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Foreign Exchange in Practice

The New Environment

Third edition

Steve Anthony

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Preface

This book is written for participants in the foreign exchange market. It attempts to explain the concepts involved in foreign exchange and the application of these concepts to a large number of day-to-day situations. Numerous worked examples appear in the text, and practice problems are set out at the end of each chapter to enable the student to test her or his understanding. The solutions to the practice problems appear in the Appendix.

The first edition of this book was written as a textbook for the Citibank Bourse Course. The Bourse Course is a course on foreign exchange markets and is based around a simulation game. Some of the examples in this book refer to the fictitious currencies used in the Bourse Game.

A recurring theme throughout the book involves the interrelationship between interest rates and exchange rates. These two markets are inextricably linked. Changes in interest rates cause changes in exchange rates and vice versa. The pricing of forward exchange rates and currency options depends on the interest rates of the two currencies. Accordingly, considerable attention is given to interest rates, particularly in Chapter 2 and Chapter 4.

Two other themes that recur throughout the book are that arbitrage forces equilibrium pricing and that break-even rates occur where the two alternatives have equal value.

Preface to the third edition

The first edition was published in 1989 and the second edition in 1997. Examples in the earlier editions use rates that prevailed at the time. The 3rd edition covers a substantial amount of new subject matter including more financial mathematics, interest rate swaps and expanded discussion on exotic options. Examples have been updated to reflect rates at the time of writing and the introduction of the euro.

Sydney
June 2002

CHAPTER 1

Exchange Rates

This chapter introduces the basic conventions used to describe exchange rates and the profits or losses that result from changes in exchange rates. In pricing physical commodities, it is apparent that a particular commodity is being priced in terms of a particular currency. In exchange rate quotations, confusion sometimes arises because both the commodity being priced and the terms in which the commodity is being priced are currencies. To avoid this potential confusion, distinction is drawn between the commodity currency and the terms currency.

Definition
An *exchange rate* is the price of one currency expressed in terms of another currency.

Exchange rates:

$$£1 = \text{US}\$1.4500$$

$$€1 = \text{US}\$0.8560$$

$$\text{US}\$ = ¥124.50$$

The word *rate* means ratio – that is, one number divided by another. Expressed in simple mathematical form:

$$\frac{\text{US}\$1.4500}{£1} = \frac{\text{US}\$1{,}450{,}000}{£1{,}000{,}000} = 1.4500$$

COMMODITY CURRENCY AND TERMS CURRENCY

In every exchange rate quotation there are two currencies. The currency on the denominator is the currency being priced. It is known as the *commodity currency* or *base currency*. The exchange rate is quoted such that a fixed number of units (usually one) of the commodity currency are expressed in terms of a variable number of units of the other currency. The numerator currency is known as the *terms currency.*

In the exchange rate quotation £1 = US$1.4500, the commodity being priced is the pound sterling. One pound is equal to 1.4500 dollars. The pound is the commodity currency; the dollar is the terms currency. The quotation is often shown as £/US$1.4500. By market convention the commodity currency is displayed before the terms currency. Using ISO currency notation this would be written as GBP/USD 1.4500. The complete ISO 4217 Currency List can be found on `http://www.xe.net/gen/iso4217.htm`.

In the exchange rate quotation US$1 = ¥124.50, the dollar is the commodity currency and the yen is the terms currency. The dollar is priced in yen terms.

Notice that the dollar is the terms currency when quoted against the pound and euro but the commodity currency when quoted against yen. There is no fixed convention which dictates which currency should be the commodity currency.

The pound sterling is usually quoted as the commodity currency from the time when it was the principal world currency. With the rise to prominence of the US economy, most exchange rates are now generally quoted with the US dollar as the commodity currency. However, the old convention still applies for some of the currencies of former British Commonwealth countries such as Australia and New Zealand. The euro is generally quoted as the commodity currency.

A good rule of thumb is that the commodity currency is the currency of which there is *one* in the exchange rate quotation.

RECIPROCAL RATES

If the price of an apple is 20 cents it is possible to express the price as five apples for a dollar. Similarly, it is possible to change the terms in which an exchange rate is expressed by taking reciprocals.

EXAMPLE 1.1

Express the exchange rate quotation US$1 = ¥124.50 with the yen as the commodity currency and the dollar as the terms currency.

	Commodity currency		*Terms currency*
Original quotation	US$1	=	¥124.50
Reciprocal rate	¥1	=	US$1/124.50
i.e.	¥1	=	US$0.008032

PRICE CHANGES

If the price of the commodity rises, it will be worth more units of the terms currency. If the price of the commodity falls, it will be worth fewer units of the terms currency. A rise in the value of the commodity is equivalent to a fall in the value of the terms currency and a fall in the value of the commodity is equivalent to a rise in the value of the terms currency.

PRICE AND VOLUME QUOTATIONS

If an exchange rate is expressed such that the foreign currency is the commodity currency and the local currency is the terms currency, this is described as a *price quotation.* In a price quotation, the foreign currency is priced in terms of the local currency.

If an exchange rate is expressed such that the foreign currency is the terms currency and the local currency is the commodity currency, this is described as a *volume quotation*. Under the volume quotation system, the local currency is priced in terms of the foreign currency.

The quotation US$1 = ¥124.50 constitutes a price quotation in Japan but a volume quotation in the United States. A rise in the price of the terms currency corresponds to a fall in the number of units in which the price is expressed. Conversely, a fall in the price of the terms currency corresponds to a rise in the number of units in which the price is expressed: see Exhibit 1.1.

EXHIBIT 1.1 Reciprocal rate relationships

Original exchange rate	*Commodity currency price rises*	*Commodity currency price falls*
US$1 = ¥124.50	US$1 = ¥124.60	US$1 = ¥124.40
Reciprocal rate	*Terms currency price falls*	*Terms currency price rises*
¥1 = US$ 0.008032	¥1 = US$ 0.008026	¥1 = US$ 0.08039

A rise in the exchange rate reflects an increase in the value of the commodity currency. Conversely, a fall in the exchange rate reflects a drop in the value of the commodity currency.

A *direct* relationship (Figure 1.1) exists between a rise or fall in the exchange rate and a rise or fall in the value of the commodity currency. An *inverse* relationship (Figure 1.2) exists between a rise or fall in the exchange rate and a rise or fall in the value of the terms currency. The terms currency

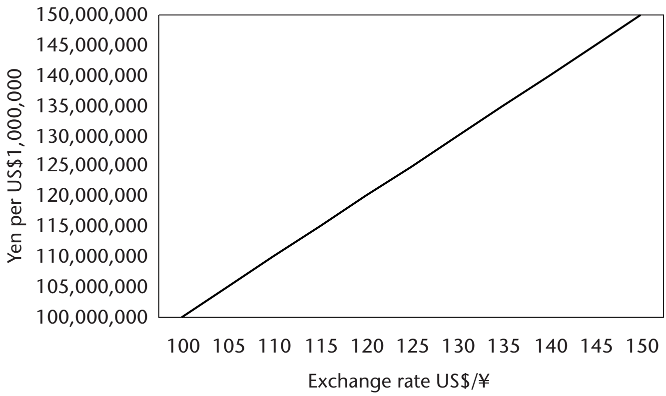


FIGURE 1.1 Direct relationship

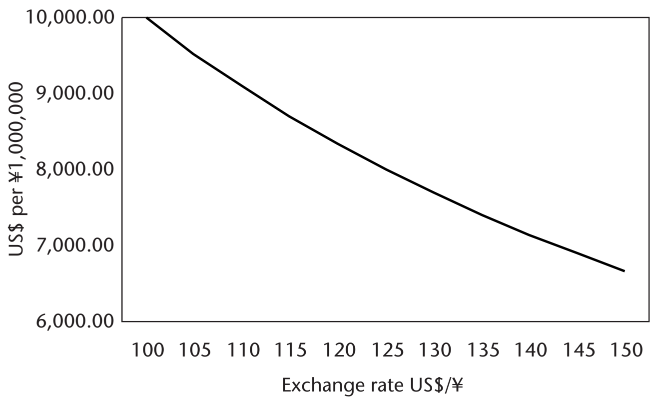


FIGURE 1.2 Inverse or reciprocal relationship

proceeds from the sale of a fixed amount of the commodity currency and the terms currency cost of purchasing a fixed amount of the commodity currency will vary directly with the exchange rate. However, if the proceeds of the sale of a fixed amount of the terms currency is measured in terms of the commodity currency, a reciprocal relationship will exist.

A direct relationship is in the form of $y = ax$, where y is the commodity currency, x is the terms currency and a is the exchange rate. An inverse relationship is in the form $y = x/b$, where y is the commodity currency, x is the terms currency and b is the exchange rate. It follows that $b = 1/a$. That is, the inverse relationship is the reciprocal of the direct relationship.

It is less confusing to work under a direct relationship than to work in reciprocals. It means for example that profit will occur when the commodity currency is purchased when the price is low and sold when the price is high: 'Buy low, sell high'. In general the concepts developed in the following chapters consider a constant amount of the commodity currency being purchased and sold for varying amounts of the terms currency. This preserves the direct relationship. In some cases the context requires the terms currency amount to be measured in units of the commodity currency. In these cases the reciprocal relationship applies. To make a profit under a reciprocal relationship it is necessary to buy high and sell low, which is counterintuitive.

CROSS RATES

Provided there is a common currency, it is possible to derive an exchange rate between two currencies from the exchange rates at which the two currencies are quoted against the common currency.

An exchange rate which is derived from two other exchange rates is known as a *cross rate*.

EXAMPLE 1.2

$$\text{US\$}1 = ¥100.00$$
$$\text{US\$}1 = \text{HK\$}7.8000$$

What is the exchange rate for Hong Kong dollars in yen terms? Algebraically:

$$\text{US\$}1 = ¥100 = \text{HK\$}7.8000$$
$$\therefore \text{HK\$}1 = \frac{100.00}{7.8000} = ¥12.82$$

EXAMPLE 1.3

$$\text{US\$}1 = ¥100.00$$
$$£1 = \text{US\$}1.5000$$

What is the exchange rate for pounds in yen terms? Algebraically:

$$£1 = ¥100.00 \times 1.5000 = ¥150.00$$

CHAIN RULE

The cross rate in Example 1.3 was calculated by multiplying the two exchange rates (100 × 1.5 = 150). The cross rate in Example 1.2 was calculated by dividing one of the exchange rates by the other (100/7.8 = 12.82).

Mathematically, cross rates are calculated by solving simultaneous equations. The chain rule provides a foolproof procedure for determining whether the exchange rates should be multiplied or divided.

1. Start with the question to be answered: How many units of the terms currency equal one unit of the commodity currency?
2. Start the next question with the currency with which the previous question finished.
3. Again, start the next question with the currency with which the previous question finished.

If the first three steps have been correctly followed, the third question will finish with the terms currency.

4. Multiply the numbers on the right-hand side and divide by the product of the numbers on the left-hand side.

Examples 1.1 and 1.2 are repeated using the chain rule.

EXAMPLE 1.2 (using the chain rule)

$$\text{¥}? = \text{HK\$}1$$
$$\text{HK\$}7.8000 = \text{US\$}1$$
$$\text{US\$}1 = \text{¥}100.00$$
$$\therefore \text{HK\$}1 = \frac{\text{¥}1 \times 1 \times 100.00}{7.8000 \times 1} = \text{¥}12.82$$

EXAMPLE 1.3 (using the chain rule)

$$\text{¥}? = \text{£}1$$
$$\text{£}1 = \text{US\$}1.5000$$
$$\text{US\$}1 = \text{¥}100.00$$
$$\therefore \text{£}1 = \frac{\text{¥}1 \times 1.5000 \times 100.00}{1 \times 1} = \text{¥}150.00$$

EXAMPLE 1.4

$$\text{A\$}1 = \text{US\$}0.5420$$
$$\text{US\$}1 = \text{SF}1.2320$$

What is the cross rate for Australian dollars in terms of Swiss francs?

Using the chain rule:

$$\begin{aligned} \text{SF?} &= \text{A\$1} \\ \text{A\$1} &= \text{US\$0.5420} \\ \text{US\$1} &= \text{SF1.2320} \\ \therefore \quad \text{A\$1} &= \frac{\text{SF1} \times 0.5420 \times 1.2320}{1 \times 1} \\ \text{i.e.} \quad \text{A\$1} &= \text{SF0.6677} \end{aligned}$$

EXAMPLE 1.5

$$\begin{aligned} \text{NZ\$1} &= \text{US\$0.4370} \\ \pounds 1 &= \text{US\$1.4500} \end{aligned}$$

What is the cross rate for New Zealand dollars in terms of pounds? Using the chain rule:

$$\begin{aligned} \pounds ? &= \text{NZ\$1} \\ \text{NZ\$1} &= \text{US\$0.4370} \\ \text{US\$1.4500} &= \pounds 1 \\ \therefore \quad \text{NZ\$} &= \frac{\pounds 1 \times 0.4370 \times 1}{1 \times 1.4500} \\ \text{i.e.} \quad \text{NZ\$} &= \pounds 0.3014 \end{aligned}$$

POINTS

It is arbitrary how many significant figures are used in an exchange rate quotation:

$$\begin{aligned} €1 &= \text{US\$0.8450} \\ \text{US\$1} &= ¥122.50 \\ \text{A\$} &= \text{US\$0.5420} \end{aligned}$$

The last decimal place to which a particular exchange rate is usually quoted is referred to as a *point* or *pip*.

In the quotations €1 = US\$ 0.8450 and A\$1 = US\$0.5420, one point means US\$0.0001 or $\frac{1}{100}$ of a US cent. In the quotation US\$1 = ¥122.50, one point means ¥ 0.01 or $\frac{1}{100}$ of a yen.

It is worth noting that all points are not of equal value. In the above example, US\$0.0001 ≠ ¥ 0.01.

US\$1 = ¥122.50. Therefore, US\$0.0001 = ¥122.50/10,000 = ¥ 0.01225. That is, one dollar point is worth more than one yen point.

CALCULATING EXCHANGE PROFITS AND LOSSES

Exchange profits and losses result from buying and selling currencies at different exchange rates. The profit or loss is calculated as the difference in the number of units of the other currency. Consequently, the method of calculating exchange profits and losses varies depending on whether the commodity currency or the terms currency is kept constant. The profit or loss will be expressed in units of the currency that is not kept constant.

EXAMPLE 1.6

Calculate the profit when £1,000,000 are purchased at a rate of £1 = US\$1.4450 and sold at a rate of £1 = US\$1.4451.

$$\begin{aligned}
\text{US\$ profit} &= \text{proceeds of sale of £1,000,000} \\
&\quad - \text{cost of purchase of £1,000,000} \\
&= 1{,}000{,}000 \times 1.4451 - 1{,}000{,}000 \times 1.4450 \\
&= 1{,}000{,}000(1.4451 - 1.4450) \\
&= 1{,}000{,}000 \times 0.0001 \\
&= \text{US\$100}
\end{aligned}$$

There is a profit of one point on £1,000,000. This is equivalent to US\$100.

EXAMPLE 1.7

Calculate the profit or loss when US\$1,000,000 are purchased at a rate of £1 = US\$1.4451 and sold at a rate of £1 = US\$1.4450.

$$\begin{aligned}
\text{£ profit} &= \text{proceeds of sale of US\$1,000,000} \\
&\quad - \text{cost of purchase of US\$1,000,000} \\
&= \frac{1{,}000{,}000}{1.4450} - \frac{1{,}000{,}000}{1.4451} \\
&= 692{,}041.52 - 691{,}993.63 \\
&= \text{£47.89}
\end{aligned}$$

There is a profit of one point on US\$1,000,000. This is equivalent to £47.89.

Notice that when dealing in reciprocals a profit is made by buying at a higher rate and selling at a lower rate.

REALIZED AND UNREALIZED PROFITS AND LOSSES

Exchange profits and losses can be either realized or unrealized. They are said to be **realized** if both the buy side and the sell side of the transaction have been *completed* and **unrealized** if only *one* side of the transaction has been completed.

EXAMPLE 1.8

Calculate the unrealized profit or loss if £1,000,000 were purchased at a rate of £1 = US$1.4450 and could be sold at a rate of £1 = US$1.4435.

$$\begin{aligned} \text{Unrealised profit} &= \text{proceeds of potential sale of £1,000,000} \\ &\quad - \text{cost of purchase of £1,000,000} \\ &= 1{,}000{,}000 \times 1.4435 - 1{,}000{,}000 \times 1.4450 \\ &= 1{,}443{,}500 - 1{,}445{,}000 \\ &= -\text{US\$}1{,}500 \end{aligned}$$

A negative profit is a *loss*.

There is an unrealized loss of 15 points or US$1,500. Until the second leg of the buy–sell transaction is complete, the profit or loss will remain unrealized. The size of the unrealized profit or loss will vary with the exchange rate.

EXAMPLE 1.9

Calculate the realized profit or loss if the exchange rate rises from £1 = US$1.4450 to £1 = US$1.4460 and the £1,000,000 are then sold.

$$\begin{aligned} \text{Realized profit} &= \text{proceeds of sale of pounds} \\ &\quad - \text{cost of purchase of pounds} \\ &= 1{,}000{,}000 \times 1.4460 - 1{,}000{,}000 \times 1.4450 \\ &= 1{,}000{,}000(1.4460 - 1.4450) \\ &= 1{,}000{,}000 \times 0.0010 \\ &= \text{US\$}1{,}000 \end{aligned}$$

Once the profit or loss is realized, the size of the profit or loss ceases to vary with the exchange rate.

HISTORY OF EXCHANGE RATE DETERMINATION

Different methods of exchange rate determination have been used at different times. A *fixed exchange rate system* means that exchange rates are kept constant. A *floating exchange rate system* means that exchange rates vary with supply and demand. Various versions of fixed exchange rate systems and the floating exchange rate system have been used over different periods.

The benefit of a fixed exchange rate system is that people know exactly what the exchange rate will be. The disadvantage is that holding exchange rates at fixed levels can require a lot of intervention through foreign exchange and/or money markets. This can create distortions in the economy and may reach a point where an adjustment (usually a devaluation) is unavoidable. When these occur they are typically large devaluations that have a major financial impact. The benefit of floating exchange rates is that the market is allowed to determine its own level. The disadvantage is that the market may set exchange rates at levels not considered desirable.

Under the Gold Standard, exchange rates were fixed to the price of gold. A British pound was originally one pound weight of gold. Under the Bretton Woods system, which operated from 1947 until it broke down in 1971, the value of the US dollar was fixed as equal to 1 oz of gold. Other currencies were given a 'parity' against the US dollar; for example, A£1 was set at US$ 3.224. Central banks held reserves, including foreign currency and large amounts of gold. They agreed to buy or sell their currencies with US dollars or gold from their reserves to keep their exchange rates fixed at the parity level. Very occasionally the parities were changed. For example, in 1949 the Australian pound (in line with sterling) was devalued by 30% against gold and the US dollar to A£1 = US$ 2.224. In 1967 the pound sterling was devalued by 14.3%, but A$, which had been decimalized in 1966, did not follow.

The Bretton Woods system ended in late 1971 and the major currencies returned to a floating rate mechanism. It was decided that the Australian dollar would be linked to the US dollar rather than the pound. Adjustments to the A$/US$ rate were made in December 1972, February 1973 and September 1973. In September 1974 the link with US$ was broken and replaced with a link to a trade-weighted basket of currencies. In November 1976 A$ was devalued by 17.5% against the trade-weighted basket and it was decided to make frequent small adjustments rather than occasional large changes.

Each morning the Reserve Bank of Australia posted a mid-rate for the day based on the closing New York exchange rates and the then-secret trade-weighted index.

The Australian dollar was floated and exchange controls were lifted on 11 December 1983.

From 1979 most European currencies joined the European Rate Mechanism (ERM), which was known as the snake. Under this arrangement, exchange rates between participating currencies were kept within a band of 2.5% of each other, but the ERM was free to move against other currencies, particularly the US dollar. As with the Bretton Woods system, realignments were made from time to time.

On 1 January 1999 the euro became the official currency for 11 European countries: Austria, Belgium, Denmark, Spain, Finland, France, Ireland, Italy, Luxemburg, Netherlands and Portugal. Greece joined the euro in June 2000.

PRACTICE PROBLEMS

1.1 Reciprocal rates
Given the following exchange rates:

$$€1 = \text{US}\$0.8420$$
$$£1 = \text{US}\$1.4565$$
$$\text{NZ}\$1 = \text{US}\$0.4250$$

(a) Calculate the reciprocal rate for US dollars in euro terms.
(b) Calculate the reciprocal rate for US dollars in pound terms.
(c) Calculate the reciprocal rate for US dollars in New Zealand dollar terms.

1.2 Cross rates
Given:

$$\text{US}\$1 = ¥123.25$$
$$£1 = \text{US}\$1.4560$$
$$\text{A}\$1 = \text{US}\$0.5420$$

(a) Calculate the cross rate for pounds in yen terms.
(b) Calculate the cross rate for Australian dollars in yen terms.
(c) Calculate the cross rate for pounds in Australian dollar terms.

1.3 Calculating profits and losses
(a) Calculate the realized profit or loss as an amount in dollars when Crowns 8,540,000 are purchased at a rate of C1 = \$1.4870 and sold at a rate of C1 = \$1.4675.
(b) Calculate the unrealized profit or loss as an amount in pesos on P17,283,945 purchased at a rate of Rial 1 = P0.5080 and that could now be sold at a rate of R1 = P0.5072.

1.4 Realized profit
Calculate the profit or loss when C\$9,360,000 are purchased at a rate of C\$1 = US\$1.4510 and sold at a rate of C\$1 = US\$1.4620.

1.5 Unrealized profit
Calculate the unrealized profit or loss on Philippine pesos 20,000,000 which were purchased at a rate of US\$1 = PHP 47.2000 and could now be sold at a rate of US\$1 = PHP 50.6000.

CHAPTER 2

Interest Rates

This chapter introduces the basic conventions used to describe interest rates. In practice, interest rates are expressed using a variety of different conventions. Measuring interest rates using different conventions makes the comparison of interest rates potentially confusing. The concept of effective interest rates is introduced as a means of comparing interest rates described under different conventions. The discussion extends to cover forward interest rates and bond pricing.

Definition

Interest is the price paid for the use of money. An *interest rate* is the ratio of the amount of interest to the amount of money. Interest rates are generally expressed in terms of per cent per annum.

NOMINAL AND EFFECTIVE INTEREST RATES

There are various conventions used in interest rate quotations. To make equivalent comparisons between two interest rates which are expressed using different conventions, it is necessary to express both interest rates in equivalent terms using a common convention. The number by which an interest rate is expressed under a particular convention is called the *nominal interest rate*. When an interest rate quotation is expressed under a different convention it is known as an *effective interest rate*.

BASIS POINTS

Interest rates are often expressed as proper fractions or decimals, e.g. $8\frac{3}{4}\%$ p.a. or 8.75% p.a. When interest rates per cent are expressed to two decimal places, one unit in the second decimal place is known as a *basis point*. The same interest rate could be expressed in decimal notation as

0.0875. In this case a basis point refers to one unit in the fourth decimal place. Exchange points and basis points do not generally have equal value.

DAY COUNT CONVENTIONS

There are 365 days to a year and 366 in leap years. Some interest rates are quoted as if there were 360 days per year; others on the basis of 365 days per year. To convert an interest rate based on 360 days per year to one based on 365 days per year, it is necessary to multiply by the factor 365/360.

For example, if a three month Eurodollar rate (which, by convention, is quoted on a 360 day year basis) is 8.25% p.a., the effective interest rate on a 365 days per year basis would be:

$$8.25 \times 365/360 = 0.0836 = 8.36\%$$

Similarly, an interest rate based on a 365 day year can be converted into a 360 days per year basis by multiplying by a factor of 360/365.

A Eurodollar refers to a US dollar deposit held in a bank in Europe. As Eurodollar deposits were first held with banks in London the rate is generally known as LIBOR, standing for London Inter Bank Offer Rate.

SIMPLE INTEREST

The amount of interest earned on an investment is a function of the amount invested (known as the *principal*), the interest rate and the period for which the investment is made.

$$\boxed{I = P \times r \times t} \tag{2.1}$$

where:

I = interest amount

P = principal sum invested

r = simple interest rate per annum

t = time period in years

EXAMPLE 2.1

\$100 is invested at a simple interest rate of 8% p.a. (365 dpy) for a period of 30 days.

$P = \$100$

$r = 8\% = 0.08$

$t = 30/365$

Therefore

$$\begin{aligned} I &= Prt \\ &= \$100 \times 0.08 \times 30/365 \\ &= \$0.66 \end{aligned}$$

The investor receives the interest at *maturity* (i.e. at the end of the investment period) together with the principal sum invested.

$$\boxed{FV = P + I} \tag{2.2}$$

where FV = final amount received at maturity, or future value of the investment.

From Example 2.1, $P = \$100$ and $I = \$0.66$, so that at the end of the 30 day period, the investor would receive the final amount (see Figure 2.1):

$$\begin{aligned} FV &= P + I \\ &= \$100 + 0.66 \\ &= \$100.66 \end{aligned}$$

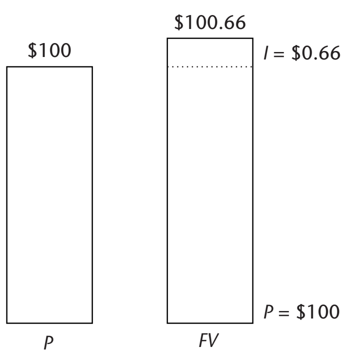


FIGURE 2.1 Simple interest: $FV = P + I$

$r = 0.08$ and $t = 30/365$

By substitution,

$$\begin{aligned} FV &= P + I \\ &= P + Prt \end{aligned}$$

$$\boxed{FV = P(1 + rt)} \tag{2.3}$$

EXAMPLE 2.1 (continued)

$$FV = P(1+rt)$$
$$= \$100(1+0.08\times 30/365)$$

The interest rate can be depicted by the slope of the line connecting the principal amount to the future value amount (Figure 2.2).

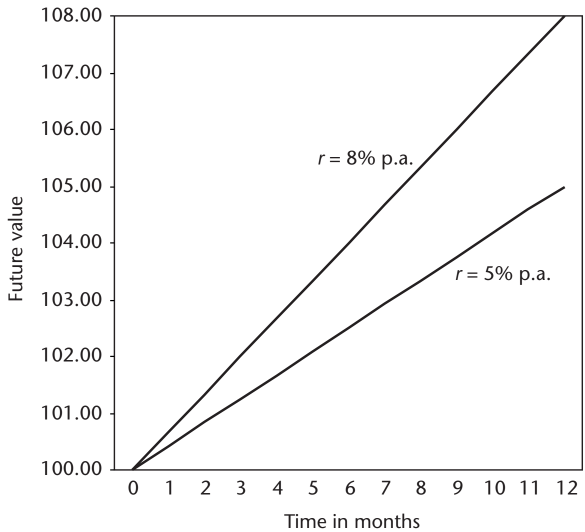


FIGURE 2.2 Simple interest

VARIABLE INTEREST

The investor can reinvest the amount of principal plus interest for subsequent periods, possibly at different interest rates. It is possible to calculate the future value of an investment under which the interest rate and/or the time period varies over time.

EXAMPLE 2.2

Calculate the future value of \$1,000,000 invested at 5.75% p.a. for 92 days and then reinvested at 6.00% p.a. for 75 days.

$$FV = P(1+r_1t_1)(1+r_2t_2)\ldots(1+r_nt_n)$$
$$= 1{,}000{,}000(1+0.0575\times 92/365)(1+0.06\times 75/365)$$
$$= 1{,}027{,}000.60$$

COMPOUND INTEREST

Compound interest assumes that the investor reinvests the amount of principal plus interest for subsequent periods at the same rate.

EXAMPLE 2.3

A principal sum of \$100 is invested for three years at an annually compounding rate of 10% p.a. At the end of year 1, the value of the investment is:

$$\begin{aligned} FV_1 &= P(1+r_1t_1) \\ &= \$100(1+0.10) \\ &= \$110 \end{aligned}$$

At the end of year 2, the value of the investment is:

$$\begin{aligned} FV_2 &= FV_1(1+r_2t_2) \\ &= \$110(1+0.10) \\ &= \$121 \end{aligned}$$

At the end of year 3, the final value of the investment:

$$\begin{aligned} FV_3 &= FV_2(1+r_3t_3) \\ &= \$121(1+0.10) \\ &= \$133.10 \end{aligned}$$

See Figure 2.3.

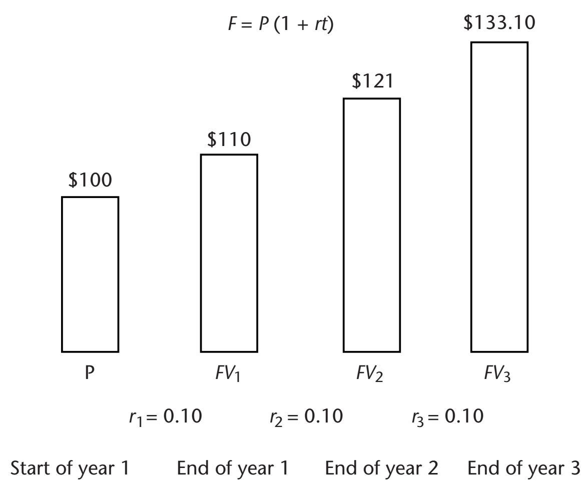


FIGURE 2.3 Compound interest

Assuming the investment is rolled over each year at the fixed rate of 10% p.a.,

$$r_1 = r_2 = r_3 = 0.10 = r$$

then

$$\begin{aligned} FV_3 &= FV_2\,(1+rt) \\ &= FV_1(1+rt)(1+rt) \\ &= P(1+rt)(1+rt)(1+rt) \\ &= P(1+rt)^3 \end{aligned}$$

In general,

$$\boxed{FV = P(1+rt)^n} \tag{2.4}$$

where

FV = future value of the investment
P = principal amount
rt = compound interest rate per period
n = number of periods

If rt is written as r/m or i, Equation (2.4) becomes:

$$\boxed{FV = P\left(1+\frac{r}{m}\right)^n = P(1+i)^n} \tag{2.5}$$

where

m = compounding frequency per year
i = periodic interest rate

Note:

$$i = r/m$$
$$n = mt$$

SEMI-ANNUAL INTEREST

The more frequently interest is paid, the faster the compounding effect will tend to occur. Interest is commonly accrued semi-annually, quarterly or monthly.

EXAMPLE 2.4

A principal sum of \$100 is invested at a semi-annually compounding rate of 5% p.a. Calculate the value of the investment after two years.

$$\begin{aligned}
P &= \$100 \\
r &= 0.05 \\
m &= 2 \\
i &= r/m = 0.05 \times 1/2 = 0.025 \\
n &= 2 \times 2 = 4 \\
FV &= P(1+i)^n \\
&= \$100(1.025)^4 \\
&= \$110.38
\end{aligned}$$

Compounding interest on quarterly, monthly or other rests simply implies different values of t, that is, a different frequency.

m		t	
1	=	1	annual rests
2	=	1/2	semi-annual rests
4	=	1/4	quarterly rests
12	=	1/12	monthly rests

FLOATING INTEREST RATES

Compound interest can be applied at different rates over different time periods.

$$\boxed{FV = P(1+i_1)^{n_1}(1+i_2)^{n_2}\ldots(1+i_n)^{n_z}} \qquad (2.6)$$

EXAMPLE 2.5

An investor earns a floating rate of interest for 3 years on an investment of \$1,500,000. The first year the interest rate applicable is 4.5% p.a. simple, the second year 5.5% p.a. semi-annually compounding and the third year 6.5% p.a. quarterly compounding.

What is the future value of the investment at the end of the third year if the interest earned in the first and second years is reinvested?

$$\begin{aligned}
FV &= 1{,}500{,}000\left(1+\frac{0.045}{1}\right)\left(1+\frac{0.055}{2}\right)^2\left(1+\frac{0.065}{4}\right)^4 \\
&= 1{,}765{,}116.79
\end{aligned}$$

EQUIVALENT INTEREST RATES

It is possible to convert a nominal annual rate compounding m_1 times per year to an effective annual rate compounding m_2 times per year by finding the rate that will produce an equal future value after 1 year (say).

$$\text{If } i = r_{m_1}/m_1, \qquad FV = \left(1+\frac{r_{m_1}}{m_1}\right)^{m_1}$$

$$\text{If } i = r_{m_2}/m_2, \qquad FV = \left(1+\frac{r_{m_2}}{m_2}\right)^{m_2}$$

So

$$\boxed{\left(1+\frac{r_{m_1}}{m_1}\right)^{m_1} = \left(1+\frac{r_{m_2}}{m_2}\right)^{m_2}} \tag{2.7}$$

EXAMPLE 2.6

Convert a nominal semi-annual interest rate of 5.50% p.a. to an effective quarterly compounding rate.

$$\left(1+\frac{0.055}{2}\right)^2 = \left(1+\frac{r_4}{4}\right)^4$$

$$\sqrt[4]{1.055756} = 1+\frac{r_4}{4}$$

$$\therefore r_4 = 0.054627 = 5.46\% \text{ p.a.}$$

INDEX ALGEBRA

The following formulae show how to multiply and divide numbers raised to powers:

$$x^a \times x^b = x^{a+b}$$

$$\text{e.g. } 3^2 \times 3^3 = 3^{2+3}$$

$$x^a \div x^b = x^{a-b}$$

$$\text{e.g. } 3^5 \div 3^2 = 3^{5-2}$$

LOGARITHMS

The *logarithm* of a number is the index to which the base must be raised to equal the number, e.g.

$$3^2 = 9$$

So

$$\log_3 9 = 2$$

e is a special number associated with exponential growth. It is defined as:

$$e = \lim\left(1 + \frac{1}{m}\right)^m \text{ as } m \to \infty \approx 2.71828\ldots$$

See Figure 2.4.

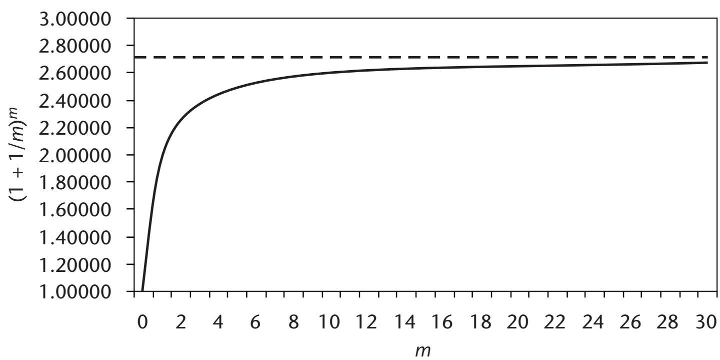


FIGURE 2.4 $e = \lim\left(1 + \frac{1}{m}\right)^m$ as $m \to \infty \approx 2.71828\ldots$

Logarithms to the base e are known as natural logarithms and are written as $\log_e$ or ln.

CONTINUOUSLY COMPOUNDING RATES

Interest rates can compound quarterly, monthly, daily etc. The limiting case is continuous compounding.

$$\text{As } m \to \infty, \quad \left(1 + \frac{r_m}{m}\right)^m = e^r$$

After n periods (remember that $n = mt$), the future value of \$1 is given by:

$$\text{As } m \to \infty, \quad \left(1+\frac{r_m}{m}\right)^{mt} = e^{rt}$$

The term e^{rt} represents the future value of \$1 continuously compounding at a rate of $r\%$ p.a. for t years:

$$\boxed{FV(X) = Xe^{rt}} \tag{2.8}$$

Whenever e^{rt} appears in a formula it can be understood that r is a continuously compounding rate (see Figure 2.5). Continuously compounding rates are not used in market practice, but they simplify the mathematics when deriving formulae such as those used for pricing options.

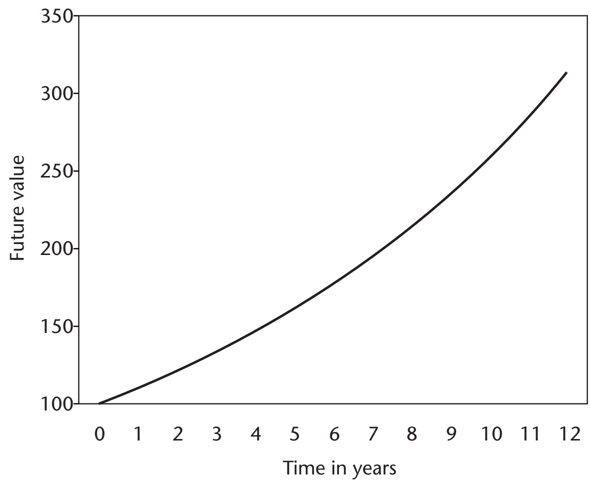


FIGURE 2.5 Continuously compounding r = 10% p.a.

EXAMPLE 2.7

Calculate the future value of an investment of \$1,000,000 compounding continuously at a rate of 5% p.a. for 736 days.

$$FV = 1{,}000{,}000e^{0.05\times736/365} = 1{,}106{,}079.65$$

It is possible to work out the continuously compounding rate that corresponds to a specified future value:

$$\text{As } m \to \infty, \quad \left(1+\frac{r_m}{m}\right)^{m} = e^{r}$$

Taking logs of both sides,

$$\boxed{r = \ln\left(1 + \frac{r_m}{m}\right) \times m} \tag{2.9}$$

This amounts to solving for r to find the continuously compounding rate that will produce the same future value after 1 year (say) as the discretely compounding rate (or simple rate i.e. $m = 1$).

EXAMPLE 2.8

Calculate the compounding continuously rate that produces the equivalent effective yield as an investment at 6.5% per annum compounding semi-annually.

$$FV(\text{semi-annually}) = FV(\text{continuously compounding})$$

$$\left(1 + \frac{0.065}{2}\right)^2 = e^r$$

$$r = 2 \times \ln\left(1 + \frac{0.065}{2}\right) = 0.0640$$

$$= 6.4\% \text{ p.a. continuous compounding}$$

FORWARD INTEREST RATES

A *forward interest rate* is an interest rate which can be determined today for a period from one future date till another future date.

EXAMPLE 2.9

If the one month interest rate is 6% p.a., and the six month interest rate is 7% p.a., calculate the forward interest rate for the period extending from one month from now to six months from now.

To describe forward interest rates, it is necessary to specify the starting date and the ending date of the period. The notation $r_{1,6}$ is used to denote the forward interest rate from one month until six months. The one month (from today) interest rate could be denoted $r_{0,1}$ and the six month interest rate $r_{0,6}$.

The future value of a one month investment of $1,000,000 would be

$$FV_1 = PV(1 + r_{0,1} \times 1/12)$$
$$= 1{,}000{,}000 \times (1 + 0.06 \times 1/12)$$
$$= 1{,}005{,}000$$

The future value of a six month investment of $1,000,000 would be

$$
\begin{aligned}
FV_6 &= PV(1+r_{0,6}\times 6/12)\\
&= 1{,}000{,}000\times(1+0.07\times 6/12)\\
&= 1{,}035{,}000
\end{aligned}
$$

See Figure 2.6.

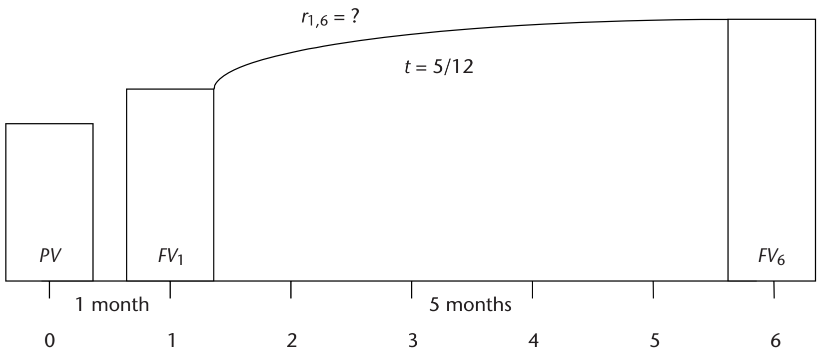


FIGURE 2.6 Forward interest rate

The forward interest rate is that rate which would make \$1,005,000 accumulate to \$1,035,000.00 over the 5 months, i.e.

$$1{,}005{,}00\times(1+r_{1,6}\times 5/12) = 1{,}035{,}000.00$$

$$
\begin{aligned}
r_{1,6} &= \left(\frac{1{,}035{,}000}{1{,}005{,}000}-1\right)\times\frac{12}{5}\\
&= 0.071642 = 7.16\%\ \text{p.a.}
\end{aligned}
$$

In general, the forward interest rate for a period from time t_1 to t_2 can be calculated by finding the rate that will make the future value at t_1 grow to the future value at t_2. Forward interest rates can be based on simple, compounding or continuously compounding rates:

Simple interest $\quad FV_1(1+rt) = FV_2$

Compound interest $\quad FV_1\left(1+\frac{r}{m}\right)^n = FV_2 \quad (2.10)$

Continuous $\quad FV_1 e^{rt} = FV_2$

EXAMPLE 2.10

Calculate the forward interest rate (expressed on a quarterly compounding basis) for the period from 2 years from now to 3 years from now if the 2 year rate is 4.5% p.a. (semi-annually compounding) and the 3 year rate is 5.0% semi-annually compounding.

$$\left(1+\frac{r_2}{2}\right)^{2\times 2}\left(1+\frac{r}{4}\right)^{1\times 4}=\left(1+\frac{r_3}{2}\right)^{3\times 2}$$

$$FV_{2\text{ years}}=\left(1+\frac{0.045}{2}\right)^{2\times 2}=1.093083$$

$$FV_{3\text{ years}}=\left(1+\frac{0.05}{2}\right)^{3\times 2}=1.159693$$

$$\therefore\left(1+\frac{r}{4}\right)^{1\times 4}=\frac{1.159693}{1.093083}$$

$$\therefore r=0.0596=5.96\%\text{ p.a.}$$

Forward interest rates are used to hedge interest rate risk as discussed in Chapter 4.

PRESENT VALUE

To compare cash flows that occur at different points of time on an apples to apples (i.e. like for like) basis, it is necessary to ascertain their equivalent values at a common point of time, say, at present. The *present value* of a future cash flow can be calculated by rearranging the relevant future value formula.

Simple interest:

$$\boxed{PV=\frac{FV}{(1+rt)}} \tag{2.11}$$

Compound interest:

$$\boxed{PV=\frac{FV}{(1+i)^n}} \tag{2.12}$$

Continuously compounding interest:

$$\boxed{PV=FVe^{-rt}} \tag{2.13}$$

The term e^{-rt} represents the present value of \$1 discounted using a continuously compounding rate of r% p.a. for t years.

EXAMPLE 2.11

Calculate the present value of a cash flow of \$800,000 due in 780 days' time using a semi-annual interest rate of 6.5% p.a.

$$FV = \$800{,}000$$
$$i = 0.065 \times 1/2 = 0.0325$$
$$n = 2 \times 780/365 = 4.273973$$
$$\therefore PV = \frac{\$800{,}000}{(1.0325)^{4.273973}}$$
$$= \$697{,}789.21$$

DISCOUNT FACTORS

A *discount factor* is a number less than one that when multiplied by the future value equals the present value.

$$\boxed{df = \frac{PV}{FV}} \tag{2.14}$$

Under simple interest: $df = \dfrac{1}{1+rt}$

Under compound interest: $df = \dfrac{1}{(1+r/m)^n}$

With continuous compounding: $df = e^{-rt}$

Discount factors are particularly useful if a series of future cash flows need to be discounted to their present values.

BONDS

Money is generally borrowed through either the money market or the capital market. In the money market the lender is typically a bank and the maturity of loans is predominantly less than one year. In the capital market money is borrowed by issuers selling securities to investors. Generally the issuers are governments or companies with high credit ratings and tenors (the periods for which the money is borrowed) range from one month to 30 years.

A *bond* is a long-term debt security. In colloquial language a bond is an IOU. The issuer promises to pay the investor the face value of the bond at maturity. The issuer also usually agrees to pay the investor an amount of interest at regular intervals throughout the life of the bond. These interim interest payments are known as *coupons*. If coupons are paid semi-annually at a rate of 7% p.a., each coupon would be represented by a cash flow of \$3.50 per \$100 face value of the bond (Figure 2.7).

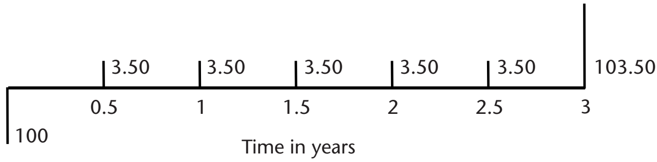


FIGURE 2.7 Cash flows of an investor who has purchased a bond

Zero coupon bonds

A *zero coupon bond* is a bond with all coupons equal to zero (Figure 2.8).

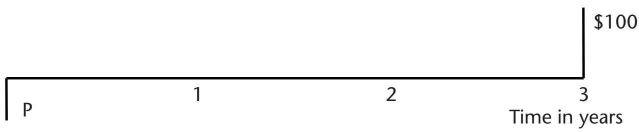


FIGURE 2.8 Cash flows of a zero coupon bond

EXAMPLE 2.12

What is the (present) value of a zero coupon bond with face value $100 due in 3 years' time if the relevant yield is 6% p.a.?

$$PV = \frac{FV}{(1+i)^n} = \frac{100}{(1+0.06/2)^6} = 100 \times 0.837484 = 83.75$$

Net Present Value

The *Net Present Value (NPV)* of an instrument is the sum of the present values of each of the cash flows.

$$\boxed{NPV = \sum_{j=1}^{n} PV(cf_j)} \tag{2.15}$$

The Σ, or sigma symbol, means 'sum of'.

EXAMPLE 2.13

Calculate the net present value of a series of cash flows of $3.50 each 6 months for 3 years if the discount factors are based on a yield to maturity of 6% p.a.

t	*Cash flows*	*Discount factor*	*PV*
0.5	3.50	0.970874	3.40
1.0	3.50	0.942596	3.30
1.5	3.50	0.915142	3.20
2.0	3.50	0.888487	3.11
2.5	3.50	0.862609	3.02
3.0	3.50	0.837484	2.93
		Net Present Value	18.96

Price of a bond

The price of a coupon bond is the sum of the net present value of the coupons and the present value of the payment at maturity.

A bond with face value \$100 maturing in 3 years' time that pays coupons of \$3.50 each six months would have a net present value equal to \$83.75 + \$18.96 = \$102.70.

In general, the price of a coupon bond is given by:

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \ldots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n}$$

$$\boxed{P = \sum_{j=1}^{n} \frac{C}{(1+i)^j} + \frac{FV}{(1+i)^n}} \tag{2.16}$$

PRACTICE PROBLEMS

2.1 Simple interest and future value

(a) Calculate the interest earned on an investment of A\$2,000 for a period of three months (92/365 days) at a simple interest rate of 6.75% p.a.

(b) Calculate the future value of the investment in 2.1(a).

2.2 Compound interest

Calculate the future value of \$1,000 compounded semi-annually at 10% p.a. for 100 years.

2.3 Equivalent rates

An interest rate is quoted as 4.80% p.a. compounding semi-annually. Calculate the equivalent interest rate compounding monthly.

2.4 Forward interest rate

Calculate the forward interest for the period from six months (180/360) from now to nine months (270/360) from now if the six month rate is 4.50% p.a. and the nine month rate is 4.25% p.a.

2.5 Present value

Calculate the present value of a cash flow of $10,000,000 due in three years' time assuming a quarterly compounding interest rate of 5.25% p.a.

2.6 Bond price

Calculate the price per $100 of face value of a bond that pays semi-annual coupons of 5.50% p.a. for 5 years if the yield to maturity is 5.75% p.a.

2.7 Compounding forward interest rate

Calculate the forward interest rate for a period from 4 years from now till 4 years and 6 months from now if the 4 year rate is 5.50% p.a. and the 4 and a half year rate is 5.60% p.a. both semi-annually compounding. Express the forward rate in continuously compounding terms.

CHAPTER 3

Cash Flows and Value Dates

In this chapter the concepts of cash flows and value dates are introduced. The T-account is presented as a simple notation for defining cash flows. The conventions used to describe value dates and the concepts of net cash flow and net exchange positions are also introduced.

SPECIFICATIONS OF CASH FLOWS

Money market and foreign exchange transactions involve cash flows. To fully define the cash flows associated with a financial transaction it is necessary to specify:

1. The direction of the cash flows
2. The currencies of the cash flows
3. The amounts of the cash flows
4. The timing of the cash flows

POSITIVE AND NEGATIVE CASH FLOWS

The receipt of cash from another party is described as an *inflow of cash* and designated a *positive cash flow*. The payment of cash to another party is described as an *outflow of cash* and designated a *negative cash flow*; see Table 3.1.

T-ACCOUNTS

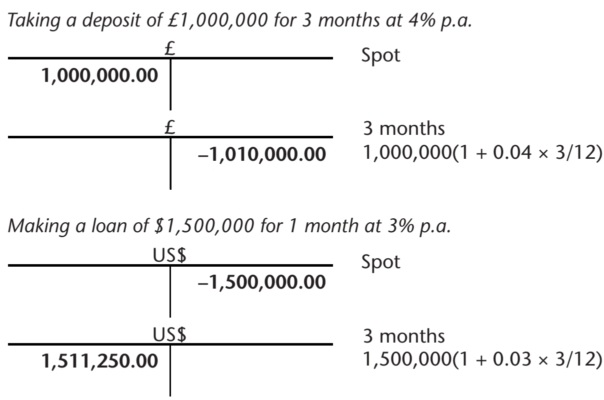
T-accounts provide a simple notation for specifying the cash flows associated with a financial transaction.

Money market transactions involve a positive and a negative cash flow in *one* currency at *different* times; see Exhibit 3.1.

Table 3.1 Examples of positive and negative cash flows of a commercial bank

Inflows (+)	**Outflows (–)**
Receipt of a deposit from a customer	Withdrawal of a deposit by a customer
Receipt of a loan repayment from a customer	Granting a loan to a customer
Purchase of a currency	Sale of a currencv
Sale of a bond	Purchase of a bond

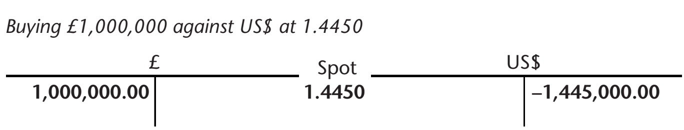
EXHIBIT 3.1 Cash flow representation of money market transactions



The amounts at the future date represent principal plus interest.

A foreign exchange transaction involves a positive and a negative cash flow in different currencies at the same time; see Exhibit 3.2.

EXHIBIT 3.2 Cash flow representation of a foreign exchange transaction



SPOT VALUE DATES

The date on which a transaction is contracted is known as the *contract date*. The dates on which the cash flows occur are known as *value dates*.

In international transactions, two business days are usually allowed between the contract date and the value date. This allows time for payments to be made to accounts in banks in other countries which are possibly in different time zones. *Spot value* refers to a payment which will be made two business days from the contract date. If there is a holiday in either or both countries in which the cash flows are to occur, the spot date moves forward to the next eligible date; see Exhibit 3.3.

EXHIBIT 3.3 Spot value dates

Jan

S	M	T	W	Th	F	S
	JA 1	J 2	3	4	5	6
7	8	9	10	11	12	13
14	H 15	16	17	18	19	20
21	22	23	24	25	A 26	27
28	29	30	31			

J	Holiday in Japan
H	Holiday in Hong Kong
A	Holiday in Australia

Spot transactions	**Contract date**	**Spot value date**
Buys US$ against €	Mon 8 Jan	Wed 10 Jan
Sells US$ against £	Thurs 4 Jan	Mon 8 Jan
Buys US$ against HK$	Thurs 11 Jan	Tues 16 Jan
Buys A$ against Y	Wed 24 Jan	Mon 29 Jan
Sells £ against €	Wed 17 Jan	Fri 19 Jan

During holiday periods it is possible for spot value dates to extend for four, five or six days. For example, if Christmas Day falls on a Monday and Boxing Day is also a holiday on the Tuesday, a transaction dealt on Friday 22 December will have spot value the following Thursday 28 December – a six day run.

The Canadian dollar trades with spot value one business day from the transaction date.

NOSTRO ACCOUNTS

Banks typically hold foreign currency accounts in each currency in which they transact. For example, a British bank would have a yen account with either its branch in Japan (if it has one) or, otherwise, with another bank referred to as its *correspondent* bank. Banks refer to their foreign currency accounts as their *nostro* (meaning 'our') accounts.

FORWARD VALUE DATES

If the cash flows associated with a transaction are to occur on a date or dates which are further into the future than spot, these are said to have *forward value*. It is common for transactions to mature some round number of months after spot. For example, if £1,000,000 is borrowed for three months from spot, there would be a receipt of pounds on the spot date and a repayment of principal plus interest three months later. If that date is a weekend or public holiday, the repayment will occur on the next available business date, unless it requires going beyond month end, in which case the forward value date would be the first available business day prior to month end; see Exhibit 3.4.

EXHIBIT 3.4 Forward value dates

JANUARY

SUN	MON	TUE	WED	THU	FRI	SAT
	JA 1	**J** 2	3	4	5	6
7	8	9	10	11	12	13
14	**H** 15	16	17	18	19	20
21	22	23	24	25	**A** 26	27
28	29	30	31			

FEBRUARY

SUN	MON	TUE	WED	THU	FRI	SAT
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	**UK** 21	**UK** 22	23	24
25	26	27	28			

1 month forward value transactions

Contract date	Spot value date	1 month forward value date
3 Jan	5 Jan	5 Feb
5 Jan	9 Jan	9 Feb
8 Jan	10 Jan	12 Feb
18 Jan	22 Jan	23 Feb
25 Jan	29 Jan	28 Feb
29 Jan	31 Jan	28 Feb

SHORT DATES

On occasions it is necessary for transactions to mature and the cash flows to occur prior to spot value. By definition there are only two eligible business days before spot value: today (*tod*) and tomorrow (*tom*). These are known as *short dates*. Transactions with cash flows which occur on the same day as the contract date are known as *value today* transactions. Similarly, transactions with cash flows which occur on the business day following the contract date are described as *value tomorrow*.

NET CASH FLOW POSITION

There is said to be a *position* if circumstances are such that a change in a rate will create a profit or loss. If cash inflows and cash outflows are unequal or have mismatched value dates, there is a *net cash flow position*. Separate net cash flow positions apply for each value date.

$$\boxed{\text{Net cash flow position} = \text{Cash inflow} - \text{Cash outflow}} \tag{3.1}$$

A positive net cash flow position reflects an excess of cash inflow over cash outflow on the relevant value date. The surplus cash will be available for investment. If interest rates rise, the return will be higher. If interest rates fall, the return will be lower.

A negative net cash flow position reflects an excess of cash outflow over cash inflow on the relevant value date. Assuming there are no idle balances, the account will become overdrawn. The shortfall of cash will require funding. If interest rates rise, it will become more expensive to fund the account. If interest rates fall, it will become less expensive to fund the account.

A negative net cash flow position also implies a *liquidity position*. There is a risk that there will be insufficient funds available for borrowing, in which case the account must remain overdrawn. Being overdrawn may involve financial and non-financial penalties. Table 3.2 summarizes net cash flow positions.

If cash inflow equals cash outflow on a particular value date, then the net cash flow position is zero. This is referred to as a *square cash flow position*. Changes in interest rates will have no net impact on profits or losses.

NET EXCHANGE POSITION

The buying and selling of foreign currencies creates exposures to changes in exchange rates. Buying a foreign currency creates an asset. The position

Table 3.2 Net cash flow positions

Cash inflow	Cash outflow	Position	
+300	–200	+100	**Positive cash flow position** Potential gain if interest rates rise Potential loss if interest rates fall
+200	–350	–150	**Negative cash flow position** Potential gain if interest rates fall Potential loss if interest rates rise
+100	–100	0	**Square cash flow position** No gain or loss from interest rate changes

is said to be *long* the foreign currency. If the foreign currency appreciates there will be an exchange gain. If the currency depreciates there will be an exchange loss.

Selling a foreign currency creates a liability. The position is said to be *short* the foreign currency. If the foreign currency depreciates there will be an exchange gain. If the foreign currency appreciates there will be an exchange loss.

The excess amount of a foreign currency which has been purchased over the amount of the same foreign currency which has been sold is described as the *net exchange position*. There is a separate net exchange position for each foreign currency.

$$\boxed{\begin{aligned}\text{Net exchange position} &= \text{Foreign currency purchased} \\ &\quad - \text{Foreign currency sold}\end{aligned}} \tag{3.2}$$

Being long a currency implies having a net exchange position which is positive. Provided the exchange rate is quoted with the foreign currency as the commodity currency, a rise in the exchange rate will yield an exchange gain and a fall in the exchange rate will result in an exchange loss.

Being short a currency implies having a net exchange position which is negative. Provided the foreign currency is the commodity currency, a rise in the exchange rate will involve an exchange loss and a fall in the exchange rate will involve an exchange gain.

If the amount of foreign currency purchased equals the amount of that currency which has been sold, then the net exchange position will be zero. This is referred to as a *square exchange position*. Changes in exchange rates will have no impact on profit or loss.

A net exchange position is created or removed at the time at which the purchase or sale of foreign currency is contracted, not at the time at which

			Potential gain if exc Potential loss if exch
+20	–40	–20	**Short foreign curre** Potential gain if exc Potential loss if exch
+10	–10	0	**Square exchange p** No gain or loss from changes

the related cash flows occur. For example, if a spot contra today to purchase US$1,000,000 against yen at a rate of the buyer immediately becomes long dollars and short y the fact that he or she will not receive the dollars nor pa until two business days later. Similarly, forward purch foreign currency immediately create or remove a net exc Table 3.3 summarizes net exchange position.

DISTINCTION BETWEEN NET EXCHANGE PO NET CASH FLOW POSITION

It is important to appreciate the distinction between a ne tion and a net cash flow position. Money market transa cash flow positions but do not create net exchange p borrowing or lending a foreign currency does not create position. Only buying or selling a currency can create a ne tion. Borrowing Swiss francs for three months will caus flow of Swiss francs now and a negative cash flow of Swis months' time, but no exposure to the exchange rate. U francs are sold (which would create a net exchange positi available to repay the loan on maturity and so exchange r bear no relevance to profit or loss.

Foreign exchange transactions create both net cash flo net exchange positions. Mismatched cash flows may be money market transactions or foreign exchange transac net exchange positions can only be offset by fo transactions.

EXAMPLE 3.1

Consider the net cash flow and net exchange implications of a corporation whose local currency is the dollar entering into a number of different transactions on 1 February.

Transaction 1: Buys foreign currency spot (Exhibit 3.5)

EXHIBIT 3.5 Buys £2,000,000 against dollars at £1 = $1.4450 value 3 February

£		Feb 3	US$		from Feb 1
2,000,000.00		1.4450		−2,890,000.00	long £2,000,000
		Cash flows			Net Exchange Position

Transaction 2: Sells foreign currency spot (Exhibit 3.6)

EXHIBIT 3.6 Sells £1,000,000 against $1,445,000 at £1 = $1.4450 value 3 February

£		Feb 3	US$		from Feb 1
	−1,000,000.00	1.4450	1,445,000.00		short £1,000,000
		Cash flows			Net Exchange Position

Transaction 3: Borrows local currency (Exhibit 3.7)

EXHIBIT 3.7 Borrows $1,000,000 at 3% p.a. from 1 February till 1 March (28 days)

Cash flows

US$		
1,000,000.00		Feb 1
	−1,002,333.33	Mar 1 1,000,000(1 + 0.03 × 28/360)

Transaction 4: Lends local currency (Exhibit 3.8)

EXHIBIT 3.8 Lends $2,000,000 at 3.5% p.a. from 1 February till 1 April (59 days)

Cash flows

US$		
	−2,000,000.00	Feb 1
2,011,472.22		Apr 1 2,000,000(1 + 0.035 × 59/360)

Transaction 5: Borrows foreign currency (Exhibit 3.9)

EXHIBIT 3.9 Borrows £2,500,000 from 3 February till 17 February (14 days) at 4% p.a.

The only net exchange position is the future obligation to pay interest (FOTPI).

Cash flows £		*Net exchange position*
2,500,000.00		Feb 3
£		Feb 17
	–2,500,000.00	
	–3,835.62	short £3,835.62 FOTPI
	–2,503,835.62	2,500,000(1 + 0.04 × 14/365)

Transaction 6: Lends foreign currency (Exhibit 3.10)

EXHIBIT 3.10 Lends £1,500,000 from 3 February till 10 February (7 days) at 4.1% p.a.

The only net exchange position is the future obligation to receive interest (FOTRI).

Cash flows £		*Net exchange position*
	–1,500,000.00	Feb 3
£		Feb 10
1,500,000.00		
1,179.45		long £1,179.45 FOTRI
1,501,179.45		1,500,000(1 + 0.041 × 7/365)

Transaction 7: Cross rate (Exhibit 3.11)

EXHIBIT 3.11 Buys £1,000,000 against yen at £1 = ¥176.29 value 3 February

Cash flows £			¥		*Net exchange position*
		Feb 3			from Feb 1
1,000,000.00		176.29		–176,290,000	long £1,000,000
					short ¥176,290,000

NET EXCHANGE POSITION SHEET

A running tally of the net exchange position in each foreign currency can be kept for successive transactions. The net exchange position can be tracked throughout the day in the manner shown in Exhibit 3.12. It is assumed that there was a square position before the first transaction.

EXHIBIT 3.12 Net exchange position sheet for 1 February

	£		**¥**	
		NEP		**NEP**
Transaction 1	£2,000,000.00	£2,000,000.00		
Transaction 2	–£1,000,000.00	£1,000,000.00		
Transaction 3		£1,000,000.00		
Transaction 4		£1,000,000.00		
Transaction 5	–£3,835.62	£996,164.38		
Transaction 6	£1,179.45	£997,343.83		
Transaction 7	£1,000,000.00	**£1,997,343.83**	–176,290,000	**–176,290,000**

After the seven transactions of Example 3.1, the net exchange position is long £1,997,343.83 and short ¥176,290,000.

Transactions 3 and 4 could be excluded because they do not impact the net exchange position. Transactions 6 and 7 only impact the net exchange position to the extent of FOTPI and FOTRI.

BLOTTER

Foreign exchange dealers who work for large banks or corporates have their net exchange positions tracked as they enter transactions into their electronic dealing system. Many dealers keep a rough copy of their net exchange sheet, known as a *blotter* (Exhibit 3.13). It is vitally important that a dealer always knows her or his net exchange position, at least approximately. Amounts are usually rounded to the nearest 100,000 units.

EXHIBIT 3.13 Blotter

	£		¥	
		NEP		NEP
Transaction 1	£2.000	£2,000		
Transaction 2	–£1.000	£1.000		
Transaction 7	£1.000	**£2.000**	–176.290	**–176.290**

Transactions 5 and 6, which are FOTPI and FOTRI entries, are excluded from the blotter as trivial.

At the end of each day the blotter should be reconciled with the complete net exchange position sheet. These days most banks have real-time systems which display accurate cash flow and net exchange positions. It is important for dealers to input deals as soon as possible once a transaction is done.

A cardinal rule of trading is for the dealer to know his or her position at all times.

NPV METHOD

A superior definition of the *net exchange position* is the net present value of foreign currency cash flows.

The present value of a future cash flow is sometimes referred to as its *delta*. This represents that amount which needs to be purchased or sold value spot to accumulate to the face value of the foreign currency amount by the future date. If the time period is sufficiently long or interest rates are sufficiently high, the delta can be significantly less than the future value.

The benefits of the NPV method will be better appreciated when forward exchange transactions have been explained in Chapter 6; see Examples 6.8 and 6.9.

Applying the NPV method for an extensive portfolio of transactions requires a reasonably sophisticated system that can continually discount future cash flows into present values.

PRACTICE PROBLEMS

3.1 Cash flow representation of a forward interest rate transaction
Show the cash flows when \$2,000,000 is borrowed from one month till six months at a forward interest rate $r_{1,6}$ of 5% p.a.

3.2 Cash flow representation of a forward exchange rate transaction
Show the cash flows when €2,000,000 are purchased three months forward against US dollars at a forward rate of €1 = US\$0.8560.

3.3 Net exchange position sheet
Prepare a net exchange position sheet for a dealer whose local currency is the US dollar who does the following five transactions. Assuming that he or she is square before the first transaction, the dealer:

1. Borrows €7,000,000 for four months at 4.00% p.a.
2. Sells €7,000,000 spot at €1 = 0.8500
3. Buys ¥500,000,000 spot at US\$1 = ¥123.00
4. Sells ¥200,000,000 spot against euro at €1 = ¥104.50
5. Buys €4,000,000 one month forward at €1 = US\$0.8470

3.4 Cash flow representation: forward investment
Show the cash flows when US\$1,000,000 is invested from 3 months for 6 months at a forward rate $r_{3,9}$ of 3.5% p.a.

3.5 Cash flow representation: forward exchange transaction
Show the cash flows when ¥4,000,000,000 is sold against euro for value 3 November at an outright rate of €1 = ¥103.60.

CHAPTER 4

Yield Curves and Gapping in the Money Market

In this chapter the concept of the yield curve is introduced. The factors which give rise to changes in the shape and position of the yield curve are examined. The practice of gapping in the money market enables profit to be made from expected changes in interest rates.

THE YIELD CURVE

At any point of time, different interest rates are likely to apply for different tenors. For example, the one month interest rate might be 4.0% p.a., the two month interest rate 4.2% p.a., the three month interest rate 4.4% p.a., the four month interest rate 4.6% p.a., and so on.

Tenor in months	*Interest rate yield% p.a.*
1	4.0
2	4.2
3	4.4
4	4.6
5	4.8
6	5.0

The curve which is obtained when yield is plotted on the vertical axis and tenor on the horizontal axis is known as the *yield curve* (Figure 4.1).

Normal yield curve

It is normal that interest rates for longer tenors are higher than for shorter tenors. A *normal yield curve* is therefore slightly upward sloping to the right. The yield curve shown in Figure 4.1 is an example of a normal yield curve. It is not necessary for a yield curve to be a straight line to be considered normal.

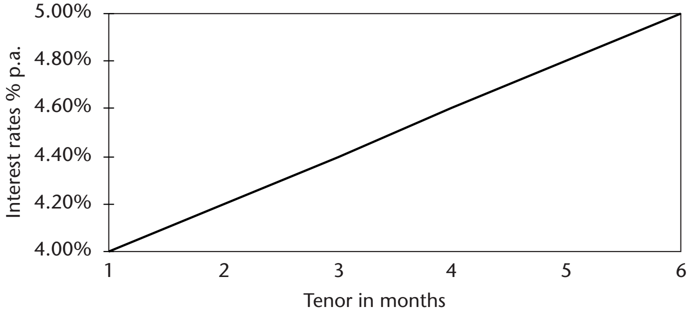


FIGURE 4.1 Yield curve

Flat yield curve
If yields were the same for all tenors, the yield curve would be horizontal. Such a yield curve would be described as a *flat yield curve* (Figure 4.2).

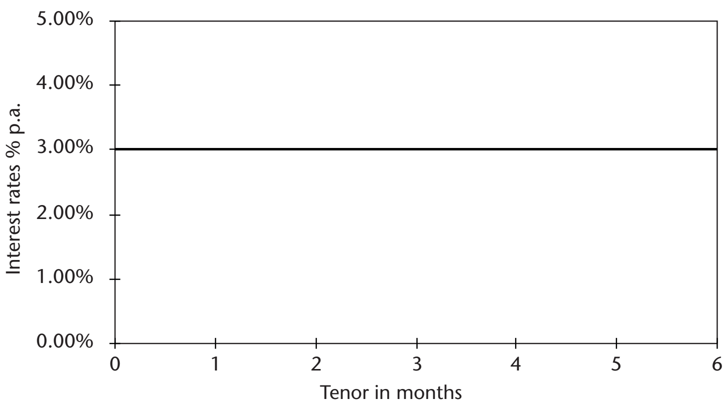


FIGURE 4.2 Flat yield curve

REASONS FOR THE NORMAL YIELD CURVE

1. Time value of money
   Interest is usually paid at the maturity of the investment. By investing for shorter periods, an investor receives interest sooner. The investor is then free to reinvest or rollover the principal plus interest for another period to enjoy the compounding of interest. The investor therefore normally requires a higher yield as an enticement to invest for longer tenors.

2. Credit risk
   When an investor lends money to a borrower, he or she takes a risk that the borrower may be unwilling or unable to repay the principal plus interest at maturity. This risk is greater the longer the period of the loan. The investor normally commands a higher yield as an enticement to extend credit for a longer term.

3. Liquidity preference
   By lending money to a borrower for a set period of time the investor foregoes the ability to spend that money until the loan matures. To compensate for the sacrifice of this liquidity the investor commands a higher yield for longer tenors.

4. Rate risk
   Interest rates change over time. When lending money to a borrower at a fixed rate, the investor takes a risk that interest rates may rise, in which case he or she will have invested at a lower return than subsequently becomes available. To compensate for taking this risk, the investor commands a higher yield for longer tenors.

IMPACT OF INTEREST RATE EXPECTATIONS

In addition to the factors that give normality to the yield curve, the shape of the yield curve is influenced by how the market expects interest rates to change.

The borrower also takes a risk when borrowing at a fixed rate for a set period of time. If interest rates fall, the borrower will be paying a higher rate of interest than if he or she had borrowed on a floating rate basis.

If market participants generally expect interest rates to rise, borrowers will be inclined to borrow for longer tenors to lock into relatively low interest rates before they rise. Consequently, longer term interest rates will tend to rise. At the same time, investors will be inclined to lend for shorter tenors in the expectation that they will be able to rollover their investments at higher interest rates in the future. Consequently, shorter term interest rates will tend to fall. The combined effect of long-term rates rising and short-term rates falling will cause the yield curve to become steeper (Figure 4.3).

Conversely, if market participants generally expect interest rates to fall, borrowers will be inclined to borrow for short periods in the expectation that they will be able to rollover their borrowings at lower interest rates in the future. Consequently, shorter term interest rates will tend to rise. At the same time investors will be inclined to lend for long periods to lock in relatively high yields before they fall. Consequently, longer term interest rates tend to fall. The combined effect of short-term rates rising and long-

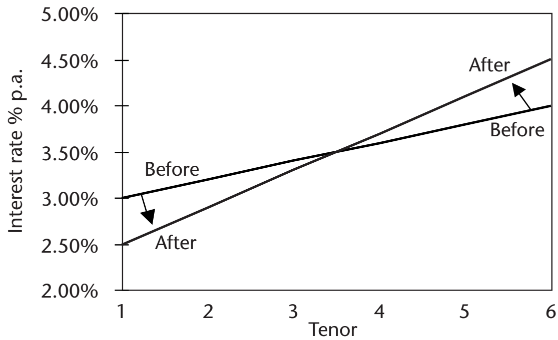


FIGURE 4.3 Market expects interest rates to rise, causing yield curve to steepen

term rates falling will cause the yield curve to become flatter. If interest rates are expected to fall more than sufficiently to outweigh the factors which give rise to normality, then the yield curve will be downward sloping to the right. Such a yield curve is described as an *inverse yield curve* (Figure 4.4).

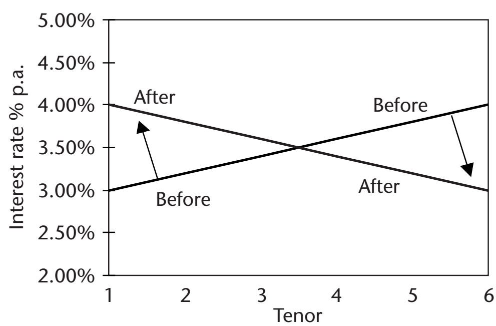


FIGURE 4.4 Market expects interest rates to fall, causing inverse yield curve

The shape of the yield curve shows what the market expects to happen to interest rates.

If the yield curve is steeper (that is, more positive) than the normal market, participants must generally expect interest rates to rise. If the yield curve is inverse, the market must generally expect interest rates to fall (Figure 4.5). Indeed, when the yield curve is flat, the market must generally expect interest rates to fall slightly, because in the absence of interest rate expectations, the yield curve would be normal.

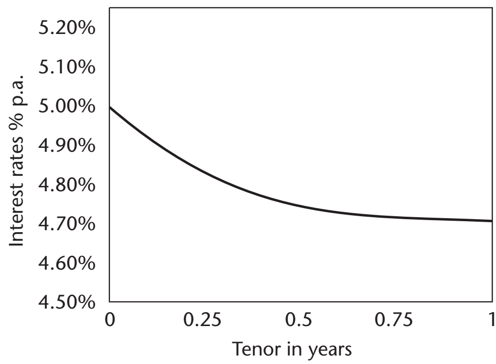


FIGURE 4.5 Inverse yield curve: Australian dollar yield curve 31 August 2001

YIELD CURVES IN PRACTICE

In practice, the slope of the yield curve is not always uniform. Market expectations for future interest rates represent a complex combination of the views of many borrowers and investors, whose expectations will vary for different tenors. Even so, the shape of the yield curve reveals a mass of market intelligence about interest rate expectations.

Figure 4.6 shows the full Australian dollar yield curve as at 31 August 2001. This yield curve is inverse out to one year and normal beyond that.

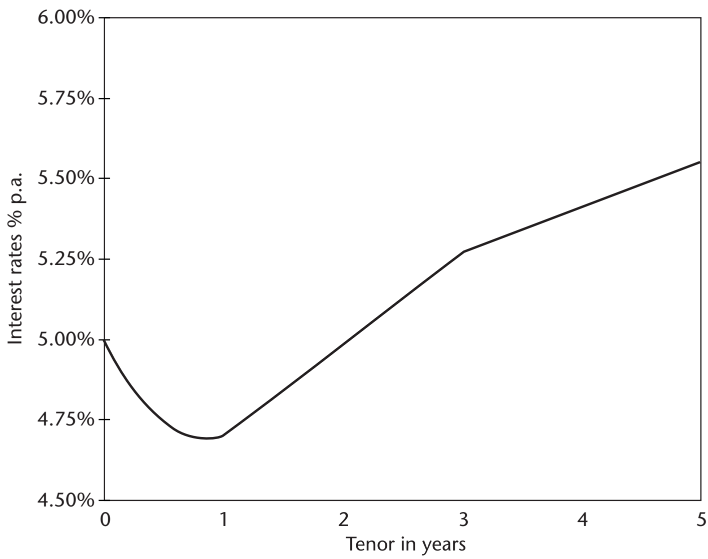


FIGURE 4.6 Australian dollar yield curve 31 August 2001

The shape of the yield curve reflects market expectations. Interest rates were expected to fall in the coming four quarters and then increase.

Interest rates do not always move in accordance with market expectations. However, the shape of the yield curve is a good barometer of what is likely to happen to interest rates. Unlike the forecasts of business commentators and economists, the yield curve reflects the views of all of the borrowers and investors participating in the market at that time.

SPREADS FOR CREDIT AND LIQUIDITY RISK

Different borrowers and issuers have to pay different rates of interest depending on their credit ratings and the liquidity of the securities that they issue. As a general rule borrowers with higher credit ratings are able to borrow at lower rates. Frequently, margins are quoted with reference to benchmark rates, such as LIBOR in the money market, or benchmark yields, which are usually government bonds in the capital markets. A small company may borrow from a bank at LIBOR + 0.50% p.a., whereas a large one may borrow at LIBOR + 0.10% p.a. Figure 4.7 illustrates how bond yields vary with the credit rating of the issuer.

The less highly rated issuers pay a higher margin that tends to widen with tenor.

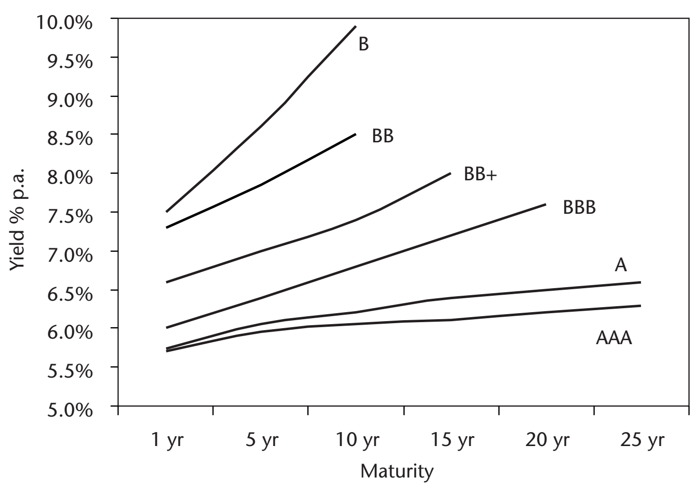


FIGURE 4.7 Yield curves for issuers with different credit ratings

YIELD CURVE MOVEMENTS

It is convenient to distinguish between parallel shifts and rotations of the yield curve. A parallel shift of the yield curve would reflect an equal increase or decrease in interest rates over a range of tenors. The slope of the yield curve is unchanged; it merely moves up or down.

Parallel shift of the yield curve
Table 4.1 shows the data plotted in Figure 4.8.

TABLE 4.1 Parallel shift of the yield curve

Tenor in months	Interest rate before shift % p.a.	Interest rate after shift % p.a.	Increase in interest rate % p.a.
1	4.0	5.0	+1.0
2	4.1	5.1	+1.0
3	4.2	5.2	+1.0
4	4.3	5.3	+1.0
5	4.4	5.4	+1.0
6	4.5	5.5	+1.0

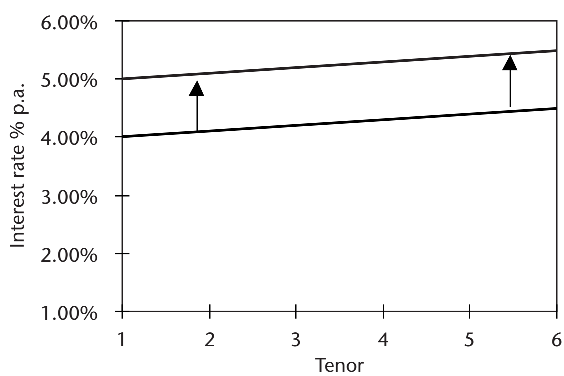


FIGURE 4.8 Parallel shift of the yield curve

A rotation of the yield curve would reflect a proportional increase or decrease in interest rates, resulting in a change in the slope of the yield curve. The yield curve would become steeper or flatter. Figure 4.3 shows an example of a rotation of a yield curve.

Rotation of the yield curve

In practice, movements of the yield curve will involve various combinations of shifts and rotations. The shape and position of the yield curve will change over time. Table 4.2 and Figure 4.9 show how the US dollar yield curve moved between 29 December 2000 and 31 December 2001.

TABLE 4.2 Shift in US dollar yield curve, 29 December 2000 and 31 December 2001

Tenor in months	Interest rate before rotation % p.a.	Interest rate after rotation % p.a.	Increase in interest rate % p.a.
1	3.0	2.5	–0.5
2	3.2	2.9	–0.3
3	3.4	3.3	–0.1
4	3.6	3.7	+0.1
5	3.8	4.1	+0.3
6	4.0	4.5	+0.5

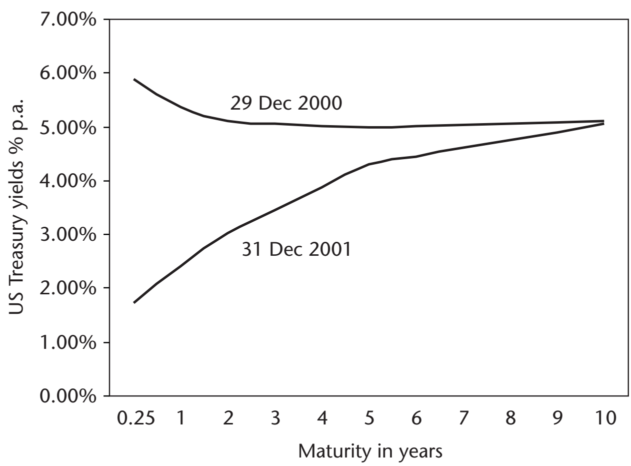


FIGURE 4.9 Shift in US dollar yield curve, 29 December 2000 and 31 December 2001

TRADITIONAL BANKING STRATEGY: RIDING THE YIELD CURVE

Traditionally, banks borrowed by taking short-term deposits and making long-term housing loans.

Whilst the yield curve remained fixed, the banks had a profitable business. They borrowed at 3% p.a. and lent at 7% p.a., making a healthy spread of 4% p.a. Taking advantage of the shape of the normal yield curve is known as 'riding the yield curve'.

During the mid-1970s, interest rates jumped dramatically in response to the inflationary shock of the oil price hike. Suddenly, banks found they had to pay, say, 10% p.a. to attract short-term deposits whilst they were locked into fixed rate mortgages yielding only 7% p.a., with perhaps several years to maturity. Mortgage loans that had traditionally provided healthy profits when the yield curve had been relatively steady were suddenly suffering losses following the jump in deposit rates.

Banks soon realized that to remain profitable in their money market activities they could no longer merely rely on riding the yield curve. In an environment where interest rates could rise or fall significantly, active interest rate management is required to ensure profitability.

GAPPING IN THE MONEY MARKET: HOW TO PROFIT FROM EXPECTED CHANGES IN INTEREST RATES

The deliberate mismatch of maturity dates of assets and liabilities is called *gapping*. The traditional banking strategy referred to as riding the yield curve was profitable while interest rates remained steady. Indeed, this strategy would have proved even more profitable had short-term deposit rates fallen. However, following this strategy involved the risk that interest rates might rise. Banks that had taken an interest rate position by borrowing short term and lending long term stood to lose when interest rates rose.

The strategy of borrowing short term and lending long term is known as opening a *negative gap*. At the maturity of the short-term deposit, the bank will find itself out of funds. It will need to borrow again by taking further deposits. If interest rates have fallen, it can borrow more cheaply, so it will make an even greater profit, but if deposit rates have risen above the rate at which it made the long-term loan, it will begin to lose.

EXHIBIT 4.1 Negative gapping

Borrow short	+ +
Lend long	– – – – – – – – –

Strategy to take advantage of an expected fall in interest rates

The opposite strategy, which involves borrowing long term and lending short term, is known as opening a *positive gap*. At the maturity of the short-term loan, the bank will find itself with surplus funds to reinvest. If yields have risen it will enjoy an increase in profit, but if yields have fallen it will suffer a decrease in profit. If yields have fallen below the rate at which the bank borrowed long term, the bank will start to incur a loss.

EXHIBIT 4.2 Positive gapping

Lend short	– –
Borrow long	+ + + + +

Strategy to take advantage of an expected rise in interest rates

OPENING A NEGATIVE GAP

EXAMPLE 4.1

For simplicity, assume the yield curve for dollar interest rates is flat at 3% p.a. for tenors from one month through to six months.

Tenor in months	*Interest rate % p.a.*
1	**3.0**
2	3.0
3	3.0
4	3.0
5	3.0
6	**3.0**

A bank expects interest rates to fall. It opens a negative gap by borrowing $1,000,000 for one month and lending $1,000,000 for six months.

For the first month the bank will break even. It pays 3% p.a. ($2,500) for its deposit and receives 3% p.a. ($2,500) on its loan (Exhibit 4.3).

Whilst the gap is open, the bank has a net cash flow position. If interest rates rise it will lose because it will have to borrow for the remaining five months at a higher rate. If interest rates fall it will gain because it will be able to borrow for the remaining five months at a lower rate.

Opening the negative gap by borrowing $1,000,000 for one month and lending $1,000,000 for six months is equivalent to forward lending $1,002,500 from one month till six months at the forward interest rate $r_{1,6}$.

EXHIBIT 4.3 Opening a negative gap

$		
1,000,000.00	**–1,000,000.00**	Today

$		
	–1,002,500.00	1 month 1,000,000(1 + 0.03 × 1/12)

$		
1,015,000.00		6 months 1,000,000(1 + 0.03 × 6/12)

Note:

$$
\begin{aligned}
(1+r_{1,6}) &= \frac{(1+0.03\times 6/12)}{(1+0.03\times 1/12)} \\
&= 1.0246828\ldots \\
\therefore r_{1,6} &= 0.029925\ldots \\
&= 2.99\%\ \text{p.a.}
\end{aligned}
$$

CLOSING A NEGATIVE GAP

Suppose the bank's expectation is fulfilled and after one month there has been a 1% p.a. downward shift in the yield curve, so that all interest rates fall from 3% p.a. to 2% p.a. The bank could then close the gap by borrowing $1,002,500 for the remaining five months.

One month later (Exhibit 4.4), by closing the gap, the bank realizes a profit of $4,145.83 and eliminates its net cash flow position. Subsequent rises or falls in interest rates will have no further impact on the bank's profit or loss.

EXHIBIT 4.4 Closing the negative gap after a parallel downward shift in a flat yield curve

$		
1,002,500.00	–1,002,500.00	Today
1,002,500.00	**1,002,500.00**	

$		
1,015,000.00	**–1,010,854.17**	5 months 1,002,500(1 + 0.02 × 5/12)
	4,145.83	Profit
1,015,000.00	**1,015,000.00**	

GAPPING WITH A NORMAL YIELD CURVE

When the yield curve is not flat at the time a gap is opened, there will be a benefit or cost involved in running the gap. If the yield curve is normal or positive there will be a benefit in running a negative gap and a cost in carrying a positive gap. Conversely, if the yield curve is inverse there will be a benefit in running a positive gap or a cost in carrying a negative gap.

The yield curve works either in favour of the gap or against it.

EXAMPLE 4.2

Consider a case where a negative gap is opened with a normal yield curve. Suppose the bank follows the identical strategy as in Example 4.1, assuming the normal yield curves as in Exhibit 4.5.

EXHIBIT 4.5 Negative gap with a normal yield curve

Tenor in months	*Interest rates when gap is opened % p.a.*	*Interest rates when gap is closed % p.a. (1 month later)*
1	**3.0**	2.0
2	3.1	2.1
3	3.2	2.2
4	3.3	2.3
5	3.4	**2.4**
6	**3.5**	2.5

Following the same strategy as in Example 4.1, the bank opens a negative gap by borrowing \$1,000,000 for one month at 3.0% p.a. and lending \$1,000,000 for six months at 3.5% p.a.

Opening the negative gap is equivalent to forward lending \$1,002,500 from one month to six months at the forward interest rate of 3.59% p.a. (Exhibit 4.6):

$$
\begin{aligned}
(1+r_{1,6}\times 5/12) &= \frac{(1+0.035\times 6/12)}{(1+0.03\times 1/12)} \\
r_{1,6} &= 0.035910 \\
&= 3.59\%\ \text{p.a.}
\end{aligned}
$$

EXHIBIT 4.6 Opening a negative gap with a normal yield curve

$		
1,000,000.00	**–1,000,000.00**	Today

$		
	–1,002,500.00	1 month 1,000,000(1 + 0.03 × 1/12)

$		
1,017,500.00		6 months 1,000,000(1 + 0.035 × 6/12)

Whilst the gap is open, the bank earns the 50 basis points difference between the borrowing rate and the lending rate. This reflects the benefit of running the negative gap with a positive yield curve. Until the gap is closed, the bank has an interest rate position. If interest rates rise, it may have to borrow for the remaining five months at a higher rate. The benefit of running the gap, which is known as *positive carry*, will reduce the unfavourable impact of a rise in rates or add to the favourable impact of a fall in rates.

Assuming that after one month there has been a 1% p.a downward shift in interest rates as portrayed in Exhibit 4.5, the bank can close the gap by borrowing $1,002,500 for five months at 2.4% p.a (Exhibit 4.7).

EXHIBIT 4.7 Closing the negative gap after a parallel shift in a normal yield curve

$		
1,002,500.00	–1,002,500.00	Today
1,002,500.00	**1,002,500.00**	

$		
1,017,500.00	**–1,012,525.00**	5 months 1,002,500(1 + 0.024 × 5/12)
	4,975.00	Profit
1,017,500.00	**1,017,500.00**	

By closing the gap, the bank realizes a profit of $4,975.00 and eliminates its net cash flow position. Subsequent rises or falls in interest rates will have no further impact on the bank's profit or loss. The difference between the profit of $4,145.83 realized in Example 4.1 and that of $4,975.00 in Example 4.2 reflects the benefit of the positive carry.

OPENING A POSITIVE GAP

A bank expecting interest rates to rise would be inclined to open a positive gap.

EXAMPLE 4.3

Consider a case where a bank expects yields to rise when the initial yield curve is as in Example 4.2. The bank might borrow \$1,000,000 for six months at 3.5% p.a. and lend \$1,000,000 for one month at 3.0% p.a. (Exhibit 4.8). For a positive gap to prove profitable when the yield curve is normal, it is necessary for interest rates to rise faster than the slope of the yield curve.

EXHIBIT 4.8 Opening a positive gap with a normal yield curve

$		
1,000,000.00	**–1,000,000.00**	Today

$		
1,002,500.00		1 month 1,000,000(1 + 0.03 × 1/12)

$		
	–1,017,500.00	6 months 1,000,000(1 + 0.035 × 6/12)

Opening the positive gap by borrowing \$1,000,000 for six months and lending \$1,000,000 for one month is equivalent to forward borrowing \$1,002,500 from one month till six months at the forward interest rate 3.59% p.a.:

$$(1+r_{1,6}\times 5/12)=\frac{(1+0.035\times 6/12)}{(1+0.03\times 1/12)}$$

$$r_{1,6}=3.59\%\text{ p. a.}$$

Whilst the gap is open the bank pays the 0.5% p.a. difference between the borrowing rate and the lending rate. This represents the cost of carrying the positive gap with a positive yield curve. The cost of carrying the gap, which is known as *negative carry*, will reduce the favourable impact of a rise in interest rates and add to the unfavourable impact of a fall in interest rates.

CLOSING A POSITIVE GAP

Assuming that after one month there has been a 1% p.a. upward shift in interest rates, as portrayed in Exhibit 4.9, the bank could close the gap by lending $1,002,500 for five months at 4.4% p.a.

By closing the gap, the bank eliminates its net cash flow position and realizes a profit of $3,379.17; refer to Exhibit 4.10. Notice that this is less than the $4,975.00 profit that would have been realized from an equivalent fall in interest rates under a negative gapping strategy (Exhibit 4.7 above). The difference reflects the impact of the normal yield curve working in favour of the negative gap and against the positive gap.

EXHIBIT 4.9 Closing a positive gap after a parallel shift in a normal yield curve

Tenor in months	*Interest rates when gap is opened % p.a.*	*Interest rates when gap is closed % p.a (1 month later)*
1	**3.0%**	4.0%
2	3.1%	4.1%
3	3.2%	4.2%
4	3.3%	4.3%
5	3.4%	**4.4%**
6	**3.5%**	4.5%

EXHIBIT 4.10 Closing a positive gap with negative carry

$		
1,002,500.00	**–1,002,500.00**	Today
1,002,500.00	**1,002,500.00**	

$		
1,020,879.17	–1,017,500.00	5 months 1,002,500(1 + 0.044 × 5/12)
	3,379.17	Profit
1,020,879.17	**1,020,879.17**	

BREAK-EVEN RATES

It is possible at the time of opening the gap to calculate the *break-even interest rate*, *b*, at which the gap will result in zero profit or loss. Assuming the normal yield curve, as in Example 4.2, the break-even five month interest rate one month after opening the gap would be:

1. Under the negative gap scenario, such that:

$$\begin{aligned}
\text{interest earned for six month period} &= \text{interest paid for} \\
&\quad\ \text{six month period} \\
(1{,}000{,}000 \times 0.03 \times 1/12) + (1{,}002{,}500 \times b \times 5/12) &= 1{,}000{,}000 \times 0.035 \times 6/12 \\
2{,}500 + 417{,}708.33 \times b &= 17{,}500 \\
b &= \frac{17{,}500 - 2{,}500}{417{,}708.33} \\
&= 0.035910 \\
&= 3.59\%\ \text{p.a.}
\end{aligned}$$

That is, the five month interest rate could rise to 3.59% p.a. before the gap would start to lose money.

The break-even rate equals the forward interest rate at the time of opening the gap.

2. Under the positive gap scenario, such that:

$$\begin{aligned}
\text{interest earned for six month period} &= \text{interest paid for} \\
&\quad\ \text{six month period} \\
1{,}000{,}000 \times 0.035 \times 6/12 &= (1{,}000{,}000 \times 0.03 \times 1/12) \\
&\quad + (1{,}002{,}500 \times b \times 5/12) \\
\text{Again,} \qquad b &= 3.59\%\ \text{p.a.}
\end{aligned}$$

That is, the five month interest rate would have to rise to 3.59% p.a. before the gap would start to make money. Again, the break-even rate equals the original forward interest rate.

Notice that the calculation of the break-even rate is identical for both the negative and positive gaps. The break-even rate is a function of the *shape* of the yield curve. It does not depend on whether the gap is positive or negative.

EARLY CLOSURE OF A GAP

It is possible to close out a gap at a date other than the maturity of the near leg. A gap can be closed prior to the maturity of the near leg by doing the reversing money market transactions.

EXAMPLE 4.4

To close out the gap opened in Example 4.1 one week after it was opened, it would be necessary to lend for three weeks and borrow for five and three quarter months. For the profit or loss from the gap to occur at the far date (i.e. six months from the date of opening the gap), the amount to be lent for three weeks should be such that the future value at the three week maturity will exactly offset the future value of the original one month borrowing, that is, \$1,002,500.00. The amount to be borrowed for five and three quarter months should exactly offset the amount being lent for three weeks.

Suppose that one week after opening the gap, the three week interest rate is 2.5% p.a. and the five and three quarter month interest rate is 2.7% p.a., i.e. the yield curve has dropped, but further at the short end than at the long end. The cash flows in Exhibit 4.11 show that closing the gap would generate a profit of \$1,114.35 at the far date.

An alternative and equivalent means of closing the gap would be to borrow \$1,002,500 at the forward interest rate of 2.73% p.a. from three weeks till five and three quarter months:

$$(1 + r_{0.75,5.75} \times 5/12) = \frac{(1 + 0.027 \times 5.75/12)}{(1 + 0.025 \times 0.75/12)}$$

$$\therefore r_{0.75,5.75} = 0.027257\ldots$$

$$= 2.73\%\ \text{p.a.}$$

EXHIBIT 4.11 Early closure of a negative gap
One week after opening:

\$		
1,000,936.04	**–1,000,936.04**	Today 1,002,500/(1 + 0.025 × 0.75/12)
1,000,936.04	**1,000,936.04**	

\$		
1,002,500.00	–1,002,500.00	0.75 months
1,002,500.00	**1,002,500.00**	

\$		
1,015,000.00	**–1,013,885.65**	5.75 months 1,000,936.04(1 + 0.027 × 5.75/12)
	1,114.35	Profit
1,015,000.00	**1,015,000.00**	

EXTENDING A GAP

To extend a gap beyond the maturity date of the near leg merely requires rolling the near leg through the money market. For example, to extend the negative gap opened in Example 4.1 for a further month simply requires rolling the original one month borrowing plus interest for an additional month. This strategy would be appropriate if interest rates were expected to fall during the coming month. Assuming that it is intended that profits be realized at the far date of the gap, it would be necessary to roll the original borrowing so as to capitalize the interest from the first month.

Assuming that one month after the gap was opened the five month interest rate was 2.7% p.a., extending the life of the gap would merely require the bank to borrow $1,002,500 for an additional month at 2.7% p.a (Exhibit 4.12).

EXHIBIT 4.12 Extending the gap
One month after opening the gap:

$		
1,002,500.00	–1,002,500.00	Today
1,002,500.00	**1,002,500.00**	

$		
	–1,004,755.63	1 month 1,002,500(1 + 0.027 × 1/12)

$		
1,015,000.00		5 months

PRACTICE PROBLEMS

4.1 Gapping: normal yield curve: expect upward shift of yield curve
The dollar yield curve is currently:

1 month	5.00%
2 months	5.25%
3 months	5.50%

Interest rates are expected to rise.

(a) What two money market transactions should be performed to open a positive gap 3 months against 1 month?

Assume that this gap was opened on a principal amount of $1,000,000 and after 1 month rates have risen such that the yield curve is then:

1 month	6.00%
2 months	6.25%
3 months	6.50%

(b) What money market transaction should be performed to close the gap?
(c) How much profit or loss would be made from opening and closing the gap?

4.2 Gapping: inverse yield curve: expect downward shift of yield curve
The dollar yield curve is currently inverse and expectations are that one month from now the yield curve will be 50 basis points below current levels, as reflected in the following table.

Tenor in months	*Current interest rates % p.a.*	*Expected interest rates % p.a.*
1	4.0	3.5
2	3.5	3.0
3	3.0	2.5

A corporation borrows \$10,000,000 for one month and lends \$10,000,000 for three months to open a negative gap position.

(a) Calculate the break-even interest rate at which it will need to be able to borrow \$10,000,000 for 2 months in one month's time.
(b) Assuming the yield curve moves according to expectation, calculate the profit or loss which will be realized on closing the gap.

4.3 Gapping: normal yield curve: expect rotation of yield curve
The crown yield curve is currently normal and expectations are that it will become steeper with the pivotal point at 6 months as reflected below:

Months	*Current rates*	*Expected rates 3 months from now*
3	5.0%	4.5%
6	5.5%	5.5%
9	6.0%	6.5%
12	6.5%	7.5%

Two gapping strategies are contemplated:

(a) Borrowing C1,000,000 for 3 months and lending C1,000,000 for 6 months.
(b) Borrowing C1,000,000 for 3 months and lending C1,000,000 for 12 months.

Assuming that interest rates move according to expectations and that the gap is closed after 3 months, which strategy will prove more profitable?

4.4 Gapping: calculation for exact number of days
On 1 July a company borrows $10,000,000 at a three month floating rate of 3.75% p.a. (360 days per year basis) This debt will be rolled on 1 October (92 days). The company also placed $10,000,000 on deposit maturing on 3 January (186 days) also at a rate of 3.75% p.a.

(a) Is the gap which the company has opened positive or negative?
(b) Would the company like the 3 month rate on 1 October to be higher or lower than at present?
(c) Calculate the profit or loss if the company rolls the floating rate borrowing for 94 days from 1 October at exactly 3.75% p.a.

4.5 Gapping: normal yield curve – expect rates to fall
The dollar yield curve is currently:

1 month	5.00%
2 months	5.25%
3 months	5.50%

Interest rates are expected to fall.

(a) What two money market transactions should be performed to open a gap 3 months against 1 month?
(b) Assume the gap was opened on a principal amount of $10,000,000 and after 1 month interest rates have fallen such that the yield curve is then:

1 month	4.75%
2 months	5.00%
3 months	5.25%

What money market transaction should be performed to close the gap?

(c) How much profit or loss would have been made from opening and closing the gap?

CHAPTER 5

Bid and Offer Rates

In this chapter, the concept of bid and offer rates is introduced. Potential confusion arises because with bid and offer quotations there are two parties, two prices and, in the case of foreign exchange rates, two currencies. Dealers use bid and offer rates to make money by making markets and exploiting arbitrage when the opportunity arises.

QUOTING BANK AND CALLING BANK

It is customary in inter-bank markets for banks to quote two-way prices. There are two parties to the quotation: the quoting bank and the calling bank. The *quoting bank* is the bank that quotes the price. The *calling bank* is the bank that calls to ask the quoting bank for a price. Having been quoted the price, the calling bank either accepts it, in which case a deal is done, or rejects it, in which case no transaction occurs.

PRICE MAKER AND PRICE TAKER

When banks quote interest rates and exchange rates to customers it is often obvious whether the customer wants to borrow, lend, buy or sell. Consequently, it is sufficient for the bank to quote a one-way price. However, on occasions customers require two-way prices. A more general description is to refer to the quoting bank as the *price maker* and the other party as the *price taker*.

Whenever two-way prices are quoted, the first rate quoted is known as the *bid rate* and the second rate quoted is known as the *offer rate*.

BID AND OFFER RATES IN THE MONEY MARKET

In the money market the *bid rate* is the rate at which the quoting bank is willing to borrow. The *offer rate* is the rate at which the quoting bank is willing to lend.

A bank quotes three month dollars at 3.00%/3.25% p.a. The bid rate is 3.00% p.a. and the offer rate 3.25% p.a. In full, the quoting bank is saying that it is willing to borrow dollars for three months at a rate of 3.00% p.a. and to lend dollars for three months at a rate of 3.25% p.a.

The price taker performs the opposite side of the transaction to the quoting bank. If the price maker borrows then the price taker lends and if the price maker lends then the price taker borrows.

If the price taker wishes to borrow dollars from the quoting bank, it must borrow at the offer rate (i.e. lending rate) of 3.25% p.a. If the price taker wishes to lend dollars to the quoting bank, it must lend them at the bid rate (i.e. borrowing rate) of 3.00% p.a. An alternative definition of the bid rate would be the rate at which the price taker can lend. Similarly, an alternative definition of the offer rate would be the rate at which the price taker can borrow; see Exhibit 5.1.

EXHIBIT 5.1 Bid and offer rates in the money market: 3 month dollars

	Bid rate	*Offer rate*
Quote	3.00% p.a.	3.25% p.a.
Price maker	Borrows	Lends
Price taker	Lends	Borrows

BID AND OFFER RATES IN THE FOREIGN EXCHANGE MARKET

In the foreign exchange market the *bid rate* is the rate at which the quoting bank is willing to buy the commodity currency.

The *offer rate* is the rate at which the quoting bank is willing to sell the commodity currency.

A bank quotes spot €1 = US$0.8410/0.8415. The bid rate is 0.8410 and the offer rate is 0.8415. In full, the quoting bank is saying that it is willing to buy euros at a rate of €1 = US$0.8410 and to sell euros at a rate of €1 = US$0.8415.

The price taker performs the opposite side of the transaction to the quoting bank. If the price maker buys euros then the price taker sells euros, and if the price maker sells euros then the price taker buys euros.

If the price taker wishes to buy euros from the quoting bank, it must buy them at the quoting bank's offer rate (i.e. selling rate) of 0.8415. If the price taker wishes to sell euros to the price maker, it must sell them at the price maker's bid rate (i.e. buying rate) of 0.8410. An alternative definition of the bid rate would be the rate at which the price taker could sell the commodity currency. Similarly, an alternative definition of the offer rate would be the rate at which the price taker could buy the commodity currency. See Exhibit 5.2.

EXHIBIT 5.2 Bid and offer rates in the foreign exchange market: commodity currency, spot €/US$

	Bid rate	*Offer rate*
Quote €1 = US$	0.8410	0.8415
Price maker	Buys euros	Sells euros
Price taker	Sells euros	Buys euros

In the quotation €1 = US$0.8410/0.8415, the euro is the commodity currency and the dollar is the terms currency. When a bank buys the commodity currency it sells the terms currency, and when it sells the commodity currency it buys the terms currency.

The bid rate could also be defined as the rate at which the price maker is willing to sell the terms currency or the rate at which the price taker is able to buy the terms currency. The offer rate could also be defined as the rate at which the price maker is willing to buy the terms currency or the rate at which the price taker is able to sell the terms currency. See Exhibit 5.3.

EXHIBIT 5.3 Bid and offer rates in the foreign market exchange: terms currency, spot €/US$

	Bid rate	*Offer rate*
Quote €1 = US$	0.8410	0.8415
Price maker	Sells dollars	Buys dollars
Price taker	Buys dollars	Sells dollars

To avoid confusion, observe the convention of concentrating on the commodity currency. When asked a question about the terms currency, convert it into a question about the commodity currency. This is referred to as the 'mental switch'. For example, if asked 'At what rate will the quoting bank buy dollars?', convert the question to 'At what rate will the quoting bank sell euros?'.

The answer is at the offer rate, US$0.8415.

EXAMPLE 5.1

A bank quotes US$1 = ¥125.50 125.60

At what rate:	*Answer:*	
1. Will the quoting bank buy dollars?	The quoting bank's bid rate:	125.50
2. Will the quoting bank sell dollars?	The quoting bank's offer rate:	125.60
3. Can a calling bank buy dollars?	The quoting bank's offer rate:	125.60
4. Can the calling bank sell dollars?	The quoting bank's bid rate:	125.50
5. Will the quoting bank buy yen? Mental switch: at what rate will the quoting bank sell dollars?	The quoting bank's offer rate:	125.60
6. Will the quoting bank sell yen? Mental switch: at what rate will the quoting bank buy dollars?	The quoting bank's bid rate:	125.50
7. Can the price taker buy yen? Mental switch: at what rate can the price taker sell dollars?	The price maker's bid rate:	125.50
8. Can the price taker sell yen? Mental switch: at what rate can the price taker buy dollars?	The price maker's offer rate:	125.60

BID OFFER SPREADS

The difference between the bid rate and the offer rate is called the *bid offer spread*.

As the quoting bank is seeking to make profit by borrowing at lower rates than it lends or buying the commodity currency more cheaply than it sells it, the bid rate will be a smaller number than the corresponding offer rate.

EXAMPLE 5.2

	Bid rate	*Offer rate*
Money market 3 month dollars	3.00% p.a.	3.25% p.a.
Foreign exchange market €1 = US$	0.8415	0.8410

If the quoting bank is able to deal equal amounts at its bid rate and its offer rate, it would 'lock in' a profit without acquiring a net cash flow position or a net exchange position.[1]

If the quoting bank borrows $1,000,000 for three months at 3.00% p.a. and lends $1,000,000 for three months at 3.25% p.a., how much profit would it make?

The quoting bank would have the cash flows shown in Exhibit 5.4.

EXHIBIT 5.4 Money market cash flows

$		
		Today
1,000,000.00		3.00%
	–1,000,000.00	3.25%
1,000,000.00	**1,000,000.00**	

$		
		3 months
	–1,007,500.00	1,000,000(1 + 0.03 × 3/12)
1,008,125.00		1,000,000(1 + 0.0325 × 3/12)
	625.00	Profit
1,008,125.00	**1,008,125.00**	

Profit = $625

If the quoting bank buys €1,000,000 at US$0.8410 and sells €1,000,000 at US$0.8415, how much profit would it make?

It would have the cash flows and net exchange position shown in Exhibit 5.5.

1 This is not exactly true, because the profit generated would create a (small) net cash flow position and if the profit is in units of a foreign currency it would create a (small) net exchange position.

EXHIBIT 5.5 Foreign exchange market cash flows

€		Spot	US$		
1,000,000.00		0.8410		**–841,000.00**	Net exchange position
	–1,000,000.00	0.8415	**841,500.00**		Square
		Profit		**500.00**	
1,000,000.00	**1,000,000.00**		**841,500.00**	**841,500.00**	

Profit = US$500

If the quoting bank widens its spread it will make a larger profit each time it is able to deal on both sides of its quote. For example, if it quoted 3.00/3.50 it would make a profit of $1,250 per $1,000,000, and if it quoted €1 = US$0.8410/0.8420, it would make a profit of US$1,000 per €1,000,000.

However, the wider the bid offer spread the less attractive the prices are to customers and calling banks. Banks that quote narrow spreads will tend to have higher turnovers than banks that quote wide spreads. See Exhibit 5.6.

EXHIBIT 5.6 Bid offer spreads: € = US$

		Bid	*Offer*	
Bank A	Aggressive quote	0.8411	0.8414	Narrow spread, high turnover
Bank B	Non-competitive quote	0.8405	0.8420	Wide spread, low turnover

Price takers will be inclined to deal with Bank A in preference to Bank B. Calling banks and customers wishing to buy euros will be inclined to buy at the lower offer rate. As Bank A's offer of US$0.8414 is lower than Bank B's offer of US$0.8420, price takers wanting to buy euros will deal with Bank A.

Similarly, customers and calling banks wishing to sell euros will be inclined to sell at the higher bid rate. As Bank A's bid of US$0.8411 is higher than Bank B's bid of US$0.8405, price takers wanting to sell euros will deal with Bank A. When quoting banks become keener to deal they narrow their bid offer spread. When they wish to become less keen to deal they widen their bid offer spread.

Narrowness of bid offer spread is not the only factor that influences the decision to deal with the quoting bank. Credit lines need to be available for the transaction to proceed. A customer or calling bank may deal with Bank B even though Bank A has a more competitive price because it has no credit lines for Bank A. Speed of quotation is also an important determinant of which bank does the deal. Waiting for a slow quote involves the risk that rates may move against the price taker.

BROKERS

Brokers facilitate the coming together of buyers and sellers by collecting prices from a variety of banks. The broker selects the highest bid rate and the lowest offer rate and combines them to establish the best two-way price available to calling banks. For example, if banks quote different prices, as reflected in Exhibit 5.7, the broker price would be €1 = US$0.8411/0.8414. Calling banks find it easier and quicker to call the broker than to call all of the banks separately.

EXHIBIT 5.7 Broking exchange rates

	Bid	*Offer*
	€1 = US$	
Bank A	0.8409	0.8414
Bank B	0.8410	0.8415
Bank C	0.8411	0.8416
Broker price	0.8411	0.8414

The fee charged by brokers for their service is known as *brokerage*.

ELECTRONIC DEALING SYSTEMS

In recent years electronic dealing has become the dominant means of communication between inter-bank dealers. Dealing systems such as Reuters 2000 enable dealers to quote and transact with each other electronically rather than by telephone or facsimile. Similarly, electronic broking systems such as EBS and Reuters 2000 have become a very popular alternative to voice brokers. Electronic dealing systems have the advantage that there is a printable record of deals done and deal capture into the back office can occur automatically. This reduces the risk of error and saves time and cost.

Essential information is displayed on the dealing screen. Figure 5.1 shows an example of an EBS screen with the broker price for spot USD/JPY for value date 7-Jan-99 being 122.05/08. A dealer can hit the bid or offer by simply clicking the relevant box. The system enables details such as credit

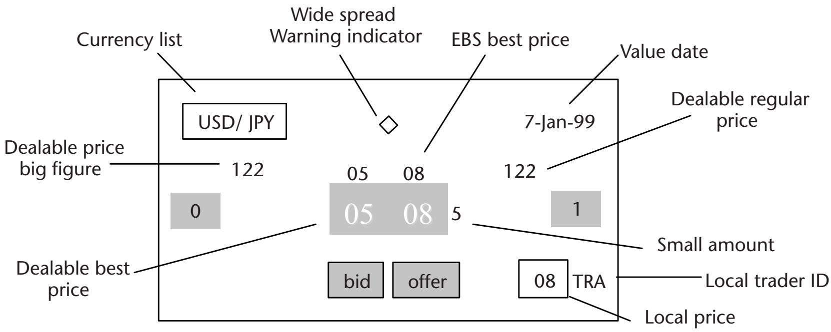


FIGURE 5.1 Electronic dealing screen

lines and settlement instructions to be linked to the screen. Consequently, deal execution, confirmation, settlement and monitoring of credit lines can occur automatically.

MARKET JARGON

Dealers tend to use market jargon when speaking to each other. Some examples of commonly used market jargon include:

- 'Mine' or 'I take' I buy the commodity currency at your offer rate
- 'Yours' I sell you the commodity currency at your bid rate
- 'Off' I am withdrawing my quote because you are taking too long to respond
- 'Your risk' I reserve the right to revise my quote because you are taking too long to respond
- 'How now?' Please make me a fresh quote

TRENDING RATES

Money markets and foreign exchange markets are dynamic. Prices change to reflect changes in supply and demand. Quoting banks move their prices to reflect changes in their willingness to borrow and lend or buy and sell.

In Exhibit 5.8, by increasing its quote from 3.00/3.25% p.a. to 3.10/3.35% p.a., the quoting bank makes its bid rate more attractive to potential lenders and its offer rate less attractive to potential borrowers.

EXHIBIT 5.8 Rising interest rates

Previous rates % p.a.	3.00	3.25
New rates % p.a.	3.10	3.35

Trending upwards or trending to the right

In Exhibit 5.9, by dropping its quote from 3.00/3.25% p.a. to 2.90/3.15% p.a., the quoting bank makes its bid rate less attractive to potential lenders and its offer rate more attractive to potential borrowers.

EXHIBIT 5.9 Falling interest rates

Previous rates % p.a.	3.00	3.25
New rates % p.a.	2.90	3.15

Trending downwards or trending to the left

In Exhibit 5.10, by increasing its quote from 0.8410/0.8415 to 0.8412/0.8417, the quoting bank makes its bid rate more attractive to potential sellers of euros and its offer rate less attractive to potential buyers of euros.

EXHIBIT 5.10 Rising exchange rates

Previous rates	€1 = US$	0.8410	0.8415
New rates	€1 = US$	0.8412	0.8417

Trending upwards or trending to the right

In Exhibit 5.11, by lowering its quote from 0.8410/0.8415 to 0.8407/0.8412, the quoting bank makes its bid rate less attractive to potential sellers of euros and its offer rate more attractive to potential buyers of euros.

EXHIBIT 5.11 Falling exchange rates

Previous rates	€1 = US$	0.8410	0.8415
New rates	€1 = US$	0.8407	0.8412

Trending downwards or trending to the left

COVERING A SPOT EXCHANGE POSITION AT MARKET RATES

EXAMPLE 5.3

If a bank quotes €1 = US$0.8410/0.8415 and a customer calls and sells the bank €1,000,000 at 0.8410, the bank becomes long €1,000,000 (short US$841,000). If the bank goes to another bank to sell euros to square its position, the first bank will find it needs to sell euros at the second bank's bid rate. Assuming that the second bank's bid rate is also 0.8410, the original bank would be square without making a profit or a loss.

To make a profit from a transaction with a customer who deals at its rates, a quoting bank should lower its bid rate or increase its offer rate.

When the market generally is quoting €1 = US$0.8410/0.8415, a bank expecting to get hit on its bid rate might quote a customer €1 = US$0.8409/0.8414. If the bank buys €1,000,000 from the customer at 0.8409 (its bid rate) and sells €1,000,000 to another bank at 0.8410 (the second bank's bid rate), the first bank would lock in a profit of one point or US$100.[2]

Similarly, a bank expecting to get hit on its offer rate might quote a customer €1 = US$0.8411/0.8416. If it sells €1,000,000 to the customer at 0.8416 (its offer rate) and buys €1,000,000 from another bank at 0.8415 (the other bank's offer rate), it would lock in a profit of one point or US$100.[3]

COVERING A SPOT EXCHANGE POSITION AT OWN RATES: JOBBING

Banks generally prefer to deal at their own prices rather than at the bid or offer rates of another bank. In practice it is rare for a quoting bank to be able to deal simultaneously on both sides of its quote for the same amount. Once a bank has been hit on one side of its quote it has a net cash flow position, and, in the case of foreign exchange transactions, a net exchange position. If market rates move against the bank, it may find that it is unable to deal on the other side of its bid offer spread at a profit.

EXAMPLE 5.4

A bank quoting €1 = US$0.8410/0.8415 gets hit for €2,000,000 on its bid rate, that is, it buys €2,000,000 at a rate of US$0.8410. It has a net exchange position long €2,000,000 (short US$1,682,000). To square its net exchange

2 Profit = 1,000,000(0.8410 – 0.8409) = US$100
3 Profit = 1,000,000(0.8416 – 0.8415) = US$100

position it will need to sell €2,000,000. If market rates rise it will gain, but if market rates fall it may lose. For example, if banks generally begin to quote 0.8400/0.8405, the highest rate at which the bank can hope to sell euros is 0.8405. It could only achieve this if it lowered its offer rate to 0.8405 and it was lucky enough to be hit there by a buyer of euros. In this best case it would lock in a loss of five points or US$1,000.[4] If it were forced to sell the euros by hitting another bank's bid rate of 0.8400, it would incur an even larger loss of ten points or US$2,000.[5]

Having purchased €2,000,000 at 0.8410, the bank might drop its quote to, say, 0.8407/0.8412. If the market generally is still quoting 0.8410/0.8415, it will now have an attractive offer and an unattractive bid. It is quite likely that its next transaction will be a sale of euros at 0.8412 and fairly unlikely that it will be a further purchase of euros at 0.8407. If the bank sells €2,000,000 at 0.8412, it will make a profit of two points or US$400, and it will have a square net exchange position.[6]

The practice of trending prices to be hit on the side which will square off a net exchange position is known as *jobbing* (Exhibit 5.12).

EXHIBIT 5.12 Jobbing exchange rates

Original quote	€1 = US$	0.8410	0.8415
Next quote	€1 = US$	0.8407	0.8412

Trending downwards to encourage potential buyers to buy euros at 0.8412

MARKET MAKING

Some banks adopt the strategy of dealing at narrow bid offer spreads in the expectation that they will deal large volumes on both sides, thereby capturing substantial profits. This strategy is known as *market making*.

An additional benefit of market making is that the bank obtains more market intelligence. Being aware of the transactions that are taking place in the market is important if the bank follows a strategy of position taking.

It is not always desirable for banks to deal at their own prices. If rates are moving quickly in one direction because there is a bias in the market, banks will be better off if they square their positions by dealing at market rates.

4 Loss = 2,000,000(0.8410 – 0.8405) = US$2,000
5 Loss = 2,000,000(0.8410 – 0.8400) = US$2,000
6 Profit = 2,000,000(0.8412 – 0.8410) = US$400

For example, if a bank is long €2,000,000 at 0.8410 and a news release comes out revealing negative news for the euro, all market participants may expect rates to fall. Banks will tend to lower their quotes and possibly to widen them as uncertainty grows. In this type of market there is no sense in a bank waiting and hoping to be hit on its offer rate. As there will be many keen sellers and possibly no keen buyers, it would be folly for a bank to wait in the vain hope of being able to sell euros at its offer rate. The prudent action to take would be to square off its position by dealing at another bank's bid rate and to trend its rates down accordingly. This will probably result in the bank taking a loss, but it is better to take a small loss now than to be caught and end up with a much larger loss later.

ARBITRAGE

Arbitrage refers to the practice of taking advantage of inconsistent pricing to lock in risk-free profits. If two banks are quoting bid and offer rates such that one bank's bid rate is higher than the other bank's offer rate, then an arbitrage opportunity exists.

EXAMPLE 5.5

	Bid	*Offer*
Bank A quotes €1 = US$	0.8410	**0.8415**
Bank B quotes €1 = US$	**0.8417**	0.8422

As Bank B's bid rate 0.8417 is higher than Bank A's offer rate 0.8415, it is possible to buy euros from Bank A at 0.8415 and to sell them to Bank B at 0.8417 for a profit of two points without creating a net exchange position. Once they realize that this situation has arisen, one or both banks will quickly trend their rates so that the arbitrage opportunity soon disappears.

CROSS RATES

Cross rate calculations with bid and offer rates involve two steps:

1. Establish which rates to cross.
2. Apply the chain rule.

EXAMPLE 5.6

$1 = ¥125.50 125.60

€ = US$0.8410 0.8415

(a) A Japanese importer needs to buy euro against yen. What is the cross rate in yen terms? Assume the bank quotes to make one point profit.

Step 1. The Japanese importer needs to *buy* euros and sell yen. The cash flows are as portrayed in Figure 5.2. If it deals with the importer, the bank would be selling euro and buying yen. In other words it is quoting an offer rate to the customer. To cover its net exchange positions with the market, the bank will need to buy euros /sell dollars at 0.8415 and buy dollars /sell yen at 125.60.

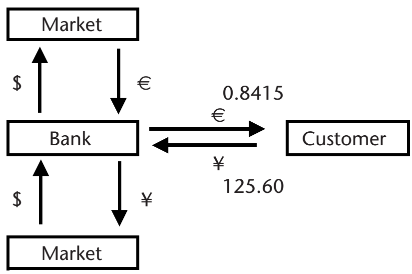


FIGURE 5.2 Flow chart representation of cross rate transaction: offer rate

Step 2. Applying the chain rule:

$$
\begin{aligned}
¥? &= €1 \\
€1 &= US\$0.8415 \\
US\$1 &= ¥125.60 \\
\therefore €1 &= \frac{1 \times 0.8415 \times 125.60}{1 \times 1} = ¥105.69
\end{aligned}
$$

This would be the market offer rate for euros in yen terms. To make one point profit the bank should quote the customer ¥105.70; that is, an offer rate one point higher than the market offer rate.

(b) A Japanese exporter needs to *sell* euros against yen. What cross rate should the bank quote to make two points profit? Express the cross rate in yen terms.

Step 1. The Japanese exporter needs to sell euros and buy yen. The cash flows would be portrayed in Figure 5.3. To cover itself the bank would need to sell euros/buy dollars at 0.8410 and to sell dollars/buy yen at 125.50.

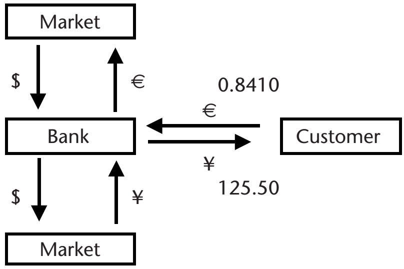


FIGURE 5.3 Flow chart representation of cross rate transaction: bid rate

Step 2. Applying the chain rule:

$$¥? = €1$$
$$€1 = US\$0.8410$$
$$US\$1 = ¥125.50$$
$$\therefore €1 = \frac{1 \times 0.8410 \times 125.50}{1 \times 1} = ¥105.55$$

This would be the market bid rate for euros in yen terms. To make two points profit the bank should quote the customer ¥105.53; that is, a bid rate which is two points lower than the market bid rate.

ARBITRAGING CROSS RATES

EXAMPLE 5.7

Bank A is quoting rates as in Example 5.5:

$1 = ¥125.50   125.60

€1 = US$0.8410   0.8415

This implies the cross rates as calculated above:

€1 = **¥105.55**   105.69

Bank B is quoting:

€1 = ¥105.40   **105.50**

What arbitrage opportunity is available?

An arbitrage opportunity exists because Bank B's offer rate is lower than Bank A's bid rate. To arbitrage:

Buy € sell ¥ with Bank B at	€1 =	¥105.50
Sell € buy ¥ with Bank A at	€1 =	¥105.55
Profit		¥ 0.05 per €

There is a potential profit of ¥0.05 per €1. On a principal amount of €1,000,000, the arbitrageur would make a profit of ¥45,500.

EXHIBIT 5.13 Cash flows in cross rate arbitrage

	Rate	€	$	¥
Bank A	0.8410	–1,000,000	+841,000	
Bank A	125.50		–841,000	+105,545,500
Bank B	105.50	+1,000,000		–105,500,000
		0	0	+ 45,500

The difference between ¥0.05 per € and ¥45,500 is due to rounding.

PRACTICE PROBLEMS

5.1 Bid offer rates
A bank quotes £1 = US$1.4020/1.4025.
(a) At what rate will the bank buy dollars?
(b) At what rate can a customer sell pounds?
(c) At what rate can a customer sell dollars?

5.2 Bid offer rates
Bank A calls and asks Bank B for a price for dollar/yen. Bank B quotes US$1 = ¥125.40/125.50. At what rate can Bank A sell yen?

5.3 Bid offer rates and spreads
A customer in Crownland asks a bank for a crown/dollar quote. The bank quotes C1 = $1.4935/1.4945.

(a) How many dollars will it cost the customer to buy C1,000,000?
(b) How many dollars will the customer receive if he or she sells C1,000,000?
(c) What is the dollar value of the 10 point bid offer spread on C1,000,000?
(d) How many crowns will it cost the customer to buy $1,000,000 ?
(e) How many crowns will the customer receive if he or she sells $1,000,000?

(f) What is the crown value of the 10 point bid offer spread on $1,000,000?

5.4 Bid offer rates
A bank quotes overnight dollars at 4.25/4.50% p.a.

(a) At what rate can a customer borrow dollars?
(b) At what rate can a customer invest dollars?

5.5 Bid offer rates and spreads
A bank quotes 7 day francs at 4.50/4.75% p.a. There are 365 days per year.

(a) Calculate the interest due if a customer borrows F1,000,000 for seven days.
(b) Calculate the interest due if a customer invests F1,000,000 for seven days.
(c) Calculate the franc value of the 25 basis point bid offer spread.

5.6 Brokers
A broker has dollar/yen prices from three banks

Bank A	US$1 = ¥125.60	125.65
Bank B	US$1 = ¥125.62	125.67
Bank C	US$1 = ¥125.63	125.68

What is the broker price?

5.7 Jobbing
A bank quotes F1 = $1.2130/1.2140. A customer calls and sells the bank F10,000,000 at its bid rate, 1.2130. The bank would like to square its position (if possible at a profit). If another bank calls a minute later asking for a price, which of the following rates should the first bank quote?

Rate A	F1 = $1.2125	1.2135
Rate B	F1 = $1.2130	1.2140
Rate C	F1 = $1.2135	1.2145

5.8 Arbitrage
Bank A quotes NZ$1 = US$0.4220 0.4225
Bank B quotes NZ$1 = US$0.4226 0.4231
What arbitrage opportunity exists? How much profit could be made by performing this arbitrage on a principal amount of NZ$10,000,000?

5.9 Cross rates

US$1	= S$ 1.7050	1.7060
€1	= US$0.8490	0.8500

A Singaporean exporter wants to sell euro and buy Singapore dollars. What is the break-even rate for euros in Singapore dollar terms?

5.10 Cross rates

US$1	= M$3.8010	3.8030
£1	= US$1.4470	1.4480

What bid and offer rates should a bank quote for pounds against ringitt in Malaysian terms to make a ten point spread on either side of the break-even rates?

5.11 Calling bank
A bank calls four other banks for dollar/Swiss franc rates.

Bank A	$1 = SF1.2430	1.2433
Bank B	$1 = SF1.2430	1.2432
Bank C	$1 = SF1.2431	1.2433
Bank D	$1 = SF1.2430	1.2433

The bank wishes to sell Swiss francs. With which bank and at what rate should it deal?

5.12 Cross rates

US$1	= ¥104.50	104.60
€1	= US$0.8550	0.8555

A Japanese importer wants to buy euros and sell yen. What is the break-even rate for euros in yen terms?

5.13 Cross rates
A customer calls and wants to buy Hong Kong dollars against Australian dollars. What rate should a bank quote for Australian dollars in terms of Hong Kong dollars to ensure a one point profit?

US$1	= HK$7.7360	7.7370
A$1	= US$0.5240	0.5245

CHAPTER 6

Forward Exchange Rates

In this chapter the concept of forward exchange rates is developed. The forward exchange rate can be calculated from the spot rate and the interest rates of two currencies. The concept is extended to cover short dates and long-term foreign exchange.

If the one year dollar interest rate is 5% p.a., $100 has a future value of $105 one year from now. It follows that if the spot price of gold is $300, the one year forward price of gold is $315. Gold could be said to command a forward premium of $15 (or 5%) as the forward price is $15 (or 5%) higher than the spot price.

Forward exchange rates are determined in the same way as the forward gold price, except that as exchange rates involve two currencies, there are two interest rates involved.[1]

Definition

A *forward exchange rate* is a rate agreed today at which one currency is sold against another for delivery on a specified future date.

For example, on 2 September, US$1,000,000 is sold against ¥118,510,000 for value 4 March at a forward rate US$1 = ¥118.51. The rate is agreed on 2 September. The currencies are exchanged 6 months later on 4 March.

CALCULATION OF FORWARD EXCHANGE RATES

Forward rates differ from spot rates to reflect the differing interest rates prevailing in the two currencies.

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1 In reality gold interest rates are not equal to zero, so the forward gold price is a function of the gold interest rate as well as spot gold and the dollar interest rate.

EXAMPLE 6.1

If the spot rate is US\$1 = ¥120 and 6 months dollar and yen interest rates are 3.00% p.a. and 0.50% p.a. respectively, calculate the 6 month forward exchange rate.

By the end of the 6 month period:

US\$1,000,000 would be worth	$1{,}000{,}000 \times (1 + 0.03 \times 6/12)$ $= \$1{,}015{,}000$
¥120,000,000 would be worth	$120{,}000{,}000 \times (1 + 0.005 \times 6/12)$ $= ¥120{,}300{,}000$

The 6 month forward exchange rate would be such that:

$$\text{US\$}1{,}015{,}000 = ¥120{,}300{,}000$$

$$\text{US\$}1 = ¥\frac{120{,}300{,}000}{1{,}015{,}000} = ¥118.52$$

EXAMPLE 6.2

In practice, it is necessary to base the forward rate calculation on the exact number of days rather than a round number of months. Working in exact number of days, the 6 month period from 4 September (spot value) till 4 March (6 months forward value) is 181 days.

On 2 September market rates are:

Spot	US\$1	=	¥120.00
US\$ interest rate			3.00% p.a. (181/360)
¥ interest rate			0.50% p.a. (181/360)

At the spot rate US\$1,000,000 = ¥120,000,000.

If US\$1,000,000 is invested for 181 days at an interest rate of 3.0% p.a., it would accumulate to a future value of \$1,015,083.33.

$$\begin{aligned}\text{Forward value} &= \text{Principal} + \text{Interest}\\ &= \text{US\$}1{,}000{,}000 + 1{,}000{,}000 \times 0.03 \times 181/360\\ &= \text{US\$}1{,}000{,}000 + 15{,}083.33\\ &= \text{US\$}1{,}015{,}083.33\end{aligned}$$

Similarly, if ¥120,000,000 is invested for 181 days at an interest rate of 0.50% p.a., it would accumulate to a future value of ¥120,310,667.

$$\begin{aligned}\text{Forward value} &= \text{Principal} + \text{Interest}\\ &= ¥120{,}000{,}000 + 120{,}000{,}000 \times 0.005 \times 181/360\\ &= ¥120{,}000{,}000 + 301{,}667\\ &= ¥120{,}301{,}667\end{aligned}$$

The market forward exchange rate is the rate at which traders would agree today to exchange dollars and yen for value 4 March. As they are willing to exchange US$1,000,000 for ¥120,000,000 value spot, it follows that they will be willing to exchange the corresponding forward values of the two currencies.

$$\text{Value 4 March} \qquad \text{US\$}1{,}015{,}083.33 = ¥120{,}301{,}667$$

$$\text{US\$}1 = ¥\,\frac{120{,}301{,}667}{1{,}015{,}083.33}$$

$$\text{US\$}1 = ¥118.51$$

In general,

$$\boxed{\text{Forward rate} = \frac{\text{Future value of terms currency}}{\text{Future value of commodity currency}}} \tag{6.1}$$

CALCULATION OF FORWARD MARGINS

The difference between the spot rate and the forward rate is known as the *forward margin*.

$$\boxed{\text{Forward margin} = \text{Forward rate} - \text{Spot rate}} \tag{6.2}$$

In Example 6.1:

$$\begin{aligned}\text{Forward margin} &= 118.52 - 120.00\\ &= -1.48\end{aligned}$$

In Example 6.2:

$$\begin{aligned}\text{Forward margin} &= 118.51 - 120.00\\ &= -1.49\end{aligned}$$

If the forward margin is positive, the commodity currency is said to be at a *forward premium*; that is, the forward rate is higher than the spot rate. If the forward margin is negative, the commodity currency is said to be at a *forward discount*.

FORWARD DISCOUNTS

In Examples 6.1 and 6.2 the forward margin is a negative number, indicating that the commodity currency (dollar) is at a forward discount against the terms currency (yen). That is, the forward price of the dollar is less than its spot price when priced in terms of yen. This reflects the fact that the dollar interest rate is higher than the yen interest rate.

The higher interest rate currency is at a forward discount against the lower interest rate currency.

EXAMPLE 6.3

Spot	€1	= US$0.9000
6 month dollar interest rate		3.50% p.a. (184/360)
6 month euro interest rate		2.50% p.a. (184/360)

At the spot rate, €1,000,000 = US$900,000.

After 6 months (184 days) €1,000,000 would accumulate to a future value of €,012,777.78.

$$
\begin{aligned}
\text{Forward value} &= \text{Principal} &+\quad& \text{Interest}\\
&= €1{,}000{,}000 &+\quad& 1{,}000{,}000 \times 0.025 \times 184/360\\
&= €1{,}000{,}000 &+\quad& 12{,}777.78\\
&= €{,}1012{,}777.78
\end{aligned}
$$

Similarly, US$900,000 would accumulate to a future value of US$916,100.00.

$$
\begin{aligned}
\text{Forward value} &= \text{Principal} &+\quad& \text{Interest}\\
&= \text{US\$}900{,}000 &+\quad& 900{,}000 \times 0.035 \times 184/360\\
&= \text{US\$}900{,}000 &+\quad& 16{,}100.00\\
&= \text{US\$}916{,}100.00
\end{aligned}
$$

Thus

$$
\text{Forward rate} = \frac{\text{Future value of terms currency}}{\text{Future value of commodity currency}}
$$

$$
= \frac{\text{US\$}916{,}100.00}{€1{,}012{,}777.78}
$$

$$
€1 = \text{US\$}0.9045
$$

$$
\begin{aligned}
\text{Forward margin} &= \text{Forward rate}\\
&= 0.9045 - 0.9000\\
&= +0.0045
\end{aligned}
$$

FORWARD PREMIUMS

In Example 6.3 the forward margin is a positive number, indicating that the commodity currency (euro) is at a forward premium against the terms currency (dollar). That is, the forward price of a euro is higher than the spot price in terms of dollars. This reflects the fact that the euro interest rate is lower than the dollar interest rate.

The lower interest rate currency is at a forward premium against the higher interest rate currency.

If one currency is at a forward premium, the other currency is at a forward discount. The euro is at a forward premium against the dollar, so it follows that the dollar is at a forward discount against the euro.

COMPENSATION ARGUMENT

Between the spot date and the value date of a forward exchange agreement, one party holds the higher yielding currency. The other party holds the lower yielding currency. In effect, the forward rate differs from the spot rate to the extent to which the holder of the higher interest currency is willing to and expected to compensate the holder of the lower interest currency.

FORWARD RATE FORMULA

The general formula can be expanded to:

$$\boxed{f = \frac{s(1 + r_{\mathrm{T}}t)}{(1 + r_{\mathrm{C}}t)}} \qquad (6.3)$$

where

f = forward exchange rate
s = spot exchange rate
r_{T} = interest rate of terms currency
r_{C} = interest rate of commodity currency
t = time period

Derivation of forward rate formula (Equation 6.3)
By definition, the forward rate is the ratio of the future value of the terms currency to the future value of the commodity currency.

$$f = \frac{FV_{\mathrm{T}}}{FV_{\mathrm{C}}}$$

Similarly, by definition the spot rate is the ratio of the present value of the terms currency to the present value of the commodity currency.

$$s = \frac{PV_T}{PV_C}$$

Using $FV = PV(1 + rt)$:

$$f = \frac{FV_T}{FV_C} = \frac{PV_T(1 + r_T t)}{PV_C(1 + r_C t)} = \frac{s(1 + r_T t)}{(1 + r_C t)}$$

If the time period is expressed in days, $t = d/dpy$, where:

$$d = \text{number of days}$$
$$dpy = \text{days per year}$$

Therefore,

$$\boxed{f = \frac{s(1 + r_T\, d/dpy_T)}{(1 + r_C\, d/dpy_C)}} \qquad (6.4)$$

Generalizing, $t = p/ppy$, where

$$p = \text{period}$$
$$ppy = \text{periods per year}$$

For example, when considering quarterly periods, $ppy = 4$; for monthly periods, $ppy = 12$; for weekly periods, $ppy = 52$; and so on.

ROLE OF PRICE EXPECTATIONS

Many people think that the forward rate is merely what the market expects the spot rate to be at the future date. In general this is not true. The forward rate differs from the spot rate only because of the interest differential. If the forward rate was not as determined by Equation (6.1), then arbitrageurs would be able to take advantage of the price inconsistencies. So, provided it is possible to borrow and lend the two currencies, the forward price is merely a mechanical function of the spot rate and the two interest rates.

In Example 6.2 the equilibrium forward rate was US\$1 = ¥118.51. If someone who expected that the spot rate would fall below US\$1 = ¥118.51 by 4 March was prepared to sell dollars against yen at a forward rate of US\$1 = ¥118.00 (i.e. 51 points below the equilibrium rate), he would find arbitrageurs willing to buy them. The arbitrageur would buy dollars value 4 March directly and sell dollars value 4 March synthetically by selling dollars spot (value 4 September) at US\$1 = ¥120.00 and borrowing dollars at 3.00% p.a. and lending yen at 0.50% p.a.

Proceeds of synthetic sale of dollars	US$1	= ¥118.51
Cost of direct purchase of dollars	US$1	= ¥118.00
Profit from arbitrage		= ¥ 0.51 per US$1

Unless artificial constraints such as exchange controls prevent one or more legs of this from happening, arbitrageurs will continue to do the above transactions until the forward rate moves into line with the equilibrium rate.

In contrast, in the market for most commodities the futures price is primarily a function of price expectations. No liquid market exists in which people can borrow and lend the commodities, so the above arbitrage process cannot happen. For example, in mid-1995 the 1 year copper futures price was around US$2,200 per tonne when the cash (i.e. spot) price was around US$3,200 per tonne. Producers knew that a number of new mines would be coming into production within the year and so the world supply of copper would increase significantly, but not in the short term. Consequently, the 1 year forward copper price was at a deep discount to the cash price.[2]

Some commodities, notably gold, do have liquid interest rate markets. For these commodities, like most exchange rates, the arbitrage process ensures that the forward price is merely a reflection of the spot rate and the two interest rates, and not a matter of price expectations.

BID AND OFFER RATES

In practice in the foreign exchange and money markets, prices are quoted with bid and offer rates. This adds a further consideration in calculating forward exchange rates. Example 6.2 can be expanded to include bid and offer pricing as follows.

EXAMPLE 6.4

Spot	US$1 =	¥120.00	120.05	
6 month US$ interest rate		2.90	3.00%	p.a. (181/360)
6 month ¥ interest rate		0.50	0.60%	p.a. (181/360)

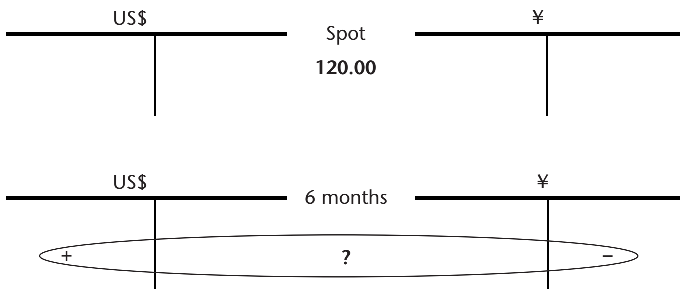
Calculate forward bid and offer rates.

2 In commodity markets forward premiums are called *contango* and forward discounts are referred to as being in *backwardation*.

Forward bid rates

The *forward bid rate* is the rate at which the quoting bank is willing to buy forward the commodity currency. A forward purchase of dollars could be represented on a cash flow diagram as in Exhibit 6.1.

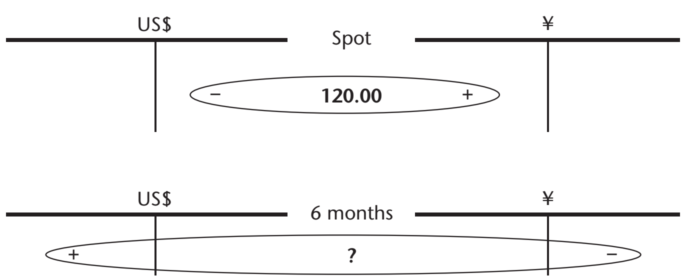
EXHIBIT 6.1 Cash flow diagram: forward purchase of dollars



If the bank purchases dollars forward it will create a net exchange position. It will become long dollars. It could square its net exchange position by selling dollars spot at the market bid rate of US$1 = ¥120.00.

Using the spot market to square the net exchange position would leave the bank with a mismatched cash flow position as in Exhibit 6.2.

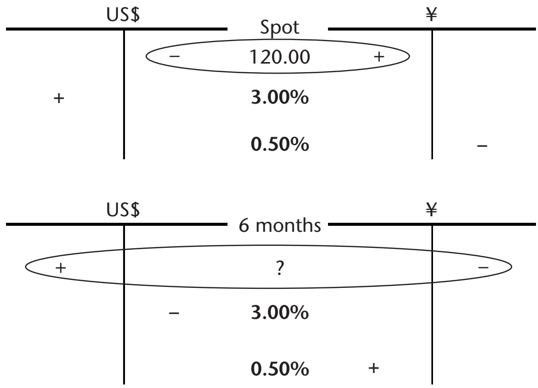
EXHIBIT 6.2 Cash flow diagram: forward purchase of dollars covered by spot sale of dollars



The cash flows could be matched by two money market transactions (Exhibit 6.3):

1. Borrow dollars for 6 months at the market offer rate of 3.00% p.a.
2. Lend yen for 6 months at the market bid rate of 0.50% p.a.

EXHIBIT 6.3 Cash flow diagram: mismatched cash flows covered through money market

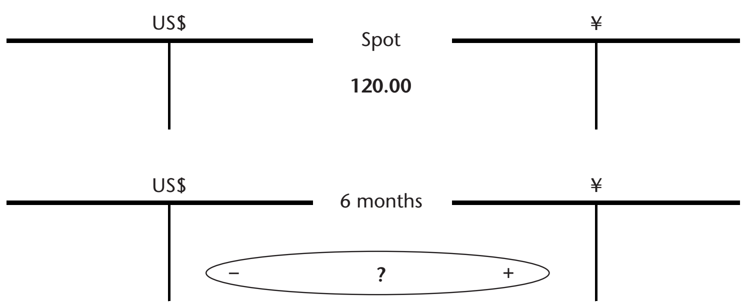


$$s = 120.00$$
$$r_T = 0.50\% \text{ p. a.}$$
$$r_C = 3.00\% \text{ p. a.}$$
$$t = 181/360$$
$$f = \frac{s(1 + r_T t)}{(1 + r_C t)}$$
$$= \frac{120.00(1 + 0.005 \times 181/360)}{(1 + 0.03 \times 181/360)}$$
$$= 118.51$$

Forward offer rates

The *forward offer rate* is the rate at which the quoting bank would sell forward the commodity currency. A forward sale of dollars could be represented on a cash flow diagram as in Exhibit 6.4.

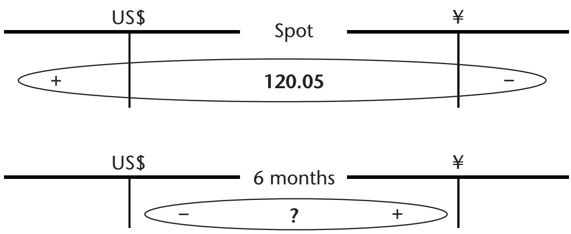
EXHIBIT 6.4 Cash flow diagram: forward sale of dollars



If the bank sells dollars forward it will create a net exchange position. It will become short dollars. It could square its net exchange postion by buying dollars spot at the market offer rate of US$1 = ¥120.05.

Using the spot market to square the net exchange position would leave the bank with a mismatched cash flow postion as shown in Exhibit 6.5.

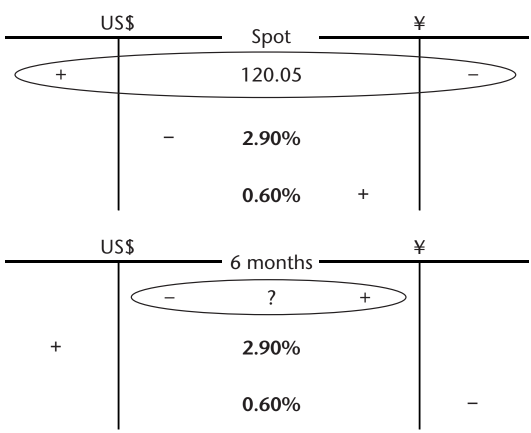
EXHIBIT 6.5 Cash flow diagram: forward sale of dollars covered by spot purchase of dollars



The cash flows could be matched by two money market transactions (Exhibit 6.6):

1. Borrow yen for 6 months at the market offer rate of 0.60% p.a.
2. Lend dollars for 6 months at the market bid rate of 2.90% p.a.

EXHIBIT 6.6 Cash flow diagram: mismatched cash flows covered through money market



$$s = 100.05$$
$$r_T = 0.60\% \text{ p.a.}$$
$$r_C = 2.9\% \text{ p.a.}$$
$$t = 181/360$$

$$\therefore f = \frac{120.05(1 + 0.006 \times 181/360)}{(1 + 0.029 \times 181/360)}$$
$$= 118.68$$

The market forward rate quoted in bid and offer terms is :

$1 = ¥118.51 118.68

If a bank wishes to quote a customer bid and offer forward rates so as to be sure of making 1 point profit if the customer deals it would widen the spread by lowering the bid rate by 1 point and raising the offer rate by 1 point.

	Bid	*Offer*
Break-even forward rate	118.51	118.68
Profit margin	–0.01	+0.01
Customer rate	118.50	118.69

FORWARD CROSS RATES

It is possible to cross two forward exchange rates to determine a forward cross rate.

EXAMPLE 6.5

Spot	US$1	= ¥120.00	and €1 =	US$0.9000
6 month forward margin		–1.50		+ 0.0050
6 month forward rate	US$1	= ¥118.50	€1 =	US$0.9050
Spot cross rate	€1	= ¥120.00 × 0.9000 =		¥108.00
Forward cross rate	€1	= ¥118.50 × 0.9050 =		¥107.24

It follows that the forward margin for € against ¥:

$$= f - s$$
$$= 107.24 - 108.00$$
$$= -0.76$$

The euro is at a forward *discount* of 76 points against the yen.

The forward margin of the cross rate cannot be determined directly from the forward margins of the currencies being crossed. This is because forward margins are not exchange rates; they are differentials.

CURRENCY FUTURES

Currency futures are forward foreign exchange contracts that are traded through a futures exchange. The same concept applies and the pricing calculation is identical except that to facilitate uniformity through the exchange currency futures are based on standard amounts, have specified maturity dates (usually one each quarter, and are net settled against the rate prevailing on the quarterly settlement date.

Currency futures contracts typically have their face value in the non-US dollar currency (e.g. each deutschemark contract is for face value DM125,000). The price is typically expressed in the non-US dollar currency (e.g. the DM June futures contract is quoted as 0.6660, meaning DM1 = US$0.6660, which in the over-the-counter market would normally be quoted as 1.5011, meaning US$1 = DM 1.5011). On the Chicago Mercantile Exchange, which is the largest centre for currency futures, contracts are traded for settlement on the third Wednesday of March, June, September and December.

Credit risk is addressed by requiring traders to place a margin deposit with the exchange and to top up if the contract moves out-of-the-money.

LONG-TERM FOREIGN EXCHANGE (LTFX)

When interest periods extend beyond one year, calculating interest will normally require using a compound interest formula. Consequently, the formula for calculating forward exchange rates for long tenors needs to provide for compounding interest.

$$\boxed{f = \frac{s(1+i_{\mathrm{T}})^n}{(1+i_{\mathrm{C}})^n}} \tag{6.5}$$

EXAMPLE 6.6

Spot	£1	=	A\$ 2.7000
3 year A\$ interest rate			5.5% p.a. (semi-annual zero coupon)
3 year £ interest rate			4.5% p.a. (semi-annual zero coupon)

$$f = 2.7000 \frac{(1+0.055/2)^{2\times 3}}{(1+0.045/2)^{2\times 3}}$$

Derivation of long-term foreign exchange formula (Equation 6.3)

$$f = \frac{FV_T}{FV_C}$$

$$FV_T = PV_T(1+i_T)^n$$

$$FV_C = PV_C(1+i_C)^n$$

$$\therefore f = \frac{PV_T(1+i_T)^n}{PV_C(1+i_C)^n}$$

$$= s\frac{(1+i_T)^n}{(1+i_C)^n}$$

Extending Example 6.6 to include bid and offer rates:

Spot	£1	=	A$ 2.7000	2.7005
3 year A$ interest rate	5.50	5.60% p.a. (semi-annual)		
3 year £ interest rate	4.50	4.60% p.a. (semi-annual)		

$$\text{Bid rate} = 2.7000\frac{(1+0.055/2)^6}{(1+0.046/2)^6} \qquad \text{Offer rate} = 2.7005\frac{(1+0.056/2)^6}{(1+0.045/2)^6}$$

$$= 2.7720 \qquad\qquad = 2.7888$$

Notice that the bid offer spread is very large for long-term foreign exchange rates (e.g. 168 points in Example 6.6). This does not constitute windfall profits for the quoting bank, but merely reflects the bid offer spread of the two interest rates applied over a long period.

ZERO COUPON DISCOUNT FACTORS

Equation (2.12) provided a formula for calculating present values using a compound interest rate. If the net present value of a series of future cash flows is calculated by discounting each of the future cash flows to present value using the same interest rate, the implication is that the yield curve is flat. However, in practice, yield curves are very rarely flat. In effect, it would be assuming that amounts of principal plus interest could always be reinvested at the same rate. The need to assume a reinvestment rate does not arise with zero coupon bonds, because the only cash flow occurs at the maturity of the bond.

Zero coupon discount factors are the discount factors that would apply to zero coupon bonds. Using zero coupon discount factors avoids the need to make any reinvestment assumption. It is possible to calculate a zero coupon discount factor for any future date on which a cash flow occurs.

All future cash flows can be discounted using their relevant discount factors.

EXAMPLE 6.7

Calculate the zero coupon discount factors corresponding to the following par curve. A bond with a coupon rate equal to its yield to maturity will have a price of 100.00 per $100 of face value and is known as a *par bond*. The *par curve* refers to the yield curve comprised of par bonds. For simplicity it is assumed that all coupons are paid at the end of each year.

1 year	5.00% p.a.
2 years	5.25% p.a.
3 years	5.50% p.a.
4 years	5.75% p.a.
5 years	6.00% p.a.

Assume that the face value of each bond is $1.00.

The 1 year bond is a zero coupon bond. The only cash flow occurs at maturity and is equal to

$$FV = 1.00 + 0.05 = 1.05$$

$$PV = 1.00$$

Discount factors can be calculated using Equation (2.14). The zero coupon discount factor for the cash flow occurring in 1 year's time would be:

$$df_1 = \frac{1.00}{1.05} = 0.952381$$

The 2 year bond has a cash flow of $0.0525 in 1 year's time and a cash flow of $1.0525 in 2 years' time.

The cash flow occurring in 1 year's time can be present valued by using df_1:

$$PV(0.0525) = 0.0525 \times 0.952381 = 0.05000$$

The 2 year par bond could be converted into a 2 year zero coupon bond by replacing the cash flow of $0.0525 at the 1 year point with one of $0.0500 today. The zero coupon bond would have:

$$FV = 1.00 + 0.0525 = 1.0525$$

$$PV = 1.00 - 0.0500 = 0.9500$$

$$\therefore df_2 = \frac{0.9500}{1.0525} = 0.902613$$

Similarly, the 3 year par bond could be converted into a zero coupon bond by discounting the cash flow of \$0.055 at the 1 year point by 0.952381 and the cash flow of \$0.055 at the 2 year point by 0.902613.

$$0.055 \times 0.952381 = 0.052381$$

$$0.055 \times 0.902613 = 0.049644$$

The 3 year zero coupon bond would have:

$$FV = 1.00 + 0.055 = 1.055$$

$$\begin{aligned} PV &= 1.00 - 0.052381 - 0.049644 \\ &= 0.897975 \end{aligned}$$

$$\therefore df_3 = \frac{0.897975}{1.055} = 0.85116$$

Similarly,

$$df_4 = 0.798483$$

$$df_5 = 0.745020$$

In general,

$$\boxed{df_n = \frac{1-(c/m)\Sigma_{j=1}^{n-1} df_j}{1+(c/m)}} \qquad (6.6)$$

NPV ACCOUNTING

It was pointed out in Chapter 3 that a superior definition of Net Exchange Position is the *net present value* of foreign currency cash flows. For transactions maturing more than a couple of months from spot the difference between the face value (i.e. future value) and the present value can be significant, so the benefits of using NPV accounting are greater.

EXAMPLE 6.8

Today is Wednesday, 5 December, 2001. A dealer has done the following transactions:

US\$ amount	*¥ amount*	*Rate*	*Maturity*	*Days*
+10,000,000	–1,234,000,000	123.40	7 Dec 01	0
–2,000,000	+243,100,000	121.55	7 Jun 02	181
–5,000,000	+596,700,000	119.34	9 Dec 02	367

To calculate the yen net exchange position in NPV terms it is necessary to calculate the present value of future yen amounts. If 6 month and 1 year yen interest rates are 0.80% p.a. and 0.95% p.a. respectively:

Face value ¥ amount	*PV*	
–1,234,000,000	$\dfrac{-1{,}234{,}000{,}000}{1}$	= –1,234,000,000
+234,100,000	$\dfrac{243{,}100{,}000}{1+0.008\times 181/360}$	= + 242,126,115
+596,700,000	$\dfrac{596{,}700{,}000}{1+0.0095\times 367/360}$	= + 590,976,556
Net exchange position		= –¥400,897,329

The net exchange position could be reduced to zero (square) by buying ¥400,897,329 value spot.

The net present value of the other currency is known as the *counter value*. In this case the counter value is the net present value of US dollar proceeds from the sale of ¥400,897,329. If 6 month and 1 year dollar interest rates are 4.50% p.a. and 4.60% p.a. respectively:

Face value US$ amount	*PV*	
+10,000,000	+10,000,000	= +10,000,000
–2,000,000	$\dfrac{-2{,}000{,}000}{1+0.045\times 181/360}$	= –1,955,751.13
–5,000,000	$\dfrac{-5{,}000{,}000}{1+0.046\times 367/360}$	= –4,776,030.69
Counter value		= US$3,268,218.18

The net exchange position converted at the spot exchange rate is known as the *close out value*.

If the current spot rate is 124.50, the close out value equals:

$$\text{Close Out Value} = 400{,}897{,}329/124.50 = \text{US\$}3{,}220{,}058.87$$

The difference between the counter value and the close out value is called the *marked-to-market*. It measures the unrealized profit or loss in NPV terms.

$$\begin{aligned}\text{MTM profit} &= \text{Counter value} - \text{Close out value}\\ &= \text{US\$}3{,}268{,}218.18 - 3{,}220{,}058.87\\ &= \text{US\$}48{,}159.31\end{aligned}$$

Only the spot transaction impacts the net cash flow position, but all spot and forward foreign exchange transactions impact the net exchange position.

EXAMPLE 6.9

A trader who starts with a square position does the following transactions based on a spot rate US$1 = ¥120.00:

Currency bought	*Currency sold*	*Rate*	*Value date*
US$1,000,000	¥117,350,000	117.35	1 year
US$4,000,000	¥455,040,000	113.76	2 years
¥319,860,000	US$3,000,000	106.62	3 years
¥101,320,000	US$1,000,000	101.32	4 years

Calculate the net exchange position using the NPV method.

Transaction	*¥ cash flows*	*Years*	*¥ rate*	*¥ NPV*
A	–117,350,000	1	0.50%	–116,765,443
B	–455,040,000	2	0.80%	–447,831,588
C	+319,860,000	3	1.00%	+310,429,912
D	+101,320,000	4	1.20%	+98,585,367
Total	–144,690,000			**–157,581,751**

Transaction	*US$ cash flows*	*Years*	*US$ rate*	*US$ NPV*
A	+1,000,000	1	2.75%	+973,057
B	+4,000,000	2	3.50%	+3,731,834
C	–3,000,000	3	5.00%	–2,586,891
D	–1,000,000	4	5.50%	–804,906
Total	+1,000,000			**+1,313,094**

Notice that it is possible for the net present value of the net exchange position and its counter value to be greater than their respective face values. This is possible because the NPV calculation involves the addition of positive and negative numbers.

	NPV position	*Revaluation*	*Profit/(Loss)*
Yen	–157,581,751	+157,581,751	**0**
Dollars	+1,313,094	–1,313,181	**–87**
Effective spot rate	120.00	120.01	

The US$87 is a trivial rounding difference in the revaluation that is a consequence of the exchange rates being rounded to two decimal places.

Suppose the same four transactions had been done except that in Transaction C US$3,000,000 was sold at a rate of US$1 = ¥106.67, implying a 5 point profit. The cash flows and NPVs would be as above except for the profit made on Transaction C.

Transaction	*¥ cash flows*	*Years*	*¥ rate*	*¥ NPV*
A	–117,350,000	1	0.50%	–116,765,443
B	–455,040,000	2	0.80%	–447,831,588
C	**+320,010,000**	**3**	**1.00%**	**+310,575,490**
D	+101,320,000	4	1.20%	+98,585,367
Total	151,060,000			**–157,436,174**

Transaction	*US$ cash flows*	*Years*	*US$ rate*	*US$ NPV*
A	+1,000,000	1	2.75%	+973,057
B	+4,000,000	2	3.50%	+3,731,834
C	–3,000,000	3	5.00%	–2,586,891
D	–1,000,000	4	5.50%	–804,906
Total	+1,000,000			**+1,313,094**

	NPV position	*Revaulation*	*Profit/(Loss)*
Yen	–157,436,174	+157,436,174	**0**
Dollars	+1,313,094	–1,311,968	**1,126**
Effective spot rate	119.90	120.00	

Note: The effective spot rate $= \dfrac{\text{¥ NPV position}}{\text{US\$ NPV counter value}}$

In NPV terms the dealer has made US$1,126 or 10 points profit on the net US$1,313,094 long position, which is equivalent to making 5 points on US$3,000,000 for value 3 years from now.

SHORT DATES

On occasions it is necessary to do foreign exchange transactions for maturities before spot value. It is possible to calculate exchange rates for value today or value tomorrow. The procedure used to engineer exchange rates for short date maturities is identical to that for forward exchange rates. However, because the maturities are prior to spot, the formula used to calculate short dates is different from Equation (6.3).

$$\boxed{t = \frac{s(1 + r_C\, d/dpy_C)}{(1 + r_T\, d/dpy_T)}} \tag{6.7}$$

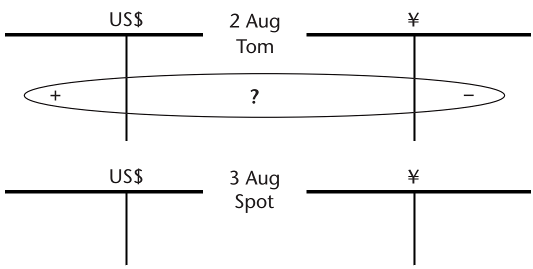
where t = today (tod) or tomorrow (tom) rate.

EXAMPLE 6.10

Spot rate (value 3 August)	US$1	=	¥124.50	124.55
Overnight US$ interest rate			3.00	3.10% p.a. (1/360)
Overnight ¥ interest rates			0.20	0.30% p.a. (1/360)

Calculate the value tom bid rate, i.e. the rate at which the quoting bank would be willing to buy dollars value tom (2 August).

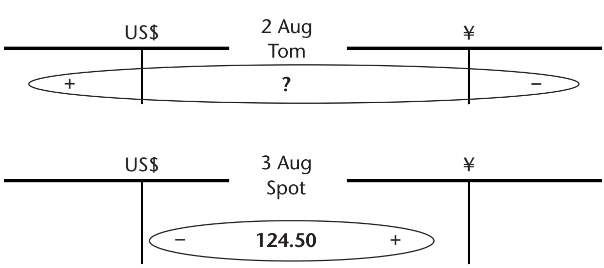
EXHIBIT 6.7 Cash flow diagram: purchase of US$ against yen value tom



If the bank purchases dollars value tom it will create a net exchange position. It will become long dollars. It could square this position by selling dollars spot at the market bid rate US$1 = ¥124.50.

Using the spot market to cover the net exchange position would leave the bank with a mismatched cash flow position as in Exhibit 6.8.

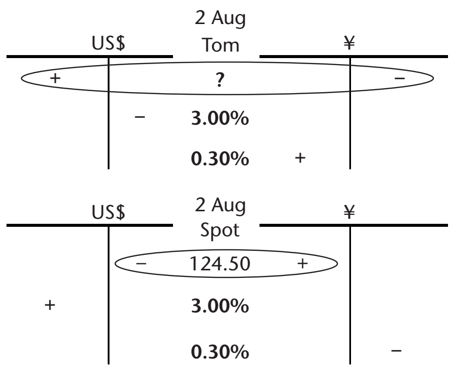
EXHIBIT 6.8 Cash flow diagram: tom purchase of dollars covered by spot sale of dollars



The cash flows could be matched by two money market transactions (Exhibit 6.9):

1. Lend dollars overnight at the market bid rate of 3.00% p.a.
2. Borrow yen overnight at the market offer rate of 0.30% p.a.

EXHIBIT 6.9 Cash flow diagram: mismatched cash flows covered through money market



We have:

$$s = 124.50$$
$$r_T = 0.30\%\ (1/360)$$
$$r_C = 6.00\%\ (1/360)$$

From Equation (6.6)

$$\text{tom} = \frac{124.50(1 + 0.03 \times 1/360)}{(1 + 0.003 \times 1/360)}$$
$$= 124.509$$

Note: Because of the size of the numbers it is common to quote short date rates to an extra decimal place.

SHORT DATE MARGINS

The tom rate is higher than the spot rate, reflecting the fact that the dollar is at a forward discount and therefore a short dated premium against the yen.

$$\begin{aligned}\text{tom margin} &= \text{tom rate} - \text{spot rate}\\ &= 124.509 - 124.50\\ &= +0.009\end{aligned}$$

The tom/next margin is +0.9 of a point.

EXHIBIT 6.10 Short date margins

	1 Aug	2 Aug	3 Aug	10 Aug	17 Aug
	Tod	Tom	Spot	Forward rates	
	124.519	124.509	124.50	124.44	124.37
Margin	+0.019	+0.009		–0.06	–0.13

Because dollar interest rates are higher than yen interest rates, the short date exchange rates are higher than spot (i.e. at a premium) and the forward exchange rates are lower than spot (i.e. at a discount). If the commodity currency is the low interest rate currency, the short date exchange rates will be lower than spot (i.e. at a discount) and the forward exchange rates will be higher than spot (i.e. at a premium).

Derivation of short date formula (Equation 6.4)

$$t = \frac{PV_T}{PV_C}$$

$$s = \frac{FV_T}{FV_C}$$

$$= \frac{PV_T\,(1 + r_T\, d/dpy_T)}{PV_C\,(1 + r_C\, d/dpy_C)}$$

$$\therefore s = \frac{t(1 + r_T\, d/dpy_T)}{(1 + r_C\, d/dpy_C)}$$

$$\therefore t = \frac{s(1 + r_C\, d/dpy_C)}{(1 + r_T\, d/dpy_T)}$$

Note: In the short date formula the factor is the reciprocal of the forward rate formula because, for short date deals, the future value is on the spot date and the present value is at the tod or tom value date.

PRACTICE PROBLEMS

6.1 Forward exchange rate and forward margin

Spot rate	£1 =	US$1.5000	
3 month US$ interest rate		2.50% p.a.	(91/360)
3 month £ interest rate		3.00% p.a.	(91/365)

(a) Calculate the three month forward rate for pounds in dollar terms.
(b) Calculate the three month forward margin.

6.2 Forward exchange rate and forward margin

Spot rate	€1	= ¥107.00
7 month euro		3.50% p.a. (212/360)
7 month yen		0.35% p.a. (212/360)

Calculate the seven month forward rate and forward margin.

6.3 Forward bid rate

Spot rate	€1	= US$0.8490 0.8500
5 month €		3.00 3.10% p.a. (152/360)
5 month US$		1.90 1.95% p.a. (152/360)

A customer wishes to buy dollars five months forward. What rate should a bank quote to make 2 points profit?

6.4 Forward offer rate
Assuming the same market rate scenario as in Problem 6.3, what rate should a bank quote a customer who wishes to sell dollars five months forward if the bank is to make two points profit?

6.5 Long-term foreign exchange

Spot rate	A$1 = US$0.5100	0.5105	
2 year A$ interest rate	5.00%	5.20%	p.a. (semi-annually)
2 year US$ interest rate	4.50%	4.70%	p.a. (semi-annually)

Calculate the break-even two year forward bid and offer rates.

6.6 Short dates

Spot rate	€1	= US$0.8780	0.8785
Overnight US$ interest rate		2.25%	2.375% p.a. (3/360)
Overnight € interest rate		3.25%	3.375% p.a. (3/360)

Calculate the break-even bid and offer rates to 5 decimal places for outright value tomorrow.

6.7 NPV accounting
A trader has done the following 3 transactions:

US$ amount	*¥ amount*	*Rate*	*Maturity*
+10,000,000	–1,075,000,000	107.50	Spot
–2,000,000	+210,610,000	105.30	6 months
–5,000,000	+512,000,000	102.40	1 year

Calculate the trader's yen net exchange position in NPV terms and marked-to-market profit or loss given the current rates:

Spot US$/¥	110.30
6 month dollar interest rate	4.20% p.a.
6 month yen interest rate	0.30% p.a.
1 year dollar interest rate	4.10% p.a.
1 year yen interest rate	0.45% p.a.

6.8 Zero coupon discount factors
Calculate the 1 year, 2 year and 3 year zero coupon discount factors given the following par curve:

1 year	2.50% p.a.
2 years	2.40% p.a.
3 years	2.60% p.a.

6.9 Short dates

Spot	NZ$ 1 =	US$	0.3940	0.3950	
Overnight NZ$			4.00%	4.15%	(1/365)
Overnight US$			2.00%	2.15%	(1/360)

Quote the bid and offer rates outright value tomorrow.

6.10 Long-term foreign exchange

Spot	US$1 =	¥127.00	
2 year dollars		5.00%	5.25%
2 year yen		1.75%	2.00%

Interest paid semi-annually in arrears.

Calculate the break-even bid and offer rate for the 2 year forward margin.

CHAPTER 7

Applications of Forward Exchange

Forward exchange rates provide a means of hedging foreign currency exposures. This chapter covers applications of forward exchange rates for importers, exporters, borrowers and investors.

FOREIGN EXCHANGE RISK

Foreign exchange risk is the risk of loss resulting from a change in exchange rates. Foreign exchange risk exists whenever there is a net exchange position.

Transaction risk is the risk of loss from the change in value of foreign currency revenues or expenses. Transaction risk exists when a payment is to be made in a foreign currency; for example, if an importer buys cars from Japan and agrees to pay the Japanese supplier in yen, or an Australian exporter agrees to receive payment for the commodities that it sells in US dollars. The importer has an exchange rate exposure upon committing to pay the supplier a fixed amount of foreign currency. The lower the exchange rate of the dollar against the yen at the time the importer purchases the fixed amount of yen the greater will be the dollar cost. The exporter has an exchange rate exposure once it agrees to accept payment in a foreign currency. The higher the A$/US$ exchange rate, the fewer Australian dollars the exporter will receive for a fixed amount of US dollar proceeds.

Translation risk is the risk of loss from a change in value of foreign currency denominated assets or liabilities. The local currency value of foreign currency denominated assets or debt change with the exchange rate. An investor that purchases shares or bonds in a foreign currency will lose in local currency terms if the foreign currency depreciates against the local currency. Similarly, it would cost a person or company that has borrowed a foreign currency more units of the local currency to repay the loan if the foreign currency appreciates against the local currency.

The distinction between transaction risk (on revenues and expenses) and translation risk (on assets and liabilities) is made because accounting standards typically require different treatment of profits and losses resulting from the different types of risk. In most countries it is necessary to report gains and losses from transaction risk as part of the profit and loss statement. However, unrealized gains and losses from translation risk are normally reported 'below the line' – that is, against a reserve account rather than as part of current year profit and loss.

Contingent risk is risk that may or may not arise. An example of contingent risk could be a European company tendering for a contact in Japan. If the tender is successful the company would have €/¥ exposure but if the company does not win the tender then it will not have the exchange risk.

HEDGING

A risk is said to be *hedged* or *covered* when the worst case is known. Hedging typically involves the use of forward transactions or the purchase of options to offset the risk.

EXAMPLE 7.1

An importer has an obligation to pay ¥1,000,000,000 to a Japanese supplier three months from now. This obligation to pay constitutes a net exchange position. The importer is short ¥1,000,000,000 against the dollar.

If the importer does not hedge the exposure, that is, the importer waits and purchases ¥1,000,000,000 later at the then prevailing rate, the dollar cost of buying the yen will increase if the dollar falls against the yen.

If the exchange rate is US$1 = ¥100.00, it would cost US$10,000,000 to purchase ¥1,000,000,000. On the other hand, if the exchange rate is US$1 = ¥125.00 it would only cost US$8,000,000 to purchase ¥1,000,000,000. The slope of the curve in Figure 7.1 reflects the extent of the risk.

$$\text{Risk} = \frac{\text{Change in US\$ cost}}{\text{Change in US\$ / ¥ rate}} = \frac{\text{US\$2,000,000}}{125.00 - 100.00} = \text{US\$80,000 per ¥}$$

If the importer expects the dollar to depreciate against the yen, he or she can hedge the exposure by using the forward exchange market to purchase the yen before the exchange rate falls.

Market rates are:

Spot rate	US$1	=	¥127.00	127.05
3 month forward margin			–2.00	–1.90
3 month forward rate	US$1	=	¥125.00	125.15

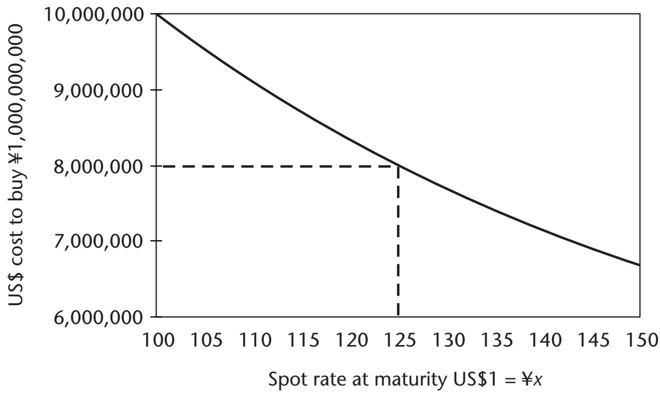


FIGURE 7.1 Unhedged imports

The importer could hedge by buying forward ¥1,000,000,000 at the forward bid rate, US$1 = ¥125.00.

$$\text{The dollar cost if hedged} = \frac{1{,}000{,}000{,}000}{125.00} = \text{US\$8,000,000}$$

Once hedged, the importer knows with certainty that it will cost US$8,000,000 to purchase the ¥1,000,000,000 to pay for the imports regardless of the spot rate in three months' time (Figure 7.2). By hedging the importer has eliminated the exposure to exchange rate movements. Figure 7.3 compares hedged and unhedged imports.

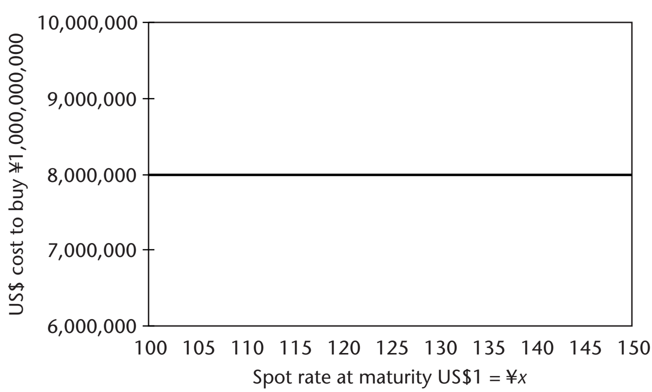


FIGURE 7.2 Hedged imports

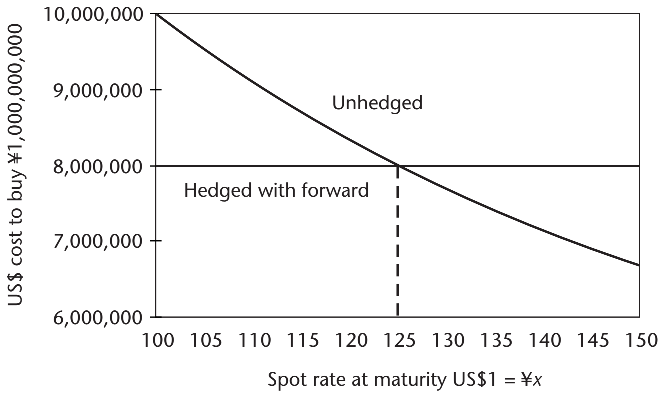


FIGURE 7.3 Hedged and unhedged imports

The forward rate US$1 = ¥125.00 represents the break-even rate between being hedged and unhedged.[1] If the spot rate at maturity turns out to be less than 125.00, the importer would be better off hedged than unhedged. For example, if the spot rate at maturity is 120.00, the cost to the importer would be US$8,000,000.00 if hedged and US$8,333,333.33 if unhedged. On the other hand, if at maturity the spot rate is greater than 125.00, the importer would be better off unhedged than hedged. For example, if the spot rate at maturity is 130.00 the cost to the importer would be US$8,000,000 if hedged and US$7,692,307.69 if unhedged.

PARTIAL HEDGING

The importer may elect to hedge part or all of the exposure. For example the importer may choose to hedge 50% (that is, ¥500,000,000) and leave the other 50% unhedged (Figure 7.4). This would reduce rather than eliminate the foreign currency exposure. If the dollar falls, the loss will be only half what it would have been if the importer was totally unhedged. It is possible to hedge any selected proportion of the exposure. Unhedged means 0% hedged. Hedged or 'fully' hedged means 100% hedged.

---

1 At the break-even rate, cost if hedged = cost if unhedged, i.e.

$$\frac{1{,}000{,}000{,}000}{f} = \frac{1{,}000{,}000{,}000}{b}$$

$$b = f = 125.00$$

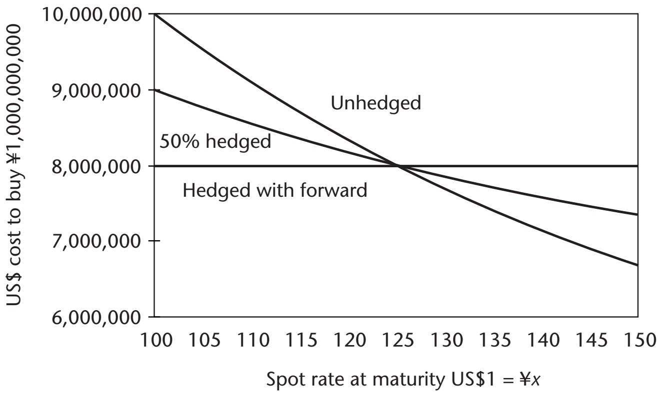


FIGURE 7.4 Partially (50%) hedged imports

HEDGING EXPORT RECEIVABLES

If the proceeds of an export sale are denominated in a foreign currency, then the exporter has a net exchange position. It would be possible to hedge this net exchange position by selling forward the future foreign currency receipts.

EXAMPLE 7.2

A French winegrower exports wine to the United Kingdom and, two months from now, will be paid an amount of £2,000,000. The French exporter is long £2,000,000 against euro.

If the pound appreciates against the euro, the unhedged exporter will receive more euros for the sale of the wine. However, if the pound depreciates against the euro, the unhedged exporter will receive fewer euros (Figure 7.5). If he or she expects the pound to depreciate, the exporter can hedge the net exchange position by selling £2,000,000 forward in the foreign exchange market before the exchange rate falls.

It should be pointed out that the reason Figure 7.5 is upward sloping to the right, whereas Figure 7.1 is downward sloping to the right, is not that Figure 7.5 relates to exports and Figure 7.1 relates to imports. Figure 7.5 has a positive slope because the exchange rate is quoted as a direct relationship and so the proceeds are being expressed in terms of the terms currency. Figure 7.1 has a negative slope because the exchange rate is quoted as an inverse relationship, so the cost is being expressed in terms of the commodity currency. If Example 7.2 referred to a British exporter measuring the pound proceeds of the sale of euros, then Figure 7.5 would be downward sloping to the right. Similarly, if Example 7.1 referred to a

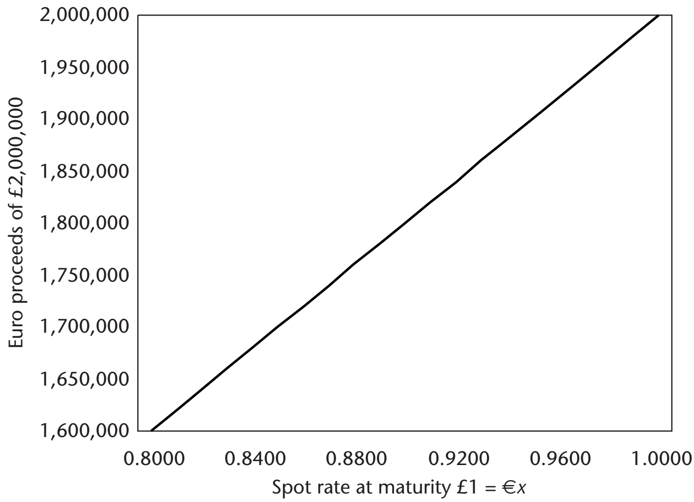


FIGURE 7.5 Unhedged exports

Japanese importer measuring the yen cost of buying the dollar, Figure 7.1 would be upward sloping to the right.

Fortunately, the analysis is unaffected by which way round the figures are drawn. This is merely pointed out to avoid confusion on the part of the reader.

Market rates are:

Spot rate	£1 =	€0.8980	0.8990
2 month forward margin		+0.0020	+0.0024
2 month forward rate	£1 =	€0.9000	0.9014

The French exporter could hedge by selling £2,000,000 forward at the market bid rate of £1 = €0.9000.

The proceeds if hedged = 2,000,000 × 0.9000 = €1,800,000 (Figure 7.6).

Once hedged, the exporter knows with certainty that he or she will receive €1,800,000 for the wine, regardless of the spot rate in two months' time. By hedging, the exporter has eliminated the exposure to the exchange rate. Figure 7.7 compares the hedged and unhedged positions.

The forward rate £1 = €0.9000 represents the break-even rate between being hedged and unhedged.[2] If at maturity the spot rate is greater than

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2 At the break-even rate, proceeds if hedged = proceeds if unhedged, i.e.

$$2{,}000{,}000 \times f = 2{,}000{,}000 \times b$$
$$\therefore b = f = 0.9000$$

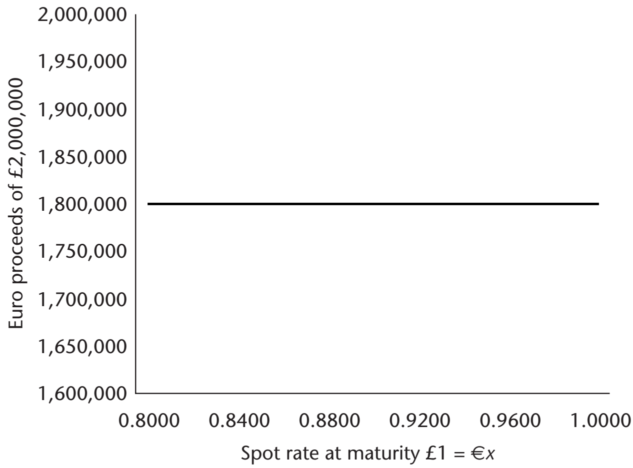


FIGURE 7.6 Hedged exports

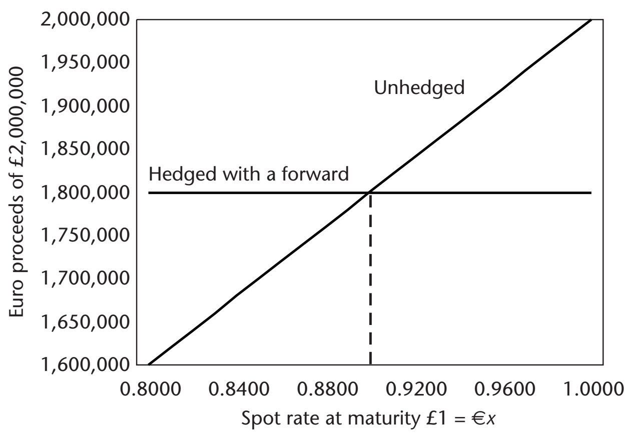


FIGURE 7.7 Hedged and unhedged exports

0.9000, the exporter would be better off unhedged than hedged. For example, if the spot rate at maturity turns out to be 1.0000, he or she would receive €2,000,000 for the wine if unhedged and €1,800,000 if hedged. On the other hand, if at maturity the spot rate is less than 0.9000, the exporter would be better off hedged than unhedged. At 0.8000, for example, the exporter would receive €1,800,000 if hedged and €1,600,000 if unhedged.

EFFECTIVE EXCHANGE RATES

The *effective exchange rate* is the exchange rate implied by the amounts of two currencies. For example, if ¥1,000,000,000 corresponds to US$8,000,000, the effective exchange rate is 125.00.

It is possible to draw the above diagrams with the effective exchange rate as the vertical axis. This is often helpful for exporters or importers who set their objectives in terms of an exchange rate level rather than a dollar amount. Figure 7.3 could be redrawn as Figure 7.8. Notice that in this case changing the unit of measurement from the dollar cost of purchasing the imports to the effective exchange rate at which the imports will be purchased changes the unhedged line from downward sloping to upward sloping.

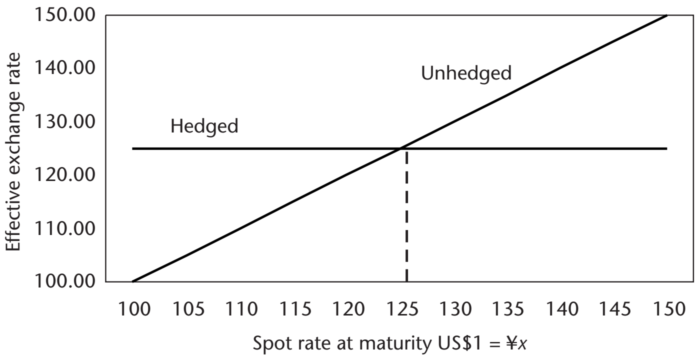


FIGURE 7.8 Effective exchange rates

BENEFITS AND COSTS OF PREMIUMS AND DISCOUNTS

In the above examples the importer would pay a cost equal to the forward discount to hedge the yen payables and the French exporter would receive a benefit equal to the forward premium to hedge the pound receivables.

If the commodity is at a discount, the buyer enjoys the benefit and the seller incurs the cost of the discount. If the commodity is at a premium, the buyer pays the cost and the seller enjoys the benefit of the premium.

The benefit or cost of hedging will reflect the forward premium or discount.

Cost to buy ¥1,000,000,000 forward at 125.00	US$8,000,000.00
Cost to buy ¥1,000,000,000 spot at 127.00	US$7,784,015.75
∴ *Cost* of hedging ¥1,000,000,000 payables	US$ 125,984.25

Cost of hedging yen payables in dollar terms:

$$= \frac{1{,}000{,}000{,}000}{125.00} - \frac{1{,}000{,}000{,}000}{127.00}$$

$$= \text{US\$}125{,}984.25$$

Proceeds of selling £2,000,000 forward at 0.9000	€1,800,000
Proceeds of selling £2,000,000 spot at 0.8980	€1,796,000
Benefit of hedging £2,000,000 receivables	€ 4,000

Benefit of hedging sterling receivables in euro terms:

$$= 2{,}000{,}000 \times 0.9000 - 2{,}000{,}000 \times 0.8980$$

$$= 2{,}000{,}000 \times 0.0020$$

$$= €4{,}000$$

When the terms currency amount is constant, the cost or benefit of hedging cannot be determined directly from the forward margin. This is because forward margins are not exchange rates; they are differentials between exchange rates.

HEDGING FOREIGN CURRENCY BORROWINGS

Borrowing a foreign currency does not create a net exchange position. However, if the foreign currency borrowed is sold for another currency, usually the local currency, the sale of the foreign currency creates an exchange rate exposure. The local currency cost of repaying the loan rises as the local currency depreciates against the foreign currency and falls as the local currency appreciates against the foreign currency.

EXAMPLE 7.3

A company wanting to borrow US$1,000,000 for a year borrows ¥100,000,000 for 1 year at an interest rate of 1.00% p.a. and converts the yen into dollars by selling them at a spot rate of US1 = ¥100.00. The yen loan provides initial proceeds of US$1,000,000.

The borrower has an obligation to pay ¥101,000,000 (principal plus interest) at the end of the year (Figure 7.9). This obligation constitutes a net exchange position similar to that of the importer with an obligation to pay foreign currency in the future. Notice that the net exchange position includes the future obligation to pay interest (FOTPI), not just the principal amount.

If the borrower remains unhedged, the dollar cost of repaying the yen loan will fluctuate with the exchange rate. If the dollar strengthens against the yen, the cost of buying the ¥101,000,000 necessary to repay the loan

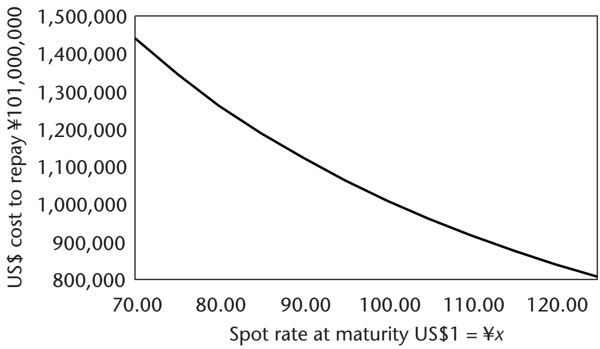


FIGURE 7.9 Unhedged yen borrowing

will fall. Similarly, if the dollar weakens against the yen, the cost of buying the ¥101,000,000 necessary to repay the loan will rise.

Borrowers gain by borrowing currencies that will weaken and lose by borrowing currencies that will strengthen.

The borrower can hedge against the risk of the dollar depreciating against the yen by purchasing ¥101,000,000 forward. If the alternative dollar borrowing rate is 4.00% p.a. (Exhibit 7.1; Figure 7.10), the repayment amount (principal plus interest) in dollars would be US\$1,040,000 and the 1 year forward rate would be US\$1 = ¥97.12. The borrower could hedge by buying ¥101,000,000 at the forward rate US\$1 = ¥97.12.

$$\text{The dollar cost if hedged} = \frac{101{,}000{,}000}{97.12} = \text{US\$}1{,}040{,}000$$

EXHIBIT 7.1 Cash flow representation of a hedged yen loan

US$		Spot	¥	
1,000,000		**100.00**		**–100,000,000**
		1.00%	**100,000,000**	
			100,000,000	**100,000,000**
US$		**1 year**	**¥**	
		P + I		**–101,000,000**
	–1,040,000	**97.12**	**101,000,000**	
			101,000,000	**101,000,000**

Once hedged, the borrower knows that it will cost US\$1,040,000 to repay the yen loan principal plus interest at the end of the year. By hedging, the borrower has eliminated the exposure to exchange rate changes. Figure 7.11 compares the hedged and unhedged loans.

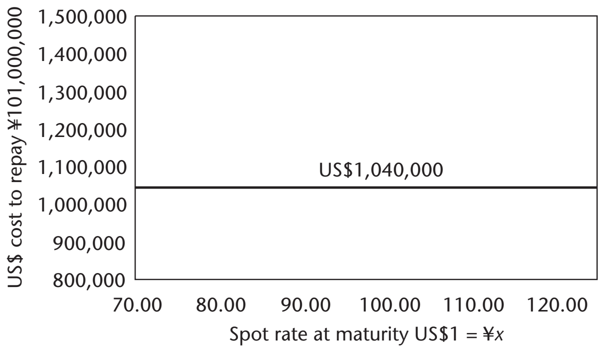


FIGURE 7.10 Hedged yen loan

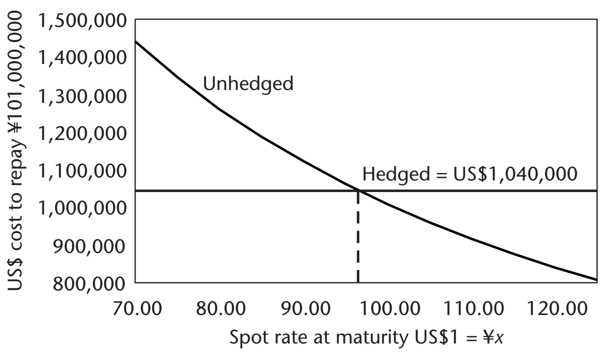


FIGURE 7.11 Hedged and unhedged yen loans

EFFECTIVE COST OF HEDGED FOREIGN CURRENCY BORROWINGS

By hedging the yen principal plus interest at the forward rate of US$1 = ¥97.12, the borrower knows it will cost exactly US$1,040,000 to repay a loan which had proceeds of US$1,000,000.

$$\text{Effective interest} = 1{,}040{,}000 - 1{,}000{,}000 = \text{US\$}40{,}000$$

$$\begin{aligned}\text{Effective interest rate \% p.a.} &= \frac{\text{interest}}{\text{principal}} \times \text{time factor} \times 100 \\ &= \frac{40{,}000}{1{,}000{,}000} \times 1 \times 100 \\ &= 4.00\%\ \text{p.a.}\end{aligned}$$

Equation (6.3) provides an algebraic means of calculating the effective borrowing cost of a fully hedged foreign currency loan.

$$f = \frac{s(1 + r_T t)}{(1 + r_C t)}$$

In Example 7.3:

$$\begin{aligned} f &= 97.12 \\ s &= 100.00 \\ r_T &= 0.01 \\ r_C &= ? \\ t &= 1 \end{aligned}$$

Hence

$$\begin{aligned} 97.12 &= \frac{100.00(1 + 0.01 \times 1)}{(1 + r_C \times 1)} \\ \therefore (1 + r_C \times 1) &= \frac{100.00}{97.12}(1.01) \\ &= 1.04 \\ \therefore r_C &= 0.04 = 4.00\% \text{ p.a.} \end{aligned}$$

See Figure 7.12.

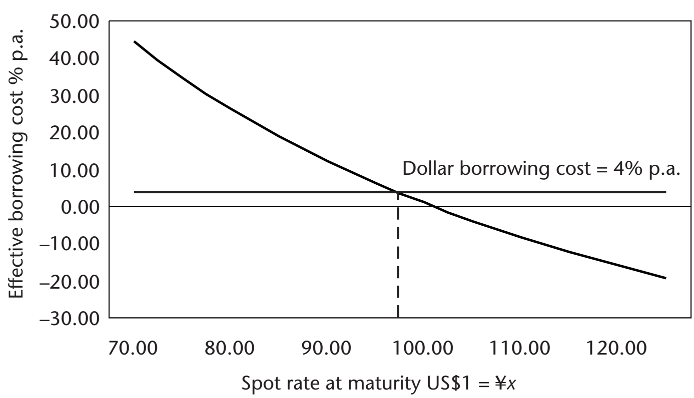


FIGURE 7.12 Effective borrowing cost

BREAK-EVEN RATES

When two different strategies result in the same rate it is known as the *break-even rate*. The forward rate US$1 = ¥97.12 represents the break-even rate between being hedged and unhedged.[3] Graphically, the break-even rate occurs where the lines intersect.

If at maturity the spot rate turns out to be less than 97.12, the borrower would be better off hedged than unhedged. For example, if the spot rate at maturity is 95.00, the cost of repayment would be US$1,040,000 if hedged and US$1,063,157.89 if unhedged. On the other hand, if at maturity the spot rate is greater than 97.12, the borrower would be better off unhedged than hedged. For example, if the spot rate at maturity turns out to be 100.00, the cost of repayment would still be US$1,040,000 if hedged but only US$1,010,000 if unhedged.

Actually, when bid and offer rates apply the break-even rate is not exactly equal to the forward rate.

Spot US$/¥	99.90/100.00
1 year US$	3.90/4.00% p.a.
1 year ¥	0.90/1.00% p.a.

To calculate the break-even rate between the strategies of borrowing on a hedged and unhedged basis it is appropriate to use the offer rates in each case. The borrower would borrow yen at 1.00% p.a. and sell yen and buy dollars at 100.00. The alternative is that it would borrow dollars at 4.00% p.a. The break-even rate using these rates is 97.12 as calculated above.

To calculate the forward offer rate it would be appropriate to use the bid rate (3.90% p.a.) for the 1 year dollars as done in Chapter 6. In this case the forward offer rate would be:

$$f = 100.00 \times \frac{1.010}{1.039} = 97.21$$

The 11 point difference between 97.12 and 97.21 reflects the 10 basis point spread between the 1 year bid and offer rates for dollar interest rates.

EXAMPLE 7.4

A US corporation borrows £7,000,000 for 38 days at an interest rate of 3.50% p.a. (365 dpy) and sells the proceeds spot at an exchange rate of £1 = US$1.4500. The corporation hedges by purchasing principal plus interest forward for the repayment date at a rate of £1 = US$1.4485.

What is the effective interest rate?

$$s = 1.4500$$

$$f = 1.4485$$

3 At the break-even rate, cost if hedged = cost if unhedged, i.e.

$$\frac{101{,}000{,}000}{f} = \frac{101{,}000{,}000}{b} \qquad \therefore b = f = 97.12$$

$r_T = ?$
$r_C = 0.035$
$t_C = 38/365$
$t_T = 38/360$

Using Equation (6.3),

$$f = \frac{s(1 + r_T t_T)}{(1 + r_C t_C)}$$

$$1.4485 = \frac{1.4500(1 + r_T \times 38/360)}{(1 + 0.035 \times 38/365)}$$

$$r_T = 0.02468 = 2.5\% \text{ p.a. (rounding)}$$

COST OF HEDGING FOREIGN CURRENCY BORROWINGS

The difference between the effective rate of a hedged foreign currency loan and the nominal foreign currency interest rate represents the cost or benefit of hedging.

Returning to Example 7.3,

Effective interest rate of hedged yen loan	4.00% p.a.
Nominal interest rate of yen loan	1.00% p.a.
∴ Cost of hedging	3.00% p.a.

In this case, hedging means buying yen forward at a premium (that is, selling dollars forward at a discount). The decision to hedge, therefore, involves a cost equivalent to adding 3.00% p.a. to the nominal interest rate. This cost reflects the forward margin of 288 exchange points. That is, in this example 288 exchange points are equivalent to 300 basis points.

Because the forward margin reflects the prevailing interest differential between the two currencies, the effective interest rate on a fully hedged foreign currency borrowing should be equal to the equivalent local currency borrowing rate.

A fully hedged foreign currency loan is equivalent to a local currency loan.

In practice, the effective interest rate on a fully hedged foreign currency loan will normally be marginally higher than the local borrowing rate. Borrowing a foreign currency may mean incurring additional costs, such as interest withholding tax. Hedging may involve paying a bid offer spread etc. In some cases it is possible to generate local currency liquidity more cheaply through a fully hedged borrowing than by borrowing the local currency through the domestic money market. This is examined further in Chapter 8 in the section dealing with covered interest arbitrage.

EFFECTIVE COST OF UNHEDGED FOREIGN CURRENCY BORROWINGS

Returning to Example 7.3, it is only possible to calculate the effective interest rate when the exchange rate at which the yen principal and interest will be purchased is known. If the borrower remains unhedged and purchases ¥101,000,000 at the prevailing spot rate at maturity, the effective borrowing cost will not be known until maturity. However, it is possible to calculate what the effective interest rate would be if the spot rate at maturity turns out to have a particular value. For example, if at maturity the spot rate at which the borrower could purchase yen is 97.00, then the effective borrowing cost will turn out to be 4.12% p.a.

This is calculated by again using the relationship

$$f = \frac{s(1 + r_T t)}{1 + r_C t}$$

where f represents the future spot rate rather than the current forward rate. Solving for r_C,

$$\begin{aligned}
f &= 97.00 \\
s &= 100.00 \\
r_T &= 0.01 \\
t &= 1 \\
\therefore 97.00 &= 100 \frac{(1 + 0.01 \times 1)}{(1 + r_C \times 1)} \\
\therefore r_C &= 0.0412 = 4.12\% \text{ p. a.}
\end{aligned}$$

By allowing f to take different values, it is possible to calculate the effective interest rates which would result from an unhedged borrowing for different possible future spot rates.

The unhedged borrower loses if the exchange rate falls. For example, if at maturity the spot rate has fallen to 90.00, the effective interest rate would have risen to 12.22% p.a. Similarly, the unhedged borrower gains if the exchange rate rises. For example, if at maturity the spot rate has risen to 102.00, the effective interest rate would have fallen to –0.98% p.a. The borrower's foreign exchange gain would exceed the nominal interest cost so that the borrower ends up repaying less than the proceeds obtained at draw down, thereby enjoying a negative effective interest rate; see Figure 7.13.

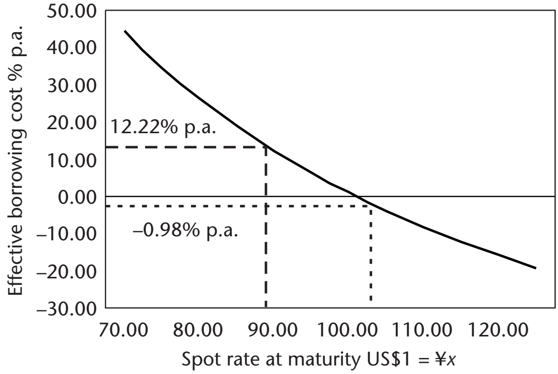


FIGURE 7.13 Effective borrowing rates

Borrowers may hedge at any time during the life of the loan. For example, they may remain unhedged for 3 months, hoping that the exchange rate will rise, and then hedge for the remaining period using the then 9 month forward rate.

Suppose that after 3 months the spot rate has risen from 100.00 to 103.00 and the 9 month forward rate (to correspond with the maturity of the loan) is then 101.00, the borrower could hedge by purchasing ¥101,000,000 at the 9 month forward rate US$1 = ¥101.00 for exactly US$1,000,000. The dollar cost to repay the loan principal plus interest would be the same as the draw down proceeds. In other words, the effective borrowing cost would be zero.

It is possible to calculate the forward rate that would make the effective borrowing cost 2.00% p.a. (say) (Figure 7.14):

$$s = 100.00$$
$$f = ?$$
$$r_T = 0.01$$
$$r_C = 0.02$$
$$t = 1$$

Again using Equation (6.3):

$$f = \frac{s(1 + r_T t)}{1 + r_C t}$$
$$= 100.00 \frac{(1 + 0.01 \times 1)}{(1 + 0.02 \times 1)}$$
$$= 99.02$$

To verify:

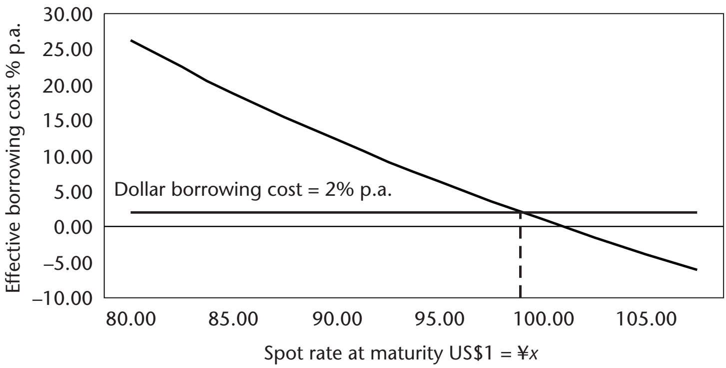


FIGURE 7.14 Locking in 2% p.a. borrowing cost

$$99.02 = \frac{100.00(1 + 0.01 \times 1)}{(1 + r_C \times 1)}$$
$$= 2.00\% \text{ p.a.}$$

It should be pointed out that the borrower might not have the opportunity to purchase yen at a forward rate of 99.02. However, he or she knows that if the forward rate reaches 99.02 then it is possible to lock in an effective borrowing cost equal to the 2.00% p.a. objective.

UNHEDGED FOREIGN CURRENCY INVESTMENTS

Investing in a foreign currency does not in itself create a net exchange position. However, purchasing the foreign currency so that it can be invested does create an exchange rate exposure. The local currency return on a foreign currency investment rises as the foreign currency appreciates against the local currency and falls as the foreign currency depreciates against the local currency.

EXAMPLE 7.5

Spot US$/¥	122.00/122.05
6 month US$	3.00/3.15% p.a. (180/360)
6 month ¥	0.50/0.60% p.a. (180/360)

An investor has US$1,000,000 to invest. The US$1,000,000 could be converted into ¥122,000,000 at a spot bid rate of 122.00 and the yen invested for 180 days at the yen bid rate of 0.50% p.a. (Exhibit 7.2).

EXHIBIT 7.2 Cash flow representation of an unhedged investment in yen

US$		Spot	¥	
1,000,000	**−1,000,000**	**122.00**	**122,000,000**	
		0.50%		**−122,000,000**
1,000,000	**−1,000,000**		**122,000,000**	**122,000,000**

US$		6 months	¥	
		P + I	**122,305,000**	

The investment will be worth ¥122,305,000 at maturity. Assuming the investor intends to sell these yen for dollars the net exchange position is similar to that of an exporter with yen receivables. Notice that the net exchange position includes the future obligation to receive interest (FOTRI) as well as the principal amount.

If the investor remains unhedged, the dollar value of the investment will fluctuate with the exchange rate (Figure 7.15). As the dollar strengthens against the yen, the value of the yen investment falls and as the dollar weakens against the yen, the value of the yen investment rises.

Investors gain by investing in currencies that will appreciate and lose by investing in currencies which will depreciate.

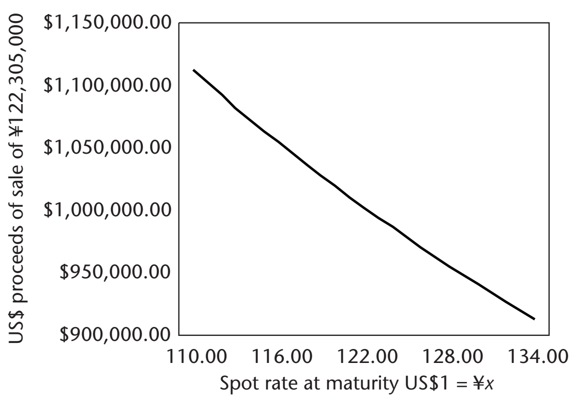


FIGURE 7.15 Unhedged investment in yen

If the investor expects the dollar to appreciate against the yen, he or she can hedge the foreign exchange exposure by selling ¥122,305,000 forward. Given the market information for Example 7.5, the 6 month forward rate would be US$1 = ¥120.41/120.61.

The investor could hedge by selling ¥122,305,000 at the forward rate US$1 = ¥120.61. The dollar proceeds if hedged are (Exhibit 7.3):

$$\frac{122{,}305{,}000}{120.61} = \text{US\$}1{,}014{,}053.56$$

EXHIBIT 7.3 Cash flow representation of a hedged yen investment

US$			¥	
		Spot		
1,000,000	–1,000,000	122.00	122,000,000	
		0.50%		–122,000,000
1,000,000	–1,000,000		122,000,000	122,000,000
US$		6 months	¥	
		P + I	122,305,000	
1,014,053.56		**120.61**		**–122,305,000**
			122,305,000	**122,305,000**

Once hedged, the investor knows that he or she will receive US$1,014,053.56 from the conversion of the yen investment principal plus interest at the end of the 6 months. By hedging the investor has eliminated the exposure to exchange rate changes. Figure 7.16 compares hedged and unhedged yen investment.

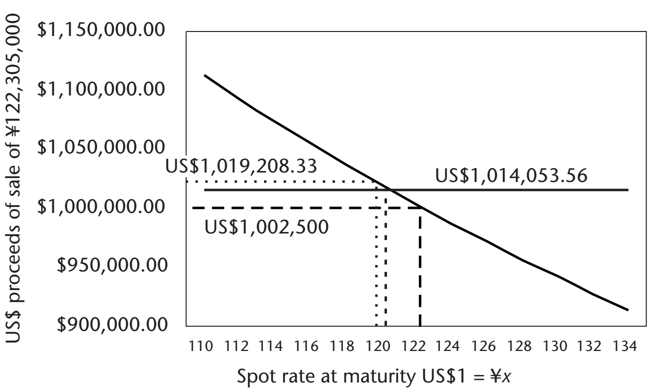


FIGURE 7.16 Hedged and unhedged yen investment

The forward rate US$1 = ¥120.61 represents the break-even rate between being hedged and unhedged.[4] If at maturity the spot rate is less than 120.61 the investor would be better off unhedged than hedged. For example, if the spot rate at maturity turned out to be 120.00, the proceeds

4 At the break-even rate, proceeds if hedged = proceeds if unhedged, $b = f =$ 120.61

would be worth US$1,019,208.33 if unhedged and US$1,014,053.56 if hedged. On the other hand, if at maturity the spot rate is higher than 120.61, the investor would be better off hedged than unhedged. For example, if the spot rate at maturity is 122.00, the proceeds would be worth US$1,014,053.56 if hedged and US$1,002,500.00 if unhedged.

EFFECTIVE YIELD ON HEDGED FOREIGN CURRENCY INVESTMENTS

By hedging the yen principal plus interest at the forward rate of US$1 = ¥120.61, the investor knows he or she will receive exactly US$1,014,053.56 from an initial investment of US$1,000,000.

$$\begin{aligned}\text{Effective interest earned} &= \text{US\$}1{,}014{,}053.56 - 1{,}000{,}000\\ &= \text{US\$}14{,}053.56\end{aligned}$$

$$\begin{aligned}\text{Effective yield \% p.a.} &= \frac{\text{interest}}{\text{principal}} \times \text{time factor} \times 100\\ &= \frac{14{,}053.56}{1{,}000{,}000} \times \frac{360}{180} \times 100\\ &= 2.81\%\ \text{p.a.}\end{aligned}$$

The difference between the nominal and effective yield on a foreign currency investment represents the benefit or cost of hedging.

Continuing Example 7.5,

Effective yield on hedged yen	2.81% p.a.
Nominal yield on yen	0.50% p.a.
∴ Benefit of hedging	2.31% p.a.

In this case hedging means selling yen at a forward premium (i.e. buying dollars at a forward discount). The decision to hedge therefore involves a benefit equivalent to adding 2.31% p.a. to the nominal yield.

Because the forward margin reflects the prevailing interest differential between the two currencies, the effective yield on a fully hedged foreign currency investment should be equal to the yield on the local currency. It will be shown in Chapter 8 that the benefit from hedging, and so the effective yield on a hedged investment in yen, can be increased by using what is called a pure swap.

A fully hedged foreign currency investment is equivalent to a local currency investment.

In practice, the effective yield on a fully hedged foreign currency investment will normally be marginally less than the corresponding local currency yield. This may be because the foreign investment attracts interest withholding tax or because hedging involves paying an additional bid offer spread. However, it is sometimes possible to engineer a higher effective yield through a hedged foreign currency investment than by investing directly through the local money market.

EFFECTIVE YIELD ON UNHEDGED FOREIGN CURRENCY INVESTMENTS

An investor may purchase a foreign currency or invest through the Eurodollar deposit market without hedging. If the investor in Example 7.4 remains unhedged till maturity, when ¥122,305,000 are sold spot, he or she will not know the effective yield until the rate at which the yen are sold is known. However, it is possible to calculate what the effective yield would be for different values of the spot rate at maturity. For example, if at maturity the spot rate turns out to be 118.00, then the effective yield will be 7.30% p.a.

Again using the relationship:

$$f = \frac{s(1 + r_T t)}{1 + r_C t}$$

$$\begin{aligned} f &= 118.00 \\ s &= 122.00 \\ r_T &= 0.005 \\ t &= 180/360 \end{aligned}$$

Solving for r_C,

$$118.00 = 122.00(1 + 0.005 \times 180/360) / (1 + r_C \times 180/360)$$

$$\therefore r_C = 0.073 = 7.3\%$$

The effective yield on the unhedged investment in yen turned out to be higher than the nominal yield on a direct investment in yen because the yen has strengthened against the dollar. The foreign exchange gain resulting from the spot rate falling from 122.00 to 118.00 is equivalent to a pick up in yield of 6.80% p.a.

Nominal yield on yen investment	0.50% p.a.
Foreign exchange gain	6.80% p.a.
Effective yield on unhedged yen	7.30% p.a.

By allowing f to take different values, it is possible to calculate the effective yield on an unhedged investment for different possible future spot rates.

The unhedged investor gains as the exchange rate falls and losses as it rises. For example, if at maturity the spot rate turns out to be 122.00, the effective yield would be 0.50% p.a. Because the spot rate at maturity turned out to be the same as the spot rate at the time of draw down there is no foreign exchange gain or loss. The effective yield on the unhedged yen investment is, therefore, equal to the nominal yield.

There is an exchange rate somewhere above 122.00 at which the effective yield from the unhedged investment would equal zero.

$$s = 122.00$$
$$r_T = 0.005$$
$$r_C = 0$$
$$t = 180/360$$

Solving for f,

$$f = 122.00 \frac{(1+0.005\times 180/360)}{(1+0\times 180/360)} = 122.31$$

The investor's exchange loss resulting from the spot rate rising from 122.00 to 122.31 would exactly offset the 0.50% p.a. nominal yield. At maturity the investor would retrieve the principal without earning any interest. If the spot rate at maturity is higher than 122.31, the yield will be negative, that is, the investor will not only earn no interest, but will also lose some of the principal. For example, if the spot rate at maturity turns out to be 126.00, the effective yield would be –5.87% p.a. (Figure 7.17). The foreign exchange loss has more than wiped out the 0.50% p.a. nominal yield. This demonstrates the risk of unhedged foreign currency investments.

An investor may hedge at any time during the life of the investment. For example, the investor may remain unhedged, hoping that the forward exchange rate will fall, and then hedge for the remaining period.

Suppose the investor in Example 7.5 hopes to achieve an effective yield of 10% p.a. for the period. What would the forward rate need to be for the investor to be able to hedge the principal plus interest at an effective yield of 10% p.a. (Figure 7.18)?

$$s = 122.00$$
$$f = ?$$
$$r_T = 0.005$$
$$r_C = 0.10$$
$$t = 180/360$$

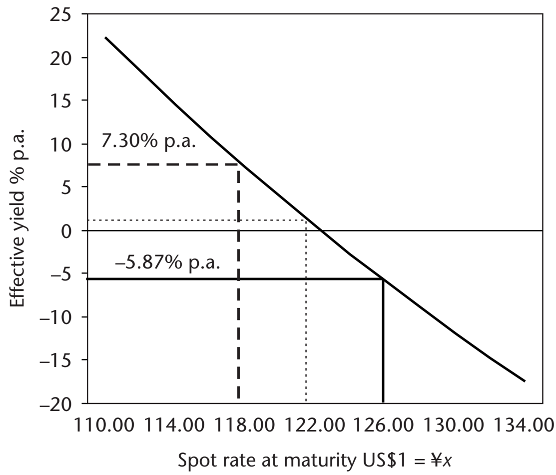


FIGURE 7.17 Effective yields on unhedged yen investment for different spot rates at maturity

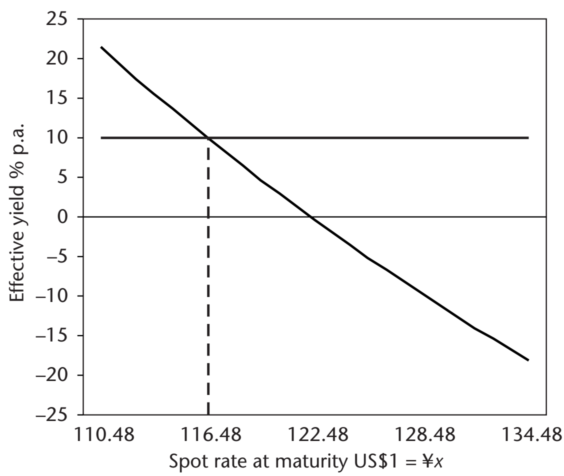


FIGURE 7.18 Locking in 10% p.a. effective yield

Solving for f,

$$f = \frac{s(1 + r_T t)}{1 + r_C t}$$

$$= \frac{122.00(1 + 0.005 \times 180/360)}{(1 + 0.10 \times 180/360)} = 116.48$$

Of course, the investor may never have the opportunity to sell yen at a forward rate of 116.48. However, the investor knows that if the forward rate reaches 116.48, then an effective yield could be locked in for the period that meets the 10.00% p.a. target.

PAR FORWARDS

The NPV methodology can be used to construct par forwards. A *par forward* is a series of forward foreign exchange rates for various maturities all at the same (average) rate.

EXAMPLE 7.6

A Japanese importer needing to buy US$5,000,000 each quarter for 8 years could hedge its foreign exchange risk by doing 8 separate forward deals in which it would buy US$5,000,000 against yen at the different forward rates for each of the 12 maturities.

Based on a spot rate US$1 = ¥125.00 and the relevant interest rates the following forward rates and zero coupon discount factors apply:

Year	*US$ amount*	*Forward*	*¥ at forwards*	*¥ zcdf*
1	5,000,000.00	122.32	611,585,366	0.994018
2	5,000,000.00	119.70	598,484,244	0.996040
3	5,000,000.00	116.80	583,981,190	0.990099
4	5,000,000.00	113.75	568,752,137	0.982226
5	5,000,000.00	110.58	552,910,694	0.970503
6	5,000,000.00	107.62	538,115,942	0.953579
7	5,000,000.00	103.93	519,657,865	0.938834
8	5,000,000.00	102.89	514,448,187	0.899134

The cost in yen of buying the successive amounts of US$5,000,000 falls with the forward date because the forward rate is lower the further forward the maturity date.

Alternatively, the exporter could hedge by buying US$5,000,000 each year for 8 years at the same (par forward) rate. The par forward rate is that rate for which the net present value of the yen cash flows is the same as the net present value for the 8 separate forward deals.

If, as a first estimate, the par forward rate was assumed to be the average of the forward rates, namely 112.20, the cost each year of buying US$5,000,000 would be ¥561,000,000. In NPV terms the cost would be only ¥4,333,406,605 under the par forward versus ¥4,341,051,011 with the 8 separate forwards. Clearly, the break-even par forward rate must be higher; that is, worse for the importer.

$$\text{The break-even par forward rate} = 112.20 \times \frac{4{,}341{,}051{,}011}{4{,}333{,}406{,}605} = 112.40$$

Year	*US$*	*¥ at forwards*	*PV(forward)*	*¥ at par forward*	*PV(par forward)*
1	5,000,000.00	611,585,366	607,926,829	561,000,000	557,644,068
2	5,000,000.00	598,484,244	596,114,108	561,000,000	558,778,310
3	5,000,000.00	583,981,190	578,199,318	561,000,000	555,445,660
4	5,000,000.00	568,752,137	558,642,968	561,000,000	551,028,620
5	5,000,000.00	552,910,694	536,601,684	561,000,000	544,452,382
6	5,000,000.00	538,115,942	513,135,839	561,000,000	534,957,587
7	5,000,000.00	519,657,865	487,872,660	561,000,000	526,686,077
8	5,000,000.00	514,448,187	462,557,606	561,000,000	504,413,901
	40,000,000.00		4,341,051,011		4,333,406,605

At 112.40, the annual cost of buying US$5,000,000 would be ¥562,000,000 and the NPV under the par forward would be virtually equal to that with the 8 separate forwards.

Year	*US$*	*¥ at forwards*	*PV(forward)*	*¥ at par forward*	*PV(par forward)*
1	5,000,000.00	611,585,366	607,926,829	562,000,000	558,638,086
2	5,000,000.00	598,484,244	596,114,108	562,000,000	559,774,350
3	5,000,000.00	583,981,190	578,199,318	562,000,000	556,435,759
4	5,000,000.00	568,752,137	558,642,968	562,000,000	552,010,846
5	5,000,000.00	552,910,694	536,601,684	562,000,000	545,422,886
6	5,000,000.00	538,115,942	513,135,839	562,000,000	535,911,166
7	5,000,000.00	519,657,865	487,872,660	562,000,000	527,624,911
8	5,000,000.00	514,448,187	462,557,606	562,000,000	505,313,034
	40,000,000.00		4,341,051,011		4,341,131,037

The ¥30,026 (equal to US$640.21) difference between the NPV amounts is due to rounding the par forward rate to 2 decimal places and would represent profit to the bank.

Under the par forward the importer pays less yen than with the separate outright forward transactions for the first four years and more yen for the final four years. For example, at the end of the year, the importer would ¥562,000,000 under the par forward versus ¥611,585,366 with an outright forward at US$1 = ¥122.32. Effectively, the bank is making an off balance sheet loan to the importer. The 20 points difference between the average rate and the par forward rate can be thought of as compensation to the bank for the extension of credit. Figure 7.19 illustrates the par forward example.

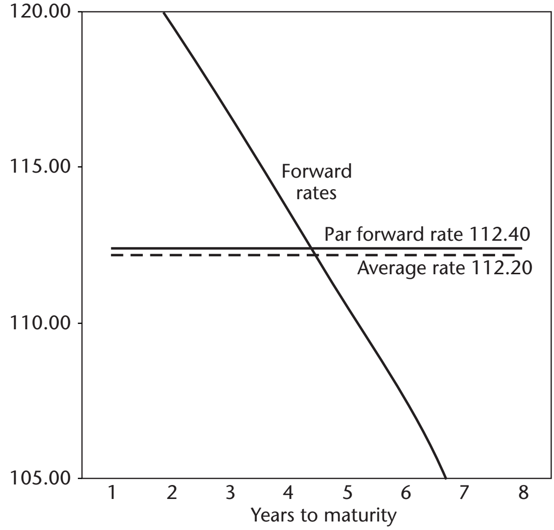


FIGURE 7.19 Par forward

PRACTICE PROBLEMS

7.1 Unhedged imports

An Australian importer has an obligation to pay ¥1,000,000,000 in 3 months' time. Calculate the cost in Australian dollars if the expected spot rate at maturity is A$1 = ¥65.20/65.30.

7.2 Hedging export receivables

A New Zealand exporter is due to receive US$4,560,000 in 2 months. The exporter considers the alternatives of remaining unhedged and selling the US dollars spot upon receiving them, or hedging by forward selling the US dollar receipts.

Spot rate NZ$1 =	US$0.4200	0.4205
2 month NZ$	3.75	3.85% p.a. (62/365)
2 month US$	2.65	2.75% p.a. (62/360)

(a) Calculate the forward rate at which the exporter could hedge
(b) If the expectation is that in 2 months' time the spot rate will be NZ$1 = US$0.4145/50, should the exporter hedge or remain unhedged?
(c) Calculate the break-even rate between being hedged and unhedged?

7.3 Hedged and unhedged exports
An Indonesian exporter expects to receive US$4,000,000 in 5 months' time.

Spot USD/IDR	10,200	10,400
5 month dollars	2.50%	2.60% p.a. (150/360)
5 month rupiah	25.00%	26.00% p.a. (150/360)

(a) At what rate could the exporter hedge its dollar receivables?
(b) How many rupiah would the exporter receive from the proceeds if it hedged?
(c) If the exporter elected not to hedge and at the end of the 5 months the spot rate turned out to be 10,600 /10,700, how many rupiah would the exporter receive?

7.4 An Australian exporter will be receiving US$5,000,000 in one year's time.

Spot	A$1 = US$0.5720	0.5725
1 year forward margin	50	45

(a) What will the A$ proceeds be if it is hedged?
(b) If at the end of the year the spot rate is A$1 = US$0.5625 /30, What would the A$ proceeds be if unhedged?
(c) Would the exporter be better off hedged or unhedged?

7.5 Unhedged foreign currency borrower
A company requires US$8,000,000 for 9 months. Two alternatives are considered:

1. Borrowing dollars domestically at an interest rate of 3.50% p.a. (272/360)
2. Borrowing euros at an interest cost of 4.00% p.a. (272/360)

(a) Calculate the effective borrowing cost if the spot rate at draw down is €1 = US$0.8650, and at repayment of principal and interest is €1 = US$0.8540.
(b) Which of the alternatives involves the lower cost?

7.6 Break-even rate unhedged borrowing
A Thai borrower has to choose between borrowing baht or borrowing dollars.

Spot rate	US$1	THB	35.7020	35.7030
3 month dollars		3.10%	3.20% p.a. (90/360)	
3 month baht		15.50%	15.75% p.a. (90/360)	

Calculate the break-even exchange rate between borrowing baht directly and borrowing US dollars on an unhedged basis.

7.7 Unhedged foreign currency investments
A funds manager has US dollars to invest for six months.

Spot rates
US$1 = ¥120.00
£1 = US$1.5000

The funds manager considers three alternatives:

1. Invest the dollars directly at 2.50% p.a.
2. Selling the dollars to buy yen to invest unhedged at 0.50% p.a.
3. Selling the dollars to buy pounds to invest unhedged at 3.20% p.a.

(a) Calculate the effective yield on the unhedged yen and unhedged pound investments if the spot rates at maturity turn out to be US$1 = ¥120.00 and £1 = US$1.4850.
(b) Which of the alternatives would have yielded the highest return on the investment?

7.8 Break-even rate on unhedged investment

Spot rate	US$1 = ¥116.50	116.60
6 month dollars	2.00%	2.25% p.a. (181/360)
6 month yen	0.10%	0.20% p.a. (181/360)

A funds manager has dollars to invest for six months.

(a) If the funds manager elects to use the dollars to buy yen for an offshore investment, what is the break-even future spot rate?
(b) If at maturity of the yen investment, the spot rate turns out to be US$1 = ¥113.30/113.40, calculate the effective yield.

7.9 Unhedged foreign investment
A money market manager considers investing in Malaysian ringgit as a way to earn a higher yield. The spot rate is currently fixed at US$/M$ 3.8000. If the money manager can access a 3 month ringgit fixed deposit rate of 8.50% p.a., what would be the effective yield in dollars if on maturity of the deposit the pegged exchange rate had been broken and the spot rate was then 4.0000/4.0100?

7.10 Par forward
An Australian exporter with receipts of US$5,000,000 each quarter for 3 years could hedge its foreign exchange risk by doing 12 separate forward deals in which it would sell US$5,000,000 against dollars at the different forward rates for each of the 12 maturities.

Based on a spot rate A$1 = US$0.5205 and the relevant interest rates the following forward rates and zero coupon discount factors apply:

Years	*Forward*	*US$ cash flow*	*A$ cash flow*	*zcdf*
0.25	0.5177	5,000,000.00	9,658,103.15	0.9895
0.50	0.5151	5,000,000.00	9,706,853.04	0.9792
0.75	0.5128	5,000,000.00	9,750,390.02	0.9688
1.00	0.5108	5,000,000.00	9,788,566.95	0.9586
1.25	0.5099	5,000,000.00	9,806,805.92	0.9476
1.50	0.5089	5,000,000.00	9,825,112.99	0.9370
1.75	0.5080	5,000,000.00	9,843,488.53	0.9266
2.00	0.5070	5,000,000.00	9,861,932.94	0.9163
2.25	0.5057	5,000,000.00	9,886,796.18	0.9051
2.50	0.5045	5,000,000.00	9,911,785.11	0.8918
2.75	0.5032	5,000,000.00	9,936,900.68	0.8806
3.00	0.5019	5,000,000.00	9,962,143.85	0.8673

Calculate the break-even par forward rate.

CHAPTER 8

Swaps

In this chapter, the concepts of currency and interest rate swaps are introduced. Currency swaps provide a powerful tool for manipulating cross currency cash flows without creating a net exchange position. They can be used to roll foreign exchange positions and to simulate foreign currency loans or investments. They can also be used to exploit arbitrage opportunities and to manage banking system liquidity. Interest rate swaps provide a means of switching interest rate exposures between fixed and floating. Cross currency swaps can be used when floating interest rates are involved.

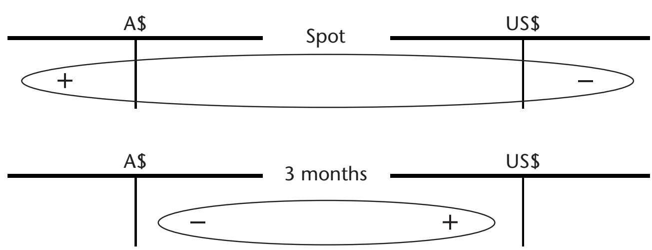
Definition

A *currency swap* is the simultaneous purchase and sale of equivalent amounts of one currency against another currency for different maturities. A currency swap does not create a net exchange position but it does create mismatched cash flows for a period of time.

A currency swap is equivalent to two money market transactions.

To illustrate this point, consider the cash flows involved when someone buys Australian dollars spot against US dollars and sells Australian dollars three months forward against US dollars (Exhibit 8.1).

EXHIBIT 8.1 Cash flow representation of a currency swap



The cash flows are the same as when borrowing Australian dollars for three months and lending US dollars for three months. The pricing of the swap must therefore reflect the interest rates applicable to these money market transactions.

The direction of the cash flows in the second leg of the swap is the opposite of those in the first leg of the swap. Consequently, as with money market transactions, currency swaps create only a temporary mismatch in cash flows. They do not create a net exchange position.

TYPES OF CURRENCY SWAP

1. A *pure swap* is a swap in which both the spot and forward transactions are done simultaneously with the *same* counterparty.
2. An *engineered swap* is a swap in which the spot and forward transactions are done with *different* counterparties.

SWAP RATES

Swap rates (previously referred to as forward margins) reflect the difference between spot rates and forward rates.

$$\boxed{\text{Swap rate} = \text{forward rate} - \text{spot rate}} \tag{8.1}$$

$$\begin{aligned}
\text{Swap rate} &= f - s \\
&= \frac{s(1 + r_T t)}{1 + r_C t} - s \\
&= s\left[\frac{1 + r_T t}{1 + r_C t} - \frac{1 + r_C t}{1 + r_C t}\right] \\
&= s\frac{(r_T - r_C)t}{1 + r_C t}
\end{aligned}$$

OUTRIGHT FORWARDS RATES

Forward exchange rates are generally referred to as *outright forwards* to distinguish them from swap rates.

Swap rates are not exchange rates. They are merely differentials. Technically they should not be referred to as 'rates' (i.e. ratios); see Exhibit 8.2. However, the term 'swap rate' is used here to conform with common practice.

EXHIBIT 8.2 Swap rates are differentials, *not* exchange rates

			Bid	*Offer*
Spot rates	A$1 =	US$	0.5200	0.5205
3 month swap rates			–0.0024	–0.0021
3 month forward rates	A$1 =	US$	0.5176	0.5184

The *swap bid rate* is the *differential* at which the quoting bank is willing to buy forward the commodity currency in the swap.

In Exhibit 8.2 the swap bid rate is –0.0024, showing that the Australian dollar is at a forward *discount* of 24 points. A bank quoting a swap bid rate of –0.0024 is willing to sell Australian dollars spot and buy Australian dollars three months forward at a differential of 24 points. By buying Australian dollars forward at 0.5176 and selling Australian dollars spot at 0.5200, the quoting bank *receives* the benefit of the 24 points forward discount.

The *swap offer rate* is the *differential* at which the quoting bank is willing to sell forward the commodity currency in the swap.

In Exhibit 8.2 the swap offer rate is –0.0021, showing that the Australian dollar is at a forward *discount* of 21 points. A bank quoting a swap offer rate of –0.0021 is willing to buy Australian dollars spot and sell Australian dollars three months forward at a differential of 21 points. By buying Australian dollars spot at 0.5205 and selling Australian dollars forward at 0.5184, the bank *pays* the 21 points forward discount.

The quoting bank will quote its bid and offer swap rates such that it receives the greater benefit or pays the lesser cost. It will buy the commodity currency at a higher forward discount (or swap benefit) and it will sell the commodity currency at a lower forward discount (or swap cost).

In bid offer quotations it is not necessary to show the sign of swap rates to identify them as forward premiums or discounts. If the swap bid rate is a larger number than the swap offer rate, the commodity currency is at a forward discount. On the other hand, if the swap bid rate is a smaller number than the swap offer rate (i.e. it looks like an exchange rate quotation in that the bid rate is less than the offer rate), then the commodity currency is at a forward premium (Exhibit 8.3).

EXHIBIT 8.3 Swap rates reflect forward premiums and discounts: 3 month swap rates

A$ Commodity currency, US$ Terms currency

Bid	*Offer*	
		A$ is at forward discount
24	21	A$ interest rate is higher than US$ interest rate

£ Commodity currency, US$ Terms currency

Bid	*Offer*	
		£ is at forward premium
10	11	£ interest rate is lower than US$ interest rate

DETERMINING THE SPOT RATE IN A SWAP

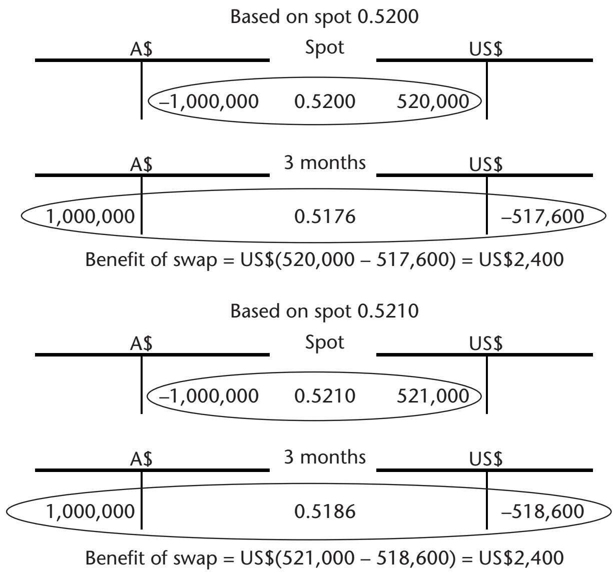
The spot rate on which a pure swap is based is virtually irrelevant within a reasonably close range. It is the swap rate reflecting the interest differential that determines the benefit or cost of the swap. Consider a swap where the quoting bank sells A$1,000,000 spot and buys A$1,000,000 at a three month forward discount of 24 points, based on slightly different spot rates 0.5200 and 0.5210 (Exhibit 8.4; overleaf).

The forward discount of 24 points is worth US$2,400 per A$1,000,000 of the swap, regardless of the spot rate upon which the swap is based. The quoting bank would marginally prefer the spot deal to be done at 0.5210, as it would have the use of an additional US$1,000 for three months. However, the incremental benefit is trivial (US$7.50, assuming the US$1,000 can be invested for 3 months at 3.0% p.a.). In practice the parties to the swap generally agree on a mutually acceptable spot rate. The quoting bank usually suggests which spot rate should be used. This is commonly the mid rate at the time.

PURE SWAPS AND ENGINEERED SWAPS

In a pure swap both legs of the swap are based on the same spot rate. In an engineered swap, because the two legs of the swap are transacted with different counterparties, the two deals will generally be based on different spot rates. Consequently, the calling bank or customer will normally find it more economical to enter into pure swaps rather than engineered swaps.

EXHIBIT 8.4 Cash flows in a swap using slightly different spot rates



If the bid offer spread is 5 points (Exhibit 8.5), the engineered swap will cost the customer an additional five points because he or she will have to pay the spot bid offer spread. In a pure swap the customer avoids the need to pay the spot bid offer spread.

EXHIBIT 8.5 Pure swaps and engineered swaps

				Bid	*Offer*
Spot rates	A$1	=	US$	0.5200	0.5205
3 month swap rates				0.0024	0.0021
3 month forward rates	A$1	=	US$	0.5176	0.5184

A customer wants to buy A$1,000,000 spot and sell A$1,000,000 three months forward.

	Pure swap			*Engineered swap*
Buy A$ spot	0.5200	or	0.5205	0.5205
Sell A$ forward	0.5176		0.5181	0.5176
Cost of swap	**0.0024**		**0.0024**	**0.0029**

SHORT DATED SWAPS

Swaps where one or both of the transactions are value today or tomorrow are known as *short dated swaps*. The swap from today until the next business day is known as the *overnight swap* and from tomorrow until the next business day (that is, spot value) is known as the *tom/next swap*. The swap rate from today until spot is determined by adding together the overnight and tom/next swap rates.

When adding or subtracting short dated swap points to determine the (outright) value today or value tomorrow exchange rates it is necessary to 'change the side and change the side'.

Because the outright rates applicable for short dates are the ratio of present values rather than future values it must be the case that the commodity currency is at a short dated premium when it is at a forward discount and at a short dated discount when it is at a forward premium.

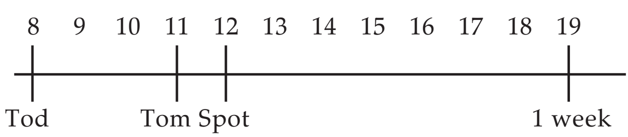
The *short dated swap bid rate* is the *differential* at which the quoting bank is willing to *buy* the commodity currency at the short date in the swap. The *short dated swap offer rate* is the *differential* at which the quoting bank is willing to *sell* the commodity currency in the short date in the swap.

To avoid confusion it is best to remember that the quoting bank always quotes so that the bid offer spread is in its favour. If the price taker is getting the benefit of the points, the smaller number of points will apply. On the other hand when the price taker is paying the cost of the points, the larger number of points will apply.

EXAMPLE 8.1

Spot	US$1	=	¥121.92	122.02
Swap rates				
Overnight (O/N)			4.3	4.1
Tom/Next (T/N)			1.4	1.3
1 week			11.0	10.0

Today is Friday 8 June. Tom is Monday 11 June. Spot is Tuesday 12 June. The 1 week forward value date is Tuesday 19 June.



To determine the rate at which a customer could sell US dollars outright value tomorrow 11 June it would be necessary to add 1.3 points to the spot bid rate.

Value tom bid rate = 121.92 + 0.013 = 121.933

Notice that short dated rates are often quoted to an extra decimal place to reflect the difference in the rates.

Because the dollar is at a forward discount it must be at a short dated premium. In other words the short dated swap points must be added to the spot rate (change the sign). The outright value tom rate is therefore higher than the spot rate. The customer is better off and the quoting bank is worse off when the customer sells the commodity currency at a higher rate. It follows that the relevant swap rate is 1.3 points rather than 1.4 points (change the side).

To determine the rate at which a customer could sell US dollars outright value today 8 June it would be necessary to add 1.3 points and 4.1 points to the spot bid rate.

Value today = 121.92 + 0.013 + 0.041 = 121.975

It is possible to do a swap in which the near leg is value today or value tomorrow. For example, a customer does a pure swap in which it sells US dollars value today and buys US dollars value 1 week (19 June)

Swap 19 Jun over 8 Jun = 1.3 + 4.1 + 10.0 = 15.4 points

The reason that the short dated swap rates are displayed on the opposite side is so that the swap points are on the same side when the swap is from a short date to a forward date. In the above example, the customer is receiving the lower interest currency first, so the bank must compensate it. The bank will quote such that it gives the customer the smaller number of points in each case.

As with swaps in general the spot rate is somewhat arbitrary. The important thing is that the difference between the rates for the short leg and the forward leg of the swap is equal to the number of swap points. If the value today deal is done at 122.00, the customer would sell US dollars value today at 122.00 and buy US dollars value 19 June at 122.00 – 0.154 = 121.846.

APPLICATIONS OF CURRENCY SWAPS

Currency swaps provide a powerful tool for manipulating cross currency cash flows without creating a net exchange position. The applications of swaps include:

1. Covering outright forward exchange positions
2. Rolling foreign exchange positions (including historic rate rollovers)
3. Simulated foreign currency loans

4. Simulated foreign currency investments
5. Covered interest arbitrage
6. Managing bank system liquidity

Swaps can also be used to open and close gaps. This is covered in Chapter 9.

COVERING OUTRIGHT FORWARD EXCHANGE POSITIONS

The interaction between swap rates and interest rates is well illustrated by examining the different ways of squaring an outright forward exchange position.

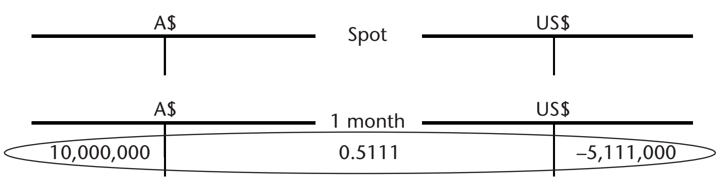
EXAMPLE 8.2

Market rates are:

1 month A$ interest rates			4.25% p.a.		4.35% p.a.	(31/365)
1 month US$ interest rates			2.50% p.a.		2.60% p.a.	(31/360)
Spot rate	A$1	=		US$	0.5120	0.5125
1 month swap rates					<u>–0.0008</u>	<u>–0.0007</u>
1 month outright forward rates	A$1	=		US$	**0.5112**	0.5118

A customer calls a bank late in the afternoon and asks for a rate at which to sell Australian dollars one month forward. Hoping to make one point profit, the bank quotes a forward bid rate A$1 = US$0.5111. The customer agrees to deal and sells the bank A$10,000,000. The bank is now long A$10,000,000/short US$5,111,000 and will have mismatched cash flows one month from now (Exhibit 8.6).

EXHIBIT 8.6 Outright forward exchange position: bank is long A$10,000,000 1 month forward



In NPV terms the bank is long A$10,000,000/(1 + 0.043 × 31/365) = A$9,963,612.34 and short US$5,111,000/(1 + 0.0255 × 31/360) = US$5,099,801.69.

The bank has three ways of squaring its net exchange and mismatched cash flow positions:

1. It could do the exact opposite outright forward transaction.
2. It could cover the net exchange position spot and square the mismatched cash flow positions through the money market.
3. It could cover the net exchange position spot and square the mismatched cash flow positions through a reverse swap.

If the bank were able to use Method 1, it would need to sell A$10,000,000 one month forward. Assuming it could deal at the market bid rate 0.5112, the bank would square its net exchange position at a one point profit (Exhibit 8.7).

EXHIBIT 8.7 Covering a forward exchange position. Method 1: Do the opposite forward transaction

A$			US$	
		Spot		
A$		1 month	US$	
10,000,000		0.5111		–5,111,000
	–10,000,000	**0.5112**	**5,112,000**	
		Profit		**1,000**
10,000,000	10,000,000		5,112,000	5,112,000

Profit = US$1,000 (that is, 1 point). In NPV terms US$997.81.

The net exchange position is *square* except for the profit.

In practice, banks do not quote one another outright forward exchange rates, so Method 1 is not a realistic alternative.

Method 2 involves the bank covering its net exchange position in the spot market at the market's bid rate 0.5120 and covering the mismatched cash flows via two money market transactions (Exhibit 8.8).

EXHIBIT 8.8 Covering a forward exchange position in the spot market

A$			US$	
		Spot		
	–10,000,000	**0.5120**	**5,120,000**	
A$		1 month	US$	
10,000,000		0.5111		–5,111,000

The net exchange position is square but the cash flows in both currencies are mismatched.

By selling Australian dollars spot and buying Australian dollars forward the bank has engineered a swap. The mismatched cash flow position can be covered through either the money market or a swap. If the bank intends to cover the forward exchange position through the money market, it would need to (Exhibit 8.9):

1. Borrow Australian dollars for one month at 4.35% p.a.
2. Lend US dollars for one month at 2.50% p.a.

EXHIBIT 8.9 Mismatched cash flows covered through money market

A$		Spot	US$	
	–10,000,000	0.5120	5,120,000	
10,000,000		**4.35%**		
		2.50%		**–5,120,000**
10,000,000	**10,000,000**		**5,120,000**	**5,120,000**

A$		1 month	US$	
10,000,000		0.5111		–5,111,000
	–10,036,945			
			5,131,022	

This would square the cash flow positions value spot and almost square them value 1 month.

To realize the profit in US dollars the bank could buy A$36,945 at 0.5112, in which case the profit would be US$(5,131,022 – 5,111,000) – 36,945 × 0.5112 = US$1,140. This amount differs slightly from the US$1,000 in Exhibit 8.8 because of rounding differences.

To realize the profit in Australian dollars the bank could sell US$(5,131,022 – 5,111,000) at 0.5112. The profit would be A$(39,175 – 36,945) = A$2,230.

Note: At 0.5112, A$2,230 = US$1,140. This profit would be realized on the 1 month date.

If the bank elects to cover the forward exchange position through a spot deal and a swap, the swap required is the reverse of that which it has engineered through the forward sale of US dollars to the customer and the purchase of US dollars from its spot counterparty. The reverse swap (Exhibit 8.10) would be:

1. Buy Australian dollars/sell US dollars spot at 0.5120.
2. Sell Australian dollars/buy US dollars forward at 0.5112.

EXHIBIT 8.10 Covering a forward exchange position. Method 3: Spot deal and a reverse swap

A$		Spot	US$	
	–10,000,000.00	0.5120	5,120,000.00	
10,000,000.00		**0.5120**		**–5,120,000.00**
10,000,000.00	**10,000,000.00**		**5,120,000.00**	**5,120,000.00**

A$		1 month	US$	
10,000,000.00		0.5111		–5,111,000.00
	–10,000,000.00	**0.5112**	**5,112,000.00**	
		Profit		**1,000.00**
10,000,000.00	10,000,000.00		5,112,000.00	5,112,000.00

Profit = US$1,000 (that is, 1 point). The net exchange position is *square*.

Provided the rates are consistent, all three methods will yield the bank the same profit with no net exchange position except for rounding differences.

In practice, the bank is most likely to use Method 3 – covering the net exchange position spot and covering the mismatched cash flows via a reverse swap. Lending in the money market involves considerably more credit risk than doing a currency swap, so Method 3 would use less of the bank's capital.

ROLLING A FOREIGN EXCHANGE POSITION

Swaps can be used to extend or shorten the maturity date of a foreign exchange position. The swap may be based either on current market or historic rates.

EXAMPLE 8.3

Three months ago a Japanese importer purchased US$10,000,000 three months forward at an outright rate of 120.00 to hedge expected US dollar payments. The original forward contract is maturing in two days time, that is, today's spot value date. The ship has been delayed and the importer will not be required to make the US dollar payment for a further month. The current interbank rate scenario is:

Spot	US$1	=	¥120.00	120.05
1 month dollars			3.00%	3.10% (30/360)
1 month yen			0.20%	0.25% (30/360)
1 month swap rate			29	27

It is initially assumed that, coincidentally, the spot rate is the same as the original outright forward rate (120.00). This is highly unlikely in practice, but nevertheless possible.

One alternative would be for the importer to take delivery of the US dollars and to place them on the money market for one month. The importer would need to borrow yen for one month to fund the purchase of the US dollars. Assuming the importer could borrow yen at 0.25% p.a. and deposit US dollars at 3.00% p.a., the resulting cash flows would be as in Exhibit 8.11.

EXHIBIT 8.11 Rolling an exchange position through the money market: importer's cash flows

US$		Spot	¥	
10,000,000		120.00		–1,200,000,000
	–10,000,000	**3.00%**		
		0.25%	**1,200,000,000**	
10,000,000	–10,000,000		1,200,000,000	1,200,000,000

US$		1 month	¥	
10,025,000				**–1,200,250,000**

$$\text{Effective forward rate} = \frac{1{,}200{,}250{,}000}{10{,}025{,}000} = 119.73$$

A second alternative is for the importer to roll the position through a swap. This would require selling US dollars spot and buying US dollars 1 month forward. Assuming the importer deals at the above market rates and the pure swap is based on the spot rate 120.00, the importer would sell dollars spot at 120.00 and buy dollars one month forward at 120.00 – 0.27 = 119.73. The cash flows would be as in Exhibit 8.12.

EXHIBIT 8.12 Rolling an exchange position through market swap: importer's cash flows

US$		Spot	¥	
10,000,000		120.00		–1,200,000,000
	–10,000,000	**120.00**	**1,200,000,000**	
10,000,000	–10,000,000		1,200,000,000	1,200,000,000

US$		1 month	¥	
10,000,000		**119.73**		**–1,197,300,000**

$$\text{Effective forward rate} = \frac{1{,}197{,}300{,}000}{10{,}000{,}000} = 119.73$$

Rolling the outright forward exchange position through a swap produces the same outcome as rolling it through two money market transactions. Both methods involved an effective new forward rate of US$1 = ¥119.73.

In practice, the importer would probably not be able to borrow at the interbank offer rate. The importer would have to borrow at a margin above the interbank offer rate. Depending on the importer's credit rating this margin might be, say, 50 basis points per annum. In that case the importer would borrow ¥1,200,000,000 at 0.75% p.a. (that is, 0.25% + 0.50%). However, the importer may be able to access the interbank borrowing rate if the position is rolled through a swap. So rolling through a swap will generally be more economical than rolling through two money market transactions.

If the importer had to borrow the yen at 0.75% the cash flows would be as in Exhibit 8.13. The additional 50 basis points would worsen the effective forward rate by 5 exchange points to US$1 = ¥119.78.

EXHIBIT 8.13 Rolling an exchange position through the money market: importer's cash flows with 50 basis point credit margin

US$		Spot	¥	
10,000,000		120.00		–1,200,000,000
	–10,000,000	**3.00%**		
		0.75%	**1,200,000,000**	
10,000,000	–10,000,000		1,200,000,000	1,200,000,000

US$		1 month	¥	
10,025,000				**–1,200,750,000**

$$\text{Effective forward rate} = \frac{1,200,750,000}{10,000,000} = 119.78$$

Rolling an exchange position through a swap also has the advantage that it is off balance sheet and will involve less credit risk.

A further complication exists if the spot rate at the time does not coincide with the original forward outright rate. Suppose, for example, that at the time of doing the rollover the spot rate has fallen to US$1 = ¥110.00. The cash flows for a market rate swap would be as in Exhibit 8.14.

EXHIBIT 8.14 Rolling an exchange position through market swap with spot 110.00: importer's cash flows

US$		Spot	¥	
10,000,000		120.00		–1,200,000,000
	–10,000,000	**110.00**	**1,100,000,000**	
		0.25%	**100,000,000**	
10,000,000	–10,000,000		1,200,000,000	1,200,000,000

US$		1 month	¥	
10,000,000		**109.75**		**–1,097,500,000**
		P + I		**–100,020,833**
				1,197,520,833

Selling US$10,000,000 value spot at a rate of 110.00 will generate proceeds of ¥1,100,000,000. This is insufficient to meet the obligation to pay ¥1,200,000,000 under the forward contract maturing at 120.00. The importer will need to borrow the ¥100,000,000 shortfall to clear its obligation. Assuming this is borrowed at 0.25%, the yen obligation due at the new forward date will be ¥1,097,500,000 (US$10,000,000 at 109.75) plus ¥100,020,833 (that is, 100,000,000 × (1 + 0.0025 × 1/12) making a total of ¥1,197,520,833.

Notice that the forward rate 109.75 is a function of the spot rate 110.00 and the prevailing 1 month interest rates.

$$\text{Effective forward rate} = \frac{1{,}197{,}520{,}833}{10{,}000{,}000} = 119.75$$

The effective forward rate in the case where the spot has fallen to 110.00 is 2 points worse than if the spot rate was 120.00. The 2 points reflect the interest cost of borrowing the ¥100,000,000 cash shortfall for the month.

If the spot rate at the time of rolling had been above the original forward rate, the effective forward rate would have improved because the spot movement would have generated a positive cash flow at the original maturity date which could have been invested and earned interest.

The impact of the spot movement on the effective forward rate will be most significant when the spot movement is large or the contract is being rolled for a long time and if the interest rate in the currency in which the cash flows occur is high.

HISTORIC RATE ROLLOVERS

A third alternative is for the importer to do an *historic rate rollover* (Exhibit 8.15). If the swap is based on a spot rate that is set equal to the historic rate, there will be no net cash flow for the importer on the spot date.

EXHIBIT 8.15 Historic rate rollover: importer's cash flows

US$		Spot	¥	
10,000,000		120.00		–1,200,000,000
	–10,000,000	**120.00**	**1,200,000,000**	
10,000,000	–10,000,000		1,200,000,000	1,200,000,000

US$		1 month	¥	
10,000,000		**?**		**?**

How is the forward rate determined in an historic rate rollover?

If at the time or rollover the market spot rate was 110.00 and the 1 month swap rate was –0.25, it would be incorrect to merely assume that the historic forward rate would be 120.00 – 0.25 = 119.75. Banks commonly deal at historic rates for their corporate customers, but they only deal interbank at market rates. Consequently, the bank will do an historic rate swap with the importer and a market rate swap with another bank. The bank's cash flows would be as in Exhibit 8.16 (overleaf).

On the spot date the bank will experience a cash shortfall of ¥100,000,000. In effect, the bank inherits the importer's loss temporarily. By funding this loss at 0.25% p.a. the bank incurs an incremental cost of ¥ 20,833 payable at the final maturity date, as shown in Exhibit 8.14. In this example the yen interest rate is so low that the effective forward rate 109.75 turns out to be equal to the original contract rate plus the then market swap rate. However, that is not true in general.

If the swap was done so that the terms currency amount is kept constant, there is a *proportionality* consideration. The value of a yen point at 110.00 is worth more than at 120.00.

$$\text{The forward rate for the historic rate rollover} = \frac{1{,}200{,}000{,}000}{10{,}933{,}941 - 911{,}439}$$
$$= 119.73$$

The bank might be justified in arguing that the importer's borrowing rate of 3.10 + 0.50 = 3.60% p.a. should be used to calculate the historic forward rate, as the bank is taking a credit risk on the importer by the extension of

EXHIBIT 8.16 Historic rate rollover with yen amount constant

Bank's cash flows with market

US$		Spot	¥	
10,000,000		120.00		–1,200,000,000
	–10,909,091	**110.00**	**1,200,000,000**	
909,091		**3.10%**		
10,909,091	10,909,091		1,200,000,000	1,200,000,000

US$		1 month	¥	
10,933,941		**109.75**		**–1,200,000,000**
	–911,439	**P + I**		

Bank's cash flows with importer

US$		Spot	¥	
10,000,000		120.00		–1,200,000,000
	–10,000,000	**120.00**	**1,200,000,000**	
10,000,000	10,000,000		1,200,000,000	1,200,000,000

US$		1 month	¥	
10,022,551		**119.73**		**–1,200,000,000**

credit for the additional month. If this is the case, the forward rate in the historic rate rollover will be equal to the effective rate for a market rate swap. This would make the $P + I$ amount equal to US$911,818 and the forward historic rate 1,200,000,000/(10,933,941 – 911,818) = 119.74.

The central banks of a number of countries have prohibited the use of historic rate rollovers because they have been used to conceal losses from foreign exchange trading. Where they are permitted, historic rate rollovers can be used as a convenient way for customers to extend hedge cover without having to manage the residual cash flows on the rollover date. Companies using HRRs should ensure that they are priced correctly as explained above. A good control to avoid inappropriate use of HRRs is to require management sign-off before they can be transacted.

EARLY TAKE-UPS

Swaps can also be used to shorten the maturity date of a foreign exchange position. As when extending the maturity date, the swap may be based on current or historic rates. A swap based on historic rates that shortens the maturity of a foreign exchange contract is known as an *early take-up* or *pre-*

delivery. As with historic rate rollovers, the pricing of an early take-up should take account of the funding cost or investment benefit of bringing profits or losses forward.

EXAMPLE 8.4

Three months ago a New Zealand exporter sold US$4,400,000 six months forward at an outright rate of 0.4400 to hedge future US dollar receivables. The contract has three months to run, but the exporter now finds she will be receiving the US$4,400,000 value spot. The exporter would like to bring the maturity of the forward contract into line with the actual cash flows.

The current market rates are:

Spot rate	NZ$1	=	US$0.4195	0.4205
3 month NZ$			4.5%	4.6% p.a. (92/365)
3 month US$			3.0%	3.1% p.a. (92/360)
3 month swap rate			–16	–10

The exporter could enter into a market rate swap in which US$4,400,000 would be sold spot at the spot mid rate 0.4200 and US$4,400,000 bought 3 months forward at 0.4200 – 0.0016 = 0.4184. The cash flows would be as in Exhibit 8.17.

EXHIBIT 8.17 Exporter's cash flows

Swap at market rates

NZ$		Spot	US$	
10,476,190		**0.4200**		**–4,400,000**
	–510,462	**4.50%**		

NZ$		3 months	US$	
10,000,000		0.4400		–4,400,000
516,252	**–10,516,252**	**0.4184**	**4,400,000**	
10,516,252	–10,516,252		4,400,000	4,400,000

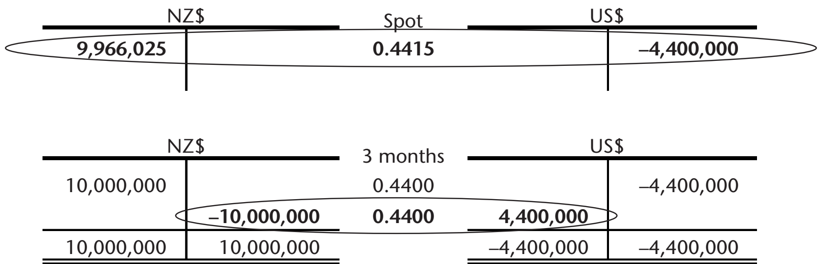
The exporter would have squared off the US dollar cash flows at the 3 month maturity date, but would have a shortfall of NZ$516,252 at that date. She would need to lend NZ$510,462 for 3 months at 4.50% p.a.

$$PV = \frac{516{,}252}{1 + 0.045 \times 92/365} = 510{,}462$$

$$\text{Effective spot rate} = \frac{4{,}400{,}000.00}{10{,}476{,}190 - 510{,}462} = 0.4415$$

Alternatively, the exporter could do an early take-up (Exhibit 8.18). Under an early take-up, the forward rate is set equal to the original forward rate so that no cash flow occurs at the forward date. The spot rate is determined by adjusting the historic forward rate for the interest cost or benefit of squaring the forward cash flows through the money market. The spot rate for the early take-up is equal to the effective spot rate calculated in Exhibit 8.17.

EXHIBIT 8.18 Early take-up: exporter's cash flows

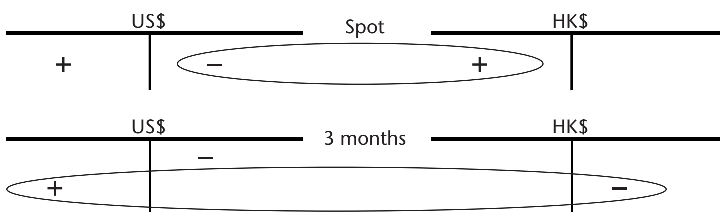


SIMULATED FOREIGN CURRENCY LOANS

Currency swaps provide a means of generating liquidity. A borrower may wish to borrow Hong Kong dollars for three months. Perhaps the borrower has no facility in place through which either the local Hong Kong dollar or Euro-Hong Kong dollar money markets can be accessed.

Provided he can borrow another currency, say US dollars, and enter into a US$/HK$ swap, the means exist to generate the Hong Kong dollars required. Generating liquidity in one currency by borrowing another currency and entering into a currency swap is known as a *simulated loan* (Exhibit 8.19).

EXHIBIT 8.19 Simulated HK$ loan



The cash flows are equivalent to borrowing Hong Kong dollars for the period.

Step 1. Borrow US$ for 3 months
Step 2. Sell US$ and buy HK$ spot. Buy US$ and sell HK$ 3 months forward

The borrower has borrowed US dollars and swapped them into Hong Kong dollars.

EXAMPLE 8.5

Spot rate	US$1	=	HK$	7.8000	7.8020
3 month	US$			2.75% p.a.	3.00% p.a. (90/360)
3 months swap rate				80	90

At what effective interest rate can the borrower generate HK$ for three months through a simulated loan?

To generate Hong Kong dollars, the borrower should:

Step 1. Borrow US dollars at 3.00% p.a.
Step 2. Swap US dollars for Hong Kong dollars for three months

That is, sell US dollars spot at 7.8010 (mid rate, say) and buy US dollars 3 months forward at 7.8100 (i.e. 7.8010 + 0.0090).

$$s = 7.8010$$

$$r_C = 0.03$$

$$r_T = ?$$

$$t = 90/360$$

$$f = 7.8100$$

$$f = \frac{s(1 + r_T t)}{1 + r_C t}$$

$$\therefore 7.8100 = 7.8010 \frac{(1 + r_T \times 90/360)}{(1 + 0.03 \times 90/360)}$$

$$\therefore r_T = 3.46\% \text{ p.a.}$$

The cash flows in a Hong Kong dollar borrowing are uneven. The repayment amount (principal plus interest) must be more than the amount borrowed (principal only). Consequently, to generate Hong Kong dollars through a simulated loan the swap needs to be for unequal amounts of Hong Kong dollars. The spot amount of Hong Kong dollars is a function of the required Hong Kong dollar principal. The principal amount of the US dollar loan is determined by converting the required number of Hong Kong dollars at the spot rate. The forward amount of Hong Kong dollars is determined by converting the forward amount of US dollars (principal plus interest) at the forward exchange rate (Exhibit 8.20).

EXHIBIT 8.20 Simulated loan HK$10,000,000 for 3 months

US$		Spot	HK$	
	–1,281,887	7.8010	10,000,000	
1,281,887		3.00%		
1,281,887	1,281,887			

US$		3 months	HK$	
	–1,291,501			
1,291,501		7.8100		–10,086,624
1,291,501	1,291,501			

The effective interest rate of the simulated Hong Kong dollars loan:

$$= \frac{\text{Interest}}{\text{Principal}} \times \text{time}$$

$$= \frac{10{,}086{,}624 - 10{,}000{,}000}{10{,}000{,}000} \times \frac{360}{90}$$

$$= 3.46\% \text{ p. a.}$$

The effective cost of borrowing a currency through a simulated loan may be less than the cost of borrowing it directly through the money market. Borrowing a foreign currency may attract interest withholding tax which adds to the effective interest cost. It may be possible to avoid incurring interest withholding tax if the company generates the foreign currency through a simulated loan involving a local currency borrowing.

In some cases exchange control restrictions may prevent non-residents from borrowing the currency at all. Provided a swap market exists, it is possible to generate liquidity in the restricted currency.

It is possible to raise local currency by borrowing a foreign currency and swapping it into local currency. This would be appropriate when the effective interest cost of local currency through a swap is cheaper than the domestic cost of funds or when domestic funds are not easily accessed.

EXAMPLE 8.6

A borrower requires euros for three months and considers two alternatives:

1. Borrowing euros directly
2. Borrowing dollars domestically and swapping them into euros

Spot rate	€1	=	US$0.8440	0.8450
3 month euros			2.50% p.a.	2.75% p.a. (90/360)
3 month dollars			3.00% p.a.	3.10% p.a. (90/360)
3 month swap rates			95	81

Alternative 1. Borrowing euros directly attracts 10% interest withholding tax. Therefore,

$$\text{Effective cost of borrowing € directly} = 2.75 \times 1.1$$
$$= 3.025\%\ \text{p.a.}$$

Alternative 2. Borrowing dollars at 3.10% p.a. and selling the dollars spot at 0.8445 then buying dollars forward at 0.8445 + 0.0005 = 0.8450 (swap based on spot mid rate); see Exhibit 8.21.

EXHIBIT 8.21 Cash flows of simulated euro loan

€		Spot	US$	
1,000,000		0.8445		–844,500
		3.10%	844,500	
			844,500	844,500

€		3 months	US$	
				–851,045
	–1,007,125	0.8450	851,045	
			851,045	851,045

$$\text{Effective euro rate} = \frac{1{,}007{,}125 - 1{,}000{,}000}{1{,}000{,}000} \times \frac{360}{90}$$
$$= 2.85\%\ \text{p.a.}$$

The effective euro borrowing rate can be calculated from Equation (6.3):

$$s = 0.8445$$
$$f = 0.8450$$
$$r_C = 0.031$$
$$t = 90/360$$
$$f = \frac{s(1 + r_T t)}{1 + r_C t}$$
$$\therefore 0.8450 = 0.8445\frac{(1 + r_T \times 90/360)}{(1 + 0.031 \times 90/360)}$$
$$\therefore r_T = 2.85\%\ \text{p.a.}$$

Alternative 2 provides a cheaper means of raising euros than Alternative 1. That is, in this example it turns out to be cheaper to raise euros through a simulated loan than borrowing euros directly and paying interest withholding tax.

It is possible to generate a floating rate foreign currency loan by using a cross currency swap, as discussed later in this chapter.

SIMULATED FOREIGN CURRENCY INVESTMENTS

Currency swaps provide the means to simulate foreign currency investments. Consider an investor with yen to place for three months. It may be possible to place them directly through the domestic yen money market or invest them in a Euro-yen deposit. Alternatively, the investor can enter into a yen/dollar swap and invest the dollars in the domestic deposit market. Investing in a foreign currency indirectly through a swap and a local currency placement is known as a *simulated foreign currency investment* (Exhibit 8.22).

EXHIBIT 8.22 Simulated investment

Step 1. Swap yen for dollars for 3 months;
that is, buy dollars and sell yen spot, and sell dollars and buy yen forward

Step 2. Invest dollars for 3 months

The cash flows are equivalent to investing yen for three months.

EXAMPLE 8.7

A Japanese fund manager has ¥100,000,000 to invest for 3 months. Calculate the effective yield of swapping into dollars and investing in dollars. Assume no interest withholding tax is payable.

Spot rate	US$1 =	¥124.95	125.05	
3 month dollars		3.50% p.a.	3.625% p.a.	(91/360)
3 month yen		0.25% p.a.	0.50% p.a.	(91/360)
3 month swap rate		105	102	

Alternative 1. Invest yen directly through the money market. Effective yield from investing yen directly = 0.25% p.a.

Alternative 2. Swap yen into dollars by buying dollars spot at 125.00 and selling dollars forward at 125.00 – 1.02 = 123.98, and then investing the dollars at 3.50% p.a. (Exhibit 8.23).

EXHIBIT 8.23 Simulated yen investment

US$		Spot	¥	
800,000.00		**125.00**		**–100,000,000**
	–800,000.00	**3.50%**		
800,000.00	800,000.00			

US$		1 month	¥	
807,077.78		**P + I**		
	–807,077.78	**123.98**	**100,061,503**	
807,077.78	807,077.78			

$$\text{Effective yield } = \frac{100{,}061{,}503 - 100{,}000{,}000}{100{,}000{,}000} \times \frac{360}{91} = 0.24\% \text{ p.a.}$$

Solving Equation (6.3) for r_T:

$$s = 125.00$$

$$f = 123.98$$

$$r_C = 0.035$$

$$t = 91/360$$

$$f = \frac{s(1 + r_T t)}{1 + r_C t}$$

$$\therefore 123.98 = 125 \frac{(1 + r_T \times 91/360)}{(1 + 0.035 \times 90/360)}$$

$$\therefore r_T = 0.24\% \text{ p.a.}$$

That is, the effective yield from the simulated yen investment is 0.24% p.a. Alternative 1 provides a higher yield than Alternative 2. That is, in this example the yield on the direct yen placement is higher than on the simulated yen investment.

Notice that a more involved structure is not necessarily better. In Example 8.5, borrowing euros indirectly through the swap market turned out to be cheaper because it avoided interest withholding tax. However, in general, unless some artificial constraint such as capital controls or withholding tax exists it is more economical to use the structure that 'crosses the least number of bid offer spreads'.

COVERED INTEREST ARBITRAGE

If swap rates are inconsistent with the prevailing spot rate and interest rates, it may be possible to profit from a risk-free set of transactions by taking advantage of the inconsistent set of rates in the market.

EXAMPLE 8.8

Spot rate	US$1 =	¥122.40	122.50	
3 month US$		3.25%	3.35% p.a. (90/360)	
3 month ¥		0.20%.	0.25% p.a. (90/360)	
3 month swap rate		96	91	

With an announcement of a cut in the dollar discount rate, a dealer checked the money market rates and observed that the three month Euro-dollar had fallen to 3.00/3.10% p.a. whilst the three month dollar/yen swap rate remained unchanged at 96/91 (Exhibit 8.24). What should the dealer do to immediately profit from the situation?

EXHIBIT 8.24 Covered interest arbitrage

	Before		*After*	
	Bid	*Offer*	*Bid*	*Offer*
3 month dollar rate	3.25% p.a.	3.35% p.a.	3.00% p.a.	3.10% p.a.
3 month ¥ rate	0.20% p.a.	0.25% p.a.	0.20% p.a.	0.25% p.a.
Available				
3 month swap rate	96	91	96	**91**
Equilibrium				
3 month swap rate	96	91	**88**	84

Because the swap rates have not yet moved to reflect the prevailing market interest rates, the swap rates available after the announcement are less than the equilibrium swap rates implied by the prevailing interest rates. In the swap market the dollar is temporarily available at a larger than market discount. The arbitrage opportunity lies in buying forward the dollar at the above equilibrium swap rate and covering through the money market (Exhibit 8.25).

EXHIBIT 8.25 Covered interest arbitrage via two money market deals
Action in the swap market

1. Sell US$ spot at 122.45
2. Buy US$ forward at 122.45 – 0.91 = 121.54   Benefit = 91 points

Action in the money market

3. Borrow dollars 3 months at 3.10% p.a. Cost = 88 points
4. Lend yen 3 months at 0.25% p.a.

Net gain = 3 points

US$		Spot	¥	
	–1,000,000.00	**122.45**	**122,450,000**	
1,000,000.00		**3.10%**		
		0.20%		**–122,450,000**
1,000,000.00	–1,000,000.00		122,450,000	122,450,000

US$		1 month	¥	
	–1,007,750.00		**122,511,225**	
1,007,750.00		**121.54**		**–122,481,935**
		Profit		29,290
1,007,750.00	1,007,750.00		122,511,225	122,511,225

Profit = ¥122,511,225 – 122,481,935 = ¥29,290 (that is, 3 points)

To crystallize the profit in yen, it is necessary to do an uneven swap for a spot amount of US$1,000,000 (principal) and a forward amount of US$1,007,750 (principal + interest).

It is to be expected that most swaps price makers will immediately adjust their swap rates to reflect the prevailing interest rates. If so, it would be possible to arbitrage the swap rates of those dealers who have not yet adjusted their prices.

	Bid	Offer
Unadjusted swap rates	99	**91**
Adjusted swap rates	**88**	84

An arbitrage opportunity exists because there is a bid rate that is higher than the offer rate. Remember, swap rates are differentials. These are negative numbers: –0.88 > –0.91 (Exhibit 8.26).

EXHIBIT 8.26 Swap arbitrage

Action in the unadjusted swap market

1. Sell US$ spot at 122.45
   Buy US$ forward at 122.45 – 0.91 = 121.54 Benefit = 91 points

Action in the adjusted swap market

2. Buy US$ spot at 122.45
   Sell US$ forward at 122.45 – 0.88 = 121.57 Cost = 88 points
   Net gain = 3 points

Assuming the above set of transactions is done for a principal amount of US$1,000,000, the profit from the arbitrage would be equal to 1,000,000 × 0.03 = ¥30,000 (Exhibit 8.27).

EXHIBIT 8.27 Swap arbitrage

US$		Spot	¥	
	–1,000,000.00	**122.45**	**122,450,000**	
1,000,000.00		**122.45**		**–122,450,000**
1,000,000.00	–1,000,000.00		122,450,000	122,450,000

US$		3 months	¥	
1,000,000.00		**121.54**		**–121,540,000**
	–1,000,000.00	**121.57**	**121,570,000**	
		Profit		**30,000**
1,000,000.00	1,000,000.00		121,570,000	121,570,000

Profit = ¥30,000 (that is, 3 points)

The small difference between the profit calculated using two money market deals and that using two swaps is merely due to rounding.

As there is no market risk involved in covered interest arbitrage, the amount of profit to be made is merely a function of the volume of the transaction. If the arbitrage could be performed with a principal amount of US$100,000,000, the profit would be ¥3,000,000. The market rates will quickly adjust toward equilibrium levels so that the arbitrage opportunity will be short lived. In practice, it may only exist for a matter of seconds. A market risk exists to the extent that unless all legs of the arbitrage can be done simultaneously, there is a risk that one or more of the rates will move so that the transaction is no longer profitable. Arbitrageurs who endeavour to perform the arbitrage on too large a volume may find that bid offer spreads widen against them or that they cannot get set for the full amount in all legs of the deal and are caught running a position which moves against them.

In the absence of regulations preventing one or more legs of the arbitrage transactions taking place, rates will return to equilibrium virtually immediately. The forces of equilibrium will tend to push swap rates towards 88/84 as they are hit on the offer or to push dollar interest rates up and yen interest rates down as the arbitrageurs borrow dollars and lend yen, or some combination of the above.

As arbitrageurs borrow dollars there may even be some upward impact on dollar interest rates. If the full adjustment to equilibrium were to occur through dollar interest rates, to what level would they need to rise?

$$s = 122.45$$
$$f = 121.54$$
$$r_C = ?$$
$$r_T = 0.20\% \text{ p.a.}$$
$$t = 90/360$$

Solving Equation (6.3) for r_C:

r_C = 3.25% p.a., which is the market offer rate

Precisely which rates move to bring the market back into equilibrium will depend on market conditions at the time. In practice, the adjustment would probably occur mostly in the swap rates.

CENTRAL BANK SWAPS

Central banks can use currency swaps to temporarily inject or withdraw liquidity from the local money market without creating a net exchange position. By offering a better than market swap rate, the central bank can entice commercial banks to enter into swaps which will have the desired effect on liquidity.

EXAMPLE 8.9

The Central Bank of Crownland wishes to inject liquidity into the local money market. It could add to the Crownland money supply by purchasing foreign currency from residents. However, an outright sale of crowns would have an undesirable effect on the exchange rate. To inject liquidity without affecting the exchange rate, the Central Bank would encourage banks to buy crowns spot from the Central Bank and to sell crowns forward.

If crowns are available in the market at a forward discount of 85 points, by offering to buy crowns in a swap at a forward discount of only 80 points, the Central Bank gives the commercial banks the opportunity to arbitrage its swap rate with the market (Exhibit 8.28).

EXHIBIT 8.28 Central bank swap

Benefit of swap with market	85 points
Cost of swap with Central Bank	80 points
Gain from CB swap arbitrage	5 points

The commercial banks can lay off the swap position with the market through a reverse swap or through the money market at a profit of five points. As deals between commercial banks and other market participants do not affect money supply (they merely transfer ownership), the Central Bank will achieve its objective of temporarily injecting liquidity.

FORWARD RATE AGREEMENTS (FRAs)

Forward interest rates are generally transacted as derivative transactions referred to as *Forward Rate Agreements* or *FRAs*. FRAs provide a way of locking in interest rates for a period of time starting sometime in the future.

The notation $FRA_{3,9}$ is referred to as the '3 by 9' FRA meaning the forward interest rate from 3 months from now until 9 months from now.

CALCULATING THE SETTLEMENT FOR AN FRA

At maturity of the FRA the borrower and the bank settle on the net difference between the FRA rate and the reference rate at maturity. For example, if the FRA rate is 3.45% p.a. and the reference rate (e.g. LIBOR) is, say, 4.00% p.a., the notional principal is $10,000,000, and the period was 92 days, the dollar amount of the settlement would be:

$$\text{FRA settlement} = \frac{10{,}000{,}000 \times (0.04 - 0.0345) \times 92/360}{1 + 0.04 \times 92/360} = \$13{,}913.39$$

The bank would be required to pay the borrower $13,913.39 because the rate at which the borrower is able to borrow is 55 basis points per annum above the contracted forward rate. If instead LIBOR had turned out to be 3.00% p.a., the borrower would have had to pay the bank an amount equal to:

$$\text{FRA settlement} = \frac{10{,}000{,}000 \times (0.0345 - 0.03) \times 92/360}{1 + 0.03 \times 92/360} = \$11{,}412.50$$

FORWARD YIELD CURVES

It is possible to calculate forward interest rates from any future time to any later time, e.g. $FRA_{3,4}$, $FRA_{3,5}$, $FRA_{3,6}$, ... The curve obtained by joining these forward rates could be called the forward yield curve.

The forward curve out of 3 months from today is shown as the higher curve in Figure 8.1. The lower curve shows the current (out of today) yield curve.

Months	Yield curve	Forward rate
3	2.50%	2.50%
6	2.75%	2.98%
9	3.00%	3.22%
12	3.25%	3.45%
15	3.50%	3.68%
18	3.75%	3.90%
21	4.00%	4.12%
24	4.25%	4.33%

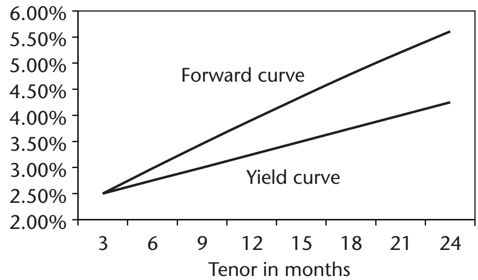


FIGURE 8.1 Forward yield curve

There is a forward yield curve starting from (or 'out of') each future date.

In the above example, the current yield curve is normal, so the forward interest rates are higher than the current yields. If the yield curve were inverse, the forward curve would lie below the yield curve.

INTEREST RATE SWAPS

An *interest rate swap* is an agreement between two parties who agree to exchange interest commitments on a notional principal over a period of time.

Swaps are effectively a strip of FRAs done at a common rate.

Consider a company with a 3 year debt facility with interest paid quarterly on a floating rate basis (say, 3 month LIBOR). The company has the risk that if interest rates rise its borrowing cost will increase. It could enter into a swap to pay a fixed rate for 3 years and to receive LIBOR (Figure 8.2).

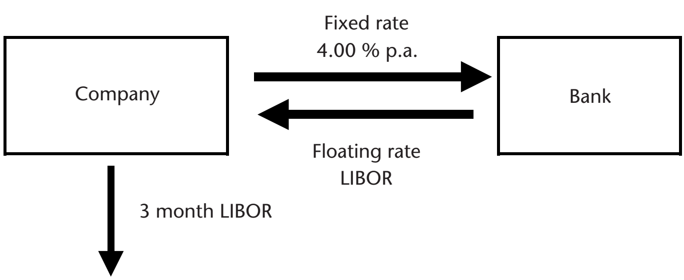


FIGURE 8.2 Interest rate swap

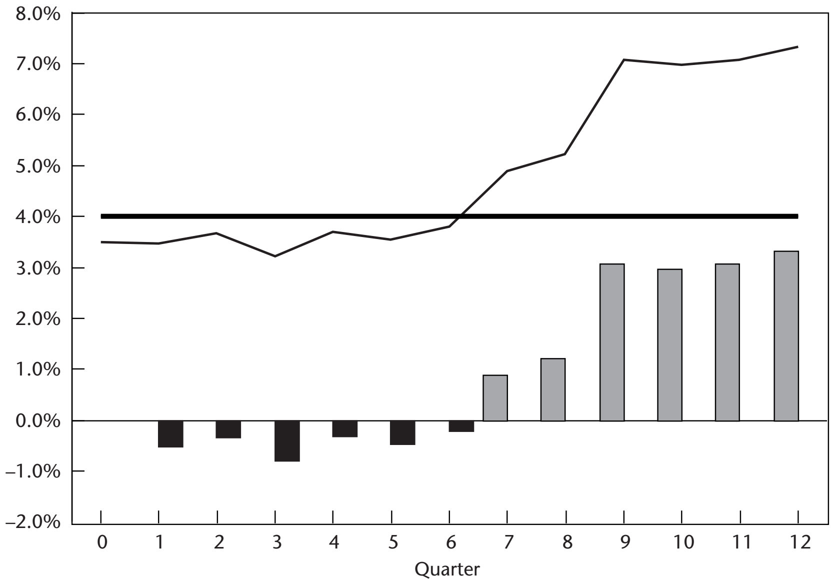


FIGURE 8.3 Swap settlements

The company has exchanged its floating rate obligations for fixed rate obligations. If interest rates rise, it will not have to pay more than the 4.00% p.a. fixed swap rate.

The swap is based on a notional principal and is often done with a different bank from the one with whom it has the debt facility. On each settlement date (quarterly in the above example) the bank and the company will net settle for the difference between the then floating rate and the fixed swap rate.

If LIBOR is below 4.00% p.a. (as in the first 6 quarters in Figure 8.3) the company will pay the bank an amount equal to the difference between 4.00% p.a. and LIBOR. For example, at the end of the first quarter LIBOR turned out to be 3.48% p.a. Assuming the notional principal of the swap is $10,000,000, the settlement amount would be:

$$\text{Swap settlement} = 10{,}000{,}000 \times (0.04 - 0.0348) \times 92/360 = \$13{,}288.89$$

On the other hand, if LIBOR is greater than 4.00% p.a. (as in the remaining 6 quarters), the bank will pay the company an amount equal to the difference between LIBOR and 4.00% p.a. For example, in the seventh quarter LIBOR turned out to be 4.68% p.a., so the settlement amount would be:

$$\text{Swap settlement} = 10{,}000{,}000 \times (0.0468 - 0.04) \times 92/360 = \$17{,}377.78$$

Interest rate swaps can also be used to swap interest obligations from fixed to floating.

A government authority that has issued a 5 year bond with fixed interest obligations could swap from fixed to floating by doing the opposite type of swap as depicted in Figure 8.4. It would be inclined to do so if it felt that interest rates would fall.

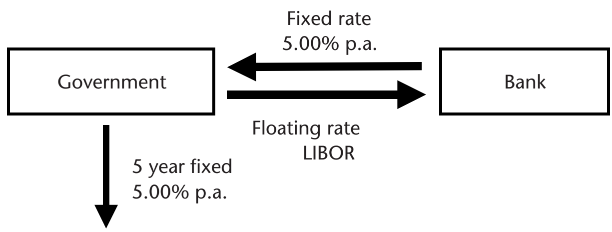


FIGURE 8.4 Swapping from fixed to floating

By receiving 5.00% p.a. in the swap the government authority ensures that it will benefit if LIBOR falls.

The government authority that has issued a 5 year bond and swapped it into floating has the benefit of long-term liquidity as well as cheaper borrowing costs if short-term interest rates fall.

PRICING INTEREST RATE SWAPS

People will be prepared to swap one set of cash flows for another set of cash flows, provided they have equal net present value. The (fixed) swap rate in a fixed/floating swap is that rate that will equate the net present value of the fixed cash flows with the net present value of the expected floating cash flows.

The expected cash flows on the floating side of the swap can be determined by calculating the forward interest rates starting from each settlement date and running for the period that corresponds to the basis of the floating side. For example, to price a 2 year quarterly swap, it is necessary to calculate the following forward rates: $FRA_{0,3}$, $FRA_{3,6}$, $FRA_{6,9}$, $FRA_{9,12}$, $FRA_{12,15}$, $FRA_{15,18}$, $FRA_{18,21}$ and $FRA_{21,24}$ (Figure 8.5). This could be described as the 3 months to go forward strip.

These forward rates represent the expected floating rates. For example, it is expected that the 3 month rate will be 2.98% p.a. in 3 months time, 3.45% p.a. in 6 months time and so on.

The expected cash flows on the floating side of the swap will be as follows.

Months	Yield curve	Forward rate
3	2.50%	2.50%
6	2.75%	2.98%
9	3.00%	3.45%
12	3.25%	3.91%
15	3.50%	4.36%
18	3.75%	4.79%
21	4.00%	5.21%
24	4.25%	5.61%

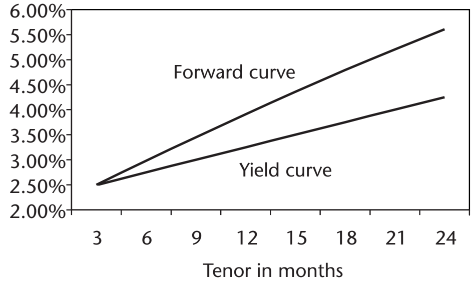


FIGURE 8.5 3 months to go forward curve

The floating rate for the first leg of the swap from today to 3 months from now is already known to be 2.50% p.a. If the principal amount of the swap is \$1,000,000, the floating cash flow on the first settlement date would be \$1,000,000 × 2.50% × 90/360 = \$6,250.

Given that the 3 month rate in 3 months time is expected to be 2.98% p.a., the expected floating cash flow on the second settlement date (6 months from now) would be \$1,000,000 × 2.98% × 92/360 = \$7,615.56 and so on, as shown in Exhibit 8.29. The present values of the expected floating cash flows are determined by multiplying the expected cash flow by the relevant zero coupon discount factors.

EXHIBIT 8.29 PV (expected floating cash flows)

Years	*Yield %*	*Forward %*	*Expected floating interest*	*zcdf*	*PV of E (floating interest)*
0.25	2.50	2.50	6,250.00	0.993846	6,211.54
0.50	2.75	2.98	7,615.56	0.986523	7,512.92
0.75	3.00	3.22	8,139.44	0.978057	7,960.84
1.00	3.25	3.45	8,816.67	0.968476	8,538.73
1.25	3.50	3.68	9,200.00	0.957809	8,811.84
1.50	3.75	3.90	9,966.67	0.946089	9,429.35
1.75	4.00	4.12	10,414.44	0.933350	9,720.32
2.00	4.25	4.33	11,065.56	0.919630	10,176.21
			71,468.33	7.683779	68,361.75

The swap rate is that rate for which the NPV of the fixed cash flows = \$68,361.75.

$$\text{NPV(Fixed)} = \text{Notional principal} \times \text{swap rate} \times \text{time period} \times \Sigma\text{zcdf}$$
$$= 1{,}000{,}000 \times \text{swap rate}/4 \times 7.683779$$

$$\therefore \text{swap rate} = \frac{68{,}361.75}{7.683779} \times \frac{4}{1{,}000{,}000} = 3.56\%\ \text{p.a.}$$

To derive a *swaps curve*, calculate the swap rate for each tenor and plot the swap rates against their corresponding tenors.

GENERAL FORMULA FOR PRICING SWAPS

The above calculation does not take account of the varying number of days in each quarter. However, it will be a very close approximation.

A general formula for pricing interest rate swaps with varying amounts of principal is shown as Equation (8.2). This formula can be used to price *amortizing* (that is, decreasing principal), *accreting* (that is, increasing principal) or *roller coaster* swaps. This enables banks to quote a fixed swap rate that equates the net present values of the fixed and floating sides of a swap to match the exact cash flows of the customer.

$$\boxed{\text{Swap rate} = \frac{\Sigma_{i=1}^{n} P_i f_i v_i}{\Sigma_{i=1}^{n} P_i v_i}} \tag{8.2}$$

where:

P_i = notional principal in period i

f_i = forward price for period i

v_i = discount factor for period i

n = number of periods during the life of the swap

Derivation of Equation (8.2)

$$\text{NPV(fixed side of swap)} = \text{Swap price} \times \sum_{i=1}^{n} P_i v_i$$

$$\text{NPV(expected floating side of swap)} = \sum_{i=1}^{n} P_i f_i v_i$$

$$\text{Swap price} \times \sum_{i=1}^{n} P_i v_i = \sum_{i=1}^{n} P_i f_i v_i$$

$$\text{Swap price} = \frac{\Sigma_{i=1}^{n} P_i f_i v_i}{\Sigma_{i=1}^{n} P_i v_i}$$

EXAMPLE 8.10

Calculate the break-even swap rate for a deferred start swap with a starting principal of $5,000,000 that amortises by $1,000,000 each quarter for 5 quarters given the forward rates and zero coupon discount factors in Exhibit 8.29.

The problem is most easily solved using a spreadsheet. The swap rate is a weighted average of the forward rates that takes account of the varying amounts of principal and the discount factors. As a first estimate, assume the swap rate is 3.90% p.a. (say). The present values would be as depicted in Exhibit 8.30.

EXHIBIT 8.30 Deferred start amortizing swap

Years	*Principal*	*Forward*	*E(floating)*	*zcdf*	*PV(floating)*	*3.90% PV(fixed)*	*360 days*
0.25		2.50%	0.00	0.993846	0.00	0	90
0.50		2.98%	0.00	0.986523	0.00	0	92
0.75		3.22%	0.00	0.978057	0.00	0	91
1.00	5,000,000	3.45%	44,083.33	0.968476	42,693.63	47,213	92
1.25	4,000,000	3.68%	36,800.00	0.957809	35,247.36	37,355	90
1.50	3,000,000	3.90%	29,900.00	0.946089	28,288.05	27,673	92
1.75	2,000,000	4.12%	20,828.89	0.933350	19,440.65	18,200	91
2.00	1,000,000	4.33%	11,065.56	0.919630	10,176.21	8,966	92
			142,677.78	7.683779	135,845.89	139,407.53	

The estimate of 3.90% p.a. is too high, because at that rate the NPV of the fixed side is greater than the NPV of the floating side. If the customer agreed to pay a fixed rate of 3.90% p.a. and receive the floating rate, it would be paying $139,407.53 to receive a set of cash flows expected to be worth only $135,845.89. The break-even swap rate is easily determined by adjusting the estimated swap rate in proportion with the relative net present values.

$$\text{Equilibrium swap rate} = 3.90\% \times \frac{135{,}845.89}{139{,}407.53} = 3.80\% \text{ p. a.}$$

The above result is verified in Exhibit 8.31 which shows the fixed cash flows when the swap rate is 3.80% p.a.

EXHIBIT 8.31 Deferred start amortizing swap

Years	*Principal*	*Forward*	*E(floating)*	*zcdf*	*PV(floating)*	*3.80% PV(fixed)*	*360 days*
0.25		2.50%	0.00	0.993846	0.00	0	90
0.50		2.98%	0.00	0.986523	0.00	0	92
0.75		3.22%	0.00	0.978057	0.00	0	91
1.00	5,000,000	3.45%	44,083.33	0.968476	42,693.63	46,003	92
1.25	4,000,000	3.68%	36,800.00	0.957809	35,247.36	36,397	90
1.50	3,000,000	3.90%	29,900.00	0.946089	28,288.05	26,964	92
1.75	2,000,000	4.12%	20,828.89	0.933350	19,440.65	17,734	91
2.00	1,000,000	4.33%	11,065.56	0.919630	10,176.21	8,736	92
			142,677.78	7.683779	135,845.89	135,832.97	

The small difference between \$135,845.89 and \$135,832.97 is due to rounding.

CROSS CURRENCY SWAPS

In general an interest rate swap is the exchange of one set of interest payments for another. When the two sets of interest payments are in different currencies they are known as *cross currency swaps*.

Figure 8.6 depicts a cross currency swap in which a company is swapping from paying floating US dollars (LIBOR) into paying a fixed euro rate of 4.00% p.a.

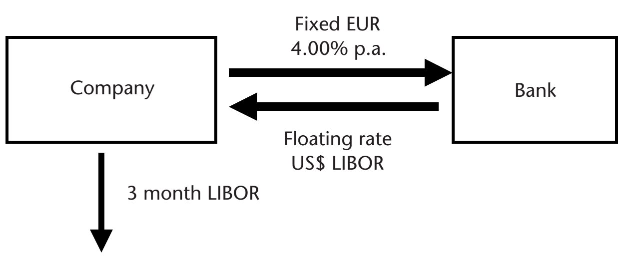


FIGURE 8.6 Cross currency swap

Cross currency swaps can be fixed to floating, floating to fixed, fixed to fixed (in the other currency) or floating to floating (known as *basis swaps*).

Cross currency (interest rate) swaps have the same applications as currency swaps and they have the advantage that they can apply for multiple maturities and either or both sides can be floating.

A par forward is a fixed/fixed cross currency swap.

VARYING MARKET CONVENTIONS

In general, interest rate swaps involve only exchange of interest payments. That is, there is no exchange of principal. In addition, interest rate swaps are settled on a net basis on each settlement date.

Cross currency swaps typically involve full flow of funds with exchange of principal at maturity. The exchange rate at which amounts of interest and the exchange of principal at maturity are converted is the spot rate at the time that the swap is contracted.

For example, if the principal amount of the swap depicted in Figure 8.6 is US\$10,000,000 and the spot rate when the swap is contracted is €1 = US\$0.9000, then the cash flows involved would be as in Exhibit 8.32, assuming that LIBOR turns out as shown.

EXHIBIT 8.32 Cash flows under a cross currency swap

End of quarter	*360 days*	*LIBOR at start*	*US\$ payments*	*Euro payments*
1	90	2.50%	\$62,500.00	€111,111.11
2	92	2.64%	\$67,466.67	€113,580.25
3	91	2.58%	\$65,216.67	€112,345.68
4	92	3.20%	\$81,777.78	€113,580.25
5	90	3.50%	\$87,500.00	€111,111.11
6	92	3.75%	\$95,833.33	€113,580.25
7	91	3.62%	\$91,505.56	€112,345.68
8	92	3.80%	\$97,111.11	€113,580.25
			\$10,000,000.00	€11,111,111.11

Exchange of principal at maturity: US\$10,000,000/0.9000 = €1,111,111.11

Fixed € interest payments = 11,111,111.11 × 0.04 × days in quarter/360

Floating LIBOR payments = 10,000,000 × LIBOR × days in quarter/360

There are 90 days in the first quarter so the company will be required to pay:

€11,111,111.11 × 0.04 × 90/360 = €111,111.11

The fixed euro interest payment each quarter will only vary with the number of days.

The floating amount of US dollar interest received will vary with LIBOR and the number of days in the interest period.

If US$ LIBOR at the start of the first quarter is 2.50% p.a., the company would receive:

$$\text{US\$10,000,000} \times 0.025 \times 90/360 = \text{US\$62,500.00}$$

If US$ LIBOR at the start of the second quarter is 2.64% p.a., the company would receive:

$$\text{US\$10,000,000} \times 0.0264 \times 92/360 = \text{US\$67,466.67}$$

and so on.

PRACTICE PROBLEMS

8.1 Using swap rates to calculate outright forward rates

Spot rates:	US$1 =	¥121.30	121.35
1 year swap		5.17	5.01

(a) At what rate can a customer buy yen outright one year forward?
(b) What is the benefit or cost to a customer of buying dollars 1 year forward and selling dollars spot in a pure swap?
(c) At what rates would a customer deal if it bought dollars 1 year forward and sold dollars spot in an engineered swap?

8.2 Pure swaps and engineered swaps

Spot rates	US$1	=	SF1.2735	1.2740
1 month swap rates			0.0030	0.0025

(a) What is the 1 month outright bid rate?
(b) What is the 1 month outright offer rate?

A customer wants to buy dollars spot and sell dollars 1 month forward.

(c) What is the benefit or cost of an engineered swap to the customer?
(d) What is the benefit or cost of a pure swap if based on a spot rate of 1.2740?

8.3 Simulated foreign currency loan
A company needs to borrow Singapore dollars for one year.

Spot rate	US$1	=	S$ 1.7500
1 year forward	US$1	=	S$ 1.7320
1 year interest rate	US$1		3.25% p.a.

Calculate the effective cost of generating Singapore dollars for one year through a swap.

8.4 Simulated foreign currency loan
An American company wants to borrow Canadian dollars for 6 months.

Spot	US$1 =	C$1.3540	1.3550	
6 month US$		5.50%	5.75%	(180/360)
6 month C$		8.00%	8.50%	(180/360)
6 month swap rate		148	168	

Is it cheaper to borrow the Canadian dollars directly or to borrow US dollars and swap them into Canadian dollars?

8.5 Simulated foreign currency investment
A fund manager has euros to invest for three months and considers two alternatives:

1. Investing euros directly at 3.5% p.a.
2. Swapping euros into US dollars and investing the dollars

Which alternative provides the higher effective yield given the prevailing market rates.

Spot	€1 =	US$0.8860	
3 month US$		3.00	3.25% p.a. (90/360)
3 month swap		11	10

10% withholding tax applies to interest earned from a direct investment in euros.

8.6 Hedging an outright forward with a swap

5 month US$ interest rates	3.25% p.a.	3.35% p.a.	(153/360)
5 month ¥ interest rates	0.20% p.a.	0.30% p.a.	(153/360)
Spot rate	US$1 = ¥	123.40	123.50
5 month swap rates		–1.63	–1.53
5 month outright forward rates	US$1 = ¥	**121.77**	121.97

A customer called a bank late in the afternoon and asked for a rate at which to sell US dollars 5 months forward. Hoping to make two points profit, the bank quoted a forward bid rate US$1 = ¥121.75. The customer agreed to deal and sold the bank US$10,000,000. The bank was then long US$10,000,000/short ¥1,217,500,000 and had mismatched cash flows on the 5 months date.

Using T-accounts, show how the bank could hedge its position with a spot deal and a swap. How much profit would the bank make?

8.7 Historic rate rollover
Three months ago a Japanese importer purchased US$10,000,000 three months forward at an outright rate of 130.00 to hedge expected

US dollar payments. The original forward contract is maturing in two days' time; that is, today's spot value date. The ship has been delayed and the importer will not be required to make the US dollar payment for a further month. The current inter-bank rate scenario is:

Spot	US$1	=	¥125.00	125.05
1 month dollars			3.15%	3.25% (30/360)
1 month yen			0.20%	0.25% (30/360)
1 month swap rate			29	31

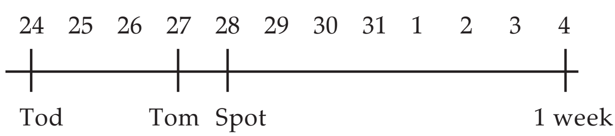
Calculate the break-even forward rate for an historic rate rollover.

8.8 Short dated swap

Spot US$1 = ¥123.56/123.61

Today is Friday 24 May. Spot value is Tuesday 28 May.

Swap rates:
O/N 2.0/1.9
T/N 0.4/0.3
S/W 7.0/6.0



(a) At what rate can a customer buy US$ outright value today (24 May)?
(b) At what swap rate could a customer buy US$ value today and sell US$ value 4 June in a pure swap?

CHAPTER 9

The FX Swaps Curve and Gapping in the Foreign Exchange Market

In this chapter the concept of the FX swaps curve is introduced. Gapping in the foreign exchange market provides a means of taking advantage of expected changes in swap rates that reflect interest differentials. The profitability of an FX gap is also affected by changes in the spot rate whilst a gap is open.

THE FX SWAPS CURVE

The term *swaps curve* generally refers to a set of interest rate swaps rates. At any point of time there is a set of currency swap rates that applies to a particular currency pair for various tenors. The curve obtained when currency swap rates are plotted against tenors is known as the *FX swaps curve*.

For example, based on a spot rate US$1 = ¥123.89, Table 9.1 shows the FX swap rates applied for the US dollar against the yen on 21 May 2002. Plotting these points produces the FX swaps curve shown in Figure 9.1.

The negative swap rates reflect the fact that the commodity currency is at a forward discount against the terms currency. Positive swap rates would indicate that the commodity currency is at a forward premium against the terms currency.

The shape of the FX swaps curve reflects the relationship between the respective yield curves of the two currencies. Parallel yield curves implying constant interest differentials generate an FX swaps curve as shown in Figure 9.2.

TABLE 9.1

Tenor	Swap rate points
T/N	+0.64
spot	0.00
1 week	–4.38
2 weeks	–9.20
3 weeks	–13.2
1 month	–19.5
2 months	–38.5
3 months	–60.0
4 months	–79.5
5 months	–102
6 months	–127
7 months	–153
8 months	–179
9 months	–205
10 months	–231
11 months	–257
12 months	–283

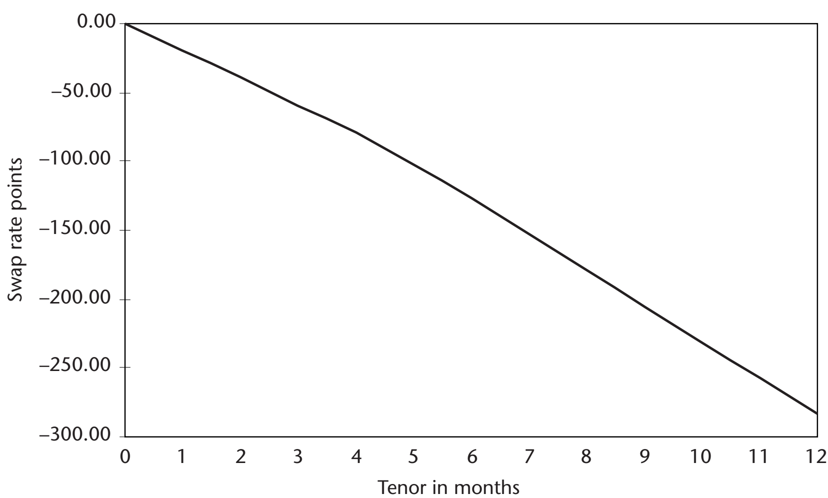


FIGURE 9.1 US$/¥ swaps curve, 21 May 2002

If the yield curves diverge, the interest differential widens with longer tenors producing a convex FX swaps curve (Figure 9.3).

If the yield curves converge the interest differential narrows with longer tenors producing a concave swaps curve (Figure 9.4).

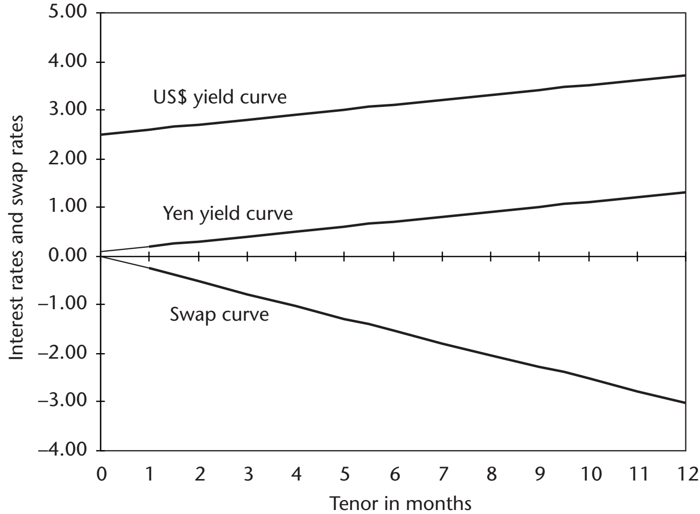


FIGURE 9.2 Constant interest differential swaps curve

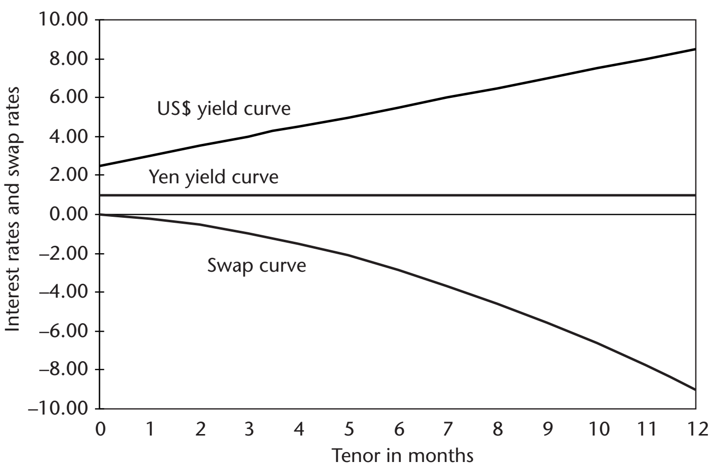


FIGURE 9.3 Convex swaps curve

Intersecting yield curves produce swaps rates that go from negative to positive at the point of intersection (Figure 9.5).

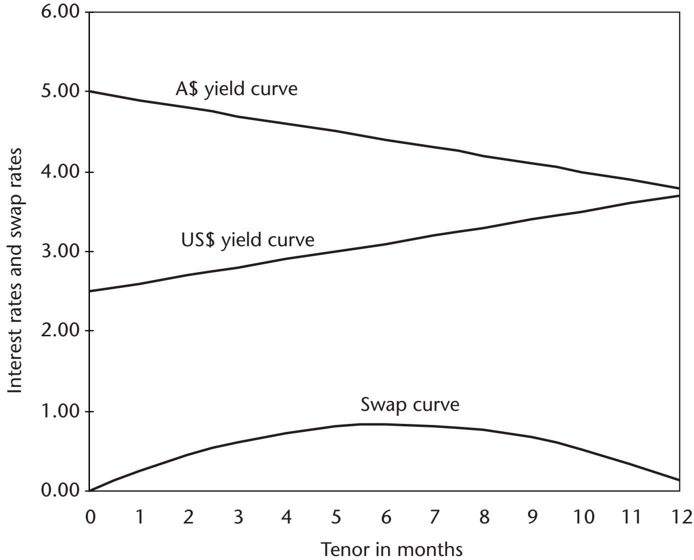


FIGURE 9.4 Concave swaps curve

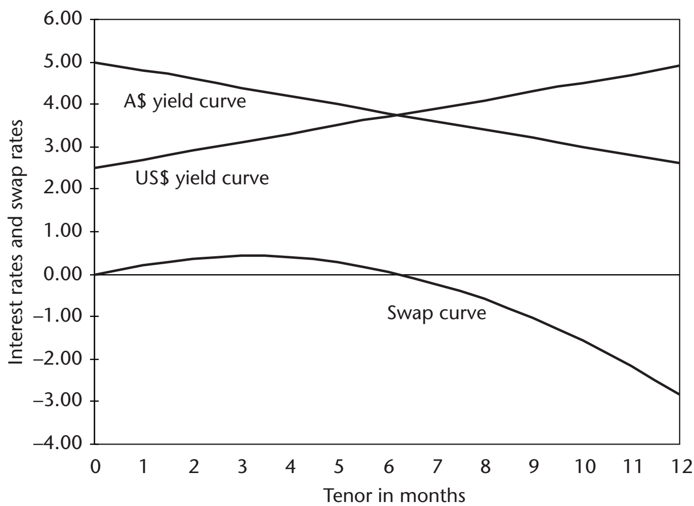


FIGURE 9.5 Intersecting yield curves

GAPPING IN THE FOREIGN EXCHANGE MARKET: HOW TO PROFIT FROM EXPECTED CHANGES IN INTEREST RATE DIFFERENTIALS

Just as it is possible to take advantage of expected changes in interest rates by opening and closing gaps through the money market, it is possible to use currency swaps to take advantage of expected changes in interest rate differentials by opening and closing gaps in the foreign exchange market. The profit or loss from opening and closing a foreign exchange gap depends on the movement in the swaps curve, the shape of the swaps curve and the change in the spot rate.

EXAMPLE 9.1

The spot exchange rate is US$1 = ¥100.00. US dollar interest rates are expected to fall relative to yen interest rates producing the following change in swap rates:

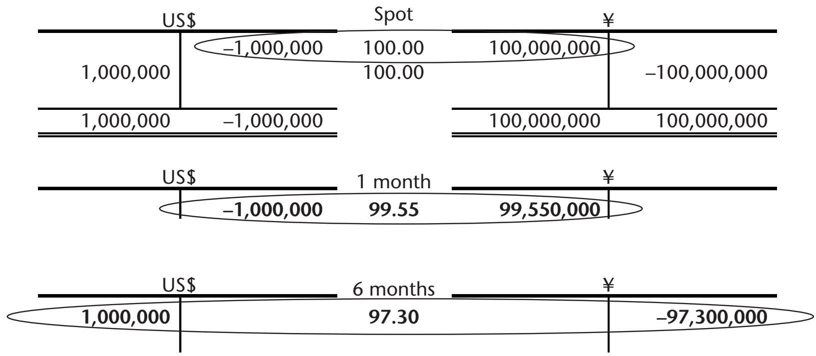
Tenor in months	*Current swap rates*	*Expected swap rates*
1	**–0.45**	–0.30
2	–0.90	–0.60
3	–1.35	–0.90
4	–1.80	–1.20
5	–2.25	**–1.50**
6	**–2.70**	–1.80

It is possible to take advantage of the expected drop in the interest differential by buying US dollars forward in a swap before the discount falls. An outright forward purchase of dollars would earn the benefit of the forward discount, but it would create a net exchange position. A gap could be opened by doing a swap to take advantage of the expected fall in the interest differential without creating a net exchange position.

To open the gap to take advantage of an expected narrowing of interest differentials, it would be necessary to buy the discount currency (dollar) for the longer tenor (e.g. 6 months) and sell the discount currency (dollar) for a shorter tenor (e.g. 1 month).

Using a swap to open a gap creates a swap position (Exhibit 9.1). If the swap rates fall because, as expected, dollar interest rates fall relative to yen interest rates, a profit will result. However, if contrary to expectations, swap rates rise reflecting a rise in dollar interest rates relative to yen interest rates, a loss will result.

EXHIBIT 9.1 Opening a gap with a swap



Opening the gap provides a benefit of 225 points or ¥2,250,000 per US$1,000,000. This is not profit because there is an open swap position. If swap rates fall, an unrealized profit will exist. If the gap is closed through a reverse swap after swap rates have fallen, then a profit will be realized.

The gap can be closed at any time through a reverse swap. To close the gap one month later, it would be necessary to buy US$1,000,000 spot and sell US$1,000,000 five months forward to match the dates of the cash flows. Assuming that swap rates have fallen in line with expectation, the gap would be closed with a profit realized. For simplicity, it is assumed that the spot rate has fallen to the original one month rate of US$1 = ¥99.55.

Narrowing interest differentials: swap rates become less negative

	When gap was opened		*When gap is closed 1 month later*	
Tenor in months	*Swap rates*	*Exchange rates*	*Swap rates*	*Exchange rates*
Spot		100.00		**99.55**
1	**–0.45**	**99.55**	–0.30	99.25
2	–0.90	99.10	–0.60	98.95
3	–1.35	98.65	–0.90	98.65
4	–1.80	98.20	–1.20	98.35
5	–2.25	97.75	**–1.50**	**98.05**
6	**–2.70**	**97.30**	–1.80	97.75

Closing the gap would involve buying US$1,000,000 spot at ¥99.55 and selling US$1,000,000 five months forward at ¥98.05. This would realize a profit of ¥750,000 as shown in Exhibit 9.2.

EXHIBIT 9.2 Closing gap after swap rates have become less negative

US$		Spot	¥	
	–1,000,000	99.55	99,550,000	
1,000,000		**99.55**		**–99,550,000**
1,000,000	–1,000,000		99,550,000	99,550,000

US$		5 months	¥	
1,000,000		97.30		–97,300,000
	–1,000,000	**98.05**	**98,050,000**	
		Profit		750,000
1,000,000	1,000,000		98,050,000	98,050,000

Consider instead the case where, contrary to expectations, dollar interest rates rise relative to yen interest rates. The interest differential widens and the swap rates become more negative (that is, the forward discount becomes larger). If the gap were closed using the higher swap rate, a loss would be realized.

Widening interest differentials: swap rates become more negative

	When gap was opened		*When gap is closed 1 month later*	
Tenor in months	*Swap rates*	*Exchange rates*	*Swap rates*	*Exchange rates*
Spot		100.00		**99.55**
1	**–0.45**	**99.55**	–0.60	98.95
2	–0.90	99.10	–1.20	98.35
3	–1.35	98.65	–1.80	97.75
4	–1.80	98.20	–2.40	97.15
5	–2.25	97.75	**–3.00**	**96.55**
6	**–2.70**	**97.30**	–3.60	95.95

Closing the gap would involve buying US$1,000,000 spot at ¥99.55 and selling US$1,000,000 five months forward at ¥96.55. This would realize a loss of ¥750,000, as shown in Exhibit 9.3.

EXHIBIT 9.3 Closing gap after swap rates have become more negative

US$		Spot	¥	
	–1,000,000	99.55	99,550,000	
1,000,000		**99.55**		**–99,550,000**
1,000,000	–1,000,000		99,550,000	99,550,000

US$		5 months	¥	
1,000,000		97.30		–97,300,000
	–1,000,000	**96.55**	**96,550,000**	
		Loss	**750,000**	
1,000,000	1,000,000		97,300,000	97,300,000

The gap would yield a profit if the 5 month swap discount after 1 month is less than 225 points and would result in a loss if the 5 month swap discount after 1 month is more than 225 points.

RIDING THE SWAPS CURVE

The shape of the swaps curve influences the attractiveness of opening a gap. The more non-linear the swaps curve is, the greater the carry associated with opening a gap.

For example, given a swaps curve like the one in Exhibit 9.4, there is an incentive to open a gap that will earn the greatest proportional benefit.

EXHIBIT 9.4 Intersecting yield curves implying non-linear swaps curve

Tenor in months	*A$ Interest rate*	*US$ Interest rate*	*Swap rate*
0	4.00%	2.00%	0.0000
1	3.80%	2.20%	0.0007
2	3.60%	2.40%	0.0010
3	3.40%	2.60%	0.0010
4	3.20%	2.80%	**0.0007**
5	3.00%	3.00%	0.0000
6	2.80%	3.20%	–0.0010
7	2.60%	3.40%	–0.0023
8	2.40%	3.60%	–0.0039
9	2.20%	3.80%	**–0.0058**

By buying Australian dollars 9 months forward and selling Australian dollars 4 months forward, it is possible to earn a gross forward discount of 65 points. If the 4 month swap rate after one month was still at +0.0007, the gap could be closed at a further benefit of 7 points. The gap would generate a profit of 72 points.

Opening the gap	*Benefit (+)/Cost (–) in points*
Buy A$9 months forward	+58
Sell A$4 months forward	+ 7
Benefit of opening the gap	+65
Benefit of closing the gap	+ 7
Profit from gap	+72

The gap would break even if the 4 month swap rate one month later was –0.0065 (65 points discount). For the gap to lose money the 4 month swap rate would have to fall by more than 72 points (ignoring movements in the spot rate, which are discussed in the following section). For this to occur, the 4 month interest differential would need to change from the A$ rate being 40 basis points higher than the US$ rate to being 440 basis points lower. Assuming that such an outcome is considered unlikely, opening the gap could be considered a 'good bet'.

Opening gaps to take advantage of positive carry implied by the shape of the swaps curve could be described as 'riding the swaps curve'.

It is not common to find steep intersecting yield curves resulting in U-shaped or inverted U-shaped swaps curves. However, less severe non-linear swaps curves are common.

CASH FLOW IMPLICATIONS OF SPOT RATE CHANGES

Movements in the spot exchange rate between when the gap is opened and closed can have a significant impact on the profitability of a gap because of timing differences in cash flows that occur at each end of the gap.

Consider the gap opened in Example 9.1. To illustrate the impact of spot rate changes, suppose that during the month in which the gap is open the spot rate rises from ¥100.00 to ¥120.00 whilst the swap rates remain as they were when the gap was opened. It might be expected that as a gap is a swap rate position rather than a net exchange position, spot rate changes would have no impact on the profitability of the gap.

Spot rate rises whilst swap rates remain unchanged

	When gap was opened		*When gap is closed 1 month later*	
Tenor in months	*Swap rates*	*Exchange rates*	*Swap rates*	*Exchange rates*
Spot		100.00		120.00
1	**–0.45**	**99.55**	–0.45	119.55
2	–0.90	99.10	–0.90	119.10
3	–1.35	98.65	–1.35	118.65
4	–1.80	98.20	–1.80	118.20
5	–2.25	97.75	**–2.25**	**117.75**
6	**–2.70**	**97.30**	–2.70	117.30

Benefit of opening the gap = 99.55 – 97.30 = 225 points
Cost of closing gap = 120.00 – 117.75 = 225 points
Loss from gap = 0

However, the above calculation overlooks the timing of cash flows. Closing the gap would involve buying US$1,000,000 spot at ¥120.00 and selling US$1,000,000 five months forward at ¥117.75. This would create the cash flows shown in Exhibit 9.5.

EXHIBIT 9.5 Closing gap after spot movement

US$		Spot	¥	
	–1,000,000	99.55	99,550,000	
1,000,000		**120.00**		**–120,000,000**
		1.00%	**20,450,000**	
1,000,000	–1,000,000		120,000,000	120,000,000

US$		5 months	¥	
1,000,000		97.30		–97,300,000
	–1,000,000	**117.75**	**117,750,000**	
		P + I		**–20,535,208**
		Loss	**85,208**	
1,000,000	1,000,000		117,835,208	117,835,208

The loss of ¥85,208 represents the interest cost of funding the cash shortfall of ¥20,450,000 for 5 months at 1% p.a.[1]

The cash shortfall is a direct consequence of the difference between the original 1 month forward rate and the spot rate at the time of closing the gap. If the spot rate when the gap is closed is above 99.55, there will be a

1 ¥20,450,000 × 0.01 × 5/12 = ¥85,208

negative cash balance that will require funding at a cost. On the other hand, if the spot rate when the gap is closed is below 99.55, there will be a positive cash balance that can be invested for a benefit. If the exchange rate movement is dramatic or the duration of the gap is long, the interest impact of the cash flow effect from spot rate changes can be substantial. Indeed, it can be more than sufficient to offset the gain or loss resulting from changes in the swap rate.

BREAK-EVEN SWAP RATE

In practice, spot rates and swap rates are continually changing. Continuing the above example, what is the break-even swap rate if at the maturity of the gap (when the near maturity date of the swap becomes spot) the spot rate is ¥120.00?

$$\text{Cash flow at spot date} = 1{,}000{,}000 \times (99.55 - 120.00) = ¥20{,}450{,}000$$

$$\text{FV(cash flow at spot date)} = 20{,}450{,}000(1 + 0.01 \times 5/12) = ¥20{,}535{,}208$$

$$\text{Cash flow at forward date} = 1{,}000{,}000(f - 97.30)$$

For the gap to break even the forward rate at which the gap is closed needs to be, f, such that:

$$1{,}000{,}000(f - 97.30) = 20{,}535{,}208$$

$$f = 117.84$$

$$\text{Break-even swap rate} = 117.84 - 120.00 = -2.16$$

In general, the break-even swap rate, b is:

$$b = f_2 + (f_1 - s)(1 + rt) - s \tag{9.1}$$

In the above example,

$$f_2 = 97.30$$

$$f_1 = 99.55$$

$$s = 120.00$$

$$r = 0.01$$

$$t = 5/12$$

giving

$$b = -2.16$$

The break-even swap rate is effectively the swap points earned or paid when the gap is opened adjusted for the interest cost or benefit from the cash flow arising from the change in the spot rate.

Observing the cash flows in Exhibit 9.5:

US$	Yen	Rate	
1,000,000	117,750,000	117.75	
	85,208	0.09	Interest
1,000,000	117,835,208	117.84	

PRACTICE PROBLEM

9.1 US dollar interest rates are higher than yen rates, so the swaps curve is negative. Over the next month, dollar interest rates are expected to rise relative to yen rates and the dollar is expected to appreciate against the yen.

	Current rates		*Expected rates (1 month from now)*	
Tenor in months	*Swap rates*	*Exchange rates*	*Swap rates*	*Exchange rates*
Spot		123.00		125.00
1	–0.20	122.80	–0.25	124.75
2	–0.40	122.60	–0.50	124.50
3	–0.60	122.40	–0.75	124.25

(a) What gap (three months against one month) should be opened to take advantage of the expected movement in rates?

(b) How much profit would be generated on a principal amount of US$1,000,000 if rates move as expected? Assume that when the gap is closed, the 2 month yen interest rate is 0.30% p.a.

CHAPTER 10

Currency Options – Pricing

This chapter introduces the concept of currency options and examines the factors that influence the price of options. Options pricing is explained intuitively using a binomial model but can alternatively be determined using closed form mathematical formula such as variations of the Black–Scholes model.

Definitions and concepts

The terminology used to describe currency options is essentially the same as that used to describe stock options.

A *currency option* is the right, without the obligation, to buy or sell one currency against another currency at a specified price during a specified period.

Calls and puts

A *call* is the right, without the obligation to *buy*, a currency. A *put* is the right, without the obligation to *sell*, a currency.

In every foreign exchange transaction, one currency is purchased and another currency sold. Consequently, every currency option is both a call and a put. An option to buy dollars against yen is both a dollar call and a yen put. For consistency in this book, unless otherwise stated, calls will refer to commodity currency calls and puts will refer to commodity currency puts.

Parties to an option

There are two parties to an option – the buyer and the seller. The *buyer* of the option enjoys the right to exercise the option and the right *not* to exercise the option (i.e. to let it lapse). The *seller* of the option has the obligation to deal at the contracted rate if the buyer elects to exercise the option. The seller is also known as the writer or grantor of the option.

Option premium

The price of the option is known as the *option premium*. The buyer pays the premium to the seller as compensation for the risk involved in writing the option. The premium is normally paid on the spot value date from the date on which the option is contracted.

Value terms

The *strike price* or *strike rate* is the exchange rate at which the option will be exercised if the buyer elects to exercise the option.

In-the-money (ITM) describes an option with a strike price that is more favourable than the current market price. *Out-of-the-money* (OTM) describes an option with a strike price that is less favourable than the current market price. *At-the-money* (ATM) describes an option with a strike price equal to the current market price. The at-the-money strike price for a European option is the forward rate (Exhibit 10.1).

EXHIBIT 10.1 Calls and puts: ATM/ITM/OTM matrix

			Call	Put
Strike	<	forward rate	ITM	OTM
Strike	=	forward rate	ATM	ATM
Strike	>	forward rate	OTM	ITM

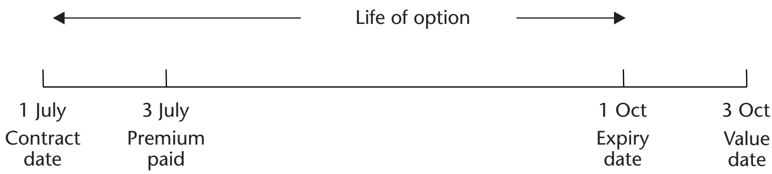
The premium will be higher if the option is in-the-money and lower if the option is out-of-the-money.

Maturity of the option

The *expiration date* or *expiry date* refers to the date on which the buyer's right to exercise ends. In practice, a specific expiry time (for example, 3 p.m. New York time on the expiry date) is agreed. If an option is exercised on the expiry date, the cash flows will occur on the then spot value date.

An *American option* refers to an option which can be exercised for spot value on any date between the contract date and the expiry date. A *European option* refers to an option which, theoretically, can only be exercised for spot value on the expiry date. In practice, writers of European options allow buyers to give notice prior to the expiry date that they will exercise the option. If a European option is 'exercised' prior to expiry date, the cash flows will occur on the spot value date following the expiry date (Exhibit 10.2).

EXHIBIT 10.2 Currency option value dates



Pay-out

The *pay-out* is the value of the option at expiry. Options have a zero minimum pay-out, because the owner of the option can choose not to exercise unless the option expires in-the-money. If an option with strike price K expires when the spot price is X, the pay-out will be:

$X - K$ for in-the-money calls

$K - X$ for in-the-money puts

Sources of options

Exchange-traded options refer to contracts traded through certain stock, futures or commodity exchanges. These contracts have strictly defined characteristics, such as standard amounts, standard expiry dates and standard strike prices. *Over-the-counter* (OTC) *options* refer to options written by banks and other institutions but not traded through public exchanges. Over-the-counter options can be tailored to suit the exact needs of the option buyer. The options referred to in this book are over-the-counter options.

CALCULATING OPTION PREMIUMS

It is common for option premiums to be expressed as either a *flat* percentage of the face value or as a fixed number of exchange points. It is possible to express the premium in terms of any currency. The dollar amount of an option premium can be determined from either the percentage of strike or number of points.

EXAMPLE 10.1

An option pricing model expresses the option premium on a 3 month dollar call/yen put as 1.25% or 145 points.

Spot rate	US$1	= ¥116.00
Strike price	US$1	= ¥115.00

Calculate the dollar value of the premium for an option with face value of US$20,000,000.

$$\text{Premium} = \text{US\$}20{,}000{,}000 \times \frac{1.25}{100} = \text{US\$}250{,}000$$

The premium is paid up-front, so to express the premium in yen it is necessary to convert at the spot rate.

$$\text{Premium} = \text{US\$}250{,}000 \times 116.00 = ¥29{,}000{,}000$$

Alternatively,

$$\text{Premium} = \text{US\$}20{,}000{,}000 \times 1.45 = ¥29{,}000{,}000$$

$$= \text{US\$}\frac{29{,}000{,}000}{116.00} = \text{US\$}250{,}000$$

PROFIT PROFILES: NAKED OPTIONS

An option which is bought or sold without any underlying foreign exchange exposure is called a *naked option*.

Bought call

EXAMPLE 10.2

A person buys a US dollar call/yen put for face value $10,000,000 with a strike price of ¥110.00 and expiry date three months for a premium of 2.00% (flat). If the spot rate is ¥112.00 and the three month dollar interest rate is 5% p.a., calculate in future value terms the profit outcome if, at expiry, the exchange rate is:

(a) 120.00
(b) 110.00
(c) 100.00

$$\text{Premium} = 10{,}000{,}000 \times 0.02$$
$$= \$200{,}000$$
$$\text{FV(Premium)} = 200{,}000(1 + 0.05 \times 3/12)$$
$$= \$202{,}500$$

The buyer of the option pays the premium and has the right to buy $10,000,000 and sell ¥1,100,000,000 at an exchange rate US$1 = ¥110.00.

(a) If at expiry the exchange rate is above the strike price, the dollar call will be in-the-money, so the option owner will exercise the option by selling ¥1,100,000,000 for US$10,000,000. If the exchange rate at expiry is ¥120.00, the option owner could simultaneously buy ¥1,100,000,000 in the market for US$9,166,666.67.

Profit from option	=	Proceeds of sale of ¥1,100,000,000 through option	–	Cost of purchase of ¥1,100,000,000 through market	–	FV(Premium)
	=	10,000,000	–	9,166,667	–	202,500
	=	U$630,833				

(b) If at expiry the exchange rate is at the strike price of ¥110.00, the dollar call will be at-the-money. The option owner will allow the option to lapse because there would be no benefit in buying dollars through the option and selling them through the market at the same rate.

$$\text{Profit from option} = -\text{FV(Premium)}$$
$$= -\text{US\$202,500}$$

(c) If at expiry the exchange rate is less than the strike price, the dollar call will be out-of-the-money so the option owner will allow the option to lapse.

$$\text{Profit from option} = -\text{FV(Premium)}$$
$$= -\text{US\$202,500}$$

The shape of Figure 10.1 represents the typical profit profile of a bought call.

The break-even rate, *b*, is the exchange rate at which the option will produce a zero profit. That is, the profit from exercising the option will be just sufficient to offset the premium.

Proceeds of sale of ¥1,100,000,000 through option	–	Cost of purchase of ¥1,100,000,000 through market	=	FV(Premium)

$$\therefore \frac{1{,}100{,}000{,}000}{110.00} - \frac{1{,}100{,}000{,}000}{b} = 202{,}500$$

$$\therefore 10{,}000{,}000 - \frac{1{,}100{,}000{,}000}{b} = 202{,}500$$

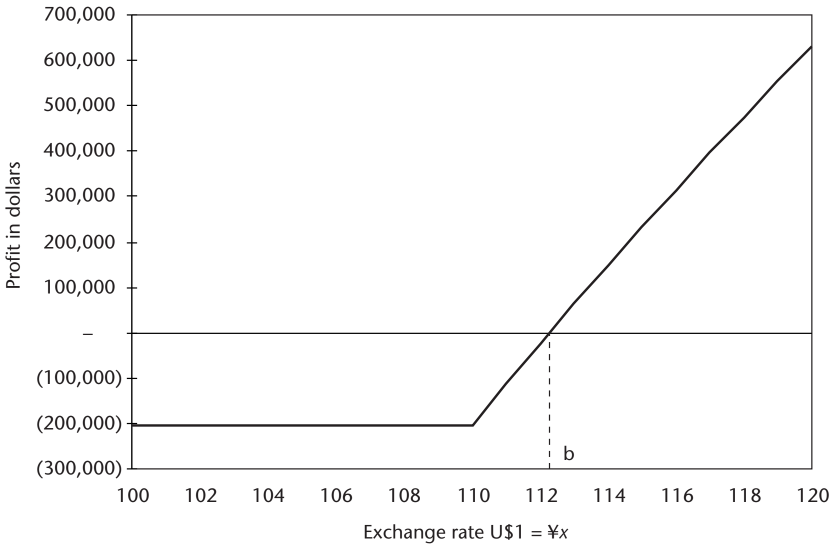


FIGURE 10.1 Profit profile: bought US$ call strike ¥110.00

$$\therefore \frac{-1{,}100{,}000{,}000}{b} = 202{,}500 - 10{,}000{,}000$$

$$\therefore b = \frac{1{,}100{,}000{,}000}{10{,}000{,}000 - 202{,}500}$$

$$= 112.27$$

Bought put

EXAMPLE 10.3

A person buys a pound put/dollar call for a face value of £10,000,000 with a strike price of US$1.5000 and expiry date three months for a premium of 3.00% (flat). If the three month pound interest rate is 5% p.a., calculate in future value terms the profit outcome if at expiry the exchange rate is:

(a) 1.4000
(b) 1.5000
(c) 1.6000

$$\text{Premium} = 10{,}000{,}000 \times 0.03$$

$$= £300{,}000$$

$$\begin{aligned}\text{FV(Premium)} &= 300{,}000(1 + 0.05 \times 3/12) \\ &= \$303{,}750\end{aligned}$$

The buyer of the option pays the premium and has the right to sell £10,000,000 and buy US$15,000,000 at an exchange rate £1 = US$1.5000.

(a) If at expiry the exchange rate is below the strike price, the pound put will be in-the-money, so the option owner will exercise the option, thereby buying US$15,000,000 for £10,000,000. If the exchange rate at expiry is 1.4000, the option owner could simultaneously sell US$15,000,000 in the market for £10,714,285.71.

Profit from option	=	Proceeds of sale of US$15,000,000 through market	–	Cost of purchase of US$15,000,000 through option	– FV(Premium)
	=	10,714,285.71	–	10,000,000	–303,750
	=	£410,535.71			

(b) If at expiry the exchange rate is 1.5000, the pound put will be at-the-money. The option owner will allow the option to lapse because there would be no benefit in selling pounds through the option and buying them through the market at the same rate.

$$\begin{aligned}\text{Profit from option} &= -\text{FV(Premium)} \\ &= -\pounds 303{,}750\end{aligned}$$

(c) If at expiry the exchange rate is 1.6000, the pound put will be out-of-the-money, so the option owner will allow it to lapse.

$$\begin{aligned}\text{Profit from option} &= -\text{FV(Premium)} \\ &= -\pounds 303{,}750\end{aligned}$$

The profit profile of a bought pound put is shown in Figure 10.2. This shape represents the typical profit profile of a bought put.

The break-even rate, b, is the exchange rate at which the option will produce a zero profit.

Proceeds of sale of US$15,000,000 through market	–	Cost of purchase of US$15,000,000 through option	= FV(Premium)

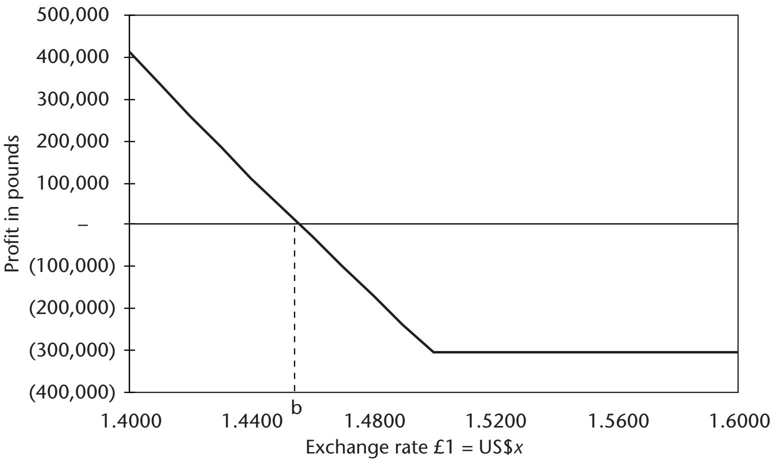


FIGURE 10.2 Profit profile: bought pound put strike US$1.5000

$$\therefore \frac{15{,}000{,}000}{b} - \frac{15{,}000{,}000}{1.5000} = 303{,}750$$

$$\therefore b = \frac{15{,}000{,}000}{10{,}000{,}000 + 303{,}750}$$

$$= 1.4558$$

EXAMPLE 10.4

A person writes a US dollar call/yen put for a face value of US$10,000,000 with a strike price of 110.00 and expiry date of three months for a premium of 2.00% (flat). If the three month dollar interest rate is 5% p.a., calculate in future value terms the profit outcome if at expiry the exchange rate is:

(a) 120.00
(b) 110.00
(c) 100.00

As in Example 10.1, FV(Premium) = US$202,500. The writer of the option receives the premium.

(a) If at expiry the exchange rate is above the strike price, the dollar call will be in-the-money, so the buyer of the option will exercise the option. If the dollar call is exercised, the writer is obliged to sell US$10,000,000 for ¥1,100,000,000. To square the net exchange position, the writer will have to sell ¥1,100,000,000 in the market. If at expiry the market rate is ¥120.00, the writer will buy ¥1,100,000,000 through the option and sell ¥1,100,000,000 for US$9,166,666.67.

Profit from option	=	Proceeds of sale of ¥1,100,000,000 through market	–	Cost of purchase of ¥1,100,000,000 through option	+	FV(Premium)
	=	9,166,666.67	–	10,000,000	+	202,500
	=	–US$630,833.33				

(b) If at expiry the exchange rate is at the strike price of ¥110.00, the owner of the dollar call will allow it to lapse so the writer merely keeps the premium.

Profit from option = FV(Premium)

= +US$202,500

(c) If at expiry the exchange rate is less than the strike price, the owner of the dollar call will allow it to lapse.

Profit from option = FV(Premium)

= +US$202,500

The shape in Figure 10.3 represents the typical profit profile of a sold call.

The break-even rate, *b*, is the exchange rate at which the option will produce zero profit. That is, the future value of the premium received will be just sufficient to offset the loss from writing the option.

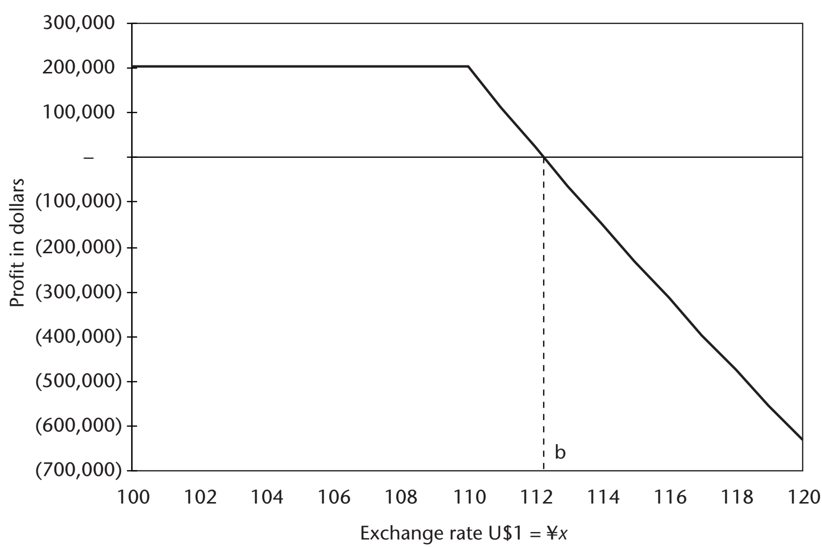


FIGURE 10.3 Profit profile: sold dollar call strike ¥110.00

Cost of purchase of ¥1,100,000,000 through option	–	Proceeds of sale of ¥1,100,000,000 through market	= FV(Premium)

$$\therefore \frac{1{,}100{,}000{,}000}{110.00} - \frac{1{,}100{,}000{,}000}{b} = 202{,}500$$

$$\therefore b = \frac{1{,}100{,}000{,}000}{10{,}000{,}000 - 202{,}500}$$

$$= 112.27$$

Sold put

EXAMPLE 10.5

A person writes (sells) a pound put/US dollar call for face value of £10,000,000 with a strike price of 1.5000 and expiry date three months for a premium of 3.00% (flat). If the three month pound interest rate is 5% p.a., calculate in future value terms the profit outcome if at expiry date the exchange rate is:

(a) 1.4000
(b) 1.5000
(c) 1.6000

As in Example 10.2, *FV*(Premium) = £303,750: the writer receives the premium.

(a) If at expiry the exchange rate is below the strike price, the pound put will be in-the-money, so the buyer of the option will exercise the option. If the option is exercised, the writer is obliged to buy £10,000,000 for US$15,000,000. To square the net exchange position, the writer will have to buy US$15,000,000 in the market. If at expiry the market rate is 1.4000, the writer will sell US$15,000,000 through the option and buy US$15,000,000 for £10,714,285.71.

Profit from option	=	Proceeds of sale of US$15,000,000 through option	–	Cost of purchase of US$15,000,000 through market	+ FV(Premium)

$$= \frac{15{,}000{,}000}{1.5000} - \frac{15{,}000{,}000}{1.4000} + 303{,}750$$

$$= -£410{,}535.71$$

(b) If at expiry the exchange rate is at the strike price of 1.5000, the owner of the pound put will allow it to lapse, so the writer merely keeps the premium.

$$\begin{aligned}\text{Profit from option} &= \text{FV(Premium)}\\ &= +\pounds 313{,}750\end{aligned}$$

(c) If at expiry the exchange rate is above the strike price, the pound put will lapse.

$$\begin{aligned}\text{Profit from option} &= \text{FV(Premium)}\\ &= +\pounds 313{,}750\end{aligned}$$

The shape in Figure 10.4 represents the typical profit profile of a sold put.

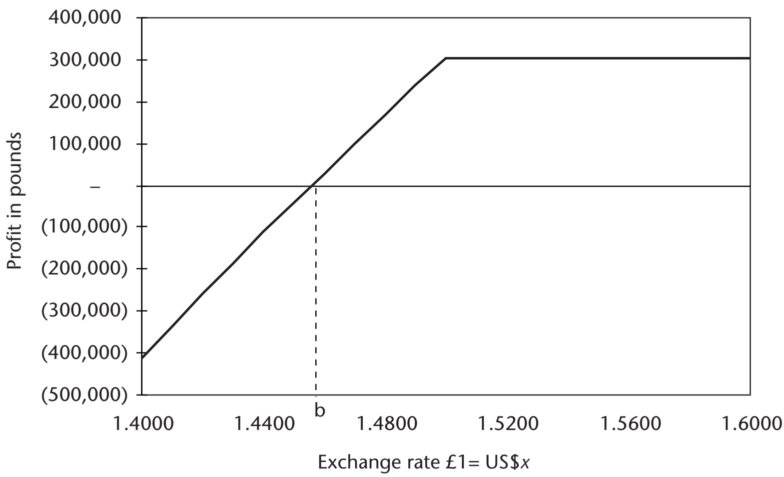


FIGURE 10.4 Profit profile: sold pound put strike US$1.5000

The break-even rate, b, is such that the future value of the premium received will be just sufficient to offset the loss from writing the option.

Cost of purchase of US$15,000,000	–	Proceeds of sale of US$15,000,000	= FV(Premium)

$$\therefore \frac{15{,}000{,}000}{b} - \frac{15{,}000{,}000}{1.5000} = 303{,}750$$

$$\therefore b = \frac{15{,}000{,}000}{10{,}000{,}000 + 303{,}750}$$
$$= 1.4558$$

OPTION PRICING

The option premium is the present value of the expected pay-out of the option.

Expected value

The *expected value* of a variable X is the probability weighted average of the possible values that X can take:

$$E[X] = \frac{\Sigma_{i=1}^{n} p_i x_i}{\Sigma_{i=1}^{n} p_i}$$

As $\Sigma_{i=1}^{n} p_i = 1$, the above expression simplifies to

$$E[X] = \sum_{i=1}^{n} p_i x_i \qquad (10.1)$$

EXAMPLE 10.6

Calculate the premium for a 6 month US dollar call/yen put with strike price ¥100.00 and face value US$1,000,000.

The pay-out of the option will be a function of where the spot rate is at expiry relative to the strike price. If, for example, the spot rate at expiry were ¥106.00, then the pay-out would be ¥6,000,000.

Pay-out = 1,000,000 × (106.00 – 100.00) = ¥6,000,000

The expected pay-out will depend on the probability of the spot rate at expiry being at various levels. A binomial tree can be used to show the set of possible outcomes. Assuming the spot rate at the start of the option period is ¥100.00 and that each month there is equal probability that the rate could move up or down, say 3 yen, then the possible paths which the rate could follow during the 6 month option period are as set out in Exhibit 10.3 (opposite).

At the end of the first month, the price will be either 100 + 3 = 103 or 100 – 3 = 97.

At the end of the second month, the price could be one of:

$$103 + 3 = 106 \quad \text{Probability} = 1/2 \times 1/2 = 1/4$$
$$103 - 3 = 100 \quad \text{Probability} = 1/2 \times 1/2 = 1/4$$
$$97 + 3 = 100 \quad \text{Probability} = 1/2 \times 1/2 = 1/4$$
$$97 - 3 = 94 \quad \text{Probability} = 1/2 \times 1/2 = 1/4$$

It is twice as probable that at the end of the second month the price will be 100 than either 94 or 106.

EXHIBIT 10.3 Binomial tree

End of month	0	1	2	3	4	5	6
							118
						115	
					112		112
				109		109	
			106		106		106
		103		103		103	
	100		100		100		100
		97		97		97	
			94		94		94
				91		91	
					88		88
						85	
							82

Possible outcomes	*No. of paths*	*Probability*
106	1	1/4
100	2	2/4 = 1/2
94	1	1/4
Total	4	4/4 = 1

and so on...

By the end of the sixth month, there will be seven possible outcomes, as shown in Exhibit 10.3, with probabilities reflecting the number of paths which get to those outcomes. There are 64 possible paths which the price could follow. (Note: $2^6 = 64$). Twenty of them have a final value of 100. Only one of the 64 paths ends at 118 (the rate has to go up every month for 6 months in a row). Similarly, only one path ends at 82 (the rate has to go down every months for 6 months in a row).

Possible outcomes	*No. of paths*	*Probability*
118	1	1/64
112	6	6/64
106	15	15/64
100	20	20/64
94	15	15/64
88	6	6/64
82	1	1/64
Total	64	64/64 = 1

COMBINATIONS AND PROBABILITIES

The number of ways of getting x results from n trials $= C_x^n = \dfrac{n!}{x!(n-x)!}$

where 'factorial n', $n! = 1 \times 2 \times 3 \times ... \times (n - 1) \times n$.

The number of paths with 0 up months out of 6

$$= C_0^6 = \frac{6\times 5\times 4\times 3\times 2\times 1}{0!\times 6\times 5\times 4\times 3\times 2\times 1} = 1$$

(Note that 0! = 1.)

The number of paths with 1 up month out of 6

$$= C_1^6 = \frac{6\times 5\times 4\times 3\times 2\times 1}{1\times 5\times 4\times 3\times 2\times 1} = 6$$

The number of paths with 2 up months out of 6

$$= C_2^6 = \frac{6\times 5\times 4\times 3\times 2\times 1}{2\times 1\times 4\times 3\times 2\times 1} = 15$$

The number of paths with 3 up months out of 6

$$= C_0^6 = \frac{6\times 5\times 4\times 3\times 2\times 1}{3\times 2\times 1\times 3\times 2\times 1} = 20$$

and so on.

If the probability of an up month = ½, then the probability of a down month = 1 – ½ = ½.

The probability of x up months from n

$$= \frac{\text{Number of paths with } x \text{ up months from } n}{\text{Total number of paths}}$$

$$= \frac{C_x^n}{2^n}$$

If $x = 2$ and $n = 6$,

Probability of 2 up months from 6 $= \dfrac{15}{64}$

Given that the strike price is 100, the pay-out for each of the possible outcomes would be:

Possible outcomes	*Pay-out in yen*
118	18,000,000
112	12,000,000
106	6,000,000
100	0
94	0
88	0
82	0

If the price at expiry is 100 or less, the owner of the option will choose not to exercise, so the pay-out will be zero.

The expected pay-out is calculated by multiplying the pay-out associated with each of the possible outcomes by the probability of that outcome.

Possible outcomes	*Pay-out in yen*	*Probability*	*Expected pay-out*
118	18,000,000	1/64	281,250
112	12,000,000	6/64	1,175,000
106	6,000,000	15/64	1,486,250
100	0	20/64	0
94	0	15/64	0
88	0	6/64	0
82	0	1/64	0
Total		1	**2,812,500**

The expected pay-out of the option is ¥2,812,500 = US$28,125.00 (at spot US$1 = ¥100.00).

If the option premium were paid at expiry, this would be its fair value. In practice, the option premium is generally paid up front. The fair value option premium is the present value of the expected pay-out at expiry. If the 6 month dollar interest rate is 5% p.a., the up-front premium would be:

$$\frac{\$28{,}125.00}{1+0.05\times 6/12}=\$27{,}439.02$$

PROBABILITY DISTRIBUTION

The distribution which shows the probability of each of the possible outcomes is known as the *probability distribution* or *probability density function* (Figure 10.5).

If the option has a different strike price, it will command a different premium. The binomial tree and the probabilities will not change but the expected pay-out will.

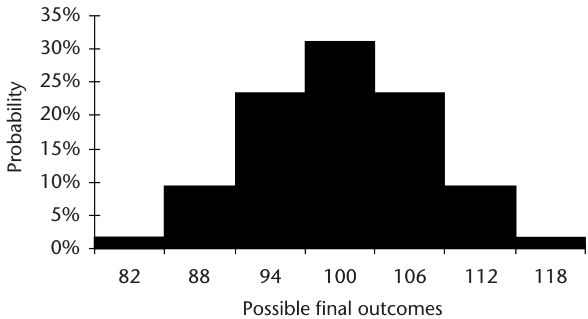


FIGURE 10.5 Probability density function

EXAMPLE 10.7

If the strike price was set at 105.00, then the expected pay-out would fall to:

Possible outcomes	*Pay-out in yen*	*Probability*	*Expected pay-out*
118	13,000,000	1/64	203,125
112	7,000,000	6/64	656,250
106	1,000,000	15/64	234,375
100	0	20/64	0
94	0	15/64	0
88	0	6/64	0
82	0	1/64	0
Total		1	**1,093,750**

The expected pay-out would now be ¥1,093,750 = $10,937.50.

The up-front premium would be

$$\frac{\$10{,}937.50}{(1+0.05\times 6/12)}=\$10{,}670.73$$

If the up-down movement changes, the option will have a different premium.

EXAMPLE 10.8

If the up–down movement were ¥2 per month, then the binomial tree would be as in Exhibit 10.4.

EXHIBIT 10.4 Binomial tree with up–down movement = ¥2

End of Month	0	1	2	3	4	5	6
							112
						110	
					108		108
				106		106	
			104		104		104
		102		102		102	
	100		100		100		100
		98		98		98	
			96		96		96
				94		94	
					92		92
						90	
							88

The expected pay-out for a dollar call with strike price at 100 would be:

Possible outcomes	*Pay-out in yen*	*Probability*	*Expected pay-out*
112	12,000,000	1/64	187,500
108	8,000,000	6/64	750,000
104	4,000,000	15/64	937,500
100	0	20/64	0
96	0	15/64	0
92	0	6/64	0
88	0	1/64	0
Total		1	**1,875,000**

The expected pay-out would now be ¥1,875,000 = $18,750.00

The up-front premium would be $18,292.68

A smaller up–down movement means less volatility. The spread of possible outcomes is less when the up–down movement is ¥2 than when it was ¥3. It can be shown that a monthly up–down movement of ¥3 represents volatility of 10.4% p.a. and a monthly up–down movement of ¥2 represents volatility of 6.9% p.a.[1]

In general, the factors determining the price of a currency option include:

1. The strike price relative to the market rate
2. The time to expiry
3. The expected volatility
4. Interest rates

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1 Refer to Example 10.10 to see how volatility is calculated.

RELATIONSHIP BETWEEN THE STRIKE PRICE AND MARKET RATE

The relationship between the strike price and the market rate will determine whether the option is at-the-money, in-the-money or out-of-the-money, in accordance with Exhibit 10.1. An option which is in-the-money has value by virtue of the fact that it is in-the-money. This is referred to as *intrinsic value*. The further an option is in-the-money, the greater will be its intrinsic value. By definition, an out-of-the-money option has no intrinsic value; see Figure 10.6.

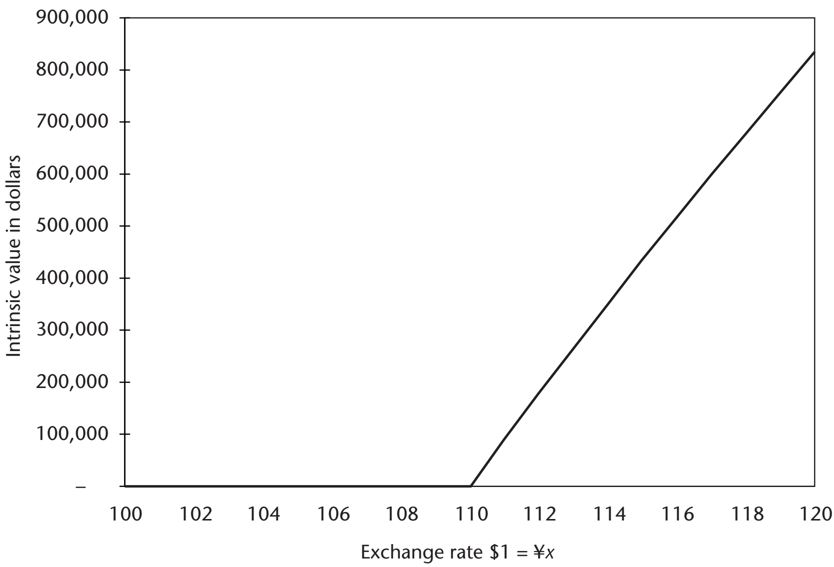


FIGURE 10.6 Intrinsic value: US dollar call strike ¥110.00

During the life of an option, its intrinsic value will vary as the market rate takes the strike price farther into-the-money or out-of-the-money. For example, a dollar call with a strike price of ¥110.00 will have more intrinsic value when the market exchange rate is 115.00 than when it is 114.00. If the face value of the option is $10,000,000, its intrinsic value in yen terms will be:

at 115.00, intrinsic value = 10,000,000 (115.00 – 110.00) = ¥40,000,000

at 114.00, intrinsic value = 10,000,000 (114.00 – 110.00) = ¥30,000,000

If the market exchange rate falls below the strike price of 110.00, its intrinsic value will fall to zero; see Figure 10.7.

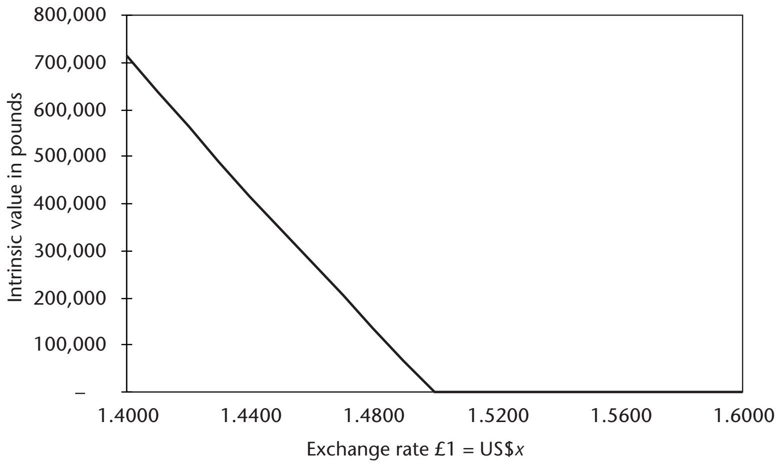


FIGURE 10.7 Intrinsic value: pound put strike US$1.5000

TIME TO EXPIRY

An option that has not expired has *time value*. An unexpired option has value because it might move (further) into-the-money before expiry. The closer the option is to expiry, the lower its time value; see Figure 10.8.

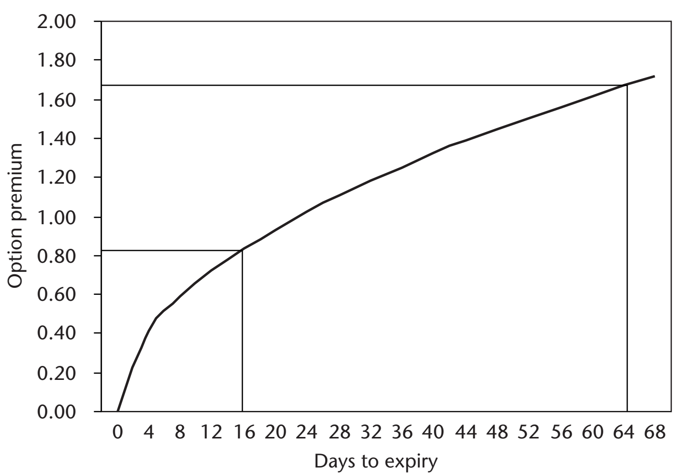


FIGURE 10.8 Time decay of an at-the-money option

The time value of an option declines as it approaches expiry. The premium of an option with a strike price that is close to at-the-money declines at an accelerating rate as it approaches expiration. At expiry, time value equals zero. The decline in the time value of an option as it approaches expiry is known as *time decay*. The time value varies with the square root of time till expiry. For example, the time value of an at-the-money option with 16 days till expiry will be half that of the corresponding option with 64 days till expiry. *Note*: $\sqrt{16} = \frac{1}{2}\sqrt{64}$.

The time value of an option will vary as the exchange rate changes. An option which is well out-of-the-money and close to expiry cannot command a high premium because it is unlikely that the option will be in-the-money before expiration.

The time value of an out-of-the-money option declines as the exchange rate takes it further out-of-the-money. For example, a dollar call with strike price at ¥110.00 is further out-of-the-money at ¥105.00 than at ¥109.00. If the at-the-money premium is 2.00% (when the spot rate is ¥112.00), the premium might be 1.90% at ¥109.00 and 0.67% at ¥105.00.

In-the-money options also have time value. An option which is in-the-money has time value because it might move further in-the-money before expiry.

Time value is always positive until the expiration date. It is maximized at the strike price, where the uncertainty as to whether it will be in or out-of-the-money is greatest (Figure 10.9).

The overall option premium is the sum of its intrinsic value and time value (Figures 10.10 and 10.11).

$$\boxed{\text{Option premium} = \text{Intrinsic value} + \text{Time value}} \tag{10.2}$$

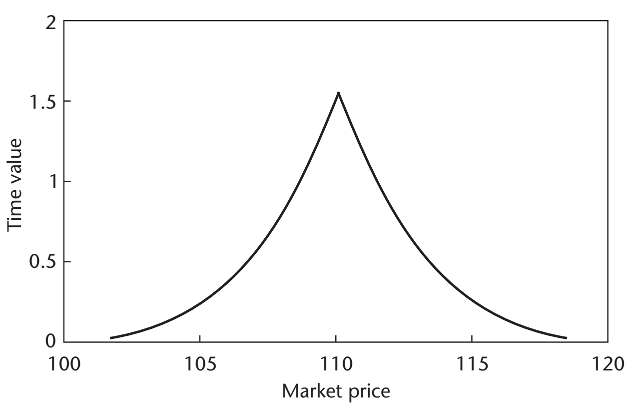


FIGURE 10.9 Time value of an option

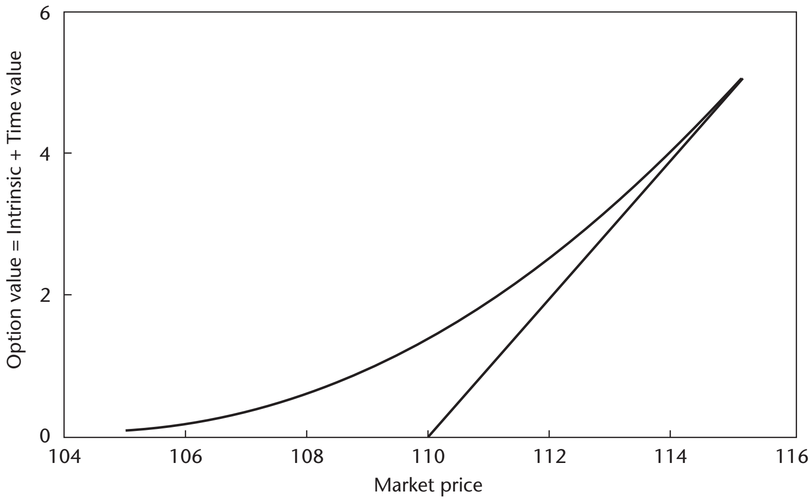


FIGURE 10.10 Premium = Intrinsic value + Time value: Call

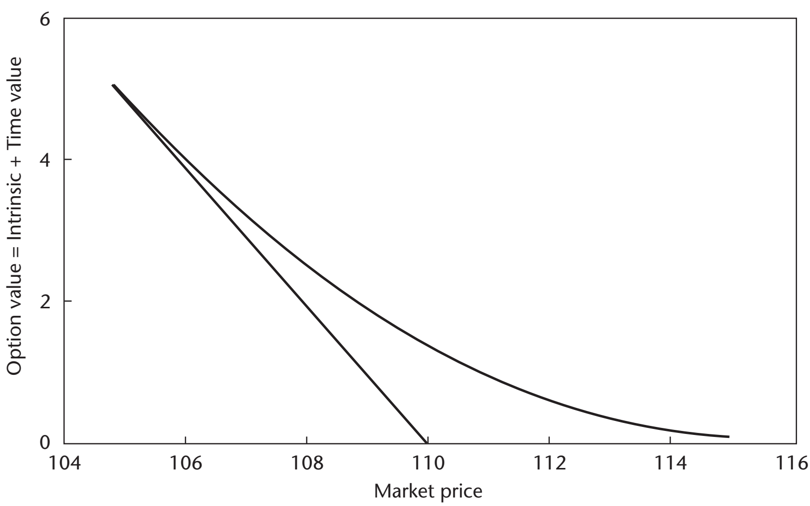


FIGURE 10.11 Premium = Intrinsic value + Time value: Put

VOLATILITY

In the binomial model the premium was greater when the up–down movement was larger. In general, the greater the expected volatility of the exchange rate, the higher the option premium will be. The more volatile the exchange rate, the higher the probability that the option will move farther into-the-money before expiry, and therefore the greater the risk on the option writer and, accordingly, the higher the option premium.

Expected volatility is set by the market and is generally a function of recent historical volatility and market expectations.

Statistically, the *variance* of a discrete random variable with mean $\bar{x}$ is defined to be

$$V[X] = E[(X-\bar{x})^2] = \sigma^2 = \sum_{i=1}^{n} p_i (X-\bar{x})^2 \qquad (10.3)$$

Variance is a measure of the 'spread' of X about its mean. In financial terms variance measures the degree of uncertainty or risk.

The square root of the variance is called the *standard deviation* of X. The lower case Greek letter sigma, σ, is used to represent the standard deviation.

$$\sigma = \sqrt{\sum_{i=1}^{n} p_i (X_i - \bar{x})^2} \qquad (10.4)$$

Volatility refers to the extent to which a price moves over time. A standard deviation provides a measure of the extent of price movements. Volatility is usually expressed in terms of percent per annum.

Historic volatility measures the spread of price changes that have occurred over some specified time in the past. Suppose the following prices occurred at close of business each day for the past 6 months:

$$s_1, s_2, s_3, \ldots, s_j, \ldots, s_n$$

For example, if there were 126 business days in the observation period, $n = 126$.

The daily price changes were $(s_2 - s_1)$ on the first day, $(s_3 - s_2)$ on the second day and so on up to $(s_n - s_{n-1})$ on the last day. On the jth day, the change in price was $(s_j - s_{j-1})$.

$$\text{The discrete proportional price change on the } j\text{th day} = \frac{s_j - s_{j-1}}{s_{j-1}}$$

$$\text{The discrete percentage price change on the } j\text{th day} = \frac{s_j - s_{j-1}}{s_{j-1}} \times 100$$

$$\text{The relative price} = \frac{s_j}{s_{j-1}}$$

The discrete proportional price change on the jth day can be written as $\dfrac{s_j}{s_{j-1}} - 1$

$$\ln\left(\frac{S_j}{S_{j-1}}\right) \text{ is the continuous rate of change of price on the } j\text{th day}$$

For example, if $S_1 = 10$ and $S_2 = 11$, the discrete rate of change = (11 – 10)/10 = 10.0% and the continuous rate of change = ln(11/10) = 0.09531 = 9.531%. (Note that 10.0% p.a. simple interest = 9.531% p.a. continuously compounding.)

If $(n + 1)$ is the number of observations, n = number of periods, where: S_j = price at the end of the jth period.

$$\text{Let } u_j = \ln\left(\frac{S_j}{S_{j-1}}\right) \text{ for } j = 1, 2, \ldots, n.$$

Raising both sides to the power of e:

$$e^{u_j} = \frac{S_j}{S_{j-1}}$$

So u_j is the continuously compounded *rate of growth* in the jth period.

The usual estimate of the standard deviation of the u_js is given by:

$$\sigma = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(u_j - \bar{u})^2}$$

where $\bar{u}$ = the mean of the u_js.

If the values of the variable over different tenors are independent, identically distributed random variables, then the volatility for a tenor, t, can be calculated from the annual volatility:

$$\sigma_t = \sigma_A \sqrt{t} \tag{10.5}$$

where:

σ_t = volatility for tenor
σ_A = annual volatility
t = time in years

If (annual) volatility is 10% p.a., volatility (for tenor) over 3 months (t = ¼) is:

$$\sigma_t = 10\sqrt{1/4} = 10\% \times 1/2 = 5\%$$

and over 2 years ($t = 2$):

$$\sigma_t = 10\sqrt{2} = 14.1\%$$

Derivation of Equation (10.5)

If daily price changes are independent:

$$\sigma_t^2 \ln \frac{s_t}{s_0} = \sigma_1^2 \ln \frac{s_1}{s_0} + \sigma_2^2 \ln \frac{s_2}{s_1} + \sigma_3^2 \ln \frac{s_3}{s_2} + \ldots + \sigma_n^2 \ln \frac{s_t}{s_{t-1}}$$

If the daily prices changes all have the same variance, σ_A^2 :

$$\sigma_t^2 = \sigma_A^2 \times t$$

EXAMPLE 10.9

Calculate historic volatility for the following exchange rates that occurred over a 10 day period.

Day	*Price* (S_j)	S_j/S_{j-1}	$u_j = \ln(S_j/S_{j-1})$	$(u_j - \bar{u})^2$
0	100.0			
1	101.5	1.01500	0.01489	0.00020139
2	102.1	1.00591	0.00589	0.00002700
3	100.6	0.98531	–0.01480	0.00024019
4	100.2	0.99602	–0.00398	0.00002192
5	101.3	1.01098	0.01092	0.00010446
6	101.7	1.00395	0.00394	0.00001502
7	100.9	0.99213	–0.00790	0.00007387
8	100.0	0.99108	–0.00896	0.00009326
9	101.6	1.01600	0.01587	0.00023030
10	100.7	0.99114	–0.00890	0.00009207
			0.00698	0.00109499

$$\text{Daily volatility} \quad \sigma = \frac{1}{n}\sqrt{\sum_{j=1}^{n}(u_j - \bar{u})^2} = \frac{\sqrt{0.00109499}}{10}$$

$$= 0.01046416 = 1.046\%$$

Given that there are 252 business days per year, t = 1/252.

The volatility percentage per annum is:

$$\sigma = 0.01046416 \times \sqrt{252} = 16.6\% \text{ p.a.}$$

The higher the volatility, the larger is the standard deviation. If the exchange rate is more volatile, there is a higher probability that it will move from the current rate of 110.00 to, say, 115.00 than if the exchange rate is less volatile. Accordingly, the time value is greater the higher the volatility; see Figure 10.12.

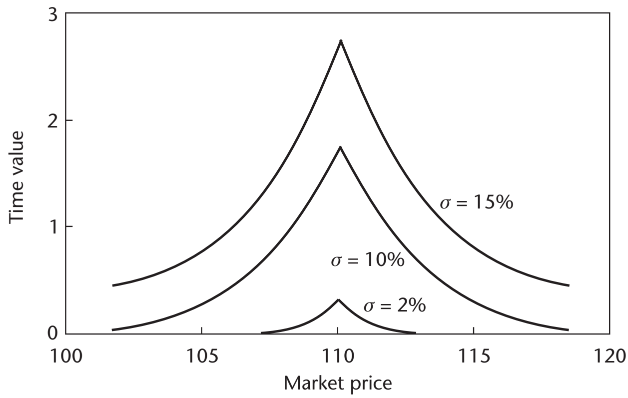


FIGURE 10.12 Time value is a function of volatility

EXAMPLE 10.10

Calculate the (annual) volatility for the distribution in Example 10.6.

Outcome	$(x - \bar{x})$	$(x - \bar{x})^2$	*Probability(p)*	$p(x - \bar{x})^2$
118	18	324	1/64	5.0625
112	12	144	6/64	13.500
106	6	36	15/64	8.4375
100	0	0	20/64	0
94	–6	36	15/64	8.4375
88	–12	144	6/64	13.500
82	–18	324	1/64	5.0625
		Total	64/64	54.000

$$\therefore \sigma_t = \sqrt{54} = 7.348469$$

$$\text{Expressed as a percentage of the spot rate} = \frac{7.348469}{100} = 7.348469\%$$

$$\text{Annualized: } \sigma_A = \frac{\sigma_t}{\sqrt{6/12}} = \frac{7.348469}{0.707107} = 10.4\% \text{ p.a.}$$

Expectations are subjective. Consequently, expected volatility is likely to vary from one market participant to another. In fact, volatility is such an important determinant of options pricing that in practice option dealers typically quote bid and offer volatilities when dealing with one another.

RISK-FREE INTEREST RATE

The price of an option also depends on the risk-free interest rate. As the option premium is paid up-front, but the currency is not delivered until the value date for which the option is exercised, the option writer has the use of the premium. The writer will adjust the premium downwards to reflect its time value. The interest rate used to discount the premium to adjust for its time value is known as the 'risk-free' interest rate, so-called because it is assumed that there is no credit risk involved in discounting the premium. The relative importance of the risk-free interest rate tends to increase the longer the term of the option.

PUT–CALL PARITY

Calls and puts are closely related. If a person bought a dollar call/yen put and sold a dollar put/yen call with the same strike price for the same amount and with the same expiry, he would have the same profit outcome at expiry as if he had bought dollars forward against yen.

EXAMPLE 10.11

A trader bought a 6 month US dollar call/yen put with strike price US$1 = ¥110.00 and sold a 6 month US dollar put/yen call also with a strike price of US$1 = ¥110.00. The pay-out at the end of the 6 month period will be the same as if he had purchased dollars forward against yen at an outright rate of ¥110.00, as set out below:

Spot rate at expiry	*Pay-out from bought $ call*	*Pay-out from sold $ put*	*Combined pay-out*
105.00	0	–5.00	–5.00
106.00	0	–4.00	–4.00
107.00	0	–3.00	–3.00
108.00	0	–2.00	–2.00
109.00	0	–1.00	–1.00
110.00	0	0	0
111.00	+1.00	0	+1.00
112.00	+2.00	0	+2.00
113.00	+3.00	0	+3.00
114.00	+4.00	0	+4.00
115.00	+5.00	0	+5.00

The relationship that a bought call and a sold put equals a long forward is known as *put–call parity*.

$$\boxed{\text{Bought call + Sold put = Bought forward}} \qquad (10.6)^2$$

It follows that a sold call and a bought put produce the same outcome as a short forward position.

PUT–CALL ARBITRAGE

Put–call parity means that there is a unique relationship between the call premium and the put premium for a given strike. If the call premium and put premium are not in accordance with put–call parity, then an arbitrage opportunity will exist.

EXAMPLE 10.12

Spot	$1	=	¥110.00
6 month forward	$1	=	¥109.00

Premium of 6 month options with strike = ¥108.00:

Dollar call	¥2.00
Dollar put	¥1.10

By buying a 108 dollar call and selling a 108 dollar put, it is possible to engineer a long dollar position against the yen at an effective exchange rate $1 = ¥108.00. This position could be covered by selling dollars 6 months forward at the forward rate $1 = ¥109.00, for a gain of ¥1.00. The net premium paid would be ¥ 0.90 (i.e. ¥ 2.00 to buy the dollar call less ¥1.10 from selling the dollar put). This represents the cost of engineering the long dollar position at 108.00. The 10 points difference between the ¥1.00 gain and the ¥ 0.90 net premium constitutes profit from the arbitrage.

Steps to arbitrage put–call disparity:

		Premium
Buy 108 dollar call		–¥2.00
Sell 108 dollar put		+¥1.10
Long dollars forward at	108.00	–¥0.90
Sell dollars forward at	109.00	
Gain from closing	¥1.00	
Net premium	–¥0.90	
Profit	¥0.10	

2 Applies to European options only.

To be more precise, the future value of the ¥0.90 net premium should be compared to ¥1.00 settlement which will occur at the end of the 6 month period. However, if the 6 month yen interest rate is as low as, say, 0.5% p.a., the difference is trivial.

Put–call parity can be restated as:

$$\boxed{\begin{aligned}&\text{Forward rate} - \text{strike rate}\\&\quad = FV(\text{ call premium} - \text{put premium})\end{aligned}} \qquad (10.7)$$

People would exploit this arbitrage opportunity until put–call parity is resumed. In practice, the arbitrage process will ensure that the put–call parity relationship applies.

For an option pricing model to be arbitrage-free, the at-the-money strike price must equal the forward rate. It follows from Equation 10.5 that the call premium will equal the put premium for at-the-money options.

The binomial model can be made arbitrage-free by centring the possible outcomes at expiry around the forward rate by assuming that the spot rate *drifts* from its current level towards the forward rate during the life of the option. This means that the up movement will have a different value from the down movement by the amount of drift.

Revisiting the binomial model to price a 6 month dollar call/yen put when the spot rate is US$1 = ¥100.00 and the 6 month forward rate is US$1 = ¥ 99.00, the binomial tree would be as in Exhibit 10.5, where the up–down movement is ¥3 +/– the allowance for drift towards the forward rate.

$$\text{Monthly drift} = \frac{\text{forward rate} - \text{spot rate}}{\text{number of months}} = \frac{99.00 - 100.00}{6} = -¥0.17$$

The up outcome for month 1 = 100.00 + 3.00 – 0.17 = ¥102.83

The down outcome for month 1 = 100.00 – 3.00 – 0.17 = ¥96.83

In general,

Up outcome in month x
= outcome in month $(x - 1)$ + up factor + drift

Down outcome in month x
= outcome in month $(x - 1)$ – down factor + drift

EXHIBIT 10.5 Binomial tree centred around forward rate \$1 = ¥99.00

End of Month	0	1	2	3	4	5	6
							117.00
						114.17	
					111.33		111.00
				108.50		108.17	
			105.67		105.33		105.00
		102.83		102.50		102.17	
	100.00		99.67		99.33		**99.00**
		96.83		96.50		96.17	
			93.67		93.33		93.00
				90.50		90.17	
					87.33		87.00
						84.17	
							81.00

The description of intrinsic value can be qualified to be the extent to which the *forward* rate is better than the strike price.

REVERSE BINOMIAL METHOD

The premium of the US dollar call in Example 10.6 could be priced using the reverse binomial method. This involves starting with the pay-out for each of the possible outcomes at expiry and working backwards through the binomial tree to determine the expected value at inception.

EXHIBIT 10.6 Reverse binomial method

Month	0	1	2	3	4	5	6	Outcome
							180,000	118
						150,000		
					120,000		120,000	112
				90,000		90,000		
			63,750		60,000		60,000	106
		43,125		37,500		30,000		
	28,125		22,500		15,000		0	100
		13,125		7,500		0		
			3,750		0		0	94
				0		0		
					0		0	88
						0		
							0	82

If at the end of the 6 months the spot rate is ¥118, the pay-out would be ¥180,000; if it is ¥112, the pay-out would be ¥120,000. It follows that if there is equal probability that the spot rate will move up or down ¥3 in the final month, if the spot rate is ¥115 at the end of the 5th month then the value of the option at that time must be (180,000 + 120,000)/2 = ¥150,000.

Similarly, because the pay-out at the end of the 6 months would be ¥120,000 at ¥112 and ¥60,000 at ¥106, it follows that the value of the option at the end of the 5th month must be ¥90,000 if the spot rate is ¥108.

By working backwards through the binomial tree it can be seen that the future value of the premium is ¥2,812,500 = US$28,125.

The reverse binomial method is useful because it shows the at-expiry value of the option at each stage of the tree. For example, if at the end of the 2nd month the spot exchange rate is ¥100, the then (future) value of the option premium would be ¥2,250,000.

AMERICAN VERSUS EUROPEAN OPTIONS

An American-style call will command a higher premium than the equivalent European-style call, provided the call currency is the high interest currency. For example, if dollar interest rates are higher than yen interest rates, the premium on an American-style dollar call/yen put would be higher than on a European-style dollar call/yen put with the same strike price and expiry date. This occurs because the holder of an American-style dollar call can exercise for spot value prior to the expiry date, thereby receiving the high-interest currency and paying the low-interest currency early. The writer raises the premium on the American option to provide for this possibility.

The premium on an American-style option is never less than the equivalent European-style option. If the call currency is the lower interest currency it will command the same premium because the holder of the American option will tend not to exercise early for spot value because of the need to fund the higher interest currency until the expiry date.

GEOMETRIC BINOMIAL MODEL

One limitation of the binomial model as used above is that if volatility is large enough it can generate outcomes with negative values. In reality exchange rates cannot be negative. A geometric binomial model overcomes this problem by generating a binomial tree using up and down movements that are constant percentage changes rather than absolute amounts. Geometric binomial models generate outcomes that are bounded by zero on the down-side, but can increase indefinitely on the up-side. The distribution of outcomes is no longer symmetrical. However, the distribution of relative changes in exchange rates is symmetrical.

EXAMPLE 10.13

Consider the following simple geometric binomial tree. The initial market price S is US1 = ¥100. The up or down factor is 20% per period.

	Period 1	Period 2
		144
	120	
100		96
	80	
		64

In the first period the spot rate could either rise by 20% from 100 to 120 or fall by 20% from 100 to 80. In the second period the spot rate could:

rise by 20% from 120	to $120 \times 1.2 = 144$
fall by 20% from 120	to $120 \times 0.8 = 96$
rise by 20% from 80	to $80 \times 1.2 = 96$
fall by 20% from 80	to $80 \times 0.8 = 64$

The forward exchange rate could be written as $f = Se^{(r-y)t}$, where:

S = spot rate
r = terms currency interest rate expressed as a continuously compounding rate
y = commodity currency interest rate expressed as a continuously compounding rate
t = time in years

If p = probability of an up-move, then $1 - p$ = probability of a down-move. In the first period the price can either move up from S to S_u or down from S to S_d. If u is the ratio of the up-price to price in the previous period, and if d is the ratio of the down-price to the price in the previous period, then, after one period the spot rate will have either gone up or down to the forward exchange rate:

$$S[pu + (1-p)d] = Se^{(r-y)t}$$

$$pu + d - pd = e^{(r-y)t}$$

$$\therefore p = \frac{e^{(r-y)t} - d}{u - d} \tag{10.8}$$

In the above example:

$$u = 120/100 = 144/120 = 96/80 = 1.2$$

$$d = 80/100 = 64/80 = 96/120 = 0.8$$

If $r = 0.05$, $y = 0.03$ and $t = 1$:

$$p = \frac{e^{0.05-0.03} - (0.8)}{0.4} = 0.5505$$

and

$$1 - p = 1 - 0.5505 = 0.4495$$

Notice that the probability of the spot rate rising in the above example is greater than its probability of falling.

In general, it is not necessary for the up factor to equal the down factor. However, the binomial tree is much less complex if it recombines. That is, the price is the same following an up-move and then a down-move as it is following a down-move and then an up-move.

The reverse binomial method can be used to price options where the tree is a geometric binomial.

The premium of a 2 year European put with strike price ¥100 using the geometric binomial tree in Example 10.13.

The intrinsic value at different stages of the tree are:

	1 year	2 years
		0
	0	
0		4
	20	
		36

The put premium at the lower node at 1 year, where the intrinsic value = 20, equals:

$$P_d = 0.9512\ (4 \times 0.5505 + 36 \times 0.4495) = ¥17.49$$

Similarly, the put premium at the higher node at 1 year equals:

$$P_u = 0.9512\ (0 \times 0.5505 + 4 \times 0.4495) = ¥1.71$$

Consequently, the put premium at time 0 must be:

$$P = 0.9512\ (1.71 \times 0.5505 + 17.49 \times 0.4495) = ¥8.37$$

In general, for a two-step binomial,

$$P = e^{-2\,r\Delta t}[p^2\, P_{uu} + 2p(1-p)P_{ud} + (1-p)^2\, P_{dd}] \tag{10.9}$$

In the above example:

$$\Delta t = 1 \text{ time step} = 1 \text{ year}$$
$$p = \text{up probability} = 0.5505$$

$$P_{uu} = \text{up–up expiry value} = 0$$
$$P_{ud} = \text{up–down expiry value} = 4$$
$$P_{dd} = \text{down–down expiry value} = 36$$
$$P = e^{-2\times1\times0.05}[0.5505^2 \times 0 + 2\times 0.5505\times 0.4495\times 4 + 0.4495^2 \times 36]$$
$$= ¥8.37$$

Similarly, for a call for a two-step binomial,

$$C = e^{-2r\Delta t}[p^2 C_{uu} + 2p(1-p)C_{ud} + (1-p)^2 C_{dd}] \tag{10.10}$$

In the case of a call:

$$C_{uu} = \text{up–up expiry value} = 44$$
$$C_{ud} = \text{up–down expiry value} = 0$$
$$C_{dd} = \text{down–down expiry value} = 0$$
$$C = e^{-2\times1\times0.05}[0.5505^2 \times 44 + 2\times 0.5505\times 0.4495\times 0 + 0.4495^2 \times 0]$$
$$= 12.07$$

In general, if there are n periods in the binomial:

$$C = e^{-nr\Delta t}\sum_{j=0}^{n}\frac{n!}{j!(n-j)!}p^j(1-p)^{n-j}\max(u^j d^{n-j}S - K, 0) \tag{10.11}$$

$$P = e^{-nr\Delta t}\sum_{j=0}^{n}\frac{n!}{j!(n-j)!}p^j(1-p)^{n-j}\max(K - u^j d^{n-j}S, 0) \tag{10.12}$$

As n approaches infinity, the distribution becomes continuous and the binomial distribution approaches a normal distribution. The properties of a normal distribution are discussed in Chapter 14.

BLACK–SCHOLES MODEL

The binomial model provides a more accurate calculation of the option premium when the time interval between steps in the binomial tree is smaller and the number of possible outcomes at expiry is larger. In 1973, Fischer Black, Myron Scholes and Robert Merton developed a mathematical model for pricing options on non-dividend paying shares:

$$C = SN(d_1) - Ke^{-rt}N(d_2) \tag{10.13}$$

where:

C = call premium
S = security price

K = strike price

r = risk-free interest rate (expressed as a continuously compounding rate)

t = time to expiry in years

$N(\,)$ = cumulative standard Normal distribution function

$$d_1 = \frac{\log_e(S/K) + (r + \tfrac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

σ = expected volatility per annum

The put–call parity relationship makes it possible to calculate the premium of puts:

$$\boxed{P = C + e^{-rt}K - S} \tag{10.14}$$

It follows that:

$$P = e^{-rt}KN(-d_2) - SN(-d_1) \tag{10.15}$$

Whilst the Black–Scholes formula may look complicated, it can easily be seen that the call premium depends on the strike price relative to the price of the security, expected volatility, time to expiry and the risk-free interest rate – the same factors which determined the option premium in the binomial model.

In effect, the Black–Scholes model is the continuous case of the geometric binomial model. There are an infinite number of paths. The probability density function becomes a log-normal distribution as shown in Figure 10.13.

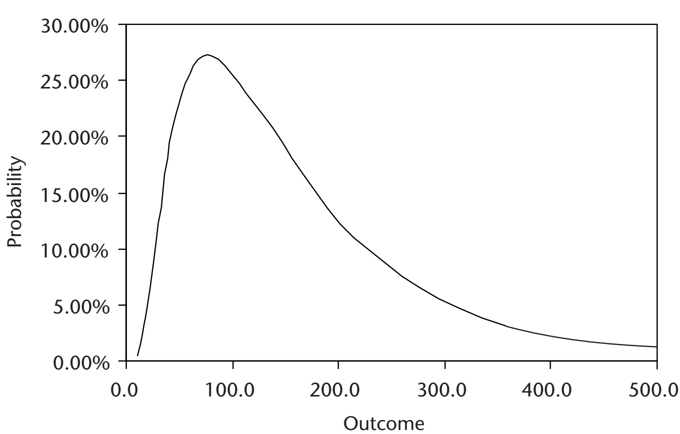


FIGURE 10.13 Log-normal probability density function

INTEREST RATE DIFFERENTIALS

The price of a currency option is related to the interest rates of the two currencies.

The writer of the option has a potential net exchange position. For example, if writing a dollar put/yen call, the writer has a risk that if the dollar depreciates, the option will be exercised and he or she will tend to lose money. To avoid this, the writer may choose to hedge all or part of the potential position by selling dollars forward before the exchange rate falls. It is common practice to hedge a variable portion of the potential position. This is known as *delta hedging* and is discussed in Chapter 14.

As the interest rate differential changes, the benefit or cost of delta hedging or unhedging will change. Consequently, the interest rate differential is a determinant of the option price.

CURRENCY OPTIONS

The Black–Scholes model was adapted for currency options in 1983 by Garmen and Kohlagen to provide for the fact that the interest rate of the commodity currency is not zero.

$$c = S\mathrm{e}^{-yt}N(d_1) - K\mathrm{e}^{-rt}N(d_2) \tag{10.16}$$

$$p = K\mathrm{e}^{-rt}N(-d_2) - S\mathrm{e}^{-yt}N(-d_1) \tag{10.17}$$

where:

r = continuously compounding terms currency interest rate

y = continuously compounding yield on commodity currency

The spot exchange rate, S, is defined as the ratio of number of units of the terms currency to one unit of the commodity currency. The spot exchange rate is assumed to follow what mathematicians call ‘Brownian motion’. This means that over any and every (infinitely small) period the spot price can move up or down randomly. The probability distribution of price changes is assumed to be normal. Consequently, small price changes have a higher probability than large ones.

PROOF OF PUT–CALL PARITY

Put–call parity can be stated as:

$$\text{call premium} - \text{put premium} = \text{PV}(\text{forward} - \text{strike})$$

$$c - p = e^{-rt}(F - K)$$

$$= e^{-rt}(Se^{rt}/e^{yt} - K)$$

$$c - p = Se^{-yt} - Ke^{-rt} \quad (10.18)$$

where:

S = spot price
K = strike price
c = European call premium
p = European put premium
r = continuously compounding interest rate
y = continuously compounding yield on the underlying
t = time in years
F = forward price = $Se^{(r-y)t}$

Consider a person who does the following four transactions simultaneously:

1. Buys a US$ call/yen put for face value US$1
2. Sells US$ put/yen call for face value US$1
3. Sells US$1 forward against yen
4. Borrows $PV(F - K)$ in yen

The cash flows associated with the four transactions will be as follows:

Transaction	*Today*	*At maturity if $S < K$*		*At maturity if $S > K$*	
	Yen	*US\$*	*Yen*	*US\$*	*Yen*
Buy US$ call	$-c$			+1	$-K$
Sell US$ put	$+p$	+1	$-K$		
Sell US$ forward		–1	$+F$	–1	$+F$
Borrows $PV(F - K)$	$+(F - K)e^{-rt}$	—	$-(F - K)$	—	$-(F - K)$
Total	$-c + p + (F - K)e^{-rt}$	0	0	0	0

The net pay-out at maturity equals zero. Unless the net cash flow today also equals zero there would be an arbitrage opportunity. Therefore,

$$-c + p + (F - K)e^{-rt} = 0$$

Restated:

$$c - p = (F - K)e^{-rt} \quad (10.19)$$

For a model pricing European options to be arbitrage-free the at-the-money strike price must equal the forward rate. It follows from the put–call parity relationship that the call premium will equal the put premium for at-the-money options.

If $K = F$,

$$\begin{aligned} c - p &= e^{-rt}(F - K) \\ &= e^{-rt}(0) \\ c &= p \end{aligned}$$

EXAMPLE 10.14

Calculate the premium for a 3 month US dollar call/yen put with strike price 108.00 given the current spot rate is US$1 = ¥110.00, 3 month dollar and yen interest rates are 5.0% and 1.0% p.a. (simple) respectively, and assuming volatility is expected to be 10% p.a.

$$\begin{aligned} S &= 110.00 \\ K &= 108.00 \\ t &= 3/12 = 0.25 \\ \sigma &= 10\% \text{ p.a.} = 0.10 \\ r_T &= 1.0\% \text{ p.a. (simple)} \quad r = 0.00999 \text{(continuous)} \\ r_C &= 5.0\% \text{ p.a. (simple)} \quad y = 0.0497 \text{(continuous)} \end{aligned}$$

$$\begin{aligned} d_1 &= \frac{\ln(110/108) + (0.00999 - 0.0497 + \tfrac{1}{2}(0.10)^2) \times 0.25}{0.1 \times 0.5} \\ &= \frac{0.018349 - 0.008678}{0.05} \\ &= 0.191983 \\ d_2 &= 0.191983 - 0.05 = 0.141983 \end{aligned}$$

$$\begin{aligned} N(d_1) &= 0.576122 \\ N(d_2) &= 0.556453 \\ \therefore c &= 110/(1 + 0.05/4) \times 0.576122 - 108/(1 + 0.01/4) \times 0.556453 \\ &= 2.65 \end{aligned}$$

INTERPRETING THE ADAPTED BLACK–SCHOLES FORMULA

$$\sigma\sqrt{t} = 0.10\sqrt{0.25} = 0.05$$

$\sigma\sqrt{t}$ is volatility for the tenor. Annual volatility of 10% is equivalent to 5.0% over 3 months.

r is the continuously compounding equivalent of 1% p.a. simple:

$$r = \ln(1 + r_T) = 0.00999 = 0.999\% \text{ p.a.}$$

y is the continuously compounding equivalent of 5% p.a. simple:

$$y = \ln(1 + r_C) = 0.0497 = 4.97\% \text{ p.a.}$$

e^{-rt} and e^{-yt} are the corresponding discount factors.

The forward rate represents the expected spot rate at expiry of the option. This assumption is necessary to ensure that the model is (put–call) arbitrage-free. Here f = 108.91. It is assumed that over the life of the option the spot rate will drift from its level at the start of the period towards the (then) forward rate.

There are many possible paths that the spot rate can follow during the life of the option. The actual path followed by the spot rate is assumed to be the result of the combination of the drift towards the forward rate and the random up or down Brownian motion (Figure 10.14).

The standard deviation of the assumed probability distribution of random price changes reflects the expected volatility.

d_2 is the number of standard deviations that the strike price is below the expected spot price at expiry assuming the percentage changes in spot prices are normally distributed with mean 110.00 and standard deviation 5.0%.

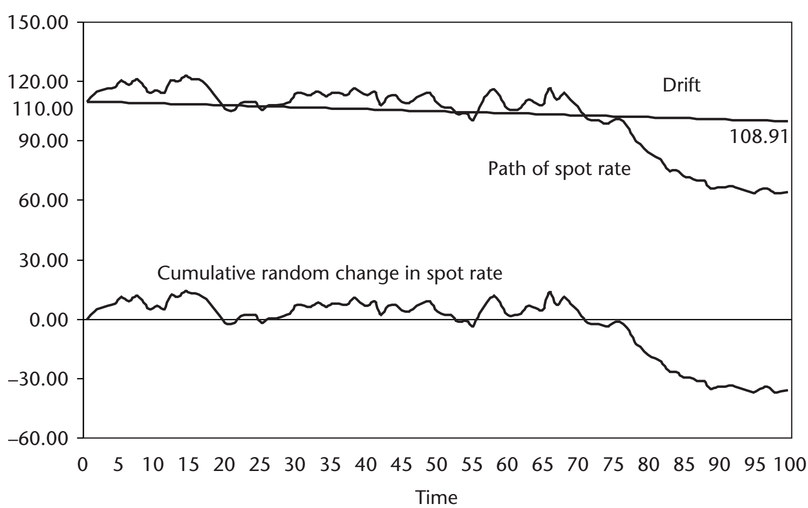


FIGURE 10.14 Assumed spot rate movement

$$d_2 = 0.141983$$
$$N(d_2) = 0.556453$$

Calls will be exercised if the spot rate at expiry is above the strike price and puts will be exercised if the spot rate at expiry is below the strike price. $N(d_2)$ is the probability that the call will be exercised and $1 - N(d_2)$ is the probability that the put will be exercised.

$KN(d_2)$ is the strike price times the probability that the strike price will be called. Multiplying by e^{-rt} discounts this to present value.

$N(d_1)$ is the call delta if $y = 0$. Here, $N(d_1) = 0.576122$. Multiplying by e^{-yt} gives the call delta when y is non-zero. $N(d_1)e^{-yt} = 0.576122 \times 0.987654 = 0.569009$. This means that if the assumptions are true, a sold call could be momentarily hedged against changes in the spot rate by buying 56.9% of the underlying commodity currency amount of the face value of the option. Delta is discussed further in Chapter 14.

BLACK'S MODEL

In 1976 Fischer Black developed a variation of the Black–Scholes model to price options from forward prices.

$$c = e^{-rt}[FN(d_1) - KN(d_2)]$$
$$d_1 = \frac{\ln(F/K) + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}} \tag{10.20}$$
$$d_2 = \frac{\ln(F/K) - \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

where F = forward price.

If $K = F$,

$$d_1 = \tfrac{1}{2}\sigma\sqrt{t}$$
$$d_2 = -\tfrac{1}{2}\sigma\sqrt{t}$$

Black's model has many applications. For example, in foreign exchange the forward rate is:

$$f = \frac{s(1 + r_T t)}{1 + r_C t}$$

It follows that the call premium formula can be expressed as:

$$c = \frac{FN(d_1) - KN(d_2)}{(1 + r_T t)}$$

Using the data from Example 10.14,

$$c = \frac{108.91 \times 0.576122 - 108 \times 0.556453}{1 + 0.01 \times 3/12} = 2.65$$

Notice that Black's model gives the same result as the modified Black–Scholes formula because it is merely an alternative way of calculating the same thing.

PRACTICE PROBLEMS

10.1 Sold euro put

A bank writes a euro put/US dollar call for €10,000,000 face value. The strike price is €1 = US$0.9000; time to expiry 4 months and the premium 2.00%.

(a) Calculate the premium in US dollars if the current spot rate is €1 = US$0.9100

(b) Calculate the pay-out if the spot rate at expiry turns out to be €1 = US$0.8950.

(c) What would the spot rate at expiry need to be for the pay-out to break-even with the future value of the premium given that the 4 month dollar interest rate is 3.00% p.a. (120/360)?

10.2 Binomial model

Use a 3-step binomial model to calculate the premium of a 3 month US$ call/S$ put given:

Spot rate	s	= 1.7000	
Forward rate	f	= 1.6940	
Strike price	k	= 1.7100	
Face value		US$1,000,000	
3 month US$ interest rate		3.0% p.a.	(90/360)
3 month S$ interest rate		1.6% p.a.	(90/360)
up–down movement		S$0.0200 per month +/– drift	

10.3 Put–call arbitrage

Identify the arbitrage opportunity available given the following prices. Articulate the actions that need to be taken to profit through the above arbitrage. Calculate the profit that could be made on a face value of £10,000,000.

Spot rate	£1	=	US$1.7000
1 year forward rate	£1	=	US$1.6950
1 year £ call (k = 1.7200) premium			US$0.0230

1 year £ put (k = 1.7200) premium	US$0.0480	
1 year US$ interest rate	4.0% p.a.	(360/360)

10.4 Black–Scholes model

(a) Use the modified Black–Scholes model to calculate the premium of a European US$ call with strike price of ¥105.00 given:

$$c = Se^{-yt}N(d_1) - Ke^{-rt}N(d_2)$$

$$d_1 = \frac{\ln(S/K) + (r - y + \tfrac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(S/K) + (r - y - \tfrac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

Spot	US$/¥	110.00	
Expected volatility		15% p.a.	
Time to expiry		3 months	(90/360)
US$ interest rate		6.50% p.a.	(90/360)
¥ interest rate		1.00% p.a.	(90/360)
Implied forward rate		108.55	

Use the z tables provided in the Appendix.

(b) Use Black's model:

$$c = e^{-rt}[FN(d_1) - KN(d_2)]$$

$$d_1 = \frac{\ln(F/K) + \tfrac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

to calculate the premium of the same option as in (a).

(c) Use put–call parity to calculate the premium of the 105.00 put with the same data as in (a).

CHAPTER 11

Applications of Currency Options

Options have many practical applications for exporters, importers, borrowers, investors and traders. This chapter examines a number of applications that involve the use of standard options. By varying one or more of the number and type of option, the strike prices, and the face value of the options, it is possible to tailor a variety of outcomes to suit the particular risk profile desired for a foreign currency exposure.

Options provide a person with a foreign exchange exposure with an extremely flexible tool for manipulating potential cash flows and the associated risk.

APPLICATIONS USING OPTIONS WHEN THERE IS AN UNDERLYING EXPOSURE

An option that is bought or sold when there is an underlying foreign exchange exposure could be called a *natural option*.

Importer

EXAMPLE 11.1

An importer whose local currency is the US dollar has an obligation to pay €10,000,000 in three months' time and considers three scenarios:

1. Remaining unhedged and purchasing the euro at the prevailing spot rate one month from now
2. Hedging by buying euros forward
3. Hedging by buying a euro call/dollar put

The prevailing market rates are:

Spot rate	€1 = US\$0.9000	
3 month forward rate	€1 = US\$0.8975	
3 month dollar interest rate	3.00% p.a.	(90/360)
3 month euro call (strike 0.8975)	1.97%	

(a) Calculate the premium payable in dollars.

$$\begin{aligned}\text{Premium} &= €10{,}000{,}000 \times 0.0197\\ &= €197{,}000\\ &= \text{US}\$177{,}300 \text{ (at spot rate 0.9000)}\\ \text{FV(Premium)} &= \text{US}\$177{,}300 \times (1 + 0.03 \times 90/360)\\ &= \text{US}\$178{,}629.75\end{aligned}$$

(b) Calculate the cost of buying €10,000,000 for value the forward date for each of the three scenarios if at maturity the spot rate is:

(i) 0.8800
(ii) 0.9000
(iii) 0.9200

1. The US dollar cost of the imports if unhedged

$= 10{,}000{,}000 \times x$ where x = spot rate at maturity

(i) $x = 0.8800$, cost $= 10{,}000{,}000 \times 0.8800 = \$8{,}800{,}000$
(ii) $x = 0.9000$, cost $= 10{,}000{,}000 \times 0.9000 = \$9{,}000{,}000$
(iii) $x = 0.9200$, cost $= 10{,}000{,}000 \times 0.9200 = \$9{,}200{,}000$

2. The dollar cost of the imports if hedged with a forward

(i) $x = 0.8800$, cost $= 10{,}000{,}000 \times 0.8975 = \$8{,}975{,}000$
(ii) $x = 0.9000$, cost $= 10{,}000{,}000 \times 0.8975 = \$8{,}975{,}000$
(iii) $x = 0.9200$, cost $= 10{,}000{,}000 \times 0.8975 = \$8{,}975{,}000$

3. The dollar cost of the imports if hedged with a bought euro call

$= 10{,}000{,}000 \times 0.8975 + \text{FV (Premium)}$ if $x \geq 0.8975$ (strike price), or
$10{,}000{,}000 \times x + \text{FV (Premium)}$ if $x < 0.8975$ (strike price)

(i) $x = 0.8800$, cost $= 8{,}800{,}000 + 178{,}629.75 = \$8{,}978{,}629.75$
(ii) $x = 0.9000$, cost $= 8{,}975{,}000 + 178{,}629.75 = \$9{,}153{,}629.75$
(iii) $x = 0.9200$, cost $= 8{,}975{,}000 + 178{,}629.75 = \$9{,}153{,}629.75$

The effect of buying the euro call is to place a ceiling on the dollar cost of the imports without limiting the potential benefit if the spot rate falls. The importer limits the cost to a worst case of US\$9,153,629.75 without limiting the minimum.

The turning point of the pay-off line occurs at the strike price.

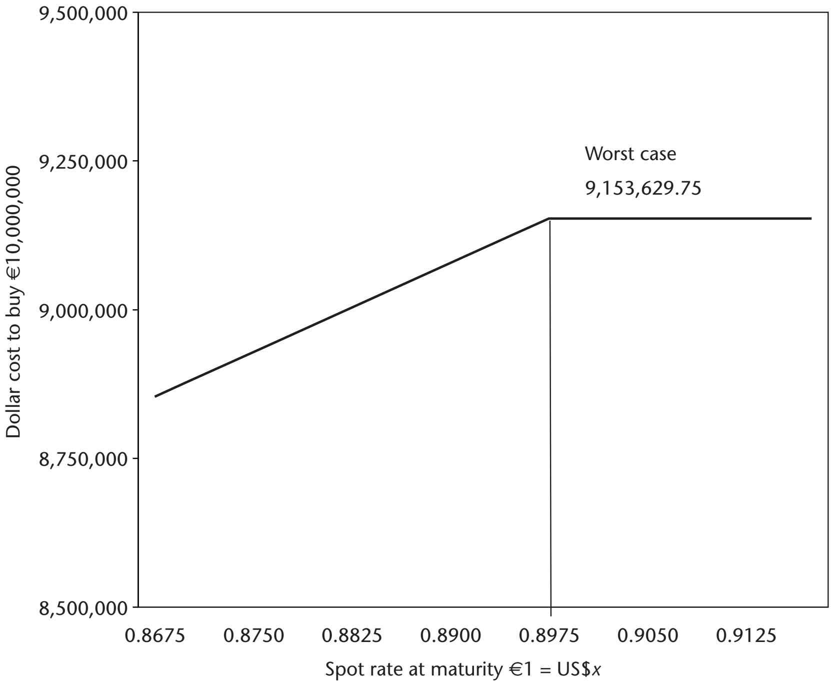


FIGURE 11.1 Imports hedged with bought euro call

(c) Calculate the break-even rates between being:
  (i) unhedged and hedged with a forward
  (ii) unhedged and hedged with the bought call
  (iii) hedged with a forward and hedged with the bought call

  (i) The break-even rate, b_1, occurs where:

  cost if unhedged = cost if hedged with a forward

$$10{,}000{,}000 \times b_1 = 10{,}000{,}000 \times 0.8975$$
$$\therefore b_1 = 0.8975 \text{ (forward rate)}$$

  (ii) The break-even rate, b_2, occurs where:

  cost if unhedged = cost if hedged with call

$$10{,}000{,}000 \times b_2 = 10{,}000{,}000 \times 0.8975 + 178{,}629.75$$
$$\therefore b_2 = 9{,}153{,}629.75/10{,}000{,}000 = 0.9154$$

(iii) The break-even rate, b_3, occurs where:

cost with forward = cost with call

$$10{,}000{,}000 \times 0.8975 = 10{,}000{,}000 \times b_3 + 178{,}629.75$$

$$\therefore b_3 = \frac{8{,}975{,}000 - 178{,}629.75}{10{,}000{,}000} = 0.8796$$

Spot rate at maturity	*Lowest cost strategy*	*Mid-cost strategy*	*Highest cost strategy*
$x < b_3$	unhedged	option	forward
$b_3 < x < b_1$	unhedged	forward	option
$b_1 < x < b_2$	forward	unhedged	option
$x > b_2$	forward	option	unhedged

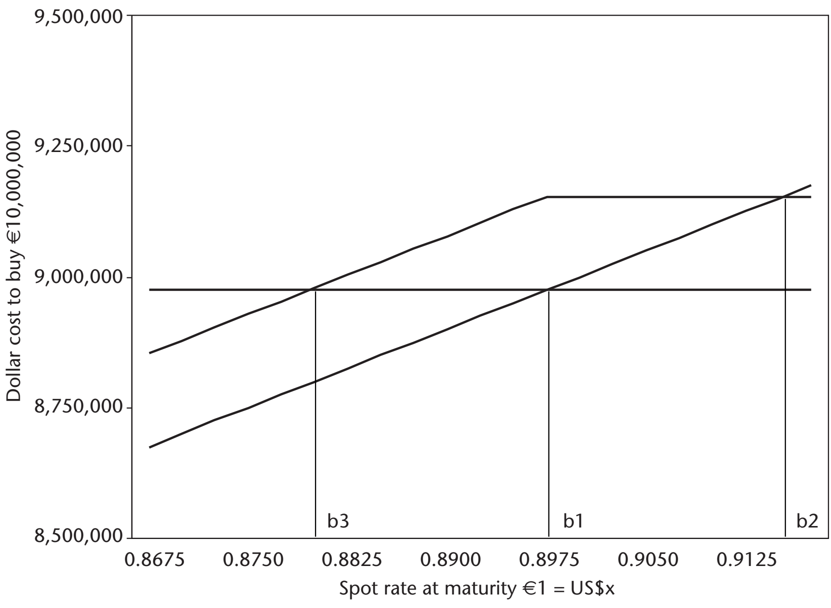


FIGURE 11.2 Importer's alternatives

Exporter

EXAMPLE 11.2

An exporter whose local currency is the US dollar will be receiving €1,000,000 in three months' time and considers three scenarios:

1. Remaining unhedged and selling the euros at the prevailing spot rate three months from now
2. Hedging by selling the euros forward
3. Hedging by buying a euro put/dollar call

The prevailing market rates are:

Spot rate	€1	=	US$0.9000	
3 month forward rate	€1	=	US$0.8975	
3 month dollar interest rate			3.0% p.a.	(90/360)
3 month euro put (strike 0.8975)			1.97%	

(a) Calculate the premium payable in dollars.

$$\begin{aligned}\text{Premium} &= €10{,}000{,}000 \times 0.0197\\ &= €197{,}000\\ &= \text{US\$}177{,}300 \text{ (at spot rate 0.9000)}\\ \text{FV(Premium)} &= \text{US\$}177{,}300 \times (1 + 0.03 \times 90/360)\\ &= \text{US\$}178{,}629.75\end{aligned}$$

(b) Calculate the proceeds from selling €10,000,000 for each of the three scenarios for the forward value date if at maturity the spot rate is:

(i) 0.8800
(ii) 0.9000
(iii) 0.9200

1. The dollar proceeds from the exports if unhedged

$= 10{,}000{,}000 \times x$, where x = spot rate at maturity

(i) $x = 0.8800$, proceeds $= 10{,}000{,}000 \times 0.8800 = \$8{,}800{,}000$
(ii) $x = 0.9000$, proceeds $= 10{,}000{,}000 \times 0.9000 = \$9{,}000{,}000$
(iii) $x = 0.9200$, proceeds $= 10{,}000{,}000 \times 0.9200 = \$9{,}200{,}000$

2. The dollar proceeds from the exports if hedged with a forward

(i) $x = 0.8800$, cost $= 10{,}000{,}000 \times 0.8975 = \$8{,}975{,}000$
(ii) $x = 0.9000$, cost $= 10{,}000{,}000 \times 0.8975 = \$8{,}975{,}000$
(iii) $x = 0.9200$, cost $= 10{,}000{,}000 \times 0.8975 = \$8{,}975{,}000$

3. The dollar proceeds from the exports if hedged with a bought euro put (Figure 11.3)

$= 10{,}000{,}000 \times 0.8975 - FV(\text{Premium})$ if $x \leq 0.8975$ (strike price), or

$10{,}000{,}000 \times x - FV(\text{Premium})$ if $x > 0.8975$ (strike price)

(i) $x = 0.8800$, proceeds $= 8{,}975{,}000 - 178{,}629.75 = \$8{,}796{,}370.25$
(ii) $x = 0.9000$, proceeds $= 9{,}000{,}000 - 178{,}629.75 = \$8{,}821{,}370.25$
(iii) $x = 0.9200$, proceeds $= 9{,}200{,}000 - 178{,}629.75 = \$9{,}021{,}370.25$

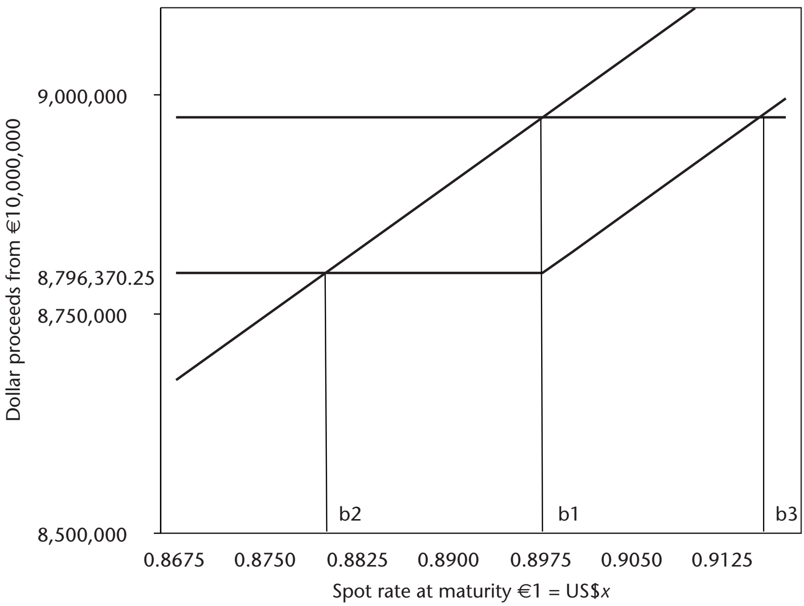


FIGURE 11.3 Exports hedged with bought euro put

The effect of buying the euro put is to limit the worst case proceeds to US$8,796,370.25 without limiting the potential benefits if the spot rate rises.

(c) Calculate the break-even rates between being:

(i) unhedged and hedged with a forward
(ii) unhedged and hedged with a bought put
(iii) hedged with a forward and hedged with a bought put

(i) The break-even rate, b_1, occurs where:

$$\text{proceeds if unhedged} = \text{proceeds with forward}$$
$$10{,}000{,}000 \times 0.8975 = 10{,}000{,}000 \times b_1$$
$$\therefore b_1 = 0.8975 \text{ (forward rate)}$$

(ii) The break-even rate, b_2, occurs where:

$$\text{proceeds if unhedged} = \text{proceeds with option}$$
$$10{,}000{,}000 \times b_2 = 10{,}000{,}000 \times 0.8975 - 178{,}629.75$$
$$\therefore b_2 = \frac{8{,}796{,}370.25}{10{,}000{,}000}$$
$$= 0.8796$$

(iii) The break-even rate, b_3, occurs where:

proceeds with foward = proceeds with option

$$10{,}000{,}000 \times 0.8975 = 10{,}000{,}000 \times b_3 - 178{,}629.75$$

$$\therefore b_3 = \frac{8{,}975{,}000 + 178{,}629.75}{10{,}000{,}000}$$

$$= 0.9154$$

The relative merit of the three strategies can be ranked depending on what the spot rate turns out to be at maturity.

Spot rate at maturity	*Lowest cost strategy*	*Mid-cost strategy*	*Highest cost strategy*
$x < b_2$	forward	option	unhedged
$b_2 < x < b_1$	forward	unhedged	option
$b_1 < x < b_3$	unhedged	forward	option
$x > b_3$	unhedged	option	forward

EFFECTIVE EXCHANGE RATE

People sometimes like to review the possible outcomes of different strategies in terms of their effective exchange rates rather than as the dollar proceeds or cost. Diagrams such as Figure 11.3 can be redrawn with the effective exchange rate as the vertical axis, as shown in Figure 11.4.

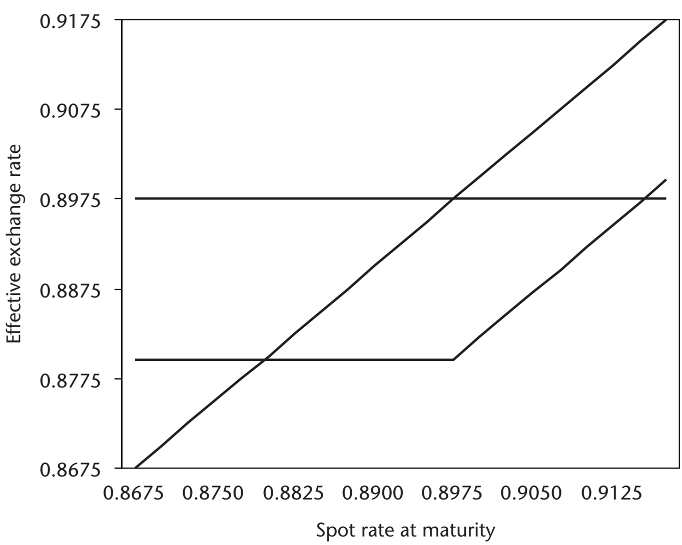


FIGURE 11.4 Effective exchange rate

The *effective exchange rate* is calculated as the terms currency proceeds (or cost) divided by the commodity amount. For example, if the spot rate at maturity turns out to be 0.9200, the dollar proceeds with a bought 0.8975 put would be US$9,021,370.25. Hence:

$$\text{Effective exchange rate} = \frac{\text{Terms currency proceeds}}{\text{Commodity currency amount}} \qquad (11.1)$$

$$= \frac{9{,}021{,}370.25}{10{,}000{,}000} = 0.9021$$

FOREIGN CURRENCY BORROWER

EXAMPLE 11.3

A real estate developer requires US$20,000,000 for six months. Upon electing to borrow Swiss francs for the period at an interest rate of 3% p.a. interest in arrears, the developer considers four scenarios:

1. Remaining unhedged and purchasing the required Swiss francs (principal plus interest) at the prevailing spot rate at maturity
2. Hedging by buying forward Swiss francs (principal plus interest)
3. Hedging by buying a dollar put/Swiss franc call
4. Remaining unhedged but writing a dollar call/Swiss franc put

The prevailing market rates are:

Spot rate	US$1 = SF1.2500	1.2510	
6 month forward rate	US$1 = SF1.2370	1.2390	
6 month dollar interest rate	5.0% p.a.	(180/360)	
6 month $ put/SF call (strike 1.2500)	3.8%	4.0%	
6 month $ call/SF put (strike 1.2500)	2.8%	3.0%	

(a) Calculate the Swiss franc liability due in six months' time.

$$\text{SF principal} = 20{,}000{,}000 \times 1.2510 = \text{SF}25{,}020{,}000$$

$$\text{SF interest} = 25{,}020{,}000 \times 0.03 \times 180/360 = \text{SF} \quad 375{,}300$$

$$\text{SF (principal + interest)} = \text{SF}25{,}395{,}300$$

(b) Calculate the dollar premium required to buy a dollar put to cover this liability and the dollar premium received from selling a dollar call for the same face value.

$$\text{Premium paid for \$ put} = \frac{25{,}395{,}300}{1.2500} \times \frac{4.0}{100} = \$812{,}649.60$$

$$FV(\text{Premium})\ \text{paid} = 812{,}649.60\ (1 + 0.05 \times 180/360) = \$832{,}965.84$$

$$\text{Premium received from \$ call } = \frac{25{,}395{,}300}{1.2500} \times \frac{2.8}{100} = \$568{,}854.72$$

$$FV(\text{Premium}) \text{ received} = 568{,}854.72 \times (1 + 0.05 \times 180/360)$$
$$= \$583{,}076.09$$

(c) Calculate the dollar cost of repaying the Swiss francs (principal + interest) under each of the four scenarios if the spot rate at maturity is:

(i) 1.1000
(ii) 1.2500
(iii) 1.4000

1. Unhedged

$$= \frac{25{,}395{,}300}{x} \quad \text{where } x = \text{spot bid rate at maturity}$$

2. Hedged with a forward

$$= \frac{25{,}395{,}300}{f} \quad \text{where } f = \text{forward bid rate}$$

3. Hedged with a bought dollar put

$$= \frac{25{,}395{,}300}{x} + 832{,}965.84 \qquad \text{if } x \geq 1.2500$$

$$= \frac{25{,}395{,}300}{1.2500} + 832{,}965.84 \qquad \text{if } x < 1.2500$$

4. Unhedged with a sold dollar call

$$= \frac{25{,}395{,}300}{1.2500} - 583{,}076.09 \qquad \text{if } x > 1.2500$$

$$= \frac{25{,}395{,}300}{x} - 583{,}076.09 \qquad \text{if } x \leq 1.2500$$

See Figure 11.5.

Dollar cost of purchasing SF25,395,300:

Spot rate at maturity:	*1.1000*	*1.2500*	*1.4000*
1. Unhedged	23,086,636.36	20,316,240.00	18,139,500.00
2. Hedged with a forward at 1.2370	20,529,749.39	20,529,749.39	20,529,74.39
3. Hedged with bought $ put strike 1.2500	21,149,205.84	21,149,205.84	18,972,465.84
4. Unhedged with a sold $ call strike 1.2500	22,503,560.27	19,733,163.91	19,733,163.91

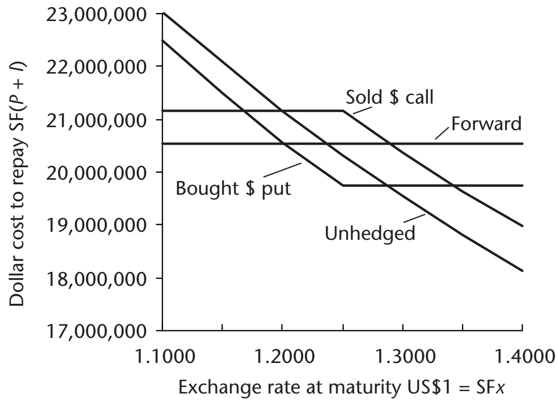


FIGURE 11.5 SF borrower's alternatives

(d) Calculate the effective borrowing cost under each of the above four scenarios.

$$\text{Effective borrowing cost} = \frac{\$ \text{ cost to repay SF}(P+I) - 20{,}000{,}000}{20{,}000{,}000} \times 200$$

Effective borrowing cost% p.a. (Figure 11.6):

Spot rate at maturity:	*1.1000*	*1.2500*	*1.4000*
1. Unhedged	30.9%	3.2%	–18.6%
2. Hedged with a forward at 1.2370	5.3%	5.3%	5.3%
3. Hedged with a bought $ put strike 1.2500	11.5%	11.5%	–10.3%
4. Unhedged with a sold $ call strike 1.2500	25.0%	–2.7%	–2.7%

(e) Determine the break-even rates between being:

(i) unhedged and hedged with a forward, b_1
(ii) unhedged and hedged with a bought dollar put, b_2
(iii) hedged with a forward and hedged with a bought dollar put, b_3
(iv) unhedged and unhedged with a sold dollar call, b_4
(v) hedged with a forward and unhedged with a sold dollar call, b_5
(vi) hedged with a bought dollar put and unhedged with a sold dollar call, b_6 and b_7

The break-even rates are:

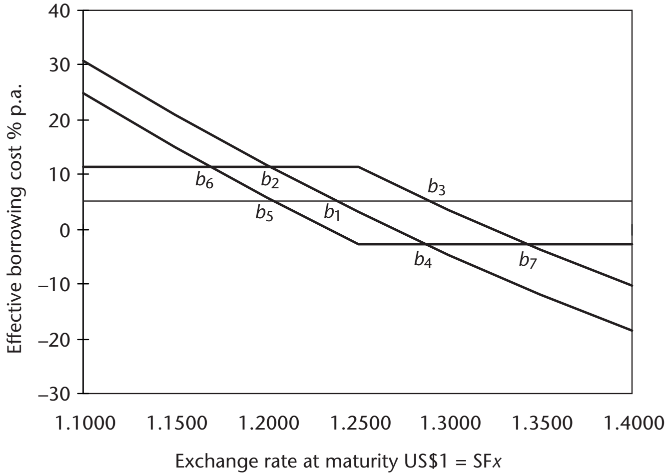


FIGURE 11.6 Effective borrowing costs

	Exchange rate	*\$ cost of SF(P + I)*	*Effective interest rate% p.a.*
b_1	1.2370	20,529,749.39	5.3%
b_2	1.2008	21,149,205.84	11.5%
b_3	1.2893	20,529,749.39	5.3%
b_4	1.2869	19,733,163.91	–2.7%
b_5	1.2028	20,529,749.39	5.3%
b_6	1.1686	21,149,205.84	11.5%
b_7	1.3437	19,733,163.91	–2.7%

FOREIGN EXCHANGE TRADER

EXAMPLE 11.4

One month ago, a foreign exchange trader bought £10,000,000 against US dollars at an outright five month forward rate of 1.4800. The spot rate has since risen to 1.5150 and the two month forward rate is now 1.5100. The four month (120/360) dollar interest rate is 5% p.a. The trader considers three strategies for taking profits:

1. Selling pounds 4 months forward
2. Buying a pound put (strike 1.5100) premium 2.0%
3. Writing a pound call (strike 1.5150) premium 1.7%

Determine the profit outcome of each of the strategies.

1. The original position is long £10,000,000 and short US$14,800,000. To square the position through a forward contract, the trader needs to sell £10,000,000 at a rate of 1.5100.

$$\text{Profit} = \begin{array}{l}\text{dollar proceeds of sale}\\ \text{of £10,000,000 through}\\ \text{reversing forward}\end{array} - \begin{array}{l}\text{dollar cost to purchase}\\ \text{£10,000,000 from}\\ \text{original position}\end{array}$$

$$= 15{,}100{,}000 - 14{,}800{,}000$$

$$= \text{US\$}300{,}000$$

2. To square the position through a bought pound put:

$$\text{Premium paid for put} = 10{,}000{,}000 \times 0.02 \times 1.5150 = \text{US\$}303{,}000$$

$$\begin{aligned}\text{FV(Premium)} &= 303{,}000 \times (1 + 0.05 \times 120/360)\\ &= \text{US\$}308{,}050.00\end{aligned}$$

$$\text{Profit} = \begin{array}{l}\text{proceeds of sale}\\ \text{of £10,000,000}\\ \text{through put}\end{array} - \begin{array}{l}\text{cost to purchase}\\ \text{£10,000,000 from}\\ \text{original position}\end{array}$$

$$= 10{,}000{,}000 \times x - 308{,}050 - 14{,}800{,}000 \quad \text{if } x \geq 1.5100$$

$$\text{or} \quad 10{,}000{,}000 \times 1.5100 - 308{,}050 - 14{,}800{,}000 \quad \text{if } x < 1.5100$$

The trader effectively gives up US$308,050 of the unrealized profit to buy an option which eliminates the possibility of further loss (if the exchange rate falls) but leaves open the opportunity for further gain if the exchange rate rises above 1.5100.

3. Another alternative is for the dealer to write a pound call that, if exercised, will lock in the profit. Writing the option will earn a premium that will add to the potential profit. However, writing the option does *not* hedge the net exchange position. If the exchange rate subsequently falls before the maturity date, the trader will lose some or all of the unrealized profit. The premium received acts as an additional buffer against a possible adverse exchange rate movement.

$$\begin{aligned}\text{Premium received for call} &= 10{,}000{,}000 \times 0.017 \times 1.5150\\ &= \text{US\$}257{,}550.00\end{aligned}$$

$$\begin{aligned}\text{FV(Premium)} &= 257{,}550 \times (1 + 0.05 \times 120/360)\\ &= \text{US\$}261{,}842.50\end{aligned}$$

$$\text{Profit} = \begin{array}{l}\text{proceeds of possible sale}\\ \text{of £10,000,000}\\ \text{through call}\end{array} - \begin{array}{l}\text{cost to purchase}\\ \text{£10,000,000 from}\\ \text{original position}\end{array}$$

$$= \quad 10{,}000{,}000 \times 1.5100 + 261{,}842.50 - 14{,}800{,}000 \qquad \text{if } x \geq 1.5100$$

$$\text{or} \quad 10{,}000{,}000 \times -261{,}842.50 - 14{,}800{,}000 \qquad \text{if } x < 1.5100$$

If the exchange rate falls below the break-even rate, b, the trader will incur a loss. The value of b is calculated by solving the profit equation with profit equal to zero.

$$\text{Profit} = 10{,}000{,}000b + 261{,}842.50 - 14{,}800{,}000$$

$$\therefore 0 = 10{,}000{,}000b - 14{,}538{,}157.50$$

$$\therefore b = 1.4538$$

FOREIGN CURRENCY INVESTOR

EXAMPLE 11.5

A funds manager wishes to maximize the yield on an investment of US$10,000,000 available for investing for 360 days. Instead of investing in the dollar money market at 5% p.a. (360/360), the funds manager elects to sell the dollars to buy ¥1,100,000,000 and to invest the yen at 1.0% p.a. (360/360). The funds manager considers two alternatives:

1. Remaining unhedged and sell ¥1,111,152,778 (principal plus interest) at the prevailing spot rate at maturity
2. Buying a dollar call/yen put at 110.00 for ¥1,111,152,778 for a premium of 3.25%.

(a) Calculate the effective yield% p.a. (360/360 basis) for each alternative if the spot rate at maturity is:

(i) 100.00
(ii) 110.00
(iii) 120.00

Unhedged

Dollar proceeds from the yen investment

$$= \frac{1{,}111{,}152{,}778}{x} \text{ where } x = \text{exchange rate at maturity}$$

(i) x = 100.00, proceeds = $11,111,527.78
(ii) x = 110.00, proceeds = $10,101,388.89
(iii) x = 120.00, proceeds = $9,259,606.48

$$\boxed{\text{Yield\% p.a.} = \frac{\$\text{ proceeds} - 10{,}000{,}000}{10{,}000{,}000} \times \frac{360}{360} \times 100}$$

(i) $x = 100.00$, yield% $= \dfrac{1{,}111{,}527.78}{10{,}000{,}000} = 11.1\%$

(ii) $x = 110.00$, yield% $= \dfrac{101{,}388.89}{10{,}000{,}000} = 1.0\%$

(iii) $x = 120.00$, yield% $= \dfrac{-740{,}393.52}{10{,}000{,}000} = -7.4\%$

Bought dollar call

$$\text{Premium} = \frac{1{,}111{,}152{,}778}{111.00} \times \frac{3.25}{100} = \$328{,}295.14$$

$$\text{FV(Premium)} = 328{,}295.14 \times \left(1 + 0.05 \times \frac{360}{360}\right) = \$344{,}709.90$$

Dollar proceeds from the yen investment

$$= \frac{1{,}111{,}152{,}778}{x} - 344{,}709.90 \qquad \text{if } x < 110.00$$

$$\text{or } \frac{1{,}111{,}152{,}778}{110.00} - 344{,}709.90 \qquad \text{if } x \geq 110.00$$

(i) $x = 100.00$, proceeds = \$10,766,817.88
(ii) $x = 110.00$, proceeds = \$9,756,678.99
(iii) $x = 120.00$, proceeds = \$9,756,678.99

(i) $x = 100.00$, yield% $= \dfrac{766{,}817.88}{10{,}000{,}000} = 7.66\%$

(ii) $x = 110.00$, yield% $= \dfrac{-243{,}321.01}{10{,}000{,}000} = -2.43\%$

(iii) $x = 120.00$, yield% $= \dfrac{-243{,}321.01}{10{,}000{,}000} = -2.43\%$

(b) Calculate the break-even rate between being unhedged and holding the bought dollar call.

The break-even rate, b, occurs where:

proceeds if unhedged = proceeds with bought dollar call

i.e.

$$\frac{1{,}111{,}152{,}778}{b} = \frac{1{,}111{,}152{,}778}{110.00} - 344{,}937.88$$

$$\therefore b = 113.89$$

(c) Calculate the spot rate at maturity required for the effective yield to be 10% p.a. with the bought dollar call.

Effective yield = 10% p.a. when:

$$\text{proceeds with bought dollar call} = 1.1 \times \$10{,}000{,}000$$

$$\text{i.e.} \quad \frac{1{,}111{,}152{,}778}{x} - 344{,}709.90 = 11{,}000{,}000$$

$$\therefore x = 97.94$$

VARYING THE STRIKE PRICE

Consider the importer in Example 11.1. Three scenarios have already been examined:

1. Remaining unhedged
2. Hedging by buying euros forward
3. Hedging by buying a euro call/dollar put with a strike price of 0.8975

Many additional outcomes become available by varying the strike price of the bought call:

	Strike	*Premium*	*FV(Premium)*
Alternative 3	0.8975	1.97%	178,629.75
Alternative 3(a)	0.9075	1.48%	134,199.00
Alternative 3(b)	0.8900	2.56%	232,128.00

The dollar cost of the imports hedged with a bought euro call with strike price, y,

$$= 10{,}000{,}000y + \text{FV(Premium)} \quad \text{if } x \geq y$$
$$= 10{,}000{,}000x + \text{FV(Premium)} \quad \text{if } x < y$$

The pay off lines bend at their respective strike prices (Figure 11.7). By buying an option which is further in-the-money, the importer will pay

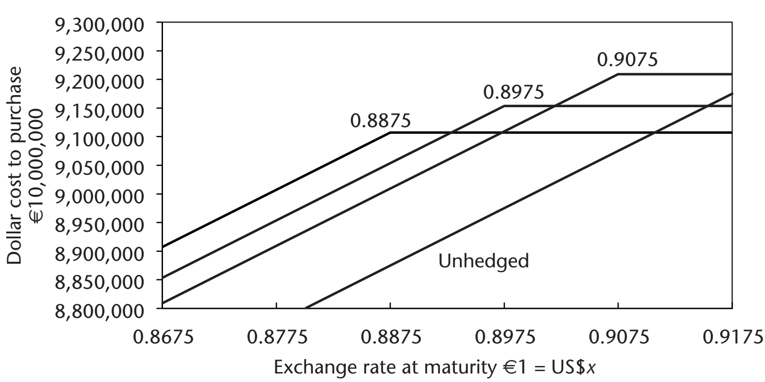


FIGURE 11.7 Imports hedged with euro calls at different strike prices

more to buy €10,000,000 if the exchange rate at expiry exceeds the lower strike price (0.8875) because of the larger premium payable on the more in-the-money option. On the other hand, the importer will be better off with the more in-the-money option if the exchange rate at expiry is less than the higher strike price (0.9075) because the strike price at which the euros will be bought will be more favourable. There are break-even rates between each of the pairs of calls at which the benefit of the lower strike price is just offset by the cost of paying the higher premium. The break-even rates correspond to the points of intersection of the pay-off lines.

The unhedged line represents the limiting case of an out-of-the-money option. As the strike price is set further out-of-the-money, the option approaches the unhedged line.

COLLARS

Another set of alternatives can be generated by considering the case where in addition to buying a call to hedge exposure, the importer writes a put (to earn a premium to pay for the bought call). Suppose that the importer decides to buy a euro call at a strike price of 0.9200 for a premium of 1.00% and to write a euro put at a strike price to be determined. The following premiums apply to euro puts:

	Strike price	*Premium*	*FV (Premium)*
Alternative 4	0.8590	0.50%	45,337.50
Alternative 4(a)	0.8760	1.00%	90,675.00
Alternative 4(b)	0.8980	2.00%	181,350.00

ZERO PREMIUM COLLAR

Alternative 4

If the importer writes a euro put with a strike price of 0.8760 for a premium of 1.00%, then he or she would receive a premium just sufficient to allow the payment of the premium on a bought euro call with a strike price of 0.9200.

$$\begin{aligned}\text{FV(Net premium)} &= \text{FV(Premium received)} - \text{FV(Premium paid)}\\ &= 90{,}675 - 90{,}675\\ &= 0\end{aligned}$$

If at expiry the spot exchange rate is above 0.9200, the call (strike 0.9200) will be in-the-money and the put (strike 0.8760) will be out-of-the-money.

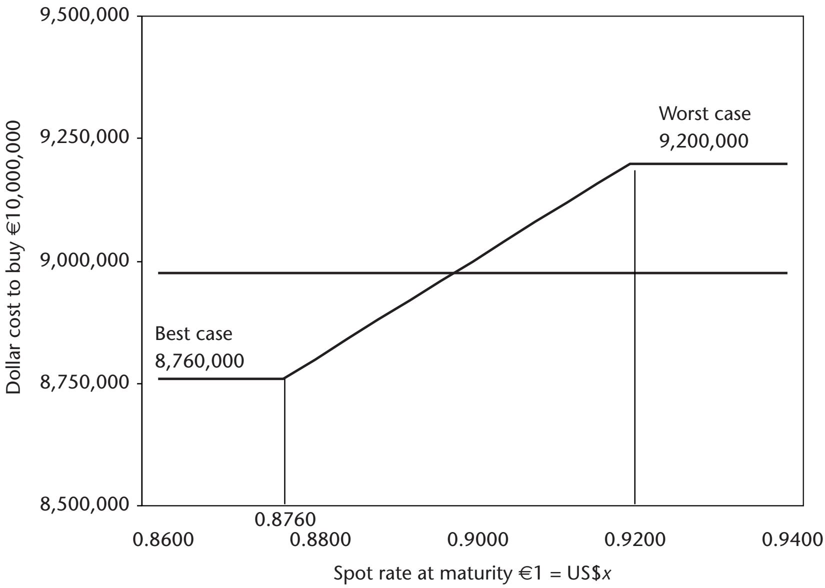


FIGURE 11.8 Zero premium collar hedging imports

The importer will exercise the call to buy €10,000,000 at 0.9200 and will find that the sold put will lapse. The importer will buy €10,000,000 at a worse case rate of 0.9200 for a cost of US$9,200,000; see Figure 11.8.

If at expiry the exchange rate is below 0.8760, the euro call (strike 0.9200) will be out-of-the-money and the euro put (strike 0.8760) will be in-the-money. The importer will be exercised on the put and will allow the call to lapse. As a result of the put being exercised, the importer will buy €10,000,000 at a best rate case of 0.8760 for a cost of US$8,760,000.

If at expiry the exchange rate is between 0.8760 and 0.9200, both options will be out-of-the-money, so both will lapse. The importer will buy €10,000,000 at the market spot exchange rate. For example, if at expiry the spot rate is 0.9000, the cost of buying €10,000,000 will be US$9,000,000.

This strategy is known as a *zero premium collar*. It is also referred to as a cylinder, range forward or fence. Different banks tend to use different names, but the concept is the same. The importer buys an out-of-the-money option that, if exercised, determines the worst case. The importer is hedged because he or she has a limited downside. The importer also writes that option that, if exercised, will provide the currency required to meet the foreign currency obligation, namely a euro put. Provided the

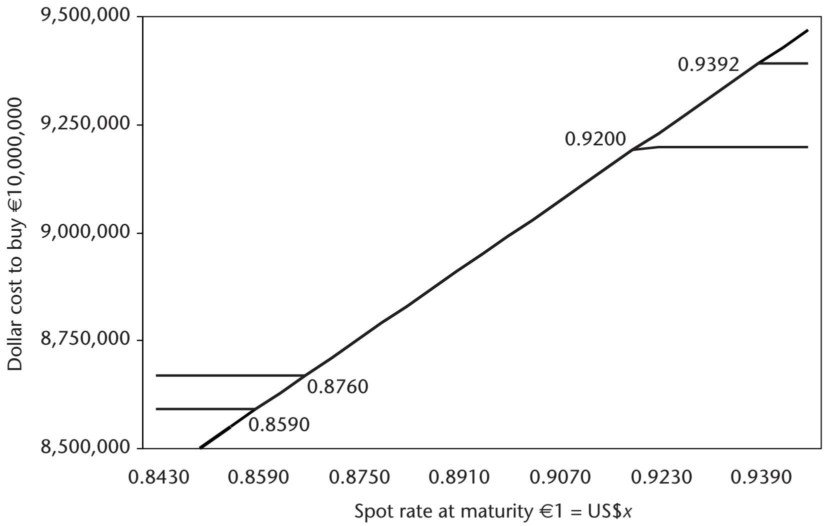


FIGURE 11.9 Zero premium collars

strike price of the call is higher than the strike price of the put, at most, one of the options will be exercised at expiry.

In effect, the importer gives up the possible benefit of the exchange rate falling below 0.8760 to avoid paying the premium on the bought call. This may represent a less painful sacrifice, because it requires no initial outlay of cash and the importer knows he or she will be buying the euros at a rate somewhere between 0.8760 (which is attractive) and 0.9200 (which is not too unattractive). In dollar terms the importer knows that the cost of buying €10,000,000 will be somewhere between US$8,760,000 and US$9,200,000 (Figure 11.9).

The above zero premium collar was constructed using a bought euro call and a sold euro put with premiums equal to 1.0%. Similar zero premium collars can be constructed, based on strike prices that imply equal offsetting premiums at other levels. The lower the premium, the wider will be the collar. For example, selling a 0.8590 put would earn a premium of 0.5% which would buy a 0.9392 call, and so on.

The zero premium collars will lie between the unhedged line and the forward line. Indeed, it can be seen that the forward rate is the limiting case of a collar where the strike price of the put equals the strike price of the call. The unhedged line is the other limiting case where the put and the call are so far out-of-the-money that their premiums are effectively zero.

In practice, bid-offer spreads will skew the collar slightly against the price-taker and in favour of the price-maker.

ILL-FITTING COLLAR

Alternative 4(a)

If a collar is constructed by buying and selling in-the-money options rather than out-of-the-money options, the collar will be reversed. For example, if the importer purchased a 0.8900 call and sold a 0.9047 put for zero net premium, the collar would be as in Figure 11.10.

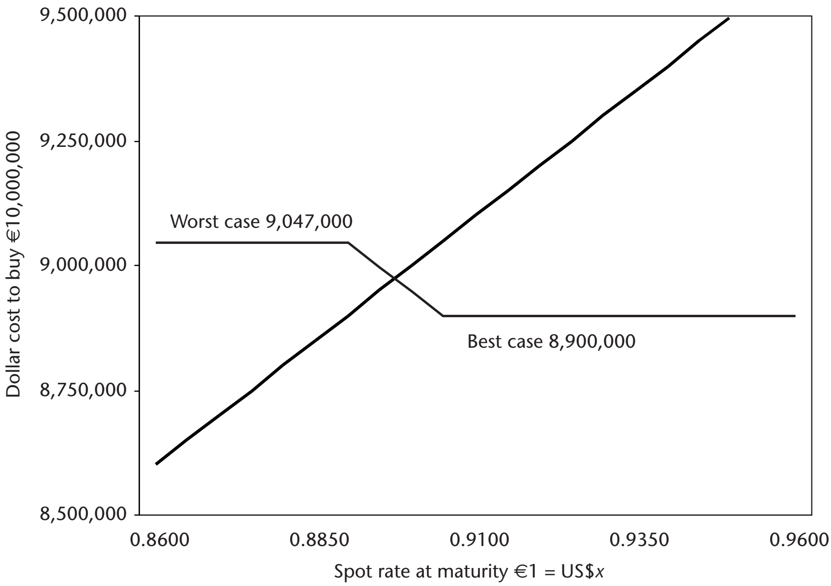


FIGURE 11.10 Ill-fitting collar

DEBIT COLLAR

Alternative 4(b)

If the premium received from the sold option is less than the premium paid for the bought option, the collar buyer will effectively pay a net premium. This is called a *debit collar* (Figure 11.11).

For example, if the importer bought a call (strike 0.9100) for 1.37% and sold a put (strike 0.8700) for 0.79%, he or she would pay a smaller net premium than if the importer had simply bought the call.

$$\begin{aligned}\text{FV(Net premium)} &= \text{FV(Premium received)} - \text{FV(Premium paid)}\\ &= 71{,}633.25 - 124{,}224.75\\ &= -52{,}591.50\end{aligned}$$

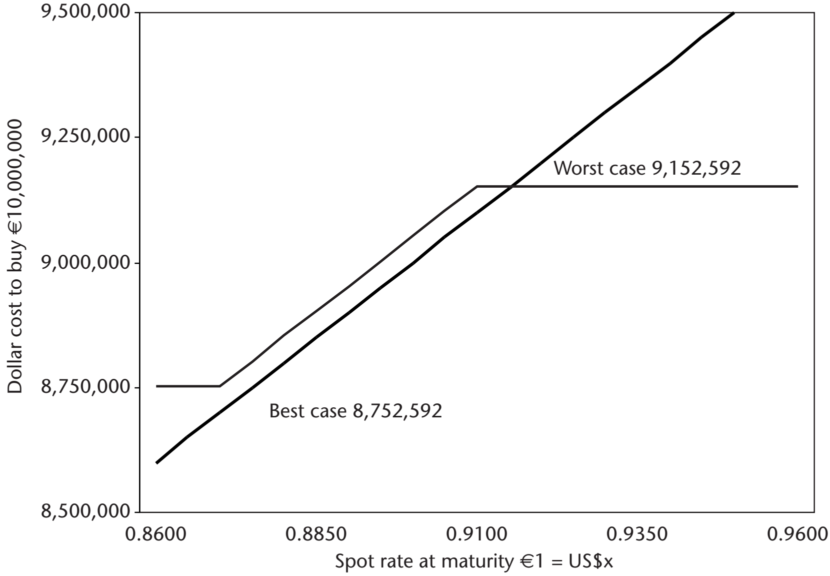


FIGURE 11.11 Debit collar hedging imports

The worst case under a debit collar will be worse than the zero premium collar worst case by the amount of the net premium payable. The best case under the debit collar will be better than the zero premium collar best case because the put is written at a higher strike price.

CREDIT COLLAR

Alternative 4(c)

If the premium received from the sold option is greater than the premium paid for the bought option, the collar buyer will effectively receive a net premium. This is called a *credit collar* (Figure 11.12).

For example, if the importer bought a call (strike 0.9300) for 0.71% and sold a put (strike 0.8900) for 1.58%, he or she would earn a net premium of 0.87%.

$$\begin{aligned}\text{FV(Net premium)} &= \text{FV(Premium received)} - \text{FV(Premium paid)}\\ &= 143{,}266.50 - 64{,}633.25\\ &= +78{,}887.25\end{aligned}$$

The worst case under a credit collar will be better than the zero premium collar worst case by the amount of the net premium received. The best case under the credit collar will be worse than under the zero premium collar because the dollar call is written at a lower strike price.

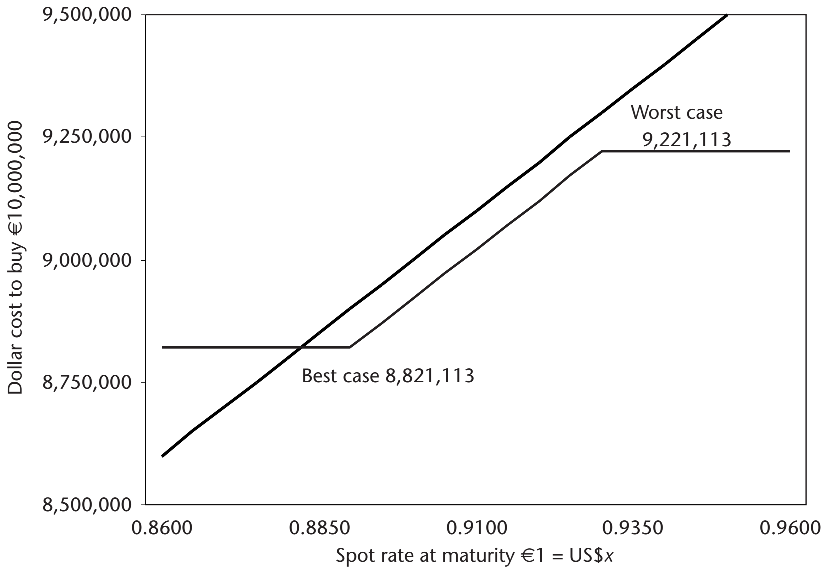


FIGURE 11.12 Credit collar hedging imports

PARTICIPATING OPTIONS

Alternative 5

Participating options can be generated by buying calls and selling puts with the same strike price, but for different face values. They are also known as participating forwards, percentage forwards and various other names.

Given that the premium to purchase a call with strike price 0.9100 is 1.37% and the premium for selling a put with the same strike price is 2.74%, the importer could hedge with a zero premium participating option by buying a 0.9100 call for face value €10,000,000 and writing a 0.9100 put for face value €5,000,000.

$$\begin{aligned} \text{FV(Premium received)} &= 5{,}000{,}000 \times 0.9100 \times 0.0274 \times (1 + 0.03/4) \\ &= \text{US\$}124{,}224.75 \\ \text{FV(Premium paid)} &= 10{,}000{,}000 \times 0.9100 \times 0.0137 \times (1 + 0.03/4) \\ &= \text{US\$}124{,}224.75 \\ \therefore \text{FV(Net premium)} &= 0 \end{aligned}$$

If at expiry the exchange rate is above the strike price (0.9100), the call will be in-the-money and the put will be out-of-the-money. The importer will exercise the call, thereby buying €10,000,000 at the worst case rate of 0.9100 for US$9,100,000 (Figure 11.13). The put will be allowed to lapse.

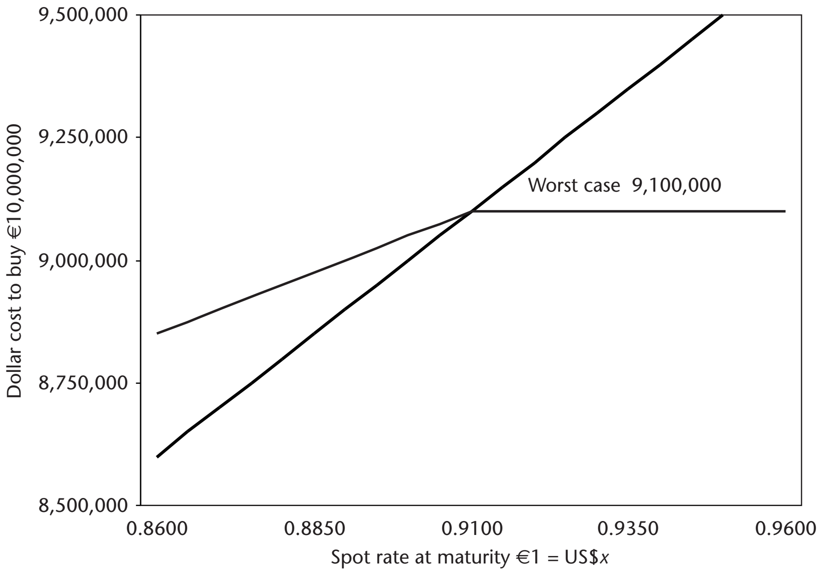


FIGURE 11.13 Zero premium participating option hedging imports

If at expiry the exchange rate is below the strike price (0.9100), the call will be out-of-the-money and the put will be in-the-money. The importer will abandon the call. The importer's counterparty will exercise the put. As a result the importer will buy €5,000,000 at the strike price 0.9100 and then buy the remaining €5,000,000 at the prevailing market spot rate, which is lower and therefore better than 0.9100.

By buying a participating option, the importer hedges against the risk of a rising exchange rate and at the same time is positioned to be able to enjoy part of the benefit of a falling exchange rate. In this case, if the exchange rate rises above 0.9100, the importer is 100% hedged, but if the exchange rate falls below 0.9100 he or she is effectively 50% unhedged. The *participation rate* is 50%.

In general, the participation rate is a function of the amounts of the two options.

$$\boxed{\begin{aligned}&\text{Participation rate\%}\\ &\quad=\frac{\text{amount of bought option} - \text{amount of sold option}}{\text{amount of bought option}}\times 100\end{aligned}} \tag{11.1}$$

$$= \frac{10{,}000{,}000 - 5{,}000{,}000}{10{,}000{,}000}\times 100$$

$$= 50\%$$

The importer pays the premium to buy an out-of-the-money call by writing an in-the-money put at the same strike price.

Similar zero premium participating options can be generated at different strike prices. The worst the strike price, the higher will be the participation rate. In the above example, setting the strike price at 0.9100 produces a participation rate of 50%. If the strike price were set at 0.9173, the respective premiums on the call and the put would be 1.09% and 3.27%. The zero premium participating option would involve buying a call for €10,000,000 and writing a put for €3,333,333. The participation rate would therefore be 66.6%.

The limiting cases of a participating option are the forward rate and a bought option. As the strike price approaches the forward rate, the participation rate approaches zero. A bought option is a participating option with 100% participation.

If the participation rate is held constant, it is possible to generate debit or credit participating options by varying the strike price.

PARTICIPATING COLLARS

Alternative 6

Another set of strategies can be generated by writing and buying opposite types of options with different strike prices and different face values. These are known as *participating collars*.

The importer could hedge with a zero premium participating collar by buying a call (strike 0.9200) for face value €10,000,000 and writing a put (strike 0.8980) for face value €5,000,000.

$$\text{FV (Premium paid)} = \text{FV (Premium received)} = \text{US\$90,675.00}$$
$$\text{FV (Net premium)} = 0$$

If at maturity the exchange rate is above 0.9200, the importer exercises the bought call and the sold put lapses. In the worst case, the importer buys €10,000,000 for US$9,200,000 (Figure 11.14). If at maturity the spot rate is in the range 0.8980 to 0.9200, the €10,000,000 would be purchased at the market spot rate. If the exchange rate is below 0.8980, the sold put will be exercised. The importer will buy €5,000,000 at 0.8980 through the put and the remaining €5,000,000 at the prevailing market rate, thereby enjoying 50% participation in any rise in the exchange rate below 0.8980.

By choosing different strike prices, the importer can generate different outcomes. By varying the strike price of the bought call, he or she can raise or lower the worst case rate. However, the strike price of the bought option can be no better than the forward rate (in which case the participating collar reduces to a forward). The importer can vary the participation rate by

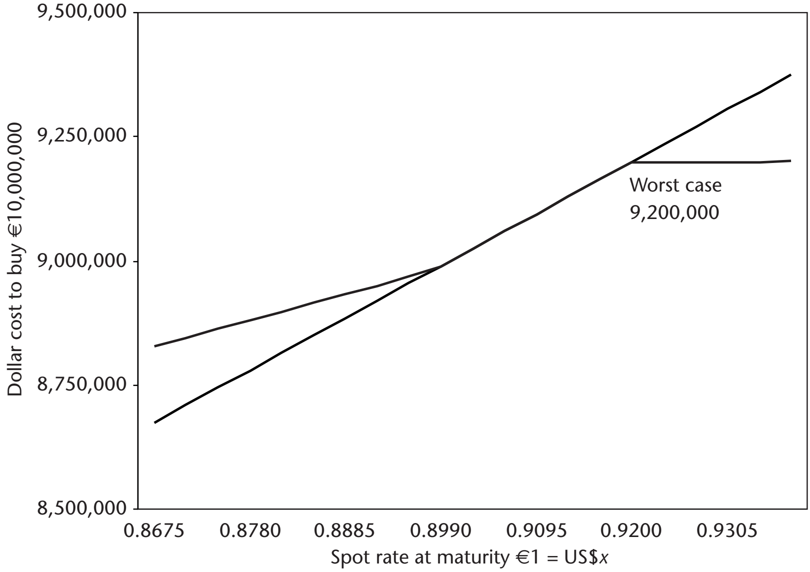


FIGURE 11.14 Participating collar hedging imports

raising or lowering the strike price of the sold option. As with the participating option, there is a trade-off between the turning point and the participation rate. The importer can enjoy a higher participation rate only by dropping 100% participation earlier.

Again, it is possible to create debit or credit participating cylinders.

There is a very large number of possible strategies that can be generated with various combinations of option products. This gives the person with a foreign currency exposure enormous flexibility in tailoring a risk profile to suit his or her particular views and needs.

Applications of option combinations for exporters, foreign currency borrowers and investors are covered in the practice problems.

PRACTICE PROBLEMS

11.1 Participating collar

An exporter with the identical exposure as in Example 11.2 enters into a participating collar to hedge euro receivables. The exporter buys a euro put/dollar call with the strike of 0.8762 for €1,000,000 at a premium of 1.0% and writes a euro call/dollar put with the strike of 0.9000 for €600,000 at a premium of 1.84%.

(a) Calculate the future value of the net premium payable in dollars.
(b) Calculate the proceeds from selling €1,000,000 if the spot rate at maturity is:

(i) 0.8662
(ii) 0.8862
(iii) 0.9062

11.2 Participating put
A foreign currency borrower with the same exposure as in Example 11.3 constructs a participating option to hedge Swiss franc liabilities. The borrower buys a US dollar put/Swiss franc call for SF25,395,300 with a strike of 1.2300 at a premium of 3.0% and writes a US dollar call/Swiss franc put for SF12,697,650 with a strike 1.2300 at a premium of 2.4%.

(a) Calculate the future value of the net premium payable in dollars.
(b) Calculate the dollar cost of repaying the Swiss franc loan principal plus interest if the spot rate at maturity is:

(i) 1.2000
(ii) 1.2400
(iii) 1.3000

(c) Calculate the effective borrowing cost in percent per annum of the Swiss franc loan if the spot rate at maturity is:

(i) 1.2000
(ii) 1.2400
(iii) 1.3000

11.3 Foreign currency investor
A funds manager with the same exposure as in Example 11.5 buys a collar by buying a dollar call at 110.00 for ¥1,111,152,778 at a premium of 3.25% and writing a dollar put at 109.00 for ¥777,806,945 at a premium of 2.00%.

(a) Calculate the effective yield if the spot rate at maturity is:

(i) 100.00
(ii) 110.00
(iii) 120.00

(b) If the spot rate at maturity is 114.00, calculate the effective yield percent per annum versus being:

(i) unhedged
(ii) invested in dollars
(iii) hedged with a bought dollar call (strike 110.00)

11.4 Foreign exchange trader – 2 for 1 strategy

A 2 *for 1 strategy* refers to the practice of buying the option required to hedge an underlying exposure and selling twice the face value of the opposite type of option (call or put) usually to earn enough premium to make the net premium zero.

One month ago, a foreign exchange trader bought £10,000,000 against US dollars at an outright 4 month forward rate of 1.4800. The spot rate has since risen to 1.5150 and the 3 month forward rate is now 1.5100. The 3 month (90/360) dollar interest rate is 3.00% p.a.

The trader considers buys a sterling put (strike 1.5100) premium 2.0% for face value £10,000,000 and sells a sterling call (strike 1.5200) premium 1.0% for twice the face value (£20,000,000).

(a) Calculate the future value of the net premium in dollars.
(b) Calculate the profit if the spot rate at expiry is:

   (i) 1.4500
   (ii) 1.5000
   (iii) 1.5500

(c) Draw the profit profile showing profit against various possible exchange rates at expiry.

CHAPTER 12

Option Derivatives

This chapter examines the pricing and applications of option derivative products such as digital options, barrier options and several other types of 'exotic' options as well as other structures that take advantage of the correlation between different variables. Options give people with currency exposures choices. Option derivative products provide even greater flexibility over how they manage their exposures.

DIGITAL OPTIONS

A *digital option* has a fixed pay-out regardless of how far it is in-the-money. Digital options are also known as binary options.

An *at-expiry* digital has its pay-out determined by the market price at its expiry. A *one-touch* digital pays out if the market price ever reaches the strike price during the life of the option. The pay-out for a one-touch option can occur at either the time of touching or at expiry depending on the terms of the contract.

Betting on the outcome of a toss of a coin is a simple everyday example of a digital option. By betting on heads a person agrees to pay a specified amount (premium), e.g. $100, if the result is tails and to receive a pay-off of the same amount ($100) if the result is heads.

Figure 12.1 shows the pay-off diagram for a person who has bet $100 on heads. The vertical line is a typical element of the pay-off for a digital option.

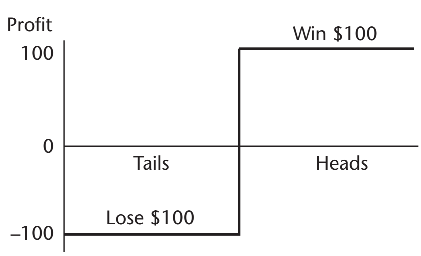


FIGURE 12.1 Digital option: heads or tails

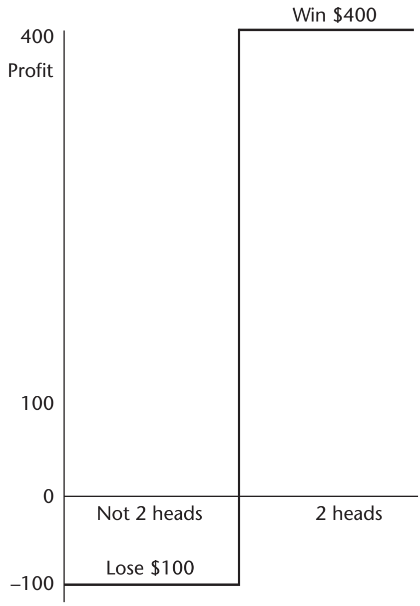


FIGURE 12.2 Digital option: 2 heads

If two coins are tossed there are three possible outcomes: 2 heads (probability ¼), 2 tails (probability ¼) and a head and a tail (probability 2/4). The pay-off for a person who bets $100 on 2 heads would be as in Figure 12.2.

The pay-out reflects the probability of the outcome occurring. The premium is the present value of the expected pay-out.

Outcome	*Pay-out*	*Probability*	*Expected value*
2 Heads	400	¼	100
2 Tails	0	¼	0
1 Head, 1 Tail	0	½	0
Total			**100**

PRICING AN AT-EXPIRY DIGITAL

At-expiry digital options can be priced using the same technique as standard calls and puts. The premium is the present value of the expected pay-out.

EXAMPLE 12.1

Calculate the premium for a 6 month US dollar call/yen put which pays US$1,000,000 if at expiry spot dollar/yen is 110 or above given the same scenario as in Example 10.6. The binomial tree and probabilities of each possible outcome will be the same as in Example 10.6, but the pay-out for the digital option will be different.

Possible outcomes	*Probability*	*Pay-out*	*Expected value*
118	1/64	1,000,000	15,625.00
112	6/64	1,000,000	93,750.00
106	15/64	0	0
100	20/64	0	0
94	15/64	0	0
88	6/64	0	0
82	1/64	0	0
Total			**109,375.00**

The expected pay-out of the option is $ 109,375.00. If the premium were paid at expiry this would be its fair value. The general practice is for the premium to be paid up-front for digital options as well as for standard calls and puts. If the 6 month dollar interest rate is 5% p.a., the up-front premium would be:

$$\frac{\$109{,}375.00}{1+0.05\times 6/12}=\$106{,}707.32$$

REVERSE BINOMIAL PRICING METHOD

Alternatively, the premium of the digital call in Example 12.1 could be priced using the reverse binomial method (Exhibit 12.1). This involves starting with the pay-out for each of the possible outcomes at expiry and working backwards through the binomial tree to determine the expected value at inception.

If at the end of the 6 months spot is $ 1 = ¥118 or ¥112, the pay-out would be $1,000,000. It follows that if there is equal probability that the price will move up or down ¥3 in the final month, if the price is ¥115 at the end of the 5th month then the value of the option at that time must be $1,000,000.

Similarly, because the pay-out at the end of the 6 months would be $1,000,000 at ¥112 and zero at ¥106, it follows that the value of the option at the end of the 5th month must be $500,000 if the price is ¥108.

By working backwards through the binomial tree it can be seen that the future value of the premium is $109,375, so the premium is $106,707.32.

EXHIBIT 12.1 Reverse binomial method

Month	0	1	2	3	4	5	6	Outcome
							1,000,000	118
						1,000,000		
					750,000		1,000,000	112
				500,000		500,000		
			312,500		250,000		0	106
		187,500		125,000		0		
	109,375		62,500		0		0	100
		31,250		0		0		
			0		0		0	94
				0		0		
					0		0	88
						0		
							0	82

The reverse binomial method is useful because it shows the at-expiry value of the option at each stage of the tree. For example, if at the end of the 2nd month the price is ¥100, the then (future) value of the option would be $62,500. It also provides a means for pricing path dependent options such as one-touch digitals.

PRICING ONE-TOUCH DIGITALS

EXAMPLE 12.2

Calculate the premium of a 6 month dollar call/yen put which pays $1,000,000 if the spot rate touches or exceeds ¥110 at any time during the life of the option given the same parameters as in Example 12.1.

The one-touch digital will pay $1,000,000 at expiry if the price at the end of the 4th month is ¥112 (because it exceeds ¥110). Accordingly, the value in the reverse binomial tree (Exhibit 12.2) at the 4th month, ¥112 outcome, which could be designated as the (4, 112) node must be $1,000,000. It follows that the values of nodes (3,109), (2, 106), (1, 103) and (0, 100) would be 625,000; 375,000; 218,750 and 125,000 respectively as shown below:

$$\text{Premium} = \text{PV}(125{,}000) = \frac{125{,}000.00}{(1 + 0.05 \times 6/12)} = \mathbf{\$121{,}951.12}$$

The premium of the one-touch digital is higher than that of the corresponding at-expiry digital because the expected pay-out is higher to the extent that it is possible for the price to touch the strike during the life of

EXHIBIT 12.2 Reverse binomial method: pricing one-touch digital

Month	0	1	2	3	4	5	6	Outcome
							1,000,000	118
						1,000,000		
					1,000,000		1,000,000	112
				625,000		500,000		
			375,000		250,000		0	106
		218,750		125,000		0		
	125,000		62,500		0		0	100
		31,250		0		0		
			0		0		0	94
				0		0		
					0		0	88
						0		
							0	82

the option but expire below it. The difference between \$121,951.12 and \$106,707.32 dimensions the extra value of the one-touch feature.

CLOSED FORM PRICING FORMULA

The formula for at-expiry digitals is simpler than the Black–Scholes formula because the pay-out is a fixed amount and the probabilities are the same as for standard calls and puts:

$$C = AN(d_2)e^{-rt} = \frac{AN(d_2)}{(1+r_T t)} \tag{12.1}$$

where A is the fixed pay-out (amount) from a call digital and the other notation is as in Equation (10.16)

As the option approaches expiry the pay-off function approaches the vertical line at the strike price (Figure 12.3).

One-touch digitals are more complicated to price because the probability of being exercised must take account of the possibility of the trigger being touched.

APPLICATIONS OF DIGITAL OPTIONS

Digital options have a number of powerful applications for people managing currency exposures.

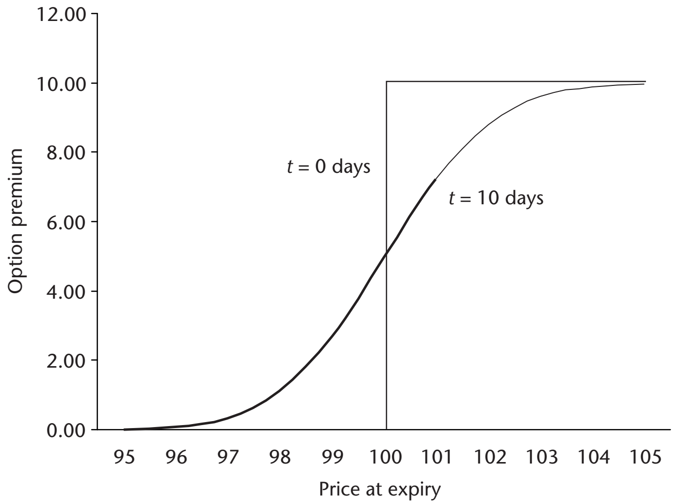


FIGURE 12.3 At expiry digital call with strike price 100 and pay-out 10

Pay-later option

A *pay-later option* is one for which the premium is only paid if the option expires in-the-money. These are also known as *contingent premium options*. If paid, the premium must be larger than the up-front premium of the corresponding standard option.

The importer in Example 11.1 could reshape the exposure by selling or buying digital options. For example, the importer could buy a 0.9200 call for a premium of US$90,000 and pay for it by selling an at-expiry digital which would pay US$200,000 if, but only if, the spot rate at expiry turns out to be 0.8910 or below (Figure 12.4).

The maximum cost for the importer to buy the €10,000,000 to pay for the imports would be US$9,200,000 if the spot rate at expiry turns out to be 0.9200 or higher. If the spot rate at expiry turns out to be 0.8910 or higher, the importer's cost would be lower than under the bought 0.9200 call by the future value of the premium saved.

If at expiry the spot rate turns out to be below 0.8910, the importer's net cost will be US$200,000 greater than the unhedged case or US$110,000 greater than under the ought 0.9200 call.

The importer may prefer the pay-later call because the premium only has to be paid when the rate has moved in the importer's favour and the worst case is not as bad as with the straight bought call.

Many variations of this structure are possible. The importer could sell digitals at other strike prices. For example, instead of selling the 0.8910 digital, the importer might fund the premium of the bought 0.9200 call by

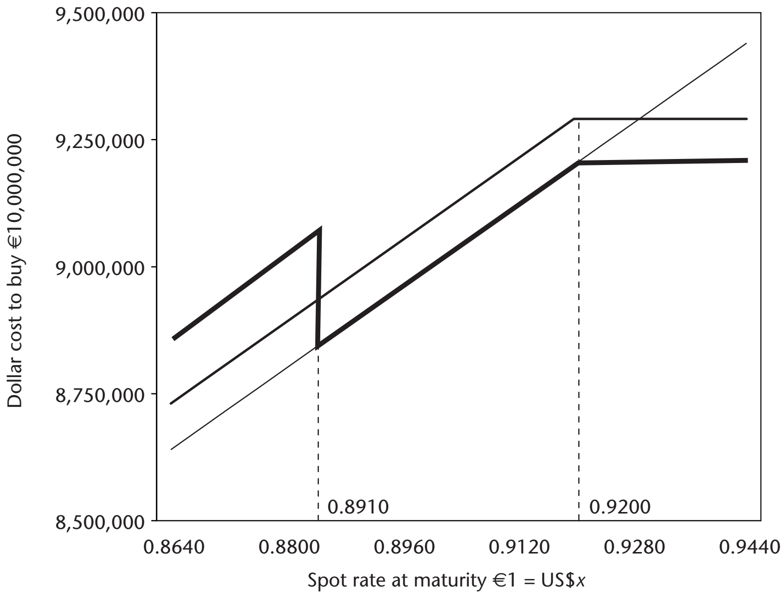


FIGURE 12.4 Pay-later call: protecting importer

selling a two digitals – one paying US$150,000 if the spot rate turns out to be less than 0.8850 and another paying US$50,000 if it turns out to be less than 0.8780. This would create two steps in the pay-off diagram.

Currency-linked note

A *currency-linked note* is an instrument for which the yield is a function of the exchange rate. Many structures are possible. A popular one gives the investor a better than market yield provided the exchange rate remains within a specified range but a lower than market yield if it moves outside of that range.

If the 6 month dollar interest rate is 5% p.a. when spot dollar/yen is 110.00, an investor might be able to purchase a currency linked note for which the yield will be 10% p.a. provided the dollar remains within a range of 100.00 to 120.00 for the entire 6 months, but only 2% p.a. if at any time the spot rate touches or moves above 120.00 or below 100.00.

The currency-linked note would normally be packaged by the bank as a single product. To construct it the bank would merely sell two one-touch digitals. The future value of the premium received would be sufficient to lift the yield to 10% p.a. provided neither level is touched. The pay-out would be such that the yield is reduced to 2% p.a. if either digital is exercised (Figure 12.5).

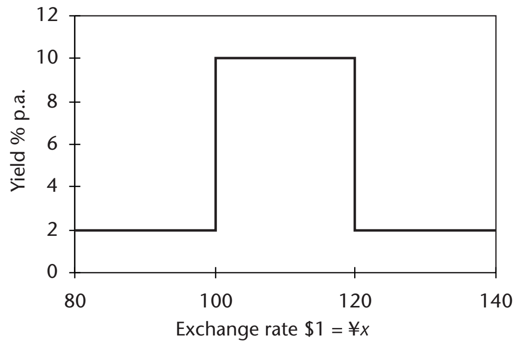


FIGURE 12.5 Currency linked note

This sort of product appeals to many investors because their capital is guaranteed and they are assured a minimal acceptable return with the possibility of a yield that is much higher than otherwise available. The investor is effectively betting that the exchange rate will be less volatile than is being priced into the options.

BARRIER OPTIONS

Path dependent options are those whose pay-off depend on the path which the market price follows through the life of the option.

Standard calls and puts as well as at-expiry digitals are not path dependent because their pay-out depends only on where the market price is at expiry versus the strike price.

One-touch digital options are one example of path dependent options. The most common group of path dependent options are known as barrier options. Other examples of path dependent options include average rate or average strike options and look-back options.

Barrier options are options which can be knocked-out or which knock-in if the market price reaches a specified level.

Knock-out (or kick-out) options have a zero pay-out if the barrier level is reached even if the market price at expiry is better than the strike price. *Knock-in (or kick-in) options* have zero pay-out unless the option expires in-the-money (i.e. the market price at expiry is better than the strike price and at sometime during the life of the option the market price has reached the barrier level).

The option portrayed in Figure 12.6 is an 8 month US dollar call/yen put with strike price at ¥102.00 but which knocks-out at ¥108.00. The solid line

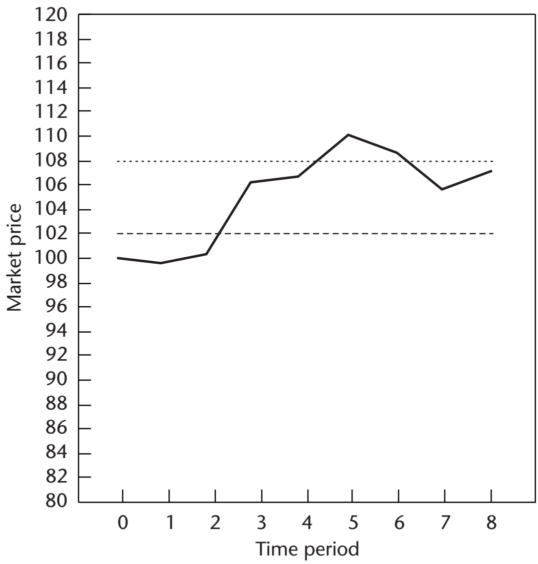


FIGURE 12.6 Knock-out dollar call/yen put: strike = ¥102, knock-out = ¥108

shows one of the many possible paths that the spot rate could follow during the life of the option. At inception the spot rate was ¥100.00. At expiry it was ¥107.90. This particular option would have been knocked-out during the fourth month when the spot rate went through ¥108.00. So, even though the spot rate at expiry was well above the ¥102.00 strike price, the pay-out would be zero.

If instead, the option in Figure 12.6 had been a knock-in at ¥108.00, the pay-out would have been:

$$
\begin{aligned}
\text{Pay out} &= \text{Expiry price} - \text{Strike price} \\
&= 107.90 - 102.00 \\
&= ¥5.90
\end{aligned}
$$

The sum of the pay-out for the knock-out and the pay-out for the knock-in will equal the pay-out for the standard option with the same strike price provide the barriers are at the same level. Only one of the knock-out and knock-in can be exercised if the barrier level is the same for both. In the above case the pay-out from the knock-out is zero and the pay-out from the knock-in is ¥5.90. The sum of the two equals the pay-out from the standard call, which would be ¥5.90.

It follows that the sum of premiums for the knock-out and the knock-in will equal the premium for the standard call provided the barrier level is the same and the options have the same strike price.

$$\boxed{\text{Knock-out premium} + \text{Knock-in premium} = \text{Call premium}} \qquad (12.2)$$

PRICING KNOCK-OUTS

The reverse binomial method can be used to price the standard dollar call/ yen put in Example 10.6.

EXHIBIT 12.3 Dollar call strike = ¥100

Month	0	1	2	3	4	5	6	Outcome
							18,000,000	118
						15,000,000		
					12,000,000		12,000,000	112
				9,000,000		9,000,000		
			6,375,000		6,000,000		6,000,000	106
		4,312,500		3,750,000		3,000,000		
	2,812,500		2,250,000		1,500,000		0	100
		1,312,500		750,000		0		
			375,000		0		0	94
				0		0		
					0		0	88
						0		
							0	82

Premium = PV(¥2,812,500) = ¥2,743,902

The reverse binomial method can also be used to price knock-out options.

EXAMPLE 12.3

Calculate the premium of a 6 month dollar call/yen put with strike price ¥100.00 which knocks-out at ¥112.00 given a monthly up–down movement of ¥3.

The binomial tree will be similar to that above, but all nodes at ¥112 or above will have zero value because these will be knocked-out. The revised binomial tree will be as below, yielding a premium for the knock-out call of ¥1,280,488.

EXHIBIT 12.4 Dollar call strike = ¥100, knock-out = ¥112

Month	0	1	2	3	4	5	6	Outcome
							0	118
						0		
					0		0	112
				1,500,000		3,000,000		
			1,875,000		3,000,000		6,000,000	106
		1,687,500		2,250,000		3,000,000		
	1,312,500		1,500,000		1,500,000		0	100
		937,500		750,000		0		
			375,000		0		0	94
				0		0		
					0		0	88
						0		
							0	82

$$\text{Premium} = \text{PV}(¥1{,}312{,}500) = ¥1{,}280{,}488$$

The premium of the ¥100 call which knocks-out at ¥112 is less than half the premium of a standard ¥100 call. The knock-out option is much cheaper but it provides much less value. The branches of the binomial tree which are knocked-out have considerable value. Furthermore, some of the branches which were not knocked-out also lost value because as the spot price approaches the ¥112 knock-out level the probability of being knocked-out increases.

PRICING KNOCK-INS

The relationship in Equation (12.2) can be used to price knock-in options. If the premium for a standard ¥100 call is ¥2,743,902 and the premium for the ¥100 call which knocks-out at ¥112 is ¥1,280,488, then the premium for a ¥100 call which knocks-in at ¥112 must be:

$$\begin{aligned}\text{Premium of knock-in} &= \text{Premium of call} - \text{Premium of knock-out}\\ &= 2{,}743{,}902 - 1{,}280{,}488\\ &= ¥1{,}463{,}414\end{aligned}$$

It follows from Equation (12.2) that options which may knock in or out always have lower premiums than their corresponding standard option. The barrier options only have value some of the time that the standard option does, so they command a lower premium.

CLOSED FORM SOLUTIONS

Knock-outs and knock-ins can also be priced using formulae. These formulae look complicated because in addition to the variables involved in pricing standard options they also need to incorporate the barrier level and whether the barrier is triggered in an upward or downward direction. The options priced in Example 12.3 are an up-and-out dollar call and an up-and-in dollar call because the barrier is triggered if the spot rate moves *up* from its current level. It is also possible to have down-and-out and down-and-in options which get triggered if the spot rate *falls* to the barrier level.

The formulae shown in this section are taken from John C. Hull: *Options, Futures & Other Derivatives*, 4th edn, published by Prentice Hall, 2000. The notation used here differs from that used by Hull to be consistent with the usage elsewhere in this book.

A down-and-out call is a regular call that ceases to exist if the exchange rate reaches a certain barrier level, B, below the initial exchange rate. The opposite type is a down-and-in call that comes into existence only if the exchange rate reaches the barrier level.

If $B \le K$, the value of a down-and-in call at time zero is

$$c_{\mathrm{di}} = Se^{-yt}(B/S)^{2\lambda} N(y) - Ke^{-rt}(B/S)^{2\lambda-2} N(y - \sigma\sqrt{t})$$

where

$$\lambda = \frac{r - y + \sigma^2/2}{\sigma^2}$$

$$y = \frac{\ln[B^2/SK]}{\sigma\sqrt{t}} + \lambda\sigma\sqrt{t}$$

Since the value of a standard call equals the value of a down-and-in call plus the value of a down-and-out call, the value of the corresponding down-and-out call is

$$c_{\mathrm{do}} = c - c_{\mathrm{di}}$$

If $B \ge K$, then

$$c_{\mathrm{do}} = SN(x_1)e^{-yt} - Ke^{-rt}N(x_1 - \sigma\sqrt{t}) - Se^{-yt}(B/S)^{2\lambda}N(y_1)$$
$$+ Ke^{-rt}(B/S)^{2\lambda-2} N(y_1 - \sigma\sqrt{t})$$

and

$$c_{\mathrm{di}} = c - c_{\mathrm{do}}$$

where

$$x_1 = \frac{\ln(S/B)}{\sigma\sqrt{t}} + \lambda\sigma\sqrt{t}$$

$$y_1 = \frac{\ln(B/S)}{\sigma\sqrt{t}} + \lambda\sigma\sqrt{t}$$

An up-and-out call is a standard call that ceases to exist if the price reaches a barrier level, B, that is higher than the current price. An up-and-in call is a regular call that comes into existence only if the barrier is reached.

When $B < K$, the value of an up-and-out call, c_{uo} is obviously zero (presumably, no such option would ever be written). The value of the up-and-in call, c_{ui}, is c.

When $B \geq K$,

$$c_{ui} = SN(x_1)e^{-yt} - Ke^{-rt}N(x_1 - \sigma\sqrt{t}) - Se^{-yt}(B/S)^{2\lambda}[N(-y) - N(-y_1)]$$
$$+ Ke^{-rt}(B/S)^{2\lambda-2}[N(-y+\sigma\sqrt{t}) - N(-y_1+\sigma\sqrt{t})]$$

and

$$C_{uo} = C - C_{ui}$$

Put barrier options are defined similarly. An up-and-out put is a put option that ceases to exist when a barrier, B, that is greater than the current price is reached. An up-and-in put is a put that comes into existence only if the barrier is reached. When the barrier, B, is greater than or equal to the strike price, K, their prices are

$$p_{ui} = -Se^{-yt}(B/S)^{2\lambda}N(-y) + Ke^{-rt}(B/S)^{2\lambda-2}N(-y+\sigma\sqrt{t})$$

and

$$p_{uo} = p - p_{ui}$$

When $B < K$,

$$p_{uo} = -SN(-x_1)e^{-yt} + Ke^{-rt}N(-x_1+\sigma\sqrt{t})$$
$$+ Se^{-yt}(B/S)^{2\lambda}N(-y_1) - Ke^{-rt}(B/S)^{2\lambda-2}N(-y_1+\sigma\sqrt{t})$$

and

$$p_{ui} = p - p_{uo}$$

A down-and-out put is a put option that ceases to exist when a barrier less than the current price is reached. A down-and-in put is a put option that comes into existence only when then barrier is reached.

When the barrier is greater than the strike price, $p_{do} = 0$ and $p_{di} = p$. When the barrier is less than the strike price,

$$p_{\mathrm{di}} = -SN(-x_1)e^{-yt} + Ke^{-rt}N(-x_1+\sigma\sqrt{t}) + Se^{-yt}(B/S)^{2\lambda}[N(y)-N(y_1)]$$
$$-Ke^{-rt}(B/S)^{2\lambda-2}[N(y-\sigma\sqrt{t})-N(y_1-\sigma\sqrt{t})]$$

and

$$p_{\mathrm{do}} = p - p_{\mathrm{di}}$$

All of the above valuations make the assumption that the probability distribution for the exchange rate at a future time is log-normal. The price of a barrier option can be quite sensitive to this log-normal assumption. Figure 12.7 is an example of up-and-out and up-and-in calls.

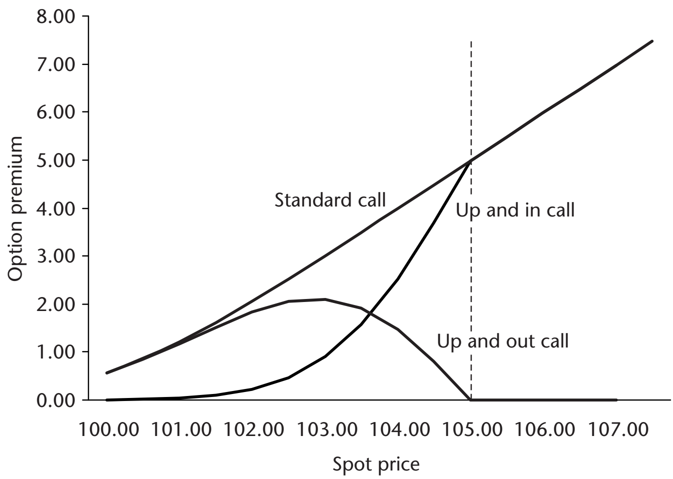


FIGURE 12.7 Up-and-out calls and up-and-in calls with strike price 100 and barrier at 105

An important issue for barrier options is the frequency with which the spot price is observed for purposes of determining whether the barrier has been reached. The analytic formulae above assume that the spot rate is observed continuously. In practice, the spot rate might be observed less frequently, say at the end the business day.

APPLICATIONS OF BARRIER OPTIONS

EXAMPLE 12.4

The importer in Example 11.1 could purchase a 3 month up-and-out euro call/US$ put with a strike price of 0.9000 which knocks-out at 0.9500 for a premium of 0.69% versus 1.84% for the standard call with the same strike price.

The advantage of the knock-out over the standard dollar call is the lower premium. The premium payable would be only US$61,850 versus US$165,600 for the standard call.

The disadvantage of the knock-out option is that if the spot rate rises to 0.9500 or above at any time during the 3 months then the importer loses the hedge. The importer could end up unhedged when the exchange rate is high and so at an unattractive level not to be hedged.

The importer might be prepared to risk having the hedge knocked-out at 0.9500 to save some of the premium if he feels that the spot rate is unlikely to breach 0.9500 during the period. Buying a standard call protects the importer against the exchange rate rising to any level above the strike price during the 3 months. If the importer were confident that the rate would not exceed 0.9500 buying a standard call would mean paying more premium than the importer considers necessary.

Alternatively, the importer could purchase a 3 month up-and-in call with a strike price of 0.9000 which knocks-in at 0.9500 for a premium of 1.15%.

Again, the advantage of the knock-in over the standard call is the lower premium. In this case the premium for the knock-in call would be US$103,150. This is US$62,450 less than the standard call.

The importer will be better off with the knock-in than with the knock-out if the spot rate ever reaches 0.9500 during the 3 month period, but worse off if it does not. With the knock-in the exporter will be hedged when he needs it most (that is, above 0.9500). The business might be able to absorb the loss of revenue if the spot rate is between 0.9000 and 0.9500 but not if it is above 0.9500. The knock-in provides disaster insurance. A similar outcome could be achieved by buying a standard call with strike at 0.9500. The pay-out from the 0.9000 call which knocks-in at 0.9500 will be greater than for the standard 0.9500 call because it would pay the difference between the price at expiry and 0.9000 whereas the standard 0.9500 call would only pay the amount in excess of 0.9500. Accordingly, the 0.9000 call that knocks-in at 0.9500 would command a higher premium than a plain vanilla 0.9500 call.

There is no best strategy before the event. The point is simply that the hedger has greater choice. Barrier options add to the flexibility available to hedgers. By varying the strike price and/or barrier level the hedger can pay for what he or she wants and no more.

KNOCK-OUT FORWARDS

The put call parity relationship (Equation 10.4) applies to barrier options.

$$\boxed{\text{Knock-out call} - \text{Knock-out put} = \text{Knock-out forward}} \qquad (12.3)$$

The strike prices and the knock-out level must correspond.

This makes it possible to create structures such as the *knock-out forward*. This enables a person to enter into a better than market forward rate that knocks-out if the spot rate breaches the barrier level. For example, if the premium of a 0.8825 call which knocks-out at 0.9500 is equal to the premium of a 0.8825 put which knocks-out at 0.9500, then it would be possible to construct a (zero premium) knock-out forward which would enable the importer to sell its euro receivables at 0.8825 unless the spot rate reaches 0.9500 during the 3 month period. If the spot rate stays below 0.9500, the importer achieves a forward rate which is 150 points better than the market forward rate of 0.8975 and no premium is payable.

The risk with using a knock-out forward is that the importer may find itself unhedged with the exchange rate above 0.9500 where it would most like to be hedged. However, in many cases the importer's alternative course of action in a rising exchange rate environment would be to remain unhedged hoping for the exchange rate to fall to a more attractive level. If so, the importer would be better off with the knock-out forward.

Reset options are options for which the strike price may change during the life of the option. *Ladder options* are reset options with multiple triggers which lock-in a new minimum amount of profit each time a trigger is reached. These are also known as ratchet, clique or lock-in options. These options are popular with traders who want to let profitable positions run but who want to avoid seeing their unrealized profits eroded or turn into losses if the rate moves against them.

Reset and ladder options are constructed by buying a series of better and better knock-in options which knock-in as the market moves further in the trader's favour and selling a series of knock-outs which knock-out as the new options knock-in.

For example, a trader who expects the euro to strengthen against the dollar buys a euro call with strike price 0.9000. To turn the call into a reset option it is necessary to also buy a call with a better strike price (say 0.8800) which knocks-in at say, 0.9500 and to sell a 0.9000 call that knocks-out at 0.9500. In this case, once the spot rate reaches 0.9500, the original 0.9000 call is effectively replaced with a 0.8800 call.

The process is repeated to create a ladder option. By buying another knock-in call with strike of say, 0.8600, which knocks-in at say, 0.9600, and selling a 0.8600 knock-out call which knocks-out at 0.9600, it is possible to construct a second step in the ladder.

The additional premium required to turn a standard option into a reset or ladder option is not usually very much if the knock-in level is far enough away from the starting rate and because the premium received from writing the knock-outs partly defrays the premium needed to buy the knock-ins.

COMBINATIONS

There are an unlimited number of possible combinations of standard calls and puts, knock-out, knock-in and digital options which can be used to shape the profile which best suits the person's view. No structure is best in all cases. There are always trade-offs. Finding the 'best' strategy is a matter of identifying a structure which best fits the needs and views of the person. It is always necessary to give up something to get something else. A lower premium can be obtained by accepting a worse strike price, by setting a knock-out level closer to the current rate or by selling some option and so on. Digital and barrier options add to the bag of tricks available to the hedger. It might not be necessary to use them, but it is worth knowing how to use them if it suits the circumstances.

OTHER PATH-DEPENDENT OPTIONS

Path-dependent options include average rate, average strike and look-back options.

An *average rate option* is one where the pay-off equals the difference between the average rate and the strike price or zero, whichever is higher. These are also known as Asian options.

The average rate is a function of the path which the exchange rate takes over the life of the option. It is usually calculated as the arithmetic average of daily rates taken at a specified time of day. If the spot rate follows the path shown in Figure 12.8 the average rate would be ¥104.00. The pay-out on a dollar call with strike price of ¥102.00 would be ¥2.00.

$$\begin{aligned}\text{Pay out} &= \text{Average rate} - \text{Strike price}\\ &= 104.00 - 102.00\\ &= ¥2.00\end{aligned}$$

An *average strike option* is one for which the strike price is the average rate. If the spot rate follows the path shown in Figure 12.8, so that the average rate is ¥104.00 and the spot rate at expiry is ¥107.00, then the pay-out of an average strike call would be ¥3.00.

$$\begin{aligned}\text{Pay out} &= \text{Expiry rate} - \text{Average rate}\\ &= 107.00 - 104.00\\ &= ¥3.00\end{aligned}$$

The sum of the pay-out from the average rate call and the corresponding average strike call equals the pay-out from the standard call with the same strike price. Similarly, the sum of the premiums of the average rate and corresponding average strike call will equal the premium of the standard

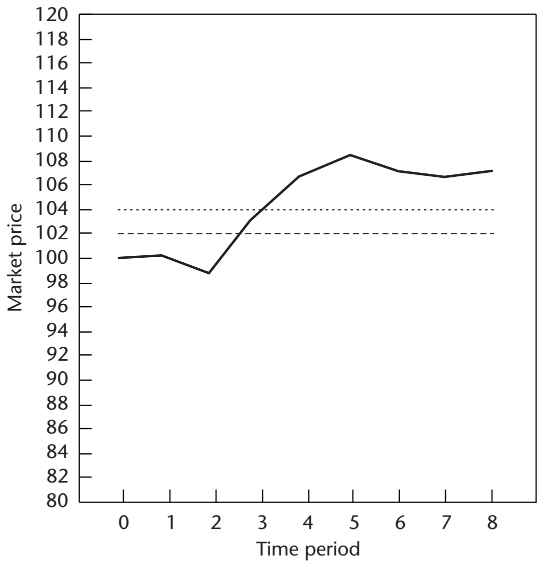


FIGURE 12.8 Average rate dollar call strike ¥102.00

call. It follows that average rate and average strike options have lower premiums than their corresponding standard options. This is to be expected because the volatility of the average must be lower than the volatility of the underlying spot rate.

Average rate and average strike options can, therefore, provide economical hedges for companies that have contractual arrangements with receipts or payments based on average exchange rates. For example, a newspaper company buys newsprint (i.e. paper) from a mill. Like most commodities paper is priced in US dollars. The newspaper company (the importer) and the mill (the exporter) have equal and opposite exchange rate exposures. They may agree that the price paid for the paper will reflect the average exchange rate during the period of shipment. If so, both would have exposure to the average exchange rate, which is less volatile than the spot rate. Both parties could use average rate or average strike options to economically and efficiently hedge their exposures.

A *look-back option* is one for which the pay-out is the peak difference between the strike price and the market price during the life of the option.

If the spot rate follows the path shown in Figure 12.9, the pay-out on a look-back dollar call with strike price ¥102.00 would be ¥10.00.

$$\begin{aligned}\text{Pay out} &= \text{Maximum value} - \text{Strike price}\\ &= 112.00 - 102.00\\ &= ¥10.00\end{aligned}$$

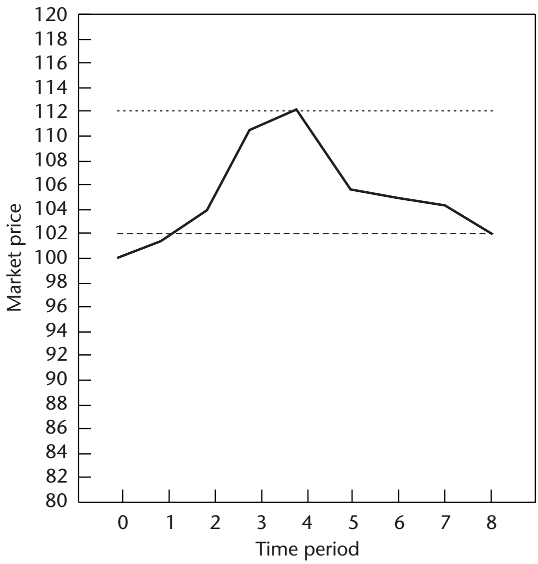


FIGURE 12.9 Look-back dollar call strike ¥102.00

Even though the spot rate at expiry in this example was ¥101.70 which is less than the strike price, the look-back call would pay-out ¥10.00.

Look-back options provide traders with the benefit of hindsight. They can achieve the best possible rate which occurred during the period. This makes look-back options sound very attractive. However, they are not used much in practice because the premium required to buy look-back options is typically large. Remember, the premium is the present value of the expected pay-out. Because the expected pay-out is high, the premium will be high.

A *shout option* gives its owner the right to lock-in a minimum pay-out equal to the intrinsic value at the time by 'shouting' i.e. declaring the intention to do so. A Japanese exporter buys a dollar call with strike price ¥102.00 to hedge his US dollar receivables which are due in 8 months' time. If the spot rate moves as in Figure 12.10 the exporter enjoys seeing the dollar rise during the first 4 months. In the fifth month the dollar starts to fall sharply. When the rate falls to ¥107.50 the exporter shouts. If the spot rate continues to fall as in Figure 12.10 then the pay-out from the option would be ¥5.50. On the other hand, if the rate at expiry is greater than ¥107.50, the pay-out would be as for a standard dollar call.

$$\begin{aligned}\text{Pay out} &= \max(\text{shout level} - \text{strike},\ \text{expiry rate} - \text{strike})\\ &= 107.50 - 102.00\\ &= ¥5.50\end{aligned}$$

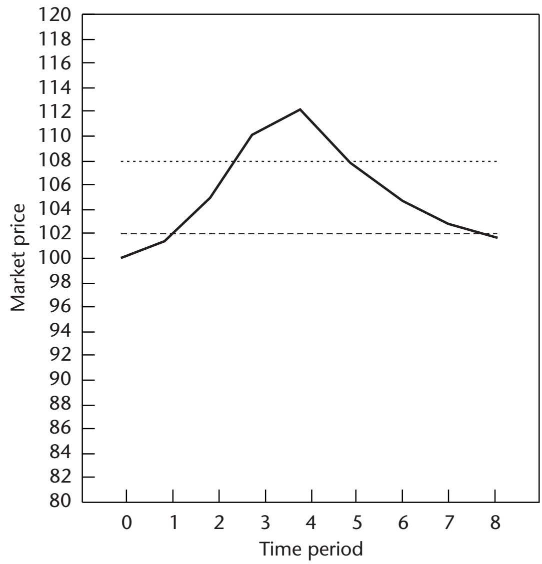


FIGURE 12.10 Shout option: dollar call with strike ¥102.00 shouted at ¥108.00

The premium for the shout option will be considerably less than for the look-back call because the expected pay-out is less. The owner of the shout option would only obtain the same pay-out as the look-back option if she shouts at exactly the peak of the market. If she shouts early she may miss out on an opportunity to shout later at a higher level. On the other hand, if she shouts late she may have missed the opportunity to lock-in a better rate than will be achieved again.

OTHER NON-PATH-DEPENDENT OPTIONS

There are other types of option, most of which are not used much in practice.

A *compound option* is an option on an option. The buyer of a compound option has the right, but not the obligation, to enter into an option with a specified strike price for a specified premium at a specified future date. Compound options are useful for companies bidding on contracts which, if won, will result in a foreign currency exposure.

A British company bidding on a Swiss contract will not know if it will have Swiss franc exposure until the contract is awarded in 3 months' time. If its bid is successful it will have Swiss franc income in a further 1 year's time. Its bid is based on prevailing exchange rates. If in 3 months' time it is awarded the bid, but by then the pound has appreciated against the franc, the project will not be as profitable as budgeted. The rate may even have moved so far that winning the bid could mean locking in a loss.

The company could purchase a 15 month SF put/sterling call for say, 5% premium. This would involve a considerable cash outlay which would be unnecessary if the contract is lost. If the contract is lost the company could sell a 12 month SF put/sterling call at the time to recover some of the 5% premium. However, the premium may be considerably lower, say 3%, because there has been 3 months' time decay, the pound may have appreciated against the franc and volatility may have fallen.

Instead, the company could purchase a compound option to enter into a 12 month SF put/sterling call with a premium of 4.5% in 3 months' time for a premium of only, say, 1% today. The initial cash outlay is only 1% versus 5%. If the company wins the contract it can purchase the 12 month SF put/sterling call for a further 4.5%. If the then premium for the 12 month SF put/sterling call is less than 4.5%, the company could allow the compound option to lapse and purchase the cheaper 12 month option in the market. If the company loses the contract it can allow the compound option to lapse, or, if the then premium on the SF put is greater than 4.5%, it could exercise the compound option and then sell a 12 month SF put to realize a profit which would offset all or part of the premium paid to buy the compound option.

Compound options give companies the ability to achieve the bought option profile at relatively little cost. They only have to pay for the bulk of the cover when they know they need it and can most afford to pay for it.

Chooser options are options which have a defined strike price and on a specified date the buyer can choose whether the option is a call or a put. They are also known as preference or double options.

Buying chooser options provides a hedge against extreme volatility. A person who is worried that the exchange rate could rise or fall dramatically, without having a feel for in which direction, could purchase a chooser option. If the price rises, he elects that the option is a call. If it falls, he can elect that it is a put.

An *instalment option* has the premium paid periodically. The owner of the option pays the premium in instalments usually at regular intervals and may elect to terminate the option at any time. Once terminated there is no obligation to pay the remaining instalments.

A 6 month option with an up-front premium of 500 points could be purchased in monthly instalments of 100 points. If the owner of the option holds it until maturity he will pay a total of 600 points in premium. However, if after say, 2 months the exchange rate has moved so that the option is well out-of-the-money, he may elect to cancel the option and will have paid only 200 points premium.

Instalment options are also known as rental options or pay-as-you-go options.

Power options have pay-outs that are a function of the difference between the strike price and the rate at expiry raised to a power greater than one. The pay-out on a standard dollar call with strike price ¥100.00 that expires with the spot rate at ¥102.00 would be ¥2.00. A power option with pay-out equal to $(X – 100)^2$ if the expiry rate, X, is greater than ¥100 would have a pay-out of ¥4.00 if the spot rate at expiry is ¥102.00.

Spot rate at expiry	*Pay-out for standard call*	*Pay-out for power option*
95	0	0
100	0	0
102	2	4
104	4	16
110	10	100

Power options involve leverage. The risk taken by the seller of power options is substantially greater than that for a standard option.

Contingent options knock-in if a specified (possibly unrelated) event occurs. For example, a currency option might knock-in if interest rates reach a certain level.

Quanto options are options for which the quantity of the face value varies with some other variable. A dollar based fund manager investing in Japanese equities has a foreign exchange exposure which varies in size with the yen value of the portfolio.

EXAMPLE 12.5

A US fund manager purchases US$10,000,000 of Japanese equities when the spot exchange rate is US$1 = ¥100.00 and the Nikkei 225 index is 15,000. The yen value of the portfolio would be initially ¥1,000,000,000 (10,000,000 × 100.00) and will vary as the prices of the equities rise or fall. Assuming that the particular basket of stocks varies approximately with the Nikkei index, if the N225 rises to 16,000, the portfolio would then be worth about ¥1,066,666,667. On the other hand, if the N225 falls to 14,000, the portfolio would be worth approximately ¥933,333,333. Buying a standard yen put/dollar call for face value ¥1,000,000,000 would provide an imperfect hedge of the currency risk. If N225 turns out to be 16,000, the investor would be under-hedged; if N225 were 14,000 the investor would be over-hedged. A quanto option for which the yen face value varies with the N225 index would provide a better hedge. It would not be perfect, to the extent that the specific portfolio might outperform or underperform the index, but it is likely to be superior to a standard (that is, constant volume) option.

CORRELATION

The price of contingent options and quantos is affected by the correlation between the exchange rate and the related variable.

Correlation describes how closely two sets of variables are related. Correlation can be measured using the following formula:

$$\boxed{\rho = \frac{n\Sigma xy - \Sigma x \Sigma y}{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}} \qquad (12.4)$$

Microsoft Excel has a function `CORREL(X1.Xn,Y1.Yn)` for calculating the correlation.

If the correlation coefficient, ρ, is positive, then when variable x rises, variable y also tends to rise. If the correlation coefficient is negative, then when x rises, y tends to fall. If the correlation coefficient equals +1, then x and y are perfectly correlated. That is, y always rises or falls proportionately with x. If the coefficient equals –1, then x and y are perfectly negatively correlated. That is, when x rises y always falls, and vice versa. If $\rho = 0$, then x and y are independent.

EXAMPLE 12.6

Over a period of 10 months the dollar/yen and dollar/SF moved as shown in Exhibit 12.5. Calculate the correlation between the two exchange rates over this period.

EXHIBIT 12.5 Calculating correlation

Month	*US\$/¥*	*US\$/SF*
1	105.60	1.7150
2	106.00	1.7230
3	107.25	1.7360
4	104.40	1.7570
5	106.20	1.7540
6	108.40	1.7680
7	107.75	1.7790
8	108.25	1.7660
9	109.00	1.7890
10	109.10	1.7995

Applying Equation (12.4), $\rho = +0.72$.

The higher the correlation coefficient, the closer the relationship between the two exchange rates. A correlation of 0.72 is high. Over the period the dollar was mostly rising against the yen and the SF. In some months the dollar rose more against the yen than it did against the franc.

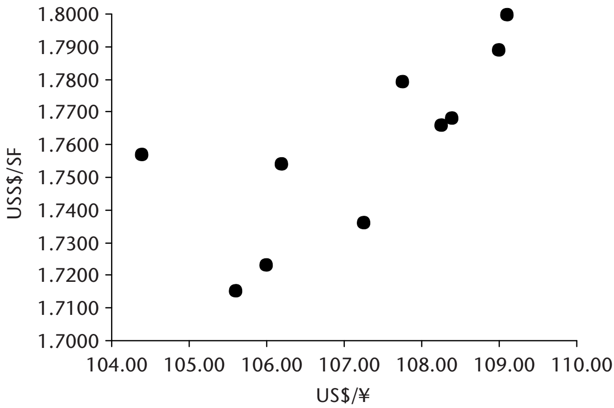


FIGURE 12.11 Correlation between US$/¥ and US$/SF = 0.72

In other months it rose more against the franc than the yen. In month 4 the dollar fell against the yen but continued rising against the franc, whereas in month 8 it fell against the franc but rose against the yen.

Considerations in the Japanese economy may affect the yen but not the SF, and some factors in Switzerland may affect the Swiss franc but not the yen. Other factors, such as developments in the US economy, will affect both exchange rates.

The correlation between two exchange rates will vary over time. However, if a sufficient number of observations are taken over a long enough time, consistent relationships between exchange rates will be reflected.

CROSS RATE VOLATILITY

The volatility of a cross rate can be derived from the volatilities of the two exchange rates and the correlation between them.

If the cross rate is the ratio of the two exchange rates:

$$\boxed{\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{12.5a}$$

If the cross rate is the product of the two exchange rates:

$$\boxed{\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2} \tag{12.5b}$$

where

σ_{12} = cross rate volatility

σ_1 = volatility of exchange rate 1

σ_2 = volatility of exchange rate 2

ρ = correlation between currency 1 and currency 2

EXAMPLE 12.7

If

$$\sigma_{\$/¥} = 12.5\%$$
$$\sigma_{\$/SF} = 10.0\%$$
$$\rho = 0.72$$

then

$$\begin{aligned}
\sigma^2_{¥/SF} &= \sigma^2_{\$/¥} + \sigma^2_{\$/SF} - 2\rho\sigma_{\$/¥}\sigma_{\$/SF} \\
&= (0.125)^2 + (0.1)^2 - 2(0.72)(0.125)(0.1) \\
&= 0.15625 + 0.01 - 0.18 \\
&= 0.007625 \\
\therefore \sigma_{¥/SF} &= \sqrt{0.007625} = 0.087 = 8.7\%
\end{aligned}$$

If the cross rate volatility is not as implied by Equation (12.5), an arbitrage opportunity exists. Arbitrageurs will buy options with below equilibrium volatility and sell options with above equilibrium volatility until the cross volatility relationship is restored.

BASKET OPTIONS

A *basket option* is an option for which the underlying is a basket of currencies.

Provided all of the currencies in the basket are not perfectly correlated, the volatility and so the premium of the basket will be less than the weighted average of the volatilities and premiums of the component currencies.

EXAMPLE 12.8

A fund manager invests US\$30,000,000 in Japanese assets and US\$70,000,000 in European assets when the respective exchange rates are US\$/¥120.00 and €/US\$0.8500. The fund manager could hedge against currency risk by purchasing a dollar call/yen put and a dollar call/euro put. Alternatively, the fund manager could purchase a dollar call/basket put.

The basket would be comprised of ¥3,600,000,000 and €82,352,941.18, as shown in Exhibit 12.6.

EXHIBIT 12.6 Basket of currencies

Currency	*Amount in dollars*	*Weight*	*Exchange rate*	*Currency amount*
Yen	US$30,000,000	0.3	120.00	¥3,600,000,000
Euro	US$70,000,000	0.7	0.8500	€82,352,941.18
Total	US$100,000,000	1.0		

Note: Sum of the weights = 1

The pay-out for the basket option will be a function of an index. The index is a number that reflects the value of the basket of currencies. The index is usually given a base number equal to 100 at the start of the option period. The value of the index can then be calculated for any set of exchange rates. For example, if the exchange rates have changed to 125.00 and 0.8000 respectively, then the index will have fallen from 100 to 94.68. The basket of currencies is only worth 94.68% as many dollars it was previously worth (Exhibit 12.7).

EXHIBIT 12.7 Basket index value at expiry

Currency amount	*Exchange rate*	*US$ value*	*Index value*
¥3,600,000,000	125.00	28,800,000	28.80
€82,352,941.18	0.8000	65,882,353	65.88
Total		94,682,353	**94.68**

An index value can be computed for a set of strike prices for exchange rates in the basket. For example, if the strike prices for the currency pairs in the basket are ¥120.00 and €0.8500, the strike index would be 100.00.

The basket put will pay out if the basket index at expiry is less than the strike index. For example, if at expiry the spot rates are ¥125.00 and €0.8000, the index value would have fallen to 94.68, so the basket put would pay out US$5,320,000.

$$\begin{aligned}\text{Pay out} &= \text{Principal (strike index} - \text{expiry index)}\\ &= 100{,}000{,}000(100.00 - 94.68)/100\\ &= \text{US\$5,320,000}\end{aligned}$$

The pay-out from a basket option will be no greater than the sum of the pay-outs of the individual options. For example, if the exchange rates at expiry were 125.00 and 0.9000, the basket index value would be 102.92, so there would be no pay-out from the basket put. However, the US$call/yen put on its own would pay US$1,200,000. The euro put would lapse because the euro has strengthened against the dollar (Exhibit 12.8).

EXHIBIT 12.8 Index at expiry

Currency amount	*Exchange rate*	*Dollar value*	*Index value*
¥3,600,000,000	125.00	28,800,000	28.80
€82,352,941.18	0.9000	74,117,647	74.12
Total		102,917,647	**102.92**

$$\text{Pay out from basket put} = 0$$

$$\text{Pay out from yen put} = \frac{3{,}600{,}000{,}000}{120.00} - \frac{3{,}600{,}000{,}000}{125.00} = \text{US\$}2{,}400{,}000$$

The investor might not particularly mind that the pay-out has not been maximized. The motivation for wanting a pay-out is to limit the amount that can be lost from the *general* weakening of the foreign currencies in which the fund has invested. The basket option achieves this objective. The fund manager may prefer to pay a lower premium for the basket option than he or she would have to pay to separately purchase a yen put and a euro put, even if the possible pay-out is less.

The premium for the basket option will be less than the sum of the premiums of the individual puts because the basket index will be less volatile than the weighted average of the volatilities of the separate exchange rates.

The volatility of the basket can be calculated using Equation (12.6).

$$\boxed{\sigma_{\text{basket}} = \sum w_i w_j \sigma_i \sigma_j \rho_{ij}} \tag{12.6}$$

If dollar/yen volatility is 13.5% p.a., euro/dollar volatility is 9.7% p.a. and the correlation between the two exchange rates is 0.33, then

$$\begin{aligned}\sigma_{\text{basket}} &= (0.3\times 0.3\times 0.135\times 0.135)+(0.3\times 0.7\times 0.135\times 0.097\times 0.33)\\ &\quad +(0.7\times 0.3\times 0.097\times 0.135\times 0.33)+(0.7\times 0.7\times 0.097\times 0.097)\\ &= 0.0898 = 9.98\%\ \text{p.a.}\end{aligned}$$

The weighted average volatility of the separate yen and euro puts would be:

$$(0.3 \times 0.135) + (0.7 \times 0.097) = 0.1084 = 10.84\%\ \text{p.a.}$$

Note that the premium for the basket option will be less than the sum of the premiums of the separate yen and euro puts.

It should be pointed out that it is possible to calculate the premium of the basket put because the market price (current value of the index), strike price (strike index) and volatility are all known.

In practice, basket options are normally based on a basket of 3 to 5 currency pairs. This typically generates a significant saving in premium without becoming too cumbersome to operate. Fund managers with exposures in many currencies tend to group them together into like sets. For example, they might use Singaporean dollars as a proxy for other South Asian currencies.

It is possible to construct collars, knock-outs, digitals etc. with basket options.

HYBRIDS

Hybrids are derivatives that establish a price based on the relationship between the two variables which might not normally be associated with each other.

EXAMPLE 12.9

An Australian nickel producer has exposure to both the price of nickel and the A$/US$ exchange rate. Profit rises with the price of nickel and falls when the exchange rate rises.

The company produces 8,000 tonnes of nickel each quarter. Like all commodities, nickel is priced in US dollars. Its annual revenue is therefore 32,000 × P where P = price of nickel.

$$\text{Revenue} = \text{US\$}32{,}000 \times P$$

The company has costs of US$25,000,000 per year. It estimates that it has an additional A$100,000,000 in expenses.

The company would like to hedge its currency risk without having to hedge its nickel price risk. The company has an exchange rate risk that its Australian dollar revenue will fall if the exchange rate rises. How many US dollars should the company sell forward to eliminate its exchange rate risk?

$$\text{US dollar exposure} = 32{,}000 \times P - 25{,}000{,}000$$

The size of the US dollar exposure can vary dramatically with the price of nickel because it is very volatile. Average annual volatility has been over 30%. The price is currently US$6,430 per tonne. Over the past decade the price has ranged between US$3,610 and US$16,350. Exhibit 12.9 shows how the company's foreign exchange risk varies with the price of nickel.

EXHIBIT 12.9 Currency risk at different nickel prices

Price of nickel	*US$ exposure*
2,000	39,000,000
4,000	103,000,000
6,000	167,000,000
8,000	231,000,000
10,000	295,000,000

The company will over-hedge if it sells more US dollars than it ends up receiving. For example, if the company sold US$180,760,000 on the assumption that the price of nickel would average US$6,430, and it fell to, say, US$4,000, it would have sold US$77,760,000 more than necessary. The company would then be exposed to loss if the exchange rate fell.

The company needs a forward exchange contract with a quanto-like feature. That is, the number of US dollars that it sells forward would vary with the nickel price. The fact that historically there has been a high positive correlation between the price of nickel and the exchange rate provides something of a natural hedge – if the price of nickel falls then it is likely that the exchange rate will also fall. This would imply that the producer should do nothing. However, the correlation is far from perfect. It will lose much more from a fall in the nickel price than it will make from the associated likely fall in the exchange rate.

Figure 12.12 shows the assumed probability surface based on the assumptions that the volatility of the US$/nickel price will be 33% p.a., the volatility of the A$/US$ exchange rate will be 12% p.a. and the correlation between US$/nickel and A$/US$ will be 40%. The chart shows the assumed probabilities of movements up to one standard deviation over 12 months. A one standard deviation move would take the nickel price up to US$8,552 or down to US$4,308. Similarly, a one standard deviation move would take the exchange rate up to 0.6048 or down to 0.4752.

The positive correlation between the nickel price and the exchange rate means that combinations of a high nickel price and a high exchange rate (for example, US$8,552 and 0.6048) or a low nickel price and a low exchange rate (for example, US$4,308 and 0.4752) are much more likely to occur than combinations of a low nickel price and a high exchange rate or a low nickel price and a high exchange rate. The surface in Figure 12.12 shows how the correlation between the nickel price and the exchange rate twists the probability of different possible outcomes.

It was possible to use this three-dimensional probability density function to construct a *hybrid transaction* to allow the producer to hedge by selling forward the variable number of US dollars each quarter for 3 years at an exchange rate which varies with the average price of nickel for the quarter according to the following relationship:

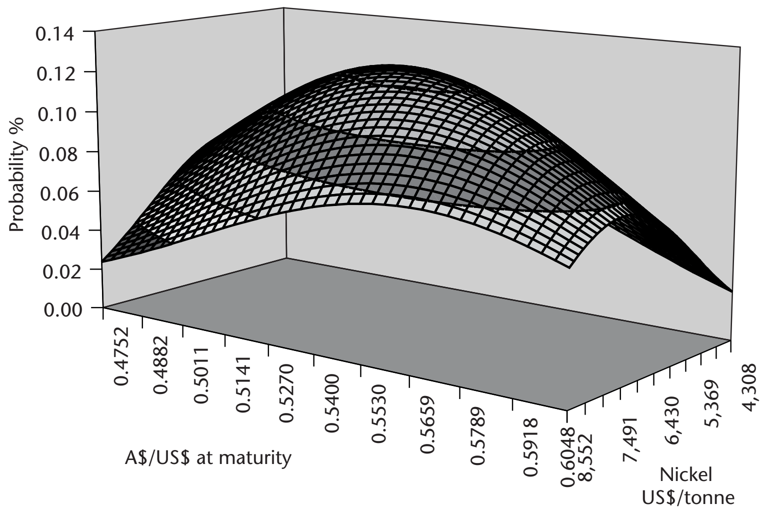


FIGURE 12.12 Probabilities of combinations of nickel price and exchange rate assuming correlation = +0.40

$$\text{Exchange rate} = 0.3100 + 0.00003 \times \text{Price of nickel}$$

Exhibit 12.10 shows the exchange rate generated by this formula for various possible nickel prices.

EXHIBIT 12.10 Nickel correlated currency hedge

Nickel price	*US$ sold*	*Exchange rate*
2,000	39,000,000	0.3700
4,000	140,000,000	0.4300
6,000	167,000,000	0.4900
8,000	231,000,000	0.5500
10,000	295,000,000	0.6100

In addition to avoiding the possibility of over-hedging the hybrid structure enables the nickel producer to totally eliminate exchange rate risk but continue to benefit from any rise in the price of nickel.

SUMMARY

Options increases the choices available to hedgers. Option derivatives and hybrids add to the number of choices. It is possible to design a structure which suits the needs and views of the person or organization with the exposure.

The list of options presented above is not a definitive list. All sorts of possible structures can be created. Some of the more exotic options are not widely used in practice. However, option derivatives such as digitals, barriers and baskets are very popular.

There is no doubt that additional products will be developed in the future and further combinations and applications of existing products will be used. Once a basic understanding of the concepts is acquired the only real constraint is imagination.

PRACTICE PROBLEMS

12.1 Digital put

Calculate the premium of an option that will pay US$1,000,000 if the A$/US$ spot rate is below 0.5300 in 90 days time given the following:

Current spot rate	A$/US$	0.5540
3 month LIBOR		3.25% p.a. (90/360)
Expected probability of spot being below 0.5300		24%

12.2 Power option

Calculate the premium of a call with a pay-out equal to $(X - 105.00)^3$ assuming the binomial tree as shown in Exhibit 10.3. The 6 month yen interest rate is 0.50% p.a. and the current spot rate is US$1 = ¥100.00.

12.3 Improving forward

A Japanese importer needs to buy US dollars at a future date. The spot rate is currently US$1 = ¥122.00 and the market forward rate is 120.30. A bank offers the importer a deal in which the rate at which the importer will buy US dollars on the forward date will be either 121.00 if the spot rate remains above 115.00 or 118.00 if the spot rate falls below 115.00 prior to the maturity date.

How does the bank engineer the improving forward?

12.4 Currency linked note

An investor places US$1,000,000 on deposit at a fixed rate of 3.5% p.a. for 6 months (180/360) and purchases a one-touch either side digital option with a pay-out of US$10,000 if the US$/¥ spot rate remains within a range of 120.00 to 130.00 for the entire 6 months. The premium of the option is US$2,948.40.

Calculate the effective yield if:

(a) the spot rate remains within the range
(b) the spot rate does not remain within the range

CHAPTER 13

Factors Affecting Exchange Rates

There are many factors that influence exchange rates and they assume different significance at different times. This chapter attempts to build a framework for examining the main factors that drive exchange rates and to provide guidelines for people managing currency exposures.

THEORIES OF EXCHANGE RATE DETERMINATION

None of the numerous theories of exchange rate determination is sufficiently comprehensive or dynamic on its own to explain exchange rate movements, let alone accurately predict the future direction and level of exchange rates. Nonetheless, there are important elements in most of the theories on what determines exchange rates. Examining a few of the most popular theories is a good starting point for building a framework to analyse exchange rate changes. Understanding why a theory fails provides further insight into the complex issue of exchange rate determination.

Current account balance

The balance of payments is a record of a country's transactions with the rest of the world. The current account records exports and imports of goods and services. If more money is received from the sale of exports than is paid for imports there is a *current account surplus*. On the other hand, if more is spent purchasing imports than is received from selling exports there is a *current account deficit*. Classical economic theory says that currencies with increasing current account surpluses or decreasing current account deficits tend to strengthen against currencies with decreasing current account surpluses or increasing current account deficits.

The capital account records cross border borrowing and investments. As a general rule, countries with current account deficits need to borrow (or obtain investments) from countries with current account surpluses.

The currency of a country with a current account surplus will tend to appreciate against the currency of a country with a current account deficit. If Country A has a current account surplus and Country B a current account deficit, then Currency A will tend to appreciate against Currency B. To purchase Country A's exports a foreigner needs to buy Currency A. The citizens of Country A need to sell Currency A to buy foreign currencies to purchase their imports. If exports exceed imports, then there will be greater demand to buy Currency A than to sell it. The exchange rate, which is the price of Currency A, will appreciate to reflect relatively high demand for the currency.

Conversely, the currency of a country with a current account deficit will tend to depreciate against the currency of a country with a current account surplus. If imports exceed exports as in Country B, then there will be more interest in selling Currency B than in buying it. So the price of Currency B will tend to fall.

It should be emphasized that it is the *change* in the current account deficit which is relevant. If a country has a diminishing current account surplus its currency will tend to depreciate, whereas a country with a shrinking current account deficit will tend to appreciate. If fewer exports or more imports are being purchased than previously, then the demand for the currency, and so the exchange rate, will tend to decline.

In practice, exchange rates do not always move to reflect relative current account figures. Over time the relationship generally holds true, but there can be sustained periods during which exchange rates move in the opposite direction.

However, the current account is *an* important determinant of exchange rates.

Purchasing power parity theory

The *purchasing power parity theory* states that in the long run exchange rates tend to reflect relative inflation rates.

Purchasing power refers to the ability of money to buy goods and services. Over time prices tend to rise. The inflation rate measures the rate of change of prices over a period of time. Purchasing power declines with inflation. The same amount of money buys less goods after prices have risen than it did before they rose.

Purchasing power parity asserts that exchange rates should change so as to reflect the relative purchasing power of the two currencies. If Country A has a higher inflation rate than Country B, then over time Country A's exports will become more competitive and Country B's exports will become less competitive, so the exchange rate must adjust to keep the real (i.e. inflation-adjusted) exchange rate constant.

The Economist published an article which used the example of Big Mac hamburgers to illustrate purchasing power parity. A Big Mac is a homogeneous product – it is identical whether you buy it in New York, London, Frankfurt or Tokyo – so it could be expected to be the same price wherever it is sold.

The exchange rate implied by the purchasing power parity theory is that reflecting the relative price of a Big Mac. For example, if a Big Mac costs US$2.19 in New York and £1.74 in London, the implied exchange rate = 1.74/1.29 = 1.2586.

In practice, it is little more than coincidence if actual exchange rates are at the levels implied by purchasing power parity. Exhibit 13.1 shows how actual exchange rates differed from the PPP level on 7 April 1992. In fact, in the ten years since then, none of these exchange rates reached the PPP level, and for much of the time the exchange rate moved away from the purchasing power parity level.

EXHIBIT 13.1 Big Mac parity

Country	*Price in local currency*	*Implied (PPP) exchange rate*	*Exchange rate 7 April 1992*
USA	$ 2.19	1.00	1.00
UK	£1.74	1.2586	1.7500
Germany	DM 4.50	2.0500	1.6400
Japan	¥ 380.00	174.00	133.00

One reason that purchasing power parity does not work is that geographical separation denies people the means to arbitrage inconsistent prices. If a person on a trans-Pacific flight was given the opportunity to buy a hamburger for either $2.19 or ¥380.00 when the exchange rate was $1 = ¥133.00, he or she would choose to pay in dollars rather than yen. Because people would always be better off paying in dollars the exchange rate would *tend* to change from 133.00 towards 174.00, at which rate the arbitrage opportunity would disappear.

A person in Tokyo would not consider going to New York to buy a Big Mac more cheaply. The time and cost would be much too great. Thus the relative price of a Big Mac can remain substantially different from the exchange rate indefinitely. Barriers to free trade, e.g. import tariffs, are another source of sustainable disparity in relative prices.

Another problem with purchasing power parity is that whilst there is a single exchange rate between two currencies at any point of time, the relative price may vary greatly depending on which goods and services are being considered. For example, a car which costs \$21,900 in the USA (i.e. 10,000 times as much as a Big Mac) might cost ¥4,560,000 in Japan (i.e. 12,000 times as much as a Big Mac). The relative price of cars implies a PPP exchange rate of 208.22, whereas the relative price of Big Macs implied 174.00.

This problem is typically addressed by assuming that exchange rate parity will reflect relative inflation rates, where inflation is measured as the change in prices *generally*. Price indices such as the CPI (Consumer Price Index) are used to estimate general price changes. Measuring variables such as CPI is extremely difficult and is subject to substantial errors of aggregation and measurement. It is important to realize that whilst inflation may be reported to the first decimal place of 1% per annum, the measure is nowhere that precise.

Empirical evidence on purchasing power parity is unconvincing except for countries which have had very high rates of inflation. Countries such as some of those in Latin America have typically had rapidly depreciating currencies associated with their very high inflation rates. Generally speaking, however, exchange rates do not reflect relative inflation rates, although there are some periods for which the relationship holds reasonably well.

The thing to remember about the purchasing power parity theory is that the relative inflation rate is another of the important factors which influence exchange rates.

Interest rate parity theory

Interest rate parity theory states that in the long run exchange rates tend to reflect relative interest rates.

Capital tends to flow between countries such that over time high interest rate currencies depreciate against low interest rate currencies. For example, over the past 15 years US interest rates have generally been significantly higher than Japanese interest rates. This has corresponded with a general weakening of the dollar against the yen.

Because the forward exchange rate reflects the interest rates of the two currencies, interest rate parity implies that over time the forward rate will be the best predictor of the future spot rate. Empirically, the forward rate has not been a good predictor of where the spot rate would go. Over some periods the spot rate has moved well beyond where the forward rate had been. In other periods it has moved in the opposite direction.

In practice, the relationship between relative interest rates and exchange rates is much more complex than implied by the interest rate parity theory. In the short term, capital tends to flow out of the low interest rate currency and into the high interest rate currency, causing the low interest rate currency to lose value against the high interest rate currency. This is the opposite of what is supposed to happen according to interest rate parity theory.

EXAMPLE 13.1

	Interest rate
Currency A	6% p.a.
Currency B	3% p.a.

In the short term people will be inclined to borrow Currency B at 3% p.a., sell it to buy Currency A and to invest in Currency A at 6% p.a. Because people will be selling Currency B and buying Currency A, Currency B will tend to depreciate against Currency A.

Whether or not this strategy turns out to be profitable will depend on what happens to the exchange rate. If the exchange rate remains unchanged, or if Currency B depreciates by less than the interest differential, then the strategy will prove profitable. On the other hand, if Currency B depreciates by more than the interest differential, then the strategy will result in a loss (i.e. the exchange rate loss will exceed the gain in the interest rate markets).

Interest rate parity theory asserts that the relative interest rates reflect the market's exchange rate expectations. If Currency A has a higher interest rate than Currency B, it means that Currency A *needs* to have a higher interest rate to induce investors to take the currency risk to invest in Currency A rather than Currency B.

It is important to understand that according to interest rate parity theory it is changes in *relative* interest rates which matters. If, for example, the interest rate of Currency A fell to 5% p.a. and that of Currency B rose to 4% p.a., that would constitute a fall in Currency A's interest rate *relative* to Currency B's. Accordingly, Currency A will tend to appreciate against Currency B.

The above argument is plausible, and in general empirical evidence supports interest rate parity theory in direction, if not by degree, but it is highly time period-dependent. There are lengthy periods in which exchange rates move in the opposite direction to that implied by interest rate parity.

The weakness in the interest rate parity argument is that in general interest rates are not determined by the investor market. Central banks actively manage interest rates and their primary objective is not usually exchange rate management. Central bank objectives may vary from country to country and they tend to change over time. In recent history (i.e. since the breakdown of the Bretton Woods agreement in the early 1970s) Central banks have generally managed interest rates to control inflation and to foster economic growth rather than to influence exchange rates.

The two lessons to take from the interest rate parity theory are that relative interest rates and exchange rate expectations are important factors that influence exchange rates.

FACTORS AFFECTING INTEREST RATES

Saying that interest rate levels affect exchange rates begs the question what factors affect interest rates?

Interest rates are determined by supply and demand. However, in most countries the Central Bank intervenes actively to manage interest rates.

Central banks tend to use interest rates to control the level of economic activity. If the economy is buoyant, the demand for goods and services, and thus the demand for money, will be high. The economy will grow quickly, but inflation is likely to be rising. If the economy is sluggish, the demand for goods and services, and thus the demand for money, will be low. The economy will be growing slowly. It may even be in recession (i.e. negative growth). Unemployment is likely to be high.

The *business cycle* (Figure 13.1) describes the pattern of alternate periods of high and low or negative growth.

During periods of recession monetary authorities are likely to lower interest rates to stimulate economic activity. During periods of high growth authorities are inclined to raise interest rates to contain inflation.

In practice, measuring the level of economic activity is not a simple task. Many economic measures are used to indicate the level of activity. Some of these are known as specific indicators which measure one aspect of economic activity. Others are general indicators which attempt to dimension the overall level of economic activity.

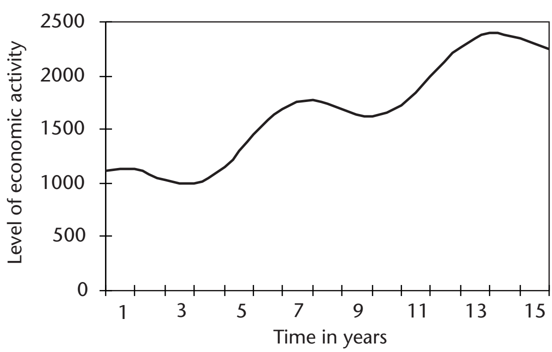


FIGURE 13.1 Business cycle

Examples of specific indicators
Retail sales
Housing starts
Motor vehicle registrations
Durable goods orders
Industrial production
Personal income
Unemployment rate

Examples of general economic indicators
GDP (Gross Domestic Product)
National income
Economic growth rate (real GDP per capita)

No measure on its own provides a complete picture. However, by observing a whole range of general and specific indicators it is possible to piece together a reasonable picture of how the economy is performing. Policy makers typically review as much economic data as is available before taking decisions to tighten (i.e. raise) or ease (i.e. lower) interest rates.

Whichever economic indicators are used, the business cycle does not follow a smooth curve as in Figure 13.1. The fact is that the economy is complex and measuring economic aggregates is prone to error. Furthermore, there are inevitable lags. It may take months to collect and aggregate data, so that by the time the data is released the level of economic activity may have changed.

Some economists calculate indices which incorporate a range of specific indicators. The most popular of these is known as the index of leading indicators, which attempts to show the direction in which the economy is heading rather than merely looking at where it has been.

There is a continual release of statistics on the economy which helps when anticipating likely changes in interest rates.

It was pointed out in Chapter 4 that the shape of the yield curve reflects where the market generally expects interest rates to be in the future.

INTERRELATIONSHIP BETWEEN INTEREST RATES AND EXCHANGE RATES

Changes in interest rates affect exchange rates. Changes in exchange rates can also affect interest rates. When a currency depreciates the cost of imports rises. If imports represent a significant component of the economy this can adversely impact the inflation rate. Monetary authorities can tighten interest rates to dampen the inflationary effect of a weakening currency.

Sometimes central banks consider exchange rate stability to be an important economic objective. If so, they may use monetary policy to help achieve their exchange rate targets. Raising interest rates will tend to attract capital, thereby supporting the exchange rate.

TIME HORIZON

Corporations wishing to manage the exchange rate risk associated with their business generally take a long-term perspective. Their business naturally has net exchange positions for extended periods. Their profitability depends on what happens to exchange rates over a period of months or years. Accordingly, their time horizon is medium or long term.

Foreign exchange traders who are market making will have a very short time horizon. They make money by carrying positions for very short periods – hours, minutes or even seconds.

Different factors affect exchange rates in the short term than those which are relevant over a longer time horizon.

As the factors considered above have longer term relevance this is a convenient point to draw together a framework analyzing the long-term exchange rate outlook.

LONG-TERM OUTLOOK

Factor	*Considerations*	*Implications*
Current account	Widening surplus or narrowing deficit	Stronger currency
	Narrowing surplus or widening deficit	Weaker currency
Inflation rates	Relatively low inflation	Stronger currency
	Relatively high inflation	Weaker currency
Interest rates	Relatively low interest rate	Stronger currency
	Relatively high interest rate	Weaker currency

The above factors are sometimes referred to as the *fundamental factors*.

The *direction* in which exchange rates are likely to move over an extended period is likely to be predominantly determined by the fundamental factors. If all three factors point in the same direction, there is a high probability that exchange rates will trend in that direction. For example, a country with a widening current account deficit with a high relative inflation rate and high relative interest rates is very likely to have a weakening currency.

The *extent* to which exchange rates can be expected to move is more difficult to predict. The more the current account imbalance, the relative inflation rate and interest rates change, the further the exchange rate is likely to move. Forward rates are one indicator, but should be used with caution. All variables keep changing and the measures are imprecise. The relative importance of the fundamental factors can change over time, depending on the current policy slant and even 'fashion'.

The recommended approach is to systematically review the fundamental factors, asking how each is likely to change and *roughly* what impact that would be expected to have on exchange rates. Forecasting a range within which the exchange rate is expected to be often makes more sense than point forecasting. This process should be reviewed regularly as new information comes to hand.

It is worth remembering that the shape of the yield curve and forward exchange rates embody the market's expectations for interest rates. It is also true that market volatilities used in option pricing embody the market's expectations for how volatile rates will be. If your forecast differs significantly from the forward rate it is worth analysing why your outlook differs from the market.

SHORT-TERM FACTORS

The fundamental factors are normally very poor predictors of short-term exchange rate movements. In the short term, exchange rates tend to be affected by a different set of factors.

Various factors influence spot rates in the short term primarily through how they affect market *expectations*.

Market flows

Dealers see the transactions being done with their customers and the orders which they are asked to watch. If on a particular day dealers observe that their customers are predominantly buying a particular currency, it is reasonable for them to expect that currency to strengthen. The logical action for them to take is to go long that currency. The combined impact of both the customers and the banks wanting to buy the currency is likely to push the exchange rate up, at least for a short time.

Many billions of dollars of foreign exchange transactions are done every day. It is not the actual volume which matters as much as the perceptions which are created. For every buyer there is a seller. On some days massive volumes are transacted with very little price movement. On other days the rate can move a long way on the back of relatively small turnover. The more closely aligned market participants become in the view of which direction the exchange rate is likely to move next, the further it is likely to move. Large transactions done by prominent players (e.g. global fund managers) tend to have the greatest impact on perceptions.

Central bank intervention

One group of prominent players is the central banks. At certain times they enter the market with the intention of moving exchange rates in a particular direction. Central bank intervention relies on the expectation that the banks will follow their lead. If the central bank buys the local currency it will be hoping that the commercial banks follow suit, thereby causing the currency to strengthen. Sometimes central banks 'jaw-bone', which means talking the exchange rate up (or down). By publically saying that they intend to purchase the currency they hope to lead the market into doing so.

Central bank intervention can back-fire. On occasions the market interprets the intervention as an indication that there is a problem. In the early 1980s the French central bank intervened by buying French francs when the franc was close to the lower band within which the EMS operated. The banks took this as a signal to sell the franc and on a number of occasions forced a devaluation. After four or five unsuccessful attempts to hold the franc within the EMS band the central bank changed its approach and instead of intervening through the foreign exchange market it used the money market. It tightened to the point where the overnight FF interest rate was 500% p.a. This made it very expensive to carry a short FF position and proved successful at keeping the exchange rate within the band.

These days central bank intervention is most successful when central banks intervene or jaw-bone on a concerted basis. If the market observes the Federal Reserve, European Central Bank, Bank of Japan and Bank of England all buying a currency, it is most unlikely to 'take them on' by selling that currency.

Release of economic statistics

The market knows in advance when various statistics will be reported. Economists put considerable effort into predicting what the figures will be. The market consensus is often collected prior to the release of important data. When the figure is released dealers are ready to pounce. If the figure is significantly different from what was expected they would have planned to buy or sell the currency in anticipation of a change in the exchange rate. This type of behaviour tends to be self-fulfilling in the short term.

Market sentiment

There is a tendency for market sentiment to develop. Certain themes become popular with market commentators. News reports tend to focus on certain issues. For this reason rates tend to overshoot. When market sentiment changes there is often a sudden adjustment or correction in the rate. Sentiment changes may occur as a result of a surprise piece of information, or they may result from a bandwagon effect.

The timings of changes in market sentiment are very difficult to predict. However, it is good practice to try to estimate to what extent expected information is incorporated (or 'discounted') into the rate.

Technical analysis

Technical analysis involves the use of empirical information to predict future price movements. Even though the author favours fundamental analysis over technical analysis, it is worth being aware of critical chart points. Because many traders use technical analysis there can be considerable market impact when support or resistance levels are broken.

The common thread through all of the short-term factors is that they work through expectations and change of perception. These are 'soft' variables. They are difficult, if not impossible, to measure and they can change quickly. There is no time for the sort of thorough analysis which is recommended for people to manage their long-term currency risk, and even if there was it would not work. Consequently, spot traders rely on their ability to 'read' the market and to react accordingly to be profitable.

SUMMARY

Different types of factors affect exchange rates in the short term from those that affect them in the longer term. Consequently, a different approach is required. It is important to understand when which factors are relevant. It is recommended that the framework of fundamental factors is used for longer term analysis.

CHAPTER 14

Value at Risk

In mathematics, *risk* exists whenever there is uncertainty. A change in the value of one variable can lead to an increase or decrease in the value of a second variable. In finance risk is taken to be the risk that the monetary value of an asset will *fall* as a result of a change in some factor. The risk that the monetary value of the asset will increase is considered opportunity.

In the real world uncertainty exists all of the time. Financial risk and opportunity are always present. In this chapter the three major categories of financial risk are examined – market price risk, credit risk and liquidity risk. These three categories of risk are separate but interdependent. Techniques to measure the value at risk and to manage each of these types of risk are discussed.

MARKET PRICE RISK

Market price risk is the risk that an asset or portfolio of assets will lose value as a result of a change in the price of a market factor. A rise in interest rates will reduce the value of a bond. A change in exchange rates may cause a foreign exchange position to result in a loss. An increase in volatility will tend to reduce the value of a short option strategy, and so on.

Interest rates, exchange rates, volatility and time are referred to as *market factors*. Other examples of market factors include equity and commodity prices and spreads or the correlation between different factors.

FACTOR SENSITIVITIES

Factor sensitivities measure the extent to which a specific change in a market factor would result in a gain or loss.

EXAMPLE 14.1

A US dollar-based company has a net short exchange position of £10,000,000 against US dollars. A 1% increase in the £/US$ rate from 1.5000 to 1.5150 would lead to a loss of US$150,000.

$$\boxed{\text{Factor sensitivity} = \frac{\text{Change in value}}{\text{Change in market factor}}} \quad (14.1)$$

$$= 10{,}000{,}000(1.5000 - 1.5150)$$

$$= -\text{US}\$150{,}000$$

How large a change in the market factor is appropriate? Should factor sensitivities be calculated for a 1% change, a 5% change or some other amount? The answer depends on the period of time for which the position is expected to remain open (known as the *defeasance period*), the tolerance to absorb losses and the expected volatility of the market factor.

A spot trader who continually jobs the market may have a defeasance period of a few minutes. In the other extreme, a corporation may carry an exchange or interest rate position for several years. Factor sensitivities should be calculated for defeasance periods that match their circumstances.

DURATION

Duration is the time-weighted average life to maturity. It reflects the sensitivity of a portfolio of cash flows to a change in interest rates.

Consider a 5 year bond in a 5% p.a. interest rate environment. It has the following cash flows with present values as shown below:

End of year (t)	*Cash flow*	*Present value*	*PV × t*
1	5,000,000	4,761,905	4,761,905
2	5,000,000	4,535,147	9,070,295
3	5,000,000	4,319,188	12,957,564
4	5,000,000	4,113,512	16,454,050
5	5,000,000	3,917,631	19,588,154
	100,000,000	78,352,617	391,763,083
		100,000,000	454,595,051

$$\text{Duration} = \frac{\Sigma PV \times t}{P} = \frac{454{,}595{,}051}{100{,}000{,}000} = 4.546 \text{ years}$$

$$\text{Modified duration} = \frac{\text{Duration}}{(1 + r/m)} \quad (14.2)$$

Modified duration gives a linear approximation for the change in the price of a bond (or value of a portfolio of bonds) for a given change in yield. A 1 percentage point increase in interest rates (from 5% p.a. to 6% p.a.) would reduce the value of the bond by approximately $4,250,000 from $100,000,000 to around $95,750,000. Similarly, a 1 percentage point fall in rates (to 4% p.a.) would increase the value of the portfolio of cash flows by approximately $4,250,000 to around $104,250,000.

Value at Risk (VaR) quantifies the risk associated with a prescribed adverse price movement in the same way that modified duration approximates the risk inherent in a 1 percentage point change in yields.

USING DISTRIBUTION THEORY

It is possible to use distribution theory to develop measures of market price risk. It is usually reasonable to assume that price changes are normally distributed implying that prices are log-normally distributed.

Distribution theory makes it possible to identify the probability that the market price will be at, above or below a specific level at a specified time in the future given that the current market price is known and making an assumption about the expected volatility of the distribution.

It is very convenient if the probability distribution can be reasonably assumed to follow a known formula such as the normal distribution because this enables powerful statements to be made regarding the probability of specific outcomes.

Normal distribution

A normal distribution is the well-known bell curve-shaped distribution, as shown in Figure 14.1.

The normal distribution is symmetrical with mean = mode = median. The formula for the normal distribution is:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\bar{x})^2/2\sigma^2} \tag{14.3}$$

where:

$$\text{Mean} = \bar{x}$$

$$\text{Variance} = \sigma^2$$

In other words, if the mean and standard deviation are known, the probability of an outcome, x, can be calculated from the formula.

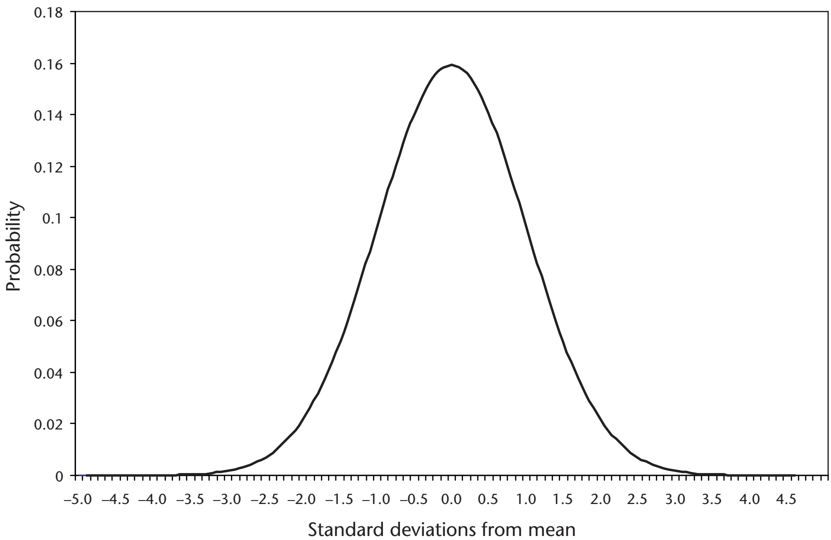


FIGURE 14.1 Normal distribution

Cumulative standard normal distribution

The probability that the outcome will be less than x is the sum of the probabilities for each of the outcomes less than or equal to x and is given by the following formula:

$$N(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{(x-\bar{x})/\sigma} e^{-(1/2)t^2} \tag{14.4}$$

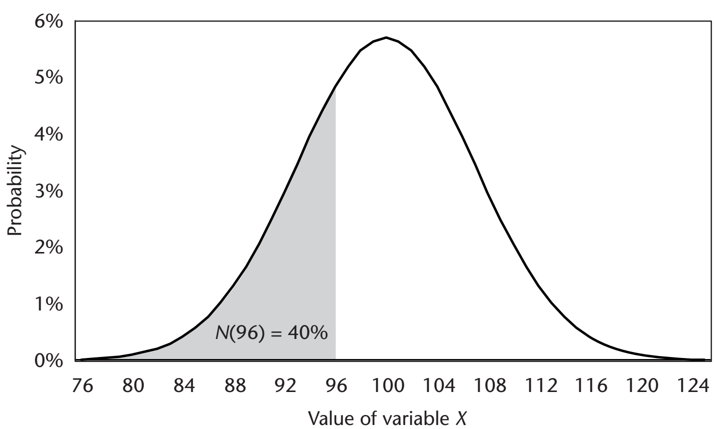


FIGURE 14.2 Cumulative normal distribution

$N(x)$ = shaded area under the normal distribution curve to the left of x. That is, the area from $-\infty$ to $x = N(x)$ (Figure 14.2).

The total area under the curve equals 1 or 100%. The area to the left of $x = 96$ equals 40%, indicating that there is a 40% probability the x will take a value of 96 or less.

The value of $N(X)$ can be drawn as a curve, as shown in Figure 14.3. For the above distribution $N(96) = 0.40$.

Using the cumulative standard normal distribution table (*z*-scores)
A standard normal distribution is one with mean = 0 and standard deviation = 1. The cumulative probabilities for normal distributions with other means and standard deviations can be determined using the cumulative standard normal distribution table by using the simple transformation:

$$z = \frac{x - \overline{x}}{\sigma} \tag{14.5}$$

where z can take any value between 0 and ∞. Refer to the table in the Appendix for sample values.

To establish the cumulative probability for a z score of 0.27, trace down the left hand column to the 0.20 line, then look across to the right to the

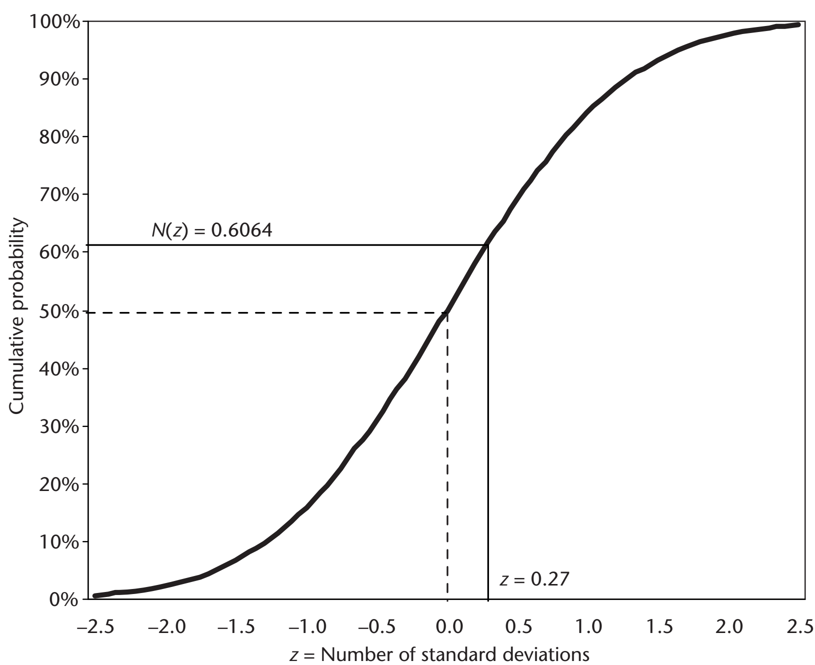


FIGURE 14.3 Cumulative standard normal distribution

0.07 column: the value of $N(z)$ is 0.6064. This means that there is a probability of 0.6064 (or approximately 60.64%) that the value will be less than or equal to 0.27 standard deviations above the mean.

If $z = 1.93$, $N(1.93) = 0.9732$. This means there is a 97.32% probability that the value will be less than or equal to 1.93 standard deviations above the mean. If $z = 0$, $N(0) = 0.5$. This is what would be expected since the normal curve is symmetrical and 50% of the total area under the curve lies on each side of the mean.

If z is negative, the table shows the probability that the value is less than or equal to z standard deviations below the mean. As the normal distribution is symmetrical, $N(-z) = 1 - N(z)$. To find $N(-z)$ subtract the value found for $N(z)$ from 1. It follows that the probability that the value will be greater than 1.93 standard deviations above the mean must be $1 - 0.9732 = 0.0268 = 2.68\%$.

Log-normal distribution

A variable has a log-normal distribution if the natural logarithm of the variable is normally distributed. That is, if $X = \ln Y$ and X has a normal distribution, then Y has a log-normal distribution.

In general, it is reasonable to assume that exchange rates are log-normally distributed. Exchange rates cannot take negative values and they can increase without limit. It follows that the continuous rate of change in price, $\ln(S_j/S_{j-1})$, is normally distributed.

It is possible to use the z scores to make statements about likely probability that the spot rate will change by a specified number of standard deviations.

The spot rate after t years is expected to be $Se^{z\sigma\sqrt{t}}$. This is sometimes referred to as the *stressed rate*.

$$\text{Stressed rate} = Se^{z\sigma\sqrt{t}} \tag{14.6}$$

For example, if the current spot exchange rate is £1 = US$1.4750 and the volatility of the spot exchange rate is expected to be 12% per annum, then:

$$S = 1.4750$$

$$\sigma = 0.12$$

If

$$z = 0.27$$

$$N(z) = 0.6064$$

$$1 - N(z) = 0.3936$$

and if

$$t = 1$$

then

$$Se^{z\sigma\sqrt{t}} = 1.4750\ e^{0.27(0.12)1} = 1.5236$$

Provided changes in spot rates are assumed to be normally distributed, it is possible to say that there is a 60.64% probability that in one year's time the spot rate will be below 1.5236 and a 39.36% probability that in one year's time the spot rate will be above 1.5236.

The stressed rate is not a worst-case scenario. Clearly, it is possible for the spot rate to move beyond the stressed rate. Indeed, it is assumed that that would happen on 39.36% of occasions.

It is simple to calculate the size of the loss that would result from a movement in the exchange rate to 1.5236.

A trader who is short £10,000,000 against the US dollar at 1.4750 would lose money if the spot rate at the end of the year turned out to be 1.5236. The loss would be 10,000,000(1.5236 – 1.4750) = US$486,000.

It could be said that there is a 39.36% chance of losing US$486,000 or more.

If z = 1, $N(z)$ = 0.8413 and 1 – $N(z)$ = 0.1587. The stressed rate = 1.6631. This implies that there is a 15.87% probability of losing 10,000,000(1.6631 – 1.4750) = US$1,881,000 or more.

If z = 2, $N(z)$ = 0.9772 and 1 – $N(z)$ = 0.0228. The stressed rate = 1.8751. This implies that there is a 2.28% probability of losing 10,000,000(1.8751 – 1.4750) = US$4,001,000 or more.

A larger value of z means that the exchange rate could be expected to move further against the net exchange position, leading to a larger loss.

Value at risk equals the amount that would be lost from a specified adverse movement in the exchange rate over a specified time period based on the assumption that the volatility of exchange rate is known and that changes in exchange rates follow a known distribution (usually assumed to be normally distributed).

EXHIBIT 14.1 Value at risk

z = No. std deviations	*Tolerance level*	*Stressed rate*	*Loss on £10,000,000 position*
0.27	39.36%	1.5236	US$ 486,000
1	15.87%	1.6631	US$1,881,000
2	2.28%	1.8751	US$4,001,000

$$\text{VaR} = \text{Amount} \times (Se^{z\sigma\sqrt{t}} - S) = \text{Amount} \times S(e^{z\sigma\sqrt{t}} - 1) \tag{14.7}$$

The value at risk increases with the size of the exposure, the assumed size of the change in exchange rate, the expected volatility and the time horizon (Figure 14.4).

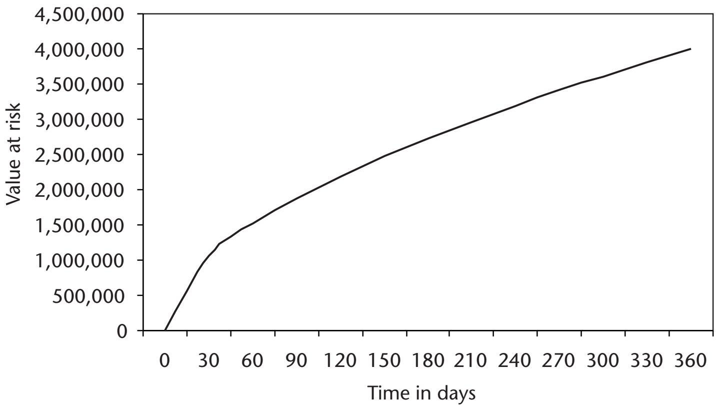


FIGURE 14.4 Value at risk increases with time

Over a longer period the spot rate is likely to move further against the position, so the potential loss would be greater. For a trader the time horizon is usually short – say, 1 day. Many banks base their VaR calculation on the assumption that there are 252 business days per year. A 1 day time horizon would be taken to be 1/252 years.

$$\text{If } z = 2 \text{ and } \sigma = 12\% \text{ p.a., stressed rate} = 1.4750e^{2*0.12/\sqrt{1.252}=}\ 1.4975$$

This implies that given the current spot rate is 1.4750, on approximately 2.28 business days in 100 the spot rate could be expected to rise to 1.4975 or higher.

$$\text{VaR} = 10{,}000{,}000 \times 1.4750(e^{2(0.12)\sqrt{1/252}} - 1) = \text{US\$}224{,}693.28$$

It is believed that the exposure would only result in a loss greater than about $US225,000 roughly 2 days in every 5 months.

It is important to realize that VaR calculations are based on some significant assumptions. The assumption that exchange rate changes are normally distributed is probably reasonable, but far from perfect. Actual volatility can vary greatly from the assumed probability. If σ = 13% rather than 12%, the stressed rate would be 1.4994 rather than 1.4975 and the VaR would be US$243,571.51. So the VaR amount should be treated as a rough indication rather than a precise number. It is the order of magnitude that is important. The trader knows that there is a small chance of losing an amount of the order of US$250,000. The trader should understand the impact of a loss of that order. It may be the case that because of the earning capacity of the firm a loss of a couple of hundred thousand dollars every now and then is to be expected. However, the situation may be very different if the possible loss was of the order of US$200,000,000.

The *tolerance level* is the assumed probability of losing the value at risk amount or more. If $z = 2$, the tolerance level is 2.28%. In other words, it is to be expected that (only) 2.28% of the time the spot rate would rise above the stressed rate, so the actual loss would be greater than the value at risk.

How many standard deviations are sufficient?

A one standard deviation adverse price movement can be expected to occur every six periods and a two standard deviations adverse movement every 44 periods. If the defeasance period is 1 year, as in Exhibit 14.1, and the exchange rate exhibits 12% annual volatility as expected, then it should be expected that in one year out of six the rate would rise above 1.6631 and in only one year in 44 to above 1.8751, starting from its current level of 1.4750.

In most circumstances measuring factor sensitivities to one or two standard deviations is more than adequate. Many organizations are content to operate at tolerance levels of 5% or 10%. The financial capacity of the individual or organization to absorb losses should be taken into account. It is not prudent to 'bet the farm' against a one or two standard deviation movement. The one day movement in the £/US$ rate on the day that sterling left the ERM was a 12 standard deviation event, so it should be remembered that, whilst highly improbable, such massive price changes could occur.

A trader with a long net exchange position in the commodity currency would have exposure to the exchange rate falling. In this case z is a negative number. The stressed rate will be a number below the spot rate and the value at risk will reflect the difference between the spot rate and the stressed rate.

MULTIPLE FACTORS

Many transactions are subject to changes in more than one market price. For example, a forward exchange contract changes value with changes in both interest rates as well as changes to the spot exchange rate. It is possible to dimension the value at risk against multiple market factors.

When spot £/US$ is 1.4750 and 1 year pound and US dollar interest rates are 3.5% p.a. and 2.5% p.a. respectively, the 1 year forward exchange rate would be 1.4612. The value at risk of a two standard deviation movement on a £10,000,000 position over 1 year would be:

Factor	*Volatility*	*Value at risk*	
£/US$	12%	10,000,000 × (1.6651 – 1.4750)	= US$4,001,000
US$ rate	18%	10,000,000 × (0.0358 – 0.025) × 1.4750	= US$159,790
£ rate	15%	10,000,000 × (0.0472 – 0.035) × 1.4750	= US$180,615

It is often the case that, as in the above example, one market factor is dominant. In this case it is the spot rate. As a general rule, the longer the tenor of the transaction, the greater the sensitivity to changes in interest rates.

Note that the volatilities of the different market factors are different.

When cash flows will occur in the future it is better to calculate VaR based on the NPV of the future cash flows. This is particularly true for longer tenor transactions.

EXAMPLE 14.2

Using the NPV method, calculate the value at risk for a 2 standard deviation increase in spot £/US$ exchange rate for a 5 year long £1,000,000 position given the following market rate scenario:

Spot	£1	=	US$1.5000
5 year dollars			5.5% p.a. (semi-annually compounding)
5 year pounds			6.0% p.a. (semi-annually compounding)
spot rate volatility			10% p.a.

When $s = 1.5000$,

$$f = 1.5000 \frac{(1.0275)^{10}}{(1.03)^{10}} = 1.4640$$

Cash flows at 5 year date:	–£1,000,000	=	+US$1,464,000
PV(£ cash flows)	–£744,093.91	=	–US$1,116,140.87
PV(US$ cash flows)			+US$1,116,150.54
NPV			9.67

Note: $1{,}000{,}000/(1.03)^{10} = 744{,}093.91$
$744{,}093.91 \times 1.5000 = 1{,}116{,}140.87$
$1{,}464{,}000/(1.0275)^{10} = 1{,}116{,}150.54$

The trivial difference US$9.67 is merely a rounding difference and can be taken to be zero.

The exposure to a long pound position is that the spot rate will fall. Consequently, the 2 standard deviations stressed rate is calculated with $z = -2$:

$$\text{Stressed rate} = 1.5000e^{-2(0.1)\sqrt{5}} = 0.9591$$

If the spot rate has fallen to £1 = US$0.9591 at the end of the 5 years:

–£1,000,000 = +US$ 959,100

PV(£ cash flows)	–£744,093.91	=	–US$1,116,140.87
PV(US$ cash flows)			+US$ 731,215.83
NPV			–US$ 384,925.04

Note: $959{,}100/(1.0275)^{10} = 731{,}215.83$

A 2 standard deviation decrease in the spot exchange rate would reduce the net present value of the 5 year outright position from zero to US$384,925.04. The value at risk is the amount by which the net present value of the exposure would decline if the spot rate falls by 2 standard deviations over the 5 year period.

VaR = US$384,925.04

If the assumptions that the spot rate changes will be normally distributed with a standard deviation equal to the expected volatility of 10% p.a., there is less than a 2.28% probability that the exposure would result in a loss of more than US$384,925.04. In other words, there is a tolerably low probability of incurring a loss of the order of around US$400,000.

THETA

The sensitivity to changes in time to maturity is known as *theta*.

$$\boxed{\text{Theta} = \frac{\text{Change in value}}{\text{Change in time to maturity}}} \tag{14.8}$$

$$= \frac{V_2 - V_1}{t_2 - t_1}$$

EXAMPLE 14.3

Calculate theta for a 1 day change in time to maturity for a 3 month forward long US$10,000,000 net exchange position.

Market rate scenario:

Spot	US$1	=	¥100.00
3 month dollars			5% p.a. (90/360)
3 month yen			1% p.a. (90/360)

At t = 90 days:

$$f_{90} = 100 \frac{(1 + 0.01 \times 90/360)}{(1 + 0.05 \times 90/360)} = 99.01$$

At t = 89 days:

$$f_{89} = 100 \frac{(1 + 0.01 \times 89/360)}{(1 + 0.05 \times 89/360)} = 99.02$$

$$\text{Theta } (\theta) = \frac{99.01 - 99.02}{90 - 89} = -0.01$$

For every day of time value the position increases by 1 point (that is, ¥100,000 or US$1,000 on a face value of US$10,000,000).

For an option,

$$\boxed{\text{Theta} = \frac{\text{Change in premium}}{\text{Change in time to expiry}}} \tag{14.9}$$

$$= \frac{P_2 - P_1}{t_2 - t_1}$$

The premium of an at-the-money ($f = k = 100.00$) US dollar call/yen put varies with the number of days to expiry as follows:

Days to expiry	*Premium*
64	1.62
16	0.81
4	0.40

Figure 14.5 shows the typical shape of time decay for an at-the-money option. As the option approaches expiry it loses value faster and faster until at expiry it has zero value. The premium halves as the time to expiry reduces by a factor of four.

For out-of-the-money and in-the-money options (Figures 14.6 and 14.7), the time decay occurs earlier than for an at-the-money option. If with, say, 20 days to go the market price is well above or below the strike price, the outcome is pretty much known, so the time value will be close to zero. The in-the-money option will decay towards its intrinsic value.

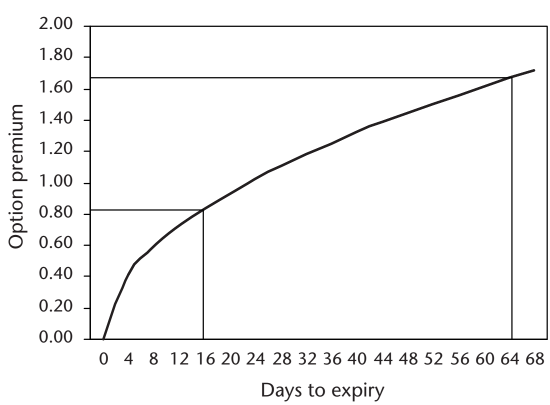


FIGURE 14.5 Time decay ATM option

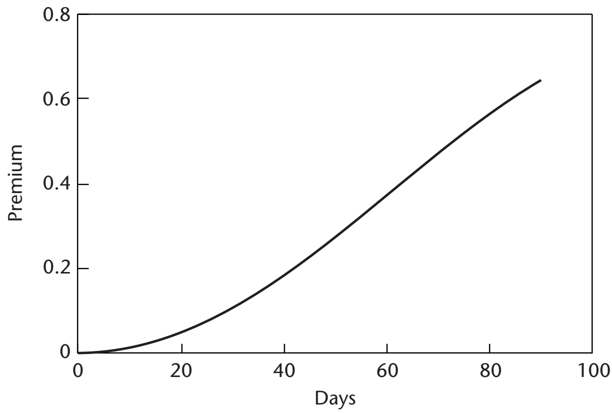


FIGURE 14.6 Time decay out-of-the-money option

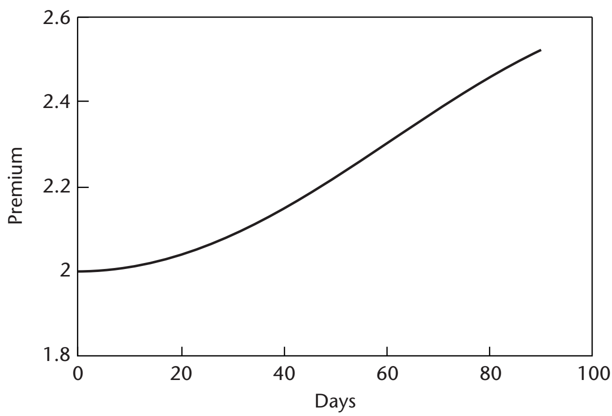


FIGURE 14.7 Time decay in-the-money option

Theta can be calculated for a single option or for a portfolio of options (and possibly, other financial instruments). Mathematically, theta is the first derivative of the option formula with respect to time:

$$\theta = \frac{\partial \Pi}{\partial T} \quad \text{where } \Pi = \text{value of portfolio}$$

For calls:

$$\theta = -\frac{SN'(d_1)\sigma}{2\sqrt{t}} - rKe^{-rt}N(d_2) \tag{14.10}$$

For puts:

$$\theta = -\frac{SN'(d_1)\sigma}{2\sqrt{t}} + rKe^{-rt}N(d_2) \tag{14.11}$$

DELTA

Delta is the change in (present) value for a given change in the market price. Delta is mostly used in relation to options.

$$\boxed{\text{Delta }(\Delta) = \frac{\text{Change in premium}}{\text{Change in market price}}} \tag{14.12}$$

$$= \frac{P_2 - P_1}{M_2 - M_1}$$

The premium of an at-the-money dollar call yen put with strike k = 100.00 with 90 days to expiry varies as follows:

Market price	*Premium*
95.00	0.09
96.00	0.18
97.00	0.33
98.00	0.56
99.00	0.89
100.00	1.33
101.00	1.89
102.00	2.55
103.00	3.31
104.00	4.14
105.00	5.03

Between 96 and 97, $\Delta = \frac{0.33 - 0.18}{97 - 96} = 0.15$

Between 104 and 105, $\Delta = \frac{5.03 - 4.14}{105 - 104} = 0.89$ (see Figure 14.8)

At-the-money, e.g. between 99 and 101,

$$\Delta = \frac{1.89 - 0.89}{101 - 99} = \frac{1.00}{2} = 0.5$$

Geometrically, delta is the slope of the pay-off diagram. It can be seen in Figure 14.9 that delta varies from approaching 0 well out-of-the-money to approaching 1 well in-the-money and equals 0.5 at-the-money.

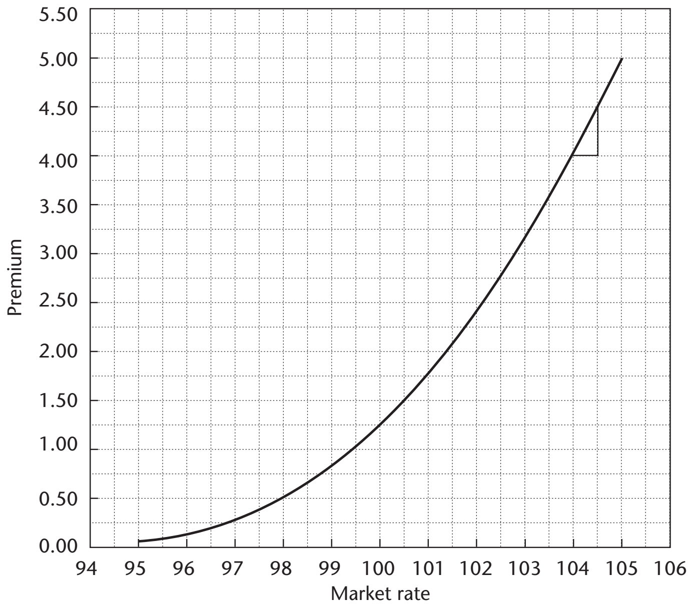


FIGURE 14.8 Delta

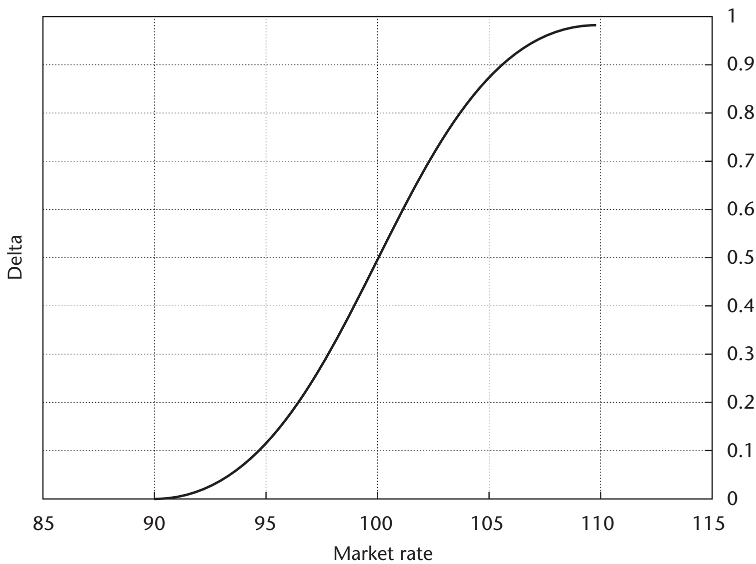


FIGURE 14.9 Delta

Delta is the equivalent of the spot position. Purchasing US$10,000,000 outright forward against yen would create a long US$10,000,000 net exchange position. Purchasing US$10,000,000 of at-the-money dollar calls creates an equivalent spot position of long US$5,000,000 because, for an at-the-money call, delta equals 0.5.

A 1% increase in the market price from 100.00 to 101.00 would increase the value of the long dollar position by US$100,000. The same 1% increase in the market price from 100.00 to 101.00 would increase the value of the long dollar call by approximately US$50,000.

Delta, therefore, shows the percentage of an option that needs to be hedged against possible spot movements. If $\Delta = 0.5$, it is possible to hedge a US$10,000,000 short dollar call against changes in the spot rate by buying US$5,000,000 spot.

If the spot rate rises from 100 to 101,

Loss from short US$10,000,000 call	=	US$50,000
Gain from long US$5,000,000 hedge	=	US$50,000
Net gain or loss	=	US$0

This process is known as *delta hedging* (Figure 14.10), and is common practice amongst option traders, who typically want to hedge themselves against potential spot changes.

Because delta changes with the underlying market rate the option trader will need to continually readjust the hedge to remain *delta neutral*. If the spot rate continues to rise to 102, delta would then be 0.61.

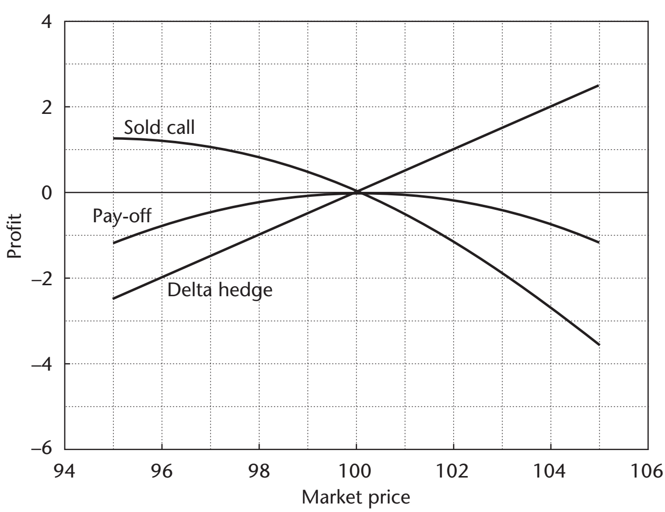


FIGURE 14.10 Delta hedging

$$\Delta = \frac{2.55 - 1.33}{102 - 100} = \frac{1.22}{2} = 0.61$$

If the hedge was unchanged,

Loss from short US$10,000,000,000 call	=	10,000,000(2.55 – 1.33)	= US$122,000
Gain from long US$5,000,000 hedge	=	5,000,000(102 – 100)/102	= US$ 98,039
Net loss			= US$ 23,961

A US$5,000,000 hedge which was appropriate at 100 when Δ = 0.5 is not sufficient at 102 because Δ is now 0.61. The short dollar call is under-hedged or *delta short*. To restore a delta neutral hedge the trader would need to adjust the delta hedge by buying an additional US$1,100,000 spot so that the hedge ratio is again equal to delta.

Mathematically, delta is the first derivative of the option formula with respect to change in the spot rate:

$$\Delta = \frac{\partial \Pi}{\partial S}$$

For European calls:

$$\Delta = e^{-yt} N(d_1) \tag{14.13}$$

For European puts:

$$\Delta = e^{-yt}(N(d_1) - 1) \tag{14.14}$$

where y = continuously compounding commodity currency interest rate.

GAMMA

Gamma measures how quickly delta changes, and so how often the hedge needs rebalancing.

$$\boxed{\text{Gamma } (\Gamma) = \frac{\text{Change in delta}}{\text{Change in market rate}}} \tag{14.15}$$

$$= \frac{\Delta_2 - \Delta_1}{P_2 - P_1}$$

As options approach expiry, gamma tends to become large around the strike price (Figures 14.11 and 14.12). If the market price is below the strike price, the probability of the call being exercised is low, so the delta (that is, the amount needing to be hedged) is small. On the other hand, if the market price is above the strike price, the probability of the call being

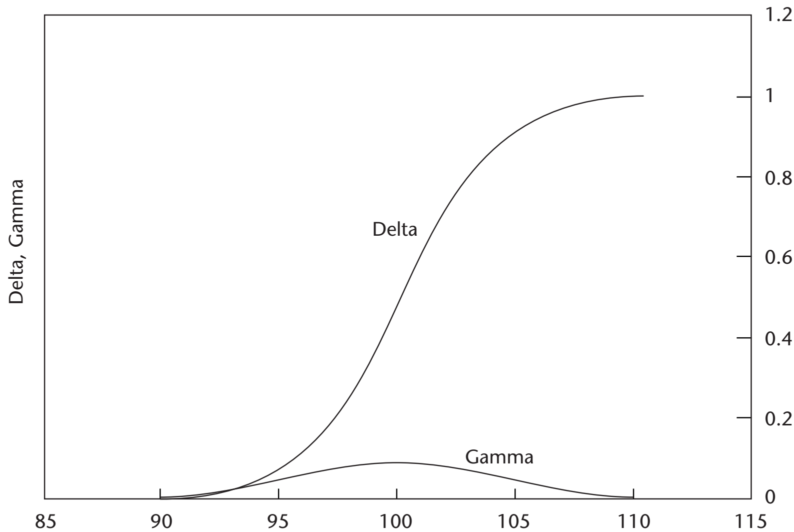


FIGURE 14.11 Delta and gamma

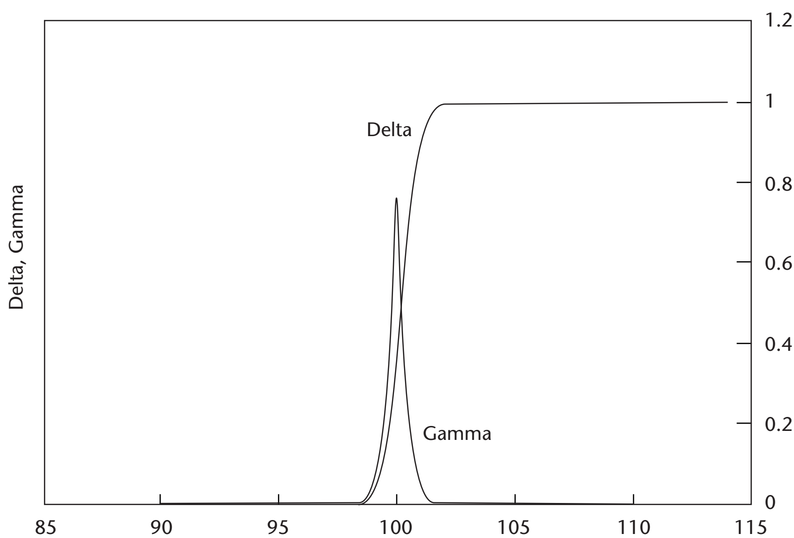


FIGURE 14.12 Gamma at t = 1 day

exercised is high, so the delta is high. A small change in the market price from just below the strike to just above the strike can require a large change in the hedge.

Traders who have purchased options have *long gamma* positions. They tend to gain from changes in the market price. Traders who have sold options have *short gamma* positions. They tend to lose from changes in the market price. Option traders try to avoid having large negative gamma positions because they expose them to possible large losses if the underlying market price moves. It is possible to reduce the size of negative gamma positions by buying options. Buying short dated options has the most impact on reducing gamma.

Another technique is to avoid strike price concentration. By structuring the book so that the options that have been written are at a wide range of strike prices, an option trader can be confident that even if the trader is exposed to high gamma risk at a certain rate he or she can be confident that his or her portfolio of sold options as a whole has only a relatively small gamma risk.

The option writer will realize a loss every time he adjusts the delta hedge. In the above example, he would realize a loss of US$23,961 when he increased the hedge from 0.50 to 0.61 when the exchange rate reached 102. If volatility turns out to equal the volatility that was used to price the option, then over the course of continually delta hedging the sold option the cumulative loss from adjusting the delta hedge would exactly equal the premium received for writing the option.

There is a trade-off between time decay that works in favour of the option seller and the expected hedging cost of following a delta neutral strategy. If actual volatility turns out to be higher than the volatility used to price the option, the option writer will lose more from delta hedging than he collected as premium. On the other hand, if actual volatility turns out to be less than the volatility used to price the option, then the cost of delta hedging will be less than the premium collected.

Traders running option portfolios, therefore, are predominantly concerned with managing their volatility risk.

Mathematically, gamma is the second derivative of the option pricing formula with respect to change in the spot rate or the first derivative of the delta with respect to spot.

$$\Gamma = \frac{\partial^2 \Pi}{\partial S^2} = \frac{\partial \Delta}{\partial S}$$
$$\Gamma = \frac{N'(d_1)e^{-yt}}{S\sigma\sqrt{t}} \tag{14.16}$$

VEGA

The sensitivity of a change in the price of an option to changes in volatility is known as *vega*.

$$\boxed{\text{Vega} = \frac{\text{Change in premium}}{\text{Change in volatility}}} \tag{14.17}$$

$$= \frac{P_2 - P_1}{\sigma_2 - \sigma_1}$$

A trader who has net purchased options will be *long vega*. She will stand to gain if volatility rises and lose if volatility falls. A trader who has net sold options will be *short vega* and will stand to gain if volatility falls and lose if volatility rises.

Mathematically, vega is the first derivative of the option pricing formula with respect to volatility:

$$v = \frac{\partial \Pi}{\partial \sigma} = S\sqrt{t}N'(d_1) \tag{14.18}$$

RHO

The price of an option also depends on interest rates. As the option premium is paid up-front, but the currency is not delivered until the value date for which the option is exercised, the option writer has the use of the premium. The writer will adjust the premium downwards to reflect its time value. The interest rate used to discount the premium to adjust for its time value is known as the risk-free interest rate, so-called because it is assumed that there is no credit risk involved in discounting the premium. The relative importance of the risk-free interest rate tends to increase the longer the term of the option.

Rho is defined as the change of the value of the option to a change in interest rates.

$$\rho = \frac{\text{Change in premium}}{\text{Change in interest rate}} = \frac{P_2 - P_1}{r_2 - r_1} \tag{14.19}$$

For currency options there are two interest rates and so two rhos.

Mathematically:

$$\rho = \frac{\partial \Pi}{\partial r} \quad \text{and} \quad \rho = \frac{\partial \Pi}{\partial y}$$

$$\text{For calls:} \quad \rho = te^{-rt}KN(d_2) \quad \text{and} \quad \rho = -te^{-yt}SN(d_1) \tag{14.20}$$

$$\text{For puts:} \quad \rho = -te^{-rt}KN(-d_2) \quad \text{and} \quad \rho = te^{-yt}SN(d_1) \tag{14.21}$$

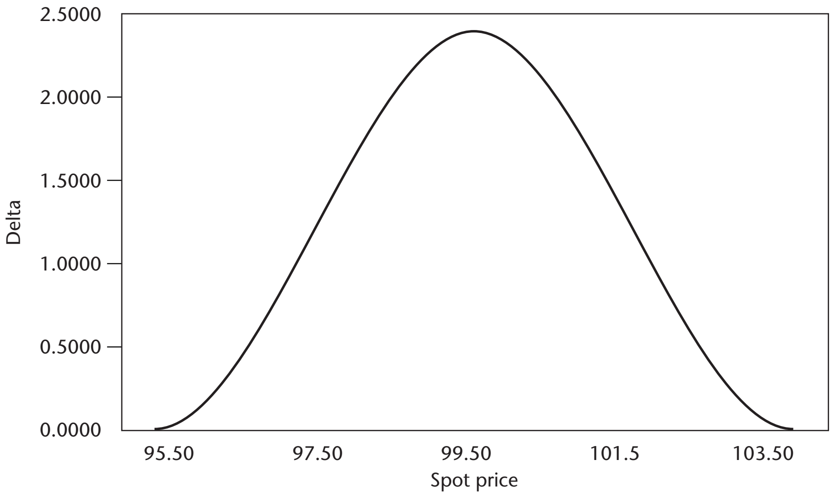


FIGURE 14.13 Delta for at-expiry digital call with strike price 100 with 10 days to expiry

The greeks for option derivatives such as digitals, knock-out, average rate options and look-back options behave differently from those for plain vanilla calls and puts.

Whereas the delta for plain vanilla calls and puts is limited to values between 0 and 1 (or –1 in the case of puts), no such restrictions apply for option derivatives (see Figure 14.13).

As the spot rate approaches the 100 strike the premium of the digital call increases quickly as the probability of a zero pay-out becomes outweighed by the probability of a full pay-out of the digital. It is possible (indeed usual) for delta to take values greater than one.

As the spot price approached the 105 barrier, the up and in call in Figure 14.14 has a delta in excess of 4.0 and the up and out call has a delta below –3.0. Once the spot price touches the 105 barrier, the up and out call knocks-out, so its delta falls to zero and the up and in call knocks-in and its delta approaches one (as it is now a standard call).

Hybrid transactions for which the pay-out is a function of a different market factor also have sensitivity to changes in the correlation between the two market factors.

VALUE AT RISK LIMITS

Organizations control the extent of market price risk by approving *limits* which specify the maximum exposure permitted. Value at risk limits can

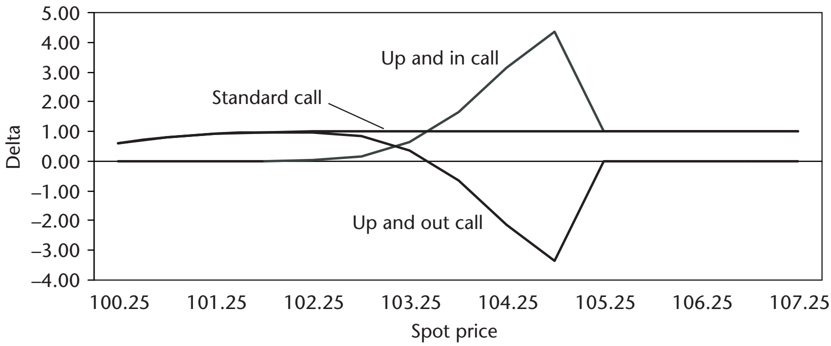


FIGURE 14.14 Delta for up and in and up and out calls with strike price 100 and barrier at 105 with 10 days to expiry

be set for as many market factors as are considered relevant. They can be set for the entire organization or for an individual dealer or both.

A foreign exchange trader dealing in US$/¥ may use a variety of products to open and hedge positions – spot, forwards, swaps, bonds, futures, FRAs and so on. The trader will have exposure to the exchange rate and interest rates in both currencies. Management may want the forwards trader to carry only a small amount of exchange rate risk with exposure predominantly to the interest rate differentials. The following limit structure might be approved to constrain the risks that the dealer may take:

US$/¥ spot	US$100,000	per 2 standard deviation change in the spot rate
US$ interest rates	US$1,000,000	per 2 standard deviation change in dollar rates
¥ interest rates	US$1,000,000	per 2 standard deviation change in yen rates

These limits may be further broken down into tenor buckets. For example, the full US$1,000,000 interest rate limit may be used out to 1 year where liquidity is good but only US$100,000 may be used for transactions maturing beyond 1 year.

It is possible to establish a full set of value at risk limits that cover whatever market price risks are relevant for the business. These can be specified by individual dealer or by business unit to suit the structure of the business.

THE PROBLEM WITH STOP-LOSS LIMITS

Stop-losses are instructions to close out an open position if a market trades through a specified level. Stop-losses are often used to limit the amount that can be lost on a position.

Stop-loss orders work fairly well for positions that are pure trading positions. They can usually be closed out at or near the stop-loss level. However, for banks that conduct market-making activities and corporations that have recurring exposures, stop-losses do not serve to limit potential losses.

A better approach is to establish *management action triggers*. These require management attention whenever accumulated losses reach a predetermined level. Once this level is triggered management reviews the causes of the adverse price movement and ascertains the likelihood of further such changes, weighs these up against their appetite for risk and decides upon a next course of action – perhaps closing or reducing the position or possibly setting a new, higher, trigger. Management action triggers provide a more dynamic approach to risk management than stop-losses.

PORTFOLIO VALUE AT RISK

It is convenient to have a measure of the overall market price risk for an entire portfolio or business.

Portfolio value at risk measures how much would be lost if all market factors moved adversely to a prescribed level of tolerance (e.g. 10% or 2 standard deviations) usually over a 1 day time horizon.

Because many of the market factors are likely to be closely correlated, portfolio value at risk should be calculated on a correlated basis.

EXAMPLE 14.4

There is a high correlation between €/US$ and US$/SF. Historically, when the dollar has strengthened against the euro it has mostly strengthened against the Swiss franc. A dealer is long US$10,000,000 against euro and short US$10,000,000 against SF. The risks associated with the two positions can be expected to largely offset one another.

Assume that the VaR associated with the long US$/short euro position is US$2,600,000 for a 2 standard deviation increase in the spot €/US$ rate and the VaR associated with the short US$/long SF position is US$2,920,000 for a 2 standard deviation increase in the spot US$/SF rate. The higher VaR for the US$/SF exposure indicates that US$/SF is expected to be more volatile than €/US$. If the US$ is likely to strengthen against the SF when it strengthens against the euro, the US$/SF rate would be expected to rise when the €/US$ rate falls. The expected correlation between the two exchange rates would be negative. If the expected correlation between US$/SF and €/US$ is –0.95, then the correlated value at risk would be much lower than the sum of the values at risk of the two separate net exchange positions.

Matrix algebra can be used to calculate the correlated value at risk.

$$\text{Correlated VaR} = \sqrt{\Sigma_{i,j}\rho_{ij}\text{VaR}_i} \times \text{VaR}_j \tag{14.22}$$

When $i = j, \rho_{i,j} = 1$. Hence

$$\begin{aligned}\text{Correlated VaR} &= \sqrt{(2{,}600{,}000)^2} - 0.95(2{,}600{,}000)(2{,}920{,}000) \\ &\quad - 0.95(2{,}920{,}000)(2{,}600{,}000) + (2{,}920{,}000)^2 \\ &= \text{US\$928{,}224}\end{aligned}$$

This is dramatically lower than the sum of the two separate VaRs (2,600,000 + 2,920,000 = 5,520,000) because of the high (negative) expected correlation.

In a large dealing room there are likely to be hundreds of different market factors to address. However, with reasonable systems it is a simple matter to calculate a correlated value at risk for the entire room's exposure.

A *value at risk limit* is a limit set by management (or possibly a regulator) that limits the size of the permissible value at risk exposure.

STRESS TESTS

Occasionally, larger spot rate movements occur than would be implied by a normal distribution of price changes. For example, the change of the £/US$ on the day that the pound left the ERM in September 1992 constituted a 12 standard deviation event if typical volatility was taken to be around 10% p.a. It is advisable to periodically conduct stress tests that dimension the possible loss that would result from an extreme price shock, such as a 4 standard deviation event. Risk managers typically perform a stress test on their portfolio a couple of times each year to assess whether the business could sustain such a shock.

CREDIT RISK

Credit risk is the risk that a counter-party will be unwilling or unable to pay. It comprises settlement risk and pre-settlement risk.

Settlement risk

Settlement risk is the risk that a counter-party takes delivery but fails to pay. Settlement risk occurs when there is a non-simultaneous exchange of items of equivalent value between counter-parties. If a customer of a firm takes delivery of a consignment of goods worth $10,000,000 and fails to pay immediately or in advance, the firm has US$10,000,000 settlement risk (or delivery risk) on the customer. If the customer fails to pay, the firm loses US$10,000,000. If the customer is late in paying (for example, pays the US$10,000,000 for the goods but not until 1 year later) the firm loses the interest on US$10,000,000 for the year. If interest rates were 5% p.a., the loss due to late settlement would be US$500,000.

For foreign exchange transactions settlement risk applies from the time that payment of one currency is made to the counter-party until good funds are received in the other currency from the counter-party. Because payments are made into bank accounts that will be in other countries, possibly in different time zones, it is frequently the case that payments cannot be made simultaneously. For example, if a company in Japan sells NZ dollars against US dollars to a British counter-party, the NZ dollars will be paid into the British company's account with a bank in New Zealand before the close of business in New Zealand, but it cannot receive the US dollars into its bank account in the USA until New York business hours. There is at least an eight hour delay. Further, by the time the US dollars are credited to the Japanese company's New York account, it will be well into the Japanese night. People in the company in Japan are not likely to be aware that they have received good funds until the next morning their time. If there is a time lag between when funds are received and confirmation that they have been received (for example, they may not receive a statement showing that the US dollars have been received until the following day), then the settlement risk applies for a longer period.

Settlement risk is predominantly operational risk. Good systems enable an organization to know at the earliest possible time that it has received payment from its counter-party.

Settlement risk on a particular counter-party can be limited by using *settlement risk limits*. For example, the credit department may limit the settlement risk that can be taken on the specified counter-party to, say, US$5,000,000, on any day. Dealers would be permitted to enter into transactions with the counter-party provided the settlement risk carried on that counter-party does not exceed US$5,000,000 on any settlement date. If the process for receiving confirmation of settlements takes more than a day, that would further restrict the permitted transactions.

Various techniques exist to reduce settlement risk.

Customers may be required to pay in advance. For example, if a foreign exchange transaction is for value 17 May, the bank may require the customer to settle its side of the deal value 16 May before releasing funds to the customer on 17 May. Normally, the bank would give good value, i.e. pay the customer a day's interest. A variation is to require the customer to post collateral.

Another technique to reduce settlement risk is to spread the settlement dates of a large transaction over a number of days. For example, a customer with a settlement risk limit of US$5,000,000 per day could do a US$20,000,000 transaction by agreeing to stagger the settlement over 4 days. This would enable both counter-parties to hold back payments to the other party until they know that the settlement due the previous day has occurred.

Yet another technique is to use a creditworthy intermediary. If a bank has no settlement risk lines available for a customer that wishes to enter into a US$10,000,000 transaction, it may be able to find another bank with whom it has lines that is prepared to stand in between the original bank and the customer. The second bank would generally expect to receive some compensation from the first bank for taking on the settlement risk on the customer. This could take the form of a small difference in the rates at which the second bank deals with the customer on the one hand and the first bank on the other.

Pre-settlement risk

Pre-settlement risk occurs when a counter-party defaults on a contractual obligation prior to settlement. The necessary condition for a credit loss to result from pre-settlement risk is that the counter-party defaults *and* the set of transactions with the counter-party has a positive replacement value.

Pre-settlement risk comprises the replacement cost and an estimate of potential exposure.

$$\boxed{\text{PSR} = \text{Replacement cost} + \text{Potential exposure}} \tag{14.23}$$

Replacement cost is the difference between the value of transaction as contracted and what it would be worth if done at current market rates for the original value dates. Replacement cost is also known as the *mark-to-market value*. It dimensions the current exposure on the counter-party.

EXAMPLE 14.5

Three months ago a customer bought ¥1,000,000,000 by selling US$10,000,000 to a bank 1 year forward at a then forward exchange rate US$1 = ¥100.00. Today the 9 month forward rate is US$1 = ¥105.00. If the customer defaulted, the bank would have to sell ¥1,000,000,000 at 105.00 to reinstate its position. The bank would receive a replacement value of only US$9,523,809.52 which is US$476,190.48 less than the US$10,000,000 which it is due to receive from the customer.

In terms of market price risk, the bank is US$476,190.48 in-the-money and the customer is US$476,190.48 out-of-the-money. In terms of *credit* risk, the bank stands to lose US$476,190.48 if its customer defaults. The bank has credit risk because the market value of its position is in-the-money and the market value of its customer's position is out-of-the-money.

From the bank's perspective, its mark-to-market value of its contract with the customer is:

$$\begin{aligned} \text{MTM} &= \frac{1{,}000{,}000{,}000}{100.00} - \frac{1{,}000{,}000{,}000}{105.00} \\ &= 10{,}000{,}000.00 - 9{,}523{,}809.52 \\ &= \text{US\$}476{,}190.48 \end{aligned}$$

Note: A positive mark-to-market value indicates that the market value of the *position* is in-the-money, so there is a positive replacement cost and, therefore, a credit risk on the counter-party.

From the customer's perspective, there is no credit risk on the bank. If the bank defaulted for some reason, which is difficult to imagine, the customer could now buy the ¥1,000,000,000 more cheaply, which would not be a problem. The customer's mark-to-market value is negative, so it would have a replacement benefit, and therefore no credit risk on the bank.[1]

NPV METHOD

A superior method of measuring replacement cost is the *present value* of the mark-to-market amount. This brings the value of the replacement cost into today's terms, which makes more sense than using future value terms, particularly if the time period is long or interest rates are high. If the 9 month dollar interest rate is 5% p.a., the mark-to-market value in present value terms would be US$458,978.78.

$$\text{PV}(476{,}190.48) = \frac{476{,}190.48}{(1 + 0.05 \times 270/360)} = 458{,}978.78$$

For bought options the replacement cost is the mark-to-market value of the option, which is the current premium. There is no pre-settlement risk on sold options unless the payment of the premium is deferred, in which case it equals the present value of the deferred premium.

POTENTIAL EXPOSURE

Continuing Example 14.5, the bank has additional credit risk on the customer because in the 9 months still to go before the contract settles it is

1 In fact the customer has potential exposure but no current exposure.

possible that the replacement cost will increase further. This would happen if the exchange rate rises above 105.00.

Potential exposure represents a provision against a possible increase in replacement cost.

It is possible to use distribution theory to estimate potential exposure. If the expected volatility of the dollar/yen exchange rate were 12% p.a., then over the 9 months remaining a 1.2825 standard deviation adverse movement (which could be expected to occur 10% of the time) would involve the exchange rate rising from 105.00 to 119.00. If the bank is comfortable having less than a 10% chance of underestimating its credit loss, it could dimension its potential exposure on the customer as the difference between ¥1,000,000,000 at 105.00 and ¥1,000,000,000 at 119.00:

$$\begin{aligned}\text{FV(Potential exposure)} &= \frac{1{,}000{,}000{,}000}{105.00} - \frac{1{,}000{,}000{,}000}{119.00}\\ &= 9{,}523{,}809.52 - 8{,}403{,}361.35\\ &= \$1{,}120{,}448.17\end{aligned}$$

$$\text{Potential exposure} = \frac{1{,}120{,}448.17}{(1 + 0.05 \times 270/360)} = \$1{,}079{,}950.04$$

Pre-settlement risk equals replacement cost plus the estimate of potential exposure.

$$\begin{aligned}\text{PSR} &= \text{Replacement cost} + \text{Potential exposure}\\ &= 458{,}978.78 + 1{,}079{,}950.04\\ &= \text{US}\$1{,}538{,}928.82\end{aligned}$$

The bank could consider itself as having US$1,538,928.82 credit risk on the customer as a result of having sold ¥1,000,000,000 to the customer at a 1 year forward rate of 100.00 three months ago.

If the mark-to-market value is negative (i.e. if the customer's position is in-the-money and the bank's position is out-of-the-money), then the bank clearly has less pre-settlement risk on the customer.

For example, if instead after 3 months the then 9 months forward rate were 95.00,

$$\begin{aligned}\text{FV(MTM)} &= \frac{1{,}000{,}000{,}000}{100.00} - \frac{1{,}000{,}000{,}000}{95.00}\\ &= 10{,}000{,}000.00 - 10{,}526{,}315.79\\ &= -\text{US}\$526{,}315.79\end{aligned}$$

$$\text{MTM} = \frac{-526{,}315.79}{10{,}000{,}000} = -\$507{,}292.33$$

So:

$$\text{PSR} = -507{,}292.33 + 1{,}079{,}950.04$$
$$= \text{US\$}572{,}657.71$$

CREDIT RISK FACTORS

A *credit risk factor* is a number that can be multiplied by the principal amount of the transaction to provide the deemed amount of potential exposure.

$$\boxed{\text{Potential exposure} = \text{CRF} \times \text{Principal}} \tag{14.24}$$

Continuing Example 14.5,

$$\text{CRF} = \frac{1{,}079{,}950.04}{10{,}000{,}000.00} = 10.8\%$$

It is possible to generate tables of credit risk factors for different tenors for exchange rates with different expected volatilities. Such tables make it easy for dealers to establish how much pre-settlement risk will be associated with a transaction. The credit risk factor will be a function of expected volatility and the square root of time to maturity.

Credit risk factors increase with the tenor of the transaction and expected volatility. The longer the transaction has till maturity, the greater will be the potential risk. In Figure 14.15, the higher curve shows the credit risk factors for currency pairs for which expected volatility is deemed to be 10% p.a. and the lower curve where expected volatility is deemed to be 5% p.a.

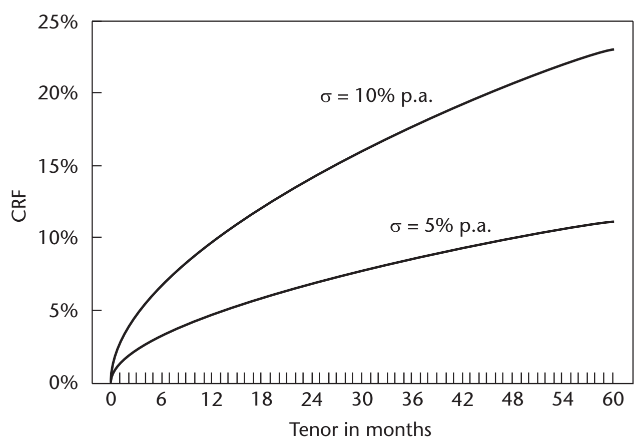


FIGURE 14.15 Credit risk factors

The 10% CRF curve shows the following credit risk factors:

Tenor	*CRF*
1 month	2.9%
3 months	5.0%
6 months	7.1%
1 year	10.1%
2 years	14.2%
5 years	22.5%

If dollar/yen is assumed to have 10% p.a. expected volatility, then the credit risk factor for a contract with 6 months to maturity would be 7.1%. The estimate of potential exposure on a US\$10,000,000 transaction would be calculated as follows:

$$\begin{aligned}\text{Estimate of potential exposure} &= CRF \times P \\ &= 0.071 \times 10{,}000{,}000 \\ &= \text{US\$710,000}\end{aligned}$$

EXAMPLE 14.6

A dealer buys US\$5,000,000 one year forward against yen at an outright rate of US\$1 = ¥106.75.

(a) If the credit risk factor for 1 year dollar/yen is 10.1%, what is the pre-settlement risk of the transaction when it is done given that dollar interest rates are 6.0% p.a.?

$$\text{PSR} = \text{MTM} + \text{Potential exposure}$$

$$\text{MTM} = 0 \quad \text{(because deal is done at market rates)}$$

$$\begin{aligned}\text{FV(Potential exposure)} &= CRF \times P \\ &= 0.101 \times 5{,}000{,}000 \\ &= \text{US\$505,000}\end{aligned}$$

$$\text{Potential exposure} = \frac{505{,}000}{(1+0.06)} = \text{US\$476,415.09}$$

$$\text{PSR} = \text{US\$476,415.09}$$

(b) One month later, the 11 month rate is US\$1 = ¥107.50 and dollar interest rates are still 6.0% p.a., calculate the pre-settlement risk if the 11 month CRF = 9.2%.

$$\begin{aligned}
\text{FV(MTM)} &= 5{,}000{,}000(106.75 - 107.50)\\
&= \frac{-3{,}750{,}000}{107.50}\\
&= -\text{US\$}34{,}883.72
\end{aligned}$$

$$\text{MTM} = \frac{-34{,}883.72}{(1 + 0.06 \times 11/12)} = -33{,}065.14$$

$$\begin{aligned}
\text{Potential exposure} &= \frac{5{,}000{,}000}{(1 + 0.06 \times 11/12)} \times 0.092\\
&= \text{US\$}436{,}018.96\\
\text{PSR} &= -33{,}065.14 + 436{,}018.96\\
&= \text{US\$}402{,}953.82
\end{aligned}$$

PRE-SETTLEMENT RISK LIMITS

Pre-settlement risk on a particular counter-party can be limited by using *pre-settlement risk limits*. For example, the credit department may limit the pre-settlement risk that can be taken on the specified counter-party to say, US$50,000,000. Dealers would be permitted to enter into transactions with the counter-party provided the pre-settlement risk carried on that counter-party never exceeds US$50,000,000. The size of the limit generally depends on the creditworthiness of the counter-party as reflected by its credit rating. Separate limits may be set for different tenors. For example, a particular counter-party may be allocated a limit of US$50,000,000 for transactions out to 2 years but only US$20,000,000 for transactions beyond 2 years. The reason for the smaller limit for longer tenors is that there is greater uncertainty that the counter-party will be creditworthy or even in existence.

TECHNIQUES TO REDUCE PSR

Credit is a scarce resource. Various techniques can be used to minimize the usage of pre-settlement risk credit lines.

Netting PSR

It is often the case that an organization will have multiple transactions with the same counter-party. The aggregate credit exposure could be calculated by simply adding together the pre-settlement risk for all of the individual transactions. However, provided netting is legally enforceable, the aggregate exposure may be substantially less than the sum of the individual PSRs.

A portfolio of three forward foreign exchange contracts with the same counter-party but different times to maturity has less exposure than the sum of the pre-settlement risks of the three contracts.

EXAMPLE 14.7

A firm has the following three foreign exchange transactions with a particular counter-party (Figure 14.16):

Transaction	*US$ amount*	*Maturity*	*Pre-settlement risk*
A	+4,000,000	6 months	244,949
B	–3,000,000	1 year	259,808
C	+1,000,000	2 years	122,474
Total without netting			627,231
Total with netting			107,616

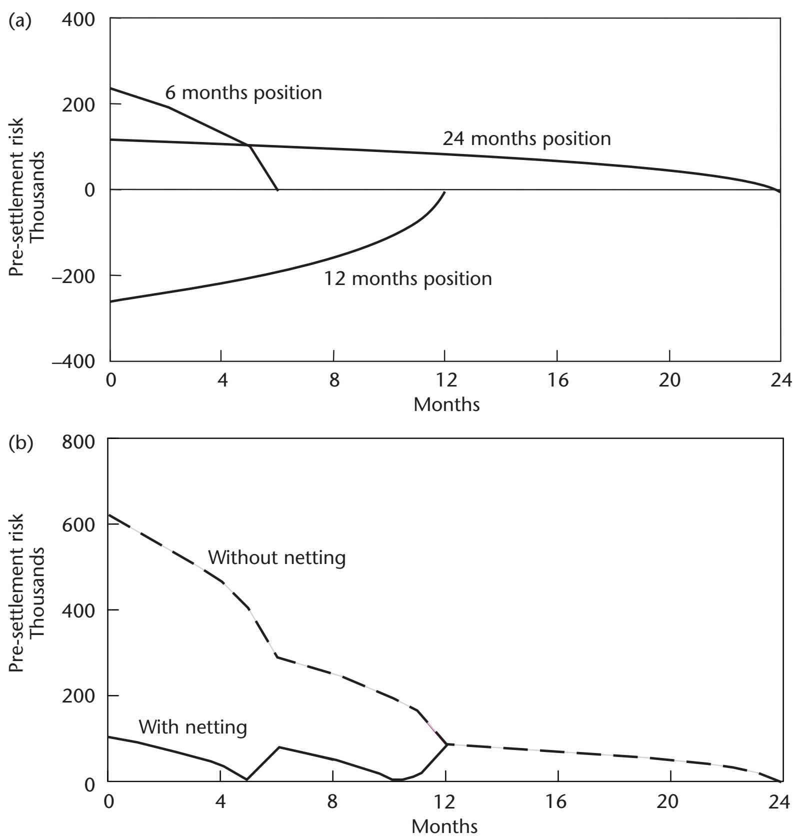


FIGURE 14.16 Pre-settlement risk on 3 separate transactions

Pre-settlement risk with and without netting

For each of the three transactions the pre-settlement risk declines as the contract approaches maturity. Because the transaction with 1 year to maturity is opposite in sign to the other two, the aggregate exposure with netting is significantly less than without netting. In this case, it is less than one sixth of what it would be without netting.

In some jurisdictions liquidators are allowed to *cherry-pick* i.e. collect on transactions that are in-the-money without having to pay out on transactions which are out-of-the-money. When this is the case there is no benefit in netting. Clearly, from the point of view of reducing credit risk, netting is preferable provided it is enforceable.

Allowing for correlation

There may be a reduction in credit risk even if the transactions with a particular counter-party are not all in the same currency pair. There is a high negative correlation between €/US$ and US$/SF. Suppose it is 0.95. A portfolio with a long dollar position against the euro and a short dollar position against the Swiss franc will have some natural offset. The loss resulting from the dollar rising against the Swiss franc is very likely to be largely offset by the euro falling against the dollar. If the two exposures are of roughly equal size, the risk on the portfolio will be only a small fraction of the risk on either leg.

Indeed, a superior method of dimensioning credit risk for a portfolio of transactions with a single counter-party is to run a Monte Carlo simulation on the entire portfolio. Input into the simulation includes the set of transactions (currencies, amounts, sides, maturities) and a variance–covariance matrix (assumed volatilities and correlations for each currency pair).

It is possible to estimate the potential change in the value of the counter-party's portfolio as a time-varying amount of risk over the life of the portfolio with a peak at some point in time. It is typically measured to two standard deviations assuming a tolerance level of 97.7% and that volatilities and correlations behave as expected.

It must be understood that in practice assumptions about volatilities and correlations do not always turn out to be correct, but provided the tolerance level is conservative, there will be little risk of credit losses exceeding approved limits. A second level of protection lies in the fact that for an organization to suffer a credit loss, the counter-party must not only be out-of-the-money but it must also fail to pay.

Market reset payments

Documentation can provide for payments equal to the then mark-to-market amount to be made at pre-specified periods. This limits the credit risk on the counter-party to the amount implied by the period at which the reset payment is required unless the payment is not made on time.

Right to break (early termination) clauses

Credit departments typically set a maximum tenor for transactions with certain counter-parties reflecting uncertainty over the possible creditworthiness of the counter-party beyond that time.

Either or both parties can be given the right to terminate contracts at specified times. This enables transactions to be done with counter-parties for tenors longer than their tenor limit.

For example, if transactions with a particular customer are limited to a maximum of 2 years, it is possible to enter into a 5 year forward with a right to break after 2 years. Provided the customer is still considered creditworthy after the first 2 years, the transaction will continue with another right to break at the 4 year point. If, on the other hand, the customer is no longer considered creditworthy at the time of the right to break, then the transaction can be unwound. The right to break need not require an event of default.

Margining

The counter-party may be required to lodge cash as a margin to provide a buffer against potential credit losses. This mechanism is widely used for exchange traded products and is commonly used in over-the-counter transactions as well.

Typically, the customer is required to lodge an initial margin amount (say, 5% of face value) and to top-up whenever the mark-to-market exposure reaches or exceeds a pre-specified level. If the party fails to make a margin payment on time, outstanding contracts are closed out.

Margining limits the credit risk to the trigger level at which the margin call will be made. There is actually an incremental amount of risk involved because the market could move further against the counter-party between the trigger being hit and the margin being received. Provided the process is efficient this will be a relatively small amount, and a provision for it is often built into the margining requirements.

Collateral

The counter-party may be permitted to lodge some asset other than cash as collateral against potential credit risk. Typically, collateral is in the form of stocks or bonds, but any asset that is acceptable to the organization taking the credit risk can be used. Because the value of the collateral is likely to change over time there needs to be a process to revalue the collateral at regular intervals in case a top-up of collateral is required to maintain adequate cover of the mark-to-market exposure.

LIQUIDITY RISK

There are two types of liquidity risk.

Funding liquidity is the ability to make payments on time. *Funding liquidity risk* is the risk of not having access to sufficient cash to be able to make payments on time. It is most important to confine funding liquidity risk, not only because there are likely to be severe interest penalties, but because counter-parties may refuse to deal with an organization that fails to meet its cash flow obligations. Even if the organization has substantial unrealizable assets, if it has not managed its cash flow so as to be able to make a payment, confidence in its business can be undermined.

Market liquidity describes the volumes that can readily be transacted in the market. *Market liquidity risk* is the risk that it may not be possible to do the full amount of a transaction without seriously impacting the market price. Spot foreign exchange in the major currencies are some of the deepest (i.e. most liquid markets) in the world. Literally billions of dollars of transactions are done every day in euro/dollar, dollar/yen and cable. However, exotic currencies can be very thin or illiquid. It can be quite difficult to find a counter-party to quote even US$10,000,000 in some South American, African or Eastern European currencies. Even in the major currencies liquidity can be scarce at certain times. At 6 p.m. New York time, which is three hours after New York inter-bank trading closes and two or three hours before the Asian market opens, it is quite hard to get a quote in some currencies that are liquid during European hours.

Market liquidity is generally reflected in the bid offer spread. If there are many banks prepared to quote two-way prices in the currency at that time of day, the bid offer spread will be narrow. However, if there are only a couple of banks quoting the currency, and/or if the amount is relatively large, the bid offer spread will be wider. If a news release that could have a significant impact on the market price is due out in a minute's time, liquidity may tend to dry up temporarily.

Like funding liquidity risk, it is important to understand and manage market liquidity risk. A dealer who trades profitably all month can have his month's profitability spoilt by having a position which he cannot get out of without taking a big loss because of lack of market liquidity.

MANAGING FUNDING LIQUIDITY RISK

Liquidity squeezes occur from time to time. Sometimes they are deliberately engineered by the central bank. Even when the market generally is liquid a particular party may find itself with a liquidity problem. This could be the consequence of its credit position deteriorating or simply a function of poor management.

Managing funding liquidity risk means ensuring that there will always be sufficient cash or credit facilities available to meet payment obligations.

Maximum cumulative outflow is the largest negative cash outflow position which will accumulate from existing commitments.

If an organization has a zero cash balance in its dollar bank account at the start of the week and has existing commitments which will cause the following cash flows during the week, then its maximum cumulative outflow would be 30 which would occur on Tuesday.

	Inflows	*Outflows*	*Balance*
Monday opening			0
Monday	+10	–20	–10
Tuesday	+30	–50	**–30**
Wednesday	+50	–20	0
Thursday	+40	–10	+30
Friday	+20	–30	+20

A *maximum cumulative outflow limit* is a limit to the permitted value of the maximum cumulative outflow. The organization may know from experience that it can easily raise, say, US$100,000,000 on any day by borrowing from banks with whom it has standby credit facilities or by selling other currencies or liquid securities. If so, its management would probably approve an MCO limit of up to US$100,000,000.

It is generally the case that it is possible to raise more money over a longer period of time. Consequently, there may be a larger MCO limit set for the first month than for the first week and so on.

The organization should have an MCO limit in every currency in which it has cash flows except for those currencies for which it follows a policy of always squaring off cash flows in the currencies in which case it effectively has a zero MCO limit in those currencies.

MCO limits should be reviewed regularly (for example, at a monthly meeting) and when liquidity is tight the limits may be reduced or reviewed more regularly (for example, every day).

It is good practice to also prepare a contingency funding plan that sets out what actions (for example, selling specific assets and arranging additional sources of funds) would be taken in the event of a liquidity problem arising.

Another good practice is to diversify funding sources so that if a particular provider of funds or class of funds providers dries up there will be sufficient cash flow available for business to proceed as usual. Management and central banks often require information on large funds providers. For example, the central bank may require a bank to report all cases where it obtains more than, say, 10% of its funding from a single entity, or management may impose limits or triggers on the percentage of funds sourced through a particular program (e.g. issuing commercial paper) or from a particular customer segment (e.g. consumer deposits). The limits or trigger levels will depend on the nature of the business.

Managing market liquidity relies on having experienced leaders who can guide junior dealers in how to quote (i.e. how much bid offer spread) and what size positions to carry in particular currencies at certain times. Because market liquidity tends to vary dramatically over time it is usually better to have the flexibility of allowing middle management judgement to react to the circumstances than to prescribe firm rules or limits. The most important thing is to know that people who understand market liquidity are always present and focused on the issue.

Interdependence of market, credit and liquidity risks

Whilst it is meaningful to measure and manage the different types of risk separately it is important to appreciate that they are closely interrelated. Less liquidity almost certainly means greater price risk. If volatility is high then both price risk and credit risk will be greater. Reducing credit limits with particular counter-parties will reduce liquidity, and thus will increase price risk and so on. The full set of risks should be viewed together as well as separately.

OTHER TYPES OF FINANCIAL RISK

Other types of risk also apply to foreign exchange and other financial transactions. These are discussed below.

Legal or documentation risk

Both parties to a transaction should have signed mutually acceptable and legally enforceable documentation, and an effective process needs to be in place to confirm the details of transactions as they are done.

Operational risk

The back-office needs to be thorough and efficient in confirming, booking and settling transactions as well as following up on any outstanding issues.

Tax and accounting risk

It is important to be aware of the tax and accounting implications of transactions. Treatment frequently varies from country to country. Even within the same country, the treatment for tax purposes may be different than for accounting. Failure to understand the tax or accounting implications of transactions or penalties for incorrect reporting can be very expensive.

Selling risk

Organizations that market financial products have a duty of care to ensure that their products are suitable for their clients and that their clients understand the risks of entering into the transaction. The more complex the transaction and the less sophisticated the client, the greater is the obligation on the selling organization. The risk is that a client may sue the bank if the bank has failed to properly identify the risks of it entering into the transaction.

As with market risk, credit risk and liquidity risk, other risks, such as legal, operational, accounting and selling risk, are best mitigated by having people who understand what risks are involved in the transactions and knowing how to manage them effectively. If the people are experienced, and there is a continuing program of education and a culture of compliance, then the likelihood of losses in excess of tolerable levels will be minimized.

PRACTICE PROBLEMS

14.1 Value at risk

Market scenario:

Spot	€1	=	US$0.9250
6 month euro			3.50% p.a. (180/360)
6 month dollars			2.75% p.a. (180/360)

$$f = 0.9250 \times \frac{(1+0.0275/2)}{(1+0.035/2)} = 0.9216$$

A dealer purchased €10,000,000 at a 6 month outright forward rate of 0.9216 and has not covered the position.

(a) Calculate the 2 standard deviations stressed rate if spot rate changes are assumed to be normally distributed and volatility is expected to be 9.2% p.a.
(b) Calculate the value at risk.

14.2 Delta hedging

On a day when the spot rate was US$1 = ¥123.50 a bank sold a US$ call/¥ put with face value US$10,000,000 and strike price 122.50. A pricing model displayed the following premiums for the sold call:

Spot rate	*Premium*
122.50	¥2.08
123.00	¥2.31
123.50	¥2.57
124.00	¥2.84

(a) Calculate the average delta between 123.00 and 124.00. What transaction should the bank do to delta hedge?

One week later the spot rate has fallen to 123.00 and the pricing model displays the following premiums:

Spot rate	*Premium*
122.50	¥2.08
123.00	¥2.31
123.50	¥2.57

(b) Calculate the revised average delta. What transaction should the bank do to adjust its delta hedge?

14.3 Credit risk
Two months ago a bank purchased A$10,000,000 from XYZ Limited at the then 5 month forward rate of A$1 = US$0.5230. The prevailing rates today are:

Spot	A$/US$	0.5620
3 month swap rate		–0.0010
3 month US$ rate		3.20% p.a. (90/360)
3 month Credit Risk Factor		5.0%

Calculate in US$ NPV terms:

(a) The bank's marked-to-market exposure on XYZ Limited
(b) The bank's estimated potential exposure on XYZ Limited
(c) The bank's pre-settlement risk on XYZ Limited

XYZ Limited wishes to sell more US dollars to the bank for the same value date.

(d) How large a deal can be done if the bank's PSR limit on XYZ is US$2,000,000?

Solutions to Practice Problems

CHAPTER 1

1.1

	Original exchange rate	Reciprocal rate	Answer
(a)	€1 = US$0.8420	US$1 = €?	**1.1876**
(b)	£1 = US$1.4565	US$1 = £?	**0.6866**
(c)	NZ$1 = US$0.4250	US$1 = NZ$?	**2.3529**

1.2 Given

$$\begin{aligned} \text{US\$1} &= ¥123.25 \\ £1 &= \text{US\$1.4560} \\ \text{A\$} &= \text{US\$0.5420} \end{aligned}$$

(a) Calculate the cross rate for pounds in yen terms.

$$\begin{aligned} ¥? &= £1 \\ £1 &= \text{US\$1.4560} \\ \text{US\$1} &= ¥123.25 \\ £1 &= 1.4560 \times 123.25 = ¥179.45 \end{aligned}$$

(b) Calculate the cross rate for Australian dollars in yen terms.

$$\begin{aligned} ¥? &= \text{A\$1} \\ \text{A\$1} &= \text{US\$0.5420} \\ \text{US\$1} &= ¥123.25 \\ \text{A\$1} &= 0.5420 \times 123.25 = ¥66.80 \end{aligned}$$

(c) Calculate the cross rate for pounds in Australian dollar terms.

$$\begin{aligned} \text{A\$}? &= £1 \\ £1 &= \text{US\$1.4560} \\ \text{US\$0.5420} &= \text{A\$1} \\ \text{A\$1} &= 1.4560/0.5420 = £2.6863 \end{aligned}$$

1.3 (a) Calculate the realized profit or loss as an amount in dollars when C8,540,000 are purchased at a rate of C1 = \$1.4870 and sold at a rate of C1 = \$1.4675.

$$\begin{aligned}\text{Realised profit} &= \text{Proceeds of sale of Crowns}\\ &\quad -\text{Cost of purchase of Crowns}\\ &= 8{,}540{,}000 \times 1.4675 - 8{,}540{,}000 \times 1.4870\\ &= \$166{,}530\end{aligned}$$

(b) Calculate the unrealized profit or loss as an amount in pesos on P17,283,945 purchased at a rate of Rial 1 = P0.5080 and that could now be sold at a rate of R1 = P0.5072.

$$\begin{aligned}\text{Unrealised profit} &= \text{Proceeds of potential sale}\\ &\quad -\text{Cost of purchase of pesos}\\ &= \frac{17{,}283{,}945}{0.5072} - \frac{17{,}283{,}945}{0.5080}\\ &= 34{,}077{,}178.63 - 34{,}023{,}513.78\\ &= \text{R}53{,}664.85\\ &= 53{,}664.85 \times 0.5072\\ &= \text{P}27{,}218.81\end{aligned}$$

1.4 Calculate the profit or loss when C\$9,360,000 are purchased at a rate of C\$1 = US\$1.4510 and sold at a rate of C\$1 = US\$1.4620.

$$\begin{aligned}\text{Realised profit} &= \text{Proceeds of sale of C\$} - \text{Cost of purchase of C\$}\\ &= 9{,}360{,}000 \times 1.4620 - 9{,}360{,}000 \times 1.4510\\ &= 9{,}360{,}000 \times (1.4620 - 1.4510)\\ &= 9{,}360{,}000 \times 0.0110\\ &= \text{US\$}102{,}960\end{aligned}$$

1.5 Calculate the unrealized profit or loss on Philippine pesos 20,000,000 which were purchased at a rate of US\$1 = PHP47.2000 and could now be sold at a rate of US\$1 = PHP50.6000.

$$\begin{aligned}\text{Unrealised profit} &= \text{Proceeds of potential sale}\\ &\quad -\text{Cost of purchase of pesos}\\ &= \frac{20{,}000{,}000}{50.6000} - \frac{20{,}000{,}000}{47.2000}\\ &= 395{,}256.92 - 423{,}728.81\\ &= -\text{US\$}28471.90\end{aligned}$$

CHAPTER 2

2.1 (a) Calculate the interest earned on an investment of A$2,000 for a period of three months (92/365 days) at a simple interest rate of 6.75% p.a.

$$\begin{aligned} I &= P \times r \times t \\ &= 2{,}000 \times \frac{6.75}{100} \times \frac{92}{365} \\ &= \$34.03 \end{aligned}$$

(b) Calculate the future value of the investment in 2.1(a).

$$\begin{aligned} FV &= P + I \\ &= 2{,}000 + 34.03 \\ &= \$2{,}034.03 \end{aligned}$$

Alternatively,

$$\begin{aligned} FV &= P(1 + rt) \\ &= 2{,}000\left(1 + \frac{6.75}{100} \times \frac{92}{365}\right) \\ &= 2{,}000 \times 1.017014 \\ &= \$2{,}034.03 \end{aligned}$$

2.2 Calculate the future value of $1,000 compounded semi-annually at 10% p.a. for 100 years.

$$\begin{aligned} FV &= P(1+i)^n \\ P &= 1{,}000 \\ i &= 0.10/2 = 0.05 \\ n &= 100 \times 2 = 200 \\ \therefore FV &= 1{,}000(1+0.05)^{200} \\ &= 100{,}000(1.06125)^4 \\ &= \$17{,}292{,}580.82 \end{aligned}$$

2.3 An interest rate is quoted as 4.80% p.a. compounding semi-annually. Calculate the equivalent interest rate compounding monthly.

$$\left(1 + \frac{r}{12}\right)^{12} = \left(1 + \frac{0.048}{2}\right)^2 = 1.048576$$

$$1 + r/12 = 1.048576^{1/12} = 1.003961$$

$$r = 0.0475 = 4.75\% \text{ p.a.}$$

2.4 Calculate the forward interest for the period from six months (180/360) from now to nine months (270/360) from now if the six month rate is 4.50% p.a. and the nine month rate is 4.25% p.a.

$$FV_6 = (1 + 0.045 \times 180/360) = 1.025$$
$$FV_9 = (1 + 0.0425 \times 270/360) = 1.0525$$
$$r_{6,9} = \frac{360}{90} \times \frac{1.03188}{1.02250} - 1$$
$$= 0.0367 = 3.67\% \text{ p. a.}$$

2.5 Calculate the present value of a cash flow of \$10,000,000 due in three years time assuming a quarterly compounding interest rate of 5.25% p.a.

$$PV = \frac{10{,}000{,}000}{(1 + 0.0525/4)^{12}} = 8{,}551{,}525.87$$

2.6 Calculate the price per \$100 of face value of a bond that pays semi-annual coupons of 5.50% p.a. for 5 years if the yield to maturity is 5.75% p.a.

Coupon t	*5.50% cf*	*YTM df*	*5.75% PV*
0.5	2.75	0.970874	2.67
1.0	2.75	0.942596	2.59
1.5	2.75	0.915142	2.52
2.0	2.75	0.888487	2.44
2.5	2.75	0.862609	2.37
3.0	2.75	0.837484	2.30
3.5	2.75	0.813092	2.24
4.0	2.75	0.789409	2.17
4.5	2.75	0.766417	2.11
5.0	102.75	0.744094	76.46
			97.87

2.7 Calculate the forward interest rate for a period from 4 years from now till 4 years and 6 months from now if the 4 year rate is 5.50% p.a. and the 4 and a half year rate is 5.60% p.a. both semi-annually compounding. Express the forward rate in continuously compounding terms.

$$\left(1 + \frac{0.055}{2}\right)^8 e^{0.5r} = \left(1 + \frac{0.056}{2}\right)^9$$
$$e^{0.5r} = \frac{1.282148}{1.242381} = 1.032009$$
$$r = 2 \times \ln(1.032009) = 0.063014 = 6.30\% \text{ p. a.}$$

CHAPTER 3

3.1 Show the cash flows when $2,000,000 is borrowed from one month till six months at a forward interest rate $r_{1,6}$ of 5% p.a.

US$		
		Spot

US$		
2,000,000.00		1 month

US$		
	–2,041,666.67	6 months 2,000,000(1 + 0.05 – 5/12)

3.2 Show the cash flows when €2,000,000 are purchased three months forward against US dollars at a forward rate of €1 = US$0.8560.

€			US$	
		Spot		
		3 months **0.8560**		**–1,712,000.00**
2,000,000.00				

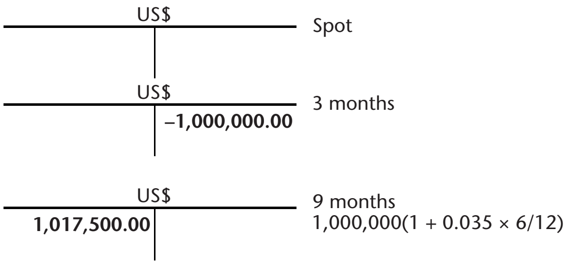
3.3 Prepare a net exchange position sheet for a dealer whose local currency is the US dollar who does the following five transactions. Assuming he or she is square before the first transaction, the dealer:

1. Borrows €7,000,000 for four months at 4.00% p.a.
2. Sells €7,000,000 spot at €1 = 0.8500
3. Buys ¥500,000,000 spot at US$1 = ¥123.00
4. Sells ¥200,000,000 spot against euro at €1 = ¥104.50
5. Buys €4,000,000 one month forward at €1 = US$0.8470.

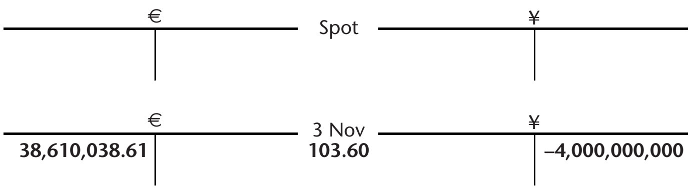
			¥	
		NEP		*NEP*
Transaction 1	–€93,333.33	€93,333.33		
Transaction 2	–€7,000,000.00	€7,093,333.33		
Transaction 3		€7,093,333.33	500,000,000	500,000,000
Transaction 4	€1,913,875.60	€5,179,457.74	–200,000,000	300,000,000
Transaction 5	€4,000,000.00	€1,179,457.74		**300,000,000**

The dealer's net exchange position is long ¥ 300,000,000 and short €1,179,457.74.

3.4 Show the cash flows when US\$1,000,000 is invested from three months for six months at a forward rate $r_{3,9}$ of 3.5% p.a.



3.5 Show the cash flows when ¥4,000,000,000 is sold against euros for value 3 November at an outright rate of €1 = ¥103.60.



CHAPTER 4

4.1 The dollar yield curve is currently:

1 month	5.00%
2 months	5.25%
3 months	5.50%

Interest rates are expected to rise.

(a) What two money market transactions should be performed to open a positive gap 3 months against 1 month?

Borrow dollars for 3 months at 5.50%, and
Lend dollars for 1 month at 5.00%

$FV_3 = (1 + 0.055 \times 3/12) = 1.013750$
$FV_1 = (1 + 0.050 \times 1/12) = 1.004167$

(b) Assume this gap was opened on a principal amount of \$1,000,000 and after 1 month rates have risen such that the yield curve is then:

1 month	6.00%
2 months	6.25%
3 months	6.50%

What money market transaction should be performed to close the gap?

Lend dollars for 2 months at 6.25%.

(c) How much profit or loss would have been made from opening and closing the gap?

$		
1,004,166.67	**–1,004,166.67**	Today
1,004,166.67	**1,004,166.67**	

$		
1,014,626.74	–1,013,750.00	2 months 1,004,166.67(1 + 0.0625 × 2/12)
	876.74	Profit
1,014,626.74	**1,014,626.74**	

Profit = 1,014,626.74 – 1,013,750.00 = $876.74

4.2 The dollar yield curve is currently inverse and expectations are that one month from now the yield curve will be 50 basis points below current levels, as reflected in the following table.

Tenor in months	*Current interest rates% p.a.*	*Expected interest rates% p.a.*
1	4.0	3.5
2	3.5	3.0
3	3.0	2.5

A corporation borrows $10,000,000 for one month and lends $10,000,000 for three months to open a negative gap position.

(a) Calculate the break-even interest rate at which it will need to be able to borrow $10,000,000 for 2 months in one month's time.

$$(1+0.04\times 1/12)(1+b\times 2/12)=(1+0.03\times 3/12)$$
$$(1+b/6)=1.0075/1.003333=1.004153$$
$$b=0.004153\times 6=0.024917$$
$$=2.49\%\text{ p.a.}$$

(b) Assuming the yield curve moves according to expectation, calculate the profit or loss which will be realized on closing the gap.

$		
10,000,000.00	**–10,000,000.00**	Today

$		
	–10,033,333.33	1 month 1,000,000(1 + 0.04 × 1/12)

$		
10,075,000.00		3 months 1,000,000(1 + 0.03 × 3/12)

Closing negative gap

$		
10,033,333.33	–10,033,333.33	Today
10,033,333.33	**10,033,333.33**	

$		
10,075,000.00	**–10,083,500.00**	2 months 10,033,333.33(1 + 0.03 × 2/12)
	–8,500.00	Profit
10,075,000.00	**10,075,000.00**	

Profit = 10,083,500 – 10,075,000 = –$8,500

Note: The gap would result in a loss because the 2 month rate at which the corporation expects to borrow 3.00% p.a. is greater than the break-even rate 2.49% p.a.

4.3 The crown yield curve is currently normal and expectations are that it will become steeper with the pivotal point at 6 months as reflected below:

Months	*Current rates*	*Expected rates 3 months from now*
3	5.0%	4.5%
6	5.5%	5.5%
9	6.0%	6.5%
12	6.5%	7.5%

Two gapping strategies are contemplated:

(a) Borrowing C1,000,000 for 3 months and lending C1,000,000 for 6 months.
Strategy (a) would result in a profit of C3,609.38.

Opening a negative gap

C		
1,000,000.00	**–1,000,000.00**	Today

C		
	–1,012,500.00	3 months 1,000,000(1 + 0.05 × 3/12)

C		
1,027,500.00		6 months 1,000,000(1 + 0.055 × 6/12)

Closing negative gap

C		
1,012,500.00	–1,012,500.00	Today
1,012,500.00	**1,012,500.00**	

C		
1,027,500.00	**–1,023,890.63**	3 months 1,012,500(1 + 0.045 × 3/12)
	3,609.38	Profit
1,027,500.00	**1,027,500.00**	

(b) Borrowing C1,000,000 for 3 months and lending C1,000,000 for 12 months.
Strategy (b) would result in a profit of C3,140.63.

Opening a negative gap

C		
1,000,000.00	**–1,000,000.00**	Today

C		
	–1,012,500.00	3 months 1,000,000(1 + 0.05 × 3/12)

C		
1,065,000.00		12 months 1,000,000(1 + 0.065 × 12/12)

Closing negative gap

C		
1,012,500.00	–1,012,500.00	Today
1,012,500.00	**1,012,500.00**	

C		
1,065,000.00	**–1,061,859.38**	9 months 1,012,500(1 + 0.065 × 9/12)
	3,140.63	Profit
1,065,000.00	**1,065,000.00**	

Assuming that interest rates move according to expectations and that the gap is closed after 3 months, which strategy will prove more profitable?

Strategy (a) would be more profitable. It would result in a larger profit at an earlier date.

Profit under Strategy (a) = C3,609.38 received after 6 months
Profit under Strategy (b) = C3,140.63 received after 12 months

To draw an exact comparison calculate the present value in each case.

Strategy (a) PV = 3,609.38 /(1 + 0.055 × 6/12) = C3,512.78
Strategy (b) PV = 3,140.63 /(1 + 0.065 × 12/12) = C2,948.95

4.4 On 1 July a company borrows \$10,000,000 at a three month floating rate of 3.75% p.a. (360 days per year basis). This debt will be rolled on 1 October (92 days). The company also placed \$10,000,000 on deposit maturing on 3 January (186 days) also at a rate of 3.75% p.a.

(a) Is the gap which the company has opened positive or negative?

The company has opened a negative gap by borrowing for a shorter period than it has lent.

(b) Would the company like the 3 month rate on 1 October to be higher or lower than at present?

The company needs to borrow at the 3 month rate on 1 October so it would like the rate to be lower. The break-even rate would be:

$$r_{1\,\text{Oct},\,3\,\text{Jan}} = \left[\frac{(1+0.0375\times 186/360)}{(1+0.0375\times 92/360)} - 1\right] \times \frac{360}{186-92} = 3.71\%\ \text{p.a.}$$

(c) Calculate the profit or loss if the company rolls the floating rate borrowing for 94 days from 1 October at exactly 3.75% p.a.

\$		
10,095,833.33	–10,095,833.33	1 October
10,095,833.33	**10,095,833.33**	

\$		
10,193,750.00	**–10,194,688.37**	3 January 10,095,833.33(1 + 0.0375 × 94/360)
	–938.37	Profit
10,193,750.00	**10,193,750.00**	

The company would lose \$938.37 because it had to borrow \$10,095,833.33 at 3.75% p.a. which is higher than the break-even rate of 3.71% p.a.

4.5 The dollar yield curve is currently:

1 month	5.00%
2 months	5.25%
3 months	5.50%

Interest rates are expected to fall.

(a) Which two money market transactions should be performed to open a gap 3 months against 1 month?

Borrow dollars for 1 month at 5. 00%, and
Lend dollars for 3 months at 5.50%

(b) Assuming the gap was opened on a principal amount of \$1,000,000 and after 1 month rates have fallen such that the yield curve is then:

1 month	4.75%
2 months	5.00%
3 months	5.25%

What money market transaction should be performed to close the gap?

Borrow dollars for 2 months at 5.00%.

(c) How much profit or loss would have been made from opening and closing the gap?

\$		
		1 month
	−1,004,166.67	FV = 1,000,000 (1 + 0.05 × 1/12)
+1,004,166.67		5.00%

\$		
		3 months
+1,013,750.00		FV = 1,000,000 (1 + 0.055 x 3/12)
	−1,012,534.73	**FV = 1,004,166.67 (1 + 0.05 x 2/12)**
	1,215.27	Profit
1,013,750.00	1,013,750.00	

CHAPTER 5

5.1 A bank quotes £1 = US\$1.4020/1.4025.

(a) The bank will buy dollars where it sells pounds; that is, at 1.4025.

(b) A customer could sell pounds at the bank's bid rate; that is, at 1.4020.
(c) At customer could sell dollars where it buys pounds; that is; at 1.4025.

5.2 Bank A calls and asks Bank B for a price for dollar/yen. Bank B quotes US$1 = ¥125.40/125.50. At what rate can Bank A sell yen?

Bank A can sell yen where it buys dollars. That is at Bank B's offer rate, 125.50.

5.3 A customer in Crownland asks a bank for a crown/dollar quote. The bank quotes C1 = $1.4935/1.4945.

(a) 1,000,000 × 1.4945 = $1,494,500
(b) 1,000,000 × 1.4935 = $1,493,500
(c) 1,494,500 – 1,493,500 = $1,000
(d) 1,000,000/1.4935 = C669,568.13
(e) 1,000,000/1.4945 = C669,120.11
(f) C669,568.13 – 669,120.11 = C448.02

5.4 A bank quotes overnight dollars at 4.25/4.50% p.a.

(a) A customer could borrow dollars at 4.50% p.a.
(b) A customer could invest dollars at 4.25% p.a.

5.5 A bank quotes 7 day francs at 4.50/4.75% p.a. There are 365 days per year.
(a) Interest = 1,000,000 × 0.0475 × 7/365 = F910.96
(b) Interest = 1,000,000 × 0.0450 × 7/365 = F863.01
(c) 910.96 – 863.01 = F47.95

5.6 A broker has dollar/yen prices from three banks:

Bank A	US$1	=	¥125.60	**125.65**
Bank B	US$1	=	¥125.62	125.67
Bank C	US$1	=	**¥125.63**	125.68

The broker price is: **125.63** **125.65.**

5.7 A bank quotes F1 = $1.2130/1.2140. A customer calls and sells the bank F10,000,000 at its bid rate 1.2130. The bank would like to square its position (if possible at a profit). If another bank calls a minute later asking for a price, which of the following rates should the first bank quote?

Rate A	**F1**	**=**	**$1.2125**	**1.2135**
Rate B	F1	=	$1.2130	1.2140
Rate C	F1	=	$1.2135	1.2145

5.8 Bank A quotes NZ\$1 = US\$0.4220 **0.4225**
Bank B quotes NZ\$1 = **US\$0.4226** 0.4231

What arbitrage opportunity exists? How much profit could be made by performing this arbitrage on a principal amount of NZ\$10,000,000?

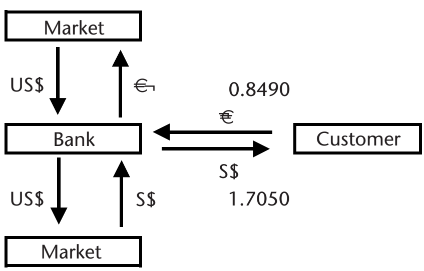
Buy NZ\$10,000,000 from Bank A at 0.4225 and sell NZ\$10,000,000 to Bank B at 0.4226.

Profit = US\$ 4,226,000 – 4,225,000 = US\$1,000

5.9

US\$1	=	S\$ 1.7050	1.7060
€1	=	US\$0.8490	0.8500

A Singaporean exporter wants to sell euro and buy Singapore dollars. What is the break-even rate for euros in Singapore dollar terms?



$$\text{S\$}\,? = €1$$

$$€1 = \text{US\$}0.8490$$

$$\text{US\$} = \text{S\$}1.7050$$

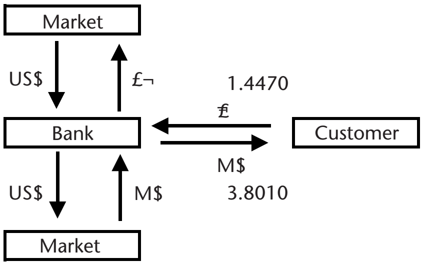
$$€1 = \text{S\$}\,\frac{1\times 0.8490\times 1.7050}{1\times 1}$$

$$€1 = \text{S\$}1.4475$$

5.10

US\$1	=	M\$ 3.8010	3.8030
£1	=	US\$1.4470	1.4480

What bid and offer rates should a bank quote for pounds against ringitt in Malaysian terms to make a ten point spread on either side of the break-even rates?



BID

$$\begin{aligned}
\text{M\$?} &= £1 \\
£1 &= \text{US\$}1.4470 \\
\text{US\$}1 &= \text{M\$}3.8010 \\
£1 &= \text{M\$}\,\frac{1.4470 \times 3.8010}{1 \times 1} \\
£1 &= \text{M\$}5.5000 \\
\text{Less spread} &\quad \underline{-0.0010} \\
£1 &= \underline{\underline{\text{M\$}5.5000}}
\end{aligned}$$

OFFER

$$\begin{aligned}
\text{M\$?} &= £1 \\
£1 &= \text{US\$}1.4480 \\
\text{US\$}1 &= \text{M\$}3.8010 \\
£1 &= \text{M\$}\,\frac{1.4480 \times 3.8010}{1 \times 1} \\
£1 &= \text{M\$}5.5067 \\
\text{Less spread} &\quad \underline{-0.0010} \\
£1 &= \underline{\underline{\text{M\$}5.5077}}
\end{aligned}$$

5.11 A bank calls four other banks for dollar/Swiss franc rates.

Bank A	\$ 1	=	SF 1.2430	1.2433
Bank B	\$ 1	=	SF 1.2430	1.2432
Bank C	\$ 1	=	SF 1.2431	1.2433
Bank D	\$ 1	=	SF 1.2430	1.2433

The bank wishes to sell Swiss francs. With which bank and at what rate should it deal?

The bank should buy dollars at the lowest offer rate which is 1.2432 from Bank B.

5.12 US$1 = ¥104.50 104.60
€1 = US$0.8550 0.8555
A Japanese importer wants to buy euros and sell yen. What is the break-even rate for euros in yen terms?

$$¥? = €1$$
$$€1 = US\$0.8555$$
$$US\$1 = ¥104.60$$
$$€1 = ¥\frac{1 \times 0.8555 \times 104.60}{1 \times 1}$$
$$€1 = ¥89.49$$

5.13 A customer calls and wants to buy Hong Kong dollars against Australian dollars. What rate should a bank quote for Hong Kong dollars in terms of Australian dollars to ensure a one point profit?

US$1	=	HK$ 7.7360	7.7370
A$1	=	US$0.5240	0.5245

$$A\$? = HK\$1$$
$$HK\$7.7360 = US\$1$$
$$US\$0.5420 = A\$1$$
$$HK\$1 = \frac{A\$1 \times 1 \times 1}{7.7360 \times 0.5420}$$
$$HK\$1 = A\$0.2467$$
$$\text{Less spread} \quad A\$0.2465$$

CHAPTER 6

6.1 Spot rate £1 = US$1.5000
3 month US$ interest rate 2.50% p.a. (91/360)
3 month £ interest rate 3.00% p.a. (91/365)

(a) 3 month forward rate

$$f = 1.5000 \times \frac{(1 + 0.025 \times 91/360)}{(1 + 0.03 \times 91/365)}$$

(b) 3 month forward margin

$$f - s = 1.4983 - 1.5000 = -0.0017$$

6.2 Spot rate €1 = ¥107.00
7 month euro 3.50% p.a. (212/360)
7 month yen 0.35% p.a. (212/360)

(a) 7 month forward rate

$$f = 107.00 \times \frac{(1 + 0.0035 \times 212/360)}{(1 + 0.0350 \times 212/360)} = 105.06$$

(b) 3 month forward margin

$$f - s = 105.06 - 107.00 = -1.94$$

6.3

Spot rate	€1	=	US$0.8490	0.8500
5 month €			3.00	3.10% p.a. (152/360)
5 month US$			1.90	1.95% p.a. (152/360)

A customer wishes to buy dollars five months forward. What rate should a bank quote to make 2 points profit?

Customer wants to buy dollars and sell euros.
Quoting bank is buying euros forward.
Quoting bank sells euros spot at 0.8490.
Quoting bank has to borrow euros at 3.10% p.a. and lend dollars at 1.90% p.a.

$$f = 0.8490 \times \frac{(1 + 0.019 \times 152/360)}{(1 + 0.031 \times 152/360)} = 0.8448$$

To make 2 points profit the bank lowers its bid rate by 2 points

Quoted rate = 0.8448 – 0.0002 = 0.8446

6.4

Spot rate	€1	=	US$0.8490	0.8500
5 month €			3.00	3.10% p.a. (152/360)
5 month US$			1.90	1.95% p.a. (152/360)

A customer wishes to sell dollars five months forward. What rate should a bank quote to make 2 points profit?

Customer wants to sell dollars and buy euros.
Quoting bank is selling euros forward.

$$f = 0.8500 \times \frac{(1 + 0.0195 \times 152/360)}{(1 + 0.0300 \times 152/360)} = 0.8463$$

To make 2 points profit the bank increases its offer rate by 2 points

Quoted rate = 0.8463 + 0.0002 = 0.8465

6.5

Spot rate	A$1	=	US$0.5100	0.5105
2 year A$ interest rate	5.00%		5.20% p.a. (semi-annually)	
2 year US$ interest rate	4.50%		4.70% p.a. (semi-annually)	

The break-even 2 year forward bid and offer rates:

Bid

$$f(1+0.052/2)^{2\times 2} = 0.5100(1+0.045/2)^{2\times 2}$$
$$f = 0.5031$$

Offer

$$f(1+0.05/2)^{2\times 2} = 0.5105(1+0.047/2)^{2\times 2}$$
$$f = 0.5075$$

2 year forward rates: A$/US$ 0.5031/0.5075

6.6

Spot rate	€1	=	US$0.8780	0.8785
Overnight US$ interest rate			2.25%	2.375% p.a. (3/360)
Overnight € interest rate	3.25%		3.375% p.a. (3/360)	

Calculate the break-even bid and offer rates to 5 decimal places for outright value tomorrow.

Bid

$$\text{tom}(1+0.02375\times 3/360) = 0.8780(1+0.0325\times 3/360)$$
$$\text{tom} = 0.87806$$

Offer

$$\text{tom}(1+0.0225\times 3/360) = 0.8785(1+0.03375\times 3/360)$$
$$\text{tom} = 0.87858$$

Outright value tomorrow €1 = US$0.87806/0.87858

6.7 A trader has done the following 3 transactions:

US$ amount	*¥ amount*	*Rate*	*Maturity*
+10,000,000	–1,075,000,000	107.50	Spot
–2,000,000	+210,610,000	105.30	6 months
–5,000,000	+512,000,000	102.40	1 year

Calculate the trader's yen Net Exchange Position in NPV terms and marked-to-market profit or loss given the current rates:

Spot US$/¥	110.30
6 month dollar interest rate	4.20% p.a.
6 month yen interest rate	0.30% p.a.

1 year dollar interest rate 4.10% p.a.
1 year yen interest rate 0.45% p.a.

¥ Amount	*PV*
−1,075,000,000	$\frac{-1{,}075{,}000{,}000}{1} = -1{,}075{,}000{,}000$
+210,610,000	$\frac{210{,}610{,}000}{1+0.003/2} = +210{,}284{,}573$
+512,000,000	$\frac{512{,}000{,}000}{1+0.0045} = +509{,}706{,}322$

Net exchange position = −355,009,105

Close out value = 355,009,105 /110.30 = $3,218,577.56

US$ Amount	*PV*
+10,000,000	$+10{,}000{,}000 = +10{,}000{,}000.00$
−2,000,000	$\frac{-2{,}000{,}000}{1+0.042/2} = -\ 1{,}958{,}863.86$
−5,000,000	$\frac{-5{,}000{,}000}{1+0.041} = -\ 4{,}803{,}073.97$

Counter value = $3,238,062.17

$$\begin{aligned}\text{MTM profit} &= \text{Counter value} - \text{Close out value}\\ &= 3{,}238{,}062.17 - 3{,}218{,}577.56\\ &= \text{US\$}19{,}484.61\end{aligned}$$

6.8 Calculate the 1 year, 2 year and 3 year zero coupon discount factors given the following par curve:

1 year 2.50% p.a.
2 years 2.40% p.a.
3 years 2.60% p.a.

$$df_1 = \frac{1.00}{1.025} = 0.975610$$

$$df_2 = \frac{1 - 0.024 \times 0.975610}{1.024} = 0.953697$$

$$df_3 = \frac{1 - 0.026 \times (0.975610 + 0.953697)}{1.026} = 0.925768$$

6.9 Spot	NZ$ 1	=	US$ 0.3940/0.3950	
Overnight NZ$			4.00%/4.15%	(1/365)
Overnight US$			2.00%/2.15%	(1/360)

Quote your bid and offer rates outright value tomorrow.

NZ$		Tom	US$	
+		**0.39402**		−
	−	4.000%		
		2.150%	+	

NZ$		Spot	US$	
	−	**0.3940**	+	
+		4.000%		
		2.150%		−

Bid

$$t(1+0.040\times 1/365) = 0.3940(1+0.020\times 1/360)$$
$$t = 0.39402$$

Offer

$$t(1+0.0415\times 1/365) = 0.3950(1+0.0215\times 1/360)$$
$$t = 0.39502$$

Outright value tomorrow NZ$1 = US$0.39402/0.39502

6.10 Spot US$1 = Yen 107.00
2 year dollars 6.00%/6.25%
2 year yen 1.75%/2.00%
Interest paid semi-annually in arrears.

Calculate the break-even bid and offer rates for the 2 year forward margins.

US$		Spot	¥	
	−	**107.00**	+	
+		6.25%		
		1.75%		−

US$		2 years	¥	
+		**?**		−
	−			
			+	

Bid

$$f(1+0.0625/2)^{2\times 2} = 107.00(1+0.0175/2)^{2\times 2}$$
$$f = 97.96$$

Forward margin bid rate = 107.00 – 97.96 = –9.04

Offer

$$f(1+0.06/2)^{2\times 2} = 107.00(1+0.02/2)^{2\times 2}$$
$$f = 98.93$$

Forward margin offer rate = 107.00 – 98.93 = –8.07

2 year forward margin: Yen 9.04/8.07

CHAPTER 7

7.1 An Australian importer has an obligation to pay ¥1,000,000,000 in 3 months' time. Calculate the cost in Australian dollars if the expected spot rate at maturity is A$1 = ¥ 65.20/65.30.

$$\text{A\$ cost} = \frac{1{,}000{,}000{,}000}{65.20} = \text{A\$}15{,}337{,}423.31$$

7.2 A New Zealand exporter is due to receive US$4,560,000 in 2 months. The exporter considers the alternatives of remaining unhedged and selling the US dollars spot upon receiving them, or hedging by forward selling the US dollar receipts.

Spot rate	NZ$1	=	US$0.4200	0.4205	
2 month NZ$			3.75	3.85%	p.a. (62/365)
2 month US$			2.65	2.75%	p.a. (62/360)

(a) Calculate the forward rate at which the exporter could hedge.

The exporter needs to buy NZ$ at the bank's forward offer rate.

Forward offer rate

s	= 0.4205	Bank buys NZ$ spot to cover its forward sale to the importer
r_C	= 3.75%	Bank lends NZ$ at the market bid rate
r_T	= 2.75%	Bank borrows US$ at the market offer rate
t	= 62/365 and 62/360	

$$f = 0.4205\frac{(1+0.0275\times 62/360)}{(1+0.0375\times 62/365)} = 0.4198$$

(b) If the expectation is that in 2 months' time the spot rate will be NZ$1 = US$0.41/4550, should the exporter hedge or remain unhedged?

The exporter would buy NZ$ at 0.4155 if unhedged. This would prove cheaper than buying them forward at 0.4198. Accordingly, the exporter should remain unhedged.

(c) Calculate the break-even rate between being hedged and unhedged?

The break-even rate will be the forward rate, 0.4198. Consequently, the exporter should buy the NZ$ forward at 0.4198 if, but only if, the expected spot offer rate is 0.4198 or higher.

7.3 An Indonesian exporter expects to receive US$4,000,000 in 5 months' time.

Spot	USD/IDR	10,200	10,400
5 month dollars		2.50%	2.60% p.a. (150/360)
5 month rupiah		25.00%	26.00% p.a. (150/360)

(a) At what rate could the exporter hedge its dollar receivables?

Exporter would sell dollars at the forward bid rate

$$f = 10{,}200 \frac{(1 + 0.25 \times 150/360)}{(1 + 0.26 \times 150/360)}$$
$$= 11{,}141.80$$

(b) How many rupiah would the exporter receive from the proceeds if it hedged?

$$\text{Hedged rupiah proceeds} = 4{,}000{,}000 \times 11{,}141.80$$
$$= 44{,}567{,}200{,}000$$

(c) If the exporter elected not to hedge and at the end of the 5 months the spot rate turned out to be 10,600/10,700, how many rupiah would the exporter receive?

$$\text{Unedged rupiah proceeds} = 4{,}000{,}000 \times 10{,}600$$
$$= 42{,}400{,}000{,}000$$

7.4 An Australian exporter will be receiving US$5,000,000 in one year's time.

Spot	A$1 = US$0.5720/25	
1 year forward margin		50/45

(a) What will the A$ proceeds be if it is hedged?

Exporter sells US$ /buys A$ at the outright offer rate:

$$\begin{array}{r} 0.5725 \\ -\underline{0.0045} \\ 0.5680 \end{array}$$

$$\text{A\$ proceeds} = \frac{5{,}000{,}000}{0.5680} = \text{A\$}8{,}802{,}816.90$$

(b) If at the end of the year the spot rate is A\$1 = US\$0.5625/30, what would the A\$ proceeds be if unhedged?

$$\text{A\$ proceeds if unhedged} = \frac{5{,}000{,}000}{0.5630} = \text{A\$}8{,}880{,}994.67$$

(c) Would the exporter be better off hedged or unhedged?

The A\$ proceeds would turn out to be greater if the exporter remained unhedged in this case.

7.5 A company requires US\$8,000,000 for 9 months. Two alternatives are considered:

1. Borrowing dollars domestically at an interest rate of 3.50% p.a. (272/360)
2. Borrowing euros at an interest cost of 4.00% p.a. (272/360)

(a) Calculate the effective borrowing cost if the spot rate at draw down is €1 = US\$0.8650, and at repayment of principal and interest is €1 = US\$0.8540.

$$0.8540 = 0.8650\frac{(1 + r \times 272/360)}{(1 + 0.04 \times 272/360)}$$
$$r = 2.27\%\ \text{p.a.}$$

(b) Which of the alternatives involves the lower cost?

It would have turned out cheaper to borrow euro unhedged at 2.27% p.a. than to borrow dollars at 3.50% p.a.

7.6 A Thai borrower has to choose between borrowing baht or borrowing dollars.

Spot	US\$1	THB 35.7020	35.7030
3 month dollars		3.10%	3.20% p.a. (90/360)
3 month baht		15.50%	15.75% p.a. (90/360)

Calculate the break-even exchange rate between borrowing baht directly and borrowing US dollars on an unhedged basis.

The borrower could borrow baht at 15.75% p.a. or borrow US dollars at 3.20% p.a. and sell the dollars spot for bath at 35.7020.

$$\text{Break-even rate } 35.7020\frac{(1+0.1575\times 90/360)}{(1+0.0320\times 90/360)}=36.8133$$

The borrower will be better off borrowing US dollars provided the spot rate remains below 36.8133 but worse off if the spot rate at maturity is above 36.8133.

7.7 Unhedged foreign currency investments
A funds manager has US dollars to invest for six months.

Spot rates
US$1 = ¥120.00
£1 = US$1.5000

The funds manager considers three alternatives:

1. Investing the dollars directly at 2.50% p.a.
2. Selling the dollars to buy yen to invest unhedged at 0.50% p.a.
3. Selling the dollars to buy pounds to invest unhedged at 3.20% p.a.

(a) Calculate the effective yield on the unhedged yen and unhedged pound investments if the spot rates at maturity turn out to be US$1 = ¥120.00 and £1 = US$1.4850.

1. Invest in dollars y_1 = 2.50%
2. Sell dollars (buy yen) at 120.00
Invest in yen at 0.50%
6 months later buy dollars at 120.00

$$120\frac{(1+0.005\times 6/12)}{(1+y/100\times 6/12)}=120$$

$$y_2 = 0.50\%\text{ p. a.}$$

3. Buy pounds (sell dollars) at 1.5000
Invest pounds at 3.20%
6 months later sell pounds at 1.4850

$$1.5000\frac{(1+y/100\times 6/12)}{(1+0.032\times 6/12)}=1.4850$$

$$y_3 = 1.17\%\text{ p. a.}$$

(b) Which of the three alternatives would have yielded the highest return on the investment?

Investing in dollars yielding 2.50% p.a. would have produced the highest return.

7.8 Break-even rate on unhedged investment

Spot rate	US$1	=	¥116.50	116.60
6 month dollars			2.00%	2.25% p.a. (181/360)
6 month yen			0.10%	0.20% p.a. (181/360)

A funds manager has US dollars to invest for six months.

(a) If the funds manager elects to use the dollars to buy yen for an offshore investment, what is the break-even future spot rate?

Sell USD spot for yen at	116.50
Invest yen for 6 months at	0.10%
Alternative yield on USD	2.00%

$$\text{Break-even rate} = 116.50\,\frac{(1+0.001\times 181/360)}{(1+0.02\times 181/360)}$$
$$= 115.39$$

(b) If at maturity of the yen investment, the spot rate turns out to be US$1 = ¥113.30/113.40, calculate the effective yield.

At maturity the investor would need to buy dollars/sell yen at 113.40. If y = effective yield

$$116.50\,\frac{(1+0.001\times 181/360)}{(1+y\times 181/360)} = 113.40$$
$$y = 5.54\%\ \text{p.a.}$$

7.9 A money market manager considers investing in Malaysian ringgit as a way to earn a higher yield. The spot rate is currently fixed at US$/M$ 3.8000. If the money manager can access a 3 month ringgit fixed deposit rate of 8.50% p.a., what would be the effective yield in dollars if on maturity of the deposit the pegged exchange rate had been broken and the spot rate was then 4.0000/4.0100?

$$3.8000\,\frac{(1+0.085\times 3/12)}{(1+r\times 3/12)} = 4.0100$$
$$r = -12.89\%\ \text{p.a.}$$

The fall in the value of the ringgit against the US dollar has much more wiped out the interest rate benefit from investing in ringgit rather than dollars.

7.10 An Australian exporter with receipts of US$5,000,000 each quarter for 3 years could hedge its foreign exchange risk by doing 12 separate forward deals in which it would sell US$5,000,000 against dollars at the different forward rates for each of the 12 maturities.

Based on a spot rate A$1 = US$0.5205 and the relevant interest rates the following forward rates and zero coupon discount factors apply:

Years	*Forward*	*US$ cash flow*	*A$ cash flow*	*zcdf*
0.25	0.5177	5,000,000.00	9,658,103.15	0.9895
0.50	0.5151	5,000,000.00	9,706,853.04	0.9792
0.75	0.5128	5,000,000.00	9,750,390.02	0.9688
1.00	0.5108	5,000,000.00	9,788,566.95	0.9586
1.25	0.5099	5,000,000.00	9,806,805.92	0.9476
1.50	0.5089	5,000,000.00	9,825,112.99	0.9370
1.75	0.5080	5,000,000.00	9,843,488.53	0.9266
2.00	0.5070	5,000,000.00	9,861,932.94	0.9163
2.25	0.5057	5,000,000.00	9,886,796.18	0.9051
2.50	0.5045	5,000,000.00	9,911,785.11	0.8918
2.75	0.5032	5,000,000.00	9,936,900.68	0.8806
3.00	0.5019	5,000,000.00	9,962,143.85	0.8673

The par forward rate is that rate for which the net present value of the Australian dollar cash flows is the same as the net present value for the 12 separate forward deals.

If the first estimate of the par forward rate is 0.5088 being the average of the forward rates:

Years	*US$*	*A$ at forwards*	*PV(forward)*	*A$ at par forward*	*PV (par forward)*
0.25	5,000,000.00	9,658,103.15	9,556,693.07	9,827,044.03	9,723,860.06
0.50	5,000,000.00	9,706,853.04	9,504,950.50	9,827,044.03	9,622,641.51
0.75	5,000,000.00	9,750,390.02	9,446,177.85	9,827,044.03	9,520,440.25
1.00	5,000,000.00	9,788,566.95	9,383,320.28	9,827,044.03	9,420,204.40
1.25	5,000,000.00	9,806,805.92	9,292,929.29	9,827,044.03	9,312,106.92
1.50	5,000,000.00	9,825,112.99	9,206,130.87	9,827,044.03	9,207,940.25
1.75	5,000,000.00	9,843,488.53	9,120,976.47	9,827,044.03	9,105,738.99
2.00	5,000,000.00	9,861,932.94	9,036,489.15	9,827,044.03	9,004,520.44
2.25	5,000,000.00	9,886,796.18	8,948,539.23	9,827,044.03	8,894,457.55
2.50	5,000,000.00	9,911,785.11	8,839,329.96	9,827,044.03	8,763,757.86
2.75	5,000,000.00	9,936,900.68	8,750,434.74	9,827,044.03	8,653,694.97
3.00	5,000,000.00	9,962,143.85	8,640,167.36	9,827,044.03	8,522,995.28
Total			109,726,138.77		109,752,358.49

If the par forward rate was 0.5088, the net present value of the par forward would be greater than the net present value of the 12 separate forwards implying that the break-even par forward rate is worse (that is, higher) than 0.5088.

$$\text{Break-even par forward rate} = 0.5088\,\frac{109{,}752{,}358.49}{109{,}726{,}138.77} = 0.5089$$

CHAPTER 8

8.1 Spot rates:	US$1 =	¥121.30	121.35
1 year swap		5.17	5.01

(a) At what rate can a customer buy yen outright one year forward?

Customer can sell dollars at the bid rate
Outright bid rate = 121.30 – 5.17 = 116.13

(b) What is the benefit or cost to a customer of buying dollars 1 year forward and selling dollars spot in a pure swap?

Customer will sell dollars spot at	121.32
Customer will buy dollars 1 year at	116.31
Benefit of the swap to customer = Cost of swap to the bank =	5.01

(c) At what rates would a customer deal if it bought dollars 1 year forward and sold dollars spot in an engineered swap?

Customer would sell dollars spot at	121.30
Customer would buy dollars 1 year at	116.34
Benefit of the swap to the customer = Cost of the swap to the bank =	4.96

8.2 Spot rates	US$1=	SF1.2735	1.2740
1 month swap rates		0.0030	0.0025

(a) What is the 1 month outright bid rate?

Outright bid rate = 1.2735 – 0.0030 = 1.2705

(b) What is the 1 month outright offer rate?

Outright offer rate = 1.2740 – 0.0025 = 1.2715

A customer wants to buy dollars spot and sell dollars 1 month forward

(c) What is the benefit or cost of an engineered swap to the customer?

The customer would buy dollars spot at 1.2740 and sell dollars forward at 1.2705.

The cost of the engineered swap to the customer = 1.2740 – 1.2705 = 0.0035.

(d) What is the benefit or cost of a pure swap if based on a spot rate of 1.2740?

The cost of a pure swap to the customer = 1.2740 – 1.2710 = 0.0030

8.3 A company needs to borrow Singapore dollars for one year.

Spot rate	US$1	=	S$ 1.7500
1 year forward	US$1	=	S$ 1.7320
1 year interest rate US$1			3.25% p.a.

Calculate the effective cost of generating Singapore dollars for one year through a swap.

$$1.7500\frac{(1+r)}{(1+0.0325)}=1.7320$$
$$r=2.19\%\text{ p.a.}$$

8.4 An American company wants to borrow Canadian dollars for 6 months.

Spot	US$1	=	C$1.3540	1.3550
6 month US$			5.50%	5.75%
6 month C$			8.00%	8.50%
6 month swap rate			148	168

Is it cheaper to borrow the Canadian dollars directly or to borrow US dollars and swap them into Canadian dollars?

Cost to borrow C$ directly	8.50% p.a.
Borrow US$	5.75% p.a.
Swap US$ into C$ by:	
Selling US$ spot at	1.3545
Buying US$ 6 months at 1.3545+0.0168 =	1.3713

Let c = effective cost:

$$1.3545\frac{(1+c\times 6/12)}{(1+0.0575\times 6/12)}=1.3713$$
$$c=8.30\text{ p.a.}$$

It would be cheaper to raise the Canadian dollars through a swap.

8.5 A fund manager has euros to invest for three months and considers two alternatives:

1. Investing euros directly at 3.5% p.a.
2. Swapping euros into US dollars and investing the dollars.

Which alternative provides the higher effective yield given the prevailing market rates.

Spot	€1	=	US$0.8860	
3 month US$			3.00	3.25% p.a. (90/360)
3 month swap			11	10

10% withholding tax applies to interest earned from a direct investment in euro.

After WHT yield on direct euro investment
= 3.50 × (1 – 0.1) = 3.15% p.a.

Alternatively, swap the euro into US dollars (sell euro spot at 0.8860 and buy euro forward at 0.8850) and lend US dollars at 3.00% p.a. Let y = effective yield with swap:

$$0.8850 = 0.8860 \frac{(1 + 0.03 \times 90/360)}{(1 + y \times 90/360)}$$

$$y = 3.46\% \text{ p.a.}$$

Investing through the swap earns a higher yield because it avoids withholding tax.

8.6 Market rates are

5 month US$ interest rates	3.25% p.a.	3.35% p.a.	(153/360)
5 month ¥ interest rates	0.20% p.a.	0.30% p.a.	(153/360)
Spot rate	US$1 = ¥	123.40	123.50
5 month swap rates		– 1.63	– 1.53
5 month outright forward rates	US$1 = ¥	**121.77**	121.97

A customer called a bank late in the afternoon and asked for a rate at which to sell US dollars 5 months forward. Hoping to make two points profit, the bank quoted a forward bid rate US$1 = ¥121.75. The customer agreed to deal and sold the bank US$10,000,000. The bank was then long US$10,000,000/short ¥1,217,500,000 and had mismatched cash flows on the 5 months date.

Using T-accounts, show how the bank could hedge its position with a spot deal and a swap. How much profit would the bank make?

US$		Spot	¥	
	–10,000,000.00	**123.40**	**1,234,000,000**	
10,000,000.00		**123.40**		**–1,234,000,000**
10,000,000.00	–10,000,000.00		1,234,000,000	1,234,000,000

US$		5 months	¥	
10,000,000.00		**121.75**		**–1,217,500,000**
	–10,000,000.00	**121.77**	**1,217,700,000**	
		Profit		**200,000**
10,000,000.00	10,000,000.00		1,217,700,000	1,217,700,000

The 2 points profit equals ¥200,000 due in 5 months time.

8.7 Three months ago a Japanese importer purchased US$10,000,000 three months forward at an outright rate of 130.00 to hedge expected US dollar payments. The original forward contract is maturing in two days time, that is, today's spot value date. The ship has been delayed and the importer will not be required to make the US dollar payment for a further month. The current inter-bank rate scenario is:

Spot	US$1 =	¥125.00	125.05	
1 month dollars		3.15%	3.25%	(30/360)
1 month yen		0.20%	0.25%	(30/360)
1 month swap rate		29	31	

Calculate the break-even forward rate for an historic rate rollover.

The importer needs to sell US$10,000,000 spot and buy US$10,000,000 one month forward. If this was done at market rates the forward leg would be done at 125.00 – 0.31 = 124.69. It would be necessary to borrow ¥50,000,000 for 1 month at 0.25% p.a. to cover the cash shortfall on the spot date. The HRR forward rate would be:

$$\frac{1{,}296{,}910{,}417}{10{,}000{,}000} = 129.69$$

as shown in the cash flow diagram opposite.

Bank's cash flows with market

US$		Spot	¥	
10,000,000		130.00		–1,300,000,000
	–10,000,000	**125.00**	**1,250,000,000**	
		0.25%	**50,000,000**	
10,000,000	10,000,000		1,300,000,000	1,300,000,000

US$		1 month	¥	
10,000,000		**124.69**		**–1,246,900,000**
		P + I		**–50,010,417**
				1,296,910,417

Bank's cash flows with importer

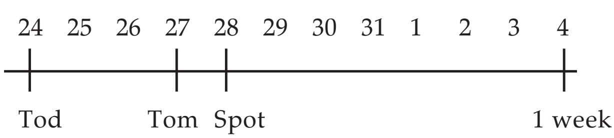
US$		Spot	¥	
10,000,000		130.00		–1,300,000,000
	–10,000,000	**130.00**	**1,300,000,000**	
10,000,000	10,000,000		1,300,000,000	1,300,000,000

US$		1 month	¥	
10,000,000		**129.69**		**–1,296,900,000**

8.8 Spot US$1 = ¥123.56/123.61

Today is Friday 24 May. Spot value is Tuesday 28 May.

Swap rates:
O/N 2.0/1.9
T/N 0.4/0.3
S/W 7.0/6.0



(a) At what rate can a customer buy US$ outright value today (24 May)?

Outright value today offer rate = 123.61 + 0.02 + 0.004
= 123.634

(b) At what swap rate could a customer buy US$ value today and sell US$ value 4 June in a pure swap?

1 week over today swap bid rate = 2.0 + 0.4 + 7.0
= 9.4 points

For example, the customer could buy US$ spot at 123.60 (say) and sell US$ value 4 June at 123.60 – 0.094 = 123.506.

CHAPTER 9

9.1 US dollar interest rates are higher than yen rates, so the swaps curve is negative. Over the next month, dollar interest rates are expected to rise relative to yen rates and the dollar is expected to appreciate against the yen.

	Current rates		***Expected rates (1 month from now)***	
Tenor in months	*Swap rates*	*Exchange rates*	*Swap rates*	*Exchange rates*
Spot		123.00		125.00
1	–0.20	**122.80**	–0.25	124.75
2	–0.40	122.60	–0.50	**124.50**
3	–0.60	**122.40**	–0.75	124.25

(a) What gap (three months against one month) should be opened to take advantage of the expected movement in rates?

Buy dollars 1 month at	122.80	20 points benefit
Sell dollars 3 months at	122.40	60 points cost
Cost of opening gap	0.40	40 points net cost

(b) How much profit would be generated on a principal amount of US$1,000,000 if rates move as expected? Assume that when the gap is closed, the 2 month yen interest rate is 0.30% p.a.

One month later...

$			¥	
		Spot		
1,000,000		122.80		122,800,000
	–1,000,000	**125.00**	**+125,000,000**	
		0.30%		**–2,200,000**
1,000,000	1,000,000		125,000,000	125,000,000

$			¥	
		2 months		
	1,000,000	122.40	122,400,000	
+1,000,000		**124.50**		**–124,500,000**
			+2,201,100	
		Profit		**101,100**
1,000,000	1,000,000		125,400,000	124,500,000

Profit = ¥101,100 = US$812.05 (at 124.50)

The profit can be thought of as:

Benefit of closing gap		
– cost of opening gap	=	¥500,000 – 400,000
	=	¥100,000
plus interest from lending		
¥2,200,000 for two months	=	¥ 1,100
	=	¥101,100

CHAPTER 10

10.1 A bank writes a euro put/US dollar call for €10,000,000 face value. The strike price is €1 = US$0.9000; time to expiry 4 months and the premium 2.00%.

(a) Calculate the premium in US dollars if the current spot rate is €1 = US$0.9100

Premium = 10,000,000 × 0.02 × 0.9100 = US$182,000

(b) Calculate the pay-out if the spot rate at expiry turns out to be €1 = US$0.8950.

Pay-out = 10,000,000 (0.9100 – 0.8950) = US$150,000

(c) What would the spot rate at expiry need to be for the pay-out to break-even with the future value of the premium given that the 4 month dollar interest rate is 3.00% p.a. (120/360)?

FV(Premium) = 182,000 (1 + 0.03 × 120/360) = US$183,820

If b = break-even rate,

$$10{,}000{,}000(0.9100 - b) = 183{,}820$$
$$b = 0.8916$$

10.2 Use a 3-step binomial model to calculate the premium of a 3 month US$ call/S$ put given:

Spot rate	s	=	1.7000
Forward rate	f	=	1.6940
Strike price	k	=	1.7100
Face value			US$1,000,000
3 month US$ interest rate			3.0% p.a. (90/360)
3 month S$ interest rate			1.6% p.a. (90/360)
up–down movement			S$0.0200 per month +/– drift

Drift = (1.6940 – 1.7000)/3 = –0.0020

Today	1 month	2 months	3 months	Pay-Off	p	E(PO)
			1.7540	0.0440	1/8	0.0055
		1.7360				
	1.7180		1.7140	0.0040	3/8	0.0015
1.7000		1.6960				
	1.6780		1.6740	0	3/8	0.0000
		1.6560				
			1.6340	0	1/8	0.0000
						0.0070

$$\text{Premium} = 0.0070/(1+0.016\times 90/360) = \text{S\$}0.006972 \text{ per US\$}$$
$$= \text{S\$}6{,}972 \text{ per US\$}1{,}000{,}000$$

10.3 Identify the arbitrage opportunity available given the following prices. Articulate the actions that need to be taken to profit through the above arbitrage. Calculate the profit that could be made on a face value of £10,000,000.

Spot rate	£1 =	US$1.7000	
1 year forward rate	£1 =	US$1.6950	
1 year £ call (k = 1.7200) premium		US$0.0230	
1 year £ put (k = 1.7200) premium		US$0.0480	
1 year US$ interest rate		4.0% p.a.	(360/360)

$$PV(F-K) = (1.6950-1.7200)/(1+0.04) = -\text{US\$}0.0240$$
$$c-p = 0.0230-0.0480 = -\text{US\$}0.0250$$

To make a profit: pay 240 points and receive 250 points.

Sell 1.72 put and buy 1.72 call	= buy £ forward at	1.7200
	sell £ forward at	1.6950
	loss	0.0250
	PV loss =	0.0240
	Net premium	0.0250
	Profit	0.0010 per £

Profit on £10,000,000 = 10,000,000 × 0.0010 = US$10,000

10.4 (a) Use the modified Black–Scholes model to calculate the premium of a European US$ call with strike price of ¥105.00 given:

Spot	US$/¥	110.00	
Expected volatility		15% p.a.	
Time to expiry		3 months	(90/360)
US$ interest rate		6.50% p.a.	(90/360)

¥ interest rate	1.00% p.a. (90/360)
Implied forward rate	108.55

$$c = Se^{-yt}N(d_1) - Ke^{-rt}N(d_2)$$

$$d_1 = \frac{\ln(S/K) + (r - y + \tfrac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(S/K) + (r - y - \tfrac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

Use the z tables provided in the Appendix:

$$\sigma\sqrt{t} = 0.15 \times \sqrt{\tfrac{1}{4}} = 0.075$$

$$r = \ln(1 + 0.01) \times 1 = 0.00995$$

$$y = \ln(1 + 0.065) \times 1 = 0.062975$$

$$e^{-rt} = 0.997516$$

$$e^{-yt} = 0.984380$$

$$\ln(S/K) = \ln(110/105) = 0.046520$$

$$(r - y + 1/2\sigma^2)t = (0.00995 - 0.062975 + 0.5(0.15)^2) \times 0.25 = -0.010444$$

$$d_1 = (0.046520 - 0.010444)/0.075 = 0.4810$$

$$d_2 = 0.481017 - 0.075 = 0.4060$$

$$N(d_1) = 0.6844 + 0.1 \times (0.6879 - 0.6844) = 0.68475$$

$$N(d_2) = 0.6554 + 0.6 \times (0.6591 - 0.6554) = 0.65762$$

$$c = 110 \times 0.984380 \times 0.68475 - 105 \times 0.997516 \times 0.65762$$

$$= 74.15 - 68.88$$

$$= 5.27$$

(b)
Use Black's model:

$$c = e^{-rt}[FN(d_1) - KN(d_2)]$$

$$d_1 = \frac{\ln(F/K) + \tfrac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

to calculate the premium of the same option as in (a):

$$\sigma\sqrt{t} = 0.15 \times \sqrt{1/4} = 0.075$$
$$r = \ln(1 + 0.01) \times 1 = 0.00995$$
$$e^{-rt} = 0.997516$$
$$\ln(F/K) = \ln(108.55/105) = 0.033251$$
$$1/2\sigma^2 t = 0.5(0.15)^2 \times 0.25 = 0.002813$$
$$d_1 = (0.033251 + 0.002813)/0.075 = 0.4810$$
$$d_2 = 0.481017 - 0.075 = 0.4060$$
$$N(d_1) = 0.6844 + 0.1 \times (0.6879 - 0.6844) = 0.68475$$
$$N(d_2) = 0.6554 + 0.6 \times (0.6591 - 0.6554) = 0.65762$$
$$c = 0.997156 \times (108.55 \times 0.68475 - 105 \times 0.65762)$$
$$= 5.27 \quad \text{as in (a)}$$

(c) Use put–call parity to calculate the premium of the 105.00 put with the same data as in (a).

$$p = c - (F - K)e^{-rt}$$
$$= 5.27 - (108.55 - 105.00) \times 0.997516 = 1.73$$

CHAPTER 11

11.1 An exporter with the identical exposure as in Example 11.2 enters into a participating collar to hedge euro receivables. The exporter buys a euro put/dollar call with the strike of 0.8762 for €1,000,000 at a premium of 1.0% and writes a euro call/dollar put with the strike of 0.9000 for €600,000 at a premium of 1.84%.

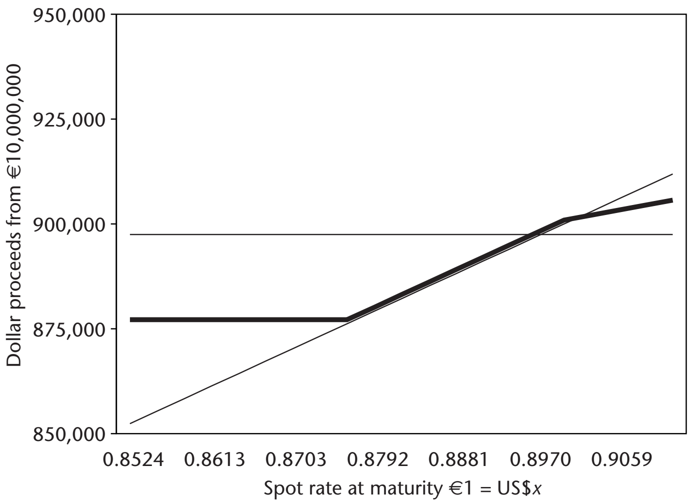
(a) Calculate the future value of the net premium payable in dollars.

Net premium payable = 1,000,000 × 0.01 – 600,000 × 0.0184
= €1.040

Note: premium received > premium paid

Net premium receivable = €1,040 = US$ 936

FV(Net premium receivable) = 936 × (1+ 0.03 × 90/360)
= US$943.02



(b) Calculate the proceeds from selling €1,000,000 if the spot rate at maturity is:

(i) 0.8662

Proceeds = 1,000,000 × 0.8762 + 943.62 = US$877,143.62

(ii) 0.8862

Proceeds = 1,000,000 × 0.8862 + 943.62 = US$887,143.62

(iii) 0.9062

Proceeds = 600,000 × 0.9000 + 400,000 × 0.9062 + 943.62
= US$903,423.62

11.2 A foreign currency borrower with the same exposure as in Example 11.3 constructs a participating option to hedge Swiss franc liabilities. The borrower buys a US dollar put/Swiss franc call for SF 25,395,300 with a strike of 1.2300 at a premium of 3.0% and writes a US dollar call/Swiss franc put for SF 12,697,650 with a strike 1.2300 at a premium of 2.4%.

(a) Calculate the future value of the net premium payable in dollars.

$$\text{Put premium} = \frac{25{,}395{,}300}{1.2500} \times 0.03 = \text{US\$}609{,}487.20$$

$$\text{Call premium} = \frac{12{,}697{,}650}{1.2500} \times 0.024 = \underline{\text{US\$}243{,}794.88}$$

$$\text{Net premium payable} = \underline{\underline{\text{US\$}365{,}692.32}}$$

$$\text{FV (Net premium)} = 365{,}692.32(1+0.05/2) = \text{US\$}374{,}834.63$$

(b) Calculate the dollar cost of repaying the Swiss franc loan principal plus interest if the spot rate at maturity is:

(i) 1.2000
Put is exercised and call lapses

$$\text{Cost} = \frac{25{,}395{,}300}{1.2300} + 374{,}834.63 = \text{US\$}21{,}021{,}420.00$$

(ii) 1.2400
Put lapses and call is exercised

$$\text{Cost} = \frac{12{,}697{,}650}{1.2300} + \frac{12{,}697{,}650}{1.2400} + 374{,}834.63 = \text{US\$}20{,}938{,}167.63$$

(iii) 1.3000
Put lapses and call is exercised

$$\text{Cost} = \frac{12{,}697{,}650}{1.2300} + \frac{12{,}697{,}650}{1.3000} + 374{,}834.63 = \text{US\$}20{,}465{,}550.39$$

(c) Calculate the effective borrowing cost in percent per annum of the Swiss franc loan if the spot rate at maturity is:

$$\text{Effective borrowing cost} = \frac{\text{US\$ cost} - 20{,}000{,}000}{20{,}000{,}000} \times 200$$

(i) 1.2000: $= \dfrac{21{,}021{,}420 - 20{,}000{,}000}{20{,}000{,}000} \times 200 = 10.21\%$ p.a.

(ii) 1.2400: $= \dfrac{20{,}938{,}167.63 - 20{,}000{,}000}{20{,}000{,}000} \times 200 = 9.38\%$ p.a.

(iii) 1.300: $= \dfrac{20{,}465{,}550.39 - 20{,}000{,}000}{20{,}000{,}000} \times 200 = 4.66\%$ p.a.

11.3 A funds manager with the same exposure as in Example 11.5 buys a collar by buying a dollar call at 110.00 for ¥1,111,152,778 at a premium of 3.25% and writing a dollar put at 109.00 for ¥777,806,945 at a premium of 2.00%.

$$\begin{aligned} \text{Net premium} &= 777{,}806{,}945 \times 0.02 - 1{,}111{,}152{,}778 \times 0.0325 \\ &= ¥20{,}556{,}326 \\ &= \text{US\$}186{,}875.69 \end{aligned}$$

$$\text{FV(Net premium)} = 186{,}875.69 \times \left(1 + 0.05 \times \frac{365}{360}\right) = \text{US\$}196{,}349.26$$

(a) Calculate the effective yield if the spot rate at maturity is:

$$\text{Effective yield} = \frac{\text{US\$ proceeds} - 10{,}000{,}000}{10{,}000{,}000} \times \frac{365}{360}$$

(i) 100.00; call lapses and put is exercised

$$\text{US\$ proceeds} = \frac{777{,}806{,}945}{109.00} + \frac{333{,}345{,}833}{100} - 196{,}349.26$$
$$= \text{US\$}10{,}272{,}952.60$$

$$\text{Effective yield} = 2.77\%\ \text{p. a.}$$

(ii) 110.00; call and put both lapse

$$\text{US\$ proceeds} = \frac{1{,}111{,}152{,}778}{110.00} - 196{,}349.26 = \text{US\$}9{,}905{,}039.63$$
$$\text{Effective yield} = -0.95\%\ \text{p. a.}$$

(iii) 120.00; exercise call, put lapses

$$\text{US\$ proceeds} = \frac{1{,}111{,}152{,}778}{110.00} - 196{,}349.26 = \text{US\$}9{,}905{,}039.63$$
$$\text{Effective yield} = -0.95\%\ \text{p. a.}$$

(b) If the spot rate at maturity is 114.00, calculate the effective yield percent per annum versus being:

Effective yield = –0.95% p.a. (again)

(i) unhedged

$$\text{US\$ proceeds} = \frac{1{,}111{,}152{,}778}{114.00} = \text{US\$}9{,}746{,}954.19$$
$$\text{Effective yield} = -2.53\%\ \text{p. a.}$$

(ii) invested in dollars

Effective yield = 0.05 × 365/360 = 5.07% p.a.

(iii) hedged with a bought dollar call (strike 110.00)

$$\text{US\$ proceeds} = \frac{1{,}111{,}152{,}778}{110.00} - 344{,}937.88 = \text{US\$}9{,}756{,}451.01$$
$$\text{Effective yield} = -2.40\%\ \text{p. a.}$$

If the spot rate at maturity turned out to be 114.00, the best outcome would have occurred if the investor was invested in US dollars.

11.4 A 2 for 1 strategy refers to the practice of buying the option required to hedge an underlying exposure and selling twice the face value of the opposite type of option (call or put) usually to earn enough premium to make the net premium zero.

One month ago, a foreign exchange trader bought £10,000,000 against US dollars at an outright 4 month forward rate of 1.4800. The spot rate has since risen to 1.5150 and the 3 month forward rate is now 1.5100. The 3 month (90/360) dollar interest rate is 3.00% p.a.

The trader considers buys a sterling put (strike 1.5100) premium 2.0% for face value £10,000,000 and sells a sterling call (strike 1.5200) premium 1.0% for twice the face value (£20,000,000).

(a) Calculate the future value of the net premium in dollars.

$$\text{Net premium} = 10{,}000{,}000 \times 0.02 - 20{,}000{,}000 \times 0.01 = 0$$

(b) Calculate the profit if the spot rate at expiry is:

US$ cost of buying £10,000,000 at 1.4800 = US$14,800,000
FV(US$14,800,000) = 14,800,000 × (1 + 0.03 × 3/12)
= US$14,911,000

This assumes that short-term pound interest rates are around 3.00% p.a.

Profit = Proceeds of sale of £10,000,000 under 2 for 1: 14,911,000

(i) 1.4500: put exercised, calls lapse

$$\text{Proceeds} = 10{,}000{,}000 \times 1.5100 = \text{US\$}15{,}100{,}000$$
$$\text{Profit} = 15{,}100{,}000 - 14{,}911{,}000 = \text{US\$}189{,}000$$

(ii) 1.5000: put exercised, calls lapse

$$\text{Proceeds} = 10{,}000{,}000 \times 1.5100 = \text{US\$}15{,}100{,}000$$
$$\text{Profit} = 15{,}100{,}000 - 14{,}911{,}000 = \text{US\$}189{,}000$$

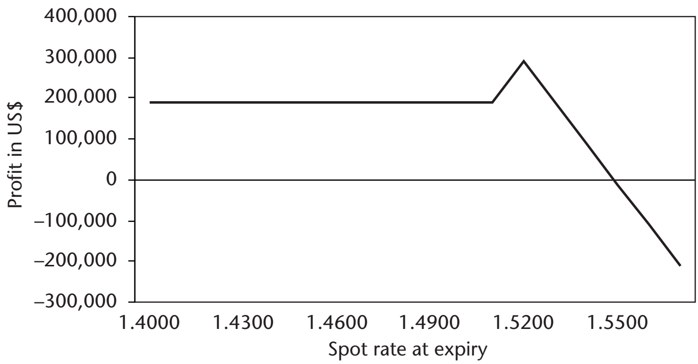
(iii) 1.5500: put lapses, calls are exercised

Trader sells £20,000,000 at 1.5200:
US$ proceeds = US$30,400,000

Trader needs to buy £10,000,000 at 1.5500:
US$ cost = US$15,500,000

$$\text{Profit} = 30{,}400{,}000 - 15{,}500{,}000 - 14{,}911{,}000 = -\text{US\$}11{,}000$$

(c) Draw the profit profile showing profit against various possible exchange rates at expiry.



CHAPTER 12

12.1 Calculate the premium of an option that will pay US$1,000,000 if the A$/US$ spot rate is below 0.5300 in 90 days time given the following:

Current spot rate	A$/US$	0.5540
3 month LIBOR		3.25% p.a. (90/360)
Expected probability of spot being below 0.5300		24%

$$\text{Digital put premium} = \frac{A(1-N(d_2))}{1+rt}$$

Here:

$$A = \text{US\$}1{,}000{,}000$$
$$1-N(d_2) = 0.24$$
$$r = 0.0325$$
$$t = 90/360$$

$$\text{Premium} = \frac{1{,}000{,}000 \times 0.24}{1+0.0325\times 90/360} = \text{US\$}236{,}162.36$$

12.2 Power option
Calculate the premium of a call with a pay-out equal to $(X - 105.00)^3$ assuming the binomial tree as shown in Exhibit 10.3. The 6 month yen interest rate is 0.50% p.a. and the current spot rate is US$1 = ¥100.00.

Outcome	*Pay-out*	*Probability*	*Expected pay-out*
118	$13^3 = 1{,}197$	1/64	¥34.33
112	$7^3 = 343$	6/64	¥32.16
106	$1^3 = 1$	15/64	¥ 0.23
100	0	20/64	0
94	0	15/64	0
88	0	6/64	0
82	0	1/64	0
	Expected pay-out		¥66.72

$$\text{Premium} = \frac{66.72}{1 + 0.005 \times 6/12} = ¥66.55$$

If the face value of the power option is US$1,000,000:

$$\begin{aligned}\text{Premium} &= ¥66{,}720{,}000 \\ &= \text{US}\$667{,}200\end{aligned}$$

12.3 Improving forward

A Japanese importer needs to buy US dollars at a future date. The spot rate is currently US$1 = ¥122.00 and the market forward rate is 120.30. A bank offers the importer a deal in which the rate at which the importer will buy US dollars on the forward date will be either 121.00 if the spot rate remains above 115.00 or 118.00 if the spot rate falls below 115.00 prior to the maturity date.

How does the bank engineer the improving forward?

Method 1

Buy a 121.00 call that knocks-out at 115.00 and sell a 121.00 put that knocks-out at 115

Buy a 118.00 call that knocks-in at 115.00 and sell a 118.00 Put that knocks-in at 115.00

If the spot never reaches 115.00, the importer has a bought 121 call and a sold 121 put = 121 forward

If the spot reaches 115.00. the importer has a bought 118 call and a sold 118 put = 118 forward and the 121 forward knocks out.

Method 2

Buy US dollars forward at 120.30 and buy a digital put with a pay-out of ¥3.00 if the spot rate falls below 115.00. The premium of the digital put must be equal to the present value of ¥0.70.

If the spot rate never reaches 115.00, the importer has effectively bought dollars at 120.30 + 0.70 = 121.00. If the spot reaches 115.00, the importer collects the ¥3.00 pay-out from the digital put to achieve an effective rate = 120.30 + 0.70 – 3.00 = 118.00.

Notice it is possible to construct the same pay-off using a forward and a digital as with four barrier options.

12.4 Currency linked note
An investor places US$1,000,000 on deposit at a fixed rate of 3.5% p.a. for 6 months (180/360) and purchases a one-touch either side digital option with a pay-out of US$10,000 if the US$/¥ spot rate remains within a range of 120.00 to 130.00 for the entire 6 months. The premium of the option is US$2,948.40.

Calculate the effective yield if:

$$\text{Interest on deposit} = 1{,}000{,}000 \times 0.035 \times 180/360 = \text{US\$}17{,}500$$

$$\text{Digital pay out} = \text{US\$}10{,}000 = 3.5\% \times \frac{10{,}000}{17{,}500} = 2.00\% \text{ p.a.}$$

$$\text{FV(Premium)} = 2{,}948.40 \times (1 + 0.035 \times 180/360) = \text{US\$}3{,}000$$

$$= 3.50\% \times \frac{3{,}000}{17{,}500} = 0.60\% \text{ p.a.}$$

(a) The spot rate remains within the range
Digital is exercised

Effective yield = 3.50% + 2.00% – 0.60% = 4.90% p.a.

(b) The spot rate does not remain within the range
Digital is not exercised

Effective yield = 3.50% – 0.60% = 2.90% p.a.

CHAPTER 14

14.1 Market scenario:

Spot	€1	=	US$0.9250
6 month euro			3.50% p.a. (180/360)
6 month dollars			2.75% p.a. (180/360)

$$f = 0.9250 \times \frac{(1 + 0.0275/2)}{(1 + 0.035/2)} = 0.9216$$

A dealer purchased €10,000,000 at a 6 month outright forward rate of 0.9216 and has not covere7d the position.

(a) Calculate the 2 standard deviations stressed rate if spot rate changes are assumed to be normally distributed and volatility is expected to be 9.2% p.a.

$$\text{Stressed rate} = 0.9250e^{-2(0.092)\times 90/360} = 0.8834$$

(b) Calculate the value at risk

$$\text{VaR} = 10{,}000{,}000(0.9250 - 0.8834) = \text{US\$416,000}$$

14.2 Delta hedging

On a day when the spot rate was US$1 = ¥123.50 a bank sold a US$ call/¥ put with face value US$10,000,000 and strike price 122.50. A pricing model displayed the following premiums for the sold call:

Spot rate	Premium
122.50	¥2.08
123.00	¥2.31
123.50	¥2.57
124.00	¥2.84

(a) Calculate the average delta between 123.00 and 124.00. What transaction should the bank do to delta hedge?

$$\text{Average delta} = \frac{2.84 - 2.31}{124 - 123} = 0.53$$

The bank loses money on the sold call as the spot rate rises, so to delta hedge the bank needs to buy US$5,300,000 against yen.

One week later the spot rate has fallen to 123.00 and the pricing model displays the following premiums:

Spot rate	Premium
122.50	¥2.08
123.00	¥2.31
123.50	¥2.57

(b) Calculate the revised average delta. What transaction should the bank do to adjust its delta hedge?

$$\text{Average delta} = \frac{2.57 - 2.08}{123.5 - 122.5} = 0.49$$

To be delta neutral the bank needs to hold US$4,900,000. Therefore, to adjust the delta hedge the bank would need to sell US$200,000.

Note: The bank would realize a loss as a result of adjusting the delta hedge. It purchased US$200,000 at 123.50 and sold them at 123.00 for a realized loss of ¥100,000 = US$813. This offset some of the premium received by the bank when it sold the option which was 10,000,000 × 2.57 = ¥25,700,000 = US$208,097.

14.3 Credit risk

Two months ago a bank purchased A$10,000,000 from XYZ Limited at the then 5 month forward rate of A$1 = US$0.5230. The prevailing rates today are:

Spot	A$/US$	0.5620
3 month swap rate		–0.0010
3 month US$ rate		3.20% p.a. (90/360)
3 month credit risk factor		5.0%

Calculate in US$ NPV terms:

(a) The bank's marked-to-market exposure on XYZ Limited

Close out rate = 0.5610

$$FV(MTM) = 10{,}000{,}000(0.5610 - 0.5230) = US\$380{,}000$$

$$MTM = 380{,}000/(1 + 0.032 \times 90/360) = US\$376{,}984$$

(b) The bank's estimated potential exposure on XYZ Limited

Estimated potential exposure = 5,230,000 × 0.05
= US$261,500.00

(c) The bank's pre-settlement risk on XYZ Limited

$$\begin{aligned} PSR &= MTM + PE \\ &= 376{,}984 + 261{,}500 \\ &= US\$638{,}484 \end{aligned}$$

XYZ Limited wishes to sell more US dollars to the bank for the same value date.

(d) How large a deal can be done if the bank's PSR limit on XYZ is US$2,000,000?

Available credit line	= 2,000,000 – 638,484	= US$1,361,516
CRF	= 5.0%	
Maximum deal size	= 1,361,516 × 100/5	= US$27,230,320

APPENDIX

Cumulative Standard Normal Distribution ($\mu = 0, \sigma = 1$)

$N(z) = p(Z < z)$ $N(-z) = 1 - N(z)$

See table overleaf.

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998
5.0	0.9999997	0.9999997	0.9999997	0.9999998	0.9999998	0.9999998	0.9999998	0.9999998	0.9999998	0.9999998
6.0	0.9999999990	0.9999999991	0.9999999991	0.9999999992	0.9999999992	0.9999999993	0.9999999993	0.9999999994	0.9999999994	0.9999999995

Glossary of Terms

accounting risk risk of loss due to incorrect accounting treatment
accreting swap an interest rate swap for which the notional principal increases towards the maturity of the swap
American option an option which can be exercised for spot value on any date between the contract date and the expiry date
amortizing swap an interest rate swap for which the notional principal decreases towards the maturity of the swap
arbitrage the practice of taking advantage of inconsistent pricing to lock in profit free of price risk
arbitrageur a person who performs an arbitrage transaction
Asian option *see* average rate option
at-expiry digital option which pays out a fixed amount if the spot rate is better than the strike price at expiry
at-the-money (ATM) an option with a strike price equal to the current market price
at-the-money spot a currency option with the strike price equal to the spot rate
average rate option option whose pay out equals the difference between the strike price and the average rate
average strike option options whose strike price is equal to the average rate during the life of the option

backwardation forward commodity price that is lower than the cash price
balance of payments record of a country's transactions with the rest of the world
barrier option an option which knocks-in or knocks-out at a specified barrier level
base currency *see* commodity currency
basis point one unit in the second decimal place of an interest rate quotation
basket option an option against a group of currencies
bid offer spread the difference between the bid rate and the offer rate
bid rate (foreign exchange) the rate at which the quoting bank is willing to buy the commodity currency
bid rate (money market) the rate at which the quoting bank is willing to borrow
binary option *see* digital option
binomial model a mathematical model which can be used for pricing options
Black–Scholes model a mathematical model for pricing European options

blotter a rough copy of a net exchange position sheet
bond a long-term note
break-even rate the rate at which the outcome will be the same under two or more different scenarios
Bretton Woods system a system of fixed exchange rates that operated from 1945 until 1971
brokerage a fee which brokers charge for their services
Brownian motion a path determined by independent random price movements
business cycle pattern of alternate growth and recession

call the right without the obligation to buy (a currency)
calling bank the bank that calls to ask the quoting bank for a price
capital market market in which investors lend money to issuers of bonds and other securities
carry cash flow cost or benefit of holding a position
central bank swaps the practice of central banks using currency swaps to inject or withdraw liquidity from the local money market without creating a net exchange positon
chain rule a procedure for calculating cross rates
cherry picking practice where liquidators can demand payment on money due without being obliged to pay money owing
chooser option option where owner can choose whether it is a call or a put
close out value profit or loss realised if a contract is reversed at current rates
closed form solution mathematical formula that provides a unique value for the price of an option
collar the strategy of buying an option and writing the opposite type of option (call/put) for the same face value and same expiry date but at a different strike price
commodity currency the currency being priced in an exchange rate quotation
compound interest interest earned when the amount of principal plus interest is reinvested for subsequent periods
compound option an option to enter into an option
contango forward commodity price that is higher than the cash price
contingent premium option *see* pay later option
contingent risk a risk that may or may not arise
continuously compounding rate interest rate that compounds continuously
contract date the date on which a transaction is contracted
correlated VaR value at risk calculated to allow for expected correlations
correlation relationship between two variables
counter-party the other party to a transaction
counter value the value of a foreign exchange transaction in the other currency
coupons interim interest payments on a bond
covered interest arbitrage the practice of transacting a currency swap and two money market transactions to make a rate risk-free profit by taking advantage of a set of inconsistent market rates
credit collar a collar where the net premium is positive
credit risk risk of loss if a counter-party is unwilling or unable to pay
Credit Risk Factor ratio of potential credit exposure to the principal of the transaction
cross rate an exchange rate derived from two other exchange rates

currency linked note investment where the yield varies with an exchange rate
currency option the right without the obligation to buy or sell one currency against another currency at a specified price during a specified period
currency swap the simultaneous purchase and sale of equal amounts of one currency against another currency for different maturities
current account balance the net outcome of receipts and payments for exports and imports of goods and services
current account deficit a negative balance reflecting an excess of import payments over export receipts
current account surplus a positive balance reflecting an excess of export receipts over import payments
cylinder *see* collar

debit collar collar where the net premium is negative
defeasance period the period over which value at risk is calculated
delivery risk the risk that a counter-party will be unable or unwilling to deliver a currency at the maturity of a contract
delta ratio of change in premium to change in market price
delta hedging the practice of hedging a variable portion of an option position
Devon model a software package for pricing and evaluating options
digital option an option with a fixed pay-out
direct relationship commodity currency increases in value when the exchange rate rises
discount factor number less than one that is the ratio of present value to future value
down and in option that knocks-in when the spot rate falls to the barrier level
down and out option that knocks-out when the spot rate falls to the barrier level
drift adjustment for assumed movement from spot rate towards the forward rate
duration time-weighted present value

early take-up a swap which shortens the maturity of a forward exchange transaction without cash flows at the original maturity date
effective interest rate an interest rate which is expressed under a different convention
engineered swap a swap in which the spot and forward transactions are done with different counter-parties
eurocurrency deposits deposits of a currency held outside the jurisdiction of that currency's government and central bank
Eurodollar US dollar deposits held outside the USA (initially in Europe)
European option an option which can only be exercised for spot value on the expiry date
exchange rate the price of one currency expressed in terms of another currency
exchange-traded options a contract traded through certain stock, futures or commodity exchanges
expected volatility the extent to which the exchange rate is expected to change during the life of the option
expiration date or expiry date the date on which the buyer's right to exercise an option ends

fixed exchange rate system system under which exchange rates are kept at fixed levels
flat yield curve a horizontal yield curve reflecting the fact that interest rates are the same for different tenors
floating exchange rate system system under which exchange rates are set by supply and demand
forward bid rate the rate at which the quoting bank is willing to buy forward the commodity currency
forward discount the excess of the spot rate over the forward rate
forward exchange agreement (FXA) a non-deliverable currency swap
forward exchange rate the rate agreed upon today at which one currency is sold against another for delivery on a specified future date
forward interest rate an interest rate which can be determined today for a period from one future date till another future date
forward offer rate the rate at which the quoting bank would sell forward the commodity currency
forward premium the excess of the forward rate over the spot rate
forward value date a value date which is farther into the future than spot value
FOTPI future obligation to pay interest
FOTRI future obligation to receive interest
FRA Forward Rate Agreement: an agreement to fix a forward interest rate
fundamental factors the economic factors that influence exchange rates in the long term
funding liquidity risk risk of loss if insufficient cash is not available to make payments on time
futures contract forward contract traded through a futures exchange
future value the value of an investment (principal plus interest) at a designated time into the life of the investment

gamma rate of change of delta
gapping the deliberate mismatch of maturity dates of assets and liabilities

hedging eliminating a risk or establishing a worst case
historical rate rollover a swap based on a spot rate which is set equal to the historic rate to avoid cash flows at the spot date
hybrid derivative transaction where the price is a function of two variables that are not normally related

indirect relationship commodity currency increases in value when the exchange rate falls
instalment option an option in which the premium is paid in installments
interbank dealings between banks
interest the price paid for the use of money
interest rate the ratio of the amount of interest to the amount of principal
interest rate parity theory a theory which purports that, in the long run, exchange rates adjust to reflect the relative interest rates of the two currencies
intervention practice of central banks transacting to influence exchange rates
in-the-money (ITM) an option with a strike price that is better than the current market price
intrinsic value the value which an option has because it is in-the-money

inverse relationship *see* indirect relationship
inverse yield curve a yield curve which is downward sloping to the right, reflecting the fact that the market generally expects interest rates to fall
ISO 4217 Currency List standard three letter representation of currencies
issuer person who raises funds by issuing a bond

J curve effect the phenomenon of the current account deficit initially widening following a depreciation of the currency
jobbing the practice of a trader moving prices to be hit on the side which will square off a position

knock-out forward better than market forward that knocks-out if the barrier rate is reached
knock-out option option that loses all value when the spot rate reaches the barrier level

LIBID London Inter-Bank Bid Rate (for Eurodollar deposits)
LIBOR London Inter-Bank Offer Rate (for Eurodollar deposits)
liquidity the ability to obtain cash
liquidity position a situation in which there is a risk that insufficient funds will be available to meet a negative cash flow commitment
liquidity risk risk of loss from insufficient funding or market liquidity
log-normal distribution a distribution where the natural logarithm of the variable is normally distributed
long a positive net exchange position
look-back option whose pay-out depends on the maximum or minimum rate reached during the life of the option
LTFX long-term foreign exchange (usually taken as beyond one year)

management action trigger system under which management make a decision when a loss level is reached
market liquidity risk risk of loss from the price moving because of the size or nature of a transaction
market making the strategy which some banks follow of dealing at narrow bid offer spreads in the expectation of doing large volumes on both sides of their prices
mark-to-market calculate the unrealised profit or loss
mean a measure of central tendency; also known as the average

naked option an option which is bought or sold without any underlying exposure
natural option an option which is bought or sold with an underlying exposure
negative carry cash flow cost of holding a position
negative cash flow a payment of cash to another party
negative gapping the strategy of borrowing short term and lending long term
net cash flow position the difference between positive and negative cash flows on a particular value date
net exchange position the difference between the amount of a foreign currency purchased and sold at a particular point of time or the net present value of that amount

net exchange position sheet a running tally of the net exchange position in a particular foreign currency as a result of a series of foreign exchange transactions
netting offsetting in-the-money valuations against the credit risk with the same counter-party
nominal interest rate a number which measures an interest rate quotation under a particular convention
normal distribution a bell-shaped distribution that follows a particular formula
normal yield curve a yield curve which is slightly upward sloping to the right, reflecting the fact that interest rates for longer tenors are 'normally' higher than for shorter tenors
NPV net present value

offer rate (foreign exchange) the rate at which the quoting bank is willing to sell the commodity currency
offer rate (money market) the rate at which the quoting bank is willing to lend
one-touch digital option with fixed pay out if a barrier level is reached at any time during the life of the option
operational risk risk of loss from the failure of a process
option premium the price of an option
out-of-the-money (OTM) an option with a strike price that is worse than the current market price
outright exchange rate a forward exchange rate (to be distinguished from a swap rate)
over-the-counter (OTC) option an option written by banks and other institutions

participating collar the strategy of buying and writing opposite types of options (call/put) with different strike prices and different face values
participating option the strategy of buying and writing opposite types of options (call/put) with the same strike price but for different face values
path-dependent option an option whose pay-out depends on the movement of the price during the life of the option
pay-as-you-go option see instalment option
pay later option option where the premium is only paid if it expires in-the-money
pay-off the option pay-out that would be received under different scenarios
pay-out the amount received if an option is exercised
pip *see* point
point one unit in the last decimal place to which a particular exchange rate is usually quoted
position a situation in which a change in a price will result in a profit or loss positive cash flow – a receipt of cash from another party
positive carry cash flow benefit from holding a position
positive gapping the strategy of borrowing long term and lending short term
positive yield curve a yield curve which is more steeply upward sloping to the right than a normal yield curve, reflecting the fact that the market generally expects interest rates to rise
power option option for which the pay-out is a function raised to a power greater than one

pre-delivery risk *see* pre-settlement risk
present value the amount of money which, if invested today, would accumulate into a future value at a designated time in the future
pre-settlement risk (PSR) risk of credit loss from a price change prior to settlement
price quotation an exchange rate which is quoted with the foreign currency as the commodity currency and the local currency as the terms currency
price-maker *see* quoting bank
price-taker the party (either customer or calling bank) that can choose to deal at the quoted price
principal the amount of money borrowed or invested in a money market transaction
probability density function *see* probability distribution
probability distribution the probability of different possible outcomes
purchasing power parity theory a theory which purports that, in the long run, exchange rates adjust to reflect the relative inflation rates of the two currencies
pure swap a swap in which both the spot and forward transactions are done simultaneously with the same counterparts
put the right without the obligation to sell (a currency)
put-call parity the relationship between the premiums of calls and puts with the same strike price

quanto an option for which the quantity of face value varies with some other variable
quoting bank the bank that quotes the price

real exchange rate the exchange rate adjusted for changes in the relative rates of inflation
real interest rate the nominal interest rate adjusted for the effect of inflation
realised profit or loss an exchange profit or loss where both the buy side and the sell side of the transaction are completed
reciprocal relationship *see* indirect relationship
reserve requirements requirements that central banks impose on commercial banks to hold a minimum specified percentage of their commercial deposits in accounts with the central bank, generally at rates below market interest rates
rho ratio of change in option premium to change in an interest rate
riding the swaps curve opening a gap to take advantage of the non-linear shape of the swaps curve
riding the yield curve gapping in the money market to take advantage of the shape of the yield curve
risk-free interest rate the interest rate used to determine the upfront premium of an option
rolling a position extending the maturity date
rotation of the swaps curve a movement of the swaps curve which reflects a proportional increase or decrease in swap rates resulting in a change in the slope of the swaps curve
rotation of the yield curve a proportional increase or decrease in interest rates resulting in a change in the slope of the yield curve

short a negative net exchange position

short dates value dates which occur sooner than spot value
shout option option that gives the owner the right to lock-in a minimum payout equal to the intrinsic value at the time
SIBID Singapore Interbank Bid Rate
SIBOR Singapore Interbank Offer Rate
simple interest the amount of interest paid at maturity on a money market transaction
simulated investment investing a currency indirectly by entering into a currency swap and investing the other currency directly
simulated loan generating liquidity by borrowing one currency and entering into a currency swap
spot value a payment which will be made two business days from the contract date
square position a situation in which a change in a rate will not cause a profit or loss
standard deviation the standard measure of spread of a distribution
stop-loss order instruction to close out an open position if the loss reaches a specified size
stressed rate the rate to which the price is assumed to have moved in a value at risk calculation
stress test calculation of value at risk for a larger than usual price movement
strike price or strike rate price at which an option would be exercised
swap bid rate the differential at which the quoting bank is willing to buy forward the commodity currency in the swap
swap offer rate the differential at which the quoting bank is willing to sell forward the commodity currency in the swap
swap rate the differential between a forward exchange rate and the spot rate
swaps curve the curve obtained when swap rates are plotted against tenors

T-accounts a simple notation for specifying the cash flows associated with financial transactions
terms currency the currency in terms of which an exchange rate is quoted
theta ratio of change in value to change in time
time decay the decline in time value as an option approaches expiry
time option the right to exercise a foreign exchange contract at any time during a specified period
time value the value that an option has because it has not expired
transaction risk risk of a change in exchange rates reducing the value of receipts or increasing the value of payments
translation risk risk of a change in exchange rates reducing the value of assets or increasing the size of liabilities
trending rates the practice of moving quotations up or down to prompt an expected movement in the market

unrealised profit or loss an (exchange) profit or loss where only one side of the transaction has been completed
up and in option that knocks-in when the spot rate rises to the barrier level
up and out option that knocks-out when the spot rate rises to the barrier level

value at risk (VAR) the risk associated with a prescribed adverse price movement
value date the date on which a cash flow occurs
value today a cash flow which occurs today
value tomorrow a cash flow which will occur tomorrow
vega ratio of change in option premium to change in volatility
volatility a measure of the extent to which the (exchange) rate changes over a given period
volume quotation an exchange rate which is quoted with the local currency as the commodity currency

yield curve the curve obtained when yield (interest rate) is plotted on the vertical axis and tenor on the horizontal axis

zero coupon bond a bond with all coupons equal to zero
zero premium collar collar where the net premium is zero

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