} \int_{Q_1^*}^{\infty} \sigma^{-1}\phi\left(\frac{Q_1-\hat{\mu}_1}{\sigma_1} \right) dQ_1 = \frac{1-\Phi(Z_1)}{1-\Phi(Z_0)} \tag{4.A3}$$

where $Z_0 = {(0-\hat{\mu}_1)}/{\sigma_1}$ and $Z_1 = {(Q_1^*-\hat{\mu}_1)}/{\sigma_1}$. It also follows that

$$\int_{Q_1^*}^{\infty} Q_1 f(Q_1) dQ_1 = \frac{1}{1-\Phi(Z_0)} \int_{Q_1^*}^{\infty} Q_1 \sigma^{-1} \phi\left(\frac{Q_1 - \hat{\mu}_1}{\sigma_1}\right) dQ_1$$

$$= \frac{\hat{\mu}_1 \left[1-\Phi(Z_1)\right] + \sigma_1 \phi(Z_1)}{1-\Phi(Z_0)} \tag{4.A4}$$

The final step is to use equation (4.4) to construct an expression for $\int_{Q_1^*}^{\infty} S^*(Q_1) f(Q_1) dQ_1$, which appears as equation (4.11) in the text.

5 Prices over form (quality)

5.1 Introduction

In the previous three chapters of this textbook, the commodity in question was assumed to be homogeneous with known quality attributes. This assumption is generally not accurate for agricultural commodities, which tend to have significant differences in product attributes such as weight, size, color, taste and general appearance. Product heterogeneity arises for a variety of reasons including differences in the underlying genetic stock, management practices, post-farm handling and an assortment of random factors attributable to weather, pests and disease. For most agricultural commodities there are a large number of dimensions for which product quality can vary, so the previous assumption that a commodity will trade at a single price is generally not realistic.

Agricultural commodities are often graded in order to create standardized subsets that can trade at different prices. Standards can exist without grades (e.g., a "free-range" designation for eggs) but grades are always assigned based on pre-established standards. Grading outcomes are subjective and will typically vary somewhat from year to year depending on the distribution of quality in the market and the specific preferences of buyers. In some cases standards and grading schemes are created and enforced by government agencies and in other cases this is done by industry associations.

A typical grading scheme will specify minimum threshold levels for key positive quality attributes such as percentage protein, and will specify maximum threshold levels for key negative quality attributes such as percent of seeds that are damaged or sprouted. For example, to achieve a No. 1 grade, French wheat must have a minimum weight of 76 kilograms per hectoliter, a maximum moisture content of 15.5 percent and a maximum of 4 percent broken kernels. The analogous values for No. 2 French wheat are 75 kilograms per hectoliter, 16 percent moisture and 5 percent broken kernels.[1]

For many agricultural commodities (e.g., grains, coffee beans and livestock), traders have an incentive to blend various quality versions together in order to achieve pre-defined grading standards at minimum cost. Blending allows a relatively low value commodity to be implicitly sold at a price that is higher than what would be the case if blending was not possible. In a competitive market, profit seeking traders will seek blending opportunities until marginal blending profits are driven to zero. Consequently, it should not be surprising that for commodities that can be blended and graded there exists a quality version of the LOP.

There are two distinct topics that relate to prices for a grade-differentiated commodity. The first concerns the pricing determinants of the different grades of the blended commodity in final markets. The second concerns the pricing determinants of the different quality versions of the unblended and ungraded commodity in primary markets. Prices for different grades of the blended commodity will depend on the distribution of supply across the various grades and the degree of substitutability across grades as perceived by buyers. The pricing of graded differentiated commodities is examined at a general level in Chapter 6 where the focus is price determination in markets with multiple sources of demand for a set of substitutable products. The focus of this chapter is on how prices are determined for different quality versions of the unblended and ungraded commodity in primary markets. Specifically, this chapter shows how the LOP allows prices in these primary markets to be derived as a function of the set of prices of the graded commodities in final markets.

To better understand the focus of this chapter, consider the following example. Suppose importer A is willing to pay \$300/tonne for wheat with a minimum protein level of 14.5 percent, and importer B is willing to pay \$250/tonne for wheat with any protein content. Wheat stocks in a country C consist of 750 tonnes of ungraded wheat with 15 percent protein, 600 tonnes of ungraded wheat with 14 percent protein and 800 tonnes of ungraded wheat with 13 percent protein. This wheat can be purchased and blended in varying proportions by competitive exporters within country C, and then resold to importers A and B. In the absence of blending costs, the value of the three classes of wheat is maximized if 50 tonnes of the 13 percent wheat is blended with all of the 14 and 15 percent wheat to create a single blend with 14.5 percent protein. An additional tonne of 15 percent wheat will allow an additional one-third of a tonne of 13 percent wheat to be added to the blend, which in turn will raise the aggregate value of the 13 percent wheat by (300 − 250)/3 = \$16.67/tonne. Consequently, a competitive exporter will bid 300 + 16.67 = \$316.67/tonne for the marginal unit of 15 percent wheat, and this bid will establish the market price for this particular type of wheat.

The previous example illustrates that, similar to prices over space and time (Chapters 2 and 3), the set of competitive prices for different quality versions of a commodity in the primary market can be derived from the outcome of a social planner's allocation problem. The prices recovered from the social planner's problem are the shadow prices associated with the supply availability constraints for the commodities that serve as inputs into the blending process. A shadow price is the amount by which net aggregate surplus will rise if an additional unit of the resource is made available. In the previous example, the shadow price of wheat with 15 percent protein is equal to \$316.67/tonne because the market value of the aggregate wheat stock will increase by this amount if 751 tonnes rather than 700 tonnes of such wheat were available for blending.

A typical blending problem is subject to many corner solutions, and so using linear programming techniques to maximize net aggregate surplus for the social planner is the best way to recover the set of competitive prices for the different quality versions of the commodity. Fortunately, Microsoft Excel automatically generates shadow prices as part of the linear programming solution, so recovering competitive prices from the social planner's problem is straightforward. Later in this chapter the shadow price technique is used to recover competitive market prices in the context of a case study involving Canadian wheat with different levels of protein.

The results of the case study analysis show that shadow prices are sensitive to the distribution of protein within the aggregate wheat stock and to the marginal value of protein in the final market. Understanding the determinants of the shadow prices for unblended commodities is important because these prices guide the marketing transactions in primary markets (e.g., commodity producers selling to rural commodity buyers). As well, commodity producers are likely to implicitly use primary market shadow prices when making production decisions, and so shadow prices should be a central feature of models of agricultural supply. Although the discussion of shadow prices is restricted to primary commodity markets with varying levels of product quality, it should be noted that shadow pricing is an important concept in a wide range of markets for which productive inputs are differentiated and in scarce supply.

Before constructing a LOP model of quality and blending, a more detailed discussion about quality is presented in Section 5.2. Included in this discussion is the presentation of quality and price data for different grades of Canadian durum wheat. The data show the distribution of price premiums across different grades of a commodity in a real-world final market and how this distribution changes over time. Both the distribution of price premiums and the evolution of these premiums over time are important determinants of the shadow prices associated with the different quality versions of the commodity that trade in the primary market.

In Section 5.3, the theoretical framework for the LOP analysis is presented in a simple two outcome framework. In Section 5.4 the wheat protein case study is introduced and a simulation model is constructed and calibrated in Excel. Section 5.5 discusses the results of the simulation exercise. Summary comments and conclusions are presented in Section 5.6.

5.2 Grading and quality-dependent price premiums

As was discussed in the Introduction, grading creates standardized commodity subsets with pre-defined quality attributes. Standardization allows buyers to make purchase decisions without visually inspecting the commodity, which in turn significantly reduces marketing transaction costs. If quality attributes are difficult to identify until after the commodity has been purchased and utilized, then grading takes on the additional role of reducing adverse selection. With lower adverse selection, commodity producers have a greater incentive to invest in quality-enhancing production techniques. For example, price premiums for wheat grades with a high minimum protein content may induce wheat producers to raise expected protein levels by adjusting fertilizer blends. Similarly, price discounts for wheat grades with a high maximum moisture content may induce wheat producers to reduce the moisture content in their grain by investing in a grain dryer.

Prices for different grades of a commodity are expected to be highly integrated over time, especially if there is a high degree of substitutability across the different quality versions of the commodity. Recall Figure 1.6 from Chapter 1 where it can be seen that the price difference between grade 1a and 1b cocoa beans in Malaysia is typically less than 5 percent, despite sizeable daily price fluctuations. In other situations, such as the parallel markets for fresh and processed fruits and vegetables, the degree of price integration is expected to be much lower. In a commodity futures market quality discounts and premiums are typically fixed by the institution and adjusted only periodically. In this case there is a near perfect degree of price integration across different quality versions of the commodity.

Another important benefit of grading is that it lowers the aggregate cost of storage and transportation. Aggregate costs are lowered because production from different producers can be mixed together shortly after the commodity leaves the farm. The cost savings associated with not preserving the identity of the commodity can be substantial because there are generally large economies of scale associated with bulk storage and shipment of commodities. Despite these scale-based cost savings, as agricultural supply chains shift toward becoming more vertically coordinated, there is a growing trend toward identity preservation from farm to consumer. Identity preservation is becoming an increasingly important issue in the production of livestock because of concerns over animal health and animal welfare.

Growing levels of vertical coordination also imply that food processors and wholesalers are continually finding new ways to differentiate commodities through use of private standards, certification and branding. Production differentiation is obviously not consistent with the standardization goal of a public grading scheme. Private standards and branding are particularly important when consumers place a high value on the credence attributes of a food product. Credence attributes such as animal welfare and pesticide free production remain unaddressed in a grading scheme because they are non-observable features of the commodity and thus not subject to grades per se. Branded beef and poultry products, eggs from free-range layers and certified organic peaches are all examples of products for which the comparative value of grading has diminished because of the increase in prominence of branding and private standards.

Grading schemes differ with respect to their visibility to the consumer. In some cases the final consumer product is graded at the consumer level (e.g., eggs, fruits and vegetables) whereas in other cases the grading scheme is applied at the processing and wholesale level (e.g., green coffee beans). Another distinguishing feature of a grading scheme is whether it is mandatory versus voluntary. Some government created schemes are mandatory (especially for grains), but the majority of government and industry grading schemes are voluntary (e.g., beef and pork in many jurisdictions). Voluntary grading schemes created by governments often serve as valuable aids for sellers and buyers in wholesale trading.

At the international level, the United Nations' Food and Agriculture Organization (FAO) manages the voluntary Codex Alimentarius (Latin for "food code"), which is a collection of internationally adopted food standards, guidelines and codes of practice. For the major commodities, countries who actively participate in global trade generally have standards and grading schemes that equal or exceed the Codex requirements. (e.g., flour, coffee beans and vegetable oil). International standards, guidelines and standards are continually changing in response to new products, new production techniques and changes in consumer tastes and preferences.

Commodity grades are often used by the processing sector to direct the commodity into an appropriate processing stream. For example, the higher grades of wheat are diverted into the supply chain for human food and the lower grades are diverted to the livestock sector. The situation is similar for apples, which are diverted to the fresh market or the processing (juice) market, depending on the grade of the apple. In cattle slaughter, the lowest grade animals are often processed into pet food rather than allowed to enter the human food supply chain.

Commodity grading has the potential to be used strategically to achieve price discrimination. Demand in domestic markets tends to be relatively inelastic whereas demand in export markets tends to be relatively elastic. Similarly, demand for fresh fruit and vegetables tends to be less elastic than demand for the processed version of the commodity. In both cases, industry revenue can be increased if supply from the inelastic market is diverted to the elastic market such that marginal revenue rather than price is equal across the two markets. One way of achieving this type of price discrimination in a competitive market place is to associate a higher grade with the inelastic market and a lower grade with the elastic market. By using the grade parameters to divert the commodity away from the inelastic market, new industry revenue from price discrimination can be generated.

Grading case study

The profitability of blending largely depends on the size of the price premiums that are associated with higher grades of the commodity. The following case study of western Canadian durum wheat demonstrates that price discounts due to various forms of seed deterioration can be significant and can change in a significant way over time. The formal analysis of blending that is presented later in this chapter is not designed to address dynamic aspects of blending incentives such as a change in grading premiums over time.

The Canadian Wheat Board (CWB) is a single-desk seller of western Canadian durum wheat that is destined for domestic human consumption or export. Because durum wheat is seldom used for feed and is largely grown for export, most Canadian durum is marketed exclusively by the CWB, and is exported from Canada via terminal elevators situated on the west coast or the eastern seaboard (i.e., the Great Lakes). Canadian durum has five primary grades ranging from #1 CWAD (highest) to #5 CWAD (lowest). The acronym CWAD stands for "Canadian western amber durum". Grades #1 and #2 are further subdivided according to protein content. Durum wheat may be downgraded from a #1 grade because of excessive moisture near harvest, which causes sprouting and bleaching, and because of various types of durum wheat diseases and insect damage.

The CWB operates by selling the durum wheat throughout the crop year (1 August to 31 July) and pooling producer receipts. After the crop year closes, aggregate producer receipts minus CWB expenses for a particular grade category is divided by the total volume of shipments within that category to obtain the final "pooled" price. The pool price is used to calculate final payments for all producers who delivered to the pool. Producers receive an initial payment when the grain is delivered, and receive the final payment (the difference between the finalized pool price and the initial price) after the close of the crop year.

Each month the CWB publishes for each grade category a "Pool Return Outlook" (PRO), which is the board's best estimate of the final pool price. The grade premiums and discounts that are associated with the pre-harvest PRO are quite speculative because the quality of the durum wheat, which is normally harvested in September, is only partially known during the pre-harvest months. The PRO estimates become increasingly accurate over the October to March period because the majority of the grain is exported during this period, and the grades are formally established at the time of export.

Table 5.1 shows the percent discount in the PRO price for #3 CWAD relative to the PRO price for #1 CWAD with 11.5 percent protein. For example, in April 2003 the price of #3 CWAD was 18.66 percent lower than the price of #1 CWAD. One year later this discount was only 10.82 percent. Table 5.1 shows that the discount typically represents between 5 and 10 percent of the value of #1 CWAD. As well, there is considerable variation in the discount across years for a particular marketing month.

Of particular interest is the stability of the #3 CWAD discount over the course of a marketing year. Table 5.1 shows that the discount decreased throughout the 2003/4 marketing year whereas it increased over time for 2004/5 and 2005/6. In 2007/8 the discount starts at about 10 percent in April but has virtually vanished by the following March. CWB PRO discounts are rather specialized because they are based on price pooling. Nevertheless it is still useful to ask why discounts tend to trend up or down over the course of a CWB marketing year.

The first explanation for a trend in the #3 CWAD discount is that the CWB selling price for durum and the ratio of #3 CWAD in overall sales may be changing over time. These changes combined with price pooling imply that the CWB's estimate of the discount for #3 CWAD will be continually changing, even if the price spread between wheat grades is constant. The price of durum wheat grew strongly throughout the 2007/8 crop year, so one explanation for the vanishing discount during that year is that sales of the #3 grade were skewed toward the latter part of the marketing year.

The second explanation for time trends in the quality discount for #3 CWAD is that information about the distribution of grades in the aggregate stock of durum becomes increasingly accurate over time. If the percentage of #3 CWAD in export sales is trending up (down) over time, then the #3 PRO discount is also expected

Table 5.1 CWB discounts in monthly durum PRO and monthly grade distributions in durum export stocks: 2003/4 to 2008/9

Percent discount in CWB PRO: #3 CWAD versus 11.5% #1 CWAD
(Fraction of #3 CWAD minus fraction of #1 CWAD in export stocks)

	2003/4	*2004/5*	*2005/6*	*2006/7*	*2007/8*	*2008/9*
April	18.66	10.82	13.61	13.14	10.31	6.93
May	18.32	10.66	13.4	13.22	10.85	7.61
June	13.47	10.5	10.77	13.14	11.01	7.46
July	13.27	10.5	10.82	12.57	8.65	7.46
August	10.84	10.5	10.99	10.05	6.71	7.65
Sept.	10.1	12.8	11.7	8.63	3.43	7.59
Oct.	9.85	13.73	13.44	8.42	3.23	8.41
	(0.298)	(−0.852)	(0.185)	(0.004)	(−0.721)	(−0.565)
Nov.	8.78	13	13.66	8.33	3.06	8.17
	(0.054)	(−0.464)	(0.018)	(−0.369)	(−0.726)	(−0.539)
Dec.	7.18	13.2	14.84	7.58	2.56	8.31
	(−0.511)	(−0.284)	(0.079)	(−0.516)	(−0.76)	(−0.223)
Jan.	7.18	13.2	15	6.54	2.44	8.19
	(−0.534)	(−0.154)	(0.326)	(−0.526)	(−0.696)	(−0.387)
Feb.	7.18	13.71	15.34	6.54	2.29	8.19
	(−0.669)	(0.054)	(0.401)	(−0.522)	(−0.662)	(−0.353)
March	6.98	13.5	15.25	6.45	2.29	9.55
	(−0.779)	(0.066)	(0.233)	(−0.471)	(−0.697)	(−0.174)

Source: Price data was downloaded from the CWB website on 16 August 2009: http://www.cwb.ca/db/contracts/pool_return/pro.nsf Terminal stock data was downloaded from the Canadian Grain Commission website on 16 August 2009: http://www.grainscanada.gc.ca/statistics-statistiques/gsw-shg/gswm-mshg-eng.htm

Note: The crop year runs from 1 August to 31 July. April to July data precede the crop year, and August to March data are within the crop year.

to trend up (down) because the size of the discount and the volume of #3 CWAD in the sales mix are normally positively related to each other.

The positive correlation between the price discounts for #3 CWAD and the fraction of #3 CWAD in export stocks is quite evident when the price data in Table 5.1 is compared with the grade distributions data that reside in parentheses below each price point for the months ranging from October to March. For example, the price discounts for #3 CWAD remained high throughout the 2004/5 crop year seemingly because #3 CWAD constituted a comparatively large fraction of export sales during this marketing year. The opposite is true for the 2007/8 marketing year.

5.3 LOP model of blending and grading

The purpose of this section is to develop a simple model of blending, grading and shadow prices in a primary commodity market. The model is quite restrictive because prices of the graded commodity in the final market are assumed to be exogenous rather than dependent on the distribution of quality in the graded commodity mix. Incorporating endogenous prices would be tricky because a differentiated demand system would be required to assign values to the quality-differentiated products. For small export oriented regions that have little influence on global commodity prices, the assumption of exogenous prices for graded commodities is not unreasonable.

Competitive traders may have an incentive to blend if two quality versions of a commodity can be sold for a higher price on average when blended versus unblended. Blending is particularly common for commodities that lend themselves to mechanical mixing at comparatively low cost (e.g., grain) and for commodities that are graded based on quantifiable features (e.g., percent protein, percent foreign material, percent sprouted kernels). In many respects, blending is a form of market arbitrage similar to the transporting of a commodity from a low price to a high price region, and similar to the storing of a commodity from a low price to a high price period of time.

The unblended commodity is assumed to be one of two quality types (low or high) and the blended commodity is sold with one of two grades (A or B). In the absence of blending, the high quality commodity receives an A grade and the low quality commodity receives a B grade. These assumptions are reasonable in situations where a crop receives either no damage or significant damage from weather (e.g., hail, frost), insects or disease. In order to qualify for a grade A, assume that the maximum percentage of low quality commodity in a commodity blend is $\gamma/_{1+\gamma}$, which is equivalent to assuming that a maximum of γ units of low quality commodity can be blended with each unit of high quality commodity. The marginal cost of blending is assumed fixed at level m per unit of the commodity blend.

Let X_H and X_L denote the respective levels of unblended high and low commodity that are available to traders in the market. As well, let the exogenous parameters P_A and P_B be the respective prices of the grade A and B commodity blend, where $P_A > P_B$. The objective of this analysis is to derive the set of prices for the low and high quality versions of the unblended commodity, P_L and P_H, in a LOP equilibrium. This set of equilibrium prices can be derived in one of two ways: (1) directly by imposing the zero profit condition on blending arbitrage activities; or (2) indirectly by solving the social planner's problem, which involves finding the allocation of X_L and X_H that maximizes net aggregate surplus subject to the grading and supply availability constraints and then recovering the shadow prices from the solution. Each approach will be illustrated in turn.

Zero arbitrage profits approach

Profits, π, earned by a trader who blends one unit of the high quality commodity with γ units of low to make $1 + \gamma$ units of a grade A blend can be expressed as $\pi = (1 + \gamma)\ (P_A - m) - (P_H + \gamma P_L)$. If at least some blending is profitable then blending by many profit-seeking blenders will cause prices P_L and P_H to adjust until $\pi = (1 + \gamma)\ (P_A - m) - (P_H + \gamma P_L) = 0$. Individual traders treat P_L and P_H as fixed parameters when making blending decisions, but for the market as a whole these two prices will depend on aggregate demand by traders and available supplies of the low and high quality commodity.

There are two solutions to the $\pi = (1 + \gamma)(P_A - m) - (P_H + \gamma P_L) = 0$ profit equation, depending on whether the high or low quality commodity is in surplus after blending is complete. If $X_L \geq \gamma X_H$ then the low quality commodity is in surplus. Specifically, γX_H units of X_L will be incorporated into the blend and $X_L - \gamma X_H$ units of surplus X_L will be sold in unblended form for price P_B. Conversely, if $X_L < \gamma X_H$ then the high quality commodity is in surplus, in which case X_L/γ units of X_H will be incorporated into the blend and $X_H - X_L/\gamma$ units of surplus X_H will be sold in unblended form for price P_A.

There are three possible scenarios to consider when deriving equilibrium values for P_L and P_H. First, marginal blending rents may be less than the marginal cost of blending, in which case no blending will take place and the equilibrium prices for the low and high quality versions of the commodity are P_B and P_A, respectively. The second outcome is that blending is profitable and the low quality commodity is in surplus, which implies $P_L = P_B$ and P_H is implied by the solution to $\pi = (1 + \gamma)(P_A - m) - (P_H + \gamma P_L) = 0$ with $P_L = P_B$. The third outcome is that blending is profitable and the high quality commodity is in surplus, which implies $P_H = P_A$ and P_L is implied by the previous zero profit condition with $P_H = P_A$.

The results from the previous paragraph can be summarized as follows:

Case 1: $X_L > \gamma X_H$ *Low quality is in surplus*

$$P_L = P_B \text{ and } P_H = \begin{cases} P_A & \text{if } P_A - P_B \leq \left(\dfrac{1+\gamma}{\gamma}\right)m \\ P_A + \gamma(P_A - P_B) - (1+\gamma)m & \text{if } P_A - P_B > \left(\dfrac{1+\gamma}{\gamma}\right)m \end{cases} \quad (5.1a)$$

Case 2: $X_L \leq \gamma X_H$ *High quality is in surplus*

$$P_H = P_A \text{ and } P_L = \begin{cases} P_B & \text{if } P_A - P_B \leq \left(\dfrac{1+\gamma}{\gamma}\right)m \\ P_A - \left(\dfrac{1+\gamma}{\gamma}\right)m & \text{if } P_A - P_B > \left(\dfrac{1+\gamma}{\gamma}\right)m \end{cases} \quad (5.1b)$$

The first expression for P_H in Case 1 and the first expression for P_L in Case 2 correspond to a corner solution where no blending takes place because the marginal cost of blending is greater than marginal revenue. The bottom expression in each case corresponds to a positive level of blending. The condition for a positive level of blending, $P_A - P_B > \dfrac{1+\gamma}{\gamma}m$, is derived in Case 1 by restricting $P_H \geq P_A$ and in Case 2 by restricting $P_L \geq P_B$.

The blending rents per unit of high quality commodity can be expressed as $\gamma(P_A - P_B) - (1 + \gamma)m$. This equation makes sense because one unit of high allows

γ units of low to be sold at price P_A rather than P_B, but the cost of blending must be subtracted from this gain. It follows from equation (5.1) that when the low quality commodity is in surplus (Case 1) then $P_L = P_B$ and $P_H = P_A$ + *blending rents*. Conversely, when the high quality commodity is in surplus (Case 2) then $P_H = P_A$ and $P_L = P_B + \text{blending rents}/\gamma$. These two equations demonstrate that blending rents are fully reflected in the price of the high (low) quality commodity when high (low) quality is scarce.

The previous results imply that if high quality commodity is in scarce supply then the gap between the equilibrium values of P_H and P_L will be comparatively large. The opposite is true if low quality commodity is in scarce supply. This outcome implies that for a fixed set of values for P_A and P_B, the prices for the ungraded commodities, P_L and P_H, are expected to be negatively correlated over time as random quality outcomes give rise to varying percentages of high and low quality in the commodity mix. In the extreme case where the unit cost of blending, m, is zero then the price of the low and high quality commodity will both equal P_A when the low quality commodity is in scarce supply.

Social planner's problem

The results of the previous section can also be derived by solving the problem facing a social planner, which is to choose the level of blending that maximizes net aggregate surplus in the market. In this simple model where there are only two quality versions of the unblended commodity and two grades of the blended commodity, the approach presented in the previous section is simpler. The social planner approach is presented in this section because it is the preferred method in more general blending problems such as the case study on wheat protein that will be examined below.

Similar to the spatial pricing problem presented in Chapter 2, the set of competitive prices for the different quality versions of the unblended commodity can be derived by calculating the optimal level of blending for the planner and then recovering the set of shadow prices associated with the resource constraints. Let Q denote the quantity of X_L that is added to the blend. The availability restriction requires $0 \le Q \le X_L$ and the grading restriction requires $0 \le Q \le \gamma X_H$. The total supply of grade A commodity is equal to $X_H + Q$ and the total supply of grade B commodity is equal to $X_L - Q$.

Let $V(Q)$ denote net aggregate market surplus for the social planner. The prices P_A and P_B are fixed, which implies perfectly elastic demand and thus zero surplus for buyers in the final goods market. Consequently, net aggregate surplus consists of the combined revenue from the sales X_L and X_H for commodity traders minus the cost of blending. The pre-optimized expression for $V(Q)$ is[2]

$$V(Q) = (X_H + Q)P_A + (X_L - Q)P_B - m\left(1 + \frac{1}{\gamma}\right)Q \tag{5.2}$$

Within equation (5.2) note that $m\left(1+\frac{1}{\gamma}\right)Q$ represents the cost of blending because one unit of low quality commodity results in $1 + {}^1\!/\gamma$ units of blend, and each unit of the blend results in cost m.

To incorporate the inequality restrictions listed above, set up the following Lagrangian function with the variables λ_1 through λ_3 serving as multipliers:

$$L(Q)=\left(X_H+Q\right)P_A+\left(X_L-Q\right)P_B-m\left(1+\frac{1}{\gamma}\right)Q$$

$$+\lambda_1\left(X_L-Q\right)+\lambda_2\left(\gamma X_H-Q\right)+\lambda_3 Q \qquad (5.3)$$

The procedure for solving this constrained optimization problem and the full solution for the problem is contained in Appendix 5.1. The results are repeated here, but to save space the results for the zero blending case ($\lambda_3 > 0$) are omitted. Because the multiplier variables, λ_1 and λ_2, play a special role in the analysis, expressions for their equilibrium values are presented here along with the expressions for the optimized value of Q:

Case 1: $\gamma X_H < X_L$ (low quality commodity is in surplus) and $Q^*>0$

$$Q^*=\gamma X_H,\ \lambda_1=\lambda_3=0 \text{ and } \lambda_2=\left[P_A-P_B-\left(1+\frac{1}{\gamma}\right)m\right]\geq 0$$

Case 2: $\gamma X_H > X_L$ (high quality commodity is in surplus) and $Q^*>0$

$$Q^*=X_L,\ \lambda_2=\lambda_3=0\ \lambda_1=\left[P_A-P_B-\left(1+\frac{1}{\gamma}\right)m\right]\geq 0$$

In Case 1 all available high quality commodity is used in the blend so the λ_1 multiplier on the $Q < X_L$ constraint takes on a value of zero and the λ_2 multiplier on the $Q \leq \gamma X_H$ constraint takes on a positive value. The opposite is true for Case 2, where all available low quality commodity is used in the blend.

Shadow prices

It is now possible to formally derive the pair of shadow prices, P_L and P_H, from the solution to the social planner's problem. As indicated above, a shadow price is a measure of the amount by which net aggregate surplus will increase if one more unit of the resource becomes available. Shadow prices are derived by differentiating the Lagrangian function given by equation (5.3) with respect to resource stock parameters, X_H and X_L. The following expressions emerge:

$$P_L=\frac{\partial L}{\partial X_L}=P_B+\lambda_1 \qquad (5.4a)$$

and

$$P_H = \frac{\partial L}{\partial X_H} = P_A + \gamma\lambda_2 \quad (5.4b)$$

The expressions for the equilibrium values of Q^*, λ_1 and λ_2, which are presented in Case 1 and Case 2 below equation (5.3), can be substituted into equation (5.4) to obtain explicit expressions for the equilibrium values of P_L and P_H. As is expected the resulting pricing expressions are identical to the expressions for P_L and P_H that were previously derived using the zero profit arbitrage approach (see equation (5.1) above). This finding formally demonstrates that equilibrium prices in a competitive market with profit-seeking traders are the same as the implicit prices that emerge when a social planner chooses a blending rule to maximize net aggregate surplus.

5.4 Wheat protein case study

The purpose of this section is to estimate the shadow prices of different protein versions of #1 Canada Western Red Spring (CWRS) wheat. CWRS is the dominant type of wheat produced in western Canada and #1 is the highest grade. Similar to western Canadian durum wheat that was discussed in Section 5.2, CWRS that is exported or shipped to Canadian flour mills is marketed exclusively by the CWB. Four separate CWB pools are maintained for #1 CWRS based on 14.5 percent, 13.5 percent, 12.5 percent and 11.5 percent minimum protein content. For the years 2001/2 to 2009/10, the average September PRO for these four protein pools was equal to \$256.44/tonne, \$245.33/tonne, \$238.00/tonne and \$232.11/tonne, respectively.[3] The average and the standard deviation of the price spread over this same period was \$11.11 and 6.92 for the 14.5 percent to 13.5 percent spread, \$7.33 and 5.24 for the 13.5 percent to 12.5 percent spread, and \$5.89 and 3.37 for the 12.5 percent to 11.5 percent spread. These values reveal that the marginal value of protein is higher for the higher protein grades, and there is considerable variation in protein premiums over time.

Table 5.2 shows the distribution of export sales of #1 CWRS for the years 1997/8 to 2008/9 broken down by three protein categories: High (14.5 percent or higher), Medium (13 to 14 percent) and Low (12.5 percent or less). The mean level of protein over the 12 year period is 13.5 percent, and only rarely do protein levels fall outside of the 11.5 to 15.5 percent range. Table 5.2 reveals that, on average, 75 percent of export sales are in the medium protein category and the remaining sales are evenly split between the high and low categories. Notice that there is considerable year-to-year variation in the distribution of protein. This outcome is expected because protein levels in #1 CWRS are largely dependent on temperatures and the distribution of rainfall during the western Canadian growing season, and both of these determinants of protein vary considerably from year to year.

Three important assumptions are made for the simulation analysis. First, data on the distribution of protein within the aggregate stock of *unblended* #1 CWRS is required to calibrate a protein blending model, but unfortunately this information

Table 5.2 Distribution of protein in export sales of #1CWRS: 1997/8 to 2008/9

		3rd and 4th quarter export sales (%)		
Year	*% Mean protein*	*High (≥ 14.5%)*	*Medium (13–14%)*	*Low (≤ 12.5%)*
1997/8	13.0	8.6	51.7	39.7
1998/9	13.7	22.2	72.7	5.1
1999/2000	13.3	14.4	57.0	28.6
2000/1	13.5	1.4	92.8	5.8
2001/2	14.0	21.2	78.8	0
2002/3	14.2	32.6	67.4	0
2003/4	14.2	41.9	58.1	0
2004/5	13.1	0	1.0	0
2005/6	13.0	0	1.0	0
2006/7	13.1	1.6	63.3	35.1
2007/8	13.6	6.9	92.7	0.4
2008/9	13.1	0	65.7	34.3
Average	13.5	12.6	75.0	12.4

Source: Tables 1, 3 and 5 of *Moisture Content, Test Weight and Other Grade Determining Factors. Atlantic and Pacific Export Cargos of Canadian Western Red Spring Wheat, Third And Fourth Quarters* (various years) and from *Quality of Western Canadian Wheat Imports*, Canadian Grain Commission (various years).

is not available. Hence, protein in the unblended stock is assumed to follow a beta distribution (the shape of the distribution is varied as part of the sensitivity analysis). The second assumption for the simulation analysis is that the annual aggregate stock of #1 CWRS can be blended at zero cost. The final assumption is that the number of protein categories is restricted to four (minimum protein levels of 14.5, 13.5, 12.5 and 11.5 percent), even though more categories with narrow protein intervals exist in reality. All three of the above assumptions imply that the simulation results should be viewed as an illustrative example rather than an estimation of a real-world scenario.

The specific objective of the simulation analysis is to calculate the set of shadow prices for the different protein versions of the wheat within the #1 CWRS stockpile while taking as given the set of graded commodity prices for this variety of wheat. As discussed above, the estimated set of shadow prices can be interpreted as the implicit prices for the different protein versions of #1 CWRS that trade in primary markets. The distribution of protein in the ungraded stocks of #1 CWRS combined with the distribution of price premiums for the four CWB protein categories give rise to a particular shape of the distribution of shadow prices after the LOP is imposed. This connection between protein scarcity, price premiums and the distribution of shadow prices is explored as part of the sensitivity analysis.

Model calibration

The beta distribution is used to model the distribution of protein within the preblended stock of #1 CWRS. Let *min* and *max* denote minimum and maximum values for the continuous beta random variable, x, which designates percent protein in the wheat.[4] The probability density function for x can be expressed as

$$f(x) = \frac{(x - \min)^{\alpha-1}(\max - x)^{\beta-1}}{\beta(\alpha,\beta)(\max - \min)^{\alpha+\beta-1}} \tag{5.5}$$

In addition to the *min* and *max* parameters, the beta distribution utilizes α and β as location and shape parameters. The beta function, β (α, β), within equation (5.5) has no interesting interpretation, and it can be generated using an Excel function. Hence, the specific equation for the beta function is not displayed here.

To calibrate the beta distribution for the simulation model, assume that $min = 0.115$ and $max = 0.157$.[5] The two remaining parameters, α and β, can be chosen to achieve a desired mean protein percentage (set equal to 0.135 for the base case) and a desired shape of the distribution of protein. An expression for the mean of equation (5.5), which is denoted $\bar{x}$, can be written as:

$$\bar{x} = \min + \frac{\alpha}{\alpha + \beta}(\max - \min) \tag{5.6}$$

Figure 5.1 shows the calibrated beta function for the base case analysis, which assumes $\alpha = 2.1$ and $\beta = 2.3$.

The top half of the blending simulation model is displayed in Figure 5.2. The parameter values listed in cells A2:B6 are the parameter values of the beta distribution that is displayed in Figure 5.1. A rather complicated formula is used in cell B9 to generate a value for the beta function, $\beta(\alpha, \beta)$, which is required to simulate equation (5.5).[6] Equation (5.6) is used to display the mean protein level in cell B10. The variable titled “surplus” in cell B11 is the social planner’s objective function (more details below).

The volumes of pre-blended wheat with protein levels ranging from 11.5 to 15.7 percent in increments of 0.00175 are displayed in columns H and A, respectively, of Figure 5.2 (starting in row 18) and Figure 5.3. The wheat volumes in column H, which are labeled “Protein Available”, were calculated using equation (5.5). The sum of the values in column H over the full distribution of protein is equal to the reciprocal of 0.00175, which is 571.43. It is therefore useful to assume that the stockpile of #1 CWRS available for blending is equal to 571.43 tonnes, and the values in column H are protein-specific tonnes of wheat that are available for blending.

The choice variables for the problem are contained in columns B through E, beginning in row 18 of Figure 5.2 and extending down to row 42 in Figure 5.3. Prior to optimization with Solver, “guess” values should be entered into these cells. These choice variables represent the social planner’s allocation of the different protein versions of the unblended wheat to the four CWB protein categories. The minimum protein content and the associated selling prices of graded

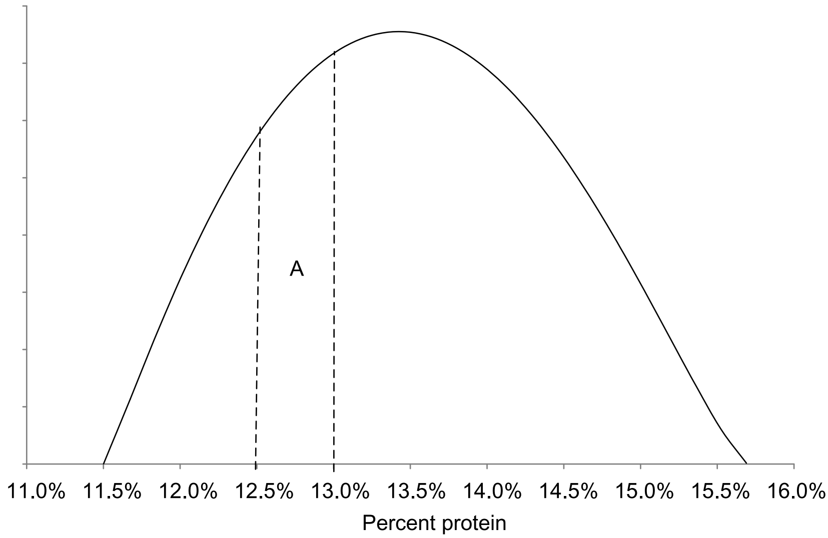


Figure 5.1 Simulated beta distribution used for the base case analysis of protein content.

Notes:
(a) Parameters are *min* = 0.115, *max* = 0.157, $\alpha = 2.1$ and $\beta = 2.3$. Mean protein for this distribution is 13.5 percent.
(b) Area A represents the fraction of total #1 CWRS production that has protein between 12.5 and 13.0 percent.

CWRS within these four categories are contained in cells B14:E14 and B15:E15 of Figure 5.2, respectively.[7] For each protein row the sum of the wheat allocated to the four CWB protein categories is contained in column G, which is labeled "Protein Used". As part of the optimization routine, Solver will be instructed to allocate wheat to the various protein categories subject to the constraint that "Protein Used" is less than or equal to "Protein Available".

The column totals for the blending choice variables are displayed in cells B43:E43 of Figure 5.3. A particular column total represents the aggregate volume of wheat delivered to that particular CWB protein category. These totals are used below to calculate total revenue from blending and to calculate the average level of protein in the blend. The aggregate volume of protein that is supplied to each category is displayed in cells B45:E45 and labeled "Protein Allocated". The array formula that is used to calculate the protein allocation multiplies each protein level in column A by the corresponding wheat volume in column B and then sums the products over all levels of protein. The minimum "Protein Required" to qualify for the designated CWB grade is reported in cells B46:E6. These values are calculated by multiplying the minimum protein percentage by the volume of wheat for each category. As part of the optimization routine, Solver will be instructed to choose values for the blending choice variables while ensuring that the "Protein

	A	B	C	D	E	F	G	H	I
1	**Parameters**								
2	min	0.115	minimum percentage protein in wheat stock						
3	max	0.157	maximum percentage protein in wheat stock						
4	alpha	2.1	parameter of beta distribution for protein						
5	beta	2.3	parameter of beta distribution for protein						
6	increment	0.00175	protein increment in table below						
7		=EXP(GAMMALN(alpha))*EXP(GAMMALN(beta))/EXP(GAMMALN(alpha+beta))							
8	**Variables**								
9	beta_ftn	0.1205	beta function required to calculate probability						
10	mean prot.	13.50%	mean level of protein in wheat stock				=min+(max-min) *alpha/(alpha+beta)		
11	surplus	$ 140,690	economic surplus per tonne of wheat						
12		=SUM(B15:E15*B43:E43)					=(A18-min)^(alpha-1)*(max-A18)^(beta-1)/(beta_ftn*(max-min)^(alpha+beta-1))		
13	**Grading**								
14	Min Protein	14.5%	13.5%	12.5%	11.5%				
15	Price/tonne	256	245	238	232				
16					=SUM(B18:E18)		Protein		
17		-------------Protein Allocation-------------					Used	Available	
18	0.11501	0.00	0.00	0.02	0.00		0.02	0.02	
19	0.11675	0.00	0.00	5.67	0.00		5.67	5.67	
20	0.1185	0.00	0.00	11.47	0.00		11.47	11.47	
21	0.12025	0.00	0.00	16.87	0.00		16.87	16.87	
22	0.122	0.00	0.00	21.73	0.00		21.73	21.73	
23	0.12375	0.00	0.00	25.98	0.00		25.98	25.98	
24	0.1255	0.00	0.00	29.60	0.00		29.60	29.60	
25	0.12725	0.00	0.00	32.55	0.00		32.55	32.55	
26	0.129	0.00	0.00	34.85	0.00		34.85	34.85	
27	0.13075	0.00	23.97	12.51	0.00		36.48	36.48	
28			*Continued in Figure 5.3*						

Figure 5.2 Setup/results for blending simulation model (top half).

	A	B	C	D	E	F	G	H
27			*Continues Figure 5.2*					
28	0.1325	0.00	37.44	0.00	0.00		37.44	37.44
29	0.13425	0.00	37.76	0.00	0.00		37.76	37.76
30	0.136	0.00	37.45	0.00	0.00		37.45	37.45
31	0.13775	0.00	36.52	0.00	0.00		36.52	36.52
32	0.1395	24.92	10.09	0.00	0.00		35.01	35.01
33	0.14125	32.93	0.00	0.00	0.00		32.93	32.93
34	0.143	30.34	0.00	0.00	0.00		30.34	30.34
35	0.14475	27.26	0.00	0.00	0.00		27.26	27.26
36	0.1465	23.76	0.00	0.00	0.00		23.76	23.76
37	0.14825	19.89	0.00	0.00	0.00		19.89	19.89
38	0.15	15.75	0.00	0.00	0.00		15.75	15.75
39	0.15175	11.43	0.00	0.00	0.00		11.43	11.43
40	0.1535	7.10	0.00	0.00	0.00		7.10	7.10
41	0.15525	1.03	2.00	0.00	0.00		3.03	3.03
42	0.1569	0.08	0.00	0.00	0.00		0.08	0.08
43	Totals	194.49	185.24	191.25	0.00			
44	**Protein**						=SUM(B18:B42)	
45	allocated	28.20	25.01	23.91	0.00			
46	required	28.20	25.01	23.91	0.00			
47								
48	=SUM($A$18:$A$42*B18		=B43*B14					
49	:B42)							

Figure 5.3 Setup/results for blending simulation model (bottom half).

Allocated" in a particular protein category is greater than or equal to the "Protein Required".

As indicated above, the objective function for the social planner is labeled "Surplus" and is reported in cell B11 of Figure 5.2. In this simple model there is no cost of blending and so the surplus to be maximized is simply the sum of aggregate wheat revenues. These revenues are calculated by multiplying the values for wheat volume in cells B43:E43 by the corresponding values for wheat price in cells B15:E15. To maximize surplus Solver must be instructed to allocate the unblended wheat to the four CWB protein categories (cells B18:E42) subject to the first constraint that the aggregate allocation of unblended wheat to the four categories (cells G18:G42) cannot exceed available supply (cells H18:H42), and subject to the second constraint that the actual protein level in each category (cells B45:E45) must equal or exceed the minimum protein requirements of that category (cells B46:E46).

When setting up Solver, be sure to select two Solver options. The first is "Assume Linear Model" and the second is "Assume Non-Negative". The first option is important because without it Excel will not generate shadow values as part of Solver's final report. Fortunately, unlike the spatial problem of Chapter 2, the blending simulation model does not require accurate "guess" values of the 100 choice variables. Dividing the allocation of available wheat evenly between the four CWB protein categories is a good way to establish initial "guess" values for the choice variables.

5.5 Simulation results

After clicking Solver's "Solve" button and then clicking "OK" when an optimal solution is returned, base case results *similar* to those shown in Figures 5.2 and 5.3 should appear. If the model is repeatedly solved, small differences in the optimal allocation may result even though the optimized level of surplus does not change. These small differences can be attributed to the lack of blending costs in the model. In the absence of blending costs, the objective function is very flat in the neighborhood of the optimized solution. The small difference in solution that emerges with each new "solve" are not important for the analysis. Fortunately, the shadow prices that are retrieved from the optimal solution appear to be highly stable.

Cells B18:E42 of Figures 5.2 and 5.3 show the base case allocation of unblended wheat to the four CWB protein categories. Notice that none of the wheat is sold in the 11.5 percent protein category. This occurs because all of the unblended wheat with protein less than 12.5 percent (i.e., the wheat stocks in rows 18 through 23) can be blended into the 12.5 percent protein category. The 12.5 percent category contains wheat ranging from the lowest level of protein (11.5 percent) to wheat with 13.075 percent protein. Similarly, the 13.5 percent category contains wheat ranging from 13.075 percent to 13.95 percent protein. Finally, the 14.5 percent category contains wheat ranging from 13.95 percent protein to 15.7 percent protein. In general, wheat with a particular level of protein is either included in a blend with similar protein wheat or is blended up to the nearest higher protein category.

To obtain the shadow prices associated with the base case solution it is necessary to select "Sensitivity Reports" in the dialogue box that appears when Solver returns a solution. After clicking "OK" a new sheet that contains the full set of shadow prices is generated. A subset of these shadow prices is shown in column E of Figure 5.4, beginning in row 113. The full set of base case shadow prices for protein ranging from 11.5 to 15.7 percent is graphed in Figure 5.5.

The heavy line in Figure 5.5 shows the base case shadow price of the wheat (vertical axis) as a function of the protein content of the unblended wheat (horizontal axis). As is expected, the base case shadow prices are an increasing function of the protein percentage in the unblended wheat. This positive marginal value of protein reflects the potential for the wheat to generate positive blending rents by either being eligible for blending into a higher protein category or by allowing lower protein wheat to be blended into an existing category.

An important feature of the base case shadow prices that are shown in Figure 5.5 is that the shadow prices of unblended wheat with 12.5, 13.5 and 14.5 percent protein are equal to the market prices of graded wheat with 12.5, 13.5 and 14.5 percent protein, respectively. In other words, blending rents vanish when there is an exact match between protein levels in the unblended and blended versions of the wheat. For all other levels of protein the price of the unblended wheat contains a premium that reflects the blending rents. Specifically, wheat with protein below 12.5 percent has a shadow price that exceeds the selling price for wheat with 11.5 percent protein, wheat with protein between 12.5 and 13.5 percent has a shadow price that exceeds the selling price for wheat with 12.5 percent protein, et cetera. The difference between the shadow price of a particular version of wheat and the price that it would sell for in an unblended form is a measure of the blending rents that accrue to that particular protein version of wheat.

To analyze the effects of protein scarcity, the model can be resolved with $\alpha = 2.2$ and $\beta = 5$ to obtain a mean level of protein equal to 12.75%. This shift to the left in the distribution of protein implies that high protein wheat is now

	A	B	C	D	E	F	G	H
110	Constraints							
111				Final	Shadow	Constraint	Allowable	Allowable
112		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
113		$G$18	Used	0.02	233.96	0.020	3.301	0.020
114		$G$19	Used	5.67	234.66	5.671	3.997	5.671
115		$G$20	Used	11.47	235.37	11.474	5.073	11.066
116		$G$21	Used	16.87	236.08	16.871	6.942	15.143
117		$G$22	Used	21.73	236.79	21.728	10.991	21.728
118		$G$23	Used	25.98	237.49	25.982	26.378	25.982
119		$G$24	Used	29.60	238.20	29.596	143.861	29.596
120		$G$25	Used	32.55	238.91	32.554	31.969	14.655
121		$G$26	Used	34.85	239.62	34.848	17.983	8.243

Figure 5.4 A subset of shadow prices for the base case.

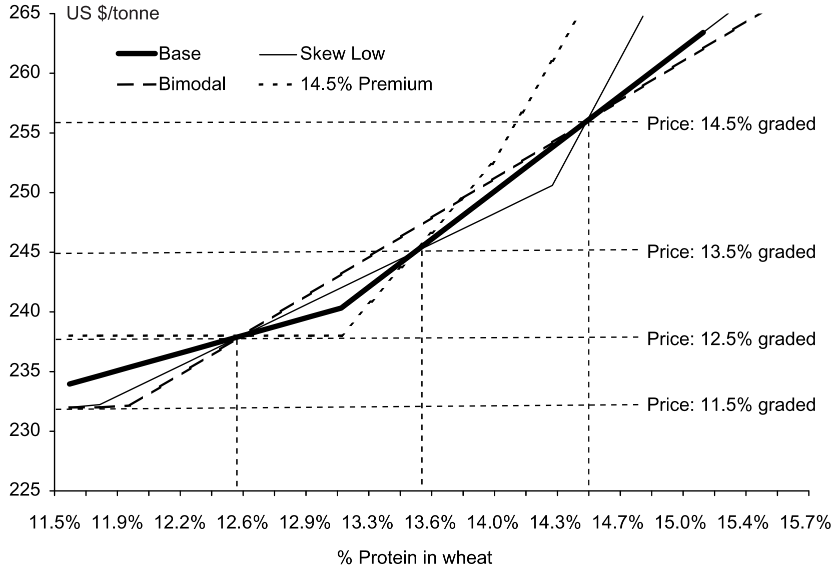


Figure 5.5 Simulated shadow prices for unblended wheat with different levels of protein.

Parameter values are as follows: (1) base [α = 2.1, β = 2.3 and mean protein = 13.5%]; (2) Skew Low [α = 2.2, β = 5 and mean protein = 12.75%]; (3) bimodal [α = β = 0.9 and mean protein = 13.6%]; and (4) 14.5% premium [base case plus CWB price for 14.5% = $266/tonne].

relatively scarce and low protein wheat is relatively plentiful. Consequently, a higher (lower) fraction of the blending rents should accrue to wheat with high (low) protein. This outcome can be observed in Figure 5.5 (the relevant graph is labeled "Skew Low"). Similar to the base case the "Skew Low" case has zero blending rents for unblended wheat with 12.5, 13.5 and 14.5 percent protein. However, relative to the base case the protein skewness decreases the shadow price for wheat with less than 12.5 percent protein and increases the shadow price for wheat with greater than 14.5 percent protein. Interestingly, for wheat with protein between 12.5 and 13.5 percent the left skewness in the distribution of protein raised the shadow price relative to the base case. The opposite is true for wheat with protein between 13.5 and 14.5 percent.

In some years growing conditions are such that the protein distribution is bimodal (U) shaped with a comparatively large amount of both high and low protein wheat, and a comparatively small amount of wheat with a medium level of protein. A bimodal (U) shaped distribution with a mean protein level of 13.6 percent can be achieved by setting $\alpha = \beta = 0.9$. Interestingly, the optimal allocation of wheat in this scenario involves dividing the wheat crop fully between the 14.5% and 12.5% protein categories. Figure 5.2 shows that relative to the base case the bimodality has no effect on the shadow price of unblended wheat with exactly 12.5 and 14.5 percent protein. However, as is expected, the shadow price is depressed relative to the base case for wheat with either less than 12.5 percent or greater than 14.5 percent protein. Moreover, for intermediate levels of protein (i.e., between 12.5 and 14.5 percent), the bimodality has increased the shadow price because this range of protein is relatively scarce.

Finally, how does the distribution of shadow prices change if price in the highest protein category increases from $256/tonne to $266/tonne? The shadow prices for this scenario (with base case values for α and β) have been plotted and are labeled "14.5% Premium" in Figure 5.5. Notice that the price increase for the 14.5 percent wheat has raised all shadow prices relative to the base case except for wheat with protein ranging from 12.5 to 13.5 percent. For wheat with protein in the range 11.5 to about 13.1 percent the shadow price is constant and equal to the CWB price for 12.5 percent wheat, presumably because wheat in this protein category is being blended to the 12.5 percent protein level. The flat shadow price schedule implies that blending rents are zero for wheat with protein in the 12.5 to 13.1 percent range. Figure 5.5 shows that the marginal value of protein for wheat with protein in excess of 13.5 percent is high relative to the base case. This result is expected given the assumed increase in the price premium for high protein graded wheat.

5.6 Concluding comments

This chapter focused on commodities that can be blended such as wheat, soybeans and coffee beans. For these commodities, the quality of the product that is delivered to the buyer can be adjusted through blending. The ability to adjust quality through blending gives rise to another version of the LOP. The LOP in the quality dimension implies that it should not be possible for a trader in a competitive market to earn profits by purchasing, blending and reselling two or more quality versions of a commodity. Similar to the analysis of prices over space in Chapter 2 and prices over time in Chapter 3, the LOP equilibrium can be derived by maximizing net aggregate surplus in the market and then recovering the set of associated prices. In this case the prices that are recovered from the social planner's problem are referred to as shadow prices.

Earlier in this chapter it was argued that grading and blending reduces marketing transaction costs. Specifically, grading is a response to the higher transaction costs that would result if buyers were required to visually inspect and negotiate unique prices for each individual batch of the commodity. The formal analysis of this chapter allows the transaction cost argument to be extended by noting that grading and blending in a competitive market gives rise to a set of implicit shadow prices for the different quality versions of a commodity. The set of shadow prices significantly reduce marketing transaction because different quality versions of a commodity will trade in primary markets based on an implicit price rather than an explicit posted price, which is costly to manage.

An interesting part of the story about shadow prices that was not addressed in this chapter involves identifying whether it is the commodity buyer or the seller who is able to extract the blending rents. The story is interesting because there are some fundamental asymmetries and institutional restrictions that determine how the blending rents are shared amongst industry participants. Suppose that a farmer with 13.3 percent protein wheat is negotiating price with a commodity buyer. Also assume that both the farmer and the buyer know that the shadow value of the wheat is $242/tonne because it can be blended and sold in the 13.5 percent price pool. However, institutional restrictions may require the buyer to offer the farmer either a 13.5 percent grade and a $245/tonne purchase price or a 12.5 percent grade and a $238/tonne purchase price. If the former is offered, the farmer captures the blending rents and if the latter is offered the blending rents accrue to the buyer. The outcome of the negotiation will depend on relative bargaining strengths of the two parties (more on this in Chapter 9).

Suppose instead the farmer is attempting to sell 15 percent protein wheat to the buyer. Because 14.5 percent is the highest available protein grade, the buyer has no choice but to offer a 14.5 percent protein grade and a $256/tonne purchase price. In this case the buyer will necessarily capture the blending rents associated with this wheat. If the farmer and buyer are in a long-term business relationship, then the farmer may agree to forfeit the blending rents to the buyer when high protein wheat is delivered, but will expect to be allocated those rents when low protein wheat is delivered. In the absence of a long-term business relationship, this system of "give-and-take" does not come into play, so negotiations between farmers and grain buyers will typically be more complex and more intense.

Questions

1 There are two quality versions of cocoa beans: low (L) and high (H). The top grade of cocoa beans (A) allows a maximum of 25 percent L in the blend. The bottom grade of cocoa beans (B) allows a maximum of 100 percent L in the blend. The price of grade A beans is $3,250/tonne and the price of grade B beans is $2,900/tonne. There are 200 tonnes of L quality beans and 500 tonnes of H quality beans available in the market. Assume that blending is performed by competitive traders and that the blending process is costless.

- a Use the zero arbitrage profits (LOP) approach to calculate the amount of quality L beans that will be blended with quality H beans. Then calculate the equilibrium prices of the L and H quality cocoa beans.
- b Demonstrate that the quantity solution to part (a) maximizes the aggregate value of the cocoa beans. Calculate the shadow value of the quality H beans by measuring the increase in the aggregate value of the beans if one more unit of H quality beans was available for blending. Does the calculated shadow price of the quality H bean correspond to the equilibrium price that you calculated in part (a)?
- c Identify the blending rents within the equilibrium prices. Explain the relationship between the equilibrium price of H quality cocoa beans and the market price of grade B beans.

2 In a stock of 500 tonnes of wheat, protein is uniformly distributed between 11 and 15 percent. High protein wheat has a protein content between 14 and 15 percent, and low protein wheat has a protein content between 11 and 14 percent. The selling price of high protein wheat is \$450/tonne and the selling price of low protein wheat is \$420/tonne. What is the market value of the wheat stock with and without blending? Assume that blending decisions are made by competitive traders and that the blending process is costless?

3 Inverse market demand is given by $P = 150 - 2Q$ for fresh potatoes and $P = 100 - Q$ for processing potatoes, where Q is quantity measured in tonnes and P is price measured in dollars per tonne. The supply side of the market consists of 100 tonnes of potatoes with individual potato weight uniformly distributed between 200 and 400 grams. A producer association that is a single desk seller of the potatoes sorts the potatoes into two piles: those weighing less than or equal to m grams and those weighing more than m grams. The low weight potatoes are sold for processing and the high weight potatoes are sold into the fresh market. When answering the following questions assume that potato prices are set competitively and sorting potatoes is costless.

  a What value will the producer association set for m if they wish to maximize the aggregate market value of the stock of potatoes?
  b With the optimal value for m, what is the pair of equilibrium prices for the fresh and processing potatoes?

4 Use the Excel programming procedures described in Chapter 5 to resolve Question 1 with the added assumption that a third, A^{minus}, grade exists. The A^{minus} grade allows a maximum of 40 percent quality L cocoa beans. The selling price of the A^{minus} grade is \$3190/tonne. Be sure to incorporate the two grade restrictions $Q_L/(Q_L + Q_H) \leq 0.25$ and $Q_L/(Q_L + Q_H) \leq 0.4$ in a linear format and check “Assume Linear Model” as a solution option. Shadow prices can be generated as part of Solver’s sensitivity report.

  a In a competitive equilibrium, how are the stocks of the L and H quality cocoa beans allocated to each of the three grades?
  b What is the equilibrium price of the L and H quality beans (i.e., what are the shadow prices)?
  c By how much can the stock of H quality beans increase before H quality beans are in surplus and blending rents therefore vanish from the price of the H quality beans?
  d Explain why the equilibrium price of the quality H beans decreases when the new grade category is added.

Appendix 5.1 Derivation of the Kuhn–Tucker conditions for equation (5.3)

Using equation (5.3), the Kuhn–Tucker conditions can be written as follows:

$$\frac{\partial L}{\partial Q} = P_A - P_B - \left(1 + \frac{1}{\gamma}\right)m - \lambda_1 - \lambda_2 + \lambda_3 \leq 0 \text{ and } \frac{\partial L}{\partial Q}Q = 0 \quad (5.A1a)$$

$$\lambda_1 (Q - X_L) = 0 \; \lambda_2 (Q - \gamma X_H) = 0 \quad \lambda_3 Q = 0 \quad (5.A1b)$$

$$0 \leq Q \leq X_H \qquad Q \leq \gamma X_H \qquad \lambda_1, \lambda_2 \text{ and } \lambda_3 \text{ non-negative} \quad (5.A1c)$$

The solution for this optimization problem has two cases (the "knife-edge" case of $\gamma X_H = X_L$ is omitted):

Case 1: $\gamma X_H < X_L$ (low quality commodity is in surplus)

In this case the $Q \leq \gamma X_H$ restriction implies that $Q < X_L$, which in turn implies $\lambda_1 = 0$ from the $\lambda_1 (Q - X_L) = 0$ expression in equation (5.A1b). Now there are two possibilities. Either, $Q = 0$, which implies $\lambda_2 = 0$ from the $\lambda_2 (Q - \gamma X_H) = 0$ expression in equation (5.A1b), or, $Q = \gamma X_H$, which implies $\lambda_3 = 0$ from the $\lambda_3 Q = 0$ expression in equation (5.A1b). Using equation (5.A1a), note that $\lambda_3 = -\left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right)m\right] \geq 0$ emerges when $Q = 0$ and $\lambda_1 = \lambda_2 = 0$. Similarly, $\lambda_2 = \left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right)m\right] \geq 0$ emerges when $Q = \gamma X_H$ and $\lambda_1 = \lambda_3 = 0$. In summary:

$$Q^* = \begin{cases} 0 & \text{if } P_A - P_B \leq \left(\frac{1+\gamma}{\gamma}\right)m \\ \gamma X_H & \text{if } P_A - P_B \geq \left(\frac{1+\gamma}{\gamma}\right)m \end{cases} \quad (5.A2)$$

Equation (5.A2) reveals that when the low quality commodity is in surplus, then the maximum allowable amount of the low quality commodity should be blended with the high quality commodity provided that the blending rents are non-negative.

Case 2: $\gamma X_H > X_L$ (high quality commodity is in surplus)

In this case the $Q \leq X_L$ restriction implies that $Q < \gamma X_H$, which in turn implies $\lambda_2 = 0$ from the $\lambda_2 (Q - \gamma X_L) = 0$ expression in equation (5.A1b). As before there are two possibilities: either $Q = 0$, which implies $\lambda_1 = 0$ from the $\lambda_1 (Q - X_L) = 0$ expression in equation (5.A1b), or $Q = X_L$, which implies $\lambda_3 = 0$ from the $\lambda_3 Q = 0$ expression in equation (5.A1b). Using equation (5.A1a) it follows that

$\lambda_3 = -\left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right)m\right] \geq 0$ emerges when $Q = 0$ and $\lambda_1 = \lambda_2 = 0$. Similarly, $\lambda_1 = \left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right)m\right] \geq 0$ emerges when $Q = X_L$ and $\lambda_2 = \lambda_3 = 0$. In summary:

$$Q^* = \begin{cases} 0 & \text{if } P_A - P_B \leq \left(\frac{1+\gamma}{\gamma}\right)m \\ X_L & \text{if } P_A - P_B \geq \left(\frac{1+\gamma}{\gamma}\right)m \end{cases} \quad (5.A3)$$

Equation (5.A3) reveals that blending all available low quality commodity with X_L/γ units of high quality commodity is optimal if the blending rents are non-negative.

6 Prices linkages across commodity markets

6.1 Introduction

Chapters 2 through 5 of this textbook have focused on the price of a single commodity with different space, time and quality attributes. This chapter examines pricing linkages for different commodities in the same location and at the same point in time. An example of a multiple commodity price linkage involves hogs, cattle and feed grains. When news about the potential seriousness of swine flu became public in April 2009, the price of hogs decreased, the price of cattle increased and the price of feed grains decreased. These changes occurred because market traders believed that consumers would substitute beef for pork in their consumption decisions and that the overall demand for feed grains would decrease. In this particular example the price linkages are both horizontal (e.g., cattle and hogs) and vertical (e.g., feed grains and hogs). This chapter focuses on horizontal price linkages and Chapter 7 focuses on vertical price linkages.

Recall from Chapter 1 that the focus of this book is on the relationship between prices at a particular point in time and the determinants of price integration. Horizontal price integration is a measure of the extent that a price change for one commodity spills over and affects the price of another commodity. Commodity prices may be horizontally integrated because they are reacting to common demand shocks (e.g., the onset of a global recession) or common production shocks (e.g., a drought that reduces the yield of all crops in the region). Of interest in this chapter is how substitution in production and utilization of a commodity allows a supply or demand shock in market A to affect the price in market B. The market B price impact serves to reduce the price response in market A according to the strength of the cross-commodity substitution. In other words, cross-commodity substitution will smooth price shocks by distributing the price impact from one market over multiple markets.

Horizontal pricing integration due to substitution effects is expected to be particularly strong for storable commodities such as corn and wheat. Storability strengthens horizontal integration because prices will jointly respond to shocks in current supply and demand, but also to news about changes in future supply and demand through inventory adjustments. For example, the release of an Australia crop report that reveals an unexpected decrease in forecasted wheat production is likely to cause the price of European corn to immediately increase. This increase occurs because traders will anticipate a higher wheat price, and will also anticipate that the higher wheat price will induce European feedlots to feed more corn and less wheat, and will induce European farmers to shift acreage out of corn and into wheat. Traders will therefore expect a higher price of corn in the future due to the anticipated increase in demand and decrease in supply. An increase in the expected future corn price implies that more corn will be stored by European traders. The higher level of storage will reduce the current supply of corn, and this reduction will have an immediate and positive impact on the European price of corn. Substitution effects, combined with the capacity of firms to adjust inventories when relative prices change, implies that news about a future supply shock in the Australian wheat market will have an immediate impact on the European price of corn.

Figure 6.1 shows daily Chicago Board of Trade (CBOT) futures prices for corn, wheat, hogs and cattle for the period January 1995 to November 2009. Corn and wheat prices appear to be well integrated, hog and cattle prices appear to be somewhat integrated, and livestock and crop prices appear to be poorly integrated.

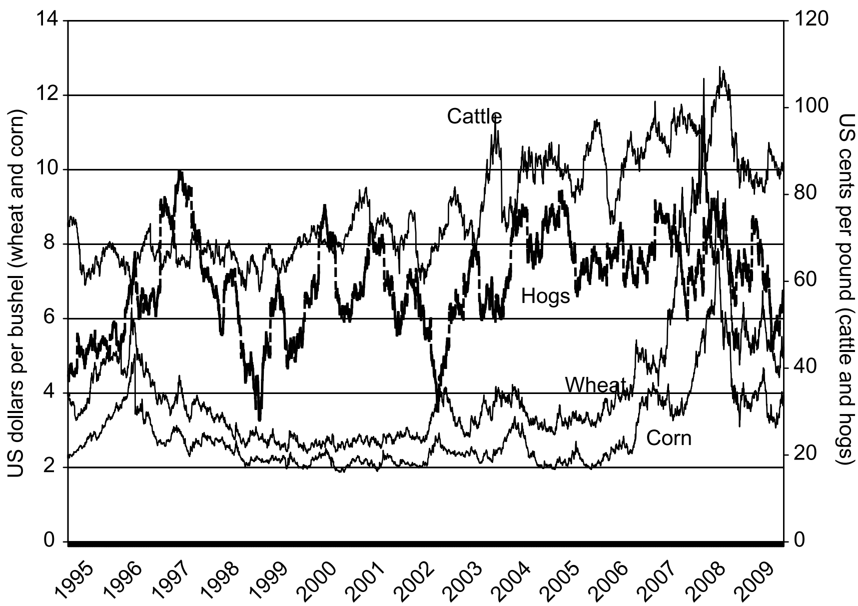


Figure 6.1 Daily CME nearest month futures prices for corn, wheat, hogs and cattle: 1995–2009.

Note: The data is a continuous rolling price of nearby CBOT contracts for corn (C2_0_10B), wheat (W2_0_10B), lean hogs (LH_0_10B) and live cattle (LC_0_10B).

A simple measure of the level of integration for a pair of prices is the correlation of the first differences of those prices.[1] The correlation results are reported in Table 6.1. Corn and wheat have the highest level of correlation (almost 50 percent) followed by the hog–cattle correlation (16 percent). The correlations for crops and livestock are quite weak (ranging between 4 and 8 percent). As was discussed above, it is useful to view the horizontal pricing integration in Figure 6.1 as the combination of substitution effects and common shock effects. However, without detailed time series analysis it is normally difficult to distinguish between these two types of effects unless the co-movement in price is particularly strong and obvious (e.g., the price crash of all commodities in early 2009 due to the global financial crisis).

The reason why the prices for cattle and hogs are less well integrated than the prices of corn and wheat can largely be explained by the limited storability of cattle and hogs, weak substitution between cattle and hogs and significant supply response lags in cattle and hog production because of breeding herd and production commitment effects. For example, a price decrease may induce cattle producers to downsize their breeding herd. This downsizing will increase current supply, which in turn will place further downward pressure on the price of cattle. However, a smaller breeding herd implies a lower future cattle supply and thus relatively high prices in the future. Slow supply response and delayed pricing feedback generally implies that hog and cattle prices cycle over time. These cycles, which were noted in Figure 1.1, are well documented in the agricultural economics literature.

Price linkages due to substitution effects are of particular importance in the “food-for-fuel” debate. The issue here is that a rising demand for ethanol and biodiesel has raised the demand for corn and soybeans. The increase in the price of corn and soybeans that is driven by growing biofuels demand has induced farmers to substitute away from other crops and toward corn and soybeans on the supply side and has induced feedlots to substitute away from corn and soybeans and toward other feed grains on the demand side. The combination of reduced supply and higher demand for crops other than corn and soybeans has placed upward pressure on prices for both non-processed and processed food products. Concerned citizens, particularly those who advocate on behalf of low income families, are

Table 6.1 Correlation table for first differences of corn, wheat, hog and cattle daily futures prices

	Corn	*Wheat*	*Hogs*	*Cattle*
Corn	1			
Wheat	0.493	1		
Hogs	0.082	0.068	1	
Cattle	0.043	0.076	0.160	1

Source: Data from Trading Blox website: http://www.tradingblox.com/tradingblox/free-historical-data.htm Downloaded on 6 November 2009.

highly critical of policies that mandate ethanol usage and inadvertently make food more expensive. Biofuels policies are particularly significant in the US where the mandated use of ethanol in gasoline blends is scheduled to equal 250 million gallons in 2011, 500 million in 2012, 1 billion in 2013 and 16 billion in 2022.[2]

The specific purpose of this chapter is to develop a simple model of commodity supply and demand for corn and "other crops" in order to examine the supply and demand substitution and price effects. The analysis focuses on long-run equilibrium pricing relationships rather than day-to-day price fluctuations. Specifically, the analysis examines how the equilibrium price of corn and "other crops" will change given a one-time permanent increase in biofuels demand for corn with all other determinants of price held constant. The assumption that all crops other than corn are aggregated together in an "other crops" category simplifies the analysis and allows for a graphical illustration of the economic forces at work. The mathematical model can easily be extended to include more than two variables.

Given the results from Chapters 2 through 5, it should not be surprising to discover that a competitive multi-market equilibrium can be derived as the solution to the problem facing a social planner who is intent on maximizing aggregate welfare across all connected markets. The first part of this chapter (Section 6.2) is devoted to demonstrating that solving the social planner's problem is equivalent to modeling the competitive market outcome where individual firms make decisions that maximize profits and market traders continually search for arbitrage opportunities. Specifically, profit maximization generates firm-level supply and demand schedules, and the set of prices that clears all markets and satisfies the market arbitrage conditions is shown to give rise to a first-best (social planner) allocation of resources. Rather than following the procedures adopted in the previous four chapters where the competitive market outcome was recovered from the solution to the social planner's problem, in this chapter the competitive market outcome is solved for directly by imposing market clearing conditions.

The second part of the analysis (Sections 6.3 and 6.4) is devoted to constructing and calibrating a multi-market model in order to illustrate the widely discussed pricing linkages in the food-for-fuel debate. In the upstream market farmers allocate land to the production of corn and a composite of all other crops, which will be referred to as the other crop composite (OCC). There are three downstream markets. First, feedlots utilize both corn and the OCC to produce livestock. Second, food processors utilize the OCC to manufacture various food products. Third, biofuels processors use corn to produce ethanol. The multi-market model is constructed using constant elasticity of substitution (CES) functions to represent the farm production technology and the livestock production technology. CES functions work well for this application because the degree of substitution between corn and the OCC can be varied by adjusting the value of a single parameter. The CES functions are calibrated using data on US crop production and utilization.

Results from the simulation are presented in Section 6.5. Of particular interest is the pricing impact of a 20 percent increase in the biofuels demand for corn. As expected, because of supply and demand substitutions, the increase in the demand for corn raises both the price of corn and the price of the OCC. This impact analysis is carried out with different assumptions about the size of the supply and demand elasticity of substitution between corn and the OCC. The main finding is that a greater degree of substitution results in a smaller price increase for both commodities. Summary comments about multi-market linkages and substitution effects are contained in Section 6.6.

6.2 Invisible hand in multi-markets

The purpose of this section is to demonstrate for the case of linked commodity markets that the competitive market outcome is identical to the social planner's outcome. Although it is straightforward to demonstrate this result with a large number of markets, attention will be restricted to the two-market case of corn and the OCC in order to keep the analysis simple. As well, the analysis will be conducted with the assumption that one representative competitive farmer is the sole supplier of corn and the OCC, and one representative competitive feedlot is the sole buyer of corn and the OCC in the feed grain market. It is not difficult to derive the main results of this chapter with multiple farmers and feedlots, but the notation is more cumbersome and little additional economic insight is gained by relaxing the representative seller/buyer assumption.

The total quantity of corn produced by the farmer is denoted C. Let C_L and C_B denote the quantity of corn that is used by the feedlot and the biofuels sector, respectively. Similarly, the total quantity of the OCC produced by the farmer is denoted X, and the quantity of the OCC that is used by the feedlot and the food processing sector, is denoted X_L and X_H, respectively. In equilibrium, $C = C_L + C_B$ and $X = X_L + X_H$.

Let $f(C, X) = K$ denote the production technology for the farmer. This function implicitly shows the combinations of C and X that can be produced using the K units of available farm capital (e.g., land and equipment). Production of both C and X is subject to diminishing marginal productivity of farm capital, which implies that the production possibility frontier (PPF) associated with K is concave to the origin (more on this below). Let $g(C_L, X_L) = Q$ define the production technology for the feedlot when it chooses to produce Q units of livestock. Feed inputs C and X are each subject to diminishing returns when used to produce livestock, which implies that each production isoquant associated with Q is convex to the origin (more on this below).

Let $M_B\ (C_B)$ denote the aggregate willingness to pay for corn by firms in the biofuels processors when C_B units of corn is allocated to this market. This function is increasing but at a decreasing rate due to an increasing marginal cost of converting corn into ethanol. The concavity of $M_B\ (C_B)$ implies that the processors' marginal willingness to pay for corn, $P_B\ (C_B) \equiv M_B'\ (C_B)$, is a downward sloping function. Let $M_H\,(X_H)$ denote the aggregate willingness to pay for the OCC by food processors when X_H units of the OCC is allocated to this market. This function is increasing but at a decreasing rate due to consumers' diminishing marginal utility of food consumption and possibly a rising marginal cost of food processing. The concavity of $M_H\ (X_H)$ implies that the processor's marginal willingness to pay for the OCC, $P_H(X_H) \equiv M_H'(X_H)$, is a downward sloping function.

Let $M_L(Q)$ denote society's valuation of the Q units of livestock that is produced by the feedlot using various combinations of corn and the OCC. The assumption of fixed livestock supply is quite restrictive because in reality one would expect the quantity of livestock produced by feedlots to change in response to changes in the price of feed grains. Assuming that Q is exogenous implies that the model can remain focused on the horizontal pricing linkage for corn and the OCC rather than also considering the vertical linkage between the price of livestock and the price of feed grains.

A second simplifying assumption is that the farmer's cost of production is fixed at level F_K and is therefore independent of the mix of crops produced by the farmer. Similarly, the feedlot's cost of production (excluding the cost of purchasing feed grains) is fixed at level F_L and is therefore independent of the mix of feed grains that is fed to the livestock. Relaxing this assumption about cost may weaken or strengthen the various substitution effects, but doing so would not change the key qualitative results of this analysis.

Social planner's problem

Consider the optimal production and allocation of corn and the OCC by a social planner whose objective is to maximize aggregate surplus across all markets. Social welfare consists in the aggregate willingness to pay for corn and the OCC by firms in the biofuels and human food processing sectors plus society's valuation of the livestock produced by the feedlot minus the farmer's cost of producing the corn and the OCC minus the feedlot's cost of producing the livestock (excluding the cost of the feed grains). If $W(C_B, X_H)$ is the measure of social welfare it then follows that:

$$W(C_B, X_H) = M_B(C_B) + M_H(X_H) + M_L(Q) - F_K - F_L \tag{6.1}$$

The social planner must choose values for the four crop allocation variables, C_L, C_B, X_L and X_H in order to maximize equation (6.1) subject to the farm level production constraint, $f(C_L + C_B, X_L + X_H) = K$, and the livestock production constraint, $g(C_L, X_L) = Q$.

To solve the social planner's problem set up the following Lagrange function:[3]

$$\begin{aligned}\ell(C_L, C_B, X_L, X_H) &= M_B(C_B) + M_H(X_H) + M_L(Q) - F_K - F_L \\ &\quad + \lambda_1 [K - f(C_L + C_B, X_L + X_H)] + \lambda_2 [Q - g(C_L, X_L)]\end{aligned} \tag{6.2}$$

Recalling that $P_B(C_B) = M_B'(C_B)$ and $P_H(X_H) = M_H'(X_H)$, it follows that the first order conditions for the social planner can be written as:

$$\frac{\partial \ell}{\partial C_L} = -\lambda_1 \frac{\partial f}{\partial C} - \lambda_2 \frac{\partial g}{\partial C_L} = 0 \tag{6.3a}$$

$$\frac{\partial \ell}{\partial C_B} = P_B\left(C_B\right) - \lambda_1 \frac{\partial f}{\partial C} = 0 \tag{6.3b}$$

$$\frac{\partial \ell}{\partial X_L} = -\lambda_1 \frac{\partial f}{\partial X} - \lambda_2 \frac{\partial g}{\partial X_L} = 0 \tag{6.3c}$$

$$\frac{\partial \ell}{\partial X_H} = P_H\left(X_H\right) - \lambda_1 \frac{\partial f}{\partial X_H} = 0 \tag{6.3d}$$

$$f\left(C_L + C_B, X_L + X_H\right) = K \tag{6.3e}$$

$$g\left(C_L, X_L\right) = Q \tag{6.3f}$$

After eliminating the λ_1 and λ_2 variables, equations (6.3a) through (6.3d) can be rewritten as:[4]

$$\frac{P_B\left(C_B\right)}{P_H\left(X_H\right)} = \frac{\partial f\left(C,X\right)}{\partial C} \Bigg/ \frac{\partial f\left(C,X\right)}{\partial X} \tag{6.4a}$$

$$\frac{P_B\left(C_B\right)}{P_H\left(X_H\right)} = \frac{\partial g\left(C_L,X_L\right)}{\partial C_L} \Bigg/ \frac{\partial g\left(C_L,X_L\right)}{\partial X_L} \tag{6.4b}$$

The solution to the social planner's resource allocation problem, which is denoted, C_L^*, C_B^*, X_L^* and X_H^*, is implied by the joint solution to equations (6.3e), (6.3f), (6.4a) and (6.4b). Simulation results presented below demonstrate that all four solution variables take on positive values for a wide range of parameter values. It is therefore reasonable to assume away any corner solutions, as was discussed above.

The solution to the social planner's problem is illustrated in Figure 6.2. The concave schedule labeled $f(C_L + C_B, X_L + X_H) = K$ is a representative PPF for the farmer. The absolute slope of the PPF, which is referred to as the marginal rate of transformation, is given by the right-hand side of equation (6.4a). The convex schedule labeled $g(C_L, X_L) = Q$ is a representative production isoquant for the livestock producer. The absolute slope of the isoquant, which is referred to as the marginal rate of technical substitution, is given by the right-hand side of equation (6.4b). The convex schedule labeled $W(C_B, X_H) = W_0$ is a social welfare indifference curve corresponding to W_0 units of social welfare. Note that this indifference curve is defined with respect to the offset horizontal axis labeled C_B and the offset vertical axis labeled X_H rather than the standard horizontal and vertical axes that intersect at the origin. The absolute slope of the social welfare indifference curve,

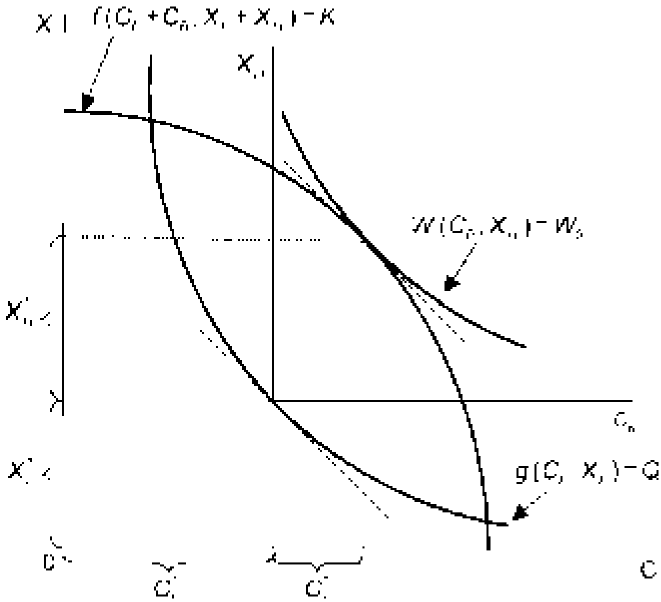


Figure 6.2 Graphical solution to the social planner's problem.

which is referred to as the marginal rate of substitution, is given by the left hand side of equations (6.4a) and (6.4b).[5]

Maximizing social welfare in Figure 6.2 implies pushing the social welfare indifference curve up and to the right as far as possible without violating the $f(C_L + C_B, X_L + X_H) = K$ and $g(C_L, X_L) = Q$ technology constraints. Figure 6.2 shows that a constrained maximum is achieved when the slopes of the three level sets are equal at the equilibrium quantities (i.e., the marginal rate of transformation is equal to the marginal rate of technical substitution, which in turn is equal to the marginal rate of substitution). Equal slopes imply that the rate at which the OCC can be transformed to corn at the farm level is equal to the rate at which corn can be substituted for the OCC at the feedlot level, which in turn is equal to the rate at which the OCC can be substituted for corn while maintaining a constant level of social welfare. The result that equal slopes are required to maximize social welfare is of course consistent with the mathematical conditions for maximizing social welfare, as given by equation (6.4).

Notice from Figure 6.2 that the degree of curvature of the $W(C_B, X_H)$, $g(C_L, X_L)$ and $f(C, X)$ level sets define the extent that corn and the OCC are substitutes in production and usage. Indeed, highly curved PPFs, isoquants and welfare indifference curves imply a relatively low degree of substitutability; the opposite is true for level sets with a small to moderate degree of curvature. The link between the degree of substitutability between corn and the OCC and the extent that the price of these two commodities are linked, is a central theme of this chapter. This theme is examined in the context of numerical simulations, which are presented below.

Competitive market outcome

The purpose of this section is to derive the multi-market equilibrium outcome with the assumption that profit maximizing firms and traders rather than a social planner are making the resource allocation decisions. To obtain the competitive market outcome it is necessary to derive market supply and demand schedules and then solve for the price of corn and the price of the OCC that results in equal market supply and aggregate market demand for these two commodities.

The supply schedules for corn and the OCC are derived from the farmer's profit maximization problem. Let P_C and P_X denote the price of corn and the OCC, respectively. Treating these prices as given, the farmer chooses C and X to maximize profits, $P_C C + P_X X - F_K$ subject to $f(C, X) = K$. Using the standard Lagrange constrained optimization procedure, the solution to the farmer's problem is the pair of values for C and X that simultaneously solve $P_C / P_X = \frac{\partial f/\partial C}{\partial f/\partial X}$ and $f(C, X) = K$. In other words, the farmer should allocate farm capital such that the marginal rate of transformation of the OCC into corn is equal to the opportunity cost of corn relative to the OCC (i.e., the ratio of the selling prices). The optimized values of C and X, which are denoted $C^S (P_C, P_X)$ and $X^S (P_C, P_X)$, can be interpreted as the market supply schedules.[6] It is straightforward to show that these two schedules have the standard supply schedule properties (i.e., an increasing function of the commodity's own price and a decreasing function of the price of the other commodity).

The derived demand for feed grains by the competitive feedlot, which takes feed grain prices as given, is obtained by minimizing the cost of production, $P_C C_L + P_X X_L + F_L$, subject to $g(C_L, X_L) = Q$. Using the standard Lagrangian procedure, the solution to the feedlot's problem is the pair of values for C_L and X_L that simultaneously solve $P_C / P_X = \frac{\partial g/\partial C_L}{\partial g/\partial X_L}$ and $g(C_L, X_L) = Q$. The solution implies that the feedlot should adjust feed grain inputs until the marginal rate of technical substitution of corn for the OCC is equal to the opportunity cost of using corn rather than the OCC (i.e., the ratio of input prices). The optimized values for C and X, which are denoted $C_L^D (P_C, P_X)$ and $X_L^D(P_C, P_X)$, can be interpreted as the derived market demand schedules for these two feed grains. Similar to the case of supply, it is straightforward to show that these demand schedules have the standard properties (i.e., a decreasing function of the commodity's own price and an increasing function of price of the other commodity).

A processor in the biofuels sector chooses C_B to maximize its surplus, which is equal to its willingness to pay for C_B units of corn, $M_B (C_B)$, minus the cost of purchasing this volume of corn, $P_C C_B$. Treating P_C as fixed, the first order condition for maximizing $M_B (C_B) - P_C C_B$ can be expressed as $M_B'(C_B) = P_C$, which is equivalent to $P_B (C_B) = P_C$. The inverse of this function, $C_B^{-1} (P_C)$, implicitly defines the market demand for C_B by the biofuels processor. Similarly, a food processor chooses X_H to maximize its surplus, which is equal to its willingness to pay for X_H units of the OCC, $M_H (X_H)$, minus the cost of purchasing this volume of the OCC, $P_X X_H$. Treating P_X as fixed, the first order condition for maximizing $M_H (X_H) - P_X X_H$ can be expressed as $M_H' (X_H) = P_X$, which is equivalent to $P_H (X_H) = P_X$. The inverse of this function, $X_H^{-1} (P_X)$, implicitly defines the market demand for X_H by the food processor.

The competitive equilibrium for this multi-market model is illustrated in Figure 6.3. In Figure 6.3a, the market demand schedules for corn used by the livestock sector and the biofuels sector are horizontally summed to give an aggregate demand for corn, which is labeled $C_B^{-1}(P_C) + C_L^D(P_C, P_X)$. The intersection of this aggregate demand schedule with the market supply of corn, $C^S(P_C, P_X)$, defines the equilibrium price of corn, P_C^*. With P_C^* established, the amount of corn allocated to the biofuels sector, C_B^*, and the feedlot, C_L^*, can be read directly off the individual demand schedules. A similar process is used to identify the equilibrium price and quantities of the OCC in the bottom panel of Figure 6.3b.

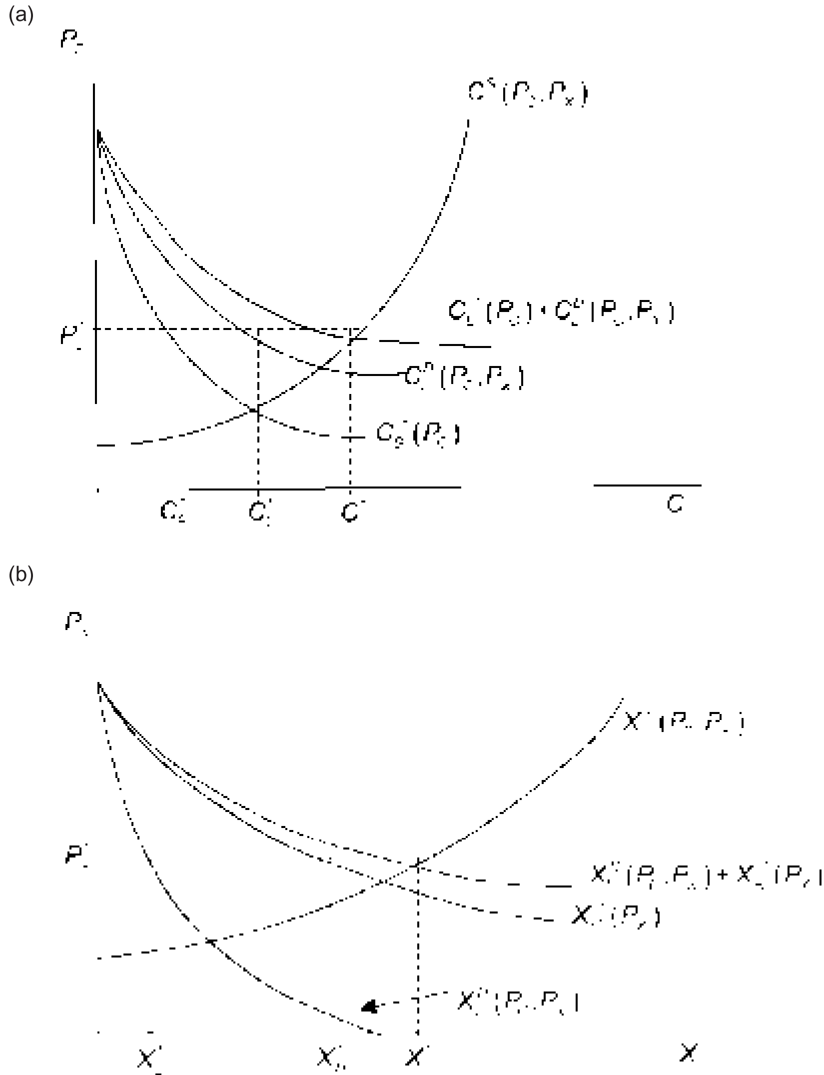


Figure 6.3 Model equilibrium in (a) the corn market and (b) the OCC market.

Two comments about Figure 6.3 are in order. First, profit-seeking traders, who are continually searching for arbitrage opportunities, ensure that markets clear in the manner illustrated in Figure 6.3. Market clearing may also be facilitated by centralized spot and futures markets, as discussed in Chapter 4. Second, it is generally not possible to use Figure 6.3 to do standard comparative static analysis such as examining the impact of an outward shift in the biofuels demand for corn on the price of corn. The reason is that an outward shift in biofuels demand for corn in Figure 6.3a will raise the price of corn, which in turn will raise the price of the OCC in Figure 6.3b due to shifts in supply and demand in the OCC market. The higher price of the OCC will cause an upward shift of the corn supply schedule in the Figure 6.3a. This shift in corn supply will have an additional positive impact on the price of the OCC, which in turn will result in additional feedback effects in both markets. In general, feedback effects of the type described above imply that standard graphical analysis in linked commodity markets is not possible.

The final step in the analysis of this section is to prove that the competitive market equilibrium outcome that is illustrated in Figure 6.3 is the same as the social planner's outcome that is illustrated in Figure 6.2. This can be accomplished by substituting the competitive market first order conditions for the biofuels and food processing sectors, $P_B\,(C_B) = P_C$ and $P_H\,(X_H) = P_X$, into the competitive market first order conditions of the farming sector and the livestock sector, $P_C / P_X = \frac{\partial f/\partial C}{\partial f/\partial X}$ and $P_C / P_X = \frac{\partial g/\partial C_L}{\partial g/\partial X_L}$. The resulting set of expressions are identical to the first order conditions for the social planner, as given by equation (6.4). Consequently, the solution to the competitive outcome where profit maximizing agents make production and resource allocation decisions while treating all prices as fixed is the same as the solution to the social planner's problem, in which case there are no explicit market prices. This outcome once again establishes the "invisible hand" result in agricultural commodity markets.

6.3 Simulation model

As discussed above, the objective of this chapter is to illustrate why commodity prices are linked because of substitution in commodity supply and demand. Of specific interest is the impact of an increase in the demand for biofuels on the price of the OCC that is destined for processing into food. Needless to say this question is central to the on-going food-for-fuel debate. To examine commodity market price linkages a simulation model is constructed in Excel and a set of simulation results are generated and analyzed. The simulation model is constructed in this section, and the simulation results are analyzed in the following section.

Before proceeding with the simulation analysis it is necessary to decide how the equilibrium is to be solved. As discussed in the previous section, the equilibrium can be obtained either by maximizing aggregate surplus across the various markets (i.e., solve the social planner's problem) or by using market supply and demand with specific functional forms to solve directly for competitive market prices. In a model with more than two commodities and with the possibility of corner solutions, the social planner approach will normally dominate. In the current analysis with only two commodities and no corner solutions, the competitive market approach is utilized in order to illustrate this particular solution technique.

It is also worth mentioning that there is a large literature that uses equilibrium displacement modeling rather than the supply and demand functional form approach when analyzing agricultural policies such as subsidies, quotas and tariffs in a multi-market setting (the annotated bibliography for this chapter discusses this literature). In an equilibrium displacement model general functions are used to specify commodity supply and demand, and various marketing clearing conditions are imposed to ensure that the system of equations implies a unique equilibrium for the endogenous price and quantity variables. The system is then totally differentiated with respect to the endogenous price and quantity variables and the policy parameter of interest. Finally, the differentiated system of equations is converted to elasticities and the results of interest (e.g., impact of a subsidy on domestic price) are isolated. The typical goal of the exercise is to determine whether the policy has a positive or negative impact, and to identify the key economic determinants of the policy impact. Results generated in an equilibrium displacement model are expected to be similar to the results presented in this chapter because in both cases the various supply and demand elasticities are assumed to be constant when prices and quantities adjust.

Farm supply and feedlot demand schedules

A generic CES function is used to derive both the farm level supply equations and the feedlot demand equations for corn and the OCC.[7] The generic CES function with constant returns to scale can be expressed as:

$$Z = H\left(a_C C^b + a_X X^b\right)^{1/b} \tag{6.5}$$

If equation (6.5) is used to derive supply functions for the farmer, then the parameter Z represents the stock of farm capital, and the variables C and X are the respective levels of farm production of corn and the OCC. Alternatively, if equation (6.5) is used to derive commodity demand schedules for the feedlot, then the parameter Z is the feedlot's level of livestock output, and the variables C and X are the respective levels of corn and the OCC feed inputs. In both scenarios H is a scale parameter, a_C and a_X are share parameters and b is a substitution parameter (more details below).

The elasticity of substitution between C and X in the generic CES function is derived by totally differentiating equation (6.5) with respect to C and X to obtain:

$$ba_C C^{b-1}\, dC + ba_X X^{b-1}\, dX = 0 \tag{6.6}$$

Equation (6.6) can be solved to give:

$$Slope \equiv \frac{dX}{dC} = -\frac{a_C}{a_X}\left(\frac{C}{X}\right)^{b-1} \tag{6.7}$$

For the case of supply, equation (6.7) measures the marginal rate of transformation, which is the slope of the PPF labeled $f(C_L + C_B, X_L + X_H) = K$ in Figure 6.2. For the case of demand, equation (6.7) measures the marginal rate of technical substitution, which is the slope of the isoquant labeled $g(C_L, X_L) = Q$ in Figure 6.2.

The elasticity of substitution of C for X, which is denoted ρ, is defined as the percent change in the C/X ratio given a one percent increase in dX/dC. The more curved the PPF or the isoquant, the smaller the absolute value of ρ because higher curvature implies that dX/dC will change more rapidly as the C/X ratio increases. To derive an expression for ρ invert equation (6.7) to obtain:

$$\frac{C}{X} = \left(\frac{a_X}{a_C}(-Slope)\right)^{\frac{1}{b-1}} \tag{6.8}$$

Using equations (6.7) and (6.8) it is straightforward to show that:

$$\rho = \frac{\partial C/X}{\partial Slope}\frac{Slope}{C/X} = \frac{1}{1-b} \tag{6.9}$$

Equation (6.9) shows that the elasticity of substitution for the CES function takes on a constant value, $1/1-b$.

The next step in the analysis is to derive the farm supply equations and feedlot demand equations for corn and the OCC using the generic CES function. As discussed in the previous section, an equilibrium requires that the slope of the farmer's PPF and the slope of the feedlot's isoquant must both be equal to the price ratio, P_C/P_X. Using equation (6.7), this equal slope condition can be expressed as

$$p_k q_k = \frac{a_k}{a_i}\left(\frac{p_k}{p_i}\right)^{\frac{1}{\rho-1}} p_k q_i \tag{6.10}$$

After some rearrangement, equation (6.10) can be written as $C = \left(\frac{a_X}{a_C}\frac{P_C}{P_X}\right)^{\frac{1}{b-1}} X$

and solved with the original CES function, $Z = K\left(a_C C^b + a_X X^b\right)^{1/b}$, to give:

$$C(P_C, P_X) = \left(\frac{P_C}{a_C}\right)^{\frac{1}{b-1}}\left[a_C\left(\frac{P_C}{a_C}\right)^{\frac{b}{b-1}} + a_X\left(\frac{P_X}{a_X}\right)^{\frac{b}{b-1}}\right]^{-\frac{1}{b}}\frac{Z}{K} \tag{6.11a}$$

and

$$X(P_C, P_X) = \left(\frac{P_X}{a_X}\right)^{\frac{1}{b-1}}\left[a_C\left(\frac{P_C}{a_C}\right)^{\frac{b}{b-1}} + a_X\left(\frac{P_X}{a_X}\right)^{\frac{b}{b-1}}\right]^{-\frac{1}{b}}\frac{Z}{K} \tag{6.11b}$$

Equation (6.11) represents a pair of farm level supply schedules for corn and the OCC if $b \geq 1$. This result emerges because $b \geq 1$ ensures that the level set of equation (6.5) is concave to the origin and thus has the properties of a PPF. Conversely, equation (6.11) represents a pair of feedlot demand schedules for corn and the OCC when $b \leq 1$. This result emerges because $b \leq 1$ ensures that the level set of equation (6.5) is convex to the origin and thus has the properties of a production isoquant.[8]

It is now possible to be specific about how the value of the elasticity of transformation/substitution parameter, ρ, determines the shape of the PPF and isoquant. Recall that $\rho = 1/_{1-b}$, which is equivalent to $b = \rho^{-1}/_{\rho}$. Notice that a negative value for ρ implies $b > 1$, which in turn implies that the CES function can be used to represent a PPF. Conversely, a positive value for ρ implies $b < 1$, which implies that the CES function can be used to represent an isoquant. In the limit $\rho \to -\infty$ implies $b \to 1$ from above, in which case the PPF converges to the perfect substitutes case on the supply side (i.e., a straight line). Similarly, $\rho \to \infty$ implies $b \to 1$ from below, in which case the isoquant converges to the perfect substitutes case on the demand side. Letting $\rho \to 0$ from below implies $b \to \infty$, which is equivalent to assuming that the fixed proportions PPF is an inverted "L" shape. Conversely, letting $\rho \to 0$ from above implies $b \to -\infty$, which is equivalent to assuming that the fixed proportions isoquant is "L" shaped. Finally $\rho \to 1$ implies $b \to 0$, in which case the isoquant reverts to the well-known Cobb–Douglas case.

Food and biofuels demand

Recall that $P_B(C_B) = M'_B(C_B)$ and $P_H(X_H) = M'_H(X_H)$ is a measure of the marginal willingness to pay for corn and the OCC by the biofuels sector and the food processing sector, respectively. These marginal functions are standard inverse demand schedules, each containing a single price-quantity combination. Assume the following constant elasticity functions for these inverse demand relationships:

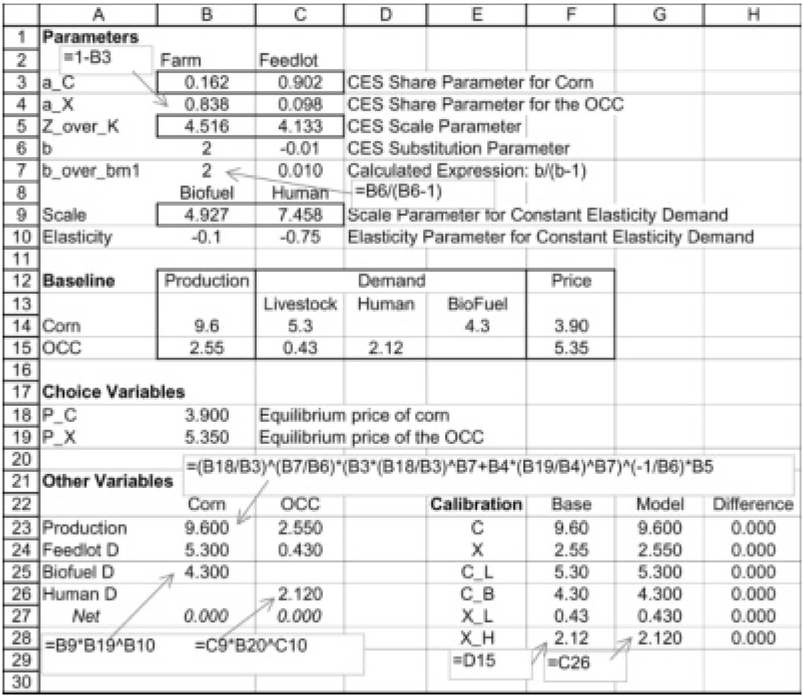
$$P_C = h_C C_B^{-\eta_C} \quad \text{and} \quad P_X = h_X X_H^{-\eta_H} \qquad (6.12)$$

6.4 Model calibration

The model described in the previous section can now be entered into an Excel spreadsheet with the parameter values chosen to represent a "realistic" market scenario. The first step in model calibration requires specifying baseline values for the production of corn and the OCC, feedlot usage of corn and the OCC, and the volume of corn and the OCC used for producing biofuels and food, respectively. Baseline prices for corn and the OCC are also required to calibrate the model.

Baseline data

The baseline data values, which are entered in cells B14:F15 in Figure 6.4, were taken from a February 2009 United States Department of Agriculture (USDA)



	A	B	C	D	E	F	G	H
1	**Parameters**							
2	=1-B3	Farm	Feedlot					
3	a_C	0.162	0.902	CES Share Parameter for Corn				
4	a_X	0.838	0.098	CES Share Parameter for the OCC				
5	Z_over_K	4.516	4.133	CES Scale Parameter				
6	b	2	-0.01	CES Substitution Parameter				
7	b_over_bm1	2	0.010	Calculated Expression: b/(b-1)				
8		Biofuel	Human	=B6/(B6-1)				
9	Scale	4.927	7.458	Scale Parameter for Constant Elasticity Demand				
10	Elasticity	-0.1	-0.75	Elasticity Parameter for Constant Elasticity Demand				
11								
12	**Baseline**	Production	Demand			Price		
13			Livestock	Human	BioFuel			
14	Corn	9.6	5.3		4.3	3.90		
15	OCC	2.55	0.43	2.12		5.35		
16								
17	**Choice Variables**							
18	P_C	3.900	Equilibrium price of corn					
19	P_X	5.350	Equilibrium price of the OCC					
20		=(B18/B3)^(B7/B6)*(B3*(B18/B3)^B7+B4*(B19/B4)^B7)^(-1/B6)*B5						
21	**Other Variables**							
22		Corn	OCC		**Calibration**	Base	Model	Difference
23	Production	9.600	2.550		C	9.60	9.600	0.000
24	Feedlot D	5.300	0.430		X	2.55	2.550	0.000
25	Biofuel D	4.300			C_L	5.30	5.300	0.000
26	Human D		2.120		C_B	4.30	4.300	0.000
27	*Net*	*0.000*	*0.000*		X_L	0.43	0.430	0.000
28	=B9*B19^B10		=C9*B20^C10		X_H	2.12	2.120	0.000
29					=D15	=C26		
30								

Figure 6.4 Model calibration.

report.[9] The specific values displayed in Figure 6.4 are 2010/11 data projections for the US production and utilization of corn and an aggregate of three other cereal grains: barley, oats and wheat. Because barley, oats and wheat can be used as a livestock feed or processed into food, these three cereal grains are assumed to collectively represent the OCC.

According to Table 8 of the USDA report, US corn production is forecast to equal 13 billion bushels in 2010/11. Of this amount, 5.3 billion bushels are expected to be utilized for livestock feed and 4.3 billion bushels are expected to be used to produce ethanol. The residual amount, 13 − 5.3 − 4.3 = 3.4 billion bushels, is destined for export and for use in minor domestic markets. To simplify the analysis, the 3.4 billion bushel residual is excluded from total corn production because there is no "other" category for corn in the existing model. Thus, production is set equal to 13 − 3.4 = 9.6 billion bushels. The projected price of corn for 2010/11 is \$3.90/bushel. These baseline corn values have been entered in cells B14:F14 of Figure 6.4. The production values have been repeated in cells F23:F28 for the purpose of model calibration (more details below).

Tables 9 through 13 of the USDA report indicate that production of the three cereal grains which comprise the OCC (barley, oats and wheat) are forecast to equal 2.55 billion bushels in 2010/11. This production is allocated to livestock feed (0.43 billion) and other uses (2.12 billion bushels), which for the purpose of this study is assumed to be food processing. The USDA projected weighted average price for the OCC is \$5.35/bushel for the year 2010/11. These baseline OCC values have been entered in cells B15:F15 of Figure 6.4. The production values are repeated in cells F23:F28 for the purpose of model calibration.

Elasticity of substitution parameters

The next step in the calibration process is to assume values for the four elasticity parameters. These four parameters include the elasticity of transformation for corn and the OCC in farm level production, the elasticity of substitution for corn and the OCC when utilized by the feedlot, the elasticity of demand for corn by the biofuels sector and the elasticity of demand for the OCC by the food processing sector. It is beyond the scope of this analysis to estimate values for these four elasticity parameters. Instead, "intermediate" values will be assigned and these four parameter values will be varied as part of the sensitivity analysis.

Equation (6.9) shows that the elasticity of substitution for a CES function is proportional to the inverse of $1 - b$. For initial model calibration assume that $b = 2$ for the farmer and $b = 0$ for the feedlot, which is equivalent to assuming an elasticity of transformation equal to −1 for the farmer and +1 for the feedlot. An elasticity of substitution or transformation equal to −1 or +1 implies the well-known Cobb–Douglas case. With Cobb–Douglas technology the absolute slope of the PPF or isoquant at a particular point is proportional to the slope of a ray through that point and through the origin of the graph. These base case values of b have been entered in cells B6:C6 of Figure 6.4. Note that the value of b for the feedlot has been set equal to −0.01 rather than 0 in order to avoid a "divide-by-zero" error. As well, to simplify the formulas in the spreadsheet, values for the $^{b}/_{b-1}$ expression have been entered in cells B7:C7.

The elasticity of demand for corn by the biofuels sector is unknown, but it is likely to be highly inelastic. Demand is thought to be inelastic because there are no cost efficient substitutes for corn, and ethanol use is mandated by US law. For this reason, a biofuels demand elasticity of −0.1 provides a reasonable starting point (see cell B10 of Figure 6.4). The elasticity of demand for the OCC by the food processing sector is assigned an intermediate value of −0.75 in cell C10.

The analysis below follows a two-stage procedure. In the first calibration stage "guess" values are entered for all of the parameters of the model except for the four elasticity values that have been specified above. Solver is then used to adjust these parameter guess values until the simulated equilibrium quantities and prices of corn and the OCC are identical to the baseline data values. The parameter values returned by Solver for this calibration stage are referred to as base case parameter values. With the base case parameter values in place, stage two of the analysis can be undertaken using standard sensitivity analysis procedures.

Specifically, one or more parameters of the model are changed, the new equilibrium of the model is obtaining using Solver, and the resulting equilibrium values are compared with the baseline values to determine how the parameter change has impacted on the equilibrium outcome.

Model construction

The model has 14 parameters, but values for the four elasticities have previously been specified. Out of the remaining ten parameters, four can be eliminated by normalizing the model. The first normalization is that equation (6.11) can be specified using a value for the Z/K ratio rather than individual values for Z and K. In cells B5:C5 of Figure 6.4 the variable "Z_over_K" represents this Z/K ratio. The second normalization is that equation (6.11) can be specified with the restriction that $a_C + a_X = 1$. To incorporate this restriction for the farm the formula "= 1 − B3" has been entered in cell B4. The restriction for the feedlot is analogous. The six non-specified parameters that require initial guess values reside in cells B3:C3, B5:C5 and B9:C9.

The main pricing model is constructed in cells B18:H28 of Figure 6.4. The initial guess values for the price of corn and the OCC reside in cells B18:B19.[10] The corn and OCC supply equations for the farmer, which are given by equation (6.11) with farm-specific parameters, have been entered into cells B23:C23. Similarly, the corn and OCC demand equations for the feedlot, which are given equation (6.11) with feedlot specific parameters, have been entered in cells B24:C24. The constant elasticity of demand equations for the biofuels and food processing sectors, which are described in equation (6.12), have been entered into cells B25 and C26, respectively. All of the equations in cells B23:C26 are linked to the pair of prices that reside in cells B18:B19. As well, the quantities that reside in cells B23:C26 have been repeated in cells G23:G28 for the purpose of model calibration (more details below).

The net supply of corn, which is equal to farm production minus feedlot demand minus biofuels demand, has been entered in cell B27. Similarly, the net supply of the OCC, which is equal to farm production minus feedlot demand minus food processing demand, is entered in cell C27. Net supply must take on a value of zero when the pair of prices in cells B18:B19 take on equilibrium values. Hence, instructing Solver to choose values for the set of prices in cells B18:B19 subject to cells B27:C27 taking on a value of zero will generate the competitive market equilibrium outcome.

Model calibration

Now that the equations for market equilibrium have been entered into the simulation model it is possible to obtain base case parameters values of the model by replacing the initial "guess" values that reside in cells B3:C3, B5:C5 and B9:C9 with values that are consistent with the baseline data. The calibration begins by replacing the price variable guess values in cells B18:B19 with the price parameter values from cells F14:F15. The next step is to instruct Solver to select values for the six unknown parameters that reside in cells B3:C3, B5:C5 and B9:C9 such that the equilibrium values for C, X, C_L, C_B, X_L and X_H are the same as the corresponding baseline values. The differences between the equilibrium values and the baseline values for these six variables are displayed in cells H23:H28 of Figure 6.4. Hence, the Solver constraint is that the values in cells H23:H28 must take on a value of zero. This calibration procedure is expected to yield a unique outcome because Solver is solving for six unknown parameter values in a system with six equations. Figure 6.4 shows the post-calibration outcome with the optimized set of parameter values in cells B3:C3, B5:C5 and B9:C9.

6.5 Simulation results

The base case results that are displayed in Figure 6.4 are not interesting to analyze because the model was constructed to ensure that the base case equilibrium values are identical to the baseline values that were previously entered in cells B14:F15. The values in Figure 6.4 will therefore serve as a benchmark for the sensitivity analysis.

The main issue of interest is the impact of an increase in the biofuels demand for corn on the prices of corn and the OCC when fully accounting for supply and demand substitutions. Suppose that with the baseline set of prices, the demand for biofuels increases by 20 percent. This situation can be simulated by increasing the value of the scale parameter in cell B9 by 20 percent (from 4.927 to 5.912) and then resolving the model. Solving the model requires instructing Solver to choose values for the pair of prices in cells B18:B19 such that the net supply values in cells B27:C27 take on a value of zero.

Figure 6.5 shows the equilibrium outcome for the model with a 20 percent higher biofuels demand for corn (i.e., with 5.912 rather than 4.927 inserted into cell B9). All changes in equilibrium prices and quantities relative to base case values can be attributed to this 20 percent increase in the biofuels demand for corn because all parameters other than the biofuels scale parameter have base case values. A comparison of cells B18:B19 in Figures 6.4 and 6.5 reveals that the increase in demand for corn by the biofuels processor has increased the price of corn from \$3.90/bushel to \$6.75/bushel, and has also increased the price of the OCC from \$5.35/bushel to \$7.49/bushel. Thus, even though the OCC is not used in the biofuels market, the increase in the biofuels demand for corn has caused the price of the OCC to increase by a significant amount. This cross-market price linkage is at the heart of the food-for-fuel debate because in a competitive market the higher price of the OCC will largely be passed on to consumers by the food processor.

The top two rows in Table 6.2 highlight the impact of the 20 percent increase in biofuels demand for corn on all relevant variables. This comparison is intended to make the substitutions at the farm level and feedlot level more explicit. At the farm level, the increase in the price of corn that results from additional corn demand induces the farmer to substitute away from the OCC and toward corn (see

	A	B	C	D	E	F
1	**Parameters**					
2		Farm	Feedlot			
3	a_C	0.162	0.902			
4	a_X	0.838	0.098			
5	Z_over_K	4.516	4.133			
6	b	2	-0.01	This value was changed from 4.927 to 5.912 to reflect the 20 percent increase in biofuel demand		
7	b_over_bm1	2	0.010			
8		Biofuel	Human			
9	Scale	5.912	7.458			
10	Elasticity	-0.1	-0.75			
11						
12	**Baseline**	Production		Demand		Price
13			Livestock	Human	BioFuel	
14	Corn	9.6	5.3		4.3	3.90
15	OCC	2.55	0.43	2.12		5.35
16						
17	**Choice Variables**					
18	P_C	6.744	Equilibrium price of corn			
19	P_X	7.489	Equilibrium price of the OCC			
20						
21	**Other Variables**					
22		Corn	OCC			
23	Production	10.075	2.166			
24	Feedlot D	5.190	0.519			
25	Biofuel D	4.885				
26	Human D		1.647			
27	*Net*	*0.000*	*0.000*			

Figure 6.5 Impact of 20 percent increase in biofuel demand on prices for corn and the OCC.

third and fourth columns). This substitution results in an increase in the production of corn from 9.60 to 10.08 billion bushels and a decrease in production of the OCC from 2.55 to 2.17 billion bushels. This change in supply will dampen the rise in the price of corn created by the higher biofuels demand, and it will also cause the price of the OCC to increase. The greater the degree of substitution of corn for the OCC the greater the magnitude of these offsetting price effects.

Now consider the feedlot. The values in the top pair of rows and the fifth and sixth columns in Table 6.2 show that the relatively high price of corn induces the feedlot to substitute OCC for corn when feeding the Q units of livestock. This substitution results in the demand for corn falling from 5.30 to 5.19 billion bushels and the demand for the OCC rising from 0.430 to 0.519 billion bushels. Similar to the case of substitution in supply, the decrease in the demand for corn by the feedlot dampens the rise in the price of corn and the increase in the demand for the OCC by the feedlot causes the price of the OCC to increase. It should now be clear that substitutions in supply and demand impact the prices of corn and the OCC in same direction.

The increase in the biofuels demand for corn has raised the amount of corn purchased by the biofuels sector from 4.30 to 4.88 billion bushels (see top pair of

Table 6.2 Simulated impact of a 20 percent increase in biofuel demand on equilibrium prices and quantities

	Farm price		*Farm supply*		*Feed demand*		*Other demand*	
	Corn	*OCC*	*Corn*	*OCC*	*Corn*	*OCC*	*Corn*	*OCC*
Scenario 1	Biofuel demand scaled up by 20 percent above baseline values							
Baseline	3.90	5.35	9.60	2.55	5.30	0.430	4.30	2.12
After	6.74	7.49	10.08	2.17	5.19	0.519	4.88	1.65
Scenario 2	Same as first except *C* and *X* are more substitutable in production							
Baseline	3.90	5.35	9.60	2.55	5.30	0.430	4.30	2.12
After	6.22	7.55	10.16	2.12	5.24	0.480	4.92	1.64
Scenario 3	Same as first except *C* and *X* are more substitutable in feeding							
Baseline	3.90	5.35	9.60	2.55	5.30	0.430	4.30	2.12
After	6.53	7.52	10.00	2.23	5.10	0.588	4.90	1.64
Scenario 4	Same as first except demand for *X* by food processor is less elastic							
Baseline	3.90	5.35	9.60	2.55	5.30	0.430	4.30	2.12
After	12.47	15.48	9.84	2.36	5.25	0.470	4.59	1.90

Note: Scenario 2: Change farm's *b* parameter to 1.5 and substitution parameter to –2.
Scenario 3: Change feedlot's *b* parameter to 0.5 and substitution parameter to 2.
Scenario 4: Change food processor's demand elasticity to –0.5.

rows and second last column in Table 6.2). This increase has been partially offset by the higher price of corn, but in this particular scenario the offset is likely to be quite small because of the inelastic nature of the biofuels demand for corn. Perhaps more importantly, the increase in the price of the OCC that results from the higher biofuels demand for corn has decreased purchases of the OCC by the food processor from 2.12 to 1.65 billion bushels (see last column). The scale of this reduction is expected because of the relatively large increase in the price of the OCC.

Elasticity of substitution effects

The extent that the increase in price of the OCC dampens the increase in the price of corn when the biofuels demand for corn shifts out depends on the degree of substitution in supply and demand. To examine this relationship, Scenario 2 in Table 6.2 (third and fourth rows) makes the same comparison as Scenario 1 except now production of corn and the OCC is more substitutable at the farm level. Specifically, the b variable for the farm level PPF has been lowered from 2 to 1.5, which implies that the elasticity of transformation along the PPF (which is equal to the inverse of $1 - b$) has increased from -1 to -2.

Prior to increasing the biofuels demand for corn and resolving the model for the new set of equilibrium prices and quantities, the model must be recalibrated with $b = 1.5$. Recall from above that recalibration involves choosing values for the other parameters that ensure that equilibrium values are consistent with the baseline values. After the model has been recalibrated the proposed 20 percent increase in biofuel demand can be incorporated by changing the scale parameter in cell B9 from 4.927 to 5.912. Solver can now be used to generate a new set of equilibrium prices and quantities.

Greater substitution at the farm level is expected to dampen the price increase of corn and further augment the price increase of the OCC that results with the higher biofuels demand. This is because the higher price of corn will result in a relatively larger shift in production away from the OCC and toward corn, and this additional shift will result in more corn and less OCC on the market. A price comparison of Scenarios 1 and 2 in Table 6.2 (first and second columns) confirms that the price of corn has indeed risen less and the price of the OCC has risen more as a result of greater substitution at the farm level. The more responsive supply of corn and the OCC has also resulted in more corn production in Scenario 2 than in Scenario 1 even though the price of corn has risen by less in the latter scenario. Similarly, the supply of the OCC has decreased by a larger amount in Scenario 2 than in Scenario 1 even though the price of the OCC is higher in the latter scenario. Finally, the more responsive supply of corn and the OCC has resulted in a higher demand for corn by the feedlot and the biofuel processor and a relatively lower demand for the OCC by the feedlot and the human food processor.

The results for Scenario 3 in Table 6.2, which assumes more substitution between corn and the OCC at the feedlot rather than at the farm, are similar to the Scenario 2 case. In this case, the substitution parameter, b, is increased in value from approximately 0 to 0.5. The difference between the two scenarios is that Scenario 3 assumes more elastic substitution at the feedlot level and Scenario 2 assumes more elastic substitution at the farm level. Specifically, $b = 0.5$, which implies an elasticity of substitution equal to $\sigma = 2$. More elastic substitution by the feedlot implies that more of the OCC will be substituted for corn when the price of corn rises due to an increase in the biofuels demand for corn. Thus, similar to Scenario 2, more elastic substitution by the feedlot implies that the price increase for corn is expected to be lower and the price increase for the OCC is expected to be higher when the biofuels demand for corn increases. Both of these results are present in Scenario 3, just as they were in Scenario 2.

Scenario 4 in Table 6.2 is the same as Scenario 1 except now the demand for the OCC by food processors is assumed have an elasticity of −0.5 rather than the base case value of −0.75. Notice for this case that the impact of an increase in biofuels demand for corn on the prices of corn and the OCC are of much larger magnitude (see first pair of columns and first pair of rows in Table 6.2). Specifically, the price of corn increases from \$3.90/bushel to \$12.47/bushel and the price of the OCC increases from \$7.49/bushel to \$15.48/bushel as a result of the higher demand for corn.

A much larger price increase emerges for this lower demand elasticity case because less OCC will be freed up by the market when the price of corn increases. A reduction in the OCC that is available for the feedlot implies that corn and the residual OCC are both relatively scare. Consequently, the feedlot and the biofuels processor will bid up the price of corn and the OCC to a relatively high level. In the food-for-fuel debate the worry is that the demand for food is relatively inelastic. Consequently, an increase in biofuels demand for corn can result in a substantial increase in the price of livestock feed and food products.

6.6 Concluding comments

The purpose of this chapter was three-fold. First, the analysis showed that in a multi-market environment where commodity prices are linked because of substitutions in supply and demand, the solution to the social planner's problem is the same as the competitive market solution. This "invisible hand" result, which has emerged repeatedly throughout this textbook, implies that equilibrium outcomes to multi-market models can be obtained either indirectly by maximizing aggregate market surplus or directly by solving for the set of market clearing prices. The latter approach was used in this chapter to highlight this particular technique.

The second purpose of this chapter was to show how the constant elasticity of substitution function can be used to represent production technologies in multi-market analysis. The CES function is used widely in real-world modeling applications because it has a comparatively small number of parameters that require estimation and because the function can readily be tailored to a wide range of market scenarios. By adjusting the value of a single parameter, the generic CES production technology can represent the two output–one input case, which involves a production possibility frontier, or it can represent the one output–two input case, which involves a production isoquant. With reasonable starting values for the endogenous variables of the multi-market model, Excel's Solver tool is highly effective at solving for the multi-market equilibrium.

The third purpose of this chapter was to examine price linkages across markets and how the strength of these linkages depends on the strength of the underlying supply and demand substitution effects. Of specific interest was how substitutions in farm level production and feedlot demand determine how an increase in the biofuels demand for corn impacts the prices of corn and a composite of other crops that is used for both livestock feed and food processing. The simulation results demonstrate that higher biofuels demand raises the price of both commodities, and that the price increases are smaller the greater the degree of substitution in supply and demand. A second conclusion is that the price impact of higher biofuels demand is much more significant if the demand for food is relatively inelastic. Although the question addressed in this chapter is at the heart of the current food-for-fuel debate, the simulation results should be interpreted with caution because of the limited scope of the analysis.

The analysis of this chapter was restricted to analyzing the change in the set of long-run equilibrium prices when demand or supply in one of the markets permanently shifts and all other parameters are held constant. In reality it is never possible to observe long-run price changes because demand and supply schedules are continually shifting. Nevertheless, when examining the co-movement in prices for multiple commodities it is useful to carefully consider the role of substitution as a determinant of the degree of pricing integration. The prices for most agricultural commodities will be somewhat integrated because of common supply and demand shocks. Commodities with the highest level of integration are usually those which have the highest degree of substitution in supply and demand.

Questions

1 Farmland in a particular region can be used to produce corn (C) and other crops (O). The production function for corn is $C = 2L_C^{0.5}$ where L_C is land devoted to corn production. The production function for other crops is $O = 4L_O^{0.5}$ where L_O is land devoted to other crop production. The total land base that can be devoted to corn and other crops is 100 units. Assume zero costs of production for corn and the other crops.

a Derive an expression for the production possibility frontier (PPF), which is the equation that implicitly shows the different combinations of C and O that can be produced on the 100 units of land. Graph the PPF with C on the horizontal axis and O on the vertical axis.

b Using the PPF from part (a), derive an expression for the marginal rate of transformation (MRT) as a function of C and O.

c Let P_C and P_O denote the price of corn and other crops respectively. Insert the revenue equation, $R = P_C C + P_O O$, on the graph that was constructed in part (a) for the special case where $P_C = 10$ and $P_O = 15$. The revenue equation should be drawn tangent to the PPF.

d Explain why the point of tangency represents the profit maximizing level of production for the two commodities (remember the assumption of zero production costs). As part of your explanation show what happens to profits if production takes place at a non-tangent point. Be sure to include in your discussion the concept of opportunity cost.

e Set the expression for the MRT that was derived in part (b) equal to the absolute slope of the revenue equation, P_C/P_O. Solve this equation together with the equation for the PPF from part (a) to obtain farm-level supply equations for corn and the other crop. Describe how the supply of C and O change with changes in P_C and P_O.

2 A cattle feedlot uses corn (C) and other crops (O) to produce 100 head of fat steers. The production function for producing 100 steers is $100 = 20C^{0.5}O^{0.5}$. Assume there are no other costs of production for the feedlot.

a Graph the production isoquant for 100 steers on a graph with C on the horizontal axis and O on the vertical axis.

b Derive an expression for the marginal rate of technical substitution ($MRTS$) as a function of C and O.

c Let P_C and P_O denote the price of corn and other crops respectively. Graph the cost equation, $Cost = P_C C + P_O O$, on the graph that was constructed in part (a) for the special case where $P_C = 10$ and $P_O = 15$. The cost equation should be drawn tangent to the production isoquant.

d Explain why the point of tangency represents the cost minimizing level of input use for the two feed grains. As part of your explanation show what happens to cost if input use occurs at a non-tangent point.

e Set the expression for the MRTS that was derived in part (b) equal to the absolute slope of the cost equation, P_C/P_O. Solve this equation together with the equation for the production function to obtain feedlot-level demand equations for corn and the other crops. Describe how the demand for these two feed grains change with changes in P_C and P_O.

3 Use your results from questions 1 and 2 to answer this question. The demand for other crops (O) by humans is equal to $O = 135P_O^{-0.5}$. Market equilibrium occurs where the combined demand for other crops (O) by the feedlot and humans is equal to the supply of other crops by farmers, and where the demand for corn (C) by the feedlot is equal to the supply of corn by farmers.

a Use Excel to solve for the equilibrium prices of corn (P_C) and other crops (P_O).

b Suppose the demand for other crops by humans shifts out to $O = 150P_O^{-0.5}$. Use your Excel model from part (b) to calculate the impact on the equilibrium prices of corn and other crops.

c Describe in general terms how the relationship between the equilibrium price of corn and the demand for other crops by humans depends on the degree of substitution between corn and other crops in both production at the farm and input use in the feedlot.

7 Marketing margins in vertical supply chains

7.1 Introduction

In the previous five chapters a dominant theme has been competitive pricing and the equivalence of the competitive outcome with the surplus maximizing outcome of a social planner. These previous chapters also focused on pricing relationships for the same commodity or a pair of substitute commodities over space, time, form and multiple markets. In this current chapter both of these assumptions are relaxed. Specifically, the purpose of this chapter is to examine vertical pricing relationships in a typical farm-to-retail food supply chain, where downstream processing firms with market power purchase the raw commodity from upstream farmers and sell processed and semi-processed differentiated products to retail consumers.

The concept of a marketing margin is central to the analysis of a food supply chain. A farm-to-retail marketing margin is the difference between the implicit value of an agricultural commodity when sold at the retail level in processed form versus the explicit value of the unprocessed commodity at the farm level. For example, if the retail price of milk is \$1/liter and the farm price of milk is \$0.35/liter then the farm-to-retail marketing margin is \$0.65/liter. In situations where the commodity moves through the supply chain in fixed proportions the marketing margin represents the net selling price for the firm that supplies the marketing service. The marketing margin must cover both the average variable cost and average fixed cost of the firm providing the marketing service. The residual margin after adjusting for average variable and fixed costs represents unit profits for the firm.

Of particular interest in this chapter is the degree of product differentiation as a determinant of the size of the marketing margin. The idea is that a processing firm that sells a more differentiated product will have more market power and so will enjoy a higher marketing margin. Selling a more differentiated product typically implies higher variable and fixed costs, so it is important to note that higher marketing margins are not necessarily indicative of higher profits for the firm. Table 7.1 shows for the years 1992 to 2000 the farm value of select fruits and vegetables as a fraction of their retail value, which is equivalent to showing the set of marketing margins.[1] Notice that the average marketing margin for six commonly

Table 7.1 USDA farm value as a percent of retail value for select fruits and vegetables: 1992–2007

	Apples	*Grapefruit*	*Strawberries*	*Tomatoes*	*Potatoes*	*Lettuce*	*6 Item average*	*Processed basket*
1992	29.0	13.2	55.6	40.8	17.1	23.4	*29.9*	*23.0*
1993	22.9	15.3	48.3	34.3	22.4	25.6	*28.1*	*19.0*
1994	23.8	13.4	49.8	28.2	20.6	27.5	*27.2*	*20.0*
1995	27.9	12.2	44.6	25.9	21.1	28.5	*26.7*	*21.0*
1996	27.1	12.3	43.0	30.5	20.6	24.4	*26.3*	*20.0*
1997	22.1	14.1	47.2	29.8	16.2	28.8	*26.4*	*19.0*
1998	20.3	16.1	40.8	26.9	18.0	22.0	*24.0*	*18.0*
1999	20.4	13.6	38.7	20.5	20.1	22.0	*22.6*	*17.0*
2000	22.1	11.8	39.3	24.9	17.7	25.5	*23.5*	*17.0*
2001	21.6	12.5	39.4	29.2	20.9	25.5	*24.9*	*16.0*
2002	26.5	14.2	37.2	28.4	26.0	27.8	*26.7*	*16.0*
2003	27.3	9.4	39.1	27.1	17.9	27.5	*24.7*	*15.0*
2004	27.9	22.0	33.1	30.2	15.9	20.2	*24.9*	*16.0*
2005	23.5	17.1	32.1	28.8	18.7	19.2	*23.2*	*17.0*
2006	26.3	14.4	33.8	28.0	21.1	21.0	*24.1*	*17.0*
2007	30.4	12.6	34.6	23.5	19.2	25.1	*24.2*	*17.0*

Source: Economic Research Service of the United States Department of Agriculture (USDA), "Price spreads from farm to consumer: at-home foods by commodity group", various tables. Data downloaded on 27 April 2010 from: http://www.ers.usda.gov/data/FarmToConsumer/pricespreads.htm#fruits

Note: The "6 Item average" is the straight average of the values in the preceding six columns. The "Processed basket" is a basket of processed fruits and vegetables.

purchased fresh fruits and vegetables (second last column) is significantly less than the marketing margin for a basket of processed fruits and vegetables (last column). Variable processing costs can explain part of the difference in these marketing margin values, but it is also likely that firms selling the processed fruits and vegetables are able to set a higher marketing margin because of greater product differentiation and market power. Notice that the margins for both the processed and non-processed fruits and vegetables are gradually rising over time, possibly the result of rising supply chain costs and increasing product differentiation.

Before formally examining marketing margins with product differentiation and non-competitive pricing it is useful to briefly discuss the competitive market benchmark. As should now be expected, in a competitive supply chain the marketing margin will conform to the law-of-one-price. Specifically, the set of prices in a competitive supply chain will be the same as the set of prices that would be chosen by a social planner. As well, the price of a commodity in a downstream position is expected to equal the upstream price plus the unit cost of shifting the commodity downstream. This cost of shifting can include processing, transportation, warehousing, marketing and other overhead expenses. The concept of efficient price transmission is also relevant when discussing a competitive marketing margin. Vertical price transmission is a measure of how quickly and how completely a downstream price adjustment causes the upstream price to adjust and vice versa.[2] Rapid and complete price transmission implies that the marketing margin will fluctuate very little over time in response to upstream and downstream price fluctuations, assuming that costs within the supply chain do not change.

By way of an example, Figure 7.1 shows the Kansas City quarterly wholesale value of flour and the farm-level cost of the wheat that is embedded in the flour for the years 2005 to 2009 (the assumed extraction rate is 73 percent). In this particular case price transmission appears to be quite rapid and complete, and so the marketing margin is quite stable over time despite large changes in the individual prices for wheat and flour. In a second example, the USDA calculated that due to an unexpected surge in the supply of green coffee beans the green bean price dropped from $0.11/ounce to $0.04/ounce over the years 1997 to 2002.[3] The price of roasted coffee beans did not drop nearly so fast, but eventually the price did drop from $0.23/ounce to $0.17/ounce. In this particular case the marketing

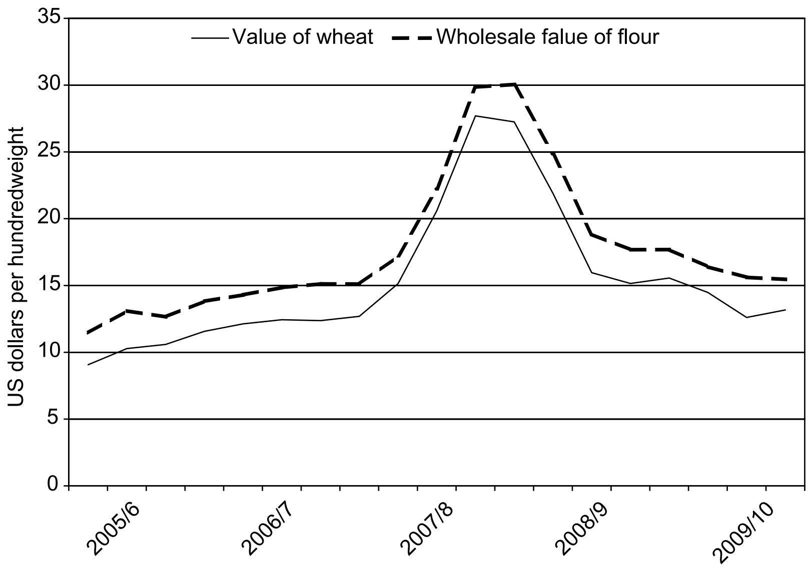


Figure 7.1 Monthly wheat and flour price relationships, Kansas City: 2005/6 to 2009/10.

Source: Table 32 – Wheat and flour price relationships, Kansas City. *Wheat Data: Yearbook Tables* (various years), Economic Research Service, USDA. Data from http://www.ers.usda.gov/data/wheat/YBtable32.asp Downloaded on 1 November 2009.

Note: Value of wheat assumes 2.28 bushels based on a 73 percent extraction rate.

margin returned (approximately) to its 1997 level and price transmission, although slow, was eventually complete.[4]

Although the speed of price transmission in a food supply chain is an interesting and important topic, it is largely an empirical issue and thus is not formally considered in this chapter. The specific purpose of this chapter is to construct a simulation model and then use the model to examine how product differentiation in the retail market and non-competitive pricing in the raw commodity market will raise the marketing margin above the competitive benchmark. Product differentiation implies that firms selling in the retail market are no longer price takers, but rather face downward sloping demand schedules. Firms take advantage of their market power by marking up price above their marginal cost of production. A single parameter in the simulation model controls the degree of product differentiation. By changing this parameter it is easy to observe the connection between product differentiation, market power and the extent that firms mark up price over marginal cost.

Public policy analysts have long worried about welfare losses that result from non-competitive marketing margins in agri-food supply chains. This issue is not addressed in this chapter because it is relatively complex to deal with and is not directly related to the theme of this textbook, which is price analysis. Certainly if firms had no fixed costs to cover, then any marketing margin set above the firm's average variable processing cost would be deemed "bad" from a social welfare perspective. In reality downstream firms in agricultural commodity supply chains operate with sizeable fixed costs, so margins must be set above marginal cost to ensure that these firms can generate sufficient revenue to cover their fixed costs. The problem arises when firms are in a position to set margins well above marginal cost and thus earn a sizeable profit. Without knowing the size of fixed costs within an industry it is impossible to know whether the marketing margin is justified for cost reasons, or whether the margin is excessive because it results in sizeable profits for the processing firm. What is known is that firms that operate in an industry with a high degree of product differentiation have a greater potential to earn positive profits.

In the next section the basic assumptions of the model are described. Central to this description is the CES utility function, which is used to derive the system of differentiated products demand equations. In Section 7.3 the equations that define the set of equilibrium prices are derived, and the full model is entered into an Excel workbook. Simulation results are presented and discussed in Section 7.4. A summary and concluding comments are contained in Section 7.5.

7.2 Demand for differentiated products

The upstream segment of the vertical market consists of producers of a homogenous raw commodity (i.e., farmers) selling their output to food manufacturing firms. These firms in turn produce processed and semi-processed versions of the commodity and then sell their differentiated food products in a downstream retail market. The variable of interest is the marketing margin, which is the difference between the retail price of the transformed commodity in the downstream market and the producer price of the raw commodity in the upstream market after making appropriate adjustments for product conversion. Of particular interest is the connection between the degree of product differentiation at the consumer level and the size of the marketing margin.

This section is devoted to the specification of consumer demand for the set of differentiated food products that are available in the downstream market. The analysis begins by describing the two-stage budgeting procedure that consumers use to allocate their food expenditures between processed and semi-processed food items. The analysis then shifts to a description of the CES utility function that is used to characterize consumer choice. Finally, consumer demand schedules for the full set of differentiated food products are derived by combining the results of the two stages of the budgeting procedure.

Two-stage budgeting

The retail market consists of a representative consumer with a fixed amount of disposal income I to be spent on a combination of n processed food products and $\hat{n}$ semi-processed (standard) food products.[5] Let q_i denote the quantity consumed and p_i denote the price of the ith processed food product. For the standard food product, the corresponding variables are $\hat{q}_i$ and $\hat{p}_i$. The objective of the representative consumer is to choose values for $q_1, q_2, \ldots, q_n$ and $\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n$ to maximize utility from consumption subject to a budget constraint, $\sum_{j=1}^{n} p_j q_j + \sum_{j=1}^{\hat{n}} \hat{p}_j \hat{q}_j = I.$ The outcome of this maximization problem is the full set of demand schedules for the $n + \hat{n}$ processed and standard food products.

The consumer's problem is simplified considerably by assuming that consumption decisions are made using two-stage budgeting. This assumption is equivalent to assuming that the consumer maximizes a "nested" utility function. In stage 1 the consumer's income is allocated between q units of a processed food basket and $\hat{q}$ units of a standard food basket. In stage 2 the budget that was previously allocated to the processed food basket is further allocated to the n processed food items. The allocation procedure is similar for the standard food basket.

The unit price of the food basket is denoted P for processed food and $\hat{P}$ for standard food. Expressions for these endogenous price variables are presented below. In stage 1 the problem facing the consumer is to choose q and $\hat{q}$ in order to maximize a stage 1 utility function, $U_1(q,\hat{q})$, subject to the stage 1 budget constraint, $qP + \hat{q}\hat{P} = I$. This utility function is assumed to be concave in both q and $\hat{q}$, and is also assumed to have the standard properties to ensure that the consumer will choose positive values for both q and $\hat{q}$. Let $q(P, \hat{P}, I)$ and $\hat{q}(P, \hat{P}, I)$ denote the solution to the consumer's stage 1 optimization problem. These two expressions allow stage 1 expenditures to be expressed as $e(P, \hat{P}, I) = Pq(P, \hat{P}, I)$ for the processed food basket and $\hat{e}(P, \hat{P}, I) = \hat{P}\hat{q}(P, \hat{P}, I)$ for the standard food basket.

The consumer faces two sub-problems in stage 2. The consumer must allocate expenditures $e(P, \hat{P}, I)$ among the n processed food items and must allocate expenditures $\hat{e}(P, \hat{P}, I)$ among the $\hat{n}$ standard food items. Specifically, the consumer chooses $q_1, q_2, \ldots, q_n$ to maximize stage 2 utility from processed food items, $U_2(q_1, q_2, \ldots, q_n)$, subject to $\sum_{j=1}^{n} p_j q_j = e(P, \hat{P}, I)$, and chooses $\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n$ to maximize stage 2 utility from standard food items, $\hat{U}_2(\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n)$, subject to $\sum_{j=1}^{\hat{n}} \hat{p}_j \hat{q}_j = \hat{e}(P, \hat{P}, I)$.

To proceed with the analysis it is necessary to be specific about the functional forms of the stage 1 and two utility functions. Assume that in both stages the utility function has a constant elasticity of substitution. Recall that a generic CES function was used in Chapter 6 to specify a production possibility frontier for the farmer and a production isoquant for the feedlot. The CES utility function that is used in this chapter is similar in structure to the CES production isoquant that was used in Chapter 6.

Stage one CES utility

In stage 1 the CES utility function can be expressed as:

$$\omega = \left(\alpha^{1-\theta} q^{\theta} + (1-\alpha)^{1-\theta} \hat{q}^{\theta}\right)^{1/\theta} \tag{7.1}$$

Within equation (7.1) the parameter $\theta \in (0,1)$ is a measure of the degree of substitution between processed and standard food products, and the parameter $\alpha \in (0,1)$ is a share parameter that measures the importance of processed food relative to standard food in consumer expenditures. A larger value for α implies more consumption of processed food items and less consumption of standard food items for a fixed set of prices.

Similar to the analysis in Chapter 6, the elasticity of substitution for equation (7.1) can be expressed as $s = \frac{1}{1-\theta}$. As $\theta \rightarrow 1$ from below the utility function becomes linear and the elasticity of substitution becomes infinitely large, the well-known Cobb–Douglas utility function emerges with $\theta \rightarrow 0$ and $s \rightarrow 1$. The case where θ takes on a negative value is not considered in this analysis because, as is shown below, $\theta < 0$ implies negative marginal revenue for the firms supplying the differentiated food products. These assumptions imply that $\theta \in (0,1)$.

Equation (7.1) can be used to obtain a specific solution to the consumer's stage 1 allocation problem. As was discussed above, the problem involves choosing q units of the processed food basket and $\hat{q}$ units of the standard food basket to maximize utility $U_1(q, \hat{q})$ subject to $qP + \hat{q}\hat{P} = I$. After substituting equation (7.1) for $U_1(q, \hat{q})$ the Lagrange function for this constrained optimization problem can be written as:

$$L_1 = \left(\alpha^{1-\theta} q^{\theta} + (1-\alpha)^{1-\theta} \hat{q}^{\theta}\right)^{1/\theta} + \lambda_1\left(I - qP - \hat{q}\hat{P}\right) \tag{7.2}$$

The solution to the constrained optimization problem that is implied by equation (7.2) is a pair of expressions for the optimal values of $q(P, \hat{P}, I)$ and $\hat{q}(P, \hat{P}, I)$. If these quantity expressions are multiplied by P and $\hat{P}$, respectively, then the resulting pair of expressions represents optimal expenditures on processed items, $e(P, \hat{P}, I) = \hat{q}(P, \hat{P}, I)P$, and standard food items, $\hat{e}(P, \hat{P}, I) = \hat{q}(P, \hat{P}, I)\hat{P}$. In Appendix 7.1 it is shown that:

$$e\left(P,\hat{P},I\right)=\frac{\alpha P^{\frac{\theta}{\theta-1}}}{\alpha P^{\frac{\theta}{\theta-1}}+(1-\alpha)\hat{P}^{\frac{\theta}{\theta-1}}}I \quad \text{and} \quad \hat{e}\left(P,\hat{P},I\right)=\frac{(1-\alpha)\hat{P}^{\frac{\theta}{\theta-1}}}{\alpha P^{\frac{\theta}{\theta-1}}+(1-\alpha)\hat{P}^{\frac{\theta}{\theta-1}}}I \tag{7.3}$$

Later in the analysis expressions for P and $\hat{P}$ are derived in order to complete the specification of the consumer's stage 1 expenditure functions.

Stage two CES utility

In stage 2 the CES utility function for the processed and standard food items can respectively be expressed as:

$$u=\left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{1/\rho} \quad \text{and} \quad \hat{u}=\left(\sum_{j=1}^{\hat{n}} \hat{a}_j^{1-\hat{\rho}} q_j^{\hat{\rho}}\right)^{1/\hat{\rho}} \tag{7.4}$$

Similar to stage 1 utility, the parameters $\rho \in (0,1)$ and $\hat{\rho} \in (0,1)$ are measures of the degree of substitutability between individual food products in the processed and standard food categories, respectively. As well, $\sum_{j=1}^{n} a_j = 1$ and $\sum_{j=1}^{\hat{n}} \hat{a}_j = 1$, where $a_i \in (0,1)$ and $\hat{a}_i \in (0,1)$, are share parameters that measure the relative importance of the ith processed food product and the ith standard food product, respectively.

Derivation of the stage 2 consumer demand system begins by constructing a pair of Lagrange functions for the constrained stage 2 utility maximization problems facing the consumer. For the processed good category, the Lagrange function can be written as

$$L_2=\left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{\frac{1}{\rho}}+\lambda_2\left(e\left(P,\hat{P},I\right)-\sum_{j=1}^{n} p_j q_j\right) \tag{7.5}$$

It is important to note that the consumer treats available expenditures $e(P, \hat{P}, I)$ as a fixed constant when choosing $q_1, q_2, \ldots, q_n$ to maximize stage 2 utility.

In Appendix 7.1 it is shown that the consumer demand functions for the ith product in the processed and standard food categories can be derived from equation (7.5) and written as:

$$q_i\left(p_1,p_2,\ldots,p_n,e\right)=\frac{a_i p_i^{\frac{1}{\rho-1}}}{\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}}}e\left(P,\hat{P},I\right) \tag{7.6a}$$

and

$$\hat{q}_i\left(\hat{p}_1,\hat{p}_2,\ldots,\hat{p}_n,\hat{e}\right)=\frac{\hat{a}_i \hat{p}_i^{\frac{1}{\hat{\rho}-1}}}{\sum_{j=1}^{\hat{n}} \hat{a}_j \hat{p}_j^{\frac{\hat{\rho}}{\hat{\rho}-1}}}\hat{e}\left(P,\hat{P},I\right) \tag{7.6b}$$

The stage 2 consumer demand system can be completed by substituting into equation (7.6) the two expressions for $e(P, \hat{P}, I)$ and $\hat{e}(P, \hat{P}, I)$, which are given by equation (7.3).

Expressions for P and $\hat{P}$

The complete consumer demand system can be finalized by deriving expressions for P and $\hat{P}$. This derivation requires expressions for the stage 2 indirect utility functions for the processed and standard goods categories. For the case of processed food equation (7.6a) can be substituted into the utility function, $u=\left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{1/\rho}$, to obtain a measure of indirect utility for the consumer, which is denoted $v(p_1, p_2, \ldots, p_n, e)$. It is straightforward to use equation (7.6a) to show that $\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}=\left(\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}}\right)^{1-\rho} e\left(P,\hat{P},I\right)^{\rho}$. If this expression is substituted into $u=\left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{1/\rho}$ then the following expression for indirect utility emerges:

$$v\left(p_1,p_2,\ldots,p_n,e\right)=\left(\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}}\right)^{\frac{1-\rho}{\rho}} e\left(P,\hat{P},I\right) \tag{7.7}$$

Equation (7.7) can be interpreted as the level of utility the consumer will enjoy if prices equal $p_1, p_2, \ldots, p_n$ and $e(P, \hat{P}, I)$ is available to spend on processed goods. However, suppose equation (7.7) is rearranged as follows:

$$e\left(P,\hat{P},I\right)=v\left(p_1,p_2,\ldots,p_n,e\right)\left(\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \tag{7.8}$$

Because $e(P, \hat{P}, I)$ represents total expenditures on the basket of processed food it follows from equation (7.8) that the variable $v(p_1, p_2, \ldots, p_n, e)$ can be interpreted as the number of units of the processed food basket that is purchased in stage 1. Moreover, the remaining expression on the right hand side of equation (7.8) can be interpreted as the unit price of the processed food basket (i.e., a quantity-weighted price index for the processed goods). Specifically,

$$P = \left(\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \tag{7.9a}$$

The analogous expression for the basket price in the standard good market can be expressed as:

$$\hat{P} = \left(\sum_{j=1}^{\hat{n}} \hat{a}_j \hat{p}_j^{\frac{\hat{\rho}}{\hat{\rho}-1}} \right)^{\frac{\hat{\rho}-1}{\hat{\rho}}} \tag{7.9b}$$

The derivation of consumer demand schedules for the n processed and $\hat{n}$ standard food products is now complete. The demand for the ith processed product and the ith standard product is given by equation (7.6) with the expressions in equation (7.3) substituting for $e(P, \hat{P}, I)$ and $\hat{e}(P, \hat{P}, I)$, and and with the expressions in equation (7.9) substituting for P and $\hat{P}$. Within this demand system the quantity demanded of each food product depends on consumer income, the prices of all food products and the full set of substitution and share parameters. Two stage budgeting is evident because demand for a processed food product depends directly on processed food prices but only indirectly (via the $\hat{P}$ variable) on standard food prices. The situation is similar for the standard food category.

7.3 Equilibrium pricing

In this section profit-maximizing food companies are added to the model. As discussed above these firms purchase a homogeneous raw commodity in the upstream market, transform it in some fashion and then sell a processed or semi-processed (standard) version of the commodity in differentiated forms to consumers in the downstream market. The analysis in this section is simplified in a number of ways in order to reduce complexity. First, assume that each of the processed and standard food products is supplied by a unique food company that takes the prices of the other food companies as given when choosing its profit-maximizing level of sales. In other words the retail side of the market is characterized by monopolistic competition. Second, food companies are assumed to operate with a fixed proportions production function. To simplify the notation assume that one unit of a raw agricultural commodity is required to produce one unit of a retail food product (e.g., fresh eggs). Third, assume that all of the firms have an identical cost structure. This latter assumption implies that differences in product demand is the only reason why prices will differ across firms when the market is in equilibrium.

The analysis below is divided into three sections. In the first section the condition for maximizing profits is derived for the *i*th firm. In the second section the firm's marginal revenue and marginal outlay functions are derived because these functions are needed to complete the profit maximization calculations. In the third section the market equilibrium is derived with the assumption that the firm is either a price taker or a monopsonist in the raw commodity market. In all three cases the analysis focuses on the *i*th firm in the processed good market. The results for a firm in the standard good market are the same after making adjustments for notation.

Profit maximization conditions

Profits for the *i*th firm that chooses to use q_i units of raw material to produce q_i units of the *i*th processed food product can be expressed as $\pi_i = (p_i - w_i - c)\, q_i$ where p_i is the firm's selling price, w_i is the raw commodity price and c is the unit cost of processing. The firm is the sole supplier of the *i*th food product, which implies that its selling price must be set according to the consumer demand schedule for the *i*th processed food product, as given by equation (7.6a). The fixed costs for the firm are not included in the profit function because in this short-run analysis there are no entry and exit considerations and so fixed costs have no implications for equilibrium pricing and marketing margins.

Two extreme assumptions are maintained regarding the firm's pricing strategy in the raw material market. In the one extreme the firm is a pure monopsonist and sets the price of the raw commodity accordingly. Specifically, the firm understands that the price it must pay to obtain q_i units of raw material is equal to $w_i^s(q_i)$, where $w_i^s(q_i)$ is the raw material industry supply schedule. The second pricing assumption is that the firm behaves as a competitive price taker in the raw commodity market because producers of the raw commodity operate a single-desk selling marketing board.[6] Real-world pricing of the raw commodity will typically lie somewhere between the two extremes that are considered in this analysis.

With these pricing assumptions in place the processing firm's condition for maximizing profit, $\pi_i = (p_i - w_i - c)q_i$, can be expressed as $d\pi_i/dq_i = mr_i(q_i) - mo_i(q_i) - c = 0$, where $mr_i(q_i) = p_i + q_i\, dp_i/dq_i$ is the firm's marginal revenue function and $mo_i(q_i) = w_i^s(q_i) + \delta_i q_i\, dw_i^s(q_i)/dq_i$ is the firm's marginal outlay function. In the marginal outlay function the indicator variable δ_i takes on a value of 0 if the firm is a price taker and a value of 1 if the food manufacturer is a monopsonist in the raw commodity market.

Marginal revenue and marginal outlay

To complete the analysis of the previous section explicit expressions for the marginal revenue and outlay schedules must be derived for the *i*th manufacturer of processed food. The derivation of marginal revenue begins by noting from equation (7.9a) that $P^{\rho/\rho-1}$ can be substituted for the denominator of equation (7.6a). The revised version of equation (7.6a) can then be inverted and written as:

$$p_i = (a_i e(P, \hat{P}, I))^{1-\rho} P^{\rho} q_i^{\rho-1} \tag{7.10}$$

Now use equation (7.10) to obtain an expression for dp_i/dq_i, which is required to compute the $mr_i(q_i) = p_i + q_i\, dp_i/dq_i$ expression. After making the substitution the following expression for $mr_i(q_i)$ emerges:

$$mr_i(q_i) = \rho(a_i e(P, \hat{P}, I))^{1-\rho} P^{\rho} q_i^{\rho-1} \tag{7.11}$$

A comparison of equations (7.10) and (7.11) reveals that $mr_i(q_i) = \rho p_i$, which has the following intuitive interpretation. The restriction $0 < \rho < 1$ implies that the firm prices where marginal revenue is positive, which is equivalent to pricing to where demand is elastic. At the one extreme where $\rho = 1$ the demand system consists of perfect substitutes and demand is infinitely elastic because $mr_i(q_i) = p_i$. At the opposite extreme where $\rho = 0$ the demand system is Cobb–Douglas and demand has an elasticity of -1 because $mr_i(q_i) = 0$. In general, decreasing ρ in the range of [0, 1] makes demand less elastic and thus provides the firm with more market power and a greater capacity to mark up price above its marginal cost.

To derive the firm's marginal outlay function it is necessary to specify a functional form for the market supply of the raw agricultural commodity, which was previously denoted $w_i^s(q_i)$. To keep things simple assume the following linear form for market supply:

$$w_i^s(q_i) = \beta_0 + \beta_1 q_i \tag{7.12}$$

Noting that $mo_i(q_i) = w_i^s(q_i) + \delta_i q_i\, dw_i^s(q_i)/dq_i$, an expression for marginal outlay can be derived from equation (7.12) and written as:

$$mo_i(q_i) = \beta_0 + (1 + \delta_i)\beta_1 q_i \tag{7.13}$$

Keep in mind that $\delta_i = 0$ implies price taking behavior and $\delta_i = 1$ implies monopsony pricing for the ith food manufacturer.

Market equilibrium condition

The condition for maximum profits for the ith firm in the processed goods market, $d\pi_i/dq_i = mr_i(q_i) - mo_i(q_i) - c = 0$, also serves as the market clearing condition for the ith market. The two conditions are equivalent because the market demand schedule and raw material supply schedule are both embedded in the $mr_i(q_i) - mo_i(q_i) - c = 0$ restriction. After substituting in equations (7.11) and (7.13) into $d\pi_i/dq_i = mr_i(q_i) - mo_i(q_i) - c = 0$, the condition for maximum profits for the ith firm can be written as:

$$\rho(a_i e\,(P, \hat{P}, I))^{1-\rho}\, P^{\rho}\, q_i^{\rho-1} - (\beta_0 + (1 + \delta_i)\,\beta_1 q_i) - c = 0 \tag{7.14}$$

If equation (7.14) holds for each of the n firms selling a processed product and if an analogous equation holds for each of the $\hat{n}$ firms selling a standard product then all food manufacturers will be maximizing profits and all markets will simultaneously be in equilibrium.

The set of equations implied by equation (7.14) are interdependent and therefore must be solved as a system rather than independently. In principle the system can be solved for the $n + \hat{n}$ equilibrium values of the q and $\hat{q}$ quantity variables. These equilibrium values can then be substituted into equation (7.10) to obtain equilibrium values for the retail price variables. Finally, the equilibrium price variables can then be substituted into equation (7.9) to obtain equilibrium values for P and $\hat{P}$.

Because many of the above equations are non-linear, the solution procedure described above cannot be used to obtain an analytical solution for the market equilibrium. In the next section the problem will be entered into Excel and values will be assigned to the various parameters in order to generate numerical solution values for the market equilibrium. This numerical procedure is implemented by manipulating the equations so that the equilibrium conditions are expressed solely in terms of the price variables. Excel's Solver tool can then be instructed to simultaneously choose values for the $n + \hat{n}$ price variables that solve the system of equilibrium conditions. Once the equilibrium price variables have been recovered it is straightforward to recover the equilibrium values for the quantity variables.

7.4 Model entry and solution procedure

The top and bottom halves of the simulation model are presented in Figures 7.2 and 7.3, respectively. The model incorporates $n = 5$ processed food items and $\hat{n} = 5$ standard food items. The a and $\hat{a}$ utility function share parameters for these ten food products are contained in cells B16:F16 and B21:F21 of Figure 7.2. For both the processed and standard food products, these stage 2 share values equal 10 percent for items 1 and 5, 20 percent for items 2 and 4, and 40 percent for item 3. The share values in cells B16:F16 represent the share of stage 2 expenditures on processed food that the consumer would allocate to the five processed products if the prices for these five products were equal. The interpretation of the share parameters for the case of standard food products is similar.

The remaining consumer demand parameters are contained in cells B3:B8 of Figure 7.2. By setting the alpha parameter in cell B3 equal to $\alpha = 0.5$ it is assumed that stage 1 expenditures on processed and standard food products will be equal if equilibrium prices are such that $P = \hat{P}$. In the simulation results presented below equilibrium expenditures on processed food are significantly less than expenditures on standard food because the equilibrium value of P is significantly higher than that of $\hat{P}$. In cell B4 the theta variable is set equal to $\theta = 0.5$, which implies an elasticity of substitution equal to $s = 1/(1 - 0.5) = 2$ for processed food and

	A	B	C	D	E	F	G	H
1	**Consumer Demand Parameters (Excluding Shares for Stage Two)**							
2	*Stage One*							
3	alpha	0.5	share of processed food in stage one CES utility function					
4	theta	0.5	substitution parameter in stage one CES utility function					
5	income	10	income of representative consumer					
6	*Stage Two*							
7	rho	0.25	stage two substitution parameter(processed items)					
8	rho_h	0.75	stage two substitution parameter(standard items)					
9	**Other Parameters**							
10	c	1	unit processing cost (excluding cost of raw commodity)					
11	beta_0	1	intercept of industry supply schedule for raw commodity					
12	beta_1	5	slope of industry supply schedule for raw commodity					
13	delta	0	indicator variable for raw commodity market structure					
14	**Stage Two Share Parameters and Choice Variables for Processed Food**							
15		item 1	item 2	item 3	item 4	item 5		
16	a	0.1	0.2	0.4	0.2	0.1		
17	p	8.773	9.409	10.450	9.409	8.773		
18	q	0.039	0.070	0.122	0.070	0.039		
19	**Stage Two Share Parameters and Choice Variables for Standard Food**							
20		item 1	item 2	item 3	item 4	item 5		
21	a_h	0.1	0.2	0.4	0.2	0.1		
22	p_h	4.195	4.665	5.218	4.665	4.195		
23	q_h	0.229	0.300	0.383	0.300	0.229		
24	=e_2_h*B21*B22^(1/(rho_h-1))/Big_P_h^(rho_h/(rho_h-1))							
25								
26	**Other Variables**							
27	Big_P	9.668	basket price for processed food items					
28	Big_P_h	4.727	basket price for standard food items					
29	e_2	3.284	stage two expenditures for processed food items					
30	e_2_h	6.716	stage two expenditures for standard food items					

Figure 7.2 Part (a) of the base case simulation model.

standard food during the stage 1 allocation of income. The $s = 2$ assumption implies "moderately" flexible consumer decision making.

Consumer income, which is set equal to 10 in cell B5, is not particularly significant for the analysis other than defining the scale for equilibrium prices and marketing margins. Setting the elasticity values in cells B7 and B8 equal to $\rho = 0.25$ and $\hat{\rho} = 0.75$, respectively, is important because these values define the degree of substitution across products within each of the two food categories. The values $\rho = 0.25$ and $\hat{\rho} = 0.75$ imply that there is a comparatively low degree of substitution for processed food items ($s = 1/(1 - 0.25) = 1.333$) and a comparatively high degree of substitution for standard food items ($s = 1/(1 - 0.75) = 4$).

The remaining parameters of the model are contained in cells B10:B13 of Figure 7.2. The cost of processing a unit of raw commodity has been set equal to $c = 1$ in B10. With $c = 1$ about 10 percent of the retail selling price of the processed food product is used to cover variable operating costs excluding the cost of the raw commodity.[7] Cells B11:B12 contain the two parameters of the market supply curve for the raw commodity: $\beta_0 = 1$ and $\beta_1 = 5$. These values give rise to a set of base case supply elasticities that vary between 0.85 and 1.05 in the processed food

	A	B	C	D	E	F	G	H
26	**Other Variables**							
27	Big_P	9.668	={SUM(B16:F16*B17:F17^(rho/(rho-1)))^((rho-1)/rho)}					
28	Big_P_h	4.727						
29	e_2	3.284	=income*alpha*Big_P^(theta/(theta-1))/(alpha*Big_P^					
30	e_2_h	6.716	(theta/(theta-1))+(1-alpha)*Big_P_h^(theta/(theta-1)))					
31								
32	**Market Equil Conditions**	=rho*B17-(beta_0+(1+delta_)*beta_1*B18)-c_						
33								
34		item 1	item 2	item 3	item 4	item 5		
35	processed	2.14E-09	3.88E-09	1.65E-07	3.88E-09	2.14E-09		
36	standard	-1.53E-08	-2.02E-08	-2.59E-08	-2.02E-08	-1.53E-08		
37	**Commodity Price and Marketing Margin**							
38		item 1	item 2	item 3	item 4	item 5		
39	*Processed*	=B17	=beta_0+beta_1*B18					
40	p	8.773	9.409	10.450	9.409	8.773		
41	w	1.193	1.352	1.612	1.352	1.193		
42	margin	7.580	8.057	8.837	8.057	7.580		
43	*Standard*		=B18-B41					
44	p	4.195	4.665	5.218	4.665	4.195		
45	w_h	2.146	2.499	2.914	2.499	2.146		
46	margin_h	2.049	2.166	2.305	2.166	2.049		

Figure 7.3 Part (b) of the base case simulation model.

market and between 1.5 and 1.9 in the standard food market. Finally, $\delta = 0$ in cell B13 implies that for the base case the raw material market is assumed to be perfectly competitive rather than monopsonistic for all $n + \hat{n}$ firms.[8]

Market equilibrium is obtained for the simulation model by initially entering "guess" values for the prices of the five processed products contained in cells B17:F17 and the five standard products contained in cells B22:F22 of Figure 7.2. Later in the analysis Solver will be used to select the specific equilibrium values for these ten prices. Equation (7.6) is used to generate the quantity variables for the ten products, which are contained in cells B18:F18 for processed food and cells B23:F23 for standard food. Equation (7.6) utilizes the individual price variables in cells B17:F17 and B22:C22, and the stage 2 expenditure variables $e(P, \hat{P}, I)$ and $\hat{e}(P, \hat{P}, I)$. These two expenditure variables, which reside in cells B29:B30 and are labeled "e_2" and "e_2_h", respectively, were generated using equation (7.3). Rather than entering the denominator of equation (7.6) in cells B18:F18 and B23:F23, the equivalent expressions $P^{\rho/\rho-1}$ and $\hat{P}^{\hat{\rho}/\hat{\rho}-1}$ have been entered instead. The P and $\hat{P}$ variables are calculated in cells B27:B28 of Figures 7.2 and 7.3 (labeled "Big_P" and "Big_P_h") using equation (7.9) expressed in array formula format.

The market equilibrium conditions, which are summarized by equation (7.14), are contained in cells B35:F36 of Figure 7.3. The market is in equilibrium only if the values in these cells are zero.[9] The retail prices that reside in cells B17:F17 and B22:F22 of Figure 7.2 are repeated in cells B40:F40 and B44:F44 of Figure 7.3 in order to more effectively present the final results. The equilibrium values of the raw commodity prices for the processed and standard goods markets are displayed in cells B41:F41 and B45:F45, respectively, of Figure 7.3. These values are calculated using the inverse supply schedules given by equation (7.12) and the quantity variables that reside in cells B18:F18 and B23:F23 of Figure 7.2. Cells B42:F42 and B46:F46 of Figure 7.3 contain the $p_i - w_i$ marketing margins for the processed food items and the standard food items, respectively. The equilibrium values of these marketing margin variables are of particular interest in the simulation results.

The final step in obtaining the equilibrium quantities, prices and marketing margins is to use Solver to choose the price variables in cells B17:F17 and B22:F22 of Figure 7.2 such that the equilibrium conditions in cells B35:F36 of Figure 7.3 take on zero values. Fortunately, Solver is able to quickly find a solution even if the initial guess values are not very accurate. The next section is devoted to analyzing the base case results that are currently displayed in Figures 7.2 and 7.3 and then performing sensitivity analysis.

7.5 Simulation results

The equilibrium prices and marketing margins for the base case are displayed in cells B40:F42 and B44:F46 of Figure 7.3 for the processed food products and standard food products, respectively. For the base case the food manufacturers are assumed to be monopolists in the retail market but behave competitively in the raw commodity market. In one of the sensitivity scenarios presented below, prices will be calculated assuming that the downstream firms exert full market power in both the retail market and the raw commodity market.

A comparison of cells B42:F42 and B46:F46 in Figure 7.3 reveals that the marketing margins for processed food are significantly higher than the marketing margins for standard food. For example, the firm selling the third standard food item earns a unit margin equal to 2.30 per unit whereas the firm selling the third processed food item earns a margin equal to 8.84 per unit. The higher margins for processed food do not depend on cost differences because the cost of purchasing the raw commodity and the unit cost of processing is the same for all firms. The difference in margins can primarily be attributed to differences in the degree of product differentiation. The standard items have a relatively low degree of retail level product differentiation and thus more substitution across items by consumers ($s = 4$) whereas the processed items have a relatively high degree of retail level product differentiation and thus less substitution across items by consumers ($s = 1.333$). The extra market power enjoyed by firms that sell the more differentiated processed food product translates into higher marketing margins.

The prices and margins displayed in Figures 7.2 and 7.3 are intended to take on realistic values. For example, item 1 for standard food has a retail selling price of 4.195, and a raw commodity price of 2.146. The approximate 100 percent mark-up of the retail price over the raw commodity price is what might be expected for non-storable fresh fruits and vegetables that have minimal handling and storage requirements. On the other hand, item 1 for processed food has a retail selling price of 8.773 and a raw commodity price of 1.193, which implies a mark-up of about 630 percent. This level of mark-up is what might be expected for a processed product such as ice cream. It is important to keep in mind that a larger marketing margin does not necessarily imply higher profits for the firm because this margin must first cover fixed costs before profits can be calculated. It is interesting to note that advertising and promotion, which is an important determinant of product differentiation, may contribute significantly to fixed costs.

Sensitivity results

Table 7.2 shows the equilibrium marketing margins for the ten standard and processed food items with different values successively assigned to four of the key parameters of the model. For each of the four sensitivity scenarios, a new set of results was generated by changing one parameter away from its base case value and then using Solver to find the new market equilibrium. The base case results are reported in the top two rows of Table 7.2 to serve as a benchmark.

In the first sensitivity scenario, which is detailed in the third and fourth rows of Table 7.2, the firm is assume to behave as a monopsonist rather than a price taker in the raw commodity market (i.e., $\delta = 1$ versus $\delta = 0$). The additional market power induces the firm to purchase a smaller amount of raw commodity in order to drive the raw commodity price down to a more profitable level. The smaller level of production implies a higher marketing margin because of a higher retail selling price and a lower raw commodity price. The results in Table 7.2 reveal that the absolute marketing margin is most impacted by monopsony pricing for those products with the largest budget share (e.g., item 3).

The second sensitivity scenario in Table 7.2 focuses on the impact of an increase in the firm's unit cost of processing from $c = 1$ in the base case to $c = 2$. In a competitive market a one unit increase in the cost of processing would result in a

Table 7.2 Simulated equilibrium marketing margins ($/unit)

	Base scenario plus:		*Item 1*	*Item 2*	*Item 3*	*Item 4*	*Item 5*
	Base case	Standard	2.05	2.17	2.30	2.17	2.05
		Processed	7.58	8.06	8.84	8.06	7.58
$\delta = 1$	Monopsony pricing	Standard	3.22	3.62	4.08	3.62	3.22
		Processed	8.34	9.31	10.80	9.31	8.34
$c = 2$	High processing cost	Standard	3.30	3.42	3.57	3.42	3.30
		Processed	11.37	11.71	12.31	11.71	11.37
$I = 15$	High income	Standard	2.18	2.32	2.48	2.32	2.18
		Processed	7.87	8.54	9.58	8.54	7.87
$\theta = 3/4$	Higher cross-category substitution	Standard	2.12	2.25	2.40	2.25	2.12
		Processed	7.29	7.54	7.99	7.54	7.29

rise in the marketing margin by one unit to ensure that the added revenue for the food manufacturer is just sufficient to cover the added cost. With monopoly pricing in the retail market and competitive pricing in the raw commodity market, the fifth and sixth rows of Table 7.2 show that the marketing margin rises by more than the one unit increase in the unit processing cost. For example, with item 3 the marketing margin for the standard item is 2.30 in the base case with $c = 1$ and 3.57 in the revised case with $c = 2$. The increase in the marketing margin of $3.57 - 2.30 = 1.27$ is greater than the one unit increase in c.

To better understand the relationship between a monopolist's processing cost and its marketing margin assume for the moment that the price of the raw commodity takes on a constant value because the supply schedule for the raw commodity is horizontal. In this simple case if the marginal cost of processing increases then the firm will reduce production to ensure that marginal revenue rises to the new higher level of marginal cost (marginal revenue equal to marginal cost maximizes profits). In the case of linear demand, the marginal revenue schedule is steeper than the demand schedule so it follows that the increase in price is *less than* the increase in the unit cost of processing. In the current model where $mr_i = \rho p_i$ with $0 < \rho < 1$, it follows that the increase in price is *greater than* the increase in the unit cost of processing. This difference in results for CES demand versus linear demand implies that for a monopoly processor there is no general relationship regarding the extent that p_i increases when c increases.

Before turning to the next sensitivity result it is useful to comment on why the marketing margin is larger for items with a larger share in the consumer's budget (e.g., item 3 versus item 1). The larger budget share items are associated with a greater volume of raw commodity purchases by the firm, which in turn implies a higher raw commodity price. This higher raw commodity price plays the same role as a higher value for c. In other words, the higher budget share items are associated with a higher marketing margin for the same reason that the equilibrium retail price rises by more than the increase in c.

The third sensitivity result that is displayed in Table 7.2 examines the impact of a higher level of consumer income on the equilibrium marketing margin. A comparison of the results in the $I = 15$ pair of rows and the top base case set of rows shows that higher consumer income results in a larger marketing margin for both the standard and processed goods categories. This result is easiest to explain in the simple case where the price of the raw commodity takes on a constant value. In this case the marginal revenue (MR) equal to marginal cost (MC) condition for maximizing profits implies that $\rho p_i = MC$ given that $mr_i = \rho p_i$ for the case of CES demand. Rearrange the $\rho p_i = MC$ condition to obtain $p_i - MC = (1 - \rho)p_i$. This expression shows that the $p_i - MC$ marketing margin is proportional to price. Higher consumer income therefore implies additional demand, which in turn implies a higher equilibrium retail price and marketing margin.

The final sensitivity result shown in Table 7.2 involves allowing a greater degree of substitution between processed and standard goods in stage 1 of the consumer's budget allocation problem. Specifically, θ is increased from 0.5 to 0.75, which results in the stage 1 elasticity of substitution increasing from

$1/(1-0.5) = 2$ to $1/(1-0.75) = 4$. The greater degree of substitution between processed and standard food products results in a more elastic demand and is therefore expected to lower the marketing margin for all food items. A comparison of the marketing margin values in the bottom pair of rows of Table 7.2 with the base case values at the top of the table confirms that this theoretical prediction holds for the case of processed goods but not for the case of standard goods. Further analysis is required to explain this unexpected outcome for the standard good category.

7.6 Concluding comments

In a competitive market the law-of-one price would ensure that the price of a commodity in successive stages of a supply chain would reflect only the added cost of transportation, processing, warehousing, etc. In most modern food supply chains the competitive model is not a realistic portrayal of the market because firms involved with processing, wholesaling, retailing, etc. are generally small in number and operate with sizeable barriers to entry. In these supply chains price is determined by a combination of supply chain costs and the market power that is distributed across firms. Relatively large marketing margins are used to cover sizeable fixed costs and generate profits for food processors, wholesalers, retailers, etc.

The CES utility function that was used for this analysis is very convenient for this type of modeling because it gives rise to realistic results and it easily accommodates two-stage budgeting. In fact, there is no need to restrict the analysis to two stages if a three- or four-stage budgeting procedure is more appropriate. One very nice feature of the CES utility function is that the degree of product differentiation can be managed with a single CES substitution parameter. This parameter indirectly determines the size of the marketing margin by controlling the level of market power that is allocated to the firm in question. In the current analysis with two-stage budgeting product substitution can be controlled at two levels. In stage 1 the extent that the consumer views processed and standard food categories as substitutes can be controlled and in stage 2 the degree of substitution amongst individual food items can be separately controlled.

The introduction of non-competitive pricing in this chapter implies that many of the results that were fundamental to competitive pricing no longer hold. For example, it was shown that an increase in the unit cost of processing by one unit will cause the retail price to increase by more than one unit. Similarly, in a competitive model an increase in consumer income would normally result in an increase in the raw material price that is approximately equal to the increase in the retail price. In this current analysis that involves non-competitive pricing there is no longer a predictable relationship between pricing at different stages of the supply chain and outside determinants of consumer demand.

There has been a great deal of research over the past 20 years with the objective of estimating marketing margins in various agri-food supply chains. Normally the costs of the firms that operate in the supply chain are unknown, so a direct calculation of the marketing margin is not possible. To get around this problem economists often study how prices and production levels change in response to external market shocks such as an increase in consumer income or raw material cost. Empirical estimation of marketing margins are important because excessively large marketing margins are often blamed for low farm prices, high consumer prices and a correspondingly high level of dead weight loss.

Questions

1 The market demand for canola oil and soybean oil is given by the following linear expenditure system (LES):

$$Q_C = \frac{\beta_C}{P_C}\left(I + \gamma_C P_C + \gamma_S P_S\right) - \gamma_C \quad \text{and} \quad Q_S = \frac{\beta_S}{P_S}\left(I + \gamma_C P_C + \gamma_S P_S\right) - \gamma_S$$

Within this pair of equations for $i \in \{C, S\}$, Q_i is quantity of oil demanded (measured in millions of tonnes), P_i is price measured in dollars per tonne, I is per-capita consumer income allocated to food, $\beta_i > 0$ is a budget share parameter and $\gamma_i > 0$ is a cross price effect parameter.

a Suppose that a monopoly food processor supplies canola oil to consumers. Invert the demand equation for canola oil to obtain the inverse demand function for canola oil that shows P_C as a function of Q_C. Use this function to derive the marginal revenue schedule facing the monopoly supplier of canola oil.

b Graph the canola inverse demand equation assuming $P_S = 2{,}500$, $\beta_C = 0.5$, $I = 15{,}000$ and $\gamma_C = \gamma_S = 10$.

2 The monopoly food processor from Question 1 incurs two types of variables costs when supplying canola oil to the retail market. The first variable cost is the cost of purchasing raw oil from canola crushers at a price W_C per tonne (assume one tonne of raw oil is required to produce one tonne of processed oil, and also assume that the processor is a price taker in the raw canola market). The second cost for the monopoly food processor is production cost of Z per tonne of processed canola oil that is supplied to the retail market.

a Write down the expression for marginal cost for the monopoly food processor. Set the expression for marginal cost equal to the expression for marginal revenue that was derived in Question 1(a) and then solve for the profit maximizing quantity of canola oil that the monopolist will supply to the retail market. Because raw and processed canola oils are used in a fixed one-to-one ratio, the expression that has been derived can also be interpreted as the derived demand for raw canola oil.

b Invert the derived demand schedule from part (a) so that W_C is written as a function of Q_C. Using the parameter values specified above along with, $Z = 200$, add this derived demand for raw canola oil to the graph that was constructed in Question 1(b).

3 A competitive canola crushing sector purchases canola seed directly from farmers and crushes the seed into canola oil and canola meal. Each tonne of canola seed yields 0.4 tonnes of canola oil and 0.6 tonnes of canola meal. Let W_F denote the price of 2.5 tonnes of canola seed (i.e., the amount required to produce one tonne of oil). The price of canola meal is fixed at \$200 per tonne, or equivalently 2.5*0.6*200 = \$300 for each tonne of processed oil. In equilibrium, the price of raw canola oil, W_C, and the farm price of canola seed, W_F, must give rise to a fixed crush margin of size M per tonne of oil. Thus, a market equilibrium requires $W_C + 300 - W_F = M$.

   a Let $W_F(X_C)$ denote the crusher's inverse derived demand for canola seed as a function of canola seed quantity. Note that X_C is measured in 2.5 million tonne units. Construct an expression for $W_F(X_C)$ using the expression for the inverse derived demand for raw canola oil from Question 2(b) together with crush margin equation, $W_C + 300 - W_F = M$.
   b Let $W_F^{tonne}(X_C) = W_F(X_C)/2.5$ denote the price of canola seed expressed on a per tonne basis. Using the parameter values specified above along with $M = 350$, graph the $W_F^{tonne}(X_C)$ expression on the existing graph from the previous questions.

4 The inverse farm supply of canola seed is given by the linear function $W_F^{tonne} = a + bX_F$ where X_F is measured in 2.5 million tonne units and W_F^{tonne} is measured in \$/tonne. Using the parameter values specified above together with $a = 100$ and $b = 125$, graph this expression on the existing graph from the previous questions.

   a Show the market equilibrium on your graph. Specifically, identify the equilibrium quantity of canola seed/oil and the various prices, which includes the price of canola seed at the farm level, the price of raw canola oil and the price of processed canola oil, all in \$/tonne. Identify all of the marketing margins and estimate their size. How much of the processor to retail marketing margin is due to monopoly pricing by the food processor?
   b Predict which schedules will shift (and in which direction) and how the various prices will be impacted by the following events:

      i An increase in consumer income
      ii An increase in the price of soybean oil
      iii An increase in the food processor's marginal production cost, Z
      iv An increase in the required crush margin
      v An upward shift in the farm level supply curve for canola seed.

Appendix 7.1 Derivation of equations (7.3) and (7.6)

The procedure for optimally allocating consumer income between the processed and standard food baskets in stage 1 is similar to the procedure for optimally allocating stage 2 expenditures amongst the individual food items in stage 2. A description of a generic procedure that is applicable for both stages of consumer demand is provided in this Appendix.

The generic problem is to choose $q_1, q_2, \ldots, q_n$ to maximize utility $u = \left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{1/\rho}$ subject to $\sum_{j=1}^{n} p_j q_j = I$. The Lagrange function for this constrained optimization problem can be expressed as:

$$L = \left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{\frac{1}{\rho}} + \lambda\left(I - \sum_{j=1}^{n} p_j q_j\right) \tag{7.A1}$$

where λ is the Lagrange multiplier. The first-order conditions are:

$$\frac{\partial L}{\partial q_l} = \frac{1}{\rho}\left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{\frac{1}{\rho}-1} a_l^{1-\rho} \rho q_l^{\rho-1} - \lambda p_l = 0 \quad \textit{for} \;\; l = 1, 2, \ldots, n$$
$$\frac{\partial L}{\partial \lambda} = I - \sum_{j=1}^{n} p_j q_j = 0 \tag{7.A2}$$

The first expression in equation (7.A2) can be solved for λ, first with $l = i$ and then with $l = k$. The two resulting equations can be set equal to each other and the terms cancelled and rearranged to give:

$$p_k q_k = \frac{a_k}{a_i}\left(\frac{p_k}{p_i}\right)^{\frac{1}{\rho-1}} p_k q_i \tag{7.A3}$$

Now substitute equation (7.A3) into the budget constraint given by the second expression in equation (7.A2):

$$I - a_i^{-1} q_i p_i^{\frac{1}{1-\rho}} \sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}} = 0 \tag{7.A4}$$

Equation (7.A4) can be solved to give an expression for consumer demand for the *i*th good:

$$q_i^d\left(p_i, P, \hat{P}, I\right) = \frac{a_i p_i^{\frac{1}{\rho-1}}}{\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}}} I \tag{7.A5}$$

Equation (7.A5) can be used to generate equation (7.3) in the text, which represents optimal stage 1 budget allocation. First, set $i = 2$ and let $a_1 = \alpha$, $a_2 = 1 - \alpha$, $p_1 = P$ and $p_2 = \hat{P}$. As well, change the notation for the substitution parameter from ρ to θ. Finally, multiply the resulting expression by p_i in order to convert equation (7.A5) into a measure of consumer expenditures on the *i*th good.

Equation (7.A5) can also be used to generate equation (7.6) in the text, which represents optimal stage 2 budget allocations for the processed and standard food categories. In this case all that needs to be done is to substitute stage 2 budget allocations, $e\ (P, \hat{P}, I)$ and $\hat{e}\ (P, \hat{P}, I)$, for the income parameter, I, in equation (7.A5).

8 Auctions and competitive bidding

8.1 Introduction

In the previous chapters of this textbook the analysis of commodity prices has assumed that buyers and sellers have perfect information when buying and selling from each other. Perfect information means that commodity sellers can seek out buyers with the highest willingness to pay. Conversely, commodity buyers can seek out sellers with the lowest reserve price. Market clearing ensures that sellers with a reserve price below the equilibrium price choose to sell the commodity and buyers with a willingness to pay above the reserve price choose to purchase the commodity. With complete information competitive price discovery is efficient and the collective welfare of market participants is maximized.

In reality buyers and sellers typically operate with private information regarding who constitutes a potential trading partner, the willingness to pay by potential buyers and the reserve prices of potential suppliers. To minimize the impact of private information a state agency that routinely purchases large volumes of a commodity may use a tendering process, first as a way to promote competition amongst commodity suppliers and second as a way to identify the lowest-cost supplier. A tendering process is similar to a first-price sealed bid auction that is commonly used when an item is to be sold to a group of buyers with heterogeneous valuations. In a standard first-price sealed bid auction a group of potential buyers submit bids to a single seller and the one with the highest bid is allowed to purchase the item for the amount of the bid. In this chapter a first-price sealed bid auction is used as part of an import tendering process whereby bids are submitted by a group of potential sellers and the seller with the lowest bid is awarded the supply contract at the bid price.

The specific purpose of this chapter is to examine the determinants of the price paid by the buyer in a commodity import tender when sellers have private information about their procurement costs. Rather than focusing on the usual intersection of market supply and demand schedules as the mechanism for price determination, the focus here is on the pricing outcome that emerges when potential suppliers strategically bid in a sealed bid tendering process. The main result from this chapter is that private information about procurement costs induces tender participants to use a bid randomization strategy in order to avoid direct price undercutting. Bid randomization implies that for a given sequence of import tenders the winning bid and thus the price paid by the importer is a random variable that bounces around within a predefined interval. Of interest in this chapter is the degree of variability in the import price that can be attributed to private information about procurement costs.

Before constructing a theoretical model of price determination with tendering it is useful to briefly describe some general features of tendering in real-world commodity trade. A sizeable fraction of grain and oilseed tendering is done by state trading enterprises (STEs). In some cases the tendering is done to maintain strategic reserves (e.g., National Food Authority in the Philippines) and in other cases the tendering is used to purchase grain and oilseed stocks on behalf of private domestic firms or for use in state-owned processing facilities (e.g., COFCO of China). In some cases the state trader has an absolute monopoly (e.g., China and Japan) and in other cases the state trader operates in conjunction with private firms. Table 8.1 provides examples of state trading importers and their host countries.

The following quote from a newspaper report nicely illustrates the broad range of multinational firms who submit bids in a typical import tender. The report describes the results of a series of import tenders that were issued by Egypt's state importer, GASC, over the period 1 January 2009 to 16 June 2009.[1]

> Among foreign firms participating directly in GASC tenders Switzerland's Glencore sold the most wheat since the start of the year with 180,000 tonnes of French wheat sold from January 2009 to date. Other foreign sellers winning GASC tenders in the same time period included:
>
> - France's Louis Dreyfus with 60,000 T of Canadian wheat
> - U.S. Bunge with 60,000 T of French wheat
> - U.S. Cargill with 30,000 T of Russian wheat
> - France's Soufflet with 60,000 T of French wheat
> - France's Lecureur with 60,000 T of French wheat and
> - France's Invivo with 60,000 T of French wheat

Table 8.1 Examples of state trading commodity importers

Country	*Agency*
China	China National Cereals, Oil and Foodstuffs Import and Export Corp. (COFCO)
Egypt	General Authority of Supply Commodities (GASC)
India	Minerals & Metals Trading Corporation of India (MMTC)
Indonesia	Badan Urusan Ligistik (BULOG)
Japan	Japanese Food Agency (JFA)
Malaysia	Padiberas Nasional Berhad (BERNAS)
Mexico	Leche Industrializada Conasupo, SA de CV (LICONSA)
Saudi Arabia	Grain Silos and Flour Milling Organization (GSFMO)

> International firms also sell wheat to private Egyptian firms which then go on to participate in GASC tenders. Since 2006, GASC has increasingly turned to companies such as Egyptian Traders, Alex Grain, Venus, Union and Horus, for its supplies

Another news report discusses the range of price bids. This report involves an Indian import tender for rice that was issued on 31 October 31 2009 by three state traders.[2]

> New Delhi Swiss trading firm Ameropa and Singapore-arm of global trading company Louis Dreyfus along with others submitted 18 bids to the rice import tenders floated by MMTC Ltd, STC and PEC to import 30,000 tonne of rice. The bids were submitted in price ranging from $ 372.70 and $ 598.75 (Rs 17,279–27,811) a tonne.

The comparatively wide range of price bids for the Indian rice tender is of particular interest in this chapter. As discussed above, the main theoretical result is that private information about procurement costs induces firms to randomize their bidding behavior. Thus, the variability in price bids within the Indian rice tender may be due to both differences in sourcing costs for the firms submitting bids and the fact that these firms are randomizing their bids over a pre-defined bidding interval.[3]

In the next section, a simple model of an import tendering process with private information is constructed. The tendering process is modeled as a one-buyer, two-seller game where each seller is endowed with either a high or low cost of sourcing the commodity. The price the buyer expects to pay is determined by solving the game for its unique Bayesian Nash equilibrium. In Section 8.2 the game focuses on the case where sellers use standard "pure" bidding strategies. In Section 8.3 the analysis is extended to include "mixed" strategies because the main result from Section 8.2 is that a pure strategy equilibrium does not exist. Numerical analysis is used in Section 8.4 to measure the impact of private information on the price the importer expects to pay, to identify the determinants of price variability and to illustrate a typical pricing profile by simulating random bidding by sellers.

8.2 Base model

A procurement agency in an importing country, I, (hereafter, the "buyer") wishes to purchase X units of a commodity from a particular exporting country, E. To reduce notation assume that the size of the units is such that $X = 1$. To facilitate the purchase the buyer announces a tender, which is equivalent to a first-price sealed bid auction. Two commodity handling firms within E (hereafter, "seller 1" and "seller 2") submit bids in this tender. In addition to various technical details such as the date and location of delivery, the bid specifies the price the seller is to receive in exchange for supplying the commodity. The winning bidder is the seller who submits the lowest price in the sealed-bid auction. In the case of a tie bid, a coin flip determines the seller who is awarded the supply contract.

The winning bidder has two options for handling the physical commodity. The first option is to purchase the commodity at export position in E and then deliver the commodity to the buyer in I. This option will be referred to as a "commercial procurement strategy". The unit cost of purchasing the commodity at export position, plus the cost of transporting the commodity from E to I, is the same for both sellers, and is equal to W. The second option for the winning bidder is to purchase the commodity from the local source of production within E, and then pay for transporting the commodity directly from this region to the buyer in I. This option will be referred to as a "private procurement strategy". The unit cost of purchasing the commodity locally, plus the cost of transporting the commodity directly from country E to I is the same for both sellers and is equal to $W - \delta$, where the constant δ takes on a positive value.

It is worth discussing why private procurement (PP) has a lower cost for the seller than commercial procurement (CP) (i.e., $\delta > 0$). When using a CP strategy, assume that T_{LE} and T_{EI} are the respective unit handling and transportation costs when the commodity is shipped from the local region to the export position in E and from the export position in E to the buyer in I. Included in these parameters are transport costs, storage, insurance and administrative fees. Based on the law-of-one-price, it follows that the price of the commodity is equal to $W - T_{EI}$ at export position in E and $W - T_{EI} - T_{LE}$ within E's local production region. With a PP strategy, the seller's total cost of shipping and handling between the local region and the buyer in I is equal to T_{LI}. It is natural to assume that $T_{LI} < T_{EI} + T_{LE}$ because direct export involves less handling and logistics than indirect export. By defining $\delta = T_{EI} + T_{LE} - T_{LI}$ as the transportation cost differential, it follows that the net cost of purchasing the commodity in the local region in E and shipping it directly to the buyer in I is equal to $W - \delta$.[4]

The obvious question is, why doesn't each seller pursue the PP strategy with cost $W - \delta$ instead of the CP strategy with cost W? Even though a PP strategy is preferred over the CP strategy, a seller may not have sufficient time or logistical capacity to pursue a PP strategy, especially if the time between the announcement of the winning bidder and the contracted delivery date is short. The feasibility of pursuing a PP strategy is assumed to be private information for each seller and as such cannot be observed by the seller's competitor. It is this assumption of private information that makes the pricing problem interesting. Although a seller's specific procurement strategy is private information, each seller knows that with probability P the rival seller has a PP strategy and with probability $1 - P$ the rival seller has a CP strategy. Maintaining symmetry in knowledge about these probabilities greatly simplifies the analysis.

The previous assumptions imply that the price the buyer should expect when initiating a tendering process should be derived as the equilibrium outcome of a game with private information, otherwise known as a Bayesian game. For the case at hand, the Bayesian game consists of two players (sellers 1 and 2) where each player is randomly assigned by nature one of two types (PP or CP) and players form beliefs about their opponent's type (PP with probability P and CP with probability $1 - P$). The Bayesian Nash equilibrium contains four equilibrium strategies because each type has a strategy and there are two players each with two possible types. The four conditional bidding strategies are used to calculate the unconditional price the buyer should expect to pay when initiating a tendering process.

Pure strategies

Consider first a game with pure bidding strategies, which means that a particular bid is associated with a particular type for each player. Specifically, if seller $i \in \{1,2\}$ has type PP and can therefore source the commodity at price $W - \delta$ then this seller will bid B^i_{PP}. Otherwise, if seller i has type CP and can therefore source the commodity at price W, then this seller will bid B^i_{CP}. To constitute a pure strategy Bayesian Nash equilibrium, B^1_{PP} must maximize expected profits for seller 1^{PP} while taking as given the bid values of seller 2, B^2_{PP} and B^2_{CP}, and while believing that seller 2 has type PP with probability P and type CP with probability $1 - P$. Similarly, B^i_{CP}. must maximize expected profits for seller 1^{CP} while taking as given seller 2's bid values and maintaining beliefs as described above. Because the problem is symmetric (i.e., the distribution of costs and beliefs for the two sellers are the same) it follows that the previous restrictions on seller 1's bidding strategies will be the same restrictions that must hold for seller 2 in a pure strategy Bayesian Nash equilibrium.

The bidding game with pure strategies is similar to standard Betrand pricing, so it is easy to rule out bids in excess of W, which is the highest possible procurement cost for each supplier. Indeed, if $B^2_{PP} \geq B^2_{CP} > W$ then, depending on the probability value P, seller 1 will choose to bid either slightly under B^2_{PP} or slightly under B^2_{CP}. This outcome implies that seller 2^{PP} would never win the auction with $B^2_{PP} \geq B^2_{CP} > W$, which in turn implies that bids by seller 2 must satisfy $B^2_{PP} < B^2_{CP}$ when $B^2_{CP} > W$. But with this type of pricing seller 1 will once again choose to bid either slightly under B^2_{PP} or slightly under B^2_{CP}. Now seller 2^{CP} would never win the auction when $B^2_{PP} < B^2_{CP}$ and $B^2_{CP} > W$. Therefore, the best seller 2^{CP} can do is set $B^2_{CP} = W$ and hope that seller 1 is also type CP and therefore bids W, in which case zero profits are split between the two sellers.[5] Given that $B^2_{CP} = W$ it follows from the previous argument that $B^2_{PP} < W$, which implies that bids in excess of W are ruled out.

Knowing that B^i_{CP} must equal W and B^i_{PP} must be below W in any Bayesian Nash equilibrium, the remainder of the analysis seeks to identify an equilibrium value for B^i_{PP}. The analysis begins by noting that seller 1^{PP} will submit a bid that is either slightly less than W or slightly less than B^2_{PP}. In particular, if $B^2_{PP} \in (W - P\delta, W)$ then seller 1^{PP} will bid slightly less than B^2_{PP} but if $B^2_{PP} \in [W - \delta, W - \delta P]$ then seller 1^{PP} will bid slightly less than W. To understand why this is the case, suppose $\hat{B}^1_{PP}$ is the bid of seller 1^{PP} that slightly undercuts B^2_{PP} . This undercutting guarantees an auction win for seller 1^{PP}, and such a win will generate surplus equal to $\hat{B}^2_{PP} - (W - \delta)$. On the other hand, bidding slightly less than W results in an auction win for seller 1^{PP} only when seller 2 bids W because her type is CP (this occurs with probability $1 - P$). In this case an auction win generates surplus for seller 1^{PP} that is slightly less than δ. Consequently, expected profit for seller 1^{PP} when bidding slightly under W is approximately equal to $(1 - P)\delta$. Comparing $\hat{B}^2_{PP} - (W - \delta)$ with $(1 - P)\delta$, it can be seen that seller 1^{PP} will choose to bid slightly less than B^2_{PP} if $B^2_{PP} \geq W - P\delta$ and will choose to bid slightly less than W if $B^2_{PP} < W - P\delta$. The bidding strategy for seller 2 is the mirror image of seller 1's bidding strategy.

In the previous paragraph it was established that the bid for each type of PP player depends on whether the opposing player's strategy calls for a bid above or below $W - P\delta$. This result can be used to demonstrate that a Bayesian Nash equilibrium for this pure strategy game is non-existent. Specifically, if seller 1^{PP} chooses a bid from the interval $(W - P\delta, W)$ then seller 2^{PP} has an incentive to slightly undercut and seller 1^{PP} will lose for sure. If instead seller 1^{PP} chooses to bid from the interval $[W - \delta, W - P\delta]$ then seller 2^{PP} will choose a bid slightly below W. But now seller 1^{PP} has an incentive to revise the bid upward to just under the bid of seller 2^{PP}. However, both bids cannot be drawn from $(W - P\delta, W)$ because when a bid is drawn from this interval the two sellers have an incentive to undercut each other. The logical conclusion is that a pure strategy Bayesian Nash equilibrium is non-existent.

8.3 Mixed strategies

Because a Bayesian Nash equilibrium does not exist when the two sellers use pure strategies, it is necessary to consider a game that allows for mixed strategies. A mixed strategy equilibrium occurs when a player randomly selects a bid from a predefined interval using a predefined probability function. This randomization prevents the pair of sellers from inferring each other's bids, which is why an equilibrium does not exist in the pure strategy game. A key feature of a mixed strategy equilibrium is that a seller must earn the same expected profits with each of the different bids from the predefined interval. Because expected profits are the same, the seller is indifferent as to which bid from the interval is submitted, and thus randomizing over all bids in the interval is a rational strategy.

It is important to note that only type PP sellers are able to use a mixed strategy. A type CP seller is not able to use a mixed strategy because it cannot profitable bid below W, and any bid above W would always be undercut by a competitor for reasons similar to those described in the pure strategy game. Thus, in a mixed strategy equilibrium a type CP seller will always submit a bid equal to W.

Knowing that type CP sellers will bid W with certainty, it is now possible to derive the mixed bidding strategy for a type PP seller. Below it is established that seller $i \in \{1, 2\}$ with type PP will randomly select a bid from the interval $[B^*_i, W]$, where $W - \delta < B^*_i < W$ is a variable to be determined. Let $G_i(B)$ be the cumulative probability distribution function for seller $i \in \{1,2\}$, which governs how seller i's bid is chosen from the interval $[B^*_i, W]$. Specifically, $G_i(\hat{B})$ is the probability that seller i's submitted bid takes on a value equal to or less than $\hat{B}$. Of course, $G_i(B^*_i) = 0$ and $G_i(W) = 1$.

Consider the expected profits for seller 1^{PP} assuming that a type CP seller 2 bids W and a type PP seller randomly selects a bid from the interval $[B_2^*, W]$ using the probability function $G_2(B)$. The relevant expression is:

$$E(\pi_1)^{PP} = (B_1 - (W - \delta))[1 - P + P(1 - G_2(B_1))] \quad (8.1)$$

Equation (8.1) shows that expected profits for seller 1^{PP} are equal to the product of the surplus that is earned if seller 1 wins the auction, $B_1 - (W - \delta)$, and the probability of winning, $1 - P + P(1 - G_2(B_1))$. The probability of seller 1^{PP} winning consists of $1 - P$, which is the probability of winning given that seller 2 has type CP, plus $P(1 - G_2(B_1))$, which is the probability of winning given that seller 2 has type PP and has chosen a bid that exceeds B_1.

To quality as a mixed strategy the probability function, $G_2(B)$, must ensure that expected profits for seller 1 take on the same value for all $B_1 \in [B_1^*, W]$. Thus, it must be the case that:

$$G_2(B_1) = \frac{B_1 - C}{P(B_1 - (W - \delta))} \quad (8.2)$$

where C is a constant, whose value is determined below. To verify that expected profits are independent of B_1 for seller 1, substitute equation (8.2) into equation (8.1) to obtain:

$$E(\pi_1)^{PP} = C - (W - \delta) \quad (8.3)$$

The next step is to simultaneously determine the equilibrium values for the unknown parameter of the probability function, C, and the lower limit of the bidding interval, B_2^*. Noting that $G_2(W) = 1$, it follows from equation (8.2) that:

$$C = W - P\delta \quad (8.4)$$

Equation (8.4) can be substituted into equation (8.2) and (8.3) to give:

$$G_2(B_1) = \frac{B_1 - (W - P\delta)}{P(B_1 - (W - \delta))} \quad (8.5)$$

and

$$E(\pi_1)^{PP} = (1 - P)\delta \quad (8.6)$$

Equation (8.5) together with the $G_2(B_2^*) = 0$ restriction implies:

$$B_2^* = W - P\delta \quad (8.7)$$

Because the problem is symmetric, it follows that $B_1^* = B_2^* = W - P\delta$.

In summary, seller 1^{PP} should randomly select from the interval $[W - P\delta, W]$ using the probability distribution function that is given by equation (8.5).

Because the two sellers have the same cost structure and beliefs, the bidding problem facing seller 2 is identical to that described above for seller 1. The "*i*" subscript on the B^* and $G(B)$ functions can therefore be dropped because these functions are the same for both sellers. The general solution to the bidding problem is that the type CP version of each seller will bid W and the type PP version of each seller will randomly choose a bid from the interval $[W - P\delta, W]$ using the cumulative probability function $G(B) = [B - (W - P\delta)]/[P(B - (W - \delta))]$.

It is important to check that a type PP seller does not have an incentive to deviate from this mixed strategy. If a type PP seller 1 bids $B_1^* - \varepsilon$ for any $\varepsilon \in (B_1^* - (W - \delta), B_1^*)$ instead of randomizing over the interval $[B_1^*, W]$, then seller 1 will win the auction for sure and earn profits equal to $B_1^* - \varepsilon - (W - \delta)$, which, after substituting in $B_1^* = W - P\delta$ from equation (8.7), reduces to $(1 - P)\delta - \varepsilon$. Comparing this expression with equation (8.6) reveals that bidding $B_1^* - \varepsilon$ rather than randomizing over the interval $[B_1^*, W]$ reduces expected profits. Similarly, bidding higher than W ensures losing the auction and thus is inferior to randomizing over the interval $[B_1^*, W]$.

Expected size of winning bid

The purpose of this section is to derive an expression for the expected size of the winning bid. This variable is obviously of interest to the buyer because the surplus earned by the buyer is inversely related to the size of the bid. Later in the analysis the expected sizes of the winning bids with and without private information are compared in order to assess the extent that a buyer's expected price is impacted by private information. The first part of this section is devoted to deriving an expression for the expected size of the winning bid conditional on the sellers' types. The second part derives an expression for the unconditional expected value of the winning bid after accounting for the distribution of types for each seller.

The winning bid is the minimum value of the pair of bids submitted by the two sellers, which is denoted B^{min}. Let $E\,(B^{min} \mid i, j)$ with $i, j \in \{CP, PP\}$ be the expected size of the winning bid in the Bayesian Nash equilibrium conditional on the sellers' types. For example, $E\,(B^{min} \mid CP, PP)$ is the expected winning bid when one seller is endowed with a CP strategy and the other seller is endowed with a PP strategy. The expected winning bid function is easy to construct when both sellers have type CP because a type CP seller always bids W. Consequently:

$$E\,(B^{min} \mid CP, CP) = W \tag{8.8}$$

It is also easy to calculate the expected size of the winning bid when one seller has type CP and the other seller has type PP because in this case the type PP seller always wins the auction. Noting that a type PP seller draws her bid from the interval (B^*, W) based on the probability density function $g\,(B)$, it follows that:

$$E\left(B^{\min} \mid CP, PP\right) = \int_{B^*}^{W} Bg\left(B\right)dB \tag{8.9}$$

An explicit expression for the probability density function, $g(B)$, can be derived from the cumulative distribution function, $G(B)$, which is given by equation (8.5). Specifically, using equation (8.5) without the subscripted "2" it follows that,

$$g(B) = \frac{dG(B)}{dB} = \frac{1-P}{P}\frac{\delta}{\left(B-\left(W-\delta\right)\right)^2} \tag{8.10}$$

Equations (8.9) and (8.10) together define an expression for $E(B^{\min} \mid CP, PP)$.

Deriving an expression for $E(B^{\min} \mid PP, PP)$, which is the expected size of the winning bid when both sellers have type PP, is somewhat more complicated because it requires calculating the expected value of the minimum of a pair of random variables. In Appendix 8.1 the theory of order statistics is used to establish that:

$$E\left(B^{\min} \mid PP, PP\right) = 2\int_{B^*}^{W} B\left(1-G(B)\right)g(B)dB \tag{8.11}$$

Equation (8.5) without the subscripted "2" and equation (8.10) can be used with equation (8.11) to obtain an expression for $E(B^{\min} \mid PP, PP)$.

Now that conditional expressions for the expected value of the winning bid have been derived, it is possible to derive an expression for the unconditional expected value of the winning bid while accounting for the fact that each seller is assigned type PP with probability P and type CP with probability $1-P$. The relevant expression is:

$$E(B^{\min}) = (1-P)^2\, E(P^{\min} \mid CP, CP) + 2P\,(1-P)\, E\,(P^{\min} \mid CP, PP) + P^2\, E(P^{\min} \mid PP, PP) \tag{8.12}$$

The expression for $E\,(B^{\min})$ can be made more explicit by substituting equations (8.8), (8.9) and (8.11) into equation (8.12). It is not possible to obtain analytical solutions to the integrals in equations (8.9) and (8.11), so numerical integration techniques will be used below to generate a solution value for $E\,(B^{\min})$.

A variable of interest in this chapter is the standard deviation of the winning bid in the context of repeated tendering. Under what conditions should the buyer expect to see highly variable bids versus closely clustered bids? Price variability across successive tenders is due to bid randomization, and bid randomization is due to private information about procurement costs. Hence, there is a close connection between the significance of the private information and the degree of price variability. Unfortunately, deriving an expression for the standard deviation of the winning bid is complex because it involves working with order statistics. Later in the analysis Monte Carlo techniques are used to generate a large quantity of simulated bid data, and an estimate of standard deviation is derived from this data set.

Before turning to the simulation model, it is useful to first derive an expression for the expected size of the winning bid with the assumption that suppliers who are competing in the import tender have complete information about their rivals' procurement costs. As discussed above, the expected size of the winning bid for this complete information case is required in order to assess the extent that private information for sellers affects the expected price for the buyer.

Calculating the expected size of the winning bid in the complete information case involves solving for a standard Betrand pricing equilibrium. Betrand pricing implies that with probability $(1 - P)^2$ the two sellers that compete in the tender will both have a CP procurement strategy, in which case the winning bid necessarily takes on a value of W. With probability $2P(1 - P)$ there will be one seller of each cost type, in which case the type PP seller, knowing that its rival has cost W, will bid slightly below W and win the tender. Finally, with probability P^2 the pair of rival sellers will each have a PP strategy. In this case competition will induce each seller to bid $W - \delta$, and so the winning bid will be of size $W - \delta$. Gathering the various bits together it is possible to conclude that the expected value of the winning bid in the complete information case can be expressed as:

$$E\,(B^{\min})^{complete} = (1 - P)^2\, W + 2P(1 - P)W + P^2\,(W - \delta) \tag{8.13}$$

8.4 Simulation model

The two key components of the simulation model that is presented in this section are: (1) Monte Carlo simulation procedures; and (2) numerical integration. Each procedure will be discussed in turn.

Monte Carlo simulation

The purpose of the Monte Carlo simulation is to generate repeating bids for pairs of randomly selected sellers. The law of large numbers implies that if the number of repeated tenders is large, then the mean and standard deviation of the winning bid that is estimated with the simulation data will be a good estimate of the theoretical mean and standard deviation. Normally tens of thousands of simulated data points should be used in order to obtain highly accurate results. For this chapter the simulation consists of 1,000 tender repetitions that are generated within an Excel workbook.[6]

Simulating a bid has two parts. In the first part an indicator variable, $\tilde{I}$, is randomly generated to identify whether the seller has type PP ($\tilde{I} = 1$) or CP ($\tilde{I} = 0$). The variable $\tilde{I}$ is assumed to take on a value of 1 with probability P and a value of 0 with probability $1 - P$. In the second part a bid, $\tilde{B}^{PP}$, is randomly drawn from the interval [B*, W] for a type PP seller according to the cumulative probability function, $G(B)$, where the expression for $G(B)$ is given by equation (8.5). After combining the two parts, the random bid, $\tilde{B}$, can be expressed as:

$$\tilde{B} = \tilde{I}\tilde{B}^{PP} + (1 - \tilde{I})\, W \tag{8.14}$$

To randomly generate an expression for $\tilde{B}^{PP}$, begin by inverting equation (8.5) to obtain:

$$B(\tilde{G}) = W - P\delta \frac{1-\tilde{G}}{1-P\tilde{G}} \tag{8.15}$$

Within equation (8.15), $\tilde{G}$ represents a cumulative probability value. According to the theory of random simulation, if $\tilde{G}$ is a uniform random variable that is drawn from the [0, 1] interval then B ($\tilde{G}$) is a random variable that is drawn from the interval [B*, W] according to the cumulative probability function, G (B). Thus, equation (8.15), together with Excel's RAND(), which randomly generates a uniform random variable, can be used to simulate bids by a type PP seller in a typical import tender. More details are supplied below.

Numerical integration

The purpose of this section is to describe the numerical integration procedure that will be used to obtain solution values for the integrals in equations (8.9) and (8.11). Numerical procedures for solving integrals vary widely with respect to accuracy and complexity. Fortunately with a high-speed computer it is possible to use a comparatively simple procedure with a small step size and a large number of steps to generate a reasonably accurate answer. For the current analysis a composite trapezoidal rule with $N = 500$ steps is used to solve the integrals in equations (8.9) and (8.11).[7] The formula for the trapezoidal approximation to $\int_a^b f(x)dx$ is given by:

$$\int_a^b f(x)dx \approx \frac{b-a}{2N}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{N-1}) + f(x_N)\right) \tag{8.16}$$

where:

$$x_k = a + k\frac{b-a}{N} \quad for\ k = 0,1,\ldots,N \tag{8.17}$$

Details about how equations (8.16) and (8.17) are used in the current analysis are provided below.

Construction of simulastion model

The simulation model is displayed in Figure 8.1. The three parameters, W, δ and P are contained in cells B2:B4. Notice that the model has been normalized by setting $W = 1$ (i.e., the cost of sourcing the commodity for a seller with a CP strategy is 1 per unit). By setting $\delta = 0.3$ in cell B3 it is assumed that the PP strategy has a 30 percent cost advantage over the CP strategy. The $P = 0.7$ assumption in cell B4 implies that the odds ratio for a PP versus CP type endowment for a particular seller is 7/3. The $P = 0.7$ assumption also implies that there exists a 9 percent probability that two CP types will compete, a 49 percent probability that two PP types will compete and a 42 probability that one CP type and one PP type will compete.

The B^* variable in cell B7 is calculated to equal 0.79 using equation (8.7) without the subscripted "2". Cells B9:B11, which correspond to equations (8.8), (8.9) and (8.11), respectively, contain the conditional expectation formulas for $E(B^{\min} \mid CP, CP)$, $E(B^{\min} \mid CP, PP)$ and $E(B^{\min} \mid PP, PP)$, respectively. The values in cells B10:B11 are obtained through numerical integration. The numerical integration procedure with $N = 500$ is displayed in cells A18:F519 of Figure 8.1 (to save space only the first six rows of results are displayed in Figure 8.1).

For numerical integration the expression for $E\left(B^{\min} \mid CP, PP\right) = \int_{B^*}^{W} Bg\left(B\right)dB$ from equation (8.9) can be rewritten using equations (8.16) and (8.17) as:

	A	B	C	D	E	F	G	H
1	**Parameters**							
2	W	1	Cost of sourcing with CP strategy					
3	delta_	0.3	Cost savings when sourcing with PP strategy					
4	P	0.7	Probability of being endowed with PP strategy					
5								
6	**Variables**		=W-P*delta_					
7	B_star	0.79	Lower limit of bidding interval with PP strategy					
8	*Conditional E(B_min)*		=W					
9	CP-CP	1	=(E20+2*SUM(E21:E519)+E520)*(W-B_star)/1000					
10	CP-PP	0.855						
11	PP-PP	0.824	=(F20+2*SUM(F21:F519)+F520)*(W-B_star)/1000					
12	E{B_min}_Priv	0.853						
13	E{B_min}_Comp	0.853	=(1-P)^2*B9+2*P*(1-P)*B10+P^2*B11					
14								
15	**Calculations of Int_1 and Int_2 variables (assumes n = 500 steps)**							
16	=B_star+A20*(W-B_star)/500		=((1-P)/P)*delta_*B20/(B20-(W-delta_))^2					
17		=(B20-(W-P*delta_))/(P*(B20-(W-delta_)))			=2*D20*E20			
18	k	B_k	G(B_k)	1-G(B_k)	B_k*g(B_k)	B_k*g(B_k)[1-G(B_k)]		
19	0	0.790	0.000	1.000	12.540	12.540		
20	1	0.790	0.007	0.993	12.430	12.348		
21	2	0.791	0.013	0.987	12.322	12.159		
22	3	0.791	0.020	0.980	12.215	11.974		
23	4	0.792	0.026	0.974	12.110	11.793		
24	5	0.792	0.033	0.967	12.006	11.615		
25	------ continues until K = 500 ------							

Figure 8.1 Main body of competitive bidding simulation model.

$$E\left(B^{\min} \mid CP, PP\right) \approx \frac{W - B^{*}}{2(500)}\left(B_0 g\left(B_0\right) + 2B_1 g\left(B_1\right) + 2B_2 g\left(B_2\right) + \ldots + 2B_{N-1} g\left(B_{N-1}\right) + B_N g\left(B_N\right)\right) \quad (8.18)$$

Within equation (8.18) the function $g(B_k)$ is given by equation (8.10) with:

$$B_k = B^{*} + k\frac{W - B^{*}}{500} \quad \text{for} \quad k = 0,1,\ldots,N \quad (8.19)$$

The procedure for numerically calculating a value for $E(B^{\min} \mid PP, PP)$ in equation (8.11) is the same as that described above except equation (8.18) should be modified by replacing the $B_k g(B_k)$ terms with $B_k(1 - G(B_k))\, g(B_k)$ terms, where the expression for $G(B)$ is given by equation (8.5) without the subscripted “2”.

The column labels in row 18 of Figure 8.1 correspond to the various pieces of the numerical integration procedure. Specifically, the B_k values in cells B19:B519 of Figure 8.1 were calculated using equation (8.19) with a link to the index variable k that resides in cells A19:A519. Values for the $G(B)$ variable in cells C19:C519, and the corresponding values for the $1 - G(B_k)$ variable in cells D19:D519, were calculated with equation (8.5) and the B_k values in column B. Values for the $B_k g(B_k)$ expression in cells E19:E519 were calculated with equation (8.10) and the B_k values in column B. In cells F19:F519 the results from cells D19:F519 and E19:E519 are combined to generate values for the $B_k\,(1 - G(B_k))\, g\,(B_k)$ expression. The numerical approximation of $E\left(B^{\min} \mid CP, PP\right) = \int_{B^{*}}^{W} Bg\left(B\right)dB$ that resides in cell B10 was calculated using equation (8.18) and the sum of the values in cells E19:E519. The numerical approximation of $E\left(B^{\min} \mid PP, PP\right) = 2\int_{B^{*}}^{W} B\left(1 - G(B)\right)g(B)dB$ that resides in cell B11 was calculated in a similar way.

The simulated value for the unconditional expected value of the minimum bid, $E(B^{\min})$, is displayed in cell B12. This value was calculated using equation (8.12) and the expressions for $E(B^{\min} \mid CP, CP)$, $E(B^{\min} \mid CP, PP)$ and $E(B^{\min} \mid PP, PP)$, whose values are displayed in cells B9:B11. Cell B13 shows the expected value of the winning bid under the assumption of complete information, which was calculated using equation (8.13).

Private information impact on expected price

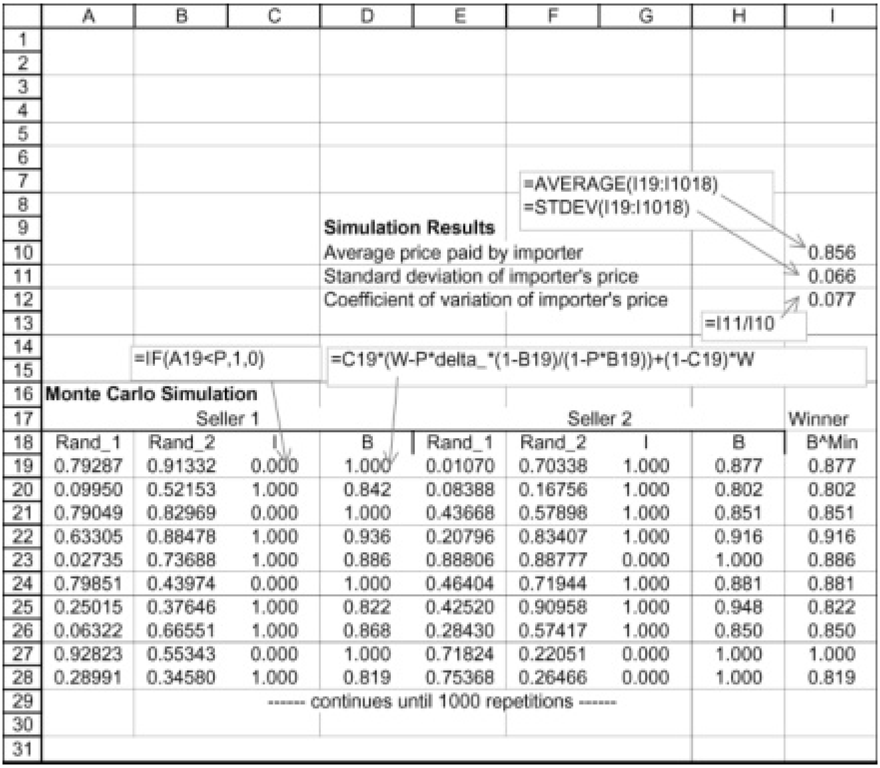
The impact of private information on price is revealed by comparing the unconditional expected value of the minimum bid with private information, which resides in cell B12 of Figure 8.1, with the analogous value for the complete information case, which was calculated in cell B13 using equation (8.13). Despite the complex formula for the private information case and the rather simple formula for the complete information case, the two 0.853 outcomes are identical. Thus, private information for the pair of sellers has no impact on the expected price for a buyer who purchases the commodity with a competitive import tender. This equivalency result holds for all feasible values of the parameters, which reside in cells B2:B4.

It is important to note that the equivalency result that is implied by the pair of results in cells B12:B13 may not hold in a more general model. The intuition of why there is not a strong link between private information and the expected price can be described as follows. Private information has both a negative and a positive impact on the bid that is submitted by a type PP seller. In this particular model the outcome that the expected price is not impacted by private information implies that the negative and positive impacts exactly offset each other. A negative impact arises because private information does not allow a type PP seller to fully capitalize on scenarios when the competing seller is of type CP. Conversely, a positive impact arises because private information prevents sellers from aggressively undercutting each other's price when the pair of rival firms both have a PP type. The standard result from the asymmetric information literature that private information results in an information rent (which would translate into a higher price paid by the buyer) does not hold for this analysis of competitive bidding.

Random bidding results

Cells D19:D28 of Figure 8.2 show the first ten out of the 1000 bids that were randomly generated for seller 1 as part of the Monte Carlo procedure. The bids account for both the variability in the seller's type (CP versus PP) and the variability in the random bid that is submitted when the type is PP. Cells H19:H28 show the analogous bids for seller 2. The minimum of the two bids, which is deemed the winning bid and the actual price that the importer pays for the commodity, is displayed in cells I18:I28. For example, in the first simulation trial the bid values are 1 for seller 1 and 0.877 for seller 2, in which case seller 2 wins the tender and the commodity price is 0.877. In the second simulation trial, the bid values are 0.842 for seller 1 and 0.802 for seller 2, in which case seller 2 once again wins the tender and the commodity price is 0.802.

The random bids contained in Figure 8.2 were generated using equations (8.14) and (8.15). For seller 1, cells A19:B28 contain two independent sets of values for a [0, 1] uniform random variable that were generated using Excel's RAND() function. The first set of random values in cells A19:A28 is used to determine whether seller 1 has type PP ($I = 1$) or type PP ($I = 0$). Specifically, the random outcome for the I indicator variable is generated using an Excel IF statement that outputs a value of $I = 1$ if the uniform random variable is less than P and outputs a value of $I = 0$ otherwise. This procedure works because the probability that a uniform random variable takes on a value less than P is equal to P. The second set of random variables in cells B19:B28 is used to calculate the level of the bid if seller 1's type is PP. Specifically, the uniform random variables from cells B19:B28 are



	A	B	C	D	E	F	G	H	I
9				**Simulation Results**					
10				Average price paid by importer					0.856
11				Standard deviation of importer's price					0.066
12				Coefficient of variation of importer's price					0.077
16	**Monte Carlo Simulation**								
17		Seller 1				Seller 2			Winner
18	Rand_1	Rand_2	I	B	Rand_1	Rand_2	I	B	B^Min
19	0.79287	0.91332	0.000	1.000	0.01070	0.70338	1.000	0.877	0.877
20	0.09950	0.52153	1.000	0.842	0.08388	0.16756	1.000	0.802	0.802
21	0.79049	0.82969	0.000	1.000	0.43668	0.57898	1.000	0.851	0.851
22	0.63305	0.88478	1.000	0.936	0.20796	0.83407	1.000	0.916	0.916
23	0.02735	0.73688	1.000	0.886	0.88806	0.88777	0.000	1.000	0.886
24	0.79851	0.43974	0.000	1.000	0.46404	0.71944	1.000	0.881	0.881
25	0.25015	0.37646	1.000	0.822	0.42520	0.90958	1.000	0.948	0.822
26	0.06322	0.66551	1.000	0.868	0.28430	0.57417	1.000	0.850	0.850
27	0.92823	0.55343	0.000	1.000	0.71824	0.22051	0.000	1.000	1.000
28	0.28991	0.34580	1.000	0.819	0.75368	0.26466	0.000	1.000	0.819
29			------ continues until 1000 repetitions ------						

Figure 8.2 Monte Carlo results for competitive bidding simulation model.

interpreted as the $\tilde{G}$ variable in equation (8.15) and the resulting values for $B(\tilde{G})$ are the simulated bids for the type PP seller. Equation (8.14) is used to combine the random outcomes for seller type, which are contained in cells C19:C28 with the simulated bids for the type PP seller into the random bid variables, which are contained in cells D19:D28. The process for generating the random bids for seller 2 that are displayed in cells H19:H28 is similar to that of seller 1.

The first 100 pair of bid values from Figure 8.2 are plotted in Figure 8.3 with seller 1 on the horizontal axis and seller 2 on the vertical axis. If a bid pair lies below the 45° line, then seller 2 is awarded the tender and the buyer must pay the price that is identified on the vertical axis. Conversely, if a bid pair lies above the 45° line, then seller 1 is awarded the tender and the buyer must pay the price that is identified on the horizontal axis.

Figure 8.3 clearly illustrates the random nature of the bidding by sellers. Vertically aligned bids on the right edge of Figure 8.3 correspond to seller 1 being endowed with a CP type. Similarly, horizontally aligned bids along the top edge of Figure 8.3 correspond to seller 2 being endowed with a CP type. Bids in the

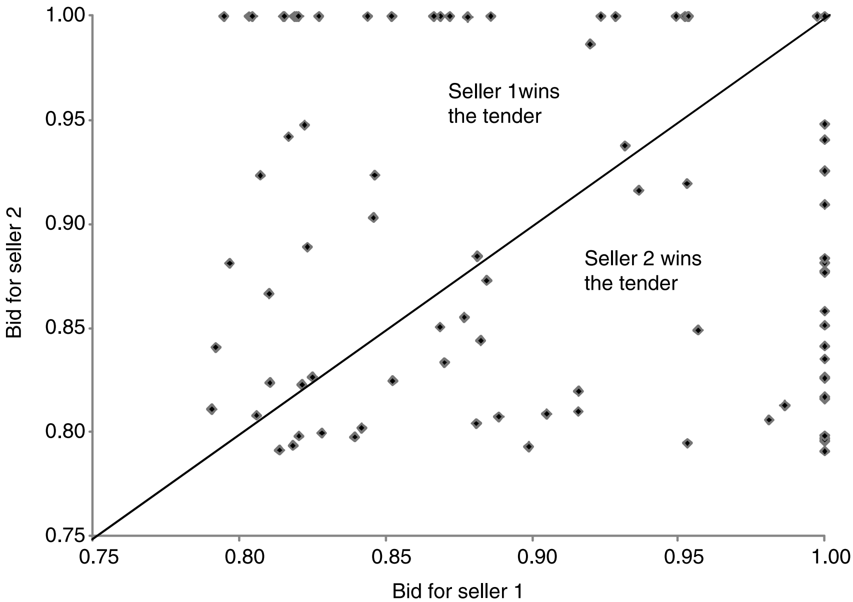


Figure 8.3 Simulated bids for 100 pairs of randomly selected sellers.

main body of the graph near the 45° line correspond to scenarios where both sellers are type PP and both submit bids that are of similar size. Bids near the top left or bottom right corners of Figure 8.3 correspond to scenarios where both sellers are type PP but both submit very different bids (one high and one low). In general, Figure 8.3 reveals considerable variation in the bids submitted by sellers who compete in a series of tenders. Of course, much of this variability in submitted bids will also be reflected as variability in the winning bid.

Determinants of bidding variability

The purpose of this section is to discuss the main determinants of bidding variability. Cells I10:I12 in Figure 8.2 display the average, standard deviation and coefficient of variation of the winning bid. These values were calculated using the 1,000 randomly generated pairs of bids that are contained in the lower portion of Figure 8.2. For the case at hand, the average bid is estimated to equal 0.856 and the coefficient of variation is estimated to equal 0.077. Thus, there is an approximate 8 percent variation in the value of the winning bid relative to the mean value across the 1,000 simulated import tenders.

The first determinant of pricing variability is the cost difference between the types CP and PP sellers. If the cost difference variable, δ, is increased from its current value of 0.3 to a revised value of 0.5, then the revised data that would appear in Figure 8.2 would reveal that the coefficient of variation increases from 0.077 to 0.145. This result is expected because the bidding interval for the type PP seller, $[W - P\delta, W]$, increases in width with a larger value for δ. In other words, with a lower cost, a type PP seller will include lower bids when randomly selecting bid values, and as a result there will be more overall variation in the bid values and the size of the winning bid.

The second determinant of pricing variability is P, which is the probability that a seller is endowed with type PP. Theory predicts that as $P \rightarrow 1$ pricing variability will vanish because all bids will take on a value of $W - \delta$ (i.e., the case of Betrand pricing with identical sellers, each with cost $W - \delta$). Conversely, as $P \rightarrow 0$ pricing variability will also vanish because all bids will take on a value of W (i.e., the case of Betrand pricing with identical sellers, each with cost W). Thus, variability in the tender price will be significant only for intermediate values of P.

Figure 8.2 can be used to examine how the coefficient of variation of the winning bid values change with a change in P. If P takes on the low value of 0.2, the coefficient of variation, which is displayed in cell I12, is equal to 0.018, which is comparatively small. The coefficient of variation climbs from 0.018 to 0.046 if P is increased from 0.2 to 0.4, and climbs again from 0.046 to 0.07 if P is increased from 0.4 to 0.6. With an additional increase in P from 0.6 to 0.8 the coefficient of variation increases only slightly from 0.07 to 0.074. Increases in P beyond 0.08 result in a reduction in the coefficient of variation for the reasons described above. Overall, price variability is at a maximum when the odds of a type PP endowment is about 75 percent.

8.5 Concluding comments

Recall from the Introduction of this chapter that when an import tender was announced by three state importers in India on 31 October 2009 the submitted bids ranged in value from US$372.70 to US$598.75/tonne. The results of this chapter help to explain this large spread. In particular, this analysis shows that private information induces sellers to randomly select bids from a predefined interval, and the size of the interval is directly related to the extent that information is private. Randomizing bid value is strategically important for sellers because it prevents rival firms from inferring bidding behavior and therefore benefiting through price undercutting.

Mixed strategies are common in many real-life situations, so it should not be surprising that employing a mixed strategy when bidding is economically efficient. For example, in the field of sports, football quarterbacks randomize between passing and ground plays and in baseball pitchers randomize over the type of pitch that will be thrown. For the case of bidding randomization is effective because when matched with a rival seller who is also randomizing, each bid has associated costs and benefits that exactly offset each other. Offsetting costs and benefits emerge because each bid implies both a particular probability of winning the tender, and an expected amount that is earned conditional on winning.

There are many sophisticated models of competitive bidding in the economics and finance literature. The model presented in this chapter was very simple, but yet the results of this analysis are qualitatively similar to the results from more general models. The standard textbook model of competitive bidding with private information assumes that the sourcing cost of each seller is a continuously distributed random variable rather than a two-point probability distribution. The seller auctions an item to two buyers, each of whom have a private valuation v, where v is drawn from a uniform distribution on the [0, 1] interval. In the Nash equilibrium, each potential buyer bids $v/2$. In most bidding models, especially those for which there are more than two bidders and a continuum of types, the solution procedure is complex because it involves numerically solving systems of differential equations with free boundary conditions.

Questions

1 Suppose two grain exporters are bidding in a grain import tender. Exporter A can source the grain and deliver it to the importer for $340/tonne. The equivalent cost for exporter B is either $345/tonne or $355/tonne. Suppose exporter A believes there is a θ percent chance that B will bid $345/tonne and a $1 - \theta$ percent chance that she will bid $355/tonne.

- a Construct a function that shows A's expected profits if he bids amount X where $X \in [340,355]$. Note that in the case of a tie bid, a coin flip is used to determine the winning bidder.
- b Graph the expected profit function from part (a) for the case of $\theta = 0.7$. Allow X to vary between $340 and $355.
- c Based on the graph from part (b), what is the optimal bid for exporter A?
- d For a particular value of θ what are the two bids that exporter A would consider submitting? For what values of θ is it optimal for exporter A to bid the lower of those two bids?

2 Consider the same scenario as Question 1 except now exporter A believes that B will draw her bid from the interval [345, 355] and all outcomes are equally likely (i.e., B's bid is drawn from a uniform distribution).

- a Calculate exporter A's expected profits as a function of A's bid value $X \in [340,355]$.
- b Graph exporter A's expected profit from part (a).
- c Based on the graph from part (b), what is the optimal bid for exporter A?

3 Consider the same scenario as Question 1 except now exporter B has cost $345/tonne with certainty, and A knows this. As well, an implicit agreement between the respective governments of exporters A and B to minimize "price wars" stipulates that bid values must be either $347/tonne or $355/tonne.

- a Set up a 2 × 2 normal form game with exporter A the "row" player and exporter B the "column" player. The top row for A and the left column for B corresponds to bidding $347/tonne. The bottom row for A and the right column for B corresponds to bidding \$355/tonne. In the event of tie bids, a coin flip determines the winning bidder. Identify all pure strategy Nash equilibrium in this game.

b Solve for the mixed strategy equilibrium for this bidding game. Specifically, let Pr_i denote the probability that exporter i bids \$347/tonne and $1 - Pr_i$ the probability that i bids \$355/tonne. Solve for the equilibrium pair of Pr_i values.

c Compare expected profits for each exporter with the mixed strategy in part (b) versus the pure strategy in part (a).

Appendix 8.1: Derivation of equation (8.11)

To derive the expression for $E\ (B^{\min} \mid PP, PP)$ note that $B^{\min} = \min\ \{B^1_{PP}, B^2_{PP}\}$, where B^1_{PP} and B^2_{PP} are independently drawn bids from the interval (B^*, W) based on the cumulative probability function, $G\ (B)$, which is given by equation (8.6). Let $F\ (\beta)$ be the cumulative probability function for $B^{\min}$ the winning bid (i.e., the probability that either $B^1_{PP} < \beta$ or $B^2_{PP} < \beta$). The probability that one bid is below and the other is above β is equal to $G\ (\beta)\ [1 - G\ (\beta)]$. The probability that both bids are below β is equal to $G\ (\beta)^2$. Noting that there are two combinations for the one below and one above outcome, it follows that:

$$F\ (\beta) = 2G\ (\beta)[1 - G\ (\beta)] + G\ (\beta)^2 \tag{8.A1}$$

There are two remaining steps to complete the derivation of equation (8.11). First, the probability density function for the winning bid when both sellers have type PP can be derived from equation (8.A1):

$$f(\beta) = \frac{dF(\beta)}{d\beta} = 2\left[1 - G(\beta)\right]g(\beta) \tag{8.A2}$$

Equation (8.A2) can be used to calculate the expected value of $B^{\min}$ as shown by equation (8.11) in the text.

9 Bargaining in bilateral exchange

9.1 Introduction

The purpose of this chapter is to examine the role of bargaining as a determinant of price in agricultural commodity markets. Bargaining is a key feature of vertically coordinated agri-food supply chains where production and marketing contracts between processors and commodity producers are common. Contract clauses that are negotiated can include simple variables such as price and the date of delivery, or complex variables such as quality-dependent pricing schedules, production processes to be followed and how various capital and operating costs are to be shared. Production contracts are common in a variety of industries including poultry in Brazil and the US, cotton and tobacco in Mozambique and Zambia and various horticultural crops in Canada. If a well functioning spot market exists for the commodity then the contract price may be linked to the spot price rather than being negotiated on a regular basis. In many industries spot markets have become too thin to serve as a reliable reference price, in which case price negotiations are a central feature of the contracting process.

Bargaining is also common when producers form cooperatives, bargaining associations or marketing boards to countervail the market power wielded by processors. In these situations the marketing or production contract will be negotiated and signed either by the agency that is representing the producers or by the individual producers with active support from the agency. In either case, for standard graded commodities such as pork and milk the main variables that are negotiated are price, quantity and the timing of delivery. Bargaining is easiest to think about and model if there is a single processor that is negotiating the price of the commodity with a single producer agency. This type of one-on-one bargaining, which is often referred to as a bilateral monopoly, arises if producers in a comparatively large geographical region are required by law to sell their production through a single marketing agency (i.e., single desk selling), and the region in question contains a single processing firm. The higher the transportation cost to move the commodity in or out of the region, the stronger the bilateral monopoly relationship for the processor and producers of the region.

The specific purpose of this chapter is to model the bilateral bargaining game between a processor and a marketing board that has single-desk selling authority when representing commodity producers at the bargaining table. The commodity in question is assumed to be homogenous and continually produced at a uniform rate with one available production technology. Consequently, the only parameter to be negotiated is price, and re-negotiation is not necessary because there is no uncertainty in the model. The model is specified for a marketing board with single-desk selling authority, but it is also relevant for a voluntary farmer cooperative or bargaining association that has a high level of producer support.

It is important to note that bargaining is the only economically meaningful way to solve for the equilibrium outcome of a bilateral monopoly scenario. Depending on the distribution of bargaining power, the equilibrium price can range from the monopsony processor outcome at the low end to the monopoly marketing board outcome at the high end. The distribution of bargaining power depends on a variety of factors including the level of "patience" and the values of the inside and outside options for the processor and marketing board. In the bargaining literature "patience" refers to the extent that future profit flows are discounted while bargaining is taking place. If the commodity in question is highly perishable, such as strawberries or milk, then the marketing board will typically have low bargaining power because it prefers a quick settlement rather than engaging in multiple rounds of back-and-forth offers and counteroffers. Conversely, if the processor has an inflexible production schedule then the bargaining power of the processor will typically be low because of a high degree of impatience during the bargaining process.

Inside and outside options refer in general terms to the opportunity cost of the commodity while bargaining. The inside option is a measure of the profit that flows to the bargaining party while the contract is being negotiated. For the current analysis the inside option for the board (processor) is the flow of profits that results when the commodity is sold (purchased) in the spot market rather than being delivered (accepted) according to the terms of the contract. The outside option is a long-run concept because it reflects the difference in profits with a successfully negotiated contract versus a permanent failure of the bargaining process. It is useful to view the inside option as a pre-contracting flow variable and the outside option as a termination stock variable.

The majority of this chapter relies on sequential non-cooperative game theory (Rubinstein's model in particular) as a way to model the outcome of a bargaining game.[1] Prior to the development of sequential bargaining theory, economists solved most bargaining problems using the Nash bargaining approach. The Nash approach solves for the contract price that satisfies a series of economically "sensible" axioms such as Pareto optimality and independence of irrelevant alternatives. The Nash bargaining solution is easy to implement, but important features of the bargaining process such as the dynamics of offers and counteroffers are not explicit. Despite the substantial differences in how the problem is specified with Nash bargaining versus non-cooperative sequential bargaining the same equilibrium outcome emerges if a reasonable set of assumptions are made. These assumptions are described in greater detail below.

Before constructing the bargaining model it is useful to describe some real-world scenarios where producer agencies bargain with processors. In the US many agricultural industries have cooperative bargaining associations as a result of enabling legislation that was passed in the late 1960s.[2] By 2002 the National Cooperative Bargaining Council (NCBC) had a membership of 40 organizations, 29 of which were processing fruit and vegetable growers. The main commodities represented by cooperative bargaining associations include select fruits and vegetables destined for California processing facilities, raw milk and sugar beets. Producers in these sectors typically operate with a processor contract, so a main function of the bargaining association is to collectively negotiate the terms of the contract. Producer participation in bargaining associations is generally voluntary. Consequently, the sectors that are most likely to succeed with collective bargaining are those with members who are geographically concentrated and who have relatively homogenous production and marketing practices.

In the Canadian province of Ontario about 60 percent of agricultural commodities (by value) is marketed through twenty-one provincial marketing boards and three representative associations.[3] The role of a board in a particular commodity sector varies widely, in some cases restricted to a specific task such as distributing market information, and in other cases involving a variety of tasks including negotiating price and other variables with the processor. Negotiation boards cover a range of commodities including apples, grapes, potatoes and processing vegetables. For example, the Ontario Asparagus Marketing Board contracts with the processor on behalf of producers, sets the price that growers will receive on an annual basis and collects payments from the processor for subsequent distribution to producers. The Ontario Pork Producers' Marketing Board gives producers the option to produce hogs with a board-negotiated processor contract or with the marketing activity transferred to the board.

Australia has recently begun to make changes that will enable a greater degree of collective bargaining by commodity producers.[4] Prior to these changes producers who wished to collectively bargain had to apply for permission from the Australian Competition and Consumer Commission (ACCC). Permission would only be granted if producers could demonstrate that collective bargaining would result in a net benefit for the Australian public. The proposed changes would allow producers to notify the ACCC of their intent to collectively bargain, and the onus would be on the ACCC to demonstrate that collective bargaining was not in the public's best interest if the ACCC wished to block the proposal. The EU is also proposing to relax various legislative requirements in order to make collective bargaining by agricultural producers easier to establish and manage. With record low milk prices for EU dairy farmers in 2009, a number of strategies to promote collective bargaining among milk producers have been proposed.

In the next section the assumptions of the basic model are described and the inside options for the board and processor are specified. In Section 9.3 the bargaining equilibrium is derived under the assumption that the processor has no outside option, and the outside and inside options are the same for the board. The equilibrium is examined first with discrete amounts of time between bargaining rounds, in which case there exists a first-offer advantage, and then with the amount of time between offers tending to zero in the limit. In Section 9.4 the limiting case of zero time between offers is shown to be identical to the bargaining outcome that would be obtained using the axiomatic Nash bargaining approach. Section 9.4 concludes with a brief examination of how an outside option for the processor changes the equilibrium outcome of the bargaining game. A bargaining case study involving Australian dairy farmers and a milk processor is presented and analyzed in Section 9.5. A summary of the analysis and concluding comments are contained in Section 9.6.

9.2 Model

A processor situated in region 1 requires a continuous flow of K units of a raw commodity to produce a retail food product. The processor can import the commodity from a competitive cash market in region 2 at a total cost of W_P per unit. Alternatively, the processor can use a long-term contract to locally purchase the commodity from a single-desk marketing board (hereafter the "board"). On the selling side of the market the board can choose to sell its members' production to the processor (with a contract), or it can instead sell in region 2's cash market at a net price of W_B per unit. To keep things simple assume that the total amount of the commodity that is supplied by producers is K units, which matches the requirements of the processor.[5]

Assume that the local cash price in region 2 is exogenously fixed at W_2 per unit. The cash market is assumed to be competitive and so it follows from the law-of-one-price that $W_B = W_2 - T_{12}$ and $W_P = W_2 + T_{21}$ where T_{12} is the transport cost when shipping from region 1 to region 2 and T_{21} is the transport cost when shipping in the opposite direction.[6] Subtracting the former expression from the latter gives $W_P - W_B = T_{12} + T_{21}$. It is this difference in the cash market opportunity cost for the processor and the board that makes these two groups prefer to contract with each other rather than utilize region 2's cash market. The $W_P - W_B = T_{12} + T_{21}$ expression shows that higher transportation costs between regions 1 and 2 results in more bargaining surplus to be divided between the board and the processor.

At date 0 the board and processor will begin bargaining over the terms of the supply contract. The only variable to be determined is the supply price, W, because both parties find it in their best interest to exchange K units. Bargaining takes place in successive rounds, and each round lasts for Δ units of time. The procedure for bargaining is "alternating offers". In round 1 the processor makes an offer, and this offer is either accepted or rejected by the board. If it is rejected by the board, the board can make a counteroffer in round 2, or the board can permanently stop the bargaining process. If the board chooses to make a counteroffer in round 2, the processor can either accept or reject the counteroffer, and if the latter option is chosen the processor can submit its own counteroffer or permanently stop the bargaining process. This alternating bargaining procedure continues until either an offer is accepted or one of the bargaining parties chooses to permanently stop the bargaining process. If the board and processor reach an agreement, a fully enforceable long-term contract is drawn up. In this case no future rounds of bargaining are needed because there is no uncertainty in the economic environment.

The processor uses a fixed proportions production technology that requires one unit of raw commodity to produce one unit of the retail product. Let P be the exogenous price of the retail product. Also, let C_P be the unit cost of production for the processor (excluding the cost of purchasing the raw commodity) and let C_B be the unit cost of production for the board.[7] Let $\pi = (P - C_P - C_B)\,K$ denote the combined flow of profits for the board and processor if an agreement is reached.[8] With a contract in place, profits for the board and processor will equal $\pi_B = (W - C_B)\,K$ and $\pi_P = (P - W - C_P)\,K$ respectively.[9]

Rather than bargaining over W, the board and processor can bargain over how π is be divided up. To see how this works, rearrange the $\pi_P = (P - W - C_P)\,K$ expression to give $W = P - C_P - {}^{\pi_P}\!/_K$ and substitute this expression into $\pi_B = (W - C_B)\,K$ together with $\pi = (P - C_P - C_B)\,K$ to give $\pi_B = \pi - \pi_P$. Thus, for given values for P, C_P and C_B, if the processor proposes π_P as either an offer or counteroffer a value for π_B is implied by $\pi_B = \pi - \pi_P$ and a value for W is implied by $W = P - C_P - {}^{\pi_P}\!/_K$. Similarly, if the board proposes π_B, a value for π_P is implied by $\pi_P = \pi - \pi_B$ and a value for W is implied by a rearranged version of $\pi_B = (W - C_B)\,K$.

Inside options

While the on-going rounds of bargaining are in progress, the board can generate a profit flow equal to $Z_B = (W_B - C_B)\,K$ by shipping the commodity to the region 2 cash market. The variable Z_B represents the board's "inside option". Similarly, while bargaining is in progress, the processor can generate a profit flow equal to $Z_P = (P - W_P - C_P)\,K$ by purchasing the commodity from the region 2 cash market. In this case Z_P represents the processor's inside option. In general bargaining theory, an inside option is the flow of utility to the player while bargaining is in progress. The inside option normally vanishes once an agreement has been reached. Noting from above that $W_P - W_B = T_{12} + T_{21}$, the combined value of the inside options for the board and processor can be expressed as $Z_B + Z_P = (P - T_{12} - T_{21} - C_B - C_P)\,K$.

Bargaining surplus (BS) is formally defined as $BS = (\pi - (Z_B + Z_P))\,K$ and can be described as the combined increase in profit flow for the board and processor when the contract is signed versus when the negotiations are taking place and the respective parties are each earning their inside option. Using the previously derived expressions, $\pi = (P - C_P - C_B)\,K$ and $Z_B + Z_P = (P - (T_{12} + T_{21}) - C_B - C_P)\,K$, it follows that an expression for bargaining surplus, $BS = (\pi - (Z_B + Z_P))\,K$, can be written as $BS = (T_{12} + T_{21})\,K$. This expression makes sense because given the opportunities for both the board and the processor to utilize the cash market in region 2, the advantage of contracting with each other is the combined transportation cost savings.

Outside options

If either the board or the processor permanently walks away from the bargaining table, then the parties will exercise their respective "outside options". Assume that the processor's outside option is to sell the processing plant to an external buyer for an amount S_P. Let r_P denote the processor's rate of discount and let $s_P = r_P S_P$ denote the flow value of the processor's outside option. The outside option is said to be binding if $s_P > (P - W_P - C_P)\,K$ (i.e., the flow value of the processor's outside option exceeds the flow value of the processor's inside option). If the outside option is binding and if bargaining between the board and the processor permanently fails, then the processor will choose to exercise its outside option rather than continue to operate with its inside option.

The board's outside option is to permanently sell its commodity in the region 2 cash market for a net price of W_B. This assumption implies that the inside option and the flow value of the outside option are the same for the board because both are equal to $s_B = Z_B = (W_B - C_B)\,K$. In a more general model that explicitly allowed for fixed costs, the board's outside option would include its shutdown decision.

9.3 Bargaining equilibrium with $S_P = 0$

In this section the bargaining problem with no outside option for the processor (i.e., $S_P = 0$) is considered. This assumption is relaxed in Section 9.4.

Equilibrium conditions

As indicated above, bargaining begins at date 0 with the processor offering π_P, where π_P is the proposed profit flow for the processor and by construction $\pi - \pi_P$ is the proposed profit flow for the board. If the board rejects the processor's offer, then the board can propose π_B as a counteroffer. As before, the board's counteroffer of π_B implies a specific profit flow, $\pi - \pi_B$, for the processor. The series of alternating offers will continue at times Δ, 2Δ, 3Δ,. . ., until either the offer is accepted by one of the participants, or the bargaining process is permanently stopped. During a single round of bargaining with time duration Δ the present value of the flow of profits from the inside option is equal to $Z_B(1 - \delta_B)/r_B$ for the board and $Z_P(1 - \delta_P)/r_P$ for the processor, where $\delta_i = e^{-r_i\Delta}$.[10]

Consider an arbitrary round of bargaining at time $t\Delta$ that begins with the processor making an offer π_P. Suppose the board decides to reject the processor's offer and plans instead to make a counteroffer, π_B, at time $(t + 1)\Delta$. If the processor accepts the board's counteroffer, then as of time $t\Delta$, the present value of the stream of current and future profits for the board is equal to $\frac{Z_B(1-\delta_B)}{r_B} + \frac{\delta_B \pi_B}{r_B}$. The first term of this expression is the date $t\Delta$ present value of the inside option for the board starting from when the board rejects the processor's initial offer and ending when the processor accepts the counteroffer that is made by the board. The second term of the previous expression is the date $t\Delta$ present value of the stream of profits for the board when an agreement is reached at time $(t + 1)\Delta$, based on the board's counteroffer.[11]

The board is indifferent between accepting the processor's offer at date $t\Delta$, the value of which in present value terms is $\frac{\pi - \pi_P}{r_B}$, and rejecting the offer and countering with π_B at date $(t+1)\Delta$, if $\frac{\pi-\pi_P}{r_B}=\frac{Z_B(1-\delta_B)}{r_B}+\frac{\delta_B\pi_B}{r_B}$. For similar reasons, at an arbitrary point in time, the processor is indifferent between accepting an offer made by the board, which has value $\frac{\pi-\pi_B}{r_\Delta}$, and countering with an offer of π_P if $\frac{\pi-\pi_B}{r_P}=\frac{Z_P(1-\delta_P)}{r_P}+\frac{\delta_P\pi_P}{r_P}$. These two conditions that make the board and processor indifferent between accepting and countering must hold for any equilibrium bargaining outcome. These two conditions imply that the board will accept any offer from the processor that satisfies $\pi_P \le \pi_P^*$, and the processor will accept any offer from the board that satisfies $\pi_B \le \pi_B^*$, where π_B^* and π_P^* are defined by:

$$\frac{\pi-\pi_P^*}{r_B}=\frac{Z_B(1-\delta_B)}{r_B}+\frac{\delta_B\pi_B^*}{r_B} \tag{9.1a}$$

and

$$\frac{\pi-\pi_B^*}{r_P}=\frac{Z_P(1-\delta_P)}{r_P}+\frac{\delta_P\pi_P^*}{r_P} \tag{9.1b}$$

Intuition of equilibrium conditions

There are subtle details associated with these equilibrium strategies that are not discussed here. Moreover, the formal derivation and proof of existence and uniqueness of a sub-game perfect Nash equilibrium for this bargaining game is beyond the scope of this analysis.[12] The following result is therefore asserted rather than formally established. If the processor makes the first offer at date 0, it will propose that the profit flow is π_P^* for itself and $\pi-\pi_P^*$ for the board. The board will immediately accept this offer. Conversely, if the board makes the first offer, it will propose that the profit flow is π_B^* for itself and $\pi-\pi_B^*$ for the processor. The processor will immediately accept this offer. Because of this immediate acceptance outcome, neither party earns profits from their inside options.

The intuition of this immediate acceptance result is that any delay in acceptance would reduce the overall surplus to be divided between the two bargaining parties by an amount $(T_{12}+T_{21})K$ per round of bargaining, and thus would be inefficient. In other words, agreement dominates permanent abandonment, and given that an agreement will be reached the two bargaining parties have an incentive to sign the contract as soon as possible because signing early maximizes the joint surplus that is available to be divided. This outcome depends on the very strong assumption that the board and processor know all of the parameters of the model, including each other's cost of production. In a more general model that realistically allowed for incomplete information and learning, the equilibrium outcome for the bargaining game will generally involve multiple rounds of bargaining before an agreement is reached.

Suppose the processor makes the first offer. Why does π_P^* and $\pi - \pi_P^*$ represent the respective equilibrium profit flows for the processor and the board? By construction, if the processor offered anything more than π_P^* then the board would reject the offer and make a counteroffer one period later. The processor cannot expect to earn more than π_P^* in present value terms by waiting for the board's counteroffer. Moreover, there is no strategic value for the processor in offering anything less than π_P^*. Consequently, the processor will offer exactly π_P^* at date 0. The board knows that any counteroffer that exceeds π_B^* will be rejected by the processor, and that it will earn the same amount in present value terms if it immediately accepts the processor's offer of $\pi - \pi_P^*$ versus countering with π_B^* one round later. Thus, the board will immediately accept the processor's date 0 offer.

Equation (9.1) can be solved to give

$$\pi_B^* = Z_B + \frac{1-\delta_P}{1-\delta_B\delta_P}\left(\pi - Z_B - Z_P\right) \tag{9.2a}$$

and

$$\pi_P^* = Z_P + \frac{1-\delta_B}{1-\delta_B\delta_P}\left(\pi - Z_B - Z_P\right) \tag{9.2b}$$

Equation (9.2a) can be interpreted as follows. If the board makes the first offer, the board's equilibrium profit flow, π_B^*, consists of the flow of profits that can be earned using the inside option, Z_B, plus a fraction of the bargaining surplus, $BS = \pi - Z_B + Z_P$. Equation (9.2b) has a similar interpretation from the perspective of the processor. Equation (9.2) shows that the relative values of the discount parameters, δ_B and δ_P, determine how the bargaining surplus is split between the board and the processor (more on this below).

First-offer advantage

An important property of the equilibrium bargaining outcome is that there exists a first-offer advantage. Indeed, the processor earns profit flow π_P^* when making the first offer and earns profit flow $\pi - \pi_B^*$ when the board makes the first offer. This situation is similar for the board. Using equation (9.2), it can be shown that:

$$\pi_P^* - \left(\pi - \pi_B^*\right) = \frac{\left(1-\delta_B\right)\left(1-\delta_P\right)}{1-\delta_B\delta_P}\left(\pi - Z_B - Z_P\right) \tag{9.3}$$

Noting that $\pi - Z_B - Z_P = (T_{12} + T_{21})\ K > 0$, it follows immediately from equation (9.3) that $\pi_P^* - (\pi - \pi_B^*) > 0$. Thus, a bargaining agent always prefers to make the first offer rather than accept the first offer.

The first-offer advantage, which depends on $\frac{1-\delta_i}{1-\delta_B\delta_P}$ in equation (9.3), disappears as the bargaining time interval, Δ, shrinks to zero. To calculate this limit, substitute $\delta_i = e^{-r_i\Delta}$ into the previous equation to obtain $\frac{1-e^{-r_i\Delta}}{1-e^{-(r_B+r_P)\Delta}}$. The limit of this expression as $\Delta \to 0$ can be derived using l'Hospital's rule. Specifically, the limit is equal to the derivative of the numerator divided by the derivative of the denominator, both with respect to Δ, and both evaluated at $\Delta = 0$. Thus, in the limit $\frac{1-\delta_i}{1-\delta_B\delta_P}$ converges to $\frac{r_i}{r_B+r_P}$. Substituting this latter expression into equation (9.2) gives expressions for the equilibrium profits earned by the board and processor when $\Delta \to 0$:

$$\pi_B^* = Z_B + \frac{r_P}{r_B+r_P}\left(\pi - Z_B - Z_P\right) \tag{9.4a}$$

and

$$\pi_P^* = Z_P + \frac{r_B}{r_B+r_P}\left(\pi - Z_B - Z_P\right) \tag{9.4b}$$

Equation (9.4) reveals that in the limiting case of $\Delta \to 0$ it is no longer necessary to identify which agent is making the offer and which one is accepting the offer because in this case $\pi - \pi_B^* = \pi_P^*$ and $\pi - \pi_P^* = \pi_B^*$. Moreover, the size of the discount rate, r_B, relative to r_P, is the only determinant of how the bargaining surplus is divided between the board and the processor. A higher discount rate implies less patience, which in turn implies a smaller share of the bargaining surplus. This result makes sense because a less patient participant in the bargaining game is more anxious to achieve a settlement to secure the higher flow of profits, and is therefore in a weaker bargaining position relative to a more patient participant. Equation (9.4) shows that the bargaining surplus is equally split in the special case where the board and the processor have the same discount rate.

To conclude this section note that equation (9.4) shows the bargaining outcome in terms of profits, but from a commodity pricing perspective the variable of interest is the equilibrium transfer price, W^*. Recall that W is implied by π_P according to the processor's rearranged profit equation, $W = P - C_P - \pi_P/K$, and is also implied by π_B according to the board's rearranged profit equation, $W = C_B + \pi_B/K$. If equations (9.4a) and (9.4b) together with $Z_P = (P - W_P - C_P)K$ and $Z_B = (W_B - C_B)K$ are respectively substituted into the two previous expressions, the following pair of expressions emerges for W^*:

$$W^* = W_P - \frac{r_B}{r_B+r_P}\left(T_{12} + T_{21}\right) \tag{9.5a}$$

and

$$W^* = W_B + \frac{r_P}{r_B + r_P}\left(T_{12} + T_{21}\right) \tag{9.5b}$$

Noting that $W_P - W_B = T_{12} + T_{21}$, it is easy to verify that equations (9.5a) and (9.5b) are equivalent expressions for W^*. Equation (9.5) confirms that if the board is more impatient than the processor, as reflected by a relatively high value for r_B, then W^* will take on a relatively low value, and if the processor is more impatient than the board, then W^* will take on a relatively high value.

9.4 Additional results

In this section the sequential bargaining equilibrium that was derived above is compared to the popular Nash bargaining solution. Following this the analysis returns to focus on the sequential game with the objective of relaxing the assumption of no outside option for the processor.

Nash bargaining solution

Economic models often utilize the "Nash bargaining solution" rather than the bargaining game described above. This substitution is acceptable because with the standard set of assumptions the two outcomes are the same in the limiting case of $\Delta \to 0$. The Nash bargaining approach is a much older theory than the sequential bargaining approach. In Nash bargaining the solution is one that satisfies a series of economically-appealing axioms. In contrast, the sequential bargaining outcome is the Nash equilibrium outcome of a well-specified non-cooperative game.[13] The purpose of this section is to show that the bargaining outcomes for the board and the processor are the same with the two alternative bargaining models in the limiting case where $\Delta \to 0$.

The asymmetric version of the Nash bargaining solution is obtained as the outcome to the following optimization problem:

$$\underset{\pi_B, \pi_P}{MAX} \left\{\left(\pi_B - D_B\right)^{\theta}\left(\pi_P - D_P\right)^{1-\theta}\right\} \text{ subject to } \pi_B + \pi_P = \pi \tag{9.6}$$

Within equation (9.6) the parameter θ reflects the relative bargaining strength of the two players and the D variables are referred to as "threat" or "disagreement" points. Maximizing the natural log of equation (9.6) subject to $\pi_B + \pi_P = \pi$ will give the desired solution and is mathematically easier to work with. Hence, the Lagrange function for this optimization problem with λ serving as the Lagrange multiplier can be written as:

$$L = \theta \ln(\pi_B - D_B) + (1 - \theta)\ln(\pi_P - D_P) + \lambda(\pi - \pi_A - \pi_B) \tag{9.7}$$

The first-order conditions for optimally choosing π_B and π_P are $dL/_{d\pi_B} = \theta\,(\pi_B - D_B)^{-1} - \lambda = 0$, $dL/_{d\pi_P} = (1-\theta)\,(\pi_P - D_P)^{-1} - \lambda = 0$, and $\pi_B + \pi_P = \pi$. Solving the first pair of equations gives $(1-\theta)(\pi_B - D_B) = \theta(\pi_P - D_P)$. This equation, together with $\pi_B + \pi_P = \pi$, results in

$$\pi_B^* = D_B + \theta(\pi - D_B - D_P) \tag{9.8a}$$

and

$$\pi_P^* = D_P + (1-\theta)(\pi - D_B - D_P) \tag{9.8b}$$

Notice that equation (9.8) has the same general structure as the expression for equilibrium profits in the sequential bargaining game as given by equation (9.4). Specifically, the D (disagreement) variables take the place of the Z (inside option) variables and the bargaining strength parameter, θ takes the place of the discount rate ratio, $r_P/(r_B + r_P)$. Thus, with an appropriate re-interpretation of the parameters of the model the Nash bargaining solution is the same as the limiting case of the sequential bargaining solution. However, it is important to keep in mind that the disagreement parameters with Nash bargaining and the inside option variables with sequential bargaining are very different. A disagreement point is similar to an outside option rather than an inside option because it is defined as the profits earned by the agent in the event that the bargain fails. As is shown next, if the outside option is included in the sequential bargaining game, then the sequential game outcome and the Nash bargaining outcome are no longer the same.

Outside option for processor

In the previous section, the outside option for the processor was suppressed by setting $s_P = 0$, where $s_P = r_P\,S_P$ is the continuous flow of returns to the processor if the plant was sold to an outside investor for price S_P. Now assume that $s_P > Z_P$ where $Z_P = (P - W_P - C_P)\,K$ to ensure that selling the plant is more profitable for the processor than earning the inside option in the event that bargaining with the board permanently fails. Throughout this section the outside option for the board continues to remain equal to the board's inside option; i.e., $s_B = Z_B = (W_B - C_B)\,K$.

To incorporate the outside option into the analysis, the equilibrium conditions given by equation (9.1) must be modified as follows:

$$\frac{\pi - \pi_P^{**}}{r_B} = \max\left\{\frac{Z_B\left(1-\delta_B\right)}{r_B} + \frac{\delta_B \pi_B^{**}}{r_B}, \frac{s_B}{r_B}\right\} \tag{9.9a}$$

and

$$\frac{\pi - \pi_B^{**}}{r_P} = \max\left\{\frac{Z_P\left(1-\delta_P\right)}{r_P} + \frac{\delta_P \pi_P^{**}}{r_P}, \frac{s_P}{r_P}\right\} \tag{9.9b}$$

The "**" double superscripts indicate an equilibrium solution with the outside option incorporated into the analysis. As before, equation (9.9) can be solved for the equilibrium bargaining outcome. These derivations are tedious and somewhat complicated, so only the solution for the limiting case of $\Delta \rightarrow 0$ is presented.[14]

The solution to the bargaining problem in the limiting case of $\Delta \rightarrow 0$ and with both inside and outside options available to the bargaining agents can be written as:

$$\left(\pi_B^{**}, \pi_P^{**}\right) = \begin{cases} \left(\pi_B^*, \pi_P^*\right) & \text{if } s_B \leq \pi_B^* \text{ and } s_P \leq \pi_P^* \\ \left(\pi - s_P, s_P\right) & \text{if } s_B \leq \pi_B^* \text{ and } s_P > \pi_P^* \\ \left(s_B, \pi - s_B\right) & \text{if } s_B > \pi_B^* \text{ and } s_P \leq \pi_P^* \end{cases} \tag{9.10}$$

For the current analysis, $s_B = Z_B < \pi_B^*$, which implies that the board's outside option does not constrain the optimal solution. Thus, attention can be restricted to the top two rows of equation (9.10). By assumption, $s_P > z_P$, which implies that there are two possibilities. If $s_P < Z_P + \frac{r_B}{r_B + r_P}\left(\pi - Z_B - Z_P\right)$ then the top row of equation (9.10) holds, in which case the processor's outside option has no impact on the bargaining outcome. Conversely, if $s_P > Z_P + \frac{r_B}{r_B + r_P}\left(\pi - Z_B - Z_P\right)$, then in the bargaining agreement, the profit flow of the processor is equal to s_p and the profit flow of the board is equal to $\pi - s_P$. In other words, if the outside option of the processor is binding, then the value of that option fully determines how π is shared by the board and processor. It is interesting to note that the board's inside option has no influence on the negotiated outcome if the processor's outside option is binding.

Finally, it is straightforward to rewrite equation (9.10) in terms of the equilibrium price, W^*, for the case of combined inside and outside options. The solution value for W^* can be obtained by substituting the equilibrium value for π_B^* from equation (9.10) into $W = C_B + \pi_B/K$ or by substituting the equilibrium value for π_P^* from equation (9.10) into $W = P - C_P - \pi_P/K$.

9.5 An example from Australia's dairy industry

The sequential bargaining model without an outside option for the processor (i.e., case $s_P = 0$ in the previous analysis) is used to examine a bargaining scenario from the Australian dairy sector. The data for this mini-case comes from a short (2007) report on Australian milk prices.[15]

By way of background, the Dairy Section of WAFarmers is an organization that represents dairy farmers in Western Australia. Over the period 2000 to 2005 this agency successfully lobbied the ACCC for legislative change that would allow Western Australian dairy farmers to collectively bargain with dairy processors over the farm gate milk price. The newly acquired bargaining strength of milk producers was put to the test in 2007 when a severe drought in Australia resulted in rapidly escalating feed costs for dairy farmers. Retail prices for milk in Western Australia were forecast to increase by A$0.25/liter to about A$3.00 for a two liter bottle (about a 20 percent increase). Dairy farmers argued that the farm gate price should rise by a similar amount to offset the higher cost of feeding their cattle.

Western Australian dairy farmers deliver their milk to one of four processing companies that operate in their state. This particular case study focuses on Fonterra, which is a large multinational dairy processing firm that is owned by New Zealand farmers. To help offset the rising cost of feed, Fonterra was proposing to provide farmers under contact with an additional A$0.035/liter, which would raise the farm gate price to A$0.305/liter. At the same time, National Foods, which is Australia's biggest milk processor, agreed to pay farmers under contract an additional A$0.0525/liter to offset higher farm production costs. Not surprisingly dairy farmers under contract with Fonterra were upset by this proposal, first because Fonterra stood to gain from a substantially higher marketing margin, and second because dairy farmers under contract with National Foods were being compensated at a higher level.

The purpose of this empirical analysis is to use the sequential bargaining model developed in the previous section to provide a possible explanation as to why Fonterra stood to benefit from the drought, while dairy farmers under contract with Fonterra stood to be worse off, both in absolute terms and relative to dairy farmers in other parts of the state. The explanation provided below should be viewed as highly speculative because there are likely to be a number of important unobservable factors that influence the farm gate milk price in Western Australia.

Data for model calibration

Limited data are available to calibrate the bargaining model, but with some assumptions a simple calibration is possible. First, the retail price of milk was schedule to rise by A$0.25/liter from A$1.375/liter to A$1.50/liter. These latter two values provide an estimate of the P variable before and after the price increase. Second, the additional A$0.0525/liter paid by National Foods to farmers under contact will be used as an estimate of the change in the W_2 parameter, where W_2 is the cash price for milk in region 2. The milk price received by dairy farmers in various locations throughout Western Australia was reported to vary between A$0.30/liter and A$0.39/liter after the increase in the farm price. If the average value of this range is used, it follows that an estimate of W_2 is 0.345 – 0.0525 = A$0.2925/liter prior to the drought and A$0.345/liter after the drought.

Estimates of the unit costs of production for the milk board, C_B and the processor, C_P, are not available. However, according to equation (9.5) these two parameters are not required to calculate the equilibrium value of W. The analysis will therefore proceed without values for these two parameters. What is needed is an estimate of the transportation cost between regions 1 and 2. Recall that the cash price in region 2 was assumed to equal A$0.2925/liter prior to the drought and A$0.345/liter after the drought. It is reasonable to assume that unit transportation costs for both directions equal A$0.04/liter because this value implies that about 12.5 percent of the farm value of the milk would be lost if it was transported between regions rather than being sold to a local processor.

Spreadsheet model

The bargaining model is displayed in Figure 9.1. Column B contains the parameters and spreadsheet formulas to generate the variables and solution value for W. The values shown in column B correspond to the bargaining outcome before the drought. For the purpose of comparison, the bargaining outcome after the drought is displayed in column C.

Cell B3 shows that the quantity flow variable has been set to $K = 1$. The before and after drought values for the region 2 cash price, W_2, that appear in cells B4:C4 have been discussed above. Similarly, the before and after drought values for the two-directional transportation cost parameters, T_{12} and T_{21}, that appear in cells B5:C6, and the retail price variable, P, that appears in cells B7:C7, have been discussed above. Cells B8:C9 are intended to display the unit production cost parameters for the dairy board and the processor. These cells are intentionally left empty for the reasons discussed above. The parameter values in cells B10:C11, which are the respective discount rates of the dairy board and the processor, are discussed below. The $W_B = W_2 - T_{12}$ and $W_P = W_2 + T_{21}$ expressions have been entered in cells B14:C15. The bargaining surplus variable, $BS = (T_{12} + T_{21})\,K$ has been entered in cells B16:C16. Equation (9.5b) has been entered in cell B20 in order to calculate the equilibrium value for W (the same result would appear if equation (9.5a) was used). This key variable is examined in detail in the next section.

	A	B	C	D
1	**Parameters (all prices and costs are in A$/liter)**			
2		Before	After	
3	K	1	1	Quanitity flow from board to processor
4	W_2	0.2925	0.3450	Wholesale price in region 2
5	T_12	0.04	0.04	Unit transport cost when shipping from region 1 to region 2
6	T_21	0.04	0.04	Unit transport cost when shipping from region 2 to region 1
7	P	1.375	1.5	Processor's unit selling price
8	C_B	*no data*	*no data*	Unit cost of production for board
9	C_P	*no data*	*no data*	Unit cost of production for processor
10	r_B	0.05	0.15	Discount rate for board
11	r_P	0.05	0.05	Discount rate of processor
12		=W_2-T_12,	=W_2+T_21	
13	**Variables**			
14	W_B	0.2525	0.3050	Cash selling price for board if no contract is signed
15	W_P	0.3325	0.3850	Cash purchase price for processor if no contract is signed
16	B_surplus	0.08	0.08	Bargaining surplus
17				
18	=(T_12+T_21)K		=W_B+(r_P/(r_B+r_P))*(T_12+T_21)	
19	**Solution**			
20	W_s	0.2925	0.3250	Equilibrium price paid by processor to board

Figure 9.1 Bargaining model for Australian dairy example.

Simulation results

Recall that there were two questions of interest. First, why was Fonterra able to increase its marketing margin by a substantial amount whereas dairy producers, who faced substantially higher feed costs, were able to negotiate only a small price increase? Second, why was the price increase for farmers under contract with Fonterra considerably smaller than the average price increase received by farmers in different parts of Western Australia? The simulation results displayed in Figure 9.1 can help shed light on the answers to these questions.

Equation (9.5) shows that the retail price, P, does not affect the bargaining surplus for Fonterra and the dairy board. This is because Fonterra will sell its output at price P regardless of whether it can successfully negotiate a contract with the dairy board or ends up purchasing milk from a distant spot market. Because the bargaining surplus is independent of P it follows that Fonterra can raise P to a level that maximizes its profits, and this price increase does not provide the dairy board with any leverage when negotiating the farm gate price with Fonterra. Of course the question remains as to why Fonterra and the other milk processors were able to raise the retail price of milk by A$0.25/liter even though their cost of purchasing milk from dairy farmers rose by a much smaller amount. One possible explanation is that during the time of the Australian drought the price of food was beginning to rapidly increase in most countries, and so raising the retail selling price of milk without an explicit cost justification may have been comparatively easy for Fonterra and the other milk processors.

Similar to P, the unit production cost for dairy producers, C_B, has no effect on bargaining surplus and thus has no effect on the bargaining outcome. This result once again emerges because the dairy farmers in question will incur cost C_B regardless of whether they will produce under contract for Fonterra, or will ship their milk to the distant spot market. This independence of C_B and the bargaining surplus implies that the increase in C_B that resulted from the drought gives no leverage to the dairy board when negotiating with Fonterra.

Equation (9.5), together with $W_B = W_2 - T_{12}$ and $W_P = W_2 + T_{21}$, make it clear that the cash price in region 2, W_2, together with the interregional transportation costs, are the key determinants of the negotiated farm gate price. Assuming that T_{12} and T_{21} were both unaffected by the drought, it follows that the size of the increase in W_2 limits the extent that farmers under contract with Fonterra can negotiate a higher selling price for their milk. If the farm gate price in all regions of Western Australia is kept low by processors, then individual dairy boards such as the one negotiating with Fonterra have little leverage when negotiating a price increase. This outcome appears to describe the post-drought situation for dairy farmers in Western Australia.

To answer the second question concerning why dairy farmers delivering to Fonterra received a price increase equal to A$0.035/liter whereas farmers in surrounding regions received a price increase of A0.0525/liter, it is useful to examine the simulation results in Figure 9.1. Cell B20 shows that before the drought the equilibrium price is $A0.2925/liter, which is the same as the cash price in region 2 (see cell B4). The reason for this outcome is that the dairy board and Fonterra have the same 5 percent discount rates (see cells B10:B11) and therefore they will equally split the A$0.08/liter bargaining surplus. The A$0.04/liter surplus allocated to the dairy board is exactly offset by the A$0.04/liter cost of shipping milk to region 2 while the dairy board is exercising its inside option. Thus, prior to the drought the equilibrium price negotiated by the dairy board is the same as the cash price in region 2.

Cells B4:C4 show that the cash price in region 2 rose by A$0.0525/liter (from A$0.2925 to A$0.3450) as a result of the drought. Based on the discussion in the previous paragraph, the equilibrium price negotiated by the dairy board and Fonterra should also have risen by A$0.0525/liter. Cells B20:C20 show that in fact the equilibrium price rose by only A$0.0325/liter. The fact that the price increase predicted by the model approximately matches the actual price increase that was identified in the news report was not coincidental. A key parameter of the model (to be discussed below) was adjusted to ensure that the model predictions were consistent with reality. The remainder of the formal analysis examines the parameter that was adjusted and discusses whether such an adjustment is reasonable given the circumstances.

The parameter adjustment in question is the dairy board's post-drought discount rate. Specifically, the drought is assumed to have caused the discount rate for farmers delivering to Fonterra to increase, and this increase reduced the board's bargaining power such that the negotiated price increase was only A$0.0325/liter instead of A$0.0525. Cells B10:C10 show that dairy board's discount rate is assumed to have increased from 5 percent before the drought to 15 percent after the drought. The remaining question is whether such an adjustment is a reasonable hypothesis given the circumstances. Without additional information the hypothesized adjustment remains highly speculative. Nevertheless, it is not unreasonable to think that the discount rate and the corresponding degree of impatience for farmers under considerable financial distress may have risen from 5 percent to 15 percent.

9.6 Concluding comments

As agri-food supply chains become increasingly coordinated, contracting is gradually replacing price discovery in traditional spot markets. Contracting necessarily involves various degrees of bargaining over key variables such as price. The purpose of this chapter was to examine the determinants of the bargaining outcome in a bilateral monopoly scenario where the only variable to be determined was price. The bargaining process was modeled as a non-cooperative game where each side is allowed to make offers and counteroffers until a deal is struck or bargaining is permanently abandoned. Key determinants of the bargaining outcome include the level of "patience" that each side has while bargaining is underway and the size of the inside and outside options for the two parties who are involved in the bargain.

An important result of the analysis is that the equilibrium price in a bargaining game and equilibrium price in a competitive spot market respond to external shocks very differently. In a competitive spot market an outward shift in consumer demand for the processed good will typically result in a higher farm gate price. Conversely, an upward shift in the farm level supply schedule will typically result in a higher consumer price. The price impacts primarily depend on the magnitudes of the elasticities of supply and demand. In contrast, in a bargaining environment the equilibrium farm gate price may not be impacted by shifts in consumer demand and the equilibrium consumer price may not be impacted by shifts in farm supply. With bargaining the change in the equilibrium price will depend on the change in the level of bargaining surplus and the size of the inside and outside options. Understanding what constitutes each party's inside and outside option is a critical component of the setup of a bargaining model.

Bargaining models are used in a variety of different contexts and for a variety of different reasons. Several different applications of bargaining in the recent food and resource economics literature is discussed in the *Annotated bibliography* that is located at the end of this book. Recent advances in bargaining theory have made it possible to solve sophisticated bargaining models such as solving for an equilibrium outcome when players bargain over multiple variables and solving the bargaining game when information is incomplete. As discussed earlier in this chapter, the incomplete information case gives rise to the realistic outcome where the two sides of the bargain will typically choose to engage in successive rounds of bargaining rather than agreeing to an immediate settlement. With incomplete information successive rounds of bargaining can be viewed as a profitable form of learning.

Many economists exploit the fact that the Nash bargaining solution and the sequential non-cooperative approach to bargaining give rise to the same outcome with a reasonable set of assumptions. In other words, even though the sequential approach to bargaining has a better theoretical foundation, the Nash bargaining solution is still being used routinely in applied research. The downside of exploiting the convergence of Nash bargaining and sequential bargaining is that it is necessary to assume an infinitely short period of time between bargaining rounds. There are likely to be many real world situations where time lapses between bargaining rounds are significant, in which case the first-offer advantage is an important consideration.

Questions

A food processor sells manufactured items to a food wholesaler. As part of their operations the processor and wholesaler jointly purchase a set of services from a food broker. A long-term contract between the processor and wholesaler calls for the processor to pay one-third and the wholesaler to pay two-thirds of the broker's $30,000 annual cost. Annual earnings equal $820,000 for the processor and $1,250,000 for the wholesaler, before subtracting the cost of the broker.

The two firms recently discovered that they can lease a warehouse and equipment for $24,000 per year that would eliminate the need for using the services of the food broker. In addition to eliminating the $30,000 annual brokerage cost, the new facility would also result in \$5,000/year efficiency cost savings for the processor and \$11,000/year efficiency cost savings for the wholesaler.

The processor and wholesaler are considering renegotiating their long-term contract. The new contract would eliminate the cost of renting the warehouse and it would also specify how the \$24,000 annual lease expense of the new facility would be shared by the two firms. Once signed, this new contract would not be subject to renegotiation unless both firms agreed (i.e., the new contract has an infinite length). Assume the annual discount rate is 5 percent for the processor and 4 percent for the wholesaler.

1 Bargaining surplus and inside option.

   a Calculate the combined earnings of the processor and wholesaler first with the existing contract and then with the proposed new contract.
   b Use your results from part (a) to calculate the bargaining surplus. What does the bargaining surplus represent?
   c Use your results from part (a) to calculate the value of the inside option for each firm.
   d Without being provided more information, what is likely to be the value of the outside option for each firm?

2 Bargaining game.

   a Show that a game where the processor and wholesaler bargain over the sharing of the \$24,000 annual warehouse rental expense is equivalent to a game where these two firms bargain over the level of net earnings that each will receive with the new contract. In the first case the processor pays C and the wholesaler pays \$24,000 − C toward the rental expense and in the second case net earnings equal π for the processor and *Combined Net Earnings* − π for the wholesaler.
   b What is the feasible range of values for π if the firms bargain over net earnings?
   c Calculate the net earnings of each firm if the two firms choose to equally split the bargaining surplus.

3 Suppose the processor starts the bargaining round by proposing earnings equal to \$820,000 for the processor and \$2,062,000 − \$820,000 = \$1,242,000 earnings for the wholesaler. Suppose also that if the wholesaler rejects this offer then the wholesaler will propose a counteroffer equal to \$1,246,000 for the wholesaler and \$2,062,000 − \$1,246,000 = \$816,000 earnings for the processor. Assume that it takes three months for the wholesaler to formulate the counteroffer and that the wholesaler believes with 100 percent certainty that the processor will accept the counteroffer.

   a Calculate the present value of the inside option that flows to the wholesaler over the three month period that it takes the wholesaler to formulate the counteroffer. When answering this question assume that annual cash flows for both firms are received in twelve equal monthly installments and that the monthly interest rate is equal to the annual rate divided by 12.

b Calculate the present value of the net earnings for the wholesaler starting from the point in time that the processor accepts the counteroffer and lasting forever. Now further discount this present value calculation to the point in time that the wholesaler receives the processor's original offer.

c Based on your answer to parts (a) and (b), should the wholesaler accept or reject the processor's initial offer that results in the wholesaler earning \$1,242,000/year?

4 Consider now the generic offer made by the processor: net returns equal π_P for the processor and \$2,062,000 − π_P for the wholesaler. As well, the counteroffer made by the wholesaler in three months' time is π_W for the wholesaler and \$2,062,000 − π_W for the processor.

a Use the procedures developed in Question 3 to construct an equation that shows the combinations of values for π_P and π_W that makes the wholesaler indifferent between accepting and rejecting the processor's initial offer.

b Repeat part (a) from the perspective of the processor assuming that the wholesaler makes the initial offer and the processor must decide whether to accept or reject the offer and make a counteroffer in three months' time. In this case let π_W denote net earnings for the wholesaler and \$2,062,000 − π_W net earnings for the processor.

c Jointly solve the pair of equations from parts (a) and (b) to identify the equilibrium values for π_P and π_W. If the processor makes the first move, the equilibrium payoffs equal π_P^* for the processor and \$2,062,000 − π_P^* for the wholesaler. If the wholesaler moves first, the equilibrium payoffs equal π_W^* for the wholesaler and \$2,062,000 − π_W^* for the processor.

d Use your answers to Questions 2(a) and 4(c) to identify how the building and equipment rental expense is shared with the new contract assuming the processor makes the first offer.

Notes

1 Introduction

1 Pfaff, William, "Speculators and Soaring Food Prices", *New York Times*, 16 April 2008.
2 Readers who are interested in long-term price trends and price volatility should consult books written on the economics of commodity price forecasting, productivity growth in agriculture and sustainable food production.

2 Prices over space

1 Throughout this chapter the term "transportation cost" will refer to all costs associated with moving the commodity from one region to another (e.g., ocean freight, terminal storage and insurance).
2 This price quote was obtained from the Market Data Center on 15 July 2009: http://data.hgca.com/demo/archive/physical/xls/Data%20Archive%20-%20Physical%20International.xls.
3 Throughout this chapter the time required for transporting the commodity is ignored (i.e., all transactions are assumed to take place on 23 November 2005).
4 The equilibrium should be viewed as a long-run concept that is relevant for when the commodity in question is perfectly homogenous and trade deals are negotiated strictly based on price. With these assumptions transportation cost differences are the primary determinants of the equilibrium outcome and trade will generally be highly specialized.
In reality, trade is not nearly so specialized because products are differentiated and non-price variables such as preferential agreements, long-term contracts and export credit packages are also important determinants of trade.
5 Examples of software which is routinely used to solve large spatial equilibrium models are GAMS and MATLAB.
6 See Huang, J. and Rozelle, S. (2006) "The emergence of agricultural commodity markets in China", *China Economic Review,* 17(3), 266–80. Their study determines that for the case of Chinese maize, price differences across a pair of regions separated by 1000 km are in the magnitude of 5 percent.
7 Shifting the import demand schedule down by an amount C_{EI} will give the same outcome.
8 In the spatial equilibrium literature the term "quasi-welfare" is often used instead of "net aggregate welfare" when referring to the aggregate area under demand minus the aggregate area under supply minus aggregate transportation costs.
9 For example, scaled revenue (*SR*) can be expressed as $SR = \hat{P}\hat{Q}$. Multiply through by z/k and k to obtain $zSR = \left(\frac{z}{k}\hat{P}\right)\left(k\hat{Q}\right) = PQ$. This equation shows that scaled revenue is

equal to non-scaled revenue divided by z. Consequently, non-scaled revenue can be recovered by multiplying scaled revenue by z.

10 See Guajardo, R. G. and Elizondo, H. A. (2003) "North American tomato market: a spatial equilibrium perspective", *Applied Economics,* 35(3), 315–322. The USDA provides additional background information: http://www.ers.usda.gov/Briefing/Vegetables/tomatoes.htm

11 The assumption that ocean freight rates for fresh tomatoes are proportional to miles transported is generally not valid. Moreover, the assumed rate of $0.03 per ton per mile was arbitrarily specified (the $0.03 value was chosen because it gives rise to a set of equilibrium prices that look "reasonable"). The results of the simulations to follow should therefore be viewed as illustrative rather than an accurate estimate of prices in the global tomato market.

12 Cell D39 contains the array formula "={SUM(B21:F25*B13:F17)}". To enter an array formula do not include the { } but it is necessary to finalize the formula with <ctrl><shift><enter> rather than a simple <enter>.

3 Prices over time (storage)

1 In other words, on a given day if a trader wished to formulate an expected price to assess the expected profitability of a forward contract, what price should that agent rationally expect in a competitive equilibrium?

2 The discount factor is equal to $\delta = 1/_{1+r}$ where r is the discount rate (e.g., the opportunity cost of a unit of capital).

3 It is straightforward to show that the second-order condition for the planner's maximization problem holds given the assumptions that $P'(x) < 0$ and $C''(S) \geq 0$.

4 If profits from investing in storage equal zero in a market equilibrium, then why do traders invest in storage? The first answer is that zero profits implies that traders earn the opportunity cost of the resources that are devoted to the storage activity. In other words storage is "as good as it gets". The second answer is that in the real world different traders will have different costs of storage, so in a market equilibrium the marginal high-cost traders will earn zero profits and inframarginal traders will earn a positive rent.

5 Solve $q_t = h + S_{t-1}$ and $q_{t+1} = h + S_t$ to obtain $q_{t+1} = q_t - (S_{t-1} - S_t)$. Now substitute $x_t = S_{t-1} - S_t$ into this equation to obtain $q_{t+1} = q_t - x_t$.

6 Wool production and price data, and the Australian stockpile data, comes from Roche, J. (1995) *The International Wool Trade*, Cambridge: Woodhead Publishing Limited.

7 A recursive calculation means that the formula in cell B30 depends on the value in cells B29 and C29, which in turn depend on the value in C28, etc.

8 How is it possible for price to continually rise and conform to the LOP when stocks decline from a positive to a negative level and then return to a value of zero as of time T? Because there is no sign restriction on the storage variable, the results in Figure 3.2 conform to the standard lifecycle model of individual consumption and savings for the case of fixed lifetime income. Discounting in the lifecycle model induces an individual to consume at a declining rate over his or her lifespan, and this declining consumption results in a continually rising marginal utility of consumption (i.e., price). Declining consumption is achieved by first running down savings at a decreasing rate, then borrowing to finance the relatively high but declining stream of consumption and finally repaying the loan by saving from current income at an increasing rate.

9 With more sophisticated programming N could be restricted to an integer value, in which case it would be possible to verify that the LOP holds exactly at the point of transition. The pricing outcomes for all periods will be slightly biased because of the integer bias associated with N.

10 In this case $a = 52.2$ and $b = 15$.

11 Consumption of 100 units implies a market price of $P = 3$. The unit cost of storage relative to this price, 0.2/3 , is 6.67 percent.
12 The weight is 1/3 because of the assumption that each production outcome is equally likely. Different probability weights can be assigned to the three different harvest outcomes by modifying this weighting scheme.
13 It is important to paste only the values (not the formulas) when transferring the values from columns P to O. It is straightforward to construct an Excel macro which will automate this copy and paste procedure.
14 To resolve the problem begin by pasting the V_T values from column N over the V_{t+1} values in column O. Then progressively transfer the values from column P to column O as discussed above.

4 Prices over time (commodity futures)

1 The basis is normally defined as the difference between the spot price and the futures price. The reverse definition is used throughout much of this chapter to avoid the confusion associated with negative basis values. As well, basis contains both an intertemporal component and a spatial component. The focus of this analysis is on the intertemporal component and so, unless otherwise indicated, the futures and spot markets are assumed to exist at the same location.
2 Substantial discussion is devoted to this topic in Williams, Jeffrey J. (1986) *The Economic Function of Futures Markets,* Cambridge and New York: Cambridge University Press. Williams argues that variations in price spreads over time is to be expected because each contract serves a unique niche of traders, and differences in supply and demand conditions for these micro markets results in a unique set of price spreads that evolve stochastically over time. He further argues that if all price spreads were equal to the carrying charge when inventories were normal, then there would be no need to simultaneously trade multiple commodities because doing so would be redundant.
3 Because of the truncation, $\hat{\mu}_1$ is generally not equal to the mean of $f(Q_1)$ and σ_1 is generally not equal to the standard deviation of $f(Q_1)$.
4 Setting $\sigma_1 = 0$ generates a calculation error in Excel, so σ_1 is set equal to 0.1 instead. This approximation does not affect the equilibrium outcome. For most of this analysis the probability of $Q_1 < 0$ is relatively small, which implies that σ_1 is approximately equal to the standard deviation of period 1 production.
5 This modeling approach assumes that $\theta^* (P_2)$ takes on an integer value, which is unlikely to be the case. This integer problem is ignored because the qualitative results are not affected by this assumption.
6 To obtain this differential first rewrite equation (4.16) as $\frac{1}{k}\sum_{i=h}^{k}(Bi - s)$ where $B = \frac{X_2(P_2)}{0.5k(k+1)}$ and $h = s/B$. Expanding results in $\frac{1}{k}\sum_{i=h}^{k}(Bi - s) = 0.5B[k(k+1) - h(h-1)] - (k+1-h)s$. Finally, substitute in the above expressions for h and B and then differentiate with respect to s to obtain equation (4.16) in the text.
7 For more details about the properties of a truncated normal distribution see Johnson, Norman L. and Kotz, Samuel (1970). *Continuous Univariate Distributions-1*, Chapter 13. John Wiley & Sons.

5 Prices over form (quality)

1 See Table 4.10 of Kent, N. L. (1983) *Technology of Cereals: An Introduction for Students of Food Science and Agriculture*, 3rd ed., Oxford and New York: Pergamon, p. 91.

2 The cost of producing the commodity is not part of the expression for net aggregate surplus because the reference point of the analysis is post harvest.
3 These price data were downloaded from the CWB website on 2 March 2010. http://www.cwb.ca/public/en/farmers/outlooks/what/
4 The beta distribution was chosen for the analysis because lower and upper limits for the protein percentage can be specified, and a variety of realistic shapes for the distribution of protein can be generated by changing the values of the two parameters of the beta function.
5 The upper limit for protein was set at 15.7 percent rather than 15.5 percent to ensure that the protein interval, *max* − *min*, can be divided into an integer number of protein categories when using a 0.175 percent increment.
6 The beta function is equal to $\Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha + \beta)$ where Γ is the gamma function. Excel has a function for the natural log of Γ so an exponential transformation is required to obtain the desired beta function.
7 The four selling prices in cells B15:E15 are the average September PRO for the years 2001/2 to 2009/10, as presented above.

6 Prices linkages across commodity markets

1 To calculate first differences, subtract the previous day's price from the current day's price. In an efficient market price is not expected to increase or decrease from one day to the next, so the mean value of the first difference data series is expected to equal zero in the long run. The correlation of first differences in prices is therefore a measure of the correlation in unexpected movements in price.
2 See Kessler, R. A. (4 February 2010) "EPA lowers 2010 US cellulosic ethanol mandate by 94%", *Recharge: The Global Source for Renewable Energy News,* http://www.rechargenews.com/energy/biofuels/article205246.ece
3 Throughout this chapter outcomes where one or more of the four choice variables take on an equilibrium value of zero (i.e., a corner solution) are not considered. This assumption means that it is not necessary to solve the planner's problem with Kuhn–Tucker programming.
4 Equations (6.3b) and (6.3d) can be solved to give equation (6.4a). Equation (6.3b) allows equation (6.3a) to be written as $-P_B - \lambda_2 \, \partial g/\partial C_L = 0$ and equation (6.3d) allows equation (6.3c) to be rewritten as $-P_H - \lambda_2 \, \partial g/\partial X_L = 0$. This pair of revised equations can be solved to give equation (6.4b).
5 The slopes of level sets such as production possibility frontiers, isoquants and welfare indifference curves are derived by totally differentiating the relevant function. For example, totally differentiate the PPF function, $f(C, X) = K$, and rearrange the differential to obtain the slope of the PPF: $dX/dC = -\dfrac{\partial f/\partial C}{\partial f/\partial X}$. When deriving the slope of the welfare indifference curve recall that $P_B(C_B) = M_B'(C_B)$ and $P_H(X_H) = M_H'(X_H)$.
6 In a more general model with a large number of competitive farmers, the farm level supply schedule for an individual farmer must be aggregated over all farmers to obtain the market supply schedule. The situation is similar when deriving the market demand for feed grains.
7 The CES function is commonly used in large-scale computable general equilibrium (CGE) models. These models typically utilize a large number of supply and demand relationships and simultaneously cover dozens of commodities and trading regions. Much of the work involved with building and managing these large-scale models centers on econometric estimation, which is not discussed in this chapter.
8 It follows from equation (6.7) with $b > 1$ that the slope of the PPF becomes more negative as C is increased and X is decreased. Conversely, with $b < 1$ the slope of the isoquant becomes less negative as C is increased and X is decreased. For $b = 1$, the PPF and isoquant are both linear.

9 See USDA (2009) *USDA Agricultural Projections to 2018*, Office of the Chief Economist, Long Term Projections Report OCE-2009-1. http://www.ers.usda.gov/briefing/corn/2009baseline.htm#US

10 The guess values do not need to be very accurate for this two-market scenario, but guess value accuracy does become increasingly important as the number of markets included in the model increases beyond two.

7 Marketing margins in vertical supply chains

1 If the values in Table 7.1 are subtracted from 100 the resulting set of values are the marketing margins expressed as a percentage of the retail selling price.

2 To be consistent with the other chapters of this textbook a measure of pricing efficiency in a vertical supply chain should be referred to as vertical price integration. The term vertical price transmission is used instead to avoid confusing readers who are familiar with the widely-used concept of vertical integration.

3 For more details see Leibtag, E., Nakamura, A., Nakamura, E. and Zerom, D. (March 2007) *Cost Pass-through in the U.S. Coffee Industry*, Economic Research Service, United States Department of Agriculture, ERR-38. http://www.ers.usda.gov/publications/err38/err38d.pdf.

4 The long-run marketing margin increased slightly from \$0.23 − \$0.11 = \$0.12/ounce to \$0.17 − \$0.04 = \$0.13/ounce. This increase can approximately be explained by an increase in the price of labor and materials, which rose from \$0.035 to \$0.055/ounce.

5 It is straightforward to include non-food items in the consumer's budget allocation problem. However, doing so would make the notation more complicated and little additional insight would be gained by considering this more general problem.

6 The marketing board is assumed to make a take-it-or-leave-it offer when selling to the processing firm. The marketing board sets price so that the demand by the processing firm is equal to the aggregate supply of producers (i.e., the competitive market outcome). Producer surplus could be increased if the marketing board raised price to the monopoly level, but it is not able to do this because there is free entry by producers in the raw commodity market (only supply restricting marketing boards have market power).

7 In the simulation results for processed food the retail selling price is about $p_i = 10$, the unit cost of the raw commodity is about $w_i = 1.5$ and the processor's unit operating cost is $c = 1$. The residual earnings of about 7.5 per unit of output is available to cover fixed costs and generate profits for the firm.

8 Certain variable names are not allowed in Excel because they are reserved for Excel functions. In these situations Excel automatically inserts an "_" after the variable name. In the current analysis the parameter "c" has been renamed "c_" and the parameter "delta" has been renamed "delta_".

9 Aggregate consumer expenditures can be calculated by multiplying and then summing equilibrium retail prices and quantities (prices reside in cells B17:F17 and B22:F22, and quantities reside in cells B18:F18 and B23:F23 of Figure 7.2). One way to check for spreadsheet accuracy is to ensure that the calculated value of aggregate consumer expenditures is equal to consumer income (cell B10) after equilibrium prices have been chosen by Solver.

8 Auctions and competitive bidding

1 See Alibaba.com, "FACTBOX-Russia/Egypt grains data, import system". Published 16 June 2009: http://news.alibaba.com/article/detail/agriculture/100119437-1-factbox-russia%252Fegypt-grains-data%252C-import-system.html

2 See ExpressIndia.com, "Eighteen bids for rice import tender". Posted 10 November 200: http://www.expressindia.com/latest-newsEighteen-bids-for-rice-import-tender/539235/

3 Bid randomization is most likely to be observed when the number of bidders is small (e.g., four or less). With 18 firms in the bidding cohort, the principles of perfect competition are more likely to be applicable.
4 The price in the local production region is $W - T_{EI} - T_{LE}$, and so with a PP strategy, the landed price in region I after paying for shipping is $W - T_{EI} - T_{LE} + T_{LI}$, which is equivalent to $W - \delta$.
5 Why would a type CP firm bother to bid if a 50 percent chance of winning zero profits is the best outcome? It is important to keep in mind that zero economic profits is an acceptable outcome for a seller because zero profits implies that the opportunity cost of all resources has just been covered.
6 For full details about Monte Carlo simulations in an Excel workbook see Chapter 11 of Render, B., Stair, R. M., Balakrishnan, N. and Smith, B. (2010) *Managerial Decision Modeling with Spreadsheets*, second Canadian edition, Toronto: Pearson.
7 The Wikipedia entry for the Trapezoid Rule describes the numerical procedure (http://en.wikipedia.org/wiki/Trapezoidal_rule). A similar description can be found in standard reference books on quantitative methods.

9 Bargaining in bilateral exchange

1 Rubinstein's classic model is essentially the only non-cooperative bargaining model that is currently being used by applied economists. More details about this model can be found in Chapter 9 of the Rubinstein entry in the *Annotated bibliography* that is at the end of this textbook. The presentation in this chapter closely follows the excellent description of bargaining theory by Muthoo, A. (1999) *Bargaining Theory with Applications*, New York: Cambridge University Press.
2 Information about the US case was taken from Hueth, B. and Marcoul, P. (2003) "An essay on cooperative bargaining in US agricultural markets", *Journal of Agricultural and Food Industrial Organization*, 1(1).
3 Information about the Ontario case was taken from an undated Ontario Ministry of Agriculture, Food and Rural Affairs website titled "Agricultural Marketing Boards in Ontario": http://www.omafra.gov.on.ca/english/farmproducts/factsheets/ag_market.htm
4 For more details see Oczkowski, E. (2006) *The Power of Collective Bargaining*, Sydney: Centre for Australian Community Organisations and Management, University of Technology: http://www.business.uts.edu.au/cacom/articles/commentaries/cbargaining.html
5 The assumption that the board produces K units, which is the exact requirement of the processor, is not as restrictive as it may appear. Suppose instead the board produces K_B units and the processor requires K_P units. If $K_P < K_B$, then the board and processor will bargain over the transaction price for the K_P units and the fact that the board's residual, $K_B - K_P$, is sold in the region 2's cash market is not relevant for the bargaining problem. The situation is similar for the case of $K_B < K_P$.
6 It is not uncommon to have asymmetric transportation costs (i.e., $T_{12} \neq T_{21}$), although the assumption of symmetric versus asymmetric bargaining costs is not important for the analysis. The price depends on overall supply and demand for transportation services, which will typically be direction specific.
7 The board's unit cost can be interpreted as an average of the unit production costs of the producers who market their commodity through the board.
8 Fixed costs for the board and processor are not included in the analysis because they do not affect the bargaining outcome (shut-down decisions are not considered). Consequently, gross profits rather than profits is a more appropriate term for the expression $\pi = (P - C_P - C_B)\,K$. However, to simplify the discussion the "gross" qualifier will not be used.

9 Profits for the board are distributed back to the farmers who produce the raw commodity. The method of distribution is assumed to have no effect on the outcome of the bargaining game.

10 This result is derived by noting that the present value of a constant flow *Z* with discount rate *r* is equal to $Z\int_0^\Delta e^{-rt}dt = Z\frac{1}{-r}\left(e^{-r\Delta}-1\right)$. The right-hand side of this expression can be rewritten as $Z(1-\delta)/r$, where $\delta = e^{-r\Delta}$.

11 The present value of the flow of profits from date $(t+1)\Delta$ onward is equal to π_B/r_B, and this amount discounted back to date $t\Delta$ is equal to $e^{-r_B\Delta}\pi_B/r_B$.

12 For a more in-depth treatment of the sequential bargaining process and equilibrium consult a reference book on bargaining theory such as Muthoo (1999) (see note #1 for the citation).

13 John Nash made important contributions to both theories of bargaining, so his name is associated with both theories, even though these theories are quite different from each other.

14 The interested reader should consult Muthoo (1999) for the details of the proof. See note #1 for the citation.

15 OzBevNet.com "(Aussie & Kiwi non-alcoholic drinks directory)", *Australian Milk Prices*, 21 July 2007: http://www.ozbevnet.com/consumer-intelligence/australian-milk-prices.html

Annotated bibliography

Chapter 2

Classic references

Samuelson, P. A. (1952) 'Spatial Price Equilibrium and Linear Programming', *The American Economic Review*, 42(3), 283–303.

This is a classic paper that shows how linear programming techniques can be used to maximize a net social welfare function in order to obtain the competitive equilibrium for a set of spatially-separated markets.

Takayama, T. and Judge, G. G. (1971) *Spatial and Temporal Price and Allocation Models, Contributions to Economic Analysis,* 73, Amsterdam: North-Holland Pub. Co.

This classic reference on spatial price analysis builds on Samuelson's linear programming approach. The quadratic quasi-welfare function that is introduced in the authors' Chapter 6 forms the basis for the theoretical and simulation model that is presented in Chapter 2 of this textbook. The multi-commodity analysis of Takayama and Judge is much more general than the single commodity case considered in this textbook.

Judge, G. G. and Takayama, T. (1973) *Studies in Economic Planning over Space and Time, Contributions to Economic Analysis,* 82, Amsterdam; New York: North-Holland Pub. Co.; American Elsevier Pub. Co.

Part III (A) of this edited collection of papers contains four specific applications of the Takayama–Judge model of spatial price analysis. Part III (B) extends the quadratic programming framework analysis by examining equilibrium prices over both space and time.

Typical 1970s applications of the spatial/intertemporal price equilibrium model

Fuchs, H. W., Farrish, R. O. P. and Bohall, R. W. (1974) 'A model of the U.S. apple industry: a quadratic interregional intertemporal activity analysis formulation', *American Journal of Agricultural Economics*, 56(4), 739–750.

Martin, L. and Zwart, A. C. (1975) 'A spatial and temporal model of the North American pork sector for the evaluation of policy alternatives', *American Journal of Agricultural Economics*, 57(1), 55–66.

Furtan, W. H., Nagy, J. G. and Storey, G. G. (1979) 'The impact on the Canadian rapeseed industry from changes in transport and tariff rates', *American Journal of Agricultural Economics*, 61(2), 238–248.

Modern applications of spatial price analysis

Kawaguchi, T., Suzuki, N. and Kaiser, H. M. (1997) 'A spatial equilibrium model for imperfectly competitive milk markets', *American Journal of Agricultural Economics*, 79(3), 851–859.

This paper generalizes Takayama and Judge's spatial equilibrium model by allowing the degree of market structure (ranging from perfect competition to monopoly) to be endogenous. The model is used to examine interregional milk shipments in Japan.

Gabriel, S. A., Vikas, S. and Ribar, D. M. (2000) 'Measuring the influence of Canadian carbon stabilization programs on natural gas exports to the United States via a "bottom-up" intertemporal spatial price equilibrium model', *Energy Economics*, 22(5), 497–525.

The Gas Systems Analysis Model (GSAM) is a spatial/intertemporal pricing equilibrium model of the North American natural gas system. The model is used to examine the impact of Canadian carbon stabilization programs on exports of natural gas to the United States.

Djunaidi, H. and Djunaidi, A. C. M. (2007) 'The economic impacts of avian influenza on world poultry trade and the US poultry industry: a spatial equilibrium analysis', *Journal of Agricultural and Applied Economics*, 39(2), 313–323.

A calibrated spatial equilibrium model is used to examine how an avian flu outbreak in major poultry producing regions of the global economy can result in substantial increases in the price of poultry in regions not directly affected by the outbreak.

Chapter 3

Classic references

Gustafson, R. L. (1958) *Carry Over Levels for Grains: A Method for Determining Amounts that are Optimal Under Specified Conditions*, Washington, DC: US Department of Agriculture.

Gustafson used optimal inventory analysis methods (an early form of dynamic programming) to calculate the socially optimal level of carryover stocks of grain. He also showed that socially optimal carryover will be the same as that stored in aggregate by risk-neutral stockholders who are interested in maximizing discounted expected profit. Unfortunately, Gustafson's path-breaking research was overlooked for a couple of decades.

Gardner, B. L. (1979) *Optimal Stockpiling of Grain*, Lexington, MA: Lexington Books.

This book was written to shed light on the debate regarding whether government stockpiling of grain should be a core component of US agricultural policy, or whether the storage decisions should be left to the private sector. Similar to Gustafson (1958), Gardner views grain stockpiling as an optimal inventory problem, but he generalizes Gustafson's analysis by incorporating supply response. The numerical methods used by Gardner to solve the commodity storage problem with production uncertainty are similar to those used in the last half of Chapter 3 of this textbook.

Newbery, D. M. G. and Stiglitz, J. E. (1981) *The Theory of Commodity Price Stabilization: A Study in The Economics of Risk*, Oxford; New York: Clarendon Press; Oxford University Press.

Chapter 30 of this book focuses on optimal commodity stockpiling rules. Newbery and Stiglitz discuss the advantages and disadvantages of three numerical solution procedures. The main focus of their work is on the welfare effects of public commodity price stabilization schemes.

Williams, J. C. and Wright, B. (1991) *Storage and Commodity Markets*, Cambridge: Cambridge University Press.

This book provides a very thorough examination of the economics of storage in competitive commodity markets. Chapters 2 and 3 describe the relationship between socially optimal storage rules, arbitrage relationships and a competitive equilibrium. The Appendix of Chapter 3 provides a detailed treatment of the three numerical solution procedures that are used to solve a general storage problem that has both supply response and production uncertainty. Chapter 4 is devoted to describing the effects of storage on production, consumption and price.

A sample of more recent research

Deaton, A. and Laroque, G. (1992) 'On the behaviour of commodity prices', *The Review of Economic Studies*, 59(1), 1–23.

This paper carries forward the idea that the non-negativity constraint for storage gives rise to a non-linear first-order Markov process for commodity prices. Deaton and Laroque examine 13 commodity price series and use their theory to explain observed skewness, autocorrelation and periodic upward spikes.

Bardsley, P. (1994) 'The collapse of the Australian wool reserve price scheme', *The Economic Journal*, 104(426), 1087–1105.

Bardsley provides a detailed description of the Australian wool storage case study that was discussed in Chapter 3 of this textbook. He also uses optimal control and option pricing techniques to examine the social value of the Australian wool stockpile and to explain the actions of industry managers that led to the wool policy crisis.

Pindyck, R. S. (2004) 'Volatility and commodity price dynamics', *Journal of Futures Markets*, 24(11), 1029–1047.

Pindyck shows how an increase in price volatility can result in higher inventory build-ups, which in turn raise the short-run price of the commodity. This finding is consistent with the simulation results for the production uncertainty case that was presented toward the end of Chapter 3 of this textbook.

Coleman, A. (2009) 'Storage, slow transport, and the law of one price: theory with evidence from nineteenth-century US corn markets', *Review of Economics and Statistics*, 91(2), 332–350.

In Coleman's model of competitive storage, traders anticipate that physical arbitrage takes time, and so localized price spikes are expected to appear with reasonable frequency in a competitive equilibrium. Data from nineteenth-century corn markets in Chicago and New York are shown to be consistent with the theoretical predications.

Chapter 4

Williams, J. C. (2001) 'Commodity futures and options' in Gardner, B. L. and Rausser, G. C., eds, *Handbook of Agricultural Economics VI(2)*, Amsterdam; New York: Elsevier, pp. 745–816.

This handbook chapter covers a broad range of topics relating to price discovery in commodity futures markets. Issues of particular interest include the stabilizing and information role of commodity futures and the theory of the price of storage, which includes convenience yield. Williams is best known for his work on the transaction demand for commodity futures (more details below).

Carter, C. A. (1999) 'Commodity futures markets: a survey', *Australian Journal of Agricultural and Resource Economics*, 43(2), 209–247.

Carter acknowledges that the literature on commodity futures is much too vast to adequately cover with a single review. He focuses on a number of key areas including risk premium and Keynes' notion of normal backwardation, the price of storage, the price stabilizing role of commodity futures, hedging and pricing efficiency.

Garcia, P. and Leuthold, R. M. (2004) 'A selected review of agricultural commodity futures and options markets', *European Review of Agricultural Economics*, 31(3), 235–272.

This paper reviews the topics similar to those highlighted by Carter (1999), but the emphasis is more heavily focused on empirical applications.

Stabilizing and information role of commodity futures

Tomek, W. G. and Gray, R. W. (1970) 'Temporal relationships among prices on commodity futures markets: their allocative and stabilizing roles', *American Journal of Agricultural Economics*, 52(3), 372–380.

This classic paper points out that futures trading for a storable commodity has the dual role of guiding inventories and discovering forward prices. In the "price of storage" theory, the full constellation of cash and futures prices move together with the arrival of new information, so it is not appropriate to view futures prices alone as a price forecast. For non-storable commodities spot and futures prices are distinct, with the latter forecasting the former. Much of the paper is devoted to testing price forecasting relationships.

Cox, C. C. (1976) 'Futures trading and market information', *The Journal of Political Economy*, 84(6), 1215–1237.

Cox asserts that futures trading will affect a firm's price expectations, which are based on incomplete information. Depending on how well informed speculators are as a group, futures trading can either stabilize or destabilize cash prices.

Peck, A. E. (1976) 'Futures markets, supply response, and price stability', *The Quarterly Journal of Economics*, 90(3), 407–423.

In this paper the classic supply response model is combined with a classic model of futures price determination. In the resulting model supply decisions are based on forward looking futures prices rather than historical spot prices.

Turnovsky, S. J. (1979) 'Futures markets, private storage, and price stabilization', *Journal of Public Economics*, 12(3), 301–327.

Turnovsky analyzes the welfare effects of price stabilization in markets that operate with a well-functioning futures market. Not surprisingly, his results hinge on the degree to which futures markets have a stabilizing versus destabilizing role.

Roll, R. (1984) 'Orange juice and weather', *The American Economic Review*, 74(5), 861–880.

With an informational efficient futures market the unpredictable component of weather should be correlated with movements in the futures price. Roll's findings support the information efficiency hypothesis for the case of frozen orange juice. In particular, he shows that the closing futures price on a given trading day is a statistically significant predictor of the forecast error of the minimum temperature later that evening.

Sumner, D. A. and Mueller, R. A. E. (1989) 'Are harvest forecasts news? USDA announcements and futures market reactions', *American Journal of Agricultural Economics*, 71(1), 1–8.

USDA harvest forecasts are kept secret until released. Sumner and Muller show that commodity futures prices respond in a significant way when the forecasts are released. Thus, USDA reports contain information that is not previously known by market traders.

Supply of storage and convenience yield

Working, H. (1949) 'The theory of price of storage', *The American Economic Review*, 39(6), 1254–1262.

Working's price of storage theory asserts that the difference between a futures price and spot price on a particular day reflects the net cost of carrying inventories. Price spreads are expected to decline with diminishing inventories and are expected to change only minimally when new information changes the set of market prices. Negative carrying charges are attributed to a convenience yield that accrues to commercial firms.

Brennan, M. J. (1958) 'The supply of storage', *The American Economic Review*, 48(1), 50–72.

Brennan generalizes the theory of the price of storage by adding a risk premium component. The difference between the futures price and spot price now has three components: marginal physical cost and marginal risk premium, both of which contribute to a positive spread, and marginal convenience yield that contributes to a negative spread.

Brennan, D., Williams, J. and Wright, B. D. (1997) 'Convenience yield without the convenience: a spatial–temporal interpretation of storage under backwardation', *The Economic Journal*, 107(443), 1009–1022.

In this paper it is argued that convenience yield may be an illusion because of the way that data has been aggregated. Wheat marketing in Western Australia is used to demonstrate that a negative price spread can exist at the port position, even though no stocks are earning monetary losses when measured at local points of delivery.

Transaction demand for commodity futures

Williams, J. (1986) *The Economic Function of Futures Markets*, Cambridge and New York: Cambridge University Press.

Williams expands on existing theories of storage arbitrage and non-linear borrowing costs by showing that hedgers will be active participants in futures markets even if they are not averse to risk. Trading futures allow commercial firms to implicitly borrow and lend the commodity without facing the high cost of owning the commodity. The demand for transactions is generally a highly interdependent, non-linear function of transaction costs and interest rate differentials.

Books

Omitted from this reading list are the many edited books on commodity futures and the books that emphasize financial futures and options. Listed below are four commonly-cited books on agricultural futures that range widely in their level of technical detail.

Atkin, M. (1989) *Agricultural Commodity Markets: A Guide to Futures Trading, Commodity Series*, London and New York: Routledge.

This highly applied book provides a non-technical overview of futures for the major agricultural commodities that trade in global markets.

Purcell, W. D. (1991) *Agricultural Futures and Options: Principles and Strategies*, New York; Toronto: Macmillan Pub. Co.

In this applied book, Purcell uses graphs and simple mathematics to emphasize key theoretical relationships in commodity futures markets. A very good overview of trading strategies using technical analysis is provided.

Tomek, W. G. and Robinson, K. L. (2003) *Agricultural Product Prices*, Ithaca, NY: Cornell University Press.

Chapters 12 and 13 provide an easy to follow overview of the main economics features of a commodity futures market.

Blank, S. C., Carter, C. A. and Schmiesing, B. H. (1991) *Futures and Options Markets: Trading in Commodities and Financials*, New Jersey: Prentice Hall.

This senior undergraduate textbook is more academic than are the books by Atkin and Purcell. The authors keep the discussion well grounded in the relevant theory and associated literature.

Stein, J. L. (1987) *The Economics of Futures Markets*, Oxford and New York: Basil Blackwell.

This book contains a sophisticated economic analysis of commodity futures trading including risk sharing, the effect of futures trading on spot price volatility, dynamic stock-flow interactions and the connections between speculation and economic welfare.

Chapter 5

Books

Hill, L. D. (1990) *Grain Grades and Standards: Historical Issues Shaping the Future*, Urbana, IL: University of Illinois Press.

Hill provides a historical perspective on the evolution of grades and standards for US agricultural commodities. For most commodities technical standards allow heterogeneous commodities to be categorized and blended into a small number of grade categories. A system of uniform grades facilitates efficient marketing, primarily because of improved information flows.

Tomek, W. G. and Robinson, K. L. (2003) *Agricultural Product Prices*, Ithaca, NY: Cornell University Press.

Chapter 7 is devoted to price differences associated with quality. The theme of the chapter is that price relationships across different grades of a commodity are special cases of price relationships across substitute commodities. Year-to-year changes in price premiums and discounts for different grades are normally due to year-to-year variations in the distribution of quality. Absolute premiums and discounts depend on a variety of factors including the own price and cross price elasticities of demand.

Kohls, R. L. and Uhl, J. N. (1997) *Marketing of Agricultural Products*, 8th ed., New Jersey: Prentice Hall.

This textbook can be read by those with little training in economics. Chapter 17 deals with standardization and grading. The general theme is that standards and uniform grading can lower marketing and other transaction costs, as well as enhance demand. Marketing efficiency is improved by allowing consumers to better signal their consumption preferences and by rewarding producers who make investments that enhance quality. Other discussion issues include mandatory versus voluntary grades and the determinants of efficient minimum quality standards.

Articles (listed chronologically)

Zusman, P. (1967) 'A theoretical basis for determination of grading and sorting schemes', *Journal of Farm Economics*, 49(1), 89–106.

Zusman is interested in the endogenous determination of grading boundaries. Key in his analysis is the set of individual consumer quality valuation functions (IQVF) and the upper envelope of these functions, which is referred to as the market quality valuation function (MQVF). In a competitive market the endogenous grade boundaries will occur at the intersections of the IQVF and MQVF. In the absence of sorting costs, the number of grades is equal to the number of modes in the MQVF.

Schruben, L. W. (1968) 'Systems approach to marketing efficiency research', *American Journal of Agricultural Economics*, 50(5), 1454–1468.

In Example B Schruben uses linear programming to numerically solve a specific grading problem. A corn merchant makes buying, blending and selling decisions based on separate buying and selling price schedules. The merchant's offers to buy are based on the corn's relative quality attributes and the implicit value of the purchased corn in a commodity blend. This approach to blending is similar to that which is pursued in Chapter 5 of this textbook.

Freebairn, J. W. (1973) 'The value of information provided by a uniform grading system', *Australian Journal of Agricultural Economics*, 17(2), 127–139.

Freebairn argues that a uniform grading scheme will alter the information set of consumers and in most cases improve market efficiency. Specifically, consumers will typically invest in information gathering to reduce ex post decision losses that arise because of incomplete information. Grading can increase producer welfare by reducing both the level of ex post decision losses and the resources devoted to information gathering.

Ladd, G. W. and Martin, M. B. (1976) 'Prices and demands for input characteristics', *American Journal of Agricultural Economics*, 58(1), 21–30.

Commodity heterogeneity is viewed as the difference in the distribution of product characteristics. The value of a particular quality version of a commodity is the sum of the buyer's valuation of the individual characteristics of that commodity. Within this hedonic framework, Ladd and Martin derive a firm's demand for a productive input, and then use comparative static analysis to determine how demand changes with a change in a quality feature of the productive input. This analysis is similar to the shadow price approach taken in Chapter 5 of this textbook.

Johnson, D. D. and Wilson, W. W. (1993) 'Wheat cleaning decisions at country elevators', *Journal of Agricultural and Resource Economics*, 18(2), 198–210.

A mathematical programming model of wheat cleaning and blending at a country elevator is used to examine how a manager's decision to clean dockage from grain depends on price discounts for excess dockage, the value of the screenings and transportation costs. Johnson and Wilson add a cleaning decision variable to a standard blending problem, similar to that which is solved in Chapter 5 of this textbook.

Hennessy, D. A. (1996) 'The economics of purifying and blending', *Southern Economic Journal*, 63(1), 223–232.

Hennessy examines the conditions that result in profitable purification and blending. A key determinant of the purification and blending outcome is the curvature of the raw material price–quality schedule, and the cost of purification and blending. For the case where costs are not excessive and the price–quality relationship is convex at low quality levels and concave at high quality levels, purification is optimal for some quality intervals, blending is optimal for other intervals and no action is optimal for the remaining intervals.

Giannakas, K., Gray, R. and Lavoie, N. (1999) 'The impact of protein increments on blending revenues in the Canadian wheat industry', *International Advances in Economic Research*, 5(1), 121–136.

Additional protein categories for western Canadian wheat have gradually been added over the past 20 years. A linear programming model is used to estimate the gain in farm revenue from these additional protein categories under the assumption that wheat is blended to maximize protein rents for grain handlers. Sensitivity results are examined with respect to the distribution of protein, the price premiums for higher protein outcomes and the protein boundaries in the grading scheme.

Mohanty, S. and Peterson, E. W. F. (1999) 'Estimation of demand for wheat by classes for the United States and the European Union', *Agricultural and Resource Economics Review*, 28(2), 158–168.

In this paper it is recognized that countries such as the US and members of the EU routinely use a variety of quality classes of domestic and imported wheat. The extent to which the different quality classes are substitutes defines the extent that equilibrium prices across the different quality classes can diverge. The demand for wheat is estimated while assuming two types of product differentiation: (1) a particular type of wheat (e.g., hard red spring) is different across countries; and (2) within a country (e.g., Canada) the different types of wheat differ in quality.

Ligon, E. (2002) 'Quality and grading risk' in Just, R. E. and Pope, R. D., eds, *A Comprehensive Assessment of the Role of Risk in US Agriculture*, Boston, MA: Kluwer Academic Publishers, p. 586.

Grading is unlikely to perfectly reflect quality differences from the perspective of the consumer, and the full scope of investments that are made by producers to enhance quality. Of particular interest is the discussion by Ligon regarding why grading may or may not add significant value to the commodity at the retail level.

Chapter 6

Constant elasticity of substitution (CES) functions

Arrow, K. J., Chenery, H. B., Minhas, B. S. and Solow, R. M. (1961) 'Capital-labor substitution and economic efficiency', *The Review of Economics and Statistics*, 43(3), 225–250.

The constant elasticity of substitution (CES) production function can be traced back to this classic paper. Arrow *et al.* derive the CES production function by first imposing reasonable restrictions on the various relationships between average and marginal product for the variable inputs. They then work backward to recover the functional form of the CES production function. The CES production function is particularly convenient to use because the degree of substitution across the two factors of production is constant and can be adjusted with a single parameter of the production function. This paper by Arrow *et al.* generated a large theoretical and empirical literature on the economics of production.

Powell, A. A. and Gruen, F. H. G. (1968) 'The constant elasticity of transformation production frontier and linear supply system', *International Economic Review*, 9(3), 315–328.

These authors generalize the CES model of Arrow *et al.* (1961) to derive the constant elasticity of transformation (CET) production possibility frontier (PPF). As was discussed in Chapter 6 of this textbook, the CET production possibility frontier is algebraically identical to the CES production isoquant except for the value of a key parameter, which determines the concavity of the function.

Equilibrium displacement models

The CES supply and demand curve approach that was used in Chapter 6 of this textbook to derive and analyze a market equilibrium is seldom used in the economics literature. With multiple commodities and multiple markets the number of parameters that would be required to fully specify the model would be excessive, and the econometric estimation of such parameters would be difficult because of the inherent non-linearity of a CES function. As well, in many cases the sensitivity results are the most interesting, so it is sufficient to focus on how the values of the endogenous variables change in response to a change in an exogenous variable rather than focusing on the values of the endogenous variables themselves. Finally, although a CES function has a very simple expression for the elasticity of substitution, the expression for the elasticity of supply or demand is relatively complex. An equilibrium displacement model is a commonly used alternative for analyzing price impacts in a multi-market, multi-commodity setting. The equilibrium displacement model is based on a relatively small number of parameters, the results are expressed as percent changes rather than levels, and the elasticity parameters are straightforward to estimate using standard econometric techniques.

Muth, R. F. (1964) 'The derived demand curve for a productive factor and the industry supply curve', *Oxford Economic Papers*, 16(2), 221–234.

Muth demonstrated how a multi-market equilibrium can be expressed in terms of the associated supply and demand elasticities and the various cost and revenue share parameters. Consider the simplest model where $Q_d = D(P, Z_d)$ is the demand curve with P representing price and Z_d representing an exogenous determinant of demand (e.g., income). The supply curve can be written as, $Q_s = S(P, Z_s)$, where Z_s represents an exogenous determinant of supply (e.g., price of a key input). Equilibrium implies $D(P, Z_d) = S(P, Z_s)$. Totally differentiate and express in elasticity form to obtain $\frac{dQ_d}{Q_d} = \left(\frac{dD}{dP}\frac{P}{Q_d}\right)\frac{dP}{P} + \left(\frac{dD}{dZ_d}\frac{Z_d}{Q_d}\right)\frac{dZ_d}{Z_d}$ and $\frac{dQ_s}{Q_s} = \left(\frac{dS}{dP}\frac{P}{Q_s}\right)\frac{dP}{P} + \left(\frac{dD}{dZ_s}\frac{Z_s}{Q_s}\right)\frac{dZ_s}{Z_s}$. Recognizing that $Q_d = Q_s = Q^*$ in equilibrium, this pair of expressions can be solved for dQ^*/Q^* and dP^*/P^* as a function of the demand and supply elasticities, the other structural elasticities and the differentials dZ_d/Z_d and dZ_s/Z_s. The resulting expressions allow the impact of an exogenous change in Z_d or Z_s on Q^* and P^* to be formally analyzed.

Gardner, B. L. (1975) 'The farm–retail price spread in a competitive food industry', *American Journal of Agricultural Economics*, 57(3), 399–409.

Gardner uses the equilibrium displacement model to analyze agri-food marketing margins. His model involves the joint equilibrium of the retail food, farm production and marketing services sectors. Gardner is particularly interested in identifying how shifts in the various supply and demand schedules quantitatively affect the retail–farm price ratio and farmers' share of retail food expenditures. His analysis is ideally suited for analyzing the biofuels demand issue, which was central to Chapter 6 of this textbook.

Gardner, B. L. (1987) *The Economics of Agricultural Policies*, New York: Macmillan.

In this book Gardner makes extensive use of a multi-factor, multi-commodity equilibrium displacement model to analyze the pricing impacts of various agricultural policies such as subsidies, quotas and tariffs in a vertical agri-food supply chain. One of the most

common uses of the equilibrium displacement model in the agricultural economics literature is agricultural policy analysis.

Wohlgenant, M. K. (1993) 'Distribution of gains from research and promotion in multi-stage production systems: the case of the US beef and pork industries', *American Journal of Agricultural Economics*, 75(3), 642–651.

The equilibrium displacement model has been used in a wide variety of applications. In this paper Wohlgenant develops a multi-stage version of the equilibrium displacement model and uses it to evaluate how the gains from research and promotion are distributed within the agri-food supply chain.

Multi-market models

Croppenstedt, A., Bellu, L. G., Bresciani, F. and DiGiuseppe, S. (2007) *Agricultural Policy Impact Analysis with Multi-Market Models: A Primer*, Agricultural and Development Economics Division of the Food and Agriculture Organization of the United Nations (FAO–ESA), Working Papers: 07–26, unpublished.

These authors describe a multi-market model as one that is somewhere between a single-market partial equilibrium model and a large-scale computable general equilibrium (CGE) model. This class of models is particularly popular for undertaking agricultural policy reform impact analysis. One reason for this popularity is that a multi-market model is more accurate than a single-market model because cross-market and other indirect effects of a policy change are accounted for. Although a multi-market model is data intensive, it is still much less data intensive and simpler to use than a general equilibrium model.

Peterson, E. B., Hertel, T. W. and Stout, J. V. (1994) 'A critical assessment of supply–demand models of agricultural trade', *American Journal of Agricultural Economics*, 76(4), 709–721.

These authors are critical of large-scale supply–demand models that are used for forecasting and policy analysis. This paper provides a nice overview of how a multi-commodity, multi-market model is built, estimated and used for policy analysis.

Positive mathematical programming

Howitt, R. E. (1995) 'Positive mathematical programming', *American Journal of Agricultural Economics*, 77(2), 329–342.

Howitt describes the positive mathematical programming approach to modeling production, consumption, trade and prices in a model with multiple regions and multiple commodities. This quadratic programming approach allows for corner solutions (similar to spatial equilibrium analysis), and it can also account for declining marginal yields and other types of non-linear relationships. An important strength of positive mathematical programming is that the base case results are always exactly replicated without imposing excessive restrictions on the changes in the endogenous variables when exogenous policy variables are adjusted.

Chapter 7

Marketing margins – general

Tomek, W. G. and Robinson, K. L. (2003) *Agricultural Product Prices*, Ithaca, NY: Cornell University Press.

Tomek and Robinson's Chapter 6 is devoted to marketing margins in a competitive agri-food supply chain. They do not allow for differentiated products and imperfect competition, but they do show the basic mechanics of farm and retail price determination when the processed food product is produced from the farm commodity in fixed proportions. The marketing margin model in Chapter 7 of this textbook also assumes a fixed proportions processing technology.

Wohlgenant, M. K. (2001) 'Marketing margins: empirical analysis' in Gardner, B. L. and Rausser, G. C., eds, *Handbook of Agricultural Economics VI(1)*, Amsterdam and New York: Elsevier, pp. 933–970.

Wohlgenant provides a very thorough review of the academic literature on marketing margins in agri-food supply chains, with particular emphasis on empirical analysis. Much of his discussion centers on marketing margin outcomes that were derived using an equilibrium displacement model. Wohlgenant discusses both competitive and non-competitive markets. His treatment of non-competitive marketing margins assumes standard homogenous goods oligopoly and oligopsony rather than differentiated products monopolistic competition.

Marketing margins – differentiated products

Wohlgenant, M. K. (1999) 'Product heterogeneity and the relationship between retail and farm prices', *European Review of Agricultural Economics*, 26(2), 219–227.

Wohlgenant relaxes the fixed proportions assumption (see previous citation) and instead examines the more general case of marketing margins when the inputs into the production of the final retail product can be used in variable proportions. This paper is one of only a small number that examine the farm–retail marketing margin in a differentiated products monopolistic competition framework.

Azzam, A. M. (1999) 'Asymmetry and rigidity in farm–retail price transmission', *American Journal of Agricultural Economics*, 81(3), 525–533.

In this paper food processors are assumed to have market power because they are spatially separated. The market power of processors, combined with costly re-pricing, results in asymmetric retail and farm gate price responses to external supply and demand shocks. Azzam's findings are consistent with the key result from Chapter 7 of this text, which is that there is no well defined relationship between the size of the marketing margin and variables such as consumer demand and the processor's unit cost of operating.

Monopolistic competition with CES utility

Keller, W. J. (1976) 'A nested CES-type utility function and its demand and price-index functions', *European Economic Review*, 7(2), 175–186.

Keller describes the basic properties of a demand system that is derived from a nested CES utility function. These properties include simple endogenous price indexes that serve as unit prices for product composites. Assuming a nested CES utility function is equivalent to assuming *n*-stage budgeting. Most applications of monopolistic competition assume two-stage budgeting for allocation of income to a numeraire product and to a composite of all other products. In Chapter 7 of this textbook two-stage budgeting involves allocating income to a composite of processed food products and a composite of standard (semi-processed) food products.

Dixit, A. K. and Stiglitz, J. E. (1977) 'Monopolistic competition and optimum product diversity', *The American Economic Review*, 67(3), 297–308.

In this highly cited article the authors derive consumer demand schedules with the assumption that consumers have CES utility and use two-stage budgeting when making their consumption decisions. Monopolistically competitive firms supply differentiated products to consumers. Prices are marked up in the equilibrium according to the firm's market power, which depends on the degree of product differentiation. Dixit and Stiglitz establish a number of results concerning the optimal level of product diversity. The market margin model developed in Chapter 7 of this textbook has many features similar to the basic monopolistic competition model of Dixit and Stiglitz.

Monopolistic competition and international trade

Johnson, P. R., Grennes, T. and Thursby, M. (1979) 'Trade models with differentiated products', *American Journal of Agricultural Economics*, 61(1), 120–127.

This paper examines trade considerations when an agricultural commodity such as wheat from different exporting countries is viewed as differentiated by international buyers. The model is based on the well-known Armington assumption of international trade. Specifically, each of n countries produces a variant of a particular good that is a close substitute in the eye of the consumer. There are m goods in total, which implies that each consumer has a choice of mn goods and there are mn individual prices. Consumers are assumed to have CES preferences and utilize two-stage budgeting (in stage 1 expenditures are allocated to the m product groups and in stage 2 the expenditure assigned to each particular group is further assigned to the products within that group from the individual countries). The Armington assumption, which is similar to the two-stage budgeting process for processed and standard goods that was used in Chapter 7 of this textbook, is the standard way to create trade linkages in large-scale CGE models.

Krugman, P. (1980) 'Scale economies, product differentiation, and the pattern of trade', *The American Economic Review*, 70(5), 950–959.

Krugman's model uses a CES monopolistic competition framework to demonstrate that gains from trade are available for two trading nations, even if the economies have identical tastes, technology and factor endowments. The gains arise because consumers value product diversity and through trade firms can benefit from increasing returns to scale.

Lanclos, D. K. and Hertel, T. W. (1995) 'Endogenous product differentiation and trade policy: implications for the US food industry', *American Journal of Agricultural Economics*, 77(3), 591–601.

Kent and Hertel analyze prices for intermediate traded goods such as agricultural commodities when consumer goods are differentiated and tariffs are applied to both traded intermediate commodities and final consumer products. A central feature of their model is a set of consumers with CES preferences and a two-stage budgeting approach to decision making.

Chapter 8

General auction theory

McAfee, R. P. and McMillan, J. (1987) 'Auctions and bidding', *Journal of Economic Literature*, 25(2), 699–738.

This review paper examines the four general styles of auctions (English, Dutch and first- and second-price sealed bid) and specific features such as reserve bids and the allowance for multiple items and multiple rounds of bidding. One common assumption is that bidders have independent *private values*, which means that each bidder believes that the item's valuation by his or her competitor is drawn from a well-defined probability distribution. A second common assumption is that the item will have the same *common value* to all bidders, but different bidders have different beliefs about the item's value. One of the most important results from auction theory is that in a simple symmetric model with independent private values, the expected selling price for the seller is the same for the four auction types described above.

Milgrom, P. (1989) 'Auctions and bidding: a primer', *The Journal of Economic Perspectives*, 3(3), 3–22.

Milgrom begins with a detailed discussion about the winner's curse, which arises when bidders in a common value auction fail to account for the fact that the winning bid is the one with the largest estimation error. If bids are formulated without accounting for the winner's curse, then the expected profits for the winning bidder are typically negative. Milgrom also discusses an auction model with affiliated valuations, which has the private value and common value outcomes as special cases. With affiliation a bidder believes that if her own valuation rises then the valuations of her competitors are also higher. When comparing an English auction and a first-price sealed-bid auction, the affiliation assumption implies that expected market surplus is the same with the two types of auctions, but the expected surplus for the seller is higher with the English auction and lower with the first-price sealed-bid auction.

Vickrey, W. (1961) 'Counterspeculation, auctions, and competitive sealed tenders', *The Journal of Finance*, 16(1), 8–37.

Several important results from auction theory trace back to Vickrey's paper. First, he observed that a second-price sealed-bid auction, where the item is awarded to the highest bidder but the price paid is the bid of the second highest bidder, leads to the same pricing outcome as an English auction. Bids should proceed up to one's reserve price in an English auction and one's reserve price should be submitted in a second-price sealed-bid auction. Second, a Dutch auction and a first-price sealed-bid auction are expected to give rise to the same pricing outcome. In both cases the bid should be less than the bidder's valuation to achieve a profit maximizing balance between the probability of winning and the amount earned when a win is achieved.

Fang, H. and Morris, S. (2006) 'Multidimensional private value auctions', *Journal of Economic Theory*, 126(1), 1–30.

When constructing an auction model an assumption must normally be made about the distribution of a bidder's valuation of the item being sold. In the most common case where values are assumed to be continuously distributed, the problem is to derive a continuous bid function that specifies the bid level for each valuation that might be drawn from the distribution. An alternative approach is to assume that the random valuation takes on a discrete number of values (e.g., two). In this case, lower and upper endpoints for a bidding interval are derived, and random bids are assumed to be drawn from this interval according to an endogenously calculated probability function. This mixed strategy approach was used in Chapter 8 of this textbook, and was also used by Fang and Morris to examine optimal bidding when a signal about an opponent's valuation can be observed.

Auction papers specific to commodity markets

Sexton, R. J. (1994) 'A survey of noncooperative game theory with reference to agricultural markets: Part 2. Potential applications in agriculture', *Review of Marketing and Agricultural Economics*, 62(2), 183–200.

Sexton provides a nice overview of the key results from auction theory and the application of these results to agricultural commodity markets. He indicates that auctions are used in competitive markets where posted prices do not work well because of a high level of price volatility (e.g., fresh fish and certain types of fresh fruits and vegetables). Auctions are also useful when posted prices do not work well because quality is variable and buyer preference for quality is uncertain (e.g., used farm equipment). Sexton indicates that most auction theory has been designed for monopoly or monopsony situations (e.g., a grain import tender by a state agency, as discussed in Chapter 8 of this textbook). He provides the details of an interesting case study involving US dairy farmers in the mid 1980s who were bidding for the right to participate in a government herd disposal program.

Bourgeon, J.-M. and Le Roux, Y. (2001) 'Traders' bidding strategies on European grain export refunds: an analysis with affiliated signals', *American Journal of Agricultural Economics*, 83(3), 563–575.

This paper examines the bidding strategies of EU exporters who wished to participate in an EU grain export refund program that operated during the 1990s (the EU paid the successful firm the difference between the EU intervention price and the prevailing market price for the grain). Bourgeon and Le Roux estimate bidding strategies assuming a multivariate distribution of traders' information and using the associated correlation data. They are particularly interested in knowing whether world grain markets should be viewed as being differentiated, in which case a private value auction is most appropriate, or homogeneous, in which case a common value auction is most appropriate.

Wilson, W. W. and Diersen, M. A. (2001) 'Competitive bidding on import tenders: the case of minor oilseeds', *Journal of Agricultural and Resource Economics*, 26(1), 142–157.

Wilson and Diersen examine detailed data from successive Egyptian import tenders for various types of vegetable oil. The past bidding behavior of rival firms is used with a Bayesian predictive density procedure to estimate a bid function and the probability of winning the auction. The authors find that bidding strategies can be quite different for different sellers. For example, when the bid value is regressed on the seller's cost, in some cases the intercept is large and the slope is small, and in other cases the opposite is true.

Van den Berg, G., van Ours, J. and Pradhan, M. (2001) 'The declining price anomaly in Dutch Dutch rose auctions', *American Economic Review*, 91(4), 1055–1062.

The authors are interested in pricing behaviour in an English auction where buyers bid on multiple items of a homogeneous commodity over multiple rounds. Rather than bidding up to their reserve price, which is the optimal strategy in a single-round English auction, buyers should lower their maximum bid price to account for the option of participating in subsequent rounds. With each passing round of the auction there are fewer remaining bidders, but also fewer items left to auction. In a simple theoretical model these two forces cancel, and so price is not expected to change over time. In this study the authors use a large data set from a Dutch flower auction and demonstrate that on average price does decline over subsequent rounds of the auction.

MacDonald, J. M., Handy, C. R. and Plato, G. E. (2002) 'Competition and prices in USDA commodity procurement', *Southern Economic Journal*, 69(1), 128–143.

The United States Department of Agriculture (USDA) is a major buyer of grain and other commodities for distribution in various domestic and international food programs (e.g., National School Lunch Program). Food products are highly standardized, and the purchase is facilitated by a regularly-scheduled first-price sealed bid auction. This paper demonstrates that the winning bids are lower than the equivalent private market transaction prices, which is the desired outcome. However, the auction price is found to be

highly sensitive to the number of bidders who participate in the auction. For some auctions the number of participants is quite low.

Other applications of auctions

Latacz-Lohmann, U. and van der Hamsvoort, C. (1997) 'Auctioning conservation contracts: a theoretical analysis and an application', *American Journal of Agricultural Economics*, 79(2), 407–418.

Farmers who wish to participate in an environmental program such as the US Conservation Reserve Program (CRP) must typically bid for the privilege to do so in a sealed-bid auction. Auctions work well for the government provision of a public good because conservation does not have a standardized value. This paper examines a conservation auction when farmers differ with respect to land quality, and face an unknown program reserve price. Successful farmers are chosen either by progressively selecting low bid farmers until the program budget is exhausted, or by selecting all farmers for which the environmental benefit per dollar of bid is sufficiently large.

Cramton, P. and Kerr, S. (2002) 'Tradeable carbon permit auctions: how and why to auction not grandfather', *Energy Policy*, 333–345.

The authors argue that auctions should be used to limit carbon emissions by firms rather than allocating emission permits based on historical emission data. Fully bankable and tradeable carbon permits would be required by large energy firms (e.g., oil refineries and coal plants) as a compliance measure. The auction would involve gradually increasing price and allowing bidding to continue until there is no more excess demand for permits. The equilibrium price of the permit would depend critically on the overall mandated reduction in emissions.

Brown, J., Cranfield, J. A. L. and Henson, S. (2005) 'Relating consumer willingness-to-pay for food safety to risk tolerance: an experimental approach', *Canadian Journal of Agricultural Economics*, 53(2–3), 249–263.

Experimental auctions are a popular way to elicit consumer preferences for specific food attributes. Brown *et al.* allow participants to bid in a second-price, sealed-bid auction for the right to upgrade their chicken sandwich from one with a low risk of pathogen contamination to another with a very low risk of contamination. Bidding data, which was collected over multiple rounds with progressive disclosure of information, was used to construct a risk tolerance index. This index was then related to characteristics of the winning bidder such as gender and response to new information about risk.

Chapter 9

Bargaining theory

Osborne, M. J. and Rubinstein, A. (1990) *Bargaining and Markets*, San Diego, CA: Academic Press.

This book is a classic reference on bargaining theory. The first three chapters are used to review the Nash bargaining model, the sequential non-cooperative bargaining model, and the relationship between the two ways of modeling a bargaining scenario. The fourth chapter introduces incomplete information, which is a necessary condition to obtain a delay in the equilibrium bargaining agreement. The remainder of Osborne and Rubinstein's analysis focuses on the relationship between bargaining and market equilibrium. Various assumptions are made about how buyers and sellers are matched over time. Of particular interest is whether a dynamic bargaining model can provide a satisfactory explanation of how markets clear and how price is established in a perfectly competitive market.

Muthoo, A. (1999) *Bargaining Theory with Applications*, New York: Cambridge University Press.

The material presented in Chapter 9 of this textbook closely follows Muthoo's classic analysis of bargaining. This book also begins with the Nash bargaining model, but the majority of the analysis is devoted to sequential bargaining, which is referred to as the "alternating offers" model or the "Rubinstein" model (Rubinstein is credited with pioneering the theory of sequential bargaining). The main difference between the book by Muthoo and the one by Osborne and Rubinstein is that Muthoo pays special attention to the concepts of inside and outside options. He also applies the bargaining results to a number of scenarios including sovereign debt renegotiation, bribery and wage quality contracts. Advanced topics include bargaining with asymmetric information and repeated bargaining situations.

Agricultural bargaining associations

Helmberger, P. G. and Hoos, S. (1963) 'Economic theory of bargaining in agriculture', *Journal of Farm Economics*, 45(5), 1272–1280.

Helmberger and Hoos point out that without a theory of bargaining the market outcome is indeterminate for a bilateral monopoly. The quantity of exchange can be identified by invoking the principle of Pareto optimality (i.e., the two parties will choose the amount to be exchanged so that one party cannot be made better off without making the other party worse off), but price is still indeterminate. The Nash bargaining solution ensures that quantity and price are both determinate. The remainder of the article is devoted to a discussion about the types of industries that are likely to benefit the most from the establishment of a bargaining association.

Sexton, R. J. (1994) 'A survey of noncooperative game theory with reference to agricultural markets: Part 2. Potential applications in agriculture', *Review of Marketing and Agricultural Economics*, 62(2), 183–200.

Sexton indicates that the assumptions of the standard sequential bargaining model are reasonable for real-world bargaining associations that operate in US agri-food markets. For example, the quantity to be exchanged is normally predefined, so price is the key bargaining variable and the "fixed pie" assumption holds. As well, the assumption of bilateral bargaining is reasonable because a bargaining association typically handles at least 50 percent of industry production and negotiates with a single dominant processor (arrangements with other processors are then based on the single negotiated agreement). The outside option for the bargaining association includes taking legal action and delivering to another market. Asymmetric information will generally favor the processor.

Hueth, B. and Marcoul, P. (2003) 'An essay on cooperative bargaining in US agricultural markets', *Journal of Agricultural and Food Industrial Organization*, 1(1), n.a.

This paper discusses industry characteristics that tend to be associated with successful bargaining associations in US agri-food markets. Associations tend to be most active when growers and commodity processors operate with bilateral contracts and the industry is geographically concentrated. As well, bargaining associations tend to be most prevalent when growers make relationship-specific investments and have limited outside options. The authors emphasize that bargaining associations provide a number of useful services such as contract assurance and cost-efficient legal counsel, but there is no clear

theoretical or empirical evidence which suggests that bargaining associations are effective at raising the long-term price for growers.

Hueth, B. and Marcoul, P. (2006) 'Information sharing and oligopoly in agricultural markets: the role of the cooperative bargaining association', *American Journal of Agricultural Economics*, 88(4), 866–881.

In markets where grower supply is relatively elastic and processed products are sufficiently differentiated, processors and growers earn higher surplus when information about product demand is shared. However, prisoner dilemma incentives imply that information may not be shared in the absence of a coordinating mechanism. Hueth and Marcoul argue that one reason for the popularity of bargaining associations in US agriculture is that information is automatically shared by the specific nature of the bargaining process.

Steiner, B. E. (2007) 'Negotiated transfer pricing: theory and implications for value chains in agribusiness', *Agribusiness*, 23(2), 279–292.

This paper examines the evolution of pricing mechanisms in agri-food supply chains, including a shift in several US industries from negotiated prices to formula prices. Pricing according to a formula is an attractive alternative to bargaining if bargaining transaction costs are high, if there are significant differences in bargaining power and if there is significant mistrust among those involved in the bilateral negotiations. Mistrust is of particular concern when firms make relationship-specific investments and are therefore subject to holdup.

Other bargaining in agriculture

Martin, L., Paarlberg, P. L. and Lee, J. G. (1999) 'Bargaining for European Union farm policy reform through US pesticide restrictions', *Agricultural and Resource Economics Review*, 28(2), 137–146.

Differences in environmental standards pose significant challenges for countries who are attempting to liberalize trade in agricultural commodities. The EU justifies price supports because of the highly restrictive pesticide policies that EU growers face. This paper uses a bargaining framework to examine whether the US will choose to tighten its pesticide policy if the EU agrees to lower support prices for agricultural commodities. Iso-welfare contours are estimated for the two regions and principles from Nash bargaining theory are used to examine the feasibility of the proposed policy shift.

Reiersen, J. (2001) 'Bargaining and efficiency in sharecropping', *Journal of Agricultural Economics*, 52(2), 1–15.

Economists have long sought to better understand why sharecropping is an important feature of agriculture in developing countries. Reiersen examines sharecropping as a two-stage bargain between the landowner and the tenant. The rental rate is negotiated in stage 1 and labor input is negotiated in stage 2. Common sharecropping arrangements emerge when particular assumptions are made about the distribution of bargaining power across the landowner and tenant.

Bandyopadhyay, S. and Bandyopadhyay, S. C. (2001) 'Efficient bargaining, welfare and strategic export policy', *Journal of International Trade and Economic Development*, 10(2), 133–149.

In the standard story of strategic export policy, the export subsidy enables the firm to commit to a higher level of output in a foreign market, and this commitment raises domestic welfare. The authors' analysis of strategic export subsidies incorporates a unionized work force that negotiates either a minimum level of employment or a

minimum wage with the firm. The minimum employment strategy raises domestic welfare because committed output in the foreign market is increased. The opposite is true for the minimum wage strategy because of the associated reduction in the output commitment.

Bargaining over natural resources

Sandler, T. (1993) 'Tropical deforestation: markets and market failures', *Land Economics*, 69(3), 225–233.

In an open market economy where natural resources are used in the production of consumer goods the joint production outcome is globally inefficient because of global public goods and positive externalities (e.g., tropical deforestation). A simple Nash bargaining model is used to show how multilateral negotiations can improve welfare by reducing the externalities. For a variety of reasons developed countries are likely to be more impatient than developing economies, and so the negotiated outcomes should favor the developing economies.

Hyndman, K. (2008) 'Disagreement in bargaining: an empirical analysis of OPEC', *International Journal of Industrial Organization*, 26(3), 811–828.

Two cartel members bargain over the allocation of production quota. The bargaining process consists of one cartel member making a take-it-or-leave-it offer to the other member. Members are different both with respect to cost and the information they possess. The main result is that if there are sufficiently large differences for the two members of the cartel, then the probability of agreement depends on both initial production and the current state of demand. Low demand and high initial production implies a low probability of agreement and vice versa.

Ansink, E. and Weikard, H.-P. (2009) 'Contested water rights', *European Journal of Political Economy*, 25(2), 247–260.

Countries that share rivers often have disputes over water allocation. Countries can choose to bargain their way to an efficient outcome, or can reject bargaining and instead spend resources on fighting to ensure that their access to the water does not erode. If this latter option is chosen, third party arbitration is eventually used to settle the dispute. These authors show that for some parameter values it is rational for countries to reject the bargaining approach because of the option value provided by third party intervention.

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