Soil Mechanics: concepts and applications 2nd edition

SOLUTIONS MANUAL

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Contents

QUESTIONS AND SOLUTIONS: CHAPTER 1

Origins and mineralogy of soils

Q1.1 Describe the main depositional environments and transport processes relevant to soils, and explain their influence on soil fabric and structure.

Q1.1 Solution
Use material in Section 1.3.1 to describe and explain

- *transport processes: water, wind, ice, ice and water*
- *depositional environment: water might be fast or slow flowing, eg upstream (fast) or downstream (slow), or ebbing floodwater (probably slow). Windborne material might be washed out of the atmosphere by rain. Material can be transported either on the top of, within or below a glacier or icesheet, or by a combination of ice and meltwater (outwash streams – possibly fast flowing) and perhaps deposited into a glacial lake (slow flowing).*
- *effect of transport mechanism and depositional environment on particle size – soils transported by wind and water are likely to be sorted, with finer particles remaining in suspension and being transported longer distances than coarse particles. Fine particles fall out of suspension where the water velocity is low, eg deltaic and flood plain deposits. Coarse particles on a river bed are left behind as terraces when a river changes course. Sand dunes migrate due to wind action; deposits of windborne dust washed out by rain may be very lightly cemented with a delicate and potentially unstable structure (loess). Material transported purely by ice tends to be less sorted (eg boulder clay typically has a very wide range of particle size). If final transport or deposition is by or through water some sorting will take place - perhaps vertically rather than horizontally, eg mixed material washed off the top of a glacier and deposited into a glacial lake will have a laminated structure as coarse material settles quickly and fine material more slowly, a pattern repeated over many seasons as the deposit accumulates.*
- *effect on particle shape – materials transported by ice are likely to be more angular, and materials transported by water more rounded.*

Q1.2 Summarize the main effects of soil mineralogy on particle size and soil characteristics.

Q1.2 Solution
Use material in Section 1.4 to describe and explain the effects of mineralogy and chemical structure on

- *particle size, flakiness and shape (clay minerals tend to be softer, more sheetlike and more easily eroded/abraded to form small, platey particles)*
- *other soil characteristics including plasticity, colloidal behaviour and capacity for cation exchange (sorption) that result from the high specific surface area, the significance of surface forces and surface chemistry effects in clays*

Phase relationships, unit weight and calculation of effective stresses

Q1.3 A density bottle test on a sample of dry soil gave the following results.

1. Mass of 50ml density bottle empty, g	25.07
2. Mass of 50ml density bottle + 20g of dry soil particles, g	45.07
3. Mass of 50ml density bottle + 20g of dry soil particles, with remainder of space in bottle filled with water, g	87.55
4. Mass of 50ml density bottle filled with water only, g	75.10

Calculate the relative density (specific gravity) of the soil particles. A 1 kg sample of the same soil taken from the ground has a natural water content of 27% and occupies a total volume of 0.52 litre. Determine the unit weight, the specific volume and the saturation ratio of the soil in this state. Calculate also the water content and the unit weight that the soil would have if saturated at the same specific volume, and the unit weight at the same specific volume but zero water content.

Q1.3 Solution

The particle relative density (grain specific gravity) G_s is defined as the ratio of the mass density of the soil grains to the mass density of water. For a fixed volume of solid - in this case, the soil particles - the specific gravity is equal to the mass of the dry soil particles divided by the mass of water they displace.

The mass of the dry soil particles is given by $(m_2-m_1) = 20.00g$

The mass of water displaced by the soil particles is given by $(m_4-m_1) - (m_3-m_2) = (50.03) - (42.48) = 7.55g$

$G_s = (m_2-m_1)/[(m_4-m_1)-(m_3-m_2)] = (20.00g)\div(7.55g) = \underline{2.65}$

For the sample of natural soil, the unit weight is equal to the actual weight divided by the total volume,

$\gamma = (1kg \times 9.81N/kg \times 0.001kN/N) \div (0.52\times10^{-3}m^3)$

$\Rightarrow \underline{\gamma = 18.865\ kN/m^3}$

The water content $w = m_w/m_s = 0.27$. For the 1kg sample, we know that $m_w+m_s = 1kg$, hence

$1.27 \times m_s = 1kg$

$\Rightarrow m_s = 0.7874kg$ *and* $m_w = 0.2126kg$

The volume of water $v_w = m_w/\rho_w = 0.2126kg \div 1kg/litre = 0.2126litre$

The volume of solids $v_s = m_s/\rho_s = 0.7874kg\div2.65kg/litre = 0.2971litre$

The specific volume v is defined as the ratio $v_t/v_s = 0.52litre/0.297litre$

$\Rightarrow \underline{v = 1.75}$

The saturation ratio is given by the volume of water divided by the total void volume, = 0.2126litre ÷(0.52litre - 0.297litre) = 0.9534

$\Rightarrow \underline{S_r = 95.34\%}$

If the soil were fully saturated, the volume of water would be (0.52litre - 0.297litre) = 0.223litre. The mass of water would be 0.223kg, and the water content would be 0.223kg ÷ 0.7874kg

$\Rightarrow \underline{w_{sat} = 28.32\%}$

The overall mass of the 0.52litre sample would be 0.223kg + 0.7874kg = 1.0104kg, and its unit weight $(1.0101kg \times 9.81N/kg \times 10^{-3}kN/N) \div (0.52 \times 10^{-3} m^3)$

$\Rightarrow \underline{\gamma_{sat} = 19.06 kN/m^3}$

If the soil were dry but had the same specific (and overall) volume, the mass would be equal to the mass of solids alone, and the unit weight would be $(0.7874kg \times 9.81N/kg \times 10^{-3}kN/N) \div (0.52 \times 10^{-3} m^3)$

$\Rightarrow \underline{\gamma_{dry} = 14.86 kN/m^3}$

Q1.4 An office block with an adjacent underground car park is to be built at a site where a 6m-thick layer of saturated clay (γ = 20 kN/m^3) is overlain by 4m of sands and gravels (γ = 18 kN/m^3). The water table is at the top of the clay layer, and pore water pressures are hydrostatic below this depth. The foundation for the office block will exert a uniform surcharge of 90 kPa at the surface of the sands and gravels. The foundation for the car park will exert a surcharge of 40 kPa at the surface of the clay, following removal by excavation of the sands and gravels. Calculate the initial and final vertical total stress, pore water pressure and vertical effective stress, at the mid-depth of the clay layer, (a) beneath the office block; and (b) beneath the car park. Take the unit weight of water as 9.81kN/m^3.

Q1.4 Solution
Initially, the stress state is the same at both locations. The vertical total stress $\sigma_v = (4m \times 18kN/m^3)$ *(for the sands and gravels) +* $(3m \times 20kN/m^3)$ *(for the clay), giving*

$\underline{\sigma_v = 132 \text{ kPa}}$

The pore water pressure $u = (3m \times 9.81kN/m^3) = \underline{29.4 \text{ kPa}}$

The vertical effective stress $\sigma'_v = \sigma_v - u = (132kPa - 29.4kPa) = \underline{102.6 \text{ kPa}}$

Finally,

(a) Beneath the office block, the vertical total stress is increased by the surcharge of 90kPa, giving

σ_v *= 132kPa + 90 kPa* $\Rightarrow$ $\underline{\sigma_v = 222\ kPa}$

The pore water pressure u is unchanged, $\Rightarrow$ $\underline{u = 29.4kPa}$

The vertical effective stress $\sigma'_v = \sigma_v - u = (222kPa - 29.4kPa)$

$\Rightarrow \underline{\sigma'_v = 192.6\ kPa}$

(b) Beneath the car park, the vertical total stress is given by
σ_v *= (40kPa) (surcharge) + (3m* $\times$ *20kPa) (for the clay)* $\Rightarrow$ $\underline{\sigma_v = 100\ kPa}$

The pore water pressure u is unchanged, $\Rightarrow$ $\underline{u = 29.4kPa}$

The vertical effective stress $\sigma'_v = \sigma_v - u = (100kPa - 29.4kPa)$

$\Rightarrow \underline{\sigma'_v = 70.6\ kPa}$

Q1.5 For the measuring cylinder experiment described in main text Example 1.3, calculate (a) the vertical effective stress at the base of the column of sand in its loose, dry state; (b) the pore water pressure and vertical effective stress at the base of the column in its loose, saturated state; (c) the pore water pressure and vertical effective stress at the base of the column in its dense, saturated state; and (d) the pore water pressure and vertical effective stress at the sand surface in the dense, saturated state. Take the unit weight of water as $9.81kN/m^3$.

Q1.5 Solution
(a) In the loose dry state, the vertical total stress is given by the unit weight of the sand $\times$ *the depth h. The depth of the sand is given by the volume,* $1200cm^3$*, divided by the cross-sectional area of the measuring cylinder,* $28.27cm^2$*, giving h = 42.448cm. Hence* $\sigma_v = 16.35kN/m^3 \times 0.4245m = 6.94kPa$*. As the sand is dry, the pore water pressure u = 0 and*

$\underline{\sigma'_v = \sigma_v = 6.94kPa}$

(Alternatively, the total weight of sand is $2kg \times 9.81 \times 10^{-3} kN/kg = 0.01962kN$*. This is spread over an area of* $(\pi \times 0.06^2 m^2) \div 4 = 0.002827m^2$*. Hence the total stress* $\sigma_v = 0.01962kN \div 0.002827m^2 = 6.94\ kPa$*.)*

(b) In the loose, saturated state, the pore water pressure $u = 0.4245m \times 9.81kN/m^3$
$\Rightarrow \underline{u = 4.164\ kPa}$

The vertical total stress σ_v = 19.99kN/m^3 $\times$ 0.4245m = 8.486kPa. Hence the vertical effective stress $\sigma'_v = \sigma_v$ - u = 8.486kPa - 4.164kPa $\Rightarrow$ $\underline{\sigma'_v = 4.322kPa}$

(c) In the dense, saturated state, the weights of water and soil grains above the base do not change. Hence the pore water pressure and the total stress are the same as before, and so also is the effective stress: $\underline{u = 4.164\ kPa;\ \sigma'_v = 4.322kPa}$

(d) The water level in the column does not change: as the sand is densified, it settles through the water. The new sample height h' is given by its volume, 1130cm^3, divided by the cross-sectional area of the measuring cylinder, 28.27cm^2, giving h' = 39.972cm. The depth of water above the new sample surface is therefore (42.448cm - 39.972cm) = 2.476cm. The pore water pressure at the new soil surface is 9.81kN/m^3 $\times$ 0.02476m $\Rightarrow$ $\underline{u = 0.243kPa}$

The effective stress at the sand surface is zero.

Particle size analysis and soil filters

Q1.6 A sieve analysis on a sample of initial total mass 294g gave the following results:

Sieve size, mm	6.3	3.3	2.0	1.2	0.6	0.3	0.15	0.063
Mass retained, g	0	0	30	39	28	28	16	11

A sedimentation test on the 117 g of soil collected in the pan at the base of the sieve stack gave:

Size, μm	<2	2-6	6-15	15-30	30-63
% of pan sample	0	48	29	14	9

Plot the particle size distribution curve and classify the soil using the system given in Table 1.5. Determine the D_{10} particle size and the uniformity coefficient U, and comment on the grading curve.

Q1.6 Solution
First, note that the total of the masses retained is 152g, which together with the 117g collected in the pan gives 269g. Thus there is a shortfall of 25g, which is presumably attributable to sieve losses.

Take the total mass of the sample as 269g.

The % by mass of the sample passing each sieve is given by the total sample mass (269g) minus the cumulative mass of soil retained on larger size sieves. Hence

Sieve size, mm	***6.3***	***3.3***	***2.0***	***1.2***	***0.6***	***0.3***	***0.15***	***0.063***
Mass retained, g	*0*	*0*	*30*	*39*	*28*	*28*	*16*	*11*
Cumulative mass retained, g	*0*	*0*	*30*	*69*	*97*	*125*	*141*	*152*
Mass passing, g	*269*	*269*	*239*	*200*	*172*	*144*	*128*	*117*
% passing	***100***	***100***	***88.8***	***74.3***	***63.9***	***53.5***	***47.6***	***43.5***

The sedimentation test data are already part-processed, with the mass of soil in each size range expressed as a percentage of the 117g collected in the pan. This is slightly different from main text Example 1.5, in which raw data are given.

The fraction of the pan sample smaller than a given size is given by 100% minus the cumulative percentage in the larger size ranges. To convert this to a percentage of the total sample, we must multiply by 117g and divide by 269g. Hence

Size, μm	*<2*	*2-6*	*6-15*	*15-30*	*30-63*
% of pan sample	*0*	*48*	*29*	*14*	*9*

Size, μm	***2***	***6***	***15***	***30***	***63***
% of pan sample smaller than this size	*(48-48) = 0*	*(77-29) = 48*	*(91-14) = 77*	*(100-9) = 91*	*100*
% of total sample smaller than this size	***0***	***20.9***	***33.5***	***39.6***	***43.5***

The particle size distribution curve is plotted in Figure Q1.6, using the data shown in bold type.

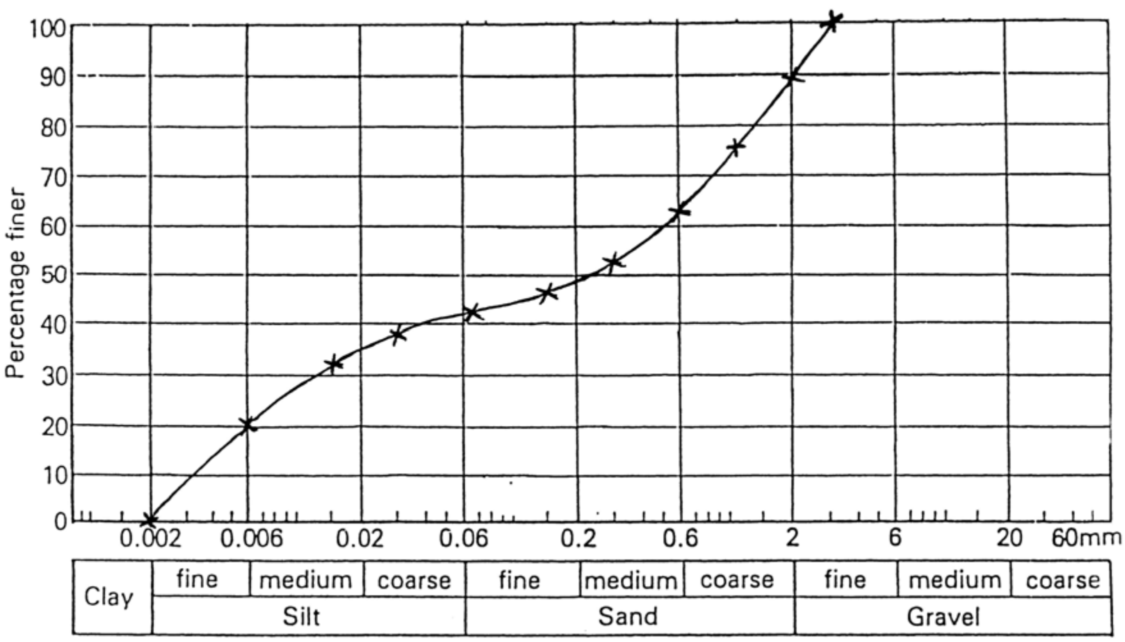


Figure Q1.6: Particle size distribution curve

Reading off from the curve, $\underline{D_{10} \approx 0.0035 \text{ mm } (3.5 \mu m)}$

$D_{60} \approx 0.52$ *mm*

Hence the uniformity coefficient $\underline{U = D_{60}/D_{10} \approx 150\ (148.6)}$

The soil is approximately 40% silt, 50% sand and 10% fine gravel: this makes it a sandy SILT according to the system given in Table 1.5.

The soil is poorly (almost gap-) graded.

Q1.7 A sieve analysis on a sample of initial total mass 411g gave the following results:

Sieve size, mm	6.3	1.2	0.3	0.063
Mass retained, g	0	60	126	92

A sedimentation test on the 121 g of soil collected in the pan at the base of the sieve stack gave:

Size, µm	<2	2-10	10-60
% of pan sample	33	24	43

Plot the particle size distribution curve and classify the soil using the system given in Table 1.5. On the PSD diagram, sketch a suitable curve for a granular filter to be used between this soil and a drainage pipe with 3 mm perforations.

Q1.7 Solution

The total of the masses retained is 278g, which together with the 121g collected in the pan gives 399g. Thus there is a shortfall of 12g, which is attributable to sieve losses. Take the total mass of the sample as 399g. The % by mass of the sample passing each sieve is given by the total sample mass (399g) minus the cumulative mass of soil retained on larger size sieves. Hence

Sieve size, mm	***6.3***	***1.2***	***0.3***	***0.063***
Mass retained, g	*0*	*60*	*126*	*92*
Cumulative mass retained, g	*0*	*60*	*186*	*278*
Mass passing, g	*399*	*339*	*213*	*121*
% passing	***100***	***85.0***	***53.4***	***30.3***

The sedimentation test data are again already part-processed, with the mass of soil in each size range expressed as a percentage of the 121g collected in the pan. The fraction of the pan sample smaller than a given size is equal to the sum of the percentages in this and the smaller *size ranges. To convert this to a percentage of the total sample, we must multiply by 121g and divide by 399g. Hence*

Size, μm	*<2*	*2-10*	*10-60*
% of pan sample	*33*	*24*	*43*

Size, μm	***2***	***10***	***60***
% of pan sample smaller than this size	*33*	*(33+24) = 57*	*(33+24+ 43)=100*
% of total sample smaller than this size	***10.0***	***17.3***	***33.0***

The particle size distribution curve is plotted in Figure Q1.7 using the data shown in bold type.

Reading from the PSD curve, the soil is approximately 10% clay, 20% silt, 60% sand and 10% fine gravel: this makes it a clayey, very silty SAND.

Also reading from the curve,

$D_{15s} \approx 0.007$ *mm,*

$D_{85s} \approx 1.2$ *mm*

Q1.7 SOLUTION

The filter PSD curve is sketched on Figure Q1.7 according to the following rules.

- $D_{15f} \leq 5 \times D_{85s}$ *(main text Equation 1.18)* $\Rightarrow D_{15f} \leq 6$ *mm (point B on Figure Q1.7)*
- $D_{15f} > 4 \times D_{15s}$ *(main text Equation 1.19)* $\Rightarrow D_{15f} > 0.028$ *mm (point A on Figure Q1.7)*
- $D_{5f} \geq 63$ *μm (main text Equation 1.18; point C on Figure Q1.7)*
- D_{10f} *~ slot width = 3 mm (point D on Figure Q1.7)*
- $D_{60f} \leq \sim 3 \times D_{10f}$ *(main text Equation 1.21)* $\Rightarrow D_{60f} \leq \sim 9$ *mm (point E on Figure Q1.7)*

Using a degree of judgement to account for the very wide range of particle size present in the natural soil, and recalling the advice given by Preene et al *(2000) that in variable ground main text Equation 1.18 should be applied to the finest soil and main text Equation 1.19 to the coarsest, a suitable PSD curve for the filter is sketched in Figure Q1.7.*

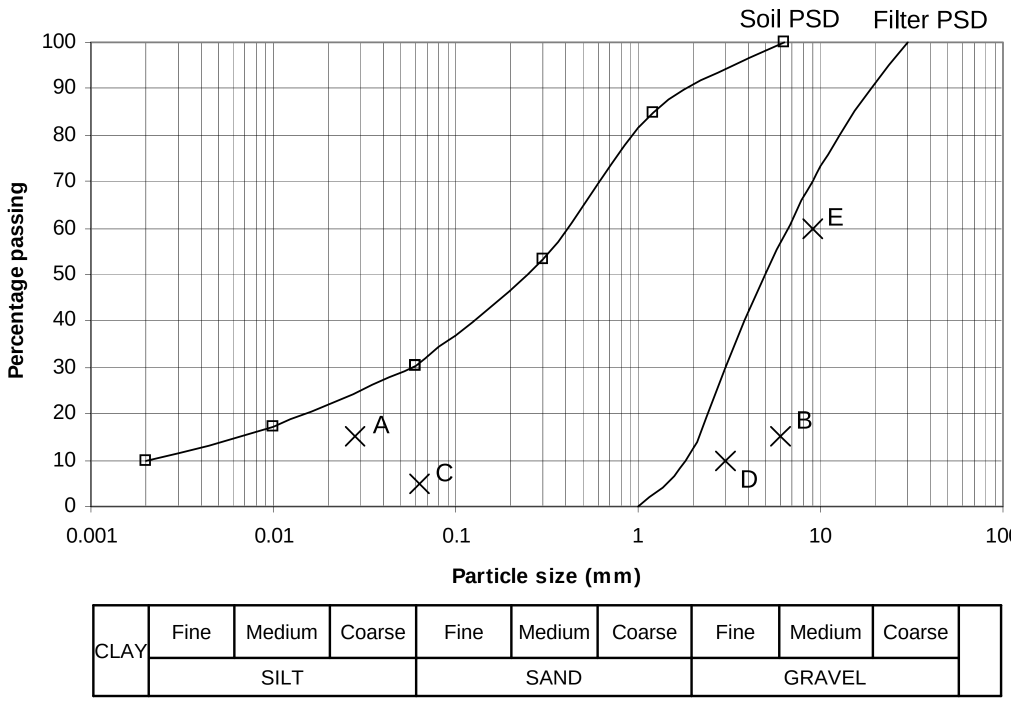


Figure Q1.7: Particle size distribution curves for natural soil and suitable filter

Index tests and classification

Q1.8 The following results were obtained from a series of cone penetrometer tests using a standard 80g, 30° cone.

Mass of tin empty, g	18.2	19.1	17.7	18.6
Mass of tin + sample wet, g	51.5	45.5	50.7	43.4
Mass of tin + sample dry, g	37.8	35.6	39.7	36.3
Cone penetration d, mm	25.0	14.2	8.5	5.1

Determine the water content w of each sample. Plot a graph of w against ln(d), and estimate the liquid limit w_{LL}. If the soil has a plastic limit of 22%, calculate the plasticity index and classify the soil using the chart given in Figure 1.15.

Q1.8 Solution

The water content is the mass of water divided by the mass of soil solids, i.e. $\{(m_s+m_w+m_t) - (m_s+m_t)\} \div \{(m_s+m_t) - (m_t)\}$:

Mass of tin empty, g (m_t)	*18.2*	*19.1*	*17.7*	*18.6*
Mass of tin + sample wet, g (m_s+m_w+m_t)	*51.5*	*45.5*	*50.7*	*43.4*
Mass of tin + sample dry, g (m_s+m_t)	*37.8*	*35.6*	*39.7*	*36.3*
m_s/m_w, %	***69.9***	***60.0***	***50.0***	***40.1***
Cone penetration d, mm	*25.0*	*14.2*	*8.5*	*5.1*
ln(d)	***3.219***	***2.653***	***2.14***	***1.63***

A graph of w against ln(d) is plotted in Figure Q1.8. The liquid limit corresponds to a cone penetration of 20mm, i.e. ln(d) = 2.996. Reading from the graph,

w_{LL}≈65%

The plasticity index PI = w_{LL} - w_{PL} = 65% - 22% ⇒ PI≈43%. By plotting the point w_{LL}=65%; PI=43% on the chart given in main text Figure 1.15, the soil can be classified as a high plasticity clay (CH).

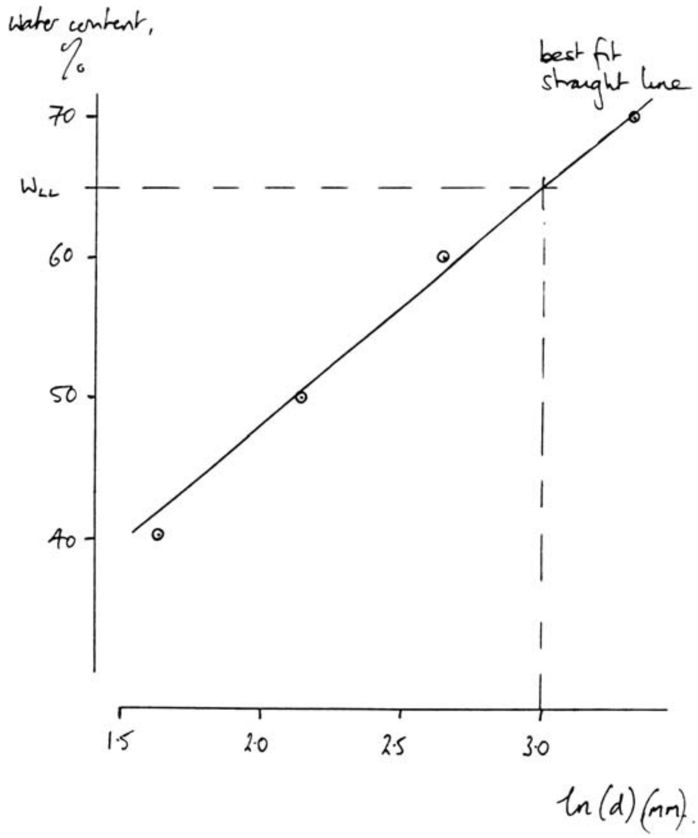


Figure Q1.8: water content against ln(cone penetration)

Compaction

Q1.9 The following results were obtained from a standard (2.5 kg) Proctor compaction test:

Mass of tin empty, g	14	14	14	14	14
Mass of tin + sample wet, g	88	68	98	94	93
Mass of tin + sample dry, g	81	62	87	82	80
Density, kg/m^3	1730	1950	2020	1930	1860

Plot a graph to determine

(i) the maximum dry density,
(ii) the optimum water content and
(iii) the actual density at the optimum water content.

If the particle relative density (grain specific gravity) G_s = 2.65, calculate
(iv) the specific volume and
(v) the saturation ratio at the maximum dry density.

Q1.9 Solution
We need to plot a graph of water content w against dry density ρ_{dry}, where

$w = m_w/m_s$ *(main text Equation 1.5)*

and

$\rho_{dry} = \rho/(1+w)$ *(main text Equation 1.27)*

The water content of each sample is calculated as in Example E1.1:

$$w = \frac{m_w}{m_s} = \frac{\{(m_t + m_s + m_w) - (m_t + m_s)\}}{\{(m_t + m_s) - (m_t)\}}$$

where

(m_t) = mass of tin, empty
$(m_t + m_s + m_w)$ = mass of tin + wet soil sample
$(m_t + m_s)$ = mass of tin + dry soil sample

Hence

Mass of tin empty, g	*14*	*14*	*14*	*14*	*14*
Mass of tin + sample wet, g	*88*	*68*	*98*	*94*	*93*
Mass of tin + sample dry, g	*81*	*62*	*87*	*82*	*80*
w, %	***10.45***	***12.5***	***15.07***	***17.65***	***19.70***
Density, kg/m^3	*1730*	*1950*	*2020*	*1930*	*1860*
Dry density, kg/m^3	***1566***	***1733***	***1755***	***1640***	***1554***

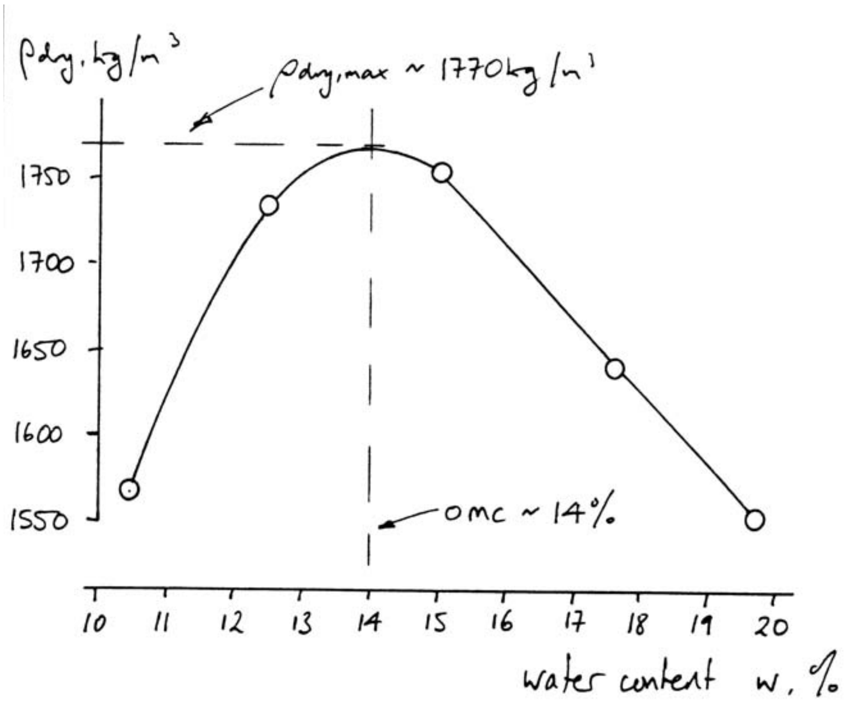


Figure Q1.9: dry density against water content

From the graph (Figure Q1.9),

- *the maximum dry density $\rho_{dry,max}$ $\underline{\approx 1770\ kg/m^3}$*

- *the optimum water content (at $\rho_{dry,max}$) $\underline{\approx 14\%}$*

- *the actual density at the optimum water content = 1770 kg/m^3 $\times$ 1.14 = $\underline{2018\ kg/m^3}$*

The specific volume v can be calculated using main text Equation 1.8,

$\gamma = [G_s.(1+w)/v].\gamma_w$ *(main text Equation 1.8)*

or

$v = G_s.(1+w).(\gamma_w/\gamma)$

hence

$v = 2.65 \times (1.14) \times (1000/2018) = \underline{1.497}$

(The void ratio $e = v - 1 = 0.497$*)*

The saturation ratio S_r *is calculated using main text Equation 1.10,*

$S_r = w.G_s/e = w.G_s/(v-1)$ *(main text Equation 1.10)*

$S_r = 0.14 \times 2.65 \div 0.497 = 0.746$ *or* $\underline{74.6\%}$

QUESTIONS AND SOLUTIONS: CHAPTER 2

The shearbox test

Q2.1 Describe with the aid of a diagram the essential features of the conventional shearbox apparatus. Stating clearly the assumptions you need to make, show how the quantities measured during the test are related to the stresses and strains in the soil sample.
[University of London 2nd year BEng (Civil Engineering) examination, King's College (part question)]

Q2.1 Solution
Diagram of shear box: See main text Figure 2.14

Assume that the stresses and strains are uniform and continuous, and that the actual deformation in shear (main text Figure 2.15a) is idealised as indicated in main text Figure 2.15b.

The known or measured quantities are

A	*the sample area on plan, assumed to remain constant during the test)*
H	*the initial height of the sample*
N	*the normal (hanger) load*
F	*the shear force*
x	*the relative horizontal displacement between the upper and lower halves of the shearbox*
y	*the upward movement of the shearbox lid.*

Consideration of main text Figure 2.15b gives strains
shear strain $\gamma = x/H$
volumetric strain $\varepsilon_{vol} = -y/H$

In terms of stresses,
shear stress on central horizontal plane $\tau = F/A$
normal stress on central horizontal plane $\sigma = N/A$

If it is further assumed that the pore water pressure u is zero (so that $\sigma' = \sigma$*) and the central horizontal plane is the plane of maximum stress obliquity* $(\tau/\sigma')_{max}$*, a Mohr circle of stress may be drawn (eg main text Figure 2.30), and the mobilised effective angle of friction is*

$$\phi'_{mob} = \tan^{-1}\{(\tau/\sigma')_{max}\}$$

Q2.2 With the aid of sketches, describe, explain and contrast the results you would expect to obtain from conventional shearbox tests on samples of dry sand which were (a) initially loose, and (b) initially dense. What factors would you take into account in selecting a soil strength parameter for use in design?
[University of London 2nd year BEng (Civil Engineering) examination, King's College (part question)]

Q2.2 Solution

Typical graphs of (a) shear stress τ against shear strain γ; (b) volumetric strain ε_{vol} against shear strain γ; and (c) specific volume v against shear strain γ are as shown in main text Figure 2.21.

In the test carried out on the initially dense sample, the shear stress gradually increases with shear strain to a peak at P, before falling to a steady value at C which is maintained as the shear strain is increased. The sample may undergo a very small compression at the start of shear, but then begins to dilate. The curve of ε_{vol} vs γ becomes steeper, indicating that the rate of dilation $-d\varepsilon_{vol}/d\gamma$ is increasing. The slope of the curve reaches a maximum at p, but with continued shear strain the curve becomes less steep until at c it is horizontal. When the curve is horizontal $d\varepsilon_{vol}/d\gamma$ is zero, indicating that dilation has ceased. The peak shear stress at P coincides with the maximum rate of dilation at p. The steady state shear stress at C corresponds to the achievement of the critical specific volume at c.

The initially loose sample displays no peak strength, but eventually reaches the same critical shear stress as the first sample. The second sample does not dilate, but gradually compresses during shear until the same critical specific volume is reached (i.e. the volumetric strain remains constant).

In both cases, a critical state, is reached in which the soil continues to shear at constant specific volume, constant shear stress and constant normal effective stress.

A dense sample displays a peak strength because additional work has to be done to overcome the effect of the initially high degree of interlocking – high, that is, relative to the equilibrium specific volume for continued shear at the vertical effective stress at which the test is carried out. The initial dense packing means that the particles are forced to "ride up" over each other ($\Rightarrow$ dilation) for deformation to occur (see the "saw blades analogy", Figure 2.24).

In design, it may be safer to use the critical state strength ϕ'_{crit} than the peak strength ϕ'_{peak}, because

- *the peak strength depends on the extent to which the soil is dense in relation to the critical state under the effective stress conditions at failure. It is not a soil constant, and is unlikely to be the same throughout the mass of soil involved in a potential failure mechanism*
- *it is unlikely that the peak strength will be mobilised simultaneously throughout the soil mass; instead, progressive failure at an average strength rather lower than the peak may occur.*

However, the factors of safety used in many traditional methods of design may well allow for these possibilities, and their use in connection with the critical state strength could lead to overconservatism.

Development of a critical state model

Q2.3 Mining operations frequently generate large quantities of fine, particulate waste known as tailings. Tailings are generally transported as slurries, and stored in reservoirs impounded by embankments or dams made up from the material itself. In order to investigate the geotechnical behaviour of a particular tailings material (G_s=2.70), an engineer carried out three slow, drained shear tests - each over a period of one day - and three fast, undrained shear tests - each over a period of two minutes - in a conventional 60mm × 60 mm shearbox apparatus.

The three samples in each group were initially allowed to come into drained equilibrium under the application of vertical hanger loads of 100 N, 200 N and 300 N. During each shear test, the hanger load was kept constant and the ultimate shear force F_{ult} recorded. Immediately after each test, a water content sample was taken from the centre of the rupture zone. All of the samples were initially saturated, and all of the tests were carried out with the sample under water in the shearbox.

Use the results of the drained tests to construct a critical state model in terms of the normal effective stress σ' and shear stress τ on the shear plane, and the specific volume v. Give the values of ϕ'_{crit}, v_o and λ. Deduce a relationship between the undrained shear strength τ_u and the normal effective stress at the start of the test, and compare its predictions with the experimental data from the undrained tests.

Test type	Vertical load V, N	Shear load F_{ult}, N	Water content w, %
slow, drained	100	53	35.1
	200	105	31.3
	300	156	29.5
fast, undrained	100	42	36.0
	200	80	32.6
	300	120	30.6

Q2.3 Solution

The critical state model must be constructed using the drained test data only, because only in these tests do we know that the pore water pressure u = 0 and that the vertical effective stress σ' is equal to the normal load divided by the sample area. We must assume that the data given for the slow tests were measured at true critical states.

For each sample,
the normal effective stress $\sigma' = V\ (kN)/A\ (m^2)$
the ultimate shear stress $t_{ult} = F_{ult}\ (kN)/A\ (m^2)$

and the specific volume v may be calculated from the water content w using main text Equation 1.10 with S_r*=1,*

$v = 1 + w.G_s$ *(main text Equation 2.12)*

Vertical load V, N	**normal effective stress σ', kPa**	**ln(σ')**	Shear load F_{ult}, N	**Shear stress τ_{ult}, kPa**	Water content w, %	**Specific volume v**
100	**27.8**	**3.325**	53	**14.7**	35.1	**1.95**
200	**55.6**	**4.018**	105	**29.2**	31.3	**1.85**
300	**83.3**	**4.422**	156	**43.3**	29.5	**1.80**

Plot graphs of τ_{ult} against σ' and v against lnσ' to determine the critical state parameters, as in main text Figure 2.28 (Example 2.2).

$\underline{\phi'_{crit} \approx 28°;\ v_O \approx 2.43;\ \lambda \approx 0.14}$

During the undrained tests, there is no overall volume change. Assuming that the specific volume is uniform throughout the sample, it must remain constant during the test. The critical state eventually reached therefore depends on the as-tested specific volume. Our model predicts that, at the critical state, the vertical effective stress σ' is related to the specific volume by the expression

$v = v_O - \lambda.\ln\sigma'$ *(main text Equation 2.11)*

or

$\sigma' = \exp\{(v_O - v)/\lambda\}$

The normal effective stress at the critical state is related to the shear stress τ_{ult} by the expression

$\tau_{ult} = \sigma'.\tan\phi'_{crit}$ *(main text Equation 2.10)*

Hence

$\tau_{ult} = \exp\{(v_O - v)/\lambda\}.\tan\phi'_{crit}$

where $v = 1 + w.G_S$. The calculated and measured values of τ_{ult} for the undrained tests are compared below:

Vertical load V, N	**normal effective stress σ', kPa**	Shear load F_{ult}, N	**Measured shear stress τ_{ult}, kPa**	Water content w, %	**Specific volume v**	**Calculated shear stress, τ_{ult} kPa**
100	**27.8**	42	**11.7**	36.0	**1.972**	**14.0**
200	**55.6**	80	**22.2**	32.6	**1.880**	**27.0**
300	**83.3**	120	**33.3**	20.6	**1.826**	**39.8**

The measured values are smaller than the theoretical values by about 16%. This is probably due to internal drainage and discontinuous sample behaviour.

Determination of peak strengths

Q2.4 The following results were obtained from a shearbox test on a 60 mm × 60 mm sample of dry sand of unit weight 18 kN/m^3.

	Reading on proving ring deflexion dial gauge (divisions)
Zero force	91
Peak shear force for a hanger load of 3kg	128
Peak shear force for a hanger load of 10kg	162
Peak shear force for a hanger load of 20kg	210

One division on the proving ring dial gauge corresponds to a force of 1.1N across the proving ring.

(a) Plot the data on a graph of shear stress against normal effective stress, and sketch the peak strength failure envelope.

(b) What is the peak resistance to shear on a horizontal plane at a depth of 3 m below the top of a dry embankment made from this soil?

(c) A model of the embankment is constructed from the same sand at a scale of 1:10. What is the peak resistance to shear on a horizontal plane at a depth of 300mm below the top of the model?

(d) Would you expect the model to behave in the same way as the real embankment?

Q2.4 Solution

(a) The normal stress on the sample is given by the hanger load (kg) × 9.81 (N/kg) ÷ the sample area, 0.06m × 0.06m = 3.6×10^{-3} m^2, ÷ 1000 to convert from Pa to kPa.

The shear force on the sample is given by 1.1 × (the number of proving ring dial divisions - the number of divisions at zero load), i.e. 1.1 × (n - 91). To convert this to the shear stress, it is necessary to divide the shear force by the area of the sample, 0.06m × 0.06m = 3.6×10^{-3} m^2, and divide by 1000 to convert from Pa to kPa.

Hanger load, kg	*Normal stress, kPa*	*Peak shear load, N*	*Peak shear stress, kPa*
3	*8.175*	*40.7*	*11.31*
10	*27.25*	*78.1*	*21.69*
20	*54.5*	*130.9*	*36.36*

These data are plotted on a graph of τ against σ' in Figure Q2.4. The peak strength failure envelope is highly non-linear, with ϕ'_{peak} = 55° at $\sigma' \approx$ 8 kPa, falling to ϕ'_{peak} = 34° at $\sigma' \approx$ 55 kPa *(b) At a depth of 3m below the top of a dry embankment made of this sand, the vertical effective stress is 3m×18kN/m^3 = 54kPa. This corresponds to a hanger load of 20kg, at which the peak shear stress is approximately 36.4 kPa*

(c) In the 1:10 scale model, the vertical effective stress at a depth of 300mm is about 0.3m × 18kN/m3 = 5.4 kPa. From Figure Q2.4, this gives a peak shear resistance of approximately 7.7kPa

(d) The model would not be expected to behave in the same way as the real embankment, because the operational values of ϕ'_{peak} at corresponding depths in the model and the real embankment are quite different.

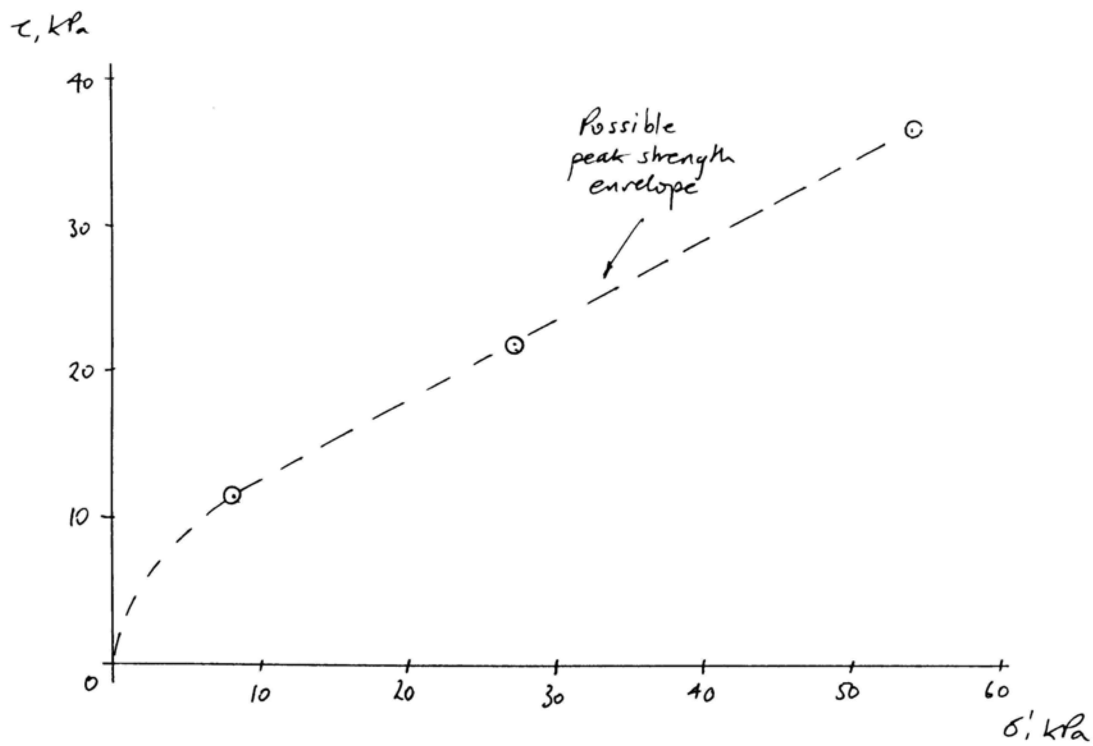


Figure Q2.4: Shear stress against normal effective stress at peak

Use of strength data to calculate friction pile load capacity

Q2.5 A friction pile, 300 mm in diameter, is driven to a depth of 25 m in dense sand of unit weight 19 kN/m^3. The ratio of horizontal to vertical effective stresses is 0.5. The angle of friction between the pile and the sand is 26° and the resistance offered at the base of the pile may be ignored. The natural water table, below which the pore water pressures are hydrostatic, is 5m below ground level. During construction works, the water table is temporarily lowered to a depth of 16m by pumping from wells. A load test on the pile is carried out while pumping to lower the groundwater level is still in progress. Calculate the ultimate load capacity of the pile (a) observed in the test, and (b) after pumping from the wells has stopped, and the water table has recovered to its natural level.

Q2.5 Solution

The vertical total stress σ_v, the pore water pressure u and the vertical (σ'_v) and horizontal (σ'_h) effective stresses all vary linearly with depth between the soil surface and the water table, and between the water table and the base of the pile.

In general at depth z, with the water table at a depth h,

$\sigma_v = \gamma.z$;

u = 0 above the water table ($z \leq h$)

$u = \gamma_w.(z - h)$ *below the water table* ($z \geq h$)

$\sigma'_v = \sigma_v - u$

$\sigma'_h = 0.5 \times \sigma'_v$

shear stress on pile $\tau = \sigma'_h \times \tan 26°$

(a) With the water table depth h = 16m. γ = 19 kN/m^3 *and* γ_w = 9.81 kN/m^3, *the following relationship between shear stress* τ *and depth z is calculated:*

	z, m	σ_v, *kPa*	*u, kPa*	σ'_v, *kPa*	σ'_h, *kPa*	τ, *kPa*
At the soil surface	*0*	*0*	*0*	*0*	*0*	*0*
At the water table	*16*	*304*	*0*	*304*	*152*	*74.14*
At the base of the pile	*25*	*475*	*88.29*	*386.71*	*193.36*	*94.31*

The frictional resistance to pile movement is given by integrating the shear stress τ *over the surface area of the pile. The surface area of the upper 16m of the pile is* ($\pi \times 0.3$)m $\times$ 16m = 15.08m^2, *and the average shear stress over this area is* 74.14kPa $\div$ 2 = 37.07kPa. *The surface area of the lower 9m of the pile is* ($\pi \times 0.3$)m $\times$ 9m = 8.48m^2, *and the average shear stress over this area is* (74.14kPa + 94.31kPa) $\div$ 2 = 84.23kPa. *Thus the overall frictional resistance is*

(15.08m^2 $\times$ 37.07kPa) + (8.48m^2 $\times$ 84.23kPa) = $\underline{1273kN}$

(b) With the water table depth h = 5m. γ = 19 kN/m^3 *and* γ_w = 9.81 kN/m^3:

	z, m	σ_v, *kPa*	*u, kPa*	σ'_v, *kPa*	σ'_h, *kPa*	τ, *kPa*
At the soil surface	*0*	*0*	*0*	*0*	*0*	*0*
At the water table	*5*	*95*	*0*	*95*	*47.5*	*23.17*
At the base of the pile	*25*	*475*	*196.2*	*278.8*	*139.4*	*67.99*

The surface area of the upper 5m of the pile is $(\pi \times 0.3)m \times 5m = 4.71m^2$*, and the average shear stress over this area is* $23.17kPa \div 2 = 11.59kPa$*. The surface area of the lower 20m of the pile is* $(\pi \times 0.3)m \times 9m = 18.85m^2$*, and the average shear stress over this area is* $(23.17kPa + 67.99kPa) \div 2 = 45.58kPa$*. Thus the overall frictional resistance is*

$(4.71m^2 \times 11.59kPa) + (18.85m^2 \times 45.58kPa) =$ *<u>914kN</u>*

Q2.6 The depth of the friction uplift pile described in main text Example 2.4 is increased to 20m, where the undrained shear strength of the clay is 40 kPa. Calculate the short- and long-term uplift resistance of the 20m pile.

Q2.6 Solution
The total shear resistance of the clay/pile interface is given by

T = average shear stress $\times$ *surface area of pile*

(a) In the short term, the average shear stress is the average undrained shear strength on the interface, so that

$T = [(0 + 40kPa) \div 2] \times [(\pi \times 0.5m) \times 20m] =$ *<u>628 kN</u>*

(b) In the long term, the ultimate shear stress on the interface is given by

$\tau_{ult} = \sigma'_h . \tan\delta$

where $\sigma'_h = 0.5 \times \sigma'_v$ *is the horizontal effective stress and* δ *is the angle of friction between the clay and the pile*

At a depth z,

$\sigma_v\ (kPa) = \{\gamma\ (kN/m^3) \times z\ (m)\} = \{18\ (kN/m^3) \times z\ (m)\}$

$u\ (kPa) = \{\gamma_w\ (kN/m^3) \times z\ (m)\} = \{9.81\ (kN/m^3) \times z\ (m)\}$*, and*

$\sigma'_v = \sigma_v - u$

As in (a), T = average shear stress $\times$ *surface area of pile*

The shear stress τ *on the soil/pile interface is now*

$0.5 \times \sigma'_v . \tan\delta$

which increases linearly from zero at the top of the pile to

$0.5 \times [(18\ kN/m^3 \times 20\ m) - (9.81\ kN/m^3 \times 20\ m)] \times \tan 20° = 29.81\ kPa$ at the base

Hence

$T = [(0 + 29.81\ kPa) \div 2] \times [(\pi \times 0.5\ m) \times 20\ m] = \underline{468\ kN}$

Stress analysis and interpretation of shearbox test data

Q2.7 A drained shearbox test was carried out on a sample of saturated sand. The normal effective stress of 41.67 kPa was constant throughout the test, and the initial sample dimensions were 60 mm × 60 mm on plan × 30 mm deep). In the vicinity of the peak shear stress, the data recorded were:

Shear stress τ, kPa	42.5	43.1	42.8
relative horizontal displacement x, mm	0.30	0.40	0.80
upward movement of shearbox lid y, mm	0.05	0.075	0.105

(a) Draw the Mohr circle of stress for the soil sample when the shear stress is a maximum, stating the assumption that you need to make. Determine ϕ'_{peak}, and the orientations of the planes of maximum stress ratio $(\tau/\sigma')_{max}$. Draw the Mohr circle of strain increment leading to the peak, and hence determine the maximum angle of dilation, ψ_{max}. Use an empirical relationship between ϕ'_{peak},ψ_{max} and ϕ'_{crit} to estimate the critical state friction angle, ϕ'_{crit}.

(b) Three further drained tests on similar samples of the same soil were carried out, at different normal effective stresses. The peak and critical state shear stresses were:

Normal effective stress, kPa	20	100	200
Peak shear stress, kPa	23.8	83.9	132.0
Critical state shear stress, kPa	12.6	63.2	126.4

For all four tests, plot the peak and critical state shear stresses τ_{peak} and τ_{crit} as a function of the normal effective stress σ'. Sketch failure envelopes for both peak and critical states, and comment briefly on their shapes. Which would you use for design, and why?
[University of London 2nd year BEng (Civil Engineering) examination, King's College (part question)]

Q2.7 Solution
(a) At τ_{max} (= 43.1 kPa), $\phi'_{peak} = \tan^{-1}\{(\tau/\sigma')_{max}\} = \tan^{-1}(43.1/41.67) = \underline{46°}$

assuming that the central horizontal plane is a plane of maximum stress ratio. The Mohr circle of stress is shown in Figure Q2.7a.

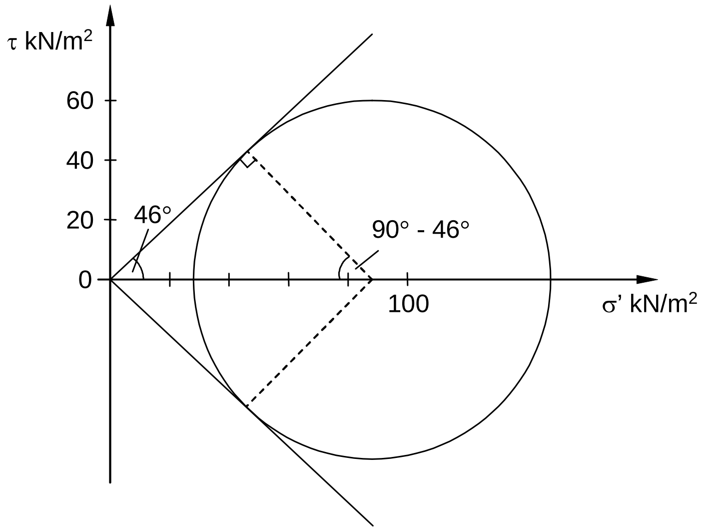


Figure Q2.7a: Mohr circle of stress

The first plane of maximum stress ratio is horizontal (this is an assumption that has to be made to draw the Mohr circle of stress). From Figure Q2.7a, the second plane of maximum stress ratio is at (90° - ϕ'_{peak}) = (90° - 46°) = 44° to the horizontal, either clockwise or anticlockwise depending on whether the shear stress on the horizontal plane plots positive or negative. (Note: the answer given in the main text is slightly ambiguous here. The planes of maximum stress ratio are horizontal and either + or - 44° to the horizontal and not, as might be interpreted from the answer given in the main text, + and - 44° to the horizontal).

The increments of shear ($\Delta\gamma$) and vertical ($\Delta\varepsilon_v$) strain leading up to peak are given by

$\Delta\varepsilon_v = \Delta y/H = 0.025/30 = 0.083\%$, *and*

$\Delta\gamma = \Delta x/H = 0.1/30 = 0.333\%$

where Δx and Δy are the incremental relative horizontal displacement of the two halves of the shearbox and the upward displacement of the shearbox lid respectively, and H = 30 mm ins the initial sample height. The increment of horizontal strain $\Delta\varepsilon_h = 0$. The Mohr circle of strain increment is shown in Figure Q2.7b, and is plotted with coordinates ($\Delta\varepsilon$, $\Delta\gamma/2$) = (0.083%, 0.167%) for the strains associated with (normal to) the horizontal plane and (0, -0.167%) for the strains associated with (normal to) the vertical plane.

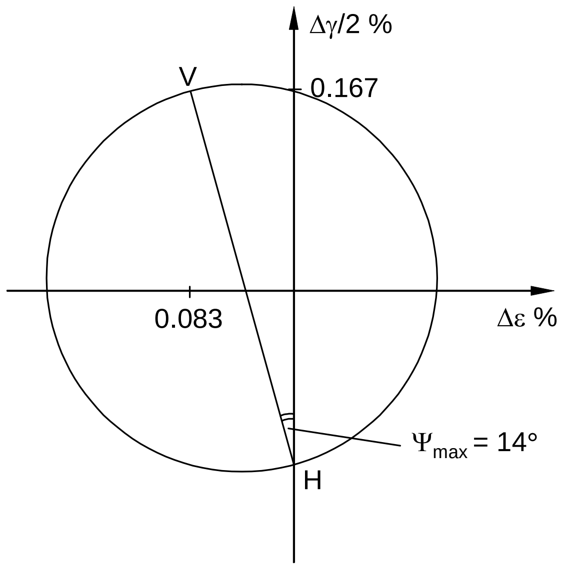


Figure Q2.7b: Mohr circle of strain increment leading up to peak

From Figure Q2.7b, the angle of dilation at peak is given by

$\psi_{max} = \Delta y/\Delta x = 2.5/10 \Rightarrow \underline{\psi_{max} = 14°}$

We might expect $\phi'_{crit} \sim \phi'_{peak} - 0.8 \times \psi_{max}$ *(main text Equation 2.14), giving*

$\phi'_{crit} \sim 46° - 11°$ *or* $\underline{\phi'_{crit} \sim 35°}$

(b) The data are plotted as τ_{peak} *and* τ_{crit} *against* σ' *in Figure Q2.7c.*

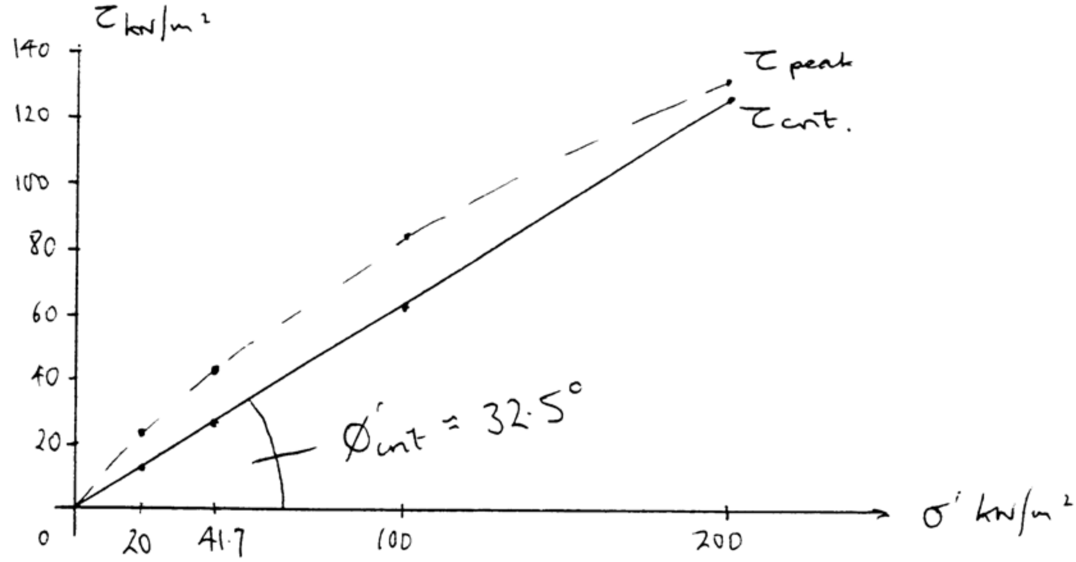


Figure Q2.7c: Failure envelopes in terms of peak and critical state strengths

The failure envelopes sketched in Figure Q2.7c show that

- *ϕ'_{crit} is constant (= 32.5°, closer to $\phi'_{peak} - \psi_{max}$ = 32° than the estimate of 35° based on $\phi'_{peak} - 0.8 \times \psi_{max}$) because there is no dilation at the critical state*
- *ϕ'_{peak} reduces as the normal effective stress σ' increases, because the amount of dilation needed to reach the appropriate (critical) specific volume is reduced.*

In design, it may be safer to use the critical state strength ϕ'_{crit} than the peak strength ϕ'peak, because

- *the peak strength depends on the extent to which the soil is dense in relation to the critical state under the effective stress conditions at failure. It is not a soil constant, and is unlikely to be the same throughout the mass of soil involved in a potential failure mechanism*
- *it is unlikely that the peak strength will be mobilised simultaneously throughout the soil mass; instead, progressive failure at an average strength rather lower than the peak may occur.*

However, the factors of safety used in may traditional methods of design may well allow for these possibilities, and their use in connection with the critical state strength could lead to overconservative design.

Q2.8 In order to investigate the drained strength of a natural silt containing thin clay laminations at a spacing of approximately 6 mm, an engineer carried out a series of shearbox tests. The clay laminations were inclined at various angles θ to the horizontal. With the laminations horizontal (θ = 0), the rupture formed entirely in the clay and the apparent angle of shearing resistance was 18°. With the laminations at an angle θ = 60°, the rupture formed entirely in the silt and the apparent angle of shearing resistance was 30°. Stating clearly the assumptions you need to make, construct Mohr circles of stress at failure for various values of apparent angle of shearing resistance, marking on each the stress state corresponding to the clay laminations. (Hint: the mobilized strength on the clay laminations must never exceed 18°). Plot a graph showing the relationship between the angle θ and the apparent angle of shearing resistance of the soil.

[University of London 1st year BEng (Civil Engineering) examination, King's College (part question)]

Q2.8Solution

When θ = 0, the shear plane forms in the clay so ϕ'_{crit} = 18° for the clay. When θ = 60°, the shear plane forms in the silt so ϕ'_{crit} = 30° for the silt.

Assume that the sample behaves as a continuum up to rupture, and that the central horizontal plane of the shearbox is the plane of maximum and apparent stress ratio (τ/σ') = $\tan\phi'_{apparent}$. The easiest procedure is to construct Mohr circles of stress for apparent ϕ' values of 21°, 24°, 27° and 30° and deduce the corresponding orientation of the clay laminations such that the stress ratio on the laminations is (τ/σ') = tan18° . Each value of $\phi'_{apparent}$ will give four possible orientations of the clay laminations (θ measured clockwise from the horizontal), as indicated in Figure Q2.8a.

Figure Q2.8a shows a general Mohr circle from which algebraic expressions for the orientations θ (measured clockwise from the horizontal) of the yellow clay laminations to give *the given value of $\phi'_{apparent}$. Remember that the rotation on the Mohr circle must be divided by 2 to give the actual rotation in the physical plane.*

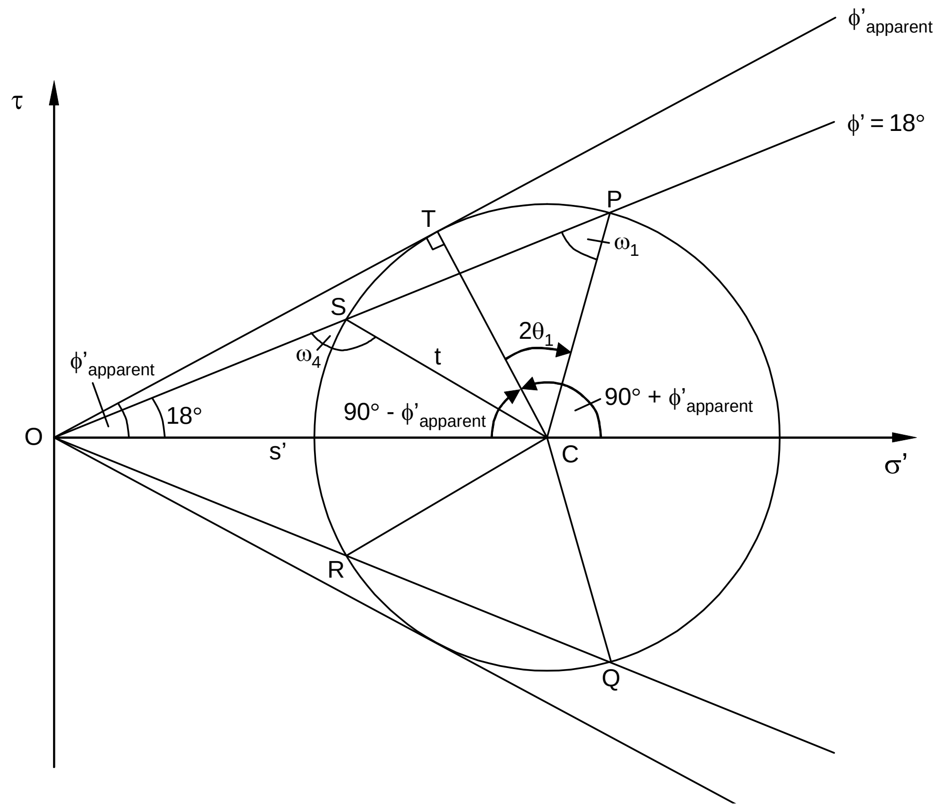


Figure Q2.8a: Mohr circle of stress

The orientations θ of the clay laminations are given by the angles clockwise from the horizontal plane θ_1, θ_2, θ_3 and θ_4, corresponding to the points P, Q, R and S respectively on Figure Q2.8a.

From triangle OTC, $t/s' = \sin\phi'_{apparent}$

From triangle OPC, angle OCP $= 180° - \omega_1 - 18°$ and angle OCP $= 2\theta_1 + (90° - \phi'_{apparent})$

Applying the sine rule to triangle OPC,

$s'/\sin\omega_1 = t/\sin 18° \Rightarrow \sin\omega_1 = \sin 18°/(t/s')$ or $\sin\omega_1 = \sin 18°/\sin\phi'_{apparent}$ (note ω_1 is acute, ie less than 90°)

Applying the sine rule to triangle OSC,

$s'/\sin\omega_4 = t/\sin 18° \Rightarrow \sin\omega_4 = \sin 18°/(t/s')$ or $\sin\omega_4 = \sin 18°/\sin\phi'_{apparent}$ (note ω_4 is obtuse, ie greater than 90°)

By considering the geometry of the Mohr circle shown in Figure Q2.8a, the values of θ_1 to θ_4 may be determined as follows.

$2\theta_1 = (90° + \phi'_{apparent}) - (\omega_1 + 18°) \Rightarrow \theta_1 = 0.5 \times (72° - \omega_1 + \phi'_{apparent})$

$2\theta_2 = (90° + \phi'_{apparent}) + (\omega_1 + 18°) \Rightarrow \theta_2 = 0.5 \times (108° + \omega_1 + \phi'_{apparent})$

$2\theta_3 = (90° + \phi'_{apparent}) + (\omega_4 + 18°) \Rightarrow \theta_3 = 0.5 \times (108° + \omega_4 + \phi'_{apparent})$

$2\theta_4 = (90° + \phi'_{apparent}) + (\omega_4 + 18°) + 2(180° - 18° - \omega_4) \Rightarrow \theta_4 = 0.5 \times (432° - \omega_4 + \phi'_{apparent})$

The values of ω_1, ω_4 and θ_1 to θ_4 for $\phi'_{apparent}$ = 21°, 24°, 27° and 30° are detailed in the table below.

$\phi'_{apparent}$	ω_1	ω_4	θ_1	θ_2	θ_3	θ_4
21	59.57	120.43	16.72	94.29	124.72	166.28
24	49.44	130.56	23.28	90.72	131.28	162.72
27	42.90	137.10	28.05	88.98	136.05	160.95
30	38.12	141.83	31.94	88.06	139.92	160.01

These values are used to construct the graph of apparent angle of shearing resistance $\phi'_{apparent}$ against orientation of the clay laminations θ shown in Figure Q2.8b: note that for orientations of the laminations θ between 32° and 88°, and between 140° and 160°, the value of $\phi'_{apparent}$ is equal to ϕ' for the silt, 30°.

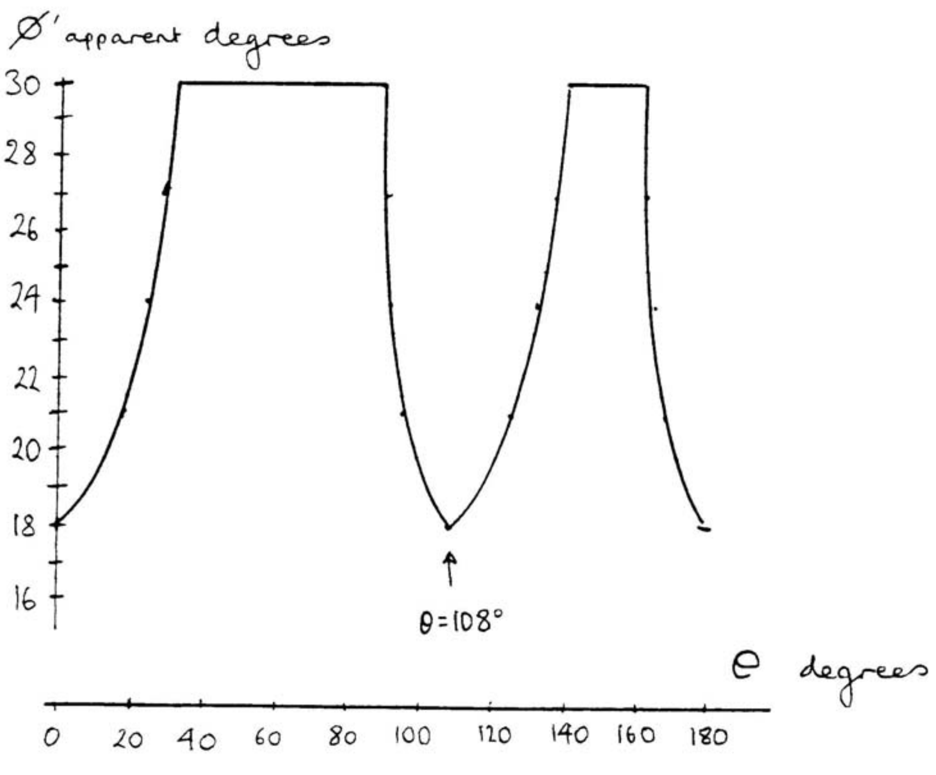


Figure Q2.8b: apparent effective angle of friction against angle of lamination inclination

Note that unless you are very confident with geometry and trigonometry, this problem is probably much more easily addressed by drawing out the four individual Mohr circles to scale and measuring off the angles θ_1 to θ_4. The principles, and hopefully the answers, are however the same.

QUESTIONS AND SOLUTIONS: CHAPTER 3

Laboratory measurement of permeability; fluidization; layered soils

Q3.1 Describe by means of an annotated diagram the principal features of a constant head permeameter. Give three reasons why this laboratory test might not lead to an accurate determination of the effective permeability of a large volume of soil in the ground. Suggest how each of these problems might be overcome.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College (part question)]

Q3.1 Solution
Diagram of constant head permeameter: see main text Figure 3.8

Inaccurate determination of the in situ *permeability might result from*

a) *sample disturbance – unrepresentative void ratio of a uniform soil*
b) *sample disturbance – destruction of soil fabric e.g. in a soil with a layered structure*
c) *large scale inhomogeneities e.g. fissures and high permeability lenses, which cannot be represented in the small scale laboratory sample*
d) *low permeability of a soil with fine particles leads to inaccurate determination of flowrate due to evaporation losses and general measurement errors*

These can be overcome by

a) *testing recompacted samples at maximum and minimum achievable void ratio to give possible limits to the* in situ *permeability*
b) *& c) carrying out field pumping tests*
c) *using a falling head permeameter*

Q3.2 Describe by means of an annotated diagram the principal features of a falling head permeameter.

Show that the water level in the top tube h would be expected to change with time t according to the following equation

$$\ln(h/h_o) = -(kA_1/A_2L).t$$

where h_o is the initial water level in the top tube, A_1 is the cross sectional area of the sample and L is its length, k is the soil permeability and A_2 is the cross sectional area of the top tube.

Give two reasons why this laboratory test might not lead to an accurate determination of the effective permeability of a large volume of soil in the ground.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College (part question)]

Q3.2 Solution
Diagram of falling head permeameter: see main text Figure 3.10

The derivation of the equation follows the main text Section 3.4.2.

At the start of the test (time t=0), the water level in the upper (small-bore) tube is at a height h_0 above the permeameter outlet. After a general time t, the water level in the upper tube has fallen to a general height h above the permeameter outlet. Applying Darcy's Law at a general time t to the soil sample in the large tube,

$$q = Aki = A_1kh/L \qquad \text{(main text Equation 3.8)}$$

In the small-bore tube, the flowrate is given by the cross-sectional area multiplied by the velocity

$$q = A_2v$$

but the velocity $v = -dh/dt$ so

$$q = -A_2.dh/dt \qquad \text{(main text Equation 3.9)}$$

(the negative sign is needed because v has been taken as positive downward, while h is measured as positive upward)

Equating (3.8) and (3.9)

$$dh/dt = -(A_1/A_2).(k/L).h$$

Integrating between limits of $h=h_0$ at $t=0$ and the general state (h, t),

$$\int_{h_0}^{h} dh/h = -\int_0^t \left(\frac{A_1}{A_2}.\frac{k}{L}\right)dt \qquad (3.10)$$

hence

$$\underline{\ln(h/h_0) = -(kA_1/A_2).(k/L).t}$$

Inaccurate determination of the in situ *permeability might result from*

- *sample disturbance – unrepresentative void ratio of a uniform soil*
- *sample disturbance – destruction of soil fabric e.g. in a soil with a layered structure*
- *large scale inhomogeneities e.g. fissures and high permeability lenses, which cannot be represented in the small scale laboratory sample*
- *the laboratory test measures the vertical permeability, while if the field the horizontal permeability is likely to dominate*

Q3.3 In the constant head permeameter test described in Example E3.2, the sample was found to fluidize in upward flow at a hydraulic gradient of 0.84. Estimate the unit weight of the soil in its loosest state.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College (part question)]

Q3.3 Solution
Consider a plug of soil on the verge of uplift (main text Figure 3.24).

Neglecting side friction, uplift will just occur when the upward force due to the pore water presure acting on the base ($A.\gamma_w.[z+h_{crit}]$) begins to exceed the weight of the block of soil ($A.\gamma.z$):

$$A.\gamma_w.[z+h_{crit}] = A.\gamma.z$$

$$z(\gamma - \gamma_w) = \gamma_w.h_{crit}$$

or $\underline{i_{crit} = h_{crit}/z = (\gamma - \gamma_w)/\gamma_w}$ *(main text Equation 3.33)*

In the present case, $i_{crit} = 0.84$. Taking $\gamma_w = 9.81\ kN/m^3$,

$(9.81\ kN/m^3 \times 0.84) = g - 9.81\ kN/m^3 \Rightarrow \underline{\gamma = 18.05\ kN/m^3}$

(taking $\gamma_w = 10\ kN/m^3$ gives $\gamma = 18.4\ kN/m^3$)

Q3.4 An engineer wishes to investigate the bulk permeability of a layered soil comprising alternating bands of fine sand (5 mm thick) and silt (3 mm thick). The engineer makes a special constant head permeameter of square cross section (internal dimensions 112 mm × 112 mm) and carries out two tests on undisturbed samples. In one test, the flow is parallel to the laminations: in the other test, the flow is perpendicular to the laminations. The data recorded in downward flow are as follows:

Hydraulic gradient i	0	1	2	5	10
Flowrate test 1, mm^3/s	0	79	158	395	-
Flowrate test 2, mm^3/s	0	-	-	16	33

Unfortunately, the engineer is not very careful in keeping a laboratory notebook, and omits to record the orientation of the sample in each test.

Estimate the permeability of the fine sand and the silt. Estimate also the flowrates at which fluidization would just occur in upward flow, both parallel and perpendicular to the laminations. Derive from first principles any formulae you use.

[University of London 1st year BEng (Civil Engineering) examination, King's College (part question)]

Q3.4 Solution

You will need to derive the formulae (main text Equations 3.21 and 3.22) for the equivalent bulk horizontal and vertical permeabilities of an alternating layer system, as in the main text Section 3.6.

For horizontal flow (ie flow parallel to the laminations), the hydraulic gradient between two vertical sections A and B is the same for both layers (see main text Figure 3.16). The total flowrate q_T is the sum of the flowrates through the individual layers. We seek an expression of the form

$q_T = A_T.k_H.i$,

where k_H is the overall (bulk) permeability in the horizontal direction and A_T is the total area available for flow. For a unit depth perpendicular to the plane of the paper,

$A_T = d_1 + d_2$

Applying Darcy's law to each layer in turn,

$$q_1 = d_1.k_1.i \text{ and } q_2 = d_2.k_2.i,$$

hence $q_T = q_1 + q_2 = (d_1.k_1 + d_2.k_2).i$,

and by comparison with the initial expression $q_T = A_T.k_H.i$, the horizontal permeability is given by

$$\underline{k_H = (d_1.k_1 + d_2.k_2)/(d_1 + d_2)} \qquad \text{(main text Equation 3.21)}$$

In vertical flow (ie flow perpendicular to the laminations), the same flow passes through each layer and the overall head drop h_T is the sum of the head drops across the individual layers (main text Figure 3.17). The hydraulic gradients across the each layer are $i_1 = \Delta h_1/d_1$, and $i_2 = \Delta h_2/d_2$. The flow area A is the same for all layers, and we seek an expression of the form

$q_T = A.k_V.i_T$,

where the overall hydraulic gradient $i_T = (\Delta h_1 + \Delta h_2)/(d_1 + d_2)$, and k_V is the overall vertical permeability. Since the flowrate through each layer is the same (and equal to q_T),

$q_T = A.k_1.\Delta h_1/d_1 = A.k_2.\Delta h_2/d_2$

and $\Delta h1 + \Delta h2 = (q_T/A).[(d_1/k_1)+(d_2/k_2)]$

hence $i_T = (\Delta h_1+\Delta h_2)/(d_1+d_2) = (q_T/A).[(d_1/k_1)+(d_2/k_2)]/(d_1+d_2)$

By comparison with the initial expression $q_T = A.k_V.i_T$, the overall vertical permeability is

$$\underline{k_V = (d_1+d_2)/[(d_1/k_1)+(d_2/k_2)]} \qquad \text{(main text Equation 3.22)}$$

Use Darcy's Law to calculate the permeability for each of the flowrates in each of the tests:

$q = A.k.i \Rightarrow k = q/Ai$, where $A = 112^2$ mm^2. (With A in mm^2 and the flowrate q in mm^3/sec, the permeability k is calculated in mm/s)

Hydraulic gradient, i	*1*	*2*	*5*	*10*
k_A, mm/s (from test 1)	6.3×10^{-3}	6.3×10^{-3}	6.3×10^{-3}	-
k_B, mm/s (from test 2)	-	-	2.55×10^{-4}	2.63×10^{-4}

Table Q3.4: Processed permeability test data

(The sample in test 1 has flow parallel to the laminations as the measured permeability is the greater)

Taking $k_H = 6.3 \times 10^{-3}$ mm/s, $k_V = 2.6 \times 10^{-4}$ mm/s, $d_1 = 5$ mm for the sand and $d_2 = 3$ mm for the silt and substituting these values into main text Equations 3.21 and 3.22 with permeabilities k_1 and k_2 for the sand and silt respectively,

$k_H = 6.3 \times 10^{-3}$ mm/s $= \{(k_1 \times 5 \text{ mm} + k_2 \times 3 \text{ mm})\} \div 8$ mm

$k_V = 2.6 \times 10^{-4}$ mm/s $= 8$ mm $\div \{(5 \text{ mm}/k_1) + (3 \text{ mm}/k_2)\}$

Rearranging the equation for k_H and working with all permeabilities in mm/s,

$k_2 = (0.0504 - 5k_1)/3$

Substituting this into the equation for k_V,

$\{5 \div k_1\} = \{9 \div (0.0504 - 5k_1)\} = \{8 \div 2.6 \times 10^{-4}\}$

Multiplying both sides by $k_1.(0.0504 - 5k_1)$,

$0.252 - 25k_1 + 9k_1 = 30769.231 \times k_1.(0.0504 - 5k_1)$

$\Rightarrow 153846.16k_1^2 \;\; -1566.77k_1 + 0.252 = 0$

$\Rightarrow k_1 = [1566.77 \pm \sqrt{(1566.77^2 - 4 \times 153846.16 \times 0.252)}] \div [2 \times 153846.16]$

$\Rightarrow k_1 = 0.01002$ mm/s or 1.63×10^{-4} mm/s

The first of these gives $k_2 = 10^{-4}$ mm/s for the silt; the second gives $k_2 = 0.0165$ mm/s. As the sand must have a greater permeability than the silt, the solution is

<u>*k_1 (sand) = 10^{-5} m/s; k_2 (silt) = 10^{-7} m/s*</u>

To estimate the flowrates at fluidization in upward flow, you will need (a) to derive main text Equation 3.33, and (b) to assume a unit weight for the soil.

Main text Equation 3.33 is derived by considering a plug of soil on the verge of uplift (main text Figure 3.24). Neglecting side friction, uplift will just occur when the upward force due to *the pore water presure acting on the base ($A.\gamma_w.[z+h_{crit}]$) begins to exceed the weight of the block of soil ($A.\gamma.z$):*

$$A.\gamma_w.[z+h_{crit}] = A.\gamma.z$$

$$z(\gamma - \gamma_w) = \gamma_w.h_{crit}$$

or $\underline{i_{crit} = h_{crit}/z = (\gamma - \gamma_w)/\gamma_w}$ *(main text Equation 3.33)*

In the present case, we will assume $\gamma = 2\gamma_w$ giving $i_{crit} = 1$. At a hydraulic gradient of 1 in upward flow, the flowrates are given by the relevant permeability $\times$ the cross sectional area of the sample, 112 mm^2.

For test 1, $q_{crit} = 6.3 \times 10^{-3}$ mm/s $\times$ 112 mm^2 = $\underline{79\ mm^3/sec}$ (parallel to the laminations)

For test 2, $q_{crit} = 2.6 \times 10^{-4}$ mm/s $\times$ 112 mm^2 = $\underline{3.3\ mm^3/sec}$ (perpendicular to the laminations)

(These answers are slightly different from those given in the main text).

Q3.5 The following data were obtained from a constant head permeameter test in downward flow on a sample of medium sand.

Measured flowrate q, cm^3/s	2.0	3.0	4.0	5.0	6.0
Head difference between manometer tappings Δh, mm	18.8	31.0	45.1	60.0	75.0
Sample height z, mm	180	175	170	165	160

Specific gravity of soil grains G_S = 2.65
Cross-sectional area of permeameter A = 8 000 mm^2
Distance between pressure tappings l = 120 mm

Prior to the test, the sample had been brought to its loosest possible state - corresponding to a sample height of 180 mm - by fluidization in upward flow. At fluidization, the upward flowrate was 11.725 cm^3/s and the head difference between the manometer tappings was 109.9 mm.

Plot a graph of flowrate q against hydraulic gradient i for downward flow, and explain its shape. Estimate the maximum and minimum permeability k and specific volume v of the sample during this part of the test.

[University of London 1st year BEng (Civil Engineering) examination, King's College (part question)]

Q3.5 Solution
The hydraulic gradient is calculated as the head difference between the manometer tappings Δh (mm) divided by the distance between them (l = 120 mm). The processed data are given in Table Q3.5 and plotted in Figure Q3.5.

Flowrate q, mm^3/sec	*2000*	*3000*	*4000*	*5000*	*6000*
Head difference Δh, mm	*18.8*	*31.0*	*45.1*	*60.0*	*75.0*
Hydraulic gradient i	*0.157*	*0.258*	*0.376*	*0.500*	*0.625*

Table Q3.5: Processed permeameter test data

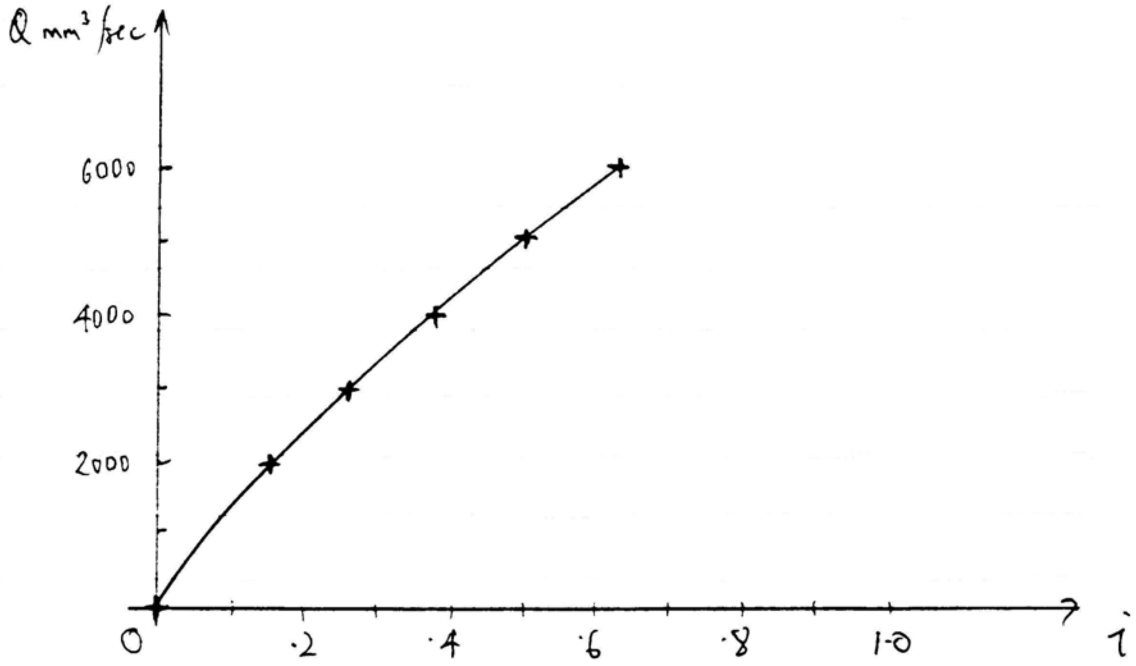


Figure Q3.5: Flowrate q against hydraulic gradient i

The graph of flowrate against hydraulic gradient is curved convex upward, indicating a permeability that decreases as the flowrate is increased (the gradient of the graph is A.k and, as the cross sectional area of the sample A is a constant, the gradually reducing slope must indicate a reducing permeability). This is because as the downward flowrate is increased the sample is compacted (evidenced by the reducing sample height), decreasing both the void ratio and the permeability.

The maximum permeability is with the sample in its loosest state, with the sample height z = 180 mm and the flowrate q = 2000 mm^3/sec. Then

k = q/Ai = 2000 mm^3/sec $\div$ (8000 mm^2 $\times$ 0.157) $\Rightarrow$ k ~ 1.6 mm/s

The minimum permeability is with the sample in its densest state, with the sample height z = 160 mm and the flowrate q = 6000 mm^3/sec. Then

k = q/Ai = 6000 mm^3/sec $\div$ (8000 mm^2 $\times$ 0.625) $\Rightarrow$ k ~ 1.2 mm/s

We can calculate the unit weight of the soil at a height of 180 mm from the data given for fluidization in upward flow, using the equation derived in the main text Section 3.11 for the critical upward hydraulic gradient,

$$i_{crit} = (\gamma - \gamma_w)/\gamma_w \qquad \textit{(main text Equation 3.33)}$$

with $i_{crit} = 109.9\text{ mm} \div 120\text{ mm} = 0.916$

Hence at fluidization, $\gamma = 1.916 \times \gamma_w$

For a saturated soil,

$$\gamma = \gamma_w.(G_s + v - 1)/(v) \qquad \textit{(main text Equation 1.11)}$$

where v is the specific volume, i.e. the ratio total volume $V_t \div$ *volume of solids* V_s

At fluidization, g/gw = 1.916 = (v + 1.65)/v

hence 0.916v = 1.65 or $\underline{v_{max} = 1.8}$

We can calculate the specific volume of the soil in the densest state, at a sample height of 160 mm, by noting that the total volume V_t *is given by*

$V_t = A.z = V_s.(V_s/V_t) = V_s.v$

so that $v/z = A/V_s = \text{constant} = v_o/z_o = 1.8/180\text{ mm} = 0.01\text{ mm}^{-1}$

Hence in the densest state with z = 160 mm, $v = v_{min} = 0.01\text{ mm}^{-1} \times 160\text{ mm} \Rightarrow \underline{v_{min} = 1.6}$

Well pumping test (field measurement of permeability)

Q3.6 A well pumping test was carried out to determine the bulk permeability of a confined aquifer. The aquifer was overlain by a clay layer 4 m thick, the depth of the aquifer was 20 m, and the initial piezometric level in the aquifer was 2m below ground level. After a period of pumping when steady-state conditions had been reached, the following observations were made.

pumped flowrate q = 1.637 litres/second
well radius = 0.1 m
drawdown just outside well = 2 m
drawdown in piezometer at 100m distance from well = 0.2 m

Deriving from first principles any equations you need to use, determine the bulk permeability of the aquifer. Would your analysis still apply for a drawdown in the well of 4 m?

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q3.6 Solution
The derivation of the equation needed to solve this problem is given in full in main text Section 3.5.1.

Consider the flowrate q through an annular ring around the well, concentric with the well and having a general radius r. The flow area at a general radius r is $2\pi rD$, where D is the thickness of the aquifer. The hydraulic gradient i = -dh/dr where h is the head measured above some convenient datum. Applying Darcy's Law,

$$q = Aki = 2\pi rD.k.dh/dr \qquad \text{(main text Equation 3.12)}$$

The negative sign has been omitted from the hydraulic gradient in Equation 3.12, because we are interested in the flow towards the well, which is in the r negative direction.

Rearranging Equation 3.12 and integrating between limits of ($h = h_w$, $r = r_w$) at the perimeter of the well and a general point (h, r) at radius r,

$$\int_{r_w}^{r} \frac{dr}{r} = \left(\frac{2\pi kD}{q}\right)\int_{h_w}^{h} dh$$

hence $\ln(r/r_w) = (2\pi Dk/q).(h-h_w)$

or $k = [q.\ln(r/r_w)] \div [2\pi D.(h-h_w)]$

We need to think about the relationships between heads and drawdowns. Taking the datum for the measurement of head h at the bottom of the aquifer, 24 m below ground level, the initial groundwater level (at a depth of 2 m below the ground surface) corresponds to a head of 24 m – 2 m = 22 m. The drawdown just outside the well of 2 m corresponds to a head measured from the base of the aquifer of 22 m – 2 m = 20m, and the drawdown of 0.2 m at a distance of 100 m from the well corresponds to a head of 22 m – 0.2 m = 21.8 m.

Substituting in the values h = 21.8 m at r = 100 m; h_w = 20 m at r_w = 0.1 m; D = 20 m and q = 1.637×10^{-3} m^3/sec gives

$k = [1.637 \times 10^{-3}\ m^3/sec \times \ln(100/0.1)] \div [2 \times \pi \times 20\ m \times (21.8\ m - 20\ m)] \Rightarrow \underline{k = 5 \times 10^{-5}\ m/s}$

If the drawdown inside the well were increased to 4 m, the aquifer would become unconfined (or strong vertical flow would occur) near the well and the analysis will not strictly be valid. In reality, the error will probably be small, but will increase with increasing drawdown.

Confined flownets; quicksand

Q3.7 Figure 3.41 shows a cross section through a square excavation at a site where the ground conditions are as indicated. Assuming that the water levels in the overlying gravels, the underlying fractured bedrock, and the medium sand outside the excavation do not change, estimate by means of a carefully-sketched flownet the capacity of the required dewatering system.

What proportion of the extracted groundwater must be recirculated through the medium sand and the gravels in order to maintain the initial groundwater level in these strata, if there is no other close source of recharge?

Do you foresee any problem concerning the stability of the base of the excavation?

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q3.7 Solution

The flownet is sketched on Figure Q3.7, according to the rules and procedures given in the main text Sections 3.8 and 3.9.

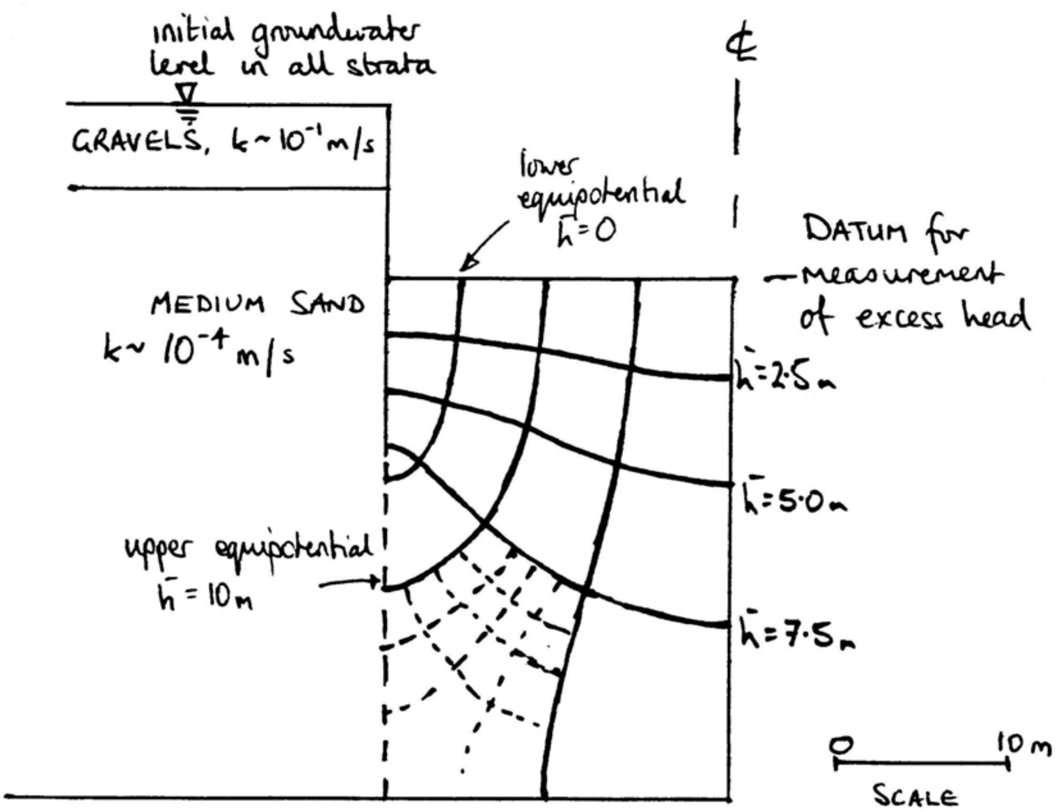


Figure Q3.7

The flowrate q is calculated from

$$q\ (m^3/s\ per\ metre) = k.H.N_F/N_H$$

where k, *the permeability of the soil* = 10^{-4} *m/s;*
H, *the overall head drop = 10 m;*
N_F, *the number of flowtubes = 4; and*
N_H, *the number of equipotential drops, = 4*

The perimeter is 8 times the half-width of the excavation, = 160 m

Hence $q = \left[10^{-4}(m/s) \times 10(m) \times \frac{4}{4}\right] \times 160m = 0.16m^3/s$

or *<u>q = 160 litre/sec</u>*

From the flownet, it may be seen that approximately $2^5/_8$ *of the 4 flowtubes start in the medium sand. The proportion of the pumped groundwater that must therefore be recharged is approximately* $2^5/_8 \div 4 \approx 66\%$. *(Note that this estimate is on the high side, as some of this flow will enter the medium sand from the bedrock).*

The upward hydraulic gradient into the excavation is, scaling from the flownet, approximately 2.5 m $\div$ *3.5 m or 0.71. While this is less than the critical value (*$i_{crit} \approx 1$ *for a soil with* $\gamma \approx 2\gamma_w$*), it is perhaps a little close for comfort – particularly in the corners of the excavation, where flow from the two adjacent sides and the plane flownet calculation is not valid – and should therefore be investigated in more detail. Note, however, that the flownet has been drawn on the basis that there is no drawdown outside the line of the retaining wall: in reality, a drawdown in the sand outside the line of the retaining wall would move the upper equipotential further from the excavation and probably reduce the upward hydraulic gradient below the excavation floor.*

Q3.8 Figure 3.42 shows a *plan* view of an excavation underlain by a confined aquifer of uniform thickness 20m. The aquifer is bounded on two sides by a river having a water level h=12m above datum level. On the third side, the effective recharge boundary to the aquifer is as indicated. A sheet pile cut-off wall is installed along the edge of the river adjacent to the excavation, extending for a certain distance on either side. The datum level for the measurement of hydraulic head is at the upper surface of the aquifer.

Estimate by means of a carefully-sketched flownet the rate at which water must be pumped from a dewatering system, in order to reduce the groundwater level at the excavation to datum level. (The permeability of the aquifer is 3.6×10^{-4} m/s).

Explain why your analysis would be invalid for drawdowns at the excavation to below datum level.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q3.8 Solution
The flownet is sketched on Figure Q3.8. Note that this is a flownet in the horizontal plane, ie on plan, but otherwise follows exactly the same principles as a more usual flownet in the

cross-sectional (vertical) plan as enumerated in the main text Sections 3.8 and 3.9. In this case, the flow domain has a finite thickness, as the aquifer is confined by impermeable layers at the top and bottom.

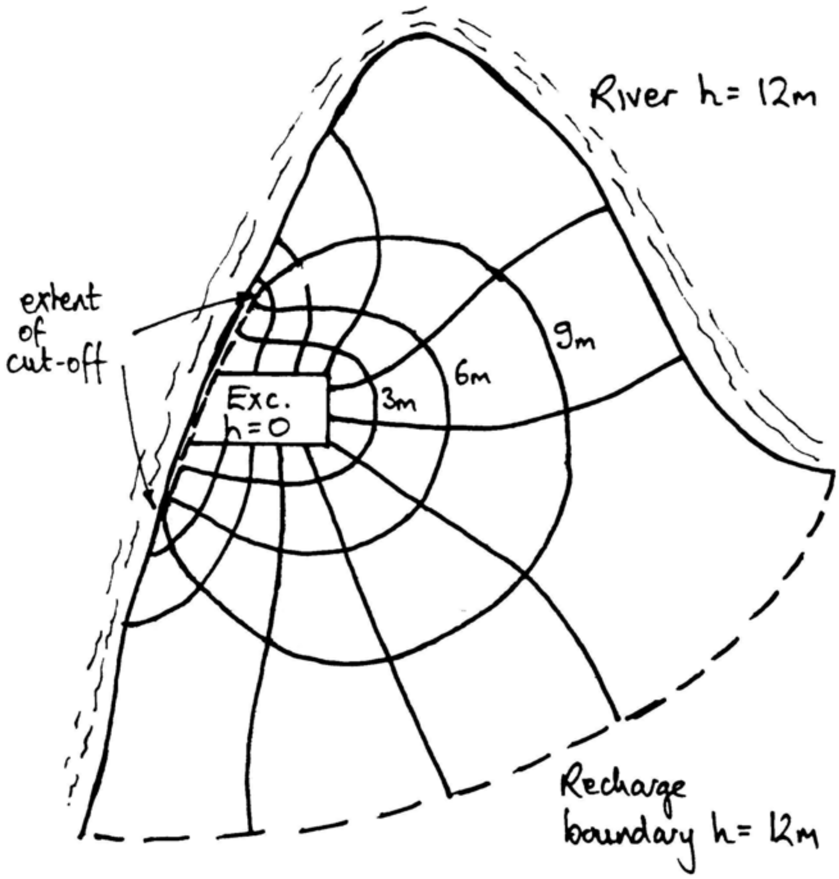


Figure Q3.8

The flowrate q is calculated from

$$q\ (m^3/s \text{ per metre thickness}) = k.H.N_F/N_H$$

where *k, the permeability of the soil =* 3.6×10^{-4} *m/s;*
H, the overall head drop = 12 m;
N_F*, the number of flowtubes = 11; and*
N_H*, the number of equipotential drops, = 4*

The thickness of the aquifer is 20 m

Hence $q = \left[3.6 \times 10^{-4}(m/s) \times 12(m) \times \frac{11}{4}\right] \times 20m = 0.238 m^3/s$

or *<u>q = 238 litre/sec</u>*

The analysis would be invalid for drawdowns below the top of the aquifer (datum level) because the flow would become unconfined. The saturated thickness of the aquifer would no longer be constant, and flow would no longer be purely horizontal and could not be represented by a two-dimensional flownet in the horizontal plane.

Unconfined flownet

Q3.9 Figure 3.43 shows a cross section through a long canal embankment. Explaining carefully the conditions you are attempting to fulfill, estimate by means of a flownet the rate at which water must be pumped from the drainage ditch back into the canal, in litres per hour per metre length.

Describe qualitatively what might happen if the drain beneath the toe of the embankment became blocked.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q3.9 Solution

The conditions that must be fulfilled in drawing the flownet are

- *equipotentials and flowlines cross at 90°*
- *elements in the flownet have the same breadth as length, forming curvilinear squares*
- *impermeable boundaries and the centreline are flowlines*
- *u = 0 on the top flowline, i.e. the x m equipotential intersects the top flowline at x m above the datum for the measurement of head (phreatic surface condition for an unconfined flownet)*

The flownet is sketched on Figure Q3.9. The phreatic surface condition is satisfied by trial and error, along with the rest of the conditions above. Note: capillary rise effects are neglected.

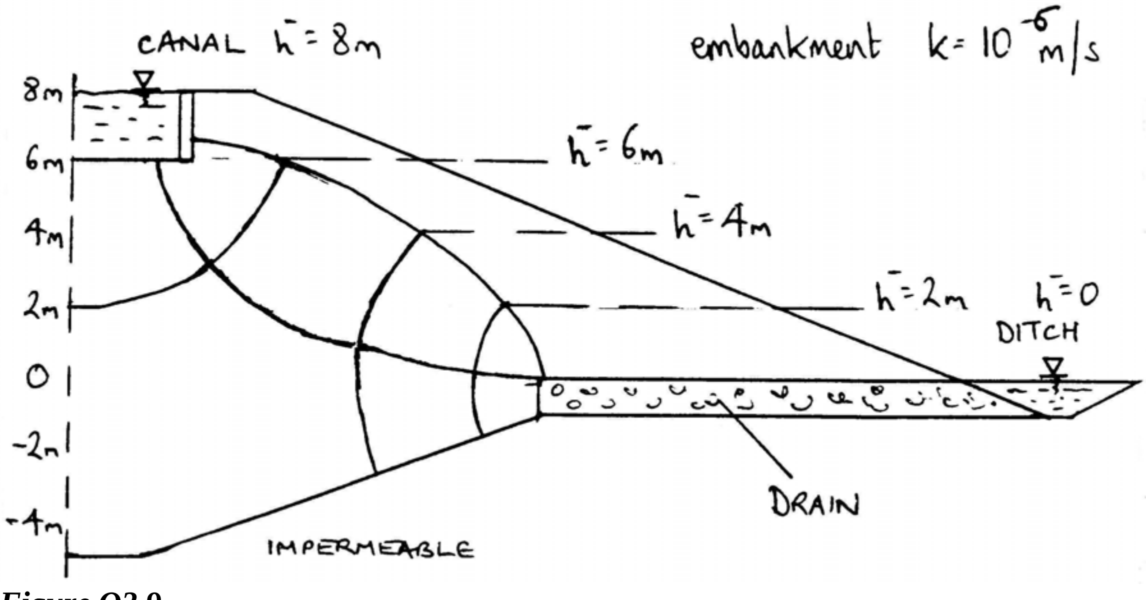


Figure Q3.9

The flowrate q is calculated from

$$q\ (m^3/s \text{ per metre length}) = k.H.N_F/N_H$$

where k, the permeability of the embankment = 10^{-6} m/s;
H, the overall head drop = 8 m ie the level of the top surface of the canal above datum, NOT 6 m which is the level of the base of the canal and a common mistake);
N_F, the number of flowtubes = 2 × 2 for symmetry = 4; and
N_H, the number of equipotential drops, = 4

Hence $q = \left[10^{-6}(m/s) \times 8(m) \times \frac{4}{4}\right] = 8 \times 10^{-6} m^3/s/m$

or <u>*q = 28.8 litre/hour per metre run*</u>

If the drain became blocked, the top flowline would rise and emerge on the downstream face of the embankment leading to erosion and failure..

Flownets in anisotropic soils

Q3.10 Figure 3.44 shows a true cross-section through a long cofferdam. It is proposed to dewater the cofferdam by lowering the water level inside it to the floor of the excavation. Investigate the suitability of this proposal by means of a carefully-sketched flownet on an appropriately-transformed cross-section (horizontal scale factor $\alpha=\sqrt{k_v/k_h}$).
How might the stability of the base be ensured?

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q3.10 Solution
The flownet must be sketched on a transformed section, with the horizontal distances reduced by a transformation factor $\alpha = \sqrt{(k_v/k_h)}$ to account for the relatively higher horizontal permeability (see main text Section 3.14).

$$\alpha = \sqrt{(k_v/k_h)} = \sqrt{(2.5 \times 10^{-5} \div 10^{-4})} = \sqrt{(0.25)} = 0.5$$

The cross section is re-drawn with the horizontal dimensions reduced by the transformation factor 0.5, and the flownet is sketched according to the rules and procedures set out in main text Sections 3.8 and 3.9, in Figure Q3.10.

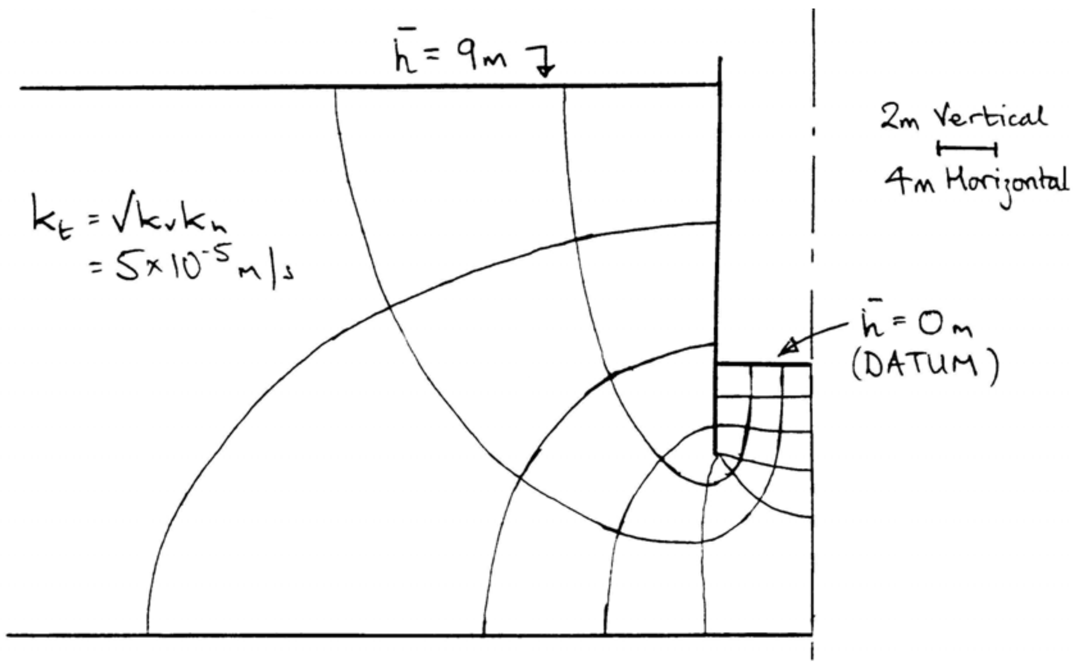


Figure Q3.10

The flowrate q is calculated from

$$q\ (m^3/s \text{ per metre length}) = k_t.H.N_F/N_H$$

where k_t*, the equivalent permeability of the transformed section, =* $\sqrt{(k_v.k_h)}$ *(see main text Section 3.14);*

$k_t = 5 \times 10^{-5}$ *m/s;*

H, the overall head drop = 9 m;

N_F*, the number of flowtubes =3* $\times$ *2 for symmetry = 6; and*

N_H*, the number of equipotential drops, = 9*

Hence $q = \left[5\times10^{-5}(m/s)\times9(m)\times\frac{6}{9}\right] = 3\times10^{-4}\,m^3/s/metre$

or <u>*q = 0.3 litre/sec per metre length*</u>

By scaling from the diagram, the vertical hydraulic gradient between the sheet piles is ~ 1, so there is a danger of base instability (quicksand or boiling).

The stability of the base could be ensured by increasing the depth of the sheet piles and/or by lowering the groundwater level inside the cofferdam to well below formation level.

Q3.11 Figure 3.45 shows a true cross-section through a sheet-piled excavation in a laminated soil of permeability $k_v = 10^{-6}$ m/s (vertically) and $k_h = 1.6\times10^{-5}$ m/s (horizontally). The laminated soil is overlain by 4 m of highly permeable gravels, and the natural groundwater level is 2 m below the soil surface. By means of a flownet sketched on a suitably-modified cross-section estimate:
(a) the minimum capacity required of the dewatering system, and
(b) the pore water pressure at the point A.
Comment briefly on the stability of the base of the excavation.
(Transformation factor $\alpha=\sqrt{k_v/k_h}$)

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q3.11 Solution
The flownet must be sketched on a transformed section, with the horizontal distances reduced by a transformation factor $\alpha = \sqrt{(k_v/k_h)}$ to account for the relatively higher horizontal permeability (see main text Section 3.14).

$\alpha = \sqrt{(k_v/k_h)} = \sqrt{(10^{-6} \div 1.6 \times 10^{-5})} = \sqrt{(1/16)} = 0.25$

The cross section is re-drawn with the horizontal dimensions reduced by the transformation factor 0.25, and the flownet is sketched according to the rules and procedures set out in main text Sections 3.8 and 3.9, in Figure Q3.11.

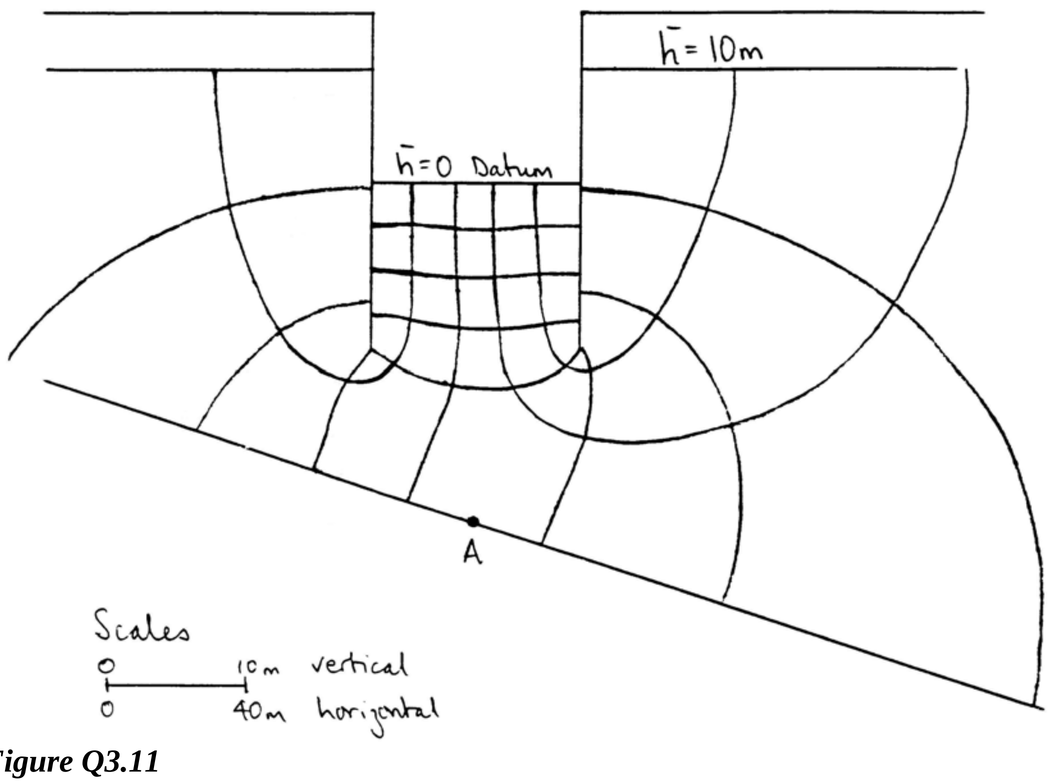


Figure Q3.11

(a) The flowrate q is calculated from

$$q\ (m^3/s \text{ per metre length}) = k_t.H.N_F/N_H$$

where k_t*, the equivalent permeability of the transformed section, =* $\sqrt{(k_v.k_h)}$ *(see main text Section 3.14);*

$k_t = 4 \times 10^{-6}$ *m/s;*

H, the overall head drop = 10 m (from the groundwater level on the retained side to the floor of the excavation);

N_F*, the number of flowtubes =5 (note the flownet is NOT symmetrical in this case); and*

N_H*, the number of equipotential drops, = 8*

$$\text{Hence } q = \left[4 \times 10^{-6} (m/s) \times 10(m) \times \frac{5}{8}\right] = 2.5 \times 10^{-5} m^3 / s / metre$$

or <u>*q = 0.025 litre/sec per metre length*</u>

(b) the point A is roughly ¼ of the way between the third and fourth equipotential lines after h = 10 m. Each equipotential drop is 10/8 = 1.25 m. Interpolating, the head at A is approximately

$h_A \approx\sim 10\ m - (3.25 \times 1.25\ m) \approx 5.94\ m$

The point A is about 24.5 m below the datum for the measurement of head, giving

<u>$u_A = \gamma_w.(24.5 + 5.94) \approx 300$ *kPa*</u>

(see main text Section 3.10 and Example 3.7 for details of the calculation of pore water pressures from flownets).

The upward hydraulic gradient between the sheet piles is (scaling from the flownet) approximately 1.25 m ÷ 3 m or 0.42, which is comfortably below the critical value of about 1. Thus our dewatering scheme, which involves installing pumped wells with sufficient capacity to draw down the groundwater level within the excavation to formation level, should be adequate to ensure the stability of the base.

QUESTIONS AND SOLUTIONS: CHAPTER 4

Analysis and interpretation of one-dimensional compression test data

Q4.1 (a) What factors govern the relevance to a given design situation of the parameters obtained from an oedometer test?

(b) Data from an oedometer test are given below. Show that the specific volume v is related to the sample height h by the expression (v/h) = constant. Plot a graph of specific volume v against the natural logarithm of the vertical effective stress, $\ln\sigma'_v$, and explain its shape. Calculate the values of κ_o and λ_o.

σ'_v, kPa	25	50	100	200	100	50
Equilibrium sample height h, mm (after consolidation has ceased)	19.86	19.56	19.27	18.48	18.79	19.08

Water content of sample at the end of the test (σ'_v=50kPa, h=19.08mm): 20.88%
Grain specific gravity G_s = 2.75

(c) Figure 4.40 shows the ground conditions at the site of a proposed new office building. The office building will have a raft foundation, the effect of which will be to increase the vertical effective stress in the clay layer by 50kPa throughout its depth. The oedometer test sample was taken from the mid-depth of the clay layer, i.e. 5m below ground level. Explaining your choice of one-dimensional modulus E'_o, estimate the eventual settlement of the clay layer. What, qualitatively, would be the effect if the foundation load were to be increased by a further 50kPa?

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q4.1 Solution
(a) The parameters (one dimensional stiffness, consolidation coefficient and by inference permeability) should have been obtained from tests that have reproduced as far as possible the initial stress state, the previous stress history and the expected loading or unloading increment of the soil in the field. Sample disturbance (leading to loss of fabric) might also reduce the reliability of laboratory test results; and the consolidation coefficient and permeability in the field might well be governed by preferential horizontal flow whereas in the oedometer test flow is vertical.

(b) The total sample volume V_t at any stage of the test is equal to the sample area A multiplied by the current sample height h,

$V_t = A.h.$

Also, the total volume is equal to the volume of voids V_v + the volume of soil grains (solids) V_s,

$V_t = V_s + V_v = V_s(1+V_v/V_s) = V_s.(1 + e) = V_s.v$

Hence

$$V_t = V_s.v = A.h, \text{ or } \frac{v}{h} = \frac{A}{V_s} = constant$$

Assuming that the sample is fully saturated at the end of the test, the final sample height h_f can be related to the final specific volume v_f by measurement of the final moisture content w_f,

$v_f = (1 + e_f) = (1 + w_f . G_s) = 1 + (0.2088 \times 2.75) = 1.5742$

$$hence\ \frac{v}{h} = \frac{A}{V_s} = constant = \frac{v_f}{h_f} = \frac{1.5742}{19.08} mm^{-1} = 0.0825 mm^{-1}$$

Convert the values of h to values of v using $v = 0.0825 \times h$,

σ'_v, kPa	25	50	100	200	100	50
h, mm	19.86	19.56	19.27	18.48	18.79	19.08
ln σ'_v	3.219	3.912	4.605	5.298	4.605	3.912
v	1.639	1.614	1.590	1.525	1.550	1.574

Plot v against lnσ'_v (Figure Q4.1)

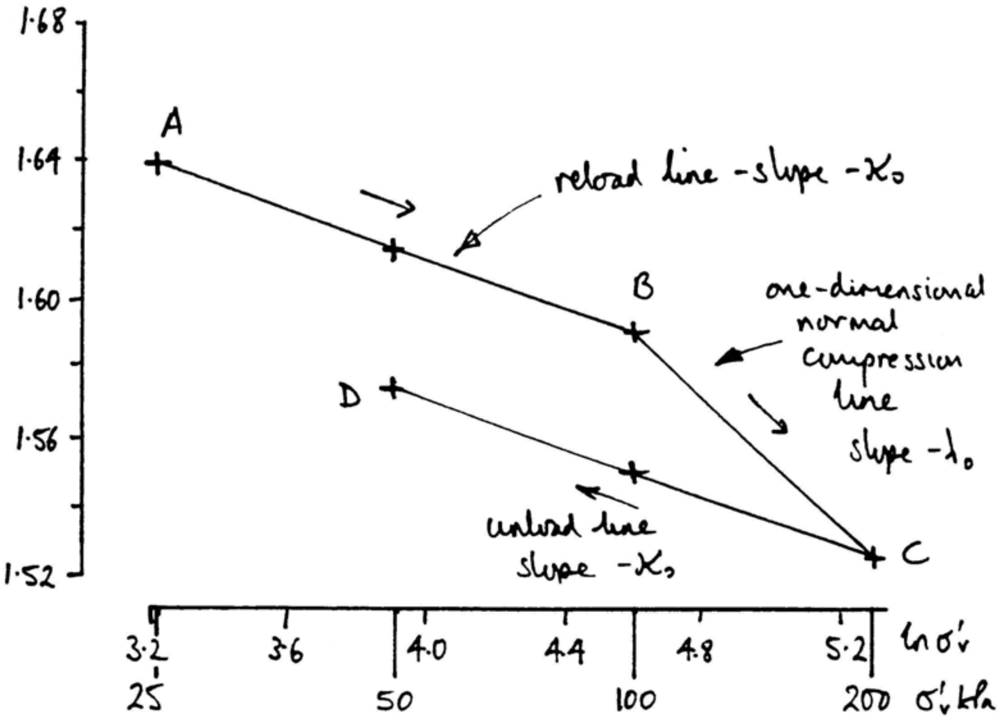


Figure Q4.1: v against ln(σ'_v)

A – B: *reloading, "elastic" (recoverable) deformation only*
B: *maximum previous preconsolidation pressure, sample moves onto normal (first) compression line*
B – C: *normal (first) compression: "elastic" plus plastic (irrecoverable) deformation. Plastic deformation is due to particle slip*
C – D: *unloading; "elastic" component of deformation is recovered*

The soil is overconsolidated along AB and CD, and normally consolidated along BC.

The slope of the reloading and unloading lines is $-\kappa_0$*; the slope of the one-dimensional normal compression line is* $-\lambda_0$

From the graph or the data,

$$\kappa_o = \frac{-\Delta v}{\Delta \ln \sigma'_v} = -\frac{1.590 - 1.639}{\ln 100 - \ln 50} = 0.035$$

(the slope of the unloading line), and

$$\lambda_o = \frac{-\Delta v}{\Delta \ln \sigma'_v} = -\frac{1.525 - 1.590}{\ln 200 - \ln 100} = 0.094$$

(the slope of the one dimensional normal compression line).

(c) The initial vertical effective stress at the centre of the clay layer is approximately (20 kN/m^3 $\times$ *5 m) – (10 kN/m*^{3} $\times$ *5 m) = 50 kPa. The vertical effective stress is then increased (after the dissipation of excess pore water pressures) to 100 kPa. The appropriate stress range is therefore 50 to 100 kPa. Over this stress increment, the sample height reduced by (19.56 – 19.27) = 0.29 mm. Assuming that the same value of one-dimensional modulus applies, the eventual settlement of a 5 m thick layer of the same clay is 0.29 mm* $\times$ *(5000 mm* $\div$ *19.56 mm) = 74 mm. (Note there is no need to calculate the value of* E'_0 *explicitly).*

If the vertical effective stress at the centre of the clay layer were increased to more than 100 kPa, the soil state would move from a (comparatively stiff) reload line to the much less stiff normal compression line, as the precompression stress was exceeded. This would lead to comparatively larger settlements (e.g. in going from 100 kPa to 200 kPa, the oedometer test sample compresses by 0.79 mm giving an equivalent settlement of the 5 m layer of $0.79 \times 5000/19.27 =$ 205 mm. Assuming a logarithmic increase in stiffness with stress, 70% of this settlement i.e. 142 mm would occur on increasing the vertical effective stress from 100 to 150 kPa).

Q4.2 (a) Describe with the aid of a diagram the important features of a conventional oedometer, and define the parameters that this apparatus is used to measure.

(b) Data from an oedometer test on a sample of clay are given below.

σ'_v, kPa	50	100	200	400	800	600	400
Equilibrium sample height h, mm	17.123	16.912	16.701	15.496	14.300	14.390	14.521

Cross-sectional area of sample = 80 000 mm^2
G_s = 2.61
Two-way sample drainage
Water content at start of test: 45.14%
Water content at end of test: 32.84%

Calculate the specific volume at the end of the test, assuming S_r=1 at this stage. What was the saturation ratio at the start of the test?

Show that the specific volume is related to the sample height by the expression $v/h = A/v_s$ = constant, where A is the cross sectional area of the sample and v_s is the volume occupied by the soil grains.

Plot a graph of the specific volume against the natural logarithm of the vertical effective stress. Explain the shape of this graph. Calculate the preconsolidation pressure and the slopes of the one-dimensional normal compression line and unloading/reloading lines.

[University of London 2nd year BEng (Civil Engineering) examination, King's College (part question)]

Q4.2 Solution
(a) A diagram of the oedometer is given in the main text Figure 4.2. The apparatus is used to measure

- i) the stiffness of the soil in one dimensional compression, E'_0, over a given stress range;
- ii) the coefficient of consolidation, $c_v = k_v.E'_0/\gamma_w$
- iii) by inference, the vertical permeability k_v.

(b) Assuming that the sample is fully saturated at the end of the test, the specific volume at this stage is given by

$v_f = (1 + e_f) = (1 + w_f . G_s) = 1 + (0.3284\times 2.61) = 1.857$

At the start of the test, the specific volume is equal to 1.857 $\times$ (17.123 $\div$ 14.521) = 2.190. The saturation ratio may be calculated using

$S_r = w.G_s/(v - 1)$ *(main text Equation 1.10)*

or

$\boldsymbol{S_r = 0.4514 \times 2.61 \div 1.190 = 99\%}$

The total sample volume V_t at any stage of the test is equal to the sample area A multiplied by the current sample height h,

$V_t = A.h.$

Also, the total volume is equal to the volume of voids V_v + the volume of soil grains (solids) V_s,

$$V_t = V_s + V_v = V_s(1+V_v/V_s) = V_s.(1 + e) = V_s.v$$

Hence

$$V_t = V_s.v = A.h,\ \ or\ \ \frac{v}{h} = \frac{A}{V_s} = constant$$

The numerical value of the constant is equal to the specific volume divided by the sample height at the end of the test,

$$\frac{v_f}{h_f} = \frac{1.857}{14.521} mm^{-1} = 0.1279 mm^{-1}$$

Convert the values of h to values of v using v = 0.1279 × h,

σ'_v, kPa	50	100	200	400	800	600	400
h, mm	17.123	16.912	16.702	15.496	14.300	14.390	14.521
ln σ'_v	3.912	4.605	5.298	5.991	6.685	6.397	5.991
v	2.190	2.163	2.136	1.982	1.829	1.840	1.857

Plot v against lnσ'_v (Figure Q4.2)

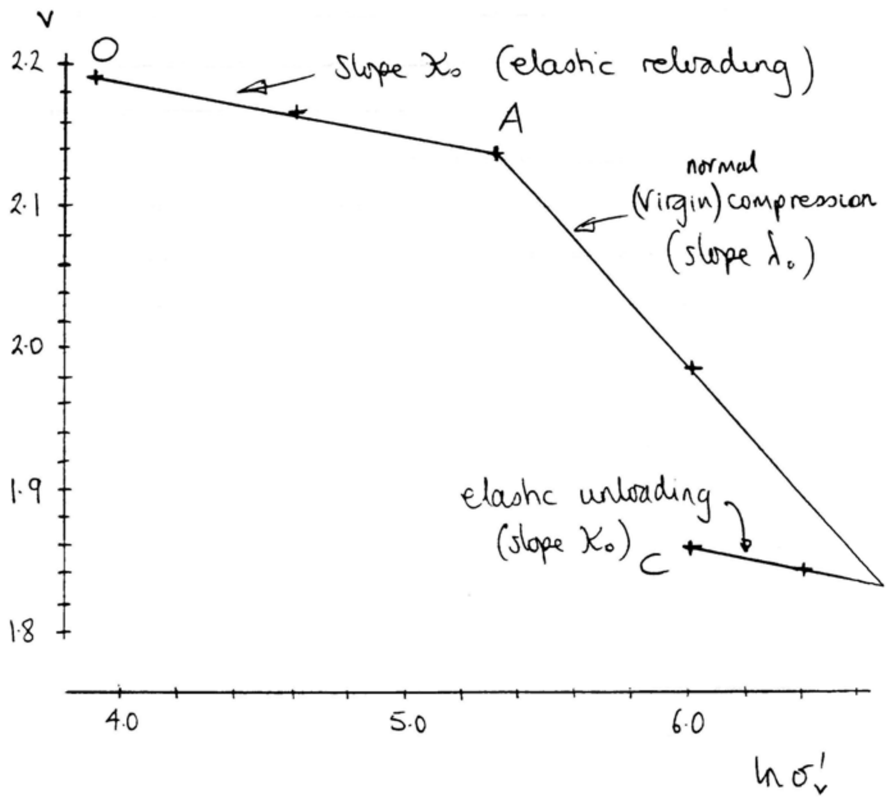


Figure Q4.2: v against ln(σ'_v)

O – A: "elastic" reloading
A: maximum previous preconsolidation pressure (200 kPa); sample moves onto normal (first) compression line
A – B: normal (first) compression: "elastic" plus plastic (irrecoverable) deformation. Plastic deformation is due to particle slip
B – C: "elastic" unloading

"Elastic" compression and swelling take place entirely as a result of particle distortion, i.e. without relative slippage of the soil particles. During normal (first) compression, plastic deformation is also occurring as the soil particles slide over each other and the soil matrix is rearranged.

The slope of the reloading and unloading lines is $-\kappa_0$; the slope of the one-dimensional normal compression line is $-\lambda_0$

From the graph or the data,

$$\kappa_o = \frac{-\Delta v}{\Delta \ln \sigma'_v} = \frac{2.190 - 2.136}{\ln 200 - \ln 50} = 0.039$$

(the slope of the unloading line), and

$$\lambda_o = \frac{-\Delta v}{\Delta \ln \sigma'_v} = \frac{2.136 - 1.829}{\ln 800 - \ln 200} = 0.221$$

(the slope of the one dimensional normal compression line).

The preconsolidation pressure (i.e. the maximum previous value of vertical effective stress, at which the soil moves from a reload line onto the one dimensional normal compression line, is

$\sigma'_{v,\ maximum\ previous}$ *= 200 kPa*

(see above)

Q4.3 For the oedometer test described in Q4.2, plot a graph of vertical effective stress σ'_v against vertical strain ε_v. For each of the loading and unloading steps, calculate the one-dimensional modulus $E'_o = \Delta\sigma'_v/\Delta\varepsilon_v$ ($\Delta\sigma'_v$ and $\Delta\varepsilon_v$ are the changes in vertical effective stress and strain that occur during the loading or unloading step.) Comment briefly on the significance of these results in the context of the selection of parameters for design.

Q4.3 Solution

For each load increment or decrement, the one-dimensional modulus E'o is defined as the change in vertical effective stress $\Delta\sigma'_v$ divided by the change in vertical strain $\Delta\varepsilon_v$. The change in vertical strain during a load increment or decrement is based on the equilibrium sample height at the start of that increment or decrement.

Load increment/ decrement, kPa	*50-100*	*100-200*	*200-400*	*400-800*	*800-600*	*600-400*
$\Delta\sigma'_v$, kPa	*50*	*100*	*200*	*400*	*-200*	*-200*
change in sample height Δh, mm	*0.211*	*0.211*	*1.205*	*1.196*	*-0.090*	*-0.131*
vertical strain during increment $\Delta\varepsilon_v = \Delta h/h_o$	*0.0123*	*0.0125*	*0.0722*	*0.0772*	*-0.0063*	*-0.0091*
$E'_o = \Delta\sigma'_v/\Delta\varepsilon_v$, MPa	***4.06***	***8.02***	***2.77***	***5.18***	***31.78***	***21.97***

Notes:

(1) The initial sample height h_o is taken as the sample height at the start of the each load increment or decrement.

(2) Negative stress and strain increments are tensile, correponding to unloading and sample heave.

(3) When calculating E'_o in MPa, remember to allow for the fact that $\Delta\sigma'_v$ is in kPa.

Even for small changes in stress, the soil stiffness is clearly dependent on the initial stress state, whether the sample is being loaded or unloaded and whether the soil is normally or over consolidated. In determining an appropriate stiffness for use in a design calculation, it is necessary to replicate the stress history, stress state and anticipated stress path of the soil in the field.

Analysis of data from the consolidation phase

Q4.4 Data from one stage of an oedometer test are given below.

Time, min	0.25	1	4	9	16	25
Settlement mm	0.063	0.075	0.103	0.133	0.160	0.185

Time, min	36	49	64	81	100	196
Settlement mm	0.210	0.228	0.240	0.250	0.258	0.265

Load increment: 25 - 50 kPa
Initial sample thickness: 20 mm
Sample diameter: 76 mm
Two-way drainage

For this load increment, estimate the one-dimensional modulus E'_o, the consolidation coefficient c_v, and the vertical permeability of the soil k_v. (It may be assumed that the initial slope of a graph of proportional settlement $R=\rho/\rho_{ult}$ against the square root of the time factor $T=c_v t/d^2$ is equal to $\sqrt{(4/3)}$, ie $R = \sqrt{(4T/3)}$.)

What factors would you take into account in the laboratory determination of E'_o c_v and k_v for use in design? What difficulties might you encounter in attempting to use oedometer test results to predict rates of settlement in the field?

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College (part question)]

Q4.4 Solution
Plot a graph of settlement against $\sqrt{}$time (Figure Q4.4):

time t, min	*0.25*	*1*	*4*	*9*	*16*	*25*	*36*	*49*	*64*	*81*	*100*	*196*
settlement ρ, mm	*.063*	*.075*	*.103*	*.133*	*.160*	*.185*	*.210*	*.228*	*.240*	*.250*	*.258*	*.265*
$\sqrt{t}$, min$^{0.5}$	*0.5*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*14*

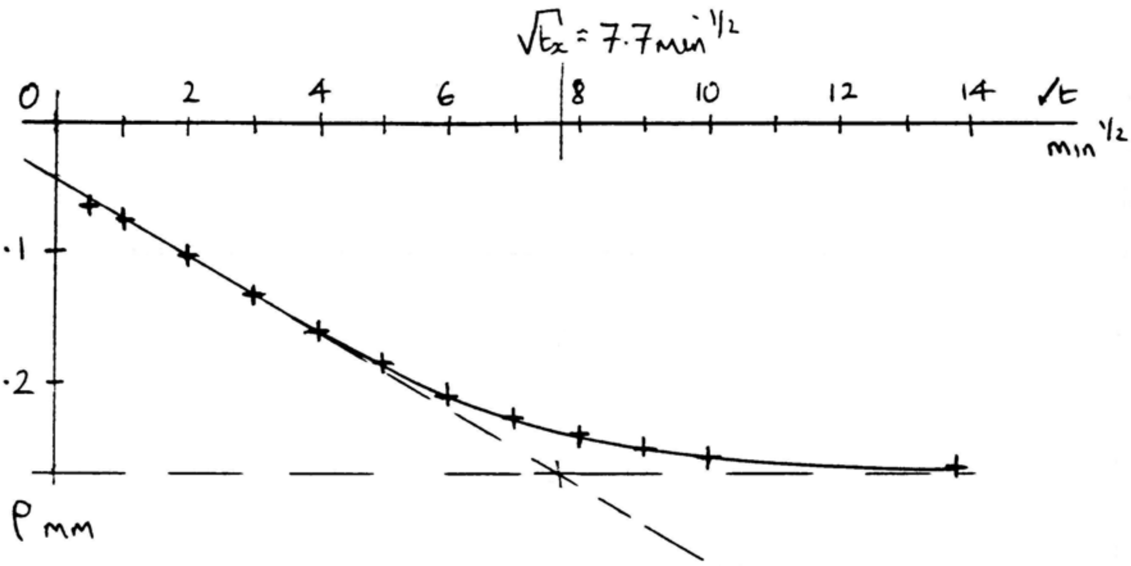


Figure Q4.4: Settlement against √time

Assume that the apparent initial settlement (c 0.045 mm) is due to trapped air.

Then $\rho_{ult} \approx 0.27 - 0.045$ *mm = 0.225 mm*

Ultimate vertical strain $\varepsilon_v = 0.225mm \div 20$ *mm = 0.01125 (1.125%)*

One dimensional modulus $E'_0 = \Delta\sigma'_v/\varepsilon_v = 25$ *kPa/0.01125 =* <u>*2.22 MPa*</u>

Initially, $R = \sqrt{({}^4/_3T)}$, *that is*

$\rho/\rho_{ult} = \sqrt{({}^4/_3)}.\sqrt{(c_v/d^2)}.\sqrt{t}$

The initial slope of the graph of ρ *against* $\sqrt{t}$ *has slope*

$d\rho/d(\sqrt{t}) = \rho_{ult}\,.\sqrt{({}^4/_3)}.\sqrt{(c_v/d^2)} = \rho_{ult}/\sqrt{t_x}$ *(see Figure Q4.4 and main text Section 4.6);*

thus $\sqrt{({}^4/_3)}.\sqrt{(c_v/d^2)} = 1/\sqrt{t_x}$ *or* $c_v = 3d^2/4t_x$

From Figure Q4.4, $\sqrt{t_x} = 7.7 \, min^{0.5} \Rightarrow t_x = 59.29$ *minutes*

drainage path length d = 10 mm (half the nominal sample height)

hence $c_v = {}^3\!/_4 \times 10^2 \, mm^2/59.29$ *minute* $= 1.265 \, mm^2$*/minute*

divide by 60×10^{-6} *to convert* mm^2*/minute to* m^2*/second*

$\Rightarrow \underline{c_v = 2.11 \times 10^{-8} \, m^2/s}$

$c_v = k.E'_0/\gamma_w$ *so* $k = c_v.\gamma_w/E'_0$

$k = 2.11 \times 10^{-8}$ m²/s $\times 9.81$ kN/m³ $\div 2222$ kN/m²

$\Rightarrow \underline{k = 9.3 \times 10^{-11} \text{ m/s}}$

(Note: the answer for permeability given at the end of Q4.4 in the
main text is not correct)

In determining parameter values for design, the following factors should be taken into account and replicated as far as possible in the laboratory test.

- *the stress state of the soil in the field*
- *the stress history of the deposit*
- *the changes in stress to which the soil will be subjected*

Difficulties in applying laboratory-determined parameters to field problems include

- *uncertainty concerning field drainage path lengths and boundary conditions*
- *E'_0 is the stiffness in one-dimensional compression – the true strain path in the field is unlikely to be one dimensional and will vary from point to point*
- *soil parameters are not constant, but vary with stress state and strain*
- *large scale fabric effects can be difficult to take into account*
- *drainage in the field is likely to be horizontal, whereas the oedometer test gives the vertical permeability of the soil.*

Q4.5 An engineer carries out an oedometer consolidation test on a sample of stiff clay, in connexion with the design of a proposed grain silo. The results from this test are as follows:

Time, min	0	0.5	1	2	4	8	16	32	64
Settlement mm	0.020	0.044	0.052	0.066	0.086	0.110	0.150	0.192	0.216

Load increment: 100 - 200 kPa
Initial sample thickness: 20 mm
Two-way drainage

Suggest a reason for the initial settlement of 0.02 mm. Estimate the one dimensional modulus E'_o and the consolidation coefficient c_v for the clay over the stress range under consideration. (It may be assumed that a graph of the consolidation settlement ρ against the square root of the elapsed time t has an initial slope of $\rho_{ult}.\sqrt{(4c_v/3d^2)}$, ie that $\rho = \rho_{ult}.\sqrt{(4c_v t/3d^2)}$.)

[University of London 2nd year BEng (Civil Engineering) examination, King's College (part question)]

Q4.5 Solution
Plot a graph of settlement against $\sqrt{}$time (Figure Q4.5):

time t, min	*0*	*0.5*	*1*	*2*	*4*	*8*	*16*	*32*	*64*
settlement ρ, mm	*.020*	*.044*	*.052*	*.066*	*.086*	*.110*	*.150*	*.192*	*.216*

$\sqrt{t}$, min$^{0.5}$	*0*	*0.707*	*1.000*	*1.414*	*2.000*	*2.828*	*4.000*	*5.657*	*8.000*

Initial settlement of 0.02 mm is probably due to trapped air.

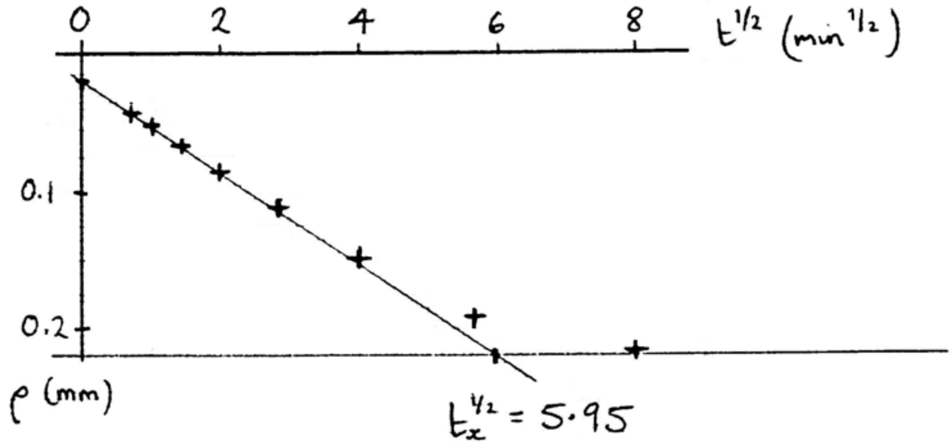


Figure Q4.5: Settlement against √time

Assume that the apparent initial settlement (0.02 mm) is due to trapped air.

Then $\rho_{ult} \approx 0.22 - 0.02$ mm = 0.2 mm

Ultimate vertical strain ε_v = 0.2 mm ÷ 20 mm = 0.01 (1.0%)

One dimensional modulus $E'_0 = \Delta\sigma'_v/\varepsilon_v$ = 100 kPa/0.01 = <u>10 MPa</u>

The initial slope of the graph of ρ against $\sqrt{t}$ is linear with slope

$d\rho/d(\sqrt{t}) = \rho_{ult} \,.\, \sqrt{(4c_v/3d^2)}$;

thus $c_v = 3d^2/4t_x$

From Figure Q4.5, $\sqrt{t_x}$ = 5.95 min$^{0.5}$ $\Rightarrow t_x$ = 35.4 minutes

drainage path length d = 10 mm (half the nominal sample height)

hence c_v = ¾ × 10^2 mm^2/35.4 minute = 2.12 mm^2/minute

divide by 60 × 10^{-6} to convert mm^2/minute to m^2/second

$\Rightarrow$ <u>c_v = 3.53 × 10^{-8} m^2/s</u>

Application of one-dimensional compression and consolidation theory to field problems

Q4.6 Figure 4.41 shows a cross section through a long sheet piled excavation. The width of the excavation is b, its depth is h and the sheet piles penetrate a further depth d to a permeable aquifer. A standpipe piezometer is driven into the aquifer, and the water in the standpipe rises to a height H above the bottom of the sheet piles.

(a) Show that the base of the excavation will become unstable if $H > H_{crit}$, where $(H_{crit}) > d.\gamma/\gamma_w$. *(Note: this is a quicksand problem: see Section 3.11.)*

(b) Some time after the excavation has been made, and steady state seepage from the aquifer to the excavation floor has been established, it is found that H is indeed very close to H_{crit}. In order to reduce the risk of base instability it is decided to reduce the head in the aquifer to H/2 by pumping. This is done very rapidly. Explain why the pore water pressures in the soil between the sheet piles cannot respond instantaneously.

(c) Taking d=10 m, γ = 20 kN/m^3 and γ_w = 10 kN/m^3, draw diagrams to show the initial and final distributions of pore water pressure with depth in the soil between the sheet piles. Draw also the initial and final distributions of excess pore water pressure with depth, and sketch in three or four isochrones at various stages in between.

(d) The soil between the sheet piles has a one dimensional modulus E'_o that increases linearly with depth from 5 MPa at the excavated surface to 45 MPa at the interface with the aquifer. Estimate the settlement which ultimately results from the reduction in pore water pressure due to pumping.

[University of London 2nd year BEng (Civil Engineering) examination, King's College]

Q4.6 Solution
(a) Upward flow between the sheet piles is one dimensional. Given that the bounding equipotentials (the top of the underlying aquifer, where the head is H above the bottom of the sheet piles; and the excavation surface, where the head is d above the bottom of the sheet piles) are parallel to each other and perpendicular to the bounding flowlines (the sheetpiles), the hydraulic gradient between the sheet piles is uniform and given by

$i = \Delta h/\Delta z = (H - d)/d = (H/d) - 1$

Instability will occur when the actual value of i reaches the critical value i_{crit}, given by

$i_{crit} = (\gamma - \gamma_w)/\gamma_w = (\gamma/\gamma_w) - 1$

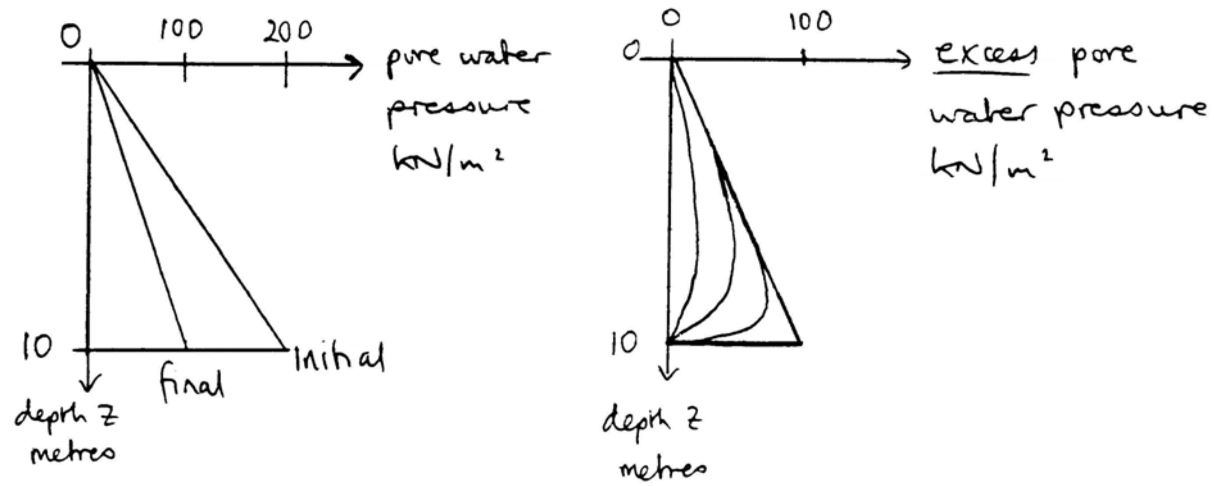
(see main text Section 3.11 for the derivation of this expression)

Hence instability will occur when $\underline{H = H_{crit} = d.\gamma/\gamma_w}$

(b) The pore water pressures in the soil between the sheet piles cannot respond instantaneously because this would require a change in effective stress (the total stress remains constant). A change in effective stress requires a change in volume, which requires

water to drain out of or into the soil, which can only occur at a rate governed by the soil permeability.

(c) Initially the pore water pressure varies linearly with depth from zero at the excavated surface (z = 0) to $\gamma_w.H_{crit} = \gamma.d$ = 200 kPa at the bottom of the sheet piles (z = 10 m). Finally, the variation in pore water pressure is linear between zero at the excavated surface and 100 kPa at depth z = 10 m (hydrostatic). The initial and final distributions of pore water pressure with depth are shown in Figure Q4.6a; subtracting the hydrostatic or steady state component, the corresponding distributions of excess pore water pressure, together with some isochrones in between, are shown in Figure Q4.6b (compare with main text Figure 4.25, and note that the excess pore water pressure in the aquifer at the lower drainage boundary can drop to zero immediately).



(a) ***(b)***

Figure Q4.6: distributions of (a) pore water pressure and (b) excess pore water pressure (with hydrostatic or steady state component subtracted) with depth

(d) Consider an element of soil of thickness δz at a depth z below the excavated surface. The compression of this element is $\delta\rho$, given by

$\delta\rho = (\Delta\sigma'_v/E'_0).\delta z$

as by definition $E'_0 = \Delta\sigma'_v/\Delta\varepsilon_v$ and $\Delta\varepsilon_v = \delta\rho/\delta z$

Now, E'_0 = (5000 + 4000z) kPa

and the eventual increase in vertical effective stress $\Delta\sigma'_v$ at depth z is equal to the reduction in pore water pressure at the same depth,

$\Delta\sigma'_v = 10z$ (ignoring friction on the sheet piles). Hence the total settlement ρ is given by

$$\rho = \int_{z=0}^{z=10m} \frac{10z}{5000 + 4000z} dz$$

Let u = 5000 + 4000z

then z = (u – 5000)/4000 and dz = du/4000, and

$$\rho = \int_{5000}^{45000} \frac{(u - 5000)/400}{u} . \frac{du}{4000}$$

$$= \int_{5000}^{45000} \left(\frac{1}{1600000} - \frac{50}{16000u} \right) du \;\; = \left[\frac{u}{1600000} - \frac{50.\ln(u)}{16000} \right]_{5000}^{45000}$$ *(settlement in metres)*

Thus ρ *= (0.028 – 0.033) – (0.003 – 0.027) m = 0.019 m*

<u>*or* ρ *= 19 mm*</u>

Q4.7 Figure 4.42 shows the ground conditions at the site of the Jubilee Line Extension station at Canary Wharf, in East London. During construction of the station, it was necessary to lower the groundwater level at the top of the Thanet Sands to 84m above site datum.

(a) Assuming that the groundwater level in the Thames Gravels is unaffected, and that the groundwater level at the top of the Chalk is reduced by only 35kPa, construct a table for the soil behind the retaining wall, showing the initial and final vertical effective stresses at the ground surface and at the interface levels between each of the strata.

(b) Using the geotechnical data given in Figure 4.42, estimate
i. the immediate settlement of the soil surface,
ii. the long term settlement of the soil surface, and
iii. using the relation between R and T given in Figure 4.18, the settlement after a period of 18 months.

Take the unit weight of water γ_w=10 kN/m^3.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q4.7 Solution

Calculate the initial and final stresses at the top and bottom of each layer:

Level, m above SD	*σ_v, kPa*	*u initial, kPa*	*σ'_v initial, kPa*	*u final, kPa*	*σ'_v final, kPa*	*$\Delta\sigma'_v$, kPa*
105	*0*	*0*	*0*	*0*	*0*	*0*
98	*126*	*0*	*126*	*0*	*126*	*0*
94	*210*	*40*	*170*	*40*	*170*	*0*
84	*420*	*95*	*325*	*0*	*420*	*+95*
70	*714*	*235*	*479*	*200*	*514*	*+35*

i) Initial settlement is due to compression of the Thanet Sands, which will occur very quickly owing to their relatively high permeability and stiffness.

$E'_0 = \Delta\sigma'_v/\varepsilon_v \quad \varepsilon_v = \rho/x$

Therefore settlement $\rho = \Delta\sigma'_v.x/E'_0$

For the Thanet Sands, $\Delta\sigma'_v$ *(average) = (95 kPa + 35 kPa)/2 = 65 kPa*

ρ *= 65 kPa × 14 m ÷ 200 MPa = <u>4.6 mm</u>*

ii) Ultimate settlement is due to compression of the Thanet Sands plus consolidation of the Woolwich & Reading Beds (now known as the Lambeth Group). For the Woolwich & Reading Beds,

$\Delta\sigma'_v$ *(average) = (95 kPa)/2 = 47.5 kPa*

ρ *= 47.5 kPa × 10 m ÷ 40 MPa = 11.875 mm*

Total ultimate settlement = 4.6 mm + 11.9 mm = <u>16.5 mm</u>

(iii) Calculate the time factor $T = c_v.t/d^2$ *after t = 18 months:*

The Woolwich & Reading Beds have two-way drainage, so drainage path length d = 10 m ÷ 2 = 5 m. Hence

$T = 4\ m^2/year \times 1.5\ year \div 52\ m^2 = 0.24$

Φρομ μαιν τεξτ Φιγυρε 4.18, τηισ χορρεσπονδσ το α προπορτιοναλ σεττλεμεντ $P = \rho/\rho_{ult} = 0.58$

Hence the settlement after 18 months = 4.55 mm (Thanet Sands) + (0.58 × 11.875 mm) (Woolwich & Reading Beds) = <u>11.4 mm</u>

QUESTIONS AND SOLUTIONS: CHAPTER 5

Interpretation of triaxial test results

Q5.1 Data from a conventional, consolidated-undrained triaxial compression test, carried out at a constant cell pressure of 400 kPa, are given below.

Axial strain ε_a, %	0	0.05	0.09	0.18	0.39	0.69	1.51
Deviator stress q, kPa	0	10.9	22.3	33.5	45.0	53.5	65.4
Pore water pressure u, kPa	274.6	280.3	284.6	290.8	300.0	307.6	314.4

Axial strain ε_a, %	3.22	4.74	6.13	7.89	9.39	11.03
Deviator stress q, kPa	79.0	85.7	89.6	91.4	93.9	94.0
Pore water pressure u, kPa	317.0	315.1	312.6	312.1	312.7	312.8

Plot graphs of mobilized strength ϕ'_{mob} and pore water change Δu against shear strain γ. Plot also the total and effective stress paths in the q,p and q,p' planes. Comment on these curves, and estimate the critical state strength ϕ'_{crit}. Is the sample lightly or heavily overconsolidated?

Q5.1 Solution

Convert the data to the required format using the following equations (for derivations and justifications, see the main text Section 5.4 etc):

$\gamma = 1.5\varepsilon_a$ *(main text Figure 5.7)*

$t = (\sigma'_1 - \sigma'_3)/2$ *(main text Equation 5.1)*

$s' = (\sigma'_1 + \sigma'_3)/2$ *(main text Equation 5.2)*

$\phi'_{mob} = \sin^{-1}(t/s')$ *(main text Figure 5.6)*

$q = \sigma'_a - \sigma'_r = \sigma_a - \sigma_r = \sigma_a - \sigma_c$ *(main text Equation 5.6)*

$p' = \sigma_c - u + q/3$ *(main text Equation 5.10)*

$p = \sigma_c + q/3.$ *(main text Equation 5.11)*

Now,

$$t = (\sigma'_1 - \sigma'_3)/2 = (\sigma_1 - \sigma_3)/2 = (\sigma_a - \sigma_c)/2 = q/2$$

and

$$s' = (\sigma'_1 + \sigma'_3)/2 = [(\sigma_1 + \sigma_3)/2] - u = [(\sigma_a + \sigma_c)/2] - u$$

hence

$s' = \sigma_c + q/2 - u$

and

$\phi'_{mob} = \sin^{-1}\{q/(2\sigma_c + q - 2u)$

Also, the change in pore water pressure $\Delta u = u - u_o$, *where* u_o *is the pore water pressure at the start of shear (= 274.6kPa in this case). Hence*

Axial strain ε_a, %	0	0.05	0.09	0.18	0.39	0.69	1.51
Shear strain $\gamma = 1.5\times\varepsilon_a$, %	**0**	**0.075**	**0.135**	**0.27**	**0.585**	**1.035**	**2.265**
Deviator stress q, kPa	0	10.9	22.3	33.5	45.0	53.5	65.4
Pore water pressure u, kPa	274.6	280.3	284.6	290.8	300.0	307.6	314.4
$\Delta u = u - u_o$, kPa	**0**	**5.7**	**10.0**	**16.2**	**25.4**	**33.0**	**39.8**
$\phi'_{mob} = \sin^{-1}\{q/(2\sigma_c + q - 2u)$, °	**0**	**2.4**	**5.1**	**7.6**	**10.6**	**13.0**	**16.0**
$p = \sigma_c + q/3$, kPa	**400**	**403.6**	**407.4**	**411.2**	**415.0**	**417.8**	**421.8**
$p' = \sigma_c + q/3 - u$, kPa	**125.4**	**123.3**	**122.8**	**120.4**	**115.0**	**110.2**	**107.4**

Axial strain ε_a, %	3.22	4.74	6.13	7.89	9.39	11.03
Shear strain $\gamma = 1.5\times\varepsilon_a$, %	**4.83**	**7.11**	**9.20**	**11.84**	**14.1**	**16.5**
Deviator stress q, kPa	79.0	85.7	89.6	91.4	93.9	94.0
Pore water pressure u, kPa	317.0	315.1	312.6	312.1	312.7	312.8
$\Delta u = u - u_o$, kPa	**42.4**	**40.5**	**38.0**	**37.5**	**38.1**	**38.2**
$\phi'_{mob} = \sin^{-1}\{q/(2\sigma_c + q - 2u)$, °	**18.8**	**19.6**	**19.8**	**20.0**	**20.5**	**20.5**
$p = \sigma_c + q/3$, kPa	**426.3**	**428.6**	**429.9**	**430.5**	**431.3**	**431.3**
$p' = \sigma_c + q/3 - u$, kPa	**109.3**	**113.5**	**117.3**	**118.4**	**118.6**	**118.5**

(Calculated values are shown in bold type)

Graphs of mobilized strength ϕ'_{mob} *and pore water change* Δu *against shear strain* γ, *and the total and effective stress paths in the q,p and q,p' planes, are plotted in Figures Q5.1a and b.*

There is no peak strength, and the sample has positive pore water pressures at failure. The sample is therefore probably wet of critical, i.e. lightly overconsolidated. The effective stress path appears to follow an undrained state boundary, but near the end veers off to the right (i.e. the pore water pressures start to reduce) to reach the critical state line at a higher value of q (and p') than might otherwise have been expected. The reason for this is unclear, but possible causes include the development of a rupture and/or sample anisotropy.

From Figure Q5.1a, $\phi'_{crit} \approx 20½°$.

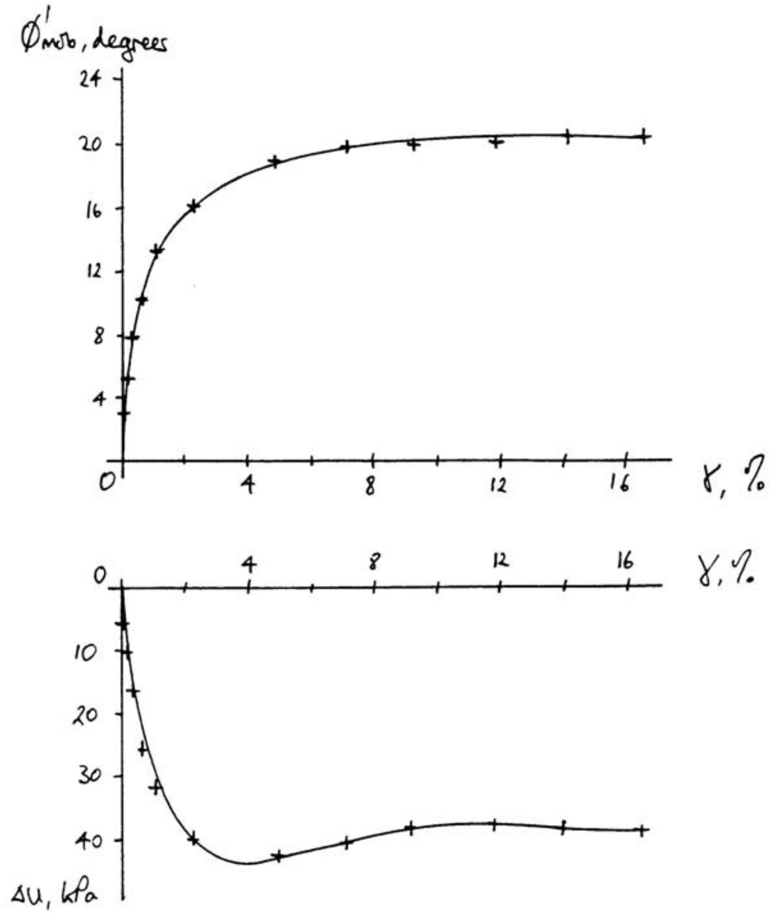


Figure Q5.1a: mobilized strength ϕ'_{mob} and change in pore water pressure Δu against shear strain γ

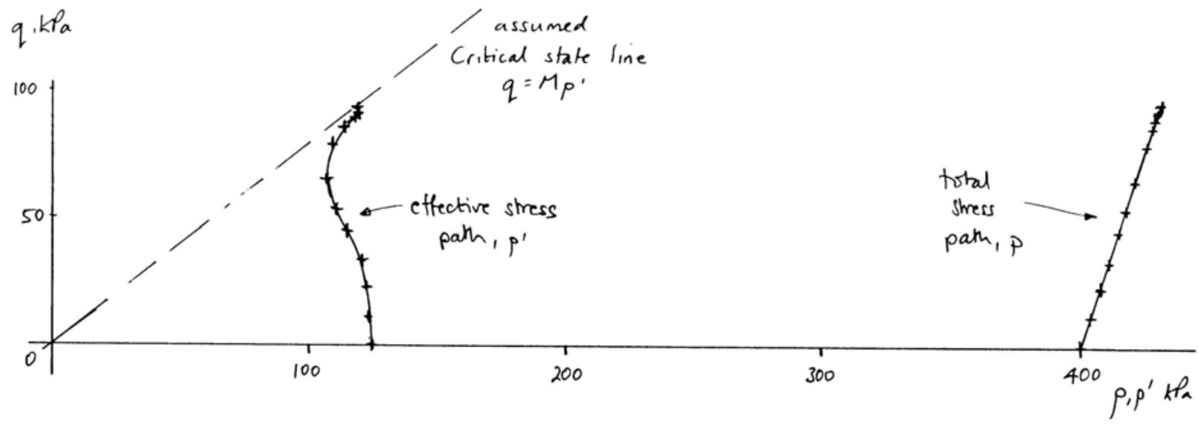


Figure Q5.1b: total and effective stress paths, q against p and q against p'

Q5.2 Two further consolidated-undrained triaxial compression tests are carried out on samples of the same clay as in Q5.1. These gave the following results.

	s' at ϕ'_{peak}	t at ϕ'_{peak}	s' at ϕ'_{crit}	t at ϕ'_{crit}
Test 2	88 kPa	35.8 kPa	90 kPa	31.5 kPa
Test 3	43 kPa	19.5 kPa	45 kPa	15.8 kPa

Using data from all three tests, plot peak and critical state strength failure envelopes on a graph of τ against σ', and comment on the data.

Q5.2 Solution
Mohr circles of effective stress may be plotted for each sample at both peak and critical states (Figure Q5.2 a and b). In each case, s' locates the centre of the circle, and t gives the radius. From the last data point for test 1 (Q5.1),

$t = q/2 = 47kPa$ *and* $s' = \sigma_C + q/2 - u = 134.2kPa.$

Hence

	s' at ϕ'_{peak}	*t at ϕ'_{peak}*	*$\phi'_{peak} = \sin^{-1}(t/s')_{peak}$*	*s' at ϕ'_{crit}*	*t at ϕ'_{crit}*	*$\phi'_{crit} = \sin^{-1}(t/s')_{crit}$*
Test 1	*134.2*	*47*	*20.5*	*134.2*	*47*	*20.5*
Test 2	*88*	*35.8*	*24.0*	*90*	*31.5*	*20.5*
Test 3	*43*	*19.5*	*27.0*	*45*	*15.8*	*20.6*

Note: t and s' are in kPa

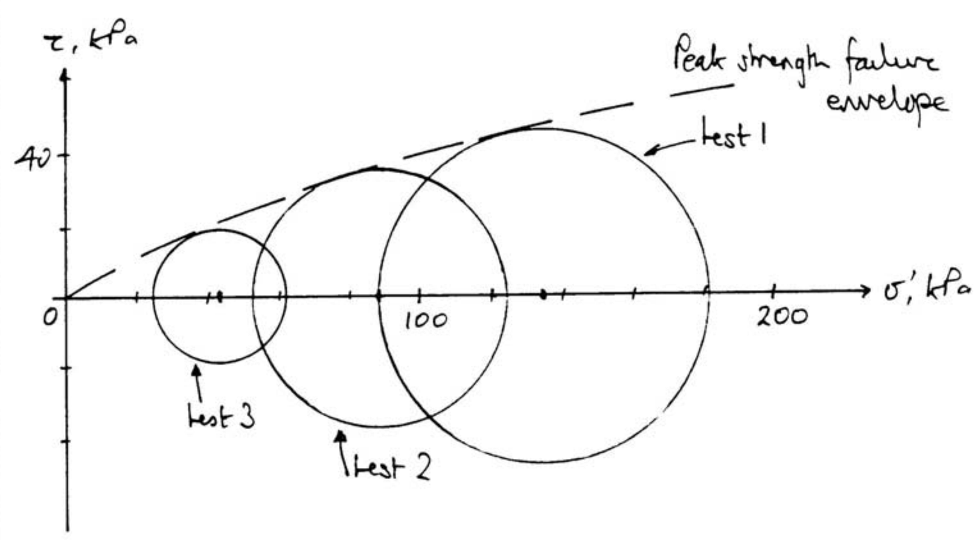


Figure Q5.2a: Mohr circles of stress at peak stress ratios $(t/s')_{peak}$

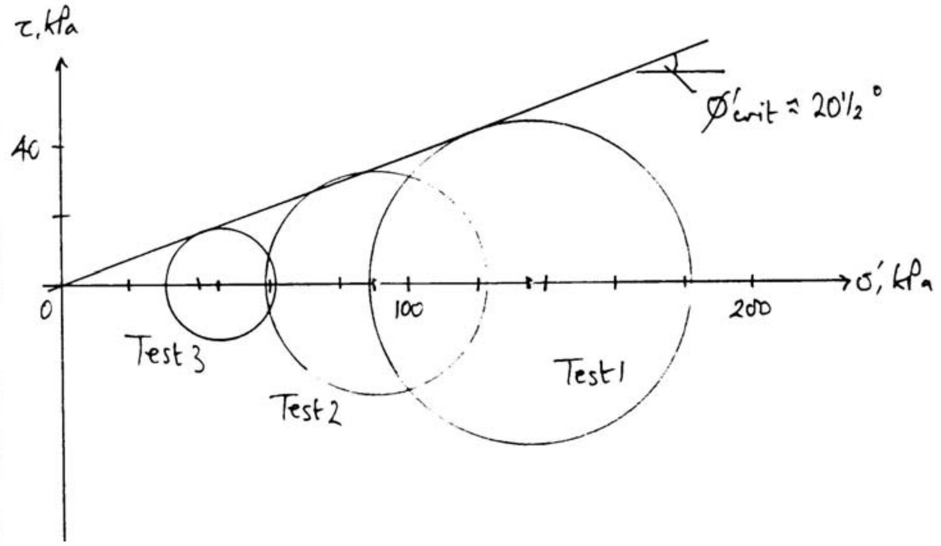


Figure Q5.2b: Mohr circles of stress at critical stress ratios $(t/s')_{crit}$

The peak strength failure envelope is curved due to the decreasing effect of the dilational component of strength as the effective stress is increased, with ϕ'_{peak} (defined as $\tan^{-1}[\tau/\sigma']_{peak} = \sin^{-1}[t/s']_{peak}$) decreasing from 27° in test 3 to 20½° in test 1.

The critical state failure envelope is a straight line through the origin with $\phi'_{crit} \approx 20½°$.

Q5.3 Using the data from Q5.1 and Q5.2, determine the equations of the critical state line in the q,p' and v,lnp' planes. (The as-tested water contents were 41.7% for sample 1; 45.5% for sample 2; and 52.0% for sample 3. Take G_S = 2.65). Hence predict the undrained shear strength of a fourth sample of the same clay, which is subjected to a conventional undrained triaxial compression test at a water content of 35%.

Q5.3 Solution
We can calculate the values of q and p' at the critical state for tests 2 and 3 from the values of t and s' given:

$t = q/2,$

$s' = (\sigma_C - u) + q/2 \Rightarrow (\sigma_C - u) = s' - q/2$

and

$p' = (\sigma_C - u) + q/3 = s' - q/6$

We can calculate the specific volume v for each test using

$v = 1 + e$

with

$e = w.G_S$

Hence, at the critical state,

	s', kPa	*t, kPa*	*p', kPa*	*q, kPa*	*v*	*lnp'*
Test 1	-	-	*118.5*	*94.0*	*2.105*	*4.775*
Test 2	*90*	*31.5*	*79.5*	*63.0*	*2.206*	*4.376*
Test 3	*45*	*15.8*	*39.7*	*31.6*	*2.378*	*3.681*

Graphs of q against p' and v against lnp' at critical states are shown in Figures Q5.3 a and b.

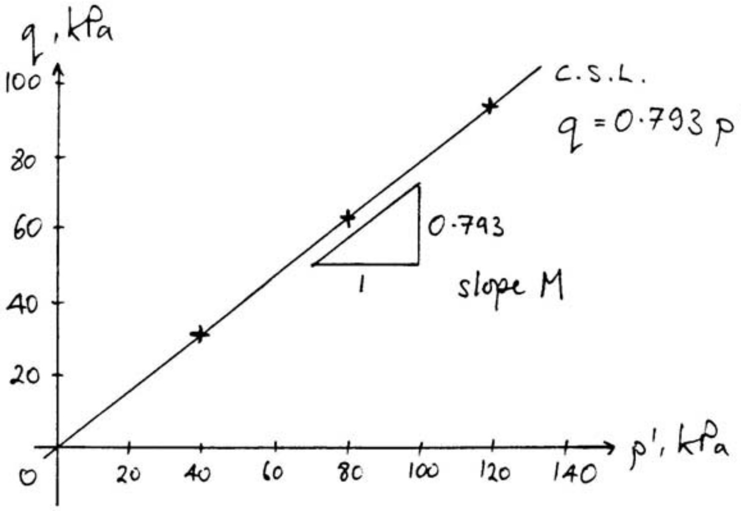


Figure Q5.3a: q against p' at critical states

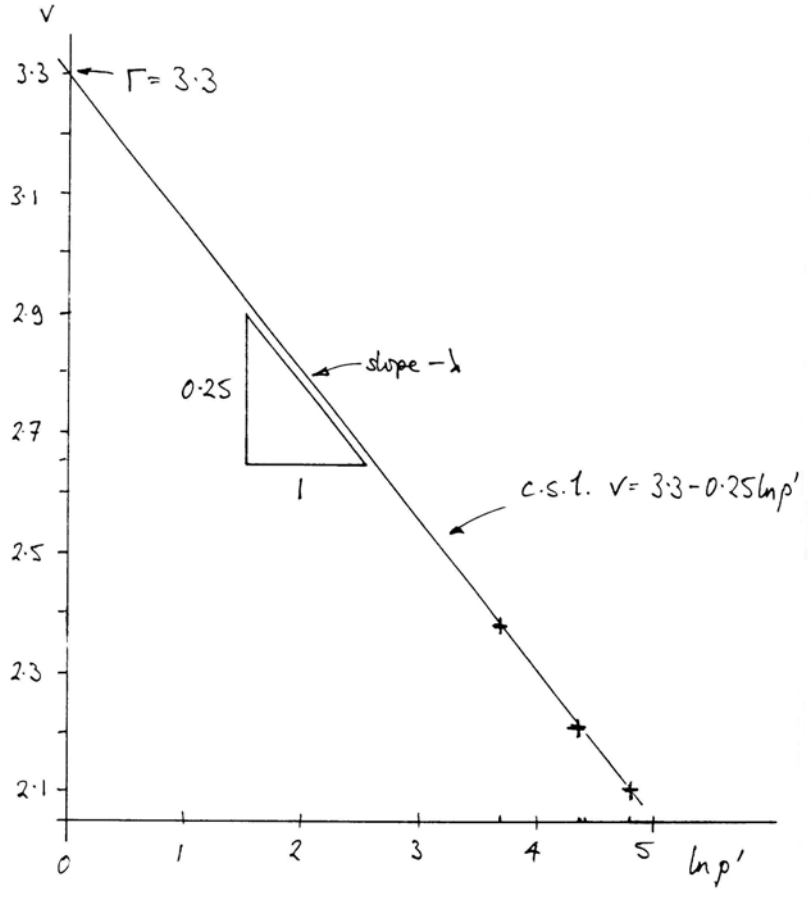


Figure Q5.3b: v against lnp' at critical states

From these graphs, the critical state line equations are

$q = 0.793 \times p'$, *i.e.* $M = 0.793$

and

$v = 3.3 - 0.25 \times \ln p'$, *i.e.* $\Gamma = 3.3$ *and* $\lambda = 0.25$

The fourth sample has $w = 35\% \Rightarrow v = 1.9275$, *giving*

$p'_{cs} = e^{\{(\Gamma - v)/\lambda\}} = e^{\{(3.3-1.9275)/0.25\}} = 242.26kPa$

Then the undrained shear strength $\tau_u = q_{cs}/2 = M.p'_{cs}/2 = 96.1\ kPa$

Determination of critical state and Cam clay parameters

Q5.4 Define in terms of principal stresses and the quantities measured during a conventional undrained compression test the parameters q, p and p'.

Data from both consolidation and shear stages of an undrained triaxial compression test on a sample of reconstituted London clay are given below. Plot the state paths followed by the sample on graphs of q against p', q against p and v against lnp', and explain their shapes.

CP,kPa	50	100	200	150	150	150	150	150
q, kPa	0	0	0	0	21	39	61	86
u, kPa	0	0	0	0	7	13	43	82
v	2.228	2.116	2.005	2.023	2.023	2.023	2.023	2.023

CP: cell pressure — q: deviator stress
u: pore water pressure — v: specific volume

Stating clearly the assumptions you need to make, estimate the soil parameters M, λ, κ and ϕ'_{crit}.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q5.4 Solution
q = deviator stress = $\sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3$

q = the ram load Q divided by the current sample area A.
In an undrained test the total volume V_t *is constant =* $A_0.h_0 = A.h = A.h_0.(1 - \varepsilon_{ax})$ *where* ε_{ax} *is the axial strain* $\Delta h/h_0$*, hence* $A = A_0/(1 - \varepsilon_{ax})$ *and* $\underline{q = Q.(1 - \varepsilon_{ax})/A_0}$

p = average total stress = $^1/_3(\sigma_1 + \sigma_2 + \sigma_3)$

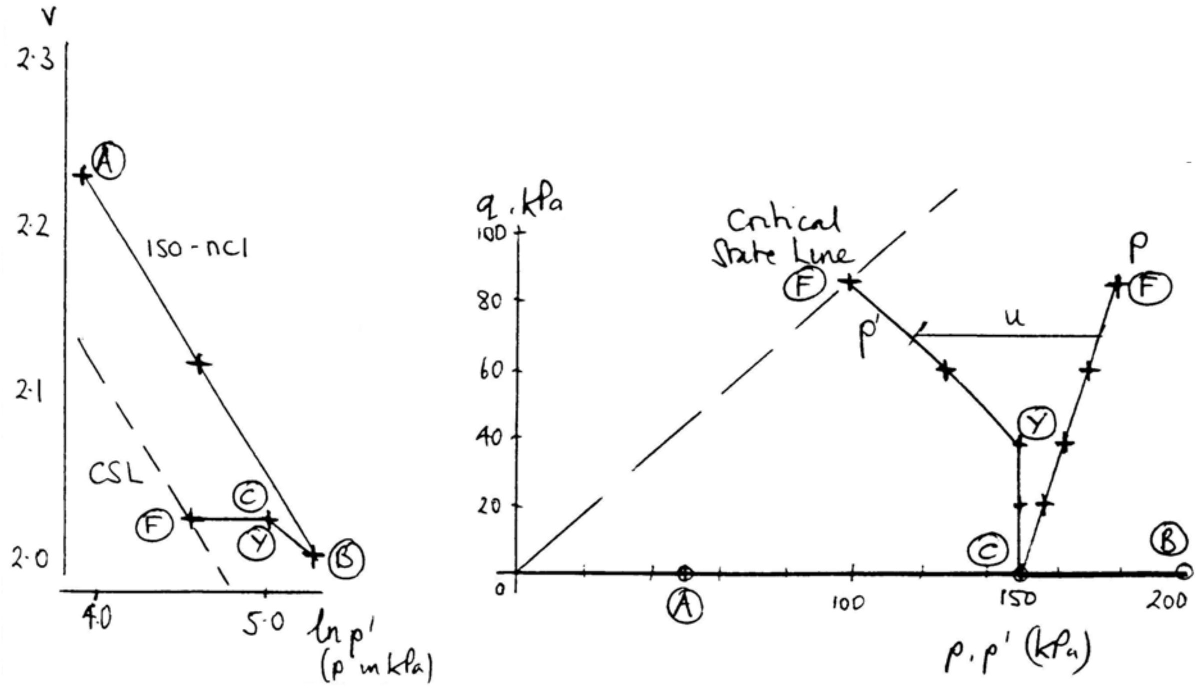
p' = average effective stress = p – u

u = pore water pressure (measured); $\sigma_2 = \sigma_3$ *= cell pressure CP (measured);* σ_1 *= cell pressure CP + q, so* $\underline{p = CP + q/3}$

The processed data (to obtain p' and lnp') are tabulated below

CP,kPa	50	100	200	150	150	150	150	150
q, kPa	0	0	0	0	21	39	61	86
u, kPa	0	0	0	0	7	13	43	82
v	2.228	2.116	2.005	2.023	2.023	2.023	2.023	2.023
p, kPa	50	100	200	150	157	163	170.3	178.7
p', kPa	50	100	200	150	150	150	127.3	96.7
lnp'(p' in kPa)	3.912	4.605	5.298	5.011	5.011	5.011	4.847	4.572
Point on graphs	A		B	C		Y		F

The state paths followed on graphs of q against p and p', and v against ln p', are plotted in Figure Q5.4.



(a) ***(b)***

Figure Q5.4: (a) q against p and p'; (b) v against ln p'

A to B is isotropic normal compression, with increasing cell pressure, drainage taps open and no deviator (shear) stress applied. It is first loading, so changes in specific volume are mainly plastic (irrecoverable)

B to C is isotropic unloading (reduction in cell pressure with the drainage taps open and no shear). The elastic component of volumetric compression that occurred during loading over the same stress range is recovered.

C to Y is undrained shear (cell pressure constant, drainage taps cloase, deviator stress applied) within the initial yield locus set up by isotropic compression to C. Behavious is "elastic" within the initial yield locus, so as there is no change in specific volume there can be no change in average effective stress and p' = constant.

At Y, the sample reaches the initial yield locus and yields. This is the transition point to plastic behaviour.

From Y to F, the sample is sheared at constant specific volume. To move the state of the sample to the appropriate point on the critical state line, the average effective stress must decrease and the pore water pressure increases to achieve this.

At F, the sample reaches the critical state appropriate to the specific volume as tested, at which continued deformation can take place at constant q, p' and v.

Assuming that A to B is on the isotropic normal compression line and that F is on the critical state line, the critical state paramaters M, λ and κ may be calculated as follows.

$M = q/p'$ at the critical state F $\Rightarrow$ $\underline{M = 86/96.7 = 0.89}$

λ the slope of the isotropic normal compression line = $-\Delta v/\Delta \ln p'$ between A and B: $\lambda = (2.228 - 2.005)/(5.298 - 3.912) \Rightarrow \underline{\lambda = 0.161}$

κ the slope of an unload/reload line = $-\Delta v/\Delta \ln p'$ between B and C:
$\kappa = (2.023 - 2.005)/(5.298 - 5.011) \Rightarrow \underline{\kappa = 0.063}$

From main text Equation 5.32a,

$\sin\phi'_{crit} = (3M)/(6 + M) = (3 \times 0.89)/(6 + 0.89) \Rightarrow \underline{\phi'_{crit} = 22.8°}$

Q5.5 Define the parameters q, p and p', in terms of principal stresses. Show also how q, p and p' are related to the quantities measured during a conventional undrained compression test.

Data from the shear stage of an undrained triaxial compression test on a sample of kaolin clay are given below. Plot the state paths followed by the sample in the q:p' and q:p planes, and explain their shapes. Stating clearly the assumptions you need to make, estimate the slope of the critical state line M and the corresponding value of ϕ'_{crit}.

q, kPa	0	13.8	27.5	41.3	53.0	59.5	63.0
u, kPa	0	4.6	9.2	13.8	33.6	48.0	59.3

Cell pressure = 100kPa   q: deviator stress   u: pore water pressure

A second, identical sample, is subjected to a drained compression test starting from a cell pressure of 100kPa. Estimate the value of q at failure, and show the effective stress path followed (in the q,p' plane) on the diagram you have already drawn for the first sample.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q5.5 Solution
q is the deviator stress = $\sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3$
p is the average total stress = $^1/_3(\sigma_1 + \sigma_2 + \sigma_3)$
p' is the average effective stress = p – u

q is given by the ram load Q divided by the current sample area A
In an undrained test the total volume V_t *is constant =* $A_0.h_0 = A.h = A.h_0.(1 - \varepsilon_{ax})$ *where* ε_{ax} *is the axial strain* $\Delta h/h_0$*, hence* $A = A_0/(1 - \varepsilon_{ax})$ *and* $\underline{q = Q.(1 - \varepsilon_{ax})/A_0}$

u is the pore water pressure (measured); $\sigma_2 = \sigma_3$ *= cell pressure CP (measured); and* σ_1 *= cell pressure CP + q, so* $\underline{p = CP + q/3}$

The processed data (to obtain p and p' from the values of q and u given, according to the relationships derived above) are given in the table below.

q, kPa	*0*	*13.8*	*27.5*	*41.3*	*53.0*	*59.5*	*63.0*
u, kPa	*0*	*4.6*	*9.2*	*13.8*	*33.6*	*48.0*	*59.3*
p, kPa	*100*	*104.6*	*109.2*	*113.8*	*117.7*	*119.8*	*121.0*
p', kPa	*100*	*100.0*	*100.0*	*100.0*	*84.1*	*71.8*	*61.7*

Graphs of q against p and p' are plotted in Figure Q5.5.

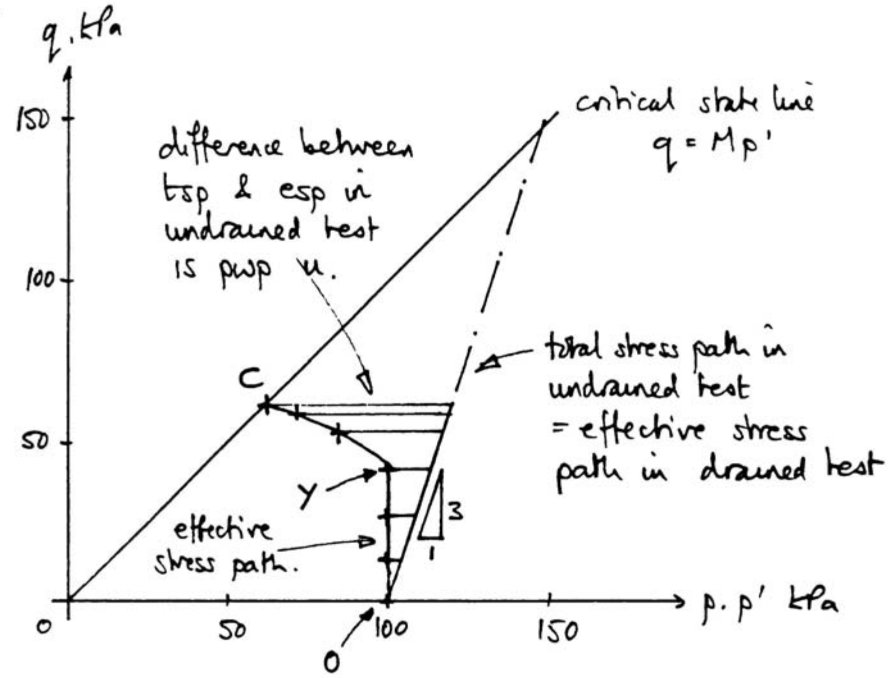


Figure Q5.5: q against p and p'

OYC is the effective stress path followed in an undrained test. Along OY, the soil is within the initial yield locus and therefore deforms “elastically” (in the sense that volume change can only occur if there is a change in effective stress) at p' = constant. At Y, the soil yields and starts to deform plastically. As it cannot compress (due to the constraint of the undrained test), excess pore water pressures are generated.

The rate of increase in pore water pressure with deviator stress, du/dq, increases suddenly at Y. The critical state corresponding to the specific volume of the sample as tested is reached (presumably) as C. At the critical state, deformatiuon could continue at constant q, p' and v (although we do not have the evidence for this in the data we are given).

The total stress path has slope dq/dp = 3, because $p = {}^{1}/_{3}(q + CP)$ *and CP = constant.*

Assuming that the sample deforms as a continuum, so that internal stresses/strains may be deduced from boundary measurements; that C is the critical state; and that the line joining critical states (the CSL) goes through the origin,

$M = q_c/p'_c = 63/61.7 \Rightarrow \underline{M = 1.02}$

A drained test starting from a cell pressure of 100 kPa has u = 0 and p' = 100 + q/3 kPa. It will reach the CSL when

$q = M.p'$ *and* $p' = 100 + q/3$, *or with* $M = 1.02$,

$p' = 100 + 1.02p'/3$

$\Rightarrow \underline{p' = 151.5\text{ kPa};\ q = 154.5\text{ kPa}}$

The effective stress path for the drained test is shown chain dotted on Figure Q5.5.

Prediction of state paths from triaxial test data using Cam clay concepts

Q5.6 A sample of saturated kaolin (G_S=2.61) was compressed isotropically in a triaxial cell to an effective cell pressure of 300 kPa. In this state, the cylindrical sample had a height of 80 mm and a diameter of 38 mm. The drainage taps were closed and the sample was subjected to a conventional undrained compression test to failure at a constant cell pressure of 300 kPa. The following values of deviator stress q and pore water pressure u were recorded.

Deviator stress q (= σ'_1-σ'_3), kPa	0	24.5	45.4	63.2	78.3	101.6	117.7	136.5
Pore water pressure u, kPa	0	30.2	53.6	76.6	97.3	132.8	161.8	211.8

At the end of the test, the water content of the sample was found to be 57.2%.

(a) Plot and explain the significance of the stress paths followed in the q,p' and q,p planes.

It was intended to prepare a second sample of kaolin in an identical manner, but the sample was accidentally over-stressed to an effective cell pressure of 320 kPa during isotropic compression. To make the water content of the second sample the same as that of the first, it was necessary to reduce the effective cell pressure to 229 kPa. During swelling from p' = 320 kPa to 229 kPa, the sample took in 618 mm^3 of water.

(b) Use all of these data to calculate the parameters Γ, λ, κ, M and ϕ'_{crit}.

(c) The second sample was subjected to a conventional undrained compression test from an effective cell pressure of 229 kPa. Sketch the stress paths followed in terms of q against p' and q against p, giving the values of q at yield and at failure.

(d) If the first sample had been subjected to a drained (rather than an undrained) test, what would have been the value of q at failure? Comment briefly on the engineering significance of this result.

[University of London 2nd year BEng (Civil Engineering) examination, King's College]

Q5.6 Solution

(a) $p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3) = \sigma'_3 + q/3$ where σ'_3 is the cell pressure (= 300 kPa in this case) minus the pore water pressure u

Hence $p' = 300 - u + q/3$; $p = p' + u = 300 + q/3$. Values of p and p' are given in the table below, and the state paths are plotted as q against p and p' in Figure Q5.6a.

q, kPa	*0*	*24.5*	*45.4*	*63.2*	*78.3*	*101.6*	*117.7*	*136.5*
u, kPa	*0*	*30.2*	*53.6*	*76.6*	*97.3*	*132.8*	*161.8*	*211.8*
p', kPa	*300*	*278*	*261.5*	*244.5*	*228.8*	*201.1*	*177.4*	*133.7*
p, kPa	*300*	*308.2*	*315.1*	*321.1*	*326.1*	*333.9*	*339.2*	*345.5*

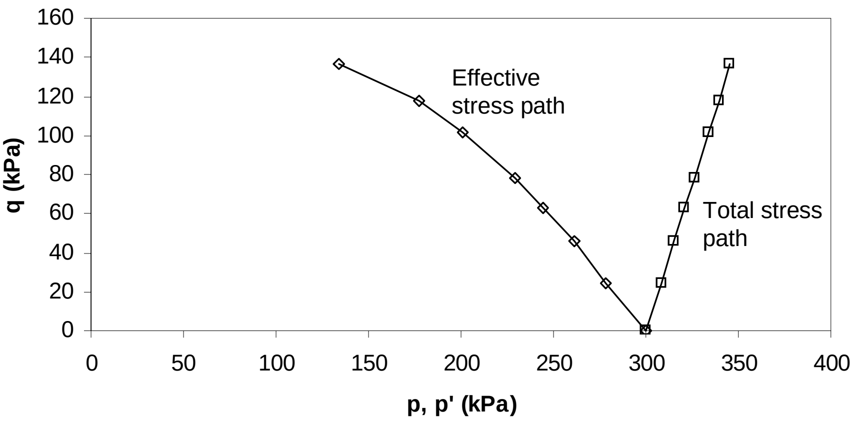


Figure Q5.6a: q against p and p'for sample 1

The total stress path (q against p) rises at a slope dq/dp = 3. The undrained stress path (q against p') is a projection of the state boundary surface at constant void ratio. Overconsolidated samples having the same void ratio will behave "elastically" (ie shear with p' = constant in an undrained test) until their stress state reaches this stress path, which will then be followed until the critical state is reached (q = M.p').

(b) We are given changes in total volume V_t, which we need to convert to changes in specific volume v.

$V_t = V_s + V_v$; $e = V_v/V_s$; so $V_t = V_s.(1 + e) = V_s.v$ and $\Delta V_t = V_s.\Delta v$

The first sample at a cell pressure of 300 kPa ($q = 0$) has a total volume V_t given by
‘
$V_t = (\pi \times 38^2\ mm^2/4) \times 80\ mm = 90729\ mm^3$

It is saturated, so $e = w.G_s$ (main text Equation 1.10); $w = 0.572$ and $G_s = 2.61$ so $e = 1.493$
Hence the volume of solids $V_s = V_t/(1 + e) = 90729\ mm^3 \div 2.493 = 36393.5\ mm^3$ (constant)

When the cell pressure was increased to 320 kPa, the change in total volume ΔV_t was -618 mm^3 giving a change in specific volume Δv of –618 mm $\div$ 36393.5 mm = -0.017.

The slope of the isotropic normal compression line on a graph of v against $\ln p'$ is $-\lambda$, where $\lambda = -\Delta v/\Delta(\ln p')$. Hence

$\lambda = 0.017 \div (\ln 320 - \ln 300) = 0.017/0.0645 \Rightarrow \underline{\lambda = 0.26}$

The same change in specific volume occurs on swelling from $p' = 320$ kPa to $p' = 229$ kPa, which is along a swelling line on the graph of v against $\ln p'$ which has slope $-\kappa$, hence

$\kappa = 0.017 \div (\ln 320 - \ln 229) = 0.017/0.3346 \Rightarrow \underline{\lambda = 0.05}$

The specific volume on the isotropic normal compression line at $p' = 1$ kPa is $(\Gamma + \lambda - \kappa)$, hence

$(\Gamma + \lambda - \kappa) - \lambda.\ln 300 = 2.493$

$(\Gamma + 0.21) - (0.26 \times 5.704) = 2.493$

$\Rightarrow \underline{\Gamma = 3.766}$

Assume that the first test ends on the critical state line at $q = M.p'$, giving

$\underline{M = 136.5 \div 133.7 = 1.02}$

(c) The second sample behaves elastically (in the sense that p' = constant during undrained shear) until the stress path reaches that of the first sample. The second sample then yields, and its stress path follows that of the first sample (the undrained state boundary) to failure at the same critical state. This is indicated in Figure Q5.6b.

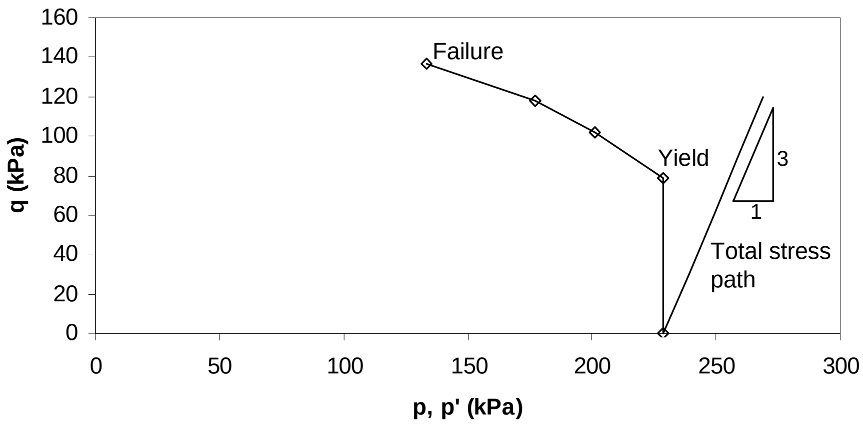


Figure Q5.6b: q against p and p'for sample 2

From the data for the first sample, we can say that for the second sample

<u>at yield, p' = 229 kPa; q = 78.3 kPa</u>

<u>at failure, p' = 133.7 kPa; q = 136.5 kPa</u>

(d) If the first sample had been subjected to a drained test, it would have followed the total stress path with u = 0,

p' = CP + q/3 = 300 + q/3 kPa. It would reach the critical state line (CSL) when

q = M.p' and p' = 300 + q/3, or with M = 1.02,

p' = 300 + 1.02p'/3 ⇒ p' = 454.4 kPa;

Hence <u>q = 463.6 kPa</u>

For a soil of this type (a normally consolidated or lightly overconsolidated clay), failure may occur during rapid (ie undrained) loading which could have been avoided if the load had been applied in smaller increments allowing drainage and consolidation to occur. An example of this is in the stage construction of embankments on soft clay, mentioned in the mein text in Section 4.3 (Figure 4.10).

Q5.7 (a) Define the triaxial invariant stress parameters p' and q in terms of the principal stresses and the quantities measured in a conventional triaxial test.

Two saturated triaxial test samples, each containing 116.3 g of dry clay powder (G_S=2.70), were prepared for a shear test by isotropic compression in the triaxial cell. For sample A, the cell pressure was gradually raised from 25 kPa to 174 kPa, with full drainage occurring throughout the process. At 174 kPa, the sample had a diameter of 40 mm and a height of 120 mm. The drainage taps were then closed, the cell pressure was increased to 274 kPa and the sample was subjected to an undrained compression test to failure. The data recorded during consolidation were:

Cell pressure, kPa	25	50	75	100	150	174
Pore water pressure, kPa	0	0	0	0	0	0
Volume of water expelled, cm^3	0	22.4	34.47	43.08	56.01	60.31

The data recorded during shear were:

Cell pressure, kPa	274	274	274	274	274	274
Pore water pressure, kPa	100	104	114	132	162	189
Deviator stress q, kPa	0	10	20	30	40	45

(b) Plot the state path followed by Sample A in the q,p' and v,lnp' planes, and comment on its significance.

Sample B was consolidated in the same manner as Sample A, but at the last increment of cell pressure was inadvertantly overstressed to 200 kPa. To achieve the same void ratio at the start of the shear test, the cell pressure was reduced to 140 kPa and the sample was allowed to swell slightly as indicated below. The drainage taps were then closed, the cell pressure was increased to 240 kPa, and the undrained shear test was commenced.

Cell pressure, kPa	150	200	140	240
Pore water pressure, kPa	0	0	0	100
Volume of water expelled, cm^3	56.01	64.62	60.31	-

(c) Predict the state paths followed by Sample B in terms of q against p' and v against lnp' during the shear test, giving values of q, p' and pore water pressure u at yield and at failure.

(d) If Sample B had been subjected to a drained shear test at a constant cell pressure of 140 kPa, estimate the values of q and p' at which failure would have occurred, and the volume of water that would have been expelled during the shear test.

[University of London 2nd year BEng (Civil Engineering) examination, King's College]

Q5.7 Solution

(a) q is the deviator stress $= \sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3$

p' is the average effective stress $= {}^1/_3(\sigma'_1 + \sigma'_2 + \sigma'_3) = {}^1/_3(\sigma_1 + \sigma_2 + \sigma_3) - u$

q is given by the ram load Q divided by the current sample area A
In an undrained test the total volume V_t is constant = $A_0.h_0 = A.h = A.h_0.(1 - \varepsilon_{ax})$ where ε_{ax} is the axial strain $\Delta h/h_0$, hence $A = A_0/(1 - \varepsilon_{ax})$ and $\underline{q = Q.(1 - \varepsilon_{ax})/A_0}$. In a drained test the total volume V_t is equal to $V_{t0}.(1 - \varepsilon_{vol})$ and $A = V_{t0}.(1 - \varepsilon_{vol})/ h_0.(1 - \varepsilon_{ax}) = A_0.(1 - \varepsilon_{vol})/(1 - \varepsilon_{ax})$ hence $\underline{q = Q/A_0 \times (1 - \varepsilon_{ax})/(1 - \varepsilon_{vol})}$ (see main text Section 5.4.3).

$\sigma_2 = \sigma_3$ = cell pressure CP (measured); u is the pore water pressure (measured); and σ_1 = cell pressure CP + q, so $\underline{p' = CP - u + q/3}$

(b) $V_t = V_s + V_v$; $e = V_v/V_s$; so $V_t = V_s.(1 + e) = V_s.v$ and $\Delta V_t = V_s.\Delta v$

The volume of solids V_s is constant and given by $V_s = m_s/\rho_s = m_s/G_s.\rho_w$ = 116.3 g $\div$ (2.70 $\times$ 10^{-3} g/mm^3) taking ρ_w = 1 g/mm^3; hence V_s = 43074 mm^3

At a cell pressure of 25 kPa (q = 0), the sample has a total volume V_t given by
‘
$V_t = (\pi \times 40^2\ \text{mm}^2/4) \times 120\ \text{mm} = 150796\ \text{mm}^3$

giving a specific volume $v = V_t/V_s = 150796/43074 = 3.501$

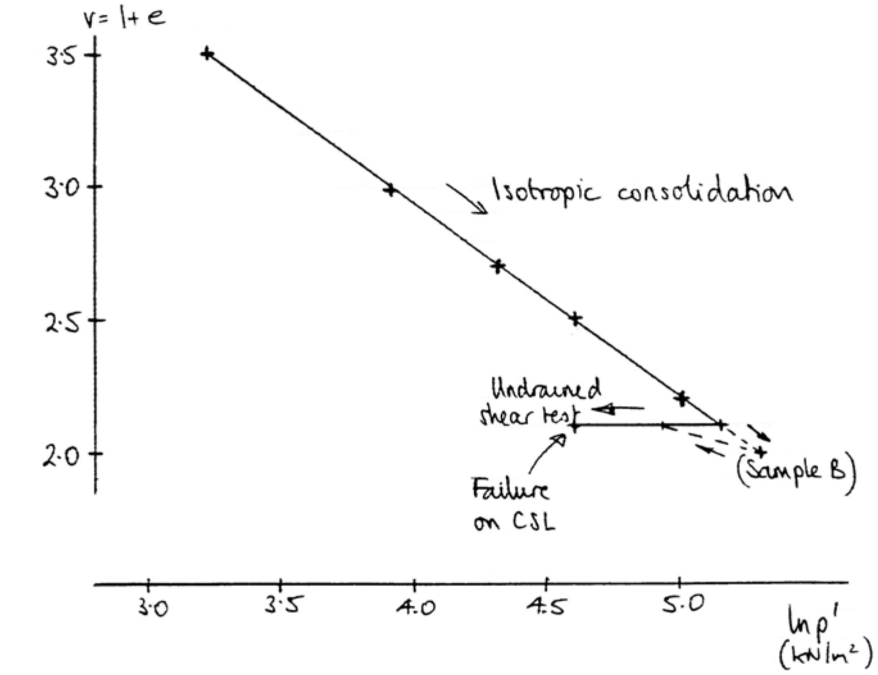
Hence we can calculate the specific volume during the consolidation stage of the test as v = 3.501 – $\Delta V_t/V_s$ as in the table below:

CP, kPa (= p')	*25*	*50*	*75*	*100*	*150*	*174*
u, kPa	*0*	*0*	*0*	*0*	*0*	*0*
ΔV_t, cm^3	*0*	*22.4*	*34.47*	*43.08*	*56.01*	*60.31*
ln p' (p' in kPa)	*3.219*	*3.912*	*4.317*	*4.605*	*5.011*	*5.159*
v	*3.501*	*2.981*	*2.701*	*2.501*	*2.201*	*2.101*

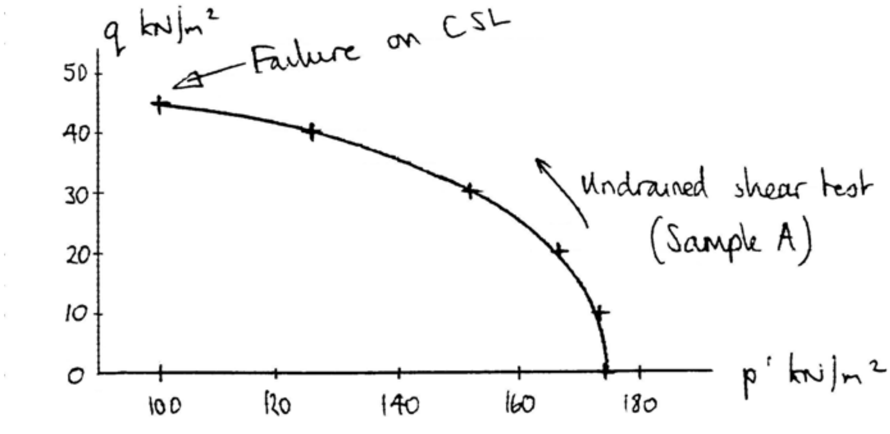
During the shear test, p' = CP – u + q/3 and the cell pressure CP = 274 kPa. Hence

CP, kPa	*274*	*274*	*274*	*274*	*274*	*274*
u, kPa	*100*	*104*	*114*	*132*	*162*	*189*
q, kPa	*0*	*10*	*20*	*30*	*40*	*45*
p', kPa	*174*	*173.3*	*166.7*	*152.0*	*125.3*	*100.0*

The state paths followed in terms of v against lnp' and q against p' are plotted in Figures Q5.7 a and b.



(a)



(b)

Figure Q5.7: (a) v against lnp' and (b) q against p'

The effective stress path shown in Figure Q5.7a represents the undrained state boundary for samples having a specific volume v = 2.101. An overconsolidated sample (eg sample B, see below) having this specific volume will follow the same effective stress path between yield and failure (see part (c) below).

(c) This enables us to predict the state path followed by sample B. On the graph of v against lnp', v = constant (because it is an undrained test) and the critical state is reached at the same value of p' as sample A. On the graph of q against p', p' = constant (= 140 kPa) until the undrained state boundary is reached. The effective stress path then follows the undrained state boundary to failure at the same point as sample A. This is shown in Figure Q5.7c (note: ln 140 = 4.94; ln200 = 5.30).

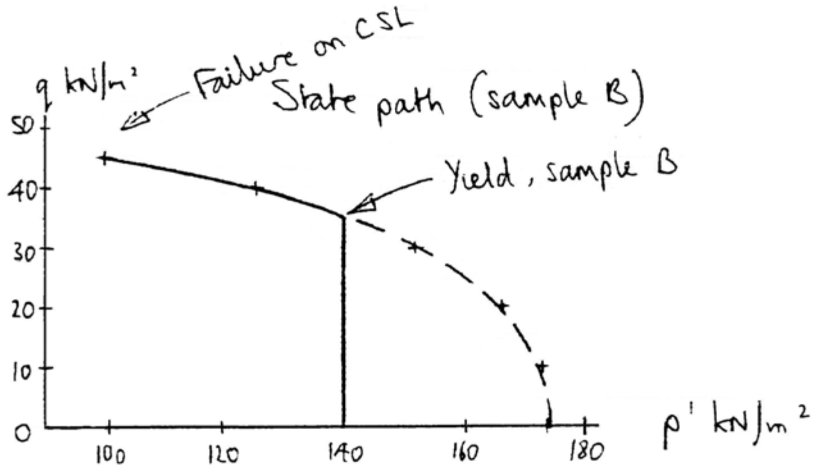


Figure Q5.7c: q against p' for sample B

For test B at yield, q = 35 kPa (scaled from Figure Q5.7c); p' = 140 kPa; p = 251.7 kPa; u = p - p' = 111.7 kPa. At failure, q = 45 kPa; p' = 100 kPa; p = 255 kPa; u = 155 kPa

(d) If sample B had been subjected to a drained shear test from a cell pressure of 140 kPa, the effective stress path would have been given by

$p' = CP + q/3$

Failure would have occurred on reaching the line joining critical states, $q = M.\, p'$

We can calculate the value of the critical state parameter M using the data for the end point of test A, $M = q/p'$ *at failure = 45 kPa/100 kPa* $\Rightarrow M = 0.45$

Thus drained failure for sample B would have occurred at

$q/0.45 = 140 + q/3 \Rightarrow \underline{q = 74.1 \text{ kPa}; p' = 164.7 \text{ kPa}}$

On the graph of v against lnp', the line joining critical states is parallel to the isotropic normal compression line and the slope λ is given by

$\lambda = -\Delta v/\Delta \ln p' = (3.501 - 2.101) \div (5.159 - 3.219) = 0.722$

from the data for the isotropic compression of sample A.

We know that the end point of test A lies on the critical state line on the graph of v against lnp' at p' = 100 kPa, and that for a drained test on sample B p' at the critical state would be 164.7 kPa. Hence the change in specific volume during a drained shear test on sample B would be

$\Delta v = 0.722\Delta(\ln p') = 0.722 \times (\ln 164.7 - \ln 100) = 0.360$

To find the actual volume of water expelled, we must multiply this by the volume of solids V_s *(because* $\Delta V_t = V_s.\Delta v$*) to give*

$\underline{\Delta V_t \approx 15.5\ cm^3}$

Prediction of triaxial state paths using the Cam clay model

Q5.8 (a) Describe by means of an annotated diagram the main features of the conventional trixial compression test apparatus.

(b) A sample of London Clay is prepared by isotropic normal compression in a triaxial cell to an average effective stress p' = 400 kPa, at which point its total volume is 86×10^3 mm^3. The drainage taps are then closed and the sample is subjected to a special compression test in which the cell pressure is reduced as the deviator stress is increased so that the average total stress p remains constant. Sketch the state paths followed, in the q,p', q,p and v,lnp' planes. Give values of cell pressure, q, p, u, p' and specific volume v at the start of the test and at failure. (You must also calculate some intermediate values in order to sketch the state paths satisfactorily.)

How do the values of undrained shear strength τ_u and pore water presure at failure compare with those that would have been measured in a conventional compression test?

Use the Cam clay model with numerical values Γ = 2.759, λ = 0.161, κ = 0.062, M = 0.89 and G_S = 2.75

[University of London 2nd year BEng (Civil Engineering) examination, King's College]

Q5.8 Solution
(a) A suitable diagram of the triaxial apparatus is given in the main text, Figure 5.1(a)

(b) Specific volume at the start of the shear test is given by

$v = (\Gamma + \lambda - \kappa) - \lambda.\ln p'$

Substituting the given values of Γ, λ and κ and p' = 400 kPa,

$v = (2.759 + 0.161 - 0.062) - (0.161.\ \ln 400) \Rightarrow \underline{v = 1.893}$

The shear test is undrained so v remains constant throughout. This defines the position reached on the critical state line, at p'_c such that

$v = 1.893 = \Gamma - \lambda.\ln p'_c = 2.759 - 0.161.\ \ln p'_c$

$\Rightarrow \ln p'_c = (2.759 - 1.893) \div 0.161 \Rightarrow \underline{p'_c = 216.8\ kPa}$

The deviator stress at the critical state q_c is given by

$q_c = M.p'_c$ *with* $M = 0.89 \Rightarrow \underline{q_c = 192.9\ kPa}$

This is a non-standard test with p = constant = 400 kPa, so the pore pressure u_c at the critical state is given by

$u_c = 400 - p'_c \Rightarrow \underline{u_c = 183.2\ kPa}$

$p = {}^1/_3(\sigma_a + 2\sigma_r) = \sigma_r + q/3 = constant$, *i.e.* $\Delta\sigma_r = -\Delta q/3$ *where sr is the cell pressure.*

Hence the cell pressure at failure = 400 kPa – 192.9 kPa/3
$\Rightarrow$ *cell pressure at failure = 335.7 kPa*

Summary table:

kPa	CP	q	p'	p	u	v
Start of shear	400	0	400	400	0	1.893
End of shear	335.7	192.9	216.8	400	183.2	1.893

We need to calculate some intermediate points. This may be done as follows.

Stress states between the start of the shear test and failure all lie on the current yield locus, which has associated with it a current value of p'_0 (which defines the isotropic pressure at the tip of the Cam clay yield locus – see main text Figure 5.26).

$q/Mp' + \ln(p'/p'_0) = 0$ (main text Equation 5.37)

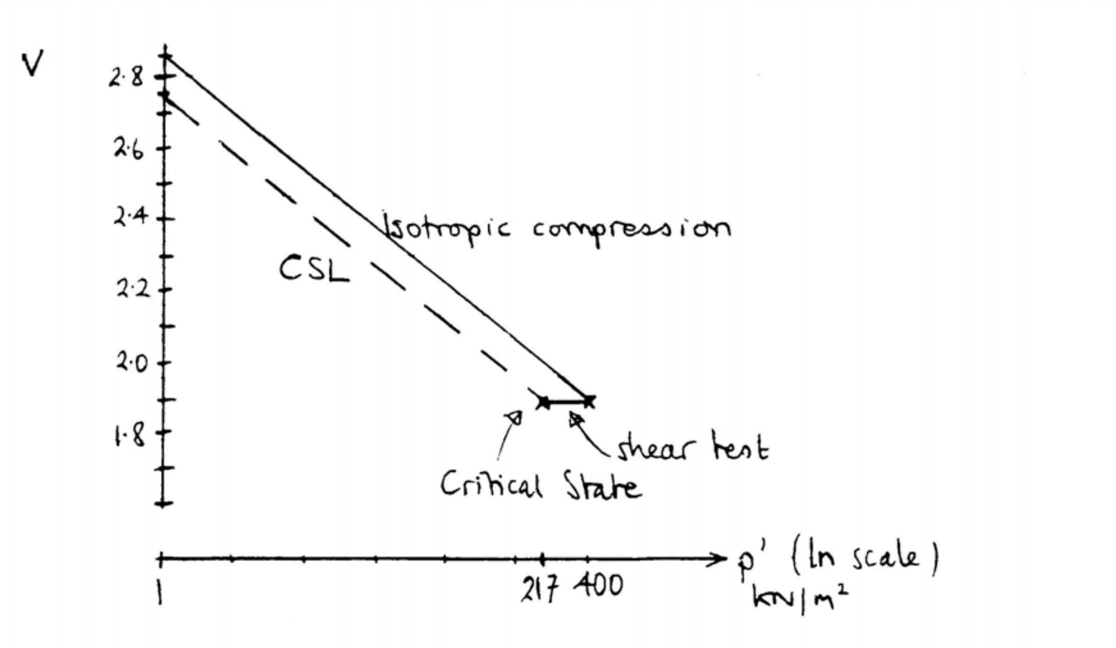
Also,
$v = 1.893 = \Gamma + \lambda - \kappa - \lambda.\ln p'_0 + \kappa.\ln(p'/p'_0)$
(Equation Q5.8a: see main text Example 5.6)

At the start of the shear test, $p'_0 = 400$ kPa. At the end of the test, knowing that $p'_c = 216.8$ kPa and v = 1.892, we can use the given values of Γ, λ and κ together with Equation Q5.8a to calculate that $p'_{0,c} = 589.2$ kPa

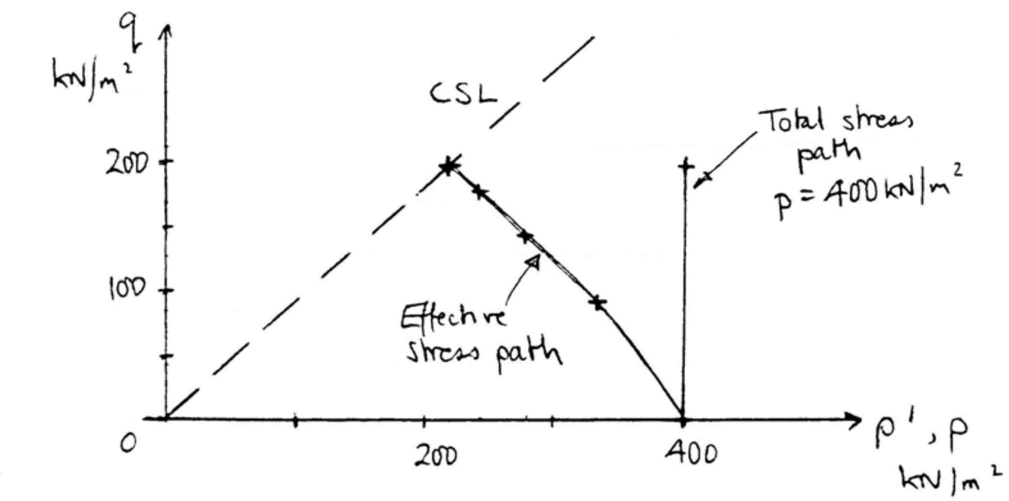
Hence we can choose some values of p'_0 between 400 kPa and 589 kPa and substitute them into Equation Q5.8a to calculate corresponding values of p'. We can then substitute the pairs of values (p'_0, p') into Equation 5.37 to calculate the corresponding value of q, as tabulated below:

p'_0, kPa	450	500	550	Chosen in range 400 to 590 kPa
p' , kPa	333.4	281.8	242.0	Calculated from Eq. Q5.8a
q, kPa	89.0	143.8	176.8	Calculated from Eq. 5.37

The stress paths followed are plotted as v against lnp' and q against p and p' in Figure Q5.8.



(a)



(b)

Figure Q5.8: (a) v against lnp'; (b) q against p and p'

The undrained shear strength τ_u (= $q_c/2$) is unaffected by the total stress path followed, as it depends only on the specific volume of the soil as sheared.

The effective stress path is also the sameas if we had performed a conventional test. The pore water pressures however are lower by the amount corresponding to the difference between the total stress paths, ie the difference between the applied values of average total stress p. This is the amount by which the cell pressure was reduced to maintain p = constant in the non-standard test described in this question, $\Delta q/3 = 64.3$ kPa.

Thus the pore water pressure that would have been observed in a conventional drained shear test carried out at a constant cell pressure of 400 kPa is 183.2 + 64.3 = <u>247.5 kPa.</u>

QUESTIONS AND SOLUTIONS: CHAPTER 6

Note: Questions 6.2 to 6.4 may be answered using either the Newmark chart (main text Figure 6.8), or Fadum's chart (main text Figure 6.14), or both. Question 6.5 is based on case study C6.1, and should therefore be answered with the aid of a Newmark chart. In these solutions, the Newmark chart method is used in all cases.

Determining elastic parameters from laboratory test data

Q6.1 (a) Write down Hooke's Law in incremental form in three dimensions and show that for undrained deformations Poisson's ratio $\nu_u = 0.5$. Assuming that the behaviour of soil can be described in terms of conventional elastic parameters, show that in undrained **plane** compression (i.e. $\Delta\varepsilon_2 = 0$), the undrained Young's modulus E_u is given by 0.75 × the slope of a graph of deviator stress q (defined as σ_1-σ_3) against axial strain ε_1. Show also that the maximum shear strain is equal to twice the axial strain, and that the shear modulus G = 0.25 × (q/ε_1).

(b) Figure 6.20 shows graphs of deviator stress q and pore water pressure u against axial strain ε_1 for an undrained plane compression test carried out at a constant cell pressure of 122 kPa. Comment on these curves and explain the relationship between them. Calculate and contrast the shear and Young's moduli at 1% shear strain and at 10% shear strain. Which would be the more suitable for use in design, and why?
[University of London 2nd year BEng (Civil Engineering) examination, King's College (part question)]

Q6.1 Solution
(a) Writing Hooke's law for an isotropic, elastic material in terms of undrained parameters E_u and ν_u, and changes in principal total stress $\Delta\sigma$ and changes in principal strain $\Delta\varepsilon$,

$$\Delta\varepsilon_1 = (1/E_u).(\Delta\sigma_1 - \nu_u\Delta\sigma_2 - \nu_u\Delta\sigma_3)$$
$$\Delta\varepsilon_2 = (1/E_u).(\Delta\sigma_2 - \nu_u\Delta\sigma_1 - \nu_u\Delta\sigma_3)$$
$$\Delta\varepsilon_3 = (1/E_u).(\Delta\sigma_3 - \nu_u\Delta\sigma_1 - \nu_u\Delta\sigma_2) \qquad \textit{(main text Equation 6.1)}$$

$$\Delta\varepsilon_{vol} = \Delta\varepsilon_1 + \Delta\varepsilon_2 + \Delta\varepsilon_3 = (1/E_u).(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3).(1 - 2.\nu_u)$$

But in an undrained test $\Delta\varepsilon_{vol} = 0$, therefore

$(1 - 2.\nu_u) = 0 \Rightarrow \underline{\nu_u = 0.5}$

In a plane compression test, σ_1 is increased while σ_3 is kept constant, i.e. $\Delta\sigma_3 = 0$. Also, the condition of plane strain $\Rightarrow \Delta\varepsilon_2 = 0$.

Hence $\quad \Delta\varepsilon_2 = (1/E_u).(\Delta\sigma_2 - \Delta\sigma_1/2) = 0$

$\Rightarrow \quad \Delta\sigma_2 = \Delta\sigma_1/2 \qquad$ *(a)*

and $\quad \Delta\varepsilon_1 = (1/E_u).(\Delta\sigma_1 - \Delta\sigma_2/2) \qquad$ *(b)*

Substituting (a) into (b),

$$\Delta\varepsilon_1 = (1/E_u).(\Delta\sigma_1 - \Delta\sigma_1/4) = 3\Delta\sigma_1/4E_u$$

or $$\Delta\sigma_1/\Delta\varepsilon_1 = 4E_u/3$$

With $\Delta\sigma_3 = 0$ *(*σ_3 *= constant),* $\Delta\sigma_1 = \Delta(\sigma_1 - \sigma_3)$ *and* $\Delta\varepsilon_1 = \varepsilon_1$ *so that*

$E_u = 0.75 \times \Delta(\sigma_1 - \sigma_3)/\Delta\varepsilon_1$ *which is 0.75 times the slope of a graph of devator stress* $(\sigma_1 - \sigma_3)$ *against axial strain* ε_1*.*

$\varepsilon_{vol} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$ *and since* $\varepsilon_2 = 0$*,* $\varepsilon_1 + \varepsilon_3 = 0$ *or* $\varepsilon_3 = -\varepsilon_1$

From the Mohr circle of strain,

$\gamma_{max}/2 = \varepsilon_1 \Rightarrow \gamma_{max} = 2\varepsilon_1$

From the Mohr circle of stress,

$\tau_{max} = (\sigma_1 - \sigma_3)/2$

Hence the shear modulus $G = \tau/\gamma = (\sigma_1 - \sigma_3)/4\varepsilon_1 = 0.25.q/\varepsilon_1$ *where q is the deviator stress* $(\sigma_1 - \sigma_3)$

(b) The deviator stress rises with (axial) strain to a peak of about 112 kPa at an axial strain of about 3.4%. The deviator stress then falls quite rapidly to as possibly bsteady value (although the test has not been continued to a high enough strain to be sure of this) of about 87 kPa. It is likely that the sudden fall in deviator stress between 3.4% and 5% is due to the formation of a rupture. The pore water pressure rises with the deviator stress until an axial strain of about 0.8%, at which point the rate of change of pore pressure with deviator stress du/dq falls abruptly: this might indicate yield. The pore water pressure reaches a maximum of about 58 kPa at an axial strain of about 2.5%, and then starts to fall just before the peak deviator stress is reached. Again, it is possiblebut not certain that steady conditions have been reached by the end of the test at an axial strain of 5%.

Using the relationships determined in part (a) and scaling values of deviator stress from the stress-strain curve at the appropriate strains,

i) *at* $\gamma = 1\%$*,* $\varepsilon_1 = \gamma/2 = 0.5\%$ *and (from the graph)* $q = (\sigma_1 - \sigma_3) \sim 67.5$ *kPa; hence using* $G = 0.25.q/\varepsilon_1$

$G_{\gamma=1\%} = 67.5\ kPa \div (4 \times 0.005) = 3375\ kPa$

ii) *at* $\gamma = 10\%$*,* $\varepsilon_1 = \gamma/2 = 5\%$ *and (from the graph)* $q = (\sigma_1 - \sigma_3) \sim 87$ *kPa; hence using* $G = 0.25.q/\varepsilon_1$

$G_{\gamma=1\%} = 87kPa \div (4 \times 0.05) = 435\ \ kPa$

The shear modulus at γ = 10% is a factor of almost ten smaller than at γ = 1%: the tangent shear modulus at γ = 10% may even be negative! The shear modulus at γ = 1% is the more suitable for use in design, as shear strains of 10% would be unacceptably large in practice (in fact, a shear strain of even 1% is quite large for a foundation or a retaining wall).

Calculation of increases in vertical effective stress below a surface surcharge

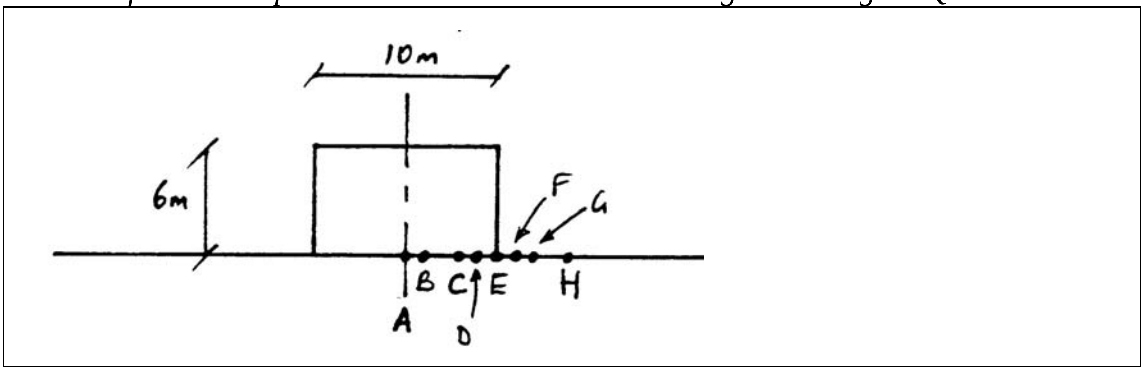
Q6.2 The foundation of a new building may be represented by a raft of plan dimensions 10 m × 6 m, which exerts a uniform vertical stress of 50kPa at founding level. A pipeline AA' runs along the edge of the building at a depth of 2m below founding level, as indicated in plan view in Figure 6.21.

Estimate the increase in vertical stress at a number of points along the pipeline AA', due to the construction of the new building. Present your results as a graph of increase in vertical stress against distance along the pipeline AA', indicating the extent of the foundation on the graph.

What is the main potential shortcoming of your analysis?
[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q6.2 Solution
Set the "scale for Z" = 2 m and draw plan views of the foundation to this scale, with the points where the increase in σ_v is to be calculated located above the centre of the chart.

The locations of points A to H are as indicated in the sketch below, and the Newmark chart with the foundation positions indicated in each case is given in Figure Q6.2a.



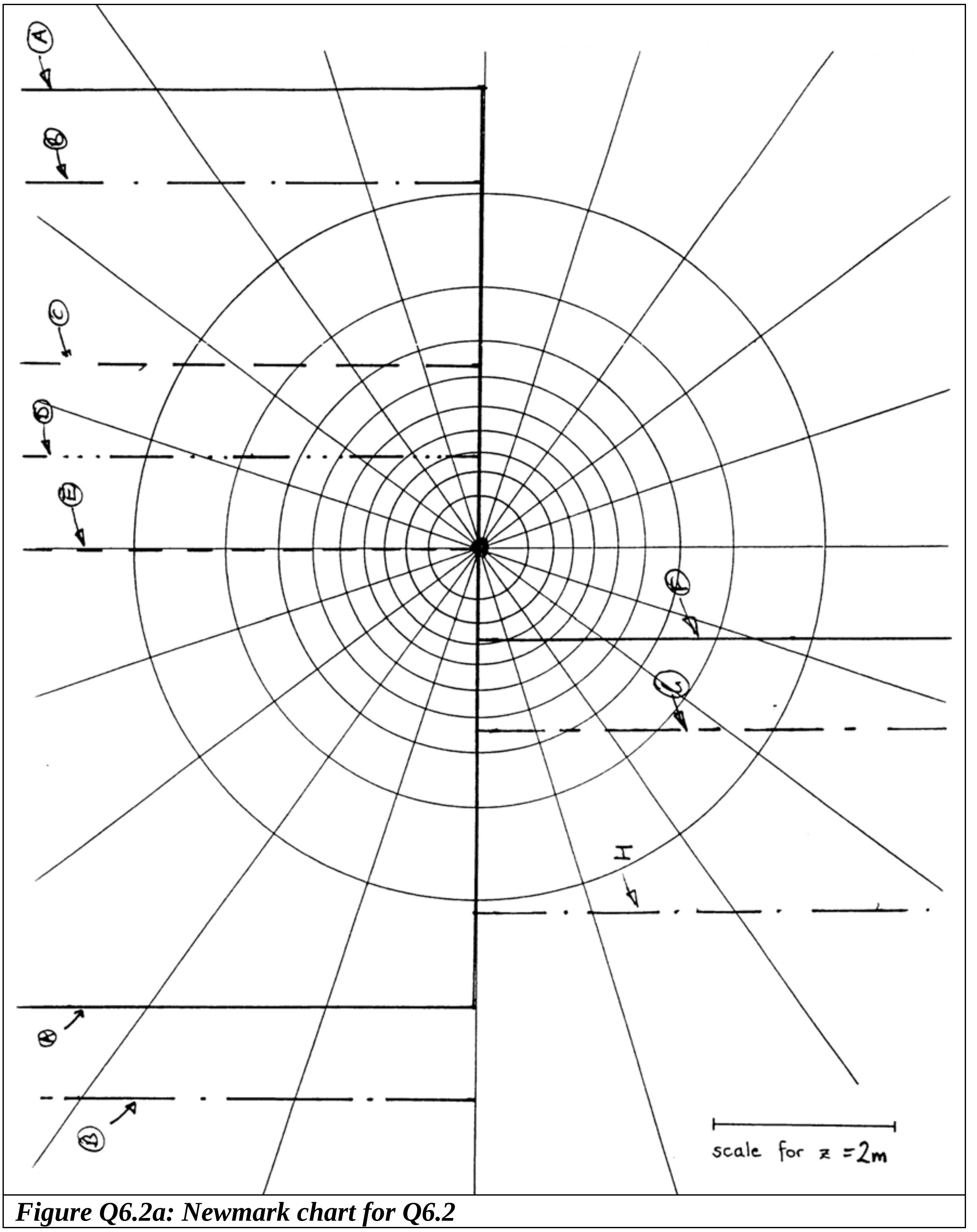


Figure Q6.2a: Newmark chart for Q6.2

The increase in vertical total stress $\Delta\sigma_v$ below each of the points A to H is given by

$\Delta\sigma_v = (n/200) \times 50$ *kPa*

where n is the number of elements on the chart covered by the foundation, as determined from the Newmark chart.

Point	A	B	C	D	E	F	G	H
Distance from centreline, m	0	1	3	4	5	6	7	9

Number of elements covered, n	98	97	91	77.5	50	22.5	9	3
$\Delta\sigma_v$, kPa	24.5	24.3	22.8	19.4	12.5	5.6	2.3	0.8

The tabulated data are used to plot a graph of increase in vertical stress against distance along the pipeline in Figure Q6.2b (note the graph is symmetrical about the centreline; only one half is shown).

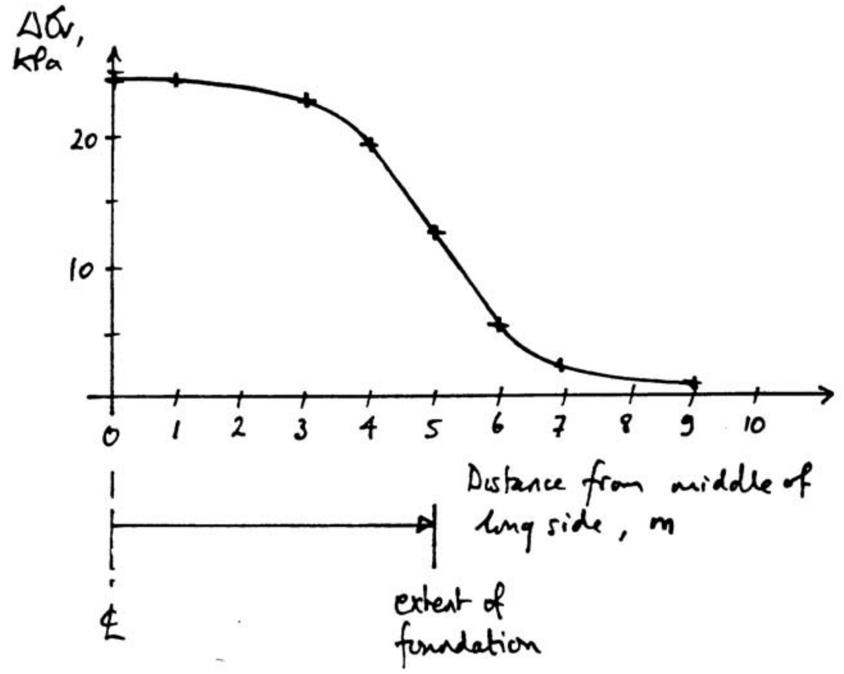


Figure Q6.2b: Increase in vertical stress against distance along the pipeline

The main potential shortcoming of the analysis is that the pipe may act as a stiff inclusion, attracting more load than indicated by the elastic stress distribution on which the Newmark chart is based.

Calculation of increases in vertical effective stress and resulting soil settlements

Q6.3 (a) In what circumstances might an elastic analysis be used to calculate the changes in stress within the body of the soil due to the application of a surface surcharge?

(b) Figure 6.22 shows a cross-section through a long causeway. Using the Newmark chart or otherwise, sketch the long-term settlement profile along a line perpendicular to the causeway. Given time, how might your analysis be refined?
[University of London 2nd year BEng (Civil Engineering) examination, King's College]

Q6.3 Solution
(a) An elastic analysis might reasonably be used for small changes in stress and strain. Also, the soil muct be overconsolidated (ie on an unload/reload line) for its behaviour to be approximately reversible – otherwise, the loading must all be in the same direction (either loading or unloading).

(b) Refer to the Newmark Charts in Figures Q6.3b, c and d: note the extensive use of symmetry to avoid repetitive calculations.

The causeway exerts a surcharge of 5 m $\times$ 20 kN/m^3 = 100 kPa on the original soil surface, which may reasonably be treated as flexible. We know that for a strip footing, the increase in vetical stress below the centreline has fallen to 90% of that at the surface at a depth of about six times the footing width (see main text page section 6.3 and Figure 6.7) – in this case about 30 m.

Divide the soil into three layers, 5 m, 10 m and 20 m thick. Calculate the increase in vertical effective stress $\Delta\sigma'_v$ at the mid-point of each layer, i.e. at depths of 2.5 m, 10 m and 25 m, and take these as the representative increases in vertical stress for each layer. The average one-dimensional stiffness of each layer is that at the centre, i.e. E'_0 = (2000 + 1000z) kPa with z = 2.5 m, 10 m and 25 m giving E'_0 = 4500 kPa, 12000 kPa and 27000 kPa respectively. To obtain a profile of settlement along a line perpendicular to the causeway, we will need to calculate the increases in vertical effective stress at each of these depths at points on the centreline of the causeway (point O on plan), halfway between the middle and the edge of the causeway (point E), the edge of the causeway (point A), and at distances of 2.5 m (point F), 5 m (point B), 10 m (point C) and 20 m (point D) from the edge (Figure Q6.3a).

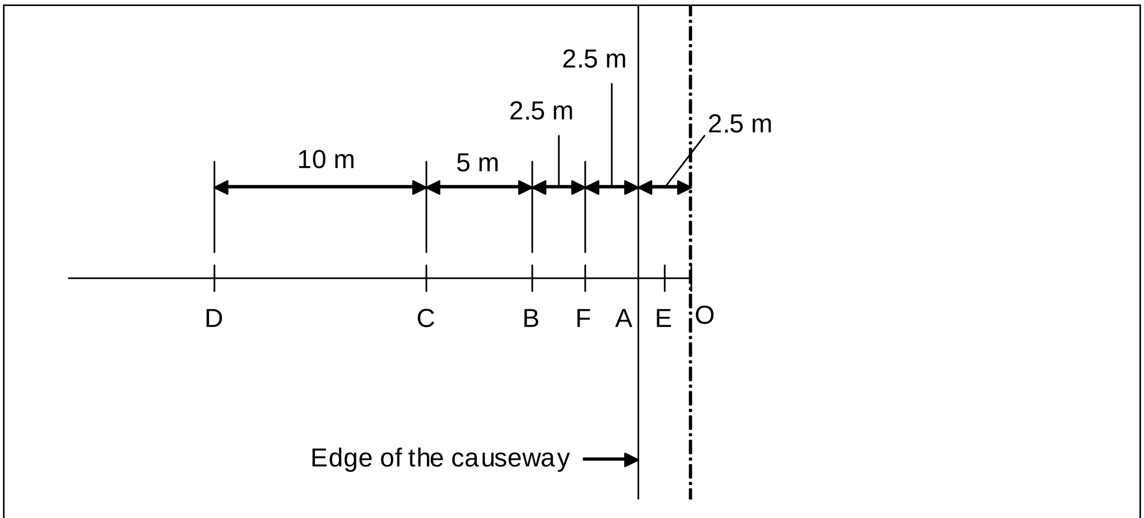


Figure Q6.3a: location of settlement calculation points relative to the centreline of the causeway

In each of the Newmark charts that follow, the causeway is drawn with the "scale for Z" set to (i) 2.5 m (Figure Q6.3b), (ii) 10 m (Figure Q6.3c) and (iii) 25 m (Figure Q6.3c), such that the point at which it is sought to calculate the increase in vertical effective stress (O, A, B etc) is located above the centre of the chart. The increase in stress is then given by $\Delta\sigma_v$ = (n/200) $\times$ 100 kPa, where n is the number of elements covered by the plan view of the causeway for the whole chart. Where symmetry is used and only a half or a quarter of the causeway is drawn, n is the number of elements counted multiplied by two or four respectively.

The compression or settlement ρ of each soil layer of thickness t is calculated from the representative increase in vertical effective stress within the layer, $\rho = \Delta\sigma'_v.t/E'_0$.

(i) Figure Q6.3b, scale for Z set to 2.5 m, E'_0 = 4500 kPa, layer thickness t = 5 m

Point	O	E	A	F	B	C	D
Number of elements covered, n	41 × 4	74 × 2	47½ × 2	9 × 2	2 × 2	~ 0	~ 0
$\Delta\sigma'_v$, kPa = (n/200) × 100 kPa	82	72	47.5	9	2	0	0
settlement $\rho = \Delta\sigma'_v.t/ E'_0$	91.1	82.2	52.8	10	2.2	0	0

(ii) Figure Q6.3c, scale for Z set to 10 m, E'_0 = 12000 kPa, layer thickness t = 10 m

Point	O	E	A	F	B	C	D
Number of elements covered, n	16½ × 4	29½ × 2	28 × 2	20 × 2	13 × 2	4½ × 2	1 × 2
$\Delta\sigma'_v$, kPa = (n/200) × 100 kPa	33	29.5	28	20	13	4.5	1
settlement $\rho = \Delta\sigma'_v.t/ E'_0$	27.5	24.6	23.3	16.7	10.8	3.8	0.8

(iii) Figure Q6.3d, scale for Z set to 25 m, E'_0 = 27000 kPa, layer thickness t = 20 m

Point	O	E	A	F	B	C	D
Number of elements covered, n	6¾ × 4	12½ × 2	12½ × 2	11½ × 2	10½ × 2	8 × 2	3½ × 2
$\Delta\sigma'_v$, kPa = (n/200) × 100 kPa	13.5	12.5	12.5	11.5	10.5	8	3.5
settlement $\rho = \Delta\sigma'_v.t/ E'_0$	10.0	9.3	9.3	8.5	7.8	5.9	2.6

Summing the settlements, we have

Point	**O**	**E**	**A**	**F**	**B**	**C**	**D**
Dist from centreline, m	**0**	**1.25**	**2.5**	**5.0**	**7.5**	**12.5**	**22.5**
Total settlement, mm	**130**	**116**	**85**	**35**	**21**	**10**	**3**

The settlement profile is plotted in Figure Q6.3e

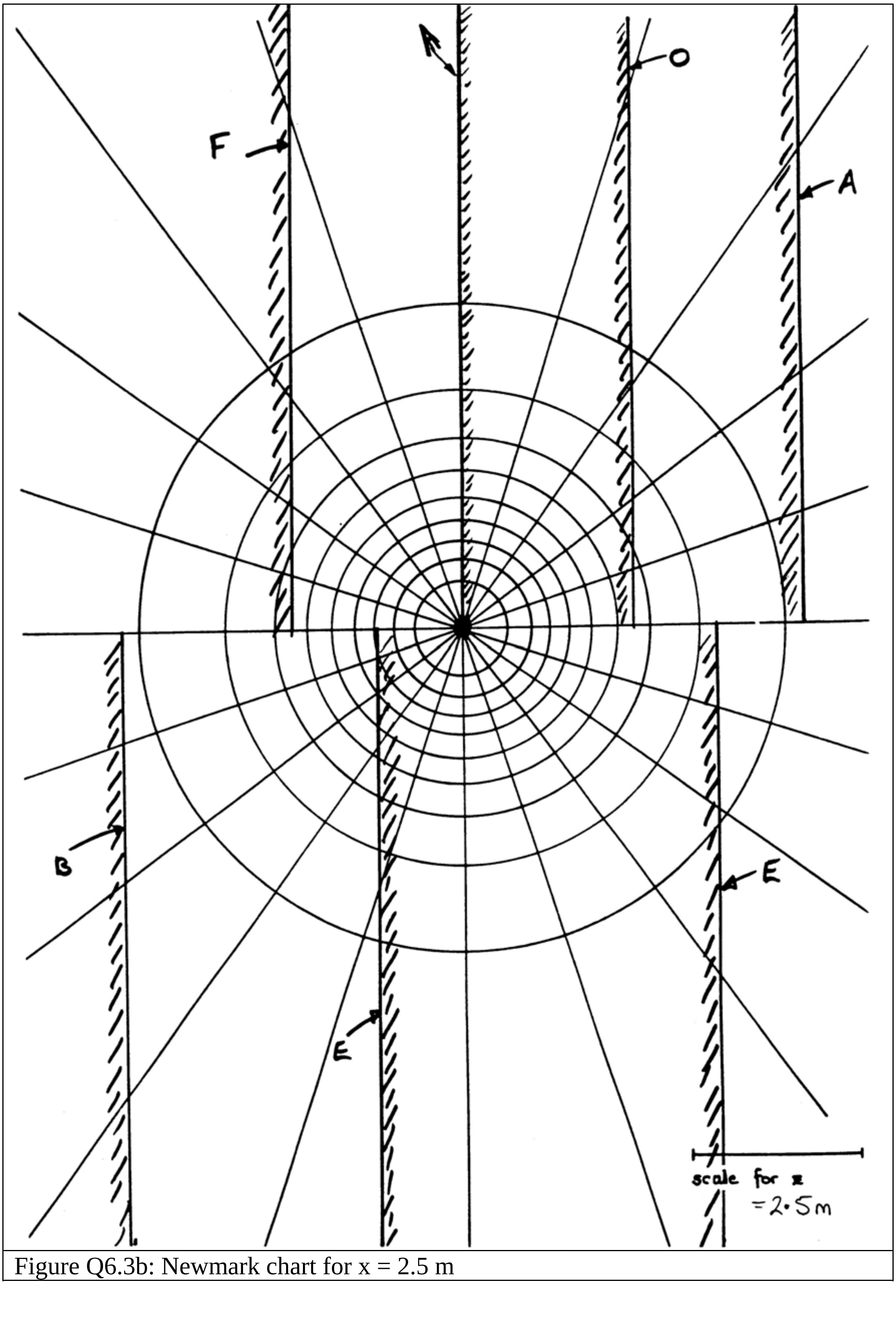


Figure Q6.3b: Newmark chart for x = 2.5 m

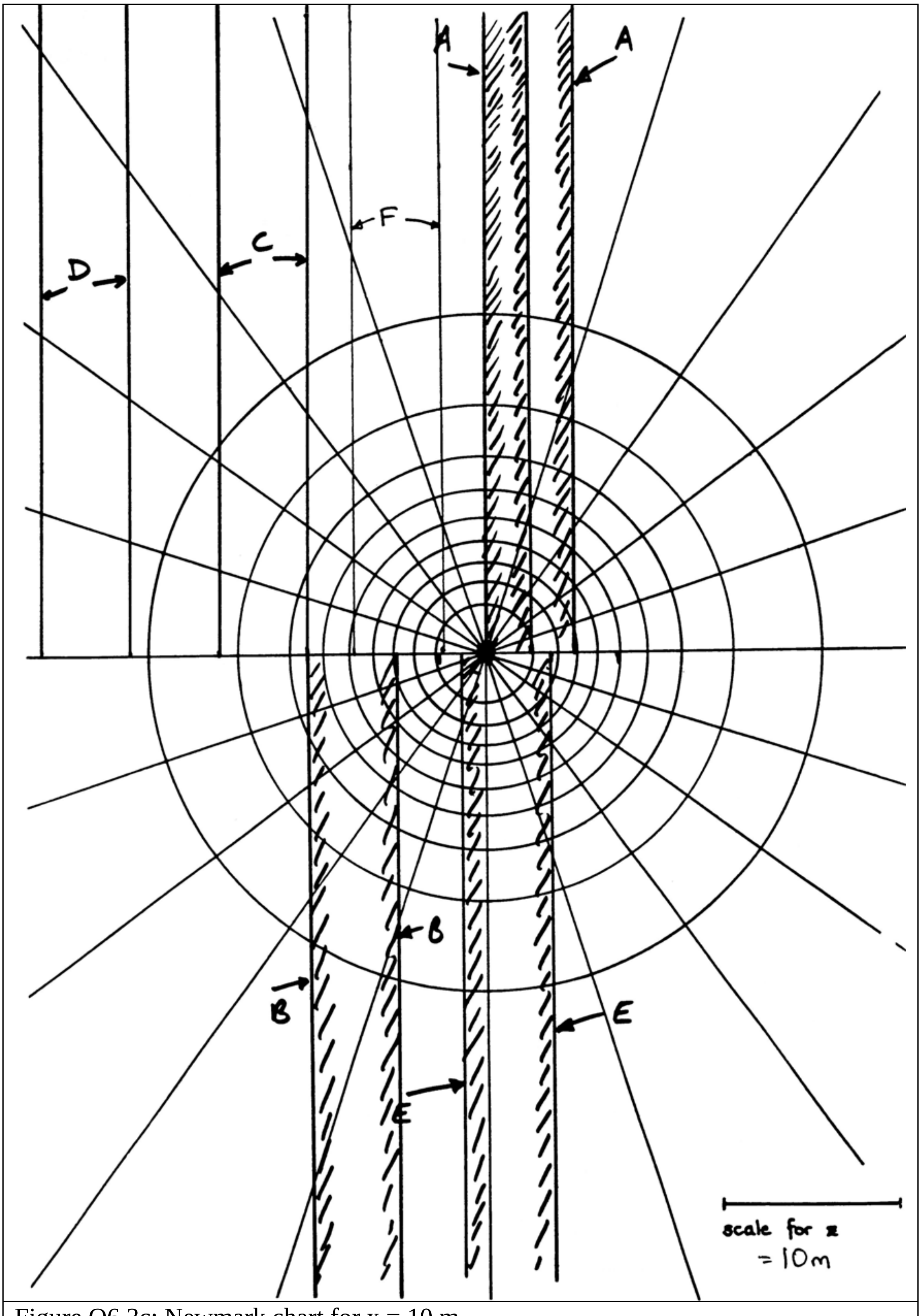


Figure Q6.3c: Newmark chart for x = 10 m

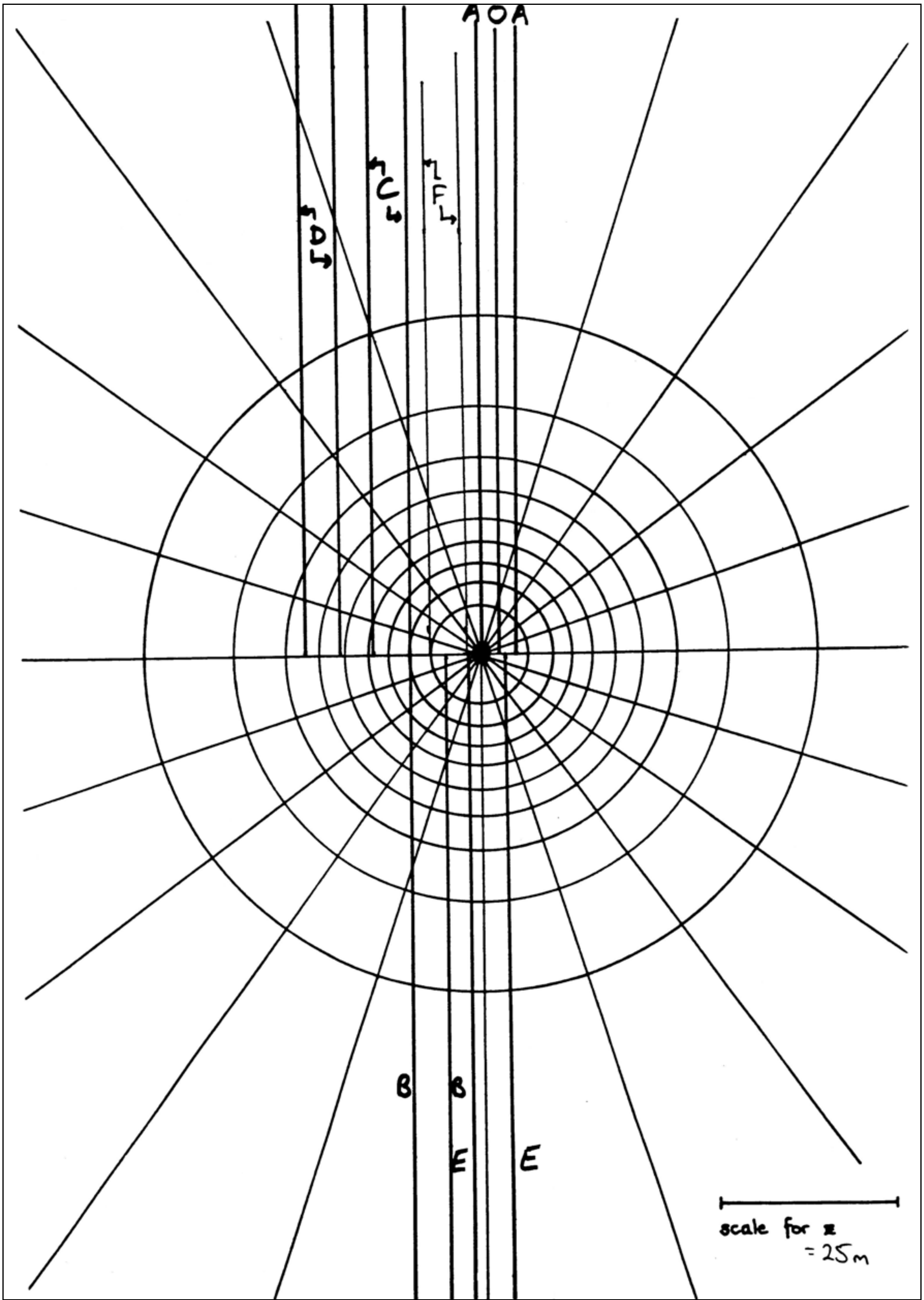


Figure Q6.3d: Newmark chart for x = 25 m

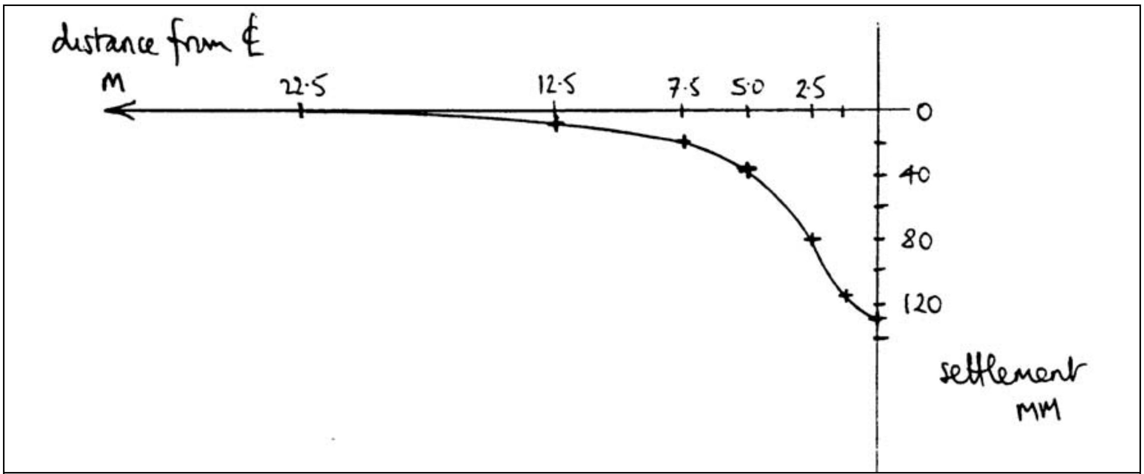


Figure Q6.3e: surface settlement profile

The analysis could be refined by dividing the soil into more layers.

Q6.4 (a) When might an elastic analysis reasonably be used to calculate the settlement of a foundation? Briefly outline the main difficulties encountered in converting stresses into strains and settlements.

(b) A square raft foundation of plan dimensions 5 m × 5 m is to carry a uniformly distributed load of 50 kPa. A site investigation indicates that the soil has a one-dimensional modulus given by E'o = (10 + 6z) MPa, where z is the depth below the ground surface in metres. Use a suitable approximate method to estimate the ultimate settlement of the raft.
[University of London 2nd year BEng (Civil Engineering) examination, King's College]

Q6.4 Solution
(a) An elastic analysis might reasonably be used if the soil is overconsolidated (on an unloading/reloading line) and the changes in stress and strain are small.

The main difficulty in attempting to convert an elastic stress distribution into strains (and settlements) is the choice of an appropriate elastic modulus that takes proper account of the stress paths followed by all the soil elements. The usual approach is to use the one-dimensional modulus on the assumption that deformations are predominantly vertical. A possible problem that then arises is that this approach can only be used to calculate long-term, drained settlements after any excess pore water pressures induced by loading have dissipated, although empirical adjustments are available to estimate the short term settlements due to shearing of the soil at constant volume.

(b) The soil nearer the surface will have more influence on settlements, as the stresses are greater and the modulus is less. The solution procedure is as follows.

1. *Divide the soil into three layers*
2. *Use the Newmark chart to calculate the increase in vertical effective stress at the middle of each layer (assuming that the surcharge can be idealised as flexible)*
3. *Use the value of E'_0 at the centre of the layer to calculate the compression of the layer (assuming deformation is primarily due to one dimensional compression).*

Recalling that the increase in vertical effective stress below the centreline falls to approximately 10% of its value at the surface at a depth of twice the footing diameter, choose layer thicknesses of 4 m (0 to 4 m below ground level – centre of layer at 2 m below ground level); 4 m (4 m to 8 m below ground level – centre of layer at 6 m below ground level); and 8 m (8 m to 16 m below ground level – centre of layer at 12 m below ground level).

Figure Q6.4 (the Newmark chart) shows plan views of the foundation with the "scale for Z" set to (i) 2 m, (ii) 6 m and (iii) 12 m, located so that the centre (solid line, one quarter of the foundation shown so that the actual number of elements covered has to be multiplied by four), a corner (chain dotted line, full foundation shown), and the middle of one side (dashed line, one half of the foundation shown so that the actual number of elements covered has to be multiplied by two) lie above the centre of the chart. The increase in stress in each case is calculated as $\Delta\sigma'_v = (n/200) \times 50$ kPa where n is the number of elements covered by the whole foundation. The compression of each layer is calculated as $\rho = t.\ \Delta\sigma'_v / E'_0$, where t is the thickness of the layer and $E'_0 = (10 + 6z)$ MPa giving $E'_0 = 22$ MPa at z = 2 m, 46 MPa at z = 6 m, and 82 MPa at z = 12 m.

The numbers of chart elements covered and the increases in vertical effective stress at each depth, and the layer and total settlements, are given in the Table below.

Scale for Z, m	*Location on foundation*	*Number of chart elements covered, n*	*Increase in vertical effective stress, $\Delta\sigma'_v = (n/200) \times 50$ kPa*	*Layer compression $\rho = t.\ \Delta\sigma'_v/E'_0$, mm*
2	*Centre*	*158*	*39.5*	*7.2*
6	*Centre*	*50*	*12.5*	*1.1*
12	*Centre*	*16*	*4*	*0.4* ***TOTAL 8.7***
2	*Corner*	*48*	*12*	*2.2*
6	*Corner*	*30*	*7.5*	*0.7*
12	*Corner*	*12*	*3*	*0.3* ***TOTAL 3.2***
2	*Mid-side*	*86*	*21.5*	*3.9*
6	*Mid-side*	*38*	*9.5*	*0.8*
12	*Mid-side*	*14*	*3.5*	*0.3* ***TOTAL 5.0***

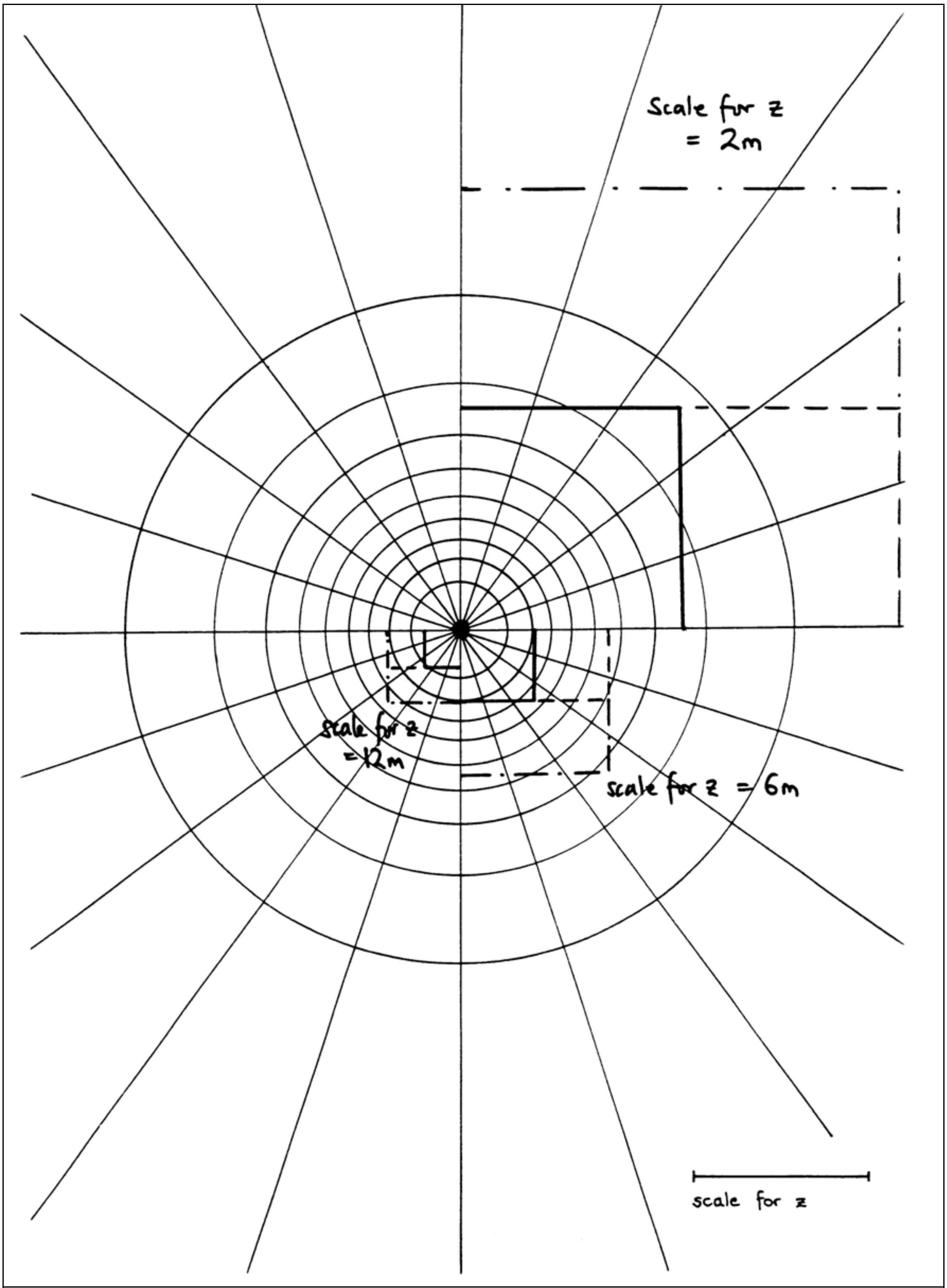


Figure Q6.4. Newmark chart for Q6.4

The total settlements below the centre, corners and mid-sides of 8.7 mm, 3.2 mm and 5.0 mm respectively would be for a prefectly flexible footing. For a rigid footing where the loading on the fooring was 50 kPa, we might estimate the average settlement as

$\rho_{rigid} \approx [(4 \times 5.0\ mm) + (4 \times 3.2\ mm) + (1 \times 8.7\ mm)] \div 9 = \underline{4.6\ mm}$

Q6.5 The foundations of a new building may be represented by a raft of plan dimensions 24 m × 32 m, which exerts a uniform vertical stress of 53.5kPa at founding level. The soil at the site comprises laminated silty clay underlain by firm rock. The estimated stiffness in one-dimensional compression E'_0 increases with depth as indicated below.

Depth below founding level, m	E'_0, MPa
0 to 4	5
4 to 10	10
10 to 20	25
below 20	very stiff

Use the Newmark chart (Figure 6.8) to estimate the increase in vertical stress at depths of 4 m, 10 m and 20 m below the centre of the raft. Hence estimate the expected eventual settlement of the centre of the foundation.

Suggest two possible shortcomings of your analysis.
[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q6.5 Solution
To calculate the increases in vertical effective stress at depths of 4 m, 10 m and 20 m, set the "scale for Z" on the Newmark chart to each of these values in turn. Draw plan views of the foundation, to these scales, with the centre of the foundation above the centre of the chart (see Figure Q6.5, and note that with Z = 4 m the foundation covers the entire chart). Using symmetry, it is necessary only to draw one quarter of the foundation in each case. The increase in stress below the centre is given by $\Delta\sigma'_v = (n/200) \times 53.5$ kPa where n is the number of elements covered by the whole foundation. The compression of each layer is $\rho = t.\ \Delta\sigma'_v / E'_0$, where t is the thickness of the layer and E'_0 is the one dimensional modulus. Hence

Depth, m	*4*	*10*	*20*
Number of elements covered, n	*50 × 4*	*40¾ × 4*	*24 × 4*
Increase in vertical effective stress $\Delta\sigma'_v = (n/200) \times 53.5$ kPa	*53.5*	*43.6*	*25.7*

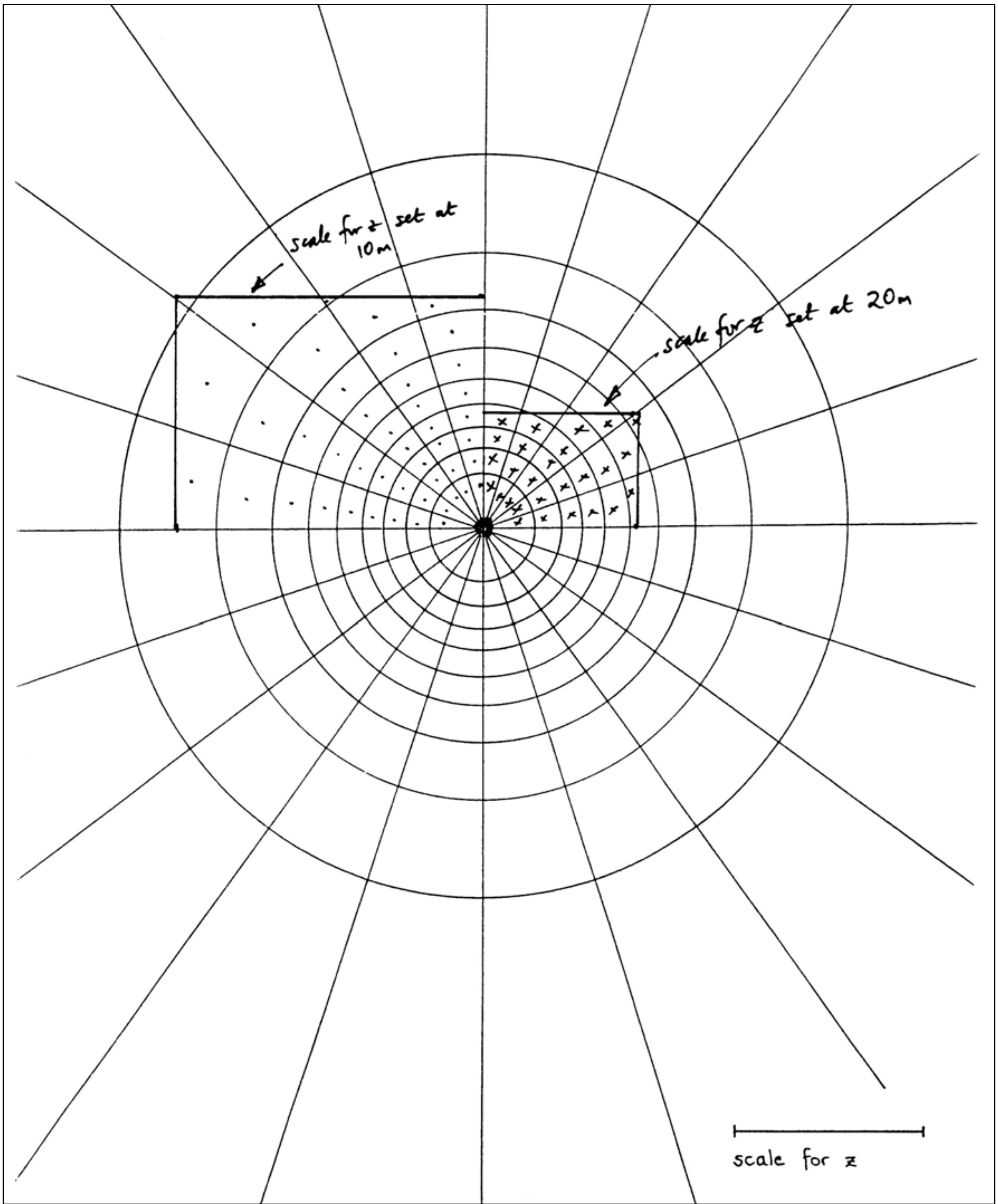


Figure Q6.5. Newmark chart

Layer depth, m below ground level	*Average increase in vertical effective stress $\Delta\sigma'_{v,average}$, kPa*	*Layer thickness t, m*	*E'_0, MPa*	*Layer compression ρ, mm*
0 – 4	*53.5*	*4*	*5*	*42.8*
4 – 10	*48.6*	*6*	*10*	*29.2*
10 – 20	*34.7*	*10*	*25*	*13.9*
below 20	*not calculated*	-	*∞*	*0*
TOTAL				***86***

The total settlement is therefore *86 mm*

The main shortcomings of the analysis are

1. *The raft foundation is stiff, so the loading transmitted to the soil will not in fact be uniform*
2. *The division of the soil into only three layers is quite crude and could be refined*
3. *It has been assumed that deformation is essentially by one dimensional compression, i.e. shear deformation at constant volume has been neglected*
4. *The use of the elastic soil model may be unrealistic*

(two only required).

Use of standard formulae in conjunction with one-dimensional consolidation theory (Chapter 4)

Q6.6 (a) To estimate the ultimate settlement of the grain silo described in Q4.5, the engineer decides to assume that the soil behaviour is elastic, with the same properties in loading and unloading. In what circumstances might this be justified?

(b) The proposed silo will be founded on a rigid circular foundation of diameter 10 m. Under normal conditions, the net or additional load imposed on the soil by the foundation, the silo and its contents will be 5 000 kN. What is the ultimate settlement due to this load? (It may be assumed that the settlement ρ of a rigid circular footing of diameter B carrying a vertical load Q at the surface of an elastic half space of one-dimensional modulus E'_0 and Poisson's ratio ν' is given by $\rho = (Q/E'_0B).[(1-\nu')^2/(1-2\nu')]$. Take $\nu' = 0.2$)

(c) To reduce the time taken for the settlement to reach its ultimate value, it is proposed to overload the foundation initially by 5000 kN, the additional load being removed when the settlement has reached 90% of the predicted ultimate value. In practice, this occurs after six months has elapsed, and the additional load is then removed. Giving two or three actual values, sketch a graph showing the settlement of the silo as a function of time. (Assume that the principle of superposition can be applied, and use the curve of R against T given in Figure 4.18.)

State briefly the main shortcomings of your analysis.
[University of London 2nd year BEng (Civil Engineering) examination, King's College (part question)]

Q6.6 Solution
(a) The assumption that the soil is elastic with the same properties in loading and unloading is justified if the soil is overconsolidated, i.e. it is on an "elastic" unload/reload line, and that the changes in stress and strain are small.

(b) The ultimate settlement when all excess pore water pressures have dissipated is given by

$$\rho = (Q/E'_0B).[(1-\nu')^2/(1-2\nu')]$$

(Note: this Equation can be recovered by writing E' and ν' in place of E and ν in main text Equation 6.14 (page 352), and substituting main text Equation 6.10 (page 344), $E' = E'_0.[(1 + \nu').(1 - 2\nu')]/(1 - \nu')$*).*

Substituting the values given of Q = 5 000 kN, B = 10 m, E'_0 *=10 000 kPa and* ν' *= 0.2,*

ρ *(m) = [5 000 kN ÷ (10 000 kPa × 10 m)) × [(0.8)*^{2} *÷ 0.6] ⇒* $\underline{\rho = 53 \text{ mm}}$

(c) Assume that the settlement vs time relationship can be based approximately on the one-dimensional comsolidation of a clay layer of thickness d with one-way drainage, in response to an increase in vertical stress of $Q/(\pi B^2/4)$*.*

The ultimate settlement with a load of 10 000 kN is 106 mm. The additional 5 000 kN load is removed when ρ *= 0.9 × 53 mm, or at* $R = \rho/\rho_{ult}$ *for the load of 10 000 kN = 0.45. From the curve of R against T given in main text Figure 4.18,* $T = (c_v.t/d^2) \approx 0.15$ *when R = 0.45.*

We can use the fact that T = 0.15 after six months to calculate the effective drainage path length (assuming one-way drainage to the surface), d:

$T = (c_v.t/d^2) = 0.15$ *when t = 6 months = 259 200 minutes, and* c_v *= 2.12 mm*^{2}*/minute from Q4.5. Thus*

$d^2 = c_v.t/0.15 = 2.12 \text{ mm}^2/\text{min} \times 259\,200 \text{ min} \div 0.15 \Rightarrow d^2 = 3.66 \times 10^6 \text{ mm}^2$*, or*

<u>d ~ 1.91 m</u>

[note that within this depth, which corresponds to about 0.2 times the silo diameter of 10 m, the increase in vertical stress is approximately constant at below most of the foundation: see main text Figure 6.7]

The analysis of the consolidation process given in main text Section 4.5 shows that the settlement ρ *is proportional to* $\sqrt{t}$ *until* $t = d^2/12c_v$ *which is in this case 100 days (t = [3.66 ×* 10^6 *mm*^{2}*] ÷ [12 × 2.12 mm*^{2}*/min] = 144 000 min = 100 days).*

After 6 months, when the overload of 5 000 kN is removed, superimpose the solutions for (A) loading of 10 000 kN at t = 0, and (B) unloading of 5 000 kN at t = 6 months, using values of T and R taken from main text Equations 4.22 and 4.23 or main text Figure 4.18.

After 12 months	$T_A = 0.30$ $T_B = 0.15$	$R_A = 0.65$ $R_B = 0.45$	ρ = (0.65 × 106 mm) – (0.45 × 53 mm)	ρ = 45 mm
After 2 years	$T_A = 0.60$ $T_B = 0.45$	$R_A = 0.86$ $R_B = 0.77$	ρ = (0.86 × 106 mm) – (0.77 × 53 mm)	ρ = 50 mm
After 3 years	$T_A = 0.90$ $T_B = 0.75$	$R_A = 0.94$ $R_B = 0.91$	ρ = (0.94 × 106 mm) – (0.91 × 53 mm)	ρ = 51 mm

A graph of settlement against time is plotted in Figure Q6.6.

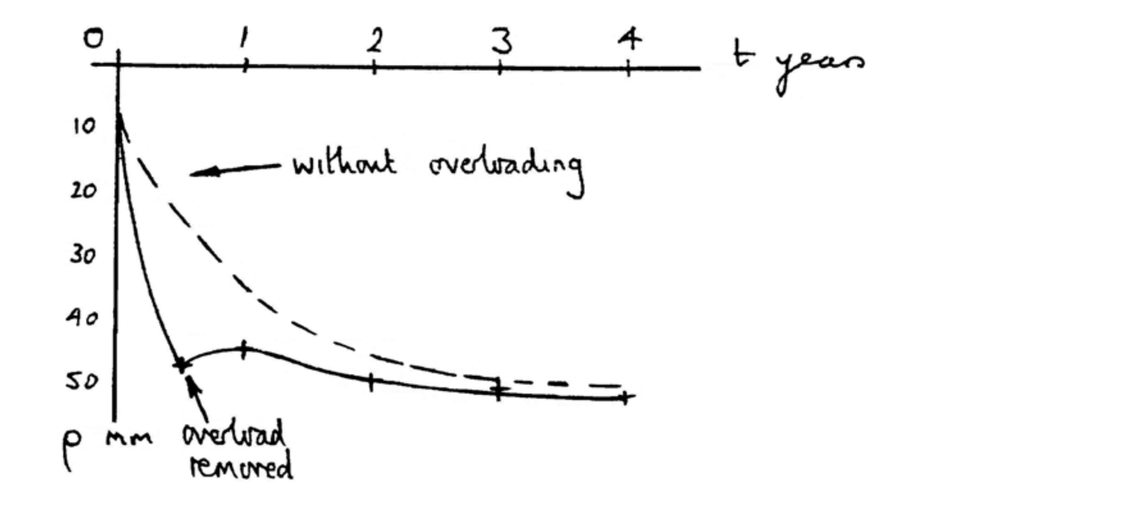


Figure Q6.6: settlement against time for grain silo

The main shortcomings of the analysis are

1. *We have assumed reversible elastic behaviour to enable us to use the principle of superposition*
2. *We have not taken into account the complex stress distributions (variation both vertically and horizontally) in the field – we have assumed a one domensional vertical stress state*
3. *Field drainage paths may well be horizontal (owing to greater horizontal than vertical soil permeability) rather than vertical, as assumed in the analysis*
4. *Soil anisotropy and inhomogeneity, and large scale structural features probably not apparent in the oedometer test sample, have been ignored*
5. *We have assumed that the parameters derived from an oedometer test covering a stress increment of 100 to 200 kPa are relevant to the actual stress changes in the order of (at the surface) 0 to 127 to 64 kPa.*

QUESTIONS AND SOLUTIONS: CHAPTER 7

Calculation of lateral earth pressures and prop loads

Q7.1 (a) Explain the terms "active" and "passive" in the context of a soil retaining wall.

(b) Figure 7.47 shows a cross section through a trench support system, which is formed of a rigid reinforced concrete U-section. Assuming that the retained soil is in the active state, and that the interface friction between the soil and the wall is zero, calculate and sketch the short-term distributions of horizontal total and effective stress and pore water pressure acting on the vertical member AB.

(c) Hence calculate the axial load (in kN per metre length of the trench) in the horizontal member BC, and the bending moment (in kNm/m) at B.

(d) Would you expect the axial load in BC and the bending moment at B to increase or decrease in the long term, and why?

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q7.1 Solution
(a) "Active": soil is on the verge of failure with lateral support being removed. Lateral stress is as small as possible for a given vertical stress, e.g. behind a retaining wall. "Passive": soil is on the verge of failure with lateral stress being increased. Lateral stress is as large as possible for a given vertical stress, e.g. in front of an embedded retaining wall.

(b) Assuming fully active conditions, i.e. the minimum possible lateral stress,

In the sandy gravel (in terms of effective stresses), $\sigma'_h = K_a.\sigma'_v$,
where $K_a = (1 - \sin\phi')/(1 + \sin\phi')$ *and with* $\phi' = 35°$, $Ka = 0.271$

In the clay (undrained in terms of total stresses), $\sigma_h = \sigma_v - 2.\tau_u$; $\tau_u = 30$ *kPa*

Hence

Depth, m	*Stratum*	σ_v*, kPa*	*u, kPa*	σ'_v*, kPa*	σ'_h*, kPa*	σ_h*, kPa*
0	*sandy gravel*	*20*	*0*	*20*	*5.4*	*5.4*
4	*sandy gravel*	*100*	*40*	*60*	*16.3*	*56.3*
4	*clay*	*100*	*(40)*	*-*	*-*	*40*
6	*clay*	*134*	*(60)*	*-*	*-*	*74*

The vertical total stress σ_v *at each depth is calculated from the surcharge (20 kPa) plus the weight of the overlying soil. The vertical effective stress* $\sigma'_v = \sigma_v - u$*. Pore pressures in brackets are those that would act in a flooded tension crack at the interface between the clay and the wall when filled with water to the level of the ground surface: provided the minimum (active) total lateral stress within the clay is greater than or equal to this value (as is the case here), a tension crack will not form.*

The resulting stress distribution is shown in Figure Q7.1.

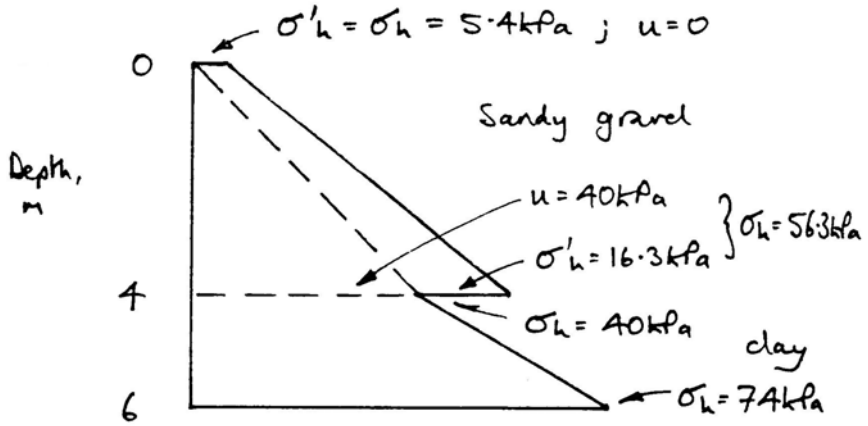


Figure Q7.1: Lateral stresses acting on retaining wall member AB, Q7.1

(c) The condition of horizontal equilibrium is used to calculate the axial load in BC, F_{BC}. Considering the total stresses,

$F_{BC} = [½ \times (5.4 + 56.3) \times 4] + [½ \times (40 + 74) \times 2]$

$\Rightarrow$ $\underline{F_{BC} = 237.4 \text{ kN/m}}$

Taking moments about B, the bending moment at B, M_B, is

$M_B = [5.4 \times 4 \times 4] + [½ \times 50.9 \times 4 \times 3.33] + [40 \times 2 \times 1] + [½ \times 34 \times 2 \times {}^2/_3]$

$\Rightarrow$ $\underline{M_B = 528 \text{ kNm/m}}$

(d) The structural loads would be expected to increase in the long term. This is because the clay has been unloaded laterally and therefore probably has negative pore pressures within it. As the negative pore pressures dissipate, the total load from the clay will increase. To calculate the long term loads, an effective stress analysis should be used.

Stress field limit equilibrium analysis of an embedded retaining wall

Q7.2 (a) Figure 7.48 shows a cross section through a smooth embedded retaining wall, propped at the crest. Show that the wall would be on the verge of failure if the strength (angle of friction) of the soil were 18°. (Take the unit weight of water γ_W=10kN/m^3.)

(b) Sketch the distributions of lateral stress on both sides of the wall, and calculate the bending moment at formation level and the prop force.

(c) If in fact the critical state strength of the soil is 24°, calculate the mobilization factor M = $\tan\phi'_{crit}/\tan\phi'_{mob}$.

[University of London 3rd year BEng (Civil Engineering) examination, Queen Mary and Westfield College (part question)]

Q7.2 Solution
(a, b) Assuming fully active conditions, i.e. the minimum possible lateral stress, with ϕ' = 18° and soil/wall friction δ = 0,

Behind the wall, conditions are active with $\sigma'_h = K_a.\sigma'_v$,
where $K_a = (1 - \sin\phi')/(1 + \sin\phi')$ and with ϕ' = 18°, K_a = 0.528

In front of the wall, conditions are passive with $\sigma'_h = K_p.\sigma'_v$,
where $K_p = (1 + \sin\phi')/(1 - \sin\phi') = 1/K_a$ = 1.894

Hence the lateral stresses acting on the wall at key depths, between which the lateral stress varies linearly, are

Behind wall

Depth, m	σ_v, *kPa*	*u, kPa*	σ'_v, *kPa*	σ'_h, *kPa*
0 (ground surface)	*0*	*0*	*0*	*0*
10 (water table)	*200*	*0*	*200*	*105.6*
25.2 (toe of wall)	*504*	*152*	*352*	*185.86*

In front of wall

Depth, m	σ_v, *kPa*	*u, kPa*	σ'_v, *kPa*	σ'_h, *kPa*
0 (ground surface)	*0*	*0*	*0*	*0*
15.2 (toe of wall)	*304*	*152*	*152*	*287.89*

The vertical total stress σ_v at each depth is calculated from the weight of the overlying soil (there is no surcharge in this case). The vertical effective stress $\sigma'_v = \sigma_v - u$.

The resulting stress distribution is shown in Figure Q7.2.

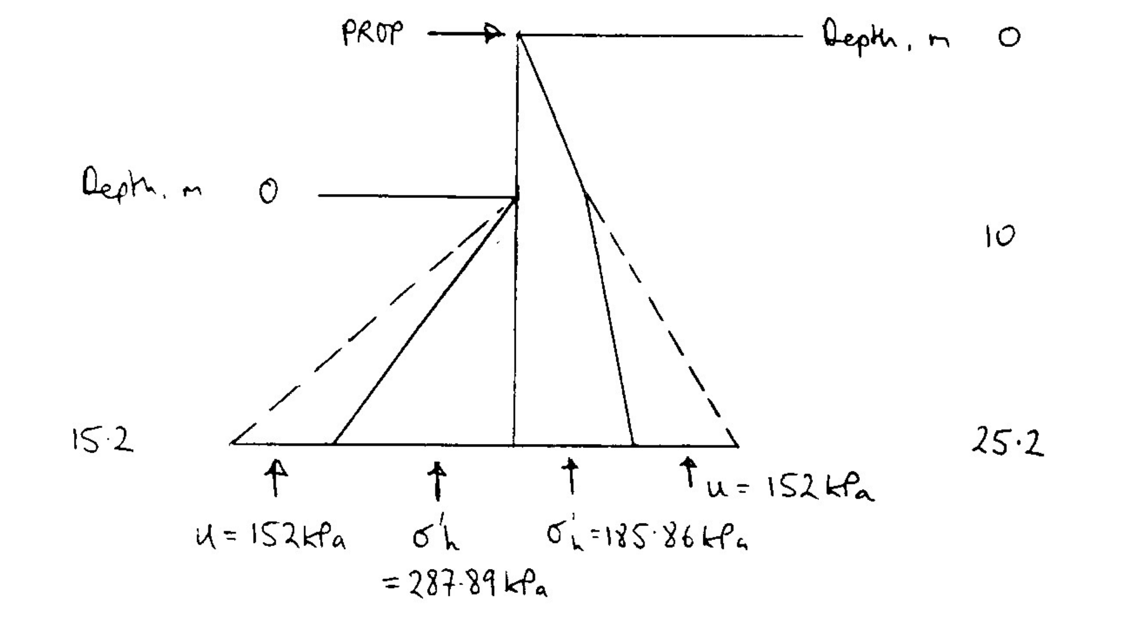


Figure Q7.2: Lateral stresses acting on retaining wall, Q7.2

Note that the pore water pressures are hydrostatic below the same groundwater level on both sides of the wall, and therefore cancel out. This is NOT a general result: this is a special case because the water table is at the level of the excavated soil surface on both sides of the wall.

Check that the stress distribution shown in Figure Q7.2 is in moment equilibrium about the prop (horizontal equilibrium will be satisfied by the prop load):

Moments clockwise, given in the format [average lateral stress × depth of stress block × lever arm about the prop] are

$[½ \times 105.6 \times 10 \times 6.67] + [105.6 \times 15.2 \times 17.6] + [½ \times 80.26 \times 15.2 \times 20.13]$

$\underline{= 44052.7\ kNm/m}$

Moments anticlockwise are

$[½ \times 287.89 \times 15.2 \times 20.13]$ $\underline{= 44043.7\ kNm/m}$

The error is 9/44000 = 0.02%, which is negligible so the condition of equilibrium is satisfied.

The prop load P is calculated from the condition of horizontal force equilibrium,

$P = [½ \times (185.86 + 105.6) \times 15.2] + [½ \times 105.6 \times 10] - [½ \times 287.89 \times 10]$

$\Rightarrow \underline{P = 555\ kN/m}$

(The pore pressures have been ignored because they are exactly the same on both sides of the wall. As already stated, this is NOT a general result and it will normally be necessary to take the pore water pressures, which will usually be different on each side of the wall, into account in the equilibrium calculation).

The bending moment at formation level is given by

M = [10 × 555.132 kN/m] – [½× 105.6 kPa × 10 m × 10 m/3] = <u>3791 kNm/m</u>

(c) The strength mobilization factor or factor of safety on soil strength is given by

<u>M = tan24° ÷ tan18° = 1.37</u>

Q7.3 (a) Figure 7.49 shows a cross section through a rough embedded retaining wall, propped at the crest. Stating any assumptions you make, estimate the long term pore water pressure distribution around the wall.

(b) Assuming that the critical state angle of soil friction of 35° is fully mobilized in the *retained* soil, calculate the earth pressure coefficient (based on effective stresses) in the soil in front of the wall required for moment equilibrium about the prop. Using Table 7.7, estimate the corresponding mobilized friction angle in the soil in front of the wall.

(c) Is the wall safe? Explain briefly your reasoning.

(Take the unit weight of water γ_W=10kN/m^3).

[University of London 3rd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q7.3 Solution
(a) Calculate the long-term pore water pressures assuming steady state seepage using the linear seepage approximation (see main text section 7.8.1, p416). The fall in head from h = 0 at the excavated soil surface to h = 4.5 m at the groundwater level on the retained side of the wall is assumed to be linear around the wall, giving a head at the toe of

$h_{toe} = (3 \text{ m} \div 10.5 \text{ m}) \times 4.5 \text{ m} = 1.286 \text{ m}$

and a pore water pressure of

$u_{toe} = \gamma_w \times (3 \text{ m} + h_{toe}) = 42.86 \text{ kPa}$

(Figure Q7.3a)

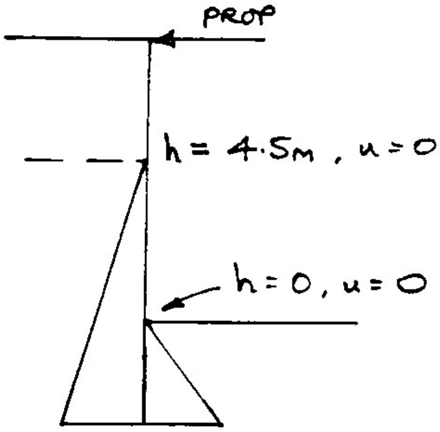


Figure Q7.3a: Pore water pressures according to the linear seepage approximation

(b) Investigate the equilibrium of the wall, assuming that the full soi strength of 35° is mobilized in the retained soil. The wall is rough, the angle of soil/wall friction $\delta = \phi' = 35°$, which gives an active earth pressure coefficient K_a of 0.2117 according to Table 7.6.

Let the mobilized earth pressure coefficient in front of the wall be K_p. The vertical and horizontal total and effective stresses and pore pressures at key depths behind and in front of the wall are calculated in the table below.

Behind wall

Depth, m	σ_v, kPa	u, kPa	σ'_v, kPa ($=\sigma_v - u$)	σ'_h, kPa ($= K_a \times \sigma'_v$)	σ_h, kPa ($=\sigma'_h + u$)
0 (ground surface)	0	0	0	0	0
3.5 (water table)	70	0	70	14.82	14.82
11 (toe of wall)	220	42.86	177.14	37.51	80.37

In front of wall

Depth, m	σ_v, kPa	u, kPa	σ'_v, kPa ($=\sigma_v - u$)	σ'_h, kPa ($= K_p \times \sigma'_v$)	σ_h, kPa ($=\sigma'_h + u$)
0 (ground surface)	0	0	0	0	0
3 (toe of wall)	60	42.86	17.14	$17.14 \times K_p$	$17.14K_p + 42.86$

The stress distribution is shown schematically in Figure Q7.3b.

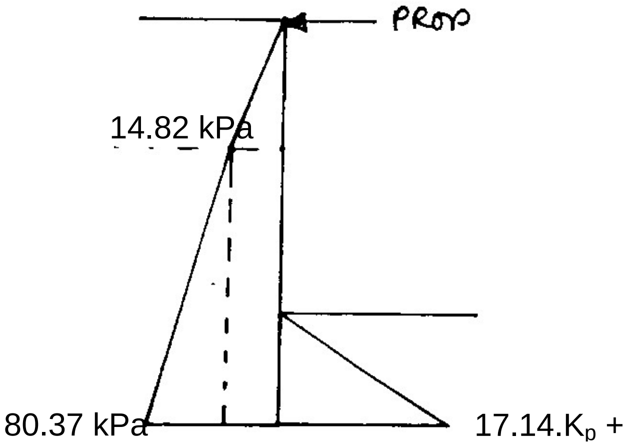


Figure Q7.3b: Total lateral stresses

Take moments about the position of the prop to calculate the value of K_p needed for equilibrium:

The overturning moment is
$M_{OT} = [½ \times 14.82 \times 3.5 \times 3.5 \times {}^2/_3] + [14.82 \times 7.5 \times (3.5 + \{7.5 \times ½\})] + [½ \times (80.37 - 14.82) \times 7.5 \times (3.5 + \{7.5 \times {}^2/_3\})] = 2955.76$ *kNm/m*

The restoring moment is
$M_{RE} = [½ \times (17.14K_p + 42.86) \times 3 \times (8 + \{3 \times {}^2/_3\})] = (257.1K_p + 642.9)$ *kNm/m*

Thus $257.1K_p = 2312.9$ *or* $\underline{K_p \sim 9}$

By interpolation from main text Table 7.7 (p 424), this requires a mobilized effective angle of friction (assuming full wall friction, $\delta = \phi'$) of just over 35.5°.

(c) The wall will probably collapse, because the required mobilized strength in front of the wall is greater than the critical state strength of the soil.

Q7.4 Figure 7.50 shows a cross-section through a long excavation whose sides are supported by propped cantilever retaining walls. Calculate the depth of embedment needed just to prevent undrained failure by rotation about the prop if the groundwater level behind the wall is
(i) below formation level; and
(ii) at original ground level.
neglect the effects of friction/adhesion at the soil/wall interface, and take the unit weight of water as 10 kN/m^3.
What is the strut load in each case?

Q7.4 Solution

In the gravel for a frictionless wall, $K_a = (1 - \sin\phi')/(1 - \sin\phi') = 0.271$ with $\phi' = 35°$.

(i) With the water table below formation level, the pore water pressures in the gravel are zero and the lateral effective stress increases from zero at original ground level to $K_a \times \sigma'_v = 0.271 \times 20\ kN/m^3 \times 2\ m = 10.84\ kPa$ at depth 2 m, the interface between the gravel and the clay.

In the clay, a dry tension crack might extend to a depth z below origilal ground level such that $\sigma_v = 2.\tau_u$, or $20z = 160\ kPa \Rightarrow z = 8\ m$ (below original ground level). Below this depth, the horizontal total stress behind the wall is given by $\sigma_h = \sigma_{active} = \gamma.z - 2\tau_u = (160 + 20.x) - 160 = 20.x\ kPa$ where x is the depth in m below the bottom of the tension crack, ie $x = (z - 8)$ where z is the depth below original ground level. Note that x is also the depth below formation level.

In front of the wall, the horizontal total stress is given by $\sigma_h = \sigma_{passive} = \gamma.z + 2\tau_u = 20.x + 160\ kPa$ where x is the depth in m below formation level.

The resulting stress distribution is shown in Figure Q7.4a: note that the triangular components of the total stress distribution on either side of the wall below formation level cancel out.

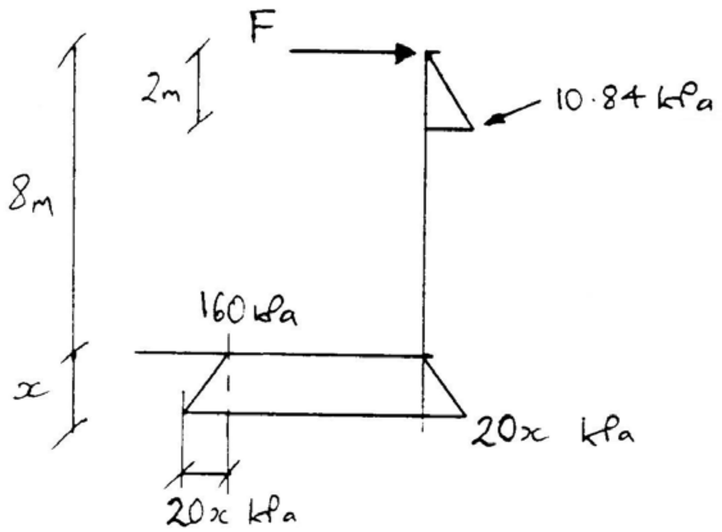


Figure Q7.4a: lateral total stresses on retaining wall with water table below formation level

Take moments about the position of the prop to find the value of x required for (moment) equilibrium:

$(½ \times 10.84\ kPa \times 2\ m) \times (^2/_3 \times 2\ m) = (160\ kPa \times x\ m) \times (8 + ½x)\ m$

$\Rightarrow 80x^2 + 1280\ x - 14.45 = 0$, *or*

$x^2 + 16\ x - 0.180625 = 0$

$\Rightarrow x = \{-16 \pm \sqrt{(16^2 + [4 \times 0.180625])}\} \div 2$

$\Rightarrow \underline{x = 0.01\ m}$ (0.01128)

The prop force F is calculated from the condition of horizontal equilibrium,

$F = (10.84 - 160\ x) = \underline{9\ kN/m}$

Note that the critical mode of failure in this case would be base instability, and the depth of embedment would need to be increased to prevent this.

(ii) With the water table at original ground level, the effect of the pore water pressures must be taken into account. Assume that the pore water pressures in the gravel are hydrostatic. The pore water pressure at 2 m depth is 2 m × 10 kN/m^3 *= 20 kPa, and the vertical total stress is 2 m × 22 kN/m*^{3} *= 44 kPa, so the vertical effective stress is 44 kPa – 20 kPa = 24 kPa. The lateral effective stress increases from zero at original ground level to* $K_a \times \sigma'_v = 0.271 \times 24$ *kPa = 16.5 kPa at 2 m depth. The lateral total stress is equal to the lateral effective stress plus the pore water pressure,* $\sigma_h = \sigma'_h + u = 26.5$ *kPa.*

In the clay, a flooded tension crack might extend to a depth z below original ground level such that $(\sigma_v - \sigma_h) = 2.\tau_u$ *with* $\sigma_v = (20\ z + 4)$ *kPa and* $\sigma_h = \gamma_w.z = 10\ z$ *kPa. Hence* $20\ z + 4 - 10\ z = 160$ *kPa* $\Rightarrow 10\ z = 156$ *m, or z = 15.6 m (below original ground level).*

In front of the wall, the horizontal total stress is given (as before) by $\sigma_h = \sigma_{passive} = \gamma.z + 2\tau_u = 20.x + 160$ *kPa where x is the depth in m below formation level.*

Assuming that the required depth of embedment x is not greater than 7.6 m (so that z ≤ 15.6 m), the resulting stress distribution is shown in Figure Q7.4b.

Take moments about the position of the prop to find the new equilibrium value of x:

$[(½ \times 6.5\ kPa \times 2\ m) \times (^2/_3 \times 2\ m)] + [(½ \times 80\ kPa \times 8\ m) \times (^2/_3 \times 8\ m)] = [(80\ kPa \times x\ m) \times (8 + ½x)\ m] + [(½ \times 10\ x\ kPa \times x\ m) \times (8 + {^2/_3}x)\ m]$

$\Rightarrow 1715.33 = 640\ x + 40x^2 + 40x^2 + 3.33x^3$

$\Rightarrow x^3 + 24\ x^2 + 192\ x = 514.6$

Solve by trial and error to give

$x \approx 2.09$

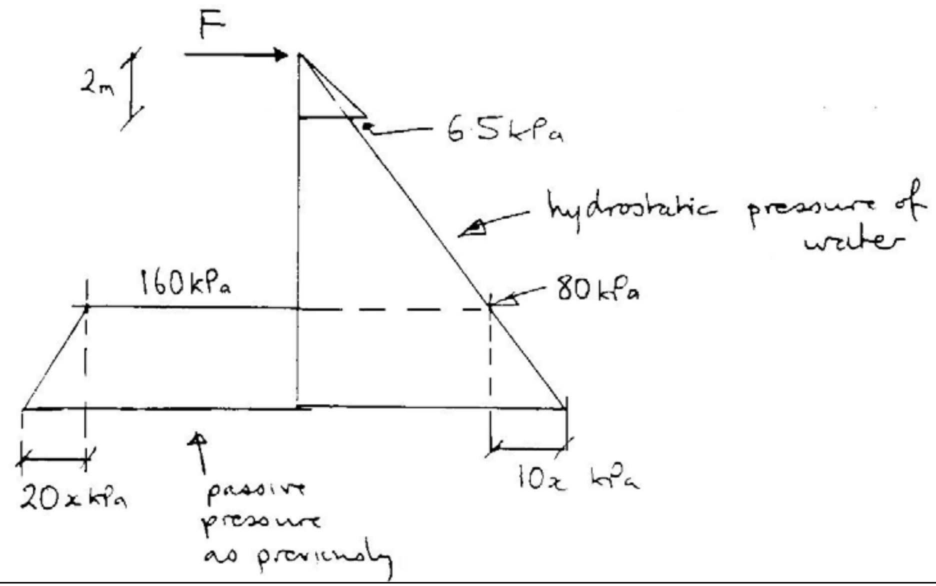


Figure Q7.4b: lateral total stresses on retaining wall with water table at original ground level

The prop load F is then given by

F = [6.5 kN/m + (½ × 80 kPa × 8 m)] – [80 kPa × x m] – [½ × 10 x kPa × x m]

⇒ <u>F = 137.5 kN/m</u> (with x = 2.09 m)

The possibility of base failure would still need to be checked.

The answers are not suitable for design because

(a) the factor of safety in the above calculations is 1 (ie the wall is on the verge of collapse)
(b) additional embedment will probably be needed to prevent base or seepage failure.

Mechanism-based limit equilibrium analysis of gravity retaining walls

Q7.5 (a) Figure 7.51 shows a cross section through a mass retaining wall. By means of a graphical construction, estimate the minimum lateral thrust which the wall must be able to resist. (Assume that the angle of friction between the soil and the concrete is equal to $0.67 \times \phi'_{crit}$).

(b) If the available frictional resistance to sliding on the base of the wall must be twice the active lateral thrust, calculate the necessary mass and width of the wall. (Take the unit weight of concrete as 24 kN/m^3).

(c) What other checks would you need to carry out before the design of the wall could be considered to be acceptable?

[University of London 3rd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q7.5 Solution

(a) The soil/wall friction angle δ is $^2/_3 \times 36° = 24°$

The succession of trial wedges is shown in Figure Q7.5a. The points A, B, C etc are marked off at horizontal distance intervals of 1 m, giving upslope distances XA = AB = BC etc of $\sqrt{1^2 + (^1/_3)^2}$ (by Pythagoras's Theorem) = 1.054 m.

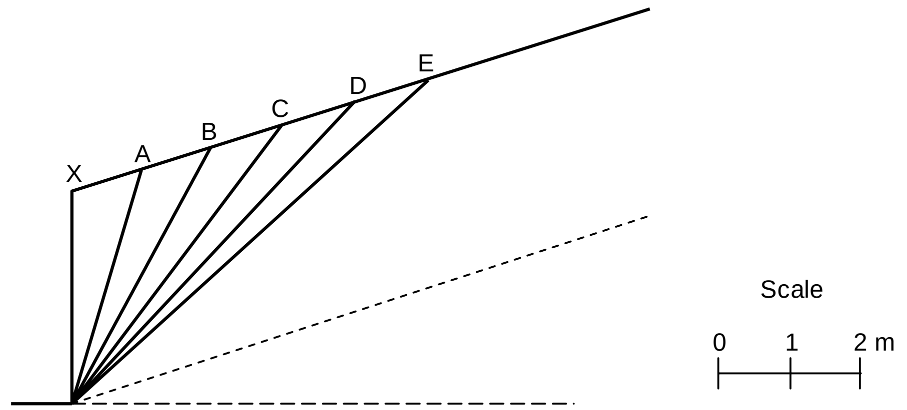


Figure Q7.5a: Succession of trial wedges

The area of each wedge OXA, OAB, OBC etc is given by ½ $\times$ base $\times$ perpendicular height = ½ $\times$ 3 m $\times$ 1 m = 1.5 m^2 (taking the baseline = 3 m as the back of the wall, the horizontal width of each wedge is its perpendicular height = 1 m). Hence the weight of each wedge is 1.5 m^2 $\times$ 20 kN/m^3 = 30 kN/m run .

Each trial rupture line OA, OB etc makes an angle θ_A, θ_B etc to the horizontal such that tan $\theta_A = (3.333 \div 1)$, tan $\theta_B = (3.666 \div 2)$ etc.

The retained soil is above the water table so we will assume zero pore water pressures. The forces acting on each wedge are

- *(i) the weight of the wedge, W, acting vertically downward*
- *(ii) the effective stress reaction from the wall, R'_W, acting at an angle δ (= 24°) to the horizontal such that the vertical component points upward (i.e., the shear stress acts so as to resist settlement of the retained soil relative to the wall)*
- *(iii) the effective stress reaction from the trial rupture, R'_R, which acts at an angle of ϕ'_{crit} (= 36°) to the normal to the rupture line with the shear component acting upwards (i.e., so as to resist sliding, i.e. at an angle of $(90° - \theta + \phi'_{crit}) = (126° - \theta)$ to the horizontal (Figure Q7.5b).*

Hence

Wedge	*OXA*	*OXB*	*OXC*	*OXD*	*OXE*
tanθ	*3.333*	*1.833*	*1.333*	*1.083*	*0.933*
θ, degrees	*73.3*	*61.4*	*53.1*	*47.3*	*43.0*

Total weight W, kN/m	*30*	*60*	*90*	*120*	*150*
Angle of R'_R *=* $(126° - \theta)$ *to the horizontal*	*52.7*	*64.6*	*72.9*	*78.7*	*83.0*

Force polygons for each wedge are shown in Figure Q7.5b, drawn to the scale indicated.

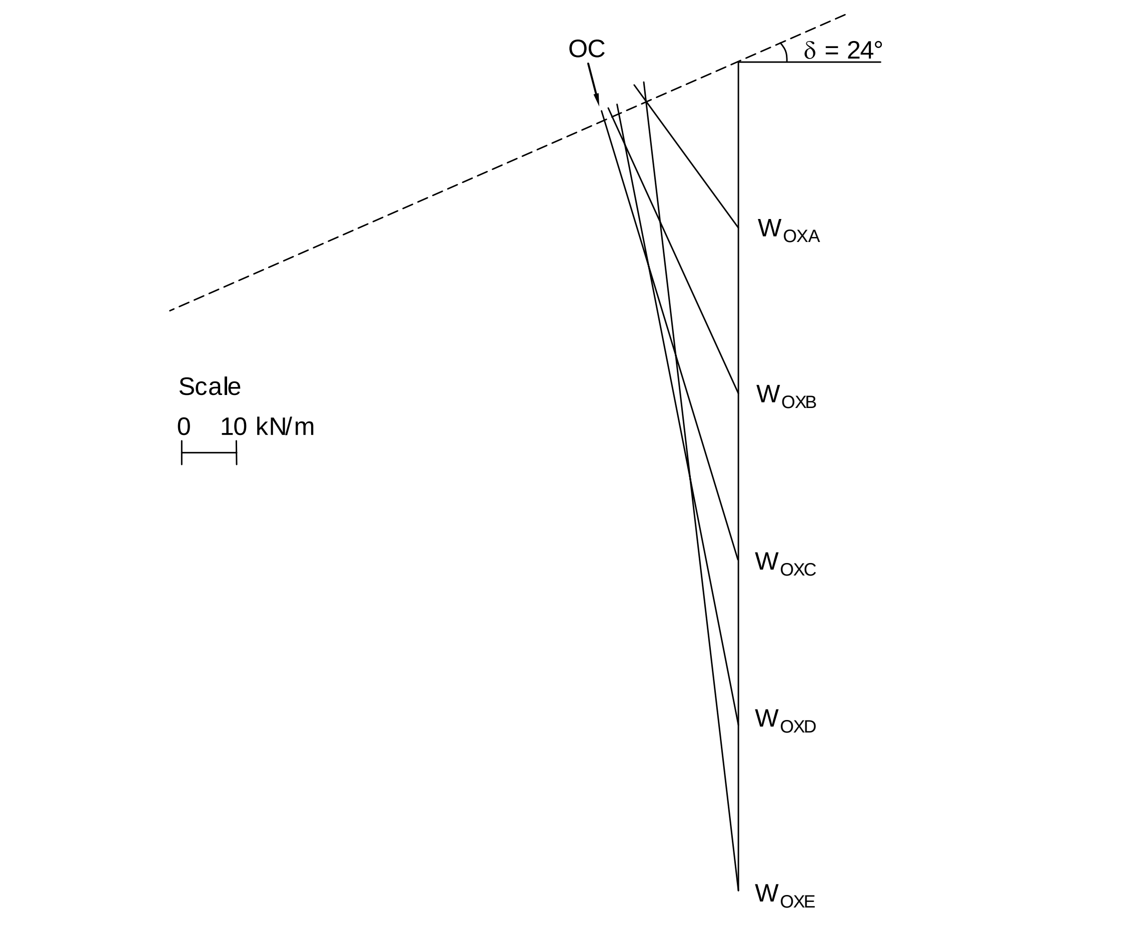


Figure Q7.5b: Force polygons for trial sliding wedges

From Figure Q7.5b, the maximum lateral thrust that must be withstood by the wall is that associated with wedge XOC , which has an inclined component (scaling from the diagram) of *<u>26.5 kN/m</u>*

(b) Figure Q7.5c shows a free body diagram for the wall.

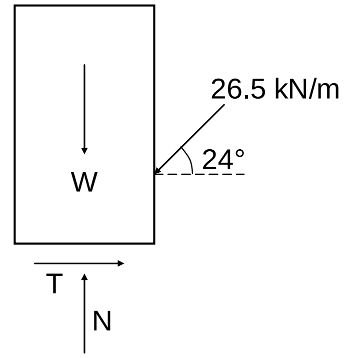


Figure Q7.5c: Free body diagram for the wall

The weight of the wall W is 3 m × b m × 24 kN/m^3 *= 72 b kN/m where b is the width of the wall (in m).*

Resolving forces vertically, $N'_B = W + 26.5 \sin 24°$

Resolving forces horizontally, $T'_B = 26.5 \cos 24° = 24.2$ *kN/m*

At failure, $T'_{B,\,failure} = N'_B.\tan 24°$ *and we require* $T'_B = (N'_B.\tan 24°) \div 2$*, hence*
$(W + 26.5 \sin 24°).\tan 24° = 48.4$ *kN/m*

$\Rightarrow W + 10.78 = 108.71$ *kN/m*

$\Rightarrow W = 72\,b \approx 98$ *kN/m* $\Rightarrow \underline{b \approx 1.36\text{ m}}$

(c) We would also have to check

- *the adequacy of the factor of safety on soil strength or strength mobilization factor*
- *safety against toppling*
- *the bearing capacity of the base*
- *structural adequacy of the wall*
- *global stability (i.e. triggering a landslip)*
- *provision for drainage of the backfill*
- *possibility of accidental surcharge loading,*

Q7.6 (a) Figure 7.52 shows a cross section through a masonry retaining wall, with a partially-sloping backfill which is subjected to a line-load of 100 kN/m as indicated. Use a graphical construction to estimate the lateral thrust which must be resisted by friction on the base of the wall, in order to prevent failure by the formation of a slip plane extending upward from the base of the wall, such as OA.

(b) Is your answer likely to be greater or less than the true value, and why?

(c) Suggest one way in which the ability of the wall to resist the thrust from the backfill could be improved.

[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q7.6 Solution

(a) The soil/wall friction angle δ is given as 25°

The succession of trial wedges is shown in Figure Q7.6a. The points A', A, B and C are marked off at horizontal distance intervals of 2 m.

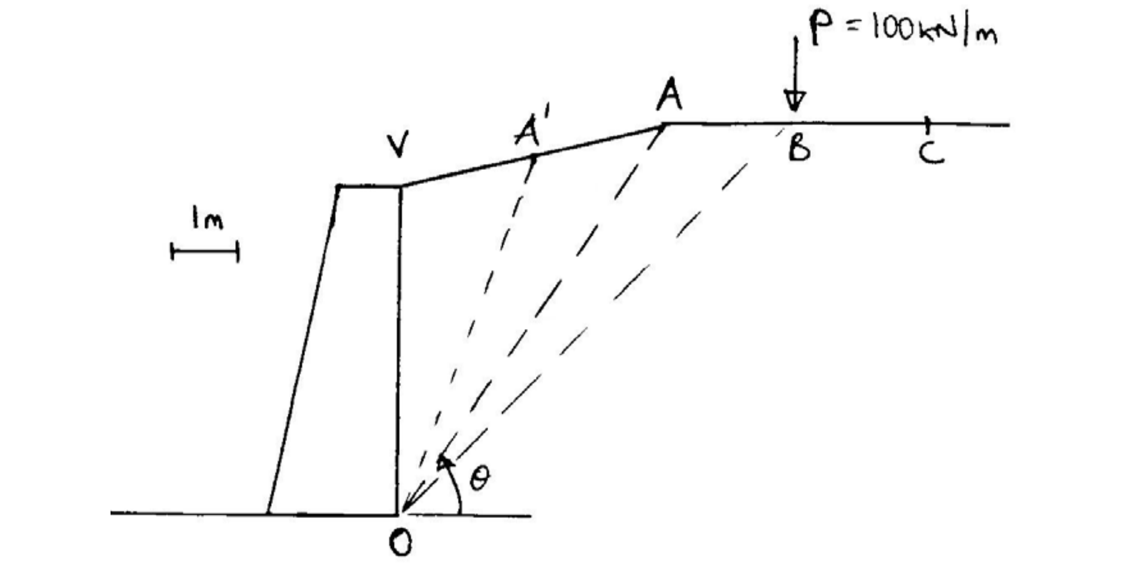


Figure Q7.6a: Succession of trial wedges

Each trial rupture line OA', OA etc makes an angle $\theta_{A'}$, θ_A etc to the horizontal such that tan $\theta_{A'}$ = (5.5 ÷ 2), tan θ_A = (6 ÷ 4), tan θ_B = (6 ÷ 6) etc.

The retained soil is above the water table so we will assume zero pore water pressures. The forces acting on each wedge are

- *(i) the weight of the wedge, W, acting vertically downward*
- *(ii) the effective stress reaction from the wall, R'_W, acting at an angle δ (= 25°) to the horizontal such that the vertical component points upward (i.e., the shear stress acts so as to resist settlement of the retained soil relative to the wall)*
- *(iii) the effective stress reaction from the trial rupture, R'_R, which acts at an angle of ϕ'_{crit} (= 30°) to the normal to the rupture line with the shear component acting upwards (i.e., so as to resist sliding, i.e. at an angle of (90° - θ + ϕ'_{crit}) = (120° - θ) to the horizontal (Figure Q7.6b)*
- *(iv) For wedges OVB and beyond, the line load P = 100 kN/m acting vertically downward.*

The area of each wedge is given by ½ × base × perpendicular height. For the wedges within the slope OVA' and OVA, the base length is the height of the wall = 5 m. For the wedges on the flat, the base is the horizontal distance AB or BC and the perpendicular height is the vertical distance to the level of the base of the wall, 6 m. Hence the areas and weights are as follows:

Area OVA' = ½ × 5 m × 2 m = 5 m^2; weight = 5 m^2 × 20 kN/m^3 = 100 kN/m

Area OVA = ½ × 5 m × 4 m = 10 m^2; weight = 10 m^2 × 20 kN/m^3 = 200 kN/m
Additional areas AOB, BOC etc = ½ × 2 m × 6 m = 6 m^2; extra weight = 120 kN/m

Hence

Wedge	*OVA'*	*OVA*	*OVB*	*OVC*
tanθ	*5.5 ÷ 2*	*6 ÷ 4*	*6 ÷ 6*	*6 ÷ 8*
θ, degrees	*70.0*	*56.3*	*45.0*	*36.9*
Total weight W, kN/m + P (= 100 kN/m) if applicable	*100*	*200*	*320 + 100*	*440 + 100*
Angle of R'_R = (120° - θ) to the horizontal	*50.0*	*56.3*	*75.0*	*83.1*

Force polygons for each wedge are shown in Figure Q7.6b, drawn to the scale indicated.

From Figure Q7.6b, the maximum lateral thrust that must be withstood by the wall is that associated with wedge VOB (which just includes the effect of the line load), which has a horizontal component (scaling from the diagram) of <u>98 kN/m</u>

(b) The answer is likely to be less than the true value, because
- (i) we have assumed that the soil is on the verge of failure, which may not be the case in reality (if the soil is not at failure, it is not mobilising its full strength and the lateral thrust on the wall will be greater)
- (ii) we have used a mechanism-based approach (“upper bound”): if we have chosen the wrong mechanism of failure, the answer will err on the unsafe side.

(c) The ability of the wall to resist sliding may be improved by
- (i) increasing its weight
- (ii) embedding it slightly
- (iii) providing a shear key
- (iv) sloping the back of the wall

Note: assuming a unit weight for the concrete γ_{conc} = 24 kN/m^3, the weight of the wall is 1.5 m × 5 m × 24 kN/m^3 = 180 kN/m, and the available base friction of (180 kN/m + 98 kN/m × tan25°) × tan25° = 105 kN/m is only just enough to prevent sliding, even taking into account the effect of the downward shear on the back of the wall (98 kN/m × tan25°).

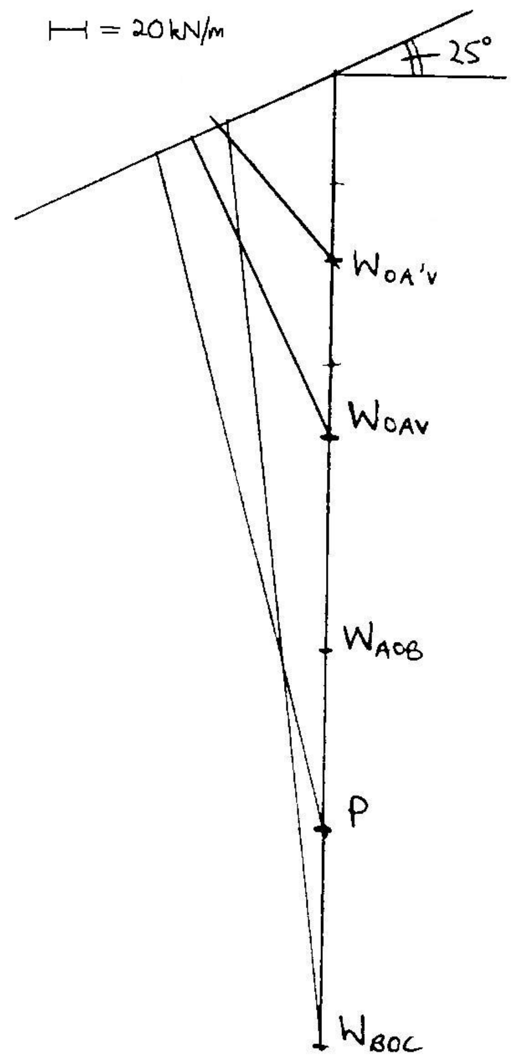


Figure Q7.6b: Force polygons for trial sliding wedges

Q7.7 Figure 7.53 shows a cross section through a mass concrete retaining wall. Estimate the minimum lateral thrust which the wall must be able to resist to maintain the stability of the retained soil. Hence investigate the safety of the wall against sliding.

[University of London 2nd year BEng (Civil Engineering) examination, King's College]

Q7.7 Solution
(a) The soil/wall friction angle δ is given as $25°$
The succession of trial wedges is shown in Figure Q7.7a. OB is 2.6 m up the slope; BC = CD = DE etc = 1.04 m up the slope.

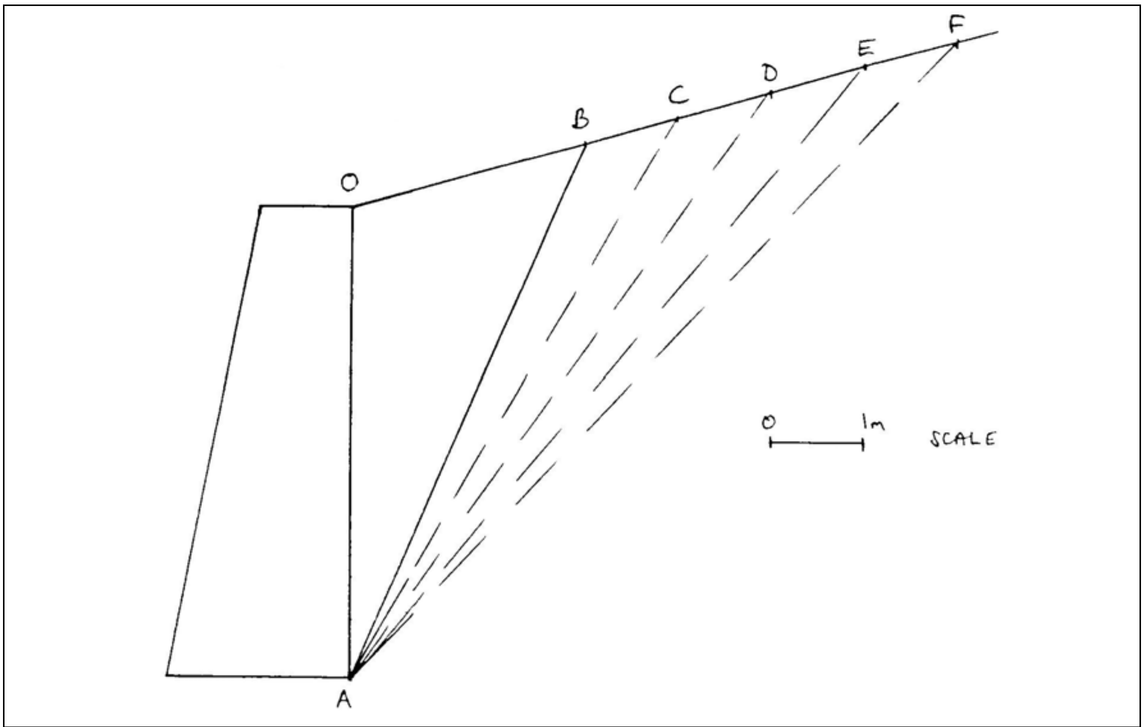


Figure Q7.7a: Succession of trial wedges

The area of each wedge OAB, OAC, OAD etc is given by ½ × base × perpendicular height = ½ × 5 m × (s.cos15°), where s (in m) is the upslope distance OB, OC etc and taking the baseline = 5 m as the back of the wall. Hence the areas and weights are as follows:

Area OAB = ½ × 5 m × 2.6 m × cos15° = 6.28 m^2; weight = 6.28 m^2 × 20 kN/m^3 = 125.6 kN/m
Area OAC = ½ × 5 m × 3.64 m × cos15° = 10 m^2; weight = 8.79 m^2 × 20 kN/m^3 = 175.8 kN/m
Additional areas CAD, DAE etc are each ½ × 5 m × 1.04 m × cos15° = 2.5 m^2; giving an extra weight of 50 kN/m

Each trial rupture line OA, OB etc makes an angle θ_B, θ_C etc to the horizontal scaled off the diagram.

The forces acting on each wedge are

- *(i) the weight of the wedge, W, acting vertically downward*
- *(ii) the effective stress reaction from the wall, R'_W, acting at an angle δ (= 25°) to the horizontal such that the vertical component points upward (i.e., the shear stress acts so as to resist settlement of the retained soil relative to the wall)*
- *(iii) the effective stress reaction from the trial rupture, R'_R, which acts at an angle of ϕ'_{crit} (= 30°) to the normal to the rupture line with the shear component acting upwards (i.e., so as to resist sliding, i.e. at an angle of (90° - θ + ϕ'_{crit}) = (120° - θ) to the horizontal*
- *(iv) The pore water pressure reaction from the wall, = ½ × 2 m × 10 kN/m^3 × 2 m = 20 kN/m acting horizontally (assuming hydrostatic conditions below the water table and taking the unit weight of water as 10 kN/m^3)*

(v) *The pore water pressure reaction from the rupture, which has a horizontal component of 20 kN/m (because the water table is level, hence no horizontal flow) and hence has a magnitude of 20 $\div \sin\theta$ kN/m acting perpendicular to the rupture surface, i.e. at an angle of (90° - θ) to the horizontal (Figure Q7.7b)*

Hence

Wedge	*OAB*	*OAC*	*OAD*	*OAE*	*OAF*
θ, degrees	*66*	*59.5*	*54*	*49.5*	*46*
Total weight W, kN/m	*125*	*175*	*225*	*275*	*325*
Angle of R'_R = (120° - θ) to the horizontal	*54*	*60.5*	*66*	*70.5*	*74*

Force polygons for each wedge are shown in Figure Q7.7b, drawn to the scale indicated.

From Figure Q7.7b, the maximum lateral thrust that must be withstood by the wall is that associated with wedge OAE , which (scaling from the diagram) has a horizontal component, including the pore water force of 78 + 20 = <u>98 kN/m</u>

The wall has weight 1.5 m × 5 m × 25 kN/m^3 = 187.5 kN/m

The downward force exerted on the wall by the soil is (78 kN/m × tan25°) = 36 kN/m

Assuming that the pore water pressure on the base of the wall varies linearly from 20 kPa at the heel (A) to zero at the toe, the pore water pressure upthrust on the base of the wall is ½ × 20 kPa × 2 m = 20 kN/m

Thus the available friction on the base is (187.5 kN/m + 36 kN/m – 20 kN/m) × tan25° = 95 kN/m, which is insufficient to resist the imposed horizontal thrust of 98 kN/m.

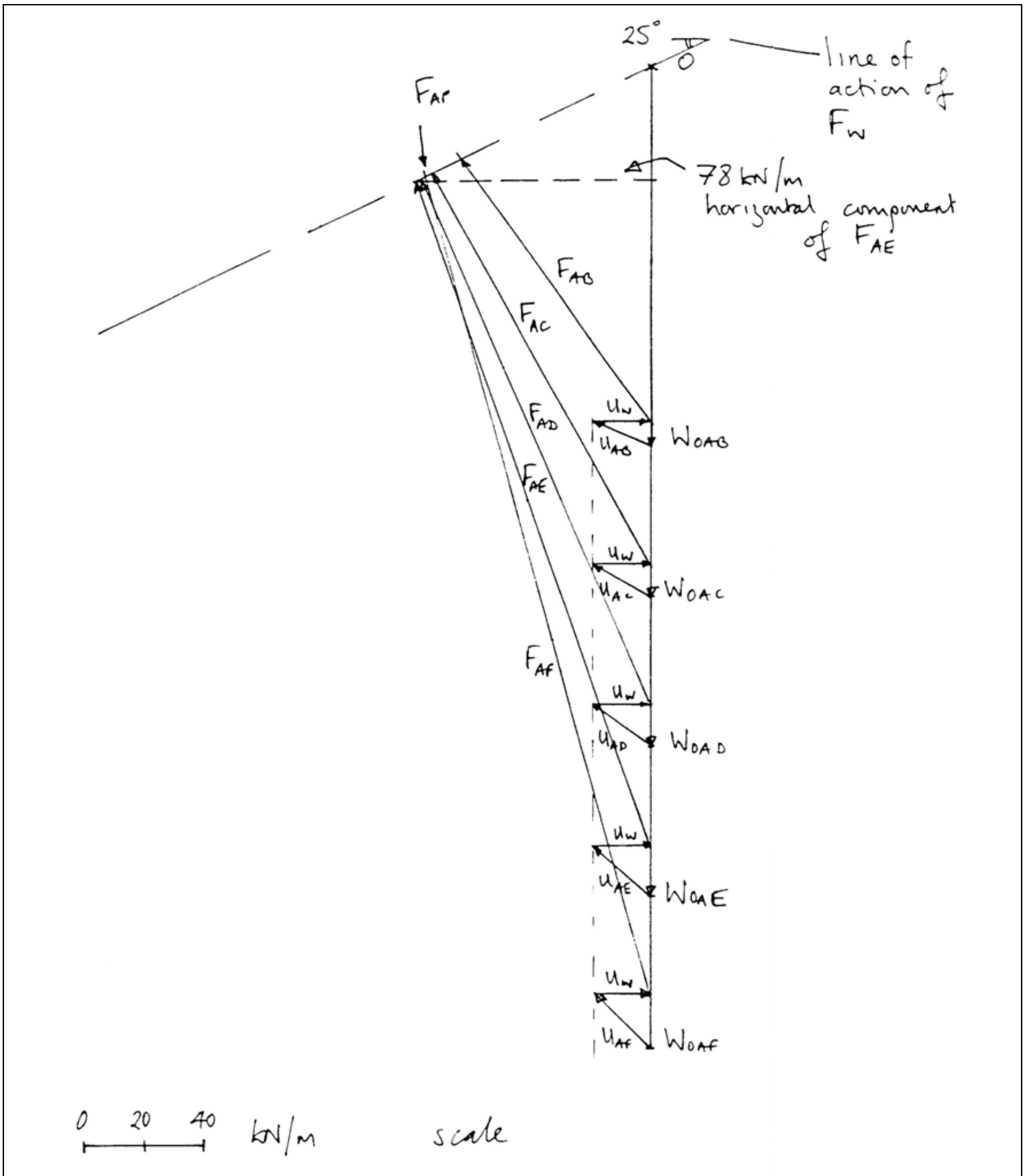


Figure Q7.7b: Force polygons for trial sliding wedges

Q7.8 (a) Figure 7.54a shows a cross-section through a gravity retaining wall retaining a partially sloping backfill of soft clay. By means of a graphical construction, estimate the minimum (active) lateral thrust that the wall must be able to resist in the short term. How does this compare with the maximum available sliding resistance on the base?
(Assume that the limiting adhesion between the wall and the clay is equal to 0.4 × the undrained shear strength τ_u, and that the angle of soil/wall friction between the wall and the underlying sand is equal to $0.67\times\phi'$.)

(b) If the thrust from the backfill acts on the back of the wall at a distance of one-third of the height of the wall above the base, and the normal total stress distribution on the base is as shown in Figure 7.54b, calculate the values of σ_L and σ_R.

(c) What further investigations would you need to carry out, before the design of the wall could be considered acceptable?

Q7.8 Solution

(a) The succession of trial wedges is shown in Figure Q7.8a. The points A, B, C and D are marked off at horizontal distance intervals of 1.5 m. The horizontal distance from the line of the wall to A is 3 m.

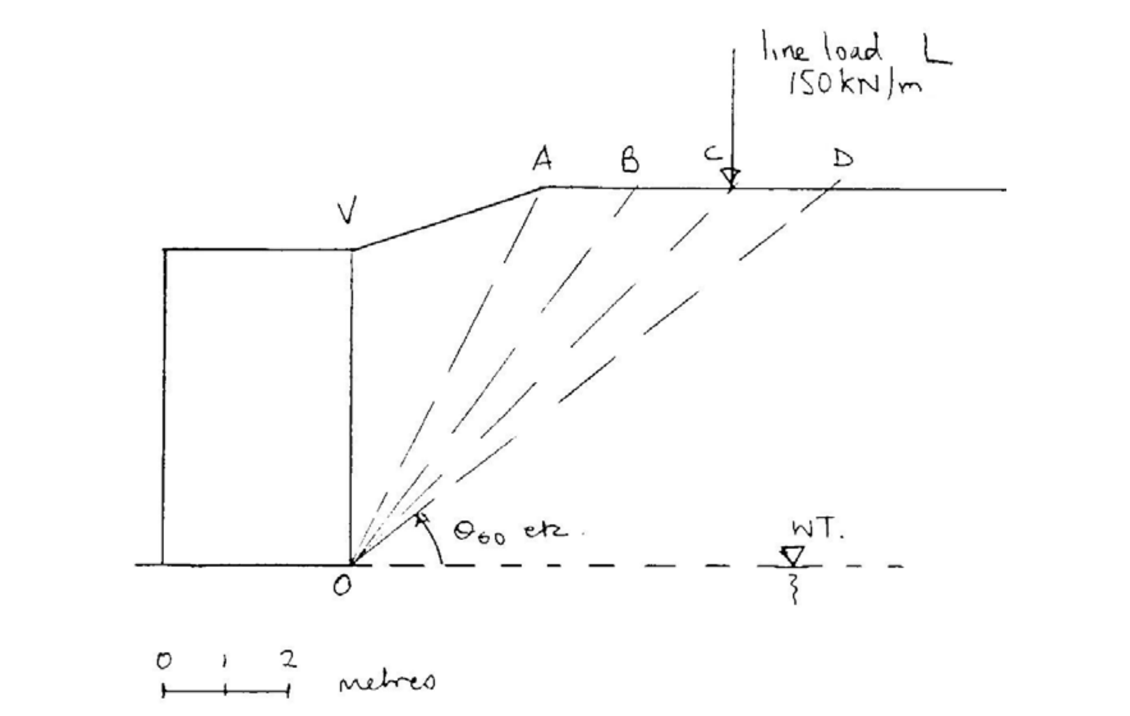


Figure Q7.8a: Succession of trial wedges

Each trial rupture line OA, OB etc makes an angle θ_A, θ_B etc to the horizontal such that tan θ_{OA} = $(6 \div 3)$, tan θ_{OB} = $(6 \div 4.5)$, tan θ_{OC} = $(6 \div 6)$ etc.

The retained soil is above the water table so we will assume zero pore water pressures. The forces acting on each wedge are

- *(i) the weight of the wedge, W, acting vertically downward*
- *(ii) the shear force T_W on the soil/wall interface, which acts vertically and is given by $\tau_w \times l_w = 0.5 \times 25$ kPa $\times 5$ m = 50 kN/m. T_R will be the same for all wedges*
- *(iii) the shear force T_R on the rupture, which acts parallel to the rupture at an angle q to the horizontal and is given by $\tau_u \times l_r$ = 25 kPa $\times l_r$ where l_r is the length of the rupture in m: $l_r^2 = 6^2 + x_A^2$; $l_r^2 = 6^2 + x_B^2$ etc where x_A, x_B etc are the horizontal distances from the line of the wall to the point A, B etc. T_R will be different for each wedge.*
- *(iv) the normal reaction from the wall, N_W, which acts horizontally but is unknown in magnitude (this is what we are trying to find)*
- *(v) the normal reaction from the rupture, N_R, which acts at right angles to the rupture, i.e. at an angle of $(90° - \theta)$ to the horizontal but is unknown in magnitude*

(vi) *For wedges OVC and beyond, the line load L = 150 kN/m acting vertically downward.*

The area of each wedge is given by ½ × base × perpendicular height. For the wedge within the slope OVA, the base length is the height of the wall = 5 m and the perpendicular height is 3 m. For the wedges on the flat, OAB, BOC, COD etc, the base length is 6 m and the perpendicular height is 1.5 m. Hence the areas and weights are as follows:

Area OVA = ½ × 5 m × 3 m = 7.5 m^2; weight = 75 m^2 × 20 kN/m^3 = 150 kN/m
Additional areas OAB, BOC, COD etc = ½ × 6 m × 1.5 m = 4.5 m^2; extra weight = 90 kN/m

Hence

Wedge	*OVA*	*OVB*	*OVC*	*OVD*
tan θ	*6 ÷ 3*	*6 ÷ 4.5*	*6 ÷ 6*	*6 ÷ 7.5*
θ*, degrees*	*63.4*	*53.1*	*45.0*	*38.7*
Total weight W, kN/m + L (= 150 kN/m) if applicable	*150*	*240*	*330 + 150*	*420 + 150*
Length of rupture $l_r = \sqrt{6^2 + x_A^2}$ *etc, m*	$\sqrt{6^2+3^2}$ = 6.71	$\sqrt{6^2+4.5^2}$ = 7.5	$\sqrt{6^2+6^2}$ = 8.48	$\sqrt{6^2+7.5^2}$ = 9.6
Shear force on rupture $T_R = 25 \times l_r$*, kN/m*	*168*	*187.5*	*212*	*240*

Force polygons for each wedge are shown in Figure Q7.8b, drawn to the scale indicated. Note that N_W *is negative for OVB, and that OVA is not show.* N_W *wuld also be negative for OVC in the absence of the line load.*

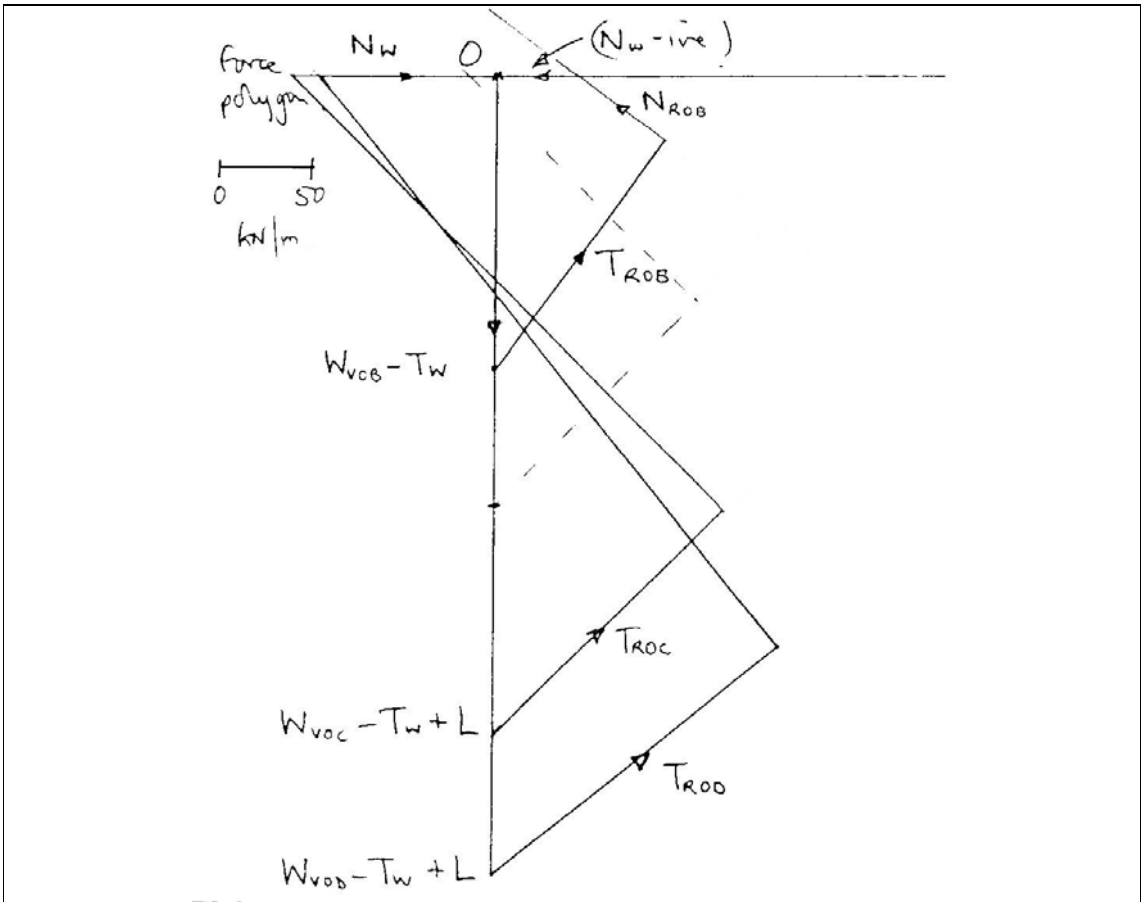


Figure Q7.8b: Force polygons for trial sliding wedges

From Figure Q7.8b, the maximum lateral thrust that must be withstood by the wall is that associated with wedge VOC including the effect of the line load, which is (by scaling from the diagram) <u>134 kN/m</u>

The maximum available sliding resistance due to friction on the base of the wall is given by

$F_{max} = (W + T_W) \times \tan\delta = (360 + 50) \times \tan 24° = \underline{182.5\ kN/m}$

where W is the weight of the wall $= 5\ m \times 3\ m \times 24\ kN/m^3 = 360\ kN/m$

This is about 36% greater than the lateral thrust calculates, so the wall should be safe against sliding at least in the short term.

(*b*) Figure Q7.8c shows a free body diagram of the wall and the forces and stresses acting on it (it is assumed that the basal shear stress T_B takes the value needed to maintain horizontal equilibrium, 134 kN/m = N_W, rather than the maximum of 182.5 kN/m calculated above)

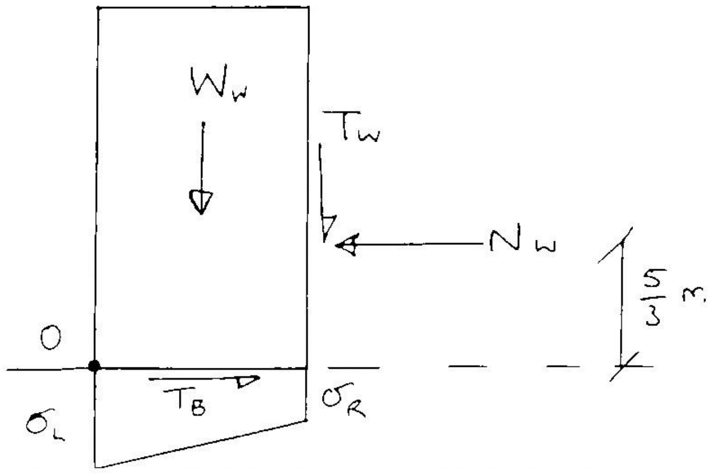


Figure Q7.8c: Free body diagram for the retaining wall in Q7.8

Taking moments about O,

$\{W \times 1.5 \text{ m}\} + \{T_W \times 3 \text{ m}\} - \{N_W \times 5 \text{ m} \div 3\} = \{\sigma_R \times 3 \text{ m} \times 1.5 \text{ m}\} + \{½(\sigma_L - \sigma_R) \times 3 \text{ m} \times 1 \text{ m}\}$

$\Rightarrow 540 + 150 - 223.33 = 3\sigma_R - 1.5.\sigma_L = 456.67$

Vertical equilibrium gives

$W + T_W = \{\sigma_R \times 3 \text{ m}\} + \{½(\sigma_L - \sigma_R) \times 3 \text{ m}\} \Rightarrow 360 + 50 = 1.5\sigma_R + 1.5.\sigma_L = 410$

Adding these to eliminate σ_L gives

$4.5\sigma_R = 866.67$ kPa

$\Rightarrow$ $\underline{\sigma_R = 192.6 \text{ kPa}; \sigma_L = 80.7 \text{ kPa}}$

(c) we would also need to check

- the stability of the slope
- the effect of a possible flooded tension crack
- the bearing capacity of the sand at the base of the wall (check against bearing failure)
- the long term stability of the wall and the slope using long-term pore water pressures and the effective stress failure criterion
- the possibility of a global landslide
- that excessive displacements would not occur

QUESTIONS AND SOLUTIONS: CHAPTER 8

Shallow foundations

Q8.1 Figure 8.38 shows a cross section through a shallow strip footing. Estimate lower and upper bounds to the vertical load Q (per metre length) that will result in the rapid (undrained) failure of the footing.
[University of London 2nd year BEng (Civil Engineering) examination, King's College]

Q8.1 Solution
Note: this question is rather trivial unless the formulae used are derived from first principles. This was expected of students in the examination, but the derivations are not repeated here.

Lower boud solution based on frictionless stress discontinuities: use the reasoning in Section 8.2.2 (page 439) of the main text to derive Equation 8.3a,

$$(\sigma_f - \sigma_0) = 4.\tau_u \qquad (8.3a)$$

More advanced students might be expected to use the reasoning given in main text Section 9.5.2 (pages 507-508) to derive Equation 9.13,

$$(\sigma_f - \sigma_0) = (2 + \pi).\tau_u \qquad (9.13)$$

In the present case,

$\tau_u = 35$ *kPa*

$\sigma_0 = 1$ *m* $\times 18$ *kN/m*^{3} = 18 *kPa on either side of the footing*

Hence

$\sigma_f = (4 \times 25$ *kPa) + 18kPa = 118 kPa using Equation 8.3a, or*

$\sigma_f = (5.14 \times 25$ *kPa) + 18kPa = 146.5 kPa using Equation 9.13*

Multiplying by the foundation width 2 m,

<u>Q = 236 kN/m</u>

using the most conservative possible approach (Equation 8.3a; the answer using Equation 9.13 is 293 kN/m)

Upper bound solution: use the reasoning in Section 8.3.1 (pages 439-443) of the main text to derive

$$(\sigma_f - \sigma_0) = 5.52.\tau_u$$

for a circular slip with its centre located above the centre of the footing (main text Figure 8.5). More advanced students might reasonably be expected to follow the reasoning given in main text Section 9.9.1(b) (pages 536-539) to derive Equation 9.47,

$(\sigma_f - \sigma_0) = (2 + \pi).\tau_u$ (9.47)

In the present case, with τ_u = 35 kPa and σ_0 = 1 m × 18 kN/m^3 = 18 kPa on either side of the footing

σ_f = (5.52 × 25 kPa) + 18 kPa = 156 kPa using the slip circle mechanism, or

σ_f = (5.14 × 25 kPa) + 18 kPa = 146.5 kPa using Equation 9.47

Multiplying by the foundation with 2 m,

Q = 312 kN/m

using the slip circle. (Equation 9.47 is the same as Equation 9.13: these upper and lower bounds are the same and the solution is therefore correct – provided of course that the conditions assumed in the analysis apply!)

Q8.2 (a) Explain briefly the essential features of upper and lower bound plasticity analyses as applied to problems in geotechnical engineering.

(b) A long foundation of depth D and width B is built on a clay soil of saturated unit weight γ_s, undrained shear strength τ_u and frictional strength ϕ'. The water table is at a depth D below the soil surface. Show that the vertical load Q, uniformly distributed across the foundation, that will cause failure is given by

$$(Q/B) \geq (\gamma_s.D + 4.\tau_u)$$

in the short term, and by

$$(Q/B) \geq (K_p^2 \gamma_s.D)$$

in the long term, where K_p is the passive earth pressure coefficient,

$$K_p = \frac{1 + \sin\phi'}{1 - \sin\phi'}$$

(c) If γ_s = 20 kN/m^3, τ_u = 25 kPa, ϕ' = 22° and D = 1.5 m, is the foundation safer in the short term or in the long term?
[University of London 2nd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q8.2 Solution
*(a) An upper bound is based on an assumed **mechanism** of collapse. If the assumed mechanism is incorrect, the analysis will err on the **unsafe** side. A lower bound solution is based on finding a system of **stresses** that can be in **equilibrium** with the applied loads without violating the **failure criterion** for the soil. It may be that a more efficient stress distribution exists, in which case the analysis will err on the **safe** side.*

(b) Use the analyses given in main text Sections 8.2.2 (page 439) and 8.2.1 (pages 437-438) to derive the short term (undrained) and long term (effective stress) bearing capacities

$(\sigma_f - \sigma_0) = 4.\tau_u$ (8.3a)

and

$(\sigma'_f/\sigma'_{f0}) = K_p^2$, *where* $K_p = (1 + \sin\phi')/(1 - \sin\phi')$ (8.1)

Substituting σ_f *or* $\sigma'_f = Q/B$ *(Q is the load per metre length of the foundation) and (with zero pore water at depth D)* σ_0 *or* $\sigma'_0 = \gamma_s.D$ *kPa, and noting that our answers are lower bounds to the actual failure loads,*

$\underline{Q/B \geq 4.\tau_u + \gamma_s.D}$ *(short term), and*

$\underline{Q/B \geq K_p^2.\gamma_s.D}$ *(long term)*

(c) Substituting $\gamma_s = 20\ kN/m^3$, $\tau_u = 25\ kPa$, $\phi' = 22°$ *and* $D = 1.5\ m$ *gives* $K_p = 2.197$ *and*

$Q/B \geq (4 \times 25\ kPa) + (20\ kN/m^3 \times 1.5\ m) = 130\ kPa$, *short term*

$Q/B \geq (2.197^2) \times (20\ kN/m^3 \times 1.5\ m) = 145\ kPa$, *long term*

Therefore the short term case is the more critical (this is usual with a foundation on a soft clay).

Q8.3 A long concrete strip footing founded at a depth of 1 m below ground level is to carry an applied load (not including its own weight) of 300 kN/m. The soil is a clay, with undrained shear strength $\tau_u = 42$ kPa, effective angle of friction $\phi' = 24°$, and unit weight $\gamma = 20\ kN/m^3$. Calculate the width of the foundation required to give factors of safety on soil strength of 1.25 (on $\tan\phi'$) and 1.4 (on τ_u). Both short-term (undrained) and long-term (drained) conditions should be considered. The water table is 1 m below ground level.

Use Equation 8.9, with $N_c = (2 + \pi)$, and a depth factor d_c as given by Skempton (Table 8.2); and Equation 8.7, with $N_q = K_p.e^{\pi\tan\phi'}$ where $K_p = (1+\sin\phi')/(1-\sin\phi')$, with d_q, N_γ, d_γ and r_γ as given by Meyerhof and Bowles (Table 8.1). Take the unit weight of concrete as 24 kN/m^3.
[University of Southampton 2nd year BEng (Civil Engineering) examination, slightly modified]

Q8.3 Solution
(a) Undrained case
The design undrained design bearing capacity is given by main text Equation 8.9,

$(\sigma_f - \sigma_o)_{design} = \{N_c \times s_c \times d_c\} \times \tau_{u,design}$ (8.9)

with $N_c = (2 + \pi) = 5.14$; $\tau_{u,design} = 42\ kPa \div 1.4 = 30\ kPa$ *and* $\sigma_o = \gamma.D = 20kN/m^3 \times 1m = 20kPa$

From Table 8.2 (Skempton), the shape factor s_c = 1 (because this is a long foundation with L >>B, whatever the value of B) and the depth factor $d_c = \{1 + 0.23\sqrt{(D/B)}\}$ assuming $(D/B) \leq 4$. The foundation width B is as yet unknown.

The actual pressure at the base of the foundation is 300 kN/m divided by the footing width B, i.e. (300/B) kPa, plus the pressure due to the concrete foundation, $\gamma_{conc}.D$ = 24 kPa (D = 1 m; γ_{conc}= 24 kN/m^3).

Equating the actual and design base pressures,

$$\sigma_{f,design} = [\{N_c \times s_c \times d_c\} \times \tau_{u,design}] + 20\ kPa = 300/B + 24\ kPa$$

$$[5.14 \times \{1 + 0.23\sqrt{(D/B)}\} \times 30\ kPa] + 20\ kPa = 300/B + 24\ kPa$$

Solve by trial and error: with B = 1.7 m, D/B = 0.588 and the depth factor dc = 1.176. The left hand side of the equation (the design base pressure) is then numerically equal to 201.4 kPa; the right hand side (the actual base pressure) is 200.5 kPa, which is close enough.

Thus the required foundation width for the short term case is approximately <u>1.7 m</u>

(b) Long term (effective stress) case
The long term (drained) design bearing capacity is given by main text Equation 8.7,

$$\sigma'_{f,design} = \{N_q \times s_q \times d_q\} \times \sigma'_o + \{N_\gamma \times s_\gamma \times d_\gamma \times r_\gamma \times [0.5\gamma B - u]\} \qquad (8.7)$$

with $N_q = K_p.e^{\pi \tan\phi'_{des}}$, $K_p = (1+\sin\phi'_{des})/(1-\sin\phi'_{des})$, and d_q, N_γ, d_γ and r_γ as given by Meyerhof and Bowles (Table 8.1).

The design strength is now given by

$$\tan\phi'_{des} = (\tan 24°) \div 1.25 \Rightarrow \phi'_{des} = 19.6°$$

ϕ'_{des} =19.6°, K_p = 2.0096 and N_q = 6.151. From Table 8.1,

$$s_q = s_\gamma = 1 \text{ (because L>>B)}$$

$$d_q = d_\gamma = 1 + 0.1 \times (D/B) \times \sqrt{K_p} = 1 + 0.142D/B$$

$$N_\gamma = (N_q - 1) \times \tan(1.4\phi'_{des}) = 5.151 \times \tan 27.44° = 2.675$$

$$r_\gamma = 1 - 0.25.\log_{10}(B/2)$$

$$\sigma'_o = \gamma.D = 20\ kPa$$

The pore water pressure u at a depth of B/2 below the bottom of the foundation = $\gamma_w.B/2$, so that $[0.5\gamma B - u]$ = 5B kPa (with B in metres)

The design effective stress on the base of the foundation is $\sigma'_{f,design}$:

$$\sigma'_{f,design} = \{N_q \times s_q \times d_q\} \times \sigma'_o + \{N_\gamma \times s_\gamma \times d_\gamma \times r_\gamma \times [0.5\gamma B - u]\}$$

or

$$\sigma'_{f,design} = \{6.151 \times (1 + 0.142D/B) \times 20 \text{ kPa}\} + \{2.675 \times (1 + 0.142D/B) \times [1 - 0.25.\log_{10}(B/2]) \times 5B\}$$

The pore water pressure acting on the base of the foundation is zero.

The actual stress applied at the base of the foundation is 300 kN/m divided by the footing width B, i.e. (300/B) kPa, plus the stress due to the weight of the concrete foundation, $\gamma_{conc}.D$. The foundation width B must be chosen so that the actual and design stresses are the same.

Equating the design and actual stresses,

$$\{6.151 \times (1 + 0.142D/B) \times 20 \text{ kPa}\} + \{2.675 \times (1 + 0.142D/B) \times [1 - 0.25.\log_{10}(B/2]) \times 5B\} = \{(300/B) + 24 \text{ kPa}\}$$

Solve by trial and error: with B = 2.18 m, D/B = 0.459, $d_q = d_\gamma = (1 + 0.142D/B) = 1.065$, and $r_\gamma = 0.99$ so that the left hand side is numerically equal to

$$\{6.151 \times 1.065 \times 20 \text{ kPa}\} + \{2.675 \times 1.065 \times 0.99 \times (5 \times 2.18) \text{ kPa}\} = 161.8 \text{ kPa}$$

The right hand side is numerically equal to (300/2.18) + 24 = 161.6 kPa,

which is near enough the same.

Thus the required foundation width for the long term case is approximately <u>2.2 m</u>

Generally, it is unusual for the drained (long term) analysis to give a more critical result than the undrained (short term) analysis.

Deep foundations

Q8.4 Figure 8.39 shows a soil profile in which it proposed to install a foundation made up of a number of circular concrete piles of 1.5m diameter and 10m depth. Using the data given below, estimate the long-term allowable vertical load for a single pile, if a factor of safety of 1.25 on the soil strength tanϕ' is required.

(Assume that the horizontal effective stress at any depth is equal to (1-sinϕ') times the vertical effective stress at the same depth, that the angle of friction δ between the concrete and the soil is equal to 0.67ϕ', and that the long-term pore water pressures are hydrostatic below the indicated water table. Take the unit weight of water as 10kN/m^3, and the unit weight of concrete as 24kN/m^3.)

Data:

Bearing capacity factor = $K_p.e^{\pi \tan\phi'}$ × depth factor × shape factor, where

$K_p = (1+\sin\phi')/(1-\sin\phi')$

Depth factor = (1+0.2[D/B]) up to a limit of 1.5

Shape factor = (1+0.2[B/L])

and the foundation has width B, length L and depth D

Comment briefly on the assumptions $\sigma'_h=(1-\sin\phi').\sigma'_v$ and $\delta = 0.67\phi'$. Why in reality might it be necessary to reduce the allowable load per pile?

[University of London 3rd year BEng (Civil Engineering) examination, Queen Mary and Westfield College, slightly modified]

Q8.4 Solution

In the sands & gravels, $\phi' = 30°$ and $\phi'_{des} = \tan^{-1}\{\tan 30° \div 1.25\} = 24.79°$. The angle of soil/wall friction $\delta_{des} = 0.67\phi'_{des} = 16.61°$. In the clay, $\phi' = 20°$; $\phi'_{des} = \tan^{-1}\{\tan 20° \div 1.25\} = 16.23°$ and $\delta_{des} = 10.88°$.

Note that the horizontal effective stresses are calculated as $\sigma'_h=(1-\sin\phi').\sigma'_v$ using the full soil strength in each stratum, as to use the design soil strength would lead to increased values of σ'_h and hence unduly optimistic increased values of skin friction shear stress τ.

The skin friction shear stress $\tau = \sigma'_h.\tan\delta_{des}$, and varies linearly between successive "key depths", i.e. the soil surface, the water table, the interface between the sands & gravels and the clay, and the base of the pile. The skin friction shear stresses at these key depths are calculated as shown in Table Q8.4. The sands & gravels have saturated unit weight $\gamma = 20$ kN/m^3; the clay has saturated unit weight $\gamma = 18$ kN/m^3. In the sands & gravels, $\sigma'_h = (1-\sin\phi').\sigma'_v$ with $\phi' = 30°$, giving $\sigma'_h = 0.5 \times \sigma'_v$. In the clays, $\sigma'_h = (1-\sin\phi').\sigma'_v$ with $\phi' = 20°$, giving $\sigma'_h = 0.658 \times \sigma'_v$.

Stratum	*Depth, m*	σ_v, *kPa* $= \Sigma\gamma.z$	*u, kPa*	$\sigma'_v = \sigma_v - u$, *kPa*	$\sigma'_h = (1-\sin\phi').\sigma'_v$ *kPa*	δ_{des}, °	$\tau = \sigma'_h.\tan\delta_{des}$, *kPa*
S & G	*0*	*0*	*0*	*0*	*0*	*16.61*	*0*
S & G	*2*	*40*	*0*	*40*	*20*	*16.61*	*5.97*
S & G	*5*	*100*	*30*	*70*	*35*	*16.61*	*10.44*
Clay	*5*	*100*	*30*	*70*	*46.1*	*10.88*	*8.86*
Clay	*10*	*190*	*80*	*110*	*72.4*	*10.88*	*13.92*

Table Q8.4: Calculation of skin friction shear stresses at key depths

The skin friction force over each section of the pile (0 – 2 m depth; 2 – 5 m depth; and 5 – 10 m depth) is given by the pile circumference × the pile section length × the average of the shear stresses at the top and bottom of the pile section. Hence the skin friction force is

SF = $[\pi \times 1.5\text{ m} \times 2\text{ m} \times ½ \times 5.97\text{ kPa}] + [\pi \times 1.5\text{ m} \times 3\text{ m} \times ½ \times (5.97 + 10.44)\text{ kPa}] + [\pi \times 1.5\text{ m} \times 5\text{ m} \times ½ \times (8.86 + 13.92)\text{ kPa}]$

$= 28.13$ kN + 116.0 kN + 268.37 kN = $\underline{412.5\text{ kN}}$

The design base bearing effective stress is given by

$\sigma'_{f,des} = K_p \times exp(\pi.tan\phi'_{des}) \times depth\ factor\ d_q \times shape\ factor\ s_q \times \sigma'_o$

σ'_o is the in situ *vertical effective stress at the depth of the base of the pile = 110 kPa.*

$K_p \times exp(\pi.tan\phi'_{des}) = \{(1+sin16.23°)/(1 - sin16.23°)\} \times exp(\pi.tan16.23°) = 4.43$

Pile depth D = 10 m, breadth (diameter) B = 1.5 m, Length (on plan, also the diameter) L = 1.5 m

Hence D/B = 8.67 and B/L = 1 $\Rightarrow$; shape factor s_q = 1.2 and depth factor d_q = 1.5

$\sigma'_f = 4.43 \times 1.2 \times 1.5 \times 110\ kPa = 877.14\ kPa$

Area of pile = $\pi \times 1.5^2 m^2/4 = 1.767\ m^2$

$\therefore$ *base bearing load = 877.14 kPa $\times$ 1.767 m^2 = 1550 kN*

The upthrust on the base due to the pore water pressure is 80 kPa $\times$ 1.767 m^2 = 141.4kN

The design load is 412.5 kN (SF) + 1550 kN (BB) + 141.4 kN (pwp) = 2103.9 kN

The weight of the foundation is (1.767 m^2 $\times$ 10 m $\times$ 24 kN/m^3) = 424.08 kN, giving a design applied load of

2104 kN – 424 kN = <u>1680 kN</u>

The in situ horizontal effective stress may well be higher than assumed by the use of $\sigma'_h = (1-sin\phi').\sigma'_h$ in the clay, especially if the clay is overconsolidated. In general, $\sigma'_h = (1-sin\phi').\sigma'_h$ is a conservative estimate, allowing perhaps for some reduction from the in situ value due to installation effects (see also the earlier note regading the use of unfactored soil strengths in calculating horizontal effective stresses).

The friction angle δ between the pile and the soil is often assumed to be $0.67\times\phi'$ in coarse materials. In clays, however, particularly if the pile is rough, any failure surface will probably form in the soil, so that $\delta = 0.67.\phi'$ is again conservative. However,the use of a bentonite slurry to support the pile bore during construction could reduce interface friction if a skin of bentonite remains between the pile and the soil.

Interaction between closely spaced piles would probably reduce the ultimate load of n piles to less than n $\times$ the ultimate load of a single pile (due eg to a tendency to block failure).

Slopes

Q8.5 A partly-complete stability analysis using the Bishop routine method is given in the Table below. The configuration of the remaining slice (slice 4) and other relevant data are given in Figure 8.40. Abstract the necessary additional data from Figure 8.40, and determine the factor of safety of the slope for this slip circle.

Slice	weight w, kN/m	u.b, kN/m	ϕ'_{crit}, °	$n_\alpha \times$ (w - u.b).tanϕ'_{crit} for F_S = 1.45, kN/m
1	390	0	25	196.5
2	635	90	25	251.8
3	691	163	25	235.1
4	?	?	30	?
5	472	130	30	198.9
6	236	20	30	137.7

[University of Southampton 2nd year BEng (Civil Engineering) examination, slightly modified]

Q8.5 Solution
The Bishop equation must be used in the form given in main text Equation 8.35(a):

$$F_s = \frac{1}{\sum w.\sin\alpha} \times \sum \left\{ \left((w - u.b).\tan\phi'_{crit} \right) \times \left(\frac{1}{\cos\alpha + \dfrac{\tan\phi'_{crit}.\sin\alpha}{F_s}} \right) \right\} \qquad (8.35a)$$

(Simplification to the form given in Equation 8.35(b) is not possible in this case, because the slices have different breadths b.)

Let $\left(\dfrac{1}{\cos\alpha + \dfrac{\tan\phi'_{crit}.\sin\alpha}{F_s}} \right) = n_\alpha$.

The solution procedure is as follows:

1. Assume a value of factor of safety F_S
2. Calculate the values of w, $\sin\alpha$*, u.b and* n_α *(which depends on* F_S*) for each slice*
3. Determine whether Equation 8.35a is satisfied
4. If not, choose a new value of F_S
5. Repeat stages 2-4 until Equation 8.35a is satisfied

The weight of slice 4 is approximately 5 m $\times$ *{(6 m+7 m)/2}* $\times$ *20 kN/m*^{3} *= 650 kN/m*

The pore water pressure at the left hand edge of slice 4 is approximately 5.4 m $\times$ *10 kN/m*^{3} *= 54 kPa. The pore water pressure at the right hand edge of slice 4 is approximately 4.6 m* $\times$ *10 kN/m*^{3} *= 46 kPa. The average pore water pressure is therefore approximately 50 kPa, acting over a width b = 5 m, giving u.b = 250 kN/m. The remainder of the calculation for* F_S *= 1.45 is tabulated below (entries show in bold have been calculated)*

Slice	weight w, kN/m	α	**$w.\sin\alpha$ kN/m**	$u.b$, kN/m	ϕ'_{crit}	**$(w-ub)\times\tan\phi'_{crit}$**	**n_α for F_S=1.45**	$n_\alpha \times (w - u.b).\tan\phi'_{crit}$ for F_S = 1.45, kN/m
1	*390*	*+46°*	***280.5***	*0*	*25°*	***181.9***	***1.080***	*196.5*
2	*635*	*+34°*	***355.1***	*90*	*25°*	***254.1***	***0.991***	*251.8*
3	*691*	*+22°*	***258.9***	*163*	*25°*	***246.2***	***0.955***	*235.1*
4	***650***	*+10°*	***112.9***	***250***	*30°*	***230.9***	***0.949***	***219.1***
5	*472*	*-8.2°*	***-8.2***	*130*	*30°*	***197.5***	***1.007***	*198.9*
6	*236*	*-11°*	***-45.0***	*20*	*30°*	***124.7***	***1.104***	*137.7*

Table Q8.5a: trial slope stability calculation for Q8.5

For F_S=1.45, $\Sigma\{n_\alpha \times (w - u.b).\tan\phi'_{crit}\}$ (i.e. the sum of the entries in the last column) = 1239.4 kN/m. Dividing this by $\Sigma\{w.\sin\alpha\}$ = 954.1 kN/m, we obtain a calculated value of F_S (according to Equation 8.35a) of 1239.4 $\div$ 954.1 = 1.299, compared with the assumed value of 1.45. The assumed value is therefore too high.

Try F_S = 1.3:

Slice	weight w, kN/m	α	**$w.\sin\alpha$ kN/m**	$u.b$, kN/m	ϕ'_{crit}	**$(w-ub)\times\tan\phi'_{crit}$**	**n_α for F_S=1.3**	**$n_\alpha \times (w - u.b).\tan\phi'_{crit}$ for F_S = 1.3, kN/m**
1	*390*	*+46°*	***280.5***	*0*	*25°*	***181.9***	***1.050***	***191.0***
2	*635*	*+34°*	***355.1***	*90*	*25°*	***254.1***	***1.030***	***261.6***
3	*691*	*+22°*	***258.9***	*163*	*25°*	***246.2***	***0.942***	***231.9***
4	***650***	*+10°*	***112.9***	***250***	*30°*	***230.9***	***0.942***	***217.5***
5	*472*	*-8.2°*	***-8.2***	*130*	*30°*	***197.5***	***1.008***	***199.1***
6	*236*	*-11°*	***-45.0***	*20*	*30°*	***124.7***	***1.115***	***139.0***

Table Q8.5b: second trial slope stability calculation for Q8.5

For F_S=1.3, $\Sigma\{n_\alpha \times (w - u.b).\tan\phi'_{crit}\}$ (i. e. the sum of the entries in the last column) = 1240.1 kN/m. Dividing this by $\Sigma\{w.\sin\alpha\}$ = 954.1 kN/m (as before), we obtain a calculated value of F_S of 1240.1 $\div$ 954.1 = 1.3. This is the same as the assumed value of 1.3, hence

$\underline{F_S = 1.3}$

Q8.6 A slope failure can be represented by the four-slice system shown in Figure 8.41. By considering the equilibrium of a typical slice (resolving forces parallel and perpendicular to the local slip surface), and assuming that the resultant of the interslice forces is zero, show that the overall factor of safety of the slope $F_S = \tan\phi'_{crit}/\tan\phi'_{mob}$ may be calculated as

$$F_s = \frac{\sum\left[\left(w.\cos\alpha - u.l\right)\tan\phi'_{crit}\right]}{\sum\left(w.\sin\alpha\right)}$$

where the symbols have their usual meaning.

If the pore pressure conditions which caused failure of the slope shown in Figure 8.41 can be represented by average pore water pressures of 15kPa, 60kPa, 70kPa and 40kPa on AB, BC, CD and DE respectively, estimate the value of ϕ'_{crit} along the failure surface DE.

[University of Southampton 2nd year BEng (Civil Engineering) examination, slightly modified]

Q8.6 Solution

A free body diagram showing the forces acting on each of the four slices, ignoring the inter-slice forces, is given in Figure Q8.6. Resolving parallel to the base of an individual slice, assuming the inter-slice forces are zero,

$T = w.\sin\alpha$

Resolving perpendicular to the base of an individual slice (again assuming that the interslice forces are zero),

$N = w.\cos\alpha$

where α is taken as positive when the base of the slice slopes up from bottom right to top left (i.e. slices 1,2 and 3)

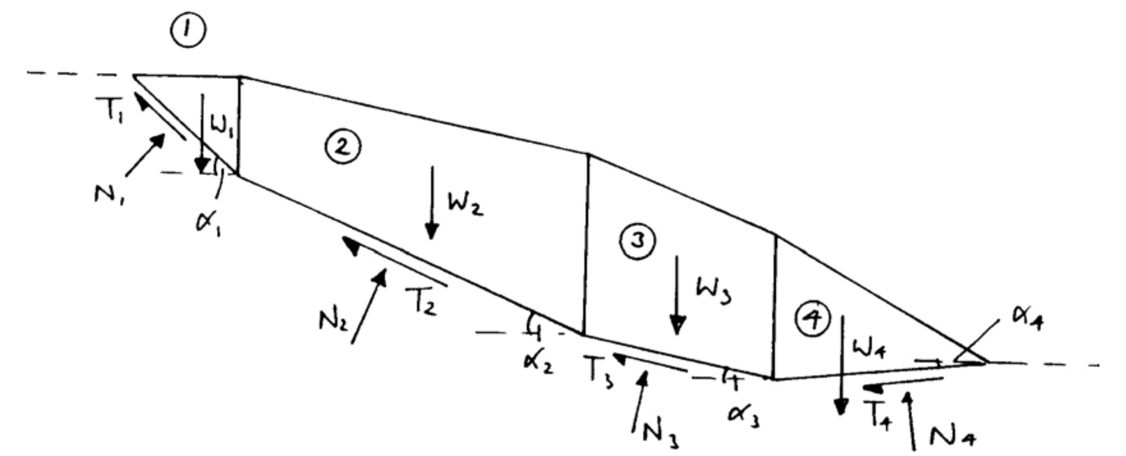


Figure Q8.6: Free body diagram showing the forces acting on each of the four slices

For each slice,

$T = (N-U).\tan\phi'_{mob} = \{(N-U).\tan\phi'_{crit}\}/F_S$

where $F_S = \tan\phi'_{crit}/\tan\phi'_{mob}$

The pore water force U acting on the base of a slice is equal to the average pore water pressure u $\times$ the base length l.

Hence for each slice,

T = w.sinα = {(w.cosα - u.l).tanϕ'_{crit}}/F_S, or

$$F_s = \frac{(w.\cos\alpha - u.l).\tan\phi'_{crit}}{w.\sin\alpha}$$

The overall factor of safety F_S for the system is given by

$$F_s = \frac{\sum[(w.\cos\alpha - u.l).\tan\phi'_{crit}]}{\sum(w.\sin\alpha)}$$

For each slice in the four slice system shown in Figure 8.41, the values of b, w, α, w.sinα, w.cosα,, u.l and (w.cosα - u.l).tanϕ'_{crit} (="NUM") are tabulated below:

Slice	*b, m*	*w, kN/m*	α	*wsinα kN/m*	*wcosα kN/m*	*l, m*	*u, kPa*	*u.l, kN/m*	ϕ'_{crit}	*NUM kN/m*
1	*8*	*640*	*47°*	*468.1*	*436.5*	*11.73*	*15*	*176.0*	*20°*	*94.8*
2	*25*	*5750*	*25°*	*2430.1*	*5211.3*	*27.58*	*60*	*1654.8*	*25°*	*1658.4*
3	*14*	*3640*	*12°*	*756.8*	*3560.5*	*14.31*	*70*	*1001.7*	*25°*	*1193.2*
4	*16*	*1920*	*-5°*	*-167.3*	*1912.7*	*16.06*	*40*	*642.4*	ϕ'_{DE}	*1270.3 $\times$tanϕ'_{DE}*

Table Q8.6: second trial slope stability calculation for Q8.6

In calculating w for slice 2, it is necessary to take account of the different unit weights of the two soil types present. The base length l of each slice is equal to b/cosα, where b is the slice width.

As the system is at failure, F_S = 1. Hence Σw.sinα = Σ{(w.cosα - u.l).tanϕ'_{crit}} = Σ{NUM}

From the table,

Σw.sinα = 3487.7 kN/m,

and

Σ{(w.cosα - u.l).tanϕ'_{crit}} = Σ{NUM} = 2946.4 + 1270.3.tanϕ'_{DE} kN/m.

Hence

3487.7 = 2946.4 + 1270.3.tanϕ'_{DE}

$\Rightarrow \ tan\phi'_{DE} = 541.3 \div 1270.3$

$\Rightarrow \underline{\phi'_{DE} = 23^{\circ}}$

QUESTIONS AND SOLUTIONS: CHAPTER 11

Modelling

Q11.1 Compare and contrast the use of physical and numerical models as aids to design. Your answer should address issues such as the assumptions that have to be made in setting up the model, limitations as to the validity of the results, and other factors which would lead to the use of one in preference to the other.
[University of London 3rd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q11.1 Solution
The answer should be in the form of a reasonably well-structured essay, illustrated with diagrams and examples as appropriate. The following notes give an indication of the expected scope.

Physical models
- *A 1/n scale model must be tested in a centrifuge at a radial acceleration of $n \times g$ so that stresses (which govern the soil stress-strain response and possibly peak and/or undrained strength) are the same at corresponding depths in the model and the field (self-weight stress σ_v at depth z is $\rho.g.z$ in the field and $\rho.ng.(z/n) = \rho.g.z$ in the model).*
- *A centrifuge model must be operated by remote control - in particular, it must be possible to simulate geotechnical processes such as excavation, embankment construction, diaphragm wall or pile installation, addition/removal of props etc.*
- *Must look carefully at scaling relationships and real-time effects of the simulated events (e.g. are they essentially drained or undrained?)*
- *Models are often plane strain, but 3-D modelling is not difficult.*

Numerical models
- *Often need to run in 2-D (plane strain or axisymmetric) because full 3-D modelling would require excessive CPU time.*
- *Plane strain modelling can be difficult to interpret, e.g. for rows of piles. (Physical modelling would enable this problem to be represented more reasonably by a line of discrete piles, even if deformation overall were constrained to be in plane strain).*
- *Results of an analysis can be critically dependent on the soil model and parameters used. Soil behaviour is still very difficult to describe mathematically. Problems can also arise in the use/omission of interface elements e.g. between soils and structures.*
- *It can be easier to follow construction processes in detail than in a physical model.*

General
- *Before using the results from either technique directly in a design, the applicability of the simplifying assumptions made in setting up the model would have to be considered very carefully.*
- *Physical modelling is useful to identify mechanisms of collapse and deformation, and to calibrate numerical models.*
- *Both can be used for parametric studies, to develop an understanding of the relative influence of different effect, and for investigating the sensitivity of the response of a system to unknown or uncertain boundary conditions or parameters in design.*

In situ testing

Q11.2 (a) Describe the principal features of the Menard and self-boring pressuremeters, and compare their advantages and limitations.

(b) Figure 11.27 shows a graph of corrected cavity pressure p as a function of the cavity strain ε_c for a self-boring pressuremeter test. The test was carried out in a borehole at a depth of 11 m in a stratum of sandy soil of unit weight 20 kN/m^3. The piezometric level was 1 m below the ground surface. Estimate

(i) the *in situ* horizontal total stress,

(ii) the coefficient of earth pressure at rest, K_o, and

(ii) the soil shear modulus, G;

Q11.2 Solution

(i) The in situ lateral total stress σ_{ho} is given by the lift-off pressure at which the cavity starts to expand. From the graph (Figure 11.27),

$\underline{\sigma_{ho} \approx 165 \text{ kPa}}$

(ii) At the test depth of 11m, the vertical total stress σ_v is 11 m $\times$ 20 kN/m3 = 220 kPa. The pore water pressure (assuming hydrostatic conditions below the piezometric surface) u is 10 m $\times$ 10 kN/m^3 = 100 kPa. Thus the vertical effective stress $\sigma'_v = \sigma_v - u = 120$ kPa; the horizontal effective stress $\sigma'_h = \sigma_h - u = 65$ kPa, and

$K_o = \sigma'_h/\sigma'_v = 65/120$

$\Rightarrow \underline{K_o = 0.54}$

(iii) The shear modulus G is obtained from the slope of the unload/reload cycle using Equation 11.24:

$$G = 0.5 \times (\rho/\rho_o) \times (dp/d\varepsilon_c) \qquad (11.24)$$

where ρ is the current cavity radius and ρ_o is the cavity radius at the start of the test (i.e. at ε_c = 0). The average cavity strain over the unload-reload cycle shown on Figure 11.27 is about 1.5%, i.e. ρ/ρ_o = 1.015 ($\approx$ 1). From the graph, the slope of the unload/reload cycle $dp/d\varepsilon_c \approx$ 500 kPa/1.1% = 45.5 MPa. Hence

$G = 0.5 \times (\rho/\rho_o) \times (dp/d\varepsilon_c) = 0.5 \times 1.015 \times 45.5$ MPa

$\Rightarrow \underline{G \approx 23 \text{ MPa}}$

Ground improvement

Q11.3 Write brief notes on:

(a) Grouting

(b) Surface compaction and heavy tamping

(c) Cement and lime stabilization

In each case, your answer should include (but not be restricted to) a discussion of the ground conditions and soil types for which the method is suitable.
[University of London 3rd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q11.3 Solution
(a) Grouting

• *Purpose: water stop (physical cut-off) or mechanical improvement (strength/stiffness) by bonding particles. Usually works by penetrating voids in between particles. Coarser soils are easier to permeate than finer soils owing to larger voids.*

• *Materials: cement based grouts are ok for fissured rocks and coarse materials (gravels). Cement particles will not penetrate a soil finer than a very coarse sand. Chemical/silicate grouts must therefore be used for medium/coarse sands. For finer soils, acrylic resin solution grouts are needed. It is, however, possible to inject grout into fissures and slip surface in clay soils to stabilize (at least temporarily) embankments and slopes.*

• *If the grout will not penetrate into the voids or pre-existing fissures, it can cause hydrofracture. Empirically, fracture pressure is ~ 2 to 6 × overburden. Long thin fractures are not helpful, but short wedge-shaped fractures can be useful in compacting the soil. Need to use pastes to achieve this.*

• *Generally, water stopping is easier than ground improvement, because it is necessary only to permeate the coarser zones. For satisfactory ground improvement all particles must be bonded, but a strong grout is not always necessary.*

• *Parameters governing the effectiveness of a grouting operation include the grout viscosity, shear resistance (shear stress as a function of strain rate), pumping pressure and flowrate into the ground: all must be carefully controlled. Viscosity varies with gel strength, and rate of gelation (setting) will depend in turn on factors including the ground temperature.*

• *Other applications include jacking up buildings, underpinning and compensation grouting (which is pre-emptive and used to prevent settlements of the ground surface due to e.g. tunnelling).*

(b) Surface compaction and heavy tamping

• ***Surface compaction*** *is most effective when applied to granular materials placed in layers. It involves the application of shear stresses (e.g. using smooth, tyred or sheepsfoot rollers); dynamic energy (e.g. using pounders or rammers); or vibration; or a combination of these.*

• *The objective is to densify the soil, increasing its (peak) strength and (more especially) its stiffness.*

• *Soils must be compacted in thin layers, generally 0.3 m to 0.5 m thick.*

• *The technique is not suitable for clays, except perhaps clay fills in clods in order to reduce the volume of air voids between the clods. In this case, there is a need to re-mould the clods by applying shear stresses: vibratory energy is ineffective.*

• *Compaction of any material - particularly a clay - requires careful monitoring and control.*

• ***Heavy tamping*** *involves dropping a large mass (up to 170 tonnes) from a height of up to 22 m in order to compact the soil. Usually, the mass is dropped onto a number of points in a grid or triangular pattern.*

• *The aim is to treat the soil at depth (up to 40 m: empirically, D (m) ~ 0.5 × √(WH) where W is the mass in tonnes and H is the height of drop in m), rather than just thin layers as in surface compaction.*
• *Originally intended for granular (free-draining materials), it can be effective in low-permeability soils because it causes fractures in the upper layers which allow water to escape in response to the excess pore water pressures generated by dropping the weight. Also, air voids can be compacted quite readily. The timing of the drops requires some thought in these materials.*
• *It is necessary to spread a 1 - 2 m thick stone blanket on the surface, to support the plant and prevent cratering.*

(c) Cement and lime stabilization
• *Both methods work by chemically bonding the soil particles. Typically, 2 - 10% cement or lime is added.*
• *Cement stabilization works with all soils (except perhaps coarse gravels where the voids are too large, and some inorganic soils). Cement and water react to form cementitious calcium silicate and aluminium hydrates which bond the soil particles together. This is the primary reaction, which releases $Ca(OH)_2$ (slaked lime) which may then react with the soil (especially clay minerals) to give a further beneficial effect.*
• *Lime stabilization essentially works on the basis of the secondary reaction with cement, and requires a substantial proportion (>35%) of fine particles (<60μm). The reaction initially involves the exchange of cations (e.g. sodium for calcium) between the lime and the clay, which causes the clay to coagulate.*
• *In the second stage of the clay/lime reaction, silica is removed from the clay lattice to form products similar to those resulting from the hydration of cement. This is the main source of "improvement", and the effectiveness of the cementation increases with particle surface area.*
• *Both processes improve volume stability, stiffness and unconfined compressive strength. Cement stabilization depends on adequate mixing and compaction, which can be difficult to achieve with clay soils.*
• *The addition of lime to clay improves workability because the plasticity index is decreased, although the exact mechanism of this (in terms of changes to w_{LL} and w_{PL}) will depend on the activity and mineralogy of the clay.*
• *The degree of cementation increases with the quantity of lime added, but the lime reaction uses the silica naturally present in the soil. There is therefore no point in adding more lime than will use up the available silica - indeed, adding further lime beyond this point can be counterproductive.*

Q11.4 Give an account of:
(a) The principal applications of grouting in geotechnical engineering
(b) The factors influencing the penetration of grouts into soils
(c) The major differences in properties and performance between cement-based grouts and low viscosity chemical grouts

[University of London 3rd year BEng (Civil Engineering) examination, Queen Mary and Westfield College]

Q11.4 Solution
The answer should be in the form of a reasonably well-structured essay, illustrated with diagrams and examples as appropriate. The following notes give an indication of the expected scope.

(a) Principal applications of grouting in geotechnical engineering

- *Prevention of groundwater flow (formation of a physical cut-off) - by blocking the soil pores*
- *Increasing soil stiffness and strength - by bonding soil particles*
- *Jacking up buildings*
- *Underpinning*
- *Compensation grouting (used to prevent settlements of the ground surface due to e.g. tunnelling)*
- *Stabilization of geotechnical structures such as tunnels, excavations and slopes*

(b) Factors influencing the penetration of grouts into soils

- *Particle/void size of the soil*
- *Viscosity and gel strength of the grout*
- *Pressure at which the grout is pumped*

(c) Major differences in properties and performance between cement-based grouts and low viscosity chemical grouts

- *Cement based grouts consist of fine cement particles in suspension, and the pore size that these grouts can penetrate is limited by the size of the cement particles. Cement grouts will penetrate into fissures and voids in coarse soils (i.e. gravels), but will not penetrate a soil finer than a very coarse sand. Grouts containing smaller particles such as sodium silicate in colloidal suspension are used for medium/coarse sands. For fine sands and silts, an acrylic resin solution grout must be used.*
- *Lower viscosity grouts are also better able to penetrate soils at a given pumping pressure, because the energy lost in overcoming the shear stresses that resist flow is reduced.*