### SELECTED PAPERS ON SOIL MECHANICS BY A. W. SKEMPTON, F.R.S.



A.W. Skemptin

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## Preface

This volume of selected papers by Professor A.W. Skempton is published to mark his 70th birthday on 4th June 1984.

In making the selection of papers we aimed to show Skempton's breadth of interest and achievement in the general field of soil mechanics and to include papers that the practising engineer, research worker and student will find of value to have to hand. For the early papers the selection was influenced by a historical interest in addition to intrinsic merit. As well as advancing the subject they reflect and are, on occasion, constrained by what was known at the time. This is well illustrated by the development of effective stress ideas in the context of the stability of slopes  $(10^*, 15, 19, 39, 53^*)^{\dagger}$ . With the more recent publications, length and availability have been a factor in the selection. Thus Skempton's 1964 Rankine Lecture (78) is not included as it is readily available, and his classic paper with D. H. MacDonald on the allowable settlements of buildings (48) proved too lengthy.

The chosen papers are reproduced chronologically, all but two falling neatly into three subject groups: soil properties, stability of slopes, and foundations. Though even a casual glance at the bibliography shows that Skempton has had a long-standing interest in many aspects of the history of civil engineering, the selection has been deliberately limited to his soil mechanics writings. The single "historical" paper that has been included, "Landmarks in early soil mechanics" (118\*), is an obvious choice. Skempton has not published many papers specifically on the topic of earth dams, yet he has made significant contributions to the design and construction of many such structures. The selection of paper (85\*) on the effect of discontinuities on the design of dams in the Mangla project is intended, in part, as a recognition of this contribution, which is discussed further in the Commentary (pp. 12–18).

In most respects the papers themselves speak for Skempton's immense and farreaching contribution to soil mechanics. However, it was felt that the inclusion of two brief essays covering his wider interests and his method of working would add an important personal touch to this volume. We are indebted to Rudolph Glossop and Bob Gibson for their articles. A brief commentary, prepared by members of the Soil Mechanics Section of Imperial College, has also been included to assist in placing the selected papers in the context of the state of knowledge at the time of publication and also to link them with papers that could not be included. We are grateful to our colleague Dr. R. J. Chandler for co-ordinating these contributions.

Permission of the following organizations to reproduce papers is gratefully acknowledged: Building Research Establishment (28\*); Comité Français de la Mécanique des Sols et des Fondations (69\*); International Commission on Large Dams (85\*); Norwegian Geotechnical Society (86\*); The Geological Society (92\*); The Royal Society (109\*); Japanese Society of Soil Mechanics and Foundation Engineering (113\*).

In 1981 Skempton presented his collection of early books and papers on soil mechanics to the Library of the Civil Engineering Department of Imperial College (120). It is entirely appropriate, therefore, that any royalties that accrue should be put towards the furtherance of this collection.

January 1984

J. B. Burland for the Soil Mechanics Section, Imperial College of Science and Technology Reference to
Skempton's papers is by
the numbers given in the
Bibliography
(pp. 277–280); an
asterisk indicates a
paper reprinted in this
volume.

# A. W. Skempton: Chronology

- 1914 4th June: born in Northampton.
- 1920 Pupil at Waynflete House Preparatory School.
- 1928 Pupil at Northampton Grammar School.
- 1932 Undergraduate in Civil Engineering Department, Imperial College, University of London.
- 1935 B.Sc. Awarded Goldsmiths' Bursary.
- 1936 M.Sc. (reinforced concrete). September: joined staff of Building Research Station.
- 1937 January: transferred to Soil Mechanics Section at Building Research Station under Dr. Cooling. August-September: field work at Chingford earth dam failure.
- 1938 Soil tests for foundations of new Waterloo Bridge.
- 1940 Investigation at Kippen foundation failure. July: married Nancy Wood, A.R.C.A.
- 1941 A.M.I.C.E. Field work at Muirhead Dam.
- 1942 Investigation at Kensal Green retaining wall.
- 1943 Field work at Naval Dockyard, Gosport.
- 1944 Investigations in the Fens at Eau Brink Cut and for flood relief channel.
- 1945 Lecture at Institution of Civil Engineers. October: started undergraduate soil mechanics course at Imperial College, on part-time secondment from Building Research Station. J. Kolbuszewski first research student.
- 1946 April: paper on Alexandre Collin read to Newcomen Society. October: appointed Senior Lecturer, Imperial College, with A.W. Bishop as assistant.
- 1947 March: appointed University Reader in Soil Mechanics, Imperial College. April: member, British National Committee of the International Society for Soil Mechanics and Foundation Engineering.
- 1948 June: first paper in *Géotechnique*. International Conference at Rotterdam. Field work at Peterlee, Co. Durham.
- 1949 Awarded D.Sc. (London).
- 1950 Started postgraduate course on soil mechanics with A. W. Bishop and D. J. Henkel.
- 1951 Investigations for Chew Stoke Dam.
- 1952 Field work at Jackfield landslide.
- 1953 European Vice-President, International Society for Soil Mechanics and Foundation Engineering (to 1957). General Reporter, International Conference at Zurich. Started course of lectures on history of civil engineering.
- 1954 Accompanied by his wife, visited U.S.A. and Canada; lectured at Harvard, M.I.T. and Urbana.
- 1955 Appointed Professor of Soil Mechanics, Imperial College.
- 1957 M.I.C.E. August: elected President of International Society of Soil Mechanics and Foundation Engineering (to 1961). October: appointed Professor of Civil Engineering, Imperial College, and Head of Department (to 1976).
- 1958 First visit to Mangla Dam, Pakistan.
- 1961 Elected F.R.S.
- 1963 Elected member of Smeatonian Society of Civil Engineers. Research on residual strength of clays in connection with Walton's Wood landslide.
- 1964 Rankine Lecture. Member, Cathedrals Advisory Committee (to 1970).

- 1965 Field investigations on Pleistocene solifluction near Sevenoaks.
- 1966 First visit to Pisa.
- 1968 D.Sc.(Hon.), Durham University. Ewing Gold Medal, Institution of Civil Engineers.
- 1970 Chairman, Geotechnique Advisory Panel (to 1972).
- 1971 Field work and design of remedial measures, M4 landslides near Swindon.
- 1972 Lyell Medal, Geological Society of London.
- 1974 Dickinson Medal, Newcomen Society. Vice-President, Institution of Civil Engineers (to 1976).
- 1976 Foreign Associate, National Academy of Engineering, U.S.A. Founder member, Fellowship of Engineering.
- 1977 President, Newcomen Society (to 1979). Field work on Mam Tor landslide, Derbyshire. Lecture on London Clay slopes, at Tokyo International Conference.
- 1978 November: Hitchcock Foundation Professor, University of California, Berkeley.
- 1979 With Charles Hadfield as co-author, published book William Jessop, Engineer.
- 1980 D.Sc.(Hon.), University of Aston.
- 1981 President, Smeatonian Society. Karl Terzaghi Award, American Society of Civil Engineers. Portrait painted by Richard Foster. October: retired as Professor of Civil Engineering. Appointed Senior Research Fellow at Imperial College.
- 1982 Gold Medal, Institution of Structural Engineers. First visit to Kalabagh Dam site, Pakistan. D.Sc.(Hon.), Chalmers University, Gothenburg.
- 1983 Research on residual strength of clays at rapid rates of displacement, for seismic design of Kalabagh Dam.

# A short biographical essay

### **R. GLOSSOP**

The circumstances which led to my first meeting with Skempton, early in 1939, were unusual. I was then an engineer working at Chingford Reservoir where I had very recently set up a small soil mechanics laboratory. One day during a party conversation with an old friend, she, knowing something of my activities said, "I met a remarkable young man the other day who should interest you. He is doing research on the properties of clay, and he bought a picture from John Tunnard's exhibition at Peggy Guggenheim's gallery." My reply was, "If such a man exists, I must meet him at once. What is his name?" She had forgotten it, so as soon as possible I went to Guggenheim Jeune, in Cork Street. Mrs. Guggenheim was away, and an assistant was in charge.

"Can you tell me the name of a young man who bought a Tunnard and is researching on clay?" I said. This simple question led to an extraordinary conversation, based on total miscomprehension, which threatened to go on for ever. At last, it dawned on me that while I was talking about clay she, not unnaturally, was talking about Klee the Swiss painter. I tried again, and suggested that we should look through the firm's books. There I found a recent purchaser called Skempton, whose address was Watford, near the Building Research Station. "That's my man," I said, and so he proved to be.

Skempton and Tunnard had known each other since 1937 when they first met in Cornwall. There they had formed a friendship which lasted until John's death in 1971. On a visit to Cadgwith in 1944, Skempton spoke to him of work on which he was then engaged—the design of a flood relief channel in the Fens. Tunnard, when at Charterhouse, had been regarded as a promising mathematician and had no difficulty in understanding it. It delighted him for he was devoted to the fenlands where he had spent his boyhood and, as can be seen from many of his pictures, he was interested in the designs which are generated by modern technology. On returning to London, Skempton sent him a drawing of the Fellenius analysis. To John, this suggested a fusion of science and landscape, which he expressed in a magnificent abstract painting, reproduced here together with Skempton's own drawing (12). It comes as near as may be to collaboration between an artist and a scientist.

To return to 1939: shortly after the episode at the Guggenheim gallery I visited the Building Research Station to study their methods of soil testing. It was then that I formed a lasting friendship with Skempton and his future wife, Nancy Wood—an artist in her own right—who shares and contributes to many of her husband's interests.

The B.R.S. laboratory had not long been in existence. In 1936 L. F. Cooling represented the Station at the Harvard Conference, and his report had convinced the Director that the small soil mechanics section, headed by Cooling himself, justified some expansion. The first recruit was A.W. Skempton, followed a few months afterwards by H. Q. Golder and, when the latter left in 1942, by W. H. Ward. These three young men, highly intelligent, hardworking and enthusiastic, created, under Cooling's direction, and with the help of two or three assistants, the first effective soil mechanics laboratory in England.

In 1937 the earthen embankment of a large reservoir under construction at Chingford, in Essex, failed. Sir George Burt, of the contractors John Mowlem and Company, was then Chairman of the Advisory Committee to the Building Research Station, and turned to its Director, Dr. R.E. Stradling, for advice. Thus it came about that Cooling and his team had the good fortune to undertake the first major soil mechanics investigation in England.

The possible consequences of this failure were so grave that Sir George Burt sought a

#### SELECTED PAPERS ON SOIL MECHANICS



Painting by John Tunnard, A.R.A. (1944)

> second opinion from Terzaghi himself. He came to England from Vienna, confirmed the findings of the Building Research Station, and redesigned the embankment. While at Chingford, he met and established a strong intellectual link with Skempton; that Terzaghi's link with his English friends also had their festive side is shown in the photograph of a picnic among the megaliths at Avebury.

> As the significance of the Chingford investigation became known, Cooling and his team were inundated by enquiries. Thereafter, they were never short of work on construction sites, and their research was planned in response to practical problems. It was in this environment of close collaboration with industry that Skempton passed the first years of his professional life and he greatly benefited by it.

By 1945 universities were planning courses in soil mechanics and he was approached by more than one. Fortunately, in 1946, he chose to return to Imperial College. There he was given his own section, and Professor A.J.S. Pippard left him with almost complete freedom to make his own decisions affecting both teaching and research.

At this very time, a chance event greatly influenced his future. Since 1938, Terzaghi had been retained as consultant by John Mowlem and Company, but he had subsequently become resident, and firmly established as a consultant, in the U.S.A. This made the relationship inconvenient to both sides and it was terminated. When the name of Skempton was mentioned to Sir George Burt as a replacement, he agreed that in spite of his youth, he was the right man. Thus Skempton was able to retain a strong link with industry and the Mowlem sites were always open to him. In a few years he built up a selective practice on works which combined practical and scientific interest, and became adviser on geotechnical matters to a number of leading consulting engineers, including Sir Alexander Gibb & Partners, and Binnie & Partners.

During the period between 1945 and 1948, leading up to the Rotterdam Conference, he was joined by A.W. Bishop, who became lecturer in soil mechanics, and a year or two later by D. J. Henkel. This group devised and established a highly effective research laboratory, and in addition to regular undergraduate teaching, research and consulting work, they started a little later the first post-graduate course in soil mechanics to be given in England.

Concurrently, Skempton served on the working party responsible for the first edition of the Code of Practice on "Site investigation"; he delivered one of the four lectures on "The principles and application of soil mechanics" (12) given at the Institution of Civil Engineers in 1945, and was one of the group of five who formed the old Geotechnical Society, and launched the journal *Géotechnique*.

#### 6

Photograph: R. C. Packer



Design of proposed flood relief channel in the Fens: the basis of Tunnard's painting

It was during this period that Skempton published the first papers in a long series principally concerned with three subjects: the variation of the geotechnical properties of sediments with depth, the stability of natural slopes, and the influence of discontinuities on the shear strength of clays. The interest of these lay in his ability to apply soil mechanics theory to complex, and hitherto ill-understood, geological phenomena. This was recognised by his election to Fellowship of the Royal Society in 1961, and the award of the Lyell Medal of the Geological Society of London in 1972, and by other academic and professional honours.

Skempton developed an interest in the history of engineering at an early age, and an important part of his life's work lies in his contribution to it. His first paper on a historical subject, published in the Transactions of the Newcomen Society for 1946 (13), was devoted to the work of Alexandre Collin, whose investigations—published exactly 100 years before—of the stability of the cuttings in clay on the Canal de Bourgoyne had been virtually forgotten.

Thereafter, he turned his attention principally to the history of civil engineering in England, and over a number of years published papers on a diversity of subjects. However, two lines of enquiry predominated; the first was on the lives and works of the earliest English engineers—a subject that had not previously been studied systematically, and which culminated in a book on John Smeaton (122) and another, written with his friend Charles Hadfield, on the neglected but important engineer William Jessop (116). Secondly, he wrote on the architectural and structural characteristics of early industrial buildings, including studies on the introduction of iron framing.

It can be asserted that this impressive body of work amounting to more than twenty papers as well as the two biographies, is one of the most significant contributions yet made to the history of English civil engineering.

During the long period of some thirty years that he has spent on such studies, he persisted with his enquiries into the history of soil mechanics, and at the Brighton Conference in 1979 published his findings in "Landmarks in early soil mechanics" (118\*). The title gives no suggestion of the range and depth of this paper, which is a minor classic of its kind. On first reading it, one wished that it could have been published as a monograph, or as a supplement to *Géotechnique*, rather than that it should lie buried in the relatively inaccessible proceedings of a conference. Fortunately its inclusion in this volume will make it better known.

Two unusual works demand attention: "Early printed reports and maps (1665–1850) in the Library of the Institution of Civil Engineers" (114), which will remain a valuable

Photograph: B. S. N. Glossop



source book for historians, and a beautifully produced little book, "A bibliographical catalogue of the collection of works on soil mechanics, 1764–1950" (120). This catalogue refers to a collection presented by Skempton to the Library of the

Department of Civil Engineering at Imperial College. It is already prized by collectors. To turn to his more personal interests: music is certainly to him one of the major pleasures in life, yet he came to it rather late, and it was only at the age of twenty-six that he discovered the music of Bach and the possibilities of the gramophone. Soon he concluded that passive listening was not enough: to understand and truly appreciate music one must be oneself a player. He chose the flute, and by intense application became competent, and after a few years more than competent. Thereafter, he regularly took the part of second flute at the Easter performances of *St. Matthew Passion* at Bury St. Edmunds Cathedral, and was then invited to join a small group—all friends and admirers of Ralph Vaughan Williams—who yearly took part in the Dorking Festival, conducted by the old master himself.

While retaining a love of music Skempton has in recent years turned more actively to book collecting, with a special interest in early printed reports by English civil engineers. Of these he has now formed perhaps the best collection in private hands, enriched by fine bindings carried out by his wife. She too has some splendid books—on wood engraving, book binding and typography. Visitors to the Skempton's flat in The Boltons, South Kensington, indeed have much to enjoy but will remember, above all, the unfailing warm hospitality which they find there.

R. Glossop, K. Terzaghi and A.W. Skempton at Avebury, Wiltshire in 1950

## Working with Skempton

### R. E. GIBSON

The Soil Mechanics Division of the Building Research Station had established by the end of the Second World War a pre-eminent position at the forefront of soil mechanics research in Britain. As a senior member of the scientific staff, Skempton had been invited in the autumn of 1945 to give a course of lectures on this new subject in the Civil Engineering Department at Imperial College. The need for this discipline to be taught was so great, and Skempton's lectures were so widely appreciated, that he was invited to join the staff at the beginning of the next academic year. Fortunately for us all he accepted. Within a year he had become the first Reader in Soil Mechanics in the University of London and Assistant Professor at Imperial College—a rare academic title which has now become defunct. Those few who held the title were invariably addressed as "Professor", so he became "Professor Skempton" at the remarkably early age of 33.

His first task was to build up a laboratory, and this was well under way when I arrived as one of his first research students in May 1947. Just after the war there was little money available for refurbishment and equipment, and in the main we had to make do with whatever could be made in our workshops or borrowed. Alan Bishop, who had been appointed as Skempton's assistant, brought on loan from the Metropolitan Water Board, a large and magnificent shear box which was put to constant use for tests on sand and gravel. In addition, a bank of three small shear boxes, also designed by Bishop, had just arrived from Farnell's, and Skempton immediately put me to work on these, testing clays. The consolidated "quick" tests were easy to carry out, but as no motor or gearbox had then arrived the drained tests required a handle to be turned smoothly, at an unbelievably slow rate, for a period of hours. Fortunately for me, all joined the queue to take their turn—Skempton included!

Those who have had an opportunity to work with Skempton on an engineering or research project have become aware first of his outstanding ability to reduce a problem to its essentials. A stimulating discussion invariably follows in which everyone joins: this seeks to identify the important questions to be answered and considers how best to go about this task. Understanding the need for opinions to be expressed freely, he guides discussion with a light touch, regarding himself merely as *primus inter pares*. Irrelevant remarks are allowed to pass; foolish suggestions call forth only mild disagreement; clever ideas are welcomed, but not unduly praised. This style reflects Skempton's innate consideration for others, his powers of judgement and his determination, rightly, to reserve for himself the final word. Conclusions are summarised succinctly and what needs now to be done is stated unequivocally.

Whenever he anticipates unusual difficulties this stage may be followed by consultation with those whose knowledge in special areas he respects. Never content merely to accept what he is told, he adopts the mantle of the student and questions them closely to reveal the path along which conclusions have been reached and, furthermore, to master for himself the details of the reasoning. He will not hesitate to acknowledge his inability initially to follow an argument, but will persist until its essence has been grasped and he has formed his own opinion. This unassuming and scholarly approach, entirely free from pomposity, makes a profound impression on his students and they, of course, warm and respond to it.

It was in this stimulating atmosphere that I had the opportunity to work with Skempton on several consulting jobs. One of these was in connection with the foundation design for a new grain silo at Durban (I.C. Report 1). This was the first chance I had to work with him in the laboratory and to learn the systematic way in which he went about his task. In this case a site visit was not possible, but U4 samples had been taken and air-freighted in. The clay was fairly firm, not unduly sensitive and arrived in good shape. Even in those days Skempton worked exclusively on plain foolscap paper and on each sheet an outline facsimile of a U4 was drawn. Within the border the individual samples taken from the tube were noted and numbered, and on the sheet all the test data were finally assembled. These sheets were carefully preserved and stored in the job file.

As each tube sample was extruded, thin discs were sliced off with a cheese-wire and carefully examined. Except close to the waxed ends and immediately around the perimeter where oxidation and disturbance had taken place, the clay was carefully husbanded and used for natural moisture contents and so on. Unconfined compression samples were obtained from an extruded length using thin-walled brass tubes, well-oiled inside and out. After these "undisturbed" samples had been tested the oily surface was carefully trimmed away with a spatula and each sample remoulded for retesting. As some loss of moisture had inevitably occurred, Skempton advocated that each sample be spat upon during remoulding—once or twice according to the remoulding time—to make good the deficit. This was the only subjective judgement I was allowed to make!

In most jobs, as these data accumulated, borehole logs with detailed sample descriptions and profiles showing the variation with depth of the measured parameters were constructed. For this, Skempton would work in his study on a drawing-board, plotting individual results in pencil on a single large sheet of graph paper. As a pattern emerged and confidence in it grew, the size of the plotted points according with it also tended to expand. Isolated points far from the trend would be carefully considered and, depending on status, either placed in parentheses or summarily erased. Every bit of information from the boreholes and the laboratory tests would be studied and evaluated, the significant data being recorded on this master plot.

While this activity was going on, the engineering problems were being thought over but would rarely be addressed explicitly—or perhaps even mentioned—until this stage had been reached. Then Skempton would usually work alone, making simple calculations and sketches and developing his own ideas. This might occupy hours or days depending on the job, but the time came when we would be summoned to listen and discuss. To act as a sounding-board would more aptly describe my role. One might even be able to contribute something worthwhile: we were certainly not denied encouragement. The equality assumed on these occasions exemplifies an entirely natural and delightful aspect of Skempton's character by which he fosters and develops young talent, deliberately ignoring rank and joining in a common quest. Where, as in a formal research project, the time-scale is long and the final objective more remote, this ability to stimulate and sustain makes him in his field one of the outstanding research supervisors of his generation.

Finally, the report would be hand-written, never dictated. All the drawings he prepared meticulously for tracing. Perched on a high stool, peering over his left shoulder as he wrote, I saw the lines emerge—apparently effortlessly. His thoughts were evidently ordered so well and clarity of expression came so naturally, that even a long report could be finished at a sitting.

In the early days I gained a great deal, of course, just by observing how this exceptionally gifted man went about his work. As my knowledge developed I was also able to benefit from the informal technical discussions which Alan Bishop, David Henkel and I had with him, often together over lunch. Talks were usually around jobs on which one or more of us were engaged, and we all came to value this regular exchange of ideas. No less rewarding were Skempton's lectures. These were prepared with great care and given with the utmost formality. The delivery itself was reflective— on occasion almost hesitant—imbued with a compelling logic and totally gripping. How often I heard the plaintive cry, "I never took any notes!"

While he laid no claim himself to special mathematical skills, Skempton, like

Terzaghi, understood well the value and limitations of analysis. He encouraged those with talent in this direction to apply it with discrimination to significant problems. I came to benefit from this in an unexpected way.

Consultants responsible for the design of an earth dam in the Usk Valley of Wales had become concerned during the early stages of construction by the high pore-water pressures recorded in the clay fill (56). Stand-pipes showed water levels well above the top of the fill and there was some anxiety that at a later stage of construction stability might be endangered. Skempton's opinion was sought. Before the risks could be assessed it was necessary to forecast future development of those pore-water pressures. Fortunately, careful records had been kept, but he immediately realised that to analyse these and to predict future trends a theory was required which took proper account of the increasing thickness of the fill during construction. This, we believed, was not then available. "Surely, this is a problem right up your street, Bob. It wouldn't be too difficult for you, would it?" Unable to resist the challenge, I soon discovered that indeed it was. But the problem nagged, and I found myself returning to it again and again until it had been mastered. I don't doubt that this is what had been intended!

It is not uncommon for an engineering don to engage in both consulting and research work, and usually these are quite closely related. However, with Skempton, the two are integrated to a quite remarkable degree; indeed, I can recall no work of his where both elements are not present. Nevertheless, they are but two facets of his total activity. His extensive bibliography demonstrates his outstanding contribution to his profession and to the world of scholarship and ideas—but it cannot convey the depth of his commitment as teacher, mentor and friend.

This is something recognised and greatly valued by those of us who have been privileged to work with him.

### Commentary

This commentary traces briefly the development of some of the major topics which have been treated by Skempton, and places the papers selected for reproduction in this volume in the context of the state of knowledge at the time of publication.

#### SOIL PROPERTIES

Whilst at the Building Research Station, Skempton was concerned with a number of problems (or "jobs" as he would call them) which involved soft clays. Some of these concerned bearing capacity or slope failures that led to the series of papers on the  $\varphi = 0$ method of stability analysis (4, 10\*, 15, 20); others, while less immediately spectacular, none the less required detailed investigations to be made of the geotechnical properties of soft clays. These investigations, coupled with a review of work on similar deposits in the U.S.A. and Sweden, led him to the inescapable conclusion that in normally consolidated clays the undrained strength increased with depth. Whilst present generations of geotechnical engineers take such relationships for granted, in 1948 when his paper on the geotechnical properties of post-glacial clays (14\*) was published, this conclusion was not generally accepted. Indeed, as Skempton mentions in that paper, no less a person than Karl Terzaghi had only recently (Terzaghi, 1947) suggested a soil fabric that might explain why the strength of soft clays was often apparently uniform with depth. Of course, with the benefit of hindsight, such observations can at least partly be explained by the difficulties in sampling. With this background information, the reader begins to appreciate more clearly the careful marshalling of the evidence and development of the various lines of argument, characteristic of all Skempton's papers, which underlines the impact that this particular paper made at the time of publication. Appropriately, it was the first paper in the very first number of Géotechnique.

In a historical context it is interesting to note that in this paper the increase of strength with depth is given in terms of the consolidated undrained angle of shearing resistance, and that although c/p values are quoted this ratio was not used to express the strength-depth relationship until a little later in 1948 (23). By 1954 Skempton had been able to collect data on the c/p ratio from some eight normally consolidated clays (41) and had demonstrated a close correlation between this ratio and plasticity index.

Also in 1948 Skempton drew attention to the interconnection between liquid limit, the proportion of clay-sized particles ("clay fraction") and mineralogy for a number of clays (21). Subsequently he realised that a linear relationship was obtained by plotting plasticity index against clay fraction, thus deriving a simple quantification of "activity" as the ratio of plasticity index to clay fraction ( $32^*$ ).

In several papers up to about this time he makes reference to Hvorslev's parameters of "true" cohesion and friction. Hvorslev's work had become known as a consequence of the Harvard International Conference in 1936 and had, rightly, attracted a good deal of attention. After the Second World War, as university research in soil mechanics developed, the avenues opened up by Hvorslev were explored, notably at Imperial College. Other engineers to whom Skempton refers most frequently in this early period are D.W. Taylor and Arthur Casagrande. For the latter in particular he had a profound admiration, movingly expressed in his obituary notice of this great pioneer of modern soil mechanics (121).

The increasing realisation in the late 1940s of the role played by effective stress in controlling the shear strength of soils led to this concept being used in design,

particularly for earth dams. Here it soon became apparent that effective stress consideration of the undrained short-term construction and rapid drawdown situations required prediction of the associated pore pressure changes. This need was fulfilled in 1954 with the introduction of the pore pressure parameters A and B (40<sup>\*</sup>), though it should be noted that the parameter A had previously been defined in a paper of 1948 (22).

In 1960 (62\*) Skempton turned his attention to the physics of the effective stress concept, and in so doing confirmed that Terzaghi's deceptively simple equation  $\sigma' = \sigma - u$  for saturated soils could for all practical purposes be assumed correct, although that conclusion did not apply to rocks or concrete. More than any other paper in the collection, this shows Skempton's ability to utilize the findings of other disciplines—from Bowden & Tabor's (1954) work on metallic friction to the results of tests on concrete, calcite and rock salt.

Another outstanding paper is that entitled "The horizontal stresses in an overconsolidated Eocene clay" (69\*). Using the data provided by a series of carefully executed oedometer and triaxial tests on London Clay from a site in Essex, four different methods are used to deduce the profile of  $K_0$  with depth. Skempton reached what at the time must have seemed the surprising conclusion that  $K_0$  could reach values as high as 2.5. This paper attracted a complimentary and enlightened discussion from Terzaghi (1961). Subsequently others have measured similar high  $K_0$  values in the London Clay, confirming the bold conclusions of the paper.

In 1967 Skempton was invited to deliver a special lecture at the European Soil Mechanics Conference at Oslo. He chose as his subject the strength along structural discontinuities in stiff clays (86\*). It was written several years after the Rankine Lecture (78) had drawn the geotechnical world's attention to the nature and problems of residual strength, but only a year or so after he had completed his work on the design of the Mangla Dam. At Mangla it had been realised that bedding plane slip during tectonic folding of the Siwalik clays had led to the development of extensive sheared surfaces. Since the Siwaliks formed the foundations of the dam and spillway, the residual strength exhibited by these surfaces posed a serious threat to stability. His Oslo paper thus provided Skempton with the opportunity to develop further the ideas expressed in the Rankine Lecture, and in doing so he presented not only a collection of interesting case records involving residual strength (including Mangla), but also the first set of data on the strength characteristics of stiff clay joint surfaces. His research assistant D.J. Petley was co-author of this paper, and of several others in this period.

With the last of this series of papers we return to Skempton's interest in normally consolidated clays. Back in 1944 he had written a paper (8) in which he considered the compressibility of clays, largely on the basis of laboratory data obtained in the oedometer, although some data relating to in situ natural clays were included. Then, in 1966 he became a member of an international panel of engineers charged with investigating the foundation conditions of the Leaning Tower of Pisa. This revived his interest in the consolidation of normally consolidated clays, culminating in the publication of his paper "The consolidation of clays by gravitational compaction" (92\*). In its general principles this paper does not add significantly to the earlier one, but it provides a treasure-house of field data on the in situ water content and strength of a series of clays with a wide range of index properties over a wide range of depths—truly a "data base" against which the reliability of soft clay data may now be tested.

#### THE STUDY OF SLOPES

At the time of Skempton's entry into soil mechanics Terzaghi had supplied the basic concepts necessary for a sound development of the subject but much, of course, remained to be explored. This was particularly the case in the area of slope stability, where Terzaghi's application of the principle of effective stress had been curiously limited. In his first publication on slope stability in 1943 (6), an appendix to the first

edition of Blyth's A geology for engineers, Skempton reflected the then current view that although sands were frictional, the strength of clays could be represented by an undrained apparent cohesion which is practically independent of the changes in normal pressure acting. The idea that in excavated slopes in stiff fissured clays the shear strength may slowly reduce through softening and bring about a slip many years after excavation is, however, also touched upon—an effect forcibly drawn to his attention by investigations in 1942 at Kensal Green (B.R.S. Report 110).

Skempton's first major paper on the subject was in 1945: "A slip in the west bank of the Eau Brink Cut" ( $10^*$ ). This has all the hallmarks we associate with his work: a comprehensive literature review, good site investigation and geological appreciation, careful laboratory testing, and critical comparison of the predictions of theoretical analyses with field behaviour. The paper is notable in particular as being the first to compare an actual slope failure with the result of stability analyses by both total stress and effective stress methods.

In Britain the Second World War had to some extent inhibited the development of soil mechanics that would otherwise have been expected to follow Terzaghi's pivotal James Forrest Lecture (1939). The Second International Conference, at Rotterdam in 1948, was thus a tremendous event, in which the pent-up energies and accumulated experience of the previous decade found their expression. This was no better exemplified than by Skempton's own contributions to the conference (15–21), which amounted to seven papers, some of which have proved to be classics and can still be read with benefit.

In the context of slopes may be mentioned the paper (15) in which the  $\varphi = 0$  analysis is examined in terms of effective stress and Hvorslev's parameters, leading to the conclusion that the analysis is strictly relevant only to short-term stability in saturated clays where no water content change has occurred. Field evidence supporting this analysis is provided in a companion paper written with H. Q. Golder (17). Another of the Rotterdam papers (19) represents Skempton's first essay on the problem presented by the delayed slips in railway cuttings in London Clay. Based on Terzaghi's concept of softening in stiff fissured clays, it uses a total stress analysis as an empirical method of establishing, from available case records of failures, a time-scale for the decay in strength following excavation.

His interest in natural slopes is expressed in a paper given in 1950 (27) on the boulder clay valleys near the New Town of Peterlee in Co. Durham. Here the concept of a limiting or ultimate slope angle is derived from extensive field observations, though as yet with little backing from soil tests or theoretical analysis.

The first advance in this aspect of the subject arose from investigations with D. J. Henkel in 1952 on a landslide at Jackfield, Shropshire, in a heavily overconsolidated clay (39). For the "long-term" conditions of this slope the authors had no hesitation in applying an effective stress analysis based on measured pore pressures and drained shear tests, using geometry defined by the observed slip surface. They found the back-analysed factor of safety to be too high by nearly 50 per cent, reasonable agreement being obtained only when the cohesion intercept was neglected, i.e. when the c' = 0 or "fully softened" strength was used. This seemed to be consistent with the observed much higher water content on the slip surface, as compared with water contents in the clay elsewhere: an effect already noted in a slip in the London Clay at Mill Lane (B.R.S. Report 138).

The Jackfield paper made an immediate impact. In an interesting aside in his contribution to the obituary tributes to Laurits Bjerrum (103), Skempton remarked that it was as a result of a stimulating evening discussion with Bjerrum, Hvorslev and Peck, on the occasion of the 1953 Zurich Conference, that he determined to apply effective stress analysis to slope problems in the fissured London Clay: a somewhat revolutionary idea at the time, that was adopted a year before publication of the Jackfield results.

The first-fruits of this decision are seen in two papers presented to the 1957 London Conference: one by Henkel on long-term slips in cuttings; the other by Skempton and

#### COMMENTARY

DeLory (53\*) on the ultimate stability of natural London Clay slopes. In both cases the fully softened strength was found to give reasonably good agreement with back analysis and a significant step forward had been made, though the long delay in cutting failures could not yet be convincingly explained. Later, of course, Skempton himself showed that in the natural slopes the residual, not the fully softened strength controlled limiting stability. None the less this paper, and also the Peterlee field observations, well illustrate his interest in geomorphology, particularly in the processes of slope development, which led to many geomorphologists being introduced to soil mechanics (e.g. Carson & Kirkby, 1972).

The next contribution was the paper in 1961 with D. J. Brown on a landslide in boulder clay at Selset, Yorkshire (72<sup>\*</sup>). This case record provided an opportunity to analyse a slip in a natural slope of essentially intact over-consolidated clay, with a longer time-scale than that represented by the earlier classic case record of a slip at Lodalen, Oslo (Sevaldson, 1956). The result supported the indication of the latter case record that in such non-fissured clays the full cohesion intercept c' is operative on the slip surface of a "first-time" failure.

In 1964 Skempton was invited to give the Fourth Rankine Lecture, for which he chose the subject which had dominated his thinking on slopes for at least the preceding ten years: "Long-term stability of slopes" (78). In this he demonstrated, for the first time, three highly significant results: first, that the strength on pre-existing shear surfaces in clay is distinctly lower than the fully softened value, due to re-orientation of the clay particles at large displacements; second, that this lower or "residual" strength as measured in the laboratory agrees well with back-analysis of reactivated landslides at Walton's Wood, in weathered Carboniferous shale, and at Sudbury Hill, in London Clay; and third, that at Walton's Wood it also agrees with the strength as measured on the shear surface itself.

The investigations on Walton's Wood landslide, carried out in conjunction with Soil Mechanics Ltd., and extended to include microscope studies of the sheared clay, were subsequently published in great detail in a paper with K. R. Early (101).

In the light of these discoveries, the Rankine Lecture went on to re-examine the Jackfield landslide and the London Clay natural slopes, arriving at much improved interpretations. In retrospect, however, it is evident that a further suggestion made in the Lecture—that even in "first-time" slides the peak strength of fissured clays tends to fall inexorably to the residual—was incorrect.

This erroneous view was later emphatically corrected in a paper (93) reaffirming that the strength in first-time slides in London Clay does not fall below the fully softened or "critical state" value, displacements up to the instant of failure not being sufficient to generate the residual state. Finally, the development of ideas on first-time slides in brown London Clay—from Terzaghi's (1936) suggestion of progressive softening along joints and fissures to Vaughan & Walbancke's (1973) concept of delayed failures being related to the dissipation of negative excess pore pressures caused by the excavation was excellently summarised by Skempton in 1977 in his Special Lecture to the Ninth International Conference in Tokyo (113\*).

Another area in which Skempton has made a distinguished contribution is that involving the relationship between geology, especially Quaternary geology, and soil mechanics. This is evident in his early papers on post-glacial clays, e.g. (14\*); but his contributions to a Royal Society meeting, organised with J.N. Hutchinson in 1975, are memorable in this connection. His approach to the investigation and understanding of natural slopes is nowhere set out more clearly than in his introductory remarks to that meeting (108), where he states unequivocally that an interdisciplinary approach, invoking the subjects of geology, soil mechanics and geomorphology, is essential. He promptly demonstrates his competence in all these fields in the paper with A. G. Weeks on the Quaternary history of the Lower Greensand escarpment and Weald Clay vale near Sevenoaks, Kent (109\*), which is outstanding in its reconstruction of the various phases of periglacial solifluction and intervening periods of erosion from the Wolstonian to the present day, and in its analysis of the resulting geotechnical problems. A companion paper (110) deals in a similar manner with the valleys of the River Avon and its tributaries in the Bath area.

#### **FOUNDATIONS**

Skempton's first published paper (1)—read before the British Association at Cambridge in 1938—discussed the settlement analysis of engineering structures. Another of his early papers (4), published in 1942, describes an investigation into the bearing capacity failure of a footing in soft clay. It is of particular interest since not only is the measured bearing capacity compared with a number of methods of analysis, but also the question of how the depth of burial affects the undrained bearing capacity factor  $N_c$  is identified.

This problem remained with Skempton for some years. In 1950 he returned to the topic in a discussion (25) on Guthlac Wilson's celebrated paper on the bearing capacity of screw piles. Referring to model tests carried out at Imperial College, and also to the cavity expansion work of R. E. Gibson, he concluded that the bearing capacity factor for a deep circular footing should be taken as equal to 9. Then in 1951 he published an important paper on the bearing capacity of clays (28\*), in which much theoretical and experimental evidence is assembled for the influence of depth and geometry on the value of  $N_c$ . The proposals put forward in figure 2 of the paper have never been seriously questioned or improved upon and are in almost universal use for the assessment of the stability of foundations in clay soils.

Skempton's 1959 paper on the design of bored piles (61\*) is used so widely that it comes as something of a surprise to find that this is his only publication on piles in clay, though he has of course made some notable contributions in discussion. This paper was written at a very opportune time, since piles of this type were being widely used as foundations for the spate of high-rise construction in London which had just begun. Machines for constructing larger capacity piles were then developing rapidly, and conflicting ideas existed on the mechanism of behaviour and design of these piles. The paper consists of a careful review of a number of records of loading tests on bored piles in London Clay, the results of which are analysed in relation to the accumulated knowledge of the undrained strength of London Clay. The mechanics of the behaviour of the shaft and the base are clearly set out, while at the same time attention is drawn to the influence of various construction practices on the pile shaft resistance. The paper made a profound impact on consultants and contractors, and subsequent detailed research on instrumented piles, mainly carried out at the Building Research Station, developed from the conclusions and recommendations of the paper.

The 1951 paper on the bearing capacity of clays also contains an important section on the settlement of foundations. The observed "immediate" settlements of a number of footings, expressed as a percentage of foundation width, are related to the proportion of net ultimate bearing capacity for various values of E/c. In a similar manner, observed "final" settlements are conveniently related to  $K_v/c$  where  $K_v = 1/m_v$ .

The analysis of settlement is another classic problem on which Skempton worked for a number of years, and to which he returned in 1955 in a paper with Peck and MacDonald (43). In this paper the final settlement was expressed conventionally as being equal to

$$\int_0^z m_{\mathbf{v}} \, \Delta \sigma_{\mathbf{v}} \, \mathrm{d}z \; (= \varrho_{\mathrm{oed}}).$$

The discussion that this paper provoked revealed much uncertainty as to whether  $\rho_{oed}$  gave the final settlement  $\rho_t$  or the consolidation settlement  $\rho_c$ . There was clearly much confusion at the time. Then in 1957, with Laurits Bjerrum, Skempton presented a simple but elegant analysis of the problem in a paper (54\*) which is a model of clarity. Consolidation settlement is shown to result from the dissipation of excess pore pressure, the magnitude of which is a function both of the properties of the clay and the applied stresses. Making use of his pore pressure parameter A and the stresses derived from simple elasticity, it is shown that the consolidation settlement  $\rho_c$  is related to  $\rho_{oed}$  by a factor  $\mu$ . The influence of A on the value of  $\mu$  is dominant, thereby demonstrating

the relation of the geological history and properties of a clay to its settlement characteristics.

Sadly, we have been able to include only three papers on foundation problems. Each of these is of outstanding importance, and together they form the basis of present practice in the design of foundations in clay. We must, however, also mention the paper, written with D. H. MacDonald, on the allowable settlements of buildings (48). The guidelines on limiting angular distortion that emerged from their study of settlement and damage in nearly 100 structures have found application not only in designing for settlement but also more generally for guidance on allowable movements in buildings.

#### **EMBANKMENT DAMS**

Embankment dams present some of the more challenging problems of geotechnical engineering and, inevitably, Skempton became involved. However, dams do not feature strongly in his published works, perhaps because they were a special interest of his colleague A.W. Bishop.

Skempton's first involvement was with the Chingford embankment failure. In the wartime paper on that failure by Cooling & Golder (1942), he is credited with recognising that the undrained strength of the foundation would have increased in the few weeks between failure and investigation, due to consolidation.

An early contribution to the stability of embankment foundations is his discussion of the paper by McLellan on the Hollowell Dam failure on a soft alluvial clay foundation (9). An analysis of the non-circular failure surface is presented, in terms of total stress, which would be entirely adequate for use today. In the 150 years of British dam building which preceded the development of modern soil mechanics, undrained failures of clay fills and foundations during construction had been a recurrent problem, culminating in the failure of Muirhead Dam in 1940 (Banks, 1948). Skempton spent two months on site investigating the failure, and it is not surprising that his early work was much concerned with the effect of consolidation during construction on the strength of embankments and their foundations, and with the acceleration of consolidation by the inclusion of drains.

A landmark is the design of the Chew Stoke Dam in Somerset, in which Skempton and Bishop were involved jointly (42). The dam, built during 1951–54, had a soft alluvial clay foundation and sand drains were used in the initial design to accelerate consolidation and ensure an adequate increase in strength. This appears to be the first time that the gain in strength due to the action of sand drains had been quantified. The analysis is also notable as it includes the effect of principal stress rotation in the foundation clay. An apparent by-product is a 1950 report to Soil Mechanics Ltd., with D. J. Henkel (I.C. Report 73), on the use of sand drains. The theories of radial consolidation by Baron (1948) and others are reviewed, and the potential benefits of accelerated settlement, gain in strength and prevention of lateral migration of excess pore pressure are all clearly treated.

Skempton's involvement with dams in Britain up to 1960 ran very much in parallel with the programme of pore pressure measurement in dams conducted by his former colleagues at B.R.S. in the post-war years. As discussed by R. E. Gibson (pp. 9–11), in 1952 piezometers installed in the boulder clay fill of the Usk Dam showed high construction pore pressures which, if extrapolated, indicated a probability of construction failure, as had previously occurred at Muirhead. Skempton and Bishop were consulted, and horizontal drainage layers were included within the fill to accelerate consolidation, again for the first time (56). These were used as a remedial measure, but became a design feature in Selset Dam, built during 1956–61 (Bishop & Vaughan, 1962). This embankment was subsequently found to have a weak foundation, and sand drains were used here also. Within five years both techniques had been widely adopted in embankment dam construction.

Skempton was closely involved with the massive embankment dam at Mangla, with its tectonically induced low strength shear surfaces in the mudstone foundation (85\*).

This interest in the effect on dam design of tectonic shears in mudstones has continued, with involvement at Mornos Dam in Greece, on flysch mudstone (I.C. Report 324), and at Kalabagh Dam in Pakistan (I.C. Report 345), which is currently under design, on foundations similar to those at Mangla.

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In conclusion, one cannot help being struck by the importance of consulting jobs in the development of Skempton's ideas, and the effective way in which these have been transmuted to research, the results of which have found wide acceptance and application. The reader will be aware from Rudolph Glossop's (pp. 5–8) appreciation that soil mechanics has received great benefit from the fortunate arrival of Skempton at a time when the subject was in a major phase of its development. Similarly, it is also probably true to say that the geology of the country in which Skempton has worked has also been of considerable benefit to the subject. Just as the main progress in soft clay studies has been made in the relatively homogeneous soft clays of Scandinavia and parts of North America, so it is not surprising that most of the concepts concerning the shear strength of stiff-fissured clays and their behaviour have been worked out in studies of one of their more homogeneous and geologically uncomplicated members the London Clay.

The papers in this volume epitomise the essential Skempton: single-mindedness to an unusual degree, great enthusiasm and ability to inspire others, a natural appreciation of the important factors in complex situations, a proper sense of the indivisibility of nature, and a rare ability to work on until a clear synoptic view has been achieved.

#### REFERENCES

Banks, J.A. (1948). Construction of Muirhead reservoir, Scotland. Proc. 2nd Int. Conf. Soil Mech., Rotterdam, 2, 24–31.

Baron, R.A. (1948). Consolidation of fine-grained soils by drain wells. Trans. Am. Soc. Civ. Engrs, 113, 718-742.

Bishop, A.W. & Vaughan, P.R. (1962). Selset Reservoir: design and performance of the embankment. *Proc. Instn Civ. Engrs*, 21, 305–346.

Bowden, F.P. & Tabor, D. (1954). Friction and lubrication of solids. Oxford.

Carson, M.A. & Kirkby, M.J. (1972). Hillslope form and process. Cambridge.

Cooling, L.F. & Golder, H.Q. (1942). The analysis of the failure of an earth dam during construction. J. Instn Civ. Engrs, 19, 38-55.

Henkel, D.J. (1957). Investigations of two long-term failures in London Clay slopes at Wood Green and Northolt. Proc. 4th Int. Conf. Soil Mech., London, 2, 315-320.

Sevaldson, R.A. (1956). The slide in Lodalen, October 6th, 1954. Géotechnique, 6, 167-182.

Terzaghi, K. (1936). Stability of slopes of natural clay. Proc. 1st Int. Conf. Soil Mech., Cambridge, Mass., 1, 161–165.

Terzaghi, K. (1939). Soil mechanics—a new chapter in engineering science. J. Instn Civ. Engrs, 12, 106–141.

Terzaghi, K. (1947). Shear characteristics of quicksand and soft clay. Proc. 7th Texas Conf. Soil Mech., Paper 5.

Terzaghi, K. (1961). Discussion on paper (69\*). Proc. 5th Int. Conf. Soil Mech., Paris, 3, 144-145.

Vaughan, P.R. & Walbancke, H.J. (1973). Pore pressure changes and the delayed failure of cutting slopes in over-consolidated clay. *Géotechnique*, 23, 531–539.

#### "A Slip in the West Bank of the Eau Brink Cut."\*

By ALEC WESTLEY SKEMPTON, M.Sc., Assoc. M. Inst. C.E.

#### INTRODUCTION.

In the autumn of 1943 a slip occurred in the west bank of the Eau Brink Cut, about  $1\frac{1}{4}$  mile south of King's Lynn, Norfolk, and it was investigated in the course of work on the stability of river and channel banks in the Fens, undertaken by the Building Research Station, Department of Scientific and Industrial Research, for the River Great Ouse Catchment Board. The slip provided an excellent opportunity for examining the reliability of the methods at present in use for analysing the stability of slopes in natural cohesive strata, and the results are presented in some detail in the hope that they may be of interest to those concerned with similar problems in engineering geology.

#### SITE EXPLORATION.

The site.—Until the early years of the nineteenth century the river Ouse made an extensive bend of about 6 miles in length from Wiggenhall St. Germans to King's Lynn, and it was suggested by Kimberley in 1751, and again by Rennie in 1809, that this bend should be cut off by a new channel in order to improve the outfall of the river. The work was finally carried out between 1817 and 1821 and this 400-foot wide channel forms the Eau Brink cut as it exists to-day.<sup>6</sup> In Fig. 1 the old course of the river Ouse is that shown by Skertchly <sup>17</sup> in his Plate I. The banks were excavated at a slope of about  $2\frac{1}{4}$ : 1 to a depth of 25 feet through the postglacial Fen clays and silts, and part of the spoil was spread in a layer several feet thick on the original ground surface for a distance of about 120 feet back from the slopes, where a small flood bank was formed.

The site of the slip is shown in Fig. 1. So far as is known, no previous movement of the bank had taken place at this point, but in September 1943 a crack and subsidence were observed near the top of the slope, together with a small displacement near the toe into the river. Material was dredged up from the lower portion of the bank and placed at the top to maintain the original levels, and new faggots were placed in position to stabilize the surface. But movement continued and turf from the saltings in front of the flood bank was placed at the top in a further effort to maintain the profile.

By the end of the year the crack had extended considerably, as shown in Fig. 2, Plate 1, and the displacement at the toe in the principal slip zone was very obvious at low water. The survey and borings were carried out in the first two weeks of January 1944, during which time measurements showed that slight movement was still in progress.

From observations immediately adjacent to the slip it was clear that considerable erosion had been taking place at approximately low water level. This is seen in Fig. 2, Plate 1, and also in cross-section 3, in Fig. 3 (c), Plate 1, and it is probable that the slip occurred from oversteepening of the slope by this erosion process.

A possible profile for the bank prior to the slip has been drawn in Fig. 4, Plate 1, from evidence supplied by cross-sections 1 and 3 of Figs. 3 (a) and 3 (c), Plate 1. The upper portions show a typical slight convexity, which is due to silt deposition, and in the lower portion it has been assumed that erosion on the same scale as that found in section 3 had taken place. The average slope of this profile is 2:1.

Boring and sampling.—One test-pit and a number of borings were made, in the positions shown in Fig. 2, Plate 1, in order to discover the sequence of strata and to obtain undisturbed samples from which the shear strengths could be determined. As the boring proceeded the material

<sup>\*</sup> Crown Copyright reserved.









encountered was continuously examined, whilst at frequent intervals small undisturbed samples  $1\frac{1}{2}$  inch diameter and 6 inches long were taken, from the lower half of which a specimen was prepared for the field compression test. After testing the specimen was placed in an airtight bottle to await transport to the laboratory for water-content and Atterberg Limit determinations.

From each stratum several cores  $4\frac{1}{2}$  inches diameter and 15 inches long were taken with a sampling-tube, driven by light blows from a 40-lb. hammer. The ratio of the area of the cutting nose of this tube to that of the sample was 19 per cent. and visual examination of the samples showed that distortion extended for not more than  $\frac{1}{4}$  inch from the outer surface. The positions of all samples are shown in Figs. 3, Plate 1.

Ground water levels were recorded in the boreholes and it was found that the changes in level during any 9-hour period were negligible, although the tidal range was about 18 feet while these observations were in progress.

Geology.—The natural strata, as revealed by the borings, are shown in Figs. 3 (b) and 4, Plate 1, and they are briefly described in Table I. The sequence is typical of the marshland region of the Fens. The following interpretation of the stratigraphy is based on the work of Godwin (1940).

All the strata are post-glacial, and from borings at St. Germans and King's Lynn it is known that they rest on a great thickness of hard Kimmeridge clay. The Fen clay was deposited between 2,000 and 3,000 years ago in a shallow but extensive area of brackish lagoons and it consists principally of the "buttery clay", a soft plastic clay, apparently nonstratified and containing traces of peat. The term "buttery clay" has long been used for describing this stratum.<sup>17</sup> The central zone of the Fen clay at this site is, however, rather silty with no peat and it was presumably laid down during a more marine phase.



SECTION 2 CROSS SECTION.

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	Elev	ation	Typical values.				
Strata.	from	to	Water- con- tent.	Liquid limit.	Plastic limit.	Clay frac- tion.	Saturated density γ lb. per cubic foot.
brown silt brown silty clay, with Fen Silt light blue veins	107	110	30	35	20	(x)	118
along old root fibres grey laminated silt .	104 99	107 104	35 40	60 45	30 25	(x) 12	118 114
Upper Peat, brown and fibrous	97.5	99	350	500		(x)	70
soft blue buttery clay, with traces of black peat and <i>phragmites</i> .	93	97.5	55	75	28	45	102
Fen Clay   clay	88	93	40	45	22	15	112
mites	80	88	65	100	36	55	100
Sand and gravel (mostly black flints) with some peat	_	80			_		_

Note: (i) Elevations refer to heights above a reference level 100.00 feet below Newlyn Ordnance Datum. Water-content is expressed as a percentage of the dry weight.

- (iii)
- (iii) Clay fraction is the percentage by weight of particles with an equivalent diameter of less than 0.002 millimetre.

(iv) (x) Not determined.

This period was terminated by an uplift of the land and on its surface the upper peat was formed; but with increasing elevation the rivers and streams eroded channels, some of an appreciable depth, and this is probably the explanation of the absence of peat in borings I and VI---an absence which was difficult to explain during the site exploration since this very distinctive stratum was clearly seen along the face of the slope and in boring V.

The next change was a rise of sea-level, causing large salt marshes of silt, filling the erosion channels and covering the peat to a depth of many feet. This Fen silt exists at the site in three distinct layers. The lowest comprises a grey silt with inclined laminations, each about  $\frac{1}{6}$  inch thick and formed of alternating sandy and clayey silt. The next layer is a brown silty clay with many light blue veins along old root fibres, and above this is a brown silt. These brown and oxidized layers extend a few feet beneath ground water level which, therefore, at some time in the past probably stood lower than it is found to-day. This may well have been the case about 1,500 years ago, when the silt lands had been elevated to a height of about 20 feet above mean sea-level and farms of Roman-British age were established in the area. This land surface has persisted





RECONSTRUCTED PROFILE AT SECTION 2 PRIOR TO SLIP

to the present day, but there is evidence which suggests that during the past 1,000 years there has been a progressive rise of sea-level, at an average rate of about 1 foot in a century, which is a detrimental factor in the important task of draining the Fens. An interesting account of the present problems in the Fens and the proposed flood protection scheme for the river Ouse has been given by Doran.<sup>7</sup>

From this general picture it may thus be concluded that the buttery clay is, for practical purposes, normally consolidated (Terzaghi <sup>20</sup>) under the weight of the existing overburden, for there was probably no opportunity for drying during the period of peat deposition and there has been no significant erosion of the silt. This is confirmed by the close agreement between the calculated weight of overburden at El.83 and the experimental determinations (following Casagrande 1) of the pre-consolidation load of a sample taken from this level (see *Fig. 5*). A similar test on the grey silt from El.102, was not capable of such accurate interpretation, but it appeared that the pre-consolidation load was rather greater than the existing overburden. This may be due to some drying which occurred when the ground water levels were lower than at present, or during the actual deposition of the silt.

The Slip Surface.—The position of the slip surface could be found with some certainty from the following indications :—(i) the tension crack at the top of the slope ; (ii) at El.86 in borehole III there was a sudden transition from grey silt to buttery clay; (iii) the very characteristic change from brown silty clay with light blue veins to grey silt occurred at El.91 in borehole III; (iv) in borehole VII the peat and upper layer of buttery clay were defined as shown in Fig. 3 (b), Plate 1; (v) a change from buttery clay to grey silty clay was found on the surface of the slope a short distance above low tide level.

By trial and error a slip circle was readily found which complied with all these conditions. If the strata shown in Fig. 4, Plate 1, are traced on to transparent paper, and if this paper is then placed over Fig. 3 (b), Plate 1, and turned about the centre  $O_{1}$ , it will be found that a rotation of about 20 degrees will bring the strata approximately into the positions shown in boreholes III and VII. The radius of this circle is 55 feet and, if the foregoing interpretation is correct, there has been a drop of about 15 feet in the original top surface of the bank.

#### SHEAR STRENGTH DETERMINATIONS

General.—It is known from the work of Hvorslev <sup>12</sup> and others that the shear strength of a clay or silt can be expressed with reasonable accuracy by the equation

where  $c_s$  and  $\phi_f$  denote the true cohesion and the angle of internal friction at the particular water-content of the specimen at the time of test; and (n - u) is the effective normal pressure on the shear plane. This pressure is equal to the difference between the total applied pressure n on the plane and the pore water pressure u, for experiments by Terzaghi <sup>19</sup>



PRE-CONSOLIDATION LOAD OF BUTTERY CLAY AT ELEVATION 83.

and Rendulic <sup>14</sup> have shown that all mechanical properties of saturated soils are controlled exclusively by this effective pressure.

The tests for the determination of  $c_{\theta}$  and  $\phi_f$  are rather elaborate and, moreover, methods of stability analysis based on these constants have not been fully developed. In practice, therefore, it is necessary to use simplified analyses based on correspondingly simplified shear strength characteristics. There are two such methods, which depend respectively upon a knowledge of the equilibrium shear constants and the existing shear strength.

Equilibrium shear constants, c and  $\phi$ .—To determine these constants three or more specimens, prepared from an undisturbed core, are placed in shear boxes of the usual pattern (Golder <sup>10</sup>) and subjected to vertical pressure. Each specimen is held between porous stones, which are in contact with water, and it is free to expand or consolidate under the vertical load, sufficient time being allowed for this process to be completed. The shear strength is then measured by applying an increasing horizontal load until failure occurs. Typical results are shown in *Fig.* 6, and it will be seen that the relation between shear strength and vertical pressure can be expressed by the equation

$$s = c + (n - u) \tan \phi \quad . \quad . \quad . \quad . \quad (2)$$

Here c is defined as the apparent cohesion and  $\phi$  as the angle of shearing resistance (Terzaghi,<sup>22</sup> p. 7). The coefficient of tan  $\phi$  has been written in the form (n - u) to emphasize that this pressure is effective. If full consolidation is not allowed to take place under the vertical pressure n there will be some definite pressure u in the pore water at the time of test and the whole of the available shear resistance will not be mobilized.

It must be pointed out, however, that although in this test sufficient time is allowed for complete consolidation, and the full increase in shear strength with increasing effective pressure normal to the shear plane is measured, yet the value of  $\phi$  must be greater than the true angle of internal friction, since the increase in strength is due not only to frictional effects but also to an increase in cohesion consequent upon the decrease in water-content under higher pressures. The values of c and  $\phi$  are therefore empirical constants and have no physical significance other than that expressed in *Figs.* 6 and equation (2).

The test results are summarized in Table II, wherein the average values used in the stability calculations are also given.

The existing shear strength, s.--The shear strength of a clay or silt as

Str <b>at</b> un	n.			Apparent cohesion c: lb. per square foot.	Angle of shearing resistance, $\phi$ : degrees.	Values used in analysis.
Brown silt Grey silt ""	•	•	• • •	120 140 130 180	32 32 34 25	$\begin{cases} c = 140 \text{ lb. per square foot.} \\ \phi = 30 \text{ degrees} \end{cases}$
Buttery clay .	:	•	•	280 200	19 22	$\begin{cases} c = 230 \text{ lb. per square foot} \\ \phi = 20 \text{ degrees} \end{cases}$
Grey silty clay	:	:	:	130 150	20 23	$\begin{cases} c = 140 \text{ lb. per square foot} \\ \phi = 21 \text{ degrees.} \end{cases}$

TABLE II.



EFFECTIVE NORMAL PRESSURE (#+14) LB. PER SQUARE FOOT.



it exists in the ground may be determined with reasonable accuracy by carrying out tests on undisturbed samples in such a manner that their natural water-content remains unchanged during the test. The most convenient method is to test a small cylindrical specimen  $1\frac{1}{2}$  inch diameter and  $3\frac{1}{2}$  inches long in the portable compression apparatus (Cooling and Golder <sup>4</sup>) and to divide this compression strength by 2; for it is known that under conditions of no water-content change the compression strength of a silt or clay is equal to twice the existing shear strength.

As a demonstration of this result, four specimens of the grey silt were prepared and completely encased in thin rubber membranes between solid end pieces. Each specimen was then subjected to a hydrostatic pressure in the triaxial compression apparatus (Casagrande and Fadum<sup>2</sup>) and its compression strength determined; but owing to the condition of test no water-content change could take place and it was found that the compression strength remained essentially constant and independent of the applied hydrostatic pressure. This conclusion was proved experimentally by Terzaghi in 1932 and has since been confirmed by tests on a wide range of saturated clays and silts.



TRIAXIAL TESTS ON GREY SILT, WITH NO CONSOLIDATION PERMITTED UNDER THE APPLIED STRESS.

The result is expressed in *Fig.* 7 by Mohr's circles of stress and it will be realized that the silt is behaving, with respect to the stress conditions at failure, as if it were a purely cohesive material with an angle of shearing resistance equal to zero.\* It therefore follows that the compression strength must be equal to twice the apparent cohesion, which is itself equal to the shear strength of the material at the particular water-content of the test specimens.

During the field investigations a number of compression tests were made on  $1\frac{1}{2}$ -inch diameter undisturbed samples and in the laboratory compression tests were carried out on two specimens prepared from each of the  $4\frac{1}{2}$ -inch diameter cores. The results are summarized in Table III.

The variations, as expressed by the standard deviation, may appear to be considerable, but it is believed that the average values are sufficiently reliable for calculation purposes. Moreover they are of the same order as the average values found for these strata at other sites in the same area of the Fens.

TABLE III.

Stratum									1-compression strength s: lb. per square foot. Average values.	Standard deviation : per cent. of average.
Brown silt and	silty	r ela	aγ					•	450	28
Grey silt	. `		٠.						500	13
Buttery clay .									470	20
Grey silty clay	•								380	30

Note : Insufficient samples were available to justify a distinction between the upper and lower buttery clay and between the brown silt and silty clay.

As a check on the compression strength measurements the existing shear strength was also determined by direct shear box tests, with no consolidation being allowed to take place in the specimen. To carry out these tests the porous stones are replaced by solid brass grids and the shear force is applied immediately after the vertical load has been placed in position. In the buttery clay it was found that the strength was, for practical purposes, independent of the vertical pressure, but in the silts there was a slight increase with increase in pressure. Under low loads, however, the silts tended to expand and by interpolation the shear strength corresponding to zero volume-change could be found. Typical results are given in Table IV, wherein it will be seen that the existing shear strength, as determined directly in the boxes, is closely equal to one-half the compression strength.

TABLE IV.

					<b>‡-compression</b> strength : lb. per square foot.	Shear-box tests : lb. per square foot.	Ratio.
Brown silt					390	430	0.91
Grey silt .					520	480	1.08
Buttery clay	·	•	·	·	450	500	0.90

\* The very slight inclination of the envelope to the Mohr's circle in Fig. 7 may be due to traces of air in the specimens.

Thus it can be seen that the clays and silts at this site, if tested under conditions of no water-content change, behave with respect to the stress conditions at failure as purely cohesive materials with  $\phi = 0$ ; and this conclusion was confidently expected from previous experience. It must, however, be clearly appreciated that the behaviour of clays and silts is, at all times and under all conditions of test, controlled by the effective stresses and by the true cohesion and angle of internal friction. This being the case, it follows from standard theory in the strength of materials that the inclination of the shear planes in a compression test specimen

must be inclined at an angle of  $\left(45 + \frac{\phi_f}{2}\right)$  degrees to the horizontal, and not at an angle of 45 degrees as would be implied by the result that the angle of shearing resistance  $\phi$  equals zero.

The lack of homogeneity in many undisturbed samples of clays and silts will often obscure this relationship, but it is of interest to note that reasonably consistent inclinations of the shear planes were observed in the majority of the test-specimens, and average values are given in Table V. The angle of shearing resistance is also included for comparison and it is seen to be in excess of the angle of friction, in accordance with the statement made on p. 273, *ante*.

TABLE V.

				Inclination of shear planes $\left(45 + \frac{\phi_{f}}{2}\right)$ : degrees.	Angle of friction, $\phi_{f}$ : degrees.	Angle of shearing resistance $\phi$ : degrees.
Brown silt Grey silt . Buttery clay	•	 •	•	60 54 51	30 18 12	32 30 20

The effect of remoulding.—A very considerable decrease in the existing shear strength was observed on completely remoulding a sample at its natural water-content. This result is well known in soft clays and silts and it emphasizes the need for care in sampling. In the buttery clay the decrease was about 70 per cent., whilst in the silts it was about 60 per cent.

It was also found, however, that the equilibrium shear constants of the grey silt were unaffected by remoulding. Fig. 6 (a) shows that the amount of consolidation taking place under each pressure was more than twice that occurring in the undisturbed specimens, and the density of the silt when remoulded and consolidated was therefore appreciably greater than the corresponding density of the undisturbed material. But this effect was apparently offset by some inherent loss of strength, due possibly to the breakdown of bond structure between the particles. It is unfortunate that this effect was discovered only when no more samples of the buttery clay were available for test and no results can therefore be quoted for this material.

#### THE ANALYSIS OF STABILITY.

General.—A theoretically correct analysis of the stability of a slope in natural cohesive strata should be based on a full knowledge of the effective pressures acting on the slip surface and on the appropriate values of true cohesion and angle of friction. This, however, is not practical at the present time and, as stated on p. 273, *ante*, two methods of analysis are in use which are based on simplified shear strength characteristics. Both of these methods have been developed from the observation that a slip in cohesive strata generally occurs along a more or less cylindrical shear surface the trace of which, on a vertical plane through the bank perpendicular to the axis of slip, is a circular arc.

The first method involves the assumption that the shear resistance along the slip circle is given by the expression  $s = c + (n - u) \tan \phi$ , where c and  $\phi$  are the equilibrium shear constants and (n - u) denotes the effective normal pressure on the slip surface at any particular point. The chief error in this method is due to the implicit assumption that  $\phi$  is a true frictional and directional factor, whereas it has been shown that the angle of friction is actually considerably less than  $\phi$ .

The second method depends upon two assumptions; firstly that the shear resistance along the slip circle is equal to the existing shear strength of the strata as measured by one-half the compression strength or other means, and secondly that the angle of shearing resistance is zero, in accordance with the evidence demonstrated in *Fig.* 7. This method, which is known as the  $\phi = 0$  analysis, is very direct and simple in its application, but in assuming that the strata behave as purely cohesive materials it will, in general, lead to an incorrect placing of the slip circle. For in so far as the shear planes in a compression specimen are inclined at

 $\left(45 + \frac{\phi_f}{2}\right)$  degrees, and not at 45 degrees, as in a truly cohesive material,

then it may be anticipated that the actual slip circle is more steeply inclined and lies more closely beneath the slope than that found from calculations based upon the assumption that the strata are purely cohesive.

The c,  $\phi$  analysis.—The principles of this analysis are shown in Fig. 8. A slip circle is chosen and the material above the arc is divided into a number of vertical slices, the forces between which will be neglected in accordance with the usual assumption. This can be justified by carrying out analyses with various statically possible systems of forces between the slices. The results will be closely equal to one another and to the special case in which the forces are neglected altogether (Terzaghi <sup>18</sup>).

The most unstable condition of the bank occurs at low water when, owing to the relatively impermeable nature of the clays and silts, all the strata above low tide level remain saturated and only the material below low water is reduced in weight by hydrostatic uplift. If, in Fig. 8, the three zones of the slice above mean tide, between mean and low tide, and below low tide level are respectively I, II, and III, then the weight of the slice may be expressed in the form  $\{W_{I} + W_{II} + W_{III}^{\bullet}\}$ , where the **asterisk** denotes a weight reduced by uplift. The component of the total weight resolved along the slip circle will be

$$\{W_{\rm I} + W_{\rm II} + W_{\rm III}^*\}\sin\alpha \ . \ . \ . \ . \ (3)$$

where  $\alpha$  denotes the inclination of the tangent to the arc at the base of the slice.

In order to estimate the shear resistance along this arc it is necessary to know the effective normal pressure, and although the total pressure will fluctuate with the rise and fall of the tide, it seems reasonable to assume that the rate of consolidation of the clays and silts is insufficient for these fluctuations to be felt as changes in effective pressure. The appropriate weight of the slice in this respect is therefore calculated from



DISTURBING FORCE =  $[W_{I} + W_{II} + W_{II}^{*}] \sin a$ . RESTORING FORCE =  $c.l + [W^{I} + W_{II}^{*} + W_{II}^{*}] \cos a \tan \phi$ .

 $W_{n}^{*}$  = weight reduced by hydrostatic uplift =  $\left[W_{n} \times \frac{\gamma - 64}{\gamma}\right]$  lb. per cubic foot, density of sea water being 64 lb. per cubic foot.

FACTOR OF SAFETY =  $\frac{\Sigma c.l + \Sigma [W_1 + W_{11} + W_{11}]}{\Sigma [W_1 + W_{11} + W_{11}]} \cos a \tan \phi.$ PRINCIPLE OF C,  $\phi$  ANALYSIS. mean tide levels, and the effective pressure normal to the arc is thus given by the expression

If the length of the arc at the base of the slice is l and the equilibrium shear constants are c and  $\phi$ , then the shear resistance is

$$cl + \{W_{I} + W_{II} + W_{III}\} \cos \alpha \tan \phi \quad . \quad . \quad . \quad (5)$$

Now the factor of safety of the bank is defined as the ratio of the sum of the moments, about the centre O, of the restoring forces and of the disturbing forces along the slip circle. Consequently

Factor of Safety = 
$$\frac{\sum cl + \sum \{W_{I} + W_{II}^{*} + W_{II}^{*}\} \cos \alpha \tan \phi}{\sum \{W_{I} + W_{II} + W_{II}^{*}\} \sin \alpha} \quad . \quad (6)$$

The factor of safety for this particular circle having been found, the calculations are then repeated for other circles until sufficient information has been gained to fix the position of the circle giving the lowest factor. In the present investigation a number of circles were analysed, all tangential to a given horizontal line, and the most dangerous centre was found by drawing contours of factors of safety as shown in *Fig. 9*. Further contours were then obtained for families of circles tangential to different horizontal lines, and in this way the circle giving the minimum factor of safety could be found.

This critical circle is shown in Fig. 10, with centre  $O_2$  and it lies rather above the actual slip surface. The calculated factor of safety is in close agreement with the true value of unity,\* but this apparent accuracy



Data from  $c,\phi$  Analyses showing Results for SLIP Circles Tangential to Elevation 80.

must be considered as partly fortuitous in view of the possibilities for error in sampling, testing, and analysis.

It is of interest to note that the factor of safety as estimated from the actual slip circle is 1.15. This gives a quantitative measure of the agreement between the theoretical and actual circles, and also shows that an error of 15 per cent. would be involved in calculations based only upon the observed slip surface.

The  $\phi = 0$  analysis.—The principles of this analysis are shown in Fig. 11. A slip circle, of radius R, is chosen and here again the analysis is carried out for the condition of low water. The area above the arc is divided into two zones I and II, above and below low water level, and the areas and centres of gravity of these zones are found. The total disturbing moment about the centre O is then

$$\{W_{\mathrm{I}}d_{\mathrm{I}} + W_{\mathrm{II}}^{*}d_{\mathrm{II}}\} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

and this is identical with the sum of the moment of the forces given in equation (3).

\* In a bank immediately prior to failure the disturbing forces are equal to the restoring forces and the factor of safety is therefore unity.

#### SLIP IN THE WEST BANK OF EAU BRINK CUT



Scale 10 5 0 10 20 30 40 50 feet

 $c,\phi$  Analysis of Stability.



MOMENT OF DISTURBING FORCES  $W_{n}d_{n} + W_{n}^{*}d_{n}$ . MOMENT OF RESTORING FORCES  $\delta R^{2}\theta$ .  $W_{n}^{*}$  denotes weight reduced by hydrostatic uplift.  $\delta$  denotes average existing shear strength along arc.

Factor of Safety =  $\frac{\sigma_{II} \cdot \sigma}{W d_1 + W_{II}^* d_{II}}$ 





 $\phi = 0$  Analysis of Stability.

The restoring force is due simply to the total existing shear strength along the arc, and the restoring moment is therefore

$$\overline{s}R^2\theta$$
 . . . . . . . . . . . (8)

where  $\bar{s}$  denotes the average shear strength along the arc and  $\theta$  denotes the angle subtended at O.

The factor of safety is thus given by the ratio

Factor of Safety = 
$$\frac{sR^{2}\theta}{\{W_{II}d_{I} + W_{II}^{*}d_{II}\}} \quad . \quad . \quad . \quad (9)$$

and a number of circles are analysed in order to find, by trial and error, that giving the minimum factor.

The results of these calculations are given in Fig. 12, where contours are shown for the family of circles tangential to the base of the buttery clay. Other families tangential to lines at higher elevations gave slightly greater factors of safety.

The critical circle giving the minimum factor is therefore the one shown with centre  $O_3$  in Fig. 12, and it will be seen to lie appreciably below and farther back from the face of the bank than the actual slip surface. In spite of this difference in position, which was anticipated on general theoretical grounds (p. 278, *ante*), the  $\phi = 0$  analysis gives a correct estimate of the factor of safety, the calculated value being equal, almost precisely, to unity. Here, again, the accuracy must be considered partly fortuitous.

The factor of safety calculated by the  $\phi = 6$  analysis along the actual slip surface is 1.30, and this discrepancy of 30 per cent. is an indication of the lack of agreement between the calculated and observed slip circles. It also shows that in this investigation an appreciable error would be involved in using only the observed surface for analysis.

**Progressive** movement.—The disturbing moment around the slip surface in the bank before failure was about 20 per cent. greater than after the slip had taken place, as may be realized by comparing the original and existing profiles shown in Fig. 4, Plate 1. Nevertheless movement continued to occur for several months and, since this indicates a factor of safety of slightly less than unity, it must be assumed that there has been a decrease in shear resistance. This decrease is probably due to remoulding in the zone immediately adjacent to the slip surface and, although in time reconsolidation may cause an increase in strength, in accordance with the experimental observations recorded on p. 277, ante, yet a temporary decrease of 20 per cent. is quite possible. A small amount of erosion may also be proceeding at the toe and removing support from the bank.

Remedial measures.—As a result of the site investigation a knowledge of the mechanism of failure has been obtained, and it can be clearly seen that the objects of remedial measures must be to increase rotational stability and to prevent erosion. A consideration of the means for attaining these objects lies, however, outside the scope of the present Paper.

#### DISCUSSION

Although no great accuracy in calculations of this nature can be expected, the results are sufficiently satisfactory to permit the general conclusions (i) that both the  $c, \phi$  and  $\phi = 0$  methods give a reasonably correct analysis of the factor of safety, but (ii) neither method gives a critical slip circle coincident with the observed slip surface.

The second conclusion may be anticipated from the simplified nature of the assumptions upon which the analyses are based, for the  $c, \phi$  method is known to over-estimate the frictional properties of the strata, whilst he assumption that  $\phi = 0$  implies the absence of all frictional effects. Therefore it is interesting to note that the actual slip surface lies between the critical circles given by the two methods and that, furthermore, it lies closer to the circle given by the  $c, \phi$  analysis. This suggests that the average true angle of friction around the slip surface is perhaps rather more than half the average angle of shearing resistance, and reference to Table V will show that a similar result has been obtained experimentally for the grey silt and buttery clay.

#### SLIP IN THE WEST BANK OF EAU BRINK CUT

Close comparisons of the slip circles are, possibly, not fully justified, since the presence of scepage forces may cause the actual slip surface to lie closer to the face of the slope than it would do if there were no tidal action. Attention may, however, be drawn to the analysis of the quay wall failure at Gothenburg harbour (Fellenius <sup>8</sup>). Here it was found that the slip surface lay between the two critical circles based respectively upon the assumptions that  $\phi = 0$  and c = 0, and Fellenius concluded that the clay behaved as if  $\phi = 4$  degrees. The Author is not acquainted with other published records on the relation between calculated and observed slip surfaces and a comparison with further data is thus not possible in this aspect of the problem.

On the other hand, several reports have been given on the calculated factors of safety of slopes in natural clay strata where slips have occurred. From these it may be concluded, at least provisionally, that the  $\phi = 0$ analysis is reliable, but that the  $c,\phi$  analysis may lead to overestimates of the stability (Terzaghi <sup>20</sup>, Cooling <sup>3</sup>). The present investigation is thus at variance in this respect with other information and it must be accepted that some element of uncertainty still exists in the problem. A partial explanation may, however, be found in the fact that the water-content of a clay consolidated in the laboratory under a given load is usually appreciably less than that of the clay in nature under the same load of overlying sediment (Terzaghi 20, Skempton 16). The lower water-content implies a greater strength, and Cooling <sup>3</sup> has given data on a soft alluvial clay from Portsmouth harbour in which the increase of existing shear strength with depth is only one-half the increase deduced from equilibrium shear tests. Moreover there is evidence that in some strata, as, for example, the post-glacial clays in the valley of the St. Lawrence river, the increase in strength with depth is inappreciable (Hough 11).

The reasons for these phenomena are not completely understood, but it is clear that in such cases the direct application of the  $c, \phi$  analysis would lead to an overestimated shear strength and factor of safety. In contrast, the shear strengths of the buttery clay and grey silt were only about 10 per cent. lower than the values compatible with the equilibrium shear tests and this may account for the satisfactory estimate of stability obtained by the  $c, \phi$  analysis in this investigation.

In general it would therefore appear that the  $\phi = 0$  method is the more reliable; but this may lead to considerable error in placing the slip circle. It is possible that in those cases where the existing shear strengths are compatible with equilibrium shear tests the  $c, \phi$  analysis may also be used with confidence, and a closer approximation to the slip surface may be obtained; but before final conclusions can be drawn it is essential to have more field evidence on slips in natural cohesive strata.

Finally, attention should be drawn to the basic assumption of the  $\phi = 0$  analysis—namely, that no volume changes take place under the applied stresses. This is well suited to the present investigation, where existing strengths were determined soon after the slip had occurred, and the chief stresses are due to rapid tidal variations in water-level. In design problems, however, some allowance should be made for progressive changes in strength with time under the altered stress conditions. In a plastic clay the decrease in strength in the banks of a cutting may be small, but in stiff fissured clays the effect is profound (Terzaghi<sup>20</sup>). It also follows from this basic assumption that the method finds its most satisfactory application in those problems where the volume change is negligible; as, for example, in the pressure exerted by cohesive strata on temporary timbering in excavations and in the ultimate bearing capacity of a clay subjected to rapid loading. Evidence concerning the reliability of the  $\phi=0$  analysis in such cases has been presented by Peck <sup>13</sup> and Skempton,<sup>16</sup> and in the investigation of the foundation failure of an earth dam, by Cooling and Golder.<sup>5</sup>

#### SUMMARY.

In the autumn of 1943 a slip occurred in the west bank of the Eau Brink Cut near King's Lynn. This cut was excavated between 1817 and 1821 to a depth of 25 feet, with side slopes of about  $2\frac{1}{2}$ : 1, through natural
cohesive strata consisting of the post-glacial Fen clays and silts. No previous movement is known to have taken place at this point, and it is probable that the slip was initiated by erosion at and below low water level, which caused an over-steepening of the slope to about 2:1.

The stability of the bank was analysed by two simplified methods, the first being based on the equilibrium shear constants of the strata, and the second being based on the existing shear strengths and the assumption that  $\phi = 0$ .

It was found that both methods gave a correct estimate of the factor of safety, namely a value equal to unity, but in neither case was the calculated critical slip circle coincident with the actual observed slip surface. In the first method the difference was not great, and may be expressed quantitatively by stating that the factor of safety calculated along the actual slip surface was 1.15; an error of 15 per cent. In the  $\phi = 0$ analysis the difference was more appreciable, the calculated slip circle lying considerably too far back from the face of the slope. The factor of safety calculated by this method along the actual slip surface was 1.30; an error of 30 per cent.

These results are discussed briefly in relation to other published data and it is concluded that more information on slips in natural cohesive strata must be presented before a complete understanding of the various factors involved is possible. For practical purposes, however, the balance of evidence seems to be in favour of the  $\phi = 0$  analysis, although it may lead to an incorrect estimate of the position of the slip circle.

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#### APPENDIX.

#### BIBLIOGRAPHY.

- 1. A. Casagrande. 1936. "The determination of the pre-consolidation load and its practical significance." Proc. Harvard Conf. on Soil Mechanics, Vol. III,
- p. 60. 2. A. Casagrande and R. E. Fadum. 1940. "Notes on soil testing for engineering

- Â. Casagrande and R. E. Fadum. 1940. "Notes on soil testing for engineering purposes." Grad. School of Eng. Harvard Univ., Publ. No. 268.
   L. F. Cooling. 1943. Discussion on :--" 'Earth pressure and shearing resistance of plastic clay." Trans. Amer. Soc. Civ. Engrs, Vol. 108, p. 1086.
   L. F. Cooling and H. Q. Golder. 1940. "A portable apparatus for compression tests on clay soil." Engineering, Vol. 149, p. 57.
   L. F. Cooling and H. Q. Golder. 1942. "The analysis of the failure of an earth dam during construction." J. Instn Civ. Engre, Vol. 19 (1942-43), p. 38 (New 1949) dam during construction." (Nov. 1942).
- 6. H. C. Darby. 1940. "The Draining of the Fens." Cambridge University Press, London.
- W. E. Doran. 1941. "The Ouse flood problem." Geogr. Journ., Vol. 97, 7. No. 4 (April), p. 217.
- No. 4 (April), p. 217.
  8. W. Fellenius. 1927. "Erdstatische Berechnungen." W. Ernst & Sohn. Berlin.
  9. H. Godwin. 1940. "Studies of the post-glacial history of British vegetation." Phil. Trans. Roy. Soc., B, Vol. 230, p. 239.
  10. H. Q. Golder. 1942. "An apparatus for measuring the shear strength of soils." Engineering Vol. 153, p. 501.
- Phil. 17000. 109.
  10. H. Q. Golder. 1942. "An apparatus for measuring Engineering, Vol. 153, p. 501.
  11. B. K. Hough. 1942. Discussion on :—"Relation of undisturbed sampling to laboratory testing." Proc. Amer. Soc. Civ. Engrs, Vol. 68, No. 10 (Dec.),

- L. Rendulic. 1937. Ein Grundgesetz der Tonmechanik und sein experimenteller Beweis. Bauinginieur, Vol. 18, Heft 31/32, p. 459.
   A. W. Skempton. 1942. "An investigation of the bearing capacity of a soft clay soil." J. Instn Civ. Engrs, Vol. 18 (1941-42), p. 307 (June 1943).
   A. W. Skempton. 1944. "Notes on the compressibility of clays." Quarterly L. Carl Stat. Vol. 100 110 clay soil." J. Instn Civ. Engrs, Vol. 18 (1941-42), p. 307 (June 1943).
  16. A. W. Skempton. 1944. "Notes on the compressibility of clays." Quarterly J. Geol. Soc., Vol. 100, p. 119.
  17. S. B. J. Skertchly. 1877. "The Geology of the Fenland." Memoir of the
- Geological Survey, London.

- K. Terzaghi. 1929. "The mechanics of shear failure on clay slopes, and the creep of retaining walls." Public Roads, Vol. 10, No. 10 (Dec.), p. 177.
   K. Terzaghi. 1932. "Tragfähigkeit der Flachgründungen." Int. Assoc. Bridge and Structural Eng. Prelim. Pub., Paris, p. 659.
   K. Terzaghi. 1936. "The stability of slopes of natural clay." Proc. Harvard Conf. on Soil Mechanics, Vol. 1, p. 161.
   K. Terzaghi. 1941. "Undisturbed clay samples and undisturbed clays." J. Boston Soc. Civ. Engrs, Vol. 27, No. 3 (July), p. 211.
   K. TEEZAGHI. 1943. "Theoretical Soil Mechanics." John Wiley & Sons, New York.

# A STUDY OF THE GEOTECHNICAL PROPERTIES OF SOME POST-GLACIAL CLAYS

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# INTRODUCTION

At the present time there are many uncertainties involved in our knowledge of the geotechnical properties of clay strata. Many of these uncertainties arise from the apparent discrepancy between the properties of samples of clay as measured in the laboratory and the behaviour of clays in nature. The method of estimating pre-consolidation load, developed by Casagrande (1936), for example, is widely used but is not accepted by all investigators as being consistent with their field observations. Yet purely experimental evidence would seem to confirm the method. Similarly some clays, which are presumed to be normally consolidated under their own weight, fail to show any appreciable decrease in water content with depth (Terzaghi 1941). Moreover, in several cases the shear strength also shows no definite tendency to increase with depth (Terzaghi 1936, Peck 1943, Housel 1943, Hough 1944). This has been taken as indicating that strength, in these clays, is independent of pressure and, therefore, that they are non-frictional. However, in the laboratory, the strength of any clay invariably increases with increasing consolidation pressure, and this gain in strength is always accompanied by a decrease in water content.

The point which has occasioned perhaps the greatest difficulty in interpretation is the fact that satisfactory estimates of earth pressure, bearing capacity and stability of slopes in homogeneous clay strata have repeatedly been obtained by using the assumption that the clay behaves as a purely cohesive material with a shear strength equal to one-half the unconfined compression strength and an angle of shearing resistance  $\phi$  equal to zero [see Skempton and Golder (1948) for a description of eleven examples from engineering practice]. Is this another indication that clays in nature have negligible frictional properties? And if this is the case, how is it that the  $\phi = 0$  analysis holds good in a wide range of clays, including those which show a definite increase in strength with depth? Above all, how can the success of this analysis be related to the fact that in laboratory tests clays are found to possess very marked frictional properties? The proof of these frictional properties is most readily seen in the inclination of the shear planes in a compression specimen. In a purely cohesive material these planes are inclined at 45°, yet in almost all clays they are inclined at angles considerably steeper than 45° (Terzaghi 1938, Skempton 1948 A). The frictional properties are, indeed, so important in clays such as the Boston clay that Taylor (1944) is inclined to the view that cohesion contributes little to their shear strength.

This brings us to another difficulty, since Hvorslev (1937) has shown that the two clays on which he carried out his classic research on shear strength both possessed appreciable cohesion.

Where, then, in all these questions is the truth to be found? A definite answer cannot possibly be given until far more data is available. I believe, however, that even at the present stage of our enquiry several of the problems can be resolved, and in order to present a reasonably consistent account of the matter I have decided to use, principally, the data which is available on five post-glacial clays : the Fens and the Gosport clays from Southern England, and the Chicago, Massena and Boston clays from the north-eastern States of America. These clays have been the subject of careful investigation and, in addition, their geological history is known.

No finality is claimed for the views which are expressed. Nevertheless it is hoped that they may prove helpful in further work on the practical and scientific problem associated with clay strata.

## GEOLOGY OF THE POST-GLACIAL CLAYS

The conditions of deposition of the post-glacial clays were controlled by two fundamental factors: (i) a world-wide *eustatic* rise in sea level, caused by the melting of the ice sheets, and (ii) local *isostatic* uplift of the land, in those areas which had been covered with ice, due to the recovery of the earth's crust after removal of the ice load.

Geological data and calculations based on the known extent of the glaciated areas all lead to the conclusion that the eustatic rise amounted to at least 200 ft. while the results of varve and pollen analyses [see Zeuner (1946) for a summary of this work] show that the major part of this increase in the depth of the sea took place between 10,000 and 4,000 years



ago. In regions such as the south coast of England, which were comparatively remote from the glaciated areas, the eustatic rise was the chief and perhaps the only geological event of any significance in the post-glacial period. Consequently we find evidence of an almost continuous deposition of sediment during the rise in sea level. Such were the conditions of deposition of the deep bed of clay at Gosport (Skempton 1948 B). The land surface here stood, in early post-glacial times, at about 60 ft. below present sea level and on this surface peat was forming. The rising sea then flooded the land and deposition of the clay commenced, see Fig. 1 (a). This process continued until the marine transgression ceased about 3,000 or 4,000 years ago (Godwin 1943).

Essentially different conditions obtained in a region such as the valley of the R. Forth. Here, immediately after deglaciation, the land lay well below sea level which, by this time, had risen considerably owing to earlier melting of the ice sheets elsewhere. Therefore, the valley became submerged and clays were deposited. Removal of the local ice load, however, also permitted isostatic recovery to take place and, in due course, this uplift exceeded the eustatic rise and the clays were raised to form dry land. The varying sea and land levels are shown diagrammatically in Fig. 1 (b). The clay at Kippen, near Stirling (Skempton 1942), is typical of those formed in this way and, as would be expected, it has undergone very considerable drying near the surface, where the strength far exceeds that of the lower layers of the clay which have never been above ground water level. The foregoing description also applies, as a simplified account, to the clays in the St. Lawrence Valley. The Massena Clay (Casagrande 1944 B) is an example of these clays which were laid down during a post-glacial marine transgression extending as far up the valley as Ottawa and Lake Ontario (Flint 1947). Subsequent isostatic recovery has lifted the clays to their present elevation.

In other regions, especially those near the edges of the great ice-sheets, the interplay of land and sea levels resulted in a more complex history. Examples are to be found at Boston and, to a smaller degree, in the English Fenland. Initially, deposition took place during a period of marine transgression : isostatic uplift then brought the new sediments above sea level and their surface was exposed to weathering and drying. A decrease in the rate of uplift then caused a re-submergence and more material was deposited on the weathered surface of the older clays, see Fig. 1 (c). But, although taking place at a slower rate, uplift was still active, whereas the eustatic rise eventually ceased. Consequently the second stratum, represented as silt in Fig. 1 (c), was finally raised above the sea to form dry land. At Boston the top of the lower post-glacial clay is revealed by a layer with a considerably greater strength than the underlying clay (as at Kippen and Massena) and at some sites it is seen to be weathered to a yellow-brown colour. It is covered with sand and silts of the second submergence, which have themselves now been lifted above sea level (Casagrande 1944 A). In the Fens the alternations in sea and land were rather small (Godwin 1940). Freshwater peat was formed on the surface of the lower clays after their uplift and it is probable that little if any drying took place in the underlying clay during this period. This clay is therefore normally consolidated under the weight of silt which was deposited during the second rise in sea level (Skempton 1945) even though deposition of the whole post-glacial series was not continuous as at Gosport.

The conditions at Chicago were probably rather similar to those at Boston : the deposits in this case resulting from the alterations in level of Lake Michigan. The strata consist of a bed of soft clay, weathered near its surface and covered with sands and silt (Terzaghi 1943 A).

#### MINERALOGY

The post-glacial clays at Gosport and the Fens were derived principally from clays and limestones of Eocene, Cretaceous and Jurassic Ages. These include the London, Oxford and Gault clays which contain normal clay minerals such as Illite, with marked colloidal properties. In Fig. 2 the liquid limits of a number of clays are plotted against the percentage *clay fraction* (particles less than 2 microns in diameter\*) and it will be seen that these two post-glacial clays are very similar to their parent materials. Moreover, the Gosport clay is known to consist of about 60 per cent. Illite and 40 per cent. Halloysite.<sup>†</sup>

In contrast the clays from Boston, Chicago and Massena all lie in the zone of "inactive" clays in which, for any given clay fraction, the liquid limit is considerably lower than in the normal clays, such as those from Gosport, and the Fens. The activity of the three North American clays is, in fact, lower than that of kaolin, which is the least colloidal of all clay minerals. The explanation of this difference is probably that the American clays were formed from the products of glacial erosion of igneous and metamorphic rocks, none of which contain any clay minerals (Nagelschmidt 1944). More evidence is required on this point, but it is possible that these clays, owing to their geological origin, have no strong colloidal properties and, as we shall see later, it does appear that they possess little if any true cohesion. In this respect they differ fundamentally from the Gosport and Fens clays, and also from the clays used by Hvorslev (1937).

#### PRE-CONSOLIDATION LOAD

Turning now to the several problems which were mentioned in the introduction, I shall deal firstly with the validity of Casagrande's method of determining pre-consolidation load. This method is based on the experimental observation that if a clay has been consolidated



under some pressure such as  $p_a$  in Fig. 3 (d), and an undisturbed sample is then taken and subjected to the usual codometer test, a p-w curve<sup>‡</sup> is obtained, as shown by curve "a," with a very marked change in slope at the *pre-consolidation load*  $p_a$ . Now in a bed of clay which has been deposited in water and has never been exposed to drying and is fully consolidated under its own weight—in, that is to say, a *normally consolidated clay*§—the effective overburden pressure increases with depth according to an approximately linear law. Con-

\*This definition of *clay fraction* does not distinguish between the actively colloidal clay minerals and relatively inactive particles which, nevertheless, are smaller than 2 microns.

<sup>†</sup>X-ray analysis by Dr. G. Nagelschmidt, quoted by Skempton (1948 B).

 $\pm i.e.$ , the relation between water content (w) and effective pressure (p).

§This term is used in the sense defined by Terzaghi (1941).



sequently it would be expected that if two undisturbed samples were

if two undisturbed samples were taken from depths a and b in such a bed of clay the pre-consolidation loads for these samples should be equal to  $p_a$  and  $p_b$  respectively.

This result has actually been obtained in the Gosport clay (Skempton 1948 B). In Fig. 4 the p--wcurves for two typical samples are shown, and in the inset to this Figure the pre-consolidation load as determined by Casagrande's method is plotted against effective over-burden pressure for all the undisturbed samples which were subjected to œdometer tests. It will be noted that a reasonable degree of conformity is found to exist.

The clay underlying the later deposit of silt in the Fens is another example of a clay which is known from geological evidence to be normally consolidated. Unfortunately samples have, up to the present, only been taken from one rather narrow zone of depth in this clay, but they show good agreement between pre-consolidation load and effective over-burden pressure (Skempton 1945).

Casagrande (1944 A and B) has given the results of a number of œdometer tests on samples of the Boston and Massena clays, see Fig. 5.



For Boston clay agreement between pre-consolidation load and present overburden pressure is found only at depths of more than about 40 ft. below the top of the clay. Above this depth, and throughout the full thickness of the Massena clay, the pre-consolidation loads exceed the overburden pressure. It is significant, however, that in each case the pre-consolidation loads progressively increase as the top of the clay is approached and Casagrande, invoking the geological history of these clays, as outlined earlier in this paper, has suggested that this phenomena is a direct consequence of the drying to which they have been exposed. It is perhaps unusual for the effects of drying to be noticeable to such depths, but the explanation is consistent with the observations and must be accepted provisionally. Had the variations in pre-consolidation load with depth been erratic, this acceptance would, of course, not be possible.

I am not aware of any published data on the variation of pre-consolidation load with depth throughout the full thickness of the Chicago clay, but Rutledge (1944) has given results on five samples, all taken in the soft clay where the overburden pressure lay between 1.0 and 1.30 kg./cm.<sup>2</sup>. The pre-consolidation loads were in good agreement with the overburden pressures.

The foregoing evidence is clearly not conclusive. Yet the apparent inconsistencies in the Boston and Massena clays can at least be related to the geological history of those strata, while the data from Gosport, where the conditions of deposition were almost ideally simple, are very promising. Further information from other sites may or may not confirm the method and at present we must keep an open mind. It is possible, for example, that the change in slope of the p-w curve in laboratory tests is related to some change in the internal structure of the clay which takes place at a certain pressure in the test not necessarily equal to the actual pre-consolidation load. However, in the following analyses of the properties of the post-glacial clays I shall *assume* that the method is valid and, as will be seen, this assumption leads to a certain conformity in what is otherwise a rather discordant set of results.





#### WATER CONTENT

Laboratory tests show that the water content of a normally consolidated clay decreases more or less exponentially with increasing pressure, as shown in Fig. 3 (d). Thus it is to be expected that in nature the water content in a clay of this type would decrease exponentially with depth. In order to investigate this effect it is necessary to determine, not only the water content, but also the liquid and plastic limits of a series of samples taken at various depths. The "corrected" water content is then calculated from the expression :—

$$w = PL + L.I. \times (LL - PL) \tag{1}$$

where LL and PL are the average limits for the whole stratum and L.I. is the actual liquidity index of the particular sample. In this way the inevitable small changes in water content due to variations in composition of the clay with depth are largely eliminated.

Using this method the water content of the Gosport clay was found to decrease with overburden pressure (or pre-consolidation load) in a manner comparable with that predicted from the laboratory test results shown in Fig. 4. Similarly, for a bed of post-glacial clay at Köping, near Stockholm, there is a definite decrease in water content in the zone beneath the top 7 ft. in which drying has taken place\*, see Fig. 9. The clarity of these cases is rather unusual and often the variations in water content over a comparatively small range of depths are barely perceptible; especially in soft clays with a highly developed structural arrangement of the particles (Terzaghi 1941 and 1947). Nevertheless, when evidence is collected together from boring records ranging in depth from a few inches to several thousand feet it is found that, for clays of the same broad mineralogical type, there is a well-marked approximately exponential decrease in water content with increasing overburden pressure (Skempton 1944).

The variations in water content with depth in the Boston and Massena clays can supply little direct data on this problem owing to the drying action which, as mentioned in the previous section, appears to extend over the top 30 to 50 ft. in these clays. It is important to note, however, that the variations in water content reflect in a general way the variations in pre-consolidation load.

We may therefore conclude (i) that in normally consolidated clays the water content decreases with increasing pressure, and (ii) that if little or no change with depth is observed

\*Information on the Köping clay is published, with permission, from data given in a personal communication from Professor W. Kjellman.

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this result is due either to the small range in pressure covered by the observations (especially if the clay has a highly developed structure) or to drying which has occurred either during or after the main period of deposition. It is, of course, not to be expected that the relation between water content and pressure in a clay consolidated slowly in nature can necessarily be reproduced exactly in laboratory tests. Comparisons of laboratory and natural p-wcurves have not frequently been made, but the indications are that the laboratory tests give water contents, under any particular pressure, which are appreciably lower than those in nature—the differences being most marked in soft clays and rather small in heavily consolidated clays (Terzaghi 1941, Skempton 1944). These indications are in accord with the fact that the processes of sampling and testing must lead to some breakdown of the delicate micro-structure in soft clays—with a consequent increase in compressibility.

# SHEAR STRENGTH

If several specimens, prepared from an undisturbed sample of homogeneous saturated clay, are tested in the triaxial apparatus under conditions of no water content change—the dw = o test—it is invariably found that the compression strength has a constant value, independent of the lateral pressure  $\sigma_1$ . The results, when plotted in the form of Mohr's circles, see Fig. 6 (a), show at once that the clay is behaving with respect to the applied stresses at failure as a purely cohesive material with an *angle of shearing resistance*\*  $\phi$  equal to zero. Consequently, the *shear strength*† c is equal to one-half the compression strength ; and it is to be noted that the compression strength may be determined simply by carrying out an unconfined compression test. The criteria of failure may be concisely stated as follows :—

$$\begin{array}{c} \frac{1}{2} \left( \sigma_{1} - \sigma_{3} \right) = c \\ \phi = 0 \end{array} \right\}$$

$$(2)$$

We may now assume that a clay has been normally consolidated under a series of pressures,  $p_a$ ,  $p_b$ ....,p, and that the shear strengths  $c_a$ ,  $c_b$ ....,c, have been determined for each sample by carrying out triaxial tests without any water content change under the applied stresses. The results of such a series of tests could be plotted, as shown by the full line circles in Fig. 7, the major and minor principal stresses being taken as (p+2c) and p, and it



\*This term defines the rate of increase in shear strength with normal pressure (Terzaghi 1943 B). fi.e., the shear strength of an ideal cohesive material with a compression strength equal to that of the clay specimen. It is also defined as the *apparent cohesion* of the clay. is found that the failure envelope is usually a close approximation to a straight line passing through the origin. The slope of this line will be defined as  $\psi$ , the angle of shearing resistance with respect to the consolidation pressure, and clearly

$$\psi = \sin^{-1}\left\{\frac{c}{p+c}\right\} \tag{3}$$

The angle  $\psi$  is a convenient measure of the increase in shear strength, as determined in dw = o tests, with increasing consolidation pressure.



Now if the previous views on the variation of pre-consolidation load with depth are correct, it follows that we should not look for the relation between shear strength and depth (or overburden pressure), but for the relation between strength and pre-consolidation load, For, in a case such as the Boston clay, for example, if the clay near the top of the stratum has been consolidated by drying, the strengths in this zone may well be greater than those deeper in the clay. In calculating  $\psi$  it is therefore necessary to use the pre-consolidation pressure in equation (3), and, as previously mentioned, I shall assume that this pressure is given, at least approximately, by the Casagrande method.

For the Gosport clay, shear strength shows a very definite increase with increasing preconsolidation load,\* as will be seen in Fig. 8, and  $\psi = 13^{\circ}$ . I do not know of any other published data giving a series of values of c and pre-consolidation load throughout a stratum of considerable depth, but from the Fens, Chicago and Massena clays one or more results are available for a set of samples taken from a relatively narrow zone of depth. By assuming the validity of equation (2) the value of  $\psi$  can therefore be calculated. In addition, on the Chicago, Boston and Massena clays,  $\psi$  was determined from a series of laboratory tests, in which undisturbed samples were normally consolidated under various pressures.

This data is collected together in Table I, and I have added the result for the Köping clay. In this case the pre-consolidation loads are not known, but the strength increases in such a regular manner with depth, as shown in Fig. 9, that we are probably justified in using overburden pressure in equation (2).

In considering these results it is to be noted that at both Gosport and Köping there is direct evidence of an increase in strength with increasing pressure. Moreover, at Gosport, the actual strength values have been checked by an analysis of a rotational slip (Skempton 1948 B). This is also true of the Fens clay (Skempton 1945), while at Chicago the shear strength of the clay was used in successful estimates of earth pressure in a deep excavation through the stratum (Peck 1943). In these two cases, however, as well as in the Massena

<sup>\*</sup>In this clay, since pre-consolidation load is equal to overburden pressure, the value of  $\psi$  is the same whichever pressure is used.

A		Average.	verage.		ue of ψ	D. (	
	ſ	w	LL	PL	Field.	Laboratory.	Reference.
		55	80	30	13°		Skempton (1948 B)
		58	85	32	14 <u></u> 2°		Skempton (1945)
		51	55	26	9°	114°	Casagrande (1942)
		66	45	25	10°	15°	Casagrande (1944 B)
•••		40	48	25		19 <u>‡</u> °	Taylor (1944)
•••		102	110	33	15 <sup>1</sup> °		Kjellman*
	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·		w         LL             55         80             51         55             66         45             40         48             102         110	w         LL         PL             55         80         30             55         26              51         55         26             66         45         25             40         48         25             102         110         33	$w$ $LL$ $PL$ Field.             55         80         30         13°             58         85         32         14 $\frac{1}{2}$ °             51         55         26         9°             66         45         25         10°             40         48         25              102         110         33         15 $\frac{1}{2}$ °	Average.         Value of $\psi$ w         LL         PL         Field.         Laboratory.             55         80         30         13°              55         80         32         14 $\frac{1}{2}°$ 51         55         26         9°         11 $\frac{1}{2}°$ 66         45         25         10°         15°             40         48         25          19 $\frac{1}{2}°$ 102         110         33         15 $\frac{1}{2}°$

Table I Values of  $\psi$  for Normally Consolidated Clays

clay, the value of  $\psi$  can be calculated only for one particular depth. Therefore, in the Fens, Chicago and Massena clays there is no direct proof of a gain in strength with pressure;\* but the fact that the values of  $\psi$ , as derived from calculations based on this assumption, are of the same order as those found for the Gosport and Köping clays, *suggests* that they do, in fact, exhibit this property. Obviously, additional data are required before more definite conclusions can be drawn.

That the values of  $\psi$  for clays consolidated in the laboratory should be greater than the corresponding "field" values is not surprising. Consolidation in the laboratory, as mentioned in the previous section, usually results in a lower water content than is produced by the same pressure in nature; whilst, in addition, some loss in strength due to sampling disturbance is almost inevitable. Correction for this latter factor would probably increase the field values of  $\psi$  for the Chicago and Massena clays to about 10° and 13° respectively.

Summarizing, it may therefore be said that so far as the foregoing cases are concerned, there is evidence of a gain in strength with consolidation pressure : definite in the case of Gosport, very probable in the case of Köping and the Fens, and probable for the Chicago and Massena clays. Now a clay from the St. Lawrence valley, which is probably similar to



\*Investigations at present in progress on samples of Fen clay with very small pre-consolidation loads give values of  $\psi$  similar to that quoted in Table I. The tests are not yet complete.

or identical with the Massena clay, has been cited by Hough (1944) as an example where there is no appreciable change in strength with depth (the top 10 or 15 ft. where oxidation and obvious drying has occurred, being excluded). But this result, far from being unexpected, is in good accord with the approximately constant pre-consolidation load throughout the lower 35 ft. of this clay, as shown in Fig. 5. The Chicago clay also shows no general tendency to increase in strength with depth. In fact, so far as the very variable nature of this clay allows any generalization to be made, the strength decreases for a depth of roughly 15 ft. below the upper surface of the stratum and then gradually increases (Peck 1943). Unfortunately no published records of pre-consolidation loads throughout the depth of the stratum are available, but this variation in strength is easily understood if the consolidation history of the clay is similar to that at Boston. A third example which appears to be contrary to the general conclusion stated above is the Detroit clay (Housel 1943). Here there is a slight increase in strength with depth, but since no pre-consolidation loads or Atterberg limits were published in relation to the investigations in this clay no deductions can be made.

It therefore appears that these clays are not necessarily anomalous. On the other hand, we cannot yet draw any final conclusions or generalize for all clay types. Indeed, data obtained from the analysis of several slips in soft clay strata in Sweden indicate that the gain in strength, even in a depth of about 100 ft., is not appreciable (Terzaghi 1936). Whether or not this behaviour can be explained in terms of the geological history of the clays I cannot say, but it is interesting to note that Terzaghi (1947) has suggested a possible micro-structural arrangement of the particles which would lead to a very small value of  $\psi$  in nature. He assumes that the major part of the overburden pressure may be transferred through the stratum by the silt grains, arranged in an open packing embedded in a matrix of fine clay particles. These particles are consequently "protected" from the full overburden pressure and develop a strength, as a result of thixotropic hardening, which is almost independent of depth. However, on taking a sample and testing it the equilibrium of the silt structure is partially or wholly destroyed and the measured strength is therefore predominantly controlled by the strength of the clay matrix.

Thus, in general, we may possibly find field values of  $\psi$  varying between almost zero and 15° or 20°. Nevertheless, the main points which emerge from the present study are (i) that before the value of  $\psi$  can be deduced it is necessary, in all except the most obvious cases, to give close consideration to the geological history and pre-consolidation loads, and (ii) that with such consideration the post-glacial clays examined in this paper give rather consistent results, irrespective of the great differences in the variations of their shear strength with depth.

#### Angle of Internal Friction

It is now necessary to consider the fundamental components of shear strength in clays. As mentioned in the introduction it has been suggested by Taylor (1944) that cohesion is negligible in many clays. Yet for the general case it is reasonable to assume that clays possess both cohesion and internal friction. This assumption leads to the Coulomb-Hvorslev equation for the shear strength on any plane in a saturated clay :—

#### $s = c_{e} + (n - u) \tan \phi_{f} \tag{4}$

In this equation  $c_e$  and  $\phi_f$  are the *true cohesion* and *angle of internal friction* at the particular water content of the clay, n is the total normal pressure on the plane and u is the pore water pressure. Both  $c_e$  and  $\phi_f$  will decrease with increasing water content.

From equation (4) it can be shown (Terzaghi 1938) that failure in an isotropic compression specimen takes place along a plane or series of planes inclined at an angle a where

$$\mathbf{x} = 45^{\circ} + \frac{1}{2} \phi_{\mathbf{f}} \tag{5}$$

as shown in Fig. 6 (b). This result is theoretically true for all conditions of test, including the case in which the clay is stressed under conditions of constant water content and the angle of shearing resistance  $\phi$  is equal to zero.

Examination of all the published data shows that  $\phi_f$ , as determined by measuring the angle  $\alpha$ , varies from about 10° to 30° for the great majority of clays. Only in bentonite, which consists almost exclusively of the clay mineral montmorillonite, is  $\phi_f$  found to be zero (Skempton 1948 A).

Hvorslev (1937) has proposed an entirely independent method of determining  $\phi_f$  and the results for the few clays which have been investigated according to this method are in approximate agreement with those determined from the inclination of the shear planes.

There can therefore be no reasonable doubt but that, in general, clays possess a definite angle of internal friction, very appreciably greater than zero.

# TRUE COHESION

From equations (4) and (5) it can be shown (Skempton 1948 A) that the compression strength of a saturated clay which has been consolidated under a pressure p and then tested in the triaxial apparatus, or in the unconfined compression apparatus, without further water content change, is given by the expression :---

$$(\sigma_1 - \sigma_2) = 2c = 2 \left\{ \frac{c_e \cos \phi_f + p \sin \phi_f}{1 + \sin \phi_f} \left( \frac{1 - 2\lambda}{1 + 2\lambda} \right) \right\}$$
(6)

where  $\lambda$  is the ratio of the expansibility to the compressibility of the clay in terms of decreasing and increasing effective pressures. From the results at present available and from physical considerations it seems probable that  $\lambda$  lies between 0 and  $\frac{1}{2}$  for normally consolidated clays.

Thus if  $\lambda$  is either measured or assigned a reasonable value, the true cohesion can readily be calculated from equation (6), since the compression strength and the consolidation pressure are known and  $\phi_f$  can be found, at least approximately<sup>\*</sup>, from the inclination of the shear planes.

Two alternative methods are available, however, which eliminate the necessity of knowing  $\lambda$ . They both involve a determination of the effective stresses at failure, either by direct measurement of the pore water pressure, or by carrying out a series of slow shear or triaxial tests in which no excess pore water pressures are allowed to develop. In the first case the effective stresses can be immediately found from the equations :---

$$\begin{array}{c} \sigma'_{1} = p + \sigma_{1} - u \\ \sigma'_{3} = p + \sigma_{3} - u \end{array}$$

$$(7)$$

The Mohr's circle corresponding to these stresses is then drawn, as shown in Fig. 6 (c), and if  $\phi_t$  is known the value of  $c_e$  can be obtained either graphically or by calculation. In the second case a series of specimens are consolidated under pressures  $p_a$  and  $p_b \ldots$  (see Fig. 7), and then tested in the triaxial apparatus with full opportunity for water content change under the applied stresses (the u=o test). Since in each of these tests these stresses are directly equal to the *effective* stresses, the Mohr's circles will theoretically share a common tangent with the circles obtained by measuring the pore water pressures. The inclination of this tangent is defined as  $\phi_s$ , the *angle of shearing resistance with respect to effective stress*. Now it can be shown (Skempton 1948 A) that the effective stresses in a dw=o test are :—

$$\sigma'_{1} = p + \sigma_{1} - u = p + (\sigma_{1} - \sigma_{3}), \quad \frac{2\lambda}{1 + 2\lambda}$$
  
$$\sigma'_{3} = p + \sigma_{3} - u = p - (\sigma_{1} - \sigma_{3}), \quad \frac{\lambda}{1 + 2\lambda}$$
(8)

<sup>\*</sup>The effects of anisotropy may preclude an exact relationship between  $\alpha$  and  $\varphi_f$ .

Consequently :--

$$\phi_{s} = \sin^{-1}\left\{\frac{c}{\sigma'_{s} + c}\right\} = \sin^{-1}\left\{\frac{\frac{c/p}{1 - \frac{c}{p}}}{1 - \frac{c}{p}}\frac{1 - 2\lambda}{1 + 2\lambda}\right\}$$
(9)

Thus, if by either of the above methods the value of  $\phi_s$  is known, the terms  $\left(\frac{1-2\lambda}{1+2\lambda}\right)$  can be evaluated and used in equation (6) in order to find the true cohesion  $c_e$ . It is to be noted that

 $c_e$  will vary with the consolidation pressure p. Hvorslev has shown, however, that the ratio  $c_e/p$  is approximately constant and this parameter is therefore evaluated in the following calculations.

For the Gosport clay  $\psi = 13^{\circ}$  (c/p = 0.29) and slow shear tests gave  $\phi_s = 23^{\circ}$ . Unfortunately at the time of the investigation I did not appreciate the importance of measuring the inclination of the shear planes. The clay is, however, very similar to the Fens clay in which  $\phi_f = 12^{\circ}$ . Assuming this value we therefore find that  $\lambda = 0.02$  and  $c_e/p = 0.14$ . Thus about one-half the strength is due to cohesion.

For the Fens clay  $\psi = 14\frac{1}{2}^{\circ}$  (c/p = 0.32) and  $\phi_f = 12^{\circ}$ . In this case no slow shear tests were carried out, but from ædometer tests  $\lambda$  is known to be small. Assuming  $\lambda = 0$  and 0.4 as reasonable limits, we find that  $c_e/p = 0.18$  and 0.12 respectively. The value of  $c_e/p$  is therefore not sensitive to  $\lambda$  and we can take  $\lambda = 0.2$  and  $c_e/p = 0.15$ , as representative average results. It should be noted that in both the Gosport and the Fens clay the shear strength c has been confirmed by a  $\phi = 0$  analysis of stability in field investigations.

For the Chicago clay we shall take the laboratory value of  $\psi = 11\frac{1}{2}^{\circ}$  (c/p = 0.25). Slow triaxial tests gave  $\phi_s = 19^{\circ}$  and from the inclination of the shear planes  $\phi_f = 21^{\circ}$ . Obviously  $\phi_f$  cannot exceed  $\phi_s$  and it must therefore be assumed that  $\phi_f$  is in error by at least 2°. If this is the case then it is at once seen that  $c_e/p$  is zero. From the values of  $\psi$  and  $\phi_s$ , it can be found that  $\lambda = 0.03$ .

For the Massena clay  $\psi = 15^{\circ}$  (c/p = 0.35),  $\phi_s = 30^{\circ}$  and  $\phi_f = 28^{\circ}$ . Consequently,  $\lambda = 0.04$  and  $c_e/p = 0.02$ .

For the Boston clay  $\psi = 19\frac{1}{2}^{\circ}$  (c/p = 0.50) and from direct measurements of pore water pressure in the triaxial tests, carried out at constant water content, it was found that  $\phi_s = 33^{\circ}$ . The inclination of the shear planes showed that  $\phi_f = 32^{\circ}$ . Thus  $\lambda = 0.37$  and  $c_e/p = 0.01$ .

No information is available on the value of  $\phi_t$  for the Köping clay.

The foregoing results are summarized in Table II, and the values obtained by Hvorslev from his work on the Wiener Tegel and Klein Belt Ton are also included. It will immediately be seen that for these two clays\* and for the Fens and Gosport clays, the values of  $c_e/p$  are of the same order and, moreover, in the latter two cases the cohesive term constituted about one-half the total strength of the clay. In contrast, the three American clays possess almost no cohesion.

<u></u>		Average values.				- / 4	Defense fr
Clay.		w.	LL	PL	îφ	$c_{e}/p$	original data.
Wiener Tegel		25	47	22	17 <u></u> *°	0.10	Hvorslev (1937)
Klein Belt Ton		60	127	36	100	0.15	Hvorslev (1937)
Gosport		55	80	30	12°	0.14	Skempton (1948 B)
Fens		58	85	32	12°	0.15	Skempton (1945)
Chicago		51	55	26	19°	0	Casagrande (1942)
Massena		66	45	25	28°	0.02	Casagrande (1944 B)
Boston		40	48	25	32°	0.01	Taylor (1944)

 TABLE II

 Values of True Cohesion and Internal Friction

\*Hvorslev used these clays in laboratory tests, consolidated from a slurry with an initial water content equal approximately to the liquid limit.

#### GEOTECHNICAL PROPERTIES OF SOME POST-GLACIAL CLAYS

The comparison between Hvorslev's clays and the two English clays is particularly interesting since the values of  $c_e/p$  have been determined by entirely different methods<sup>\*</sup> and since, in addition, all four of these clays exhibit a very similar relationship between clay fraction and liquid limit, falling in the zone of "normal" clays, see Fig. 2. On the other hand, it will be recalled that the three American clays fall in the zone of "inactive" clays, and this difference between the two groups may well be the explanation of the very different degrees of cohesion which they appear to possess.

#### The $\phi = 0$ Analysis of Stability

We have now seen that in these post-glacial clays there is an angle of internal friction varying from 12° to 32° and that some possess true cohesion while in others this is negligible. We have also seen that, both in the field and in the laboratory, the shear strength of these clays increases with increasing consolidation pressure; and that this increase may be represented by the angle  $\psi$  which lies between 9° and 20°. It has, finally, been shown that the angle of shearing resistance of the clays, measured in terms of the effective stresses at failure, varies between 23° and 33°.

The results are reasonably concordant and although, as I have previously emphasized, no finality can be claimed for the assumptions which have been used to obtain these conclusions most of the problems mentioned in the Introduction seem to have been resolved. However, there remains the very important practical question concerning the validity of the  $\phi=0$  analysis. In this analysis the assumption is made that the clay behaves as a purely cohesive material, with a shear strength equal to one-half the unconfined compression strength and an angle of shearing resistance  $\phi$  equal to zero. The procedure adopted is to obtain undisturbed samples throughout the depth of the clay which is influenced by construction, measure their unconfined compression strength, take one-half of this value as representing the shear strength of the clay at the particular depths of the sample and then calculate the earth pressure, bearing capacity or slope stability on the assumption that the clay is frictionless. The problem is to relate the success of this method with the undoubted presence of internal friction in the clays and their increase in strength with consolidation pressure. Now it was mentioned earlier, in the section on shear strength, that if a saturated clay is tested under conditions of no water content change it behaves with respect to the applied stresses at failure as a purely cohesive  $\phi = 0$  material. It follows, therefore, that the  $\phi = 0$  analysis of stability applies precisely to those cases in which the clay in the ground is stressed under conditions of no water content change.

Moreover, this condition is frequently encountered in practice owing to the very low permeability of soft homogeneous clays.<sup>†</sup> Thus, in the construction of a building the changes in water content of the underlying clay will usually be almost negligible during the period of construction [see, for example, Skempton (1942)]. Similarly, in a job such as the Chicago Subway (Peck 1943) the struts in the excavation have to support the adjacent ground only for a period of a few weeks and the conditions are therefore ideal for the application of the  $\phi=0$  analysis. Therefore, the success of the analysis is due simply to the fact that in many cases the essential condition for its validity is satisfied. This is independent of any increase, or otherwise, in the strength of the clay with depth or with pre-consolidation load; provided representative samples are obtained. Conversely, it follows that the successful application of the  $\phi=0$  analysis can give no information on the value of  $\psi$ ,  $\phi_s$  or  $\phi_f$  for the clay.

Even with no water content change, however, the  $\phi = 0$  analysis will not lead to a correct estimate of the position of the slip surface. For although the clay behaves, with respect to the applied stress at failure, as a purely cohesive material, yet we have seen that the shear

<sup>\*</sup>Hvorslev's method is not applicable to the data at present available on the post-glacial clays.

<sup>†</sup>It is evident that there are, nevertheless, many practical cases in which the analysis is not directly applicable. Stiff fissured clays and varved clays, for example, require special treatments, a consideration of which lies outside the scope of the present paper.

surfaces are, in fact, controlled by the angle of internal friction  $\phi_{f}$ . Consequently, although the  $\phi = 0$  analysis leads to a correct estimate of earth pressure, for example, and does so by implicitly assuming that failure takes place on a  $45^{\circ}$  shear plane, the actual plane will be inclined at  $(45^\circ + \frac{1}{2}\phi_i)$ . This discrepancy has been observed in several practical cases (Skempton and Golder 1948).

#### SUMMARY

Considerations based on the properties and geological history of several post-glacial clays lead to the following tentative conclusions :-

(i) It appears that Casagrande's method of estimating pre-consolidation loads is at least approximately correct for these clays.

(ii) Assuming the validity of this method it is found that, in the clays considered in the present paper, shear strength (as measured in the unconfined compression test) increases with increasing consolidation pressure in the ground, although in some of the cases this does not imply that the strength increases with depth.

(iii) All the clays possess considerable internal friction. In the American clays, which appear to be colloidally inactive, the great majority of the strength is composed of internal friction, while in the English clays about 50 per cent of the strength is contributed by cohesion.

The foregoing conclusions are not intended as general statements valid for a wide range of clays. However, the principles used in this study, and the results obtained, may prove helpful in further research on the properties of clay strata.

(iv) In addition, it is shown from general considerations that in all cases the  $\phi = 0$ analysis will give reasonably correct estimates of earth pressure, bearing capacity and slope stability in homogeneous saturated clays, provided the analysis is applied to conditions in which water content changes in the clay are negligible. The success of this analysis is entirely independent of whether the strength is constant with depth or increases; and can lead to no conclusion other than the fact that the condition of no water content change held good in the case under consideration.

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#### BIBLIOGRAPHY

CASAGRANDE, A. (1936). "The Determination of the Pre-consolidation Load and its Practical Significance. Proc. 1st Int. Conf. Soil Mechanics. Vol. III, p. 60. Harvard. CASAGRANDE, A. (1942). "Fifth Progress Report on Triaxial Shear Research." Harvard University

Publication.

CASAGRANDE, A. (1944). A. Closing discussion on "Application of Soil Mechanics in Designing Building Foundations." Tr. Am. Soc. C.E. Vol. 109, p. 463. CASAGRANDE, A. (1944). B. "Seventh Progress Report on Triaxial Shear Research." Harvard

University Publication

" Development and Scope of Soil Mechanics." The Principles and Application COOLING, L. F. (1946). of Soil Mechanics. Inst. Civ. Eng. London.

FLINT, R. F. (1947). "Glacial Geology and the Pleistocene Epoch." John Wiley. New York. GODWIN, H. (1940). "Studies of the Post-Glacial History of British Vegetation. IV-Post-Glacial Changes of Relative Land and Sea Level in the English Fenland." Phil. Trans. Roy. Soc., B. Vol. 230, p. 285.

Godwin, H. (1943). "Coastal Peat Beds of the British Isles and North Sea." Journ. Ecol., Vol. 31, p. 199.

Ноидн, В. К. (1944). Discussion on "Relation of Undisturbed Sampling to Laboratory Testing." Tr. Am. Soc. C.E., Vol. 109, p. 1184. JSEL, W. S. (1943). "Earth Pressure on Tunnels." Tr. Am. Soc. C.E., Vol. 108, p. 1037.

HOUSEL, W. S. (1943). "Earth Pressure on Tunnels." Tr. Am. Soc. C.E., Vol. 108, p. 1037. HVORSLEV, M. J. (1937). "Uber die Festigkeitseigenschaften gestörter bindiger Boden." Ingeniorridenskabelige Skriften. A. No. 45. NAGELSCHMIDT, G. (1944). "The Mineralogy of Soil Colloids." Tech. Comm., No. 42. Imperial

Bureau of Soil Science, Harpenden.

- РЕСК, R. B. (1943). "Earth Pressure Measurements in Open Cuts, Chicago Subway." Tr. Am. Soc. C.E. Vol. 108, p. 1008.
- RUTLEDGE, P. C. (1944). Closing discussion on "Relation of Undisturbed Sampling to Laboratory Testing." Tr. Am. Soc. C.E. Vol. 109, p. 1208.
   RUTLEDGE, P. C. (1947). "Triaxial Shear Research." U.S. Waterways Expt. Station, Vicksburg.
- (Note: This contains a summary of the work of CASAGRANDE (1942) and (1944) B, on the shear characteristics of the Chicago and Massena clays.)
- Skempton, A. W. (1942). "An Investigation of the Bearing Capacity of a Soft Clay Soil." Journ. Inst. Civ. Eng. Inst. Civ. Eng. Vol. 18, p. 307. SKEMPTON, A. W. (1944.) "Notes on the Compressibility of Clays." Quart. Journ. Geol. Soc. Vol.
- 100, p. 119.
- (1945). "A Slip in the West Bank of the Eau Brink Cut." Journ. Inst. Civ. Eng. SKEMPTON, A. W. Vol. 24, p. 767.
- SKEMPTON, A. W. (1948). A. "A Study of the Immediate Triaxial Test on Cohesive Soils." Proc. 2nd Int. Conf. Soil Mechanics, Vol. I, p. 192. Rotterdam. SKEMPTON, A. W. (1948). B. "A Deep Stratum of Post-Glacial Clay at Gosport." Proc. 2nd Int.

- SKEMPTON, A. W. (1946). B. A Deep Stratum of Post-Giacial Clay at Gosport." Proc. 2nd Int. Conf. Soil Mechanics, Vol. I, p. 145. Rotterdam.
  SKEMPTON, A. W., and H. Q. GOLDER. (1948). "Practical Examples of the φ=0 Analysis of Stability in Clays." Proc. 2nd Int. Conf. Soil Mechanics, Vol. II, p. 63. Rotterdam.
  TAYLOR, D. W. (1944). "Tenth Progress Report on Triaxial Shear Research." M.I.T. Publication. TERZAGHI, K. (1936). "Stability of Slopes in Natural Clay." Proc. 1st Int. Conf. Soil Mechanics. Vol. I. p. 161. Vol. I, p. 161.
- TERZAGEI, K. (1938). "Die Coulombsche Gleichung für den Scherwiderstand bindiger Boden." Bautech. Vol. 16. p. 343. TERZAGHI, K. (1941). "Undisturbed Clay Samples and Undisturbed Clays." Journ. Boston Soc. C.E.
- Vol. 28, p. 211.
- TERZAGHI, K. (1943). A. "Liner Plate Tunnels on the Chicago Subway." Tr. Am. Soc. C.E. Vol. 108, p. 970.
- TERZAGHI, K. (1943). B. "Theoretical Soil Mechanics." John Wiley, New York. TERZAGEI, K. (1947). "Shear Characteristics of Quicksand and Soft Clay." Proc. 7th Texas Conf. Soil Mech., Univ. of Texas.
- ZEUNER, F. E. (1946). "Dating the Past: An Introduction to Geochronology." Methuen, London.

# The Bearing Capacity of Clays

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# 1. Introduction

The first criterion which must be satisfied in any successful foundation design is that there should be an adequate factor of safety against a complete shear failure in the underlying soil. This is obviously a necessary condition but, in general, it is not a sufficient condition. In addition, the foundations should be designed in such a way that the settlements, and particularly the differential settlements\* of the structure, remain within tolerable limits.

Except for footings or piers with a breadth of only a few feet, the settlement criterion controls the allowable pressures on sands and gravels. Consequently, methods for estimating the ultimate bearing capacity of cohesionless soils have a somewhat restricted value. In contrast, the possibility of a complete shear failure in clays is a very real one, and frequently in practice it is considered necessary, for economic reasons, to work with factors of safety against ultimate failure of not more than 3. Therefore, since these factors are of a similar magnitude to those used in structural materials such as steel and reinforced concrete, it is desirable to possess methods of calculating the ultimate bearing capacity of clays with the same order of accuracy as the methods used in structural design. But in many cases the use of a low factor of safety on the failure criterion leads to very considerable settlements, and it is necessary for the designer to be aware at least of the order of the settlements. He can then adopt a suitable type of structure which can safely withstand the deformations consequent upon the movement Yet the modern forms of conof the foundations. struction involving continuous beams, portal frames, reinforced concrete shells and rigid or semi-rigid frames are sensitive to differential settlements. And these structural forms are usually more economical in materials and more elegant in design than the older forms; particularly in steel and reinforced concrete bridges. Thus it is often more satisfactory to restrict the settlements by using a higher factor of safety. This will increase the cost of the foundations, but will not necessarily increase the cost of the whole structure. Moreover, so far as buildings are concerned, the interior plastering and exterior panelling are themselves sensitive to settlement. By reducing the deformations, the occurrence of unsightly cracking in these elements of the building is also prevented, thereby reducing maintenance charges and enhancing the appearance.

# 2. General Considerations

On opening up the excavation, the pressure at foundation level is reduced to zero from its original value p (equal to the weight per unit area of the soil and water above this level, see Fig. 1). This release of pressure causes the soil to rise by an amount  $\rho_r$ . When the structural load becomes equal to p the original state of stress existing in the ground under the foundation, prior to excavation, is restored. The settlement taking place under the foundation pressure p is  $\rho_P$  and, if the ground were perfectly elastic and no water content changes had occurred, then  $\rho_P$  would be equal to  $\rho_r$ , and, moreover, these movements could be calculated. However, the magnitude of  $\rho_r$  is controlled by many practical factors, and even approximate estimations are difficult. But as a very rough rule it may be said that  $\rho_P$  is of the same order as  $\rho_r$  (see point b in Fig. 1).

Once the foundation pressure exceeds p the ground is subjected to stresses in excess of those existing prior to excavation, and it is the settlements resulting from these excess stresses that are calculated by the present methods of settlement analysis. Similarly, the factor of safety against ultimate failure must be expressed in terms of the so-called "nett pressure"; that is, the pressure at foundation level in excess of the original overburden p.

At the end of construction the nett settlement may be considered as being made up of two parts :

- (i) the "immediate" settlement, due to deformation of the soil taking place without change in water content;
- (ii) the "consolidation" settlement, due to a volume reduction caused by the extrusion of some of the pore water from the soil.

Owing to the presence of the extremely small particles of which clays are composed, the rate of consolidation is very slow and, in general, the elastic settlement is considerably the greater of the two components at the end of construction. There is, nevertheless, a small decrease in water content in the clay beneath the foundation, and this will cause a corresponding small increase in strength. But for the purpose of estimating the factor of safety against shear failure, the assumption is generally made that this increase in strength is negligible. That assumption is not only conservative but it also leads to a great simplification in the calculation. For saturated clays (and most clays are saturated) behave with respect to applied stresses as if they were purely cohesive, non-frictional materials ; provided that no water content change takes place under the applied stresses. That is to say, they exhibit an angle of shearing resistance  $\Phi$  equal to zero.

The assumption that  $\Phi = 0$  forms the basis of all normal calculations of ultimate bearing capacity in clays. Only in special cases, with prolonged loading periods or with very silty clays, is the assumption sufficiently far from the truth to justify a more elaborate analysis.

In the course of time, however, the consolidation becomes important, and leads to the characteristic feature of foundations on clays: namely the longcontinued settlements increasing, although at a decreasing rate, for years or decades after construction. The principal objects of a settlement analysis are therefore to obtain (i) a reasonable estimate of the nett "final" settlement  $\rho_n$ , corresponding to a time when consolidation is virtually complete, and (ii) at least an approximate estimate of the progress of settlement with time. The settlement at the end of construction is of minor consequence in most problems. All settlement calculations are, at the present time, based on the classical consolidation theory of Terzaghi, or on extensions of this theory.

<sup>\*</sup>For the relation between average and differential settlement see the important paper by Terzaghi<sup>1</sup>. Limitations of space restrict the present discussion to average settlements.

q .

# 3. Ultimate Bearing Capacity of Clays $(\Phi = 0)$ General

In the general case, the allowable foundation pressure may be expressed in the form<sup>2</sup>:--

$$IIowable = \frac{I}{F} \left[ c \cdot N_{e} + p_{o} \cdot (N_{q} - I) + \frac{IB}{2} \cdot N \Upsilon \right] + p$$
(I)

where F = the desired factor of safety.

- c = apparent cohesion of the soil.
- $p_{o} =$ effective overburden pressure at foundation level.
- p = total overburden pressure at foundation level.
- $\Upsilon$  = density of soil beneath the foundation (submerged density if foundation is below water level).
- B = breadth of foundation.
- $N_{\rm e}, N_{\rm q}, N_{\rm \gamma} =$  factors depending upon the angle of shearing resistance  $\Phi$  of the soil, the ratio of length L to breadth B of the foundation and the ratio of the depth D to the breadth of the foundation B (see Fig. 1).
- $q_{nf}$  = the term in square brackets, is the nett ultimate bearing capacity.



With the condition that  $\Phi = 0$  the factors  $N_q$  and  $N_{\gamma}$  are equal to unity and zero respectively. Thus equation (1) reduces to the simple form :

$$q_{\texttt{allowable}} = rac{c}{F} \cdot N_{\texttt{c}} + p \quad \dots \quad \dots \quad \dots \quad \dots \quad (2) \texttt{a}$$

and the ultimate bearing capacity is

 $q_t = c. N_e + p \dots \dots \dots \dots \dots \dots \dots (2)b$ The problem of calculating the ultimate bearing capacity of clays is therefore solved when the apparent cohesion c (usually referred to as the "shear strength") of the clay has been determined and the factor  $N_e$ has been evaluated for the particular values of B, Land D.

#### Measurement of c

To determine the shear strength of the clay undisturbed samples are taken from boreholes, which should extend either to the bottom of the clay stratum or to a depth where the stresses caused by the foundation pressures are negligible. Unconfined compression tests or, preferably, undrained triaxial tests\*, are then carried out on specimens cut from these cores; and if  $\sigma_1$  and  $\sigma_3$  are the major and minor principal stresses at failure, then

It is not possible, in this summary, to discuss in detail the procedure for estimating the value of c to be used where the strength varies appreciably with depth. It must suffice to mention that if the shear strength within a depth of approximately 2/3 B beneath foundation level does not vary by more than about  $\pm$  50 per cent. of the average strength in that depth, then this average value of c may be used in equation (2). Value of  $N_c$ 

The values suggested for the factor  $N_c$  are given in Fig. 2. As an example consider a foundation with B = 15 ft., L = 23 ft. and D = 9 ft. Then the value of  $N_c$  for a square footing with D/B = 9/15 = 0.60 is 7.2, from the upper curve in Fig. 2. Thus the required  $N_c$  for the actual rectangular footing with B/L=15/23=0.65 is given by the expression

 $N_{\rm e} = (0.84 \pm 0.16 \times 0.65) \ 7.2 = 6.8$ 



Since an important part of the research work leading to these values of  $N_{e^*}$  is of recent origin, a discussion of their derivation will be given in the following section of the paper. Before doing so, however, it will be convenient to consider the available field evidence on the ultimate bearing capacity of clays. This is assembled in Table I, and it will at once be seen that the evidence, although limited to six cases and although

<sup>\*</sup>For a recent account of triaxial testing methods, and interpretation, see a paper by Skempton and Bishop<sup>6</sup>.

	Din	nension	s of foundation	Approx.	۰ 1۹	Nett	Average	Value of $N_{e}$		Reference
Location and structure	B ft.	L ft.	D <sup>°</sup> ft.	settlement at failure. Fr inches	$\frac{\overline{B}}{\text{per}}$ cent.	pressure at failure. $q_{nt} \text{ ton/ft.}^2$	of clay beneath foundation. c. ton/ft. <sup>2</sup>	$\frac{q_{nf}}{c}$	From Fig. 2	for original data
Hagalund loading tests	1.3	6.5	o (lower limit) 1 (upper limit)		3	0.43	0.074 (vane) 0.067 (compr.)	5.8 6.4	5.4 6.5	Odenstad (1948) <sup>32</sup> Cadling and Odenstad (1950) <sup>5</sup>
Kippen spread footing	8	9	5.5	10	10	0.95 (with side friction) 1.15 (no side friction)	0.16	6.0 7.2	7.2	Skempton (1942) <sup>33</sup>
Loch Ryan screw cylinder	8	8	50	II	128	1.9	0.22	8.6	9.0	Morgan (1944) <sup>23</sup> Skempton (1950) <sup>34</sup>
Newport screw cylinder	8	8	6 (in clay) 20 (total depth)	14	15 <sup>8</sup>	2.9	0.36	8.0	7·4 8.6	Wilson (1950) <sup>13</sup>
Shellhaven oil tank A	25	25	0	_		0.84	0.135 <sup>b</sup>	6.2	6.2	Nixon (1949) <sup>21</sup>
oil tank B	52	52	0	30	5	0.83	0.140 <sup>b</sup>	5.9	6.2	Nixon (personal comm.)
Tunis warehouse	50	125	10	40	7°	· · · · · · · · · · · · · · · · · · ·		_		Fountain (1907) <sup>35</sup>
Transcona grain elevator	76	195	I2	140	15°	2.2	_			Allaire (1916) <sup>36</sup>

TABLE 1.—Field data on ultimate bearing capacity of clays

a: extrapolation from load-settlement curve.

b: in a depth  $z_1 = 2/3B$ .

c: failure by tilting.

subject to the usual lack of precision inherent in any field observations, provides a satisfactory confirmation of the suggested values of  $N_{\rm c}$ . Further, indirect confirmation will be considered in the section on load-settlement curves.

In most cases it is possible to use Fig. 2 directly in the estimation of bearing capacity. But for some purposes it is desirable to have a set of simple rules which can easily be remembered. The following rules may be put forward :

- (i) At the surface, where D = 0,
  - $N_{co} = 5$  for strip footings;  $N_{co} = 6$  for square or circular footings.
- (ii) At depths where  $D/B < 2\frac{1}{2}$ :
- $N_{cD} = (\mathbf{I} + 0.2 D/B) N_{co}.$ (iii) At depths where  $D/B > 2\frac{1}{2}$ :
- $N_{eD} = 1.5 N_{e2}$ . (iv) At any depth the bearing capacity of a rectangular footing is

$$N_{e}$$
 (rectangle) =  $\left[ 1 + 0.2 B/L \right] N_{e}$  (strip).

# 4. Derivation of the Bearing Capacity Factors N<sub>c</sub> Theoretical Results

The analysis of the bearing capacity of strip footings on the surface (D = 0) is due to Prandtl' who showed that  $N_e = 2 + \pi = 5.14$ . The mechanism of failure assumed in this analysis is that the footing pushes in front of itself a "dead" wedge of clay which, in its turn, pushes the adjacent material sideways and upwards. Model tests in the laboratory indicate that this mechanism is a reasonable approximation.

When the footing is placed at a considerable depth the slip surfaces no longer rise up to ground level. Meyerhof<sup>\*</sup> has evolved a modified form of Prandtl's analysis in which the slip surfaces curve back on to the sides of the foundation. For strip footings the corresponding value of  $N_e$  is 8.3; but this is, clearly, an upper limit since it involves too great a length of shear surface. It may be noted that the values of  $N_e$  for strip footings are independent of the amount of shear mobilised along the base of the footing.

For circular footings with a smooth base, on the surface of a clay, a rigorous solution has been obtained by Ishlinsky<sup>•</sup>,  $N_e$  being 5.68. The more practical condition of a rough-based circular footing on the surface of a clay stratum has been solved by Meyerhof, using an approximate\* analysis. This leads to the result  $N_e=6.2$ .

For circular footings located at a considerable depth beneath the surface three solutions are available. With assumptions concerning slip surfaces similar to these mentioned above, Meyerhof finds that  $N_c = 9.3$ . For the reason given earlier  $N_c = 9.3$  is almost certainly an upper limit. A completely different approach is that originated by Bishop, Hill and Mott<sup>10</sup>, for metals, and extended to clays by Gibson<sup>11</sup> using the largestrain theory of Swainger<sup>12</sup>. In this analysis it is assumed that the penetration of the footing, at ultimate failure, is equivalent to expanding a spherical hole in the clay, of diameter equal to the diameter of the footing. If E is the Young's modulus of the clay, then

a plastic zone is developed, of radius 
$$\frac{B}{2} \sqrt[s]{\frac{E}{c}}$$
 beyond

which the clay is still in the elastic state. The expression for  $N_{e}$ , according to Gibson, is

$$N_{c} = \frac{4}{3} \left[ \log_{c} \frac{E}{c} + \mathbf{I} \right] + \mathbf{I} \qquad \dots \qquad (4)$$

For materials with stress-strain curves of the type exhibited by clays it is convenient to define E as the secant modulus at a stress equal to one-half the yield value (see Fig. 3). With this convention the range of E/c for the great majority of undisturbed clays

<sup>\*</sup>An indication of the error involved in this analysis is given by the fact that the same form of solution leads to the result  $N_c = 5.71$  for the smooth base circle, as compared with Ishlinsky's  $N_c = 5.68$ .



Fig. 3.—Ultimate bearing capacity factors for deeply buried circular footings in  $\phi = 0$  materials Theoretical values and laboratory tests



is from 50 to 200. The corresponding values of  $N_{e}$ in equation (4) are 7.6 and 9.4. Thus, even with this four-fold variation in E/c the change in  $N_{e}$  is only  $\pm$  10 per cent., and it is therefore sufficiently accurate to say that the Mott-Gibson theory leads to the result  $N_{\rm e} = 8.5$  for undisturbed clays.

Finally, Guthlac Wilson<sup>19</sup> has approached the problem of the bearing capacity of a clay loaded at depth by a rigid circular plate, by finding the foundation pressure necessary to bring about the merging of the two plastic zones originating from the edges of the footing. The result depends to a slight degree on the depth of the footing and on the original state of stress in the clay, as indicated by the coefficient of earth pressure at rest  $K_{o}$ , but for practical purposes  $N_{e}$  may be taken as 8.0, when D is greater than 4B.

Each of these three approaches to the problem is by no means an exact analysis. And, indeed, the difficulties in the way of producing a rigorous solution for the bearing capacity factor for deep foundations are great. Yet it is remarkable that all three theories lead to values for  $N_c$  within the  $\pm$  10 per cent. range covered by the Mott-Gibson analysis for clays.

#### Experimental Results

The first published results obtained from model footing tests on clay, the shear strength of which was also measured, appear to be those of Golder<sup>14</sup>. These were carried out on footings 3 inches square and 3 inches  $\times$  18 inches long, on the surface of remoulded London clay. The tests were of a preliminary nature, but they showed that  $N_e$  was about 6.7 for the square footings and 5.2 for the long footings. More recently, model tests have been carried out

at Imperial College by Meigh<sup>15</sup> and Yassin<sup>16</sup> on both

remoulded and undisturbed clays. Careful corrections were made for the effects of small decreases of water content in the clay beneath the footings, due to the diffusion of the high pore pressures set up by the load, and for the effects of different rates of strain in the loading tests and the unconfined compression tests. It was found that, if the load-settlement curves were plotted in the dimensionless form shown in Fig. 4, then these curves were almost identical for all sizes of footings used in the experiments and for all values of the shear strength of the clay under investigation. Secondly, it was found that after penetrating about four or five diameters the footings continued to settle under a constant nett pressure. The ratio of this pressure to the shear strength of the clay is clearly the value of  $N_{\rm e}$ for circular footings at a considerable depth beneath the surface, and the experimental results are plotted in Fig. 3. Of the clays, "Horten" and "London"\* were remoulded and "Shellhaven" was undisturbed. The value for plasticine was obtained by Meyerhof (private communication) and that for copper was determined by Bishop, Hill and Mott<sup>10</sup>. For simplicity, only the Mott-Gibson theory and Meyerhof's upper limit of  $N_c = 9.3$  have been shown in Fig. 3. The six experimental points all lie in the zone bounded by these two theories and, for the practical range of E/c for undisturbed clays (50 to 200), it will be seen that, as previously suggested (Skempton 1950), a value of  $N_{\rm e} = 9.0$  is a very reasonable average from the theoretical and experimental results.

Similarly, for deep buried strip footings,  $N_e = 7.5$  is a reasonable average value.

A typical relation between 
$$q/c$$
 and  $\frac{\text{penetration}}{B}$ 

– for

a footing pushed into the clay from the surface is shown by the line O a<sup>1</sup> b<sup>1</sup> c<sup>1</sup> d<sup>1</sup> in Fig. 4. This line is also the envelope of all loading tests for footings initially



Fig. 5.—Load settlement curves for model footings in remoulded London clay

buried at any depth D; the load-settlement curves for such footings being b b<sup>1</sup> c<sup>1</sup> d<sup>1</sup> e, c c<sup>1</sup> d<sup>1</sup> e and d d<sup>1</sup> e. It is evident that, for the test starting at  $D = \frac{1}{2}B$ , the shear strength of the clay is progressively mobilised as the pressure is raised from zero until, at the point b<sup>1</sup>

<sup>\*</sup>Tests on model screw-cylinders, with blade diameters of two, four and six inches, by Wilson<sup>13</sup> also show an average value of  $N_e$  of about 9.5 for remoulded London clay. But this result is probably a little too high, since no corrections were made for pore pressure diffusion from the clay immediately under the blades. The actual strength of the clay was therefore somewhat greater than that measured by compression tests on samples taken from the bulk of clay in the test container on samples taken from the bulk of clay in the test container.

on the envelope, the strength is fully mobilised. Similarly, for the test starting at D = 1.5 B the shear strength of the clay is fully mobilised at point c1. Moreover, it will be seen that the "envelope" may be extrapolated to the axis of zero penetration at a value of q/c = 6.2. This is Meyerhof's value of  $N_e$ for a circular footing on the surface, and in his theory, as in that of Prandtl for a strip footing, it is tacitly assumed that failure occurs at deformations negligibly small compared with the breadth of the footing. The experimental results in Fig. 5 therefore confirm\* the theoretical surface values of 6.2, and so also do the tests on strip footings ; the envelope in these experiments extrapolating back to q/c = 5.2.

Nevertheless, since the penetrations required to mobilise full shear in the clay are, in the laboratory tests, equal to about 0.4 B, it is logical to take the values of q/c at the points  $a^1 b^1$  and  $c^1$  as the values of  $N_c$  for the appropriate foundation depths D = 0, 0.5 B and 1.5 B. In this way the relation between  $N_e$  and D/B shown by the "step-back" curve in Fig. 4 is obtained. Thus, for a surface circular footing on remoulded London clay ultimate failure occurs (i.e. the full shear strength of the clay is mobilised), when  $q/c = N_c = 6.8$ ; and similarly for any other value of D.

But, as will be seen from Table 1, ultimate failure takes place in some undisturbed clays at a penetration of only 0.1 B or even less. Therefore, although the "step-back" curve in Fig. 4 is undoubtedly the logical interpretation of the particular test results expressed in that graph, yet in practice it may be an error not on the side of safety to assume that such high values of  $N_c$  can be used. Clearly, the most conservative assumption is to use the "envelope" itself since this implies that full shear strength is mobilised after negligible penetration of the footing.

It may, of course, well be true that with more brittle clays the envelope is itself higher than that obtained for the remoulded London clay. But the tests on undisturbed Shellhaven clay did not indicate any substantial difference. Consequently the most reasonable procedure, for the present at least, until more evidence is forthcoming, is to take the average envelope from the available test data and assume that this gives the required relation between  $N_e$  and depth of the This average envelope for circular footings is, footing. in fact, that shown by the upper curve in Fig. 2. It may be noted that laboratory tests<sup>15</sup> (Meigh 1950) showed no significant difference between square and circular footings.

The information on strip footings is less complete, the tests so far carried out being limited to London clay. But, since the ratio of  $N_e$  for the strip to that for the circle is 0.84 both at depth and at the surface, it is unlikely that any appreciable error will be involved in the assumption that this ratio applies for all values of D/B. The ordinates of the "strip" curve in Fig. 2 are therefore simply 0.84  $\times$  N<sub>e</sub> (square).

It is further assumed that the value of  $N_c$  for a rectangular footing may be obtained by linear interpolation according to the formula:

$$N_{\rm c} \text{ (rectangle)} = \left[0.84 + 0.16 \frac{B}{L}\right] N_{\rm c} \text{ (square)}$$
(5)

Summary

Clearly there is scope for developing a more satisfactory theory for the bearing capacity of deep footings in clay, but the semi-empirical values of 9.0 and 7.5 for circular and strip footings are probably sufficiently accurate for practical purposes. Also the interpolation formula, equation (5), requires experimental and theoretical investigation. More important, the values of  $N_e$  given in Fig. 2 are probably somewhat conservative, and future work may lead to improvement in this respect. Nevertheless, the comparison of the bearing capacity factors as given in Fig. 2, with the available field data, in Table 1, is decidedly encouraging.

#### 5. Load-Settlement Curves

In Fig. 5 some of the observed points on the individual load-settlement curves aa1, bb1 and dd1 (shown in Fig. 4) are plotted with a common origin; the ordinates being expressed as the ratio of the pressure q to the ultimate bearing capacity  $q_1$ , as represented by points  $a^1$ ,  $b^1$  etc. The results of a typical test on a strip footing (B/L = 0.1) are also plotted in the same manner. As a rough approximation, all the points lie on the same curve, and it is interesting to examine the measure of agreement between these experimental points and the load-settlement curve as predicted from simple theoretical considerations.

Now, from the theory of elasticity it is known that the mean settlement of a foundation, of breadth  $B_{i}$ on the surface of a semi-infinite solid is given by the expression

where q = foundation pressure.  $I_{\rho} =$  influence value depending upon the shape and rigidity of the foundation.

= Poisson's ratio of the solid.

E = Young's modulus of the solid.

For the present purpose equation (5) is more conveniently written in the form

$$\frac{\rho}{B} = \frac{q}{q_t} \cdot \frac{q_t}{c} \cdot I \rho \frac{\mathbf{I} - \mu^z}{E/c} \qquad \dots \qquad \dots \qquad \dots \qquad (6)$$

In saturated clays with no water content change under applied stress (the  $\Phi = 0$  condition) Poisson's ratio is equal to  $\frac{1}{2}$ , and for a rigid circular footing on the surface  $I\rho = \pi/4$ . Moreover, from the experiments previously described,  $q_t/c = 6.8$ . Thus for the model tests with circular footings on the surface

$$\frac{P_1}{B} = \frac{4}{E/c} \cdot \frac{q}{q_t} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (7)$$

With footings buried at some depth below the surface the influence value  $I_{\rho}$  decreases (Fox<sup>17</sup>), but the bearing capacity factor  $N_{\rm e} = q_{\rm nt}/c$  increases as shown in Fig. 2, and to a first approximation the product  $I_{\rho}$ .  $N_{e}$  remains constant. Therefore equation (7) holds good for all the circular footing tests.

Further, in an undrained compression test the axial strain under a deviator stress  $(\sigma_1 - \sigma_3)$  is given by the expression

$$\varepsilon = \frac{(\sigma_1 - \sigma_3)}{\Gamma} \dots (8)$$

where E is the secant Young's modulus at the stress  $(\sigma_1 - \sigma_3).$ 

<sup>\*</sup>Cone tests approximate to the conditions implied in Meyerhof's theory but difficulties are present in carrying out cone-penetration tests with high accuracy. The shear mobilised along the surface of the cone, the high rate of strain in the carly stages of the test, the dissipation of pore pressure and the depression or elevation of the clay surface during penetration, all influence the results. The most that can be said at present is that the values of  $N_e$  deduced from cone tests (in which an attempt has been made to apply these corrections) lie in the range 5.0 to 7.0 for most clays.



undisturbed clays (E/c = 100)

As before, equation (8) is more conveniently written in the form

$$\varepsilon \qquad \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)t} \qquad \frac{(\sigma_1 - \sigma_3)t}{c} \qquad \frac{1}{E/c} \qquad \dots \qquad (9)$$

In saturated clays with no water content change  $\frac{(\sigma_1 - \sigma_3)t}{2 - 2.0} = 2.0.$  Thus

under applied stress -

$$\varepsilon = \frac{2}{E/c} \cdot \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)t} \quad \dots \quad \dots \quad \dots \quad \dots \quad (10)$$

From a comparison of equations (7) and (10) it will therefore be seen that, for the same ratio of applied stress to ultimate stress, the strain in the loading tests is related to that in the compression test by the equation

The average stress-strain curve for all the compression tests carried out on the remoulded London clay used in the model loading tests is shown in Fig. 6 (a). From this curve the values of  $\rho_1/B$  can immediately be calculated from equation (11); and the result is shown by the dotted line in Fig. 5. The agreement with the experimental points is moderately good except for high values of  $q/q_t$ . But the simple theory leading to equation (11) cannot be expected to yield accurate results in this range, since at loads near the ultimate bearing capacity a considerable zone of the clay beneath the footing is subjected to strains greater than those at the ultimate stress in the compression test.

The container in which the circular footings were tested had a depth of at least 8B. Theoretically' the settlements should therefore be about 7 per cent. less than the values calculated from equation (II). This is of no consequence, in view of the very approximate nature of the derivation of the strain relationship. However, the container in which the strip footings were tested had a depth of about 6B. This is adequate for investigating ultimate failure; but the settlements would be 30 per cent. less than the values calculated from the theory of semi-infinite elastic solids, and the corresponding value of  $I_{\rho}$ .  $N_{e}$  is only about 20 per cent. greater than that used in equation (7), whereas, on the assumption of a semi-infinite solid, the product



Fig. 7.----'' Immediate '' settlements in field loading tests on saturated clays ( $\Phi = 0$ )

 $I_{\rho}.N_{e}$  is about 65 per cent. greater for the 10: I strip than for the circle. Hence the observed fact of roughly equal settlements, at the same factors of safety, for the two types of footing, which might at first glance seem to be anomalous, is accounted for within the limits of accuracy of the few tests carried out on strips.

In applying the foregoing conceptions to full-scale foundations it is necessary to take into account the probability that the great majority of the settlement is due to strains in the clay within a depth of not more than about 4B below the base of the footing. At greater depths the shear stresses are less than about 5 per cent. of the nett foundation pressure, and the corresponding value of E/c is typically 50 to 80 per cent. greater than that calculated at  $\sigma = \frac{1}{2}\sigma_{\rm f}$ . Moreover, the strength of the clay usually increases appreciably with depth. Thus the strains at relatively great depths are one-half, or even less, of those according to simple elastic theory, with a shear strength, in equation (6), equal to that within a depth of 2/3 B beneath the footing.

From the values of  $I_{\rho}$  given by Terzaghi<sup>2</sup> and Timoshenko<sup>18</sup> the following results are obtained for the mean settlement of uniformly loaded areas, if strains below 4B are neglected.

TABLE 2

L/B	I <sub>ρ</sub>	N <sub>c</sub>	$\begin{array}{ c c c }\hline 3 & & \\ \hline -N_{\rm c} \cdot I_{\rm p} \\ 4 & \\ \end{array}$	ρ <sub>1</sub> . <u>B</u> . ε
circle	0.73	6.2	3.4	1.7
2:1	1.00	5.2 5.7	4.3	2.1
5:1 10:1	1.22 1.26	5 · 4 5 · 3	4.9 5.0	2.4 2.5

Thus, to a degree of approximation ( $\pm$  20 per cent.) comparable with the accuracy of the assumptions, it may be taken that equation (II) applies to a circular or any rectangular footing.

In order to investigate this relationship in practice, it is necessary to know the shape of the stress-strain curves for undisturbed clays, and to compare the calculated settlements with field observations. For this purpose the stress-strain curves of a number of clays were plotted in the form shown in Fig. 6 (b) and, apart from a few exceptional cases, all the "equivalent" stress-strain curves were found to lie within the shaded zone shown in this graph. The load-settlement curve calculated from equation (II) and from the average equivalent stress-strain curve indicated by the solid line in Fig. 6 (b), is plotted in Fig. 7. This load-settlement curve is therefore a crude estimate of the theoretical curve for undisturbed clays with E/c = 100. The settlements at any given factor of safety  $(= q_{nt}/q_n)$  will be inversely proportional to E/c, and the curves for E/c = 50 and 200 are also shown in Fig. 7.

The author is aware of loading tests at six sites for which sufficient data are available to enable the results to be plotted in Fig. 7. Three of these tests were taken to failure, and  $q_n/q_{nt}$  is therefore known directly. In the other three cases  $q_{n1}$  has been calculated from Fig. 2 and the shear strength of the clay. The most valuable tests were those carried out by Sir John Hawkshaw on the piers of his bridges over the Thames at Charing Cross and Cannon Street. The former is only a few hundred yards away from Waterloo Bridge, where extensive investigations were recently made on the London clay<sup>19</sup>. Each of the cylinders forming the piers of Charing Cross Bridge were loaded with 450 tons or 700 tons, before building the deck, and the settlements were observed. In addition, the skin friction was measured during the sinking of the

cylinders. Similarly at Cannon Street Bridge the cylinders were test-loaded with 850 tons. For undisturbed London clay E/c = 50 and the stress-strain curve is closely similar to that shown by the full line in Fig. 6 (b). It is therefore interesting to note the reasonable degree of comparison between the field observations and the approximate theoretical loadsettlement curve in these cases. The clay at Boston<sup>20</sup> had an E/c of about 40 or 50, whereas the loading test indicates a value of the order 80. This discrepancy may be due partly to the fact that the test was carried out at the bottom of a 40 ft. shaft, and the clay had therefore been considerably "pre-stressed": the test being, in effect, a re-loading of the clay. At Shellhaven<sup>21</sup>, it is difficult to make any direct comparison, since the oil tank rested on a 5 ft. crust of hard clay overlying The crust had little effect on the ultimate soft clay. failure of the tank, but it would appreciably reduce the settlements. Moreover, the soft clay is extra-sensitive and the laboratory value of E/c = 80 may well be considerably too low on account of sampling disturbance<sup>22</sup>. For the tests on the screw cylinder at Newport<sup>13</sup> the results agree reasonably well with the actual E/c for the clay, which was about 60. No value of E/c is available for the clay beneath the cylinder tested by Morgan<sup>23</sup> but the load-settlement result indicates about 90 and this is of the order often measured in normally consolidated silty clays.

Summarising this field evidence, it may therefore be said that none of the data is seriously at variance with the approximate theory expressed by equation (11) and Table 2, while the tests on the Thames bridges appear to confirm this theory and also, by implication, the bearing capacity factors given in Fig. 2 for circular foundations at a depth of about  $1\frac{1}{2}B$  to  $2\frac{1}{2}B$ .

#### 6. Factor of Safety

As a minimum requirement for the stability criterion it is usual to specify a factor of safety of not less than 2. But, for general purposes, experience has indicated that it is desirable to use a factor of safety of 3 (Terzaghi and Peck<sup>24</sup>). Thus, quite distinct from any settlement criteria, the allowable nett pressure should not exceed one-third of the nett pressure causing ultimate failure. Yet with a factor of safety of 3, although there can be no possibility of complete failure, or even of any appreciable over-stressing\* in the clay, the settlements may be excessive. Consequently, it is necessary to give at least a brief consideration to the settlement problem if 'the subject of bearing capacity is to be seen in proper perspective.

#### 7. Final Settlement

Where the clay exists as a relatively thin layer beneath the foundation, or where the foundation rests on sand or gravel underlain by clay, the "immediate" settlements are small, owing to the lateral restraint imposed on the clay by the adjacent rigid or comparatively rigid materials. In such cases the final settlement, and also the rate of settlement, can be calculated with sufficient accuracy from Terzaghi's theory of one-dimensional consolidation. The procedure for calculating settlements by this theory can be found in the standard text-books, such as Terzaghi and Peck<sup>24</sup>, and need not be considered further in this paper.

<sup>\*</sup>If the nett foundation pressure is one-third of that causing ultimate failure, the maximum shear stress in the clay does not exceed about 65 per cent. of the shear strength. Thus, a factor of safety of 3 on ultimate failure corresponds to a factor of safety of at least  $1\frac{1}{2}$  on over-stressing (neglecting isolated stress concentrations.)

Where the foundation rests directly on a relatively thick bed of clay the problem is more complicated. As a first approximation, however, the nett final settlement (*including both* "immediate." and "consolidation" settlement) may be calculated from the equation l<sup>z<sub>1</sub></sup>

$$\varphi_{n} = \int_{0}^{m_{v}} g_{z} dz \dots \dots \dots \dots (12)$$
  
=  $m_{v}, q_{n}, B, I \varphi \dots \dots \dots \dots \dots \dots (13)$ 

 $= m_v$ ,  $q_0.B. I_{\rho}$  ... ... (13) where  $m_v$  is the compressibility of the clay at a depth z beneath the foundation as measured in oedometer tests on undisturbed samples; the compressibility being determined over the range of pressure from  $p_0$ , the original effective overburden pressure at depth z, to  $(p_0 + \sigma z)$  where  $\sigma_z$  is the increment of vertical pressure set up at this depth by the nett foundation pressure. Also, in these equations  $z_1$  is the maximum depth of the clay beneath the foundation or, if the clay is very thick,  $z_1$  is some depth such as 4B beneath which the settlements are negligible, and  $I_{\rho}$  is the influence value for settlements in a depth  $z_1$ .

If the clay structure was elastic then this conventional method would underestimate the final settlement, since it implies the assumption that Poisson's ratio  $\mu_{\theta}$  is zero. But the compressibility  $C_{e}$  of the clay structure is greater than the expansibility  $C_{\theta}$  (both expressed in terms of effective stress) and if this fact is taken into account<sup>25</sup> it is found that the conventional method leads to final settlements, which may be either lower or higher than those calculated from more comprehensive theory; but not differing by more than  $\pm$  40 per cent., as shown\* in Table 3. The "theoretical" final settlements have, so far, only been evaluated for the centre of a uniformly loaded circular footing, and the determination of  $C_{e}$ ,  $C_{a}$  and  $\mu_8$  for the clay structure is experimentally a difficult matter. The purpose of the theory is therefore not to provide a method of settlement analysis, but merely to enable the order of error in the conventional analysis to be examined.

Since, in practice, structural design often does not justify an attempt to predict settlements with an accuracy greater than that implied by the results in Table 3, it may be concluded that the conventional method (equations 12 and 13) is adequate for estimating the final settlement of foundations on deep beds of clay. Field observations justify this conclusion<sup>27</sup>, <sup>28</sup>, <sup>29</sup>.

In order to obtain a relationship between final settlement (from the conventional method) and factor of safety against ultimate failure, equation (13) may be written in the form

$$\frac{\rho_{n}}{P} = \frac{q_{n}}{m_{v}} \cdot \frac{q_{n}}{m_{v}} \cdot c \cdot I \rho \dots \dots \dots \dots \dots \dots \dots (14)$$

B  $q_{nt}$  c or, if  $K_v = 1/m_v$ , where  $K_v =$  modulus of compressibility as measured in oedometer,

and equation (15) is analogous to the corresponding equation (6) for "immediate" settlements; except that equation (15) cannot be expected to hold good for values of  $q_n/q_{nt}$  of more than about 0.5, since at greater values of this ratio the clay will be overstressed.

Values of  $I_{\rho}$  can be found from data given by Terzaghi<sup>2</sup> and Timoshenko<sup>18</sup>, and values of  $q_{nt}/c$  are

\*The few tests at present available show that the compressibility ratio  $\lambda$  lies in the range 0.1 to 0.5 (Skempton<sup>24</sup>).

	TA	BLE 3					
C	Conventional final settlement						
$\frac{C_{\mathbf{s}}}{C} = \lambda$	theor	retical final settlen	nent				
C <sub>c</sub>	$\mu_8 = 0.3$	$\mu_{s} = 0.35$	$\mu_{s}=0.4$				
0.I	¥.4	1.4	1.3				
0.25	1.2	Ι.Ι	0.9				
0.5	1.0	1.9	0.7				
I.O (elastic)	0.8	0.7	0.6				

given in Fig. 2. From these values it can be shown that the order of the average nett final settlement is given by the expression 5

Equation (16) enables a study to be made of the relationship between the factor of safety against ultimate failure and the average nett final settlement of a foundation on a deep bed of clay. In evaluating equation (16) it is, however, essential to know the value of the ratio  $K_v/c$ . A preliminary examination of the published data indicates that for over-consolidated clays  $K_v/c$  lies approximately in the range from 70 to 200, while for normally-consolidated clays the range is approximately from 25 to 80. In each class  $K_v/c$  tends to be higher for clays with a lower liquid limit. These values must be taken as being only indicative, but they enable certain interesting de-ductions to be made. In order to clarify the basis of these deductions, equation (16) has been plotted in Fig. 8 for several typical values of  $K_{v}/c$ . Also on this graph points have been plotted representing the results of field observations on ten structures.

The first inference from Fig. 8 is that the field observations in the six cases where  $K_v/c$  is known, agree roughly with equation (16.) The second inference is that, for any given clay, the settlement is approximately proportional to the width B, at the same factor of safety. This result was first predicted by Terzaghi<sup>30</sup>, and there is considerable supporting evidence from loading tests. But the observations summarised in Fig. 8 show that it holds good also for the final settlement of large foundations. It therefore follows that, conversely, the allowable nett foundation pressure on any given clay will decrease in direct proportion to the foundation width, if it is required to restrict the settlement to some specified magnitude.

The factors of safety corresponding to various settlements for several typical values of  $K_v/c$  are given

TABLE 4

		IND			
Nett	Width		Factor o	of safety	
Pn inches	B ft.	$\frac{K_{\rm v}}{c} = 200$	$\frac{K_{\rm v}}{c} = 100$	$\frac{K_{\rm v}}{c} = 50$	$\frac{K_{\rm v}}{c} = 25$
I	5	(3)	3	6	12
	10	3	6	12	24
	20	6	12	24	48
	40	12	24	48	96
3	5	(3)	(3)	(3)	4
	10	(3)	(3)	4	8
	20	(3)	4	8	16
	40	4	8	16	32
6	5	(3)	(3)	(3)	(3)
	10	(3)	(3)	(3)	4
	20	(3)	(3)	4	8
	40	(3)	4	8	16

†On this point see an excellent general treatment by Taylor<sup>a1</sup>.



Fig. 8 .--- "Final" settlements of foundations in saturated clays

in Table 4. Where the factor of safety as given by equation (16) is less than 3, the stability criterion controls the design. These cases are distinguished in Table 4 by the number 3 in brackets. If it is desired to limit the average settlement to one inch, then it will be seen that the stability criterion is relevant only for small footings on over-consolidated clays. In all other cases the design is governed by settlement With a limiting average settlement considerations. of three inches, the stability criterion applies to all footings on over-consolidated clays and to small footings on most normally-consolidated clays. But for raft foundations the settlement criterion is still of controlling importance in all clays except those which are overconsolidated, with high values of  $K_{\rm v}/c$ .

Settlements of more than three inches are not usually tolerated in buildings, but in bridge design settlements of six inches or more are often permissible, especially where provision exists for maintaining the correct elevation of the deck by means of jacks (as at Waterloo Bridge and elsewhere). In such cases, the factor of safety depends upon stability considerations in all clays except those with a very low value of  $K_v/c$ , unless the piers are unusually wide. The importance of width in controlling the design of foundations on clay is therefore clearly demonstrated, and also the interdependence of the two criteria. But a further inference may be made from an examination of Table 4, namely that the factors of safety necessary to limit the settlements to a few inches on normally-consolidated clays, with all but the smallest footings, are so large as to be outside practical possibility. Therefore, unless settlements of many inches, or even a few feet can be tolerated, it is not feasible to found directly on such clays, especially if the liquid limit is high. This point has previously been made by Terzaghi and Peck<sup>24</sup>, but it requires re-emphasis, since there appears to be

an increasing tendency to accept a factor of safety of 3 as being adequate for the design of footings of any clay. Table 5 shows that this is not even approximately correct for clays with low values of  $K_v/c$  if the settlements are to be restricted to a reasonably small magnitude.

#### Conclusion

In conclusion, it may be said that, so far as the present evidence is concerned, the values of  $N_e$  given in Fig. 2 are sufficiently accurate for the determination of the ultimate bearing capacity of deep beds of relatively homogeneous clay. A factor of safety of at least 3 is desirable in estimating allowable bearing capacity. But in many cases the foundation design will be controlled by settlement considerations, and the engineer may be compelled to use factors of safety very considerably greater than 3, in order to restrict the settlement to a magnitude compatible with structural requirements.

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#### References

<sup>1</sup>Terzaghi, K., 1935. The Actual Factor of Safety in Foundations. STRUCT. ENG. 13: 126. <sup>2</sup>Terzaghi, K., 1943. Theoretical Soil Mechanics. New York :

John Wiley.

<sup>3</sup>Carlson, L., 1948. Determination in situ of the Shear Strength of Undisturbed Clay by Means of a Rotating Auger. PROC. 2ND

Soll Mech. 1: 265.
Skempton, A. W., 1948. Vane Tests in the Alluvial Plain of the River Forth Near Grangemouth. GEOTECHNIQUE. 1: 111.
Cadling, L. and S. Odenstad, 1950. The Vane Borer. PROC.

ROY. SWEDISH GEOT. INST. No. 2.
"Skempton, A. W. and Bishop, A. W., 1950. The Measurement of the Shear Strength of Soils. GEOTECHNIQUE. 2:90.
"Prandtl, L, 1920. Uber die Harte platischer Korper. NACH. GESELL. WISS. GOTTINGEN. MATH-PHYS. KL. P. 74.

\*Meyerhof, G. G., 1950. A General Theory of Bearing Capacity.

BUILDING RESEARCH STATION NOTE NO. C. 143. \*Ishlinsky, A. J., 1944. The Axial Symmetrical Problem in Plasticity and the Brinell Test. J. APP. MATH. AND MECH. U.S.S.R.

8: 201. <sup>10</sup>Bishop, R. F. Hill, R. and N. F. Mott, 1945. The Theory Number of Teste PROC. PHYS. SOC. 57: 147.

of Indentation and Hardness Tests. PROC. PHYS. SOC. 57: 147. <sup>11</sup>Gibson, R. E., 1950. Discussion on Paper by Guthlac Wilson.

<sup>11</sup>Gibson, R. E., 1950. Discussion on Paper by Guthlac Wilson. (1950). J. INST. C.E. 34: 382.
<sup>13</sup>Swainger, K. H., 1947. Stress-Strain Compatibility in Greatly Deformed Engineering Metals. PHIL. MAG. 38: 422.
<sup>13</sup>Wilson, G., 1950. The Bearing Capacity of Screw Piles and Screwcrete Cylinders. J. INST. C.E. 34: 4.
<sup>14</sup>Golder, H. Q., 1941. The Ultimate Bearing Pressure of Rectangular Footings. J. INST. C.E. 17: 161.
<sup>15</sup>Meigh, A. C., 1950. Model Footing Tests on Clay. Thesis: M.Sc. (Eng.), Univ. of London Library.
<sup>16</sup>Yassin, A. A., 1950. Model Studies on Bearing Capacity of Piles. Thesis: Ph.D. (Eng.), University of London Library.
<sup>17</sup>Fox, E. N., 1948. The Mean Elastic Settlement of a Uniformly Loaded Area at a Depth Below the Ground Surface. PROC. 2ND INT. CONF. SOIL MECH. 1: 129.

PROC. 2ND INT. CONF. SOIL MECH. 1: 129. <sup>18</sup>Timoshenko, S., 1934. Theory of Elasticity. New York: McGraw-Hill.

<sup>19</sup>Cooling, L. F. and A. W. Skempton, 1942. A Laboratory

Study of London Clay. J. INST. C.E. 17:251. <sup>20</sup>Fadum, R. E., 1948. Concerning the Physical Properties of Clay. PROC. 2ND INT. CONF. SOIL MECH. 1:250. <sup>21</sup>Nixon, I. K., 1949.  $\Phi = 0$  Analysis. GEOTECHNIQUE. 1:208

and 1: 274. <sup>22</sup>Rutledge, P. C., 1944. Relation of Undisturbed Sampling to Laboratory Testing. TR. AM. SOC. C.E. 109: 1155.

<sup>23</sup>Morgan, H. D., 1944. The Design of Wharves on Soft Ground.

J. INST. C.E. 22:5. <sup>24</sup>Terzaghi, K. and R. B. Peck, 1948. Soi Engineering Practice. New York: John Wiley. Soil Mechanics in

<sup>25</sup>Skempton, A. W., 1939. A Theory of Settlement. BUILD.

RES. STATION, M.S. FILE : 24.4.39. <sup>24</sup>Skempton, A. W., 1948. The Effective Stresses in Saturated Clays Strained at Constant Volume. PROC. 7TH INT. CONG. APP. месн. 1:378.

<sup>47</sup>Tschebotarioff, G. P. and J. R. Schuyler, 1948. Com-parison of the Extent of Disturbance Produced by Driving Piles into Plastic Clay to the Disturbance Caused by an Unbalanced Excavation. PROC. 2ND INT. CONF. SOIL MECH. 2: 199.

<sup>28</sup>Dawson, R. F. and W. E. Simpson, 1948. Settlement Records of Structures in the Texas Gulf Coast Area. PROC. Settlement

Records of Structures in the Texas Gulf Coast Area. PROC. 2ND INT. CONF. SOIL MECH. 5: 125. <sup>29</sup>Hanna, W. S., 1950. Settlement Studies in Egypt. GEOTECHNIQUE. 2: 33. <sup>30</sup>Terzaghi, K., 1929. The Science of Foundations, Its Present and Future. TR. AM. SOC. C.E. 93: 270. <sup>31</sup>Taylor, D. W., 1948. Fundamentals of Soil Mechanics. New York : John Wiley. <sup>32</sup>Odenstad. S., 1948. Loading Tests on Clay. PROC. 2ND

<sup>32</sup>Odenstad, S., 1948. Loading Tests on Clay. PROC. 2ND
 INT. CONF. SOIL MECH. 1: 299.
 <sup>33</sup>Skempton, A. W., 1942. An Investigation of the Bearing

Capacity of a soft clay soil. J. INST. C.E. 18: 307.

<sup>34</sup>Skempton, A. W., 1950. Discussion on Paper by Guthlac Wilson (1950). J. INST. C.E. 34: 76.

<sup>35</sup>Fountain, H. P., 1907. (Warehouses in Tunis). ENG. NEWS. 57: 458. <sup>36</sup>Allaire, A.

<sup>34</sup>Allaire, A. 1916. The Failure and Righting of a Million-Bushel Grain Elevator. TR. AM. SOC. C.E. 80: 799.

<sup>37</sup>Hayter, H., 1863. The Charing Cross Bridge. PROC. INST.

C.E. 22: 512.
<sup>38</sup>Barry, J. Wolfe, 1868. The City Terminus Extension of the Charing Cross Railway. PROC. INST. C.E. 27: 410.
<sup>39</sup>Cooling, L. F., 1948. Settlement Analysis of Waterloo Bridge. PROC. 2ND INT. CONF. SOIL MECH. 2: 130.

<sup>40</sup>Buckton, E. J. and H. J. Fereday, 1938. The Reconstruction of Chelsea Bridge. J. INST. C.E. 3: 383.
<sup>41</sup>Tschebotarioff, G. P., 1940. Settlement Studies of Structures in Egypt. TR. AM. SOC. C.E. 105: 919.

<sup>42</sup> Jeppesen, A., 1948. The Foundation Problems of the Piers of the Little Belt Bridge. PROC. 2ND INT. CONF. SOIL MECH.

4:5. <sup>43</sup>Glick, G. W., 1936. Foundations of the New Telephone Building, Albany, N.Y. proc. 1st int. conf. soil mech. 1: 278.

# The Colloidal "Activity" of Clays

L'Activité colloïdale des argiles

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#### Summary

In any particular clay stratum the ratio of the plasticity index to the clay fraction content is approximately constant, and may be defined as the "activity" of the clay. Values of activity are given for many clays and also for the more common minerals. It is shown that activity is related to the mineralogy and geological history of clays, and to the proportion of their shear strength contributed by true cohesion. Field data is presented which indicates that the difficulties of taking satisfactory undisturbed samples in deep beds of sensitive clay are restricted to those clays with an activity of less than 0.75.

#### Sommaire

Dans une couche d'argile le rapport de l'indice de plasticité à la proportion d'argile (moins de 2 microns) est presque constant et peut être défini comme «l'activité» de la couche. Les valeurs de cette activité sont données pour diverses argiles et aussi pour les minéraux les plus répandus. Il est démontré que l'activité dépend de la minéralogie et de la géologie des argiles et de leur résistance au cisaillement due à la cohésion vraie. Les observations sur le terrain montrent que l'extraction d'échantillons satisfaisants, dans de profondes couches d'argile sensitive, ne présentent des difficultés que pour les argiles dont l'activité est inférieure à 0,75.

# Introduction

The properties of a clay are determined fundamentally by the physico-chemical characteristics of the various constituent minerals and by the relative proportions in which the minerals are present. The determination of these characteristics is a lengthy and difficult process requiring the use of an X-ray spectrometer, thermal analysis, etc., and it is evident that such techniques can never become part of the normal laboratory procedure in soil mechanics. Some simple tests are therefore required that give a quantitative measure of the composite effects of all the basic properties of a clay and, as is well known, the Atterberg limits fulfill this function in large measure. But they are not wholly sufficient, and in the present paper evidence is given which shows that valuable additional information is provided by an index property combining the Atterberg limits and the particle size distribution of a clay; yet requiring for its determination only the results of these routine tests.

# The Ratio: PI/Clay Fraction

If a number of samples are taken from a particular clay stratum and the clay fraction content (percentage by weight of particles finer than 2 microns) and the Atterberg plasticity index (PI) are determined for each sample, then there is gene-

rally a quite wide range in the numerical values for both properties. Yet if the plasticity index is plotted against clay fraction it will be found that the points lie about a straight line which extrapolates back to the origin. Typical sets of results obtained from such tests on four clays are given in Fig. 1. The degree of scatter about the mean line is presumably a measure of the variations in composition within the stratum.

Now it is widely recognised that the higher the plasticity index the more pronounced are the colloidal properties of a clay. Moreover the colloidal properties are contributed largely by the finest particles and, in particular, by the "clay fraction". But reference to Fig. 1 will at once show that two clays which may have the same content of clay fraction can have widely different plastic indexes, and it would seem logical to assume that the clay with the higher *PI*, for a given clay fraction content, is more colloidally active than the clay with a lower *PI*, for the same given clay fraction content.

The direct linear relationship between *PI* and clay fraction content for any particular clay enables this degree of colloidal activity to be expressed very simply by the ratio:—

$$activity = \frac{plasticity index}{clay fraction}$$

This ratio is, in fact, the slope of the lines such as those in Fig. 1; and it provides a convenient single-valued parameter for any particular clay.

The above definition of activity was given by the author in 1950, and is a development of an earlier conception (*Skempton*, 1948c) in which liquid limit was plotted against clay fraction.



Fig. 1 Relation Between Plasticity Index and Clay Fraction Relation entre l'indice de plasticité et le pourcentage d'argile

In the 1948 paper three classes of clay were recognised, from this point of view, namely "inactive", "normal" and "active". Data obtained subsequently has not lead to any essential change in this classification which, in terms of the ratio *PI*/clay fraction, may be stated as follows:—

inactive clays — activity 
$$<0.75$$
  
normal clays — activity 0.75 to 1.25  
active clay — activity  $>1.25$ 

The relation between liquid limit and clay fraction, although linear, is not one of direct proportion and is therefore less convenient than the ratio *PI*/clay fraction. For the idea of plotting clay fraction against plasticity index rather than liquid limit, the author is indebted to a graph in a paper by *A. Casagrande* and *Shannon* (1948).

#### Activity of Various Minerals

In examining more fully the significance of activity it is in the first place of interest to discover the values of PI/clay fraction for the commonly occurring minerals in clays. The principal data are assembled in Table 1.

Table 1 Values of PI/Clay Fraction for some Clay-Minerals

Mineral	Activity	Reference
Quartz	0.0	von Moos (1938)
Calcite	0.18	von Moos (1938)
Mica (muscovite)	0.23	von Moos (1938)
Kaolinite	(0.33 0.46	Northey (1950) Samuels (1950)
Illite	0.90	Northey (1950)
Ca-montmorillonite	1.5	Samuels (1950)
Na-montmorillonite	7.2	Samuels (1950)

The three minerals quartz, calcite and mica, tested by von Moos, were ground to a very small particle size and the PI then determined on the fraction finer than 2 microns. The activity of these minerals is low, as might be expected from their relatively simple crystal structure. Of the true clay minerals so far examined kaolinite has the lowest activity. Illite is probably the most widespread of all clay minerals but it usually occurs in conjunction with other minerals. Fortunately, however, a clay shale exists in Illinois, the clay fraction of which consists almost entirely of illite. A large sample if this material was kindly sent by Professor Grim and the average result of tests carried out by Dr. Northey, in the author's laboratory, is given in Table 1. The clay known as bentonite consists almost exclusively of the mineral montmorillonite. In its natural state bentonite is usually a sodium clay and, in this state, it has a very exceptionally high activity. By effecting a base exchange from sodium (monavalent) to calcium (bivalent), Samuels (1950) has shown by repeated tests that the activity is considerably lowered; although even the Ca-bentonite has a high activity. He has also shown that bentonite carrying a tri-valent base Al has an activity of about 1.3. In contrast, Samuels (1950) found that base exchange has only a minor influence on kaolinite. No base exchange tests appear to have been made on illite but, since this mineral shows moderate activity, the effect would probably be appreciable.

It is clear from the above results that activity is, broadly speaking, related to the structural complexity of the minerals; ranging from quartz through kaolin up to montmorillonite.

# Activity and Geological History

Information concerning 27 clays is given in Table 2, from which it may be deduced that there is some degree of correlation between activity and the mineralogy and geological history of a clay.

The "inactive" clays (activity less than 0.75) seem to possess one or more of the following characteristics:—

- (a) clay fraction either consists predominantly of kaolinite, or contains little true clay mineral;
- (b) deposition in fresh water;
- (c) deposition in salt water, but subsequently leached by percolation of fresh water.

Clays combining the characteristics (a) and (b) or (a) and (c) form the least active group I (activity less than 0.5). Apart from kaolin the typical members of this group are late-glacial clays derived largely by mechanical erosion of non-argillaceous rocks by ice-sheets, and deposited in ice-dammed lakes; and post-glacial marine or estuarine clays which have subsequently been leached by fresh water, usually following isostatic uplift. There is evidence, both from the field and the laboratory, that many of the extra sensitive clays belong to this category of leached post-glacial marine deposits (*Rosenquist*, 1946; *Skempton* and *Northey*, 1952). Clays formed by normal weathering and deposited in fresh water seem to fall into the group 2 with activities between about 0.5 and 0.75.

The largest group is that with activities between 0.75 and 1.25, and it includes the marine and estuarine clays with illite as the predominant clay mineral. Only 8 examples are given in Table 2, but many more clays could be included ranging in geological age from the Jurassic to the post-glacial periods.

Group 4, the members of which may be described as "active" clays, consists of deposits which contain an appreciable amount of organic colloids, although in other respects they would be classed as "normal". It may be expected that clays containing

Group	Range of	Location	Geology	Minero Clay F	Mineralogy of Clay Fraction		Authority
	Activity			Major	Minor		
Inactive 1	less than	St. Thuribe, near Quebec	Post Glacial marine or estuarine, leached	Q	Mi	0.33	Peck et al., Grim
	0.5	Cornwall, England Chicago, U.S.A. Boston, U.S.A. Horten, Norway Detroit U.S.A	Formed in situ by pneumatolysis (kaolin) Late Glacial, lacustrine Late Glacial, marine Post Glacial, marine, leached Late Glacial lacustrine	k Q Mii MiiC	  mo k	0.39 0.41 0.42 0.42 0.42	Northey Rutledge Taylor Hansen, Northey, Grim Peck Grim
Inactive 2	0.5 to 0.75	Wrexham, Wales R. Lidan, Sweden Weald (various sites), England Reading, England Seagrove Bay, I.O.W., Engl. Grangemouth, Scotland	Late Glacial, probably lacustrine Post Glacial, probably as Horten Weald Clay, Cretaceous, lacustrine Reading Clay, Eocene, fresh-water Oligocene, fresh-water Late Glacial, Estuarine	i k	vermiculite	0.54 0.58 0.63 0.72 0.73 0.74	B.R.S. Cadling B.R.S., A.O.R.G. B.R.S. Skempton Skempton
Normal 3	0.75 to 1.25	Peterborough, England Gosport, England Grundy County, Ill., U.S.A. Aylesbury, England London (various sites) Various sites, S.E. England Norfolk Fens, England Vienna, Austria Klein-Belt, Denmark	Oxford Clay, Jurassic, marine Post Glacial, marine Upper Carboniferous (illite.) Kimmeridge Clay, Jurassic, marine London Clay, Eocene, marine Gault Clay, Cretaceous, marine Post Glacial, marine and estuarine Wiener Tegel, Miocene, marine Klein-Belt-Ton, Eocene, marine	i i i k 	h  k mo  mo 	0.86 0.88 0.90 0.93 0.95 0.96 1.06 1.08 1.18	B.R.S. Skempton, Nagelschmit Northey, Grim B.R.S. Cooling, Skempton, Grim B.R.S., A.O.R.G. B.R.S. Hvorslev Hvorslev
Active 4	1.25 to 2.0	Shellhaven, England La Guardia Airport, New York R. Shannon, Eire Belfast, N. Ireland Chingford, England Panama, Central America	Post Glacial, organic and estuarine Post Glacial, organic, marine Recent river alluvium, organic Post Glacial, organic, estuarine Recent river alluvium, organic Recent organic, marine	i   	k   	1.33 1.45 1.5 1.6 1.7 1.75	Skempton, Grim Harris et al. B.R.S. B.R.S. B.R.S. Casagrande
Active 5	more than 2.0	Mexico City Wyoming, U.S.A.	Bentonite Clay Bentonite	mo mo		4.3 6.3	Marsal et al. Samuels, Northey
	<u> </u>	C = Calcite Mi = Mica Q = Quartz	h = Halloysite i = Illite   clay k = Kaolinite   minerals mo = Montmorillonite	s .	— negligi not de	ble termined	

Table 2 Correlation between Activity and the Mineralogy and Geology of Some Clays

Ca-montmorillonite would also fall into this group, but the author does not know of any data on such materials. Group 5 includes only bentonitic clays, no others are known with such high activity values; and the reason is immediately apparent from the fact that they consist predominantly of Na-mont-morillonite (see Table 1).

Boulder clays have not been given in Table 2 since they can vary between Groups 1 to 4, depending upon the nature of the ground from which the glacier or ice-sheet derived the material. Thus the boulder clays of East Anglia, being derived from the Jurassic and Cretaceous clays of the southern and eastern Midlands, fall into group 3. But some of the boulder clays of northern England and Scotland fall into groups 1 or 2 as they consist largely of finely ground rock minerals with little if any true clay minerals incorporated in the matrix.

#### Acitivity and True Cohesion

The shear strength of a clay is made up of two parts, the cohesion  $c_r$  and the coefficient of internal friction tan  $\varphi_r$ , according to the expression (*Hvorslev*, 1937)

 $\tau_f = c_r + {\sigma_n}' \tan \varphi_r$ 

where  $\sigma_n'$  is the effective pressure normal to the shear plane. If a clay is normally-consolidated from a slurry under a pressure  $\sigma_n'$  and is then sheared sufficiently slowly for all the pore water pressure to be fully dissipated (a "drained" shear test), then:

$$\sigma_f = \sigma_n' \tan \varphi_d$$

where  $\varphi_d$  is the angle of shearing resistance in the "drained" state. If, moreover,  $c_r$  is the cohesion of the clay at the water content at failure in the drained shear test, then the proportion of the shear strength due to cohesion is

$$\frac{c_r}{\sigma_n' \tan \varphi_d}$$

and the proportion due to internal friction is

$$\frac{\tan \varphi_r}{\tan \varphi_d} = 1 - \left[\frac{c_r}{\sigma_n' \tan \varphi_d}\right]$$

In Fig. 2 the components of shear strength in 8 normallyconsolidated materials are plotted against their activity. It is

#### COLLOIDAL "ACTIVITY" OF CLAYS

not to be expected that there would be an exact correlation, but the results show beyond doubt that the greater the activity the greater the contribution of cohesion to the shear strength. Of these tests two clays were investigated by *Hvorslev* in his



Fig. 2 Relation Between the Components of Shear Strength and the Activity of Normally Consolidated Clays Relation entre les composantes de la résistance au cisaillement et l'activité d'argiles normalement consolidés

classic research (1937) and the others have been studied by *Gibson* (1951), working at Imperial College.

Fig. 2 provides evidence supporting the suggestion made in an earlier paper (*Skempton*, 1948c) that there is likely to be a correlation between the cohesion of clays and their mineralogy and, in particular, that the true cohesion in some of the North American clays (Boston Clay, Massena Clay, Chicago Clay, etc.) is probably only a small proportion of their shear strength. This conclusion may go far towards resolving an apparent conflict in viewpoint between some investigators working in America and those working in Europe; the former maintaining that in normally consolidated clays true cohesion





Fig. 3 Relation Between Sensitivity and Liquidity Index Relation entre la sensitivité et l'indice de liquidité

#### Activity and Sampling Difficulties

From the investigations of *Carlson* (1948), *Skempton* (1948 b) and especially *Cadling* and *Odenstad* (1950), it is known that there are a number of normally-consolidated clays in which it seems to be impossible to take satisfactory samples from depths of more than about 20 or 30 ft.; even with the best available sampling techniques. In contrast, cases have been reported by *Skempton* (1948 a), *Harris, Mueser* and *Porter* (1948) and others where it proved to be possible to obtain satisfactory samples from depths of 40 ft. to 70 ft. in normally-consolidated clays. It may be mentioned that no difficulties in this respect have been encountered in any over-consolidated clays.



Fig. 4 Relation Between Depth, Liquidity Index and Activity and Feasibility of Sampling Relation entre la profondeur, l'indice de liquidité, l'activité et la possibilité d'obtenir des échantillons

Now although in all normally-consolidated clays (including those from which satisfactory samples cannot be obtained at depth) the in-situ vane test gives a sufficiently correct measure of undrained shear strength, it is nevertheless generally desirable to take samples in order to carry out tests for the determination of properties other than the undrained shear strength. Consequently it is important to understand as far as possible the reasons for the sampling difficulties mentioned above. This problem was briefly considered in an earlier paper (Skempton, 1948 b) and subsequently some valuable data from Sweden has become available (Cadling and Odenstad, 1950) which can be used to throw more light on the subject. The most obvious suggestion is that satisfactory sampling at depths of more than about 20 ft. to 30 ft. becomes increasingly difficult as the sensitivity<sup>1</sup>) of the clay increases. As shown in Fig. 3 the liquidity index may be used as a simple measure of sensitivity, where (Terzaghi, 1936)

liquidity index =  $\frac{\text{water content} - \text{plastic limit}}{\text{plasticity index}}$ 

If the liquidity index of a sample is plotted against the depth from which the sample was taken, and if the point is given a symbol showing whether the sample was satisfactory or not, then it becomes clear that the unsatisfactory samples all lie to the right of the heavy line shown in Fig. 4. But it is also found that in this zone there are a number of perfectly satisfactory samples. Some other factor must therefore be involved, and this appears to be the activity of the clay since, in the cases known to the author, the unsatisfactory samples all have an activity of less than 0.75, while the satisfactory samples lying in the zone to the right of the line in Fig. 4 all have an activity of more than 1.0. In order to make this apparent the data has been separated, in Fig. 4, into two graphs (a) for inactive and (b) for normal and active clays.

The information at present available is not sufficient to enable any detailed or final deductions to be made, but the evidence does at least suggest that the sampling difficulties at depth, in normally consolidated clays, may be restricted to sensitive clays of low activity; and that neither sensitivity or activity are by themselves a sufficient criterion. The importance of sensitivity is evident, and the influence of activity may perhaps be explained by the proportionately low cohesion in clays of low activity, see Fig. 2. It seems not unreasonable to assume that in sampling two clays of the same sensitivity, and at the same depth, more difficulty would be experienced with the clay in which the majority of the strength derived from internal friction, and less difficulty with the clay between the particles of which there were appreciable cohesion forces. The clarification of the problem, however, awaits further research and the publication of additional case records of field investigations.

<sup>1</sup>) Defined by *Terzaghi* (1944) as undisturbed strength remoulded strength

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#### References

- Bjerrum, L. (1951): Fundamental considerations on the shear strength of soil. Géotechnique, Vol. 2, p. 209.
- Cadling, L. and Odenstad, S. (1950): The vane borer. Proc. Roy. Swedish Geot. Inst. No. 2 (Stockholm).
- Carlson, L. (1948): Determination in situ of the shear strength of undisturbed clay. Proc. 2nd Int. Conf. S.M., Vol. I, p. 265.
- Casagrande, A. and Shannon, W. L. (1948): Research on stress-deformation and strength under transient loading. Grad. School Eng. Publ. No. 447. (Harvard Univ.)
- Gibson, R. E. (1951): An investigation of the fundamental shear strength characteristics of clays. Ph.D. Thesis Faculty of Eng. University of London. (See also paper to 3rd Int. Conf. Soil Mechanics.)
- Grim, R. E. (1949): Mineralogical composition in relation to the properties of certain soils. Géotechnique, Vol. 1, p. 139.
- perties of certain soils. Géotechnique, Vol. 1, p. 139. Harris, F. R., Mueser, W. H. and Porter, O. J. (1948): Report of Board of Consultants . . . on La Guardia Field Test Section. Port of New York Authority (New York).
- Hvorslev, M. J. (1937): Über die Festigkeitseigenschaften gestörter bindiger Böden. Ingeniorridenskabelige Skr. A No. 5 (Copenhagen).
- Moos, A. von (1938): Geotechnische Eigenschaften und Untersuchungsmethoden der Lockergesteine. Erdbaukurs der E.T.H. No. 4 (Zurich).
- Northey, R. D. (1950): An experimental study of the structural sensitivity of clays. Ph.D. Thesis Faculty of Science. University of London.
- Rosenquist, I. T. (1947): Om de norske kvikkleirers egenskaper og mineralogiske sammensetning. N.I.M. 3 Forhaudlingar, Vol. 10, p. 1. (Stockholm).
- Samuels, S. G. (1950): The effect of base exchange on the engineering properties of soils. Building Res. Station Note, No. C176 (Watford).
- Skempton, A. W. (1948a): A deep stratum of post-glacial clay at Gosport. Proc. 2nd Int. Conf. S.M., Vol. I, p. 45.
- Skempton, A. W. (1948b): Vane tests in the alluvial plane of the River Forth near Grangemouth. Géotechnique, Vol. I, p. 111.
- Skempton, A. W. (1948c): A possible relationship between true cohesion and the mineralogy of clays. Proc. 2nd Int. Conf. S.M., Vol. 7, p. 45.
- Skempton, A. W. (1950): Soil Mechanics in relation to geology. Yorkshire Geological Soc. (in press).
- Skempton, A. W. and Bishop, A. W. (1950): The measurement of the shear strength of soils. Géotechnique, Vol. 2, p. 90.
- Skempton, A. W. and Northey, R. D. (1952): The sensitivity of clays. Géotechnique, Vol. 3, p. 30.
- Taylor, D. W. (1943): Discussion of factors affecting shearing strengths of clays. Ninth Progress Report, Triaxial Shear Research Supplement. M.I.T. (Cambridge, Mass.).
- Terzaghi, K. (1936): Stability of slopes of natural clay. Proc. 1st Int. Conf. S.M., Vol. 1, p. 161.
- Terzaghi, K. (1944): Ends and means in soil mechanics. Grad. School Eng. Publ. No. 402 (Harvard University).

# THE PORE-PRESSURE COEFFICIENTS $\boldsymbol{A}$ AND $\boldsymbol{B}$

by

A. W. SKEMPTON, D.Sc., A.M.I.C.E.

#### SYNOPSIS

In a number of problems involving the undrained shear strength of soils (especially in the design of earth dams) the change in pore pressure  $\Delta u$  occurring under changes in total stresses must be known. The equation  $\Delta u = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]$  is derived, and some typical values of the experimentally determined pore-pressure coefficients Aand B are given. Some practical applications of these coefficients have been outlined by Bishop (1954). Pour un certain nombre de problèmes comportant la résistance au cisaillement à teneur en eau constante des sols (en particulier, pour le calcul des barrages en terre), il est nécessaire de connaître les changements dans la pression interstitielle  $\Delta u$  qui se produisent lors des changements dans les contraintes totales. L'équation  $\Delta u = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]$  est dérivée et certaines valeurs typiques des coefficients A et B de pression interstitielle obtenues expérimentalement sont données dans cet article. Certaines applications pratiques de ces coefficients ont été exposées par Bishop (1954).

# INTRODUCTION

In problems concerning the undrained shear strength of soils, it has been found convenient to express the pore-pressure change  $\Delta u$ , which occurs under changes in the principal stresses  $\Delta \sigma_1$  and  $\Delta \sigma_3$ , by the following equation:

$$u = \boldsymbol{B}[\varDelta \sigma_3 + \boldsymbol{A}(\varDelta \sigma_1 - \varDelta \sigma_3)],$$

where A and B are "pore-pressure coefficients." These coefficients are measured experimentally in the undrained triaxial test, and the values of  $\Delta \sigma_1$  and  $\Delta \sigma_3$  are, in general, chosen to represent the changes in principal stress occurring in the practical problem under consideration.

If the sample in the test, or if an element of soil in the ground or in an earth dam, is originally in equilibrium under an all-round \* effective pressure p (which may in certain cases

be close to zero), then the application of the stresses  $\Delta \sigma_1$  and  $\Delta \sigma_3$  can be considered as taking place in two stages (see Fig. 1). Firstly, the element is subjected to an equal all-round increment  $\Delta \sigma_3$  and, secondly, it is subjected to a deviator stress  $(\Delta \sigma_1 - \Delta \sigma_3)$ . Corresponding to each of these stages there will be pore-pressure changes  $\Delta u_a$  and  $\Delta u_d$ , where:



01	Drained			
ssure	test	$\Delta V_a$	$\Delta V_d$	$\Delta V = \Delta V_a + \Delta V$
			Fig. 1	
$\Delta u = \Delta u$	$u_a + \Delta u_d$ .			

# THE COEFFICIENT B

The relation between  $\Delta u_a$  and  $\Delta \sigma_3$  for a typical test on a partially saturated soil is shown in Fig. 2 (a). The increase in effective stress in the test is :

$$\Delta \sigma' = \Delta \sigma_3 - \Delta u_a$$

and, if  $C_c$  is the compressibility of the soil structure, then the volume change is :

$$\Delta V_c = -C_c \cdot V(\Delta \sigma_3 - \Delta u_a),$$

where V is the original volume of the sample. And, if  $C_v$  is the compressibility of the fluid (air and water) in the voids and if n is the porosity of the soil, then the change in volume in the void space is :

$$\Delta V_{\boldsymbol{v}} = -C_{\boldsymbol{v}} \, . \, nV \, . \, \Delta u_a$$

<sup>\*</sup> The all-round pressure condition is assumed for simplicity of presentation. The case of an element consolidated under p and  $K_p$  can also be treated by the pore-pressure coefficients.



But these two changes in volume are identical and, hence,

$$\frac{\Delta u_a}{\Delta \sigma_3} = \boldsymbol{B} = \frac{1}{1 + \frac{nC_v}{C_c}}$$

Now, in saturated soils (zero air voids),  $C_v/C_c$  is approximately equal to zero, since the compressibility of water is negligible compared with that of the soil structure. Consequently, for such soils,

B = 1, when the degree of saturation = 1.

An experimental confirmation of this result, for a saturated clay, is given below in Table 1.

$\Delta \sigma_3$	Δua	В	
0	0		
15 lb/sg. in.	14.7 lb/sq. in.	0.980	
30	29.5	0.984	
45	45.0	1.000	
60	59.8	0.996	

If, in contrast, the soil is dry, then  $C_v/C_c$  approaches infinity, since the compressibility of air is far greater than that of the soil structure. Hence, for dry soils,

 $\boldsymbol{B} = 0$ , when the degree of saturation = 0.

For partially saturated soils, O < B < 1 and, at the Proctor optimum water content and density, the values of **B** range typically from about 0.1 to 0.5. The relation between **B** and the degree of saturation, for a clay gravel, is shown in Fig. 2 (b).

#### THE COEFFICIENT A

The changes in pore pressure during the application of a deviator stress are shown, for two compacted clay soils, in Fig. 3. At any time when the increment of deviator stress is  $(\Delta \sigma_1 - \Delta \sigma_3)$ , the pore pressure due to this increment is  $\Delta u_d$  and the corresponding changes in the principal effective stresses are :

$$\begin{aligned} \Delta \sigma_1' &= (\Delta \sigma_1 - \Delta \sigma_3) - \Delta u_d \\ \Delta \sigma_3' &= -\Delta u_d. \end{aligned}$$

and

If, for the moment, it is assumed that the soil behaves in accordance with elastic theory, the volume change of the soil structure under the increment of deviator stress is :

$$\Delta V_e = -C_e \cdot V \cdot \frac{1}{3} (\Delta \sigma_1' + 2\Delta \sigma_3')$$
  
$$\Delta V_e = -C_e \cdot V \cdot \frac{1}{3} (\Delta \sigma_1 - \Delta \sigma_2) - 3\Delta$$

or

$$\Delta V_c = -C_c \cdot V \cdot \frac{1}{3} [(\Delta \sigma_1 - \Delta \sigma_3) - 3 \Delta u_d].$$

And the volume change in the void space is :

$$\Delta V_v = -C_v . nV . \Delta u_d$$

But, as before, these two volume changes are identical and, hence,

$$\Delta u_d = \frac{1}{1 + \frac{nC_v}{C_c}} \cdot \frac{1}{3} (\Delta \sigma_1 - \Delta \sigma_3)$$

or

In general, however, the behaviour of soils is by no means in accordance with elastic theory and the above expression must be written in the form :

$$\Delta u_d = \boldsymbol{B} \cdot \boldsymbol{A} (\Delta \sigma_1 - \Delta \sigma_3),$$

where  $\boldsymbol{A}$  is a coefficient to be determined experimentally.

Combining the expressions for the two components of pore pressure, we have :

 $\Delta u_d = \boldsymbol{B} \cdot \frac{1}{3} (\Delta \sigma_1 - \Delta \sigma_3).$ 

$$\Delta u = \boldsymbol{B}[\Delta \sigma_3 + \boldsymbol{A}(\Delta \sigma_1 - \Delta \sigma_3)],$$

which is the equation given at the beginning of the Paper. It may be noted that for the important particular case of fully saturated soils, where B = 1, the equation becomes :

$$\Delta u = \Delta \sigma_3 + \mathbf{A} (\Delta \sigma_1 - \Delta \sigma_3).$$

This expression was given by the author in 1948. Test results for a saturated clay are plotted in Fig. 4.



Fig. 3. Undrained triaxial tests on two compacted clay-gravels
For any given soil, the coefficient A varies with the stresses and strains. Its value may be quoted at failure (maximum deviator stress), at maximum effective principal stress ratio, or at any other required point. At failure, the values of A for various clay soils, with positive total stress increments, may be summarized approximately as in Table 2. With decreasing total stresses, A will have different values in general, but the data for this case are scanty.

Table 2

Type of Clay		A
Clays of high sensitivity		$+\frac{3}{4}$ to $+1\frac{1}{2}$
Normally consolidated clays		$+\frac{1}{2}$ to $+1^{2}$
Compacted sandy clays		$+\frac{1}{4}$ to $+\frac{3}{4}$
Lightly over-consolidated clays		$\hat{0}$ to $+\frac{1}{2}$
Compacted clay-gravels	• •	$-\frac{1}{4}$ to $+\frac{1}{4}$
Heavily over-consolidated clays		$-\frac{1}{2}$ to 0

ALTERNATIVE FORMS OF THE PORE-PRESSURE EQUATION

$$\Delta u = \boldsymbol{B}[\Delta \sigma_3 + \boldsymbol{A}(\Delta \sigma_1 - \Delta \sigma_3)]$$

may be written in several alternative forms, each of which has some particular advantages. In the normal laboratory undrained test, the pore pressures under  $\Delta \sigma_3$  and under  $(\Delta \sigma_1 - \Delta \sigma_3)$  are measured and, hence, the coefficients determined directly from the test are those in the following equation :

 $\Delta u = \boldsymbol{B} \cdot \Delta \sigma_3 + \bar{\boldsymbol{A}} (\Delta \sigma_1 - \Delta \sigma_3).$ 



Fig. 4. Undrained triaxial tests on two samples of remoulded saturated clay

In evaluating A from  $\overline{A}$ , care must be taken to use a value of B appropriate to the pressure range in the deviator part of the test.

For earth-dam problems, it is convenient to write the basic equation in the forms :

$$\Delta \boldsymbol{u} = \boldsymbol{B}[\Delta \sigma_1 - (1 - \boldsymbol{A}) (\Delta \sigma_1 - \Delta \sigma_3)]$$

and

$$\frac{\Delta u}{\Delta \sigma_1} = \overline{B} = B \left[ 1 - (1 - A) \left( 1 - \frac{\Delta \sigma_3}{\Delta \sigma_1} \right) \right].$$

The "overall" coefficient  $\overline{B}$  is a useful parameter, especially in stability calculations involving rapid draw-down, and it can be measured directly in the laboratory for the relevant values of stress-change in any particular problem.

From a physical point of view, the porepressure equation is best written in the form :  $\Delta u =$ 

$$B\left[\frac{1}{3}(\varDelta\sigma_1+2\varDelta\sigma_3)+\frac{3A-1}{3}(\varDelta\sigma_1-\varDelta\sigma_3)\right],$$

since this shows that, for a material behaving in accordance with elastic theory, with  $A = \frac{1}{3}$ , the pore pressure depends solely on

#### PORE-PRESSURE COEFFICIENTS A AND B

the mean principal stress, whereas in soils with  $A \neq \frac{1}{3}$  the pure shear stress has a marked influence on the pore pressures.

#### APPLICATIONS

During the past few years a number of practical problems have been encountered in which the pore-pressure coefficients have proved to be helpful. Bishop (1954) has described briefly some of these applications.

# ACKNOWLEDGEMFNT

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#### REFERENCES

BISHOP, A. W., 1954. The Use of Pore-Pressure Coefficients in Practice. Géotechnique, 4:4:148. SKEMPTON, A. W., 1948. The Effective Stresses in Saturated Clays Strained at Constant Volume. Proc. 7th Int. Congr. App. Mech., 1:378.

# Stability of Natural Slopes in London Clay

Stabilité des Talus Naturelles en Argile Londonienne

# by A. W. SKEMPTON, D.Sc., A.M.I.C.E., and F. A. DELORY, Ph.D., Imperial College, University of London, England

#### Summary

In several areas of the London clay north of the Thames the natural hillsides are not yet in final equilibrium. Where the ground water level reaches the surface in winter, slips can occur on 10 degree slopes, but all slopes flatter than 10 degrees are stable, although subject to soil creep. This critical slope is in agreement with an analysis of stability based upon the laboratory value for the angle of shearing resistance  $\phi' = 20$  degrees, provided the cohesion intercept c' is taken as zero.

#### Introduction

A number of landslides in natural clay slopes have been analysed and the results published; these have, however, been treated necessarily as individual cases and no data appear to be available concerning a wide survey of the slopes in any particular clay stratum. The London clay provides excellent opportunities for such a survey, owing to the great area which it covers and its exceptional uniformity, coupled with the fact that both stable and unstable slopes can be found. The present paper summarizes the information so far obtained on natural slopes in the London clay. It is to be regarded as an interim report since work is still in progress, but already a remarkably clear and significant pattern has emerged from this survey.

In 1953 an analysis of a landslide in the stiff-fissured clays of the Coal Measures in the River Severn valley in Shropshire led to the conclusion that stability depended solely upon the angle of shearing resistance  $\phi'$  and that the clay behaved as if its cohesion intercept c' was zero (HENKEL and SKEMPTON, 1954). This analysis, carried out in terms of effective stresses, also showed the great importance of the position of the water table; and if the water table rises to ground surface then, with c' = 0, the maximum stable slope is equal to about  $\frac{1}{2}\phi'$ .

At the same time drained tests were being made on London clay, and it was found that  $\phi' = 20$  degrees for this material.

Now a few isolated observations had already indicated that the limiting slope was about 10 degrees in the London clay (SKEMPTON, 1945). Previously the quantitative significance of this result had not been apparent, but it was now realized that this slope was in fact closely equal to  $\frac{1}{2}\phi'$ . It therefore became worth while to undertake a more systematic survey.

#### Areas Studied

Reconnaissance by GRAVES (1954) revealed three promising localities, near Elstree, at Sewardstone (between Chingford and Waltham Abbey) and near Potters Bar. These are all north of the Thames, and more recently Mr W. H. Ward has drawn the authors' attention to another northern area of great value, at Childerditch near Brentwood in Essex. A foundation investigation on the slope of Sydenham Hill (SKEMPTON and HENKEL, 1955) provided a locality south of the river, and Telegraph Hill, near Chessington, appeared to present a second interesting site in the south.

The detailed investigations at Sydenham, however, had

#### Sommaire

Dans quelques régions du London clay au nord de la Tamise le talus naturel ne se trouve pas encore en équilibre définitif. Aux endroits où le niveau phréatique arrive jusqu'à la surface en hiver des glissements peuvent se produire sur les pentes de 10 degrés. Toutes les pentes de moins de 10 degrés sont stables, bien que sujettes au fluage. Cette pente critique est en accord avec une analyse de stabilité calculée d'après la valeur au laboratoire de l'angle de la résistance au cisaillement  $\phi' = 20$  degrés; pourvu que la cohésion soit zéro.

revealed that the upper and steeper parts of the slope consisted of the sandy clays of the Claygate Beds, although these were not plotted on the 6 in. Geological Survey map. Telegraph Hill is likewise plotted as London clay, but it stands out so conspicuously in the landscape, and its slopes are comparatively so steep, that we felt it also might owe its existence to a capping of Claygate Beds; and a hand auger hole proved this to be the case. Consequently, as the steepest parts of the sites south of the Thames had to be eliminated, we have obtained no critical data from this region. Indeed, the topography of the London clay south of the Thames is in general noticeably flatter than north of the river.

The steeper slopes in the northern part of the London clay are probably due to the valleys having been deeply eroded by melt water from the ice sheets which, about 150,000 years ago, deposited the 'Older Drift' boulder clay much of which can still be seen a few miles north of the localities mentioned above. The melt water, however, would enter the Thames and therefore not influence the country to the south. The whole area was probably heavily eroded again, under periglacial conditions, about 50,000 years ago, following the period of the 'Newer Drift' boulder clays of northern England; and possibly the slope adjustment which is still occurring today may result partly from a fresh disturbance caused by the de-forestation which has taken place since mediaeval times. These later effects would be superimposed on the major topographical pattern left by the older interglacial erosion.

#### **Field Observations**

Inclinations were measured with a vernier clinometer, one observer sighting on the eyes of another standing not less than 50 yards away down the slope. At least 2 and often 4 or 5 observations were made at each site and the average value recorded, as well as the precise (numbered) location, on the 6 in. maps.

At selected sites a hand boring was made and the water content and Atterberg limits determined on a sample from a depth of about 5 ft.

#### The Clay

All the slopes recorded in this paper are in the 'brown' London clay, the weathered zone about 20 ft. to 30 ft. thick, of the London clay. The clay in these slopes has the following average properties:

water content		w = 33
liquid limit		LL = 80
plastic limit		PL = 29
density .		$\nu = 119 \text{ lb./cu. ft}$

Values of c' and  $\phi'$  have been measured in drained tests on undisturbed samples from a number of localities (see Appendix) with the following results:

angle of shearing resist	tance		$\phi' = 20$ degrees
cohesion intercept	•	•	c' = 250  lb./sq. ft.

The average water content and Atterberg limits of these samples are not significantly different from those of the clay in the slopes, and the above values of c' and  $\phi'$  are therefore sufficiently accurate for our purpose.

The London clay has a total thickness of at least 100 ft. It is a stiff fissured clay, as defined by TERZAGHI (1936).

#### **Stability Analysis**

Slips which take place in the relatively flat natural clay slopes are typically quite shallow and take the form essentially of a sheet of material sliding down hill on a slip surface parallel to the ground. Moreover the length of the slipping mass, measured up the slope, is usually great compared with its depth.



For limiting equilibrium

$$yz \sin\beta\cos\beta = c' + (y - my_w)z \cos\beta \tan\phi'$$

If c'=0:

$$\tan\beta = \frac{\gamma - m\gamma w}{\gamma} \tan\phi'$$

Hence the problem can be treated on the assumption that the slip surface is a plane.

Referring to Fig. 1, let the inclination of the slope be  $\beta$ , the depth to the slip surface z and the depth of ground water above the slip surface mz. Then the shear stress acting on the slip surface is

$$\tau = \gamma z \sin \beta \cos \beta \qquad \dots \qquad (1)$$

where  $\gamma$  is the saturated density of the clay. The effective normal pressure on the slip surface is

$$\sigma' = (\gamma - m \cdot \gamma_w) z \cos^2 \beta$$

where  $\gamma_{\psi}$  is the density of water. Consequently, if the shear strength parameters of the clay, in terms of effective stresses, as measured in the drained test, are c' and  $\phi'$  then the shear resistance that can be mobilized on the slip surface is

$$s = c' + (\gamma - m \cdot \gamma_w) z \cos^2 \beta \cdot \tan \phi' \qquad \dots (2)$$

Thus the factor of safety is

$$F = \frac{c' + (\gamma - m.\gamma_w)z\cos^2\beta.\tan\phi'}{\gamma z\sin\beta\cos\beta} \qquad \dots \qquad (3)$$

and for the special case where c' = 0 the critical slope is given by the expression

$$\tan \beta_c = \frac{\gamma - m \cdot \gamma_w}{\gamma} \cdot \tan \phi' \qquad \dots \quad (4)$$

If, in addition, m = 1 (i.e. ground water level coincides with ground surface), then

$$\tan \beta_c = \frac{\gamma'}{\gamma} \cdot \tan \phi' \qquad \dots \quad (5)$$

where  $\gamma'$  is the submerged density of the clay. With the foregoing values of  $\gamma$  and  $\phi'$ , but assuming that c' = 0, the critical slope of London clay in accordance with equation 5 is  $9\frac{3}{4}$ degrees. When ground water level is below surface the maximum stable slope will be greater than this value. For example if  $m = \frac{3}{4}$ , then  $\beta_c = 12\frac{1}{2}$  degrees.

#### **Unstable Slopes**

A certain number of the slopes which have been measured are difficult to classify with regard to their stability. Some of these, although apparently stable, show signs of having slipped in the distant past and others show a somewhat wavy surface,



Fig. 2 Elstree No. 1. 10 degree slope, slump Talus à 10 degrés, affaisement

probably indicative of soil creep. There are, however, seven cases of definite instability and these are listed in Table 1.

Table 1 Unstable slopes in London clay Talus instables dans l'argile de Londres

Site	Slope ' degrees	w	LL	PL	Remarks
Childerditch 5	113	36	78	30	Moderately large slip
9 1 Sewardstone 6a	10 <del>1</del> 10 <del>1</del> 10 <del>1</del>	33	78	29	High G. W. L. Active slip Slip in 1913
Childerditch 2 Elstree 1 Sewardstone 1	10 <del>1</del> 10 10	31 36	80 84	28 28	G. W. L. at surface G. W. L. at surface

The last two cases in Table 1 are of especial interest since there can be little doubt that in winter the water table at both these sites is very close to the ground surface. At each locality a cattle pond has been formed at the top of the slope and marshy grass is widespread. At Elstree No. 1 the slope is extremely uneven and the movement may well be described as a slump (Fig. 2). At Sewardstone No. 1 there is a definite slip, the toe of which forms a sharp ridge several feet high.

At the other five sites recorded in Table 1 we cannot be

certain that the ground water coincides with the surface, although it is likely that in the winter it is not more than 2 or 3 ft.



Fig. 3. Childerditch No. 1. 10½ degree slope, active slip Talus à 10½ degrés, glissement en course

deep. At sites Nos. 1, 2 and 5 at Childerditch, instability takes the form of a very pronounced slip (Fig. 3) and Sewardstone



Fig. 4 Sewardstone No. 6. 10½ degree slope, old slip Talus à 10½ degrés, ancien glissement

No. 6 was the scene of a large slip in 1913, although this has now been partially obscured by ploughing and there has been



Fig. 5. Sewardstone No. 5. 10 degree slope, stable Talus à 10 degrés, stable

no recent movement (Fig. 4). All the sites in Table 1 are in grass pasture.

#### Stable slopes in London clay Talus stables dans l'argile de Londres Slope Site LL PL Remarks W degrees Childerditch 6 Wooded slope 10 10 12 10<del>1</del> High G. W. L. Sewardstone 60 101 7 5 10<del>1</del> 10 4t Probably low G. W. L. 27 29 79 28 2 8ь **3**0 81 Elstree 4 3 Probably high G. W. L. Potters Bar 2a G. W. L. about 3 ft. 31 90 30 Sydenham A(5) deep Sewardstone 8a 8 8 7<del>1</del> 7<del>1</del> Potters Bar 5 2b $\overline{2}$ Elstree There are a vast number of stable slopes flatter than 7 degrees Potters Bar 7<del>1</del> 71 71 1 3 4

Details concerning 20 slopes which are undoubtedly stable

Table 2

Stable Slopes

will be found in Table 2.

Childerditch No. 6, with an inclination of  $11\frac{1}{2}$  degrees, is the steepest stable slope recorded and, significantly, it is heavily wooded. The other slopes in Table 2 are all in pasture fields. There are five cases with slopes of 10 to  $10\frac{1}{2}$  degrees and their



Fig. 6 Summary of observations Resumé des observations

stability is probably due to the fact that although the water table may be high it is not at the surface and, as we have seen, if the factor m falls from 1 to  $\frac{3}{4}$ , the maximum stable slope increases from  $9\frac{3}{4}$  to  $12\frac{1}{2}$  degrees.

Most of the slopes recorded in Table 2 are essentially plane

over a substantial length (Fig. 5). The well known S-shaped profile seems to be a characteristic of slopes which are more mature than the majority of those found in the present survey.

#### Conclusions

The data in Tables 1 and 2 are expressed graphically in Fig. 6. The three most striking features of this survey are: (1) no slopes have yet been found which are steeper than 12 degrees; (2) all the unstable slopes have inclinations between 10 and 12 degrees; and (3) all slopes flatter than 10 degrees are stable.

It therefore appears that 10 degrees is a critical angle for natural slopes in the London clay, and this agrees with the value obtained theoretically on the assumption that the water table is at the surface and that c' = 0. It will be recalled that this result was also obtained from the analysis of the landslide in Shropshire.

There is, consequently, rather strong evidence suggesting that, on a geological time scale, stiff-fissured clays in natural slopes behave as if c' = 0. More investigations are necessary before this can be held to be a result of general validity, but the very consistent data obtained from the present study indicate that further work of this nature is likely to prove fruitful.

#### Appendix

Slow drained triaxial tests have been made, under the supervision of the authors' colleague Mr D. J. Henkel, on undisturbed samples of brown London clay from five localities. The specimens were  $1\frac{1}{2}$  in. diameter and 3 in. long, with a porous plate at the lower end and filter strips up the sides, to accelerate the dissipation of pore water pressures. The rate of strain was arranged so that the time to failure was about 2 days.

Location	w	LL	PL	<i>c'</i> lb./sq. ft.	φ' degrees	
Chingford Sydenham Hill Uxbridge Queen Victoria Street Gresham Street	34 32 28 33 29	82 83 82 89 86	29 26 28 26 27	230 220 280 220 280	19 19 21 22 19	
Average	31	84	27	250	20	

#### References

GRAVES, A. C. (1954). An Investigation into the Rate of Softening in London Clay. Unpublished dissertation. Imperial College, 1954

London Clay. Unpublished dissertation. Imperial College, 1954
 HENKEL, D. J. and SKEMPTON, A. W. (1954). A landslide at Jackfield, Shropshire, in a heavily over-consolidated clay. Proc. European Conference on Stability of Slopes, Vol. 1, pp. 90–101, reprinted in Géotechnique, 5, 131, 1955
 SKEMPTON, A. W. (1945). Earth pressure and the stability of slopes in The Principles and Application of Soil Mechanics. Inst. Civ. Eng. London

Eng., London - and HENKEL, D. J. (1955). Report on Foundations for Sydenham

Hill Flats. Unpublished report to Ove Arup and Partners TERZAGHI, K. (1936). Stability of slopes of natural clay. Proc. 1st International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, pp. 161–165

# A CONTRIBUTION TO THE SETTLEMENT ANALYSIS OF FOUNDATIONS ON CLAY

#### by

#### PROFESSOR A. W. SKEMPTON and DR L. BJERRUM

#### SYNOPSIS

The settlement of a foundation on a saturated clay is composed chiefly of the "immediate" settlement due to deformations taking place at constant volume, and the "consolidation" settlement due to of pore pressures. This latter component is therefore dependent on the pore pressures set up by the foundation load, and these pore pressures are themselves dependent on the type of clay. An approximate theory is described which takes account of the type of clay; and comparisons with observed settlements in a number of practical cases show the theory to be an improvement on existing methods of calculation. Le tassement d'une fondation sur une argile saturée se compose principalement du tassement "immédiat" dû aux déformations ayant lieu suivant une ampleur constante, et le tassement "de consolidation" dû à la diminution d'ampleur par suite de la dissipation de la pression interstitielle. Cette dernière composante dépend donc des pressions interstitielles provoquées par la charge de fondation, et ces pressions interstitielles dépendent elles-mêmes du type d'argile. On décrit une théorie approximative qui tient compte du type d'argile; et des comparaisons avec les tassements observés dans un nombre de cas pratiques démontrent cette théorie comme étant une amélioration des méthodes de calcul existantes.

#### INTRODUCTION

If we imagine a footing on a saturated clay to be loaded quite rapidly, then during the load application the clay will be deformed and pore pressures will be set up in the clay. Owing to the extremely low permeability of clays little if any water will be squeezed out of the clay during the load application, and the deformations therefore take place without change in volume. The deformations have both lateral and vertical components, and the vertical component constitutes what is known as the "immediate settlement".

In the course of time, however, some of the pore-water drains out of the clay, leading to a volume decrease, and the vertical component of this volume change is known as the "consolidation settlement".

Now the consolidation of a clay results from the dissipation of pore pressure, with an accompanying increase in effective pressures. But a given set of stresses will cause different pore pressures in different clays. Thus if we have two identical footings, carrying identical loads, and these footings rest on two clays with identical compressibilities, yet if the pore pressures set up in the two cases are different, the consolidation settlements will also be different. And this is true in spite of the fact that no difference would be seen in the results of the oedometer test.

This may seem paradoxical. But the explanation is that in the oedometer test no lateral strains are permitted and, under this special condition, the pore pressure set up in a saturated clay by an applied pressure is always equal precisely to that applied pressure irrespective of the type of clay; provided only that it is fully saturated.

It also follows that if, in practice, the conditions are such that no lateral strains can take place during the load application then, other things being equal, the pore pressures will be the same in all saturated clays, and the consolidation settlement (there will be no immediate settlement if there can be no lateral strains) will be directly proportional to the compressibility of the clays. The condition of no lateral strain is approximately true for at least two practical cases : (a) that of a thin layer of clay lying between beds of sand or between sand and rock, and (b) that of a loaded area of horizontal extent which is great compared with the thickness of the underlying clay, when the lateral strain will be negligible except near the edges of the loaded area. In cases such as these the consolidation settlement can be estimated with reasonable accuracy by a direct application of the oedometer test results, and it is exactly for such cases that Terzaghi (1925) developed his one-dimensional theory of consolidation, the essential data for which are derived from the oedometer.

But in the more general case where lateral deformations can occur the pore pressures set up by the stresses depend upon the type of clay, as well as on the stresses themselves; and the consolidation settlements therefore also depend on the type of clay. Any method of calculating consolidation settlements which does not enable some allowance to be made for this effect is bound to be unsatisfactory in principle.

In 1939 the senior Author developed a theoretical approach to this problem but it was expressed in fundamental parameters which cannot readily be measured in ordinary laboratory tests. More recently the concept of pore pressure coefficients has been introduced which, together with simplifications in the analytical work, has made possible a solution yielding results of practical value.\* Philosophically, the solution should be regarded as illustrative or semi-empirical.

#### IMMEDIATE SETTLEMENTS

By definition, the immediate settlement takes place without dissipation of the pore pressures. Consequently it is possible to measure the relevant properties of the clay in an undrained triaxial test.

According to the theory of elasticity, the settlement of a loaded area is given by the classical expression :

$$\rho = q.b.\frac{1-\nu^2}{E}.\mathbf{I}_{\rho} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

where q = net foundation pressure

b = breadth or diameter of the loaded area

 $\nu = \text{Poisson's ratio}$ 

E = Young's modulus

 $I_{\rho} =$  Influence value, depending on the shape of the loaded area and the depth of the clay bed.

For saturated clays there is no volume change so long as there is no dissipation of pore pressure. Consequently in the calculation of immediate settlements  $\nu = 0.50$ . The value of *E* can be found from the stress-strain curve obtained in the undrained triaxial test, although experience has shown that *E* is sensitive to sampling disturbance, especially in normally consolidated clays, and a correction may often be necessary (for example, Peck and Uyanik, 1955; Simons, 1957).

#### CONSOLIDATION SETTLEMENTS

If  $\Delta \sigma_1$  and  $\Delta \sigma_3$  are the increases in the principal stresses at any point, caused by loading the footing, then the excess pore pressure set up in the clay at this point may be represented by the expression

$$u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

where A and B are the pore-pressure coefficients (Skempton, 1954).

For saturated clays  $\hat{B} = 1$ . The value of A can be determined from pore pressure measurements in undrained triaxial tests (for details, see Bishop and Henkel, 1957). In general, the coefficient A is not a constant for a given clay, but depends on the magnitude of the applied stresses. Nevertheless for our present purpose a range of values can be quoted for various

\* Preliminary statements outlining this solution, and giving equation (9) of the present Paper, were published by the Authors in 1956 (see References on p. 178).

types of clay, as in Table 1. The value of A depends primarily on the geological history of the clay.

Table 1	aiont A fan
the working range of stress below four	idations
Type of clay	A

Type of clay		А
Very sensitive soft clays Normally-consolidated clays Overconsolidated clays	•••	$> 1 \\ \frac{1}{2} - 1 \\ \frac{1}{4} - \frac{1}{2} \\ 0 - \frac{1}{4}$

For simplicity in the analysis only points on the axis of symmetry below a foundation will be considered. The directions of principal stress are then vertical and horizontal.

If at the point under consideration  $p_1$  is the vertical effective stress before the foundation load is applied, then the vertical effective stress immediately after load application is :

$$\sigma_1' = p_1 + \Delta \sigma_1 - u$$

As the pore pressure gradually dissipates to zero during the consolidation process, Poisson's ratio (in terms of total stresses) decreases from 0.50 to some smaller value. But this has little effect on the vertical stresses and therefore, so long as the foundation pressures have not changed, the vertical effective stress when consolidation is completed is :

$$\sigma_1' = p_1 + \Delta \sigma_1$$

Hence the change in vertical effective stress during consolidation is equal to  $u_i$ , and this is wholly an increase above the original stress  $p_1$ .

If  $p_3$  is the horizontal effective stress before the foundation load is applied, then immediately after load application the stress is :

$$\sigma_3' = p_3 + \Delta \sigma_3 - u$$

But u is greater than  $\Delta\sigma_3$  (see equation 2). Hence the horizontal effective stress is reduced by the load application. During the earlier stage of consolidation, as the pore pressure dissipates, the clay is therefore subjected to a recompression in the horizontal direction, under a stress increase equal to  $(u - \Delta\sigma_3)$ . Owing to the comparatively low compressibility of clays in recompression the strains associated with this effective stress increase are small. However, in the later stage of consolidation, after the horizontal effective stress has regained its original value  $p_3$ , the clay will be subjected to a net increase in horizontal effective stress, from  $p_3$  to  $p_3 + (\Delta\sigma_3 - \delta\sigma_3)$ , where  $\delta\sigma_3$  is the decrease in horizontal total stress due to the decrease in Poisson's ratio as consolidation takes place.

Numerical investigations of an approximate nature show that as a consequence of the foregoing effects, the lateral strains during consolidation are so small that they may be neglected without involving an error of more than roughly 20% in the value of the vertical consolidation movements. And this result holds good no matter how important the lateral strains may have been during the "immediate" stage, when the clay was deforming under undrained conditions.

Now, in the oedometer test the vertical compression of the clay is measured under the condition of no lateral strain and if a vertical compression  $\rho$  is caused by consolidation, in this test, under an increase in effective pressure  $\Delta \sigma'$ , then :

$$\rho = m_v . \Delta \sigma' . h$$

where h is the thickness of the sample and  $m_v$  is defined as the coefficient of compressibility in the oedometer test.\*

Hence, since the consolidation of an element of clay, beneath a foundation, takes place without appreciable lateral strain, the vertical compression of an element during consolidation can be expressed approximately by the analogous equation :

$$d\rho_c = m_v.u.dz$$

where dz is the thickness of the element. And the consolidation settlement  $\rho_c$  of the centre of a foundation, resting on a bed of clay of thickness Z is therefore :

$$\rho_c = \int_0^z m_v \cdot u \, dz \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

But, from equation (2), with B = 1,

$$\cdot u = \Delta \sigma_1 \left[ A + \frac{\Delta \sigma_3}{\Delta \sigma_1} (1 - A) \right]$$

Hence :

This expression completes the first part of our investigation, for the "immediate" and "consolidation" settlements are given in equations (1) and (4) in terms of soil properties which can be measured in ordinary laboratory tests.<sup>†</sup>

# CONSOLIDATION SETTLEMENTS: A FURTHER SIMPLIFICATION

It is possible, however, to reduce equation (4) to a more practical form.

In the case of one-dimensional consolidation, in which the lateral strains are zero throughout the loading period as well as subsequently, the settlement is :

This settlement is given the suffix *oed* to indicate that it is the value obtained by a straightforward application of the oedometer test results. It can be readily computed as a matter of routine, and we may repeat that for thin layers of clay, or for loaded areas that are wide as compared with the thickness of the underlying clay, the settlement given by equation (5) is a reasonable approximation to the total settlement which, in these cases, is composed almost entirely of consolidation.<sup>‡</sup>

But a comparison of equations (4) and (5) will show that there is a broad similarity between the "oedometer" settlement and the approximate "consolidation" settlement for the general case of a footing on a deep bed of clay. And it seems reasonable to postulate that the two can be related by a factor  $\mu$  as shown in the following equation :

If the value of  $\mu$  can be found in any particular case without resource to elaborate testing or

<sup>\*</sup> For corrections to be applied to the oedometer test results to allow for sample disturbance, see Terzaghi and Peck (1948).

if in general there will also be a "secondary consolidation", additional to the "immediate" and the "consolidation" settlements. But in most clays the secondary consolidation is small and, moreover, it in no way affects the problem considered here.

the howay anects the problem considered here.  $\ddagger$  Even where the foundation bears directly on a thick bed of clay the final settlement is often given with tolerable accuracy by  $\rho_{0ed}$ . This is known as the "conventional" method of calculating final settlements. It has been discussed in detail, with references to practical examples, by MacDonald and Skempton (1955), and by Skempton, Peck, and MacDonald (1955).

computations, then the consolidation settlement can be derived quite simply from the standard calculation of  $\rho_{oed}$ .

From equations (4) and (5) it follows that :

$$\mu = \frac{\int_0^z m_v \cdot \Delta \sigma_1 \left[ A + \frac{\Delta \sigma_3}{\Delta \sigma_1} (1 - A) \right] \cdot dz}{\int_0^z m_v \cdot \Delta \sigma_1 \cdot dz} \quad \dots \quad \dots \quad (7)$$

And by assuming constant values of  $m_v$  and A with depth, it is possible to express  $\mu$  by the simple equation:

where

The coefficient  $\alpha$  depends only on the geometry of the problem, since Poisson's ratio is 0.5 for all saturated clays during load application; and this coefficient has been computed for circular and strip footings, with various ratios of the thickness of clay Z to the breadth of footing b. The results are given in Table 2.

Table 2 Column of x in the equation $y = 4 + x(1 - 4)$				
	$\begin{array}{c} \text{ Circular} \\ \end{array}$	$A + \alpha(1 - A)$		
Z/b	footing	footing		
	α	α		
0	1.00	1.00		
0.25	0.67	0.74		
0.5	0.20	0.53		
1	0.38	0.37		
<b>2</b>	0.30	0.26		
4	0.28	0.20		
10	0.26	0.14		
ŝ	0.25	0		

Values of  $\mu$  have been plotted against the pore-pressure coefficient A in Fig. 1 with the ratio Z/b as an independent parameter. Reference to this graph will show that for normally-consolidated clays the factor  $\mu$  is typically rather less than 1, while for overconsolidated clays  $\mu$  is in the region of  $\frac{1}{2}$ . In the more extreme cases of heavily overconsolidated sandy clays  $\mu$  can be as low as  $\frac{1}{4}$ , and in very sensitive clays the consolidation settlement can even exceed the oedometer settlement, that is,  $\mu$  is greater than 1.0.

The pore-pressure coefficient, which itself depends greatly on the geological history of a clay, is thus seen to be a vital factor in settlement analysis.

#### PRACTICAL PROCEDURE

It must be remembered that this analysis has been derived in terms of the consolidation settlement of the centre of a foundation. But it does not seem unreasonable to assume that the results apply, broadly, to other points. If this be granted, then the procedure to be used in practice may be summarized as follows :

The net final settlement is :

# $ho_{final} = ho_i + ho_c$

and, in some clays, the "secondary "settlement must also be considered.



The net "immediate" settlement  $\rho_i$  can be calculated for any point in a foundation, on saturated clay, from the equation :

$$\rho_i = q.b.\frac{3}{4E}.\mathbf{I}_{\rho}$$

where  $I_{\rho}$  is the appropriate influence value, as given by Steinbrenner (1934), q is the net pressure, b is the width of the foundation, and E is determined from undrained compression tests with a correction for sampling disturbance if necessary.

The net " consolidation " settlement  $\rho_c$  can be calculated from the equation :

$$\rho_c = \mu \cdot \rho_{oed}$$

$$\rho_{oed} = \int_0^z m_v \cdot \Delta \sigma_1 \cdot dz$$

$$\mu = A + \alpha (1 - A)$$

and

Values of  $\Delta \sigma_1$  have been tabulated by Jurgenson (1934), Newmark (1935), and others. Oedometer tests, again with corrections if necessary, give  $m_v$ ; and A can be found from undrained triaxial tests with pore pressure measurements. Values of  $\alpha$  are given in Table 2. For many purposes, however, a knowledge of the geological history of the clay together with Fig. 1, is sufficient to enable an approximate value of  $\mu$  to be chosen.

The net settlement at any time t after the load application is :

where U is the degree of consolidation as evaluated from the theory of consolidation.

Finally, it may be noted that if the loaded area is very wide compared with the thickness of clay, or if the clay exists as a layer between beds of sand, the influence value  $I_{\rho}$  tends to zero and the factor  $\mu$  tends to unity. Hence, in these cases :

$$\rho_t = U \cdot \rho_{oed}$$
 . . . . . . . . . . (10)

This equation expresses Terzaghi's theory of one-dimensional consolidation, which applied strictly to the cases mentioned above. Equation (10) is also the basis of the "conventional" method of settlement analysis for footings on clay.

#### EXAMPLES OF APPLICATION

The settlements of an oil tank 144 ft in diameter, on 90 ft of normally consolidated silty clay, have been published by Cooling and Gibson (1955) together with the calculated settlements. The observed immediate settlement at the centre of the tank was just over 2 in. and the consolidation settlement, although not quite complete at the time of publication, could be extrapolated to a final value of about 19 in.

The calculated immediate settlement was 3 in., and the oedometer settlement was 18.5 in. The clay had a moderate sensitivity and A was about 0.65.\* With Z/b = 0.63,  $\alpha = 0.46$  and the value of  $\mu$  is 0.8. Consequently the calculated consolidation settlement is equal to  $0.8 \times 18.5 = 15.0$  in. The comparison between observed and calculated settlements are set out in Table 3 and the agreement is reasonably satisfactory.

 Table 3

 Oil tank, Isle of Grain. Comparison of calculated and observed settlements at centre of tank

			Calculated (in.)	Observed (in.)
Immediate		• •	 3	2
Consolidation			 15	19
Final	••	••	 18	21

Settlement records have also been published, with the appropriate soil properties, for three buildings founded on the normally-consolidated Chicago clay (Skempton, Peck, and MacDonald, 1955). In those cases it is not possible to deduce the observed immediate settlement with precision, but the final settlements are known with some accuracy.

The clay has a depth of roughly 50 ft and the buildings a width of the order of 100 ft. The ratio Z/b is thus about 0.5 and hence  $\alpha$  is also about 0.5. The A value for Chicago clay is probably in the region 0.7 to 0.9, and consequently  $\mu$  lies between 0.85 and 0.95. We will take  $\mu = 0.9$ , and the calculated consolidation settlements are then as given in Table 4. The immediate settlements are those quoted in the above-mentioned Paper.

 
 Table 4

 Calculated and observed final settlements of three buildings on Chicago Clay (Settlements in in.)

		Pi	$ = \begin{array}{c} \rho_c \\ 0.9 \ \rho_{oed} \end{array} $	Pfinal calc.	Pfinal observed
Masonic Temple	• •	3	8	11	10
Monadnock Block	• •	6	14·5	20·5	22
Auditorium tower	• •	6·5	19·5	26	24

The calculated final settlements are seen to be in good agreement with the observations. And, although the rate of settlement is not our present concern, it is interesting to note that

\* This value of A has been derived from a preliminary analysis of pore pressure observations at three different depths in the clay beneath the tank (private communication from Dr R. E. Gibson).



the time-settlement curves, for which U in equation (9) is calculated from the theory of onedimensional consolidation, are also in tolerable agreement with the observations (see Fig. 2).

Three structures in London provide good examples of settlements on overconsolidated clay (Skempton, Peck, and MacDonald, 1955). For the London Clay, A is about  $\frac{1}{3}$  in the stress range under foundations.\* The clay is moderately deep, compared with the foundation width, and thus the factor  $\mu$  has a value of about  $\frac{1}{2}$ . The calculated and observed settlements  $\dagger$  are given in Table 5. Here again there is satisfactory agreement. However, the calculated time-settlement curves are widely different, in the early stages of consolidation, from the observations (Fig. 3); and further work is obviously required in this aspect of the problem.

\* At failure, A has a negative value in London Clay.

 $\dagger$  The final net settlement of Waterloo Bridge has been given as 3.4 in. (Cooling and Gibson, 1955). But this is derived by subtracting the average heave from the total settlement. It seems more in keeping with the usual definition of net settlement to subtract the settlement when the excavation load has been replaced. In this way the figure of 3.7 in Table 5 is obtained.



Fig. 3. Structures on London Clay

Another example of settlements on a heavily overconsolidated clay is provided by the Peterborough grain silo (Cooling and Gibson, 1955). The wings of the silo have a width of b = 35 ft and are founded on Oxford Clay with a thickness of about 0.9 b. Thus  $\alpha$  is approximately 0.4. The A value may well be rather less than for London Clay, possibly about 0.25

 
 Table 5

 Calculated and observed final settlements of three structures on London Clay (Settlements in in.)

Structure	$\rho_i$	$\begin{array}{c} \rho_{cons} \\ = \frac{1}{2} \rho_{oed} \end{array}$	<sup>p</sup> final calc.	Pfinal observed
Fire Testing Station, Elstree         Chelsea Bridge          Waterloo Bridge	0·25 0·8 0·9	$0.35 \\ 1.65 \\ 2.6$	0.6 2.45 3.5	0.7 2.1 3.7

The factor  $\mu$  is therefore about 0.55. Cooling and Gibson give the calculated immediate and oedometer settlements as 0.35 in. and 1.0 in. respectively. Hence on the basis of the present Paper the calculated consolidation settlement is 0.55 in. These calculated settlements are compared with the observations in Table 6, from which the agreement is seen to be excellent.

#### Table 6 Comparison of calculated and observed settlements, Peterborough grain silo

			Calculated (in.)	Observed (in.)
Immediate	• •	 	$0.35 \\ 0.55 \\ 0.9$	0·25
Consolidation	• •	 		0·65
Final	• •	 		0·9

# SUMMARY

A method of calculating the net settlements of foundations on clay is given, which takes into account the pore pressures set up in the clay when the incoduction load is applied. The final settlement is expressed by the equation :

$$P_{final} = P_i + \mu \cdot P_{oed}$$

where  $\rho_i$  is the immediate settlement,  $\rho_{oed}$  the settlement calculated in the usual manner from oedometer test results and  $\mu$  is a factor depending chiefly on the pore-pressure coefficient A.

The observed final settlements for eight structures are given in Table 7 where the values calculated from the above equation are also given, in column (a). The comparison between calculated and observed settlements is satisfactory in all cases.

The conventional method of estimating settlement is expressed by the equation :

# $ho_{final} = ho_{oed}$

The settlements obtained in this way are given in Table 7, column (b). The method is in error in three of the eight cases ; and it tends to underestimate the settlements on normally-

Structure	Net observed	Net calculated settlement (in.)						
Structure	(in.)	$\stackrel{(a)}{_{\rho_i+\mu\cdot\rho_{oed}}}$	$\frac{\text{calc.}}{\text{obs.}}$	(b) Poed	calc. obs.	(c) $\rho_i + \rho_{oed}$	calc. obs.	
Normally-consolidated clays : Oil tank, Isle of Grain Masonic Temple, Chicago Monadnock Block, Chicago Auditorium tower, Chicago	21 10 22 24	18 11 20·5 26	$ \begin{array}{c} 0.86\\ 1.10\\ 0.93\\ 1.08\\ \hline 0.99 \end{array} $	18.5 9 16 22	0.88 0.90 0.73 0.92 0.81	21.5 12 22 28.5	1.02 1.20 1.00 1.18 1.10	
Overconsolidated clays : Fire Testing Station, Elstree Chelsea Bridge, London Waterloo Bridge, London Grain silo, Peterborough	0.7 2.1 3.7 0.9	0.6 2.45 3.5 0.9	0.86 1.17 0.95 1.00 0.99	0.65 3.3 5.2 1.0	0.93 1.57 1.40 1.11 1.25	0·9 4·1 6·1 1·35	1.28 1.95 1.65 1.50 1.59	

 Table 7

 Calculations of net final settlement, by three methods

consolidated clays, and overestimate the settlements on the overconsolidated clays. Nevertheless the conventional method is very simple, and is not without use as providing a basis for the approximate estimation of settlements in a wide range of clays (MacDonald and Skempton, 1955).

In some publications it has been suggested that the settlement can be calculated from the expression :

$$\rho_{final} = \rho_i + \rho_{ord}$$

Reference to column (c) in Table 7 will show, however, that this method leads to severe errors in overconsolidated clays, although it is fairly satisfactory for normally-consolidated clays (see also, Simons, 1957).

The new method of settlement calculation can therefore be regarded as an improvement on existing methods. It is, however, semi-empirical in nature and there is ample room for a more exact theoretical analysis.

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#### REFERENCES

BISHOP, A. W., and D. J. HENKEL, 1957. "The triaxial test." Arnold, London.
BJERRUM, L., 1956. Discussion. Conf. Corr. Calc. and Obs. Str. and Displ. Struct. Final vol., pp. 494-95.
COOLING, L. F., and R. E. GIBSON, 1955. "Settlement studies on structures in England." Conf. Corr. Calc. and Obs. Str. and Displ. Struct. Prelim. vol., pp. 295-317.
JURGENSON, L., 1934. "The application of theories of elasticity and plasticity to foundation problems." J. Boston Soc. Civ. Eng, 21: 206-41.
MACDONALD, D. H., and A. W. SKEMPTON, 1955. "A survey of comparisons between calculated and observed settlements of structures on clay." Conf. Corr. Calc. and Obs. Str. and Displ. Struct. Prelim. vol., pp. 318-37. vol., pp. 318-37. NEWMARK, N. M., 1935. "Simplified computations of vertical pressures in elastic foundations." Univ.

Illinois Eng. Expl. Station Bull., No. 24. PECK, R. B., and M. E. UYANIK, 1955. "Observed and computed settlements of structures in Chicago."

Univ. Illinois Eng. Expt. Station Bull., No. 429. SIMONS, N., 1957. "Settlement studies on two structures in Norway." 4th Int. Conf. Soil Mech. London, 1: 431-36.

London, 1: 431-36. SKEMPTON, A. W., 1954. "The pore pressure coefficients A and B." Géotechnique, 4: 143-47. SKEMPTON, A. W., R. B. PECK, and D. H. MACDONALD, 1955. "Settlement analyses of six structure Chicago and London." Proc. Instn civ. Engrs, 4(1): 525-44. SKEMPTON, A. W., 1956. Correspondence on above paper. Proc. Instn civ. Engrs, 5(1): 170-72. STEINBRENNER, W., 1934. "Tafeln zur Setzungsberechnung" Die Strasse, 1: 121-24. TERZAGHI, K., 1925. "Erdbaumechanik" Deuticke, Vienna. TERZAGHI, K., and R. B. PECK, 1948. "Soil mechanics in engineering practice." Wiley, New York. "Settlement analyses of six structures in

# CAST IN-SITU BORED PILES IN LONDON CLAY

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#### SYNOPSIS

The records of loading tests on bored piles in London Clay from ten sites are examined. The ratio ( $\alpha$ ) of the adhesion developed on the shaft, to the average undisturbed shear strength of the clay within a depth equal to the embedded length of the pile in the clay, is found to be typically about 0.45. The value of  $\alpha$  is less than unity, chiefly because the clay immediately adjacent to the pile shaft absorbs water during the drilling operations and from the concrete. With favourable geological conditions and careful workmanship  $\alpha$  can be as high as 0.6, but with unfavourable conditions  $\alpha$  falls to about 0.3. There is no evidence that the end bearing capacity factor decreases with increasing depth.

Extensive data are summarized for the relation between strength and water content, and for the increase in strength with depth in the London Clay. On y examine les dossiers des épreuves de charge sur pieux forés dans l'argile londonienne sur dix chantiers. On trouve le rapport ( $\alpha$ ) caractéristique moyen de 0.45 entre l'adhésion développée sur le puits et la moyenne intacte de résistance au cisaillement de l'argile à une profondeur égale à la longueur d'enfoncement du pieu dans l'argile.

La valeur de  $\alpha$  est inférieure à l'unité principalement parce que l'argile immédiatement adjacente au putts du pieu a absorbé de l'eau pendant les opérations de forage et aussi du ciment. Dans des conditions géologiques favorables et avec un travail soigné  $\alpha$  peut atteindre 0.6, mais dans des conditions défavorables  $\alpha$  tombe à 0.3 environ. Il n'y a aucune évidence montrant que le coefficient de capacité de résistance à la pointe décroit avec l'accroissement de la profondeur. On y résume de nombreux renseignements sur la

On y résume de nombreux renseignements sur la relation entre la résistance et la teneur en eau, et sur l'accroissement de la résistance avec la profondeur dans l'argile londonienne.

# INTRODUCTION

Cast in-situ bored piles are widely used for the foundations of buildings in London. Typically they range from 14 to 24 in. dia. and from 20 to 50 ft long, depending on the loads which have to be carried, but with the appearance of high buildings in the London architectural scene it is probable that considerably larger piles, or piers, will be employed to an increasing extent. Construction is already completed on the foundations of the 27-storey Shell building on the South Bank, consisting in part of concrete piers  $4\frac{1}{2}$  ft dia. belled out to 9 ft dia. at the base, 80 ft below the surface of the London Clay, while work has recently commenced (June 1959) on the site of a 32-storey building at Millbank for which concrete piles are being cast 3 ft dia. extending to 65 ft deep in the clay.

Comparatively little has been published on the behaviour of bored piles in London Clay, however, and the time seems opportune for making a survey of the data; especially as somewhat conflicting suggestions for the design of this type of foundation have been made from time to time, both in print and verbally.

Through the courtesy of a number of consulting engineers and contractors the Author has been able to use information from ten different sites in the London area. On three of these he has acted in an advisory capacity, in conjunction with his colleagues, Dr D. J. Henkel and Dr A. W. Bishop, to whom he is indebted for much helpful discussion and for evaluation of the results. Mr A. E. Insley, a research student at Imperial College, has collected data from the other sites for which no records have been published, and has been of assistance in various ways in the preparation of the Paper.

#### GENERAL PRINCIPLES OF DESIGN

The design procedure consists of three steps: (i) the calculation of the *end bearing capacity*, and (ii) the calculation of the *shaft bearing capacity*: the sum of these two terms is the *ultimate bearing capacity* of an individual pile; (iii) the *working load* of the pile has then to be deduced from the ultimate by applying a factor of safety, as well as a reduction factor to allow for the interaction of the piles within a group.

(i) The end bearing capacity is derived from the formula:

where W is the weight of the pile,  $A_p$  the area of its base or point,  $c_p$  the shear strength of the clay at the level of the point (more accurately the average shear strength within a depth below the base equal to about two-thirds its diameter), N is a bearing capacity factor,  $\gamma$  is the average density of the soil within the total length H of the pile below ground level.

To a sufficiently close approximation, in most cases, W is equal to  $A_p \cdot \gamma H$  and thus one may write:

Both theory and experiment show that N is equal to 9.0 for circular areas loaded at a considerable depth within a mass of saturated clay (Skempton, 1951). This value is generally accepted, although small variations have been suggested; but the idea has also been put forward that N may decrease with increasing ratio of depth to diameter of the pile, owing to the tendency for the clay around the pile base to settle in consequence of the load transferred from the shaft. For longer piles this load is greater, and there may be a corresponding tendency for the relative settlement of the pile base and the adjacent clay to become less, and therefore a tendency for the shear strength to be less fully mobilized at the pile base. There is, however, little evidence to support this hypothesis.

(ii) The shaft bearing capacity is calculated from the formula:

where  $A_s$  is the area of the shaft of the pile in the clay and  $c_a$  is the average adhesion between the clay and the pile shaft. If the pile penetrates gravel, lying above the clay, an additional term for shaft friction can be included, but it is usually advisable to neglect any contribution from made ground or fill, or from gravel overlying a layer of soft alluvial clay.

If c is the average undisturbed shear strength of the clay within the depth of penetration of the pile in the clay, then there is no doubt that  $c_a$  is appreciably less than c and, in general, may be written:

where  $\alpha$  is less than unity, and not necessarily a constant.

The adhesion is also less than the strength of the clay in the case of driven piles (Tomlinson, 1957), but whereas the reduction there may be attributed partly to lack of contact between the pile and the clay due to lateral "whipping" of the pile during driving, for cast in-situ piles the reduction in adhesion seems to be caused essentially by a softening of the clay immediately adjacent to the contact surface. Meyerhof and Murdock (1953) have measured the water contents of the clay immediately adjacent to the shaft of a bored pile (S2) at Southall and they found an increase of nearly 4% at the contact surface, although at a distance of 3 in. from the shaft the water contents had not been altered. It is recorded that the hole for this pile was drilled by hand, and took 2-3 days to complete. This is an exceptionally long period and the clay may have had unusual opportunity for softening. But the effect of various increases in water content are set out in Table 1 using the curve relating strength and water content shown in Fig. 1. In this graph the points A-P are results obtained from block samples taken from various tunnels in London (Ward, Samuels, and Butler, 1959). All the other points represent averages of groups of samples taken from boreholes. The data for Paddington, Victoria, and South Bank are published (Skempton and Henkel, 1957). The remainder of the information comes from sites where bored piles have been constructed (see later in the present Paper), except for the new Bank of England office block near St Paul's Cathedral. For this latter case the Author is indebted to Mr Stanley Serota of Foundation Engineering Ltd.

#### CAST IN-SITU BORED PILES IN LONDON CLAY



Fig. 1. Relation between shear strength and water content for London Clay (liquid limit, 70-85)

If there is perfect adhesion between the clay and the concrete, the effect of softening will be as given in the fourth column of Table 1. As mentioned later, both field and laboratory tests indicate, however, that the adhesion is about 80% of the shear strength adjacent to the concrete. Using this factor one arrives at the rough estimate of the ratio ( $\alpha$ ) of adhesion to the original strength given in the fifth column of Table 1. In deriving these figures a natural water content of 28 has been used. Slightly different values of  $\alpha$  would be obtained for other natural water contents, but the differences are not significant.

An increase in water content in the clay adjacent to the pile may be due to any or all of four causes: (a) water flowing out of the clay itself during the process of boring; this effect being more marked the more fissured the clay and the higher the ground-water level; (b) migration of water from the body of the clay towards the less highly stressed zone around the borehole; (c) water tipped into the boring to facilitate operation of the cutting tool; and (d) water derived from the concrete, which for practical reasons has to be placed at a water-cement ratio greater than that required solely for hydration of the cement.

Of these causes, (c) can be eliminated by good drilling technique and (a) can be minimized by carrying out the drilling and concreting operations as rapidly as possible. But (b) and (d)would seem to be inevitable. Moreover, since a variation of only 1% in the water content is responsible for a 20% change in  $\alpha$  it is obvious that considerable variations are likely to occur with different techniques, and from one job to another where the same technique is employed and even on any one site. The main object of the present study is to find the values of  $\alpha$ obtaining in practice.

The curious fact may be noted that the two published recommendations for calculating the adhesion represent almost the extreme upper and lower limits. Thus, Golder and Leonard (1954) suggested that  $\alpha = 0.7$  for piles more than 30 ft long, which implies only a very small increase in water content; and this value of  $\alpha$  is, in fact, slightly greater than any found in the present survey. On the other hand Meyerhof and Murdock (1953) proposed that the adhesion should be taken as equal to the strength of the clay after it has been allowed to soften fully under zero (or negligible) pressure. They also state that under this condition the increase in

Water	Increase	Shear	Relative	Adhesion
content	in water content	strength: lb/sq.ft	strength ·	Original strength: (a)
28	0	3 300	1.00	0.80
28.5	0.5	2,900	0.88	0.70
29	i	2,550	0.77	0.62
30	2	2,000	0.61	0.48
31	3	1,640	0.50	0.40
32	4	1,320	0.40	0.32
33	5	1,050	0.32	0.26
34	6	850	0.26	0.21

Table 1Effect of softening on adhesion

water content is from 5-8%, although subsequent experience (see under the notes on the Galway Street site, p. 167) shows that in general the increase is probably rather less. If the maximum increase in water content is taken to be typically in the region of 5-6% then the softened strengths will be about 30% of the original strength (see Table 1). On the basis of the proposal by Meyerhof and Murdock, therefore, the lowest values of  $\alpha$  will also be about 0.3. And, as will be seen from the case records, this is approximately the minimum result obtained. But the average is about 50% higher, and in several instances the pile tests show that  $\alpha$  is twice as great.

It is worth mentioning that softening will also take place in the clay just beneath the pile base. But this will have a negligible effect on the end bearing capacity owing to the comparatively great mass of clay involved in the shearing process when the base of the pile is forced to penetrate the clay. In contrast, the shearing process which develops in the clay alongside the shaft of the pile is probably restricted essentially to the narrow softened zone.

(iii). The working load  $Q_w$  is calculated from the formula:

where  $Q_u$  is the ultimate load of the individual pile (=  $Q_p + Q_s$ ),  $\beta$  is a group factor and F is the factor of safety. For an isolated pile  $\beta$  is equal to 1.0, but when n piles are in a closelyspaced group the ultimate bearing capacity of the group is less than  $n.Q_u$  owing to the fact that the clay adjacent to any one pile has to support, in addition, part of the load transferred by adhesion from the adjacent piles. In an important Paper on this subject, with a full bibliography, Whitaker (1957) has reported experimental investigation on the group effect

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and he finds, for example, that with piles spaced at 3 diameters  $\beta$  lies between 0.7 and 0.8, depending on the number of piles and the ratio of length to diameter. In these tests, however, a very soft clay was used and its shear strength was fully mobilized along the shaft. With bored piles in London Clay only about one-half the shear strength is developed as adhesion, and  $\beta$  will probably be greater than in Whitaker's experiments.

The factor of safety serves two functions. Chiefly it is necessary to keep the settlement within safe limits, but also it ensures that even with the inevitable variations from the calculated ultimate load, any one pile will be able to pass the acceptance test or proof loading which the district surveyor and consulting engineer require before approving the proposed pile or pier foundation. A typical criterion is that the pile should not have a permanent set of more than 0.25 in. after having been loaded under  $1.5 Q_w$  for 24 hours, or for a shorter period if the settlement has already become constant. Experience indicates that the working load of an individual pile is typically between 45 and 50% of the ultimate. But uncertainties amounting to 10 or 20% will arise in calculating the ultimate. Hence a factor of safety of not less than 2.5 should be used on the calculated ultimate load for piles of the usual dimensions.

For larger diameter piers, however, the factor of safety should be greater in order to restrict the settlements. And, indeed, for all pile or pier foundations the overall settlement due to shear strain and consolidation of the clay beneath the groups must be calculated, since this is often the controlling feature of the design. Settlement considerations, however, lie outside the scope of this Paper, which is chiefly concerned with the ultimate bearing capacity of the individual pile as used for heavy foundations.

#### LOADING TESTS AND SETTLEMENTS

An examination of pile loading tests shows that in many cases the load has not been taken up to the ultimate, and that what is often called the ultimate load is at least 20% below the true value.

The ultimate bearing capacity is reached when the pile first continues to settle at a steady rate under constant load.\* At lower loads the rate of settlement decreases with time and eventually becomes zero. Admittedly, greater loads than the ultimate can sometimes be sustained by the pile, but only with still greater rates of settlement than that at the ultimate.

In Table 2 the settlements of a number of piles at the ultimate are given; that is to say, the settlements at the *beginning* of the stage at which the rate of movement becomes constant. The Author has also given the settlement at 90% of the ultimate load, since this can occasionally be used to estimate the ultimate where the test has not been carried quite far enough for the ultimate to be determined directly. It will be seen that for piles between 12–24 in. dia. the settlement at the ultimate is approximately equal to 1 in. per ft dia., or 0.085 *B*, where *B* is the pile diameter, and that the length of pile has little if any influence. At 90% of the ultimate the settlement averages about 0.04 *B* and ranges from 0.03 *B* to 0.06 *B*.

The Author is indebted to Dr Golder for the results of loading tests carried out on 21-in.-dia. concrete pads at the bottom of a lined borehole at Kensal Green, at three different depths. Great care was taken to avoid bedding errors, and the average settlements at the ultimate and at 90% of the ultimate were 0.10 B and 0.04 B respectively; results which, it will be noticed, are closely equal to those for the piles in Table 2. The Author is also indebted to Mr Tomlinson of George Wimpey Ltd and Mr Weeks of Pressure Piling Ltd for the results of loading tests on special piles constructed with their base clear of the bottom of the borehole. These tests, at Southall and West Hill, Wandsworth, were on piles 12 and 17 in. dia. and about 40 ft long. They showed that the shaft adhesion was fully mobilized after a settlement of about 0.4 in. in both cases. For normal piles of these diameters the ultimate load would require settlements of about 1.0 and 1.5 in., respectively.

\* Or the maximum load in a constant rate of strain test.

Location and	P	ile	At ultin	nate	At $0.9 \times \text{ultimate}$		
pile No.	Dia: B in.	Length: ft	Settlement: $\rho_u$ in.	Pu B	Settlement: $\rho_{\cdot 9}$ in.	$\frac{\rho \cdot g}{B}$	
Southall S1 , S3 Wembley Brandon 5 Kensal Green B5 , B33 Millbank 2	12 12 15 17 18 24 24	20 30 36 38 34 23 65	$ \begin{array}{c} 1 \cdot 0 \\ 1 \cdot 0 \\ 1 \cdot 25 \\ 1 \cdot 75 \\ 1 \cdot 25 \\ 1 \cdot 75 \\ 2 \cdot 0 \end{array} $	0.083 0.083 0.083 0.103 0.070 0.073 0.083	$ \begin{array}{c} 0.4\\ 0.3\\ 0.6\\ 1.0\\ 0.5\\ 0.9\\ 1.1 \end{array} $	$\begin{array}{c} 0.035\\ 0.025\\ 0.04\\ 0.06\\ 0.03\\ 0.04\\ 0.045\end{array}$	
Average				0.085		0.04	

$\mathbf{T}_{i}$	aDI	e 2	
Settlements	in	loading	tests

It is therefore clear that the shaft adhesion is mobilized at an early stage in the loading, and that the settlement at the ultimate is essentially controlled by the settlement required fully to mobilize the end bearing capacity.

#### SHEAR STRENGTH OF LONDON CLAY

The use of ultimate loads which are too low has led to some confusion in this subject; but another source of trouble has been the use of shear strengths which are too low. These two errors can, in some instances, partially balance each other in the analysis of loading tests, but they are by no means always operative in the same case.

The London Clay is an unusually constant geological material, at least within the region of the city and its suburbs, but even so there are appreciable variations from place to place. It will therefore always be necessary to determine the strengths at any particular site. Nevertheless it is possible, from an examination of test results at many localities, to give at least a rough estimate of the probable strength at any depth, and if the test results at a particular site are substantially less there are good grounds for doubting their accuracy.

Two cases have to be distinguished. First, where the London Clay extends to ground surface, and secondly where it is covered by the gravels and alluvial clay of one or other of the Thames terrace deposits. In Fig. 2 are plotted the strengths (averaged in groups of not less than two and usually of at least four samples) for the London Clay extending to the surface. At all the sites represented in this Figure good commercial sampling and testing methods were used; and there is little reason to doubt the general accuracy of the results, especially within the upper 50 or 60 ft. At greater depths sampling disturbance may lead to some loss in strength, possibly due to softening of the clay as a consequence of the migration and absorption of water during the drilling operations. But in the Author's opinion the basic line in Fig. 2 can be considered sufficiently reliable as a check on sampling and testing methods.

Where the London Clay has been eroded and later covered with alluvial gravel and clay, the strengths at depths of more than about 50 ft below ground level seem not to have been altered. At smaller depths, however, some softening has occurred (Skempton and Henkel, 1957) and between 10 ft and 35 ft this amounts to approximately 800 lb/sq. ft, as will be seen from Fig. 3. Exceptionally low strengths are occasionally reported for the London Clay immediately below the alluvial gravel, but in some cases these are probably due to difficulties in sampling in this zone.

In plotting Fig. 3 the "effective" depths have been computed by reducing the actual depths of fill, and of alluvial clay, in the ratio of their density to that of the London Clay and gravel; both of which have a density of about 125 lb/cu. ft. The six sites represented in Fig. 3 with fill over the alluvium are in the older parts of London and the fill has been in position for

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at least a century. At Wembley (Fig. 2) it appears that the fill has had no appreciable influence on the strength of the underlying London Clay, so it has been neglected.

#### END BEARING CAPACITY FACTOR

An essential step in the analysis of pile loading tests, in order to establish the shaft adhesion, is the calculation of the end bearing capacity. This is derived from equation (2) which, in turn, depends on the factor N.

During the loading test there will be very little consolidation of the clay beneath the pile base and thus for saturated clays  $\phi$  can be taken as zero. Under these conditions theory and model tests indicate that N is approximately equal to 9.0. The evidence for this conclusion is given by Skempton (1951), but it may be useful to recall that the two principal theories are those of Mott, as modified by Gibson (1950), and Meyerhof (1951). In the Mott-Gibson theory

 $N = \frac{4}{3} \left( \log \frac{E}{c} + 1 \right) + 1$ , where E and c are the Young's modulus and shear strength of the

clay. For undisturbed London Clay E/c = 140 (Skempton and Henkel, 1957). Hence N = 8.9. Using plastic theory, Meyerhof showed that N = 9.3 when the shaft adhesion is zero, as in plate loading tests in boreholes, and N = 9.7 for the extreme case of shaft adhesion  $(c_a)$  equal to the shear strength of the clay at the pile base  $(c_p)$ . In practice  $c_a$  is usually less than  $0.3 c_p$ . Hence the influence of shaft adhesion on the value of N is negligible. Meyerhof's theory gives an upper limit for N and it is therefore clear that N = 9.0 is a reasonable approximation for the theoretical value.

It is now necessary to investigate the degree to which this value of N is justified in practice in London Clay.

At Southall tests were carried out at three depths in a boring at the bottom of which a 12-in.-dia. steel plate was loaded to failure (Rodin and Tomlinson, 1953, and personal communication). The results (see Table 3) can be analysed in accordance with the following equation where, in these tests, W is the weight of the loading equipment:

The value of N at the greatest depth was redetermined by casting a special pile with its base above the bottom of the boring and observing the increase in bearing capacity as the end of the pile penetrated the clay. There was no substantial difference between the result of this test and the plate test; and the average value of N at the three depths is 9.1.

At Kensal Green gas works six end bearing tests were carried out by Soil Mechanics Ltd and have been described by Golder and Leonard (1954). The plates consisted of cast in-situ

Type of test	Dia.: in.	Depth: ft	$c_p$ tons/sq. ft	$q_{p}$ tons/sq. ft	γH tons/sq. ft	N
Plate	$\begin{array}{c}12\\12\\12\\12\end{array}$	22 32 40	$1 \cdot 1$ $1 \cdot 3$ $1 \cdot 6$	$     \begin{array}{r}       10.6 \\       16.9 \\       14.2     \end{array} $	$1 \cdot 2 \\ 1 \cdot 7 \\ 2 \cdot 2$	8.6 11.7 7.5
Pile	12	40	1.6	16.2	2.2	8.7

Table 3 End bearing tests at Southall

Mean = 9.1

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concrete blocks about 6 in. thick and approximately 15 and 21 in. dia., each size being tested successively at 14.5 ft, 21 ft, and 27.7 ft below ground level at the bottom of two adjacent boreholes. The values of N given in the original Paper had not been corrected for the overburden term in equation (6); but allowing for this the average results at the three depths are 8.8, 10.9, and 10.3. These figures are based chiefly on the strengths of cores taken a short distance below the final level of a loading test, and it is probable that the clay had suffered some disturbance due to the shear strains set up by the test; for each plate was allowed to settle about 6 in. under the ultimate load.

Dr Golder has kindly allowed the Author to examine the relevant test results, and by comparison with the curve in Fig. 1 it is clear that, in fact, several of the samples must have been partially disturbed. Neglecting these, the remaining tests are plotted in Fig. 4.

Also in this Figure are given the results of a series of triaxial tests carried out at Imperial College on specimens  $1\frac{1}{2}$  in. and 4 in. dia. prepared from cores taken just *above* the position of a loading test and, in the case of the deepest samples, at a depth of 3 ft 6 in. below the final position of the test plate.

From these tests (and each point in Fig. 4 is the average of two cores) the relation between strength and depth is found to be approximately linear, within the range of levels of the samples, and converges to the basic line (Fig. 2) at a depth of about 30 ft. At shallower depths, however, the strengths are somewhat greater than those given by the basic line, and this is probably caused by the loading and heating of the upper parts of the clay due to the presence of an old retort house which had been demolished before the tests were made.



Fig. 4. Shear strengths at site of Kensal Green loading tests (1951)

The ultimate bearing capacity<sup>\*</sup>, the overburden pressure, and the shear strength of the clay as found from Fig. 4, are given in Table 4, together with the calculated values of the end bearing capacity factor N (mean = 9.2).

The end bearing tests at Kensal Green and at Southall are therefore in close agreement with each other and with theory. The value of N = 9.0 is thus established beyond reasonable doubt for practical purposes.

#### PILE LOADING TESTS

A number of loading tests on piles in the London Clay will now be analysed, with a view to evaluating the shaft adhesion. In all cases the procedure is, first of all, to establish a reliable strength-depth curve for the clay at the location of the tests; using Fig. 1 and Figs

<sup>\*</sup> The total load  $(Q_p + W)$  in equation (6) has to be reduced to allow for the adhesion on the side of the 6-in.-thick test plate, before calculating  $q_p$ . This has been done by assuming  $\alpha = 0.5$ . The correction is small, and the exact value of  $\alpha$  is of little consequence.

Average depth: ft	Dia.: in.	c <sub>p</sub> tons/sq. ft	$q_p$ tons/sq. ft	γH tons/sq. ft	N
14·5 21·0 27·7	$ \begin{array}{r} 20.5 \\ 14.5 \\ 23.7 \\ 17.0 \\ 20.5 \\ 14.5 \end{array} $	$ \begin{array}{r} 1 \cdot 13 \\ 1 \cdot 13 \\ 1 \cdot 35 \\ 1 \cdot 35 \\ 1 \cdot 35 \\ 1 \cdot 53 \\ 1 \cdot 53 \\ 1 \cdot 53 \\ \end{array} $	10·3 12·0 12·8 13·5 15·7 16·8	0.8 0.8 1.1 1.1 1.5 1.5	$\begin{array}{c} 8 \cdot 4 \\ 9 \cdot 8 \\ 9 \cdot 8 \\ 9 \cdot 3 \\ 9 \cdot 3 \\ 9 \cdot 3 \\ 10 \cdot 0 \\ \end{array} \begin{array}{c} 9 \cdot 1 \\ 9 \cdot 0 \\ 9 \cdot 3 \\ 10 \cdot 0 \\ \end{array} $
				Mean	= 9.2

		TaD	04	•		
End	hearing	tests	at	Kensal	Green	

2 or 3 as a check. The end bearing capacity is then calculated from the equation  $Q_p = 9.c_p.A_p$ . The load-time-settlement diagrams are next studied to decide the ultimate load of the pile  $Q_u$ . Hence the shaft load is known, since  $Q_s = Q_u - Q_p$ , and  $c_a$  can be calculated from the equation  $c_s = Q_s/A_s$ . The average shear strength of the clay in the length of the pile  $(\bar{c})$  is then determined from the strength-depth curve, and the ratio  $\alpha = c_a/\bar{c}$  can thus be determined.

#### Southall (1950)

Seven piles were tested at Southall by George Wimpey & Co., in conjunction with the Building Research Station. One failed in the concrete and this was then excavated and the water contents of the clay, which have previously been mentioned, were determined. Of the six other piles three were 12 in. and three were 14 in. dia., and each size of pile was tested in lengths of 20 ft, 30 ft, and 40 ft beneath ground level. The results are given by Rodin and Tomlinson (1953) and will be found in Table 5. The values of  $c_a$  range from 550 to 800 lb/sq. ft and  $\alpha$  lies within the limits 0.21-0.42. At each depth one hole was drilled by hand and the other by power auger; the average values of  $\alpha$  being 0.28 and 0.32, respectively, a difference which probably reflects the shorter period required for drilling by power.

The water content observations within a depth of 18 ft are summarized in Table 6 and they show that,  $\frac{1}{8}$  in. away from the pile shaft, there was an increase of 3.5% from the average natural water content of 29.5 existing 3 in. or more from the shaft.

Now from Fig. 1 this increase in water content causes a drop in strength from 2,300 to 1,050 lb/sq. ft. The relative strength of the softened clay in a depth of 20 ft is therefore 1,050/2,300 = 0.46. But, considering the two 20-ft test piles, the values of  $\alpha$  are 0.42 for pile S1 (power auger) and 0.35 for pile S5 (hand auger). Of these the latter is more comparable to pile S2, the only difference being the diameter of 14 in. instead of 12 in., as in S2. It may therefore be inferred that the ratio of adhesion to softened shear strength is 0.35/0.46 = 0.76or if the average for the two piles ( $\alpha = 0.385$ ) is taken, the ratio is 0.84. These figures are closely equal to the results determined directly by Meyerhof and Murdock (1953) from shear tests between clay and stone with a surface texture similar to that of smooth concrete. In these tests the clay had shear strengths ranging from 2,000 to 6,000 lb/sq. ft and an adhesion coefficient of 0.8 was measured for applied pressures of less than 1 ton/sq. ft. At higher pressures the coefficient approached unity. But the residual adhesion, after large shear strains such as occur along the pile shaft at the ultimate load, fell to values corresponding to a coefficient between 0.5 and 0.8. At lower shear strengths it is possible that the coefficient even after large strains is rather greater. Thus, until more evidence is available, 0.8 may be taken as an approximate value for this coefficient.

# slay wed around of pile

Table 5 Data on loading tests; bored piles; London Clay

$\alpha$ Remarks		<ul> <li>42 Power auger</li> <li>55 Hand "</li> <li>25 Power "</li> <li>22 Hand "</li> <li>23 Hand "</li> <li>21 Hand "</li> <li>30 Power "</li> </ul>	-61	$\begin{array}{c c} -32 \\ -32 \\ -46 \\ -46 \\ -46 \\ -33 \\ -33 \\ -33 \\ -33 \\ -33 \\ -33 \\ -33 \\ -33 \\ -33 \\ -33 \\ -33 \\ -31 \\ -31 \\ -31 \\ -31 \\ -31 \\ -31 \\ -31 \\ -32$	1-56 1-60	1.34	1.52 ) c in fill taken as 400 lb/ft <sup>2</sup> and the corresponding shaft load	1.51 subtracted from ultimate to give $Qu$	<ul> <li>46</li> <li>Fill and gravel</li> <li>57</li> <li>129</li> <li>120</li> <li></li></ul>	$\begin{array}{c c} +48 \\ +57 \\ +57 \\ +57 \\ +56 \\ +56 \\ +56 \\ +56 \\ +66 \\ +66 \\ +66 \\ +16 \\ +66 \\ +16$	$\cdot 45$ Fill and gravel sleeved off
	lb/ft2	750 620 560 550 780 780 0 780	870 0 540 0	350 500 660 430 410 860 0 410 0 380 0 0 380 0 0 310 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	,440 ,320 0	960 0	940 0	1,050 0 1;220 0	730 970 920 920 1,020 0	1,660 0 1,970 0 2,100 0	2,240
80 8	tons	5213304000 5213304000	14 9	8 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9	103 85	82	39	63 73	12 54 56 69 1	1002	281 327
đ	tons	8 119 119 119 119 119 119 119	10	0.004 0.004 0.004 0.004 0.004	27 40	28	13	24 24	13 26 31 28 31 28	8888	883
nð	tons	229 352 70446 332 29	24 19	$\begin{array}{c} 7.1\\ 11.2\\ 13.2\\ 12.2\\ 12.2\\ 12.2\\ 12.2\\ 13.4\\ 13.4\\ 15.7\\ 15.7\end{array}$	130 125	110	52	87 97	$25 \\ 80 \\ 82 \\ 95 \\ 95 \\ 95 \\ 95 \\ 95 \\ 95 \\ 95 \\ 9$	$102 \\ 110 \\ 130 $	320 410
10	lb/ft <sup>2</sup>	1,770 2,240 2,620	1,420	1,080 ,,, 1,250 ,,, 1,300	$2,600 \\ 2,220$	2,850	1,800	2,050	1,600 2,700 3,150 2,920	$3,470 \\ 3,450 \\ 3,580$	4,900
d y	lb/ft²	2,820 3,600 ,, 4,100	2,310	1,750 1,950 2,180	3,800 3,150	4,400	2,700	3,000	2,100 4,120 4,880 4,480 4,480	4,600 4,500 4,750	6,600
Dia.:	in.	242424	14	12101410141	18 24	17	15	19 ,,	2 : : : :	.: 17	16 <b>4</b> 24
Effective depth:	ft	20 30 40 30 30 30 30 30 30 30 30 30 30 30 30 30	10	8 : : 12: : : 8	34 23	43	24	27 ,,	20 40 50 45	39 42 46	66
Length in London	Clay: ft	19 2,9 3,0 ,,	10	8 * 10 * 15 * *	34 23	43	24	27 ''	33 85 85 85 33 85 85 85 86 85 86 86 85 86 85 86 86 86 86 86 86 86 86 86 86 86 86 86	22 21 24	65 64
Depth of (clay) and	gravel: ft	0 : : : : :	0:	0::::::::	0:	0	0	00	∞ : : : :	$(2) + 10 \\ (2) + 10 $	(9) + 13
Depth of fill:	ft	0:::::	0:	0 * * * * * * * *	• :	•	12	en en	4	$\frac{5}{11}$	L :
Pile No.		SSSSS2	28		B5 B33			0160	2 (b) 3 4 (c) 6	5 12	n co a
Location		Southall (1950)	Barnet (1950)	Hendon (1950)	Kensal Green (1951)	Wimbledon (1951)	Wembley (1956)	Finsbury (1957)	Brandon (1957)	Galway St (1958)	Millbank (1959)

CAST IN-SITU BORED PILES IN LONDON CLAY

Depth: ft	Water	content:	Increase
	Natural	At pile shaft	
2	27.3	29.6	2.3
4	29.5	31.5	2.0
6	28.2	30.8	2.6
8	28.0	32.4	4.4
10	30.2	33.7	3.5
14	29.8	32.0	$2 \cdot 2$
16	31.0	35.0	4.0
18	30.8	36.5	5.7
Mean	29.5	33.0	3.5

	Т	abl	le 6			
Water	contents	at	pile	S2,	Southall	

The fully softened strengths were also determined, the average values for depths of 20, 30, and 40 ft being 500, 520, and 620 lb/sq. ft. The corresponding adhesion for each of these depths would therefore be 400, 420, and 500 lb/sq. ft, and the lowest adhesion deduced from the pile tests is 550 lb/sq. ft for pile S4 (hand auger, 40 ft deep); only 10% more than the fully softened adhesion.

The Southall piles were constructed for research purposes, and a longer period was required for drilling the holes than is usual in practice owing to the time spent on sampling. The hand auger holes, in particular, took much longer to drill and these results have therefore been excluded in plotting the graphs, Figs 5–7, which summarize the field data for the London Clay sites. Nevertheless these tests are of the greatest importance since, by exaggerating the softening of the clay they enabled this phenomenon to be studied more accurately.

The piles at Southall were normally tested 1 month after casting, but three, S1 (power), S5 (hand), and S6 (hand), were retested 6 months and 18 months later. An increase in bearing capacity of about 5% was observed after 6 months, while in two of the piles the increase after 18 months was roughly 10%, although the third showed a small decrease. It therefore appears



Fig. 5

# CAST IN-SITU BORED PILES IN LONDON CLAY

that very little reconsolidation of the softened clay took place; although a greater effect might be expected with a pile which had been held under load, as would be the case during the construction of a building.

# Barnet (1950)

In an attempt to reduce the softening action, tests were made at Barnet using a water/ cement ratio of only 0.2 (instead of 0.4 as at Southall). A test pit alongside one of these piles showed that, in fact, the gain in water content in the clay was very small: but all the piles failed by crushing the concrete (Meyerhof and Murdock, 1953).

Two other piles have also been tested at Barnet, however, and these were 14 in. dia. and 10 ft long. The results (Rodin and Tomlinson, 1953) are given in Table 5. The average adhesion is 700 lb/sq. ft, compared with  $\bar{c} = 1,420$  ( $\alpha = 0.50$ ). The fully softened strength was 500 lb/sq. ft.



#### Hendon (1950)

The Building Research Station tested forty-five piles at Hendon in their programme on short bored piles for house foundations (Ward and Green, 1952; Rodin and Tomlinson, 1953). The piles were 10, 12, and 14 in. dia. and 6, 8, 10, and 12 ft deep. The results at 6 ft were erratic, but the average values at the other depths are  $c_a = 430$ , 480, and 370 ( $\alpha = 0.39$ , 0.39, and 0.28). The minimum adhesion was recorded on the 14-in. piles 12 ft deep, the mean value for this group being 310 lb/sq. ft (see Table 5). At this depth the fully softened strength is 360 lb/sq. ft and the corresponding adhesion is roughly  $0.8 \times 360 = 290$  lb/sq. ft. The absolute lower limit was therefore very nearly reached at Hendon, as at Southall.

To avoid over-weighting the general picture with the results of these very short piles, only the average values for each depth are plotted in Figs 5–7.

#### Kensal Green (1951)

At the same location as the end bearing tests, Soil Mechanics Ltd carried out two pile tests as part of a foundation investigation for the North Thames Gas Board. The piles were 18 in. dia., 34 ft long (B5) and 24 in. dia., 23 ft long (B33). The ultimate loads and the settlement diagram for B33, are given by Golder and Leonard (1954). From the strength-depth curve in Fig. 4 the shaft adhesion in the two cases is found to be 1,440 and 1,320 lb/sq. ft, and the values of  $\alpha$  are 0.56 and 0.60, respectively. There is some uncertainty as to the strengths in the top 10 ft, but as a lower limit for  $\overline{c}$ , and hence to obtain upper limits for  $\alpha$  the strengths given by the basic line could be considered (see Fig. 4). The values of  $\alpha$  are then 0.59 and 0.66 for piles B5 and B33. There can be no doubt that the true values are rather lower, and the results given above, namely 0.56 and 0.60, are probably sufficiently accurate. These figures are less than those given by Golder and Leonard, which were 0.73 and 0.64. The difference arises from the rather higher shear strengths which the Author has used.

The average value of  $\alpha = 0.58$  at Kensal Green is the highest recorded in the present survey. The workmanship was good and the boreholes appeared to be dry before concreting. The site conditions were perhaps unusually favourable, but it will be noted that values of  $\alpha$ averaging 0.55 have been obtained at Finsbury Park and Galway Street, with individual piles at each site attaining  $\alpha = 0.59$ .

#### Wimbledon Park (1951)

In connexion with foundation work for the Wimbledon Borough Council the Pressure Piling Co. made and tested a 17-in.-dia. pile 43 ft deep in the London Clay, and the Author is grateful to Mr V. J. Weeks of this firm for supplying the data. Although the test was not taken quite far enough for the ultimate load to be clearly defined, it could be estimated with reasonable accuracy. A number of samples were tested by LeGrand, Sutcliff, and Gell, and from the results the points in Fig. 2 have been plotted. There is a considerable scatter, but the strengthdepth relationship evidently differs little from the basic line. This has therefore been used in the analysis and, as will be seen from Table 5, the adhesion is 960 lb/sq. ft with  $\alpha = 0.34$ .

#### Wembley (1956)

A test pile at this site was made and tested by Soil Mechanics Ltd, who also carried out the site investigation. The Author is indebted to Mr R. P. Milner of the Ministry of Works for the results which appear in Fig. 2 and Table 5. The ultimate load was closely approached in the test, but an estimate has to be made of the shaft adhesion in the 12 ft of fill over the London Clay. The Author has assumed this to be 400 lb/sq. ft, and the load transferred to the fill is then 8 tons. The ultimate load of the pile was 60 tons. Thus the load taken by the clay is 52 tons and the corresponding values of  $c_a$  and  $\alpha$  are 940 lb/sq. ft and 0.52. Owing to the presence of the fill these figures are approximate, but it may be noted that a variation of  $\pm$  50% in the adhesion of the fill would cause a change of only  $\pm$  10% in the calculated adhesion in the clay. Thus it is unlikely that  $\alpha$  lies outside the limits 0.47 and 0.57.

#### Finsbury Park (1957)

Six piles were made and tested by the Cementation Company in connexion with the foundations for a building for the MacFisheries Company. The site exploration was carried out by Ground Explorations Ltd and the data have very kindly been given by the consulting engineers, Clarke, Nicholls, and Marcel. Four of the test piles penetrated a considerable thickness of fill before entering the London Clay, but at two of the piles there was only 3 ft of fill. The adhesion between the fill and the pile has again been taken as 400 lb/sq. ft, but in this case the load transferred to the fill is only about 3% of the measured ultimate; so the possible error due to the assumed adhesion in the fill is negligible. The two piles were identical in length and diameter, and the values of  $\alpha$  are 0.51 and 0.59.

# Brandon Estate (1957)

At the site of a group of 18-storey flats in Southwark, built by the London County Council, several piles were made and tested by the Pressure Piling Co. The site investigation was carried out partly by the L.C.C. and partly by George Wimpey & Co. The late Mr Felix Samuely was consulting engineer, and Dr Henkel and the Author worked with him in an advisory capacity.

The London Clay was covered by 12 ft of fill and gravel, but five of the piles were isolated from these materials by a sleeve, so that the whole load was transferred to the clay. The piles were carefully constructed but, apart from pile 2(b) which penetrated 8 ft into the clay, the average value of  $\alpha$  was only 0.34, and in one case  $\alpha = 0.29$ , as will be seen from the details given in Table 5. A possible explanation of these low results is that the clay contained lenses of silty material which might be unusually sensitive to changes in water content, and which might also allow more ground-water than usual to flow into the borehole during the drilling operations.

It is interesting that a further pile, 2(a), was tested under identical conditions to pile 2(b) except that the gravel and fill were not sleeved off. Pile 2(a) failed at 80 tons, compared with 25 tons in pile 2(b). The gravel was 8 ft thick, overlain by 4 ft of fill; and it will be seen that these materials contributed 55 tons, which corresponds to a shaft friction of 2,300 lb/sq. ft. Since the fill was probably responsible for little of this extra load, the influence of the gravel is very great, and obviously any pile test in which the gravel is not excluded is worthless from the point of view of assessing the bearing capacity of the clay. In another test at Brandon, pile 4(a), 7 ft of gravel was left in position. Otherwise this pile was identical with piles 3 and 4(c), the ultimate loads of which were 80 and 82 tons. The ultimate of pile 4(a) was not defined, but it must have been at least 100 tons. Thus, in this case, 7 ft of gravel contributed at least 20 tons with a corresponding shaft friction of more than 1,500 lb/sq. ft. This pile was later excavated and the shaft was found to be fairly smooth. It is possible, however, that the very high shaft friction deduced from pile 2(a) may have been caused by a "collar" on the pile, where the concrete had displaced the gravel and thereby increased the diameter.

#### Galway Street (1951)

At the site of a building for the Finsbury Borough Council, about  $\frac{1}{2}$  mile north of Aldersgate, the Pressure Piling Co. tested three piles. Mr Samuely was again the consulting engineer and George Wimpey Ltd made the site investigation. The London Clay is overlain by alluvial gravels and clay, and by fill, but in the pile tests these materials were virtually isolated by jacketing the upper part of the pile with soft mud. For the calculations the adhesion of the mud has been taken as 200 lb/sq. ft. The piles all penetrated about 23 ft into the London Clay; the average values of  $c_a$  and  $\alpha$  being 1,900 lb/sq. ft and 0.55. This is almost the highest result for  $\alpha$  at any of the sites, yet the construction technique was practically identical with that used at the Brandon Estate.

During the site investigation, tests were carried out on specimens cut from four samples, from depths between 3 and 20 ft in the clay, to determine the fully softened strengths under negligible pressure. The water content increased from 26.5 to 30 with a decrease in strength from 3,900 to 1,600 lb/sq. ft. The relative strength is therefore 0.41 (almost exactly equal to that deduced from Fig. 1 for this water content increase) and with an adhesion coefficient of 0.8, the corresponding value of  $\alpha$  is  $0.8 \times 0.41 = 0.33$ . This is appreciably less than the lowest value of  $\alpha$  recorded at Galway Street (0.48), although the lowest adhesion (1,660 lb/sq. ft) is practically the same as the fully softened strength: the apparent discrepancy being due to the fact that the average undisturbed strength of the four samples happens to be about 15% higher than the site average used in analysing the pile tests. On the other hand, this value of  $\alpha = 0.33$  is comparable with the lowest result at Brandon ( $\alpha = 0.29$ ) from which it seems that full softening had occurred at this pile.

#### Millbank (1959)

An important series of loading tests, and the related site investigation, has recently been completed by Soil Mechanics Ltd at a new 32-storey building on Millbank. The Author is indebted to Mr R. Glossop of John Mowlem & Co., the main contractors (to whom Dr Bishop and the Author acted as advisers) and to Mr G. W. Kirkland of Travers, Morgan, & Partners, for permission to quote the results. Three plain shaft piles were loaded to the ultimate, each with a length of about 65 ft in the London Clay, and with diameters of  $16\frac{1}{2}$ , 24, and 36 in. The overlying fill and alluvium were sleeved off, and the entire test load was therefore transferred to the clay. The average values of  $c_a$  and  $\alpha$  were found to be 1,960 lb/sq. ft and 0.39.

These piles at Millbank are considerably deeper in the London Clay than any others so far tested, and therefore contribute exceptionally valuable data to the problem of the bearing capacity of cast in-situ piles. The drilling operations were carried out most efficiently with a rotary earth drill and every care was taken in construction. Moreover, the site conditions appeared perfectly normal. Nevertheless the values of  $\alpha$  ranged from 0.36 to 0.45.

At first glance these figures appear incompatible with the foregoing statements. But the value of  $\bar{c}$  for these piles is nearly 5,000 lb/sq. ft and there may be an upper limit to  $c_a$  irrespective of the shear strength of the clay. For driven piles a limit of  $c_a = 2,200$  lb/sq. ft has been suggested (Meyerhof, 1951) and if this applies to cast in-situ piles then  $\alpha$  cannot exceed about 0.45 for depths such as those at Millbank. In addition, the clay around the deeper parts of the shaft has been subjected to a release of total pressure amounting to 4 or 5 tons/sq. ft when the borehole was made. Consequently there will be a very marked tendency for swelling and softening to occur; and in a deep boring some small inflow of water is almost inevitable apart, of course, from the water which can be absorbed from the concrete. It is therefore possible that  $\alpha = 0.45$  is the highest value that could be expected for these very deep piles. Even so, the 36-in.-dia. piles, which are being used for this building, provide a safe and economical foundation unit with an ultimate bearing capacity of over 600 tons.

#### SUMMARY OF RESULTS

Twenty-five test results have been plotted in Figs 5–7, representing piles ranging from those 8 ft deep and 10 in. dia. at Hendon, to the 90-ft-deep piles 36 in. dia. at Millbank. From Fig. 5 will be seen unmistakable evidence (i) that the adhesion increases with increasing shear strength, and (ii) that  $\alpha$  lies between the limits 0·3 and 0·6. Exceptional cases will be found where  $\alpha$  lies outside this range, but it is improbable that  $\alpha$  can be less than about 0·25 or more than about 0·7. These figures are based on the following physical reasoning.

The lowest possible strength of the clay is that measured after it has been allowed to soften fully under zero load. If this strength is  $c_0$  then the minimum relative strength is  $c_0/\bar{c}$ , and with an adhesion coefficient of 0.8 the corresponding minimum value of  $\alpha$  is  $0.8 \cdot c_0/c$ . The ratio  $c_0/\bar{c}$  has been determined at four sites. The results are given in Table 7 and  $\alpha = 0.25$  is a rough approximation in all cases. And from Table 1 this is seen to be equivalent to an increase of 5% in water content. Moreover, as mentioned earlier, it is probable that in three cases full

Site	Softening tests							
	co lb/sq. ft	$\frac{c_o}{\bar{c}}$	α					
Hendon Barnet Southall Galway St	360 500 550 1,600	0·29 0·35 0·25 0·41	0.23 0.28 0.20 0.33					
	Mean	0.32	0.26					

softening had occurred. Data for these piles, which give the three lowest values of  $\alpha$  in Table 5, are summarized in Table 8; and  $\alpha = 0.25$  is again the representative result.

Table 8Minimum field values of  $\alpha$ Site and<br/>pile No.Pile dimensions<br/>Dia.: in. $c_a$ <br/>lb/sq. ft $\alpha$ Hendon, --<br/>Southell S414123100.24

 Southall, S4
 12
 40
 550
 0.21

 Brandon, 5
 17
 50
 920
 0.29

 Mean
 0.25
 0.25

It should be explained, however, that the Hendon piles are situated in the London Clay near the surface where it is weathered and heavily fissured, and the Southall pile was hand augered. They cannot therefore be considered as characteristic of normal construction for heavy foundations, and  $\alpha = 0.3$  is probably the best lower limit for practical purposes.

Regarding the upper limit,  $\alpha = 0.7$  corresponds to an increase in water content of only about 0.5% (see Table 1) and since some absorption of water by the clay is inevitable, this would seem to be the highest value of  $\alpha$  that could be expected<sup>\*</sup>. The available data nevertheless suggests that it is difficult to achieve more than  $\alpha = 0.6$  (equivalent to an increase of 1% in water content) and, as discussed in the notes on Millbank,  $\alpha = 0.45$  may well be the maximum for very deep piles. From the considerable number of cases where  $\alpha$  lies between

\* The Author has information on one pile test, however, which shows a very high value of  $\alpha$ . Unfortunately the associated shear tests are not reliable; but by making reasonable assumptions it appears that  $\alpha$  cannot be less than about 0.9. This result is so entirely different from any other that the Author suspected a collar had been formed on the pile, perhaps where some claystones fell out of the sides of the boring. A second pile at the same site gave  $\alpha = 0.5$  with identical assumptions concerning shear strength.

about 0.5 and 0.6 (for piles less than 50 ft deep) it is probable, however, that this range corresponds to a combination of favourable geological conditions and careful workmanship.

Since the shear strength increases with depth so also will the adhesion and in Fig. 6, to demonstrate this point,  $c_{\alpha}$  has been plotted against depth. Approximate limits are shown, but these cannot be clearly defined, owing to the considerable variation in strength at any particular depth. For the same reason the relation between  $\alpha$  and depth (Fig. 7) is little more than illustrative.

# CALCULATION OF ULTIMATE LOAD

Having presented what is believed to be a representative selection of field evidence, the Author now uses this data to derive, and check, a method for calculating the ultimate load of individual cast in-situ concrete piles in London Clay.

In Fig. 8 the mean values of  $c_a$  and  $\overline{c}$  are plotted for each of the ten sites, and the points scatter around a line corresponding to:

$$c_a = 0.45 \ \overline{c} \text{ or } \alpha = 0.45$$

with a maximum variation of  $\pm$  30%. But this variation is only in the shaft adhesion, and in calculating the ultimate load the end bearing capacity has to be added. Fortunately there is little doubt that the latter is given with sufficient accuracy by the expression:

$$Q_p = 9.c_p.A_p$$
  
Now the ultimate load is:

$$Q_u = Q_p + Q_s$$
$$Q_s = c_a \cdot A_s$$

where

and it seems worth while to examine whether the formula  $c_a = 0.45 \bar{c}$  leads to results which are of adequate reliability for practical purposes. It is prudent, however, to assume for the present that  $c_a$  cannot exceed 2,000 lb/sq. ft (see Fig. 8). In other words, the formula  $c_a = 0.45 \bar{c}$  should be used only when  $\bar{c}$  is less than 4,500 lb/sq. ft.



In Table 9 the ultimate loads for twenty-five piles are calculated by this method, and compared with the observed values. This comparison is also shown graphically in Fig. 9.

If the calculated load is taken as 100, then in all but three cases the observed load lies within the range  $\pm 22\%$ , and the only serious error is the Southall pile S3, where the observed load is 35% less than the calculated value. The standard deviation is  $\pm 17\%$ . These limits appear to be sufficiently close for the method to be useful in practice.

The working load as usually defined is typically a little less than one-half the ultimate. Thus, if a factor of safety of 2.5 is applied to the ultimate load as calculated, only in exceptional cases will the pile fail to pass the acceptance test.

With good site conditions and workmanship  $\alpha$  can lie between 0.5 and 0.6 (with the same restriction that  $c_a$  should not be taken as greater than 2,000 lb/sq. ft). Improved drilling techniques, which are already coming into use, may lead to a more widespread adoption of such values of  $\alpha$  but they should be applied in the design of piles at any particular site only after checking by at least two loading tests.

#### CONCLUSIONS

(1) The settlement at the ultimate load is roughly 1 in. per ft dia. of the pile. This corresponds to the full mobilization of end bearing capacity; the shaft adhesion being mobilized at smaller settlements.

Site	Pile	Dia.:	Effective		Calculated	l	Observed
		111.	ft	Qp	Q <b>s</b>	Qu	Qu
Southall	S1 S3 S7	12 12 14	$\begin{array}{r}20\\30\\40\end{array}$	9 11 18	21 43 78	30 54 96	29 35 70
Barnet	7 8	14 14	10 10	10 10	10 10	20 20	24 19
Hendon		$\begin{cases} 10\\ to\\ 14 \end{cases}$	8 10 12	5·7 6·4 7·1	5.5 7.6 10.0	11·2 14·0 17·1	$     \begin{array}{r}       10.5 \\       13.0 \\       13.3     \end{array} $
Kensal Green	B5 B33	18 24	34 23	27 40	83 64	110 104	130 125
Wimbledon		17	43	28	108	136	110
Wembley		15	24	13	34	47	52
Finsbury	2 3	19 19	27 27	24 24	56 56	80 80	87 97
Brandon	2(b) 3 4(c) 5 6	17 17 17 17 17	20 40 40 50 45	13 26 26 31 28	12 68 68 107 86	25 94 94 138 114	25 80 82 100 95
Galway St	5 7 12	17 17 17	39 42 46	29 28 30	68 65 77	97 93 107	102 110 130
Millbank	5 2 6	$16\frac{1}{2}$ 24 36	91 90 90	39 83 185	251 359 537	290 442 722	320 410 650

 Table 9

 Comparison between calculated and observed ultimate loads (tons)


(2) The end bearing capacity is given with sufficient accuracy by the expression:

$$Q_p = 9.c_p.A_p$$

(3) As a general rule, subject to the variations mentioned in conclusion (6), the shaft load in the clay can be calculated from the expression:

 $Q_s = c_a \cdot A_s$ 

where, for  $\bar{c} < 4,500$  lb/sq. ft

and for greater values of  $\bar{c}$ ,

 $c_a = 2,000 \text{ lb/sq. ft.}$ 

 $c_a = 0.45 \,\overline{c}$ 

(4) The ultimate load contributed by the clay is:

$$Q_u = Q_p + Q_{s}.$$

(5) For the working load a factor of safety of 2.5 is advisable. But settlement considerations and the group effect must be taken into account.

#### CAST IN-SITU BORED PILES IN LONDON CLAY

(6) With short bored piles for house foundations, where close control of construction may not be practicable and where the clay is likely to be heavily fissured, the shaft adhesion may be about  $0.3\bar{c}$ . This corresponds approximately to the fully softened strength, which can be used as an alternative basis of design; and adhesion should be neglected in the zone of seasonal shrinkage. With deeper piles for heavy foundations the shaft adhesion may attain values in the region  $0.6\overline{c}$  under favourable conditions; but an adhesion of this magnitude should be adopted only after checking by pile loading tests and, in any case, it is unwise to use values exceeding 2,000 lb/sq. ft until more data are available.

(7) Care should be taken to avoid using water in drilling, and the construction of any particular pile must be carried out with the minimum possible delay.

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#### REFERENCES

GIBSON, R. E., 1950. Discussion on Paper, "The bearing capacity of screw piles and screwcrete cylinders." J. Instn civ. Engrs, 34:382.

GOLDER, H. Q., and M. W. LEONARD, 1954. "Some tests on bored piles in London Clay." Géotechnique, 4:32-41.

MEVERHOF, G. G., 1951. "The ultimate bearing capacity of foundations." *Géotechnique*, 2:301-332. MEVERHOF, G. G., and L. J. MURDOCK, 1953. "An investigation of the bearing capacity of some bored

MEYERHOF, G. G., 1951. "The ultimate bearing capacity of foundations." Geotechnique, 2:301-332.
MEYERHOF, G. G., and L. J. MURDOCK, 1953. "An investigation of the bearing capacity of some bored and driven piles in London Clay." Géotechnique, 3: 267-282.
RODIN, S., and M. J. Tomlinson, 1953. "Recherches sur le frottement lateral des pieux forés et battus dans l'argile" ("Investigations on the skin friction of bored piles and driven piles in clay soils"). Ann. Inst. Technique Batim., 6: 343-351.
SKEMPTON, A. W., 1951. "The bearing capacity of clays." Build. Res. Cong., 1:180-189.
SKEMPTON, A. W., and D. J. HENKEL, 1957. "Tests on London Clay from deep borings at Paddington, Victoria, and the South Bank." Proc. 4th Int. Conf. Soil Mech., 1:100-106.
TOMLINSON, M. J., 1957. "The adhesion of piles driven in clay soils." Proc. 4th Int. Conf. Soil Mech., 2:66-71.

2:66-71.

WARD, W. H., and H. GREEN, 1953. "House foundations: the short-bored pile." Publ. Wks. Cong. and Exhib., London, 1952, 373-388.
 WARD, W. H., S. G. SAMUELS, and M. E. BUTLER, 1959. "Further studies of the properties of London Clay." Géotechnique, 9:33-58.

Géotechnique, 9:33–58. T., 1957. "Experiments with model piles in groups." Géotechnique, 7:147–167.

WHITAKER, T., 1957.

# Effective Stress in Soils, Concrete and Rocks

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#### Introduction

The pore space in soils generally contains both air and water, the pressures in which may be denoted by  $u_a$  and  $u_w$ . If the 'total' stress acting in a given direction at any point in the soil is  $\sigma$ , then it is a fundamental problem in soil mechanics to determine in what manner the 'effective' stress is related to the three known stresses  $\sigma$ ,  $u_a$  and  $u_w$ ; where the effective stress, denoted by  $\sigma'$ , is, by definition, the stress controlling changes in volume or strength of the soil.

For saturated soils in which the voids are filled with water, TERZAGHI (1923) showed experimentally that

$$\sigma' = \sigma - u_w \qquad \dots (1)$$

and subsequent work has confirmed this expression with a sufficiently high degree of accuracy for engineering purposes.

For partially saturated soils, however, tests carried out chiefly during the past decade have shown that this equation can be appreciably in error. And in the present paper experimental evidence will be given supporting a more general expression for effective stress, first suggested by BISHOP in 1955,

$$\sigma' = \sigma - [u_a - \chi(u_a - u_w)]$$

where  $\chi$  is a parameter related to the degree of saturation and equalling unity when the soil is fully saturated. In this latter case Bishop's equation then becomes identical with Terzaghi's.

These equations can be used in all, or certainly in most, practical soil mechanics problems. But it is of philosophical interest to examine the fundamental principles of effective stress, since it would seem improbable that an expression of the form  $\sigma' = \sigma - u_w$  is strictly true. And, indeed, when one turns to other porous materials such as concrete or limestone, this equation is not always adequate. We may therefore anticipate that, even for fully saturated porous materials, the general expression for effective stress is more complex, and that Terzaghi's equation has the status of an excellent approximation in the special case of saturated soils.

In fact this view has long been held; the common opinion being that the effective stress is actually the intergranular stress acting between the particles comprising the porous material. It can readily be shown that this stress is

$$\sigma_g = \sigma - (1 - a)u_w$$

where a is the area of contact between the particles, per unit gross area of the material. And hence, on this hypothesis,

$$\sigma' = \sigma - (1 - a)u_w$$

Now there can be little doubt that a is very small in soils, and this expression is therefore virtually identical with Terzaghi's equation. Hence the intergranular concept is superficially attractive. But recent high-pressure consolidation tests on lead shot have proved that the stress controlling volume change is by no means equal to  $\sigma_g$ ; whilst triaxial compression tests on Marble indicate that the effective stress controlling shear strength changes in rocks is also not equal to  $\sigma_g$ . Thus it is desirable to examine the physics of effective stress more closely in the hope of obtaining a theory which is reasonably consistent with all the available experimental data. Such a theory would have four advantages:

(a) it would provide a satisfactory explanation for the validity of Terzaghi's equation; although this advantage is admittedly of no more than academic interest in most soil problems;

(b) it would be of practical benefit in certain cases, such as the consolidation of deep beds of geological sediments, where the pressures are sufficiently great to establish appreciable contact areas between the particles;

(c) it would help to resolve the difficulties of interpreting triaxial and other tests on concrete which have been made with the express purpose of determining the contact area in this material;

(d) it would be relevant in various engineering and geophysical problems in rock mechanics.

#### PART I

#### Shear Strength of Saturated Materials

When porous materials are tested under the condition of zero pore pressure their shear strength  $\tau_d$  is found to increase with increasing applied pressure  $\sigma'$  normal to the shear surface; and, within the range of stresses used in practice, the relationship between  $\tau_d$  and  $\sigma'$  may be expressed by the Coulomb equation

$$\tau_d = c' + \sigma' \tan \phi'$$

where c' and  $\phi'$  are the apparent cohesion and the angle of shearing resistance. The problem now under consideration is to obtain an expression for  $\sigma'$  when the material is subjected to a pore-water pressure  $u_w$  as well as an applied pressure  $\sigma$ .

Three solutions to this problem will be given, but it is first necessary to set out the equations of equilibrium between the external forces and the stresses acting at the contact between any two particles or grains of the porous material.

Throughout the first two Parts of this paper the pore pressure will for simplicity be denoted by u, and this may be regarded as the pressure in any single-phase pore fluid.

Equilibrium Equations—With reference to Fig. 1 consider two particles in solid contact on a statistical plane of area  $A_s$ and occupying a gross area A in a plane parallel to the contact. Then the contact area ratio a is defined by the expression

$$a=\frac{A_s}{A}$$

If the total force normal to the contact plane is P and if the shear force is T, then the normal total stress  $\sigma$  and the shear stress  $\tau$ are

$$\sigma = \frac{P}{A}$$
$$\tau = \frac{T}{A}$$

and the stresses at the interfacial contact are

$$\sigma_s = \frac{P_s}{A_s} \, \tau_s = \frac{T_s}{A_s}$$

where  $P_s$  and  $T_s$  denote the normal and the shear forces acting between the particles. In addition there is a pressure u in the pore water.



 $P = P_s + (A - A_s)u$ 

 $\sigma = a \cdot \sigma_s + (1 - a)u$ 

Hence and

$$a = \frac{\sigma - u}{\sigma_s - u}$$

For equilibrium parallel to the plane

Hence

$$\tau = a.\tau_s$$

 $T = T_s$ 

In the above equation  $\tau$  is the shear stress per unit gross area occupied by the particles. But in an assemblage of particles the shear strength  $\tau_f$  per unit gross area of the material as a whole will be greater than the limiting value of  $\tau$ . If, for example, the coefficient of friction at the interfacial content is  $\mu$ , then CAQUOT (1934) has shown that in a non-cohesive granular material, sheared at constant volume,

$$\tan \phi' = \frac{\pi}{2} \cdot \mu$$

and BISHOP (1954) has derived rather similar expressions for plane strain and triaxial stress conditions. Thus we may write

$$\tau_f = m.\tau = a.m.\tau_s$$

where m is a factor greater than unity.

Theory I-The shear strength at the contact is assumed to be

$$\tau_s = c_s + \sigma_s \tan \phi_s$$

Thus, when this strength is fully mobilized, and failure takes place,

$$\tau = a.c_s + a.\sigma_s.\tan\phi_s$$

But

$$a \cdot \sigma_s = \sigma - (1 - a)u$$
  
$$\tau = a \cdot c_s + [\sigma - (1 - a)u] \tan \phi_s$$

or

$$\tau_f = a.m.c_s + [\sigma - (1 - a)u]m.\tan\phi_s$$

Now, when  $\sigma = u = 0$ ,  $\tau_f = c'$  $\therefore c' =$ 

$$c' = a.m.c_s$$

and in 'drained' tests (on soils) or 'jacketed' tests (on concrete

and rocks), when u = 0,

$$\tau_d = c' + \sigma . \tan \phi'.$$
  
$$\therefore \quad \tan \phi' = m . \tan \phi_s = m . \mu$$

Hence,

and we see that the effective stress  $\sigma'$  is given by the expression  $\sigma' = \sigma = (1 - \sigma)u$ 

 $\tau_f = c' + [\sigma - (1 - a)u] \tan \phi'$ 

$$\sigma' = \sigma - (1 - a)u$$

It is to be noted that the so-called 'intergranular stress'  $\sigma_g$  is defined as  $\sigma_g = \frac{P_s}{4}$ 

$$\sigma_g = \frac{P - (A - A_s)\nu}{A} = \sigma - (1 - a)u$$

Therefore Theory I leads to the conclusion that the effective stress is equal to the intergranular stress.

Theory II—In 1925 Terzaghi included in Erdbaumechanik a highly original treatment of the physical nature of friction. If two surfaces are lightly touching each other then, no matter how flat they may appear to be, the actual area of molecular contact at the interface is extremely small. If a load W is now applied normal to the surfaces the microscopic irregularities will yield and the total area of contact will attain a value  $A_s$ such that

$$W = A_s . \sigma_y$$

where  $\sigma_y$  is the yield stress of the material in these asperities. Terzaghi then makes the implicit assumption that the solid material is purely cohesive, with a shear strength k. The maximum tangential force which can be applied to the surfaces is therefore

$$F = A_s.k$$

But the coefficient of friction  $\mu$  is defined by the ratio  $\mu = F/W$ . Hence

$$\mu = \frac{\kappa}{\sigma_y}$$

Now for many substances  $\mu$  is roughly 0.5. Hence  $\sigma_y$  is of the order 2k. And this is the compression strength; which is not an unreasonable value for the yield stress of the asperities.

It will thus be seen that Terzaghi presented a concept of friction based on the hypothesis of a purely cohesive material. For many years the significance of the relevant page in *Erdbaumechanik* was overlooked. But, quite independently, the above equation was put forward by BOWDEN and TABOR in 1942 and broadly substantiated in a brilliant series of experiments, chiefly on metallic friction<sup>\*</sup>.

Two modifications in this simple theory must be made, however, if a more accurate model is to be established (TABOR, 1959). Firstly, the shear strength at the interfacial contact is generally less than that of the body of the solid material. Thus we should write

$$\tau_s = \beta . k$$

where  $\beta$  is less than unity. Secondly, the yield stress  $\sigma_{r}$  is not an independent strength property but will decrease as the shear stress increases, probably in accordance with an equation of the type

$$\sigma_{\nu}^2 + \alpha^2 \tau_s^2 = \alpha^2 k^2$$

Hence when  $\tau_s = 0$ ,  $\sigma_y = \alpha . k$ . But when failure takes place  $\tau_s = \beta . k$  and

$$\sigma_y = \alpha \sqrt{1 - \beta^2} \cdot k = N \cdot k$$

where N is a parameter depending on the geometry of the asperities and on the stress-strain characteristics. If for example  $\beta = \frac{2}{3}$ , then  $\sigma_y = \frac{3}{4}\alpha k$  and the contact area will increase

\* The greater part of this work is described in BOWDEN and TABOR (1954).

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by 30 per cent during shear. This is the phenomenon of an 'junction growth'.

Now it will be shown in the next section of this paper that the shear strength of solid substances can be expressed by the equation

$$\tau_i = k + \sigma . \tan \psi$$

where k is the intrinsic cohesion and  $\psi$  as the angle of intrinsic friction of the solid<sup>\*</sup>. For most metals  $\psi$  is not more than 5° (or tan  $\psi = 0.1$ ) whilst for minerals  $\psi$  appears to be typically in the range 3° to 10°. The physical reason for  $\psi$  being greater than zero is possibly associated in part with the closing up of internal flaws, under increasing pressure. And it may perhaps be considered that an ideal or 'perfect' solid is one in which  $\psi = 0$ .

In fact, the softer metals approximate to this case; but the minerals, with which we are concerned, depart appreciably from the perfect solid. Nevertheless it will be a matter of considerable theoretical interest to derive the equations for effective stress in porous materials comprised of ideal solid particles. The basic assumptions will therefore be

$$\tau_i = k \quad \psi = 0$$
$$\tau_s = \beta . k$$

If under zero external pressure and zero pore pressure the contact area is  $A_0$  then the corresponding shear strength of the porous material will be

$$\tau_0 = c' = a_0.m.\beta.k$$

where  $a_0 = A_0/A$ . Under an external pressure  $\sigma$  and pore pressure u the contact area ratio will change, and if its value is a then

$$\tau_f = a.m.\beta.k$$
  
$$\tau_f = c' + (a - a_0)m.\beta.k$$

Now with zero pore pressure the yield stress at the contact will be  $\sigma_y = Nk$ . But a pore-water pressure will support the sides of the particles and the asperities, and just as in a triaxial test on a purely cohesive material,

$$\sigma_1 = 2c + \sigma_2$$

so in the present case

$$\sigma_y = N.k + u$$

The initial contact area  $A_0$  may be associated with an internal force  $P_0$  such that

$$\frac{P_0}{A_0} = N.k$$

And if the corresponding pressure per unit gross area is  $p_0$  then

$$p_0 = \frac{P_0}{A} = \frac{A_0}{A} \cdot \frac{P_0}{A_0} = a_0 \cdot N \cdot k$$

The addition of an external pressure  $\sigma$  and a pore-water pressure u will increase the normal load at the contact by  $P_s$  and thus

$$\frac{P_0 + P_s}{A_s} = N.k + u$$
  
$$\therefore p_0 + a.\sigma_s = a(N.k + u)$$

But

$$a.\sigma_s - a.u = \sigma - u$$
  
 $\therefore p_0 + \sigma - u = a.N.k$ 

or

Hence

$$a = \frac{p_0 + \sigma - u}{N.k}$$

$$a-a_0=\frac{\sigma-u}{N.k}$$

\* The angle of intrinsic friction is a formal concept, merely expressing in a convenient manner the increase in strength with pressure. It might more properly be defined as the 'angle of intrinsic shearing resistance'. and therefore

or

Thus

And, finally,

$$r_f = c' + (\sigma - u) \frac{m \cdot \beta}{N}$$

 $\tau_f = c' + \frac{\sigma - u}{N.k} \cdot m.\beta.k$ 

In drained (or jacketed) tests where u = 0

÷.

 $\tau_{i}$ 

$$a = c' + \sigma \cdot \frac{m \cdot \beta}{N}$$

$$\tan \phi' = \frac{m.\beta}{N} = m.\mu$$
$$\tau_f = c' + (\sigma - u) \tan \phi$$

$$\sigma'=\sigma-u$$

We therefore find that if the particles have the property of a perfect cohesive solid ( $\psi = 0$ ), the effective stress in shear strength problems is given rigorously by Terzaghi's equation. The formal demonstration of this fact is here given for the first time, but the result may be considered as intuitively evident and has been previously stated by BISHOP (1955) and CHUGAEV (1958)\*.

Theory III—Shear tests on many solids, under very high pressures, have been made by BRIDGMAN (1935, 1936). For metals, the tensile strength of the pure polycrystalline material is also available (SETTZ and READ 1941, Landolt-Bornstein tables etc.). Combining these data for aluminium and plotting the results in the form of Mohr circles (Fig. 2(a)) it is seen that the strength increases linearly with pressure, according to the equation

$$\tau_i = k + \sigma \tan \psi$$

This type of relationship is found to hold good for many metals, at least as an approximation, although there are some in which a polymorphic transition under high stresses causes a change in  $\psi$ . We are not concerned with details, however, and the values of k and  $\psi$  given in Table 1 can be taken as representative.

Table 1 Intrinsic Shear Parameters

Solid	k kg/cm²	ψ			
Lead	100	<sup>1</sup> / <sub>1</sub>			
Zinc	600	1 <sup>1</sup> / <sub>4</sub> °			
Aluminium	500	3°			
Copper	1200	4 <u>1</u> / <sub>2</sub> °			
Nickel	1800	7 <u>1</u> °			
Rock Salt	450	3 <del>1</del> °			
Calcite	1900	8°			
Quartz	9500	13 <del>1</del> °			

Strength data for minerals are far less abundant, and I have been able to determine k and  $\psi$  only for Calcite, Rock Salt and Quartz. In triaxial tests on Marble, which is a porous material consisting of practically pure Calcite, VON KARMAN (1911) found that under a cell pressure of 2500 kg/cm<sup>2</sup>, when the deviatoric compression strength (corrected for area increase) was about 5000 kg/cm<sup>2</sup>, the macroscopic voids were virtually eliminated by plastic flow. The corresponding stress circle, applicable essentially to solid Calcite, is shown in Fig. 2(b). Two sets of high-pressure tests have been made on Calcite by BRIDGMAN (1936, and in GRIGGS 1942), and the mean results are also

\* And probably in *Trans. Res. Inst. Hydrotechnics* (U.S.S.R.), 1947, but I have not been able to check this reference.



Fig. 2 Intrinsic strength data

or

plotted in this figure. Up to about 30,000 kg/cm<sup>2</sup> the strength increases linearly with pressure and  $\psi = 8^{\circ}$ , but under greater pressures the slope of the failure envelope (or 'intrinsic line') decreases. From compression tests on discs of Rock Salt by KING and TABOR (1954) a stress circle at comparatively low pressure can be obtained which, combined with BRIDGMAN's tests (1936), shows that  $\psi = 3\frac{1}{2}^{\circ}$  for pressures up to about 20,000 kg/cm<sup>2</sup>. For Quartz crystals, triaxial tests have been carried out by GRIGGS and BELL (1938) and by BRIDGMAN (1941)\*. The results are shown in Fig. 2(c) from which it will be seen that  $\psi = 13\frac{1}{4}^{\circ}$ .

It is therefore evident, even from these fragmentary data, that  $\psi$  is likely to differ significantly from zero for soils, concrete and rocks. Hence for these materials we should assume that the intrinsic strength of the particles is given by the equation

$$r_i = k + \sigma \tan \psi$$

The shear strength at the contact will then be

$$\tau_s = \beta . k + \sigma_s . \tan \psi_s$$

where  $\psi_s$  is the angle of friction at the interface which may, in general, be rather less than  $\psi$ .

Under zero external pressure and zero pore-water pressure the contact area ratio is  $a_0$  and

$$\tau_0 = c' = a_0.m.\beta.k$$

When the external pressure is increased, and a pore-water pressure established, the contact area will change, and if the normal stress at the interface is  $\sigma_s$  then

But 
$$\tau_f = a.m.\beta.k + a.\sigma_s.m.\tan\psi_s$$
$$a\sigma_s = \sigma - u + au$$

$$\therefore \tau_f = c' + (a - a_0)m\beta k + (\sigma - u)m \tan \psi_s + aum \tan \psi_s$$

In order to obtain an expression for the change in contact area the yield stress of the solid material must be known. If the asperities have the form of a cylinder and if junction growth

\* At cell pressures of 13,000 and 19,500 kg/cm<sup>2</sup> the tests by Griggs and Bell show a very marked increase in strength, which was not found by Bridgman, using an improved technique, even with cell pressures of 25,000 kg/cm<sup>2</sup>. These anomalous results were probably due to experimental difficulties, and they have been omitted from Fig. 2(c). Possibly  $\psi$  would be less, for Calcite as well as Quartz, if a gas was used as the cell fluid in triaxial tests.

during shear is negligible, then

$$\sigma_y = k \cdot \frac{2 \cos \psi}{1 - \sin \psi} + u \cdot \frac{2 \sin \psi}{1 - \sin \psi} + u$$
$$\sigma_y = k \cdot \frac{2 \cos \psi}{1 - \sin \psi} \left( 1 + \frac{u}{k} \cdot \tan \psi \right) + u$$

More generally, with any shape of asperities and with some junction growth, we may write

$$\sigma_y = k \cdot M \left( 1 + \frac{u}{k} \cdot \tan \psi \right) + u$$

where *M* is a coefficient similar to *N* in Theory II and numerically equal to *N* if  $\psi = 0$ . But as an approximation *u*.tan  $\psi/k$ may be neglected in comparison with unity. If, for example,  $k = 1000 \text{ kg/cm}^2$  and  $\psi = 10^\circ$ , then even if  $u = 100 \text{ kg/cm}^2$ (which is equivalent to a head of 3000 ft of water), this term is equal to 0.018. Hence it is sufficiently accurate for most problems to assume that

$$\sigma_y = M.k + u$$

Thus, by direct analogy with the relevant passage in Theory II,

$$a-a_0=\frac{\sigma-a}{M.k}$$

Hence  

$$\tau_f = c' + (\sigma - u) \left(\frac{m.\beta}{M} + m.\tan\psi_s\right) + a.u.m.\tan\psi_s$$

In drained (or jacketed) tests u = 0

$$\therefore \tau_d = c' + \sigma \left(\frac{m.\beta}{M} + m.\tan\psi_s\right)$$

Thus

or

$$\tan \phi' = \frac{m.\beta}{M} + m.\tan \psi_s = m.\mu$$

$$\therefore \tau_f = c' + (\sigma - u) \tan \phi' + a.u.m.\tan \psi_s$$

But since m is greater than unity  $(m = \pi/2 \text{ in Caquot's analysis})$ and  $\psi_s$  is less than  $\psi$  it is unlikely that any serious error will arise in writing m.tan  $\psi_s = \tan \psi$ . In that case

$$\sigma_f = c' + (\sigma - u) \tan \phi' + a \cdot u \cdot \tan \psi$$

$$\tau_f = c' + \left[\sigma - \left(1 - \frac{a \cdot \tan \psi}{\tan \phi'}\right)u\right] \tan \phi'$$

#### Hence

$$\sigma' = \sigma - \left(1 - \frac{a \cdot \tan \psi}{\tan \phi'}\right)u$$

The above equation is a general expression for effective stress in relation to the strength of saturated porous materials, and it includes the two previous theories as special cases.

Strength of porous materials—Before examining the foregoing theories in the light of experimental evidence, it is necessary to consider an aspect of the strength properties of porous material which does not appear hitherto to have been recognized. It is well known that the failure envelope for concrete and rocks, and to some extent for sands, becomes progressively flatter with increasing pressure. This is clearly shown in Fig. 3 where the whole series of triaxial (jacketed) tests on Marble by von Karman (*loc. cit.*) have been plotted. But it also becomes

that when 
$$\sigma =$$

$$c' = a_0.m.\beta.k$$

0

and since  $m.\beta$  is of the order unity, the initial area ratio is approximately

$$a_0 = \frac{c}{k}$$

Further, if  $m \tan \psi_s$  is assumed to equal  $\tan \psi$  then under a pressure  $\sigma$  the drained strength is approximately

or  
$$\tau_d = c' + (a - a_0)k + \sigma \tan \psi$$
$$\tau_d = a \cdot k + \sigma \cdot \tan \psi$$

And, for all positive values of  $\sigma$ 

 $\tau_i = k + \sigma . \tan \psi$ Hence

$$a=1-\frac{\tau_i-\tau_d}{k}$$



Fig. 3 Triaxial tests on Marble (after von KARMAN, 1911)

clear that the failure envelope is tending towards the intrinsic line of the solid substance comprising the particles of the porous material and, finally, at a pressure sufficiently high to cause complete yield of the particles, when the voids are eliminated, the failure envelope becomes coincidental with the intrinsic line, as determined from Bridgman's high-pressure tests.

The shape of the failure envelope is therefore controlled, to an important extent, by changes in the contact area under pressure. When the porosity is comparatively high a given pressure increment  $\Delta\sigma$  causes a correspondingly high increase in contact area  $\Delta a$ ; but as the porosity is progressively reduced  $\Delta a/\Delta\sigma$  also becomes smaller and eventually, at a pressure  $\sigma^*$ when the porosity is zero and a = 1,  $\Delta a/\Delta\sigma$  becomes zero and the slope of the failure envelope falls to the value  $\psi$ . This behaviour is illustrated diagrammatically in Fig. 4 where  $\tau_i$  is the intrinsic line for the solid, and  $\tau_d$  is the failure envelope from drained (jacketed) tests on the porous material. When  $\sigma$ is small compared with  $\sigma^*$  the failure envelope may be considered as linear, and Coulomb's equation is applicable. Thus  $\tau_d = c' + \sigma \tan \phi'$  when  $\sigma/\sigma^*$  is small. But we have shown For solids a = 1 throughout the entire pressure range whilst, at the other extreme, for cohesionless porous materials such as sand and lead shot,  $a_0 = 0$  (see Fig. 4). The variation of a from  $a_0 = 0.18$  to  $a^* = 1$  for von Karman's tests on Marble, is shown in Fig. 3. At a pressure equal to that on the shear plane in the unconfined compression test on this material, a = 0.25.

It is probable that this concept of the strength of porous materials is capable of interesting development. But for our present purpose its importance lies in the fact that it provides a method for determining a in two calcitic limestones for which, from the results of unjacketed triaxial tests, the value of a can also be deduced from the theories of effective stress. And in this way at least a rough check can be obtained on the theories of effective stress.

Unjacketed tests on rocks and concrete—In these tests the specimen is not covered with a membrane and the cell fluid can penetrate fully into the voids of the material. Thus, with the usual notation,  $\sigma_3 = u$  and the procedure consists of measuring the deviatoric compression strength under various cell pressures.

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Now according to Theory I

$$\tau_f = c' + [\sigma - (1 - a)u] \tan \phi$$

This is the shear strength on a plane on which the normal applied stress is  $\sigma$ . In the triaxial test where, in general, the specimen is subjected to applied principal stresses  $\sigma_1$  and  $\sigma_3$  and there is a pore pressure u in the specimen,

and

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta$$
$$\sigma = \sigma_3 + \frac{1}{2}(\sigma_1 - \sigma_3)\cos^2\theta$$

where  $\theta$  is the inclination of the shear plane to the direction of  $\sigma_3$ .

For the condition that  $(\sigma_1 - \sigma_3)$  is a minimum  $\theta = 45^\circ + \phi'/2$ and it then follows that

$$(\sigma_1 - \sigma_3)_f = c' \cdot \frac{2\cos\phi'}{1 - \sin\phi'} + (\sigma_3 - u) \frac{2\sin\phi'}{1 - \sin\phi'} + a \cdot u \cdot \frac{2\sin\phi'}{1 - \sin\phi'}$$

Thus, in the unjacketed tests where  $\sigma_3 = u$ 

$$\frac{\Delta(\sigma_1-\sigma_3)_u}{\Delta u}=a\cdot\frac{2\sin\phi'}{1-\sin\phi'}$$

It is evident that according to Theory II there is no increase in strength in the unjacketed test, but in Theory III

$$\tau_f = c' + (\sigma - u) \tan \phi' + a.u.\tan \psi$$
  
As before  $\theta = 45^\circ + \phi'/2$  and hence

$$(\sigma_1 - \sigma_3)_f = c' \cdot \frac{2\cos\phi'}{1 - \sin\phi'} + (\sigma_3 - u) \frac{2\sin\phi'}{1 - \sin\phi'} + a \cdot u \cdot \frac{\tan\psi}{\tan\phi'} \cdot \frac{2\sin\phi'}{1 - \sin\phi'}$$

Thus in the unjacketed tests

$$\frac{\varDelta(\sigma_1-\sigma_3)_u}{\varDelta u}=a.\,\frac{\tan\psi}{\tan\phi'}\frac{2\sin\phi'}{1-\sin\phi'}$$

Now in all three theories the shear strength in drained or jacketed tests, in which the pore-water pressure is zero, will be

$$\tau_d = c' + \sigma \tan \phi'$$

Hence

$$\frac{\Delta(\sigma_1 - \sigma_3)_d}{\Delta\sigma_3} = \frac{2\sin\phi'}{1 - \sin\phi'}$$

Therefore, provided  $\phi'$  is known, the area ratio can be computed either from Theory I or Theory III, given the rate of increase in unjacketed strength with increasing cell pressure.

GRIGGS (1936) has carried out such tests on Marble and Solenhofen Limestone. At a cell pressure of 10,000 kg/cm<sup>2</sup> an extraordinary increase in strength was obtained in both materials, and to a lesser extent in the Solenhofen also at 8000 kg/cm<sup>2</sup>. These results may be due to experimental errors as in the case of Quartz, previously mentioned; but the tests at smaller pressures show a definite increase in strength with pore pressure and there seems to be no reason to doubt this general trend; which is evidently at variance with Theory II. From Fig. 5 it will be seen that approximate values for  $\Delta(\sigma_1 - \sigma_3)/\Delta u$ 



Fig. 5. Unjacketed triaxial tests on Marble and Solenhofen Limestone (GRIGGS, 1936)

are 0.08 and 0.14 for the Marble and Solenhofen respectively. Unfortunately Griggs does not give sufficient data to determine the failure envelope for drained or jacketed tests, but the stress circles for the unconfined compression tests are plotted in Fig. 6 together with the failure envelope for von Karman's tests. Since both of the materials in Griggs' tests consist of pure Calcite, the intrinsic line will presumably be the same, or very similar, to that previously used in analysing von Karman's tests. Thus it is not difficult to sketch the probable failure envelopes for the Marble and Solenhofen tested by Griggs. From Fig. 6 it will be seen that the corresponding values of  $\phi$ are 34° and 24°; and  $\psi$  is 8°. The area ratios can then be calculated, see Table 2, and are found to be 0.03 and 0.14 (Theory I), or 0.15 and 0.45 (Theory III). But from the Mohr's circles in Fig. 6 and from the intrinsic line it is possible to estimate that the area ratios at the normal stress  $\sigma$  acting on the shear plane in the unconfined compression tests are 0.16 and 0.52 for the Marble and Solenhofen respectively. These latter values of a are not precise, but they tend to disprove Theory I and broadly to substantiate Theory III.

We may now turn to the tests which have been made on concrete with the object of determining the area ratio, and concerning which there has been for many years some uncertainty and controversy.

Triaxial tension tests on cement by FILLUNGER (1915), using unjacketed specimens, show little change in strength with increasing pore pressure, but more elaborate tests by LELIAVSKY (1945) on concrete prove the existence of a small but quite



Fig. 6 Probable failure envelopes for Griggs' tests on Marble and Solenhofen Limestone

definite increase in strength. The analysis of tension tests, however, is beyond the scope of the present paper and it will merely be noted that Leliavsky concludes that an average value for the contact area ratio is 0.09 for concrete failing in tension.

Table 2Evaluation of Contact Area Ratio

Parameter	Marble	Solenhofen
$k \qquad (kg/cm^2)$	1900 8°	1900 8°
'c' (kg/cm²) φ'	210 34°	780 24°
q = unconfined comp. strength $\sigma = \frac{1}{2}q(1 - \sin \phi')$ in unconfined	800 180	2400 700
$\tau_{d} = \frac{1}{2}q \cdot \cos \phi' $ $\tau_{i} = k + \sigma \cdot \tan \psi $ $\tau_{i} = k + \sigma \cdot \tan \psi $ $\tau_{i} = k + \sigma \cdot \tan \psi $	330 1930	1090 2000
$D_{\sigma} = \Delta(\sigma_1 - \sigma_3)/\Delta\sigma_3$ $D_u = \Delta(\sigma_1 - \sigma_3)/\Delta u$ $\tan \phi'/\tan \psi$	2·5 0·08 4·8	0·20 3·2
Theory I $a = \frac{D_u}{D_\sigma}$	0.03	0.14
Theory III $a = \frac{D_u}{D_\sigma} \cdot \frac{\tan \phi'}{\tan \psi}$	0.12	0.45
$a=1-\frac{\tau_i-\tau_d}{k}$	0.16	0.52

Triaxial compression tests on concrete by TERZAGHI and RENDULIC (1934) also indicate small increases in strength in unjacketed specimens, although the scatter of individual results is too great for any exact deductions to be made. But a comprehensive series of triaxial compression tests on a  $1:2\frac{1}{2}:2\frac{1}{2}$ concrete has been reported by MCHENRY (1948) and the results are plotted in Fig. 7, each point being the average of at least three tests. The unconfined compression strength appears to be somewhat anomalous and if this is neglected we find from the jacketed tests that  $\phi' = 52^{\circ}$  or  $2 \sin \phi'/(1 - \sin \phi') = 7.4$ . The increase in unjacketed strength with increasing pore pressure is expressed by the ratio  $\Delta(\sigma_1 - \sigma_3)/\Delta u = 0.25$ . Thus according to Theory I, a = 0.25/7.4 = 0.035.

If the unconfined test is included  $\phi' = 48^{\circ}$ , but statistically the unjacketed strength then decreases with increasing pore pressure. This is not physically possible, however, and since the unjacketed tests under the eight different pressures show a fairly steady increase (Fig. 7) it seems more reasonable to accept the foregoing value of  $\Delta(\sigma_1 - \sigma_3)/\Delta u$ . Whether  $\phi'$  is taken as  $48^{\circ}$  or  $52^{\circ}$  is of little consequence and, for consistency, the latter will be used: this being equivalent to neglecting the unconfined strength in both types of test.



Fig. 7 Triaxial tests on concrete (MCHENRY, 1948)

Now an area ratio of 0.035 (it merely rises to 0.045 if  $\phi' = 48^{\circ}$ ) appears to be remarkably small for a concrete which, in this series of tests, has a compression strength of about 5000 lb/in<sup>2</sup>. Unfortunately there is no evidence from an intrinsic line which would enable a to be estimated independently, as with the calcitic limestones. But it may readily be assumed that  $\psi$  can hardly be greater than the value for Quartz, namely about 13°. In that case  $\tan \frac{\phi'}{\tan \psi}$  is 5.5 and hence according to Theory III a would be not less than  $5.5 \times 0.035 = 0.19$ . Any further refinement would not be justified and, in round figures, this analysis of McHenry's tests may be summarized by saying that the contact area ratio is 0.04 or 0.2 depending on whether Theory I or Theory III is adopted. The Marble and Solenhofen tests have already shown that the latter is to be preferred and it seems that Theory III also leads to a more reasonable result for concrete.

Soils—So far as soils are concerned no critical tests, necessarily involving high pore pressures, have been carried out. But there is ample evidence, from work by RENDULIC (1937), TAYLOR (1944), BISHOP and ELDIN (1950) and others, that within the range of pore pressures encountered in practice Terzaghi's equation  $\sigma' = \sigma - u$  involves no significant error for fully saturated sands and clays. If Theory III is correct, however, the effective stress is more properly given by the expression

$$\sigma - \left(1 - \frac{a \tan \psi}{\tan \phi'}\right) u$$

and the validity of Terzaghi's equation must therefore depend upon the small magnitude of the term  $a \cdot \tan \psi / \tan \phi'$ .

For sands, which typically consist chiefly of Quartz particles,  $\psi$  will be about 13°. And  $\phi'$  is usually within the range 30° to 40°. Thus  $\tan \psi/\tan \phi'$  may be expected to be of the order 0.3. This is far from negligible, as compared with unity, and hence the validity of the equation  $\sigma' = \sigma - u$  in shear strength problems in sands must be dependent upon the small magnitude of the contact area ratio at normal engineering pressures. Owing to the high-yield stress of Quartz, and probably of other minerals found in sands, there is no difficulty in accepting this conclusion.

For clay minerals it is likely that  $\psi$  has quite low values, but clay soils contain appreciable quantities of silt and often some fine sand as well. The average values of  $\psi$  for clays may therefore be very roughly in the range 5° to 10°. In these soils  $\phi'$ usually lies between 20° and 30°. Thus tan  $\psi/\tan \phi'$  is probably not less than about 0.15 for clays, and possibly a more representative value is about 0.25. Thus it seems that in clays, as well as in sands, the physical basis for Terzaghi's equation in shear strength problems is the small area of contact between the particles.

Lead Shot—Triaxial tests on lead shot show that  $\phi' = 24^{\circ}$ (BISHOP 1954)\*. But as we have seen,  $\psi$  for lead is only  $\frac{3}{4}^{\circ}$ . Thus tan  $\psi/\tan \phi' = 0.03$ . Consequently even at pressures sufficiently high virtually to eliminate the voids, when the particles will approximate to polyhedra and *a* is approaching unity, an error of not more than 3 per cent would be involved in using the expression  $(\sigma - u)$  for effective stress in relation to shear strength. It may therefore be anticipated that Terzaghi's equation would be proved valid in shear tests on lead shot over a very wide pressure range; but, in this case, owing to the fact that tan  $\psi/\tan \phi'$  is small.

# PART II

#### **Compressibility of Saturated Materials**

When porous materials are subjected to an increase in allround pressure, under the condition of zero pore pressure (i.e. in jacketed or drained consolidation tests), their volume decreases, and if for any comparatively small increase in pressure from p' to  $p' + \Delta p'$  the volume changes from V to  $V + \Delta V$ , where  $\Delta V$  is negative, then the compressibility C of the material for this particular pressure increment is defined by the equation

$$-\left(\frac{\Delta V}{V}\right)_d = C.\Delta p'$$

Compressibility is not a constant, but decreases with increasing pressure and eventually, under a pressure sufficiently high to eliminate the voids, C will fall to the value  $C_s$  where  $C_s$  is the compressibility of the solid particles. This behaviour is analogous to the progressive flattening of the slope of the failure envelope with increasing pressure, until it is eventually equal to  $\psi$ , and is clearly illustrated by the jacketed tests on Vermont

\* Bishop has also shown that for lead  $\mu = 0.26$ . Thus in the equation  $\tan \phi' = m \cdot \mu$  the coefficient m = 1.70. This may be compared with Caquot's value  $m = \pi/2 = 1.57$ .

Marble and a Quartzitic sandstone (ZISMAN, 1933) shown in Fig. 8. It will be seen that, under the highest pressure in these tests, namely 600 kg/cm<sup>2</sup>, the compressibilities have decreased to values only about 15 per cent greater than the compressibilities of Calcite and Quartz, respectively.



Fig. 8 Compressibility tests (ZISMANN, 1933 and BRIDGMAN, 1928)

The problem under consideration, however, is to obtain an expression for  $\Delta p'$  when a saturated porous material is subjected to a pore-pressure change  $\Delta u$  as well as a change in the applied pressure  $\Delta p$ .

Theory I—As in the case of shear strength, the usually accepted theory is based on the assumption that the effective stress is the intergranular pressure

$$\sigma_g = \sigma - (1 - a)u$$

On this hypothesis it therefore follows at once that

$$-\frac{\Delta V}{V} = C[\Delta p - (1 - a)\Delta u]$$

$$\Delta p' = \Delta p - (1 - a) \Delta u$$

and

Theory II—It has already been suggested that in a 'perfect' solid  $\psi = 0$ . We may now add that such a solid will also be incompressible, or  $C_s = 0$ . Thus in a porous material consisting of perfect solid particles a change in applied stress together with an equal change in pore pressure will cause no volume change or deformation in the particles, nor any change in the contact area and shear strength. Consequently a decrease in volume, just as an increase in shear strength, can only result from an increase in  $(\sigma - u)$  or, in the present case, from

an increase in  $(\Delta p - \Delta u)$ . Therefore

and

$$\Delta p' = \Delta p - \Delta u$$

 $-\frac{\Delta V}{V} = C[\Delta p - \Delta u]$ 

Theory III—In fact, however, the solid particles have a finite compressibility  $C_s$  and an angle of intrinsic friction  $\psi$ . If the porous material is subjected to equal increases in applied pressure and pore pressure, as in an unjacketed test with an equal all-sided cell pressure and no additional axial stress, then each particle will undergo cubical compression under a hydrostatic pressure  $\Delta u$ . And the material will decrease in volume by an amount\*

$$-\left(\frac{\Delta V}{V}\right)_{s}=C_{s}.\Delta u$$

If now, for the moment, we assume that  $\psi$  is zero, or negligible, then a net pressure increment  $(\Delta p - \Delta u)$  will cause a volume change exactly equal to that which would result from an application of an identical pressure in the absence of pore pressure. Thus the total volume change of the porous material must be

$$-\Delta V/V = C(\Delta p - \Delta u) + C_s \Delta u$$

and it will be noted that the contact area ratio is not involved.

A similar line of reasoning has been expressed by CHUGAEV (1958 and probably 1947—see previous footnote). But the above equation was first given, so far as the author is aware, by A. W. Bishop in a letter to A. S. Laughton (24 Nov. 1953) following discussions at Imperial College between Dr Laughton, Dr Bishop and the author.

The existence of an angle of intrinsic friction implies, however, that when a porous material is subjected to an increase in pressure  $(\Delta p - \Delta u)$  there will be a small increase in resistance to particle rearrangement and deformation, associated with the increase in pore pressure. An approximate analysis suggests that a more complete expression is of the form

$$-\frac{\Delta V}{V} = C(\Delta p - \Delta u)[1 - \eta \cdot \Delta u \cdot \tan \psi] + C_s \cdot \Delta u$$

when  $\eta$  is a function depending on the relative contributions by particle rearrangement and deformation to the total volume change, and involving the parameters a,  $1/\tan \phi'$  and 1/k. But this analysis also indicates that the  $\eta$  term is numerically unimportant in most cases<sup>†</sup>. Thus it is sufficiently accurate to write  $-\frac{\Delta V}{V} = C \left[ \Delta p - \left(1 - \frac{C_s}{C}\right) \Delta u \right]$ 

or

$$\Delta p' = \Delta p - \left(1 - \frac{C_s}{C}\right)\Delta u$$

Some representative values of  $C_s/C$  for rocks (from data by Zisman *loc. cit.*) and for soils (data from author's files) are given in Table 3.

It will at once be seen that for soils the ratio  $C_s/C$  is negligible and hence Terzaghi's equation is acceptable to a high degree of approximation. In contrast, the compressibility ratio is so considerable for rocks and concrete that important errors could be involved in using the equation  $\Delta p' = \Delta p - \Delta u$  for such materials.

Unjacketed tests on rocks—In these tests the specimen is not covered with a membrane and the cell fluid can penetrate fully

\* This equation may be confirmed by analysing a formal arrangement of spherical particles. It is to be noted that the total volume change of the particles themselves, not of the porous material, is equal to  $-C_s(1-n)V$ .  $\Delta u$  where n is the porosity.

equal to  $-C_1(1-n)V \cdot \Delta u$  where *n* is the porosity.  $\dagger$  For example, the  $\eta$  term will, in effect, typically reduce *C* by the order of 3 per cent for Marble and for Lead Shot. into the voids of the material. Thus  $\Delta p = \Delta u$ , and the procedure consists of measuring the volume changes under various cell pressures.

Table 3

Compressibilities at  $p = 1 \text{ kg/cm}^2$ Water  $C_w = 48 \times 10^{-6} \text{ per kg/cm}^2$ 

Material	Compre. per kg/cm	$\frac{C_s}{c}$	
	С	Cs	
Quartzitic sandstone Quincy granite (100 ft deep) Vermont marble Concrete (approx. values) Dense sand Loose sand London Clay (over-cons.) Gosport Clay (normally- cons.)	5.8 7.5 17.5 20 1,800 9,000 7,500 60,000	2·7 1·9 1·4 2·5 2·7 2·7 2·0 2·0	0.46 0.25 0.08 0.12 0.0015 0.0003 0.00025 0.00003

According to Theory I the volume change in an unjacketed test is

$$-(\Delta V/V)_u = C.a.\Delta u$$

$$-\frac{(\Delta V/V)_u}{\Delta u}=C.a$$

According to Theory II the volume change in these tests will be zero, but from Theory III even in its most general form including the  $\eta$  term,

$$-\frac{(\Delta V/V)_u}{\Delta u} = C_s$$

Unjacketed tests on a number of rocks have been made by Zisman (*loc. cit.*) and those on Marble and on a Quartzitic sandstone are especially valuable since the particles of each of these materials consist essentially of a pure mineral of known compressibility; namely Calcite and Quartz respectively. In Fig. 8 the results of the unjacketed tests are plotted, together with the compressibility of the relevant mineral, and it will be seen that the observed values of  $-(\Delta V/V)/\Delta u$  are almost indistinguishable from the values of  $C_s$  (as determined by BRIDGMAN, 1925, 1928).

This evidence is strongly in favour of Theory III, but it is not sufficient to rule out Theory I since the values of a which could be deduced from these tests are quite reasonable.

Lead shot-It is therefore fortunate that further information is available from a special oedometer test on lead shot (grade 10) carried out by LAUGHTON (1955 and personal communication). An increment of total pressure was applied together with a known increment of pore pressure, and after a sufficient time for equilibrium to be attained the volume change was noted. The pore pressure was then reduced to zero and, after the same time interval, the further consolidation was observed. From a number of such increments a curve can be obtained (Fig. 9) relating void ratio and pressure, with zero pore pressure. And since this curve gives the effective pressure for a known void ratio, the effective pressure corresponding to any of the particular combinations of total pressure and pore pressure can be deduced. Moreover, at the end of the test the lead shot was removed and by measurement with the microscope of the facets where the particles had been pressed together, an approximate value of a was obtained. Separate tests were also made to determine a in this way for two of the intermediate stages of loading.

The author is indebted to Dr Laughton for the results of these experiments, from which the data set out in Table 4 have been computed. Perhaps the most striking point to note is





that the use of Terzaghi's equation would involve quite small errors over the entire pressure range, in spite of the fact that the particles had been so distorted under the highest pressure that the area ratio was approaching unity. But the last four increments in this test provide the most critical evidence for checking the validity of the various theories of effective stress. The pressures and void ratios, and the changes in these quantities during the penultimate increment, are given for convenience in Table 5, together with approximate values of a obtained by interpolation from the microscope measurements.

Table 4 Consolidation Test on Lead Shot

Total pressure	Pore pressure	Void ratio	Effective pressure p' kg/cm <sup>2</sup>		p - u kg/cm <sup>2</sup>	a Observed
kg/cm <sup>2</sup>	kg/cm <sup>2</sup>	e	Observed	Deduced	0,	
17	0	0.675	17			
27	8	0.670		19	19	
27	0	0.650	27			0.03
60	16	0.601	]	42	44	
60	0	0.549	60			
128	32	0.452		94	96	
128	0	0.379	128			0.11
256	64	0.262		195	192	
256	0	0.190	256			
512	128	0.105		385	384	
512	0	0.069	512			
1024	256	0.032		780	768	
1024	0	0.020	1024			0.95
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р	и	е	p'	a
512 1024	0 256	0·069 0·032	512 780	0·7 0·9
$\Delta p = 512$	$\Delta u = 256$	$\varDelta e = 0.037$	$\Delta p' = 268$	a = 0.8

The volume change is

$$-\frac{\Delta V}{V} = \frac{\Delta e}{1+e} = \frac{0.037}{1.069} = 0.035$$

and this is caused by an increase in effective stress of  $268 \text{ kg/cm}^2$ . Thus, since the three-dimensional compressibility C can be taken to be of the same order as the one-dimensional compressibility (SKEMPTON and BISHOP, 1954),

$$C = \frac{0.035}{268} = 130 \times 10^{-6} \text{ per kg/cm}^2$$

The compressibility of lead, as determined by EBERT (1935), is  $C_s = 2.45 \times 10^{-6}$  per kg/cm.<sup>2</sup> Hence  $C_s/C = 0.02$ .

Before calculating the effective stress it should be mentioned that high precision is not possible in the oedometer, owing to friction between the specimen and the walls of the apparatus, and owing to the unknown lateral pressures. These factors will, however, reduce both  $\Delta p$  and  $\Delta p'$ . Consequently the usefulness of the test as a comparative check will not be greatly impaired.

Now according to Theory I

$$\Delta p' = \Delta p - (1 - a)\Delta u$$
  
:.  $\Delta p' = 512 - (1 - 0.8) 256 = 460 \text{ kg/cm}^2$ 

But the value of  $\Delta p'$  deduced from the test is 268 kg/cm. In this case Theory I therefore involves a discrepancy of 70 per cent, which is far in excess of experimental error.

According to Theory II

$$\Delta p' = \Delta p - \Delta u$$
  
:.  $\Delta p' = 512 - 256 = 256 \text{ kg/cm}$ 

and in the simpler form of Theory III

 $\Delta p' = \Delta p - \left(1 - \frac{{}^{t}C_{s}}{C}\right)\Delta u$ 

 $\therefore \Delta p' = 512 - (1 - 0.02) 256 = 261 \text{ kg/cm}^2$ 

These two results differ from  $268 \text{ kg/cm}^2$  by less than 5 per cent, which is probably insignificant in view of the inaccuracies associated with the test.

Similar calculations have been made for the other highpressure increments, and the results are summarized in Table 6. The apparently excellent agreement between Theory III and the test values is to some extent fortuitous, but there can be no doubt that Theory I is altogether unacceptable; whilst Terzaghi's equation (Theory II) is a good approximation even when the contact-area ratio exceeds 50 per cent.

Table 6

,			C	$\Delta p' \text{ kg/cm}^2$			
kg/cm <sup>2</sup>	kg/cm <sup>2</sup>	(approx.)	$\frac{C_{3}}{C}$	Theory I	Theory II	Theory III	Experi- mental
256	0		0.005	150	100	100	
512	128	0.35	0.002	170	128	129	129
	1.20	0.6	0.01	50	128	127	127
512	0	0.8	0.02	460	256	261	769
1024	256	0.0	0.02	400	250	201	200
1021		0.9	0.02	20	256	243	244
1024	0						

The foregoing experiments, together with the unjacketed compressibility tests on Marble and Quartzite, therefore confirm that the effective stress controlling volume changes in porous materials is given with sufficient accuracy by the equation

$$\Delta p' = \Delta p - \left(1 - \frac{C_s}{C}\right) \Delta u$$

And since this expression does not include the contact-area ratio, it follows that this parameter cannot be determined from volume change tests.

Globigerina ooze—In the past there has been some doubt as to the effective pressure in very deep beds of sediment. This uncertainty has arisen from the use of Theory I and the probability that under high-pressures the contact area ratio must be considerable. But it will now be seen that the only relevant question is whether the ratio  $C_s/C$  becomes significant. From compressibility data on clays (SKEMPTON, 1953) it would seem that at pressures of the order 200 kg/cm<sup>2</sup> C is unlikely to be less than about  $250 \times 10^{-6}$  per kg/cm<sup>2</sup> and  $C_s/C$  is therefore still not more than 0.01. In other words Terzaghi's equation should be applicable to depths of several thousand feet of sediments.



Fig. 10 Consolidation test on Globigerina ooze (LAUGHTON, 1955)

In order to confirm this conclusion Laughton (op. cit.) carried out a consolidation test on a sample of Globigerina ooze in the same apparatus as that used for the lead shot tests, and with the same procedure. The sample was obtained from the bed of the eastern Atlantic ocean, during the 1952 cruise of R.R.S. Discovery II at Station 2994 (Lat. 49° 08' N; Long. 17° 37' W). The results are plotted in Fig. 10 and it will be seen that up to pressures of at least 800 kg/cm<sup>2</sup> the effective stress is given within reasonably close limits by the expression (p - u). This would be expected, since at 800 kg/cm<sup>2</sup> the compressibility of the material is about  $150 \times 10^{-6}$  per kg/cm<sup>2</sup> and  $C_s/C$  is therefore less than 2 per cent.

#### PART III

#### Partially Saturated Soils

When the pore space of a soil contains both air and water. the soil is said to be partially saturated, and the degree of saturation is defined by the ratio

$$S_r = \frac{\text{vol. water}}{\text{vol. pore space}}$$

Owing to surface tension the pore-water pressure  $u_w$  is always less than the pore-air pressure,  $u_a$ . And if we take the case of a soil with a rather low degree of saturation, the pore water will be present chiefly as menisci in the vicinity of the interparticle contacts, as sketched in Fig. 11. It is then possible to consider that the pore-water pressure acts over an area  $\chi$  per unit gross area of the soil\*, and that the pore-air pressure acts over an area  $(1 - \chi)$ . The equivalent pore pressure will thus be

$$\chi . u_w + (1 - \chi) u_a$$

When the degree of saturation is small, and the voids are chiefly filled with air, this expreession is better written in the

\* Strictly,  $\chi$  is not an area (Aitchison and Donald 1956). But this assumption leads to a simple model of the problem and a correct form of the expression for equivalent pore pressure.



form

expressed in the form

since this represents directly the concept that the equivalent pore pressure is less than the pore-air pressure only by the pressure difference  $(u_a - u_w)$  acting over a small area  $\chi$ . Similarly when the soil is almost fully saturated, and  $\chi$  is approaching unity, the equivalent pore pressure is better

 $u_a - \chi(u_a - u_w)$ 

$$u_w + (1 - \chi)(u_a - u_w)$$

Now it has been shown that for fully saturated soils the effective stress is given to a very close approximation by Terzaghi's equation

$$\sigma' = \sigma - u_{\mu}$$

Thus it is reasonable to presume that for partially saturated soils the effective stress would similarly be given by the equation, first suggested by BISHOP in 1955,

$$\sigma' = \sigma - [u_a - \chi(u_a - u_w)]$$

There is no reason, however, why the coefficient  $\chi$  should be identical in problems of shear strength and consolidation, and for a given degree of saturation the value of  $\chi$  must be determined experimentally in both types of test. Nevertheless it is evident that  $\chi = 1$  when  $S_r = 1$  and that  $\chi = 0$  when  $S_r = 0$ , for all conditions of test. Consequently, at these two limits, we have

$$\sigma' = \sigma - u_w$$
 when  $S_r = 1$   
 $\sigma' = \sigma - u_a$  when  $S_r = 0$ 

An important special case arises when  $u_a$  is equal to atmospheric pressure. For, since all pressures are normally expressed in relation to atmospheric pressure as a base,  $u_a$  is then zero and

$$\sigma' = \sigma - \chi . u_w$$

It is to be noted that in most engineering problems the degree of saturation is more nearly unity than zero; and since it is the pore-water pressure that is usually measured, in the laboratory and in the field, a more convenient form of the effective stress equation for partially saturated soils is

 $[u_w + (1 - \chi)(u_a - u_w)]$ 

$$\sigma' = \sigma -$$

If we

or

then

forms

$$\sigma' = \sigma - \left[1 + (1 - \chi)\frac{u_a - u_w}{u_w}\right]u_w$$
  
e write

 $S_{\chi} = 1 + (1 - \chi) \frac{u_a - u_w}{u_w}$ 

$$\sigma' = \sigma - S_{\chi}.u_w$$

and as  $S_r \to 1$ , so does  $S_x \to 1$ . Further, if  $u_a = 0$ , then  $S_x = \chi$ . Moreover the equations for effective stress derived in Parts I and II of the present paper may now be expressed in the general

$$\sigma' = \sigma - \left(1 - \frac{a \cdot \tan \psi}{\tan \phi'}\right) S_x \cdot u_w$$
$$\sigma' = \sigma - \left(1 - \frac{C_s}{C}\right) S_x \cdot u_w$$

Experimental verification of Bishop's equation—In order to check the validity of the equation

$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

a triaxial test was carried out recently at Imperial College under Dr Bishop's direction by Mr Ian Donald on a specimen of silt with a degree of saturation of about 45 per cent. In this test the cell pressure, the pore-water pressure and the pore-air pressures could be varied, and measured, independently but simultaneously. Throughout the early part of the test  $\sigma_3$ ,  $u_a$ and  $u_w$  were held at steady values. When failure was being approached, however, and the slope of the stress-strain curve was therefore becoming comparatively small, all three pressures were varied; but in such a manner that  $(\sigma_3 - u_a)$  and  $(u_a - u_w)$ remained constant. If Bishop's equation is correct, then these variations should cause no change in the effective stress and, consequently, there should be no change in the stress-strain curve. The results are plotted in Fig. 12, from which it is seen that this prediction is verified within the limits of experimental accuracy.

Equally important, however, is the fact that the effective stress is not equal to  $(\sigma - u_w)$ ; as can be proved from the following data. The sample for which the results in Fig. 12 were



Fig. 12 Triaxial test on partially saturated silt (Bishop and Donald)  $\sigma_3 - u_a = 2.0 \text{ lb/in}^2$  throughout test  $u_a - u_w = 8.5 \text{ lb/in}^2$  throughout test

obtained was a normally consolidated silt, from Braehead (LL = 29, PL = 23, clay fraction = 6 per cent). At failure the deviatoric compression strength  $(\sigma_1 - \sigma_3)_f = 19\cdot2$  lb/in<sup>2</sup>, the void ratio = 0.86, and the degree of saturation  $S_r = 43$  per cent. Now a series of four drained triaxial tests on the same material and with the same void ratio at failure, but in the fully saturated condition, gave the results c' = 0 and  $\phi' = 33\frac{1}{2}^{\circ}$ . There is no reason why this value of  $\phi'$  should not apply closely

to the partially saturated test, since the void ratio was identical, and hence the effective minor principal stress  $\sigma_3'$  in the partially saturated test can be calculated from the equation

$$(\sigma_1 - \sigma_3)_f = \sigma'_3 \cdot \frac{2\sin\phi'}{1 - \sin\phi}$$

With  $\phi' = 33\frac{1}{2}^{\circ}$  and  $(\sigma_1 - \sigma_3)_f = 19.2$  lb/in<sup>2</sup> it is at once found that  $\sigma'_3 = 7.8$  lb/in<sup>2</sup>.

But throughout the test on the partially saturated silt,  $(\sigma_3 - u_w)$  was held constant at 10.5 lb/in<sup>2</sup>. Thus, if Terzaghi's equation were used, the effective stress would be given as 10.5 lb/in<sup>2</sup> instead of the actual value of 7.8 lb/in.<sup>2</sup>

Also, throughout the test,  $(\sigma_3 - u_a) = 2.0$  lb/in<sup>2</sup> and  $(u_a - u_w) = 8.5$  lb/in<sup>2</sup>. Hence, since  $\sigma'_3 = 7.8$  lb/in<sup>2</sup>, it follows from the equation

$$\sigma'_3 = \sigma_3 - u_a + \chi(u_a - u_w)$$

that  $\chi = 0.68$  at the particular degree of saturation of the sample.

#### Summary

It is usually assumed that the effective stress controlling changes in shear strength and volume, in saturated porous materials, is given by the equation

$$\sigma' = \sigma - (1 - a)u_w \qquad \dots (1)$$

where a is the area of contact between the particles, per unit gross area of the material. Experimental evidence is presented however, which shows that equation 1 is not valid; and theoretical reasoning leads to the conclusion that more correct expressions for effective stress in fully saturated materials are (i) for shear strength

for shear strength

$$\sigma' = \sigma - \left(1 - \frac{a \cdot \tan \psi}{\tan \phi'}\right) u_{\psi} \qquad \dots \qquad (2)$$

(ii) for volume change

where  $\psi$  and  $C_s$  are the angle of intrinsic friction and the compressibility of the solid substance comprising the particles, and  $\phi'$  and C are the angle of shearing resistance and the compressibility of the porous material.

For soils  $\tan \psi/\tan \phi'$  may be about 0.15 to 0.3 but *a* is very small at pressures normally encountered in engineering and geological problems. Also, under these low pressures,  $C_s/C$  is extremely small. Thus, for fully saturated soils, equations 2 and 3 both degenerate into the form

$$\sigma' = \sigma - u_w \qquad \dots \qquad (4)$$

which is Terzaghi's equation for effective stress.

If we define as a 'perfect' solid a substance which is incompressible and purely cohesive, then  $C_s = 0$  and  $\psi = 0$ . In that case Terzaghi's equation is rigorously true. And the comparison may be made with Boyle's Law for a 'perfect' gas; when equations 2 and 3 become analogous to the more complex expression derived by Van der Waals in which the attractions between the molecules, and the volume of the molecules, are taken into account.

But if Terzaghi's equation has the status of an excellent approximation for saturated soils, this cannot be said to be generally true for saturated rocks and concrete. For in these materials  $C_s/C$  is typically in the range 0.1 to 0.5, whilst tan  $\psi/\tan \phi'$  may be of the order 0.1 to 0.3 and *a* is not negligible<sup>\*</sup>. Equation 1 is, however, also not correct and where this expression has been used to determine the area ratio from triaxial

\* Nevertheless it must be emphasized that in most practical shear strength problems, where u rarely exceeds  $\frac{1}{2}\sigma$ , little error will also be involved in taking  $\sigma' = \sigma - u$ .

For partially saturated soils Bishop has suggested the following expression for effective stress

$$\sigma' = \sigma - [u_a - \chi(u_a - u_w)] \qquad \dots (5)$$

where  $\chi$  is a coefficient to be determined experimentally and, at a given degree of saturation, has not necessarily the same values in relation to shear strength and volume change. Nevertheless  $\chi$  is always equal to unity in saturated soils and is always equal to zero for dry soils. In both these limiting conditions equation 5 therefore becomes identical with equation 4, since this latter equation can apply to any single-phase pore fluid with a pressure *u*; when

$$\sigma'=\sigma-u$$

Bishop's equation, which has been confirmed experimentally, can be written in the form

$$\sigma' = \sigma - S_{\chi} . u_{w} \qquad \dots \qquad (6)$$

$$S_{\chi} = 1 + (1 - \chi) \frac{u_a - u_w}{u_w}$$
 .... (7)

and in the important special condition of  $u_a = 0$ 

$$\sigma' = \sigma - \chi . u_w$$

Since  $S_{x}$ .  $u_{w}$  can be considered as an equivalent pore pressure. the effective stresses in partially saturated porous materials may be expressed by the general equations

$$\sigma' = \sigma - \left(1 - \frac{a \cdot \tan \psi}{\tan \phi'}\right) S_{\chi} \cdot u_{\psi} \qquad \dots \qquad (8)$$

$$\sigma' = \sigma - \left(1 - \frac{C_s}{C}\right) S_{\chi}.u_w \qquad \dots \qquad (9)$$

These equations define the influence of pore pressure on the stress controlling shear strength and volume change in porous materials.

#### References

AITCHISON, G. D. and DONALD, I. B. (1956). Effective Stresses in Michael, S. D. and Dorall, A. B. (1990). Effective Stresses in unsaturated soils. Proc. 2nd Australia-New Zealand Conf. Soil Mech. pp. 192-199
 BISHOP, A. W. (1954). Letter in Geotechnique, 4, 43-45
 BISHOP, A. W. (1955). Lecture delivered in Oslo, entitled 'The prin-ticle's Generic stress' Deleted in The Ulaskia Mar 20 (1960)

- ciple of effective stress'. Printed in *Tek. Ukeblad*, No. 39 (1959). BISHOP, A. W. and ELDIN, G. (1950). Undrained triaxial tests on saturated sands and their significance in the general theory of shear strength. Geotechnique, 2, 13-32.

- BOWDEN, F. P. and TABOR, D. (1942). The mechanism of metallic friction. *Nature*, **150**, 197-199.
  BOWDEN, F. P. and TABOR, D. (1954). Friction and Lubrication of
- Solids. Oxford. BRIDGMAN, P. W. (1925). The linear compressibility of fourteen

- BRIDGMAN, P. W. (1925). The initial compressionty of routreen natural crystals. Amer. J. Sci., 10, 483-498.
   BRIDGMAN, P. W. (1928). The compressibility of thirteen natural crystals. Amer. J. Sci., 15, 287-296.
   BRIDGMAN, P. W. (1935). Effects of high shearing stress combined with high hydrostatic pressure. Phys. Rev., 48, 825-847.
- BRIDGMAN, P. W. (1936). Shearing phenomena at high pressures. Proc. Amer. Acad. Arts Sci., 71, 387-459.
   BRIDGMAN, P. W. (1941). Explorations towards the limit of utilizable
- pressures. J. Appl. Phys., 12, 461–469. CAQUOT, A. (1934). Equilibre des Massifs à Frottement Interne, Paris. CHUGAEV, R. R. (1958). Influence of uplift pressure on deformation
- and stressed conditions of dams. Proc. 6th Cong. Large Dams, Paper 129.
- EBERT, H. (1935). Versuche bis zu Drucken von 5000 kg/sq. cm.
- Phys. Zeits., 36, 385–392. FILLUNGER, P. (1915). Versuche über die Zugfestigkeit bei all-seitigem Wasserdruck. Osterr. Wochenschr. Offentlich Baudienst. Wien., 443-448.
- GRIGGS, D. T. (1936). Deformation of rocks under high confining pressure. J. Geol., 44, 541-577.
   GRIGGS, D. T. (1942). Strength and plasticity. Handbook of Phy-configuration Constants. Second Second American Specific Papers, No. 26
- sical Constants. Geol. Soc. America. Special Paper, No. 36. 107-130.
- GRIGGS, D. T. and BELL, J. F., (1938). Experiments bearing on the orientation of quartz in deformed rocks. Bull. Geol. Soc. Amer., 49, 1723-1746.
- KARMAN, TH. VON (1911). Festigkeitsversuche unter allseitigem Druck. Ziet. Vereines Deutsch. Ing., 55, 1749-1757
   KING, R. F. and TABOR, D. (1954). The strength properties and frictional behaviour of brittle solids. Proc. Roy. Soc. A, 223, 225
- 225-238
- AUGHTON, A. S. (1955). The Compaction of Ocean Sediments. Ph.D. Thesis, University of Cambridge. LAUGHTON,
- LELIAVASKY, S. (1945). Experiments on effective uplift area in gravity dams. *Proc. Amer. Soc. Civ. Engrs.*, 71, 1474–1517 MCHENRY, D. (1948). The effect of uplift pressure on the shearing strength of concrete. *Proc. 6th Congr. Large Dams*, Paper 48. RENDULIC, L. (1937). Ein Grundgesetz der Tonmechanik. *Bauing.*, 18, 459–467
- 18, 459-467 SETZ, F. and READ, T. A. (1941). The theory of the plastic properties of solids. J. Appl. Phys., 12, 538-554.
   SKEMPTON, A. W. (1953). Soil mechanics in relation to geology.
- Proc. Yorkshire Geol. Soc., 29, 33-62. SKEMPTON, A. W. and BISHOP, A. W. (1954). Soils. Building Materials: their Elasticity and Inelasticity, ed. M. Reiner, Amster-
- Materials. their Elasticity and Inelasticity, ed. M. Keiner, Amster-dam (see p. 441)
   TABOR, D. (1959). Junction growth in metallic friction. Proc. Roy. Soc. A., 251, 378-393.
   TAYLOR, D. W. (1944). Tenth progress report on shear research.
- Mass. Inst. Tech.
- TERZAGHI, K. (1923). Die Berechnung der Durchlässigkeitsziffer des Tones aus dem Verlauf der hydrodynamischen Spannungser-scheinungen. Sitz. Akad. Wissen. Wien Math-naturw Kl. Abt. IIa, 132, 105-124
- 132, 105-124
  TERZAGHI, K. (1925). Erdbaumechanik auf bodenphysikalischer Grundlage. Leipzig (see pp. 50-52).
  TERZAGHI, K. and RENDULIC, L. (1934). Die wirksame Flächen-porositat des Betons. Zeit. österr. Ing. Arch. Ver., 86, 1-9
  ZISMAN, W. A. (1933). Compressibility and anisotropy of rocks at and near the earth's surface. Proc. Nat. Acad. Sci., 19, 666-679

# Horizontal Stresses in an Over-Consolidated Eocene Clay

Contraintes horizontales dans une argile de l'éocène surconsolidée

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#### Summary

Investigations of a short-term slip in a 40 ft. deep excavation at Bradwell, in Essex, provided an opportunity for determining  $K_o$  in the over-consolidated London Clay at a locality where no complications existed due to under-drainage or adjacent foundations. The capillary pressure in undisturbed samples was found directly by suction measurements, and indirectly by swelling tests and by strength tests. All three methods were in agreement, and showed that the capillary pressure was about twice the vertical effective overburden pressure. From this fact it could be deduced that  $K_o$  within the depth of the excavation was approximately 2.5. The same result was found from an analysis of the slip. The strength tests indicated, however, that  $K_o$  tended to decrease with depth, falling to about 1.5 at a depth of 100 ft. From these observations, and previously obtained correlations for normally-consolidated clays, it has been possible to reconstruct the stress history of the London Clay at this site.

#### Introduction

The vertical and horizontal effective stresses at any depth z beneath a level ground surface are :

$$\sigma_{v}' = \overline{\gamma}z - u_{o}$$
  
$$\sigma_{h}' = K_{o} \cdot \sigma_{v}'$$

where  $\bar{\gamma}$  is the average density of the overlying material,  $u_o$  is the pore pressure at the point considered and, by definition,  $K_o$  is the coefficient of earth pressure at rest. The vertical stress, or effective overburden pressure (usually denoted by p, for convenience) is readily found from density measurements and piezometer observations. But the determination of  $K_o$  is not easy and, indeed, very little field data on the values of this coefficient are available. Yet without a knowlodge of  $K_o$  the stress state of a particular soil cannot be properly defined, nor can a number of practical problems be solved. In particular, an effective stress analysis of the short-term stability of clays in open cuts and strutted excavations depends fundamentally upon this parameter, whilst the pressures acting on a rigid tunnel-lining are probably controlled to an important extent by the original stresses in the ground, and hence by  $K_o$ .

In the laboratory sufficient tests have been made (TERZAGHI 1925, BISHOP, 1958, et. al.) to suggest that for normallyconsolidated soils  $K_o$  lies between the limits 0.3 and 0.8 and, moreover, that an approximate indication of the value of  $K_o$  for such materials may be obtained from the semiempirical expression given by JAKY (1944):

$$K_o = 1 - \sin \Phi$$

Nevertheless it is not yet possible to say whether these laboratory results can be applied with accuracy to normallyconsolidated clays in the field.

#### Sommaire

Des recherches sur un glissement rapide, dans une fouille de 40 pieds de profondeur à Bradwell, en Essex, ont permis de déterminer  $K_o$  dans l'argile londonienne surconsolidée à un endroit où n'existait aucune complication due au drainage ou aux fondations voisines. On a trouvé directement la pression capillaire dans des échantillons intacts par des mesures de succion et indirectement par des essais de gonflement et des essais de résistance.

Ces trois méthodes concordaient et montraient que la pression capillaire était environ deux fois la pression de surcharge effective verticale. On a pu déduire de ce fait que  $K_o$ , dans les limites de la profondeur de la fouille était de 2.5 environ.

On a trouvé le même résultat par une analyse du glissement. Les essais de résistance ont indiqué pourtant que  $K_0$  tendait à décroître avec la profondeur et atteignait 1.5 environ à 100 pieds de profondeur. Grâce à ces observations et aux relations obtenues précédemment pour des argiles normalement consolidées, on a pu reconstituer l'histoire de la tension de l'argile londonienne sur ce chantier.

Laboratory tests have also been made on over-consolidated sand, and KJELLMAN (1936) was able to show that  $K_{\alpha}$  progressively increased with increasing over-consolidation ratio, finally reaching values of about 1.5 when the sand was practically unloaded. Similar results have been obtained by Bishop (loc. cit.). But it appears that no direct laboratory determinations of  $K_o$  have been published for over-consolidated clays\*. Nor is the situation much better regarding field observations. Qualitatively the existence of large horizontal pressures was noticed in the Fort Union clay-shale at Garrison Dam, North Dakota, (Smith and RedLinger, 1953), whilst pressure observations in a tunnel 65 ft. deep in the Bearpaw clay-shale in south Saskatchewan (PETER-SON, 1954) and at a depth of 120 ft. in the Argile plastique at Provins, France (LANGER, 1936), indicate that  $K_o$  may possibly be of the order 2 to 4 in these heavily over-consolidated deposits.

Recently, however, the opportunity has arisen of making an intensive study of a short-term failure in a deep open cut in the London Clay. Details of this research will be published elsewhere (SKEMPTON and LA ROCHELLE, 1961) but as an essential step in the analysis it was necessary to determine  $K_o$ and the results are given in the present paper. In brief, it has been found that in the upper 30 ft. of the London Clay  $K_o$ is almost certainly not less than 2.0 and is more probably about 2.5. It has also been found that  $K_o$  tends to decrease with increasing depth, and falls to a value of approximately 1.5 at 100 ft. below ground level. In addition, an attempt has been made, for the first time, to reconstruct quantitati-

\* The idea that  $K_0$  can exceed unity in over-consolidated soils seems to have been originated by Samsioe in the early thirties, and he suggested (1936) on the basis of ædometer tests, that  $K_0$  was 2.2 in the Devonian clay-shales at the Svir 3 power station in Russia.

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vely the stress history of an over-consolidated clay and to relate this history to the changes in volume and shear strength.

#### Geology of the Bradwell Site

The investigations were carried out at the site of the Bradwell nuclear power station, at the mouth of the Blackwater estuary on the Essex coast, some 50 miles northeast of London (Fig. 1). Before construction, a number of borings were



Fig. 1 Approximate Coasts Lines of the London Clay Sea at its Maximum Extent.

Limites, approximatives de la transgression marine ayant donné naissance à l'argile de Londres.

made, mostly between 40 and 100 ft. deep, but one penetrated to a depth of 200 ft. From sub-surface contours of the Chalk (WOODLAND, 1946) it is possible to complete the relevant geological section at this location. In Fig. 2 it will be seen



Coupe géologique.

that the uppermost stratum consists of 9 ft. of soft Marsh clay (post-glacial). The London Clay extends to a depth of 165 ft., and the upper 24 ft. of this stratum is the usual brown weathered material, whilst the deeper clay is grey. Between 165 ft. and 180 ft. there occurs a layer of silty sand with occasional clay lenses, and then the hard clays of the Woolwich and Reading Beds are encountered. The Thanet Sand is probably a few feet thick, and the top of the Chalk is situated at a depth of about 240 ft. Its thickness amounts to many hundreds of feet. The strata dip very gently in a southerly direction.

The surface is at + 6 ft. O. D. (i.e. 6 ft. above mean sea level) and the site is protected from the sea by an earth bank. Ground water level is about 4 ft. below the surface, and a piezometer installed in the London Clay at a depth of 19 ft. showed a water level corresponding exactly to ground water level. Moreover, pumping from the Chalk in the London area and from various wells in Essex has not affected the water levels in this stratum so far east as Bradwell, and the piezometric head in the Chalk in the neighbourhood is approximately at mean sea level (WOODLAND, *op. cit.*). The effective vertical pressure at any point in the London Clay is therefore given by the expression :

$$p = \sigma_v' = \bar{\gamma}z - \gamma_w h = \sigma_v - u_o$$

where h is the depth of the point below ground water level and  $\bar{\gamma}$  is the average density of the clay in the depth z. Values of p at various depths are given in Table I.

The London Clay is a marine formation of Lower Eocene age<sup>\*</sup>. Its full thickness in Essex, amounting to about 500 ft. has been preserved at Ingatestone 20 miles west of Bradwell (Woodland, *op. cit.*) and perhaps in later geological periods a further net thickness of 150 ft. of sediment was deposited. Since the London Clay at Bradwell now extends to a depth of only 165 ft., it seems that erosion has accounted for the removal of about 500 ft. of sediments. If at the time of maximum deposition ground water level was near the surface, the reduction in effective overburden pressure, due to erosion, therefore amounts to approximately 15 ton/sq.ft. or 30,000 lb./sq.ft. This is no more than a rough estimate, but it is in reasonable agreement with the pre-consolidation load as determined from an edometer test on a sample taken from a depth of 20 ft.

#### Geotechnical Properties of the London Clay

- (i) Water content, density and Atterberg limits were determined on a large number of samples taken from boreholes, and the results are plotted in Fig. 3. With very few exceptions the liquid limits lie between 90 and 100 throughout the thickness of London Clay which was sampled (about 100 ft.) and the plastic limits lie between 25 and 35. Average values are LL = 95, PL = 30. The water content decreases steadily with depth from a value of 36 just below the Marsh Clay, to 29 at a depth of 100 ft. The density correspondingly increases from 118 to 123 lb./cu.ft.; the clay being fully saturated and the specific gravity of the particles being 2.75. The activity of the London Clay is 1.25 and the average value of the clay fraction content (particles less than 2 microns) is 52 per cent by weight;
- (ii) The undrained shear strength was determined by triaxial tests since, when the cell pressure is equal to or greater than the total overburden pressure the the weakening effect of fissures is virtually eliminated, and the clay shows  $\Phi u = 0$ . The time to failure in

\* The maximum extent of the London Clay sea is shown in Fig. 1 (after Davis and Elliott 1957).



Fig. 3 Variations of properties with depth. Variations des propriétés du sol en fonction de la profondeur.

these tests was usually about 15 mintes. In Fig. 3 the circular points represent tests on samples from the boreholes whilst the square points are the results for tests on specimens cut from block samples taken by the author and Dr. LaRochelle during excavation for the foundations of the various structures of the power station. Every point is the average of at least three tests; and the scatter, approximately  $\pm$  20 per cent, is typical of the London Clay. It is also typical that a few samples showed strengths (not plotted in Fig. 3) more than double the average at a given depth. The reason for these high strengths is not understood, but it might be due to slight cementation.

(*iii*) If the reduction in vertical effective stress consequent upon erosion is 30,000 lb./sq.ft. then the average pre-consolidation load for the depth of clay investigated is about 33,000 lb./sq.ft. From previous field studies of normally-consolidated clays the relation between water content (or void ratio) and pressure for a clay of the Bradwell type can be estimated (SKEMPTON, 1953) and so can the relation between undrained shear strength and pressure (SKEMPTON and HENKEL, 1953) It is thus possible to re-construct the consolidation and shear strength cycles for this clay, as shown in Figs. 4 (a) and (b); where the data in Table I have been plotted as full lines, and

Properties of the London Clay at Bradwell LL = 95 PL = 30 c' = 380 lb./ft.<sup>2</sup>  $\Phi' = 20^{\circ}$ 

Total Depth ft.	p lb./ft. <sup>2</sup>	Over- consolidation ratio *	Water Content	c lb./ft.²	$\frac{P_k}{\text{lb./ft.}^2}$	$\frac{p_k}{p}$	K <sub>0</sub>	$\sigma_{h}' = p K_{0}$ lb./ft. <sup>2</sup>
- 10	690	44	36.0	950	1260	1.82	2.17	1500
15	970	32	34.3	1290	2080	2.15	2.64	2560
20	1260	25	33.0	1610	2850	2.26	2.80	3530
30	1830	17	32.0	2100	4050	2.22	2.74	5000
40	2420	13	31.1	2500	5010	2.07	2.53	6120
50	3030	11	30-4	2800	5750	1.90	2.29	6950
60	3610	9	29.9	3070	6390	1.77	2.10	7580
70	4210	8	29.6	3300	6960	1.65	1.93	8100
80	4800	7	29.3	3500	7450	1.55	1.79	8600
90	5410	6.5	29.0	3700	7920	1.47	1.67	9050
100	6020	6	28.8	3860	8320	1.38	1.54	9280
110	6620	5-5	28.7	4020	8700	1.32	1.46	9700

Assuming that erosion has reduced effective overburden pressure by 30,000 lb./ft.3 and expressed as ratio of vertical effective pressures.



Fig. 4 Geotechnical history of the London Clay at Bradwell Essex.

Caractéristiques géotechnique de l'argile de Londres en fonction de la charge appliquée.

the estimated relationships for the normally-consolidated state are shown by dotted lines. The extrapolation from the observations to the pre-consolidation pressure is shown by chain-dotted lines.

(iv) Drained tests were carried out in the triaxial apparatus(18 tests) and in the shear box (10 tests) on specimenscut from the block samples. The average results were

$$c' = 380 \text{ lb./sq.ft. } \Phi' = 20.0^{\circ}$$

and the corresponding shear strengths have been plotted in Fig. 4 (c). The time to failure was about 3 or 4 days in each test.

(v) A comparable number of triaxial consolidated-undrained tests were also made, the time to failure in individual cases varying from 15 minutes to 8 days. Details of these tests will be given elsewhere (SKEMP- TON and LAROCHELLE, 1961) but the principal results may be summarised as follows :

(a) Failure occured in a narrow shear zone.

(b) In the slow tests considerable migration of pore water took place into the shear zone from the less heavily strained adjacent portions of the specimen although there was, of course, no overall change in water content in the specimen as a whole. The pore pressures, in contrast, were virtually constant throughout the specimen.

(c) In these slow tests c' and  $\Phi'$  were practically identical with the parameters measured in the drained tests, and at failure the pore pressure coefficient  $A_f$  was about 0.55, as defined by the equation (SKEMPTON 1954).

$$\Delta u = \frac{1}{3} \left( \Delta \sigma_1 + 2 \Delta \sigma_3 \right) + \left( A - \frac{1}{3} \right) \left| \Delta \sigma_1 - \Delta \sigma_3 \right|$$

for fully saturated soils.

(d) In the quick tests (15 minutes to failure) there was no migration of pore water, but the pore pressure coefficient  $A_f$ , in the shear zone, was about 0.25. Outside the shear zone pressures were very appreciably higher. Since, in these tests, there was no opportunity for migration or for increase in pore pressure in the shear zone, the undrained strength in the quick tests was greater than in the slow tests, the difference being about 20 per cent.

# **Determination of Capillary Pressure**

The capillary pressure in the samples, which will be denoted by  $p_k$ , was determined by four methods.

(i) In the consolidation stage of the drained tests volume changes were measured. Under low cell pressures the clay swelled, and under high pressures it consolidated. By interpolation the "swelling pressure" corresponding to zero volume change could readily be found (Fig. 5), and the results are set out in Table III for samples from three different depths.





(ii) The swelling pressure can also be measured, though with less accuracy, by finding the load which just prevents a volume change when a specimen in the oedometer is immersed under water. Two tests of this kind were made on specimens from the same depth, the results being 1 400 and 2 100 lb./sq.ft. The average figure of 1 700 lb./sq.ft. is almost identical with the result from method (i) for this sample, but the close agreement is probably fortuitous.

- (iii) The two foregoing methods depend upon the assumtion that the swelling pressure equals the capillary pressure. In the third method, however, the capillary pressure was measured more directly, by determining the pore water suction. A specimen was set up in the triaxial apparatus under a cell pressure of 25 lb./sq. in. and the pore pressure measured under conditions of no volume change. After about 10 hours a steady reading of 6.7 lb./sq.in. was recorded. The effective stress in the specimen was therefore 25.0-6.7 = 18.3 lb./sq. in. or 2 630 lb./sq.ft. A repeat test gave 2 570 lb./sq.ft. Thus the capillary pressure could be taken as 2 600 lb./sq.ft. The swelling pressure as measured in method (i) on an immediately adjacent sample was 2 300 lb./sq.ft. (Fig. 5). Hence the two methods are in accordance.
- (*iv*) The capillary pressure can also be decuded from the conventional (15 minute) undrained strength. If this strength is  $c = \frac{1}{2} (\sigma_1' \sigma_3')_f$  then (Fig. 6).



Fig. 6 Effective stresses in undrained test. Contraintes effectives dans un essai non drainé.

$$c = c' \cdot \frac{\cos \Phi'}{1 - \sin \Phi'} + \sigma_3' \cdot \frac{\sin \Phi'}{1 - \sin \Phi'}$$

where  $\sigma_3'$  is the minor principal effective stress at failure. And the effective stress in the specimen before shearing, which is the capillary pressure, must be

$$p_k = \sigma_3' + \Delta u_f$$

where  $\Delta u_f$  is the change in pore pressure during shear. But, since  $\Delta \sigma_3 = 0$  in this test,

$$\Delta u_f = A_f \left[ \Delta \sigma_1 - \Delta \sigma_3 \right]_f$$

and hence, as  $|\Delta\sigma_1 - \Delta\sigma_3|_f = 2c$ , the capillary pressure can at once be found from the equation

$$p_k = \sigma_3' + A_f \cdot 2c$$

Taking  $A_f = 0.25$  and usings the values of c' and  $\Phi'$  for individual samples as obtained in drained tests<sup>\*</sup>, the results given in Table II are obtained.

\* Strictly, a small correction should be applied, since c' and  $\Phi'$  may be slightly greater in tests lasting only 15 minutes. But the correction is probably of the same order of magnitude as the experimental errors and scatter in the drained tests.

Table	n

Samples	Equivalent depth ft.	c lb./ft.²	c' lb./ft.²	Ф'	p <sub>k</sub> lb./ft.²
5 & 6	11	1180	370	19·2°	1900
7 & 8	20	1640	480	20·7°	2500
1	23	1730	340	21·3°	3000

Values of the capillary pressure found by the swelling, suction and strength tests are collected together in Table III, where the effective overburden pressure p is also given.

Table III

•	Equi-		Capillary pressure $p_k$ (lb./ft. <sup>2</sup> )					
Sam- ples	valent depth ft.	p lb./ft.²	Swel- ling tests	Suction tests	Strength tests	Averagc	$\frac{p_k}{p}$	
5 & 6 7 & 8 1	11 20 23	740 1310 1420	1700 2300 2500	2600	1900 2500 3000	1800 2500 2750	2·4 1·9 1·9	

Since the method of determining  $p_k$  from strength tests gives results which are reasonably consistent with the other methods, the variation of strength with depth (Fig. 3), together with the average values of c' = 380 lb./sq. ft. and  $\Phi' = 20^{\circ}$  and  $A_f = 0.25$ , may be used to estimate approximately the capillary pressure throughout the depth of the London Clay in which samples were taken. The results are given in Table I, from which it will be seen that, after an increase in the top 10 ft. of the clay, the ratio  $p_k/p$  decreases steadily with increasing depth. The accuracy of these figures depends, however, on the assumption that  $A_f$  remains constant with depth. The value of  $A_f$  has been obtained in tests on samples from depths of 11 ft. to 20 ft. But it is possible that at greater depths this coefficient may increase, with decreasing overconsolidation ratio (as found in clays over-consolidated in the laboratory, HENKEL, 1956) and  $p_k$  would be correspondingly greater than the values given in Table I; although this effect is not expected to be of great significance.

#### Relation Between $K_o$ and $p_k/p$

The vertical and horizontal total stresses acting at a depth z below the surface are

$$\sigma_v = \bar{\gamma} \cdot z$$
  
$$\sigma_h = (\sigma_v - u_o) K_o + u_o = p \cdot K_o + u_o$$

where  $u_o$  is the piezometric pore pressure. After sampling, the total stresses are zero, but there is a pore pressure u in the sample. Hence the effective stress in the sample (i.e. the capillary pressure) is

$$p_k = O - u$$

where u is negative (suction). If  $\Delta u$  is the change in pore pressure on sampling, and if  $A_s$  is the relevant pore pressure coefficient, then

$$u = u_o + \Delta u = u_o + \frac{1}{3}(A\sigma_1 + 2\Delta\sigma_3) + (A_s - \frac{1}{3})|\Delta\sigma_1 - \Delta\sigma_3|$$
  
Now

$$\Delta \sigma_1 = -\sigma_v$$
 and  $\Delta \sigma_3 = -\sigma_v$ 

and if  $K_o > 1$ , which is the case at Bradwell,

Hence

or

$$|\varDelta\sigma_1 - \varDelta\sigma_3| = \sigma_h -$$

 $p_k = p \left[ K_o - A_s \left( K_o - 1 \right) \right]$ 

 $u = -p \left[ K_o - A_s \left( K_o - 1 \right) \right]$ 

In order to find the order of  $A_s$  a special triaxial test was carried out, in which a specimen cut from block 8 was consolidated under the following stresses :

σ,,

$$\sigma_1 = 30$$
  $\sigma_2 = \sigma_3 = 40$   $u = 20$  lb./sq.in.

Then, without allowing any volume change, the total stresses (were all reduced to 25 lb./sq.in., when the pore pressure was observed to fall to 8 lb./sq.in. Thus  $\Delta u = -12$ ,  $\Delta \sigma_1 = -5$ ,  $\Delta \sigma_3 = -15$  lb./sq.in. and hence, in this test : A = +0.3. Further tests of this type should be made, especially on deeper samples but, for the time being,  $A_s$  will be taken as 0.3.

# Variation of K<sub>0</sub> with depth

Thus from a knowledge of the ratio  $p_k/p$  the value of  $K_o$  can be computed, and the results for various depths are given in Table I, together with the corresponding horizontal effective stress. At the greater depths these values may be rather too small, owing to the possibility that  $p_k$  (and hence  $K_o$ ) may have been underestimated; as previously mentioned. But the general tendency for  $K_o$  to decrease with depth is unmistakeable. Superimposed on this trend, however, there is a reduction in  $K_o$  in the upper 10 ft. of the London Clay; probably associated with weathering and softening prior to deposition of the post-glacial Marsh Clay.

Within the depth of the slip the average value of  $K_o$  will be seen to be about 2.5, whilst at 100 ft.  $K_o$  is probably not less than 1.5. These results are unexpectedly high for a clay which has not been very heavily over-consolidated and it is therefore of interest to examine the degree to which they approach the upper limit, which presumably must be the coefficient of passive earth pressure  $K_p$ .

As an approximation, the effective horizontal passive pressure  $p_p$  at a depth where the effective vertical overburden pressure is p will be given by the equation :

$$p_p = c' \cdot \frac{2 \cos \Phi'}{1 - \sin \Phi'} + p \cdot \frac{1 + \sin \Phi'}{1 - \sin \Phi'}$$

But  $K_p = p_p/p$ , hence

$$K_p = \frac{c'}{p} \cdot \frac{2\cos\Phi'}{1-\sin\Phi'} + \frac{1+\sin\Phi'}{1-\sin\Phi'}$$

Values of  $K_p$  at various depths have been computed, with the parameters c' = 380 lb./sq.ft. and  $\Phi' = 20^\circ$ , and are plotted in Fig. 3. The remarkable result will at once be seen that between depths of about 25 ft. to 40 ft. the coefficient  $K_o$ is almost identical, numerically, with  $K_p$ . Too much emphasis should not be placed on this coincidence, since the values of  $K_o$  are not very precise, but it certainly tends to confirm the suggestion that  $K_o$  is extraordinarily high. Indeed, the passive pressures might be considered to throw some doubt on the  $K_o$  values. But, as will be shown in the next section of the paper, a completely independent set of calculations based on a stability analysis of the slip also leads to the conclusion that  $K_o$  is of the order 2.5 in the zone, extending to a depth of about 40 ft., affected by this failure.

If, therefore, the  $K_o$  values in Table I are accepted, at least as approximations, they can be plotted as shown in

Fig. 4 (d). Moreover, from Jaky's expression  $K_o = 1 - \sin \Phi'$ it may be expected that  $K_o$  for the London Clay ( $\Phi' = 20^\circ$ ) in its normally-consolidated state would be about 0.65. This value has also been plotted in Fig. 4 (d) and the Bradwell results can be extrapolated in a reasonable manner to meet the point represented by the pre-consolidation pressure of 33,000 lb./sq.ft. and  $K_o = 0.65$ .

## Stability Analysis of the Slip

While excavating to a depth of 37 ft. for the foundations of the Reactor a slip occured five days after the bottom of the cut had been reached at the particular site of the failure. An effective stress analysis of this slip has been made, and is described in detail by Skempton and LaRochelle (1961). For the present purpose it is sufficient to mention that from a knowledge of the total stresses acting on the slip surface, and of the shear strength which must have been mobilised along that surface, the pore pressure can be deduced. But the pore pressure at the time of the slip depends upon the original pore pressure, which is definitely known, and the change in pore pressure during excavation. And this change in pore pressure depends upon the change in stresses during excavation \*. Thus these stress changes can be calculated, for any given value of  $A_{t}$ . Hence, since the final stresses are known, the original stresses can also be estimated. And since the original vertical stresses are unambigous, the original horizontal stresses can be deduced; and hence  $K_o$ .

The whole problem, in essence, can be expressed as a necessary relationship between  $A_f$  and  $K_o$ . This relationship is plotted in Fig. 7 for c' = 380 lb./sq.ft. and for c' = 300 lb./sq.ft.; this latter value allowing for a reduction in strength on account of fissures. Fortunately the value of  $K_o$  for a given value of  $A_f$  is not sensitive to a change in c' of this magnitude.

Now in the undrained triaxial tests, on specimens only 3 ins. long, sufficient migration of pore water into the shear zone took place (in about 3 days) for the pore pressure coefficient to rise to a value of 0.55. In the field there would presumably be a greater opportunity for migration to take place and  $A_f$ may be expected to be rather more than 0.55; certainly it could hardly be less. But from Fig. 7 it will be seen that even with  $A_f = 0.55$ ,  $K_o$  must be 2.0 whilst if  $A_f$  is 0.65,  $K_o$ is 2.5. This is the value previously obtained from the independent measurements of capillary pressure; and since a pore pressure coefficient of 0.65 is quite reasonable for the conditions of the clay in the slip, there seems to be no reason to doubt that the stability analysis has in fact tended to confirm the tests and calculations leading to the conclusion that, in the upper 30 ft. of the London Clay at Bradwell,  $K_o$  is about 2.5.

#### Stress History of the London Clay at Bradwell

During the course of this investigation the changes in water content, shear strength and  $K_o$  have been obtained throughout the loading and unloading cycles to which the London Clay has been subjected. To those results can now be added the fundamentally important changes in horizontal effective stress, as shown in Fig. 4 (e). The entire stress history of this clay has therefore been reconstructed. The general picture, as shown in Fig. 4, is somewhat idealised, since there

\* In these calculations the intermediate stress, after excavation, has been taken as the mean of the vertical and horizontal stresses, and the pore pressure change has been related to the principal components of stress change by the equation

 $\Delta u = \frac{1}{3} \left( \Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3 \right) + \alpha \sqrt{\left( \Delta \sigma_1 - \Delta \sigma_2 \right)^2 + \left( \Delta \sigma_2 - \Delta \sigma_3 \right)^2 + \left( \Delta \sigma_3 - \Delta \sigma_1 \right)^2}$ In the triaxial test, where  $\Delta \sigma_2 = \Delta \sigma_3$ ,  $A_f - \frac{1}{3} = \alpha \sqrt{2}$ 





Relation entre  $K_o$  et  $A_f$  d'après le calcul de la stabilité du glissement.

have probably been several minor cycles of unloading and re-loading, and the numerical values of some of the parameters are no more than approximations. Nevertheless it is believed that Fig. 4 represents a reasonably correct analysis of the geotechnical history of this clay.

It is planned to continue this research in the hope of establishing more detailed and more accurate data.

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## References

- [1] A. W. BISHOP (1958). Test requirements for measuring the coefficient of earth pressure at rest. Proc. Conf. Earth Pressure Problems, vol. 1, pp2-14 (Brussels).
- [2] A. G. DAVIS and G. F. ELLIOTT (1957). The palaeogeography of the London Clay sea. Proc. Geol. Assoc. vol. 68, pp. 255-77.
- [3] D. J. HENKEL (1956). The effect of overconsolidation on the behaviour of clays during shear. *Geotechnique*, vol. 6, pp. 139-50.
- [4] J. JAKY (1944). The coefficient of earth pressure at rest (paper in Hungarian, Magyar Mernok es Epitesz Egylet Kozlonye). See also Proc. 2nd Int. Conf. Soil Mech., vol. 1, pp. 103-07 (1948).
- [5] W. KJELLMAN (1936). Report on an apparatus for consummate investigation of the mechanical properties of soils. *Proc. 1st Int. Conf. Soil Mech.*, vol. 2, pp. 16-20.
- [6] K. LANGER (1936). Discussion by K. Terzaghi reporting observations at Provins, France. Proc. 1st Int. Conf. Soil Mech., vol. 3, pp. 152-55.
- [7] R. PETERSON (1954). Studies of Bearpaw shale at a damsite in Saskatchewan. Proc. A.S.C.E. Separates No. 476 & 759.
- [8] A. F. SAMSIOE (1936). Report on the investigation of the compressibility of the ground of the hydro-electric power plant Svir 3. Proc. 1st Int. Conf. Soil Mech., vol. 1, pp. 41-47.
- [9] A. W. SKEMPTON (1953). Soil mechanics in relation to geology. Proc. Yorks. Geol. Soc., vol. 29, pp. 33-62.
- [10] A. W. SKEMPTON and D. J. HENKEL (1953). The post-glacial clays of the Thames estuary at Tilbury and Shellhaven. *Proc. 3rd Int. Conf. Soil. Mech.*, vol. 1, pp. 302-308.
- [11] A. W. SKEMPTON (1954). The pore-pressure coefficients A and B. Geotechnique, vol. 4, pp. 143-147.
- [12] A. W. SKEMPTON and P. LAROCHELLE (1961). The Bradwell slip: a short-term failure in London Clay. *Geotechnique*, vol. 11 (in the press).
- [13] C. K. SMITH and J. F. REDLINGER (1953). Soil properties of Fort Union clay shale. Proc. 3rd Int. Conf. Soil Mech., vol. 1, pp. 62-66.
- [14] K. TERZAGHI (1925). Principles of soil mechanics. Eng. News Rec., vol. 95, pp. 796-800.
- [15] A. W. WOODLAND (1946). Water supply from underground sources of Cambridge-Ipswich District. H. M. Geol. Survey (London).

# A LANDSLIDE IN BOULDER CLAY AT SELSET, YORKSHIRE

by

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#### SYNOPSIS

An analysis has been made of a long-term slip in a valley slope of the River Lune, near Middleton-in-Teesdale. The slip was entirely within a deposit of heavily overconsolidated intact boulder clay. Although a complete picture of the ground-water flow net was not obtained, sufficient information was nevertheless available to show that the field value of the cohesion intercept c' was approximately equal to the laboratory value measured in drained triaxial tests. This is in marked contrast to the results found from long-term slips in overconsolidated fissured clays.

Une analyse d'un glissement à long terme a été faite sur une pente de la vallée de la Lune, rivière voisine de Middleton-in-Teesdale. Le glissement s'est produit entièrement dans un dépôt d'argile de moraine à blocs intacts surconsolidée. Quoique l'on n'ait pas obtenu des renseignements complets sur le débit de l'eau souterraine, on a néanmoins rassemblé assez de renseignements pour montrer que la valeur sur le terrain de la section de cohésion c' est à peu près égale à la valeur de laboratoire mesurée lors d'essais triaxiaux. Ceci est en opposition nette avec les résultats obtenus avec des glissements à long terme dans des argiles fissurées surconsolidées.

#### INTRODUCTION

Several long-term slips in cuttings in overconsolidated fissured clays have been analysed (Henkel, 1957; DeLory, 1957). They all show that the cohesion intercept c' in the clay is appreciably less than the value measured in laboratory tests on samples taken outside the narrow zone of shear, and that the field values of c' tend to decrease with time. Indeed, there is evidence that c' is virtually zero in landslips which take place in natural hill slopes in fissured clays where the time scale is perhaps to be measured in centuries, as shown by Skempton and DeLory (1957), for London Clay, and by Henkel and Skempton (1955), for a clay in the Carboniferous series where the slip surface was located at a depth of 17 ft, well below the zone of weathering.

Only one case is known to the Authors, however, where an accurate stability analysis has been made of a slip in non-fissured, or intact, overconsolidated clay. This slip occurred in a railway cutting at Lodalen, near Oslo, in 1954, and it was found that the full value of c' must have been acting (Sevaldson, 1956). But the Lodalen slip took place only 6 years after excavations for widening the cutting, although the first excavations at the site were made in 1925. The slope at Lodalen was about 58 ft high with a slope of 1:2 (26·5°) and the clay, which was lightly overconsolidated, had the following average properties:

$$w = 31$$
  $w_L = 36$   $w_P = 18$   
clay fraction (< 2 $\mu$ ) = 30–50%  
c' = 210 lb/sq. ft  $\phi' = 27^\circ$ 

Using these values of c' and  $\phi'$  the calculated factor of safety was 1.00 on the critical slip circle (almost coincident with the actual surface of slip), while if c' was taken as zero the calculated factor of safety on the same critical circle fell to 0.7. The investigations at Lodalen were remarkable for the comprehensive measurements of pore pressures in the slope, and for the excellent samples which were obtained. Moreover, the laboratory measurements of c'and  $\phi'$  showed standard deviations of only 20% and 8%, respectively, on ten samples.

In spite of this highly important case record, it is nevertheless very desirable that a slip in non-fissured overconsolidated clay should be analysed where the time scale is greater than that at Lodalen.

The opportunity for obtaining such data arose during site investigations in connexion with

the design of an earth dam, known as Selset Dam, located upstream from Grassholme Reservoir in the Lune Valley  $4\frac{1}{2}$  miles south-west of Middleton-in-Teesdale. There a slip was discovered in the boulder clay forming the valley side, in a slope 42 ft high with an inclination of 28° or 1:1.9. The length of the slip was about 180 ft with well-defined tension cracks (Fig. 1). The general character of the slip suggested a relatively deep movement, although there was also evidence of surface instability on the slope.

The field work was carried out during 1955–57 and 1958–60. Eight samples were obtained on which drained triaxial tests were carried out, with reasonably consistent results, and the bed-rock profile was established with some accuracy from observations made during the construction in 1958 of a drainage gallery situated in the immediate vicinity of the slip, and from the drilling of relief wells from this gallery carried out in 1959. Five shallow piezometers



Fig. 1. Site plan

were installed, and one deep piezometer in the rock. Two moderately deep borings in the boulder clay were also made but, through an oversight these were lined with open-jointed clay pipes, and no piezometric readings could be obtained.

It has therefore not been possible to draw an accurate flow net. Fortunately, however, sufficient information was gained for a reasonably satisfactory analysis to be carried out and this shows that approximately the full value of c' is required in order to obtain a factor of safety of unity. And, if c' were zero, the calculated factor of safety would be as low as 0.65.

# THE SITE

The slip is located in the south slope of the River Lune valley, 200 yd upstream of Selset Weir at the head of Grassholme Reservoir, which was constructed in 1910 (Fig. 1).

In general the river has changed its course appreciably during recent times,\* but at the site of the slip it has been running alongside the toe of the hill slope for at least a century and,

\* The positions of the river in 1856 and 1912, as shown in Fig. 1, are taken from Ordnance Survey maps of those dates; the first being to a scale of 6 in. to the mile, the second to a scale of 25 in. to the mile. The course of the river in 1955, and other topographical details, are taken from a survey made preparatory to the construction of Selset Dam, while a detailed survey of the slip area was made in 1958.

when in flood, some erosion undoubtedly takes place. The conditions at the particular locality have therefore changed little for a considerable time and the slope has almost certainly been at or near a state of limiting equilibrium for many years, with movements taking place from time to time.

Bed-rock, consisting chiefly of sandstone, shale, and limestone strata of the Lower Carboniferous, is found at a depth of about 30 ft beneath the valley floor. The rocks dip in a southeasterly direction at an angle of about  $6^{\circ}$ . They are overlain by massive deposits of heavily overconsolidated boulder clay which extend, in general, up to an elevation of roughly 1,000 ft O.D. or about 80 ft above the valley floor. At higher elevations the rocks are exposed or covered with a thin layer of soil.

The boulder clay was probably formed during an early stage of the Last Glaciation and may be contemporary with the so-called Irish Sea ice sheets. If so, then it was deposited about 45,000-50,000 years ago.\* Subsequently the river has cut down through this material and is now running in its own flood plain which is about 500 ft wide and consists of alluvial gravels about 6 ft thick. Naturally, where the river is undercutting the valley, at the edges of the flood plain, slips are still taking place and the valley slopes are quite steep.

Fig. 2 gives a section through the three principal borings Nos 23, 25, and 26 (which are shown in plan in Fig. 1) and through the drainage gallery. This section is immediately upstream of the slip. Five other profiles of the slope, including one just downstream of the slip, were also drawn and an average section through the slip could then be obtained. The stability analyses were made on this composite section.

Observations of piezometer readings and ground-water levels were made during 1956 and 1957. The highest water levels recorded are plotted in Fig. 2, and these occurred during the winter months, as would be expected. There was, however, a maximum seasonal variation of only 3 ft.

During the drilling operations appreciable flows of water were encountered in the rock, particularly in a bed of limestone, and there can be no question that the rock is considerably more permeable than the boulder clay. Moreover, the water levels in the rock are artesian with respect to the valley floor near the toe of the slope. At borehole 23, however, which is situated 10 ft back from the top of the slope, the piezometric height in the rock was found to be 14 ft below surface; although this is 28 ft above the flood plain. The piezometric height in the rock continues to rise as one moves further from the valley, but the only observation was some distance upstream of the slip and cannot be plotted in Fig. 2.

#### LABORATORY TESTS

Eight samples were taken, at the positions shown in Fig. 2, and subjected to the usual classification tests (see Table 1). From each sample at least three specimens were prepared for drained triaxial tests. Owing to the presence of occasional large stones it was necessary, in some cases, to form the specimens by packing the material into brass tubes, without change in water content and with zero air voids. As usual with boulder clay no essential difference in strength was found between these specimens and those cut from the undisturbed cores.

The specimens, which were  $1\frac{1}{2}$  in. dia. and 3 in. long, had filter strips up the sides and porous stones at the ends. With samples *a* to *f* the deviatoric stress was applied at a rate such that failure occurred after not less than 5 hours. This corresponds to a 98% dissipation of pore pressure, as calculated from the formula given by Gibson and Henkel (1954), with a coefficient of consolidation of the Selset boulder clay which is of the order 0.01 sq. in/min. However, to be certain that the tests were for all practical purposes fully drained, samples *g* and *h* were taken to failure in 2 days. The results were not significantly different from those obtained in

\* According to radio-carbon dating, kindly communicated to the senior Author by Professor F. W. Shotton, F.R.S. (in lit. 18 September, 1958).





the 5-hour tests. In all cases at least a day was allowed for consolidation under the cell pressure.

Of the specimens from sample g, five were tested in the usual manner, by increasing the axial stress, and four were tested by decreasing the radial stress. All nine Mohr's circles are plotted in Fig. 3(a) and the common failure envelope is drawn. Two typical stress-strain curves and the corresponding volume change relationships are shown in Fig. 3(b) and Fig. 3(c). The tests with decreasing radial stress are more analogous to the conditions in nature, and it will be seen that in these tests a considerable increase in volume takes place. The rate of volume change at failure is, however, roughly the same in both types of test. It should be noted that there is only a small decrease in strength at large strains.

The values of c' and  $\phi'$  for all eight samples are given in Table 1, and the standard deviations of these parameters are 30% and 7%, respectively.

There do not appear to be any important variations between the samples, and the following average properties may therefore be taken as representative of the boulder clay:

natural water content = 12 clay fraction  $(< 2\mu) = 17\%$  on whole sample  $w_L = 26 \ w_P = 13 \ Ip = 13$ clay fraction  $(< 2\mu) = 25\%$  on matrix passing 36 sieve  $c' = 180 \text{ lb/sq. ft} \quad \phi' = 32^{\circ}$ bulk density  $\gamma = 139 \text{ lb/cu. ft}$ 

Undrained triaxial tests also were carried out on Sample *h* using undisturbed and remoulded specimens. Remoulding had no measurable effect, and from both series of tests it was found that  $\phi_u = 0$  and  $c_u = 3,000$  lb/sq. ft. The water content of these specimens was a little higher than the average figure of 12%, but there is no reason to suppose that this undrained strength is not reasonably typical for the boulder clay. At a site in County Durham, for example, the following results were obtained (average of ten samples) for a similar boulder clay (Skempton 1949).

$$w = 14$$
  $w_L = 28$   $w_P = 14$   
clay fraction = 17% (whole sample)  
 $c_u = 2,600$  lb/sq. ft  $\gamma = 137$  lb/cu. ft

It may also be mentioned that the values of  $c_u$  showed no significant variation within a depth of 60 ft at this location.

# FLOW NETS

The free water surface is well defined by the shallow piezometers and open boreholes. Except at the top of the slope, water level is practically at ground surface. The piezometric level in the rock at borehole 23 was measured and, as mentioned earlier, the water levels in the rock beneath the toe of the slope are artesian with respect to the flood plain. The magnitude of this artesian head is not known with certainty, but it was proved to exceed 8 ft and is probably about 10-15 ft.

With these fragmentary data it is clearly impossible to construct an accurate flow net. Recourse was therefore made to sketching two flow nets corresponding to the extreme assumptions that the rock is either: (a) of the same permeability as the boulder clay, and (b) infinitely permeable as compared with the boulder clay. These two flow nets are shown in Figs 4 and 5 and they both imply that, theoretically, there is a shallow pool of water standing on the flat ground behind the top of the slope.





Fig. 3. Drained triaxial tests on sample g

	Undisturbed or remoulded		R	U	U	R	R	U	к	ж	
sst results on samples of boulder clay, Selset	Drained shear strength parameters	φ' degrees	34	32	32	34	29	33	30	32	
		°′ lb∕sq. ft	170	120	200	80	190	270	260	160	
	Clay fraction: $< 2\mu$ per cent		1	14	15	15	18	17	25		ticles $= 2.70$
	Bulk density: lb/cu. ft		140	139	135	141	134	141	140	141	ic gravity of par
	Water content: w		11-9	12.0	15.1	11-2	14-4	9. <del>0</del>	11.1	10-4	Average specif
F	mits	$_{IP}$	12	15	14	13	13	14	13		
	Atterberg li	<b>d</b> at	11	13	12	12	15	12	14		
		Тт	23	28	26	25	28	26	27		
	Depth: ft		9	9	6	25	42	12	10	<i>с</i> ,	
	Sample		a	f	, o	0	à	e	ø	Ņ	
	Bouhole		23	24	25	25	25	26	ļ		

Table 1 results on samples of boulder clay

# SELECTED PAPERS ON SOIL MECHANICS

It will be seen that the two assumptions lead to no very great differences in the pore pressures in the area including the slip and, in fact, the flow nets in this region are chiefly controlled by the free water surface.

Two further, very simple, ground-water conditions have also been analysed: (c) horizontal flow, and (d) flow parallel to the slope: the water-table in both cases being assumed to be at ground surface. The former is roughly equivalent to the flow nets, but the latter is very different. It represents the case where the clay in the area of slip is more permeable than the main body of the stratum. It is the most favourable condition, from the point of view of stability, that can be considered at all realistic and will undoubtedly lead to an extreme upper limit for the calculated factor of safety.

#### STABILITY ANALYSIS

During 1957, before the rock contours were fully ascertained, preliminary analyses were carried out with a flow net differing slightly from either of those already shown. Eight circles were analysed, using the method proposed by Bishop (1955), and the lowest factor of safety was found to be 1.03.

Five of these slip circles were later analysed (Fig. 6) using the flow nets in Figs 4 and 5. The critical circle, which is the same for both cases, and has an average depth of 12.5 ft, is shown in Fig. 7. The minimum factors of safety are 1.01 and 1.00 for flow nets A and B respectively. Details for the other slip circles are given in Table 2, together with the results obtained when c' = 0.

These latter calculations show that the cohesion contributes about 30% to the total shear strength.

Table 2 Stability analysis; factors of safety

Flow net A		Flow net B		
c' = 180 lb/sq. ft	c'=0	c' = 180  lb/sq. ft	c' = 0	
1.05	0.71	1.03	0.70	
1.02	0.67	1.01	0.65	
1.09	0.83	1.09	0.83	
1.01	0.71	1.00	0.70	
1.03	0.77	1.02	0.76	
	Flow net A c' = 180  lb/sq. ft 1.05 1.02 1.09 1.01 1.03	Flow net A           c' = 180 lb/sq. ft         c' = 0 $1.05$ $0.71$ $1.02$ $0.67$ $1.09$ $0.83$ $1.01$ $0.71$ $1.03$ $0.77$	Flow net AFlow net B $c' = 180$ lb/sq. ft $c' = 0$ $c' = 180$ lb/sq. ft $1.05$ $0.71$ $1.03$ $1.02$ $0.67$ $1.01$ $1.09$ $0.83$ $1.09$ $1.01$ $0.71$ $1.00$ $1.03$ $0.77$ $1.02$	

The foregoing slip circles all pass through the toe of the slope. In addition, thirteen analyses were made with circles passing at various depths beneath the toe but these all gave higher factors of safety, typically about 1.05 with c' = 180 lb/sq. ft. The depth factor is therefore equal to unity. Two of these deeper circles are shown in Fig. 6.

In all the slip circles a tension crack was assumed to extend to a depth:

$$z_0 = \frac{2c'}{\gamma} \tan\left(45 + \frac{\phi'}{2}\right)$$

With the appropriate values of c',  $\phi'$ , and  $\gamma$  the depth  $z_0$  is about 4.5 ft. It will be seen from Fig. 7 that a tension crack of this depth penetrates just below the free-water surface, and the use of effective stress parameters, together with the full density  $\gamma$ , is therefore justified.

The conditions of (c) horizontal flow and (d) flow parallel to the slope can be readily evaluated from the stability coefficients published by Bishop and Morgenstern (1960).

The basic data are: c' = 180 lb/sq. ft,  $\phi' = 32^{\circ}$ ,  $\gamma = 139$  lb/cu. ft, inclination  $\beta = 28^{\circ}$ , height H = 42 ft, depth factor = 1.0.



Fig. 4. Flow net A



Fig. 5 Flow net B

For horizontal flow the pore pressure at any depth z is  $\gamma_w z$  and the total pressure is  $\gamma z$ . Thus, the pore-pressure ratio is:

$$r_u = \frac{\gamma_w}{\gamma} = \frac{62}{139} = 0.45$$

Referring to Plate 1 in the Paper by Bishop and Morgenstern, if  $c'/\gamma H = 0.05$  and  $\beta = 28^{\circ}$  and  $\phi' = 32^{\circ}$ , then m = 1.97 and n = 1.70. Now  $F = m - r_u n$ . Hence:

$$F = 1.97 - 0.45 \times 1.70 = 1.20$$

From Plate 2, if  $c'/\gamma H = 0.025$ , then m = 1.65 and n = 1.62. Hence:

$$F = 1.65 - 0.45 \times 1.62 = 0.92$$

But for c' = 180 lb/sq. ft,  $c'/\gamma H = 0.031$ . Thus, by interpolation, the factor of safety is:

$$F = 0.92 + \frac{0.031 - 0.025}{0.025} (1.20 - 0.92) = 0.99$$

For flow parallel to the surface the pore pressure at any depth z is  $\gamma_w z \cos^2 \beta$ . Thus:

$$r_u = \frac{\gamma_w \cos^2 \beta}{\gamma} = 0.35$$

Proceeding in exactly the same manner as before, but with this new value of  $r_u$ , the factor of safety will be found to be 1.14. As previously mentioned, this may be taken as an extreme upper limit.

It will be noted from Table 2 and Fig. 6 that as the slip circles move nearer the face of the slope the factors of safety with c' = 0 become progressively smaller. With the flow nets and the free-water surface lying below the top of the slope it is probable that Circle 2 may give a minimum factor of safety (about 0.65) for circles encompassing the full height of the slope. But a more critical case for c' = 0 will be a very shallow slip, approximating to a plane parallel to the slope, situated in the lower two-thirds of the slope where free-water coincides with ground surface (Figs 4 and 5). For this condition:

$$F = (1 - r_u \sec^2 \beta) \frac{\tan \phi'}{\tan \beta}$$

and with  $r_u$  equal to 0.45 and 0.35 for horizontal flow and flow parallel to the slope, the factors of safety with c' = 0 are 0.50 and 0.65, respectively.

Although horizontal flow corresponds roughly with the flow nets, this may be slightly too severe a condition even in the body of the slip. But it is almost certainly unrealistic at shallow depths where some weathering must have occurred due to frost and seasonal water content changes. In such a weathered zone the conditions of flow parallel to the slope is not a limiting case, as for the deep slip, but is the most realistic approximation. It therefore appears that a factor of safety of about 0.65 applies to the c' = 0 case both for relatively deep slip circles such as No. 2 (in Fig. 6) and for shallow slip planes.

This conclusion would seem to indicate that c' cannot possibly be equal to zero and, indeed, within the unweathered clay there is no reason to doubt the result. It is necessary, however, to give further consideration to a shallow slip since we might expect that in the weathered zone c' is, in fact, greatly reduced below the laboratory value measured on unweathered clay. In other words, whatever the factor of safety may be for c' = 0 along deep slip circles, it should presumably be approximately equal to unity along very shallow slip surfaces. Yet the foregoing calculations appear to be at variance with this expectation. Taking the condition of flow parallel to the slope, and with water level coincident with the surface, the factor of safety along a slip plane which is also parallel to the surface (Skempton and DeLory, 1957) is:

$$F = \frac{c' + (\gamma - \gamma_w)z\cos^2\beta \tan \phi'}{\gamma z\sin\beta\cos\beta}$$

where z is the depth of the slip plane. The zone of weathering is unlikely to extend to a depth of more than 4 ft, and probably only to 2 or 3 ft. For F = 1 the above equation leads to the following values of c' and  $\phi'$ :

	c' (lb/sq. ft) for $F = 1$						
φ	z = 2 ft	z = 3 ft	z = 4 ft				
44° 38° 32°	0 20 40	0 30 60	0 40 80				

Thus we see that whereas with c' = 0 an angle of shearing resistance of 44° is required (or, with  $\phi' = 32^\circ$ , the factor of safety is 0.65) yet with a value of c' of not more than about 60 lb/sq. ft limiting equilibrium is obtained with  $\phi' = 32^\circ$ . Moreover, it is not impossible that at very small effective pressures  $\phi'$  may be rather more than 32°. If, for example,  $\phi' = 38^\circ$  then c' in the shallow weathered zone could be as low as 30 lb/sq. ft.

The stability analyses may therefore be summarized as follows:

- (i) With c' = 180 lb/sq. ft and  $\phi' = 32^{\circ}$  the calculated factor of safety is about 1.05, ranging from 0.99 to 1.14 depending on the flow pattern.
- (ii) The actual factor of safety must, of course, be equal to 1.0, and the corresponding value of c' is about 160 lb/sq. ft. The difference between this figure and the laboratory value of 180 lb/sq. ft is within experimental and natural variations.
- (iii) With c' = 0 and  $\phi' = 32^{\circ}$  the calculated factor of safety on relatively deep slip circles, corresponding to the probable actual slip surface, is about 0.65.
- (iv) In order for the slope to stand at 28° with ground-water at the surface the value of c' in the shallow weathered zone must be roughly 60 lb/sq. ft, with  $\phi' = 32^{\circ}$ . This is about 30% of the laboratory value of c' measured on unweathered clay.

Finally, it is of interest to find the factor of safety as calculated from a  $\phi_u = 0$  analysis, using the undrained shear strength. This method is illogical for long-term stability problems, and especially in overconsolidated clays; but its use in such conditions is not unknown, even at the present time.

With a clay having an essentially constant strength with depth the critical slip circle, on the  $\phi_u = 0$  basis, will tend to be as deep as possible for slopes flatter than 52° (Taylor, 1937). At Selset the clay extends beneath the toe of the slope to a depth of 26 ft (Fig. 7). The relevant depth factor is thus 1.6. From Taylor's stability coefficients the corresponding value of the parameter  $c_u/\gamma H$ , for  $\beta = 28^\circ$ , is 0.165. The shear strength  $c_u$  required for limiting equilibrium is therefore:

## $c_u = 0.165 \times 139 \times 42 = 960$ lb/sq. ft.

The actual value of  $c_u$  is about 3,000 lb/sq. ft, and consequently the factor of safety based on undrained shear strengths and a  $\phi_u = 0$  analysis is approximately 3.0 if no allowance is made for tension cracks. With tension cracks extending to a depth of one-half the height of



Fig. 6 Typical slip circles used in analysis (the factors of safety are for flow net B)

the slope, the length of slip circle (on the  $\phi_u = 0$  basis) is reduced by only 12%. Thus the lowest factor of safety by this method is about 2.7.

# CONCLUSIONS

The Lodalen slip has already shown that even on a long-term basis the full cohesion intercept c' is operative on the actual slip surface in the lightly overconsolidated intact clay at that location. The present investigation at Selset has led to the same conclusion, but for a very long-term slip in heavily overconsolidated intact clay.

Moreover, in both cases, if c' is zero the calculated factors of safety fall to about 0.7, or even less. This is in strong contrast to the results of several analyses of very long-term slips in overconsolidated fissured clays, where the movements could only occur if c' was equal or close to zero.

The principal cause of this difference in behaviour is not fully understood, but it may be associated with local overstressing and softening of the clay in the vicinity of the fissures.



Fig. 7. Slip circle No. 7, flow net B

At Selset some movement was also taking place in the shallow zone of weathering. Calculations suggest that in this zone the cohesion intercept may be roughly 30% of the value measured on samples of the unweathered clay.

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# REFERENCES

BISHOP, A. W., 1955. "The use of the slip circle in the stability analysis of slopes." Géotechnique, 5:1:7-

BISHOP, A. W., and N. MORGENSTERN, 1960. "Stability coefficients for earth slopes." Géolechnique 10:4:129-150.

DELORY, F. A., 1957. "Long-term stability of slopes in overconsolidated clays." Ph.D. Thesis, University of London.

of London.
GIBSON, R. E., and D. J. HENKEL, 1954. "Influences of duration of tests at constant rate of strain on measured 'drained' strength." Géotechnique, 4:1:6-15.
HENKEL, D. J., 1957. "Investigations of two long-term failures in London Clay slopes at Wood Green and Northolt." Proc. 4th Int. Conf. Soil Mech., 2:315-320.
HENKEL, D. J., and A. W. SKEMPTON, 1955. "A landslide at Jackfield, Shropshire, in a heavily over-consolidated clay." Géotechnique, 5:2:131-137.
SEVALDSON, R. A., 1956. "The slide in Lodalen, October 6th 1954." Géotechnique, 6:4:167-182.
SKEMPTON, A. W., 1949. Final Report on the Geological and Engineering Problems... at Peterlee. Unpublished Report, Imperial College.

SKEMPTON, A. W., 1945. Final Report of the Geological and Engineering Problems ... at Peterlee. Unpublished Report, Imperial College.
 SKEMPTON, A. W., and F. A. DELORY, 1957. "Stability of natural slopes in London Clay." Proc. 4th Int. Conf. Soil Mech., 2: 378-381.
 TAYLOR, D. W., 1937. "Stability of earth slopes." J. Boston Soc. civ. Engrs, 24: 197-246.
# THE EFFECT OF DISCONTINUITIES IN CLAY BEDROCK ON THE DESIGN OF DAMS IN THE MANGLA PROJECT

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## INTRODUCTION

Mangla Dam Project is part of the Indus Basin Scheme, the construction of which forms the basis of the treaty signed in 1960 between India and Pakistan settling the disputes on water rights between them which had arisen following Partition. The scheme has been aided financially by a number of friendly countries, under the trusteeship of the World Bank. The West Pakistan Water and Power Development Authority (WAPDA) is responsible for the execution of works in West Pakistan for the Indus Basin Scheme.

## MANGLA DAM PROJECT

Mangla Dam is situated in the north of West Pakistan on the Jhelum River. The project layout is shown on Figs. 1 and 2. The Project consists of Mangla Dam (maximum height 116 metres, crest length 3,300 metres) across the Jhelum, with the Main Spillway (a gated structure) and the Emergency Spillway (an overflow structure) on the right bank. On the left bank, as an extension of Mangla Dam, is the Intake Embankment founded on a natural saddle. Beneath the Intake Embankment, through the bedrock, five tunnels lead to the Power Station and Tailrace and new irrigation canal known as the Bong Canal. These tunnels are used for diversion during construction of Mangla Dam and subsequently for power generation. Also on the left bank, beyond the Intake Embankment, Sukian Dam (maximum height 30 metres, crest length 5,200 metres) is constructed on a natural ridge to raise the rim of the reservoir. Some 22 kilometres to the east, Jari Dam (maximum height 71 metres, crest length 4,600 metres) closes another gap in the perimeter of the reservoir. Construction of the Project involves a total of 91,000,000 cubic metres of necessary excavation and 110,000,000 cubic metres of fill.

The heights of the dams given above are those for the present construction. The Project has been designed to allow crest levels of the dams to be raised 12 metres in the future with arise in reservoir conservation level of 15 metres.

## GENERAL GEOLOGY

The Project Area is formed of the Siwalik Formation which consists of overconsolidated stiff fissured clays and siltstones, weakly cemented sandstones and some gravels. The formation also contains a few thin beds of bentonitic clay usually associated with volcanic ash. The Siwalik Formation runs parallel to the Himalaya Mountains and has been subjected to folding and faulting during a period of uplift of the Himalayas, in Plio-Pleistocene times.

The site lies on one limb of an asymetrical anticline, the Changar anticline, the axis of which runs north west — south east just down-



Principal Works and Geology of Mangla Dam Project. Ouvrages principaux et géologie de l'aménagement de Mangla.

stream of the Project area (see Fig. 1). Within the Mangla site, the beds dip north-east at  $10^{\circ}$  to  $15^{\circ}$  but at Jari the dip of the beds is approximately  $45^{\circ}$ . South of the anticline the dip is almost vertical, this structure probably being associated with a fault.

The beds at the site are lenticular and vary in extend from a few metres to several kilometres. In general the lenses are large in area in comparison with the size of potential slip surfaces investigated in design, and for this purpose have therefore been considered as continuous beds.

The total thickness of the Siwalik Series is 4,600 to 6,000 metres of which approximately 1,800 metres has been eroded at Mangla and 1,200 metres at Jari.

## Gravels.

The gravel beds in the Siwalik series do not, in general, occur under the foundations of any of the structures in the Project. They have been used as a source of fill material for the construction of Jari Dam.

#### Sandstones.

The sandstones are friable, fine-to medium-grained and fissured and jointed. They are weakly cemented with calcium carbonate and clayey silt. In places, isolated lenses or nodules of well cemented material occur forming up to 10% of the rock by volume. Some of the younger sandstone beds outcropping at Jari appear to be uncemented and are better described as beds of sand.

### Clays.

The clays and siltstones have been grouped together for design purposes and referred to as clays. They are stiff, fissured and heavily overconsolidated. 141



#### CONTRACT DESIGN

In June 1961, tenders for the construction of the Project were invited. Designs for the structures had been prepared based on the geology as then known. Site investigations up to this time had consisted principally of mapping of outcrops and drilling of exploratory boreholes; some 6,500 metres having been drilled. The investigation had led to a general appreciation of the structure of the site and of the constituent bedrock as outlined above.

Samples from boreholes had been obtained and tested in the laboratory using both consolidated drained tests and consolidated undrained tests with pore pressure measurements. These tests were carried to moderately large strains in an effort to obtain residual effective shear strength parameters. Subsequent development of testing techniques have shown that these tests were not taken to sufficiently large strains and that the parameters obtained from them were in fact higher than the residual. Design parameters for bedrock obtained from these early tests are listed in Table 1.

The site investigations prior to the preparation of tender designs had also been directed to finding and proving sources of fill material. For Mangla Dam and the Intake Embankment, it was intended that

	Та	BLE 1		
Contract	Design	Bedrock	Par	rameters
		BEDBOCK		SANDSTON

DOCTOR	CLAY BEDR	OCK	SANDSTONE BEDROCK		
POSITION	c' (Kg/cm²)	φ'	c' (Kg/cm <sup>2</sup> )	φ'	
Mangla Dam Intake Embankment Sukian Dam Jari Dam & Rimworks	0.244 0 0 0	32° 30° 31° 28°	} •	38°	

the major portion of the rolled clay and rolled sandstone should be derived from necessary excavations such as the Intake, the Tailrace and the two Spillways. Gravel materials were intended to come from borrow areas in the river bed alluvium and terrace gravels. For Sukian Dam and Jari Dam, little, if any, material from necessary excavations would be useful as fill material. The effective shear strength parameters used in design for fill materials are listed in Table 2.

TABLE 2Contract Design Fill Material Parameters

MATERIAL	MANGLA DAM & INTAKE EMBANKMENT		SUKIAN DAM		JARI DAM	
	c' (Kg/cm²)	φ'	c' (Kg/cm²)	φ'	c' (Kg/cm²)	φ'
Rolled clay	0.22	22°	0.22	22°		
Rolled sandstone	0	36°	0	36°		
Sandstone/clay	0.22	20°		—	—	-
Gravel fill	0	40°	—	-	0	40°
Washed gravel fill .	0	40°	—	—		

Factors of safety agreed for the Contract designs for the dams are listed in Table 3.

# TABLE 3

Design Factors of Safety

CONDITION	FACTOR OF SAFETY (equal or greater than)	
Upstream		
Rapid drawdown	1.4	
Rapid drawdown with earthquake	1.0	
Downstream		
End of construction	1.4	
Earthquake (surfaces breaching the core)	1.2	
Earthquake (surfaces not breaching the core)	1.0	

Design of the dams was carried out using Bishop's analysis for circular arc slip surfaces [1] as programmed for an electronic computer by Little and Price [2]. In the majority of cross sections analysed, the critical surfaces only just penetrated the bedrock.

The topographically obvious site for a dam to close the eastern gap in the reservoir rim is the saddle near the town of Mirpur (see Fig. 1). This site is underlain by loess-silt some 300 metres thick. Because of doubts concerning the behaviour of the silt when saturated and subjected to an earthquake, this site was finally abandoned in favour of the Jari Dam site. This decision was not made until early in 1960. The investigations of the Jari Dam site were not complete at the time tender documents were prepared and the design of Jari Dam included in these documents was preliminary.

## DISCOVERY OF SHEAR ZONES

Site investigations continued throughout the tendering period. In January 1962 the Contract for the construction of the Civil Work was awarded to Mangla Dam Contractors, a consortium of United States contractors sponsored by Guy F. Atkinson Co. of San Francisco. After the Contractors had started stripping the foundations and exposing the bedrock, the pace of the investigations was greatly increased. Because of the hitherto preliminary nature of the design of Jari Dam, site investigations were concentrated at Jari at that time. It was here that a proper appreciation of the discontinuities in clay bedrock was first made. It was initially thought that these discontinuities were confined to Jari, where the tectonic folding as characterised by the dip had been greater than elsewhere on the site. It was subsequently discovered that such discontinuities were present almost everywhere in the Project area.

The discontinuities within the clay bedrock have been discussed in detail elsewhere [3, 4, 5, 6]. Here it is sufficient to describe them briefly under three headings.

i) Random Fissures. Such fissures, which are typical of most overconsolidated clays, had been observed and appreciated from the beginning of investigations. They can be up to one metre long and are polished and occasionally slickensided. Laboratory test results on carefully selected samples show that the strength of the clay bounding the fissure has been reduced almost to the residual value. Although these fissures are termed " random " there is some evidence that they occur with a preferred orientation and dip near structural features such as faults [5].



Une zone de cisaillement typique dans l'argile Siwalik à Jari montrant les lentilles de cisaillement et le plan principal de cisaillement.

ii) Thrust Shear Joints. These joints occur as a result of beds adjusting to folding under compressive stresses. Typically, they occur in conjugate sets (although one set is more pronounced than the other) dipping at about 20° to the bedding and extending from top to bottom of a bed. The strike of the joints is approximately parallel to the strike of the bedrock. The surfaces of the joints are polished, and laboratory tests on undisturbed samples show that the material on the surface of the joint has no more than its residual strength.

iii) Shear zones. Shear zones are planes within clay strata where the strains produced by folding have been concentrated, producing relative movement along bedding planes. They are a special form of fault. It has not been possible to measure the relative movement but it is probably several metres. The shear zones vary in thickness from a few millimetres to over one metre. In the thinnest of the zones, the relative movement is concentrated into one plane. In others the movement has caused the bedrock to be sheared into a number of lenses [6]. Detailed mapping has shown that shear zones often contain a continuous or 'principal' shear plane running parallel to the bedding for many metres (Fig. 3). Shear zones have been found in approximately two out of three clay beds and some beds contain more than one zone. They have been traced over distances up to 500 metres without encountering an end to them, and they are therefore assumed to be continuous throught the full extent of a lens or bed.

## THE EFFECT OF DISCONTINUITIES ON DESIGN

Shear Zones. The presence of shear zones resulted in the following design assumptions :

- i) Unless positive proof to the contrary has been obtained, all clay beds contain a shear zone.
- ii) All shear zones are continuous.
- iii) The clay in a shear zone has been reduced to the residual condition.

Fissures and Joints. The presence of random fissures and thrust shear joints was taken into account in assessing the probable shear strength parameters applicable to that part of a potential failure surface within a clay bed in any direction other than parallel to the bedding plane. For the major structures in the Project, potential failure surfaces usually passed through more than one clay bed. It was assumed for design purposes that, of that part of the potential slip surface passing through clay bedrock, half would be in intact clay and half would follow joints or fissures. Thus the residual factor R as defined by Skempton [7], was taken to be 50 %. This represents an estimate of the present condition. It was assumed for design purposes that this condition would, in the long term, remain unchanged in foundations which were substantially loaded by the construction as, for example, under the three dams.

However, there are on the site a large number of steep naturally occurring bedrock slopes. The country is arid and the water table is approximately at river level. These slopes probably maintain their present stability by negative pore water pressures. The conditions of such slopes which form part of the reservoir rim will be altered after the filling of the reservoir. The shear stress in the bedrock will increase with the increase in pore water pressure. In addition, annual drawdown of the reservoir will induce a cyclic loading. Both these phenomena were considered sufficient to justify the assumption that the clay bedrock would deteriorate to the residual condition with time [7]. For design purposes therefore, where the bedrock occurs on a slope, either cut artificially, as for example at the Intake Embankment, or cut naturally, as for example at the rimworks and abutments of Jari Dam, it has been assumed that, with time, the clay bedrock will deteriorate from its estimated present condition (R = 50%) to the residual condition (R = 100%).

The effective shear strength parameters of bedrock used in the final design have been derived from tests carried out since the Contract was awarded. Both consolidated drained triaxial tests and slow shear box tests with reversals have been made on specimens prepared from



Fig. 4 Correlations between Residual Angle of Friction and Clay Fraction for Clay Bedrock at Mangla and Jari. Corrélation entre l'angle de frottement résiduel et la fraction argile pour l'assise en argile à Mangla et à Jari.

block samples obtained from necessary excavations. For clay bedrock, specimens of both intact and natural sheared material were tested and the parameters thus obtained were correlated with clay fraction. Such correlations were obtained separately for Mangla and Jari and are shown in Fig. 4. In order to determine design parameters for each structure, very large numbers of clay fraction tests were done. Samples for these tests were obtained from scrapes both across beds and along bedding planes. The latter were almost entirely confined to shear zones. Inspection of the results from these tests enabled representative clay fractions to be determined for each area both along shear zones and across the bed. Invariably, the shear zones were found to have higher average clay fractions (and hence lower parameters) than the rest of a clay bed.

POSITION	CLAY BED Along bedding		BDROCK Across bedding		SANDSTONE BEDROCK	
	c' (Kg/cm <sup>2</sup> )	φ′	c' (Kg/cm <sup>2</sup> )	φ'	c' (Kg/cm <sup>2</sup> )	φ'
Mangla Dam	0 (R = 100 %)	18°	0.422 (R = 50 %)	26°	h	
Intake Embankment	0 (R = 100 %)	18°	$(\mathbf{R} = 100 \%)$	24°		
Sukian Dam (Western Half)	0 (R = 100 %)	18°	0.422 (R = 50 %)	25°	$\left  \right  = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	38°
Sukian Dam (Eastern Half)	0 (R = 100 %)	16°	0.422 (R = 50 %)	25°	(R = 100%)	
Jari Dam	(R = 100 %)	13°	0.35 (R = 50 %)	20°		
Jari Rimworks	(R = 100 %)	13°	$(\mathbf{R} = 100 \%)$	16°	J	

TABLE 4Final Design Bedrock Parameters

Compared with those obtained prior to tendering, the parameters derived for the final design are lower. Firstly, the average clay fraction is higher than the average of specimens obtained from boreholes during the initial investigations. Secondly, the improved testing techniques have shown that for clays with the same clay fraction, the residual parameters are lower than were measured originally. The actual parameters used in the final design are listed in Table 4.

Across bedding, clay bedrock parameters and sandstone parameters were combined for each surface considered in accordance with the mapped geology.

Routine triaxial testing of fill materials on undisturbed samples obtained from the dams during construction showed that the parameters for fill used in the initial design, and listed in Table 2, did not require revision for the final design.

#### METHOD OF ANALYSIS

The use of circular arc analysis is inherently unsuitable for material with the degree of cross anisotropy exhibited by the Mangla bedrock. Final design was therefore based on non-circular slip paths. In general, such analyses were carried out in a conventional manner by graphical methods assuming the interwedge total force acts at an angle to the horizontal of 12°. For the design of Sukian Dam, analyses were made using the method developed by Dr. N. R. Morgenstern and programmed for an electronic computer by Dr. V. E. Price [8]. This analysis, while making the same basic assumptions concerning the mode of failure, solves all the equations of static equilibrium, whereas the analysis done by hand does not solve the equations of moments. At the time of making these analyses, the programme was still under development and meaningful physical answers were obtained for only some of the surfaces considered. This deficiency in the programme, which has since been overcome, led to its abandonment for the design of Sukian Dam. However, the results from this form of analysis gave some support to the selection of 12° for the direction of the interwedge force.

All the dams in the Project, with the exception of Sukian Dam, have been designed so that raising of the crest level will be carried out by the addition of fill on the downstream shoulder and crest only; the upstream profile remaining unchanged. For this reason, all analyses of upstream dam slopes have been made for the raised dam while downstream analyses have been confined to the unraised dam.

In general, stability analyses have been carried out on cross sections taken normal to the dam centreline. However, cases arise where significantly lower factors of safety are given by sections taken at other angles to the dam centreline. It has been found that this occurs only when a change in cross section involves a significant change in topography; changes in apparent dip of the bedrock have only a minor effect on the factor of safety but so far as possible this effect has been taken into account in the final design of the dams.

#### CHANGES IN DESIGN

Following the discovery of shear zones and thrust shear joints and the consequent reassessment of shear strength parameters, changes in design were found necessary to provide adequate stability for the various structures. The measures taken were determined largely by the state of construction at the time and by the overriding consideration that neither the completion of the project nor the intermediate stage of diversion of the river should be delayed.

All major structures on the project required redesign to a greater or lesser extent. This paper describes only the changes found necessary for the dams on the Project. W. A. P. D. A. were naturally concerned about the increasing costs of the Project and therefore decided that the various measures proposed by the Project Consultants (Binnie & Partners) should be reviewed by their General Consultants (Harza Engineering Co.) assisted by an Advisory Panel. The review took place in November 1964. Although this review was restricted to the detailed proposals for the Intake Embankment and the Main Spillway, the principles used in design, which were endorsed by the Panel, were applicable to the other structures in the Project.

Jari Dam. As explained previously the Contract design for Jari Dam was preliminary. The design included a core of rolled silt derived from local deposits just upstream of the site, gravel shoulders and a triangular zone of silt downstream of the core. The discovery of shear zones and of clay bedrock with shear strength parameters lower than had been assumed in the Contract design, led to a revised design which was completed in January 1963. The dam has been constructed to this design. For a cross-section of Jari Dam see Fig. 5 in the paper by Eldridge and Little submitted to the present Congress [11].

The necessary increase in stability was achieved by a rearrangement of the position of the core; only at the abutments and in the adjacent reservoir rimworks was it found necessary to flatten the slopes. This design used clay bedrock parameters intermediate in value between the Contract design (Table 1) and the final design (Table 4); the final parameters being decided in 1965. Since then a design check has



been carried out using the final parameters. On mapping of the excavated foundations, the percentage of sandstone within the bedrock proved to be greater than assumed in the 1963 design. This has compensated for the decrease in strength of clay bedrock so that the dam and rimworks, as 'constructed, are considered to possess adequate stability.

Intake Embankment. The Intake Embankment as originally designed is shown in Fig. 5a. The tunnels were lined with a cast-in-place concrete lining except for the downstream third of their length where a steel lining has been inserted to resist the bursting pressure. The concrete linings were expected to leak under service so that pore water pressure in the surrounding bedrock would be approximately equal to the head of water in the tunnels.

At the time of redesign of the Intake Embankment, excavation of the intake and tailrace was substantially complete; as were the tunnels, the lower portions of the intake structures and the foundations of the power station. As a consequence, slope flattening was not possible. Having regard to problems of gate operation, it was in any case questionable as to whether a solution satisfactory in all respects could have been obtained by flattening the upstream slope. Design changes were therefore directed towards reducing pore water pressures in the bedrock.

The first step was to waterproof the tunnel linings and this was achieved by inserting steel linings over those portions of the tunnels lying within bedrock which were not already so lined. As the concrete linings had already been placed, the new steel linings were a smaller diameter; 7.9 metres as against the original 9.15 metres. These extra linings were inserted in four tunnels only. The other tunnel will be kept closed at the upstream end until such time as the power station is extended.

The diameter of the tunnels was determined by diversion requirements and not by their subsequent use for power generation. During diversion, one tunnel was in use with a reduced diameter and the loss in discharging capacity was compensated by a modest increase in height of the closure dam, thus increasing the head [9].

A blanket of rolled sandstone was placed over the bedrock. Rolled sandstone was found to have a permeability of one tenth to one hundredth of that of the fissured sandstone bedrock as measured *in* situ [10]. Less permeable materials such as clay and silt were considered, but because of their lower strengths these would have produced unacceptable factors of safety for failure surfaces passing along the blanket.

An extra drainage adit and wells were constructed on the upstream side. This adit was extended downstream on both flanks, so that, with the original adit, the drainage curtain surrounds the centre portion of the intake embankment on all four sides.

A limited amount of extra excavation of bedrock has been undertaken to remove some portions of critical clay beds. In addition, after diversion, an upstream toe weight placed under water will be added to resist potential deep slips.

All these additional works can be seen in Fig. 5b.

Sukian Dam. Sukian Dam is sited on a natural ridge so that the ground slopes away from the dam both upstream and downstream. The design of a dam in such an unusual position presented certain difficulties.

The Contract design of Sukian consisted of a rolled clay core with a slight upstream slope and rolled sandstone shoulders. Following the discovery of shear zones at Jari early in the Contract, the Sukian site was re-investigated by detailed mapping of outcrops supplemented by shallow trenches and the testing of specimens taken from block samples. This revealed the presence of low strength clays, but shear zones were not found. A design was therefore prepared assuming R = 50 % for clay bedrock both along and across bedding planes. This design was submitted to the Advisory Panel in 1964. The Contractors started excavation of the foundations a few days before the Panel met at Mangla and almost immediately shear zones were recognized in the foundations. This invalidated the design presented and the Panel did not, therefore, comment upon it.

The design presented to the Panel differed from the Contract design for Sukian in that rolled sandstone was substituted for clay in the core and the upstream slope was flatter. To achieve the desired stability after the discovery of shear zones, which reduced the along bedding clay bedrock parameters from R = 50 % to R = 100 %, excavation of clay bedrock beneath the foundations was carried out. The excavations, which were as much as 50 feet deep, were backfilled with rolled sandstone.

Sukian Dam is the only Project structure in the foundations of which have been found bentonitic clay seams. These seams, one of which is known to contain a shear zone, are not all continuous, although they have been assumed to be continuous for design purposes. R = 100 % parameters equal to c' = 0,  $\varphi = 12^{\circ}$  have been used.

The final design of Sukian Dam fails to comply with the agreed criteria for rapid drawdown with earthquake on the upstream side [11] (Table 3). It was not found possible to achieve this without unjustified expense. The probable movement along a slip surface during a design earthquake has been calculated to be under 30 cms using Newmark's method [12]. Factors of safety less than 1.0 have therefore been accepted on the grounds that should such a movement take place, the dam, being situated on a hill, can be repaired before the reservoir is again filled to conservation level. A typical cross-section of Sukian Dam will be found in Fig. 6 of Ref. 11.

Mangla Dam. A cross-section of Mangla Dam is shown on Fig. 6. The closure dam forming a berm on the upstream side exists only on the left half of the dam as can be seen in Fig 2. The closure dam was designed to be built across the river during the period from the end of one monsoon until the beginning of the next so that the river section of Mangla Dam could be built behind it. From the start of construction of the closure dam across the river, water was diverted



through the tunnels under the Intake Embankment.

Redesign of Mangla Dam using the final design parameters has shown the necessity of adding upstream and downstream toe weights in order to achieve the agreed factors of safety. Where the closure dam exists, no further weight is required. Elsewhere upstream toe weights are required as shown in plan in Fig 2. At the time of writing, the downstream toe weights have not been finally designed and have not therefore been shown on Fig. 2. It is anticipated that the downstream toe weights will be incorporated in, and form part of, the dam when it is raised.

#### SUMMARY

The design of the dams in the Mangla Project prepared for the Contract used residual effective shear strength parameters for clay bedrock, derived from laboratory tests on intact samples obtained from boreholes. Further investigation of the bedrock after exposure of the foundations of the dams by the Contractor's excavation operations revealed the existence of shear zones running parallel to the bedding and thrust shear joints across the bedding within clay bedrock. This led to a reassessment of shear strength parameters for use in design. The final parameters selected were lower than had been assumed in the original design and reflected the anisotropic nature of the bedrock. The designs of the dams were modified during the construction period to increase their stability.

## RÉSUMÉ

Le projet des barrages dans l'Aménagement de Mangla préparé pour le marché utilisait des paramètres de force de cisaillement résiduelle effective pour l'assise en argile, obtenus à partir d'essais de laboratoire sur des échantillons intacts retirés de forages. Un autre examen de l'assise, après que les fondations des barrages aient été exposées par les travaux de déblaiement de l'entrepreneur révéla l'existence de zones de cisaillement dirigées parallèlement à la couche d'assise et aux joints de cisaillement de charriage en travers de l'assise à l'intérieur de l'assise en argile. Ceci conduisit à une réévaluation des paramètres de force de cisaillement furent inférieurs à ce qui avait été supposé dans le projet d'origine et indiquèrent la nature anisotropique de l'assise. Les projets des barrages furent modifiés pendant la période de la construction pour augmenter leur stabilité.

#### REFERENCES

- [1] A.W. BISHOP: "The Use of the Slip Circle in the Stability Analysis of Slopes" Geotechnique March 1955.
- [2] A. L. LITTLE and V. E. PRICE: "The Use of an Electronic Computer for Slope Stability Analysis" Geotechnique, Sept. 1958.
- [3] P. G. FOOKES and D. D. WILSON: "The Geometry of Discontinuities and Slope Failures in Siwalik Clay" Geotechnique (in print).
- [4] A.W. SKEMPTON, P. G. FOOKES and K. E. HEAD: "The Nature and Occurrence of Tectonic Shear Zones in the Siwalik Clays" (in preparation).
- [5] P. G. FOOKES: "Orientation of Fissures in Stiff Overconsolidated Clay of the Siwalik System". Geotechnique June 1965.
- [6] A. W. SKEMPTON : "Some Observations on Tectonic Shear Zones" 1st International Congress on Rock Mechanics Lisbon 1966 (in print).

- [7] A.W. SKEMPTON: "The Long Term Stability of Clay Slopes" Geotechnique June 1964.
- [8] N. R. MORGENSTERN and V. E. PRICE : "The Analysis of the Stability of General Slip Surfaces". Geotechnique March 1965.
- [9] A. R. THOMAS and J. R. GWYTHER: "Diversion of the River Jhelum during Construction of Mangla Dam". Transactions Ninth Congress on Large Dams 1967.
- [10] A. L. LITTLE, J. C. STEWART, P. G. FOOKES : "Bedrock Grouting Tests at Mangla Dam, West Pakistan "Grouts and Drilling Muds in Engineering Practice, London 1963.
- [11] J. G. ELDRIDGE and A. L. LITTLE: "The Seismic Design of Earth Dams of the Mangla Project". Transactions Ninth Congress on Large Dams 1967.
- [12] N. M. NEWMARK : " Effect of Earthquakes on Dams and Embankments ". Geotechnique, June 1965.

# The Strength along Structural Discontinuities in Stiff Clays

La résistance au cisaillement le long des discontinuités structurales dans les argiles dures

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#### Summary

Field and laboratory investigations are described in which measurements have been made of the shear strength along principal slip surfaces, minor shears and joint surfaces in clay strata. In all cases the clays are over-consolidated, with liquidity indices ranging from +0.1 to -0.4. Along principal slip surfaces, in landslides and tectonic shear zones, the strength is at or very close to the residual (typically R > 0.95). For the investigations reported here this result appears to hold good even where there has been no renewal of movement during the past 10,000 years or more. Along minor shears, with somewhat irregular surfaces on which the relative movements have been small (probably of the order of millimetres), the strength is appreciably above the residual (R = 0.7 in the case investigated). Joint surfaces display a 'brittle-fracture' texture, and along these surfaces relative (shearing) movements have been zero or some exceedingly small amount. The few tests at present available on joint surfaces (in London Clay) indicate that the fracture which produced the joint virtually destroyed the cohesion intercept of the clay, but reduced the value of  $\phi'$  by only 1.5°. Movements of not more than 5 mm, however, are sufficient to bring the strength along the joint to the residual and polish the joint surface. All these discontinuities are therefore surfaces of weakness which reduce the strength of the clay mass far below the strength of the 'intact' material.

#### INTRODUCTION

In common with other sedimentary rocks, stiff clays contain numerous discontinuities in the form of bedding surfaces and joints. If, in addition, they have been sheared by landsliding or tectonic forces, shear zones will be formed containing minor shears and, usually, one or more principal slip surfaces (Skempton, 1966). These discontinuities (see Table I for a tentative classification) are surfaces of weakness the presence of which can reduce the strength of the clay mass, at least in certain directions, to values much below the strength of the 'intact' clay. Thus in many civil engineering works design is controlled largely or entirely by the strength along such discontinuities.

The significance of small non-systematic joints, known as 'fissures', has been recognised since Terzaghi (1936) first drew attention to them but the larger structural discontinuities in stiff clays, systematic joints and principal slip surfaces, have been strangely neglected in the literature. Their importance was forcibly drawn to the attention of the senior author during the years 1961–63 in connection with

## Résumé

Des études en terrain et au laboratoire au cours desquelles la résistance au cisaillement le long de surfaces principales de glissement, de surfaces secondaires et de joints dans les argiles sont décrites. Dans tous les cas étudiés les argiles sont sur-consolidées, avec des indices de liquidité variant de +0.1 à -0.4. Le long des surfaces principales de glissement dans les glissements de terrain et dans les zones de cisaillement tectonique la résistance est résiduelle ou très proche de la valeur résiduelle (typiquement R > 0.95). Les études décrites ici montrent que ce résultat semble tenir même quand il n'y a eu aucun renouvèlement du mouvement depuis au moins 10,000 ans. Le long des discontinuités secondaires à surfaces quelque peu irrégulières et qui n'ont subies que de faibles mouvements (probablement de l'ordre de quelques milimètres), la résistance est nettement au-dessus de la valeur résiduelle (R = 0.7, dans le cas étudié). Les joints ont une surface dont la texture a une allure de "rupture cassante", et le long de ces surfaces le mouvement relatif (de cisaillement) a été nul ou très petit. Les quelques résultats d'essais que l'on possède sur les surfaces de joints dans le London Clay indiquent que la rupture qui a produit le joint a détruit l'intercept de cohésion, mais n'a réduit la valeur de  $\phi'$  que de 1.5°. Des mouvements de moins de 5 mm sont cependant suffisants pour réduire la résistance le long des joints à la valeur résiduelle et pour polir la surface. Toutes ces discontinuités sont donc des surfaces de faible résistance qui réduisent la résistance de la masse bien en-dessous de la résistance "intacte" de l'argile.

TABLE I. Partial Classification of Discontinuities in Stiff Clays.

Group	Туре	Occurrence	Relative movement	
Depositional or Diagenetic	BEDDING SURFACES	Bedding planes Laminations Partings	Zero	
Structural	JOINTS 'Brittle fracture' surface	Systematic joints 'Fissures'	Practically Zero	
	MINOR SHEARS Non-planar, slickensided	Small displace- ment-shears Riedel- and thrust-shears	Less than 1 cm	
	PRINCIPAL DIS- PLACEMENT- SHEARS Subplanar, polished	Principal slip surfaces in : landslides faults bedding-plane slips	More than 10 cm	

works on the Mangla Dam Project and on the M6 motorway. A programme of research was then initiated embracing three interrelated topics:

- (i) naturalistic studies of joints and shear zones in the field, supplemented by model tests on shear zones in the laboratory,
- (ii) investigations of the fabric of clays, especially sheared clays, using the optical microscope and thin sections prepared by the Carbowax technique,
- (iii) measurements of the strength and stress-strain characteristics along discontinuities, and, for comparison, in the adjacent 'intact' clay.

Work on these lines is still actively in progress, but some results on the first two aspects have been published by Skempton (1964, 1966), Fookes (1965), Fookes & Wilson (1966) and Morgenstern & Tchalenko (1967). So far as the third item is concerned, at the time of the Rankine Lecture in 1964 data were available for the strength along slip surfaces only in landslides at Walton's Wood and in the brown London Clay; although in both cases it could be shown that the strength was equal to the residual strength of the clay. Indeed, from general considerations it seemed that under ideal conditions this result was necessarily true; and the concept of residual strength might well provide the unifying principle in the study of strength along discontinuities in clays and rocks (Skempton, 1964). Since then additional information has been obtained from landslides at Guildford and Sevenoaks and tectonic shear zones at Mangla, where the shearing has been caused by folding of the strata. In addition, at Wraysbury, tests have been made for the first time on joint surfaces in a stiff clay.

As a result of this work, carried out during the past four years, we are now able to present data for slip surfaces in six clays and for one set of minor shears and one set of joint surfaces. The results are summarised in Table II.

No investigations can yet be reported on the strength along bedding surfaces in stiff clays, or on the laminations in compaction shales. Information is also lacking on the strength along faults, although there is no reason to suppose that the principal slip surfaces in a fault differ essentially from those in landslides or in the tectonic shear zones at Mangla.

#### **Methods of Testing**

The tests reported here were carried out in standard (6 cm) shear boxes or in the triaxial compression apparatus (specimens  $1^{1}/_{2}$  ins diameter  $\times$  3ins high) using ball bearings on the top cap and rotating bushes to reduce piston friction. The specimens were consolidated for two days and sheared

TABLE II. Summary of Test Results.

Residual Consistency Clay Peak fraction  $\phi'_r$  if  $c'_r=0$ Site Clay Ref. Condi-PI $\Delta s \; \frac{-1}{s \; - \; s_r}$ CF $\phi'_r$ LL PL PI LIф Rtion w  $\overline{CF}$ Walton's Wood Weathered T.P.3 intact\* 29 57 26 31 +0.1069 0.45 320 21  $13^{\circ}$ Carboniferous slip 32 +0.1910 0 13° 13° 9 0.014 0.98 mudstone Guildford Brown London Clay 2, K 51 + 0.02intact\* 33 83 32 55 0.91 330  $20^{\circ}$ +0.0457 12° 12°  $15^{\circ}$ 11 0.026 0.97 slip 34 70 60 Guildford Brown London Clay 33 87 31 56 + 0.04 56 1.0060  $12^{\circ}$ 50  $12^{\circ}$  $14^{\circ}$ 10 0.023 0.98 slips Dedham Sudbury Hill 18° Sevenoaks Atherfield Clav WB1 intact\* 34 75 29 46 +0.1157 0.79 430 59  $10.5^{\circ}$ 70 10.5°  $12^{\circ}$ 9 0.015 0.98 slip 36 +0.1580 WB2 27 69 29 40 -0.05 47 0.82 720 intact\* 24 30 +0.03 51 100  $15^{\circ}$ 30 14° 15° 70 0.068 0.93 slip 16° 0 Reworked Ather-F 2 slip 34 71 31 40 +0.08 58 0.69 20  $16^{\circ}$  $16^{\circ}$ 18 field Clay Weathered Weald 317 slip 27 69 27 42 0.0071 0.59 60 15 50 15 15.5 10 24 62 26 36 -0.0650 0.72 70 15.5° 40 15.5 16.5 Clay 300 slip 26 Upper Siwalik Clay intact 15 53 25 28 -0.36 45 0.62 1000 23 Jari 12° Mangla Project D 17 59 25 34 -0.2345 0.75 95 0 12° 12° 95 0.034 0.96 slip (b) Joints and Minor Shears. 0.39 52 Sukian Upper Siwalik Clay intact 15 58 31 0.60 1200  $22^{\circ}$ Mangla Project 512 minor 16 60 28 32 --0.38 52 0.62 360 16° 0 14 14<sup>c</sup> 450 0.300 0.70 shear Blue London Clay Wraysbury intact 28 69 26 43 +0.02 58 0.74 650 20  $18.5^{\circ}$ 1 A 28 73 28 45 0.00 55 0.81 70 30 16°  $17^{\circ}$ 160 0.200 joint

(a) Principal Slip Surfaces.

Notes (i) Values of c',  $c'_r$  and  $\Delta s$  given in  $lb/ft^2$  (205  $lb/ft^2 = 1$   $t/m^2$ ).

(ii) For Sukian and Wraysbury  $\Delta s$  is quoted at  $\sigma_n = 2,000 \text{ lb/ft}^2 (\simeq 1 \text{ kg/cm}^2)$ .

(iii) Intact\* refers to clay a few inches from the shear zone; in other cases 'intact clay' refers to samples from the site with index properties similar to those of the clay in the shear zone.

in a period of at least 3 to 5 days; except for reversals in the shear box tests, each of which was accomplished in 3 days. All the clays were fully saturated in their natural state and in the laboratory tests.

Throughout the paper the strength parameters are expressed in terms of the effective normal pressure on the shear plane as given by Terzaghi's law

$$\sigma'_n = \sigma_n - \iota$$

where  $\sigma_n$  is the total pressure acting normal to the plane and u is the pore water pressure in the clay.

In the shear boxes one traverse in 2 days corresponds to a rate of movement of about 4 mm per day. Ancillary experiments by the authors on London Clay, briefly reported by Skempton (1965), show that the residual strength decreases very slightly with decreasing rates of shear; but for most purposes the effect can be neglected, even if the rate is reduced by two orders of magnitude. This result has been confirmed by Kenney (1967) for a wide range of clays.

Samples containing a discontinuity are taken in such a manner that the test specimen can be set up with the discontinuity coinciding as nearly as possible with the seperation plane in the shear box or, in the triaxial test, with the discontinuity inclined at about  $50^{\circ}$  to the horizontal and lying well clear of the end caps. In the triaxial test, corrections for area change and the strength of the rubber envelope and filter strips are of great importance; and the results are evaluated by calculating the normal and tangential stresses on the failure plane, the inclination of which is measured in each test.

In general the triaxial test is to be preferred for measuring the strength along discontinuities, as the kinematic restraints are minimal; although in some cases, particularly in hard clays, the shear box is more practicable from the point of view of preparing the specimens, and it has the additional advantage that if the residual strength is not reached in the first traverse it can be determined by subsequent reversals.

When testing principal slip surfaces and joints, however, we found almost invariably that the residual strength was attained before reaching the limit of the triaxial test or before completing the first traverse of the shear box. Nevertheless, it was satisfactory to be able to prove this by subsequent reversals in the shear box. Moreover, when testing the minor shears from one of the Mangla shear zones a single traverse of the box was insufficient to define the residual and so, in this case, the triaxial test would have had severe limitations.

The peak strength of 'intact' clays can be measured in either type of apparatus, but the maximum strain which can be applied to the specimen in the triaxial test is usually far too small to permit the determination of residual strength in an originally 'intact' clay. In all cases reported here this strength was measured by reversal shear box tests; four to six reversals generally being required before the strength eventually falls to a value closely approximating to the residual.

It must be emphasised that by the peak strength of intact' clay we refer simply to the average peak strength as measured in drained tests on specimens not containing an obvious macroscopic discontinuity; the specimens having the dimensions  $6 \times 6 \times 2$  cm in the shear box or  $1^{1}/_{2} \times 3$  ins  $(3.8 \times 7.6 \text{ cm})$  in the triaxial test. It is best to regard these tests as giving no more than a conventional measure of the 'intact' strength, which is certainly lower than the strength of the lumps of apparently unfissured material (although even these may contain micro-fissures) and a little higher than the strength of the clay mass in those parts unaffected by systematic joints and shears (see, for example, the tests by Marsland & Butler (1967) on Barton Clay).

In the investigations reported here the specimens of 'intact' clay were generally taken a short distance away from the shear zone but at Mangla, where the shear zones are often limited by lithological boundaries, the strength of intact clay having similar index properties to those in a shear zone under investigation was found by interpolation among test results on samples from other, unsheared clay beds in the vicinity.

#### Symbols

Except where noted in Fig. 11, the symbols used on all the graphs conform to the following system:



#### **General Considerations**

When a clay is subjected to simple shear five successive stages can be recognised as the deformations increase (Skempton, 1966). The first stage, before the peak is reached, is one of continuous non-homogeneous strain. In the second stage, which occurs at or just before the peak, 'Riedel shears' are formed (Fig. 1). They lie *en echelon* at an inclination usually between  $10^{\circ}$  and  $30^{\circ}$  to the direction of general movement (the a axis), and conjugate shears  $R^*$  are sometimes seen. With further movement a third stage is soon reached at which slip along the Riedel shears is no longer kinematically possible, and the clay is compelled to develop new slip surfaces parallel or subparallel to the a axis. These are the 'displacement shears.' With greater movements the displacement shears extend and eventually, in the fourth stage, some of them link up to form a 'principal displacement shear' or 'slip surface.' This surface is undulating, since the shears involved were not originally all in line. 'Thrust shears,' typically inclined at about 160° to the a axis, also tend to develop in the third and fourth stages. In the fifth stage the slip surface undergoes appreciable flattening as a result of still greater movements.

The Riedel-, thrust- and displacement-shears of limited extent are here collectively grouped under the heading of 'minor shears.' The relative movements along individual minor shears are small, probably of the order of a few millimetres at most. Once a principal slip surface has formed, subsequent movements are mostly concentrated along it; and large displacements can then take place without a fundamental change in the pattern of shears. We have, as yet, no exact knowledge of the movements necessary to reach the fifth stage, although it certainly amounts to several centimetres and may be more than 10 cm.



Fig. 1. Successive stages in the development of a shear zone in clay, from laboratory tests (Skempton 1966). Etapes successives dans le développement d'une zone de cisaillement dans l'argile, d'après des essais au laboratoire. (Skempton 1966).

The development of shears in clays is accompanied by particle orientation. This has been observed in laboratory experiments by Weymouth & Williamson (1953) and other authors, whilst studies of natural clays by Mr. K. R. Early and Morgenstern & Tchalenko (1967) have shown that a principal slip 'surface' in fact consists of a band, typically 10 to 50 microns wide, in which the clay particles are strongly orientated more or less in the direction of movement. Particle orientation is presumably the cause of the 'polish' so often seen on shear surfaces in clays.

A flattened principal slip surface in which the particles have attained their maximum degree of orientation must possess the minimum possible resistance to shear, and this is the 'residual strength' of the clay. At any given effective normal pressure its magnitude depends upon the amount and the nature of the clay minerals present (Skempton 1964, Kenney, 1967).

If a test is carried out on a flat slip surface with full particle orientation we should expect to find a stress-strain curve of the type shown by line (1) in Fig. 2; provided the sense of movement in the test is the same as that which had produced the slip surface in nature. This curve shows no peak and at a small displacement reaches the residual strength  $s_r$ . It will be shown in the present paper that such curves are frequently obtained in tests on principal slip surfaces.

Quite often, however, the test shows a small peak; although with further displacement the strength soon falls to the residual – see curve (2) in Fig. 2. The difference between the peak and the residual strength is denoted by  $\Delta s$ . The reasons for  $\Delta s$  being greater than zero in tests on





Courbes contrainte-déplacement pour des essais sur une discontinuité et dans l'argile "intacte".

principal slip surfaces include the following 'real' factors: the slip surface is not planar or there may be some asperities; all the clay particles may not be fully orientated; or some 'bonding' effect may have developed during the period since movement last occurred. In addition there can be 'fictitious' reasons arising from errors in setting up the specimen or in applying shear movements in the wrong direction. Even with the most careful testing procedures, however, when experimental errors are minimal, it may be found that  $\Delta s$  is not zero. But on principal slip surfaces the value of  $\Delta s$  is rarely more than 5 per cent of the difference between the residual strength  $s_r$  and the peak strength of the 'intact,' unsheared clay s. In other words, on principal slip surfaces the residual factor R is rarely less than 0.95 and, for curves of type (1) in Fig. 2, R is equal to 1.0.

The complex of minor shears and slip surfaces constitutes a shear zone. In the landslides described in this paper the shear zones are between 0.5 and 5 cm wide. The tectonic shear zones at Mangla, which occur in much harder clays, are about one order of magnitude larger and the minor shears are frequently several centimetres apart, forming the boundaries of well defined shear lenses. On this scale it is possible to prepare specimens including a single minor shear. Tests on such specimens show values of  $\Delta s$  between 20 and 40 per cent of  $(s-s_r)$ , or values of R between 0.6 and 0.8. Clearly, in the minor shears, the strength is still well above the residual; due largely to their decidedly nonplanar surfaces and the small relative movements which have taken place along them in nature. Moreover, two reversals in the shear box tests were usually necessary to reach the residual strength.

The origin of systematic joints is not fully understood. They can extend for lengths of several metres, or even tens of metres, and a set of such joints may maintain an essentially constant direction for long distances. They are, typically, normal to bedding and their surfaces are characterised by a 'brittle fracture' texture often displaying delicate conchoidal ridges and plumose structure (Hodgson, 1961). It seems clear from this texture that relative shear movements along the joints must have been zero or exceedingly small but, at the same time, there has undoubtedly been a complete fracture. An analogy can perhaps be found between joints and the 'axial cleavage' or splitting failure observed in compression tests on rocks (Gramberg, 1965).

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The study of joints in sedimentary strata has not, until very recently, included stiff clays; although systematic joints similar to those in sandstones, shales and mudstones certainly occur in the London Clay (this paper) and in Pliocene clays in Italy (Esu, 1966) and probably in many others.

Microscopic examination of the fabric of clays adjacent to a joint surface has yet to be carried out. But it is reasonable to suppose (i) that the fracture must have destroyed the cohesion (any 'apparent' cohesion being due to interlocking on the slight irregularities of the joint surface) and (ii) that the formation of the joint would have caused scarcely any orientation of clay particles parallel to the surface. Preliminary investigations reported here on strength tests on systematic joints in the London Clay are consistent with these suppositions, indicating an almost complete loss of the cohesion intercept but a relatively high value of  $\phi'$ . After the peak, however, a relative shearing movement of not more than 5 mms is sufficient to reduce the strength to the residual and impart a 'polish' to the joint surface.

## **CASE RECORDS**

The following test results have been obtained in the course of investigations mainly at five sites. It is hoped eventually to publish a full account of each site but, for our present purpose, attention is focussed on the tests carried out on the discontinuities, with brief descriptions of the geological setting. The case records are considered under three headings: principal slip surfaces, minor shears and joint surfaces.

#### **Principal Slip Surfaces**

In most landslides and tectonic shear zones large movements have taken place, chiefly concentrated on 'principal slip surfaces.' Typically these surfaces are polished and subplanar, with striations in the direction of movement, and they may extend over considerable areas. Tests on such surfaces are reported from four sites.

Walton's Wood. At the locality of Walton's Wood in Staford shire the M6 motorway crosses a large landslide in reworked weathered mudstone of Upper Carboniferous age (Fig. 3). The toe of the slope was eroded by a glacial drainage channel during a retreat phase of the Weichselian glaciation; and landsliding has been active intermittently ever since; until stabilisation works were undertaken a few years ago. In the early stages mass movement was probably accentuated by solifluction.

Field investigations with numerous borings and exploration pits were carried out during 1962–63 by Mr. K. R. Early of Soil Mechanics Ltd. Shear zones containing principal slip surfaces were discovered at each of the points marked by a short heavy line in Fig. 3. The bulk of the landslide material comprises a moderately stiff clay matrix with fragments of sandstone and mudstone; but within a distance of several centimetres from a shear zone only clay (including some harder clay pellets) is encountered. This is the 'intact' clay.

Examination of thin sections by Mr. Early showed little or no preferred orientation in the intact clay; but he discovered that the slip surface consists of a band about 20 to 30 microns wide in which the clay particles are strongly orientated approximately along the a axis, and this band lies within a shear zone up to about 2 cm wide containing many minor shears. Subsequent work by Dr. J. S. Tchalenko has shown that the clay particles in the shear zone (excluding those involved in minor shears) are generally orientated between  $145^{\circ}$  and  $160^{\circ}$  to the *a* axis, forming a 'compression texture' probably lying normal to the maximum principal stress. In addition, the clay particles within the 20 to 30 micron bands constituting the main slip surfaces are orientated at about  $160^{\circ}$  to  $170^{\circ}$ , with exceedingly thin boundary layers in which the particles may be almost parallel to the a axis. A section showing these various features is given in Fig. 4.

From Fig. 3 it will be seen that the overall displacement of the landslide is large. This displacement is distributed among many slip surfaces, but even so the movement in any one shear zone may amount to several metres.

Shear box and triaxial tests made in the laboratories at Soil Mechanics Ltd. and at Imperial College gave the following parameters for the peak strength of the 'intact' clay:

$$c' = 320 \text{ lb/ft}^2 (1.55 \text{ t/m}^2) \qquad \phi' = 21^\circ$$



Fig. 3. Geological section through Walton's Wood landslide, Staffordshire.

Coupe géologique du glissement de terrain à Walton's Wood, Staffordshire.

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Tests on the slip surfaces gave a very consistent set of results, corresponding to the parameters

$$c'_r = 0 \qquad \phi'_r = 13^\circ$$

The results of all these tests are plotted in Fig. 5, and two typical stress-strain curves, for intact clay and a slip surface test respectively, are shown in Fig. 6.

In all seven tests on slip surfaces  $\Delta s$  was either zero, as in Fig. 6 (b), or some small quantity. Individual values are plotted in the lower diagram in Fig. 5; and the average value of  $\Delta s$  is only 9 lb/ft<sup>2</sup> (4.5 gm/cm<sup>2</sup>). In other words the stress-strain curves in the slip surface tests either conform exactly with the type (1) curve in Fig. 2 or show a very small peak effect.

Since, at the time of the investigations, the landslide was still subject to intermittent movements its minimum factor of safety must have been equal to 1.0. The average shear strength and effective normal pressure on a given slip surface could therefore be estimated. The results of two such calculations are plotted as triangular symbols in Fig. 5 and they indicate strengths only slightly higher than those corresponding to c'=0,  $\phi'=13^\circ$ . Thus, regarding the landslide as a shear test, carried out in nature on a very large scale, we obtain values of strength closely comparable to those measured in the laboratory.



Fig. 4. Walton's Wood; details of shear zone. Walton's Wood; détails de la zone de cisaillement.

The great difference between the strength of the intact clay and the strength along the slip surfaces remained for some time an unresolved problem. Before the end of 1963, however, not only had the significance of the particle orientation been appreciated but we were able to prove experimentally that, at large strains, the strength of originally intact clay fell to values approximately equal to the strength as measured on the natural slip surfaces; as shown in Fig. 5 by the four points represented by dotted crosses. These tests were carried out in the shear boxes; five or six traverses being required to reduce the strength to the residual – see Fig. 6 (a). A photograph of the slip surface produced in one of the reversal tests is reproduced by Skempton (1964).

The investigations at Walton's Wood are of outstanding importance as they revealed for the first time all the features



Fig. 5. Walton's Wood; shear test results (1963). Walton's Wood; résultats d'essais (1963).



Fig. 6. Walton's Wood; typical stress-displacement curves. Walton's Wood; courbes contrainte-déplacement typiques.

essential to the concept of residual strength in clays. These may be summarised as follows:

- (i) the strength along a principal slip surface is at or very close to the residual,
- (ii) this residual strength may be much lower than the (peak) strength of the intact clay,
- (iii) after displacements of several centimetres the strength of the intact clay falls approximately to the residual value;
- (iv) the clay particles are strongly orientated along the slip surface, practically in the direction of movement.

The index properties showed rather too great a scatter for any certain indication of significant differences between the intact clay and the shear zone. Average values are:

$$LL = 57$$
  $PL = 26$   $PI = 31$   
clay fraction ( $< 2\mu$ ) = 69  
activity = 0.45

The natural water contents, however, did show a more consistent pattern; average values being 29 for the 'intact' clay (corresponding to a liquidity index of +0.10) and 32 for the shear zones. An increased water content in shear zones has been noted in the London Clay (Skempton, 1964) and at Sevenoaks (present paper) and is a characteristic feature of stiff clays, due to their tendency to dilate when sheared.

X-ray analysis of the minerals by Professor R. E. Grim, in December 1963, showed no essential difference between the intact clay and the clay in a shear zone, the quantitative estimates being (in percentages)

A high kaolinite content is consistent with the activity of

Surrey.

0.45, but chemical analyses, reported by Mr. R. H. S. Robertson, suggest a somewhat lower percentage of this mineral balanced by a rather higher quartz content together with a small amount of goethite and other minerals.

Guildford. The north face of Stag Hill near Guildford, Surrey, is the site of a large landslide in brown London Clay on a  $9^{\circ}$  slope (Fig. 7). Following the decision to construct an important building project at this location, field investigations were carried out by the senior author with Mr. Nicholas Paine and Dr. J. N. Hutchinson in 1965–66, with laboratory tests by the junior author.

The depth of weathering (oxidation), shown by the brown clay, varies from nearly 50 ft. (15 m) in the upper part of the slope to about 30 ft. (9 m) at the bottom. These depths are considerable and indicate that the slope is receding very slowly indeed. Nevertheless this particular landslide is comparatively recent. Certainly it must have occurred more than 110 years ago, since this was the age of a tree on the back scarp when cut down in 1966, but the scarp is still a clearly expressed feature (in some places more pronounced than in Fig. 7) which is unlikely to be more than a few centuries old, and there are signs of fresh movements at the toe.

The main slip surface, which was highly polished, could be seen in each of the three exploration pits. A thin section examined under the microscope (Morgenstern & Tchalenko, 1967) showed that the slip surface consists of a band roughly 50 microns wide in which the particles are strongly orientated. It is located at or near the lower boundary of a shear zone about 0.6 cm in width containing numerous minor shears (Fig. 8). Outside the shear zone the degree of particle orientation is slight.

Tests on specimens of 'intact' clay, taken several centimetres away from the shear zone, showed peak strengths expressed by the parameters (Fig. 9)

$$c' = 330 \text{ lb/ft}^2 (1.6 \text{ t/m}^2) \qquad \phi' = 20^\circ$$



Fig. 7. Section through landslide at Stag Hill, near Guildford, Coupe du glisse

A.W.S. & N.P. August 1965

Coupe du glissement de terrain à Stag Hill, près de Guildford, Surrey.

Fourteen tests were carried out on slip-surface specimens. Nine of these gave stress-strain curves of type (1), as shown in Fig. 10 (b). In the other five tests small peak effects were observed, but subsequent displacements of a few millimetres were sufficient to reduce the strength to the residual value. The best linear fit to the fourteen residual strength points gives the following parameters

$$c'_r = 60 \text{ lb/ft}^2 (0.3 \text{ t/m}^2) \qquad \phi'_r = 12^\circ$$

but a more correct representation of the results is probably given by a curved line as shown in Fig. 9.

Reversal shear box tests on the 'intact' clay gave residual strengths approximately equal to those measured on the slip surface; see Fig. 9 and Fig. 10 (a).

Average values of the Atterberg limits are

$$LL = 83$$
  $PL = 32$   $PI = 51$ 

Particle size determinations on small samples taken to include as much as possible of the slip surface indicated a slightly greater clay fraction than in samples of intact clay (57 per cent as compared with 55 per cent). The average water content of the intact clay is 33, corresponding to a



Fig. 8. Guildford; details of shear zone, Pit 11. Guildford; détails de la zone de cisaillement.



Fig. 9. Guildford; test results. Blocks 2 & K. Guildford; résultats d'essais.

liquidity index of +0.02. In the shear zone the average water content is 34. The activity is about 0.9; a value compatible with the fact that illite is the predominant clay mineral in London Clay (Skempton, 1953).

The residual strengths as measured on the Guildford specimens are re-plotted on a larger scale graph in Fig. 11, together with three points obtained from tests made in 1964 at Imperial College and by Mr. W. Watson in the laboratory of Soil Mechanics Ltd. on samples taken from the slip surface in a landslide in brown London Clay at Dedham, Essex. Also in this graph is shown a point determined by calculations of shear strength and effective normal pressure on the slip surface of a landslide in a railway cutting at Sudbury Hill, Middlesex (Skempton, 1964). Here, as at Walton's Wood, intermittent movements were taking place at the time of the field investigations, and a full scale shear test was thus in progress in nature.

The seventeen points in Fig. 11 cover a pressure range from 600 lb/ft<sup>2</sup> (0.3 kg/cm<sup>2</sup>) to 5,000 lb/ft<sup>2</sup> (2.5 kg/cm<sup>2</sup>). Within this range the best linear fit is expressed by the parameters

$$c'_r = 50 \text{ lb/ft}^2 (0.25 \text{ t/m}^2) \qquad \phi'_r = 12^\circ$$

There is a strong suggestion, however, that the envelope is non-linear, especially at low normal pressures. More data are required before the precise relation can be established, but for pressures between 600 and 1,200 lb/ft<sup>2</sup> reasonable linear approximations are

or 
$$c'_r = 30 \text{ lb/ft}^2 (0.15 \text{ t/m}^2)$$
  $\phi'_r = 13^\circ$   
 $\phi'_r = 0$   $\phi'_r = 15^\circ$ 

and for pressures between 1,200 and 5,000 lb/ft<sup>2</sup>

$$c'_r = 70 \text{ lb/ft}^2 (0.35 \text{ t/m}^2) \qquad \phi'_r = 11.5^\circ$$

Finally, it may be noted that the average value of  $\Delta s$  for the sixteen tests on slip surfaces from Guildford and Dedham is only 10 lb/ft<sup>2</sup> (5 gm/cm<sup>2</sup>).

Sevenoaks. The escarpment south of Sevenoaks, overlooking the Weald of Kent, is composed of Hythe Beds



Fig. 10. Guildford; typical stress-displacement curves. Guildford; courbes contrainte-déplacement typiques.

STRENGTH ALONG STRUCTURAL DISCONTINUITIES IN STIFF CLAYS



Fig. 11. Summary of residual strengths on principal slip surfaces in brown London Clay.

(weak sandstones) overlying Atherfield Clay and Weald Clay, all of Lower Cretaceous age (Fig. 12). The Sevenoaks By-Pass is now under construction across the escarpment. In connection with this work, field studies during the past two years by Mr. Alan Weeks of the Kent County Council and the senior author, with assistance from Dr. N. R. Morgenstern and Mr. D. J. Shearman, have revealed the following features: (i) cambering and landsliding of the Hythe Beds, accompanied by 'valley bulging' in the Weald and Atherfield clays and silts; (ii) a widespread sheet of solifluction material extending more than 1/2 mile from the escarpment, dating from the Weichselian glaciation; (iii) solifluction lobes of smaller extent, overlying the main sheet and dating almost certainly from Zone III of the Late-Glacial sequence. All the tests reported here on samples from the Sevenoaks were carried out by the junior author.

The escarpment landslides are associated with a principal slip surface in the Atherfield Clay. Tests on specimens cut from a large block sample, taken from the position shown in Fig. 13, gave the following parameters for the peak strength of the 'intact' clay

$$c' = 430 \text{ lb/ft}^2 (2.1 \text{ t/m}^2) \qquad \phi' = 18^{\circ}$$

Résumé des résistances sur les discontinuitiés principales dans le brown London Clay.

With the single exception of a shear box test in which the slip surface was presumably not aligned correctly, tests on slip surface specimens gave values of  $\Delta s$  equal to zero or some very small quantity. The residual strengths, when plotted against effective normal pressure, show an envelope with marked curvature at pressures less than about 2 kg/cm<sup>2</sup> (Fig. 14); approximate parameters in this range being

or 
$$c'_r = 70 \text{ lb/ft}^2 (0.35 \text{ t/m}^2)$$
  $\phi'_r = 10.5^\circ$   
 $\phi'_r = 0$   $\phi'_r = 12^\circ$ 

At higher pressures (2 to 7 kg/cm<sup>2</sup>) the envelope becomes linear with a slope of  $8^{\circ}$ .

Reversal shear box tests lead to values of residual strength practically identical with those measured on the slip surface, as shown by the dotted crosses in Fig. 14.

Average values of the Atterberg limits are

LL = 75 PL = 29 PI = 46

The water content of the intact clay is 34, giving a liquidity index of +0.11, and the water content near the slip surface is 36. The clay fractions of the intact and sheared clays are



Fig. 12. Geological section of escarpment south of Sevenoaks, Kent. Section through lobe 'F'

A.W. 5. 6. A.O.W. 15. 5. 07.

Coupe géologique de la falaise au sud de Sevenoaks, Kent.

respectively 57 and 59 per cent, corresponding to an activity of about 0.8.

Mineralogical analyses of this clay, and other samples from the Sevenoaks investigations, are not yet completed.

Another block sample from a similar situation has been taken in a stiffer and rather more silty facies of the Atherfield Clay. The test results are summarised in Fig. 15 and Table II, and call for no special comment except that the average value of  $\Delta s$  in the slip surface specimens is unusually high, at about 70 lb/ft<sup>2</sup> (35 gm/cm<sup>2</sup>). This may be attributed to minor irregularities in the slip surface and partly to the greater strength of the clay.

The solifluction sheets are characterised by an abundance of angular chert fragments, often up to 20 cm in size, brought down from the Hythe Beds along with pieces of sandstone, and set in a highly variable matrix consisting of silt, sand

and clay. Exploration pits in one of the solifluction lobes showed that the material had moved downhill, on a slope of about 7°, over a layer of reworked clay probably derived mainly from the Atherfield. Details of the section in Pit F2 (the position of which is shown in Fig. 12) are given in Fig. 16. At this location a fossil soil was found between the upper and lower solifluction sheets immediately beneath the layer of reworked clay. The organic matter in the soil has been dated by C14 analysis at approximately 12,200 years B.P. This indicates that the soil was forming during an interstadial in the Late-Glacial stage, and leads to the conclusion that the overlying lobe originated in Zone III, a cold phase at the close of the Late-Glacial well known as a period of renewed periglacial activity in south-east England. There are signs of instability at the toe, so intermittent movements are probably still taking place, at least in this part of the lobe.



Fig. 13. Sevenoaks: section showing valley bulging and escarpment landslide at Pit WB1.



Sevenoaks: coupe montrant le bombement en vallée ("valley bulge") et le glissement de falaise à la fouille WB1



Fig. 14. Sevenoaks: test results on Atherfield Clay at Pit WB1. Sevenoaks: résultats d'essais sur l'argile d'Atterfield à la fouille WB1.



Fig. 15. Sevenoaks: test results on Atherfield Clay at Pit WB2. Sevenoaks: résultats d'essais sur l'argile d'Atterfield à la fouille WB2. A thin section of the reworked clay from Pit F2 when viewed under the microscope (Fig. 17) shows inclusions of small pieces of sandstone and iron-stained nodules and numerous clay pellets typically 1 mm in diameter. The shear zone is about 2 cm wide, with many minor shears, and displays a moderate degree of particle orientation at approximately 160° to the *a* axis. The principal slip surface varies in width up to about 80 microns, and shows a strong orientation of clay particles generally in the direction of movement (Morgenstern & Tchalenko, 1967). A view of part of the slip surface in Pit F2 is shown in Fig. 18.

Six triaxial tests were carried out on the slip surface. Four of these gave stress-strain curves of type (1) and two had a flat-topped peak with  $\Delta s$  equal to about 50 lb/ft<sup>2</sup> (25 gm/cm<sup>2</sup>). Examples of both types of curve are shown in Fig. 22. It is possible that the presence of clay pellets may have been responsible for the rather high values of  $\Delta s$  observed in tests of type (2).

The residual strengths, plotted in Fig. 19, lie on a line represented by the parameters

$$c'_r = 0$$
  $\phi'_r = 16^{\circ}$ 

The index properties of the clay in the shear zone are

LL = 71 PL = 31 PI = 40water content = 34, liquidity index = +0.08clay fraction = 58, activity = 0.7.

The lower solifluction sheet has no obvious topographical expression, but its presence has been proved not only in



Fig. 17. Sevenoaks: shear zone in reworked clay in Pit F2. Sevenoaks: zone de cisaillement dans l'argile remaniée à la fouille F2.



Fig. 16. Sevenoaks: section through upper and lower solifluction sheets at Pit F2. Sevenoaks: coupe des nappes supérieures et inférieures de solifluxion à la fouille F2.

A.W.S., D.J.S., A.G.W., N.R.M. 25. 10. 65 - 4. 11. 65.

the section revealed in Pit F2 (Fig. 16) but also by the discovery of numerous chert fragments within a depth of 5 to 15 ft. below the ground surface in several borings and pits far beyond the lobes. Although of considerable extent the sheet is nevertheless confined to the lower ground of the embayments and valley floors. Further south, however, thin spreads of chert gravel are found capping the higher ground and interfluves. These are probably the remnants of a much older solifluction sheet formed in the Saale glaciation (Bird, 1963). Erosion, leading more or less to the present day drainage pattern, then took place during the Last Interglacial, to be followed by our lower solifluction sheet in the Weichselian glaciation and, after a relatively short interval, by the lobes in Zone III.

Exploration pits through the lower solifluction sheet at positions well south of the lobes, where the ground slopes



Fig. 18. Sevenoaks : view of slip surface in Pit F2. Sevenoaks : vue de la surface de glissement à la fouille F2.



Fig. 19. Sevenoaks: test results on reworked clay in Pit F2. Sevenoaks: résultats d'essais sur l'argile remaniée à la fouille F2.

at only 3 or  $4^{\circ}$ , have shown several slip surfaces in the underlying weathered Weald Clay (Fig. 20). There is no evidence of any recent movement in the lower sheet and indeed it is highly probable that, on such flat slopes, there has been no movement throughout Post-Glacial times; that is, for the past 10,000 years.

It is therefore interesting to note that of ten tests on these slip surfaces (Fig. 21) five showed stress-strain curves of type (1) and in the other five the values of  $\Delta s$  were very small; the average  $\Delta s$  from all tests being 18 lb/ft<sup>2</sup> (9 gm/cm<sup>2</sup>). Examples of the curves are given in Fig. 22.

Average results for these slip surfaces in the weathered Weald Clay can be summarized as follows:

LL = 65 PL = 26 PI = 39water content = 25 liquidity index = -0.03clay fraction = 60 activity = 0.65

Residual strength, for effective normal pressures less than about  $2 \text{ kg/cm}^2$ :

$$c'_r = 40 \text{ lb/ft}^2 (0.2 \text{ t/m}^2) \qquad \phi'_r = 15.5^\circ$$

At higher pressures:

 $\phi'_r = 13^\circ$ 

Among the tests available from the Sevenoaks investigations it is possible to recognise two sub-groups in each of the two main types of stress-strain curves characteristic of



Fig. 20. Sevenoaks: section through lower solifluction sheet at Pit 317H. Sevenoaks: coupe de la nappe inférieure de solifluxion à la fouille 317H.

principal slip surfaces (Fig. 22). The shape of the curves seems to depend in part on the volume changes during the test. If these are small the stress-strain curves tend to be in group (a); i.e. either 1(a) or 2(a). If the volume of the specimen decreases appreciably then there is a tendency for the curves to rise less steeply to the residual (1b) or to exhibit a flat-topped peak (2b). In all cases, however, the values of  $\Delta s$  are zero (type 1) or small (type 2), corresponding in the latter type to a residual factor R > 0.9 and, usually, to values of R > 0.95.

Jari, Mangla Dam Project. At the site of the Mangla Project, in West Pakistan, the strata consist of alternating sandstones and hard clays of the Upper Siwaliks, ranging in age from Upper Pliocene to Lower Pleistocene. These strata have been folded (Fig. 23) and, in the process, bedding-plane slip has taken place in the clays. The resulting shear zones are often as wide as 10 to 20 cms and are known to extend for many tens of metres in the dip and strike directions. These shear zones have had an important influence on the design of the dams and the spillway and intake excavations (Binnie, Clark & Skempton, 1967).

Field work on the shear zones has been carried out during 1961-67 by the senior author with Mr. P. G. Fookes, Mr. D. D. Wilson and Dr. M. S. Money; and the tests were made in the Mangla laboratory by Mr. K. H. Head and Mr. M. Arshad, and also at Imperial College by the junior author.

Most, but not all, of the shear zones contain one and sometimes two or three principal slip surfaces. From the stratigraphic spacing of the zones and the dip of the strata it can be demonstrated by simple geometry that the maximum shearing movements along the slip surfaces must have been several metres; and the slip surfaces examined were



Fig. 21. Sevenoaks: test results on weathered Weald Clay in Pits 317H and 300H. Sevenoaks: résultats d'essais sur l'argile de Weald altérée aux fouilles 317H et 300H.

Shear zones are particularly frequent and well developed in the locality of Jari Dam at the eastern end of the Project, where the beds are dipping at  $30^{\circ}$  to  $50^{\circ}$ . A short section of one of these zones (shear zone D) is shown in Fig. 24. There are numerous minor shears, dividing the clay into 'shear lenses', and two principal slip surfaces. The lower one extends throughout the entire length of the exposure, for a distance of at least 20 m, and the merging of the upper into the lower slip surface can be seen towards the bottom left hand corner in Fig. 24.

Ten tests were made on specimens from these slip surfaces, and the residual strength (Fig. 25) can be expressed by the parameters

$$c'_r = 0 \qquad \phi'_r = 12$$

In four of the tests no peak effect was observed, the stress strain curves being of type (1); whilst in the other six tests the values of  $\Delta s$  were small, as shown by a typical result plotted in Fig. 26 (b).

Average values of the index properties for clay near the slip surfaces are:

$$LL = 59$$
  $PL = 25$   $PI = 34$   
water content = 17 liquidity index =  $-0.23$   
clay fraction = 45 activity =  $0.75$ 



Fig. 22. Sevenoaks: typical stress-displacement curves for triaxial tests on natural slip surfaces. Sevenoaks: courbes contrainte-déplacement typiques.

X-ray analyses of three samples of clay from Jari, by Dr. R. M. S. Perrin, indicate the following composition of the clay fraction:

Chemical analyses reported by Mr. Robertson suggest a higher percentage of montmorillonite with a correspondingly lower content of illite + mica.

Three sets of tests on 'intact' clay from Jari, having similar Atterberg limits and the same average clay fraction as the slip surface specimens, were selected to represent the unsheared clay. The peak strength parameters (Fig. 24) are:

$$c' = 1,000 \text{ lb/ft}^2 (4.9 \text{ t/m}^2) \quad \phi' = 23^\circ$$

and the residual strengths as obtained after three or four traverses in the shear box, see Fig. 26 (a), differ little from those measured on the slip surfaces (see Fig. 25).

Similar results have been found from tests on another shear zone at Jari and also from heavily sheared clay in the vertical limb of the Changhar monocline (Fig. 23). In all three cases the average values of  $\Delta s$  are not more than 5 per cent of the difference between the residual and the peak strength of intact clay. Thus, although it is almost certain that there has been no relative movement along the slip surfaces during Upper Pleistocene times (that is, for at least the past 100,000 years) the strength actually existing on these surfaces corresponds to an average value of the residual factor R = 0.95, and in many individual tests the strength is precisely at the residual, with R = 1.0.

*Principal Slip Surfaces: Conclusions.* The slip surfaces recorded here include examples from landslides, from solifluction mass-movements and from tectonic shear zones. In all cases the surfaces are polished and subplanar with striations or shallow grooves aligned in the direction of shearing. Relative movements along the slip surfaces are not known with any precision, but certainly amount at

sw



syncline Suruh Unah Jari Changhar Mirpur anticline anticline thrust fault 8 monocline 12,000 Ramkot monocline ft Fossil locality Jari shear zones LOWER PLEISTOCENE 8000 loess - silt Mirpur UPPER PLEISTOCENE 4000 Plains UPPER SIWALIKS MIDDLE SIWALIKS PLIOCENE 5 7 9 10 11 2 3 4 6 8 12 miles

Fig. 23. Diagrammatic geological section through the Mangla region, West Pakistan.

Coupe géologique schématique de la région de Mangla, Pakistan Occidental.



Fig. 24. Mangla: shear zone D at Jari. Mangla: zone de cisaillement D à Jari.



Fig. 25. Mangla : test results on shear zone D at Jari. Mangla : résultats d'essais sur la zone de cisaillement D à Jari.

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least to several tens of centimetres. Stress-strain curves show either no peak effect (R=1.0) or a small value of  $\Delta s$ usually corresponding to a value of R > 0.95; and in the latter cases a subsequent displacement of a few millimetres is sufficient to reduce the strength to the residual.

Where the residual strength envelope is linear (within the range of pressure used in the tests) it appears to pass through the origin; as at Walton's Wood, Sevenoaks Pit F2 and Jari shear zone D. More generally the residual strength envelope shows some curvature, particularly at low effective normal pressures, and it is difficult to say whether or not there is a small cohesion intercept. The parameters quoted in Table II are those giving the best linear fit for pressures up to about 2 kg/cm<sup>2</sup>. At higher pressures the tangent values of  $\phi'_r$  can be appreciably lower; a significant point in relation to large scale landslides and tectonic shearing, where values of  $\phi'_r$  not exceeding 10° may be operative in clays of quite moderate activity and clay fraction content.

From the tests reported in this paper, there is no evidence that the strength on principal slip surfaces may increase with time after shearing movements have ceased. Indeed, at Jari and in the Weald Clay at Sevenoaks it is practically certain that no such increase in strength has occurred.

The data at present available show that the water content in a shear zone is typically a few per cent higher than in the adjacent 'intact' clay, and also suggest that the clay fraction content on the slip surfaces is slightly greater, indicating some degree of clay enrichment which may result from physical breakdown of aggregations or particles in the silt grade, or migration of coarser grains from the shear zone. Of far greater importance is the undoubted orientation of clay particles, more or less in the direction of shearing, within a band usually between 10 and 50 microns in width which, in fact, constitutes the slip 'surface.'



Fig. 26. Mangla: typical stress-displacement curves for clays at Jari. Mangla: courbes contrainte-déplacement typiques pour les argiles à Jari.

## **Minor Shears**

In addition to principal slip surfaces, shear zones contain numerous minor shears of limited extent on which the relative movements have been small, probably of the order of millimetres. Sukian, Mangla Project. A large block sample from a shear zone at Sukian, on the Mangla Dam project, was sent to London and tests were carried out in 1965 by the junior author on specimens prepared in such a way as to include one or other of the minor shear surfaces (Fig. 27). These surfaces were slickensided and somewhat irregular, in contrast to the flat, polished surfaces of the principal slip planes observed in the Jari shear zones.

There is some scatter among the results, as would be expected, but all eight tests show peak strengths appreciably above the residual, corresponding to an average value of R = 0.7. Moreover, it was necessary to apply two or three reversals in the shear box before the strength fell to the residual, as shown in Fig. 28.

Index properties for the clay in this sample are:

LL = 60 PL = 28 PI = 32water content = 16 liquidity index = -0.38clay fraction = 52 activity = 0.6

By interpolation among tests on samples in the locality, the peak strength parameters of intact clay having the same clay fraction and similar Atterberg limits can be taken as

$$c' = 1,200 \text{ lb/ft}^2 (5.8 \text{ t/m}^2) \qquad \phi' = 22$$

The peak strength on the minor shears is given approximately by

$$c' = 360 \text{ lb/ft}^2 (1.75 \text{ t/m}^2) \qquad \phi' = 16^\circ$$

and the residual strength from reversal shear box tests has the parameters

$$c'_r = 0$$
  $\phi'_r = 14^\circ$ 

Expressed in another way, the peak strength on these discontinuities at an effective normal pressure of 1 kg/cm<sup>2</sup> exceeds the residual strength by an amount  $\Delta s = 2.2$  t/m<sup>2</sup>.



 Fig. 27. Mangla: test results on minor shears in shear zone Bed 512 at Sukian.
 Mangla: résultats d'essais sur des discontinuités secondaires au Lit 512 à Sukian.

This may be contrasted with an average value of  $\Delta s = 0.45$  t/m<sup>2</sup> for the peak strengths measured on the principal slip surfaces in the Jari shear zone, corresponding to R = 0.96.

At Sukian the dip of the strata is between  $10^{\circ}$  and  $12^{\circ}$ . Consequently the shearing action due to bedding-plane slip has been considerably less than at Jari, where the folding is more intense. Some of the shear zones at Sukian certainly include principal slip surfaces, but in others the general displacement has not been sufficient and these latter shear zones are in the third stage of development or perhaps, in places, just entering the fourth stage (Fig. 1).

In such shear zones the strength is clearly greater than the residual, not only because the strength on the minor shears shows an appreciable value of  $\Delta s$ , but also because there must be a significant degree of imbrication. Nevertheless it seems reasonable to suppose that a progressive failure mechanism could readily be initiated and therefore, in practice, the designer might be well advised only to rely upon the residual strength; possibly accepting a somewhat



Fig. 28. Mangla: typical stress-displacement curves for clays at Sukian. Bed 512. Mangla: courbes contrainte-déplacement typiques pour les argiles de Sukian.

lower factor of safety than he would use when the shear zones are known to contain extensive, flattened principal slip surfaces.

#### Joint Surfaces

In sedimentary rocks the joints are characterised by a 'brittle-fracture' type of surface along which the relative (shearing) movements, if not zero, must have been exceedingly small. Such joints are found in stiff clays, and may be classified broadly in two groups: (i) small, non-systematic joints or 'fissures' usually in a more or less random arrangement; often with irregular, curved or conchoidal surfaces, and dimensions typically ranging from 1 to 10 or 20 cms, and exceptionally up to 40 or 50 cms. (ii) systematic joints, occurring in ordered sets, with subplanar surfaces the length of which may be as great as several metres.

Wraysbury. At Wraysbury in south Buckinghamshire the blue London Clay is found under widespread Upper Pleistocene river gravels of the Thames, usually between 3 and 6 m in thickness. In this western part of the London Basin the clay during its geological history has been consolidated under a maximum thickness of sediments estimated very approximately at 400 m (Bishop, Webb & Lewin, 1965). Immediately below the gravels the clay is heavily fissured; but at depths of more than 1 or 2 metres below the interface the typical blue (unweathered) London Clay is encountered, with fissures up to about 10 cms in size. At this site the clay has a thickness of about 40 m.



Fig. 29. A joint surface in the blue London Clay at Wraysbury, Buckinghamshire. Une surface d'un joint dans le blue London Clay à Wraysbury, Buckinghamshire.

#### STRENGTH ALONG STRUCTURAL DISCONTINUITIES IN STIFF CLAYS

Excavations carried 6 m below the gravel, and covering a wide area, have revealed a prominent set of systematic joints approximately normal to bedding (which is almost horizontal) with a predominant strike direction at N 55 W. These joints are commonly about 1 metre deep and spaced at 1 to 2 m intervals; although both the depth and spacing can vary considerably. In length the joints have been observed to extend up to 5 or 6 metres. Other joints are found striking between N 20 E and N 50 E.

In some cases there have been small 'posthumous' movements, but in most of the joints there is no evidence of any shearing displacements, the surfaces displaying a delicate 'brittle-fracture' texture as shown in Fig. 29 – a photograph of the joint surface in one of the specimens taken in the manner indicated in Fig. 30.

Work at this site is still in progress by Professor A. W. Bishop, Mr. P. G. Fookes and the authors, with assistance in the laboratory by Mr. K. B. Agarwal. So far only four tests have been made on the joint surfaces, two from the position shown in Fig. 30 and another two from a similar joint nearby. These tests show peak strengths in excess of the residual; the best linear fit being expressed by the parameters (Fig. 31)



Fig. 30. Wraysbury: sampling at a joint in the London Clay. Wraysbury: échantillonage d'un joint dans le London Clay.



Fig. 31. Wraysbury: test results on joint sufaces. Wraysbury: résultats d'essais.

$$c' = 70 \text{ lb/ft}^2 (0.35 \text{ t/m}^2) \qquad \phi' = 18.5^\circ$$

It is particularly to be noted, however, that the strength falls to the residual after a subsequent relative movement of not more than 5 mm, as shown in Fig. 32 (b), and the surface has then become polished and striated (Fig. 33). The residual strength parameters are

$$c'_r = 30 \text{ lb/ft}^2 (0.15 \text{ t/m}^2) \qquad \phi'_r = 16^\circ$$

Index properties of the clay adjacent to the joint surfaces have the following average values:

$$LL = 73$$
  $PL = 28$   $PI = 45$   
water content = 28 liquidity index = 0.0  
clay fraction = 55 activity = 0.8

Shear box tests have been made on 'intact' clay from the same site, with similar index properties but a somewhat closer spacing of fissures. The peak strengths are represented by the parameters

$$c' = 650 \text{ lb/ft}^2 (3.2 \text{ t/m}^2)$$
  $\phi' = 20^\circ$ 

and it will be noted from Fig. 31 that no systematic differences exist between the peak strengths of samples tested horizontally and those tested vertically. Reversal shear box tests are not always easy to interpret in this clay, see Fig. 32 (a), but the residual strengths obtained by these tests are at least comparable with the residual strengths on the joint surfaces (Fig. 31).

More tests are required properly to define the peak strength on the joints, and thin sections must be examined; but from the information at present available we may draw the following tentative conclusions: (i) the brittle fracture, which produced the joints, virtually destroyed the cohesion intercept of the clay (c' reduced by 90 per cent from the 'intact' value) without causing much re-orientation of the particles ( $\phi'$  reduced by only 1.5°); (ii) small shearing



Fig. 32. Wraysbury: typical stress-displacement curves. Blue London clay. Wraysbury: courbes contrainte-déplacement typiques.

movements of the order of a few millimetres, given the original fracture, are sufficient fully to orientate the clay particles within a thin boundary layer, reducing  $\phi'$  to its residual value and polishing the surface (with a further, slight reduction in c'; (iii) the amount of work required to bring the strength along a joint surface to the residual is so small that, in practice, the available strength along a joint in stiff clay can scarcely be considered as being greater than the residual, especially at low pressures; (iv) since very small movements subsequent to the formation of a joint in stiff clay can result in polishing and striations, an observer could readily arrive at the misleading decision that the joint had originally been caused by small shearing displacements.



Fig. 33. Wraysbury: joint surface after shearing displacement of 5 mm in laboratory test. Wraysbury: surface d'un joint après un déplacement au

cisaillement de 5 mm dans un essai de laboratoire.

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#### REFERENCES

- Binnie, G. M., Clark, J. F. F. and Skempton, A. W., 1967. The effect of discontinuities in clay bedrock on the design of dams in the Mangla Project. Trans. 9th Int. Congr. Large Dams (Istanbul), Vol. 1, p. 165-83. Bird, E. C. F., 1963. Denudation of the Weald Clay Vale in West
- *Kent.* Proc. Geol. Assoc., Vol. 74, p. 445-55. Bishop, A. W., Webb, D. L. and Lewin, P. I., 1965. *Undisturbed*
- samples of London Clay from the Ashford Common shaft: strengtheffective stress relationships. Géotechnique, Vol. 15, p. 1-31.
- Esu, F., 1966. Short-term stability of slopes in unweathered jointed clays. Géotechnique, Vol. 16, p. 321-28. Fookes, P. G., 1965. Orientation of fissures in stiff over-consolidated
- clay of the Siwalik system. Géotechnique, Vol. 15, p. 195-206. Fookes, P. G., and Wilson, D. D., 1966. The geometry of discon-
- tinuities and slope failure in Siwalik clay. Géotechnique, Vol. 16, p. 305-20.
- Gramberg, J., 1965. Axial cleavage fracturing, a significant process
- Granderg, J., 1965. And checkage fracturing, a significant process in mining and geology. Eng. Geol., Vol. 1, p. 31-72.
  Hodgson, R. A., 1961. Classification of structures on joint surfaces. Amer. Journ. Sci., Vol. 259, p. 493-502.
  Kenney, T. C., 1967. The influence of mineral composition on the residual strength of natural soils. Proc. Geotech. Conf. (Oslo), Vol. 1, p. 123-29. Vol. 1, p. 123-29.
- Marsland, A. and Butler, M. E., 1967. Strength measurements on stiff fissured Barton Clay from Fawley, Hampshire. Proc. Gcotech. Conf. (Oslo), Vol. 1, p. 139-45. Morgenstern, N. R. and Tchalenko, J. S., 1967. Microstructural
- observations on shear zones from slips in natural clays. Proc. Geotech. Conf. (Oslo), Vol. 1, p. 147-52. Skempton, A. W., 1953. The colloidal activity of clays. Proc. 3rd
- Int. Conf. Soil Mech., (Zurich), Vol. 1, p. 57-61.
- Skempton, A. W., 1964. Long-term stability of clay slopes. Géotechnique, Vol. 14, p. 77-101. Skempton, A. W., 1965. Discussion in Proc. 6th Int. Conf. Soil

Mech. (Montreal), Vol. 3, p. 551-52.

- Skempton, A. W., 1966. Some observations on tectonic shear zones. Proc. 1st Int. Conf. Rock Mech. (Lisbon), Vol. 1, p. 329-35
- Proc. 1st Int. Cont. Rock Meen. (Elsoon), vol. 1, p. 327-53.
  Terzaghi, K., 1936. Stability of slopes in natural clay. Proc. 1st Int. Conf. Soil Mech., (Harvard), Vol. 1, p. 161-65.
  Weymouth, J. H. and Williamson, W. O., 1953. The effects of extrusion and some other processes on the micro-structure of clay. Amer. Journ. Sci., Vol. 251, p. 89-108.

# The consolidation of clays by gravitational compaction

## ALEC WESTLEY SKEMPTON

## SUMMARY

Sedimentation compression curves, relating void ratio to effective overburden pressure, are presented for a wide lithological range of argillaceous deposits. These curves show the progressive changes from recently deposited muds on the sea floor, to Quaternary clays at depths of several tens of metres, and finally to hard clays and mudstones of Pliocene age at depths extending to about 3 000 m. Twelve localities are examined in some detail and information is also given from another eight previously published sites. In all cases the data are derived from 'normally-consolidated' deposits, strata which have never been under greater pressures than those existing at the present time. This procedure eliminates the difficulties of estimating the effect of pressure reduction by erosion. Clays containing high proportions of carbonates and organic matter are not included in this study.

The water content (or void ratio or porosity) of any particular clay in the normally-consolidated condition is controlled by the effective overburden pressure  $p_0$ , given by Terzaghi's law  $p_0 = \sigma - u$ , where  $\sigma$  is the total vertical pressure exerted by all the material (particles and water) above the point considered, and uis the pore water pressure at that point. This law is shown to hold good even at porosities as low as 15 per cent.

At any particular effective overburden pressure the water content of a normally-consolidated clay is directly related to the amount of clay minerals present and to their colloidal activity. The combined influence of these two factors can be indicated quantitatively by the Atterberg limits; and at a given value of p the water content is found to be a function of the Atterberg (liquid and plastic) limits for all inorganic non-calcareous clays except those with an extremely unstable microstructure, such as the so-called 'quick clays' of Scandinavia. Moreover, the water contents of muds on the sea bed or in tidal flats can also be expressed approximately by single-value parameters in terms of these limits. Thus if the water content, effective overburden pressure and Atterberg limits are known for an individual layer of normally-consolidated clay, it is possible to reconstruct the entire sedimentation compression curve for that clay with a reasonable degree of certainty; and hence an estimate can be made of the compaction which has occurred in the clay under its own weight and under the load of any overlying strata.

At some of the sites, in addition to data relating to compaction, information is given on the increase in strength with depth and the rate of deposition as deduced from radiometric dating.

## 1. Introduction

ABOUT twenty-five years ago on the initiative of the late Professor O. T. Jones a paper was presented to the Society on the compressibility of clays, based on laboratory experiments (Skempton 1944). This work established the broad pattern of response of different types of clay to compaction and, in particular, demonstrated the value of the Atterberg limits as quantitative index properties of clays. Except for a single case record from Gosport, however, information from the field at that time was fragmentary.

During the next seven or eight years a little more field data emerged, and was summarized in a paper published by the Yorkshire Geological Society (Skempton 1953A), but around 1958 the pace began to accelerate and a considerable amount of field evidence is now available, which has been used in the present paper to build up a reasonably complete account of the compaction of clays.

## (A) CONSOLIDATION AND COMPACTION

*Consolidation* may be defined as the result of all processes causing the progressive transformation of an argillaceous sediment from a soft mud (as originally deposited) to a clay and finally to a mudstone or shale. These include inter-particle bonding, desiccation, cementation and, above all, the squeezing out of pore water under increasing weight of overburden. The latter process is *gravitational compaction*, in the terminology of Hedberg (1936). It dominates to such a degree in clays that consolidation and compaction are almost synonymous terms in the present context.

### (B) EFFECTIVE OVERBURDEN PRESSURE

Referring to Fig. 1, the total overburden pressure at any given depth z is denoted by  $\sigma_v$ . It is simply the vertical pressure exerted by the weight of all materials above



FIG. 1. A typical distribution of total overburden pressure  $\sigma_v$  pore pressure u and effective overburden pressure  $p_0$ .

the point considered. Thus, if the average density of these materials is  $\bar{\gamma}$ , then

$$\sigma_v = \bar{\gamma} z$$

A piezometer inserted in the clay will usually show a positive head of water h, indicating a pore pressure

$$u = \gamma_w h$$

where  $\gamma_w$  is the density of water. Each particle of clay at the depth z can be considered as being subjected to an all-round pressure u and, in the vertical direction, to a pressure

$$p_0 = \sigma_v - u$$

It was clearly recognized by Terzaghi (1923), and formally stated by him in

1936, that a variation in the all-round pressure u can, by itself, have little or no influence on any mechanical properties of the clay, and that compaction must be a function of the pressure  $p_0$  which he therefore defined as the *effective pressure*. This principle is practically self-evident when the clay is submerged and highly porous, since a change in the depth of sea water, for instance, would scarcely be expected to cause any change in the compaction of clay underlying the sea-bed; a conclusion implicit in the writings of Lyell (1871, pp. 41-2). But when the porosity is low the principle is not so obvious, and theoretical investigations by Professor A. W. Bishop have revealed that a more exact formulation of effective pressure should be written

$$p_0 = \sigma_v - \left(\mathbf{I} - \frac{C_s}{C}\right)u$$

where  $C_s$  is the compressibility of the particles and C is the compressibility of the clay structure. In practice, however, even for clays and mudstones with a porosity as low as 15 per cent, the ratio  $C_s/C$  is so small (less than 1 per cent) that in all cases studied here, to depths of at least 3000 m in normally-consolidated argillaceous sediments, Terzaghi's equation  $p_0 = \sigma_v - u$  is satisfactory to a high degree of approximation (Skempton 1960, and Appendix to this paper).

Since the pore pressure is an important factor in determining effective overburden pressure it must wherever possible be measured by piezometers. If ground water conditions are hydrostatic, piezometric level corresponds to the free water table; but frequently piezometric levels are either lower or higher than the water table, indicating respectively a downward or upward component of flow in the ground water.

Above the water table negative pore pressures are encountered, due to capillary suction in the soil, and effective pressures of more than  $1 \text{ kg/cm}^2$  can readily be developed in clays within this zone of desiccation; reaching a maximum value of the order 10 kg/cm<sup>2</sup> when the vegetation wilts (Coleman & Farrar 1966).

#### (C) SEDIMENTATION COMPRESSION CURVES

When freshly deposited on the sea-bed clay (in the form of mud) will have a high void ratio or porosity,<sup>1</sup> represented by point (a) in Fig. 2. As deposition continues the effective overburden pressure increases and the clay will compact, for example to point (b) or (c); and the line relating void ratio to effective pressure is known as the *sedimentation compression curve* for that clay (Terzaghi 1941). The main objective in this paper is to present such curves for a representative selection of argillaceous deposits (Fig. 21).

Sedimentation compression curves are built up from a series of points. Each point is obtained by taking an undisturbed sample from a known depth in a deposit of known average density and determining its void ratio. Void ratio is calculated from measurements of the density and water content w of the sample and the specific gravity of the particles  $\rho$ . Obviously the natural water content must be preserved unaltered from the moment of sampling. In almost all cases the clay is found to be fully saturated, (i.e. the voids are filled with water, no air being present) and the void ratio is then given by the expression

$$e = \rho \cdot w$$
.

The pore pressure at this depth must also be known in order that the effective overburden pressure can be calculated. It is also desirable to know the geological

<sup>1</sup> Void ratio  $e = \frac{\text{vol. voids}}{\text{vol. solids}}$ , porosity  $n = \frac{\text{vol. voids}}{\text{total volume}}$ . Hence  $e = \frac{n}{1-n}$  and  $n = \frac{e}{1+e}$ .

history of the deposit and its age, and in particular it is necessary to establish that the clay is normally-consolidated; in other words that its present condition is represented by a point on the line (a)-(c) in Fig. 2 and not on a rebound curve such as (c)-(d). Finally the clay must be classified by visual examination and also by quantitative index property tests such as particle-size distribution and the Atterberg limits (see section 1 (E)).



FIG. 2. Normally- and over-consolidated clay.

## (D) NORMALLY-CONSOLIDATED CLAYS

According to the definition of Terzaghi (1941) a clay is normally-consolidated if it has never been under a pressure greater than the existing effective overburden load. Clays in this condition would be represented by any point such as (b) on the curve (a)-(c) in Fig. 2. If the existing effective overburden pressure is less than the maximum effective pressure to which the clay has been subjected in the past, the clay is said to be *over-consolidated* and it would be represented by a point such as (d) in Fig. 2.

Now it is well known from laboratory experiments and field experience that the compaction caused by a given increase in pressure acting on a normally-consolidated clay is much greater than the expansion caused by a numerically equal reduction in pressure. Thus the two clays at points (b) and (d), which are under the same effective pressure and are in all respects identical except for their consolidation history, will have very different void ratios. In order to construct a sedimentation compression curve it is therefore necessary to establish that any particular sample is normally-consolidated.

#### CONSOLIDATION OF CLAYS BY GRAVITATIONAL COMPACTION

In a few cases this can be deduced from the known conditions of formation but, in general, the sample must be tested in the oedometer (Skempton 1953A). For normally-consolidated clays the line relating void ratio and the effective pressures applied in the test (plotted on a logarithmic scale) shows strong curvature at pressures corresponding approximately to the overburden pressure which had been acting on the sample when in the ground, as shown in Fig. 3. Moreover it is possible from the test results to obtain upper and lower limits of this pressure. The



FIG. 3. Typical results of an oedometer test on normally-consolidated clay.

FIG. 4. Relation between undrained shear strength c and effective overburden pressure  $p_0$  for a normally-consolidated clay.

upper limit, indicated by  $p_c$  in Fig. 3, is derived from a simple graphical construction due to Casagrande (1936), who defined the pressure  $p_c$  as the pre-consolidation load. The lower limit indicated by  $p_i$  is found by extrapolating the linear portion of the  $e - \log p$  curve back to the line representing the *in situ* void ratio  $e_0$ . If the clay is over-consolidated to any appreciable degree then both  $p_i$  and  $p_c$  are greater than the existing overburden pressure  $p_0$ , but if the clay is normallyconsolidated  $p_0$  will generally lie between  $p_i$  and  $p_c$ .

Another useful check on the state of consolidation can be obtained by plotting the undrained shear strength c of a clay against the effective overburden pressure  $p_0$ . For normally-consolidated clays the points will lie approximately on a straight line, as shown in Fig. 4, with a slope  $(dc/dp_0)$  ranging from 0.2 to 0.5 for most types
of clay (Skempton 1954) and often with a small intercept of the order 10 to 50 g/cm<sup>2</sup>, the so-called 'origin cohesion' (Jakobson 1953). In contrast the relation between c and  $p_0$  for over-consolidated clays is non-linear, and even at low effective pressures the value of c can be in excess of 300 to 500 gm/cm<sup>2</sup>.

#### (E) ATTERBERG LIMITS AND SENSITIVITY

Two index properties known as the *liquid limit* and *plastic limit* were introduced by Atterberg (1911) to provide an empirical but quantitative measure of the degree of plasticity of clays. Their merits for classification purposes were emphasized by Terzaghi (1925, 1926) and, with modifications due to Casagrande (1932), the tests for measuring these properties have become internationally standardized in all soil mechanics laboratories. For details, reference may be made to the latest relevant British Standards Institution publication (B.S. 1377:1967, *Methods of testing soils*).

Briefly, the liquid limit (LL) of a clay is the water content at which, in the remoulded state, it passes from the plastic to an almost liquid condition; and the plastic limit (PL) is the water content at which the remoulded clay passes from the plastic to a friable or brittle condition. The plasticity index (PI) is defined as the difference between the two limits:

$$PI = LL - PL$$

In Fig. 5 average values of LL and PI are plotted for all the clays studied here. The points lie within a narrow band just above the 'A-line' of Casagrande (1948), a relationship typical of sedimentary clays having low contents of carbonate and organic material. Thus as a convenient approximation any of these clays can be characterized by a single parameter, the liquid limit. In addition the Atterberg limits have two significant advantages. In the first place they reflect both the amount and the type of minerals present in a given clay (Skempton 1953B) and are therefore functions of such properties as cation-exchange capacity and total surface area (Farrar & Coleman 1967). Secondly, they are expressed as water contents, and the natural water content of a clay can thus be compared directly with its Atterberg limits by a ratio defined as the *liquidity index* (LI) where

$$LI = \frac{w - PL}{LL - PL}$$

It will be shown later that for a wide variety of normally consolidated clays the liquidity index lies inside a rather narrow range of values, at any given effective pressure, although the corresponding natural water contents of the clays may vary between very wide bounds. This result is useful in several ways, but in particular it enables a reduction to be made in the scatter of results due to small random variations in clay type within in single stratum. The procedure, which has been adopted in plotting Fig. 21, involves (i) calculating LI for each sample; (ii) finding the average values of LL and PL for the stratum; (iii) evaluating a 'corrected natural water content' for each sample, using the actual LI of that sample and the average liquid and plastic limits of the stratum.

The importance of the liquidity index is made clear by the simple example illustrated in Fig. 6. Here two samples at different depths have the same natural water content and, without a knowledge of the Atterberg limits, it might be inferred that the greater pressure acting on the deeper sample had caused no additional compaction. Very probably, however, the liquid limit of the lower sample would be found to be greater than that of the upper sample and the *liquidity index* would decrease with depth.



FIG. 5. Plasticity chart for the clays designated in Figs 11 and 21.

The Atterberg limits, then, are of great value; but they give no information on the structure of a clay or on its strength in the undisturbed condition. Most normally consolidated clays having relatively high water contents, at depths to the order of 30 m, show marked *sensitivity*; defined as the ratio of undisturbed to remoulded strength. Indeed it is usual to find clays at shallow depths with a sensitivity  $(S_t)$  of 4 or 5, and sensitivities as high as 8 are by no means uncommon. Thus, although a clay with a natural water content equal to the liquid limit will (by definition) possess very little strength when remoulded, it may nevertheless show an appreciable strength in its natural state. For the same reason it is not surprising to find clays at depths of several metres below the sea bed with water contents above the liquid limit. This does not imply that they exist in the form of a fluid mud, but it is of course true that they would be reduced to such a condition when remoulded. None of the clays examined in the present investigation has a sensitivity of more than 10. Without pursuing the point in detail it is worth noting that there is a tendency for the liquidity index at a given effective pressure to increase with increasing sensitivity. This result might be expected since, other things being equal, a high sensitivity indicates a more open structural arrangement of the particles. Fortunately the effect is not pronounced within the range of clays considered here. Nevertheless the influence of structure can be very great in a special class of clays, the so-called 'quick-clays', the sensitivity of which can be of the order of 100.



FIG. 6. Natural water content, liquid and plastic limits and liquidity index for two samples in a clay stratum.

These quick-clays are found typically in regions such as Norway and the St Lawrence valley where post-glacial marine deposits have been subjected to strong leaching following isostatic uplift. Their properties are exceptional in many respects and their sedimentation compression curves are very different from those of more normal sedimentary clays (Skempton & Northey 1952). They have therefore been excluded from the present study. Diatomaceous clays and those containing more than 5 per cent of organic carbon, as well as clays with a carbonate content of more than 25 per cent, have also been excluded. The fragmentary data available on such materials suggest that they require separate investigation.

Inorganic clays having a low carbonate and organic content are here classified in the following groups:

Silty clays: liquid limit between 20 and 30. In this group the plasticity index is less than 15 and the clay fraction is typically less than 10 per cent.<sup>1</sup> Sediments with

<sup>&</sup>lt;sup>1</sup> Clay fraction (CF) is the proportion by weight of particles smaller than 2 microns. For the relation between Atterberg limits, clay fraction and mineralogy see Skempton (1953B).

a liquid limit below 20 are non-plastic and behave essentially as fine-grained granular materials.

Clays of low plasticity: liquid limit between 30 and 50. The plasticity index is typically between 10 and 30.

*Clays of medium plasticity:* liquid limit between 50 and 90. This is the largest single group; the plasticity index typically lies between 25 and 65, and the clay fraction is usually between 20 and 70 per cent.

*Clays of high plasticity*: liquid limit greater than 90. In this group the plasticity index is typically greater than 55 and the clay fraction higher than 50 per cent. Inorganic clays with a liquid limit exceeding 120 are uncommon, and those with a liquid limit above 140 consist almost exclusively of bentonitic clays with a high proportion of montmorillonite.

#### (F) RATES OF DEPOSITION

Pore pressures in excess of hydrostatic values can be developed in clays which are deposited very rapidly or attain a great thickness by continuous sedimentation. This condition of partial consolidation, if unrecognized, may give rise to highly misleading interpretations of field data.

Theoretically a clay can be fully consolidated only with infinitely slow deposition, but in practice 95 per cent consolidation can be taken as a sufficiently close approximation to final equilibrium. From work by Gibson (1958) it can be shown that for this degree of consolidation to be reached in a clay, with a uniform rate of deposition r, the ratio  $c_v/rh$  must be more than 10; where h is the thickness of the layer and  $c_v$  is the coefficient of consolidation of the clay, a typical value being  $c_v = 1 \text{ m}^2/\text{yr}$ .

Now a typical rate of deposition of estuarine Holocene clays is about 2 m/1000 yr (see Table 1) and such clays will therefore be more than 95 per cent consolidated provided their thickness is less than 50 m, a condition satisfied by the majority of sediments in this category, quite apart from the fact that deposition has usually been slower in the past 3000 years owing to a decrease in the rate of rise of sea-level during this recent period (Jelgersma 1966). In contrast, deposition in modern deltas may be so rapid that the clays are partially consolidated. Thus, in the Orinoco delta Kidwell & Hunt (1958) reported excess pore pressures of at least 0.5 kg/cm<sup>2</sup> at depths of 30 to 50 m in sediments which, from various radiocarbon dates, are known to have been deposited at a rate of about 8 m/1000 yr; and, as an extreme case, McClelland (1967) has published the results of two borings in the Mississippi delta where the clays show almost no consolidation effects throughout the entire thickness of 50 or 60 m of interdistributary sediments. At these localities the rate of deposition is rather more than 100 m/1000 yr. At the other extreme are the deep-sea sediments. A rate of deposition of the order 0.03 m/1000 yr has been reported by Rosholt et al. (1961, 1962) for three cores from the Caribbean Sea, with specimens dated back to 150 000 years by the Pa/Th method. A similar result, but with a much shorter time-scale, can be deduced from data published by Emery (1960, 249-53) for sediments in marine basins off the California coast; while Ku (1965), also using the Pa/Th method of dating, has found rates of

	Thickness of deposit m	Rate of deposition m/1000 yr	Reference
DELTAIC			
Mississippi, Holocene Rhone, Holocene Orinoco, Holocene	55 65 40	120 17 8	McClelland (1967) Lagaaij ( <i>in litt.</i> ) Kidwell & Hunt (1958)
ESTUARINE	40	-	(-995)
Avonmouth, Holocene Tilbury, Holocene Pisa, Holocene	13 16 10	2·5 2 2·5	present paper present paper present paper
MARINE, shallow water			
Oslofjord, Holocene Po Valley, Pleistocene Po Valley, Pliocene Kambara, Pliocene	2000 3000 2600	0.8 1.2 1.0 0.9	Richards ( <i>in litt.</i> ) present paper present paper present paper
MARINE, deep-sea Caribbean, Pleistocene	-	0.03	Rosholt et al. (1961)

TABLE 1: Rate of deposition and thickness of some Quaternary and Pliocene argillaceous sediments

deposition one order of magnitude less for cores of red clay from the Atlantic and Indian Oceans. It can readily be accepted that such slow accumulations could scarcely give rise to any excess pore water pressures.

With the great thickness of shallow-water marine deposits often encountered in deep borings the problem is more complex. In the Po Valley, for example, Pliocene and Quaternary sediments extend to a depth of 5000 m (Fig. 19) and have been deposited at an average rate of the order 1 m/1000 yr. Now if these sediments were argillaceous throughout, then, on theoretical grounds, high excess pore pressures would be expected; and it is well known that such 'abnormal' pressures do indeed exist in some formations (Tkhostov 1963). Nevertheless pore pressures have been measured in the Pliocene deposits of the Po Valley (see section 4 (E)) and they are not more than about 15 to 20 per cent above hydrostatic values. The explanation of these relatively low excess pressures is to be sought, almost certainly, in the presence of sand and silt beds which permit more rapid consolidation. Under such conditions calculations are exceedingly difficult and the pore pressures in deep formations should be measured in the field if reliable values of effective overburden pressure are required.

### 2. Recent sediments

In this section of the paper summaries will be given of case records for several argillaceous sediments recently deposited on the sea-bed and in tidal flats.

#### (A) CORE A-31, MEDITERRANEAN SEA

Richards (1961, 1962) and Richards & Hamilton (1967) have presented the results of tests on a number of cores collected by the U.S. Navy Hydrographic Office, and of these Core A-31 has been selected for illustration (Fig. 7). This core was taken from the sea-bed in the western Mediterranean near Gibraltar, where the water depth is 400 m, using an open-barrel gravity sampler. Depths within the core have been corrected on the assumption of linear core shortening, following the



FIG. 7. Core A-31 from the western Mediterranean near Gibraltar (data from Richards 1961, 1962; Richards & Hamilton 1967).

investigations of Emery & Dietz (1941) and Emery & Hülseman (1964). The sediment in Core A-31 consists of a grey terrigeneous mud, the Atterberg limits of which remain more or less constant with depth. The clay fraction also varies little, indicating an essentially uniform mineralogy. Oedometer tests show that the sediment is normally-consolidated within the limits of accuracy expected; water content and liquidity index decrease with depth in a remarkably consistent manner, and the shear strength bears a linear relationship to effective over-burden pressure (except in the top 30 cm) with an 'origin cohesion' of about 15 gm/cm<sup>2</sup>.

Values of void ratio are plotted against overburden pressure in Fig. 21, the void ratios being calculated from water contents corrected to correspond to a constant liquid limit of 63 and a plastic limit of 26 (average values for the core). Each point

in this figure is the mean of several determinations; a procedure which is followed for most of the case records in order to avoid overcrowding and a confusion of data from different sites.

Results from three other ocean cores (Richards 1962) are also included in Fig. 21, namely Cores A-33 and B-87 from the western Mediterranean and C-18 from the Norwegian Sea between the Shetland and Faroe Isles. At these locations the water depth is about 100 m but, as in Core A-31, the sediments are of terrigeneous origin and have been deposited on the continental slope.



from Richards *in litt.*).

### (B) OSLOFJORD

Dr A. F. Richards has kindly permitted the publication of results which he has obtained from four cores taken in close proximity at a site near Hvitsten in Oslofjord, where the depth of water is about 220 m. The cores were taken with a piston sampler developed by the Norwegian Geotechnical Institute (Andresen *et al.* 1965). There is clear evidence for a break in the depositional sequence at a depth of 3.6 m (Fig. 8), probably due to a turbidity current, and the test results within a thickness of about 0.8 m above this break have been ignored as they reveal a gradation from sandy to silty material quite distinct from the otherwise uniform dark green-grey clay with liquid limits around 100 and a clay fraction of about 60 per cent. In the disturbed zone the shear strength is relatively high and the liquidity index low, as would be expected for material which has been disturbed and subsequently re-consolidated. It is possible that the turbidity current accompanied or was preceded by bed erosion, but an oedometer test shows that the clay at a depth of 0.3 m below the depositional break is normally-consolidated under the present overburden pressure. This result is in no way surprising, since it shows simply that the thickness of sediment now existing above the break is greater (perhaps much greater) than that temporarily removed by erosion. Except in the disturbed zone the decrease of the



FIG. 9. Intertidal flats at Gosport, Hampshire. Cores E, F and G taken near low water mark neap tides (Skempton, unpublished report).

liquidity index with depth is strikingly uniform, and the shear strength increases linearly with overburden pressure except, as usual, in the uppermost layers (Fig. 8). These cores therefore provide excellent data for a compression curve of a high liquid limit clay between depths of 1 and 7 m (Fig. 21).

A radiocarbon date of 6000 yrs B.P. determined on organic material at a depth of 5.7 m indicates an average rate of deposition (uncorrected for compaction) of about 0 95 m/1000 yr. This may be rather too high as a representative figure owing to the possibility of a short period of rapid deposition during the supposed turbidity current phase, and it is interesting to note that a date of 9500 yrs B.P. has been obtained at a similar depth in another core from the Oslofjord where the depth of water is 250 m. Taking these two determinations, and making some allowance for compaction, a typical rate of deposition in this region is estimated to be of the order 0.8 m/1000 yr.

Additional information on clays of high plasticity at small overburden pressures is provided by an investigation in St Andrew Bay, Florida (Keller 1964). Two cores, nos 4 and 8, obtained at locations where the depth of water is about 10 m lead to the results plotted in Fig. 21.

#### (C) GOSPORT AND AVONMOUTH TIDAL FLATS

In 1942 three adjacent cores were taken from the tidal flats near the Royal Clarence Yard at Gosport, Hampshire, near the level of low water neap tides. The material within the top 30 cm (the length of the cores) is a uniform soft blue-grey clay the liquidity index of which changes very little with depth, and the shear strength shows only a slight increase (Fig. 9). Oedometer tests on two samples indicate a pre-consolidation pressure of about 100 gm/cm<sup>2</sup>, a figure substantially greater even than the total overburden pressure. A core taken just above low water neaps in the tidal flats at Avonmouth, Gloucestershire (Smotrych<sup>1</sup> shows almost identical results (Fig. 10).

The simplest explanation of these relatively high pre-consolidation pressures is that they are caused by desiccation, perhaps during a few days in summer, when the tide is exceptionally low and the flats become dry. There may be other processes involved, however, and for the present it is sufficient merely to record the observations whilst noting that although similar results have also been found in cores of red clay taken from the sea-bed in great depths of water (Hamilton 1964, Richards & Hamilton 1967) the effect is attributed to inter-particle bonding which can develop at extremely slow rates of deposition.

#### (D) DEPOSITIONAL WATER CONTENTS

In Fig. 11 the mean water content in the top 25 cm of clay at each of the six seabed locations mentioned previously has been plotted against the appropriate liquid limit, and the points lie close to a line representing a liquidity index equal to 1.75. Thus, if the Atterberg limits of a marine clay are known it is possible to estimate the depositional water content as expressed by the mean water content in the uppermost 25 cm of the sediment (which, incidentally, is about the same as the water content at a depth of 10 cm).

The points for the Gosport and Avonmouth intertidal flats are also plotted in Fig. 11 together with a result obtained from Stanpit Marsh near Christchurch, Hampshire, on a core of brown sandy clay (LL = 35, PL = 18) taken by the author at about mean tide level. From these three sites it seems that the water content of clays deposited in tidal flats is equal approximately to the liquid limit (liquidity index = 1.0); but more field work is necessary before this conclusion can be established.

<sup>&</sup>lt;sup>1</sup> SMOTRYCH, S. W. The physical and mechanical properties of the Avonmouth clay. Unpublished thesis, University of London, 1968.



FIG. 10. Intertidal flats at Avonmouth, Gloucestershire. Core taken about 1.5 m above low water mark neap tides (data from Smotrych, op. cit., 1968).

# 3. Late Quaternary clays

At many places along the coast, and particularly in estuaries, thick beds of clay have been deposited during the rise of sea-level consequent upon the melting of the ice-sheets of the Würm glaciation. Most of these clays are normally-consolidated and they can provide valuable information on sedimentation compression curves, usually in the pressure range 0.5 to 2.0 kg/cm<sup>2</sup> (a maximum depth of the order 30 m).

#### (A) AVONMOUTH AND GOSPORT

A detailed study of the post-glacial (Flandrian) clay at Avonmouth has been carried out by S. W. Smotrych and the author. The results are summarized in Fig. 12. In the top 4 m the pre-consolidation pressures exceed the overburden pressures, possibly as a consequence of intermittent drying during formation, but from 4 m to the base of the deposit at a depth of 13.5 m the clay is normally-consolidated under existing effective overburden loads. The importance of measuring pore water pressures by piezometers is well illustrated here, since the



FIG. 11. Depositional water contents of clays below the sea-bed and intertidal flats.

piezometric level at a depth of 8.5 m is approximately 2 m lower than the free water table (Fig. 12) although the ground water condition is approximately hydrostatic to a depth of about 5 m.

In the normally-consolidated zone the shear strength increases in direct proportion to the effective overburden pressure  $p_0$  and the liquidity index shows a consistent decrease with depth. Values of void ratio, derived from liquidity index and the average Atterberg limits, are plotted against  $p_0$  in Fig. 21; and the line is extrapolated back to a point representing the void ratio and pre-consolidation pressure in the core from the tidal flats at Avonmouth.

A radiocarbon date of 7090 yrs B.P. [GX 1112] has been obtained from the top of the basal peat found in the Avonmouth boring. Other borings in the coastal plain, further inland, show four peat beds above the basal layer. These have also

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been dated and the ages are given in Fig. 12 at the appropriate depths. The surface of the plain is almost precisely flat, at the level of high water springs, and it seems probable that throughout the entire thickness of the Flandrian clays intertidal sedimentation kept pace more or less with marine transgression. This suggestion is supported by an examination of the foraminifera, kindly made by Dr John Haynes at Aberystwyth, and by the fact that the radiocarbon dates when plotted against the relevant depth below high water fall approximately on the



FIG. 12. Borehole at St Andrew's Road in the coastal plain north of Avonmouth, Gloucestershire (data from Smotrych, op. cit., 1968).

curve (Jelgersma 1966) representing the post-glacial eustatic rise in sea level. These points indicate an average rate of deposition of 2.5 m/1000 yr during the period from 7000 to 2500 B.P. followed by a markedly lower rate up to recent times.

Data on another post-glacial estuarine clay, at Gosport, have been published previously (Skempton 1948A). The clay, which has a thickness of 15 m and is normally-consolidated, lies on a fresh-water peat formed in pollen zone IV (Godwin 1945) at a depth of 20 m below high tide level. The sedimentation compression curve is shown in Fig. 21 and, as at Avonmouth, the line is continued back to the point representing the intertidal deposit.

### (B) TILBURY

Tests on samples from several borings made in the reclaimed marsh about 1.5 miles east of Tilbury Docks, on the north shore of the Thames Estuary, have been

published (Skempton & Henkel 1953) and are presented with some amendments in Fig. 13. Overlying a sandy gravel there are three clay beds separated by peat layers, and the upper clay includes a discontinuous peat. The peats lie at approximately the same levels here as at Tilbury Docks where samples have been dated by radiocarbon analysis (Godwin *et al.* 1965), and from these results it is possible to estimate an average rate of deposition of about 2 m/1000 yr.



FIG. 13. Test results from borings in the marsh east of Tilbury Docks, Essex (modified after Skempton & Henkel 1953).

Below 3 m the clays are normally-consolidated except for a zone extending to a depth of about 50 cm beneath each of the peat layers, where the shear strength shows a significant increase (Fig. 13). This may indicate some desiccation during the later stages of deposition of the clays, or during the periods of peat formation, but the effect might be due to subsequent adsorption of organic cations from the peat.

Pore-water pressures in the gravel reflect tidal changes, and in calculating effective overburden pressures it has been assumed that piezometric levels in the clays decrease linearly with depth from the water table to a mean of several observations made in the gravel at low tide. The sedimentation compression curve plotted in Fig. 21 refers only to the upper and lower clays at Tilbury, which are essentially inorganic.



#### (C) ÄLVÄNGEN

Valuable data from a deep deposit of post-glacial clay have been obtained by Kallstenius (1963 and *in litt.*) at Älvängen near Gothenburg, Sweden, from a boring located 15 m from the bank of the Göta River (Fig. 14). The two piezometers give clear evidence of a small upward component of ground-water flow. Oedometer tests show a sharp point of curvature in the e-log p line. The values of  $p_i$  and  $p_c$  are therefore almost identical and they practically coincide with the effective overburden pressure  $p_0$ . Shear strengths, measured *in situ* by the vane, increase linearly with  $p_0$  in a regular manner throughout the entire thickness (about 24 m) of the clay which was tested, and there is a marked decrease in liquidity index with depth.

The liquid limit of the Älvängen clay (LL = 95) is similar to that of the Oslofjord clay (LL = 98) described in section 2 (B), and it is interesting to note from Fig. 21 that the sedimentation compression curve for Älvängen is virtually an extension of the line previously plotted from the Oslofjord tests. It can also be seen from this Figure that the Älvängen line lies slightly above the sedimentation compression curve for Tilbury, although the clay at the latter site has a greater liquid limit. This is a consequence of the higher sensitivity, and therefore the more open structure, of the Älvängen clay.

### (D) PISA

Recent investigations at the Tower of Pisa have revealed a complex series of Late Quaternary deposits underlying the coastal plain at this locality. The investigations have been carried out by the Geotechnical Group (of which the author was a member) of a Commission set up by the Ministero dei Lavori Pubblici. The results will be published in detail elsewhere but a summary of the data relevant to the present paper is shown in Fig. 15.

Underlying about 3 m of made ground dating from Roman and mediaeval times there are silts, in which some Bronze Age Pottery has been found, and a bed of sand containing fragments of wood. Professor F. W. Shotton has obtained a radiocarbon date of 3880 B.P. [Birm 42] on a sample of this wood from a depth of 9.4 m. Beneath the sand lies a soft grey clay about 10 m thick deposited in a shallow brackish-water lagoon and containing intact shells of Cardium. The clay is lightly over-consolidated in the top 2 m but is normally-consolidated throughout the lower 8 m. It rests on much firmer clays showing yellow or brown mottling, with small calcareous nodules and occasional plant remains some of which, from a depth of 23 m, have been dated at 9500 B.P. [GX 1007]. Clearly these mottled clays are over-consolidated, as revealed by the low liquidity index and relatively high strength (Fig. 15), and there can be no doubt that they were subjected to desiccation before the overlying clay was deposited. This fact, together with the radiocarbon age of the underlying clay (mentioned in the next paragraph), strongly suggests that the mottled clays were formed during the period of low sea level in the late or main Würm, probably under alluvial conditions.

Beneath these mottled clays there is a layer of silty sand devoid of organic remains. This overlies another bed of grey clay containing mainly unbroken shells of *Cardium (Cerastoderma)* sp. and fragments of several other species. After pretreatment with HCl Professor Shotton has determined a radiocarbon date of 36 400 B.P. on the *Cardium* shells from a depth of 30 m [Birm 76b], while shell fragments from the same depth have yielded a date of 35 900 B.P. without pretreatment [Birm 76a]. It is therefore certain that the clay was deposited during the mid-Würm interstadial, and a comparison with the elevation of the upper *Cardium* clay indicates that sea-level during the interstadial was roughly 20 m to 25 m lower than at present. The lower portion of the clay is normally-consolidated and similar in its physical properties to the upper clay; thus providing an extension of the Pisa sedimentation compression curve to effective pressures of about 3 kg/cm<sup>2</sup> (Fig. 21).

A second bed of over-consolidated mottled clays is then encountered, with calcareous nodules and traces of organic matter, overlying a thick deposit of sand. The quantity of organic material unfortunately proved to be insufficient for dating, but desiccation during a phase of low sea-level is again indicated; probably in the early Würm glacial period.

An approximate estimate of the rate of deposition of the upper grey clay can be obtained. The radiocarbon dates previously mentioned imply that its formation began about 9000 B.P. and ended about 4000 B.P. The water content at present averages 50 (void ratio  $e_2 = 1.35$ ) but before compaction the water content would



have been probably rather more than the liquid limit, say about 80 (void ratio  $e_1 = 2 \cdot 15$ ). The existing thickness  $(h_2)$  of 10 m would therefore be equivalent to an original thickness

$$h_1 = h_2 \frac{1 + e_1}{1 + e_2} = 13 \text{ m}$$

and the corresponding rate of deposition is 2.5 m/1000 yr.

Piezometers at three different depths show pore pressures significantly less than the values corresponding to hydrostatic ground water conditions. This result was not unexpected, as some pumping for water supply takes place from the lower sand, but it emphasizes once more the importance of measuring piezometric levels in order to establish correct quantitative values of effective overburden pressure.

#### (E) OTHER SITES

Sedimentation compression curves for other Late Quaternary clays have been plotted in Fig. 21 from data published elsewhere. The first of these is derived from investigations at BH.6 on the continental shelf in the Gulf of Mexico 45 miles off the Louisiana coast, where nearly 30 m of normally-consolidated clay lie beneath the sea bed (Fisk & McClelland 1959). Another valuable set of results have been obtained by Bjerrum (1967) from a post-glacial marine clay of medium plasticity at Drammen in Norway. He has also published information on a late-glacial clay from the same location; and a clay similar in age and lithology occurs below the carse lands of the River Forth near Grangemouth (Skempton 1948B). These two cases provide useful data on clays with liquid limits around 40, in the category of clays of low plasticity. Thick beds of uniform silty clay appear to be rare, and one of the few documented records for such deposits is found in a paper by Wu (1958) relating to a glacial lake clay of late Wisconsin age in the Detroit area. His work is of particular interest as it helps to define the lower boundary of the compression curves for argillaceous sediments (Fig. 21).

# 4. Pliocene and early Pleistocene clays and mudstones

#### (A) MILAZZO

Through the courtesy of Professor A. Croce of Naples University a summary can be given of preliminary results from a boring in clay of Piacenzian (Upper Pliocene) age at Milazzo, Sicily. The clay underlies 40 m of Quaternary sands and the boring, which is located near the sea coast, extends to a depth of 120 m (Fig. 16). Oedometer tests show that the clay is to a close approximation normallyconsolidated in relation to effective overburden pressures calculated on the assumption of hydrostatic ground-water conditions. The clay appears to be moderately uniform throughout the depth investigated and free from the minor structural discontinuities ('fissures') characteristic of many over-consolidated sedimentary clays. The water contents show a consistent decrease with depth, corresponding to a change in liquidity index from about 0.5 near the top to values of about 0.25 at 120 m.



# (B) SAN JOAQUIN VALLEY

Pumping from deep aquifers in the San Joaquin Valley, California, has resulted in regional subsidence of such magnitude that extensive investigations have been carried out for quantitative studies and predictions of future settlements (Gibbs 1959, Miller 1961). The strata (Fig. 17) comprise silts and clays of the San Joaquin formation overlain by two units of lenticular sands and clays separated by a bed of relatively impermeable diatomaceous lacustrine clay, the Corcoran clay (Davis &





Poland 1957). The lower unit and the Corcoran clay together make up the greater part of the Tulare formation, the base of which is certainly not later than Late Pliocene (Woodring 1952). The sands and clays were deposited on alluvial fans and flood plains by streams (Meade 1961) and radiocarbon dates ranging from 10 000 to 27 000 B.P. on wood fragments at depths between 6 and 20 m indicate that deposition has continued until recent times (Ives *et al.* 1967).

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Intensive pumping from the sands in the Tulare formation, below the Corcoran clay, began about 1944; and by 1956 the piezometric level had fallen approximately 80 m below ground water level in the upper sands. Sampling of the clays took place in 1957-58 when consolidation was evidently still in progress. Thus the effective overburden pressure in the clays at this time must be intermediate between the minimum values, operative before pumping began, and the maximum values which will be acting when consolidation is complete. These limits are shown in Fig. 17 together with the pre-consolidation pressures derived from oedometer tests. With due allowance for sample disturbance in cores taken from such great depths, it will be seen that the clays in the Tulare formation may be considered as normally-consolidated under the mean effective overburden pressures.

Deposition on flood plains and alluvial fans is an intermittent process. Most of the clays have therefore been subjected to desiccation, but it is clear from the oedometer test results that the overburden pressures now acting at depths of more than 100 m are greater than the effective stresses set up when the clays were exposed to drying. Hence the upper clays below this depth, as well as the Tulare clays, are normally-consolidated.

The values of liquidity index plotted in Fig. 17 include two results from another boring in the locality, making a total of twelve observations distributed over a range of depth from about 100 m to 570 m in which the liquidity index falls from approximately 0.3 to zero. In terms of void ratio the San Joaquin results (see Fig. 21) form a valuable extension of the sedimentation compression curve from Milazzo, the average liquid limits of the clays from these two sites being almost identical.

### (C) BAKU

Tests on numerous samples taken from a boring extending 220 m beneath the sea bed in the Alyaty Sea, south of Baku peninsular, have been published by Monyouchko (1963) and average values for groups of samples at four different depths are given in Table 2. Also included in this table are two sets of results extracted from a summary of tests from deeper borings in the same region (Korobanova *et al.* 1963). The clays are mostly of Quaternary age, probably reaching into the Pliocene at the greatest depth quoted in Table 2.

It is unfortunate that Korobanova and her co-workers and Monyouchko do not report any oedometer tests or piezometric levels. Effective overburden pressures have therefore been calculated on the most probable assumption for sub-aqueous deposits, namely the assumption of hydrostatic pore pressure, but the values of  $p_0$ in Table II can only be regarded as reasonable estimates. Nevertheless the data from Baku are of considerable interest as they provide approximate information on the sedimentation compression curve for clay of low plasticity over a very wide range of pressure, and greatly extend the results previously mentioned for clays having similar index properties from Grangemouth and Drammen (see Fig. 21).

#### (D) KAMBARA

In 1963 a borehole was drilled to a depth of 3700 m at Kambara, Japan, near the centre of the Niigata sedimentary basin where geosynclinal deposition has been

more or less continuous since Miocene times. The strata consist chiefly of sandstones and mudstones (Fig. 18). Porosities were determined on samples which had been sealed immediately after coring, to preserve the natural water content, and drilling-mud densities were recorded as the borehole advanced (Nagumo 1965A, B). It seemed, therefore, that the Kambara site was likely to yield data of unique importance; with reliable porosity measurements in extremely deep beds of normally-consolidated argillaceous sediments, in which the pore pressures were known with some accuracy.

	$M = Monyouchko (1963)$ $\rho = 2.75$		$K = Korobanova \ et \ al.$ (1963) $CF = 42 \ per \ cent$				
Ref.	Depth m	∲₀ kg/cm²	w	LL	PL	$\frac{w - PL}{PI}$	e <sub>0</sub>
M	31	2.8	31	41	22	0.48	0.82
Μ	66	6.3	25	38	19	0.32	<b>0.7</b> 3
K	8o	7.8	24	39	20	0.51	0.67
Μ	113	10	23	37	19	0.25	0.67
Μ	190	19	22	41	19	0.14	0.62
К	575	62	18	43	22	-0.13	<b>0·4</b> 5
			average:	40	20		

TABLE 2: Summary of test results on samples from the Alyaty Sea, south of Baku peninsula

In response to requests for further information Dr Nagumo generously sent the results of the tests on porosity and grain density, and stated that the weight of the drilling-mud had been adjusted to exert a fluid pressure, at a given depth, about 10 per cent greater than the corresponding *in situ* pore pressures. The pore pressures calculated on this basis are plotted in Fig. 18. They are hydrostatic in the upper sands, to a depth of about 800 m, but reach values about 15 per cent above hydrostatic towards the bottom of the borehole. Dr Nagumo also sent a revised stratigraphical analysis prepared by Dr Yasufumi Ishiwada, chief petroleum geologist of the Geological Survey of Japan, and a rather detailed description of the lithology from which the interesting point emerged that the mudstones displayed no fissility. Finally, Professor Fuyuji Takai very kindly entered into correspondence concerning the broad correlation of the formations in the Niigata basin with their European equivalents. His conclusions are set out in Table 3.

From these correlations it can be deduced that the Pliocene strata at Kambara attain a thickness of 2600 m. The average porosity of the Pliocene mudstones is about 25 per cent (void ratio = 0.35) and the sandstones have a similar porosity. Assuming the sediments to be composed of approximately equal total thicknesses of mudstone and sandstone, and taking the depositional porosities to be 40 and 65 per cent respectively, then the original thickness of the Pliocene would have been



4500 m. Now the base of the Pliocene in Italy is dated at about 7 m.y. (Tongiorgi & Tongiorgi 1964) and recent determinations of the date of the Plio-Pleistocene boundary are converging on a figure of about 2 m.y. (Mathews & Curtis 1966, Berggren *et al.* 1967). Thus the average rate of deposition at Kambara during Pliocene times was approximately 0.9 m/1000 yr.

In Fig. 18 the void ratios of the mudstones are plotted, and a summary of these values is given in Fig. 21 in the form of a sedimentation compression curve. Unfortunately no index properties are available, but since the Kambara results continue the line already established at Milazzo and San Joaquin Valley it is reasonable to group the Kambara mudstones with the clays from these sites.

	TABLE 3	
	Europe	Niigata
Pleistocene	Calabrian and later stages	Uonuma
Middle and Upper Pliocene	Piacenzian	Haizume Nishiyama
Lower Pliocene	Pontian Meotian	Hamatsuda
Upper Miocene (in part)	Chersonian	Shiiya

#### (E) PO VALLEY

Storer (1959) has published many density measurements on samples of mudstone taken from deep borings in the Po Valley. A typical set of results from a borehole at Camposanto is shown in Fig. 19 together with a geological section through this locality. The stratigraphy was given by Rocco & Jaboli (1958) who also recorded piezometric observations to depths of 3000 m at a number of sites in the region. These observations are assembled in Fig. 20, and they indicate that the pore pressures at depths below 1500 m are 15 to 20 per cent above hydrostatic values. The measurements of density were made on samples after drying in the laboratory. Consequently the densities can be used for calculating *in situ* porosity or void ratio only when the sample in its natural state already had a water content lower than the shrinkage limit. For most clays this means a water content not exceeding about 12, corresponding to a void ratio of 0 32 or a porosity of 25 per cent (Terzaghi 1926).

Values of density and void ratio are plotted in Fig. 20, each point being the mean of at least six measurements. These mean void ratios are further averaged in pairs when plotting the sedimentation-compression curve in Fig. 21. As at Kambara, no index properties are available for the Po Valley sediments but it appears from Fig. 21 that they may be classed in the same group as the clays from Baku.

From maps published by Lucchetti (1959) a typical thickness of Pliocene deposits in the Po Valley is 3000 m, near the centre of the basin, and the overlying Quaternary attains a thickness of about 2000 m. The Pliocene mudstones have a porosity of approximately 20 per cent and, assuming that they originated as clays



FIG. 19. Boring no. 1 at Camposanto, Italy (test results from Storer 1959, stratigraphy from Rocco & Jaboli 1959).



FIG. 20. Summary of results from borings in the Po Valley (data from Storer 1959, Rocco & Jaboli 1958).

of low plasticity, their porosity when first laid down might have been about 55 per cent (see Fig. 11). The sands seem to have compacted very little, their porosity having been reduced from a possible value of 40 per cent to about 30 per cent (Fig. 20). Borehole records in the Pliocene (Rocco & Jaboli 1959) showed mudstones predominating over sands, but the exact ratio is of minor consequence as any proportion above 3:1 leads to the conclusion that the original thickness was roughly 5000 m and, therefore, that the average rate of deposition of the Pliocene sediments



FIG. 21. Sedimentation compression curves for normally consolidated argillaceous sediments.

was approximately 1 m/1000 yr. In the Quaternary sections sands predominate, and with reasonable assumptions the original thickness is estimated to be 2400 m; corresponding to an average rate of deposition of about 1.2 m/1000 yr.

# 5. Summary of results

Sedimentation compression curves for normally consolidated argillaceous deposits from twenty sites are assembled in Fig. 21. In this figure the *in situ* void ratio  $e_0$  is plotted against effective overburden pressure  $p_0$  using a logarithmic scale for pressure; the values of water content are calculated assuming full saturation and a grain density of 2.7; a scale of porosities will be found along the right-hand margin and an approximate conversion from pressure to depth is set out along the bottom of the graph. These field data confirm a result already well known in the laboratory, namely that the relation between  $e_0$  and  $\log p_0$  is essentially linear for any particular clay, although at very high pressures the lines would presumably become curved asymptotes to the axis of zero void ratio.

It is also clear from Fig. 21 that the void ratio of a normally consolidated clay, at a given overburden pressure, depends upon the nature and the amount of clay minerals present, as indicated by the liquid limit. Indeed it is possible in this figure to define approximate boundaries between clays of low, medium and high plasticity



FIG. 22. Relation between liquidity index and effective overburden pressure for the clays designated in Fig. 21.

represented by liquid limits of 30, 50, 90 and 140. Similarly if the points in Fig. 21 are replotted in terms of liquidity index, rather than void ratio, they lie within a moderately narrow band as shown in Fig. 22. Moreover an examination of the position of the various clays in this band reveals that those with a high sensitivity (e.g. Drammen and Älvängen) lie towards the upper part of the zone while those with a low sensitivity (e.g. Gosport and Tilbury) have a relatively low liquidity index.

Various correlations, then, could readily be derived between void ratio, pressure, Atterberg limits and sensitivity; but it is probably best to leave the data in the form of the sedimentation compression curves of Fig. 21. This expresses the field observations in the most direct manner possible and establishes with sufficient clarity the volumetric response of argillaceous sediments to gravitational compaction.

# 6. References

ANDRESEN, A., SOLLIES, S. & RICHARDS, A. F. 1965. N.G.I. gas-operated sea-floor sampler. Proc. 6th Int. Conf. Soil Mech. 1, 8-11.

ATTERBERG, A. M. 1911. Die Plastizitat der Tone. Int. Mitt. Bodenkunde 1, 4-37.

BERGGREN, W. A., PHILLIPS, J. D., BERTELS, A. & WALL, D. 1967. Late Pliocene-Pleistocene stratigraphy in deep sea cores from the south-central North Atlantic. *Nature*, Lond. 216, 253-5.

BIRCH, F. 1966. Compressibility; Elastic Constants. In S. P. Clark (Ed.) Handbook of physical Constants Mem. geol. Soc. Am. no. 97.

BJERRUM, L. 1967. Engineering geology of Norwegian normally-consolidated marine clays as related to settlement of buildings. *Géotechnique* 17, 83-117.

CASAGRANDE, A. 1932. Research on the Atterberg limits of soils. Public Roads 13, 121-36.

---- 1936. The determination of the pre-consolidation load and its practical significance. Proceedings of the 1st International Conference on Soil Mechanics (Harvard), vol. 3, pp. 60-4.

---- 1948. Classification and identification of soils. Trans. Amer. Soc. civ. Eng. 113, 901-30.

COLEMAN, J. D. & FARRAR, D. M. 1966. Soil moisture deficit and water-table depth in London Clay beneath neutral grassland. Q. Jl R. met. Soc. 92, 162-8.

DAVIS, G. H. & POLAND, J. F. 1957. Ground-water conditions in the Mendota-Huron area, Fresno and Kings Counties, California. Water-Supply Pap. U.S. geol. Surv. 1360-G.

EMERY, K. O. 1960. The sea off southern California. New York (Wiley), 249-43.

---- & DIETZ, R. S. 1941. Gravity coring instrument and mechanics of sediment coring. Bull. geol. Soc. Am. 52, 1685-1714.

---- & HÜLSEMAN, J. 1964. Shortening of sediment cores collected in open barrel gravity corers. Sedimentology 3, 144-54.

FARRAR, D. M. & COLEMAN, J. D. 1967. The correlation of surface area with other properties of nineteen British clay soils. J. Soil Sci. 18, 118-24.

FISK, H. N. & McClelland, B. 1959. Geology of continental shelf off Louisiana: its influence on offshore foundation design. Bull. geol. Soc. Am. 70, 1369-94.

GIBBS, H. J. 1959. A laboratory testing study of land subsidence. Proceedings of the 1st Pan-American Conference on Soil Mechanics, vol. 1, 13-36.

— & LARCOM, F. B. 1958. Laboratory studies on drill-core samples for deep subsidence investigations: San Joaquin Valley. U.S. Bureau of Reclamation Earth Lab. Report EM-531.

GIBSON, R. E. 1958. The progress of consolidation in a clay layer increasing in thickness with time. Géotechnique 8, 171-82.

GODWIN, H. 1945. A submerged peat bed in Portsmouth harbour. New Phytologist 44, 152-5.

----, WILLIS, E. H. & SWITSUR, V. R. 1965. Cambridge University natural radiocarbon measurements-VII. Radiocarbon 7, 205-12.

HAMILTON, E. L. 1964. Consolidation characteristics and related properties of sediments from experimental Mohole (Guadalupe site). J. geophys. Res. 69, 4257-69.

HEDBERG, H. D. 1936. Gravitational compaction of clays and shales. Am. J. Sci. 31, 241-87.

Ives, P. C., LEVIN, B., OMON, C. L. & ROBIN, M. 1967. U.S. Geological Survey radiocarbon dates—IX. Radiocarbon 9, 505-29.

JAKOBSON, B. 1953. Origin cohesion of clay. Proceedings of the 3rd International Conference on Soil Mechanics (Zurich) vol. 1, pp. 35-7.

JELGERSMA, S. 1966. Sea-level changes during the last 10,000 years. Proceedings of the International Symposium on World Climate 8000 to 0 B.C. London (Royal Meteorological Society), pp. 54-69.

KALLSTENIUS, T. 1963. Studies of clay samples taken with standard piston sampler. Proc. Swedish geotech. Inst. no. 21.

KELLER, G. H. 1964. Investigation of the application of standard soil mechanics techniques and principles to bay sediments. U.S. Naval Oceanographic Office, Informal MS. Report 0-4-64.

KIDWELL, A. L. & HUNT, J. M. 1958. Migration of oil in recent sediments at Pedernales, Venezuela., In L. G. Weeks (Ed.) Habitat of oil. Tulsa (American Association of Petroleum Geologists), pp. 790-817.

- KOROBANOVA, I. G., KOPYLOVA, A. K. & KOVALEVA, A. P. 1963. Physicomechanical properties of argillaceous deposits of the Baku archipelago during lithifaction. *Dokl. Akad. Sci. USSR.* (Earth Science section) 149, 163-5.
- Ku, T. L. 1965. Dating of deep-sea sediments by <sup>230</sup>Th and <sup>231</sup>Pa methods. Trans. Am. geophys. Un. 46, p. 168.
- LUCCHETTI, L. 1959. Tettonica padona. In I Giacementi Gassiferi dell'Europa Occidentale vol. 2. Rome (Accad. Naz. dei Lincei), pp. 17–28.
- LYELL, C. 1871. Students' elements of geology. London (John Murray).
- MCCLELLAND, B. 1967. Progress of consolidation in delta front and prodelta clays of the Mississippi River. In A. F. Richards (Ed.) Marine Geotechnique. Urbana (University of Illinois Press), pp. 22-40.
- MATHEWS, W. H. & CURTIS, G. H. 1966. Date of the Pliocene-Pleistocene boundary in New Zealand. Nature, Lond. 212, 979-80.
- MEADE, R. H. 1961. Compaction of montmorillonite-rich sediments in Western Fresno County, California. Prof. Pap. U.S. geol. Surv. 424D, 89-92.
- MILLER, R. E. 1961. Compaction of an aquifer system computed from consolidation tests and decline in artesian head. Prof. Pap. U.S. geol. Surv. 424B, 54-8.
- MONYOUCHKO, A. M. 1963. The change in physico-mechanical properties of marine clays by lithogenic processes, in the region of the Baku archipelago. In *The development of engineering*geological properties of clays during lithogenesis Moscow (Nat. Acad. Sci. USSR), pp. 5-21 [In Russian].
- NAGUMO, S. 1965A. Compaction of sedimentary rocks: a consideration by the theory of porous media. Bull. Earthquake Res. Inst. 43, 339-48.
- ---- 1965B. An estimation of tectonic stresses from compaction of sedimentary rock strata. Bull. Earthquake Res. Inst. 43, 565-76.
- RICHARDS, A. F. 1961. Investigations of deep-sea sediment cores—I. Shear strength, bearing capacity and consolidation. U.S. Navy Hydrographic Office Tech. Rep. TR-63.
- —— 1962. Investigations of deep-sea sediment cores—II. Mass physical properties. U.S. Navy Hydrographic Office Tech. Rep. TR-106.

RICHARDS, A. F. & HAMILTON, E. L. 1967. Investigations of deep-sea sediment cores—III. Consolidation. *Marine Geotechnique*. Urbana (University of Illinois), pp. 93-117.

ROCCO, T. & JABOLI, D. 1958. Geology and hydrocarbons of the Po Basin. In L. G. Weeks (Ed.) Habitat of oil Tulsa (American Association of Petroleum Geologists), pp. 1153-67.

— & — 1959. Descrizione dei giacementi gassiferi padani. In I Giacementi Gassiferi dell'Europa Occidentale vol. 2 Rome (Accad. Naz. dei Lincei), pp. 134-406.

ROSHOLT, J. N., EMILIANI, C., GEISS, J., KOCZY, F. F. & WANGERSKY, P. J. 1961. Absolute dating of deep-sea cores by the <sup>231</sup>Pa/<sup>230</sup>Th method. J. Geol. **69**, 162-85.

-----, ----, -----, & ----- 1962. <sup>231</sup>Pa/<sup>230</sup>Th dating and <sup>18</sup>O/<sup>16</sup>O temperature analysis of Core A254-BR-C. J. geophys. Res. 67, 2907-11.

SKEMPTON, A. W. 1944. Notes on the compressibility of clay. Q. Jl geol. Soc. Lond. 100, 119-35. — 1948A. The geotechnical properties of a deep stratum of post-glacial clay at Gosport.

Proceedings of the 2nd International Conference on Soil Mechanics (Rotterdam), 1, 145-50.

- ----- 1948B. Vane tests in the alluvial plain of the River Forth, near Grangemouth. *Géotechnique* 1, 111-24.
- ---- 1953A. Soil mechanics in relation to geology. Proc. Yorks. geol. Soc. 29, 33-62.
- ---- 1953B. The colloidal activity of clays. Proceedings of the 3rd International Conference on Soil Mechanics (Zurich), 1, 57-61.
- ----- 1954. The structure of inorganic soil (discussion). Proc. Amer. Soc. civ. Eng. 80, separate 478, 19-22.
- ----- 1960. Effective stress in soils, concrete and rocks. Conference on Pore Pressure & Suction in Soils (London), pp. 4-16.
- & HENKEL, D. J. 1953. The post-glacial clays of the Thames estuary at Tilbury and Shellhaven. Proceedings of the 3rd International Conference on Soil Mechanics (Zurich), **1**, 302–8.
- ----- & NORTHEY, R. D. 1952. The sensitivity of clays. Géotechnique 3, 30-53.

- STORER, D. 1959. Costipamento dei sedimenti argillosi nel Bacino Padano. In I Giacementi Gassiferi dell'Europa Occidentale, vol. 2 Rome (Accad. Naz. dei Lincei), pp. 519-36.
- TERZAGHI, K. 1923. Die Berechnung der Durchlässigkeitsziffer des Tones aus dem Verlauf der hydrodynamischen Spannungserscheinungen. Sitz. Akad. Wissen. Wien Math-naturw. Kl. 132, 105-24.
  - 1925. Erdbaumechanik auf bodenphysikalischer Grundlage. Vienna (Deuticke).
  - -- 1926. Simplified soil tests for subgrades and their physical significance. Public Roads 7, 153-62.
- ----- 1936. The shearing resistance of saturated soils. Proceedings of the 1st International Conference on Soil Mechanics (Harvard) 1, 54-6.
- ---- 1941. Undisturbed clay samples and undisturbed clays. J. Boston Soc. civ. Eng. 28, 211-31.
- TKHOSTOV, B. 1963. Initial Rock Pressures in Oil and Gas Deposits. (Trans. R. A. Ledward) London (Pergamon Press).
- TONGIORGI, E. & TONGIORGI, M. 1964. Age of the Miocene-Pliocene limit in Italy. Nature, Lond. 201, 365-7.
- WOODRING, W. P. 1952. Pliocene-Pleistocene boundary in California coast ranges. Am. J. Sci. 250, 401-10.
- WU, T. H. 1958. Geotechnical properties of glacial lake clays. Proc. Am. Soc. civ. Eng. 84, SM3, Paper 1732.

Since the preparation of this paper additional information on the San Joaquin Valley clays has been published:

JOHNSON, A. I., MOSTON, R. P. & MORRIS, D. A. 1968. Physical and hydrologic properties of water-bearing deposits in subsiding areas in Central California. *Prof. Pap. U.S. geol. Surv.* 497-A.

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# 7. Appendix: a numerical investigation of Bishop's expression for effective overburden pressure

In a letter to Dr A. S. Laughton dated 24 November 1953 Professor A. W. Bishop gave the following expression for effective overburden pressure:

$$p_0 = \sigma_v - \left(\mathbf{I} - \frac{C_s}{C}\right) u$$

where  $\sigma_v$  is the total overburden pressure, u is the pore pressure and  $C_s$  and C are respectively the compressibilities of the mineral particles and of the sediment. The derivation of this expression and its experimental confirmation by Laughton are briefly described by Skempton (1960) who showed that the term  $C_s/C$  is numerically significant for rocks of very low porosity but unimportant in sands and clays. The data given in the present paper from the Po Valley and Kambara, in the Niigata basin, now permit an assessment to be made of the ratio  $C_s/C$  for mudstones having a porosity of about 15 per cent and occurring at depths of 2000 to 3500 m in normally-consolidated conditions. A representative upper limit for the compressibility of mineral particles is approximately  $C_s = 2.5 \times 10^{-6} \text{ cm}^2/\text{kg}$ ; some individual values in these units being quartz 2.7, calcite 1.4, mica 2.3, orthoclase 2.1 (Birch 1966). To evaluate the compressibility of the mudstones it is necessary to plot void ratio  $e_0$  against effective pressure  $p_0$ . The compressibility at any given value of  $p_0$  is then derived from the expression

$$C = \frac{\Delta e}{\Delta p_0(\mathbf{1} + e_0)}$$

where  $\Delta e$  is the decrease in void ratio corresponding to an increase in effective pressure  $\Delta p_0$ .

At Kambara a porosity of 15 per cent is reached near the bottom of the boring at a depth of about 3500 m in mudstones of Upper Miocene age (Fig. 18). If it is assumed that  $p_0 = \sigma_v - u$  then C is approximately 750  $\times$  10<sup>-6</sup> cm<sup>2</sup>/kg. The ratio  $C_s/C$  therefore has the value 0.003, and the assumption that the term  $[1 - (C_s/C)]$ is equal to unity is justified within very close limits. Conversely if it is assumed that  $p_0 = \sigma_v$  the compressibility at this depth is about 350  $\times$  10<sup>-6</sup> cm<sup>2</sup>/kg and the ratio  $C_s/C$  is still only 0.007; which shows that the assumption  $p_0 = \sigma_v$  is incorrect.

Very similar results are obtained from calculations based on the Po Valley borings (Fig. 20). Here, typically, a porosity of 15 per cent is found at depths between 2000 and 2500 m near the base of the Upper Pliocene, and the compressibility at this porosity is approximately  $850 \times 10^{-6}$  cm<sup>2</sup>/kg assuming that  $p_0 = \sigma_v - u$ . The ratio  $C_s/C$  is again about 0 003 and clearly the effective overburden pressure can be taken as  $p_0 = \sigma_v - u$  for all practical purposes.

The compressibility C increases as the pressure decreases and therefore at smaller depths the effective overburden pressure, in normally consolidated argillaceous deposits, is given to a still higher degree of approximation by Terzaghi's expression

$$p_0 = \sigma_v - u$$

# The Quaternary history of the Lower Greensand escarpment and Weald Clay vale near Sevenoaks, Kent

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In the neighbourhood of Sevenoaks Weald many of the small hills and ridges standing up to 20 or 30 m above the streams in the clay vale south of the Lower Greensand escarpment are capped by Head deposits consisting of angular chert fragments, and other stones derived from the Greensand, set in a clay matrix. These deposits extend for a distance of at least 2 km from the escarpment, forming dissected remnants of what were originally extensive sheets, inclined at gradients of about  $1.5^{\circ}$ . The available evidence suggests they are periglacial solifluction deposits of Wolstonian age. Probably at about the same period large-scale structural disturbances occurred in what are now spurs of the escarpment; massive blocks of the Hythe Beds subsided into the underlying Atherfield and Weald Clays, and the clays were forced up at the foot of the scarp in the form of bulges.

Following this stage considerable erosion took place in the vale, accompanied by retreat of the escarpment within embayments between the spurs. On the eroded landscape solifluction debris moved up to 1 km from the scarp face during the Devensian period. This deposit again consists predominantly of clay with embedded angular chert fragments. It is about 2 m thick, with a minimum gradient of a little more than  $2^{\circ}$ , and overlies brecciated Weald Clay which typically contains several slip surfaces in its uppermost layers. Landslips in the escarpment within the embayments probably occurred at about the same time.

Not long afterwards, in the Late-Devensian Interstadial, around  $12\,000$  radiocarbon years B.P., a soil formed of which traces can be found buried beneath a lobate solifluction sheet. The lobes extend over the lower sheet for distances of 300 m from the scarp foot at an average slope of about 7°. In the subsequent Postglacial period only minor changes have taken place; some escarpment landslips have been reactivated and the streams in the vale have eroded small channels or valleys not more than 4 m deep.

Based on thaw-consolidation theory, and by using measured properties of the clays, calculations are presented which provide a reasonable explanation, in terms of soil mechanics principles, for solifluction movements of the active layer above permafrost on slopes inclined at angles as low as 1.5 or 2°. Under temperate conditions, mass movements are possible only on slopes steeper than about 8°.

The paper includes an account of the longitudinal profiles and stratigraphy of the Eden and Medway river terraces.

#### 1. INTRODUCTION

In connection with the design of a new road (A 21) by-passing the town of Sevenoaks, in Ken site investigations were made in the Lower Greensand escarpment and the clay vale lying the south. During the course of these investigations the opportunity was taken of amplifying the field work to provide additional data of scientific interest. The main area of study, centred on the village of Sevenoaks Weald (National Grid Reference TQ 528509), is shown in figure 1. and further illustrated by the sections in figures 2 and 3. Most of the trial pits and borings and associated laboratory tests were carried out in the years 1965–6, but several visits have been made subsequently to examine features of the country down to the River Eden, and further information on the escarpment was obtained in 1969 from deep borings near Bayley's Hill and Ide Hill, respectively  $1\frac{1}{2}$  and 4 km west of Sevenoaks Weald.



FIGURE 1. Map of the district around Sevenoaks Weald.

The regional setting is shown on the geological sketch map, figure 4. This is based on the Institute of Geological Sciences One-inch map of the country around Sevenoaks and Tonbridge (Sheet 287, first published 1950, partly revised 1971) and on the manuscript Six-inch maps (I.G.S. Library, South Kensington) modified in a few particulars by our own observations. Mr S. Buchan and the late Mr H. G. Dines surveyed the area covered by figure 4 between 1932 and 1936, and the southern part of the map was revised by Dr C. R. Bristow in 1965–6. Many details of the geology of the district are given in the Memoir accompanying Sheet 287 (Dines, Buchan, Holmes & Bristow 1969).

When mapping south of the escarpment, Dines and Buchan were impressed by the drifts of flint and chert-bearing gravels capping numerous small, isolated hills and ridges which rise above the general level of the vale and extend over a wide tract of land nearly to the northern



limits of the belt of river terraces. In an important paper published jointly with other officers of the I.G.S. (Dines *et al.* 1940), on the subject of Head deposits, they identified the high-level chert gravels as the dissected remnants of a widespread sheet of debris which had been derived from the Lower Greensand and moved south by periglacial solifluction down the gentle slopes of the then existing landscape. They also recognized a dissected fan of Head, composed of flints and flint pebbles with the addition of chert, extending down to Edenbridge from gaps in the escarpment south of Limpsfield. The source of the flints and pebbles has long been attributed to gravels, of which remains still exist, near Limpsfield (Topley 1875); these gravels in turn having been derived mostly from the Chalk escarpment further north.



FIGURE 3. Section X-X through Sevenoaks Weald.

The Head deposits were later examined by Bird (1964), who agreed with this interpretation of the chert gravels but suggested that the flint-bearing gravels had been transported by streams, possibly fed by heavy rain or melting snow, rather than by solifluction. More recently, in the Sevenoaks Memoir, high-level flint gravels have been identified above the north bank of the Eden as far east as How Green (1.3 km north of Hever).

Dr Bird also considered the age of the high-level gravels and arrived at the tentative conclusion that they were formed after the development of a valley floor equivalent to the Boyn Hill terrace in the London Basin, which is generally accepted as being of Hoxnian Interglacial age. We have further examined the relation between these gravels and the river terraces, and while broadly agreeing with Bird's conclusion we can show that the gravels are older than the Last (Ipswichian) Interglacial. It therefore seems probable that the period of their deposition can be correlated with the Wolstonian glacial stage. Stratigraphically the high-level chert and flint gravels will be classified as Older Head deposits.

Before the present investigations little information had been available on periglacial solifluction deposits of the Last (Devensian) Glaciation in the area. But evidence can now be given for the existence of chert-bearing gravels, typically 2 m in thickness, spreading off the foot of the scarp face as an almost continuous sheet within the embayment between Hubbard's Hill and River Hill (figure 1) and extending at least 500 m to the south. These gravels can also be traced for considerable distances down some of the valleys. In addition there is a lobate sheet overlying the lower sheet of chert gravels; and in several places beneath the lobes a fossil soil has been found, developed in slope-wash deposits on the lower sheet. The soil is dated by radiocarbon assay to 12200 years B.P. and its formation can therefore be correlated with the Late-glacial Interstadial of the Devensian. The Lower Solifluction Sheet and the Lobes may be grouped together as the Younger Head deposits.



FIGURE 4. Sketch map of the Weald Clay Vale and the Eden-Medway Terraces

Our investigations have also demonstrated structural disturbances at Ide Hill and Hubbard's Hill and landslips in the scarp face within the embayment. The structural disturbances take the form of deep gulls and block subsidence in the Hythe Beds associated with bulging of the underlying clays. It is probable that the greater part of these disruptions of the strata occurred under periglacial conditions during the Wolstonian. Subsequent to that stage there has been much erosion in the clay vale, accompanied by retreat of the scarp face within the embayment. The embayment landslips may have originated in periods of excessively high ground water level brought about by melting snow in the Devensian.

Changes in the landscape of the vale during Postglacial (Flandrian) times appear to have been restricted to erosion by the streams of shallow channels cut in or through the lower solifluction gravels and the formation of small valleys in the Weald Clay, not more than 4 m in depth and floored by narrow belts of alluvium.

Preliminary accounts of the Devensian solifluction deposits and fossil soil, and also of the large-scale disturbances at Hubbard's Hill, have previously been published (Skempton & Petley 1967; Weeks 1969).

#### 2. STRATIGRAPHY: CRETACEOUS

The general stratigraphy is set out in table 1. More detailed correlations of the Quaternary deposits will be given later, and are summarized in table 5.

### (a) Hythe Beds

The Hythe Beds form a plateau at an altitude of a little over 200 m above Ordnance Datum. From the northern limit of the plateau the land falls gently in a northerly direction and the southern edge is defined by an escarpment. At intervals along the escarpment there are spurs of relatively stable ground, such as River Hill and Hubbard's Hill (figure 1), which continue to the south as ridges standing above the lower landscape of the clay vale. The old roads of the Weald ascend along these ridges. Between the spurs, or 'hills', the escarpment has been caused to retreat by erosion and takes the form of a wide embayment. Within the embayment between the two hills just mentioned the scarp face is steep, inclined at average angles of 15-25°, and has been subjected to landslipping.

TABLE 1. GENERAL STRATIGRAPHY

Quaternary	Alluvium Terraced River Gravels Older and Younger He	ad deposits
Cretaceous	Lower Greensand	Hythe Beds Atherfield Clay
	Wealden Beds	Weald Clay Hastings Beds

The Hythe Beds have a thickness of about 45 m beneath the escarpment crest. They consist of sands and sandstones, buff or pale greenish grey in colour (known as 'Hassock'), sandy limestone (the 'Kentish Rag', much used in the past as building stone) and, in the upper parts of the formation, bands of chert up to 15 cm in thickness. The chert is extremely resistant to weathering; it is found as broken, angular or sub-angular fragments in the Head deposits and river gravels. The sandy beds are uncemented but can have a considerable clay content, especially near the base of the formation. Index properties of two representative samples are given in table 2; the clayey sand is typical Hythe Beds material between two Ragstone layers, and the other sample was taken near the base.

In logging the trial pits and borings the transition from Hythe Beds to the underlying Atherfield Clay was assumed to occur at a change from the buff or khaki coloured basal clay to a more plastic grey-brown clay. To check the validity of this and other lithological boundaries Mr D. J. Carter, of the Department of Geology at Imperial College, kindly carried out a micropalaeontological examination of two continuous cores taken through the basal layers of the Hythe Beds down to the Weald Clay. He reports (private communication) that so far as the Hythe/Atherfield junction is concerned the true boundary is marked by an alteration from foraminifera indicating a muddy, perhaps brackish water environment in the Atherfield Clay to a fully marine fauna in the Hythe Beds. In both cores this boundary occurred about 40 cm lower than the lithological change noted above. To this extent, then, there is probably a consistent error in plotting the base of the Hythe Beds (e.g. in figures 19–21).
# (b) Atherfield Clay

Where the Atherfield Clay is not greatly affected by structural disturbance or landslipping near the foot of the scarp face, borings show that it is about 8 m in thickness, reaching a maximum of 12 m at Ide Hill. Its outcrop is generally obscured by Head deposits.

Two distinct facies can be recognized: an upper clay of medium to high plasticity, and a lower silt. Both are grey in colour. The lithological difference is clear and, in the cores examined by Mr Carter, the microfauna indicate a change at this boundary from marine conditions in the silt to a muddy or brackish water environment in the clay. Index properties were determined on a number of samples from both facies; typical values are given in table 2 together with the range of liquid limit. The clay facies has a 'clay fraction' (particles smaller than  $2 \mu m$ ) of about 65 % of the dry mass.

	water	liquid	plastic	plasticity
	content	limit	limit	index
Head deposits				
Slope-wash	35	65 (60-75)	28	37
Chert Gravels matrix				
Lobe	25	42 (40 - 45)	<b>24</b>	18
Lower sheet	28	52 (40-65)	26	<b>26</b>
Flint Gravels matrix	-	27	17	10
Hythe Beds				
clayey sand	19	31	<b>24</b>	7
Basal bed	28	64	24	40
Atherfield Clay				
clay	30	85 (65-95)	30	55
silt	19	35 (30-40)	18	17
Weald Clay				
brecciated	27	60 (40-80)	23	37
undisturbed	23	60 (40-80)	23	37

**TABLE 2. INDEX PROPERTIES** 

#### (c) Weald Clay

The Weald Clay is a massive deposit, up to 300 m in thickness, the outcrop of which forms the clay vale of the northern Weald of Kent. The vale extends 4–6 km from the escarpment towards the Rivers Eden and Medway. In this distance its altitude falls from about 150 to 40 m above O.D. The regional dip is northerly and varies from 2° or 3° under most of the vale to almost zero under the Hythe Beds plateau. A fault separates the Weald Clay from the outcrop of the Hastings Beds, which rise to form high land south of the rivers.

Characteristically the clays are brackish or freshwater deposits displaying fine laminations or silt partings which often show steep bedding planes and other sedimentary structures. But the laminated clays can alternate with more homogeneous, fissured clays of higher plasticity, presumably laid down in deeper water and there are occasional layers of siltstone and limestone. Ironstone modules also occur. In its unweathered and undisturbed state the Weald Clay is dark grey in colour with water contents usually at, or slightly below, the plastic limit. Where it has been exposed to weathering the clay is oxidized to a brown colour.

Beneath the solifluction gravels, for depths of several metres, the clay is brecciated. In this state it consists of small lumps of intact clay in a matrix of almost completely reworked, softer clay. At some sections the uppermost layers beneath the gravels show intense brecciation, virtually none of the original clay structure being visible. The small-scale disturbances of the clay are probably associated chiefly with the melting out of ice lenses.

Index properties from a representative selection of Weald Clay samples are given in table 2. Individual test results are plotted in figure 5 from which it will be seen that, for a given liquid limit, the water content of the brecciated clay is generally higher than in the undisturbed material. An average value for the clay fraction is about 55% of the dry mass.



FIGURE 5. Index properties of Weald Clay (individual samples) and Head Deposits (average values).

# 3. STRATIGRAPHY: QUATERNARY (a) Terraced river gravels

Terrace gravels of the Rivers Eden and Medway consist of sand and sandy clay (loam) with flints, flint pebbes, cherts and pieces of sandstone. They have been mapped by the I.G.S. on the One-inch sheets of the Sevenoaks, Maidstone and Chatham districts (Sheets 287, 288 and 272), and details are given in the accompanying Memoirs. But in order to establish relations between the terraces and the high-level gravels, and to establish a stratigraphy of the terraces themselves, it is necessary to construct a longitudinal profile of the rivers with the adjacent gravels plotted in their correct positions. Altitudes of various points on the gravels, alluvium and Head deposits can be found from Ordnance Survey spot heights. To supplement this information in the critical area between Edenbridge and Tonbridge additional levels have been obtained with a surveying aneroid. In using this instrument care was taken to make frequent checks on bench marks and spot heights in the vicinity.

The longitudinal profile is given in figure 6 and the places referred to in the following discussion are shown in figure 7.

#### No. 1 Terrace

The First Terrace forms a well-defined feature 3 or 4 m above alluvium at several places between Edenbridge and Penshurst. It appears again in the neighbourhood of Yalding and Aylesford, where its height above the floodplain is about 1 m. Further downstream the terrace seems to merge with gravels at the top of the Buried Channel series containing cool-temperate



mammalian fauna, with advanced Mousterian implements at a site near Rochester (J. N. Carreck in lit. 14 May 1975).

The evidence suggests a mid-Devensian age, and it is probable that No. 1 Terrace is the equivalent of the lowest terrace in several Midland rivers, for example the Tame, Nene and Ouse, dated radiometrically to a period around 30000-40000 years B.P. (Coope & Sands 1966; Morgan 1969; Bell 1970).



FIGURE 7. Map of the Rivers Eden and Medway.

# No. 2 Terrace

The Second Terrace is seen frequently between Edenbridge and Tonbridge at heights of 10–13 m above alluvium. No exposures have been found in a long stretch of the Medway downstream of Tonbridge, but extensive deposits of sand and gravel at Aylesford with a surface 11 m above alluvium can safely be assigned to the Second Terrace. The gravels there rest on a bench at about 10 m o.D. and have a surface level around 15 m o.D. (Worssam 1963). Very probably the same terrace is represented by a spread of sandy gravel on the Isle of Grain at the mouth of the Medway, where the river joins the Thames estuary: the surface of the gravel lies at 12 to 13 m o.D., or 10 m above alluvium (Dines, Holmes & Robbie 1954).

In the Aylesford Terrace a fossil mammalian fauna has been found in association with Palaeolithic implements. The fauna, as reported by Carreck (1964), includes the species listed in table 3. This assemblage matches closely the fauna from the gravel and brickearth bordering the Thames at Crayford (Kennard 1944). The Thames brickearth, or loam, which is found also at Ilford and Aveley, has a surface elevation at approximately the same height above alluvium as no. 2 Terrace at Aylesford and dates from some part, probably towards the end, of the Ipswichian Interglacial (West, Lambert & Sparks 1964). Moreover, Mr Carreck kindly informs us that the Palaeolithic industry in the Aylesford terrace, apart from derived artefacts, is Late Acheulian showing Levalloisian technique, which agrees with an Ipswichian dating.

TABLE 3. AYLESFORD NO. 2 TERRACE: FAUNAL LIST

lion	Panthera leo
mammoth	Mammuthus primigenius
straight-tusked elephant	Palaeoloxodon antiquus
horse	Equus caballus
woolly rhinoceros	Coelodonta antiquitatis
pig	Sus scrofa
red deer	Cervus elaphus
giant deer	Megaceros giganteus
bison	Bison priscus
giant ox	Bos primigenius

# No. 3 and 4 Terraces

Correlations of individual occurrences of the First and Second Terraces along the length of the rivers present little difficulty, but problems arise with the higher terraces. The interpretation shown in figure 6 is an attempt to bring the available field observations into reasonable order.

In the neighbourhood of Tonbridge and Yalding the pattern appears to be simple. No. 3 Terrace at both localities lies 18–20 m above alluvium, and no. 4 Terrace is distinctly higher: 25 m above the floodplain near Tonbridge and 27 m at Nettlestead Green. Around Aylesford there are several patches of gravel about 26 m above alluvium, and it seems evident that they should be regarded as belonging to the Fourth Terrace. They are mapped as no. 3 Terrace on the Maidstone sheet, but it is noted in the Memoir (Worssam 1963) that the Third Terrace at Yalding is appreciably lower relative to the floodplain.

At Frindsbury a large quarry exposes sections of two terraces with surface levels at approximately 18 and 30 m O.D., or 15 and 27 m above alluvium, and the gravels rest on benches cut in the Chalk at about 14 and 24 m O.D. (Cook & Killick 1924). In the Chatham Memoir (Dines *et al.* 1954) they are referred to the 2nd and 3rd Terraces respectively; but these terms are used in a very broad sense and, in the present context, it is clear that the 100 m terrace at Frindsbury can be correlated with no. 4 Terrace further upstream. The 18 m terrace is too high, both in its surface level and bench height, to be a continuation of the Aylesford Second Terrace and it may be assigned tentatively to no. 3.

On the interpretation adopted here, no. 4 Terrace has an almost constant altitude of 30 m o.p. downstream of Aylesford, in what is today the tidal reach of the Medway. This altitude immediately invites a comparison with the Boyn Hill Terrace at Swanscombe, on the south side of the Thames estuary only 14 km from Frindsbury; and the benches at both locations are almost identical in height. Studies of the fauna, flora and Palaeolithic industries at Swanscombe have shown that part, if not the whole, of the river sands and gravels date from the Hoxnian Interglacial (see Wymer 1974 for a recent summary of the evidence). It addition the Swanscombe terrace is overlain by a loam, which is covered and disturbed by solifluction gravel, containing advanced Acheulian implements of Wolstonian age, while at Frindsbury there are implements,

described by Mr Carreck (personal communication) as Late Acheulian, in solifluction deposits which are later than the 30 m terrace but closely associated with it.

The suggestion can therefore be made that the Fourth Terrace of the Medway is equivalent to the Swanscombe terrace of the Thames, and broadly of Hoxnian age. Certainly it would not be easy to sustain arguments for a earlier date, while the presence of no. 3 Terrace, intermediate between no. 4 and the Ipswichian no. 2, precludes a substantially later date. The clear implication is that no. 3 Terrace can be correlated with the Wolstonian.

## High terraces

Near Maidstone, and also downstream of Rochester, there are remnants of terraces (not shown in figure 6) at altitudes around 45 and 60 m o.p. There is no direct evidence of their age, though they are obviously older than no. 4.

# (b) Older Head deposits

The high-level drifts always occur as deposits capping small isolated hills, ridges or spurs in the country between the escarpment and the belt of river terraces (figure 4). They fall into two groups: (i) gravels free from flints, with abundant chert fragments derived from the Hythe Beds, set in a fine-grained matrix derived partly from the sandy clays of the Hythe Beds but chiefly from the Atherfield and Weald Clays, and (ii) gravels containing flints and flint pebbles, usually with cherts as well, and typically with a more sandy or silty matrix.

It is generally agreed (following Dines *et al.* 1940) that the high-level chert gravels are Head deposits which have been transported from the escarpment and across the clay vale by periglacial solifluction. In contrast, it appears from their terrace-like distribution, northwest and east of Edenbridge, that the flint gravels were carried along pre-existing valleys of the Eden and its tributaries, Kent Brook and Gibbs Brook; and it is known (Sevenoaks Memoir 1969) that the flints were derived from the Limpsfield gravel north of the escarpment. Flint gravels which lie about 20 m above the river Bourne no doubt have an analogous origin and history.

In principle, then, the distinction between the two groups is clear. But good sections of the deposits are rare, and intensive field work, involving pits and borings, would be necessary to establish full details of the stratigraphy. The following notes are therefore provisional and may have to be modified in some respects in the light of future research.

## High-level flint gravels

The main occurrences of high-level flint gravels identified by the I.G.S. on the Six-inch maps and in the Sevenoaks Memoir are shown in figure 4, together with two additional exposures in the district north of Hever. Murchison (1851), describing the gravels in this district, says they 'are spread out in what the country-people call the "plains", which are, in fact, plateaux 60 or 80 feet above the adjacent valley' of the Eden. The largest of these is at How Green (475462). Another spread of flint gravel occurs 800 m to the northwest, near Whistlers (468467). One of the new exposures is a short distance to the east (472468) near Meachlands, at the same altitude as How Green. Cherts were recorded here by the I.G.S. but in a recently cut roadside ditch we found flint pebbles mixed with the usual fragments of sandstone, ironstone and chert. The stones were closely packed with a silt filling the interstices. The silt had a liquid limit as low as 27 (table 2) and a clay fraction of only 10%. It may be noted that Bird (1964) maps chert gravels at all three sites. The other revision applies to flint gravels at Bough Beech (489466). These were originally identified as Head and later as no. 2 Terrace. However, they lie at least 6 m higher than the Second Terrace in the locality and should be classified either as high-level flint gravels, or possibly as no. 3 Terrace. Indeed it will be seen in figure 6 that the flint gravels from this point up to Edenbridge lie between 19 and 24 m above alluvium, approximately at altitudes which might be expected for the Third Terrace, and clearly well above no. 2.

Further east, on a spur at Somerden Farm (501469), it seems that cherts merge into flint gravels, also at a level considerably higher than the Second Terrace, although towards the southern end of the same spur the gravels appear to have been moved down slope, perhaps by later solifluction, almost to the terrace level. Similarly it is reported, in the Sevenoaks Memoir, that cherts on Camp Hill (523468) merge into flint gravel on the south side of the hill, and again there has been later solifluction or down-washing. Flint pebbles have also been noted at the northwest end of Camp Hill. It may therefore be assumed that chert and flint drifts are in contact at Somerden and Camp Hill. No exact levels or boundaries can be given for the flint gravels at these sites, but their positions are not very different from the associated cherts plotted in figure 6 at 5.8 and 7.4 km downstream of Edenbridge.

The high-level flint gravels along the north side of the Eden, then, can be correlated with the Wolstonian, since they are of about the same age as the Third Terrace, or slightly older. In a different category are the gravels containing flints, flint pebbles and large angular cherts at Starvecrow (602502), 3.5 km north of Tonbridge. They are situated at an altitude around 75 m o.D., which is higher than no. 4 Terrace, to the south, and higher than the flint gravels to the north along the river Bourne. Very tentatively it may be suggested that they belong to an older drift, possibly of Anglian age.

## High-level chert gravels

Characteristically the presence of these deposits is revealed by an abundance of angular chert fragments in clayey soil on hill-tops standing above the clay vale. The cherts are very obvious in freshly ploughed fields but the larger fragments can also be seen in grass. They make a striking contrast with the lower parts of the vale, where cherts are usually absent or scarce. Bird (1964) reports that cherts are sometimes found to depths of 2 m, but as a rule the deposit is less than 1 m thick. He describes a section from a pit in the middle of Camp Hill, where cherts in a matrix of silty clay to between 1 and 1.5 m were seen to be underlain by 3 m of weathered clay with contortions indicative of freeze-and-thaw action, and then unweathered Weald Clay and bedded siltstone *in situ*. Apparently there were no flints in this section but, as previously noted, flints do exist on the north and south ends of the hill.

All the high-level chert gravels identified by the I.G.S. are included in figure 4, apart from the deposit near Meachlands, now classified as a flint gravel, and with the addition of a small spread of cherts on the ridge leading from Hubbard's Hill to the church of Sevenoaks Weald (529514). There is little doubt that this gravel at one time continued up the ridge on to the slopes of the hill; indeed many cherts can be seen in a field at Anchor Cottage (figure 1), though a nearby roadside section indicates that they are confined to a thin veneer. South of the church the ridge is deeply dissected by a stream, but chert gravels appear again at Weald village and, after crossing a col, on the ridge south of Pitts Farm. From the profile shown in figure 8 it is evident that these gravels very probably form part of what was originally a continuous spread, extending southwards for a distance of 2 km, the average gradient of which is about  $1.4^{\circ}$ .



FIGURE 8. Section from Hubbard's Hill to Camp Hill and across the River Eden.



FIGURE 9. Section from River Hill to Summerthorn Wood.

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Chert gravels capping the ridge below River Hill, as shown in figures 1 and 2, have a similar mode of occurrence (though they have not been traced up to the foot of the hill itself) and exhibit a gradient of about 1.7°. Cherts in the several exposures mapped in figure 1 are often up to 10 or 20 cm in length.

No process at present operating can release cherts of this size from the escarpment and transport them in a clay matrix many hundreds of metres down slopes inclined at less than 2°. Vigorous frost-shattering and periglacial solifluction clearly have to be invoked.

Between Hubbard's Hill and River Hill ridges, all traces of the high-level cherts have been removed by subsequent erosion, except for occasional fragments lying on the adjacent hill slopes well below their original position. From the sections in figures 2 and 3 it will be seen that the maximum depth of dissection is of the order 20-30 m.

Further south, exposures of high-level chert gravel become less frequent. They are found in association with flints at Somerden and Camp Hill; but there seems to be no ambiguity at Summerthorn Wood (542477), north of Leigh, where the deposit is described on the Six-inch map as clay with cherts and sandstone fragments, and at Little Trench (595485), north of Tonbridge. The gravels at all these sites lie between 4 and 5 km south of the escarpment, at heights of 10-20 m above the local streams. They appear to lie on a continuation of the same profile as the high-level gravels further north; but the gradients leading to the southerly gravels, as at Camp Hill (figure 8) and Summerthorn Wood (figure 9), are about  $0.6-0.8^{\circ}$ .

## Dating the Older Head deposits

The flint gravels east of Edenbridge lie well above the Second Terrace, of Ipswichian age, and at about the same height above river alluvium as no. 3 Terrace. They were therefore deposited at some period during the Wolstonian. The most southerly chert gravels, at Somerden, Camp Hill and Little Trench, are slightly higher but, judging by the latter locality, they do not attain altitudes above no. 4 Terrace. As the Fourth Terrace is very probably not older than Hoxnian it follows that these chert gravels can also be correlated with the Wolstonian.

The high-level chert gravels further north, in the vicinity of Sevenoaks Weald, are unquestionably older than Ipswichian. This can be determined from the facts that deposits known to be of Devensian age lie at much lower altitudes in a totally different topographic situation  $(\S 3c)$  and, of course, that periglacial deposition could not have taken place in the temperate climate of the Ipswichian. Three hypotheses then have to be considered regarding the age of these more northerly high-level drifts.

(i) They were deposited later than the chert gravels further south. In this case, for reasons given above, they could only be of later Wolstonian age.

(ii) They were deposited at the same time. This assumption is the simplest to make; it implies that the high-level cherts were all deposited in Wolstonian times and originally formed parts of continuous sheets spreading off the escarpment (in its contemporary position) and reaching down to, or near to, the rivers in their Wolstonian locations.

(iii) They were deposited earlier than the chert gravels further south. What has to be considered here is not the question of their formation in an earlier phase of the Wolstonian, which is little more than a variant of (ii), but the radically different case for an Anglian age. In order to sustain this correlation, however, it is necessary to explain why no solifluction deposits of intermediate age (i.e. Wolstonian on the present assumption) have been found between the abundant high-level cherts around Sevenoaks Weald and the Devensian drift, while chert gravels of Wolstonian age have been preserved further south.

As no such explanation appears to be readily available the most probable conclusion is that the northerly high-level drift is Wolstonian. Bird (1964), acknowledging the 'formidable difficulties' involved, and using somewhat different lines of reasoning, arrives at basically the same result; namely that the high-level cherts were deposited during a 'solifluction episode after the development of a valley floor equivalent to the Boyn Hill terrace in the London Basin'.

Nevertheless a question remains, for it is conceivable that the most southerly cherts might have been brought down from the escarpment by solifluction in the Anglian stage and subsequently redeposited in their present positions by further solifluction in Wolstonian times. This implies the existence originally of extensive Anglian deposits which have since been lost by erosion, a possibility which cannot entirely be ruled out. Consequently it has to be admitted that although the high-level chert gravels, as shown in figure 8 for example, were formed in Wolstonian times, those nearest to the river might have been redeposited at that stage; in which case it would not be correct to deduce that solifluction had occurred on gradients as low as  $0.6^{\circ}$ .

# (c) Younger Head deposits

The Younger Head deposits have been studied in some detail within the embayment between Hubbard's Hill and River Hill (figures 10 and 11). A complete section of the deposits was exposed in Pit F2 (535520), where the following sequence could be established (figure 15): overlying brecciated Weald Clay is the chert gravel of the Lower Solifluction Sheet, followed by slope-wash which grades up into a dark grey organic clay dated radiometrically to 12200 years B.P.; this organic horizon (fossil soil) is in turn covered by gouge clay containing a highly developed slip surface; over the gouge clay comes another layer of chert gravel (lobe F) followed again by slope-wash on which the modern topsoil has formed.

The upper and lower slope-wash deposits are practically identical materials. They consist of soft silty clays, buff or pale-grey or sometimes light reddish brown in colour, with a few very small stones. Little difference was noted in the field between the upper and lower chert gravels, but laboratory tests show that the upper gravel has a more sandy matrix. The matrix of the lower gravel, in pit F2 and in other borings and pits, is a silty clay of medium plasticity, usually brown or light grey in colour.

Index properties are summarized in table 2 and average values of the existing water contents are plotted in figure 5. The water contents are seen to be relatively high, but even so they have certainly been reduced by consolidation from still higher values at the time of deposition. Determinations of the clay fraction were made on seven samples of the chert gravels matrix; the results lay within the range 14-45% of the dry mass, mean values for the matrix of the Lower Sheet and of lobe F being 33 and 24%, respectively.

Clay particles in the chert gravel matrix of the lobes have been derived from the basal bed and clayey sands of the Hythe Beds. In the Lower Solifluction Sheet, as in the high-level chert gravels, the clay content is supplemented by material derived from the Atherfield and Weald Clays. Apart from rare ironstone fragments, in the lower gravel, which come from the Weald Clay, the stones in both the upper and lower solifluction layers consist wholly of chert, sandstone and Rag from the Hythe Beds, with chert strongly predominating.





#### Lower Solifluction Sheet

The Lower Solifluction Sheet has no topographic expression but its presence has been proved by borings and pits beneath the Lobes and beyond their limits for a distance of at least 500 m from the foot of the scarp face. The row of borings from 300H through 317H to 340H (figure 10) all show about 2 m of chert gravel overlying brecciated Weald Clay, on ground sloping at  $3-4^{\circ}$ , and the east-west continuity of this sheet is interrupted only by the small ridge on which Panthurst Farm is built.

The southern boundary of the sheet cannot be determined from the available evidence but there is no doubt that the gravel deposits moved for considerable distances down pre-existing valleys. Cherts embedded in silty clay to a depth of 1.5 m can be seen, for example, in the sides of a stream channel at a point (533513) 500 m east of Sevenoaks Weald church, and very probably they fan out across gently sloping land to the southeast; cherts were certainly seen in abundance in a ploughed field near the 76 m spot height shown in figure 1, almost 1 km from the escarpment.



FIGURE 13. Section and borehole at pit 317H 100 m south of lobe E.

From point 533513, looking east, the Morleys Farm ridge is seen, capped by high-level chert gravels (figure 12, plate 5), and in a dry valley on the other side of the ridge cherts have been proved to a depth of 2.5 m, over Weald Clay, in a boring (543513) 250 m northeast of the farm (figure 1).

There is no reason to question the idea that the chert gravels have moved by solifluction to these positions, as the longitudinal gradient of the valley leading to point 533513 is about 2.4°, while the valley east of Morleys Farm is slightly steeper. From point 533513 down to the 76 m spot height the gradient averages 2.1°. To the south only occasional chert fragments can be seen, and it may be noted that in this area the gradient rapidly decreases, falling to 1° about 500 m to the south.



FIGURE 12. View from point 533513 looking towards the ridge north of Morleys Farm. Devensian chert gravel is exposed in a stream channel just behind the view point and high-level (Wolstonian) gravel caps the ridge.

FIGURE 14. Section of core from a depth of 3.5 m in borehole 317H, showing brecciated Weald Clay. The core diameter is 10 cm.

A representative section through the Lower Solifluction Sheet is shown by the pit and adjacent boring at location 317H (figure 13). Chert gravel with a silty clay matrix extends to a depth of 2 m. It is underlain by 2.7 m of brecciated Weald Clay, the upper half of which is partially weathered to a brown colour, with small lumps of intact clay in a mass of soft, reworked clay (figure 14, plate 5). The ground at this location slopes at 4°, and is completely stable under present climatic conditions. Nevertheless there are several well-defined subhorizontal slip surfaces in the brecciated Weald Clay down to 3 m below ground level (the main slip only being shown in figure 13), and it therefore seems that the solifluction movement occurred chiefly by shearing on these slip surfaces under periglacial conditions when the 'active layer' of the permafrost penetrated the top 2–3 m.



FIGURE 15. Section of pit F2 in lobe F.

Similar sections have been observed in the pits and borings at locations 300H, 305H and 309H where cherts were found to depths of 1.5–2 m, with shears in the underlying clay and, in one case, also within the chert gravel matrix.

In pit F2 the Weald Clay was seen to be dragged up into the chert gravel as a tongue, about 2 m in length, containing a trail of ironstone fragments (figure 15). Contortions of the same type, though smaller in scale, were also observed in pit F3, nearby. No slip surfaces in the Weald Clay could be detected in these pits. But when an excavation was made through lobe G a large mass of material on the uphill side began sliding on a shear surface in the Weald Clay, below the chert gravel, and calculations showed the strength to be at its residual value. In other

words, the excavation led to reactivation of movement on a pre-existing slip surface at the base of the lower sheet.

The largest chert fragment encountered in the investigations was found in the lower sheet in pit F2. It is drawn to scale in figure 15, and has a length of 60 cm. The other stones were frequently up to 10 cm and occasionally 20 cm in size.

The amount of slope-wash recorded in pit F2, overlying the lower chert gravel, appears to be exceptional and may have been deposited, and preserved, in a local depression or furrow. In pit F3 it was seen only as a pocket of clay beneath the organic horizon, while in the pits further south, beyond the lobes, this material if it exists at all is no thicker than the modern topsoil.

# Fossil soil

As seen in pit F2 the lower slope-wash grades up into a dark grey silty organic clay rich in humus. Sample 15, taken in this soil at the position shown in figure 15, was submitted to Geochron Laboratories for radiocarbon assay. The sample was inspected for rootlet contamination and digested in hot HCl to remove carbonates. The age proved to be  $12250 \pm 280$  radiocarbon years B.P. (GX 0793). This dates the soil firmly within the Late-glacial Interstadial of the Devensian as defined by Coope (1975). Calculations of the radiocarbon age were based upon the Libby half-life (5570 years) for <sup>14</sup>C, using as a standard 95% of the activity of N.B.S. oxalic acid (Geochron Laboratories Report 21 October 1966).

TABLE 4. POLLEN ANALYSIS OF FOSSIL SOIL, PIT F2

Betula	<b>25</b>
Pinus	11
	36 AP total
Gramineae	17
Cyperaceae	6
Artemisia	23
Compositae	8
Epilobium	13
Rubiceae	4
Thalictrum	5
Umbelliferae	4
	80 NAP total
Botrychium	4
Lycopodium	1
Ophioglossum	1
• •	6 spore total

A block of the soil taken immediately adjacent to sample 15 was sent to Dr R. G. West, F.R.S., at the Botany School, Cambridge, and we are very grateful to him for carrying out a pollen analysis. The results (West *in lit.* 15 Aug. 1966) are given in table 4. Dr West commented that the pollen was scarce  $(33 \times 2 \text{ cm traverses})$ , and the spectrum is of a type which may well be Late Devensian. This conclusion is of course in agreement with the radiocarbon date.

The organic clay horizon was found also in pit F3 and in several of the borings such as nos. 118 and 122 in figure 11. But these borings through the lobes, made in a routine fashion, for the installation of piezometers, were not logged in detail and the absence of any record of the soil is not necessarily significant.



FIGURE 16. Pit F2. Slip surface in gouge clay beneath the upper chert gravel and overlying Late Devensian interstadial fossil soil.

FIGURE 22. Postglacial stream erosion in the valley east of Dale Farm.

FIGURE 17. Lobe F, looking northwest. The Lower Solifluction Sheet extends beyond the edge of the lobe, in the foreground.

## Upper Lobate Sheet

The upper layer of chert gravel spreads off the escarpment over the Lower Solifluction Sheet. Characteristically it takes the form of lobes, but these appear to be extensions, perhaps in a slightly later stage of development, from a more continuous sheet. The bench forming the margin of this sheet can be seen between lobes D and E and between lobes F and G (figure 10). Engineering works prior to the investigations may have removed evidence for the sheet elsewhere, just as subsequent works have obliterated almost all traces of lobes D, G and H. The southern parts of lobes E and F remain, however, and stand out as conspicuous features of the landscape north of the new by-pass road (figure 17, plate 6). They extend about 300 m from the scarp foot. Similar lobes have been mapped to the west of Hubbard's Hill and on the east of River Hill (figure 1), and no doubt others could be found further afield.

A section through lobe F is shown in figure 11, with a detailed profile in figure 15. There is some evidence of flow structure within the chert gravel, indicating very low strengths at the time of formation, but clearly the main component of downhill movement has taken place by shearing on a polished and striated slip surface within a thin layer of gouge clay beneath the gravel (figure 16, plate 6).

The surface inclination of the lobes is about 6 or 7° on average, with a tendency in some cases to show a slight concave profile ranging in slope from 9° near the scarp face to 5° at the toe. The edges of the lobes are quite steep in places and exhibit signs of recent instability.

Where the thickness of the lobe material, above the organic horizon, could be determined in borings and pits F2 and F3 it varied from 3 to 5 m; the upper metre, approximately, being slope-wash clay. The stones, all derived from the Hythe Beds, were found to be often up to 10 cm in size and occasionally as large as 20 cm, embedded in a matrix of sandy clay.

# Dating the Younger Head deposits

It is evident that the lower chert gravels were deposited on a landscape which has changed little since the time of their deposition; in marked contrast to the high-level gravels. This fact, coupled with their obviously periglacial origin, strongly suggests a Devensian age. Moreover, there are no signs of weathering beween the gravel and the overlying slope-wash which, inturn, grades up into the fossil soil proved to be of Late-glacial Interstadial age. Thus there is no difficulty in correlating the Lower Solifluction Sheet with the Devensian.

From the abundance of large chert fragments in the Lobes, these too are judged to have a periglacial origin and can therefore be correlated with the Late-glacial Zone III period (10800–10000 years B.P.): the only periglacial phase following the Interstadial and preceding the return to temperate conditions in Postglacial times. The upper slope-wash is later than the chert gravel of the Lobes, but no exact date can be given for its formation.

# 4. THE MECHANICS OF PERIGLACIAL MUDFLOWS

The solifluction sheets can best be described as periglacial mudflows or mudslides (following the terminology of Chandler 1972), in which a clayey gravel moves down slope principally by shearing on a slip surface at or near the top of the underlying Weald Clay. In temperate conditions such a mechanism would not be possible at slopes of less than about 8°. But it can be hown that during seasonal thawing of the active layer above permafrost sufficiently high pores pressures may be developed at the base of the layer for movement to take place on slopes of less than  $2^{\circ}$ .

To outline the mechanics of this problem, consider a layer of material of depth z sliding on a plane parallel to the slope and inclined at an angle  $\beta$ . The unit weight of the material is  $\gamma$ . It will be assumed that the length of the sliding mass is large compared to its depth and, further, that the shear strength on the slip plane can be represented in terms of the effective stress  $\sigma' = (\sigma - u)$  by the Coulomb-Terzaghi expression

$$s = c' + (\sigma - u) \tan \phi',$$

where c' and  $\phi'$  are the apparent cohesion and angle of shearing resistance of the clay,  $\sigma$  is the total stress normal to the plane and u is the pore water pressure acting at depth z.

For limiting equilibrium, when movement is just possible, the shear stress on the plane is equal to the shear strength, or

$$\gamma z \sin \beta \cos \beta = c' + (\gamma z \cos^2 \beta - u) \tan \phi'. \tag{1}$$

If the piezometric height is h, then  $u = \gamma_w h$ , where  $\gamma_w$  is the unit weight of water (9.8 kN/m<sup>3</sup>); and it is convenient to express the pore pressure in terms of the ratio

$$r_u = u/\gamma z = \gamma_w h/\gamma z.$$

Equation (1) can then be rewritten in the form

$$\sin\beta\cos\beta = c'/\gamma z + (\cos^2\beta - r_u)\tan\phi'.$$
 (2)

Under temperate conditions the highest pore pressures will exist when groundwater level is at the surface. This is the full 'hydrostatic' case; and as  $\gamma$  is usually about  $2\gamma_w$  the corresponding value of  $r_u$  is approximately 0.5. The extreme upper limit is the 'geostatic' value  $r_u = 1.0$ , which implies a piezometric level rising above the slope to a height approximately equal to the depth z; a state clearly impossible under ordinary conditions, but theoretically possible if frozen soil is thawed so rapidly that the entire weight of overburden is transferred to the pore water, without any of the water being able to escape.

The time required for thawing to proceed from the surface to a depth of, say, 2 m is several months (McRoberts 1975). In a sandy material much consolidation could occur in this period, but in a clay the rate of consolidation may be sufficiently low to prevent any substantial dissipation of pore pressure at the depth of the slip plane. It is therefore of significance in the present context to note (i) that sliding occurred chiefly within the brecciated Weald Clay and (ii) that the overlying chert gravel has a clay matrix, and is a matrix dominated material.

Morgenstern & Nixon (1971) have presented an analysis of the problem of calculating the excess pore pressure at a depth D in soil having a coefficient of consolidation  $c_v$ , when t is the time required for thawing to penetrate to this depth. The results are expressed as a function of the thaw-consolidation ratio

$$R = D/(2\sqrt{(c_{\rm v}t)}).$$

Laboratory tests on samples of brecciated Weald Clay and of the chert gravel matrix, from a boring adjacent to Pit 309H, show a coefficient of consolidation virtually identical for both materials and equal to about  $2.5 \text{ m}^2$ /year. It will be assumed that thawing reached a depth of 2 m, the typical depth of slip surfaces beneath the Lower Solifluction Sheet, in a period of 3 months, which seems to be a reasonable time from the annual temperature curve deduced by Williams (1975) for the colder parts of the Devensian. With these assumptions the thaw-consolidation ratio has a value of 1.3; and from the numerical solution given by Morgenstern & Nixon this corresponds to 22% dissipation of excess pore pressure, or  $r_u = 0.89$ .

It must be emphasized that the calculation is approximate as t is not known precisely and the tests were carried out on the clays as they exist today;  $c_v$  may have been rather different when the soil had just thawed out. To make some allowance for these uncertainties R will be varied by  $\pm 25 \%$ , in which case the upper and lower limits of  $r_u$  are 0.93 and 0.83.

The next step is to determine the shear strength parameters. Tests have been made to measure the strength on slip surfaces in samples taken from pits 300H and 317H (Skempton & Petley 1967), while the residual strength of brecciated Weald Clay with similar index properties, from Arlington in Sussex, has been measured in ring shear tests (Bishop *et al.* 1971). The failure envelope is slightly curved but can with little error be linearized over a moderately wide range of effective stress. Thus for values of  $\sigma'$  between 20 and 50 kN/m<sup>2</sup>, which is a typical range for shallow landslides in temperate climates,

$$c' = 1 \text{ kN/m^2}, \phi' = 14^{\circ}.$$

Extrapolating the failure envelope back towards zero effective stress we find that for values of  $\sigma'$  between 5 and 20 kN/m<sup>2</sup>

$$c' = 0.2 \text{ kN/m^2}, \quad \phi' = 16^\circ,$$

but for effective stresses less than  $5 \text{ kN/m}^2$  there is some uncertainty. The above set of parameters might also apply, or if the true cohesion is zero the appropriate parameters would be

$$c' = 0, \phi' = 18^{\circ}.$$

It is now possible to find from equation (2) the minimum or limiting slope  $\beta$  for any given value of  $r_u$  assuming the thickness of the sliding mass to be z = 2 m and taking the unit weight  $\gamma = 20$  kN/m<sup>3</sup>.

In the first place we note that under temperate conditions, with a maximum  $r_u$  equal to 0.5 (and therefore with  $\sigma' \ge 20 \text{ kN/m}^2$ ), sliding can occur only on slopes steeper than about 8°. And the same result applies in a thaw-consolidation process with 100% dissipation of excess pore pressures.

Under periglacial conditions and in a clay soil, however, it has just been shown that  $r_u$  might be as high as 0.93, in which case  $\sigma' \approx 3 \text{ kN/m}^2$  and the limiting slopes is  $1.3^\circ$  (c' = 0,  $\phi' = 18^\circ$ ) or  $1.4^\circ$  (c' = 0.2,  $\phi' = 16^\circ$ ). By using the calculated value of  $r_u = 0.89$  the effective stress is about 4.5 kN/m<sup>2</sup> and the limiting slope is  $2.0^\circ$  (with either set of strength parameters), while for a lower bound of  $r_u = 0.83$  the slope is  $3.0^\circ$  ( $\sigma' \approx 7 \text{ kN/m}^2$ , c' = 0.2,  $\phi' = 16^\circ$ ).

The Devensian lower solifluction gravels moved on a minimum gradient of just over  $2^{\circ}$ , while the high-level drift moved on slopes of about  $1.5^{\circ}$ . These field observations can readily be explained on the basis of the foregoing calculations; but in order to account for movements on slopes of  $0.6^{\circ}$  or  $0.8^{\circ}$ , as in the profiles down to Camp Hill and Summerthorn Wood, values of  $r_u$  around 0.97 or 0.96 have to be invoked. These correspond to about 6 % dissipation of excess pore pressure, an extremely low value though not impossible if  $c_v$  is less than 1 m<sup>2</sup>/year.

It is also of interest, following Hutchinson (1974), to estimate the water contents of the clays at the time when movements were taking place. The shear strength at the base of a sheet 2 m thick moving on a slope of  $2^{\circ}$ , for example, is about 1.5 kN/m<sup>2</sup>. Now, to a first approximation,

the strength of remoulded clays is uniquely related to the liquidity index, as defined by the expression

$$LI = \frac{w - PL}{LL - PL},$$

and for a strength of 1.5 kN/m<sup>2</sup> the value of LI is around 0.9 (Skempton & Northey 1952). For Weald Clay, with liquid and plastic limits of 60 and 23 respectively (table 2), the corresponding water content is about 55. Similarly the water content of the chert gravel clay matrix would be approximately 50. These water contents are of course much higher than the present values. They refer to the state of the clays just after thawing, when the effective stresses were very small.

So far as the lobes are concerned, analyses based on measured shear strength parameters of the slip surface from samples taken in pit F2, and on observed piezometric levels, show that they are marginally stable under present-day conditions. An increase in pore water pressure up to the full hydrostatic value might be sufficient to cause movement. Thus the existence of permafrost is not necessarily implied. On the other hand severe frost-shattering must have been active in order to release the large fragments of chert and other stones from the Hythe Beds; and the material in the lobes may well have been softened by freezing and thawing, with the development of some excess pore pressures.

## 5. The escarpment

Studies have been made at four sites in the Lower Greensand escarpment between River Hill and Ide Hill. They indicate that large-scale structural disturbance, in the form of block subsidence in the Hythe Beds accompanied by bulging in the Atherfield and Weald Clays, is confined to older parts of the escarpment now left as 'hills' or spurs; while mass movements within the embayments are characterized by landslips.

## (a) Hubbard's Hill

A plan of this locality is shown in figure 18. Section I (figure 19) records the investigations made at the site of Weald Bridge. The southern part of pit WB1 showed Weald Clay at a higher elevation than had been expected, and also a 'pinching out' of the Atherfield silt. These observations suggested the presence of a bulge at the foot of the escarpment, somewhat analogous to the well known 'valley bulges'. To check this hypothesis borehole 25 was drilled. It proved the Weald Clay at a level 5 m lower than the exposure in pit WB1, and also revealed two siltstone layers. As the siltstones could act as excellent marker bands, additional borings were made below the pit, and beyond; and pit WB4 was excavated to provide further information. The results, as shown in figure 19, confirmed the existence of a sharp bulge in the Weald Clay, rising 5 m and continuing, with some minor distortions, at least 20 m to the south. The new borings and pit also confirmed the almost total absence of Atherfield silt above the bulge.

Throughout the length of the pits a slip surface, with many secondary shears, was traced in the Atherfield clay. At the northern end of pit WB1 the slip surface appears to be associated with minor landslip movements in the Hythe Beds, but at other sites subhorizontal shears have been found in the Atherfield clay well behind the scarp face. Although the slip surface was not specifically noted in borehole 25 it may continue northwards under the Hythe Beds, and has merely been involved in, not caused by, the landslips at the scarp toe. A sheet of Head sweeps down the slope, with a smooth surface profile and an approximately uniform thickness of about 2 m. It is entirely unaffected by the bulging and truncates the small landslips. Possibly this sheet is the upper part of lobe D or, more probably, it continues as the Lower Solifluction Sheet proved in pits 300H and 305H (figure 18).

Two other sections through the escarpment at Hubbard's Hill are shown in figures 20 and 21. The Weald Clay bulge is seen again. It has much the same amplitude as in section I, but is less sharply defined.



FIGURE 18. Hubbard's Hill: plan.

An important feature in sections II and III is the presence of deep fissures or gulls. The gulls are filled with silt and stones, but they can be discerned as depressions in the ground surface. When first observed they were taken as indicating that the Hythe Beds had been cambered. Certainly the Hythe Beds are disturbed, with many minor faults, but the evidence now available from the borings and pit 283/A (figure 21) shows that a large block up to 25 m thick has subsided and moved forward, essentially without rotation. It should be noted, however, that the toe of this mass of rock has been forced upwards, as indicated by the section between boreholes 15 and 10 in figure 20, and confirmed by a pit excavated nearby.

Observations at Ide Hill, shortly to be described, leave little doubt that block subsidence and bulging in the clays at the foot of the scarp are closely related. Further, it is very unlikely that such disturbances could take place under temperate conditions. Deep freezing and thawing seem to have been necessary to reduce the strength of the clays, and the most readily acceptable explanation of the forward thrusting invokes ice-wedges acting in joints within the Hythe Beds.



#### LOWER GREENSAND ESCARPMENT AND WEALD CLAY VALE

To put the various events at Hubbard's Hill into a chronological sequence, the first point to recall is that high-level cherts were found near Sevenoaks Weald church on the ridge which runs south from the hill. Subsequent to the deposition of this gravel widespread erosion has taken place, though the central part of the hill and the land now forming the ridge have remained with little change; and it is on the slopes of this eroded landscape that the Devensian solifluction sheets and lobes have been formed. Clearly the bulging of the Weald Clay, as seen in sections I–III, pre-dates the solifluction material and the slopes over which it flowed. The conclusion can therefore be drawn that block subsidence in the Hythe Beds and bulging of the Weald Clay occurred during a periglacial stage earlier than the Devensian; though the gulls may have been widened, at least in their upper parts, during this period.



## (b) Ide Hill

Further evidence on the phenomenon of 'block subsidence' in the Hythe Beds escarpment is provided by two groups of borings near the village of Ide Hill. In this area there is a large reentrant, or coombe like feature, in which several landslides occurred following exceptionally heavy rainfall in the winter 1968–9. The borings about to be described were made, among others, in connection with works to stabilize the slopes.

In the first group, centred on location 486515, three borings were carried down through the Hythe Beds and the Atherfield Clay into the Weald Clay; they were situated between 200 and 300 m behind the main, south-facing escarpment, and proved a thickness of the Atherfield clay and silt varying from 8 to 12 m.

In the second group one boring was located at the crest of the scarp and another about 40 m behind. After penetrating 20 m of Hythe Beds both showed only 2 m of Atherfield deposits over the Weald Clay. But a third boring in this group, drilled beyond the toe of the Hythe Beds, proved 10 m of Atherfield clay and silt, underlying Head deposits. It is remarkable that, at this location, the upper surface of the Weald Clay remained at a practically constant elevation (154 m o.p.).

The conclusion is inescapable that a vast block of Hythe Beds, 20 m thick and more than 40 m wide, has subsided into the Atherfield Clay and in doing so has squeezed this material out



\_\_\_\_\_ 236 beyond the foot of the scarp face in the form of a 'bulge'. The vertical component of subsidence is probably at least 6 m.

The recent landslide at this site took place entirely within the solifluction deposits.

It is further to be noted that two of the borings in the first group were drilled through the scarp face within the re-entrant; but they revealed no sign of disturbance in the Atherfield or Weald Clays, and the slope movements in 1969 simply reactivated an older landslip in the toe of the escarpment beyond these two borings. Thus, despite the steepness of the scarp (averaging  $25^{\circ}$ ) and the great thickness of Hythe Beds (42 m below the scarp crest), there is no evidence here of block subsidence or other types of deformation, except landslipping.

The marked difference between the two sites may perhaps be explained on the assumptions (i) that the re-entrant has been formed, or enlarged and deepened, by erosion subsequent to the (Wolstonian) period when block subsidence occurred in the main escarpment, and (ii) that periglacial conditions during the Devensian stage, though capable of producing the Head deposits which blanket the slopes within the re-entrant, and also capable of initiating large landslips, were not sufficiently severe to cause major structural disturbances.

# (c) Bayley's Hill

At the head of a small embayment east of Bayley's Hill (515518) there is an old landslip the reactivation of which in 1969 caused considerable damage. The slip involves most of the scarp face and the slip surface passes through the Atherfield Clay. The scarp is again fairly steep and the Hythe Beds have a thickness of about 35 m.

Borings through the scarp face show no disturbances in the Atherfield or Weald Clays, apart from those associated with the landslip.

## (d) Embayment between Hubbard's Hill and River Hill

Geomorphological mapping, carried out at the request of the senior author by Mr R. P. Martin, under the direction of Dr Denys Brunsden, has revealed three areas of major landslipping in the escarpment between Hubbard's Hill and River Hill. These are in the thickly wooded slopes of Beechmont Bank, at the positions indicated in figure 10. The degraded toes of the landslip masses can just be discerned beyond the southern boundary of the wood, and their very subdued topographic expression suggests that they have been modified by solifluction. Indeed they may have provided a source of material for the upper lobate sheet. It therefore seems probable that the landslips were initiated in Devensian times before, and possibly during, Zone III. A Wolstonian date cannot be postulated, owing to the retreat of the embayment escarpment subsequent to that period.

Unfortunately it was not possible to make borings through the escarpment in Beechmont Bank and only three boreholes were taken through the Hythe Beds south of the scarp foot (figure 10). These show nothing abnormal, however, and there is no reason to suppose that conditions in the embayment differ essentially from those observed near Bayley's Hill.

#### 6. POSTGLACIAL CHANGES IN THE LANDSCAPE

In Postglacial times only slight changes have occurred in the escarpment and in the clay vale. At several places old landslips have been reactivated, both in the escarpment proper and in the solifluction material blanketing the steeper slopes beneath the scarp foot. The streams have eroded channels, typically up to 2 m in depth, in or through the Lower Solifluction Sheet, and in the Weald Clay they have excavated new, small valleys the sides of which sometimes show signs of continued instability. A view of one of these little valleys, to the east of Dale Farm (figure 1), is shown in figure 22, plate 6. Further downstream, narrow bands of alluvium have been deposited and the maximum depth of incision is about 3-4 m. The present floodplains of the Rivers Eden and Medway have been formed during this period by the deposition of alluvium.

## 7. SUMMARY AND CONCLUSIONS

The Quaternary correlations are set out in table 5 and the main conclusions may be summarized as follows:

(i) South of the Lower Greensand escarpment in the vicinity of Sevenoaks Weald many of the small hills and ridges, standing up to 20 or 30 m above the streams of the clay vale, are capped by deposits consisting of chert fragments, and other stones derived from the Greensand, set in a clay matrix. These deposits are the dissected remnants of originally continuous sheets which spread off the escarpment and moved down the gentle slopes of the then existing landscape for distances of at least 2 km at an average gradient of about 1.5°. The cherts are angular and often up to 20 cm in length. For stones of this size to be released from the Greensand in large quantities and transported in a clay matrix for such distances on slopes of less than 2°, intense frost shattering and periglacial solifluction have to be invoked. There is reason to believe that the formation of these high-level chert gravels very probably occurred during the Wolstonian stage.

(ii) In some cases the solifluction debris may have moved as far as 4 km from the escarpment on gradients falling to about  $0.8^{\circ}$ .

(iii) Bordering the northern limits of the belt of river terraces of the Eden and Medway, at distances of 4-5 km from the escarpment, there are several exposures of high-level chert gravel of Wolstonian age. If in primary positions they would represent the most southerly limits of the solifluction sheets, and near the limits the gradients would have been not more than about  $0.6^{\circ}$ . But it is possible that these gravels were brought down from the escarpment in Anglian times and redeposited in their present situations during the Wolstonian.

#### TABLE 5. QUATERNARY CORRELATIONS

	vale	escarpment	rivers
Flandrian	_	_	alluvium
Zone III Devensian L-g Inst.	Upper lobate sheet fossil soil		
ł	Lower Solifluction Sheet	landslips in embayments	no. 1 Terrace
Ipswichian		_	no. 2 Terrace
Wolstonian	high-level chert gravels	block subsidence and and bulging	no. 3 Terrace

(iv) Following deposition of the Wolstonian gravels considerable erosion took place in the clay vale, accompanied by retreat of the escarpment within embayments between spurs.

(v) On this eroded landscape, in Devensian times, sheets of solifluction debris spread off the escarpment within the embayments and moved at least 500 m, while streams of the debris extended down valleys for a total distance of about 1 km from the foot of the scarp face. The typical gradient on which movement occurred is 3 or  $4^\circ$ , falling to a minimum value of just over 2°. This lower solifluction sheet is about 2 m in thickness, consisting of clay with embedded angular chert fragments, overlying brecciated Weald Clay in the upper layers of which there are several slip surfaces. Brecciation of the clay is attributed to disturbance caused by the meltout of ice lenses.

(vi) During the Late Devensian Interstadial (ca. 12000 radiocarbon years B.P.) a soil formed on slope-wash clay covering the lower solifluction sheet.

(vii) Soon afterwards, in Zone III of the Late-glacial sequence, the soil was buried beneath a lobate sheet of solifluction debris spreading for a distance of up to 300 m from the scarp foot. The lobes have an average slope of about 7°.

(viii) Based on thaw-consolidation theory, and on measured properties of the Weald Clay and the clay matrix of the chert gravels, a rational explanation in terms of soil mechanics principles can be established for the movement of the active layer above permafrost on gradients as low as  $1.5^{\circ}$  or  $2^{\circ}$ . Under temperate conditions such slopes are completely stable; movement could occur only on gradients steeper than about  $8^{\circ}$ .

(ix) In the spurs of the escarpment there are large-scale structural disturbances; massive blocks of the Hythe Beds subsided into the underlying Atherfield and Weald Clays, and the clays were forced up at the foot of the scarp in the form of bulges. These disturbances probably occurred in the Wolstonian stage.

(x) Large landslips are present in the escarpment within the embayments. Field evidence suggests that they took place in the Devensian before and perhaps during the final phase of solifluction which produced the upper lobate sheet.

(xi) In Postglacial times some landslips have been reactivated, and the streams in the clay vale have eroded small channels or valleys not more than 4 m in depth.

(xii) Finally it may be noted that in the region of Haslemere, 50–60 km west of Sevenoaks Weald, three groups of solifluction deposits can be recognized south of the Lower Greensand escarpment: a younger Head of Devensian age; an older Head capping small hills and interfluves; and the earliest Head existing as a few remnants at still higher altitudes (Thurrell, Worssam & Edmonds 1968). Originally the older Head was correlated with the Wolstonian and the earliest Head assigned tentatively to the Anglian. But according to Kellaway, Worssam, Holmes & Kerney (1973) all the high-level drifts in the Haslemere area may be of Anglian age (i.e. pre-Hoxnian). This view reflects a major uncertainty concerning the status of the Wolstonian stage; a question which pervades Quaternary studies in England today. The present investigations support the more traditional interpretation, namely that the Wolstonian includes a period or periods of intense glacial and periglacial activity.

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#### **REFERENCES** (Skempton & Weeks)

Bell, F. G. 1970 Late Pleistocene floras from Earith, Huntingdonshire. Phil. Trans. R. Soc. Lond. B 258, 347-378.

Bird, E. C. F. 1964 Denudation of the Weald Clay vale in West Kent. Proc. Geol. Ass. 74, 445-455.

- Bishop, A. W., Green, G. E., Garga, V. K., Andresen, A. & Brown, J. D. 1971 A new ring shear apparatus and its application to the measurement of residual strength. *Géotechnique* 21, 273-328.
- Carreck, J. N. 1964 Field meeting to the Medway valley, Kent from Maidstone to Rochester. Proc. Geol. Ass. 75, 357-360.

Chandler, R. J. 1972 Periglacial mudslides in Vestspitsbergen and their bearing on the origin of fossil solifluction shears in low angled clay slopes. Q. J. Eng. Geol. 5, 223-241.

Cook, W. H. & Killick, J. R. 1924 On the discovery of a flint-working site of Palaeolithic date in the Medway valley at Rochester. Proc. Prehist. Soc. East Anglia 4, 133-154.

Coope, G. R. & Sands, C. H. S. 1966 Insect faunas of the last glaciation from the Tame Valley, Warwickshire. Proc. R. Soc. Lond. B 165, 389-412.

Coope, G. R. 1975 Climatic fluctuations in northwest Europe since the Last Interglacial. Ice Ages: Ancient and Modern. Geol. J. Spec. Issue No. 6, pp. 153-168.

- Dines, H. G., Hollingworth, S. E., Edwards, W., Buchan, S. & Welch, F. B. A. 1940 The mapping of head deposits. *Geol. Mag.* 77, 198-226.
- Dines, H. G., Buchan, S., Holmes, S. C. A. & Bristow, C. R. 1969 Geology of the country around Sevenoaks and Tonbridge. London: H.M.S.O.
- Dines, H. G., Holmes, S. C. A. & Robbie, J. A. 1954 Geology of the country around Chatham. London: H.M.S.O.
- Hutchinson, J. N. 1974 Periglacial solifluction: an approximate mechanism for clayey soils. Géotechnique 24, 438-443.
- Kellaway, G. A., Worssam, B. C., Holmes, S. C. A., Kerney, M. P. & Shephard-Thorn, E. R. 1973 South-east England. In Mitchell et al. A correlation of Quaternary deposits in the British Isles. Geol. Soc. Spec. Rep. No. 4.
- Kennard, A. S. 1944 The Crayford brickearths. Proc. Geol. Ass. 55, 121-167.

McRoberts, E. C. 1975 Field observations of thawing in soils. Can. Geotech. J. 12, 126-130.

- Morgan, M. A. 1969 A Pleistocene fauna and flora from Great Billing, Northamptonshire. Opuscula Entom. 34, 109-129.
- Morgenstern, N. R. & Nixon, J. F. 1971 One-dimensional consolidation of thawing soils. Can. Geotech. J. 8, 558-565.
- Murchison, R. I. 1851 On the distribution of the flint drifts of the south-east of England. Q. J. Geol. Soc. 7, 349-398.
- Skempton, A. W. & Northey, R. D. 1952 The sensitivity of clays. Géotechnique 3, 30-53.
- Skempton, A. W. & Petley, D. J. 1967 The shear strength along structural discontinuities in stiff clays. Proc. Geotech. Conf. (Oslo) 2, 29-46.
- Thurrell, R. G., Worssam, B. C. & Edmonds, E. A. 1968 Geology of the country around Haslemere. London: H.M.S.O.
- Topley, W. 1875 The geology of the Weald. London: H.M.S.O.
- Weeks, A. G. 1969 The stability of natural slopes in south-east England as affected by periglacial activity. Q. J. Eng. Geol. 2, 49-61.
- West, R. G., Lambert, C. A. & Sparks, B. W. 1964 Interglacial deposits at Ilford, Essex. Phil. Trans. R. Soc. Lond. B 247, 185-212.
- Williams, R. B. G. 1975 The British climate during the Last Glaciation; an interpretation based on periglacial phenomena. Ice Ages: ancient and modern. Geol. J. Spec. Issue No. 6, pp. 95-120.

Worssam, B. C. 1963 Geology of the country around Maidstone. London: H.M.S.O.

Wymer, J. J. 1974 Clactonian and Acheulian industries in Britain – their chronology and significance. Proc. Geol. Ass. 85, 391–421.

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# Slope Stability of Cuttings in Brown London Clay

Stabilité des Pentes de Voies en Tranchées au London Clay

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SYNOPSIS In this paper a summary is presented of research on first-time slides in cuttings in the brown London Clay: a classic example of the geological materials classified by Tergaghi (1936) as 'stiff fissured clays'. The two principal conclusions may be summarised as follows: (i) Failure generally occurs many years after excavation; and field evidence is now available to indicate that the main reason for this delay is a very slow rate of pore pressure equilibration, despite the fissured structure of the clay. (ii) Back analysis of typical long-term slips shows that the strength of the clay at failure corresponds rather closely to the 'fully softened' condition, or to the fissure strength; and that the field strength is greater than the residual but smaller than the peak strength, even as measured on large samples.

#### INTRODUCTION

Slope failures, or 'slips' as they are traditionally called, in London Clay cuttings present three inter-related problems (i) determining the shear strength at failure,(ii) deciding on the best means of measuring or predicting this strength in laboratory tests, and (iii) finding an explanation for the long delayed failures so characteristic of the slips.

Research into these problems has progressed from an initial stage in which the rate of softening of the clay was deduced, in terms of undrained strength, from the analyses of various case records; through a stage in which the analyses were carried out in terms of effective stress, but with an inadequate knowledge of the pore pressure; and finally to the current stage where the importance of the very slow rate of pore pressure equilibration, after excavation, has come to be recognised.

Further research is still required before the processes involved in first-time slides are fully understood, especially concerning the strength properties at failure.

#### BROWN LONDON CLAY

Where the London Clay extends up to ground level, or is covered only by a thin mantle of drift deposits, it is oxidised to a brown colour to depths of 5 to 15m. At greater depths the clay is blue-grey in colour. Small joints and fissures occur throughout the clay. The fissures are more numerous, and therefore of smaller size, in the upper zone. Studies by Skempton, Schuster & Petley (1969) show an average fissure size in the brown London Clay of around 4cm, ranging from a maximum of 10cm down to about 1cm. The fissures exhibit little preferred orientation in azimuth, but their dip angles tend to be either fairly steep or rather flat; not many dips are recorded around 45°. The joints are nearly always steeply dipping.

Throughout the London area, from which the case records have been obtained, the clay exhibits only minor variations in index properties. Thus a direct comparison can be made between any one site and the rest, and all the sites contribute towards establishing a unified interpretation of the phenomena.

Typical index properties of the brown London Clay, as quoted by Chandler & Skempton (1974), are given in Table I.

#### TABLE I

#### Typical properties of brown London Clay

water content	=	31
liquid limit	=	82
plastic limit	=	30
plasticity index	=	52
clay fraction	=	55 per cent
unit weight	=	18.8 kN/m <sup>3</sup>

Below the depth of seasonal variation, 1.5 to 2m beneath ground surface, the undrained shear strengths increase from roughly 70  $kN/m^2$  to 160  $kN/m^2$  at a depth of 10m (Skempton 1959).

The London Clay is of Eocene age and, except for the lowest and highest parts of the stratum, it was deposited in a moderately deep marine environment. It has been overconsolidated by the erosion of at least 150m of sediments. In its natural state, away from excavations, the horizontal stresses exceed the vertical pressures, with values of  $K_0$  which have been estimated to be more than 2.0 in the top 15m (Skempton 1961). Subsequent to its deposition the clay, in the London area, has been subjected to gentle folding; bedding dips of more than 2<sup>o</sup> are rare. Stratification is usually not apparent and, when considered on the scale of a mass several cubic meters in size, the clay is a remarkably uniform material.

The mineralogy of the clay fraction (particles smaller than two microns) may be summarised from the work of Burnett & Fookes (1974) approximately as follows:

illite	47	per	cent
montmorillonite	35	-	
kaolinite	15		
chlorite	3		

#### FIRST-TIME SLIDES

With a single exception, which has been included to point a contrast, all the slope failures mentioned here are 'first-time slides'. That is to say there has been no previous instability and the slip does not take place on a pre-existing shear surface.

#### EXAMPLES OF DELAYED FAILURES

(a) <u>New Cross</u> One of the earliest deep cuttings in London Clay was excavated in 1838 on the London & Croydon Railway. The line was opened in June 1839. On 2 November 1841, without warning, and in the course of four hours, nearly 40,000 cubic metres of clay slipped into the position shown in Fig. 1. Work was still in hand to clear the line when a similar large slip occurred in the opposite side of the cutting. The cutting was eventually stabilised by forming wide benches and making the slopes between the benches at 2:1, a total volume of 200,000 cubic metres of clay having been removed.

Gregory (1844) gives a good description of the slips and the geology of the cutting, and he records that the slip surface passed along the base of the brown clay. The fact that slips in cuttings do not penetrate any appreciable depth into the blue clay has been noted in several cases, for example at Northolt (Henkel 1957), and demonstrates that the





strength of the blue clay must be greater than that of the brown clay.

(b) <u>Wembley Hill</u> The cutting, with a retaining wall, shown in Fig. 2 was completed in 1905 at Wembley Hill on the Neasden-Northolt line of the Great Central Railway. Movement of the permanent way appeared in February 1918 and a few days later the wall slid forward 6m in less than half an hour (Anon, 1918). Owing to the presence of the wall the slip surface was forced into the blue clay. Piezometers were installed at this site in 1956, but it is now realised that the pore pressures measured then, 51 years after construction, are not relevant to an analysis of the failure in 1918.



(c) <u>Kensal Green</u> This site lies to the east of Kensal Green station and tunnel. A cutting was first made in 1836 for the London & Birmingham Railway, renamed, after an amalgmation, the London & North Western Railway. The cutting was widened in 1875 and again in 1912, to accomodate the Euston to Watford electric lines, when the retaining wall shown in Fig. 3 was built. The wall extends west to the station, where it is higher, and a failure occurred there in 1927. After remedial measures had been carried out accurate surveys were made along the entire length of wall and repeated at regular intervals. They showed a gradually accelerating movement until, at the section in Fig. 3, a tension crack appeared in the clay 6m behind the wall and the wall itself had slid forward by 30cm. This was in April 1941. Eight months later the movement had increased to about 40cm, the wall cracked, and remedial works were started.



During these works, in January 1942, the writer investigated the clay. He noted the fissured structure and found slip surfaces behind and in front of the wall. It was also noted that the clay on the slip surface and adjacent to some fissures was much softer than in the main body of the stratum (Skempton 1942).

Next year, with his colleage W.H. Ward, the writer examined another retaining wall failure in London Clay, at Mill Lane, and detailed water content determinations were made across the slip surface. The results are plotted in Fig. 4 together with similar observations made by D.J. Henkel in 1955 at Uxbridge and Northolt.



The Kensal Green investigations established that there must be a reduction in strength of clay with time, and it was an easy matter to calculate the undrained strength at failure ( $16 \text{ kN/m}^2$ ) using the Ø = O analysis. In the same way the strengths at Mill Lane and several other sites were determined and, by

plotting these strengths against the age of the cutting, a rough time-scale for the softening or strength reduction could be derived (Skempton 1948). The explanation of the softening process was assumed to be as given by Terzaghi in 1936; namely the infiltration of ground water into fissures opened in consequence of lateral movements following stress release during excavation.

(d) Sudbury Hill In 1953 the writer decided to examine the problem of slope stability in stiff fissured clays in terms of effective stress. Work began in the autumn of 1954 with D.J. Henkel and the able assistance of F.A. DeLory. New sites were found, piezometers were installed and laboratory tests carried out to determine the effective stress shear-strength parameters. By 1956 it was clear that the cohesion intercept c' at the time of failure was very considerably smaller than the value in laboratory tests. Moreover the data could be interpreted as showing a decrease in c' with time, approaching c' = 0 after several decades.

One of the newly discovered sites was a cutting at Sudbury Hill on the same line as Wembley Hill. The cutting dates from about 1903 and a slip occurred on the south side in 1949 (Fig. 5). As will be shown later, stability analyses of the first-time slide and also of the post-slip movements yield valuable results; but for the present it is sufficient to note that we had a 'long-term' case record of a slip in which pore pressures were measured only a few years after the event, and which showed that c' was very small.



It is perhaps worth mentioning that the germ of the idea that clays may loose their 'cohesion' can be found in Rankine (1862) and several other works on civil engineering in the 19th century; while experience had shown that London Clay cuttings were generally not stable at slopes steeper than 3:1 (Baker 1881).

#### PORE PRESSURES

The most convenient parameter for characterising the piezometric conditions in a slope is the average pore pressure ratio  $\bar{r}_{\rm u}$ introduced by A.W. Bishop in 1960 and defined in Fig. 6.



For the south side of Sudbury Hill cutting (Fig. 5) the value of  $\bar{r}_u$  along the slip surface is about 0.26. Piezometers were also installed on the north side (Fig. 7) and they gave an average value on a typical surface of 0.28, despite the fact that some trench drains had been placed in this slope two years before the piezometers; though the piezometers were of course located mid-way between the drains.



From these and a few other rather fragmentary observations it was decided by 1970, when P.M. James completed the next research thesis on London Clay slopes, that the long term value of  $\overline{r}_u$  could be taken as lying between 0.25 and 0.35, and that no important errors would be involved in taking 0.3 as a typical figure for back analysis (or design) in the absence of reliable piezometric data at any given site.

Under the direction of N.R. Morgenstern and the writer, James had discovered some more sites and analysed these slides, as well as most of the earlier ones, using the method of Morgenstern & Price (1965) for noncircular slip surfaces. The results, more numerous and precise than those previously available, appeared to confirm the interpretation regarding the decrease in c' with time. They also led to a conclusion of major importance: namely that the lower bound of all the strengths calculated from back-analysis of first-time slides lay well above the residual strength (Skempton 1970).

However, by 1973 a radical change had occurred in our concept of the physical process responsible for delayed failures. Already it was known that the rate of pore pressure equilibration in clay fill embankments could be extremely slow. Observations made in 1971 under the direction of P.R. Vaughan had shown, for example, that negative pore pressures were still existing in a 16m high dam, built in 1963, with clay having average liquid and plastic limits of 47 and 22 respectively, even in the upstream shoulder six years after impounding (Walbancke 1973). This result brought forcibly to mind the fact that no field studies were available on the rate of equilibration in London Clay cuttings; the tacit assumption had been that, owing to the fissured structure and the possibility (suggested by Terzaghi) of the fissures opening up during excavation, the in-situ permeability would be relatively high.

It was then decided to install piezometers in a London Clay cutting which had been excavated in recent times. The only one we knew was on the Hendon Motorway (M1) at Edgwarebury. In 1972 the Science Research Council made a grant for the work to be done, and Miss Walbancke obtained the first reliable readings in the early months of 1973. These showed negative pore pressures at each of the five piezometers (Fig. 8) although the cutting had been completed nine years earlier in 1964 (Vaughan & Walbancke 1973). The piezometers were of the twin-tube hydraulic type developed by A.W. Bishop, with high air-entry ceramic filters (Bishop et al 1960).



It has of course long been known that the removal of load by excavating a cutting would cause an immediate reduction in pore pressure. What caused surprise at Edgwarebury was that the pore pressures were still negative after 9 years, and calculations indicated a coefficient of swelling not dissimilar in magnitude from values measured in the laboratory on small undisturbed samples. Clearly in this case the fissures had little effect on in-situ permeability of the clay mass after excavation.



New Side (19 yrs.)	Piez.	r <sub>u</sub>	Piez.	r.	Old Side (125 yrs.)
	1	0.06	8	0.31	]
$r_u = 0.15$	2	0.18	9	0 34	mean   r <sub>u</sub> = 0.32
l	5	0 21	11	0.31	]] -
	4	0 09	10	0.32	

#### POTTERS BAR CUTTING

after Walbancke (1976)

(9)

Unfortunately the Edgwarebury cutting is predominately in blue London Clay. It therefore became a matter of great interest to see whether the same conclusion applied in the brown clay. After an intensive search an ideal site was discovered at Potters Bar. A cutting here, on the main line from King's Cross to York, had been made in 1850, to a depth of 11m, entirely in the brown clay. But so recently as 1956 the cutting was widened on the west side, leaving the old east side unaltered except for deepening by 1 metre and the construction of a small toe wall (Fig. 9). Piezometers were installed in 1974, and the observations for 1975 are summarised in Fig. 9. There can be no doubt that in the east side, after 125 years, the pore water pressures have reached a state of equilibrium; and it is interesting to note that the average value of  $\bar{r}_u$  along a representative (imaginary)slip surface is 0.30 (Fig. 10). In striking contrast the pore pressures of the west side, after 19 years, are only about one-half of the equilibrium values, although there is no essential difference between the two sides other than age.

Taking all the pore pressure evidence into account it seems that equilibration is indeed a slow process; at least 40 to 50 years, for practical purposes, being required for its completion in typical cases (Fig. 11). If this is correct, the conclusion is inescapable that we have here the principal reason for delayed failures in London Clay cuttings.





1

#### SHEAR STRENGTH

(a) Back analysis, first-time slides From the available case records five have been chosen as being 'long term' (more than 45 years to failure) and as covering a reasonably wide range of depth (and therefore of effective pressure). The basic data are set out in Table II. Each slip is analysed taking a factor of safety = 1.0, and assuming  $F_u = 0.30$ , to find the average shear strength and average normal effective pressure along the slip surface. The results are shown by solid points in Fig. 12. Effective pressures are also calculated for  $\bar{r}_u = 0.25$  and 0.35, as indicated in this graph.

The best fit to the points is a line defined by the parameters  $c' = 1 \text{ kN/m}^2$   $\emptyset' = 20^\circ$ and a lower limit is given by c' = 0  $\emptyset' = 20^\circ$ 



TABLE	II	

First-time	slides	in	brown	London	Clay

	Site	Date of cutting	Date of slip	Time to failure years	Height m	Slope
Inter- mediate	New Cross Kingsbury St Helier Cuffley	1838 1931 1930 1918	1841 1947 1952 1953	3 16 22 35	17.0 6.0 7.0 7.2	1½:1 2¼4:1 2:1 2 <sup>3</sup> /4:1
Long- term	Sudbury Hill Crews Hill Grange Hill West Acton Hadley Wood widened	1903 1901 1902 1916 1850) 1916	1949 1956 1950 1966 1947	46 47 48 50 c.65	7.0 6.2 12.2 4.9 10.4	3:1 3 <sup>1</sup> /3:1 3 <sup>1</sup> /4:1 3:1 3 <sup>2</sup> /3:1

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(b) <u>Back analysis, post-slip movements</u> After the slip in 1949 at Sudbury Hill (Fig.5) no remedial works were undertaken; the toe of the slip was merely trimmed back. Further small movements occurred in succeeding winters, and were similarly treated. Piezometer levels during these post-slip movements are known and it is therefore possible to calculate with some accuracy the residual shear strength and average effective normal pressure. The results are compared in Fig. 13 with those obtained from back-analysis of the first time slide.

From Fig. 13 it is seen that the strength mobilised in a first-time slide is significantly greater than the residual strength; a conclusion which has been emphasised already (Skempton 1970) but is not widely appreciated.



13

(c) Laboratory tests The peak strength parameters of brown London Clay as measured in 6cm shear box tests or 38mm diameter triaxial tests, are:

 $c' = 14 \text{ kN/m}^2 \qquad \phi' = 20^\circ$ 

Samples of this size are too small to include a representative assemblage of fissures, so tests have been made on the largest triaxial samples (250mm diameter) which can be handled at all conveniently in the laboratory (Sandroni 1977). The resulting parameters

$$c' = 7 kN/m^2 \qquad \phi' = 20^{\circ}$$

certainly show a marked reduction from those obtained in the usual small sized samples; but even so the strength is around 30 to 70 per cent in excess of the field values obtained from back analysis (Fig. 14).

Laboratory tests to measure the residual strength on natural slip surfaces in the brown London Clay (Skempton & Petley 1967) give the parameters

$$c' = 1.4 \text{ kN/m}^2 \quad \phi' = 13^\circ$$



These are in good agreement with the analysis of post-slip movements at Sudbury Hill (Fig.13) but of no relevance to first-time slides, while ring shear tests give even lower results (Bishop et al 1971).

We therefore return to the conclusion (Henkel 1956, Scholfield & Wroth 1968, Skempton 1970) that the most appropriate laboratory parameters are those for the 'fully softened' or 'critical state' condition, which can be determined by measuring the strengths of remoulded, normally consolidated clay. For London Clay (Gibson 1953, Bishop et al 1965, Petley1966) these parameters are approximately

$$p' = 0 \quad p' = 20^{\circ}$$
 (Fig. 15)

However, a few tests have been carried out to measure the strength of joints and fissures in London Clay (Skempton et al 1969) and the parameters representing the lower limit of the results are also approximately c' = 0,  $\emptyset' = 20^{\circ}$ . (Fig. 15).

Conclusions It appears that the dis-(d) placements preceding a first-time slide are sufficient to cause some progressive failure, reducing the strength towards the fully softened or the lower limit of fissure strength; but the displacements are not so large as to reduce the strength to the residual value. Field evidence in support of these conclusions is provided (i) by the observation at the Uxbridge retaining wall failure (Watson 1956) that a continuous, single slip surface had not yet been developed with a total movement of about 40cms, and (ii) by the fact that after the slip at Sudbury Hill, when the strength had fallen to the residual, the displacements were about 1.5 to 2m.

Some explanation is also required for the progressive failure mechanism which takes the clay past its peak strength, and this can probably be attributed in part to the presence of local stress concentrations at the fissures.


## RATE OF EQUILIBRATION

If the assumption is made that the shear strength parameters  $c^{\,\prime}$  = 1  $kN/m^2$  and  $\phi' = 20^{\circ}$  apply at failure in all cases, it is possible to evaluate the average pore pressure ratio  $\bar{r}_u$  for each of the first-time slides for which adequate data are available (Table II). The results, when plotted against time to failure (Fig. 11), show a value at New Cross (3 years), though small positive values at St Helier and Cuffley (around 20 years) up to the full equilibrium value of  $\bar{r}_{11}$  = 0.30 after about 50 years. It is of particular interest that the point representing the observed pore pressures in the west side of Potters Bar cutting  $(\bar{r}_u = 0.14 \text{ on a typical slip surface})$  lies practically on the line deduced from back analysis of the slope failures, and thus provides an independent check on the validity of this method of calculating the pore pressures.

It is of course apparent that the rate of change of pore pressure will depend to some extent on the dimensions of the cutting. For this reason the slides used in deriving Fig. 11 have been selected to exclude cuttings of unusually shallow depth; in these it would be expected that equilibration is achieved on a shorter time scale, but the number of such cases is too small for a graph to be drawn.

Slips in the zone of seasonal variation have also been excluded. They take place after exceptionally heavy rainfall, especially following a prolonged dry season. NATURAL SLOPES

Finally it may be pointed out that the stability of natural slopes in brown London Clay is a different and distinct problem, in which the residual strength is the controlling factor. Information on this subject is given by Skempton & Delory (1957), Skempton (1964), Hutchinson (1967 and 1974) and Hutchinson & Gostelow (1976).

#### CONCLUSIONS

(i) The shear strength parameters of the brown London Clay relevant to first-time slides are:

 $c' = 1 \text{ kN/m}^2$  and  $\emptyset' = 20^\circ$ 

(ii) The peak strength, even as measured on large samples, is considerably higher; so some progressive failure mechanism appears to be involved.

(iii) The in-situ strength is given approximately by the 'fully softened' value and also by the lower limit of strength measured on structural discontinuities (joints and fissures).

(iv) The residual strength is much smaller than this and corresponds to the strength mobilised <u>after</u> a slip has occurred, with large displacements of the order 1 or 2m.

(v) It is a characteristic feature of firsttime slides in London Clay that they generally occur many years after a cutting has been excavated.

(vi) The principal reason for this delay is the very slow rate of pore pressure equilibration; a process which in typical cuttings is not completed, for practical purposes, until 40 or 50 years after excavation.

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#### Chronological Bibliography

relating to London Clay

GREGORY, C.H. (1844) On railway cuttings and embankments; with an account of some slips in the London Clay, on the line of the London and Croydon Railway. <u>Min. Proc. Inst.</u> <u>C.E.</u> <u>3</u>, 135-145.

BAKER, B. (1881) The actual lateral pressure of earthwork. <u>Min. Proc. Inst. C.E</u>. <u>65</u>, 140-186.

ANON, (1918) Reconstruction of a retaining wall on the Great Central Railway. The Engineer 126, 536-537.

SKEMPTON, A.W. (1942) The Failure of a Retaining Wall at Kensal Green. BRS Soil Mechanics Record No. 3, Loc. 110.

SKEMPTON, A.W. (1946) Earth pressure and the stability of slopes. <u>Principles and Appli-</u> cation of Soil Mechanics. (London: Inst. C.E.). 31-61.

SKEMPTON, A.W. (1948) The rate of softening in stiff fissured clays, with special reference to London Clay. <u>Proc. 2nd Int. Conf.</u> <u>Soil Mech.</u> (Rotterdam)  $\frac{2}{2}$ , 50-53.

GOLDER, H.Q. (1948) Measurement of pressure in timbering of a trench in clay. Proc. 2nd Int. Conf. Soil Mech. (Rotterdam)  $\frac{2}{2}$ , 76-81.

GIBSON, R.E. (1953) Experimental determination of the true cohesion and true angle of internal friction in clays. <u>Proc. 3rd Int</u>. <u>Conf. Soil Mech</u>. (Zurich) <u>1</u>, <u>126-130</u>.

WATSON, J.D. (1956) Earth movement affecting L.T.E. Railway in deep cutting east of Uxbridge. <u>Proc. Inst. C.E. Part II</u> 5, 302-316.

HENKEL, D.J. (1956) Discussion on Watson (1956). Proc. Inst. C.E. Part II <u>5</u>, 320-323. DELORY, F.A. (1957) Long-term Stability of Slopes in Over-consolidated Clays. Ph.D. Thesis, University of London.

HENKEL, D.J. (1957) Investigations of two long-term failures in London Clay slopes at Wood Green and Northolt. <u>Proc. 4th Int. Conf.</u> Soil Mech. (London) 2, 315-320.

SKEMPTON, A.W. & DELORY, F.A. (1957) Stabil ity of natural slopes in London Clay. <u>Proc.</u> <u>4th Inst. Conf. Soil Mech</u>. (London) <u>2</u>, <u>378-381</u>.

SKEMPTON, A.W. (1959) Cast in-situ bored piles in London Clay. <u>Geotechnique</u> 9, 153-173.

SKEMPTON, A.W. & HENKEL, D.J. (1960) Field observations on pore pressures in London Clay. <u>Conf. on Pore Pressures and Suction</u> <u>in Soils</u> (London) pp. 81-84.

SKEMPTON, A.W. (1961) Horizontal stresses in an over-consolidated Eocene clay. Proc. 5th Int. Conf. Soil Mech. (Paris) 1, 351-357.

SKEMPTON, A.W. (1964) Long-term stability of clay slopes. Geotechnique 14, 77-101.

BISHOP, A.W., WEBB, D.L. & LEWIN, P.I. (1965) Undisturbed samples of London Clay; strengtheffective stress relationships. <u>Geotechnique</u> <u>15</u>, 1-31.

PETLEY, D.J. (1966) <u>The Shear Strength of</u> <u>Soils at Large Strains</u>. Ph.D. Thesis, University of London.

HUTCHINSON, J.N. (1967) The free degradation of London Clay cliffs. <u>Proc. Geotech. Conf</u>. (Oslo) 1, 113-118.

SKEMPTON, A.W. & PETLEY, D.J. (1967) The strength along structural discontinuities in stiff clays. <u>Proc. Geotech. Conf</u>. (Oslo) <u>2</u>. 29-46.

SKEMPTON, A.W., SCHUSTER, R.L. & PETLE., D.J. (1969) Joints and fissures in the London Clay at Wraysbury and Edgware. <u>Geotechnique</u> <u>19</u>, 205-217.

JAMES, P.M. (1970) <u>Time Effects and Progres</u> sive Failure in Clay <u>Slopes</u>. Ph.D. Thesis, University of London.

SKEMPTON, A.W. (1970) First-time slides in over-consolidated clays. <u>Geotechnique 20</u>, 320-324.

BISHOP, A.W., GREEN, G.E., GARGA, V.K., ANDRESEN, A. & BROWN, J.D. (1971) A new ring shear apparatus and its applications to the measurement of residual strength. <u>Geotechnique</u> 21, 273-328. VAUGHAN, P.R. & WALBANCKE, H.J. (1973) Pore pressure changes and the delayed failure of cutting slopes in over-consolidated clay. <u>Geotechnique</u> 23, 531-539.

BURNETT, A.D. & FOOKES, P.G. (1974) A regional engineering geology study of the London Clay in the London and Hampshire basins. Q.J. Eng. Geol. 7, 257-295.

HUTCHINSON, J.N. (1974) Periglacial solifluction: an approximate mechanism for clayey soils. <u>Geotechnique</u> 24, 438-443.

CHANDLER, R.J. & SKEMPTON, A.W. (1974) The design of permanent cutting slopes in stiff fissured clays. <u>Geotechnique</u> 24, 457-466.

WALBANCKE, H.J. (1976) Pore Pressures in Clay Embankments and Cuttings. Ph.D. Thesis, University of London.

HUTCHINSON, J.N. & GOSTELOW, T.P. (1976) The development of an abandoned cliff in London Clay at Hadleigh, Essex. <u>Phil.Trans</u>. <u>Roy, Soc</u>. <u>A.283</u>, 557-604.

SANDRONI, S.S. (1977) <u>The Strength of</u> London Clay in Total and <u>Effective Stress</u> Terms. Ph.D. Thesis, University of London. Other References

RANKINE, W.J.M. (1862) <u>A Manual of Civil</u> Engineering. London: Griffin & Bohn.

TERGAGHI, K. (1936) Stability of slopes of natural clay. Proc. Int. Conf. Soil Mech. (Harvard) <u>1</u>, 161-165.

BISHOP, A.W. & MORGENSTERN, N.R. (1960) Stability coefficients for earth slopes. <u>Geotechnique</u> <u>10</u>, 129-150.

BISHOP, A.W., KENNARD, M.F. & PENMAN, A.D.M. (1960) Pore pressure observations at Selset Dam. <u>Proc. Conf. Pore Pressure</u> (London) 91-102.

MORGENSTERN, N.R. & PRICE, V.E. (1965) The analysis of the stability of general slip surfaces. <u>Geotechnique</u> <u>15</u>, 79-93.

SCHOFIELD, A.N. & WROTH, C.P. (1968) Critical State Soil Mechanics. London: McGraw-Hill

WALBANCKE, H.J. (1973) Discussion on principles of measurement. Symp. on Field Instrumentation in Geotech. Eng. (London) 552-555.

## Landmarks in early soil mechanics

Jalons dans les premiers siècles de la mécanique des sols

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#### INTRODUCTION

Modern soil mechanics came into being around the years 1910-30, largely as a result of work by Terzaghi (Bjerrum et al., 1960) and by Fellenius and his colleagues in Sweden (Bjerrum and Flodin, 1960). The term 'soil mechanics' first came into general use after a series of articles by Terzaghi (1925) had been published in 'Engineering News Record.'

However, the subject existed in all but name long before 1925 even if it had not yet been welded together and recognized as a coherent discipline. Rather, it existed as a set of somewhat isolated topics, such as earth pressure theory and practical knowledge of slips in clay slopes, with little correlation between field observations and theoretical analysis, and lacking above all the unifying principle of effective stress. Nevertheless, theoretical and practical contributions of importance were made in this period of early soil mechanics, and it is with some of the more interesting of these contributions that I shall be concerned in the present lecture.

Of the early masters, Coulomb's celebrated essay of 1773 has been reproduced and translated, with a very thorough discussion, by Heyman (1972), Collin's work on clay slips has been translated (Schriever, 1956) with a biographical and analytical commentary by myself, Rowe (1969) gives an account of Osborne Reynolds' experiments on dilatancy, and Boussinesq's great treatise of 1885 has appeared in a reprint (1969) with a preface by Caquot. A valuable survey of early French contributions is given by Kerisel (1956), a résumé of earth pressure tests can be found in Feld (1923), and Flodin and Broms (1977) provide much historical information on geotechnical problems in soft clays; on the practical aspects, excellent studies have been published by Peck (1948) on Chicago foundations and by Glossop (1960, 1961) on grouting and (1976) on the use of compressed air in shaft and tunnel construction. But of the 30 items selected for presentation here, some are very little known and all of them have been examined afresh.

## THE FIRST PHASE

Several attempts to tackle soil mechanics problems were made before Coulomb, in what may be called the pre-classical period; examples can readily be found in foundation and earth dam practice of rational designs based on sound engineering judgement.  GAUTIER, H. (1717). Dissertation sur l'epaisseur des culées des ponts ... sur l'effort et la pesanteur des arches ... et sur les profils de maconnerie qui doivent supporter de chaussées, des terrasses et des remparts. Paris: Cailleau
 In his chapter on retaining walls Gautier says

that earth fills can be classified as sand, ordinary earth, or clay (glaise); he determined the natural slope of soils in the first two categories, by measuring the angle at which they stand, when tipped in a heap, as 5:3 ( $31^{\circ}$ ) for clean dry sand and 1:1 ( $45^{\circ}$ ) for a freshly dug and crumbled earth. He made no tests on clay but states that when well compacted it will exert smaller pressures than earth or sand. He also gives the following unit weights: water 63, sand 116, earth 84, masonry 126 lb/ft<sup>3</sup>.

In presenting a theory for retaining wall design, he starts by observing that as earth will stand unsupported at its natural slope CA (Fig. 1) the function of a wall is clearly to retain the wedge of earth ABC lying above the natural slope. Assume that the wall has been built to some height CE. Then the height of earth remaining above this level would stand at its natural slope FG and therefore requires a base extending out to F, where  $EF = {}^{1}_{2}BE$ , and of course the slope JG would be stable if the triangle EFJ were replaced by masonry. This argument follows for any level between C and B; hence the theoretical profile for a wall is the triangle BCD in which  $CD = \frac{1}{2}BC$ . In practice the wall must have a finite top width, and it is usually built with a front batter of, say, 1/5:1. As a simple rule, therefore, the wall can be made with this batter and the same cross-sectional area as the triangle BCD; in other words the mean thickness of the wall should be a quarter of its height. Gautier gives a table of wall dimensions, and



If BC = 20 ft mean wall thickness = 5 ft for  $\beta_p = 45^{\circ}$ 

#### Gautier (1717)



#### Biography

Henri Gautier (1660-1737), after working as a royal engineer in the maritime service, took a senior post in the Ponts et Chaussées on its formation in 1715; his 'Traite de la construction des chemins' (Paris, 1715) and 'Traité des ponts' (Paris, 1716) were well-known practical text books (Nouvelle biographie générale, 1857).

2. BELIDOR, B.F. (1729). La science des ingenieurs dans la conduite des travaux de fortifications et d'architecture civil. Paris: Jombert

In this famous treatise for military and civil engineers Belidor deals at length with the design of masonry retaining walls. He adopts Gautier's threefold classification of earth fills and takes for standard calculations the case of ordinary earth with a natural slope of  $45^{\circ}$ . If earth had no friction the thrust on a vertical wall with horizontal back-fill would be equal to the weight of the wedge ABC (Fig. 1), but real soils have some strength (tenacité) and it is reasonable to take the thrust as one half of the weight. Thus Belidor is postulating that

$$P_a = \frac{1}{2} \frac{\gamma H^2}{2}$$
 or  $K_a = 0.5$ 

and he shows that the resultant acts at the lower third point of the wall height. Wall friction is neglected.

The analysis is extended to walls carrying a surcharge slope and to walls with internal counterforts or a battered back face, the principle of design being to balance moments of the earth thrust and wall weight about the toe, taking the unit weight of masonry as  $1.5\gamma$ , and then to increase the wall thickness by 25%. This factor of safety is particularly desirable for quay walls and walls supporting a road, he says.

For walls with a vertical back face and a front batter of 1/5:1 the mean thickness (allowing for the 25% increase) is one third of the height if the backing is horizontal. Belidor is very aware that this method of design implies a good foundation. If the ground is not hard, a piled foundation must be used.

Several further attempts to tackle the earth pressure problems were made during the next forty years, but without showing much advance on Belidor. This work has been fully discussed by Mayniel (1808).

#### Biography

Bernard Forest de Belidor (1671-1761), professor of mathematics at the military college at La Fère, saw active service as an engineer during the War of the Austrian Succession, and then settled in Paris with the rank of Brigadier; he was a member of the Academy of Sciences and a Fellow of the Royal Society. His 'Architecture hydraulique' (4 vols, Paris, 1737-53) was the standard reference for French and British civil engineers up to the early 19th century (Dictionary of scientific biography, 1970).

3. GRUNDY, J. (1766). Report and estimates for an earth dam at Grimsthorpe in Lincolnshire. In Surveys ... reports ... and estimates in works of draining, navigation, and other business in engineering, by John Grundy of Spalding, Engineer, pp. 143-151 (Library, Institution of Civil Engineers.

In 1748 a large ornamental lake, 32 acres in extent, was formed on the Duke of Ancaster's estate at Grimsthorpe by building an earth dam. Designed by John Grundy, the dam (which is still in existence) had a height of 18 ft and a central clay core, as we learn from a report of his written in 1758. No original drawings appear to have survived, but in 1766 Grundy produced estimates and a cross-section for a dam to be constructed further downstream, which would add an extra 20 acres to the lake. The estimates and section are undated, but there is a report on the scheme dated 1766 and a plan, drawn by Grundy in 1767, showing the existing lake and the site of the proposed new dam (see Binnie, 1976).

The dam was to have a maximum height of 25 ft, an upstream slope of 3½:1, a downstream slope of 2:1, a 20 ft crest width, and a clay core wall 6 ft wide (Fig. 2). The banks were to be made of rammed earth, using locally available material, and the core is specified to be of clay 'well rammed and watered' after being 'tempered'; the clay was to be brought from a selected site some distance away. Grundy allows 2 in. settlement per foot, and says that the height as built would have to be 29 ft.

These are the earliest known engineering details of an English earth dam, and they probably apply with little variation to the dam built in 1748. A particularly interesting feature is the clay core wall. Similar clay cores were used in dams built by canal engineers in the late 18th century; and 'puddle clay' cores, usually of tapered section, were a standard feature in resevoir dams of the 19th century.

Grundy's section and estimates provide a good example of rational design based on experience, observation and engineering judgement without the use of analysis, in a period when analysis was either non-existent or of insufficient power to be capable of application.

#### Biography

John Grundy (1719-83), was a well-known consulting engineer working chiefly on fen drainage and river navigations in eastern and northeastern England (Skempton, A.W. and Wright, E.





1971. Early members of the Smeatonian Society of Civil Engineers. Trans. Newcomen Soc. vol 44, pp. 23-47).

 PERRONET, J.R. (1769). Mémoire sur l'éboulement qui arrive quelquefois a des portions de montagnes et autres terreins eleves; et sur les moyens de prévenir ces éboulements et de s'en garantir dans plusieurs circonstances. Paris (no imprint). Reprinted in Oeuvres de M. Perronet, 2nd edn. Paris: Didot, 1788, pp. 631-643.

In this 12 page memoir Perronet initiates the engineering study of slope stability.

Slopes can exist in natural, intact soils *(terre vierge)* or in fills. Natural slopes which have remained stable for a long time will continue in that condition unless changes are introduced. Such changes may be brought about (i) by loading the upper part of a slope with earth or a heavy structure, (ii) by excavation at the foot of the slope, or (iii) by the infiltration of water which reduces the strength.

Cuttings can be made in strong intact soils with vertical sides (Perronet must be speaking here of temporary excavations), but in soft earths and dry sands the slopes, even in intact masses, will form themselves at an inclination of about 30° to the horizon.

Fills made of earth which has been dug some time ago and has lost much of its cohesion, or freshly turned earth, which has still less cohesion, will stand at angles ranging from about  $35^{\circ}$  for the strongest soils (not  $45^{\circ}$  as commonly asserted) to about  $30^{\circ}$  for sands and soft earth, exactly as in natural slopes, and to angles as low as  $18^{\circ}$  or even less for wet clays, although coarse gravel and broken rock can form slopes of  $40-45^{\circ}$ .

These angles apply to banks of moderate height, the limiting slopes of which are nearly linear. In banks of great height the slopes tend to be concave and stand at somewhat flatter inclinations than those just mentioned. An example is the rock-fill embankment in the valley of the Bois de la Haie, on the Paris-Nancy road; it is 142 ft high and with slopes averaging 1½:1 from crest to toe but having a concave profile with a versed sine of 6 ft 8 in.

Before making a cutting in hilly country, it is desirable to investigate the nature of the ground by probes (sondes) and auger holes (trous de tarière) or by pits; if beds of clay are found inclined towards the proposed cutting, the engineer should not hesitate to search for a safer route. For in such conditions slips can take place on quite small inclinations. Moreover, instability may result even without making a cut if water penetrates the slope and reduces the friction. Good drainage is therefore important, and Perronet briefly describes two case records, at Marly in 1758 and near Croix-Fontaine in 1756, where drainage measures proved effective in stabilizing slips on inclined clay beds. Four notes should be added.

 (i) The first edition of the 'Ouvres de M. Perronet' was published in folio: two volumes in 1782-83 and a supplementary volume in 1789. The second edition of 1788 has the full text in quarto with a folio atlas of plates.

- (ii) The original printed memoir on éboulements (in the Bibliothèque Nationale) is dated 5 July, 1769. It was reprinted in 1788, and also in the supplement of 1789, but without date. I am indebted to Professor Kerisel for sending me photocopies of the 1769 memoir and of a manuscript version (also dated) in the École des Ponts et Chausées library. The manuscript has eight accompanying explanatory drawings.
- (iii) It has been said on more than one occasion that Perronet, in this memoir, is the first to describe curved slip surfaces. Actually he is referring only to the curved (concave) surface of the slope itself. This is clear from the text and proved by one of the drawings already mentioned.
- (iv) As we learn from Perronet's memoir on piled foundations (printed in the 1782 and 1788 volumes) probes were made with a 2 in. dia. iron rod driven into the ground. At 1 ft intervals along its length the rod had cavities or pockets, inclined downwards, with a lip projecting on their lower edge; these retaining small samples of soil captured at the start of withdrawal.

#### Biography

Jean-Rodolphe Perronet (1708-94) was chief engineer of the Ponts et Chaussées and director of the Ecole for 47 years from its inception in 1747 until his death; he was a member of the Academy of Sciences and a Fellow of the Royal · Society and the most eminent civil engineer in France in the 18th century; he is especially renowned for his bridges, the best known of which are the Pont de Neuilly, Pont Sainte-Maxence and the Pont de la Concorde (Dictionary of scientific biography, 1975).

CLASSICAL SOIL MECHANICS 1776-1845 This section deals with Coulomb's method of limit equilibrium analysis in soil mechanics, its subsequent development and some earth pressure tests.

5. COULOMB, C.A. (1776). Essai sur une application des règles de maximis et minimis à quelques problèmes de statique, relatifs à l'architecture. Mém. Acad. R. Sci., vol. 7, pp. 343-382

Coulomb read his paper to the Academy on 10 March and 2 April, 1773. It was refereed, a year later, by Bossut and Borda and published in 1776. A contribution of fundamental importance in civil engineering science, the paper deals with the shear strength of masonry and soils, earth pressure, stability of arches and the strength of beams. The main points in soil mechanics are as follows.

(i) Coulomb introduces the idea that the shear resistance S which can be developed on an area a of masonry or soil, on which the normal force is N, is the sum of cohesion and a frict-ion component

$$S = ca + \frac{1}{n} N$$
 (1)

where c is the (non-directional) cohesion per unit area and 1/n is the coefficient of internal friction.

(1)

(ii) He also introduces the principle of searching for a critical slip surface, using equation (1), which gives (for example) the maximum thrust on a retaining wall or the minimum compression strength of a column. In general the slip surface on which shearing takes place can be curved, but in the problems analysed in the paper it is assumed for simplicity to be a plane.

(iii) He then shows that the compression strength Q of a short vertical column of cross-sectional area A is

$$Q = \frac{CA}{\cos\alpha(\sin\alpha - \frac{1}{p}\cos\alpha)}$$
(2)

where failure occurs by shearing on a plane inclined at  $\alpha$  to the horizontal and

ana = 
$$\frac{1}{\sqrt{(1 + 1/n^2)} - 1/n}$$
 (3)

(iv) For a purely cohesive material (with l/n = O),  $\alpha$  =  $45^\circ$  and

$$Q = 2cA \tag{4}$$

(v) Using principles (i) and (ii) Coulomb finds that in a condition of limiting equilibrium, the earth fill behind a vertical retaining wall of height H fails on a plane inclined to the horizontal at this same angle  $\alpha$ , and the total thrust P<sub>a</sub> on the wall is

$$P_{2} = mH^{2} - C\ell H$$
 (5)

where  $m = \gamma/\tan \alpha$  and  $\ell = 2/\tan \alpha$ .

(vi) Further, the unit pressure at depth z is

$$\mathbf{b}_{a} = 2m\mathbf{z} - c\mathbf{l} \tag{6}$$

(vii) By integrating the moment of the pressure p dz about the base, between the limits  $z = 0^{a}$  and z = H, the overturning moment on the wall is found to be

$$M = \frac{1}{3}mH^{3} - \frac{1}{2}c\ell H^{2}$$
(7)

(viii) The angle  $\alpha$  is not the natural slope of the earth fill, as all previous investigations had assumed, and it is independent of cohesion.

(ix) By putting  $P_a = 0$  in equation (5) the limiting height of an unsupported vertical face of soil is immediately recovered

$$H_{\rm C} = \frac{C}{m} = \frac{4C}{\gamma} \tan \alpha \tag{8}$$

(x) In the foregoing analysis the upper surface of the soil or earth fill is horizontal. To illustrate the results numerically, Coulomb takes the natural slope of a freshly-tipped earth fill as  $45^{\circ}$  (i.e. n = 1 if c = 0) and, following Belidor, increases the wall width by 25% as a factor of safety. He then arrives at the practical rule for stability against overturning that, for a wall having a front batter of 1/6:1, the top width should be H/7 and thus the mean (mid-height) width is just under 0.25H.

(xi) He realizes, without proceeding to analysis, the existence of passive pressure if the wall is pressed against the earth. It must be greater than  $\frac{1}{2}\gamma H^2$ , just as the active pressure must be less than this fluid pressure.

(xii) Finally, Coulomb tackles the effect of wall friction and derives expressions for active pressure in terms of slip plane angle and for

$$\int_{\frac{1}{1}}^{\frac{1}{2}} s = c + \sigma \tan \phi$$

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 $\delta =$ 

$$q = \frac{2 c \cos \phi}{1 - \sin \phi} = \frac{2 c}{\tan \varepsilon}$$
(2)

$$\alpha = 45^{\circ} + \phi/2 = 90^{\circ} - \varepsilon \tag{3}$$

If 
$$\phi = 0$$
  $q = 2 c$   $\alpha = 45^{\circ}$  (4)

$$P_{a} = \frac{1}{2} \gamma H^{2} \frac{1 - \sin \phi}{1 + \sin \phi} - 2cH \frac{\cos \phi}{1 + \sin \phi}$$

$$H = \frac{1}{2} \gamma H^{2} \tan^{2} \varepsilon - 2cH \tan \varepsilon$$

$$p_{a} = \gamma z \tan^{2} \varepsilon - 2 c \tan \varepsilon$$
(6)

$$M = \int_{0}^{H} p_{a}(H-z) dz$$
(7)

$$H_{c} = \frac{4 c}{\gamma} \frac{\cos \phi}{1 - \sin \phi} = \frac{4 c}{\gamma \tan \epsilon}$$
(8)



Fig. 3. Results in terms of  $\phi$  and  $\varepsilon = 45^{\circ} - \phi/2$  (Coulomb, 1776)

this angle itself (which is rather less than  $\boldsymbol{\alpha}$ with no wall friction). In his notation both expressions are cumbersome, but if we substitute  $\tan\phi$  = 1/n, and for simplicity take c = 0, equation (9) as given in Fig. 3 is obtained. In a numerical example, taking n = 1 and c = 0 and wall friction as equal to the internal friction, Coulomb finds that the horizontal component of earth pressure (P cos $\delta$ ) is 0.125 $\gamma H^2/2$ . This, he says, is too low for design, as the friction of soil on masonry is not so large as internal friction. Also water can percolate into the fill, reducing its internal friction and, even with drainage provision, exerting some hydrostatic pressure on the wall. So, in practice, he returns to the conclusion that walls should have a mean width about one quarter of their height.

Even today, Coulomb's paper is not easy reading. At the time of its publication the originality of his reasoning, the difficulties of notation and the then extraordinary conclusion that the slip plane was much steeper than the natural slope, hindered recognition of its fundamental importance.

The substitution of  $\tan \beta_n = 1/n$  was made by Reinhard Woltman (1753-1837), in his 'Beyträge zur hydraulischen Architectur', vol. 3 (Göttingen, 1794) and it is he who first gives the familiar expression (if c = 0)

$$P_{a} = \frac{1}{2}\gamma H^{2} \frac{1 - \sin\beta_{n}}{1 + \sin\beta_{n}}$$

Another step towards the acceptance of Coulomb's theory was the publication of 'Recherches sur la poussée des terres' (Paris, 1802) by G. C. M. Riche de Prony (1755-1830), head of

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Fig. 4

the École des Ponts et Chaussées and professor of mechanics at the École Polytechnique. He introduced the parameter  $\eta$  where tan $\eta$  equals Coulomb's n. Thus  $\eta = 90 - \phi$  and with a horizontal fill and zero wall friction the slip plane is inclined at  $\varepsilon = \eta/2$  to the back of the wall, i.e. the slip plane bisects the angle between the wall and the natural slope if c = 0. In Fig. 3 the various solutions obtained by

Coulomb are given in terms of  $\phi$  and  $\varepsilon$ .

#### Biography

Charles Augustin Coulomb (1736-1806) studied at the École du Génie at Mézières under Bossut, served as a military engineer from 1762 to 1781 at Brest, Martinque, Bouchain, Cherbourg, Besancon and Rochefort, and was then stationed in Paris. He retired in 1791. He was a member of the Academy of Sciences, and is famous for his researches on electricity and magnetism carried out in the late 1780s, In addition to the essay of 1776, he published other contributions to engineering (Dictionary of scientific biography, 1971).

 MAYNIEL, J.-H. (1808) Traité expérimentale, analytique et practique de la poussée de terres et des murs de revêtement. Paris: Colas

Some earth pressure tests were carried out in the 18th century, and are described by Mayniel who spared no effort in discovering printed and manuscript accounts of theoretical and experimental work in this subject. But the first really significant tests were those made by him in 1806 and 1807 using the apparatus shown in Fig. 4. The box, 3 m long, 1.5 m wide and 1.5 m high, has a bottom hinged door at one end.

Before making a test the wooden vessel M is filled with water and sufficient weight placed on it to resist the lateral pressure on the door caused by tipping sand or earth into the box. Water is then allowed to run out of M until the door yields and a slip surface develops in the fill. The movement of the door and the outcrop of the slip having been noted, the door is fixed and the strut K removed. By means of a rope, pulley and scale-pan the force required to slide M is measured and this is, of course, equal to the horizontal component of earth pressure on the door when yield occurs.

Tests were made on loosely tipped and compacted earth, on earth mixed with gravel, and on loose sand, level with the top of the box or with a surcharge slope, and the strut was placed at various heights above the hinge. From these latter tests Mayniel satisfied himself that the centre of pressure was located at the lower third point, at least for the loosely tipped materials. In the tests on earth fill the slip plane was inclined at about 62° to the horizontal and the active pressure developed after the top of the door had moved about 10 cm.

I analyse here only the tests on loose earth and sand, without a surcharge. For those materials the natural slope and unit weight, and the horizontal earth pressure (Pcosô) per metre width, are given in Table 1, together with values  ${}^{1}_{2}\gamma H^{2}$  and Pcosô/ ${}^{2}_{2}\gamma H^{2}$  = Kcosô.

The values of Kcos $\delta$  agree almost exactly with Coulomb's theory, as given by equation (9) in Fig. 3, if  $\beta_n = \phi = \delta$ . But in Mayniel's otherwise admirable test arrangement, the width of the box is too small (compared with its height) to avoid appreciable side friction. If, for example, the measured horizontal thrust is too low by 10%, due to this effect, the resulting corrected values of Kcos $\delta$  agree closely with Coulomb's theory with  $\delta = (2/3)\phi$ .

However, it must be emphasized that Mayniel himself only indicated how equation (9) could be obtained, without deriving a solution. Moreover, by a procedure which I am unable to understand, he deduces coefficients of internal friction considerably smaller than  $\tan\beta_n$ . This mistake unfortunately reduces the significance of his contribution, but the tests themselves remain a valuable record.

Table l.	Results	of	earth	pressure	tests	by
Mayniel						

		Earth	Sand
β <sub>n</sub>		45 <sup>0</sup>	370
Υ,	$t/m^3$	1.10	1.36
Pcosδ,	kg	155	288
<sup>1</sup> 2γH <sup>2</sup> ,	kg	1240	1530
Kcosδ		0.125	0.188

## SELECTED PAPERS ON SOIL MECHANICS

#### Biography

Jean-Henri Mayniel (1760-1809) started his career in the Ponts et Chaussées but transferred to the army engineers in 1792; he rose to the rank of Chef de Bataillon and died in Spain during the Peninsular War (information through M. Armand Mayer from the Archives du Génie).

7. FRANÇAIS, J.F. (1820) Recherches sur la poussée des terres, sur la forme et les dimensions des murs de revêtement, et sur les talus d'excavation. Mémorial de l'officier du Genie No. 4, pp. 157-206

Français begins his paper by extending Coulomb's analysis to obtain the active pressure on walls having an inclined back face, with zero wall friction and a horizontal fill, and arrives at a correct solution.

Next comes a discussion on the design of retaining walls, including factors of safety, but the most interesting part of the paper concerns the stability of clay slopes. Français considers a cutting of depth H with a slope inclined at  $\beta$  to the horizontal, and shows that for limiting equilibrium on a plane slip surface

 $H = H_{c1 - \cos(\beta - \phi)}^{\sin\beta(1 - \sin\phi)}$ (9) where H is Coulomb's expression for critical vertical height

 $H_{c} = \frac{4c}{\gamma} \frac{\cos\phi}{1 - \sin\phi}$ 

Thus equation (1) can be written in the form

$$H = \frac{4c}{\gamma} \frac{\sin\beta\cos\phi}{1 - \cos(\beta - \phi)}$$
(10)

and this is the first analytical solution relating to the stability of clay slopes, other than for  $\beta = 90^{0}$ .

A further point of interest is that Français recognises three basic soil properties: the unit weight  $\gamma$ , the natural slope  $\phi$  (he actually uses  $\eta = 90 - \phi$ ) and the critical height H<sub>c</sub>. The natural slope is measured by tipping the soil in a loose state, when c = 0. To determine critical height he suggests making vertical cuts of various depths, leaving them exposed to all climatic conditions for several months, and taking for H<sub>c</sub> the greatest depth to withstand the test. Careful determination of these three parameters for each type of ground should be made, Francais says, and the results would be most useful.

This approach was adopted by Navier (1785-1836) in his famous 'Résume des leçons données a l'École des Ponts et Chaussées sur l'application de la mécanique à l'establi**s**sement des constructions' (Paris, 1833) and he gives some values of c based on critical heights for clays ranging from 1 m to 4 m. However, little significant development followed along these lines.

#### **Biography**

Jacques-Fréderic Français (1775-1833) studied at the École Polytechnique, served as an army officer from 1801 and in 1811 became professor of fortifications and surveying at the military college at Metz (information through M. Armand Mayer from the Archives du Génie).

8. GARIDEL (1840). Note sur la poussée et la butée des terres. In Mémoire sur la stabilité des revêtements et de leurs fondations, by J.-V. Poncelet. Paris: Bachelier. (No. 9) Appendix.

In 1836 Poncelet discussed his research on earth pressure theory with M. le capitaine du genie de Garidel' who, it transpired, had already arrived at analogous results by an analysis similar to that of Prançais. The note which Garidel wrote at this time appealed so strongly to Poncelet that he included a verbatim transcript in his treatise.

The main point is that Garidel gives an expression for passive pressure (butée). Like Français, he deals with an inclined wall, zero wall friction and a horizontal fill, and still uses n rather than  $\phi$ . His expression for active pressure is equivalent to that previously obtained by Français, but in a more elegant form. For a vertical wall the passive pressure is

$$P_{p} = \frac{1}{2}\gamma H^{2} \frac{1 + \sin\phi}{1 - \sin\phi} + 2cH \frac{\cos\phi}{1 - \sin\phi}$$

Biography

I am informed by Mlle L. Lacrocq that in the Archives du Génie the only officer at the relevant period to whom this work can be attributed is Bruno Charles François Garidel-Thoron (1807- ). He studied at the École Polytechnique and at Metz, and held the rank of Capitaine du Génie until his resignation in 1843.

# 9. PONCELET, J.-V. (1840). Mémoire sur la stabilité des revêtements et de leurs fondations. Paris: Bachelier

This classic treatise on the design of retaining walls and their foundations, based on Coulomb's wedge theory and the c = 0 hypothesis, contains much new material. Only a summary of some of the principal contributions to soil mechanics can be given here.

Poncelet derives solutions for active and passive pressure in the general case with wall friction, sloping back fill and inclined walls. He also presents a graphical method for determining the critical slip plane and earth pressure. This became very well known and can still be found in modern textbooks, e.g. Taylor (1948). The original analytical expressions were rewritten by Müller-Breslau (1906) and in this form are given as equations (1) and (2) in Fig. 5 for a vertical wall. Equation (9) in Fig. 3 corresponds to the special case for  $\beta = 0$ .



 $\delta$  and  $\beta$  shown + ve

 $P_{p}^{a} \cos \delta = \frac{\gamma H^{2}}{2} \frac{\cos^{2} \phi}{\left\{1 \pm \sqrt{\left[\frac{\sin(\phi + \delta)\sin(\phi \mp \beta)}{\cos \delta\cos \beta}\right]}\right\}^{2}} (1)$ 

after Poncelet (1840)

Fig. 5



Fig. 6. Failure of a retaining wall at Soissons, 1837 (Poncelet, 1840)

Poncelet emphasizes the practical importance of foundation stability against sliding and large vertical displacements, and tackles the theoretical basis of these problems for the first time. He preceeds his analysis by quoting from a report by Vauban, written in 1699, where this great engineer mentions the failure at Ypres of walls, 20-25 ft in height, by sliding forward on a slip surface in a soapy clay which, after rains, could scarcely maintain itself on a 2:1 slope. Similar failures occurred at Bergues in 1778, and Poncelet illustrates the sliding of a wall at Soissons, also on clay, in 1837 (Fig. 6).

It is therefore absolutely necessary either to take the wall sufficiently deep to develop adequate passive pressure on the front face of the foundation, and the calculations in poor ground should be based, for instance, on  $\phi =$ 26° (cot 26 = 2.0), or if a piled foundation has to be used the piles must be raked at an angle of, say, 10° to the vertical.

The horizontal component of the resultant force at foundation level having been neutralized by passive pressure in the depth D, it is additionally necessary to guard against shearing failure in the ground beneath the foundation caused by the vertical component Q of the resultant (Fig. 7). Such failures may be represented by active and passive pressure wedges with a vertical interface extending to a depth z below the front edge.

The passive pressure which can be developed on this interface (assuming zero wall friction) is

$$P_{p} = \frac{\gamma}{2\tan^{2}\varepsilon} \left[ (D + z)^{2} - D^{2} \right] = \frac{\gamma}{\tan^{2}\varepsilon} \left[ \frac{z^{2}}{2} + Dz \right]$$

where as usual  $\varepsilon = (45 - \phi/2)$ . Similarly, if q is the vertical foundation pressure (assumed to be uniform) on the active wedge, then

$$P_{a} = \gamma \tan^{2} \varepsilon \left[ \frac{z^{2}}{2} + \frac{qz}{\gamma} \right]$$

but for limiting equilibrium  $P = P_p$  and hence the foundation pressure causing failure is

$$q_{f} = \frac{\gamma D}{\tan^{4} \varepsilon} + \frac{\gamma z}{2} \left[ \frac{1}{\tan^{4} \varepsilon} - 1 \right]$$
(11)

With a properly designed foundation on comp-



Poncelet, 1840

Fig. 7

ressible soil, the vertical force Q will be at midpoint of the base width B, in order to avoid tilting. In this case  $z = B/2tan\epsilon$ . More generally, if Q acts at a distance b from the front edge then  $z = b/tan\epsilon$  and

$$q_{f} = \frac{\gamma D}{\tan^{4} \varepsilon} + \frac{\gamma b}{2} \frac{1}{\tan \varepsilon} \left[ \frac{1}{\tan^{4} \varepsilon} - 1 \right]$$
(12)

where for a symmetrically loaded foundation b = B/2.

Reverting to equation (11) it will be seen that a maximum value of D is obtained if z = 0, when

$$D = \frac{q}{2} \tan^{4} \varepsilon$$
 (13)

This implies failure of an element of soil beneath the front edge and, of course, the same result can immediately be obtained by considering the equilibrium of stresses on such an element. However, equation (13) provides a prudent design method for determining D. In the same way it is prudent to check that B, or more generally b, satisfies equation (12) when D = 0. If both these design rules are followed the settlement of the foundation will simply be that due to the compressibility of the soil, all danger of a shear failure having been eliminated.

In modern terminology, we see that Poncelet in equation (12), for a centrally loaded foundation, is stating that

$$N_{q} = \frac{1}{\tan^{4} \varepsilon}$$

$$N_{\gamma} = \frac{1}{2 \tan \varepsilon} \left[ \frac{1}{\tan^{4} \varepsilon} - 1 \right]$$

and numerically, for  $25^{\circ} < \phi < 35^{\circ}$ , his results lie slightly above the values of these coefficients given by Terzaghi (1943) for local shear failure.

Poncelet makes the interesting point, supported by field evidence, that counterforts reduce earth pressure by arching action.

Finally, he uses the symbol  $\phi$  for the angle of internal friction of the soil, perhaps for the first time in French literature; his

justification for taking c = 0 in clays is that cohesion ceases to exist after the instant preceding failure, and thus when failure is actually taking place the shear strength is  $\sigma \tan \phi$ .

#### Biography

Jean-Victor Poncelet (1788-1867), after serving as an army engineer, became professor of mechanics at Metz in 1824, and at the Sorbonne from 1837. He was a member of the Academy of Sciences and made fundamental contributions in pure geometry, applied mechanics and hydraulics (Dictionary of scientific biography, 1975).

10. HOPE, C.W. (1845). Experiments carried on at Chatham by the late Lieutenant Hope, Royal Engineers, on the pressure of earth against revêtements, and the best form of retaining walls. Papers on subjects connected with the duties of the Corps of Royal Engineers, vol. 7, pp. 69-86.

In 1842-43, as a young officer at the Royal Engineers' establishment, Lieut. Hope carried out experiments on earth pressure and on the comparative strength of large-scale model retaining walls. He died before writing up the results but as a tribute these papers, based on his notes and records of the tests, were published.

The experiments were made on dry sand, having a natural slope of  $35^{\circ}$  and a unit weight of 91 lb/ft<sup>3</sup> in the loose state, placed in a wooden box 2 ft long and 1 ft square in section. Like Mayniel, whose book he had read, Hope decreased the horizontal force applied to the wall, forming one end of the box, until yield occurred, but with a greatly improved experimental technique he measured the horizontal and vertical components of earth pressure simultaneously. Thus he actually determined the wall friction tan $\delta$ . The mean results of two tests with a smooth wall and seven tests with a roughened wall are given in Table 2, where Kcos $\delta$  = Pcos $\delta/A_2\gamma H^2$ .

The measured values of Kcos $\delta$  are approximately 10% lower than those calculated from equation (9) in Fig. 3 - a difference which may be attributed partly to the effects of side friction in the box and partly to the possibility that the angle of internal friction  $\phi$  is slightly greater than the natural slope  $\beta_n$ . Additional experiments were carried out with a

Additional experiments were carried out with a surcharge sloping at  $\beta_n$ , but such tests are illconditioned as the theoretical values of Kcos $\delta$  vary very rapidly as  $\beta \rightarrow \phi$ .

Hope also made experiments with layers of

Table	2.	Results	of	earth	pressure	tests	by
Lieut.	HO	pe					-

		Smooth wall	Rough wall
Pcosδ,	lb	10.0	9.4
δ		c.8 <sup>0</sup>	27 <sup>0</sup>
<b>Ϟ</b> γΗ <sup>2</sup> ,	lb	45.5	45.5
Kcosð		0.220	0.207

different coloured sand, and a glass side to the box, which clearly demonstrated the line of rupture when the wall was allowed to move forward; it was sometimes quite straight but more frequently a little concave. In these experiments, with various inclinations of the wall, the slip surface was found to be rather steeper than Coulomb's theory would indicate.

In his model tests, Hope built a wall having a rectangular section, another with counterforts and a third also with counterforts but battered back and front 1/5:1, in each case using the same number of bricks per course. The walls were raised in height, earth filling being placed behind as each course was added, until failure occurred. The vertical rectangular wall 1 ft 11 in. wide failed when 10 ft high, the vertical counterforted wall reached nearly 13 ft, and the battered wall collapsed after reaching a height of 21 ft. This last result fully confirms the economical nature of the battered walls introduced by Jessop and Rennie in their great dock works in London more than 40 years earlier.

#### Biography

Charles William Hope (1824-44), youngest son of the distinguished soldier General Sir John Hope, was commissioned in the Royal Engineers in June 1842 and died at his mother's home in Edinburgh in March 1844 aged 19 (Army lists; Gentleman's Mag., 1844, New series, vol. 21, p. 557).

#### FIELD WORK, 19TH CENTURY

In the 19th century great advances were made in practical soil mechanics and geotechnical engineering. Of the many aspects which could be considered, I shall concentrate on only three topics: (i) the stability of clay slopes, including observations of slip surfaces and remedial measures involving drainage, (ii) the realization that long-term settlements of foundations and embankments on clay are due to consolidation, and (iii) the design of weirs on permeable foundations. Here I shall simply mention grouting of alluvium and rock, the use of compressed air in tunnelling and caisson foundations, groundwater lowering, the evolution of pile-driving formulae, measurements of side friction on piles and cylinder foundations, and improvements in earth dam design.

That advances were made in such topics is clear from the civil engineering works themselves, railways, dams, tunnels, docks and bridges all being built on an ever-increasing scale and, on the whole, with reasonable success. Of course, there were failures, as well as triumphs, in some cases due to bad engineering judgement but in others due to a still very inadequate knowledge of soil properties. This now seems obvious, but the problems required the genius of Terzaghi for their solution and, more than anything else, it was his research on soil properties combined with an insistence on relating theory to practice which led to the creation of modern soil mechanics.

11a. JESSOP, W. and TELFORD, T. (1810). Report, 18 October, 1809. In Seventh report of the Commissioners for ... the Caledonian Canal, Parliamentary Papers, pp. 22-24



Fig. 8. Sketch section of Clachnaharry sea lock 1808-12, Caledonian Canal

11b. Telford, T. (1821). Inland navigation. Edinburgh Encyclopaedia, vol. 15, pp. 209-315

Clachnaharry Sea Lock, forming the eastern entrance to the Caledonian Canal, near Inverness, was built in 1808-12 under the direction of Telford with Jessop as consulting engineer. The lock (180 ft in length between gates, 40 ft in internal width, and with 25 ft of water on the outer sill at spring tides) is located 400 yd out from high water mark. Probings showed 55 ft of soft silty clay beneath foundation level (Fig. 8). To avoid having to build a very large cofferdam in difficult conditions, and also to improve the soft clay, the site was preloaded by a fill of boulder clay and quarry waste within which, after six months, excavation for the lock took place.

In their first report on this subject (dated 14 November, 1808, in the sixth Commissioners' report, printed 1809) Jessop and Telford say the fill would serve 'The purpose of so compressing or squeezing out the mud as to make a firm foundation'. Here they are making an analogy with squeezing out a sponge, as becomes clear when we read in their next report, of 18 October, 1809, that they had now decided to raise the fill temporarily above its final design height 'in order, by this additional weight, the more speedily and effectually to consolidate the mud'. In his 1821 article Telford writes that this loading was 'done for the purpose of squeezing out the water, and consolidating the mud, by laying on a greater weight than the masonry of the lock'.

Thus by 1809 Jessop and Telford understood in broad terms the process of clay consolidation and the principle of preloading, and were using these concepts on a major engineering work. Telford's 1821 statement, although brief, is absolutely explicit. The progress of this lock, which was satisfactorily completed, may be summarized as follows: fill placed November 1808 to December 1809, settlement practically ceased by May 1810, excavation June 1810 to June 1811, and masonry construction June 1811 to August 1812.

#### **Biographies**

William Jessop (1745-1814) was trained by the great 18th century engineer John Smeaton; he was responsible for the design and construction of many important works including Rochdale Canal, Grand Junction Canal, West India Docks in London and Bristol Floating Harbour (Hadfield, C. and Skempton, A.W. (1979). William Jessop, Engineer. Newton Abbot: David & Charles). Thomas Telford (1757-1834), FRS, was the first President of the Institution of Civil Engineers. During a long and varied career of the utmost distinction, he excelled as an engineer of bridges, roads, canals and harbours (Gibb, Sir Alexander (1935). The Story of Telford. London: Maclehose).

12. PARNELL, H.B. (1833). A treatise on roads; wherein the principles on which roads should be made are explained and illustrated, by the plans, specifications, and contracts made use of by Thomas Telford, Esq. on the Holyhead Road. London: Longman Through long experience as a Commissioner of the London-Holyhead road, Parnell gained much knowledge of road construction. Of particular interest are the figures for the safe slopes of cuttings in various strata, as set out in Table 3.

#### Biography

Sir Henry Brooke Parnell (1776-1842), MP, political economist and honorary member of the Institution of Civil Engineers, was primarily responsible for establishing the Holyhead Road Commission (Dictionary of national biography, 1895; Rickman, J. (ed.) (1838). Life of Thomas Telford. London: printed by Hansard).

13. GREGORY, C.H. (1844). 'On railway cuttings and embankments; with an account of some slips in the London Clay, on the line of the London and Croydon Railway'. Min. Proc. Instn. Civ. Engrs, vol. 3, pp. 135-173.

## Table 3. Safe slopes for cuttings (Sir Henry Parnell, 1833)

London Clay	3:1	18½ <sup>0</sup>
Oxford Clay	2:1	26 <sup>1</sup> 2 <sup>0</sup>
Oxford Clay in deep cuttings	3:1	18½ <sup>0</sup>
Chalk	1:1	45 <sup>0</sup>
Limestone or sandstone	1 <sub>4</sub> :1	76 <sup>0</sup>
Limestone or sandstone interbedded with clay: horizontal bedding	1½:1	33 <sup>1</sup> 2 <sup>0</sup>
If bedding is inclined, slopes may have to be as flat as	4:1	14 <sup>0</sup>



#### Fig. 9. New Cross cutting (after Gregory, 1844)

This is the earliest record of a first-time slide in London Clay. The slide occurred without warning on 2 November, 1841, near New Cross Station, three years after excavation of the cutting. In four hours about 50,000  $yd^3$  of clay moved on a 'glass like' slip surface passing along the junction of the brown and blue clay (Fig. 9). Work to clear the tracks was still progressing on 26 November when a similar slip took place on the opposite side.

The slopes were made originally at the extraordinarily steep inclination of  $l_2$ :1, despite the nature of the ground and great depth (75 ft maximum) of the cut. Remedial measures involved reducing the slopes to 2:1 with wide berms, resulting in an average inclination around  $2l_2$ :1. But other slips on the railway, of which details are not given, were treated by the use of gravel-filled trenches (or counterforts) dug down below the slip surface, at right angles to the line, and linked by a gravel footing at the toe.

From the lengthy and interesting discussion on the paper, it emerged that Robert Stephenson had used counterforts for stabilizing slips on the London and Birmingham Railway in 1839, notably in the Lias in the Blisworth cutting near Northampton. There was general agreement that counterforts acted as deep drains and provided some increase in strength as gravel or stone-fill buttresses founded below the slip surface.

In this discussion, as in the paper itself,

reference is made to 'natural joints and fissures' in the London Clay (Gregory) and that it 'abounded with fissumes in all directions' (De la Bèche).

#### Biographies

Charles (later Sir Charles) Hutton Gregory (1817-98), trained under Robert Stephenson and James Walker; he was resident engineer for William Cubitt on the London and Croydon Railway, he succeeded Brunel in 1846 as engineer of the Bristol and Exeter Railway, and went on to become an international authority on railway engineering. He was also President of the Institution of Civil Engineers (Obituary. Min. Proc. Instn. Civ. Engrs, 1898, vol. 132, pp. 377-382).

Sir Henry Thomas De la Bèche (1796-1855), FRS, was the first Director of the Geological Survey from 1835 (Dictionary of scientific biography, 1971).

## 14. COLTHURST, J. (1844). Discussion On railway cuttings and embankments, by C.H. Gregory (no. 13). Min. Proc. Inst Civ. Engrs, vol. 3, pp. 163-168.

Colthurst gives the first printed description of a foundation failure of an embankment. The gravel embankment, on the Great Western Railway near Hanwell, was 54 ft high with slopes of  $1\frac{1}{5}$ :1 on 4 ft of alluvial clay and 3-10 ft of gravel, overlying London Clay (Fig. 10). The slip, which occurred in May 1837, caused a subsidence of 30 ft accompanied by an upheaval of 10 ft over a width of about 80 ft, along a length of 400 ft. Minor disturbances extended up to 220 ft from the embankment.

Immediately after the slip, I.K. Brunel, chief engineer of the GWR, directed a loading berm or 'terrace' to be built, and trenches were also dug at right angles to the line up to the foot of the embankment. These enabled sections of the disturbed ground to be studied, and the trenches were probably back-filled with gravel to act as drains. Sections beneath the embankment were deduced from the observed dip and subsidence at the top of the slip. The berm proved to be an effective stabilizing expedient.



Fig. 10. Brent embankment at Hanwell, Great Western Railway



Fig. 11. Slip in foundation trench, Grosbois Dam (Collin, 1846)

#### Biographies

Joseph Colthurst (1812-82) was resident engineer on the Hanwell-Iver section of the GWR and subseqently on the line near Didcot; later he was engaged on several railway works in England and abroad (Obituary. Min. Proc. Instn Civ. Engrs, 1883, vol. 73, pp. 356-358).

Isambard Kingdom Brunel (1806-59) was the most famous of Victorian railway engineers. He was a Fellow of the Royal Society and Vice President of the Institution of Civil Engineers (Rolt, L.T.C. (1957), Isambard Kingdom Brunel. London: Longmans).

15. COLLIN, A. (1846). Recherches expérimentales sur les glissements spontanés des terrains argilleux. Paris: Carilian-Goeury and Dalmont

Collin makes an outstanding contribution to knowledge on the stability of clay slopes, principally in terms of first-time slides in cuttings, embankments and earth dams. Briefly the main points are as follows.

(i) Failures typically occur on a curved slip surface (suface de glissement), approximately cycloidal in section, which is soapy, polished and striated, and extends down to about the level of the foot of the slope. A slip is often preceded by cracks opening at the crest. Sometimes a second or third movement follows quickly after the first, giving rise to multiple (retrogessive) slips.

(ii) Deep slips, as described above, occur when the cohesion is just exceeded by the gravitational force. They can take place during or immediately after construction, or they may be delayed for several years while cohesion is being reduced by infiltration of water.

(iii) Post-slip movements continue until a new position of equilibrium is attained, and the final slope takes an S-shaped profile. These movements are resisted by friction, cohesion having been destroyed. Erictional resistance can be reduced by rain penetrating the disturbed mass of clay.

(iv) In general a slip surface is the result, not the cause, of failure. The possibility of pre-existing slip surfaces is recognized, but Collin considers them to be highly exceptional and rejects Girard's explanation (1831) in these terms of slips in the Bois de Saint-Dennis. (v) There are also shallow, superficial slides in which cohesion is destroyed or greatly reduced by repeated rain and drought, frost and thaw.

(vi) Collin describes in detail, with measured sections, ten deep slips. He also refers, with sketch sections, to a considerable number of other slips. The slip surface in the 1833 failure of the foundation trench of Grosbois Dam (Fig. 11) is probably the first ever to be accurately measured; the earliest to be published (by Minard, 1841) was Collin's survey of the third (1836) slip in Cercey Dam.

(vii) Much attention is paid to remedial measures, including drainage and recompaction, but stone filled counterforts are specially emphasized. Spaced at 9 m or 10 m centres and 2 m or 2½ m wide, these are taken below the slip surface and act primarily as internal buttresses. They were used to repair the 1835 slip in Cercey Dam (apparently an early example in France) and subsequently on most of the slips encountered in the Canal de Bourgogne works. For superficial slides, piling, fascine mattresses and shallow drains can be effective, and planting of trees.

(viii) Tests to measure cohesion were made in the shear box shown in Fig. 12, using samples with a cross-sectional area of  $16 \text{ cm}^2$  or  $1 \text{ cm}^2$ for very hard clays. Several different clays, remoulded at two degrees of consistency, and also allowed to dry in the air, were taken to failure in less than 30 s. The results may be summarized as shown in Table 4. These show the immense influence of water content on shear

Table 4. Shear strength tests by Collin

Soft clay, such as results from the infiltration of spring or rain water	1.8-2.2t/m <sup>2</sup>
Clay at the consistency of material used in embankments or dams	3.2-5.9t/m <sup>2</sup>
Hard clay of the consistency found in compact virgin clay of the Secondary or Tertiary eras	40-60t/m <sup>2</sup>

#### SELECTED PAPERS ON SOIL MECHANICS



## Fig. 12

strength. Collin realizes the tests were too quick in practice, and attempts to measure the 'permanent' strength by finding the maximum shear stress the samples could withstand without apparent deformation. This seems to be roughly 25% of the 'instantaneous' strength.

(ix) Friction tests were made by sliding one block of clay on another. Coefficients of friction around 0.6-1.0 emerged, but on wetting the surfaces Collin found coefficients of about 0.2 ( $\phi = 12^{\circ}$ ).

(x) In view of the effects of water and of time it is difficult to assign a limit to the cohesion of a clay for permanent stability. But if such a value could be given, a simple analysis will show whether a slope is safe or what force is required from counterforts to make it so. This force Q, expressed in tonnes per metre width, can be derived approximately from the equation

$$Q = W \sin \alpha - cL \tag{14}$$

where W is the weight of the mass above a potential cycloidal slip surface,  $\alpha$  is the inclination of the surface at a point vertically below the centre of gravity of the mass, L is the length of the slip surface and Q acts at the angle  $\alpha$ . For a condition of limiting equilibrium Q = 0 and

 $Wsin\alpha = cL$  (15)

In a numerical example for a slope of 1.5:1 and 10 m high, with  $\gamma = 1.725 \text{ t/m}^3$ , the value of c is 1.7 t/m<sup>2</sup> (which may be compared with c = 2.4 by Taylor's circular arc analysis). Similarly, after a slip has occured

 $Q = W \sin \alpha - W (\cos \alpha) f$ (16)

where f is the coefficient of friction. Equation (15) may be regarded as a first step towards the  $\phi = 0$  analysis of slope stability, and Collin's shear box tests are the first attempts to measure the undrained strength of clays. However, Collin himself attached little importance to the analysis and regarded the tests as illustrating the effects of water rather than determining shear strength in practice. Moreover, he never refers to the possibility of testing undisturbed samples, or

that of determining shear strength by the backanalysis of actual slips.

In fact, no further advances along these lines were made for many years, but the practical importance of Collin's field work and his general reasoning was immediately realized by French engineers.

#### Biography

Alexandre Collin (1808-90) graduated from the Ecole Polytechnique, entered the Ponts et Chaussées, and was appointed to a post on the works of the Canal de Bourgogne in 1833. He started writing his treatise on clay slips in 1836. From 1855 he lived at Orleans, as Ingenieur en Chef de la Loire. He published papers on grouting, irrigation, hydrology, etc., and retired in 1873 (Skempton, A.W. (1946). Alexandre Collin, 1808-1890, pioneer in soil mechanics. Trans. Newcomen Soc. vol. 25, pp. 91-103).

16. COMOY, G.-E. (1875). Notice sur divers travaux de consolidation de terrains éboulés. Annls Ponts Chauss., 5th series, vol. 10, pp.8-51

In his retirement Comoy wrote this account of remedial works, mostly carried out in the period 1856-66, of several slips *(éboulements)* in clay slopes. The works include counterforts in a railway embankment and a cutting, and in the downstream slope of an earth dam, and a retaining wall at the toe of a cutting, but the most interesting cases are those primarily involving drainage alone.

In the winter of 1856-57 exceptional rainfall reactivated old landslides in the  $9-10^{\circ}$  slopes of the River Allier valley near Vichy, in middle Tertiary clays; the movements extended 150-200 m up slope, typically on slip surfaces 3-7 m in depth. In five of the slips a single trench was dug up the middle of the sliding mass. Each trench had a bottom width of about 1 m and was filled with gravel or broken stone to a thickness of 2 m, the remainder being back-filled with earth. In a sixth slip, of exceptional width, four such transverse drains were made at 50 m spacing and connected to a longitudinal drain near the foot of the slope. The trenches reached down below the slip surface and where, in one instance, the depth exceeded 6 m over a length of 60 m, an adit was constructed. These works were carried out during 1857-62.

The same rains reactivated an old (1825) slip in a hill slope above the Canal de Roanne à Digoin near Avrilly in the Loire valley. Here four trenches, running up slope about 40 m apart, were cut down to the slip surface (in this case with a maximum depth of 7 m), the bottom part of each being filled with granular material to a depth of 2 m. A longitudinal drain was again provided, 3 m deep and 2 m wide, but the four transverse trenches were left with open slopes above the granular filling. After the completion of the work in 1857 no further

movements occurred up to the time when Comoy wrote his paper in November 1874.

The third case is a slip on the Bayonne-Irun railway near Biarritz involving an embankment built on sloping ground. The slip surface, extending as far as 13 m below ground, could not be reached by trenches, but nevertheless the engineers thought that sufficient stability would be provided by two trench drains 30 m apart cut to a maximum depth of 10 m and backfilled with about 3 m of granular material followed by compacted earth. The works, carried out in 1863-64, proved to be entirely satisfactory.

Comroy attributes the success of the remedial measures at all three sites partly to a subdivision of the sliding mass and partly, or chiefly, to the provision of adequate drainage of the 'internal waters' by which means the cohesion is restored. He speaks warmly of the contributions 'par notre camarade M. Collin' to the better understanding of clay slips.

## Biography

Guillaume-Emmanuel Comoy (1803-85) studied at the École Polytechnique and the École des Ponts et Chaussées; he worked from 1828 to 1856 on the Canal du Centre and the lateral canal of the Loire, and from 1856 to 1861 he was director of flood control works in the Loire district. He wrote more than a dozen papers (Annls Ponts et Chauss., 6th series, (1885) vol. 10, pp. 441-157).

## 17a. SOOY SMITH, W. (1892). Chigago buildings and foundations. Engng News, vol. 28, pp 343-345

Civ. Engrs, vol. 128, pp. 1-27 In downtown Chicago soft to medium soft clay, about 40 ft thick, lies over a stronger clay and has a thin drying crust under 12-14 ft of sand and fill. By 1890, when Shankland was designing the 20 storey Masonic Temple (later known as the Capitol Building), the Chicago engineers had learnt to keep the pressures beneath the spread footings of their tall steel-frame buildings to about  $l_2^1$  ton/ft<sup>2</sup> and to expect settlements amounting, over a period of several years, to at least 6 in. Moreover, although the view was perhaps not yet generally held, Sooy Smith knew that 'the slow progressive settlements result from the squeezing out of the water from the earth'; words, in his 1892 paper, which echo Telford's statement of 1821 (No. 11). Shankland, in 1897, says with equal clarity that 'settlement is due to the compression of the clay, the water being pressed out of it'.

What proved to be the first long-term settlement record in Chicago, and probably the first to be made anywhere, was started by Sooy Smith in 1887 during construction of the Auditorium, and continued for 50 years (Peck, 1948). Another long record began in 1890 at an early stage of building the Monadnock North Block, in this case by Shankland; his record of the Masonic Temple settlements (Fig. 13) beginning in May 1891, shortly after completion of the foundations, was the first to be published, in 1897, with observations over 4½ years. A complete set of levels taken in 1913 enabled a contour plan to be drawn showing the settlement pattern after 22 years. By that time the average settlement was



around 10 in., compared with the 8 in. that Shankland had allowed for in design (Peck, 1948).

#### Biographies

William Sooy Smith (1830-1916), apart from serving in the Civil War, and attaining the rank of brigadier-general, worked from 1854 on railway and bridge construction and from 1887 as the foremost consultant on Chicago building foundations (Dictionary of American biography, 1937; Peck, 1948).

Edward C. Shankland (1854-1924), after graduating from Rensselear Polytechnic, worked on the Missouri river improvements and from 1883 on bridge design. In 1889 he became an engineer, and from 1894 to 1900 partner, in the firm of Burnham & Root in Chicago; later he set up his own consulting practice. He was the structural designer of many of the early steel frame Chicago buildings (Information from records of the Institution of Civil Engineers; Randall, F.A. (1949). The development of Chicago building construction. University of Illinois Press).

#### 18. WEIRS ON PERMEABLE FOUNDATIONS

In the design of weirs and barrages on relatively deep beds of sand, where a complete cut off cannot be achieved, it is essential to guard against failure by piping and excessive uplift pressures. The hydraulic gradient theory, evolved in the years around 1900, provided the first quantitative solution to this problem. Referring to Fig. 14, the theory assumes (i) that the loss of head h' at any point on the base of the foundation is proportional to the length of percolation path to the point in guestion, and (ii) that for safety the percolation factor C, which is the ratio of the total percolation path to the total head H, should not be less than certain values (as given in Fig. 14) depending on the type of soil.

These design values of C were published by Bligh (1910) in his paper 'Dams, barrages and weirs on porous foundations' (Engng News, vol. 64, pp. 708-710), in which he codified existing practice as developed in India and Egypt during the previous twelve years.

The idea of preventing piping by means of partial cut offs or 'curtain walls' at the upstream and downstream ends of the floor of hydraulic structures on sand, goes back in India to the 1830s in works by Sir Proby Cautley on the Eastern Jumna Canal and by Sir Arthur Cotton on the Cauvery Delta System. A further



Fig. 14. Hydraulic gradient theory of weir design

safety measure was introduced in 1874 when Colonel James Western constructed the first impervious upstream clay blanket in remedial works at Jaoli Falls on the Ganges Canal. Upstream blankets were used at several other sites during the next twenty years and became standard practice from about 1898. Up to that time, however, designs appear to have been entirely empirical.

The hydraulic gradient theory emerged from tests made by John Clibborn at Roorkee in 1896-97, following the piping failure of Khanki Weir on the Chenab river. His experiments showed (i) that underseepage tends to hug an impermeable boundary, as indicated by the arrows in Fig. 14, (ii) that piping could start in fine sand at horizontal hydraulic gradients of about 0.1 (i.e. with values of C less than about 10) if there were no downstream curtain wall, and (iii) that a curtain wall of moderate depth causes only a small increase in the length of percolation path but introduces a significant extra margin of safety, since it forces the water to percolate upwards at exit: clearly a more stable condition than if the flow is essentially horizontal, as it would be without a curtain. Finally, Clibborn presented diagrams with hydraulic gradient lines (he called them pressure slopes) similar to Fig. 14. Moreover, in 1897 or 1898, J. S. Beresford made experiments which demonstrated for the first time the effect of an inverted filter in providing an additional safequard against piling.

Clibborn's report, written in 1897, was published under the title 'Experiments made on the passage of water through sand of the Chenab River from the Khanki Weir site', in his 'Roorkee treatise on civil engineering: irrigation works in India' (Roorkee: Thomason College, 1901). It also appeared, together with a brief note on Beresford's experiments, in 1902 as Government of India, Technical Paper No. 97. This Technical Paper included a report by Beresford on the first pressure-pipe observations, made in 1898, on the floor of the Narora Weir on the Ganges. He used two pipes as represented in Fig. 14.

By an extraordinary chance, the floor of this weir, in a neighbouring bay, blew up only a few days later. Like Khanki Weir, it was founded on fine sand; the failure at both sites occurred with C values around 9. Both weirs were rebuilt (Khanki was completed in 1898 and Narora in 1900) with an upstream clay blanket incorporating a curtain wall which increased C to about 15 or 16. Also in 1900 Jamroa Weir in Sind, on fine sand as well, was built with C = 15 and proved to be satisfactory.

However, Coleroon Weir in the Cauvery Delta System, which had failed with C = 8 in 1837 (a year after construction), was rebuilt with C =12 and remained stable, but it was founded on coarse sand. In contrast, the Delta Barrage on fine silty sand of the Nile became secure only after extensive remodelling had increased C to a figure of about 20. This work, initiated by Colonel Western, reached completion in the early 1890s.

In 1898, combining experience from India and Egypt, W. J. Wilson produced a superb design for the foundation of Asyut Barrage on the Nile.

He (i) ensured a C value of 20 under normal operating heads, and 16 under the maximum possible head, (ii) introduced interlocking iron sheet piles for the partial cut offs, (iii) placed a two-layer inverted filter beneath the pitching immediately downstream of the downstream row of sheet piles, and (iv) increased the floor thickness, on Beresford's advice, from 2 m to 3 m.

Asyut Barrage, built 1898-1902, may be regarded as opening a new epoch in the rational design of hydraulic structures on permeable foundations. It was followed by Zifta Barrage in the Nile delta, built in 1902-03, and Rasul Weir on the Jhelum river in the Punjab, built in 1899-1901. All were on fine or silty sand, had inverted filters, downstream curtains or sheet piles, and C values not less than 16.

With the exception of Jamroa Weir, which is mentioned only briefly, all the structures referred to in this section are fully described in Buckley's classic 'The irrigation works of India' (London: Spon, 1905). Buckley also gave a clear explanation of the hydraulic gradient theory and an account of the Narora pressure observations.

Such, then, is the background experience drawn on by Bligh in his 1910 paper. In the same year he published the second, and much revised, edition of 'The practical design of irrigation works' (London: Constable, 1910). This also includes the C values but, as a text book covering the whole field of design, gives less detail of the case records on which these values are based.

The hydraulic gradient theory, with or without some relatively minor modifications, found world-wide acceptance. More recently, design methods based on flow nets and exit gradients have been introduced and the hydraulic gradient theory has fallen into disrepute. But this should not detract from the merit of those engineers who took the first steps towards understanding an important branch of soil mechanics, and who, in difficult conditions, built major structures of benefit to millions of people.

#### Biographies

Lt-Col. John Clibborn (1847-1938) took his degree at Trinity College, Dublin, and entered the Indian Staff Corps, Irrigation Department, in 1872. He was principal of Thomason Civil Engineering College, Roorkee from 1892 to 1901 (Who Was Who 1929-40).

John Stuart Beresford (1845-1926), after studying at Queen's University, Belfast, joined the India Public Works Department in 1867. He became chief engineer in the Central Provinces in 1893 and in the Punjab in 1896, and was inspector general of irrigation in 1898-1900. Subsequently he was in consulting practice (Who Was Who 1916-28).

William John Wilson (1851-1900) trained at the Royal Indian Engineering College, Coopers Hill; he was in the Public Works Department in 1874-92 and then moved to Egypt and was inspector general of irrigation in Upper Egypt until his sudden death from meningitis in August 1900 (Obituary. Min. Proc. Instn Civ. Engrs, 1900, vol. 142, pp. 383-384).

Robert Burton Buckley (1847-1927) was a

Whitworth Scholar; he went to India in 1869 and became chief engineer of the Public Works Department (Who Was Who 1916-28).

William George Bligh (1846-1923) served in the India Public Works Department in 1869-89. He transferred to irrigation work in Burma and in about 1908 moved to Toronto as inspecting engineer in the Department of Interior, Canada (records of Institution of Civil Engineers).

#### CLASSICAL SOIL MECHANICS 1857-1910

In the fifty years covered in this part of the lecture several new lines of research were developed which are of great interest. Outstanding are the contributions by Rankine, Boussinesq and Résal to stress field analysis, and the latter's correct treatment of the tension crack problem in clays which led, inter alia, to what is called the lower bound solution for the critical height of a vertical cut. Also outstanding are the experiments by Darwin and Osborne Reynolds from which the concept of dilatancy clearly emerged, Darcy's experiments on permeability, theoretical work by Dupuit on groundwater flow, and the introduction by Richardson and Forchheimer of flow net analysis.

A point of special relevance is the attempt by Rankine in 1862 to work out a unified approach to slope stability and earth pressure for sand and for clays in the long-term condition, using a simple and very practical approach. And of course there is Boussinesq's classic solution of the stress distribution problem.

Other topics, which cannot be considered here, include Mohr's stress circle and failure envelope, graphical trial and error methods for earth pressure calculations by Culmann and Engesser, Winkler's modulus of subgrade reaction and Kötter's analysis of slip lines.

To sum up, by 1910 much of the applied mechanics of our subject had been worked out, and again we see that the next fundamental advance had to depend on an understanding of soil properties.

19. RANKINE, W.J.M. (1857). On the stability of loose earth. Phil. Trans. R. Soc., vol. 147, pp. 9-27

Just as Coulomb introduced the limit equilibrium method, so Rankine established the general principles of stress field (slip-line) analysis in soil mechanics. Dealing in two dimensions with cohesionless granular materials, he gives the equations of equilibrium

$$\frac{d\sigma_{\mathbf{x}}}{d\mathbf{x}} + \frac{d\tau}{d\mathbf{z}} = 0$$

$$\left. \begin{array}{c} 17 \\ \frac{d\sigma_{\mathbf{z}}}{d\mathbf{z}} + \frac{d\tau}{d\mathbf{x}} = \gamma \end{array} \right|$$

and the failure condition

$$\frac{(\sigma_{\mathbf{x}} - \sigma_{\mathbf{z}})^2 + 4\tau^2}{(\sigma_{\mathbf{x}} + \sigma_{\mathbf{z}})^2} = \sin^2\phi \qquad (18)$$

where  $\sigma$  and  $\tau$  are the normal and shear stresses on the element and  $\phi$  is the angle of repose defined by the shear strength relationship s =  $\sigma_n \tan \phi$ . In terms of the principal stresses



Fig. 15. Stress field, active state (Rankine, 1857)

equation (18) becomes

$$\frac{\sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \sin\phi \tag{19}$$

and the slip lines (planes of rupture) are

inclined at  $\pm$  (45 -  $\phi/2$ ) to the direction of  $\sigma_1$ . Considering a semi-infinite mass with a free surface sloping at  $\beta$  to the horizontal (Fig. 15) a plane at a constant depth z is a surface of equal pressure, and this pressure is  $\gamma z cos \beta.$ Therefore the conjugate pressure must be parallel to the slope. Rankine then shows that its minimum value consistent with stability is

$$\mathbf{P}_{a} = \gamma z \cos\beta \left[ \frac{\cos\beta - \sqrt{(\cos^{2}\beta - \cos^{2}\phi)}}{\cos\beta + \sqrt{(\cos^{2}\beta - \cos^{2}\phi)}} \right] \quad (20)$$

and the inclination of  $\sigma_{1}$  to the vertical is

$$\psi = 45 - \frac{\beta}{2} - \frac{1}{2}\cos^{-1}\frac{\sin\beta}{\sin\phi}$$
(21)

For the extreme case of  $\beta = \phi$ 

$$\mathbf{P}_{a} = \gamma z \cos \phi \tag{22}$$

and the axis of  $\sigma_1$  bisects the acute angle between the vertical and the slope; so the slip lines are vertical and parallel to the slope.

More generally, if the surface carries a uniform vertical pressure q, then  $(\gamma z + q)$  can be substituted for  $\gamma z\,.\,$  Hence in the case with  $\beta = 0$ 

$$\mathbf{b}_{a} = (\gamma z + q) \frac{1 - \sin\phi}{1 + \sin\phi}$$
(23)

and the axis of  $\sigma_1$  is vertical; so the slip lines make angles of  $(45 - \phi/2)$  with the vertical.

For the maximum conjugate pressure consistent with stability, the signs in the foregoing equations are reversed; e.g. if  $\beta = 0$ 

$$\mathbf{p}_{p} = (\gamma z + q) \frac{1 + \sin\phi}{1 - \sin\phi}$$
(24)

and the axis of  $\sigma_1$  is horizontal.

On a vertical plane penetrating to a depth H below the surface, the total thrust or resistance is obtained by integrating p.dz from z = 0 to z = H. Thus for  $\beta = O$ 

$$P_{p}^{a} = (\frac{1}{2}\gamma H^{2} + qH) \frac{1 + \sin\phi}{1 \pm \sin\phi}$$
(25)  
(26)

Finally Rankine considers the equilibrium of horizontal sresses on a vertical plane beneath the edge of a foundation. If the foundation is at a depth D beneath level ground, and if  $q_f$  is the foundation pressure causing failure at any depth z below the base, then

$$(q_{f} + \gamma z)\frac{1 - \sin\phi}{1 + \sin\phi} = (\gamma D + \gamma z)\frac{1 + \sin\phi}{1 - \sin\phi}$$

or

$$q_{f} = \gamma D \left[ \frac{1 + \sin \phi^{2}}{1 - \sin \phi} + \gamma z \frac{4 \sin \phi}{(1 - \sin \phi)^{2}} \right]$$
(27)

Clearly the minimum value occurs at z = 0, when

$$q_{f} = \gamma D \left[ \frac{1 + \sin \phi}{1 - \sin \phi} \right]^{2}$$
(28)

But we note that this corresponds to local failure at the foundation edge. Rankine has therefore obtained only a partial stress field, and his solution is the same as Poncelet's limiting case, (see No. 9).

#### Biography

William John Macquorn Rankine (1820-72), studied at Edinburgh University, and worked as a civil engineer in Ireland and Scotland from 1838 to 1855 when he was appointed Regius professor of engineering at Glasgow. He was a fellow of the Royal Society and author of fundamental papers on thermodynamics and strength of materials, and of standard text books on applied mechanics (1858), steam engines (1859), civil engineering (1862) and machines (1869); he exerted a profound influence on British engineering education. His paper 'On the stability of loose earth' was read to the Royal Society on 19 June, 1856 (Dictionary of scientific biography, 1975; Sutherland, H.B. (1973). Rankine, his life and times. London, Institution of Civil Engineers).

20. RANKINE, W.J.M. (]862). A manual of civil engineering. London: Griffin & Bohn This book immediately became a standard text and continued in use, with only minor additions, for at least half a century. The main points in soil mechanics may be summarized as follows.

- Earthwork fails by the slipping or (i) sliding of its parts on each other. The resistance to a shearing force, in general, arises from friction between the grains and their mutual adhesion i.e. cohesion.
- (ii) Adhesion in soils is gradually destroyed by the action of air and moisture and weathering.
- (iii) However, adhesion is useful in temporary works in enabling the sides of a cutting to stand for a time with a vertical face to a certain depth. That depth depends on  $c/\gamma$  and ranges from zero for dry sand

to 3-6ft for ordinary earth and 10-16ft for clay.

- (iv) The permanent stability of earth, due to friction alone, maintains the sides of an embankment or cutting at a slope, the inclination of which is the angle of repose  $\phi$ , where f = tan $\phi$ . This is called the natural slope.
- (v) Adhesion and friction of earth are so variable that the engineer should never trust to books or tables 'when he has it in his poser to obtain the necessary data either by observation of existing earthworks in the same stratum, or by experiment'.
- (vi) Nevertheless, as a guide, Rankine tabulates ranges of  $\phi$  in different types of soil. There is a misprint (21<sup>°</sup> instead of 31<sup>°</sup> for the lower limit for sands) but ignoring this I have set out in Table 5 some typical values. It will be seen that for clays these angles of repose relate to long-term or permanent stability with c = 0.
- (vii) In general the cohesion of rock can be depended on and the sides of excavations in it can be made at steep angles. But shales are liable to soften by weathering, and some sandstones are scarcely stronger than earth, requiring slopes of l:l or l<sup>1</sup><sub>2</sub>:l. Chalk, if hard, stands at <sup>1</sup><sub>2</sub>:l but if lacking soundness may require slopes of l<sup>1</sup><sub>2</sub>:l.
- (viii) The expressions, already derived in his 1857 paper (see No.19), for active and passive pressure and for bearing capacity are given.
- (ix) In addition he introduces the concept of an equivalent unit weight  $\gamma_e$  such that  $\frac{1}{2}^2$

$$P_a = \frac{2H}{2} \gamma_e$$

 $\mathbf{or}$ 

$$\gamma_e = \gamma K_a$$

Some values of  $\gamma_{\rho}$  for the case of a vertical wall and horizontal fill will be found in Table 5.

(x) As a further addition to the 1857 paper Rankine formulates the limiting load T which can be applied by a horizontal landtie to an anchor beam. If the top and bottom edges of the beam are at depths  $D_1$  and  $D_2$  below level ground, then

$$T = \frac{1}{2}\gamma (D_2^2 - D_1^2) \frac{4\sin\phi}{\cos^2\phi}$$

This follows directly from the integration of  $(p_p - p_a)dz$ .

- (xi) Retaining walls should be designed so that
  - (a) the resultant force passes within the middle third of the base, to avoid tension or uplift at the heel
  - (b) the obliquity of the resultant does not exceed  $\phi_1$  where this is the angle of friction between the foundation and the underlying earth (Rankine gives values of  $\phi_1$  of 18° for moist clay and 27° for dry clay)

Table 5. Some typical values of unit weight  $\gamma$  (lb/ft<sup>3</sup>) and (long-term) angle of repose  $\phi$  with corresponding values of earth pressure coefficients  $K_a$  and  $K_p$  (for  $\beta = 0$  and vertical walls) and equivalent unit weight  $\gamma_e = \gamma K_a$  (after Rankine, 1862)

	Υ	ф	cotø	K a	Υ <sub>e</sub>	к р
Wet clay	120	16 <sup>0</sup>	3½	0.57	68	1.8
Dry Clay	130	27 <sup>0</sup>	2	0.38	50	2.6
Sand	100	33 <sup>0</sup>	15	0.29	29	3.5
Gravel	100	38 <sup>0</sup>	14	0.23	23	4.3

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(c) the maximum foundation pressure does not exceed the safe bearing capacity.(xii) Foundations of walls and buildings can

.) Foundations of walls and buildings can be classified as being in

- (a) rock
- (b) firm earth such as sand, gravel and hard clay
- (c) soft earth

For sound rocks, bearing pressures up to 10 ton/ft<sup>2</sup> can be used, although in the weakest sandstones 2 ton/ft<sup>2</sup> may be the limit. In firm earth, foundations should be taken 3-4 ft below ground level to avoid the disintegrating effects of frost and drought, and the pressures are usually limited to about  $l_2^{1}$  ton/ft<sup>2</sup>. In soft earth a timber platform or concrete footing may be employed to maximize the spread of load, but more frequently a pile foundation is adopted. This can take the form of a group of short piles, 6-12 ft long, driven as close together as possible, effectively to bring the foundation load that much deeper, or more widely spaced bearing piles each supporting its share of the load. Bearing piles may be driven through a soft stratum to reach firm material, or if that is impracticable they may be supported by friction in the soft stratum.

(xiii) Embankments on soft ground may be formed with slopes not exceeding the angle of repose  $\phi_1$  of the ground; alternatively the ground may be excavated to a depth  $h_1$  and replaced by banking material,  $h_1$ being determined from the expression

$$h_{1} \left[ k^{2} \gamma_{1} - \gamma \right] = \gamma h$$

where  $\gamma_1$  and  $\gamma$  are the unit weights of the ground and bank, h is the height of bank, and  $k = (1 + \sin\phi)/(1 - \sin\phi)$ . The excavation slopes will be at  $\phi_1$  and the bank slopes formed accordingly.

(xiv) The best method of ascertaining the nature of the ground is to sink one or more shafts, combined with borings which for cuttings (or tunnels) should be 200-300 yd apart. The borings provide only disturbed samples (either fragmented or softened by the drilling water) but they will show whether any changes in strata occur sufficient to make it advisable to sink additional shafts.

Rankine has been criticized in modern times for ignoring cohesion and, by his great authority, perpetuating a grossly oversimplified theory. He was, of course, aware of the c,  $\phi$  analysis of earth pressure, but says that 'for want of precise experimental data its practical utility is doubtful'. In 1862 this remark is justified; neither he nor anyone else could give reliable figures for c and  $\phi$  combined. Instead he adopts what may be called a semiempirical approach based on observed 'permanent' slopes in clays and using values of  $\phi$  deduced therefrom on the assumption of c = o. He clearly recognized that this method could not be applied to short-term problems in clay, but the logic of taking zero cohesion in clay foundations can be questioned.

For cohesionless soils a valid criticism is his neglect of wall friction. He also failed to deal with hydrostatic pressure due to groundwater, although as will be seen in Baker's paper (No.24), this presented no difficulty in pervious soils.

Rankine includes a brief description of shell and auger drilling tools. This is based on the detailed account by Haskoll (1846), but the technique was already well established and records of site investigation borings go back at least to the 1770s in England.

## 21. DARCY, H.P.G. (1856). Les fontaines publiques de la ville de Dijon. Paris: Dalmont

Graded sand filters for water purification had been introduced by James Simpson at Chelsea Waterworks in 1829 and were adopted at some half dozen places in Great Britain and France during the next two decades. Darcy gives details of these installations but realized that data relating to their performance did not allow any general law to be deduced for the flow of water through sand. He therefore decided to investigate the subject experimentally and carried out 36 tests at Dijon between October 1855 and February 1856.

His apparatus consisted of a vertical tube 35 cm in diameter equipped with a fine mesh grating near the lower end, supporting a bed of sand, and mercury manometers reading water pressure above and below the sand. With 58 cm of sand in the tube, the flow was measured under differential heads ranging from 1 m to 10 m, each test lasting about 20 minutes. The experiments were then repeated with 110 cm and 170 cm of sand.

Referring to Fig. 16, if the peizometric heights above and below a sand bed of thickness L and area A are (h + L) and  $h_O$  respectively, and if Q/A is the flow per unit area (which is also the velocity of flow v), then Darcy found that for all values of h,  $h_O$  and L in his experiments the results conformed with the relation

$$\frac{\Omega}{A} = v = k \frac{(h+L) - h}{L} = k \frac{\Delta h}{L}$$
(29)

where k is a constant which he defined (using this symbol) as 'a coefficient depending on the



## $\frac{Q}{A} = v = k \frac{(h + L) - h_o}{L} = k \frac{\Delta h}{L}$

Tests at constant head show

where k = coef. of permeability

Falling head, with h constant

v = -dh/dt

$$\log_{e} (\Delta h_1 / \Delta h_2) = k (t_2 - t_1) / L$$

## Darcy, 1856

Fig. 16

permeability of the sand bed'. Equation (29) is known as Darcy's Law. It is analogous to the law of flow in capillary tubes established by Poiseuille in 1841.

In his experiments Darcy used a medium-coarse sand with a porosity of 38%. The value of k was about 0.03 cm/s.

In any particular test, h and h were maintained constant during the measurement of Q. But Darcy also considered the case in which a given quantity of water is placed on top of a sand bed and allowed to percolate under gravity. The depth of water h will then decrease with time, and the velocity of flow at any instant (with h constant) is

V = -dh/dt

Combining this expression with equation (29) he shows that

$$\log(\Delta h_1/\Delta h_2) = k(t_2 - t_1)/L$$
(30)

which is the falling head equation of flow. Darcy confirmed equation (30) experimentally.

#### Biography

Henri Philibert Gaspard Darcy (1803-1858), inspecteur général of the Ponts et Chaussées, designed and directed construction of the water supply of Dijon (1834-42) and that part of the Paris-Lyon railway passing through the Côte d'Or (1842-48). He then became chief engineer of the municipal service of Paris, and in 1850 divisional engineer, but from 1855, returning to Dijon, devoted himself entirely to hydraulic research (Dictionnaire biographie française, 1970).

22. DUPUIT, A.J.E.J. (1863). Etudes théoriques et pratiques sur le mouvement des eaux dans les canaux découverts et à travers les terrains perméables. Paris: Dunod In this enlarged edition of an earlier work (1848) Dupuit deals for the first time with the steady state gravity flow of groundwater in permeable soils. He makes the simplifying assumptions that the hydraulic gradient at any vertical section is equal to the slope of the free water surface and remains constant at all depths in that section. These approximations, as he realizes, are valid only for small gradients. For a permeable soil overlying a horizontal impervious boundary the flow per unit width can then be expressed as

q = -kh.dh/dx

where h is the depth from the free surface to the boundary and k is Darcy's coefficient of permeability.

Dupuit starts by considering the case of twodimensional flow. If h is the depth at x = oand h the depth (h > h) at a distance x from the origin, then by integration of equation (31)

$$q = k(h^2 - ho^2)/2x$$
 (32)

This can be applied directly to a bank with vertical faces, such as a cofferdam. Let h be the depth of water downstream (at x = 0) and H the depth upstream. Then

$$q = k(H^2 - h_w^2)/2L$$
 (33)

where L is the width of the dam. Moreover from equations (32) and (33) we see at once that the equation for the free surface within the dam is

$$h^2 = h_w^2 + (X/L) (H^2 - h_w^2)$$
 (34)

To analyse the three-dimensional flow to a pumped well, Dupuit imagines the well, of radius  $r_w$ , to be centrally placed in a sand island of radius R with vertical sides outside which the water level is maintained at a depth H. He then shows that the total flow towards the well is

$$Q = \mathbf{k} \frac{\pi (H^2 - h_w^2)}{\log (R/r_w)}$$
(35)

where h is the depth of water in the well; and the free surface equation is

$$h^{2} = h_{w}^{2} + (H^{2} - h_{w}^{2}) \frac{\log(r/r_{w})}{\log(R/r_{w})}$$
(36)

Similarly, the flow to an artesian well pumping from a confined, horizontal aquifer of thickness D is

$$Q = k \frac{2\pi D (H - h_w)}{\log (R/r_w)}$$
(37)

where H and h are now the heights of the piezometric surface at the outer boundary and at the well. These equations apply to a well penetrating the full thickness of the sand.

For subsequent investigations on seepage towards wells see Hall (1954). Four points may be noted here.

- (a) The permeable stratum is likely to be of almost unlimited extent compared with its depth, but R cannot be given an infinite value. This difficulty was resolved in principle by Adolph Thiem who, in 1870, realized that drawdown beyond a certain distance is negligible or zero, as replenishment of groundwater by rainfall tends to equalize the amount drained by the well; so H can be taken as the original depth of water at some large, but finite, value of R known as the radius of influence.
- (b) For steep gradients, the free surface equations (34) and (36) are crude approximations but, surprisingly, equations (33) and (35) are found to be mathematically exact expressions for the quantity of flow. This point is explained by Polubarinova-Kochina (1962).
- (c) The problem of flow towards a line or

group of wells, important in the process of groundwater lowering, was tackled by Forchheimer in 1898 and later papers.

(d) The use of field pumping tests to determine insitu permeability, adapting equation (35) and using standpipe or piezometer observations not too close to the well, is due to Thiem (1906).

#### Biography

Arsène Jules Etienne Juvénal Dupuit (1804-66), entered the Ponts et Chaussées in 1824 after graduating at the Ecole Polytechnique. He was chief engineer of the Maine-et-Loire from 1842 and of the municipality of Paris from 1850. He wrote papers and books on several aspects of civil engineering (Dictionnaire biographie française, 1970).

- 23a. BOUSSINESQ, J.V. (1876). Essai théorique sur l'equilibre d'elasticité des massifs pulvérulents et sur la poussée des terres sans cohésion. Mem.Acad.R.Belg; vol.40 (reprinted with slightly different title -Paris: Gauthier-Villars, 1876).
- 23b. BOUSSINESQ, J.V. (1885). Sur l'intégration par approximations successive d'une equation ... dont dependent les pressions intérieures d'un massif de sable à l'état ébouleux'. In Application des potentiels à l'étude de l'equilibre et du mouvement des solides élastiques. Paris: Gauthier-Villars (No.27), pp. 705-712)

Rankine's analysis of the stresses in a semiinfinite mass of cohesionless soil (No.19) is exact, so far as it goes, but suffers from a serious limitation when applied to retaining walls since it does not take wall friction into account as an independent variable. Indeed, it implies that the earth pressure acts on the wall in a direction parallel to the free surface. The retaining wall problem, in terms of stress field analysis, engaged the attention of Maurice Levy in 1867 and of Barre de Saint-Venant in 1870 (Heyman, 1972) and significant advances were made by Boussinesq. His investigations began in 1873 and he presented his essay to the Academie Royale des Sciences de Belgique in June 1874. It was published two years later.

Boussinesq starts by considering a mass of dry sand, with a sloping surface, in a state of elastic equilibrium under its own body weight. After remarking that his analysis is based on the assumptions of negligible compressibility and a shear modulus  $\mu$  proportional to the mean pressure, I shall not enter further into this part of the paper except to note that in an opening paragraph Boussinesq says atmospheric pressure can be neglected 'as this acts in all directions within the mass and around each grain ... It therefore has no influence on their mutual action, and consequently does not modify the supplementary normal and tangential forces which the contacts of the grains produce on unit area of an element. These supplementary forces alone have to be considered.' Here we have a clear, if restricted, statement of the principle of effective stress.

Turning to the earth pressure problem, Boussinesq assumes that there has been sufficient lateral movement for the entire mass of sand to be in a state of plastic or limiting



Fig. 17

equilibrium. The difficulty is then to combine Rankine's basic equations (17) and (18) with the boundary condition  $\tau = \sigma$  tano on the plane representing the back of the retaining wall.

Boussinesq takes the general case of an inclined wall and sloping back-fill, and succeeds in finding an approximate solution for the earth pressure acting at an angle  $\sigma$  to the normal of the wall. For simplicity, considering a vertical wall and horizontal fill (Fig. 17), there is a discontinuity in the stress field between the Rankine state in zone OAQ and the wedge OMQ, and the slip lines in the wedge must be curved.

Writing the earth pressure at any depth z on the x = 0 plane as

$$p = \gamma z K \tag{38}$$

he finds as a first approximation

$$K = \tan^2 \varepsilon \underline{\cos \varepsilon}$$
$$\cos(\varepsilon - \delta)$$

$$K\cos\delta = a^2$$
(39)  
1 + atan $\delta$ 

where

01

 $a = tan(45 - \phi/2) = tan\epsilon$ 

Values of Kcos $\delta$  from equation (39) are set out in Table 6.

Boussinesq returned to this problem several times and finally, in a supplementary chapter in 'Application des potentiels' obtained a second approximation which for vertical walls and horizontal fill can be written in the form

$$K\cos\delta = a^2 \frac{1}{1 + \tan\delta} C$$
 (40)

where

$$C = 1 + \frac{1}{\sin\sigma} \left[ \frac{\operatorname{atan} \delta}{1 + \operatorname{atan} \delta} \right]^2 \log \frac{4}{e}$$

He thought this gave an upper limit but the results (Table 6) are numerically almost identical with the best limit analysis solution at present available (Chen, 1975) and lie slightly below the stress field values obtained by Caquot and Kerisel (1949) and Sokolovski (1960). Table 6. Values of Kcos $\delta$  where  $P_a = \frac{1}{2}\gamma H^2 K$  for vertical walls and horizontal fill

	δ = Ο				
ф			Boussi	nesq	Caquot
	Rankine	Coulomb	1876	1885	and Kerisel
30 <sup>0</sup>	0.333	0.257	0.250	0.262	0.267
35 <sup>0</sup>	0.271	0.305	0.199	0.208	0.213
40 <sup>0</sup>	0.217	0.161	0.156	0.164	0.168
45 <sup>0</sup>	0.172	0.125	0.121	0.127	0.131

#### Biography:

Joseph Valentin Boussinesq (1842-1929), a selftaught applied mathematician of power and originality, started his career as an obscure schoolmaster, and became a member of the Academy of Sciences. He was a professor at Lille University from 1873 and at the Sorbonne from 1886 (Dictionary of scientific biography, 1970; Mayer A.(1954). Obituary. Geotechnique, vol.4, pp.3-5).

24. BAKER, B. (1881). The actual lateral pressure of earthwork. Min. Proc. Instn. Civ. Engrs; vol. 65, pp. 140-186 Baker takes the conventional c = 0,  $\delta = 0$  earth pressure theory, expressing the results in terms of equivalent unit weight (see No.20), and shows that by this theory the lateral thrust and overturning moment on experimental walls tested by Hope (No. 10) and others are overestimated. Curiously, he does not allow for wall friction, but in correspondence on the paper (published in the same volume) Flamant and Boussinesq take account of this factor and show that it largely resolves the discrepancies.

Baker next describes some of the retaining walls on the Metropolitan Railway in London. A wall 23 ft high retaining dry sand and designed for  $\gamma = 20$  lb/ft<sup>3</sup> remained stable, although as  $\phi = 3^{70}$  the wall should have failed. (Here again we note that wall friction accounts for the facts.) At another site where 20 ft of made ground and gravel overlies London Clay a 30 ft wall failed by rotational slip. It was rebuilt with a wider and somewhat deeper section, to withstand about 55 lb/ft<sup>3</sup>, and remained stable. A second wall nearby, designed for a rather greater equivalent unit weight, tilted slightly and moved forward, but there was a leaking sewer not far behind the wall.

Some 20 examples of dock walls are then described. Where the equivalent fluid design pressures are given, these range from an exceptionally low value of 30 lb/ft<sup>3</sup> to what seems to be a more usual 50-60 lb/ft<sup>3</sup> or even 80 lb/ft<sup>3</sup> where a wall was backed by very soft clay. Only one of the walls failed by tilting, but several failed by sliding forward on clay. At least two of these appear to have been designed with a base friction of  $27^{0}$ , i.e. a factor of safety of 1.0 on Rankine's higher figure for  $\phi_1$  in clay, and generally the foundations were too shallow for any appreciable development of



in pervious soils

after Baker, 1881

#### Fig. 18

passive pressure; a feature strongly advocated by Poncelet (No. 9) but seemingly forgotten by some engineers in the later 19th century.

Taking a broad view of all field experience, Baker concludes that the conventional theory is of limited practical use. But this opinion seems not to be fully justified, and we might say that if proper attention had in all cases been given to safety against sliding, the values of earth pressure indicated by Rankine's theory, using sensible angles of respose, would be a quite good approximate design basis for long-term stability in clays.

One of the most interesting points in the paper arises in connection with locks, where water level in front of the wall is often considerably lower than groundwater behind (Fig.18). Here Baker says that below groundwater level the total pressure is the sum of the full hydrostatic pressure and the lateral pressure of the fill with its unit weight reduced by buoyancy. He gives an example of rubble filling, with a porosity of 35%. In air this weighs 100 lb/ft<sup>3</sup>, but below water level its lateral thrust will be that due to the submerged weight of 59 lb/ft<sup>3</sup>. This is perfectly correct but the same reasoning would probably not have been applied to clays, which are considered to be impervious.

#### Biography

Sir Benjamin Baker (1840-1907), FRS, LLD, President of the Institution of Civil Engineers, joined the staff of Sir John Fowler and from 1869 was chief assistant on the Metropolitan Railway construction. Later he became a partner and with Fowler designed many of the extensions of the London Underground system, and the Forth Bridge, and was a consultant on irrigation works in Egypt, etc (Obituary. Min.Proc. Instn. Civ. Engrs; 1907, vol.170,pp.377-383).

## 25. DARWIN, G.H. (1883). On the horizontal thrust of a mass of sand. Min. Proc. Instn. Civ. Engrs; vol.71, pp. 350-378

Experiments were made to measure the overturning noment on a bottom-hinged door, the containing box being 22 cm long and 30 cm wide, with depths of sand ranging from 18 cm to 34 cm. Careful tests on the uniform, dry and fine-grained sand showed an angle of repose  $\beta_n = 35^\circ \pm 1^\circ$  and a density in loose packing of 1.40. The inner face

of the door had a thin coating of sand glued to it. After filling the box to any particular depth H the force restraining the door was gradually reduced until yielding of the sand occurred, typically when the top of the door had moved about  $1-1\frac{1}{2}$ mm.

If the overturning moment at yield is M, and if the horizonatal component of earth pressure is  $P\cos\sigma$  then, assuming the centre of pressure lies at the lower third point

 $M = P\cos\delta(H/3)$ 

and writing  $P = \frac{1}{2}\gamma H^2 K$  we have

 $M = (\gamma H^3 K \cos \delta) / 6$ 

In the first series of tests the box was filled with loose sand placed in horizontal layers. The results show that M is indeed proportional to  $H^3$  and Kcos $\delta$  = 0.18, this being the mean of 16 tests.

After each test the sand was thoroughly stirred up with a stick and the sides of the box were hammered. In this dense packing the sand had a density of 1.55 and Kcos $\delta$  = 0.132. Other experiments were made with loose sand tipped in sloping layers, and also with the upper surface inclined at  $\pm\beta_{\rm p}$ .

at  $\pm \beta_n$ . Taking  $\phi = \beta_n$  Darwin compares his result with Rankine's formula and, assuming  $\delta = \phi$ , with Boussinesq's 1876 formula (see No.23) and a slightly incorrect wedge theory worked out by Darwin himself in ignorance of the solution obtained by Poncelet and Scheffler. For the first series of tests Boussinesq and wedge theory both give Kcos $\delta$  about 0.2 but Rankine (with  $\delta =$ 0 if  $\beta = 0$ ) gives 0.27.

Darwin made some tests to determine the effect of side friction in the box, and in his calculations of M, and hence of K, makes an allowance for this, although probably not quite enough. But within the accuracy that could be expected, the conclusion must be that the theories of Boussinesq and Coulomb (when correctly applied) give satisfactory results for loose sand.

For compact sand, however, there is a major discrepancy; Darwin gets very near the truth when he points out that the angle of internal friction is not necessarily the same as the angle of repose. In particular he realizes that in dense sand there must be a gradual unsettlement of the sand 'in which one grain after another partially rotates, assumes a more open order, and causes the whole mass to occupy a larger volume'. Moreover he says this effect 'would almost certainly take place along certain surfaces or narrow regions which ultimately form the seat of sliding'. This leads him to conclude that the wedge theory is physically preferable to theories which treat the sand as a continuum.

Here we have a clear appreciation of dilatancy and the consequent formation of shear zones. The phenomenon of dilatancy was subsequently named and demonstrated experimentally in sand by Osborne Reynolds (No.26).

Darwin sums up by saying, 'no mass of sand can be put together without some history, and that history will determine the nature of its limiting equilibrium': a point, he says, foreseen by the late Professor Clerk Maxwell in discussions when he (Darwin) was beginning the experiments. These, it may be added, were made in 1877 and Baker's paper afforded the inducement to publish, as there appeared to be 'a singular deficiency of experimental data' for testing the accuracy of earth pressure theories.

He considers that the 'historical element', including dilatancy, 'essentially alludes mathematical treatment'. But Boussinesq (1883) fully understands that the interior angle of friction  $\phi$  will in general be greater than the angle of friction in the uppermost layers of a tipped slope (i.e. the angle of repose) and points out that  $\phi$  can be evaluated by backanalysis from the experiments e.g. in the second series, with dense sand,  $\phi = 43^0$  using equation (39). Boussinesq also suggests that an approximate value of  $\phi$  could be determined directly by placing the sand in a rough-bottomed box in any desired state of compaction, and tilting the box until slipping begins.

Darwin's paper is of outstanding importance in pure soil mechanics, but his experiments were not well designed for the accurate determination of earth pressure, wall friction and so on. The classic tests in the period under review were those by Muller-Breslau reported in his 'Erddruck und Stutzmauern'. (Stuttgart, 1906).

#### **Biographies**

Sir George Howard Darwin (1845-1912), FRS, started his scientific work in 1875 as a Fellow of Trinity College, Cambridge, and became Plumian professor of astronomy and experimental philosopy in 1883. He was an applied mathematician working chiefly in geophysics and cosmological dynamics (Dictionary of scientific biography, 1971).

Heinrich Franz Bernhard Muller-Breslau (1851-1925), professor of bridge design at Hanover and from 1888 professor of structural engineering at Charlottenburg, is considered to be the founder of the modern school in this subject in Germany (Dictionary of scientific biography, 1974).

26a. REYNOLDS, O. (1885). On the dilatancy of media composed of rigid particles in contact; with experimental illustrations. Phil. Mag; 5th series, vol. 20, pp. 469-481

26b. REYNOLDS, O. (1887). Experiments showing dilatancy, a property of granular material, possibly connected with gravitation. Proc. R.Instn. Gt.Br; vol. 11, pp. 354-363 Reynolds starts his 1885 paper with three general points.

(i) Dilatancy, a term introduced here for the first time, is the change in volume consequent upon a change of relative positions of the grains in a mass of granular material. A simple illustration is the dilation of a group of spherical particles changing from a tetrahedral to a cubical packing: an increase of about 30% in volume.

(ii) Dilatancy within a granular mass, being essentially a geometrical phenomenon, will be largely if not entirely independent of intergranular friction.

(iii) It may perhaps be assumed that principal stresses at failure are related by Rankine's expression

 $\frac{\sigma_3}{\sigma_1} = \frac{1 - \sin\phi}{1 + \sin\phi}$ 

where  $\phi$  is the angle of repose, but it cannot be assumed, Reynolds says, that tan $\phi$  has any relation to the actual friction between the particles, the value of  $\phi$  being a matter of their arrangement.

This third point, advanced as a by-product of his thoughts on dilatancy, is exceptionally interesting, for until recently the general view has been to the contrary, namely that  $\phi$  depends to an important degree on intergranular friction. But Reynolds' physical insight was correct. In cohesionless materials  $\phi$  is controlled almost exclusively by particle shape and packing, and hardly at all by the coefficient of friction between the grains, as shown in a beautiful series of tests by Skinner (1969).

Reynolds then describes some experiments on lead shot which clearly demonstrate the effects of dilatancy, but I shall immediately turn to his second paper dealing with experiments on sand. These were performed during an evening discourse at the Royal Institution in February 1886, and two of them are classic.

In the first, a thin rubber bag containing 6 pints of dense sand, the interstices of which are full of water, is connected to a long glass tube. On squeezing the bag between two boards, water is drawn in (not out, as would be the case with a sponge) and, at maximum dilation, the volume has increased by 15%. At this stage dilatancy has brought the material to its minimum density under the circumstances of the test, although further loading tends to cause alternate contraction and expansion on a minor scale.

In the second experiment the bag is again filled with dense, fully saturated sand but now the neck of the bag is closed with a mercury manometer. On loading, the mercury rises on the side connected with the bag. The sand is almost rigid, as no volume change can occur (except by a small amount from the manometer) and, with 200 lb on the boards, the pressure in the bag is less by 27 in. of mercury than the pressure of the atmosphere. On opening the neck to allow the entrance of water the bag at once yields even to a slight load, changing shape, but this change immediately stops when the supply is cut off.

We note that Reynolds' experiments provide direct proof that a dense sand when subjected to shear forces shows an increase in volume due to dilatancy in an open (drained) system, and a decrease in pore pressure in a closed (undrained) system, accompanied by a gain in strength.

He then proceeds to explain the temporary firmness of damp beach sand when trodden on (resulting from dilatancy and capillary tensions), the softening of the sand if you continue to stand in one position (as water flows in from adjacent parts and so reduces the tensions) and the softness, at all times, of sand when dry or fully submerged. He ends with some highly speculative thoughts on a possible role of dilatancy in gravitation theory.

A further discussion of these and some other experiments is given by Rowe (1969).

#### Biography

Osborne Reynolds (1842-1912), a fellow of the Royal Society and Royal Medallist, after serving an engineering apprenticeship, read mathematics at Cambridge and, following a brief period in practice, became professor of engineering at Manchester in 1868. There he carried out experimental work of remarkable brilliance, particularly in hydraulics and lubrication. He retired because of ill health in 1905, (Dictionary of scientific biography, 1975).

27. BOUSSINESQ, J. (1885). Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques. Paris: Gauthier-Villars

The whole of this famous treatise has been examined in detail by Todhunter and Pearson (1893), and the results of importance in soil mechanics are summarized by Terzaghi (1943). Here I shall merely recall that Boussinesq for the first time

- gives the stresses and deformations at any (a) point within, and on the surface of, a semi-infinite elastic solid due to a vertical point load on its (horizontal) surface
- in a chapter headed 'Calcul des dépressions (b) que produisent, à la surface d'un sol horizontal... des pressions extérieures normales', gives the mean, centre and edge settlements of a circular loaded area for uniform, parabolic and inverted parabolic pressure distributions
- in the same chapter derives the settlement (c) of a rigid circular area and the corresponding counterpressure distribution.
- 28. RICHARDSON, L.F. (1908). The lines of flow of water in saturated soils. Scient. Proc. R. Dubl. Soc; vol. 11, pp. 295-316

Taking a hint from Clerk Maxwell's 'Elementary treatise on electricity' (Oxford, 1881), Richardson developed the trial and error method of solving two-dimensional flow nets. He read a paper on 'A freehand graphic way of determining stream lines and equipotentials' to the Physical Society in November 1907, (Phil.Mag; 1908, vol. 15, pp. 237-269); and followed this by the Dublin paper, which appeared in print in May 1908

In his Dublin paper he employs the method to determine a relation between the spacing of drainage ditches and the height to which the saturating water will rise, with a given rainfall and a known permeability. The solutions are applied to the problem of draining peat mosses. One of his several carefully constructed flow nets is illustrated in Fig.19.

Richardson deals very thoughly with the mathematical principles and the boundary conditions on a free surface, and with the technique of drawing the nets.

The first use of square-meshed nets as a graphical means of flow pattern determination in pure hydraulic problems is attributed by Rouse and Ince (1957) to Franz Prasil of E.T.H. Zurich in his 'Technische hydrodynamik' (Berlin, 1913), but the wide acceptance of this technique, and especially its development in seepage studies by Terzaghi and Casagrande, stems from Forchheimer's 'Hydraulik' (1st edn, Leipzig, 1914).



#### = 0.06 S.D. $= 0.05 \text{ Kg}\rho$

#### Fig. 19

#### **Biographies**

Lewis Fry Richardson (1881-1953), FRS, followed his work on flow nets by originating in 1910 the finite difference method of solving differential equations of the Laplace type with given boundary conditions. He worked on meteorology in 1907-20 (except for service in the Friends Ambulance Unit in 1916-18); subsequently he was in charge of the physics department at Westminster College and from 1929-40 was principal of Paisley Technical College (Roy. Soc. Obits, 1954, vol.9, pp. 217-235).

Philipp Forchheimer (1852-1933) was professor of hydraulics at Aachen and subsequently at Graz (Rouse and Ince, 1957). During the First World War he reorganized engineering education in Istanbul and was responsible in 1916 for bringing Terzaghi (whom he remembered as a student at Graz) to his first teaching post at the Technical University there (Casagrande in Bjerrum et al;1960).

#### 29. RESAL, J. (1910). Poussée des terres. Deuxième partie: Théorie des terres

cohérentes. Paris: Béranger Résal agrees with Rankine that clays can lose their cohesion as a result of alternate drying and wetting, and freeze and thaw, but points out that these effects are limited to the shallow zone of seasonal variations. It is also possible, he says, that cohesion will be lost at greater depths if a clay is subjected to high intersticial hydrostatic pressure, or if it contains voids or fissures which water can penetrate easily. However, in practice, care is taken properly to compact clay fills and to drain cuttings, so there must be many cases in which cohesion can be relied upon as a permanent component of strength.

It is therefore necessary to return to Coulomb's criterion of failure, and to obtain reliable data on c and  $\phi$  in clays, preferably by measuring shear strength under a range of normal pressures. In the absence of such information Résal takes  $\gamma = 1.8 \text{ t/m}^3$ , c = 3.6 t/m<sup>2</sup> and  $\phi$  = 15<sup>0</sup> as parameters for quantitative illustration. In a major contribution to classical soil mechanics he then proceeds to examine four basic problems.

#### Stress-field analysis

First, he extends Rankine's stress field

analysis to include cohesive soils. For details see Terzaghi (1943), but here we may note the following.

In the active state there is tension to a depth

$$z_{o} = \frac{2c}{\gamma} \tan(45 + \frac{\phi}{2}) = \frac{2c}{\gamma} \frac{\cos\phi}{1 - \sin\phi}$$
(41)

and this depth is independent of the slope  $\beta$ . For  $\beta$  = 0 the slip lines are straight and coincide with Rankine's solution at all depths in the passive state, and at depths greater

than z in the active. For  $\beta > 0$  the lines are curved, and if  $\beta > \phi$  both the active and passive lines become tangential to a plane inclined at  $\beta$  lying at a depth

$$z_{1} = \frac{c}{\gamma} \frac{\cos\phi}{\sin(\beta - \phi)\cos\beta}$$
(42)

In the tension zone the slip lines merge into lines of tensile failure which are normal to the surface.

Vertical cut

Coulomb's solution for the critical height of a vertical cut in horizontal ground

$$H_{c} = \frac{4c}{\gamma} \frac{\cos\phi}{1 - \sin\phi}$$

implies that the soil can withstand tensile forces, i.e. there is no tension crack. This is contrary to experience, and to determine a lower limit more reasonable in practice Résal assumes the soil to be incapable of taking any tension. He finds the critical height in that case to be

$$H_{o} = \frac{2c}{\gamma} \frac{\cos\phi}{1 - \sin\phi}$$
(43)

The same assumption leads to the conclusion that tension cracks will penetrate to the depth  $z_{0}$  which equals H. A thin vertical slab will then fail if the cut is made any deeper. With the parameters mentioned above H<sub>0</sub> = 5.2m.

#### Slope stability

Equation (43) applies if  $\beta = 90^{0}$  and, at the other limit, the slope can stand to an infinite height when  $\beta = \phi$ . For intermediate cases Résal obtains an approximate solution

$$H = \frac{2c}{\gamma} \frac{\sin\beta \cos\phi}{1 - \cos(\beta - \phi)}$$
(44)

based on a slip surface which starts at the bottom of a tension crack of depth z just behind the crest, becomes tangential to a plane inclined at  $\beta$  and lying at a depth z, and then curves out through the foot of the slope, where it makes an angle (45 -  $\phi/2$ ) with the face.

In Table 7 values of the stability factor  $\gamma H/c$  as given by equation (44) are compared with those from the circular arc analysis (Taylor,

Table 7. Values of  $\gamma H/c$  for  $\phi = 15^{\circ}$ 

β	90 <sup>0</sup>	60 <sup>0</sup>	45 <sup>0</sup>	30 <sup>0</sup>	
Résal	2.6	5.7	10.2	28	
Taylor	2.6	6.5	10.5	21	



Fig. 20. Active pressure in clay

1948) to which I have applied an approximate correction allowing for the effect of tension cracks (of depth z ) as suggested by Taylor himself.

#### Retaining Walls

Résal handles the general case of active pressure on an inclined wall, with wall friction, and a sloping back-fill. For simplicity, however, let us consider a vertical wall with  $\delta = 0$  and a horizontal fill. Then the standard solution is

$$P_a = \gamma z tan^2 \varepsilon - 2 c tan \varepsilon$$

where  $\varepsilon = (45 - \phi/2)$ . But for z < z this equation yields negative values of P which are not admissible in a soil devoid of tensile strength. Thus  $p_a$  must be taken as zero in the tension zone, and the total pressure is

$$P_{a} = \int_{z_{o}}^{H} p_{a} dz = \frac{1}{2} \gamma (H - z_{o})^{2} \tan^{2} \varepsilon$$

$$(45)$$

This expression corresponds exactly to the result obtained by a wedge analysis in which tension cracks extend to the depth z. Conversely if p\_dz is integrated from z = 0 t

Conversely if p dz is integrated from z = 0 to z = H he shows that the result corresponds exactly with the Coulomb wedge analysis, which implies no tension cracks (Fig. 20).

Equation (45) is given for the first time by Résal, but it was derived independently by Cain (1911), who also explains the points in the foregoing paragraph with great clarity.

Résal also deals with passive pressure and an analysis of stability against sliding, but these matters call for little comment other than that he chooses to ignore wall friction in passive pressure calculations.

When the book was about to go to press he added a summary of shear test results obtained in July and August 1910 by Frontard on the clay fill of an earth dam at Charmes in the Haute-Marne (see No. 30). The tests, made at Résal's instigation, confirmed the Coulomb criterion  $s = c + \sigma tan\phi$  and typically showed  $c = 2t/m^2$ and  $\phi = 8^0$ . As the tests approximated to the undrained condition a value of  $\phi = 8^0$  is not surprising, but at the time it was regarded with some astonishment: (see the closing remarks of Cain in the discussion on his paper, Resal's book having by that time arrived in the United States).

#### **Biographies**

Jean Résal (1854-1919) was distinguished as a bridge designer and from 1896 as professor of

the strength of materials at the Ecole des Ponts et Chaussées (Annls Ponts Chauss; 10th series, 1920, vol.55, pp. 147-168).

William Cain (1847-1930), after experience on railway works, entered academic life and from 1888 to 1920 was professor of engineering at the University of North Carolina (Trans Am. Soc. Civ. Engrs; 1931, vol.95, pp. 1467-1472).

30. Notes on shear tests by Frontard and Bell, and the period 1910-26

The tests by Jean Frontard in 1910 (see No.29) are described in his paper 'Notice sur l'accident de la digue de Charmes' (Annls 9th series, 1914, vol.22, pp.173-280). A year later, Arthur Langtry Bell built the first practical shear box apparatus and in 1911-12 made tests on eight clays, two of them in the form of undisturbed samples. Like Frontard's, the tests were fairly quick, approximating to the undrained condition, and as the clays were fully saturated, or nearly so, the  $\phi$  values tended to be even lower than the 8<sup>0</sup> obtained by Frontard; indeed for the softer clays  $\phi$  approached zero.

Undaunted by these results, Bell used them in designing a large dock wall at Rosyth and, in 1913, checked the calculated active pressure by direct observation with pressure cells. His paper 'Lateral pressure and resistance of clay and the supporting power of clay foundations' was published in 1915 (Min. Proc. Inst. Civ. Engrs; vol. 199, pp. 233-272). It stands on the threshold of modern soil mechanics.

Frontard's tests and Bell's important work at Rosyth are fully discussed by Skempton (1958), and will not be considered further here as they properly belong to the opening phase of a new era, which lies beyond the scope of the present lecture.

This new phase of activity includes the research by Atterberg on soil plasticity and particle size (his paper on liquid and plastic limits appeared in 1911), the introduction of slip-circle analysis in 1916 (see the excellent historical account by Petterson, 1955), and the field and laboratory work carried out between 1914 and 1920 by the Geotechnical Commission of the Swedish State Railways (see Bjerrum and Flodin, 1960).

Then came Terzaghi's fundamental experiments, published 1921-24, from which emerged the principle of effective stress (Skempton, 1960) and his monumental 'Erdbaumechanik' (Vienna, 1925), followed in the same year by the 'Engineering News Record' articles referred to in the Introduction. These appeared in book form in 1926, under the title 'Principles of soil mechanics', exactly 150 years after the publication of Coulomb's paper.

#### REFERENCES

- BINNIE, G.M. (1976). The evolution of British dams. Trans. Newcomen Soc., vol. 47, pp. 207-224.
- BJERRUM, L., CASAGRANDE, A., PECK, R.P. and SKEMPTON, A.W. (1960). From theory to practice in soil mechanics. New York: Wiley.
- BJERRUM, L. and FLODIN, N. (1960). The development of soil mechanics in Sweden. Géotechnique, vol. 10, pp. 1-18.

- BOUSSINESQ, J. (1883). Note on Mr G.H. Darwin's paper on the horizontal thrust of a mass of sand. Min. Proc. Instn Civ. Engrs, vol. 72, pp. 262-271.
- BOUSSINESQ, J. (1885). Reprint of Application des potentiels, with preface by A. Caquot. Paris: Blanchard, 1969.
- CAIN, W. (1911). Experiments on retaining walls and pressures on tunnels. Trans. Am. Soc. Civ. Engrs, vol. 72, pp. 403-448.
- CAQUOT, A. and KERISEL, J. (1949). Traité de mécanique des sols. Paris: Gauthier-Villars.

CHEN, W.F. (1975). Limit analysis and soil plasticity. Amsterdam: Elsevier.

FELD, J. (1923). The accurate experimental determination of lateral earth pressure, together with a résumé of previous experiments. Trans. Am. Soc. Civ. Engrs, vol. 86, pp. 1448-1505.

- FLODIN, N. and BROMS, B.B. (1977). Historical development of civil engineering in soft clay. International symposium on soft clay, Bangkok.
- GIRARD, P.S. (1831). Mémoires sur la canal de l'Ourcq. Paris: Carilian-Goeury.
- GLOSSOP, R. (1960). The invention and development of injection processes. Part I: 1802-1850. Géotechnique, vol. 10, pp. 91-100.
- GLOSSOP, R. (1961). The invention and development of injection processes. Part II: 1850-1960. Géotechnique, vol. 11, pp. 255-279.
- GLOSSOP, R. (1976). The invention and early use of compressed air to exclude water from shafts and tunnels during constructior. Géotechnique, vol. 26, pp. 253-280.
- HALL, H.P. (1954). A historical review of investigations of seepage toward wells. J. Boston Soc. Civ. Engrs, vol. 41, pp. 251-311.
- HASKOLL, W.D. (1846). The assistant engineer's railway guide in boring. London: Williams.
- HEYMAN, J. (1972). Coulomb's memoir on statics. Cambridge University Press.
- KERISEL, J. (1956). Historique de la mécanique des sols en France jusqu'au 20e siècle. Géotechnique, vol. 6, pp. 151-166.
- MAYNIEL, J.H. (1808). Traité experimentale analytique et practique de la poussée des terres et des murs de revêtement. Paris: Colas.
- MINARD, C.J. (1841). Cours de construction des ouvrages qui etablissent la navigation des rivières et des canaux. Paris: Carilian-Goeury et Dalmont.
- MÜLLER-BRESLAU, H.F.B. (1906). Erddruck auf Stützmauern. Stuttgart: Kreuner.
- PECK, R.B. (1948). History of building foundations in Chicago. Bull. Ser. Univ. Ill. Engng Exp. Stn., no. 373.
- PETTERSON, K.E. (1955). The early history of circular sliding surfaces. Géotechnique, vol. 5, pp. 275-296.

POLUBARINOVA-KOCHINA, P.Y. (1962). Theory of ground water movement (translated by R. De Weist). Princeton University Press.

- ROUSE, H. and INCE, S. (1957). History of hydraulics. Iowa State University.
- ROWE, P.W. (1969). Osborne Reynolds and dilatancy. Géotechnique, vol. 19, pp. 1-5.
- SCHRIEVER, W.R. (1956). Translation of Landslides in clays, by A. Collin (1846), with memoir on Collin by A.W. Skempton. Toronto University Press.

- SKEMPTON, A.W. (1958). Arthur Langtry Bell (1874-1956) and his contribution to soil mechanics. Géotechnique, vol. 8, pp. 143-157.
- SKEMPTON, A.W. (1960). Terzaghi's discovery of effective stress. In From theory to practice in soil mechanics, by L. Bjerrum et al., pp. 42-53. New York: Wiley.
- SKINNER, A.E. (1969). Note on the influence of interparticle friction on the shearing strength of a random assembly of spherical particles. Géotechnique, vol. 19, pp. 150-157.
- SOKOLOVSKI, W.V. (1960). Statics of soil media (translated by D.H. Jones and A.N. Schofield). London: Butterworths.

- TAYLOR, D.W. (1948). Fundamentals of soil mechanics. New York: Wiley.
- TERZAGHI, K. (1925). Principles of soil mechanics. Engng News Rec., vol. 95, 8 articles between p. 742 and p. 1068. Reprinted in 1926. New York: McGraw-Hill.
- TERZAGHI, K. (1943). Theorectical soil mechanics. New York: Wiley.
- TIMOSHENKO, S.P. (1953). History of strength of materials. New York: McGraw-Hill.
- TODHUNTER, I. and PEARSON, K. (1893). A history of the theory of elasticity and of the strength of materials. Cambridge University Press.

## Bibliography

## PRINTED BOOKS AND PAPERS

An asterisk indicates that the paper is reprinted in the main body of the text.

## 1938

1. Settlement analysis of engineering structures. Engineering, 1938, 146, 402–406.

#### 1941

(With L. F. Cooling.) Some experiments on the consolidation of clay. J. Instn Civ. Engrs, 1941, 16, 381-398.

#### 1942

- 3. (With L. F. Cooling.) A laboratory study of London Clay. J. Instn Civ. Engrs, 1942, 17, 251-272. (Reprinted in A century of soil mechanics, pp. 209-234. London: Institution of Civil Engineers, 1969.)
- 4. An investigation of the bearing capacity of a soft clay soil. J. Instn Civ. Engrs, 1942, 18, 307-321. (Reprinted in A century of soil mechanics, pp. 265-279. London: Institution of Civil Engineers, 1969.)
- 5. Some principles of foundation behaviour. J. R. Inst. Br. Archit., 1942, 50, 3-6. (Reprinted in Building science, edited by D. D. Harrison, pp. 41-51. London: Allen and Unwin, 1948.)

#### 1943

- 6. Soil mechanics. In A geology for engineers, by F. G. H. Blyth, pp. 260–276. London: Arnold, 1943.
- (Measurements of load on the lining of a deep tunnel in London Clay.) Discussion on paper by L. G. Groves on tunnel linings. J. Instn Civ. Engrs, 1943, 20, 53-56.

## 1944

- 8. Notes on the compressibility of clays. Q. J. Geol. Soc. Lond., 1944, 100, 119-135.
- (Stability analysis of Hollowell dam.) Discussion on paper by A. G. McLellan on the Hollowell Reservoir scheme for Northampton. *Trans. Instn Wat. Engrs*, 1944, 49, 204–208.

## 1945

- 10.\* A slip in the west bank of the Eau Brink Cut. J. Instn Civ. Engrs, 1945, 24, 267–287.
- (With R. Glossop.) Particle size in silts and clays. J. Instn Civ. Engrs, 1945, 25, 81–105. (Reprinted in A century of soil mechanics, pp. 381–407. London: Institution of Civil Engineers, 1969.)
- 12. Earth pressure and the stability of slopes. In The principles and application of soil mechanics, pp. 31-61. London: Institution of Civil Engineers, 1946.

## 1946

Alexandre Collin (1808–90). Pioneer in soil mechanics. Trans. Newcomen Soc., 1950 for 1945–47, 25, 91–104. French translation by J. Kerisel in Annls Ponts Chauss., 1956, 126, 317–340.

1948

- 14.\* A study of the geotechnical properties of some postglacial clays. *Géotechnique*, 1948, **1**, 7–22.
- 15. The  $\varphi = 0$  analysis of stability and its theoretical basis. Proc. 2nd Int. Conf. Soil Mech., Rotterdam, 1948, 1, 72–78.
- 16. The geotechnical properties of a deep stratum of post-glacial clay at Gosport. *Ibid*, 1, 145-150.
- 17. (With H.Q. Golder.) The angle of shearing resistance in cohesive soils for tests at constant water content. *Ibid*, **1**, 185–192.
- 18. A study of the immediate triaxial test on cohesive soils. *Ibid*, 1, 192–196.
- 19. The rate of softening in stiff-fissured clays, with special reference to London Clay. *Ibid*, 2, 50-53.
- 20. (With H. Q. Golder.) Practical examples of the  $\varphi = 0$  analysis of stability of clays. *Ibid*, 2, 63-70.
- 21. A possible relationship between the true cohesion and the mineralogy of clays. *Ibid*, 7, 45-46.
- 22. The effective stresses in saturated clays strained at constant volume. Proc. 7th Int. Congr. Appl. Mech., London, 1948, 1, 378–392.
- 23. Vane tests in the alluvial plain of the River Forth near Grangemouth. *Géotechnique*, 1948, 1, 111-124.

#### 1949

- 24. Alexandre Collin, a note on his pioneer work in soil mechanics. *Géotechnique*, 1949, 1, 216-221.
- 25. (Bearing capacity of screw cylinders in clay.) Discussion on paper by Guthlac Wilson. J. Instn Civ. Engrs, 1949, 34, 76-81.

## 1950

- 26. (With A. W. Bishop.) The measurement of the shear strength of soils. *Géotechnique*, 1950, **2**, 90–108.
- 27. Soil mechanics in relation to geology. Proc. Yorks. Geol. Soc., 1953 for 1950, 29, 33-62.

## 1951

28.\* The bearing capacity of clays. Building Research Congress, London, 1951, 1, 180-189.

## 1952

29. (With R. D. Northey.) The sensitivity of clays. *Géotechnique*, 1952, **3**, 30-53.

- 30. (With W. H. Ward.) Investigations concerning a deep cofferdam in the Thames estuary clay at Shellhaven. *Géotechnique*, 1952, **3**, 119–139.
- (With A. A. Yassin and R. E. Gibson.) Théorie de la force portante des pieux dans la sable. *Annls Inst. Tech. Bâtim.*, 1953 for 1952, 16, 285–290.

- 32\* The colloidal "activity" of clays. Proc. 3rd Int. Conf. Soil Mech., Zurich, 1953, 1, 57-61.
- 33. (With D. J. Henkel.) The post-glacial clays of the Thames estuary at Tilbury and Shellhaven. *Ibid*, 1, 302–308.
- 34. Earth pressure, retaining walls, tunnels and strutted excavations (General Report). *Ibid*, 2, 353-361.
- 35. (Settlement of pile groups in sand.) Discussion. *Ibid*, **3**, 172.
- 36. The engineers of the English river navigations, 1620–1760. *Trans. Newcomen Soc.*, 1958 for 1953–55, **29**, 25–54.
- 1954
- A foundation failure due to clay shrinkage caused by poplar trees. Proc. Instn Civ. Engrs, Part 1, 1954, 3, 66-86.
- 38. (With A. W. Bishop.) Soils. In *Building materials*, edited by M. Reiner, pp. 417–482. Amsterdam: North Holland, 1954.
- 39. (With D. J. Henkel.) A landslide at Jackfield, Shropshire, in a heavily over-consolidated clay. *Proc. Eur. Conf. Stability Earth Slopes, Stockholm*, 1954, **1**, 90–101. (Reprinted in *Géotechnique*, 1955, **5**, 131–137.)
- 40.\* The pore-pressure coefficients A and B. Géotechnique, 1954, 4, 143-147.
- (Sensitivity of clays, and the c/p ratio in normally consolidated clays.) Discussion on paper by T. W. Lambe on the structure of inorganic soil. Proc. Am. Soc. Civ. Engrs, 1954, Separate 478, 19-22.

- 42. (With A. W. Bishop.) The gain in stability due to pore-pressure dissipation in a soft clay foundation. *Proc. 5th Int. Congr. Large Dams, Paris*, 1955, 1, 613–638.
- 43. (With R. B. Peck and D. H. MacDonald.) Settlement analyses of six structures in Chicago and London. Proc. Instn Civ. Engrs, 1955, Part 1, 4, 525-544.
- 44. Foundations for high buildings. Proc. Instn Civ. Engrs, Part III, 1955, 4, 246–269.
- 45. (With D. H. MacDonald.) A survey of comparisons between calculated and observed settlements of structures on clay. Proceedings of conference on the correlation between calculated and observed stresses and displacements in structures, Preliminary volume, pp. 318-337. London: Institution of Civil Engineers, 1955.
- 46. Soil mechanics and its place in the university (Inaugural Lecture). London: Imperial College, 1955, 22 pp.

1956

47. Alexandre Collin (1808–1890) and his pioneer work in soil mechanics. In *Landslides in clays*, by Alexandre Collin, 1846, translated by W. R. Schriever, pp. xi-xxxiv. Toronto University Press, 1956.

- (With D. H. MacDonald.) The allowable settlements of buildings. Proc. Instn Civ. Engrs, 1956, Part III, 5, 727-768.
- The origin of iron beams. Actes du VIII Congr. Int. d'Histoire des Sciences, Florence, 1956, 3, 1029–1039.
- 50. (With H. R. Johnson.) William Strutt's cotton mills. Trans. Newcomen Soc., 1960 for 1955–57, 30, 179–205.
- 51. Particle separation in clays. *Géotechnique*, 1956, **6**, 193-194.
- 1957
- 52. (With D. J. Henkel.) Tests on London Clay from deep borings at Paddington, Victoria and the South Bank. Proc. 4th Int. Conf. Soil Mech., London, 1957, 1, 100–106.
- 53\* (With F. A. DeLory.) Stability of natural slopes in London Clay. *Ibid*, 2, 378-381.
- 54.\* (With L. Bjerrum.) A contribution to the settlement analysis of foundations on clay. *Géotechnique*, 1957, 7, 168–178.
- 55. Canals and river navigations before 1750. In *A history of technology*, vol. 3, edited by Charles Singer *et al.*, pp. 438–470. Oxford: Clarendon Press, 1957.
- 56. (Pore pressure observations.) Discussion on paper by G. A. R. Sheppard and L. B. Aylen on the Usk scheme for water supply of Swansea. *Proc. Instn Civ. Engrs*, 1957, 7, 267–269.
- (Further data on the c/p ratio in normally consolidated clays.) Discussion on paper by Henry Grace and J. K. M. Henry on Hong Kong airport. *Ibid*, 305-307.

1958

 Arthur Langtry Bell (1874–1956) and his contribution to soil mechanics. Géotechnique, 1958, 8, 143–157.

1959

- 59. The evolution of steel-frame buildings. *Guilds' Engr*, 1959, **10**, 37–51.
- 60. Heave of London Clay: correlation with laboratory tests. *Géotechnique*, 1959, **9**, 145–146.
- 61.\* Cast in-situ bored piles in London Clay. Géotechnique, 1959, 9, 153–173.

- 62\* Effective stress in soils, concrete and rocks. In Pore pressure and suction in soils, pp. 4-16. London: Butterworths, 1961.
- 63. (With D. J. Henkel.) Field observations on pore pressures in London Clay. *Ibid*, pp. 81-84.
- 64. (Co-editor with A. Casagrande, L. Bjerrum and R. B. Peck.) From theory to practice in soil mechanics. Selections from the writings of Karl Terzaghi. New York: Wiley, 1960, viii + 425 pp.
- 65. Terzaghi's discovery of effective stress. *Ibid*, pp. 42-53.
- 66. The pore-pressure coefficient in saturated soils. *Géotechnique*, 1960, **10**, 186–187.
- 67. The Boat Store, Sheerness (1858–60) and its place in structural history. *Trans. Newcomen Soc.*, 1961 for 1959–60, **32**, 57–75.

<sup>1953</sup> 

<sup>1955</sup> 

## BIBLIOGRAPHY

279

- (With Eric de Maré.) The Sheerness Boat Store 1858–60. J. R. Inst. Br. Archit., 1961, 68, 318–324.
- 69.\* Horizontal stresses in an over-consolidated Eocene clay. Proc. 5th Int. Conf. Soil Mech., Paris, 1961, 1, 351-357.
- Presidential Address to the 5th International Conference on Soil Mechanics and Foundation Engineering. *Ibid*, 3, 39–42.
- 71. Discussion on cohesion intercept in stability analysis. *Ibid*, **3**, 349–350.
- 72.\* (With D. J. Brown.) A landslide in boulder clay at Selset, Yorkshire. *Géotechnique*, 1961, **11**, 280-293.

- 73. (With H. R. Johnson.) The first iron frames. Archit. Rev. Lond., 1962, 131, 175–186.
- 74. Portland cements, 1843–1887. Trans. Newcomen Soc., 1964 for 1962–63, 35, 117–151.

1963

- 75. (With P. Catin.) A full-scale alluvial grouting test at the site of Mangla Dam. In *Grouts and drilling muds*, pp. 131–135. London: Butterworths, 1963.
- (With V. A. Sowa.) The behaviour of saturated clays during sampling and testing. *Géotechnique*, 1963, 13, 269-290.

1964

- 77. (With D. J. Henkel, J. L. Knill and D. G. Lloyd.) Stability of the foundations of Monar Dam. *Trans.* 8th Int. Congr. Large Dams, Edinburgh, 1964, 1, 425-441.
- Long-term stability of clay slopes (Fourth Rankine Lecture). Géotechnique, 1964, 14, 77–101. (Reprinted in Milestones in soil mechanics, pp. 81–107. London: Institution of Civil Engineers, 1975.)

## 1965

- 79. (With P. LaRochelle.) The Bradwell slip; a shortterm failure in London Clay. *Géotechnique*, 1965, 15, 221-242.
- (Discussion on relevance of laboratory tests to slope stability problems.) Proc. 6th Int. Conf. Soil Mech., Montreal, 1965, 3, 278–280.
- 81. (Effect of rate of displacement on residual strength of London Clay.) Discussion on paper by R. Haefeli on creep and progressive failure. *Ibid*, **3**, 551–552.

#### 1966

- 82. Bedding-plane slip, residual strength and the Vaiont landslide. *Géotechnique*, 1966, **16**, 82–84.
- Some observations on tectonic shear zones. Proc. 1st Congr. Int. Soc. Rock Mech., Lisbon, 1966, 1, 329-335.
- Large bored piles (Summing up). In Large bored piles, pp. 155–157. London: Institution of Civil Engineers, 1966.

1967

- 85\* (With G. M. Binnie and J. F. F. Clark.) The effect of discontinuities in clay bedrock on the design of dams in the Mangla Project. *Trans. 9th Int. Congr. Large Dams, Istanbul*, 1967, 1, 165–183.
- 86\* (With D. J. Petley.) The strength along structural discontinuities in stiff clays. Proc. Geotech. Conf., Oslo, 1967, 2, 29-46.

1968

 (Geology of the Mangla Dam project.) Discussion on paper by G. M. Binnie *et al.* on engineering of Mangla. Proc. Instn Civ. Engrs, 1968, 41, 133–137.

## 1969

- 88. (With R. L. Schuster and D. J. Petley.) Joints and fissues in the London Clay at Wraysbury and Edgware. *Géotechnique*, 1969, **19**, 205–217.
- (With J. N. Hutchinson.) Stability of natural slopes and embankment foundations (General Report). *Proc. 7th Int. Conf. Soil Mech.*, *Mexico City*, 1969, 3, 151–155.
- 90. (With J. N. Hutchinson.) Stability of natural slopes. *Ibid*, State of the art volume, 291–340.
- 91. (Co-editor with L. F. Cooling and A. L. Little.) A century of soil mechanics. London: Institution of Civil Engineers, 1969, vi + 482 pp.

1970

- 92.\* The consolidation of clays by gravitational compaction. Q. J. Geol. Soc. Lond., 1970, 125, 373-412.
- 93. First-time slides in over-consolidated clays. Géotechnique, 1970, 20, 320-324.
- 94. (With D. J. Petley.) Ignition loss and other properties of peats and clays from Avonmouth, Kings Lynn and Cranberry Moss. Géotechnique, 1970, 20, 343–356.
- 95. Alfred John Sutton Pippard, 1891–1969. Biogr. Mem. Fellows R.Soc., 1970, 16, 463–478.

1971

- 96. The Smeatonians. Duo-centenary notes on the Society of Civil Engineers 1771–1971. London: Institution of Civil Engineers, 1971, 18 pp.
- The publication of Smeaton's reports. Notes Rec. R. Soc. Lond., 1971, 26, 135–155.
- 98. The Albion Mill foundations. *Géotechnique*, 1971, **21**, 203–210.
- 99. Samuel Wyatt and the Albion Mill. Archit. Hist., 1971, 14, 53-73.
- 100. (With E. C. Wright.) Early members of the Smeatonian Society of Civil Engineers. *Trans. Newcomen Soc.*, 1974 for 1971–72, 44, 23–42.

1972

 (With K. R. Early.) Investigations of the landslide at Walton's Wood, Staffordshire. Q. J. Engng Geol., 1972, 5, 19-42.

1973

- 102. (With J. Brown.) John and Edward Troughton, mathematical instrument makers. Notes Rec. R. Soc. Lond., 1973, 27, 233-262.
- 103. Laurits Bjerrum 1918–1973: a tribute. *Géotechnique*, 1973, **23**, 315.

1974

- 104. William Chapman (1749–1832), civil engineer. Trans. Newcomen Soc., 1976 for 1973–74, 46, 45–82.
- 105. (With R. J. Chandler.) The design of permanent cutting slopes in stiff fissured clays. *Géotechnique*, 1974, 24, 457-466.

1975

106. A history of the steam dredger. Trans. Newcomen Soc., 1977 for 1974-76, 47, 97-116.

<sup>1961</sup> 

<sup>1962</sup> 

107. Notes on the origins and early years of the British Geotechnical Society. Géotechnique, 1975, 25, 635-646.

1976

- 108. Introductory remarks. A discussion on valley slopes and cliffs in southern England: morphology, mechanics and Quaternary history. *Phil. Trans. R. Soc.*, Series A, 1976, 283, 423–426.
- 109<sup>\*</sup> (With A. G. Weeks.) The Quaternary history of the Lower Greensand escarpment and Weald Clay vale near Sevenoaks, Kent. *Phil. Trans. R. Soc.*, Series A, 1976, **283**, 493–526.
- 110. (With R. J. Chandler, G. A. Kellaway and R. J. Wyatt.) Valley slope sections in Jurassic strata near Bath, Somerset. *Phil. Trans. R. Soc.*, Series A, 1976, **283**, 527–556.

1977

- 111. The engineers of Sunderland harbour, 1718–1817. Ind. Archaeol. Rev. 1977, 1, 103–125.
- 112. Dr. Leonard Frank Cooling, 1903–1977. Géotechnique, 1977, 27, 265–270.
- 113.\* Slope stability of cuttings in brown London Clay. Proc. 9th Int. Conf. Soil. Mech., Tokyo, 1977, 3, 261-270.
- 114. Early printed reports and maps (1665–1850) in the Library of the Institution of Civil Engineers. London: Institution of Civil Engineers, 1977, xx + 84 pp.

1978

115. (With A. Andrews.) Cast iron edge rails at Walker Colliery 1798. Trans. Newcomen Soc., 1978, 48, 119–122.

- 1979
- 116. (With C. Hadfield.) William Jessop, Engineer. Newton Abbot: David and Charles, 1979, 315 pp.
- 117. Engineering in the Port of London 1789–1808. *Trans. Newcomen Soc.*, 1980 for 1978–79, 50, 87–108.
- 118.\* Landmarks in early soil mechanics. Proc. 7th Eur. Conf. Soil. Mech., Brighton, 1979, 5, 1-26.

1980

119. Telford and the design for a new London Bridge. In *Thomas Telford, Engineer*, edited by A. Penfold, pp. 62–83. London: Thomas Telford Ltd, 1980.

1981

- 120. A bibliographical catalogue of the collection of works on soil mechanics 1764–1950. London: Imperial College, 1981, 73 pp.
- 121. Casagrande: one of the great civil engineers (Obituary notice). New Civ. Engr, 1981, 1 Oct., p. 14.
- 122. (Editor.) John Smeaton, FRS. London: Thomas Telford Ltd, 1981, vi + 291 pp. (Includes chapters by Skempton on fen drainage, harbours, Smeaton's papers and drawings, a list of his works and (jointly with T. Turner) Smeaton's life.)

1982

123. Engineering in the Port of London, 1808–1834. *Trans. Newcomen Soc.*, 1983 for 1981–82, 53, 73–94.

1983

124. The engineering works of John Grundy (1719–1783). J. Lincolnshire Hist. Archaeol., 1984.

#### REPORTS

Selected from the archive of reports kept in the Civil Engineering Department, Imperial College. Listed under job numbers.

**ISSUED FROM BUILDING RESEARCH STATION** 

- 66. Note on settlement analysis for a proposed gas holder at Weston-super-Mare. For Sir Cyril Kirkpatrick & Partners. December 1937.
- 71. Report of investigation on samples of foundation soils from the site of the Calcutta main pier of the new Howrah Bridge. For Messrs Rendel, Palmer & Tritton. September 1938.
- 72. Report of investigation on samples of London Clay from beneath pier 4 of the new Waterloo Bridge. For Messrs Rendel, Palmer & Tritton. July 1938.
- 94. Report of investigation on a foundation failure of a stores building at Kippen, near Stirling, and an analysis of the revised foundation designs. For Messrs Kennedy & Donkin. August 1940.
- 99. Report of investigation on a foundation failure at the Royal Ordnance Factory, Wrexham. For the Ministry of Works and Buildings. June 1941.
- 104. Report of investigation on the stability of the earth dam at Muirhead Reservoir, Dalry, Ayrshire. For Messrs Babtie, Shaw & Morton. July 1942.
- 110. Soil Mechanics Record No. 3. The failure of a retaining wall at Kensal Green. June 1942.
- 129. Report of investigation on the stability of the proposed flood relief channel for the Great Ouse Catchment Board. April 1944.

- 138. Soil Mechanics Record No. 14. An investigation of the failure of a retaining wall at Mill Lane, Cricklewood. April 1943.
- 156. Report of investigation on a landslip at Seagrove Bay, Isle of Wight. For F. E. Wentworth-Sheilds, Esq., O.B.E., M.I.C.E. January 1944.

## **ISSUED FROM IMPERIAL COLLEGE**

- 1. Report on foundations for a new grain silo at Durban. For H. J. Gough, C.B., F.R.S. November 1947.
- 16. Report on the design of a cofferdam for the new dock at Lackenby on the River Tees. For Soil Mechanics Ltd. May 1947.
- 29. An investigation of a slip in the boulder clay cliffs at Percy Square, Tynemouth. For Mr. M. Nachshen. August 1948.
- 56. Final report on the geological and engineering problems associated with the Denes at Peterlee. For the Peterlee Development Corporation. August 1949.
- 63. Report on the pump house cofferdam at Shellhaven. For John Mowlem & Co. May 1949.
- 65. Report on an investigation relating to the design of an earth dam for Hanningfield Reservoir, South Essex. For Messrs Binnie, Deacon & Gourley and T. & C. Hawksley. February 1950.
- 73. (With D. J. Henkel.) Report on the use of sand

107. Notes on the origins and early years of the British Geotechnical Society. Géotechnique, 1975, 25, 635-646.

1976

- 108. Introductory remarks. A discussion on valley slopes and cliffs in southern England: morphology, mechanics and Quaternary history. *Phil. Trans. R. Soc.*, Series A, 1976, 283, 423–426.
- 109<sup>\*</sup> (With A. G. Weeks.) The Quaternary history of the Lower Greensand escarpment and Weald Clay vale near Sevenoaks, Kent. *Phil. Trans. R. Soc.*, Series A, 1976, **283**, 493–526.
- 110. (With R. J. Chandler, G. A. Kellaway and R. J. Wyatt.) Valley slope sections in Jurassic strata near Bath, Somerset. *Phil. Trans. R. Soc.*, Series A, 1976, **283**, 527–556.

1977

- 111. The engineers of Sunderland harbour, 1718–1817. Ind. Archaeol. Rev. 1977, 1, 103–125.
- 112. Dr. Leonard Frank Cooling, 1903–1977. Géotechnique, 1977, 27, 265–270.
- 113.\* Slope stability of cuttings in brown London Clay. Proc. 9th Int. Conf. Soil. Mech., Tokyo, 1977, 3, 261-270.
- 114. Early printed reports and maps (1665–1850) in the Library of the Institution of Civil Engineers. London: Institution of Civil Engineers, 1977, xx + 84 pp.

1978

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- 1979
- 116. (With C. Hadfield.) William Jessop, Engineer. Newton Abbot: David and Charles, 1979, 315 pp.
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- 118.\* Landmarks in early soil mechanics. Proc. 7th Eur. Conf. Soil. Mech., Brighton, 1979, 5, 1-26.

1980

119. Telford and the design for a new London Bridge. In *Thomas Telford, Engineer*, edited by A. Penfold, pp. 62–83. London: Thomas Telford Ltd, 1980.

1981

- 120. A bibliographical catalogue of the collection of works on soil mechanics 1764–1950. London: Imperial College, 1981, 73 pp.
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1983

124. The engineering works of John Grundy (1719–1783). J. Lincolnshire Hist. Archaeol., 1984.

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- 110. Soil Mechanics Record No. 3. The failure of a retaining wall at Kensal Green. June 1942.
- 129. Report of investigation on the stability of the proposed flood relief channel for the Great Ouse Catchment Board. April 1944.

- 138. Soil Mechanics Record No. 14. An investigation of the failure of a retaining wall at Mill Lane, Cricklewood. April 1943.
- 156. Report of investigation on a landslip at Seagrove Bay, Isle of Wight. For F. E. Wentworth-Sheilds, Esq., O.B.E., M.I.C.E. January 1944.

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- 1. Report on foundations for a new grain silo at Durban. For H. J. Gough, C.B., F.R.S. November 1947.
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- 29. An investigation of a slip in the boulder clay cliffs at Percy Square, Tynemouth. For Mr. M. Nachshen. August 1948.
- 56. Final report on the geological and engineering problems associated with the Denes at Peterlee. For the Peterlee Development Corporation. August 1949.
- 63. Report on the pump house cofferdam at Shellhaven. For John Mowlem & Co. May 1949.
- 65. Report on an investigation relating to the design of an earth dam for Hanningfield Reservoir, South Essex. For Messrs Binnie, Deacon & Gourley and T. & C. Hawksley. February 1950.
- 73. (With D. J. Henkel.) Report on the use of sand

drains for the acceleration of consolidation in clays and silts. For Messrs Soil Mechanics Ltd. March 1950.

- 74. Auckland Harbour Bridge foundations. Report for Freeman Fox and Partners. March 1951.
- 75. Tilbury Generating Station. Report on the River Bank and proposed jetty. For Sir Alexander Gibb & Partners. July 1951.
- 86. (With A. W. Bishop.) Final report on the foundations of Chew Stoke Dam, Bristol Waterworks Company. For Thomas Hawksley, Esq. March 1956.
- 92. (With A. W. Bishop.) Report on the design of the inner block dam for the reconstruction of Gallions Lock, Royal Albert Dock, Port of London. For John Mowlem & Co. February 1952.
- 108. Report on laboratory tests and stability analysis for Sasumua Dam, Kenya. For Howard Humphreys & Sons. June 1953.
- 129. (With A. W. Bishop.) Report on the design of Glascarnoch Dam. For Sir Alexander Gibb & Partners. February 1954.
- 155. (With D. J. Henkel.) Report on foundations for Sydenham Hill flats. For Ove Arup & Partners. January 1955.
- 169. Investigations concerning the foundations for a proposed 25-storey office building near Victoria Station, Westminster. For Holland, Hannen & Cubitts Ltd. June 1955.
- 171. (With D. J. Henkel.) Mombasa Harbour. Berths 9 and 10. Second report for Coode & Partners. March 1956.
- 175. (With D. J. Henkel.) Report on the landslips and remedial works on Bredon Hill. For Norman Pugh, Esq., Water Engineer, Coventry Corporation. May 1958.
- 191. (With D. J. Henkel.) *The London Clay at Bradwell*. Report for G.E.C. Simon-Carves Atomic Energy Group. February 1957.
- 195. Report on settlements and remedial measures at the Research Laboratories Building, Avonmouth. For the Imperial Smelting Corporation. June 1957.
- 201. (With D. J. Henkel.) Report on the stability of the cliffs of the Avon Gorge in the vicinity of the Clifton Docks Railway. For the Bristol City Engineer. April 1959.
- 205. (With D. J. Henkel.) Report on the foundations for the 18-storey blocks of flats on the Brandon Estate, Southwark. For Felix J. Samuely & Partners. July 1959.
- 211. Mangla Dam Report on visit to site and preliminary consideration on design. For G. M. Binnie, Esq., April 1958.
- 211. Mangla and Jari Dams. Site visit (including note on shear zones at Jari). For Binnie & Partners. November 1962.

- 211. Notes on a site visit to Mangla. For Binnie & Partners. February 1967.
- 211. Interim report on Siwalik stratigraphy in the Mangla Region. April 1967.
- 211. Jari fossil locality. Letter to Dr. M. S. Money, Mangla Dam Project. February 1968.
- 273. (With C. C. Guidi, A. Croce and E. Schultze.) Caratteristiche Geotecniche del Sottosuolo della Torre. Printed report in *Ricerche e Studi su la Torre Pendente di Pisa*, vol. 1, pp. 179–199. Florence: Ministero dei Lavori Pubblici Commissione per il Consolidamento della Torre Pendente di Pisa, 1971.
- 279. (With D. J. Petley.) Sevenoaks By-pass. Shear tests on clays. Report for Kent County Council. October 1967.
- 296. Report on the foundations of St. Paul's Cathedral and the underlying Potters Clay. For Freeman Fox & Partners. December 1972.
- 299. (With S. W. Smotrych, F. A. Hibbert and J. R. Haynes.) Holocene stratigraphy and sea-level changes near Avonmouth, Gloucestershire. 1969.
- 300. (With J. N. Hutchinson.) Slope problems on the north-east side of Cave Hill, near Belfast. For the Ministry of Development. March 1970.
- 301. Report on the investigations and remedial works at Burderop and Hodson landslides on the M4 motorway near Swindon. For Sir Alexander Gibb & Partners. July 1972.
- 324. (With G. M. Binnie, N. Ambraseys, A. Bozovic and D. Deere.) Mornos Dam and Reservoir. Reports 1-5 of Board of Advisers. For the Ministry of Public Works, Athens. July 1975 to July 1978.
- 324. Note on the clay breccias at Mornos Dam site. For the Ministry of Public Works, Athens. June 1976.
- 337. (With R. J. Chandler.) Report on the Mam Tor landslide. For Derbyshire County Council. September 1981.
- 338. (With N. Ambraseys and A. Bozovic.) *Bramianos* Dam. First and Second Reports of the Board of Advisers. For the Ministry of Public Works, Athens. June and July 1980.
- 339. Avalon Lakes, Somerset Levels. Letter to B. A. Tinkler, Esq., Wessex Water Authority. July 1983.
- 344. Salisbury Cathedral. Report on foundation settlements and bearing capacity. For Gifford & Partners. November 1983.
- 345. (With A. L. Little.) Report on Kalabagh Dam proposals. For Kalabagh Consultants. April 1982.
- 345. (With A. L. Little). Kalabagh Dam project. Report on site visit. For Kalabagh Consultants. June 1983.
- 345. Kalabagh Dam project. Report on tests on sample 910, with some considerations on the strengths to be used in design for seismic conditions. November 1983.