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HEAT TRANSFER APPLICATIONS FOR THE PRACTICING ENGINEER

LOUIS THEODORE

WILEY

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To
Jack Powers

My friend,
a future basketball Hall-of-Famer,
and
a true quality individual

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Preface

We should be careful to get out of an experience only the wisdom that is in it—and stop there; lest we be like the cat that sits down on a hot stove-lid. She will never sit down on a hot stove-lid again—and that is well; but also she will never sit down on a cold one anymore.

Mark Twain (Samuel Langhorne Clemens 1835–1910),
Pudd'nhead Wilson, Chapter 19

This project was a rather unique undertaking. Heat transfer is one of the three basic tenants of chemical engineering and engineering science, and contains many basic and practical concepts that are utilized in countless industrial applications. The author therefore considered writing a practical text. The text would hopefully serve as a training tool for those individuals in industry and academia involved directly, or indirectly, with heat transfer applications. Although the literature is inundated with texts emphasizing theory and theoretical derivations, the goal of this text is to present the subject of heat transfer from a strictly pragmatic point-of-view.

The book is divided into four Parts: Introduction, Principles, Equipment Design Procedures and Applications, and ABET-related Topics. The first Part provides a series of chapters concerned with introductory topics that are required when solving most engineering problems, including those in heat transfer. The second Part of the book is concerned with heat transfer principles. Topics that receive treatment include steady-state heat conduction, unsteady-state heat conduction, forced convection, free convection, radiation, boiling and condensation, and cryogenics. Part Three—considered by the author to be the "meat" of the book—addresses heat transfer equipment design procedures and applications. In addition to providing a detailed treatment of the various types of heat exchangers, this part also examines the impact of entropy calculations on exchanger design, operation, maintenance and inspection (OM&I), plus refractory and insulation effects. The concluding Part of the text examines ABET (Accreditation Board for Engineering and Technology)-related topics of concern, including environmental management, safety and accident management, ethics, numerical methods, economics and finance, and open-ended problems. An appendix is also included. An outline of the topics covered can be found in the Table of Contents.

The author cannot claim sole authorship to all of the problems and material in this text. The present book has evolved from a host of sources, including: notes, homework problems and exam problems prepared by several faculty for a required one-semester, three-credit, "Principles II: Heat Transfer" undergraduate course offered at Manhattan College; I. Farag and J. Reynolds, "Heat Transfer", A Theodore Tutorial, East Williston, N.Y., 1994; J. Reynolds, J. Jeris, and L. Theodore, "Handbook of

Chemical and Environmental Engineering Calculations", John Wiley & Sons 2004, and J. Santoleri, J. Reynolds, and L. Theodore's "Introduction to Hazardous Waste Incineration", 2nd edition, John Wiley & Sons, 2000. Although the bulk of the problems are original and/or taken from sources that the author has been directly involved with, every effort has been made to acknowledge material drawn from other sources.

It is hoped that this writing will place in the hands of industrial, academic, and government personnel, a book covering the principles and applications of heat transfer in a thorough and clear manner. Upon completion of the text, the reader should have acquired not only a working knowledge of the principles of heat transfer operations, but also experience in their application; and, the reader should find himself/herself approaching advanced texts, engineering literature, and industrial applications (even unique ones) with more confidence. The author strongly believes that, while understanding the basic concepts is of paramount importance, this knowledge may be rendered virtually useless to an engineer if he/she cannot apply these concepts to real-world situations. This is the essence of engineering.

Last, but not least, I believe that this modest work will help the majority of individuals working and/or studying in the field of engineering to obtain a more complete understanding of heat transfer. If you have come this far, and read through most of the Preface, you have more than just a passing interest in this subject. I strongly suggest that you try this text; I think you will like it.

My sincere thanks are extended to Dr. Jerry Maffia and Karen Tschinkel at Manhattan College for their help in solving some of the problems and proofing the manuscript, and to the ever reliable Shannon O'Brien for her valuable assistance.

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Introductory Comments

Prior to undertaking the writing of this text, the author (recently) co-authored a text entitled "Thermodynamics for the Practicing Engineer". It soon became apparent that some overlap existed between thermodynamic and heat transfer (the subject of this text). Even though the former topic is broadly viewed as a science, heat transfer is one of the unit operations and can justifiably be classified as an engineering subject. But what are the similarities and what are the differences?

The similarities that exist between thermodynamics and heat transfer are grounded in the three conservation laws: mass, energy, and momentum. Both are primarily concerned with energy-related subject matter and both, in a very real sense, supplement each other. However, thermodynamics deals with the transfer of energy and the conversion of energy into other forms of energy (e.g., heat into work), with consideration generally limited to systems in equilibrium. The topic of heat transfer deals with the transfer of energy in the form of heat; the applications almost exclusively occur with heat exchangers that are employed in the chemical, petrochemical, petroleum (refinery), and engineering processes.

The aforementioned transfer of heat occurs between a hot and a cold body, normally referred to as the source and receiver, respectively. (The only exception is in cryogenic applications.) When this transfer occurs in a heat exchanger, some or all of the following 10 topics/subjects can come into play:

1. The class of heat exchanger
2. The physical surface arrangement of the exchanger
3. The quantity or rate of heat transferred
4. The quantity or rate of heat "lost" in the application
5. The temperature difference between the source and receiver
6. The prime mover(s) required in the application (e.g., pump, fan, etc.)
7. The entropy gain (i.e., the quality energy lost in the application)
8. The cost to design, construct, and start up a new application
9. The cost to operate the exchanger
10. The cost to maintain the exchanger

Each of the above topics receive treatment once or several times in this text.

Part One

Introduction

Part One serves as the introductory section to this book. It reviews engineering and science fundamentals that are an integral part of the field of heat transfer. It consists of six chapters, as noted below:

1. History of Heat Transfer
2. History of Chemical Engineering: Transport Phenomena vs Unit Operations
3. Process Variables
4. Conservation Laws
5. Gas Laws
6. Heat Exchanger Pipes and Tubes

Those individuals with a strong background in the above area(s) may choose to bypass all or some of this Part.

Chapter 1

History of Heat Transfer*

INTRODUCTION

After a review of the literature, the author has concluded that the concept of heat transfer was first introduced by the English scientist Sir Isaac Newton in his 1701 paper entitled "Scala Graduum Caloris."(1) The specific ideas of heat convection and Newton's Law of Cooling were developed from that paper.

Before the development of kinetic theory in the middle of the 19th century, the transfer of heat was explained by the "caloric" theory. This theory was introduced by the French chemist Antoine Lavoisier (1743–1794) in 1789. In his paper, Lavoisier proposed that caloric was a tasteless, odorless, massless, and colorless substance that could be transferred from one body to another and that the transfer of caloric to a body increased the temperature, and the loss of calorics correspondingly decreased the temperature. Lavoisier also stated that if a body cannot absorb/accept any additional caloric, then it should be considered saturated and, hence, the idea of a saturated liquid and vapor was developed.(2)

Lavoisier's caloric theory was never fully accepted because the theory essentially stated that heat could not be created or destroyed, even though it was well known that heat could be generated by the simple act of rubbing hands together. In 1798, an American physicist, Benjamin Thompson (1753–1814), reported in his paper that heat was generated by friction, a form of motion, and not by caloric flow. Although his idea was also not readily accepted, it did help establish the law of conservation of energy in the 19th century.(3)

In 1843, the caloric theory was proven wrong by the English physicist James P. Joule (1818–1889). His experiments provided the relationship between mechanical work and the nature of heat, and led to the development of the first law of thermodynamics of the conservation of energy.(4)

The development of kinetic theory in the 19th century put to rest all other theories. Kinetic theory states that energy or heat is created by the random motion of atoms and molecules. The introduction of kinetic theory helped to develop the concept of the conduction of heat.(5)

*Part of this chapter was adapted from a report submitted by S. Avais to L. Theodore in 2007.

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The earlier developments in heat transfer helped set the stage for the French mathematician and physicist Joseph Fourier (1768–1830) to reconcile Newton's Law of Cooling, which in turn led to the development of Fourier's Law of Conduction. Newton's Law of Cooling suggested that there was a relationship between the temperature difference and the amount of heat transferred. Fourier took Newton's Law of Cooling and arrived at a convection heat equation.(6) Fourier also developed the concepts of heat flux and temperature gradient. Using the same process as he used to develop the equation of heat convection, Fourier subsequently developed the classic equation for heat conduction that has come to be defined as Fourier's law.(7)

Two additional sections complement the historical contents of this chapter. These are:

Peripheral Equipment

Recent History

PERIPHERAL EQUIPMENT

With respect to heat transfer equipment, the bulk of early equipment involved the transfer of heat across pipes. The history of pipes dates back to the Roman Empire. The ingenious "engineers" of that time came up with a solution to supply the never-ending demand of a city for fresh water and then for disposing of the wastewater produced. Their system was based on pipes made out of wood and stone, and the driving force of the water was gravity.(8) Over time, many improvements have been made to the piping system. These improvements include material choice, shape, and size of the pipes: pipes are now made from different metals, plastic, and even glass, with different diameters and wall thicknesses. The next challenge was the connection of the pipes and that was accomplished with fittings. Changes in piping design ultimately resulted from the evolving industrial demands for specific heat transfer requirements and the properties of fluids that needed to be heated or cooled.(9)

The movement of the fluids to be heated or cooled was accomplished with prime movers, particularly pumps. The first pump can be traced back to 3000 B.C., in Mesopotamia, where it was used to supply water to the crops in the Nile River Valley.(10) The pump was a long lever with a weight on one side and a bucket on the other. The use of this first pump became popular in the Middle East and was used for the next 2000 years. At times, a series of pumps would be put in place to provide a constant flow of water to crops far from the source. The most famous of these early pumps is the Archimedean screw. The pump was invented by the famous Greek mathematician and inventor Archimedes (287–212 B.C.). The pump was made of a metal pipe in which a helix-shaped screw was used to draw water upward as the screw turned. Modern force pumps were adapted from an ancient pump that featured a cylinder with a piston "at the top that create[d] a vacuum and [drew] water upward."(10) The first force pump was designed by Ctesibus (285–222 B.C.) of Alexandria, Egypt. Leonardo Da Vinci (1452–1519) was the first to come up with the idea of lifting water by means of centrifugal force; however, the operation of the centrifugal pump was first described scientifically by the French physicist Denis Papin (1647–1714) in 1687.(11) In 1754, Leonhard Euler further developed the principles on which centrifugal pumps operated; today, the ideal pump performance term, "Euler head," is named after him.(12)

RECENT HISTORY

Heat transfer, as an engineering practice, grew out of thermodynamics at around the turn of the 20th century. This arose because of the need to deal with the design of heat transfer equipment required by emerging and growing industries. Early applications included steam generators for locomotives and ships, and condensers for power generation plants. Later, the rapidly developing petroleum and petrochemical industries began to require rugged, large-scale heat exchangers for a variety of processes. Between 1920 and 1950, the basic forms of the many heat exchangers used today were developed and refined, as documented by Kern.(13) These heat exchangers still remain the choice for most process applications. Relatively speaking, there has been little since in terms of "new" designs. However, there has been a significant amount of activity and development regarding peripheral equipment. For example, the 1930s saw the development of a line of open-bucket steam traps, which today are simply referred to as steam traps. (*Note*: Steam traps are used to remove condensate from live steam in heat exchangers. The trap is usually attached at the bottom of the exchanger. When condensate enters the steam trap, the liquid fills the entire body of the trap. A small hole in the top of the trap permits trapped air to escape. As long as live steam remains, the outlet remains closed. As soon as sufficient condensate enters the trap, liquid is discharged. Thus, the trap discharges intermittently during the entire time it is in use.)

Starting in the late 1950s, at least three unrelated developments rapidly changed the heat exchanger industry.

1. With respect to heat-exchanger design and sizing, the general availability of computers permitted the use of complex calculational procedures that were not possible before.
2. The development of nuclear energy introduced the need for precise design methods, especially in boiling heat transfer (see Chapter 12).
3. The energy crisis of the 1970s severely increased the cost of energy, triggering a demand for more-efficient heat utilization (see Chapter 21).(14)

As a result, heat-transfer technology suddenly became a prime recipient of large research funds, especially during the 1960s and 1980s. This elevated the knowledge of heat-exchanger design principles to where it is today.(15)

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Chapter 2

History of Chemical Engineering: Transport Phenomena vs Unit Operations

INTRODUCTION

Although the chemical engineering profession is usually thought to have originated shortly before 1900, many of the processes associated with this discipline were developed in antiquity. For example, filtration operations were carried out 5000 years ago by the Egyptians. During this period, chemical engineering evolved from a mixture of craft, mysticism, incorrect theories, and empirical guesses.

In a very real sense, the chemical industry dates back to prehistoric times when people first attempted to control and modify their environment. The chemical industry developed as any other trade or craft. With little knowledge of chemical science and no means of chemical analysis, the earliest "chemical engineers" had to rely on previous art and superstition. As one would imagine, progress was slow. This changed with time. The chemical industry in the world today is a sprawling complex of raw-material sources, manufacturing plants, and distribution facilities which supplies society with thousands of chemical products, most of which were unknown over a century ago. In the latter half of the 19th century, an increased demand arose for engineers trained in the fundamentals of chemical processes. This demand was ultimately met by chemical engineers.

Three sections complement the presentation for this chapter. They are:

History of Chemical Engineering
Transport Phenomena vs Unit Operations
What is Engineering?

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HISTORY OF CHEMICAL ENGINEERING

The first attempt to organize the principles of chemical processing and to clarify the professional area of chemical engineering was made in England by George E. Davis. In 1880, he organized a Society of Chemical Engineers and gave a series of lectures in 1887 which were later expanded and published in 1901 as "A Handbook of Chemical Engineering." In 1888, the first course in chemical engineering in the United States was organized at the Massachusetts Institute of Technology (MIT) by Lewis M. Norton, a professor of industrial chemistry. The course applied aspects of chemistry and mechanical engineering to chemical processes.(1)

Chemical engineering began to gain professional acceptance in the early years of the 20th century. The American Chemical Society was founded in 1876 and, in 1908, organized a Division of Industrial Chemists and Chemical Engineers while authorizing the publication of the *Journal of Industrial and Engineering Chemistry*. Also in 1908, a group of prominent chemical engineers met in Philadelphia and founded the American Institute of Chemical Engineers.(1)

The mold for what is now called chemical engineering was fashioned at the 1922 meeting of the American Institute of Chemical Engineers when A. D. Little's committee presented its report on chemical engineering education. The 1922 meeting marked the official endorsement of the unit operations concept and saw the approval of a "declaration of independence" for the profession.(1) A key component of this report included the following:

> Any chemical process, on whatever scale conducted, may be resolved into a coordinated series of what may be termed "unit operations," as pulverizing, mixing, heating, roasting, absorbing, precipitation, crystallizing, filtering, dissolving, and so on. The number of these basic unit operations is not very large and relatively few of them are involved in any particular process . . . An ability to cope broadly and adequately with the demands of this (the chemical engineer's) profession can be attained only through the analysis of processes into the unit actions as they are carried out on the commercial scale under the conditions imposed by practice.

The key unit operations were ultimately reduced to three: Fluid Flow,(2) Heat Transfer (the subject title of this text), and Mass Transfer.(3) The Little report also went on to state that:

> Chemical Engineering, as distinguished from the aggregate number of subjects comprised in courses of that name, is not a composite of chemistry and mechanical and civil engineering, but is itself a branch of engineering, . . .

A time line diagram of the history of chemical engineering between the profession's founding to the present day is shown in Figure 2.1. As can be seen from the time line, the profession has reached a crossroads regarding the future education/curriculum for chemical engineers. This is highlighted by the differences of Transport Phenomena and Unit Operations, a topic that is discussed in the next section.

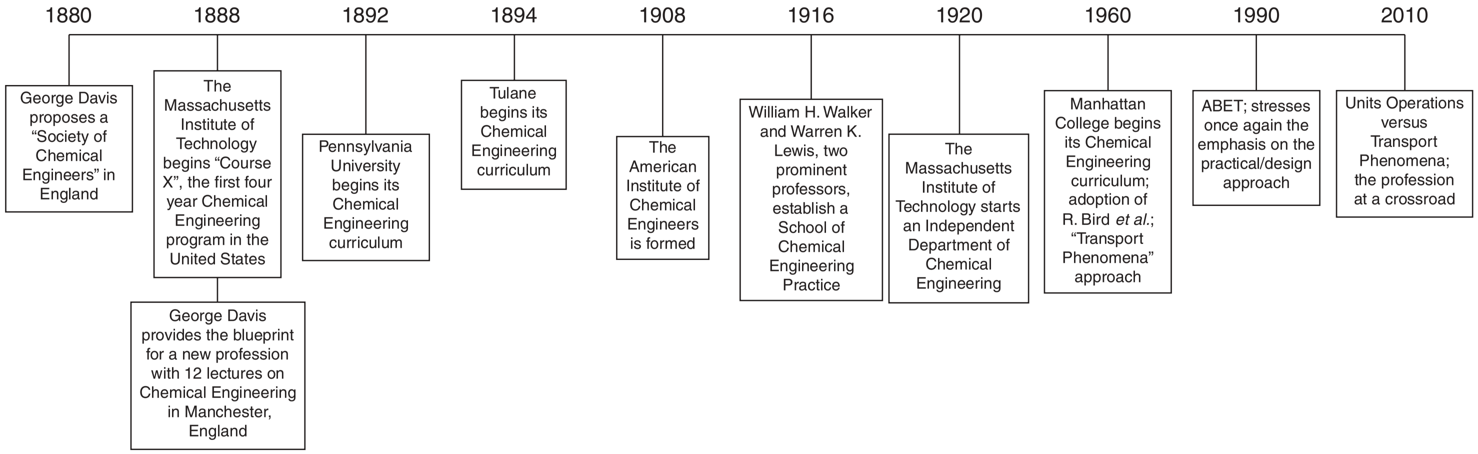

Figure 2.1 Chemical engineering time-line.

TRANSPORT PHENOMENA VS UNIT OPERATIONS

As indicated in the previous section, chemical engineering courses were originally based on the study of unit processes and/or industrial technologies. It soon became apparent that the changes produced in equipment from different industries were similar in nature (i.e., there was a commonality in the fluid flow operations in the petroleum industry as with the utility industry). These similar operations became known as the aforementioned Unit Operations. This approach to chemical engineering was promulgated in the Little report, as discussed earlier in the previous section, and to varying degrees and emphasis, has dominated the profession to this day.

The Unit Operations approach was adopted by the profession soon after its inception. During the many years since 1880 that the profession has been in existence as a branch of engineering, society's needs have changed tremendously and, in turn, so has chemical engineering.

The teaching of Unit Operations at the undergraduate level has remained relatively static since the publication of several early-to-mid 1900 texts. Prominent among these was one developed as a result of the recommendation of an advisory committee of more than a dozen educators and practicing engineers who recognized the need for a chemical engineering handbook. Dr. John H. Perry of Grasselli Chemical Co. was persuaded to undertake this tremendous compilation. The first edition of this classic work was published in 1934; the latest edition (eighth) was published in 2008. (The author of this text has served as an editor and author of the section on Environment Management for the past three editions). However, by the middle of the 20th century, there was a slow movement from the unit operation concept to a more theoretical treatment called transport phenomena. The focal point of this science was the rigorous mathematical description of all physical rate processes in terms of mass, heat, or momentum crossing boundaries. This approach took hold of the education/curriculum of the profession with the publication of the first edition of the Bird *et al.*(5) book. Some, including the author of this text, feel that this concept set the profession back several decades since graduating chemical engineers, in terms of training, were more applied physicists than traditional chemical engineers.

There has fortunately been a return to the traditional approach of chemical engineering in recent years, primarily due to the efforts of the Accreditation Board for Engineering and Technology (ABET). Detractors to this approach argue that this type of practical education experience provides the answers to 'what' and 'how' but not 'why' (i.e., a greater understanding of both physical and chemical processes). However, the reality is that nearly all practicing engineers are in no way presently involved with the 'why' questions; material normally covered here has been replaced, in part, with a new emphasis on solving design and open-ended problems. This approach is emphasized in this text.

One can qualitatively describe the differences between the two approaches discussed above. Both deal with the transfer of certain quantities (momentum, energy, and mass) from one point in a system to another. Momentum, energy, and mass are all conserved (see Chapter 4). As such, each quantity obeys the conservation law within a system:

$$\left\{\begin{matrix}\text{quantity}\\\text{into}\\\text{system}\end{matrix}\right\}-\left\{\begin{matrix}\text{quantity}\\\text{out of}\\\text{system}\end{matrix}\right\}+\left\{\begin{matrix}\text{quantity}\\\text{generated in}\\\text{system}\end{matrix}\right\}=\left\{\begin{matrix}\text{quantity}\\\text{accumulated}\\\text{in system}\end{matrix}\right\} \tag{2.1}$$

This equation may also be written on a time rate basis:

$$\left\{\begin{matrix}\text{rate}\\\text{into}\\\text{system}\end{matrix}\right\}-\left\{\begin{matrix}\text{rate}\\\text{out of}\\\text{system}\end{matrix}\right\}+\left\{\begin{matrix}\text{rate}\\\text{generated in}\\\text{system}\end{matrix}\right\}=\left\{\begin{matrix}\text{rate}\\\text{accumulated}\\\text{in system}\end{matrix}\right\} \tag{2.2}$$

The conservation law may be applied at the macroscopic, microscopic, or molecular level. One can best illustrate the differences in these methods with an example. Consider a system in which a fluid is flowing through a cylindrical tube (see Figure 2.2), and define the system as the fluid contained within the tube between points 1 and 2 at any time.

If one is interested in determining changes occurring at the inlet and outlet of the system, the conservation law is applied on a "macroscopic" level to the entire system. The resultant equation describes the overall changes occurring *to* the system (or equipment). This approach is usually applied in the Unit Operation (or its equivalent) courses, an approach which is highlighted in this text. The resulting equations are almost always algebraic.

In the microscopic approach, detailed information concerning the behavior within a system is required and this is occasionally requested of or by the engineer. The conservation law is then applied to a differential element within the system which is large compared to an individual molecule, but small compared to the entire system. The resulting equation is usually differential, and is then expanded via an integration to describe the behavior of the entire system. This has been defined as the transport phenomena approach.

The molecular approach involves the application of the conservation laws to individual molecules. This leads to a study of statistical and quantum mechanics—both of

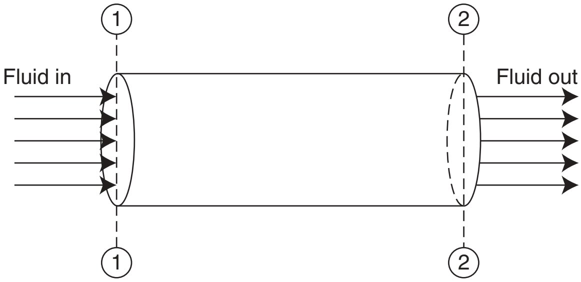

Figure 2.2 Flow through a cylinder.

which are beyond the scope of this text. In any case, the description of individual particles at the molecular level is of little value to the practicing engineer. However, the statistical averaging of molecular quantities in either a differential or finite element within a system can lead to a more meaningful description of the behavior of a system.

Both the microscopic and molecular approaches shed light on the physical reasons for the observed macroscopic phenomena. Ultimately, however, for the practicing engineer, these approaches may be valid but are akin to killing a fly with a machine gun. Developing and solving these equations (in spite of the advent of computer software packages) is typically not worth the trouble.

Traditionally, the applied mathematician has developed the differential equations describing the detailed behavior of systems by applying the appropriate conservation law to a differential element or shell within the system. Equations were derived with each new application. The engineer later removed the need for these tedious and error-prone derivations by developing a general set of equations that could be used to describe systems. These are referred to as the transport equations. In recent years, the trend toward expressing these equations in vector form has also gained momentum (no pun intended). However, the shell-balance approach has been retained in most texts, where the equations are presented in componential form—in three particular coordinate systems—rectangular, cylindrical, and spherical. The componential terms can be “lumped” together to produce a more concise equation in vector form. The vector equation can in turn, be re-expanded into other coordinate systems. This information is available in the literature.(5,6)

WHAT IS ENGINEERING?

A discussion on chemical engineering is again warranted before proceeding to the heat transfer material presented in this text. A reasonable question to ask is: What is Chemical Engineering? An outdated but once official definition provided by the American Institute of Chemical Engineers (AIChE) is:

> Chemical Engineering is that branch of engineering concerned with the development and application of manufacturing processes in which chemical or certain physical changes are involved. These processes may usually be resolved into a coordinated series of unit physical operation and chemical processes. The work of the chemical engineer is concerned primarily with the design, construction, and operation of equipment and plants in which these unit operations and processes are applied. Chemistry, physics, and mathematics are the underlying sciences of chemical engineering, and economics is its guide in practice.

The above definition has been appropriate up until a few decades ago since the profession grew out of the chemical industry. Today, that definition has changed. Although it is still based on chemical fundamentals and physical principles, these principles have been de-emphasized in order to allow the expansion of the profession to other areas (biotechnology, semiconductors, fuel cells, environment, etc.). These areas include environmental management, health and safety, computer applications, and economics and finance. This has led to many new definitions of chemical engineering, several of which are either too specific or too vague. A definition proposed by the author is simply “chemical engineers solve problems.” This definition can be extended to all engineers and thus “engineers solve problems.”

Obviously, the direction of the engineering profession, and chemical engineering in particular, has been a moving target over the past 75 years. For example, a distinguished AIChE panel in 1952 gave answers to the question: “Whither, chemical engineering as a science?” The panel concluded that the profession must avoid freezing concepts into a rigid discipline that leaves no room for growth and development. The very fluidity of chemical engineering must continue to be one of its most distinguishing aspects. In 1964, J. Hedrick of Cornell University (at an AIChE Tri-Section Symposium in Newark, NJ) posed the question “Will there still be a distinct profession of chemical engineering twenty years from now?” The dilemma has surfaced repeatedly in the past 50 years. More recently Theodore$^{(7)}$ addressed the issue; here is part of his comments:

> One of my goals is to keep in touch with students following graduation. What I have learned from graduates in the workforce is surprising—approximately 75% of them use little to nothing of what was taught in class. Stoichiometry? Sometimes. Unit operations? Sometimes. Kinetics? Not often. Thermodynamics? Rarely. Transport Phenomena? Forget about it. It is hard to deny that the chemical engineering curriculum is due for an overhaul.
>
> The traditional chemical engineers who can design a heat exchanger, predict the performance of an adsorber, specify a pump, etc., have become a dying breed. What really hurts is that I consider myself in this category. Fortunately (or perhaps unfortunately), I’m in the twilight of my career.
>
> Change won’t come easy. Although several universities in the U.S. are pioneering new programs and course changes aimed at the chemical engineer of the furture, approval by the academic community is not unanimous. Rest assured that most educators will do everything in their power to protect their “turf”.
>
> But change really does need to come. Our profession owes it to the students.

The main thrust of these comments can be applied to other engineering and science disciples.

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Chapter 3

Process Variables

INTRODUCTION

The author originally considered the title "State, Physical, and Chemical Properties" for this chapter. However, since these three properties have been used interchangeably and have come to mean different things to different people, it was decided to employ the title "Process Variables." The three aforementioned properties were therefore integrated into this all-purpose title and eliminated the need for differentiating between the three.

This third chapter provides a review of some basic concepts from physics, chemistry, and engineering in preparation for material that is covered in later chapters. All of these topics are important in some manner to heat transfer. Because many of these are unrelated to each other, this chapter admittedly lacks the cohesiveness that chapters covering a single topic might have. This is usually the case when basic material from such widely differing areas of knowledge as physics, chemistry, and engineering is surveyed. Though these topics are widely divergent and covered with varying degrees of thoroughness, all of them will find use later in this text. If additional information of these review topics is needed, the reader is directed to the literature in the reference section of this chapter.

Three additional sections complement the presentation for this chapter. They are:

Units and Dimensional Consistency
Key Terms and Definitions
Determination of Dimensionless Groups

ILLUSTRATIVE EXAMPLE 3.1

Discuss the traditional difference between chemical and physical properties.

***SOLUTION*:** Every compound has a unique set of *properties* that allows one to recognize and distinguish it from other compounds. These properties can be grouped into two main

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categories: physical and chemical. *Physical properties* are defined as those that can be measured without changing the identity and composition of the substance. Key physical properties include viscosity, density, surface tension, melting point, boiling point, and so on. *Chemical properties* are defined as those that may be altered via reaction to form other compounds or substances. Key chemical properties include upper and lower flammability limits, enthalpy of reaction, autoignition temperature, and so on.

These properties may be further divided into two categories—intensive and extensive. *Intensive properties* are not a function of the quantity of the substance, while *extensive properties* depend on the quantity of the substance. ■

UNITS AND DIMENSIONAL CONSISTENCY

Almost all process variables are dimensional (as opposed to dimensionless) and there are units associated with these terms. It is for this reason that a section on units and dimensional consistency has been included with this chapter. The units used in the text are consistent with those adopted by the engineering profession in the United States. One usually refers to them as the English or engineering units. Since engineers are often concerned with units and conversion of units, both the English and SI system of units are used throughout this book. All quantities, including physical and chemical properties, are expressed using either of these two systems.

Equations are generally dimensional and involve several terms. For the equality to hold, each term in the equation must have the same dimensions (i.e., the equation must be dimensionally homogeneous or consistent). This condition can be easily proved. Throughout the text, great care is exercised in maintaining the dimensional formulas of all terms and the dimensional consistency of each equation. The approach employed will often develop equations and terms in equations by first examining each in specific units (e.g., feet rather than length), primarily for the English system. Hopefully, this approach will aid the reader and will attach more physical significance to each term and equation.

Consider the example of calculating the perimeter, P, of a rectangle with length, L, and height, H. Mathematically, this may be expressed as $P = 2L + 2H$. This is about as simple as a mathematical equation can be. However, it only applies when P, L, and H are expressed in the same units.

A conversion constant/factor is a term that is used to obtain units in a more convenient form. All conversion constants have magnitude and units in the term, but can also be shown to be equal to 1.0 (unity) with *no* units. An often used conversion constant is

$$12\,\text{inches/foot}$$

This term is obtained from the following defining equation:

$$12\,\text{in} = 1.0\,\text{ft}$$

If both sides of this equation are divided by 1 ft, one obtains

$$12\,\text{in/ft} = 1.0$$

Note that this conversion constant, like all others, is also equal to unity without any units. Another defining equation (Newton's Law) is

$$1\ \text{lb}_\text{f} = 32.2\ \frac{\text{lb} \cdot \text{ft}}{\text{s}^2}$$

If this equation is divided by lb_f, one obtains

$$1.0 = 32.2\ \frac{\text{lb} \cdot \text{ft}}{\text{lb}_\text{f} \cdot \text{s}^2}$$

This serves to define the conversion constant g_c, which of necessity is also equal to unity with no units. Other conversion constants are given in Appendix A.6 and B.1.

ILLUSTRATIVE EXAMPLE 3.2

Convert the following:

1. 8.03 yr to seconds (s)
2. 150 miles/h to yards/h
3. 100.0 m/s^2 to ft/min^2
4. 0.03 g/cm^3 to lb/ft^3

***SOLUTION*:**

The conversion factors needed include:

365 day/yr
24 h/day
60 min/h
60 s/min

1. The following is obtained by arranging the conversion factors so that units cancel to leave only the desired units

$$(8.03\ \text{yr})\left(\frac{365\ \text{day}}{\text{yr}}\right)\left(\frac{24\ \text{h}}{\text{day}}\right)\left(\frac{60\ \text{min}}{\text{h}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right) = 2.53 \times 10^8\ \text{s}$$

2. In a similar fashion,

$$\left(\frac{150\ \text{miles}}{\text{h}}\right)\left(\frac{5280\ \text{ft}}{\text{mile}}\right)\left(\frac{\text{yd}}{3\ \text{ft}}\right) = 2.6 \times 10^5\ \text{yd/h}$$

3. $(100.0\ \text{m/s}^2)\left(\frac{100\ \text{cm}}{\text{m}}\right)\left(\frac{\text{ft}}{30.48\ \text{cm}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right)^2 = 1.181 \times 10^6\ \text{ft/ min}^2$

4. $(0.03\ \text{g/cm}^3)\left(\frac{\text{lb}}{454\ \text{g}}\right)\left(\frac{30.48\ \text{cm}}{\text{ft}}\right)^3 = 2.0\ \text{lb/ft}^3$ ■

Terms in equations must also be constructed from a "magnitude" viewpoint. Differential terms cannot be equated with finite or integral terms. Care should also be exercised in solving differential equations. In order to solve differential equations to obtain a description of the pressure, temperature, composition, etc., of a system, it is necessary to specify boundary and/or initial conditions for the system. This information arises from a description of the problem or the physical situation. The number of boundary conditions (BC) that must be specified is the sum of the highest-order derivative for each independent differential position term. A value of the solution on the boundary of the system is one type of boundary condition. The number of initial conditions (IC) that must be specified is the highest-order time derivative appearing in the differential equation. The value for the solution at time equal to zero constitutes an initial condition. For example, the equation

$$\frac{d^2T}{dz^2} = 0 \tag{3.1}$$

requires 2 BCs (in terms of z). The equation

$$\frac{dT}{dt} = 0; \qquad t = \text{time} \tag{3.2}$$

requires 1 IC. And finally, the equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}; \qquad \alpha = \text{thermal diffusivity} \tag{3.3}$$

requires 1 IC and 2 BCs (in terms of y).

Problems are frequently encountered in heat transfer and other engineering work that involve several variables. Engineers are generally interested in developing functional relationships (equations) between these variables. When these variables can be grouped together in such a manner that they can be used to predict the performance of similar pieces of equipment, independent of the scale or size of the operations, something very valuable has been accomplished. More details on this topic are provided in the last section.

Consider, for example, the problem of establishing a method of calculating the power requirements for heating liquids in open tanks. The obvious variables would be the depth of liquid in the tank, the density and viscosity of the liquid, the speed of the agitator, the geometry of the agitator, and the diameter of the tank. There are therefore six variables that affect the power, or a total of seven terms that must be considered. To generate a general equation to describe power variation with these variables, a series of tanks having different diameters would have to be set up in order to gather data for various values of each variable. Assuming that ten different values of each of the six variables were imposed on the process, 10^6 runs would be required. Obviously, a mathematical method for handling several variables that requires considerably less than one million runs to establish a *design method* must be available. In fact, such a method is available and it is defined as *dimensional analysis.*(1)

Dimensional analysis is a powerful tool that is employed in planning experiments, presenting data compactly, and making practical predictions from models without detailed mathematical analysis. The first step in an analysis of this nature is to write down the units of each variable. The end result of a dimensional analysis is a list of pertinent dimensionless numbers,(1) details of which are presented in the last section.

Dimensional analysis is a relatively "compact" technique for reducing the number and the complexity of the variables affecting a given phenomenon, process or calculation. It can help obtain not only the most out of experimental data but also scale-up data from a model to a prototype. To do this, one must achieve similarity between the prototype and the model. This similarity may be achieved through dimensional analysis by determining the important aforementioned dimensionless numbers, and then designing the model and prototype such that the important dimensionless numbers are the same in both.(1)

KEY TERMS AND DEFINITIONS

This section is concerned with key terms and definitions in heat transfer. Since heat transfer is an important subject that finds wide application in engineering, the understanding of heat transfer jargon is therefore important to the practicing engineer. It should also be noted that the same substance in its different phases may have various properties that have different orders of magnitude. As an example, heat capacity values are low for solids, high for liquids, and usually intermediate for gases.

Fluids

For the purpose of this text, a fluid may be defined as a substance that does not permanently resist distortion. An attempt to change the shape of a mass of fluid will result in layers of fluid sliding over one another until a new shape is attained. During the change in shape, shear stresses (forces parallel to a surface) will result, the magnitude of which depends upon the viscosity (to be discussed shortly) of the fluid and the rate of sliding. However, when a final shape is reached, all shear stresses will have disappeared. Thus, a fluid at equilibrium is free from shear stresses. This definition applies for both liquids and gases.

Temperature

Whether in a gaseous, liquid, or solid state, all molecules possess some degree of kinetic energy, i.e., they are in constant motion—vibrating, rotating, or translating. The kinetic energies of individual molecules cannot be measured, but the combined effect of these energies in a very large number of molecules can. This measurable quantity is known as *temperature*; it is a macroscopic concept only and as such does not exist at the molecular level.

Temperature can be measured in many ways; the most traditional method makes use of the expansion of mercury (usually encased inside a glass capillary tube) with increasing temperature. (However, thermocouples or thermistors are more commonly employed in industry.) The two most commonly used temperature scales are the Celsius (or Centigrade) and Fahrenheit scales. The Celsius scale is based on the boiling and freezing points of water at 1-atm pressure; to the former, a value of 100°C is assigned, and to the latter, a value of 0°C. On the older Fahrenheit scale, these temperatures correspond to 212°F and 32°F, respectively. Equations (3.4) and (3.5) illustrate the conversion from one scale to the other:

$$^{\circ}\mathrm{F} = 1.8(^{\circ}\mathrm{C}) + 32 \tag{3.4}$$

$$^{\circ}\mathrm{C} = (^{\circ}\mathrm{F} - 32)/1.8 \tag{3.5}$$

where $^{\circ}\mathrm{F}$ = a temperature on the Fahrenheit scale and $^{\circ}\mathrm{C}$ = a temperature on the Celsius scale.

Experiments with gases at low-to-moderate pressures (up to a few atmospheres) have shown that, if the pressure is kept constant, the volume of a gas and its temperature are linearly related (see Charles' law in Chapter 5) and that a decrease of 0.3663% or (1/273) of the initial volume is experienced for every temperature drop of 1°C. These experiments were not extended to very low temperatures, but if the linear relationship were extrapolated, the volume of the gas would *theoretically* be zero at a temperature of approximately −273°C or −460°F. This temperature has become known as *absolute zero* and is the basis for the definition of two *absolute* temperature scales. (An *absolute* scale is one that does not allow negative quantities.) These absolute temperature scales are the Kelvin (K) and Rankine (°R) scales; the former is defined by shifting the Celsius scale by 273°C so that 0 K is equal to −273°C. The Rankine scale is defined by shifting the Fahrenheit scale by 460°. Equation (3.6) shows this relationship for both absolute temperatures:

$$\begin{aligned} \mathrm{K} &= {}^{\circ}\mathrm{C} + 273 \\ {}^{\circ}\mathrm{R} &= {}^{\circ}\mathrm{F} + 460 \end{aligned} \tag{3.6}$$

Pressure

There are a number of different methods used to express a pressure term or measurement. Some of them are based on a force per unit area, e.g., pound·force per square inch, dyne, psi, etc., and others are based on fluid height, e.g., inches of water, millimeters of mercury, etc. Pressure units based on fluid height are convenient when the pressure is indicated by a difference between two levels of a liquid. Standard barometric (or atmospheric) pressure is 1 atm and is equivalent to 14.7 psi, or 33.91 ft of water, or 29.92 in of mercury.

Gauge pressure is the pressure relative to the surrounding (or atmospheric) pressure and it is related to the absolute pressure by the following equation:

$$P = P_a + P_g \tag{3.7}$$

where P is the absolute pressure (psia), P_a is the atmospheric pressure (psi) and P_g is the gauge pressure (psig). The absolute pressure scale is absolute in the same sense that the absolute temperature scale is absolute (i.e., a pressure of zero psia is the lowest possible pressure theoretically achievable—a perfect vacuum).

In stationary fluids subjected to a gravitational field, the *hydrostatic pressure difference* between two locations A and B is defined as

$$P_{\mathrm{A}} - P_{\mathrm{B}} = -\int_{z_{\mathrm{A}}}^{z_{\mathrm{B}}} \rho g \, dz \tag{3.8}$$

where z is positive in the vertical upward direction, g is the gravitational acceleration, and ρ is the fluid density. This equation will be revisited in Chapter 10.

Expressed in various units, the standard atmosphere is equal to 1.00 atmosphere (atm), 33.91 feet of water (ft H_2O), 14.7 pound·force per square inch absolute (psia), 2116 pound·force per square foot (psfa), 29.92 inches of mercury (in Hg), 760.0 millimeters of mercury (mm Hg), and 1.013×10^5 Newtons per square meter (N/m^2). The pressure term will be reviewed again in several later chapters.

Vapor pressure, usually denoted p', is an important property of liquids and, to a much lesser extent, of solids. If a liquid is allowed to evaporate in a confined space, the pressure in the vapor space increases as the amount of vapor increases. If there is sufficient liquid present, a point is eventually reached at which the pressure in the vapor space is exactly equal to the pressure exerted by the liquid at its own surface. At this point, a dynamic equilibrium exists in which vaporization and condensation take place at equal rates and the pressure in the vapor space remains constant.(2) The pressure exerted at equilibrium is called the vapor pressure of the liquid. The magnitude of this pressure for a given liquid depends on the temperature, but not on the amount of liquid present. Solids, like liquids, also exert a vapor pressure. Evaporation of solids (called *sublimation*) is noticeable only for those with appreciable vapor pressures.

ILLUSTRATIVE EXAMPLE 3.3

Consider the following pressure calculations.

1. A liquid weighing 100 lb held in a cylindrical column with a base area of 3 in^2 exerts how much pressure at the base in lb_f/ft^2?
2. If a pressure reading is 35 psig (pounds per square inch gauge), what is the absolute pressure?

***SOLUTION*:**

1. See an earlier section in this chapter.

$$\begin{aligned} F &= mg/g_c = 100\,\text{lb}(1\,\text{lb}_\text{f}/\text{lb}) \\ &= 100\,\text{lb}_\text{f} \end{aligned}$$

Note: As already discussed, g_c is a conversion factor equal to 32.2 $\text{lb}\cdot\text{ft}/\text{lb}_\text{f}\cdot\text{s}^2$; g is the gravitational acceleration, which is equal, or close to, 32.2 ft/s^2 on Earth's surface.

Therefore,

$$\begin{aligned} P &= F/\text{area} = 100\,\text{lb}_\text{f}/3\,\text{in}^2 \\ &= 33.33\,\text{lb}_\text{f}/\text{in}^2 \\ &= 4800\,\text{lb}_\text{f}/\text{ft}^2 \end{aligned}$$

2. $P = P_\text{g} + P_\text{a} = 35 + 14.7$
$= 49.7$ psia

This assumes the surrounding pressure to be atmospheric. ■

Moles and Molecular Weights

An atom consists of protons and neutrons in a nucleus surrounded by electrons. An electron has such a small mass relative to that of the proton and neutron that the weight of the atom (called the *atomic weight*) is approximately equal to the sum of the weights of the particles in its nucleus. Atomic weight may be expressed in *atomic mass units (amu) per atom* or *in grams per gram · atom*. One gram · atom contains 6.02×10^{23} atoms (Avogadro's number). The atomic weights of the elements are available in the literature.(3)

The *molecular weight* (MW) of a compound is the sum of the atomic weights of the atoms that make up the molecule. Atomic mass units per molecule (amu/molecule) or grams per gram · mole (g/gmol) are used for molecular weight. One gram · mole (gmol) contains an Avogadro number of molecules. For the English system, a pound · mole (lbmol) contains $454 \times 6.023 \times 10^{23}$ molecules.

Molal units are used extensively in heat transfer calculations as they greatly simplify material balances where chemical (including combustion) reactions are occurring. For mixtures of substances (gases, liquids, or solids), it is also convenient to express compositions in mole fractions or mole percentages instead of mass fractions. The mole fraction is the ratio of the number of moles of one component to the total number of moles in the mixture. Equations (3.9)–(3.12) express these relationships:

$$\text{moles of A} = \frac{\text{mass A}}{\text{molecular weight of A}}$$

$$n_\text{A} = \frac{m_\text{A}}{(\text{MW})_\text{A}} \tag{3.9}$$

$$\text{mole fraction A} = \frac{\text{moles A}}{\text{total moles}}$$

$$y_A = \frac{n_A}{n} \tag{3.10}$$

$$\text{mass fraction A} = \frac{\text{mass A}}{\text{total mass}}$$

$$w_A = \frac{m_A}{m} \tag{3.11}$$

$$\text{volume fraction A} = \frac{\text{volume A}}{\text{total volume}}$$

$$v_A = \frac{V_A}{V} \tag{3.12}$$

The reader should note that, in general, mass fraction (or percent) is not equal to mole fraction (or percent).

ILLUSTRATIVE EXAMPLE 3.4

If a 55-gal tank contains 20.0 lb of water,

1. How many pound · moles of water does it contain?
2. How many gram · moles does it contain?
3. How many molecules does it contain?

***SOLUTION*:** The molecular weight of the water (H_2O) is

$$\begin{aligned}\text{MW} &= (2)(1.008) + (15.999) = 18.015\ \text{g/gmol}\\ &= 18.015\ \text{lb/lbmol}\end{aligned}$$

Therefore,

1. $(20.0\ \text{lb})\left(\dfrac{\text{lbmol}}{18.015\ \text{lb}}\right) = 1.11\ \text{lbmol water}$

2. $(20.0\ \text{lb})\left(\dfrac{454\ \text{g}}{1\ \text{lb}}\right)\left(\dfrac{\text{gmol}}{18.015\ \text{g}}\right) = 504\ \text{gmol water}$

3. $(504\ \text{gmol})\left(\dfrac{6.023\times10^{23}\ \text{molecules}}{1\ \text{gmol}}\right) = 3.036\times10^{26}\ \text{molecules}$

Note that the volume of the tank does not impact the calculations. ■

Mass and Volume

The *density* (ρ) of a substance is the ratio of its mass to its volume and may be expressed in units of pounds per cubic foot (lb/ft^3), kilograms per cubic meter (kg/m^3), and so on. For solids, density can be easily determined by placing a known mass of the substance in a liquid and determining the displaced volume. The density of a liquid can be measured by weighing a known volume of the liquid in a volumetric flask. For gases, the ideal gas law, to be discussed in Chapter 5, can be used to calculate the density from the pressure, temperature, and molecular weight of the gas.

Densities of pure solids and liquids are relatively independent of temperature and pressure and can be found in standard reference books.(3,4) The *specific volume* (v) of a substance is its volume per unit mass (ft^3/lb, m^3/kg, etc.) and is, therefore, the inverse of its density.

The *specific gravity* (SG) is the ratio of the density of a substance to the density of a reference substance at a specific condition:

$$\text{SG} = \rho/\rho_{\text{ref}} \tag{3.13}$$

The reference most commonly used for *solids* and *liquids* is water at its maximum density, which occurs at 4°C; this reference density is 1.000 g/cm^3, 1000 kg/m^3, or 62.43 lb/ft^3. Note that, since the specific gravity is a ratio of two densities, it is dimensionless. Therefore, any set of units may be employed for the two densities as long as they are consistent. The specific gravity of *gases* is used only rarely; when it is, air at the same conditions of temperature and pressure as the gas is usually employed as the reference substance.

Another dimensionless quantity related to density is the API (American Petroleum Institute) gravity, which is often used to indicate densities of fuel oils. The relationship between the API scale and specific gravity is

$$\text{degrees API} = {}^\circ\text{API} = \frac{141.5}{\text{SG}(60/60^\circ\text{F})} - 131.5 \tag{3.14}$$

where SG(60/60°F) = specific gravity of the liquid at 60°F using water at 60°F as the reference.

Petroleum refining is a major industry. Petroleum products serve as an important fuel for the power industry, and petroleum derivatives are the starting point for many syntheses in the chemical industry. Petroleum is also a mixture of a large number of chemical compounds. A list of the common petroleum fractions derived from crude oil and °API is given in Table 3.1.

Table 3.1 API Values for Crude Oil Fractions

Fractions from crude oil	Approximate °API
Light ends and gases	114
Gasoline	75
Naphtha	60
Kerosene	45
Distillate	35
Gas oil	28
Lube oil	18–30
Fuel oil (residue)	25–35

ILLUSTRATIVE EXAMPLE 3.5

The following information is given:

Specific gravity of liquid (methanol) = 0.92 (at 60°F)
Density of reference substance (water) = 62.4 lb/ft^3 (at 60°F)

Determine the density of methanol in lb/ft^3.

***SOLUTION*:** Calculate the density of methanol in English units by multiplying the specific gravity by the density of water [see Equation (3.13)]:

$$\begin{aligned}\text{Density of methanol} &= (\text{specific gravity})(\text{density of water})\\ &= (0.92)(62.4)\\ &= 57.4\ \text{lb/ft}^3\end{aligned}$$

The procedure is reversed in order to calculate specific gravity from density data. As noted above, the notation for density is usually, but not always, ρ. The notation ρ_V and ρ_G are also occasionally employed for gases while ρ_L may be employed for liquids. ■

Viscosity

Viscosity is a property associated with a fluid's resistance to flow. More precisely, this property accounts for the energy losses that result from the shear stresses that occur between different portions of a fluid moving at different velocities. The *absolute* or *dynamic viscosity* (μ) has units of mass per length · time; the fundamental unit is the *poise* (*P*), which is defined as 1 g/cm · s. This unit is inconveniently large for many practical purposes, and viscosities are frequently given in *centipoises* (0.01 poise), which is abbreviated cP. The viscosity of pure water at 68.6°F is 1.00 cP. In English units, absolute viscosity is expressed either as pounds (mass) per foot · second (lb/ft · s) or pounds per foot · hour (lb/ft · h). The absolute viscosity depends primarily on temperature and to a lesser degree on pressure. The *kinematic viscosity* (ν) is the absolute viscosity divided by the density of the fluid and is useful in certain fluid flow and heat transfer problems; the units for this quantity are length squared per time (e.g., square foot per second (ft^2/s) or square meters per hour (m^2/h)). A kinematic viscosity of 1 cm^2/s is called a *stoke*. For pure water at 70°F, $\nu = 0.983$ cS (centistokes). Unlike the absolute viscosity, the kinematic viscosity of gases is a strong function of pressure due to its effect on the density. Because fluid viscosity changes rapidly with temperature, a numerical value of viscosity has no significance unless the temperature is specified.

Liquid viscosity is usually measured by the amount of time it takes for a given volume of liquid to flow through an orifice. The *Saybolt universal viscometer* is the most widely used device in the United States for the determination of the viscosity of fuel oils and liquids. It should be stressed that Saybolt viscosities, which are expressed in *Saybolt seconds* (SSU), are not even approximately proportional to absolute viscosities except in the range above 200 SSU; hence, converting units from Saybolt seconds to other units requires the use of special conversion tables. As the time of flow decreases, the deviation becomes more marked. In any event, viscosity is an important property because of potential flow problems that are associated with viscous liquids and/or petroleum products.

The viscosities of air at atmospheric pressure and water are presented in Tables 3.2 and 3.3, respectively, as functions of temperature. Viscosities of other substances are available in the literature.(3,4) Finally, the notation for the viscosity of a gas is usually μ (but occasionally μ_V or μ_G); μ_L is often employed for liquids.

Table 3.2 Viscosity of Air at 1 Atmosphere

T (°C)	Viscosity, micropoise (μP)
0	170.8
18	182.7
40	190.4
54	195.8
74	210.2
229	263.8

$1\,\text{P} = 100\ \text{cP} = 10^6\ \mu\text{P}$; $1\,\text{cP} = 6.72 \times 10^{-4}\ \text{lb/ft} \cdot \text{s}$.

Table 3.3 Viscosity of Water

T (°C)	Viscosity, centipoise (cP)
0	1.792
5	1.519
10	1.308
15	1.140
20	1.000
25	0.894
30	0.801
35	0.723
40	0.656
50	0.594
60	0.469
70	0.406
80	0.357
90	0.317
100	0.284

ILLUSTRATIVE EXAMPLE 3.6

What is the kinematic viscosity of a gas, if the specific gravity and absolute viscosity are 0.8 and 0.02 cP, respectively?

***SOLUTION*:** Convert to engineering units.

$$\mu = \left(\frac{0.02\,\text{cP}}{1}\right)\left(\frac{6.720 \times 10^{-4}\,\text{lb/ft}\cdot\text{s}}{1\,\text{cP}}\right) = 1.344 \times 10^{-5}\,\text{lb/ft}\cdot\text{s}$$

$$\rho = (\text{SG})(\rho_{\text{ref}}) = (0.8)(62.43\,\text{lb/ft}^3) = 49.94\,\text{lb/ft}^3$$

Substituting,

$$\begin{aligned} \nu &= \mu/\rho = (1.344 \times 10^{-5}\,\text{lb/ft}\cdot\text{s})/(49.94\,\text{lb/ft}^3) \\ &= 2.691 \times 10^{-7}\,\text{ft}^2/\text{s} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 3.7

To illustrate the use of a monogram, calculate the absolute viscosity of a 98% sulfuric acid solution at 45°C.

***SOLUTION*:** Referring to Figure C.1 (see Appendix C), the coordinates of 98% H_2SO_4 are given as $X = 7.0$ and $Y = 24.8$ (number 97). Locate these coordinates on the grid and call it point A. From 45°C, draw a straight line through point A and extend it to cut the viscosity axis. The intersection occurs at approximately 12 centipoise (cP). Therefore,

$$\mu = 12\,\text{cP} = 0.12\,\text{P} = 0.12\,\text{g/cm}\cdot\text{s}$$

■

Heat Capacity

The *heat capacity* of a substance is defined as the quantity of heat required to raise the temperature of that substance by one degree on a unit mass (or mole) basis. The term *specific heat* is frequently used in place of *heat capacity*. This is not strictly correct, because specific heat has been traditionally defined as the ratio of the heat capacity of a substance to the heat capacity of water. However, since the heat capacity of water is approximately $1\,\text{cal/g}\cdot{}^\circ\text{C}$ or $1\,\text{Btu/lb}\cdot{}^\circ\text{F}$, the term *specific heat* has come to imply heat capacity.

For gases, the addition of heat to cause a 1° temperature rise may be accomplished either at constant pressure or at constant volume. Since the amounts of heat necessary are different for the two cases, subscripts are used to identify which heat capacity is being used—c_p for constant pressure and c_v for constant volume. For liquids and solids, this distinction does not have to be made since there is little difference between the two. Values of heat capacity are available in the literature.(2–5)

Heat capacities are often used on a *molar* basis instead of a *mass* basis, in which case the units become cal/gmol · °C or Btu/lbmol · °F. To distinguish between the two bases, uppercase letters (C_P, C_V) are used in this text to represent the molar-based heat capacities, and lowercase letters (c_p, c_v) will be used for the mass-based heat capacities or specific heats.

Heat capacities are functions of both the temperature and pressure, although the effect of pressure is generally small and is neglected in almost all engineering calculations. The effect of temperature on C_P can be described by

$$C_P = \alpha + \beta T + \gamma T^2 \tag{3.15}$$

or

$$C_P = a + bT + cT^{-2} \tag{3.16}$$

Values for α, β, γ, and a, b, c, as well as average heat capacity information are provided in tabular form by Theodore et al.$^{(2)}$ *Average* or *mean* heat capacity data over specific temperature ranges are also available. It should be noted that heat capacities for solids are a weak function of both temperature and pressure. Liquids are less dependent on pressure than solids, but are slightly influenced by temperature. Gases exhibit a strong temperature dependence while the effect of pressure is small, except near the critical state where the pressure dependence diminishes with increasing temperature.

ILLUSTRATIVE EXAMPLE 3.8

The following is given:

$$\text{Heat capacity of methanol} = 0.61\ \text{cal/g}\cdot{}^\circ\text{C (at } 60^\circ\text{F)}$$

Convert the heat capacities to English units.

***SOLUTION*:** Note that 1.0 Btu/lb · °F is equivalent to 1.0 cal/g · °C. This also applies on a mole basis, that is

$$1\ \text{Btu/lbmol}\cdot{}^\circ\text{F} = 1\ \text{cal/gmol}\cdot{}^\circ\text{C}$$

The heat capacity can also be converted from units of cal/g · °C to Btu/lb · °F using appropriate conversion factors:

$$\left(\frac{0.61\ \text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)\left(\frac{454\ \text{g}}{\text{lb}}\right)\left(\frac{\text{Btu}}{252\ \text{cal}}\right)\left(\frac{{}^\circ\text{C}}{1.8{}^\circ\text{F}}\right) = 0.61\ \text{Btu/lb}\cdot{}^\circ\text{F}$$

■

Thermal Conductivity

Experience has shown that when a temperature difference exists across a solid body, energy in the form of heat will transfer from the high-temperature region to the low-temperature region until thermal equilibrium (same temperature) is reached. This

Table 3.4 Typical Thermal Conductivity Values

Material	k, W/m · °C
Solid metals	15–400
Liquids	0.1–10
Gases	0.01–0.2

mode of heat transfer, where vibrating molecules pass along kinetic energy through the solid, is called *conduction*. Liquids and gases may also transport heat in this fashion. The property of *thermal conductivity*, k, provides a measure of how fast (or how easily) heat flows through a substance. It is defined as the amount of heat that flows per unit time through a unit surface area of unit thickness as a result of a unit difference in temperature. It is also a property of the material. Typical units for conductivity are $\mathrm{Btu \cdot ft/h \cdot ft^2 \cdot {}^\circ F}$ or $\mathrm{Btu/h \cdot ft \cdot {}^\circ F}$ or $\mathrm{W/m \cdot {}^\circ C}$. Typical values of k are given in Table 3.4.

Thermal conductivities of solids are primarily dependent on temperature. In general, thermal conductivities for a pure metal decrease with temperature; alloying elements tend to reverse this trend. Liquids are for the most part temperature dependent but insensitive to pressure. The thermal conductivities of most liquids decrease with increasing temperature. It should be noted that water has the highest thermal conductivity of all liquids except the so-called liquid metals. The thermal conductivity of a gas increases with increasing temperature; it is a weak function of pressure for pressures close to atmospheric. However, the effect of pressure is significant for high pressures. Also note that the thermal conductivity of steam exhibits a strong pressure dependence.

With regard to heat transfer applications, this particular property finds extensive application in designing heat exchangers (see Chapters 7 and 8, and Part III).

ILLUSTRATIVE EXAMPLE 3.9

The following data is given:

$$\text{Thermal conductivity of methanol} = 0.0512\ \mathrm{cal/m \cdot s \cdot {}^\circ C}\ (\text{at } 60^\circ\mathrm{F})$$

Convert the thermal conductivity to English units.

***SOLUTION*:** The factor for converting $\mathrm{cal/m \cdot s \cdot {}^\circ C}$ to $\mathrm{Btu/ft \cdot h \cdot {}^\circ F}$ can be shown to be 2.419. The thermal conductivity of methanol can therefore be converted to $\mathrm{Btu/ft \cdot h \cdot {}^\circ F}$ from $\mathrm{cal/m \cdot s \cdot {}^\circ C}$ as follows:

$$\begin{aligned} k &= \left(\frac{0.0512\,\mathrm{cal}}{\mathrm{m \cdot s \cdot {}^\circ C}}\right)\left(\frac{\mathrm{Btu}}{252\,\mathrm{cal}}\right)\left(\frac{0.3048\,\mathrm{m}}{\mathrm{ft}}\right)\left(\frac{3600\,\mathrm{s}}{\mathrm{h}}\right)\left(\frac{{}^\circ\mathrm{C}}{1.8^\circ\mathrm{F}}\right) \\ &= (0.0512)(2.419) \\ &= 0.124\,\mathrm{Btu/ft \cdot h \cdot {}^\circ F} \end{aligned}$$

Note that the usual engineering notation for thermal conductivity is k and that 1 Btu/h · ft · °F = 1.7307 W/m · K. ■

ILLUSTRATIVE EXAMPLE 3.10

Estimate the thermal conductivity (W/m · K) of air at ambient conditions.

***SOLUTION*:** Refer to Table D.7 in the Appendix. The thermal conductivity is approximately 0.025 W/m · K. ■

Thermal Diffusivity

A useful combination of terms already considered is the *thermal diffusivity*, α; it is defined by

$$\alpha = \frac{k}{\rho c_p} \tag{3.17}$$

As with the kinematic viscosity, the units of α are ft^2/h or m^2/s. As one might expect, thermal energy diffuses rapidly through substances with high α and slowly through those with low α. There is a strong dependence of α for gases for both pressure and temperature.

Reynolds Number

The Reynolds number, Re, is a dimensionless number that indicates whether a moving fluid is flowing in the laminar or turbulent mode. *Laminar* flow is characteristic of fluids flowing slowly enough so that there are no eddies (whirlpools) or macroscopic mixing of different portions of the fluid. (*Note*: In any fluid, there is always *molecular* mixing due to the thermal activity of the molecules; this is distinct from *macroscopic* mixing due to the swirling motion of different portions of the fluid.) In laminar flow, a fluid can be imagined to flow like a deck of cards, with adjacent layers sliding past one another. *Turbulent* flow is characterized by eddies and macroscopic currents. In practice, moving gases are generally in the turbulent region. For flow in a pipe, a Reynolds number above 2100 is an indication of turbulent flow.

The Reynolds number is dependent on the fluid velocity, density, viscosity, and some *length* characteristic of the system or conduit; for pipes, this characteristic length is the inside diameter:

$$\text{Re} = DV\rho/\mu = DV/\nu \tag{3.18}$$

where Re = Reynolds number
D = inside diameter of the pipe (ft)
V = fluid velocity (ft/s)
ρ = fluid density (lb/ft^3)
μ = fluid viscosity (lb/ft · s)
ν = fluid kinematic viscosity (ft^2/s)

Any consistent set of units may be used with Equation (3.18). In addition, the reader should note that both v and V are employed to represent velocity while volume is solely designated V (see next chapter).

ILLUSTRATIVE EXAMPLE 3.11

Calculate the Reynolds number for a fluid flowing through a 5-inch diameter pipe at 10 fps (feet per second) with a density of 50 lb/ft^3 and a viscosity of 0.65 cP. Is the flow turbulent or laminar?

***SOLUTION*:** By definition

$$\mathrm{Re} = DV\rho/\mu \tag{3.18}$$

Substitution yields

$$\begin{aligned}\mathrm{Re} &= \left(\frac{50\,\mathrm{lb}}{\mathrm{ft}^3}\right)\left(\frac{10\,\mathrm{ft}}{\mathrm{s}}\right)\left(\frac{5\,\mathrm{in}}{1}\right)\left(\frac{1\,\mathrm{ft}}{12\,\mathrm{in}}\right)\left(\frac{1}{0.65\,\mathrm{cP}}\right)\left(\frac{1\,\mathrm{cP}}{6.720\times10^{-4}\,\mathrm{lb/ft\cdot s}}\right)\\ &= (50\,\mathrm{lb/ft^3})(10\,\mathrm{ft/s})[(5/12)\mathrm{ft}]/(0.65\times6.72\times10^{-4}\,\mathrm{lb/ft\cdot s})\\ &= 477{,}000\end{aligned}$$

The Reynolds number is >2100; therefore, the flow is turbulent. ■

Kinetic Energy

Consider a body of mass, m, that is acted upon by a force, F. If the mass is displaced a distance, dL, during a differential interval of time, dt, the energy expended is given by

$$dE_{KE} = m\left(\frac{a}{g_c}\right)dL \tag{3.19}$$

Since the acceleration is given by $a = dV/dt$,

$$dE_{KE} = \left(\frac{m}{g_c}\right)\left(\frac{dV}{dt}\right)dL = \left(\frac{m}{g_c}\right)\left(\frac{dL}{dt}\right)dV \tag{3.20}$$

Noting that $V = dL/dt$, the above expression becomes:

$$dE_{KE} = m\left(\frac{V}{g_c}\right)dV \tag{3.21}$$

If this equation is integrated from V_1 to V_2, the change in energy is

$$\Delta E_{KE} = \frac{m}{g_c}\int_{V_1}^{V_2} V\,dV = \frac{m}{g_c}\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) \tag{3.22}$$

or

$$\Delta E_{KE} = \left(\frac{mV_2^2}{2g_c} - \frac{mV_1^2}{2g_c}\right) = \Delta\left(\frac{mV^2}{2g_c}\right) \tag{3.23}$$

The term above is defined as the change in kinetic energy.

The reader should note that for flow through pipes and tubes, the above kinetic energy term can be retained as written if the velocity profile is uniform i.e., the local velocities at all points in the cross-section are the same. Ordinarily, there is a velocity gradient across the passage; this introduces an error in the above calculation, the magnitude of which depends on the nature of the velocity profile and the shape of the cross section. For the usual case where the velocity is approximately uniform (e.g., turbulent flow), the error is not serious and, since the error tends to cancel because of the appearance of kinetic terms on each side of any energy balance equation, it is customary to ignore the effect of velocity gradients. When the error cannot be ignored, the introduction of a correction factor is needed.(1)

Potential Energy

A body of mass m is raised vertically from an initial position z_1 to z_2. For this condition, an upward force at least equal to the weight of the body must be exerted on it, and this force must move through the distance $z_2 - z_1$. Since the weight of the body is the force of gravity on it, the minimum force required is given by Newton's law:

$$F = \frac{ma}{g_c} = m\left(\frac{g}{g_c}\right) \tag{3.24}$$

where g is the local acceleration of gravity. The minimum work required to raise the body is the product of this force and the change in vertical displacement, that is,

$$\Delta E_{PE} = F(z_2 - z_1) = m\left(\frac{g}{g_c}\right)(z_2 - z_1) = \Delta\left(m\frac{g}{g_c}z\right) \tag{3.25}$$

The term above is defined as the change in the potential energy of the mass.

ILLUSTRATIVE EXAMPLE 3.12

As part of a heat transfer course, a young environmental engineering major has been requested to determine the potential energy of water before it flows over a waterfall 10 meters in height above ground level conditions.

***SOLUTION*:** The potential energy of water depends on two considerations:

1. the quantity of water, and
2. a reference height.

For the problem at hand, take as a basis 1 kilogram of water and assume the potential energy to be zero at ground level conditions. Apply Equation (3.25). Based on the problem statement, set $z_1 = 0\,\text{m}$ and $z_2 = 10\,\text{m}$, so that

$$\Delta z = 10\,\text{m}$$

At ground level conditions,

$$PE_1 = 0$$

Therefore

$$\begin{aligned} \Delta(PE) &= PE_2 - PE_1 = PE_2 \\ PE_2 = E_{PE_2} &= m(g/g_c)z_2 \\ &= (1\,\text{kg})(9.8\,\text{m/s}^2)(10\,\text{m}) \\ &= 98\,\text{kg}\cdot\text{m}^2/\text{s}^2 \\ &= 98\,\text{J} \end{aligned}$$

■

DETERMINATION OF DIMENSIONLESS GROUPS

To scale-up (or scale-down) a process, it is necessary to establish geometric and dynamic similarities between the model and the prototype. These two similarities are discussed below.

Geometric similarity implies using the same geometry of equipment. A circular pipe prototype should be modeled by a tube in the model. Geometric similarity establishes the scale of the model/prototype design. A 1/10th scale model means that the characteristic dimension of the model is 1/10th that of the prototype.

Dynamic similarity implies that the important dimensionless numbers must be the same in the model and the prototype. For a flowing fluid, which is being heated in a tube of an exchanger, it has been shown that the friction factor, *f*, is a function of the dimensionless Reynolds number (defined earlier). By selecting the operating conditions such that Re in the model equals the Re in the prototype, then the friction factor in the prototype will equal the friction factor in the model.(1)

It should now be apparent that dimensionless numbers, as well as dimensionless groups, play an important role in the engineering analysis of heat transfer phenomena. The Buckingham π (pi) theorem may be used to determine the number of independent dimensionless groups required to obtain a relation describing a physical phenomenon. The theorem requires that equations relating variables must be dimensionally homogenous (consistent). The approach can be applied to the problem of correlating experimental heat transfer data for a fluid flowing across (or through) a heated tube. This is defined as a convective process that requires expressing a heat transfer coefficient, a term and topic to be defined and treated in Chapter 9, in terms of its dependent variables. The units of this coefficient are energy/time · area · temperature. For this process, it is reasonable to expect that the physical quantities listed below are pertinent to the description of this system. These have been expressed in terms of the four primary dimensions—length (L), mass (M), time (t), and temperature (T).

The aforementioned heat transfer coefficient is assumed a function of the following variables

$$h = \psi(D, k, V, \rho, \mu, C) \tag{3.26}$$

Each variable is expressed below in terms of the primary dimensions

Tube diameter	D	L
Thermal conductivity	k	$ML/t^2 \cdot T$
Fluid velocity	V	L/t
Fluid density	ρ	M/L^3
Fluid viscosity	μ	$M/L \cdot t$
Heat capacity	C	$L^2/t^2 \cdot T$
Heat transfer coefficient	h	$M/t^3 \cdot T$

Note that energy can be expressed as $M \cdot L^2/t^2$, heat capacity is represented with a capital C, and velocity with a capital V.

The following procedure is employed for the Buckingham π theorem:

$$\pi = D^a k^b V^c \rho^d \mu^e C^f h^g \tag{3.27}$$

Substituting primary units gives

$$\pi = (L)^a (M \cdot L/t^2 \cdot T)^b (L/t)^c (M/L^3)^d (M/L \cdot t)^e (L^2/t^2 \cdot T)^f (M/t^3 \cdot T)^g \tag{3.28}$$

For π to be dimensionless, the exponents of each primary dimension must separately add up to zero, i.e.,

$$M: \quad b + d + e + g = 0$$
$$L: \quad a + b + c - 3d - e + 2f = 0$$
$$t: \quad -3b - c - e - 2f - 3g = 0$$
$$T: \quad -b - f - g = 0$$

There are seven unknown but only four equations at this point. Since h is the dependent variable, set $g = 1$. A trial-and-error procedure, based to some extent on experience is now applied. Set $c = d = 0$.

For this condition (there are now four equations and four unknowns), resulting in

$$a = 1; \qquad b = -1; \qquad e = f = 0 \tag{3.29}$$

and the first dimensionless group, π, is

$$\pi_1 = \frac{hD}{k} = \text{Nusselt number} = \text{Nu} \tag{3.30}$$

For π_2, once again set $g = 0$, and $a = 1, f = 0$. The solution now becomes

$$b = 0; \qquad c = d = 1; \qquad e = -1 \tag{3.31}$$

with

$$\pi_2 = \frac{DV\rho}{\mu} = \text{Reynolds number} = \text{Re}$$

For π_3, set $e = 1$, and $c = g = 0$. The solution now becomes

$$\pi_3 = \frac{C\mu}{k} = \text{Prandtl number} = \text{Pr} \tag{3.32}$$

Thus, the function relationship can be written as

$$\text{Nu} = f(\text{Re}, \text{Pr}) \tag{3.33}$$

Another dimensionless number that appears in combined natural and forced convection systems as well as just natural convection (Chapter 10) systems is the Graetz number. There are several defining equations for this number:

$$\text{Gz} = (\text{Re})(\text{Pr})\left(\frac{D}{L}\right); \quad L = \text{characteristic length} \tag{3.34}$$

Table 3.5 Dimensionless Numbers

Biot number (Bi)	$\frac{hL}{k}$
Graetz number (Gz)	$\frac{\dot{m}c_p}{kL}$
Grashof number (Gr)	$\frac{L^3\rho^2 g\beta\Delta T}{\mu^2}$
Liquid Jacob number (Ja)	$\frac{c_p\Delta T_e}{h_{\text{vap}}}$
Nusselt number (Nu)	$\frac{hD}{k}$
Peclet number (Pe)	$\frac{DV\rho c_p}{k}$
Prandtl number (Pr)	$\frac{c_p\mu}{k}$
Reynolds number (Re)	$\frac{DV\rho}{\mu}, \frac{DG}{\mu}$
Stanton number (St)	$\frac{h}{c_pV\rho}, \frac{h}{c_pG}$

$$\mathrm{Gz} = \frac{4}{\pi}\frac{\dot{m}c_p}{kL}; \quad \dot{m} = \text{mass flow rate} \tag{3.35}$$

However, it should be noted that some define the Graetz number by

$$\mathrm{Gz} = \frac{\dot{m}c_p}{kL} \tag{3.36}$$

These dimensionless numbers, as well as others, will appear with regular consistency in both this and the next two Parts. Extensive reference to these terms is made in Chapter 9.

There are several dimensionless numbers encountered in heat transfer applications. Some of the more important and most commonly used numbers are provided in alphabetical order in Table 3.5, with some of the terms to be defined later. With reference to Table 3.5, also note that

$$\mathrm{Nu} = (\mathrm{St})(\mathrm{Re})(\mathrm{Pr}) \tag{3.37}$$

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Chapter 4

Conservation Laws

INTRODUCTION

In order to better understand heat transfer, it is necessary to first understand the theory underlying this science. How can one predict at what temperature products will be emitted from cooled or heated effluent streams? At what temperature must a thermal device be operated? How much energy in the form of heat is given off during a combustion process? Is it economically feasible to recover that heat? Is the feed high enough in heating value, or must additional fuel be added to assist in the thermal process? If so, how much fuel must be added? The answers to these questions are rooted in the various theories of heat transfer, including thermodynamics, thermochemistry, and to a lesser degree, phase equilibrium and chemical reaction equilibrium.

One of the keys necessary to answer the above questions is often obtained via the application of one or more of the conservation laws. The contents of this chapter deal with these laws. The four topics covered are:

The Conservation Laws
The Conservation Law for Momentum
The Conservation Law for Mass
The Conservation Law for Energy

Obviously, at the heart of this chapter is topic (**4**) and, to a lesser extent, topic (**3**); however, some momentum considerations also come into play in part of this book.

Four important terms are defined below before proceeding to the conservation laws:

1. A *system* is any portion of the universe that is set aside for study.
2. Once a system has been chosen, the rest of the universe is referred to as the *surroundings*.
3. A *system* is described by specifying that it is in a certain state.
4. The *path*, or series of values certain variables assume in passing from one state to another, defines a process.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

THE CONSERVATION LAWS

Momentum, energy, and mass are all conserved. As such, each quantity obeys the conservation law below as applied within a system (see also Chapter 2):

$$\left\{\begin{array}{c}\text{quantity}\\\text{into}\\\text{system}\end{array}\right\} - \left\{\begin{array}{c}\text{quantity}\\\text{out of}\\\text{system}\end{array}\right\} + \left\{\begin{array}{c}\text{quantity}\\\text{generated}\\\text{in system}\end{array}\right\} = \left\{\begin{array}{c}\text{quantity}\\\text{accumulated}\\\text{in system}\end{array}\right\} \tag{4.1}$$

This equation may also be written on a *time* rate basis:

$$\left\{\begin{array}{c}\text{rate of}\\\text{quantity}\\\text{into}\\\text{system}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of}\\\text{quantity}\\\text{out of}\\\text{system}\end{array}\right\} + \left\{\begin{array}{c}\text{rate of}\\\text{quantity}\\\text{generated}\\\text{in system}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of}\\\text{quantity}\\\text{accumulated}\\\text{in system}\end{array}\right\} \tag{4.2}$$

The above conservation law may be applied at the macroscopic, microscopic, or molecular level.$^{(1,2)}$ Chapter 2 has illustrated the differences between these methods. As noted earlier, this text departs from the molecular approach even though this method has a great deal to commend it. Experience has indicated that the engineer possessing a working knowledge of the conservation laws is likely to obtain a more integrated and unified picture of heat transfer by applying the macroscopic approach. However, the microscopic approach receives some treatment in Chapters 7–9.

THE CONSERVATION LAW FOR MOMENTUM

The general conservation law for momentum on a rate basis is first applied to a volume element:

$$\begin{array}{ccccccc}
\left\{\begin{array}{c}\text{rate of}\\\text{momentum}\\\text{in by}\\\text{convection}\end{array}\right\} & - & \left\{\begin{array}{c}\text{rate of}\\\text{momentum}\\\text{out by}\\\text{convection}\end{array}\right\} & + & \left\{\begin{array}{c}\text{rate of}\\\text{momentum}\\\text{in by}\\\text{molecular}\\\text{diffusion}\end{array}\right\} & - & \left\{\begin{array}{c}\text{rate of}\\\text{momentum}\\\text{out by}\\\text{molecular}\\\text{diffusion}\end{array}\right\}\\
(1) & & (2) & & (3) & & (4)\\
\multicolumn{3}{c}{\downarrow\;\downarrow\;\downarrow\;\downarrow\;\downarrow\;\downarrow\;\downarrow} & & \multicolumn{3}{c}{\downarrow\;\downarrow\;\downarrow\;\downarrow\;\downarrow\;\downarrow\;\downarrow}\\
\multicolumn{3}{c}{\text{bulk flow}} & & \multicolumn{3}{c}{\text{velocity gradients}}
\end{array}$$

$$\begin{array}{cccc}
+ & \left\{\begin{array}{c}\text{external forces}\\\text{exerted on fluid}\end{array}\right\} & = & \left\{\begin{array}{c}\text{rate of momentum}\\\text{accumulation}\end{array}\right\}\\
& (5) & & (6)\\
& \downarrow\;\downarrow\;\;\downarrow\;\downarrow & & \downarrow\;\downarrow\;\downarrow\;\downarrow\\
& \text{surface forces and/or body forces} & & \text{inventory}
\end{array}$$

The above enumerates the rate of momentum and forces acting on a moving fluid in the volume element of concern at any time t. Each rate of momentum or force term in the above equation can be expressed in units of lb_f in order to maintain dimensional consistency. It is suggested that the reader refer to the literature for more information.(1,2)

The application of this conservation law finds extensive use in the field of fluid mechanics or fluid flow.(3) Applications in the field of heat transfer, as indicated above, are limited.

ILLUSTRATIVE EXAMPLE 4.1

A 10-cm-diameter horizontal line carries saturated steam at 420 m/s. Water is entrained by the steam at a rate of 0.15 kg/s. The line has a 90° bend. Calculate the force components in the horizontal and vertical directions required to hold the bend in place due to the entrained water.

***SOLUTION*:** A line diagram of the system is provided in Figure 4.1. Select the fluid in the bend as the system and apply the conservation law for mass (see next Section):

$$\dot{m}_1 = \dot{m}_2$$

Since the density and cross-sectional area are constant,

$$V_1 = V_2$$

where $\dot{m}_1, \dot{m}_2$ = mass flowrate at 1 and 2, respectively
V_1, V_2 = velocity at 1 and 2, respectively

A linear momentum ($\dot{M}$) balance in the horizontal direction provides the force applied by the channel wall on the fluid in the x-direction, F_x:

$$\begin{aligned} F_x g_c &= \dot{M}_{x,\text{out}} - \dot{M}_{x,\text{in}} \\ &= \frac{d}{dt}(mV)_{x,\text{out}} - \frac{d}{dt}(mV)_{x,\text{in}} \end{aligned}$$

Note that the above equation assumes that the pressure drop across the bend is negligible.

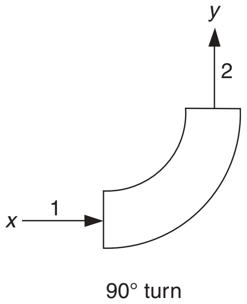

Figure 4.1 Diagram for Illustrative Example 4.1.

Since $V_{x,\text{out}} = 0$ and $dm/dt = \dot{m}$,

$$F_x g_c = 0 - \dot{m}V_{x,\text{in}} = -\frac{(0.15)(420)}{(1)} = -63\,\text{N} = -14.1\,\text{lb}_\text{f}$$

The x-direction supporting force acting on the 90° elbow is 14.1 lb_f acting toward the left.

A linear momentum balance in the vertical direction results in

$$\begin{aligned} F_y g_c &= \dot{M}_{y,\text{out}} - \dot{M}_{y,\text{in}} \\ &= \dot{m}V_{y,\text{out}} - \dot{m}V_{y,\text{in}} \\ &= \dot{m}V_2 - 0 = (0.15)(420) = 63\,\text{N} = 14.1\,\text{lb}_\text{f} \end{aligned}$$

The y-direction supporting force on the 90° elbow is 14.1 lb_f acting upwards. ■

ILLUSTRATIVE EXAMPLE 4.2

Refer to Illustrative Example 4.1. Calculate the magnitude and direction of the resultant force.

***SOLUTION*:** The resultant supporting force is given by:

$$F_\text{res} = \sqrt{F_x^2 + F_y^2}$$

Substituting,

$$F_\text{res} = \sqrt{(-63)^2 + (63)^2} = 89.1\,\text{N} = 19.1\,\text{lb}_\text{f}$$

The direction is given by

$$\begin{aligned} \tan\theta &= \frac{F_y}{F_x} = \frac{63}{-63} = -1 \\ \theta &= 135^\circ \end{aligned}$$

where θ is the angle between the positive x axis and the direction of the force. The counterclockwise rotation of the direction from the x axis is defined as positive.

The supporting force is therefore 19.1 lb_f acting in the “northwest” direction. ■

THE CONSERVATION LAW FOR MASS

The *conservation law* for mass can be applied to any process or system. The general form of this law is given as:

$$\text{mass in} - \text{mass out} + \text{mass generated} = \text{mass accumulated} \tag{4.4}$$

or, on a time rate basis, by

$$\left\{\begin{matrix}\text{rate of}\\ \text{mass in}\end{matrix}\right\}-\left\{\begin{matrix}\text{rate of}\\ \text{mass out}\end{matrix}\right\}+\left\{\begin{matrix}\text{rate of mass}\\ \text{generated}\end{matrix}\right\}=\left\{\begin{matrix}\text{rate of mass}\\ \text{accumulated}\end{matrix}\right\} \tag{4.5}$$

In heat transfer-related processes, it is often necessary to obtain quantitative relationships by writing mass balances on the various elements in a system. This equation may be applied either to the total mass involved or to a particular species on either a mole or mass basis. This law can be applied to steady-state or unsteady-state (transient) processes and to batch or continuous systems. As noted earlier, in order to isolate a system for study, it is separated from the surroundings by a boundary or envelope. This boundary may be real (e.g., the walls of a heat exchanger) or imaginary. Mass crossing the boundary and entering the system is part of the *mass in* term in Equation 4.5, while that crossing the boundary and leaving the system is part of the *mass out* term. Equation 4.5 may be written for any compound whose quantity is not changed by chemical reaction and for any chemical element whether or not it has participated in a chemical reaction. It may be written for a heat exchanger, one piece of equipment, around several pieces of equipment, or around an entire process. It may be used to calculate an unknown quantity directly, to check the validity of experimental data, or to express one or more of the independent relationships among the unknown quantities in a particular problem situation.

A *steady-state* process is one in which there is no change in conditions (pressure, temperature, composition, etc.) or rates of flow with time at any given point in the system. The accumulation term in Equation 4.5 is then zero. (If there is no chemical or nuclear reaction, the generation term is also zero.) All other processes are *unsteady state*.

In a *batch* process, a given quantity of feed is placed in a container or heat exchanger and a change in temperature can be made to occur. At the end of the process, the container or exchanger holds the heated or cooled product(s). In a *continuous* process, heated or cooled feed is continuously introduced into a heat exchanger, a piece of equipment, or several pieces in series, and products are continuously removed from one or more locations. A continuous process may or may not be steady-state. A coal-fired power plant, for example, operates continuously. However, because of the wide variation in power demand between peak and slack periods, there is an equally wide variation in the rate at which the coal is fired. For this reason, power plant problems may require the use of average data over long periods of time. However, most, but not all, heat transfer operations are assumed to be steady-state and continuous.

As indicated previously, Equation 4.5 may be applied to the total mass of each stream (referred to as an *overall* or *total material balance*) or to the individual component(s) of the stream (referred to as a *componential* or *component material balance*). The primary task in preparing a material balance in heat transfer calculations is often to develop the quantitative relationships among the streams.

Four important processing concepts are *bypass*, *recycle*, *purge*, and *makeup* (see Figure 4.2). With *bypass*, part of the inlet stream is diverted around the heat exchanger or equipment to rejoin the (main) stream after the unit. This stream effectively moves

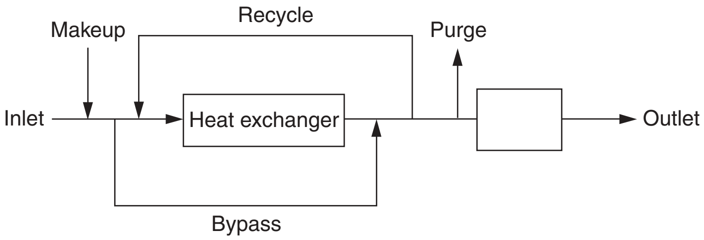

Figure 4.2 Recycle, bypass, and purge.

in parallel with the stream passing through the equipment. In *recycle*, part of the heated or cooled product stream is sent back to mix with the feed stream. If a small quantity of nonreactive material is present in the feed to a process that includes recycle, it may be necessary to remove the nonreactive material in a *purge* stream to prevent it from building up to above a maximum tolerable value. This can also occur in a process without recycle; if a nonreactive material is added in the feed and not totally removed in the products, it will accumulate until purged. The purging process is sometimes referred to as *blowdown. Makeup*, as its name implies, involves adding or making up part of a stream that has been removed from a process. Makeup may be thought of as the opposite of purge and/or blowdown.

ILLUSTRATIVE EXAMPLE 4.3

Fuel is fed into a boiler at a rate of 10,000 lb/h in the presence of 20,000 lb/h of air. Due to the low heating value of the fuel, 2000 lb/h of methane is added to assist in the combustion of the fuel. At what rate (lb/h) do the product gases exit the incinerator?

***SOLUTION*:** Apply the conservation law for mass to the boiler. Assume steady-state conditions to apply:

$$\text{Rate of mass in } (\dot{m}_{\text{in}}) = \text{rate of mass out } (\dot{m}_{\text{out}}) \tag{4.5}$$

Substituting,

$$\begin{aligned}\dot{m}_{\text{in}} &= (10{,}000 + 20{,}000 + 2000)\\ &= 32{,}000\,\text{lb/h}\end{aligned}$$

Therefore,

$$\dot{m}_{\text{out}} = 32{,}000\,\text{lb/h}$$

■

ILLUSTRATIVE EXAMPLE 4.4

A proposed heat exchanger design requires that a packed column and a spray tower are to be used in series for the removal of HCl from a gas feed stream. The spray tower is operating at an efficiency of 65% and the packed column at an efficiency of 98%. Calculate the mass flow rate of

HCl leaving the spray tower, the mass flow rate of HCl entering the packed tower, and the overall fractional efficiency of the removal system if 76.0 lb of HCl enters the system every hour.

***SOLUTION*:** By definition, the efficiency, *E*, is given by

$$E = (\dot{m}_{\text{in}} - \dot{m}_{\text{out}})/\dot{m}_{\text{in}}$$

$$\dot{m}_{\text{out}} = (1 - E)(\dot{m}_{\text{in}}) \tag{4.6}$$

For the spray tower:

$$\begin{aligned}\dot{m}_{\text{out}} &= (1 - 0.65)(76.0)\\ &= 26.6\,\text{lb/h HCl}\end{aligned}$$

Note that the mass flow rate of HCl leaving the spray tower equals the mass flow rate of HCl entering the packed column.

For the packed column:

$$\begin{aligned}\dot{m}_{\text{out}} &= (1 - 0.98)(26.6)\\ &= 0.532\,\text{lb/h HCl}\end{aligned}$$

The overall fractional efficiency is therefore

$$\begin{aligned}E &= (\dot{m}_{\text{in}} - \dot{m}_{\text{out}})/\dot{m}_{\text{in}}\\ &= (76.0 - 0.532)/76.0\\ &= 0.993 = 99.3\%\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 4.5

Consider the flow diagram in Figure 4.3 for a wastewater treatment system. The following flowrate data are given:

$$\begin{aligned}\dot{m}_1 &= 1000\,\text{lb/min}\\ \dot{m}_2 &= 1000\,\text{lb/min}\\ \dot{m}_4 &= 200\,\text{lb/min}\end{aligned}$$

Calculate the amount of water lost by evaporation in the operation, $\dot{m}$.

***SOLUTION*:** Apply a material balance around the treatment system to determine the value of $\dot{m}_5$. The value of $\dot{m}_5$ is given by:

$$\begin{aligned}\dot{m}_4 + \dot{m}_5 &= \dot{m}_3\\ \dot{m}_4 + \dot{m}_5 &= \dot{m}_1 + \dot{m}_2\\ 200 + \dot{m}_5 &= 1000 + 1000\\ \dot{m}_5 &= 1800\,\text{lb/min}\end{aligned}$$

Similarly (for tank 2),

$$\begin{aligned}\dot{m}_6 &= \dot{m}_2\\ \dot{m}_6 &= 1000\,\text{lb/min}\end{aligned}$$

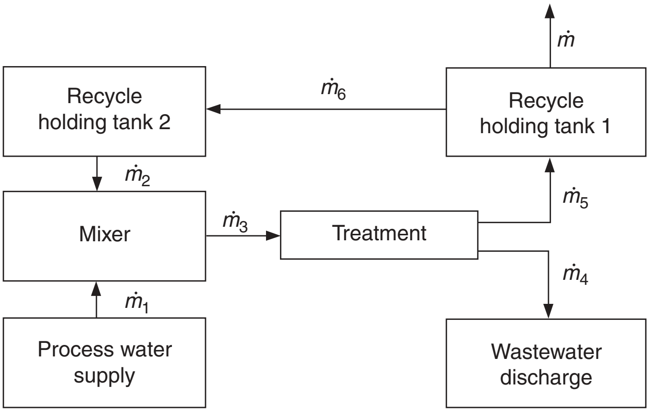

Figure 4.3 Flow diagram for Illustrative Example 4.5.

Thus (for tank 1),

$$\begin{aligned} \dot{m}_5 + \dot{m} &= \dot{m}_6 \\ 1800 - \dot{m} &= 1000 \\ \dot{m} &= 800\,\text{lb/min} \end{aligned}$$

Therefore, 800 lb of water per minute is lost in the operation. ■

ILLUSTRATIVE EXAMPLE 4.6

Consider the system shown in Figure 4.4. The following volumetric flowrate and phosphate concentration (volume basis) data have been provided by the plant manager. Are the data correct and/or consistent?

$$\begin{array}{ll} q_1 = 1000 \text{ gal/day} & C_1 = 4 \text{ ppm} \\ q_2 = 1000 \text{ gal/day} & C_2 = 0 \text{ ppm} \\ q_3 = 2000 \text{ gal/day} & C_3 = 2 \text{ ppm} \\ q_4 = 200 \text{ gal/day} & C_4 = 20 \text{ ppm} \\ q_5 = 1800 \text{ gal/day} & C_5 = 0 \text{ ppm} \\ q_6 = 1000 \text{ gal/day} & C_6 = 0 \text{ ppm} \end{array}$$

***SOLUTION*:** A componential balance around the mixer (in lb/day) gives (with the conversion factor for water of 120,000 gal/10^6 lb)

$$\begin{aligned} C_1q_1 + C_2q_2 &= C_3q_3 \\ (4)(1000/120{,}000) + (0)(1000/120{,}000) &= (2)(2000/120{,}000) \\ 4000 &= 4000 \qquad \text{OK} \end{aligned}$$

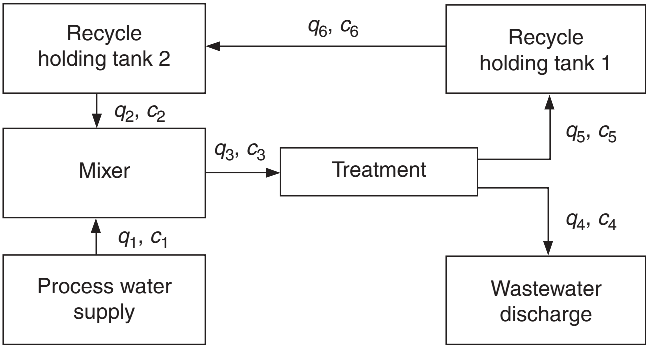

Figure 4.4 Flow diagram for Illustrative Example 4.6.

A balance around the treatment tank gives

$$C_3q_3 = C_4q_4 + C_5q_5$$
$$(2)(2000/120{,}000) = (20)(200/120{,}000) + (0)(1800/120{,}000)$$
$$4000 = 4000 \qquad \text{OK}$$

A balance around holding tank 1 gives

$$C_5q_5 = C_6q_6$$
$$(0)(1800) = (0)(1000)$$
$$0 = 0 \qquad \text{OK}$$

A balance around holding tank 2 gives

$$C_2q_2 = C_6q_6$$
$$(0)(1000) = (0)(1000)$$
$$0 = 0 \qquad \text{OK}$$

The data appear to be consistent. ■

THE CONSERVATION LAW FOR ENERGY

From the early recognition of energy, man has studied its effects upon objects and its transfer from object to object. This field of study is an integral part of heat transfer. Before proceeding to the first law of thermodynamics, often referred to as the conservation law for energy, certain important terms are defined as follows:

1. *Isothermal* is constant temperature.
2. *Isobaric* is constant pressure.
3. *Isochoric* is constant volume.
4. *Adiabatic* specifies no transfer of heat to or from system(s).

The first law of thermodynamics specifies that energy is conserved. In effect, this law states that energy is neither created nor destroyed. Thus, the change in energy of a system is exactly equal to the negative of the change in the surroundings. For a system of constant mass (a closed system), the only way the system and surroundings may interchange energy is by work and heat. Work and heat are defined as energy in transit. They are not properties and cannot be stored in a system. Two common forms of work are expansion and electrical. Heat is energy in transit because of a temperature difference. This heat transfer may take place by conduction, convection, or radiation, topics to be discussed in Part II.

The energy balance makes use of the conservation law to account for all the energy in a chemical process, or in any other process for that matter. After a system is defined, the energy balance considers the energy entering the system across the boundary, the energy leaving the system across the boundary, and the accumulation of energy within the system. This may be written in a simplified equation form as

$$\text{Energy in} - \text{Energy out} = \text{Energy accumulation} \tag{4.7}$$

This expression has the same form as the general law of conservation of mass as well as the conservation law for momentum. It may also be written on a time rate basis.

All forms of energy must be included in an energy balance. In many processes, certain energy forms remain constant and changes in them may be neglected. However, these forms should be recognized and understood before their magnitude and constancy can be determined. Some forms of energy are easily recognized in everyday life: the energy of a moving object, the energy given off by a fire, and the energy content of a container of hot water. Other forms of energy are less easily recognized. However, the five key energy terms are kinetic, potential, internal, heat, and work. These are briefly described below.

1. *Kinetic energy* (also defined in Chapter 3). The energy of a moving object is called "kinetic energy." A baseball thrown by a pitcher possesses a definite kinetic energy as it travels toward the catcher. A pound of flowing fluid possesses kinetic energy as it travels through a duct.
2. *Potential energy* (also defined in Chapter 3). The energy possessed by a mass by virtue of its position in the Earth's gravitational field is called "potential energy." A boulder lying at the top of a cliff possesses potential energy with reference to the bottom of the cliff. If the boulder is pushed off the cliff, its potential energy is transformed into kinetic energy as it falls. Similarly, a mass of fluid in a flowing system possesses a potential energy because of its height above an arbitrary reference level.
3. *Internal energy*. The component molecules of a substance are constantly moving within the substance. This motion imparts "internal energy" to the material. The molecules may rotate, vibrate, or migrate within the substance. The addition of heat to a material increases its molecular activity and, hence, its internal energy. The temperature of a material is a measure of its internal energy.

4. *Heat.* As noted above, when energy if transferred between a system and its surroundings, it is transferred either as work or as heat. Thus, heat is energy in transit. This type of energy transfer occurs whenever a hot body is brought into contact with a cold body. Energy flows as heat from the hot body to the cold body until the temperature difference is dissipated—that is, until thermal equilibrium is established. For this reason, heat may be considered as energy being transferred due to a temperature difference.
5. *Work.* Work is also energy in transit. Work is done whenever a force acts through a distance.

The first law of thermodynamics may be stated formally—as opposed to equation form—in many ways. One of these is as follows: although energy assumes many forms, the total quantity of energy is constant, and when energy disappears in one form, it must appear simultaneously in other forms.

As noted earlier, application of the conservation law for energy gives rise to the first law of thermodynamics. This law, in steady-state equation form for batch and flow processes, is presented below.

For *batch* processes:

$$\Delta U = Q + W \tag{4.8}$$

For *flow* processes:

$$\Delta H = Q + W_s \tag{4.9}$$

where potential, kinetic, and other energy effects have been neglected and

Q = energy in the form of heat transferred across the boundaries of the system

W = energy in the form of work transferred across the boundaries of the system

W_s = energy in the form of mechanical work transferred across the boundaries of the system

U = internal energy of the system

H = enthalpy of the system (defined below)

ΔU, ΔH = changes in the internal energy and enthalpy, respectively, during the process.

Contrary to an earlier convention, both Q and W (or W_s) are considered positive if added/transferred *to* the system. Also note that for a flow process.

$$\Delta H = Q \tag{4.10}$$

if $W_s = 0$.

The internal energy and enthalpy in Equations (4.8) and (4.9), as well as the other equations in this section, may be on a *mass* basis (i.e., for 1 g or 1 lb of material), on a *mole* basis (i.e., for 1 gmol or 1 lbmol of material), or represent the total internal energy and enthalpy of the entire system. As long as these equations are dimensionally consistent, it makes no difference. For the sake of clarity, the same convention that is used for heat capacities will be employed throughout this text—uppercase letters (e.g., H, U, C_P) represent properties on a mole basis, while lowercase letters (e.g., h, u, c_p) represent properties on a mass basis. Properties for the entire system will rarely be used and therefore require no special symbols.

Perhaps the most important thermodynamic function that the engineer works with is the aforementioned *enthalpy*. The enthalpy is defined by:

$$H = U + PV \tag{4.11}$$

where P = pressure of the system

V = volume of the system

ILLUSTRATIVE EXAMPLE 4.7

A lake is located at the top of a mountain. A power plant has been constructed at the bottom of the mountain. The potential energy of the water traveling downhill can be used to spin turbines and generate electricity. This is the operating mode in the daytime during peak electrical demand. At night, when demand is reduced, the water is pumped back up the mountain. The operation is shown in Figure 4.5.

Using the method of power "production" described above, determine how much power (Watts) is generated by the lake located at an elevation of 3000 ft above the power plant. The flowrate of water is 500,000 gpm. The turbine efficiency is 30%. Neglect friction effects.

Note: This programmed-instructional problem is a modified and edited version (with permission) of an illustrative example prepared by Marie Gillman, a graduate mechanical engineering student at Manhattan College.

***SOLUTION*:** First, convert height and flowrate to SI units in order to solve for the power in Watts:

$$(3000\ \text{ft})(0.3048\ \text{m/ft}) = 914.4\ \text{m}$$

$$(500{,}000\ \text{gal/min})(0.00378\ \text{m}^3/\text{gal}) = 1890\ \text{m}^3/\text{min}$$

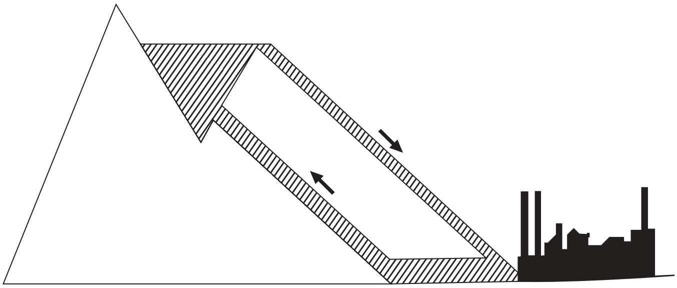

Figure 4.5 Schematic for Illustrative Example 4.7.

The mass flow rate of the water in kilograms/second is

$$\frac{(1890\,\text{m}^3/\text{min})(1000\,\text{kg/m}^3)}{60\,\text{s/min}} = 31{,}500\,\text{kg/s}$$

The loss in potential energy, ΔPE, of the water flow is given by

$$\Delta\text{PE} = \frac{mg\Delta z}{g_c}$$

Substituting yields

$$\begin{aligned}\Delta\text{PE} &= (31{,}500\,\text{kg/s})(9.8\,\text{m/s}^2)(914.4\,\text{m})\\ &= 2.82 \times 10^8\,\text{kg}\cdot\text{m/s}^3\\ &= 2.82 \times 10^8\,\text{N/s}\\ &= 282\ \text{MW}\end{aligned}$$

Note that $g_c = 1$ in the SI system of units.

Assuming that the potential energy decrease is entirely converted to energy input to the turbine, the actual power output AP is

$$\text{AP} = (0.30)(282) = 84.6\,\text{MW}$$

This is enough power for a small town. No pollutants or greenhouse gases are generated because no fossil fuel is required. The initial construction expense could be quite high, but the long-term cost of producing electricity might be economical. ■

The flow of heat from a hot fluid to a cooler fluid through a solid wall is a situation often encountered in engineering equipment; examples of such equipment (to be discussed in Part III) are heat exchangers, condensers, evaporators, boilers, and economizers. The heat absorbed by the cool fluid or given up by the hot fluid may be sensible heat, causing a temperature change in the fluid, or it may be latent heat, causing a phase change such as vaporization or condensation. In a typical heat exchanger (for example, a waste heat boiler) hot flue gas gives up heat to water through thin metal tube walls separating the two fluids. As the flue gas loses heat, its temperature drops. As the water gains heat, its temperature quickly reaches the boiling point where it continues to absorb heat with no further temperature rise as it changes into steam. The rate of heat transfer between the two streams, assuming no heat loss due to the surroundings, may be calculated by the enthalpy change of either fluid:

$$\dot{Q} = \dot{m}_h(h_{h1} - h_{h2}) = \dot{m}_c(h_{c1} - h_{c2}) \tag{4.12}$$

where $\dot{Q}$ is the rate of heat flow (Btu/h); $\dot{m}_h$, mass flow rate of hot fluid (lb/h); $\dot{m}_c$, mass flow rate of hot fluid (lb/h); h_{h1}, enthalpy of entering hot fluid (Btu/lb); h_{h2}, enthalpy of exiting hot fluid (Btu/lb); h_{c1}, enthalpy of entering cold fluid (Btu/lb); h_{c2}, enthalpy of exiting cold fluid (Btu/lb). The reader should also note that upper and lower case letters are employed in the text to represent the hot fluid, i.e., *H* and *h* respectively; both *C* and *c* are employed for the cold fluid.

Equation (4.12) is applicable to the heat exchange between two fluids whether a phase change is involved or not. In the aforementioned waste heat boiler example, the enthalpy change of the flue gas is calculated from its sensible temperature change:

$$\dot{Q} = \dot{m}_h(h_{h1} - h_{h2}) = \dot{m}_h c_{ph}(T_{h1} - T_{h2}) \tag{4.13}$$

where c_{ph} is the heat capacity of the hot fluid (Btu/lb·°F); T_{h1}, temperature of the entering hot fluid (°F); and T_{h2}, temperature of the existing hot fluid (°F). The enthalpy change of the water, on the other hand, involves a small amount of sensible heat to bring the water to its boiling point plus a considerable amount of latent heat to vaporize the water. Assuming all of the water is vaporized and no superheating of the steam occurs, the enthalpy change is

$$\dot{Q} = \dot{m}_c(h_{c2} - h_{c1}) = \dot{m}_c c_{pc}(T_{c2} - T_{c1}) + \dot{m}_c \Delta h_{\text{vap}} \tag{4.14}$$

where c_{pc} is the heat capacity of the cold liquid water (Btu/lb·°F), T_{c1}, temperature of the entering liquid water (°F), T_{c2}, temperature of the exiting steam (°F), and Δh_{vap}, heat of vaporization at T_{c2} of the water (Btu/lb).$^{(4)}$

ILLUSTRATIVE EXAMPLE 4.8

If 111.4 lbmol/min of an air stream is heated from 200°F to 600°F, calculate the heat transfer rate required to bring about this change in temperature. Use the following enthalpy and average heat capacity data:

$$H_{200^\circ\text{F}} = 1170\ \text{Btu/lbmol}$$
$$H_{600^\circ\text{F}} = 4010\ \text{Btu/lbmol}$$

***SOLUTION*:** Calculate the heat transfer rate, $\dot{Q}$, using enthalpy data:

$$\dot{Q} = \dot{n}\Delta H = 111.4(4010 - 1170) = 3.16 \times 10^5\ \text{Btu/min}$$

The heat transfer rate could also be calculated using the average heat capacity data. This is addressed in the next example. ■

ILLUSTRATIVE EXAMPLE 4.9

Given the mass flow rate of a fluid and its heat capacity, determine the required heat rate to change the fluid from one temperature to another. Data are provided below:

$$\dot{n} = 600\ \text{lbmol/min}$$
$$\bar{C}_{P,\text{AV}} = 0.271\ \text{Btu/lbmol}\cdot{}^\circ\text{F (over the 200–600}^\circ\text{F range)}$$
$$T_1 = 200^\circ\text{F}$$
$$T_2 = 600^\circ\text{F}$$

***SOLUTION*:** Write the equation describing the required heat rate, $\dot{Q}$, and solve:

$$\dot{Q} = \dot{n}\bar{C}_{P,\text{AV}}\Delta T = (600)(0.271)(600 - 200) = 65{,}000\ \text{Btu/min}$$ ■

ILLUSTRATIVE EXAMPLE 4.10

Obtain the heat transfer rate in an exchanger if equal mass liquid flow rates are used. The hot fluid is cooled from 94°C to 82°C while the cold fluid is initially at 20°C.

***SOLUTION*:** The final temperature of the cold liquid can be found from direct application of Equation (4.12):

$$\dot{m}_c c_{pc}(T_{c,2} - T_{c,1}) = \dot{m}_h c_{ph}(T_{h,2} - T_{h,1})$$
$$\dot{m}_c c_{pc}(T_{c,2} - 20) = \dot{m}_h c_{ph}(94 - 82)$$

Since

$$\dot{m}_c = \dot{m}_h$$
$$c_{ph} = c_{pc}$$

one can conclude that the temperature change for both liquids are equal. This yields

$$T_{c,2} = 32^\circ\mathrm{F}$$

This and the two previous examples will be revisited in Chapter 14. ■

ILLUSTRATIVE EXAMPLE 4.11

As a gas flows through a cooler, 5.5 MW of heat is transferred from the gas. The average heat capacity of the gas is 1090 J/(kg · °C), the gas mass flow rate, $\dot{m}$, is 9 kg/s and the gas inlet temperature, T_1, is 650°C. For this example, kinetic and potential energy effects are again neglected. Furthermore, there is no shaft work. Determine the gas outlet temperature.

***SOLUTION*:** Since there are no kinetic, potential, or shaft work effects in this flow process, Equation (4.13) applies

$$\dot{Q} = \Delta H$$

where

$$\dot{Q} = \Delta H = \dot{m}\bar{c}_p\,\Delta T = \dot{m}\bar{c}_p(T_2 - T_1)$$

Solving for the gas outlet temperature, T_2,

$$T_2 = \frac{\dot{Q}}{\dot{m}\bar{c}_p} + T_1 = \frac{-5.5 \times 10^{-6}}{9(1090)} + 650 = 89^\circ\mathrm{C}$$

Note that the sign of $\dot{Q}$ is negative since the heat is transferred out from the gas. ■

ILLUSTRATIVE EXAMPLE 4.12

A heat pump takes in 3500 gpm of water at a temperature of 38°F and discharges it back to the lake at 36.2°F. How many Btu are removed from the water per day [C_P for $H_2O = 75.4$ J/(gmol · °C), $\rho = 62.4$ lb/ft^3]?

***SOLUTION*:** The following equation is employed to calculate the heat load:

$$\dot{Q} = \dot{m}c_p(T_2 - T_1) \tag{4.13}$$

where

$$\dot{m} = \frac{(3500\ \text{gal/min})(62.4\ \text{lb/ft}^3)(1440\ \text{min/day})}{7.48\ \text{gal/ft}^3}$$
$$= 4.20 \times 10^7\ \text{lb/day}$$

The heat capacity is converted into consistent units and placed on a mass bases as follows:

$$c_p = \frac{[75.4\ \text{J/(gmol}\cdot{}^\circ\text{C)}](454\ \text{g/lb})}{(1054\ \text{J/Btu})(18\ \text{g/gmol})(1.8^\circ\text{F}/^\circ\text{C})} = 1.00\ \text{Btu/(lb}\cdot{}^\circ\text{F)}$$

Therefore,

$$\dot{Q} = (4.20 \times 10^7\ \text{lb/day})[1.0\ \text{Btu/(lb}\cdot{}^\circ\text{F)}](38 - 36.2^\circ\text{F})$$
$$= 1.36 \times 10^8\ \text{Btu/day}$$

■

ILLUSTRATIVE EXAMPLE 4.13

Determine the percentage of a river stream's flow available to an industry for cooling in order that the river temperature does not increase more than 10°F. Fifty percent of the industrial withdrawal is lost by evaporation and the industrial water returned to the river is 60°F warmer than the river.

Note: This problem is a modified and edited version (with permission) of an illustrative example prepared by Ms. Marie Gillman, a graduate mechanical engineering student at Manhattan College.

***SOLUTION*:** Draw a flow diagram representing the process as shown in Figure 4.6. Express the volumetric flow lost by evaporation from the process in terms of that entering the process:

$$q_{\text{lost}} = 0.5q_{\text{in}}$$

Express the process outlet temperature and the maximum river temperature in terms of the upstream temperature:

$$T_{\text{out}} = T_{\text{up}} + 60^\circ\text{F}$$
$$T_{\text{mix}} = T_{\text{up}} + 10^\circ\text{F}$$

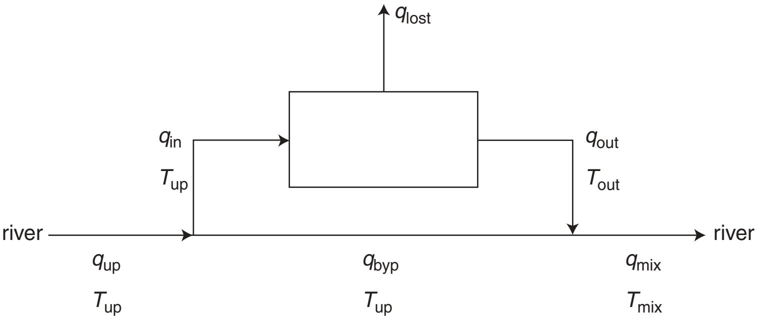

Figure 4.6 Flow diagram for Illustrative Example 4.13.

Using the conservation law for mass, express the process outlet volumetric flow in terms of the process inlet flow. Also express the flow bypassing the process in terms of the upstream and process inlet flows:

$$q_{out} = 0.5q_{in}$$
$$q_{byp} = q_{up} - q_{in}$$
$$q_{mix} = q_{up} - 0.5q_{in}$$

The flow diagram with the expressions developed above are shown in Figure 4.7.

Noting that the enthalpy of any stream can be represented by $qc_p\rho(T - T_{ref})$. An energy balance around the downstream mixing point leads to

$$(q_{up}-q_{in})c_p\rho(T_{up} - T_{ref}) + 0.5q_{in}c_p\rho(T_{up} + 60 - T_{ref}) = (q_{up} - 0.5q_{in})c_p\rho(T_{up} + 10 - T_{ref})$$

Note that T_{ref} is arbitrary and indirectly defines a basis for the enthalpy. Setting $T_{ref} = 0$ and assuming that density and heat capacity are constant yields

$$(q_{up} - q_{in})T_{up} + 0.5q_{in}(T_{up} + 60) = (q_{up} - 0.5q_{in})(T_{up} + 10)$$

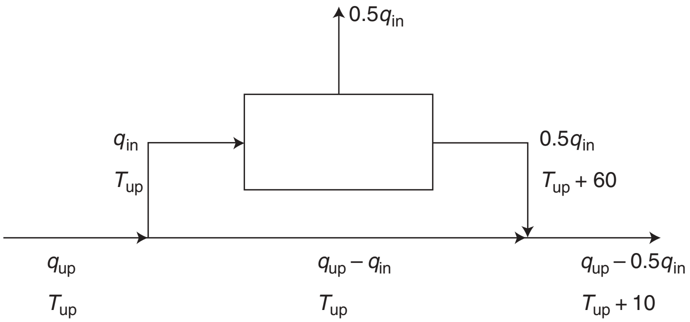

Figure 4.7 Flow diagram after applying mass balances.

The equation may now be solved for the inlet volumetric flow to the process in terms of the upstream flow:

$$\begin{aligned} q_{\text{up}}T_{\text{up}} - q_{\text{in}}T_{\text{up}} + 0.5q_{\text{in}}T_{\text{up}} + 30q_{\text{in}} \\ = q_{\text{up}}T_{\text{up}} + 10q_{\text{up}} - 0.5q_{\text{in}}T_{\text{up}} - 5q_{\text{in}} \end{aligned}$$

Canceling terms produces

$$\begin{aligned} 35q_{\text{in}} &= 10q_{\text{up}} \\ q_{\text{in}} &= 0.286q_{\text{up}} \end{aligned}$$

Therefore, 28.6% of the original flow, q_{up}, is available for cooling.

Note that the problem can also be solved by setting $T_{\text{ref}} = T_{\text{up}}$. Since for this condition, $T_{\text{ref}} - T_{\text{up}} = 0$, the solution to the problem is greatly simplified. ■

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Chapter 5

Gas Laws

INTRODUCTION

Observations based on physical experimentation often can be synthesized into simple mathematical equations called *laws*. These laws are never perfect and hence are only an approximate representation of reality. There is no universal all-purpose equation of state to describe the PVT (pressure–volume–temperature) behavior of a pure fluid. (An equation of state is an equation that relates pressure, volume, and temperature for any pure fluid.) Since most equations of state deal with gases, this chapter primarily highlights this phase. It should also be noted that liquids and solids exhibit little PVT variation in heat transfer applications; understandably, engineers and scientists are therefore rarely concerned with the PVT behavior of either of these two phases.

The *ideal gas law* (IGL) was derived from experiments in which the effects of pressure and temperature on gaseous volumes were measured over moderate temperature and pressure ranges. This law works well in the pressure and temperature ranges that were used in taking the data; extrapolations outside of the ranges have been found to work well in some cases and poorly in others. As a general rule, this law works best when the molecules of the gas are far apart (i.e., when the pressure is low and the temperature is high). Under these conditions, the gas is said to behave *ideally* and its behavior is a close approximation to the so-called *perfect* or *ideal gas*, a hypothetical entity that obeys the ideal gas law exactly. For engineering calculations, the ideal gas law is almost always assumed to be valid since it generally works well (usually within a few percent of the correct result) up to the highest pressures and down to the lowest temperatures used in most heat transfer applications. Nonetheless, the chapter concludes with comments on non-ideal behavior.

Five sections complement the presentation for this chapter. Topics include:

Boyle's and Charles' Laws
The Ideal Gas Law
Standard Conditions
Partial Pressure and Partial Volume
Non-Ideal Gas Behavior

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BOYLE'S AND CHARLES' LAWS

The two precursors of the ideal gas law were *Boyle's* and *Charles'* laws. Boyle found that the volume V of a given mass of gas is inversely proportional to the *absolute* pressure if the temperature is kept constant:

$$P_1V_1 = P_2V_2 \tag{5.1}$$

where $V_1 =$ volume of gas at absolute pressure P_1 and temperature T
$V_2 =$ volume of gas at absolute pressure P_2 and temperature T

Charles found that the volume of a given mass of gas varies directly with the *absolute* temperature at constant pressure:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \tag{5.2}$$

where $V_1 =$ volume of gas at pressure P and absolute temperature T_1
$V_2 =$ volume of gas at pressure P and absolute temperature T_2

Boyle's and Charles' laws may be combined into a single equation in which neither temperature nor pressure need be held constant:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \tag{5.3}$$

For Equation (5.3) to hold, the mass of gas must be constant as the conditions change from (P_1, T_1) to (P_2, T_2). This equation indicates that for a given mass of a specific gas, PV/T has a constant value. Since, at the same temperature and pressure, volume and mass must be directly proportional, this statement may be extended to

$$\frac{PV}{mT} = C \tag{5.4}$$

where $m =$ mass of a specific gas
$C =$ constant that depends on the gas

Note that volume terms may be replaced by volume rate (or volumetric flow rate), q.

ILLUSTRATIVE EXAMPLE 5.1

What is the final (f) volumetric flow rate of a gas that is heated at constant pressure in a heat exchanger from 100 to 300°F if its initial (i) flow is 3500 actual cubic feet per minute (acfm).

***SOLUTION*:** Apply Charles' law on a volume rate basis. See Equation (5.2) and be sure to employ absolute temperature units:

$$\begin{aligned} q_f &= q_i(T_f/T_i) \\ &= 3500[(300+460)/(100+460)] \\ &= 4750\,\text{acfm} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 5.2

What is the volumetric flow rate of the gas (100°F, 1 atm) in the previous example if it is compressed isothermally (constant temperature) to 3 atm?

***SOLUTION*:** Apply Boyle's law. See Equation (5.1),

$$\begin{aligned} q_f &= q_i(P_i/P_f) \\ &= 3500(1.0/3.0) \\ &= 1167\,\text{acfm} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 5.3

What is the volumetric flow rate of the gas in the previous example if the final temperature is 300°F.

***SOLUTION*:** Using the combined gas law provided in Equation (5.3),

$$\begin{aligned} q_f &= q_i(P_i/P_f)(T_f/T_c) \\ &= 3500(1/3)(760/560) = 1167(760/560) \\ &= 1583\,\text{acfm} \end{aligned}$$

■

THE IDEAL GAS LAW

Experiments with different gases showed that Equation (5.4) could be expressed in a far more generalized form. If the number of moles (n) is used in place of the mass (m), the constant is the same for all gases:

$$\frac{PV}{nT} = R \tag{5.5}$$

where R = universal gas constant.

Equation (5.5) is called the ideal gas law. Numerically, the value of R depends on the units used for P, V, T, and n (see Table 5.1). In this text, heat transfer applications involving gases are generally assumed to approximate ideal gas behavior. As is usually the case in engineering practice, unless otherwise stated the ideal gas law is assumed to

Table 5.1 Values of R in Various Units

R	Temperature scale	Units of V	Units of n	Units of P	Unit of PV (energy)
10.73	°R	ft^3	lbmol	psia	–
0.7302	°R	ft^3	lbmol	atm	–
21.85	°R	ft^3	lbmol	in Hg	–
555.0	°R	ft^3	lbmol	mm Hg	–
297.0	°R	ft^3	lbmol	in H_2O	–
0.7398	°R	ft^3	lbmol	bar	–
1545.0	°R	ft^3	lbmol	psfa	–
24.75	°R	ft^3	lbmol	ft H_2O	–
1.9872	°R	–	lbmol	–	Btu
0.0007805	°R	–	lbmol	–	hp · h
0.0005819	°R	–	lbmol	–	kW · h
500.7	°R	–	lbmol	–	cal
1.314	K	ft^3	lbmol	atm	–
998.9	K	ft^3	lbmol	mm Hg	–
19.32	K	ft^3	lbmol	psia	–
62.361	K	L	gmol	mm Hg	–
0.08205	K	L	gmol	atm	–
0.08314	K	L	gmol	bar	–
8314	K	L	gmol	Pa	–
8.314	K	m^3	gmol	Pa	–
82.057	K	cm^3	gmol	atm	–
1.9872	K	–	gmol	–	cal
8.3144	K	–	gmol	–	J

be valid for all illustrative examples and assigned problems. If a case is encountered in practice where the gas behaves in a very non-ideal fashion (e.g., a high-molecular-weight gas, such as a chlorinated organic under high pressure), one of the many *real gas* correlations available in the literature should be used.(1–3)

Other useful forms of the ideal gas law are shown in Equations (5.6) and (5.7). Equation (5.6) applies to a gas volume flow rate rather than to a gas volume confined in a container:(4)

$$Pq = \dot{n}RT \tag{5.6}$$

where q = gas volumetric flow rate (ft^3/h)
P = absolute pressure (psia)
$\dot{n}$ = molar flow rate (lbmol/h)
T = absolute temperature (°R)
$R = 10.73\ psia \cdot ft^3/lbmol \cdot {}^\circ R$

Equation (5.7) combines n and V from Equation (5.5) to express the law in terms of density:

$$P(MW) = \rho RT \tag{5.7}$$

where $MW =$ molecular weight of gas (lb/lbmol)
$\rho =$ density of gas (lb/ft^3)

ILLUSTRATIVE EXAMPLE 5.4

What is the density of air at 75°F and 14.7 psia? The molecular weight of air is approximately 29.

***SOLUTION*:** This example is solved using the ideal gas law:

$$PV = nRT = \left(\frac{m}{MW}\right)RT \tag{5.5}$$

For the density, apply Equation (5.7).

$$\begin{aligned}\rho = \frac{P(MW)}{RT} &= \frac{(14.7\ \text{psia})(29\ \text{lb/lbmol})}{(10.73\ \text{ft}^3 \cdot \text{psi/lbmol} \cdot {}^\circ\text{R})(75 + 460)} \\ &= 0.0743\ \text{lb/ft}^3\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 5.5

Calculate the volume (in ft^3) of 1.0 lbmol of any ideal gas at 60°F and 14.7 psia.

***SOLUTION*:** Solve the ideal gas law for V and calculate the volume:

$$\begin{aligned}V = \frac{nRT}{P} &= \frac{(1)(10.73)(60 + 460)}{14.7} \\ &= 379\ \text{ft}^3\end{aligned}$$

This result is an important number to remember in many heat transfer calculations—1 lbmol of any (ideal) gas at 60°F and 1 atm occupies 379 ft^3. ■

ILLUSTRATIVE EXAMPLE 5.6

Calculate the density of a gas ($MW = 29$) in g/cm^3 at 20°C and 1.2 atm using the ideal gas law.

***SOLUTION*:** Calculate the density of the gas again using the ideal gas law:

$$PV = nRT = \left(\frac{m}{MW}\right)RT \tag{5.5}$$

$$\frac{m}{V} = \rho = \frac{P(MW)}{RT}\frac{(1.2)(29)}{(82.06)(20 + 273)}$$

$$= 0.00145\ \text{g/cm}^3$$

The effects of pressure, temperature, and molecular weight on density can be obtained directly from the ideal gas law equation. Increasing the pressure and molecular weight increases the density; increasing the temperature decreases the density. ■

ILLUSTRATIVE EXAMPLE 5.7

A certain pure-component two-element ideal gas has a specific volume v of 10.58 ft^3/lb at 70°F and 14.7 psia. Determine the molecular weight of the gas and state its name. (*Hint*: The gas is acidic and soluble in water.)

***SOLUTION*:** Rewrite the ideal gas law equation in terms of *MW*,

$$MW = \left(\frac{m}{V}\right)\frac{RT}{P} = \frac{RT}{vP};\ v = \text{specific volume, volume/mass} \tag{5.7}$$

Substituting,

$$MW = \frac{(10.73)(460 + 70)}{(10.58)(14.7)}$$

$$= 36.56\ \text{lb/lbmol}$$

It appears that the gas is HCl (i.e., hydrogen chloride). ■

STANDARD CONDITIONS

Volumetric flow rates are often not given at the actual conditions of pressure and temperature but rather at arbitrarily chosen standard conditions (*STP, standard temperature and pressure*). To distinguish between flow rates based on the two conditions, the letters *a* and *s* are often used as part of the units. The units acfm and scfm stand for actual cubic feet per minute and standard cubic feet per minute, respectively. The ideal gas law can be used to convert from *standard* to *actual* conditions, but since there are many standard conditions in use, the STP being used must be known or specified. Standard conditions most often used are shown in Table 5.2. The reader is cautioned on the incorrect use of acfm and/or scfm. The use of standard conditions is a convenience; actual conditions *must* be employed when predicting the performance of or designing heat transfer equipment. Designs based on standard conditions can lead to disastrous results, with the unit usually underdesigned. For example, for

Table 5.2 Common Standard Conditions

System	Temperature	Pressure	Molar volume
SI	273 K	101.3 kPa	22.4 m^3/kmol
Universal scientific	0°C	760 mm Hg	22.4 L/gmol
Natural gas industry	60°F	14.7 psia	379 ft^3/lbmol
American engineering	32°F	1 atm	359 ft^3/lbmol
Hazardous waste incineration industry	60°F	1 atm	379 ft^3/lbmol
	70°F	1 atm	387 ft^3/lbmol

a flue gas stream at 2140°F, the ratio of acfm to scfm (standard temperature = 60°F) for a thermal application is 5.0.

Equation (5.8), which is a form of Charles' law, can be used to convert flow rates from standard to actual conditions:

$$q_a = q_s\left(\frac{T_a}{T_s}\right) \tag{5.8}$$

where q_a = volumetric flow rate at actual conditions (ft^3/h)
q_s = volumetric flow rate at standard conditions (ft^3/h)
T_a = actual absolute temperature (°R)
T_s = standard absolute temperature (°R)

The reader is again reminded that absolute temperatures and pressures must be employed in all ideal gas law calculations.

ILLUSTRATIVE EXAMPLE 5.8

Data from a heat exchanger indicate a volumetric flow rate of 30,000 scfm (60°F, 1 atm). If the operating temperature and pressure of the unit are 1100°F and 1 atm, respectively, calculate the flow rate in actual cubic feet per minute (acfm).

SOLUTION**:** Since the pressure remains constant, the standard cubic feet per minute using Charles' law is calculated as:

$$q_a = q_s\left(\frac{T_a}{T_s}\right) \tag{5.8}$$

Substituting,

$$\begin{aligned} q_a &= 30{,}000\left(\frac{1100 + 460}{60 + 460}\right) \\ &= 90{,}000 \text{ acfm} \end{aligned}$$

The reader is again cautioned on the use of acfm and/or scfm. Predicting the performance of and designing heat exchangers should always be based on *actual* conditions. ■

ILLUSTRATIVE EXAMPLE 5.9

The exhaust gas flow rate from a condenser is 1000 scfm. All of the gas is vented through a small stack that has an inlet area of 2.0 ft^2. The exhaust gas temperature is 300°F. What is the velocity of the gas through the stack inlet in feet per second? Assume standard conditions to be 70°F and 1.0 atm. Neglect the pressure drop across the stack.

***SOLUTION*:** The actual flow rate, in acfm, using Charles' law is calculated as:

$$q_a = q_s\left(\frac{T_a}{T_s}\right) \tag{5.8}$$

Substituting,

$$= 1000\left(\frac{460 + 300}{460 + 70}\right)$$

$$= 1434 \text{ acfm}$$

Note that since the gas from the condenser is vented through the stack to the atmosphere, the pressure is 1.0 atm. The velocity v of the gas is calculated as follows:

$$v = \frac{q_a}{A} = \frac{1434}{2.0}$$

$$= 717 \text{ ft/min} = 12 \text{ ft/s}$$

■

ILLUSTRATIVE EXAMPLE 5.10

C_6H_5Cl is fed into a thermal oxidizer at a rate of 5000 scfm (60°F, 1 atm) and is combusted in the presence of air fed at a rate of 3000 scfm (60°F, 1 atm). Both streams enter the oxidizer at 70°F. The products are then cooled from 2000°F and exit the cooler at 180°F. At what rate (lb/h) do the products exit the cooler? The molecular weight of C_6H_5Cl is 112.5; the molecular weight of air is 29.

***SOLUTION*:** First convert the scfm feed to acfm using Charles' law:

$$5000 \text{ scfm}\left(\frac{460 + 70}{460 + 60}\right) = 5096 \text{ acfm of } C_6H_5Cl$$

$$3000 \text{ scfm}\left(\frac{460 + 70}{460 + 60}\right) = 3058 \text{ acfm of air}$$

One lbmol of any ideal gas occupies 387 ft^3 at 70°F and 1 atm (see Table 5.2). Therefore, the molar flow rate ($\dot{n}$) may be calculated by dividing these results by 387:

$$\dot{n}(C_6H_5Cl) = \frac{5096}{387}$$

$$= 13.17 \text{ lbmol/min}$$

$$\dot{n}(\text{air}) = \frac{3058}{387}$$

$$= 7.90 \text{ lbmol/min}$$

The mass flow rate is obtained by multiplying these results by the molecular weight

$$\begin{aligned}\dot{m}(C_6H_5Cl) &= (13.17)(112.5)(60)\\ &= 88{,}898\ \mathrm{lb/h}\\ \dot{m}(\mathrm{air}) &= (7.90)(29)(60)\\ &= 13{,}746\ \mathrm{lb/h}\end{aligned}$$

Since mass is conserved, $\dot{m}_{\mathrm{in}}$ is equal to $\dot{m}_{\mathrm{out}}$:

$$\dot{m}_{\mathrm{out,total}} = \dot{m}_{\mathrm{in,total}} = 88{,}898 + 13{,}746 = 102{,}644\ \mathrm{lb/h}$$ ■

PARTIAL PRESSURE AND PARTIAL VOLUME

Mixtures of gases are more often encountered than single or pure gases in engineering practice. The ideal gas law is based on the *number* of molecules present in the gas volume; the *kind* of molecules is not a significant factor, only the number. This law applies equally well to mixtures and pure gases alike. Dalton and Amagat both applied the ideal gas law to mixtures of gases. Since pressure is caused by gas molecules colliding with the walls of the container, it seems reasonable that the total pressure of a gas mixture is made up of pressure contributions due to each of the component gases. These pressure contributions are called *partial pressures*.

Dalton defined the partial pressure of a component as the pressure that would be exerted if the same mass of the component gas occupied the same total volume *alone* at the same temperature as the mixture. The sum of these partial pressures would then equal the total pressure:

$$P = p_A + p_B + p_C + \cdots + p_n = \sum_{i=1}^{n} p_i \tag{5.9}$$

where P = total pressure

n = number of components

p_i = partial pressure of component i.

Equation (5.9) is known as *Dalton's law*. Applying the ideal gas law to one component (A) only,

$$p_A V = n_A RT \tag{5.10}$$

where n_A = number of moles of component A.
Eliminating R, T, and V between Equation (5.5) and (5.10) yields

$$\frac{p_A}{P} = \frac{n_A}{n} = y_A$$

or

$$p_A = y_A P \tag{5.11}$$

where y_A = mole fraction of component A.

Amagat's law is similar to Dalton's. Instead of considering the total pressure to be made up of partial pressures, where each component occupies the total container volume, Amagat considered the total volume to be made up of partial volumes in which each component is at the total pressure. The definition of the *partial volume* is therefore the volume occupied by a component gas alone at the same temperature and pressure as the mixture. For this case:

$$V = V_A + V_B + V_C + \cdots + V_n = \sum_{i=1}^{n} V_i \tag{5.12}$$

Applying Equation (5.5), as before,

$$\frac{V_A}{V} = \frac{n_A}{n} = y_A \tag{5.13}$$

where V_A =partial volume of component A.

It is common in heat exchanger applications to describe low concentrations of corrosive components in gaseous mixtures in parts per million (ppm) by volume. Since partial volumes are proportional to mole fractions, it is only necessary to multiply the mole fraction of the component by 1 million (10^6) to obtain the concentration in parts per million. [For liquids and solids, parts per million (ppm) is also used to express concentration, although it is usually on a *mass* basis rather than a *volume* basis. The terms ppmv and ppmw are sometimes used to distinguish between the volume and mass bases, respectively.]

ILLUSTRATIVE EXAMPLE 5.11

The exhaust to the atmosphere from a thermal device has a SO_3 concentration expressed as 0.15 mm Hg partial pressure. Calculate the parts per million of SO_3 in the exhaust.

***SOLUTION*:** First calculate the mole fraction (y). By definition,

$$y_{SO_3} = p_{SO_3}/P \tag{5.11}$$

(Strictly speaking, this equation only applies to ideal gas mixtures.) Since the exhaust is discharged into the atmosphere, the atmospheric pressure (760 mm Hg) is the total pressure (P). Thus,

$$y_{SO_3} = (0.15)/(760) = 1.97 \times 10^{-4}$$

As noted above, partial pressures may be converted to ppm (parts per million) by multiplying by 10^6:

$$\begin{aligned} \text{ppm} &= (y_{SO_3})(10^6) = (1.97 \times 10^{-4})(10^6) \\ &= 197\ \text{ppm} \end{aligned}$$

Note: Since the concentration of a *gas* is involved, it is understood (unless otherwise specified) that ppm is on a volume basis. ■

NON-IDEAL GAS BEHAVIOR

As noted earlier, an *ideal* gas is a hypothetical entity that obeys the ideal gas law perfectly. But, in heat transfer applications, one almost always deals with *real* gases. Although most heat exchanger applications involving gases occur at conditions approaching ideal gas behavior, there are rare occasions when the deviation from ideality is significant. Detailed calculation procedures are available$^{(1,2,5)}$ to account for these deviations.

Although no real gas obeys the ideal gas law exactly, the "lighter" gases (hydrogen, oxygen, air, and so on) at ambient conditions approach ideal gas law behavior. The "heavier" gases such as sulfur dioxide and hydrocarbons, particularly at high pressures and low temperatures, deviate considerably from the ideal gas law. Despite these deviations, the ideal gas law is routinely used in not only heat transfer but also engineering calculations.

As noted earlier, deviations from ideality increase at higher pressures and lower temperatures, where the density increases. The aforementioned law does not describe the behavior of real gases in some cases because the fluid molecules themselves occupy a finite volume, and they exert forces of attraction and repulsion on each other.

Numerous attempts have been made to develop an all-purpose gas law. Although it is beyond the scope of this book to review these theories in any great detail, a brief outline and discussion is presented below.

One (and perhaps the most popular) approach to account for deviations from ideality is to include a "correction factor," Z, which is defined as the *compressibility coefficient* or *factor*,

$$PV = ZnRT \tag{5.13}$$

Note that Z approaches 1.0 as P approaches a vacuum. For an ideal gas, Z is exactly unity. This equation may also be written as

$$Pv = ZRT \tag{5.14}$$

where v is now the *specific* volume (not the total volume or velocity) with units of volume/mole. Details on calculating Z are available in the literature.$^{(1,2,5)}$

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Chapter 6

Heat Exchanger Pipes and Tubes

INTRODUCTION

This chapter is primarily concerned with providing information on pipes and tubes. Both serve as the “arteries” and “veins” that supply the fluids enabling energy transfer to occur in heat exchangers.

Many types of heat exchangers and heat exchanger applications involve the transfer of heat between and/or across a metallic surface and a fluid that is either being heated or cooled. The metallic surface in question usually takes the form of a cylindrical pipe or tube. Although the terms pipe and tube have come to mean different things to different people, the two are routinely used interchangeably. However, there are differences. In a relative sense, pipes are generally “heavier,” of shorter length, larger diameter, rougher internally, and can be threaded. The one common denominator is that both are almost always metallic. Some key properties of metals, many of which are employed in the making of pipes and tubes, are presented in Table 6.1.(1) Note that in addition to providing information on density (ρ), heat capacity (c_p), and thermal diffusivity (α) at 20°C, the table contains thermal conductivity (k) data over a wide temperature range.

Chapter contents include a separate treatment of pipes and tubing as well as sections on valves and fittings, non-circular conduits, and flow considerations.

PIPES

As noted in the Introduction, fluids are usually transported in pipes or tubes. Pipes are specified in terms of their diameter and wall thickness. The nominal diameters range from $\frac{1}{8}$ to 30 inches for steel pipe. Standard dimensions of steel pipe are provided in Table 6.2(2) and are known as IPS (iron pipe size) or NPS (nominal pipe size). The wall thickness of the pipe is indicated by the schedule number (SN), which can be approximated from

$$\mathrm{SN} = 1000(P/S) \tag{6.1}$$

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Table 6.1 Property Values for Metals

	Properties at 20°C				Thermal conductivity k, $\mathrm{W/m \cdot K}$ (or $\mathrm{W/m \cdot {}^\circ C}$)									
Metal	ρ, $\mathrm{kg/m^3}$	c_p, $\mathrm{kJ/kg \cdot K}$	k, $\mathrm{W/m \cdot K}$	α, $\mathrm{m^2/s}$ $(\times 10^5)$	−100°C −148°F	0°C 32°F	100°C 212°F	200°C 392°F	300°C 572°F	400°C 752°F	600°C 1112°F	800°C 1472°F	1000°C 1832°F	1200°C 2192°F
Aluminum:														
Pure	2707	0.896	204	8.418	215	202	206	215	228	249				
Al-Cu (Duralumin), 94–96% Al 3–5% Cu trace Mg	2787	0.883	164	6.676	126	159	182	194						
Al-Si (Silumin, copper-bearing), 86.5% Al 1% Cu	2659	0.867	137	5933	119	137	144	152	161					
Al-Si (Alusil), 78–80% Al 20–22% Si	2627	0.854	161	7.172	144	157	168	175	178					
Al-Mg-Si, 97% Al 1% Mg 1% Si 1% Mn	2707	0.892	177	7.311		175	189	204						
Lead	11,373	0.130	35	2.343	36.9	35.1	33.4	31.5	29.8					
Iron:														
Pure	7897	0.452	73	2.034	87	73	67	62	55	48	40	36	35	35
Wrought iron, 0.5% C	7849	0.46	59	1.626		59	57	52	48	45	36	33	33	33
Steel (C max ≈ 1.5%): Carbon steel,														
C ≈ 0.5%	7833	0.465	54	1.474		55	52	48	45	42	35	31	29	31
1.0%	7801	0.473	43	1.172		43	43	42	40	36	33	29	28	29
1.5%	7753	0.486	36	0.970		36	36	36	35	33	31	28	28	29

(*Continued*)

Table 6.1 *Continued*

	Properties at 20°C				Thermal conductivity k, W/m · K (or W/m · °C)									
Metal	ρ, kg/m^3	c_p, kJ/kg · K	k, W/m · K	α, m^2/s ($\times 10^5$)	−100°C −148°F	0°C 32°F	100°C 212°F	200°C 392°F	300°C 572°F	400°C 752°F	600°C 1112°F	800°C 1472°F	1000°C 1832°F	1200°C 2192°F
Nickel steel,														
Ni ≈ 0%	7897	0.452	73	2.206										
20%	7933	0.46	19	0.526										
40%	8169	0.46	10	0.279										
80%	8618	0.46	35	0.872										
Invar, 36% Ni	8137	0.46	10.7	0.286										
Chrome steel,														
Cr = 0%	7897	0.452	73	2.026	87	73	67	62	55	48	40	36	35	36
1%	7865	0.46	61	1.665		62	55	52	47	42	36	33	33	
5%	7833	0.46	40	1.110		40	38	36	36	33	29	29	29	
20%	7689	0.46	22	0.635		22	22	22	22	24	24	26	29	
Cr-Ni (chrome-nickel):														
15% Cr 10% Ni	7865	0.46	19	0.527										
18% Cr 8% Ni (V2A)	7817	0.46	16.3	0.444		16.3	17	17	19	19	22	27	31	
20% Cr 15% Ni	7833	0.46	15.1	0.415										
25% Cr 20% Ni	7865	0.46	12.8	0.361										
Tungsten steel,														
W = 0%	7897	0.452	73	2.206										
1%	7913	0.448	66	1.858										
5%	8073	0.435	54	1.525										
10%	8314	0.419	48	1.391										
Copper:														
Pure	8954	0.3831	386	11.234	407	386	379	374	369	363	353			
Aluminum bronze, 95% Cu 5% Al	8666	0.410	83	2.330										

(*Continued*)

Table 6.1 *Continued*

Metal	Properties at 20°C				Thermal conductivity k, W/m · K (or W/m · °C)									
	ρ, kg/m^3	c_p, kJ/kg · K	k, W/m · K	α, m^2/s (× 10^5)	−100°C −148°F	0°C 32°F	100°C 212°F	200°C 392°F	300°C 572°F	400°C 752°F	600°C 1112°F	800°C 1472°F	1000°C 1832°F	1200°C 2192°F
Bronze, 75% Cu 25% Sn	8666	0.343	26	0.859										
Red brass, 85% Cu 9% Sn 6% Zn	8714	0.385	61	1.804		59	71							
Brass, 70% Cu 30% Zn	8522	0.385	111	3.412	88		128	144	147	147				
German silver, 62% Cu 15% Ni 22% Zn	8618	0.394	24.9	0.733	19.2		31	40	45	48				
Constantan, 60% Cu 40% Ni	8922	0.410	22.7	0.612	21		22.2	26						
Magnesium:														
Pure	1746	1.013	171	9.708	178	171	168	163	157					
Mg-Al (electrolytic), 6–8% Al 1–2% Zn	1810	1.00	66	3.605		52	62	74	83					
Molybdenum	10,220	0.251	123	4.790	138	125	118	114	111	109	109	102	99	92
Nickel:														
Pure (99.9%)	8906	0.4459	90	2.266	104	93	83	73	64	59				
Ni-Cr, 90% Ni, 10% Cr	8666	0.444	17	0.444		17.1	18.9	20.9	22.8	24.6				
80% Ni 20% Cr	8314	0.444	12.6	0.343		12.3	13.8	15.6	17.1	18.0	22.5			
Silver:														
Purest	10,524	0.2340	419	17.004	419	417	415	412						
Pure (99.9%)	10,525	0.2340	407	16.563	419	410	415	374	362	360				
Tin, pure	7304	0.2265	64	3.884	74	65.9	59	57						
Tungsten	19,350	0.1344	163	6.271		166	151	142	133	126	112	76		
Zinc, pure	7144	0.3843	112.2	4.106	114	112	109	106	100	93				

Table 6.2 Dimensions, Capacities, and Weights of Standard Steel Pipes

Nominal pipe size, in	Schedule number	Outside diameter, in	Wall thickness, in	Inside diameter (ID), in	Cross-sectional area of metal, in^2	Inside sectional area, ft^2	Pipe weight, lb/ft
$\frac{1}{8}$	40	0.405	0.068	0.269	0.072	0.00040	0.24
	80		0.095	0.215	0.093	0.00025	0.31
$\frac{1}{4}$	40	0.540	0.088	0.364	0.125	0.00072	0.42
	80		0.119	0.302	0.157	0.00050	0.54
$\frac{3}{8}$	40	0.675	0.091	0.493	0.167	0.00133	0.57
	80		0.126	0.423	0.217	0.00098	0.74
$\frac{1}{2}$	40	0.840	0.109	0.622	0.250	0.00211	0.85
	80		0.147	0.546	0.320	0.00163	1.09
$\frac{3}{4}$	40	1.050	0.113	0.824	0.333	0.00371	1.13
	80		0.154	0.742	0.433	0.00300	1.47
1	40	1.315	0.133	1.049	0.494	0.00600	1.68
	80		0.179	0.957	0.639	0.00499	2.17
$1\frac{1}{4}$	40	1.660	0.140	1.380	0.668	0.01040	2.27
	80		0.191	1.278	0.881	0.00891	3.00
$1\frac{1}{2}$	40	1.900	0.145	1.610	0.800	0.01414	2.72
	80		0.200	1.500	1.069	0.01225	3.63
2	40	2.375	0.154	2.067	1.075	0.02330	3.65
	80		0.218	1.939	1.477	0.02050	5.02

(*Continued*)

Table 6.2 *Continued*

Nominal pipe size, in	Schedule number	Outside diameter, in	Wall thickness, in	Inside diameter (ID), in	Cross-sectional area of metal, in^2	Inside sectional area, ft^2	Pipe weight, lb/ft
$2\frac{1}{2}$	40	2.875	0.203	2.469	1.704	0.03322	5.79
	80		0.276	2.323	2.254	0.02942	7.66
3	40	3.500	0.216	3.068	2.228	0.05130	7.58
	80		0.300	2.900	3.016	0.04587	10.25
$3\frac{1}{2}$	40	4.000	0.226	3.548	2.680	0.06870	9.11
	80		0.318	3.364	3.678	0.06170	12.51
4	40	4.500	0.237	4.026	3.17	0.08840	10.79
	80		0.337	3.826	4.41	0.07986	14.98
5	40	5.563	0.258	5.047	4.30	0.1390	14.62
	80		0.375	4.813	6.11	0.1263	20.78
6	40	6.625	0.280	6.065	5.58	0.2006	18.97
	80		0.432	5.761	8.40	0.1810	28.57
8	40	8.625	0.322	7.981	8.396	0.3474	28.55
	80		0.500	7.625	12.76	0.3171	43.39
10	40	10.75	0.365	10.020	11.91	0.5475	40.48
	80		0.594	9.562	18.95	0.4987	64.40
12	40	12.75	0.406	11.938	15.74	0.7773	53.36
	80		0.688	11.374	26.07	0.7056	88.57

where P is the maximum internal service pressure (psi) and S is the allowable bursting stress in the pipe material (psi). (The S value varies by material, grade of material, and temperature; allowable S values may be found in standard piping handbooks.) As noted above, the internal walls of a pipe are generally “rough,” as opposed to tubes whose walls are generally “smooth.”

ILLUSTRATIVE EXAMPLE 6.1

Obtain the inside diameter in inches of 2 inch schedule 40 nominal steel pipe.

***SOLUTION*:** Refer to Table 6.2. The answer is 2.067 in. ■

ILLUSTRATIVE EXAMPLE 6.2

Determine the inside diameter (ID), outside diameter (OD), wall thickness, and pipe weight (in lb/ft) of 3 inch schedule 40 steel pipe.

***SOLUTION*:** Obtain the pipe inside diameter, outside diameter, wall thickness, and weight using Table 6.2.

ID = 3.068 in
OD = 3.5 in
Wall thickness = 0.216 in
Pipe weight = 7.58 lb/ft ■

ILLUSTRATIVE EXAMPLE 6.3

The following information is provided for a steel pipe:

ID = 0.957 in
OD = 1.315 in
Wall thickness = 0.179 in
Pipe weight = 2.17 lb/ft

Determine the nominal size and schedule number of the pipe.

***SOLUTION*:** A quick check of Table 6.2 indicates that the steel pipe is 1 inch schedule 80. ■

TUBES

Generally speaking, tubes are thin-walled and often come in coils. Tube sizes are indicated by the outside diameter. The wall thickness is usually given a BWG (Birmingham Wire Gauge) number. The smaller the BWG, the heavier the tube.

Table 6.3 Dimensions of Heat Exchanger Tubes

Tube OD, in	BWG gauge	Thickness, in	Tube inside diameter (ID), in	Flow area, in^2	Surface area, per foot of length, ft	
					External	Internal
$\frac{1}{4}$	22	0.028	0.194	0.0295	0.0655	0.0508
$\frac{1}{4}$	24	0.022	0.206	0.0333	0.0655	0.0539
$\frac{1}{2}$	18	0.049	0.402	0.1269	0.1309	0.1052
$\frac{1}{2}$	20	0.035	0.430	0.1452	0.1309	0.1126
$\frac{1}{2}$	22	0.028	0.444	0.1548	0.1309	0.1162
$\frac{3}{4}$	10	0.134	0.482	0.1825	0.1963	0.1262
$\frac{3}{4}$	14	0.083	0.584	0.2679	0.1963	0.1529
$\frac{3}{4}$	16	0.065	0.620	0.3019	0.1963	0.1623
$\frac{3}{4}$	18	0.049	0.652	0.3339	0.1963	0.1707
1	8	0.165	0.670	0.3526	0.2618	0.1754
1	14	0.083	0.834	0.5463	0.2618	0.2183
1	16	0.065	0.870	0.5945	0.2618	0.2278
1	18	0.049	0.902	0.6390	0.2618	0.2361
$1\frac{1}{4}$	8	0.165	0.920	0.6648	0.3272	0.2409
$1\frac{1}{4}$	14	0.083	1.084	0.9229	0.3272	0.2838
$1\frac{1}{4}$	16	0.065	1.120	0.9852	0.3272	0.2932
$1\frac{1}{4}$	18	0.049	1.152	1.042	0.3272	0.3016
2	11	0.120	1.760	2.433	0.5236	0.4608
2	12	0.109	1.782	2.494	0.5236	0.4665
2	13	0.095	1.810	2.573	0.5236	0.4739
2	14	0.083	1.834	2.642	0.5236	0.4801

(1 in = 25.4 mm; 1 in^2 = 645.16 mm^2; 1 ft = 0.3048 m; 1 ft^2 = 0.0929 m^2).

Table 6.3(2) lists the sizes and wall thicknesses of condenser and heat exchanger tubes. Standard exchanger tubing is $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, and 2 inch outside diameter. The most commonly used tubes in the chemical process industry and petroleum refineries are $\frac{3}{4}$ and 1 inch outside diameter. Standard tube lengths are 8, 10, 12, 16, and 20 ft with 16 ft as the most common. Longer lengths are usually coiled. It is common practice to specify a heat exchanger's surface in terms of the total external square feet of tubing. Tubing can be easily drawn and is usually set to minimum wall specifications.

When corrosive atmospheres or concern with temperature excursions limit the use of a single metal for the tubes, bimetallic (or duplex) tubes may be employed. These tubes can be made from almost any possible combination of metals. For thin gauges, the wall thickness is generally divided equally between the two components. As one might expect with heavier gauges, the more expensive component usually comprises from a fifth to a third of the total thickness.

ILLUSTRATIVE EXAMPLE 6.4

Determine the inside diameter, wall thickness, outside diameter, and external area per foot (EA) of a $\frac{3}{4}$ inch 16 BWG tube.

***SOLUTION*:** From Table 6.3,

$$\begin{aligned}
&\text{ID} = 0.620\ \text{in}\\
&\text{Wall thickness} = 0.065\ \text{in}\\
&\text{OD} = 0.620 + (2)(0.065) = 0.75\ \text{in}\\
&\text{EA} = 0.1963\ \text{ft}
\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 6.5

The tube weight of 2.0 inch 16 BWG tubing is 1.699 lb/ft. Comment on the weight of comparably-sized pipes and tubes.

***SOLUTION*:** With reference of Table 6.1, the pipe weight of 2.0 inch schedule 80 pipe is 5.02 lb/ft. The pipe is three ($2.95 = 5.02/1.699$) times heavier than the tubing! ■

VALVES AND FITTINGS

As indicated earlier, pipes and tubes, as well as other conduits, are used for the transportation of gases, liquids, and slurries. These ducts are often connected and may also contain a variety of valves and fittings, including expansion and contraction joints. The two major types of connecting conduits include:

1. Threaded
2. Welded

Extensive information on these two classes of connections is available in the literature.(1)

Details on valves and fittings are provided in the following two subsections. Changes in cross-sectional area receive treatment in Illustrative Examples 6.6 and 6.7.

Valves(2,3)

Because of the diversity of the types of systems, fluids, and environments in which valves(2) must operate, a vast array of valve types employed in exchangers have been developed. Examples of the common types are the globe valve, gate valve, ball valve, plug valve, pinch valve, butterfly valve, and check valve. Each type of valve has been designed to meet specific needs and are almost always located external to the exchanger. Some valves are capable of throttling the flow of a fluid to be heated or cooled, other valve types can only stop flow, others work well in corrosive systems, and others handle high pressure fluids. Each valve type has advantages and disadvantages. Understanding these differences and how they affect the valve's application or operation is necessary for the successful operation of a heat exchanger.

Valves have two main functions in an exchanger: to control the amount of flow or to stop the flow completely. Of the many different types of valves, the most commonly used are the gate valve and the globe valve. The gate valve contains a disk that slides at right angles to the flow direction. This type of valve is used primarily for on–off control of a liquid flow. Because small lateral adjustments of the disk can cause extreme changes in the flow cross-sectional area, this type of valve is not suitable for accurately adjusting flow rates. As the fluid passes through the gate valve, only a small amount of turbulence is generated; the direction of flow is not altered and the flow cross-sectional area inside the valve is often only slightly smaller than that of the pipe. As a result, the valve causes only a minor pressure drop. Problems with abrasion and erosion of the disk can arise when the valve is used in positions other than fully open or fully closed.

Unlike the gate valve, the globe valve—so called because of the spherical shape of the valve body—is designed for more sensitive flow control. In this type of valve, the fluid to be heated or cooled passes through the valve in a somewhat circuitous route. In one form, the seal is a horizontal ring into which a plug with a slightly beveled edge is inserted when the stem is closed. Good control of flow is achieved with this type of valve, but at the expense of a higher pressure loss than a gate valve.

Stop valves are used to shut off or, in some cases, partially shut off the flow of fluid to the exchanger. Stop valves are controlled by the movement of the valve stem. Stop valves can be divided into four general categories: globe, gate, butterfly, and ball valves. Plug valves and needle valves may also be considered stop valves.

Ball valves, as the name implies, are stop valves that use a ball to stop or start the flow of fluid. The ball performs the same function as the disk in the globe valve. When the valve handle is operated to open the valve, the ball rotates to a point where the hole through the ball is in line with the valve body inlet and outlet. When the valve is shut, which requires only a 90-degree rotation of the handwheel for most valves, the ball is rotated so that the hole is perpendicular to the flow openings of the valve body, and flow is stopped.

A plug valve is a rotational motion valve, similar to a ball valve, used to stop or start fluid flow. The name is derived from the shape of the disk, which resembles a plug. The simplest form of a plug valve is the petcock. The body of a plug valve is machined to receive the tapered or cylindrical plug. The disk is a solid plug with a bored passage at a right angle to the longitudinal axis of the plug. In the open position, the passage in the plug lines up with the inlet and outlet ports of the valve. When the plug is turned 90° from the open position, the solid part of the plug blocks the ports and stops fluid flow.

Finally, check valves are designed to prevent the reversal of flow in a heat exchanger. These valves are activated by the flowing material in the exchanger. The pressure of the fluid passing through the piping/tubing system opens the valve, while any reversal of flow will close the valve. Closure is accomplished by the weight of the check mechanism, by back pressure, by a spring, or by a combination of these means. The general types (classification) of check valves are swing, tilting-disk, piston, butterfly, and stop.

Valves are also sometimes classified according to the resistance they offer to flow. The low resistance class of valves includes the straight-through flow units (e.g., gate, ball, and plug valves). Valves having a change in direction are high resistance valves; an example is the globe valve.

Fittings[2,3]

A fitting is a piece of equipment that has for its function one or more of the following:

1. The joining of two pieces of straight pipe or tube (e.g., couplings and unions).
2. The changing of flow direction (e.g., elbows and Ts).
3. The changing of pipe or tube diameter (e.g., reducers and bushings).
4. The terminating of a pipe or tube (e.g., plugs and caps).
5. The joining of two streams (e.g., Ts and Ys).

Since fittings find application primarily with pipes—as opposed to tubes—the next paragraph will key on pipe fittings.

A coupling is a short piece of pipe threaded on the inside and used to connect straight sections of pipe with no change in direction or size. When a coupling is opened, a considerable amount of piping must usually be dismantled. A union is also used to connect two straight sections but differs from a coupling in that it can be opened conveniently without disturbing the rest of the exchanger. An elbow is an angle fitting used to change flow direction, usually by 90°, although 45° elbows are also available. In addition, a T (shaped like the letter T) can be used to change flow direction; this fitting is more often used to combine two streams into one, i.e., when two branches of piping are to be connected at the same point. A reducer is a coupling for two pipe sections of different diameter. A bushing is also a connector for pipes of different diameter, but, unlike the reducer coupling, is threaded on both the inside and outside; the larger pipe screws onto the outside of the bushing and the smaller pipe screws into the inside of the bushing. Plugs, which are threaded on the outside, and caps, which are threaded on the inside, are used to terminate the flow of the heated or cooled fluid in a heat exchanger. Finally, a Y (shaped like the letter Y) is similar to the T and is used to combine two streams.

Fittings may be classified as reducing, expanding, branching, or deflecting. Reducing or expanding fittings are ones that change the area for flow; these include reducers, bushings, and sudden expansions and contractions. Branch fittings are Ts, crosses, or side outlet elbows. Deflecting fittings change the direction of flow, e.g., Ys, elbows and bends.

ILLUSTRATIVE EXAMPLE 6.6

Discuss when expansion and contraction effects can come into effect with heat exchangers.

***SOLUTION*:** Contraction effects can come into play when the fluid enters the exchanger. Expansion effects can appear as the fluid exits the exchanger. ■

ILLUSTRATIVE EXAMPLE 6.7

Provide equations that can be employed to calculate the pressure drop associated with:

1. a rapid (sudden) expansion
2. a rapid (sudden) contraction

SOLUTION: If the cross-section of a conduit enlarges gradually so that the flowing fluid velocity in the heat exchanger does not undergo any disturbances, energy losses (pressure drop) are minor and may be neglected. However, if the change is sudden, as in a rapid expansion, it can result in friction losses. For such sudden enlargement/expansion situations, the pressure loss as the fluid exits the exchanger can be represented by

$$h_{f,e} = \frac{V_1^2 - V_2^2}{2g_c}; \quad e = \text{sudden expansion} \tag{6.2}$$

where $h_{f,e}$ is the loss in head, V_2 is the velocity at the larger cross-section and V_1 is the velocity at the smaller cross-section. When the cross-section of the conduit is reduced suddenly (a contraction), the loss may be expressed by:

$$h_{f,c} = \frac{KV_2^2}{2g_c}; \quad c = \text{sudden contraction} \tag{6.3}$$

where V_2 is the velocity in the small cross-section and K is a dimensionless loss coefficient that is a function of the ratio of the two cross-sectional areas. Both of the above calculations receive extensive treatment in the literature.$^{(2-4)}$ ■

NONCIRCULAR CONDUITS

Some heat transfer applications involve the flow of a fluid in a conduit or channel that does not have a circular cross-section. When the flow is turbulent, it is recommended that the flow is treated as if the flow occurs in a pipe. For this situation, a hydraulic radius, r_h, is defined as:

$$r_h = \frac{\text{cross-sectional area perpendicular to flow}}{\text{wetted perimeter}} = \frac{A_f}{P_w} \tag{6.4}$$

For flow in a circular tube of diameter D

$$r_h = \frac{(\pi D^2/4)}{\pi D} = \frac{D}{4}$$

and

$$D = 4r_h \tag{6.5}$$

One may extend this concept to any cross-section such that

$$D_h = 4r_h = \frac{4A_f}{P_w} \tag{6.6}$$

It is then possible to use this hydraulic or equivalent diameter in any circular pipe expression for pressure drop. Interestingly, the hydraulic diameter approach is usually valid for laminar flow but always valid for turbulent flow.

ILLUSTRATIVE EXAMPLE 6.8

A common non-circular geometry is the flow in a concentric tube annulus. For this application, the fluid passes through the annulus formed by the concentric tubes. Express the hydraulic diameter in terms of the inside diameter of the outer pipe, D_o, and the outside diameter of the inner pipe, $D_{i.}$

***SOLUTION*:** In this case the hydraulic diameter, D_h, is

$$D_{eq} = D_h = \frac{4A_f}{P_w} \tag{6.6}$$

Substituting,

$$D_h = \frac{4(\pi/4)(D_o^2 - D_i^2)}{\pi D_o + \pi D_i} = D_o - D_i$$

■

ILLUSTRATIVE EXAMPLE 6.9

Determine the area and perimeter of the following conduits:

1. Rectangle of length b and width a.
2. Parallelogram of height h and base b with angle between ab denoted by θ.
3. Trapezoid of height h and parallel sides a and b with base angles θ and ϕ.
4. Regular polygon of n sides each of length b.

***SOLUTION*:**

1. Area $= ab$

   Perimeter $= 2a + 2b$
2. Area $= bh = ab \sin\theta$

   Perimeter $= 2a + 2b$
3. Area $= \frac{1}{2}h(a + b)$

   $$\text{Perimeter} = a + b + h\left(\frac{1}{\sin\theta} + \frac{1}{\sin\phi}\right)$$

   $$= a + b + h(\csc\theta + \csc\phi)$$

4. Area $= \frac{1}{4}nb^2 \cot\frac{\pi}{n} = \frac{1}{4}nb^2 \frac{\cos(\pi/n)}{\sin(\pi/n)}$

Perimeter $= nb$ ■

ILLUSTRATIVE EXAMPLE 6.10

Refer to Illustrative Example 6.9. Outline how to determine the hydraulic diameter for cases (1–4).

***SOLUTION*:** Apply Equation (6.6) for each case, i.e.,

$$D_h = \frac{4A_f}{P_w}$$

For example, for (1)

$$D_h = \frac{4(ab)}{2a + 2b} = \frac{2ab}{a + b}$$

■

FLOW CONSIDERATIONS

Fluid flow through circular tubes (or pipes) is common in many heat exchanger applications. Such flow is always accompanied by frictional losses (e.g., between the fluid and the stationary wall and between portions of the fluid moving at different velocities), which cause a pressure drop in the direction of flow.

Flow in a pipe may be laminar or turbulent. Some characteristics of the two types of flow in terms of the dimensionless Reynolds number are given in Table 6.4. The Reynolds number is given by

$$\text{Re} = DV\rho/\mu = \frac{DV}{\nu}; \quad \text{dimensionless} \tag{6.7}$$

where D = diameter
V = average fluid velocity
ρ = fluid density
μ = fluid viscosity
ν = kinematic viscosity

Table 6.4 Flow Characteristics

Flow type	Characteristics
Laminar (Re $<$ 2300)	– Flow moves in axial direction – No motion normal to the tube axis
Turbulent (Re $>$ 4000)	– No net flow normal to the tube axis – Strong, local, oscillating motion (or eddy) normal to the tube axis

The above Reynolds number is an important dimensionless number that plays a major role in determining a host of heat transfer coefficients to be introduced and discussed in the next Part.

The aforementioned pressure drop ΔP may be calculated from any of a host of equations,$^{(1,2,4)}$ including

$$\Delta P = \frac{4fLV^2}{2g_cD} \tag{6.8}$$

where $L =$ pipe length
$f =$ fraction factor

The friction factor is a function of Re and the internal roughness of the pipe. Details are available in the literature.$^{(1,2,4)}$

Another important concept is that referred to as a "calming," "entrance," or "transition" length. This is the length of conduit required for a velocity profile to become fully developed following some form of disturbance in the conduit. This disturbance can arise because of a valve, a bend in the line, an expansion in the line, etc. This is an important concern when measurements are conducted in the cross-section of the pipe or conduit. An estimate of this "calming" length, L_c, for laminar flow is

$$\frac{L_c}{D} = 0.05\,\mathrm{Re} \tag{6.9}$$

For turbulent flow, one may employ

$$L_c = 50D \tag{6.10}$$

ILLUSTRATIVE EXAMPLE 6.11

An air-conditioning duct has a rectangular cross-section of 1 m by 0.25 m. If the kinematic viscosity of the air is approximately $1 \times 10^{-5}\,\mathrm{m^2/s}$, determine the maximum air velocity before the flow becomes turbulent. Assume the critical Reynolds number is 2300.

***SOLUTION*:** Compute the equivalent or hydraulic diameter (see Illustrative Example 6.10).

$$D_{eq} = D_h = \frac{2ab}{a+b} = \frac{2(1)(0.25)}{1+0.25}$$
$$= 0.4\,\mathrm{m}$$

The equation for the "critical" Reynolds number is

$$\mathrm{Re}_{\mathrm{crit}} \approx 2300 = \frac{D_hV}{\nu} \tag{6.7}$$

Substitute and solve for V.

$$V = 2300\frac{1 \times 10^{-5}}{0.4} = 0.0575\,\text{m/s}$$
$$= 5.8\,\text{cm/s}$$

■

ILLUSTRATIVE EXAMPLE 6.12

A circular 2-inch diameter horizontal tube contains a cold liquid fluid of density and viscosity $70\ \text{lb/ft}^3$ and $0.1806\ \text{lb/ft}\cdot\text{s}$, respectively. If, the flow rate is $0.486\ \text{ft}^3/\text{s}$, determine if the flow is laminar.

***SOLUTION*:** Apply the continuity equation to obtain the flow velocity:

$$q = VS = V(\pi D^2/4)$$
$$V = \frac{4q}{\pi D^2} = \frac{4(0.486)}{\pi(0.1667)^2}$$
$$= 22.3\ \text{ft/s}$$

Check on the assumption of laminar flow:

$$\text{Re} = \frac{DV\rho}{\mu} = \frac{(0.1667)(22.3)(70)}{0.1806} = 1440 < 2100$$

The flow is laminar. ■

ILLUSTRATIVE EXAMPLE 6.13

Refer to Illustrative Example 6.12. How long must the pipe be to ensure a fully developed flow?

***SOLUTION*:** The pipe must be longer than the entrance length to have fully developed flow. Since the flow is laminar, apply Equation (6.9):

$$L_c = (0.05)(D)(\text{Re})$$

Substituting,

$$L_c = (0.05)(2/12)(1440)$$
$$= 12\ \text{ft}$$

This is an abnormally long calming length for a pipe (or tube) in a heat exchanger. ■

ILLUSTRATIVE EXAMPLE 6.14

Calculate the average velocity of water flowing in a 2-inch schedule 40 standard pipe at 60°F for which the flow is viscous (laminar).

***SOLUTION*:** For laminar flow, assume Re < 2100, so the equation

$$\mathrm{Re} = \frac{DV\rho}{\mu} \leq 2100$$

can be solved for the velocity term.

$$V \leq \frac{2100\mu}{D\rho}$$

For water, $\mu = 6.72 \times 10^{-4}\,\mathrm{lb/ft \cdot s}$, $\rho = 62.4\,\mathrm{lb/ft^3}$. In addition, from Table 6.2, $D =$ 2.067 in. Therefore,

$$V \leq \frac{2100(6.72 \times 10^{-4})}{(2.067/12)(62.4)} = 0.13\ \mathrm{ft/s}$$

■

ILLUSTRATIVE EXAMPLE 6.15

Discuss some of the problems associated with pipes and tubing.

***SOLUTION*:** Some of the usual piping and tubing problems facing the practicing engineers are:

1. Choosing the correct material.
2. Selecting the most economical pipe size.
3. Selection of suitable means for joining piping and sealing joints.
4. Selection of anchors, hangars, and other supports.
5. Specifying an economic and appropriate insulation where required.
6. Determining proper pipe colors where required.
7. Be familiar with tubing and piping codes provided by:
   1. ASME (American Society of Mechanical Engineers)
   2. ASTM (American Society for Testing Materials)
   3. API (American Petroleum Institute)

■

REFERENCES

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Part Two

Principles

The purpose of this Part of the text is to present the principles of heat transfer. Although a host of principles are introduced, the primary focus will be on the three heat transfer mechanisms: conduction, convection, and radiation. A brief description of these mechanisms is given in the next paragraph.

A difference in temperature between two bodies in close proximity, or between two parts of the same body, results in a flow of heat from the higher temperature to the lower temperature. There are three different mechanisms by which this heat transfer can occur: the aforementioned conduction, convection, and radiation. When heat transfer is the result of molecular motion (e.g., the vibrational energy of molecules in a solid being passed along from molecule to molecule), the mechanism of transfer is *conduction*. An example is the conduction of heat through the brick wall of a furnace. When the heat transfer results from macroscopic motion, such as currents in a fluid, the mechanism is *convection*. An example is the flow of heat when hot air flows across a bank of cold tubes. If fluid flow is the result of the temperature difference in the fluid (which causes buoyancy currents), the solid-to-fluid heat transfer is termed *free* or *natural* convection. If a mechanical device (e.g., fan, pump, or compressor) causes the motion of the fluid, it is termed *forced* convection. The rate of heat transfer is proportional to the temperature difference in both conduction and convection. When heat is transferred by electromagnetic waves, the mechanism is *radiation*. Radiation may be transmitted, reflected, or absorbed, depending upon the medium and the receiving surface. A transmittance example is the incidence of solar radiation on glass. Most of the solar energy is transmitted through glass as electromagnetic waves. Generally, radiation becomes important only at high temperatures.

Summarizing, heat is the transfer of energy occurring as a result of a driving force referred to as a temperature difference. There are three mechanisms by which heat transfer can occur:

1. Conduction—Heat is transferred by the energy of motion between adjacent molecules.
2. Convection—Heat is transferred due to both motion and mixing of macroscopic elements of materials. Because the motion of a fluid is involved, heat transfer by convection is partially governed by the laws of fluid mechanics. If convection is induced by a density difference which arises due to temperature differences, then the mechanism is referred to as natural or free convection.

If convection occurs due to an external force (e.g., pump or fan), the mechanism is defined as forced convection.

3. Radiation—This mechanism transfers energy by electromagnetic radiation or photons having a certain range of wavelength. Although energy can be transferred by radiation through gases, liquids, and solids, these media absorb some or all of the energy. Therefore, energy is radiated most efficiently through empty space. The most obvious example of the radiation mechanism is the transport of heat from the Sun to the Earth; the contributions from conduction and convection are obviously negligible because of the distance separating the two.

The remainder of this part deals with heat transfer principles and fundamentals. Chapter topics include:

Steady-State Heat Conduction: Chapter 7
Unsteady-State Heat Conduction: Chapter 8
Forced Convection: Chapter 9
Free Convection: Chapter 10
Radiation: Chapter 11
Condensation and Boiling: Chapter 12
Refrigeration and Cryogenics: Chapter 13

The reader should note that sections detailing the microscopic approach are included within the first three chapters (i.e., steady-state conduction, unsteady-state conduction, and forced convection). This will complement the presentation for some. However, this material can be bypassed, particularly by those not interested or concerned with a more theoretical treatment of these topics.

Chapter 7

Steady-State Heat Conduction

INTRODUCTION

As the temperature of a solid increases, the molecules that make up the solid experience an increase in vibrational kinetic energy. Since every molecule is bonded in some way to neighboring molecules, usually by electrical force fields, this energy can be passed through the solid. Thus, the transfer of heat may be viewed as thermal energy in transit due to the presence of a temperature difference. Heating a wire at one end eventually results in raising the temperature at the other. This type of heat transfer is called conduction—the subject title of this chapter—and is the principle mechanism by which solids transfer heat.

Fluids are capable of transporting heat in a similiar fashion. Conduction in a stagnant liquid, for example, occurs by the movement of not only vibrational kinetic energy but also translational kinetic energy as the molecules move throughout the body of the stationary liquid. It is also important to note that there is *no* bulk motion with conduction. The ability of a fluid to flow, mix, and form internal currents on a macroscopic level (as opposed to the molecular mixing just described) allows fluids to carry heat energy by convection as well, a topic that receives treatment in Chapters 9 and 10.

Finally, a microscopic analysis of heat conduction is presented in the last section with additional development material (based on the Monte Carlo method) in Chapter 26.

FOURIER'S LAW

The rate of heat flow by conduction is given by Fourier's law$^{(1)}$

$$\begin{aligned} \dot{Q} &= -kA\frac{dT}{dx} = kA\left(-\frac{dT}{dx}\right) \\ \dot{Q}' &= \frac{\dot{Q}}{A} = k\left(-\frac{dT}{dx}\right) \end{aligned} \tag{7.1}$$

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

where (in English units) $\dot{Q}$ is the heat flow rate (Btu/h), x is the direction of heat flow (ft), k is the thermal conductivity (Btu/h · ft · °F), (see also Chapter 3), A is the heat transfer area perpendicular to the flow of heat (ft^2), and T is the temperature (°F). The term $(-dT/dx)$ is defined as the temperature gradient and carries a negative sign since T is assumed to decrease with increasing x for $\dot{Q}$ being transferred in the positive x direction. Fourier derived his law from experimental evidence and/or physical phenomena, not from basic principles (i.e., theory). The reader should also note that Equation (7.1) is the basis for the definition of thermal conductivity as given in Chapter 3 (i.e., the amount of heat (Btu) that flows in a unit of time (1 h) through a unit area of surface (1 ft^2) of unit thickness (1 ft) by virtue of a difference in temperature (1°F)).

Refer to Figure 7.1. Once again, the negative sign in Equation (7.1) reflects the fact that heat flow is from a high to low temperature and therefore the sign of the derivative (dT/dx) is opposite to that of the heat flow. If the thermal conductivity (k) can be considered constant over a limited temperature range (ΔT), Equation (7.1) can be integrated to give

$$\dot{Q} = \frac{kA(T_H - T_C)}{(x_H - x_C)} = \frac{kA\Delta T}{\Delta x} = \frac{kA\Delta T}{L} \tag{7.2}$$

where T_H is the higher temperature at point x_H (°F), T_C is the lower temperature at point x_C(°F), ΔT is the difference between T_H and T_C, L is the distance between points x_H and x_C(ft) $= \Delta x$, and A(ft^2) is the area across which the heat is flowing.

It should be noted that the thermal conductivity (k) is a property of the material through which the heat is passing and, as such, does vary somewhat with temperature.

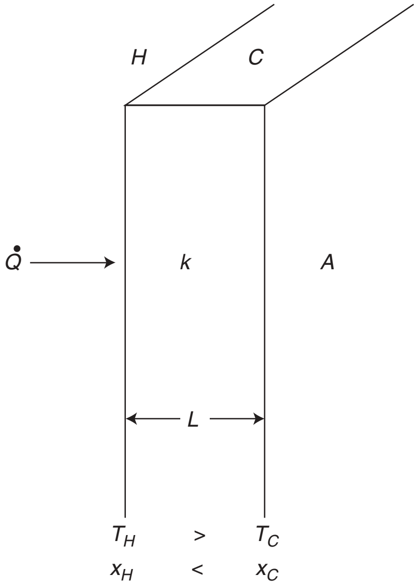

Figure 7.1 Heat flow through a solid wall.

Table 7.1 Thermal Conductivities of Three Common Insulating Materials

	k (Btu/h · ft · °F)
Asbestos–cement boards	0.430
Fiber, insulating board	0.028
Glass wool, 1.5 lb/ft^3	0.022

Equation (7.2) should therefore be strictly employed only for small values of ΔT with an average value of the conductivity used for k.

Values of k for three insulating solids are given in Table 7.1. (A separate chapter on insulation is provided in Part Three.) Approximate values of k for materials at different states are provided in Table 7.2 employing SI units.

As can be seen from Table 7.2, the thermal conductivities of liquids and of gases are very small in comparison with the thermal conductivities of most solids. For example, at 212°F the thermal conductivity (in English units) of silver is 240 Btu · ft/h · ft^2 · °F; that of a building brick is about 0.8, that of water is about 0.35, and of air is 0.017.

Thermal conductivities, as well as other properties, of various liquids and gases are listed in the Appendix (see Tables B.2–B.8). Properties of metals are provided in Chapter 6.

In general, the variation of thermal conductivity with temperature is small, but a fair approximation is to assume that this variation is linear, i.e.,

$$k = a + bT \tag{7.3}$$

where a and b are constants and T is the temperature. The reader is left the option of determining appropriate and consistent units for a and b. The effect of pressure is almost always negligible, particularly for gases approaching ideal gas behavior.

If a plane wall of thickness L has steady-state temperatures of T_H and T_C on the hot and cold faces, respectively, one can derive an expression for the steady-state one-dimensional heat transfer rate per unit area, $\dot{Q}/A$, in terms of T_H, T_C, a, b, and L. Substituting Equation (7.3) into Equation (7.1) gives

$$\dot{Q} = (a + bT)A\frac{dT}{dx} \tag{7.4}$$

Table 7.2 Thermal Conductivities for Materials of Different States

	K(W/m · °C)
Solid metals	15–400
Liquids	0.0–100
Gases	0.01–0.2

Note that 1 Btu/h · ft · °F = 1.7307 J/s · m · °C = 1.7307 W/m · °C.

or

$$\frac{\dot{Q}}{A}\,dx = (a + bT)\,dT \tag{7.5}$$

This equation may be integrated to give

$$\left(\frac{\dot{Q}}{A}\right)L = a(T_H - T_C) + \frac{b}{2}(T_H^2 - T_C^2) \tag{7.6}$$

Another thermal quantity useful in heat transfer calculations is the thermal diffusivity, α. It is defined as

$$\alpha = \frac{k}{\rho c_P} \tag{7.7}$$

where ρ is the material density and c_P is the heat capacity of the material. The term α has the units of m^2/s or ft^2/s, which is the same units as the kinematic viscosity, ν. When the thermal conductivity is constant and independent of position, the material is termed *homogeneous*. Materials with properties such as thermal conductivity that are independent of the direction of heat transfer are termed *isotropic* (i.e., in rectangular coordinates, $k_x = k_y = k_z = k$). (Refer to the last section of this chapter for additional details.)

CONDUCTIVITY RESISTANCES

Equation (7.2) may be written in the form of the general rate equation:

$$\text{rate} = \frac{\text{(driving force)(area)}}{\text{resistance}} = \frac{\Delta T}{L/kA} \tag{7.8}$$

Note that the area term in Equation (7.8) can be combined with the *resistance*, L/k, to form the term L/kA. Since Q in Equation (7.2) is the heat flow rate and ΔT the driving force, the L/kA term may be viewed as the resistance to heat flow. This approach is useful when heat is flowing by conduction through different materials in sequence and of varying heat transfer areas.

Consider, for example, a flat incinerator wall made up of three different layers: an inside insulating layer, a; a steel plate, b; and an outside insulating layer, c. Refer to Figure 7.2.

The total resistance to heat flow through the incinerator wall is the sum of the three individual resistances:

$$R = R_a + R_b + R_c; \qquad R_i = \frac{L_i}{k_i A_i} \tag{7.9}$$

Note that it is assumed that the heat transferred across these resistances experiences negligible resistance at each interface, an assumption that may not always be true.

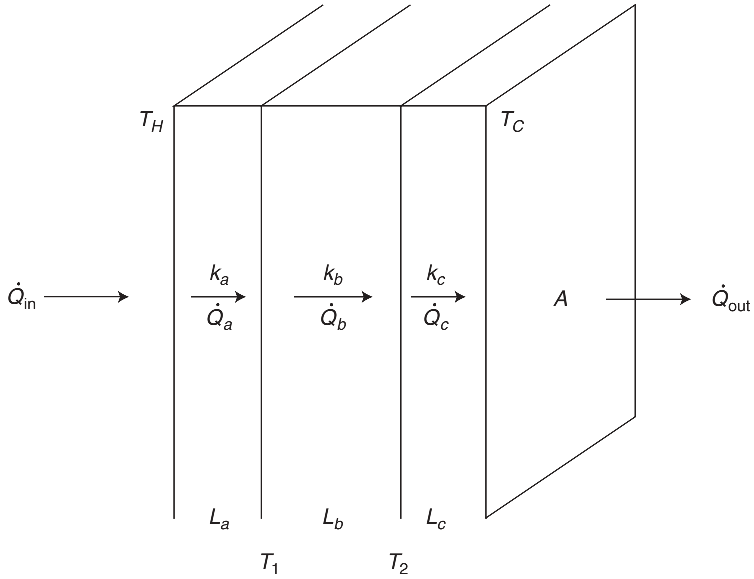

Figure 7.2 Heat transfer through an insulated incinerator wall.

For steady-state conditions

$$\dot{Q}_{\text{in}} = \dot{Q}_a = \dot{Q}_b = \dot{Q}_c = \dot{Q}_{\text{out}} = \dot{Q} \tag{7.10}$$

with

$$\dot{Q} = k_a \frac{A(T_H - T_1)}{L_a} = k_b \frac{A(T_1 - T_2)}{L_b} = k_c \frac{A(T_2 - T_C)}{L_c} \tag{7.11}$$

Since the overall temperature difference $(T_H - T_C)$ is easily measured and usually known, one can solve for it by noting

$$(T_H - T_1) = \dot{Q}\frac{L_a}{k_a A} \tag{7.12}$$

$$(T_1 - T_2) = \dot{Q}\frac{L_b}{k_b A} \tag{7.13}$$

$$(T_2 - T_C) = \dot{Q}\frac{L_c}{k_c A} \tag{7.14}$$

Adding and solving yields

$$(T_H - T_C) = \dot{Q}\left(\sum_i \left(\frac{L_i}{k_i A}\right)\right) \tag{7.15}$$

or

$$\dot{Q} = (T_H - T_C) \Big/ \sum_i \left(\frac{L_i}{k_i A} \right) \tag{7.16}$$

Thus, at steady-state, the rate of heat flow through the wall is given by

$$\dot{Q} = \frac{T_H - T_C}{(L_a/k_a A_a) + (L_b/k_b A_b) + (L_c/k_c A_c)} \tag{7.17}$$

where for engineering units k_a, k_b, k_c are the thermal conductivities of each section (Btu/h · ft · °F), A_a, A_b, A_c are the areas of heat transfer of each section (ft^2) (these are equal for a constant cross-section of heat conduction), L_a, L_b, L_c are the thickness of each layer (ft), T_H is the temperature at the inside surface of the insulating wall a (°F), and T_C is the temperature at the outside surface of the insulating wall c (°F).

In the above example, the heat is flowing through a slab of constant cross-section (i.e., all the areas available for heat transfer are equal). In many cases of industrial importance, however, this is not the case. For example, in heat flow through the walls of a cylindrical vessel (see Figure 7.3), such as a pipe or tube, the heat transfer area increases with radial distance from the center of the cylinder. The heat flow in this case is given by

$$\dot{Q} = \frac{k A_{\text{lm}} \Delta T}{L} \tag{7.18}$$

The term A_{lm} in this equation represents an average heat transfer area, or more accurately, the log-mean average heat transfer area. This log-mean average is calculated by

$$A_{\text{av}} = A_{\text{lm}} = \frac{A_2 - A_1}{\ln(A_2/A_1)} \tag{7.19}$$

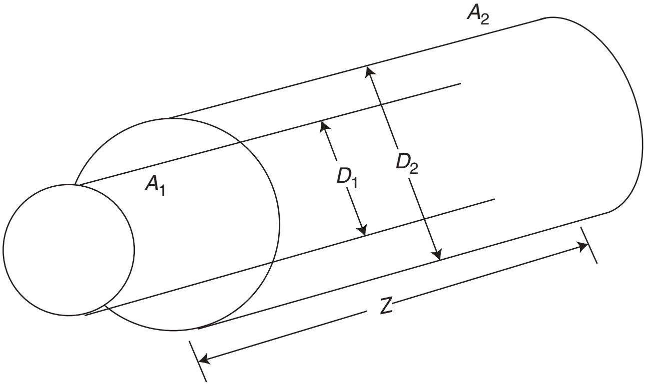

Figure 7.3 Log mean area figure.

where A_2 is the outer surface area of the cylinder (ft^2) and A_1 is the inner surface area of the cylinder (ft^2). The log-mean averaging of terms will be discussed again later in Part Three. When the value of A_2/A_1 does not exceed 2.5, the arithmetic-mean area may be employed; that is, $A_{\mathrm{av}} = (A_1 + A_2)/2$. The geometric mean area may also be used; that is, $A_{\mathrm{av}} = (A_1A_2)^{0.5}$. Since

$$A = \pi DZ; \quad Z = \text{length of pipe} \tag{7.20}$$

Equation (7.19) may be rewritten as

$$A_{\mathrm{lm}} = \frac{\pi Z(D_2 - D_1)}{\ln(D_2/D_1)} \tag{7.21}$$

For heat flowing through different materials in sequence, the companion to Equation (7.9) is

$$R = R_a + R_b + R_c = \frac{L_i}{k_iA_{i,\mathrm{lm}}} \tag{7.22}$$

with (once again)

$$\dot{Q} = \frac{\Delta T}{\sum R_i} \tag{7.23}$$

$$= \frac{T_H - T_C}{\dfrac{L_a}{(k_a)(A_{a,\mathrm{lm}})} + \dfrac{L_b}{(k_b)(A_{b,\mathrm{lm}})} + \dfrac{L_c}{(k_c)(A_{c,\mathrm{lm}})}} \tag{7.24}$$

ILLUSTRATIVE EXAMPLE 7.1

A glass window is 1 cm thick and has an area of 3 m^2. The temperature at the outer surface is 10°C. The glass has a conductivity of 1.4 W/m · K. The heat transfer rate is 3 kW. Calculate the temperature at the inner surface.

***SOLUTION*:** Write the equation for one-dimensional heat conduction:

$$\dot{Q} = \frac{-kA(T_C - T_H)}{L}; \quad T_H > T_C \tag{7.2}$$

Solve for the unknown temperature T_1 at the wall,

$$T_1 = T_2 + \frac{\dot{Q}L}{kA}$$

For this illustrative example, $\dot{Q} = 3000$ W and $L = 0.01$ m.

Substitute the values provided and compute T_H.

$$T_H = (273 + 10) + \frac{(3000)(0.01)}{(1.4)(3)} = 290.1\ \mathrm{K} = 17.1^\circ\mathrm{C}$$

Note that there are two kinds of SI temperatures used in this solution: Celsius and Kelvin (absolute). To be safe, one should compute using the absolute temperature, K (K = °C + 273). It is also important to remember that a change in temperature in Celsius is equal to the change in Kelvin, that is, $\Delta T(\text{K}) = \Delta T(^\circ\text{C})$. ■

ILLUSTRATIVE EXAMPLE 7.2

A new 1 ft thick insulating material was recently tested for heat resistant properties. The data recorded temperatures of 70°F and 210°F on the cold and hot sides, respectively. If the thermal conductivity of the insulating material is 0.026 Btu/ft · h · °F, calculate the rate of the heat flux, Q/A, through the wall in $\text{Btu/ft}^2 \cdot \text{h}$. Resolve the problem in SI units.

***SOLUTION*:** For this test,

$$\Delta T = 210 - 70 = 140^\circ\text{F}; \quad \Delta x = L = 1\ \text{ft}$$

Substitution into Equation (7.2) leads to

$$\dot{Q} = kA\frac{\Delta T}{\Delta x}$$

$$\frac{\dot{Q}}{A} = (0.026)\left(\frac{140}{1.0}\right) = 3.64\ \text{Btu/ft}^2 \cdot \text{h}$$

In SI units,

$$\frac{\dot{Q}}{A} = (3.64)\left(\frac{252\ \text{cal/h}}{1\ \text{Btu/h}}\right)\left(\frac{1\ \text{ft}^2}{0.093\ \text{m}^2}\right) = 9863\ \text{cal/m}^2 \cdot \text{h} = 9.863\ \text{kcal/m}^2 \cdot \text{h}$$

■

ILLUSTRATIVE EXAMPLE 7.3

The following information is provided. A rectangular furnace wall is made of fire clay (height, $H = 3$ m, width, $W = 1.2$ m, thickness, $L = 0.17$ m). Temperature of inside surface (area = $H \times W$), T_H, is 1592 K, and of the outside surface (area = $H \times W$), T_C, is 1364 K. Determine the temperature gradient, the heat transfer rate, and the heat transfer flux.

***SOLUTION*:** Determine the temperature at which the wall properties should be calculated. Thermophysical properties are usually calculated at the average wall temperature,

$$\begin{aligned} T_{\text{av}} &= (T_H + T_C)/2 \\ &= (1592 + 1364)/2 \\ &= 1478\ \text{K} \end{aligned}$$

Obtain the fire clay properties (see Table in the Appendix). For $T = 1478$ K,

$$\begin{aligned} \rho &= 2645\ \text{kg/m}^3 \\ k &= 1.8\ \text{W/m} \cdot \text{K} \\ c_p &= 960\ \text{J/kg} \cdot \text{K} \end{aligned}$$

Calculate the thermal diffusivity noting that

$$1\,\text{ft}^2/\text{s} = 0.0929\,\text{m}^2/\text{s}$$

and

$$\alpha = \frac{k}{\rho c_p} \tag{7.7}$$

Substituting

$$\begin{aligned}\alpha &= 1.8/[(2645)(960)]\\ &= 7.09 \times 10^{-7}\text{m}^2/\text{s}\\ &= 7.09 \times 10^{-7}/0.0929\\ &= 7.63 \times 10^{-6}\,\text{ft}^2/\text{s}\end{aligned}$$

Since the temperature gradient may be approximated by

$$\frac{dT}{dx} = \frac{T_H - T_C}{\chi_C - \chi_H} = \frac{T_H - T_C}{L}$$

calculate the temperature gradient.

$$\begin{aligned}\frac{dT}{dx} &= \frac{1364 - 1592}{0.17}\\ &= -1341\,\text{K/m}\\ &= -1341^\circ\text{C/m}\end{aligned}$$

The heat transfer area is

$$\begin{aligned}A &= (H)(W)\\ &= (3)(1.2)\\ &= 3.6\,\text{m}^2\end{aligned}$$

Calculate the heat transfer rate using Equation (7.2):

$$\dot{Q} = \frac{kA(T_H - T_C)}{L}$$

Substituting

$$\begin{aligned}\dot{Q} &= \frac{(1.8)(3.6)(1592 - 1364)}{0.17}\\ &= 8691\,\text{W}\\ &= 8.7\,\text{kW; note that } 1\,\text{W} = 3.412\,\text{Btu/h}\\ &= (8691)(3.412)\\ &= 29{,}653\,\text{Btu/h}\end{aligned}$$

Calculate the heat transfer flux, $\dot{Q}'$, where

$$\dot{Q}' = \dot{Q}/A$$

Substituting

$$\begin{aligned}\dot{Q}' &= 8691/3.6\\ &= 2414\ \mathrm{W/m^2};\ \text{note that } 1\ \mathrm{W/m^2} = 0.3171\ \mathrm{Btu/ft^2\cdot h}\\ &= (2414)(0.3171)\\ &= 765.5\ \mathrm{Btu/h\cdot ft^2}\end{aligned}$$

Calculate the thermal resistance. The thermal resistance, R, is given by Equation (7.9),

$$R = \frac{L}{kA}$$

Substituting

$$\begin{aligned}R &= \frac{0.17}{(1.8)(3.6)}\\ &= 0.0262\ \mathrm{K/W}\\ &= 0.0262^\circ\mathrm{C/W}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 7.4

A concrete wall has a surface area of 30 $\mathrm{m^2}$ and is 0.30 m thick. It separates warm room air from cold ambient air. The inner surface of the wall is known to be at a temperature of 25°C, while the outer surface is at −15°C. The thermal conductivity of the concrete is 1.0 $\mathrm{W/m\cdot K}$.

1. Describe the conditions that must be satisfied in order for the temperature distribution in the wall to be linear.
2. What is the driving force for the transfer of heat?
3. What is the heat loss through the wall?

***SOLUTION*:**

1. The conditions are:
   a. steady-state heat transfer
   b. a homogeneous isotropic medium
2. The driving force for heat transfer is

$$\Delta T = T_H - T_C = 25 - (-15) = 40^\circ\mathrm{C}$$

3. Finally, calculate the heat loss, $\dot{Q}$, through the wall. The thermal resistance due to conduction is

$$R_t = \frac{L}{kA} \tag{7.9}$$

Substituting,

$$R = \frac{0.3}{(1.0)(30)} = 0.01^\circ\text{C/W} = 0.0005275^\circ\text{F} \cdot \text{h/Btu}$$

Apply Equation (7.8) and substitute.

$$\begin{aligned}\dot{Q} &= \frac{\text{thermal driving force}}{\text{thermal resistance}}\\ &= \frac{40^\circ\text{C}}{0.01^\circ\text{C/W}} = 4000\,\text{W} = 4\,\text{kW}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 7.5

The inside and outside temperatures of the walls of a room are 27°C and 68.7°C, respectively. The room walls consist of 6-inch concrete (C), 8-inch cork-board (B), and 1-inch wood (W) with corresponding thermal conductivities of 0.762, 0.0433, and 0.151 W/m · K, respectively. Calculate the heat transfer rate across the wall and determine the temperature at the interface between the wood and the cork-board.

***SOLUTION*:** Based on the data provided

$$\begin{aligned}T_C &= T_1 = 27^\circ\text{C}\\ T_H &= T_4 = 68.7^\circ\text{C}\\ L_C &= \Delta x_\text{C} = (6)(0.0254) = 0.1524\,\text{m}\\ L_B &= \Delta x_\text{B} = (8)(0.0254) = 0.2032\,\text{m}\\ L_W &= \Delta x_\text{W} = (1)(0.0254) = 0.0254\,\text{m}\end{aligned}$$

Perform the calculations to follow on 1.0 m^2 of area, so that $A = 1.0$. From Equation (7.9),

$$\begin{aligned}R_\text{C} &= \frac{\Delta x_\text{C}}{k_\text{C}} = \frac{L_\text{C}}{k_\text{C}} = \frac{0.1524}{0.762} = 0.200\,\text{K/W}\\ R_\text{B} &= \frac{\Delta x_\text{B}}{k_\text{B}} = \frac{L_\text{B}}{k_\text{B}} = \frac{0.2032}{0.0433} = 4.693\,\text{K/W}\\ R_\text{W} &= \frac{\Delta x_\text{W}}{k_\text{W}} = \frac{L_\text{W}}{k_\text{W}} = \frac{0.0254}{0.151} = 0.1682\,\text{K/W}\end{aligned}$$

Equation (7.17) is employed to calculate the heat transfer rate

$$\dot{Q} = \frac{T_1 - T_4}{R_\text{C} + R_\text{B} + R_\text{W}} = \frac{27 - 68.7}{0.200 + 4.693 + 0.1682} = -8.239\,\text{W}$$

The negative sign indicates that the heat flows from the hot to the cold surface. If the wall area is 4.0 m^2, the heat rate is 32.96 W.

The interface temperature between the wood and cork-board, T_2, can now be calculated,

$$\dot{Q} = \frac{T_1 - T_2}{R_W}$$

$$-8.239 = \frac{27 - T_2}{0.1682}$$

$$T_2 = 28.4^\circ\text{C} = 301.4\,\text{K}$$

■

ILLUSTRATIVE EXAMPLE 7.6

A 3-inch outside diameter steel pipe is covered with a $\frac{1}{2}$-inch layer of asbestos, (*a*), which in turn is covered with a 2-inch layer of glass wool, (*b*). Determine the steady-state heat transfer per foot of pipe, *Z*, if the pipe outer surface temperature is 500°F and the glass wool outer temperature is 100°F.

Assume an asbestos–glass wool interfacial temperature of 300°F and an average asbestos (*a*) temperature of 200°F and glass wool (*b*) temperature of 400°F. Based on a literature review, asbestos and glass wool thermal conductivity values have been estimated to be:

$$k_{(a)}(200^\circ\text{F}) = 0.120\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$$
$$k_{(b)}(400^\circ\text{F}) = 0.0317\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$$

***SOLUTION*:** Apply Equation (7.24) to the materials, asbestos (*a*) and glass wool (*b*),

$$\dot{Q} = \frac{T_H - T_C}{\dfrac{L_a}{(k_a)(A_{a,\text{lm}})} + \dfrac{L_b}{(k_b)(A_{b,\text{lm}})}}$$

For the asbestos (*a*), apply Equation (7.21) while maintaining dimensional consistency:

$$A_{a,\text{lm}} = \frac{\pi Z(D_2 - D_1)/12}{\ln(D_2/D_1)}; \quad Z = \text{pipe length}$$

Substituting

$$A_{a,\text{lm}} = \frac{\pi Z(4.0 - 3.0)/12}{\ln(4.0/3.0)}$$
$$= 0.910z;\ \text{ft}^2$$

Similarly, for glass wool (*b*)

$$A_{b,\text{lm}} = \frac{\pi Z(8.0 - 4.0)/12}{\ln(8.0/4.0)}$$
$$= 1.51z;\ \text{ft}^2$$

Substituting into the equation above gives

$$\dot{Q} = \frac{(500 - 100)}{\dfrac{(0.5/12)}{(0.120)(0.910Z)} + \dfrac{(2.0/12)}{(0.0317)(1.51Z)}}$$

$$= \frac{400}{\dfrac{0.382}{Z} + \dfrac{3.48}{Z}}$$

Factoring out Z gives

$$\frac{\dot{Q}}{Z} = \frac{400}{3.864} = 103.5\,\text{Btu/h}\cdot\text{ft}$$

■

ILLUSTRATIVE EXAMPLE 7.7

Refer to the previous example. Calculate the outer asbestos temperature, T_i (i.e., the interfacial temperature between the asbestos and glass wool).

***SOLUTION*:** Since the heat transfer rate is now known, a single-layer equation can be used to determine the interfacial temperature. For the glass wool layer, Equation (7.24) is then

$$500 - T_i = \frac{(2.0/12)(103.5)}{(0.0317)(1.51)}$$

$$T_i = 500 - 360 = 140^\circ\text{F}$$

Alternatively, for the asbestos layer,

$$T_i - 100 = \frac{(103.5)(0.5/12)}{(0.120)(0.910)}$$

$$T_i = 100 + 39.5 = 139.5^\circ\text{F}$$

The interfacial temperatures are in agreement. ■

MICROSCOPIC APPROACH

As noted in the Introduction to Part Two, this "Microscopic Approach" section has been included to complement the material in this chapter, particularly for those readers interested in a more theoretical approach and treatment of conduction. This section also provides the derivations and/or justifications for the describing equations employed in the earlier macroscopic treatment. Readers are encouraged to compare both approaches while noting that several of the Illustrative Examples to follow were drawn from the literature.(2,3)

The analysis for Fourier's law was presented previously for a rather simple system. Generally, a fluid will possess three temperature gradients with corresponding heat-flux components in a given coordinate system. When this is the case, three heat-flux

Table 7.3 Heat-Flux Components

Component	Rectangular coordinates	Cylindrical coordinates	Spherical coordinates
$\dot{Q}_1'$	$\dot{Q}_x' = -k\dfrac{\partial T}{\partial x}$	$\dot{Q}_r' = -k\dfrac{\partial T}{\partial r}$	$\dot{Q}_r' = -k\dfrac{\partial T}{\partial r}$
$\dot{Q}_2'$	$\dot{Q}_y' = -k\dfrac{\partial T}{\partial y}$	$\dot{Q}_\phi' = -k\dfrac{1}{r}\dfrac{\partial T}{\partial \phi}$	$\dot{Q}_\theta' = -k\dfrac{1}{r}\dfrac{\partial T}{\partial \theta}$
$\dot{Q}_3'$	$\dot{Q}_z' = -k\dfrac{\partial T}{\partial z}$	$\dot{Q}_z' = -k\dfrac{\partial T}{\partial z}$	$\dot{Q}_\phi' = -k\dfrac{1}{r\sin\theta}\dfrac{\partial T}{\partial \phi}$

terms, $\dot{Q}_x'$, $\dot{Q}_y'$, $\dot{Q}_z'$, arise at every point in a system described in rectangular coordinates. These $\dot{Q}/H$ flux terms are given by

$$\dot{Q}_x' = -k\frac{dT}{dx} \tag{7.25}$$

$$\dot{Q}_y' = -k\frac{dT}{dy} \tag{7.26}$$

$$\dot{Q}_z' = -k\frac{dT}{dz} \tag{7.27}$$

All components of the heat flux in a given coordinate system can be expressed in terms of the thermal conductivity of the fluid and a temperature gradient. These are presented in Table 7.3 for rectangular, cylindrical, and spherical coordinates.(2,3) The equations are applicable to all fluids obeying Fourier's law.

ILLUSTRATIVE EXAMPLE 7.8

The temperature of the solid slab pictured in Figure 7.4 is given by the expression

$$T = 100\cos\left(\frac{\pi z}{2h}\right)$$

Calculate the heat flux at $z = 0$ and $z = h$ in terms of the thermal conductivity and h. Comment on the results.

***SOLUTION*:** This problem is solved using rectangular coordinates. First note that $\dot{Q}_x'$ and $\dot{Q}_y'$ are zero. From Table 7.3,

$$\begin{aligned}\dot{Q}_z' &= -k\frac{\partial T}{\partial z}\\ &= -k\frac{dT}{dz}, \text{ since } T \text{ is solely a function of } z.\end{aligned}$$

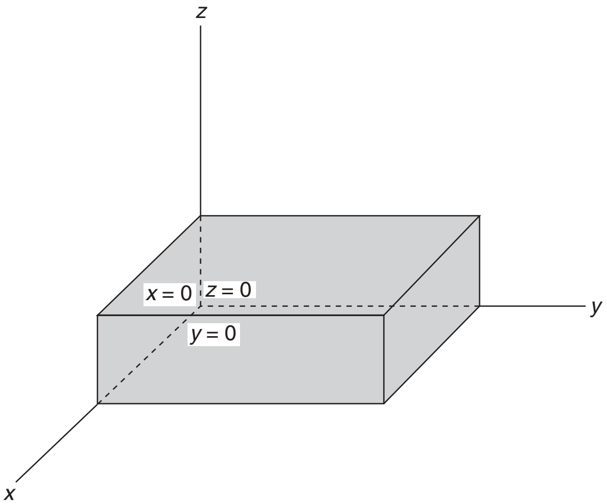

Figure 7.4 Heat-flux components in a rectangular slab.

If

$$T = 100\cos\left(\frac{\pi z}{2h}\right)$$
$$\frac{dT}{dz} = -100\left(\frac{\pi}{2h}\right)\sin\left(\frac{\pi z}{2h}\right)$$
$$\dot{Q}_z' = -k\frac{dT}{dz} = +\frac{50\pi k}{h}\sin\left(\frac{\pi z}{2h}\right)$$

so that

$$\dot{Q}_z' = +\frac{50\pi k}{h}\sin\left(\frac{\pi z}{2h}\right)$$

For the two locations,

$$\dot{Q}_z'|_{z=0} = 0$$
$$\dot{Q}_z'|_{z=h} = +\frac{50\pi k}{h}; \quad \text{with units Btu/s} \cdot \text{ft}^2$$

The above results indicate that energy is generated within the solid since *no* energy enters the solid from the bottom surface (it is insulated) and *finite* energy leaves from the top surface. ■

The equation describing energy transfer in *stationary* solids serves as an excellent starting point for the microscopic presentation of the general equation for energy transfer. The energy-transfer equation for solids is developed by applying the conservation law for energy on a time-rate basis to a fixed-volume element. The derivation is available in the literature.(3,4)

The resulting equation describes the temperature variation in solids due to energy transfer, subject to the assumptions in the development. It applies to either steady-state or unsteady-state systems and is independent of any particular coordinate system.

Table 7.4 Steady-State Energy-Transfer Equation for Stationary Solids

Rectangular coordinates:

$$\alpha\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{A}{\rho c_p}=0; \quad A = \text{source term} \tag{1}$$

Cylindrical coordinates:

$$\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{A}{\rho c_p}=0 \tag{2}$$

Spherical coordinates:

$$\alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right]+\frac{A}{\rho c_p}=0 \tag{3}$$

A = rate of heat generated in solid; energy/time · volume

The describing equations can be expanded into rectangular, cylindrical, and spherical coordinates for *steady-state conduction*. (The *unsteady-state* conduction equation can be found in the next chapter.) The results are presented in Table 7.4 for a solid that behaves in accordance with Fourier's law.

Table 7.4 provides various differential equations that describe steady-state energy transfer in solids. Integration constants must be specified before a complete solution to the differential equations may be obtained. These integration constants are usually obtained from information at specified locations at the boundary and/or knowledge of initial conditions. The most common conditions specified for these energy transfer problems are:

1. The temperature is specified at a solid boundary or surface.
2. The heat flux is specified at a boundary.
3. The temperatures on either side of an interface are equal.
4. The heat flux at either side of an interface are equal. The gradients on either side of an interface may also be specified.
5. The value of the temperature at time zero is given (for unsteady-state systems, see next chapter).

APPLICATIONS

This section contains six illustrative examples adopted from the earlier work of Theodore.(3) The last two are rather lengthy. One solution involves the separation of variables method. Another example introduces the reader to Bessel functions.

ILLUSTRATIVE EXAMPLE 7.9

A long hollow *cylinder* has its inner and outer surfaces maintained at temperatures T_b and T_a, respectively. The inner and outer radii are b and a, respectively. Calculate the temperature profile in the solid section of the cylinder and determine the flux at both surfaces. Comment on the results. Assume steady-state conditions.

SOLUTION**:** This system is represented pictorially in Figure 7.5. This problem is solved using cylindrical coordinates. If end effects are neglected, T is not a function of ϕ and z. Equation (2) in Table 7.4 reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

or

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0, \quad \text{since } T \text{ depends solely on } r$$

Integrating once yields

$$r\frac{dT}{dr} = C$$

Integrating again gives

$$T = C\ln r + B \tag{1}$$

The boundary conditions (BC) are

$$\text{BC(1): } T = T_b \quad \text{at } r = b$$
$$\text{BC(2): } T = T_a \quad \text{at } r = a$$

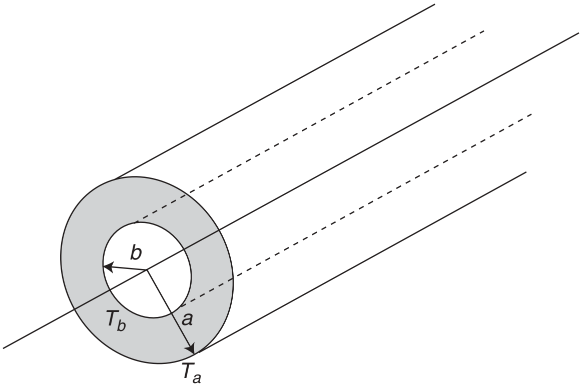

Figure 7.5 Heat conduction, hollow cylinder.

Apply the BC to Equation (1).

$$T_b = C \ln b + B \tag{2}$$

$$T_a = C \ln a + B \tag{3}$$

Solving Equations (2) and (3) simultaneously gives

$$C = \frac{T_a - T_b}{\ln\left(\frac{a}{b}\right)}$$

$$B = T_b - \frac{(T_a - T_b)}{\ln\left(\frac{a}{b}\right)} \ln b$$

Therefore

$$T = T_b + \frac{(T_a - T_b)}{\ln\left(\frac{a}{b}\right)} \ln\left(\frac{r}{b}\right) \tag{4}$$

The heat flux is given by

$$\dot{Q}_r' = -k\frac{dT}{dr}$$

But

$$\frac{dT}{dr} = \frac{C}{r}$$

so that

$$\begin{aligned}\dot{Q}_r' &= -\frac{kC}{r}\\ &= -\frac{k(T_a - T_b)}{\ln\left(\frac{a}{b}\right)}\left(\frac{1}{r}\right)\end{aligned}$$

Therefore

$$\dot{Q}_r'|_{r=a} = -\frac{k(T_a - T_b)}{\ln\left(\frac{a}{b}\right)}\left(\frac{1}{a}\right)$$

$$\dot{Q}_r'|_{r=b} = -\frac{k(T_a - T_b)}{\ln\left(\frac{a}{b}\right)}\left(\frac{1}{b}\right)$$

Although the fluxes are not equal, one can easily show that the rate of energy transfer, $\dot{Q}$, at both surfaces are equal. If the reader so desires, he/she is invited to review Illustrative Examples 7.6 and 7.7 in order to examine and better appreciate how the microscopic approach can be employed in generating the describing equations for this system. ■

ILLUSTRATIVE EXAMPLE 7.10

A constant rate of energy per unit volume A in the form of heat is uniformly generated in a long solid cylinder. The temperature at the outer surface of the cylinder ($r = a$) is maintained at T_a. Calculate the temperature profile and the heat flux at the outer wall under steady-state conditions.

***SOLUTION*:** This problem is solved once again using cylindrical coordinates. Based on the problem statement and physical grounds, it is concluded that the temperature is solely a function of r. Equation (2) in Table 7.4 reduces to

$$\frac{\alpha}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{A}{\rho c_P} = 0$$

Integrating this equation once and rearranging the coefficients gives

$$r\frac{dT}{dr} = -\frac{Ar^2}{2k} + B$$

Integrating again yields

$$T = -\frac{Ar^2}{4k} + B\ln r + C \tag{1}$$

The BC are now written

$$\begin{aligned} &\text{BC(1): } T = T_a && \text{at } r = a \\ &\text{BC(2): } T \neq -\infty && \text{at } r = 0 \end{aligned}$$

or

$$\frac{dT}{dr} = 0 \quad \text{at } r = 0\text{, due to symmetry}$$

Substituting the BC into Equation (1) above gives

$$B = 0$$

$$C = T_a + \left(\frac{A}{4k}\right)a^2$$

so that

$$T = T_a + \frac{A}{4k}(a^2 - r^2) \tag{2}$$

The heat flux at the wall is given by

$$\dot{Q}'_r|_{r=a} = -k\frac{dT}{dr}\bigg|_{r=a}$$

However,

$$\frac{dT}{dr} = -\frac{Ar}{2k}$$

so that

$$\dot{Q}'_r|_{r=a} = \frac{Aa}{2}$$

The maximum temperature is obtained at $r = 0$, i.e.,

$$T_{\max} = T_a + \frac{Aa^2}{4k} \qquad \blacksquare$$

ILLUSTRATIVE EXAMPLE 7.11

A hollow solid *sphere* has its inner ($r = b$) and outer ($r = a$) surfaces maintained at temperature T_b and T_a, respectively. Calculate the temperature profile in the solid at steady-state conditions. Determine the heat flux at both surfaces and comment on the results.

***SOLUTION*:** This system is represented pictorially in Figure 7.6. This problem is solved using spherical coordinates. Based on the problem statement and symmetry, it is concluded that the temperature is not a function of θ and ϕ. Equation (3) in Table 7.4 reduces to

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

Integrating once yields

$$r^2 \frac{dT}{dr} = B$$

Integrating again yields

$$T = -\frac{B}{r} + C \tag{1}$$

The BC are

$$\text{BC(1): } T = T_a \quad \text{at } r = a$$
$$\text{BC(2): } T = T_b \quad \text{at } r = b$$

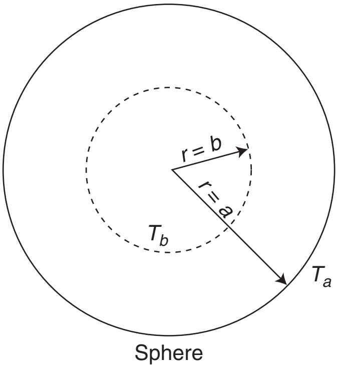

Figure 7.6 Hollow sphere.

Substituting the BC into Equation (1) above gives

$$T_a = -\frac{B}{a} + C \tag{2}$$

$$T_b = -\frac{B}{b} + C \tag{3}$$

Solving Equations (2) and (3) simultaneously results in

$$B = (T_b - T_a)\left(\frac{ba}{b-a}\right)$$

$$C = T_a + (T_b - T_a)\left(\frac{b}{b-a}\right)$$

so that

$$T = T_a + (T_b - T_a)\left(\frac{b}{b-a}\right)\left(1 - \frac{a}{r}\right)$$

The heat flux is given by

$$\dot{Q}_r' = -k\frac{dT}{dr}$$

But,

$$\frac{dT}{dr} = \frac{B}{r^2}$$

so that

$$\dot{Q}_r' = -k\left(\frac{T_a - T_b}{a-b}\right)\left(\frac{ab}{r^2}\right)$$

Therefore

$$\dot{Q}_r'|_{r=a} = -k\left(\frac{T_a - T_b}{a-b}\right)\left(\frac{b}{a}\right)$$

$$\dot{Q}_r'|_{r=b} = -k\left(\frac{T_a - T_b}{a-b}\right)\left(\frac{a}{b}\right)$$

The reader can easily show that the rate of heat transfer across both surfaces are equal by multiplying the flux by the appropriate area. ■

ILLUSTRATIVE EXAMPLE 7.12

An infinite horizontal slab of uniform width h has its lower surface placed in direct contact with the top surface of another slab of the same uniform width. The thermal conductivity of one slab is twice the other. The temperature of the bottom surface of the lower slab is zero, while the temperature of the top surface of the upper slab is T_h. Obtain the temperature profile in both solids at steady-state conditions. Comment on the temperature profile.

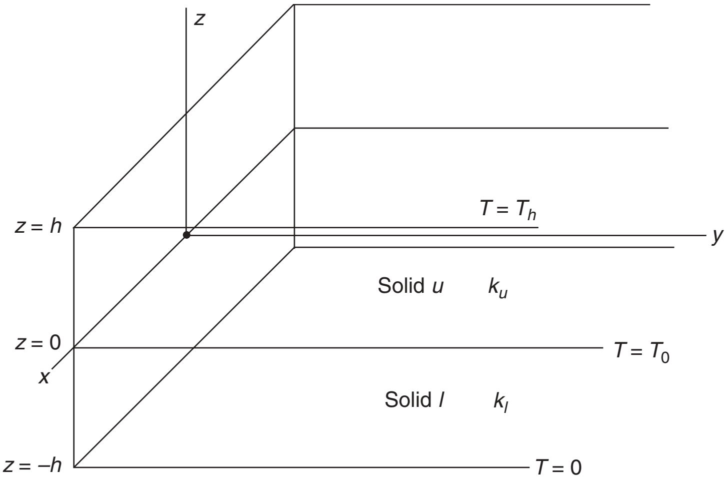

Figure 7.7 Heat conduction, two slabs.

***SOLUTION*:** This system is pictured in Figure 7.7. Select rectangular coordinates with the origin of the z-axis located at the interface of the two solids. A differential equation (DE) is obtained for both solid u and solid l from Table 7.4. The describing equations are given by

$$\frac{d^2T_u}{dz^2} = 0; \text{ solid } u$$

$$\frac{d^2T_l}{dz^2} = 0; \text{ solid } l$$

Integrating once yields

$$\frac{dT_u}{dz} = A_u \tag{1}$$

$$\frac{dT_l}{dz} = A_l \tag{2}$$

The BC are now specified. At $z = 0$, the temperature of u must equal that of l. In addition, the heat flux for both solids must be equal; otherwise, an accumulation of energy at $z = 0$ would result—this is a physical impossibility. Therefore, the BC are

BC(1): at $z = -h$, $T_l = 0$

BC(2): at $z = +h$, $T_u = T_h$

BC(3): at $z = 0$, $T_l = T_u = T_0$

BC(4): at $z = 0$, $q_z|_u = q_z|_l$

or

$$k_u \frac{dT_u}{dz} = k_l \frac{dT_l}{dz}$$

Substituting BC(4) into Equations (1) and (2) above gives

$$k_u \frac{dT_u}{dz} = k_u A_u$$

$$k_l \frac{dT_l}{dz} = k_l A_l$$

or

$$k_u A_u = k_l A_l$$

Integrating Equations (1) and (2) again gives

$$T_u = A_u z + B_u \tag{3}$$

$$T_l = A_l z + B_l \tag{4}$$

Substituting BC(1), BC(2), and BC(3) into Equations (3) and (4) above yields

$$A_u = \frac{T_h}{h}\left(\frac{k_l}{k_l + k_u}\right)$$

$$A_l = \frac{T_h}{h}\left(\frac{k_u}{k_l + k_u}\right)$$

$$B_u = B_l = T_h\left(\frac{k_u}{k_l + k_u}\right)$$

and

$$T_u = T_h\left(\frac{k_l}{k_l + k_u}\right)\left(\frac{z}{h} + \frac{k_u}{k_l}\right)$$

$$T_l = T_h\left(\frac{k_u}{k_l + k_u}\right)\left(\frac{z}{h} + 1\right)$$

If $k_u = 2k_l$, then

$$T_u = \frac{T_h}{3}\left(\frac{z}{h} + 2\right); \quad 0 \leq z \leq h$$

$$T_l = \frac{2T_h}{3}\left(\frac{z}{h} + 1\right); \quad -h \leq z \leq 0$$

At $z = 0$,

$$T_u = T_l = T_0 = \frac{2}{3}T_h$$

The temperature profile is presented in Figure 7.8. ■

ILLUSTRATIVE EXAMPLE 7.13

Consider the two-dimensional problem of a very thin solid bounded by the y-axis ($z = 0$), the lines $y = 0$ and $y = l$, and extending to infinity in the z-direction. The temperature of the vertical

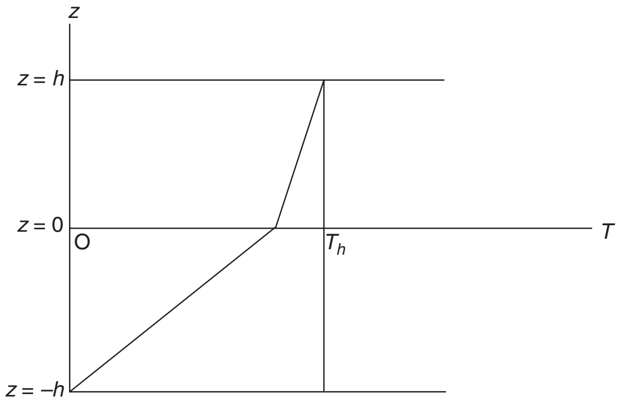

Figure 7.8 Two slab temperature profile for Illustrative Example 7.12.

edge at $y = 0$ and $y = l$ is maintained at zero. The temperature at $z = 0$ is T_0. Determine the steady-state temperature profile in the solid. Assume both plane surfaces of the solid are insulated.

***SOLUTION*:** A two-dimensional view of this system is presented in Figure 7.9. This problem is solved in rectangular coordinates. Based on the problem statement, T is not a function of x. "Extract" the following equation from Table 7.4.

$$\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{1}$$

This partial differential equation (PDE) is solved using the separation-of-variables method. Since the temperature of the solid is a function of y and z, assume that the solution can be separated into the product of one function $\psi(y)$, which depends solely on y and a second function $\Phi(z)$ that depends only on z.

$$\begin{aligned} T &= T(y, z) \\ &= \psi(y)\Phi(z) \\ &= \psi\Phi \end{aligned} \tag{2}$$

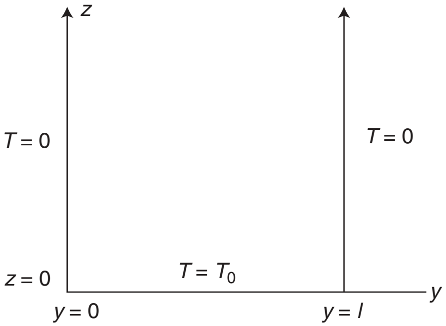

Figure 7.9 Two-dimensional heat transfer; Illustrative Example 7.13.

The first term of Equation (1) above is

$$\frac{\partial^2 T}{\partial y^2} = \Phi \psi'' \tag{3}$$

Note that the double prime represents the second derivative. The second term takes the form

$$\frac{\partial^2 T}{\partial z^2} = \psi \Phi'' \tag{4}$$

Substituting Equations (3) and (4) into Equation (1) gives

$$\Phi \psi'' + \psi \Phi'' = 0$$

or

$$\underbrace{\frac{\psi''}{\psi}}_{\substack{\downarrow\downarrow\downarrow\downarrow \\ \text{function} \\ \text{of } y}} = \underbrace{-\frac{\Phi''}{\Phi}}_{\substack{\downarrow\downarrow\downarrow\downarrow \\ \text{function} \\ \text{of } z}} \tag{5}$$

Since both terms are functions of different independent variables, it is concluded that both terms in Equation (5) are equal to the same constant, which is noted as $-\lambda^2$.

$$\frac{\psi''}{\psi} = -\frac{\Phi''}{\Phi} = -\lambda^2 \tag{6}$$

The reader can verify that a positive constant or zero does not contribute to the solution of the problem. The solution to Equation (6) takes the form

$$\psi = a'_\lambda \sin \lambda y + b'_\lambda \cos \lambda y; \quad a'_\lambda \text{ and } b'_\lambda \text{ are constants that depend on the value of } \lambda$$

$$\Phi = Ce^{\lambda z} + De^{-\lambda z}$$

Since

$$\begin{aligned} T &= \psi \Phi \\ T &= (a'_\lambda \sin \lambda y + b'_\lambda \cos \lambda y)(Ce^{\lambda z} + De^{-\lambda z}) \end{aligned} \tag{7}$$

The BC for the solid are:

BC(1): $T = 0$ at $y = 0$

BC(2): $T = 0$ at $y = l$

BC(3): $T = T_0$ at $z = 0$

BC(4): $T = 0$ at $z = \infty$

Based on physical grounds, BC(4) gives

$$C = 0$$

Therefore

$$
\begin{aligned}
T &= (a'_\lambda \sin \lambda y + b'_\lambda \cos \lambda y) D e^{-\lambda z} \\
&= (a_\lambda \sin \lambda y + b_\lambda \cos \lambda y) e^{-\lambda z};\ a_\lambda = a'_\lambda D
\end{aligned}
$$

BC(1) gives

$$b_\lambda = 0$$

and

$$T = a_\lambda (\sin \lambda y) e^{-\lambda z} \tag{8}$$

BC(2) gives

$$0 = a_\lambda (\sin \lambda l) e^{-\lambda z}$$

The RHS of this equation is zero if $a_\lambda = 0$ or $\sin \lambda l = 0$. If a_λ is zero, no solution results. Therefore

$$\sin \lambda l = 0$$

and

$$
\begin{aligned}
\lambda l &= n\pi; \quad n = 1, 2, 3, 4, \ldots . \\
\lambda &= \frac{n\pi}{l}
\end{aligned}
$$

Equation (8) now becomes

$$T = a_n e^{-(n\pi/l)z} \sin\left(\frac{n\pi y}{l}\right)$$

The constant a_n is evaluated from BC(3),

$$T_0 = a_n \sin\left(\frac{n\pi y}{l}\right)$$

A Fourier series analysis$^{(3,4)}$ gives

$$a_n = \frac{2T_0}{n\pi}[(-1)^{n+1} + 1]$$

Conforming all of the above gives

$$T = \sum_{n=1}^{\infty} \frac{2T_0}{n\pi}[(-1)^{n+1} + 1] e^{-(n\pi/l)z} \sin\left(\frac{n\pi y}{l}\right) \tag{9}$$

■

ILLUSTRATIVE EXAMPLE 7.14

Energy is being absorbed in a long solid cylinder of radius a. The temperature at the outer surface of the cylinder is maintained at T_a. Calculate the temperature profile in the solid at steady-state conditions. Assume the energy generation term A is a linear function of the temperature (i.e., $A = -ST$, where S is a positive constant).

***SOLUTION*:** This system is presented in Figure 7.10, and the problem is solved in cylindrical coordinates. The appropriate DE is "extracted" from Table 7.4. Since

$$T = T(r) \neq T(\phi, z, t)$$

the following equation is obtained

$$\frac{\alpha}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{A}{\rho c_p} = 0 \tag{1}$$

where

$$A = -ST$$

Substituting and expanding the above DE,

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} - \left(\frac{S}{k}\right)T = 0 \tag{2}$$

This is one form of Bessel's equation. The solution is given by

$$T = BI_0\left[r\left(\frac{S}{k}\right)^{1/2}\right] + CK_0\left[r\left(\frac{S}{k}\right)^{1/2}\right] \tag{3}$$

where I_0 = "modified" Bessel function; first kind, zero order
K_0 = "modified" Bessel function; second kind, zero order
B, C = integration constants

The BC are now written

BC(1): at $r = a$, $T = T_a$

BC(2): at $r = 0$, $\frac{dT}{dr} = 0$ due to symmetry

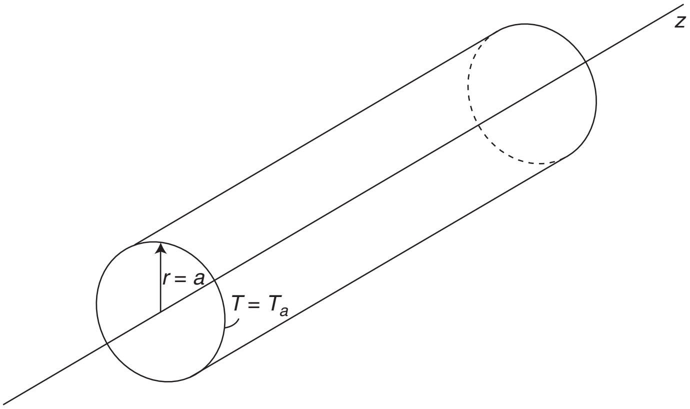

Figure 7.10 Heat absorption in a solid cylinder; Illustrative Example 7.14.

or

$$\text{at } r = 0,\ T = \text{finite}$$

Taking the derivative of Equation (3) leads to

$$\frac{dT}{dr} = B\left(\frac{S}{k}\right)^{1/2} I_1\left[r\left(\frac{S}{k}\right)^{1/2}\right] - C\left(\frac{S}{k}\right)^{1/2} K_1\left[r\left(\frac{S}{k}\right)^{1/2}\right] \tag{4}$$

The above Bessel functions have the property

$$\begin{aligned} I_0(0) &= \text{finite} \\ K_0(0) &= \infty \\ I_1(0) &= 0 \\ K_1(0) &= \infty \end{aligned}$$

Applying BC(2) gives

$$C = 0$$

Applying BC(1) gives

$$T_a = BI_0\left[a\left(\frac{S}{k}\right)^{1/2}\right]$$

so that

$$B = \frac{T_a}{I_0\left[a\left(\frac{S}{k}\right)^{1/2}\right]}$$

Hence, the solution to Equation (1) is

$$T = T_a \frac{I_0\left[r\left(\frac{S}{k}\right)^{1/2}\right]}{I_0\left[a\left(\frac{S}{k}\right)^{1/2}\right]} \tag{5}$$

■

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Chapter 8

Unsteady-State Heat Conduction

INTRODUCTION

Engineers and scientists spend a good deal of time working with systems that are operating under steady-state conditions. However, there are many processes that are transient in nature, and it is necessary to be able to predict how process variables will change with time, as well as how these effects will impact the design and performance of these systems. The prediction of the unsteady-state temperature distribution in solids is an example of one such process. It can be accomplished very effectively using conduction equations; the energy balance equations can usually be easily solved to calculate the spatial and time variation of the temperature within the solid.

In a very real sense, this material is an extension of that presented in the previous chapter. The relationships developed in the preceding chapter applied only to the steady-state conditions in which the heat flow and spatial-temperature profile were constant with time. *Unsteady-state* processes are those in which the heat flow, the temperature, or both, vary with time at a fixed point in space. Batch heat-transfer processes are typical unsteady-state processes. For example, heating reactants in a tank or the startup of a cold furnace are two unsteady-state applications. Still other common examples include the rate at which heat is conducted through a solid while the temperature of the heat source varies, the daily periodic variations of the heat of the Sun on various solids, the quenching of steel in an oil or cold water bath, cleaning or regeneration processes, or, in general, any process that can be classified as intermittent.

A good number of heat transfer conduction problems are time dependent. These *unsteady*, or *transient*, situations usually arise when the boundary conditions of a system are changed. For example, if the surface temperature of a solid is changed, the temperature at each point in the solid will also change. For some cases, the changes will continue to occur until a *steady-state* temperature distribution is reached.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

Transient effects also occur in many industrial heating and cooling processes involving solids. The solids are generally designated in one of three physical categories:

1. Finite
2. Semi-infinite
3. Infinite

Each of the above three geometries receives treatment later in this chapter.

The remaining three sections of this chapter address a general all-purpose topic that classifies a host of application categories. The chapter concludes with a treatment of heat conduction from a microscopic viewpoint and is followed, in turn, by applications that are based on these microscopic equations.

The reader should note that the solution of heat conduction problems using Monte Carlo methods is treated in Chapter 26.

CLASSIFICATION OF UNSTEADY-STATE HEAT CONDUCTION PROCESSES

In order to treat common applications of batch and unsteady-state heat transfer, Kern(1) defined processes as either liquid (fluid) heating or cooling and solid heating or cooling. These real world examples are outlined below.

1. Heating and cooling liquids
    a. Liquid batches
    b. Batch reactors
    c. Batch distillation
2. Heating and cooling solids
    a. Constant solid temperature
    b. Periodically varying temperature
    c. Regenerators
    d. Granular solids in stationary beds
    e. Granular solids in fluidized beds

The *physical* representation of several solid systems is provided below.

1. Finite wall (or slab or plate)
2. Semi-infinite solid
3. Semi-infinite flat wall
4. Infinite flat wall
5. Finite rectangular parallelepiped
6. Finite hollow rectangular parallelepiped
7. Semi-infinite rectangular parallelepiped
8. Infinite rectangular parallelepiped

9. Short finite cylinder
10. Long finite cylinder
11. Short finite hollow cylinder
12. Long finite hollow cylinder
13. Semi-infinite cylinder
14. Semi-infinite hollow cylinder
15. Infinite cylinder
16. Infinite hollow cylinder
17. Sphere
18. Hollow sphere

The reader should also note that most of these geometric systems are employed to describe not only conduction systems but also forced convection (Chapter 9), free convection (Chapter 10), and radiation (Chapter 11) systems. Although a comprehensive treatment of all of the above is beyond the scope of this text, the reader is referred to the classic work of Carslaw and Jaeger(2) for a truly all-encompassing treatment of nearly all of these systems.

The above categories can be further classified to include specific applications involving unsteady-state heat conduction in *solids*. These include:

1. Walls of furnaces
2. Structural supports
3. Mixing elements
4. Cylindrical catalysts
5. Spherical catalysts
6. Fins (Chapter 17)
7. Extended surfaces (Chapter 17)
8. Insulating materials (Chapter 19)

Transient heat transfer in infinite plates, infinite cylinders, finite cylinders, spheres, bricks, and other composite shapes has been studied extensively in the literature. Farag and Reynolds(3) provide an excellent review that is supplemented with numerous worked illustrative examples; a semi-theoretical approach to describing the time-position variations in these systems has been simplified by use of a host of figures and tables.

MICROSCOPIC EQUATIONS

As noted in the Introduction to Part Two, this “Microscopic Equations” section has been included to complement the qualitative material presented earlier in this chapter. It should serve the needs of those readers interested in a more theoretical approach and treatment of conduction. This section also provides the results of the derivations

Table 8.1 Unsteady-State Energy-Transfer Equation for Stationary Solids

Rectangular coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{A}{\rho c_p} \tag{1}$$

Cylindrical coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{A}{\rho c_p} \tag{2}$$

Spherical coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right] + \frac{A}{\rho c_p} \tag{3}$$

and/or justifications for the describing equations employed in the "Applications" section to follow.

The microscopic equations describing transient heat conduction can be found in Table 8.1.$^{(4,5)}$ The equations are provided for rectangular, cylindrical, and spherical coordinates, and are valid subject to the assumptions specified for Table 7.4 in the previous chapter.

APPLICATIONS

This section contains six illustrative examples adopted from the earlier work of Theodore.$^{(4)}$ The last example is rather lengthy. One solution re-examines the separation of variables method. Another example introduces the reader to the error function.

ILLUSTRATIVE EXAMPLE 8.1

A constant rate of energy per unit volume (A) is *uniformly* liberated in a solid of arbitrary shape. The solid is insulated. Obtain the temperature of the solid as a function of position and time if the initial temperature of the solid is everywhere zero.

***SOLUTION*:** Based on the problem statement and physical grounds, it is concluded that the temperature of the solid is *not* a function of position. Therefore, the describing equation(s) are independent of the coordinate system. The equation(s) in Table 8.1 reduce to

$$\frac{\partial T}{\partial t} = \frac{A}{\rho c_p}$$

or

$$\frac{dT}{dt} = \frac{A}{\rho c_p}$$

since T is solely a function of t. Integrating this equation gives

$$T = \frac{A}{\rho c_P} t + B$$

The initial condition (IC) is

$$T = 0 \quad \text{at } t = 0$$

Therefore, $B = 0$ and

$$T = \left(\frac{A}{\rho c_p}\right) t$$

The temperature of the solid will increase linearly with time. ■

ILLUSTRATIVE EXAMPLE 8.2

The temperature of an infinite horizontal slab of uniform width h is everywhere zero. The temperature at the top of the slab is then set and maintained at T_h, while the bottom surface is maintained at zero. Determine the temperature profile in the slab as a function of position and time.

***SOLUTION*:** The "initial" one-dimensional profile of this system is described in Figure 8.1(a)–(b). The steady-state solution takes the form (see Chapter 7). See also Fig. 8.1(d).

$$T = T_h\left(\frac{z}{h}\right)$$

During the transient period (see Figure 8.1(c)), T is a function of t as well as z, that is,

$$T = T(z, t)$$

This problem is solved in rectangular coordinates. Now proceed to "extract" the following equation from Table 8.2:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} \tag{1}$$

Note that the partial derivative signs may *not* be replaced by ordinary derivatives since T is a function of two variables, z and t. The separation-of-variables method is again applied. Assume that the solution for T may be separated into the product of one function $\psi(z)$ that depends solely on z, and by a second function $\theta(t)$ that depends only on t,

$$\begin{aligned} T &= T(z, t) \\ &= \psi(z)\theta(t) \\ &= \psi\theta \end{aligned} \tag{2}$$

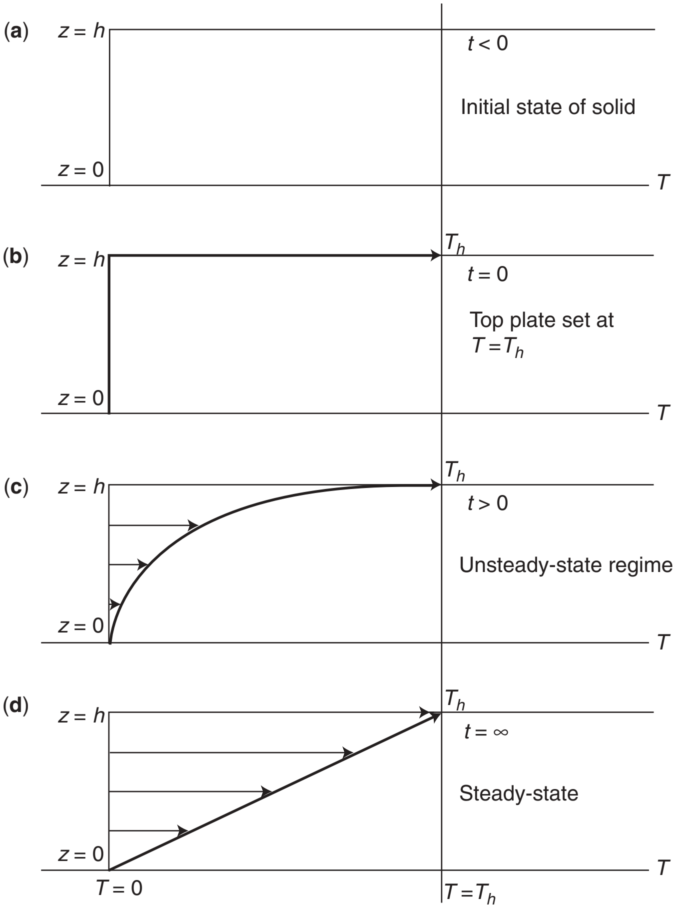

Figure 8.1 Temperature profile; Illustrative Example 8.2.

The left-hand side of Equation (1) above becomes

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{\partial}{\partial t}(\psi\theta) \\ &= \psi\theta'\end{aligned} \tag{3}$$

The right-hand side of Equation (1) is given by

$$\alpha\frac{\partial^2 T}{\partial z^2} = \alpha\theta\psi'' \tag{4}$$

Combining Equations (3) and (4)

$$\frac{\theta'}{\alpha\theta} = \frac{\psi''}{\psi} = -\lambda^2 \tag{5}$$

where $-\lambda^2$ is a constant. The BC and/or IC are now written:

$$\begin{aligned}
&\text{BC(1): at } z = 0,\ T = 0 \ \text{ for } t \geq 0\\
&\text{BC(2): at } z = h,\ T = T_h \ \text{ for } t > 0\\
&\text{IC: } \quad\ \text{at } t = 0,\ T = 0 \ \text{ for } 0 \leq z \leq h
\end{aligned}$$

The following solution results if $-\lambda^2$ is zero:

$$T_0 = C_1 + C_2 z \tag{6}$$

Substituting BC(1) and BC(2) into Equation (5) gives

$$T_0 = T_h\left(\frac{z}{h}\right) \tag{7}$$

Note that Equation (7) is the steady-state solution to the partial differential equation (PDE). If the constant is nonzero, one obtains

$$T_{-\lambda^2} = e^{-\alpha\lambda^2 t}(a_\lambda \sin \lambda z + b_\lambda \cos \lambda z) \tag{8}$$

Equations (7) and (8) are both solutions to Equation (1). Since Equation (1) is a linear PDE, the sum of Equations (7) and (8) also is a solution, that is,

$$\begin{aligned}
T &= T_0 + T_{-\lambda^2}\\
&= T_h\left(\frac{z}{h}\right) + e^{-\alpha\lambda^2 t}(a_\lambda \sin \lambda z + b_\lambda \cos \lambda z)
\end{aligned} \tag{9}$$

Resubstitution of the BC gives

$$T = T_h\left(\frac{z}{h}\right) + a_n e^{-\alpha(n\pi/h)^2 t} \sin\left(\frac{n\pi z}{h}\right) \tag{10}$$

The constant a_n is evaluated using the IC.

$$a_n = \frac{2T_h}{n\pi}(-1)^n$$

Combining the above gives

$$T = \underbrace{T_h\left(\frac{z}{h}\right)}_{\text{steady-state solution}} + \underbrace{\frac{2T_h}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n} e^{-\alpha(n\pi/h)^2 t} \sin\left(\frac{n\pi z}{h}\right)}_{\text{transient solution}} \tag{11}$$

■

ILLUSTRATIVE EXAMPLE 8.3

Another system, somewhat similar but slightly different from the previous application is examined in this illustrative example.

Consider the insulated cylindrical copper rod pictured in Figure 8.2. If the rod is initially ($t = 0$) at T_A and the ends of the rod are maintained at T_S at $t \geq 0$, provide an equation that describes the temperature (profile) in the rod as a function of both position and time.

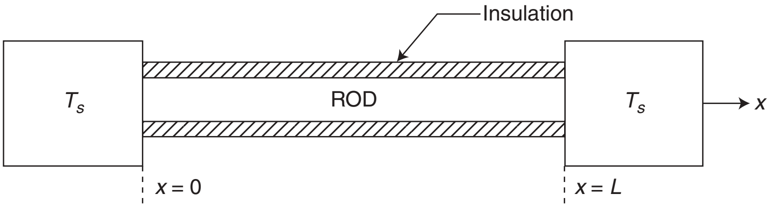

Figure 8.2 Transient rod temperature system; Illustrative Example 8.3.

***SOLUTION*:** Although the rod is of cylindrical form, the geometry of the system is best described in rectangular coordinates. The describing equation once again takes the form

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

However, the BC and/or IC are different and given by

$$\begin{aligned} T &= T_A \quad \text{at } t = 0 \quad \text{(IC)} \\ T &= T_S \quad \text{at } x = 0 \quad \text{(BC)} \\ T &= T_S \quad \text{at } x = L \quad \text{(BC)} \end{aligned}$$

The solution to this equation can again be obtained via the separation of variables technique. The solution is given by[(4)]

$$T = T_S + (T_A - T_S)\left[\sum_{n=1}^{\infty} 2\left[\frac{(-1)^{n+1} + 1}{n\pi}\right] e^{-\alpha(n\pi/L)^2 t} \sin\left(\frac{n\pi x}{L}\right)\right] \tag{2}$$

Additional details are provided in Chapter 26. ■

ILLUSTRATIVE EXAMPLE 8.4

Refer to Illustrative Example 8.3. If T_S represents saturated steam at 15 psig, the initial temperature T_A is 71°F, and the rod is 20 inches in length of stainless steel with $k = 9.1\ \text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$, $\rho = 0.29\ \text{lb/in}^3$, and $c_p = 0.12\ \text{Btu/lb} \cdot {}^\circ\text{F}$, calculate the temperature 0.875 inches from one of the ends after 30 minutes. Note: This unit was designed by the author and is located in the Unit Operations Laboratory of Manhattan College.

***SOLUTION*:** First calculate α

$$\alpha = \frac{9.1}{(0.29)(17.28)(0.12)} = 0.151 \approx 0.15$$

From the steam tables in the Appendix, at $\text{P} = 15 + 14.7 = 29.7$ psig, $T_S = 249.7^\circ\text{F}$. Assuming only the first term (i.e., $n = 1$), in the infinite series in Equation (2) above contributes significantly to the solution, one obtains

$$T = 249.7 + (71 - 249.7)\left[\frac{(2)(-1)^2 + 1}{(1)(\pi)}\right] e^{-0.15[(1)(\pi)/(20/12)]^2 0.5} \sin\left[\frac{(1)(\pi)(0.875)}{20}\right]$$
$$= 232^\circ\text{F}$$

The reader is left the exercise of determining the effect on the calculation by including more terms in the infinite series.

The finite difference method of solving the differential equation describing this system can be found in Chapter 26. As indicated earlier, Chapter 26 also details how the Monte Carlo method can be applied in the solution to some partial differential equations. ■

ILLUSTRATIVE EXAMPLE 8.5

Outline how to verify that Equation (2) in Illustrative Example 8.3 describes the system.

***SOLUTION*:** Equation (2) may be verified if it satisfies the boundary and initial conditions *and* if the equation can be differentiated in order to satisfy Equation (1). For example, for BC(1),

$$T = T_S \quad \text{at} \quad x = L$$

$$\begin{aligned} T &= T_S + (T_A - T_S)\left[\sum_{n=1}^{\infty} \frac{4}{n\pi} e^{-\alpha(n\pi/L)^2 t} \sin\left(\frac{n\pi L}{L}\right)\right] \\ &= T_S + (T_A - T_S)(0) \\ &= T_S \end{aligned}$$

For BC(2),

$$T = T_S \quad \text{at } x = 0$$

so that

$$\begin{aligned} T &= T_S + (T - T_S)\left[\sum_{n=1}^{\infty} \frac{4}{n\pi} e^{-\alpha(n\pi/L)^2 t} \sin(0)\right] \\ &= T_S + (T - T_S)(0) \\ &= T_S \end{aligned}$$

Differentiating Equation (2) to determine if it satisfies Equation (1) is left as an exercise for the reader. ■

ILLUSTRATIVE EXAMPLE 8.6

A plane membrane, impervious to the transfer of heat, separates an infinite solid into two equal parts. One half of the solid's temperature is initially at T_0 while the other half is at zero. At time $t = 0$, the membrane is removed and the solids brought into direct contact with each other. Calculate the temperature in the solid as a function of position and time.

***SOLUTION*:** The initial temperature profile in the system is shown in Figure 8.3. The PDE describing this system can easily be shown to be

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{1}$$

where $T = T(y, t)$.

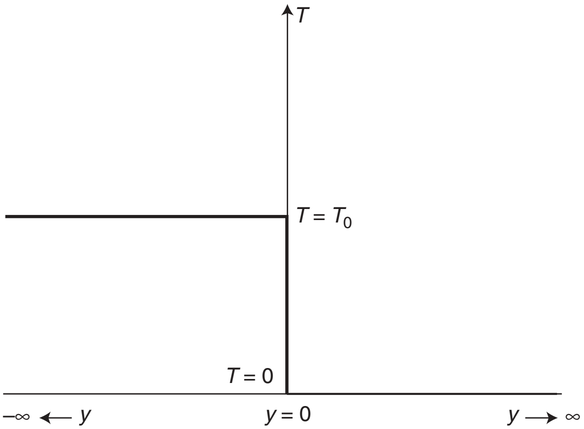

Figure 8.3 Infinite solid separated by a membrane: initial profile; Illustrative Example 8.6.

The BC and IC are

BC(1): $T = \text{finite}$ at $y = \infty$, for all t and approaching $\dfrac{T_0}{2}$ as $t \to \infty$

BC(2): $T = \text{finite}$ at $y = -\infty$, for all t and approaching $\dfrac{T_0}{2}$ as $t \to \infty$

IC: $T = T_0$ at $t = 0$, for $-\infty \le y \le 0$

$T = 0$ at $t = 0$, for $0 < y \le +\infty$

The separation-of-variables method is usually applicable to finite-media systems. However, this technique often produces meaningless results when used for semi-infinite or infinite media. Equation (1) can be solved by using Laplace transforms or Fourier integrals. An outline of the Laplace transform method of solution is presented below.

Begin by multiplying both sides of Equation (1) by $e^{-pt}\,dt$ and integrating from 0 to ∞

$$\int_0^\infty e^{-pt}\frac{\partial T}{\partial t}dt = \int_0^\infty \alpha\frac{\partial^2 T}{\partial y^2}e^{-pt}dt \tag{2}$$

The left-hand side (LHS) of Equation (2) is integrated by parts to give

$$\begin{aligned}\text{LHS} &= e^{-pt}T|_0^\infty + p\int_0^\infty e^{-pt}T\,dt \\ &= -T(0) + p\overline{T};\ T(0) = \text{initial temperature}\end{aligned} \tag{3}$$

where the integral

$$\int_0^\infty e^{-pt}\,T\,dt \tag{4}$$

is represented by $\overline{T}$ and defined as the Laplace transform of T. $\overline{T}$ is a function of p and y but not of t. The right-hand side (RHS) of Equation (2) becomes

$$\text{RHS} = \alpha \frac{\partial^2}{\partial y^2} \int_0^\infty T e^{-pt} \, dt \tag{5}$$

$$= \alpha \frac{\partial^2 \overline{T}}{\partial y^2} = \alpha \frac{d^2 \overline{T}}{dy^2} \tag{6}$$

The resulting equation is

$$-T(0) + p\overline{T} = \alpha \frac{d^2 \overline{T}}{dy^2} \tag{7}$$

Note that this operation has converted the PDE(1) to an ODE(7) and eliminated time as a variable. It is an order of magnitude easier to integrate. This ordinary DE can now be solved subject to a revised BC. The remainder of this solution is not presented. Instead, the following simpler method of solution is discussed. First note that Laplace's source solution satisfies$^{(2,4)}$ the above PDE(1), that is,

$$T(y, t) = \frac{1}{\sqrt{t}} e^{-(y^2/4\alpha t)} \tag{8}$$

One can easily verify that this is a solution to the equation, since

$$\frac{\partial T}{\partial t} = -\frac{1}{2t^{3/2}} e^{-\xi^2} + \frac{1}{t^{1/2}} e^{-\xi^2} \left(\frac{y^2}{4\alpha t^2} \right) \tag{9}$$

$$\frac{\partial T}{\partial y} = -\frac{1}{t^{1/2}} e^{-\xi^2} \left(\frac{y}{2\alpha t} \right) \tag{10}$$

$$\frac{\partial^2 T}{\partial y^2} = -\frac{1}{2\alpha t^{3/2}} e^{-\xi^2} + \frac{y^2}{4\alpha t^{5/2}} e^{-\xi^2} \tag{11}$$

where

$$\xi^2 = \frac{y^2}{4\alpha t}$$

The solution

$$T(y, t) = \frac{B}{\sqrt{4\pi\alpha t}} e^{-(y^2/4\alpha t)} \tag{12}$$

where

$$B = \text{constant} = \frac{Q}{\rho c_p A} \tag{13}$$

also satisfies Equation (1). Q/A is a source of heat (Q per unit area A) located at the point $y = 0$ (origin) at $t = 0$. If the location of the source is changed from point $y = 0$ to $y = y'$, the solution becomes

$$T(y, t) = \frac{Q}{\rho c_p A\sqrt{4\pi\alpha t}} e^{-[(y-y')^2/4\alpha t]} \tag{14}$$

The quantity of heat located at $y = y'$ must be differential, dQ, since it is located at a point, the dimensions of which are differential. If the rest of the system is initially at zero temperature, this heat will produce a differential temperature at any other y and t, that is,

$$dT(y, t) = \frac{dQ}{\rho c_p A\sqrt{4\pi\alpha t}} e^{-[(y-y')^2/4\alpha t]} \tag{15}$$

The volume element at y' is $A\,dy'$. The amount of heat present in the volume element at y' is initially

$$dQ = T(y')A\rho c_P\,dy' \tag{16}$$

where $T(y')$ is the initial temperature at y'. Therefore,

$$dT(y, t) = \frac{T(y')}{\sqrt{4\pi\alpha t}} e^{-[(y-y')^2/4\alpha t]}\,dy' \tag{17}$$

This is the equation describing the temperature at any y and t for a source initially located at y'. Therefore, to calculate $T(y, t)$ for all sources between $-\infty$ and $+\infty$, sum the above equation (integrate) over all values of y', that is,

$$T(y, t) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{4\pi\alpha t}} e^{-[(y-y')^2/4\alpha t]} T(y')\,dy' \tag{18}$$

Note that y and t are constants and y' varies in the integration of Equation (18). The reader can verify that Equation (18) is also a solution to Equation (1). Equation (18) is a solution to systems with infinite media providing $T(y')$ is the *initial* temperature profile in the system. Physically, the above solution represents the temperature at any t and any y due to the presence of an initial temperature at y'. The summation, or integral, of all the initial contributions gives rise to $T(y, t)$.

The initial temperature distribution in this problem is

$$\begin{aligned} T &= T_0 \quad \text{for } -\infty \le y < 0 \\ T &= 0 \quad\ \ \text{for } 0 < y \le \infty \end{aligned}$$

Therefore

$$T = \frac{1}{2\sqrt{\alpha\pi t}} \int_{y'=-\infty}^{y'=0} T_0 e^{-[(y-y')^2/4\alpha t]}\,dy' \tag{19}$$

$$+ \frac{1}{2\sqrt{\alpha\pi t}} \int_{y'=0}^{y'=\infty} (0) e^{-[(y-y')^2/4\alpha t]}\,dy'$$

$$= \frac{T_0}{2\sqrt{\alpha\pi t}} \int_{-\infty}^{0} e^{-[(y-y')^2/4\alpha t]}\,dy' \tag{20}$$

Introducing a new variable,

$$\xi = \frac{y - y'}{\sqrt{4\alpha t}} \tag{21}$$

so that

$$y' = y - \sqrt{4\alpha t}\,\xi \tag{22}$$

$$dy' = -\sqrt{4\alpha t}\,d\xi \tag{23}$$

and

$$\text{at } y' = 0, \quad \xi = \frac{y}{\sqrt{4\alpha t}}$$

$$\text{at } y' = -\infty, \quad \xi = +\infty$$

Therefore, (after reversing limits),

$$\begin{aligned}
T &= -\frac{T_0}{\sqrt{4\pi\alpha t}} \int_{\xi=\infty}^{\xi=y/\sqrt{4\alpha t}} e^{-\xi^2} \sqrt{4\alpha t}\,d\xi \\
&= -\frac{T_0}{\sqrt{\pi}} \int_{\infty}^{y/\sqrt{4\alpha t}} e^{-\xi^2}\,d\xi \\
&= -\frac{T_0}{\sqrt{\pi}} \int_{y/\sqrt{4\alpha t}}^{\infty} e^{-\xi^2}\,d\xi \\
&= \frac{T_0}{\sqrt{\pi}} \left[\int_0^{\infty} e^{-\xi^2}\,d\xi - \int_0^{y/\sqrt{4\alpha t}} e^{-\xi^2}\,d\xi \right]
\end{aligned} \tag{24}$$

$$\underset{I(1)}{\downarrow\;\downarrow\;\downarrow\;\downarrow} \qquad \underset{I(2)}{\downarrow\;\downarrow\;\downarrow\;\downarrow}$$

For the first integral, $I(1)$, proceed as follows. Set

$$\xi^2 = x; \quad \xi = x^{1/2} \tag{25}$$

so that

$$2\xi\,d\xi = dx$$

$$d\xi = \frac{dx}{2\xi}$$

$$d\xi = \tfrac{1}{2}x^{-1/2}\,dx$$

and

$$I(1) = \frac{1}{2}\int_0^{\infty} e^{-x}x^{-1/2}\,dx \tag{26}$$

$$= \frac{\Gamma(\frac{1}{2})}{2} = \frac{\sqrt{\pi}}{2} \tag{27}$$

where $\Gamma =$ gamma function.

For $I(2)$, first define the error function, erf,

$$\operatorname{erf} z = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\xi^2}\, d\xi \tag{28}$$

Therefore,

$$\begin{aligned} T &= \frac{T_0}{2} - \frac{T_0}{2} \operatorname{erf} z \\ &= \frac{T_0}{2}\left[1 - \operatorname{erf}\left(\frac{y}{\sqrt{4\alpha t}}\right)\right] \end{aligned} \tag{29}$$

The error function is tabulated in most advanced mathematics texts.$^{(6,7)}$ A plot of erf z against z is presented in Figure 8.4. Note that this function has the properties

$$\begin{aligned} \operatorname{erf}(\infty) &= 1.0 \\ \operatorname{erf}(0) &= 0 \\ \operatorname{erf}(-\infty) &= -1.0 \end{aligned}$$

and

$$\operatorname{erf}(-z) = -\operatorname{erf}(z)$$

It can also be evaluated numerically by expanding $e^{-\xi^2}$ in a power series,

$$e^{-\xi^2} = 1 - \xi^2 + \tfrac{1}{2}\xi^4 - \cdots \tag{30}$$

The RHS of this equation can then integrated term by term to give

$$\operatorname{erf} z = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \tag{31}$$

This power series converges for all values of z. However, it converges very slowly for large z.

The reader can verify that Equation (29) is a solution to Equation (1). A check of the BC and/or IC yields

$$\text{BC: at } t = \infty \text{ and any } y,\ T = \frac{T_0}{2}$$

$$\text{IC: at } t = 0 \text{ and } y > 0,$$

$$\begin{aligned} T &= \frac{T_0}{2}[1 - \operatorname{erf}(\infty)] \\ &= \frac{T_0}{2}[1 - 1] \\ &= 0 \end{aligned}$$

$$\text{at } t = 0 \text{ and } y < 0,$$

$$\begin{aligned} T &= \frac{T_0}{2}[1 - \operatorname{erf}(-\infty)] \\ &= \frac{T_0}{2}[1 - (-1)] \\ &= T_0 \end{aligned}$$

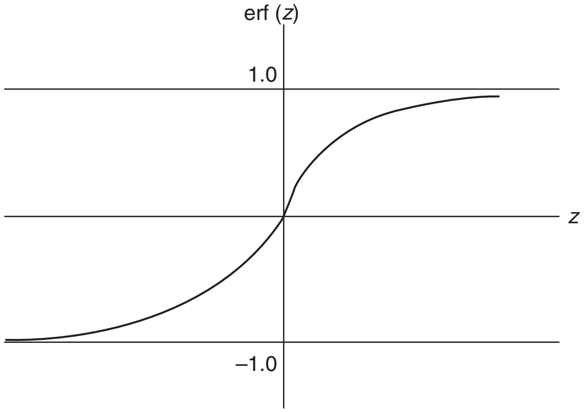

Figure 8.4 Error function.

The reader should note that the analysis above can be extended to include systems with *continuous* sources of heat. The equation describing the temperature due to an *instantaneous* source of heat at $t = 0$ and $y = 0$ was given by [see also Equation (15)].

$$dT(y, t) = \frac{dQ}{\rho c_p A\sqrt{4\pi\alpha t}} e^{-[(y^2)/4\alpha t]} \tag{32}$$

If the source is liberated at $t = t'$ rather than $t = 0$, the solution can be shown to be

$$dT(y, t) = \frac{dQ}{\rho c_p A\sqrt{4\pi\alpha(t - t')}} e^{-[(y^2)/4\alpha(t-t')]} \tag{33}$$

The quantity of heat liberated at the instant dt' is given by

$$dQ = \dot{Q}(t')dt' \tag{34}$$

where $\dot{Q}(t')$ is the rate of heat liberated at t'. The temperature arising due to a *continuous* source of heat from $t = 0$ to $t = t'$ is obtained by adding (integrating) the contributions from each instantaneous source, i.e.,

$$T(y, t) = \int_{t'=0}^{t'=t'} \frac{\dot{Q}(t')e^{-[(y^2)/4\alpha(t-t')]}}{\rho c_p A\sqrt{4\pi\alpha(t - t')}} dt' \tag{35}$$

The term y^2 is replaced by $(y - y')^2$ in the above solution if the source is located at y' rather than the origin. A more detailed presentation is provided by Carslaw and Jaeger$^{(2)}$.

Most of the material in the literature on this subject is concerned with semi-infinite media. The analysis presented above can be applied to these systems as well. ■

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Chapter 9

Forced Convection

INTRODUCTION

When a pot of water is heated on a stove, the portion of water adjacent to the bottom of the pot is the first to experience an increase in temperature. Eventually, the water at the top will also become hotter. Although some of the heat transfer from bottom to top is explainable by conduction through the water, most of the heat transfer is due to a second mechanism of heat transfer, *convection*. Agitation produced by a mixer, or the equivalent, adds to the convective effect. As the water at the bottom is heated, its density becomes lower. This results in convection currents as gravity causes the low density water to move upwards while being replaced by the higher density, cooler water from above. This macroscopic mixing is occasionally a far more effective mechanism for transferring heat energy through fluids than conduction. This process is called *natural* or *free* convection because no external forces, other than gravity, need be applied to move the energy in the form of heat. In most industrial applications, however, it is more economical to speed up the mixing action by artificially generating a current by the use of a pump, agitator, or some other mechanical device. This is referred to as *forced* convection and practicing engineers are primarily interested in this mode of heat transfer (i.e., most industrial applications involve heat transfer by convection).

Convective effects described above as *forced convection* are due to the bulk motion of the fluid. The bulk motion is caused by external forces, such as that provided by pumps, fans, compressors, etc., and is essentially independent of "thermal" effects. *Free* convection is the other effect that occasionally develops and was also briefly discussed above. This convective effect is attributed to buoyant forces that arise due to density differences within a system. It is treated analytically as another external force term in the momentum equation. The momentum (velocity) and energy (temperature) effects are therefore interdependent; consequently, both equations must be solved simultaneously. This treatment is beyond the scope of this text, but is available in the literature.(1) Also note that *both* forced and free convection may exist in some applications.

In order to circumvent the difficulties encountered in the analytical solution of microscopic heat-transfer problems, it is common practice in engineering to write

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Heat Transfer Applications for the Practicing Engineer. Louis Theodore

the rate of heat transfer in terms of a heat transfer coefficient h, a topic that will receive extensive treatment in Part Three. If a surface temperature is T_S, and T_M represents the temperature of a fluid medium at some distance from the surface, one may write that

$$\dot{Q} = hA(T_S - T_M) \tag{9.1}$$

Since h and T_S are usually functions of the area A, the above equation may be rewritten in differential equation form

$$d\dot{Q} = h(T_S - T_M)\,dA \tag{9.2}$$

Integrating for the area gives

$$\int_0^{\dot{Q}} \frac{d\dot{Q}}{h(T_S - T_M)} = \int_0^A dA \tag{9.3}$$

This concept of a heat-transfer coefficient is an important concept in heat transfer, and is often referred to in the engineering literature as the *individual film* coefficient.

The expression in Equation (9.1) may be better understood by referring to heat transfer to a fluid flowing in a conduit. For example, if the resistance to heat transfer is thought of as existing only in a laminar film$^{(2)}$ adjacent to the wall of the conduit, the coefficient h may then be viewed as equivalent to $h/\Delta x_e$, where Δx_e is the equivalent thickness of a stationary film that offers the same resistance corresponding to the observed value of h. This is represented pictorially in Figure 9.1. In effect, this simply replaces the real resistance with a hypothetical one.

The reader should note that for flow past a surface, the velocity at the surface (s) is *zero* (no slip). The only mechanism for heat transfer at the surface is therefore conduction. One may therefore write

$$\dot{Q} = -kA\left.\frac{dT}{dx}\right|_s \tag{9.4}$$

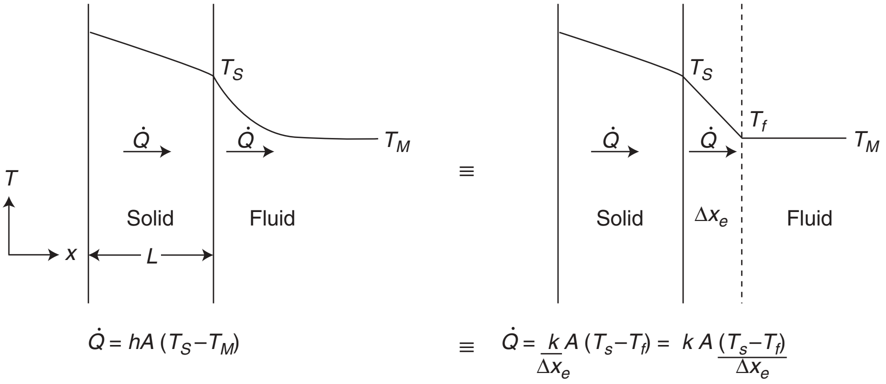

Figure 9.1 Convection temperature profile.

If the temperature profile in the fluid can be determined (i.e., $T = T(x)$), the gradient, dT/dx, can be evaluated at all points in the system including the surface, $dT/dx\big|_s$.

The heat transfer coefficient, h, was previously defined by

$$\dot{Q} = hA(T_S - T_M) \tag{9.1}$$

Therefore, combining Equations (9.1) and (9.4) leads to

$$-kA\frac{dT}{dx}\bigg|_s = hA(T_S - T_M) \tag{9.5}$$

Since the thermal conductivity, k, of the fluid is usually known, information on h may be obtained. This information is provided later in this chapter.

To summarize, the transfer of energy by convection is governed by Equation (9.1) and is referred to as *Newton's Law of Cooling*,

$$\dot{Q} = hA(T_S - T_M) \tag{9.1}$$

where $\dot{Q}$ is the convective heat transfer rate (Btu/h); A, the surface area available for heat transfer (ft^2); T_S, the surface temperature (°F); T_M, the bulk fluid temperature (°F); and h is the aforementioned convection heat transfer coefficient, also termed the *film coefficient* or *film conductance* ($\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$ or $\text{W/m}^2 \cdot \text{K}$). Note that $1\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} = 5.6782\ \text{W/m}^2 \cdot \text{K}$.

The magnitude of h depends on whether the transfer of heat between the surface and the fluid is by forced convection or by free convection, radiation, boiling, or condensation (to be discussed in later chapters). Typical values of h are given in Tables 9.1 and 9.2.

Topics covered in this chapter include:

Convective Resistances

Heat Transfer Coefficients: Qualitative Information

Heat Transfer Coefficients: Quantitative Information

Microscopic Approach

Table 9.1 Typical Film Coefficients

	h	
Mode	$\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$	$\text{W/m} \cdot \text{K}$
Forced convection		
Gases	5–50	25–250
Liquids	10–4000	50–20,000
Free convection (see next chapter)		
Gases	1–5	5–25
Liquids	10–200	50–1000
Boiling/condensation	500–20,000	2500–100,000

Table 9.2 Film Coefficients in Pipes[a]

	h, Inside pipes	h, Outside pipes[b,c]
Gases	10–50	1–3 (n), 5–20 (f)
Water (liquid)	200–2000	20–200 (n), 100–1000 (f)
Boiling water[d]	500–5000	300–9000
Condensing steam[d]		1000–10,000
Nonviscous fluids	50–500	50–200 (f)
Boiling liquids[d]		200–2000
Condensing vapor[d]		200–400
Viscous fluids	10–100	20–50 (n), 10–100 (f)
Condensing vapor[d]		50–100

[a] $h = \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$.

[b] (n) = natural convection (see next chapter).

[c] (f) = forced convection.

[d] Additional details in Chapter 12.

CONVECTIVE RESISTANCES

Consider heat transfer across a flat plate, as pictured in Figure 9.2. The total resistance (R_t) may be divided into three contributions: the inside film (R_i), the wall (R_w), and the outside film (R_o),

$$\sum R = R_t = R_i + R_w + R_o \tag{9.6}$$

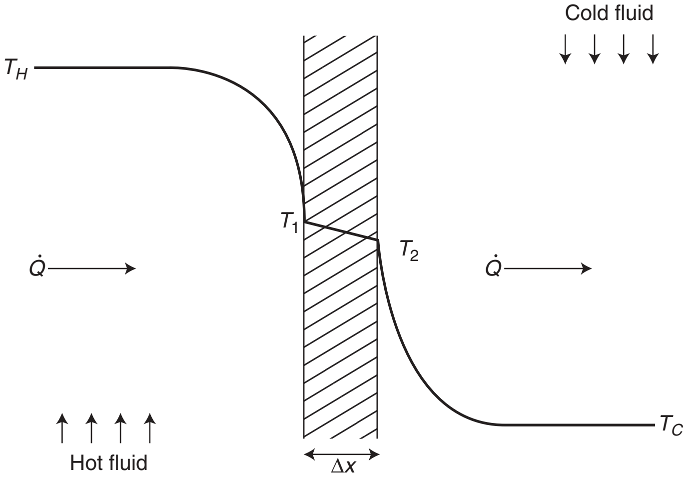

Figure 9.2 Flow past a flat plate.

where the individual resistances are defined by

$$R_t = \frac{1}{h_i A} + \frac{\Delta x}{kA} + \frac{1}{h_o A} \tag{9.7}$$

The term h_i is the *inside* film coefficient (Btu/h · ft^2 · °F); h_o, the *outside* film coefficient (Btu/h · ft^2 · °F); A, the surface area (ft^2); Δx, the thickness (ft); and, k, the thermal conductivity (Btu/h · ft · °F).

ILLUSTRATIVE EXAMPLE 9.1

Consider a closed cylindrical reactor vessel of diameter $D = 1$ ft, and length $L = 1.5$ ft. The surface temperature of the vessel, T_1, and the surrounding temperature, T_2, are 390°F and 50°F, respectively. The convective heat transfer coefficient, h, between the vessel wall and surrounding fluid is 4.0 Btu/h · ft · °F. Calculate the thermal resistance in °F · h/Btu.

***SOLUTION*:** Write the heat transfer rate equation:

$$\dot{Q} = hA(T_1 - T_2) \tag{9.1}$$

Since $D = 1$ ft and $L = 1.5$ ft, the total heat transfer area may be determined.

$$A = \pi(1)(1.5) + \frac{(2)\pi(1)^2}{4} = 6.28 \text{ ft}^2$$

Calculate the rate of heat transfer.

$$\dot{Q} = (4.0)(6.28)(390 - 50) = 8545 \text{ Btu/h}$$

Finally, calculate the thermal resistance associated with the film coefficient (see Equation 9.7).

$$R = \frac{1}{hA} = \frac{1}{(4.0)(6.28)} = 0.0398°\text{F} \cdot \text{h/Btu}$$

■

ILLUSTRATIVE EXAMPLE 9.2

Referring to the previous example, convert the resistance to K/W and °C/W.

***SOLUTION*:** First note that

$$1 \text{ W} = 3.412 \text{ Btu/h}$$

and a 1°C change corresponds to a 1.8°F change. Therefore,

$$\begin{aligned} R &= (0.0398)(3.412)/1.8 \\ &= 0.075°\text{C/W} \\ &= 0.075 \text{ K/W} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.3

Hot gas at 530°F flows over a flat plate of dimensions 2 ft by 1.5 ft. The convection heat transfer coefficient between the plate and the gas is 48 $\text{Btu/ft}^2 \cdot \text{h} \cdot {}^\circ\text{F}$. Determine the heat transfer rate in Btu/h and kW from the air to one side of the plate when the plate is maintained at 105°F.

SOLUTION**:** Assume steady-state conditions and constant properties. Write Newton's law of cooling to evaluate the heat transfer rate. Note that the gas is hotter than the plate. Therefore, $\dot{Q}$ will be transferred from the gas to the plate,

$$\dot{Q} = hA(T_S - T_M) \tag{9.1}$$

Substituting

$$\begin{aligned}\dot{Q} &= (48)(3)(530 - 105) = 61{,}200\,\text{Btu/h}\\ &= 61{,}200/3.4123 = 17{,}935\,\text{W}\\ &= 17.94\,\text{kW}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.4

The glass window shown in Figure 9.3 of area 3.0 m^2 has a temperature at the outer surface of 10°C. The glass has conductivity of 1.4 $\text{W/m} \cdot \text{K}$. The convection coefficient (heat transfer coefficient) of the air is 100 $\text{W/m}^2 \cdot \text{K}$. The heat transfer is 3.0 kW. Calculate the bulk temperature of the fluid.

SOLUTION**:** Write the equation for heat convection. Equation (9.1) is given by

$$\dot{Q} = hA(T_S - T_M)$$

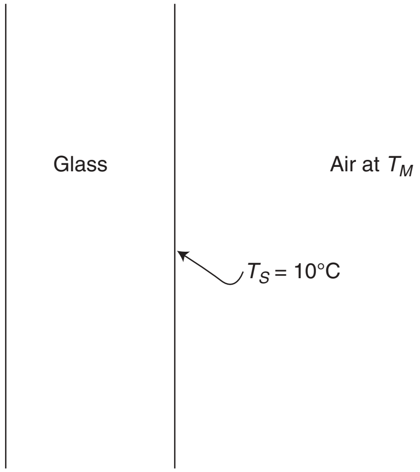

Figure 9.3 Convective glass.

where T_S = surface temperature at the wall and T_M = temperature of the bulk fluid. Solving for the unknown, the air temperature T_M, and substituting values yields

$$T_M = T_S - \frac{\dot{Q}}{hA}$$

$$T_M = (273 + 10) - \frac{3000}{(100)(3)} = 273\,\text{K} = 0°\text{C}$$ ■

ILLUSTRATIVE EXAMPLE 9.5

Refer to Illustrative Example 9.1. If the convective heat transfer coefficient, h, between the vessel wall and the surrounding fluid is constant at 4.0 Btu/h · ft^2 · °F and the plant operates 24 h/day, 350 days/yr, calculate the steady-state energy loss in Btu/yr.

***SOLUTION*:** The operating hours per year, N, are

$$N = (24)(350) = 8400\,\text{h/yr}$$

The yearly energy loss, $\dot{Q}$, is therefore

$$\dot{Q} = (8545)(8400) = 7.18 \times 10^7\,\text{Btu/yr}$$ ■

Returning to Newton's law, the above development is now expanded and applied to the system pictured in Figure 9.2. Based on Newton's law of cooling,

$$\dot{Q}_1 = h_1 A(T_H - T_1) \tag{9.8}$$

$$\dot{Q}_2 = \frac{kA}{\Delta x}(T_1 - T_2) \tag{9.9}$$

$$\dot{Q}_3 = h_2 A(T_2 - T_C) \tag{9.10}$$

Assuming steady-state ($\dot{Q} = \dot{Q}_1 = \dot{Q}_2 = \dot{Q}_3$), and combining the above in a manner similar to the development provided in Chapter 7 yields:

$$\dot{Q} = \frac{T_H - T_C}{(1/h_1 A) + (\Delta x/kA) + (1/h_2 A)} \tag{9.11}$$

$$= \frac{T_H - T_C}{R_i + R_w + R_o} = \frac{T_H - T_C}{R_t} \tag{9.12}$$

HEAT TRANSFER COEFFICIENTS: QUALITATIVE INFORMATION

The heat transfer coefficient h is a function of the properties of the flowing fluid, the geometry and roughness of the surface, and the flow pattern of the fluid. Several methods are available for evaluating h: analytical methods, integral methods, and dimensional analyses. Only the results of these methods are provided in the next section. These may take the form of local and/or average values. Their derivation is not included. As will be demonstrated in the next Part, it is convenient to know these values for design purposes.

As one might suppose, heat transfer coefficients are higher for turbulent flow than with laminar flow, and heat transfer equipment are usually designed to take advantage of this fact. The heat transferred in most turbulent streams occurs primarily by the movement of numerous microscopic elements of fluid (usually referred to as eddies) in the system. Although one cannot theoretically predict the behavior of these eddies quantitatively with time, empirical equations are available for the practicing engineer.

Surface roughness can also have an effect on the heat transfer coefficient. However, the effect in laminar flow is, as one might expect, negligible. Except for a slight increase in the heat transfer area, the roughness has little to no effect on heat transfer with laminar flow. When the flow is turbulent, the roughness may have a significant effect. In general, the coefficient will not be affected if the rough "elements" do not protrude through the laminar sub-layer.

The heat transfer problems most frequently encountered have to do with the heating and cooling of fluids in pipes. Although the entrance effects can significantly affect overall performance in short pipes and conduits, this is normally neglected.

Before proceeding to a quantitative review of the equations that can be employed to calculate heat transfer film coefficients, it should be noted that a serious error can arise if the viscosity of the fluid is strongly dependent on temperature. The temperature gradient is normally greatest near the surface wall of the conduit and it is in this region that the velocity gradient is also greatest. The effect of temperature on the viscosity of the fluid at or near the wall may therefore have a pronounced effect on both the viscosity and temperature profiles. This effect is reflected in the heat transfer coefficient equations. If μ and μ_s represent the fluid viscosity at the average bulk fluid temperature and the viscosity at the wall temperature, respectively, the dimensionless term μ/μ_s is an empirical correction factor for the distortion of the velocity profile that results from the effect of temperature on viscosity.

A summary of the describing equations employed to predict convective heat transfer coefficients is presented in the next section. Most of the correlations contain dimensionless numbers, many of which were introduced in Chapter 3; some are redefined and reviewed again in this section. A host of illustrative examples complement the presentation.

HEAT TRANSFER COEFFICIENTS: QUANTITATIVE INFORMATION

Many heat transfer film coefficients have been determined experimentally. Typical ranges of film coefficients are provided in Tables 9.3–9.5. Explanatory details are provided in the subsections that follow. In addition to those provided in Tables 9.3–9.5, many empirical correlations can be found in the literature for a wide variety of fluids and flow geometries.$^{(1)}$

Table 9.3 Summary of Forced Convection Heat Transfer Correlations for External Flow

Geometry	Correlation	Conditions
Flat plate: laminar	Hydrodynamic boundary layer, $\delta/x = 5\,\mathrm{Re}_x^{-0.5}$	Laminar flow, properties at the fluid film temperature, T_f
	Local friction factor, $f_x = 0.664\,\mathrm{Re}_x^{-0.5}$	Laminar, local, T_f
	Local Nusselt number $\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{0.333}$	Laminar, local, T_f, $0.6 \le \mathrm{Pr} \le 50$
	Thermal boundary layer thickness, $\delta_t = \delta\,\mathrm{Pr}^{-0.333}$	Laminar, T_f
	Average friction factor, $\bar{f} = 1.328\,\mathrm{Re}_x^{-0.5}$	Laminar, average, T_f
	Average Nusselt number, $\overline{\mathrm{Nu}}_x = 0.644\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{0.333}$	Laminar, average, T_f, $0.6 \le \mathrm{Pr} \le 50$
Flat plate: liquid metal	Liquid metal, $\mathrm{Nu}_x = 0.565\,\mathrm{Pr}_x^{0.5}$	Laminar, local, T_f, $\mathrm{Pr} \le 0.05$
Flat plate: turbulent flow	Local friction factor, $f_x = 0.0592\,\mathrm{Re}_x^{-0.2}$	Turbulent, local, T_f, $\mathrm{Re}_x \le 10^8$
	Hydrodynamic boundary layer, $\delta/x = 0.37\,\mathrm{Re}_x^{0.2}$	Turbulent, local, T_f, $\mathrm{Re}_x \le 10^8$
	Local Nusselt number, $\mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.333}$	Turbulent, local, T_f, $\mathrm{Re}_x \le 10^8$, $0.6 \lesssim \mathrm{Pr} \lesssim 60$
	Average Nusselt number, $\overline{\mathrm{Nu}} = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}$	Turbulent, average, T_f, $\mathrm{Re} > \mathrm{Re}_{x,c} = 5 \times 10^5$, $0.6 < \mathrm{Pr} < 60$, $L \gg x_c$
	Average friction factor, $\bar{f} = 2\,j_H = 0.064\,\mathrm{Re}_x^{-0.2}$	Turbulent, average, T_f
Flat plate: mixed flow	Average friction factor, $\bar{f} = 0.074\,\mathrm{Re}_L^{-0.2} - 1472\,\mathrm{Re}_L^{-1}$	Mixed average, T_f, $\mathrm{Re}_{x,c} = 5 \times 10^5$, $\mathrm{Re}_L \le 10^8$
	Average Nusselt number, $\overline{\mathrm{Nu}} = (0.037\,\mathrm{Re}_L^{4/5} - 871)\,\mathrm{Pr}^{1/3}$	Mixed average, T_f, $\mathrm{Re}_{x,c} = 5 \times 10^5$, $\mathrm{Re}_L \le 10^8$, $0.6 < \mathrm{Pr} < 60$
Cylinder	Kundsen and Katz equation of average Nusselt number $\overline{\mathrm{Nu}}_D = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3}$ (Table 9.4 provides C and m information)	Average, T_f, $0.4 < \mathrm{Re}_d < 4 \times 10^5$, $\mathrm{Pr} \gtrsim 0.7$

(*Continued*)

Table 9.3 *Continued*

Geometry	Correlation	Conditions
Sphere	Whitaker equation for the average Nusselt number, $\overline{\mathrm{Nu}}_D = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}(\mu/\mu_s)^{1/4}$	Average, properties at T_∞, $3.5 < \mathrm{Re}_D < 7.6 \times 10^4$, $0.71 < \mathrm{Pr} < 380$, $1.0 < (\mu/\mu_s) < 3.2$
Falling drop	$\overline{\mathrm{Nu}} = 2 + 0.6\,\mathrm{Re}_D^{1/2}\mathrm{Pr}^{1/3}[25(x/D)^{-0.7}]$	Average, T_∞
Packed bed of spheres	$\varepsilon\,\bar{j}_H = \varepsilon\,\bar{j}_m = 2.06\,\mathrm{Re}_D^{-0.575}$	Average, $\overline{T}$, $90 \le \mathrm{Re}_D \le 4000$, $\mathrm{Pr} \approx 0.7$

Notes: These correlations assume isothermal surfaces.

$\bar{j}_H$ = Colburn j_H factor $= \mathrm{St}\,\mathrm{Pr}^{2/3} = \mathrm{Nu}\,\mathrm{Pr}^{-1/3}$

ε = void fraction or porosity

For packed beds, properties are evaluated at the average fluid temperature, $\overline{T} = (T_i + T_o)/2$, or the average film temperature, $\overline{T}_f = (T_s + \overline{T})/2$.

Table 9.4 Constants of Knudsen and Katz for Heat Transfer to Cylinders in Cross Flow

Re	C	m
0.04–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

One of the more critical steps in solving a problem involving convection heat transfer is the estimation of the convective heat transfer coefficient. Cases considered in this section involve forced convection, while natural convection is considered in the next chapter. During forced convection heat transfer, the fluid flow may be either external to the surface (e.g., flow over a flat plate, flow across a cylindrical tube, or across a spherical object) or inside a closed surface (e.g., flow inside a circular pipe or a duct).

When a fluid flows over a flat plate maintained at a different temperature, heat transmission takes place by forced convection. The nature of the flow (laminar or turbulent) influences the forced convection heat transfer rate. The correlations used to calculate the heat transfer coefficient between the plate surface and the fluid are usually presented in terms of the average Nusselt number, $\overline{\mathrm{Nu}}_x$, the average Stanton number, $\overline{\mathrm{St}}$, (or, equivalently, the Colburn j_H factor), the local Reynolds number, Re_x, and the Prandtl number, Pr. In some cases, the Peclet number, Pe, is also used.

Correlation details as presented in Tables 9.3 and 9.4, apply in this section for:

1. Convection from a plane surface
2. Convection in circular pipes
   a. laminar flow
   b. turbulent flow
3. Convection in non-circular conduits
4. Convection normal to a cylinder
5. Convection normal to a number of circular tubes
6. Convection for spheres
7. Convection between a fluid and a packed bed

Topics 1 and 2 receives the bulk of the treatment. In addition, heat transfer to liquid metals is also discussed.

Flow Past a Flat Plate

For external flow, the fluid is assumed to approach the surface with a uniform constant velocity, V, and a uniform temperature, T_∞. The stagnant surface has a constant

Table 9.5 Summary of Forced Convection Correlations for Internal Flow

Correlation		Conditions
Darcy friction factor:(2)		
$f = 64/\mathrm{Re} = 64/\mathrm{Re}$	(1)	Laminar, fully developed
$\mathrm{Nu} = 4.364$	(2)	Laminar, fully developed, constant Q', UHF, $\mathrm{Pr} \geq 0.6$
$\mathrm{Nu}_D = 3.658$	(3)	Laminar, fully developed, constant T_s, UWT, $\mathrm{Pr} \geq 0.6$
Seider and Tate equation:		Laminar, combined entry length, properties at mean bulk temperature of the fluid
$\overline{\mathrm{Nu}} = 1.86\left(\mathrm{Re}_D \, \mathrm{Pr}\dfrac{D}{L}\right)^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$ $= 1.86\ \mathrm{Gz}^{0.333}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(4)	$\left[\left(\mathrm{Re}\,\mathrm{Pr}\dfrac{L}{D}\right)^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}\right] \geq 2$ Constant wall temperature, T_s, $0.48 < \mathrm{Pr} < 16{,}700$, $0.0044 < (\mu/\mu_s) < 9.75$
$f = 0.316\,\mathrm{Re}_D^{-0.25}$	(5)	Turbulent, fully developed, smooth tubes, $\mathrm{Re} \leq 2 \times 10^4$
$f = 0.184\,\mathrm{Re}_D^{-0.2}$	(6)	Turbulent, fully developed, smooth tubes, $\mathrm{Re} \geq 2 \times 10^4$
Dittus–Boelter equation:		Turbulent, fully developed, $0.6 \leq \mathrm{Pr} \leq 160$, $\mathrm{Re} \geq 10{,}000$, $L/D \geq 10$
$\mathrm{Nu} = 0.023\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^n$	(7)	$n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$

(Continued)

Table 9.5 *Continued*

Correlation		Conditions
Seider and Tate turbulent equation: $\mathrm{Nu} = 0.027\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.333}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8)	Turbulent, fully developed, $0.7 \leq \mathrm{Pr} \leq 16{,}700$, $\mathrm{Re}_D \geq 10{,}000$, $L/D \gtrsim 10$, large change in fluid properties between the wall of the tube and the bulk flow
Nusselt turbulent entrance region equation: $\mathrm{Nu}_D = 0.036\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.333}\left(\dfrac{D}{L}\right)^{0.055}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(9)	Turbulent, entrance region, $10 < L/D < 400$, $0.7 \leq \mathrm{Pr} \leq 16{,}700$
Chilton–Colburn analogy: $j_H = \mathrm{St}\,\mathrm{Pr}^{2/3} = \mathrm{Nu}\,\mathrm{Re}^{-1}\,\mathrm{Pr}^{-1/3} = f/8$	(10)	Rough tubes, turbulent flow, St is based on the fluid bulk temperature, while Pr and f are based on the film temperature
Skupinski equation for liquid metals: $\mathrm{Nu}_D = 4.82 + 0.0185(\mathrm{Re}_D\,\mathrm{Pr})^{0.827}$ $= 4.82 + 0.0185\,\mathrm{Pe}_D^{0.827}$	(11)	Liquid metals, $0.003 < \mathrm{Pr} < 0.05$, turbulent, fully developed, constant $\dot{Q}_S$, $3.6 \times 10^3 < \mathrm{Re}_D < 9.05 \times 10^5$, $10^2 < \mathrm{Pe}_D$ (Peclet number based on diameter) $< 10^4$
Seban and Shimazaki equation for liquid metals: $\mathrm{Nu}_D = 5.0 + 0.025(\mathrm{Re}_D\,\mathrm{Pr})^{0.8}$ $= 5.0 + 0.025\,\mathrm{Pe}_D^{0.8}$	(12)	Liquid metals, turbulent, fully developed, constant T_S, $\mathrm{Pe}_D > 100$, properties at the average bulk temperature

Notes: Properties in Equations 2, 3, 7, 8, 11 and 12 are based on the mean fluid temperature T_m; properties in Equations 1, 5, and 6 are based on the average film temperature, $T_f \equiv (T_s + T_m)/2$; properties in Equation 4 are based on the average of the mean temperature of fluids entering and leaving $\overline{T} \equiv (T_{m,i} + T_{m,o})/2$.

$\mathrm{Re}_D \equiv D_h V/\nu$; D_h = hydraulic diameter $\equiv 4A/P_w$; $V \equiv \dot{m}/\rho A$.

UHF ≡ uniform heat flow; UWT ≡ uniform wall temperature.

temperature of T_S. Since a fluid is a material that cannot slip, the velocity of the fluid in contact with the surface is zero. A retardation layer forms near the stagnant surface. This layer is termed the "boundary-layer", or "hydrodynamic boundary layer". Its thickness, δ, increases as the fluid moves down the surface.

The dimensionless Reynolds number for flow over a flat plate depends on the distance, x, from the leading edge of the plate. It is termed the local Reynolds number, Re_x, and is defined by

$$\text{Re}_x = xV/\nu = xV\rho/\mu \tag{9.13}$$

A value of Re_x less than 500,000 indicates that the boundary layer is in laminar flow. Critical Reynolds numbers above 500,000 indicate that most of the boundary layer is in turbulent flow. (Note that, even when the boundary layer is turbulent, the thin layer adjacent to the plate must still be in laminar flow.)

For laminar flow, the thickness of the laminar boundary layer, δ, is given by the Blasius equation,

$$\frac{\delta}{x} = \frac{5}{\sqrt{\text{Re}_x}}, \qquad \text{or} \qquad \delta = 5\sqrt{\frac{\nu x}{V}} \tag{9.14}$$

where

δ = boundary layer thickness
ν = kinematic viscosity of the fluid = μ/ρ
μ = dynamic or absolute viscosity of the fluid
ρ = fluid density
V = free stream velocity of the fluid (outside the boundary layer)

Similar correlations for other types of external flow are listed in Table 9.3 with the coefficients for the Knudsen and Katz equation given in Table 9.4. When the fluid is a gas, the correlations are based on the assumption of incompressible flow, an assumption that is valid for Mach numbers less than 0.3. The Mach number is defined by the equation,

$$\text{Ma} = V/c \tag{9.15}$$

where

c = speed of sound in the gas = $\sqrt{kRT/(MW)}$
k = ratio of gas heat capacities = c_p/c_v
c_p = constant pressure heat capacity
c_v = constant volume heat capacity
R = universal (ideal) gas constant
MW = molecular weight of the gas

For air, $k = 1.4$, $MW = 28.9$, $R = 8314.4\ \mathrm{m^2/s^2 \cdot K}$; the speed of sound in air from Equation (9.15) is therefore,

$$\begin{aligned} c_{\mathrm{air}} &= 20\sqrt{T\ (\text{in K})};\ \mathrm{m/s} \\ &= 49\sqrt{T\ (\text{in }^\circ\mathrm{R})};\ \mathrm{ft/s} \end{aligned} \tag{9.16}$$

The heat transfer between the surface and the fluid is by convection. The heat transfer coefficient, h, will vary with position and is termed the local heat transfer coefficient, h_x. At any distance, x, on the plate surface, the heat flux, $\dot{Q}'$ (i.e., the heat flow rate through a unit cross-section, $\dot{Q}/A$) is

$$\dot{Q}' = \frac{\dot{Q}}{A} = h_x(T_S - T_\infty) \tag{9.17}$$

Since the fluid is stagnant at the plate surface, then (from a conduction heat transfer point-of-view)

$$\dot{Q}' = -k\left(\frac{dT}{dy}\right)_S \tag{9.18}$$

where k is the thermal conductivity of the fluid and y is the coordinate perpendicular to the direction of flow.

For a small area of the surface, dA, the heat transfer rate, $d\dot{Q}$, is

$$d\dot{Q} = h_x(T_S - T_\infty)\,dA \tag{9.19}$$

Integration of Equation (9.19) yields

$$\dot{Q} = (T_S - T_\infty)\int h_x\,dA = \bar{h}A(T_S - T_\infty) \tag{9.20}$$

where $\bar{h}$ is now the *average* heat transfer coefficient.

For flow over a flat plate of width, b, with the distance x measured from the leading edge of the plate, the surface area for heat transfer, A, is $A = bx$ so that $dA = b\,dx$. Combining this result and noting that the plate length, L, is

$$L = \int_0^L dx \tag{9.21}$$

yields the following relation for the average heat transfer coefficient over the whole length, L, of the plate:

$$\bar{h} = \frac{1}{L}\int_0^L h_x\,dx \tag{9.22}$$

For the entire plate, then,

$$\dot{Q}' = \frac{\dot{Q}}{A} = \bar{h}(T_S - T_\infty) \tag{9.23}$$

The local coefficient h_x for laminar flow is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = (0.332)\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \tag{9.24}$$

The average coefficient over the interval $0 < x < L$ can be shown to be

$$\bar{h} = 2h_x|_{x=L}$$

so that

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = (0.664)\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3} \tag{9.25}$$

Flow in a Circular Tube

Many industrial applications involve the flow of a fluid in a conduit (e.g., fluid flow in a circular tube). When the Reynolds number exceeds 2100, the flow is turbulent. Three cases are considered below. For commercial (rough) pipes, the friction factor is a function of the Reynolds number and the relative roughness of the pipe, ε/D.(2) In the region of complete turbulence (high Re and/or large ε/D), the friction factor depends mainly on the relative roughness.(2) Typical values of the roughness, ε, for various kinds of new commercial piping are available in the literature.(2)

1. *Fully Developed Turbulent Flow in a Smooth Pipe*
   When the temperature difference is moderate, the Dittus and Boelter(3) equation may be used

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^n \tag{9.26}$$

where $\mathrm{Nu} = $ Nusselt number $= hD/k$, $\mathrm{Re} = DV/\nu = DV\rho/\mu$, and $\mathrm{Pr} = c_p\mu/k$. The properties in this equation are evaluated at the average (or mean) fluid bulk temperature, T_m, and the exponent n is 0.4 for heating (i.e., $T_1 < T_{s1}$, $T_2 < T_{s2}$) and 0.3 for cooling (i.e., $T_1 > T_{s1}$, $T_s > T_{s2}$). The equation is valid for fluids with Prandtl numbers, Pr, ranging from about 0.6 to 100.

If wide temperature differences are present in the flow, there may be an appreciable change in the fluid properties between the wall of the tube and the central flow. To take into account the property variations, the Seider and

Tate$^{(1,4)}$ equation should be used:

$$\mathrm{Nu} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}(\mu/\mu_s)^{0.14} \tag{9.27}$$

All properties are evaluated at bulk temperature conditions, except μ_s which is evaluated at the wall surface temperature, T_S.

2. *Turbulent Flow in Rough Pipes*
   The recommended equation is the Chilton–Colburn j_H analogy between heat transfer and fluid flow. It relates the j_H factor ($=\mathrm{St}\,\mathrm{Pr}^{2/3}$) to the Darcy friction factor, f:

$$j_H = \mathrm{St}\,\mathrm{Pr}^{2/3} = \mathrm{Nu}\,\mathrm{Re}^{-1}\,\mathrm{Pr}^{-1/3} = f/8$$

or

$$\mathrm{Nu} = (f/8)\,\mathrm{Re}\,\mathrm{Pr}^{1/3} \tag{9.28}$$

where f is obtained from the Moody chart.$^{(1-4)}$ The average Stanton number (St) is based on the fluid bulk temperature, T_m, while Pr and f are evaluated at the film temperature, i.e., $(T_S + T_{\mathrm{av}})/2$.

Internal flow film coefficients are provided in Table 9.5. Special conditions that apply are provided in the note at the bottom of the Table.

Liquid Metal Flow in a Circular Tube

Liquid metals have small Prandtl numbers because they possess large thermal conductivities. This makes it possible for liquid metals to remove larger heat fluxes than other fluids. These fluids remain liquid at much higher temperatures than water or other substances. On the down side, liquid metals are difficult to handle, corrosive, and react violently with water. Despite their disadvantages, liquid metals are used in high heat flux systems. For example, they are commonly used to cool the cores of nuclear reactors.

For liquid metals, the Nusselt number has been found to depend on the product of the Reynolds and Prandtl numbers. This product is termed the aforementioned Peclet number:

$$(\mathrm{Re})(\mathrm{Pr}) = \mathrm{Pe} = DV/\alpha = DV\rho c_p/k \tag{9.29}$$

Table 9.5 lists the equations to use for predicting the heat transfer in liquid metals in smooth pipes. The Seban and Shimazaki equation is used for uniform wall temperature (UWT), or constant surface temperature, T_s, fully developed flow ($L/D > 60$) and a Peclet number, $\mathrm{Pe} > 100$.

$$\mathrm{Nu} = hD/k = 5.0 + 0.025\,\mathrm{Pe}^{0.8} \tag{9.30}$$

All properties are evaluated at the the average bulk temperature. The correlation of Srupinski is used for a uniform heat flux (UHF), Q_s', turbulent flow, fully developed flow, $0.003 < \mathrm{Pr} < 0.05$, $3.6 \times 10^3 < \mathrm{Re} < 9.05 \times 10^5$, and $100 < \mathrm{Pe} < 10{,}000$.

$$\mathrm{Nu} = 4.82 + 0.0185(\mathrm{Re\ Pr})^{0.827} = 4.82 + 0.0185\,\mathrm{Pe}^{0.827} \tag{9.31}$$

Convection Across Cylinders

Forced air coolers and heaters, forced air condensers, and cross-flow heat exchangers (to be discussed in Part Three) are examples of equipment that transfer heat primarily by the forced convection of a fluid flowing across a cylinder. For engineering calculations, the average heat transfer coefficient between a cylinder (at temperature T_s) and a fluid flowing across the cylinder (at a temperature T_∞) is calculated from the Knudsen and Katz equation (see Table 9.4)

$$\overline{\mathrm{Nu}}_D = \bar{h}D/k = C\,\mathrm{Re}^m\,\mathrm{Pr}^{1/3} \tag{9.32}$$

All the fluid physical properties are evaluated at the *mean film temperature* (the arithmetic average temperature of the cylinder surface temperature and the bulk fluid temperature). The constants C and m depend on the Reynolds number of the flow and are given in Table 9.4.

ILLUSTRATIVE EXAMPLE 9.6

Identify the following three dimensionless groups:

1. $h_f L/k_f$ (subscript f refers to fluid)
2. $h_f L/k_s$ (subscript s refers to solid surface)
3. (Reynolds number)(Prandtl number), i.e., (Re)(Pr)

***SOLUTION*:** As noted above,

1. $h_f L/k_f =$ Nusselt number $=$ Nu
2. $h_f L/k_s =$ Biot number $= Bi$ (see also Table 3.5 and Equation 9.35 later in Chapter)
3. $(\mathrm{Re})(\mathrm{Pr}) = \left(\dfrac{LV\rho}{\mu}\right)\left(\dfrac{c_p\mu}{k}\right) = \dfrac{LV\rho\,c_p}{k} =$ Peclet number $=$ Pe ■

ILLUSTRATIVE EXAMPLE 9.7

For a flow of air over a horizontal flat plate, the local heat transfer coefficient, h_x, is given by the equation

$$h_x = \frac{25}{x^{0.4}},\ \mathrm{W/m^2 \cdot K}$$

where h_x is the local heat transfer coefficient at a distance x from the leading edge of the plate and x is the distance in meters. The critical Reynolds number, $\mathrm{Re}_{\mathrm{cr}}$, which is the Reynolds number at which the flow is no longer laminar, is 500,000.

Consider the flow of air at $T_\infty = 21°\mathrm{C}$ ($c_p = 1004.8\ \mathrm{J/kg\cdot K}$, $\nu = 1.5 \times 10^{-5}\ \mathrm{m^2/s}$, $k = 0.025\ \mathrm{W/m\cdot K}$, $\mathrm{Pr} = 0.7$), at a velocity of 3 m/s, over a flat plate. The plate has a thermal conductivity $k = 33\ \mathrm{W/m\cdot K}$, surface temperature, $T_S = 58°\mathrm{C}$, width, $b = 1$ m, and length, $L = 1.2$ m. Calculate

1. the heat flux at 0.3 m from the leading edge of the plate
2. the local heat transfer coefficient at the end of the plate
3. the ratio $\bar{h}/h_x$ at the end of the plate

***SOLUTION*:**

1. Calculate the local heat transfer coefficient, h_x, at $x = 0.3$ m:

$$\begin{aligned} h_x &= 25/x^{0.4} = 25/(0.3)^{0.4} \\ &= 40.5\ \mathrm{W/m^2\cdot K} \end{aligned}$$

Calculate the heat flux, $\dot{Q}'$, at $x = 0.3$ m:

$$\begin{aligned} \dot{Q}' &= h_{x=0.3}(T_S - T_\infty) \\ &= 40.5(58 - 21) \\ &= 1497\ \mathrm{W/m^2} \end{aligned}$$

2. Calculate h_x at the end of the plate. Since $x = L$,

$$\begin{aligned} h_L &= 25/(1.2)^{0.4} \\ &= 23.2\ \mathrm{W/m^2\cdot K} \end{aligned}$$

3. Calculate the average heat transfer coefficient, $\bar{h}$. Apply Equation (9.22).

$$\begin{aligned} \bar{h} &= (1/L)\int_0^L h_x\,dx \\ &= (1/L)\int_0^L (25/x^{0.4})\,dx \\ &= (25/L)(1/0.6)(x^{0.6})|_0^L \\ &= 41.67L^{0.6}/L = 41.67/L^{0.4} = 41.67/(1.2)^{0.4} \\ &= 38.7\ \mathrm{W/m^2\cdot K} \end{aligned}$$

Calculate the ratio $\bar{h}/h_x$ at the end of the plate:

$$\bar{h}/h_L = 38.7/23.2 = 1.668$$ ■

ILLUSTRATIVE EXAMPLE 9.8

Refer to the previous example. Calculate the rate of heat transfer over the whole length of the plate.

***SOLUTION*:** Calculate the area for heat transfer for the entire plate

$$A = bL = (1)(1.2) = 1.2\,\text{m}^2$$

Apply Equation (9.23).

$$\begin{aligned}\dot{Q} &= \bar{h}A(T_s - T_\infty)\\ &= (38.7)(1.2)(58 - 21)\\ &= 1720\,\text{W}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.9

Air with a mass rate of 0.075 kg/s flows through a tube of diameter $D = 0.225$ m. The air enters at 100°C and, after a distance of $L = 5$ m, cools to 70°C. Determine the heat transfer coefficient of the air. The properties of air at 85°C are approximately, $c_p = 1010\,\text{J/kg}\cdot\text{K}$, $k = 0.030$ $\text{W/m}\cdot\text{K}$, $\mu = 208 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2$, and $\text{Pr} = 0.71$.

***SOLUTION*:** The Reynolds number is

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{(4)(0.075)}{(\pi)(0.225)(208 \times 10^{-7}\,\text{N})} = 20{,}400$$

Apply either Equation (7) in Tables 9.5 and/or Equation (9.26), with $n = 0.3$ for heating,

$$\text{Nu} = hD/k = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(20{,}400)^{0.8}(0.71)^{0.3} = 58.0$$

Thus

$$\begin{aligned}h = \left(\frac{k}{D}\right)\text{Nu} &= (0.03/0.225)58.0\\ &= 7.73\,\text{W/m}^2\cdot\text{K}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.10

Calculate the average film heat transfer coefficient (Btu/h · ft² · °F) on the water side of a single pass steam condenser. The tubes are 0.902 inch inside diameter, and the cooling water enters at 60°F and leaves at 70°F. Employ the Dittus–Boelter equation and assume the average water velocity is 7 ft/s. Pertinent physical properties of water at an average temperature of 65°F are:

$\rho = 62.3\,\text{lb/ft}^3$
$\mu = 2.51\,\text{lb/ft}\cdot\text{h}$

$c_p = 1.0\ \text{Btu/lb} \cdot {}^\circ\text{F}$
$k = 0.340\ \text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$

***SOLUTION*:** For heating, Equation (9.26) applies:

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4}$$

or

$$h = \frac{k}{D} 0.023 \left(\frac{DVe}{\mu}\right)^{0.8} \left(\frac{c_p \mu}{k}\right)^{0.4}$$

The terms $\text{Re}^{0.8}$ and $\text{Pr}^{0.4}$ are given by

$$\begin{aligned}
\text{Re}^{0.8} &= [(0.902/12)(7)(62.4)/(2.51/3600)]^{0.8} \\
&= (47{,}091)^{0.8} = 5475 \\
\text{Pr}^{0.4} &= [(1.0)(2.51)/(0.340)]^{0.4} \\
&= (7.38)^{0.4} \\
&= 2.224
\end{aligned}$$

Therefore,

$$\begin{aligned}
h &= \left(\frac{0.340}{0.0752}\right)(0.023)(5475)(2.224) \\
&= 1266\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}
\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.11

Air at 1 atm and 300°C is cooled as it flows at a velocity 5.0 m/s through a tube with a diameter of 2.54 cm. Calculate the heat transfer coefficient if a constant heat flux condition is maintained at the wall and the wall temperature is 20°C above the temperature along the entire length of the tube.

***SOLUTION*:** The density is first calculated using the ideal gas law in order to obtain the Reynolds number. Apply the appropriate value of *R* for air on a *mass* basis.

$$\rho = \frac{P}{RT} = \frac{1(1.0132 \times 10^5)}{(287)(573)} = 0.6161\ \text{kg/m}^3$$

The following data is obtained from the Appendix assuming air to have the properties of nitrogen:

$$\mathrm{Pr} = 0.713$$

$$\mu = 1.784 \times 10^{-5}\,\mathrm{kg/m\cdot s}$$
$$k = 0.0262\,\mathrm{W/m\cdot K}$$
$$c_p = 1.041\,\mathrm{kJ/kg\cdot K}$$

Thus,

$$\mathrm{Re} = \frac{DV\rho}{\mu} = \frac{(0.0254)(5)(0.6161)}{1.784 \times 10^{-5}} = 4386$$

Since the flow is turbulent, Equation (7) in Table 9.5 and/or Equation (9.26) applies:

$$\mathrm{Nu} = \frac{\mathrm{hD}}{\mathrm{k}} = 0.023\,\mathrm{Re}^{0.8}\mathrm{Pr}^{0.3} = 0.023(4386)^{0.8}(0.713)^{0.3} = 17.03$$

Thus,

$$h = \left(\frac{k}{D}\right)\mathrm{Nu} = \left(\frac{0.0262}{0.0254}\right)(17.03) = 17.57\,\mathrm{W/m^2\cdot K}$$

■

ILLUSTRATIVE EXAMPLE 9.12

Water flows with an average velocity of 0.355 m/s through a long copper tube (inside diameter = 2.2 cm) in a heat exchanger. The water is heated by steam condensing at 150°C on the outside of the tube. Water enters at 15°C and leaves at 60°C. Determine the heat transfer coefficient, h, for the water. (Adapted from Griskey.(5))

***SOLUTION*:** First evaluate the average bulk temperature of the water which is

$$(15 + 60)/2 = 37.5^\circ\mathrm{C}$$

Evaluating water properties (from the Appendix) at this temperature yields,

$$\rho = 993\,\mathrm{kg/m^3}$$
$$\mu = 0.000683\,\mathrm{kg/m\cdot s}$$
$$c_p = 4.17 \times 10^3\,\mathrm{J/kg\cdot K}$$
$$k = 0.630\,\mathrm{W/m\cdot K}$$

Calculate the Reynolds and Prandtl numbers for the flowing water:

$$\mathrm{Re} = \frac{DV\rho}{\mu} = \frac{(0.022)(0.355)(993)}{6.83 \times 10^{-4}} = 11{,}350$$

$$\mathrm{Pr} = \frac{c_p\mu}{k} = \frac{(4170)(6.83 \times 10^{-4})}{0.630} = 4.53$$

The flow is turbulent and, since the tube is a long one (i.e., no L/D effect), one may use the Seider and Tate turbulent relation [Equation (8) in Table 9.5 and/or Equation (9.25)] for internal flow:

$$\frac{hD}{k} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

All of the quantities in the above equation are known except μ_w. To obtain this value, the fluids' average wall temperature must be determined. This temperature is between the fluid's bulk average temperature of 37.5°C and the outside wall temperature of 150°C. Once again, use a value of 93.75°C (the average of the two). At this temperature (see Appendix)

$$\mu_w = 0.000306\,\mathrm{kg/m\cdot s}$$

Then

$$h = \frac{0.630}{0.022}(0.027)(11{,}350)^{0.8}(4.53)^{0.33}\left(\frac{0.000683}{0.000306}\right)^{0.14} = 2498.1\,\mathrm{W/m^2\cdot K}$$

■

An important dimensionless number that arises in some heat transfer studies and calculations is the Biot number, *Bi*. It plays a role in conduction calculations that also involve convection effects, and provides a measure of the temperature change within a solid relative to the temperature change between the surface of the solid and a fluid.

Consider once again the heat transfer process described in Figure 9.1. Under steady-state conditions, one may express the heat transfer rate as

$$\dot{Q} = \frac{kA}{L}(T - T_S) = hA(T_S - T_M) \tag{9.33}$$

The above equation may be rearranged to give

$$\frac{T - T_S}{T_S - T_M} = \frac{hA}{kA/L} = \frac{L/kA}{1/hA} = \frac{R_{\mathrm{cond}}}{R_{\mathrm{conv}}} = Bi = \frac{hL}{k} \tag{9.34}$$

The above equation indicates that, for small values of *Bi*, one may assume that the temperature across the solid is (relatively speaking) constant. If $Bi = 1.0$, the resistances are approximately equal. If $Bi \ll 1.0$, the resistance to conduction within the solid is much less than the resistance to convection across the fluid boundary layer. The reverse applies if $Bi \gg 1.0$. Generally, if $Bi < 2.0$, one may assume that for the scenario provided in Figure 9.1, the temperature profile in the solid is constant.

ILLUSTRATIVE EXAMPLE 9.13

Refer to Illustrative Example 9.7. Calculate the average Biot number.

***SOLUTION*:** Since the thickness of the flat plate is specified, the Biot number can be calculated.

$$Bi = \frac{hL}{k} = \frac{(38.7)(1.2)}{0.025} = 1858$$

■

ILLUSTRATIVE EXAMPLE 9.14

The surface temperature T_s of a circular conducting rod is maintained at 250°C by the passage of an electric current. The rod diameter is 10 mm, the length is 2.5 m, the thermal conductivity is 60 $\mathrm{W/m \cdot K}$, the density is 7850 $\mathrm{kg/m^3}$, and the heat capacity is 434 $\mathrm{J/kg \cdot K}$. The rod is in a fluid at temperature $T_f = 25°\mathrm{C}$, and the convection heat transfer coefficient is 140 $\mathrm{W/m^2 \cdot K}$. The thermal conductivity of the fluid is 0.6 $\mathrm{W/m \cdot K}$.

1. What is the thermal diffusivity of the bare rod?
2. What is the Nusselt number of the fluid in contact with the bare rod?
3. What is the Biot number of the bare rod?
4. Calculate the heat transferred from the rod to the fluid.

***SOLUTION*:**

1. The thermal diffusivity, α, of the bare rod is

$$\alpha = k/(\rho c_p) = 60/[(7850)(434)] = 1.76 \times 10^{-5}\ \mathrm{m^2/s}$$

2. The Nusselt number of the fluid is

$$\mathrm{Nu} = hD/k_f = (140)(0.01)/0.6 = 2.33$$

3. The Biot number of the bare rod is

$$Bi = hD/k_f = (140)(0.01)/60 = 0.0233$$

Finally, calculate $\dot{Q}$ for the bare rod:

$$\begin{aligned}\dot{Q}_{\mathrm{bare}} &= h(\pi DL)(T_s - T_f) = (140)(\pi)(0.01)(2.5)(250 - 25)\\ &= 2474\ \mathrm{W}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.15

Comment on result (3) in the previous Illustrative example.

***SOLUTION*:** Since $Bi = 0.0233$, which is $\ll 2.0$, one may assume that the temperature profile in the electric current conducting rod is relatively constant. ■

MICROSCOPIC APPROACH

The microscopic (transport) equations employed to describe heat transfer in a flowing fluid are presented in Table 9.6.(6,7) The equations describe the temperature profile in a moving fluid. Note that this table is essentially an extension of the microscopic material presented in the previous two chapters. The six illustrative examples that follow, drawn from the work of Theodore,(7) involve the application of this table.

ILLUSTRATIVE EXAMPLE 9.16

An incompressible fluid enters the reaction zone of an insulated tubular (cylindrical) reactor at temperature T_0. The chemical reaction occurring in the zone causes a rate of energy per unit volume to be liberated. This rate is proportional to the temperature and is given by

$$(S)(T)$$

where S is a constant. Obtain the steady-state equations describing the temperature in the reactor zone if the flow is laminar. Neglect axial diffusion.

***SOLUTION*:** This problem is solved in cylindrical coordinates. The pertinent equation is “extracted” from Table 9.6. Based on the problem statement, the temperature is a function of *both* z and r, and the only velocity component is v_z:

$$v_z \frac{\partial T}{\partial z} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + \frac{A}{\rho c_p}$$

Table 9.6 Energy-Transfer Equation for Incompressible Fluids(7)

Rectangular coordinates:

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{A}{\rho c_p} \tag{1}$$

Cylindrical coordinates:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\phi}{r} \frac{\partial T}{\partial \phi} + v_z \frac{\partial T}{\partial z} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{A}{\rho c_p} \tag{2}$$

Spherical coordinates:

$$\begin{aligned} &\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \\ &= \alpha \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right] + \frac{A}{\rho c_p} \end{aligned} \tag{3}$$

Note. Lowercase v employed for velocity.

If one neglects axial diffusion

$$\frac{\partial^2 T}{\partial z^2} = 0$$

The source term A is given by

$$A = (S)(T)$$

The resulting equation is

$$v_z \frac{\partial T}{\partial z} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right] + \left(\frac{S}{\rho c_p} \right) T \qquad (1)$$

If the flow is laminar$^{(2)}$,

$$v_z = V \left[1 - \left(\frac{r}{a} \right)^2 \right]$$

where a = radius of cylinder and V = maximum velocity, located at $r = 0$. Equation (1) now takes the form

$$V \left[1 - \left(\frac{r}{a} \right)^2 \right] \frac{\partial T}{\partial z} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right] + \left(\frac{S}{\rho c_p} \right) T$$

■

ILLUSTRATIVE EXAMPLE 9.17

Refer to the previous example. Obtain the describing equation if the flow is plug.

***SOLUTION*:** If the flow is plug, v_z is constant$^{(2)}$ across the area of the reactor. Based on physical grounds, the radial-diffusion term is zero and T is solely a function of z. Equation (1) then becomes

$$v_z \frac{dT}{dz} = \left(\frac{S}{\rho c_p} \right) T$$

■

ILLUSTRATIVE EXAMPLE 9.18

Obtain the temperature profile in the reactor of Illustrative Example 9.17 if the flow is plug.

***SOLUTION*:** The describing DE is obtained from Illustrative Example 9.17,

$$v_z \frac{dT}{dz} = \left(\frac{S}{\rho c_p} \right) T$$

The equation is rewritten as

$$\frac{dT}{T} = \left(\frac{S}{\rho c_p v_z} \right) dz$$

Integrating the above equation gives

$$\ln T = \left(\frac{S}{\rho c_p v_z}\right) z + B$$

The BC is

$$T = T_0 \quad \text{at } z = 0$$

so that

$$B = \ln T_0$$

Therefore

$$\ln\left(\frac{T}{T_0}\right) = \left(\frac{S}{\rho c_p v_z}\right) z$$

or

$$T = T_0 e^{(S/\rho c_p v_z)z}$$

If energy is absorbed due to chemical reaction, the above solution becomes

$$T = T_0 e^{-(S/\rho c_p v_z)z}$$

■

ILLUSTRATIVE EXAMPLE 9.19

A fluid is flowing (in the y direction) through the reaction zone of a rectangular parallelepiped reactor. Chemical reaction is occurring in the zone and energy is liberated at a constant rate. Obtain the equation(s) describing the steady-state temperature in the reaction zone if the flow is laminar. Assume the height (in the z direction) of the reactor h to be very small compared to its width w. Do not neglect diffusion effects.

***SOLUTION*:** For laminar flow,

$$T = T(y, z)$$
$$v_y = v_y(z)$$

From Table 9.6, Equation (1),

$$v_y \frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{A}{\rho c_p}$$

In addition, Theodore(7) has shown that for this application,

$$v_y = \frac{6q}{wh^3}(zh - z^2)$$

where q = volumetric flow rate. ■

ILLUSTRATIVE EXAMPLE 9.20

Refer to Illustrative Example 9.19. Obtain the describing equation if the flow is plug.

***SOLUTION*:** For plug flow,

$$T = T(y)$$
$$v_y = \text{constant}$$

From Table 9.6, Equation (1),

$$v_y \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} + \frac{A}{\rho c_p}$$

■

ILLUSTRATIVE EXAMPLE 9.21

Consider the motion of a fluid past a thin flat plate as pictured in Figure 9.4. The boundary-layer thickness for energy transfer δ_T is arbitrarily defined at the points where $T = 0.99T_0$. The temperature of the fluid upstream from the plate is T_0. The temperature at every point on the plate is maintained at zero. Obtain the steady-state equations describing the temperature in and outside the boundary layer.

***SOLUTION*:** The describing DE is obtained from Equation (1) in Table 9.6. Since the flow is two-dimensional,

$$v_x = 0$$
$$v_y \neq 0$$
$$v_z \neq 0$$

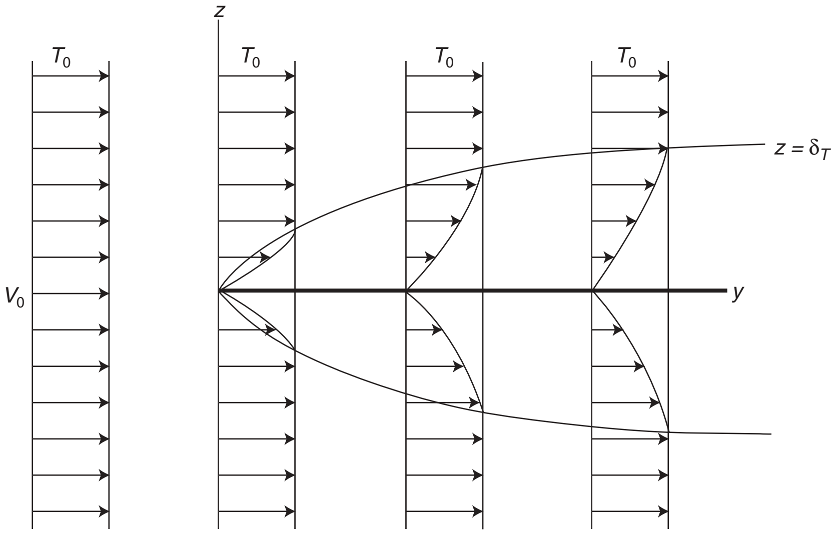

Figure 9.4 Boundary layers.

Based on the problem statement

$$T = T(y, z)$$

Since the flow is "primarily" in the y-direction, neglect thermal diffusion effects in that direction, i.e.,

$$\frac{\delta^2 T}{\delta y^2} = 0$$

The following equation results from Table 9.6:

$$v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} \tag{1}$$

The equation of continuity in rectangular coordinates for this application is$^{(2,7)}$

$$\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{2}$$

The BC are

$$\begin{aligned} T = 0\ ,\quad & v_y = 0 \quad \text{at } z = 0 \\ T = T_0,\quad & v_y = V_0 \quad \text{at } z = \infty \end{aligned}$$

Equations (1) and (2) can now be solved to give the temperature profile in the system. The complete solution is not presented. However, the profile outside the boundary layer is (naturally)

$$T = T_0$$

■

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Chapter 10

Free Convection

INTRODUCTION

Convective effects, previously described as *forced convection*, are due to the bulk motion of the fluid. The bulk motion is caused by external forces, such as that provided by pumps, fans, compressors, etc., and is essentially independent of "thermal" effects. *Free convection* is another effect that occasionally develops and was briefly discussed in the previous chapter. This effect is almost always attributed to buoyant forces that arise due to density differences within a system. It is treated analytically as another external force term in the momentum equation. The momentum (velocity) and energy (temperature) effects are therefore interdependent; consequently, both equations must be solved simultaneously. This treatment is beyond the scope of this text but is available in the literature.(1)

Consider a heated body in an unbounded medium. In natural convection, the velocity is zero (no-slip boundary condition) at the heated body. The velocity increases rapidly in a thin boundary adjacent to the body, and ultimately approaches zero when significantly displaced from the body. In reality, both natural convection and forced convection effects occur simultaneously so that one may be required to determine which is predominant. Both may therefore be required to be included in some analyses, even though one is often tempted to attach less significance to free convection effects. However, this temptation should be resisted since free convection occasionally plays the more important role in the design and/or performance of some heated systems.

As noted above, free convection fluid motion arises due to buoyant forces. Buoyancy arises due to the combined presence of a fluid density gradient and a body force that is proportional to density. The body force is usually gravity. Density gradients arise due to the presence of a temperature gradient. Furthermore, the density of gases and liquids depends on temperature, generally decreasing with increasing temperature, i.e., $(\partial\rho/\partial T)_P < 0$.

There are both industrial and environmental applications. Free convection influences industrial heat transfer from and within pipes. It is also important in transferring heat from heaters or radiators to ambient air and in removing heat from the coil of a

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Heat Transfer Applications for the Practicing Engineer. Louis Theodore

refrigeration unit to the surrounding air. It is also relevant to environmental sciences and engineering where it gives rise to both atmospheric and oceanic motion, a topic treated in the last section of this chapter.

In addition to this Introductory section, this chapter addresses the following three topics:

Key dimensionless numbers
Describing equations
Environmental applications$^{(2)}$

KEY DIMENSIONLESS NUMBERS

If a solid surface at temperature, T_s, is in contact with a gas or liquid at temperature, T_∞, the fluid moves solely as a result of density variations in natural convection. It is the fluid motion that causes the so-called natural convection. The nature of the buoyant force is characterized by the coefficient of volumetric expansion, β. For an ideal gas, β is given by

$$\beta = \frac{1}{T} \tag{10.1}$$

where T is the *absolute* temperature; this is an important term in natural convection theory and applications.

Semi-theoretical equations for natural convection use the following key dimensionless numbers, some of which have been discussed earlier:

$$\text{Gr} = \text{Grashof number} = \frac{L^3 g\beta\Delta T}{\nu^2} = \frac{L^3\rho^3 g\beta\Delta T}{\mu^2} \tag{10.2}$$

$$\text{Nu} = \text{Nusselt number} = hL/k \tag{10.3}$$

$$\text{Ra} = \text{Rayleigh number} = (\text{Gr})(\text{Pr}) = \frac{L^3 g\beta\Delta T}{\nu\alpha} = \frac{L^3 g\beta\Delta T\rho^2 c_p}{\mu k} \tag{10.4}$$

$$\text{Pr} = \text{Prandtl number} = \frac{\nu}{\alpha} = \frac{c_p\mu}{k} \tag{10.5}$$

where ν = kinematic viscosity
μ = absolute viscosity
α = thermal diffusivity $= k/\rho c_p$
ρ = fluid density
c_p = fluid heat capacity
k = thermal conductivity
L = characteristic length of system
ΔT = temperature difference between the surface and the fluid $= |T_s - T_\infty|$

The above Rayleigh number is used to classify natural convection as either laminar or turbulent:

$$\text{Ra} < 10^9 \quad \text{laminar free convection} \tag{10.6}$$

$$\text{Ra} > 10^9 \quad \text{turbulent free convection} \tag{10.7}$$

In the previous chapter on forced convection, the effects of natural convection were neglected, a valid assumption in many applications characterized by moderate-to-high-velocity fluids. Free convection may be significant with low-velocity fluids. A measure of the influence of each convection effect is provided by the ratio

$$\frac{\text{Gr}}{\text{Re}^2} = \frac{\text{buoyancy force}}{\text{inertia force}} = \frac{\rho g \beta L(\Delta T)}{V^2}; \; V = \text{fluid velocity} \tag{10.8}$$

This dimensionless number is represented by *LT* by the author,(2) so that

$$\frac{\text{Gr}}{\text{Re}^2} = LT \tag{10.9}$$

For $LT > 1.0$, free convection is important. The regimes of these convection effects are:

Free convection predominates, i.e., $LT \gg 1.0$ or $\text{Gr} \gg \text{Re}^2$ (10.10)

Forced convection predominates, i.e., $LT \ll 1.0$ or $\text{Gr} \ll \text{Re}^2$ (10.11)

Both effects contribute; mixed free and forced convection, i.e.,

$LT \approx 1.0$or $\text{Gr} \approx \text{Re}^2$ (10.12)

Combining these three convection regimes with the two flow regimes—laminar and turbulent—produces six subregimes of potential interest.

ILLUSTRATIVE EXAMPLE 10.1

The Grashof and Reynolds numbers for a system involved in a heat transfer process are approximately 100 and 50, respectively. Can free convection effects be neglected.

***SOLUTION*:** Employ Equation (10.9).

$$LT = \frac{\text{Gr}}{\text{Re}^2}$$

Substituting

$$LT = \frac{100}{50^2} = 0.04$$

Since $LT \ll 1.0$, free convection effects can be neglected. ■

DESCRIBING EQUATIONS

Free convection is an important consideration in the calculation of heat transfer rates from pipes, transmission lines, electronic devices, and electric baseboards. The average Nusselt number, $\overline{\mathrm{Nu}}$ and the Rayleigh number can be related through the following semi-theoretical correlation:

$$\overline{\mathrm{Nu}} = \bar{h}L/k = c\,\mathrm{Ra}^m \tag{10.13}$$

with all fluid properties evaluated at the film temperature, T_f,

$$T_f = (T_S + T_\infty)/2 \tag{10.14}$$

Generally, but with some exceptions,

$$\begin{aligned} m &= \tfrac{1}{4} \quad \text{for laminar free convection} \\ m &= \tfrac{1}{3} \quad \text{for turbulent free convection} \end{aligned} \tag{10.15}$$

As one might expect, the characteristic length, L, depends on the geometry. For a vertical plate, L is the plate height and for a horizontal plate, the plate length. For a horizontal cylinder, L is the diameter, D, and for a horizontal disk, L is given by: $L = 0.9D$. The constants C and m to be used with Equation (10.13) are listed in Table 10.1 for several geometries and a wide range of Rayleigh numbers.

Another correlation that can be used to calculate the heat transfer coefficient for natural convection from spheres is Churchill's equation,

$$\overline{\mathrm{Nu}} = 2 + \frac{0.589\,\mathrm{Ra}^{0.25}}{\left[1 + \left(\frac{0.469}{\mathrm{Pr}}\right)^{9/16}\right]^{4/9}} \tag{10.16}$$

Table 10.1 Coefficients for Equation (10.13)

Geometry	Gr Pr = Ra	c	m
Vertical planes and cylinders	10^4–10^9	0.59	0.25
	10^9–10^{13}	0.10	0.3333
Horizontal cylinders	0–10^{-5}	0.4	0
	10^{-5}–10^4	0.85	0.188
	10^4–10^9	0.53	0.25
	10^9–10^{12}	0.13	0.3333
Spheres	0–10^{12}	0.60	0.25
Upper surface of horizontal heated plates; plate is hotter than surroundings ($T_S > T_\infty$) or lower surface of horizontal cooled plates ($T_S < T_\infty$)	2×10^4–8×10^6	0.54	0.25
	8×10^6–10^{11}	0.15	0.3333
Lower surface of horizontal heated plates ($T_S > T_\infty$) or upper surface of horizontal cooled ($T_S < T_\infty$) plates	10^5–10^{11}	0.58	0.2

The Rayleigh number, Ra, and the Nusselt number, Nu, in Equation (10.16) are based on the diameter of the sphere. Churchill's equation is valid for $\mathrm{Pr} \geq 0.7$ and $\mathrm{Ra} \leq 10^{11}$.

There are also *simplified* correlations for natural (or free) convection in air at 1 atm. The correlations are dimensional and are based on the following SI units: $h =$ heat transfer coefficient, $\mathrm{W/m^2 \cdot K}$; $\Delta T = T_S - T_\infty$, °C; $T_s =$ surface temperature, °C; $T_\infty =$ surroundings temperature, °C; $L =$ vertical or horizontal dimension, m and $D =$ diameter, m. These correlations are presented in Table 10.2.

The average heat transfer coefficient can be calculated once the Nusselt number has been determined. Rearranging Equation (10.13) gives

$$\bar{h} = \overline{\mathrm{Nu}}\, k/L \tag{10.17}$$

The heat transfer rate, $\dot{Q}$, is given by the standard heat transfer equation:

$$\dot{Q} = \bar{h}A(T_S - T_\infty) \tag{10.18}$$

If the air is at a pressure other than 1 atm, the following correction may be applied to the reference value at 1 atm:

$$h = h_{\mathrm{ref}}(P \text{ in atmospheres})^n \tag{10.19}$$

where

$$\begin{aligned} n &= \tfrac{1}{2} \text{ for laminar cases } (\mathrm{Ra} < 10^9) \\ n &= \tfrac{2}{3} \text{ for turbulent cases } (\mathrm{Ra} > 10^9) \end{aligned} \tag{10.20}$$

Table 10.2 Free Convection Equation in Air

Geometry	$10^4 < \mathrm{Gr\,Pr} = \mathrm{Ra} < 10^9$ Laminar	$\mathrm{Gr\,Pr} = \mathrm{Ra} > 10^9$ Turbulent
Vertical planes and cylinders	$h = 1.42\ (\Delta T/L)^{0.25}$	$h = 0.95\ (\Delta T)^{0.333}$
Horizontal cylinders	$h = 1.32\ (\Delta T/D)^{0.25}$	$h = 1.24\ (\Delta T)^{0.333}$
Upper surface of horizontal heated plates; plate is hotter than surroundings ($T_S > T_\infty$) or lower surface of horizontal cooled plates ($T_S > T_\infty$)	$h = 1.32\ (\Delta T/L)^{0.25}$	$h = 1.43\ (\Delta T/L)^{0.333}$
Lower surface of horizontal heated plates or upper surface of horizontal cooled plates ($T_S < T_\infty$) plates	$h = 0.61\ (\Delta T/L^2)^{0.2}$	

ILLUSTRATIVE EXAMPLE 10.2

The heat flux rate incident on a vertical flat plate at 110°C is $800\ \mathrm{W/m^2}$. The plate is 2 m wide and 3.5 m high and is well insulated on the back side. The ambient air temperature is 30°C. All the incident radiation ($800\ \mathrm{W/m^2}$) on the plate is absorbed and dissipated by free convection to the ambient air at 30°C. Determine the Grashof and Rayleigh numbers.

***SOLUTION*:** Obtain ν, k, and Pr from the Appendix for air at the film temperature, T_f.

$$\begin{aligned} T_f &= (T_S + T_\infty)/2 \\ &= (110 + 30)/2 \\ &= 70^\circ\mathrm{C} = 343\ \mathrm{K} \end{aligned}$$

$$\begin{aligned} \nu &= 2.0 \times 10^{-5}\ \mathrm{m^2/s} \\ k &= 0.029\ \mathrm{W/m \cdot K} \\ \mathrm{Pr} &= 0.7 \end{aligned}$$

Calculate the coefficient of expansion β from Equation (10.1).

$$\begin{aligned} \beta &= \frac{1}{343} \\ &= 0.0029\ \mathrm{K^{-1}} \end{aligned}$$

Calculate the Grashof and Rayleigh numbers:

$$\mathrm{Gr} = g\beta\Delta T L^3/\nu^2 \tag{10.2}$$

$$\begin{aligned} &= \frac{(9.807)(0.0029)(80)(3.5)^3}{(2.0 \times 10^{-5})^2} \\ &= 2.44 \times 10^{11} \end{aligned}$$

$$\mathrm{Ra} = (\mathrm{Gr})(\mathrm{Pr}) \tag{10.4}$$

$$\begin{aligned} &= (2.44 \times 10^{11})(0.7) \\ &= 1.71 \times 10^{11} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 10.3

Refer to Illustrative Example 10.2. Determine the type of natural convection (flow regime).

***SOLUTION*:** Determine the free convection flow type. Refer to Table 10.2.

Since $\mathrm{Ra} > 10^9$, the convection flow catagory is turbulent.

■

ILLUSTRATIVE EXAMPLE 10.4

Refer to Illustrative Example 10.2. Determine the average heat transfer coefficient.

***SOLUTION*:** Calculate the average Nusselt number. From Table 10.1, $c = 0.1$ and $m = \frac{1}{3}$. Therefore,

$$\overline{\text{Nu}} = c\,\text{Ra}^m \tag{10.13}$$

Substituting,

$$\begin{aligned} &= 0.1(1.71 \times 10^{11})^{1/3} \\ &= 555.0 \end{aligned}$$

Calculate the average heat transfer coefficient employing Equation (10.13):

$$\begin{aligned} \bar{h} &= (\overline{\text{Nu}})(k)/(L) \\ &= (555.1)(0.029)/3.5 \\ &= 4.6\,\text{W/m}^2\cdot\text{K} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 10.5

Comment on the calculations performed in the previous example.

***SOLUTION*:** The approximate correlations for natural air convection provided in Table 10.2 could simplify the calculations of the heat transfer rate. This is left as an exercise for the reader. [Hint: the answer in approximately 4.0 $\text{W/m}^2\cdot\text{K}$] ■

ILLUSTRATIVE EXAMPLE 10.6 (adapted from Holman(3))

Calculate the air heat transfer film coefficient for a horizontal 6 inch diameter pipe whose surface temperature is 200°F in a room containing air at 70°F.

***SOLUTION*:**

$$T_{\text{av}} = (200 + 70)/2 = 135^\circ\text{F}$$

The fluid volume density is obtained directly from the ideal gas law:

$$v_f = RT/P = (0.73)(135 + 460)/(1.0) = 434\ \text{ft}^3/\text{lbmol}$$

The mass density of the fluid is

$$\rho_f(\text{air, MW} = 29) = 29/434 = 0.0668\,\text{lb/ft}^3$$

At 70°F (from the Appendix)

$$\begin{aligned} \mu &= 0.019\,\text{cP} = 1.28 \times 10^{-5}\,\text{lb/ft}\cdot\text{s} \\ k &= 0.016\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F} \end{aligned}$$

Calculate the Grashof number; employ Equation (10.2). Since

$$\beta = 1/T_{\text{av}} = 1/(135 + 460) = 0.0017\,^{\circ}\text{R}^{-1}$$

$$\text{Gr} = \frac{D^3\rho^2 g\beta\Delta T}{\mu^2} = \frac{(0.5)^3(0.0668)^2(32.174)(0.017)(660 - 530)}{(1.28 \times 10^{-5})^2} = 24.2 \times 10^7$$

The Prandtl number may also be calculated. At 135°F, $c_p = 0.25$ Btu/lb · °F. Substituting into Equation (10.5),

$$\text{Pr} = \frac{c_p\mu}{k} = \frac{(0.25)(1.28 \times 10^{-5})}{0.016/3600} = 0.72$$

Note that for air, Pr is generally taken as 0.7 (see earlier examples). The following term is calculated:

$$\log_{10}[(\text{Gr})(\text{Pr})] = 8.24$$

From Holman,(3) at this value

$$\log_{10}(\text{Nu}) \cong 1.5$$

so that

$$\text{Nu} = 31.6$$

Since

$$\text{Nu} = \frac{hD}{k} \tag{10.13}$$

$$h = \text{Nu}\left(\frac{k}{D}\right) = 31.6\left(\frac{0.016}{0.5}\right) = 1.01\ \text{Btu/h} \cdot \text{ft} \cdot {}^{\circ}\text{F}$$

■

ILLUSTRATIVE EXAMPLE 10.7

Calculate the free convection heat transfer coefficient for a plate 6 ft high and 8 ft wide at 120°F that is exposed to nitrogen at 60°F.

***SOLUTION*:** The mean film temperature is

$$T_f = (120 + 60)/2 = 90^{\circ}\text{F} = 550^{\circ}\text{R}$$

From the Appendix,

$$\rho = 0.0713\ \text{lb/ft}^3$$
$$k = 0.01514\ \text{Btu/h} \cdot \text{ft} \cdot {}^{\circ}\text{F}$$
$$\nu = 16.82 \times 10^{-5}\ \text{ft}^2/\text{s}$$
$$\text{Pr} = 0.713$$

In addition,

$$\beta = 1/T = 1/550 = 1.818 \times 10^{-3}\,^{\circ}\text{R}^{-1}$$

Therefore

$$\mathrm{Gr} = \frac{g\beta(T_S - T_\infty)L^3}{\nu^2} \tag{10.2}$$

Substituting,

$$\mathrm{Gr} = \frac{(32.2\ \mathrm{ft/s^2})(1.818 \times 10^{-3}\ {}^\circ\mathrm{R}^{-1})[(120 - 40)^\circ\mathrm{R}](6\ \mathrm{ft})^3}{(16.82 \times 10^{-5})^2\ \mathrm{ft^4/s^2}} = 3.576 \times 10^{10}$$

In addition, from Equation (10.4),

$$\mathrm{Ra} = (\mathrm{Gr})(\mathrm{Pr}) = (3.576 \times 10^{10})(0.713) = 2.549 \times 10^{10}$$

The flow is therefore turbulent (see Table 10.2). Equation (10.13) applies, with appropriate constants from Table 10.1, to give

$$\begin{aligned}\frac{\overline{h}L}{k} &= (0.10)(\mathrm{Ra})^{1/3}\\ \overline{h} &= \frac{0.01514\,\mathrm{Btu/h\cdot ft\cdot {}^\circ F}}{6\ \mathrm{ft}}(0.10)(2.549 \times 10^{10})^{1/3}\\ &= 0.743\ \mathrm{Btu/h\cdot ft^2\cdot {}^\circ F}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 10.8

Calculate the heat loss in the previous example.

***SOLUTION*:** Apply Equation (10.18).

$$\begin{aligned}\dot{Q} &= \overline{h}A(T_S - T_\infty)\\ &= \big(0.743\,\mathrm{Btu/h\cdot ft^2\cdot {}^\circ F}\big)[(6 \times 8)\,\mathrm{ft^2}][(120 - 60)^\circ\mathrm{F}]\\ &= 2140\,\mathrm{Btu/h}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 10.9

Calculate the heat transfer from a 100-W light bulb at 113°C to 31°C ambient air. Approximate the bulb as a 120-mm-diameter sphere.

***SOLUTION*:** For this example,

$$T_f = (T_S + T_\infty)/2 = 72^\circ\mathrm{C}$$

From the Appendix,

$$\begin{aligned}\nu &= (22.38 \times 10^{-5})(0.0929) = 2.079 \times 10^{-5}\,\text{m}^2/\text{s}\\ \text{Pr} &= 0.70\\ k &= (0.01735)(1.729) = 0.0300\,\text{W/m}\cdot\text{K}\\ \beta &= 1/T = 1/345 = 2.899 \times 10^{-3}\,\text{K}^{-1}\end{aligned}$$

Employ the characteristic length as the diameter of the sphere, D, in Equation (10.2).

$$\begin{aligned}\text{Gr} &= \frac{g\beta(T_S - T_\infty)D^3}{\nu^2}\\ &= \frac{(9.80\,\text{m/s}^2)(2.899 \times 10^{-3}\,\text{K}^{-1})(113 - 31)(\text{K})(0.060\,\text{m})^3}{(2.079 \times 10^{-5})^2\,\text{m}^4/\text{s}^2}\\ &= 1.16 \times 10^6\end{aligned}$$

Apply Equation (10.13) with constants drawn from Table 10.1, i.e.,

$$\begin{aligned}\frac{\bar{h}D}{k} &= (0.60)(\text{Ra})^{1/4}\\ \bar{h} &= \frac{0.0300\,\text{W/m}\cdot\text{K}}{0.060\,\text{m}}(0.60)[(1.16 \times 10^6)(0.7)]^{1/4}\\ &= 9.01\,\text{W/m}^2\cdot\text{K}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 10.10

Refer to Illustrative Example 10.9. Calculate the heat transfer lost by free convection from the light bulb.

***SOLUTION*:** Once again, apply Equation (10.18)

$$\begin{aligned}\dot{Q} &= \bar{h}A(T_S - T_\infty)\\ &= \left(9.01\,\text{W/m}^2\cdot\text{K}\right)\pi(0.060\,\text{m})^2(82\,\text{K})\\ &= 8.36\,\text{W}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 10.11

With reference to Illustrative Examples 10.9–10.10, what percentage of the energy is lost by free convection.

***SOLUTION*:**

$$\text{Energy lost} = \frac{8.36}{100}(100\%) = 8.36\% = 0.0836$$

■

ILLUSTRATIVE EXAMPLE 10.12

How else could Illustrative Examples 10.9–10.11 have been solved.

***SOLUTION*:** Employ Equation (10.16) rather than Equation (10.13). This is left as an exercise for the reader ■

ENVIRONMENTAL APPLICATIONS$^{(2)}$

Two applications that involve the environment comprise the concluding section of this chapter. The first is concerned with lapse rates and the other with plume rise.

Lapse Rates

The concept behind the so-called *lapse rate* that has found application in environmental science and engineering can be best demonstrated with the following example. Consider the situation where a fluid is enclosed by two horizontal plates of different temperature ($T_1 \neq T_2$), see Figure 10.1. In case (*a*), the temperature of the lower plate exceeds that of the upper plate ($T_2 > T_1$) and the density decreases in the (downward) direction of the gravitational force. If the temperature difference exceeds a particular value, conditions are termed *unstable* and buoyancy forces become important. In Figure 10.1(a), the gravitational force of the cooler and denser fluid near the top plate exceeds that acting on the lighter hot fluid near the bottom plate and the circulation pattern as shown on the right-hand side of Figure 10.1(a) will exist. The cooler heavier fluid will descend, being warmed in the process, while the lighter hot fluid will rise, cooling as it moves. However, for case (*b*) where $T_1 > T_2$, the density no longer decreases in the direction of the gravitational force. Conditions are now

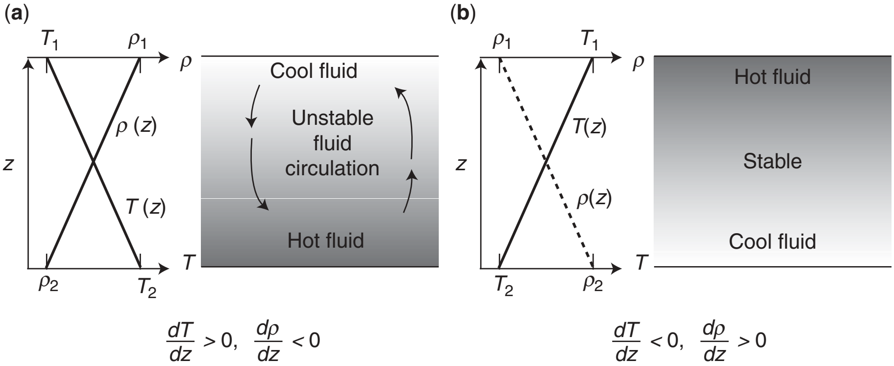

Figure 10.1 A fluid contained between two horizontal plates at different temperatures. (**a**) Unstable temperature gradient. (**b**) Stable temperature gradient.

reversed and are defined as *stable* since there is no bulk fluid motion. In case (a), heat transfer occurs from the bottom to the top surface by free convection; for case (b), any heat transfer (from top to bottom) occurs by conduction. These two conditions are similar to that experienced in the environment, particularly with regard to the atmosphere. The extension of the above development is now applied to the atmosphere but could just as easily be applied to oceanographic systems.(2)

Apart from the mechanical interference of the steady flow of air caused by buildings and other obstacles, the most important factor that determines the degree of turbulence, and hence how fast diffusion in the lower air occurs, is the variation of temperature with height above the ground (i.e., the aforementioned "lapse rate"). Air is a good insulator. Therefore, heat transfer in the atmosphere is caused by radiative heating or by mixing due to turbulence. If there is no mixing, an air parcel rises adiabatically (no heat transfer) in the atmosphere.

The Earth's atmosphere is normally treated as a perfect gas mixture. If a moving air parcel is chosen as a control volume, it will contain a fixed number of molecules. The volume of such an air parcel must be inversely proportional to the density. The perfect gas law is

$$PV \propto RT$$

or

$$P \propto \rho RT \tag{10.21}$$

The air pressure at a fixed point is caused by the weight of the air above that point. The pressure is highest at the Earth's surface and decreases with altitude. This is a hydrostatic pressure distribution with the change in pressure proportional to the change in height.(3)

$$dP = -\rho g \, dz \tag{10.22}$$

Since air is compressible, the density is also a decreasing function of height. An air parcel must have the same pressure as the surrounding air and so, as it rises, its pressure decreases. As the pressure drops, the parcel must expand adiabatically. The work done in the adiabatic expansion ($P\,dV$) comes from the thermal energy of the air parcel.(4) As the parcel expands, the internal energy then decreases and the temperature decreases.

If a parcel of air is treated as a perfect gas rising in a hydrostatic pressure distribution, the rate of cooling produced by the adiabatic expansion can be calculated. The rate of expansion with altitude is fixed by the vertical pressure variation described in Equation (10.22). Near the Earth's surface, a rising air parcel's temperature normally decreases by 0.98°C with every 100-m increase in altitude.

The vertical temperature gradient in the atmosphere (the amount the temperature changes with altitude, dT/dz) is defined as the *lapse rate*. The dry *adiabatic lapse rate*

(DALR) is the temperature change for a rising parcel of dry air. The dry adiabatic lapse rate is approximately $-1^{\circ}\mathrm{C}/100\,\mathrm{m}$ or $dT/dz = -10^{-2}\,{}^{\circ}\mathrm{C/m}$ or $-5.4^{\circ}\mathrm{F}/1000\,\mathrm{ft}$. Strongly stable lapse rates are commonly referred to as *inversions*; $dT/dz > 0$. The strong stability inhibits mixing. Normally, these conditions of strong stability only extend for several hundred meters vertically. The vertical extent of the inversion is referred to as the inversion height. Thus, a positive rate is particularly important in air pollution episodes because it limits vertical motion (i.e., the inversion traps the pollutants between the ground and the inversion layer).

Ground-level inversions inhibit the downward mixing of pollutants emitted from automobiles, smoke stacks, etc. This increases the ground-level concentrations of pollutants. At night the ground reradiates the solar energy that it received during the day. On a clear night with low wind speeds, the air near the ground is cooled and forms a ground-level inversion. By morning, the inversion depth may be 200–300 m with a 5–10° temperature difference from bottom to top. Clouds cut down the amount of heat radiated by the ground because they reflect the radiation back to the ground. Higher wind speeds tend to cause more mixing and spread the cooling effect over a larger vertical segment of the atmosphere, thus decreasing the change in lapse rate during the night.(4)

Since temperature inversions arise because of solar radiation, the effects of nocturnal radiation often results in the formation of frost. When the Sun is down, some thermal radiation is still received by the Earth's surface from space, but the amount is small. Consequently, there is a net loss of radiation energy from the ground at night. If the air is relatively still, the surface temperature may drop below 32°F. Thus, frost can form, even though the air temperature is above freezing. This frost is often avoided with the presence of a slight breeze or by cloud cover.

Plume Rise

Smoke from a stack will usually rise above the top of the stack for a certain distance (see Figure 10.2). The distance that the plume rises above the stack is called the *plume rise*. It is actually calculated as the distance to the imaginary centerline of the plume rather than to the upper or lower edge of the plume. Plume rise, normally denoted Δh, depends on the stack's physical characteristics and on the effluent's (stack gas) characteristics. For example, the effluent characteristic of stack gas temperature T_s in relation to the surrounding air temperature T_a is more important than the stack characteristic of height. The difference in temperature between the stack gas and ambient air determines plume density, and this density affects plume rise. Therefore, smoke from a short stack could climb just as high as smoke from a taller stack.

Stack characteristics are used to determine momentum, and effluent characteristics are used to determine buoyancy. The *momentum* of the effluent is initially provided by the stack. It is determined by the speed of the effluent as it exits the stack. As momentum carries the effluent out of the stack, atmospheric conditions begin to affect the plume.

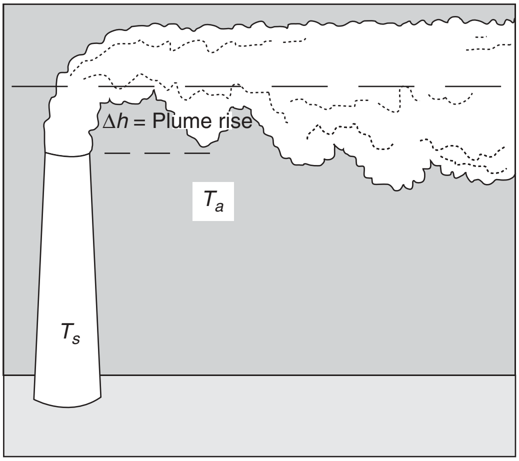

Figure 10.2 Plume rise.

The condition of the atmosphere, including the winds and temperature profile along the path of the plume, will primarily determine the plume's rise. As the plume rises from the stack, the wind speed across the stack top begins to tilt (or bend) the plume. Wind speed usually increases with distance above the Earth's surface. As the plume continues upward, stronger winds tilt the plume even farther. This process continues until the plume may appear to be horizontal to the ground. The point where the plume appears level may be a considerable distance downwind from the stack.

Plume buoyancy is a function of temperature. When the effluent's temperature, T_s, is warmer than the atmosphere's temperature, T_a, the plume will be less dense than the surrounding air. In this case, the density difference between the plume and air will cause the plume to rise. The greater the temperature difference, ΔT, the more buoyant the plume. As long as the temperature of the pollutant remains warmer than the atmosphere, the plume will continue to rise. The distance downwind where the pollutant cools to atmospheric temperature may also be quite displaced from its original release point.

Buoyancy is taken out of the plume by the same mechanism that tilts the plume over—the wind. The faster the wind speed, the faster this mixing with outside air takes place. This mixing is called *entrainment*. Strong wind will "rob" the plume of its buoyancy rapidly and, on windy days, the plume will not climb significantly above the stack.

Many individuals have studied plume rise over the years. The most popular plume rise formulas in use are those of Briggs.(5) Most plume rise equations are used on plumes with temperatures greater than the ambient air temperature. The equations proposed by Briggs are presented below in Illustrative Example 10.13. Plume rise formulas determine the imaginary *centerline* of the plume; the centerline is located where the *greatest concentration* of pollutant occurs at a given downward distance. Finally, plume rise is a linear measurement, usually expressed in feet or meters.

ILLUSTRATIVE EXAMPLE 10.13(6,7)

If a waste source emits a gas with a buoyancy flux of 50 $\mathrm{m^4/s^3}$, and the wind speed averages 4 m/s, find the plume rise at a distance of 750 m downward from a stack that is 50 m high under unstable atmospheric conditions. Several plume rise equations are available. Use the equation proposed by Briggs.

Briggs(5) used the following equations to calculate the plume rise:

$$\begin{aligned}
\Delta h &= 1.6F^{1/3}u^{-1}x^{2/3}; \quad x < x_f \\
&= 1.6F^{1/3}u^{-1}x_f^{2/3}; \quad \text{if } x \geq x_f \\
x^* &= 14F^{5/8}; \quad \text{when } F < 55\,\mathrm{m^4/s^3} \\
&= 34F^{2/5}; \quad \text{when } F \geq 55\,\mathrm{m^4/s^3} \\
x_f &= 3.5x^*
\end{aligned} \tag{10.23}$$

where Δh = plume rise, m
F = buoyancy flux, $\mathrm{m^4/s^3} = 3.7 \times 10^{-5}\dot{Q}_H$
u = wind speed, m/s
x^* = downward distance, m
x_f = distance of transition from first stage of rise to the second stage of rise, m
$\dot{Q}_H$ = heat emission rate, kcal/s

If the term $\dot{Q}_H$ is not available, the term F may be estimated by

$$F = (g/\pi)q(T_S - T)/T_S \tag{10.24}$$

where g = gravity term 9.8 $\mathrm{m/s^2}$
q = stack gas volumetric flowrate, $\mathrm{m^3/s}$ (actual conditions)
T_S, T = stack gas and ambient air temperature, K, respectively

***SOLUTION*:** Calculate x_f to determine which plume equation applies

$$\begin{aligned}
x^* &= 14F^{5/8}, \quad \text{since } F \text{ is less than } 55\,\mathrm{m^4/s^3} \\
&= (14)(50)^{5/8} \\
&= 161.43\,\mathrm{m} \\
x_f &= 3.5x^* \\
&= (3.5)(161.43) \\
&= 565.0\,\mathrm{m}
\end{aligned} \tag{10.23}$$

The plume rise is therefore ($x > x_f$)

$$\begin{aligned}
\Delta h &= 1.6F^{1/3}u^{-1}x_f^{2/3}, \quad \text{since } x \geq x_f \\
&= (1.6)(50)^{1/3}(4)^{-1}(565)^{2/3} = 101\,\mathrm{m}
\end{aligned} \tag{10.23}$$

■

ILLUSTRATIVE EXAMPLE 10.14

Briefly discuss other plume rise equations.

***SOLUTION*:** Many other plume rise equations may be found in the literature.(4,7,8) The Environmental Protection Agency (EPA) is mandated to use Brigg's equations to calculate plume rise. In past years, industry has often chosen to use the Holland or Davidson–Bryant equation. The Holland equation is(4,7,8)

$$\Delta h = D_s(v_s/u)[1.5 + 2.68 \times 10^{-3}P(\Delta T/T_s)/d_s] \tag{10.25}$$

where D_s = inside stack diameter, m

v_s = stack exit velocity, m/s

u = wind speed, m/s

P = atmospheric pressure, mbar

T_s, T = stack gas and ambient temperature, respectively, K

$\Delta T = T_S - T$

Δh = plume rise, m

The Davidson–Bryant equation is(9)

$$\Delta h = D_s(v_s/u)^{1.4}\{1.0 + [(T_s + T)/T_s]\}$$

The reader should also note that the "plume rise" may be negative in some instances due to surrounding structures, topography, and so on. ■

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Chapter 11

Radiation

INTRODUCTION

In addition to conduction and convection, heat can be transmitted by radiation. Conduction and convection both require the presence of molecules to “carry” or pass along energy. Unlike conduction or convection, radiation does not require the presence of any medium between the heat source and the heat sink since the thermal energy travels as electromagnetic waves. This radiant energy (thermal radiation) phenomena is emitted by every body having a temperature greater than absolute zero. Quantities of radiation emitted by a body are a function of both temperature and surface conditions, details of which will be presented later in this chapter. Applications of thermal radiation include industrial heating, drying, energy conversion, solar radiation, and combustion.

The amount of thermal radiation emitted is not always significant. Its importance in a heat transfer process depends on the quantity of heat being transferred simultaneously by the other aforementioned mechanisms. The reader should note that the thermal radiation of systems operating at or below room temperature is often negligible. In contrast, thermal radiation tends to be the principal mechanism for heat transfer for systems operating in excess of 1200°F. When systems operate between room temperature and 1200°F, the amount of heat transfer contributed by radiation depends on such variables as the convection film coefficient and the nature of the radiating surface.

Radiation heat transfer in an industrial boiler from the hot gases to most solid surfaces inside the combustion chamber is considerable. However, in most heat exchangers, the contribution of radiation as a heat transfer mechanism is usually minor.

In the heat transfer mechanisms of conduction and convection discussed in Chapters 7–10, the movement of energy in the form of heat takes place through a material medium—a fluid in the case of convection. Since a transfer medium is not required for this third mechanism, the energy is carried by electromagnetic radiation. Thus, a piece of steel plate heated in a furnace until it is glowing red and then placed several inches away from a cold piece of steel plate will cause the temperature of the cold steel to rise, even if the process takes place in an evacuated container.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

As noted above, radiation generally becomes important as a heat transfer mechanism only when the temperature of the source is very high. As will become clear later in the chapter, the energy transfer is approximately proportional to the fourth power of the *absolute* temperature of the radiating body. However, the driving force for conduction and convection is simply the temperature difference (driving force) between the source and the receptor; the actual temperatures have only a minor influence. For these two mechanisms, it does not matter whether the temperatures are 120°F and 60°F or 520°F and 460°F. Radiation, on the other hand, is strongly influenced by the temperature level; as the temperature increases, the extent of radiation as a heat transfer mechanism increases rapidly. It therefore follows that, at very low temperatures, conduction and convection are the major contributors to the total heat transfer; however, at very high temperatures, radiation is often the prime contributor.

ILLUSTRATIVE EXAMPLE 11.1

Qualitatively discuss radiation.

***SOLUTION*:**

1. Radiation is energy transported in the form of electromagnetic waves at the velocity of light.
2. In contrast to conduction and convection, radiation does not require matter for the transfer of energy; for example, radiation can be transferred through a vacuum.
3. Molecules and atoms emit electromagnetic waves when at high energy levels corresponding to high temperatures; this release of energy allows for the atom or molecule to return to a lower energy state. ■

Knowledge of the mechanism of heat transfer is essential in selecting an appropriate heat transfer equation, which is the first step in designing heaters, coolers, condensers, evaporators, and so on. In analyzing the performance of a thermal system, the engineer must therefore be able to identify the relevant heat transfer process (there are generally more than one). Only then can the system behavior be properly quantified. These mechanisms are discussed in the following illustrative example.

ILLUSTRATIVE EXAMPLE 11.2

Identify the pertinent heat transfer processes for the following systems.

1. A heat exchanger made of metal tubing with the fluid inside the tube hotter than the outside fluid.
2. An insulation blanket placed around a tank of Liquefied Nitrogen Gas (LNG).
3. Air flowing across a heated radiator.

4. The heat transfer process that determine the temperature of an asphalt pavement on a breezy summer's day.
5. A thermocouple junction is used to measure the temperature of a hot gas stream flowing through a channel by inserting the junction into the mainstream of the gas. Identify the heat transfer process associated with the junction surface.
6. With respect to (5), will the junction sense a temperature that is less than, equal to, or greater than the gas temperature?

***SOLUTION*:**

1. In scenario 1, metallic solids transfer heat by conduction. The metal tube wall conducts heat from the hot fluid, through the metal wall, to the cold fluid.
2. In scenario 2, solid insulators also transfer heat by conduction. The insulation blanket conducts heat from the warmer outside air to the colder metal of the tank. Because of the high thermal resistance of the blanket, the rate of transfer is slow.
3. In scenario 3, heat is transferred by natural convection. If the air currents are caused by an external force, the heat transfer from the radiator surface to the air is by forced convection.
4. Pathways for energy to and from asphalt paving for (4) include incident solar radiation (a large portion of which is absorbed by the asphalt). Asphalt emits heat by radiation to the surroundings and also loses heat by convection to the air.
5. Pathways for energy to and from the thermocouple with respect to scenario (5) include convection of heat from the hot gas to the thermocouple junction and radiation from the thermocouple junction to the wall.
6. Finally, in (6), since the thermocouple is losing heat by radiation, it will indicate a temperature lower than that of the actual gas temperature. ■

Several additional examples of heat transfer mechanisms by radiation have appeared in the literature. Badger and Banchero$^{(1)}$ provided the following explanation of radiation: "If radiation is passing through empty space, it is not transformed to heat or any other form of energy and it is not diverted from its path. If, however, matter appears in its path, it is only the absorbed energy that appears as heat, and this transformation is quantitative. For example, fused quartz transmits practically all the radiation which strikes it; a polished opaque surface or mirror will reflect most of the radiation impinging on it; a black surface will absorb most of the radiation received by it (as one can experience on a sunny day while wearing a black shirt) and will transform such absorbed energy quantitatively into heat." The relationship between the energy transmitted, reflected, and absorbed is discussed in the next section; see also Equation (11.7). Bennett and Meyers$^{(2)}$ provide an additional example involving the operation of a steam "radiator."

Characteristic wavelengths of radiation are provided in Table 11.1. Note that light received from the Sun passes through the Earth's atmosphere which absorbs some of the energy and thus affects the quality of visible light as it is received. The units of wavelength may be expressed in meters (m), centimeters (cm), micrometers (μm), or Angstroms (1.0 Å = 10 μm), with the centimeter being the unit of choice.$^{(3)}$ The speed of electromagnetic radiation is approximately 3×10^8 m/s in a vacuum. This

Table 11.1 Characteristic Wavelengths$^{(3)}$

Type of radiation	$\lambda \times 10^8$
Gamma rays	0.01–0.15 cm
X-rays	0.06–1000 cm
Ultraviolet	100–35,000 cm
Visible	3500–7800 cm
Infrared	7800–4,000,000 cm
Radio	0.01–0.15 cm

velocity (c) is given by the product of the wavelength (λ) and the frequency (ν) of the radiation, that is,

$$c = \lambda\nu; \quad \text{consistent units} \tag{11.1}$$

As noted earlier, the energy emitted from a "hot" surface is in the form of electromagnetic waves. One of the types of electromagnetic waves is thermal radiation. Thermal radiation is defined as electromagnetic waves falling within the following range:

$$0.1\ \mu\text{m} < \lambda < 100\ \mu\text{m}; \quad 1.0\ \mu\text{m} = 10^{-6}\ \text{m} = 10^{-4}\ \text{cm}$$

However, most of this energy is in the interval from 0.1 to 10 μm. The visible range of thermal radiation lies within the narrow range of 0.4 μm (violet) $< \lambda <$ 0.8 μm (red).

The remainder of the chapter consists of five additional sections:

Energy and Intensity
Radiant Exchange
Kirchoff's Law
Emissivity Factors
View Factors

An additional 15 illustrative examples complement the presentation.

ENERGY AND INTENSITY

A body at a given temperature will emit radiation over a range of wavelengths, not a single wavelength. Information is available on the *intensity* of the radiant energy I (Btu/h·ft^2·μm) as a function of the wavelength, λ (μm). In addition, at any given temperature, a wavelength exists at which the amount of energy given off is a maximum. For the same body at a lower temperature, the maximum intensity of radiation is obviously less; however, it is also significant that the wavelength at which the maximum intensity exists is higher in value.

Since the $I - \lambda$ curve for a single temperature depicts the amount of energy emitted at a given wavelength, the sum of all the energy radiated by the body at all its wavelengths is simply the area under a plot of I vs. λ. This quantity of radiant energy (of all wavelengths) emitted by a body per unit area and time is defined as the *total emissive power E* (Btu/h·ft^2). Given the intensity of the radiation at any wavelength, I, one may calculate the total emissive power, E, from

$$E = \int_0^\infty I\,d\lambda \tag{11.2}$$

ILLUSTRATIVE EXAMPLE 11.3

The intensity of radiation as a function of wavelength ($\lambda = \mu$m) is specified as

$$I = 40e^{-\lambda^2}; \quad \text{Btu/h} \cdot \text{ft}^2 \cdot \mu\text{m}$$

Calculate the total emissive power.

***SOLUTION*:** Apply Equation (11.2):

$$E = \int_0^\infty I\,d\lambda$$

Substitute,

$$E = \int_0^\infty 40e^{-\lambda^2}\,d\lambda = 40\int_0^\infty e^{-\lambda^2}\,d\lambda$$

The above integral is calculated as follows.(4) Set

$$\lambda^2 = x; \quad \lambda = x^{1/2}$$

so that

$$\begin{aligned} 2\lambda\,d\lambda &= dx \\ d\lambda &= \frac{dx}{2\lambda} \\ &= \tfrac{1}{2}x^{-1/2}\,dx \end{aligned}$$

Insertion into the above integral gives

$$E = \frac{40}{2}\int_0^\infty e^{-x}x^{-1/2}\,dx$$

This integral is the gamma function of $\frac{1}{2}$, that is, $\Gamma(\frac{1}{2})$. Since

$$\begin{aligned} \Gamma(\tfrac{1}{2}) &= \pi^{1/2} = \sqrt{\pi} \\ E &= 20\sqrt{\pi} \\ &= 35.5 \text{ Btu/h} \cdot \text{ft}^2 \end{aligned}$$

■

Maxwell Planck was the first to fit the I vs. λ relationship to equation form, as

$$I_\lambda = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1} \tag{11.3}$$

where I_λ = intensity of emission ($\mathrm{Btu/h \cdot ft^2 \cdot \mu m}$) at λ
λ = wavelength (μm)
C_1 = 1.16×10^8 (dimensionless)
C_2 = 25,740 (dimensionless)
T = temperature of the body (°R)

It was later shown that the product of the wavelength of the maximum value of the intensity of emission and the absolute temperature is a constant. This is referred to as Wien's displacement law,

$$\lambda T = 2884\ \mu\mathrm{m} \cdot {}^\circ\mathrm{R} \approx 5200\ \mu\mathrm{m} \cdot \mathrm{K} \tag{11.4}$$

One can derive Equation (11.4) from (11.3) as follows. Since $dI_\lambda/d\lambda = 0$ at the maximum value of the intensity

$$\frac{dI_\lambda}{d\lambda} = d\left(\frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}\right)\Big/ d\lambda = 0 \tag{11.5}$$

After differentiation,

$$(-5C_1\lambda^{-6})(e^{C_2/\lambda T} - 1) + C_1\lambda^{-5}(e^{C_2/\lambda T})/(e^{C_2/\lambda T} - 1)^2 = 0$$

which can be reduced to

$$(-5C_1\lambda^{-6})(e^{C_2/\lambda T} - 1) + C_1\lambda^{-5}(e^{C_2/\lambda T})\left(\frac{C_2}{\lambda^2 T}\right) = 0$$

This ultimately simplifies to

$$(-5 + \frac{C_2}{\lambda T})e^{C_2/\lambda T} + 5 = 0;\ C_2 = 25{,}740 \tag{11.6}$$

The reader is left with the exercise of showing that the first term equals -5 when $\lambda T = 2884$.

Atmospheric data indicates that the maximum intensity, I, of the Sun is experienced around 0.25 μm wavelength. This accounts for the predominance of blue in the visible spectrum and the high ultraviolet content of the Sun's rays.

ILLUSTRATIVE EXAMPLE 11.4

Estimate the Sun's temperature. Employ equation (11.4).

SOLUTION: Assuming a wavelength of 0.25 μm, substitute into Equation (11.4):

$$\lambda T = 2884\,\mu\text{m}\cdot{}^{\circ}\text{R},\ \lambda = 0.25\,\mu\text{m}$$
$$T = 2884\,\mu\text{m}\cdot{}^{\circ}\text{R}/0.25\,\mu\text{m}$$
$$T = 11{,}500^{\circ}\text{R} \approx 11{,}000^{\circ}\text{F}$$

■

RADIANT EXCHANGE

The conservation law of energy indicates that any radiant energy incident on a body will partially absorb, reflect, or transmit stored energy. An energy balance around a receiving body on which the total incident energy is assumed to be unity gives

$$\alpha + \rho + \tau = 1 \tag{11.7}$$

where the absorptivity α is the fraction absorbed, the reflectivity ρ is the fraction reflected, and the transmissivity τ is the fraction transmitted. It should be noted that the majority of engineering applications involve opaque substances having transmissivities approaching zero (i.e., $\tau = 0$). This topic receives additional treatment below in a later paragraph.

When an ordinary body emits radiation to another body, it will have some of the emitted energy returned to itself by reflection. Equation (11.7) assumes that none of the emitted energy is returned; this is equivalent to assuming that bodies having zero transmissivity also have zero reflectivity. This introduces the concept of a perfect "black body" for which $\alpha = 1$. Not all substances radiate energy at the same rate at a given temperature. The theoretical substance to which most radiation discussions refer to is called a "black body." This is defined as a body that radiates the maximum possible amount of energy at a given temperature. Much of the development to follow is based on this concept.

To summarize, when radiation strikes the surface of a semi-transparent material such as a glass plate or a layer of water, three types of interactive effects occur. Some of the incident radiation is reflected off the surface, some of it is absorbed within the material, the remainder is transmitted through the material. Examining the three fates of the incident radiation, one can see that α and ρ depend on inherent properties of the material; it is for this reason that they are referred to as *surface* properties. The transmissivity, τ, on the other hand, depends on the amount of the material in question; it is therefore referred to as a *volumetric* property.

It is appropriate to examine a few common surfaces. An opaque surface, the most commonly encountered surface type, has $\tau \approx 0$. Because of this, Equation (11.7) becomes:

$$\alpha + \rho = 1 \tag{11.8}$$

or

$$\rho = 1 - \alpha \tag{11.9}$$

Equation (11.9) may also be applied to gases; this may be counterintuitive since most gases are invisible. With respect to the reflectivity term, ρ, surfaces may have either *specular* reflection, in which the angle of incidence of the radiation is equal to the angle of reflection, or *diffuse* reflection, in which the reflected radiation scatters in all directions. In addition, a *gray* surface is one for which the absorptivity is the same as the emissivity, ε, at the temperature of the radiation source. For this case,

$$\alpha = \varepsilon \tag{11.10}$$

$$\rho = 1 - \varepsilon \tag{11.11}$$

Reflectivity and transmissivity are characteristics experienced in the everyday world. Polished metallic surfaces have high reflectivities and granular surfaces have low reflectivities. Reflection from a surface depends greatly on the characteristics of the surface. If a surface is very smooth, the angles of incidence and reflection are essentially the same. However, most surfaces encountered in engineering practice are sufficiently rough so that some reflection occurs in all directions. Finally, one may state that a system in thermal equilibrium has its absorptivity equal to the emissivity.

KIRCHOFF'S LAW

Consider a body of given size and shape placed within a hollow sphere of constant temperature, and assume that the air has been evacuated. After *thermal equilibrium* has been reached, the temperature of the body and that of the enclosure (the sphere) will be the same, inferring that the body is absorbing and radiating heat at equal rates. Let the total intensity of radiation falling on the body be I (now with units of Btu/h·ft^2), the fraction absorbed α_1, and the total emissive power E_1 (Btu/h·ft^2). Kirchhoff noted that the energy emitted by a body of surface A_1 at thermal equilibrium is equal to that received, so that:

$$E_1 A_1 = I \alpha_1 A_1 \tag{11.12}$$

or simply,

$$E_1 = I \alpha_1 \tag{11.13}$$

If the body is replaced by another of identical shape, then:

$$E_2 = I \alpha_2 \tag{11.14}$$

If a third body that is a black body is introduced, then:

$$E_b = I \tag{11.15}$$

Since the absorptivity, α, of a black body is 1.0, one may write:

$$I = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = E_b \tag{11.16}$$

Thus, at thermal equilibrium, the ratio of the total emissive power to the absorptivity for all bodies is the same. This is referred to as Kirchhoff's law. Since $\alpha = \varepsilon$, the above equation may also be written

$$\frac{E_1}{E_b} = \alpha_1 = \varepsilon_1 \quad \text{and}$$

$$\frac{E_2}{E_b} = \alpha_2 = \varepsilon_2 \tag{11.17}$$

where the ratio of the actual emissive power to the black-body emissive power is defined as the aforementioned emissivity, ε. Values of ε for various bodies and surfaces are given in Table 11.2.

If a *black* body radiates energy, the total radiation may be determined from Planck's law:

$$I = \frac{C_1\lambda^{-5}}{e^{C_2/\lambda T} - 1} \tag{11.3}$$

Integration over the entire spectrum at a particular temperature yields:

$$E_b = \int_0^\infty \frac{C_1\lambda^{-5}}{e^{C_2/\lambda T} - 1}\, d\lambda \tag{11.18}$$

The evaluation of the previous integral can be shown to be:

$$E_b = 0.173 \times 10^{-8} T^4 = \sigma T^4; \quad T \equiv {}^\circ\mathrm{R} \tag{11.19}$$

Thus, the total radiation from a perfect black body is proportional to the fourth power of the absolute temperature of the body. This is also referred to as the Stefan–Boltzmann law. The constant $0.173 \times 10^{-8}\,\mathrm{Btu/h{\cdot}ft^2{\cdot}{}^\circ R}$(2) is known as the Stefan–Boltzmann constant, usually designated by σ. Its counterpart in SI units is $5.669 \times 10^{-8}\,\mathrm{W/m^2{\cdot}K^4}$. However, note that this equation was derived for a perfect black body.

If the body is non-black, the emissivity is given by

$$E = E_b\varepsilon \tag{11.16}$$

Substituting Equation (11.16) into Equation (11.19) gives

$$E = \varepsilon\sigma T^4 \tag{11.20}$$

or, since $E = \dot{Q}/A$,

$$\frac{\dot{Q}}{A} = \varepsilon\sigma T^4 \tag{11.21}$$

Thus, when the law is applied to a real surface with emissivity, ε, the total emissive power of a real body is given by Equation (11.21). Typical values for the emissivity were provided in Table 11.2.

Table 11.2 Total Emissivity of Various Sources

Surface	T, °F	Emissivity, ε
Aluminum		
Anodized		0.76
Highly polished plate, 98.3% pure	440–1070	0.039–0.057
Commercial sheet	212	0.09
Heavily oxidized	299–940	0.20–0.31
Al-surfaced roofing	100	0.216
Brass		
Highly polished		
73.2% Cu, 26.7% Zn	476–674	0.028–0.031
62.4% Cu, 36.8% Zn, 0.4% Pb, 0.3% Al	494–710	0.033–0.037
82.9% Cu, 17.0% Zn	530	0.030
Hard-rolled, polished, but with the direction of polishing visible	70	0.038
Dull plate	120–660	0.22
Chromium		
Polished	100–2000	0.08–0.38
Copper		
Polished	242	0.028
Plate, heated a long time, and covered with thick oxide layer	77	0.78
Gold		
Pure, highly polished	440–1160	0.018–0.035
Iron and steel (not including stainless)		
Steel, polished	212	0.066
Iron, polished	800–1800	0.14–0.38
Cast iron, newly turned	72	0.44
Turned and heated	1620–1810	0.60–0.70
Milled steel	450–1950	0.20–0.32
Oxidized surfaces		
Iron plate, pickled, then rusted red	68	0.61
Iron, dark gray surface	212	0.31
Rough ingot iron	1700–2040	0.87–0.95
Sheet steel with strong, rough oxide layer	75	0.80
Lead		
Unoxidized, 99.96% pure	260–440	0.057–0.075
Gray oxidized	75	0.28
Oxidized at 300°F	390	0.63
Magnesium		
Magnesium oxide	530–1520	0.055–0.20

(*Continued*)

Table 11.2 *Continued*

Surface	T, °F	Emissivity, ε
Molybdenum		
Filament	1340–4700	0.096–0.202
Massive, polished	212	0.071
Monel metal		
Oxidized at 1110°F	390–1110	0.41–0.46
Nickel		
Polished	212	0.072
Nickel oxide	1200–2290	0.59–0.86
Nickel alloys		
Copper–nickel, polished	212	0.059
Nichrome wire, bright	120–1830	0.65–0.79
Nichrome wire, oxidized	120–930	0.95–0.98
Platinum		
Polished plate, pure	440–1160	0.054–0.104
Silver		
Polished, pure	440–1160	0.020–0.032
Polished	100–700	0.022–0.031
Stainless steels		
Polished	212	0.074
Type 301; B	450–1725	0.54–0.63
Tin		
Bright tinned iron	76	0.043–0.064
Tungsten		
Filament	6000	0.39
Zinc		
Galvanized, fairly bright	82	0.23
Alumina; effect of mean grain size (μm)		
10		0.18–0.30
50		0.28–0.39
100		0.40–0.50
Asbestos		
Board	74	0.96
Brick		
Red, rough, but no gross irregularities	70	0.93
Fireclay	1832	0.75

(*Continued*)

Table 11.2 *Continued*

Surface	T, °F	Emissivity, ε
Carbon		
T-Carbon 0.9% ash, started with an emissivity of 0.72 at 260°F but on heating changed to given values	260–1160	0.79–0.81
Filament	1900–2560	0.526
Rough plate	212–608	0.77
Lampblack, rough deposit	212–932	0.78–0.84
Concrete tiles	1832	0.63
Enamel		
White fused, on iron	66	0.90
Glass		
Smooth	72	0.94
Pyrex, lead and soda	500–1000	0.85–0.95
Rubber		
Hard, glossy plate	74	0.94
Water	32–212	0.95–0.963

ILLUSTRATIVE EXAMPLE 11.5

Estimate the increase in heat transferred by radiation of a black body at 1500°F relative to one at 1000°F.

***SOLUTION*:** Convert to *absolute* temperatures:

$$T_1 = 1500^\circ\text{F} = 1960^\circ\text{R}$$
$$T_2 = 1000^\circ\text{F} = 1460^\circ\text{R}$$

The ratio of the quantity/rate of heat transferred from Equation (11.19) is:

$$\frac{T_1^4}{T_2^4} = \frac{1960^\circ\text{R}^4}{1460^\circ\text{R}^4} \approx 3.25$$

This represents a 225% increase in heat transfer. ■

Now consider the energy transferred *between* two *black* bodies. Assume the energy transferred from the hotter body and the colder body is E_H and E_C, respectively. All of the energy that each body receives is absorbed since they are black bodies. Then, the net exchange between the two bodies maintained at two constant temperatures, T_H and T_C, is therefore

$$\frac{\dot{Q}}{A} = E_H - E_C = \sigma(T_H^4 - T_C^4) \tag{11.22}$$

$$= 0.173\left[\left(\frac{T_H}{100}\right)^4 - \left(\frac{T_C}{100}\right)^4\right] \tag{11.23}$$

ILLUSTRATIVE EXAMPLE 11.6

Two large walls are required to be maintained at constant temperatures of 800°F and 1200°F. Assuming the walls are black bodies, how much heat must be removed from the colder wall to maintain a steady-state, constant temperature?

***SOLUTION*:** For this application,

$$T_1 = 1200°\text{F} = 1660°\text{R} \qquad T_2 = 800°\text{F} = 1260°\text{R}$$

Apply Equation (11.23) and substitute:

$$\begin{aligned}\frac{\dot{Q}}{A} &= 0.173\left[\left(\frac{1660}{100}\right)^4 - \left(\frac{1260}{100}\right)^4\right]\\ &= (0.173)(75{,}900 - 25{,}200)\\ &= 8770\ \text{Btu/h}\cdot\text{ft}^2\end{aligned}$$

Note once again that the above solution applies to black bodies. ■

EMISSIVITY FACTORS

If the two large walls in Illustrative Example 11.6 are not black bodies, and instead (each) have an emissivity ε, then the net interchange of radiant energy is given by

$$\frac{\dot{Q}}{A} = \varepsilon\sigma(T_H^4 - T_C^4) \tag{11.24}$$

This equation can be "verified" as follows.(2) If the two planes discussed above are not black bodies and have *different* emissivities, the net exchange of energy will be different. Some of the energy emitted from the first body will be absorbed and the remainder radiated back to the other source. For two parallel bodies of infinite size, the radiation of each body can be accounted for. If the energy emitted from the first body is E_H with emissivity ε_H, the second body will absorb $E_H\varepsilon_C$ and reflect $1-\varepsilon_C$ of it, i.e., $E_H(1-\varepsilon_C)$. The first body will then receive $E_H(1-\varepsilon_C)\varepsilon_H$ and again radiate to the cold body, but in the amount $E_H(1-\varepsilon_C)(1-\varepsilon_H)$. The exchanges for the two bodies are therefore:

Hot body
Radiated: E_H
Reflected back: $E_H(1-\varepsilon_C)$
Radiated: $E_H(1-\varepsilon_C)(1-\varepsilon_H)$
Reflected back: $E_H(1-\varepsilon_C)(1-\varepsilon_H)(1-\varepsilon_C)$
etc.

Cold body
Radiated: E_C
Reflected back: $E_C(1-\varepsilon_H)$
Radiated: $E_C(1-\varepsilon_H)(1-\varepsilon_C)$
Reflected back: $E_C(1-\varepsilon_H)(1-\varepsilon_C)(1-\varepsilon_H)$
etc.

For a non-black body, ε is not unity and must be included in Equation (11.19), that is,

$$E = 0.173\varepsilon\left(\frac{T}{100}\right)^4 \tag{11.25}$$

When Equation (11.25) is applied to the above infinite series analysis, one can show that Equation (11.26) results:

$$E = \frac{\dot{Q}}{A} = \frac{\sigma}{\left[\left(\frac{1}{\varepsilon_H}\right) + \left(\frac{1}{\varepsilon_C}\right) - 1\right]}(T_H^4 - T_C^4) \tag{11.26}$$

ILLUSTRATIVE EXAMPLE 11.7

If the two bodies from Illustrative Example 11.6 have emissivities of 0.5 and 0.75, respectively, what is the net energy exchange (per unit area)? Assume that the temperatures remain constant at 1660°R and 1260°R, and the two bodies are of infinite size.

SOLUTION: Apply Equation (11.26) and substitute:

$$\begin{aligned}\frac{\dot{Q}}{A} &= \frac{0.173}{\left[\left(\frac{1}{0.5}\right) + \left(\frac{1}{0.75}\right) - 1\right]}\left[(16.6)^4 - (12.6)^4\right] \\ &= 3760\,\text{Btu/h}\cdot\text{ft}^2\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 11.8

Compare and discuss the results of the last two illustrative examples.

***SOLUTION*:** The black bodies had an energy exchange of 8770 Btu/h·ft^2, while the non-black bodies had an exchange of $3760\,\text{Btu/h}\cdot\text{ft}^2$. The percent difference (relative to the black body calculation) is:

$$\begin{aligned}\%\text{ difference} &= \left(\frac{8770-3760}{8770}\right)100\\ &= 57.1\%\end{aligned}$$

■

The radiation between a sphere and an enclosed sphere of radii R_H and R_C, respectively, may be treated in a manner similar to that provided above. The radiation emitted initially by the inner sphere is $E_H A_H$, all of which falls on A_C. Of this total, however, $(1-\varepsilon_C)E_H A_H$ is reflected back to the hot body. If this analysis is similarly extended as before, the radiant exchange will again be represented by an infinite series whose solution may be shown to give

$$\begin{aligned}E_H = \frac{\dot{Q}}{A_H} &= \frac{\sigma_H(T_H^4 - T_C^4)}{\left[\dfrac{1}{\varepsilon_H} + \left(\dfrac{A_H}{A_C}\right)\left(\dfrac{1}{\varepsilon_C} - 1\right)\right]}\\ &= \frac{\sigma_H(T_H^4 - T_C^4)}{\left[\dfrac{1}{\varepsilon_H} + \left(\dfrac{R_H}{R_C}\right)^2\left(\dfrac{1}{\varepsilon_C} - 1\right)\right]}\end{aligned} \tag{11.27}$$

A similar relation applies for infinitely long concentric cylinders except that A_H/A_C is replaced R_H/R_C, not R_H^2/R_C^2.

In general, an emissivity correction factor, F_ε, is introduced to account for the exchange of energy between different surfaces of different emissivities. The describing equation takes the form

$$E_H = \frac{\dot{Q}}{A_H} = F_\varepsilon \sigma (T_H^4 - T_C^4) \tag{11.28}$$

Values of F_ε for the interchange between surfaces are provided in Table 11.3 for the three cases already considered, i.e., (a), (b), (c), plus three additional cases (d), (e), (f).

Table 11.3 Values of F_ε

Condition	F_ε
a Surface A_H small compared with the totally enclosing surface A_C	ε_H
b Surfaces A_C and A_H of infinite parallel planes or surface A_H of a completely enclosed body is small compared with A_H	$\dfrac{1}{\left(\dfrac{1}{\varepsilon_H}+\dfrac{1}{\varepsilon_C}\right)-1}$
c Concentric spheres or infinite concentric cylinders with surfaces A_H and A_C	$\dfrac{1}{\dfrac{1}{\varepsilon_H}+\left(\dfrac{A_H}{A_C}\right)\left(\dfrac{1}{\varepsilon_C}-1\right)}$
d Surfaces A_H and A_C of parallel disks, squares, 2 : 1 rectangles, long rectangles (see Figures 11.2 and 11.4 later)	$\varepsilon_H\varepsilon_C$
e Surface A_H and A_C of perpendicular rectangles having a common side (see Figure 11.3 later)	$\varepsilon_H\varepsilon_C$
f Surface A_H and parallel rectangular surface A_C with one corner of rectangle above A_H	$\varepsilon_H\varepsilon_C$

ILLUSTRATIVE EXAMPLE 11.9

Calculate the radiation from a 2-inch IPS cast iron pipe (assume polished) carrying steam at 300°F and passing through the center of a 1 ft × 1 ft galvanized zinc duct at 75°F and whose outside is insulated.

***SOLUTION*:** Base the calculation on 1 ft of pipe/duct. For a 2-inch pipe $A_H = 0.622\ \text{ft}^2$ of external surface per foot of pipe (see Table 6.2 in Part One). The emissivity of oxidized steel from Table 11.2 is $\varepsilon_H = 0.44$. The surface of the duct is $A_C = 4(1)(1) = 4.0\ \text{ft}^2$, and for galvanized zinc, $\varepsilon_C = 0.23$. Assume condition (c) in Table 11.3 applies with the physical representation of the duct replaced by a cylinder of the same area. Therefore,

$$F_\varepsilon = \frac{1}{\left[\dfrac{1}{\varepsilon_H}+\left(\dfrac{A_H}{A_C}\right)\left(\dfrac{1}{\varepsilon_C}-1\right)\right]} = \frac{1}{\left[\dfrac{1}{0.44}+\left(\dfrac{0.622}{4.0}\right)\left(\dfrac{1}{0.23}-1\right)\right]} = 0.358$$

Apply Equation (11.28):

$$\dot{Q}_H = \dot{Q} = F_\varepsilon A\sigma(T_H^4 - T_C^4)$$

Substituting,

$$\begin{aligned}\dot{Q} &= (0.358)(0.622)(0.173\times10^{-8})(760^4 - 535^4)\\ &= 97.85\ \text{Btu/h}\cdot\text{ft}\end{aligned}$$

■

Finally, when a heat source is small compared to its enclosure, it is customary to assume that some of the heat radiated from the source is reflected back to it. Such is often the case on the loss of heat from a pipe to surrounding air. For these applications, it is convenient to represent the net radiation heat transfer in the same form employed for convection, i.e.,

$$\dot{Q} = h_r A(T_H - T_C) \tag{11.29}$$

where h_r is the *effective* radiation heat transfer coefficient.
When $T_H - T_C$ is less than 120°C (120 K or 216°R), one may calculate the radiation heat transfer coefficient using

$$h_r = 4\varepsilon\sigma T_{\text{av}}^3 \tag{11.30}$$

where $T_{\text{av}} = (T_H + T_C)/2$.

ILLUSTRATIVE EXAMPLE 11.10

The outside temperature of a 10 ft^2 hot insulated pipe is 140°F and the surrounding atmosphere is 60°F. The heat loss by free convection and radiation is 13,020 Btu/h, and the combined coefficient of heat transfer is estimated to be 2.10 $\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$. How much of the heat loss is due to radiation? Assume the pipe emissivity is approximately 0.9.

***SOLUTION*:** For this example,

$$T_H = 140 + 460 = 600^\circ\text{R} \qquad T_C = 60 + 460 = 520^\circ\text{R}$$

Apply Equation (11.21) and substitute:

$$\dot{Q} = \dot{Q}_{\text{rad}} = (0.9)(10)(0.173)\left[\left(\frac{600}{100}\right)^4 - \left(\frac{520}{100}\right)^4\right] = 880\,\text{Btu/h}$$

■

ILLUSTRATIVE EXAMPLE 11.11

With reference to Illustrative Example 11.10, calculate the radiation heat transfer coefficient, h_r.

***SOLUTION*:** Apply Equation (11.29):

$$h_r = \frac{\dot{Q}}{A(T_H - T_C)}$$

Substituting,

$$h_r = \frac{880}{10(600 - 520)} = 1.10\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

■

ILLUSTRATIVE EXAMPLE 11.12

A small oxidized horizontal metal tube is placed in a very large furnace enclosure with firebrick walls. The metal tube has an outside diameter of 1 inch, a length of 2 ft, a surface emissivity of 0.6, and its surface is maintained at 600°F. The hot air in the furnace is at 1500°F and the furnace brick walls are at 1350°F. The convection heat transfer coefficient for the horizontal tube is 2.8 Btu/h · ft^2 · °F. Calculate

1. The convective, radiative, and total heat transferred to the metal tube;
2. The percent of total heat transferred by radiation;
3. The radiation heat transfer coefficient;
4. Is it appropriate to use the approximate equation presented in Equation (11.30) for (3)?

***SOLUTION*:**

1. Calculate the tube area:

$$A = \pi DL = \pi(0.0833)(2) = 0.524\,\text{ft}^2$$

The convective heat transfer (from air to metal tube) is therefore

$$\dot{Q}_{\text{conv}} = hA(T_{\text{air}} - T_{\text{tube}}) = (2.8)(0.524)(1500 - 600) = 1320\,\text{Btu/h}$$

Calculate the radiation heat transfer (from wall to metal tube). Apply Equation (11.24):

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A(T_{\text{air}}^4 - T_{\text{tube}}^4);\ T_{\text{air}} = 1350°\text{F},\ T_{\text{tube}} = 600°\text{F}\\ &= (0.6)(0.1713 \times 10^{-8})(0.524)(1810^4 - 1060^4)\\ &= 5100\ \text{Btu/h}\end{aligned}$$

Determine the total heat transfer:

$$\dot{Q} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} = 1320 + 5100 = 6420\ \text{Btu/h}$$

2. The radiation percent contribution to the total heat transfer rate is

$$(5100/6420) \times 100\% = 79.4\% = 80\%$$

3. Determine the radiation heat transfer coefficient. Apply Equation (11.29):

$$\begin{aligned}h_r &= \frac{\dot{Q}}{A(T_{\text{wall}} - T_{\text{tube}})}\\ &= \frac{5100}{(0.524)(750)} = 13.1\,\text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}\end{aligned}$$

4. Finally, since $|T_{\text{tube}} - T_{\text{wall}}| = 1350 - 600 = 750°\text{F} > 200°\text{F}$, the use of the approximation Equation (11.30), $h_r = 4\varepsilon\sigma T_{\text{av}}^3$, is not appropriate. ■

ILLUSTRATIVE EXAMPLE 11.13

The filament of a light bulb is at a temperature of 900°C and emits 5 W of heat toward the glass bulb. The interior of a light bulb can be considered a vacuum and the temperature of the glass bulb is 150°C. Ignoring heat transfer to the room and assuming the emissivity of the filament is 1.0, calculate the surface area of the filament in cm^2.

***SOLUTION*:** Write the equation for radiation heat transfer, see Equation (11.24),

$$\dot{Q} = \varepsilon\sigma A(T_1^4 - T_2^4)$$

Solve for the unknown, surface A,

$$A = \frac{\dot{Q}}{\varepsilon\sigma(T_1^4 - T_2^4)}$$

Substitute known values and compute A:

$$\begin{aligned} A &= \frac{\dot{Q}}{\varepsilon\sigma(T_1^4 - T_2^4)} \\ &= \frac{5.0}{\{(1)(5.669 \times 10^{-8})[(273 + 900)^4 - (273 + 150)^4]\}} \\ &= 4.74 \times 10^{-5}\ \text{m}^2 \\ &= 0.47\ \text{cm}^2 \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 11.14

A system consists of an uninsulated steam pipe made of anodized aluminum with a diameter $D = 0.06$ m and a length $L = 100$ m. The surface temperature is $T_1 = 127°\text{C}$ and the surface emissivity of anodized aluminum is $\varepsilon = 0.76$. The pipe is in a large room with a wall temperature $T_2 = 20°\text{C}$. The air in the room is at a temperature $T_3 = 22°\text{C}$. The pipe convective heat transfer coefficient is $h = 15\ \text{W/m}^2 \cdot \text{K}$.

Estimate the emissive power, the total heat transfer by convection and radiation, and the radiation heat transfer coefficient. Assume steady-state operation, constant properties, and a room surface area much larger than the pipe surface area.

***SOLUTION*:** Calculate the emissive energy of the pipe surface assuming it is a blackbody:

$$\begin{aligned} T_1 &= 127°\text{C} = 400\ \text{K} \\ E_b &= \sigma T_1^4 = (5.669 \times 10^{-8})(400)^4 = 1451\ \text{W/m}^2 \end{aligned}$$

Employ Equation (11.17). Calculate the emissive power from the surface of the pipe:

$$E = \varepsilon E_b = (0.76)(1451) = 1103\ \text{W/m}^2$$

The total heat transfer, $\dot{Q}$, from the pipe to the air and walls is

$$\dot{Q} = \dot{Q}_c + \dot{Q}_r$$

With reference to the convection equation,

$$\dot{Q}_c = hA(T_1 - T_3)$$

Calculate the surface area of the pipe:

$$A = \pi(0.06)(100) = 18.85\,\mathrm{m}^2$$

The convective heat transfer to the air is therefore

$$\dot{Q}_c = (15)(18.85)(127 - 22) = 29{,}700\,\mathrm{W} = 29.7\,\mathrm{kW}$$

With reference to the radiation heat transfer rate, $\dot{Q}_r$, apply Equation (11.24):

$$\begin{aligned}\dot{Q}_r &= \varepsilon\sigma A(T_1^4 - T_2^4)\\ &= (0.76)(5.669 \times 10^{-8})(18.85)(400^4 - 293^4)\\ &= 14{,}800\,\mathrm{W} = 14.8\,\mathrm{kW}\end{aligned}$$

The total heat transfer rate is then

$$\dot{Q} = \dot{Q}_c + \dot{Q}_r = 29{,}700 + 14{,}800 = 44{,}500\,\mathrm{W} = 151{,}834\,\mathrm{Btu/h}$$

Since $(T_1 - T_2) = 107^\circ\mathrm{C}$, which is less than 120°C, it is valid to use the approximation equation,

$$h_r = 4\varepsilon\sigma T_{\mathrm{av}}^3 \tag{11.30}$$

where

$$T_{\mathrm{av}} = (T_H + T_C)/2$$

in order to calculate the radiation heat transfer coefficient. First calculate T_{av}:

$$T_{\mathrm{av}} = (T_H + T_C)/2 = [(127 + 273) + (20 + 273)]/2 = 346.5\,\mathrm{K}$$

Substituting into Equation (11.30), and solving for h_r

$$h_r = 4\varepsilon\sigma T_{\mathrm{av}}^3 = 4(0.76)(5.669 \times 10^{-8})(346.5)^3 = 7.2\ \mathrm{W/m^2 \cdot K}$$ ■

ILLUSTRATIVE EXAMPLE 11.15

Refer to Illustrative Example 11.14. How much does radiation contribute to the total heat transfer?

***SOLUTION*:** Since the convection heat transfer coefficient is 15.0 $\mathrm{W/m^2 \cdot K}$ and the radiation heat transfer coefficient is 7.2 $\mathrm{W/m^2 \cdot K}$,

$$\%\ \text{heat transfer by radiation} = \frac{7.2}{15.0 + 7.2} \times 100 = 32.4\%$$

Therefore, radiation accounts for approximately one-third of the total heat transfer. ■

VIEW FACTORS

As indicated earlier, the amount of heat transfer between two surfaces depends on geometry *and* orientation of the two surfaces. Again, it is assumed that the intervening medium is non-participating. The previous analyses were concerned with sources that were situated so that every point on one surface could be "connected" with every surface on the second . . . in effect possessing a perfect view. This is very rarely the case in real-world engineering applications, particularly in the design of boilers and furnaces. Here, the receiving surface, such as a bank of tubes, is cylindrical and may partially obscure some of the surfaces from "viewing" the source. These systems are difficult to evaluate. The simplest cases are addressed below; however, many practical applications must resort to the use of empirical methods.

To introduce the subject of view factors, the reader should note that the flow of radiant heat is analogous to the flow of light (i.e., one may follow the path of radiant heat as one may follow the path of light). If any object is placed between a hot and cold body, a light from the hot body would cast a shadow on the cold body and prevent it from receiving all the light leaving the hot one.

As noted above, the computation of actual problems involving "viewing" difficulty is beyond the scope of this book. One simple problem may illustrate the nature of these complications. Contemplate two black bodies, with surfaces consisting of parallel planes of finite size and separated by a finite distance (see Figure 11.1). Then, a small differential unit of surface *dA* can see the colder body only through solid angel β, and any radiation emitted by it through solid angles γ and γ^1 will fall elsewhere. To evaluate the heat lost by radiation from body *H*, one must integrate the loss from element *dA* over the whole surface of *A*.

The previously mentioned problem is sufficiently complicated. However, it can become more complicated. If the colder body is not a black body, then it will reflect some of the energy imparted upon it. Some of the reflected energy will return to the hot body. Since the hot body is black, it will absorb the reflected energy, tending to raise its temperature. One can envision even more complicated scenarios.

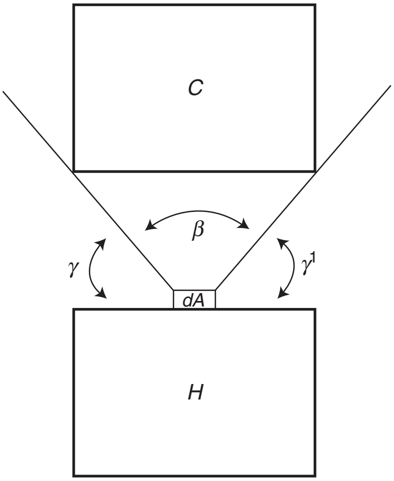

Figure 11.1 View factor illustration.

Table 11.4 Values of F_v

Condition	F_v
a Surface A_H small compared with the totally enclosing surface A_C	1.0
b Surfaces A_H and A_C of infinite parallel planes or surface A_H of a completely enclosed body is small compared with A_H	1.0
c Concentric spheres or infinite concentric cylinders with surfaces A_H and A_C	1.0
d Surfaces A_H and A_C of parallel disks, squares, 2 : 1 rectangles, long rectangles	Figure 11.2
e Surfaces A_H or A_C of perpendicular rectangles having a common side	Figure 11.3
f Coaxial parallel disks	Figure 11.4

In order to include the effect of "viewing", Equation (11.28) is expanded to

$$\frac{\dot{Q}}{A_H} = F_v F_\varepsilon \sigma (T_H^4 - T_C^4) \tag{11.31}$$

with the inclusion of view factor, F_v. View factors for the cases considered in Table 11.3 are presented in Table 11.4 in conjunction with Figures 11.2–11.3. View factors for parallel disks are provided in Figure 11.4. Note that $F_{i,j}$ represents the fraction of energy leaving surface i that strikes surface j. Other information and applications involving view factors are available in the literature.$^{(5-7)}$

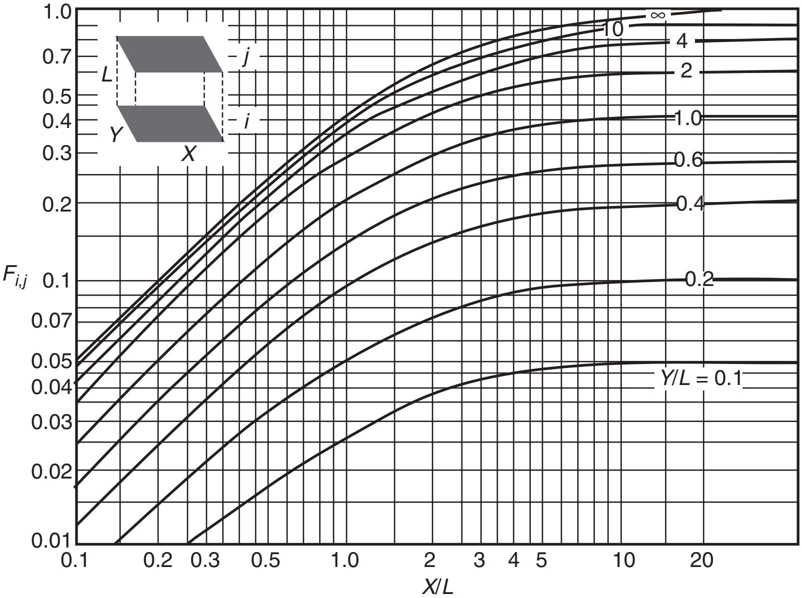

Figure 11.2 View factors for aligned parallel rectangles.$^{(7)}$

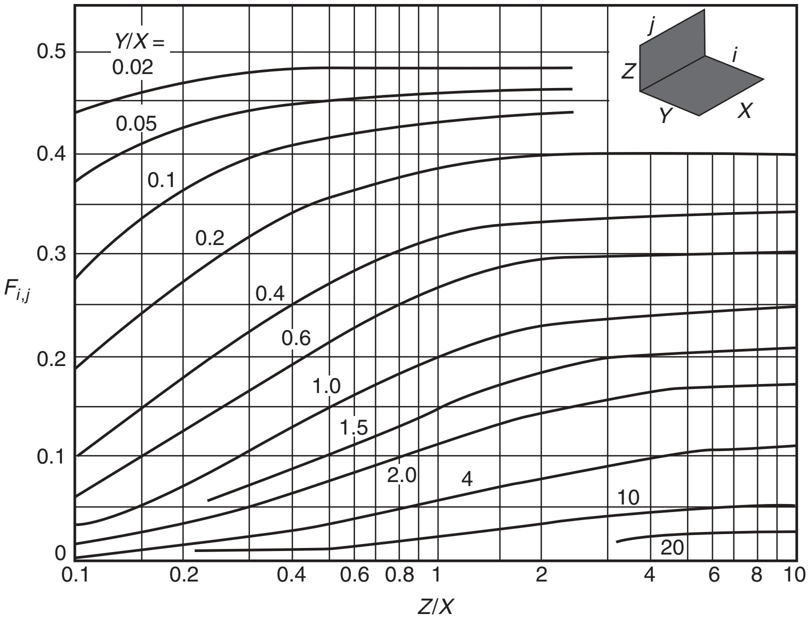

Figure 11.3 View factors for perpendicular rectangles with a common edge.(7)

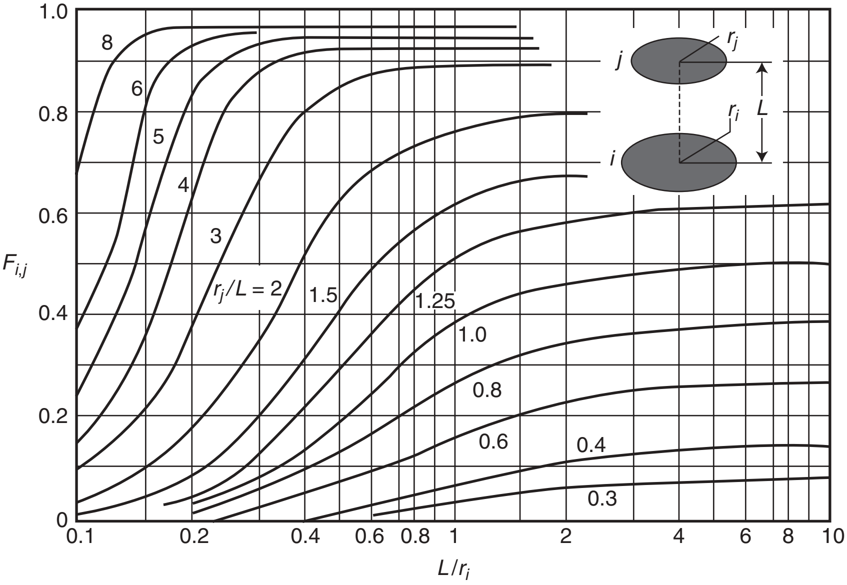

Figure 11.4 View factor for coaxial parallel disks.(7)

ILLUSTRATIVE EXAMPLE 11.16

Refer to Illustrative Example 11.9. Calculate the heat transfer rate if $F_v = 1.0$.

***SOLUTION*:** Apply Equation (11.31):

$$\dot{Q} = F_v F_\varepsilon \sigma A (T_H^4 - T_C^4)$$

Since $F_v = 1.0$, the solution remains unchanged:

$$\dot{Q} = 97.85\,\text{Btu/h}\cdot\text{ft}$$

■

ILLUSTRATIVE EXAMPLE 11.17

Two parallel rectangular *black* plates 0.5 m by 2.0 m are spaced 1.0 m apart. One plate is maintained at 1000°C and the other at 2000°C. What is the net radiant heat exchange between the two plates?

***SOLUTION*:** Figure 11.2 is to be employed. For this application, $L = 1.0$, $X = 0.5$ and $Y = 2.0$. Therefore,

$$\frac{Y}{L} = \frac{2.0}{1.0} = 2.0$$

and

$$\frac{X}{L} = \frac{0.5}{1.0} = 0.5$$

From Figure 11.2,

$$F_v \cong 0.18$$

The net heat transfer exchange rate is calculated employing Equation (11.31), noting that $F_\varepsilon = 1.0$ and $\sigma = 5.669 \times 10^{-8}$ (when employing temperatures in K):

$$\begin{aligned}\dot{Q} &= F_v F_\varepsilon \sigma A (T_H^4 - T_C^4)\\ &= (0.18)(1.0)(5.669 \times 10^{-8})(0.5)(2.0)(2273^4 - 1273^4)\\ &= 245{,}600\ \text{kW} = 8.37 \times 10^8\ \text{Btu/h}\end{aligned}$$

■

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Chapter 12

Condensation and Boiling

INTRODUCTION

It should be noted that phase-change processes involve changes (sometimes significantly) in density, viscosity, heat capacity, and thermal conductivity of the fluid in question. The heat transfer process and the applicable heat transfer coefficients for boiling and condensation is more involved and complicated than that for a single-phase process. It is therefore not surprising that most real-world applications involving boiling and condensation require the use of empirical correlations.

The transfer of heat, which accompanies a change of phase, is often characterized by high rates. Heat fluxes as high as 50 million $\mathrm{Btu/h \cdot ft^2}$ have been obtained in some boiling systems. This mechanism of transferring heat has become important in rocket technology and nuclear-reactor design where large quantities of heat are usually produced in confined spaces. Although condensation rates have not reached a similar magnitude, heat transfer coefficients for condensation as high as 20,000 $\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$ have been reported in the literature.(1) Due to the somewhat complex nature of these two phenomena, simple pragmatic calculations and numerical details are provided later in this chapter.

This chapter addresses phenomena associated with the change in phase of a fluid. The processes almost always occur at a solid–liquid interface and are referred to as *boiling* and *condensation*. The change from liquid to vapor due to boiling occurs because of heat transfer from the solid surface; alternatively, condensation of vapor to liquid occurs due to heat transfer to the solid surface.

Phase changes of substances can only occur if heat transfer is involved in the process. The phase change processes that arise include:

1. Boiling (or evaporation)
2. Condensation
3. Melting (or thawing)
4. Freezing (or fusion)
5. Sublimation.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

The corresponding heat of transformation arising during these processes are:

1. Enthalpy of vaporization (or condensation)
2. Enthalpy of fusion (or melting)
3. Enthalpy of sublimation.

These common phase change operations are listed in Table 12.1. Other phase changes besides these are possible. During any phase change, it is usual (but not necessary) to have heat transfer without an accompanying change in temperature.

The applications of phase change involving heat transfer are numerous and include utility units where water is boiled, evaporators in refrigeration systems where a refrigerant may be either vaporized or boiled, or both, and condensers that are used to cool vapors to liquids. For example, in a power cycle, pressurized liquid is converted to vapor in a *boiler*. After expansion in a turbine, the vapor is restored to its liquid state in a *condenser*; it is then pumped to the boiler to repeat the cycle. Evaporators, in which the boiling process occurs, and condensers are also essential components in vapor-compression refrigeration cycles. Thus, the practicing engineer needs to be familiar with phase change processes.

Applications involving the solidification or melting of materials are also important. Typical examples include the making of ice, freezing of foods, freeze-drying processes, solidification and melting of metals, and so on. The freezing of food and other biological matter usually involves the removal of energy in the form of both sensible heat (enthalpy) and latent heat of freezing. A large part of biological matter is liquid water, which has a latent enthalpy of freezing, h_{sf}, of approximately 335 kJ/kg (144 Btu/lb or 80 cal/g). When meat is frozen from room temperature, it is typically placed in a freezer at -30°C, which is considerably lower than the freezing point. The sensible heat to cool any liquids from the initial temperature to the freezing point is first removed, followed by the latent heat, h_{sf}, to accomplish the actual freezing. Once frozen, the substance is often cooled further by removing some sensible heat of the solid.

Dimensionless parameters arise in boiling and condensation. The Nusselt and Prandtl numbers, presented earlier in convection analyses appear once again. The new dimensionless parameters are the Jakob number, Ja, the Bond (no relation to James) number, Bo, and the condensation number, Co. The Jakob number is the ratio of the maximum sensible energy absorbed by the liquid (vapor) to the latent energy absorbed by the liquid (vapor). However, in many applications, the sensible

Table 12.1 Phase Change Operations

Process	Description
Solidification	Change from liquid to solid
Melting	Change from solid to liquid
Boiling	Change from liquid to vapor
Condensation	Change from vapor to liquid
Sublimation	Change from solid to vapor

energy is much less than the latent energy and Ja has a small numerical value. The Bond number is the ratio of the buoyancy force to the surface tension force and finds application in some nucleate boiling equations. The condensation number, as one might suppose, is employed in condensation calculations.

ILLUSTRATIVE EXAMPLE 12.1

It was noted earlier in the section that "other phase changes besides these are possible" (see Table 12.1). Provide an example of a solid-to-solid phase change.

***SOLUTION*:** Iron–carbon undergoes a solid-to-solid phase change at a temperature of 727°C. ■

The objectives of this chapter are to develop an understanding of the physical conditions associated with boiling and condensation and to provide a basis for performing related heat transfer calculations. The remaining chapter contents are highlighted with four sections:

Condensation Fundamentals
Condensation Principles
Boiling Fundamentals
Boiling Principles

CONDENSATION FUNDAMENTALS

Although several earlier chapters dealt with situations in which the fluid medium remained in a single phase, a significant number of real-world engineering applications involve a phase change that occurs simultaneously with the heat transfer process. As discussed earlier, the process of condensation of a vapor is usually accomplished by allowing it to come into contact with a surface where the temperature is maintained at a value lower than the saturation temperature of the vapor for the pressure at which it exists. The removal of thermal energy from the vapor causes it to lose its latent heat of vaporization and, hence, to condense onto the surface.

The appearance of the liquid phase on the cooling surface, either in the form of individual drops or in the form of a continuous film, offers resistance to the removal of heat from the vapor. In most applications, the condensate is removed by the action of gravity. As one would expect, the rate of removal of condensate (and the rate of heat removal from the vapor) is greater for vertical surfaces than for horizontal surfaces. Most condensing equipment consists of an assembly of tubes around which the vapor to be condensed is allowed to flow. The cool temperature of the outer tube surface is maintained by circulating a colder medium, often water, through the inside of the tube.

There are primarily three types of condensation processes:

1. *Surface Condensation.* This type of condensation occurs when vapor is in contact with a cool surface. This process is common in industrial applications and is discussed below.
2. *Homogeneous Condensation.* Homogeneous condensation occurs when the vapor condenses out as droplets in the gas phase.
3. *Direct Contact Condensation.* This process occurs when vapor is in contact with a cold liquid.

Surface condensation may occur in one of two modes depending upon the conditions of the surface.

1. *Film Condensation.* When the surface is clean and uncontaminated, the condensed vapor forms a liquid film that covers the entire condensing surface; this film contributes an additional resistance to heat transfer.
2. *Dropwise Condensation.* When the surface is coated with a substance that inhibits wetting, the condensed vapor forms drops in cracks and cavities on the surface. The drops often grow and coalesce and "drop" from the surface available for heat transfer. Thus, the additional resistance to heat transfer can be either reduced or eliminated. Up to 90% of the surface can be covered by drops. This topic receives a superficial treatment later in the chapter, with film condensation receiving the bulk of the analysis.

Liquid condensate provides resistance to heat transfer between the vapor and the surface. This resistance naturally increases as the thickness of the condensate later increases. For design purposes, it is usually desirable to have the condensation occur on surfaces that discourage the formation of thick liquid layers (e.g., short vertical surfaces or horizontal cylinders, or tube bundles, through which a coolant liquid flows). Dropwise condensation has a lower thermal resistance (and therefore a higher heat transfer rate) than filmwise condensation. In industrial practice, surface coatings that inhibit wetting are often used. Examples of such coatings include Teflon, silicones, waxes, and fatty acids. The drawback is that these coatings lose their effectiveness over time, which results in the condensation mode eventually changing from dropwise to filmwise. Obviously, all other things being equal, dropwise condensation is preferred to film condensation. In fact, steps are often put in place to induce this "dropwise" effect on heat transfer surfaces.

Finally, it should be noted that the local heat transfer coefficient varies along a flat surface and along the length of a vertical tube and around the perimeter of a horizontal tube (i.e., the local coefficient of heat transfer for a vapor condensing in a horizontal tube is a function of position, just like a vertical tube). As one would suppose, the highest coefficient is located at the top of a tube where the condensate film is thinnest. The condensing temperature in the application of the equations to be presented in the next section involving film coefficients is taken as the temperature at the vapor–liquid interface.

Phase Equilibrium$^{(2)}$

The *degrees of freedom*, *F*, or the *variance* of a system is defined as the *smallest number* of independent variables (such as pressure, temperature, concentration) that must be specified in order to completely define (the remaining variables of) the system. The significance of the degrees of freedom of a system may be drawn from the following examples. In order to specify the density of gaseous (vapor) steam, it is necessary to state both the temperature and pressure to which this density corresponds. For example, the density of steam has a particular value at 150°C and 1 atm pressure. A statement of this density at 150°C without mention of pressure does not clearly define the state of the steam, for at 150°C the steam may exist at many other possible pressures. Similarly, mention of the pressure without the temperature leaves ambiguity. Therefore, for the complete description of the state of the steam, two variables must be given, and this phase, when present alone in a system, possesses two degrees of freedom, or the system is said to be *bivariant*. When liquid water and steam exist in equilibrium, however, the temperature and the densities of the phases are determined only by the pressure, and a statement of some arbitrary value of the latter is sufficient to define all the other variables. The same applies to the choice of temperature as the independent variable. At each arbitrarily chosen temperature (within the range of existence of the two phases), equilibrium is possible only at a given pressure, and once again the system is defined in terms of one variable. Under these conditions, the system possesses only one degree of freedom or it is *monovariant*.

There is a definite relation in a system between the number of degrees of freedom, the number of components, and the number of phases present. This relationship was first established by J. Willard Gibbs in 1876.(3) This relation, known as the *Gibbs Phase Rule*, is a principle of the widest generality. It is one of the most often used rules in thermodynamic analyses, particularly in the representation of equilibrium conditions existing in heterogeneous systems.

To arrive at a mathematical description of the phase rule, consider a system of C components in which P phases are present. The problem is to determine the total number of variables upon which such a system depends. First, the state of the system will depend upon the pressure and the temperature. Again, in order to define the composition of each phase, it is necessary to specify the concentration of $C - 1$ constituents of the phase, the concentration of the remaining component being determined by difference. Since there are P phases, the total number of concentration variables will be $P(C - 1)$, and these along with the temperature and pressure constitute a total of $[P(C - 1) + 2]$ variables.

Recall from algebra that when a system possesses n independent variables, n independent equations are necessary in order to solve for the value of each of these. Similarly, in order to define the $[P(C - 1) + 2]$ variables of a system, this number of equations relating these variables would have to be available. The next question is then: How many equations involving these variables can possibly be written from the conditions describing the system? To answer this, recourse must be had to thermodynamics. Equilibrium thermodynamic principles makes it possible to write *one* *equation* among the variables *for each constituent distributed between any two phases*.(2) When P phases are present, $P - 1$ equations are available for each constituent, and for C constituents a total of $C(P - 1)$ equations arise. If this number of equations is equal to the number of variables, the system is completely defined. However, this will generally not be the case, and the number of variables will exceed the number of equations by F, where

$$\begin{aligned} F &= \text{number of variables} - \text{number of equations} \\ &= [P(C-1)+2] - [C(P-1)] \\ &= C - P + 2 \end{aligned} \tag{12.1}$$

Equation (12.1) is the celebrated Gibbs Phase Rule (GPR). The F term is the number of degrees of freedom of a system and provides the number of variables whose values must be specified (arbitrarily) before the state of the system can be completely and unambiguously characterized. According to the phase rule, the number of degrees of freedom of a system whose pressure and temperature are specified is therefore given by the difference between the number of components and the number of phases present (i.e., by $C - P$).

It is assumed in the above derivation that each component is present in every phase. If a component is missing from a particular phase, however, the number of concentration variables is decreased by one. But, at the same time, the number of possible equations is also decreased by one. Hence the value of $C - P$, and therefore F, remains the same whether each constituent is present in every phase or not. This effectively means that the phase rule is not restricted by this assumption and is generally valid under all conditions of distribution provided that equilibrium exists in the system.

The simplest case of GPR is one in which only a single component in a single phase is present, as with ice or steam. When more than one component and/or phase is present in a system, the number of degrees of freedom correspondingly increases in accordance with Equation (12.1). Most (but not all) of the applications in this chapter involve one-component one-phase systems.

ILLUSTRATIVE EXAMPLE 12.2

Calculate the number of degrees of freedom for a one-component, one-phase system.

***SOLUTION*:** Refer to Equation (12.1)

$$F = C - P + 2$$

Since $C = 1$ and $P = 1$,

$$\begin{aligned} F &= 1 - 1 + 2 \\ &= 2 \end{aligned}$$

Thus, two independent variables must be specified to completely define the system. ■

From a thermodynamic point-of-view, condensation of a condensable vapor in a condensable vapor–noncondensable vapor mixture can be induced by either increasing the pressure or decreasing the temperature, or both. Condensation most often occurs when a vapor mixture contacts a surface at a temperature lower than the saturation (dew point) temperature of the mixture. The dew point(2) of a vapor mixture is the temperature at which the vapor pressure exerted by the condensable component(s) is equal to the(ir) partial pressure in the vapor. For example, consider an air–water mixture at 75°F that is 80% saturated with water (or has a relative humidity, RH, of 80%):

$$\%\,\mathrm{Sat} = \%\,\mathrm{RH} = \frac{p_{H_2O}}{p'}(100) \tag{12.2}$$

where p_{H_2O} = partial pressure of water
p' = vapor pressure of water = 0.43 psia at 75°F (see Steam Tables, Appendix, Table C.1)

For this condition, $p_{H_2O} = (0.8)(0.43) = 0.344$ psia.

Pure vapors condense (or vaporize) at their vapor pressure at a given temperature. For example, water at 212°F and 1 atm will vaporize to steam (or steam will condense to water). A vapor or a mixture of vapors in a non-condensable gas is more difficult to analyze. A typical example is steam (water) in air or a high molecular weight organic in air.

There are two key vapor–liquid mixtures of interest to the practicing engineer: air–water and steam–water (liquid). Information on the former is available on a psychometric chart while steam tables provide information on the later. A discussion on both follows.

Psychrometric Chart

A vapor–liquid phase equilibrium example involving raw data is the psychrometric or humidity chart.(2) A humidity chart is used to determine the properties of moist air and to calculate moisture content in air. The ordinate of the chart is the absolute humidity $\mathcal{H}$, which is defined as the mass of water vapor per mass of bone-dry air. (Some charts base the ordinate on moles instead of mass.) Based on this definition, Equation (12.3) gives $\mathcal{H}$ in terms of moles and also in terms of partial pressure:

$$\mathcal{H} = \frac{18 n_{H_2O}}{29(n_T - n_{H_2O})} = \frac{18 p_{H_2O}}{29(P - p_{H_2O})} \tag{12.3}$$

where n_{H_2O} = number of moles of water vapor
n_T = total number of moles in gas
p_{H_2O} = partial pressure of water vapor
P = total system pressure

Curves showing the *relative humidity* (ratio of the mass of the water vapor in the air to the maximum mass of water vapor that the air could hold at that temperature, i.e., if the air were saturated) of humid air also appear on the charts. The curve for 100% relative humidity is also referred to as the *saturation curve*. The abscissa of the humidity chart is air temperature, also known as the *dry-bulb* temperature (T_{DB}). The *wet-bulb* temperature (T_{WB}) is another measure of humidity; it is the temperature at which a thermometer with a wet wick wrapped around the bulb stabilizes. As water evaporates from the wick to the ambient air, the bulb is cooled; the rate of cooling depends on how humid the air is. No evaporation occurs if the air is saturated with water; hence, T_{WB} and T_{DB} are the same. The lower the humidity, the greater the difference between these two temperatures. On the psychrometric chart, constant wet-bulb temperature lines are straight with negative slopes. The value of T_{WB} corresponds to the value of the abscissa at the point of intersection of this line with the saturation curve.

Given the dry bulb and wet bulb temperatures, the relative humidity (along with any other quantity on the chart) may be determined by finding the point of intersection between the dry bulb abscissca and wet bulb ordinate. The point of intersection describes all humidity properties of the system.

Steam Tables

The steam tables comprise a tabular representation of the thermodynamic properties of water. These tables are divided into three separate categories:

1. The *saturated*-steam tables provide the value of the enthalpy, specific volume, and entropy of saturated steam and saturated water as functions of pressures and/or temperatures (condensation or boiling points). Changes in these extensive properties during the evaporation of 1 lb of the saturated liquid are also tabulated. The tables normally extend from 32 to 705°F, the temperature range where saturated liquid and vapor can coexist.
2. The *superheat* tables list the same properties in the superheated-vapor region. Degrees superheat or number of degrees above the boiling point (at the pressure in question) are also listed.
3. The Mollier chart (enthalpy–entropy diagram) for water is so frequently used in engineering practice that it also deserves mention. This diagram is useful since the *entropy function stays constant during any reversible adiabatic expansion or compression.*(2)

The steam tables are located in the Appendix.

ILLUSTRATIVE EXAMPLE 12.3

A large *enclosed* main contains steam at 500 psia and 233°F superheat. Find the temperature and pressure in the main after half the steam has condensed.

***SOLUTION*:** From the saturated steam tables in the Appendix, the condensation temperature is 467°F at 500 psia. Therefore, the steam temperature is $467 + 233 = 700°\text{F}$. The corresponding initial specific volume is approximately 1.30 ft^3/lb. After condensation, only half of the steam vapor remains. Therefore, the final specific volume will be(4) $(1.30)/0.5 = 2.60\ \text{ft}^3/\text{lb}$. Since two phases exist at equilibrium, Gibbs phase rule indicates that

$$F = (C - P) + 2 = (1 - 2) + 2 = 1$$

Employing the saturated tables, one notes (by interpolation)(4) that $T = 370^\circ\text{F}$ and $P = 175\,\text{psia}$ as the final state. The specific volume of the liquid is approximately $0.018\ \text{ft}^3/\text{lb}$. ■

ILLUSTRATIVE EXAMPLE 12.4

Refer to the previous illustrative example. How much heat is removed from the system during this process?

***SOLUTION*:** This is a constant volume system since the main is enclosed. From the first law(2)

$$Q = \Delta U = U_2 - U_1;\ U = \text{internal energy}$$

From the steam tables(4)

$$\begin{aligned} U_1 &= 1237.1\,\text{Btu/lb} \\ U_{2,g} &= 1112.2\,\text{Btu/lb} \\ U_{2,l} &= 343.15\,\text{Btu/lb} \end{aligned}$$

Substituting,

$$\begin{aligned} Q &= 0.5(1112.2 + 343.15) - (1)(1237.1) \\ &= 727.7 - 1237.1 \\ &= -509.4\,\text{Btu/lb (heat is removed)} \end{aligned}$$

■

Regarding solid–vapor equilibrium, a solid, like a liquid, has a definite vapor pressure at each temperature (some solids cannot exist in contact with vapor). Examples are helium below the critical temperature and various forms of ice; this pressure may be extremely small but it is nevertheless finite. The vapor pressure of a solid increases with temperature and the variation can be represented by a curve similar to that for a liquid; it is generally called a *sublimation curve*, with the term *sublimation* used to indicate the direct conversion of solid to vapor without the intervention of liquid. The change from solid to vapor is accompanied, like the analogous change from liquid to vapor, by an absorption of heat; this is referred to as the latent enthalpy of sublimation.

CONDENSATION PRINCIPLES

As noted earlier, condensation occurs when the temperature of a vapor is reduced below its saturation temperature and results from contact between the vapor and a cooler surface. The latent enthalpy of the vapor is released, heat is transferred to the surface, and the condensate forms.

As noted in the Introduction to this chapter, there are three basic types of condensation: *surface condensation*, *homogeneous condensation*, and *direct contact condensation*. The corresponding modes are: *film condensation* and *dropwise condensation*. Although it is desirable to achieve dropwise condensation in real-world applications, it is often difficult to maintain this condition. Although convection coefficients for film condensation are smaller than those for the dropwise case, condenser design calculations may be and usually are based on the assumption of film condensation. It is for this reason that the paragraphs to follow in this section focus on film condensation.

Filmwise condensation may be laminar or turbulent, depending on the Reynolds number of the condensate. The condensate Reynolds number is defined as

$$\mathrm{Re} = \frac{4\rho_L v_{mL}\delta}{\mu_L} = \frac{4\dot{m}'}{\mu_L} = \frac{4\dot{m}}{W\mu_L} \tag{12.4}$$

where $\dot{m}'$ is the mass flow rate of condensate per unit width of the surface; $\dot{m}$, the mass flow rate of condensate; v_{mL}, the mean velocity of the condense liquid film; W, the width of the condensing surface; δ, the average thickness of the condensed liquid film; ρ_L, the density of the liquid condensate; and μ_L, the absolute viscosity of the liquid condensate.

Laminar condensation occurs when $\mathrm{Re} < 1800$, while turbulent condensation occurs when the $\mathrm{Re} > 1800$. Since the flow may not occur in a circular conduit, the hydraulic diameter, D_h, must be used. The hydraulic diameter, D_h (as defined earlier), is:

$$D_h = \frac{4A_C}{P_W} = \frac{4\dot{m}}{\rho_L v_{mL} P_W} \tag{12.5}$$

where A_C is the area of the conduit and P_W is the wetted perimeter. Assuming that the vapor is at its saturation temperature, T_{sat}, and letting T_S represent the surface temperature, the rate of heat transfer, $\dot{Q}$, can be related to the rate of condensation, $\dot{m}'$, through an energy balance, as shown in the following equations:

$$\dot{Q} = hA(T_{\text{sat}} - T_S) = \dot{m}h_{\text{vap}} \tag{12.6}$$

with A representing the heat transfer area. Rearranging Equation (12.6) leads to

$$\dot{m}' = \frac{\dot{m}}{P_W} = \frac{hA(T_{\text{sat}} - T_S)}{P_W h_{\text{vap}}} \tag{12.7}$$

Substituting Equation (12.7) for $\dot{m}'$ in the condensate Reynolds number definition in Equation (12.4) gives

$$\mathrm{Re} = \frac{4\dot{m}'}{\mu_L} = \frac{4hA(T_{\text{sat}} - T_S)}{P_W h_{\text{vap}} \mu_L} \tag{12.8}$$

The values of A, P_W and the ratio of A/P_W for several geometries are summarized in Table 12.2.

Table 12.2 Condensation Geometries

Geometry	Wetted perimeter, P_W	Heat transfer area, A	A/P_W
Vertical plate ($W \times Z$)	W	WZ	Z
Vertical tube	πD	πDZ	Z
Horizontal tube	Z	πDZ	πD
Horizontal tube bank n_1 rows $\times$ n_2 columns	$n_2 Z$	$n_1 n_2 \pi DZ$	$\pi n_1 D$

Note: W = width; Z = height; D = diameter.

The following correlations are available for calculating the heat transfer coefficient during condensation.

1. For laminar film condensation on a vertical plate or vertical cylinder:

$$\text{Co} = 1.874\,\text{Re}_f^{-1/3} \tag{12.9}$$

where Co is the average condensation number and is given by

$$\text{Co} = \frac{\bar{h}}{k}\left(\frac{\nu_L^2}{g(1-[\rho_v/\rho_L])}\right)^{1/3}; \; g = \text{acceleration due to gravity} \tag{12.10}$$

This equation can also be written in the dimensional form:

$$\bar{h} = 1.13\left[\frac{\rho_L(\rho_L-\rho_v)gh_{\text{vap}}k_L^3}{(A_h/P_W)\mu_L(T_{\text{sat}}-T_S)}\right]^{0.25} \tag{12.11}$$

where k_L is the thermal conductivity of the liquid and ρ_L is the density of the liquid.

2. For laminar film condensation on horizontal tubes of diameter D, the Nusselt equation is:

$$\text{Co} = 1.514\,\text{Re}_f^{-1/3} \tag{12.12}$$

In dimensional form:

$$\bar{h} = 0.725\left[\frac{\rho_L(\rho_L-\rho_v)gh_{\text{vap}}k_L^3}{D\mu_L(T_{\text{sat}}-T_S)}\right]^{0.25} \tag{12.13}$$

3. For turbulent film condensation, the Kirkbride equation is:

$$\text{Co} = 0.0077\,\text{Re}_f^{0.4} \tag{12.14}$$

with Co once again given by Equation (12.10).

A superheated vapor occurs when the temperature of the vapor is higher than that of the boiling point at the corresponding pressure. When a superheated vapor enters a condenser, the sensible heat of superheat and the latent heat of condensation must be transferred through the cooling surface. The condensation mechanism is therefore somewhat different if the condensing vapor is superheated rather than saturated. Experimental results have shown that, in most cases, the effect of superheat may be ignored and the equations for saturated vapors may be used with negligible error. It should be noted that

$$T_{\text{sat}} - T = \Delta T \tag{12.15}$$

is still the temperature difference driving force, and that the actual superheated vapor temperature does not enter into the calculations.

For mixed vapors (i.e., if the vapor contains two or more volatile components), the condensation temperature is no longer constant at a given pressure unless the mixture is azeotropic. If the cooling surface temperature is low, the vapor may condense and the condensate may be assumed to be the same as that of the original vapor. If the cooling surface temperature is straddled by the condensation temperatures of the components, part of the vapor will condense and some vapor must be vented from the condenser.

Regarding dropwise condensation, heat transfer coefficients are an order of magnitude larger than those for film condensation. If other thermal resistances in the system are significantly larger than that due to condensation, the condensation resistance may be neglected. The following expressions are recommended for estimation purposes for dropwise condensation:

$$\bar{h} = 51{,}000 + 2000T_{\text{sat}} \qquad 22^\circ\text{C} < T_{\text{sat}} < 100^\circ\text{C} \tag{12.16}$$

$$\bar{h} = 255{,}000 \qquad 100^\circ\text{C} < T_{\text{sat}} \tag{12.17}$$

where the heat transfer coefficient h has units of $\text{W/m}^2 \cdot \text{K}$.

ILLUSTRATIVE EXAMPLE 12.5

A vertical rectangular plate is 0.2 m wide and 0.4 m high. One surface of the plate is exposed to saturated steam at atmospheric pressure. The plate surface is maintained at 98°C. Filmwise condensation may be assumed. Calculate the type of condensation (flow type) and the condensation heat transfer coefficient. Assume steady-state, constant properties, an isotropic homogeneous surface, and that the properties of the water condensate may be evaluated at a film temperature of 99°C. The thermal conductivity of water may be assumed equal to 0.68 $\text{W/m} \cdot \text{K}$.

***SOLUTION*:** Obtain the latent heat of steam, h_{vap}, at the saturation temperature. From the steam tables (see Appendix), at 100°C,

$$h_{\text{vap}} = 970.3\,\text{Btu/lb} = 2.255 \times 10^6\ \text{J/kg}$$

Obtain the density of steam at the mean film temperature. From the same steam tables,

$$\rho_v = 0.577\,\text{kg/m}^3 \ \text{ at } 99^\circ\text{C}$$

Also, obtain the properties of liquid water condensate (ρ_L, μ_L, k_L) at the mean film temperature. From the Appendix, at 99°C (some interpolation required),

$$\rho_L = 960\,\text{kg/m}^3$$
$$\mu_L = 2.82 \times 10^{-4}\,\text{kg/m}\cdot\text{s}$$
$$k_L = 0.68\,\text{W/m}^2\cdot\text{K}$$

Obtain the ratio, A/P_W for a vertical plate from Table 12.2:

$$\begin{aligned} Z &= \text{height} = 0.4\,\text{m} \\ P_W &= W = 0.2\,\text{m} \\ A/P_W &= Z = 0.4\,\text{m} \end{aligned}$$

Generate an expression for the condensation number, Co, in terms of the average heat transfer coefficient using Equation (12.10):

$$\begin{aligned} \nu_L &= \frac{\mu_L}{\rho_L} = \frac{2.82 \times 10^{-4}}{960} = 2.94 \times 10^{-7}\,\text{m}^2/\text{s} \\ \text{Co} &= \frac{\bar{h}}{k}\left(\frac{\nu_L^2}{g(1-\rho_v/\rho_L)}\right)^{1/3} = \frac{\bar{h}}{0.68}\left(\frac{(2.94 \times 10^{-7})^2}{9.807(1-0.577/960)}\right)^{1/3} \\ &= 3.038 \times 10^{-5}\bar{h} \end{aligned}$$

Also, generate the Reynolds number in terms of the average heat transfer coefficient using Equation (12.8), noting that $A/P_W = 0.4$:

$$\text{Re} = \frac{4\bar{h}(0.4)(100-98)}{(2.82 \times 10^{-4})(2.255 \times 10^6)} = 5.03 \times 10^{-3}\bar{h}$$

Assume a flow type (laminar or turbulent) and determine Co in terms of Re_L. First, assume turbulent condensation on a vertical plate so that Equation (12.14) applies:

$$\text{Co} = 0.0077\,\text{Re}^{0.4}$$

Obtain a value for $\bar{h}$ by substituting into the equations above:

$$3.038 \times 10^{-5}\bar{h} = (0.0077)(5.03 \times 10^{-3})^{0.4}(\bar{h})^{0.4}$$
$$\bar{h} = 298\,\text{W/m}^2\cdot\text{K}$$

Check the Reynolds number to verify the assumption of flow type:

$$\text{Re} = (5.03 \times 10^{-3})\bar{h} = (5.03 \times 10^{-3})(298) = 1.50 < 1800$$

The assumption of turbulent condensation is not valid. Therefore, assume laminar flow and employ the expression for Co in terms of Re in Equation (12.9):

$$\text{Co} = 1.874\,\text{Re}^{-1/3}$$

Once again, obtain a value for $\bar{h}$ by substituting into the equation above

$$3.038 \times 10^{-5}\bar{h} = (1.874)(5.03 \times 10^{-3})^{-1/3}(\bar{h})^{-1/3}$$
$$\bar{h} = 14{,}700\ \text{W/m}^2 \cdot \text{K}$$

Check the Reynolds number to verify the assumption of flow type:

$$\text{Re} = (5.03 \times 10^{-3})\bar{h} = (5.03 \times 10^{-3})(14{,}700) = 73.9 < 1800$$

The assumption of laminar flow is valid. ■

ILLUSTRATIVE EXAMPLE 12.6

Refer to Illustrative Example 12.5. Calculate

1. the mass flow rate of condensate, $\dot{m}'$
2. the heat transfer rate $\dot{Q}$.

***SOLUTION*:** First calculate $\dot{m}'$:

$$\text{Re} = \frac{4\dot{m}'}{\mu_L} \tag{12.4}$$
$$\dot{m}' = \frac{\text{Re}\mu_L}{4}$$

Substituting,

$$\dot{m}' = \frac{(73.9)(2.82 \times 10^{-4})}{4}$$
$$= 0.0052\,\text{kg/m} \cdot \text{s}$$

Calculate $\dot{m}$ employing Equation (12.7):

$$\dot{m} = P_W\dot{m}' = W\dot{m}'$$
$$= (0.2)(0.0052)$$
$$= 0.00104\ \text{kg/s}$$

Calculate Co using the result from the previous illustrative example:

$$\text{Co} = (3.0377 \times 10^{-5})\bar{h} = (3.0377 \times 10^{-5})(14{,}700) = 0.4465$$

Finally, calculate $\dot{Q}$:

$$\begin{aligned}\dot{Q} &= hA(T_{\text{sat}} - T_S)\\ &= (14{,}700)(0.2)(0.4)(100 - 98)\\ &= 2350\ \text{W}\\ &= 2.35\,\text{kW}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 12.7

A horizontal 4-inch OD tube is surrounded by saturated steam at 2.0 psia. The tube is maintained at 64°F. What is the average heat-transfer coefficient? Assume laminar flow.

SOLUTION**:** Assuming laminar flow, the average heat-transfer coefficient is given by Equation (12.13). The liquid properties are evaluated at the mean film temperature, $T_f = (T_{\text{sat}} + T)/2 = (126 + 64)/2 = 95°\text{F}$. The approximate values (at 100°F) of key properties are:

$$h_{\text{vap}} = 1022\,\text{Btu/lb (at } T_{\text{sat}})$$

$$\rho_v = 0.00576\,\text{lb/ft}^3\ \text{(at } T_{\text{sat}})$$

$$\rho_L = 62.03\,\text{lb/ ft}^3$$

$$k_L = 0.364\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$$

$$\mu_L = 4.26 \times 10^{-4}\,\text{lb/ ft}\cdot\text{s}$$

Solving,

$$\bar{h} = 0.725\left[\frac{\rho_L(\rho_L - \rho_v)gh_{\text{vap}}k_L^3}{\mu_L D(T_{\text{sat}} - T_S)}\right]^{1/4} \tag{12.13}$$

$$= 0.725\left[\frac{(62.03)(62.03 - 0.00576)(32.2)(1022)(0.364)^3}{(4.26 \times 10^{-4})(4/12)(126 - 64)/(3600)}\right]^{1/4}$$

$$= 911.6\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

■

BOILING FUNDAMENTALS

The process of converting a liquid into a vapor is also of importance to practicing engineers. The production of steam for electrical power generation is a prime example. Many other processes, particularly in the refining of petroleum and the manufacture of chemicals, require the vaporization of a liquid.

Boiling is the opposite of condensation. Boiling occurs when a liquid at its saturation temperature, T_{sat}, is in contact with a solid surface at a temperature, T_S, which is above T_{sat}. The excess temperature, ΔT_e, is defined as:

$$\Delta T_e = T_S - T_{\text{sat}} \tag{12.18}$$

The ΔT_e driving force causes heat to flow from the surface into the liquid, which results in the formation of vapor bubbles that move up through the liquid. Because of bubble formation, the surface tension of the liquid has an impact on the rate of heat transfer.

Boiling may be classfied as *pool boiling* or *forced convection boiling.* In pool boiling, the liquid forms a "pool" in a container while submerged surfaces supply the heat. The liquid motion is induced by the formation of bubbles as well as density variations. In forced convection, the liquid motion is induced by external means (e.g., pumping of liquids through a heated tube).

Another common classification for types of boiling is based on the relationship of the liquid temperature to its saturated temperature. Boiling is *subcooled* (also known as *local*) when the liquid temperature is below the saturation temperature (i.e., $T < T_{\text{sat}}$). When the liquid is at the saturation temperature, the boiling is *saturated* boiling.

The explanation of the strange behavior of boiling systems lies in the fact that boiling heat transfer occurs by several different mechanisms and the mechanism is often more important in determining the heat rate than the temperature-difference driving force. Consider, for example, the heating of water in an open pot. Heat is initially transferred within the water by natural convection. As the heat rate is further increased, the surface temperature at the base of the pot increases to and above 212°F. Bubbles begin to form and then rise in columns from the heating surface creating a condition favorable to heat transfer. As the temperature increases further, more sites become available until the liquid can no longer reach the heating surface at a sufficient rate to form the required amount of vapor. This ends the *nucleate boiling* stage (to be discussed later). At this point, the mechanism changes to *film boiling* since the heating surface is now covered with a film of vapor. The temperature of the base of the pot rises (even though the heat rate is constant). Vaporization takes place at the liquid–vapor interface and the vapors disengage from the film in irregularly-shaped bubbles at random locations.

Although the phenomena of superheating occurs in most boiling systems, the temperature of the boiling liquid, measured some distance from the heated surface, is higher than the temperature of the vapor above the liquid (which is at the saturation temperature). The liquid superheat adjacent to the heated surface may be as high as 25 to 50°F. Most of the temperature change occurs in a narrow thin film from the surface. This superheat occurs because the internal pressure in the vapor bubble is higher due to surface tension effects.

The formation, growth, and release of bubbles is an extremely rapid sequence of events. The rapid growth and departure of vapor bubbles causes turbulence in the liquid, especially in the aforementioned zone of superheat near the heated surface. This turbulence assists the transport of heat from the heated surface to the liquid evaporating at the bubble surface. The rapid growth of bubbles and the turbulence in the liquid complement each other, resulting in high heat transfer coefficients.

As the superheat of a boiling liquid is further increased, the concentration of active centers on the heating surface increases and the heat rate correspondingly increases. The mass rate of the vapor rising from the surface must be equal to the mass rate of liquid proceeding toward the surface if steady-state conditions prevail. As the boiling rate increases, the rate of liquid influx must increase since the area available for flow decreases with the increasing number of bubble columns; in addition, the liquid velocity must also increase. At this limiting condition, the liquid flow toward the heated surface cannot increase and the surface becomes largely blanketed with vapor. If the heat rate to the surface is held constant, the surface temperature will rise to a high value at which point heat is transmitted to the fluid by the mechanism of film boiling. Film boiling occurs when the superheat is sufficiently high to keep the heated surface completely blanketed with vapor. Heat may then be transmitted through the gas film by conduction, convection, and radiation.

Finally, the *Leidenfrost* phenomenon is one of the many complex properties of boiling. If a liquid is dropped on a surface that has a temperature significantly higher than the liquids boiling point, the liquid will skitter across the surface and evaporate at a slower rate than expected. This is due to an instantaneous layer of vapor, suspending the liquid, which acts as insulation between the hot surface and the drop of liquid. At temperatures closer to boiling, the liquid will evaporate at a much faster rate.

Interestingly, there is a unique relationship between a liquid's normal boiling point and critical point. The describing equation is given by:

$$\frac{T_b}{T_c} \approx 0.6 \tag{12.19}$$

The relationship is demonstrated in Table 12.3.

Boiling point elevation is another interesting phenomenon. When a solute (e.g., NaOH) is dissolved in water, the vapor pressure of the aqueous solution is less than that of water at the same temperature. Thus, at a given pressure, the solution boils at a higher temperature than water. This increase is termed the boiling point elevation (BPE) of the solution. Dilute solutions of organic compounds exhibit small (negligible) BPEs. For aqueous solutions of inorganic compounds, the BPE may be as

Table 12.3 Ratio of Boiling Point to Critical Temperature

Substance	T_b, K	T_c, K	T_b/T_c Ratio
Nitrogen	77.3	126.0	0.61
Oxygen	90.1	154.3	0.58
Carbon dioxide	194.6	304.2	0.64
Ammonia	239.7	405.5	0.59
Water	373.2	647	0.58
Ethane	185	305	0.61
n-Pentane	309	470	0.66
Benzene	353	562	0.63
Carbon tetrachloride	350	556	0.63
Ethyl alcohol	351	516	0.68
Acetic acid	391	594	0.66

high as 150°F. This BPE represents a loss in the thermal driving force and has to be accounted for in some calculations.

ILLUSTRATIVE EXAMPLE 12.8

The rate of heat flow per unit area of heat transfer surface to a boiling liquid

- **(a)** increases with the temperature of the heating medium.
- **(b)** decreases with the temperature of the heating medium.
- **(c)** increases with the temperature of the boiling liquid.
- **(d)** decreases with the temperature of the boiling liquid.
- **(e)** may increase or decrease with the temperature of the heating medium.
- **(f)** None of the above.

Which statement is correct?

***SOLUTION*:** The answer is (e) due to the aforementioned "bubble" effect. ■

BOILING PRINCIPLES

It has been found that the heat trasfer coefficient, h, in boiling systems depends on the excess temperature, ΔT_e. For free convection boiling ($\Delta T_e < 5°\text{C}$):

$$h \propto (\Delta T_e)^n \tag{12.20}$$

where $n = \frac{1}{4}$ laminar free convection conditions
$\quad = \frac{1}{3}$ turbulent free convection conditions

The boiling heat flux, $\dot{Q}'_s$, is calculated as:

$$\dot{Q}'_s = h(T_S - T_{\text{sat}}) \tag{12.21}$$

and

$$\Delta T_e = T_S - T_{\text{sat}} \tag{12.22}$$

The dimensionless number of importance in boiling heat transfer is the liquid Jakob number, Ja_L. It is defined as

$$\text{Ja}_L = \frac{c_p \Delta T_e}{h_{\text{vap}}} \tag{12.23}$$

The mechanism of pool boiling heat transfer depends on the excess temperature, ΔT_e, where, once again, ΔT_e is the difference between surface temperature and the fluid saturation temperature. A plot of the boiling heat transfer flux, $\dot{Q}'_S$, versus the excess temperature, is shown in Figure 12.1. When $\Delta T_e > 5°\text{C}$, bubbles start to

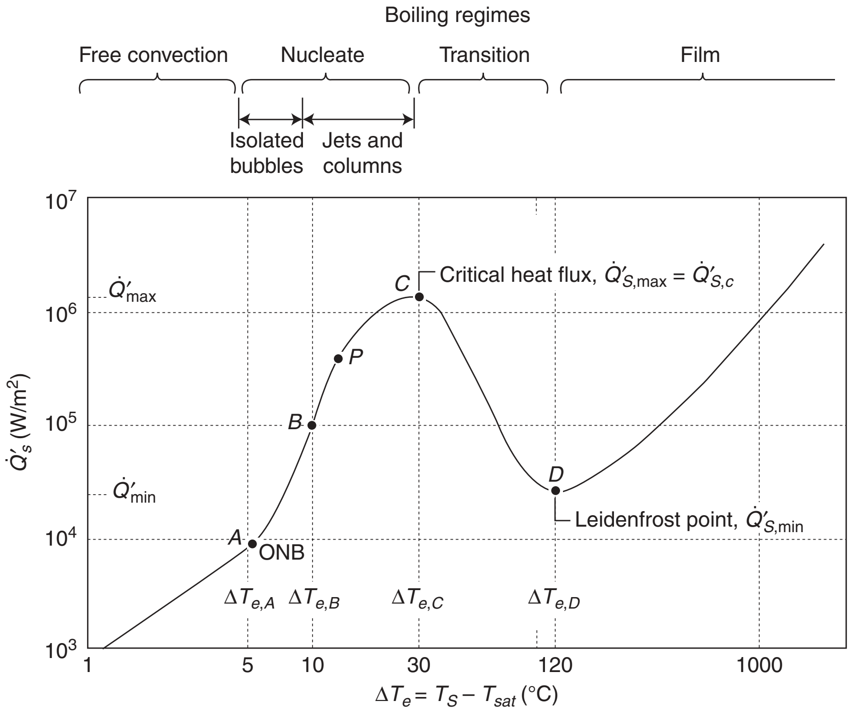

Figure 12.1 Boiling curve. (Adapted from Incroprera and De Witt, *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, 1981.)

form at the onset of nucleate boiling (ONB). As noted earlier, the bubbles rise and are dissipated in the liquid; the resulting stirring causes an increase in the heat flux. This regime was termed *nucleate* boiling. The heat transfer coefficient in this regime varies with ΔT_e as follows:

$$h \propto (\Delta T_e)^n \tag{12.20}$$

where n varies between 3 or 4. As ΔT_e increases, the rate of bubble formation increases. This causes h to increase, but at higher rates of bubble formation some bubbles cannot diffuse quickly enough and form a blanket around the heating element. This blanket increases the heat transfer resistance and slows down the increase in both h and the heat flux. These two factors balance out at approximately $\Delta T_e = 30°\text{C}$ where the flux (introduced below) reaches its maximum. The peak flux is termed the *critical heat flux*, $\dot{Q}'_{S,c}$. For boiling water, $\dot{Q}'_{S,c}$ is greater than 1 MW/m^2. As ΔT_e increases still further, the gas film formed by the coalescing bubbles around the element prevents the liquid from coming in and causes a further increase in the heat transfer resistance. This mechanism is defined as *film* boiling.

For the approximate 30°C $< \Delta T_e <$ 120°C range, the mechanism becomes *unstable film* boiling, also known as *transition film* boiling, or *partial film* boiling. The heat transfer coefficient decreases with increasing ΔT_e until a stable value is reached. At these high surface temperatures, T_s, thermal radiation contributes significantly to the heat transfer. This contribution causes the heat flux to increase again. The minimum in the curve is referred to as the *Leidenfrost point* (see previous section) and the heat flux is given by $\dot{Q}'_{S,\min}$.

At the point of peak (or critical) flux, $\dot{Q}'_{S,c}$, a small increase in ΔT_e causes $\dot{Q}'_S$ to decrease. However, with the lower heat flux, the energy from the heating surface cannot be completely dissipated, causing a further increase in ΔT_e. Eventually, the fluid temperature can exceed the melting point of the heating medium. The critical heat flux (at point C on Figure 12.1) is therefore also termed the *boiling crisis point* or the *burnout point*.

Boiling equipment should be operated in the nucleate boiling region, close to the critical flux, but never exceeding it. To calculate the boiling heat flux in the nucleate boiling regime, one may use the Rohsenow equation:(5)

$$\dot{Q}'_s = \mu_L h_{\text{vap}} \sqrt{\frac{g(\rho_L - \rho_v)}{\sigma}} \left(\frac{\text{Ja}_L}{C_{sf}\,\text{Pr}_L^n} \right)^3 \tag{12.24}$$

where σ is the liquid surface tension (lb$_f$/ft or N/m), μ_L is the dynamic viscosity of the liquid (lb/ft · h or kg/m · s), C_{sf} is the correlation coefficient that depends on the surface–fluid combination, n is the Prandtl number exponent that depends on the surface–fluid combination and Ja_L is the liquid Jakob number, as presented in Equation (12.23). Numerical values of C_{sf} and n can be obtained from Table 12.4. Note that $n = 1$ for water and 1.7 for other liquids.

The critical heat flux (or burnout flux) is calculated from the Zuber equation:(6,7)

$$\dot{Q}'_{S,c} = \frac{\pi}{24} h_{\text{vap}} \rho_v \left(1 + \frac{\rho_v}{\rho_L} \right)^{1/2} \left(\frac{\sigma g(\rho_L - \rho_v)}{\rho_v^2} \right)^{1/4} \tag{12.25}$$

If $\rho_v \ll \rho_L$, the Zuber equation simplifies to:

$$\dot{Q}'_{S,\max} = \dot{Q}'_{S,c} = \frac{\pi}{24} h_{\text{vap}} (\sigma g \rho_L / \rho_v^2)^{1/4} \tag{12.26}$$

At the so-called *Leidenfrost point*, the heating surface is covered by a stable vapor blanket and the heat flux reaches its minimum condition, $\dot{Q}'_{S,\min}$. Zuber developed the following relationship to calculate the minimum heat flux from a large horizontal plate:

$$\dot{Q}'_{S,\min} = 0.09 h_{\text{vap}} \rho_v \left(\frac{\sigma g(\rho_L - \rho_v)}{(\rho_L + \rho_v)^2} \right)^{1/4} \tag{12.27}$$

Table 12.4 Values of C_{sf} for Various Fluid–Surface Combinations to be Used with the Rohsenow Equation

Fluid–surface combination	C_{sf}	n
Water–copper		
Scored	0.0068	1.0
Polished	0.0130	1.0
Water–stainless steel		
Chemically etched	0.0130	1.0
Mechanically polished	0.0130	1.0
Ground and polished	0.0060	1.0
Water–brass	0.0060	1.0
Water–nickel	0.0060	1.0
n-Pentane–copper		
Polished	0.0154	1.7
Lapped	0.0049	1.7
Benzene–chromium	0.0101	1.7
Ethyl alcohol–chromium	0.0027	1.7

If $\rho_v \ll \rho_L$, the Zuber minimum flux equation simplifies to:

$$\dot{Q}'_{S,\,\mathrm{min}} = 0.09 h_{\mathrm{vap}} \rho_v \left(\frac{\sigma g}{\rho_L} \right)^{1/4} \tag{12.28}$$

Water is the most common fluid used in boiling applications. To facilitate calculations involving water, approximate heat transfer relations have been developed to calculate the surface flux, $\dot{Q}'_S$, for the boiling of water during both pool boiling and forced convection. The surface flux, $\dot{Q}'_S$, is given by:

$$\dot{Q}'_S = h(T_s - T_w) \tag{12.29}$$

For forced convection boiling inside vertical tubes, the heat transfer coefficient is approximated by the dimensional relation:

$$h = 2.54(\Delta T_e)^n e^{(P/1.55)} \tag{12.30}$$

and for *pool* boiling:

$$h = C(\Delta T_e)^n (P/P_a)^{0.4} \tag{12.31}$$

where h is the heat transfer coefficient from the surface to the boiling water, P is the absolute pressure in mega pascals (MPa), and P_a is the ambient absolute pressure (e.g., 1 atm = 0.1013 MPa). The coefficients C and n are constants that depend on the orientation of the surface and the magnitude of the surface heat flux, $\dot{Q}'_S$. Numerical values of C and n are given in Table 12.5.

Table 12.5 Constants for Use with the Approximate Pool Boiling Heat Transfer Coefficient Correlation for Water

Surface orientation	Surface flux, $\dot{Q}'_s$, kW/m^2	C	n
Horizontal	$\dot{Q}'_s < 15.8$	1040	$\frac{1}{3}$
	$15.8 < \dot{Q}'_s < 236$	5.56	3
Vertical	$\dot{Q}'_s < 3.15$	539	7
	$3.15 < \dot{Q}'_s < 63.1$	7.95	3

Since the surface flux is often unknown, the calculation of h involves trial-and-error. A typical procedure is to:

1. assume the range of the surface flux, $\dot{Q}'_s$
2. read C and n from Table 12.5
3. calculate h using Equation (12.30) or Equation (12.31)
4. calculate $\dot{Q}'_s$ from Equation (12.29), and
5. determine if the assumption made in (1) is valid.

For heat transfer coefficients involving boiling water, the effect of pressure on the heat transfer coefficient may be estimated from

$$h = h_{1\,\text{atm}}\left(\frac{P}{P_a}\right)^{0.4} \tag{12.32}$$

This pressure correction is recommended for other liquids if another correlation is not available.

ILLUSTRATIVE EXAMPLE 12.9

Boiling water at 100°C and 1 atm is supplied with heat from a surface at 102°C. The heat flux is 9800 W/m^2. If the surface temperature is raised to 103°C, determine the new heat flux.

At 1 atm, the saturation temperature of the water is 100°C. The liquid water properties (SI units) are: density = 957.9 kg/m^3, heat capacity = 4217 J/kg · K, latent heat of vaporization = 2.26×10^6 J/kg. Assume steady-state operation, constant properties, and laminar conditions.

***SOLUTION*:** Calculate the heat transfer coefficient at 102°C using Equation (12.21)

$$h = \frac{\dot{Q}'_s}{T_S - T_L} = \frac{\dot{Q}'_s}{T_S - T_{\text{sat}}} = \frac{9800}{102 - 100} = 4900\ \text{W/m}^2 \cdot \text{K}$$

The new heat transfer coefficient may now be calculated, see Equation (12.20). Since $h \propto (\Delta T_e)^n$ and $n = 0.25$, for laminar flow;

$$h_2 = h_1 \left(\frac{\Delta T_{e1}}{\Delta T_{e2}} \right)^{1/4} = 4900 \left(\frac{3}{2} \right)^{1/4} = 5421\,\mathrm{W/m^2 \cdot K}$$

The new heat flux is:

$$\dot{Q}_s' = h(T_S - T_L) = 5421(103 - 100) = 16{,}260\,\mathrm{W/m^2}$$

■

ILLUSTRATIVE EXAMPLE 12.10

With respect to Illustrative Example 12.9, determine the mechanism of boiling heat transfer. Is the earlier assumed boiling mechanism still valid?

***SOLUTION*:** Calculate the original excess temperature:

$$\Delta T_e = T_S - T_{\mathrm{sat}} = 102 - 100 = 2^\circ\mathrm{C}$$

Calculate the new excess temperature (for $T_S = 103^\circ\mathrm{C}$).

$$\Delta T_e = T_S - T_{\mathrm{sat}} = 103 - 100 = 3^\circ\mathrm{C}$$

The assumption of the free convection mechanism is valid since $\Delta T_e < 5^\circ\mathrm{C}$. ■

ILLUSTRATIVE EXAMPLE 12.11

Refer to Illustrative Example 12.9 and calculate the liquid Jakob number.

***SOLUTION*:** The liquid Jakob number may be calculated from Equation (12.22):

$$\mathrm{Ja}_L = \frac{c_p \Delta T_e}{h_{\mathrm{vap}}} = \frac{4127(3)}{2.26 \times 10^6} = 0.00548$$

■

ILLUSTRATIVE EXAMPLE 12.12

An electric heater 0.3 m in diameter at 106°C is used to heat water. The water is at 1 atm and 100°C. Determine

1. the boiling regime, and
2. The heat transfer coefficient.

Assume steady-state operation and constant properties.

***SOLUTION*:** Calculate the excess temperature, ΔT_e,

$$\Delta T_e = T_S - T_{\text{sat}} = 106 - 100 = 6^\circ\text{C}$$

See Figure 12.1. Since ΔT_e is between 5°C and 30°C, nucleate boiling applies.

Assume a range for the surface flux, $\dot{Q}'_S$, and obtain the corresponding values of C and n. Assuming that $15.8 < \dot{Q}'_S < 236\,\text{kW/m}^2$, then, from Table 12.5, $C = 5.56$ and $n = 3$. For pool, rather than forced convection, the water boiling heat transfer coefficient is given by Equation (12.31):

$$\begin{aligned} h &= C(\Delta T_e)^n (P/P_a)^{0.4} \\ &= 5.56(6)^3(1)^{0.4} \\ &= 1200\,\text{W/m}^2 \end{aligned}$$

Calculate the surface flux using Equation (12.29) and check the assumption made above:

$$\begin{aligned} \dot{Q}'_S &= h\Delta T_e = (1200)(6) \\ &= 7200\,\text{W/m}^2 \\ &= 7.2\,\text{kW/m}^2 \end{aligned}$$

Since $\dot{Q}'_S < 15.8$, the assumption above is *not* correct. Therefore, assume a new range for the surface flux (e.g., that $\dot{Q}'_S < 15.8\,\text{kW/m}^2$). From Table 12.5, $C = 1040$ and $n = \frac{1}{3}$.

Once again, calculate h and $\dot{Q}'_S$:

$$\begin{aligned} h &= 1040(6)^{1/3}(1)^{0.4} = 1890\,\text{W/m}^2\cdot\text{K} \\ \dot{Q}'_S &= h(\Delta T_e) \\ &= (1890)(6) \\ &= 11{,}340\,\text{W/m}^2 \\ &= 11.34\,\text{kW/m}^2 \end{aligned}$$

The second assumption is valid. ■

ILLUSTRATIVE EXAMPLE 12.13

Refer to the previous example. Calculate the rate of heat transfer.

***SOLUTION*:** To calculate the heat transfer rate, $\dot{Q}'_S$,

$$\dot{Q}_S = \dot{Q}'_S A = \dot{Q}'_S\left(\frac{\pi D^2}{4}\right) = (11{,}340)\left[\frac{\pi(0.3)^2}{4}\right] = 801\,\text{W}$$

■

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Chapter 13

Refrigeration and Cryogenics*

INTRODUCTION

Refrigeration and cryogenics have aroused considerable interest among workers in the fields of engineering and science. All refrigeration processes involve work, specifically, the extraction of heat from a body of low temperature and the rejection of this heat to a body willing to accept it. Refrigeration generally refers to operations in the temperature range of 120 to 273 K, while cryogenics usually deals with temperatures below 120 K where gases, including methane, oxygen, argon, nitrogen, hydrogen, and helium, can be liquefied.

In addition to being employed for domestic purposes (when a small "portable" refrigerator is required), refrigeration and cryogenic units have been used for the storage of materials such as antibiotics, other medical supplies, specialty foods, etc. Much larger cooling capacities than this are needed in air conditioning equipment. Some of these units, both small and large, are especially useful in applications that require the accurate control of temperature. Most temperature-controlled enclosures are provided with a unit that can maintain a space below ambient temperature (or at precisely ambient temperature) as required. The implementation of such devices led to the recognition that cooling units would be well suited to the refrigeration of electronic components and to applications in the field of instrumentation. Such applications usually require small compact refrigerators, with a relatively low cooling power, where economy of operation is often unimportant.

One of the main cost considerations when dealing with refrigeration and cryogenics is the cost of building and powering the equipment. This is a costly element in the process, so it is important to efficiently transfer heat so that money is not wasted in lost heat in the refrigeration and cryogenic processes. Since the cost of equipment can be expensive, there are a number of factors to be considered when choosing equipment. Equipment details are discussed in later sections.

Cryogenics plays a major role in the chemical processing industry. Its importance lies in the recovery of valuable feedstocks from natural gas streams, upgrading the heat

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content of fuel gas, purifying many process and waste streams, producing ethylene, as well as other chemical processes.

Cryogenic air separation provides gases (nitrogen, oxygen, and argon) used in

1. the manufacturing of metals such as steel,
2. chemical processing and manufacturing industries,
3. electronic industries,
4. enhanced oil recovery, and
5. partial oxidation and coal gasification processes.

Other cryogenic gases, including hydrogen and carbon monoxide, are used in chemical and metal industries while helium is used in welding, medicine, and gas chromatography.

Cryogenic liquids have their own applications. Liquid nitrogen is commonly used to freeze food, while cryogenic cooling techniques are used to reclaim rubber tires and scrap metal from old cars. Cryogenic freezing and storage is essential in the preservation of biological materials that include blood, bone marrow, skin, tumor cells, tissue cultures, and animal semen. Magnetic resonance imaging (MRI) also employs cryogenics to cool the highly conductive magnets that are used for these types of non-intrusive body diagnostics.(1)

As will become apparent throughout this chapter, there is a wide variety of applications, uses, and methods to produce and to utilize the systems of refrigeration and cryogenics. Multiple factors must be considered when dealing with these practices, including the choice of refrigerant or cryogen, the choice of equipment and methods of insulation, and all hazards and risks must be accounted for to ensure the safest environment possible.

Topics covered in this chapter include:

1. Background material
2. Equipment
3. Materials of construction
4. Insulation and heat loss
5. Storage and transportation
6. Hazards, risks, and safety (see also Chapter 24)
7. Basic principles and applications

BACKGROUND MATERIAL

Refrigeration

The development of refrigeration systems was rapid and continuous at the turn of the 20th century, leading to a history of steady growth. The purpose of refrigeration, in a general sense, is to make materials colder by extracting heat from the material. As described in earlier chapters, heat moves in the direction of decreasing temperature (i.e., it is transferred from a region of high temperature to one of a lower temperature). When the opposite process needs to occur, it cannot do so by itself, and a refrigeration system (or its equivalent) is required.(2)

Refrigeration, in a commercial setting, usually refers to food preservation and air conditioning. When food is kept at colder temperatures, the growth of bacteria and the accompanying spoiling of food is prevented. People learned early on that certain foods had to be kept cold to maintain freshness and many kept these foods in ice boxes where melting ice usually absorbed the heat from the foods. Household refrigerators became popular in the early 1900s and only the wealthy could afford them at the time. Freezers did not become a staple part of the refrigerator until after World War II when frozen foods became popular.

The equipment necessary in refrigeration is dependent upon many factors, including the substances and fluids working in the system. One very important part of refrigeration is the choice of refrigerant being employed and the refrigerant choice obviously depends on the system in which it will be used. The following criteria are usually considered in refrigerant selection:

1. practical evaporation and condensation pressures
2. high critical and low freezing temperatures
3. low liquid and vapor densities
4. low liquid heat capacity
5. high latent heat of evaporation
6. high vapor heat capacity.

Ideally, a refrigerant should also have a low viscosity and a high coefficient of performance (to be defined later in this chapter). Practically, a refrigerant should have:

1. a low cost,
2. chemical and physical inertness at operating conditions,
3. no corrosiveness toward materials of construction,
4. low explosion hazard, and
5. be non-poisonous and non-irritating.

Solid refrigerants are not impossible to use but liquid refrigerants are most often used in practice. These liquid refrigerants include hydrocarbon and non-hydrocarbon refrigerants. The most commonly used hydrocarbon refrigerants include:

1. propane,
2. ethane,
3. propylene, and
4. ethylene.

Non-hydrocarbon liquid refrigerants include:

1. nitrogen,
2. oxygen,
3. neon,
4. hydrogen, and
5. helium.

Cryogenics

Cryogenics is not, in itself, an integral field. It is merely the extension of many other fields of science that delve into the realm of the thermodynamic variable of temperature. When compared to room temperature, the properties of most substances change dramatically at extremely low temperatures. From a molecular perspective, the atoms in any substance at a lower temperature, while still vibrating, are compressed closer and closer together. Depending on the phase of the substance, various phenomena and changes to physical and chemical characteristics occur at these lower temperatures.

There are many accepted definitions of cryogenics. Some classify it simply as a "temperature range below −240°F." Another more elaborate explanation defines it as: "is the unusual and unexpected property variations appearing at low temperatures and which make extrapolations from ambient to low temperature reliable."(3) Webster's dictionary defines cryogenics as "the branch of physics that deals with the production and effects of very low temperatures."(4) Cryogenics has also been referred to as: "all phenomena, processes, techniques, or apparatus occurring or using temperatures below 120 K."(5) Combining all of these definitions, the contributing author of this chapter(6) has provided the all-purpose definition that "cryogenics is the study of the production and effects of materials at low temperatures."

There have been uses for cryogenic technologies as far back as the latter part of the 19th century. It became common knowledge in the 1840s that in order to store food at low temperatures for long periods of time, it needed to be frozen, a technology that is still utilized today.

At the beginning of the 20th century, the scientist Carl von Linde produced a double distillation column process that separated air into pure streams of its basic components of 78% nitrogen, 21% oxygen, and 1% argon. By 1912, it was discovered that minor modifications to the double distillation column process could separate many other gases from the input stream. Since there are trace amounts of neon, krypton, and xenon in air, the aforementioned distillation process, with minor modifications, was found capable of separating these gases into relatively pure streams. In the 1930s, the development of the sieve tray brought changes in cryogenic technology. A sieve tray is a plate, utilized in distillation columns, with perforated holes about 5–6 mm in diameter, which enhances mass transfer.(7) These trays were highly popular in cryogenics due to their simplicity, versatility, capacity, and cost effectiveness.

When the "Space Race" hit the United States in the 1960s, cryogenic technologies were utilized to develop a process known as cryopumping which is based on the freezing of gases on a cold surface. This process helped produce an ultrahigh vacuum here on Earth that would be similar to what was to be experienced in outer space. This led to many other discoveries, including rocket propulsion technologies which enabled astronauts to better prepare for their voyage(s) into space. Unfortunately, the space race initialized the Cold War and the subsequent development of missiles and nuclear weapons.

The *cryopreservation* process is based on the same principles as food storage (i.e., using extremely low temperatures to preserve a perishable item). Cryopreservation has become more and more popular because of its appeal in preserving living cells. Whole cells and tissue can be preserved by this technique by stopping biological activity at extremely low temperatures. The preservation of organs by cryogenics has been a stepping stone for *cryosurgery* which relies on the cold temperatures to insure clean and precise incisions. More recently, *cryobiology* has been applied well below freezing temperatures to living organisms to observe how they "react." Cryobiology has also provided developing technologies in order to help cure such fatal illnesses as Parkinson's disease.(8) Most recently, incorporating electronic systems with cryo-technologies has provided valuable information on superconductivity. Extremely low temperatures have also led to systems that contain near-zero resistance throughout the wires.

Liquefaction

Liquefaction is the process for converting a gaseous substance to a liquid. Depending on the liquefied material, various steps are employed in an industrial process. Common to each is the use of the Joule–Thomson effect(9) (where the temperature changes as a fluid flows through a valve), heat exchangers, and refrigerants to achieve the cryogenic temperatures. Generally, the methods of refrigeration and liquefaction used include:

1. vaporization of a liquid,
2. application of the Joule–Thomson effect (a throttling process), and
3. expansion of a gas in a work producing engine.

Liquid nitrogen is the best refrigerant for hydrogen and neon liquefaction systems while liquid hydrogen is usually used for helium liquefaction.

The largest and most commonly used liquefaction process involves the separation of air into nitrogen and oxygen. The process starts by taking air compressed initially to 1500 psia through a four-stage compressor with intercoolers.(10) The air is then compressed again to 2000 psia and cooled down to about the freezing point of water in a precooler. The high-pressure air is then further cooled by ammonia to about −70°F. The air is then split into two streams after this cooling stage. One stream leads to heat exchangers that cool most of the air by recycled cold gaseous nitrogen. This proceeds to an expansion valve which condenses most of the air, absorbing heat in the process.

The other stream goes to a booster expansion engine which compresses the air and then allows for expansion to further cool and then condense it (by means of the aforementioned Joule–Thomson effect). This liquid air is mixed and filtered and then introduced to the first of two fractional distillation columns. 100% nitrogen exits the top of the first column, and when condensed, contains less than 7 ppm oxygen. The oxygen rich mixture is pumped out of the bottom of the first column and is introduced to the second column. The oxygen leaving the bottom of the second column is usually 99.6% oxygen or higher. The remaining 0.4% is argon and requires a subsequent separation process. The columns implemented are similar to distillation columns and function to separate the nitrogen and oxygen.(7)

The major application of these liquefied gases is that they can act as refrigerants for other substances.

Cryogens

The most widely used cryogenic liquids include oxygen, nitrogen, air, hydrogen, and helium. There are multiple sources that contain reference and thermodynamic data for these cryogens. Table 13.1(11) shows typical data for a select number of cryogenic liquids and some details are discussed below for the most common of these. Most of the fluids, excluding helium, may be considered “classical” fluids in that their behavior can be predicted using accepted mechanical and thermodynamic principles.(1)

Liquid oxygen presents some safety problems not characteristic of some of the other cryogens (i.e., it is slightly magnetic, is extremely reactive, and may be a contributing factor to fire and explosions). Additional details are provided in a later section. Liquid nitrogen is more often used as a refrigerant in cryogenic laboratories than oxygen because it has a lower cost, is easily produced, and has an ease of handling that is certainly not the case with oxygen. Liquid nitrogen is often used in industrial and laboratory settings to:

1. maintain an intermediate heat sink between room temperature and lower temperature areas,
2. provide precooling in liquefiers and refrigerators,
3. precool equipment for use at low temperatures, and
4. cool adsorbents used in gas purification.

While liquid air was once the most commonly used cryogen due to it being the principal product of liquefiers, this is not the case anymore. The ease of producing liquid nitrogen from air has gradually decreased the use of this particular cryogen. Liquid hydrogen, which exists in its *ortho* and *para* states, is also widely used, especially as a rocket fuel, although, like oxygen, it is flammable. Hydrogen is slightly less hazardous because it is only necessary to isolate it from oxygen, while liquid oxygen must be isolated from a number of substances. Liquid helium is a unique cryogen and is available in two isotopes (helium-3 and helium-4), with helium 4 being the more

Table 13.1 Properties of Cryogenic Fluids

	Boiling point at 1 atm, K	Critical temperature, K	Critical pressure, atm	Freezing point, K	Density		Heat of vaporization, cal/g
					Liquid,[a] g/L	Gas,[b] g/L	
Helium 4	4.26	5.29	2.26	<0.80	0.125	0.178	5.20
Hydrogen	20.43	33.22	12.80	13.99	0.071	0.090	107.00
Deuterium	23.60	38.30	16.30	–	–	–	75.60
Neon	27.21	44.42	26.85	24.44	1.202	0.900	20.70
Nitrogen	77.35	126.00	33.50	63.27	0.807	1.250	47.00
Carbon monoxide	81.10	134.00	35.00	74.00	0.803	1.250	51.56
Fluorine	85.02	144.00	55.00	53.54	1.100	1.700	41.20
Argon	87.40	150.70	48.00	83.83	1.390	1.780	37.90
Oxygen	90.13	154.31	49.71	54.73	1.140	1.430	51.00
Methane	111.70	190.60	45.80	89.90	0.420	0.720	–
Krypton	120.20	210.50	54.30	116.50	2.400	3.730	–
Ozone	161.00	285.30	54.60	–	–	–	71.00
Xenon	166.00	289.60	58.20	161.60	3.060	5.890	–
Ethylene	169.40	282.80	50.90	103.70	0.570	1.260	–

[a] Liquid density at boiling point.

[b] Gas density at 273 K.

common choice because of the rarity of helium 3. It is most commonly used in systems that work with temperatures lower than liquid nitrogen.

EQUIPMENT

Highly specialized equipment is used to achieve the extremely low temperatures necessary for refrigeration and cryogenic processes. Compressors, expanders, heat exchangers, storage containers, and transportation devices (as a means of moving materials and end products) are just a few of the specialty pieces of equipment that are required in these processes. Since the drastically cold temperatures that are reached can damage most equipment, it is important for all devices to be made of materials that are durable and that can withstand any massive temperature and pressure excursions. In addition, it is also essential to incorporate temperature, pressure, and density measurement devices/gauges.

Compressor power makes up approximately 80% of the total energy used in processes involving the production of industrial gases and the liquefaction of natural gases. Therefore, in order to operate a cryogenic facility at optimum efficiency, the compressor choice is an important factor. Selection of a compressor relies on the cost of the equipment and the cost of installation, the energy and fuel costs associated with the equipment, and the cost of maintenance. There are several different types of compressors, including reciprocating compressors and centrifugal compressors. Reciprocating compressors adapt to a wide range of volumes and pressures and operate with high efficiency. Centrifugal compressors are ideal for high-speed compression; these compressors are highly efficient and reliable, especially when dealing with low pressure cryogenics.(10)

In refrigeration and cryogenics, expansion valves, often referred to as expanders, serve to reduce the temperature of a gas being expanded to provide refrigeration. Fluid expansion to produce refrigeration is performed by two unique methods: in an expansion valve where work is produced, or by a Joule–Thomson valve where no work is produced. Mechanical expansion valves generally work very much like a reciprocating compressor, while a Joule–Thomson valve provides constant enthalpy (isenthalpic) cooling of the flowing gas.(2)

There are a number of different methods used to measure temperature when dealing with refrigerants and cryogens. Gas, vapor pressure, metallic resistance, and nonmetallic resistance thermometry are among a few of the methods used. Each of the thermometry techniques requires a number of specific equipment parts but all include a measurement of temperature across a wide range. When choosing a type of equipment for a refrigeration or cryogenic process, it is important to select a unit that can withstand not only the variety of temperatures but also the pressures that are experienced when expanding and compressing a gas into a liquid.

Typical Heat Exchangers

Low-temperature operation has varied effects on equipment; therefore, sophisticated heat exchangers must be implemented for optimum efficiency in heat transfer. The following guidelines should be followed when designing low-temperature heat exchangers:

1. small temperature differences between the inlet and exit streams
2. large surface area to volume ratio
3. high heat transfer
4. low mass flow rates
5. multichannel capability
6. high pressure potential
7. minimal pressure drop
8. minimal maintenance.

Some of the most common heat exchanger designs (see Part III for details) used for cryogenic processes include:

1. coiled tube exchangers,
2. plate-fin exchangers,
3. reversing exchangers, and
4. regenerators.

Again, these units are primarily used because of their high efficiency in extremely low temperatures. Details on each of these follows.

A coiled tube heat exchanger is especially important to the cryogenic process because of its unique abilities. The large number of tubes that are wound in helices around a core tube can have varying spacing patterns that allow for equalized pressure drops in any stream. Systems that desire simultaneous heat transfer between multiple streams employ the coiled tube heat exchanger. The coiled tube heat exchanger is specifically useful in cryogenic processes because of the typically high demand of heat transfer and high operating pressures that are required.

Plate-fin exchangers, reversing exchangers, and regenerators are all second in popularity to the coiled tube heat exchanger. However, each of these exchangers has its own advantages and can be used for a number of the same reasons that a coiled tube exchanger would be employed. The plate-fin exchanger is compact and light-weight and allows for large temperature and pressure excursions. The primary purpose for a reversing exchanger is to remove impurities in a stream before it is cooled and condensed. Similar to the previously mentioned heat exchangers, a regenerator uses a method to simultaneously cool and purify any given stream. Calculation details on heat exchangers are provided in the next Part of this book.

MATERIALS OF CONSTRUCTION

It is common for basic construction materials to contract and become distorted as temperature decreases. This can result in unnecessary stresses on accompanying equipment including expanders, pumps, piping, etc. Many materials have temperature limitations and because of this, when it comes to these concerns, more exotic materials must be considered and possibly implemented. It is important to know the properties and behavior of the various materials for proper design considerations. Some combinations of materials, with each other as well as with the refrigerant or cryogen, can be hazardous to the fluid in question or the outside environment.

Equipment considerations that are taken into account when choosing a construction material are thermal conductivity, thermal expansivity, and density. Some materials exhibit the effect of superconductivity at very low temperatures. This phenomenon affects the heat capacity, thermal conductivity, electrical resistance, magnetic permeability, and thermoelectric effect of the material. These superconductive materials need to be strictly analyzed before use in these systems because high temperature superconductors usually have a brittle ceramic structure.

Plastics actually increase in strength as temperature decreases but decrease in impact resistance, which is not a desirable property. Teflon and glass-reinforced plastics retain impact resistance upon decreasing temperature but for some situations, these materials may not be ideal. Usually, for the double-walled vessel to be discussed later, carbon steel or aluminum is used for the outer shell but it should be thick enough to withstand collapsing or buckling.

About 9% nickel steels are often utilized for high boiling cryogens (>75 K) while many aluminum alloys and austenitic steels are structurally suitable for the entire cryogenic temperature range. While aluminum alloys are acceptable, pure aluminum is not recommended across the insulation space because of its high thermal conductivity.(12)

INSULATION AND HEAT LOSS

Insulation must certainly be considered an integral part of any refrigeration or cryogenic unit. The extent of the problem of keeping heat out of a storage vessel containing a liquid refrigerant or a cryogenic liquid varies widely. Generally, one must decide on the permissible and/or allowable heat losses (leaks) since insulation costs money, and an economic analysis must be performed. Thus, the main purpose of insulation is to minimize radiative and convective heat transfer and to use as little material as possible in providing the optimal insulation. When choosing appropriate insulation, the following factors are taken into consideration:

1. ruggedness
2. convenience
3. volume and weight
4. ease of handling
5. thermal effectiveness
6. cost.

The thermal conductivity (k) of a material is a major consideration in determining the thermal effectiveness of the insulation material. Different types of insulation obviously have different k values and there are five categories of insulation. These include:

1. vacuum insulation which employs an evacuated space that reduces radiant heat transfer
2. multilayer insulation, referred to by some as superinsulation, which consists of alternating layers of highly reflective material and low conductivity insulation in a high vacuum
3. powder insulation which utilizes finely divided particulate material packed between surfaces
4. foam insulation which employs non-homogeneous foam whose thermal conductivity depends on the amount of insulation
5. special insulation which includes composite insulation that incorporates many of the advantageous qualities of the other types of insulation.

It should also be noted that multilayer insulation has revolutionized the design of cryogenic refrigerant vessels.

In a double-walled vessel, typical of cryogen storage to be discussed in the next section, heat is usually transferred to the inner vessel by three methods:

1. conduction through the vessel's "jacket" by gases present in this space
2. conduction along solid materials touching both the inner and outer containers
3. radiation from the outer vessel.

It was discovered in 1898, that the optimum material to place inside the space created by the double-walled vessel is "nothing" (i.e., a vacuum). This Dewar vessel, named after Sir James Dewar, is still one of the most widely used insulation techniques for cryogenic purposes.

Insulation practices and techniques can be applied to all types of insulation, whether it is to keep heat in or to prevent heat from entering a system. The following three illustrative examples, although based on a furnace, are applicable to any problem where insulation is employed. In a very real sense, they involve extensions of the illustrative examples presented in Chapter 7 of this Part. Also note that a separate Chapter (19) in Part Three is devoted to insulation and refractory.

ILLUSTRATIVE EXAMPLE 13.1

The vertical flat walls of the combustion chamber of a furnace consist of 7.5 in of refractory, $k_R = 0.75$; 3 in of insulation, $k_I = 0.08$; and 0.25 in of steel, $k_S = 26.0$. The units of k are Btu/h·ft·°F. The average surface temperature of the inner face of the refractory and the outer face of the steel are 2000°F and 220°F, respectively.

Calculate the heat loss expressed as Btu/h·ft^2.

***SOLUTION*:** Figure 13.1 depicts the system described in the problem statement. The heat loss per hour can be calculated using the following equation, presented in Chapter 7.

$$\frac{\dot{Q}}{A} = \frac{\sum \Delta T}{\left(\frac{L}{k_R}\right)_R + \left(\frac{L}{k_I}\right)_I + \left(\frac{L}{k_S}\right)_S} \tag{13.1}$$

The thicknesses of the wall components must first be converted to feet before this calculation can proceed. Therefore,

$$L_R = 0.625 \text{ ft}$$
$$L_I = 0.25 \text{ ft}$$
$$L_S = 0.021 \text{ ft}$$

ΔT is defined as

$$\Delta T = T_R - T_S = 2000 - 220 = 1780$$

Substituting,

$$\frac{\dot{Q}}{A} = \frac{1780}{\left[\left(\frac{0.625}{0.75}\right) + \left(\frac{0.25}{0.08}\right) + \left(\frac{0.021}{26.0}\right)\right]}$$
$$= 450 \text{ Btu/h} \cdot \text{ft}^2$$

■

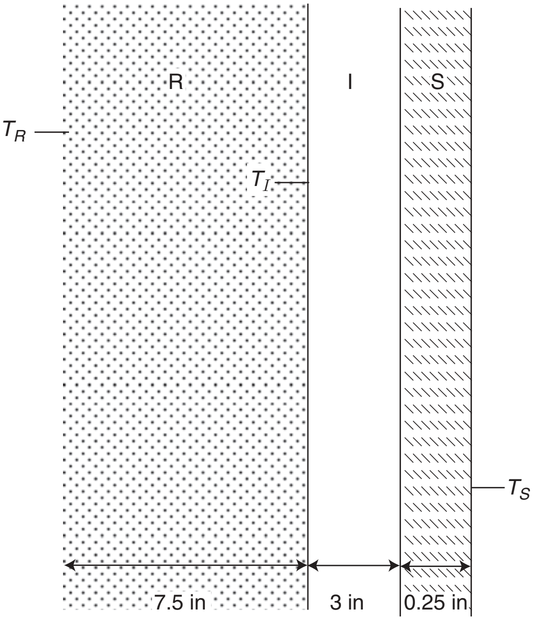

Figure 13.1 Insulation on furnace wall; Illustrative Example 13.1.

ILLUSTRATIVE EXAMPLE 13.2

For the combustion chamber of the furnace in the previous example, calculate the temperature of the boundary where the refractory meets the insulation, T_I.

***SOLUTION*:** The heat transferred across all three components is equal to that across each individual material. Therefore, since

$$\dot{Q} = \frac{\Delta T}{\sum R}$$

and the resistance in a solid is defined as

$$R = \frac{L}{kA} \tag{13.2}$$

apply the equation from Illustrative Example 13.1 to calculate T_I.

$$\frac{\dot{Q}}{A} = \frac{T_R - T_S}{\left(\dfrac{L}{k_{\text{av}}}\right)_R + \left(\dfrac{L}{k_{\text{av}}}\right)_I + \left(\dfrac{L}{k_{\text{av}}}\right)} = \frac{T_R - T_I}{\left(\dfrac{L}{k_{\text{av}}}\right)_R}$$

Substituting,

$$450 = \frac{2000 - T_I}{\dfrac{0.625}{0.75}}$$

$$T_I = 1625°\text{F}$$

■

ILLUSTRATIVE EXAMPLE 13.3

Using the scenario described in Illustrative Examples 13.1 and 13.2, calculate the area available for heat transfer if the total heat loss is 70,000 Btu/h.

***SOLUTION*:** The heat loss, $\dot{Q}$, is provided in the problem statement.

$$\dot{Q} = 70,000 \text{ Btu/h}$$

From the two previous examples,

$$\frac{\dot{Q}}{A} = 450 = \frac{70,000}{A}$$

$$A = 155.6 \text{ ft}^2$$

■

STORAGE AND TRANSPORTATION

Storage and transportation needs must also be considered in order to use cryogenic processes in food production. As noted in the previous section, the storage and transfer of these products is dependent on the storage tanks and how they are insulated; the heat gain in a storage tank could jeopardize the cryogenic state and could potentially ruin any product that was contained in the tank. Therefore, to safely store a product in a tank, the structure of the tank needs to be protected against any heat gain in the form of radiation and conduction through the insulation itself, as well as conduction though the inner shell, the supports, and the other openings and valves in the storage tank. Furthermore, the material that the storage tank is made out of needs to be non-reactive to the material being stored.

The choice of storage vessel(s) is dependent on the material being kept in the vessel and they range in type from low performance containers with minimal insulation to high performance vessels with multilayer insulation. When storing liquid refrigerants, caution must be exercised. It is crucial to avoid overpressure inside the vessel, which manufacturers try to account for by providing "bursting disks" that prevent actual disasters from occurring (see following Hazards, Risks, and Safety section).(13) Superinsulated vessels normally have an inner stainless steel vessel and are kept at the temperature of the stored material. The outer vessel, or vacuum jacket, for these units maintain a vacuum necessary for effective insulation and prevents the condensation of water or other materials on the inner vessel's cold surface.

Three major processes are employed to transport a cryogen, i.e., a cryogenic liquid:

1. a self-pressurized container
2. an external gas pressurization
3. a mechanical pumping system.

Self-pressurization is a process in which some of the fluid is removed from a container, vaporized, and then the vapor is reintroduced to the excess space and displaces the contents. *External gas pressurization* uses an external gas to displace the contents of a vessel. In *mechanical pumping*, a cryogenic pump, located at the liquid drain line, removes the contents of a container.

The use of trucks, railroad cars, and airplanes has been utilized to transport cryogens. More recently, there have been barges used to transport the cryogens via waterways. Specialized equipment is also used to measure the temperature and pressure of the cryogen while stored in these tanks and transporting units. For pressure measurements, a gauge line is run from the point of interest to a point with ambient pressure, and thus the two pressures are compared using a device such as a Bourdon gauge.(10) This procedure is not void of thermal oscillations which make it less than perfect; however, these problems can be avoided by insulating pressure transducers at the point of measurement.

HAZARDS, RISKS, AND SAFETY

While there are many risks and hazards associated with refrigerant and cryogenic fluids, time and experience has proven that under proper conditions, these materials can be used safely in industrial and laboratory applications. In these environments, all facilities and equipment, as previously discussed, must be properly designed and maintained; and, the personnel in these areas must be sufficiently trained and supervised. The primary hazards associated with cryogenic fluids are those dealing with human response to the fluids, as well as risks related to the interaction of cryogenic fluids and their surroundings. Constant attention and care must be exercised in order to avoid most, if not all, conceivable hazards that may be encountered in this field (see also Chapter 24, Part Four).(14)

Physiological Hazards

There are a few sources of personal hazard in the field of cryogenics. If the human body were to come in contact with a cryogenic fluid or a surface cooled by a cryogenic fluid, severe "cold burns" could result. Cold burns inflict damage similar to a regular burn, causing stinging sensations and accompanying pain. But, with cryogenics, the skin and/or tissue is essentially frozen, significantly damaging or destroying it. As with any typical burn or injury, the extent and "brutality" of a cold burn depends on the area and time of contact; medical assistance is strongly advised when one receives a burn of this type.

Protective insulated clothing should be worn during work with low temperature atmospheres to prevent "frost bite" when dealing with cryogenic liquids. Safety goggles (or in some cases, face shields), gloves and boots are integral parts of these uniforms. The objective of these precautions is to prevent any direct contact of the skin with the cryogenic fluid itself or with surfaces in contact with the cryogenic liquid. All areas in which cryogenic liquids are either stored or used should be clean and organized in a manner to prevent any avoidable accidents or fires and explosions that could result; this is especially true when working with systems using oxygen.

ILLUSTRATIVE EXAMPLE 13.4

Briefly discuss the routes by which a chemical, specifically a hazardous cryogen, can enter the body.(15)

***SOLUTION*:** To protect the body from hazardous cryogens, one must know the route of entry into the body. All chemical forms, including cryogens and refrigerants, may be inhaled. After a cryogen is inhaled into the nose or mouth, it may be ingested, absorbed into the bloodstream, or remain in the lungs. Various types of personal protective equipment (PPE) such as dust (particulate) masks and respirators prevent hazardous chemicals from entering the body through inhalation. Ingestion of cryogens can also be prevented by observing basic housekeeping rules, such

Table 13.2 Chemical Routes of Entry into the Body

Chemical form	Principal danger
Solids and fumes	Inhalation, ingestion, and skin absorption
Dusts and gases	Inhalation into lungs
Liquids, vapors, and mists	Inhalation of vapors and skin absorption

as maintaining separate areas for eating and chemical use or storage, washing hands before handling food products, and removing gloves when handling food products. Wearing gloves and protective clothing prevent hazardous chemicals from entering the body through skin absorption.

The route of entry of a chemical is often determined by the physical form of the chemical. After a chemical has entered the body, the body may decompose it, excrete it, or the chemical may remain deposited in the body. Most cryogens are liquids. Their chemical forms and the routes of entry are summarized in Table 13.2. ■

Physical Hazards

There are multiple possible hazards associated with high pressure gases in cryogenic situations because their stored energy may be considerable. During gas compression in liquefaction and refrigeration, liquids are pumped to high pressure and then evaporated. These high pressure liquids and gases are eventually stored. When these materials are stored, they could experience breaks or ruptures in the line that can cause significant force upon storage vessels and their environments, or spills that could have disastrous effects. If a spill of a cryogenic fluid occurs, the heat in a room will readily vaporize the fluid into a gas. The primary hazard that occurs when dealing with non-oxygen cryogenic fluids is asphyxiation. The rapidly expanding gas can fill the room or area and displace the oxygen that was in the room. With a lack of oxygen in the room, the environment is extremely dangerous for humans.

ILLUSTRATIVE EXAMPLE 13.5

Briefly describe the role that respirators can play in the health risk management of cryogens and refrigerants.(16)

***SOLUTION*:** Respirators provide protection against inhaling harmful materials. Different types of respirators may be used depending in the level of protection desired. For example, supplied-air respirators (e.g., a self-contained breathing apparatus) may be required in situations where the presence of highly toxic substances is known or suspected and/or in confined spaces where it is likely that toxic vapors may accumulate. On the other hand, a full-face or half-face air-purifying respirator may be used in situations where measured air concentrations of identified substances will be reduced (by the respirator) below the substance's threshold limit value (TLV) and the concentration is within the service limit of the respirator.(13)

Other sources of hazard include the use of portable high pressure storage cylinders, which if knocked over or not handled correctly, can result in serious accidents. Manufacturers recommend inspection and testing of high pressure storage cylinders every five years to prevent the previously stated dangerous consequences. All storage cylinders should also be adequately secured (e.g., with chains, etc.) even if they are empty. Simply put, it is necessary to give considerable thought and energy to the design and implementation of all storage containers for refrigerants and cryogens. Designers must also attempt to foresee all possible consequences(13) from leaks and blockages and take the appropriate steps to avoid them. ■

ILLUSTRATIVE EXAMPLE 13.6

List the types of process events that can result in a plant accident and discuss the various kinds of equipment failure that can occur in a process plant where cryogenic substances are implemented.

***SOLUTION*:** The types of process events that can result in a plant accident are:

Abnormal temperatures
Abnormal pressures
Material flow stoppage

Equipment failures that can occur in a process plant implementing cryogens may be described within the major equipment categories of heat exchangers, vessels, pipes and valves, pumps and compressors. The failures associated with these categories are discussed below:

Heat Exchangers

(a) *Fouling*: Heat transfer is reduced when deposited materials accumulate in the heat exchanger and foul the heat exchanger.
(b) *Tube rupture*: Tube rupture may be caused by fouling, which leads to high fluid velocities, tube vibration, corrosion, or erosion.
(c) *Leakage*: Leakage is usually caused by corrosion of baffles in the heat exchanger.

Vessels

(a) *Pressure/temperature excursions*: High pressure and temperature in a vessel may result in tank rupture, especially when the condition repeats itself.
(b) *Stability*: Vessels, tanks, and phase separators(7) are designed according to specifications. Factors such as fabrication, construction, and corrosion must be considered when selecting a vessel for a particular service.

Pipes and Valves

(a) Insufficient or ineffective supports
(b) Poor weld quality
(c) Temperature stress
(d) Overpressure
(e) Dead ends
(f) Material compatibility

Pumps

(a) Gland failure

(b) *Dead-heading*: Dead-heading in a pump is a condition where the pump is allowed to run against a closed valve. When this occurs, the temperature rise may lead to seal damage and process fluid leakage compressors.

Compressors

See pumps above. ■

Chemical Hazards

Hazards associated with the chemical properties of cryogenic fluids can give rise to fires or explosions. In order for a fire or explosion to occur, there must be a fuel and/or an oxidant, and an ignition source. Because oxygen and air are prime candidates for cryogenic fluids, and are present in high concentrations, the chances of disasters occurring dramatically increases, as oxygen will obviously act as the oxidizer. A source of fuel can range from a noncompatible material to a flammable gas, or even a compatible material under extreme heat. An ignition source could be any electrical or mechanical spark or flame, any undesired thermodynamic event, or even a chemical reaction.(14–16)

ILLUSTRATIVE EXAMPLE 13.7

What is the difference between a hazardous chemical and a toxic chemical?(13)

***SOLUTION*:** The term hazardous chemicals encompasses a broad category of terms that includes chemicals that may be (according to the EPA) toxic, flammable, corrosive, explosive, or harmful to the environment. A toxic chemical is one type of a hazardous chemical. Toxic chemicals cause adverse health effects, including severe illness or death when ingested, inhaled, or absorbed by a living organism. ■

BASIC PRINCIPLES AND APPLICATIONS

As noted earlier, refrigeration systems are cyclic and operate using the following main components: a compressor, condenser, expander, and evaporator (see Figure 13.2). If the object to be refrigerated is a reservoir of heat at some low temperature and the object or reservoir where the heat is rejected is at a higher temperature, the continuous refrigeration cycle depicted in Figure 13.2 is a simplistic representation of the process.

ILLUSTRATIVE EXAMPLE 13.8

Discuss the details of a basic refrigeration cycle as seen in Figure 13.2.(17)

***SOLUTION*:** The cycle begins when a refrigerant enters the compressor as a low pressure gas (1). Once compressed, it leaves as a hot high pressure gas. Upon entering the condenser (2), the

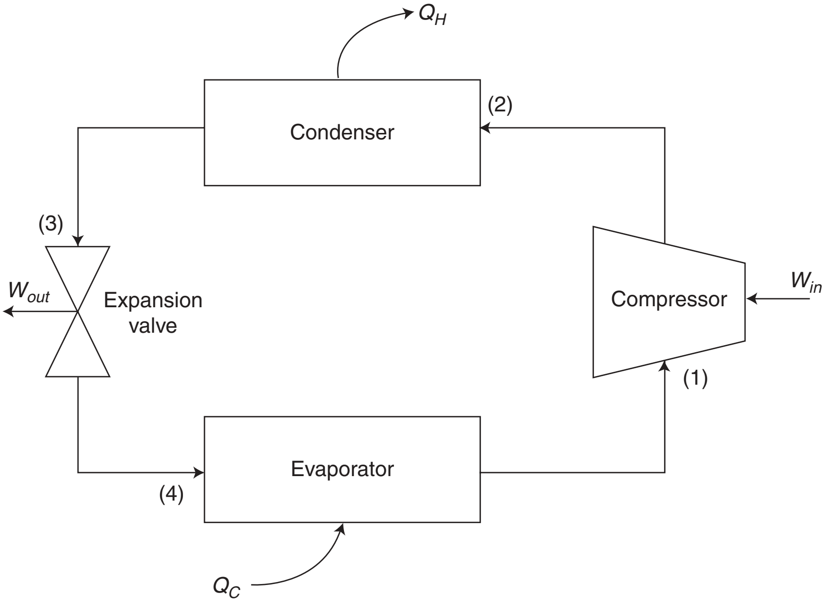

Figure 13.2 Basic Components of a Refrigeration System.

gas condenses to a liquid and releases heat to the outside environment, which may be air or water. The cool liquid then enters the expansion valve at a high pressure (3); the flow is restricted and the pressure is lowered. In the evaporator (4), heat from the source to be cooled is absorbed and the liquid becomes a gas. The refrigerant then repeats the process (i.e., the cycle continues).

In a refrigerator, the working fluid enters the evaporator in a wet condition and leaves dry and saturated (or slightly superheated). The heat absorbed, Q_C, by the evaporator can therefore be estimated by multiplying the change in the fluid's entropy, Δs, as it passes through the evaporator by the fluid's saturation temperature, T_S, at the evaporator pressure since the fluid's temperature will be constant while it is in a wet condition at constant pressure. Thus,

$$Q_C = T_S \Delta s \tag{13.2}$$

where Q_C = heat absorbed by the evaporator (e.g., kJ/kg)
T_S = fluid saturation temperature at evaporator temperature (e.g., K)
Δs = fluid entropy change (e.g., kJ/kg·K)

■

ILLUSTRATIVE EXAMPLE 13.9

A refrigerator's evaporator is at 0.2 MPa and has a working fluid that enters the unit as liquid with an enthalpy, h_{in}, of 230 kJ/kg. If the refrigerator's working fluid (see Table 13.3) exits the evaporator as dry and saturated vapor, calculate the heat absorbed by the evaporator, Q_C.

Table 13.3 Fluid Data for Illustrative Example 13.9

P, MPa	T_{sat}, °C	h_f, kJ/kg	h_g, kJ/kg	h at 20°C superheated, kJ/kg
0.2	−10	190	390	410
0.6	20	230	410	430

SOLUTION**:** Determine the enthalpy of the fluid that exits from the evaporator, h_{out}. From the problem statement and data,

$$\begin{aligned} h_{\text{out}} &= h_g \text{ at } 0.2\text{ MPa} \\ &= 390\text{ kJ/kg} \end{aligned}$$

Calculate the heat absorbed by the evaporator, Q_C, using the change in enthalpy across the evaporator:

$$\begin{aligned} Q_C &= h_{\text{out}} - h_{\text{in}};\ h_{\text{in}} = h_l \\ &= 390 - 230 \\ &= 160\,\text{kJ/kg} \end{aligned}$$ ■

ILLUSTRATIVE EXAMPLE 13.10

As a follow-up to Illustrative Example 13.9, what is the fluid's change in entropy across the evaporator?

SOLUTION**:** Determine the fluid's saturation temperature expressed in Kelvin (K). From the problem statement and data,

$$T_S = -10^\circ\text{C} = 263\,\text{K}$$

Calculate the fluid's change in entropy from Equation (13.2) and using the solution in the previous example.

$$\begin{aligned} \Delta s &= \frac{Q_C}{T_S} = \frac{160}{263} \\ &= 0.61\,\text{kJ/kg}\cdot\text{K} \end{aligned}$$ ■

Coefficient of Performance

The performance and energy saving ability of a refrigerator is measured in terms of the system's Coefficient of Performance (C.O.P.). This is defined as the heat removed at a low temperature, i.e., the cooling effect, Q_C, divided by the work input, W_{in}, into the system:

$$\text{C.O.P.} = \frac{Q_C}{W_{\text{in}}} \tag{13.3}$$

where C.O.P. = coefficient of performance, dimensionless

Q_C = cooling effect (e.g., kJ/kg)

W_{in} = work input (e.g., kJ/kg)

The traditional system of units used in refrigeration are English units (i.e., Btu, etc.) but SI units are also acceptable and are used in the following two illustrative examples. The cooling effect, Q_C, is equal to the change in enthalpy of the working fluid as it passes through the evaporator, and the work input (W_{in}) is equal to the increase in the working fluid's enthalpy as it passes through the compressor.

ILLUSTRATIVE EXAMPLE 13.11

The working fluid in a refrigerator enters a compressor at dry-saturated conditions at a pressure of 0.2 MPa and exits the compressor 20°C superheated at a pressure of 0.6 MPa. Given the fluid data from Illustrative Example 13.9, and given that the fluid leaves the condenser wet-saturated, what is the heat absorbed by the evaporator of the refrigerator?

***SOLUTION*:** Employ the subscripts associated with Figure 13.2. Determine the fluid enthalpy on entering the compressor, h_1. From the problem statement and data,

$$h_1 = 390 \text{ kJ/kg}$$

Determine the fluid enthalpy on leaving the compressor, h_2. From the problem statement and data,

$$h_2 = 430 \text{ kJ/kg}$$

Finally, determine the fluid enthalpy on leaving the condenser, h_3. From the problem statement and data, and noting that the enthalpy change across the expansion value is approximately zero,(2)

$$h_4 = h_3 = 230 \text{ kJ/kg}$$

Calculate the heat rejected from the condenser, Q_H, using the change in enthalpy across the condenser:

$$\begin{aligned} Q_H &= h_2 - h_3 \\ &= 430 - 230 \\ &= 200 \text{ kJ/kg} \end{aligned}$$

Calculate the work input, W_{in}, using the change in enthalpy across the compressor:

$$\begin{aligned} W_{\text{in}} &= h_2 - h_1 \\ &= 430 - 390 \\ &= 40 \text{ kJ/kg} \end{aligned}$$

Calculate the heat absorbed by the evaporator, Q_C, using the first law of thermodynamics:(2)

$$\begin{aligned} Q_C &= Q_H - W_{\text{in}} \\ &= 200 - 40 \\ &= 160 \text{ kJ/kg} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 13.12

From Illustrative Example 13.11, what is the refrigerator's C.O.P.?

***SOLUTION*:** Determine the C.O.P using Equation (13.3) and the values of Q_C and W_{in} from the previous example:

$$\begin{aligned} \text{C.O.P.} &= \frac{Q_C}{W_{\text{in}}} \\ &= \frac{160}{40} \\ &= 4.0 \end{aligned}$$

Comment: The C.O.P. for a refrigerator is defined in terms of the cooling load, Q_C; however, the C.O.P. for a heat pump is defined in terms of the heating load, Q_H. ■

Thermal Efficiency

The performance of a steam power plant process can be measured in a manner somewhat analogous to the C.O.P. for a refrigeration system. The thermal efficiency, η_{th}, of a work-producing cycle is defined as the ratio of work produced to heat added. Thus,

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{in}}} \tag{13.4}$$

where η_{th} = thermal efficiency, dimensionless
W_{net} = net work produced by the cycle (e.g., J/kg)
Q_{in} = heat added to the cycle (e.g., J/kg)

This can be rewritten as:

$$\eta_{\text{th}} = \frac{W_{\text{out}} - W_{\text{in}}}{Q_{\text{in}}} \tag{13.5}$$

where W_{out} = work produced by the cycle (e.g., J/kg)
W_{in} = work consumed by the cycle (e.g., J/kg)

For this type of cycle, the compressor, evaporator, and expansion value in Figure 13.2 are replaced by a turbine, boiler, and pump, respectively, with both Q_C and Q_H as well as W_{in} and W_{out} reversed. See Figure 13.3. When no velocity information is provided, velocity effects can be neglected and this equation can be expressed in terms of enthalpies at points on entry and exit to the boiler, turbine, and pump, which for a simple power cycle is:

$$\eta_{\text{th}} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)} \tag{13.6}$$

where h_1 = enthalpy on entry to the boiler (e.g., J/kg)
h_2 = enthalpy on exit from the boiler, on entry to the turbine (e.g., J/kg)
h_3 = enthalpy on exit from the turbine (e.g., J/kg)
h_4 = enthalpy on entry to the pump (e.g., J/kg)

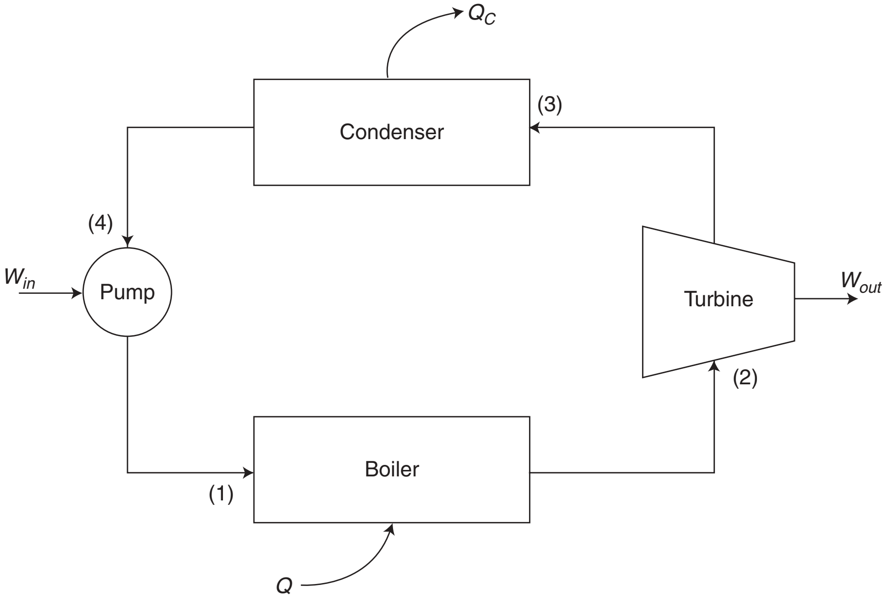

Figure 13.3 Basic Components of a Steam Power System.

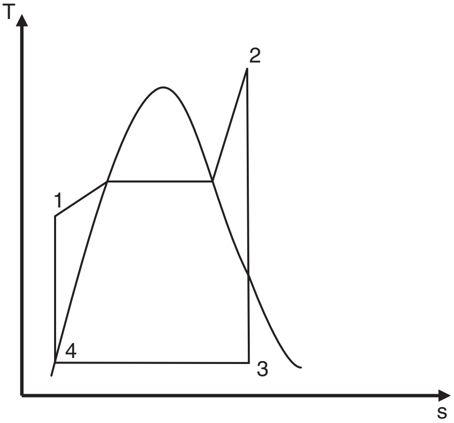

Figure 13.4 Rankine Cycle T-s Diagram.

Note that the change in enthalpy across the pump is often neglected since it is close to zero relative to the other enthalpy changes. In effect, $h_1 - h_4 \approx 0$.

ILLUSTRATIVE EXAMPLE 13.13

A Rankine cycle has a T-S diagram as shown in Figure 13.4, and rejects 2043 kJ/kg of heat during the constant pressure process between points 3 and 4. The data provided in Table 13.4 is also associated with this cycle. Calculate the enthalpy change across the boiler.

Properties associated with the condenser are as follows:

$P = 0.1235$ bar
$h_f = 209$ kJ/kg
$h_g = 2592$ kJ/kg
$s_f = 0.7038$ kJ/kg · K
$s_g = 8.0763$ kJ/kg · K

Table 13.4 T-S Data for Illustrative Example 13.13

Point	x	h, kJ/kg	s, kJ/kg·K
1		548	
2		3989	7.5190
3	0.9575	2491	7.7630
4			1.4410

Note: x = mass fraction vapor.

***SOLUTION*:** Determine the steam enthalpy at the entry and exit to the boiler (points 1 and 2), turbine (points 2 and 3), and pump (points 4 and 1) from the problem statement and data:

$$\begin{aligned} h_1 &= 548\ \text{kJ/kg} \\ h_2 &= 3989\ \text{kJ/kg} \\ h_3 &= 2491\ \text{kJ/kg} \end{aligned}$$

In addition,

$$h_4 = h_3 - Q_H$$

where Q_H = heat rejected by the condenser

Thus:

$$\begin{aligned} h_4 &= 2491 - 2043 \\ &= 448\ \text{kJ/kg} \end{aligned}$$

Calculate the heat added to the boiler (which is equal to the enthalpy change across the boiler):

$$\begin{aligned} Q_{\text{boiler}} &= h_2 - h_1 \\ &= 3989 - 548 \\ &= 3441\ \text{kJ/kg} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 13.14

What is the thermal efficiency of the cycle in Illustrative Example 13.13?

***SOLUTION*:** Calculate the work produced by the turbine by determining the enthalpy change across the turbine:

$$\begin{aligned} W_{\text{turbine}} &= h_2 - h_3 \\ &= 3989 - 2491 \\ &= 1498\ \text{kJ/kg} \end{aligned}$$

Calculate the work used by the pump, which is equal to the enthalpy change across the pump:

$$\begin{aligned} W_{\text{pump}} &= h_1 - h_4 \\ &= 548 - 448 \\ &= 100\ \text{kJ/kg} \end{aligned}$$

Calculate the net work by subtracting the pump work from the turbine work:

$$\begin{aligned} W_{\text{net}} &= W_{\text{turbine}} - W_{\text{pump}} \\ &= 1498 - 100 \\ &= 1398\ \text{kJ/kg} \end{aligned}$$

Calculate the thermal efficiency, η_{th}, from Equation (13.4):

$$\begin{aligned}\eta_{\text{th}} &= \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{W_{\text{net}}}{Q_{\text{boiler}}}\\ &= \frac{1398}{3441}\\ &= 0.406 = 40.6\%\end{aligned}$$

Comment: Unless otherwise specified, it is assumed that the turbine and pump operate adiabatically. ■

Entropy and Heat

The Clausius inequality states that, for a reversible process, the entropy change associated with the process is equal to the integral of the heat transferred divided by the temperature. Thus,

$$\Delta s = \int \frac{dQ_{\text{rev}}}{T} \tag{13.7}$$

where Δs = process change in entropy (e.g., J/kg)
dQ_{rev} = total heat transferred (e.g., J/kg)
T = temperature (e.g., K)

Consequently, for a constant temperature process,

$$\int dQ_{\text{rev}} = Q_{\text{rev}} = T\Delta s \tag{13.8}$$

ILLUSTRATIVE EXAMPLE 13.15

Using the Rankine cycle T-s diagram, in Figure 13.4, and the data from Illustrative Example 13.13, calculate the temperature at point 3.

***SOLUTION*:** Identify the known properties at point 3, which are x, h, and s from Table 13.4:

$$\begin{aligned}x_3 &= 0.9575\\ h_3 &= 2491\ \text{kJ/kg}\\ s_3 &= 7.7630\ \text{kJ/kg}\cdot\text{K}\end{aligned}$$

Identify a process that is associated with point 3, which is inside the vapor dome and therefore will be a constant pressure process. The heat rejection of

$$h_3 - h_4 = Q_{\text{out}} = 2043\ \text{kJ/kg} \approx Q_{\text{rev}}$$

is associated with a constant temperature, constant pressure condensation process.

Determine the relationship between the process and the point in question. Since the entropy and the heat transfer associated with the condenser are known, a relationship therefore exists that allows the temperature to be determined by rearranging Equation (13.8).

$$T_3 = \frac{Q_{\text{out}}}{\Delta s} = \frac{Q_{\text{out}}}{s_3 - s_4}$$

Substitute the known values.

$$\begin{aligned} T_3 &= \frac{2043}{7.7630 - 1.4410} \\ &= 323\,\text{K} = 50^\circ\text{C} \end{aligned}$$

Comment: The Clausius inequality is only applicable to *reversible* processes. Therefore, it does not apply to *irreversible* processes. ■

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Part Three

Heat Transfer Equipment Design Procedures and Applications

The purpose of this Part is primarily to apply the principles presented in the previous Part to the design, operation, and predictive calculations of several types of heat exchangers. Although the presentation will focus on double-pipe, shell and tube (tube and bundle), and finned heat exchangers, information on other classes of exchangers will be addressed. Peripheral topics such as insulation, entropy analysis, operation, maintenance and inspection, etc., also receives treatment.

As described earlier, heat exchangers are devices used to transfer heat from a hot fluid to a cold fluid. They can be classified by their function. An abbreviated summary of these functions is provided in the first chapter in this Part. The design of these exchangers is concerned with the quantities of heat to be transferred, the rates at which they are transferred, the temperature driving force, the extent and arrangement of the surface(s) separating the heat source and receiver, the area requirements, and the amount of mechanical energy (pressure drop) that is required to facilitate the heat transfer. Since heat transfer involves an *exchange* in a *system*, the loss of heat by one body (solid or fluid) will equal (for steady processes) the heat absorbed by another *within the confines of the same system*. These issues are addressed in this Part of book.

Chapter headings for this Part are listed below:

Chapter 14: Introduction to Heat Exchangers
Chapter 15: Double Pipe Heat Exchangers
Chapter 16: Shell and Tube Heat Exchangers
Chapter 17: Fins and Extended Surfaces
Chapter 18: Other Heat Exchange Equipment

Chapter 19: Insulation and Refractory
Chapter 20: Operation, Maintenance, and Inspection (OM&I)
Chapter 21: Entropy Considerations and Analysis
Chapter 22: Design Principles and Industrial Applications

Chapter 14

Introduction to Heat Exchangers

INTRODUCTION

Heat exchangers are defined as equipment that effect the transfer of thermal energy in the form of heat from one fluid to another. The simplest exchangers involve the direct mixing of hot and cold fluids. Most industrial exchangers are those in which the fluids are separated by a wall. The latter type, referred to by some as a *recuperator*, can range from a simple plane wall between two flowing fluids to more complex configurations involving multiple passes, fins, or baffles. Conductive and convective heat transfer principles are required to describe and design these units; radiation effects are generally neglected.

Heat exchangers for the chemical, petrochemical, petroleum, paper, and power industries encompass a wide variety of designs that are available from many manufacturers. Equipment design practice first requires the selection of safe operable equipment. The selection and design process must also seek a cost-effective balance between initial (capital) installation costs, operating costs, and maintenance costs.

The proper application of heat exchange principles can significantly minimize both the initial cost of a plant and the daily operating and/or utility costs. Each heat exchange application may be accomplished by the use of many types of heat exchange equipment. To perform these applications, their design and materials of construction must be suitable for the desired operating conditions; the selection of materials of construction is primarily influenced by the operating temperature and the corrosive nature of the fluids being handled.

A more detailed description of heat exchanger equipment is warranted before proceeding to the development of the heat exchanger equation. The presentation in this chapter keys not only on the various heat exchanger equipment (and their classification) but also on the log mean temperature difference driving force, ΔT_{lm}, and the overall heat transfer coefficient, U. The development of both ΔT_{lm}

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

and U ultimately leads to the classic heat exchanger equation. Topics addressed in this chapter include:

Energy relationships

Heat exchange equipment classification

The log mean temperature difference (LMTD) driving force

Overall heat transfer coefficients

The heat transfer equation

ENERGY RELATIONSHIPS

The flow of heat from a hot fluid to a cooler fluid through a solid wall is a situation often encountered in engineering equipment; examples of such equipment are the aforementioned heat exchangers, condensers, evaporators, boilers, and economizers. The heat absorbed by the cool fluid or given up by the hot fluid may be sensible heat, causing a temperature change in the fluid, or it may be latent heat, causing a phase change such as vaporization or condensation. In a typical waste heat boiler, for example, the hot flue gas gives up heat to water through thin metal tube walls separating the two fluids. As the flue gas loses heat, its temperature drops. As the water gains heat, its temperature quickly reaches the boiling point where it continues to absorb heat with no further temperature rise as it changes into steam. The rate of heat transfer between the two streams, assuming no heat loss due to the surroundings, may be calculated by the enthalpy change of either fluid:

$$\dot{Q} = \dot{m}_h(h_{h1} - h_{h2}) = \dot{m}_c(h_{c1} - h_{c2}) \tag{14.1}$$

where $\dot{Q}$ is the rate of heat flow (Btu/h), $\dot{m}_h$ is the mass flow rate of hot fluid (lb/h), $\dot{m}_c$ is the mass flow rate of cold fluid (lb/h), h_{h1} is the enthalpy of entering hot fluid (Btu/lb), h_{h2} is the enthalpy of exiting hot fluid (Btu/lb), h_{c1} is the enthalpy of entering cold fluid (Btu/lb), and h_{c2} is the enthalpy of exiting cold fluid (Btu/lb).

Equation (14.1) is applicable to the heat exchange between two fluids whether a phase change is involved or not. In the above waste heat boiler example, the enthalpy change of the flue gas is calculated from its sensible temperature change:

$$\dot{Q} = \dot{m}_h(h_{h1} - h_{h2}) = \dot{m}_c c_{ph}(T_{h1} - T_{h2}) \tag{14.2}$$

where c_{ph} is the heat capacity of the hot fluid (Btu/lb · °F), T_{h1} is the temperature of the entering hot fluid (°F), and T_{h2} is the temperature of the exiting hot fluid (°F). The enthalpy change of the water, on the other hand, involves a small amount of sensible heat to bring the water to its boiling point plus a considerable amount (usually) of latent heat to vaporize the water. Assuming all of the water is vaporized and no superheating of the steam occurs, the enthalpy change is

$$\dot{Q} = \dot{m}_c(h_{c2} - h_{c1}) = \dot{m}_c c_{pc}(t_{c2} - t_{c1}) + \dot{m}_c \Delta h_{\text{vap}} \tag{14.3}$$

where c_{pc} is the heat capacity of the cold fluid (Btu/lb · °F), t_{c1} is the temperature of the entering cold fluid (°F), t_{c2} is the temperature of the exiting cold fluid (°F), and Δh_{vap} is the heat of vaporization of the cold fluid (Btu/lb). Note that, wherever possible, lower case t and upper case T will be employed to represent the cooler fluid temperature and hotter fluid temperature, respectively; in addition, lowercase c/uppercase C and lowercase h/uppercase H will be employed to represent the cold and hot fluid, respectively.

ILLUSTRATIVE EXAMPLE 14.1

If 20,000 scfm (32°F, 1 atm) of an air stream is heated from 200°F to 2000°F, calculate the heat transfer rate required to bring about this change in temperature. Use the following enthalpy and average heat capacity data:

$$H_{200^\circ\text{F}} = 1170\ \text{Btu/lbmol}$$

$$H_{2000^\circ\text{F}} = 14{,}970\ \text{Btu/lbmol}$$

$$\overline{C}_{p,\text{av}} = 7.53\ \text{Btu/lbmol}\cdot{}^\circ\text{F (over the 200–2000}^\circ\text{F range)}$$

***SOLUTION*:** Convert the flow rate of air to a molar flow rate, $\dot{n}$:

$$\dot{n} = \frac{\text{scfm}}{359} = \frac{20{,}000}{359} = 55.7\ \text{lbmol/min}$$

Note that at standard conditions of 32°F and 1 atm, 1.0 lbmol of an ideal gas occupies 359 ft^3 or 22.4 (see also Chapter 3).

Calculate the heat transfer rate: $\dot{Q}$, using enthalpy data:

$$\dot{Q} = \dot{n}\Delta H = 55.7(14{,}970 - 1170) = 7.69 \times 10^5\ \text{Btu/min}$$

Also, calculate the heat transfer rate using the average heat capacity data:

$$\dot{Q} = \dot{n}\overline{C}_{p,\text{av}}\Delta T = 55.7(7.53)(2000 - 200) = 7.55 \times 10^5\ \text{Btu/min}$$

Both approaches provide near identical results. ■

ILLUSTRATIVE EXAMPLE 14.2

Given the mass flow rate of a gas stream and its heat capacity, determine the required heat rate to change the gas stream from one temperature to another. Data are provided below:

$$\dot{n} = 1200\ \text{lbmol/min}$$

$$\overline{C}_{P,\text{av}} = 0.26\ \text{Btu/lbmol}\cdot{}^\circ\text{F (over the 200–2000}^\circ\text{F range)}$$

$$T_1 = 200^\circ\text{F}$$

$$T_2 = 1200^\circ\text{F}$$

***SOLUTION*:** Write the equation describing the required heat rate, $\dot{Q}$, and solve:

$$\dot{Q} = \dot{n}\overline{C}_{P,\text{av}}\Delta T = (1200)(0.26)(1200 - 200) = 3.12 \times 10^5\ \text{Btu/min}$$ ■

ILLUSTRATIVE EXAMPLE 14.3

Obtain the heat transfer rate in an exchanger if equal similar liquid flow rates are used. The hot fluid is cooled from 84°C to 72°C while the cold fluid is initially at 25°C.

***SOLUTION*:** The final temperature of the cold liquid can be found from direct application of Equation (14.2):

$$\dot{m}_c c_{pc}(t_{c,2} - t_{c,1}) = \dot{m}_h c_{ph}(T_{h,2} - T_{h,1})$$
$$\dot{m}_c c_{pc}(t_{c,2} - 25) = \dot{m}_h c_{ph}(84 - 72)$$

Since

$$\dot{m}_c = \dot{m}_h$$
$$c_{ph} = c_{pc}$$

one can conclude that the temperature *change* for both liquids is equal. This yields

$$t_{c,2} = 37^\circ\mathrm{F}$$

■

HEAT EXCHANGE EQUIPMENT CLASSIFICATION

There is a near infinite variety of heat exchange equipment. These can vary from a simple electric heater in the home to a giant boiler in a utility power plant. A limited number of heat transfer devices likely to be encountered by the practicing engineer have been selected for description in this Part. Most of the units transfer heat from one fluid to another fluid, with the heat passing through a solid interface such as a tube wall that separates the two (or more) fluids. The size, shape, and material employed to separate the two fluids is of course important. Another problem in a heat exchanger is the method of confining one or both of the two fluids involved in the heat transfer process.

As noted above, heat exchangers are devices used to transfer heat from a hot fluid to a cold fluid. They are classified by their functions, as shown in the Table 14.1. In a general sense, heat exchangers are classified into three broad types:

1. Recuperators or through-the-wall non-storing exchangers (e.g., double-pipe heat exchangers, shell-and-tube heat exchangers)
2. Direct contact non-storing exchangers
3. Regenerators, accumulators, or heat storage exchangers

Through-the-wall non-storing exchangers can be further classified into:

1. Double pipe heat exchangers
2. Shell-and-tube heat exchangers
3. Cross-flow exchangers

Exchangers (1) and (2) receive treatment in the next two chapters.

Table 14.1 Heat Exchanger Equipment

Equipment	Function
Chiller	Cools a fluid to a temperature below that obtainable if only water were used as a coolant. It often uses a refrigerant such as ammonia or freon.
Condenser	Condenses a vapor or mixture of vapors, either alone or in the presence of a non-condensable gas.
Cooler	Cools liquids or gases by means of water.
Exchanger	Performs a double function: (1) heats a cold fluid and (2) cools a hot fluid. Little or none of the transferred heat is normally lost.
Final condenser	Condenses the vapors to a final storage temperature of approximately 100°F. It uses water cooling, which means the transferred heat is often lost to the process.
Forced-circulation reboiler	A pump is used to force liquid through the reboiler (see reboiler below).
Heater	Imparts sensible heat to a liquid or a gas by means of condensing steam or some other hot fluid (e.g., Dowtherm).
Partial condenser	Condenses vapors at a temperature high enough to provide a temperature difference sufficient to preheat a cold stream of process fluid. This saves heat and eliminates the need for providing a separate preheater.
Reboiler	Connected to the bottom of a fractionating tower, it provides the reboil heat necessary for distillation. The heating medium may be either steam (usually) or a hot process fluid.
Steam generator	Generates steam for use elsewhere in the plant by using available high-level heat, e.g., from tar or a heavy oil.
Superheater	Heats a vapor above its saturation temperature.
Thermosiphon reboiler	Natural circulation of the boiling medium is obtained by maintaining sufficient liquid head to provide for circulation (see reboiler).
Vaporizer	A heater which vaporizes all or part of the liquid.
Waste-heat boiler	Produces steam; similar to a steam generator, except that the heating medium is a hot gas or liquid produced in a chemical reaction.

The flow in the double pipe heat exchanger(1) may be countercurrent or parallel (co-current). In countercurrent flow, the fluid in the pipe flows in a direction opposite to the fluid in the annulus. In parallel/co-current flow, the two fluids flow in the same direction. The variations of fluid temperature within the heat exchanger depend on whether the flow is parallel (co-current) or countercurrent. The definitions below are employed in the development to follow:

t_1 = temperature of fluid entering the inside tube

t_2 = temperature of fluid exiting the inside tube

T_1 = temperature of fluid entering the annulus (space between the two tubes)

T_2 = temperature of fluid exiting the annulus.

The difference between the temperature of the tube side fluid and that of the annulus side is the temperature difference driving force, ΔT.

In a parallel heat exchanger, both hot and cold fluids enter on the same side and flow through the exchanger in the same direction. The *temperature approach* is defined as the temperature difference driving force at the heat exchanger entrance, ΔT_1 or $(T_1 - t_1)$. This driving force drops as the streams approach the exit of the exchanger. At the exit, the temperature difference driving force is ΔT_2 or $(T_2 - t_2)$. The heat exchanger is more effective at the entrance than at the exit because of the higher driving force.

In a countercurrent flow exchanger, the two fluids exchanging heat flow in opposite directions. The temperature approach at the tube entrance end, ΔT_1 or $(T_1 - t_2)$, and at the annulus entrance end, ΔT_2 or $(T_2 - t_1)$ are usually roughly the same. The thermal driving force is normally relatively constant over the length of the exchanger.

THE LOG MEAN TEMPERATURE DIFFERENCE (LMTD) DRIVING FORCE

When heat is exchanged between a surface and a fluid, or between two fluids flowing through a heat exchanger, the local temperature driving force, ΔT, usually varies along the flow path. This effect is treated through the log mean temperature difference approach, as discussed below.

The concept of a log-mean temperature is first developed before proceding to the log-mean temperature difference driving force. Consider an absolute temperature profile $T(x)$ that is continuous and where T_1 is the absolute temperature at some point x_1 with T_2 at x_2. By definition, the mean value of the reciprocal temperatures $(1/T)$ between x_1 and x_2 is given by

$$\begin{aligned}\overline{\left(\frac{1}{T}\right)} &= \int_{T_1}^{T_2} \frac{1}{T}\,dT \Big/ \int_{T_1}^{T_2} dT = \left(\frac{1}{T_2 - T_1}\right) \int_{T_1}^{T_2} \frac{1}{T}\,dT \\ &= \frac{\ln(T_2/T_1)}{T_2 - T_1}\end{aligned} \tag{14.4}$$

If the average of the reciprocal is a reasonable approximation to the reciprocal of the average, i.e.,

$$\overline{\left(\frac{1}{T}\right)} \approx \frac{1}{\overline{\overline{T}}} \tag{14.5}$$

then one can combine the above two equations to give

$$\frac{1}{\overline{\overline{T}}} \approx \frac{\ln(T_2/T_1)}{T_2 - T_1}$$

or

$$\overline{T} \approx \frac{T_2 - T_1}{\ln(T_2/T_1)} \tag{14.6}$$

The right-hand side of Equation (14.6) is *defined* to be the log-mean temperature between T_1 and T_2, that is,

$$T_{\text{lm}} = \frac{T_2 - T_1}{\ln(T_2/T_1)} \tag{14.7}$$

Note once again that absolute temperatures must be employed.

Assuming steady-state operation and constant properties independent of the temperature, an overall energy balance is first applied to a fluid in a conduit (e.g., a pipe with inlet and exit temperatures t_1 and t_2, respectively), and heated by a source at temperature T_s (see also Figure 14.1):

$$\dot{Q} = \dot{m}c_p(t_2 - t_1) = \dot{m}c_p[(T_s - t_1) - (T_s - t_2)] = \dot{m}c_p(\Delta T_1 - \Delta T_2) \tag{14.8}$$

where ΔT_1, or $(T_s - t_1)$, is the temperature difference driving force (also termed the *approach*) at the fluid entrance, and ΔT_2, or $(T_s - t_2)$, is the temperature driving force, or approach, at the fluid exit. The differential balance is

$$d\dot{Q} = \dot{m}c_p\,dt = \dot{m}c_p d(\Delta T) \tag{14.9}$$

One may also apply an energy balance to a differential fluid element of cross-sectional area, A, and a thickness of dx. Set $x = 0$ at the pipe entrance and $x = L$ at the pipe exit. The pipe diameter is D, so

$$A = \frac{\pi D^2}{4} \tag{14.10}$$

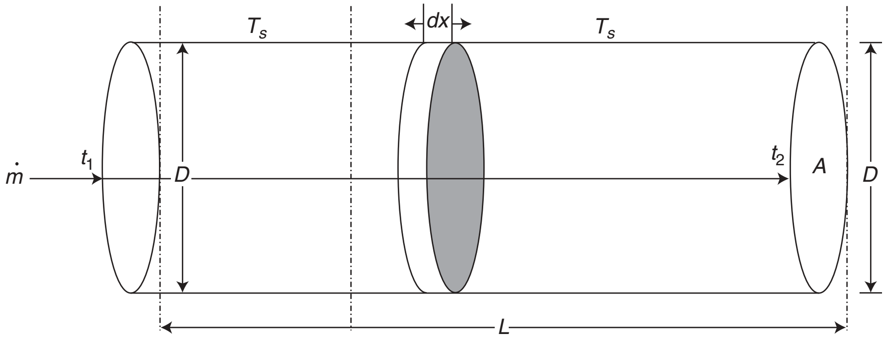

Figure 14.1 Energy balance of a differential element of a pipe.

The outer surface differential area of the element is $dA = \pi D(dx)$. One may also apply the energy balance across the walls of the unit

$$d\dot{Q} = h(T_s - t)(\pi D)(dx) = \dot{m}c_p\ dt \tag{14.11}$$

where h is the heat transfer coefficient of the fluid defined earlier in Chapter 9. Separation of variables in Equation (14.11) yields

$$\frac{dt}{T_s - t} = \frac{h}{\dot{m}c_p}(\pi D)(dx) \tag{14.12}$$

Integrating from $x = 0$, where $t = t_1$, to $x = L$, where $t = t_2$, yields

$$\int_{t_1}^{t_2} \frac{dt}{T_s - t} = \frac{h}{\dot{m}c_p}(\pi D)\int_0^L (dx) \tag{14.13}$$

$$\ln\left(\frac{T_s - t_1}{T_s - t_2}\right) = \frac{h}{\dot{m}c_p}(\pi D)L \tag{14.14}$$

Letting $\Delta T_1 = T_s - t_1$ and $\Delta T_2 = T_s - t_2$ leads to

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \frac{h}{\dot{m}c_p}(\pi D)L \tag{14.15}$$

Thus,

$$\dot{m}c_p = \frac{h(\pi D)L}{\ln(\Delta T_1/\Delta T_2)} \tag{14.16}$$

Combining Equations (14.4) and (14.16) gives

$$\dot{Q} = h(\pi D)L\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = hA\Delta T_{\text{lm}} \tag{14.17}$$

where ΔT_{lm} = log mean temperature difference, or log mean temperature approach, and is defined as

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{14.18}$$

For the special case of $\Delta T_1 = \Delta T_2$,

$$\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2 \tag{14.19}$$

Heat transfer calculations using ΔT_{lm} are convenient when terminal temperatures are known. If the temperature of the fluid leaving the tube or exchanger is not known, the procedure may require a trial-and-error calculation.

ILLUSTRATIVE EXAMPLE 14.4

Steam at 1 atm outside a pipe is heating a fluid inside a pipe whose temperature may be assumed uniform at 25°C. Calculate the LMTD.

***SOLUTION*:** At 1 atm, the steam temperature is 100°C. For this case, Equation (14.19) applies and

$$\Delta T_{\mathrm{lm}} = 100 - 25 = 75^{\circ}\mathrm{C}$$

■

ILLUSTRATIVE EXAMPLE 14.5

Repeat the calculation in the previous example if the fluid is heated from 25°C and 80°C.

***SOLUTION*:** For this case, Equation (14.18) applies with

$$\Delta T_1 = 100 - 25 = 75^{\circ}\mathrm{C}$$
$$\Delta T_2 = 100 - 80 = 20^{\circ}\mathrm{C}$$

Therefore,

$$\Delta T_{\mathrm{lm}} = \frac{75 - 20}{\ln(75/20)} = 43.9^{\circ}\mathrm{C}$$

■

Temperature Profiles

Consider a heated surface, as represented in Figure 14.2, where temperatures are plotted against distance along the surface. The temperature drop at the left-hand side of the figure is much greater than that at the right-hand side. Heat is being transferred more rapidly at the left-hand end since the hot fluid and the cold fluid enter at the same end of the unit and flow parallel to each other. This arrangement is known as *parallel flow* or *co-current* flow.

The alternative method is to feed the hot fluid at one end of the unit and the cold fluid at the other, allowing the fluids to pass by each other in opposite directions. Such an arrangement is called *counterflow* or *counter-current flow*. The temperature gradients for this case are provided in Figure 14.3. In this case of counter-current operation, the temperature drop along the length of the unit is relatively more constant than in parallel flow.

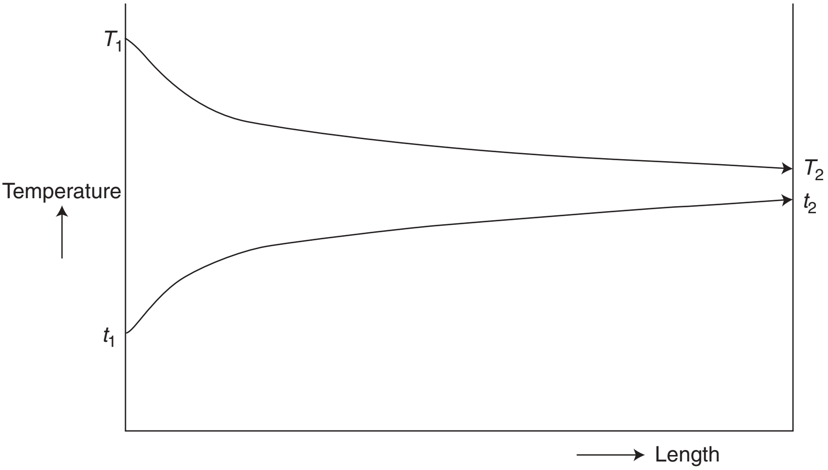

Figure 14.2 Heat transfer with parallel flow.

ILLUSTRATIVE EXAMPLE 14.6

A liquid-to-liquid *counterflow* heat exchanger is used to heat a cold fluid from 120°F to 310°F. Assuming that the hot fluid enters at 500°F and leaves at 400°F, calculate the log mean temperature difference (driving force) for the heat exchanger.

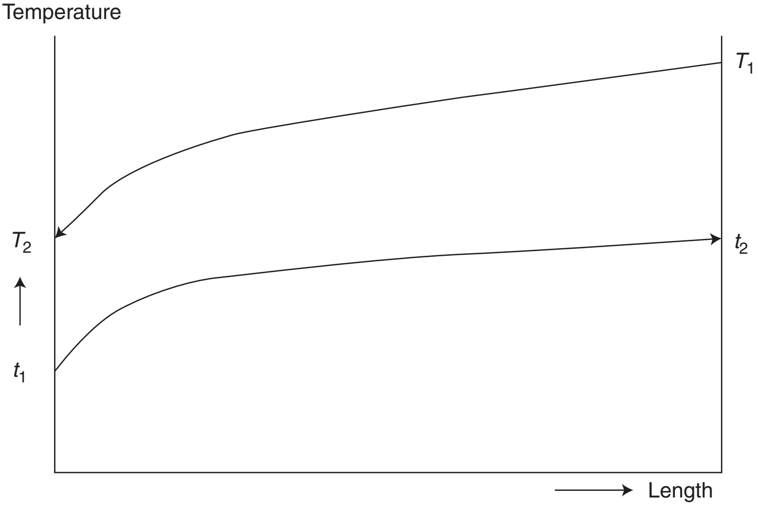

Figure 14.3 Heat transfer with counterflow.

***SOLUTION*:** For this application

$$\Delta T_1 = T_1 - t_2 = 500 - 310 = 190^\circ\mathrm{F}$$
$$\Delta T_2 = T_2 - t_1 = 400 - 120 = 280^\circ\mathrm{F}$$

Therefore,

$$\Delta T_{\mathrm{lm}} = \frac{190 - 280}{\ln(190/280)} = 232^\circ\mathrm{F}$$

■

ILLUSTRATIVE EXAMPLE 14.7

A heavy hydrocarbon oil with heat capacity, $c_p = 0.55\ \mathrm{Btu/lb \cdot {}^\circ F}$, is being cooled from $T_1 = 210^\circ\mathrm{F}$ to $T_2 = 170^\circ\mathrm{F}$. The oil flows inside a tube at a rate of 8000 lb/h and the tube surface temperature is maintained at 60°F. Calculate the heat transfer rate and the LMTD in °F.

***SOLUTION*:** Calculate the heat transferred from the heavy oil:

$$\dot{Q} = \dot{m}c_p\Delta T = (8000)(0.55)(170 - 210) = -176{,}000\ \mathrm{Btu/h}$$

The negative sign is a reminder that heat is lost by the oil. The thermal driving forces, or the temperature approaches, at the pipe entrance and exit are

$$\Delta T_1 = T_1 - t = 210 - 60 = 150^\circ\mathrm{F}$$
$$\Delta T_2 = T_2 - t = 170 - 60 = 110^\circ\mathrm{F}$$

The LMTD is therefore

$$\Delta T_{\mathrm{lm}} = \frac{150 - 110}{\ln(150/110)} = 129^\circ\mathrm{F}$$

Note once again that the use of ΔT_{lm} is valid for heating or cooling. ■

ILLUSTRATIVE EXAMPLE 14.8

An oil cooler is to operate with an inlet of 138°F and outlet of 103°F, and the coolant entering at 88°F is to be allowed to rise to 98°F. What is the LMTD for this unit, if it is considered as

1. counter-current flow unit, and
2. a parallel flow unit.

***SOLUTION*:**

For the counter-current flow case,

$$\Delta T_1 = T_1 - t_2 = 138 - 98 = 40^\circ\mathrm{F}$$
$$\Delta T_2 = T_2 - t_1 = 103 - 88 = 15^\circ\mathrm{F}$$

Therefore,

$$\Delta T_{\text{lm}} = \frac{40 - 15}{\ln(40/15)} = 25.5^{\circ}\text{F}$$

For the parallel flow case,

$$\Delta T_1 = T_1 - t_1 = 138 - 88 = 50^{\circ}\text{F}$$
$$\Delta T_2 = T_2 - t_2 = 103 - 98 = 5^{\circ}\text{F}$$

Therefore,

$$\Delta T_{\text{lm}} = \frac{50 - 5}{\ln(50/5)} = 19.5^{\circ}\text{F}$$

■

ILLUSTRATIVE EXAMPLE 14.9

Comment on the results of the previous example.

***SOLUTION*:** Note that the counter-current flow unit clearly provides the greater driving force for heat transfer. ■

OVERALL HEAT TRANSFER COEFFICIENTS

In order to design heat transfer equipment and calculate the required energy, one must know more than just the heat transfer rate calculated by the enthalpy (energy) balances described previously. The rate at which heat can travel from the hot fluid at T_H, through the tube walls, into the cold fluid at t_C, must also be considered in the calculation of certain design variables (e.g., the contact area). The slower this rate is, for given hot and cold fluid flow rates, the more contact area is required. The rate of heat transfer through a unit of contact area was referred to earlier as the *heat flux* and, at any point along the area or the tube length, is given by

$$\frac{d\dot{Q}}{dA} = U(T_H - t_C) \tag{14.20}$$

where $d\dot{Q}/dA$ is the local heat flux (Btu/h · ft^2) and U is defined as the local *overall heat transfer coefficient* (Btu/h · ft^2 · °F), a term that provides a measure (inversely) of the resistance to heat transfer.

The use of the above overall heat transfer coefficient (U) is a simple, yet powerful concept. In most applications, it combines both conduction and convection effects, although heat transfer by radiation can also be included. In actual practice, it is not uncommon for vendors to provide a numerical value for U. For example, a typical value for U for estimating heat losses from an incinerator is approximately 0.1 Btu/h · ft^2 · °F. Methods for calculating the overall heat transfer coefficient are presented later in this section.

With reference to Equation (14.20), the temperatures T_H and t_C are actually local average values. As described in the previous section, when a fluid is being heated or cooled, the temperature will vary throughout the cross-section of the stream. If the fluid is being heated, its temperature will be highest at the tube wall and will decrease with increasing distance from the tube wall. The average temperature across the stream cross-section is therefore t_C; i.e., the temperature that would be achieved if the fluid at this cross-section was suddenly mixed to a uniform temperature. If the fluid is being cooled, on the other hand, its temperature will be lowest at the tube wall and will increase with increasing distance from the wall.

In order to apply Equation (14.20) to an entire heat exchanger, the equation must be integrated. This cannot be done unless the geometry of the exchanger is first defined. For simplicity, one of the simplest geometries will be assumed here—the double pipe heat exchanger to be discussed in the next chapter. This device consists of two parallel concentric pipes. The outer surface of the outer pipe is well insulated so that no heat exchange with the surroundings may be assumed. One of the fluids flows through the center pipe and the other flows through the annular channel (known as the annulus) between the pipes. The fluid flows may be either co-current where the two fluids flow in the same directions, or countercurrent where the flows are in the opposite directions; however, the countercurrent arrangement is more efficient and is more commonly used. For this heat exchanger, integration of Equation (14.20) along the exchanger area or length, and applying several simplifying assumptions, yields:

$$\dot{Q} = UA\Delta T \tag{14.21}$$

The above equation was previously applied to heat transfer across a plane wall in Chapter 9 and it was shown that

$$\dot{Q} = \frac{T_H - T_C}{\dfrac{1}{h_iA} + \dfrac{\Delta x}{kA} + \dfrac{1}{h_0A}} \tag{14.22}$$

where the h's represent the individual heat transfer coefficients, also discussed in Chapter 9. Since $\dot{Q} = UA\Delta T$

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{\Delta x}{k} + \dfrac{1}{h_0}} \tag{14.23}$$

Since A is a constant,

$$\frac{\dot{Q}}{\Delta T} = UA = \frac{1}{\dfrac{1}{h_iA} + \dfrac{\Delta x}{kA} + \dfrac{1}{h_0A}} \tag{14.24}$$

For a tubular unit, $\dot{Q} = UA\Delta T$ still applies, but A is now a variable. Equation (14.24) is rewritten for this geometry as

$$\frac{\dot{Q}}{\Delta T} = \frac{1}{\dfrac{1}{h_i A_i} + \dfrac{\Delta D}{2kA_{\mathrm{lm}}} + \dfrac{1}{h_o A_o}}; \ \Delta D = D_0 - D_i = 2(r_0 - r_i) \tag{14.25}$$

where the subscripts i and o refer to the inside and outside of the tube, respectively. In addition,

$$A_{\mathrm{lm}} = \frac{A_o - A_i}{\ln(A_o/A_i)} = \frac{\pi L \Delta D}{\ln(D_o/D_i)} \tag{14.26}$$

and L = tube length (ft). Thus,

$$\frac{\dot{Q}}{\Delta T} = UA = \frac{1}{\dfrac{1}{h_i A_i} + \dfrac{\ln(D_o/D_i)}{2\pi L k} + \dfrac{1}{h_o A_o}} \tag{14.27}$$

Equation (14.27) may again be written as

$$\frac{\dot{Q}}{\Delta T} = UA = \frac{1}{R_i + R_w + R_o} \tag{14.28}$$

The term U in Equation (14.27) may be based on the inner area (A_i) or the outer area (A_o) so that

$$\frac{\dot{Q}}{\Delta T} = U_i A_i = U_o A_o = \frac{1}{\dfrac{1}{h_i A_i} + \dfrac{\ln(D_o/D_i)}{2\pi L k} + \dfrac{1}{h_o A_o}} \tag{14.29}$$

Dividing by either A_i or A_o yields an expression for U_i and U_o:

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(D_o/D_i)}{2\pi L k} + \dfrac{A_i}{h_o A_o}} \tag{14.30}$$

$$U_o = \frac{1}{\dfrac{A_o}{h_i A_i} + \dfrac{A_o \ln(D_o/D_i)}{2\pi L k} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{D_o}{h_i D_i} + \dfrac{D_o \ln(D_o/D_i)}{2k} + \dfrac{1}{h_o}} \tag{14.31}$$

Numerical values of U can range from as low as 2 $\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$ (10 $\mathrm{W/m^2 \cdot K}$) for gas-to-gas heat exchangers to as high as 250 $\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$ for water-to-water units. Additional values are provided in the literature.(1)

Fouling Factors

During heat exchange operation with liquids and/or gases, a "dirt" film gradually builds up on the exchanger surface(s). This deposit may be rust, boiler scale, silt, coke, or any number of things. Its effect, which is referred to as *fouling*, is to increase the thermal resistance, R, which results in decreased performance. The nature of the rate of deposit is generally difficult to predict *a priori*. Therefore, only the performance of clean exchangers is usually guaranteed. The fouling resistance is often obtained from field, pilot, or lab data, or from experience.(2) This unknown factor enters into every design.

The scale of fouling is dependent on the fluids, their temperature, velocity, and to a certain extent, the nature of the heat transfer surface and its chemical composition. Due to the unknown nature of the assumptions, these fouling factors can markedly affect the design of heat transfer equipment.

The effect (resistance) of fouling, R_f, can be obtained from the following equation:

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \tag{14.32}$$

where U_{dirty} is the overall heat transfer coefficient for design (with fouling) and U_{clean} is the overall heat transfer coefficient for the clean condition (without fouling). Thus, in general, Equation (14.28) can be rewritten as

$$UA = \frac{1}{R_i + R_{fi} + R_w + R_{fo} + R_o} = \frac{1}{R_t} \tag{14.33}$$

where R_t = total resistance.

If U is based on the outside area of the tube, A_o,

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{A_o}{A_i}\right) + R_{fi}\left(\frac{A_o}{A_i}\right) + R_w + R_{fo} + \frac{1}{h_o}} \tag{14.34}$$

where R_w is once again the wall resistance and given by (see also Equation (14.31))

$$R_w = \frac{A_o \ln(D_o/D_i)}{2\pi L k} \tag{14.35}$$

The two fouling factors, R_{fi} and R_{fo}, are sometimes combined and treated as one resistance, R_f. For this condition,

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{A_o}{A_i}\right) + \frac{A_o \ln(D_o/D_i)}{2\pi L k} + R_f + \frac{1}{h_o}} \tag{14.36}$$

Table 14.2 Fouling Coefficients (English Units)

Fluid	R_f, h · ft^2 · °F/Btu
Alcohol vapors	0.0005
Boiler feedwater	0.001
Fuel oil	0.005
Industrial air	0.002
Quench oil	0.004
Refrigerating liquids	0.001
Seawater	0.0008
Steam, non oil-bearing	0.0005

Typical values of the fouling coefficient, h_f, are shown in Table 14.2. Additional values are available.(1) Note that

$$h_f = R_f^{-1} \tag{14.37}$$

One of the main contributors to fouling is corrosion. Therefore, corrosion protection is important and the selection of equipment materials with reasonable corrosion rates must be balanced against initial and replacement costs. Materials chosen for corrosion resistance in a given process service may include carbon steels, stainless steels, aluminum, copper alloys, nickel alloys, graphite, glass, and other non-metallics. Corrosion effects can be reduced without costly materials by means of special design and fabrication techniques; among these are limitations on flow velocities and use of impingement baffles.

As noted above, fouling factors are usually determined experimentally by generating data for both the dirty and clean unit. This topic will be revisited in the next chapter.

The Controlling Resistance

The reader should note that the numerical value of the overall heat transfer coefficient may be primarily governed by only one of the individual coefficients (and, therefore, only one of the resistances). That coefficient is then referred to as the *controlling* resistance. As one might expect, the resistance due to conduction (the wall) is often negligible relative to the convective resistances.(2)

ILLUSTRATIVE EXAMPLE 14.10

A rectangular plane window glass panel is mounted on a house. The glass is 0.125 inch thick and has a surface area of 1.0 m^2; its thermal conductivity, k_2, is 1.4 W/m · K. The inside house temperature, T_1, and the outside air temperature, t_4, are 25°C and −14°C, respectively. The heat transfer coefficient inside the room, h_1, is 11.0 W/m^2 · K and the heat transfer coefficient from the window to the surrounding cold air, h_3, is 9.0 $\mathrm{W/m^2 \cdot K}$. Calculate the overall heat transfer coefficient in $\mathrm{W/m^2 \cdot K}$.

***SOLUTION*:** Calculate the internal convection resistance, R_1:

$$R_1 = \frac{1}{h_1 A} = \frac{1}{(11)(1)} = 0.0909\ \mathrm{K/W}$$

Calculate the conduction resistance through the glass panel:

$$L_2 = \Delta x = 0.125\ \text{inch}\left(\frac{0.0254\ \text{m}}{\text{inch}}\right) = 0.00318\ \text{m}$$

$$R_2 = \frac{\Delta x}{k_z A} = \frac{0.00318}{(1.4)(1)} = 0.00227\ \mathrm{K/W}$$

Also calculate the outside convection resistance:

$$R_3 = \frac{1}{h_3 A} = \frac{1}{(9)(1)} = 0.111\ \mathrm{K/W}$$

The total thermal resistance is therefore, from Equation (14.33),

$$R_t = 0.0909 + 0.00227 + 0.111 = 0.204\ \mathrm{K/W}$$

The overall heat transfer coefficient may now be calculated:

$$U = \frac{1}{AR_t} \tag{14.33}$$

Substituting,

$$U = \frac{1}{(1)(0.204)} = 4.9\ \mathrm{W/m^2 \cdot K}$$

■

ILLUSTRATIVE EXAMPLE 14.11

A heat exchanger wall consists of a copper plate 0.049 inch thick. If the two surface film coefficients are 208 and 10.8 $\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$, respectively, calculate the overall heat transfer coefficient in $\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$.

***SOLUTION*:** Since information on the internal and external area is not given, neglect the effect of area. Equation (14.24) may be applied:

$$U = \frac{1}{\dfrac{1}{h_1} + \dfrac{\Delta x}{k} + \dfrac{1}{h_2}}$$

Substituting,

$$U = \frac{1}{\dfrac{1}{208} + \dfrac{(0.049/12)}{220} + \dfrac{1}{10.8}} = 10.26\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

■

ILLUSTRATIVE EXAMPLE 14.12

A coolant flows through a steel pipe (inside diameter of 0.05 m, outside diameter of 0.06 m) at a velocity of 0.25 m/s. What is the overall heat transfer coefficient for the system based on the pipe's outside surface area if the inside and outside coefficients are 2000 $\text{W/m}^2 \cdot \text{K}$ and 8.25 $\text{W/m}^2 \cdot \text{K}$, respectively. Assume the "resistance" term for the steel, that is,

$$R = \frac{D_o \ln(D_o/D_i)}{2k}$$

is $1.33 \times 10^{-4}\ \text{m}^2 \cdot \text{K/W}$.

***SOLUTION*:** For this example, Equation (14.31) applies. Therefore,

$$\begin{aligned} U_o &= \frac{1}{\dfrac{D_o}{h_i D_i} + \dfrac{D_o \ln(D_o/D_i)}{2k} + \dfrac{1}{h_o}} \\ &= \frac{1}{\dfrac{0.06}{(2000)(0.05)} + 1.33 \times 10^{-4} + \dfrac{1}{8.25}} \\ &= \frac{1}{0.0006 + 1.33 \times 10^{-4} + 0.121} = 8.21\,\text{W/m}^2 \cdot {}^\circ\text{K} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 14.13

With reference to the previous example, determine the controlling resistance.

***SOLUTION*:** Obviously, the controlling resistance is located outside (external to) the pipe ($1/h_o = 0.121$). ■

ILLUSTRATIVE EXAMPLE 14.14

Steam at 247°F is flowing through a pipe exposed to air but covered with 1.5 inch thick insulation. The following data is provided:

1. Pipe diameter, inside = 0.825 inch
2. Pipe diameter, outside = 1.05 inch
3. Surrounding air temperature = 60°F

4. Thermal conductivity, pipe = 26 Btu/h · ft · °F
5. Thermal conductivity, insulation = 0.037 Btu/h · ft · °F
6. Steam film coefficient = 800 Btu/h · ft^2 · °F
7. Air film coefficient = 2.5 Btu/h · ft^2 · °F

Calculate the overall heat transfer coefficient based on the inside area of the pipe.

***SOLUTION*:** Obviously, cylindrical coordinates are employed again. Assume a basis 1.0 ft of pipe length. The inside (i) and outside (o) areas of the pipe (P) plus the insulation (I) areas are

$$A_i = 0.2157\ \text{ft}^2$$

$$A_o = 0.2750\ \text{ft}^2$$

$$A_I = 1.060\ \text{ft}^2$$

The log mean area for the steel pipe (P) and insulation (I) are, therefore

$$A_{P\text{lm}} = \frac{A_0 - A_i}{\ln(A_0/A_i)} = \frac{0.2750 - 0.2157}{\ln(0.2750/0.2157)} = 0.245\ \text{ft}^2$$

$$A_{I\text{lm}} = \frac{A_I - A_0}{\ln(A_I/A_0)} = \frac{1.060 - 0.2750}{\ln(1.060/0.2750)} = 0.582\ \text{ft}^2$$

A slightly modified form of Equation (14.25) and (14.30) is employed to calculate the overall heat coefficient based on the inside area:

$$U_i = \frac{1}{A_i \sum R} = \frac{1}{A_i(R_i + R_P + R_o + R_I)} = \frac{1}{A_i\left(\dfrac{1}{h_i A_i} + \dfrac{r_o - r_i}{k_P A_{P\text{lm}}} + \dfrac{1}{h_o A_I} + \dfrac{r_1 - r_o}{k_I A_{I\text{lm}}}\right)}$$

$$= \frac{1}{0.2157\left(\dfrac{1}{(800)(0.2157)} + \dfrac{(0.525 - 0.412)/12}{(26)(0.245)} + \dfrac{1}{(2.5)(1.060)} + \dfrac{(2.025 - 0.525)/12}{(0.037)(0.582)}\right)}$$

$$= \frac{1}{0.2157(0.005795 + 0.001478 + 0.37736 + 5.80477)}$$

$$= 0.749\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

Obviously, the major resistance to heat transfer is the insulation. ■

ILLUSTRATIVE EXAMPLE 14.15

Calculate the rate of heat transfer in the previous example.

***SOLUTION*:** The above result can be extended to calculate the heat transfer rate.

$$A_i = \pi D_i L = \pi(0.825/12)(1) = 0.2157\ \text{ft}^2 \text{ (see previous example)}$$

$$\dot{Q} = U_i A_i (T_s - t_a) = 0.7492(0.2157)(247 - 60) = 30.2\ \text{Btu/h}$$

■

ILLUSTRATIVE EXAMPLE 14.16

The following information is provided for a heat exchanger:

$h_w(\text{water}) = 200\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$
$h_o(\text{oil}) = 50\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$
$h_f(\text{fouling}) = 1000\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$
$\Delta T_{\text{lm}} = 90^\circ\text{F}$
$A = 15.0\ \text{ft}^2$

Calculate the heat transfer rate for the exchanger if the wall resistance can be neglected.

***SOLUTION*:**
Apply Equation (14.34) for a constant *A*,

$$\frac{1}{U} = \frac{1}{h_w} + \frac{1}{h_o} + \frac{1}{h_f} \tag{14.34}$$

Substituting,

$$\begin{aligned}\frac{1}{U} &= \frac{1}{200} + \frac{1}{50} + \frac{1}{1000}\\ &= 0.005 + 0.02 + 0.001\\ &= 0.026\end{aligned}$$

Therefore,

$$U = 1.0/0.026 = 38.46\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

The heat transfer rate is

$$\begin{aligned}\dot{Q} &= UA\Delta T_{\text{lm}} \qquad (14.21)\\ &= (38.46)(15.0)(90)\\ &= 51{,}920\ \text{Btu/h}\end{aligned}$$

■

Varying Overall Heat Transfer Coefficients

The calculation of ΔT_{lm} is valid for heating or cooling, i.e., whether ΔT_1 and ΔT_2 are both positive or both negative. It is not possible that ΔT_1 and ΔT_2 have opposite signs since it would constitute a violation of the Second Law of Thermodynamics. Finally, the LMTD should not be used when the overall heat transfer coefficient, *U*, changes appreciably through the unit. When *U* varies linearly with the temperature difference over the entire heating surface, the rate of heat transfer may be estimated from the following equation:

$$\dot{Q} = A\left[\frac{U_2\Delta T_1 - U_1\Delta T_2}{\ln\left(\dfrac{U_2\Delta T_1}{U_1\Delta T_2}\right)}\right] = A(U\Delta T)_{\text{lm}} \tag{14.38}$$

where U_1 is the overall heat transfer coefficient at the fluid entrance side and U_2 is the overall heat transfer coefficient at the fluid exit side. Equation (14.38) requires the use of a log mean value of the product of U and ΔT. Note that the U at one end of the heater is multiplied by the temperature difference at the other end.

ILLUSTRATIVE EXAMPLE 14.17

A brine solution at 10°F in a food processing plant is heated by flowing through a heated pipe. The pipe surface is maintained at 80°F. The pipe surface area for heat transfer is 2.5 ft^2. The brine solution (with a density of 62.4 lb/ft^3 and a heat capacity of 0.99 Btu/lb · °F) flows at a rate of 20 lb/min. The overall heat transfer coefficient varies linearly with the temperature approach, with values of 150 Btu/h · ft^2 · °F at the brine solution entrance (where the brine temperature is 10°F) and 140 Btu/h · ft^2 · °F at the brine solution exit. Determine the temperature approach at the brine inlet side, the exit temperature of the brine solution, and the rate of heat transfer, $\dot{Q}$.

***SOLUTION*:** Set T as the surface temperature and t_1 and t_2 as the brine inlet and outlet temperatures, respectively. Calculate the temperature approach at the pipe entrance:

$$\Delta T_1 = T - t_1 = 80 - 10 = 70^\circ\text{F} = 70/1.8 = 38.9^\circ\text{C}$$

Note that ΔT_2, which is equal to $(T - t_2)$, cannot be calculated since t_2 is not known. Apply an energy balance to the brine solution across the full length of the pipe:

$$\begin{aligned}\dot{Q} &= \dot{m}c_p(t_2 - t_1) = \dot{m}c_p(\Delta T_1 - \Delta T_2)\\ &= (1200)(0.99)(70 - \Delta T_2) = 1188(70 - \Delta T_2)\end{aligned}$$

The equation for the LMTD is

$$\Delta T_{\text{lm}} = (70 - \Delta T_2)/\ln(70/\Delta T_2)$$

Write the equation for the heat transfer rate. Note that U varies linearly with ΔT. See Equation (14.38):

$$\dot{Q} = A\left\{\frac{U_2(\Delta T_1) - U_1(\Delta T_2)}{\ln\left(\dfrac{U_2\Delta T_1}{U_1\Delta T_2}\right)}\right\} = 2.5\left\{\frac{140(70) - 150(\Delta T_2)}{\ln\left(\dfrac{(140)(70)}{(150)(\Delta T_2)}\right)}\right\}$$

Combine the previous equations and eliminate $\dot{Q}$:

$$1188(70 - \Delta T_2) = 2.5\left\{\frac{(140)(70) - (150)(\Delta T_2)}{\ln\left(\dfrac{(140)(70)}{(150)(\Delta T_2)}\right)}\right\}$$

This equation is non-linear with one unknown (ΔT_2). Solve this equation by trial-and-error. Note that $0 \leq \Delta T_2 \leq 70°\text{F}$. Solution gives $\Delta T_2 \simeq 51.6°\text{F} \simeq 28.7°\text{C}$.

Calculate the discharge temperature of the brine solution:

$$\begin{aligned}\Delta T_2 &= 51.6 = T - t_2 = 80 - t_2\\ t_2 &= 80 - 51.6 = 28.4°\text{F}\\ &= (28.4 - 32)/1.8 = -2°\text{C}\end{aligned}$$

Finally, calculate the heat transfer rate, $\dot{Q}$. Using the earlier equation for $\dot{Q}$,

$$\begin{aligned}\dot{Q} &= 1188(70 - 51.6) = 21{,}860\,\text{Btu/h}\\ &= 21{,}860/3.412 = 6407\,\text{W}\end{aligned}$$

■

THE HEAT TRANSFER EQUATION

The chapter concludes with a (very) short section entitled the heat transfer equation. By now the reader should have concluded that

$$\dot{Q} = UA\Delta T_{\text{lm}} \tag{14.21}$$

is that equation. It will find extensive use in the remainder of the text, particularly with the remaining chapters in this Part.

Summarizing,$^{(3)}$ for a plane wall,

$$\begin{aligned}U &= \frac{1}{R_i + R_w + R_o} \qquad (14.23)\\ &= \frac{1}{\dfrac{1}{h_i} + \dfrac{\Delta x}{k} + \dfrac{1}{h_o}}\end{aligned}$$

If the area is constant, this equation may be written as

$$UA = \frac{1}{\dfrac{1}{h_iA} + \dfrac{\Delta x}{kA} + \dfrac{1}{h_oA}} \tag{14.24}$$

For a tubular wall, Equation (14.24) remains the same, but which A should be employed? For inside/wall/outside resistances present, and employing the radius rather than the diameter leads to:

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{\Delta r}{k} + \dfrac{1}{h_o}} \tag{14.40}$$

In addition (and employing radii),

$$UA = \frac{1}{\dfrac{1}{h_i A_i} + \dfrac{\Delta r}{k A_{\text{lm}}} + \dfrac{1}{h_o A_o}} \tag{14.41}$$

with

$$A_{\text{lm}} = \frac{A_o - A_i}{\ln(A_o/A_i)} = \frac{2\pi L \Delta r}{\ln(r_o/r_i)} \tag{14.42}$$

Equation (14.41) may therefore be written as

$$UA = \frac{1}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi kL} + \dfrac{1}{h_o A_o}} \tag{14.43}$$

U may be based on (i) or (o). For $U_i A_i$,

$$U_i A_i = \frac{1}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi kL} + \dfrac{1}{h_o A_o}} \tag{14.44}$$

or

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi kL} + \dfrac{1}{h_o}\left(\dfrac{A_i}{A_o}\right)} \tag{14.45}$$

with a similar equation for U_o.

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Chapter 15

Double Pipe Heat Exchangers

INTRODUCTION

Of the various types of heat exchangers that are employed in industry, perhaps the two most fundamental are the *double pipe* and the *shell and tube*. Despite the fact that shell and tube heat exchangers (see next chapter) generally provide greater surface area for heat transfer with a more compact design, greater ease of cleaning, and less possibility of leakage, the double pipe heat exchanger still finds use in practice.

The double pipe unit consists of two concentric pipes. Each of the two fluids—hot and cold—flow either through the inside of the inner pipe or through the annulus formed between the outside of the inner pipe and the inside of the outer pipe. Generally, it is more economical (from a heat efficiency perspective) for the hot fluid to flow through the inner pipe and the cold fluid through the annulus, thereby reducing heat losses to the surroundings. In order to ensure sufficient contacting time, pipes longer than approximately 20 ft are extended by connecting them to *return bends*. The length of pipe is generally kept to a maximum of 20 ft because the weight of the piping may cause the pipe(s) to sag. Sagging may allow the inner pipe to touch the outer pipe, distorting the annulus flow region and disturbing proper operation. When two pipes are connected in a "U" configuration by a return bend, the bend is referred to as a *hairpin*. In some instances, several hairpins may be connected in series.

Double pipe heat exchangers have been used in the chemical process industry for over 85 years. The first patent on this unit appeared in 1923. The unique (at that time) design provided "the fluid to be heated or cooled and flow longitudinally and transversely around a tube containing the cooling or heating liquid, etc." The original design has not changed significantly since that time.

Although this unit is not extensively employed in industry (the heat transfer area is small relative to other heat exchangers), it serves as an excellent starting point from an academic and/or training perspective in the treatment of all the various heat exchangers reviewed in this Part.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

Topics covered in this chapter include:

Equipment Description
Describing Equations
Effectiveness Factor and Number of Transfer Units
Wilson's Method

EQUIPMENT DESCRIPTION

As discussed in the Introduction, the simplest kind of heat exchanger, which confines both the hot and cold fluids, consists of two concentric tubes, or pipes. When conditions are such that only a few tubes per pass are required, the simplest construction is the double pipe heat interchanger shown in Figure 15.1. This consists of special fittings that are attached to standard iron (typically) pipe so that one fluid (usually liquid) flows through the *inside* pipe and the second fluid (also usually a liquid) flows through the *annular* space between the two pipes. Such a heat interchanger will usually consist of a number of passes which are almost invariably arranged in a vertical stack. If more than one pipe per pass is required, the proper number of such stacks is connected in parallel. Although they do not provide a large surface area for heat transfer, double pipe heat exchangers are at times used in industrial settings. A schematic of this unit and flow classification(s) is provided in Figure 15.2.

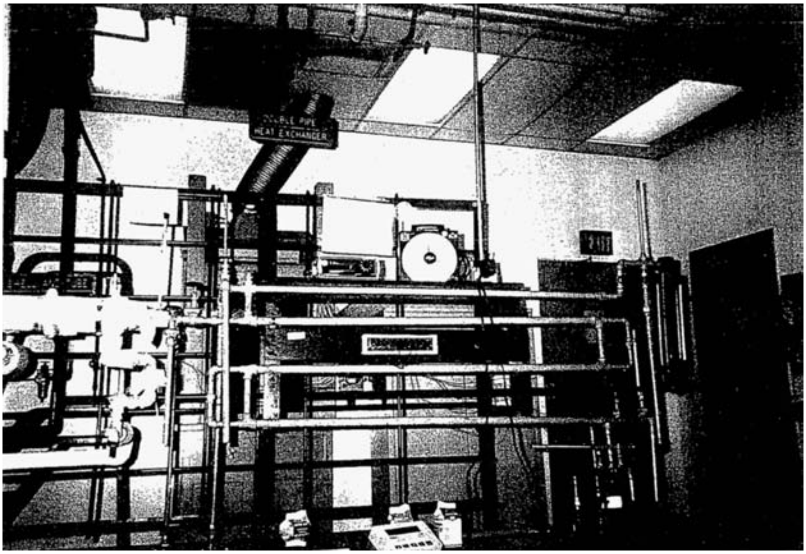
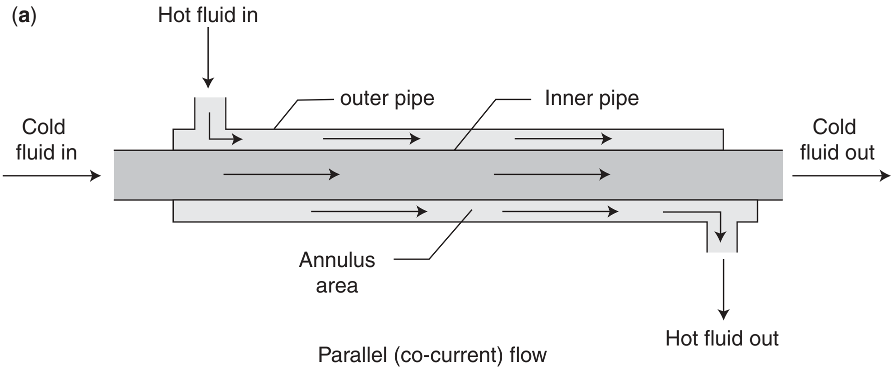
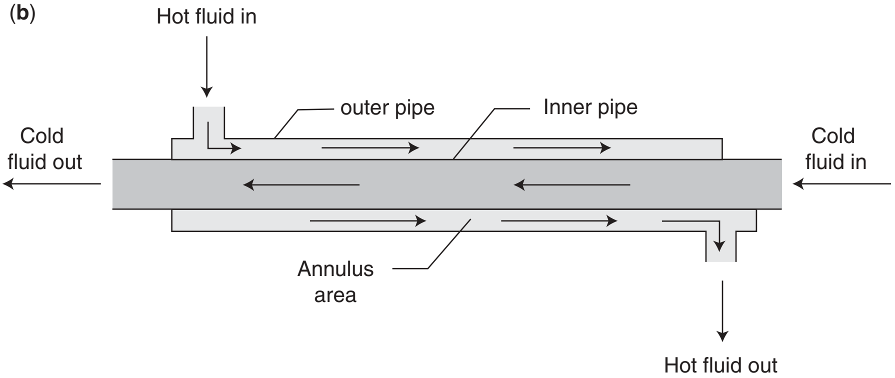


Figure 15.1 Double pipe heat exchanger (Manhattan College Unit Operations Laboratory).



Parallel (co-current) flow



Countercurrent flow

Figure 15.2 Double pipe schematic.

Recommended standard fittings for double pipe interchangers are provided in the literature.$^{(1)}$ Additional details are provided in Chapter 6 of Part One. Other types of tubes or pipes available include:

1. Plain tubes
2. Duplex tubes
3. Finned tubes

Plain tubes are used when the corrosion potential on either the tube or shell side (see next chapter) are approximately the same. Duplex tubes have walls of two different metals and are used when it is impossible to find a single metal that can adequately handle corrosion on both sides of the unit. Finned tubes (to be discussed in Chapter 17) are usually employed for heat transfer to gases where film coefficients are very low. Tube materials include: carbon steel, carbon alloy steels, stainless steels, brass and alloys, cupro-nickel, nickel, monel, glass, reinforced fiberglass plastic (RFP), etc.

As noted in Chapter 6, calculations for flow in the annular region referred to above require the use of a characteristic or *equivalent diameter*. By definition, this diameter,

D_{eq}, is given by 4 times the area available for flow divided by the "wetted" perimeter. Note that there are two wetted perimeters in an annular area. This reduces to

$$D_{\mathrm{eq}} = 4\left(\frac{\pi}{4}\right)(D_{o,i}^2 - D_{i,o}^2)\big/\pi(D_{o,i} + D_{i,o}) \tag{15.1}$$

where $D_{o,i}$ is the inside diameter of the outer pipe and $D_{i,o}$ is the outside diameter of the inner pipe. This equation reduces to

$$D_{\mathrm{eq}} = D_{o,i} - D_{i,o} \tag{15.2}$$

which is four times the hydraulic radius, i.e.,

$$D_{\mathrm{eq}} = 4r_H \tag{15.3}$$

with

$$r_H = \tfrac{1}{4}(D_{o,i} - D_{i,o}) \tag{15.4}$$

The flow in a double pipe heat exchanger may be countercurrent or parallel (co-current). In countercurrent flow, the fluid in the pipe flows in a direction opposite to the fluid in the annulus. In parallel flow, the two fluids flow in the same direction. The variations of fluid temperature within the heat exchanger depend on whether the flow is parallel or countercurrent (see Figure 15.2).

The definitions below are employed in the development to follow (see also Figure 15.2):

T_1 = temperature of the hot fluid entering the inside pipe/tube

T_2 = temperature of the hot fluid exiting the inside pipe/tube

t_1 = temperature of the cold fluid entering the annulus

t_2 = temperature of the cold fluid exiting the annulus

The difference between the temperature of the tube side fluid and that of the annulus side is the *temperature difference driving force* (TDDF), ΔT.

As noted in the previous chapter, in a co-current flow heat exchanger, both hot and cold fluids enter on the same side and flow through the exchanger in the same direction. The *temperature approach* is defined as the temperature difference driving force at the heat exchanger entrance, ΔT_1 or $(T_1 - t_1)$. This driving force drops as the streams approach the exit of the exchanger. At the exit, the temperature difference driving force is ΔT_2 or $(T_2 - t_2)$. Thus, the heat exchanger is more effective at the entrance than at the exit. In a countercurrent flow exchanger, the two fluids exchange heat while flowing in opposite directions. The temperature approach at the tube entrance end ΔT_1 or $(T_1 - t_2)$ and at the annular entrance end, ΔT_2 or $(T_2 - t_1)$ is usually roughly the same. In addition, the thermal driving force is normally relatively constant over the length of the exchanger. The temperature profiles for both parallel and countercurrent systems are presented in Figure 15.3.

Summarizing, the double pipe heat exchanger employed in practice consists of two pipes: an inner and an outer pipe. Hot fluid normally flows in the inner pipe and the cold fluid flows in the annulus between the outer diameter of the inner pipe

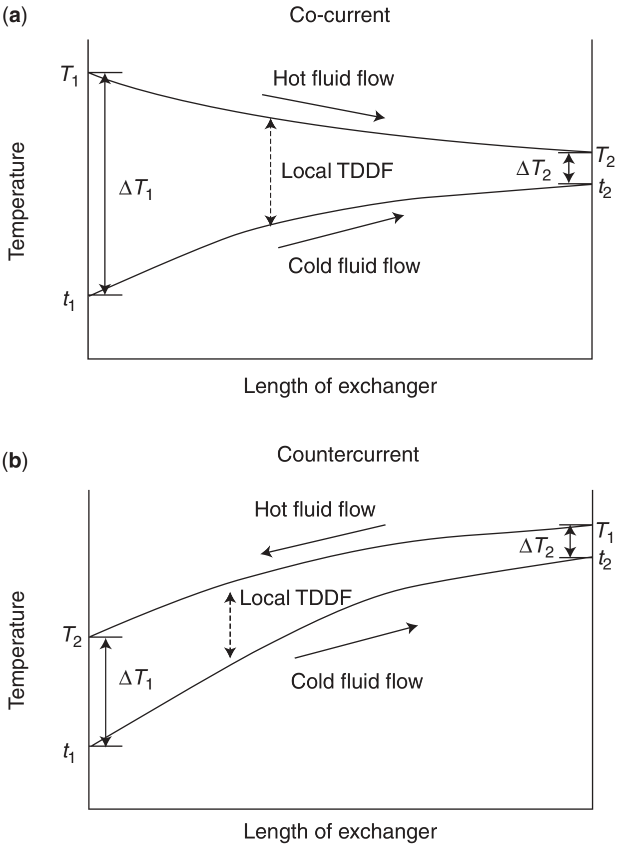


Figure 15.3 Co-current and countercurrent flow in a heat exchanger.

and inner diameter of the outer pipe. Heat transfer occurs from the inner pipe to the outer pipe as the fluids flow through the piping system. By recording both inner and outer fluid temperatures at various points along the length of the exchanger, it is possible to calculate heat exchanger duties and heat transfer coefficients (to be discussed shortly).

ILLUSTRATIVE EXAMPLE 15.1

Explain why the cold fluid flows in the annular region in nearly all double pipe heat exchanger applications.

***SOLUTION*:** As discussed above, heat losses to the environment are lower (except in cryogenic applications) with the hot fluid flowing in the inside pipe. Any losses are thus transferred to the cold fluid and not the surroundings. ■

DESCRIBING EQUATIONS

In designing a double pipe heat exchanger, mass balances, heat balances, and the applicable heat transfer equation(s) are used. The steady-state heat balance equation is

$$\begin{aligned}\dot{Q} &= \Delta\dot{h}_H = \Delta\dot{h}_C \\ &= \dot{m}_H c_{pH}(T_1 - T_2) = \dot{m}_C c_{pC}(t_2 - t_1)\end{aligned} \tag{15.5}$$

This equation assumes steady state, no heat loss, no viscous dissipation, and no heat generation.(1) The rate equation used to design an exchanger is the design equation, which includes the previously developed log mean temperature difference (ΔT_{lm} or LMTD) and overall heat transfer coefficient, U

$$\dot{Q} = UA\Delta T_{\text{lm}} \tag{15.6}$$

where $\dot{Q}$ is the heat load, U is the overall heat transfer coefficient, A is the heat transfer area, and ΔT_{lm} is the log mean (or global) temperature difference driving force. If the temperature difference driving forces are ΔT_1 and ΔT_2 at the entrance and exit of the heat exchanger, respectively, then

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{15.7}$$

If $\Delta T_1 = \Delta T_2$, then $\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2$. The overall heat transfer coefficient, U, is usually based on the inside area of the tube; the A term in Equation (15.6) should then be based on the inside surface area. Typical values of U for new, clean exchangers are available in the literature and, as noted earlier, are often assigned the symbol, U_{clean}. See also Equation (14.32).

If the heat duties are known (or have been calculated), values for the overall heat transfer coefficient, U, can be calculated as follows:

$$U_o = \frac{\dot{Q}_C}{A_o\Delta T_{\text{lm}}} \tag{15.8}$$

$$U_i = \frac{\dot{Q}_H}{A_i\Delta T_{\text{lm}}} \tag{15.9}$$

where A = surface area for heat transfer (outside or inside), ft^2
$\dot{Q}$ = average heat duty (cold or hot), Btu/h
U_o = overall heat transfer coefficient based on the outside area, $\text{Btu/ft}^2 \cdot \text{h} \cdot {}^\circ\text{F}$
U_i = overall heat transfer coefficient based on the inside area, $\text{Btu/ft}^2 \cdot \text{h} \cdot {}^\circ\text{F}$

In addition, one may write

$$\frac{1}{UA} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} \tag{15.10}$$

As described in the previous chapter, the overall heat transfer coefficient, U, for flow in a tube is related to the individual coefficients by the following equation:

$$\frac{1}{UA}=\frac{1}{h_iA_i}+\frac{R_{f,i}}{A_i}+\frac{\ln(D_o/D_i)}{2\pi kL}+\frac{R_{f,o}}{A_o}+\frac{1}{h_oA_o} \tag{15.11}$$

where h_i = inside heat transfer coefficient, $\text{Btu/ft}^2\cdot\text{h}\cdot{}^\circ\text{F}$
h_o = outside heat transfer coefficient, $\text{Btu/ft}^2\cdot\text{h}\cdot{}^\circ\text{F}$
$R_{f,i}$ = fouling factor based on inner tube surfaces
$R_{f,o}$ = fouling factor based on outer tube surfaces
D_i/D_o = diameter of pipe (inside or outside), ft
k = pipe thermal conductivity, $\text{Btu/ft}\cdot\text{h}\cdot{}^\circ\text{F}$
L = tube length, ft

If the fouling factors, $R_{f,i}$ and $R_{f,o}$, and the tube wall resistance (middle term in right-hand side of Equation (15.11)) are negligible, then the relationship between the overall heat transfer coefficient and the individual coefficients simplifies as follows:

$$\frac{1}{U}=\frac{D_o}{h_iD_i}+\frac{1}{h_o} \tag{15.12}$$

or

$$\frac{1}{U_i}=\frac{1}{h_i}+\frac{D_i}{h_oD_o}$$

Individual coefficients, h_i and h_o, can be calculated by using empirical equations provided in Part Two, some of which is repeated below. Except for the viscosity term at the wall temperature, all of the physical properties in the equations that follow are evaluated at bulk temperatures. For the hot stream in the inner tube, the bulk temperature is $T_{H,\text{bulk}}=(T_{H,i}+T_{H,o})/2$. For the cold stream in the annulus, the bulk temperature is $t_{C,\text{bulk}}=(t_{C,i}+t_{C,o})/2$. Employing viscosity values at the wrong temperature can lead to substantial errors; however, the density and thermal conductivity of liquids do not vary significantly with temperature.

The Reynolds number for both the cold and hot process streams must be found in order to determine whether the flow rate for each stream is in the laminar, turbulent, or transition region. In all of the equations that follow, calculations for the annulus require that the aforementioned equivalent or hydraulic diameter, $D_{\text{eq}}=(D_{o,i}-D_{i,o})$, replace the tubular diameter, D. Thus, the Reynolds numbers, Re_i and Re_o, are defined as follows:

$$\text{Inner pipe:}\quad \text{Re}_i=\frac{4\dot{m}_i}{\pi D_{i,i}\mu_i} \tag{15.13}$$

$$\text{Annulus between pipes:}\quad \text{Re}_o=\frac{4\dot{m}_o}{\pi(D_{o,i}+D_{i,o})\mu_o}=\frac{D_{\text{eq}}\dot{m}_o}{\mu_oS} \tag{15.14}$$

Table 15.1 Reynolds Number Values vs. Type of Flow

Reynolds number, Re	Flow region
$\mathrm{Re} < 2100$	Laminar
$2100 < \mathrm{Re} < 10{,}000$	Transitional
$\mathrm{Re} > 10{,}000$	Turbulent

where $D_{i,i}$ = inside diameter of inner pipe, ft
$D_{i,o}$ = outside diameter of inner pipe, ft
$D_{o,i}$ = inside diameter of outer pipe, ft
D_{eq} = equivalent diameter, ft
S = cross-sectional annular area, ft^2
μ = viscosity of hot or cold fluid at bulk temperature, $\mathrm{lb/ft \cdot h}$

Flow regimes for various Reynolds numbers appear in Table 15.1.

Similarly, Nusselt numbers, Nu_i and Nu_o, are defined by the following equations:

$$\text{Inner pipe:} \quad \mathrm{Nu}_i = \frac{h_i D_{i,i}}{k_i} \tag{15.15}$$

$$\text{Annulus between pipes:} \quad \mathrm{Nu}_o = \frac{h_o(D_{o,i} - D_{i,o})}{k_o} \tag{15.16}$$

where k = thermal conductivity at bulk temperature of hot or cold fluid, $\mathrm{Btu/h \cdot ft \cdot {}^\circ F}$
Nu = Nusselt number, inside or outside

For *laminar flow*, the Nusselt number equals 4.36 for uniform surface heat flux ($\dot{Q}/A$) or 3.66 for constant surface temperature.(2,3) This value should only be used for Graetz numbers, Gz, less than 10. For laminar flow with Graetz numbers from 10 to 1000, the following equation applies:(3)

$$\mathrm{Nu} = 2.0\ \mathrm{Gz}^{1/3}\left(\frac{\mu}{\mu_{\mathrm{wall}}}\right)^{0.14}; \quad \mathrm{Gz} = \frac{\dot{m}c_p}{kL} \tag{15.17}$$

where μ_{wall} = viscosity at wall temperature, $\mathrm{lb/ft \cdot h}$
L = total length of tubular exchanger, ft
Gz = Graetz number

For *turbulent flow* ($\mathrm{Re} > 10{,}000$), the Nusselt number may be calculated from the Dittus–Boelter equation if $0.7 \leq \mathrm{Pr} \leq 160$ or the Sieder–Tate equation if $0.7 \leq \mathrm{Pr} \leq 16{,}700$ where Pr is the Prandtl number. Both equations are valid for L/D greater than 10.(3)

Dittus–Boelter equation:

$$
\begin{aligned}
Nu &= 0.023\ \mathrm{Re}^{0.8}\,\mathrm{Pr}^{n} \\
\mathrm{St} &= 0.023\ \mathrm{Re}(\mathrm{Re})^{-0.2}(\mathrm{Pr})^{-0.667} \\
\mathrm{Pr} &= \frac{c_p\mu}{k} \\
n &= 0.4 \ \text{ for heating or } 0.3 \text{ for cooling}
\end{aligned}
\tag{15.18}
$$

Sieder–Tate equation:

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_{\mathrm{wall}}}\right)^{0.14} \tag{15.19}$$

Note that Equation (15.18) is often written in terms of the Stanton number, St (occassionally referred to as the modified Nusselt number), where $\mathrm{St} = \mathrm{Nu}/\mathrm{RePr} = h/\rho V c_p$.

The Dittus–Boelter equation should only be used for small to moderate temperature differences. The Sieder–Tate equation applies for larger temperature differences.(2,3) Errors as large as 25% are associated with both equations.(2) Other empirical equations with more complicated formulas and less error are available in the literature.(2–6)

Either Equation (15.17) or (15.19) requires a trial-and-error solution between the hot and cold streams because these equations include a viscosity term that is evaluated at the wall temperature, T_{wall}. An educated guess can be made for the wall temperature, or it can be estimated to be the average of the four known temperatures:

$$T_{\mathrm{wall}} = \frac{t_{C,i} + t_{C,o} + T_{H,i} + T_{H,o}}{4}. \tag{15.20}$$

After the individual heat transfer coefficients are calculated, a new wall temperature can be calculated with the following equation:(3)

$$\Delta T_i = \frac{1/h_i}{(1/h_i) + (D_{i,i}/D_{i,o})h_o}(T_{H,\mathrm{bulk}} - t_{C,\mathrm{bulk}}) \tag{15.21}$$

with

$$T_{\mathrm{wall}} = T_{H,\mathrm{bulk}} - \Delta T_i \tag{15.22}$$

The entire calculation is repeated several times until T_{wall} converges. As previously stated, all of the terms in the operation above may be evaluated at the bulk temperatures of the hot and cold streams, except for μ_{wall} which is estimated at the wall temperature. Note that this correction factor is included to account for the distortion to the velocity profile that arises because of the viscosity variation with temperature.

Table 15.2 Representative Fouling Factors

Fluid	R_f, $\mathrm{m^2 \cdot K/W}$
Seawater and treated boiler feedwater (below 50°C)	0.0001
Seawater and treated boiler feedwater (above 50°C)	0.0002
River water (below 50°C)	0.0002–0.0001
Fuel oil	0.0009
Quenching oil	0.0007
Refrigerating liquid	0.0002
Steam (not oil bearing)	0.0001

After several months of use, the tubes in an exchanger can become fouled by scale or dirt. This scale adds an extra resistance to heat transfer and causes a decrease in the overall heat transfer coefficient from U_{clean} to U_{dirty}. The relation between these two values of U—see previous chapter, Equation (14.32)—is given by

$$\frac{1}{U_{\text{dirty}}} = \frac{1}{U_{\text{clean}}} + R_f \tag{15.23}$$

where R_f is the fouling or dirt factor in typical units of $\mathrm{ft^2 \cdot h \cdot {}^\circ F/Btu}$ or $\mathrm{m^2 \cdot K/W}$. Typical values of these fouling factors are given in Table 15.2. Information is also available in the previous chapter.

In designing a double pipe heat exchanger, it is often desirable to estimate the minimum required pipe length. This is accomplished by using the pipe conduction resistance as the only resistance to heat transfer and taking the larger of the two temperature differences at the ends of the exchanger, $\Delta T_{\max}$ (i.e., either $T_1 - t_2$ or $T_2 - t_1$ for countercurrent flow and $T_1 - t_1$ for co-current flow), to be the driving force. For this condition,

$$\dot{Q} = \frac{2\pi k L_{\min}}{\ln(r_o/r_i)} \Delta T_{\max} \tag{15.24}$$

This finds limited use in some real world applications.

ILLUSTRATIVE EXAMPLE 15.2

The following data is provided for a double pipe heat exchanger

$$\dot{Q} = 12{,}000\ \mathrm{Btu/h}$$

$$U = 48.0\ \mathrm{Btu/ft^2 \cdot h \cdot {}^\circ F}$$

$$\Delta T_{\text{lm}} = 50^\circ\mathrm{F}$$

Calculate the area of the exchanger.

***SOLUTION*:** Apply Equation (15.6):

$$\dot{Q} = UA\Delta T_{\text{lm}}$$

Substitute the data provided and solve for A.

$$\begin{aligned} A &= \frac{\dot{Q}}{U\Delta T_{\text{lm}}} \\ &= \frac{12{,}000}{(48.0)(50)} \\ &= 5.0 \text{ ft}^2 \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 15.3

Calculate the inlet cold water temperature flowing at a rate of 14.6 lb/min in a double pipe heat exchanger given the following data. Assume co-current operation.

$$\begin{aligned} t_1 &= t_{c,i} = 63^\circ\text{F} \\ T_1 &= T_{H,i} = 164^\circ\text{F} \\ T_2 &= T_{H,o} = 99^\circ\text{F} \\ U &= 35.35\,\text{Btu/ft}\cdot\text{h}\cdot{}^\circ\text{F} \\ \dot{Q} &= 56{,}760\,\text{Btu/h} \\ A &= 32.1 \text{ ft}^2 \end{aligned}$$

***SOLUTION*:** Apply Equation (15.6) once again and solve for ΔT_{lm}.

$$\begin{aligned} \dot{Q} &= UA\Delta T_{\text{lm}} \\ \Delta T_{\text{lm}} &= \frac{\dot{Q}}{UA} = \frac{56{,}760}{(35.35)(32.1)} \\ &= 50^\circ\text{F} \end{aligned}$$

Apply Equation (15.7).

$$\begin{aligned} \Delta T_{\text{lm}} &= \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \\ \Delta T_1 &= 164 - 63 = 101 \\ \Delta T_2 &= 99 - t_{c,o} = 99 - t_2 \end{aligned}$$

Rearrange and solve for $t_{c,o}$.

$$t_{c,o} = t_2 = 79^\circ\text{F}$$

■

ILLUSTRATIVE EXAMPLE 15.4

As Shakespeare once said: “Something is rotten in the state of . . .” Comment on whether the information provided in the previous example is “consistent.”

SOLUTION**:** In addition to satisfying the heat transfer rate equation, the information provided must also satisfy the energy transfer (or conservation) equation. For this example,

$$\begin{aligned}\dot{Q} &= \dot{m}c_{pC}(t_{C,o} - t_{C,i}) = \dot{m}c_{pC}(t_2 - t_1)\\ &= [(14.6)(1.0)(79 - 63)]\\ &= 233.6.\ \text{Btu/min}\\ &= 14{,}016\ \text{Btu/h}\end{aligned}$$

This result does not agree with the $\dot{Q}$ provided in the problem statement. Shakespeare is right, something is indeed rotten. ■

ILLUSTRATIVE EXAMPLE 15.5

A heavy hydrocarbon oil with a heat capacity, $c_p = 0.55$ Btu/lb · °F, is being cooled in a double pipe heat exchanger from $T_1 = 210$°F to $T_2 = 170$°F. The oil flows inside a tube at a rate of 8000 lb/h and the tube surface temperature is maintained at 60°F. The overall heat transfer coefficient $U = 63$ Btu · h · ft^2 · °F. Calculate the required heat transfer area, A, in ft^2.

SOLUTION**:** First calculate the LMTD:

$$\begin{aligned}\Delta T_1 &= 210 - 60 = 150°\text{F}\\ \Delta T_2 &= 170 - 60 = 110°\text{F}\end{aligned}$$

Apply Equation (15.7).

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(150) - (110)}{\ln(150/110)} = 129°\text{F}$$

Calculate the heat transferred or the duty. Apply Equation (15.5):

$$\dot{Q} = \dot{m}c_p\Delta T = 8000(0.55)(210 - 170) = 176{,}000\ \text{Btu/h}$$

Write the describing equation for the area and substitute:

$$\dot{Q} = UA\Delta T_{\text{lm}} \tag{15.6}$$

$$A = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{176{,}000}{63(129)} = 21.65\ \text{ft}^2$$

■

ILLUSTRATIVE EXAMPLE 15.6

Two variations of the design of a heat exchanger have been proposed. The unit is to cool a hot water stream from 140°F to 110°F through indirect contact with a cold water stream that is being heated from 60°F to 90°F. The water flow rate is 100 lb/min and the overall heat transfer coefficient may be assumed equal to 750 $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$. Calculate the area requirements for the following two exchanger conditions:

1. double pipe co-current flow
2. double pipe countercurrent flow

***SOLUTION*:**

1. For co-current flow,

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(140 - 60) - (110 - 90)}{\ln(80/20)} = 43.3^\circ\text{F}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p\Delta T = (100)(1)(30) = 3000\,\text{Btu/min} = 180{,}000\,\text{Btu/h}$$

The area for parallel flow is therefore

$$A_p = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{180{,}000}{750(43.3)} = 5.55\ \text{ft}^2$$

2. For countercurrent flow,

$$\Delta T_{\text{lm}} = 50^\circ\text{F (a constant)}$$

and

$$A_c = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{180{,}000}{750(50)} = 4.80\ \text{ft}^2$$

■

ILLUSTRATIVE EXAMPLE 15.7

Compare and comment on the area requirements for the two flow conditions in the previous example.

***SOLUTION*:** As expected, the countercurrent exchanger yields the smaller (more compact) design due to the higher driving force. ■

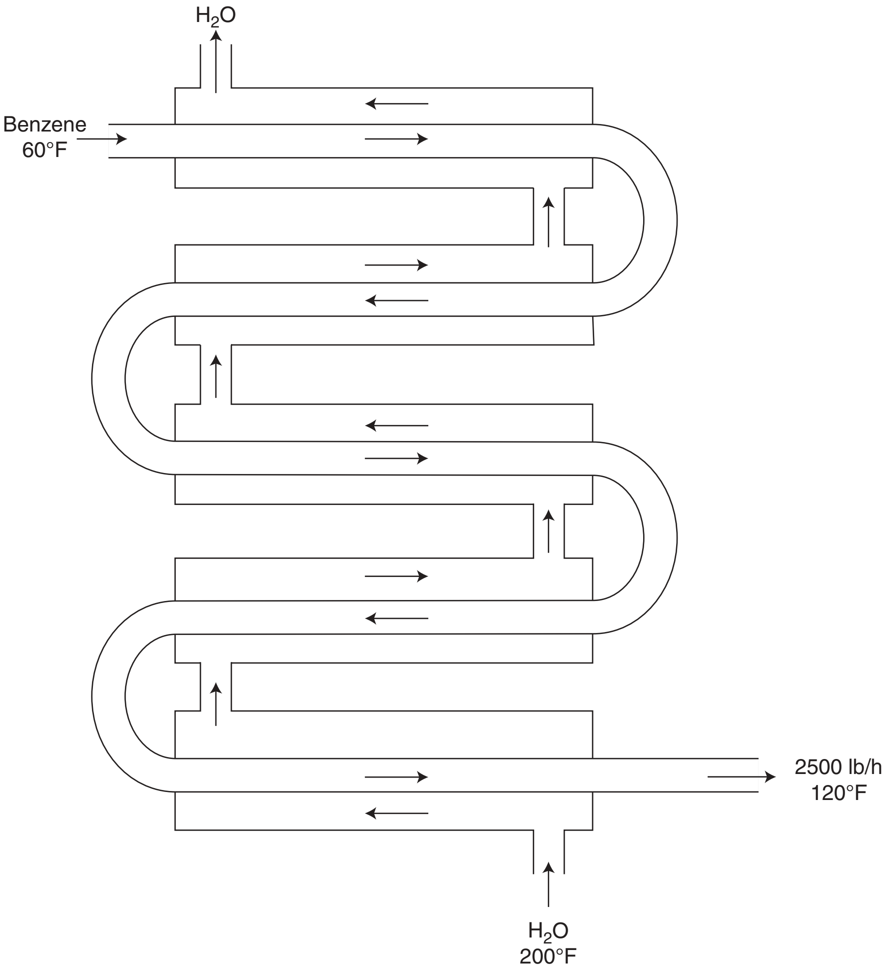


Figure 15.4 Trombone heat exchanger.

ILLUSTRATIVE EXAMPLE 15.8(7)

The double pipe heat exchanger shown in Figure 15.4 is used for heating 2500 lb/h benzene from 60°F to 120°F. Hot water at 200°F from a boiler is available for heating purposes in amounts up to 4000 lb/h. Use schedule 40 brass pipe and an integral number of 15 foot long sections are to be employed in the design of the exchanger.

The design is to be based on achieving a Reynolds number of 13,000 (turbulent flow) in both the inner pipe and annular region. As an additional constrain because of pressure drop consideration, the width of the annular region must be at least equal to one fourth the outside diameter of the inner pipe.

Note that one can obtain $\dot{Q}$ by calculating the duty of the benzene stream, and a heat balance will yield the exit temperature of the water, T_w. However, the area or the overall heat transfer coefficient is not known. Obtaining U becomes more complicated since information on flow variables or a pipe diameter is not given.

Data is provided below for this problem, but can be estimated or obtained from Tables in the Appendix:
Benzene at 90°F:

$$\begin{aligned}\mu &= 0.55\,\text{cP} = 3.70 \times 10^{-4}\,\text{lb/ft}\cdot\text{s}\\ \text{SG} &= 0.879\ (\text{at } 68.8^\circ\text{F})\\ \rho &= 0.879(62.4) = 54.8\,\text{lb/ft}^3\\ c_p &= 0.415\,\text{Btu/lb}\cdot{}^\circ\text{F}\\ k &= 0.092\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}\ (\text{at } 86^\circ\text{F})\end{aligned}$$

Water at 200°F:

$$\begin{aligned}\mu &= 0.305\,\text{cP} = 2.05 \times 10^{-4}\,\text{lb/ft}\cdot\text{s}\\ \rho &= 60.13\,\text{lb/ft}^3\\ c_p &= 1.0\,\text{Btu/lb}\cdot{}^\circ\text{F}\\ k &= 0.392\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}\end{aligned}$$

***SOLUTION*:** For flow inside tubes (pipes)

$$\text{Re} = \frac{4\dot{m}}{\pi D_i \mu} \tag{15.13}$$

Substituting,

$$D_i = \frac{4\dot{m}}{\text{Re}\,\pi\mu} = \frac{4(2500/3600)}{(13{,}000)\pi(3.7 \times 10^{-4})} = 0.184\ \text{ft} = 2.20\,\text{in}$$

The closest standard pipe with $D_i = 2.20$ in is 2-inch schedule 40 pipe (see also Chapter 6). Thus,

$$D_i = 2.067\,\text{in}$$

This results in

$$\text{Re} = 13{,}874$$

For 2-inch schedule 40 pipe

$$A_i = 0.541\ \text{ft}^2/\text{ft}$$

$$A_o = 0.622\ \text{ft}^2/\text{ft}$$

$$D_i = 2.067\,\text{in}$$

$$D_o = 2.375\,\text{in}$$

$$S_i = 0.0233\ \text{ft}^2(\text{inside area})$$

$$\Delta x_w = 0.154\,\text{in} = 0.0128\ \text{ft}$$

For flow in the annular region

$$\text{Re} = \frac{D_{eq}V\rho}{\mu} = \frac{D_{eq}\dot{m}}{\mu S} \tag{15.14}$$

with

$$D_{eq} = 4r_H = \frac{4S}{L_p} = \frac{4S}{\pi(D_{o,i} + D_{i,o})}$$

Therefore,

$$\text{Re} = \frac{4\dot{m}}{\pi(D_{o,i} + D_{i,o})\mu}$$

However, note the constraint:

$$0.5(D_{i,o} - D_{o,i}) \geq 0.25 D_{o,i}$$

or

$$D_{i,o} \geq 1.5 D_{o,i} = 3.562\,\text{in}$$

The smallest pipe, which satisfies this constraint, is 4-inch schedule 40 pipe ($D_{i,o} = 4.026\,\text{in}$). The outer pipe is therefore 4-inch schedule 40 pipe (see Chapter 6 once again).

For Re = 13,000, the water flow is:

$$\begin{aligned}\dot{m} &= \frac{\text{Re}\ \pi\mu(D_{o,i} + D_{i,o})}{4} \\ &= \frac{(13{,}000)\pi(2.05 \times 10^{-4})(3600)[(2.375/12) + (4.026/12)]}{4} \\ &= 4019\,\text{lb/h}\end{aligned}$$

This is close enough to 4000 lb/h.

The outlet water temperature is obtained from a heat balance:

$$\begin{aligned}\dot{Q} &= \dot{m}_C c_{pC} \Delta t_C = \dot{m}_H c_{pH} \Delta T_H \\ &= (2500)(0.415)(60) \\ &= 62{,}250\,\text{Btu/h}\end{aligned}$$

From a water heat balance, with $c_{pH} = 1.0$,

$$\Delta T_H = 62{,}250/4000 = 15.6^\circ\text{F}$$

$$T_{H,\text{out}} = 200 - 15.6 = 184.4^\circ\text{F}$$

$$T_{H,\text{av}} = (200 + 184.4)/2 = 192.2^\circ\text{F}$$

Properties of water at 192°F are estimated from the data at 200°F. At 200°F,

$$\begin{aligned} \mu &= 0.322\,\text{cP} = 2.16 \times 10^{-4}\,\text{lb/ft}\cdot\text{s} \\ \rho &= 60.13\,\text{lb/ft}^3 \\ c_p &= 1.0\,\text{Btu/lb}\cdot{}^\circ\text{F} \\ k &= 0.390\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F} \end{aligned}$$

The Dittus–Boelter equation is employed to determine the heat transfer coefficients:

$$\text{St} = 0.023(\text{Re})^{-0.2}(\text{Pr})^{-0.667}$$

For the inside pipe (benzene)

$$\begin{aligned} \text{Re} &= 13{,}000 \\ \text{Pr} &= \frac{c_p\mu}{k} = \frac{0.415(3.70 \times 10^{-4})}{0.092/3600} = 6.01 \\ \text{St} &= 0.023(13{,}000)^{-0.2}(6.01)^{-0.667} = 0.00105 \\ h_i &= \frac{(\text{St})c_p\dot{m}}{S} = \frac{(0.00105)(0.415)(2500)}{0.0233} = 46.7\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F} \end{aligned}$$

For the annular region (water)

$$\begin{aligned} \text{Re} &= \frac{4\dot{m}}{\pi\mu(D_{o,i} + D_{i,o})} = \frac{4(4000/3600)}{\pi(2.16 \times 10^{-4})[(2.375/12) + (4.026/12)]} = 12{,}279 \\ \text{Pr} &= \frac{c_p\mu}{k} = \frac{1(2.16 \times 10^{-4})}{0.390/(3600)} = 1.99 \\ \text{St} &= 0.023(12{,}278)^{-0.2}(1.99)^{-0.667} = 0.00221 \\ S_{\text{annulus}} &= \frac{\pi}{4}(D_{i,o}^2 - D_{o,i}^2) = \frac{\pi}{4}[(4.026)^2 - (2.375)^2]/144 = 0.0576\ \text{ft}^2 \\ h_o &= \frac{0.00221 c_p\dot{m}}{S_{\text{annulus}}} = \frac{0.00221(1)(4000)}{0.0576} = 153\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F} \end{aligned}$$

Neglecting fouling,

$$\frac{1}{U_o} = \frac{D_o}{D_i h_i} + \frac{\Delta x_w D_o}{k_w D_{\text{lm}}} + \frac{1}{h_o} \tag{15.11}$$

For the pipe,

$$\begin{aligned} D_{\text{lm}} &= \frac{2.375 - 2.067}{\ln(2.375/2.067)} = 2.217\,\text{in} \\ \frac{1}{U_o} &= \frac{2.375}{(2.067)(46.7)} + \frac{(0.0128)(2.375)}{(26)(2.217)} + \frac{1}{153} \\ &= 0.0246 + 0.0005274 + 0.00654 \\ &= 0.0316 \end{aligned}$$

Therefore

$$U_o = 31.6\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

Finally, to obtain the length, use

$$\dot{Q} = U_o A_o \Delta T_{\text{lm}}$$
$$\Delta T_{\text{lm}} = \frac{124.4 - 80}{\ln(24.4/80)} = 100.6^\circ\text{F}$$

In addition,

$$A_o = 0.622L$$

The heat exchanger equation is again employed to calculate the length

$$\begin{aligned} 62{,}250 &= (31.6)(0.622)(L)(100.6) \\ L &= 31.5\ \text{ft} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 15.9

With reference to the previous example, how many sections should be required in the design.

***SOLUTION*:** Two sections would do the job. However, three sections might be recommended; the unit would then be somewhat over-designed. ■

CALCULATION OF EXIT TEMPERATURES(7)

The equations describing exit temperatures for parallel flow heat exchangers follows. These equations are based on three assumptions:

1. Streams do not experience phase change from inlet to exit.
2. Heat capacities of both streams are constant.
3. A single average U applies for the entire exchanger.

For parallel (p) flow, one notes (with capital and lowercase letters representing the hot and cold fluid, respectively, as per Famularo's(7) notation)

$$\begin{aligned} \Delta T_1 &= T_1 - t_1 \\ \Delta T_2 &= T_2 - t_2 \end{aligned} \qquad (15.25)$$

In addition,

$$\dot{Q}_p = \dot{M}C(T_1 - T_2) = \dot{m}c(t_2 - t_1) \qquad (15.26)$$

or

$$\frac{T_1 - T_2}{B} = \frac{t_2 - t_1}{b}$$

with

$$B = \frac{1}{\dot{M}C}$$
$$b = \frac{1}{\dot{m}c}$$

Combining the above equation with the heat exchanger equation ($\dot{Q} = UA\Delta T_{\text{lm}}$) leads to

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -(B + b)UA \tag{15.27}$$

Combining Equations (15.26) and (15.27) gives

$$T_2 = \frac{[(b/B) + e^{-UA(B+b)}]T_1 + [1 - e^{-UA(B+b)}]t_1}{1 + (b/B)} \tag{15.28}$$

The cold fluid outlet temperature may be calculated by replacing T_2, T_1, and t_1 in Equation (15.28) by t_2, t_1, and T_1, respectively.$^{(7)}$

For countercurrent (c) flow, one notes

$$\begin{aligned} \Delta T_1 &= T_1 - t_2 \\ \Delta T_2 &= T_2 - t_1 \end{aligned} \tag{15.29}$$

Once again,

$$\dot{Q}_c = \dot{M}C(T_1 - T_2) = \dot{m}c(t_2 - t_1) \tag{15.30}$$

or

$$\frac{T_1 - T_2}{B} = \frac{t_2 - t_1}{b}$$

Combining the above equations with the heat transfer equation leads to

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -(B - b)UA \tag{15.31}$$

Combining Equation (15.30) and (15.31) gives

$$T_2 = \frac{[(B/b) - 1]T_1 + \left[(B/b)e^{UA(B-b)} - (B/b)\right]t_1}{(B/b)e^{UA(B-b)} - 1} \tag{15.32}$$

The cold fluid outlet temperature may be calculated by replacing T_2, T_1, and t_1 in Equation (15.32) by t_2, t_1, and T_1, respectively.$^{(7)}$

Another application involves a simplified case in which the fluid temperature on one side of the dividing wall (tube/pipe) is constant. If one can assume that

$$\dot{M}C \ggg \dot{m}c$$

or

$$\frac{1}{\dot{M}C} \lll \frac{1}{\dot{m}c} \tag{15.33}$$

and $T_2 = T_1 = T$ and $B \lll b$, the co-current case reduces to

$$\ln\left(\frac{T - t_2}{T - t_1}\right) = -bUA \tag{15.34}$$

In addition, the countercurrent case reduces to

$$\ln\left(\frac{T - t_1}{T - t_2}\right) = bUA \tag{15.35}$$

If the cold stream is flowing inside the inner pipe, then

$$UA = U_i A_i \tag{15.36}$$

and

$$t_2 = T - (T - t_1)e^{-U_i A_i/\dot{m}c} \tag{15.37}$$

This equation applies to both the co-current and countercurrent case. There are two other situations in which the fluid temperature is constant, and the above equation applies:

1. Condensation of a vapor at T_s without subcooling. Set $T_s = T$ in Equation (15.37).
2. Outside flow at T_∞ normal to a tube. Set $T_\infty = T$ in Equation (15.37).

ILLUSTRATIVE EXAMPLE 15.10

Calculate T_2 and t_2 in (consistent units)

$$\begin{aligned} MC &= \dot{m}_H c_{pH} = 2000 \\ mc &= \dot{m}_C c_{pC} = 1000 \\ U &= 2000 \\ A &= 10 \\ T_1 &= 300 \\ t_1 &= 60 \end{aligned}$$

Calculate the exit temperatures if the flow is *countercurrent* by employing Equation (15.32).(7)

***SOLUTION*:** For this system:

$$B = 0.001, \quad b = 0.0005, \quad \text{and} \quad B/b = 2.0$$

In addition,

$$UA(B - b) = 2000(0.001 - 0.0005) = 1.0$$

so that

$$e^{1.0} = 2.7183$$

Substituting into Equation (15.32) gives

$$\begin{aligned} T_2 &= \frac{(2 - 1)300 + 2(2.7183 - 1)60}{2(2.7183) - 1} \\ &= \frac{506.2}{4.4366} = 114.1^\circ\text{F} = 114^\circ\text{F} \end{aligned}$$

Employ a revised Equation (15.32) or an overall energy balance to generate t_2,

$$\begin{aligned} t_2 &= t_1 + \frac{T_1 - T_2}{(B/b)} = 60 + \frac{300 - 114.1}{2} \\ &= 152.95^\circ\text{F} = 153^\circ\text{F} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 15.11

The following information is provided for a *co-current* flow double pipe heat exchanger. The pipe consists of 200 ft of 2-inch schedule 40 pipe, $k = 25\,\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$. The hot and cold film coefficients are 1200 and 1175 $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$, respectively, In addition,

$$\begin{aligned} MC &= \dot{m}_H c_{pH} \\ mc &= \dot{m}_C c_{pC} = 22{,}300\,\text{Btu/h} \cdot {}^\circ\text{F} \\ T_1 &= 300^\circ\text{F} \\ t_1 &= 60^\circ\text{F} \end{aligned}$$

Calculate T_2 and t_2.

***SOLUTION*:** First calculate the overall inside heat transfer coefficient. For the pipe

$$\begin{aligned} \text{ID} &= 2.067\,\text{in} \\ \text{OD} &= 2.375\,\text{in} \\ \Delta x &= 0.154\,\text{in} \end{aligned}$$

$$\begin{aligned} A_i' &= 0.541\ \text{ft}^2/\text{ft} \\ k &= 25\,\text{Btu/h} \cdot\ \text{ft} \cdot {}^\circ\text{F} \end{aligned}$$

Thus,

$$U_i = \cfrac{1}{\cfrac{1}{1200} + \cfrac{0.154}{(25)(12)} + \cfrac{1}{1175(2.375/2.067)}}$$
$$= 482\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

In addition,

$$\begin{aligned} A_i &= (A_i')(L) \\ &= (0.541)(200) \\ &= 108.2\ \text{ft}^3 \end{aligned}$$

There are two key equations—one based on an energy balance and one based on rate considerations. These two equations are:

$$\begin{aligned} \dot{Q}_H &= \dot{Q} = MC(300 - T_2) \\ &= (30{,}000)(300 - T_2) \qquad (1a) \end{aligned}$$

$$\dot{Q}_C = 22{,}300(t_2 - 60) \qquad (1b)$$

with

$$\dot{Q}_C = \dot{Q}_H$$

and

$$\begin{aligned} \dot{Q} &= U_i A_i \Delta T_{\text{lm}} \qquad (2) \\ &= (482)(108.2)\left\{\frac{(300 - 60) - (T_2 - t_2)}{\ln[(300 - 60)/(T_2 - t_2)]}\right\} \end{aligned}$$

There are also the two unknowns—T_2 and t_2. These can be solved by any suitable numerical method. By trial-and-error, one obtains

$$\begin{aligned} T_2 &= 200^\circ\text{F} \\ t_2 &= 195^\circ\text{F} \end{aligned}$$

with

$$\dot{Q} = 3.0 \times 10^6\ \text{Btu/h}$$

■

ILLUSTRATIVE EXAMPLE 15.12

Solve the previous example employing the equations above provided by Famularo.(7) Also calculate the discharge temperature, the LMTD, and $\dot{Q}$ for the *countercurrent* case.

***SOLUTION*:** Equation (5.28) applies for the cocurrent case

$$T_2 = \frac{[(b/B) + e^{-UA(B+b)}]T_1 + [1 - e^{-UA(B+b)}]t_1}{1 + (b/B)}$$

For $B = 3.33 \times 10^{-5}$, $b = 4.48 \times 10^{-5}$, $A = 108.2$ and $U = 482$, one obtains

$$T_2 = 199.4 \approx 200°\text{F}$$

Apply an energy balance to both fluids.

$$30{,}000(300 - T_2) = 22{,}300(t_2 - 60)$$

For $T_2 = 200°$F, one obtains

$$t_2 = 195°\text{F}$$

The agreement between both methods is excellent. Note that temperature t_2 may also be calculated directly from Equation (15.28) with the temperature(s) appropriately reversed.

Substitute into Equation (15.32) to obtain T_2 for the countercurrent case:

$$T_2 = \frac{(B/b)[e^{UA(B-b)} - 1]t_1 + [(B/b) - 1]T_1}{(B/b)e^{UA(B-b)} - 1}$$

Once again, for $B = 3.33 \times 10^{-5}$, $b = 4.48 \times 10^{-5}$, $A = 108.2$ and $U = 482$, one obtains

$$T_2 = 164.20 = 164°\text{F}$$

Substitute into Equation (15.32) with the temperature reversed to generate t_2:

$$t_2 = \frac{(b/B)[e^{UA(b-B)} - 1]T_1 + [(b/B) - 1]t_1}{(b/B)e^{UA(b-B)} - 1}$$

Substituting gives

$$t_2 = 242.85 = 243°\text{F}$$

For the LMTD,

$$\Delta T_{\text{lm}} = \frac{(T_2 - t_1) - (T_1 - t_2)}{\ln\left(\dfrac{T_2 - t_1}{T_1 - t_2}\right)}$$

Substituting

$$\begin{aligned} T_2 - t_1 &= 164 - 60 = 104 \\ T_1 - t_2 &= 300 - 243 = 57 \\ \Delta T_{\text{lm}} &= \frac{104 - 57}{\ln(104/57)} \\ &= 78.2°\text{F} \end{aligned}$$

Apply the standard heat transfer equation to calculate $\dot{Q}$:

$$\begin{aligned} \dot{Q} &= UA\Delta T_{\text{lm}} \qquad (15.6) \\ &= \left(482\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}\right)(108.2\ \text{ft}^2)(78.2°\text{F}) \\ &= 4{,}078{,}000\ \text{Btu/h} \end{aligned}$$

This result may be checked by employing the heat load equation:

$$\begin{aligned}\dot{Q} &= MC(T_1 - T_2)\\ &= (30{,}000\ \text{Btu/h}\cdot{}^\circ\text{F})(300 - 164.20)\\ &= 4{,}074{,}000\ \text{Btu/h}\end{aligned}$$

The two results are once again in reasonable agreement. ■

EFFECTIVENESS FACTOR AND NUMBER OF TRANSFER UNITS

The effectiveness factor (or effectiveness) and number of transfer units are two approaches that have been employed in heat exchanger design and analysis. The effectiveness, ε, of a heat exchanger is defined as

$$\varepsilon = \frac{\text{actual heat transfer rate}}{\text{maximum heat transfer rate}} = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} \tag{15.38}$$

A maximum rate, $\dot{Q}_{\text{max}}$, is calculated for the hot side and the cold side (tube and annulus side, respectively) for the double pipe heat exchanger. For example, assuming countercurrent flow,

$$\dot{Q}_{\text{max,tube}} = (\dot{m}c_p)_{\text{tube}}|T_1 - t_1| = C_{\text{tube}}|T_1 - t_1| \tag{15.39}$$

$$\dot{Q}_{\text{max,annulus}} = (\dot{m}c_p)_{\text{annulus}}|T_1 - t_1| = C_{\text{annulus}}|T_1 - t_1| \tag{15.40}$$

The C term is defined as the thermal capacitance rate and is simply $\dot{m}c_p$. The lower $\dot{Q}_{\text{max}}$ value is used in evaluating the effectiveness.

The effectiveness factor, ε, is applied in simulation (predictive) studies rather than design. It enables one to calculate exit temperatures and heat transfer rates for an existing unit. The effectiveness factor is provided below for both systems (parallel, countercurrent) and for both hot and cold side. Note the development of these equations assumes that the hot fluid is flowing in the inside tube.

Cocurrent flow:

$$\begin{aligned}\text{tube: } \varepsilon_H &= \frac{T_1 - T_2}{T_1 - t_1}\\ \text{annulus: } \varepsilon_C &= \frac{t_2 - t_1}{T_1 - t_1}\end{aligned} \tag{15.41}$$

Countercurrent flow:

$$\begin{aligned} \text{tube: } \varepsilon_H &= \frac{T_1 - T_2}{T_1 - t_1} \\ \text{annulus: } \varepsilon_C &= \frac{t_2 - t_1}{T_1 - t_1} \end{aligned} \tag{15.42}$$

Note that subscripts 1 and 2 refer to inlet and outlet conditions, respectively.

Thus, if the effectiveness factor is known, one could calculate the exit temperature T_2 or t_2 (and the corresponding $\dot{Q}$). The question that remains is whether to use ε_H or ε_C. The fluid to employ, hot or cold, is that which undergoes the maximum temperature change (or difference) or the corresponding minimum value of C:

$$\dot{Q} = (\dot{m}c_p)\Delta T = (\dot{m}c_p)_H(T_1 - T_2) = (\dot{m}c_p)_C(t_2 - t_1) \tag{15.43}$$

If the temperatures are unknown and the flow rates are known, the minimum value of $\dot{m}c_p$ is obtained from flow rate data. The $\dot{m}c_p$ value for the fluid in question thus requires the use of the appropriate ε_H or ε_C, and this equation calculates the exit temperature. This general topic will be reviewed again in the next chapter, particularly as it applies to shell and tube heat exchangers.

Another term employed in heat exchanger studies is the number of transfer units, NTU, of a heat exchanger. It is defined as

$$\text{NTU} = \frac{UA}{(\dot{m}c_p)_{\min}} = \frac{UA}{C_{\min}} \tag{15.44}$$

where $C_{\min}$ is the minimum thermal capacitance rate. NTU is determined by calculating C for each fluid and choosing the lower value.

ILLUSTRATIVE EXAMPLE 15.13

Is an absolute maximum value of 1.0 for ε achievable?

***SOLUTION*:** The answer is no since ΔT would go to zero and therefore the area required would be infinite; this would be a violation of the Second Law of Thermodynamics.(8) ■

ILLUSTRATIVE EXAMPLE 15.14

A double pipe heat exchanger is employed to heat 228 kg/h of water from 25°C to 73°C. Water is on the tube side and the flow is countercurrent. The heating fluid is saturated steam at 1 atm, with 55 kg/h of steam condensing on the annular side. No subcooling of condensed water takes place. The inner pipe of the double pipe heat exchanger has an inner radius ($r_{i,i}$) of 1.3 cm and an outer radius ($r_{o,i}$) of 1.9 cm. The inner radius of the outer tube ($r_{i,o}$) is 3.2 cm. Both tubes are 0.5% carbon steel. The overall heat transfer coefficient for the new heat exchanger, U_{clean}, was originally determined to be 3994 $\text{W/m}^2 \cdot \text{K}$, based on the inside area of the inner pipe. After a period of use, the fouling factor, R_f, was estimated to be 0.0002 $\text{m}^2 \cdot \text{K/W}$. Determine:

1. the temperature profile of the water and steam along the length of the exchanger,
2. whether the steam has enough energy to heat the water,
3. the minimum length of the heat exchanger, and
4. the dirty overall heat transfer coefficient.

Water properties at the average bulk temperature $(25 + 73)/2 = 49°\text{C}$ are: heat capacity, $c_p = 4174\ \text{J/kg} \cdot \text{K}$; density, $\rho = 993\ \text{kg/m}^3$; absolute viscosity, $\mu = 6.82 \times 10^{-4}\ \text{kg/m} \cdot \text{s}$; kinematic viscosity, $\nu = 6.87 \times 10^{-7}\ \text{m}^2/\text{s}$, and the Prandtl number, $\text{Pr} = 4.53$. The properties of water at the "average" film temperature $(100 + 49)/2 = 74.9°\text{C}$ are: heat capacity, $c_p = 4190$ $\text{J/kg} \cdot \text{K}$; density, $\rho = 975\ \text{kg/m}^3$; absolute viscosity, $\mu = 3.8 \times 10^{-4}\ \text{kg/m} \cdot \text{s}$; kinematic viscosity, $\nu = 3.89 \times 10^{-7}\ \text{m}^2/\text{s}$, and the Prandtl number, $\text{Pr} = 2.53$.

For saturated steam at 100°C: latent heat of condensation $= 2.26 \times 10^6\ \text{J/kg}$, vapor density $= 0.596\ \text{kg/m}^3$. The thermal conductivity for 0.5% carbon steel at an average temperature of $(100 + 25)/2 = 62.5°\text{C}$ is $k = 54\ \text{W/m} \cdot \text{K}$

Assumptions include steady-state operation, incompressible fluids, Newtonian fluids, constant properties, homogeneous isotropic materials, and there are negligible heat losses. Notations 1 and 2 denote the water inlet and outlet, respectively, and *s* the stream.

***SOLUTION*:** Organize the given information by aligning the data in tabular form as shown in Table 15.3. The temperature profile in the heat exchanger is provided in Figure 15.5.

Calculate the temperature driving forces at both ends of the heat exchanger and ΔT_{lm}.

$$\Delta T_1 = (T_s - t_1) = 100 - 25 = 75°\text{C} = 75\ \text{K}$$
$$\Delta T_2 = (T_s - t_2) = 100 - 73 = 27°\text{C} = 27\ \text{K}$$
$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{75 - 27}{\ln(75/27)} = 47°\text{C}$$

Table 15.3 Key Parameters for Illustrative Example 15.14

	Stream			
Parameter	1	2	s	s
Type	Water		Steam	
Side	Tube		Annular	
ρ, kg/m^3	993		0.596	
c_p, $\text{J/kg} \cdot \text{K}$	4174		N/A	
μ, $\text{kg/m} \cdot \text{s}$	6.82×10^{-4}		N/A	
k, $\text{W/m} \cdot \text{K}$	0.63		N/A	
h_{vap}, J/kg	N/A		2.26×10^6	
t, T_s, °C	25	73	100	100
$\dot{m}$, kg/s	0.0633		0.01528	

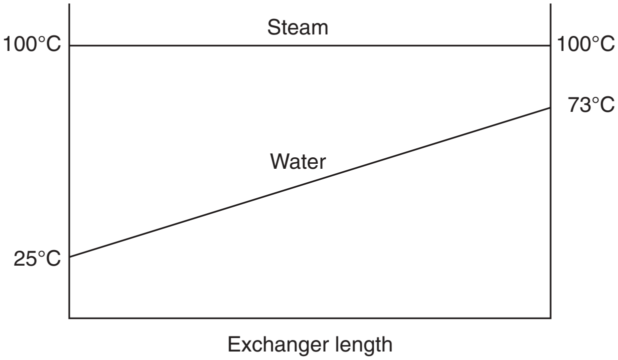


Figure 15.5 Temperature profile; Illustrative Example 15.14.

Calculate both the capacitance rate ($\dot{m}c_p$) of water and the heat load:

$$C_w = \dot{m}c_p = (228)(4174)(1\,\text{h}/3600\,\text{s}) = 264\,\text{W/K}$$

$$\dot{Q} = \dot{m}c_p(t_2 - t_1) = C_w(t_2 - t_1) = (264)(73 - 25) = 12{,}700\,\text{W} = 4568\,\text{kJ/h}$$

Determine the maximum heat term from the water stream:

$$\dot{Q}_{\max,w} = C_w(T_s - t_1) \tag{15.39}$$

Substituting

$$\begin{aligned}\dot{Q}_{\max,w} &= (264)(100 - 25)\\ &= 19{,}800\,\text{W} = C_w(T_s - t_1)\end{aligned}$$

Also, determine the maximum heat term for the steam assuming that all the steam condenses to liquid:

$$\dot{Q}_{\max,st} = \dot{m}_{\text{steam}}h_{\text{vap}} = (55)(2.26 \times 10^6)(1\,\text{h}/3600\,\text{s}) = 34{,}500\,\text{W}$$

Select the appropriate $\dot{Q}_{\max}$ for calculating the effectiveness. Since $\dot{Q}_{\max,w}$ is the smaller value, $\dot{Q}_{\max} = \dot{Q}_{\max,w} = 19{,}800\,\text{W}$.

The effectiveness, ε, from Equation (15.38) is then

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{12{,}700}{19{,}800} = 0.641$$

The minimum required length of the heat exchanger is (see Equation (15.24)):

$$\dot{Q} = \frac{2\pi k L_{\min}(T_s - t_1)}{\ln(r_{o,i}/r_{i,i})}$$

$$L_{\min} = \frac{\dot{Q}\ln(r_{o,i}/r_{i,i})}{2\pi k(T_s - T_1)} = \frac{(12{,}700)\ln(1.9/1.3)}{2\pi(54)(100 - 25)} = 0.189\,\text{m}$$

Determine the effects of fouling. Employ Equation (15.23):

$$\begin{aligned}R_f &= 0.0002\,\mathrm{m}^2\cdot\mathrm{K/W}\\ \frac{1}{U_{\text{dirty}}} &= \frac{1}{U_{\text{clean}}}+R_f=\frac{1}{3994}+0.0002=0.00025+0.0002\\ &= 0.00045\end{aligned}$$

Inverting,

$$U_{\text{dirty}} = 2220\,\mathrm{W/m}^2\cdot\mathrm{K}$$ ■

ILLUSTRATIVE EXAMPLE 15.15

Refer to the previous example. Calculate the area and the length of the heat exchanger.

***SOLUTION*:** The heat transfer area may be calculated directly from Equation (15.6) employing U_{dirty}.

$$A=\frac{\dot{Q}}{U_{\text{dirty}}\Delta T_{\text{lm}}}=\frac{12{,}700}{2220(47)}=0.1217\,\mathrm{m}^2=1.31\ \mathrm{ft}^2$$

The tube length (based on the inner radius) is then

$$L=\frac{A}{2\pi r_{i,i}}=\frac{0.1217}{2\pi(0.013)}=1.49\,\mathrm{m}$$ ■

ILLUSTRATIVE EXAMPLE 15.16

With reference to Illustrative Example 15.14, calculate the number of transfer units (NTU) of the exchanger.

***SOLUTION*:** The effectiveness was calculated in the previous example. The number of transfer units may also be calculated from Equation (15.44):

$$\mathrm{NTU}=\frac{U_{\text{dirty}}A}{C_w}=\frac{2220(0.1217)}{264}=1.02$$ ■

ILLUSTRATIVE EXAMPLE 15.17

How would the answers differ if the flow were co-current in Illustrative Example 15.14?

***SOLUTION*:** Since T_s is constant, $\Delta T_1=\Delta T_2=\Delta T_{\text{lm}}$. Thus, the answers would be the same for a co-current heat exchanger. ■

WILSON'S METHOD

There is a procedure for evaluating the *outside* film coefficient for a double pipe unit. Wilson's method$^{(9)}$ is a graphical technique for evaluating this coefficient. The inside coefficient is a function of the Reynolds and Prandtl numbers via the Dittus–Boelter equation presented in Equation (15.18), i.e.,

$$h_i = f(\text{Re}^{0.8}\text{Pr}^{0.3}) \tag{15.45}$$

A series of experiments can be carried out on a double pipe exchanger where all conditions are held relatively constant except for the velocity (V) of the cooling (in this case) inner stream. Therefore, for the proposed experiment:

$$h_i = f(\text{Re}^{0.8}) = f(V^{0.8}) = f(\dot{m});\ \text{Re} = DV\rho/\mu \tag{15.46}$$

or, in equation form,

$$h_i = aV^{0.8} \tag{15.47}$$

where a is a constant. Equation (15.47) can be substituted into the overall coefficient equation

$$\frac{1}{U_oA_o} = R_o + R_w + R_i$$

so that

$$= \frac{1}{h_oA_o} + \frac{\Delta x}{kA_{\text{lm}}} + \frac{1}{aV^{0.8}A_i} \tag{15.48}$$

Data can be taken at varying velocities. By plotting $1/U_oA_o$ versus $1/V^{0.8}$, a straight line should be obtained since the first two terms of Equation (15.48) are constants. The intercept of this line corresponds to an infinite velocity and an inside resistance of zero. Thus, the above equation may be rewritten as

$$\left(\frac{1}{U_oA_o}\right)_{\text{intercept}} = \frac{1}{h_oA_o} + \frac{\Delta x}{kA_{\text{lm}}} \tag{15.49}$$

The second term on the right-hand side is known and/or can be calculated and h_o can then be evaluated from the intercept (details on this calculational scheme are provided later in the illustrative examples). Fouling coefficients, *f*, can be estimated by the Wilson method if the outside fluid coefficient, h_o, can be predicted or is negligible. Note that the fouling resistance is normally included in the intercept value.

ILLUSTRATIVE EXAMPLE 15.18

In 1975, Arthur Andrews$^{(10)}$—a chemical engineering senior at Manhattan College—conducted a double pipe heat exchanger experiment in the Unit Operations Laboratory. Some of the

Table 15.4 Wilson Method Information: Illustrative Example 15.18

Run	$1/U_oA_o$, °F · h/Btu	T, °F Average temperature	$1/V$, h/ft
1	1.2176×10^{-3}	124.5	2.29×10^{-4}
2	0.9454×10^{-3}	125	1.65×10^{-4}
3	0.9366×10^{-3}	129	1.29×10^{-4}
4	0.8618×10^{-3}	121.5	1.117×10^{-4}
5	0.7966×10^{-3}	122	0.962×10^{-4}

co-current flow data and calculations submitted are presented in Table 15.4. Employing Andrew's data and Wilson's method, estimate h_o for the exchanger fluid. For this lab unit (see Figure 15.1), $A_o = 1.85$ ft^2.

***SOLUTION*:** A "best" straight line representation plot of $1/U_oA_o$ versus $1/V^{0.8}$ is provided in Figure 15.6. The intercept $(1/h_oA_o)$ is approximately 0.560×10^{-3}. This value may now be used to estimate h_o:

$$h_o = 1/(0.560 \times 10^{-3})(1.85) = 965 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

■

ILLUSTRATIVE EXAMPLE 15.19

Another experiment$^{(11)}$ on the double pipe heat exchanger in the Unit Operation Laboratory at Manhattan College produced the following data and calculated results for U (overall heat transfer coefficient) and $\dot{m}_C$ (inside cold water flow rate); see Table 15.5.

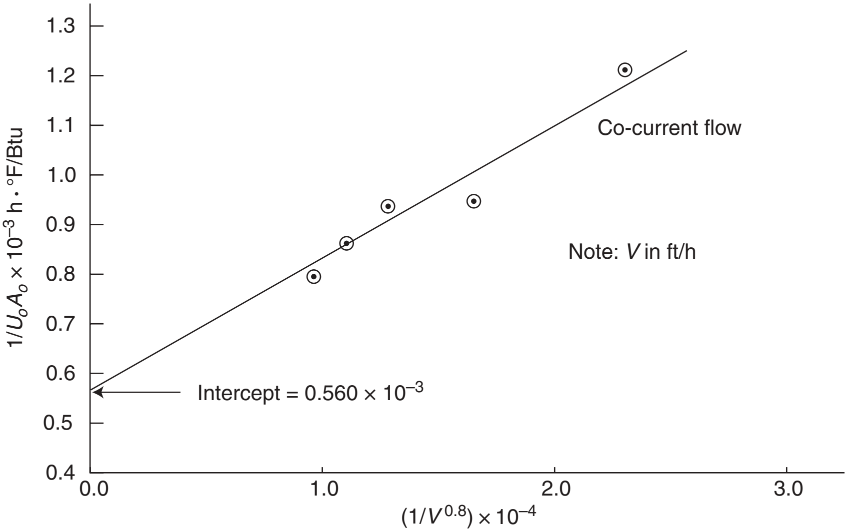


Figure 15.6 Wilson method plot: co-current flow.

Table 15.5 Wilson Experiment Data and Results

U, Btu/h · ft^2 · °F	$\dot{m}_C$, lb/h	$\dot{m}_C^{-0.8}$
687.98	500.97	144.49
740.74	1001.94	251.58
748.10	1502.90	347.97
757.08	2003.87	438.02
756.31	2204.26	472.73

Using the Wilson method, estimate the outside fluid film coefficient. Once again, neglect the effects of scale and pipe wall.

SOLUTION$^{(12)}$**:** Noting once again that $\dot{m}_C$ is linearly related to V, regress the data in columns (2) and (5) of Table 15.6 to the equation

$$\frac{1}{U} = a + b\dot{m}_C^{-0.8}$$

For this data (see Figure 15.7)

$$a = 0.00126$$
$$b = 0.0276$$

The term a represents the inverse of the h_o so that

$$h_o = 1.0/a = 1.0/0.00126$$
$$= 794\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

■

ILLUSTRATIVE EXAMPLE 15.20

Refer to the previous illustrative example. Perform the same calculation but include the resistance associated with the tube (Admiralty metal) if the inside and outside diameters are 1.00 and 0.902 in, respectively; k for the tube is 60 Btu/h · ft^2 · °F.

Table 15.6 Wilson Experiment Data and Results

U, Btu/h · ft^2 · °F	$1/U$, h · ft^2 · °F	$\dot{m}_C$, lb/h	$\dot{m}_C^{0.8}$	$\dot{m}_C^{-0.8}$
687.98	0.0014535	500.97	144.49	0.00692071
740.74	0.00135	1001.94	251.58	0.003974904
748.10	0.0013367	1502.90	347.97	0.002873796
757.08	0.0013209	2003.87	438.02	0.002282992
756.31	0.0013222	2204.26	472.73	0.002115387

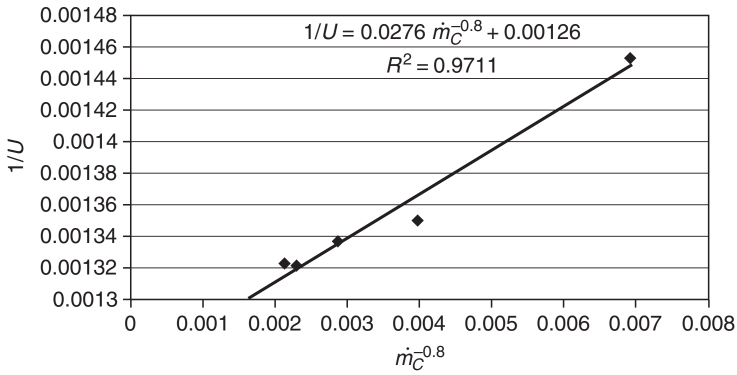


Figure 15.7 Wilson Plot for Illustrative Example 15.19.

***SOLUTION*:** From the concept of resistances in series, it is clear that the total resistance is equal to the sum of the individual resistances (i.e., $1/U = R_o + R_w + R_s + R_i$). If the scale is neglected, the *a* term in the previous example represents the outside and wall resistances.

From Equation (15.49), the resistance of the wall is (neglecting any area correction) is

$$\begin{aligned} R_w &= \frac{\Delta r}{k} = \frac{(1.00 - 0.902)/2}{(12)(60)} \\ &= 0.681 \times 10^{-4}\ \mathrm{Btu/h \cdot {}^\circ F} \end{aligned}$$

Therefore, the contribution of the wall resistance to the term *a* is 0.681×10^{-4}. The revised h_o is then

$$\begin{aligned} h_o &= \frac{1.0}{0.00126 - 0.0000681} \\ &= 839\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 15.21

Data from experiments based on Wilson's method produced the following results:

$$\text{Clean tube:} \quad \frac{1}{U_{\text{clean}}} = 0.00044 + \frac{0.0050}{(\dot{m})^{0.8}}$$

$$\text{Tube with scale:} \quad \frac{1}{U_{\text{dirty}}} = 0.00089 + \frac{0.0050}{(\dot{m})^{0.8}}$$

Calculate the scale film coefficient if the wall resistance can be neglected.

***SOLUTION*:** The overall resistance has been subdivided into component resistances. By comparing the two equations, one obtains the resistance associated with the scale to be:

$$\begin{aligned} R_{\text{dirty}} &= R_s = 0.00089 - 0.00044 \\ &= 0.00045 \end{aligned}$$

Thus

$$h_s = \frac{1}{0.00045} = 2222\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$ ■

ILLUSTRATIVE EXAMPLE 15.22

With reference to the Wilson method, discuss why the resistance of the wall is often neglected.

***SOLUTION*:** The resistance of the wall is usually small (due to the high conductivity of the metal) in comparison to the resistances of the two flowing fluids. ■

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Chapter 16

Shell and Tube Heat Exchangers

INTRODUCTION

Shell and tube (also referred to as tube and bundle) heat exchangers provide a large heat transfer area economically and practically. The tubes are placed in a bundle and the ends of the tubes are mounted in tube sheets. The tube bundle is enclosed in a cylindrical shell, through which the second fluid flows. Most shell and tube exchangers used in practice are of welded construction. The shells are built as a piece of pipe with flanged ends and necessary branch connections. The shells are made of seamless pipe up to 24 inches in diameter; they are made of bent and welded steel plates, if above 24 inches. Channel sections are usually of built-up construction, with welding-neck forged-steel flanges, rolled-steel barrels and welded-in pass partitions. Shell covers are either welded directly to the shell, or are built-up constructions of flanged and dished heads and welding-neck forged-steel flanges. The tube sheets are usually nonferrous castings in which the holes for inserting the tubes have been drilled and reamed before assembly. Baffles can be employed to both control the flow of the fluid outside the tubes and provide turbulence.

There are vast industrial uses of shell and tube heat exchangers. These units are used to heat or cool process fluids, either through a single-phase heat exchanger or a two-phase heat exchanger. In single-phase exchangers, both the tube side and shell side fluids remain in the same phase that they enter. In two-phase exchangers (examples include condensers and boilers), the shell side fluid is usually condensed to a liquid or heated to a gas, while the tube-side fluid remains in the same phase.

Generally, shell and tube exchangers are employed when double pipe exchangers do not provide sufficient area for heat transfer. Shell and tube exchangers usually require less materials of construction and are consequently more economical when compared to double pipe and/or multiple double pipe heat exchangers in parallel.

Chapter topics include:

Equipment Description

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

Describing Equations
The “F” Factor
Effectiveness Factor and Number of Transfer Units

EQUIPMENT DESCRIPTION

There are more than 280 different types of shell and tube heat exchangers that have been defined by the Tubular Exchanger Manufacturers Association (TEMA). The simplest shell and tube heat exchanger has a single pass through the shell and a single pass through the tubes. This is termed a 1–1 shell and tube heat exchanger. A photograph of this unit is provided in Figure 16.1. A schematic (line diagram) of a 1–2 unit can be found in Figure 16.2. A side view of the tubes in a typical exchanger is shown in Figure 16.3. Fluids that flow through tubes at low velocity result in low heat transfer coefficients and low pressure drops. To increase the heat transfer rates, multipass operations may be used. As noted earlier, baffles are used to divert the fluid within the distribution header. An exchanger with one pass on the shell side and four tube passes is termed a 1–4 shell and tube heat exchanger. It is also possible to increase the number of passes on the shell side by using dividers. A 2–8 shell and tube heat exchanger has two passes on the shell side and 8 passes on the tube side.

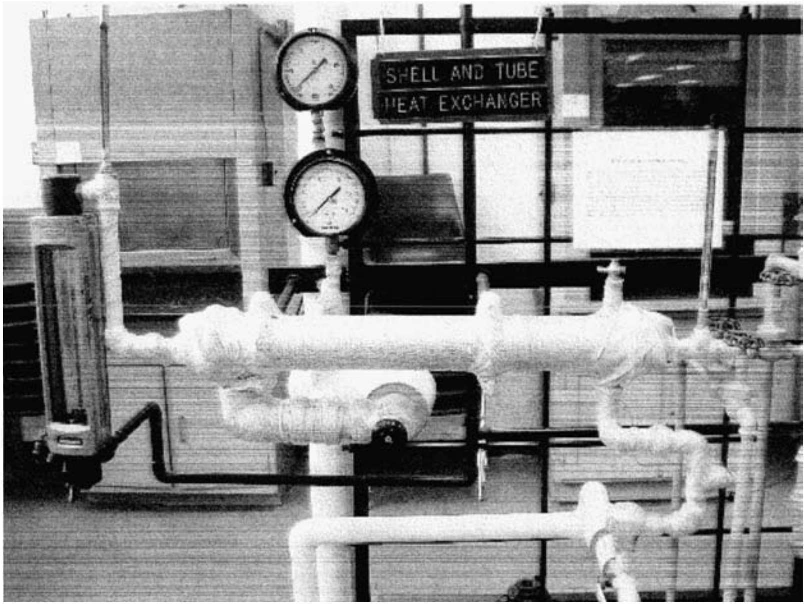


Figure 16.1 Shell and tube heat exchanger—Manhattan College Unit Operations Laboratory.

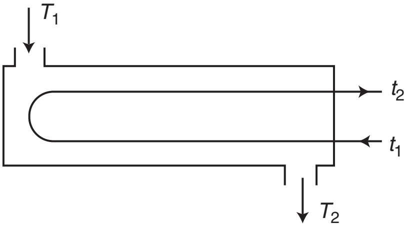


Figure 16.2 One shell pass, 2 tube passes schematic.

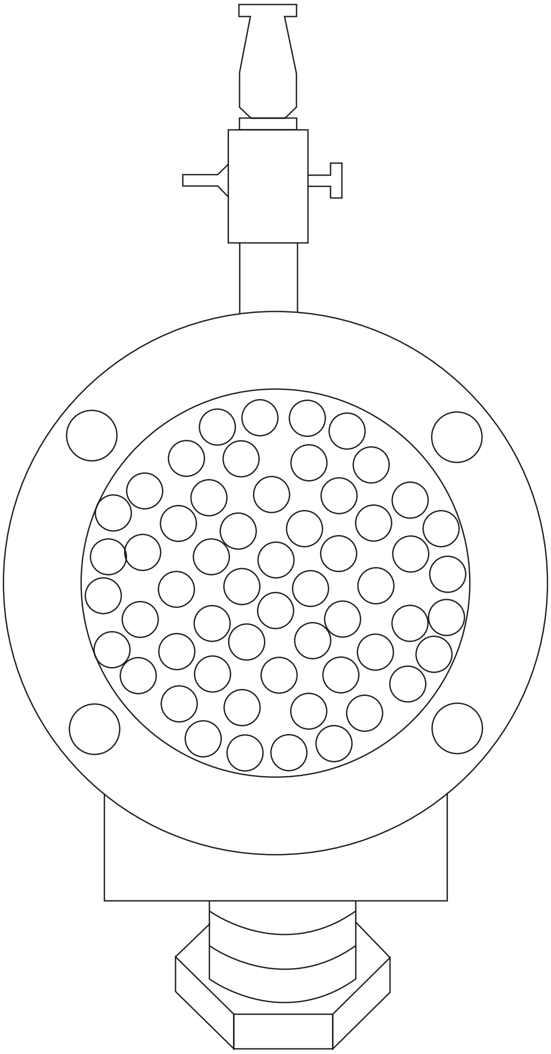


Figure 16.3 Side view of tube layout in the shell and tube heat exchanger.

ILLUSTRATIVE EXAMPLE 16.1

1. Determine the number of passes on the shell side and the number of passes on the tube side for a 4–6 shell and tube heat exchanger.
2. Provide a line diagram of a 3–6 shell and tube heat exchanger.

***SOLUTION*:**

1. First consider how many passes there are on the shell side. There are four passes on the shell side. Finally, there are six passes on tube side. The number of shell passes depends, on the inner design of the shell while the number of tube passes depends on how many hairpins are formed by the bundle of tubes within the shell.
2. See Figure 16.4 for line diagram of a 3–6 shell and tube heat exchanger. ■

ILLUSTRATIVE EXAMPLE 16.2

What is the purpose of baffles? Select one answer.

- **a.** To decrease the turbulence of the shell side fluid and therefore increase the contact between the shell side fluid and tube.
- **b.** To decrease the turbulence of the tube side fluid and therefore increase the contact between the shell side fluid and tube.
- **c.** To change the direction of flow from co-current to countercurrent and therefore increase the heat transfer coefficient.
- **d.** To increase the turbulence of the tube side fluid and lower the effect of fouling.
- **e.** To increase the turbulence of the shell side fluid and therefore provide better contact between the shell side fluid and tube.

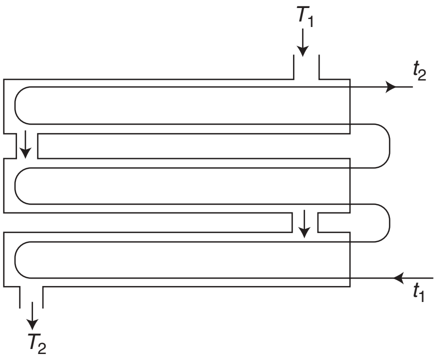


Figure 16.4 3–6 Exchanger.

***SOLUTION*:** Baffles are included in heat exchangers to increase the turbulence of the shell side fluid and therefore increase contact between the shell side fluid and the tube. The more contact that takes place, the greater the heat transfer.

Therefore, the correct answer is (e). ■

ILLUSTRATIVE EXAMPLE 16.3

What is the difference between passes and crosses on the shell side? Select one answer.

- **a.** Passes are the number of times a fluid crosses the tube bundle while crosses are the number of times a fluid crosses an imaginary plane parallel to the tubes.
- **b.** Passes are the number of times a fluid crosses an imaginary plane parallel to the tubes while crosses are the number of times a fluid crosses the tube bundle.
- **c.** Passes are the number of times a fluid crosses an imaginary plane perpendicular to tubes while crosses are the number of times a fluid crosses the tube bundle.
- **d.** Passes are the number of times a fluid crosses the tube bundle while crosses are the number of times a fluid crosses an imaginary plane perpendicular to tubes.
- **e.** There is no difference; they are synonymous.

***SOLUTION*:** Passes are the number of times the fluid crosses an imaginary plane perpendicular to the tubes. Crosses are the number of times the fluid crosses the tube bundle.

Therefore, the correct answer is (c). ■

When designing a shell and tube heat exchanger, the layout of the inner tube bundle is the main physical design criterion and includes:

1. Tube diameter
2. Tube wall thickness
3. Tube length
4. Tube layout
5. Tube corrugation
6. Baffle design.

Each are detailed below.

1. *Tube Diameter*

   The fouling nature of the tube-side fluid determines the correct tube diameter. Smaller diameter tubes allow for compact and economical design; however, mechanical cleaning of smaller diameter tubes can be difficult. For cleaner fluids that do not foul, tubes may have an outside diameter as small as $\frac{1}{2}$ inch, while shell and tube heat exchangers that handle tars may have tubes that have outside diameters of 3 inches. Most shell and tube heat exchangers have tubes with an outside diameter that range from $\frac{3}{4}$ to 1 inch.(1)

2. *Tube Wall Thickness*

   The tube wall thickness is governed with six different variables in mind. First and foremost, the tube must have enough thickness to handle the external pressure exerted by the shell-side fluid and the internal pressure exerted by the tube-side fluid. In most cases, pressure is not a factor. Other factors that the tube wall thickness must provide are listed below.

   a. Allow enough margin for corrosion of the tube from use, or if any of the process fluids have corrosive properties.
   b. Provide enough resistance against flow-induced vibrations.
   c. Provide enough axial strength to reduce sagging or bending throughout the length of the tube.
   d. Meet standards for ease in ordering replacement tubes.
   e. Ordering thinner or thicker tubes must justify the associated extra costs.(1)

3. *Tube Length*

   Generally, the most economical shell and tube heat exchangers have a small shell diameter and long tube length. Therefore, manufacturers strive to produce longer shell and tube heat exchangers. Long tubular bundles may prove difficult to replace. In practice, the maximum tube length for a fixed-bundle of tubes (individual tubes may not be withdrawn and replaced) may be 9 m with a total weight of about 20 tons. If a fixed tube sheet (individual tubes may be withdrawn and replaced) exchanger is used, then the tube length should be limited to about 15 m although tubes 22 m in length have been employed to meet process demands in industrial plants and are not uncommon.(1)

4. *Tube Layout*

   When laying out a bundle of tubes, the tube pitch, i.e., the distance from the center of one tube to the next, should not exceed 1.25 times the outside diameter of the tubes. There are generally four types of tube bundle arrangements, as shown in Figure 16.5. The fouling properties of the shell-side fluid must also be considered when determining the tube pitch. A tube pitch angle of 30° or 60° can accommodate 15% more tubes than their 45° and 90° counterparts; however, 30° triangular pitch and 60° triangular pitch do not allow for external cleaning of the outer tubes. When the shell-side fluid is in the laminar region, then a 45° square pitch tube layout has better heat transfer rates as well as lower pressure drops across the exchanger. When the shell-side fluid is in the turbulent region, then the 90° square pitch provides for better heat transfer rates and a lower pressure drop across the exchanger.

5. *Tube Corrugation*

   Roped, fluted, and finned (see next chapter) tubes can be used in the design of a shell and tube heat exchangers to either increase the area available for heat transfer or ensure that the shell-side fluid remains in the turbulent regime. Finned tubes increase the area available for heat transfer in a shell and tube exchanger, while fluted and roped tubes induce turbulent flow in the shell-side fluid.(1)

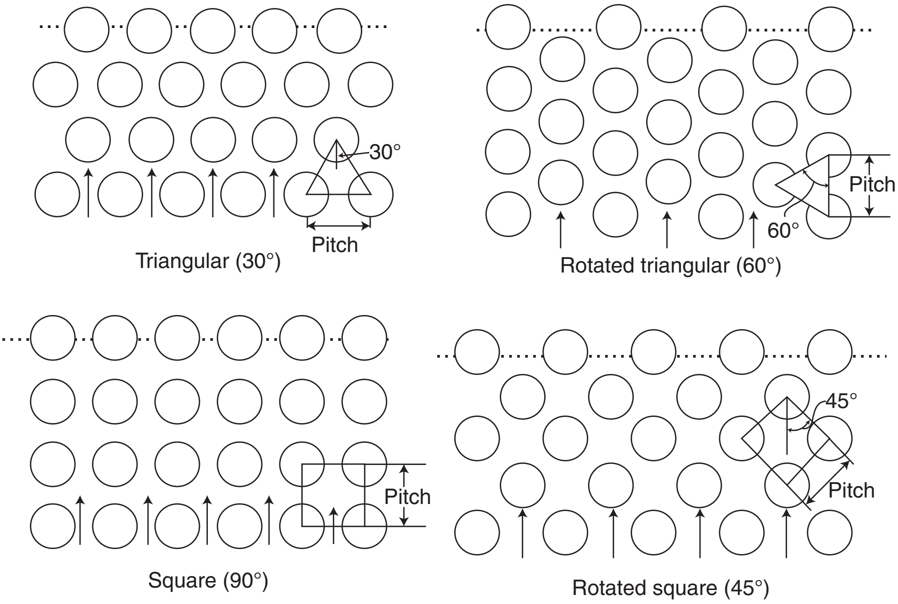


Figure 16.5 Types of tube pitches used in shell and tube heat exchangers.(1)

6. *Baffle Design*

   Baffles in a shell and tube heat exchanger serve two purposes. First, they allow the shell-side fluid to flow across all of the tubes, preventing dead spaces occurring within the heat exchanger. Secondly, they provide support for longer tubes to prevent damage from sagging or flow-induced vibrations. Baffles can either be designed in a single-, double-, or triple-segmental pattern, as shown in order from top to bottom in Figure 16.6.(1) Single segmental baffles usually have a total length of 15–40% of the shell's inner diameter while the total length of double segmental baffles is 20–30% of the shell's inner diameter. Baffle spacing affects pressure drop; therefore, calculations of a pressure drop with certain baffle spacing must be performed to ensure that the pressure drop is reasonable. A general rule of thumb in industry is that baffles should not be spaced closer than one-fifth of the shell's inner diameter or two inches apart, whichever is greater.(1)

Finally, the choice of which fluid should flow in the tubes is an important design decision. If one of the fluids is particularly corrosive, it is typically introduced on the tube side since more expensive resistant tubes can be purchased. If one of the fluids is more likely to form scale and/or deposits, it should be the tube side fluid since the inside of the tubes are much easier to clean than the outside of the tubes. In addition, viscous fluids normally flow on the shell side in order to help induce turbulence. The impact on the overall heat transfer coefficient and pressure drop should also be considered when determining the flow locations of the fluids.

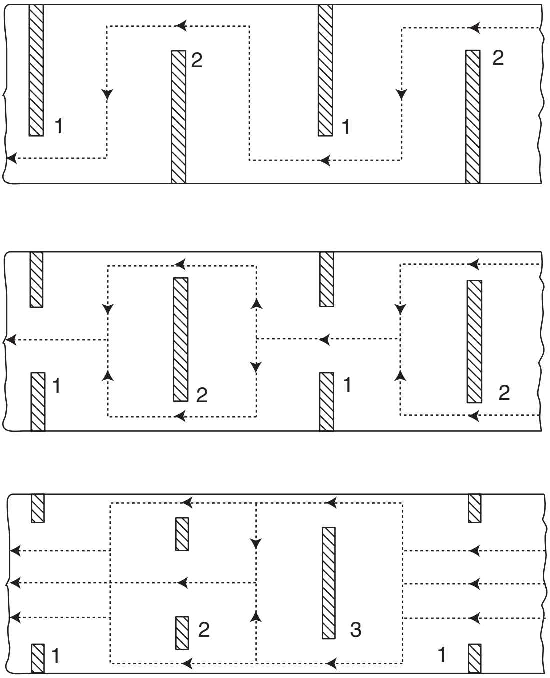


Figure 16.6 Baffle arrangements in shell and tube heat exchangers.(1)

ILLUSTRATIVE EXAMPLE 16.4

Discuss the effect of pressure drop in the design of shell and tube exchangers.

***SOLUTION*:** Shell and tube heat exchangers are often designed to meet upper limit specifications on shell and tube side pressure drops. Many exchangers are designed for pressure drops less than 5 psi. For flow of low viscosity fluids (1–35 cP) through tubes, a good trial velocity is 6 fps. Equations are available in the literature(2,3) that estimate pressure drops for both the tube and shell side. ■

In summary, shell and tube heat exchangers are among the most widely used exchangers due to the large surface area available for heat transfer. Countercurrent operation is the norm in industry but co-current operation is sometimes desired.

DESCRIBING EQUATIONS

When two process fluids at different temperatures pass one another as in a shell and tube exchanger, heat transfer occurs due to the temperature difference between the two streams. The energy required to accomplish this heat transfer, i.e., the heat exchanger duty, can be determined for both process streams. For a well-insulated exchanger, the two duties should equal one another. The describing equations for the two duties are similar to those presented for the double pipe heat exchanger (see previous chapter). However, the following equation gives the heat exchanger duty of the hot process stream when saturated steam is introduced on the shell side (enthalpy balances between steam and condensate may also be used):

$$\dot{Q}_h = \dot{m}_s\lambda + \dot{m}_s c_{p,hw}(T_{s,i} - T_{s,o}) = \dot{m}_s\lambda + \dot{m}_s c_{p,hw}(T_1 - T_2) \tag{16.1}$$

where $\dot{Q}_h = \dot{Q}_H$ = rate of energy lost by steam, Btu/h
$\dot{m}_s$ = total mass flow rate of steam condensate, lb/h
$h_{\text{vap}} = \lambda$ = heat of vaporization, Btu/lb
$c_{p,hw}$ = average heat capacity of condensate (hot water), Btu/lb · °F
$T_{s,i} = T_1$ = steam inlet temperature, °F
$T_{s,o} = T_2$ = condensate outlet temperature, °F

The cold process stream duty is calculated similarly:

$$\dot{Q}_c = \dot{m}_c c_{p,cw}(t_{c,o} - t_{c,i}) = \dot{m}_c c_{p,cw}(t_2 - t_1) \tag{16.2}$$

where $\dot{Q}_c = \dot{Q}_C$ = rate of energy gained by cooling water, Btu/h
$\dot{m}_c$ = total mass flow rate of the cold water (*all* tubes), lb/h
$c_{p,cw}$ = average heat capacity of the cold process stream, Btu/lb · R
$t_{c,o} = t_2$ = cold water outlet temperature, °F
$t_{c,i} = t_1$ = cold water inlet temperature, °F

The reader should note that much of the material that follows was presented in the last chapter. However, as each chapter is written on a stand-alone basis, the applicable equations for shell and tube exchangers receive treatment.

If the heat exchanger duty is known, the overall heat transfer coefficient is calculated from either of the two equations provided below:

$$U_o = \frac{\dot{Q}_h}{A_o \Delta T_{lm}} \tag{16.3}$$

$$U_i = \frac{\dot{Q}_c}{A_i \Delta T_{lm}} \tag{16.4}$$

where U_o = overall heat transfer coefficient based on hot (h) process stream, Btu/ft^2 · h · °F
U_i = overall heat transfer coefficient based on cold (c) process stream, Btu/ft^2 · h · °F
A_o = exchanger transfer area based on outside tube surfaces, ft^2

A_i = exchanger transfer area based on inside tube surface, ft^2

ΔT_{lm} = log-mean temperature difference, °F

Typical values for the *overall* heat transfer coefficient in steam condensers with water in the tubes range from $1000{-}6000\,\text{W/m}^2\cdot\text{K}$.(2) *Perry's Chemical Engineers' Handbook*(2) reports coefficients ranging from $400{-}1000\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{R}$.(3) These values are approximately the same after conversion of units.

For tubular heat exchangers, as with double pipe heat exchangers, the overall heat transfer coefficient is related to the individual coefficients by the following equation:

$$\frac{1}{UA} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} \tag{16.5}$$

$$= \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o} \tag{16.6}$$

where U = overall heat transfer coefficient, $\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$

A_i, A_o = inside and outside surface areas (πD_iL or πD_oL), ft^2

A = average of the two areas, ft^2

U_i, U_o = coefficients based on inner and outer tube surfaces, $\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$

h_i, h_o = inside and outside heat transfer coefficients, $\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$

$R_{f,i}$, $R_{f,o}$ = fouling factors based on inner and outer tube surfaces

D_i, D_o = inside and outside pipe diameters, ft

k = thermal conductivity, $\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$

L = tube length, ft

If the fouling factors, $R_{f,i}$ and $R_{f,o}$, and the tube wall resistance, i.e., the middle terms on the right-hand side of Equation (16.6) are negligible, then the relationship between the overall heat transfer coefficient and the individual coefficients simplifies once again to:

$$\frac{1}{U_o} = \frac{D_o}{h_iD_i} + \frac{1}{h_o} \tag{16.7}$$

The overall heat transfer coefficient, U, is an average value based on the average duty or $\dot{Q} = (\dot{Q}_h + \dot{Q}_c)/2$, and an average heat transfer area or $A = (A_i + A_o)/2$.

The individual coefficients above, h_i and h_o, are almost always calculated using empirical equations. First, the Reynolds number, Re, for the cold process fluid must be found in order to determine whether the flow is in the laminar, turbulent, or transition region. Refer to Table 15.1 for additional details. Except for the viscosity term at the wall temperature, all of the physical properties of the tube-side fluid are evaluated at the average bulk temperature, $t_{c,\text{bulk}} = (t_{c,i} + t_{c,o})/2$.

$$\text{Re} = \frac{4\dot{m}_c}{\pi D_i \mu_c} \tag{16.8}$$

where $\dot{m}_c$ = mass flow rate of tube-side fluid through *one* tube, lb/h
D_i = inside diameter of *one* tube, ft
μ_c = viscosity of tube-side fluid at bulk temperature, $\text{lb/ft} \cdot \text{h}$
Re = Reynolds number

The Nusselt number, Nu, can be determined from empirical equations. For *laminar flow* in a circular tube ($\text{Re} < 2100$), the Nusselt number equals 4.36 for uniform surface heat flux (Q/A) or 3.66 for constant surface temperature.(2,4) This value should only be used for Graetz numbers, Gz, less than 10.(4) For laminar flow with Graetz numbers from 10 to 1000, the following equation applies:(4)

$$\text{Nu} = 2\,\text{Gz}^{1/3}\left(\frac{\mu_c}{\mu_{c,\text{wall}}}\right)^{0.14}; \qquad \text{Nu} = \frac{h_i D_i}{k_c}, \qquad \text{Gz} = \frac{\dot{m}_c c_{p,c}}{k_c L} \tag{16.9}$$

where $\mu_{c,\text{wall}}$ = viscosity of tube-side fluid at wall temperature, $\text{lb/ft} \cdot \text{h}$
k_c = thermal conductivity of tube-side fluid at bulk temperature, $\text{Btu/ft} \cdot \text{h} \cdot {}^\circ\text{F}$
$c_{p,c}$ = heat capacity of tube-side fluid at bulk temperature, $\text{Btu/lb} \cdot {}^\circ\text{F}$
L = length of tubes, ft
Nu = Nusselt number
Gz = Graetz number

For *turbulent flow* ($\text{Re} > 10{,}000$), the Nusselt number may be calculated from the Dittus–Boelter equation if $0.7 \leq \text{Pr} \leq 160$ or the Sieder–Tate equation if $0.7 \leq \text{Pr} \leq 6700$, where Pr is the Prandtl number. Both equations are valid for L/D greater than 10.(2,4)

$$\text{Dittus–Boelter equation: } \text{Nu} = 0.023\,\text{Re}^{4/5}\,\text{Pr}^n \tag{16.10}$$

$$\text{Pr} = \frac{c_{p,c}\mu_c}{k_c}, \quad n = 0.4 \text{ for heating or 0.3 for cooling}$$

$$\text{Sieder–Tate equation:} \quad \text{Nu} = 0.023\,\text{Re}^{4/5}\,\text{Pr}^{1/3}\left(\frac{\mu_c}{\mu_{c,\text{wall}}}\right)^{0.14} \tag{16.11}$$

The Dittus–Boelter equation should only be used for small to moderate temperature differences. The Sieder–Tate equation applies for larger temperature differences.(2–4) Errors as large as 25% are associated with both equations. Other empirical equations with more complicated formulas and less error are available in the literature.$^{(2-4)}$

The above empirical equations do not apply for flows in the *transition region* (between 2100 and 10,000). For the transition region, one should review the literature to determine the inside heat transfer coefficient. The reader is cautioned to use the correct mass flow rate; i.e., flow through a *single* tube rather than the total flow rate.

The outside heat transfer coefficient, h_o, is calculated using empirical correlations for condensation of a saturated vapor on a cold surface if steam is employed in the shell. Under normal conditions, a continuous flow of liquid is formed over the surface (film condensation) and condensate flows downward due to gravity. In most cases, the motion of the condensate is laminar and heat is transferred from the vapor–liquid interface to the surface by conduction through the film. Heat transfer coefficients for laminar film condensation on a single horizontal tube or on a vertical tier of N horizontal tubes can be calculated using appropriate equations from the literature$^{(2-4)}$ or by reviewing Chapter 12.

As noted earlier, the shell and tube heat exchanger usually consists of a series of vertical tiers of horizontal tubes. In a vertical tier of N horizontal tubes, the average heat transfer coefficient for the stack of tubes is less than for a single tube. The equation for the average heat transfer coefficient for steam condensing on a vertical stack of horizontal tubes (laminar film condensation) follows:

$$h_o = 0.725\left[\frac{k_f^3\rho_f^2 g\lambda}{N(T_s - T_{\mathrm{w}})D_o\mu_f}\right]^{1/4} \tag{16.12}$$

where k_f = thermal conductivity of condensate at reference temperature, Btu/ft · h · °F

ρ_f = density of condensate at reference temperature, lb/ft^3

N = average number of tubes in a vertical tier or stack

T_s = temperature of condensing vapor, °F

T_{w} = temperature of outside surface of wall, °F

D_o = outside tube diameter, ft

μ_f = viscosity of condensate at reference temperature, lb/ft · h

T_f = reference temperature, °F

Figure 16.3 is a drawing of the header of the shell and tube heat exchanger consisting of 56 tubes, N_{total}. The average number of tubes in a vertical tier (stack) can be determined by drawing a series of vertical lines down through the diagram of the header, then counting the tubes in each column and averaging the results. This procedure results in 11 vertical tiers of horizontal tubes with $N \approx 5$ tubes per tier. The correct value or $N \approx 5$ must be used in Equation (16.12).

Equation (16.12) is limited to cases where the steam-side Reynolds number, Re_s, is less than 2100 (laminar flow of shell-side fluid).$^{(2)}$ Other correlations are provided in the literature for condensation heat transfer in the turbulent region.$^{(2-4)}$

Re_s is calculated as follows:

$$\mathrm{Re}_s = \frac{4\Gamma_b}{\mu_L}; \qquad \Gamma_b = \frac{\dot{m}_s/N_{\text{total}}}{L} \tag{16.13}$$

where Γ_b = condensate loading per unit length of tube, lb/h · ft

N_{total} = total number of tubes, 56 tubes in the discussion above

L = length of tubes in Equation (16.12)

The physical properties are evaluated at a reference temperature, T_f. A reasonable value for the reference temperature in the range between $T_{c,i}$ and $T_{s,i}$ can be assumed, or an average of the four known temperatures, $(t_{c,i} + t_{c,o} + T_{s,i} + T_{s,o})/4$, can be used for a first estimate. Note that $t_{c,i}$ and $t_{c,o}$ represent the inlet and outlet cold liquid temperatures, respectively.

A trial-and-error calculation is required because the wall temperature and the reference temperature are both unknown and the wall temperature, T_{w}, depends on h_o and h_i. Details are provided in the previous chapter.

There are additional calculations for shell and tube exchangers. The following notation (consistent units) is employed in the shell and tube development to follow:$^{(2)}$

A = heat transfer area

D_t = inside diameter of tube

L = tube length

$\dot{m}_{1,\text{tube}}$ = mass flow rate of tube-side fluid per tube

$\dot{m}_1$ = mass flow rate of tube-side fluid

N_{tpp} = number of tubes per pass on tube side

N_p = number of passes on the tube side

N_t = total number of tubes

U = overall heat transfer coefficient

V = average velocity of fluid in the tube

The mass flow rate per tube is given by

$$\dot{m}_{1,\text{tube}} = \frac{\rho V \pi D_t^2}{4} \tag{16.14}$$

The number of tubes per pass is obtained from the total mass flow of the fluid on the tube side.

$$N_{tpp} = \frac{\dot{m}_1}{\dot{m}_{1,\text{tube}}} \tag{16.15}$$

The total number of tubes and heat transfer area in the heat exchanger are:

$$N_t = N_{tpp} N_p \tag{16.16}$$

with

$$A = N_t \pi D_t L \tag{16.17}$$

Illustrative examples involving the equations presented in this section are provided in the two sections that follow.

THE "F" FACTOR

Some heat exchangers use true countercurrent flow. However, these heat exchangers are not as economical as multipass and crossflow units. In multipass exchangers, the flow alternates between co-current and countercurrent between the different sections of the exchanger. As a result, the driving force is not the same as a true countercurrent or a true co-current exchanger. The aforementioned ΔT_{lm} (log mean temperature difference driving force or LMTD) for these exchangers is almost always less than that of countercurrent flow and greater than that of co-current flow.

The analysis of these units requires including an F factor in the describing equation (i.e., the heat transfer equation). For those multipass and crossflow exchangers, the log mean temperature difference (LMTD) method is applicable, i.e.,

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\text{lm}\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} \tag{16.18}$$

or

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\text{lm}\left(\dfrac{\Delta T_2}{\Delta T_1}\right)} \tag{16.19}$$

For the special case of

$$\Delta T_1 = \Delta T_2 = \Delta T \tag{16.20}$$

One may employ

$$\Delta T_{\text{lm}} = \Delta T \tag{16.21}$$

The heat transfer rate is still given by

$$\dot{Q} = UA\Delta T_{\text{lm}} \tag{16.22}$$

except that ΔT_{lm} must be corrected by a geometry factor, F. This correction factor accounts for portions of multipass heat exchangers where the flow is not countercurrent (e.g., hairpin turns). Equation (16.22) is now written as

$$\dot{Q} = UAF\Delta T_{\text{lm},cp} \qquad \Delta T_{\text{lm}} = F\Delta T_{\text{lm},cp} \tag{16.23}$$

where $\Delta T_{\text{lm},cp}$ is the LMTD for ΔTs based on "ideal" countercurrent operation and F is the geometry factor, or correction factor, applied to a different flow arrangement with the same hot and cold fluid temperature. Bowman et al.$^{(5)}$ coordinated the results of

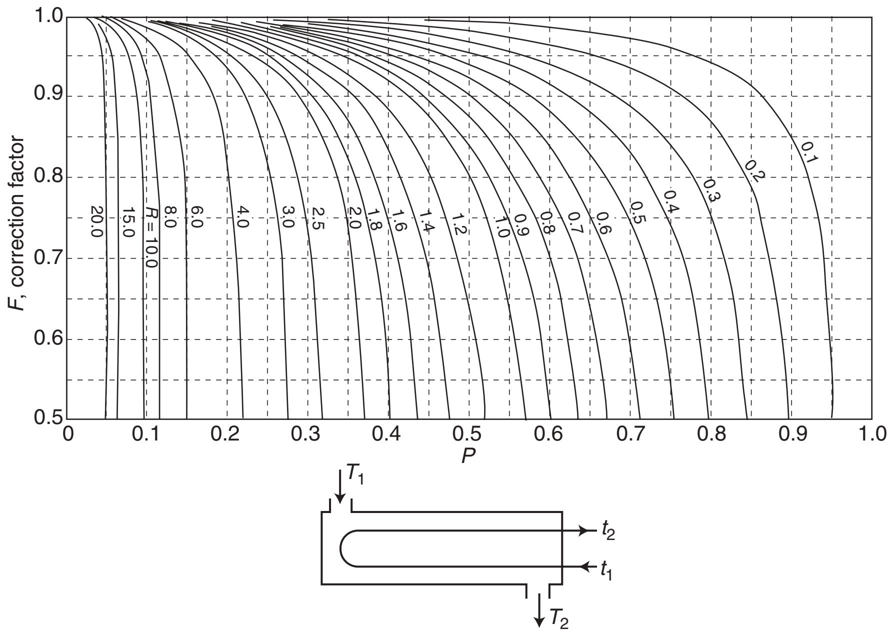


Figure 16.7 One shell pass; 2 or more tube passes.

earlier studies in his day regarding the ΔT_{lm} for exchangers that experience neither co-current or countercurrent flow to provide as complete a picture as possible of the various arrangements of surface conditions and flow directions. Shell and tube heat exchangers that experience any number of passes on the shell and tube side were covered in his work as well as crossflow exchangers with different pass arrangements. The results for the various classifications follow.

1. *One Pass Shell Side; Two Passes Tube Side*:

   The correction factor, F, for multipass heat exchangers having one shell pass and two tube-side passes is shown in Figure 16.7 for the 1–2 pass exchanger versus P and R, where P and R are both dimensionless ratios given by:

$$P = \frac{t_2 - t_1}{T_1 - t_1} \tag{16.24}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} \tag{16.25}$$

   This graphical data is based on the integrated equations obtained by Underwood.$^{(6)}$ The integrated equation for ΔT_{lm} is:

$$\Delta T_{\mathrm{lm}} = \frac{\sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}}{\log\left\{\dfrac{T_1 + T_2 - t_1 - t_2 + \sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}}{T_1 + T_2 - t_1 - t_2 - \sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}}\right\}} \tag{16.26}$$

The above equation was transposed into the following equation, in which the F factor for a 1–2 exchanger is expressed in terms of the aforementioned dimensionless ratios of P and R:

$$F_{1,2} = \frac{\sqrt{R^2+1}}{R-1}\log\left(\frac{1-P}{1-PR}\right) \Big/ \log\left[\frac{(2/P)-1-R+\sqrt{R^2+1}}{(2/P)-1-R-\sqrt{R^2+1}}\right] \tag{16.27}$$

Note that the term in the expression above

$$\frac{1}{R-1}\log\left(\frac{1-P}{1-PR}\right) \tag{16.28}$$

becomes indeterminate when $R = 1$. However, the usual treatment for such indeterminates reduces Equation (16.28) to:

$$\frac{P}{2.3(1-P)} \tag{16.29}$$

2. *One Pass Shell Side; Four Passes Tube Side*

The correction factor for 1–4 heat exchangers is slightly less than that for 1–2 exchangers. The correction factor is given by:

$$F_{1,4} = \frac{\sqrt{4R^2+1}}{2(R-1)}\log\left(\frac{1-P}{1-PR}\right) \Big/ \log\left[\frac{1+V\left(\sqrt{4R^2+1}-2R\right)}{1-V\left(\sqrt{4R^2+1}+2R\right)}\right] \tag{16.30}$$

where

$$V = \frac{t_2 - t_1}{4T_1 - (t_1 + 2t_i + t_2)} \tag{16.31}$$

and t_i is the intermediate temperature of the fluid where it leaves the second tube-side pass and enters the third tube-side pass. The value of t_i may be obtained using a trial-and-error solution of the following equation:

$$\left(\frac{t_i - t_1}{t_2 - t_i}\right)^{\sqrt{4R^2+1}} = \frac{1+V\left(\sqrt{4R^2+1}-2R\right)}{1-V\left(\sqrt{4R^2+1}+2R\right)} \tag{16.32}$$

3. *One Pass Shell Side; Six Passes Tube Side*

Nagle$^{(7)}$ found that the correction factors for 1–6 exchangers agree rather closely with those of 1–2 exchangers operating with the same terminal temperatures. Therefore, the curves of the 1–2 exchanger graph may be used to determine the correction factor for 1–6 exchangers in this specific case. Therefore, from this discussion above, one can conclude that the correction factor, F, for heat exchangers with one shell-side pass and two, four, six and infinite tube-side passes are approximately the same.

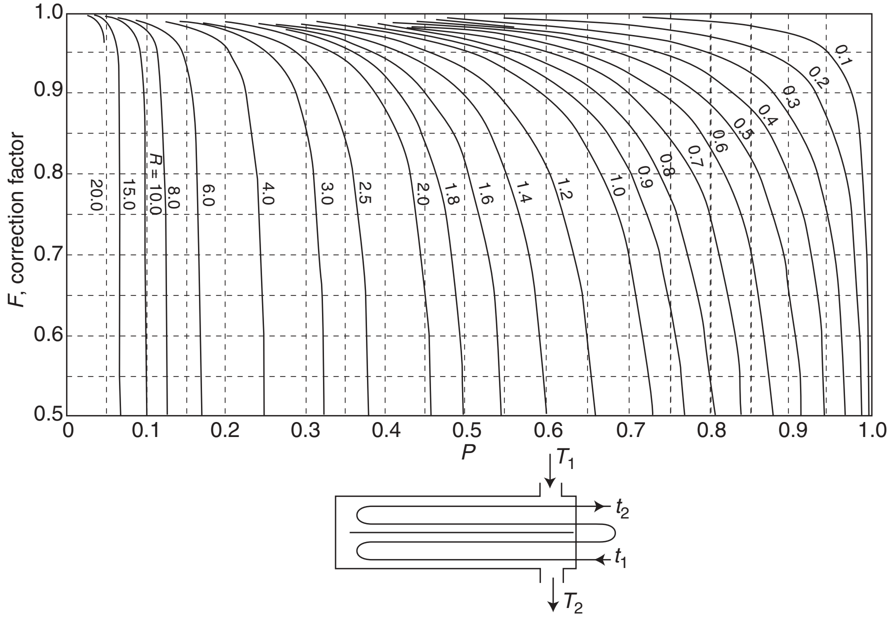


Figure 16.8 Two shell passes; four or more tube passes.

4. *Two Passes Shell Side; Four Passes Tube Side*

The correction factor for 2–4 exchangers designed to approach countercurrent flow is given in Figure 16.8. Underwood’s algebraic equation for 2–4 heat exchangers is given by

$$F_{2,4} = \frac{\sqrt{R^2+1}}{2(R-1)} \log\left(\frac{1-P}{1-PR}\right) \Bigg/ \log\left[\frac{(2/P)-1-R+(2/P)\sqrt{(1-P)(1-PR)}+\sqrt{R^2+1}}{(2/P)-1-R+(2/P)\sqrt{(1-P)(1-PR)}-\sqrt{R^2+1}}\right] \tag{16.33}$$

5. *Three and More Shell-Side Passes*

A general method was developed by Bowman et al.$^{(5,8)}$ to calculate the correction factor, F, for the 3–6, 4–8, and 6–12, etc., exchangers. This method uses the correction factors from the 1–2 exchangers to help estimate the correction factor for three or more shell-side passes. At any given values of F and R, the value of P for an exchanger having N shell-side and $2N$ tube-side passes is related to P for a 1–2 exchanger by the equation:

$$P_{N,2N} = \frac{1-\left(\dfrac{1-P_{1,2}R}{1-P_{1,2}}\right)^N}{R-\left(\dfrac{1-P_{1,2}R}{1-P_{1,2}}\right)^N} \tag{16.34}$$

For the special case of $R = 1$, Equation (16.34) becomes indeterminate and reduces to:

$$P_{N,2N} = \frac{P_{1,2}N}{P_{1,2}N - P_{1,2} + 1} \tag{16.35}$$

Graphical results for four and six shell passes are provided in Figures 16.9 and 16.10, respectively.

6. *Fluids Mixed*

   Graphical results for single pass, crossflow exchangers with both fluids unmixed (compartmentalized), and with one fluid mixed and the other unmixed are provided in Figures 16.11 and 16.12, respectively.

From the above analysis, the correction factor approaches unity as the number of shell-side passes is increased for any value of P and R. This is to be expected since a multipass exchanger with several shell-side passes approaches the ideal countercurrent heat exchanger more closely than one shell-side and two or more tube passes. In addition, one notes that

$$F_{1-2} \approx F_{1-4} \approx F_{1-6} \tag{16.36}$$

$$F_{1-3} \approx F_{1-6} \approx F_{1-9} \tag{16.37}$$

$$F_{2-4} \approx F_{2-8} \approx F_{2-12} \tag{16.38}$$

$$F_{3-6} \approx F_{3-12} \approx F_{3-18} \tag{16.39}$$

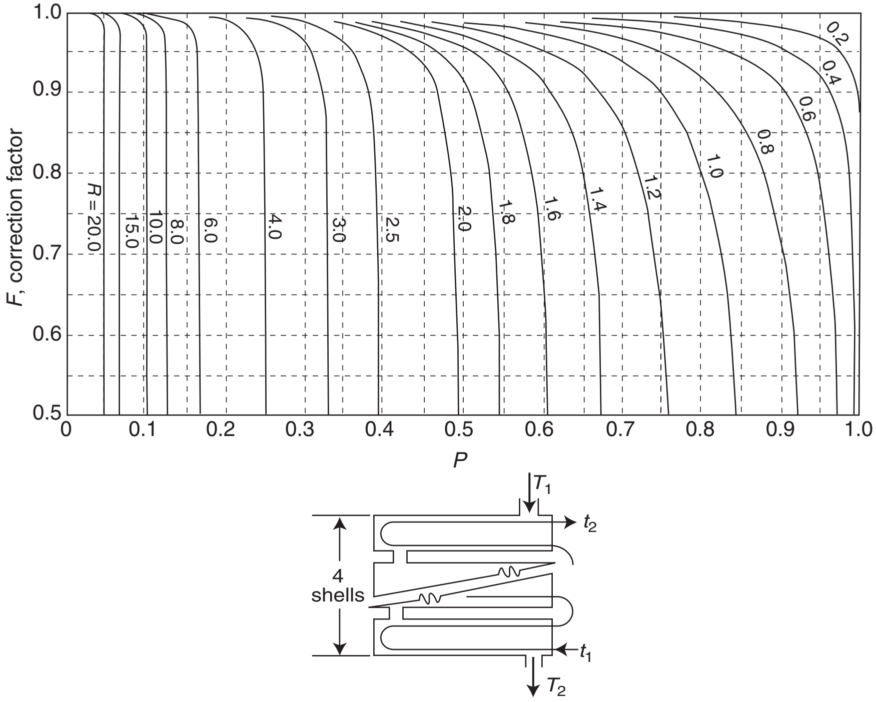


Figure 16.9 Four shell passes; eight or more tube passes.

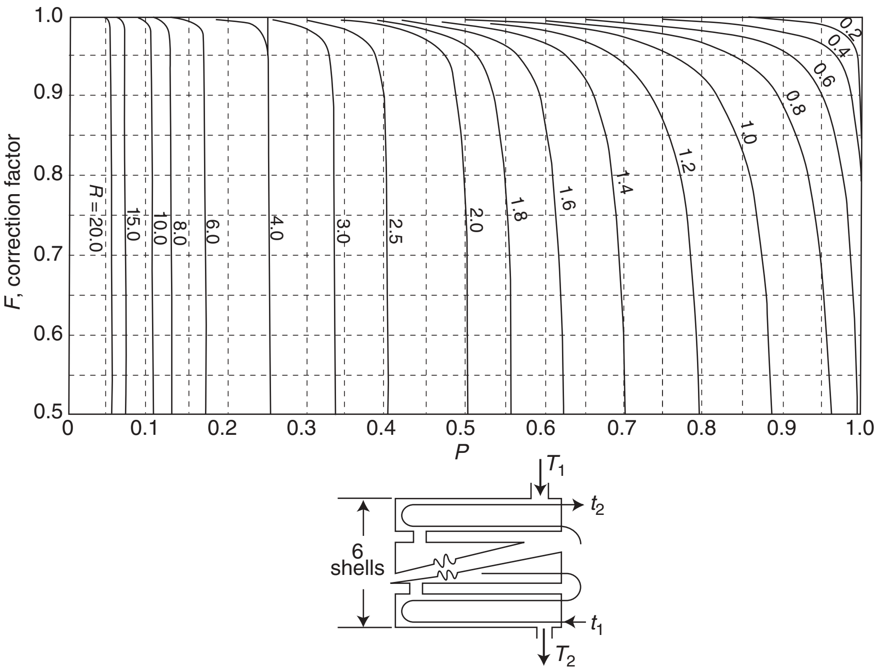


Figure 16.10 Six shell passes; 12 or more tube passes.

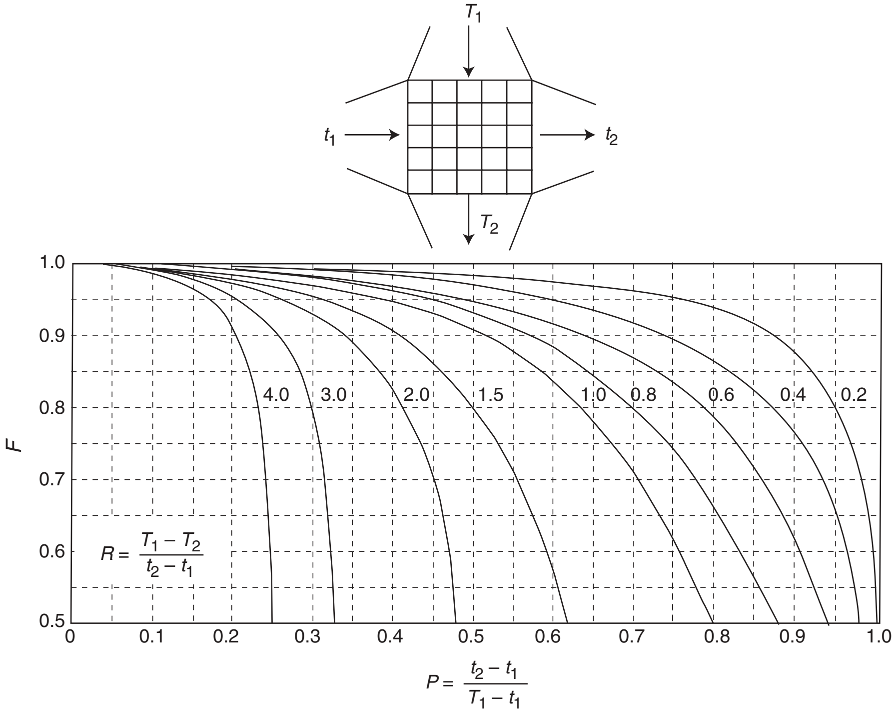


Figure 16.11 Correction factor for a single pass, cross-flow heat exchanger with both fluids unmixed.

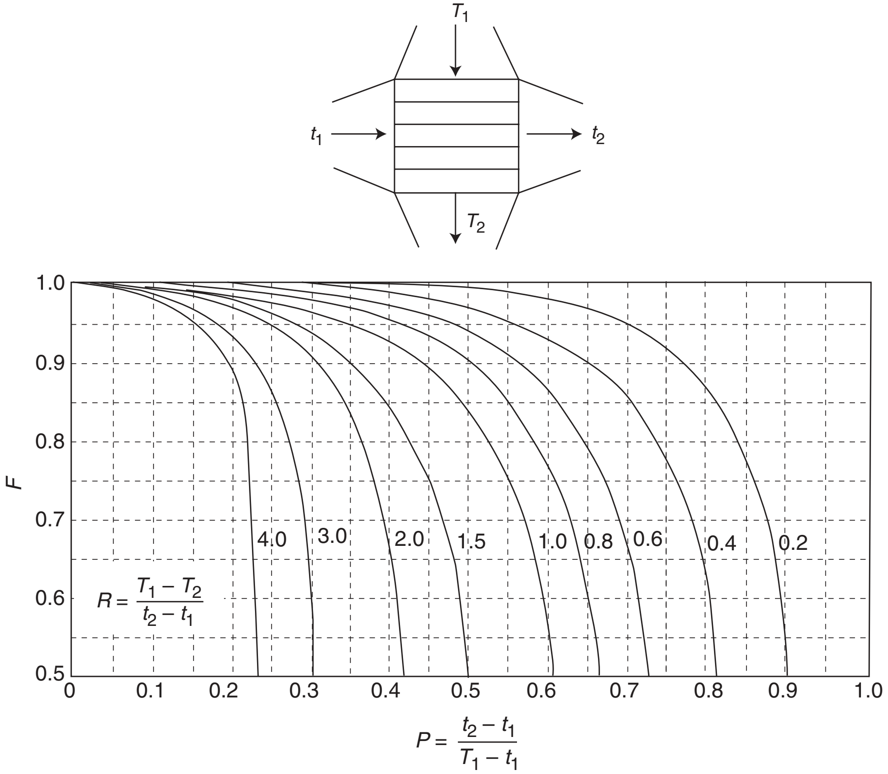


Figure 16.12 Correction factor for a single pass, crossflow heat exchanger with one fluid mixed and the other unmixed (t_1, t_2).

ILLUSTRATIVE EXAMPLE 16.5

Consider the shell and tube heat exchanger in Figure 16.13. Calculate the log mean temperature driving force. Also perform the calculation for the exchanger described in Figure 16.14.

***SOLUTION*:** The system is first treated as an ideal countercurrent system as shown in Figure 16.13. For this ideal countercurrent system,

$$\text{LMTD} = \Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{16.18}$$

$$= \frac{(150 - 80) - (100 - 50)}{\ln(70/50)} = 59.4^\circ\text{F}$$

Figure 16.13 Ideal countercurrent heat exchanger; Illustrative Example 16.5.

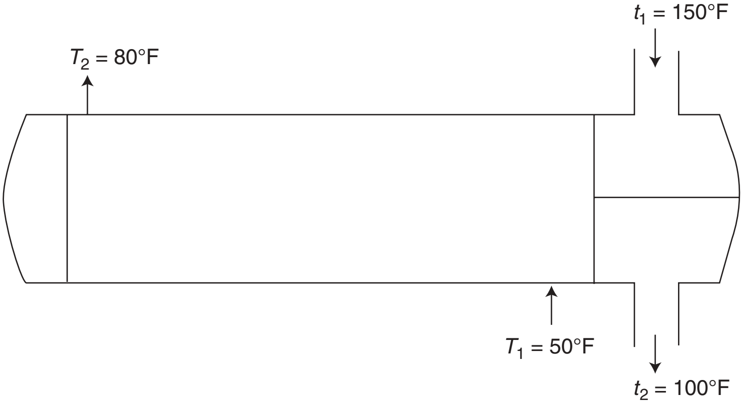


Figure 16.14 Shell and tube exchanger; Illustrative Example 16.5.

For the system in Figure 16.14,

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{80 - 50}{150 - 50} = 0.30$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{150 - 100}{80 - 50} = 1.67$$

From Figure 16.7, $F = 0.925$. Therefore,

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm},cp} = (0.925)(59.4) = 54.9^\circ\text{F}$$

This somewhat lower value represents the LMTD for the actual (real) system shown in Figure 16.14. ■

ILLUSTRATIVE EXAMPLE 16.6

A shell and tube heat exchanger having two shell passes and four tube passes is being used for cooling. The shell-side fluid enters at 400°F and leaves at 250°F and the tube-side fluid enters at 100°F and leaves at 175°F. What is the log mean temperature difference between the hot fluid and the cold fluid?

***SOLUTION*:** The log mean temperature difference (driving force) based ideal upon ideal (or true) countercurrent operation is first evaluated:

$$\text{LMTD} = \Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln \dfrac{\Delta T_1}{\Delta T_2}} \tag{16.18}$$

$$= \frac{(400 - 250) - (175 - 100)}{\ln\left(\dfrac{400 - 250}{175 - 100}\right)} = \frac{150 - 75}{\ln \dfrac{150}{75}} = 108^\circ\text{F}$$

The F factor can be obtained from Figure 16.8,

$$P = \frac{75}{350} = 0.214$$

$$R = \frac{150}{75} = 2.0$$

From Figure 16.8,

$$F = 0.985$$

Thus,

$$\begin{aligned}\Delta T_{\text{lm}} &= F\Delta T_{\text{lm,cp}} = (0.985)(108) \\ &= 106.4^\circ\text{F}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 16.7

Recalculate the results of the two previous examples using the equation upon which the graphs are based.

***SOLUTION*:** For Illustrative Example 16.5,

$$P = 0.30$$
$$R = 1.67$$

Applying Equation (16.27) gives

$$F = 0.92$$

Therefore,

$$\begin{aligned}\Delta T_{\text{lm}} &= F\Delta T_{\text{lm,cp}} \\ &= (0.92)(59.4) \\ &= 54.65^\circ\text{F}\end{aligned} \tag{16.23}$$

For Illustrative Example (16.6),

$$P = 0.214$$
$$R = 2.0$$

Applying Equation (16.33) leads to

$$F \simeq 0.985$$

so that

$$\begin{aligned}\Delta T_{\text{lm}} &= (0.985)(108)\\ &= 106.4^{\circ}\text{F}\end{aligned}$$

The results are in excellent agreement with the two previous examples. ■

ILLUSTRATIVE EXAMPLE 16.8

A shell and tube heat exchanger has one pass on the shell side and two passes on the tube side (i.e., a 1–2 shell and tube heat exchanger). It is being used for oil cooling. The oil flows in the tube side. It enters at 110°C and leaves at 75°C. The shell-side fluid is water at a flowrate of 1.133 kg/s, entering at 35°C and leaving at 75°C. The heat capacity of the water is 4180 J/kg · K. The overall heat-transfer coefficient for the heat exchanger is 350 $\text{W/m}^2 \cdot \text{K}$. The geometry factor F has been previously estimated to be 0.965. Calculate the heat-transfer area requirement for this unit in square meters.(9)

***SOLUTION*:** The heat load is

$$\begin{aligned}\dot{Q} &= \dot{m}c_p(T_{\text{out}} - T_{\text{in}}) = \dot{m}c_p(t_2 - t_1)\\ &= (1.133)(4180)(75 - 35)\\ &= 189{,}400\ \text{W}\end{aligned}$$

The countercurrent log-mean temperature difference is

$$\begin{aligned}\Delta T_1 &= 110 - 75 = 35^{\circ}\text{C}\\ \Delta T_2 &= 75 - 35 = 40^{\circ}\text{C}\\ \Delta T_{\text{lm,cp}} &= \frac{35 - 40}{\ln(35/40)}\\ &= 37.4^{\circ}\text{C}\\ &= 310.6\ \text{K}\end{aligned}$$

The corrected log-mean temperature difference is calculated employing the correction factor, F:

$$\begin{aligned}\Delta T_{\text{lm}} &= F\Delta T_{\text{lm},cp}\\ &= (0.965)(310.6)\\ &= 299.7\ \text{K}\end{aligned}$$

The required heat-transfer area is then

$$\begin{aligned} A &= \frac{\dot{Q}}{U\Delta T_{\mathrm{lm}}} \\ &= \frac{189{,}400}{(350)(299.7)} \\ &= 1.806\ \mathrm{m}^2 \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 16.9

Discuss the problem that can arise in employing the "*F*" factor.

SOLUTION**:** The LMTD method is adequate when the terminal temperatures are known. If the heat-transfer area is given and the exit temperatures are unknown, the problem often requires solution by trial and error. This is discussed in the next section. ■

ILLUSTRATIVE EXAMPLE 16.10

A shell and tube heat exchanger is to be designed for heating 10,000 kg/h of water from 16°C to 84°C by a new high heat capacity hot engine oil flowing through the shell. The oil makes a single shell pass, entering at 160°C and leaving at 94°C. The water flows through 11 brass tubes of 22.9 mm inside diameter and 25.4 mm outside diameter with each tube making four passes through the shell. The overall heat transfer coefficient (including the fouling resistance effect) is approximately 350 $\mathrm{W/m^2 \cdot {}^\circ C}$ and the thermal conductivity of brass is 137 $\mathrm{W/m \cdot {}^\circ C}$. Calculate

1. the heat load, in MW,
2. the countercurrent flow log mean temperature difference, and
3. the *F* correction factor and the corrected log mean temperature difference.

SOLUTION**:** Calculate the average bulk temperatures of the water and oil:

$$\begin{aligned} T_{w,\mathrm{av}} &= (16 + 84)/2 = 50^\circ\mathrm{C} \\ T_{\mathrm{oil,av}} &= (160 + 94)/2 = 127^\circ\mathrm{C} \end{aligned}$$

Obtain the properties of both fluids and organize this information in tabular form (see Table 16.1):

for water: $\rho = 987\ \mathrm{kg/m^3},\ c_p = 4176\ \mathrm{J/kg \cdot {}^\circ C}$

for oil: $\rho = 822\ \mathrm{kg/m^3},\ c_p = 4820\ \mathrm{J/kg \cdot {}^\circ C}$

Table 16.1 Data/Information for Illustrative Example 16.10

	Stream			
Property	Inlet	Outlet	Inlet	Outlet
Fluid	Water (being heated)		Hot oil	
Side/passes	Tube (four passes)		Shell (one pass)	
t, T, °C	16	84	160	94
Average t, T, °C	50		127	
ρ, kg/m^3	987		822	
c_p, J/kg · °C	4176		4820	
$\dot{m}$, kg/s	2.778		2.480	
Capacitance rate, $C = \dot{m}c_p$, W/°C	11,600		11,950	

Calculate the heat load, $\dot{Q}$, from the water-side information:

$$\dot{m}_1 = \dot{m}_w = 10{,}000/3600 = 2.778\,\text{kg/s}$$

$$\dot{Q} = \dot{m}_1 c_{p,w}(t_2 - t_1) = (2.778)(4176)(84 - 16) = 788{,}800\,\text{W} = 0.788\,\text{MW}$$

Calculate the oil flow rate, $\dot{m}_3$:

$$\dot{Q} = 788{,}800 = (c_{p,\text{oil}})(\dot{m}_{\text{oil}})(T_1 - T_2) = (4820)(\dot{m}_{\text{oil}})(160 - 94)$$

$$\dot{m}_2 = \dot{m}_{\text{oil}} = 2.480\ \text{kg/s}$$

Calculate the log mean temperature difference based on the countercurrent flow, $\Delta T_{\text{lm},cp}$:

$$\Delta T_1 = 94 - 16 = 78°\text{C}$$

$$\Delta T_2 = 160 - 84 = 76°\text{C}$$

$$\Delta T_{\text{lm},cp} = (78 - 76)/\ln(78/76) = 77°\text{C}$$

Calculate P and R:

$$t_1 = 16°\text{C}, \quad t_2 = 84°\text{C}, \quad T_1 = 160°\text{C}, \quad T_2 = 94°\text{C}$$

$$P = (84 - 16)/(160 - 16) = 0.472$$

$$R = (160 - 94)/(84 - 16) = 0.971$$

Using Figure 16.7, for a 1–4 shell and tube heat exchanger, $F \simeq 0.965$. Therefore,

$$\begin{aligned}\Delta T_{\text{lm}} &= F\Delta T_{\text{lm},cp}\\ &= (0.965)(77)\\ &= 74.3°\text{C}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 16.11

Refer to the previous example. Calculate the area and the length of the tubes required for this heat exchanger.

***SOLUTION*:** The heat transfer area may now be calculated:

$$\dot{Q} = UA\Delta T_{\text{lm}} \tag{16.22}$$

$$A = \dot{Q}/(U\Delta T_{\text{lm}}) = 788{,}800/[(350)(74.3)] = 30.33\ \text{m}^2$$

Calculate the tube length, L:

$$\begin{aligned} N_t &= \text{total number of tubes} \\ &= (\text{number of tubes per pass, } N_{tpp})(\text{number of passes, } N_p) \\ &= N_{tpp}\, N_P \qquad (16.16) \\ &= (11)(4) \\ &= 44 \text{ tubes} \end{aligned}$$

Therefore,

$$A = \pi D_i L N_t$$

$$L = (30.34)/[(\pi)(0.0229)(44)] = 9.58\,\text{m} = 31.3\,\text{ft}$$ ■

ILLUSTRATIVE EXAMPLE 16.12

Steam enters the shell-side of a shell and tube heat exchanger at 450°F and exits at a temperature of 300°F. An organic material that has fluid properties similar to water enters the tube-side of the same heat exchanger at 200°F and is heated by the steam to an exit temperature of 300°F. Draw the diagram of each of the following specifications and/or heat exchangers:

- **(a)** Countercurrent flow
- **(b)** Co-current flow
- **(c)** One shell-side pass and two tube-side passes
- **(d)** One shell-side pass and four tube side passes
- **(e)** Two shell-side passes and four tube-side passes
- **(f)** Six shell-side passes and twelve tube-side passes

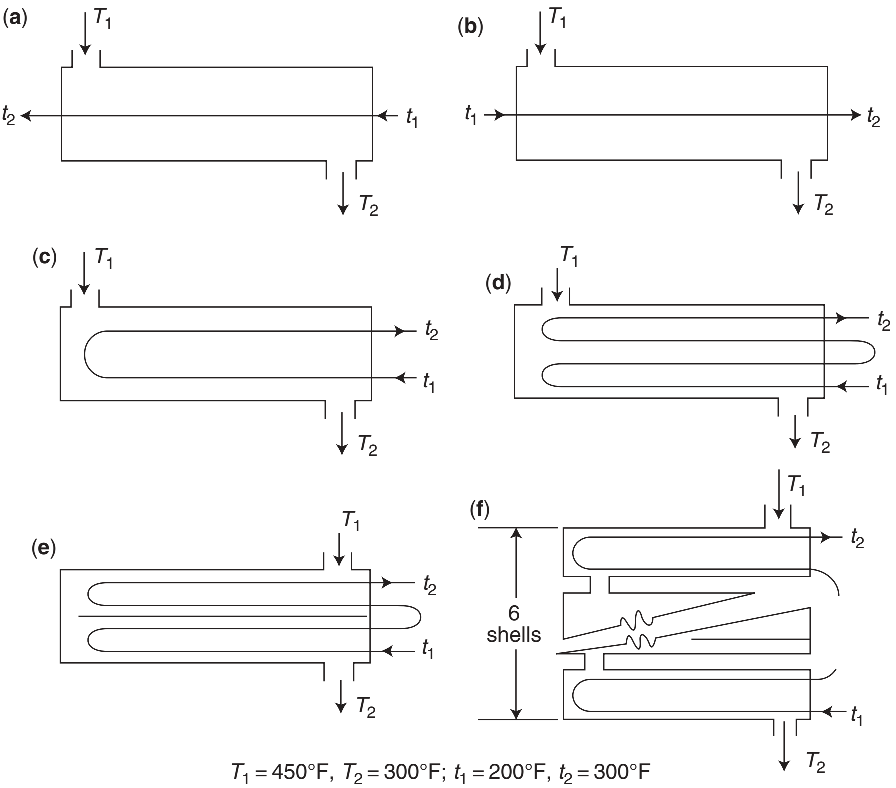
***SOLUTION*:** The appropriate labeling of one of the heat exchanger examples is as follows:

$$T_1 = 450^\circ\text{F} \qquad T_2 = 300^\circ\text{F}$$

and

$$t_1 = 200^\circ\text{F} \qquad t_2 = 300^\circ\text{F}$$

The diagrams of each specification and/or heat exchanger are shown in Figure 16.15a–f. ■



$T_1 = 450°\text{F}$, $T_2 = 300°\text{F}$; $t_1 = 200°\text{F}$, $t_2 = 300°\text{F}$

Figure 16.15 Diagrams of shell tube passes; Illustrative Example 16.12.

ILLUSTRATIVE EXAMPLE 16.13

Refer to the previous example. Calculate the correction factor, F, and the log mean temperature, ΔT_{lm}, for each specification employing the figures provided earlier.

***SOLUTION*:** The F and ΔT_{lm} values obtained using the figures for the given exchanger specifications are:

		F	ΔT_{lm}
a	Countercurrent Flow	1.000	123.3
b	Co-current Flow	0	0
c	1–2 Multipass	0.810	99.9
d	1–4 Multipass	0.805	99.3
e	2–4 Multipass	0.960	118.4
f	6–12 Multipass	1.000	123.3

■

ILLUSTRATIVE EXAMPLE 16.14

Use the "Bowman" equations to solve the previous example.

***SOLUTION*:** The F and ΔT_{lm} values generated using the equations for the given exchanger specifications are:

		F	ΔT_{lm}
a	Countercurrent Flow	1.000	123.3
b	Co-current Flow	0	0
c	1–2 Multipass	0.803	99.1
d	1–4 Multipass	0.799	98.5
e	2–4 Multipass	0.957	118.1
f	6–12 Multipass	1.000	123.3

■

ILLUSTRATIVE EXAMPLE 16.15

Comment on the results of the two previous examples.

***SOLUTION*:** The agreement between the two approaches is excellent. ■

ILLUSTRATIVE EXAMPLE 16.16

Provide some sample calculations for Illustrative Example 16.14.(10)

***SOLUTION*:** These sample calculations show the different steps to calculating the correction factor, F, and log mean temperature, ΔT_{lm}, for the different specifications and/or heat exchangers.

The first log mean temperature, ΔT_{lm}, is calculated assuming ideal countercurrent flow. This log mean temperature, ΔT_{lm}, calculated will be used throughout the example with the correction factor, F, for each of the different heat exchangers examined.

$$\begin{aligned}\Delta T_{\text{lm}} &= \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\dfrac{T_1 - t_2}{T_2 - t_1}\right)} \\ &= \frac{(450^\circ\text{F} - 300^\circ\text{F}) - (300^\circ\text{F} - 200^\circ\text{F})}{\ln\left(\dfrac{450^\circ\text{F} - 300^\circ\text{F}}{300^\circ\text{F} - 200^\circ\text{F}}\right)} \\ &= \frac{(150^\circ\text{F}) - (100^\circ\text{F})}{\ln\left(\dfrac{150^\circ\text{F}}{100^\circ\text{F}}\right)} \\ &= 123.3^\circ\text{F}\end{aligned}$$

For ideal countercurrent flow, the correction factor F is assumed to be equal to one. Therefore, the log mean temperature of countercurrent flow is 123.3°F.

The calculation of the correction factor, F, for a 2–4 multipass exchanger follows. First calculate P and R.

$$P = \frac{100}{250} = 0.4$$

$$R = \frac{150}{100} = 1.5$$

In addition,

$$F_{2,4} = \frac{\sqrt{R^2+1}}{2(R-1)}\log\left(\frac{1-P}{1-PR}\right) \Big/ \log\left[\frac{(2/P)-1-R+(2/P)\sqrt{(1-P)(1-PR)}+\sqrt{R^2+1}}{(2/P)-1-R+(2/P)\sqrt{(1-P)(1-PR)}-\sqrt{R^2+1}}\right] \tag{16.33}$$

Substituting,

$$\begin{aligned} F_{2,4} &= \frac{\sqrt{1.5^2+1}}{2(1.5-1)}\log\left(\frac{1-0.4}{1-0.4\times 1.5}\right) \\ &\Big/ \log\left[\frac{(2/0.4)-1-1.5+(2/0.4)\sqrt{(1-0.4)(1-0.4\times 1.5)}+\sqrt{1.5^2+1}}{(2/0.4)-1-1.5+(2/0.4)\sqrt{(1-0.4)(1-0.4\times 1.5)}-\sqrt{1.5^2+1}}\right] \\ &= 0.957 \end{aligned}$$

For this case,

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm,cp}} \tag{16.23}$$

Substituting,

$$\begin{aligned} \Delta T_{\text{lm,corrected}} &= (0.957)(123.3) \\ &= 118.1^\circ\text{F} \end{aligned}$$

The calculation of the correction factor for a 1–4 multipass exchanger requires an iterative calculation of the intermediate temperature, t_i. In order to determine this value, the dimensionless constant, V, is calculated and used in the steps to follow. The goal seek function in Microsoft Excel can be used to perform the final calculation of the intermediate temperature.

Calculation of the dimensionless constant, V:

$$V = \frac{t_2 - t_1}{4T_1 - (t_1 + 2t_i + t_2)} \tag{16.31}$$

Substituting,

$$V = \frac{300 - 200^\circ\text{F}}{4 \times 450^\circ\text{F} - (200^\circ\text{F} + 2t_i + 300^\circ\text{F})}$$

Below is the (iteration) equation for the intermediate temperature, t_i:

$$\left(\frac{t_i - t_1}{t_2 - t_i}\right)^{\sqrt{4R^2+1}} = \frac{1 + V\left(\sqrt{4R^2+1} - 2R\right)}{1 - V\left(\sqrt{4R^2+1} + 2R\right)} \tag{16.32}$$

Substituting,

$$\left(\frac{t_i - 200°\text{F}}{300 - 200°\text{F}}\right)^{\sqrt{4R^2+1}} = \frac{1 + V\left(\sqrt{4\times 1.5^2+1} - 2\times 1.5\right)}{1 - V\left(\sqrt{4\times 1.5^2+1} + 2\times 1.5\right)}$$

$$0 = \frac{1 + \left(\dfrac{300 - 200°\text{F}}{4\times 450°\text{F} - (200°\text{F} + 2t_i + 300°\text{F})}\right)\left(\sqrt{4\times 1.5^2+1} - 2\times 1.5\right)}{1 - \left(\dfrac{300 - 200°\text{F}}{4\times 450°\text{F} - (200°\text{F} + 2t_i + 300°\text{F})}\right)\left(\sqrt{4\times 1.5^2+1} + 2\times 1.5\right)} - \left(\frac{t_i - 200°\text{F}}{300 - 200°\text{F}}\right)^{\sqrt{4R^2+1}}$$

Solving,

$$t_i = 262.4°\text{F}$$

for which

$$V = 0.129$$

The calculation for the correction factor F follows:

$$F_{1,4} = \frac{\sqrt{4R^2+1}}{2(R-1)}\log\left(\frac{1-P}{1-PR}\right) \Big/ \log\left[\frac{1 + V\left(\sqrt{4R^2+1} - 2R\right)}{1 - V\left(\sqrt{4R^2+1} + 2R\right)}\right] \tag{16.30}$$

Substituting,

$$\begin{aligned} F_{1,4} &= \frac{\sqrt{4\times 1.5^2+1}}{2(1.5-1)}\log\left(\frac{1-0.4}{1-0.4\times 1.5}\right) \Big/ \log\left[\frac{1 + 0.129\left(\sqrt{4\times 1.5^2+1} - 2\times 1.5\right)}{1 - 0.129\left(\sqrt{4\times 1.5^2+1} + 2\times 1.5\right)}\right] \\ &= 0.799 \end{aligned}$$

The corrected driving force may now be calculated

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm,cp}} \tag{16.23}$$

Substituting,

$$\begin{aligned} \Delta T_{\text{lm}} &= (0.799)(123.3°\text{F}) \\ &= 98.5°\text{F} \end{aligned}$$

■

EFFECTIVENESS FACTOR AND NUMBER OF TRANSFER UNITS

The effectiveness factor number of transfer units (or ε-NTU) method described in the previous chapter for double pipe exchangers is again preferable to use when only inlet temperatures to the shell and tube heat exchanger are known. The following notation (different from the previous chapter since the colder fluid is normally on the tube side) will be used for this shell and tube exchanger development:

$\dot{m}_1$ = mass flow rate of fluid entering or leaving the tube side
$\dot{m}_2$ = mass flow rate of fluid entering or leaving the shell side
T_1, T_2 = inlet and exit temperatures of the shell side fluid, respectively
t_1, t_2 = inlet and exit temperatures of the tube side fluid, respectively
$\dot{Q}$ = heat duty, heat load, or heat transfer rate
$\dot{Q}_{\max}$ = maximum possible heat transfer rate
c_1, c_2 = heat capacity of fluids on the tube and shell sides, respectively
C_1, C_2 = capacitance rate of fluids on the tube and shell sides, respectively

As noted earlier, the heat duty of a shell and tube exchanger is calculated from the energy balance on the tube side or on the shell side:

$$\dot{Q} = \dot{m}_1 c_1 (T_1 - T_2) = \dot{m}_2 c_2 (T_1 - T_2) = C_1 (t_2 - t_1) = C_2 (T_1 - T_2) \tag{16.40}$$

This assumes that the fluid on the tube side is being heated ($t_2 > t_1$) and the shell side fluid is being cooled ($T_1 > T_2$). To determine $\dot{Q}_{\max}$, the maximum heat transfer rates for both the tube, $\dot{Q}_{\max,t}$, and shell side fluids, $\dot{Q}_{\max,s}$, are first determined and the lower of the two values is used. For the tube side, $\dot{Q}_{\max,t}$ occurs when the exit fluid temperature, t_2, is the same as the temperature of the incoming fluid on the shell side (i.e., $t_2 = T_1$), so that

$$\dot{Q}_{\max,t} = C_1 (T_1 - t_1) \tag{16.41}$$

Similarly, for the shell side fluid, $\dot{Q}_{\max,s}$ occurs when $T_2 = t_1$:

$$\dot{Q}_{\max,s} = C_2 (T_1 - t_1) \tag{16.42}$$

The lower of the two capacitance rates, C_1 and C_2, determines $C_{\min}$ and the permissible maximum heat load, $\dot{Q}_{\max}$. If the capacitance rate $C_1 < C_2$, then $C_{\min} = C_1$ and $C_{\max} = C_2$. In this case,

$$\dot{Q}_{\max} = C_1 (T_1 - t_1) = C_{\min} (T_1 - t_1) \tag{16.43}$$

Using the earlier definition of the effectiveness, ε, and combining Equations (16.40) and (16.43) yields

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{t_2 - t_1}{T_1 - t_1} \tag{16.44}$$

If the shell side fluid has the lower heat capacity rate (i.e., $C_2 < C_1$), then $C_{\min} = C_2$, $C_{\max} = C_1$, and

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{T_1 - T_2}{T_1 - t_1} \tag{16.45}$$

For any heat exchanger, it can be shown that

$$\varepsilon = f\left(\text{NTU}, \frac{C_{\min}}{C_{\max}}\right); \qquad \text{NTU} = \frac{UA}{C_{\min}} \tag{16.46}$$

Several graphs in which ε is plotted against the NTU, with the $C_{\min}/C_{\max}$ ratio C_r as the curve parameter, are presented below. Six graphs are available for six different heat exchanger geometries.$^{(2)}$ These are:

Figure 16.16: for a parallel flow heat exchanger.

Figure 16.17: for a counterflow heat exchanger.

Figure 16.18: for a shell and tube heat exchanger with one shell pass and any even multiple of two tube passes (two, four, etc.).

Figure 16.19: for a shell and tube heat exchanger with two shell passes and any even multiple of four tube passes (four, eight, etc.).

Figure 16.20: for a single pass, crossflow heat exchanger with both fluids unmixed.

Figure 16.21: for a single pass, crossflow heat exchanger with one fluid mixed and the other unmixed.

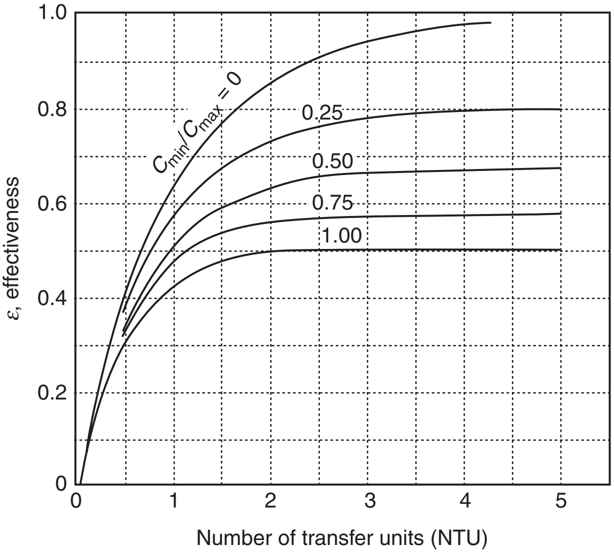


Figure 16.16 Effectiveness of transfer units for a parallel-flow heat exchanger.

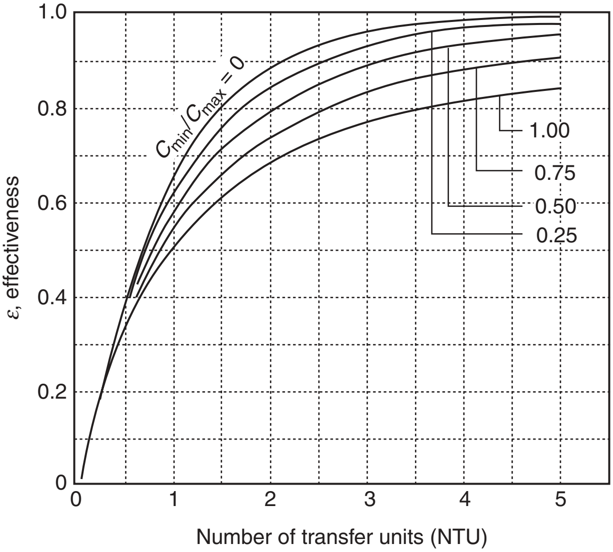


Figure 16.17 Effectiveness of transfer units for a counterflow heat exchanger.

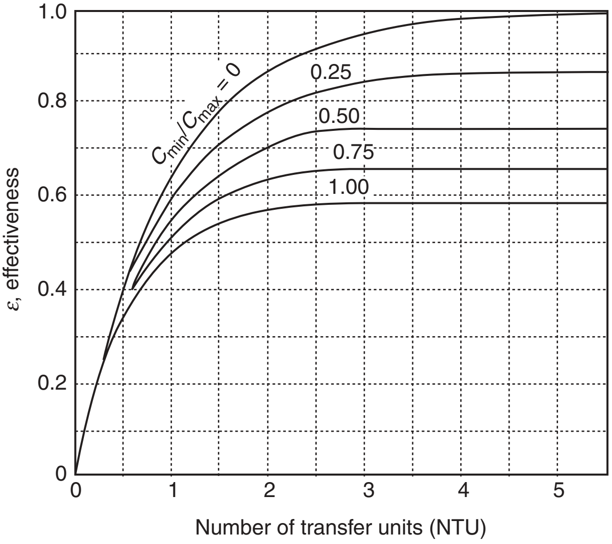


Figure 16.18 Effectiveness of transfer units for a shell and tube heat exchanger with one shell pass and any even multiple of two tube passes.

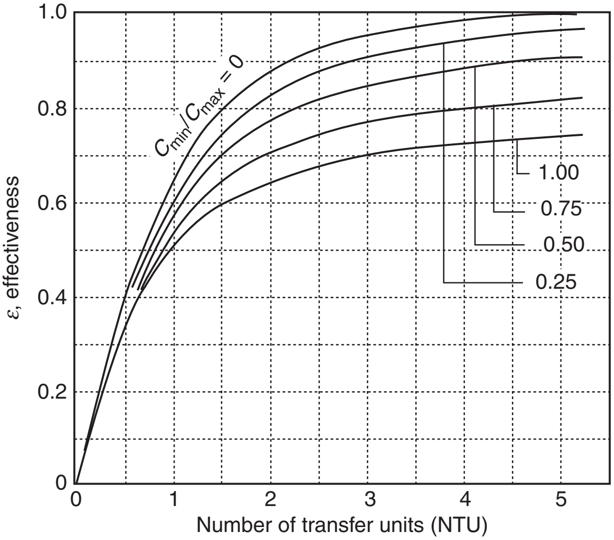


Figure 16.19 Effectiveness of transfer units for a shell and tube heat exchanger with two shell passes and any even multiple of four tube passes.

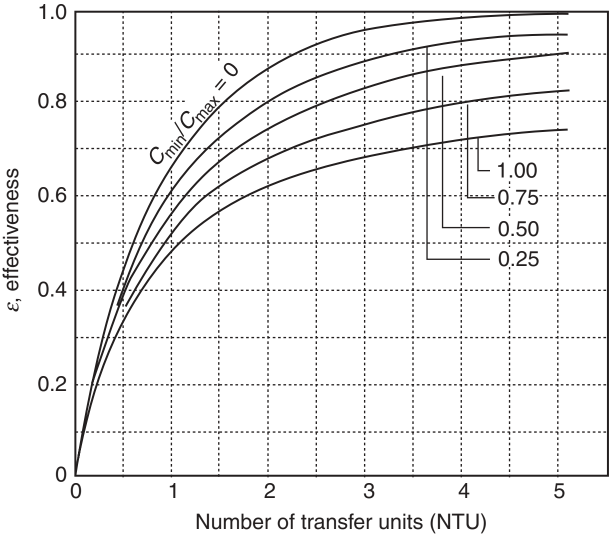


Figure 16.20 Effectiveness of transfer units for a single pass, crossflow heat exchanger with both fluids unmixed.

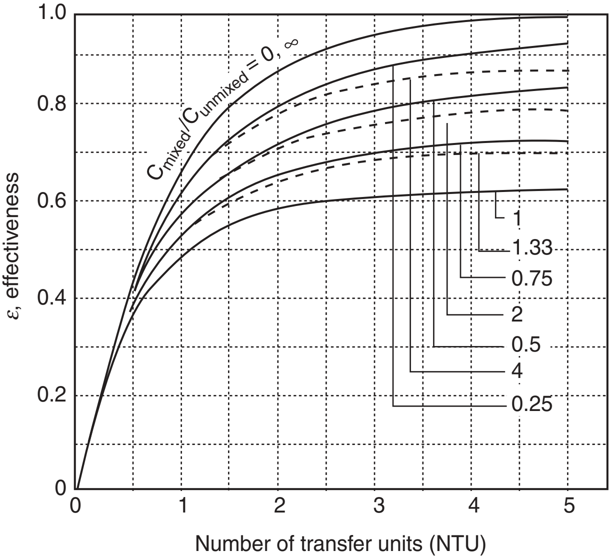


Figure 16.21 Effectiveness of transfer units for a single pass, crossflow heat exchanger with one fluid mixed and the other unmixed. The solid curves correspond to $C_{\min}$ mixed and $C_{\max}$ unmixed while the dashed curves correspond to $C_{\min}$ unmixed and $C_{\max}$ mixed.

The figures show that when the capacitance rate ratio $C_{\min}/C_{\max} = C_r = 0$, all heat exchangers have the same effectiveness. Moreover, if NTU < 0.25, all heat exchangers have the same effectiveness regardless of the value of C_r. More generally, for $C_r > 0$ and NTU > 0.25, the counterflow exchanger is the most effective.

ILLUSTRATIVE EXAMPLE 16.17

Describe the relationship between the maximum and minimum values of the effectiveness and C_r.

***SOLUTION*:** Examining Figures (16.16–16.20) indicates that the maximum and minimum value of the effectiveness are associated with $C_r = 0$ and $C_r = 1$, respectively.

Griskey(11,12) has also provided effectiveness relationships in equation form. These are provided in Table 16.2. Barkwill(13) has also provided these relationships in equation form as can be seen in Table 16.3. However, the equations of Griskey, if applicable, are recommended. ■

ILLUSTRATIVE EXAMPLE 16.18

Refer to Illustrative Example 16.10. Determine the effectiveness and number of transfer units.

Table 16.2 Heat Exchanger Effectiveness Relations$^{(11,12)}$

System	Effectiveness relation
Double pipe	
Parallel flow (Figure 16.16)	$\varepsilon = \dfrac{1 - \exp[-N(1+C)]}{1+C}$
Countercurrent flow (Figure 16.17)	$\varepsilon = \dfrac{1 - \exp[-N(1+C)]}{1 - C\exp[-N(1-C)]}$
Cross-flow	
Both fluids unmixed (Figure 16.20)	$\varepsilon = 1 - \exp\{C/n[\exp(-NCn) - 1]\}$ where $n = N^{-0.22}$
$C_{\max}$ mixed, $C_{\min}$ unmixed (Figure 16.21)	$\varepsilon = (1/C)\{1 - \exp[C(1 - e^{-N})]\}$
$C_{\max}$ unmixed, $C_{\min}$ mixed (Figure 16.21)	$\varepsilon = 1 - \exp\{(1/C)[1 - \exp(-NC)]\}$
Shell and tube	
One shell pass; 2, 4, 6 tube passes (Figure 16.18)	$\varepsilon = 2\{1 + C + (1+C^2)^{1/2}\left\{\dfrac{1+\exp[-N(1+C^2)^{1/2}]}{1-\exp[-N(1+C^2)^{1/2}]}\right\}\}^{-1}$

where N is NTU or $UA/(\dot{m}c_p)_{\min}$ and $C = C_r = (\dot{m}c_p)_{\min}/(\dot{m}c_p)_{\max}$

Table 16.3 Regression Equations: Heat Exchanger Effectiveness Relations$^{(13)}$

System	Effectiveness relation
Parallel flow (Figure 16.16)	$y = [-\exp(-x) + 1] - \left[(0.625)(z)\left(\dfrac{0.85 \times x^2}{x^2 + 1.5}\right)\right]$
Counterflow (Figure 16.17)	$y = [-\exp(-x) + 1] - \left[(0.22)(z)\left(\dfrac{x^2}{x^2 + 3.5}\right)\right]$
One shell pass + multiple of 2 tube passes (Figure 16.18)	$y = [-\exp(-x) + 1] - \left[(0.40)(z)\left(\dfrac{x^2}{x^2 + 1}\right)\right]$
Two shell pass + multiple of 4 tube passes (Figure 16.19)	$y = [-\exp(-x) + 1] - \left[(0.29)(z)\left(\dfrac{x^2}{x^2 + 2}\right)\right]$
Single pass, no mixed fluids (Figure 16.20)	$y = [-\exp(-x) + 1] - \left[(0.29)(z)\left(\dfrac{x^2}{x^2 + 3}\right)\right]$
Single pass mixed fluids, (Figure 16.21)	$y = [-\exp(-x) + 1] - \left[(0.40)(z)\left(\dfrac{x^2}{x^2 + 2}\right)\right]$

Note: y = effectiveness number, x = NTU, $z = C_{\min}/C_{\max}$.

***SOLUTION*:** $C_1 = C_w =$ capacitance rate of water $= \dot{m}_1 c_{p,w} = (2.778)(4176) = 11{,}600\ \mathrm{W/^\circ C}$ and $C_2 = C_{\text{oil}} = \dot{m}_2 c_{p,\text{oil}}(2.480)(4820) = 11{,}950\ \mathrm{W/^\circ C}$.

To determine the effectiveness, ε, note that tube-side water has the minimum capacitance rate. Thus,

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{water,max}}} = \frac{C_w(84 - 16)}{C_w(160 - 16)} = \frac{84 - 16}{160 - 16}$$
$$= 0.472$$

The number of transfer units, NTU is

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{(350)(30.33)}{11{,}600}$$
$$= 0.915$$

Finally, calculate the capacitance rate ratio:

$$\frac{C_{\min}}{C_{\max}} = \frac{C_w}{C_{\text{oil}}} = \frac{11{,}600}{11{,}950} = 0.971$$

■

ILLUSTRATIVE EXAMPLE 16.19

The feedwater heater for a boiler supplies 10,000 kg/h of water at 65°C. The raw feedwater enters the tubes with an inlet temperature of 20°C and is to be heated in a single-shell, two-tube pass heat exchanger by condensing steam at 1.30 bars. Water is on the tube side and steam is on the shell side. The overall heat transfer coefficient is $2000\ \mathrm{W/m^2 \cdot K}$. Determine the required heat transfer area for the exchanger. Use both the LMTD method and the ε-NTU approach to obtain the results.

***SOLUTION*:** Organize the information as presented in Table 16.4.

Table 16.4 Data and Information for Illustrative Example 16.19

	Water (1)		Steam (2)	
Properties	Inlet	Outlet	Inlet	Outlet
Side	Tube		Shell	
P, bars	N.A.		1.3	
t, T, °C	20	65	107.3	107.3
$\dot{m}$, kg/s	2.778			
c_p, $\mathrm{J/kg \cdot K}$	4179		Latent heat of condensation, $h_{\text{vap}} = 2.238 \times 10^6\ \mathrm{J/kg}$	
Capacitance rate, C, W/°C	11,608.3		∞	
Capacitance rate, C, classification	$C_{\min}$		$C_{\max}$	

Calculate the heat load, $\dot{Q}$:

$$\dot{Q} = C_w(t_2 - t_1) = 11{,}608.3(65 - 20) = 5.224 \times 10^5 \text{ W}$$

Calculate the steam condensation rate:

$$\dot{Q} = \dot{m}_2 h_{\text{vap}}$$

$$\dot{m}_2 = (5.224 \times 10^5)/(2.238 \times 10^6) = 0.2334 \text{ kg/s}$$

Calculate the LMTD ($\Delta T_{\text{lm},cp}$):

$$\begin{aligned} \Delta T_1 &= 107.3 - 20 = 87.3^\circ\text{C} = 87.3 \text{ K} \\ \Delta T_2 &= 107.3 - 65 = 42.3^\circ\text{C} = 42.3 \text{ K} \\ \Delta T_{\text{lm},cp} &= (87.3 - 42.3)/\ln(87.3/42.3) = 62.1^\circ\text{C} = 62.1 \text{ K} \end{aligned}$$

Calculate the correction factor, F. Since the shell side has a constant temperature, $F = 1$:

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm},cp} = 62.1^\circ\text{C}$$

Calculate the heat transfer area, A (i.e., by the LMTD method):

$$\dot{Q} = UA\Delta T_{\text{lm}}$$

$$A = (5.224 \times 10^5)/[(2000)(62.1)] = 4.21 \text{ m}^2 = 45.32 \text{ ft}^2$$

Calculate $C_{\min}/C_{\max}$ and ε using the effectiveness-NTU (number of transfer units) method:

$$C_{\min}/C_{\max} = 0$$

Since water has the lower capacitance rate, the effectiveness is based on the temperature change of water:

$$\varepsilon = \frac{\text{actual temperature change of water}}{\text{maximum temperature change of water}} = \frac{65 - 20}{107.3 - 20} = 0.515$$

From the ε vs. NTU figure for a shell and tube heat exchanger with one pass on the shell side and two passes on the tube side (Figure 16.18), one estimates

$$\text{NTU} \simeq 0.7$$

Calculate the heat transfer area, A by the ε-NTU method:

$$\text{NTU} = UA/C_{\min} \tag{16.46}$$

$$A = \frac{(\text{NTU})/(C_{\min})}{U} = \frac{(0.7)(11{,}608.3)}{2000} = 4.1 \text{ m}^2$$

■

ILLUSTRATIVE EXAMPLE 16.20

Compare the results obtained using the two approaches in the previous example.

***SOLUTION*:** The NTU result is very close to the area result obtained from the LMTD method. This was to be expected. ■

ILLUSTRATIVE EXAMPLE 16.21

A shell and tube heat exchanger has one pass on the shell side and two passes on the tube side (i.e., 1–2 shell and tube heat exchanger). It is being used for oil cooling. The oil flows in the tube side. It enters at 110°C and leaves at 75°C. The shell-side fluid is water at a flow rate of 1.133 kg/s, entering at 35°C and leaving at 75°C. The oil flows in the tubes at an average velocity of 0.3 m/s. The tube inside diameter is $\frac{3}{4}$ inch or 0.01905 m. The overall heat transfer coefficient for a clean heat exchanger is 350 $\text{W/m}^2 \cdot \text{K}$. The fouling factor, R_f, is 0.00027 $\text{m}^2 \cdot \text{K/W}$. The oil properties are: $\rho_{\text{oil}} = 850\ \text{kg/m}^3$ and $c_{\text{oil}} = 1900\ \text{J/kg} \cdot \text{K}$. Calculate:

1. the mass flow rate of the oil
2. the minimum and maximum heat capacity rate
3. the heat load, $\dot{Q}$
4. the total number of tubes
5. the tube length.

***SOLUTION*:** Organize the information in the problem statement as shown in Table 16.5.

Calculate the water capacitance rate:

$$\begin{aligned} C_w &= \dot{m}_w c_w;\ C_w = C_{p,w} \\ &= (1.133)(4180) \\ &= 4736\ \text{W/K} \end{aligned}$$

Calculate the heat load:

$$\begin{aligned} \dot{Q} &= \dot{m}_w c_w (t_2 - t_1) = C_w (t_2 - t_1) \\ &= (4736)(75 - 35) \\ &= 189{,}440\ \text{W} \end{aligned}$$

Table 16.5 Data and Information for Illustrative Example 16.21

	Stream			
Parameter	Inlet	Outlet	Inlet	Outlet
Fluid	Oil		Water	
Side	Tube		Shell	
ρ, kg/m^3	850		995	
C_p, $\text{J/kg} \cdot \text{K}$	1900		4180	
$\dot{m}$, kg/s	*		1.133	
Kinematic viscosity, ν, m^2/s	0.0008		N/A	
Capacitance rate, $C = \dot{m}$, W/K	*		4736	
T, t, °C	110	75	35	75
V, m/s	0.3		N/A	

*To be determined later.

Calculate the oil capacitance rate and mass flow rate:

$$\begin{aligned}
\dot{Q} &= \dot{m}_{\text{oil}} c_{\text{oil}}(T_1 - T_2); \qquad c_{\text{oil}} = C_{p,\text{oil}} \\
&= C_{\text{oil}}(T_1 - T_2) \\
C_{\text{oil}} &= \dot{Q}/(T_1 - T_2) \\
&= 189{,}440/(110 - 75) \\
&= 5412.6 \text{ W/K} \\
\dot{m}_{\text{oil}} &= 5412.6/1900 \\
&= 2.85 \text{ kg/s}
\end{aligned}$$

Determine the minimum and maximum capacitance rate:

$$\begin{aligned}
C_{\min} &= C_w = 4736 \text{ W/K} \\
C_{\max} &= C_{\text{oil}} = 5412.6 \text{ W/K}
\end{aligned}$$

Calculate the oil flow rate per tube:

$$\begin{aligned}
\dot{m}_{\text{oil,tube}} &= \rho V(\pi/4)D^2 \\
&= (850)(0.3)(\pi/4)(0.01905)^2 \\
&= 0.0727 \text{ kg/s}
\end{aligned}$$

Determine the number of tubes:

$$\begin{aligned}
N_{tpp} &= \dot{m}_{\text{oil,total}}/\dot{m}_{\text{oil,tube}} && (16.15) \\
&= 2.85/0.0727 \\
&= 39.2 \\
&= 40 \text{ tubes per pass} \\
N_t &= N_{tpp}(\text{number of passess, } N_p) && (16.16) \\
&= (40)(2) \\
&= 80 \text{ tubes}
\end{aligned}$$

For countercurrent flow in a double-pipe heat exchanger, calculate the log mean temperature difference:

$$\begin{aligned}
\Delta T_1 &= 110 - 75 = 35^\circ\text{C} \\
\Delta T_2 &= 75 - 35 = 40^\circ\text{C} \\
\Delta T_{\text{lm}} &= \frac{35 - 40}{\ln(35/40)} = 37.44^\circ\text{C} = 37.44 \text{ K}
\end{aligned}$$

Determine the parameters *P* and *R* needed to obtain the correction factor, *F*, from the appropriate *F* chart or equation:

$$\begin{aligned} P &= (t_2 - t_1)/(T_1 - t_1) && (16.24) \\ &= (75 - 35)/(110 - 35) \\ &= 0.533 \\ R &= (T_1 - T_2)/(t_2 - t_1) && (16.25) \\ &= (110 - 75)/(75 - 35) \\ &= 0.875 \end{aligned}$$

From Figure 16.7, $F = 0.81$. The corrected log mean temperature difference is therefore

$$\begin{aligned} \Delta T_{\text{lm}} &= F \Delta T_{\text{lm},cp} \\ &= (0.81)(37.44) \\ &= 30.33^\circ\text{C} \end{aligned}$$

The "dirty" overall heat transfer coefficient is

$$\begin{aligned} \frac{1}{U_{\text{dirty}}} &= \frac{1}{U_{\text{clean}}} + R_f \\ &= \frac{1}{350} + 0.00027 \\ U_{\text{dirty}} &= 320\ \text{W/m}^2 \cdot \text{K} \end{aligned}$$

Calculate the required heat transfer area:

$$\begin{aligned} A &= \dot{Q}/[U_{\text{dirty}} \Delta T_{\text{lm}}] \\ &= 189{,}440/[(320)(30.33)] \\ &= 19.5\ \text{m}^2 \end{aligned}$$

Finally, calculate the tube length:

$$\begin{aligned} A &= N_t \pi D L \\ L &= A/(N_t \pi D) \\ &= 19.5/[(80)(\pi)(0.01905)] \\ &= 4.1\ \text{m} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 16.22

Determine the effectiveness and NTU for the previous example.

***SOLUTION*:** For the effectiveness of the exchanger, the actual water temperature change $= 75 - 35 = 40$°F. The maximum water temperature change $= 110 - 35 = 75$°F. Therefore

$$\varepsilon = \frac{40}{75} = 0.533$$

Calculate the NTU using earlier results:

$$\begin{aligned} \text{NTU} &= UA/C_{\min};\ C_{\min} = C_w \qquad (16.46) \\ &= (320)(19.5)/4736 \\ &= 1.318 \end{aligned}$$

■

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Chapter 17

Fins and Extended Surfaces

INTRODUCTION

Consider the case of a heat exchanger that is heating air outside the tubes by means of steam inside the tubes. The steam-side coefficient will be very high and the air-side coefficient will be extremely low. Therefore, the overall heat transfer coefficient will approximate that of the air side (i.e., the air is the controlling resistance). One method of increasing the heat transfer rate is to increase the surface area of the heat exchanger. Therefore, if the surface of the metal on the air side could be increased, it would increase the area term without putting more tubes in the exchanger. This can be accomplished by mounting metal fins on a tube in such a way that there is good metallic contact between the base of the fin and the wall of the tube. If this contact is secured, the temperature throughout the fins will approximate that of the temperature of the heating (cooling) medium because of the high thermal conductivity of most metals used in practice. Consequently, the surface will be increased without more tubes.(1)

Additional metal is often added to the outside of ordinary heat transfer surfaces such as pipes, tubes, walls, etc. These extended surfaces, usually referred to as fins, increase the surface available for heat flow and result in an increase of the total transmission of heat. Examples of fin usage includes auto radiators, air conditioning, cooling of electronic components, and heat exchangers. They are primarily employed for heat transfer to gases where film coefficients are very low.

The remaining sections of this chapter include:

Fin Types
Describing Equations
Fin Effectiveness and Performance
Fin Considerations

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

FIN TYPES

Extended surfaces, or fins, are classified into longitudinal fins, transverse fins, and spine fins. *Longitudinal fins* (also termed straight fins) are attached continuously along the length of the surface (see Figure 17.1). They are employed in cases involving gases or viscous liquids. As one might suppose, they are primarily employed with double pipe heat exchangers. *Transverse* or *circumferential fins* are positioned approximately perpendicular to the pipe or tube axis and are usually used in the cooling of gases (see Figure 17.2). These fins find their major application with shell and tube exchangers. Transverse fins may be continuous or discontinuous (segmented). *Annular fins* are examples of continuous transverse fins. *Spine* or *peg fins* employ cones or cylinders, which extend from the heat transfer surface, and are used for either longitudinal flow or cross flow.

Fins are constructed of highly conductive materials. The optimum fin design is one that gives the highest heat transfer for the minimum amount of metal. The metal used in their manufacture has a strong influence on fin efficiency. Table 17.1 compares the volume and mass of three different metals required to give the same amount of heat transfer for fins with identical shapes. The values in the volume and mass columns are relative to the volume and mass of a copper fin.

ILLUSTRATIVE EXAMPLE 17.1

Consider the longitudinal (rectangular) fin pictured in Figure 17.1. Estimate the fin face area, neglecting the (top) area contribution associated with the fin thickness if $w = 1$ ft and $L = 1.5$ in. The fin thickness is 0.1 in. Also calculate the total area of the fin.

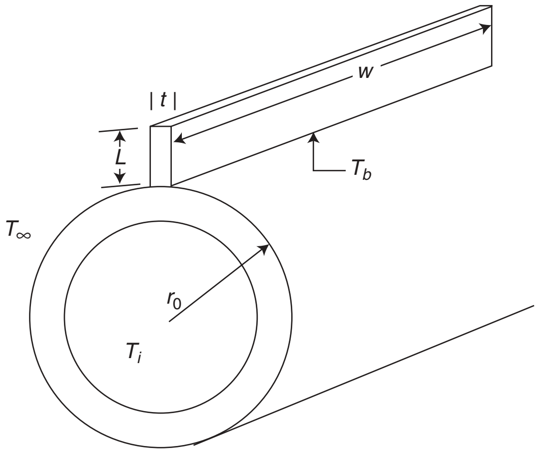

Figure 17.1 Longitudinal fins.

Table 17.1 Fin Metal Data

Metal	Thermal conductivity	Specific gravity	Relative volume	Relative mass
Copper	400	8.9	1.00	1.00
Aluminum	210	2.7	1.83	0.556
Steel	55	7.8	7.33	0.43

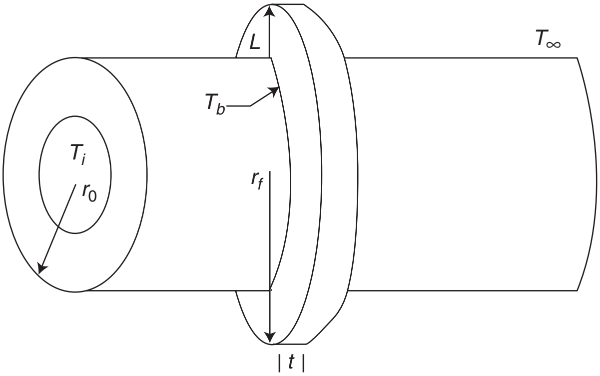

Figure 17.2 Transverse or circumferential fins.

***SOLUTION*:** The face area of the fin is given as

$$A_f = 2wL = 2(1.5)(12) = 36\,\text{in}^2 = 0.25\ \text{ft}^2$$

The total area is given by

$$A_{f,t} = 2wL + Lw = 36 + 12(0.1) = 36 + 1.2 = 37.2\,\text{in}^2 = 0.258\ \text{ft}^2 \qquad \blacksquare$$

ILLUSTRATIVE EXAMPLE 17.2

Comment on the results of the previous example.

***SOLUTION*:** Note that the external "rim" area, *wt*, does not contribute significantly to the total area and is normally neglected in fin calculations. ■

ILLUSTRATIVE EXAMPLE 17.3

Refer to Figure 17.2. Estimate the fin area, neglecting the area contribution associated with the fin thickness if $r_0 = 4.0$ in, $r_f = 6.0$ in, and $t = 0.1$ in.

***SOLUTION*:** This is an example of a circumferential (annular) fin. For a circumferential fin, the face area is given by

$$A_f = 2\pi(r_f^2 - r_0^2) = 2\pi[(6)^2 - (4)^2] = 125.7\,\text{in}^2 = 0.873\,\text{ft}^2$$

The total area is given by

$$A_{f,t} = A_f + 2\pi r_f(t) = 125.7 + 2\pi(6)(0.1) = 125.7 + 3.8 = 129.5\,\text{in}^2 = 0.899\ \text{ft}^2$$

Once again, the external "rim" area does not contribute significantly to the total area. ■

DESCRIBING EQUATIONS

There are two problems in calculating the heat transfer coefficient for smooth finned tubes:

1. The mean surface temperature of the fin is lower than the surface temperature of a smooth tube under the same conditions (due to the flow of heat through the metal of the fin).
2. There is a question as to whether or not the flow of the fluid outside the tube is as great at the bottom of the space between the fins as in the unobstructed space. Both these factors depend on the size and thickness of the fins, their spacing, and the conditions of flow.(1)

To analyze the heat transfer in extended surfaces, the following assumptions are usually made:(2)

- **a.** Steady-state operation
- **b.** Constant properties
- **c.** Constant surrounding air temperature of T_∞
- **d.** Homogeneous isotropic material, with thermal conductivity, k
- **e.** One-dimensional heat transfer by conduction in the radial direction
- **f.** No internal heat generation
- **g.** Heat transfer coefficient, h, is uniform along the fin surface
- **h.** Negligible thermal radiation
- **i.** Fin perimeter at any cross section is P
- **j.** The fin cross-sectional area is A_f
- **k.** The temperature of the heat transfer surface (exposed and unexposed) at the base of the fin is constant, T_b
- **l.** The maximum temperature driving force for convection is $T_b - T_\infty$

The maximum rate of heat transfer, $\dot{Q}_{f,\max}$, from a fin will occur when the entire fin surface is isothermal at $T = T_b$. In the case of a fin of total surface area A_f, $\dot{Q}_{f,\max}$

is written as

$$\dot{Q}_{f,\max} = hA_f(T_b - T_\infty) = hA_f\theta_b \tag{17.1}$$

where θ, termed the excess temperature, is defined as $(T - T_\infty)$. Thus, θ_b is the excess temperature at the base of the fin, and T_∞ is the fluid temperature.(2)

Since the fin has a finite thermal conductivity, a temperature gradient will exist along the fin. The actual heat transfer rate from the fin to the outside fluid, $\dot{Q}_f$, will be less than $\dot{Q}_{f,\max}$. The *fin efficiency*, η_f, is a measure of how close $\dot{Q}_f$ comes to $\dot{Q}_{f,\max}$ and is defined as:

$$\eta_f = \frac{\dot{Q}_f}{\dot{Q}_{f,\max}} = \frac{\dot{Q}_f}{hA_f\theta_b} \tag{17.2}$$

From Equations (17.1) and (17.2),

$$\dot{Q}_f = \eta_f hA_f\theta_b = \frac{T_b - T_\infty}{(1/\eta_f hA_f)} = \frac{T_b - T_\infty}{R_{t,f}} \tag{17.3}$$

where

$$R_{t,f} = \text{fin thermal resistance} = \frac{1}{\eta_f hA_f} \tag{17.4}$$

Figure 17.3 is a plot of the efficiency of straight (longitudinal) fins ($\eta\%$) versus the following dimensionless group,

$$L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} \tag{17.5}$$

where L_c is the corrected fin length and A_p is the profile area of the fin. Table 17.2 provides expressions to calculate the corrected length, L_c, the projected or profile area, A_p, and the surface area, A_f, in terms of the fin length, L, fin thickness at the base, t, and the fin width, w, for various fin types.

The fin effficiency figures are valid for fin Biot number ≤ 0.25, i.e.,

$$\mathrm{Bi}_f = \frac{h(t/2)}{k} \leq 0.25 \tag{17.6}$$

Barkwill et al.(3) recently converted the graphical results presented in Figure 17.3 into equation form. His results are provided in Table 17.3.

Figure 17.4 shows the fin efficiency ($\eta_f\%$) of annular fins of rectangular profile. The variables L_c and A_p used in the abscissa of the graph are related to the fin height, L,

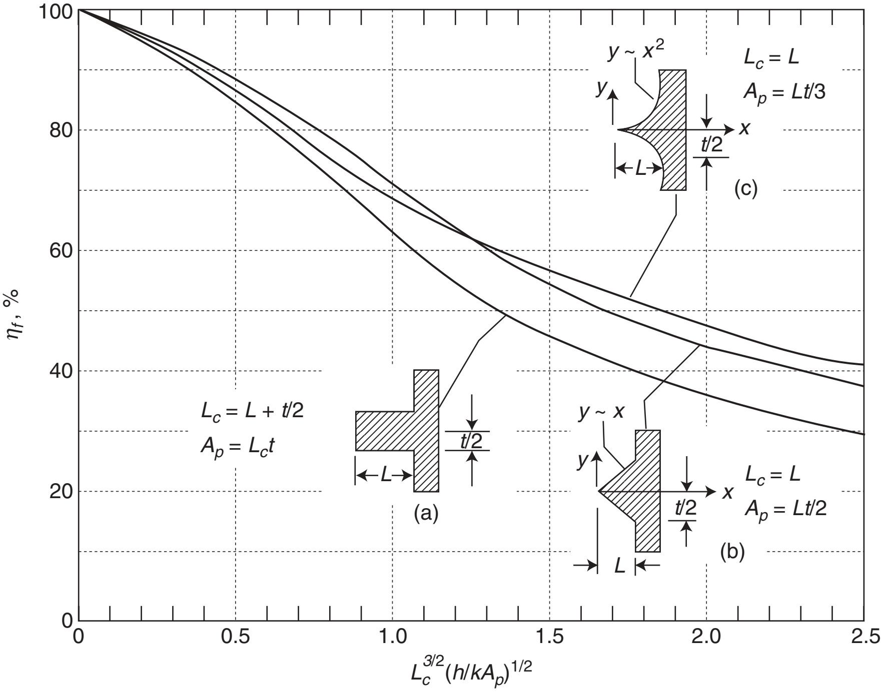

Figure 17.3 Efficiency of straight rectangular fins (a), triangular fins (b), and parabolic profile fins (c). (Adapted from Incropera and De Witt, *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, 1981, Figure 3.17.)

pipe radius, r_o, and fin outside radius, r_f, i.e.,

$$\begin{aligned}
L &= \text{ fin height} = r_f - r_o \\
r_{2c} &= \text{corrected outside radius} = r_f + (t/2) \\
L_c &= \text{corrected height} = L + t/2 \\
A_p &= \text{ profile (cross-sectional) area} = L_c t \\
A_f &= \text{ fin surface area} = 2\pi(r_{2,c}^2 - r_o^2)
\end{aligned}$$

Table 17.2 Fin Data

Variable	Rectangular fin	Triangular fin	Parabolic fin
L_c = corrected height	$L + t/2$	L	L
A_p = profile area	$L_c t$	$Lt/2$	$Lt/3$
A_f = fin surface area of length w	$2wL_c$	$2w\sqrt{L^2 - (t/2)^2}$	$2.05w\sqrt{L^2 - (t/2)^2}$

Table 17.3 Efficiency of Straight Rectangular Fins, Triangular Fins, and Parabolic Profile Fins

For rectangular fins:

$$Y = (4.5128 \times X^3 - 10.079 \times X^2 - 31.413 \times X + 101.47) \quad \text{(A)}$$

For triangular and parabolic profile fins:

$$Y = (3.1453 \times X^3 - 7.5664 \times X^2 - 25.536 \times X + 101.18) \quad \text{(B)}$$

Note: $Y = \eta_f$, %

$$X = L_c^{3/2}(h/kA_p)^{1/2} = \sqrt{\frac{L_c^3 h}{kA_p}}$$

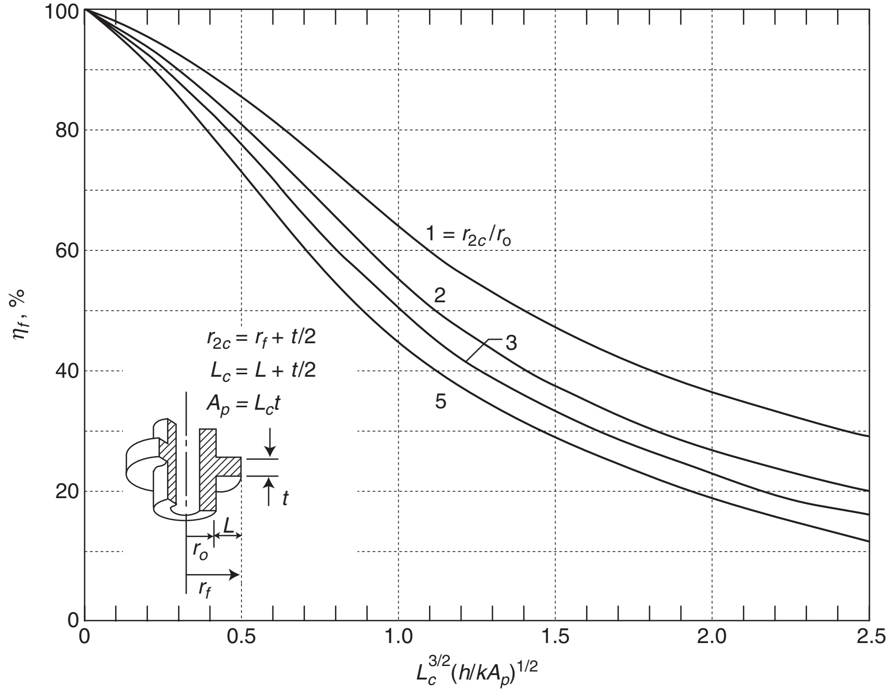

Figure 17.4 Efficiency of annular fins of rectangular profiles. (Adapted from Incroprera and De Witt, *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, 1981, Figure 3.18.)

Table 17.4 Efficiency of annular fins of rectangular profiles

$$Y = \left[(-5.1459)(X^4) + (30.478)(X^3)\right] - \left[(51.613)(X^2) - (9.3683)(X) + (100.08)\right]$$
$$- \left\{\ln\left[(-20X^3) + (95)(X^2) + (77)(X) + (0.5)(Z)\right]\ln(Z)^2\right\}$$

Note:

$$Y = \eta_f, \%$$

$$X = L_c^{3/2}(h/kA_p)^{1/2} = \sqrt{\frac{L_c^3 h}{kA_p}}$$

$$Z = r_{2c}/r_o$$

The parameter of the curves is the ratio, r_{2c}/r_o. Barkwill et al.$^{(3)}$ also converted the results of Figure 17.4 into equation form. His results are provided in Table 17.4.

For a straight fin (one of uniform cross-section as opposed to one that, for example, tapers down to a point), the heat transfer from the fin may be represented mathematically by

$$\dot{Q} = \sqrt{hPkA_c}\,\theta_c \tanh(mL_c) \tag{17.7}$$

where P is the fin cross-section perimeter, A_c is the (cross-sectional) area of the fin, and $m = \sqrt{hP/kA_c}$.

ILLUSTRATIVE EXAMPLE 17.4

The following information is provided for a straight rectangular fin: $h = 15\ \mathrm{W/m^2 \cdot K}$, $k = 300\ \mathrm{W/m \cdot K}$, $L = 3$ in, and $t = 1$ in. Estimate the fin efficiency.

SOLUTION: Refer to Figure 17.3. For a rectangular fin:

$$L_c = L + t/2 = 3 + (1/2) = 3.5\,\mathrm{in} = 0.0889\,\mathrm{m}$$

and

$$A_p = L_c t = 3.5(1) = 3.5\,\mathrm{in}^2 = 0.00226\,\mathrm{m}^2$$

Generate the x-coordinate of Figure 17.3:

$$\sqrt{\frac{L_c^3 h}{kA_p}} = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = (0.0899)^{3/2}\left(\frac{15}{300(0.00226)}\right)^{1/2}$$
$$= (0.0265)(22.124)^{1/2}$$
$$= 0.1246$$

Reading off Figure 17.3,

$$\eta_f \approx 98\%$$

■

ILLUSTRATIVE EXAMPLE 17.5

Estimate the fin efficiency in the previous example using the equation developed by Barkwill et al.$^{(3)}$

***SOLUTION*:** Apply Equation (A) from Table 17.3:

$$Y = 4.5128X^3 - 10.079X^2 - 31.413X + 101.47$$

with

$$X = 0.1246$$

and

$$Y = \eta_f$$

Substituting,

$$\begin{aligned}\eta_f &= 4.5128(0.1246)^3 - 10.079(0.1246)^2 - 31.413(0.1246) + 101.47\\ &= 0.008730 - 0.15648 - 3.91406 + 101.47\\ &= 0.974\\ &= 97.4\%\end{aligned}$$

The two results are in agreement with each other. ■

ILLUSTRATIVE EXAMPLE 17.6

A set of micro-fins is designed to cool an electronic circuit. Each micro-fin has a square cross-section of 0.2 cm by 0.2 cm and a length of 1 cm. The conductivity of the fin material is 400 $\mathrm{W/m \cdot K}$ and the air heat transfer coefficient is 16 $\mathrm{W/m^2 \cdot K}$. The circuit temperature is 100°C and the air temperature is 25°C. Calculate the heat transfer from each micro-fin in W.

***SOLUTION*:** Write the equation for the heat transfer application given above:

$$\dot{Q} = \sqrt{hPkA_c}\,\theta_b \tanh(mL_c) \tag{17.7}$$

Substitute known values and compute the heat transfer:

$$P = (4)(0.2) = 0.8\,\text{cm}$$

$$L_c = L + A_c/P = 1.0 + (0.2)(0.2)/(4.0)(0.2) = 1.05\,\text{cm} = 0.0105\,\text{m}$$

$$m = [hP/kA_c]^{1/2} = [(16)(4)(0.002)/(400)(0.002)(0.002)]^{1/2} = 8.95\,\text{m}^{-1}$$

$$\dot{Q} = \sqrt{(16)(4)(0.002)(400)(0.002)(0.002)}(100 - 25)\tanh[(8.95)(0.0105)]$$
$$= 0.10\,\text{W}$$

■

ILLUSTRATIVE EXAMPLE 17.7

Calculate the total heat transfer from the set of micro-fins in the previous example.

***SOLUTION*:** Although the heat transfer from one micro-fin is known, the total number of fins in the set is *not* known. Therefore, the total heat transfer cannot be calculated. ■

ILLUSTRATIVE EXAMPLE 17.8

Air and water are separated by a 1.5 mm plane wall made of steel ($k = 38\ \text{W/m}\cdot\text{K}$; density, $\rho = 7753\ \text{kg/m}^3$; heat capacity, $c_p = 486\ \text{J/kg}\cdot\text{K}$). The air temperature, T_1, is 19°C, and the water temperature, T_4, is 83°C. Denote the temperature at the air–wall interface T_2 and let T_3 be the temperature at the wall–water interface. The air-side heat transfer coefficient, h_1, is $13\ \text{W/m}^2\cdot\text{K}$ and the water side heat transfer coefficient, h_3, is $260\ \text{W/m}^2\cdot\text{K}$. Assume an area of the wall that is 1 m high and 1 m wide as a basis.

1. Show whether the conduction resistance may be neglected.
2. What is the rate of heat transfer from water to air?

To increase the rate of heat transfer, it is proposed to add steel fins to the wall. These straight rectangular steel fins will be 2.5 cm long, 1.3 mm thick, and will be spaced such that the fin pitch, S, is 1.3 cm between centers.

3. Calculate the percent increase in steady-state heat transfer rate that can be realized by adding fins to the air side of the plane wall.
4. Calculate the percent increase in steady-state heat transfer rate that can be realized by adding fins to the water side of the plane wall.

***SOLUTION*:** The base wall area, A, is $1.0\ \text{m}^2$.

1. R_1 = air resistance = $1/(h_1A) = 1/13 = 0.0769°\text{C/W}$
   R_2 = conduction resistance = $L_2/(k_1A) = 0.0015/38 = 3.95 \times 10^{-5}°\text{C/W}$
   R_3 = water resistance = $1/(h_4A) = 1/260 = 0.00385°\text{C/W}$
   Clearly R_2 (conduction) $\ll R_1$ or R_3 and may be neglected.
2. $R_{\text{tot}} = R_1 + R_3 = 0.0769 + 0.00385 = 0.0807°\text{C/W}$
   Therefore,

$$\dot{Q} = (T_4 - T_1)/R_{\text{tot}} = (83 - 19)/0.0807 = 793.1\,\text{W}$$

This represents the total heat transfer from the base pipe without fins, i.e. $\dot{Q}_{t,w/o,f}$.

3. For the case where fins have been added on the air side, write an expression for the total heat transfer rate, $\dot{Q}_t$:

$$\begin{aligned}\dot{Q}_t &= \dot{Q}_{be} + \dot{Q}_{ft} = \dot{Q}_{be} + N_f\dot{Q}_f \\ &= \dot{Q}_{be} + N_f\eta_f\dot{Q}_{f,\,\max} \\ &= h\theta_b(A_{be} + N_f\eta_f A_f)\end{aligned}$$

(See also Equations 17.10–17.13 in the next section for notation/nomenclature.)
For a unit area, 1 m in length and 1 m in width, calculate the number of fins, N_f, and the exposed base surface area, A_{be}:

$$\begin{aligned}N_f &= \text{number of fins} = 1/0.013 = 77\,\text{fins};\ S = 1.3\,\text{cm} \\ L_{be} &= \text{unfinned exposed base surface} = w - N_f t;\ t = 1.3\,\text{mm} \\ &= 1 - (77)(0.0013) = 1 - 0.1 = 0.9\,\text{m} \\ A_{be} &= wL_{be} = (1)(0.9) = 0.9\,\text{m}^2\end{aligned}$$

For the rectangular fin, calculate the corrected length, L_c, the profile area, A_p, and the fin surface area, A_f:

$$\begin{aligned}L_c &= \text{corrected length} = L + t/2 = 0.025 + 0.0013/2 = 0.02565\,\text{m} \\ A_p &= \text{profile area} = L_c t = (0.02565)(0.0013) = 3.334 \times 10^{-5}\,\text{m}^2 \\ A_f &= \text{fin surface area} = 2wL_c = 2(1)(0.02565) = 0.0513\,\text{m}^2\end{aligned}$$

(See also next section for additional details.)
Check the Biot number to verify that the use of the fin efficiency figure is valid:

$$\text{Bi}_f = \frac{h(t/2)}{k_f} = \frac{(13)(0.001312/2)}{38} = 2.2 \times 10^{-4}$$

Since $\text{Bi}_f < 0.25$, the use of the figure is valid.

Calculate the abscissa of the fin efficiency diagram and obtain the fin efficiency:

$$\begin{aligned}\sqrt{\frac{L_c^3 h}{kA_p}} &= L_c^{3/2}(h/kA_p)^{1/2} \\ &= \sqrt{\frac{(0.02565)^3(13)}{(13)(3.334 \times 10^{-5})}} = \sqrt{0.1731} = 0.416\end{aligned}$$

From Figure 17.3, $\eta_f \simeq 0.88$.

Calculate the air thermal resistances:

$$\begin{aligned}R_{\text{base}} &= \frac{1}{hA_{be}} = \frac{1}{(13)(0.9)} = 0.0855^\circ\text{C/W} \\ R_{\text{fins}} &= \frac{1}{hN_f A_f \eta_f} = \frac{1}{(13)(77)(0.0513)(0.88)} \\ &= 0.0221^\circ\text{C/W}\end{aligned}$$

The total resistance of the fin array is therefore given by

$$\frac{1}{R_{\text{tot}}} = \frac{1}{R_{\text{base}}} + \frac{1}{R_{\text{fins}}} = \frac{1}{0.0855} + \frac{1}{0.0221}$$
$$= 11.69 + 45.2 = 56.9\,\text{W/°C}$$
$$R_{\text{tot}} = 0.0176\text{°C/W (due to finned surface)}$$

This is the *outside* resistance; the water side resistance remains the same.

Determine the total resistance to heat transfer and the heat transfer rate:

$$R_{\text{tot}} = 0.0176 + 0.00385 = 0.0214\text{°C/W}$$
$$\dot{Q}_t = (T_1 - T_4)/R_{\text{tot}} = (83 - 19)/0.0214 = 2867\,\text{W}$$

Calculate the percent increase in $\dot{Q}$ due to the fins on the air side:

$$\%\ \text{increase} = 100\left(\frac{2867}{793.1} - 1\right) = 261.5\%$$

Therefore, $\dot{Q}$ will increase by 261.5% when fins are added on the air side.

4. For the case where fins have been added on the water side, determine the new fin efficiency:

$$\text{Abscissa} = L_c^{3/2}(h/kA_p)^{1/2}\sqrt{\frac{L_c^3 h}{kA_p}}$$
$$= \sqrt{\frac{(0.02565)^3(260)}{(38)(3.334 \times 10^{-5})}} = \sqrt{3.46} = 1.86$$

From Figure 17.3, η_f is about 38%.

Calculate the thermal resistance of the base, fins, and the total resistance of the finned surface:

$$R_{\text{base}} = \frac{1}{hA_{be}} = \frac{1}{(260)(0.9)} = 0.00427\text{°C/W}$$
$$R_{\text{fins}} = \frac{1}{hN_f A_f \eta_f} = \frac{1}{(260)(77)(0.0513)(0.38)} = 0.00256\text{°C/W}$$
$$R_t\ \text{(finned surface)} = \frac{1}{\dfrac{1}{0.00427} + \dfrac{1}{0.00256}} = \frac{1}{624.6} = 0.0016\text{°C/W}$$

This resistance is on the water side; the air resistance remains the same (i.e., as it was before fins were added to the air side):

$$R_{\text{tot}} = 0.0769 + 0.0016 = 0.0785^\circ\text{C/W}$$

$$\dot{Q}_t = (83 - 19)/(0.0785) = 815.3\,\text{W}$$

Finally, calculate the percent increase in $\dot{Q}$ due to water-side fins:

$$\%\ \text{increase} = 100\left(\frac{815.3}{793.1} - 1\right) = 2.8\%$$

■

ILLUSTRATIVE EXAMPLE 17.9

Comment on the results of the previous example.

***SOLUTION*:** It is concluded that fins should be added on the air side; the fins on the water side are practically useless. ■

ILLUSTRATIVE EXAMPLE 17.10

A circular tube has an outside diameter of 2.5 cm and a surface tempeature, T_b, of 170°C. An annular aluminum fin of rectangular profile is attached to the tube. The fin has an outside radius, r_f, of 2.75 cm, a thickness, t, of 1 mm, and a thermal conductivity, k, of 200 W/m · K. The surrounding fluid is at a temperature $T_\infty = 25^\circ\text{C}$ and the associated heat transfer coefficient, h, is 130 $\text{W/m}^2 \cdot \text{K}$. Calculate the heat transfer rate without the fin, $\dot{Q}_{\text{w/o},f}$, the corrected length, L_c, the outer radius, r_{2c}, the maximum heat transfer rate from the fin, $\dot{Q}_{f,\max}$, the fin efficiency, η_f, the fin heat transfer rate, q_f, and the fin thermal resistance, $R_{\text{t},f}$.

***SOLUTION*:** Determine the area of the base of the fin:

$$r_o = D_o/2 = 0.025/2 = 0.0125\,\text{m}$$

$$A_b = 2\pi r_o t = (2\pi)(0.0125)(0.001) = 7.854 \times 10^{-5}\,\text{m}^2$$

Calculate the excess temperature at the base of the fin.

$$\theta_b = T_b - T = 170 - 25 = 145\,\text{K}$$

The total heat transfer rate without the fin, $\dot{Q}_{\text{w/o},f}$ is then (see Equation 17.8)

$$\dot{Q}_{t,\text{w/o},f} = \dot{Q}_{\text{w/o},f} = hA_b\theta_b = (130)(7.854 \times 10^{-5})(145) = 1.48\,\text{W} = 5.0\,\text{Btu/h}$$

Calculate the Biot number. Determine if it is valid to use the fin efficiency figures provided in this section since the Biot number must be less than 0.25:

$$\mathrm{Bi}_f = \frac{h(t/2)}{k_f} = \frac{130(0.001/2)}{200} = 3.25 \times 10^{-4} < 0.25$$

The fin efficiency figures may be used.

Calculate the fin height, L. Since

$$\begin{aligned} r_f &= 0.0275\,\mathrm{m} \\ L &= r_f - r_o = 0.0275 - 0.0125 = 0.015\,\mathrm{m} \end{aligned}$$

Calculate the corrected radius and height:

$$\begin{aligned} r_{f,c} &= r_f + t/2 = 0.0275 + 0.001/2 = 0.028\,\mathrm{m} \\ L_c &= L + t/2 = 0.015 + 0.001/2 = 0.0155\,\mathrm{m} \end{aligned}$$

Calculate the profile area and the fin surface area:

$$\begin{aligned} A_p &= L_c t = (0.0155)(0.001) = 1.55 \times 10^{-5}\,\mathrm{m}^2 \\ A_f &= 2\pi(r_{f,c}^2 - r_0^2) = (2\pi)(0.028^2 - 0.0125^2) = 3.94 \times 10^{-3}\,\mathrm{m}^2 \end{aligned}$$

The maximum fin heat transfer rate, $\dot{Q}_{f,\max}$, is therefore:

$$\dot{Q}_{f,\max} = hA_f\theta_b = (130)(3.94 \times 10^{-3})(145) = 74.35\,\mathrm{W}$$

Calculate the abscissa and the curve parameter for the fin efficiency figure in Figure 17.3:

$$\text{Abscissa} = L_c^{3/2}(h/kA_p)^{1/2} = \left(\frac{L_c^3 h}{kA_p}\right)^{1/2} = \sqrt{\frac{(0.0155)^3(130)}{(200)(1.55 \times 10^{-5})}} = 0.40$$

$$\text{Curve parameter} = 0.028/0.0125 = 2.24$$

From Figure 17.4 (interpolating),

$$\eta_f = 86\% = 0.86$$

Therefore, the fin heat transfer rate, $\dot{Q}_f$, is given by Equation (17.2),

$$\dot{Q}_f = \eta_f \dot{Q}_{f,\max} = (0.86)(74.35) = 64\,\mathrm{W} = 218\,\mathrm{Btu/h}$$

The corresponding fin resistance, from Equations (17.2)–(17.4), is

$$R_{t,f} = \frac{\theta_b}{\dot{Q}_f}; \quad \theta_b = T - T_\infty$$

Substituting,

$$R_{t,f} = \frac{145}{64} = 2.27\,\mathrm{K/W} \qquad \blacksquare$$

FIN EFFECTIVENESS AND PERFORMANCE

Another dimensionless quantity used to assess the benefit of adding fins is the *fin effectiveness*, ε_f, or *fin performance coefficient*, FPC. It is defined as

$$\varepsilon_f = \mathrm{FPC} = \frac{\dot{Q}_f}{\dot{Q}_{\mathrm{w/o},f}} \tag{17.8}$$

where $\dot{Q}_{\mathrm{w/o},f}$ is the rate of heat transfer without fins, i.e.,

$$\dot{Q}_{\mathrm{w/o},f} = hA_b\theta_b \tag{17.9}$$

Usually, fins are not justified unless ε_f or FPC ≥ 2.

The fin efficiency, η_f, and the effectiveness, ε_f, characterize the performance of a single fin. As indicated above, arrays of fins are attached to the base surface in many applications. The distance from the center of one fin to the next one along the same tube surface is termed the *fin pitch*, S. In this case, the total heat transfer area, A_t, includes contributions due to the fin surfaces and the exposed (unfinned) base surface, that is,

$$A_t = A_{be} + N_f A_f \tag{17.10}$$

The total heat transfer area without fins, $A_{t,\mathrm{w/o},f}$, is

$$A_{t,\mathrm{w/o},f} = A_{be} + N_f A_b \tag{17.11}$$

where N_f = number of fins, A_f = surface area per fin, A_{be} = total exposed (unfinned) base area of the surface, and A_b = the base area of one fin. Equation (17.11) is often used to determine A_{be} from knowledge of the surface geometry, fin base area, and number of fins.

The total heat transfer rate, without fins, $\dot{Q}_{t,\mathrm{w/o},f}$ is

$$\dot{Q}_{t,\mathrm{w/o},f} = hA_{t,\mathrm{w/o},f}\theta_b = h(A_{be} + N_f A_b)\theta_b \tag{17.12}$$

The total heat transfer rate from the finned surface, $\dot{Q}_t$, is

$$\begin{aligned} \dot{Q}_t &= \dot{Q}_{be} + \dot{Q}_{ft} \\ &= \dot{Q}_{be} + N_f\dot{Q}_f \\ &= hA_{be}\theta_b + N_f hA_f\eta_f\theta_b \\ &= h(A_{be} + N_f A_f\eta_f)\theta_b \end{aligned} \tag{17.13}$$

where $\dot{Q}_{ft}$ is the heat transfer rate due to all (total) fins and $\dot{Q}_f$ is that due to a single fin.

The maximum heat transfer rate, $\dot{Q}_{t,\max}$, of the surface occurs when $n_f = 1.0$, i.e., when the temperature of the base and all the fins is T_b, and is given by

$$\dot{Q}_{t,\max} = h\theta_b(A_{be} + N_f A_f) = hA_t\theta_b \tag{17.14}$$

The *overall fin efficiency*, $\eta_{o,f}$, is defined as

$$\eta_{o,f} = \frac{\dot{Q}_t}{\dot{Q}_{t,\max}} = \frac{A_{be} + N_f A_f \eta_f}{A_t} \tag{17.15}$$

Substituting from Equation (17.10) into (17.15) yields

$$\eta_{o,f} = 1 - \left(\frac{N_f A_f}{A_t}\right)(1 - \eta_f) \tag{17.16}$$

Finally, the *overall surface effectiveness*, $\varepsilon_{o,f}$, is defined as

$$\varepsilon_{o,f} = \frac{\dot{Q}_t}{\dot{Q}_{t,\mathrm{w/o},f}} \tag{17.17}$$

ILLUSTRATIVE EXAMPLE 17.11

Refer to Illustrative Example 17.10. Calculate the fin effectiveness, ε_f, and whether the use of the fin is justified.

***SOLUTION*:** Calculate the fin effectiveness or performance coefficient using Equation (17.8):

$$\varepsilon_f = \frac{\dot{Q}_f}{\dot{Q}_{\mathrm{w/o},f}} = \frac{64}{1.48} = 43.2$$

Since,

$$\varepsilon_f = 43.2 \gg 2.0$$

the use of the fin is justified. ■

ILLUSTRATIVE EXAMPLE 17.12

If the tube described in Illustrative Example 17.10 had a length of one meter and fin pitch of 10 mm, what would be the total surface area for heat transfer, the exposed tube base total heat transfer rate, $\dot{Q}_t$, the overall efficiency of the surface, and the overall surface effectiveness?

Note: This problem is an extension of Illustrative Example 17.10. The information is obtained from it and its solution is used in the solution of this problem.

***SOLUTION*:** Calculate the number of fins in the tube length:

$$N_f = w/S = 1/0.01 = 100\,\text{fins};\ S = 1.0\,\text{mm} = 0.01\,\text{cm}$$

Calculate the unfinned base area:

$$w_{be} = \text{unfinned (exposed) base length} = w - N_f t = 1 - (100)(0.001) = 0.9\,\text{m}$$

$$A_{be} = \text{unfinned base area} = 2\pi r_o w_{be} = (2\pi)(0.0125)(0.9) = 0.0707\,\text{m}^2$$

The total transfer surface area, A_t, may now be calculated:

$$A_{t,\text{w/o},f} = A_t = A_{be} + N_f A_f = 0.0707 + (100)(3.94 \times 10^{-3}) = 0.465\,\text{m}^2$$

Equation (17.12) is used to obtain the total heat rate without fins:

$$\dot{Q}_{t,\,\text{w/o},f} = h(2\pi r_o w)\theta_b = (130)(2\pi)(0.0125)(1)(145) = 1480\,\text{W}$$

Calculate the heat flow rate from the exposed tube base:

$$\begin{aligned}\dot{Q}_{be} &= hA_{be}\theta_b \\ &= (130)(0.0707)(145) = 1332.7\,\text{W} = 4548\,\text{Btu/h}\end{aligned} \tag{17.13}$$

Calculate the heat flow rate from all the fins:

$$\begin{aligned}\dot{Q}_{ft} &= N_f\dot{Q}_f \\ &= (100)(64) = 6400\,\text{W}\end{aligned} \tag{17.13}$$

The total heat flow rate may now be calculated:

$$\begin{aligned}\dot{Q}_t &= \dot{Q}_{be} + \dot{Q}_{ft} \\ &= 1332.7 + 6400 = 7732.7\,\text{W}\end{aligned} \tag{17.13}$$

Calculate the maximum heat transfer rate from Equation (17.14):

$$\dot{Q}_{t,\max} = hA_t\theta_b = (130)(0.465)(145) = 8765.3\,\text{W}$$

By definition, the overall fin efficiency is given in Equation (17.15):

$$\eta_{o,f} = \frac{\dot{Q}_t}{\dot{Q}_{t,\,\max}} = \frac{7732.7}{8765.3} = 0.882$$

The corresponding overall effectiveness is therefore given by Equation (17.17):

$$\varepsilon_{o,f} = \frac{\dot{Q}_t}{Q_{t,\,\text{w/o},f}} = \frac{7732.7}{1480} = 5.22$$

Finally, calculate the thermal resistance:

$$R_{\text{base}} = \frac{1}{hA_{be}} = \frac{1}{(130)(0.0707)} = 0.109\,\text{K/W}$$

$$R_{\text{fins}} = \frac{1}{hN_f A_f \eta_f} = \frac{1}{(130)(100)(3.94 \times 10^{-3})(0.82)} = 0.0238\,\text{K/W}$$

■

ILLUSTRATIVE EXAMPLE 17.13

Consider the case of aluminum fins of triangular profile that are attached to a plane wall with a surface temperature is 250°C. The fin base thickness is 2 mm and its length is 6 mm. The system is in ambient air at a temperature of 20°C and the surface convection coefficient is $40\ \mathrm{W/m^2 \cdot K}$. Consider a 1 m width of a single fin. Determine:

1. the heat transfer rate without the fin,
2. the maximum heat transfer rate from the fin, and
3. the fin efficiency, thermal resistance, and effectiveness.

Properties of the aluminum may be evaluated at the average temperature, $(T_b + T_\infty)/2 = (250 + 20)/2 = 135°\mathrm{C} = 408\ \mathrm{K}$, where $k \simeq 240\ \mathrm{W/m \cdot K}$.

***SOLUTION*:** Determine the base area of the fin:

$$A_b = (t)(w) = (0.002)(1) = 0.002\,\mathrm{m}^2$$

Calculate the excess temperature at the base of the fin:

$$\theta_b = T_b - T_\infty = 250 - 20 = 230°\mathrm{C} = 230\,\mathrm{K}$$

Calculate the heat transfer rate without a fin, $\dot{Q}_{\mathrm{w/o},f}$, employing Equation (17.9).

$$\dot{Q}_{\mathrm{w/o},f} = hA_b\theta_b = (40)(0.002)(230) = 18.4\,\mathrm{W}$$

Also, calculate the maximum heat transfer rate, $\dot{Q}_{f,\max}$:

$$\dot{Q}_{f,\max} = hA_f\theta_b = (40)(0.012)(230) = 110\,\mathrm{W};\ A_f = 0.0118 \simeq 0.012\,\mathrm{m}^2$$

Check the Biot number criterion to determine if the use of the fin efficiency figure is valid:

$$\mathrm{Bi}_f = \frac{h(t/2)}{k} = \frac{(40)(0.002/2)}{240} = 1.67 \times 10^{-4} < 0.25$$

Since $\mathrm{Bi}_f < 0.25$, the use of the figure is permitted.

Calculate the fin L_c, A_p, and P, noting that this is a triangular fin:

L_c = corrected length = $L = 0.006\ \mathrm{m}$
A_p = profile area = $Lt/2 = (0.006)(0.002)/2 = 6 \times 10^{-6}\ \mathrm{m}^2$
A_f = fin surface area = $2w\sqrt{L^2 - (t/2)^2} \approx 0.012\ \mathrm{m}^2$

Determine the abscissa for the fin efficiency figure:

$$\text{Abscissa} = \sqrt{\frac{L_c^3 h}{kA_p}} = \sqrt{\frac{(0.006)^3(40)}{(240)(6 \times 10^{-6})}} \simeq 0.0775$$

Obtain the fin efficiency from Figure 17.4 for triangular profile straight fins:

$$\eta_f \approx 0.99$$

Calculate the fin heat transfer rate, $\dot{Q}_f$ employing Equation (17.15).

$$\dot{Q}_f = \eta_f \dot{Q}_{f,\max} = (0.99)(110) = 108.9\,\text{W}$$

The fin thermal resistance, $R_{t,f}$, is therefore

$$\begin{aligned} R_{t,f} &= \theta_b/\dot{Q}_f \\ &= 230/108.9 = 2.1^{\circ}\text{C/W} \end{aligned} \tag{17.3}$$

■

ILLUSTRATIVE EXAMPLE 17.14

Determine whether the use of the fin is justified in the previous example.

***SOLUTION*:** Calculate the fin effectiveness, ε_f, or performance coefficient, FPC, employing Equation (17.8).

$$\varepsilon_f = \text{FPC} = \dot{Q}_f/\dot{Q}_{\text{w/o},f} = 108.9/18.4 = 5.92$$

Is the use of the fin justified? Since $\varepsilon_f\,(= \text{FPC}) = 5.94 > 2$, the use of the fin is justified. Note that the triangular fin is known to provide the maximum heat transfer per unit mass. ■

ILLUSTRATIVE EXAMPLE 17.15

Annular aluminum fins of rectangular profile are attached to a circular tube. The outside diameter of the tube is 50 mm and the temperature of its outer surface is 200°C. The fins are 4 mm thick and have a length of 15 mm. The system is in ambient air at a temperature of 20°C and the surface convection coefficient is $40\,\text{W/m}^2 \cdot \text{K}$. The thermal conductivity of aluminum is $240\,\text{W/m} \cdot \text{K}$. What are the efficiency, thermal resistance, effectiveness, and heat transfer rate of a single fin? Is the use of the fin justified?

***SOLUTION*:** Determine the base area of the fin:

$$A_b = 2\pi r_o t = (2\pi)(0.025)(0.004) = 6.283 \times 10^{-4}\,\text{m}^2$$

Calculate the excess temperature at the fin base:

$$\theta_b = T_b - T_\infty = 200 - 20 = 180^{\circ}\text{C} = 180\,\text{K} = 324^{\circ}\text{R} = 324^{\circ}\text{F}$$

Calculate the heat transfer rate without a fin, $\dot{Q}_{\text{w/o},f}$, using Equation (17.9).

$$\dot{Q}_{\text{w/o},f} = hA_b\theta_b = (40)(6.283 \times 10^{-4})(180) = 4.52\,\text{W}$$

Check the Biot number criterion to determine if the use of the fin efficiency figure is valid:

$$\text{Bi}_f = \frac{h(t/2)}{k} = \frac{(40)(0.004/2)}{240} = 3.33 \times 10^{-4} < 0.25; \quad \text{OK}$$

Calculate the fin corrected radius and length, profile area, and surface area:

$$\begin{aligned} r_f &= r_0 + L = 0.025 + 0.015 = 0.04\,\text{m} \\ r_{f,c} &= r_f + t/2 = 0.04 + 0.002 = 0.042\,\text{m} \\ L_c &= L + t/2 = 0.015 + 0.002 = 0.017\,\text{m} \\ A_p &= L_c t = (0.017)(0.004) = 6.8 \times 10^{-5}\,\text{m}^2 \\ A_f &= 2\pi(r_{2,f}^2 - r_0^2) = 2\pi((0.042)^2 - (0.025)^2) = 7.157 \times 10^{-3}\,\text{m}^2 \end{aligned}$$

Calculate the maximum heat transfer rate from a *single* fin using Equation (17.14).

$$\dot{Q}_{f,\max} = hA_f\theta_b = (40)(7.157 \times 10^{-3})(180) = 51.53\,\text{W}$$

Determine the abscissa and curve parameter for the efficiency figure (for annular rectangular fins):

$$r_{f,c}/r_0 = 0.042/0.025 = 1.68$$

$$\text{Abscissa} = \sqrt{\frac{L_c^3 h}{kA_p}} = \sqrt{\frac{(0.0017)^3(40)}{(240)(6.8 \times 10^{-5})}} = 0.11$$

Read the fin efficiency from the figure for annular rectangular straight fins:

$$\eta_f = 0.97$$

The fin heat transfer rate, $\dot{Q}_f$, is therefore

$$\dot{Q}_f = \eta_f \dot{Q}_{f,\max} \tag{17.2}$$

Substituting,

$$\begin{aligned} \dot{Q}_f &= (0.97)(51.53) \\ &= 50\,\text{W} \end{aligned}$$

Calculate the fin resistance and effectiveness:

$$\begin{aligned} R_{t,f} &= \theta_b/\dot{Q}_f = 180/50 \\ &= 3.6^\circ\text{C/W} \end{aligned}$$

$$\begin{aligned} \varepsilon_f &= \dot{Q}_f/\dot{Q}_{\text{w/o},f} = 50/4.52 \\ &= 11.06 \end{aligned}$$

Since $\varepsilon_f = \text{FCP} > 2$, the use of the fin is justified. ■

ILLUSTRATIVE EXAMPLE 17.16

What is the rate of heat transfer per unit length of tube in the previous illustrative example, if there are 125 such fins per meter of tube length? Also calculate the total efficiency and effectiveness.

***SOLUTION*:** For an array of 125 fins per meter ($N_f = 125$), calculate the unfinned base area:

$$w_{be} = \text{unfinned exposed base length} = w - N_f t = 1 - (125)(0.004) = 0.5\,\text{m}$$

$$A_{be} = (2\pi)(0.025)(0.5) = 0.0785\,\text{m}^2$$

Calculate the total heat transfer surface area, A_t:

$$A_t = A_{be} + N_f A_f = 0.0785 + (125)(7.157 \times 10^{-3}) = 0.973\,\text{m}^2$$

Calculate the heat rate without fins:

$$\dot{Q}_{\text{w/o},f} = h(2\pi r_o w)\Delta T_b = (40)(2\pi)(0.025)(1)(180) = 1131\,\text{W}$$

Calculate the heat rate from the base and fins, and the total heat rate:

$$\dot{Q}_{be} = hA_{be}\theta_b = (40)(0.0785)(180) = 565.2\,\text{W}$$

$$\dot{Q}_{ft} = N_f\dot{Q}_f = (125)(50) = 6250\,\text{W}$$

$$\begin{aligned}\dot{Q}_t &= 565.2 + 6250 = 6815.2\,\text{W}\\ &= 6.82\,\text{kW}\end{aligned}$$

Calculate the maximum heat transfer rate:

$$\dot{Q}_{\max} = hA_t\theta_b = (40)(0.973)(180) = 7005.6\,\text{W}$$

The overall fin efficiency is therefore

$$\begin{aligned}\eta_{o,f} &= \dot{Q}_t/\dot{Q}_{t,\max} \\ &= 6815.2/7008.6 = 0.973\end{aligned} \tag{17.15}$$

The corresponding overall fin effectiveness is

$$\begin{aligned}\varepsilon_{o,f} &= \dot{Q}_t/\dot{Q}_{t,\text{w/o},f} \\ &= 6815.2/1131 = 6.03\end{aligned} \tag{17.17}$$

Finally, the thermal resistances are

$$R_{\text{base}} = \frac{1}{hA_{be}} = \frac{1}{(40)(0.0785)} = 0.318^\circ\text{C/W}$$

$$\begin{aligned}R_f &= \frac{1}{hN_fA_f\eta_f} \\ &= \frac{1}{(40)(125)(7.157 \times 10^{-3})(0.97)} = 0.0288^\circ\text{C/W}\end{aligned} \tag{17.4}$$

■

ILLUSTRATIVE EXAMPLE 17.17

A metal fin 1 inch high and $\frac{1}{8}$ inch thick has a thermal conductivity, k, of 25 Btu/h · ft · °F and a uniform base temperature of 250°F. It is exposed to an air stream at 60°F with a velocity past the fin such that the convection coefficient of heat transfer, h, is 15 Btu/h · ft · °F. Calculate the temperature at the tip of the fin and the heat transfer from the fin per foot of fin. Solve the problem analytically.

***SOLUTION*:** Set the fin height, thickness, and length equal to L, t, and w, respectively. Bennett and Meyers$^{(4)}$ have shown that the equation describing the temperature profile of this system is:

$$\frac{d^2T}{dx^2} - \frac{2h}{\mathrm{k}t}(T - 70) = 0; \; x = \text{height coordinate}$$

Let

$$\alpha^2 = \frac{2h}{(\mathrm{k})(t)} = (2)(15)/(25)(1/96) = 115$$

so that

$$\frac{d^2(T - 70)}{dx^2} - \alpha^2(T - 70) = 0; \quad \alpha = 10.7$$

The solution to this ordinary differential equation is:

$$(T - 70) = c_1 e^{\alpha x} + c_2 e^{-\alpha x}$$

or

$$T = 70 + c_1 e^{\alpha x} + c_2 e^{-\alpha x}$$

In addition,

$$\frac{dT}{dx} = \alpha c_1 e^{\alpha x} - \alpha c_2 e^{-\alpha x}$$

and

$$\frac{d^2T}{dx^2} = \alpha^2 c_1 e^{\alpha x} + \alpha^2 c_2 e^{-\alpha x}$$

The reader is left the exercise of showing that this solution satisfies the above second order ordinary differential equation. The solution must also satisfy the two boundary conditions (BC) below that are employed to evaluate the two integration constants c_1 and c_2.

BC(1): $T = 250$ at $x = 0$ (fin base)

BC(2): $-kA\dfrac{dT}{dx} = hA(T - 70); \quad A = (t)(w)$

or

$$\frac{dT}{dx} = -\frac{h}{k}(T - 70) \quad \text{at } x = L = 1/12\,\text{ft}$$

From BC(1),

$$180 = c_1 + c_2$$

From BC(2),

$$-k(\alpha c_1 e^{\alpha x} - \alpha c_2 e^{-\alpha x}) = h(c_1 e^{\alpha x} + c_2 e^{-\alpha x})$$

Solving these two equations simultaneously gives

$$c_1 = 25.9 \quad c_2 = 154.1$$

The fin top $\left(x = L = \frac{1}{12}\ \text{ft}\right)$ temperature is therefore

$$\begin{aligned}(T - 70) &= (25.9)(2.4392) + (154.1)(0.40997)\\ &= 63.2 + 63.2\\ &= 126.4\\ T &= 70 + 126.4 = 196.4^\circ\text{F}\end{aligned}$$

The heat transfer rate is:

$$\begin{aligned}\dot{Q} &= -k(A_{\text{base}})\left.\frac{dT}{dx}\right|_{x=0}; \quad \left.\frac{dT}{dx}\right|_{\lambda=0} = \alpha(c_1 - c_2)\\ &= -(25)(1/8)(1/12)(w)(10.7)(25.9 - 154.1); \quad L = 1.0\\ &= 357.2\,\text{Btu/h}\cdot\text{ft of fin length}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 17.18

Estimate the fin efficiency in the previous example, if the efficiency is defined (as noted earlier) as the actual heat transfer divided by the heat rate of the entire fin with the same temperature as its base.

***SOLUTION*:** For the entire fin at $T = 250^\circ$F,

$$\begin{aligned}\dot{Q} &= hA(T - 70)\\ &= (15)\frac{[(1)(w) + (1)(w) + (1/8)w]}{12}(T - 70)\end{aligned}$$

For $w = 1$ ft

$$\begin{aligned}\dot{Q} &= (15)(2.125/12)(250 - 70)\\ &= 478\,\text{Btu/h}\cdot\ \text{ft of fin length}\end{aligned}$$

The fin efficiency from Equation (17.15) is therefore

$$\begin{aligned}\eta_f &= (357.2/478)100\\ &= 74.7\%\end{aligned}$$

■

FIN CONSIDERATIONS

The selection of suitable fin geometry requires an overall comparison of

1. economics,
2. mass of fin,
3. space (if available),
4. pressure drop (if applicable), and
5. the heat transfer characteristics of the fin.

Generally, fins are effective with gases; are less effective with liquids in forced (or natural) convection; are very poor with boiling liquids; and, are extremely poor with condensing vapors.

As discussed earlier, fins should be placed on the side of the heat exchanger surface where h is the lowest. Thin, closely spaced fins (subject to economic constraints) are generally superior to fewer thicker fins. Their thermal conductivity, k, should obviously be high.

In summary, the same basic heat exchanger equation applies for fins:

$$\dot{Q} = UA\Delta T \tag{17.18}$$

with

$$\frac{1}{U} = \frac{1}{h_o} + \frac{\Delta x}{k} + \frac{1}{h_i} \tag{17.19}$$

For many applications, $h_i \ggg h_o$ and $\Delta x/k \ggg h_o$ so that

$$\frac{1}{U} \simeq \frac{1}{h_o} \tag{17.20}$$

The outside coefficient is generally the controlling resistance and it may be large. One way to increase Q is to increase A. As noted earlier, this may be accomplished by the addition of fins to the appropriate heat transfer surface and these can be mounted longitudinally or circumferentially.

REFERENCES

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2. I. Faraq and J. Reynolds, *Heat Transfer*, A Theodore Tutorial, Theodore Tutorials, East Williston, NY, 1996.
3. B. Barkwill, M. Spinelli, and K. Valentine, project submitted to L. Theodore, 2009.
4. C. Bennett and J. Meyers, *Momentum, Heat, and Mass Transfer*, McGraw-Hill, New York City, NY, 1962.

Chapter 18

Other Heat Exchange Equipment

INTRODUCTION

The purpose of this chapter is to extend the material presented in the three previous chapters to the design, operation, and predicative calculations of several other types of heat exchangers. The presentation will focus on: evaporators, waste heat boilers, condensers, and quenchers.

As described earlier, heat exchangers are devices used to transfer heat from a hot fluid to a cold fluid. They can be classified by their functions. Faraq and Reynolds(1) provide an abbreviated summary of these units and their functions (see Table 14.1). To this list can be added a host of other exchangers, including parallel corrugated plates, plate units, etc.(2) Also to be included in the list are direct-contact coolers and speciality condensers.(3)

The four general classifications of direct-contact gas–liquid heat transfer operations are:

1. simple gas cooling,
2. gas cooling with vaporization of coolant,
3. gas cooling with partial condensation, and
4. gas cooling with total condensation.

Most of the direct-exchange applications listed above are accomplished with the following devices:

1. Baffle-tray columns.
2. Spray chambers.
3. Packed columns.
4. Crossflow-tray columns.
5. Pipeline contactors.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

ILLUSTRATIVE EXAMPLE 18.1

Which of the following is NOT a heat exchanger?

- **(a)** Reboiler
- **(b)** Condenser
- **(c)** Absorber
- **(d)** Superheater

***SOLUTION*:** A reboiler is connected to the bottom of a fractionating tower and provides the reboil heat necessary for distillation. The heating medium, which may be steam or a hot process fluid, transfers heat to the bottoms product of the column. Therefore, a reboiler is classified as a heat exchanger.

A condenser (see later section) condenses a vapor or mixture of vapors either alone or in the presence of a non-condensable gas. The cooling medium, which may be cooling water or air, absorbs heat from the hot process vapor. Therefore, a condenser is classified as a heat exchanger.

An absorber(4) is a mass transfer device, used for separating components from either a liquid or gaseous stream. Therefore, an absorber is NOT a heat exchanger.

A superheater heats a vapor above its saturation temperature. Therefore a superheater is classified as a heat exchanger.

Therefore, (**c**) is the correct answer. ■

The following topics are treated in this chapter:

Evaporators
Waste heat boilers
Condensers
Quenchers

Cooling towers do not receive treatment; however, material is available in the literature.(4)

EVAPORATORS

The vaporization of a liquid for the purpose of concentrating a solution is a common operation in the chemical process industry. The simplest device is an open pan or kettle that receives heat from a coil or jacket or by direct firing underneath the pan. Perhaps the traditional unit is the horizontal-tube evaporator in which a liquid (to be concentrated) in the shell side of a closed, vertical cylindrical vessel is evaporated by passing steam or another hot gas through a bundle of horizontal tubes contained in the lower part of the vessel. The liquid level in the evaporator is usually less than half the height of the vessel; the empty space permits disengagement of entrained liquid from the vapor passing overhead.(5)

The describing equation of an evaporator, like that of any heat exchanger, is given by

$$\dot{Q} = UA\Delta T_{\mathrm{lm}} \tag{18.1}$$

The term $(UA)^{-1}$ is equal to the sum of the individual resistances of the steam, the walls of the tubes, the boiling liquid, and any fouling that may be present.

Consider Figure 18.1.(5) Assume F lb of feed to the evaporator per hour, whose solid content is x_F. (The symbol x is employed for weight fraction.) Also, assume the enthalpy of the feed per lb to be h_F. L lb of thick liquor, whose composition in weight fraction of solute is x_L and whose enthalpy is h_L leaves from the bottom of the evaporator. V lb of vapor, having a solute concentration of y_V and an enthalpy of h_V Btu/lb, leaves the unit. In most evaporators, the vapor is pure water, and therefore y_V is zero.

The material balance equations for this evaporator are relatively simple. A total material balance gives

$$F = L + V \tag{18.2}$$

A componential balance leads to

$$Fx_F = Lx_L + Vy_V \tag{18.3}$$

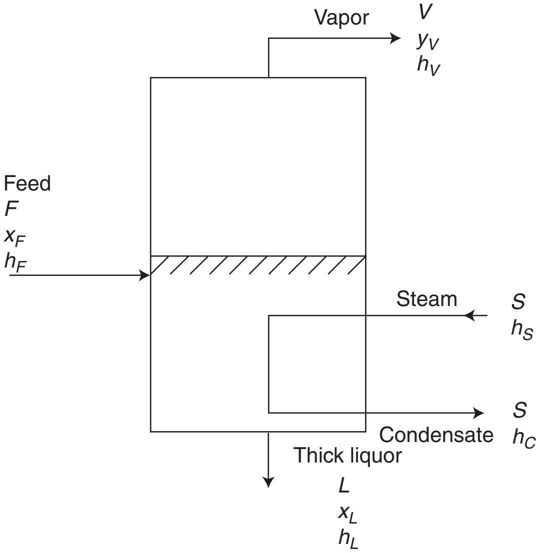

Figure 18.1 Material and enthalpy balance for a single-effect evaporator.

In order to furnish the heat necessary for evaporation, S lb of stream is supplied to the heating surface with an enthalpy of h_S Btu/lb; S lb of condensate with an enthalpy of h_C Btu/lb is leaving as condensate. One simplifying assumption usually made is that, in an evaporator, there is very little cooling of the condensate. This is usually less than a few degrees in practice; the sensible heat recovered from cooling the condensate is so small compared to the latent heat of the steam supplied to the heating surface that the condensate will leave at the condensing temperature of the steam. The enthalpy balance equation is therefore,

$$Fh_F + Sh_S = Vh_V + Lh_L + Sh_C \tag{18.4}$$

Both Equations (18.1) and (18.4) are applied in tandem when designing and/or predicting the performance of an evaporator.

Mixing Effects

The calculations for an evaporator can be complicated because of an adiabatic solution temperature change. When two or more pure substances are mixed to form a solution, a heat effect usually results. Many have experienced this effect on mixing concentrated sulfuric acid with water. This heat of mixing is defined as the enthalpy change that occurs when two or more pure substances are mixed at constant temperature and pressure to form a solution.

Enthalpy concentration diagrams offer a convenient way to calculate enthalpy of mixing effects and temperature changes associated with this type of process. These diagrams, for a two-component mixture, are graphs of the enthalpy of a binary solution plotted as a function of composition (mole fraction or weight fraction of one component), with the temperature as a parameter. For an ideal solution, isotherms or an enthalpy-concentration diagram would be straight lines. For a real solution, the actual isotherm is displaced vertically from the ideal solution isotherm at a given point by the value of Δh at that point, where Δh is the enthalpy of mixing. With reference to the enthalpy concentration diagram in Figure 18.2, Δh is negative over the entire composition range. This means that heat must be evolved whenever the pure components at the given temperature are mixed to form a solution at the same temperature. Such a system is said to be exothermic. An endothermic system is one for which the heats of solution are positive, i.e., solution at constant temperature is accompanied by the absorption of heat. Organic mixtures often fit this description.$^{(5)}$

ILLUSTRATIVE EXAMPLE 18.2

Calculate the temperature when 50 lb of pure H_2SO_4 at 25°C is mixed adiabatically with 200 lb of a 50% by weight aqueous H_2SO_4 solution at 25°C. Also calculate the heat effect if the final mixture is at 25°C.

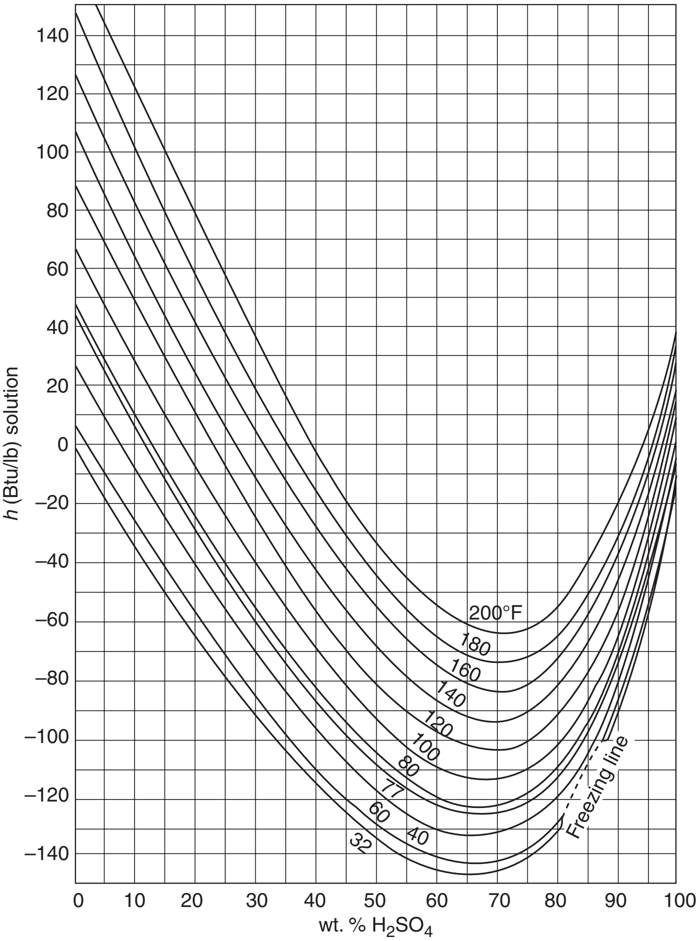

Figure 18.2 Enthalpy-concentration diagram for H_2SO_4–H_2O. (Adapted from the data of W. D. Ross, *Chem. Eng. Progr.*, 43, 314, 1952.)

***SOLUTION*:** Calculate the weight (or mass) percent H_2SO_4 in the final H_2SO_4–H_2O mixture:

$$H_2SO_4\text{: } 50 + 100 = 150\,\text{lb}$$

$$H_2O\text{: } 100\,\text{lb}$$

$$\%H_2SO_4 = (150/250)100 = 60\%$$

Construct a straight line between the 50% solution and pure H_2SO_4 at 25°C (77°F) on Figure 18.2 in order to estimate the final temperature in °F at 60% H_2O:

$$T = 140^{\circ}\text{F}$$

Calculate the heat, Q, that needs to be transferred to return the mixture to 25°C. From Figure 18.2,

$$h \text{ at } 140^\circ\text{F} = -86\,\text{Btu/lb}$$
$$h \text{ at } 77^\circ\text{F} = -121.5\,\text{Btu/lb}$$

so that

$$Q = m\Delta h = m(h_2 - h_1)(250)(-121.5 + 86) = -8875\,\text{Btu}$$

This represents the amount of heat that must be removed since Q is negative.

There is also a heat effect when gases or solids are dissolved in liquids. This effect is normally small but can be significant in some applications, particularly acidic gases in water. This type of calculation should be included in not only evaporator calculations but also for some absorber systems when temperature changes through the unit can be significant. ■

ILLUSTRATIVE EXAMPLE 18.3

A single-effect evaporator is to concentrate 10,000 lb/h of a 10% NaOH solution to 75%. The feed enters at 120°F and the evaporator is to operate at an absolute pressure of 14.7 psi. The 75% NaOH solution leaves at the evaporator equilibrium temperature. For what heat transfer rate (Btu/h) should the evaporator be designed? Also calculate the area requirement in the evaporator if the overall heat transfer coefficient is 500 $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$ and 103 psig (340°F) saturated steam is employed in the steam chest. The $NaOH{-}H_2O$ enthalpy-concentration diagram is provided in Figure 18.3. The enthalpy of saturated steam at 14.7 psia is approximately 1150 Btu/lb.

***SOLUTION*:** Assume a basis of one hour of operation. Calculate the flow rate of steam, V, and the 75% $NaOH{-}H_2O$ solution, L, leaving the evaporator. From a NaOH balance,

$$(10{,}000)(0.1) = (0.75)(L)$$
$$L = 1333.3\,\text{lb/h}$$

From an overall material balance,

$$F = L + V$$
$$V = 10{,}000 - 1333 = 8667\,\text{lb/h}$$

Estimate the enthalpy of solution entering the unit, h_F, in Btu/lb, see Figure 18.3,

$$h_F = 81\,\text{Btu/lb solution}$$

Estimate the enthalpy of the 75% NaOH solution, h_L, leaving the unit:

$$h_L = 395\,\text{Btu/lb solution}$$

The evaporator heat required, $\dot{Q}$, in Btu/h is then

$$\dot{Q} = \Sigma \dot{m}_i h_i = (8667)(1150) + (1333)(395) - (10{,}000)(81) = 9{,}683{,}600\,\text{Btu/h}$$

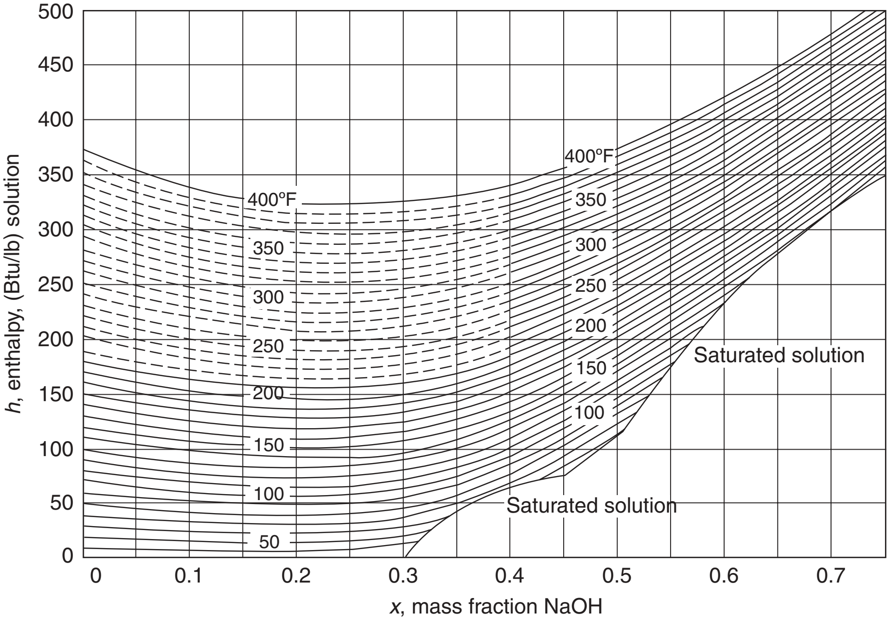

Figure 18.3 Enthalpy-concentration diagram for NaOH–H_2O. [Adapted from W. L. McCabe, *Trans. AIChE*, 31, 129, 1935; R. H. Wilson and W. L. McCabe, *Ind. Eng. Chem.*, 34, 558, 1942.]

Finally, the area of the evaporator in ft^2 may be calculated:

$$\dot{Q} = UA\Delta T \tag{18.1}$$

Rearranging and substituting,

$$A = 9{,}683{,}600/(500)(340 - 212) = 151.3 \text{ ft}^2$$

■

One of the principal operating expenses of evaporators is the cost of steam for heating. A considerable reduction in those costs can be achieved by operating a battery of evaporators in which the overhead vapor from one evaporator (or "effect") becomes the heating medium in the steam chest of the next evaporator, thus saving both the cost on condensing the vapor from the first unit and supplying heat for the second. Several evaporators may operate in a battery in this fashion.(2,5,6)

Basically, a multiple-effect evaporator may be thought of as a number of resistances, in series, to the flow of heat. The main resistances are those to heat transfer across the heating surface of each effect in the evaporator and across the final condenser if a surface condenser is used. The resistances of the heating surfaces are equivalent to the reciprocal of the product of area and overall heat transfer coefficient ($1/UA$) for each effect.

Neglecting all resistances but those noted above and assuming they are equal, it can be seen that if the number of resistances (effects) is doubled, the flow of heat (steam consumption) will be cut in half. With half as much heat, each effect will evaporate about half as much water. But since there are twice as many effects, the total evaporation will be the same. Thus, under these simplifying assumptions, the same output would be obtained regardless of the number of effects (each of equal resistances), and the steam consumption would be inversely proportional to the number of effects or the total heating surface installed.

This resistance concept is also useful in understanding the design and operation of such units. The designer places as many resistances (effects) in series as can be afforded in order to reduce the steam consumption. One can also show that the lowest total area required arises when the ratio of temperature drop to area is the same for each effect.

ILLUSTRATIVE EXAMPLE 18.4

Given the triple-effect evaporator information below, calculate the temperature drop across the heating surface in the first effect:

Effect	U, Btu/h · ft^2 · °F	A, ft^3
1	240	125
2	200	150
3	125	160

The condensing stream temperature in the first effect and the vapor leaving the third effect are 400°F and 120°F, respectively.

***SOLUTION*:** Assuming the resistances across each effect are the same,

$$R_1 = \frac{1}{U_1A_1} = \frac{1}{(240)(125)} = 3.333 \times 10^{-5}$$

$$R_2 = \frac{1}{U_2A_2} = \frac{1}{(200)(150)} = 3.333 \times 10^{-5}$$

$$R_3 = \frac{1}{U_3A_3} = \frac{1}{(125)(160)} = 5.0 \times 10^{-5}$$

$$\sum R = 11.666 \times 10^{-5}$$

One may therefore write,

$$\frac{\Delta T_1}{\Delta T_{\text{TOTAL}}} = \frac{R_1}{\sum R}$$

Substituting

$$\Delta T_1 = \frac{(400 - 120)(3.333 \times 10^{-5})}{(11.666 \times 10^{-5})}$$

$$= 80°\text{F}$$

■

ILLUSTRATIVE EXAMPLE 18.5

Qualitatively describe what an evaporator does.

SOLUTION**:** This is an open-ended question that could just have easily been placed in Chapter 23. Standiford(7) provides an answer to this example.

The requirements for the correct functioning of any evaporator are:

1. It must transfer a great deal of heat—on the order of 1000 Btu/lb of water evaporated. This, more than anything else, determines the type, size, and cost of each effect of the evaporator.
2. It must efficiently separate the vapor from the residual liquid. What is efficient in one evaporator may be many orders of magnitude different from what is efficient in another (e.g., from loss of salt value at only a few dollars a ton, to lithium chemicals valued at a dollar a pound, to radioactive waste). Separation may be important solely because of the value of product lost, pollution problems, or because of fouling or corrosion in the equipment in which the vapor is condensed or in which the condensate is subsequently used.
3. It must make as efficient use of the available heat or mechanical energy as is economically feasible. This means using the vapor evaporated in one part (effect) of the evaporator as the heating steam in another effect that is operating at a lower temperature (as in a multiple-effect evaporator), or compressing the vapor evolved so that it can be used as the heating medium in the same evaporator, or by employing a combination of these. As in most other cases, efficiency is usually gained only as capital cost is increased and, for evaporators, the designer has a wide range to choose from.
4. It must meet conditions dictated by the characteristics of either the liquid being evaporated or the product. In a crystallizing evaporator,(4) crystal size, shape, and purity may be of the utmost importance. If a salting or scaling liquid is to be handled, the evaporator type selected must be capable of dealing with it. Product quality characteristics that may also be important are corrosiveness and degradation at high temperature, long holdup time, or contact with certain metals.

Other considerations are the size of the operation, the foaming characteristics of the liquor, the need for special types of materials of construction such as polished stainless steel (required for many food products), and easy access for cleaning. ■

ILLUSTRATIVE EXAMPLE 18.6 (adapted from Badger and Banchero(8))

An evaporator is to be fed with 5000 lb of solution containing 2% solids by weight. The feed, F, is at a temperature of 100°F. It is to be concentrated to a solution of 5% solute by weight in an evaporator operating at a pressure of 1 atm in the vapor space. In order to carry out the evaporation, the heating surface is supplied with steam at 5 psig (227°F) and the overall heat transfer coefficient of the evaporator, U, is $280\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$. What is the mass of vapor produced, the total mass of steam required, and the surface area required? *Neglect* enthalpy of solution effects.

SOLUTION**:** In order to simplify the solution, it will be assumed that the solution is so dilute that its boiling point is the same as the boiling point of water and that its heat capacity and latent enthalpy are the same as that of water. Under these circumstances, the thermal properties of the solution (both feed and product) and of the steam can be taken from the steam tables. This results in the following values for pertinent quantities on a per hour basis (see Figure 18.1):

$$\begin{aligned}
F &= 5000\,\text{lb}\\
x_F &= 0.02\\
\text{Total solids in feed} &= (5000)(0.02) = 100\,\text{lb (componential balance)}\\
\text{Total water in feed} &= 5000 - 100 = 4900\,\text{lb (componential balance)}\\
T_F &= 100^\circ\text{F}\\
h_F &= 68\,\text{Btu/lb (estimated from steam tables at }100^\circ\text{F)}\\
x_L &= 0.05\\
\text{Total solids in liquor} &= (5000)(0.02) = 100\,\text{lb}\\
L &= 100/(0.05) = 2000\,\text{lb}\\
T_L &= 212^\circ\text{F (at 1 atm)}\\
h_L &= 180\,\text{Btu/lb (estimated from steam tables)}\\
V &= 5000 - 2000 = 3000\ \text{(overall balance)}\\
T_V &= 212^\circ\text{F}\\
h_V &= 1156\,\text{Btu/lb (estimated from steam tables at 1.0 atm)}\\
T_S &= 227^\circ\text{F}\\
h_S &= 1156\,\text{Btu/lb (estimated from steam tables at }227^\circ\text{F}\\
&\quad\ \text{and 5 psig)}\\
T_C &= 227^\circ\text{F (condensate)}\\
h_C &= 195\,\text{Btu/lb (estimated from steam tables)}
\end{aligned}$$

The following enthalpy (energy) balance results (S representing steam):

$$F(h_F) + S(h_S) = V(h_V) + L(h_C) + S(h_C)$$

Substituting and solving yields

$$\begin{aligned}
(5000)(68) + 1156(S) &= (3000)(1150) + (2000)(180) + 195(S)\\
S &= 3611\,\text{lb}
\end{aligned}$$

The total heat requirement is

$$Q = (3611)(1156 - 195) = 3{,}470{,}000\,\text{Btu}$$

The required area is (assuming all of the above is based on one hour)

$$A = Q/U\Delta T = 3{,}470{,}000/(280)(227 - 212) = 826\,\text{ft}^2$$

■

ILLUSTRATIVE EXAMPLE 18.7 (adapted from Badger and Banchero[(8)])

An evaporator is fed with 5000 lb/h of a 20% solution of sodium hydroxide at 100°F. This is to be concentrated to a 40% solution. The evaporator is supplied with saturated steam at 5 psig. Although the unit operates with the vapor space at a pressure of 4 in Hg absolute, the boiling temperature of the solution in the evaporator is 198°F (due to the superheat created by the exposed heating element). The overall heat transfer coefficient is $400\,\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$. Calculate the steam rate and the required heat transfer area.

***SOLUTION*:** Assume a basis of one hour. The following data is known:

$$F = 5000\,\mathrm{lb}; \quad x_F = 0.20$$
$$T_F = 100^\circ\mathrm{F}; \quad x_L = 0.40$$
$$x_V = y_V = 0.00$$

From the steam tables (see Appendix) at 228°F and 5 psig,

$$h_S = 1156\,\mathrm{Btu/lb}$$
$$h_C = 196\,\mathrm{Btu/lb}$$
$$h_{\mathrm{vap}} = 1156 - 196 = 960\,\mathrm{Btu/lb}$$

In addition,

Boiling point of water at 4 in Hg absolute = 125.4°F
Enthalpy of saturated steam at 125°F = 1116 Btu/lb (estimated from steam tables)

$$h_F(100^\circ\mathrm{F}) \simeq 55\,\mathrm{Btu/lb}\ \text{(from Figure 18.3)}$$
$$h_L(198^\circ\mathrm{F}) \simeq 177\,\mathrm{Btu/lb}\ \text{(from Figure 18.3)}$$

A componential material balance yields

$$Fx_F = Lx_L + Vy_V;\ y_V = 0.0$$
$$(5000)(0.20) = (0.40)(L)$$
$$L = 2500\,\mathrm{lb}$$
$$V = 2500\,\mathrm{lb}$$

An enthalpy balance yields

$$Fh_F + S(h_S - h_C) = Vh_V + Lh_L$$

In calculating h_V, the enthalpy of vapor leaving the solution, it should be remembered that this vapor is in equilibrium with the boiling solution at a pressure of 4 in Hg absolute and therefore, is superheated in comparison with vapor in equilibrium with water at the same pressure. Since the heat capacity of superheated steam in this range may be assumed to be $0.46\,\mathrm{Btu/lb \cdot {}^\circ F}$, then

$$h_V = 1116 + (0.46)(198.0 - 125) = 1150\,\mathrm{Btu/lb}$$

Substituting into the enthalpy balance gives

$$5000(55.0) + S(1156 - 196) = (2500)(1150) + (2500)(177)$$

Solving for S, the steam rate, yields

$$S = 3170\,\mathrm{lb/h}$$

The area may now be calculated using the standard heat exchanger equation.

$$\dot{Q} = UA\Delta T$$
$$(3170)(1156 - 196) = 400(A)(228 - 198)$$
$$A = 254\ \text{ft}^2$$

■

WASTE HEAT BOILERS

Energy has become too valuable to discard. As a result, waste heat and/or heat-recovery boilers are now common in many process plants. As the chemical processing industries become more competitive, no company can afford to waste or dump thermal energy. This increased awareness of economic considerations has made waste-heat boilers one of the more important products of the boiler industry. The term "waste heat boiler" includes units in which steam is generated primarily from the sensible heat of an available hot flue or hot gas stream rather than by solely firing fuel.

An obvious by-product of incineration processes is thermal energy—in many cases, a large amount of thermal energy. The total heat load generated by a typical hazardous waste incinerator, for example, is in the range of 10 to 150 million Btu/h. While waste heat boilers are capable of recovering 60–70% of this energy, the effort may or may not be justified economically. In assessing the feasibility of recovery, a number of factors must be taken into account; among these are the amount of heat wasted, the fraction of that heat that is realistically recoverable, the irregularity in availability of heat, the cost of equipment to recover the heat, and the cost of energy. The last factor is particularly critical and may be the most important consideration in a decision involving whether or not to harness the energy generated by a particular incinerator. Other important considerations are the incinerator capacity and nature of the waste/fuel being handled. Generally, heat recovery on incinerators of less than 5 million Btu/h may not be economical because of capital cost considerations. Larger capacity incinerators may also be poor candidates for heat recovery if steam is not needed at the plant site or if the combustion gases are highly corrosive; in the latter case, the maintenance cost of the heat recovery equipment may be prohibitive.

The main purpose of a boiler is to convert a liquid, usually water, into a vapor. In most industrial boilers, the energy required to vaporize the liquid is provided by the direct firing of a fuel in the combustion chamber. The energy is transferred from the burning fuel in the combustion chamber by convection and radiation to the metal wall separating the liquid from the combustion chamber. Conduction then takes place through the metal wall and conduction/convection into the body of the vaporizing liquid. In a waste heat boiler, no combustion occurs in the boiler itself; the energy for vaporizing the liquids is provided by the sensible heat of hot gases which are usually product (flue) gases generated by a combustion process occurring elsewhere in the system. The waste heat boilers found at many facilities make use of the flue gases for this purpose.

In a typical waste heat boiler installation, the water enters the unit after it has passed through a water treatment plant or the equivalent. This boiler feed water is sent to heaters/economizers and then into a steam drum. Steam is generated in the boiler by indirectly contacting the water with hot combustion (flue) gases. These hot gases may be around 2000°F. The steam, which is separated from the water in the steam drum, may pass through a superheater, and is then available for internal use or export. The required steam rate for the process or facility plus the steam temperature and pressure are the key design and operating variables on the water side. The inlet and outlet flue gas temperatures also play a role, but it is the chemical properties of the flue gas that can significantly impact boiler performance. For example, acid gases can arise due to the presence of any chlorine or sulfur in the fuel or waste. The principal combustion product of chlorine is hydrogen chloride, which is extremely corrosive to most metal heat transfer surfaces. This problem is particularly aggravated if the temperature of the flue gas is below the dewpoint temperature of HCl (i.e., the temperature at which the HCl condenses). This usually occurs at temperatures of about 300°F. In addition to acid gases, problems may also arise from the ash of incineration processes; some can contain a fairly high concentration of alkali metal salts that have melting points below 1500°F. The lower melting point salts can slag and ultimately foul (and, in some cases, corrode) boiler tubes and/or heat transfer surfaces.

Boilers may be either *fire tube* or *water tube* (water-wall). Both are commonly used in practice; the fire-tube variety is generally employed for smaller applications ($<15 \times 10^6$ Btu/h). In the fire-tube waste heat boiler, the hot gases from the process are passed through the boiler tubes. The bundle of tubes is immersed in the water to be vaporized; the vaporizing water and tube bundles are encased in a large insulated container called a shell. The steam generated is stored in a surge drum, usually located above the shell and connected to the shell through vertical tubes called risers. Because of construction constraints, steam pressure in fire-tube boilers is usually limited to around 1000 psia.

Fire-tube boilers are compact, low in initial cost, and easy to modularize based on plant requirements. However, they are also slow to respond to changes in demand for steam (load) compared to water-tube boilers, and the circulation is slower. Also, stresses are greater in fire-tube boilers because of their rigid design and subsequent inability to expand and contract easily. Fire-tube boilers usually range in size from less than 2 to 50 million Btu/h. Most industrial fire-tube boilers are either horizontal return tube (HRT), scotch marine, or firebox. Fire-tube boilers are usually directly fired with fuels, either liquid or gaseous. They also serve to recover heat from incinerators fired on waste fuels.

In the water-tube waste heat boiler, the water is contained inside the tubes and the hot flue gases flow through the tube bundle, usually in a direction perpendicular to the tubes (cross-flow). Because of the increased turbulence that accompanies cross-flow, the overall heat transfer coefficient for water-tube boilers is higher than that for fire-tube boilers. This advantage is somewhat offset, however, because it is more difficult to clean the outside surfaces of the tubes than the inside surfaces. As a result, heat transfer losses and maintenance problems due to flue gas fouling tend to be greater in water-tube boilers.

Water-tube boilers can be physically divided into two sections, the furnace and the convection pass. Furnaces (fireboxes, combustion chambers, etc.) will vary in configuration and size, but their function is to contain the flaming combustion gases and transfer the heat energy to the water-cooled walls. The convection pass contains the superheaters, reheater, economizer, and air preheater heat exchangers, where the heat of the combustion flue gases is used to increase the temperature of the steam, water, and combustion air. The *superheaters* and *reheaters* are designed to increase the temperature of the steam generated within the tubes of the furnace walls. Steam flows inside the tubes and flue gas passes along the outside surface of the tubes. The *economizer* is normally a counterflow heat exchanger designed to recover energy from the flue gas after the superheater and the reheater. The boiler economizer is a tube bank type, hot-gas-to-water heat exchanger. It increases the temperature of the water entering the steam drum. The *air heater* is not a portion of the steam–water circuit, but serves a key role in the steam generator system to provide additional heat transfer and efficiency. In many cases, especially in a high pressure boiler, the temperature of the flue gas leaving the economizer is still quite high. The air heater recovers much of this energy and adds it to the combustion air. Heating the combustion air prior to its entrance to the furnace reduces fuel usage.

Water-tube boilers respond quickly to changes in demand for steam due to improved water circulation. They can withstand much higher operating pressures and temperatures than fire-tube boilers. In addition, the water-tube boiler design is safer. They can also burn a wide variety of fuels as well as wastes and have the ability to expand and contract more easily than fire-tube boilers. The major drawback is that water-tube boilers are more expensive to install. They also require more complicated furnaces and repair techniques.

Describing Equations

The design of waste heat boilers involve calculations that are based on energy balances and estimates of the rates of heat transfer. Although some units operate in an unsteady-state or cyclical mode, the calculation procedures are invariably based on steady-state conditions.

In heat transfer equipment, there is no shaft work, and potential and kinetic energy effects are small in comparison with the other terms in an energy balance equation. Heat flow to or from the surroundings is not usually desired in practice and is usually reduced to a small magnitude by suitable insulation. It is therefore customary to consider this heat loss or gain negligible in comparison to the heat transfer through the walls of the tubes from the hot combustion gases to the water in the boilers. Thus, all the sensible heat lost by the hot gases may be assumed transferred to the steam.

The heart of the heat transfer calculation is the heat flux, which is based on the area of the heating surface and is a function of the temperature difference driving force and the overall heat transfer coefficient, as discussed earlier. This relatively simple equation can be used to estimate heat transfer rates, area requirements, and temperature changes in a number of heat transfer devices. However, problems develop if more exact calculations are required. The properties of the fluid (viscosity, thermal conductivity, heat capacity, and density) are important parameters in these calculations. Each of these properties, especially viscosity, is temperature dependent. Since a temperature profile, in which the temperature varies from point to point, exists in a flowing stream undergoing heat transfer, a problem arises in the choice of temperature at which the properties should be evaluated. When temperature changes within the stream become large, the difficulty of calculating heat transfer quantities is increased. Because of these effects, the entire subject of heat transfer to fluids with phase change (as in a waste heat boiler) is complex and, in practice, is treated empirically rather than theoretically.(3)

The rigorous design and/or performance evaluation of a waste heat boiler is an involved procedure. Fortunately, several less rigorous methods are available in the literature. One approach that is fairly simple and yet reasonably accurate has been devised by Ganapathy.(9) This method provides a technique for sizing waste heat boilers of the fire-tube type and involves the use of a performance evaluation chart (see Figure 18.4) that is based on fundamental heat transfer equations plus some simplifying assumptions.

Details regarding Ganapathy's methods are now discussed, employing this author's notation. Earlier equations may be combined and applied to the boiler where the temperature outside the tubes (T_c) can be assumed constant due to the

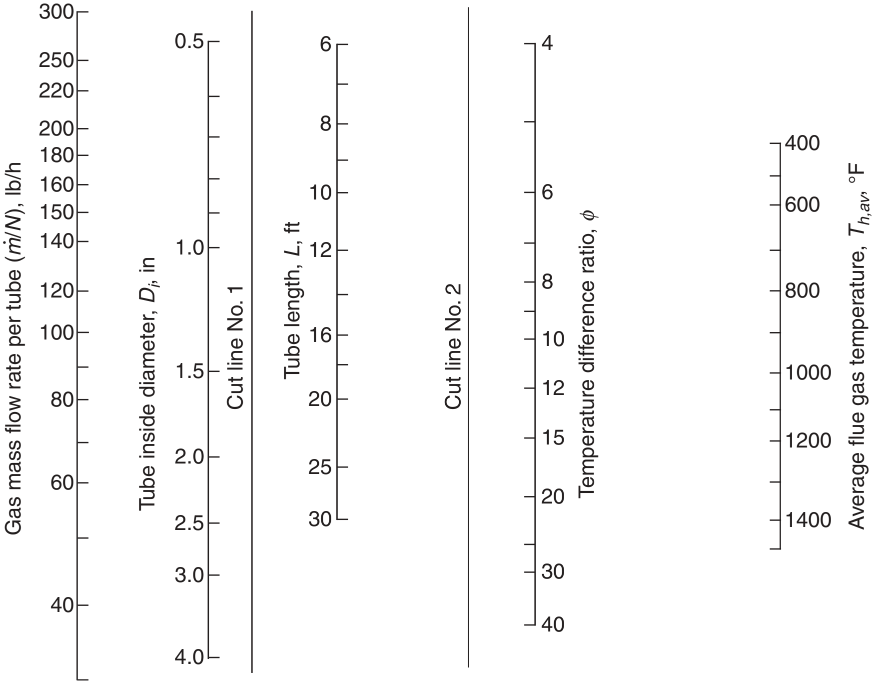

Figure 18.4 Waste heat boiler performance evaluation chart.(9)

water-to-steam phase change. The result is once again,

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h1} - T_{h2}) = U_i A_i \frac{(T_{h1} - T_c) - (T_{h2} - T_c)}{\ln[(T_{h1} - T_c)/(T_{h2} - T_c)]} \tag{18.5}$$

where (employing Ganapathy's notation) T_c is the boiling temperature of the water. Note that U_iA_i was used in Equation (18.5) instead of U_oA_o. The two expressions are equal. In this procedure, it is slightly more convenient to use the inside heat transfer area rather than the outside as the basis for U. Since the outside film coefficient, h_o, associated with a boiling liquid is much greater than the inside coefficient for the flue gas, the inside resistance predominates and is responsible for about 95% of the total resistance. The inside overall heat transfer coefficient may therefore be simplified by the approximate relationship

$$U_i = 0.95h_i \tag{18.6}$$

The total inside heat transfer area for the boiler is given by

$$A_i = \pi D_i N L \tag{18.7}$$

where N is the number of tubes. The Dittus–Boelter equation may be used to solve explicitly for h_i (the inside film coefficient):

$$h_i = 0.023 \frac{k^{0.6} G^{0.8} c_p^{0.4}}{D_i^{0.2} \mu^{0.4}} \tag{18.8}$$

and the gas mass flux, G, may be represented in terms of $\dot{m}_h$

$$G = \frac{4\dot{m}_h}{D_i^2 N} \tag{18.9}$$

Equations (18.5) through (18.9) may be combined to give

$$\ln\left(\frac{T_{h1} - T_c}{T_{h2} - T_c}\right) = \frac{U_i A_i}{\dot{m}_h c_{ph}} = \frac{(C)(L)[F(T)]}{D_i^{0.8}(\dot{m}_h/N)^{0.2}} \tag{18.10}$$

where $F(T) = k^{0.6}/c_{ph}^{0.6}\mu^{0.4}$ and C is a constant equal to 0.0833 when D_i is in feet, or 0.608 when D_i is in inches.

Keeping in mind that this procedure is used for engineering design purposes and is not intended for rigorous analytical calculations, the value of $F(T)$ does not, in practice, vary over a very large range for most hot gas streams. The performance evaluation chart shown in Figure 18.4 is based on Equation (18.10) for typical values of $F(T)$.

The following describes a procedure for the use of the chart as a design tool. It is assumed in this procedure that the inside tube diameter and the number of tubes have been chosen and the tube length is to be determined. Two later Illustrative Examples further demonstrate the use of Figure 18.4.

1. From the inlet and desired outlet gas temperature, and the water saturation (boiling) temperature, calculate the arithmetic average gas temperature, $T_{h,\mathrm{av}}$, and the value of the temperature difference ratio, ϕ, where

$$\phi = \frac{T_{h1} - T_c}{T_{h2} - T_c} \tag{18.11}$$

   Mark those points on the appropriate axes.
2. Draw a straight line connecting these points and extend the line to the left to cut line No. 2. Mark this point B.
3. Mark the value of the flue gas mass flow rate per tube and the value of the inside diameter on the appropriate axes. (For design purposes, a good starting value of $\dot{m}_h/N$ is 80–150 lb/h.)
4. Draw a straight line connecting these points and extend the line to the right to cut line No. 1. Mark this point A.
5. Connect points A and B by a straight line. The intersection of this line with the L axis yields the appropriate tube length.

The chart could be used in similar fashion as a performance evaluation tool. In this case, the outlet gas temperature would be unknown and hence would have to be estimated in order to determine the average gas temperature. In order to avoid a time consuming trial-and-error procedure involving the gas outlet temperature and the average gas temperature, Equation (18.12) may be used to estimate $T_{h,\mathrm{av}}$ without too much loss of accuracy:

$$T_{h,\mathrm{av}} = 0.5(T_{h1} + T_c) \tag{18.12}$$

ILLUSTRATIVE EXAMPLE 18.8

What factors should be considered when assessing whether it is feasible to implement energy recovery using a waste heat boiler?

***SOLUTION*:** As discussed above, important factors include:

1. the amount of heat wasted,
2. the fraction of heat that could realistically be recovered,
3. irregularity in availability of heat (scheduling),
4. cost of equipment,
5. cost of energy,
6. incinerator capacity, and
7. nature of waste handled. ■

ILLUSTRATIVE EXAMPLE 18.9

Discuss the following with respect to waste heat boilers:

1. The effect of fouling on the water/steam side.
2. The effect of fouling on the gas side.
3. Monitoring procedures to follow to account for fouling and/or scaling.

***SOLUTION*:**

1. The exit gas temperature will increase, thus resulting in loss of energy recovery and reduced steam production. Tube wall temperatures can increase significantly, leading to tube failures due to scale formation.
2. This leads to loss of steam output and can increase the gas side pressure drop. However, it does not significantly increase the tube wall temperatures.
3. In monitoring the boiler performance, one should be aware of increases in exit gas temperatures, tube wall temperatures or loss of steam production for the same gas inlet conditions. ■

ILLUSTRATIVE EXAMPLE 18.10

Using Ganapathy's method, determine the required "length" of a waste heat boiler to be used to cool hot gases (average heat capacity = 0.279 Btu/lb · °F) from 2000 to 550°F and generate 30,000 lb/h of steam at 330°F from water at 140°F. The boiler contains 800 1.5-inch ID tubes.

***SOLUTION*:** Calculate the temperature difference ratio, ϕ:

$$\phi = \frac{2000 - 330}{550 - 330} = 7.59$$

The average gas temperature is

$$T_{h,\text{av}} = 0.5(T_{h1} + T_{h2}) = (0.5)(2000 + 550) = 1275^\circ\text{F}$$

From the steam tables, see Appendix:

$$h_{\text{steam}} = 1187.7\ \text{Btu/lb}$$
$$h_{\text{water}} = 107.89\ \text{Btu/lb}$$

Therefore,

$$\dot{Q} = \dot{m}(h_{\text{steam}} - h_{\text{water}}) = 30{,}000(1187.7 - 107.89) = 32.39 \times 10^6\ \text{Btu/h}$$

$$\dot{m}_h = \dot{Q}/c_p(T_{h1} - T_{h2}) = 32.39 \times 10^6/(0.279)(2000 - 550) = 80{,}070\ \text{lb/h}$$

$$\dot{m}_h/N = 80{,}700/800 = 100\ \text{lb/h}\ \text{per tube}$$

From Figure 18.5, the tube length is $L = 15$ ft. ■

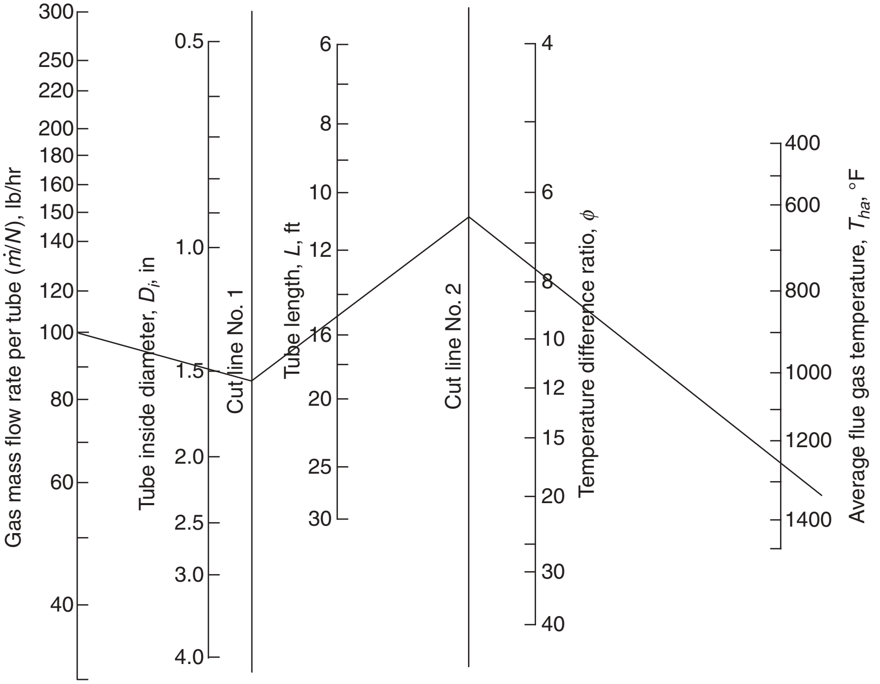

Figure 18.5 Chart for Illustrative Example 18.10.

ILLUSTRATIVE EXAMPLE 18.11

Refer to Illustrative Example 18.10. Provide the details of the calculations and results on Ganapathy's performance chart (Figure 18.4).

***SOLUTION*:** Refer to Figure 18.5 for details. ■

ILLUSTRATIVE EXAMPLE 18.12

120,000 lb/h of flue gas from an incinerator is to be cooled from 1800 to 500°F in a waste heat boiler. If 2-inch ID tubes and a flow rate of 150 lb/h through each tube is to be used, estimate the required heat transfer area, tube length, heat duty, and water mass flow rate. Water at 200°F is available for the steam generator; saturated steam at 80 psia is needed. The average heat capacity of the flue gas is 0.26 Btu/lb · °F. Use Ganapathy's method to solve the problem.

***SOLUTION*:** From the steam tables (see Appendix), the boiling point of water at 80 psia is 312°F. The temperature difference ratio, ϕ, is

$$\phi = \frac{T_{h1} - T_c}{T_{h2} - T_c} = \frac{1800 - 312}{500 - 312} = 7.9$$

The average gas temperature is

$$T_{h,\text{av}} = (T_{h1} + T_{h2})/2 = (1800 + 500)/2 = 1150^\circ\text{F}$$

On the nomograph (see Figure 18.4), connect $(\dot{m}/N) = 150$ with $D_i = 2$ and extend the line to intersect cut line No. 1 at point A. Connect $\phi = 7.9$ with $T_{h,\text{av}} = 1150$ and extend the line to intersect cut line No. 2 at point B. The line connecting points A and B intersects the L line close to 21.5 ft. The number of tubes, N, is

$$N = \frac{\dot{m}}{\dot{m}/N} = 120{,}000/150 = 800$$

The total heat transfer area (inside) is given by

$$A_i = NLD_i = (800)(21.5)(2/12) = 2866\,\text{ft}^2$$

Note: This is only one design; several alternatives are possible by changing $\dot{m}/N$ or L. If L has to be limited because of space constraints, the nomograph may be used in reverse to calculate $\dot{m}/N$.

The heat duty is

$$\dot{Q} = \dot{m}c_p(T_{h1} - T_{h2}) = (120{,}000)(0.26)(1800 - 500) = 4.06 \times 10^7\ \text{Btu/h}$$

The required water flow rate is obtained from an enthalpy balance. The enthalpy of water at 200°F is 168.1 Btu/lb. The value for the water's enthalpy was obtained from the saturated steam tables. Note that, since pressure has very little effect on the enthalpy of liquids, the enthalpy may be assumed the same as the enthalpy of liquid water at 200°F and its saturation pressure. The enthalpy of steam at 80 psia is 1183.1 Btu/lb. The required water flow is, therefore,

$$\dot{m} = \frac{\dot{Q}}{h_2 - h_1} = \frac{4.06 \times 10^7}{1183.1 - 168.1} = 40{,}000\,\text{lb/h}$$

■

ILLUSTRATIVE EXAMPLE 18.13

The sulfuric acid dew point for a flue gas discharge from a boiler is 285°F. Is the downstream economizer safer from corrosion if the feed water temperature is 230°F and flue gas is above 600°F or when the feed water is at 275°F and flue gas temperature is 350°F? Provide a qualitative answer.

SOLUTION: In a typical economizer, the water-side film heat transfer coefficient is 50–150 times higher than the gas side film coefficient. Therefore, the tube wall temperature will be close to the water temperature. Therefore, the gas temperature (600°F or 350°F) will not be a factor. Condensation is more likely to occur when the feed water temperature is at 275°F; a slight temperature excursion can lead to problems, e.g., tube failure due to corrosion. ■

CONDENSERS(10,11)

Condensation can be accomplished by increasing pressure or decreasing temperature (removing heat), or both. In practice, condensers operate through extraction of heat. Condensers differ in the means of removing heat and the type of device used. The two different means of condensing are direct contact (or contact), where the cooling medium with vapors and condensate are intimately mixed and combined, and indirect (or surface), where the cooling medium and vapors/condensate are separated by a surface area of some type. The reader is referred to Chapter 12 for additional details.

Contact condensers are simpler, less expensive to install, and require less auxiliary equipment and maintenance. The condensate/coolant from a contact condenser has a volume flow 10 to 20 times that of a surface condenser. This condensate cannot be reused and may pose a waste disposal problem unless the dilution of any pollutant is sufficient to meet regulatory requirements. Some typical contact condensers are shown in Figure 18.6.

Surface condensers form the bulk of the condensers used in industry. Some of the applicable types of surface condensers are: shell and tube, double pipe, spiral plate, flat plate, air-cooled, and various extended surface tubular units. This section focuses on shell and tube condensers (see also Chapter 16) because they are so widely used in industry and have been standardized by the Tubular Exchanger Manufacturers Association (TEMA).

Condensing can be accomplished in either the shell or tubes. The economics, maintenance, and operational ramifications of the allocation of fluids are extremely important, especially if extended surface tubing is being considered. The designer should be given as much latitude as possible in specifying the condenser.

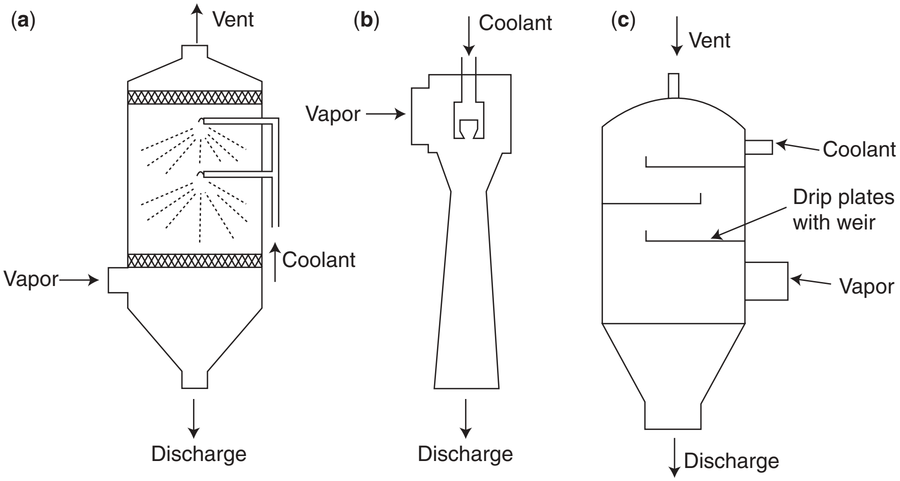

Figure 18.6 Contact condensers: (a) spray; (b) jet; (c) barometric.

Air-cooled condensers consist of a rectangular bank of high finned tubes, a fan, a plenum for even distribution of air to the rectangular face of the tube bank, a header for vapor inlet and condensate outlet, and a steel supporting structure. Fins are usually aluminum, 0.5 to 0.625 inch high, applied to a bare tube by tension winding, or soldering, or cold-extruded from the tube itself. Bimetallic tubes can be used to provide process corrosion resistance on the inside and aluminum extruded fins on the outside. Headers for the tube side frequently contain removable gasketed plugs corresponding to each tube end and are used for access to the individual tube-to-tube sheet joint for maintenance.

Nearly all tubular exchangers employ roller expanding of the tube ends into tube holds drilled into tube sheets as a means of providing a leak-proof seal. TEMA requires that tube holes be of close tolerance and contain two concentric grooves. These grooves give the tube joint greater strength but do not increase sealing ability and may actually decrease it. Research is currently being done to clarify what is the best procedure to follow for a given set of materials, pressures, temperatures, and loadings. Welding or soldering of tubes to tube sheets may be performed for additional leak-tightness or in lieu of roller expanding.

The design of contact condensers involves calculating the quantity of coolant required to condense and subcool the vapor, and proper sizing of the discharge piping and hotwell. The calculation of coolant flow rate is available in the literature.(11) Use of a contact condenser requires consideration of coolant availability, any liquid waste disposal, or treatment facilities. Contact condensers are relatively efficient scrubbers as well as condensers and have the lowest equipment cost. Reputable manufacturers of the various types of contact condensers should be consulted for sizing and layout recommendations.

The heat transfer for a surface condenser is governed by the standard relationship:

$$\dot{Q} = U_o A \Delta T_{lm} \tag{18.13}$$

where $\dot{Q}$ = total heat load, Btu/h

U_o = overall heat-transfer coefficient, $\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$

ΔT_{lm} = log mean temperature difference, °F

A = surface area, $\mathrm{ft^2}$

The first determination is the heat load. In the simplest case, it is the latent heat to condense plus the subcooling load. If a condensing range exceeding 10–20°F exists, gas cooling or desuperheating and intermediate liquid subcooling must be accounted for. The next step is specifying the coolant flow rate and temperatures to balance the vapor heat load. Temperature crosses where the outlet coolant temperature is higher than the vent and condensate outlet temperature should be avoided. Fluids are usually in counterflow, but in some cases, as with a fluid near its freezing point, parallel flow is used to afford a greater degree of safety from freezing the condenser. Frequently, there is a combination of parallel and counterflow as in a two-pass tube side, single-pass shell tubular condenser. The procedure is to apply a correction (described in Chapter 16) to the LMTD. Strictly, the correction factor applies only to the subcooling or desuperheating zones, but it is sometimes applied to condensers.

ILLUSTRATIVE EXAMPLE 18.14

Discuss the effect of a non-condensable gas in a vapor mixture.

***SOLUTION*:** If there is non-condensable gas present or a vapor mixture of miscible and immiscible components with a large condensing range, diffusion of the condensing vapor through another vapor or a gas must be considered. The presence of a noncondensable gas in a vapor stream reduces the film coefficient; the amount of reduction is related to the size of the gas cooling load relative to the total load. The greater the ratio of sensible heat to the total heat, the closer the heat-transfer coefficient approaches that of cooling the gas only. The calculation procedure is essentially that established by Colburn and Hougen.(12) It is a trial-and-error method performed for at least five points along the condensing curve. ■

ILLUSTRATIVE EXAMPLE 18.15

Provide some typical condensation heat transfer coefficients.

***SOLUTION*:** Some typical condensing coefficients are presented in Table 18.1.(10) ■

ILLUSTRATIVE EXAMPLE 18.16

Discuss coolant selection.

Table 18.1 Condensation Heat Transfer Coefficient

Fluid	h, $\text{Btu/ft}^2 \cdot \text{h} \cdot {}^\circ\text{F}$
Steam	1500
Steam/10% gas	600
Steam/20% gas	400
Steam/40% gas	220
Pure light hydrocarbons	250
Mixed light hydrocarbons	175
Medium hydrocarbons	100
Medium hydrocarbons with steam	125
Pure organic solvents	250

***SOLUTION*:** The choice of a coolant will depend upon the particular plant and the efficiency required of the condenser. The most common coolant is the primary plant coolant, usually cooling tower or river water. It has been shown(13) that the vapor outlet temperature is critical to the efficiency of a condenser. In some instances, use of chilled brine or a boiling refrigerant can achieve a collection efficiency that will be sufficient without additional devices. It is not unusual to specify multiple-stage condensing, where condensers, usually two, connected in series use cooling mediums with successively lower temperatures. For example, a condenser using cooling-tower water can be used prior to a unit using chilled water or brine—thereby achieving maximum efficiency while minimizing use of chilled water. ■

QUENCHERS

Hot gases must often be cooled before being discharged to the atmosphere or entering another device(s) which normally is not designed for very high temperature operation (>500°F). These gases are usually cooled either by recovering the energy in a waste heat boiler, as discussed in an earlier section of this chapter, or by quenching. Both methods may be used in tandem. For example, a waste heat boiler can reduce an exit gas temperature down to about 500°F; a water quench can then be used to further reduce the gas temperature to around 200°F, as well as saturate the gas with water. This secondary cooling and saturation can later eliminate the problem of water evaporation and can also alleviate other potential problems.(3)

Although quenching and the use of a waste heat boiler are the most commonly used methods for gas cooling applications, there are several other techniques for cooling hot gases. All methods may be divided into two categories: *direct-contact* and *indirect-contact* cooling. The direct-contact cooling methods include (a) dilution with ambient air, (b) quenching with water, and (c) contact with high heat capacity solids. Among the indirect contact methods are (d) natural convection and radiation from ductwork, (e) forced-draft heat exchangers, and (f) the aforementioned waste heat boilers.

With the *dilution* method, the hot gaseous effluent is cooled by adding sufficient ambient air that results in a mixture of gases at the desired temperature. The *water quench* method uses the heat of vaporization of water to cool the gases. When water is sprayed into the hot gases under conditions conducive to evaporation, the energy contained in the gases evaporates the water, and this results in a cooling of the gases. The hot exhaust gases may also be quenched using submerged exhaust quenching. This is another technique employed in some applications. In the *solids contact* method, the hot gases are cooled by giving up heat to a bed of ceramic elements. The bed in turn is cooled by incoming air to be used elsewhere in the process. *Natural convection and radiation* occur whenever there is a temperature difference between the gases inside a duct and the atmosphere surrounding it. Cooling hot gases by this method requires only the provision of enough heat transfer area to obtain the desired amount of cooling. In *forced-draft heat exchangers*, the hot gases are cooled by forcing cooling fluid past the barrier separating the fluid from the hot gases. These methods are further discussed below.

Dilution with Ambient Air

The cooling of gases by dilution with ambient air is the simplest method that can be employed. Essentially, it involves the mixing of ambient air with a gas of known volume and temperature to produce a low-temperature mixture that can be admitted to another device. In designing such a system, the amount of ambient air required to provide a gaseous mixture of the desired temperature is first determined.

The quantity of air required for cooling may be calculated directly from an enthalpy balance, i.e., the heat "lost" by the gas is equal to that "gained" by the dilution air. The design of the vessel or duct to accomplish this mixing process is based on a residence time of approximately 0.8–1.5 s (based on the combined flow rate) at the average temperature.

Quenching with Liquids

When a large volume of hot gas is to be cooled, a method other than dilution with ambient air should be used. This is usually the case and the cooling method most often used is liquid quenching.

Cooling by liquid quenching is essentially accomplished by introducing the hot gases into a liquid (usually water) contacting device. When the water evaporates, the energy necessary to vaporize the water is obtained at the expense of the hot gas, resulting in a reduction in the gas temperature. The temperature of the gases discharged from the quencher is at the adiabatic saturation temperature of the gas if the operation is adiabatic and the gas leaves the quencher saturated with water vapor.(3) (A saturated gas contains the maximum water vapor possible at that temperature; any increase in water content will result in condensation.) Simple calculational and graphical procedures are available for estimating the adiabatic saturation temperature of a gas.(3)

There are four types of liquid quenchers that may be employed: spray towers, venturi scrubbers, submerged exhaust tanks, and packed towers (generally a poor choice). The venturi scrubber is actually an air pollution control device but can also be used simultaneously for both particulate removal from, and quenching of, the hot gases.(11)

Contact with High Heat Capacity Solids

Inert ceramics with high heat capacities have, in a few cases, been used to cool hot effluent gases from one unit to temperatures suitable for another process unit. Besides cooling the gases by absorbing heat, this method also allows a certain amount of energy recovery by using the heat absorbed by the solids to subsequently preheat an airstream being introduced to another process unit. In one such system, the solid elements are packed into beds called *regenerative thermal oxidizers* (*RTOs*).(13) These chambers operate on a *regenerative* principle—alternately absorbing, storing, and recycling heat energy. The chambers are thus used in a cyclic fashion. In the first half-cycle, the process air is passed through the previously heated ceramic elements in a particular chamber, simultaneously cooling the bed and transferring heat into the gas that is then moved on to another unit. At any point during the cycle, the temperature at the bed varies over 1000°F from the cold side (i.e., the inlet side for the ambient air) to the hot side. In the second half-cycle, hot gases are introduced to the hot side of the bed, flow through the bed in the opposite direction of the cold flow in the first half-cycle, the heat lost by the gases is absorbed by the bed that is brought to its maximum average temperature for the start of the next cycle. Typical cycle times range from 2–10 min. Manufacturers claim over 90% thermal energy recovery with this device.

Natural Convection and Radiation

When a hot gas flows through a duct, the duct becomes hot and heats the surrounding air. As the air becomes heated, natural drafts are formed, carrying the heat away from the duct. This phenomenon is *natural convection*. Heat is also discharged from the hot duct to its surroundings by *radiant energy*. Both of these modes of heat transfer have already been discussed in Chapters 12 and 13, respectively.

The rate of heat transfer, $\dot{Q}$, is determined by the amount of heat to be removed from the hot gaseous effluent entering the system. For any particular basic process, the mass flow rate of gaseous effluent and its maximum temperature are fixed. The cooling system must therefore be designed to recover sufficient heat to lower the effluent temperature to the operating temperature of the device to be used. The rate of heat transfer can be calculated by the enthalpy difference of the gas at the inlet and outlet of the cooling system.

Forced-Draft Cooling

As discussed in Chapters 9 and 10, heat transfer by convection is due to fluid motion. Cold fluid adjacent to a hot surface receives heat, which is imparted to the bulk of the fluid by mixing. With *natural convection*, the heated fluid adjacent to the hot surface rises and is replaced by colder fluid. By agitating the fluid, mixing occurs at a much higher rate than with natural currents, and heat is taken away from the hot surface at a much higher rate. In most process applications, this agitation is induced by circulating the fluid at a rapid rate past the hot surface. This method of heat transfer is called *forced convection*. Since forced convection transfers heat much faster than natural convection, most process applications use forced-convection heat exchangers. Whenever possible, heat is exchanged between the hot and cold streams to reduce the heat input to the process. There are, however, many industrial applications where it is not feasible to exchange heat in this fashion, and a cooling fluid such as water or air is used, and the heat removed from the stream by the coolant is transferred to the environment. When water is used, the heat is taken from the process stream in a shell-and-tube cooler, and the heat picked up by the water is transferred to the atmosphere in a cooling tower. When air is used as the cooling medium in either shell-and-tube or fin-tube coolers, the heated air is discharged to the atmosphere and is not recirculated through the cooler. Because there are so many different types of forced-convection heat exchangers, it is impossible to present a specific design procedure unless a specific type is chosen for discussion.

ILLUSTRATIVE EXAMPLE 18.17

Discuss the equipment employed for most convection–radiation cooling systems.

***SOLUTION*:** For most convection–radiation systems, the only equipment used is sufficient ductwork to provide the required heat transfer area. Unless the temperature of the gases discharged is exceptionally high, or if there are corrosive gases or fumes present, black iron ductwork is generally satisfactory. The temperature of the duct wall, T_w, can be determined for any portion of the ductwork. If T_w proves to be greater than black iron can withstand, either a more heat-resistant material should be used for that portion of the system or a portion of the cooled gas should be recirculated to lower the gas temperature at the cooling system inlet. ■

ILLUSTRATIVE EXAMPLE 18.18

It is proposed to cool 144,206 lb/h of a hot flue combustion gas by using ambient air at 70°F. Calculate the quantity (mass, mole, and volume basis) of air required to cool the gases from 2050°F to an acceptable temperature of 560°F. Assume an average flue gas heat capacity of 0.3 Btu/lb·°F.

***SOLUTION*:** Under adiabatic conditions,

$$\dot{Q}_{\text{flue}} = \dot{Q}_{\text{air}}$$

$$\begin{aligned}\dot{Q}_{\text{flue}} &= \dot{m}_{\text{flue}}\bar{c}_p\,\Delta T = (144{,}206)(0.3)(2050 - 560)\\ &= 64.5 \times 10^6\ \text{Btu/h}\end{aligned}$$

Since $\dot{Q}_{\text{flue}} = \dot{Q}_{\text{air}}$,

$$64.5 \times 10^6 = \dot{m}_{\text{air}}(0.243)(560 - 70)$$

or

$$\begin{aligned}\dot{m}_{\text{air}} &= 541{,}700\ \text{lb/h};\ \text{MW} = 29\\ &= 18{,}680\ \text{lb mol/h}\\ &= 7{,}215{,}000\ \text{ft}^3/\text{h}\ (70^\circ\text{F})\end{aligned}$$

Note: The combined mass (and volume) flow of the gas is now significantly higher. This can adversely impact on the economics for the downstream equipment. ■

ILLUSTRATIVE EXAMPLE 18.19

Design the air quench tank in the previous example if a 1.5-s residence time is required.

***SOLUTION*:** The total mass flow rate of the gas is now

$$\begin{aligned}\dot{m} &= 144{,}200 + 541{,}700\\ &= 685{,}900\ \mathrm{lb/h}\end{aligned}$$

The volumetric flow at 560°F (assuming air) is

$$\begin{aligned}q &= \dot{m}_t RT/P(\mathrm{MW}) = (685{,}900)(0.73)(1020)/(1.0)(29)\\ &= 1.76\times 10^7\ \mathrm{ft}^3/\mathrm{h}\\ &= 4890\ \mathrm{f}^3/\mathrm{s}\end{aligned}$$

The volume of the tank is, therefore,

$$\begin{aligned}V_t &= (4890)(1.5)\\ &= 7335\ \mathrm{ft}^3\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 18.20

Calculate the physical dimensions of the tank in the previous example.

***SOLUTION*:** The physical dimensions of the tank are usually set by minimizing surface (materials) cost. The total surface is given by

$$S = 2(\pi D^2/4) + \pi DH; \quad H = \text{tank height}$$

To minimize the total surface, the derivative of S with respect to D is set equal to zero:

$$\frac{dS}{dD} = \pi D + \pi H = 0$$

or

$$D = H$$

The dimensions of the tank may now be calculated:

$$\begin{aligned}V_t &= (\pi D^2/4)(H) = (\pi D^3/4)\\ D &= 21.06\ \mathrm{ft}\\ H &= 21.06\ \mathrm{ft}\end{aligned}$$

Note: The size of the tank is excessive. This is another reason why air quenching is rarely used. ■

ILLUSTRATIVE EXAMPLE 18.21

It is proposed to reduce the temperature of 10% of the 144,206 lb/h of flue gas described in Illustrative Example 18.18 to 550°F using a *solids contact* method that operates on a 1-h cooling cycle. Assume the average heat capacities of the flue gas and solid to be 0.3 and 0.88 Btu/lb · °F, respectively, over the temperature range in question. The initial temperature of the solids is 70°F. Assuming an approach temperature of 40°F, what mass of solid must be provided in order to cool the flue gas to the required temperature during each hour of operation.

***SOLUTION*:** Set up an enthalpy balance for both the flue gas and solid:

$$\Delta h_{\text{flue}} = \Delta h_{\text{solid}}$$

For the flue gas, the enthalpy change for one hour of operation is

$$\begin{aligned}\Delta h_f &= mc_p(T_2 - T_1) = (0.1)(144{,}206)(0.3)(2050 - 550)\\ &= 6{,}489{,}000\ \text{Btu}\end{aligned}$$

For the solids,

$$\Delta h_s = m_s(0.88)(510 - 70)$$

Since $\Delta h_s = \Delta h_f$,

$$m_s = 6{,}489{,}000/(0.88)(440) = 16{,}759\ \text{lb solids}$$ ■

ILLUSTRATIVE EXAMPLE 18.22

With reference to the previous example, design a radiative heat exchanger to cool the entire combustion gases (MW = 28.27) from 2050 to 180°F. The ambient air temperature is 60°F and an overall heat transfer coefficient for the cooler of 1.5 Btu/h · ft^2 · °F may be assumed to apply.

***SOLUTION*:** Based on the data provided in the previous examples,

$$\begin{aligned}\dot{Q} &= (144{,}206)(0.3)(2050 - 180)\\ &= 80.9 \times 10^6\ \text{Btu/h}\end{aligned}$$

The long-mean temperature difference is given by

$$\begin{aligned}\Delta T_{\text{lm}} &= [(2050 - 60) - (180 - 60)]/\ln[(2050 - 60)/(180 - 60)]\\ &= 666^\circ\text{F}\end{aligned}$$

The radiative surface area required is

$$\begin{aligned}A &= \dot{Q}/U\Delta T_{\text{lm}} = 80.9 \times 10^6/(1.5)(666)\\ &= 80{,}980\ \text{ft}^2\end{aligned}$$

The volumetric flow, q_a, at inlet conditions is given by

$$\begin{aligned} q_a &= (144{,}206/28.27)(379)(2050 + 460)/(60 + 460) \\ &= 9{,}332{,}000\ \mathrm{ft}^3/\mathrm{h} \end{aligned}$$

Duct or pipe velocities of 60 ft/s for this type of application are typical. Assume this value to apply at inlet (2050°F) conditions. The duct area and diameter may now be calculated:

$$\begin{aligned} A_{\mathrm{duct}} &= (9{,}332{,}000)/(3600)(60) \\ &= 43.2\ \mathrm{ft}^2 \\ D &= 7.42\ \mathrm{ft} \end{aligned}$$

The length of required heat exchange ducting is then

$$\begin{aligned} L = A/\pi D &= 80{,}980/(\pi)(7.42) \\ &= 3476\ \mathrm{ft} \end{aligned}$$

The practicality of this method of cooling is questionable. ■

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Chapter 19

Insulation and Refractory

INTRODUCTION

The development presented in earlier chapters may be expanded to include insulation plus refractory and their effects. Industrial thermal insulation usually consists of materials of low thermal conductivity combined in a way to achieve a higher overall resistance to heat flow. Webster(1) defines insulation as: “to separate or cover with a non-conducting material in order to prevent the passage or leakage of . . . heat . . . etc.” Insulation is defined in Perry’s(2) in the following manner: “Materials or combinations of materials which have air- or gas-filled pockets or void spaces that retard the transfer of heat with reasonable effectiveness are thermal insulators. Such materials may be particulate and/or fibrous, with or without binders, or may be assembled, such as multiple heat-reflecting surfaces that incorporate air- or gas-filled void spaces.” Refractory materials also serve the chemical process industries. In addition to withstanding heat, refractory also provides resistance to corrosion, erosion, abrasion, and/or deformation.

This chapter contains three remaining sections:

Describing Equations
Insulation
Refractory

However, the bulk of the material keys on insulation since it has found more applications than refractories, particularly in its ability to reduce heat losses. The reader should note that there is significant overlap of common theory, equations, and applications with Chapters 7 and 8, both of which are concerned with heat conduction.

DESCRIBING EQUATIONS

When insulation is added to a surface, the heat transfer between the wall surface and the surroundings will take place by a two-step steady-state process (see Figure 19.1 for flow past a base flat plate and Figure 19.2 for flow past an insulated plate): conduction from the wall surface at T_0 through the wall to T_1 and through the insulation from T_1 to

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Heat Transfer Applications for the Practicing Engineer. Louis Theodore

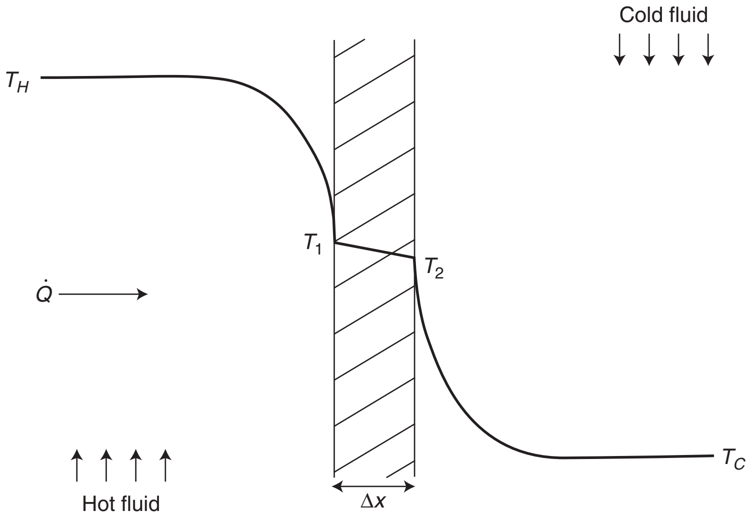

Figure 19.1 Flow past a flat plate.

T_2, and convection from the insulation surface at T_2 to the surrounding fluid at T_3. The temperature drop across each part of the heat flow path in Figure 19.2 is given below. The temperature drop across the wall and insulation is

$$(T_0 - T_1) = \dot{Q}R_0 \tag{19.1}$$

$$(T_1 - T_2) = \dot{Q}R_1 \tag{19.2}$$

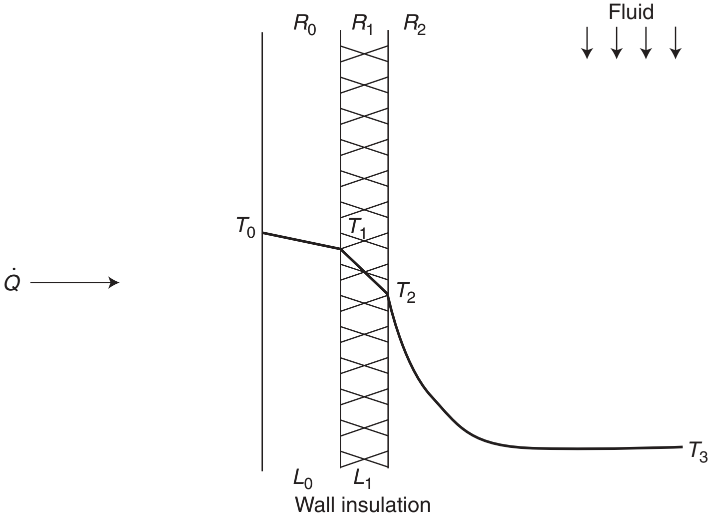

Figure 19.2 Flow past a flat insulated plate.

The temperature drop across the fluid film is

$$(T_2 - T_3) = \dot{Q}R_2 \tag{19.3}$$

where R_0 is the thermal resistance due to the conduction through the pipe wall (L_0/k_0A_0); R_1, the thermal resistance due to the conduction through the insulation (L_1/k_1A_1); and R_2, the thermal resistance due to the convection through the fluid $(1/h_2A_2)$.

As with earlier analyses, the same heat rate, $\dot{Q}$, flows through each thermal resistance. Therefore, Equations (19.1) through (19.3) can be combined to give

$$(T_0 - T_3) = \dot{Q}R_t \tag{19.4}$$

The heat transfer is then

$$\dot{Q} = \frac{(T_0 - T_3)}{R_t} = \frac{\text{total thermal driving force}}{\text{total thermal resistance}} \tag{19.5}$$

Note that for the case of plane walls, the areas A_0, A_1, and A_2 are the same (see Figure 19.2). Dividing Equation (19.2) by Equation (19.3) yields

$$\frac{(T_1 - T_2)}{(T_2 - T_3)} = \frac{R_1}{R_2} = \frac{h_2L_1}{k_1}\frac{\text{conduction insulation resistance}}{\text{convection resistance}} \tag{19.6}$$

The group h_2L_1/k_1 is a dimensionless number termed earlier as the *Biot number*, Bi (refer to Table 9.1 in Part II),

$$\text{Bi} = \frac{\text{(fluid convection coefficient)(characteristic length)}}{\text{(thermal conductivity of insulation surface)}} = \frac{hL}{k} \tag{19.7}$$

ILLUSTRATIVE EXAMPLE 19.1

The following data is provided: a rectangular plane room wall, 2.5 m high and 4 m wide, has an outside surface temperature, $T_1 = 24°\text{C}$; the outside air temperature is $T_3 = -15°\text{C}$. Calculate the heat transfer rate. The convective heat transfer coefficient between the outside surface and the air is $11\ \text{W/m}^2\cdot\text{K}$.

If loosely packed wool with $k = 0.04\ \text{W/m}\cdot\text{K}$ and a thickness of 7.62 mm (3 in.) is used for insulation on the outer wall, calculate a revised heat transfer rate.

***SOLUTION*:** Heat is transferred by convection from the outer wall surface at T_1 to the surrounding air at T_3. Therefore, Equation (19.3) may be applied with R replaced by $1/hA$:

$$\dot{Q} = hA(T_1 - T_3)$$

The heat transfer area is

$$A = (2.5)(4) = 10\,\mathrm{m}^2$$

First, calculate $\dot{Q}_b$ for the base pipe without the insulation:

$$\begin{aligned}\dot{Q}_b &= hA(T_1 - T_3)\\ &= (11)(10)[24 - (-15)]\\ &= 4290\,\mathrm{W} = 4.29\,\mathrm{kW} = 14{,}639\,\mathrm{Btu/h}\end{aligned}$$

Now consider the addition of insulation. Calculate the two thermal resistances (i.e., those due to conduction and convection). Since the insulation thickness, L_I, is 0.00762 m, the insulation resistance, R_I, is

$$R_I = \frac{L_I}{kA} = \frac{0.00762}{(0.04)(10)} = 0.0191\,\mathrm{K/W}$$

In addition, the convective resistance, R_c, is

$$R_c = \frac{1}{hA} = \frac{1}{(11)(10)} = 0.0091\,\mathrm{K/W}$$

The total resistance is

$$R_t = R_I + R_c = 0.0191 + 0.0091 = 0.0282\,\mathrm{K/W}$$

The heat transfer rate is therefore

$$\begin{aligned}\dot{Q}_I &= \frac{T_1 - T_3}{R_t}\\ &= \frac{[24 - (-15)]}{0.0282}\\ &= 1383\,\mathrm{W} = 1.38\,\mathrm{kW} = 4583\,\mathrm{Btu/h}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 19.2

Refer to Illustrative Example 19.1. Calculate the temperature at the wall-insulation surface.

***SOLUTION*:** The temperature at the outer surface of the insulation, T_2, is given by

$$T_2 = T_1 - \dot{Q}R_I \tag{19.3}$$

Substitution gives

$$\begin{aligned}T_2 &= 24.0 - 1383(0.0191)\\ &= 24.0 - 26.4\\ &= -2.4^\circ\mathrm{F}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 19.3

Calculate the Biot number in the previous example and comment on the results.

***SOLUTION*:** First calculate the Biot number. See Equations (19.6) and (19.7).

$$\mathrm{Bi} = hL/k$$

Substituting

$$\begin{aligned}\mathrm{Bi} &= \frac{(11)(0.00762)}{0.04}\\ &= 2.1\end{aligned}$$

This indicates that the conduction resistance is more than twice the convective resistance. ■

ILLUSTRATIVE EXAMPLE 19.4

One wall of an oven has a 3-inch insulation cover. The temperature on the inside of the wall is at 400°F; the temperature on the outside is at 25°C. What is the heat flux (heat flow rate per unit area) across the wall if the insulation is made of glass wool ($k = 0.022$ Btu/h · ft · °F)?

***SOLUTION*:** Once again, the thermal resistance associated with conduction is defined as

$$R = L/kA$$

where R = thermal resistance
k = thermal conductivity
A = area across which heat is conducted
L = length across which heat is conducted

The rate of heat transfer, $\dot{Q}$, is then

$$\dot{Q} = \Delta T/R \tag{19.4}$$

Thus,

$$\dot{Q} = kA\Delta T/L$$

Since 25°C is approximately 77°F and L is 0.25 ft,

$$\begin{aligned}\frac{\dot{Q}}{A} &= \frac{0.022(400-77)}{0.25}\\ &= 28.4\,\mathrm{Btu/h\cdot ft^2}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 19.5

A cold-storage room has a plane rectangular wall 8 m wide (w) and 3 m high (H). The temperature of the outside surface of the wall T_1 is -18°C. The surrounding air temperature T_3 is 26°C.

The convective heat transfer coefficient between the air and the surface is $21\ \text{W/m}^2 \cdot \text{K}$. A layer of cork board insulation (thermal conductivity, $k = 0.0433\ \text{W/ m} \cdot \text{K}$) is to be attached to the outside wall to reduce the cooling load by 80%.

1. Calculate the rate of heat flow through the rectangular wall without insulation. Express the answer in tons of refrigeration (1 ton of refrigeration = 12,000 Btu/h). Which direction is the heat flowing?
2. Determine the required thickness of the insulation board.

***SOLUTION*:** 1. Calculate the heat transfer area, A:

$$A = (w)(H) = (8)(3) = 24\,\text{m}^2$$

Calculate the rate of heat flow in the absence of insulation. Heat is transferred by convection from the wall surface to the surroundings. Applying Newton's law of cooling,

$$\begin{aligned}\dot{Q} &= hA(T_1 - T_3)\\ &= (21)(24)(-18 - 26) = -22{,}176\,\text{W} = -22.18\ \text{kW}\\ &= (-22{,}176)(3.4123) = -75{,}671\,\text{Btu/h}\\ &= -75{,}671/12{,}000 = -6.3\ \text{ton of refrigeration}\end{aligned}$$

The negative sign indicates heat flow from the surrounding air into the cold room.

2. Calculate the heat rate with insulation. Since the insulation is to reduce $\dot{Q}$ by 80%, then

$$\dot{Q} = (0.2)(-22{,}176) = -4435.2\,\text{W}$$

Calculate the total thermal resistance:

$$\begin{aligned}R_{\text{tot}} &= (T_1 - T_3)/\dot{Q} = (-18 - 26)/(-4435.2) = 0.00992^\circ\text{C/W}\\ &= R_1 + R_2\end{aligned}$$

Calculate the convection thermal resistance, R_2:

$$R_2 = 1/hA = 1/[(21)(24)] = 0.00198^\circ\text{C/W}$$

Also calculate the insulation conduction resistance, R_1:

$$R_1 = R_{\text{tot}} - R_2 = 0.00992 - 0.00198 = 0.00794^\circ\text{C/W}$$

The required insulation thickness, L_I, is given by

$$R_I = L_I/k_I A$$

Substituting

$$\begin{aligned}L_I &= R_I k_I A = (0.00794)(0.0433)(24)\\ &= 0.00825\ \text{m} = 8.25\,\text{mm}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 19.6

Refer to the previous example.

1. Calculate the temperature at the interface between the cork board and the air.
2. Calculate the Biot number, Bi.
3. What can one conclude?

***SOLUTION*:** Calculate the interface temperature, T_2:

$$T_2 - T_3 = \dot{Q}R_2 \tag{19.3}$$

$$\begin{aligned} T_2 &= T_3 + \dot{Q}R_2 = 26 + (-4435.2)(0.00198) \\ &= 26 - 8.78 = 17.22^\circ\text{C} \end{aligned}$$

Also, calculate the Biot number:

$$\begin{aligned} \text{Bi} &= hL_1/k_1 \\ &= (21)(0.00825)/(0.0433) = 4.00 \end{aligned} \tag{19.7}$$

The result that Bi $\approx$ 4 indicates that the conduction resistance is four times as large as the convection resistance.

As will be demonstrated later in this chapter, if this dimensionless number is applied to insulation around a given pipe of diameter D_o, there is critical thickness (of outer diameter D_c or outer radius r_c) of insulation for which any increase in this value will result in a decrease in $\dot{Q}$, the heat loss. This critical length can be determined from Equation (19.8).

$$Bi = 2.0 = \frac{hD_c}{k}$$

or

$$Bi = 1.0 = \frac{hr_c}{k} \tag{19.8}$$

In effect, if the outer radius of the insulation is greater than the value obtained from Equation (19.8), any further increase in the insulation thickness will result in a corresponding decrease in heat transfer. Interestingly, for "small" diameter pipes, i.e., $D_o < D_c$, an increase in insulation initially produces an increase in heat loss until the insulation diameter achieves a value equal to D_c. Further increases in insulation produces a decrease in heat loss. ■

ILLUSTRATIVE EXAMPLE 19.7

A hypodermic needle with an external diameter D_1 of 0.50 mm is to be used to transfer a reactant preheated to 95°C in a laboratory reactor. To reduce the heat loss from the transfer line, the hypodermic needle is threaded through the center of a solid rubber insulating tube (thermal conductivity, $k_2 = 0.2\ \text{W/m}\cdot\text{K}$) with a diameter of D_2 of 2 mm.

1. Calculate the rate of the heat loss from the hypodermic needle with and without the rubber insulation.
2. Calculate the Biot and Nusselt numbers for the uninsulated needle taking the characteristic dimension to be the diameter of the needle.

The stainless steel needle has a thermal conductivity, k_1, of 16 $\text{W/m} \cdot \text{K}$. The ambient air temperature is 20°C. The thermal conductivity of the air, k_3, is 0.0242 $\text{W/m} \cdot \text{K}$. The heat transfer coefficient, h_3, from the outside surface of the transfer line to the surrounding air is primarily due to natural convection and is approximately equal to 12 $\text{W/m}^2 \cdot \text{K}$; it may also be assumed independent of the radius or the temperature.

***SOLUTION*:** Neglect the resistance of the metal needle. Determine the value of R_{tot} for the case of no insulation. In the absence of insulation,

$$r_2 = r_1 = 2.5 \times 10^{-3}\ \text{m} \quad \text{and} \quad R_I = 0$$

Therefore,

$$\begin{aligned} R_{\text{tot}} &= R_3 \\ &= 1/[h_3(2\pi r_2 L)] \\ &= 1/[12(2\pi)(0.25 \times 10^{-3})(1)] \\ &= 53.05^\circ\text{C/W} \end{aligned}$$

Calculate $\dot{Q}$ without insulation:

$$\begin{aligned} \dot{Q} &= (T_1 - T_3)/R_{\text{tot}} \\ &= (95 - 20)/53.05 \\ &= 1.41\ \text{W} \end{aligned}$$

Calculate the Biot number:

$$\begin{aligned} \text{Bi} &= h_3 D_2/k_{\text{needle}};\ k_{\text{needle}} = k_1 \\ &= (12)(0.5 \times 10^{-3})/16 \\ &= 0.000375 \end{aligned}$$

Calculate the Nusselt number of the air:

$$\begin{aligned} \text{Nu} &= h_3 D_2/k_{\text{air}} \\ &= (12)(0.5 \times 10^{-3})/0.0242 \\ &= 0.248 \end{aligned}$$

Now consider the insulated needle. Calculate the thermal resistances:

$$D_3 = 2\ \text{mm} \qquad r_3 = 1\ \text{mm} = 0.001\ \text{m}$$

Note that cylindrical coordinates apply (see Chapters 8 and 9):

$$\begin{aligned} R_I = R_2 &= \frac{\ln(r_2/r_1)}{2\pi k_2 L} \\ &= \frac{\ln(0.001/0.00025)}{2\pi(0.2)(1)} \\ &= 1.103^\circ\mathrm{C/W} \\ R_3 &= 1/[h_3(2\pi r_2 L)] \\ &= 1/[(12)(2\pi)(0.001)(1)] \\ &= 13.26^\circ\mathrm{C/W} \\ R_{\mathrm{tot}} &= R_2 + R_3 \\ &= 14.37^\circ\mathrm{C/W} \end{aligned}$$

Calculate the rate of heat loss:

$$\begin{aligned} \dot{Q} &= (T_1 - T_3)/R_{\mathrm{tot}} \\ &= (95 - 20)/(14.37) \\ &= 5.22\,\mathrm{W} \end{aligned}$$

Check on the critical radius of insulation. Since

$$\mathrm{Bi}_c = 2.0 = h\,D_c/k \tag{19.7}$$

or

$$\begin{aligned} r_c &= k_2/h_3 \\ &= 0.2/12 = 0.0166\ \mathrm{m} = 16.6\,\mathrm{mm} \\ D_c &= 33.3\,\mathrm{mm} \end{aligned}$$

Repeat the calculations for other insulation diameters and present the results in tabular form (see Table 19.1). The table indicates that the addition of insulation increases the rate of heat loss

Table 19.1 Heat Rates for Different Insulators

Outside pipe insulation diameter, D_3, mm	Insulation resistance, R_2, °C/W	Convection resistance R_3, °C/W	Total resistance $R_{\mathrm{tot}} = R_2 + R_3$, °C/W	Heat rate $\dot{Q}$, W
0.5	0	53.05	53.05	1.41
2.0	1.103	13.26	14.37	5.22
4.0	1.654	6.63	8.29	9.05
6.0	1.977	4.42	6.39	11.72
8.0	2.206	3.32	5.52	13.58
10.0	2.384	2.65	5.036	14.89
12.0	2.529	2.21	4.739	15.82
14.0	2.651	1.895	4.546	16.49
$33.3 = D_c$	3.342	0.796	4.138	18.13
50.0	3.3667	0.531	4.198	17.87

because the increase in the conduction resistance is proportionally less than the decrease in the convection resistance. The total resistance therefore decreases for this “small” diameter pipe but increases when $D > D_c$. ■

ILLUSTRATIVE EXAMPLE 19.8

Comment further on the results of the previous example.

SOLUTION**:** As shown in Table 19.1, the maximum value of the heat loss (at the critical radius) is 18.13 W. The initial increase in heat loss may be viewed as occurring because of what the author refers to as a “radius of curvature” effect for small radii. In effect, the area increases at a faster rate than the resistance decreases. More details follow in the next section. The reader may choose to verify that $\dot{Q} \rightarrow 0$ in the limit as $D \rightarrow \infty$. ■

ILLUSTRATIVE EXAMPLE 19.9

The surface temperature of a circular conducting rod is maintained at 200°C (T_1) by the passage of an electric current. The rod diameter is 10 mm, the length is 2.5 m, the thermal conductivity is $60\ \text{W/m}\cdot\text{K}$, the density is $7850\ \text{kg/m}^3$, and the heat capacity is $434\ \text{J/kg}\cdot\text{K}$. A bakelite coating (thermal conductivity $= 1.4\ \text{W/m}\cdot\text{K}$) is applied to the rod. The rod is in a fluid at 25°C (T_3), and the convection heat transfer coefficient is $140\ \text{W/m}^2\cdot\text{K}$. The thermal conductivity of the fluid is $0.6\ \text{W/m}\cdot\text{K}$.

1. Calcualte the rate of heat transfer for the bare rod?
2. What is the critical radius associated with the bakelite coating? What is the heat transfer rate at the critical radius?
3. If the bakelite insulation thickness is 55 mm, determine the fractional reduction in heat transfer rate relative to the case of a bare rod.

SOLUTION**:**

1. Calculate $\dot{Q}$ for the bare rod:

$$\begin{aligned}\dot{Q}_{\text{bare}} &= h(\pi D_1 L)(T_1 - T_3) = (140)(\pi)(0.01)(2.5)(200 - 25)\\ &= 1924\ \text{W}\end{aligned}$$

2. Calculate the critical bakelite (subscript 2) radius:

$$\begin{aligned}\text{Bi}_i &= \text{critical Biot number} = h_3 D_{2,c}/k_2 = 2.0\\ D_{2,c} &= 2k_2/h_3 = 2(1.4)/140 = 0.02\ \text{m} = 20\ \text{mm}\\ r_{2,c} &= D_{2,c}/2 = 0.01\ \text{m} = 10\ \text{mm}\end{aligned}$$

Since $r_1 < r_{2,c}$, the addition of bakelite will increase $\dot{Q}$ until $r_1 = r_{2,c}$.

$$R_1 = \frac{\ln(r_2/r_1)}{2\pi k_2 L} = \frac{\ln(0.01/0.005)}{(2\pi)(1.4)(2.5)} = 0.0315^\circ\text{C/W}$$

$$R_2 = \text{convection resistance} = \frac{1}{h_3(2\pi r_2 L)} = \frac{1}{(140)(2\pi)(0.01)(2.5)} = 0.0455^\circ\text{C/W}$$

$$R_{\text{tot}} = R_1 + R_2 = 0.077^\circ\text{C/W}$$

$$\dot{Q}_{\text{crit}} = (T_1 - T_3)/R_{\text{tot}} = (200 - 25)/0.077 = 2273\,\text{W}$$

3. For insulation of thickness = 55 mm = 0.055 m, calculate $\dot{Q}_{\text{insul}}$:

$$r_2 = r_1 + 0.055 = 0.06\ \text{m}$$

$$R_1 = \text{conduction (bakelite) resistance} = \frac{\ln(0.06/0.005)}{2\pi(1.4)(2.5)} = 0.113^\circ\text{C/W}$$

$$R_2 = \text{convection resistance} = 1/[(140)(2\pi)(0.06)(2.5)] = 0.0076^\circ\text{C/W}$$

$$R_{\text{tot}} = 0.12^\circ\text{C/W}$$

$$\dot{Q} = \dot{Q}_{\text{insul}} = (T_1 - T_3)/R_{\text{tot}} = (200 - 25)/0.12 = 1451.3\,\text{W}$$

Since $r_2 > r_{2,c}$, $\dot{Q}_{\text{insul}}$ decreases. Therefore, the percent reduction in $\dot{Q}$ relative to the bare case is:

$$\begin{aligned}\text{Reduction} &= \left(\frac{\dot{Q}_{\text{bare}} - \dot{Q}_{\text{insul}}}{\dot{Q}_{\text{bare}}}\right)100\% \\ &= \left(\frac{1924 - 1451.3}{1924}\right)100\% \\ &= 24.6\%\end{aligned}$$

The results are in agreement with the discussion presented earlier for a small diameter pipe, i.e.,

$$Q_{\text{crit}} > Q_{\text{bare}}$$

and

$$Q_{\text{insul}} < Q_{\text{crit}} \text{ if } r > r_{\text{c}}$$

■

ILLUSTRATIVE EXAMPLE 19.10

A stainless steel tube carries hot ethylene glycol at 124°C (T_1). The surrounding air outside the tube is at 2°C (T_5). To reduce the heat losses from the ethylene glycol, the tube is surrounded by asbestos insulation. For a 1 m length of the tube, calculate:

1. the rate of heat transfer without insulation,
2. the rate of heat transfer with insulation, and

3. the overall heat transfer coefficient based on the inside area of the tube,
4. the overall heat transfer coefficient based on the outside area of the insulation,
5. the temperature, T_3, at the steel–insulation interface,
6. the inside and outside Biot numbers, and the outside Nusselt number, and
7. the log mean radius of insulation.

The following data is provided.

Stainless steel pipe: inside radius, $r_1 = 1.1$ cm; outside radius, $r_2 = 1.3$ cm; thermal conductivity, $k_2 = 19\ \mathrm{W/m \cdot K}$; heat transfer coefficient from ethylene glycol to the stainless steel pipe, $h_1 = 190\ \mathrm{W/m^2 \cdot K}$.

Asbestos insulation: inside radius, $r_2 = 1.3$ cm; outside radius, $r_3 = 3.8$ cm; thermal conductivity, $k_3 = 0.2\ \mathrm{W/m \cdot K}$.

The outside heat transfer coefficient from the air to the surface of the insulation (or of the pipe, in the case of no insulation), $h_3 = 14\ \mathrm{W/m^2 \cdot K}$. The thermal conductivity, k_4, of the air is $0.0242\ \mathrm{W/m \cdot K}$.

***SOLUTION*:** Calculate the inside and outside areas of heat transfer. Let A_1 and A_2 represent the inside and outside surface areas of the stainless steel tube, respectively, and A_3 represent the outside surface area of the insulation:

$$\begin{aligned} A_1 &= 2\pi r_1 L = (2\pi)(0.011)(1) \\ &= 0.0691\ \mathrm{m^2} \\ A_2 &= 2\pi r_2 L = (2\pi)(0.013)(1) \\ &= 0.0817\ \mathrm{m^2} \\ A_3 &= 2\pi r_3 L = (2\pi)(0.038)(1) \\ &= 0.239\ \mathrm{m^2} \end{aligned}$$

1. Calculate the inside convection resistance:

$$\begin{aligned} R_1 &= 1/[(190)(0.0691)] \\ &= 0.0762^\circ\mathrm{C/W} \end{aligned}$$

Calculate the conduction resistance through the tube (see Chapters 7 and 14):

$$\begin{aligned} R_2 &= \frac{\ln(r_2/r_1)}{2\pi k_2 L} \\ &= \frac{\ln(0.013/0.011)}{(2\pi)(19)(1)} \\ &= 0.0014^\circ\mathrm{C/W} \end{aligned}$$

For the case of no insulation, calculate the outside convection resistance:

$$\begin{aligned} R_3 &= 1/h_4 A_2 \\ &= 1/[(14)(0.0817)] \\ &= 0.874^\circ\mathrm{C/W} \end{aligned}$$

Calculate the heat transfer rate:

$$\begin{aligned} R_{\text{tot}} &= 0.0762 + 0.0014 + 0.874 \\ &= 0.952^{\circ}\text{C/W} \end{aligned}$$

$$\begin{aligned} \dot{Q} &= (T_1 - T_5)/R_{\text{tot}} \\ &= (124 - 2)/0.952 \\ &= 128.2\,\text{W} \end{aligned}$$

2. For the case of insulation, calculate the conduction resistance associated with the insulation:

$$\begin{aligned} R_3 &= \frac{\ln(r_3/r_2)}{2\pi k_3 L} \\ &= \frac{\ln(0.038/0.013)}{2\pi(0.2)(1)} \\ &= 0.854^{\circ}\text{C/W} \end{aligned}$$

Calculate the outside convection resistance:

$$\begin{aligned} R_4 &= 1/h_4 A_3 \\ &= 1/[(14)(0.239)] \\ &= 0.3^{\circ}\text{C/W} \end{aligned}$$

Calculate the total resistance:

$$\begin{aligned} R_{\text{tot}} &= 0.0762 + 0.0014 + 0.854 + 0.3 \\ &= 1.23^{\circ}\text{C/W} \end{aligned}$$

Calculate the heat transfer rate:

$$\begin{aligned} \dot{Q} &= (T_1 - T_5)/R_{\text{tot}} \\ &= (124 - 2)/(1.23) \\ &= 99.2\,\text{W} \end{aligned}$$

3. Calculate the overall heat transfer coefficient, U, based on the inside area:

$$\begin{aligned} U_1 &= 1/R_{\text{tot}} A_1 \\ &= 1/[(1.23)(0.0691)] \\ &= 11.76\,\text{W/m}^2 \cdot \text{K} \end{aligned}$$

4. Calculate U based on the outside area:

$$\begin{aligned} U_3 &= 1/R_{\text{tot}} A_3 \\ &= 1/[(1.23)(0.239)] \\ &= 3.4\,\text{W/m}^2 \cdot \text{K} \end{aligned}$$

5. Calculate the temperature T_3:

$$\begin{aligned} T_3 &= T_1 - (R_1 + R_2)\dot{Q} \\ &= 124 - (0.076 + 0.0014)(99.2) \\ &= 116.3^\circ\text{C} \end{aligned}$$

6. Calculate the outside Biot number:

$$\begin{aligned} \text{Bi}_o &= h_4(2r_3)/k_3 \\ &= (14)(2)(0.038)/0.2 \\ &= 5.32 \end{aligned}$$

Calculate the inside Biot number:

$$\begin{aligned} \text{Bi}_i &= h_1(2r_1)/k_3 \\ &= (190)(2)(0.011)/19 \\ &= 0.22 \end{aligned}$$

Calculate the Nusselt number of the air:

$$\begin{aligned} \text{Nu} &= \frac{h_{\text{air}} D_3}{k_{\text{air}}} \\ &= \frac{h_4 D_3}{k_4} \\ &= \frac{(190)(2)(0.011)}{0.0242} \\ &= 172.7 \end{aligned}$$

7. Calculate the log mean radius of the insulation:

$$\begin{aligned} r_{\text{lm}} &= \frac{r_3 - r_2}{\ln(r_3/r_2)} \\ &= \frac{0.038 - 0.013}{\ln(0.038/0.013)} \\ &= 0.0233\ \text{m} \\ &= 2.33\ \text{m} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 19.11

A recently developed synthetic oil is stored in a vertical tank 10 feet in diameter and 30 feet high which is insulated with a 2-inch layer of insulation ($k = 0.039$ Btu/h · ft · °F). To prevent freezing of the oil, it is maintained at a temperature of 115°F by a heating coil consisting of an 18-gauge, $\frac{3}{4}$-inch copper tube ($k = 224$ Btu/h · ft · °F) containing saturated steam at 10 psig.

Assuming that the minimum outdoor temperature is 5°F and that the oil temperature is uniform, calculate of copper tubing required in feet to maintain the tank at 120°F in the coldest weather. Neglect upper and lower tank surface heat losses. The following additional information is provided:

$$\text{Steam condensing inside coil, } h = 800\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$\text{Oil outside coil, } h = 40\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$\text{Oil inside tank wall, } h = 40\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$\text{Outer tank wall to ambient air, } h = 2.0\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

***SOLUTION*:** First consider the tank (t). Since wall thermal conductivity data is not provided, assume the wall resistance is negligible, and any area corrections can be neglected because of the large tank diameter (i.e., $A_i = A_o = A_{\text{lm}} = A_t$). Thus,

$$U_o = \frac{1.0}{\dfrac{1}{40} + \dfrac{2.0}{(12)(0.039)} + \dfrac{1}{2.0}}$$
$$= 0.21\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

The heat transfer rate lost from the tank can now be calculated:

$$\begin{aligned}\dot{Q}_{\max} = \dot{Q} &= U_o A_t \Delta T_t\\ &= (0.21)(\pi)[10 + (4/12)](30)(120 - 5)\\ &= 23{,}000\,\text{Btu/h}\end{aligned}$$

For the coil, also neglect any area correction factors, i.e.,

$$A_i = A_o = A_{\text{lm}} = A_c$$

From Table 6.3, $\Delta x_c = 0.049$ in. Thus,

$$U_o = \frac{1}{\dfrac{1}{40} + \dfrac{0.049}{(12)(224)} + \dfrac{1}{800}}$$
$$= 38.0\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

For 10 psia (24.7 psia) steam, $T_{\text{st}} = 240$°F. Therefore,

$$\begin{aligned}\dot{Q} &= U_o A_c \Delta T_c\\ 23{,}000 &= (38.0)(A_c)(240 - 120);\ c = \text{copper tubing}\\ A_c &= 5.04\,\text{ft}^2\end{aligned}$$

The area of the copper tube is $0.1963\,\text{ft}^2/\text{ft}$. The length of tube, L, is then

$$L = \frac{5.04}{0.1963}$$
$$= 25.7\,\text{ft}$$

■

ILLUSTRATIVE EXAMPLE 19.12

Ricci and Theodore (R&T) Consultants have been assigned the job of selecting insulation for all the plant piping at the local power plant. Included in the plant piping are 8000 ft of 1-inch schedule 40 steel (1% C) pipe carrying steam at 240°F. It is estimated that the heat transfer coefficient for condensing steam on the inside of the pipe is 2000 $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$. The air temperature outside of the pipe can drop to 20°F, and with wind motion the outside heat transfer coefficient can be as high as 100 $\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$.

R&T have decided to use a fiberglass insulation having a thermal conductivity of 0.01 $\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$. It is available in 6 ft lengths in the four thicknesses listed below:

$\frac{3}{8}$-inch thick	\$1.51/6 ft length
$\frac{1}{2}$-inch thick	\$3.54/6 ft length
$\frac{3}{4}$-inch thick	\$5.54/6 ft length
1-inch thick	\$8.36/6 ft length

Calculate the energy saved per dollar of insulation investment in going from $\frac{3}{8}$-inch to $\frac{1}{2}$-inch thick insulation. Repeat this calculation in going from $\frac{1}{2}$-inch to $\frac{3}{4}$-inch and $\frac{3}{4}$-inch to 1-inch. Express the results in units of Btu/h per dollar.

***SOLUTION*:** Employ cylindrical coordinates. Calculate the outside log-mean diameter of the pipe. For the 1-inch pipe schedule 40:

$$D_i = 1.049\,\text{inch} = 0.0874\,\text{ft}$$
$$D_o = 1.315\,\text{inch} = 0.1096\,\text{ft}$$

The log-mean diameter of the pipe is therefore

$$\begin{aligned}\bar{D} &= \frac{D_o - D_i}{\ln(D_o/D_i)} \\ &= \frac{0.1096 - 0.0874}{\ln(0.1096/0.0874)} = 0.09808\,\text{ft}\end{aligned}$$

Provide an expression for the insulation outside diameter in terms of Δx_I, the insulation thickness in inches:

$$D_I = \frac{1.315 + 2(\Delta x_I)}{12\,\text{in/ft}}$$

Thus,

Thickness, inch	D_I, ft
$\frac{3}{8}$	0.172
$\frac{1}{2}$	0.193
$\frac{3}{4}$	0.235
1	0.276

Obtain the relationship between the insulation thickness and the insulation log-mean diameter of the pipe:

$$\bar{D}_I = \frac{D_I - 0.1096}{\ln(D_I/0.1096)}$$

Therefore,

Thickness, inch	$\overline{D}_I$, ft
$\frac{3}{8}$	0.139
$\frac{1}{2}$	0.147
$\frac{3}{4}$	0.164
1	0.180

The pipe wall resistance and pipe thickness are:

$$R_w = \frac{\Delta x_w}{k\pi \bar{D}_w L}$$

$$\Delta x_w = \frac{0.1096\,\text{ft} - 0.0874\,\text{ft}}{2} = 0.0111\,\text{ft}$$

For steel:

$$k = 24.8\,\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$$

Therefore, for an 8000 ft pipe, the wall resistance is:

$$R_w = \frac{0.0111\,\text{ft}}{(24.8\,\text{Btu/hr} \cdot\ \text{ft} \cdot {}^\circ\text{F})\pi(0.09808\,\text{ft})(8000\,\text{ft})}$$
$$= 1.816 \times 10^{-7}(\text{Btu/h} \cdot {}^\circ\text{F})^{-1}$$

The inside steam convection resistance is:

$$R_i = \frac{1}{h_i \pi D_i L}$$
$$= \frac{1}{(2000\,\text{Btu/h} \cdot\ \text{ft}^2 \cdot {}^\circ\text{F})\pi(0.0874\,\text{ft})(8000\,\text{ft})}$$
$$= 2.28 \times 10^{-7}(\text{Btu/h} \cdot {}^\circ\text{F})^{-1}$$

Express the insulation resistance in terms of Δx_I:

$$R_I = \frac{\Delta x_I}{k_I \pi \bar{D}_I L}$$

Thickness, inch	R_I, $(\mathrm{Btu/h \cdot {}^\circ F})^{-1}$
$\frac{3}{8}$	8.978×10^{-4}
$\frac{1}{2}$	1.125×10^{-3}
$\frac{3}{4}$	1.514×10^{-3}
1	1.839×10^{-3}

Also express the outside air convection resistance in terms of the thickness, Δx_I:

$$R_o = \frac{1}{h_o \pi D_I L}$$

Thickness, inch	R_o, $(\mathrm{Btu/h \cdot {}^\circ F})^{-1}$
$\frac{3}{8}$	2.312×10^{-6}
$\frac{1}{2}$	2.062×10^{-6}
$\frac{3}{4}$	1.696×10^{-6}
1	1.440×10^{-6}

The total resistance, R, is

$$R = R_i + R_w + R_I + R_o$$

The total resistance for each thickness is therefore:

Thickness, inch	R, $(\mathrm{Btu/h \cdot {}^\circ F})^{-1}$
$\frac{3}{8}$	9.005×10^{-4}
$\frac{1}{2}$	1.128×10^{-3}
$\frac{3}{4}$	1.156×10^{-3}
1	1.841×10^{-3}

The overall outside heat transfer coefficient, U_o, is given by

$$U_o = \frac{1}{R \pi D_I L}$$

so that

Thickness, inch	U_o, $(\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F})$
$\frac{3}{8}$	0.257
$\frac{1}{2}$	0.183
$\frac{3}{4}$	0.112
1	0.078

The inside overall heat transfer coefficient, U_i, is

$$U_i = \frac{1}{R\pi D_i L}$$

so that

Thickness, inch	U_i, (Btu/h · ft^2 · °F)
$\frac{3}{8}$	0.506
$\frac{1}{2}$	0.404
$\frac{3}{4}$	0.300
1	0.247

The energy loss is

$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T$$

with

$$A_i = \pi D_i L = \pi(0.0874)(8000) = 2195\,\text{ft}^2$$
$$\Delta T = 240 - 20 = 220$$

Therefore,

Thickness, inch	$\dot{Q}$, Btu/h
$\frac{3}{8}$	2.443×10^5
$\frac{1}{2}$	1.951×10^5
$\frac{3}{4}$	1.451×10^5
1	1.195×10^5

Finally, calculate the energy saved per dollar of insulation investment going from $\frac{3}{8}$-inch to $\frac{1}{2}$-inch, from $\frac{1}{2}$-inch to $\frac{3}{4}$-inch, and from $\frac{3}{4}$-inch to 1-inch.

$$\frac{\text{Energy}}{\$} = \frac{\Delta\dot{Q}}{(8000)(\Delta\text{Unit cost per 6 ft})}$$

From $\frac{3}{8}$-inch to $\frac{1}{2}$-inch:

$$\frac{\text{Energy}}{\$} = \frac{(2.443 \times 10^5\,\text{Btu/h} - 1.951 \times 10^5\,\text{Btu/h})}{(8000\,\text{ft})(\$3.54 - \$1.51)/(6\,\text{ft})} = 18.1\,\text{Btu/h} \cdot \$$$

From $\frac{1}{2}$-inch to $\frac{3}{4}$-inch:

$$\frac{\text{Energy}}{\$} = \frac{(1.951 \times 10^5\,\text{Btu/h} - 1.451 \times 10^5\,\text{Btu/h})}{(8000\,\text{ft})(\$5.54 - \$3.54)/(6\,\text{ft})} = 18.7\,\text{Btu/h} \cdot \$$$

From $\frac{3}{4}$-inch to 1-inch:

$$\frac{\text{Energy}}{\$} = \frac{(1.451 \times 10^5\,\text{Btu/h} - 1.195 \times 10^5\,\text{Btu/h})}{(8000\,\text{ft})(\$8.36 - \$5.54)/(6\,\text{ft})} = 6.8\,\text{Btu/h} \cdot \$$$

■

ILLUSTRATIVE EXAMPLE 19.13

Comment on the results of the previous example.

***SOLUTION*:** The energy recovery quotient is the highest when going from $\frac{1}{2}$-inch pipe to $\frac{3}{4}$-inch pipe. Obviously, the insulation should be in this range based on the calculations. Additional calculations may be necessary employing different insulation thicknesses since either of these pipes may not be the choice for the most cost effective system. In order to decide which system is the best, the total annual capital and operating costs should also be calculated. The total annual capital cost also depends upon the plant life and the rate of return (see also Chapter 27). In addition to cost minimization, a safety factor must be taken into consideration. The insulation must be adequate to prevent exposed surfaces from exceeding the temperature that would be safe for the person working nearby. Some of these factors are considered in the next (and last) Illustrative Example in this chapter.

Additional economic and finance illustrative examples are provided in Chapter 27 ■

INSULATION

Fiber, *powder*, and *flake-type* insulation consist of finely dispersed solids throughout an air space. The ratio of the air space to the insulator volume is called the *porosity* or *void fraction*, ε. In *cellular* insulation, a material with a rigid matrix contains entrapped air pockets. An example of such rigid insulation is *foamed insulation* which is made from *plastic* and *glass* material. Another type of insulation consists of multi-layered thin sheets of *foil* of high reflectivity. The spacing between the foil sheets is intended to restrict the motion of air. This type of insulation is referred to as *reflective* insulation.

Most thermal insulation systems consist of the insulation *and* a so-called "finish." The finish provides protection against water or other liquid entry, mechanical damage, and ultraviolet degradation; it can also provide fire protection. The finish usually consists of any form of coating (e.g., polymeric paint material, etc.), a membrane (e.g., felt, plastic laminate, foil, etc.), or a sheet material (e.g., fabric, plastic, etc.). Naturally, the finish must be able to withstand any potential temperature excursion in its immediate vicinity.

Critical Insulation Thickness

It would normally seem that the thicker the insulation, the less the heat loss, i.e., increasing the insulation should reduce the heat loss to the surroundings. But, and as discussed in the previous section, this is not always the case. There is a "critical insulation thickness" below which the system will experience a greater heat loss due to an increase in insulation. This situation arises for "small" diameter pipes when the increase in area increases more rapidly then the resistance opposed by the thicker insulation.

Consider the system shown in Figure 19.3. As noted in Chapters 7 and 14, the area terms for the heat transfer equations in rectangular coordinates are no longer equal in cylindrical coordinates (e.g., for the inside surface, the heat transfer area is given by $2\pi r_i$L). Applying the above development to a pipe/cylinder system leads to

$$\dot{Q} = \frac{T_i - T_o}{\dfrac{1}{2\pi r_i L}\left(\dfrac{1}{h_i}\right) + \dfrac{\Delta x_w}{k_w 2\pi L r_{\text{lm},w}} + \dfrac{\Delta x_i}{k_i 2\pi L r_{\text{lm},i}} + \dfrac{1}{2\pi r L}\left(\dfrac{1}{h_o}\right)}$$

$$= \frac{2\pi L(T_i - T_o)}{\dfrac{1}{r_i h_i} + \dfrac{\ln(r_o/r_i)}{k_w} + \dfrac{\ln(r/r_o)}{k_i} + \dfrac{1}{r h_o}} = \frac{2\pi L(T_i - T_o)}{f(r)} \tag{19.9}$$

where

$$f(r) = \frac{1}{r_i h_i} + \frac{\ln(r_o/r_i)}{k_w} + \frac{\ln(r/r_o)}{k_i} + \frac{1}{r h_o} \tag{19.10}$$

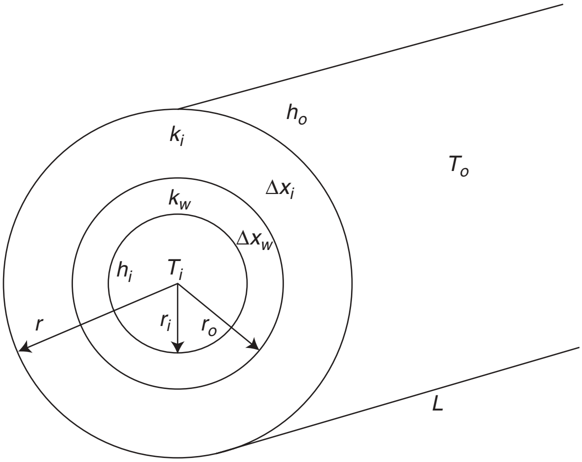

Figure 19.3 Critical insulation thickness for a pipe.

Assuming that $\dot{Q}$ goes through a maximum or minimum as r is varied, l'Hôpital's rule can be applied to Equation (19.10):

$$\frac{d\dot{Q}}{dr} = 2\pi L(T_i - T_o)\left[\frac{-\dfrac{df(r)}{dr}}{f(r)^2}\right] = 0 \tag{19.11}$$

with

$$-\frac{df(r)}{dr} = -\frac{1}{rk_i} + \frac{1}{r^2 h_0}$$

For $d\dot{Q}/dr = 0$, one may therefore write

$$\frac{d\dot{Q}}{dr} = -\frac{d(fr)}{dr} = \frac{1}{rk_i} + \frac{1}{r^2 h_o} = 0 \tag{19.12}$$

For this maximum/minimum condition, set $r = r_c$ and solve for r_c.

$$r_c = \frac{k_i}{h_o} \tag{19.8}$$

The second derivative of $d\dot{Q}_r/dr$ of Equation (19.11) provides information as to whether $\dot{Q}$ experiences a maximum or minimum at r_c.

$$\frac{d}{dr}\left(\frac{d\dot{Q}_r}{dr}\right) + \frac{1}{r^2 k_i} - \frac{2}{r^3 h_o} = \frac{h_o^2}{k^3} - \frac{2h_o^2}{k^3} = \frac{h_o^2}{k^3}(1-2) \tag{19.13}$$

Clearly, the second derivative is a negative number; $\dot{Q}$ is therefore a *maximum* at $r = r_c$. $\dot{Q}$ then decreases monotonously as r is increased beyond r_c. However, one should exercise care in interpreting the implications of the above development. This result applies *only* if r_o is *less* than r_c, i.e., it generally applies to "small" diameter pipes/tubes. Thus, r_c represents the outer radius (not the thickness) of the insulation that will maximize the heat loss and at which point any further increase in insulation thickness will result in a decrease in heat loss.

A graphical plot of the resistance R versus r is provided in Figure 19.4. (The curve is inverted for the plot of $\dot{Q}$ or r.) In other words, the maximum heat loss from a pipe occurs when the critical radius equals the ratio of the thermal conductivity of the insulation to the surface coefficient of heat transfer. This ratio has the dimension of length (e.g., ft). Note that once again the equation for r_i can be rewritten in terms of a dimensionless number defined earlier:

$$\frac{k_i}{h_o r_c} = \mathrm{Bi}^{-1} \tag{19.7}$$

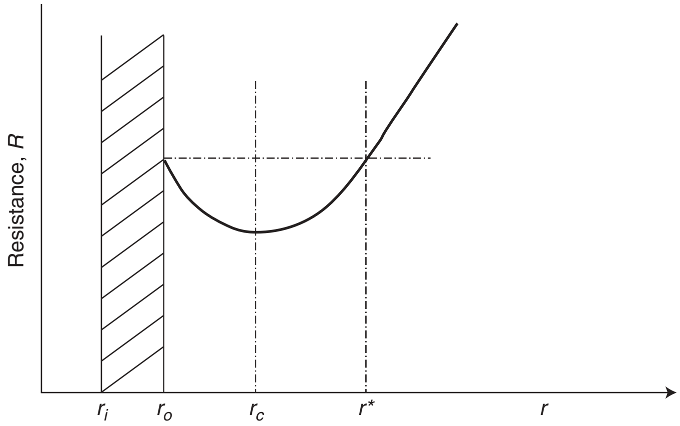

Figure 19.4 Resistance associated with the critical insulation thickness for a bare surface.

To reduce $\dot{Q}$ below that for a bare wall ($r = r_0$), r must be greater than r_c, i.e., $r > r_c$. The radius at which this occurs is denoted as r^*. The term r^* may be obtained by solving the equation

$$\dot{Q}_{\text{bare}} = \dot{Q}_{I,r^*} \tag{9.14}$$

so that

$$\frac{2\pi L(T_i - T_o)}{\dfrac{1}{r_i h_i} + \dfrac{\ln(r_o/r_i)}{k_w} + \dfrac{1}{r_o h_o}} = \frac{2\pi L(T_i - T_o)}{\dfrac{1}{r_i h_i} + \dfrac{\ln(r_o/r_i)}{k_w} + \dfrac{\ln(r^*/r_o)}{k_i} + \dfrac{1}{r^* h_o}} \tag{19.15}$$

One may now solve for r^* using a suitable trial-and-error procedure.

The above development applies when r is less than r_c. If r_o is larger than r_c, the above analysis again applies, but only to the results presented for $r > r_c$, i.e., $\dot{Q}$ will decrease indefinitely as r increases. Note that there is *no* maximum/minimum (inflection) for this case since values of $\ln(r/r_o)$ are indeterminate for $r < r_o$. Once again, $\dot{Q}$ approaches zero in the limit as r approaches infinity.

ILLUSTRATIVE EXAMPLE 19.14

Explain why it is important to determine the critical radius.

***SOLUTION*:** As noted above, as the thickness of the insulation is increased, the cost associated with heat lost decreases but the insulation cost increases. The optimum thickness is determined by the minimum of the total costs. Thus, as the thickness of the insulation is increased, the heat loss reaches a maximum value and then decreases with further increases in insulation. Reducing this effect can be accomplished by using an insulation of low conductivity. ■

ILLUSTRATIVE EXAMPLE 19.15

It has come to the attention of a young engineer that there is a bare pipe that is releasing a significant amount of heat into the atmosphere. List factors that should be considered in selecting the optimum insulation diameter for the pipe.

***SOLUTION*:** Two important factors include durability and maintainability. If it is determined that cost is the number one factor, and a cheaper insulation is chosen, it would be wise to investigate these two factors for that insulation. The end result might be that the cheaper insulation does not have a long life and might have to be maintained much more often than a more expensive one. This could cause the more expensive one to be more cost effective than the cheaper insulation.

Temperature difference will also play a role in determining the insulation diameter. It may be imperative to keep the temperature of the material in the pipe just above freezing, or it may be that the temperature needs to be 60° above freezing. These different situations call for different insulation diameters. Another factor that falls under the temperature category is the location of the pipe. It would be extremely different if a pipe is insulated in New York, Alaska, or Tahiti. All of these places have different climates and it is imperative that these be investigated in order to know how large or small the diameter of the insulation needs to be.

Finally, the other factors that need to be looked at, and may be as important as the first, are whether the materials that constitute the insulation are harmful. First and foremost, there is asbestos, and due to recent studies, insulation with no or very little asbestos should be used. Another insulation material is fiberglass. A good number of insulators are made with fiberglass. When the insulation is cut, fiberglass escapes into the air and workers should not breathe this harmful material. ■

ILLUSTRATIVE EXAMPLE 19.16

Calculate the outer critical radius of insulation on a 2.0-inch OD pipe. Assume the air flow coefficient to be 1.32 $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$ and the loosely-packed insulation's thermal conductivity to be 0.44 $\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$. Comment on the effect of insulation on the heat rate lost from the pipe.

***SOLUTION*:** Employ Equation (19.8):

$$\begin{aligned} r_c &= \frac{k_i}{h_o} \\ &= \frac{0.44}{1.32} = 0.333\ \text{ft} \\ &= 4.0\ \text{in} \end{aligned}$$

The critical insulation thickness is therefore:

$$L_I = \Delta x_I = 4.0 - (2.0/2) = 3.0\ \text{in}$$

Since $r_o = 1.0$ in, $r_o < r_c$, so that the heat loss will increase as insulation is added, but start to decrease when the radius of the insulation increases above r_c, i.e., when $r > r_c$. ■

REFRACTORY

Webster[(1)] defines refractory as: “resistant to heat, hard to melt or work . . . not yielding to treatment . . . a heat resistant material used in lining furnaces, etc.” Refractory materials must obviously be chemically and physically stable at high temperatures. Depending on the operating environment, they must also be resistant to thermal shock, chemically inert, and resistant to wear. Refractories normally require special warm-up periods to reduce the possibility of thermal shock and/or drying stresses.

The oxides of aluminum (alumina), silicon (silica), and magnesium (magnesia) are the most common materials used in the manufacture of refractories. Another oxide usually found in refractories is the oxide of calcium (lime). Fireclays are also widely used in the manufacture of refractories. Additional details are provided in Tables 19.2 and 19.3.

Refractories are selected based primarily on operating conditions. Some applications require special refractory materials. Zirconia is used when the material must withstand extremely high temperatures. Silicon carbide and carbon are two other refractory materials used in some very severe temperature conditions, but they cannot be used in contact with oxygen, as they will oxidize and burn.

Table 19.2 Properties of Non-metals

Structural and heat-resistant materials					
Substance	Temperature, °C	k, W/m · °C	ρ, kg/m^3	c_p, kJ/kg · °C	α, m^2/s × 10^7
Asphalt	20–55	0.74–0.76			
Brick					
Building brick, common	20	0.69	1600	0.84	5.2
face		1.32	2000		
Carborundum brick	600	18.5			
	1400	11.1			
Chrome brick	200	2.32	3000	0.84	9.2
	550	2.47			9.8
	900	1.99			7.9
Diatomaceous earth,	200	0.24			
molded and fired	870	0.31			
Fire clay, burnt	500	1.04	2000	0.96	5.4
2426°F	800	1.07			
	1100	1.09			
burnt 2642°F	500	1.28	2300	0.96	5.8
	800	1.37			
	1100	1.40			
Fireclay brick	200	1.0	2645	0.96	3.9
	655	1.5			

(*Continued*)

Table 19.2 *Continued*

Structural and heat-resistant materials					
Substance	Temperature, °C	k, W/m · °C	ρ, kg/m^3	c_p, kJ/kg · °C	α, m^2/s × 10^7
	1205	1.8			
Missouri	200	1.00	2600	0.96	4.0
	600	1.47			
	1400	1.77			
Magnesite	200	3.81		1.13	
	650	2.77			
	1200	1.90			
Cement, Portland		0.29	1500		
Mortar	23	1.16			
Concrete, cinder	23	0.76			
Stone, 1-2-4 mix	20	1.37	1900–2300	0.88	8.2–6.8
Glass, window	20	0.78 (avg)	2700	0.84	3.4
Borosilicate	30–75	1.09	2200		
Plaster, gypsum	20	0.48	1440	0.84	4.0
Metal lath	20	0.47			
Wood lath	20	0.28			
Stone					
Granite		1.73–3.98	2640	0.82	8–18
Limestone	100–300	1.26–1.33	2500	0.90	5.6–5.9
Marble		2.07–2.94	2500–2700	0.80	10–13.6
Sandstone	40	1.83	2160–2300	0.71	11.2–11.9
Wood (across the grain)					
Balsa, 8.8 lb/ft^3	30	0.055	140		
Cypress	30	0.097	460		
Fir	23	0.11	420	2.72	0.96
Maple or oak	30	0.166	540	2.4	1.28
Yellow pine	23	0.147	640	2.8	0.82
White pine	30	0.112	430		
Asbestos					
Loosely packed	−45	0.149			
	0	0.154	470–570	0.816	3.3–4
	100	0.161			
	20	0.74			
Asbestos–cement boards	20	0.74			
Sheets	51	0.166			
Felt					
40 laminations/in	38	0.057			
	150	0.069			
	260	0.083			
20 laminations/in	38	0.078			

(*Continued*)

Table 19.2 *Continued*

Substance	Temperature, °C	k, W/m · °C	ρ, kg/m^3	c_p, kJ/kg · °C	α, m^2/s × 10^7
Structural and heat-resistant materials					
	150	0.095			
	260	0.112			
Corrugated, 4 piles/in	38	0.087			
	93	0.100			
	150	0.119			
Asbestos cement	–	2.08			
Balsam wood, 2.2 lb/ft^3	32	0.04	35		
Cardboard, corrugated	–	0.064			
Celotex	32	0.048			
Corkboard, 10 lb/ft^3	30	0.043	160		
Cork, regranulated	32	0.045	45–120	1.88	2–5.3
Ground	32	0.043	150		
Diatomaceous earth (Sil-o-cel)	0	0.061	320		
Felt, hair	30	0.036	130–200		
Wool	30	0.052	330		
Fiber, insulating board	20	0.048	240		
Glass wool, 1.5 lb/ft^3	23	0.038	24	0.7	22.6
Insulex, dry	32	0.064			
		0.144			
Kapok	30	0.035			
Magnesia, 85%	38	0.067	270		
	93	0.071			
	150	0.074			
	204	0.080			
Rock wool, 10 lb/ft^3	32	0.040	160		
Loosely packed	150	0.067	64		
	260	0.087			
Sawdust	23	0.059			
Silica aerogel	32	0.024	140		
Wood shavings	23	0.059			

There is no single design and selection procedure for refractories. Three general rules can be followed:

1. Design for compressive loading.
2. Allow for thermal expansion.
3. Take advantage of the full range of materials, forms, and shapes.

These apply to whether the design is essentially brickwork and masonry construction or whether the refractory is one that might have been made of speciality metal.

Table 19.3 Physical Properties of Some Common Low-Melting-Point Metals

Metal	Melting point, °C	Normal boiling point, °C	Temperature, °C	Density, $kg/m^3 \times 10^{-3}$	Viscosity, $kg/m \cdot s \times 10^3$	Heat capacity $kJ/kg \cdot K$	Thermal conductivity, $W/m \cdot K$	Prandtl number
Bismuth	271	1477	316	10.01	1.62	0.144	16.4	0.014
			760	9.47	0.79	0.165	15.6	0.0084
Lead	327	1737	371	10.5	2.40	0.159	16.1	0.024
			704	10.1	1.37	0.155	14.9	0.016
Lithium	179	1317	204	0.51	0.60	4.19	38.1	0.065
			982	0.44	0.42	4.19		
Mercury	−39	357	10	13.6	1.59	0.138	8.1	0.027
			316	12.8	0.86	0.134	14.0	0.0084
Potassium	63.8	760	149	0.81	0.37	0.793	45.0	0.0066
			704	0.67	0.14	0.754	33.1	0.0031
Sodium	97.8	883	93	0.93	0.67	1.38	86.2	0.011
			204	0.90	0.43	1.34	80.3	0.0072
			704	0.78	0.18	1.26	59.7	0.0038
Sodium potassium								
22% Na	19	826	93.3	0.848	0.49	0.946	24.4	0.019
			760	0.69	0.146	0.883		
56% Na	−11	784	93.3	0.89	0.58	1.13	25.6	0.026
			760	0.74	0.16	1.04	28.9	0.058
Lead bismuth								
44.5% Pb	125	1670	288	10.3	1.76	0.147	10.7	0.025
			649	98.4	1.15			

Example: For lead at 371°C, density = 10,500 kg/m^3, viscosity = 0.0024 $kg/m \cdot s$, heat capacity = 159 $J/kg \cdot K$.

ILLUSTRATIVE EXAMPLE 19.17

Provide melting point temperature ranges for various refractories.

***SOLUTION*:**

Refractory metals: 3500–6000°F
Refractory oxides: 4500–6000°F
Refractory ceramics: 4000–7000°F
Refractory cermets (ceramic–metals): 3500–5500°F ■

ILLUSTRATIVE EXAMPLE 19.18

A flat incinerator wall with a surface area of 480 ft^2 consists of 6 inches of firebrick with a thermal conductivity of 0.61 Btu/h · ft · °F and an 8-inch outer layer of rock wool insulation with a thermal conductivity of 0.023 Btu/h · ft · °F. If the temperature of the insulation of the inside face of the firebrick and the outside surface of the rock wool insulation are 1900 and 140°F, respectively, calculate the following:

1. The heat loss through the wall in Btu/h.
2. The temperature of the interface between the firebrick and the rock wool.

***SOLUTION*:** As noted earlier

$$\dot{Q} = \frac{\Delta T}{\sum R} \tag{19.4}$$

The individual resistances are:

$$\begin{aligned} R_{\text{firebrick}} &= \frac{L_{\text{f}}}{k_{\text{f}}A} \\ &= \frac{0.5}{(0.61)(480)} \\ &= 0.0017\,\text{h}\cdot{}^\circ\text{F/Btu} \\ R_{\text{rock wool}} &= \frac{L_{\text{rw}}}{k_{\text{rw}}A} \\ &= \frac{0.67}{(0.023)(480)} \\ &= 0.0604\,\text{h}\cdot{}^\circ\text{F/Btu} \end{aligned}$$

Thus,

$$\sum R = 0.0017 + 0.0604 = 0.0621\,\text{h}\cdot{}^\circ\text{F/Btu}$$

The heat loss through the wall is then

$$\dot{Q} = \frac{1900 - 140}{0.0621}$$
$$= 28{,}341 \text{ Btu/h}$$ ■

ILLUSTRATIVE EXAMPLE 19.19

Heat is flowing from steam on one side of a 0.375-inch thick vertical steel sheet to air on the other side. The steam heat-transfer coefficient is 1700 $\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$ and that of the air is 2.0 $\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$. The total temperature difference is 120°F. How would the rate of heat transfer be affected if:

1. the wall was copper rather than steel,
2. by increasing the steam coefficient to 2500, and
3. by increasing the air coefficient to 12.0?

Note that the thermal conductivities, k, for steel and copper are 26 and 218 $\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$, respectively.

***SOLUTION*:** The describing equation is once again

$$\dot{Q} = \frac{\sum \Delta T}{\sum R} \tag{19.4}$$

For the existing application, assume a basis of 1.0 ft^2. Therefore,

$$R_{\text{steam}} = \frac{1}{hA} = \frac{1}{(1700)(1)} = \frac{1}{1700}$$
$$R_{\text{air}} = \frac{1}{hA} = \frac{1}{(2)(1)} = \frac{1}{2}$$
$$R_{\text{steel}} = \frac{\Delta k}{kA} = \frac{0.375/12}{(26)(1)}; \quad k_{\text{steel}} = 26 \text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$$
$$\therefore \sum R = \frac{1}{1700} + \frac{1}{2} + \frac{0.375/12}{(26)}$$
$$= 0.502 \text{ h}\cdot{}^\circ\text{F/Btu}$$

1. If copper ($k = 218$ $\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$) is employed,

$$\sum R = \frac{1}{1700} + \frac{1}{2} + \frac{0.375/12}{(218)}$$
$$= 0.50 \text{ h}\cdot{}^\circ\text{F/Btu}$$

Thus, the rate of heat transfer is essentially unaffected.

2. If h_{steam} is 2500 Btu/h · ft² · °F,

$$\sum R = \frac{1}{2500} + \frac{1}{2} + \frac{0.375/12}{(26)}$$
$$= 0.50 \text{ h} \cdot {}^\circ\text{F/Btu}$$

   The rate is again unaffected.

3. However, if h_{air} is 12 Btu/h · ft² · °F,

$$\sum R = \frac{1}{1700} + \frac{1}{12} + \frac{0.375/12}{(26)}$$
$$= 0.0852 \text{ h} \cdot {}^\circ\text{F/Btu}$$

   The rate is affected for this case. Thus, it can be concluded that the air is the *controlling* resistance. ■

ILLUSTRATIVE EXAMPLE 19.20[3]

An incinerator is 30 ft long, has a 12-ft ID and is constructed of $\frac{3}{4}$-inch carbon steel. The inside of the steel shell is protected by 10 in. of firebrick ($k = 0.608$ Btu/h · ft · °F) and 5 inches of Sil-o-cel insulation ($k = 0.035$ Btu/h · ft · °F) covers the outside. The ambient air temperature is 85°F and the average inside temperature is 1800°F. The present heat loss through the furnace wall is 6% of the heat generated by combustion of a fuel. Calculate the thickness of Sil-o-cel insulation that must be added to cut the losses to 3%.

***SOLUTION*:** A diagram of this system is presented in Figure 19.5. Although the resistance of the steel can be neglected, the other two need to be considered. For cylindrical systems, the effect of the radius of curvature must once again be included in the resistance equations. These take the form presented below:

$$R_{\text{firebrick}} = \frac{\ln(r_{\text{fo}}/r_{\text{fi}})}{2\pi L k_{\text{f}}}$$
$$= \frac{\ln(6.000/5.167)}{2\pi(30)(0.608)}$$
$$= 1.304 \times 10^{-3} \text{h} \cdot {}^\circ\text{F/Btu}$$
$$R_{\text{Sil-o-cel}} = \frac{\ln(r_{\text{so}}/r_{\text{si}})}{2\pi L k_{\text{s}}}$$
$$= \frac{\ln(6.479/6.063)}{2\pi(30)(0.035)}$$
$$= 10.059 \times 10^{-3} \text{ h} \cdot {}^\circ\text{F/Btu}$$

Thus,

$$\sum R = 11.363 \times 10^{-3} \text{ h} \cdot {}^\circ\text{F/Btu}$$

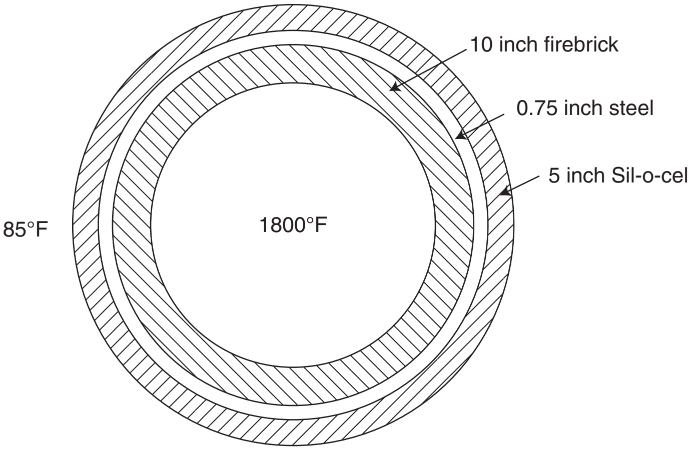

Figure 19.5 Diagram for Illustrative Example 19.20.

To cut the heat loss in half, R must be doubled. The additional Sil-o-cel resistance is therefore 11.363×10^{-3} h · °F/Btu. The new outside radius, r_o, is calculated from:

$$R_{\text{addedSil-o-cel}} = 11.363 \times 10^{-3} = \frac{\ln(r_o/6.479)}{2\pi(30)(0.035)}$$

$$r_o = 6.983\ \text{ft}$$

The extra thickness is $6.983 - 6.479 = 0.504$ ft $= 6.05$ in ■

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Chapter 20

Operation, Maintenance, and Inspection (OM&I)

INTRODUCTION

This chapter is primarily concerned with operation, maintenance, and inspection issues as they apply to heat exchangers. These issues obviously vary with the type of heat exchanger under consideration. For the purposes of this chapter, the presentation will primarily address condensers since most of the exchangers reviewed up to this point in the text can be employed for condensation operations. In effect, this material can be applied virtually to all heat exchangers which for decades have been used in process operations—shell and tube, double pipe, air-cooled, flat plate, spiral plate, barometric jet, spray, etc.

Chapter contents include:

Installation Procedures
Operation
Maintenance and Inspection
Testing
Improving Operation and Performance

Note that the bulk of the material for this chapter has been drawn from the literature.(1,2)

INSTALLATION PROCEDURES

The preparation of a condenser or heat exchanger for installation begins upon receipt of the unit from the manufacturer. Condensers are shipped domestically using skids for complete units, and boxes or crates for bare tube bundles. Units are normally removed from trucks using a crane or forklift. Lifting devices should be attached to lugs provided for that purpose (i.e., for lifting of the complete unit as opposed to individual

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parts), or used with slings wrapped around the main shell. Shell supports are acceptable lugs for lifting, provided that the complete set of supports are used together; nozzles should not be used for attachment of lifting cables. Upon receipt of the unit, the general condition should be noted to determine any damage sustained during transit. Any dents or cracks should be reported to the manufacturer prior to attempting to install the unit. Flanged connections are usually blanked with plywood, masonite, or equivalent covers, and threaded connections are blanked with suitable pipe plugs. These closures are to avoid entry of debris into the unit during shipping and handling, and should remain in place until actual piping connections are made.

Clearance Provisions

Sufficient clearance is required for at least inspection of the unit or in-place maintenance. Inspection of heat exchangers requires minimal clearances for the following: access to inspection parts if provided, removal of channel or bonnet covers, and inspection of tube sheets and tube-to-tube sheet joints. If the removal of tubes or tube bundles in place is anticipated, provision should be provided in the equipment layout. Actual clearance requirements can be determined from the condenser setting plan.

Foundations

Heat exchangers must be supported on structures of sufficient rigidity to avoid imposing excessive strains due to settling. Horizontal units with saddle-type shell supports are normally supplied with slotted holes in one support to allow for expansion. Foundation bolts in these supports should be loose enough to allow movement.

Leveling

Heat exchangers should be carefully leveled and squared to ensure proper drainage, venting, and alignment with piping. On occasions, these units are purposely angled to facilitate venting and drainage, and alignment with piping becomes the prime concern.

Piping Considerations

The following guidelines for piping are necessary to avoid excessive strains, mechanical vibration, and access for regular inspection.

1. Sufficient support devices are required to prevent the weight of piping and fittings from being imposed on the unit.
2. Piping should have sufficient expansion joints or bends to minimize expansion stresses.

3. Forcing the alignment of piping should be avoided so that residual strains will not be imposed on any nozzles, if applicable.
4. If external forces and moments are unavoidable, their magnitude should be determined and made known to the manufacturer so that a necessary stress analysis can be performed.
5. Surge drums or sufficient length of piping to the condenser should be provided to minimize pulsations and mechanical vibrations.
6. Valves and bypasses should be provided to permit inspection or maintenance in order to isolate the condenser during periods other than complete system shutdown.
7. Plugged drains and vents are normally provided at low and high points of shell-tube sides not otherwise drained or vented. These connections are functional during startup, operation, and shutdown, and should be piped up for either continuous or periodic use and never left plugged.
8. Instrument connections should be provided either on condenser nozzles or in the piping close to the condenser. Pressure and temperature indicators should be installed to validate the initial performance of the unit as well as to demonstrate the need for inspection or maintenance.

OPERATION

The maximum allowable working pressures and temperatures are normally indicated on the heat exchanger's nameplate. These values must not be exceeded. Special precautions should be taken if any individual part of the unit is designed for a maximum temperature lower than the unit as a whole. The most common example is some copper-alloy tubing with a maximum allowable temperature lower than the actual inlet gas temperature. This is done to compensate for the low strength levels of some brasses or other copper alloys at elevated temperatures. In addition, an adequate flow of the cooling medium must be maintained at all times.

Condensers are designed for a particular fluid throughput. Generally, a reasonable overload can be tolerated without causing damage. If operated at excessive flow rates, erosion or destructive vibration could result. Erosion could occur at normally acceptable flow rates if other conditions, such as entrained liquids or particulates in a gas stream or abrasive solids in a liquid stream, are present. Evidence of erosion should be investigated to determine the cause. Vibration can be propagated by other than flow overloads (e.g., improper design, fluid maldistribution, or corrosion/erosion of internal flow-directing devices such as baffles). Considerable study and research have been conducted in recent years to develop a reliable vibration analysis procedure to predict or correct damaging vibration. At this point in time, the developed correlations are considered "state of the art," yet most manufacturers have the capability of applying some type of vibration check when designing an exchanger. Vibrations can produce severe mechanical damage, and operation should not be continued when an audible vibration disturbance is evident.

Startup

Exchangers should be warmed up slowly and uniformly; the higher the temperature ranges, the slower the warm-up should be. This is generally accomplished by introducing the coolant and bringing the flow rate to the design level and gradually adding the vapor. For fixed-tube-sheet units with different shell-and-tube material, consideration should be given to differential expansion of shell and tubes. As fluids are added, the respective areas should be vented to ensure complete distribution. A procedure other than this could cause large differences in temperature between adjacent parts of the unit and result in leaks or other damage. It is recommended that gasketed joints be inspected after continuous full-flow operation has been established. Handling, temperature fluctuations, and yielding of gaskets or bolting may necessitate retightening of the bolting.

Shut Down

Cooling down is generally accomplished by shutting off the vapor stream first and then the cooling stream. Again, fixed-tube-sheet units require consideration of differential expansion of the shell and tubes. Condensers containing flammable, corrosive, or high-freezing-point fluids should be thoroughly drained for prolonged outages.

MAINTENANCE AND INSPECTION

Recommended maintenance of exchangers requires regular inspection to ensure the mechanical soundness of the unit and a level of performance consistent with the original design criteria. A brief general inspection should be performed on a regular basis while the unit is operating. Vibratory disturbance, leaking gasketed joints, excessive pressure drop, decreased efficiency indicated by higher gas outlet temperatures or lower condensate rates, and intermixing of fluids are all signs that thorough inspection and maintenance procedure are required.

Complete inspection requires a shutdown of the condenser for access to internals and pressure testing and cleaning. Scheduling can only be determined from experience and general inspections. Tube internals and exteriors, where accessible, should be visually inspected for fouling, corrosion, or damage. The nature of any metal deterioration should be investigated to properly determine the anticipated life of the equipment or possible corrective action. Possible causes of deterioration include general corrosion, intergranular corrosion, stress cracking, galvanic corrosion, impingement, or erosion attack.

Cleaning

Fouling of exchangers occurs because of the deposition of foreign material on the interior or exterior of tubes. Evidence of fouling during operation is increased pressure drop and a general decrease in performance. Fouling can be so severe that tubes are completely plugged, resulting in thermal stresses and the subsequent mechanical damage of equipment.

The nature of the deposited fouling determines the method of cleaning. Soft deposits can be removed by steam, hot water, various chemical solvents, or brushing. Cooling water is sometimes treated with four parts of chlorine per million to prevent algae growth and the consequent reduction in the overall heat transfer coefficient of the exchanger. Plant experience usually determines the method to be used. Chemical cleaning should be performed by contractors specialized in the field who will consider the deposit to be removed and the materials of construction. If the cleaning method involves elevated temperatures, consideration should be given to thermal stresses induced in the tubes; steaming-out individual tubes can loosen the tube-to-tube sheet joints.

Mechanical methods of cleaning are useful for both soft and hard deposits. There are numerous tools for cleaning tube interiors: brushes, scrapers, and various rotating cutter-type devices. The exchanger manufacturer or suppliers of tube tools can be consulted in the selection of the correct tool for the particular deposit. When cutting or scraping deposits, care should be exercised to avoid damaging tubes.

Cleaning of tube exteriors is generally performed using chemicals, steam, or other suitable fluids. Mechanical cleaning is performed but requires that the tubes be exposed, as in a typical air-cooled condenser, or capable of being exposed, as in a removable bundle shell-and-tube condenser. The layout pattern of the tubes must provide sufficient intersecting empty lanes between the tubes, as in a square pitch. Mechanical cleaning of tube bundles, if necessary, requires the utmost care to avoid damaging tubes or fins.

TESTING

Proper maintenance requires testing of a condenser to check the integrity of the following: tubes, tube-to-tube sheet joints, welds, and gasketed joints. The normal procedure consists of pressuring the shell with water or air at the nameplate-specified test pressure and viewing the shell welds and the face of the tube sheet for leaks in the tube sheet joints or tubes. Water should be at ambient temperature to avoid false indications due to condensation. Pneumatic testing requires extra care because of the destructive nature of a rupture or explosion, or fire hazards when residual flammable materials are present. Condensers of the straight-tube floating-head construction require a test gland to perform the test. Tube bundles without shells are tested by pressuring the tubes and viewing the length of the tubes and back face of the tube sheets.

Corrective action for leaking tube-to-tube sheet joints requires expanding the tube end with a suitable roller-type tube expander. Good practice calls for an approximate 8% reduction in wall thickness after metal-to-metal contact between the tube and tube hole. Tube expanding should not extend beyond $\frac{1}{8}$ inch of the inner tube-sheet face to avoid cutting the tube. Care should be exercised to avoid over-rolling the tube, which can cause work-hardening of the material, an insecure seal, and/or stress-corrosion cracking of the tube.

Defective tubes can either be replaced or plugged. Replacing tubes requires special tools and equipment. The user should contact the manufacturer or a qualified repair contractor. Plugging of tubes, although a temporary solution, is acceptable provided that the percentage of the total number of tubes per tube pass to be plugged is not excessive. The type of plug to be used is a tapered one-piece or two-piece metal plug suitable for the tube material and inside diameter. Care should be exercised in seating plugs to avoid damaging the tube sheets. If a significant number of tube or tube joint failures are clustered in a given area of the tube layout, their location should be noted and reported to the manufacturer. A concentration of failures is usually caused by something other than corrosion (e.g., impingement, erosion, or vibration).

IMPROVING OPERATION AND PERFORMANCE

Within the constraints of the existing system, improving operation and performance refers to maintaining operation and original or consistent performance. There are several factors previously mentioned which are critical to the design and performance of a condenser: operating pressure, amount of noncondensable gases in the vapor stream, coolant temperature and flow rate, fouling resistance, and mechanical soundness. Any pressure drop in the vapor line upstream of the condenser should ordinarily be minimized. Deaerators or similar devices should be operational where necessary to remove gases in solution with liquids. Proper and regular venting of equipment and leakproof gasketed joints in vacuum systems are all necessary to prevent gas binding and alteration of the condensing equilibrium. Coolant flow rate and temperatures should be checked regularly to ensure that they are in accordance with the original design criteria. The importance of this can be illustrated merely by comparing the winter and summer performance of a condenser using cooling-tower or river water. Decreased performance due to fouling will generally be exhibited by a gradual decrease in efficiency and should be corrected as soon as possible. Mechanical malfunctions can also be gradual, but will eventually be evidenced by a near total lack of performance.

Fouling and mechanical soundness can only be controlled by regular and complete maintenance. In some cases, fouling is much worse than predicted and requires frequent cleaning regardless of the precautions taken in the original design. These cases require special designs to alleviate the problems associated with fouling. For example, a leading PVC manufacturer found that carryover of polymer reduced the efficiency of its monomer condenser and caused frequent downtime. The solution was providing polished internals and high condensate loading in a vertical down flow shell-and-tube condenser. In an another example, a major pharmaceutical intermediate manufacturer had catalyst carryover to a vertical downflow shell-and-tube condenser which accumulated on the tube internals. The solution was to recirculate condensate to the top of the unit and spray it over the tube-sheet face to create a film descending down the tubes to rinse the tubes clean.

Most condenser manufacturers will provide designs for alternate conditions as a guide to estimating the cost of improving efficiency via other coolant flow rates and temperatures as well as alternate configurations (i.e., vertical, horizontal, shell side, or tube side).

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Chapter 21

Entropy Considerations and Analysis

INTRODUCTION

The law of conservation of energy is defined by many as the first law of thermodynamics. Its application allows calculations of energy relationships associated with various processes. The second law of thermodynamics is referred to as the "limiting law." Historically, the study of the second law was developed by individuals such as Carnot, Clausius and Kelvin in the middle of nineteenth century. This development was made purely on a macroscopic scale and is referred to as the "classical approach" to the second law.

Environmental concerns involving conservation of energy issues gained increasing prominence during and immediately after the OPEC oil embargo of 1973. In addition, global population growth has led to an increasing demand for energy. Although the use of energy has resulted in great benefits, the environmental and human health impact of this energy use has become a concern. One of the keys to reducing and/or eliminating this problem will be achieved through what has come to be referred to as meaningful energy conservation.

The first law of thermodynamics is a conservation law concerned with energy transformations. Regardless of the types of energy involved in processes—thermal, mechanical, electrical, elastic, magnetic, etc.—the change in the energy of a system is equal to the difference between energy input and energy output. The first law also allows free convertibility from one form of energy to another, as long as the overall quantity is conserved. Thus, this law places no restriction on the conversion of work into heat, or on its counterpart—the conversion of heat into work. However, the second law is another matter.

Chapter contents include:

Qualitative Review of the Second Law

Describing Equations

The Heat Exchanger Dilemma

Applications

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The reader should note that flow arrangements for increased/optimum heat recovery receives additional treatment in Chapter 22 as well as in Part Four, particularly Illustrative Examples 27.12 and 28.11.

QUALITATIVE REVIEW OF THE SECOND LAW

The brief discussion of energy conversion above leads to an important second-law consideration—energy has "quality" as well as quantity. Because work is 100% convertible to heat whereas the reverse situation is not true, work is a more valuable form of energy than heat. Although it is not as obvious, it can also be shown through second law arguments that heat also has "quality" in terms of the temperature at which it is discharged from a system. The higher the temperature, the greater the possible energy transformation into work. Thus, thermal energy stored at high temperatures is generally more useful to society than that available at lower temperatures. While there is an immense quantity of energy stored in the oceans, for example, its present availability to society for performing useful tasks is quite low. This implies, as noted above, that thermal energy loses some of its "quality" or is degraded when it is transferred by means of heat transfer from one temperature to a lower one. Other forms of energy degradation include energy transformations due to frictional effects and electrical resistance. Such effects are highly undesirable if the use of energy for practical purposes is to be maximized.(1–3)

The second law provides some means of measuring this energy degradation through a thermodynamic term referred to as entropy and it is the second law (of thermodynamics) that serves to define this important property. It is normally designated as *S* with units of energy per absolute temperature (e.g., Btu/°R or cal/K). Furthermore, entropy calculations can provide quantitative information on the "quality" of energy and energy degradation.(2,3)

In line with the discussion regarding the "quality" of energy, individuals at home and in the workplace are often instructed to "conserve energy." However, this comment, if taken literally, is a misnomer because energy is automatically conserved by the provisions of the first law. In reality, the comment "conserve energy" addresses only the concern associated with the "quality" of energy. If the light in a room is not turned off, energy is degraded although energy is still conserved; i.e., the electrical energy is converted to internal energy (which heats up the room). Note, however, that this energy transformation will produce a token rise in temperature of the room from which little, if any, "quality" energy can be recovered and used again (for lighting or other useful purposes).(1)

There are a number of other phenomena that cannot be explained by the law of conservation of energy. It is the second law of thermodynamics that provides an understanding and analysis of these diverse effects. However, among these considerations, it is the second law that can allow the measuring of the aforementioned "quality" of energy, including its effect on the design and performance of heat exchangers.

DESCRIBING EQUATIONS

Key equations pertinent to entropy calculations and heat exchanger design receive treatment in this section.

If ΔS_{syst} and ΔS_{surr} represent the entropy change of a system and surroundings, respectively, it can be shown$^{(1-3)}$ that for a particular process (and as a consequence of the second law), the total entropy change ΔS_{tot} is given by:

$$\Delta S_{\text{tot}} = \Delta S_{\text{syst}} + \Delta S_{\text{surr}} \geq 0 \tag{21.1}$$

In effect, the second law requires that for any real processes, the total entropy change is positive; the only exception is if the process is reversible (the driving force for heat transfer is at all times zero) and then:

$$(\Delta S_{\text{tot}})_{\text{rev}} = 0 \tag{21.2}$$

Thus, no real process can occur for which the total entropy change is zero or negative. The fundamental facts relative to the entropy concept are that the entropy change of a system may be positive (+), negative (−), or zero; the entropy change of the surroundings during this process may likewise be positive, negative, or zero.

To re-examine the concept of "quality" energy, consider the insulated space pictured in Figure 21.1(A)–(B). Space (A) contains air and steam that are separated; space (B) contains the resulting mixture when both components are mixed. Both spaces are insulated ($Q = 0$) in this closed system with no work term ($W = 0$), so that one can conclude from the first law ($Q + W = \Delta U$) that:

$$\begin{aligned} &\Delta U = 0 \\ &U_A = U_B;\ U \equiv \text{internal energy} \end{aligned} \tag{21.3}$$

Although the energy levels are the same, one notes that system (A) has the capability of doing useful work (because of the high-temperature high-pressure steam) while

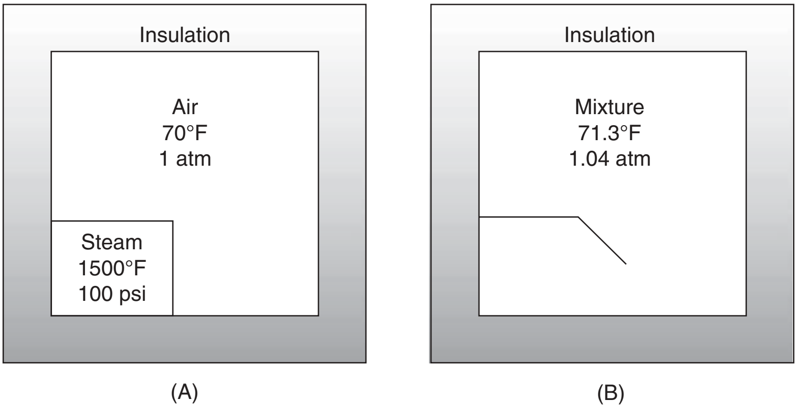

Figure 21.1 Entropy analysis.

system (B) does not. If an entropy analysis is performed (see later illustrative example) on both systems (A) and (B), one would deduce that (as noted earlier):

$$S_A < S_B \tag{21.4}$$

In effect, the entropy level has increased for the system that has lost its ability to do useful work due to the irreversible nature of the mixing process. It is in this manner that the concept of entropy can be used to determine a system's ability to either do useful work or lose its ability to do useful work. Thus, the second law leads to the conclusion that the greater the irreversibility of a process, the greater the (rate of) entropy increase and the greater the amount of energy that becomes unavailable for doing useful work.

Consider now the entropy change of gases. The entropy change of an ideal gas undergoing a change of state from pressure P_1 to P_2 at a constant temperature T is given by:

$$\Delta S_T = R \ln\left(\frac{P_1}{P_2}\right) \tag{21.5}$$

where R is the ideal gas law constant in consistent units. The entropy change of an ideal gas or liquid undergoing a change of state from T_1 to T_2 at a constant pressure is given by:

$$\Delta S_P = C_P \ln\left(\frac{T_2}{T_1}\right) \tag{21.6}$$

where C_P is the heat capacity at constant pressure in consistent units. Correspondingly, the entropy change for an ideal gas undergoing a change from (P_1, T_1) to (P_2, T_2) is:

$$\Delta S = R \ln\left(\frac{P_1}{P_2}\right) + C_P \ln\left(\frac{T_2}{T_1}\right) \tag{21.7}$$

Heat exchanger design equations are again reviewed in light of the development to follow. If $\dot{Q}$ represents the rate of heat transfer between a hot and cold fluid flowing in a heat exchanger, application of the conservation on law for energy gives

$$\dot{Q}_H = \dot{m}_H c_{p,H}(T_{HI} - T_{HO}) \tag{21.8}$$

and

$$\dot{Q}_C = \dot{m}_C c_{p,C}(T_{CO} - T_{CI}) \tag{21.9}$$

where the subscripts H and C refer to the hot and cold fluids, respectively; I and O refer to the fluid inlet and outlet temperature, respectively; $\dot{m}$ represents the mass flow rate, and c_p is once again the heat capacity at constant pressure (assumed constant). In addition, if there is no heat lost from the exchanger to the surroundings,

$$\dot{Q}_H = \dot{Q}_C \tag{21.10}$$

As noted numerous times earlier in the text, the following important equation relates $\dot{Q}$ to the average temperature difference between the hot and cold fluids:

$$\dot{Q} = UA\Delta T_{lm} \tag{21.11}$$

This is the aforementioned heat exchanger design equation. The terms U, A, and ΔT_{lm} represent the overall heat transfer coefficient (a function of the resistance to heat transfer), the area for heat transfer, and the log mean temperature difference driving force (LMTD), respectively. For some exchangers, the latter term is given by

$$\Delta T_{lm} = \text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \tag{21.12}$$

where ΔT_2 and ΔT_1 represent the temperature difference between the hot and cold fluid at each end of the exchanger, respectively. If $\Delta T_1 = \Delta T_2 = \Delta T$, then $\Delta T_{lm} = \Delta T$. For purposes of the analysis to follow, Equation (21.11) is rearranged in the form

$$\frac{\dot{Q}}{U\Delta T_{lm}} = A \tag{21.13}$$

THE HEAT EXCHANGER DILEMMA$^{(4,5)}$

One of the areas where the aforementioned meaningful energy conservation measures can be realized is in the design and specification of process (operating) conditions for heat exchangers. This can be best accomplished by the inclusion of second law principles in the analysis. The quantity of heat recovered in an exchanger is not alone in influencing size and cost. As the energy temperature difference driving force (LMTD) in the exchanger approaches zero, the "quality" heat recovered increases.

Most heat exchangers are designed with the requirements/specification that the temperature difference between the hot and cold fluid be at all times positive and be at least 20°F. This temperature difference or driving force is referred to by some as the approach temperature. However, and as it will be demonstrated in the Illustrative Examples to follow, the corresponding entropy change is also related to the driving force, with large temperature difference driving forces resulting in large irreversibilites and the associated large entropy changes.

The individual designing a heat exchanger is faced with two choices. He/she may decide to design with a large LMTD that results in both a more compact (smaller area) design (see Equation (21.13)) and a large entropy increase that is accompanied by the loss of "quality" energy. Alternately, a design with a small driving force results in both a larger heat exchanger and a smaller entropy change/larger recovery of "quality" energy.

Regarding the cooling medium for a given heat transfer duty, the design engineer has the option of circulating a large quantity with a small temperature change or a small quantity with a large temperature change. The temperature change (or range) of the coolant affects the LMTD. If a large coolant quantity is used, the LMTD is larger and less heat transfer area A is required as a result of the large LMTD. Although this will reduce the original investment and fixed charges (capital and operating costs are discussed in Chapter 27), the amount of "quality" energy recovered will also be smaller, owing to the greater quantity of coolant employed. It is therefore apparent that an optimum must exist between the two choices: too much coolant, smaller surface, and the recovery of less "quality" energy *or* too little coolant, larger surface, and the recovery of more quality energy. In the limit, as the LMTD $\rightarrow$ 0 the area requirement $A \rightarrow \infty$, the entropy change $\Delta S \rightarrow 0$ and the aforementioned recovered "quality" energy increases. Clearly, cost must be minimized, but just as clearly, the "quality" energy recovered must be included in the analysis. This dilemma is addressed below.

Consider first the modes of operation for the three heat exchangers shown in Figure 21.2. Note that for the purpose of analysis, $m_C = m_H = 1.0$ lb and $c_P = 1.0$ Btu/lb · °F.

For operation (A):

$$\begin{aligned}\Delta T_{lm,A} &= 540 - 300 = 300 - 60\\ &= 240^\circ\mathrm{F}\end{aligned}$$

The entropy change for the hot fluid is

$$\begin{aligned}\Delta S_H &= m_H c_p \ln\frac{T_2}{T_1}\\ &= (1)(1)\ln\left(\frac{300+460}{540+460}\right)\\ &= \ln\left(\frac{760}{1000}\right) = -0.2744\ \mathrm{Btu/^\circ R}\end{aligned}$$

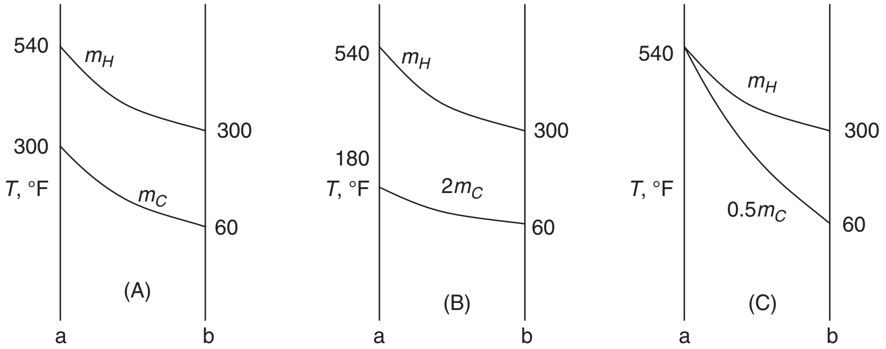

Figure 21.2 Heat exchanger operation.

and

$$\begin{aligned}\Delta S_C &= m_C c_p \ln\left(\frac{300+460}{540+460}\right)\\ &= \ln\left(\frac{760}{520}\right)\\ &= 0.3795\,\text{Btu}/^\circ\text{R}\end{aligned}$$

The total entropy increase is therefore

$$\begin{aligned}\Delta S_{T,A} &= -0.2744 + 0.3795\\ &= 0.1054\,\text{Btu}/^\circ\text{R}\end{aligned}$$

For operation (B):

$$\begin{aligned}\Delta T_{lm,B} &= (360 - 240)/\ln(360/240)\\ &= 296^\circ\text{F}\end{aligned}$$

The entropy change can be calculated in a manner similar to that of operation (A):

$$\Delta S_H = -0.2744\,\text{Btu}/^\circ\text{R}$$

and,

$$\begin{aligned}\Delta S_C &= (2)(1)\ln\left(\frac{180+460}{60+460}\right)\\ &= 2\ln\left(\frac{640}{520}\right)\\ &= 0.4153\,\text{Btu}/^\circ\text{R}\end{aligned}$$

The total entropy increase for operation (B) is therefore

$$\begin{aligned}\Delta S_{T,B} &= -0.2744 + 0.4153\\ &= 0.1409\,\text{Btu}/^\circ\text{R}\end{aligned}$$

For operation (C):

$$\begin{aligned}\Delta T_{lm} &= 240/\infty\\ &= 0^\circ\text{F}\end{aligned}$$

The entropy change for the hot fluid is again

$$\Delta S_H = -0.2744\,\text{Btu}/^\circ\text{R}$$

while

$$\begin{aligned}\Delta S_C &= (0.5)(1)\ln\left(\frac{540+460}{60+460}\right)\\ &= (0.5)\ln\left(\frac{1000}{520}\right)\\ &= 0.3270\,\text{Btu}/^\circ\text{R}\end{aligned}$$

The total entropy change for (C) is therefore

$$\begin{aligned}\Delta S_{T,C} &= -0.2744 + 0.3270\\ &= 0.0526\,\text{Btu}/^\circ\text{R}\end{aligned}$$

A summary of the results for operation A, B, and C plus the heat exchanger area requirement (A) and quality energy (QE) analysis is provided in Table 21.1.

One concludes that as the ΔT_{lm} (or LMTD) increases, the area requirement decreases (see Equation (21.13)); however, the QE available correspondingly decreases. Alternatively, if ΔT_{lm} decreases, both A and QE increase.

Consider now the operation of heat exchangers A and B, as provided in Figure 21.3. For Case I, one notes, using the same analysis as above, that

$$\begin{aligned}\text{LMTD}_A &= \text{LMTD}_B\\ A_A &= A_B;\, A_A = A_B = A\\ \Delta S_A &= \Delta S_B\end{aligned}$$

End result: Two m_C streams (m_{CA} and m_{CB}) are heated to 300°F where $m_{CA} = m_{CB} = m_C$. Two m_H streams are cooled to 300°F where $m_{HA} = m_{HB} = m_H$.

Consider Case II described by Figure 21.4. Here the coolant discharge from exchanger C serves as the inlet coolant to exchanger D. For Case II, one notes that

$$\begin{aligned}\text{LMTD}_C &> \text{LMTD}_D; \quad \text{LMTD}_D = 0\\ A_C &< A_D; \quad A_D = \infty\\ \Delta S_C &> \Delta S_D; \quad \Delta S_D = 0\end{aligned}$$

Table 21.1 Heat Exchanger-Entropy Analysis Results

	ΔT_{lm},°F	ΔS_T, Btu/°R	A	QE
Operation A	0.1054	240	Moderate	Moderate
Operation B	0.1409	296	Lower	Lower
Operation C	0.0526	0	∞	Higher

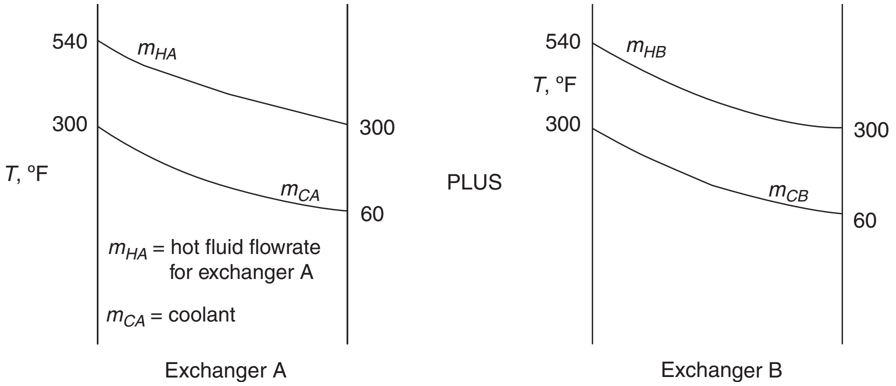

Figure 21.3 Heat exchanger comparison; Case I.

Also,

$$\mathrm{LMTD}_A = \mathrm{LMTD}_\mathrm{B} = \mathrm{LMTD}_C$$
$$A_A = A_B = A_C$$

End result: One m_C (m_{CC}) results at 540°F.

Case I/Case II comparison:

One m_C stream at 540°F (II) vs. two m_C at 300°F (I)

$$A = \infty(\mathrm{II}) \text{ vs. } 2A(\mathrm{I})$$
$$\Delta S(\mathrm{II}) < \Delta S(\mathrm{I})$$

Thus, for Case II, stream m_{CD} can heat, for example, another fluid to 520°F while stream m_{CA} and/or m_{CB} for Case I cannot.

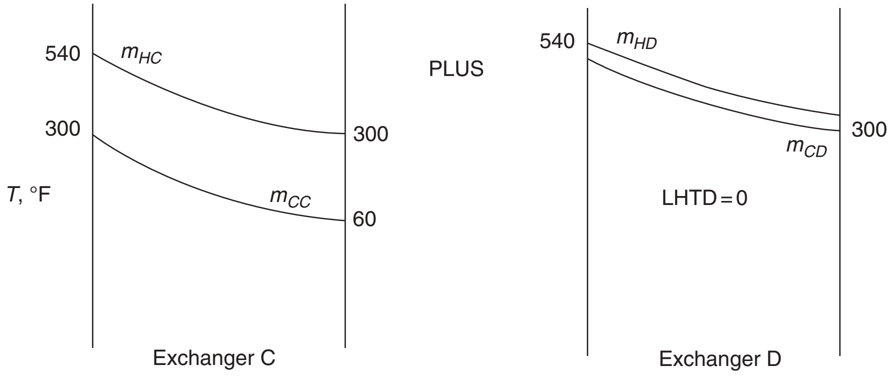

Figure 21.4 Heat exchanger comparison; Case II.

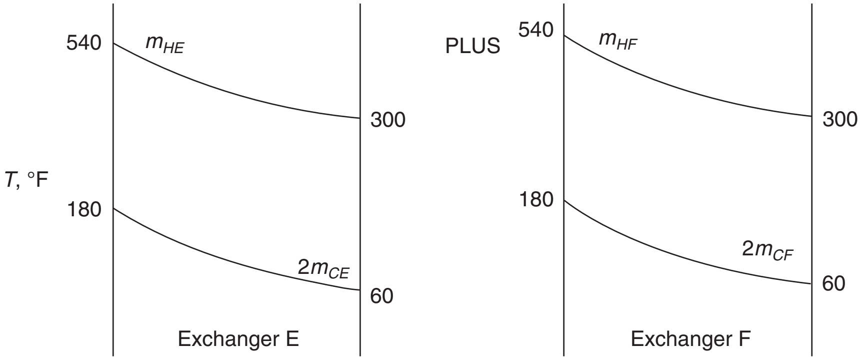

Figure 21.5 Heat exchanger comparison; Case III.

Consider Case III (see Figure 21.5). Here twice the coolant is employed in each exchanger. For Case III, one notes that:

LMTD has increased.

A has decreased.

ΔS has increased.

Also,

$$\begin{aligned} \text{LMTD}_F = \text{LMTD}_E &> \text{LMTD}_C = \text{LMTD}_A = \text{LMTD}_B \\ A_F = A_E &< A_A = A_B = A_C \\ \Delta S_F = \Delta S_E &> \Delta S_A = \Delta S_B = \Delta S_C \end{aligned}$$

End result: Four m_C at 180°F, *A* has decreased and ΔS has increased.

APPLICATIONS

The above analysis is extended to the four examples that follow.

ILLUSTRATIVE EXAMPLE 21.1

Refer to Case I, Figure 21.3, above. Calculate the entropy change of the two exchangers. For the purposes of analysis, arbitrarily assume $c_p = 1.0\ \text{Btu/lb}\cdot{}^\circ\text{F}$ and $m_H = m_C = 1.0$ lb.

***SOLUTION*:** For the hot fluid,

$$\begin{aligned} \Delta S_H &= mc_p \ln\left(\frac{300+460}{540+460}\right) \\ &= (1)(1)(-0.2744) \\ &= -0.2744\ \text{Btu/}^\circ\text{F} \end{aligned}$$

For the cold fluid,

$$\Delta S_C = mc_p \ln\left(\frac{300+460}{60+460}\right)$$
$$= (1)(1)(0.3795)$$
$$= 0.3795\ \text{Btu}/^\circ\text{F}$$

Therefore, for one exchanger, say A:

$$\Delta S_A = \Delta S_H + \Delta S_C$$
$$= -0.2744 + 0.3795$$
$$= 0.1051\ \text{Btu}/^\circ\text{F}$$

Since there are two similar exchangers

$$\Delta S_A = \Delta S_B$$

and

$$\Delta S_{\text{tot},I} = (2)(0.1051)$$
$$= 0.2102\ \text{Btu}/^\circ\text{F}$$

As expected, there is a positive entropy change. ■

ILLUSTRATIVE EXAMPLE 21.2

Refer to Case II, Figure 21.4. Calculate the entropy change of exchangers C and D.

***SOLUTION*:** Consider exchanger C first.

$$\Delta S_C = \Delta S_{HC} + \Delta S_{CC}$$

This is given by the result in the previous example, that is,

$$\Delta S_C = 0.1051\ \text{Btu}/^\circ\text{F}$$

Consider exchanger D. Since the temperature difference driving force is zero, the operation is reversible. Therefore,

$$\Delta S_D = 0$$

The total entropy change is then

$$\Delta S_{\text{tot},II} = \Delta S_C + \Delta S_D$$
$$= 0.1051 + 0.0$$
$$= 0.1051\ \text{Btu}/^\circ\text{F}$$

■

ILLUSTRATIVE EXAMPLE 21.3

Refer to Case III, Figure 21.5. Calculate the entropy change in exchangers E and F.

***SOLUTION*:** Consider exchanger E,

$$\begin{aligned}\Delta S_E &= \Delta S_{HE} + \Delta S_{CE}\\ &= -0.2744 + (2)(1)\ln\left(\frac{180+460}{60+460}\right)\\ &= -0.2744 + 0.4153\\ &= 0.1409\ \text{Btu}/^\circ\text{F}\end{aligned}$$

Since there are two similar exchangers,

$$\Delta S_E = \Delta S_F$$

and

$$\begin{aligned}\Delta S_{\text{tot},III} &= \Delta S_E + \Delta S_F\\ &= (2)(0.1409)\\ &= 0.2818\ \text{Btu}/^\circ\text{F}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 21.4

Comment on the results of Illustrative Examples 21.1, 21.2, and 21.3.

***SOLUTION*:** The calculated results of the three cases again paint a clear picture. As the LMTD decreases, the area cost requirement (per ft^2 of heat exchanger area) increases. In addition, the entropy change decreases, and the "quality" energy increases. Thus, from a "conservation of energy" perspective, second law considerations mandate its inclusion in heat exchanger applications with appropriate economic considerations; i.e., both the cost of the exchanger and the economic factors associated with the "quality" of the recovered energy must be considered in the analysis. Note that pressure drop, materials of construction, etc., are not included in this analysis.

The inclusion of economics in the analysis presented in Illustrative Examples 21.1–21.4 is treated in Chapter 27. The interested reader is encouraged to review Illustrative Example 27.12. ■

ILLUSTRATIVE EXAMPLE 21.5

Briefly discuss the use of interconnected networks of heat exchangers.

***SOLUTION*:** Highly interconnected networks of exchangers can save a great deal of energy in a chemical plant. The more interconnected they are, however, the harder the plant is to operate, control, start-up, and shut-down. Often, auxiliary heat and cooling sources must be included in the plant design in order to ensure that the plant can operate smoothly. An excellent illustrative example is provided by Theodore et al.(4) in Illustrative Example 28.11 in Chapter 28, Part Four. ■

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Chapter 22

Design Principles and Industrial Applications

INTRODUCTION

Current design practices for some heat exchangers usually fall into the category of state-of-the-art and pure empiricism. Past experience with similar applications is commonly used as the sole basis for the design procedure. The vendor maintains proprietary files on past exchanger installations; these files are periodically revised and expanded as new orders are evaluated. In designing a new exchanger, the files are consulted for similar applications and old designs are heavily relied on.

By contrast, the engineering profession in general, and the chemical engineering profession in particular, has developed well-defined procedures for most "standard" heat exchangers (e.g., double pipe, tube and bundle, boilers, etc.). These techniques, tested and refined for nearly a century, are routinely used by today's engineers.

The purpose of this chapter is to introduce the reader to some heat exchange process design principles and industrial applications. Such an introduction, however sketchy, can provide the reader with a better understanding of the major engineering aspects of a heat exchanger, including some of the operational, economics, controls and instrumentation for safety requirements, and any potential environmental factors associated with the unit. No attempt is made in the sections that follow to provide extensive coverage of this topic; only general procedures and concepts are presented and discussed.

Chapter contents include:

General design procedures
Process schematics
Purchasing a heat exchanger
Applications

The last section contains 24 Illustrative Examples that are concerned with heat exchanger design principles and/or industrial applications.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

GENERAL DESIGN PROCEDURES

There are usually five conceptual steps to be considered with the design of any equipment and they naturally apply to heat exchangers. These are:

1. Identification of the parameters that must be specified.
2. Application of the fundamentals underlying theoretical equations or concepts.
3. Enumeration, explanation, and application of simplifying assumptions.
4. Possible use of correction factors for non-ideal behavior.
5. Identification of other factors that must be considered for adequate equipment specification.

Calculation procedures for most of these have been presented earlier in this Part. Since design calculations are generally based on the maximum throughput capacity for the heat exchanger, these calculations are never completely accurate. It is usually necessary to apply reasonable safety factors when setting the final design. Safety factors vary widely and are a strong function of the accuracy of the data involved, calculational procedures, and past experience. Attempting to justify these is a difficult task.(1,2)

Unlike many of the problems encountered and solved by the engineer, there is no absolutely correct solution to a design problem; however, there is usually a *better* solution. Many alternative exchanger designs when properly implemented will function satisfactorily, but one alternative will usually prove to be economically more efficient and/or attractive than the others.

Overall and componential material balances have already been described in rather extensive detail in Part One. Material balances may be based on mass, moles, or volume, usually on a rate (time rate of change) basis. However, the material balance calculation should be based on mass or volume rates since both play an important role in equipment sizing calculations.

Some design calculations in the chemical process industry today include transient effects that can account for process upsets, startups, shutdowns, and so on. The describing equations for these time-varying (unsteady-state) systems are differential. The equations usually take the form of a first-order derivative with respect to time, where time is the independent variable. However, design calculations for almost all heat exchangers assume steady-state conditions, with the ultimate design based on worst-case or maximum flow conditions. This greatly simplifies these calculations since the describing equations are no longer differential, but rather algebraic.

The safe operation of the exchanger requires that the controls keep the system operating within a safe operating envelope. The envelope is based on many of the design, process, and (if applicable) regulatory constraints. The control system should also be designed to vary one or more of the process variables to maintain the appropriate conditions for the exchanger. These variations are often programmed into the system based on past experience with the specific unit. The operational parameters that may vary include the flow rates, temperatures, and system pressure.

The control system may be subjected to (extensive) analysis on operational problems and items that could go wrong. A hazard and operational (*HAZOP*) analysis (see Chapter 24) can be conducted on the system to examine and identify all possible failure mechanisms. It is important that all of the failure mechanisms have appropriate response reactions by the control system. Several of the failure mechanisms that must be addressed within the appropriate control system response are excess (excursion) or minimal temperature, excessive or subnormal flow rate, equipment failure, component failure, and broken circuits.

In the case of a heat exchanger design, data on similar existing units are normally available and economic estimates and/or process feasibility are determined from these data. It should be pointed out again that most heat exchangers in real practice are designed by duplicating or *mimicking* similar existing systems. Simple algebraic correlations that are based on past experience are the rule rather than the exception. This stark reality is often disappointing and depressing to students and novice engineers involved in design.

PROCESS SCHEMATICS

To the practicing engineer, particularly the process engineer, the process flow sheet is the key instrument for defining, refining, and documenting a process unit. The process flow diagram is the authorized process blueprint and the framework for specifications used in equipment designation and design; it is the single, authoritative document employed to define, construct, and operate the unit and/or process.(3)

Beyond equipment symbols and process stream flow lines, there are several essential constituents contributing to a detailed process flow sheet. These include equipment identification numbers and names, temperature and pressure designations, utility designations, flow rates for each process stream, and a material balance table pertaining to process flow lines. The process flow diagram may also contain additional information such as major instrumentation and physical properties of the process streams. When properly assembled and employed, a process schematic provides a coherent picture of the process. It can point out some deficiencies in the process that may have been overlooked earlier in the study. But basically, the flow sheet symbolically and pictorially represents the interrelation between the various flow streams and the exchanger (or any other equipment), and permits easy calculations of material and energy balances.

There are a number of symbols that are universally employed to represent equipment, equipment parts, valves, piping, and so on. Some of these are depicted in the schematic in Figure 22.1. Although there are a significant number of these symbols, only a few are needed for even the most complex heat exchanger unit. These symbols obviously reduce, and in some instances replace, detailed written descriptions of the unit or process. Note that many of the symbols are pictorial, which helps in better describing process components and information.

The degree of sophistication and details of an exchanger flow sheet usually vary with time. The flow sheet may initially consist of a simple free-hand block diagram with limited information that includes only the equipment; later versions may include

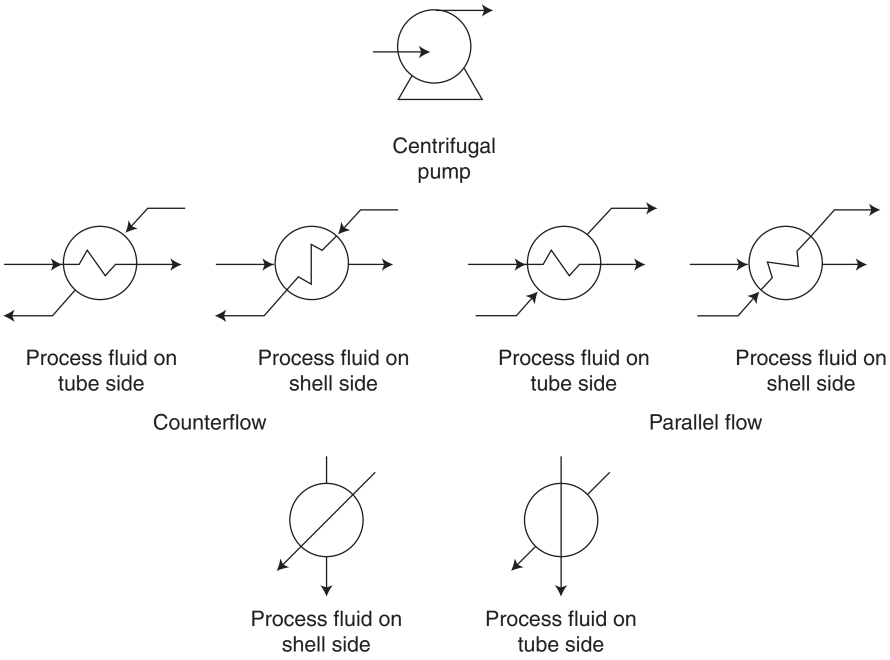

Figure 22.1 Selected flow sheet symbols.

line drawings with pertinent process data such as overall and componential flow rates, temperatures, pressures, and instrumentation. During the later stages, the flow sheet can consist of a highly detailed P&I (piping and instrumentation) diagram; this aspect of the design procedure is beyond the scope of this text; the reader is referred to the literature$^{(4)}$ for information on P&I diagrams.

In summary, industrial plant flow sheets are the international language of the engineer, particularly the chemical engineer. Chemical engineers conceptually view a (chemical) plant as consisting of a series of interrelated building blocks that are defined as *units* or *unit operations* (the heat exchanger is one such unit). The plant essentially ties together the various pieces of equipment that make up the process. Flow schematics follow the successive steps of a process by indicating where the pieces of equipment are located and the material streams entering and leaving each unit.

PURCHASING A HEAT EXCHANGER

Prior to the purchase of a heat exchanger, experience has shown that the following points should be emphasized:

1. Refrain from purchasing any heat exchanger without reviewing *certified independent test data* on its performance under a similar application.

Request the manufacturer to provide performance information and design specifications.

2. In the event that sufficient performance data are unavailable, request that the equipment supplier provide a small pilot model for evaluation under existing conditions.
3. Prepare a good set of specifications. Include a *strong performance guarantee* from the manufacturer to ensure that the heat exchanger will meet all design criteria and specific process conditions.
4. Closely review the overall process, other equipment, and economic fundamentals.
5. Make a careful material balance study.
6. Refrain from purchasing any heat exchanger until *firm* installation cost estimates have been added to the cost. *Escalating installation costs are the rule rather than the exception*.
7. Give operation and maintenance costs high priority on the list of exchanger selection factors.
8. Refrain from purchasing any heat exchanger until a solid commitment from the vendor(s) is obtained. Make every effort to ensure that the exchanger is compatible with the (plant) process.
9. The specification should include written assurance of *prompt* technical assistance from the supplier. This, together with a completely understandable operating manual (with parts list, full schematics, consistent units and notations, etc.), is essential and is too often forgotten in the rush to get the heat exchanger operating.
10. Schedules can be critical. In such cases, delivery guarantees should be obtained from the manufacturers and penalties identified.
11. The heat exchanger should be of fail-safe design with built-in indicators to show when performance is deteriorating.
12. Perhaps most importantly, withhold 10–15% of the purchase price until satisfactory operation is clearly demonstrated.

The usual design, procurement, installation, and/or startup problems can be further compounded by any one or a combination of the following:

1. Unfamiliarity of process engineers with heat exchangers.
2. New suppliers, frequently with unproven heat exchanger equipment.
3. Lack of industry standards with some designs.
4. Compliance schedules that are too tight.
5. Vague specifications.
6. Weak guarantees.
7. Unreliable delivery schedules.
8. Process reliability problems.

Proper selection of a particular heat exchanger for a specific application can be extremely difficult and complicated. The final choice in heat exchanger selection is usually dictated by that unit capable of achieving the aforementioned design criteria and required process conditions at the lowest uniform annual cost (amortized capital investment plus operation and maintenance costs—see also Chapter 27, Part Four).

In order to compare specific exchanger alternatives, knowledge of the particular application and site is also essential. A preliminary screening, however, may be performed by reviewing the advantages and disadvantages of each type or class of unit. However, there are many other situations where knowledge of the capabilities of the various options, combined with common sense, will simplify the selection process.

APPLICATIONS

The 24 Illustrative Examples to follow serve the function of merging many of the design procedures and industrial applications presented earlier in the text (including this chapter) into a more complete package.

ILLUSTRATIVE EXAMPLE 22.1

List four construction considerations/measures that should be followed during the design of a process/plant heat exchanger.

***SOLUTION*:**

1. Keep hot steam lines away from workers.
2. Insulate the entire exchanger so it is not hot to the touch.
3. Make sure all the flows entering the exchanger are turned off when it is being serviced.
4. Without compromising efficiency, allow adequate space inside the exchanger so that it can be easily cleaned. ■

ILLUSTRATIVE EXAMPLE 22.2

Discuss some overall mechanical considerations in heat exchanger design.

***SOLUTION*:** Choosing the most economical exchanger requires consideration of several mechanical specifications, but it is of prime importance to use standard designs. This narrows the choice but has the advantage of simplifying the job. The only exception is usually special designs. ■

ILLUSTRATIVE EXAMPLE 22.3

Discuss the difference between simulation and design.

***SOLUTION*:** Both simulation and design approaches are examined in this text.

Two factors should be considered in simulation (predicting performance):

1. Will the existing exchanger(s) transfer the duty required? This involves the calculation of all temperatures and heat transfer rates.
2. Will the pressure drop be within acceptable limits?

Thus, a simulation (or rating) problem involves analyzing the performance of an existing heat exchanger of known length and area.

In design, the exchanger does not exist. The duty or heat transfer rate is usually specified. The size and configuration of the exchanger must be specified subject to a number of other constraints. Thus, when the problem involves calculating the heat exchanger area needed to accomplish a set temperature change or a specific duty, it is termed a *design* problem. ■

ILLUSTRATIVE EXAMPLE 22.4

Discuss some key design specifications for combustion devices.

***SOLUTION*:** A key design specification for any combustion device, including heat exchangers, boilers, and incinerators, is the operating temperature. Materials of construction must withstand the operating temperature without experiencing any damage for safe and efficient plant operation. In addition, the volumetric flow rate is a strong function of temperature (see Chapter 5) with gaseous applications and plays an important role in properly sizing combustion/reactor equipment. ■

ILLUSTRATIVE EXAMPLE 22.5

List factor(s) that need to be considered when a heat exchanger is selected as an off-the-shelf unit or designed specifically for a particular application.

***SOLUTION*:**

1. Heat transfer rate requirements such as $\dot{Q}$.
2. Temperature limitations (safety and materials of construction).
3. Capital costs.
4. Operation costs.
5. Physical size.
6. Space limitations.
7. Pressure drop constraints (one large tube or many little tubes). ■

ILLUSTRATIVE EXAMPLE 22.6

It is desired to evaporate 1000 lb/h of 60°F water at 1 atm at a power plant. Utility superheated steam at 40 atm and 1000°F is available, but since this steam is to be used elsewhere in the plant, it cannot drop below 20 atm and 600°F. What mass flowrate of the utility steam is required? Assume that there is no heat loss in the evaporator.

From the steam tables:

$$\begin{array}{lll} P = 40\,\text{atm} & T = 1000^\circ\text{F} & h = 1572\,\text{Btu/lb} \\ P = 20\,\text{atm} & T = 600^\circ\text{F} & h = 1316\,\text{Btu/lb} \end{array}$$

For saturated steam:

$$P = 1\,\text{atm} \qquad h = 1151\,\text{Btu/lb}$$

For saturated water:

$$T = 60^\circ\text{F} \qquad h = 28.1\,\text{Btu/lb}$$

***SOLUTION*:** A detailed flow diagram of the process is provided in Figure 22.2. Assuming the process to be steady-state and noting that there is no heat loss or shaft work, the energy balance on a rate basis is

$$\dot{Q} = \Delta\dot{h} = 0$$

This equation indicates that the sum of the enthalpy changes for the two streams must equal zero.

The change in enthalpy for the vaporization of the water stream is

$$\begin{aligned} \Delta\dot{h}_{\text{vaporization}} &= (1000\,\text{lb/h})(1151 - 28.1\,\text{Btu/lb}) \\ &= 1.123 \times 10^6\,\text{Btu/h} \end{aligned}$$

The change of enthalpy for the cooling of the superheated steam may now be determined. Since the mass flowrate of the steam is unknown, its Δh must be expressed in terms of this mass flowrate, which is represented in Figure 22.2 as $\dot{m}$ lb/h.

$$\Delta\dot{h}_{\text{cooling}} = \dot{m}(1572 - 1316) = (256)\dot{m}\ \text{Btu/h}$$

Since the overall $\Delta\dot{H}$ is zero, the enthalpy changes of the two streams must total zero. Thus,

$$\begin{aligned} \Delta\dot{h}_{\text{vaporization}} + \Delta\dot{h}_{\text{cooling}} &= 0 \\ 1.123 \times 10^6 &= (256)\dot{m} \\ \dot{m} &= 4387\,\text{lb/h} \end{aligned}$$

■

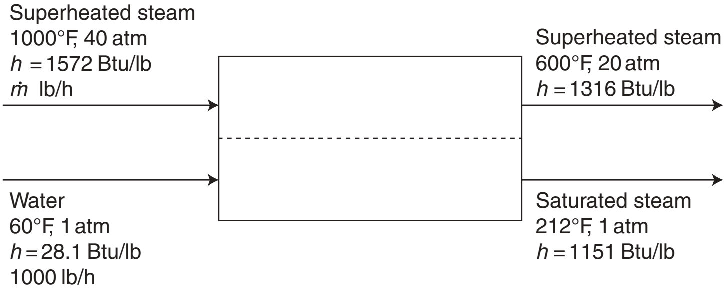

Figure 22.2 Flow diagram for Illustrative Example 22.6.

ILLUSTRATIVE EXAMPLE 22.7

Determine the total flowrate of cooling water required for the services listed below if a cooling tower system supplies the water at 90°F with a return temperature of 115°F. How much fresh water makeup is required if 5% of the return water is sent to “blowdown?” Note that the cooling water heat capacity is 1.00 Btu/(lb · °F), the heat of vaporization at cooling tower operating conditions is 1030 Btu/lb, and the density of water at cooling tower operating conditions is 62.0 lb/ft^3. Process data is provided in Table 22.1.

***SOLUTION*:** The required cooling water flowrate, q_{CW}, is given by the following equation:

$$q_{\text{CW}} = \dot{Q}_{\text{HL}}/[(\Delta T)(c_p)(\rho)]$$

where $\dot{Q}_{\text{HL}}$ = heat load, Btu/min

ΔT = change in temperature = 115°F − 90°F = 25°F

c_p = heat capacity = 1.00 Btu/(lb · °F)

ρ = density of water = (62.0 lb/ft^3)(0.1337 ft^3/gal) = 8.289 lb/gal

The heat load is

$$\begin{aligned}\dot{Q}_{\text{HL}} &= (12 + 6 + 23.5 + 17 + 31.5)(10^6\ \text{Btu/h})/60\ \text{min/h}\\ &= 1{,}500{,}000\ \text{Btu/min}\end{aligned}$$

Thus,

$$q_{\text{CW}} = \frac{1{,}500{,}000\ \text{Btu/min}}{(25^\circ\text{F})(1.00\ \text{Btu/lb}\cdot{}^\circ\text{F})(8.289\ \text{lb/gal})} = 7250\ \text{gpm}$$

The blow-down flow, q_{BD}, is given by the following:

$$q_{\text{BD}} = (\text{BDR})(q_{\text{CW}})$$

where BDR is the blow-down rate = 5% = 0.05. Thus,

$$q_{\text{BD}} = (0.05)(7250\ \text{gpm}) = 362.5\ \text{gpm}$$

■

Table 22.1 Data for Illustrative Example 22.7

Process unit	Heat duty, Btu/h	Required temperature, °F
1	12,000,000	250
2	6,000,000	200–276
3	23,500,000	130–175
4	17,000,000	300
5	31,500,000	150–225

ILLUSTRATIVE EXAMPLE 22.8

Determine how many pounds per hour of steam are required for the following situation if steam is provided at 500 psig, and if steam is provided at both 500 and 75 psig pressures. The plant has the heating requirements given in Table 22.2. Also note the properties of saturated steam in Table 22.3.

***SOLUTION*:** The total required flowrate of 500 psig steam, $\dot{m}_{\mathrm{BT}}$, is given by:

$$\dot{m}_{\mathrm{BT}} = \dot{m}_{\mathrm{B1}} + \dot{m}_{\mathrm{B2}} + \dot{m}_{\mathrm{B3}} + \dot{m}_{\mathrm{B4}}$$

For the above equation:

$$\dot{m}_{\mathrm{B1}} \text{ (mass flowrate of 500 psig steam through unit 1)} = \dot{Q}_1/h_{\mathrm{vap}} = 13{,}320\,\mathrm{lb/h}$$

$$\dot{m}_{\mathrm{B2}} \text{ (mass flowrate of 500 psig steam through unit 2)} = \dot{Q}_2/h_{\mathrm{vap}} = 10{,}655\,\mathrm{lb/h}$$

$$\dot{m}_{\mathrm{B3}} \text{ (mass flowrate of 500 psig steam through unit 3)} = \dot{Q}_3/h_{\mathrm{vap}} = 15{,}980\,\mathrm{lb/h}$$

$$\dot{m}_{\mathrm{B4}} \text{ (mass flowrate of 500 psig steam through unit 4)} = \dot{Q}_4/h_{\mathrm{vap}} = 26{,}635\,\mathrm{lb/h}$$

Thus,

$$\dot{m}_{BT} = \Sigma \dot{m}_{Bi} = 66{,}590\,\mathrm{lb/h}$$

■

ILLUSTRATIVE EXAMPLE 22.9

A feed stream to a distillation column processes of 28,830 gals/day of a light oil, and is fed to the column through a 6-inch I.D. pipeline. The temperature of the feed is usually 27°C. You are asked to explore the possibility of using either a countercurrent or a parallel (co-current) double pipe heat exchanger as an auxiliary heater for the feed stream. The double-pipe exchanger is to heat the oil from 23.5°C to 27°C. Water at 8406 gals/day and 93°C is available for heating the oil. The necessary physical properties are provided in Table 22.4. The inner diameter of the pipe forming the annular region is 20.3 cm and the outer surface of that pipe is well insulated. For a counter-current flow arrangement, calculate the following using the SI system:

1. The lowest temperature the exiting heating water could reach. Is there sufficient energy available from the water to raise the oil temperature? (If there is not enough energy, there is no point in designing the exchanger.)

Table 22.2 Process Data for Illustrative Example 22.8

Process unit	Unit heat duty ($\dot{Q}$), Btu/h	Required temperature, °F
1	10,000,000	250
2	8,000,000	450
3	12,000,000	400
4	20,000,000	300

Table 22.3 Steam Data

Pressure provided, psig	Saturation temperature, °F	Enthalpy of vaporization (h_{vap}), Btu/lb
75	320	894
500	470	751

2. The log mean temperature difference.
3. The overall heat transfer coefficient, U, for a new clean exchanger based on the inside area is $35.4\ \text{W/m}^2 \cdot \text{K}$. Correct the U for the fouling of the heat exchanger. Use a fouling factor, R_f, of quenching oil, which is $0.0007\ \text{m}^2 \cdot \text{K/W}$.
4. The length of the double pipe heat exchanger.
5. The effectiveness of the exchanger and the NTU.

***SOLUTION*:** Convert oil properties to SI units:

$$\rho_{\text{oil}} = 53\ \text{lb/ft}^3 = (53)(16.0185) = 849\ \text{kg/m}^3$$
$$c_{\text{oil}} = c_{p,\text{oil}} = 0.46\ \text{Btu/lb} \cdot {}^\circ\text{F} = (0.46)(4186.7) = 1926\ \text{J/kg} \cdot {}^\circ\text{C}$$
$$\mu_{\text{oil}} = 150\ \text{cP} = 0.15\ \text{kg/m} \cdot \text{s}$$
$$\nu_{\text{oil}} = \mu_{\text{oil}}/\rho_{\text{oil}} = 0.15/849 = 1.767 \times 10^{-4}\ \text{m}^2/\text{s}$$
$$k_{\text{oil}} = 0.11\ \text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F} = (0.11)(1.7303) = 0.19\ \text{W/m} \cdot {}^\circ\text{C}$$
$$q_{\text{oil}} = \text{volumetric flow rate of oil} = (28{,}830)(4.381 \times 10^{-8}) = 0.001263\ \text{m}^3/\text{s}$$
$$\dot{m}_{\text{oil}} = \text{mass flow rate of oil} = \rho_{\text{oil}} q_{1,\text{oil}} = (849)(0.001263) = 1.072\ \text{kg/s}$$

Also convert water properties to SI units:

$$\rho_w = 964\ \text{kg/m}^3$$
$$c_w = c_{p,w} = 4204\ \text{J/kg} \cdot {}^\circ\text{C}$$
$$\begin{aligned}\mu_w &= 0.7\ \text{lb/ft} \cdot \text{h} \\ &= 0.0001944\ \text{lb/ft} \cdot \text{s} \\ &= (0.0001944)(1.4881) \\ &= 2.89 \times 10^{-4}\ \text{kg/m} \cdot \text{s}\end{aligned}$$
$$\nu_w = \mu_w/\rho_w = 3 \times 10^{-7}\ \text{m}^2/\text{s}$$
$$k_w = 0.678\ \text{W/m} \cdot {}^\circ\text{C}$$

Table 22.4 Property Data for Illustrative Example 22.9

Oil (27°C)	Water (93°C)	Pipe
$\rho = 53\ \text{lb/ft}^3$	$\rho = 964\ \text{kg/m}^3$	ID = 6.0 in
$c_p = 0.46\ \text{Btu/lb} \cdot {}^\circ\text{F}$	$c_p = 4204\ \text{J/kg} \cdot {}^\circ\text{C}$	OD = 168 mm
$\mu = 150\ \text{cP}$	$\mu = 0.7\ \text{lb/h} \cdot \text{ft}$	$k = 45\ \text{W/m} \cdot {}^\circ\text{C}$
$k = 0.11\ \text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$	$k = 0.678\ \text{W/m} \cdot {}^\circ\text{C}$	

In addition,

$$\begin{aligned} q_w &= (8406)(4.38 \times 10^{-8}) = 3.683 \times 10^{-4}\,\text{m}^3/\text{s} \\ \dot{m}_w &= \text{mass flow rate of water} \\ &= \rho_w q_w \\ &= (964)(3.683 \times 10^{-4}) \\ &= 0.355\,\text{kg/s} \end{aligned}$$

Organize the above properties in tabular form (see Table 22.5).
Calculate the duty of the exchanger. Use the information on the oil side:

$$\dot{Q} = \dot{m}_{\text{oil}} c_{\text{oil}}(t_2 - t_1) = (2064.7)(27 - 23.5) = 7227.2\,\text{W}$$

1. Determine the lowest possible temperature of the water. This corresponds to the inlet temperature of the oil:

$$T_{2,\text{min}} = t_1 = 23.5^\circ\text{C}$$

2. Check the maximum $\dot{Q}$ to be given off by the hot water:

$$\dot{Q}_{w,\text{max}} = \dot{m}_w c_w(T_1 - T_{2,\text{min}}) = (1492.4)(93 - 23.5) = 1.037 \times 10^5\,\text{W}$$

Since $\dot{Q} = 7227.2\,\text{W}$, there is enough energy in the water to heat the oil.
Calculate the exit temperature, T_2, of the water:

$$\begin{aligned} \dot{Q} &= 7227.2 = (1492.4)(T_1 - T_2) = (1492.4)(93 - T_2) \\ T_2 &= 88.16^\circ\text{C} \end{aligned}$$

Calculate the log mean temperature difference:

$$\begin{aligned} \Delta T_1 \text{ (inlet)} &= 88.16 - 23.5 = 64.66^\circ\text{C} \\ \Delta T_2 \text{ (outlet)} &= 93 - 27 = 66^\circ\text{C} \end{aligned}$$

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \frac{(64.66 - 66)}{\ln\left(\dfrac{64.66}{66}\right)} = 65.33^\circ\text{C}$$

Table 22.5 Key Properties and Data for Illustrative Example 22.9

	Stream			
Parameter	1	2	1	2
Type	Oil (being heated)		Water (being cooled)	
Side	Tube		Shell (annulus)	
ρ, kg/m^3	849		964	
c, J/kg · K	1926		4204	
ν, m^2/s	1.767×10^{-4}		3×10^{-7}	
$\dot{m}$, kg/s	1.072		0.355	
t, T, °C	23.5	27	93	
Capacitance rate, $C = \dot{m}c$, W/K	2064.7		1492.4	

3. Include the effects of fouling in the overall heat transfer coefficient U; use a fouling resistance, $R_f = 0.0007\ \text{m}^2 \cdot \text{K/W}$:

$$\begin{aligned}\frac{1}{U_{\text{dirty}}} &= \frac{1}{U_{\text{clean}}} + R_f \\ &= \frac{1}{35.4} + 0.0007 = 0.02822 + 0.0007 = 0.02892 \\ U_{\text{dirty}} &= 34.6\ \text{W/m}^2 \cdot {}^\circ\text{C (based on the inside area)}\end{aligned}$$

4. Calculate the required heat transfer area and the length of the heat exchanger:

$$\begin{aligned}\dot{Q} &= UA\Delta T_{\text{lm}} \\ 7227.2 &= (34.6)(A)(65.33) \\ A &= 3.197\ \text{m}^2 = \pi D_1 L = \pi(0.1524)L \\ L &= 6.68\ \text{m}\end{aligned}$$

5. Calculate the maximum amount of heat absorbed by the oil. The maximum amount of heat is absorbed when the oil is heated to the maximum possible temperature, which is 93°C:

$$\dot{Q}_{\text{max,oil}} = (2064.7)(93 - 23.5) = 1.435 \times 10^5\ \text{W}$$

Select the $\dot{Q}_{\text{max}}$ for calculating the effectiveness. The values of $\dot{Q}_{\text{max,oil}}$ and $\dot{Q}_{\text{max},w}$ should be compared. The lower value is $\dot{Q}_{\text{max}}$.

$$\dot{Q}_{\text{max}} = \dot{Q}_{\text{max},w} = 1.037 \times 10^5\ \text{W}$$

The effectiveness, ε, is therefore:

$$\begin{aligned}\varepsilon &= \dot{Q}/\dot{Q}_{\text{max}} \\ &= 7227.2/(1.037 \times 10^5) = 0.0696 = 6.96\%\end{aligned}$$

Calculate the number of transfer units, NTU, and the ratio of $(\dot{m}c)_{\text{max}}/(\dot{m}c)_{\text{min}}$:

$$\begin{aligned}\text{NTU} &= U_{\text{dirty}}A/(\dot{m}c)_{\text{min}} \\ &= (34.6)(3.197)/1492.4 \\ &= 0.0741\end{aligned}$$

where the ratio of $(\dot{m}c)_{\text{max}}/(\dot{m}c)_{\text{min}} = \text{C}_{\text{max}}/\text{C}_{\text{min}} = 2064.7/1492.4 = 1.383$. ■

ILLUSTRATIVE EXAMPLE 22.10

Refer to Illustrative Example 22.9. How would the answer to Parts 2 and 4 be affected if the flow were parallel?

***SOLUTION*:** If the flow was cocurrent (parallel) in Illustrative Example 22.9,

$$\Delta T_1 = 93 - 23.5 = 69.5^\circ\text{C}$$
$$\Delta T_2 = 88.16 - 27 = 61.16^\circ\text{C}$$
$$\Delta T_{\text{lm}} = \frac{69.5 - 61.16}{\ln\left(\dfrac{69.5}{61.66}\right)} = 65.24^\circ\text{C}$$

and

$$\dot{Q} = UA\Delta T_{\text{lm}}$$
$$7227.2 = (34.6)A_h(65.24)$$
$$A = 3.2\,\text{m}^2 = \pi(0.1524)L$$
$$L = 6.69\,\text{m}$$

The tube length would increase slightly. ■

ILLUSTRATIVE EXAMPLE 22.11

How would the results of Illustrative Example 22.9 be affected if this flow were co-current?

***SOLUTION*:** Since co-current flow is the equivalent of parallel flow, the answer to the previous example applies. ■

ILLUSTRATIVE EXAMPLE 22.12

A brine solution at 10°F is heated in a food processing plant by flowing through a heated pipe. The pipe surface is maintained at 80°F. The pipe surface area for heat transfer is 2.5 ft^2. The brine solution (with a density of 62.4 lb/ft^3 and a heat capacity of 0.99 $\text{Btu/lb}\cdot{}^\circ\text{F}$) flows at a rate of 20 lb/min. The overall heat transfer coefficient varies linearly with the temperature approach, with values of 150 $\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$ at the brine solution entrance (where the brine temperature is 10°F) and 140 $\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$ at the brine solution exit. Determine:

1. the temperature approach at the brine inlet side
2. the exit temperature of the brine solution.

***SOLUTION*:** Calculate the temperature approach at the pipe entrance:

$$\Delta T_1 = T - t_1 = 80 - 10 = 70^\circ\text{F} = 70/1.8 = 38.9^\circ\text{C}$$

Note that ΔT_2 $(T - t_2)$ cannot be calculated since t_2 is not known. Apply an energy balance to the brine solution across the full length of the pipe:

$$\dot{Q} = \dot{m}c(t_2 - t_1) = \dot{m}c(\Delta T_1 - \Delta T_2)$$
$$= (1200)(0.99)(70 - \Delta T_2) = 1188(70 - \Delta T_2)$$

Express the equation for the LMTD:

$$\Delta T_{\mathrm{lm}} = (70 - \Delta T_2)/\ln(70/\Delta T_2)$$

Also express the equation for the heat transfer rate:

$$\dot{Q} = (A)\frac{U_2(\Delta T_1) - U_1(\Delta T_2)}{\ln\left(\dfrac{U_2\Delta T_1}{U_1\Delta T_2}\right)} \tag{14.38}$$

$$= (2.5)\frac{140(70) - 150(\Delta T_2)}{\ln\left(\dfrac{(140)(70)}{(150)(\Delta T_2)}\right)}$$

Note that U varies linearly with ΔT.

Combine the above two equations and eliminate $\dot{Q}$:

$$1188(70 - \Delta T_2) = (2.5)\frac{(140)(70) - (150)(\Delta T_2)}{\ln\left(\dfrac{(140)(70)}{(150)(\Delta T_2)}\right)}$$

This equation is non-linear with one unknown (ΔT_2). This equation may be solved by trial-and-error. (Note that $0 \leq \Delta T_2 \leq 70°\mathrm{F}$). Solution gives

$$\Delta T_2 \approx 51.6°\mathrm{F} \approx 28.7°\mathrm{C}$$

The temperature of the brine solution may now be calculated:

$$\Delta T_2 = 51.6 = T - t_2 = 80 - t_2$$

$$t_2 = 80 - 51.6 = 28.4°\mathrm{F}$$

$$= (28.4 - 32)/1.8 = -2°\mathrm{C}$$

■

ILLUSTRATIVE EXAMPLE 22.13

Refer to Illustrative Example 22.12. Calculate:

1. the rate of heat transfer, $\dot{Q}$
2. the log mean temperature difference.

***SOLUTION*:** The heat transfer rate, $\dot{Q}$, is

$$\dot{Q} = 1188(70 - 51.6) = 21{,}860\ \mathrm{Btu/h}$$

$$= 21{,}860/3.412 = 6407\ \mathrm{W}$$

and the LMTD is

$$\Delta T_{\mathrm{lm}} = (70 - 51.6)/\ln(70/51.6) = 60.3°\mathrm{F}$$

$$= 60.3/1.8 = 33.5°\mathrm{C}$$

■

ILLUSTRATIVE EXAMPLE 22.14

Describe the various types of heat exchangers that might be employed with a distillation unit.

***SOLUTION*:** See Figure 22.3. The five types include:

Standard exchanger
Heater
Condensor
Chiller
Reboiler ■

ILLUSTRATIVE EXAMPLE 22.15

Discuss industrial applications involving exchangers employed in parallel and series.(4)

***SOLUTION*:** In plants where large numbers of exchangers are used, certain size standards (total number of tubes, pass arrangements, baffle spacing) are established for 1–2 exchangers

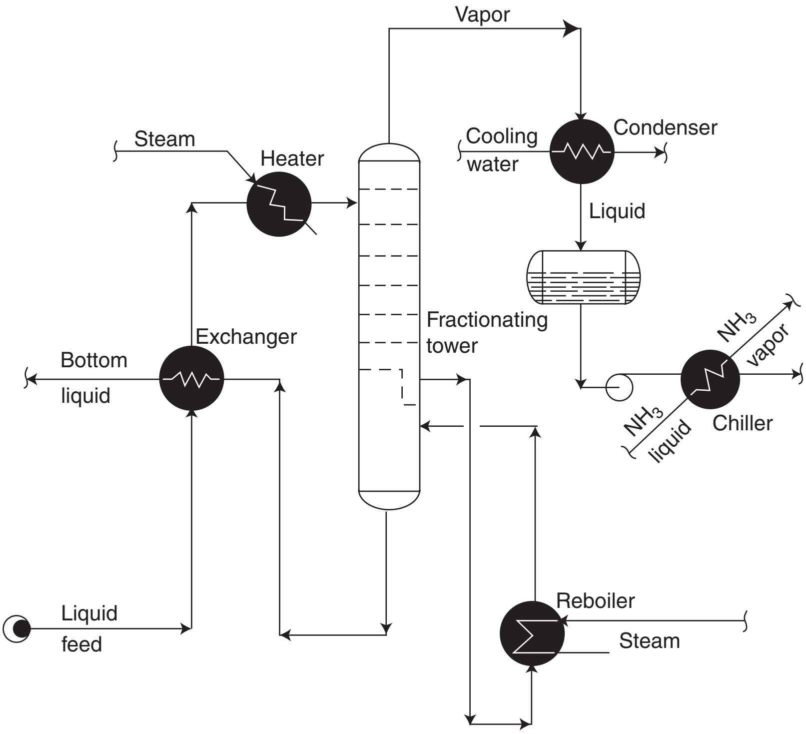

Figure 22.3 Heat exchanger use in a distillation unit.

so that a majority of future services can be fulfilled by an arrangement of a number of standard exchangers in series or parallel. Although this can sometimes cause a problem because of the impossibility of utilizing the equipment most efficiently, it has the advantage of reducing the type and number of replacement parts, tubes, tools, etc. When an exchanger has become obsolete in these plants, it is customary to find a number of exchangers of similar size available for other uses. If the tube bundle is simply retubed, the exchanger will frequently be as serviceable as when new. When two exchangers are connected in series on *both* the shell and tube sides, they form a temperature arrangement which has been shown to be similar to the 2–4 exchanger. When a temperature cross involves a correction factor for an arrangement which approximates true counterflow more closely than that possible in a 1–2 exchanger, it can be met by a series arrangement of a number of 1–2 exchangers. The 2–4, 3–6, 4–8, etc., arrangements are all based upon shells and channels being connected in series. Any arrangement which is an even multiple of two shell passes such as 2–4, 4–8, etc., may be employed by using a number of 1–2 exchangers or half as many 2–4 exchangers. ■

ILLUSTRATIVE EXAMPLE 22.16

Walas(5) has provided some simple "rules of thumb" for selecting and designing heat exchangers. Provide an outline of his suggestions.

***SOLUTION*:**

1. Take true countercurrent flow in a shell-and-tube exchanger as the basis for comparison.
2. Standard tubes are $\frac{3}{4}$-in OD, 1-in triangular spacing, 16 ft long; a shell 1 ft in diameter accommodates 100 ft^2; 2 ft diameter, 450 ft^2; 3 ft diameter, 1100 ft^2.
3. Tube side is for corrosive, fouling, scaling, and high-pressure fluids.
4. Shell side is for viscous and condensing fluids.
5. Pressure drops are approximately 1.5 psi for boiling liquids and 3–9 psi for other services.
6. Minimum temperature-approach is 20°F with normal coolants, 10°F or less with refrigerants.
7. Water inlet-temperature is 90°F, maximum outlet 120°F.
8. For estimating, use the following heat-transfer coefficients ($\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$): water-to-liquid, 150; condensers, 150; liquid-to-liquid, 50; liquid-to-gas, 5.0; gas-to-gas, 5.0; reboiler, 200. For the maximum flux in reboilers, use 10,000.
9. Double-pipe exchangers are competitive at duties requiring 100–200 ft^2.
10. Compact (plate or finned-tube) exchangers have 350 ft^2 of surface area/ft^3 of volume and about four times the heat transfer per cubic foot of shell-and-tube units.
11. Plate-and-frame exchangers are suited for sanitary (environmental) service and, in stainless steel, are 25–50% cheaper than shell and tube units.
12. For air coolers: tubes are 0.75–1.00 in OD; total finned surface is 15–20 ft^2/ft^2 of bare surface; the overall heat-transfer coefficient, $U = 80–100\ \text{Btu/h} \cdot \text{ft}^2$ bare surface $\cdot\ {}^\circ\text{F}$; fan power-input is 2–5 hp/million Btu · h; the approach is 50°F or more.
13. For fired heaters: radiant rate is 12,000 $\text{Btu/h} \cdot \text{ft}^2$; convection rate, 4000 $\text{Btu/h} \cdot \text{ft}^2$; cold oil-tube velocity, 6 ft/s; thermal efficiency, 70–75%; flue gas temperature,

250–350°F above feed inlet; stack-gas temperature 650–950°F. Approximately equal heat transfer occurs in both sections. ■

The following information applies to Illustrative Examples 22.17–22.22 below. A 1–2 shell and tube heat exchanger is employed to cool 1000 lb/h of hot process fluid. The hot process fluid is liquid water entering the unit at 212°F and is located on the shell side. The cold process fluid is liquid water, flowing at 2000 lb/h, entering at 35°F, and is located on the tube side. The tubes are $\frac{3}{4}$-inch OD, 18 BWG, 16 ft long and on a $\frac{15}{16}$-inch triangular pitch. The insulated shell is 15.25-inch ID and 25% percent segmental baffles are spaced 12 inches apart. The allowable fouling "factor" is 0.003 $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$ and the allowable pressure drop is 10 psi on both sides. Select the correct answers.

ILLUSTRATIVE EXAMPLE 22.17

What happens when the insulation is removed from the outside of the shell? Select the correct answer.

(a) The hot fluid gains heat from the environment and the cold fluid exit temperature decreases.

(b) The hot fluid gains heat from the environment and the cold fluid exit temperature increases.

(c) The hot fluid loses heat to the environment and the cold fluid exit temperature decreases.

(d) The hot fluid loses heat to the environment and the cold fluid exit temperature increases.

(e) The hot fluid loses heat to the environment, but the cold fluid exit temperature remains the same.

***SOLUTION*:** Since insulation protects against heat loss, the hot fluid loses heat to the environment and the cold fluid exit temperature decreases. Therefore, the correct answer is (**c**). ■

ILLUSTRATIVE EXAMPLE 22.18

What happens when the baffle spacing is decreased to 3.5 inches? Select the correct answer.

(a) The pressure drop on the tube side increases.

(b) The cold fluid flow becomes more turbulent.

(c) The pressure drop on the shell side decreases.

(d) The hot fluid experiences a phase change.

(e) The pressure drop on the shell side increases.

***SOLUTION*:** When baffles are moved closer, the fluid takes a more "torturous" path and a larger number of sudden changes in direction, resulting in a greater pressure drop. Therefore, the answer is (**e**). ■

ILLUSTRATIVE EXAMPLE 22.19

Which process and/or design scenario is more acceptable?

1. $R_f = 0.0001$ (consistent units) and $\Delta P_{\text{total}} = 4$ psi
2. $R_f = 0.0032$ (consistent units) and $\Delta P_{\text{total}} = 12$ psi

Select the correct answer.

(a) Scenario (1)
(b) Scenario (2)
(c) Neither scenario
(d) Both scenarios
(e) Not enough information provided

***SOLUTION*:** Scenario (**2**) is marginally acceptable because the total pressure drop is only 20% higher than allowed and the fouling factor varies by less than 7%. However, the fouling factor in scenario (**1**) is an order of magnitude lower than allowed. Therefore, the correct answer is (**a**). ■

ILLUSTRATIVE EXAMPLE 22.20

What happens if the hot process fluid is replaced by an equal amount of steam? Select the correct answer.

(a) The exit temperature of the cold fluid decreases.
(b) The exit temperature of the cold fluid increases.
(c) There is less heat transfer to the cold fluid.
(d) There is no effect.
(e) Not enough information provided.

***SOLUTION*:** The temperature of the steam is higher and, as the steam condenses, there is a larger temperature driving force resulting in a greater heat transfer. As a result, the exit temperature of the cold fluid must increase. Therefore, the correct answer is (**b**). ■

ILLUSTRATIVE EXAMPLE 22.21

How would the design have changed if the original hot process fluid had been 1000 lb/h of steam? Select the correct answer.

(a) The number of tube passes would be increased.
(b) A shorter heat exchanger is needed.
(c) There is less heat transfer to the cold fluid.
(d) A longer heat exchanger is needed.
(e) No change is needed.

***SOLUTION*:** Using steam provides a greater temperature difference driving force and would require less heat transfer area, resulting in the design of a shorter heat exchanger. Therefore, the correct answer is (**b**). ■

ILLUSTRATIVE EXAMPLE 22.22

Is the design of a shell and tube heat exchanger affected by the direction of flow of the process fluids? Select the correct answer.

- **(a)** Yes, because the heat transfer area is affected by the log-mean temperature difference, which depends on the direction of flow.
- **(b)** Yes, because the heat transfer area is a function of fouling, which is dependent on the direction of flow.
- **(c)** No, because the direction of flow only affects the number of tube passes.
- **(d)** No, there is no effect.
- **(e)** It depends on numerous other factors.

***SOLUTION*:** Yes, because the heat transfer area is affected by the log-mean temperature difference, which depends on the direction of flow. Countercurrent flow results in a larger temperature driving force. Therefore the correct answer is (**a**). ■

ILLUSTRATIVE EXAMPLE 22.23

Heating or cooling of liquids in batch processes is used in a number of commercial applications. Some reasons for using a batch rather than a continuous heat transfer operation include: the liquid is not continuously available, the liquid cleaning and regeneration is a significant part of the operation, or batch operation is simpler and cheaper.

For example, vapor degreasers are widely used for cleaning metal parts. A degreaser consists of a tank partially filled with a solvent. The tank is equipped with a heating coil to heat the solvent close to its boiling point. The vapor of the solvent occupies the remaining volume of the tank, forming the "solvent vapor zone." When a metal part is placed in the solvent vapor zone, the solvent condenses on the metal part and then drips off, taking contaminants with it. For ease of use, vapor degreasers are often open to the atmosphere. This makes it easier to introduce and remove the metal parts. It has been a common practice to use a halogenated hydrocarbon for such cleaning since they are excellent solvents, volatile, and non-flammable; however, they can be toxic and the open tank of a degreaser can be a significant source of solvent emissions or volatile organic components (VOCs).

When an agitated liquid batch, initially at T_i, is heated, the temperature, T, of the liquid at any time, t, can be assumed to be uniform if the liquid is well stirred. Three different types of calculations involving the batch heating of liquids are outlined in Table 22.6.

The assumptions in these analyses include:

1. constant properties for the duration of the process,
2. constant overall heat transfer coefficient, U, for the process,
3. a constant temperature, T_∞, of the heating medium,

Table 22.6 Batch Heating of Liquids

Case	Description	Calculate
Design	It is desired to heat the liquid to a specified temperature within a given time.	The surface area required.
Simulation	It is desired to heat the liquid for a specified time; the heat transfer surface area is known.	The final temperature, *T*.
Time	The heat transfer surface area and the final temperature are known.	The batch time, *t*.

4. a vigorously stirred unit (and therefore a uniform temperature),
5. negligible heat losses, and
6. no phase change.

The describing equation for these batch systems can be developed. Consider a liquid batch that has a volume, V, density, ρ, and heat capacity, c. The initial temperature is T_i, and the temperature at any time, t, is T. The energy balance on the liquid in the agitated tank simplifies to

$$UA(T_\infty - T) = \rho V c_p \frac{dT}{dt} \tag{1}$$

An excess temperature, θ, can be defined as

$$\theta = T_\infty - T; \quad \theta_i = T_\infty - T_i, \quad T_i = \text{initial temprature} \tag{2}$$

Substituting this expression into Equation (1) and integrating with respect to t yields

$$\frac{\theta}{\theta_i} = e^{-t/\tau} \tag{3}$$

where τ_t is the thermal time constant that is given by

$$\tau = \rho V c_p / UA \tag{4}$$

Design a degreaser employing the development provided above; i.e., calculate the heating surface area requirement for a vapor degreaser using 1,1,1-trichloroethane, $C1_3C\text{-}CH_3$ (TCA) as the solvent. The degreaser tank has a rectangular cross section (6 ft × 3 ft) and a height of 5 ft. Liquid TCA at 18°C is poured into the tank to a height of 1 ft. The solvent vapor zone will occupy the remaining 4 ft. At time $t = 0$, saturated steam is passed into the heating coil. The steam temperature is 100°C. The overall heat transfer coefficient, U, from the steam to the liquid TCA, based on the outside area of the coil, is $200\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$. It is desired to heat the liquid TCA from 18°C to its boiling point (74°C) without evaporation in 180 seconds. The approximate properties of TCA are: density, $\rho = 87.4\ \mathrm{lb/ft^3}$, heat capacity, $c_p = 0.23\ \mathrm{Btu/lb \cdot {}^\circ F}$, and viscosity $= 0.56$ cP. The design should include:

1. the required surface area of the heating coil
2. the total heat added to the liquid TCA.

***SOLUTION*:** Calculate the initial and final excess temperatures:

$$\theta_i = T_\infty - T_i = 100 - 18 = 82^\circ\mathrm{C} = 147.6^\circ\mathrm{F}$$
$$\theta_f = T_\infty - T_f = 100 - 74 = 26^\circ\mathrm{C} = 46.8^\circ\mathrm{F};\ T_f = \text{final temperature}$$

Calculate t/τ_t and the thermal time constant:

$$\begin{aligned} t/\tau &= \ln(\theta_i/\theta_f) \qquad (3)\\ &= \ln[(8.2)/(2.6)]\\ &= 1.149\\ \tau &= t/1.149\\ &= 180/1.149\\ &= 156.7\ \mathrm{s} \end{aligned}$$

The required heating area is given by

$$\begin{aligned} A &= \rho V c_p / U\tau\\ &= (87.4)(18)(0.23)/(3600)(200)(156.7)\\ &= 41.6\ \mathrm{ft}^3 \end{aligned}$$

This is the required area of the outside surface of the heating oil. Convert the final and initial temperatures to °F:

$$\begin{aligned} T_f &= 18^\circ\mathrm{C} = 46.8^\circ\mathrm{F}\\ T &= 74^\circ\mathrm{C} = 147.6^\circ\mathrm{F} \end{aligned}$$

The total amount of heat added is then:

$$\begin{aligned} Q &= \rho V c_p (T_f - T_i)\\ &= (87.4)(18)(0.23)(147.6 - 46.8)\\ &= 36{,}469\ \mathrm{Btu} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 22.24

62,000 lb/h of pure ethyl alcohol ($h_{\text{vap}} = 365$ Btu/lb; specific gravity, s = 0.79) at 2.0 psig is to be condensed by water ($\rho = 62.5\ \mathrm{lb/ft^3}$) entering at 85°F and exiting at 120°F. A 1–2 horizontal condenser consists of 700(N_t) one-inch outside diameter, 14 BWG tubes, 16-inch long on a 1.25-inch triangular pitch. There are four (n) tube passes. Assume that the flow is countercurrent and that the alcohol is on the shell side. A fouling factor of 0.003 $\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$ is recommended. The cooling water inside film coefficient has been previously determined to be 862.4 $\mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$. Neglecting any pressure drop considerations, determine if the exchanger is suitable. Additional information is provided in Table 22.7.

Table 22.7 Exchanger Information for Illustrative Example 22.24

Shell	Tubes
ID = 39 inch	Pitch = 1.25 inch
Baffle spacing = 39 inch	Flow area/tube = 0.546 in^2
Flow area = 2.11 ft^2	ID = 0.834 inch
Condensation temperature, $t_c = 173^\circ\text{F}$	Wall thickness = 0.083 inch

For film condensation on horizontal tubes (see also Chapter 12), the average heat transfer coefficient is to be calculated from the equation (consistent units):

$$\bar{h} = 1.51\left[\frac{k_f \rho_f^2 g \mu_f}{\mu_f^2 4G'}\right]^{1/3}$$

where k_f = thermal conductivity
ρ_f = fluid density
g = acceleration due to gravity
μ_f = viscosity of the fluid
G' = loading, $G' = (\dot{m}_{\text{alcohol}}/LN_t^{2/3})$
$\dot{m}_{\text{alcohol}}$ = condensate mass flow
L = tube length
N_t = number of tubes in bundle
V = velocity

For a clean (unused) tube, the overall heat transfer coefficient can be calculated from the following equation:

$$U_C = U_{\text{clean}} = \frac{h_{io}h_o}{h_{io} + h_o}$$

where $U_C = U_{\text{clean}}$ = heat transfer coefficient of a clean (new/unused) tube
h_{io} = corrected inside heat transfer coefficient to the outside diameter, with $h_{io} = h_i(D_i/D_o)$

The dirt fouling factor may be calculated from:

$$R_D = R_{\text{dirty}} = \frac{U_c - U}{U_c U} \tag{14.32}$$

where $R_D = R_{\text{dirty}}$ = dirt (fouling) factor
U = heat transfer coefficient calculated from the actual heat transfer

***SOLUTION*:** Calculate heat loss from the alcohol:

$$\dot{Q}_{\text{alcohol}} = \dot{m}_{\text{alcohol}} h_{\text{vap}}$$
$$\dot{Q}_{\text{alcohol}} = (62{,}000\ \text{lb/h})(365\ \text{Btu/lb}) = 22{,}630{,}000\ \text{Btu/h}$$

Also, calculate the water mass flow rate. Since the heat lost from alcohol equals the heat gained by water:

$$\begin{aligned}\dot{m}_{\mathrm{H_2O}} &= \frac{\dot{Q}_{\mathrm{H_2O}}}{c_p \Delta t} \\ &= \frac{22{,}630{,}000\,\mathrm{Btu/h}}{(1.0\,\mathrm{Btu/lb\cdot{}^\circ F})(120^\circ\mathrm{F} - 85^\circ\mathrm{F})} = 646{,}571\,\mathrm{lb/h}\end{aligned}$$

Calculate the log-mean temperature difference:

$$\mathrm{LMTD} = \frac{88^\circ\mathrm{F} - 53^\circ\mathrm{F}}{\ln(88^\circ\mathrm{F}/53^\circ\mathrm{F})} = 69^\circ\mathrm{F}$$

Determined the total flow area for both the shell side and the tube side. The flow area for the shell side is given:

$$a_s = 2.11\ \mathrm{ft}^2$$

The total flow area for the tube side can be calculated from the following equation:

$$\begin{aligned}a_t &= \frac{N_t a_t}{144(n)} \\ &= \frac{(700\,\mathrm{tubes})(0.546\,\mathrm{in}^2/\mathrm{tube})}{(144\,\mathrm{in}^2/\mathrm{ft}^2)(4\,\mathrm{passes})} \\ &= 0.664\,\mathrm{ft}^2\end{aligned}$$

Use the above two areas to calculate the mass velocity of the flow in the shell side and the tube side:

$$\begin{aligned}G_s &= \frac{\dot{m}_{\mathrm{alcohol}}}{a_s} \\ &= \frac{62{,}000\,\mathrm{lb/h}}{(2.11\,\mathrm{ft}^2)} \\ &= 29{,}384\,\mathrm{lb/h\cdot ft^2}\end{aligned}$$

$$\begin{aligned}G_t &= \frac{\dot{m}_{\mathrm{H_2O}}}{a_t} \\ &= \frac{646{,}571\ \mathrm{lb/h}}{0.664\,\mathrm{ft}^2} \\ &= 973{,}752\,\mathrm{lb/h\cdot ft^2}\end{aligned}$$

Calculate the velocity of water in the tubes:

$$\begin{aligned}\mathrm{V}_{\mathrm{H_2O}} &= \frac{G_t}{3600\rho} \\ &= \frac{973{,}752\,\mathrm{lb/h\cdot ft^2}}{(3600\,\mathrm{s/h})(62.4\,\mathrm{lb/ft^3})} \\ &= 4.33\,\mathrm{ft/s}\end{aligned}$$

Also, calculate loading G':

$$
\begin{aligned}
G' &= \frac{\dot{m}_{\text{alcohol}}}{LN_t^{2/3}} \\
&= \frac{62{,}000\,\text{lb/h}}{(16\,\text{ft})(700\,\text{tubes})^{2/3}} \\
&= 49.15\,\text{lb/h} \cdot \text{ft}
\end{aligned}
$$

Determine the heat transfer coefficient for the shell side:

$$
\bar{h} = 1.51\left[\frac{k_f^3 \rho_f^2 g \mu_f}{\mu_f^2 4G'}\right]^{1/3}
$$

For the alcohol:

$$
\begin{aligned}
k_f &= 0.105\,\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F} \\
\mu_f &= (0.55)(2.42) = 1.331\,\text{lb/ft} \cdot \text{h} \\
s_f &= 0.79 \\
\rho_f &= (0.79)(62.5\,\text{lb/ft}^3) = 49.375\,\text{lb/ft}^3
\end{aligned}
$$

Substituting,

$$
\begin{aligned}
\bar{h} &= 1.51\left[\frac{(0.105\,\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F})^3(49.375\,\text{lb/ft}^3)^2(32.4\,\text{ft/s}^2)(3600\,\text{s/h})^2}{(1.331\,\text{lb/ft} \cdot \text{h})(4)(49.15\,\text{lb/ft} \cdot \text{h})}\right]^{1/3} \\
\bar{h} &= h_o = 249.8\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}
\end{aligned}
$$

In this case, the average heat transfer coefficient of the film is the outside heat transfer coefficient of the tube bundle. Use this result to calculate the overall heat transfer coefficient for a new (clean) heat exchanger

$$
\begin{aligned}
U_C &= \frac{h_{io} h_o}{h_{io} + h_o} \\
&= \frac{(249.8\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(862.4\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})}{(249.8\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}) + (862.4\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})} \\
&= 193\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}
\end{aligned}
$$

Calculate the design (D) overall heat transfer coefficient:

$$
A = (700\,\text{tubes})(16\,\text{ft})(0.2618\,\text{ft}) = 2932\,\text{ft}^2
$$

$$
U_D = \frac{\dot{Q}}{A\Delta T}
$$

Substituting

$$U_D = \frac{22{,}630{,}000\,\text{Btu/h}}{(2932\,\text{ft}^2)(69^\circ\text{F})}$$
$$= 111.86\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

Finally, calculate the dirt (d) factor and determine whether the heat exchanger is suitable for the process conditions using the equation provided:

$$R_D = \frac{U_C - U_D}{U_C U_D} \tag{14.32}$$
$$= \frac{(193\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}) - (111.86\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})}{(193\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(111.86\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})}$$
$$= 0.0038\,(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})$$

Therefore, the exchanger as specified is unsuitable for these process conditions since the fouling factor is above the recommended value. Cleaning is recommended. ■

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Part Four

Special Topics

This last part of the book is concerned with six topics that the Accreditation Board for Engineering and Technology (ABET) has recently indicated should be included in any engineering curriculum. The contents of these chapters are briefly described below.

Chapter 23 is concerned with Environmental Management; this chapter contains a broad discussion of environmental issues facing today's engineers and scientists, and presents some of the more recent technology to deal with the issues at hand. Chapter 24 is concerned with Accident and Emergency Management; it deals with ways to ensure both employee and public safety, the determination of the severity of accidents, and determining the causes and potential causes of accidents. Chapter 25 is concerned with Ethics; the case study approach is employed to make the reader think about the ethical questions involved, to reflect on their past decisions, and to project forward to their future decisions with a higher degree of thought and insight when faced with an ethical dilemma. Chapter 26 is concerned with Numerical Methods; a brief overview of numerical methods is given to provide the practicing engineer and scientist with some insight into what many of the currently used software packages (MathCad, Mathematica, MatLab) are actually doing. Chapter 27 is concerned with Economics and Finance; this subject area provides material that can ultimately dictate the decisions made by the practicing engineer and his/her company. Finally, Chapter 28 introduces the reader to Open-Ended Questions; although engineers at their very essence are problem solvers, most problems in real life do not come fully defined with a prescribed methodology to arrive at a solution. These open-ended questions are exercises in using brain power—and like any muscle, you use it or lose it. The author believes that those who conquer or become adept with this topic will have taken the first step toward someday residing in an executive suite.

Chapter 23

Environmental Management

INTRODUCTION

In the past four decades, there has been an increased awareness of a wide range of environmental issues covering all resources: air, land, and water. More and more people are becoming aware of these environmental concerns and it is important that professional people, many of whom do not possess an understanding of environmental problems, have the proper information available when involved with environmental decisions. All professionals should have a basic understanding of the technical and scientific terms related to these issues. In addition to serving the needs of the professional, this chapter examines how one can increase his or her awareness of and help solve the environmental problems facing both industry and society.

This chapter provides a broad discussion of environmental issues facing today's engineers and presents some of the more recent technology to deal with the issues at hand. Some of the topics covered in this chapter include air pollution, water pollution, solid waste, etc., and several illustrative examples dealing with these topics are presented.

This chapter is not intended to be all-encompassing. Rather, it is to be used as a starting point. Little is presented on environmental regulations because of the enormity of the subject matter; in a very real sense, it is a moving target that is beyond the scope of this text. Further, the material primarily keys on traditional environmental topics. Although much of the material is qualitative in nature, some quantitative material and calculations are presented in the illustrative examples that appear in the last section (Applications).

ENVIRONMENTAL MANAGEMENT HISTORY

BANG! The Big Bang. In 1948, physicist G. Gamow proposed the big bang theory to explain the origin of the universe. He believed that the universe was created in a gigantic explosion as all mass and energy were created in an instant of time. Estimates on the age of the universe at the present time range between 7 and 20 billion years, with 13.5

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

billion years often mentioned as the age; 4.5 billion years is generally accepted as the planet Earth's age.

The bang occurred in a split second and within a minute the universe was approximately a trillion miles wide and expanding at an unbelievable rate. Several minutes later, all the matter known to humanity had been produced. The universe as it is known today was in place. Environmental problems, as they would later relate to living organisms and humans, were born.

Flash forward to the present. More than any other time in history, the 21st century will be a turning point for human civilization. Human beings may be facing ecological disasters that could affect their ability to survive. These crises could force them to re-examine the value system that has governed their lives for the past two million years (approximately) of existence.

The year 1970 was a cornerstone year for modern environmental policy. The National Environmental Policy Act (NEPA), enacted on January 1, 1970, was considered a "political anomaly" by some. NEPA was not based on specific legislation; instead, it referred in a general manner to environmental and quality-of-life concerns. The Council for Environmental Quality (CEQ), established by NEPA, was one of the councils mandated to implement legislation. April 22, 1970, brought Earth Day, where thousands of demonstrators gathered all around the nation. NEPA and Earth Day were the beginning of a long, seemingly never-ending debate over environmental issues.

The Nixon Administration became preoccupied with not only trying to pass more extensive environmental legislation but also with implementing the laws. Nixon's White House Commission on Executive Reorganization proposed in the Reorganizational Plan #3 of 1970 that a single, independent agency be established, separate from the CEQ. The plan was sent to Congress by President Nixon on July 9, 1970, and this new U.S. Environmental Protection Agency (EPA) began operation on December 2, 1970. The EPA was officially born.

The aforementioned EPA works with the states and local governments to develop and implement comprehensive environmental programs. Federal laws such as the Clean Air Act, the Safe Drinking Water Act, the Resource Conservation and Recovery Act, the Comprehensive Environmental Response, Compensation, and Liability Act, etc., all mandate involvement by state and local government in the details of implementation. These laws, in a very real sense, have dictated the environmental management policies and procedures that are presently in place and serve as the subject matter for this chapter.

A waste management timetable that provides information on environmental approaches since World War II is provided in Table 23.1.

ENVIRONMENTAL MANAGEMENT TOPICS

There are two dozen major topics that the author considers to be integral parts of environmental management (there are, of course, more). Reviewing each subject area in any detail is beyond the scope of this text; the reader is referred to a key reference in the literature(1) for an extensive review of the entire field of environmental

Table 23.1 Waste Management Timetable

Time frame	Control
Prior to 1945	No control
1945–1960	Little control
1960–1970	Some control
1970–1975	Greater control (EPA founded)
1975–1980	More sophisticated control
1980–1985	Beginning of waste reduction management
1985–1990	Waste reduction management
1990–1995	Pollution Prevention Act
1995–2000	Sophisticated pollution prevention approaches
2000–2010	Green chemistry and engineering; sustainability
2010–	?????

management. Additional and more specific references for each of the topics referred to above are provided below:

1. Air pollution control equipment, etc.(2–4)
2. Atmospheric dispersion modeling(5)
3. Indoor air quality(6)
4. Industrial wastewater management(6,7)
5. Wastewater treatment technologies(6,7)
6. Wastewater treatment processes(5–7)
7. Solid waste management(5–8)
8. Superfund(5,6,8)
9. Municipal solid waste management(5,6,8)
10. Hospital waste management(5,6,8)
11. Nuclear waste management(5,6)
12. Pollution prevention(9–15)
13. Multimedia analysis and lifecycle cost analysis(6,7)
14. Noise(5,6,16,17)
15. ISO 14000(5,6,18,19)
16. Environmental justice(5,6,20–22)
17. Electromagnetic fields(5,6)
18. Acid rain(5,6)
19. Greenhouse effect and global warming(5,6)
20. Public perception of risk(5,6,20–22)
21. Health risk assessment(5–7,23)

22. Hazard risk assessment$^{(5-7,24)}$
23. Risk communication$^{(5,6,20-22)}$
24. Environmental implication of nanotechnology$^{(25,26)}$

APPLICATIONS

This last section is devoted to environmental management applications. Nine illustrative examples complement the presentation; much of this material has been drawn from Natural Science Foundation (NSF) literature$^{1-5}$ plus several other sources$^{(6-9,\ 25-26)}$.

ILLUSTRATIVE EXAMPLE 23.1

List at least three examples of actions that can be taken as a means of conserving energy from home cooling applications.(13)

***SOLUTION*:**

1. Install air conditioners that have been properly sized for the target area.
2. Installing ceiling fans can magnify the cooling effects of air conditioners with minimal added energy consumption.
3. Hang shades or blinds to block direct sunlight from entering the house.
4. Plant leafy trees in the east/west faces of the house for added shade.
5. Make sure that attics are both well ventilated during the summer months and sufficiently insulated during the winter.
6. During hot weather, use outdoor barbecue grills and microwaves rather than kitchen stoves to reduce the heat load in the kitchen. ■

ILLUSTRATIVE EXAMPLE 23.2

List at least three examples of actions that can be taken as a means of conserving energy from heating and cooling applications in the workplace.(14)

***SOLUTION*:**

1. The location of a new office or building should be considered with respect to environmental elements like sunlight, wind exposure, etc.
2. Carefully select materials of construction that will enhance energy conservation effects.
3. Regularly maintain the building's heating and cooling systems.
4. Use timers to adjust the thermostat for periods of high and low usage.
5. Institute summer hours when applicable.
6. Encourage employee participation in energy conservation practices. ■

ILLUSTRATIVE EXAMPLE 23.3

Describe several of the environmental impacts that can be attributed to energy consumption.

***SOLUTION*:** Energy production facilities that generate energy through the combustion of fossil fuels emit pollutants like CO_2, SO_2, and NO_x, which can lead to urban smog, acid rain, deforestation, water pollution, and (possibly) global warming. Generators can also produce large amounts of solid waste in the form of ash, slag, coke, and nuclear waste that can affect land and groundwater quality. ■

ILLUSTRATIVE EXAMPLE 23.4

The offensive odor of butanol (C_4H_9OH) can be removed from stack gases by its complete combustion to carbon dioxide and water. It is of interest that the incomplete combustion of butanol actually results in a more serious odor pollution problem than the original one. Write the equations showing the two intermediate malodorous products formed if butanol undergoes incomplete combustion.

***SOLUTION*:** The malodorous products are butyraldehyde (C_4H_8O) and butyric acid (C_3H_7COOH), which can be formed sequentially as follows:

$$\begin{aligned} C_4H_9OH + \tfrac{1}{2}O_2 &\longrightarrow C_4H_8O + H_2O \\ C_4H_8O + \tfrac{1}{2}O_2 &\longrightarrow C_3H_7COOH \end{aligned}$$

or the acid can be formed directly as follows:

$$C_4H_9OH + O_2 \longrightarrow C_3H_7COOH + H_2O$$

For complete combustion:

$$C_4H_9OH + 6O_2 \longrightarrow 4CO_2 + 5H_2O$$

■

ILLUSTRATIVE EXAMPLE 23.5

Even with an aggressive energy conservation program, the growing population will continue to demand increasing amounts of electricity. Identify and describe the environmental impacts, both positive and negative, of the two means of power generation: coal-fired steam boilers and nuclear power.

***SOLUTION*:** The following answer itemizes the positive and negative aspects of each of the two energy generation methods from the standpoint of their impact on the environment.

A contemporary coal-fired boiler and electric generation facility requires three primary raw materials: coal (the energy source), water (for steam, cooling, and probably emissions control), and limestone (for emissions control of SO_2). Therefore, the potential impacts of raw materials suppliers and waste management, as well as the potential impacts of coal combustion, must be considered. Some negative impacts usually include:

1. Air pollution caused by SO_2, NO_x, particulate matter, and CO_2 (*potential* global warming).
2. Water pollution from boiler operations (thermal pollution).
3. Surface or groundwater contamination from mining of coal and limestone.
4. Land pollution from mining wastes and disposal of scrubber sludge (i.e., calcium sulfate).

Some positive impacts can include:

1. Producing huge amounts of electricity at one location where highly efficient environmental controls are cost effective.
2. Producing a potentially useful waste/by-product in the form of calcium sulfate.
3. Producing potentially useful surplus heat (e.g., hot water, low-pressure steam, etc.).

The principal raw materials for a nuclear power facility are uranium and water (for cooling). The potential impacts of nuclear fission must be considered, as well as the potential impacts of uranium mining and processing. Some negative impacts may include:

1. Accidental release of radiation to the environment (as recently encountered in Japan following the tsunami of 2011).
2. Thermal pollution of the cooling water supply.
3. Voluminous uranium mining and processing wastes since only a very small percentage of uranium bearing ore is beneficially used.
4. Difficult and costly storage and disposal of spent nuclear fuel, with a potentially continuous, indefinite threat to the environment.

Some positive impacts may include:

1. Producing huge amounts of electricity at one facility, although highly toxic waste volumes are relatively small.
2. Virtually contaminant-free stack emissions if the plant is operating properly. No particulate emissions, heavy metals from fuel combustion, etc., are generated from nuclear power.
3. No waste materials generated in the treatment of gas streams, so that the impact of nuclear power plants to the land are minimal when operated properly. ■

ILLUSTRATIVE EXAMPLE 23.6

In 1900, it took about 20,000 Btu fuel input to produce 1 kW · h of electricity. Estimate the efficiency of conversion and compare it with a typical value for today's power industry.

***SOLUTION*:** The solution to this problem is based, in part, on unit conversions. From standard conversion tables, 1 kW · h is equivalent to 3412 Btu. Since only 1 kW · h was being produced in 1900 from 20,000 Btu, the energy requirement (ER) was

$$\begin{aligned}\text{ER} &= (20{,}000\,\text{Btu})/(3412\,\text{Btu/kW}\cdot\text{h})\\ &= 5.86\,\text{kW}\cdot\text{h}\end{aligned}$$

Since only 1 kW · h was being produced, the efficiency (E) of energy conversion is

$$\begin{aligned} E &= \frac{\text{Actual energy produced}}{\text{Energy production potential}} \\ &= \frac{1\,\text{kW}\cdot\text{h}}{5.86\,\text{kW}\cdot\text{h}} \\ &= 0.171 \\ &= 17.1\% \end{aligned}$$

Today's energy conversion efficiency has improved over this value from a century ago (thankfully!), but not as much as many would like. Typical values range from 30 to 35% efficiency, or approximately 100% better than before. However, as one can see, there is significant room for improvement. ■

ILLUSTRATIVE EXAMPLE 23.7

The James David University runs it own coal-fired power plant, consuming Utah bituminous coal with an energy content (in the combustion literature, energy content is defined as the lower heating value, LHV) of 25,000 kJ/kg. The coal contains, on average, 1.0 wt% sulfur and 1.2 wt% ash (based on the total mass of the coal). The power plant is 35% efficient (indicating that 35% of the energy in the coal is actually converted to electrical energy), and is operated at a 2.0-MW average daily load (ADL).

Assume that the coal is completely burned during combustion, and also that the power plant captures 99% of the ash and 70% of the sulfur dioxide produced during combustion. After a U.S. Environmental Protection Agency (EPA) Green Lights energy audit, James David found that it could install energy-efficient lighting and reduce its average daily electrical generating needs by 25%.

Using the information given above, calculate the average reduction in electrical load and the new average daily load for the power plant.

***SOLUTION*:** The new electrical load will be 75% of the old electrical load with a 25% reduction in electrical load resulting from the implementation of energy conservation measures.

For a 2.0-MW power plant, the new ADL will be

$$\begin{aligned} \text{new ADL} &= (2.0\,\text{MW})(0.75) \\ &= 1.5\,\text{MW} \end{aligned}$$

The average reduction (AR) in electrical load becomes

$$\begin{aligned} \text{AR} &= (\text{new ADL}) - (\text{old ADL}) \\ &= 2.0\,\text{MW} - 1.5\,\text{MW} \\ &= 0.5\,\text{MW} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 23.8

Describe the health risk assessment process.

***SOLUTION*:** Health risk assessments provide an orderly, explicit way to deal with scientific issues in evaluating whether a health problem exists and what the magnitude of the problem may be. This evaluation typically involves large uncertainties because the available scientific data are limited and the mechanisms for adverse health impacts or environmental damage are only imperfectly understood.

When examining risk, how does one decide how safe is "safe" or how clean is "clean"? To begin with, one has to look at both sides of the risk equation; that is, both the toxicity of a pollutant and the extent of public exposure. Information is required for both the current and the potential exposure, considering all possible exposure pathways. In addition to human health risks, one needs to look at potential ecological or other environmental effects. It should be remembered that there are always uncertainties in conducting a comprehensive health risk assessment and these assumptions must be included in the analysis.

In recent years, several guidelines and handbooks have been produced to help explain approaches for doing health risk assessments. As discussed by a special National Academy of Sciences committee convened in 1983, most human or environmental health problems can be evaluated by dissecting the analysis into four parts: hazard identification, dose-response assessment or hazard assessment, exposure assessment, and risk characterization (see Figure 23.1). For some perceived health problems the risk assessment might stop with the first step, identification, if no adverse effect is identified, or if an agency elects to take regulatory action without further analysis.(5) Regarding health problem identification, a problem is defined as a toxic agent or a set of conditions that has the potential to cause adverse effects to human health or the environment. The identification process involves an evaluation of various forms of information in order to identify the different health problems. Dose-response or toxicity assessment is required in an overall assessment; responses/effects can vary widely since all chemicals and contaminants vary in their capacity to cause adverse effects. This step frequently requires that assumptions be made to relate experimental data from animals and humans.

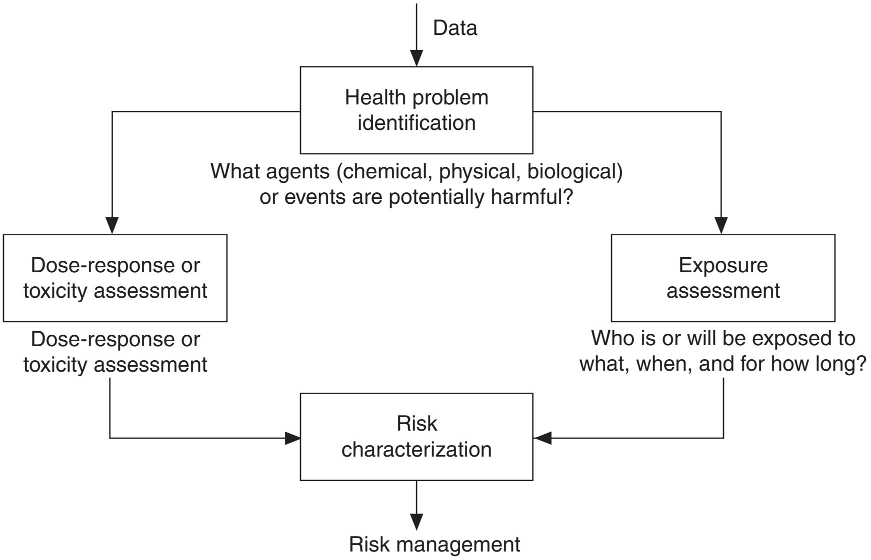

Figure 23.1 The health risk evaluation process.(5−7)

Exposure assessment is the determination of the magnitude, frequency, duration, and routes of exposure of human populations and ecosystems. Finally, in risk characterization, toxicology, and exposure data/information are combined to obtain qualitative or quantitative expressions of risk.

Risk assessment also involves the integration of the information and analysis associated with the above four steps to provide a complete characterization of the nature and magnitude of risk and the degree of confidence associated with this characterization. A critical component of the assessment is a full elucidation of the aforementioned uncertainties associated with each of the major steps. All of the essential problems of toxicology are encompassed under this broad concept of risk assessment. Risk assessment takes into account all of the available dose-response data. It should treat uncertainty not by the application of arbitrary safety factors, but by stating them in quantitatively and qualitatively explicit terms, so that they are not hidden from decision-makers. Risk assessment, defined in this broad way, forces an assessor to confront all the scientific uncertainties and to set forth in explicit terms the means used in specific cases to deal with these uncertainties.(4) In effect, risk characterization is the process of estimating the incidence of a health effect under the various conditions of human or animal exposure described in the exposure assessment. As noted above, it is performed by combining the exposure and dose-response assessments, and the summary effects of the uncertainties in the preceding steps should also be described in this step.

Risk assessment and risk management are two different processes, but they are intertwined. Risk assessment and risk management give a framework not only for setting regulatory priorities but also for making decisions that cut across different environmental areas. Risk management refers to a decision-making process that involves such considerations as risk assessment, technology feasibility, economic information about costs and benefits, statutory requirements, public concerns, and other factors. Therefore, risk assessment supports risk management in that the choices on whether and how much to control future exposure to a suspected problem may be determined.

Regarding both risk assessment and risk management, this illustrative example primarily addresses this subject from a health perspective; the next chapter will primarily address this subject from a safety and accident perspective.

The reader should note that two general types of potential risks exist. These are classified as:

1. *Acute*. Exposures that occur for relatively short periods of time, generally from minutes to one or two days. Concentrations of (toxic) air contaminants are usually high relative to their protection criteria. In addition to inhalation, airborne substances might directly contact the skin, or liquids and sludges may be splashed on the skin or into the eyes, leading to adverse health effects. This subject area falls, in a general sense, in the domain of hazard risk assessment (HZRA), a topic treated in the next chapter.
2. *Chronic*. Continuous exposure occurring over long periods of time, generally several months to years. Concentrations of inhaled (toxic) contaminants are usually relatively low. This subject area falls in the general domain of health risk assessment (HRA) and it is this subject that is addressed in this example. Thus, in contrast to the acute (short-term) exposures that predominate in hazard risk assessment, chronic (long-term) exposures are the major concern in health risk assessments.

Finally, there are two major types of risk: maximum individual risk and population risk. Maximum individual risk is defined exactly as it implies (i.e., the maximum risk to an individual person). This person is considered to have a 70-yr lifetime of exposure to a process or a chemical. Population risk is basically the risk to a population. It is expressed as a certain number of deaths per thousand or per million people. These risks are often based on very conservative assumptions that may yield too high a risk. ■

ILLUSTRATIVE EXAMPLE 23.9

Provide a more detailed presentation on dose-response and/or toxicity.

***SOLUTION*:** Dose-response assessment is the process of characterizing the relation between the dose of an agent administered or received and the incidence of an adverse health effect in exposed populations, and estimating the incidence of the effect as a function of exposure to the agent. This process considers such important factors as intensity of exposure, age pattern of exposure, and other possible variables that might affect responses such as sex, lifestyle, and other modifying factors. A dose-response assessment usually requires extrapolation from high to low doses and extrapolation from animals to humans, or one laboratory animal species to a wildlife species. A dose-response assessment should describe and justify the methods of extrapolation used to predict incidence, and it should characterize the statistical and biological uncertainties in these methods. When possible, the uncertainties should be described numerically rather than qualitatively.

Toxicologists tend to focus their attention primarily on extrapolations from cancer bioassays. However, there is also a need to evaluate the risks of lower doses to see how they affect the various organs and systems in the body. Many scientific papers focus on the use of a safety factor or uncertainty factor approach, since all adverse effects other than cancer and mutation-based developmental effects are believed to have a threshold—a dose below which no adverse effect should occur. Several researchers have discussed various approaches to setting acceptable daily intakes or exposure limits for developmental and reproductive toxicants. It is thought that an acceptable limit of exposure could be determined using cancer models, but today they are considered inappropriate because of thresholds.(5–7)

Dangers are not necessarily defined by the presence of a particular chemical, but rather by the amount of that substance one is exposed to, also known as the dose. A dose is usually expressed in milligrams of chemical received per kilogram of body weight per day. For toxic substances other than carcinogens, a threshold dose must be exceeded before a health effect will occur, and for many substances, there is a dosage below which there is no harm; i.e., a health effect will occur or at least will be detected at the threshold. For carcinogens, it is assumed that there is no threshold, and, therefore, any substance that produces cancer is assumed to produce cancer at any concentration. It is vital to establish the link to cancer and to determine if that risk is acceptable. Analyses of cancer risks are much more complex than those for non-cancer risks.

For a variety of reasons, it is difficult to precisely evaluate toxic responses caused by acute exposures to hazardous materials. First, humans experience a wide range of acute adverse health effects including irritation, narcosis, asphyxiation, sensitization, blindness, organ system damage, and death. In addition, the severity of many of these effects varies with intensity and duration of exposure. Second, there is a high degree of variation in response among individuals in a typical population. Third, for the overwhelming majority of substances encountered in industry, there is insufficient data on toxic responses of humans to permit an accurate or precise assessment of the substance's health potential. Fourth, many releases involve multiple components. There are presently no rules on how these types of releases should be evaluated. Fifth, there are no toxicology testing protocols that exist for studying episodic releases on animals. In general, this has been a neglected area of toxicology research. There are many useful measures available to employ as benchmarks for predicting the likelihood that a release event will result in serious injury or death.

Not all contaminants or chemicals are equal in their capacity to cause adverse effects. Thus, clean-up standards or action levels are based in part on the compounds' toxicological properties. Toxicity data employed are derived largely from animal experiments in which the animals (primarily mice and rats) are exposed to increasingly higher concentrations or doses. As described above, responses or effects can vary widely from no observable effect to temporary and reversible effects, to permanent injury to organs, to chronic functional impairment, to, ultimately, death. ■

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Chapter 24

Accident and Emergency Management

INTRODUCTION

Accidents are a fact of life, whether they are a careless mishap at home, an unavoidable collision on the freeway, or a miscalculation at a chemical plant. Even in prehistoric times, long before the advent of technology, a club-wielding caveman might have swung at his prey and inadvertently toppled his friend in what can only be classified as an "accident."

As man progressed, so did the severity of his misfortunes. The "Modern Era" has brought about assembly lines, chemical manufacturers, nuclear power plants, and so on, all potentially capability of causing disasters. To keep pace with the changing times, safety precautions must constantly be upgraded. It is no longer sufficient, as with the caveman, to shout the warning, "Watch out with that thing!" Today's problems require more elaborate systems of warnings and controls to minimize the chance of serious accidents.

Industrial accidents occur in many ways—a chemical spill, an explosion, a nuclear power plant melt-down, and so on. There are often problems in transport, with trucks overturning, trains derailing, or ships capsizing. There are "acts of God," such as earthquakes, tsunamis and storms. The one common thread through all of these situations is that they are rarely expected and frequently mismanaged.

Most industrial process plants are safe to be around. Plant management, aided by reliable operators, who are in turn backed up by still-more-reliable automatic controls, does its best to keep operations moving along within the limits usually considered reasonably safe to man and machine. Occasionally, however, there is a "whoosh" or a "bang" that is invariably to the detriment of the operation, endangering investment and human life, and rudely upsetting the plant's loss expectancy.(1)

Accidents have occurred since the birth of civilization. Anyone who crosses a street, rides in a car, or swims in a pool, runs the risk of injury through carelessness, poor judgment, ignorance, or other circumstances. This has not changed throughout history. Current legislation plus a number of accidents and disasters that took place

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Heat Transfer Applications for the Practicing Engineer. Louis Theodore

before the advances of modern technology will be examined in the following pages. This will be followed by a number of illustrative examples, several of which draw heavily on probability and statistics.

LEGISLATION(4–7)

The concern for emergency planning and response is reflected in the legislation(2–4) summarized in this section. Although the Clean Air Act does not cover emergency planning and response in a clear and comprehensive manner, certain elements of the act are particularly significant. These include implementation plans and national emission standards for hazardous air pollutants. The Clean Water Act as well as other legislation pertaining to water pollution provides emergency planning and response that is more developed than it is for air. The Resource Conservation and Recovery Act (RCRA) and the Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA) are two important pieces of legislation that are concerned with preventing releases, and with the requirements for the cleanup of hazardous and toxic sites. RCRA and CERCLA contain specific sections that address emergency planning and response. The Superfund Amendments and Reauthorization Act (SARA) is another important piece of legislation. SARA deals with the cleanup of hazardous waste sites as well as emergency planning and response. Title III, which is the heart of SARA, establishes requirements for emergency planning and "community right to know" for federal, state, and local government, as well as industry. Title III is a major stepping-stone in the protection of the environment, but its principal thrust is to facilitate planning in the event of a catastrophe. The Occupational Safety and Health Act (OSHAct) was enacted by Congress in 1970 and established the Occupational Safety and Health Administration (OSHA), which addressed safety in the workplace. Both EPA and OSHA are mandated to reduce the exposure of hazardous substances over land, sea, and air. The OSHAct is limited to conditions that exist in the workplace, where its jurisdiction covers both safety and health. Frequently, both agencies regulate the same substances but in a different manner as they are overlapping environmental organizations. Developed under the Clean Air Act's (CAA's) Section 112(r), the Risk Management Program (RMP) rule (40 CFR Part 68) is designed to reduce the risk of accidental releases of acutely toxic, flammable, and explosive substances. A list of the regulated substances (138 chemicals) along with their threshold quantities is provided in the Code of Federal Regulations at 40 CFR 68.130.

A brief overview of CERCLA, SARA, OSHA, and RMP is provided in the next four subsections.

Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA)

The Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA) of 1980 was the first major response to the problem of abandoned hazardous waste sites throughout the nation. CERCLA was the beginning of the remediation of hazardous waste sites. This program was designed to:

1. Develop a comprehensive program to set priorities for cleaning up the worst existing hazardous waste sites.
2. Make responsible parties pay for these cleanups wherever possible.
3. Set up (initially) a $1.6 billion *Hazardous Waste Trust Fund*, known as the "Superfund," for the two-fold purpose of performing remedial cleanups when responsible parties could not be held accountable and responding to emergency situations involving hazardous substances.
4. Advance scientific and technological capabilities in all aspects of hazardous waste management, treatment, and disposal.

CERCLA requires the person in charge of a process or facility to notify the *National Response Center* (NRC) immediately when there is a release of a designated hazardous substance in an amount equal to or greater than a reportable quantity. CERCLA establishes the reportable quantity for releases of designated hazardous substances at one pound, unless otherwise specified. Such releases require notification to government officials to ensure that the need for response can be evaluated and any response can be undertaken in a timely fashion.

The development of the emergency planning and response actions under CERCLA is based primarily on a national contingency plan that was developed under the *Clean Water Act*. Although the actions of CERCLA have the capabilities to handle hazardous and toxic releases, the act was primarily directed toward the cleanup of abandoned hazardous waste sites.

Under Section 7003 of the RCRA legislation (1984), private citizens are authorized to bring legal action against companies, governmental entities, or individual citizens if past or present hazardous waste management practices are believed to pose an imminent danger. Section 7003 applies to past generators as well as to situations or sites where past acts or failures to act may have contributed to a present endangerment of human health and the environment. Citizen rights to sue are limited, however: (1) if the EPA or the state government is diligently bringing and prosecuting a related action under Section 7003 of RCRA or Section 106 of CERCLA, or (2) if the EPA or the state has settled a related action by entering into a consent decree. CERCLA was amended by the *Superfund Amendments and Reauthorization Act* (SARA) in 1986.

Superfund Amendments and Reauthorization Act of 1986 (SARA)

The *Superfund Amendments and Reauthorization Act of 1986* renewed the national commitment to correcting problems arising from previous mismanagement of hazardous wastes. While SARA was similar in many respects to the original law (i.e., CERCLA), it also contained new approaches to the program's operation. The 1986 Superfund legislation:(5)

1. Reauthorized the program for five more years and increased the size of the cleanup fund from $1.6 billion to $8.5 billion.
2. Set specific cleanup goals and standards, and stressed the achievement of permanent remedies.
3. Expanded the involvement of states and citizens in decision-making.
4. Provided for new enforcement authorities and responsibilities.
5. Increased the focus on human health problems caused by hazardous waste sites.

The new law is more specific than the original statute with regard to remedies to be used at Superfund sites, public participation, and the accomplishment of cleanup activities. The most important part of SARA with respect to public participation is Title III, which addresses the important issues of community awareness and participation in the event of a chemical release.

As mentioned earlier, Title III of SARA addresses hazardous materials release; its subtitle is the *Emergency Planning and Community Right-to-Know Act of 1986*. Title III establishes requirements for emergency planning, hazardous emissions reporting, emergency notification, and "community right-to-know." The objectives of Title III are to improve local chemical emergency response capabilities, primarily through improved emergency planning and notification, and to provide citizens and local governments with access to information about chemicals in their localities. The major sections of Title III that aid in the development of contingency plans are as follows:

1. Emergency Planning (Sections 301–303).
2. Emergency Notification (Section 304).
3. Community Right To Know Reporting Requirements (Sections 311 and 312).
4. Toxic Chemicals Release Reporting—Emissions Inventory (Section 313).

Title III also developed time frames for the implementation of the Emergency Planning and Community Right-to-Know Act of 1986.

Sections 301–303 of Title III, which are responsible for emergency planning, are designed to develop state and local governments' emergency response and preparedness capabilities through better coordination and planning, especially within local communities.

Occupational Safety and Health Act (OSHA)

Congress intended that OSHA be enforced through specific standards in an effort to achieve a safe and healthy working environment. A "general duty clause" was added to attempt to cover those obvious situations that were admitted by all concerned but for which no specific standard existed. The OSHA standards are an extensive compilation of regulations, some that apply to all employers (such as eye and face protection) and some that apply to workers who are engaged in a specific type of work (such as welding or crane operation). Employers are obligated to familiarize themselves with the standards and comply with them at all times.

Health issues, most importantly, contaminants in the workplace, have become OSHA's primary concern. Health problems are complex and difficult to define. Because of this, OSHA has been slow to implement health standards. To be complete, each standard requires medical surveillance, record keeping, monitoring and physical reviews. On the other side of the ledger, safety hazards are aspects of the work environment that are expected to cause death or serious physical harm immediately or before the imminence of such danger can be eliminated.

Probably one of the most important safety and health standards ever adopted is the OSHA hazard communication standard, more properly known as the "right to know" laws. The hazard communication standard requires employers to communicate information to the employees on hazardous chemicals that exist within the workplace. The program requires employers to craft a written hazard communication program, keep *material safety data sheets* (MSDSs) for all hazardous chemicals at the workplace and provide employees with training on those hazardous chemicals, and assure that proper warning labels are in place.

USEPA's Risk Management Program

In the RMP rule, EPA requires a *Risk Management Plan* that summarizes how a facility is to comply with EPA's RMP requirements. It details methods and results of hazard assessment, accident prevention, and emergency response programs instituted at the facility. The hazard assessment shows the area surrounding the facility and the population potentially affected by accidental releases. EPA requirements include a three-tiered approach for affected facilities. A facility is affected if a process unit

Table 24.1 RMP Approach

Program	Description
1	Facilities submit RMP, complete registration of processes, analyze worst-case release scenario, complete 5-year accident history, coordinate with local emergency planning and response agencies; and, certify that the source's worst-case release would not reach the nearest public receptors.
2	Facilities submit RMP, complete registration of processes, develop and implement a management system; conduct a hazard risk assessment; implement certain prevention steps; develop and implement an emergency response program; and, submit data on prevention program elements.
3	Facilities submit RMP, complete registration of processes, develop and implement a management system; conduct a hazard risk assessment; implement prevention requirements; develop and implement an emergency response program; and, provide data on prevention program elements.

manufactures, processes, uses, stores, or otherwise handles any of the listed chemicals at or above the threshold quantities. The RMP approach is summarized in Table 24.1.

HAZARD RISK ASSESSMENT(7–10)

There are many definitions for the word risk. It is a combination of uncertainty and damage; a ratio of hazards to safeguards; a triplet combination of event, probability, and consequences; or even a measure of economic loss or human injury in terms of both the incident likelihood and the magnitude of the loss or injury. People face all kinds of risks everyday, some voluntarily and others involuntarily. Therefore, risk plays a very important role in today's world. Studies on cancer caused a turning point in the world of risk because it opened the eyes of risk engineers and health science professionals to the world of risk assessments.

Risk evaluation of accidents serves a dual purpose. It estimates the probability that an accident will occur and also assesses the severity of the consequences of an accident. Consequences may include damage to the surrounding environment, financial loss, or injury to life. This section is primarily concerned with the methods used to identify hazards and the causes and consequences of accidents. Issues dealing with health risks have been explored in the previous chapter. Risk assessment of accidents provides an effective way to help ensure either that a mishap does not occur or that the likelihood of an accident is reduced. The result of a hazard risk assessment allows concerned parties to take precautions to prevent an accident before it happens.

Regarding definitions, the first thing an individual needs to know is what exactly is an accident. An accident is an unexpected event that has undesirable consequences. The causes of accidents have to be identified in order to help prevent accidents from occurring. Any situation or characteristic of a system, plant, or process that has the potential to cause damage to life, property, or the environment is considered a hazard. A hazard can also be defined as any characteristic that has the potential to cause an accident. The severity of a hazard plays a large part in the potential amount of damage a hazard can cause if it occurs. Hazard risk is the probability that human injury, damage to property, damage to the environment, or financial loss will occur. An acceptable risk is a risk whose probability is unlikely to occur during the lifetime of the plant or process. An acceptable risk can also be defined as an accident that has a high probability of occurring, but with negligible consequences. Risks can be ranked qualitatively in categories of high, medium, and low. Risk can also be ranked quantitatively as an annual number of fatalities per million affected individuals. This is normally denoted as a number times one millionth, for example, 3×10^{-6}. This number indicates that on average three will die every year out of one million individuals. Another quantitative approach that has become popular in industry is the Fatal Accident Rate (FAR) concept. This determines or estimates the number of fatalities over the lifetime of 1000 workers. The lifetime of a worker is defined as 10^5 hours, which is based on a 40-hour work week for 50 years. A reasonable FAR for a chemical plant is 3.0 with 4.0 usually taken as a maximum. A FAR of 3.0 means that there are 3 deaths for every 1000 workers over a 50-year period. Interestingly, the FAR for an individual at home is

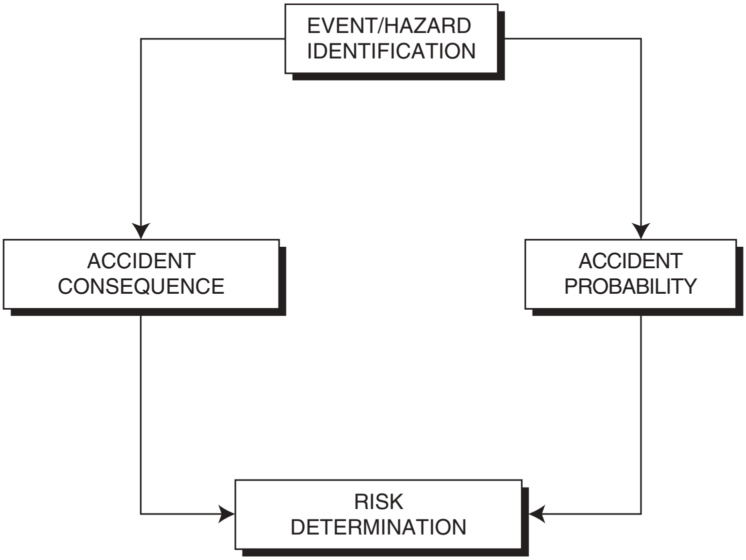

Figure 24.1 Hazard risk assessment flowchart.

approximately 3.0. Some of the illustrative examples later in this chapter compliment many of the concepts described below with technical calculations and elaborations.

As with Health Risk Assessment (HRA), there are four key steps involved in a Hazard Risk Assessment (HZRA). These are presented in Figure 24.1. A more detailed flowchart is presented in Figure 24.2 if the system in question is a chemical plant. These steps are detailed below:

1. A brief description of the equipment and chemicals used in the plant is needed.
2. Any hazard in the system has to be identified. Hazards that may occur in a chemical plant include:
   a. Fire
   b. Toxic vapor release
   c. Slippage
   d. Corrosion
   e. Explosions
   f. Rupture of pressurized vessel
   g. Heat exchanger failure
   h. Runaway reactions
3. The event or series of events that will initiate an accident has to be identified. An event could be a failure to follow correct safety procedures, improperly repaired equipment, or a safety mechanism.
4. The probability that the accident will occur has to be determined. For example, if a chemical plant has a given life, what is the probability that the temperature in a heat exchanger will exceed the specified temperature range? The

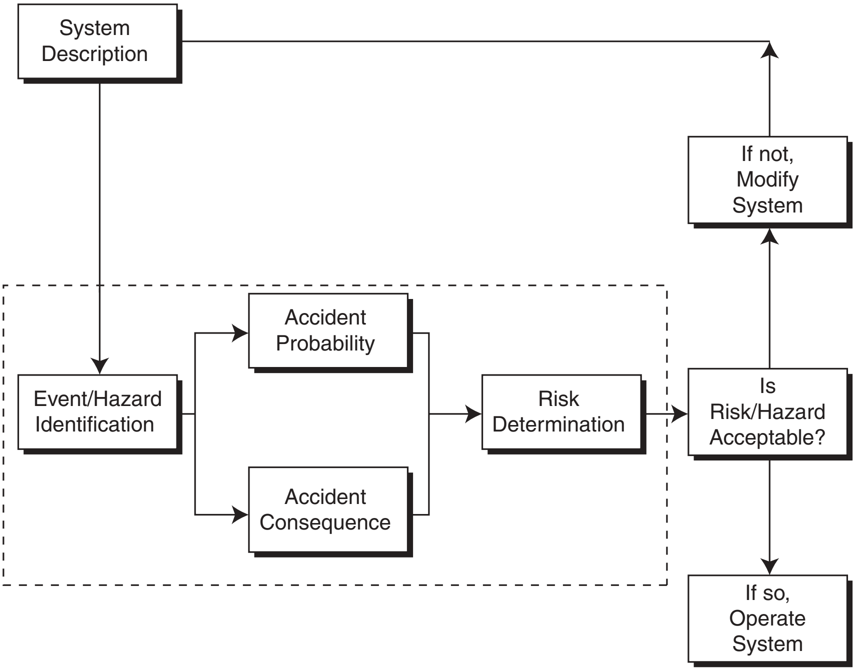

Figure 24.2 Chemical plant hazard risk assessment flowchart.

probability can be ranked from low to high. A low probability means that it is unlikely for the event to occur during the life of the plant. A medium probability suggests that there is a possibility that the event will occur. A high probability means that the event will probably occur during the life of the plant.

5. The severity of the consequences of the accident must be determined.
6. The information from (4) and (5) are combined. If the probability of the accident and the severity of its consequences are low, then the risk is usually deemed acceptable and the plant should be allowed to operate. If the probability of occurrence is too high or the damage to the surroundings is too great, then the risk is usually unacceptable and the system needs to be modified to minimize these effects.

The heart of the hazard risk assessment algorithm provided is enclosed in the dashed box of Figure 24.2. The algorithm allows for re-evaluation of the process if the risk is deemed unacceptable (the process is repeated starting with either step one or two).

As evident in the lessons from past accidents, it is essential for industry to abide by stringent safety procedures. The more knowledgeable the personnel, from the management to the operators of a plant, and the more information that is available to them, the less likely a serious incident will occur. The new regulations, and especially Title III of 1986, help to ensure that safety practices are up to standard. However, these regulations should only provide a minimum standard. It should be up to the companies, and specifically the plants, to see that every possible measure is taken to ensure the safety and well-being of the community and the environment in the surrounding area. It is also up to the community itself, under Title III, to be aware of what goes on inside local industry, and to prepare for any problems that might arise.

APPLICATIONS

The remainder of this chapter is devoted to illustrative examples, many of which contain technical developmental material. A good number of applications have been drawn from National Science Foundation (NSF) literature,(11–16) and two other sources.(6,18)

ILLUSTRATIVE EXAMPLE 24.1

Consider the release of a toxic gas from a storage tank. List and discuss possible causes for the release.

***SOLUTION*:** Some possible causes for a toxic gas release from a storage tank are:

1. Rupture in storage tank.
2. Fire in tank farm.
3. Explosion of storage tank.
4. Collapse of tank due to earthquake.
5. Rupture in main line.
6. Leak in line or from tank. ■

ILLUSTRATIVE EXAMPLE 24.2

What are potential consequences if a pinhole leak develops in a tube in a reboiler of a distillation column?

***SOLUTION*:** Potential consequences of a pinhole leak include:

1. Changes in pressure.
2. Changes in temperature.
3. Chemical reaction, with accompanying over-pressure, over-temperature, and formation of other phases.
4. Leakage of toxics/flammables to an undesirable location.
5. Corrosion, embrittlement, or similar effect. ■

ILLUSTRATIVE EXAMPLE 24.3

Discuss the three major factors that often influence equipment (such as heat exchangers) failure rates.

***SOLUTION*:** The variation of equipment failure rate with time-in-service is usually represented by three regions:

1. At initial start up, the rate of equipment failure is high due to factors such as improper installation or problems as a result of defective equipment.
2. The rate of failure declines when the equipment is under normal operation. At this point, failures are chance occurrences.
3. The rate of failure increases as the equipment ages. This can be termed as wear-out failure.

This failure rate with time is discussed later in Illustrative Example 24.8; its graphical representation is known as the "bathtub curve" (or Weibull distribution to statisticians) because of its shape.(16) ■

ILLUSTRATIVE EXAMPLE 24.4

Calculate the upper flammability limit (UFL) and the lower flammability limit (LFL) of a gas mixture that consists of 30% methane (*m*), 50% ethane (*e*), and 20% pentane (*p*) by volume. Employ the following equation:

$$\text{FL(mixture, } n \text{ components)} = \sum_{i=1}^{n} \frac{1}{(f_i/\text{FL}_i)}; \quad f_i = \text{mole fraction } i \tag{24.1}$$

***SOLUTION*:** Use the expanded form of the equation provided above:

$$\text{LFL(mix)} = \frac{1}{(f_m/\text{LFL}_m) + (f_e/\text{LFL}_e) + \left(f_p/\text{LFL}_p\right)}$$

Substituting,

$$\text{LFL(mix)} = \frac{1}{(0.30/0.046) + (0.5/0.35) + (0.2/0.014)} = 0.0285 = 2.85\%$$

$$\text{UFL(mix)} = \frac{1}{(f_m/\text{UFL}_m) + (f_e/\text{UFL}_e) + \left(f_p/\text{UFL}_p\right)}$$

Substituting,

$$\text{UFL(mix)} = \frac{1}{(0.30/0.142) + (0.5/0.151) + (0.2/0.078)} = 0.125 = 1.25\%$$

■

ILLUSTRATIVE EXAMPLE 24.5

Two boiler tubes are drawn in succession from a lot of 100 tubes, of which 10 are defective. What is the probability that both tubes are defective if (a) the first is replaced before the second is drawn and (b) the first is not replaced before the second is drawn.

***SOLUTION*:** The probability of event *A*, *P*(*A*), can be interpreted as a theoretical relative frequency; i.e., a number about which the relative frequency of event *A* tends to cluster as *n*, the number of times a random experiment (on the event) is performed, increases indefinitely. This is the objective interpretation of probability. Probability can also be interpreted subjectively as a measure of the degree of belief, on a scale from 0 to 1, that the event *A* occurs. This interpretation is frequently used in ordinary conversation. For example, if someone says, "The probability I (the author) will go to the racetrack today is 90%," then 90% is a measure of the person's belief that he or she will go to the racetrack. This interpretation is also used when, in the absence of concrete data needed to estimate an unknown probability on the basis of observed relative frequency, the personal opinion of an expert is sought to provide the estimate.

The conditional probability of event B given A is denoted by $P(B \mid A)$ and defined as follows:

$$P(B \mid A) = P(AB)/P(A) \tag{24.2}$$

where $P(AB)$ is the probability that events A and B will occur. $P(B \mid A)$ can be interpreted as the proportion of A occurrences that also feature the occurrence of B.

First, determine the probability that the first tube is defective, $P(A)$. Since 10 out of 100 are defective:

$$P(A) = (10/100)$$

Determine the probability that the second tube is defective if the first is replaced, $P(B)$. Since the first tube is replaced, the probability for the defective tube is the same:

$$P(B) = (10/100)$$

Determine the probability that the two tubes are defective if the first is replaced, that is, $P(AB)$:

$$\begin{aligned} P(AB) &= P(A)P(B) \\ &= (10/100)(10/100) \\ &= 1/100 \end{aligned}$$

Determine the probability that the second tube is defective if the first tube is not replaced, that is, $P(B \mid A)$. Since the remaining lot contains 99 tubes:

$$P(B \mid A) = 9/99$$

Finally, determine the probability that both tubes are defective if the first tube is not replaced, that is, $P'(AB)$:

$$\begin{aligned} P'(AB) &= P(A)P(B \mid A) \\ &= (10/100)(9/99) \\ &= 1/110 \end{aligned}$$

Conditional probability can be used to formulate a definition for the independence of two events A and B. Event B is defined to be independent of event A only if $P(B \mid A) = P(B)$. Similarly, event A is defined to be independent of event B if and only if $P(A \mid B) = P(A)$. From the definition of conditional probability, one can deduce the logically equivalent definition of the independence of event A and event B if and only if $P(AB) = P(A) \times P(B)$. ■

ILLUSTRATIVE EXAMPLE 24.6

The difference between the magnitude of a large earthquake at a nuclear power plant, on the Richter scale, and the threshold value of 3.25, is a random variable X having the following probability distribution function (pdf):

$$\begin{aligned} f(X) &= 1.7\exp(-1.7X); \quad && X > 0 \\ &= 0; && \text{elsewhere} \end{aligned}$$

Find the probability that X will have a value between 2 and 6; that is, $P(2 < X < 6)$.

***SOLUTION*:** The probability distribution of a random variable concerns the distribution of probability over the range of the random variable. The distribution of probability is specified by the probability distribution function (pdf). The random variable may be discrete or continuous. Special pdfs finding application in risk analysis are considered in later problems. The pdf of a continuous random variable X has the following properties:

1. $\int_a^b f(x)\,dx = P(a < X < b)$

2. $f(x) \geq 0$

3. $\int_{-\infty}^{\infty} f(x)\,dx = 1$ (24.3)

where $P(a < X < b)$ = probability assigned to an outcome or an event corresponding to the number x in the range of X between a and b

$f(x)$ = pdf of the continuous random variable X.

Property (1) indicates that the pdf of a continuous random variable generates probability by integration of the pdf over the interval whose probability is required. When this interval contracts to a single value, the integral over the interval becomes zero. Therefore, the probability associated with any particular value of a continuous random variable is zero. Consequently, if X is continuous,

$$\begin{aligned} P(a \leq X \leq b) &= P(a < X \leq b) \\ &= P(a < X < b) \end{aligned} \tag{24.4}$$

Property (2) restricts the values of $f(x)$ to non-negative numbers. Property (3) follows from the fact that:

$$P(-\infty < X < \infty) = 1 \tag{24.5}$$

Calculate the probability that X will have a value between 2 and 6:

$$\begin{aligned} P(2 < X < 6) &= \int_2^6 f(x)\,dx \\ &= \int_2^6 1.7\exp(-1.7x)\,dx \end{aligned}$$

Since $\int \exp(ax)\,dx = (1/a)\exp(ax)$,

$$\begin{aligned} P(2 < X < 6) &= -\exp(-1.7x)\Big|_2^6 \\ &= \exp[(-1.7)(2)] - \exp[(-1.7)(6)] \\ &= 0.0333 \end{aligned}$$ ■

The pdf of a discrete (rather than a continuous) random variable X is specified by $f(x)$ where $f(x)$ has the following essential properties:

1. $f(x) = P(X = x)$
$=$ probability assigned to the outcome corresponding to the number x in the range of X

2. $f(x) \geq 0$ (24.6)

ILLUSTRATIVE EXAMPLE 24.7

A coolant sprinkler system in a reactor has 20 independent spray components each of which fails with a probability of 0.1. The coolant system is considered to "fail" only if four or more of the sprays fail. What is the probability that the sprinkler systems fails?

***SOLUTION*:** Several probability distributions figure prominently in reliability calculations. The binomial distribution is one of them. Consider n independent performances of a random experiment with mutually exclusive outcomes which can be classified "success" or "failure". The words "success" and "failure" are to be regarded as labels for two mutually exclusive categories of outcomes of the random experiment. They do not necessarily have the ordinary connotation of success or failure. Assume that P, the probability of success on any performances of the random experiment, is constant. Let $q = 1 - P$ be the probability of failure. The probability distribution of x, the number of successes in n performances of the random experiment is the binomial distribution with probability distribution function specified by:

$$f(x) = \frac{P^x q^{n-x} n!}{x!(n-x)!}; \quad x = 0, 1, 2, \ldots, n \tag{24.7}$$

where $f(x)$ is the probability of x successes in n performances and n is the number of independent performances of a random experiment. The binomial distribution can therefore be used to calculate the reliability of a redundant system. A redundant system consisting of n identical components is a system which fails only if more than r components fail. Typical examples include single-usage equipment such as missile engines, short-life batteries, and flash bulbs which are required to operate for one time period and are not reused.

Assume that the n components are independent with respect to failure, and that the reliability of each is $1 - P$. One may associate "success" with the failure of a component. Then x, the number of failures, has a binomial pdf and the reliability of the random system is:

$$P(x \leq r) = \sum_{x=0}^{r} \frac{P^x q^{n-x} n!}{x!(n-x)!} \tag{24.8}$$

Let x denote the number of components which fail. Identify the value of n, P, and q from the problem statement.

$$n = 20$$
$$P = 0.1$$
$$q = 0.9$$

Calculate the probability that the sprinkler system fails (i.e., $P(X \geq 4)$) by using the binomial distribution equation:

$$P(X \geq 4) = \sum_{x=4}^{20} \{P^x q^{n-x} n!/[x!(n-x)!] \tag{24.8}$$

Note that calculation can be simplified by the fact that $P(X \geq 4) = 1 - P(X \leq 3)$. Therefore,

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 1 - \sum_{x=0}^{3} (0.1)^x (0.9)^{20-x} 20!/[x!(20-x)!] \\ &= 0.13 = 13\% \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 24.8

Assume the time to failure (in hours), t, of a tube in a heat exchanger has a Weibull distribution with $\alpha = 1.3 \times 10^{-3}$ and $\beta = 0.77$. Find the probability that a tube in a heat exchanger will fail in 1000 hours.

***SOLUTION*:** Frequently, and as discussed earlier, the failure rate of equipment exhibits three states: a break-in stage with a declining failure rate, a useful life stage characterized by a fairly constant failure rate, and a wear out period characterized by an increasing failure rate. A failure rate curve exhibiting these three phases is called a *bathtub curve*. The Weibull distribution provides a mathematical model of all three states of the bathtub curve. The probability distribution function is given by:(16)

$$\begin{aligned} f(t) &= \alpha\beta t^{\beta-1} e^{-\int_0^t \alpha\beta t^{\beta-1} dt} \\ &= \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}; \quad t > 0, \alpha > 0, \beta > 0 \end{aligned} \tag{24.9}$$

where a and b are constants.

The pdf for the heat exchanger tube is then defined as

$$f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}} = 1.3 \times 10^{-3}(0.77)(t^{0.77-1})(e^{(-1.3\times10^{-3})(t^{0.77})}); \quad t > 0$$

The probability that a heat exchanger will fail within 100 hours (i.e., $P(t < 1000)$), can now be calculated

$$P(t < 1000) = \int_0^{1000} f(t)\,dt = 1 - e^{(-1.3\times10^{-3})(1000^{0.77})} = 0.23$$

■

ILLUSTRATIVE EXAMPLE 24.9

The measurement of the pitch diameter of an acceptable thread of a boiler tube is normally distributed with mean of 0.4008 inch and standard deviation of 0.0004 inch. The specifications are given as 0.4000 ± 0.0010. What is the probability of a "defect" occurring?

***SOLUTION*:** When time to failure, t, has a normal distribution, its pdf is given by:

$$f(t) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-0.5[(t-\mu)/\sigma]^{1/2}} \tag{24.10}$$

where μ is the mean value of t and σ is the standard deviation of t. Thus, if t is normally distributed with mean μ and standard deviation σ, then the random variable, $(t - \mu)/\sigma$, is also normally distributed with mean 0 and standard deviation 1. The term $(t - \mu)\sigma$ is called a standard normal variable (designated by Z) and the graph of its pdf is called a standard normal curve. Table 24.2 provides a tabulation of areas under a standard normal curve to the right of Z_0 for non-negative values of Z_0.(16) From this table, probabilities about a standard normal variable, Z, can be determined. This may now be applied to the solution at hand.

In order to calculate the probability of a defect occurring, the probability of meeting the specification must first be calculated. Determine the standard normal variable, Z:

$$\begin{aligned} Z &= (X - \mu)/\sigma \\ &= (X - 0.4008)/0.0004 \end{aligned} \tag{24.11}$$

Determine the lower and upper limits of the probability of meeting specification:

$$\text{Lower limit (LL)} = 0.4000 - 0.0010 = 0.3990$$
$$\text{Upper limit (UL)} = 0.4000 + 0.0010 = 0.4010$$

Determine the probability of meeting specification, P_s, from the area under the standard normal curve between the lower and upper limits:

$$\begin{aligned} P_s &= P[(LL - \mu)/\sigma < Z < (UL - \mu)/\sigma] \\ &= P\{[(0.3990 - 0.4008)/(0.0004)] \\ &\quad < Z < [(0.4010 - 0.4008)/(0.0004)]\} \\ &= P(-4.5 < Z < 0.5) \end{aligned}$$

From Table 24.2

$$P_s = 0.5 + 0.191 = 0.61 = 61\%$$

Table 24.2 Standard Normal Variables

	Next decimal place of Z_0									
Z_0	0	1	2	3	4	5	6	7	8	9
0.0	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0.1	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.4	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.7	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
0.8	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.189	0.166	0.164	0.161
1.0	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
1.1	0.136	0.133	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.100	0.099
1.3	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.7	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037

(*Continued*)

Table 24.2 *Continued*

	Next decimal place of Z_0									
Z_0	0	1	2	3	4	5	6	7	8	9
1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2.0	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.3	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
Z_0	Detail of tail ($0._{2}135$, for example, means 0.00135)									
2	$0._{1}228$	$0._{1}179$	$0._{1}139$	$0._{1}107$	$0._{2}820$	$0._{2}621$	$0._{2}466$	$0._{2}347$	$0._{2}256$	$0._{2}187$
3	$0._{2}135$	$0._{3}968$	$0._{3}687$	$0._{3}483$	$0._{3}337$	$0._{3}233$	$0._{3}159$	$0._{3}108$	$0._{4}723$	$0._{4}481$
4	$0._{4}317$	$0._{4}207$	$0._{4}133$	$0._{5}854$	$0._{5}541$	$0._{5}340$	$0._{5}211$	$0._{5}130$	$0._{6}793$	$0._{6}479$
5	$0._{6}287$	$0._{6}170$	$0._{7}996$	$0._{7}579$	$0._{7}333$	$0._{7}190$	$0._{7}107$	$0._{8}599$	$0._{8}332$	$0._{8}182$
0	1	2	3	4	5	6	7	8	9	

Finally, calculate the probability of a defect occurring, P_d:

$$\begin{aligned} P_d &= 1 - P_s \\ &= 1 - 0.69 \\ &= 0.31 = 31\% \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 24.10

Three thermometers (A, B, C) are positioned near the outlet of a heat exchanger. Assume that the individual thermometer component lifetimes are normally distributed with means and standard deviations provided in Table 24.3.

Using the following random numbers, simulate the lifetime (time to failure) of the temperature recording system and estimate its mean and standard deviation. The lifetime to failure is defined as the time (in weeks) for one of the thermometers to "fail".

Monte Carlo simulation is a procedure for mimicking observations on a random variable that permits verification of results that would ordinarily require difficult mathematical calculations or extensive experimentation. The method normally uses computer programs called *random number generators*. A *random number* is a number selected from the interval (0, 1) in such a way that the probabilities that the number comes from any two subintervals of equal length are equal. For example, the probability the number is in the sub-interval (0.1, 0.3) is the same as the probability that the number is in the subinterval (0.5, 0.7). Thus, random numbers are observations on a random variable X having a uniform distribution on the interval (0, 1). This means that the pdf of X is specified by:

$$\begin{aligned} f(x) &= 1; \quad 0 < x < 1 \\ &= 0; \quad \text{elsewhere} \end{aligned}$$

The above pdf assigns equal probability to subintervals of equal length in the interval (0, 1). Using random number generators, Monte Carlo simulation can generate observed values of a random variable having any specified pdf. For example, to generate observed values of T, the time to failure, when T is assumed to have a pdf specified by $f(t)$, first use the random number generator to generate a value of X between 0 and 1. The solution is an observed value of the random variable T having pdf specified by $f(t)$. Additional information on Monte Carlo methods is provided in Chapter 26.

***SOLUTION*:** Let T_A, T_B, and T_C denote the lifetimes of thermometer components A, B, and C, respectively. Let T_S denote the lifetime of the system. First determine the values of standard normal variable Z and T_A for component A using the 10 random numbers given. Note that the random number generated is the cumulative probability shown in Table 24.4. Also note that $T = \mu + (\sigma)(Z)$.

Table 24.3 Thermometer Failure Data; Illustrative Example 24.10

	A	B	C
Mean (weeks)	100	90	80
Standard deviation (weeks)	30	20	10

Table 24.4 Random Numbers; Illustrated Example 24.10

For A		For B		For C	
0.52	0.01	0.77	0.67	0.14	0.90
0.80	0.50	0.54	0.31	0.39	0.28
0.45	0.29	0.96	0.34	0.06	0.51
0.68	0.34	0.02	0.00	0.86	0.56
0.59	0.46	0.73	0.48	0.87	0.82

Random No.		Z	T_A
0.52	From Table 24.4;	0.05	$T_A = 100 + 30(0.05) = 102$
0.80		0.84	125
0.45		-0.13	96
0.68		0.47	114
0.59		0.23	107
0.01		-2.33	30
0.50		0.00	100
0.29		-0.55	84
0.34		-0.41	88
0.46		-0.10	97

Next, determine the values of standard normal variable Z and T_B for component B using the 10 random numbers given.

Random No.		Z	T_B
0.77	From Table 24.4;	0.74	$T_B = 90 + 20(0.74) = 105$
0.54		0.10	92
0.96		1.75	125
0.02		-2.05	49
0.73		0.61	102
0.67		0.44	99
0.31		-0.50	80
0.34		-0.41	82
0.00		-3.90	12
0.48		-0.05	89

Determine the values of standard normal variable Z and T_C for component C using the 10 random numbers given.

Random No.		Z	T_C
0.14	From Table 24.4;	-1.08	$T_C = 80 + 10(-1.08) = 69$
0.39		-0.28	77
0.06		-1.56	64
0.86		1.08	91
0.87		1.13	91
0.90		1.28	93

0.28	−0.58	74
0.51	0.03	80
0.56	0.15	81
0.82	0.92	89

For each random value (10) of each component (A, B, C), determine the system lifetime, T_S.

T_A	T_B	T_C	T_S	
102	105	69	69	(minimum of A, B, and C)
125	92	77	77	
96	125	64	64	
114	49	91	49	
107	102	91	91	
30	99	93	30	
100	80	74	74	
84	82	80	80	
88	12	81	12	
97	89	89	89	
			Total = 635	

Calculate the mean value of T_S.

$$\begin{aligned}\text{Mean} &= 635/10\\ &= 63.5\,\text{weeks}\end{aligned}$$

Calculate the standard deviation of T_S.

T_S	$(T_S - \mu)^2$
69	$(69 - 63.5)^2 = 30.25$
77	182.25
64	0.25
49	210.25
91	756.25
30	1122.25
74	110.25
80	272.25
12	2652.25
89	650.25
	Total = 5987.00

By definition,

$$\text{Standard deviation} = \left(\frac{\sum (T_s - \mu)^2}{n-1}\right)^{0.5};\ n = 10 = (5987/9)^{0.5}$$

$$= 25.8\,\text{weeks}$$

■

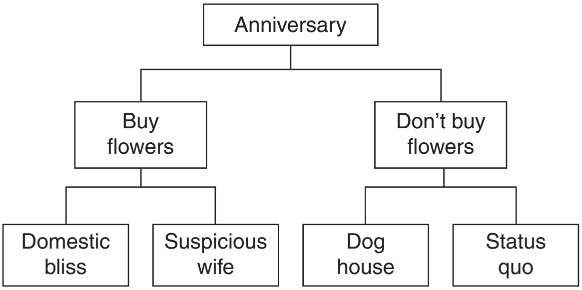

Figure 24.3 Event tree; Illustrative Example 24.11.

ILLUSTRATIVE EXAMPLE 24.11

Construct a decision tree, given the following sequence of events.

Date: a married couple's anniversary
Husband's decisions: buy flowers or do not buy flowers
Consequences (buy flowers): domestic bliss or suspicious wife
Consequences (do not buy flowers): status quo or wife in tears/husband in doghouse

SOLUTION**:** A *decision tree* provides a diagrammatic representative of sequences that begin with a so-called initiating event and terminate in one or more consequences. It finds application in many engineering and accident management calculations.

The bottom four events provided in Figure 24.3 evolve from what may be defined as *resolution of uncertainty points*. This is an example of an "event" tree. In contrast to a "fault" tree which works backward from a consequence to possible causes, an event tree works forward from the initiating (or top) event to all possible consequences. ■

ILLUSTRATIVE EXAMPLE 24.12

If a building fire occurs, a smoke alarm sounds with probability 0.9. The sprinkler system functions with probability 0.7 whether or not the smoke alarm sounds. The consequences are minor

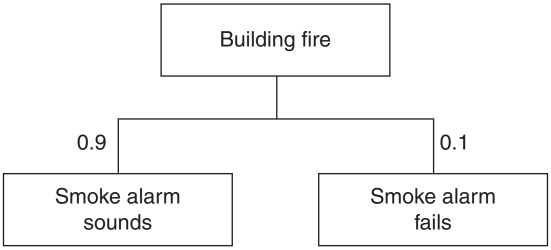

Figure 24.4 Event tree I; Illustrative Example 24.12.

fire damage (alarm sounds, sprinkler works), moderate fire damage with few injuries (alarm sounds, sprinkler fails), moderate fire damage with many injuries (alarm fails, sprinkler works), and major fire damage with many injuries (alarm fails, sprinkler fails). Construct an event tree and indicate the probabilities for each of the four consequences.

***SOLUTION*:** As described in the previous illustrative example, an event tree provides a diagrammatic representation of event sequences that begin with a so-called *initiating* event and terminate in one or more undesirable consequences. In contrast to a fault tree (considered in the next illustrative example) which works backward from an undesirable consequence to possible causes, an event tree works forward from the initiating event to possible undesirable consequences. The initiating event may be equipment failure, human error, power failure or some other event that has the potential for adversely affecting the environment or an ongoing process and/or equipment.

1. Determine the first consequence(s) of the building fire and list the probabilities of the first consequence. See Figure 24.4.
2. Determine the second consequence(s) of the building fire and then list the probabilities of the consequence(s). See Figure 24.5.
3. Determine the final consequences and calculate the probabilities of minor fire damage, moderate fire damage with few injuries, moderate fire damage with many injuries, and major fire damage with many injuries. See Figure 24.6.

Note that for each branch in an event tree, the sum of probabilities must equal 1.0. Note again that an event tree includes the following: (1) works forward from the initial event, or an event that has the potential for adversely affecting an ongoing process, and ends at one ore more undesirable consequences, (2) is used to represent the possible steps leading to a failure or accident, (3) uses a series of branches which relate the proper operation and/or failure of a system with the ultimate consequences, (4) provides a quick identification of the various hazards which could result from a single initial event, (5) is beneficial in examining the possibilities and consequences of a failure, (6) usually does not quantify (although it can) the potential of the event occurring, and (7) can be incomplete if all the initial events are not identified.

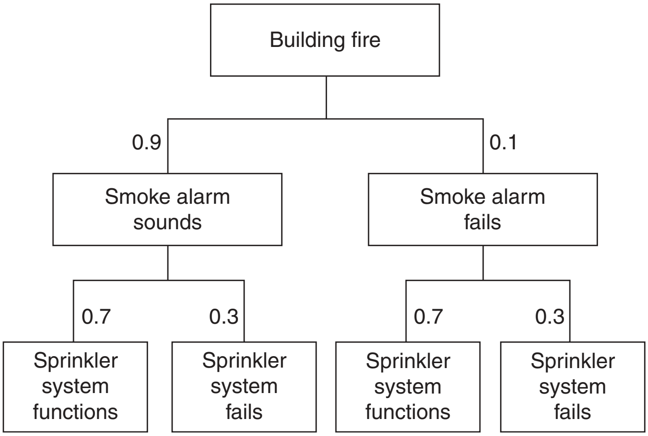

Figure 24.5 Event tree II; Illustrative Example 24.12.

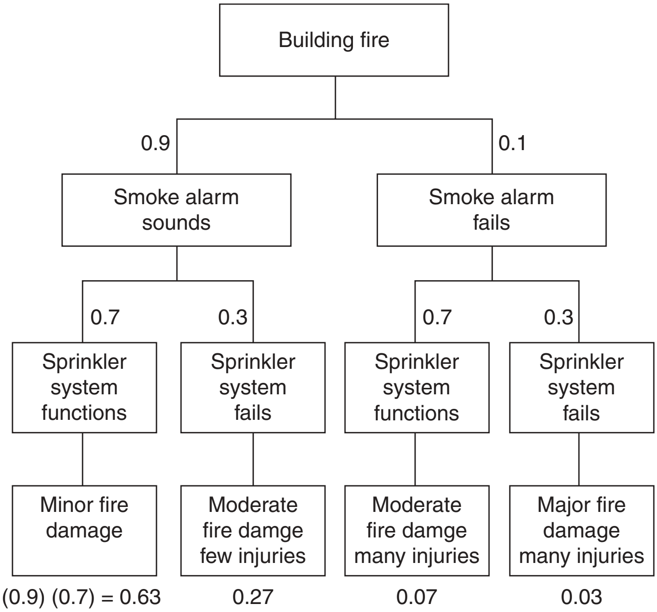

Figure 24.6 Event tree III; Illustrative Example 24.12.

Thus, the use of an event tree is sometimes limiting for hazard analysis because it lacks the capability of quantifying the potential of the event occurring. As noted, it may also be incomplete if all the initial occurrences are not identified. Its use is beneficial in examining, rather than evaluating, the possibilities and consequences of a failure. For this reason, a fault tree analysis should supplement this model to establish the probabilities of the event tree branches. This topic is introduced in the next illustrative example. ■

ILLUSTRATIVE EXAMPLE 24.13

A runaway chemical reaction can occur if coolers fail (A) *or* there is a bad chemical batch (B). Coolers fail only if both cooler #1 fails (C) *and* cooler #2 fails (D). A bad chemical batch occurs if there is a wrong mix (E) *or* there is a process upset (F). A wrong mix occurs if there is an operator error (G) *and* instrument failure (H). Construct a fault tree. If the following annual probabilities are provided by the plant engineer. Also calculate the probability of a runaway chemical reaction occurring in a year's time.

$$P(C) = 0.05$$
$$P(D) = 0.08$$
$$P(F) = 0.06$$
$$P(G) = 0.03$$
$$P(H) = 0.01$$

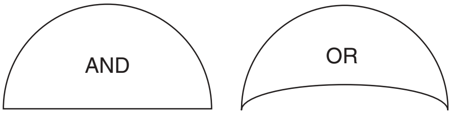

Figure 24.7 AND and OR Gates.

Fault tree analysis seeks to relate the occurrence of an undesired event to one or more antecedent events. The undesired event is called the “top event” and the antecedent events called “basic events.” The top event may be, and usually is, related to the basic events via certain intermediate events. The fault tree diagram exhibits the casual chain linking of the basic events to the intermediate events and the latter to the top event. In this chain, the logical connection between events is illustrated by so called “logic gates.” The principal logic gates are the AND and OR gates, the symbols for which are shown in Figure 24.7.

SOLUTION**:** Construct the fault tree. See Figures 24.8–24.11.

Obtain the first branch of the fault tree, applying the logic gates.

Obtain the second branch of the fault tree, applying the logic gates.

Obtain the third branch of the fault tree, applying the logic gates.

Calculate the probability that the runaway reaction will occur.

$$P = (0.05)(0.08) + (0.01)(0.03) + 0.06 = 0.0643$$

Note that the major contribution to the probability comes from F (process upset).

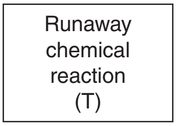

Figure 24.8 Fault tree I; Illustrative Example 24.13.

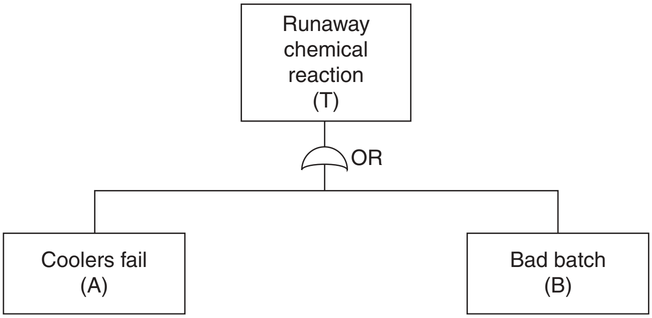

Figure 24.9 Fault tree II; Illustrative Example 24.13.

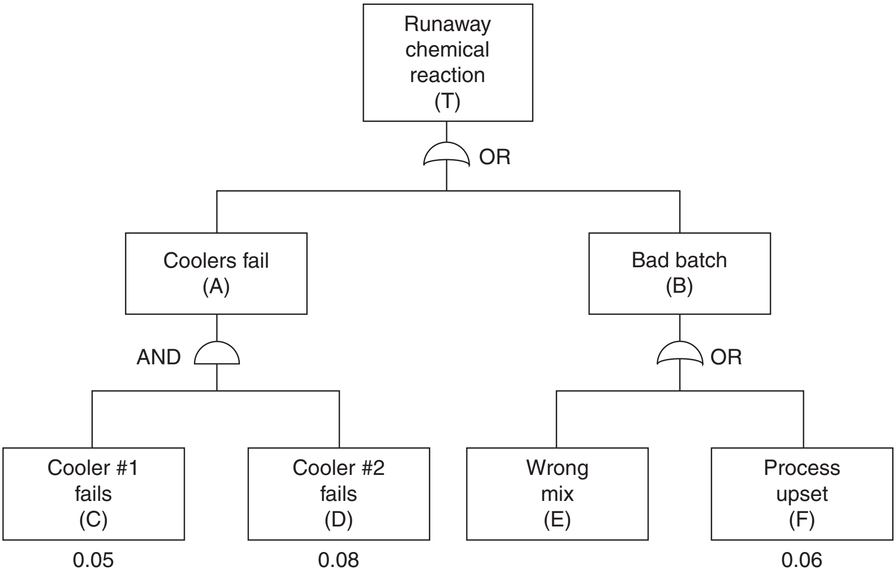

Figure 24.10 Fault tree III; Illustrative Example 24.13.

The reader should note that a fault tree includes the following:

1. Works backward from an undesirable event or ultimate consequence to the possible causes and failures.
2. Relates the occurrence of an undesired event to one or more preceding events.
3. "Chain links" basic events to intermediate events which in turn is connected to the top event.
4. Is used in the calculation of the probability of the top event.
5. Is based on the most likely or credible events which lead to a particular failure or accident.
6. Analysis includes human error as well as equipment failures. ■

ILLUSTRATIVE EXAMPLE 24.14

Discuss the HAZOP (Hazard and Operability) procedure.

***SOLUTION*:** Specific details regarding this procedure are available in the literature.(4,17) The overall HAZOP method, however, is summarized in the following steps:

1. Define objective(s).
2. Define plant limits.
3. Appoint and train a team.
4. Obtain (generate) complete preparative work.

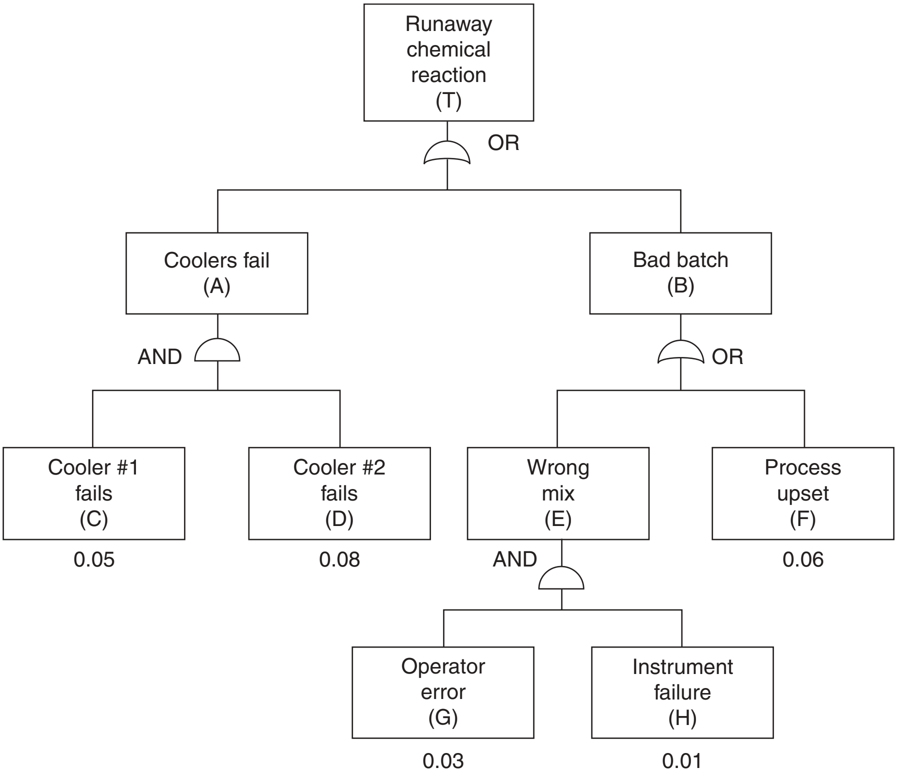

Figure 24.11 Fault tree IV; Illustrative Example 24.13.

5. Conduct examination meetings in order to:
   - **a.** Select a manageable portion of the process.
   - **b.** Review the flowsheet and operating instructions.
   - **c.** Agree on how the process is intended to operate.
   - **d.** State and record the intention.
   - **e.** Search for possible ways to deviate from the intention, utilizing the HAZOP "guide" words.(2,4,16,17)
   - **f.** Determine possible causes for the deviation.
   - **g.** Determine possible consequences of the deviation.
   - **h.** Recommend action(s) to be taken.
6. Issue meeting reports.
7. Follow up on recommendations.

After the serious hazards have been identified with a HAZOP study or some other type of qualitative approach, a quantitative examination should be performed. Hazard quantification or hazard analysis (HAZAN) involves the estimation of the expected frequencies or probabilities of events with adverse or potentially adverse consequences. It logically ties together historical occurrences, experience, and imagination. To analyze the sequence of events that lead to an accident or failure, event and fault trees are used to represent the possible failure sequences. ■

ILLUSTRATIVE EXAMPLE 24.15

A heat exchanger is located in a relatively large laboratory with a volume of 1100 m^3 at 22°C and 1 atm. The exchanger can leak as much as 0.75 gmol of hydrocarbon (HC) from the flowing liquid into the room if the exchanger ruptures. A hydrocarbon mole fraction in the air greater than 425 parts per billion (ppb) constitutes a health and safety hazard.

Suppose the heat exchanger fails and the maximum amount of HC is released instantaneously. Assume the air flow in the room is sufficient to cause the room to behave as a continuously stirred tank reactor (CSTR$^{(17-19)}$); i.e., the air composition is spatially uniform. Calculate the ppb of hydrocarbon in the room. Is there a health risk? From a treatment point-of-view, what can be done to decrease the environmental hazard or to improve the safety of the exchanger?

***SOLUTION*:** Calculate the total number of gmols of air in the room, n_{air}. Assuming that air is an ideal gas, 1 gmol of air occupies 22.4 liters (0.0224 m^3) at standard temperature and pressure (273 K, 1 atm). Since the room temperature is not 273 K,

$$\begin{aligned} n_{\text{air}} &= (1100\,\text{m}^3)\left(\frac{1\,\text{gmol}}{0.0224\,\text{STP}\,\text{m}^3}\right)\left(\frac{273\,\text{K}}{295\,\text{K}}\right) \\ &= 45{,}445\,\text{gmol} \end{aligned}$$

Note: STP m^3 indicates the volume (in m^3) that the gas would have at a specified standard temperature and pressure.

The mole fraction of hydrocarbon in the room, x_{HC}, is

$$x_{\text{HC}} = \frac{0.75\,\text{gmol HC}}{45{,}445\,\text{gmol air} + 0.75\,\text{gmol HC}} = 16.5\,\text{ppm} = 16{,}500\,\text{ppb}$$

Since 16,500 ppb $\gg$ 850 ppb, the hazard presents a significant health risk.

To implement safety measures, the potential rupture area should be vented directly into a hood or a duct to capture any leakage in the event of a rupture. Another alternative is input liquid substitution, a source reduction measure;$^{(6)}$ input substitution is the replacement of the fluid flowing in the heat exchanger. ■

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Chapter 25

Ethics

INTRODUCTION

The primary responsibility of an engineering professional is to protect public health and safety. However, engineering professionals also have a responsibility to their employers or clients, to their families and themselves, and to the environment. Meeting these responsibilities will challenge the practicing engineer to draw upon a system of ethical values.

Well, what about ethics? Ethics means "doing the right thing" as opposed to "what you have the right to do." But doing the right thing is not always obvious or easy. In fact, ethical decisions are often difficult and may involve a certain amount of self-sacrifice. Doing the right thing for a practicing engineer can be especially challenging. Furthermore, the corporate and government world has confused this concept by developing ethics programs that emphasize only what you have the right to do. An organization, for example, may have a list—often called a Code of Ethics or Code of Conduct—of what an employee can and cannot get away with. Employees are required to sign an acknowledgement that they have read and understood the list. The company unfortunately calls this "ethics training."(1)

One difficulty in some situations is recognizing when a question of ethics is involved. Frequently, in the area of environmental management, a breach of ethics involves a practice that endangers public health and safety or covers up a violation of a rule or regulation. Occasionally, however, a breach may involve a case of the exact opposite. This might seem an unlikely scenario. How can someone be too honest, too caring or too professional?

Regarding the above, one example is lying to save a life. Suppose you are standing on a street and a woman runs past you chased by two men. She screams, "They are trying to attack me!" as she dashes into the entry of a building around a corner. The men ask you, "Which way did she go?" What do you tell them? Clearly, the right thing is to lie. In this case, the value of caring overrides the value of honesty. This situation is exaggerated to illustrate that sometimes it is appropriate to violate certain values to protect public health and safety. In doing the right thing, ideally one should not have to make snap decisions and should take the time to investigate all of the facts (e.g., whether or not the woman was a thief and the men were police).

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

Sometimes, one must decide how much to sacrifice to ensure public health and safety. In establishing environmental regulations, the regulating agency must decide how safe and how stringent to make the regulations. For example, in the case of air toxic regulations, one standard may result in 10 cancer cases per one million people. But why isn't it for or one or none? Who should decide?(2)

TEACHING ETHICS

Professionals are often skeptical about the value or practicality of discussing ethics in the workplace. When students hear that they are required to take an ethics course or if they opt for one as an elective in their schedules, they frequently wonder whether ethics can be taught. They share the skepticism of the practitioners about such discussion. Of course, both groups are usually thinking of ethics as instruction in "goodness", and they are rightly skeptical, given their own wealth of experience with or knowledge of moral problems. They have seen enough already to know that you cannot change a person's way of doing things simply by teaching about correct behavior.

The teaching of ethics is not a challenge if ethics is understood *only* as a philosophical system. Parks notes that teaching ethics is important, but "if we are concerned with the teaching of ethics is understood as the practice of accountability to a profession vital to the common good, the underlying and more profound challenge before all professional schools (and other organizations) is located in the question: How do we foster the formation of leadership characterized, in part, by practice of moral courage?"(3)

Moral courage requires knowing *and* acting. College and university educators, as well as those charged with ethics training in the private sector, develop a sense of uneasiness when topics such as "fostering leadership formation," "moral courage," or "knowing *and* acting on that knowledge" are mentioned. Such terms resurrect images of theological indoctrination, Sunday school recitations, or pulpit sermonizing. These images contrast sharply with what the present-day professor envisions as the groves of academic freedom and dispassionate analysis. Perhaps out of fear of disrespecting the dignity of students and devaluing their critical reasoning powers or their ability to understand where the truth lies, faculty take a dim view of academic goals that go beyond those strictly cognitive. The consequence of such values among the professoriate is the further erosion of a moral commons where an agreed-upon set of values and beliefs allows for discourse on ethics. Of course, the erosion has continued steadily from the inception of the Enlightenment Project in the seventeenth century until the present day wherever industrialized and postindustrialized societies have been subject to rapid cultural, economic, political, and technological change. It is not simply an erosion in the realm of higher education. Practitioners in the engineering and scientific communities experience the same erosion of the moral commons taking place in society as a whole.

The author is certainly in agreement with other colleagues in higher education and those who do ethics training in the private sector, that individuals are not to be manipulated or indoctrinated. However, the author is also convinced that not only should students and other participants in ethics analysis have a body of knowledge but also that they have a responsibility for the civic life of society. Such responsibility requires leadership, moral courage, and action. Of course, none of these characteristics can be demanded or forced, only elicited. That is the great, yet delicate challenge facing the professoriate and all those charged with ethics training in other sectors. Eliciting a sense of civic responsibility as a goal of ethics analysis can be realized only as a derivative of cognitive processes and not as a direct goal. In sum, the formation of personal character and the practice of virtue are not to be subject to external control and the diminution of individual freedom through manipulation or indoctrination.

The reader should note that the bulk of the material to follow has been drawn from the work of Wilcox and Theodore.(4)

THE CASE STUDY APPROACH

The author believes that the case study method is a valuable way to take seriously Parks's response to the question "Can ethics be taught?" He also considers the method to be an important tool in investigating the relationship between assumptions, values, and moral life, as well as an ethical reflection on those three aspects of life. The author is also convinced that the case study method is one of the most useful ways of teaching ethics and of achieving the goals of ethics education outlined below by the Hastings Center.(5)

1. *Stimulating the Moral Imagination*. The concreteness of the case study appeals very much to the learning style of most people. While a certain amount of ambiguity is essential to evoke interest and discussion, it is also a stimulus for enlivening knowledge. Hopefully, the participant will begin to appreciate the moral complexity of a situation, which in the past might have been thought of only as a technical or managerial problem. Practice in the art of case discussion has the larger intent of leading the individual to bring an ethical frame of reference to bear on the variety of problems faced in the discipline studied. Stimulating the moral imagination is similar to putting on a pair of glasses that are tinted. The result is that the whole world is seen through that tint. As a consequence of the case study method, the authors of the cases to follow hope that each individual will see his or her field of study through the interpretive glasses of engineering and environmental ethics. He or she would then routinely ask: "What is the moral issue here?"
2. *Recognizing Ethical Issues*. The case analyst should not be content with a good "imagination." The further challenge is the recognition of specific moral problems and how they differ from one another in terms of immediacy or urgency. Concreteness is an important asset of the case study and clearly assists in achieving this second goal. Comparing and contrasting a variety of cases through discussion is essential to recognition and leads to achievement of the next goal.

3. *Developing Analytical Skills.* Differentiation, comparison, contrasts—all of these must be related to an enhanced ability to solve the problem. To achieve this goal, the student of ethics is taught to bring the skills developed in his or her major field of study to bear on the ambiguous situation, the moral dilemma, or the competing values that must be addressed. Analytic skills are best honed through the use of examples or cases. The technical ability to analyze all dimensions of an environmental spill will have an impact on how the moral aspect of the problem is understood in terms of resolving the problem. Of course, ethical systems that emphasize the importance of consequences, the obligations inherent in a duty-based ethic, as well as theories of justice or virtue, will enhance the ability to use technical or discipline-based analytic skills in resolving the problem. Knowing, however, is related to acting. This leads to the fourth goal.
4. *Eliciting a Sense of Moral Obligation and Personal Responsibility.* Much has already been said about the importance of this goal. However, it should be clear that a sense of moral obligation does not mean that there is one set of absolute answers. Dictating a solution is quite different from an internalization process whereby the individual commits himself or herself to be a "seeker"—one who takes personal responsibility for addressing and resolving the moral problems facing engineers or scientists. Both professions constitute the "guardians of the system" in the technical community. They are the first line of response to the problems and dilemmas facing the professions as such. To point to the Environmental Protection Agency (EPA), the Occupational Safety and Health Administration (OSHA), the Federal Bureau of Investigation (FBI), congressional formulators of public policy, or other sovereign countries as the parties responsible for resolving acute problems is to abnegate one's moral responsibility as a professional. To say this is not to dictate solutions, but to alert individuals to their personal responsibility for the integrity of their respective field. Eliciting a sense of responsibility depends on an assessment of the assumptions or "images at the core of one's heart." Assessment of ethical systems or normative frames of reference must be connected to the actual assumptions or images that constitute a person's worldview. Challenging the individual to examine that worldview in relation to a case and ethical system(s) is the first step in joining doing to knowing. Closely related to the achievement of this goal is the following one.
5. *Tolerating and Resisting—Disagreement and Ambiguity.* An essential component of case discussion is the willingness to listen carefully to the points-of-view held by others. Cases, by their nature, are ambiguous. They are bare-boned affairs meant more to be provocative than to lead to a clear-cut jury decision. The purpose of the case is to stimulate discussion and learning among individuals. As a result, there will be much disagreement surrounding the ethical issues in the case and the best option for resolving it. Toleration does not mean "putting up with people with whom I disagree." Respect for the inherent dignity of the person and a willingness to understand not only

another position but also a person's reasons for or interest in that point-of-view, should be part of the case discussion. Toleration does not mean all opinions must be of equal value and worth. It is true that respect for and listening to another person's argument may lead one to change a position. However, a careful description and discussion of the other person's position may also lead to a greater conviction that one's own position is correct. What is clearly of central concern is the belief that the free flow of ideas and carefully wrought arguments, presented from all sides without fear of control, manipulation, threat, or disdain, is at the core of human understanding and development. This hallowed concept of academic freedom is the catalyst that allows human communities to be committed to the search for truth, without at the same time declaring absolute possession of the truth.

APPLICATIONS

The three Illustrative Examples below have been drawn from the work of Wilcox and Theodore,(4) keying primarily on heat transfer issues. Each example is presented in case-study format, containing both a fact pattern and finally, questions for discussion. These questions are by no means definitive. While they will help individuals focus on the case, the issues raised will make the most sense if they lead to a wide-ranging discussion among all readers. As noted, analysis of ethics cases comes alive in group work. Answering the questions individually is a helpful first step, but one's understanding of ethical problems and dilemmas generally improves dramatically in group discussion.

ILLUSTRATIVE EXAMPLE 25.1

Fact Pattern A chemical plant on the West Coast recently experienced a serious reactor explosion that resulted in several injuries and a fatality. There is an ongoing investigation to find out what caused the explosion. Injured workers and their families are angry and anxious to blame someone for this unfortunate accident.

Stan, one of the head executives at this chemical plant, is trying to get to the bottom of the situation with Terry, a plant manager.

Stan asks, "So, did you find out anything new?"

Terry responds, "Well, the reactor that exploded was recently upgraded to increase production. Everything was supposedly checked out, but someone overlooked a minor factor. The reactor eventually overloaded and exploded."

Stan asks, "Well, who's responsible for that mistake?"

Terry disappointingly says, "Well, remember Laura? She was one of the chemical engineers working on that production line. She quit about a month ago. Everyone seems to be pointing fingers at her, saying that she was in charge of upgrading that reactor."

"I have all of these angry people ready to seek revenge for this explosion. Terry, the reputation of this company is at stake. Not only have we caused injuries, but we have polluted the environment. Now you tell me the person at fault isn't even here anymore. How is that supposed to help anything?" shouts Stan.

Reluctantly, Terry suggests, "Well, we could always blame someone who is dispensable. For example, Rob, one of the janitors that works here, has been acting up lately. His supervisors have warned him about slacking off. He has quite a temper and tends to scream threats at people. We could try to blame him."

Stan asks, "But how would we do that? Wouldn't people know that it's not his fault? Can we get away with that?"

"Well, most people wouldn't be surprised Rob got mad about something and decided to mess up one of the reactors. He was working the night the explosion occurred. In fact, the explosion happened during one of his shifts. Maybe he was at the other side of the plant when it occurred, which is probably why he was not hurt," says Terry.

"But isn't this blatantly wrong? We cannot blame an innocent man for this," Stan comments.

"Stan, we are in a serious mess here. Our company's reputation is on the line. You said so yourself. We may all lose our jobs. No one will care if we blame a rowdy janitor for this. He was bound to be fired anyway. His attitude was not going to be tolerated much longer. I think that this is the best solution for everyone," says Terry.

After pondering the situation for a few minutes, Stan agrees. "After weighing all of the pros and cons, I think you're right, Terry. Our blaming Rob for the explosion may not be the moral thing to do, but we have to try to solve this problem by hurting as few people as possible. If we do it this way, we might be able to recover from this disaster."

Questions for Discussion

1. What are the facts in this case?
2. What other options do Terry and Stan have to remedy the situation?
3. Was the decision to blame Rob justified?
4. Do you think Terry and Stan can get away with blaming Rob for the explosion?
5. If the truth was revealed, what do you think would happen to Stan, Terry, Rob, and the rest of the company? ■

ILLUSTRATIVE EXAMPLE 25.2

Fact Pattern Curts Inc. is a company specializing in applications involving power plants. Recently, their business has taken a jump forward, and the company has started obtaining contracts faster than they can hire and train individuals.

John is a mechanical engineer for Curts Inc. Because of the recent lack of employees, he has been juggling two or more projects at the same time. One project that he neglected for a while is a contract with the local utility company to modify an existing burner to use a more economical fuel and still produce the same amount of energy.

Now, the project is near the end, and he just got started on his assignment of giving a detailed report about the fuel to be used. Initial information was gathered by a team of engineers who concluded the fuel will be easy to obtain if imported from South America and that it will produce the same amount of energy. John finds out that the fuel, which is a very crude (tarlike) oil, is suspected of being carcinogenic when burned, according to a study done on this type of fuel.

When he brings this to the attention of his peers, they accuse him of trying to destroy all of their work at the last minute.

John takes a moment to think over his options: Should he destroy the whole team's work, or forget about that *one* study and finish the project?

Questions for Discussion

1. What are the facts in this case?
2. What is the ethical problem John faces?
3. Does John have more than two options?
4. Which course of action would you recommend to John? ■

ILLUSTRATIVE EXAMPLE 25.3

Fact Pattern Ricardo is beginning his fourth year as an engineering student at a university in Boston and is now in charge of a team of students who are determined to build the school's first solar-electric car. In order to gain corporate donations and university funding, the car must gain a great deal of exposure. It is decided by the team that the car will race in the Tour de Sol in May 2010. This gives the students a goal and provides ample justification for investment.

The time frame from the team's inception of the idea to the date of the race is less than a year; this fact poses the obvious problem of feasibility. To overcome this difficulty, the group presents the solar car project to all of the senior design classes in every discipline of engineering. The idea is to split the car into many smaller projects; teams would finish their concentrated assignments and present them as senior projects. The idea is also pitched to marketing, business, and communications seniors, who can design investment solicitation plans and help the team gain access to local and national media.

Within the next few weeks, teams are set up for all known aspects of the project, and Ricardo is to coordinate everything—creating communication links between teams, setting goals, and negotiating timelines for the integration of all the projects. Key to all aspects of the project are safety considerations. In addition to all government safety regulations, the car has to abide by several safety guidelines set by the Tour de Sol. In order to obtain money and materials, the team has to convince every sponsor that all Tour de Sol requirements will be met on time.

Over the following few weeks, car body designs are submitted, altered, and reworked until a final design is agreed upon. At that point a massive effort is made to obtain the materials and components necessary to construct it. With those in hand and a procedure plan laid out, groups of five to twenty people work in rotation twenty-four hours a day for the next month to complete the monocoque of the car (the shell made of all advanced composite materials). At this point, every team has the huge task of integrating the final design of its product to meet the actual specifications of the monocoque and the preliminary specs of other product groups. During this process the interior design group discovers that miscommunication between their group and the body team has led to a monocoque design that is not tall enough at certain critical points. There is a specific regulation regarding the height from the bottom of the lowest part of the seat to the ceiling of the car. The height of the car at this point is about an inch too short, and the seat configuration also takes an additional, unexpected inch of height. This causes a two-inch disparity between the actual and the designed configuration, and a breach of regulation by about an inch. More important, this change also directly reduces the distance between the top of the driver's helmet and the ceiling, which means that, in the event of even a minor crash, the driver's chances of serious injury to the neck are increased. The interior group analyzes the situation and determines that the seat angle cannot be changed to reduce the height because they have designed the seat based on the largest ergonomically feasible angle to minimize the height in the first place.

A meeting is held including Ricardo and all of the team members involved in the process. It is determined that there is nothing that can be done in the interior of the car without breaking several safety regulations directly related to the driver. The only possible way to bring the car within these regulations is to increase the height of the monocoque, which will require the reconstruction of the entire monocoque and is impossible from both time and resources standpoints. Ricardo is faced directly with either enforcing a redesign that would effectively scrap the project or keeping a design that breaks the height regulation. He decides to postpone the decision by allowing everyone to work as scheduled and assigning a person from each of the two relevant teams to try to find an alternative solution. Weeks pass, and the deadline is approaching. The two team members report that there is no way, outside of a new monocoque, to change the height without causing more severe problems. They do report, however, that this regulation was never actually checked by a third party in past races and is used more as a guideline than as a hard-and-fast rule. Since there seems no alternative and apparently no punishment is rendered for breaking the rule, all of the group members agree to allow the "violation" to go unchanged. Ricardo despises the predicament and the decision, but does not want to let his entire project go down the tubes for a technicality. The decision holds and the car is finished barely in time, yet Ricardo is still uneasy in the final days before the race.

Questions for Discussion

1. What are the facts in this case?
2. What is the ethical problem facing Ricardo?
3. How could Ricardo have anticipated the problem before it was too late?
4. What are Ricardo's options in handling the situation?
5. What do you think should be done?
6. What do you think Ricardo's actions should have been upon learning of the situation? ■

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Chapter 26

Numerical Methods

INTRODUCTION

Chapter 26 is concerned with Numerical Methods. This subject was taught in the past as a means of providing engineers and scientists with ways to solve complicated mathematical expressions that they could not solve otherwise. However, with the advent of computers, these solutions have become readily obtainable.

A brief overview of Numerical Methods is given to provide the practicing engineer with some insight into what many of the currently used software packages (MathCad, Mathematica, MatLab, etc.) are actually doing. The author has not attempted to cover all the topics of Numerical Methods. There are several excellent texts in the literature that deal with this subject matter in more detail.(1,2)

Ordinarily, discussion of the following eight numerical methods would be included in this chapter:

1. Simultaneous linear algebraic equations.
2. Nonlinear algebraic equations.
3. Numerical integration.
4. Numerical differentiation.
5. Ordinary differential equations.
6. Partial differential equations, including Monte Carlo methods.
7. Regression analysis.
8. Optimization.

However, because of the breadth of the subject matter, the reader should note that only the last three numerical methods receive treatment in the chapter. The remaining five methods are to be found in the literature.(3,4) It should be noted that the Applications Section contains heat transfer material dealing with the later three topics.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

HISTORY

Early in one's career, the engineer/scientist learns how to use equations and mathematical methods to obtain exact answers to a large range of relatively simple problems. Unfortunately, these techniques are often not adequate for solving real-world problems. The reader should note that one rarely needs exact answers in technical practice. Most real-world solutions are usually inexact because they have been generated from data or parameters that are measured, and hence represent only approximations. What one is likely to require in a realistic situation is not an exact answer but rather one having reasonable accuracy from an engineering point of view.

The solution to an engineering or scientific problem usually requires an answer to an equation or equations, and the answer(s) may be approximate or exact. Obviously an exact answer is preferred, but because of the complexity of some equations, often representing a system or process, exact solutions may not be attainable. For this condition, one may resort to another method that has come to be defined as a numerical method. Unlike the exact solution, which is continuous and in closed form, numerical methods provide an inexact (but reasonably accurate) solution. The numerical method leads to discrete answers that are almost always acceptable.

The numerical methods referred to above provide a step-by-step procedure that ultimately leads to an answer and a solution to a particular problem. The method usually requires a large number of calculations and is therefore ideally suited for digital computation.

High-speed computing equipment has had a tremendous impact on engineering design, scientific computation, and data processing. The ability of computers to handle large quantities of data and to perform mathematical operations at tremendous speeds permits the examination of many more cases and more engineering variables than could possibly be handled on the slide rule—the trademark of engineers of yesteryear. Scientific calculations previously estimated in lifetimes of computation time are currently generated in seconds and, in many instances, microseconds.(5)

One of the earliest modern day computers (perhaps more appropriately called a calculator) appeared on the scene in 1914. A new patented indicator or runner for slide rules, called the "frameless," was perfected by Keuffel & Esser Co. The "framless" was unique in that every figure on the rule was clearly visible at all times such there was no side piece(s) to the metal holder of the glass indicator. In 1968, Hewlett-Packard announced the development of a calculator that could "deftly handle arithmetically large numbers"; the calculator weighted 40 lbs and cost nearly $5000.

A procedure-oriented language (POL) is a way of expressing commands to a computer in a form somewhat similar to such natural languages as English and mathematics. The instructions that make up a program written in a POL are called the source code. Because the computer understands only machine language (or object code), a translator program must be run to translate the source code into an object code. In terms of input, processing, and output, the source code is the input to the translator program, which processes (translates) the code. The output is the object code. It is the object code that is actually executed in order to process data and information.

The first POL to be widely used was FORTRAN, an acronym that was coined from the words "FORmula TRANslation." FORTRAN was designed initially for use on problems of a mathematical nature and it is still used for solving some problems in mathematics, engineering, and science.

PASCAL is a POL designed by Niklaus Wirth in 1968. The motivation behind its design was to provide a language that encouraged the programmer to write programs according to the principles of structured programming. An important aspect of the PASCAL design philosophy is that it is a "small" language. The purpose of this is to provide the programmer with a language that can be easily learned and retained. PASCAL is also most commonly used in mathematics, engineering and science.

BASIC is an acronym for Beginner's All-purpose Symbolic Instruction Code. J. G. Kemeny and T. E. Kurtz developed BASIC in 1967 to give students a simple language for learning programming. BASIC is an interactive language; i.e., the programmer sees an error or output as soon as it occurs. The simplicity of BASIC makes it easy to learn and use. Many versions of BASIC have been written since the late 1960s. BASIC can be used effectively for a variety of business and scientific applications.

Two types of translator programs—compilers and interpreters—are used to convert program statements to a machine-readable format. A compiler first translates the entire program to machine language. If any syntax or translation errors are encountered, a complete listing of each error and the incorrect statement is given to the programmer. After the programmer corrects the errors, the program is compiled again. When no errors are detected, the compiled code (object code) can be executed. The machine-language version can then be saved separately so that the compiling step need not be repeated each time the program is executed unless the original program is changed. Compiled programs run much faster than the interpreted ones. An interpreter translates and executes one source code instruction of a program at a time. Each time an instruction is executed, the interpreter uses the key words in the source code to call pre-written machine-language routines that perform the functions specified in the source code. The disadvantage of an interpreter is that the program must be translated each time it is executed.

The general availability of computers has been followed by a near infinite number of application programs, promising relief from the aforementioned slide rule. Many of these programs deal with design optimization for heat exchangers as well as other process equipment and full scale plant applications. Today, many powerful commercial mathematical applications are available and widely used in academia and industry. Some of these programs include MathCad, Matlab, Mathematica, etc. These user-friendly programs allow engineers and scientists to perform mathematical calculations without knowing any programming. In addition, new programs, for example, Visual Basic, .NET, JAVA, C++, etc., are constantly evolving. These software packages are the modern day slide rule for the 21st century engineer and the numerical methods listed below provide the "instructions" on how to properly use this new slide rule.

With commercial mathematical application's powerful computing ability, learning the numerical procedures involved does little for the practicing engineer in terms of interpreting and analyzing answers. For example, instead of performing a numerical integration using the trapezoid rule, one can use one of the programs mentioned above to do the calculation. However, the job of the engineer then becomes that of assessing if the answer given by the computer program seems reasonable. If it does, one can be fairly certain the numbers are correct (providing the rest of the input was correct), but if the answer seems unreasonable, the engineer's job is to figure out what technique the program is using and what might cause it to return an incorrect answer (or, more precisely, a correct answer to the equation, but not to the problem).

PARTIAL DIFFERENTIAL EQUATIONS

Many practical problems in engineering involve at least two independent variables; that is, the dependent variable is defined in terms of (or is a function of) more than one independent variable. The derivatives describing these independent variables are defined as partial derivatives. Differential equations containing partial derivatives are referred to as partial different equations (PDEs).

Contrary to a widely accepted myth, an engineer's mathematical obligations do not end after formulating a problem, where it may be given to a mathematician to solve. Even if such an ideal situation should exist, it is still necessary for engineers to have a reasonable understanding of the mathematical methods and their limitations in order to interpret results.

It has been said that "the solution of a partial differential equation is essentially a guessing game." In other words, one cannot expect to be given a formal method that will yield exact solutions for all partial differential equations.(6) Fortunately, numerical methods for solving these equations were developed during the middle and latter part of the 20th century.

The three main PDEs encountered in engineering practice are briefly introduced below employing T (e.g., the temperature as the dependent variable) with t (time) and x, y, z (position) the independent variables.

The parabolic equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} \tag{26.1}$$

The elliptical equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{26.2}$$

The hyperbolic equation:

$$\frac{\partial^2 T}{\partial t^2} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{26.3}$$

The preferred numerical method of solution involves finite differencing. Only the parabolic and elliptical equations are considered below.

Parabolic PDE

Examples of parabolic PDEs include

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{26.4}$$

and (the two-dimensional)

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \tag{26.5}$$

Ketter and Prawler,(2) as well as many others, have reviewed the finite difference approach to solving Equation (26.4). This is detailed below.

Consider the (t, x) grid provided in Figure 26.1. The partial derivatives may be replaced by

$$\frac{\partial T}{\partial t} \cong \frac{\Delta T}{\Delta t} = \frac{-T_4 + T_2}{2(\Delta t)} = \frac{-T_4 + T_2}{2k}; \ \Delta t = k \tag{26.6}$$

and

$$\frac{\partial^2 T}{\partial x^2} \cong \frac{\Delta}{\Delta x}\left(\frac{\Delta T}{\Delta x}\right) = \frac{T_3 - 2T_0 - T_1}{h^2}; \ \Delta x = h \tag{26.7}$$

Substituting Equations (26.6) and (26.7) into Equation (26.4) leads to

$$\frac{-T_4 - T_2}{2k} = \frac{T_3 - 2T_0 - T_1}{h^2} \tag{26.8}$$

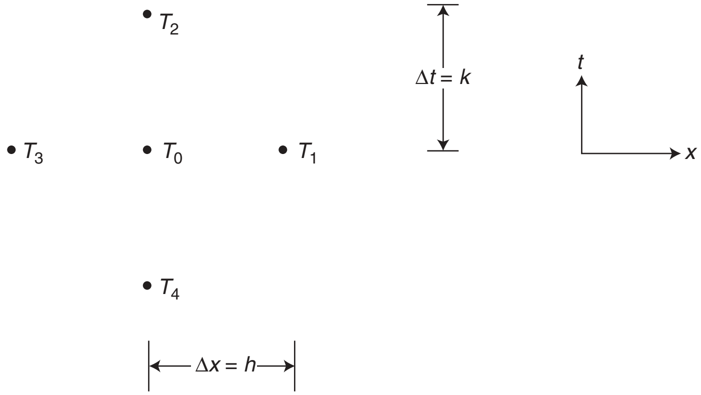

Figure 26.1 Parabolic grid.

Solving for T_2:

$$T_2 = T_4 + 2r(T_3 - 2T_0 - T_1) \tag{26.9}$$

where $r = k/h^2$,

Thus, T_2 may be calculated if T_0, T_1, T_3, and T_4 are known. Unfortunately, stability and error problems arise employing the above approach. These can be removed by replacing the central difference term in Equation (26.6) by a forward difference term, that is,

$$\frac{\partial T}{\partial t} \cong \frac{\Delta T}{\Delta t} = \frac{-T_0 + T_2}{(\Delta t)} = \frac{-T_0 + T_2}{k} \tag{26.10}$$

With this substitution, Equation (26.9) becomes

$$T_2 = T_0 + r(T_3 - 2T_0 + T_1) \tag{26.11}$$

It can be shown that the problem associated with the central difference derivative is removed if $r \leq 0.5$.

An unsteady-state heat conduction experiment conducted at Manhattan College's Unit operations laboratory is concerned with the application of this equation. In the experiment, a 316 stainless steel rod is heated at each end with steam at temperature T_S. The initial temperature of the rod is T_A. The describing equation and Ba/o IC(s) for this system are

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{26.1}$$

BC(1): $T = T_S$ at $x = 0$

BC(2): $T = T_S$ at $x = L$

IC: $T = T_A$ at $t = 0$, $0 \leq x \leq L$

The solution to this equation can be shown$^{(7)}$ to be:

$$T = T_S + (T_A - T_S)\left\{\sum_{n=1}^{\infty} 2\left[\frac{(-1)^{n+1} + 1}{n\pi}\right] e^{-\alpha(n\pi/2)^2 t} \sin\left(\frac{n\pi x}{2}\right)\right\} \tag{26.12}$$

The reader may choose to (again) review this essentially same problem in Illustrative Example 8.3.

Elliptical PDE

For this equation, examine the grid in Figure 26.2. Using finite differences in Equation (26.2) ultimately leads to

$$T_0 = \frac{1}{4}(T_1 + T_2 + T_3 + T_4);\ \Delta x = \Delta y \tag{26.13}$$

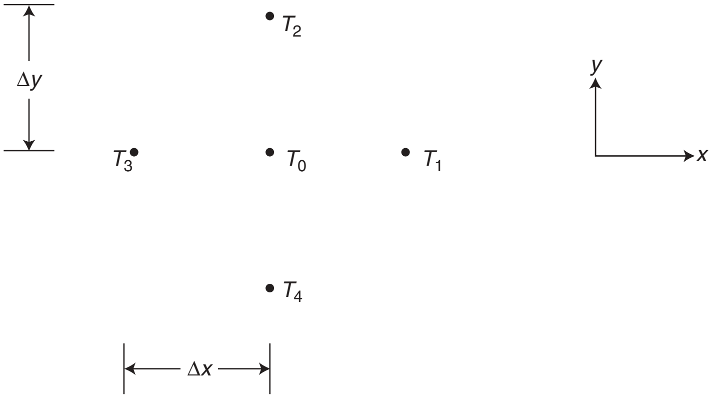

Figure 26.2 Elliptic grid.

In effect, each T value reduces to the average it its four nearest neighbors in the square grid. This difference equation may then be written at each interior grid point, resulting in a linear system of N equations, where N is the number of grid points. The system can then be solved by one of several methods provided in the literature.(1,2,7)

Another method of solution involves applying the Monte Carlo approach, requiring the use of random numbers.(7) Considered the square pictured in Figure 26.3. If the describing equation for the variation of T within the grid structure is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{26.14}$$

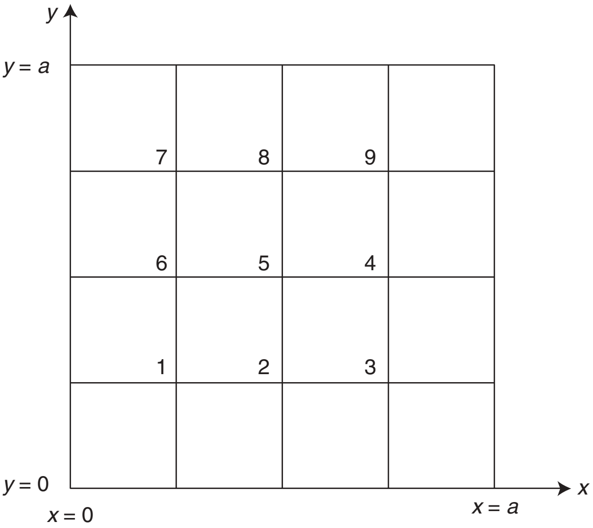

Figure 26.3 Monte Carlo grid.

with specified boundary conditions (BC) for $T(x, y)$ of $T(0, y)$, $T(a, y)$, $T(x, 0)$, and $T(x, a)$ one may employ the following approach.

1. Proceed to calculate T at point 1 (i.e., T_1).
2. Generate a random number between 00 and 99.
3. If the number is between 00–24, move to the left. For 25–49, 50–74, 75–99, move upward, to the right, and downward, respectively.
4. If the move in step 3 results in a new position that is at the outer surface (boundary), terminate the first calculation for point 1 and record the T value of the boundary at the new position. However, if the move results in a new position that is not at a boundary, and is still at one of the other eight interval grid points, repeat steps (2) and (3). This process is continued until an outer surface or boundary is reached.
5. Repeat steps (2–4) numerous times, e.g., 1000 times.
6. After completing step (5), sum all the T values obtained and divided this value by the number of times steps (2–4) have been repeated. The resulting value provides a reasonable estimate of T_1.
7. Return to step (1) and repeat the calculation for the remaining eight grid points.

This method of solution is not limited to square systems. The general energy equation for heat transfer in solids is once again presented below for rectangular, cylindrical, and spherical coordinate systems. See also Chapters 7 and 8.

Rectangular coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] \tag{26.15}$$

Cylindrical coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] \tag{26.16}$$

Spherical coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right] \tag{26.17}$$

ILLUSTRATIVE EXAMPLE 26.1(8)

Consider the two-dimensional problem of a very thin solid bounded by the y-axis ($z = 0$), the lines $y = 0$ and $y = 1$, and extending to infinity in the z-direction. The temperature T of the

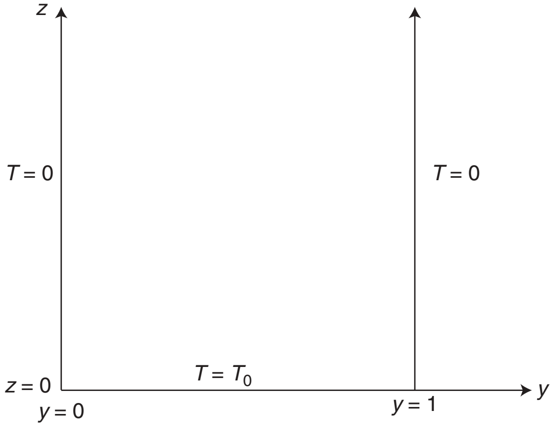

Figure 26.4 System in Illustrative Example 26.1.

vertical edge at $y = 0$ and $y = 1$ is maintained at zero. T at $z = 0$ is T_0. Obtain the equation describing the steady-state profile of T in the solid. Refer to Figure 26.4.

***SOLUTION*:** Select rectangular coordinates. Based on the problem statement, T is not a function of x. Thus, T is solely a function of y and z. Thus, $T = T(y, z)$. The following equation is extracted from Equation (26.15) provided above.

$$\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

One can show that the analytical solution to the describing equation, subject to the boundary conditions specified is:$^{(9)}$

$$T = \sum_{n=1}^{\infty} \frac{2T_0}{n\pi}[(-1)^{n+1} + 1]e^{-(n\pi/1)z} \sin\left(\frac{n\pi y}{1}\right)$$

■

ILLUSTRATIVE EXAMPLE 26.2

Consider the system pictured in Figure 26.5. If the system is solid and the variable T is the temperature, the elliptical equation provided in Equation (26.2) applies. For this system, the temperature at each location is given by the average temperature of its four neighboring points. Using the Monte Carlo procedure provided earlier in this section and in Chapter 24 (see Illustrative Example 26.10 later), generate the temperature profile in the solid.

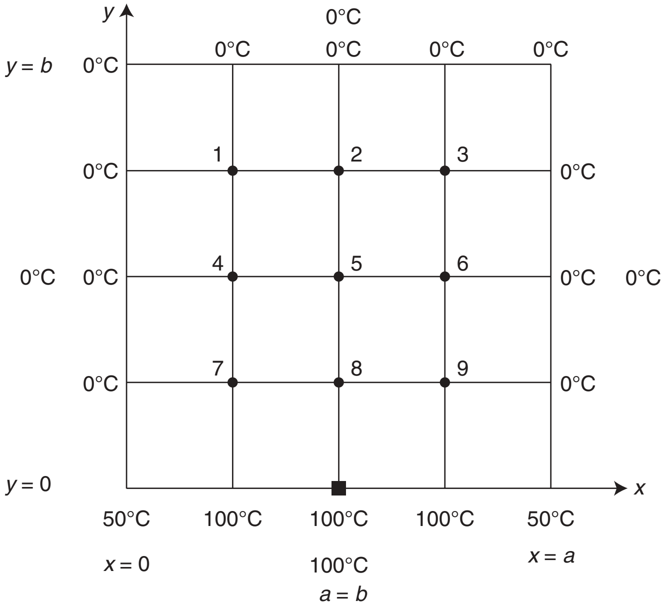

Figure 26.5 Temperature grid for a square.

***SOLUTION*:** Using the Monte Carlo algorithm provided and a random number generator leads to the following results

$$T_1 = 7.07^\circ\text{C}$$
$$T_2 = 9.80^\circ\text{C}$$
$$T_3 = 7.16^\circ\text{C}$$
$$T_4 = 18.67^\circ\text{C}$$
$$T_5 = 25.12^\circ\text{C}$$
$$T_6 = 18.77^\circ\text{C}$$
$$T_7 = 42.93^\circ\text{C}$$
$$T_8 = 52.57^\circ\text{C}$$
$$T_9 = 42.80^\circ\text{C}$$

■

This procedure can be extended to rectangles; it is not limited to squares. If a rectangle is subjected to the following (somewhat similar) boundary conditions.

$$T(x = 0, \text{any } y) = T_0$$
$$T(x = b, \text{any } y) = 0$$
$$T(\text{any } x,\ \ y = +a) = 0$$
$$T(\text{any } x, y = -a) = 0$$

The analytical solution for the above rectangle ($2a$ by b) is

$$T = 2T_0 \sum_{n=0}^{\infty} \left\{ \frac{(-1)^n}{\lambda_n a} \right\} \left\{ \frac{\sinh[\lambda_n(b-x)]}{\sinh(\lambda_n b)} \right\} \cos(\lambda_n y)$$

where $\lambda_n = \dfrac{(2n+1)\pi}{2a}$; $n = 0, 1, 2, 3 \ldots$

ILLUSTRATIVE EXAMPLE 26.3

Solve Illustrative Example 26.2 using the Gauss-Elimination method.

***SOLUTION*:**(9) The familiar finite difference approach will be employed. For example,

$$T_5 \equiv (1/4)[T_2 + T_4 + T_6 + T_8]$$

This form of the equation may be applied to the nine points, which sets each T equal to the average of temperature of its four neighboring points. This produces a set of nine equations with nine unknowns. Employing Mathematica, Mathcad, Excel or a similar program leads to

$T_1 = 7.14°\text{C}$
$T_2 = 9.82°\text{C}$
$T_3 = 7.14°\text{C}$
$T_4 = 18.75°\text{C}$
$T_5 = 25°\text{C}$
$T_6 = 18.75°\text{C}$
$T_7 = 42.86°\text{C}$
$T_8 = 52.68°\text{C}$
$T_9 = 42.86°\text{C}$ ■

ILLUSTRATIVE EXAMPLE 26.4

Comment on the results of the two previous examples.

***SOLUTION*:** As expected the results are in reasonable agreement with each other. In addition,

$$T_8 > T_7 > T_5 > T_4 > T_2 > T_1$$

with,

$$T_1 \cong T_3$$
$$T_4 \cong T_6$$
$$T_7 \cong T_9$$

■

ILLUSTRATIVE EXAMPLE 26.5

Outline how to solve the previous example if the face surface is a rectangle solid (see Figure 26.6).

***SOLUTION*:** The outline of the calculation presented in the previous examples remains the same. ■

ILLUSTRATIVE EXAMPLE 26.6

A rectangular walled stack (see Figure 26.7) serves as the discharge conduit for flue gases from an incinerator. If the inner and outer (atmosphere) steady-state temperatures are 1700°F and 1400°F respectively, calculate the temperature profile in the stack wall.

***SOLUTION*:** Due to symmetric only the local temperatures in the bottom part of Figure 26.7, i.e., points (nodes) 4–12, will be calculated. First note, that

$$T_1 = T_2 = T_3 = T_4 = 1700^\circ\text{F}$$
$$T_{13} = T_{14} = T_{15} = T_{16} = T_{17} = T_{18} = 1400^\circ\text{F}$$

Neglecting "curvature" effects, one notes that the temperature at nodes below points 3 and 4 are approximately equal. Applying the appropriate equation at each node leads to 9 equations

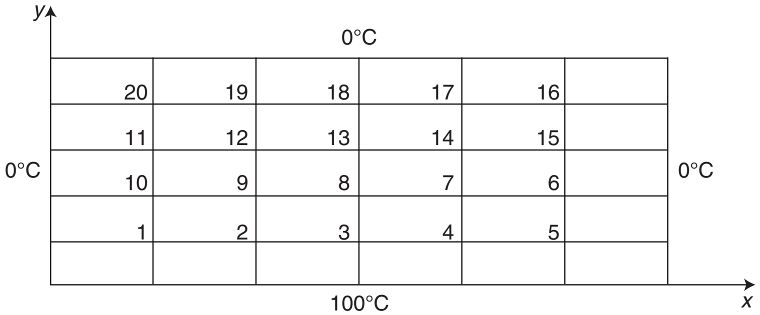

Figure 26.6 Rectangular grid; Illustrative Example 26.5.

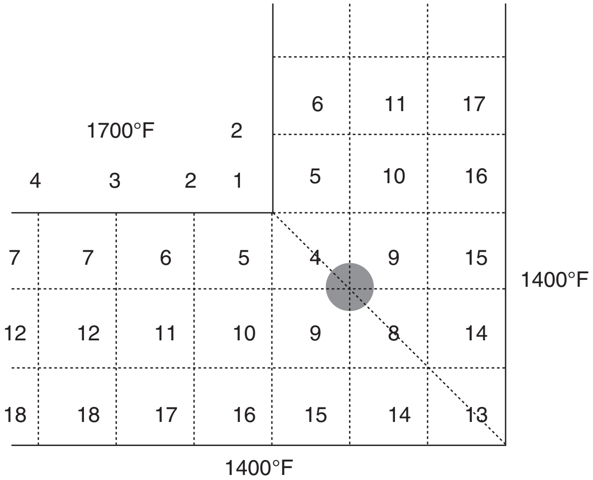

Figure 26.7 Stack temperature profile; Illustrative Example 26.6.

and 9 unknowns

$$\begin{aligned}
4:&\quad T_4 = (2T_5 + 2T_9)/4\\
5:&\quad T_5 = (T_4 + T_6 + T_{10} + 1700)/4\\
6:&\quad T_6 = (T_5 + T_7 + T_{11} + 1700)/4\\
7:&\quad T_7 = (2T_6 + T_{12} + 1700)/4\\
8:&\quad T_8 = (2T_9 + 1400 + 1400)/4\\
9:&\quad T_9 = (T_4 + T_8 + T_{10} + 1400)/4\\
10:&\quad T_{10} = (T_5 + T_9 + T_{11} + 1400)/4\\
11:&\quad T_{11} = (T_6 + T_{10} + T_{12} + 1400)/4\\
12:&\quad T_{12} = (T_7 + 2T_{11} + 1400)/4
\end{aligned}$$

Employing a suitable numerical method proposed in an earlier example leads to:

$$\begin{array}{lll}
T_4 = 312^\circ\text{F} & T_7 = 392^\circ\text{F} & T_{10} = 278^\circ\text{F}\\
T_5 = 370^\circ\text{F} & T_8 = 227^\circ\text{F} & T_{11} = 290^\circ\text{F}\\
T_6 = 388^\circ\text{F} & T_9 = 254^\circ\text{F} & T_{12} = 292^\circ\text{F}
\end{array}$$

■

ILLUSTRATIVE EXAMPLE 26.7

Comment on the results of the previous illustrative example.

***SOLUTION*:** The results are consistent. As one would expect

$$\begin{aligned} T_7 &> T_{12} \\ T_6 &> T_{11} \\ T_5 &> T_{10} \\ T_4 &> T_9 > T_8 \end{aligned}$$

In addition,

$$\begin{aligned} T_6 &\simeq T_7 \\ T_{11} &\simeq T_{12} \end{aligned}$$

■

REGRESION ANALYSIS

The application of statistics in practice during the past several decades is so extensive that every practicing engineer should posses a basic knowledge of this branch of mathematics. In many applications, statistics enables one to obtain the most information out of his/her data. Although statistical methods are often a poor substitute for rigorous mathematical analyses, they frequently can be used where too little is known about a process or system to permit a rigorous mathematical treatment.

Regression analysis is a useful statistical technique for developing a quantitative relationship between a dependent variable and one or more independent variables. It usually utilizes experimental data on the pertinent variables to develop a numerical equation showing the influence of the independent variables on the dependent variable of the system. A simple correlation problem can arise when one asks whether there is any relationship between smoking and heart ailments, between beauty and brains, between pressure drop and velocity, between particle drag force and fluid density, etc. For example, in a tubular flow reactor involving a complex chemical reaction, regression methods have been used to develop an equation relating the yield of a desired product to entering concentrations, temperature, pressure, and residence time.(10)

Consider two random variables x and y and the problem of determining the extent to which they are related. The investigation of the relationship between two such variables, based on a set of n pairs of measurements $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, usually begins with an attempt to discover the approximate form of the relationship by graphing the data as n points in the x, y-plane. Such a graph is called a *scatter diagram*. Use of a scatter diagram allows one to quickly discern whether there is any pronounced relationship and, if so, whether the relationship may be treated as approximately linear.(11)

One of the problems of fitting a curve to a set of points in some efficient manner is essentially that of estimating the parameters of the curve in an efficient manner. Although there are numerous methods for performing the estimation of such parameters, the best known and most popular method is known as the method of least squares.

Since the desired equation is generally used for estimating or predicting purposes, it is reasonable to expect that it leads to small errors of estimation. An error of estimation or prediction is defined as the difference between an observed value of y and the corresponding fitted curve value of y. If the value of the variable to be estimated is denoted by y and the corresponding curve value by y', then the error of estimation, E, is given by $y{-}y'$. Since the errors may be positive or negative and might add up to a small value for a poorly fitted curve, it will not do to merely require that the sum of the errors be as small as possible. This difficulty can be removed by requiring that the sum of the absolute values of the errors be as small as possible. However, sums of absolute values are not convenient to work with mathematically; consequently, the difficulty is avoided by requiring that the sum of the squares of the errors be minimized. The values of the parameters obtained by this minimization procedure determines what is known as the best fitting curve in the sense of least squares.(11)

The method of least squares is based on the assumption that the sum of the squares of the deviations between the assumed function and the given data, i.e., the sum of these errors, must be a minimum. If the ordinate of the data point is designated as y_i and the value of the approximating function at the same value of x_i is $f(x_i)$, the error at x_i can be written as

$$E_i = y_i - f(x_i) \tag{26.18}$$

Mathematically, the method of least squares requires that

$$\sum_{i=1}^{n} E_i^2 = \sum_{i=1}^{n} [\,y_i - f(x_i)]^2 = \text{minimum} \tag{26.19}$$

where n is the number of data points.

For illustration, assume a second-order polynomial, as given by Equation (26.20).

$$f(x) = a_0 + a_1 x + a_2 x^2 \tag{26.20}$$

The error, E_i, at each data point, i, is therefore

$$E_i = a_0 + a_1 x + a_2 x^2 - y_i \tag{26.21}$$

The sum of the errors squared, S, is

$$S = \sum E_i^2 = \sum (a_0 + a_1 x + a_2 x^2 - y_i)^2 \tag{26.22}$$

The requirement for this function, S, to be a minimum is given by

$$\left.\frac{\partial S}{\partial a_2}\right|_{a_1,a_0} = 0; \quad \left.\frac{\partial S}{\partial a_1}\right|_{a_2,a_0} = 0; \quad \left.\frac{\partial S}{\partial a_0}\right|_{a_2,a_1} = 0 \tag{26.23}$$

These three equations can be shown to reduce to$^{(12)}$ (for Equation (26.20))

$$\begin{aligned}
\sum_{i=1}^{n} 2x_i^2(a_2x_i^2 + a_1x_i + a_0 - y_i) &= 0 \\
\sum_{i=1}^{n} 2x_i(a_2x_i^2 + a_1x_i + a_0 - y_i) &= 0 \\
\sum_{i=1}^{n} 2(a_2x_i^2 + a_1x_i + a_0 - y_i) &= 0
\end{aligned} \tag{26.24}$$

Equation (26.24) represents a set of three equations containing unknowns a_0, a_1, and a_2. Recasting these in matrix form gives

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_iy_i \\ \sum_{i=1}^{n} x_i^2y_i \end{bmatrix} \tag{26.25}$$

Equation (26.25) may then be solved for the regression coefficients a_0, a_1, and a_2.$^{(12)}$

The above development can easily be extended to other types of polynomials. Other functions are listed below.

$$\begin{aligned}
\text{0th degree polynomial: } & y = a_0 \\
\text{1st degree polynomial: } & y = a_0 + a_1x_1 \\
\text{2nd degree polynomial: } & y = a_0 + a_1x_1 + a_2x_2^2 \\
\text{mth degree polynomial: } & y = a_0 + a_0x_1 + \cdots + a_0x_m^m
\end{aligned} \tag{26.26}$$

The above can also be applied to functions $f_i(x)$ that are complex. For example, if

$$y = a_1f_1(x) + a_2f_2(x) + a_3f_3(x) \tag{26.27}$$

then Equation (26.27) takes the form:

$$\begin{bmatrix} \sum f_1^2 & \sum f_1f_2 & \sum f_1f_3 \\ \sum f_1f_2 & \sum f_2^2 & \sum f_2f_3 \\ \sum f_1f_3 & \sum f_2f_3 & \sum f_3^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y_if_1 \\ \sum y_if_2 \\ \sum y_if_3 \end{bmatrix} \tag{26.28}$$

The reader is referred to Illustrative Example 28.4 for additional details.

Correlation Coefficient

When data is plotted to determine any possible relationship, it is common to try to fit the data to some sort of curve (typically a line). The correlation coefficient is a measure of the amount of deviation between the variables. A correlation coefficient of 1.0 indicates a perfect association between the variables; a correlation coefficient of 0.0 indicates a completely random relation. Nothing is said about the dependence or independence of the variables, and nothing is said about the nature of the relation between the variables.

Although the correlation coefficient provides an answer as to how well the model fits the data, it does not provide an answer as to whether it is the best and/or correct model. In fact, it can often provide misleading information. For example, if one where interesting in fitting a model to the four data points presented in Figure 26.8, one could apply a zero, first, second or third order model to the data. These four models are superimposed on the figure. The zero order polynomial would provide the worst correlation coefficient while the third order polynomial (which exactly passes through each data point) would correlate perfectly with a corresponding correlation coefficient of 1.0. Obviously, the latter is almost certainly *not* the "correct" or "best" model.(12)

To obtain information on the "best" model, one must resort to a statistical technique referred to as ANOVA, an acronym referring to Analysis Of Variance. This topic is beyond the scope of this text; however, information is available in the literature.(12,13)

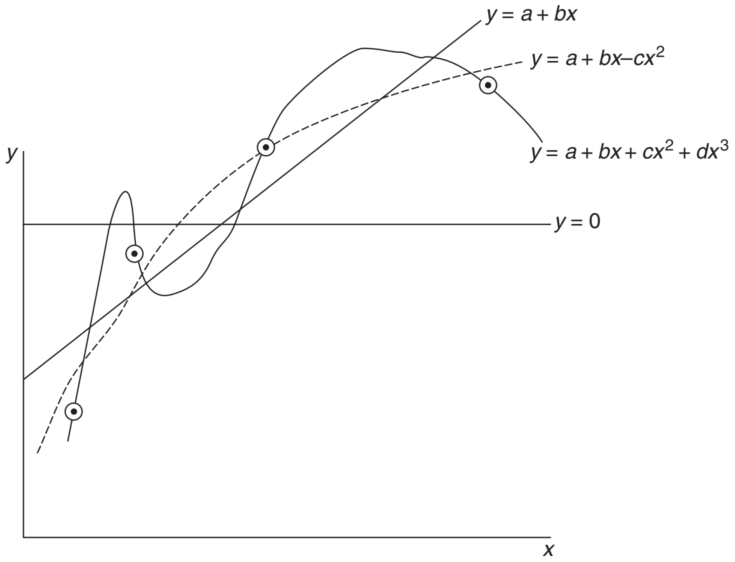

Figure 26.8 Regression models.

ILLUSTRATIVE EXAMPLE 26.8(14)

Species A is undergoing a reaction in a tubular flow unit. The following data (see Table 26.1) have been obtained for the rate, $-r_A$, versus concentration, C_A. Using this data, estimate the coefficient k_A and α in the equation below.

$$-r_A = k_A C_A^{\alpha}$$

***SOLUTION*:** Linearize the describing equation

$$\ln(-r_A) = \ln(k_A) + \alpha \ln(C_A)$$

Change the variables to Y and X.

$$Y = A + BX$$

where

$$Y = \ln(-r_A)$$
$$A = \ln(k_A)$$
$$B = \alpha$$

Substitute the above data.

$$\ln(3) = A + B\ln(2)$$
$$\ln(12) = A + B\ln(4)$$
$$\ln(27) = A + B\ln(6)$$
$$\ln(48) = A + B\ln(8)$$

Generate the linear coefficients A and B using any convenient method:

$$A = -0.2878$$
$$B = 2.0$$

Therefore,

$$k_A = 0.75$$
$$\alpha = 2.0$$

and the equation for the rate of reaction is

$$-r_A = 0.75C_A^2$$

Table 26.1 Rate Data; illustrative Example 26.8

$-r_A$, lbmol/ft$^3 \cdot$ s	C_A, lbmol/ft^3
48	8
27	6
12	4
3	2

Obviously, the regression of data has been greatly simplified through the use of software packages. ■

ILLUSTRATIVE EXAMPLE 26.9

The viscosity of air as a function of temperature is provided below in Table 26.2. Assuming a linear relationship, obtain the viscosity as a function of temperature.

***SOLUTION*:** For a linear model,

$$\mu = A + BT$$

Regressing the data using the method of least squares gives

$$A = 1.7456$$
$$B = 0.0083$$

Therefore,

$$\mu = 1.7456 + 0.0083T$$

with μ in $\mathrm{kg/m \cdot s} \times 10^5$ and T in °C.

Table 26.2 Viscosity–Temperature Data; Illustrative Example 26.9

Temperature, T, °C	Viscosity, μ, $\mathrm{kg/m \cdot s} \times 10^5$
−40	1.51
−20	1.61
0	1.71
10	1.76
12	1.81
30	1.86
40	1.90
50	1.95
60	2.00
80	2.09
100	2.17
150	2.38
200	2.57
250	2.75
300	2.93
400	3.25
500	3.55

■

OPTIMIZATION

Optimization has come to mean different things to different people. However, one might offer the following generic definition: "*Optimization* is concerned with determining the 'best' solution to a given problem". This process is required in the solution of many problems that involves the maximization or minimization of a given function.

A significant number of optimization problems face the practicing engineer. The optimal design of industrial processes as well as heat exchangers has long been of concern to the practicing engineer, and indeed, for some, might be taken as a definition of the function and goal of applied engineering. The practical attainment of an optimum design is generally a result of factors that include mathematical analysis, empirical information, and both the subjective and objective experience of the engineer.

In a general sense, optimization problems can be divided into four categories:

1. The number of independent variables involved.
2. Whether the optimization is "constrained."
3. Time-independent systems.
4. Time-dependent systems.

In addition, if no unknown factors are present, the system is defined as *deterministic* while a system containing experimental errors and/or other random factors is defined as *stochastic*.

Perturbation Studies in Optimization

Once a particular heat exchange or process scheme has been selected, it is common practice to optimize the process from a capital cost and O&M (operation and maintenance) standpoint. There are many optimization procedures available, most of them too detailed for meaningful application to a heat exchanger analysis. These sophisticated optimization techniques, some of which are also routinely used in the design of conventional chemical and petrochemical plants, invariably involve computer calculations. Use of these techniques in heat exchanger analysis is not always warranted, however.

One simple optimization procedure that is recommended is the *perturbation study*. This involves a systematic change (or *perturbation*) of variables, one by one, in an attempt to locate the optimum design from a cost and operation viewpoint. To be practical, this often means that the engineer must limit the number of variables by assigning constant values to those process variables that are known beforehand to play an insignificant role, Reasonable guesses and simple or short-cut mathematical methods can further simplify the procedure. Much information can be gathered from this type of study since it usually identifies those variables that significantly impact on the overall performance of the heat exchanger or process and also helps identify the major contributors in the procedure.

More detailed and sophisticated optimization procedures are available, most of which are located in the literature.(15,16) Three illustrative examples follow, two additional illustrative examples that outline the methodology employed in optimization calculations can be found later in Chapter 27. As is often the case, both examples involve an economic analysis.

ILLUSTRATIVE EXAMPLE 26.10

Qualitatively describe the objective of rigorous optimization techniques.(15)

SOLUTION: Formal optimization techniques have as their goal the development of procedures for the attainment of an optimum in a system which can be characterized mathematically. The mathematical characterization may be:

1. partial or complete
2. approximate or exact, and/or
3. empirical or theoretical.

The resulting optimum may be a final implementable design or a guide to practical design and a criterion by which practical designs are to be judged. In either case, the optimization techniques should serve as an important part of the total effort in the design of the units, structure, and control of a industrial system, including heat exchangers. ■

ILLUSTRATIVE EXAMPLE 26.11

A refinery has two catcrackers that can produce various grades of hydrocarbon products. Because of consumer fuel demand, the production of gasoline, home heating oil and diesel must be limited. This information is provided in Table 26.3.

The profit on processing U.S. crude oil is \$2.00/gal and on Venezuelan crude is \$1.60/gal. Find the approximate daily processing rate of the two crudes in order to maximize profits.

SOLUTION: Set

$$q_1 = \text{gallons of U.S. crude}$$
$$q_2 = \text{gallons of Venezuelan crude}$$

The objective function for the daily profit, P, that is to be maximized is:

$$P = 2.0q_1 + 1.6q_2$$

Table 26.3 Cat Cracker Information

Product grades	U.S. crude (% distribution)	Venezuelan crude (% distribution)	Max. production rate (gal/day)
Gasoline	8	11	1500
Home heating oil	29	54	5500
Diesel oil	63	35	11,000

The constraints are:

$$\begin{aligned} 0.08q_1 + 0.11q_2 &\leq 1500 \\ 0.29q_1 + 0.54q_2 &\leq 5500 \\ 0.63q_1 + 0.35q_2 &\leq 11{,}000 \\ q_1 \geq 0,\ q_2 &\geq 0 \end{aligned}$$

The solution to the above from Excel is: 16,820 gal of U.S. crude per day should be processed and 1152 gal per day should be derived from Venezuelan crude. This results in a total daily profit of \$35,484.(17) ■

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Chapter 27

Economics and Finance

INTRODUCTION

Chapter 27 is concerned with Economics and Finance. These two topics can ultimately dictate the decisions made by the practicing engineer and his/her company. For example, a company may decide that due to the rising price of fuel they will explore the possibility of recovering the energy in a hot process stream instead of discharging it to the environment. A decision will then be based on whether it makes sense economically in the short- and long-term to purchase and install a heat exchange. Furthermore, economic evaluations are a major part of process and plant design.

This chapter provides introductory material, including the need for economic analyses, to this vast field within engineering. The next section discusses the need for economic analyses. The following section is devoted to definitions. This is followed with an overview of accounting principles. The chapter concludes with Illustrative Examples in the Applications section.

Both the qualitative and quantitative viewpoint is emphasized in this chapter although it is realized that the broad subject of engineering economics cannot be fitted into any rigid set of formulas. The material presented falls into roughly three parts: namely, general principles, practical information, and applications. The presentation starts with simple situations and proceeds to more complicated formulations and techniques that may be employed if there are sufficient data available. Other texts in the literature provide further details on the subject.

THE NEED FOR ECONOMIC ANALYSES

A company or individual hoping to increase its profitability must carefully assess a range of investment opportunities and select the most profitable options from those available. Increasing competitiveness also requires that efforts be expended to reducing costs of existing processes. In order to accomplish this, engineers should be fully aware of not only technical factors but also economic factors, particularly those that have the largest effect on profitability.

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

In earlier years, engineers concentrated on the technical side of projects and left the financial studies to the economist. In effect, engineers involved in making estimates of the capital and operating costs have often left the overall economic analysis and investment decision-making to others. This approach is no longer acceptable.

Some engineers are not equipped to perform a financial and/or economic analysis. Furthermore, many engineers already working for companies have never taken courses in this area. This shortsighted attitude is surprising in a group of professionals who normally go to great lengths to get all the available technical data before making an assessment of a project or study. The attitude is even more surprising when one notes that data are readily available to enable an engineer to assess the economic prospects on both his/her own company and those of his/her particular industry.(1)

As noted above, the purpose of this chapter is to provide a working tool to assist the student or engineer in not only understanding economics and finance but also in applying technical information to economic design and operation. This applies to both equipment (e.g., heat exchangers) and also processes and plants. The material to follow will often focus on industrial and/or plant applications. Hopefully, this approach will provide the reader with a better understanding of some of the fundamentals and principles.

Bridging the gap between theory and practice is often a matter of experience acquired over a number of years. Even then, methods developed from experience all too often must be re-evaluated in the light of changing economic conditions if optimum designs are to result. The approach presented here therefore represents an attempt to provide a consistent and reasonably concise method for the solution of these problems involving economic alternatives.(2)

The term "economic analysis" in engineering problems generally refers to calculations made to determine the conditions for realizing maximum financial return for a design or operation. The same general principles apply, whether one is interested in the choice of alternatives for completing projects, in the design of plants so that the various components are economically proportioned, or in the economical operation of existing equipment and plants.

General considerations that form the framework on which sound decisions must be made are often simple. Sometimes their application to the problems encountered in the development of a commercial enterprise involves too many intangibles to allow exact analysis, in which case judgment must be intuitive. Often, however, such calculations may be made with a considerable degree of exactness. This chapter will attempt to develop a relatively concise method for applying these principles.

Finally, concern with maximum financial return implies that the criterion for judging projects involved is profit. While this is usually true, there are many important objectives which, though aimed at ultimate profit increase, cannot be immediately evaluated in quantitative terms. Perhaps the most significant of these is increased concern with environmental degradation and sustainability. Thus, there has been some tendency in recent years to regard management of commercial organizations as a profession with social obligations and responsibilities; in effect, considerations other than the profit motive may govern business decisions. However, these additional social objectives are for the most part often not inconsistent with the economic goal of satisfying human wants with the minimum effort. In fact, even in the operation of primarily nonprofit organizations, it is still important to determine the effect of various policies on profit.(2)

As noted above, the next section is devoted to definitions. This is followed with an overview of accounting principles and applications. The chapter concludes with applications.

DEFINITIONS

Before proceeding to the applications, it would be wise to provide the reader with certain key definitions in the field. Fourteen concepts that often come into play in an economic analysis are given below. The definitions have been drawn from the literature.(3)

Simple Interest

The term interest can be defined as the money paid for the use of money. It is also referred to as the value or worth of money. Two terms of concern are simple interest and compound interest. Simple interest is always computed on the original principal. The basic formula to employ in simple interest calculations is:

$$S = P(1 + ni) \tag{27.1}$$

where P = original principal

n = time in years

i = annual interest rate

S = sum of interest and principal after n years

Normally, the interest period is one year, in which case i is often referred to as the *effective* interest rate.

Compound Interest

Unlike simple interest, with compound interest, interest is added periodically to the original principal. The term conversion or compounding of interest simply refers to the addition of interest to the principal. The interest period or conversion period in compound interest calculations is the time interval between successive conversions of interest and the interest period is the ratio of the stated annual rate to this number of interest periods in one year. Thus, if the given interest rate is 10% compounded semiannually, the interest period is 6 months and the interest rate per interest period is 5%. Alternately, if the given interest rate is 10% compounded quarterly, then the interest period is 3 months and the interest rate per interest period is 2.5%. One should always assume the interest is compounded annually unless otherwise stated. The basic formula to employ for compound interest is:

$$S = P(1 + i)^n \tag{27.2}$$

If interest payments become due m times per year at compound interest, $(m)(n)$ payments are required in n years. A nominal annual interest rate, i', may be defined by:

$$S = P\left(1 + \frac{i'}{m}\right)^{mn} \tag{27.3}$$

In this case, the effective annual interest, i, is:

$$i = \left(1 + \frac{i'}{m}\right)^{m} - 1 \tag{27.4}$$

In the limit (as m approaches infinity), such payments may be considered to be required at infinitesimally short intervals, in which case, the interest is said to be compounded continuously. Numerically, the difference between continuous and annual compounding is small. However, annual compounding may be significant when applied to very large sums of money.

Present Worth

The present worth is the current value of a sum of money due at some later time n and at interest rate i. This equation is the compound interest equation solved for the present worth term, P.

$$P = S(1 + i)^{-n} \tag{27.5}$$

Evaluation of Sums of Money

The value of a sum of money changes with time because of interest considerations. \$1000 today, \$1000 ten years from now, and \$1000 ten years ago all have different meanings when interest is taken into account. \$1000 today would be worth more ten years from now because of the interest that could be accumulated in the interim. On the other hand, \$1000 today would have been worth less ten years ago because a smaller sum of money could have been invested then so as to yield \$1000 today. Therefore, one must refer to the date as well as the sum of money when discussing money.

Summarizing, evaluating single sums of money requires multiplying by $(1 + i)^n$ if the required date of evaluation is after the date associated with the obligation or multiplying by $(1 + i)^{-n}$ if the required date of evaluation is before the date associated with the obligation. The term n is always the time in periods between the date associated with the obligation and the date of evaluation.

The evaluation of sums of money may be applied to the evaluation of a uniform series of payments. A uniform series is a series of equal payments made at equal intervals. Suppose R is invested at the end of every interest period for n periods. The total value of all these payments, S, as of the date of the last payment, may be calculated from the equation

$$S = R[(1 + i)^n - 1]/i \tag{27.6}$$

The term S is then called the amount of the uniform series.

Depreciation

The term depreciation refers to the decrease in the value of an asset. Two approaches that can be employed are the straight line and sinking fund method. In the straight line method of depreciation, the value of the asset is decreased each year by a constant amount. The annual depreciation amount, D, is given by

$$D = (\text{Original cost} - \text{Salvage value})/(\text{Estimated life in years}) \tag{27.7}$$

In the sinking fund method of depreciation, the value of the asset is determined by first assuming that a sinking fund consisting of uniform annual payments had been set up for the purpose of replacing the asset at the end of its estimated life. The uniform annual payment (UAP) may be calculated from UAP = (Original cost − Salvage value)(SFDF) where SFDF is the sinking fund deposit factor and is given by

$$\text{SFDF} = i/[(1+i)^n - 1] \tag{27.8}$$

The value of the asset at any time is estimated to be the difference between the original cost and the amount that would have accumulated in the sinking fund. The amount accumulated in the sinking fund is obtained by multiplying the UAP by the compound amount factor (CAF) where

$$\text{CAF} = [(1+i)^n - 1]/i \tag{27.9}$$

Fabricated Equipment Cost Index

A simple process is available to estimate the equipment cost from past cost data. The method consists of adjusting the earlier cost data to present values using factors that correct for inflation. A number of such indices are available; one of the most commonly used is the fabricated equipment cost index (FECI):

$$\text{Cost}_{\text{year B}} = \text{Cost}_{\text{year A}}\left(\frac{\text{FECI}_{\text{year B}}}{\text{FECI}_{\text{year A}}}\right) \tag{27.10}$$

Given the cost and FECI for year A, as well as the FECI for year B, the cost of the equipment in year B can be estimated. Similar methods for estimating the cost of equipment (e.g., heat exchangers), are available in the literature. However, actual quotes from vendors is preferred and should be used if greater accuracy is required.

Capital Recovery Factor

In comparing alternative processes or different options for a particular process from an economic point-of-view, one recommended procedure to follow is that the total capital cost can be converted to an annual basis by distributing it over the projected lifetime of the facility (or heat exchanger, etc). The sum of both the annualized capital cost (ACC), including installation, and the annual operating cost (AOC), is called the total annualized cost (TAC) for the project or facility. The economic merit of the proposed facility, process, or scheme can be examined once the total annual cost is available.

The conversion of the total capital cost (TCC) to an ACC requires the determination of an economic parameter known as the capital recovery factor (CRF). This parameter can be found in any standard economics textbook or calculated directly from the following equation:

$$\mathrm{CRF} = i(1+i)^n/[(1+i)^n - 1] = i/[1-(1+i)^{-n}] \tag{27.11}$$

where n = projected lifetime of the system

i = annual interest rate (as a fraction)

The CRF is a positive, fractional number. Once this factor has been determined, the ACC can be calculated from the following equation:

$$\mathrm{ACC} = (\mathrm{TCC})(\mathrm{CRF}) \tag{27.12}$$

The annualized capital cost reflects the cost associated with recovering the initial capital expenditure over the depreciable life of the system.

Present Net Worth

There are various approaches that may be employed in the economic selection of the best of several alternatives. For each alternative in the present net worth (PNW) method of economic selection, a single sum is calculated that would provide for all expenditures over a common time period. The alternative having the least PNW of expenditures is selected as the most economical. The equation to employ is

$$\mathrm{PNW} = \mathrm{CC} + \mathrm{PN} + \mathrm{PWD} - \mathrm{PWS} \tag{27.13}$$

where CC = capital cost

PN = future renewals

PWD = other disbursements

PWS = salvage value

If the estimated lifetimes differ for the various alternatives, one should employ a period of time equal to the least common multiple of the different lifetimes for review purposes.

Perpetual Life

Capitalized cost can be viewed as present worth under the assumption of perpetual life. Computing capitalized cost involves, in a very real sense, finding the present worth of an infinite series of payments. To obtain the present worth of an infinite series of payments of $\$R$ at the end of each interest period forever, one needs simply to divide R by i, where i is the interest rate (fractional basis) per interest period. Thus, to determine what sum of money, P would have to be invested at 8.0% to provide payments of

\$100,000 at the end of each year forever, P would have to be such that the interest on it each period would be \$100,000. Withdrawal of the interest at the end of each period would leave the original sum intact to again draw \$100,000 interest at the end of the next period. For this example,

$$\begin{aligned} P &= 100{,}000/0.08 \\ &= \$1{,}250{,}000 \end{aligned}$$

The \$1,250,000 would be the present worth of an infinite series of payments of \$100,000 at the end of each year forever, assuming money is worth 8%.

To determine the present worth of an infinite series of payments of \$$R$ at the end of each n periods forever, first multiply by the SFDF to convert to an equivalent single period payment and then divide by i to obtain the present worth.

Break-Even Point

From an economic point-of-view, the break-even point of a process operation is defined as that condition when the costs (C) exactly balance the income (I). The profit (P) is therefore,

$$P = I - C \tag{27.14}$$

At break-even, the profit is zero.

Approximate Rate of Return

Rate of return can be viewed as the interest that will make the present worth of net receipts equal to the investment. The approximate rate of return (ARR), denoted by some as p, may be estimated from the equation below:

$$p = \text{ARR} = \text{Average annual profit or earnings/Initial total investment} \tag{27.15}$$

To determine the average annual profit, simply divide the difference between the total money receipts (income) and the total money disbursements (expenses) by the number of years in the period of the investment.

Exact Rate of Return

Using the approximate rate of return as a guide, one can generate the exact rate of return (ERR). This is usually obtained by trial-and-error and interpolation calculations of the rate of interest that makes the present worth of net receipts equal to the investment. The approximate rate of return will tend to overestimate the exact rate of return when all or a large part of the receipts occur at the end of a period of investment. The approximate rate will tend to underestimate the exact rate when the salvage value is zero and also when the salvage value is a high percentage of the investment.

Bonds

A bond is a written promise to pay both a certain sum of money (redemption price) at a future date (redemption date) and equal interest payments at equal intervals in the interim. The holder of a \$1000, 5% bond, redeemable at 105 (bond prices are usually listed without the last zero) in 10 years, with interest payable semiannually would be entitled to semiannual payments of \$1000 (0.025) or \$25 for 10 years and 105% of \$1000, that is \$1050, at the end of 10 years when the bond is redeemed.

The interest payment on a bond is found by multiplying the face value of the bond by the bond interest rate per period. From above, the face value is \$1000 and the bond interest rate per period is 0.025. Therefore, the periodic interest payment is \$25. Redeemable at 105 means that the redemption price is 105% of the face value of the bond.

The purchase price of a bond depends on the yield rate; i.e., the actual rate of return on the investment represented by the bond purchase. Therefore, the purchase price of a bond is the present worth of the redemption price plus the present worth of future interest payments, all computed at the yield rate. The bond purchase price formula is:

$$V = C(1+i)^{-n} + R[1-(1+i)^{-n}]/i \tag{27.16}$$

where $V =$ purchase price
$C =$ redemption price
$R =$ periodic interest payment
$n =$ time in periods to maturity
$i =$ yield rate (fractional basis)

Incremental Cost

By definition, the average unit increment cost is the increase in cost divided by the increase in production. Only those cost factors which vary with production can affect the average unit increment cost. In problems involving decisions as to whether to stay in production or (temporarily) shut down, the average unit increment cost may be compared with the unit increment cost or the unit selling price.

Optimization

As noted in the previous chapter, optimization is often applied in several areas of engineering science and technology. The two key areas involve process design and plant operation. The term “optimization” may be viewed as a process involving the selection of the best option among different solutions. Optimization in process design involves maximizing annual profit, while minimizing total annual cost and environmental degradation. Optimization in plant operation generally involves maximizing product and product quality subject to economic, environmental, and energy constraints. Two heat transfer illustrative examples dealing with optimization are provided at the end of the Applications Section.

PRINCIPLES OF ACCOUNTING(3)

Accounting is the science of recording business transactions in a systematic manner. Financial statements are both the basis for and the result of management decisions. Such statements can tell a manager or an engineer a great deal about a company, provided that one can interpret the information correctly.

Since a fair allocation of costs requires considerable technical knowledge of operations in the chemical process industries, a close liaison between the senior process engineers and the accountants in a company is desirable. Indeed, the success of a company depends on a combination of financial, technical and managerial skills.

Accounting is also the language of business and the different departments of management use it to communicate within a broad context of financial and cost terms. The engineer who does not take the trouble to learn the language of accountancy denies oneself the most important means available for communicating with top management. He/she may be thought by them to lack business acumen. Some engineers have only themselves to blame for their lowly status within the company hierarchy since they seem determined to displace themselves from business realities behind the screen of their specialized technical expertise. However, more and more engineers are becoming involved in decisions that are business related.

Engineers involved in feasibility studies and detailed process evaluations are dependent on financial information from the company accountants, especially information regarding the way the company intends to allocate its overhead costs. It is vital that the engineer should correctly interpret such information and that he/she can, if necessary, make the accountant understand the effect of the chosen method of allocation.

The method of allocating overheads can seriously affect the assigned costs of a project and hence the apparent cash flow for that project. (Note: *Cash flow* is an algebraic monitary quantity whose numerical value represents the amount of money transferred. If money is received (inflow), the cash flow is positive; if the money is disbursed, the cash flow is negative.) Since these cash flows are often used to assess profitability by such methods as PNW, unfair allocation of overhead costs can result in a wrong choice between alternative projects.

In addition to understanding the principles of accountancy and obtaining a working knowledge of its practical techniques, the engineer should be aware of possible inaccuracies of accounting information in the same way that he/she allows for errors in any technical data.

At first acquaintance, the language of accountancy appears illogical to most engineers. Although the accountant normally expresses information in tabular form, the basis of all practice can be simply expressed by:

$$\text{Capital} = \text{Assets} - \text{Liabilities} \tag{27.17}$$

or

$$\text{Assets} = \text{Capital} + \text{Liabilities} \tag{27.18}$$

Capital, often referred to as net worth, is the money value of the business, since assets are the money value of things the business *owns* while liabilities are the money value of the things the business *owes*.

Most engineers have great difficulty in thinking of capital (also known as ownership) as a liability. This is easily overcome once it is realized that a business is a legal entity in its own right, owing money to the individuals who own it. This realization is absolutely essential when considering large companies with stockholders, and is used for consistency even for sole ownerships and partnerships. If a person (say FR) puts up \$10,000 capital to start a business, then that business has a liability to repay \$10,000 to that person.

It is even more difficult to think of profit as being a liability. Profit is the increase in money available for distribution to the owners, and effectively represents the interest obtained on the capital. If the profit is not distributed, it represents an increase in capital by the normal concept of compound interest. Thus, if the aforementioned business makes a profit of \$5000, the liability is increased to \$15,000. With this concept in mind Equation (27.18) can be expanded to:

$$\text{Assets} = \text{Capital} + \text{Liabilities} + \text{Profit} \tag{27.19}$$

where the capital is considered the cash investment in the business and is distinguished from the resultant profit in the same way that principal and interest are separated.

Profit (as referred to above) is the difference between the total cash revenue from sales and the total of all costs and other expenses incurred in making those sales. With this definition, Equation (27.19) can be further expanded to:

$$\begin{aligned}\text{Assets} + \text{Expenses} &= \text{Capital} + \text{Liabilities} + \text{Profit}\\ &\quad + \text{Revenue (from sales)}\end{aligned} \tag{27.20}$$

Some engineers have the greatest difficulty in regarding an expense as being equivalent to an asset, as is implied by Equation (27.20). However, consider FR's earnings. During the period in which he made a profit of \$5000, his total expenses excluding his earnings were \$8000. If he assessed the worth of his labor to the business at \$12,000, then the revenue required from sales would be \$25,000. Effectively, FR has made a personal income of \$17,000 in the year but he has apportioned it to the business as \$12,000 expense for his labor and \$5000 return on his capital. In larger businesses, there will also be those who receive salaries but do not hold stock and therefore, receive no profits, and stockholders who receive profits but no salaries. Thus, the difference between expenses and profits is very practical.

The period covered by the published accounts of a company is usually one year, but the details from which these accounts are compiled are often entered daily in a *journal*. The journal is a chronological listing of every transaction of the business, with details of the corresponding income or expenditure. For the smallest businesses, this may provide sufficient documentation but, in most cases, the unsystematic nature of the journal can lead to computational errors. Therefore, the usual practice is to keep accounts that are listings of transactions related to a specific topic such as "Purchase of Heating Oil Account." This account would list the cost of each purchase of heating oil, together with the date of purchase, as extracted from the journal.

The traditional work of accountants has been to prepare balance sheets and income statements. Nowadays, accountants are becoming increasingly concerned with forward planning. Modern accountancy can roughly be divided into two branches: financial accountancy and management or cost accountancy.

Financial accountancy is concerned with stewardship. This involves the preparation of balance sheets and income statements that represent the interest of stockholders and are consistent with the existing legal requirements. Taxation is also an important element of financial accounting.

Management accounting is concerned with decision-making and control. This is the branch of accountancy closest to the interest of most (process) engineers. Management accounting is concerned with standard costing, budgetary control, and investment decisions.

Accounting statements only present facts that can be expressed in financial terms. They do not indicate whether a company is developing new products that will ensure a sound business future. A company may have impressive current financial statements and yet may be heading for bankruptcy in a few years' time if provision is not being made for the introduction of sufficient new products or services.

APPLICATIONS

The remainder of the chapter is devoted to illustrative examples, many of which contain technical developmental material. A good number of heat transfer related applications have been drawn from the National Science Foundation (NSF) literature(4–8) and two other key sources.(9,10)

ILLUSTRATIVE EXAMPLE 27.1

List the major *fixed* capital costs for the chemical process industry.

***SOLUTION*:**

1. Major process equipment (i.e., heat exchangers, reactors, tanks, pumps, filters, distillation columns, etc.).
2. Installation of major process equipment.
3. Process piping.
4. Insulation.
5. Instrumentation.
6. Auxiliary facilities (i.e., power substations, transformers, boiler houses, fire-control equipment, etc.).

7. Outside lines (i.e., piping external to buildings, supports and posts for overhead piping, electric feeders from power substations, etc.).
8. Land and site improvements.
9. Building and structures.
10. Consultant fees.
11. Engineering and construction (design and engineering fees plus supervision of plant erection).
12. Contractors' fees (administrative). ■

ILLUSTRATIVE EXAMPLE 27.2

List the major *working* capital costs for the chemical process industry.

***SOLUTION*:**

1. Raw materials for plant startup.
2. Raw material, intermediate and finished product inventories.
3. Cost of handling and transportation of materials to and from sites.
4. Cost inventory control, warehouse(s), associated insurance, security arrangements, etc.
5. Money to carry accounts receivable (i.e., credit extended to customers) less accounts payable (i.e., credit extended by suppliers).
6. Money to meet payrolls when starting up.
7. Readily available cash for emergency.
8. Any additional cash required to operate the process or business.
9. Expenses associated with new hirees.
10. Startup consultant fees. ■

ILLUSTRATIVE EXAMPLE 27.3

Answer the following three questions:

1. Define the straight-line method of analysis that is employed in calculating depreciation allowances.
2. Define the double-declining balance (DDB) method of analysis.
3. Define the sum-of-the-year's digits (SYD) method of analysis.

***SOLUTION*:** 1. The straight-line rate of depreciation is a constant equal to $1/n$ where n is the life of the facility for tax purposes. Thus, if the life of the plant is 10 yr, the straight-line rate of depreciation is 0.1. This rate, applied over each of the 10 yr, will result in a depreciation reserve equal to the initial investment.

2. A declining balance rate is obtained by first computing the straight-line rate and then applying some multiple of that rate to each year's unrecovered cost rather than to the original investment. Under the double-declining balance method, twice the straight-line rate is applied to each year's remaining unrecovered cost. Thus, if the life of a facility is 10 years, the

straight-line rate will be 0.1, and the first year's double-declining balance will be 0.2. If the original investment is I, the depreciation allowance the first year will be $0.2I$. For the second year, it will be $0.16I$, or 0.2 of the unrecovered cost of $0.8I$. The depreciation allowances for the remaining years are calculated in a similar manner until the tenth year has been completed. Since this method involves taking a fraction of an unrecovered cost each year, it will never result in the complete recovery of the investment. To overcome this objection, the U.S. Internal Revenue Service, allowed the taxpayer in the past to shift from the DDB depreciation method to the straight-line method any time after the start of the project.

3. The rate of depreciation for the sum-of-the-year's digits method is a fraction. The numerator of this fraction is the remaining useful life of the project at the beginning of the tax year, while the denominator is the sum of the individual digits corresponding to the total years of life of the project. Thus, with a project life, n, of 10 years, the sum of the year's digits will be $10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55$. The depreciation rate the first year will be $10/55 = 0.182$. If the initial cost of the facility is I, the depreciation for the first year will be $0.182I$, $9/55 = 0.164I$ for the second year, and so on until the last year. The SYD method will recover 100% of the investment at the end of n years. However, shift from SYD to straight-line depreciation cannot be made once the SYD method has been started. ■

ILLUSTRATIVE EXAMPLE 27.4

Compare the results of the three methods discussed in Illustrative Example 27.3.

***SOLUTION*:** A tabular summary of the results of depreciation according to the straight-line, double-declining, and sum-of-the-year's digits methods are shown in Table 27.1. ■

ILLUSTRATIVE EXAMPLE 27.5

A heat exchanger costing \$60,000 has an estimated lifetime of 9 years and a salvage value of \$500. What uniform annual payment must be made into a fund at the end of the year to replace

Table 27.1 Comparative Methods of Analysis

Year	Straight-line	Double-declining	Sum-of-the-year's digits
0	1.000	1.000	1.000
1	0.900	0.800	0.818
2	0.800	0.640	0.655
3	0.700	0.512	0.510
4	0.600	0.410	0.383
5	0.500	0.328	0.274
6	0.400	0.262	0.183
7	0.300	0.210	0.110
8	0.200	0.168	0.056
9	0.100	0.134	0.018
10	0.000	0.108	0.000

the exchanger if the fund earns 3.375%? What would be the appraisal value of the exchanger at the end of the fifth year based on straight line depreciation?

***SOLUTION*:** Write the equation for the uniform annual payment (UAP) in terms of the cost (P) and salvage value (L), using a sinking fund model. See Equation (27.8).

$$\text{UAP} = (P - L)(\text{SFDF})$$

Calculate the sinking fund depreciation factor, SFDF

$$\begin{aligned}\text{SFDF} &= \frac{i}{(1+i)^n - 1} \qquad (27.8)\\ &= \frac{0.03375}{(1+0.03375)^9 - 1} = 0.0969\end{aligned}$$

Thus,

$$\text{UAP} = (\$60{,}000 - \$500)(0.0969) = \$5765$$

In determining the appraisal value B where the straight line method of depreciation is used, the following equation applies:

$$B = P - \left(\frac{P - L}{n}\right)x \qquad (27.21)$$

The term n refers to the years to the end of life, and x refers to any time (in years) from the present before the end of usable life. One can employ this equation for the appraisal value and solve for B_5 after five years.

$$B_5 = \$60{,}000 - \left(\frac{\$60{,}000 - \$500}{9}\right)(5) = \$26{,}945$$

This problem assumed that the depreciation of the heat exchanger followed a sinking fund method, while the appraisal value of the exchanger followed a straight line depreciation trend. For the depreciation calculation, it is assumed that the exchanger will remain in operation for all of its nine years of usable life. For this reason, the depreciable amount of the exchanger may be thought of as being deposited into a sinking fund to be applied toward the replacement of the heat exchanger after nine years.

The appraisal value of the exchanger after the fifth year is calculated as part of the appraisal calculation. This value takes into account the fact that the heat exchanger, even one year after it is purchased, is no longer worth what was paid for it. Since the appraisal had little to do with the fund for its replacement, the exchanger was assumed to follow a straight line depreciation model. ■

ILLUSTRATIVE EXAMPLE 27.6

The annual operation costs of an outdated heat exchanger/boiler system is $75,000. Under a proposed new design, the installation of a new system will require an initial cost of $150,000 and an annual operating cost of \$15,000 for the first five years. Determine the annualized cost for the new heating system by assuming the system has only five years (n) operational life. The interest rate (i) is 7%. The capital recovery factor (CRF) or annual payment of a capital investment can be calculated as follows:

$$\text{CRF} = \left(\frac{A}{P}\right)_{i,n} = \frac{i(1+i)^n}{(1+i)^n - 1} \tag{27.11}$$

where A is the annual cost and P is the present worth.

Compare the costs for both the outdated and proposed operations.

***SOLUTION*:** The annualized cost for the new system is determined based on the following input data:

Capital cost = \$150,000
Interest, $i = 7\%$
Term, $n = 5$ yr

For $i = 0.07$ and $n = 5$, the CRF is

$$\begin{aligned}\text{CRF} &= \frac{0.07(1+0.07)^5}{(1+0.07)^5 - 1}\\ &= 0.2439\end{aligned}$$

The total annualized cost for the heat exchanger is then

$$\begin{aligned}AC &= IC + OC\\ &= (0.2439)(\$150{,}000) + \$15{,}000 = \$51{,}585\end{aligned}$$

Since this cost is lower than the annual cost of \$75,000 for the old process, the proposed plan should be implemented. ■

ILLUSTRATIVE EXAMPLE 27.7

Plans are underway to construct and operate a commercial hazardous waste facility in Dumpsville in the sate of Egabrag. The company is still undecided as to whether to install a double pipe or shell-and-tube heat exchanger at the plant site to recover energy. The double pipe (DP) unit is less expensive to purchase and operate than a comparable shell-and-tube (ST) system. However, projected energy recover income from the ST unit is higher since it will handle a larger quantity and different temperature steam.

Based on economic and financial data provided in Table 27.2, select the heat exchanger or that will yield the higher annual profit.

Calculations should be based on an interest rate of 12% and a process lifetime of 12 years for both exchangers.

Table 27.2 Costs/Credits Data

Costs/credits	Double pipe (DP)	Shell-and-tube (ST)
Capital ($)	2,625,000	2,975,000
Installation ($)	1,575,000	1,700,000
Operation ($/yr)	400,000	550,000
Maintenance ($/yr)	650,000	775,000
Income ($/yr)	2,000,000	2,500,000

***SOLUTION*:** Calculate the capital recovery factor CRF:

$$\begin{aligned} \text{CRF} &= i/[1-(1+i)^{-n}] \qquad (27.11)\\ &= 0.12/[1-(1+0.12)^{-12}]\\ &= 0.1614 \end{aligned}$$

Determine the annual capital and installation costs for the DP unit:

$$\begin{aligned} \text{DP costs} &= (\text{Capital} + \text{Installation})(\text{CRF})\\ &= (2{,}625{,}000 + 1{,}575{,}000)(0.1614)\\ &= \$677{,}880/\text{yr} \end{aligned}$$

Determine the annual capital and installation costs for the ST unit:

$$\begin{aligned} \text{ST costs} &= (\text{Capital} + \text{Installation})(\text{CRF})\\ &= (2{,}975{,}000 + 1{,}700{,}000)(0.1614)\\ &= \$754{,}545/\text{yr} \end{aligned}$$

The information below in Table 27.3 provides a comparison of the costs and credits for both exchangers.

The profit is the difference between the total annual cost and the income credit.

$$\begin{aligned} \text{DP(profit)} &= 2{,}000{,}000 - 1{,}728{,}000 = \$272{,}000/\text{yr}\\ \text{ST(profit)} &= 2{,}500{,}000 - 2{,}080{,}000 = \$420{,}000/\text{yr} \end{aligned}$$

A shell-and-tube heat exchanger should therefore be selected based on the above economic analysis.

Table 27.3 Comparison of Results; Illustrative Example 27.7

	Double pipe	Shell-and-tube
Total installed ($/yr)	678,000	755,000
Operation ($/yr)	400,000	550,000
Maintenance ($/yr)	650,000	775,000
Total annual cost ($/yr)	1,728,000	2,080,000
Income credit ($/yr)	2,000,000	2,500,000

■

ILLUSTRATIVE EXAMPLE 27.8

Based on an outgrowth of a 2011 energy audit study for a new process, it is necessary to heat 50,000 lb/h of an organic liquid form 150 to 330°F. The liquid is at a pressure of 135 psia. A simple steam-heated shell-and-tube foating-head carbon steel exchanger is the preferred equipment choice. Steam is available at 150 psia (135 psig) and 300 psia (285 psig). The higher pressure steam should result in a smaller heat exchanger but the steam will cost more. Which steam choice would be better?

Data:

The heat capacity of the organic liquid is 0.6 Btu/lb · °F.

The plant on-stream operation factor is expected to be 90%.

Steam properties are:

	150 psia	300 psia
Saturation temperature, °F	358.0	417.0
Latent heat (enthalpy), Btu/lb	863.6	809.0
Cost, $/1000 lb	5.20	5.75

Heat exchanger cost correlation (1998 basis):

Base cost (BC) = 117 $A^{0.65}$

Installation factor (IF) = 3.29

Pressure factors (PF):

100 to 200 psig = 1.15

200 to 300 psig = 1.20

Cost indexes (CI):

1998 = 230

2011 = 360

Capital cost (C) = (BC)(IF)(PF)(CI)

***SOLUTION*:** Calculate the overall heat duty:

$$\begin{aligned}\dot{Q} &= \dot{m}c_p(T_2 - T_1) = (50{,}000\ \text{lb/h})(0.6\ \text{Btu/lb}\cdot{}^\circ\text{F})(330 - 150^\circ\text{F})\\ &= 5{,}400{,}000\ \text{Btu/h}\end{aligned}$$

Calculate the log-mean temperature difference for each case:

	150 psia case	300 psia case
ΔT_1	358 − 150 = 208	417 − 150 =267
ΔT_2	358 − 330 = 28	417 − 330 = 87
LMTD	89.8°F	160.5°F

Calculate the required heat transfer area for each case:

$$A_{150} = (5{,}400{,}000)/[(138)(89.8)] = 436\ \text{ft}^2$$

$$A_{300} = (5{,}400{,}000)/[(138)(160.5)] = 244\ \text{ft}^2$$

Determine the capital cost for each case:

$$\text{Cost}_{150} = (117)(436)^{0.65}(360/230)(3.29)(1.15) = \$36{,}000$$

$$\text{Cost}_{300} = (117)(244)^{0.65}(360/230)(3.29)(1.20) = \$25{,}800$$

Obtain the steam requirement for each case in lb/yr:

$$\begin{aligned}\text{St}_{150} &= (5{,}400{,}000\ \text{Btu/h})(8760 \times 0.9\ \text{h/yr})/(863.6\ \text{Btu/lb}) \\ &= 49.3\ \text{million lb/yr} \\ \text{St}_{300} &= (5{,}400{,}000\ \text{Btu/h})(8760 \times 0.9\ \text{h/yr})/(809.0\ \text{Btu/lb}) \\ &= 52.6\ \text{million lb/yr}\end{aligned}$$

Use the above calculation to determine the annual steam cost for each case:

$$\text{StCost}_{150} = (49.3 \times 10^6\ \text{lb/yr})(0.00520\ \$/\text{lb}) = \$256{,}000/\text{yr}$$

$$\text{StCost}_{300} = (52.6 \times 10^6\ \text{lb/yr})(0.00575\ \$/\text{lb}) = \$303{,}000/\text{yr}$$

The 300-psia exchanger costs \$10,200 less to purchase and install, but it costs \$47,000 per year more to operate. Choosing the more expensive, 150-psia exchanger is the obvious choice. ■

ILLUSTRATIVE EXAMPLE 27.9

Two small commercial power plant designs are under consideration. The first design involves a traditional boiler (TB) and the second a fluidized fed (FB). For the TB system, the total capital cost (TCC) is \$2.5 million, the annual operating costs (AOC) are \$1.2 million, and the annual revenue generated from the facility (*R*) is \$3.6 million. For the FB system, TCC, AOC and R are \$3.5, 1.4 and 5.3 million, respectively. Using straight-line depreciation and the discounted cash flow method, which unit is more attractive? Assume a 10-yr facility lifetime and a 2 yr construction period. Note that the solution involves the calculation of the rate of return for each of the two proposals.$^{(11)}$

***SOLUTION*:** For TB system, calculate the depreciation *D*, the working capital WC, the taxable income TI, the income tax to be paid IT, and the annual after tax cash flow *A*. The depreciation is (straight-line):

$$\begin{aligned}D &= 0.1(\text{TCC}) \\ &= (0.1)(\$2{,}500{,}000) \\ &= \$250{,}000\end{aligned}$$

The WC is set at 10% of the TCC:

$$\begin{aligned}\text{WC} &= 0.1(\text{TCC})\\ &= (0.1)(\$2{,}500{,}000)\\ &= \$250{,}000\end{aligned}$$

In addition,

$$\begin{aligned}\text{TI} &= R - \text{AOC} - D\\ &= \$3{,}600{,}000 - \$1{,}200{,}000 - \$250{,}000\\ &= \$2{,}150{,}000\end{aligned}$$

and one may estimate that

$$\begin{aligned}\text{IT} &= (0.5)\text{TI}\\ &= (05)(\$2{,}150{,}000)\\ &= \$1{,}075{,}000\end{aligned}$$

The after-tax cash flow is calculated using

$$\begin{aligned}A &= R - \text{AOC} - \text{IT}\\ &= \$3{,}600{,}000 - \$1{,}200{,}000 - \$1{,}075{,}000\\ &= \$1{,}325{,}000\end{aligned}$$

The rate of return, *i*, for the TB unit is also calculated. This rate of return can be computed by solving the equation below:

$$\left[\frac{(1+i)^{10}-1}{I(1+i)^{10}}\right]A + \left[\frac{1}{(1+i)^{10}}\right]\text{WC}$$
$$= \text{WC} + (0.5)\text{TCC} + (0.5)\text{TCC}(1+i)^{1}$$

or

$$\left[\frac{(1+i)^{10}-1}{I(1+i)^{10}}\right](1.325\times 10^{6}) + \left[\frac{1}{(1+i)^{10}}\right](0.250\times 10^{6})$$
$$= (0.250\times 10^{6}) + (0.5)(1.250\times 10^{6}) + (0.5)(1.250\times 10^{6})(1+i)^{1}$$

By trial and error (assume *i*),

$$i = 39.6\% \simeq 40\%$$

For the FB system,

$$\begin{aligned}\text{WC} = D &= (0.1)(\$3{,}500{,}000)\\ &= \$350{,}000\\ \text{TI} &= \$5{,}300{,}000 - \$1{,}400{,}000 - \$350{,}000\\ &= \$3{,}550{,}000\\ \text{IT} &= (0.5)(\$3{,}550{,}000)\\ &= \$1{,}775{,}000\end{aligned}$$

The annual after-tax cash flow is

$$\begin{aligned} A &= \$5{,}300{,}000 - \$1{,}400{,}000 - \$1{,}775{,}000 \\ &= \$2{,}125{,}000 \end{aligned}$$

The rate of return equation for the FB unit becomes

$$\begin{aligned} &\left[\frac{(1+i)^{10}-1}{i(1+i)^{10}}\right](2.125\times10^6)+\left[\frac{1}{(1+i)^{10}}\right](0.3650\times10^6) \\ &= (0.350\times10^6)+(0.5)(1.750\times10^6)+(0.5)(1.750\times10^6)(1+i)^1 \end{aligned}$$

By trial and error,

$$i = 44.8\%$$

Hence, by the discounted cash flow method, the rate of return on the initial capital investment is approximately 5% greater for the FB system than the TB system. From a purely financial standpoint, the FB system is the more attractive option. ■

ILLUSTRATIVE EXAMPLE 27.10

A stream of 100,000 acfm flue gas from a utility facility is to be cooled in an air preheater. You have been requested to find the best unit to install to cool the flue gas and preheat the combustion air feed to the boiler. A reputable vendor has provided information on the cost of three units, as well as installation, operating, and maintenance costs. Table 27.4 summarizes all the data you have collected. Determine what preheater you would select in order to minimize costs on an annualized basis.

***SOLUTION*:** The first step is to convert the equipment, installation, and operating costs to total costs by multiplying each by the total gas flow, 100,000 acfm. Hence, for the finned exchanger, the total costs are

$$\begin{aligned} \text{Equipment cost} &= 100{,}000\text{ acfm }(\$3.1/\text{acfm}) = \$310{,}000 \\ \text{Installation cost} &= 100{,}000\text{ acfm }(\$0.80/\text{acfm}) = \$80{,}000 \\ \text{Operating cost} &= 100{,}000\text{ acfm }(\$0.06/\text{acfm}\cdot\text{yr}) = \$6000/\text{yr} \end{aligned}$$

Table 27.4 Preheater Cost Data

	Finned	4-pass	2-pass
Equipment cost	\$3.1/acfm	\$1.9/acfm	\$2.5/acfm
Installation cost	\$0.80/acfm	\$1.4/acfm	\$1.0/acfm
Operating cost	\$0.06/acfm-yr	\$0.06/acfm-yr	\$0.095/acfm-yr
Maintenance cost	\$14,000/yr	\$28,000/yr	\$9500/yr
Lifetime of equipment	20 yr	15 yr	20 yr

Costs are based on comparable performance. Interest rate is 10% and there is zero salvage value.

Table 27.5 Preheater Cost Calculations

	Finned	4-pass	2-pass
Equipment cost	\$310,000	\$190,000	\$250,000
Installation cost	\$80,000	\$140,000	\$100,000
CRF	0.11746	0.13147	0.11746
Annual equipment cost	\$36,412	\$24,980	\$29,365
Annual installation cost	\$9397	\$18,405	\$11,746
Annual operating cost	\$6000	\$6000	\$9500
Annual maintenance cost	\$14,000	\$28,000	\$9500
Total annual cost	\$65,809	\$77,385	\$60,111

Note that the operating costs are on an annualized basis. The equipment cost and the installation cost must then be converted to an annual basis using the CRF. From Equation (27.11)

$$\begin{aligned}\text{CRF} &= (0.1)(1+0.1)^{20}/[(1+0.1)^{20}-1]\\ &= 0.11746\end{aligned}$$

The annual costs for the equipment and the installation is given by the product of the CRF and the total costs of each:

$$\begin{aligned}\text{Equipment annual cost} &= \$310{,}000(0.11746)\\ &= \$36{,}412/\text{yr}\\ \text{Installation annual cost} &= \$80{,}000(0.11746)\\ &= \$9397/\text{yr}\end{aligned}$$

The calculations for the 4-pass and the 2-pass exchangers are performed in the same manner. The three preheaters can be compared after all the annual costs are added. The tabulated results are provided in Table 27.5. According to the analysis, the 2-pass exchanger is the most economically attractive device since the annual cost is the lowest. ■

ILLUSTRATIVE EXAMPLE 27.11

A new large diameter steam line, carrying steam at 450 K is being designed. It is desired to calculate the optimal insulation thickness for this line. Outline the calculations required.

***SOLUTION*:** The optimum thickness of insulation is arrived at by employing an economic approach. If a bare pipe were to carry a hot fluid, there would be a certain hourly loss of heat whose value could be determined from the cost of producing the energy. The lower the heat loss the greater the thickness and initial cost of the insulation and the greater the annual fixed charges (maintenance and depreciation) which must be added to the annual heat loss. Assuming a number of thicknesses of insulation and adding the fixed charges to the value of the heat lost, a minimum cost can be obtained and the thickness corresponding to it will represent the optimum economic thickness of the insulation. The form of a graphical solution is provided in Figure 27.1. ■

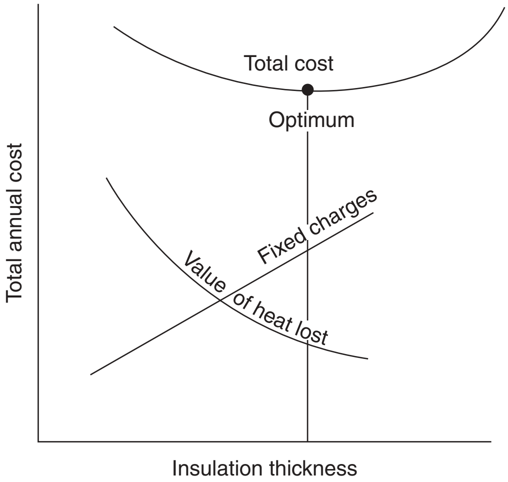

Figure 27.1 Optimum thickness of insulation.

ILLUSTRATIVE EXAMPLE 27.12

Shannon O'Brien, a recent graduate from Manhattan College's prestigious chemical engineering program was given an assignment to design the most cost-effective heat exchanger to recover energy from a hot flue gas at 500°F. The design is to be based on pre-heating 100°F incoming air (to be employed in the boiler) to a temperature that will result in the maximum annual profit to the utility. A line diagram of the proposed countercurrent exchanger is provided in Figure 27.2.

Having just completed a heat transfer course with Dr. Flynn and a thermodynamics course with the infamous Dr. Theodore, Shannon realizes that their are two costs that need to be considered:

1. the heat exchanger employed for energy recovery, and
2. the "quality" (from an entropy perspective) of the recovered energy—refer to Chapter 20 for additional details.

She also notes that the higher the discharge temperature of the heated air, t, the smaller will be the temperature difference driving force, and the higher the area requirement of the exchanger and the higher the equipment cost. Alternatively, with a higher t, the "quality" of the recovered energy is higher, thus leading to an increase in recovered profits (by reducing fuel costs).

Based on similar system designs, Ricci Consultants (RC) has provided the following annual economic models:

$$\begin{aligned} \text{Recovered energy profit:}\quad & A(t - t_c); \qquad A = \$/\text{yr} \cdot {}^\circ\text{F} \\ \text{Exchange cost:}\quad & B/(T_H - t); \quad B = \$ \cdot {}^\circ\text{F}/\text{yr} \end{aligned}$$

For the above system, RC suggests a value for the coefficients in the cost model be set at:

$A = 10$
$B = 100{,}000$

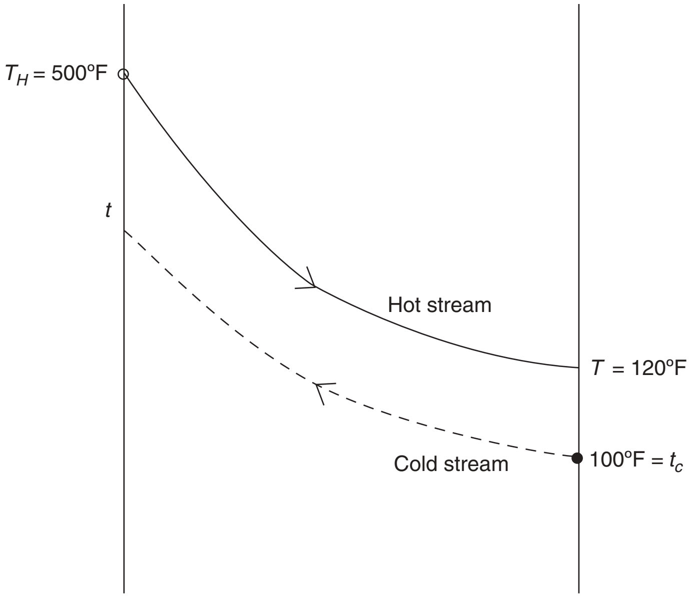

Figure 27.2 Proposed countercurrent exchanger.

Employing the above information, Shannon has been asked to calculate a t that will

1. provide breakeven operation
2. maximize profits

She is also required to perform the calculation if $A = 10$, $B = 4000$, and $A = 10$, $B = 400{,}000$. Finally, an analysis of the results is requested.

***SOLUTION*:** Since there are two contributing factors to the cost model, one may write the following equation for the profit, P

$$P = A(t - t_c) - B/(T_H - t); \quad T_H = 500 \quad \text{and} \quad t_c = 100$$

For breakeven operation, set $P = 0$ so that

$$(t - t_c)(T_H - t) = B/A$$

This may be rewritten as

$$t^2 - (T_H + t_c)T + [(B/A) + T_H t_c] = 0$$

The solution to this quadratic equation for $A = 10$ and $B = 100{,}000$ is

$$\begin{aligned} t &= \frac{600 \pm \sqrt{(600)^2 - (4)(1)(10{,}000 + 50{,}000)}}{2} \\ &= \frac{600 \pm 346}{2} \\ &= 473^\circ\text{F},\ 127^\circ\text{F} \end{aligned}$$

To maximize the profit, take the first deriative of P with respect to t and set it equal to zero, i.e.,

$$\frac{dP}{dt} = A - \frac{B}{(T_H - t)^2} = 0$$

Solving,

$$\begin{aligned}(T_H - t)^2 &= B/A \\ T_H - t &= \sqrt{B/A} \\ &= \sqrt{10{,}000} \\ &= 100 \\ t &= 400^\circ\mathrm{F}\end{aligned}$$

Upon analyzing the first derivative with t values greater than and less than 400°F, Shannon observes that the derivative changes sign from $+ \rightarrow -$ about $t = 400$, indicating a relative maximum.

Similarly, for $A = 10$, $B = 4000$,

$$\begin{aligned}t_{\mathrm{BE}} &= 499^\circ\mathrm{F},\ 101^\circ\mathrm{F} \\ t_{\max} &= 480^\circ\mathrm{F}\end{aligned}$$

For $A = 10$, $B = 400{,}000$,

$$\begin{aligned}t_{\mathrm{BE}} &= 300^\circ\mathrm{F} \\ t_{\max} &= 300^\circ\mathrm{F}\end{aligned}$$

Graphical results for the three scenarios is shown in Figure 27.3. ■

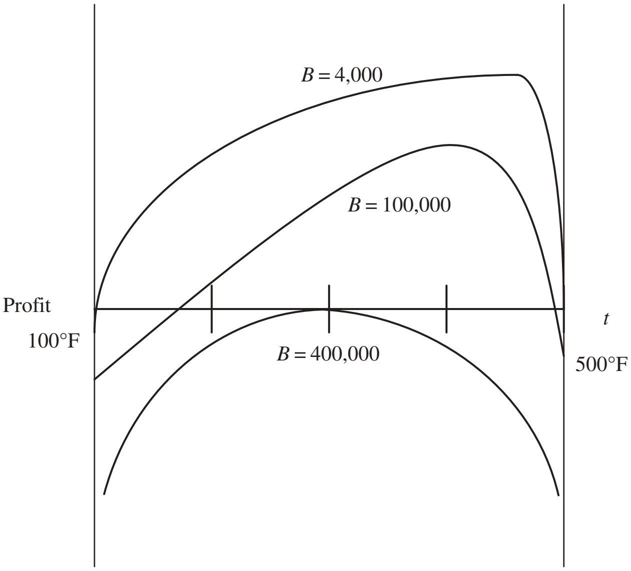

Figure 27.3 Heat exchanger results.

ILLUSTRATIVE EXAMPLE 27.13

Two hot liquid (essentially water) process streams are discharged from sources *A* and *B* in a plant. A young engineer has proposed to recover energy from the two streams by employing two heat exchangers that are currently available on site: a double pipe (DP) exchanger and a shell-and-tube (ST) exchanger. Due to diameter differences in the two lines that discharge the hot liquid from source *A*, there is a 60/40 split (mass basis) of the flowrate. There are also two lines discharging liquid from source *B* with a 75/25 split to the ST/TB exchangers. Due to the location of the exchangers relative to the two sources, the 40% flow from *A* and 75% flow from *B* can be diverted (fed) to the ST exchanger. The remaining discharge can only be sent to the DP exchanger.

The young engineer has obtained additional information. The maximum flowrate that exchanger ST and DP can accommodate to 12,000 ft^3/day and 6000 ft^3/day respectively. In addition, the maximum flow that be drawn from sources *A* and *B* is 8000 ft^3/day and 6000 ft^3/day respectively.

Prepare a line diagram of a system to recover the energy from sources $A + B$.

***SOLUTION*:** The line diagram is provided in Figure 27.4. ■

ILLUSTRATIVE EXAMPLE 27.14

Refer to Illustrative Example 27.13. The profit value of the recovered energy from sources *A* and *B* are \$1.70/ft^3 and \$2.00/ft^3, respectively. *Develop* the describing equations that will provide values of the (volumetic) flowrates from *A* and *B*, i.e., q_A and q_B, that will maximize the profit *P* for the proposed process.

***SOLUTION*:** The discribing equations are as follows. The objective function of the profit is

$$P = (1.70)q_A + (2.00)q_B$$

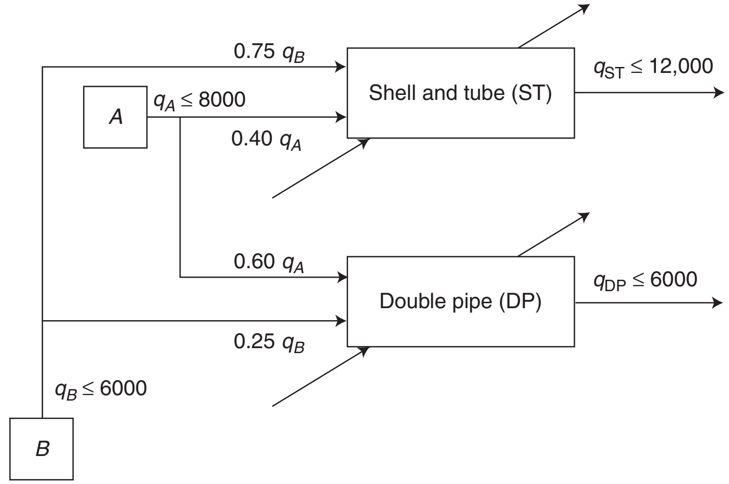

Figure 27.4 Shell-and-tube and double pipe exchangers.

The constraints are

$$\begin{aligned} q_A &\le 8000 \\ q_B &\le 6000 \\ 0.75q_A + 0.40q_B = q_{ST} &\le 12{,}000 \\ 0.60q_A + 0.25q_B = q_{DP} &\le 6000 \\ q_A \ge 0,\ q_B &\ge 0 \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 27.15

Refer to the two previous examples. Calculate values of q_A and q_B that will maximize the profit from this process. Also calculate the annual (365 day basis) profit based on this condition.

***SOLUTION*:** Employing a suitable optimization program gives:

$$\begin{aligned} q_A &= 7500\ \text{ft}^3/\text{day} \\ q_B &= 6000\ \text{ft}^3/\text{day} \\ P &= 24{,}750\ \$/\text{day} \\ &= \$8{,}910{,}000\ \$/\text{yr} \end{aligned}$$

Note that operating, maintenance, depreciation, etc., costs have not been included in the analysis. ■

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Chapter 28

Open-Ended Problems

INTRODUCTION

The education literature provides frequent references to individuals, particularly engineers and scientists, that have different learning styles. And, in order to successfully draw on these different styles, a variety of approaches can be employed. One such approach involves the use of open-ended problems.

The term "open-ended problem" has come to mean different things to different people. It basically describes an approach to the solution of a problem and/or situation for which there is usually not a unique solution. The author of this text has applied this somewhat unique approach by including numerous open-ended problems in several course offerings at Manhattan College. Student comments for the graduate course "Accident and Emergency Management" were recently tabulated. Student responses (unedited) to the question, "What aspects of this course were most beneficial to you?" are listed below:

1. "The open-ended questions gave engineers a creative license. We don't come across many of these opportunities."
2. "Open-ended questions allowed for candid discussions and viewpoints the class may not have been otherwise exposed to."
3. "The open-ended questions gave us an opportunity to apply what we were learning in class with subjects we have already learned which gave us a better understanding of the course."
4. "Much of the knowledge that was learned in this course can be applied to everyday situations and in our professional lives."
5. "Open-ended problems made me sit down and research the problem to come up with ways to solve them."
6. "I thought the open-ended problems were inventive and got me to think about problems in a better way."
7. "I felt that the open-ended problems were challenging. I, like most engineers, am more comfortable with quantitative problems vs qualitative."

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

There is a tale$^{(1)}$ that appeared in print many years ago that dissected the value of an open-ended approach to a particular problem. The story (adapted for purposes of this text) is presented below. This is a tale that occurred late in 1960 that may or may not serve to justify the value of an open-ended approach to a particular engineering/science problem. That story, as recounted by an engineering educator, follows.

Some time ago, I received a call from a colleague named Lou Theodore who asked if I would be willing to serve as the referee on the grading of an examination question. He was about to give a student a zero for his answer to a heat transfer question, while the student claimed he should receive a perfect score and would if the system were not rigged against the student. During discussions with my colleague, it became apparent that he felt the student was a "loser and a liability, and would ultimately serve as an embarrassment to the department in later years." He also complained that the student "was Irish, often showed up late for class, was occasionally inebriated, and used many of my lectures as a sleeping pill."

I went to my colleague's office and read the examination question: "As part of a QA/QC (quality assurance/quality control) test, outline how one can ascertain the readings on this long stem mercury thermometer employed in the department's lab is correct." My colleague felt that the student should have provided an answer that reflected the lecture material. For example, the mercury in the narrow tube expands as it gets hotter due to its coefficient of expansion. In effect, the mercury expands by an amount that linearly depends on the temperature so that a rise of 10 degrees produces twice the expansion than a 5 degree increase. In any event, my colleague (Dr. Theodore) and the student agreed to submit this to an impartial arbiter, and I had been selected.

The student had answered the exam as follows: "Place several other thermometers along the one of concern and check the readings. If all the readings are in close agreement with one another, one can conclude that the lab thermometer reading is correct."

I pointed out to Dr. Theodore that the student really had a strong case for full credit since he had answered the question completely and correctly. On the other hand, if full credit was given, it could well contribute to a high grade for the student in his heat transfer course. A high grade is supposed to certify competence in the subject of concern, but the answer did not confirm this. I suggested that the student have another try at answering the question. I was not surprised that Dr. Theodore agreed, but I was surprised that the student did.

I gave the student ten minutes to answer the question with the warning that the answer should show some knowledge of heat transfer. At the end of five minutes, he had not written anything. I asked if he wished to give up, but he said no. He had many answers to this problem; he was just thinking of the best one. I excused myself for interrupting him and asked him to please go on. In the next minute, he dashed off his answer which read:

> *Call the local airport and weather bureau and inquire about the ambient temperature. Then take the thermometer outside. If the reading is in close proximity to that reported by the two agencies, one can conclude that the reading is correct.*

At this point, I asked my colleague it he would give up. He conceded and I gave the student almost full credit.

In leaving Dr. Theodore's office, I recalled that the student had said he had many other answers to the problem, so I asked him what they were. "Oh yes" said the student. "There are a great many ways of checking the validity of the thermometer readings. For example, you could mail the thermometer back to the vendor and request that it be recalibrated to reassure the reading is valid."

"Fine" I asked, "And the others?"

"Yes," said the student. "There is a very basic measurement method that you will like. In this method you take the thermometer and place it in running tap water. Since the tap water in the lab is in the 62–64°F range, a temperature reading of the lab thermometer in that range would signify validity."

"Of course, if you want an even simpler method, you can take the thermometer to the lab technician. Speak to him as follows: 'Mr. Technician, here I have a fine thermometer. If you tell me if the thermometer is properly calibrated, I will give you this thermometer.'"

Finally, he concluded there are many other ways of solving the problem. "Probably the best but perhaps a somewhat unorthodox way is to use the traditional method to take Dr. Theodore's temperature. If the thermometer registers something in the 98.6°F area, it must be in working order."

At this point I asked the student if he really did know the conventional answer to this question. He admitted that he did, but said that he was fed up with traditional education and college professors trying to teach him how to think using the "scientific method", i.e., to explore the deep inner logic of the subject in a pedantic way, as is often done in the new mathematics, rather than teaching him the structure of the subject.

Interestingly, the student did manage to graduate despite the efforts of my dear colleague. The student married soon after graduation, and his two children—Michelle and Mark—later served as babysitters for the children of my colleague who had married a beautiful woman of Irish decent later in life. The student? John D. McKenna, who went on to earn a law degree and a PhD. He is currently President and CEO of ETS International in Roanoke, VA.

So much for the decision-making capabilities of some of our educators. Need more be said?

The remainder of this chapter addresses a host of topics involved with open-ended problems. The remaining sections are entitled:

Developing Students' Power of Critical Thinking

Creativity

Brainstorming

Inquiring Minds

The chapter concludes with open-ended applications primarily in the heat transfer field. Applications in other areas can be found in the literature.(2,3)

DEVELOPING STUDENTS' POWER OF CRITICAL THINKING(4)

It has often been noted that today's society is living in the middle of an information revolution. That revolution has had an effect on teaching and learning for nearly two decades. Teachers are hard-pressed to keep up with the advances in their fields. Often their attempts to keep the students informed are limited by the difficulty of making new material available.

The basic need of both teacher and student is to have useful information readily accessible. Then comes the problem of how to use this information properly. The objectives of both teaching and studying such information are: to assure comprehension of the material and to integrate it with the basic tenets of the field it represents; and, to use comprehension of the material as a vehicle for critical thinking, reasoning, and effective argument.

Information is valueless unless it is put to use; otherwise it becomes mere data. To use information most effectively, it should be taken as an instrument for understanding. The process of this utilization works on a number of incremental levels. Information can be: absorbed, comprehended, discussed, argued in reasoned fashion, written about, and integrated with similar and contrasting information.

The development of critical and analytical thinking is the key to the understanding and use of information. It is what allows the student to discuss and argue points of opinion and points of fact. It is the basis for the student's formation of independent ideas. Once formed, these ideas can be written about and integrated with both similar and contrasting information.

CREATIVITY

Engineers bring mathematics and science to bear on practical problems, molding materials and harnessing technology for human benefit. Creativity is often a key component in this synthesis; it is the spark that motivates efforts to devise solutions to novel problems, design new products, and improve existing practices. In the competitive marketplace, it is a crucial asset in the bid to win the race to build better machines, create new materials, decrease product delivery times, and anticipate the needs of future generations.(5)

Although one of the keys to success for an engineer or a scientist is the ability to generate fresh approaches, processes, and products, they need to be creative. Gibney(5) has detailed how some schools and institutions are attempting to use certain methods that essentially share the same objective: to open students' minds to their own creative potential.

Gibney(5) provides information on "The Art of Problem Definition" developed by Rensselaer Polytechnic Institute. To stress critical thinking, they teach a seven step methodology for creative problem development. These steps are:

1. Define the problem
2. State the objective(2)

3. Establish functions
4. Develop specifications
5. Generate multiple alternatives
6. Evaluate alternatives
7. Build

In addition, Gibney(5) identified the phases of the creative process set forth by psychologists. They essentially break the process down into five basic stages:

1. Immersion
2. Incubation
3. Insight
4. Evaluation
5. Elaboration

Psychologists have ultimately described the creative process as recursive. At any one of these stages, a person can double back, revise ideas, or gain new knowledge that reshapes his or her understanding. For this reason, being creative requires patience, discipline, and hard work.

Finally, Delle Femina(6) recently outlined five secrets regarding the creative process:

1. Creativity is ageless
2. You don't have to be Einstein
3. Creativity is not an eight hour job
4. Failure is the mother of all creativity
5. Dead men don't create

The reader is left with a thought from Theodore:(7) Creativity usually experiences a quick and quiet death in rooms that house large conference tables.

BRAINSTORMING

Panitz(8) has demonstrated how brainstorming strategies can help engineering students generate an outpouring of ideas. Brainstorming guidelines include:

1. Carefully define the problem upfront
2. Allow individuals to consider the problem before the group tackles it
3. Create a comfortable environment
4. Record all suggestions
5. Appoint a group member to serve as a facilitator
6. Keep brainstorming groups small

A checklist for change was also provided:

1. Adapt
2. Modify
3. Magnify
4. Minify
5. Put to other uses
6. Substitute
7. Rearrange
8. Reverse
9. Combine

INQUIRING MINDS

In an exceptional and well-written article by Lih(9) titled *Inquiring Minds*, Lih commented on Inquiring Minds saying “You can’t transfer knowledge without them.” His thoughts (which have been edited) on the inquiring or questioning process follow.

1. Inquiry is an attitude—a very important one when it comes to learning. It has a great deal to do with curiosity, dissatisfaction with the status quo, a desire to dig deeper, and having doubts about what one has been told.
2. Questioning often leads to believing. There is a saying that has been attributed to Confucius: “Tell me, I forget. Show me, I remember. Involve me, I understand.” It might also be fair to add: “Answer me, I believe.”
3. Effective inquiry requires determination to get to the bottom of things.
4. Effective inquiry requires wisdom and judgment. This is especially true for a long-range intellectual pursuit that is at the forefront of knowledge.
5. Inquiry is the key to successful life-long learning. If one masters the art of questioning, independent learning is a breeze.
6. Questioning is good for the questionee as well. It can help clarify issues, uncover holes in an argument, correct factual and/or conceptual errors, and eventually lead to a more thoughtful outcome.
7. Teachers and leaders should model the importance of inquiry. The teacher/leader must allow and encourage questions and demonstrate a personal thirst for knowledge.

APPLICATIONS

Twelve open-ended illustrative examples follow.

ILLUSTRATIVE EXAMPLE 28.1

Using terms a liberal arts major could understand, briefly explain the concept of:

1. Unit conversion(s) or conversion factors
2. Dimensional analysis
3. Dimensionless numbers
4. Kinetic energy
5. Potential energy
6. Energy conservation

***SOLUTION*:** Qualitative questions like these are often difficult to answer. Perhaps (1) is the easiest of the six. For this case, one might draw an analogy between 5 pennies and 1 nickel, or 2 nickels and 1 dime, or 12 inches and 1 foot, etc. ■

ILLUSTRATIVE EXAMPLE 28.2

The Alaska Department of Environmental Protection (DEP) has requested your consulting firm to provide "the best estimate" of the oil requirements to sustain a proposed remote outpost in the middle of a pristine region. If both the heating and electrical requirements for the site are known, *outline* how to calculate the average daily oil needs. Assume the energy value of the oil is given. Indicate if any additional information is required. Note that no calculations are required.(10)

***SOLUTION*:** When calculating the average daily oil needs for the outpost, several factors come into play. The most important is knowing the heating and electrical requirements for the site and the heating value of the oil. It is also important to take into account how the oil will be transported to the outpost. It is one thing to purchase the oil and another entirely different thing to transport and store the oil at the outpost. As this nation knows, oil prices are governed by international and national events as well as the demand for the oil. Thus, the fluctuations in the oil prices also need to be accounted for. The thermal conversion efficiencies are also required to be known. The following four-step approach is suggested.

Step 1: Break down the given heating and electrical requirements into a per month basis and from this the average daily oil needs can be obtained. (Take the total monthly requirements and divided it by the number of days in a month to get the average daily requirements.) One reason for this is that winter months tend to use more electricity, heating and oil than summer months. Also, one must know when the "peak hours are." This is what is referred to as "demand."

Step 2: Use conversion factors to convert the heating and electrical units into oil units. Oil is usually sold by the barrel. Once all the units are converted into Btu units, the heating value of oil can be used (such as one barrel = 6.287 million Btu) as a way to see how many barrels of oil are required.

Step 3: Find out the cost of oil after calculating how much the daily and monthly consumption will be.

Step 4: Decide how the oil will be transported to the outpost and how/where it will be stored.

Include all these factors in the analysis. ■

Table 28.1 P, V, T Data

P, atm	V, m^3	T, K	z
1.0	10.0	200	1.0
5.0	4.0	400	1.0
20.0	2.3	800	1.15
40.0	1.3	800	1.3
100.0	1.6	1,000	3.2

ILLUSTRATIVE EXAMPLE 28.3

The P, V, T (plus the compressibility coefficient, z) data for a recently synthesized gas is given in Table 28.1 ($MW = 609$ gmol). Based on either statistical or theoretical principles (or both), develop an equation describing the P, V, T behavior of this gas. State the limitations of the approach and/or final equations.

***SOLUTION*:** The sky is the limit regarding what approach to employ. See Illustrative Example 28.4 for suggestions. ■

ILLUSTRATIVE EXAMPLE 28.4

The heat capacity variation with temperature of a fluid are given in Table 28.2. As part of a simulation project, your immediate supervisor has requested that you develop an algebraic equation describing the c_p variation with T. Outline how to solve the problem and justify your approach.

***SOLUTION*:** As noted in Chapter 26, regression analysis is a useful statistical technique for developing a quantitative relationship between a dependent variable and one or more independent variables. It usually utilizes experimental data on the pertinent variable to develop a numerical equation showing the influence of the independent variable(s) on the dependent variable of the system.

As noted in Chapter 26, a simple correlation problem can arise when one asks whether there is any relationship between smoking and heart ailments, between beauty and brains, between kinetic energy and velocity, between drag force and pressure, etc. For example, in a thermal combustion unit involving a complex chemical reaction, regression methods have been used to develop an equation relating the conversion to entering concentrations, temperature, pressure, and residence time.(11,12) One of the problems of fitting a curve to a set of points in some efficient manner is essentially that of estimating the parameters of a proposed equation. Although there

Table 28.2 Heat Capacity Variation with Temperature

C_P, cal/gmol · K	T, K
0.203	200
0.238	300
0.258	400
0.274	500

are numerous methods for performing the estimation of such parameters, the best known and most popular method is referred to as the method of least squares.(12)

The least-squares regression development can be applied to several types of polynomials. Four of these functions are:

$$\text{0th degree polynomial: } y = a_0 \tag{28.1}$$

$$\text{1st degree polynomial: } y = a_0 + a_1x_1 \tag{28.2}$$

$$\text{2nd degree polynomial: } y = a_0 + a_1x_1 + a_2x_2^2 \tag{28.3}$$

$$n\text{th degree polynomial: } y = a_0 + a_1x_1 + \cdots + a_nx_n^n \tag{28.4}$$

Applying these equations to the four data points is left as an exercise for the reader. However, some information is provided in the Numerical Methods chapter (see Chapter 26).

Finally, to obtain information on the "best" model, one must resort to a statistical technique referred to as ANOVA, an acronym referring to ANalysis Of VAriance. This topic is beyond the scope of this text; however, information is available in the literature.(12) ■

ILLUSTRATIVE EXAMPLE 28.5

It has been proposed to use the sensible heat from the flue gas of a boiler to reduce the energy need of the combustion process. A young engineer, recently graduated from Manhattan College's prestigious #2 ranked Chemical Engineering Department has proposed to recover the heat in a heat exchanger and use the recovered heat to preheat the air required for combustion. Comment on the advantages and disadvantages of the young engineer's proposal.

***SOLUTION*:** The total energy content of the gas should be taken into consideration before implementing the proposal since the mass involved is (relatively speaking) small (i.e., the density of the gas is low). ■

ILLUSTRATIVE EXAMPLE 28.6

Over time, the heat required to keep a catalytic cracker in an oil refinery at its operating temperature of 1200°F has increased by a factor of 2. Rather than purchasing a new reactor, you are asked to recommend other possible ways to decrease the amount of energy required to heat the reactor to 1200°F.

***SOLUTION*:**

1. Schedule a routine maintenance during a plant shutdown to clear any possible carbon buildup in the reactor.
2. Preheat the crude oil to a higher temperature before entering the reactor so less heat is required for heating to 1200°F.
3. Paint the reactor black to absorb heat to assist the heating of the reactor contents. ■

ILLUSTRATIVE EXAMPLE 28.7

Provide reasons for choosing a batch heat-transfer operation rather than a continuous one.(4)

***SOLUTION*:**

1. The liquid being processed for the product is not continuously available.
2. The heating or cooling medium to the equipment is not continuously available.
3. Reaction-time or treating-time requirements necessitate holdup.
4. The economics of intermittently processing a large batch is often justified.
5. Cleaning or regeneration procedures take up a significant part of the total operating period.
6. Simplified operation of most batch processes. ■

ILLUSTRATIVE EXAMPLE 28.8

As a plant manager and as part of a project concerned with energy conservation, you have asked your plant personnel to examine and provide recommendations in order to reduce the use of utilities, i.e., fuel, steam, water, etc. A young upstart on your staff has requested that you detail exactly what she should be looking for. List recommendations for this individual.

***SOLUTION*:**(13)

Fouling

An expression for the overall heat transfer coefficient of a heat exchanger is:

$$\frac{1}{UA} = \frac{1}{(\eta hA)_c} + \left(\frac{R_f}{\eta A}\right)_c + \frac{1}{(\eta hA)_h} + \left(\frac{R_f}{\eta A}\right)_h + R_w$$

The assumption is made that the heat exchangers in the plant have been in service and have set operating conditions and surface areas. It is clear from this equation that fouling of a heat exchanger's surfaces will increase the resistance for heat transfer between fluids. Fouling in a heat exchanger may be reduced as follows:

1. Periodically draining and cleaning the heat exchanger (especially fuel oil heaters)
2. Reducing the temperature increase when using an impure coolant (i.e., fouling factors are significantly increased with sea and river waters raised above 50°C).
3. When using oils, use high-grade oil that will not suffer thermal breakdown.
4. Oil purifiers can be incorporated into the closed loop oil cooling systems.

Fluid Combinations

Use different fluid combinations to improve heat transfer. For example, an average heat transfer coefficient between oil and water is $230\ \text{W/m}^2 \cdot \text{K}$. If temperatures allow using water on both sides of the heat exchanger, the efficiency will improve because the heat transfer coefficient between water and water is approximately $1275\ \text{W/m}^2 \cdot \text{K}$.

Performance Analysis

The performance of a concentric tube or shell and tube heat exchanger can be improved by simply changing the flow characteristics of the fluids. The equation used for heat exchanger analysis is (see Chapter 14):

$$Q = UA\Delta T_{\text{lm}}$$

One of the most important observations that can be made using this equation is that the log mean temperature difference for a counterflow heat exchanger will be greater than that for a parallel flow heat exchanger. This means that for similar inlet and outlet conditions, and the same surface area, a counterflow exchanger will provide greater heat transfer. On the design end, the surface area requirements to affect a prescribed transfer rate would be smaller for a counterflow exchanger than for a parallel flow exchanger. Thus, simply changing the flow of fluids will make a difference in the heat transfer rate of an exchanger.

Insulation

Loss of heat from heat exchangers, tanks and piping can be reduced by properly insulating equipment. Heat loss or gain to the atmosphere may counteract the work of a heat exchanger and reduce the net heat transfer.

If heat transfer rates are increased, then less utilities, steam and other forms of energy are needed to achieve desired heat transfer rates. ■

ILLUSTRATIVE EXAMPLE 28.9

List a key factor that should be included in the design of a heat exchanger.

***SOLUTION*:** There are a number of factors that should be included in the design of a heat exchanger.

One of the more important is that the exchanger should be designed for appropriate film temperatures and high turbulence. This reduces the inefficiencies of heat transfer by the exchanger. This reduction takes place since there is a decrease in cling (which appears on the process side) or scale (which appears on the utility side). In addition, the exchanger is able to operate more efficiently with less cling or scale and therefore, transfers more energy. ■

ILLUSTRATIVE EXAMPLE 28.10

A heat exchanger is designed to cool a discharging water stream to a required 105°F. However, the unit is currently only cooling the stream to the 110 to 115°F range. Rather than replace the heat exchanger you have been asked to list or briefly describe what steps can be taken to get the heat exchanger back to design specifications.

***SOLUTION*:** Depending on the type of heat exchanger, the approaches to bring the heat exchanger back to design specification are different. However, most of them include the following:

1. Cleaning of heat exchanger
2. Installation or removal of insulation
3. Correction of leakage

4. Upgrading or replacing of gaskets
5. Increasing the mass flow rate of the cooling fluid
6. Change the cooling medium (e.g., have colder water pass through the heat exchanger)

■

ILLUSTRATIVE EXAMPLE 28.11(14)

A plant has three streams to be heated (see Table 28.3) and three streams to be cooled (see Table 28.4). Cooling water (90°F supply, 155°F return) and steam (saturated at 250 psia) are available. Note that the saturated steam at 250 psia has a temperature of 401°F. Calculate the heating and cooling duties and indicate what utility (or utilities) should be employed.

***SOLUTION*:** The sensible heating duties for all streams are first calculated. The results are shown in Table 28.5.

The total heating and cooling duties can now be computed and compared.

$$\begin{aligned}&\text{Heating: } 7{,}475{,}000 + 6{,}612{,}000 + 9{,}984{,}000 = 24{,}071{,}000\ \text{Btu/h}\\&\text{Cooling: } 12{,}600{,}000 + 4{,}160{,}000 + 3{,}150{,}000 = 19{,}910{,}000\ \text{Btu/h}\\&\text{Heating} - \text{Cooling} = 24{,}071{,}000 - 19{,}910{,}000 = 4{,}161{,}000\ \text{Btu/h}\end{aligned}$$

As a minimum, 4,431,000 Btu/h will have to be supplied by steam or another hot medium. ■

Table 28.3 Streams to be Heated in Illustrative Example 28.11

Stream	Flowrate, lb/h	c_p, Btu/lb · °F	T_{in}, °F	T_{out}, °F
1	50,000	0.65	70	300
2	60,000	0.58	120	310
3	80,000	0.78	90	250

Table 28.4 Streams to be Cooled in Illustrative Example 28.12

Stream	Flowrate, lb/h	c_p, Btu/lb · °F	T_{in}, °F	T_{out}, °F
4	60,000	0.70	420	120
5	40,000	0.52	300	100
6	35,000	0.60	240	90

Table 28.5 Duty Requirements in Illustrative Example 28.11

Stream	Duty, Btu/h
1	7,475,000
2	6,612,000
3	9,984,000
4	12,600,000
5	4,160,000
6	3,150,000

ILLUSTRATIVE EXAMPLE 28.12

Refer to the previous Illustrative Example. Devise a network of heat exchangers that will make full use of heating and cooling streams against each other, using utilities only if necessary.

***SOLUTION*:** Figure 28.1 represents a system of heat exchangers that will transfer heat from the hot streams to the cold ones in the amounts desired. It is important to note that this is but one of many possible schemes. The optimum system would require a trial-and-error procedure that would examine a host of different schemes. Obviously, the economics would come into play.

It should also be noted that in many chemical and petrochemical plants there are cold streams that must be heated and hot steams that must be cooled. Rather than use steam to do all the heating and cooling water to do all the cooling, it is often advantageous, as demonstrated in this problem, to have some of the hot streams heat the cold ones. The problem of optimum heat exchanger networks to accomplish this has been extensively studied and is available in the literature. This example provides one simple illustration.

Finally, highly interconnected networks of exchangers can save a great deal of "quality" energy (referred to in Chapter 21) in a chemical plant. An economic analysis should be included

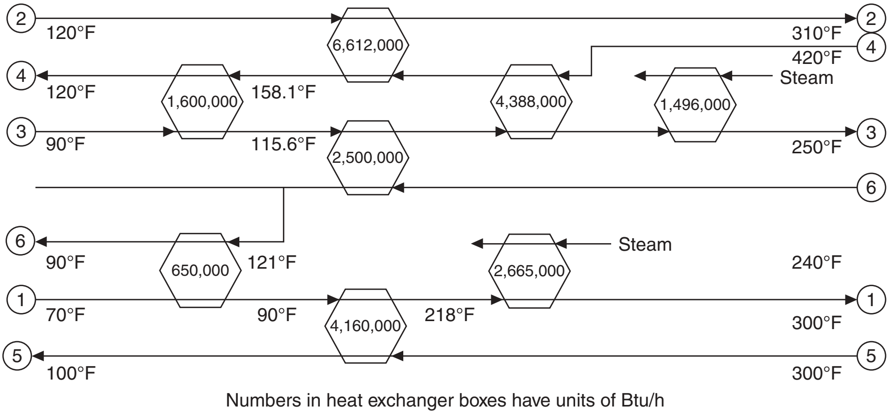

Figure 28.1 Flow diagram for Illustrative Example 28.12.

in any study.(15) The more interconnected they are, however, the harder the plant is to control, start-up, and shut down. Often auxiliary heat sources and cooling sources must be included in the plant design in order to ensure that the plant can operate smoothly. ■

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Appendix A

UNITS

A.1 The Metric System

The need for a single worldwide coordinated measurement system was recognized over 300 years ago. In 1670, Gabriel Mouton, Vicar of St. Paul's church in Lyon, proposed a comprehensive decimal measurement system based on the length of one minute of arc of a great circle of the Earth. In 1671, Jean Picard, a French astronomer, proposed the length of a pendulum beating seconds as the unit of length. (Such a pendulum would have been fairly easy to reproduce, thus facilitating the widespread distribution of uniform standards.) Other proposals were made, but over a century elapsed before any action was taken.

In 1790, in the midst of the French Revolution, the National Assembly of France requested the French Academy of Sciences to "deduce an invariable standard for all the measures and weights." The Commission appointed by the Academy created a system that was, at once, simple and scientific. The unit of length was to be a portion of the Earth's circumference. Measures for capacity (volume) and mass (weight) were to be derived from the unit of length, thus relating the basic units of the system to each other and to nature. Furthermore, the larger and smaller versions of each unit were to be created by multiplying or dividing the basic units by 10 and its multiples. This feature provided a great convenience to users of the system by eliminating the need for such calculating and dividing by 16 (to convert ounces to pounds) or by 12 (to convert inches to feet). Similar calculations in the metric system could

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be performed simply by shifting the decimal point. Thus, the metric system is a *base-10* or *decimal* system.

The Commission assigned the name *metre* (which is now spelled *meter*) to the unit of length. This name was derived from the Greek word *metron* meaning “a measure.” The physical standard representing the meter was to be constructed so that it would equal one ten-millionth of the distance from the north pole to the equator along the meridian of the Earth running near Dunkirk in France and Barcelona in Spain.

The metric unit of mass, called the *gram*, was defined as the mass of one cubic centimeter (a cube that is $1/100$ of a meter on each side) of water at its temperature of maximum density. The cubic decimeter (a cube $1/10$ of a meter on each side) was chosen as the unit of fluid capacity. This measure was given the name *liter*.

Although the metric system was not accepted with enthusiasm at first, adoption by other nations occurred steadily after France made its use compulsory in 1840. The standardized character and decimal features of the metric system made it well suited to scientific and engineering work. Consequently, it is not surprising that the rapid spread of the system coincided with an age of rapid technological development. In the United States, by Act of Congress in 1866, it was made “lawful throughout the United States of America to employ the weights and measures of the metric system in all contracts, dealings, or court proceedings.”

By the late 1860s, even better metric standards were needed to keep pace with scientific advances. In 1875, an international treaty, the “Treaty of the Meter,” set up well-defined metric standards for length and mass, and established permanent machinery to recommend and adopt further refinements in the metric system. This treaty, known as the *Metric Convention*, was signed by 17 countries, including the United States.

As a result of the treaty, metric standards were constructed and distributed to each nation that ratified the Convention. Since 1893, the internationally agreed to metric standards have served as the fundamental weights and measures standards of the United States.

By 1900, a total of 35 nations—including the major nations of continental Europe and most of South America—had officially accepted the metric system. Today, with the exception of the United States and a few small countries, the entire world is using predominantly the metric system or is committed to such use. In 1971, the Secretary of Commerce, in transmitting to Congress the results of a 3-year study authorized by the Metric Study Act of 1968, recommended that the U.S. change to predominant use of the metric system through a coordinated national program.

The International Bureau of Weights and Measures located at Sevres, France, serves as a permanent secretariat for the Metric Convention, coordinating the exchange of information about the use and refinement of the metric system. As measurement science develops more precise and easily reproducible ways of defining the measurement units, the General Conference of Weights and Measures—the diplomatic organization made up of adherents to the Convention—meets periodically to ratify improvements in the system and the standards.

A.2 The SI System

In 1960, the General Conference adopted an extensive revision and simplification of the system. The name *Le Systeme International d'Unites* (International System of Units), with the international abbreviation SI, was adopted for this modernized metric system. Further improvements in and additions to SI were made by the General Conference in 1964, 1968, and 1971.

The basic units in the SI system are the *kilogram* (mass), *meter* (length), *second* (time), *Kelvin* (temperature), *ampere* (electric current), *candela* (the unit of luminous intensity), and *radian* (angular measure). All are commonly used by the practicing engineer and scientists. The Celsius scale of temperature ($0^\circ\text{C}-273.15$ K) is commonly used with the absolute Kelvin scale. The important derived units are the *newton* (SI unit of force), the *joule* (SI unit of energy), the *watt* (SI unit of power), the *pascal* (SI unit of pressure), the *hertz* (unit of frequency). There are a number of electrical units: *coulomb* (charge), *farad* (capacitance), *henry* (inductance), *volt* (potential), and *weber* (magnetic flux). As noted above, one of the major advantages of the metric system is that larger and smaller units are given in powers of ten. In the SI system, a further simplification is introduced by recommending only those units with multipliers of 10^3. Thus for lengths in engineering, the *micrometer* (previously referred to as *micron*), *millimeter*, and *kilometer* are recommended, and the *centimeter* is generally avoided. A further simplification is that the decimal point may be substituted by a comma (as in France, Germany, and South Africa), while the other number, before and after the comma, is separated by spaces between groups of three. More details are provided below.

A.3 Seven Base Units

a. Length—meter (m)

The meter (common international spelling, *metre*) is defined as 1,650,763 wavelengths in vacuum of the orange-red line of the spectrum of krypton-86. The SI unit of area is the *square meter* (m^2). The SI unit of volume is the *cubic meter* (m^3). The *liter* (0.001 cubic meter), although not an SI unit, is commonly used to measure fluid volume.

b. Mass—kilogram (kg)

The standard for the unit of mass, the *kilogram*, is a cylinder of platinum–iridium alloy kept by the International Bureau of Weights and Measures at Paris. A duplicate in the custody of the National Bureau of Standards serves as the mass standard for the United States. This is the only base unit still defined by an artifact. The SI unit of force is the *newton* (N). One newton is the force which, when applied to a 1 kilogram mass, will give the kilogram mass an acceleration of 1 (meter per second) per second ($1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$). The SI unit for pressure is the *pascal* (Pa) ($1\text{ Pa} = 1\text{ N/m}^2$). The SI unit for work and energy of any kind is the *joule* (J) ($1\text{ J} = 1\text{ N}\cdot\text{m}$). The SI unit for power of any kind is the *watt* (W) ($1\text{ W} = 1\text{ J/s}$).

c. Time—second (s)
The *second* is defined as the duration of 9,192,632,770 cycles of the radiation associated with a specified transition of the cesium-133 atom. It is realized by tuning an oscillator to the resonance frequency of cesium-133 atoms as they pass through a system of magnets and a resonant cavity into a detector. The number of periods or cycles per second is call *frequency*. The SI unit for frequency is the *hertz* (Hz). One hertz equals one cycle per second. The SI unit for speed is the *meter per second* (m/s). The SI unit for acceleration is the (*meter per second*) *per second* (m/s^2).

d. Electric current—ampere (A)
The *ampere* is defined as that current which, if maintained in each of two long parallel wires separated by one meter in free space, would produce a force between the two wires (due to their magnetic fields) of 2×10^{-7} newton for each meter of length. The SI unit of voltage is the *volt* (V) ($1\ V = 1\ W/A$). The SI unit of electrical resistance is the *ohm* (Ω) ($1\ \Omega = 1\ V/A$).

e. Temperature—Kelvin (K)
The *Kelvin* is defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. The temperature 0 K is called *absolute zero*. On the commonly used Celsius temperature scale, water freezes at about 0°C and boils at about 100°C. The °C is defined as an interval of 1 K, and the Celsius temperature 0°C is defined as 273.15 K. 1.8 Fahrenheit scale degrees are equal to 1.0°C or 1.0 K; the Fahrenheit scale uses 32°F as a temperature corresponding to 0°C.

f. Amount of substance—mole (mol) (aka gram · mole)
The *mole* is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. The SI unit of concentration (of amount of substance) is the *mole per cubic meter* (mol/m^3).

g. Luminous intensity—candela (cd)
The *candela* is defined as the luminous intensity of 1/600,000 of a square meter of a blackbody at the temperature of freezing platinum (2045 K). The SI unit of light flux is the *lumen* (lm). A source having an intensity of 1 candela in all directions radiates a light flux of 4π lumens.

A.4 Two Supplementary Units

a. Phase angle—radian (rad)
The *radian* is the plane angle with its vertex at the center of a circle that is subtended by an arc equal in length to the radius.

b. Solid angle—steradian (sr)
The *steradian* is the solid angle with its vertex at the center of a sphere that is

subtended by an area of the spherical surface equal to that of a square with sides equal in length to the radius.

A.5 SI Multiples and Prefixes

Multiples and submultiples		Prefixes	Symbols
1 000 000 000 000	10^{12}	tera (ter'a)	T
1 000 000 000	10^{9}	giga (ji'ga)	G
1 000 000	10^{6}	mega (meg'a)	M
1 000	10^{3}	kilo (kil'o)	k
100	10^{2}	hecto (hek'to)	h
10	10^{1}	deka (dek'a)	da
Base unit 1	10^{0}		
0.1	10^{-1}	deci (des'i)	d
0.01	10^{-2}	centi (sen'ti)	c
0.001	10^{-3}	milli (mil'i)	m
0.000 001	10^{-6}	micro (mi'kro)	μ
0. 000 000 001	10^{-9}	nano (nan'o)	n
0.000 000 000 001	10^{-12}	pico (pe'ko)	p
0.000 000 000 000 001	10^{-15}	femto (fem'to)	f
0.000 000 000 000 000 001	10^{-18}	atto (at'to)	a

A.6 Conversion Constants

To convert from	To	Multiply by
Length		
m	cm	100
m	mm	1000
m	microns (μm)	10^{6}
m	angstroms (Å)	10^{10}
m	in	39.37
m	ft	3.281
m	mi	6.214×10^{-4}
ft	in	12
ft	m	0.3048
ft	cm	30.48
ft	mi	1.894×10^{-4}

(*Continued*)

Conversion Constants (*continued*)

To convert from	To	Multiply by
Mass		
kg	g	1000
kg	lb	2.205
kg	oz	35.24
kg	ton	2.268×10^{-4}
kg	grains	1.543×10^{4}
lb	oz	16
lb	ton	5×10^{-4}
lb	g	453.6
lb	kg	0.4536
lb	grains	7000
Time		
s	min	0.01667
s	h	2.78×10^{-4}
s	day	1.157×10^{-7}
s	week	1.653×10^{-6}
s	yr	3.171×10^{-8}
Force		
N	$kg \cdot m/s^2$	1
N	dynes	10^5
N	$g \cdot cm/s^2$	10^5
N	lb_f	0.2248
N	$lb \cdot ft/s^2$	7.233
lb_f	N	4.448
lb_f	dynes	4.448×10^{5}
lb_f	$g \cdot cm/s^2$	4.448×10^{5}
lb_f	$lb \cdot ft/s^2$	32.17
Pressure		
atm	N/m^2 (Pa)	1.013×10^{5}
atm	kPa	101.3
atm	bars	1.013
atm	$dynes/cm^2$	1.013×10^{6}
atm	lb_f/in^2 (psi)	14.696
atm	mm Hg at 0°C (torr)	760
atm	in Hg at 0°C	29.92
atm	ft H_2O at 4°C	33.9
atm	in H_2O at 4°C	406.8
psi	atm	6.80×10^{-2}
psi	mm Hg at 0°C (torr)	51.71

(*Continued*)

Conversion Constants (*continued*)

To convert from	To	Multiply by
psi	in H_2O at 4°C	27.70
in H_2O at 4°C	atm	2.458×10^{-3}
in H_2O at 4°C	psi	0.0361
in H_2O at 4°C	mm Hg at 0°C (torr)	1.868
Volume		
m^3	L	1000
m^3	cm^3 (cc, mL)	10^6
m^3	ft^3	35.31
m^3	gal (U.S.)	264.2
m^3	qt	1057
ft^3	in^3	1728
ft^3	gal (U.S.)	7.48
ft^3	m^3	0.02832
ft^3	L	28.32
Energy		
J	$N \cdot m$	1
J	erg	10^7
J	$dyne \cdot cm$	10^7
J	$kW \cdot h$	2.778×10^{-7}
J	cal	0.2390
J	$ft \cdot lb_f$	0.7376
J	Btu	9.486×10^{-4}
cal	J	4.186
cal	Btu	3.974×10^{-3}
cal	$ft \cdot lb_f$	3.088
Btu	$ft \cdot lb_f$	778
Btu	$hp \cdot h$	3.929×10^{-4}
Btu	cal	252
Btu	$kW \cdot h$	2.93×10^{-4}
$ft \cdot lb_f$	cal	0.3239
$ft \cdot lb_f$	J	1.356
$ft \cdot lb_f$	Btu	1.285×10^{-3}
Power		
W	J/s	1
W	cal/s	0.2390
W	$ft \cdot lb/s$	0.7376
W	kW	10^{-3}
kW	Btu/s	0.949
kW	hp	1.341
hp	$ft \cdot lb/s$	550

(*Continued*)

Conversion Constants (*continued*)

To convert from	To	Multiply by
hp	kW	0.7457
hp	cal/s	178.2
hp	Btu/s	0.707
Concentration		
μg/m^3	lb/ft^3	6.243×10^{-11}
μg/m^3	lb/gal	8.346×10^{-12}
μg/m^3	gr (grain)/ft^3	4.370×10^{-7}
gr/ft^3	μg/m^3	2.288×10^{6}
gr/ft^3	g/m^3	2.288
lb/ft^3	μg/m^3	1.602×10^{10}
lb/ft^3	μg/L	1.602×10^{-8}
lb/ft^3	lb/gal	7.48
Viscosity		
P (poise)	g/cm · s	1
P	cP (centipoise)	100
P	kg/m · h	360
P	lb/ft · s	6.72×10^{-2}
P	lb/ft · h	241.9
P	lb/m · s	5.6×10^{-3}
lb/ft · s	P	14.88
lb/ft · s	g/cm · s	14.88
lb/ft · s	kg/m · h	5.357×10^{3}
lb/ft · s	lb/ft · h	3600
Heat Capacity		
cal/g · °C	Btu/lb · °F	1
cal/g · °C	kcal/kg · °C	1
cal/g · °C	cal/gmol · °C	Molecular weight
cal/gmol · °C	Btu/lbmol · °F	1
J/g · °C	Btu/lb · °F	0.2389
Btu/lb · °F	cal/g · °C	1
Btu/lb · °F	J/g · °C	4.186
Btu/lb · °F	Btu/lbmol · °F	Molecular weight

A.7 Selected Common Abbreviations

Å, A	angstrom unit of length
abs	absolute
amb	ambient

app. MW, M	apparent molecular weight
atm	atmospheric
at. wt.	atomic weight
b.p.	boiling point
bbl	barrel
Btu	British thermal unit
cal	calorie
cg	centigram
cm	centimeter
cgs system	centimeter · gram · second system
conc	concentrated, concentration
cc, cm^3	cubic centimeter
cu ft, ft^3	cubic feet
cfh	cubic feet per hour
cfm	cubic feet per minute
cfs	cubic feet per second
m^3, M^3 (rarely)	cubic meter
°	degree
°C	degree Celsius, degree Centigrade
°F	degree Fahrenheit
°R	degree Rankine, degree Reamur (rarely)
ft	foot
ft · lb	foot pound
fpm	feet per minute
fps	feet per second
fps system	foot · pound · second system
f.p.	freezing point
gr	grain
g, gm	gram
h	hour
in	inch
kcal	kilocalorie
kg	kilogram
km	kilometer
liq	liquid
L	liter
log	logarithm (common)
ln	logarithm (natural)
m.p.	melting point
m, M (rarely)	meter
μm	micrometer (micron)

mks system	meter · kilogram · second system
mph	miles per hour
mg	milligram
ml	milliliter
mm	millimeter
mμ	millimicron
min	minute
mol wt, MW, M	molecular weight
oz	ounce
ppb	parts per billion
pphm	parts per hundred million
ppm	parts per million
lb	pound
psi	pounds per square inch
psia	pounds per square inch absolute
psig	pounds per square inch gage
rpm	revolutions per minute
s	second
sp gr	specific gravity
sp ht	specific heat
sp wt	specific weight
sq	square
scf	standard cubic foot
STP	standard temperature and pressure
t	time, temperature
T, temp.	temperature
wt	weight

Appendix B

TABLES

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

Table B.1 Common Engineering Conversion Factors

Length

1 ft = 12 in = 0.3048 m, 1 yard = 3 ft

1 mi = 5280 ft = 1609.344 m

1 nautical mile (nmi) = 6076 ft

Mass

1 slug = 32.174 lb = 14.594 kg

1 lb = 0.4536 kg = 7000 grains

Acceleration and Area

1 $\text{ft/s}^2 = 0.3048\ \text{m/s}^2$

1 $\text{ft}^2 = 0.092903\ \text{m}^2$

Mass Flow and Mass Flux

1 slug/s = 14.594 kg/s, 1 lb/s = 0.4536 kg/s

1 $\text{kg/m}^2 \cdot \text{s} = 0.2046\ \text{lb/ft}^2 \cdot \text{s} = 0.00636\ \text{slug/ft}^2 \cdot \text{s}$

Pressure

1 $\text{lb}_\text{f}/\text{ft}^2 = 47.88$ Pa, 1 torr = 1 mm Hg

1 psi = 144 psf, 1 bar = 10^5 Pa

1 atm = 2116.2 psf = 14.696 psi = 101,325 Pa = 29.9 in Hg = 33.9 ft H_2O

Power

1 hp = 550 $(\text{ft} \cdot \text{lb}_\text{f})/\text{s} = 745.7$ W

1 $(\text{ft} \cdot \text{lb}_\text{f})/\text{s} = 1.3558$ W

1 W = 3.4123 Btu/h = 0.00134 hp

Specific Weight

1 $\text{lb}_\text{f}/\text{ft}^3 = 157.09\ \text{N/m}^3$

Viscosity

1 $\text{slug}/(\text{ft} \cdot \text{s}) = 47.88\ \text{kg}/(\text{m} \cdot \text{s}) = 478.8$ poise (P)

1 P = 1 $\text{g}/(\text{cm} \cdot \text{s}) = 0.1\ \text{kg}/(\text{m} \cdot \text{s}) = 0.002088\ \text{slug}/(\text{ft} \cdot \text{s})$

Temperature Scale Readings

$°\text{F} = (9/5)°\text{C} + 32$

$°\text{C} = (5/9)(°\text{F} - 32)$

$°\text{R} = °\text{F} + 459.69$

$\text{K} = °\text{C} + 273.16$

$°\text{R} = (1.8)\text{K}$

Heat Capacity or Gas Constant*

1 $(\text{ft} \cdot \text{lb}_\text{f})/(\text{slug} \cdot °\text{R}) = 0.16723\ (\text{N} \cdot \text{m})/(\text{kg} \cdot \text{K})$

1 $\text{Btu}/(\text{lb} \cdot °\text{R}) = 4186.8\ \text{J}/(\text{kg} \cdot \text{K})$

Volume

1 $\text{ft}^3 = 0.028317\ \text{m}^3 = 7.481$ gal, 1 bbl = 42 U.S. gal

1 U.S. gal = 231 in^3 = 3.7853 L = 4 qt = 0.833 Imp. gal

1 L = 0.001 $\text{m}^3 = 0.035315\ \text{ft}^3$ = 0.2642 U.S. gal

Density

1 $\text{slug/ft}^3 = 515.38\ \text{kg/m}^3$, 1 $\text{g/cm}^3 = 1000\ \text{kg/m}^3$

1 $\text{lb/ft}^3 = 16.0185\ \text{kg/m}^3$, 1 $\text{lb/in}^3 = 27.68\ \text{g/cm}^3$

Velocity

1 ft/s = 0.3048 m/s, 1 knot = 1 nmi/h = 1.6878 ft/s

1 mi/h = 1.4666666 ft/s (fps) = 0.44704 m/s

Volume Flow

1 gal/min = 0.002228 ft^3/s = 0.06309 L/s

1 million gal/day = 1.5472 $\text{ft}^3/\text{s} = 0.04381\ \text{m}^3/\text{s}$

Force and Surface Tension

1 $\text{lb}_\text{f} = 4.448222$ N = 16 oz, 1 dyne = 1 $\text{g} \cdot \text{cm/s}^2 = 10^{-5}$ N

1 $\text{kg}_\text{f} = 2.2046\ \text{lb}_\text{f} = 9.80665$ N

1 U.S. (short) ton = 2000 lb_f, 1 N = 0.2248 lb_f

1 N/m = 0.0685 $\text{lb}_\text{f}/\text{ft}$

Energy and Specific Energy

1 $\text{ft} \cdot \text{lb}_\text{f} = 1.35582$ J, 1 $\text{hp} \cdot \text{h} = 2544.5$ Btu

1 Btu = 252 cal = 1055.056 J = 778.17 $\text{ft} \cdot \text{lb}_\text{f}$

1 cal = 4.1855 J, 1 $\text{ft} \cdot \text{lb}_\text{f}/\text{lb}_\text{m} = 2.9890$ J/kg

Heat Flux

1 $\text{W/m}^2 = 0.3171\ \text{Btu}/(\text{h} \cdot \text{ft}^2)$

Kinematic Viscosity

1 $\text{ft}^2/\text{h} = 2.506 \times 10^{-5}\ \text{m}^2/\text{s}$, 1 $\text{ft}^2/\text{s} = 0.092903\ \text{m}^2/\text{s}$

1 stoke (st) = 1 $\text{cm}^2/\text{s} = 0.0001\ \text{m}^2/\text{s} = 0.001076\ \text{ft}^2/\text{s}$

Thermal Conductivity*

1 $\text{cal}/(\text{s} \cdot \text{cm} \cdot °\text{C}) = 242\ \text{Btu}/(\text{h} \cdot \text{ft} \cdot °\text{R})$

1 $\text{Btu}/(\text{h} \cdot \text{ft} \cdot °\text{R}) = 1.7307\ \text{W}/(\text{m} \cdot \text{K})$

*Note that the intervals in absolute (Kelvin) and °C are equal. Also, 1°R = 1°F.

Latent heat: 1 J/kg = 4.2995×10^{-4} Btu/lb = 10.76 $\text{lb}_\text{f} \cdot \text{ft/slug} = 0.3345\ \text{lb}_\text{f} \cdot \text{ft/lb}$, 1 Btu/lb = 2325.9 J/kg.

Heat transfer coefficient: 1 $\text{Btu}/(\text{h} \cdot \text{ft}^2 \cdot °\text{F}) = 5.6782\ \text{W}/(\text{m}^2 \cdot °\text{C})$.

Heat generation rate: 1 $\text{W/m}^3 = 0.09665\ \text{Btu}/(\text{h} \cdot \text{ft}^3)$.

Heat transfer per unit length: 1 W/m = 1.0403 $\text{Btu}/(\text{h} \cdot \text{ft})$.

Mass transfer coefficient: 1 $\text{lbmol}/(\text{h} \cdot \text{ft}^2) = 0.0013562\ \text{kgmol}/(\text{s} \cdot \text{m}^2)$.

Table B.2 Properties of Selected Liquids at 1 atm and 20°C (68°F)

Liquid	Density, ρ kg/m^3	Dynamic viscosity, μ, kg/m · s ($\times 10^4$)	Kinematic viscosity, ν, m^2/s ($\times 10^6$)	Surface tension, σ N/m ($\times 10^2$)	Vapor pressure, p' kPa	Sound velocity, c m/s
Acetone	785	3.16	0.403	2.31	27.6	1174
Ammonia	608	2.20	0.362	2.13	910.0	
Benzene	881	6.51	0.739	2.88	10.1	1298
Carbon disulfide	1272					
Carbon tetrachloride	1590	9.67	0.608	2.70	1.20	924
Castor oil	970	9000	927.8			1474
Crude oil	856	72	8.4	3.0		
Engine oil (unused)	888	7994	900.2			
Ethanol (or ethyl alcohol)	789	11	1.4	2.28	5.7	1144
Ethylene glycol	1117	214	19.16	3.27		1644
Freon-12	1330	2.63	0.198	1.58		
Fuel oil, heavy	908	1324	145.9			
Fuel oil, medium	854	32.7	3.82			
Gasoline	680	2.92	0.429	2.16	55.1	
Glycerin	1260	14,900	1183	6.33	0.14	1909
Kerosene	804	1.92	0.239	2.8	3.11	1320
Mercury	13,550	15.6	0.115	48.4	1.1×10^{-6}	1450
Methanol	791	5.98	0.756	2.25	13.4	1103
Milk (skimmed)	1041	14	1.34			
Milk (whole)	1030	21.2	2.06			
Olive oil	919	840	91.4			
Pentane	624					
Soybean oil	919	400	43.5			
SAE 10 oil	917	1040	113.4	3.6		
SAE 30 oil	917	2900	316.2	3.5		
Seawater	1025	10.7	1.04	7.28	2.34	1535
Turpentine	862	14.9	1.73			
Water	998	10.0	1.06	7.28	2.34	1498

Example: At 20°C, the properties liquid methanol are: density = 791 kg/m^3 (or SG = 0.791), dynamic or absolute viscosity = 0.000598 kg/m · s (or 0.598 cP), kinematic viscosity = 0.756×10^{-6} m^2/s (0.756 cP = 8.14×10^{-6} ft^2/s), surface tension = 0.0225 N/m (0.00154 lb$_f$/ft), vapor (or saturation) pressure = 13,400 Pa (1.943 psi).

Table B.3 Properties of Selected Gases at 1 atm and 20°C (68°F)

Gas	Molecular weight, MW	Density, ρ kg/m^3	Viscosity: Dynamic, μ, $kg/m \cdot s\ (\times 10^5)$	Viscosity: Kinematic, ν, $m^2/s\ (\times 10^6)$	Ratio of heat capacities, k	T_{crit}, K	P_{crit}, atm
Acetylene	26	1.09	0.97	8.3	1.30	309.5	61.6
Air (dry)	28.96	1.20	1.80	15.0	1.40	133	37
Ammonia	17.03	0.74	1.01	13.6	1.31	405	111.3
Argon	39.944	1.66	2.24	13.5	1.67		
Butane	58.1	2.49			1.11	425.2	37.5
Carbon dioxide	44.01	1.83	1.48	8.09	1.30	304	72.9
Carbon monoxide	28.01	1.16	1.82	15.7	1.40	133	34.5
Chlorine	70.91	2.95	1.03	3.49	1.34	417	76.1
Ethane	30.07	1.25	0.85	6.8	1.19	305	48.2
Ethylene	28	1.17	0.97	8.3	1.22	283.1	50.5
Helium	4.003	0.166	1.97	118.7	1.66	5.26	2.26
Hydrogen	2.016	0.0838	0.905	108.0	1.41	33	12.8
Hydrogen chloride	36.5	1.53	1.34	8.76	1.41	324.6	81.5
Hydrogen sulfide	34.1	1.43	1.24	8.67	1.30	373.6	88.9
Methane	16.04	0.667	1.34	20.1	1.32	190	45.8
Methyl chloride	50.5	2.15			1.20	416.1	65.8
Natural gas	19.5	0.804			1.27		
Nitrogen	28.02	1.16	1.76	15.2	1.40	126	33.5
Nitrogen oxide (NO)	30.01	1.23	1.90	15.4	1.40	179	65.0
Nitrous oxide (N_2O)	44.02	1.83	1.45	7.92	1.31	309	71.7
Oxygen	32.0	1.36	2.00	14.7	1.40	154	49.7
Propane	44.1	1.88			1.15	369.9	42.0
Sulfur dioxide	64	2.66	1.38	5.2	1.29	430	77.8
Water vapor	18.02	0.749	1.02	13.6	1.33	647	218.3

Example: At 20°C, the properties argon gas are: molecular weight = 39.944, density = 1.66 kg/m^3 (0.00322 $slug/ft^3$ = 0.104 lb/ft^3), dynamic or absolute viscosity = 0.0000224 $kg/m \cdot s$ (0.0224 cP = 4.68×10^{-7} $slug/ft \cdot s$ = 1.51×10^{-5} $lb/ft \cdot s$), kinematic viscosity = 13.5×10^{-6} m^2/s (13.5 cSt = 1.45×10^{-4} ft^2/s = 0.523 ft^2/h), heat capacity ratio = 1.67.

Table B.4 Properties of Water at 1 atm (Critical Point 374°C, 22.09 MPa)

Temperature		Density, ρ		Dynamic (absolute) viscosity, μ		Kinematic viscosity, ν		Surface tension, σ, N/m	Vapor pressure, p', kPa
°C	°F	$\mathrm{kg/m^3}$	$\mathrm{slug/ft^3}$	$\mathrm{kg/m \cdot s}\ (\times 10^3)$	$\mathrm{slug/ft \cdot s}\ (\times 10^5)$	$\mathrm{m^2/s}\ (\times 10^6)$	$\mathrm{ft^2/s}\ (\times 10^5)$		
0	32	1000	1.940	1.788	0.373	1.788	1.925	0.0756	0.611
5	41	1000	1.940	1.518	0.317	1.519	1.635	0.0749	0.87
10	50	1000	1.940	1.307	0.273	1.307	1.407	0.0742	1.227
15	59	999	1.938	1.139	0.238	1.139	1.226	0.0735	1.70
20	68	998	1.937	1.003	0.209	1.005	1.082	0.0728	2.337
25	77	997	1.934	0.890	0.186	0.893	0.961	0.0720	3.17
30	86	996	1.932	0.799	0.167	0.802	0.864	0.0712	4.242
40	104	992	1.925	0.657	0.137	0.662	0.713	0.0696	7.375
50	122	988	1.917	0.548	0.114	0.555	0.597	0.0679	12.34
60	140	983	1.908	0.467	0.975	0.475	0.511	0.0662	19.92
70	158	978	1.897	0.405	0.846	0.414	0.446	0.0644	31.16
80	176	972	1.886	0.355	0.741	0.365	0.393	0.0626	47.35
90	194	965	1.873	0.316	0.660	0.327	0.352	0.0608	70.11
100	212	958	1.859	0.283	0.591	0.295	0.318	0.0589	101.33

Example: At 50°C (122°F) $\rho = 988\ \mathrm{kg/m^3}$ ($1.917\ \mathrm{slug/ft^3}$), $\mu = 0.548 \times 10^{-3}\ \mathrm{kg/m \cdot s}$ ($0.114 \times 10^{-5}\ \mathrm{slug/ft \cdot s}$), $\nu = 0.555 \times 10^{-6}\ \mathrm{m^2/s}$ ($0.597 \times 10^{-5}\ \mathrm{ft^2/s}$), $\sigma = 0.0679\ \mathrm{N/m}$ ($0.00465\ \mathrm{lb_f/ft}$), vapor pressure = 12,340 Pa (1.79 psi).

Table B.5 Properties of Saturated Liquids

Temperature, T, °C	Density, ρ, kg/m^3	Heat capacity, c_p, kJ/kg · °C	Kinematic viscosity, ν, m^2/s	Thermal conductivity, k, W/m · °C	Diffusivity, α, m^2/s	Prandtl number, Pr	Thermal expansion coefficient, β, K^{-1}
Ammonia, NH_3							
−50	703.69	4.463	0.435×10^{-6}	0.547	1.742×10^{-7}	2.60	
−40	691.68	4.467	0.406	0.547	1.775	2.28	
−30	679.34	4.476	0.387	0.549	1.801	2.15	
−20	666.69	4.509	0.381	0.547	1.819	2.09	
−10	653.55	4.564	0.378	0.543	1.825	2.07	
0	640.10	4.635	0.373	0.540	1.819	2.05	
10	626.16	4.714	0.368	0.531	1.801	2.04	
20	611.75	4.798	0.359	0.521	1.775	2.02	2.45×10^{-3}
30	596.37	4.890	0.349	0.507	1.742	2.01	
40	580.99	4.999	0.340	0.493	1.701	2.00	
50	564.33	5.116	0.330	0.476	1.654	1.99	
Carbon Dioxide, CO_2							
−50	1156.34	1.84	0.119×10^{-6}	0.0855	0.4021×10^{-7}	2.96	
−40	1117.77	1.88	0.118	0.1011	0.4810	2.46	
−30	1076.76	1.97	0.117	0.1116	0.5272	2.22	
−20	1032.39	2.05	0.115	0.1151	0.5445	2.12	
−10	983.38	2.18	0.113	0.1099	0.5133	2.20	
0	926.99	2.47	0.108	0.1045	0.4578	2.38	
10	860.03	3.14	0.101	0.0971	0.3608	2.80	
20	772.57	5.0	0.091	0.0872	0.2219	4.10	14.00×10^{-3}
30	597.81	36.4	0.080	0.0703	0.0279	28.7	

(Continued)

Table B.5 *Continued*

Temperature, T, °C	Density, ρ, kg/m^3	Heat capacity, c_p, kJ/kg · °C	Kinematic viscosity, ν, m^2/s	Thermal conductivity, k, W/m · °C	Diffusivity, α, m^2/s	Prandtl number, Pr	Thermal expansion coefficient, β, K^{-1}
Sulfur Dioxide, SO_2							
−50	1560.84	1.3595	0.484×10^{-6}	0.242	1.141×10^{-7}	4.24	
−40	1536.81	1.3607	0.424	0.235	1.130	3.74	
−30	1520.64	1.3616	0.371	0.230	1.117	3.31	
−20	1488.60	1.3624	0.324	0.225	1.107	2.93	
−10	1463.61	1.3628	0.288	0.218	1.097	2.62	
0	1438.46	1.3636	0.257	0.211	1.081	2.38	
10	1412.51	1.3645	0.232	0.204	1.066	2.18	
20	1386.40	1.3653	0.210	0.199	1.050	2.00	1.95×10^{-3}
30	1359.33	1.3662	0.190	0.192	1.035	1.83	
40	1329.22	1.3674	0.173	0.185	1.019	1.70	
50	1299.10	1.3683	0.162	0.177	0.999	1.61	
Dichlorodifluoromethane (Freon), CCl_2F_2							
−50	1546.75	0.8750	0.310×10^{-6}	0.067	0.501×10^{-7}	6.2	2.63×10^{-3}
−40	1518.71	0.8847	0.279	0.069	0.514	5.4	
−30	1489.56	0.8956	0.253	0.069	0.526	4.8	
−20	1460.57	0.9073	0.235	0.071	0.539	4.4	
−10	1429.49	0.9203	0.221	0.073	0.550	4.0	
0	1397.45	0.9345	0.214×10^{-6}	0.073	0.557×10^{-7}	3.8	
10	1364.30	0.9496	0.203	0.073	0.560	3.6	
20	1330.18	0.9659	0.198	0.073	0.560	3.5	
30	1295.10	0.9835	0.194	0.071	0.560	3.5	
40	1257.13	1.0019	0.191	0.069	0.555	3.5	
50	1215.96	1.0216	0.190	0.067	0.545	3.5	

(*Continued*)

Table B.5 *Continued*

Temperature, T, °C	Density, ρ, kg/m^3	Heat capacity, c_p, kJ/kg · °C	Kinematic viscosity, ν, m^2/s	Thermal conductivity, k, W/m · °C	Diffusivity, α, m^2/s	Prandtl number, Pr	Thermal expansion coefficient, β, K^{-1}
Engine Oil (unused)							
0	899.12	1.796	0.00428	0.147	0.911×10^{-7}	47,100	
20	888.23	1.880	0.00090	0.145	0.872	10,400	0.70×10^{-3}
40	876.05	1.964	0.00024	0.144	0.834	2870	
60	864.04	2.047	0.839×10^{-4}	0.140	0.800	1050	
80	852.02	2.131	0.375	0.138	0.769	490	
100	840.01	2.219	0.203	0.137	0.738	276	
120	828.96	2.307	0.124	0.135	0.710	175	
140	816.94	2.395	0.080	0.133	0.686	116	
160	805.89	2.483	0.056	0.132	0.663	84	
Ethylene Glycol, $C_2H_4(OH)_2$							
0	1130.75	2.294	57.53×10^{-6}	0.242	0.934×10^{-7}	615	
20	1116.65	2.382	19.18	0.249	0.939	204	0.65×10^{-3}
40	1101.43	2.474	8.69	0.256	0.939	93	
60	1087.66	2.562	4.75	0.260	0.932	51	
80	1077.56	2.650	2.98	0.261	0.921	32.4	
100	1058.50	2.742	2.03	0.263	0.908	22.4	

(*Continued*)

Table B.5 *Continued*

Temperature, T, °C	Density, ρ, kg/m^3	Heat capacity, c_p, kJ/kg · °C	Kinematic viscosity, ν, m^2/s	Thermal conductivity, k, W/m · °C	Diffusivity, α, m^2/s	Prandtl number, Pr	Thermal expansion coefficient, β, K^{-1}
Glycerin, $C_3H_5(OH)_3$							
0	1276.03	2.261	0.00831	0.282	0.983×10^{-7}	84.7×10^3	
10	1270.11	2.319	0.00300	0.284	0.965	31.0	
20	1264.02	2.386	0.00118	0.286	0.947	12.5	0.50×10^{-3}
30	1258.09	2.445	0.00050	0.286	0.929	5.38	
40	1252.01	2.512	0.00022	0.286	0.914	2.45	
50	1244.96	2.583	0.00015	0.287	0.893	1.63	
Mercury, Hg							
0	13,628.22	0.1403	0.124×10^{-6}	8.20	42.99×10^7	0.0288	
20	13,759.04	0.1394	0.114	8.69	46.06	0.0249	1.82×10^{-4}
50	13,505.84	0.1386	0.104	9.40	50.22	0.0207	
100	13,384.58	0.1373	0.0928	10.51	57.16	0.0162	
150	13,264.68	0.1365	0.0853	11.49	63.54	0.0134	
200	13,144.94	0.1570	0.0802	12.34	69.08	0.0116	
250	13,025.60	0.1357	0.0765	13.07	74.06	0.0103	
315.5	12,857	0.134	0.0673	81.5	0.0083		

Table B.6 Properties of Water (Saturated Liquid)

Temp. °F	Temp. °C	Heat capacity, c_p, kJ/kg · K	Density, ρ, kg/m^3	Viscosity, μ, kg/m · s	Thermal conductivity, k, W/m · °C	Prandtl number, Pr	Free convection coefficient, $\frac{g\beta\rho^2 c_p}{\mu k}$ 1/m^3 · °C
32	0	4.225	999.8	1.79×10^{-3}	0.566	13.25	1.91×10^{9}
40	4.44	4.208	999.8	1.55×10^{-3}	0.575	11.35	1.91×10^{9}
50	10	4.195	999.2	1.31×10^{-3}	0.585	9.40	6.34×10^{9}
60	15.56	4.186	998.6	1.12×10^{-3}	0.595	7.88	1.08×10^{10}
70	21.11	4.179	997.4	9.8×10^{-4}	0.604	6.78	1.46×10^{10}
80	26.67	4.179	995.8	8.6×10^{-4}	0.614	5.85	1.91×10^{10}
90	32.22	4.174	994.9	7.65×10^{-4}	0.623	5.12	2.48×10^{10}
100	37.78	4.174	993.0	6.82×10^{-4}	0.630	4.53	3.3×10^{10}
110	43.33	4.174	990.6	6.16×10^{-4}	0.637	4.04	4.19×10^{10}
120	48.89	4.174	988.8	5.62×10^{-4}	0.644	3.64	4.89×10^{10}
130	54.44	4.179	985.7	5.13×10^{-4}	0.649	3.30	5.66×10^{10}
140	60	4.179	983.3	4.71×10^{-4}	0.654	3.01	6.48×10^{10}
150	65.55	4.183	980.3	4.3×10^{-4}	0.659	2.73	7.62×10^{10}
160	71.11	4.186	977.3	4.01×10^{-4}	0.665	2.53	8.84×10^{10}
170	76.67	4.191	973.7	3.72×10^{-4}	0.668	2.33	9.85×10^{10}
180	82.22	4.195	970.2	3.47×10^{-4}	0.673	2.16	1.09×10^{11}
190	87.78	4.199	966.7	3.27×10^{-4}	0.675	2.03	
200	93.33	4.204	963.2	3.06×10^{-4}	0.678	1.90	
220	104.4	4.216	955.1	2.67×10^{-4}	0.684	1.66	
240	115.6	4.229	946.7	2.44×10^{-4}	0.685	1.51	
260	126.7	4.250	937.2	2.19×10^{-4}	0.685	1.36	
280	137.8	4.271	928.1	1.98×10^{-4}	0.685	1.24	
300	148.9	4.296	918.0	1.86×10^{-4}	0.684	1.17	
350	176.7	4.371	890.4	1.57×10^{-4}	0.677	1.02	
400	204.4	4.467	859.4	1.36×10^{-4}	0.655	1.00	
450	232.2	4.585	825.7	1.20×10^{-4}	0.646	0.85	
500	260	4.731	785.2	1.07×10^{-4}	0.616	0.83	
550	287.7	5.024	735.5	9.51×10^{-5}			
600	315.6	5.703	678.7	8.68×10^{-5}			

Note: $\mathrm{Gr}_x\mathrm{Pr}$ = Rayleigh number, $\mathrm{Ra}_x = \left(\frac{g\beta\rho^2 c_p}{\mu k}\right)L^3\Delta T$.

Table B.7 Properties of Air at 1 atm

Temperature, °C	Density, ρ, kg/m^3	Dynamic viscosity, μ, kg/m · s ($\times 10^5$)	Kinematic viscosity, ν, m^2/s ($\times 10^5$)	Capacity heat, c_p, J/kg · K	Thermal conductivity, k, W/m · K ($\times 10^2$)	Thermal expansion coefficient, β, K (10^3)	Prandtl number, Pr
−40	1.52	1.51	0.98		2.0		
−20	1.40	1.61	1.15	1004.8	2.21		
0	1.29	1.71	1.32	1004.8	2.42	3.65	0.715
10	1.248	1.76	1.41	1004.8	2.49	3.53	0.713
20	1.205	1.81	1.50	1004.8			
30	1.165	1.86	1.60	1004.8			
40	1.128	1.90	1.68	1004.8	2.7		
50	1.09	1.95	1.79	1007.0	2.8		
60	1.060	2.00	1.87	1009.0			
80	1.000	2.09	2.09	1009.0			
100	0.946	2.17	2.30	1009.0	3.12		
150	0.835	2.38	2.85	1017.0	3.53		
200	0.746	2.57	3.45	1025.8	3.88		0.686
250	0.675	2.75	4.07	1034.1	4.24		0.680
300	0.616	2.93	4.76				
400	0.525	3.25	6.19				
500	0.457	3.55	7.77		5.73		0.709

Example: At 50°C, the air properties are: density = 1.09 kg/m^3 (0.00211 slug/ft^3 = 0.679 lb/ft^3), dynamic or absolute viscosity = 0.0000195 kg/m · s (4.073 × 10^{-7} slug/ft · s = 1.31 × 10^{-5} lb/ft · s), thermal conductivity, $k = 0.028$ W/m · K, coefficient of thermal expansion, $\beta = 1/T = 1/(273 + 50) = 0.0031$ K^{-1}. The Prandtl number, $\mathrm{Pr} = c_p\mu/k = 0.7$.

Table B.8 Properties of Gases at 1 atm[a]

Temperature, T, K	Density, ρ, kg/m^3	Heat capacity, c_p, kJ/kg · °C	Dynamic viscosity, μ, kg/m · s	Kinematic viscosity, ν, m^2/s	Thermal conductivity, k, W/m · °C	Thermal diffusivity, α, m^2/s	Prandtl number, Pr
Helium							
144	0.3379	5.200	125.5×10^{-7}	37.11×10^{-6}	0.0928	0.5275×10^{-4}	0.70
200	0.2435	5.200	156.6	64.38	0.1177	0.9288	0.694
255	0.1906	5.200	181.7	95.50	0.1357	0.1375	0.70
366	0.13280	5.200	230.5	173.6	0.1691	2.449	0.71
477	0.10204	5.200	275.0	269.3	0.197	3.716	0.72
589	0.08282	5.200	311.3	375.8	0.225	5.215	0.72
700	0.07032	5.200	347.5	494.2	0.251	6.661	0.72
800	0.06023	5.200	381.7	634.1	0.275	8.774	0.72
Hydrogen							
150	0.16371	12.602	5.595×10^{-6}	34.18×10^{-6}	0.0981	0.475×10^{-4}	0.718
200	0.12270	13.540	6.813	55.53	0.1282	0.772	0.719
250	0.09819	14.059	7.919	80.64	0.1561	1.130	0.713
300	0.08185	14.314	8.963	109.5	0.182	1.554	0.706
350	0.07016	14.436	9.954	141.9	0.206	2.031	0.697
400	0.06135	14.491	10.864	177.1	0.228	2.568	0.690
450	0.05462	14.499	11.779	215.6	0.251	3.164	0.682
500	0.04918	14.507	12.636	257.0	0.272	3.817	0.675
550	0.04469	14.532	13.475	301.6	0.292	4.516	0.668
600	0.04085	14.537	14.285	349.7	0.315	5.306	0.664
700	0.03492	14.574	15.89	455.1	0.351	6.903	0.659
800	0.03060	14.675	17.40	569	0.384	8.563	0.664
900	0.02723	14.821	18.78	690	0.412	10.217	0.676

(Continued)

Table B.8 *Continued*

Temperature, T, K	Density, ρ, kg/m^3	Heat capacity, c_p, kJ/kg · °C	Dynamic viscosity, μ, kg/m · s	Kinematic viscosity, ν, m^2/s	Thermal conductivity, k, W/m · °C	Thermal diffusivity, α, m^2/s	Prandtl number, Pr
Oxygen							
150	2.6190	0.9178	11.490×10^{-6}	4.387×10^{-6}	0.01367	0.05688×10^{-4}	0.773
200	1.9559	0.9131	14.850	7.593	0.01824	0.10214	0.745
250	1.5618	0.9157	17.87	11.45	0.02259	0.15794	0.725
300	1.3007	0.9203	20.63	15.86	0.02676	0.22353	0.709
350	1.1133	0.9291	23.16	20.80	0.03070	0.2968	0.702
400	0.9755	0.9420	25.54	26.18	0.03461	0.3768	0.695
450	0.8682	0.9567	27.7	31.99	0.03828	0.4609	0.694
500	0.7801	0.9722	29.91	38.34	0.4173	0.5502	0.697
550	0.7096	0.9881	31.97	45.05	0.04517	0.641	0.700
Nitrogen							
200	1.7108	1.0429	12.947×10^{-6}	7.568×10^{-6}	0.01824	0.10224×10^{-4}	0.747
300	1.1421	1.0408	17.84	15.63	0.02620	0.22044	0.713
400	0.8538	1.0459	21.98	25.74	0.03335	0.3734	0.691
500	0.6824	1.0555	25.70	37.66	0.03984	0.5530	0.684
600	0.5687	1.0756	29.11	51.19	0.04580	0.7486	0.686
700	0.4934	1.0969	32.13	65.13	0.05123	0.9466	0.691
800	0.4277	1.1225	34.84	81.46	0.05609	1.1685	0.700
900	0.3796	1.1464	37.49	91.06	0.06070	1.3946	0.711
1000	0.3412	1.1677	40.00	117.2	0.06475	1.6250	0.724
1100	0.3108	1.1857	42.28	136.0	0.06850	1.8591	0.736
1200	0.2851	1.2037	44.50	156.1	0.07184	2.0932	0.748
Carbon Dioxide							
220	2.4733	0.783	11.105×10^{-6}	4.490×10^{-6}	0.010805	0.05920×10^{-5}	0.818
250	2.1657	0.804	12.590	5.813	0.012884	0.07401	0.793

(*Continued*)

Table B.8 *Continued*

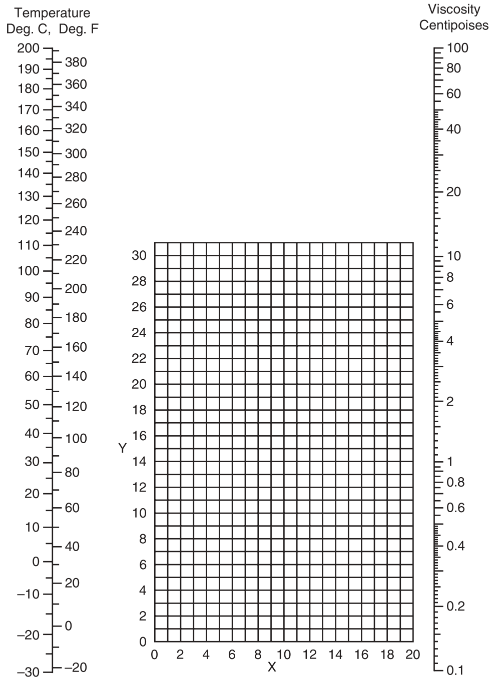
Temperature, T, K	Density, ρ, kg/m^3	Heat capacity, c_p, $kJ/kg \cdot °C$	Dynamic viscosity, μ, $kg/m \cdot s$	Kinematic viscosity, ν, m^2/s	Thermal conductivity, k, $W/m \cdot °C$	Thermal diffusivity, α, m^2/s	Prandtl number, Pr
300	1.7973	0.871	14.958	8.321	0.016572	0.10588	0.770
350	1.5362	0.900	17.205	11.19	0.02047	0.14808	0.755
400	1.3424	0.942	19.32	14.39	0.02461	0.19463	0.738
500	1.0732	1.013	23.26	21.67	0.03352	0.3084	0.702
550	0.9739	1.047	25.08	25.74	0.03821	0.3750	0.685
600	0.8938	1.076	26.83	30.02	0.04311	0.4483	0.668
Ammonia							
273	0.7929	2.177	9.353×10^{-6}	1.18×10^{-5}	0.0220	0.1308×10^{-4}	0.90
323	0.6487	2.177	11.035	1.70	0.0270	0.1920	0.88
373	0.5590	2.236	12.886	2.30	0.0327	0.2619	0.87
423	0.4934	2.315	14.672	2.87	0.0391	0.3432	0.87
473	0.4405	2.395	16.49	3.74	0.0467	0.4421	0.84
Water Vapor							
380	0.5863	2.060	12.71×10^{-6}	2.16×10^{-5}	0.0246	0.2036×10^{-4}	1.060
400	0.5542	2.014	13.44	2.42	0.0261	0.2338	1.040
450	0.4902	1.980	15.25	3.11	0.0299	0.307	1.010
500	0.4405	1.985	17.04	3.86	0.0339	0.387	0.996
550	0.4005	1.997	18.84	4.70	0.0379	0.475	0.991
600	0.3652	2.026	20.67	5.66	0.0422	0.573	0.986
650	0.3380	2.056	22.47	6.64	0.0464	0.666	0.995
700	0.3140	2.085	24.26	7.72	0.0505	0.772	1.000
750	0.2931	2.119	26.04	8.88	0.0549	0.883	1.005
800	0.2739	2.152	27.86	10.20	0.0592	1.001	1.010
850	0.2579	2.186	29.69	11.52	0.0637	1.130	1.019

[a] Values of dynamic viscosity μ, thermal conductivity k, specific heat c_p, and Prandtl number Pr, are not strongly pressure-dependent for He, H_2, O_2, and N_2 and may be used over a fairly wide range of pressures.

Appendix C

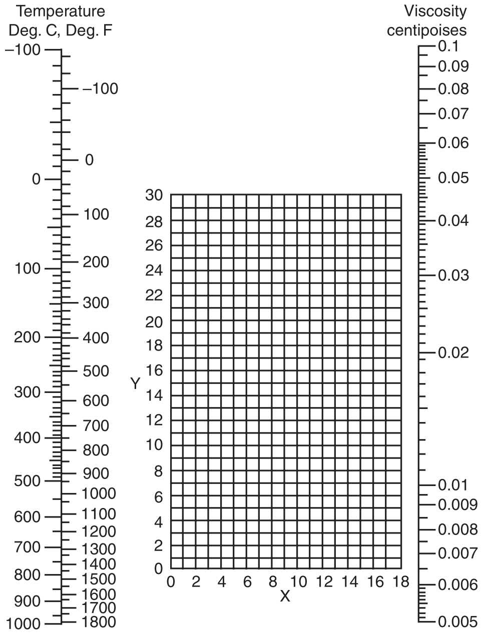
FIGURES

Heat Transfer Applications for the Practicing Engineer. Louis Theodore



No	Liquid	X	Y	No	Liquid	X	Y
1	Acetaldehyde	15.2	4.8	56	Freon-22	17.2	4.7
2	Acetic acid, 100%	12.1	14.2	57	Freon-113	12.5	11.4
3	Acetic acid, 70%	9.5	17.0	58	Glycerol, 100%	2.0	30.0
4	Acetic anhydride	12.7	12.8	59	Glycerol, 50%	6.9	19.6
5	Acetone, 100%	14.5	7.2	60	Heptane	14.1	8.4
6	Acetone, 35%	7.9	15.0	61	Hexane	14.7	7.0
7	Allyl alcohol	10.2	14.3	62	Hydrochloric acid, 31.5%	13.0	16.6
8	Ammonia, 100%	12.6	2.0	63	Isobutyl alcohol	7.1	18.0
9	Ammonia, 26%	10.1	13.9	64	Isobutyric acid	12.2	14.4
10	Amyl acetate	11.8	12.5	65	Isopropyl alcohol	8.2	16.0
11	Amyl alcohol	7.5	18.4	66	Kerosene	10.2	16.9
12	Aniline	8.1	18.7	67	Linseed oil, raw	7.5	27.2
13	Anisole	12.3	13.5	68	Mercury	18.4	16.4
14	Arsenic trichloride	13.9	14.5	69	Methanol, 100%	12.4	10.5
15	Benzene	12.5	10.9	70	Methanol, 90%	12.3	11.8
16	Bimethyl oxalate	12.3	15.8	71	Methanol, 40%	7.8	15.5
17	Biphenyl	12.0	18.3	72	Methyl acetate	14.2	8.2
18	Brine, $CaCl_2$, 25%	6.6	15.9	73	Methyl chloride	15.0	3.8
19	Brine, NaCl, 25%	10.2	16.6	74	Methyl ethyl ketone	13.9	8.6
20	Bromine	14.2	13.2	75	Naphtalene	7.9	18.1
21	Bromotoluene	20.0	15.9	76	Nitric acid, 95%	12.8	13.8
22	Butyl acetate	12.3	11.0	77	Nitric acid, 60%	10.8	17.0
23	Butyl alcohol	8.6	17.2	78	Nitrobenzene	10.6	16.2
24	Butyric acid	12.1	15.3	79	Nitrotoluene	11.0	17.0
25	Carbon dioxide	11.6	0.3	80	Octane	13.7	10.0
26	Carbon disulfide	16.1	7.5	81	Octyl alcohol	6.6	21.1
27	Carbon tetrachloride	12.7	13.1	82	Pentachloroethane	10.9	17.3
28	Chlorobenzene	12.3	12.4	83	Pentane	14.9	5.2
29	Chloroform	14.4	10.2	84	Phenol	6.9	20.8
30	Chlorosulfonic acid	11.2	18.1	85	Phosphorus tribromide	13.8	16.7
31	*o*-Chlorotoluene	13.0	13.3	86	Phosphorus trichloride	16.2	10.9
32	*m*-Chlorotoluene	13.3	12.5	87	Propionic acid	12.8	13.8
33	*p*-Chlorotoluene	13.3	12.5	88	Propyl alcohol	9.1	16.5
34	*m*-Cresol	2.5	20.8	89	Propyl bromide	14.5	9.6
35	Cyclohexanol	2.9	24.3	90	Propyl chloride	14.4	7.5
36	Dibromoethane	12.7	15.8	91	Propyl iodide	14.1	11.6
37	Dichloroethane	13.2	12.2	92	Sodium	16.4	13.9
38	Dichloromethane	14.6	8.9	93	Sodium hydroxide, 50%	3.2	25.8
39	Diethyl oxalate	11.0	16.4	94	Stannic chloride	13.5	12.8
40	Dipropyl oxalate	10.3	17.7	95	Sulfur dioxide	15.2	7.1
41	Ethyl acetate	13.7	9.1	96	Sulfuric acid, 110%	7.2	27.4
42	Ethyl alcohol, 100%	10.5	13.8	97	Sulfuric acid, 98%	7.0	24.8
43	Ethyl alcohol, 95%	9.8	14.3	98	Sulfuric acid, 60%	10.2	21.3
44	Ethyl alcohol, 40%	6.5	16.6	99	Sulfuryl chloride	15.2	12.4
45	Ethyl benzene	13.2	11.5	100	Tetrachloroethane	11.9	15.7
46	Ethyl bromide	14.5	8.1	101	Tetrachloroethylene	14.2	12.7
47	Ethyl chloride	14.8	6.0	102	Titanium tetrachloride	14.4	12.3
48	Ethyl ether	14.5	5.3	103	Toluene	13.7	10.4
49	Ethyl formate	14.2	8.4	104	Trichloroethylene	14.8	10.5
50	Ethyl iodide	14.7	10.3	105	Turpentine	11.5	14.9
51	Ethyl glycol	6.0	23.6	106	Vinyl acetate	14.0	8.8
52	Formic acid	10.7	15.8	107	Water	10.2	13.0
53	Freon-11	14.4	9.0	108	*o*-Xylene	13.5	12.1
54	Freon-1	16.8	5.6	109	*m*-Xylene	13.9	10.6
55	Freon-21	15.7	7.5	110	*p*-Xylene	13.9	10.9

Figure C.1 Dynamic or absolute viscosity of liquids.



No	Gas	X	Y	No	Gas	X	Y
1	Acetic acid	7.7	14.3	29	Freon-113	11.3	14.0
2	Acetone	8.9	13.0	30	Helium	10.9	20.5
3	Acetylene	9.8	14.9	31	Hexane	8.6	11.8
4	Air	11.0	20.0	32	Hydrogen	11.2	12.4
5	Ammonia	8.4	16.0	33	$3H_2 + N_2$	11.2	17.2
6	Argon	10.5	22.4	34	Hydrogen bromide	8.8	20.9
7	Benzene	8.5	13.2	35	Hydrogen chloride	8.8	20.9
8	Bromine	8.9	19.2	36	Hydrogen cyanide	9.8	14.9
9	Butene	9.2	13.7	37	Hydrogen iodide	9.0	21.3
10	Butylene	8.9	13.0	38	Hydrogen sulfide	8.6	18.0
11	Carbon dioxide	9.5	18.7	39	Iodine	9.0	18.4
12	Carbon disulfide	8.0	16.0	40	Mercury	5.3	22.9
13	Carbon monoxide	11.0	20.0	41	Methane	9.9	15.5
14	Chlorine	9.0	18.4	42	Methyl alcohol	8.5	15.6
15	Chloroform	8.9	15.7	43	Nitric oxide	10.9	20.5
16	Cyanogen	9.2	15.2	44	Nitrogen	10.6	20.0
17	Cyclohexane	9.2	12.0	45	Nitrosyl chloride	8.0	17.6
18	Ethane	9.1	14.5	46	Nitrous oxide	8.8	19.0
19	Ethyl acetate	8.5	13.2	47	Oxygen	11.0	21.3
20	Ethyl alcohol	9.2	14.2	48	Pentane	7.0	12.8
21	Ethyl chloride	8.5	15.6	49	Propane	9.7	12.9
22	Ethyl ether	8.9	13.0	50	Propyl alcohol	8.4	13.4
23	Ethylene	9.5	15.1	51	Propylene	9.0	13.8
24	Fluorine	7.3	23.8	52	Sulfur dioxide	9.6	17.0
25	Freon-11	10.6	15.1	53	Toluene	8.6	12.4
26	Freon-12	11.1	16.0	54	2,3,3-Trimethylbutane	9.5	10.5
27	Freon-21	10.8	15.3	55	Water	8.0	16.0
28	Freon-22	10.1	17.0	56	Xenon	9.3	23.0

Figure C.2 Dynamic or absolute viscosity of gases and vapors at 1 atm.

Appendix D

STEAM TABLES

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

Table D.1 Saturated Steam*

Temperature, T, °F	Absolute pressure, P, $\mathrm{lb_f/in^2}$	Specific volume, $\mathrm{ft^3/lb}$			Enthalpy, Btu/lb			Entropy, Btu/lb · °R		
		Saturated liquid, v_l	Evaporation difference v_{vap}	Saturated vapor, v_g	Saturated liquid, h_l	Evaporation difference h_{vap}	Saturated vapor, h_g	Saturated liquid, s_l	Evaporation difference s_{vap}	Saturated vapor, s_g
32	0.08854	0.01602	3306	3306	0.00	1075.8	1075.8	0.0000	2.1877	2.1877
35	0.09995	0.01602	2947	2947	3.02	1074.1	1077.1	0.0061	2.1709	2.1770
40	0.12170	0.01602	2444	2444	8.05	1071.3	1079.3	0.0162	2.1435	2.1597
45	0.14752	0.01602	2036.4	2036.4	13.06	1068.4	1081.5	0.0262	2.1167	2.1429
50	0.17811	0.01603	1703.2	1703.2	18.07	1065.6	1083.7	0.0361	2.0903	2.1264
60	0.2563	0.01604	1206.6	1206.7	28.06	1059.9	1088.0	0.0555	2.0393	2.0948
70	0.3631	0.01606	867.8	867.9	38.04	1054.3	1092.3	0.0745	1.9902	2.0647
80	0.5069	0.01608	633.1	633.1	48.02	1048.6	1096.6	0.0932	1.9428	2.0360
90	0.6982	0.01610	468.0	468.0	57.99	1042.9	1100.9	0.1115	1.8972	2.0087
100	0.9492	0.01613	350.3	350.4	67.97	1037.2	1105.2	0.1295	1.8531	1.9826
110	1.2748	0.01617	265.3	265.4	77.94	1031.6	1109.5	0.1471	1.8106	1.9577
120	1.6924	0.01620	203.25	203.27	87.92	1025.8	1113.7	0.1645	1.7694	1.9339
130	2.2225	0.01625	157.32	157.34	97.90	1020.0	1117.9	0.1816	1.7296	1.9112
140	2.8886	0.01629	122.99	123.01	107.89	1014.1	1122.0	0.1984	1.6910	1.8894
150	3.718	0.01634	97.06	97.07	117.89	1008.2	1126.1	0.2149	1.6537	1.8685
160	4.741	0.01639	77.27	77.29	127.89	1002.3	1130.2	0.2311	1.6174	1.8485
170	5.992	0.01645	62.04	62.06	137.90	996.3	1134.2	0.2472	1.5822	1.8293
180	7.510	0.01651	50.21	50.23	147.92	990.2	1138.1	0.2630	1.5480	1.8109
190	9.339	0.01657	40.94	40.96	157.95	984.1	1142.0	0.2785	1.5147	1.7932
200	11.526	0.01663	33.62	33.64	167.99	977.9	1145.9	0.2938	1.4824	1.7762
210	14.123	0.01670	27.80	27.82	178.05	971.6	1149.7	0.3090	1.4508	1.7598

(*Continued*)

Table D.1 *Continued*

Temperature, T, °F	Absolute pressure, P, $\mathrm{lb_f/in^2}$	Specific volume, $\mathrm{ft^3/lb}$ Saturated liquid, v_l	Evaporation difference v_{vap}	Saturated vapor, v_g	Enthalpy, Btu/lb Saturated liquid, h_l	Evaporation difference h_{vap}	Saturated vapor, h_g	Entropy, Btu/lb · °R Saturated liquid, s_l	Evaporation difference s_{vap}	Saturated vapor, s_g
212	14.696	0.01672	26.78	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566
220	17.186	0.01677	23.13	23.15	188.13	965.2	1153.4	0.3239	1.4201	1.7440
230	20.780	0.01684	19.365	19.382	198.23	958.8	1157.0	0.3387	1.3901	1.7288
240	24.969	0.01692	16.306	16.323	208.34	952.2	1160.5	0.3531	1.3609	1.7140
250	29.825	0.01700	13.804	13.821	218.48	945.5	1164.0	0.3675	1.3323	1.6998
260	35.429	0.01709	11.746	11.763	228.64	938.7	1167.3	0.3817	1.3043	1.6860
270	41.858	0.01717	10.044	10.061	238.84	931.8	1170.6	0.3958	1.2769	1.6727
280	49.203	0.01726	8.628	8.645	249.06	924.7	1173.8	0.4096	1.2501	1.6597
290	57.556	0.01735	7.444	7.461	259.31	917.5	1176.8	0.4234	1.2238	1.6472
300	67.013	0.01745	6.449	6.466	269.59	910.1	1179.7	0.4369	1.1980	1.6350
310	77.68	0.01755	5.609	5.626	279.92	902.6	1182.5	0.4504	1.1727	1.6231
320	89.66	0.01765	4.896	4.914	290.28	894.9	1185.2	0.4637	1.1478	1.6115
330	103.06	0.01776	4.289	4.307	300.68	887.0	1187.7	0.4769	1.1233	1.6002
340	118.01	0.01787	3.770	3.788	311.13	879.0	1190.1	0.4900	1.0992	1.5891
350	134.63	0.01799	3.324	3.342	321.63	870.7	1192.3	0.5029	1.0754	1.5783
360	153.04	0.01811	2.939	2.957	332.18	862.2	1194.4	0.5158	1.0519	1.5677
370	173.37	0.01823	2.606	2.625	342.79	853.5	1196.3	0.5286	1.0287	1.5573
380	195.77	0.01836	2.317	2.335	353.45	844.6	1198.1	0.5413	1.0059	1.5471
390	220.37	0.01850	2.0651	2.0836	364.17	835.4	1199.6	0.5539	0.9832	1.5371
400	247.31	0.01864	1.8447	1.8633	374.97	826.0	1201.0	0.5664	0.9608	1.5272
410	276.75	0.01878	1.6512	1.6700	385.83	816.3	1202.1	0.5788	0.9386	1.5174
420	308.83	0.01894	1.4811	1.5000	396.77	806.3	1203.1	0.5912	0.9166	1.5078
430	343.72	0.01910	1.3308	1.3499	407.79	796.0	1203.8	0.6035	0.8947	1.4982

(*Continued*)

Table D.1 *Continued*

Temperature, T, °F	Absolute pressure, P, $\mathrm{lb_f/in^2}$	Specific volume, $\mathrm{ft^3/lb}$			Enthalpy, Btu/lb			Entropy, Btu/lb · °R		
		Saturated liquid, v_l	Evaporation difference v_{vap}	Saturated vapor, v_g	Saturated liquid, h_l	Evaporation difference h_{vap}	Saturated vapor, h_g	Saturated liquid, s_l	Evaporation difference s_{vap}	Saturated vapor, s_g
440	381.59	0.01926	1.1979	1.2171	418.90	785.4	1204.3	0.6158	0.8730	1.4887
450	422.6	0.0194	1.0799	1.0993	430.1	774.5	1204.6	0.6280	0.8513	1.4793
460	466.9	0.0196	0.9748	0.9944	441.4	763.2	1204.6	0.6402	0.8298	1.4700
470	514.7	0.0198	0.8811	0.9009	452.8	751.5	1204.3	0.6523	0.8083	1.4606
480	566.1	0.0200	0.7972	0.8172	464.4	739.4	1203.7	0.6645	0.7868	1.4513
490	621.4	0.0202	0.7221	0.7423	476.0	726.8	1202.8	0.6766	0.7653	1.4419
500	680.8	0.0204	0.6545	0.6749	487.8	713.9	1201.7	0.6887	0.7438	1.4325
520	812.4	0.0209	0.5385	0.5594	511.9	686.4	1198.2	0.7130	0.7006	1.4136
540	962.5	0.0215	0.4434	0.4649	536.6	656.6	1193.2	0.7374	0.6568	1.3942
560	1133.1	0.0221	0.3647	0.3868	562.2	624.2	1186.4	0.7621	0.6121	1.3742
580	1325.8	0.0228	0.2989	0.3217	588.9	588.4	1177.3	0.7872	0.5659	1.3532
600	1542.9	0.0236	0.2432	0.2668	617.0	548.5	1165.5	0.8131	0.5176	1.3307
620	1786.6	0.0247	0.1955	0.2201	646.7	503.6	1150.3	0.8398	0.4664	1.3062
640	2059.7	0.0260	0.1538	0.1798	678.6	452.0	1130.5	0.8679	0.4110	1.2789
660	2365.4	0.0278	0.1165	0.1442	714.2	390.2	1104.4	0.8987	0.3485	1.2472
680	2708.1	0.0305	0.0810	0.1115	757.3	309.9	1067.2	0.9351	0.2719	1.2071
700	3093.7	0.0369	0.0392	0.0761	823.3	172.1	995.4	0.9905	0.1484	1.1389

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[*] Abridged from *Thermodynamic Properties of Steam*, by Joseph H. Keenan and Frederick G. Keyes. Copyright 1936, by Joseph H. Keenan and Frederick G. Keyes. Published by John Wiley & Sons, Inc., Hoboken, NJ.

Table D.2 Superheated Steam

Absolute pressure, lb_f/in^2 (saturated temperature)		Temperature, °F												
		200	220	300	350	400	450	500	550	600	700	800	900	1000
	v	392.6	404.5	452.3	482.2	512.0	541.8	571.6	601.4	631.2	690.8	750.4	809.9	869.5
1	h	1150.4	1159.5	1195.8	1218.7	1241.7	1264.9	1288.3	1312.0	1335.7	1383.8	1432.8	1482.7	1533.5
(101.74)	s	2.0512	2.0647	2.1153	2.1444	2.1720	2.1983	2.2233	2.2468	2.2702	2.3137	2.3542	2.3923	2.4283
	v	78.16	80.59	90.25	96.26	102.26	108.24	114.22	120.19	126.16	138.10	150.03	161.95	173.87
5	h	1148.8	1158.1	1195.0	1218.1	1241.2	1264.5	1288.0	1311.7	1335.4	1383.6	1432.7	1482.6	1533.4
(162.24)	s	1.8718	1.8857	1.9370	1.9664	1.9942	2.0205	2.0456	2.0692	2.0927	2.1361	2.1767	2.2148	2.2509
	v	38.85	40.09	45.00	48.03	51.04	54.05	57.05	60.04	63.03	69.01	74.98	80.95	86.92
10	h	1146.6	1156.2	1193.9	1217.2	1240.6	1264.0	1287.5	1311.3	1335.1	1383.4	1432.5	1482.4	1533.2
(193.21)	s	1.7927	1.8071	1.8595	1.8892	1.9172	1.9436	1.9689	1.9924	2.0160	2.0596	2.1002	2.1383	2.1744
	v		27.15	30.53	32.62	34.68	36.73	38.78	40.82	42.86	46.94	51.00	55.07	59.13
14.696	h		1154.4	1192.8	1216.4	1239.9	1263.5	1287.1	1310.9	1334.8	1383.2	1432.3	1482.3	1533.1
(212.00)	s		1.7624	1.8160	1.8460	1.8743	1.9008	1.9261	1.9498	1.9734	2.0170	2.0576	2.0958	2.1319
	v			22.36	23.91	25.43	26.95	28.46	29.97	31.47	34.47	37.46	40.45	43.44
20	h			1191.6	1215.6	1239.2	1262.9	1286.6	1310.5	1334.4	1382.9	1432.1	1482.1	1533.0
(227.96)	s			1.7808	1.8112	1.8396	1.8664	1.8918	1.9160	1.9392	1.9829	2.0235	2.0618	2.0978
	v			11.040	11.843	12.628	13.401	14.168	14.93	15.688	17.198	18.702	20.20	21.70
40	h			1186.8	1211.9	1236.5	1260.7	1284.8	1308.9	1333.1	1381.9	1431.3	1481.4	1532.4
(267.25)	s			1.6994	1.7314	1.7608	1.7881	1.8140	1.8384	1.8619	1.9058	1.9467	1.9850	2.0214
	v			7.259	7.818	8.357	8.884	9.403	9.916	10.427	11.441	12.449	13.452	14.454
60	h			1181.6	1208.2	1233.6	1258.5	1283.0	1307.4	1331.8	1380.9	1430.5	1480.8	1531.9

(*Continued*)

Table D.2 *Continued*

Absolute pressure, lb_f/in^2 (saturated temperature)		Temperature, °F												
		200	220	300	350	400	450	500	550	600	700	800	900	1000
(292.71)	s			1.6492	1.6830	1.7135	1.7416	1.7678	1.7926	1.8162	1.8605	1.9015	1.9400	1.9762
	v				5.803	6.220	6.624	7.020	7.410	7.797	8.562	9.322	10.077	10.830
80	h				1204.3	1230.7	1256.1	1281.1	1305.8	1330.5	1379.9	1429.7	1480.1	1531.3
(312.03)	s				1.6475	1.6791	1.7078	1.7346	1.7598	1.7836	1.8281	1.8694	1.9079	1.9442
	v				4.592	4.937	5.268	5.589	5.905	6.218	6.835	7.446	8.052	8.656
100	h				1200.1	1227.6	1253.7	1279.1	1304.2	1329.1	1378.9	1428.9	1479.5	1530.8
(327.81)	s				1.6188	1.6518	1.6813	1.7085	1.7339	1.7581	1.8029	1.8443	1.8829	1.9193
	v				3.783	4.081	4.363	4.636	4.902	5.165	5.683	6.195	6.702	7.207
120	h				1195.7	1224.4	1251.3	1277.2	1302.5	1327.7	1377.8	1428.1	1478.8	1530.2
(341.25)	s				1.5944	1.6287	1.6591	1.6869	1.7127	1.7370	1.7822	1.8237	1.8625	1.8990
	v					3.468	3.715	3.954	4.186	4.413	4.861	5.301	5.738	6.172
140	h					1221.1	1248.7	1275.2	1300.9	1326.4	1376.8	1427.3	1478.2	1529.7
(353.02)	s					1.6087	1.6399	1.6683	1.6945	1.7190	1.7645	1.8063	1.8451	1.8817
	v					3.008	3.230	3.443	3.648	3.849	4.244	4.631	5.015	5.396
160	h					1217.6	1246.1	1273.1	1299.3	1325.0	1375.7	1426.4	1477.5	1529.1
(363.53)	s					1.5908	1.6230	1.6519	1.6785	1.7033	1.7491	1.7911	1.8301	1.8667
	v					2.649	2.852	3.044	3.229	3.411	3.764	4.110	4.452	4.792
180	h					1214.0	1243.5	1271.0	1297.6	1323.5	1374.7	1425.6	1476.8	1528.6
(373.06)	s					1.5745	1.6077	1.6373	1.6642	1.6894	1.7355	1.7776	1.8167	1.8534
	v					2.361	2.549	2.726	2.895	3.060	3.380	3.693	4.002	4.309

(*Continued*)

Table D.2 *Continued*

Absolute pressure, lb_f/in^2 (saturated temperature)		Temperature, °F												
		200	220	300	350	400	450	500	550	600	700	800	900	1000
200	*h*					1210.3	1240.7	1268.9	1295.8	1322.1	1373.6	1424.8	1476.2	1528.0
(381.79)	*s*					1.5594	1.5937	1.6240	1.6513	1.6767	1.7232	1.7655	1.8048	1.8415
	v					2.125	2.301	2.465	2.621	2.772	3.066	3.352	3.634	3.913
220	*h*					1206.5	1237.9	1266.7	1294.1	1320.7	1372.6	1424.0	1475.5	1527.5
(389.86)	*s*					1.5453	1.5808	1.6117	1.6395	1.6652	1.7120	1.7545	1.7939	1.8308
	v					1.9276	2.094	2.247	2.393	2.533	2.804	3.068	3.327	3.584
240	*h*					1202.5	1234.9	1264.5	1292.4	1319.2	1371.5	1423.2	1474.8	1526.9
(397.37)	*s*					1.5319	1.5686	1.6003	1.6286	1.6546	1.7017	1.7444	1.7839	1.8209
	v						1.9183	2.063	2.199	2.330	2.582	2.827	3.067	3.305
260	*h*						1232.0	1262.3	1290.5	1317.7	1370.4	1422.3	1474.2	1526.3
(404.42)	*s*						1.5573	1.5897	1.6184	1.6447	1.6922	1.7352	1.7748	1.8118
	v						1.7674	1.9047	2.033	2.156	2.392	2.621	2.845	3.066
280	*h*						1228.9	1260.0	1288.7	1316.2	1369.4	1421.5	1473.5	1525.8
(411.05)	*s*						1.5464	1.5796	1.6087	1.6354	1.6834	1.7265	1.7662	1.8033
	v						1.6364	1.7675	1.8891	2.005	2.227	2.442	2.652	2.859
300	*h*						1225.8	1257.6	1286.8	1314.7	1368.3	1420.6	1472.8	1525.2
(417.33)	*s*						1.5360	1.5701	1.5998	1.6268	1.6751	1.7184	1.7582	1.7954
	v						1.3734	1.4923	1.6010	1.7036	1.8980	2.084	2.266	2.445
350	*h*						1217.7	1251.5	1282.1	1310.9	1365.5	1418.5	1471.1	1523.8
(431.72)	*s*						1.5119	1.5481	1.5792	1.6070	1.6563	1.7002	1.7403	1.7777
	v						1.1744	1.2851	1.3843	1.4770	1.6508	1.8161	1.9767	2.134
400	*h*						1208.8	1245.1	1277.2	1306.9	1362.7	1416.4	1469.4	1522.4
(444.59)	*s*						1.4892	1.5281	1.5607	1.5894	1.6398	1.6842	1.7247	1.7623

Table D.3 Saturated Steam–Ice

Temperature, *T*, °F	Absolute pressure, *P*, $\mathrm{lb_f/in^2}$	Specific volume, $\mathrm{ft^3/lb}$		Enthalpy, Btu/lb			Entropy, Btu/lb · °R		
		Saturated ice, v_i	Saturated steam, $v_g \times 10^{-3}$	Saturated ice, h_i	Sublimation difference h_{sub}	Saturated steam, h_g	Saturated ice, s_i	Sublimation difference s_{sub}	Saturated steam, s_g
32	0.0885	0.01747	3.306	−143.35	1219.1	1075.8	−0.2916	2.4793	2.1877
30	0.0808	0.01747	3.609	−144.35	1219.3	1074.9	−0.2936	2.4897	2.1961
20	0.0505	0.01745	5.658	−149.31	1219.9	1070.6	−0.3038	2.5425	2.2387
10	0.0309	0.01744	9.05	−154.17	1220.4	1066.2	−0.3141	2.5977	2.2836
0	0.0185	0.01742	14.77	−158.93	1220.7	1061.8	−0.3241	2.6546	2.3305
−10	0.0108	0.01741	24.67	−163.59	1221.0	1057.4	−0.3346	2.7143	2.3797
−20	0.0062	0.01739	42.2	−168.16	1221.2	1053.0	−0.3448	2.7764	2.4316
−30	0.0035	0.01738	74.1	−172.63	1221.2	1048.6	−0.3551	2.8411	2.4860

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