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Multicriteria Analysis

Applications to Water and Environment Management

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Preface

Decision making is an essential part of our private and professional life. The consequences of our decisions are sometimes very simple, but very often our decisions affect our life and future significantly. For example, selecting a dessert after a dinner is a simple decision, however applying for a new job or choosing a retirement plan could have significant effect on our life.

Every decision making problem has three major components: the decision makers, the decision alternatives, and the consequences of our decisions. The decision maker can be a single person or a group of people, who are sometimes called the stakeholders. The decision alternatives are the options from which we can choose from. In selecting the feasible alternative set we have to take into account all of the technical, economic, environmental, regulatory, etc., constraints. If the consequences of a decision making problem can be characterized by a single criterion (such as profit), then the problem can be modeled as a single-objective optimization problem. Depending on the types of the objective function and the feasible set of alternatives the mathematical model can be linear programming, or a nonlinear, discrete, mixed programming problem or even dynamic or stochastic optimization to mention only the most frequently used model variants. There are many textbooks discussing these model types and the most important solution methodology. Most of the practical decision making problems cannot be described by a single criterion. Water resources and environmental management problems always have to consider several criteria simultaneously, social and other economic factors have to be considered among others. Multi Criteria Decision Analysis (MCDA) is the usual methodology to model and solve such problems. This book attempts to introduce the modeling and solving of MCDA problems with illustrative case studies in water resources and environmental management. Chapter 1 presents the major components and modeling of MCDA problems. The hierarchy of the criteria is the subject of Chapter 2. The most important methods for solving discrete problems are introduced in Chapter 3, and their counterparts for solving continuous problems are discussed in Chapter 4. Social choice methodology is often used if there are several stakeholders with conflicting priorities in the decision making process when some of the criteria are hard to or cannot be quantified. Chapter 5 is devoted to this subject. Conflict resolution concepts and procedures are introduced in Chapter 6 including symmetric and non-symmetric bargaining. All models and methods discussed in the first six chapters assume complete and perfect knowledge of all criteria and constraints. However, in reality most decision making problems are faced with uncertain environmental and economical conditions. Chapter 7 introduces the main concepts of modeling uncertainty and the corresponding solution methodology.In addition to introducing and discussing modeling concepts and mathematical methodology with simple classroom-size numerical examples, several case studies are selected to illustrate how they work in reality. These case studies include project selection, inter-basin water transfer, urban water management, water allocation, groundwater quality as environmental health risk, forestry treatment selection, multi-reservoir irrigation planning, water distribution network design, and long-term watershed management. These studies are chosen from different regions and countries including Hungary, India, Iran, Mexico, USA and Vietnam.

This book is a result of the 4-year long cooperation of the authors which started with a 1-year scholarship of the first author at the University of Arizona, Tucson. This visit was followed by several meetings, conferences and short courses, when the authors could exchange ideas and earlier drafts of different parts of the manuscript of this book.

We hope that the material of this book will be helpful for graduate students in mathematics, engineering and economics in their studies in decision making. We also hope that engineers, managers and all others facing with practical decision making problems will find the material of this book useful in their work. The methodology and the concrete application studies might suggest new ideas, interesting and important research topics for students and scientists.

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Chapter 1
Introduction to Multicriteria Decision Analysis

1.1 Decision Analysis

Decision analysis is the science and art of designing or choosing the best alternatives based on the goals and preferences of the decision maker (DM). Making a decision implies that there are alternative choices to be considered. In such cases, we do not want to identify only as many of these alternatives as possible but we want to choose the one that best fits our goals, desires, lifestyle, values, and so on (Harris 1997). In other words, decision analysis is the science of choice. For example, selecting the best technology for urban water supply, developing flood protection alternatives, or optimizing the operation of a reservoir are all the problems of choice.

To describe the preferences of a DM we may use one of the terms of Goals, Objectives, Criteria, and Attributes. However, there are differences among their meanings. Goals are useful for clearly identifying a level of achievement to strive toward (Keeney and Raiffa 1993). Goals relate to desired performance outcomes in the future, while an objective is something to be pursued to its fullest level or it may generally indicate the direction of desired change. Criteria are more specific and measurable outcomes. A criterion generally indicates the direction in which we should strive to do better. In all decision problems, we want

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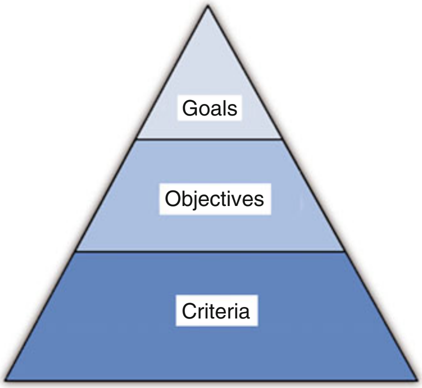

Fig. 1.1 Relation among the goals, objectives and criteria

to accomplish or avoid certain things. To what degree we accomplish our goals or avoid unfavorable consequences should be among the criteria. They are either achieved, or surpassed or not exceeded (Hwang and Yoon 1981). The relation among these three terms is indicated in Fig. 1.1. The attributes are also performance parameters, components, factors, characteristics, and properties. In this book, we will use the term criteria, instead of objectives and attributes, which is closer to the meaning of what is usually used by the DMs in water resources and environment management.

Example 1.1. The Common Agricultural Policy (CAP) absorbs roughly 45% of the total budget of the European Union. The CAP is a widely debated policy, in terms of both its budget and the instruments being used (Gomez-Limon and Atance 2004). The hierarchy of its goal, main objectives and criteria used to evaluate the objectives is shown in Table 1.1.

The management of water resources and the environment takes place in a multicriteria framework when it is necessary to consider the technical, environmental and social implications of the water resources projects, in addition to the economic criteria to ensure sustainable decisions and favorable decision outcomes. The traditional cost–benefit analysis, used for many decades in water resources planning and environmental management,

Table 1.1 Goal, objectives and criteria for the CAP project

Goal	Objectives	Criteria
Improving the welfare of residents in European countries	Social objectives	1. To safeguard family agricultural holdings
		2. To maintain villages and improving the quality of rural life
		3. To conserve traditional agricultural products (typical local products)
	Environmental objectives	1. To encourage agricultural practices compatible with environmental conservation
		2. To contribute to the maintenance of natural areas
		3. To maintain traditional agricultural landscapes
	Economic objectives	1. To ensure reasonable prices for consumers
		2. To ensure safe and healthy food
		3. To encourage competitiveness of farms
		4. To provide adequate income for farmers
		5. To guarantee national food self-sufficiency

transformed the different types of impacts into a single monetary metric. Once that was done, the task was to find the plan or policy that maximized the difference between the benefits and costs. If the maximum difference between the benefits and costs was positive, then the best plan or policy was found. However, not all system performance criteria can be easily expressed in monetary units. Even if monetary units are used to describe each objective, then they do not address the distributional issues of who benefits, who pays, and how much (Loucks and van Beek 2005). To overcome this inefficiency multicriteria decision analysis (MCDA) techniques are applied. The most important advantages of using these methods for water resources management are:

- To cope with limited water, financial and human resources
- To allow the combination of multiple criteria instead of a single criterion
- To avoid opportunity costs of delay in decision-making
- To resolve conflict among stakeholders
- To simplify the administration of the projects

1.2 The Components of MCDA Problems

Any MCDA problem has three main components: decision maker/s (DMs), alternatives and criteria. These three elements can be shown as the three basis of a triangle (Fig. 1.2).

The classification of an MCDA problem depends on the types of these elements. The definitions of the three components are as follow:

- *Decision maker/s*. The first element is identifying the DMs. For a particular problem, we might have a single person who is responsible for deciding what to do or several people or organizations being involved in the decision-making process. In the first case, we have only one DM; in the second case, we have multiple DMs. When more than one DM is present, then they might have different preferences, goals, objectives and criteria, so no decision outcome is likely to satisfy every decision maker equally. In such cases, a collective decision has to be made when the outcome depends on how the different DMs take the interests of each others into account. In other words, the outcome depends on their willingness to cooperate with each other. In the case of multiple decision makers, we might consider the problem as an MCDA problem, where the criteria of the different decision makers are considered the criteria of the problem (Karamouz et al. 2003). In the case of a single DM and one criterion, we have a single-objective optimization problem. The applied methods depend on the type of the problem (linear programming, nonlinear programming, integer or mixed programming, dynamic optimization, stochastic programming, etc.). Typical MCDA problems arise when a single decision maker considers several criteria simultaneously. In the presence

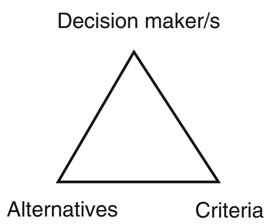

Fig. 1.2 The elements of an MCDA problem

of multiple DMs the problem can be modeled by MCDA as mentioned above, or in the case of conflicting priorities and desires of the DMs, game theory can be used. MCDA is often considered as the most powerful methodology of solving game theoretical problems with cooperating players.

- *Alternatives*. These are the possibilities one has to choose from. Alternatives can be identified (that is, searched for and located) or even developed (created where they did not previously exist). The set of all possible alternatives is called the decision space. In many cases, the decision space has only a finite number of elements. For example, selecting a technology from four possibilities results in a decision space with four alternatives. In many other cases, the decision alternatives are characterized by continuous decision variables that represent certain values about which the decision has to be made. For example, reservoir capacity can be any real value between the smallest feasible value and the largest possibility.
- *Criteria*. These are the characteristics or requirements that each alternative must possess to a greater or lesser extent. The alternatives are usually rated on how well they possess the criteria.

Since we have to make a choice from a given set of feasible alternatives we need to measure how good those alternatives are. The goodness of any alternative can be characterized by its evaluations with respect to the criteria. These evaluations can be described by crisp numbers, linguistic values, random or fuzzy numbers. A criterion is called positive, if better evaluation is indicated by larger values. Similarly a criterion is called negative, if better evaluation is shown by a smaller value. Regarding the types of the alternatives, we have two major classes of MCDA problems.

Before proceeding further, some comments are in order. In the case of one criterion the problem is given as

$$\begin{gathered}\text{Maximize } f(x)\\ \text{subject to } x \in X.\end{gathered}$$

Here x represents an alternative and X is the set of all feasible alternatives. All values of $f(x)$ when x runs through the feasible set, X, are located on the real line. The optimal solution has therefore the following properties:

1. The optimal solution is at least as good as any other solution.
2. There is no better solution than the optimal solution.
3. All optimal solutions are equivalent, i.e., they have the same objective value.

In the case of one criteria, any two decisions $x^{(1)}$ and $x^{(2)}$ can be compared since either $f(x^{(1)}) > f(x^{(2)})$, or $f(x^{(1)}) = f(x^{(2)})$, or $f(x^{(1)}) < f(x^{(2)})$. In the case of multiple criteria, this is not true. For example, in the case of two positive criteria the following two outcomes cannot be compared:

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ 1 \end{pmatrix},$$

since the first outcome is better in the second criterion and worse in the first criterion.

1.3 Classification of MCDA Problems

1.3.1 Discrete Case

If the decision space is finite, then the construction of the feasible decision space is very simple. We have to check the feasibility of each alternative by determining whether or not it satisfies all restrictions. We can show the discrete alternatives, criteria and the evaluations of the alternatives with respect to the criteria in a matrix, called the evaluation or decision matrix. In a decision matrix, the (i, j) element indicates the evaluation of alternative j with respect to criteria i, as it will be explained in the following example.

Example 1.2. Table 1.2 represents an evaluation matrix. The problem is to choose the best scheme for inter basin water transfer from five alternatives (what can we do). The four criteria (what

Table 1.2 Evaluation matrix of Example 1.2

Criteria	Weights	Alternatives				
		A_1	A_2	A_3	A_4	A_5
C_1	0.2	1.3	1.4	1.1	1.7	1.2
C_2	0.1	High	Low	Medium	High	Very high
C_3	0.4	Easy	Easy	Difficult	Difficult	Medium
C_4	0.3	70	90	75	40	55

we get) are benefit–cost ratio, environmental sustainability, easy operation/maintenance, and compliance with former water rights in the watershed (in subjective judgment on a scale between 0 and 100). These criteria show and indicate the Integrated Water Resources Management (IWRM) principles.

If we quantify the linguistic values on a 0 through 100 scale, then the evaluation values of the alternatives with respect to criteria C_2 and C_3 might become

80 10 50 80 100

and

90 90 10 10 50.

The decision space of the problem has five elements, the five alternatives: A_1, A_2, A_3, A_4 and A_5. The consequence of selecting any one of the alternatives is characterized by the simultaneous values of the criteria, which is a four-element vector. So the objective space consists of five points in the four dimensional space: (1.3, 80, 90, 70), (1.4, 10, 90, 90), (1.1, 50, 10, 75), (1.7, 80, 10, 40) and (1.2, 100, 50, 55). The decision space shows our possible choices. That is, it represents what can be done. The objective space shows the simultaneous criteria values, that is, what we can get.

If we compare these alternatives, then we see that neither of them can be improved in all criteria simultaneously by selecting another alternative. In this case all of these alternatives can be considered reasonable choices. In order to choose only one of them which could be considered as the "best", additional preference information is needed from the DM. The preferences of the DM can be represented by many different ways, for example, by specifying relative importance weights. These values are shown in the second column of Table 1.2.

Figure 1.3 represents the steps of formulating and solving a mathematical model for a discrete MCDA problem. As it is shown in this procedure, the decision making process has recursive nature.

1.3.2 Continuous Case

If the decision alternatives are characterized by continuous variables then the problem is considered to be continuous. In this case, the alternatives satisfying all constraints are feasible, and the set of all feasible alternatives is the feasible decision space. The constraints are usually presented as certain equalities or inequalities containing the decision variables.

In the classical optimization models, we have only one criterion to optimize. However, in most decision-making problems we are faced with several criteria that might conflict with each other. For example, treatment cost and water quality are conflicting criteria, as better quality requires higher cost. We can assume

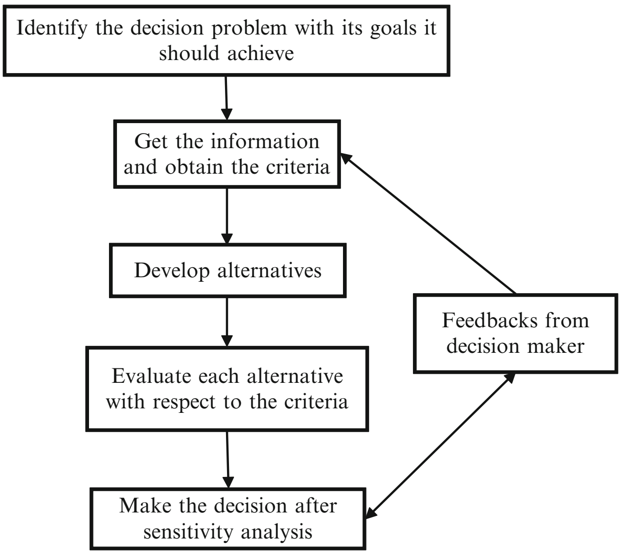

Fig. 1.3 The steps of formulating and solving a discrete MCDA problem

that all criteria are maximized, otherwise each negative criterion can be multiplied by -1.

Example 1.3. The water demand of an urban area can be supplied from two sources, from groundwater and also from surface water. The decision variables (alternatives) are how much water should be pumped from the groundwater resource, x_1, and how much water should be transformed from the reservoir, x_2. The DM wants to minimize the total cost of satisfying the demand. The unit cost of water supply from groundwater and surface water supply are 3 and 2, respectively. The DM also desires to maximize the reliability of the supply, which can be identified by a numerical scale. The groundwater is more reliable than surface water in this area and then according to the knowledge of an expert, the reliability can be shown by the numbers of 5 and 3 for groundwater and surface water, respectively. The minimum amount of total supplied water should be at least 5 units. The groundwater can supply at most 4 units/year and the surface water can supply maximum 3 units/year in average. The corresponding continuous MCDA problem can be formulated as follows:

$$\text{Minimize } f_1 = 3x_1 + 2x_2 \tag{1.1}$$

and

$$\begin{aligned} \text{Maximize} \quad f_2 &= 5x_1 + 3x_2 \\ \text{subject to } x_1 + x_2 &\geqslant 5 \\ x_1 &\leqslant 4 \\ x_2 &\leqslant 3 \\ x_1, x_2 &\geqslant 0. \end{aligned} \tag{1.2}$$

The decision space of this problem is shown in Fig. 1.4. The decision alternatives should be chosen from this space. So, the set of alternatives (possible supply designs) allows infinitely many different choices.

The criteria (f_1 and f_2) are functions of the decision variables (x_1 and x_2). These criteria are clearly in conflict: a low cost water

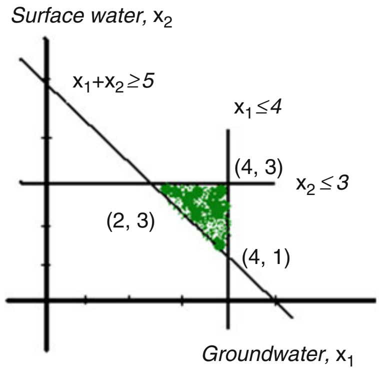

Fig. 1.4 The decision space for Example 1.3

supply scheme will certainly have low reliability. Figure 1.4 shows the set of feasible alternatives, it shows only what we can do. In order to see the consequences of the decisions we have to find and illustrate the set of the feasible criteria values, which is called the objective space. In order to do this, we have to express the decision variables as functions of the criteria values by solving (1.1) and (1.2) for unknowns x_1 and x_2:

$$x_1 = -3f_1 + 2f_2 \tag{1.3}$$

and

$$x_2 = 5f_1 - 3f_2. \tag{1.4}$$

By substituting these expressions into the constraints of the original decision model, the corresponding constraints for the criteria values become as follows:

$$\begin{aligned} 2f_1 - f_2 &\geqslant 5 \\ -3f_1 + 2f_2 &\leqslant 4 \\ 5f_1 - 3f_2 &\leqslant 3 \\ -3f_1 + 2f_2 &\geqslant 0 \\ 5f_1 - 3f_2 &\geqslant 0. \end{aligned}$$

The feasible set of these inequalities is the objective space, which is shown in Fig. 1.5.

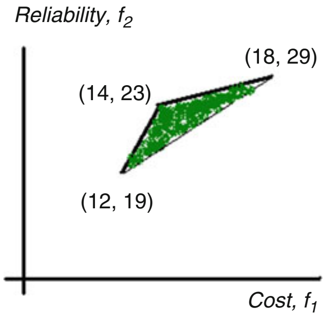

Fig. 1.5 The objective space for Example 1.3

Any point of the broken line with segments connecting the point (12, 19) with (14, 23) and (14, 23) with (18, 29), shown in Fig. 1.5, is reasonable since none of the criteria can be improved without worsening the other. The choice of a single "best" point from this infinite set should be based on additional preference and tradeoff information obtained from the DM.

The steps of formulating and solving a mathematical model of a continuous MCDA problem are presented in Fig. 1.6.

The continuous case is a special case of infinite problems. There are many decision making problems with infinitely many alternatives where some of the alternatives cannot be described by continuous variables. For example this is the case if some variables have only integer values. Consider the case when we decide on doing something or not doing it at all. In this case, the MCDA model has variables with discrete (0 or 1) values and some other variables with continuous scales. These mixed problems can be solved by combining discrete and continuous methods. These types of decision problems are very rare in the water resources modeling and environmental management problems. In this book, we restrict our discussions to the purely discrete and continuous models.

Notice that regardless of the type of the MCDA problem, decision making is usually an iterative and continuous process. That is, most decisions are made by moving back and forth between choosing the criteria and identifying the alternatives and the preferences of the DM. The available alternative set often influences the choice of the criteria we use to evaluate

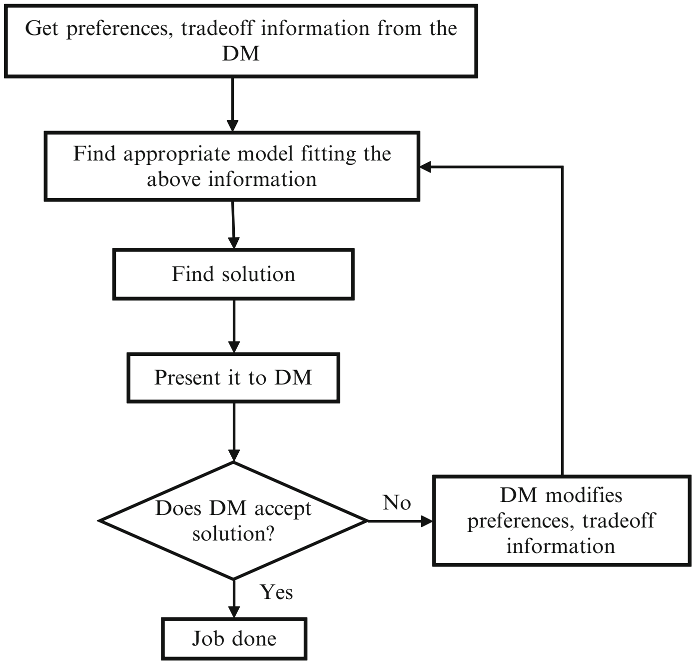

Fig. 1.6 The steps of formulating and solving a continuous MCDA problem

them, and similarly the criteria set might also influence the selection of the alternatives. After a computed solution is presented to the DM, it is either accepted or the DM makes some changes and modifications in the model. Then the new solution is computed, which is shown again to the DM. This interactive process continues until a satisfactory solution is obtained.

Chapter 2
The Hierarchy of the Criteria

2.1 Introduction

It has become more and more difficult to assess the consequences of water resources and environmental decisions in a single-dimensional way and to use only one criterion when judging their characteristics and making comparisons. However in practice the water and environmental managers always compare, rank, and then select decision alternatives with respect to only a specific criterion. A single criterion of choice can be fully satisfactory only in very simple and straightforward situations. For example, they may select the largest water transfer scheme from a lake, but they might worry whether the largest water transfer scheme is the least expensive, the most reliable, and the most suitable for the environment.

In this chapter we will first discuss the most important and most common criteria in water and environmental management in the scope of sustainable development. Sustainable water resources systems are developed and managed to fully contribute to the aims and objectives of the society, for now as well as for the future, while maintaining their ecological, environmental, and hydrological integrity (ASCE 1998; UNESCO 1999). A method of criteria selection to be used in structuring the hierarchy of the criteria will be the subject of this chapter. A case study will illustrate the methodology.

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2.2 Criteria

In the following subsections various social, economic, and environmental criteria are introduced and discussed. These criteria are essential in securing sustainable development (Fig. 2.1).

2.2.1 Social Criteria

The social welfare of the region is the fundamental objective of every water resources development plan and the social performance of any project has a huge effect in the life of the society. Some important social aspects and externalities of these projects can be listed as follows:

- Considering equity in water resources allocation is very important. The upper regions of a watershed should be treated in the same way as the lower parts and vice versa, otherwise social conflicts will arise. This equity will promote public consensus and participation, which is vital for the success of a project.
- Traditional water right holders are very sensitive to keep their rights. Serious social conflicts will be developed if a water plan violates the already confirmed treaties. New water transfers should also satisfy these people, otherwise they will oppose the plan.
- Alternative projects that create more job opportunities and reduce the poverty at the area are more preferred.
- The new water and environmental plans should be consistent with the cultural and religious customs of the people.

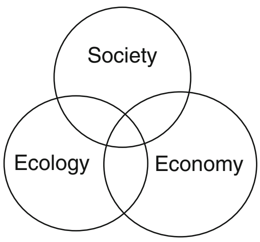

Fig. 2.1 Main pillars of sustainable development

- Reservoir construction, water transfer pipelines or floodplain restoration always require that some people have be relocated from their homes, resulting in various mental and emotional tensions. Therefore projects with less resettlement are more preferred.
- Dam constructions and their corresponding facilities may damage historical sites such as heritage buildings. This damage has to be kept in minimal level.

Social criteria are very important. However their modeling is usually very difficult since they are evaluated and measured by only qualitative data without quantitative measures.

2.2.2 Economic Criteria

Economic criteria are the basic measures in evaluating the alternative water resources development projects. There are useful measures to quantify the economic and financial outcomes of a project in its operation period. For example, we may maximize the difference of the total benefits of a project and its costs. This analysis is based on two economic concepts of scarcity and substitution (Loucks and van Beek 2005). Since the various resources are limited, people are willing to pay for them, therefore they should be utilized efficiently. Instead of using the difference of benefit and cost, the ratio of benefit to cost may also be used. In these cases, the amount of benefit or cost is not important by itself since only their difference or ratio is important. Another measure to evaluate the investment alternatives is the internal rate of return, which is the interest rate at which the total cost of the investment is equal to the benefit of the investment.

In most multi-purpose water and environmental projects, trade-offs should be considered with respect to the economic criteria. As an example, consider an offshore aquifer that can be withdrawn for irrigation. More withdrawal will lower the groundwater table resulting in sea water intrusion. The salt-water intrusion may destroy water quality, which might result in less agricultural area.

2.2.3 Environmental Criteria

In recent years, regular water resources were severely damaged because of their unsustainable usage. In the future, the situation could become much worse due to the growing demands and some other factors like the impact of climate change. They will degrade the condition of the rivers, lakes, groundwater supply and wetlands. This requires the consideration of environmental criteria in all decision making. Some of the important factors, which should be considered in the planning of water resources and environmental projects, are as follows (World Bank 2007):

- Loss of aquatic habitat and elimination of species
- Water quality degradation in terms of oxygen depletion, nutrient loading, elevated turbidity, effluent discharge of contaminants, and risk of groundwater contamination
- Change of hydrologic regimes, groundwater table shortfall, saltwater intrusion into ground and surface water supplies
- Some problems such as soil erosion and its instability, land subsidence due to extra groundwater pumping have increasing effects in human, animal and agricultural diseases due to extended growing seasons

Because of these common and complex problems, environmental criteria should be included into every MCDA model in water resources and environmental management, in order to find sustainable development for now and for the future human generations and for the environment.

2.3 Constructing the Hierarchy of the Criteria

Developing the hierarchy of the criteria has been already carried out in the literature for many disciplines. According to Keeney and Raiffa (1993), any hierarchy of the criteria has to fulfill all of the following conditions. It has to be complete, operational, decomposable, non-redundant, and minimum size. Although sometimes compromises have to be made among these requirements. For example, the effect of the attribute “time of construction” can be combined and embodied within the calculation of the financial attributes such as “benefit cost ratio”. Roy (1994) studied different options of criteria selection, aiming to promote an overall analysis. In a case study of Southern France, Netto et al. (1996) used 13 criteria, which were divided into three general categories: vulnerability, reliability and adaptability. Constructing a hierarchy of criteria is an iterative process. It continues until the stakeholders find a commonly approved criteria set. The hierarchy of the criteria has to comprise a reasonable simplification of the real life (Aravossis et al. 2003).

2.3.1 Value Management

Value Management is dedicated to motivate people, develop skills and promote innovation, with the aim of maximizing the overall performance of the system. Value Management has evolved based on previous methods using the concept of value and functional approach. This approach was pioneered by Miles in the 1940s and the 1950s who has developed the technique of Value Analysis. In the early stages, this method was principally used to identify and eliminate unnecessary costs (IVM 2005). It is suggested that the concept of value relies on a relationship between the satisfaction of the different criteria and their cost of implementation in a model. Both the use of less resources and larger satisfaction of needs result in greater value:

$$Value = \frac{Satisfaction\ of\ the\ needs}{Use\ of\ resources}. \tag{2.1}$$

The following steps of Value Management can be used to structure the hierarchy of criteria:

1. *Information:* Identify the preliminary hierarchy of criteria.
2. *Function analysis:* Identify the primary and secondary functions of the criteria and their associated benefit–cost relationships.

Primary functions are the initial and main goals and the secondary functions are the externalities resulting in using the criteria. For example, when we define an attribute as "Diversification of Financial Resources" the primary function is the reduction of the Governmental budget of a project. The secondary function of this attribute is lowering the economical risk of the project by attaching the financial needs to its users.

3. *Generating ideas:* Generate new criteria for value improvement through innovation mostly based on brainstorming.
4. *Evaluation:* Prioritize the new criteria according to their contribution in improving the total value.
5. *Action plan:* Identify the actions/strategies required to achieve the Value Analysis outcomes and to provide ongoing management frameworks for project progression. For example, in the case study described in the next subsection the strategy is to achieve IWRM in ranking national water resources projects.
6. *Analysis and reports:* Prepare the final report, which includes the description of the process outcomes and the developed hierarchy of criteria.

2.3.2 Case Study

In the following case study, the above-described methodology will be used as a basis for extracting the effective attributes in sustainable ranking of water resources projects in Iran. The Government of the Islamic Republic of Iran has approved some important acts for water resources management. Based on the principle no. 138 of the Islamic Republic of Iran's constitution, the cabinet approved "Long-Term Development Strategies for Iran's Water Resources". It comprises IWRM principles and water governance principles in 18 parts (MOE 2003). However, these acts are only at vision, strategy and policy levels. To evaluate water resources projects, it is necessary to construct a hierarchy of criteria from the acts. As one of the main goals of this study, a hierarchy of criteria has been developed. In the first step similar watershed plans of 20 countries were analyzed, including Pakistan, Turkey, India, Kenya, Sweden, United States and Brazil. Then, based on the state-of-the-art review and the national acts of Iran, a preliminary hierarchy of the criteria was introduced (Ardakanian and Zarghami 2004). In order to revise and finalize the preliminary hierarchy, 30 experts conducted the revision. Stakeholders were invited from the government,

Objectives	Criteria
Social	Employment and Migration
	Public Participation
	Social Equity
	Recreation, Tourism and Aditional Facilities
	Social Casualties and Damages of Dam Project
	Natural Disasters Management "Flood and Drought"
	More Settlement in Border Regions
	Priority of Shared Waters
	Reducing the Confilicts among Stakeholders
Economics and Finance	Priority of Usages
	Benefit minus Cost
	Benefit/Cost Ratio
	Extent of Investments
	Risk of Investments
	Development and Improvement Ratio in Agricultural Area
	Base for Supplimentary Projects
	Diversification of Financial Resources
	Level of Construction Technology
	Capabilties of Phased Operation
	Simplicity of Operation and Maintanance
	Level of Studying Phases
Environmental	Consistancy with Climate
	Less Damages to Ancient and Cultural Heritage
	Range of Environmental Impacts
	Studies of Watershed Conservation
	Balancing of Water Resources
	Studies on Supply and Demand Management
Comprehensive criteria	Consistancy with Policies
	Consistancy with Logistic Plan
	Impacts on Other Projects
	Management Capacities in Basin
	Comprehensive Study in Basins

Fig. 2.2 Hierarchy of criteria for evaluation of water resources projects in Iran

consulting companies, universities and non-governmental organizations. They participated in several sessions applying Value Management methodology. The revised hierarchy is indicated in Fig. 2.2, consisting of 4 objectives and 32 criteria. The main objectives include social (cultural, political, security and legal), economic and financial, environmental and comprehensive management affairs. In the future, the Government should organize suitable acts and legislations to support the use of this hierarchy. It is very general, so it can be used to evaluate any kind of water resources projects.

Chapter 3
Solution of Discrete MCDA Problems

3.1 Introduction

In this chapter we will introduce the most popular and most frequently used solution methods for deterministic discrete MCDA problems. In deterministic models the evaluations of the alternatives, the criteria weights and all other parameters are assumed to be certain and known. Models with uncertainty will be discussed later in Chap. 7.

Assume that the number of decision alternatives is m which are evaluated by n criteria. Let a_{ij} denote the evaluation of alternative j with respect to criterion i, then the goodness of this alternative can be characterized by the evaluation vector $X_j = (a_{1j}, a_{2j},\ldots, a_{nj})$. In the case of a single-objective optimization problem each alternative is evaluated by only one criterion, so vector X_j has only one element, $X_j = (a_j)$. If j_1 and j_2 are two alternatives, then their evaluations can be easily compared, since either $a_{j_1} > a_{j_2}$, $a_{j_1} < a_{j_2}$ or $a_{j_1} = a_{j_2}$. We can always assume that larger a_j values indicates better evaluation, otherwise we can multiply it by (-1). So in the first case, when $a_{j_1} > a_{j_2}$, alternative j_1 is preferred to j_2, in the second case j_2 is preferred to j_1 and in the third case they are equally preferred. So in the case of one criterion, any two alternatives can be directly compared. Unfortunately this is not the case in the presence of multiple criteria. For instance, in the case of $n = 2$ assume that the evaluation vector of these alternatives

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are $X_{j_1} = (1,\ 2)$ and $X_{j_2} = (2,\ 1)$. Alternative j_1 is better than j_2 in the second criterion but worse in the first. So these alternatives cannot be compared directly. For a DM in order to choose between alternatives j_1 and j_2 it is important to decide whether a unit loss in one criterion is compensated by a unit gain in the other or not. This kind of decision becomes much more complicated if the gains and losses are given in different units and more than two criteria are present.

This chapter will give an overview of the different methods being used for best alternative selection. There are many different ways how a DM can express his/her priorities and preferences. For each such way a particular solution method can be suggested.

3.2 Dominance Method

Let $X_j = (a_{1j}, a_{2j}, \ldots, a_{nj})$ denote the evaluation vector of alternative j ($j = 1, 2, \ldots, m$). We say that an alternative dominates another if it results in an equal or superior value in all criteria and in at least one criterion it is strictly better. Mathematically the property that alternative j dominates alternative l can be expressed as $a_{ij} \geq a_{il}$ for all criteria i and there is at least one criterion i such that

$$a_{ij} > a_{il}.$$

It is however very seldom the case that one alternative dominates all others. In such cases the dominating alternative is the choice, and there is no need for further study. In many practical problems however we can find alternative pairs that one alternative dominates the other even if no alternative dominates all others. If an alternative j_1 dominates j_2, then there is no need to consider alternative j_2 in the further selection process, so it can be eliminated and the number of alternatives decreases by one. After all dominated alternatives are eliminated and still we have more than one alternative left, then we have to continue the process with the application of another method.

Example 3.1. The best choice of the location of a dam, which to be built in a watershed, is based on three criteria: net benefit (in million dollars), number of beneficiaries (in thousands of people), and geological stability (in subjective scale between 0 and 100). The location alternatives are shown in Fig. 3.1. The evaluation vectors for the four alternative locations are given in the columns of Table 3.1. None of the alternatives dominates all others.

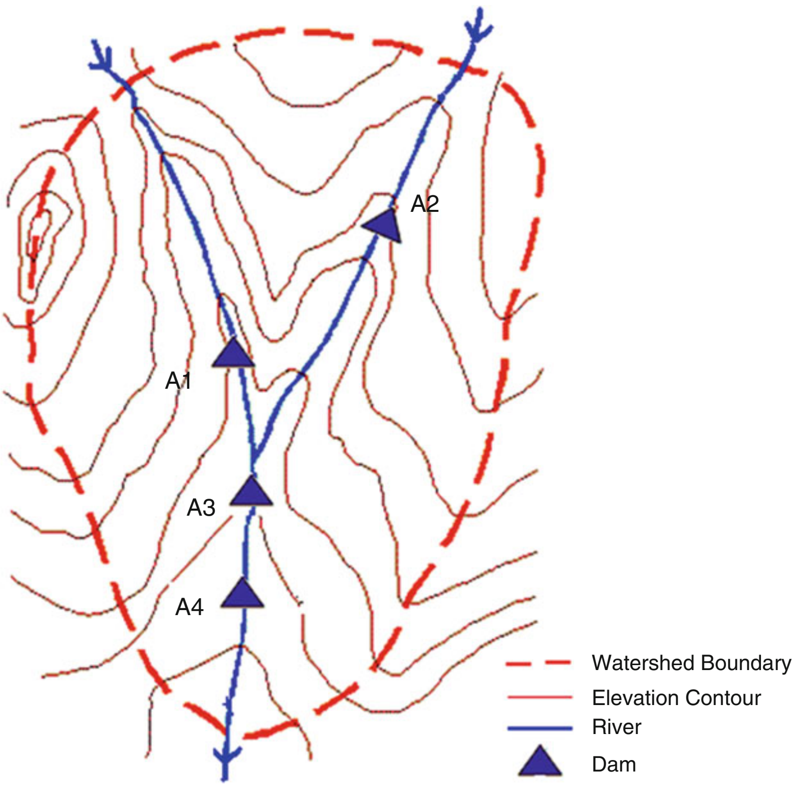

Fig. 3.1 A schematic map of the four alternative dam locations

Table 3.1 Evaluation table for Example 3.1

Criteria	Alternatives			
	A_1	A_2	A_3	A_4
C_1	99.6	85.7	101.1	95.1
C_2	4	19	40	50
C_3	70	50	10	20

Moreover, if we change the $a_{11} = 99.6$ value to $a_{11} = 102$ and the value of a_{21} from 4 to 51, then alternative A_1 will dominate all others, so it would be the choice.

3.3 Sequential Optimization (SO)

This method is based on the ordinal preferences of the criteria. It is assumed that the DM can identify his/her most important criterion, i_1, the second most important criterion, $i_2, \ldots$, and finally the least preferred criterion i_n. The DM wants to satisfy first the most important criterion as well as possible, and then to satisfy the second with keeping the first at its most favorable level, if some choices are still possible. After the second most important criterion is satisfied in its best possible level and no single best alternative is found, then the third criterion is optimized, and so on. The procedure terminates if either a single best alternative is found in any of the steps, or the least preferred criterion is already optimized. If a single optimal alternative is found at the end of this procedure, then it is the choice. Otherwise any one of the optimal alternatives can be selected, since they have identical evaluations in all criteria, so there is no difference between them as far as the originally selected criteria are concerned. In many cases the DM can add one or more new criteria and repeat the process with only the optimal alternatives in order to reach a unique decision.

Example 3.2. Consider again the dam location selection problem given in Table 3.1. If criterion 1 is the most important for the DM, then alternative A_3 is his/her choice. If C_2 is the most important, then A_4 is the choice, and if C_3 is the most important, then alternative A_1 is the best. Assume now that the preference order of the criteria is $C_1 \succ C_2 \succ C_3$, where $C_{i_1} \succ C_{i_2}$ means that criterion i_1 is more important than i_2. Assume that in addition, the DM has to rank the alternatives in addition to select his/her best choice. In the first step the DM considers criterion C_1 and selects the best alternative with respect to this criterion: A_3. Then this alternative is eliminated from the table, and C_1 is considered again with the remaining three alternatives. The best C_1 value is obtained at alternative A_1. After this is also eliminated and only alternatives A_2 and A_4 remain in the table, they are compared with respect to criterion C_1 again and A_4 is found to be better. So the ranking of the alternatives based on this approach becomes

$$A_3 \succ A_1 \succ A_4 \succ A_2.$$

Notice that the ranking of the alternatives was very simple in the above example, since there was always a unique best alternative in each step, which is not always the case. In general, we say that alternative j_1 is considered better than j_2 if either $a_{ij_1} > a_{ij_2}$ for all criteria, or there is a k $(1 \leq k \leq n-1)$ such that $a_{i_l j_1} = a_{i_l j_2}$ for $l = 1, 2, \ldots, k$ and $a_{i_{k+1} j_1} > a_{i_{k+1} j_2}$, where $i_1, i_2, \ldots, i_n$ is the preference order of the criteria.

3.4 The ε-Constraint Method (εCM)

In the application of the sequential optimization method it is very often the case that the procedure terminates before all criteria are considered. It was the case earlier in Example 3.2, when the optimization with respect to the most important criteria gave a unique solution and no further investigation with the less important criteria was needed. There is however the possibility that some or all less important criteria give very poor, or unacceptable evaluation to the selected alternatives. In order to avoid this possibility the DM identifies his/her most important criteria and gives minimum acceptable levels to all other criteria. This information and requirement must guarantee that the selected best choice will not give worse values in any criterion than the specified lower level.

The procedure consists of two steps. In the first step all alternatives which violate the lower bound conditions are eliminated.

In the second step only the most important criterion is considered and the remaining alternative with the best evaluation number is selected.

Example 3.3. Consider again the problem of Table 3.1. Assume that C_1 is the most important criterion and the minimum acceptance value for C_2 is 15 and for C_3 is 20. Based on these lower bounds the alternatives A_1 and A_3 have to be eliminated, and only A_2 and A_4 remain in the table. In the second step they are compared with respect to the most important criterion C_1, and in the comparison A_4 has larger value, so it is the choice.

3.5 Simple Additive Weighting (SAW)

In applying this method the DM has to specify the relative importance weights of all criteria. If the criteria altogether represent 100% of the interest of the DM, then let w_i denote the percentage of interest for criterion i. It is assumed that for all i, $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$. In the simplest version of the method the DM constructs a new objective function, which is the weighted average of the evaluation values with respect to the different criteria. That is, the weighted average

$$F_j = \sum_{i=1}^{n} w_i a_{ij} \tag{3.1}$$

is assigned as the "overall" evaluation of alternative j. This method is known as Simple Additive Weighting (SAW).

In most applications the a_{ij} values represent very different phenomena, such as dollars, number of people and geologic suitability in our earlier examples. In such cases the objective function (3.1) has no direct meaning, since we added different things. Another difficulty of applying this objective functions is the fact that by changing the unit of any of the objectives, its weight changes automatically. For example, if we change the unit of criterion C_1 (net benefit) from million dollars to ten millions, then the corresponding evaluation numbers will be divided by 10, which is equivalent to giving one tenth less weight to this criterion. In order to overcome these difficulties we have to normalize the evaluation numbers. We can use a simple linear transformation

$$\bar{a}_{ij} = \frac{a_{ij} - m_i}{M_i - m_i}, \tag{3.2}$$

where m_i and M_i are the computed or estimated minimum and maximum values of criterion i. Clearly $\bar{a}_{ij}$ will be always between 0 and 1 with zero worst value and unit best value. Instead of this simple linear transformation we can introduce utility functions $u_i(a_{ij})$ for all criteria showing the satisfaction levels (in percentages) of the values a_{ij} for criterion i. Then the weighted average

$$F_j = \sum_{i=1}^{n} w_i u_i(a_{ij}) \tag{3.3}$$

shows the average satisfaction level of alternative j. The linear transformation (3.1) corresponds to the linear utility function

$$u_i(a_{ij}) = \frac{a_{ij} - m_i}{M_i - m_i}. \tag{3.4}$$

The alternative with the largest objective value is selected as the best choice.

Example 3.4. Returning to the dam selection problem with data given in Table 3.1, we can select the minimum criteria values as

$$m_1 = 85.7,\ m_2 = 4,\ m_3 = 10$$

and maximum values

$$M_1 = 101.1,\ M_2 = 50,\ M_3 = 70.$$

So the normalized criteria can be computed in the following way

$$\bar{a}_{1j} = \frac{a_{1j} - 85.7}{101.1 - 85.7} = \frac{a_{1j} - 85.7}{15.4},$$

$$\bar{a}_{2j} = \frac{a_{2j} - 4}{50 - 4} = \frac{a_{2j} - 4}{46},$$

and

$$\bar{a}_{3j} = \frac{a_{3j} - 10}{70 - 10} = \frac{a_{3j} - 10}{60}$$

for $j = 1, 2, 3, 4$. The normalized values are shown in Table 3.2, where the weights of the criteria are also indicated.

The weighted average satisfaction values of the four alternatives are as follows:

$$F_1 = 0.2(0.903) + 0.3(0) + 0.5(1) = 0.681$$

$$F_2 = 0.2(0) + 0.3(0.326) + 0.5(0.667) = 0.431$$

$$F_3 = 0.2(1) + 0.3(0.783) + 0.5(0) = 0.435$$

$$F_4 = 0.2(0.610) + 0.3(1) + 0.5(0.167) = 0.506.$$

Clearly the first alternative is the best. The complete ordering of the alternatives can be done by ordering the alternatives in decreasing F_j values. In our case

$$\mathrm{A}_1 \succ \mathrm{A}_4 \succ \mathrm{A}_3 \succ \mathrm{A}_2.$$

Table 3.2 Normalized evaluations and weights for Example 3.1

Criteria	Alternatives				
	Weights	A_1	A_2	A_3	A_4
C_1	0.2	0.903	0.000	1.000	0.610
C_2	0.3	0.000	0.326	0.783	1.000
C_3	0.5	1.000	0.667	0.000	0.167

3.6 Distance Based Methods (DBM)

There are two fundamentally different versions of this method. In the first case the DM specifies (or we compute) the ideal point, the components of which are the subjective or computed best values of the different criteria. The ideal point is an n-dimensional vector, and the evaluation vector $\boldsymbol{X}_j$ of each alternative is compared to the ideal point by computing their distance. The alternative with the smallest distance is considered the best. In the second approach the DM specifies (or we compute) the nadir, the components of which are the subjective or computed worst values of the criteria. The nadir also has n components. Each alternative j will be compared to the nadir by computing the distance of the evaluation vector $\boldsymbol{X}_j$ from the nadir. The alternative with the largest distance is then selected as the best choice. In order to avoid the difficulties resulting from the different units of the criteria, all criteria are normalized, so the components of the ideal point, the nadir and the evaluation vectors are all normalized. In most applications the weighted Minkowski-distance is used. Let a_i^* denote the ith component of the ideal point and a_{i*} the ith component of the nadir, and assume that linear transformation is used for normalizing. Then the distance of alternative j from the ideal point is given by

$$D_j^p = \left\{ \sum_{i=1}^{n} \left(w_i \frac{a_i^* - a_{ij}}{a_i^* - a_{i*}} \right)^p \right\}^{\frac{1}{p}}, \tag{3.5}$$

where $p \geq 1$ is a positive user-selected model parameter. Similarly the distance of alterative j from the nadir is defined by relation

$$d_j^p = \left\{ \sum_{i=1}^{n} \left(w_i \frac{a_{ij} - a_{i*}}{a_i^* - a_{i*}} \right)^p \right\}^{\frac{1}{p}}. \tag{3.6}$$

The selection of parameter p is very important, since it has a significant effect on the final choice. The case of $p = 1$ corresponds to simple averaging, $p = 2$ to squared averaging, and $p = \infty$ is selected if only the largest deviation is considered. According to Tecle et al. (1998), "Varying the parameter p from 1 to infinity, allows one to move from minimizing the sum of individual regrets (i.e., having a perfect compensation among the objectives) to minimizing the maximum regret (i.e., having no compensation among the objectives) in the decision making process. The choice of a particular value of this compensation parameter p depends on the type of problem and desired solution. In general, the greater the conflict between players, the smaller the possible compensation becomes".

Two particular methods are especially popular in applications: Compromise Programming (CP) and Technique for Order Performance by Similarity to Ideal Solution (TOPSIS). In the case of compromise programming (Zeleny 1973) the distance (3.5) is minimized. It is illustrated by the following example.

Example 3.5. Consider again the data of the previous problem. For all criteria, the ideal point components are the maximum values, and the components of the nadir are the actual minimum values. So the ideal point and the nadir are (101.1, 50, 70) and (85.7, 4, 10) respectively. Using the distance formula (3.5) with $p = 2$ and weights ($w_1 = 0.2$, $w_2 = 0.3$, $w_3 = 0.5$) as before, the D_j^2 distances become

$$D_1^2 = \left\{0.2^2\left(\frac{101.1-99.6}{101.1-85.7}\right)^2 + 0.3^2\left(\frac{50-4}{50-4}\right)^2 + 0.5^2\left(\frac{70-70}{70-10}\right)^2\right\}^{\frac{1}{2}}$$

$$\approx 0.301,$$

$$D_2^2 = \left\{0.2^2\left(\frac{101.1-85.7}{101.1-85.7}\right)^2 + 0.3^2\left(\frac{50-19}{50-4}\right)^2 + 0.5^2\left(\frac{70-50}{70-10}\right)^2\right\}^{\frac{1}{2}}$$

$$\approx 0.330,$$

$$D_3^2 = \left\{0.2^2\left(\frac{101.1-101.1}{101.1-85.7}\right)^2 + 0.3^2\left(\frac{50-40}{50-4}\right)^2 + 0.5^2\left(\frac{70-10}{70-10}\right)^2\right\}^{\frac{1}{2}}$$
$$\approx 0.504,$$

$$D_4^2 = \left\{0.2^2\left(\frac{101.1-95.1}{101.1-85.7}\right)^2 + 0.3^2\left(\frac{50-50}{50-4}\right)^2 + 0.5^2\left(\frac{70-20}{70-10}\right)^2\right\}^{\frac{1}{2}}$$
$$\approx 0.424.$$

The first alternative gives the smallest distance, so it is the best choice. The ranking of the alternatives can be also obtained by ordering them in increasing D_j^2 values. In our case

$$\mathrm{A}_1 \succ \mathrm{A}_2 \succ \mathrm{A}_4 \succ \mathrm{A}_3.$$

Notice that in the case of $p = 1$, minimizing distance (3.5), maximizing distance (3.6) and the SAW methods with normalized objectives are equivalent to each other, they result in the same best choice and ranking of the alternatives. This observation can be shown easily by noticing that

$$\begin{aligned} D_j^1 &= \sum_{i=1}^n w_i \frac{a_i^* - a_{ij}}{a_i^* - a_{i*}} = \sum_{i=1}^n w_i \left(\frac{-a_{ij} + a_{i*}}{a_i^* - a_{i*}} + \frac{a_i^* - a_{i*}}{a_i^* - a_{i*}}\right) \\ &= -\sum_{i=1}^n w_i \frac{a_{ij} - a_{i*}}{a_i^* - a_{i*}} + 1 \end{aligned}$$

and

$$d_j^1 = \sum_{i=1}^n w_i \frac{a_{ij} - a_{i*}}{a_i^* - a_{i*}}.$$

Therefore if F_j denotes the weighted average normalized evaluation numbers of alternatives j, then clearly

$$D_j^1 = 1 - F_j \text{ and } d_j^1 = F_j.$$

Another version of distance based methods is known as the TOPSIS method which combines the distances D_j^1 and d_j^1 from the ideal point and from the nadir into one combined measure

$$F_j^p = \frac{d_j^p}{D_j^p + d_j^p}. \tag{3.7}$$

Notice that $0 \leqslant F_j^p \leqslant 1$ and $F_j^p = 0$ if and only if $d_j^p = 0$, that is, if alternative j is the nadir. Similarly $F_j^p = 1$ if and only if $D_j^p = 0$, that is, if alternative j is the ideal point. Therefore larger F_j^p value indicates better alternative, so the alternative with the largest F_j^p value is considered the best, and the ranking of the alternatives is done by ordering them with decreasing F_j^p values.

Example 3.6. Consider again the decision problem of the previous problem. Table 3.1 shows the evaluation numbers, the weights are $w_1 = 0.2$, $w_2 = 0.3$, $w_3 = 0.5$, the ideal point is (101.1, 50, 70) and the nadir is (85.7, 4, 10). The distances D_j^2 from the ideal point were already determined earlier in Example 3.5, so we need now to compute only the distances from the nadir. They are as follows:

$$d_1^2 = \left\{0.2^2\left(\frac{99.6-85.7}{101.1-85.7}\right)^2 + 0.3^2\left(\frac{4-4}{50-4}\right)^2 + 0.5^2\left(\frac{70-10}{70-10}\right)^2\right\}^{\frac{1}{2}}$$
$$\approx 0.532,$$

$$d_2^2 = \left\{0.2^2\left(\frac{85.7-85.7}{101.1-85.7}\right)^2 + 0.3^2\left(\frac{19-4}{50-4}\right)^2 + 0.5^2\left(\frac{50-10}{70-10}\right)^2\right\}^{\frac{1}{2}}$$
$$\approx 0.347,$$

$$d_3^2 = \left\{0.2^2\left(\frac{101.1-85.7}{101.1-85.7}\right)^2 + 0.3^2\left(\frac{40-4}{50-4}\right)^2 + 0.5^2\left(\frac{10-10}{70-10}\right)^2\right\}^{\frac{1}{2}}$$
$$\approx 0.308,$$

$$d_4^2 = \left\{ 0.2^2 \left(\frac{95.1 - 85.7}{101.1 - 85.7} \right)^2 + 0.3^2 \left(\frac{50 - 4}{50 - 4} \right)^2 + 0.5^2 \left(\frac{20 - 10}{70 - 10} \right)^2 \right\}^{\frac{1}{2}}$$
$$\approx 0.334.$$

Notice that the quantities inside the parentheses are the normalized evaluation numbers listed in Table 3.2. The combined "goodness" measures of the TOPSIS method can now be obtained by using formula (3.7):

$$F_1^2 = \frac{0.532}{0.532 + 0.301} \approx 0.639,$$

$$F_2^2 = \frac{0.347}{0.347 + 0.330} \approx 0.513,$$

$$F_3^2 = \frac{0.308}{0.308 + 0.504} \approx 0.379,$$

$$F_4^2 = \frac{0.334}{0.334 + 0.424} \approx 0.441.$$

Since larger F_j^p value indicates better alternative, the ranking of the four dam locations is as follows:

$$\mathrm{A}_1 \succ \mathrm{A}_2 \succ \mathrm{A}_4 \succ \mathrm{A}_3.$$

3.7 The Analytic Hierarchy Process (AHP)

In the application of the previously discussed methods the priority of the DM is expressed by a vector $(w_1, w_2, \ldots, w_n)$ of importance weights. In many applications the assessment of such weights in not easy. The analytic hierarch process we discuss in this section is based on pair-wise comparisons. In this task the DM is asked about the relative importance of criterion

i in comparison to criterion j for each criteria pair (i, j). In answering the question, the DM has to concentrate on only two criteria and not on the entire set of the criteria. The answer α_{ij} of the DM gives an estimate of the ratio w_i/w_j. Since the DM concentrates on only two criteria at each time and he/she does not think of the relations between these criteria and the others, his/her answers are usually inconsistent. If they were consistent, then the following relations should be satisfied:

(i) $\alpha_{ij} = \frac{1}{\alpha_{ji}}$ for all i and j, since

$$\alpha_{ij} = \frac{w_i}{w_j} = \frac{1}{\frac{w_j}{w_i}} = \frac{1}{\alpha_{ji}},$$

(ii) $\alpha_{ij} \cdot \alpha_{jk} = \alpha_{ik}$ for all i, j and k, since

$$\alpha_{ij} \cdot \alpha_{jk} = \frac{w_i}{w_j} \cdot \frac{w_j}{w_k} = \frac{w_i}{w_k} = \alpha_{ik}.$$

Assume first that the answers are consistent. Then the matrix $\mathrm{A} = (\alpha_{ij})$ clearly satisfies the following relation:

$$\mathrm{A}\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = \begin{pmatrix} \frac{w_1}{w_1} & \cdots & \frac{w_1}{w_n} \\ \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \cdots & \frac{w_n}{w_n} \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = n\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix},$$

and if we use the notation $w = (w_1, w_2, \ldots, w_n)^T$ then

$$\mathrm{A}w = n\mathrm{w} \tag{3.8}$$

meaning that n is an eigenvalue of matrix A with the associated eigenvector w. Matrix A is nonnegative and its rank is unity, since row k of the matrix is the w_k/w_1-multiple of the first row. Therefore it has one positive eigenvalue and all other eigenvalues are equal to 0. The Perron–Frobenius theory implies that n is the principal eigenvalue of A and vector w is unique except with a constant multiplier. We can normalize the components of this vector by dividing them by their sum as:

$$\overline{w}_i = \frac{w_i}{\sum_{j=1}^{n} w_j} \tag{3.9}$$

to get the normalized weights of the criteria.

In order to obtain the weights we do not need to compute the eigenvectors, a good approximation can be obtained in a simple approach. Notice that the sum of the elements of the different columns equals

$$\frac{\sum_{l=1}^{n} w_l}{w_1}, \frac{\sum_{l=1}^{n} w_l}{w_2}, \ldots, \frac{\sum_{l=1}^{n} w_l}{w_n},$$

respectively, and by dividing each column by the sum of its elements the modified $\boldsymbol{A}$ matrix becomes

$$\begin{pmatrix} \bar{w}_1 & \bar{w}_1 & \cdots & \bar{w}_1 \\ \bar{w}_2 & \bar{w}_2 & \cdots & \bar{w}_2 \\ \vdots & \vdots & & \vdots \\ \bar{w}_n & \bar{w}_n & \cdots & \bar{w}_n \end{pmatrix}, \tag{3.10}$$

that is, it has identical columns.

As mentioned earlier, the α_{ij} estimates obtained from the DM usually do not satisfy the consistency requirements, therefore the normalized matrix (3.10) based on the estimates will not have identical columns. So we have to find the column vector which is the best overall approximation of the different columns of the obtained normalized matrix. As it is well known from statistics, it is the simple algebraic average of the columns. This average vector gives the estimates of the normalized weights. Based on this observation the procedure can be described as follows:

Step 1. Construct the $\boldsymbol{A}$ matrix from pair-wise comparisons.

Step 2. Normalize each column of this matrix.

Step 3. Compute the algebraic average of the columns of the normalized matrix.

The elements of this vector give the weights.

Example 3.7. We return now to the dam selection problem examined in some of our earlier examples. There are four alternatives to select from based on three criteria. From the questionnaire given to the representatives of the DM company, the following approximating pair-wise comparison matrix was obtained:

$$\mathbf{A} = \begin{pmatrix} 1 & 1/3 & 5 \\ 3 & 1 & 7 \\ 1/5 & 1/7 & 1 \end{pmatrix}.$$

The sums of the elements of the columns are 21/5, 31/21 and 13, respectively. The normalized comparison matrix is obtained by dividing all elements of each column by the column sum. The resulting normalized matrix becomes

$$\begin{pmatrix} 5/21 & 7/31 & 5/13 \\ 15/21 & 21/31 & 7/13 \\ 1/21 & 3/31 & 1/13 \end{pmatrix},$$

and the algebraic average of the three columns gives the approximating weight-vector:

$$w = \frac{1}{3}\left\{ \begin{pmatrix} 5/21 \\ 15/21 \\ 1/21 \end{pmatrix} + \begin{pmatrix} 7/31 \\ 21/31 \\ 3/31 \end{pmatrix} + \begin{pmatrix} 5/13 \\ 7/13 \\ 1/13 \end{pmatrix} \right\} = \begin{pmatrix} 0.2828 \\ 0.6434 \\ 0.0738 \end{pmatrix}.$$

We can also check the level of inconsistency of the DM. We have to find first a good approximation of the principal eigenvalue of matrix $\boldsymbol{A}$, which clearly differs from the theoretical value, n. If we multiply each element of the normalized weight vector $\boldsymbol{w}$ by the corresponding column sum of the true comparison matrix and add these products, then the result becomes

$$\sum_{l=1}^{n} w_l \cdot \frac{\sum_{i=1}^{n} w_i}{w_l} = \sum_{l=1}^{n} 1 = n,$$

which is the true principal eigenvalue. We can obtain its good estimation $\lambda_{\max}$ based on the approximating comparison matrix by doing the same: adding the products of the column sums and the corresponding components of the normalized weights vector. Then the inconsistency index can be obtained by the principal formula:

$$ICI = \frac{\lambda_{\max} - n}{(n-1)RI},$$

where the value of RI depends on the size of the problem. It is tabulated and its values are given in Table 3.3.

Example 3.8. In the case of the previous example $n = 3$, so $RI = 0.58$. It is easy to see that

$$\lambda_{\max} = \frac{21}{5}(0.2828) + \frac{31}{21}(0.6434) + 13(0.0738) \approx 3.0969,$$

so

$$ICI = \frac{3.0969 - 3}{(3-1)0.58} \approx 0.084,$$

which is less than 10% so we consider the DM consistent.

In many applications this procedure is used first to find the weights, and then another (such as weighting or distance based) method is used with the obtained weights to find the best solution.

Table 3.3 Values of *RI*

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The AHP can be also used to find final decisions. In this case the problem has to be decomposed into several hierarchy levels. The highest level is the level of the numerical criteria, the lowest level consists of the alternatives, and there might be several levels in between the elements of which are combined in obtaining the next higher level, for example by using a multi-objective method. The pair-wise comparisons are repeated in each level and the results are combined by simple linear combinations.

Example 3.9. Returning to the previous example we have two levels, the alternatives and the criteria. The hierarchic structure is shown in Fig. 3.2. Pair-wise comparisons are performed for the four alternatives with respect to the three criteria. The comparison matrices are shown in Tables 3.4–3.6.

Then we can compute the combined goodness measure of each alternative based on the weights of the corresponding level for the

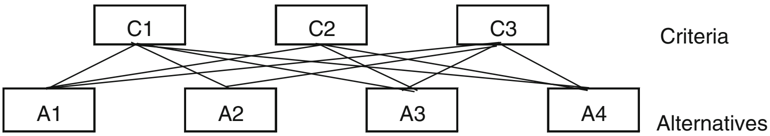

Fig. 3.2 Hierarchical structure for Example 3.9

Table 3.4 Pair-wise comparison matrix level 2 with respect to C_1

Alternatives	A_1	A_2	A_3	A_4	Weight vector
A_1	1	1	7	5	0.45
A_2	1	1	3	5	0.37
A_3	0.14	0.33	1	0.33	0.07
A_4	0.11	0.2	3	1	0.11
Sum	2.25	2.53	14	11.33	1

$\lambda_{max} = 4.17$, $ICI = 6.3\% < 10\%$ (acceptable)

Table 3.5 Pair-wise comparison matrix level 2 with respect to C_2

Alternatives	A_1	A_2	A_3	A_4	Weight vector
A_1	1	5	3	5	0.53
A_2	0.2	1	0.33	3	0.13
A_3	0.33	3	1	5	0.27
A_4	0.2	0.33	0.2	1	0.07
Sum	1.73	9.33	4.53	14	1

$\lambda_{max} = 4.33$, $ICI = 12.2\%$~10% (marginally acceptable)

Table 3.6 Pair-wise comparison matrix level 2 with respect to C_3

Alternatives	A_1	A_2	A_3	A_4	Weight vector
A_1	1	0.2	0.33	1	0.09
A_2	5	1	3	5	0.54
A_3	3	0.33	1	5	0.28
A_4	1	0.2	0.2	1	0.09
Sum	10	1.73	4.53	12	1

$\lambda_{\max} = 4.18$, $ICI = 6.6\% < 10\%$ (acceptable)

three criteria. The overall goodness measures are obtained as the following linear combinations:

$$
\begin{aligned}
F_1 &= 0.29 \times 0.45 + 0.64 \times 0.53 + 0.07 \times 0.09 = 0.48,\\
F_2 &= 0.29 \times 0.37 + 0.64 \times 0.13 + 0.07 \times 0.54 = 0.23,\\
F_3 &= 0.29 \times 0.07 + 0.64 \times 0.27 + 0.07 \times 0.28 = 0.21,\\
F_4 &= 0.29 \times 0.11 + 0.64 \times 0.07 + 0.07 \times 0.09 = 0.08.
\end{aligned}
$$

The multipliers 0.29, 0.64 and 0.07 are the resulted weights of the three criteria obtained in Example 3.7. The second part of each term is taken from the computed components of the weight vectors of Tables 3.4–3.6. The final ranking is therefore $A_1 \succ A_2 \succ A_3 \succ A_4$.

There are user-friendly softwares available to solve MCDA problems by using the AHP method. For example, the reader may refer to *Expert Choice* which can be downloaded from http://www.expertchoice.com/.

3.8 Other Methods

There are other important methods reported in the literature to solve MCDA problems and several case studies have been solved by using these methods. Some of them are ELECTRE (Figueira et al. 2005), MAUT (Keeney and Raiffa 1993), and PROMETHEE (Brans et al. 1986). For a complete review and analysis, the reader can refer to Hajkowicz and Collins (2007).

3.9 Case Studies

3.9.1 Inter-basin Water Transfer

The mean annual rainfall of Iran is about 250 mm that is about 30% of the world's average. The increasing water demand has also caused a decrease in annual per capita water resources. The uneven distribution of water across the country and the growth of population have led to the present water shortages in the major parts of the country, especially in the central and the Southeastern regions. The country is divided into six main hydrological basins as shown in Fig. 3.3. The per capita water resources potential in basin 3 is four times the potential in basin 4.

Water transfer between regions is an effective way to decrease water shortages. In this study transfers to the Zayanderud sub-basin are examined. The importance of the most appropriate project selection is due to the high cost, long tunnels, large quantity and good quality of water transfer and the induced social conflicts. The Zayanderud sub-basin is located west of the

Fig. 3.3 Main basins of Iran and the position of the water transfer projects

Central main basin, and it is denoted by number 4. The Zayanderud River in this basin flows through Isfahan which is a main tourist city in Iran. In recent years this basin had extensive growth in quantity and quality. To decrease water shortages in this basin, there are several possibilities for inter-basin water transfer (IBWT) projects and we have selected four of them to evaluate: Kuhrang III, Cheshmelangan, Beheshtabad and Gukan. Data for the evaluation of these four alternatives with respect to the attributes are presented in Table 3.7. They were gathered from experts of the DM company for these projects (adopted from Zarghami et al. 2007). In the decision matrix, the following seven criteria were considered:

- *Consistency with policies*: The DM has been questioned to rate the alternatives in view of their consistency with national, regional and local policies, especially those of the water authorities. The DM answered in linguistic variables as very high, high, fairly high, medium, fairly low, low and very low.
- *Resettlement of people*: Each of the IBWT projects in this study needs a reservoir. The reservoir requires the costly

Table 3.7 Decision matrix for the evaluation of the IBWT projects (adopted from Zarghami et al. 2007)

Criteria	Weights	Alternatives			
		A_1	A_2	A_3	A_4
C_1 Consistency with policies	High	High	Very high	Very high	High
C_2 Resettlement of people (negative)	Fairly high	0	0	200	4,000
C_3 Public participation	Low	Fairly high	Medium	High	Very high
C_4 Benefit/cost	Medium	1.5	1.4	1.1	1.6
C_5 Diversification of financial resources (%)	Medium	5	0	3	4
C_6 Allocation of water to prior usages	Very high	Fairly high	High	High	High
C_7 Range of the negative environmental impacts (negative)	Fairly low	Low	Medium	Fairly low	Fairly high

resettlement of people from some villages. It is a negative attribute. This attribute has been measured by the number of the resettled people.

- *Public participation*: The IBWT projects generate social conflicts in the entire transportation line. If the people have higher participation in planning and organizing the resettlement process and in selling their lands, then higher participation in increased labour opportunities and in designing reduced water rights could make the project successful. It is difficult to define a quantitative index for this attribute, therefore the DM has been asked to rate the alternatives by linguistic variables only.
- *Benefit/cost*: The financial studies give the benefit/cost ratio for each alternative.
- *Diversification of financial resources*: The Governmental budget for the construction of the projects is limited and also uncertain. It is therefore an advantage of a project if it has other financial sources from private companies or foreign funds. The index for measuring this attribute is the percentage of non-governmental funds in all financial resources.
- *Allocation of water to prior usages*: Priorities of water usage for transferred waters to the Zayanderud basin are domestic users (high), industry (fairly high), agriculture (fairly low) and environmental and recreational needs (low).
- *Range of the negative environmental impacts*: According to the environmental impact assessment studies, the DM declared the range of environmental impacts for each project by linguistic variables. This criterion is also negative.

These criteria satisfy the general test imposed by Cox (1999) which evaluates the economic productivity impacts, environmental quality impacts, socio-cultural impacts and benefit distribution considerations.

After completion of the decision matrix, the relative weights of the criteria have been obtained from the DM by a direct method using linguistic variables. We added the word “negative” for criteria, for which higher value means worse performance.

Based on these data different methods were applied.

3.9.1.1 Dominance Method

There is no alternative that dominates all other projects with respect to all of the criteria, so this method does not provide solution for the problem.

3.9.1.2 Sequential Optimization Method

The criterion C_6 has the highest weight. Then using the sequential optimization method, with respect to this criterion A_2, A_3 and A_4 are selected as the best alternatives. Since multiple best choice is found we have to continue the method with the second most important criterion. C_1 has the second highest weight. Then among A_2, A_3 and A_4, alternatives A_2 and A_3 are the best. To compare A_2 and A_3 we use the third most important criterion, C_2. With respect to this criterion, A_2 dominates A_3. So the final ranking becomes $A_2 \succ A_3 \succ A_4 \succ A_1$.

3.9.1.3 ε-Constraint Method

The most important criterion is C_6, and in applying this method the DM should give minimum acceptance levels for positive criteria and maximum acceptance levels for negative criteria. Let's assume that $\varepsilon_1 = \text{High}$, $\varepsilon_2 = 200$, $\varepsilon_3 = \text{Medium}$, $\varepsilon_4 = 1.1$, $\varepsilon_5 = 0$, and $\varepsilon_7 = \text{Medium}$. Then A_4 is omitted since it violates the constraints of ε_2 and ε_7. Based on the most important criterion C_6, the ranking of the alternatives become $A_2 = A_3 \succ A_1$ and the least preferred one is A_4.

3.9.1.4 The SAW, CP and TOPSIS Methods

Before applying the SAW, CP and TOPSIS methods, the original data of Table 3.7 have to be synthesized by the following steps.

Step 1. The linguistic data should be quantified by numerical values. A typical scale is shown in Table 3.8.

Table 3.8 Linguistic variables and equivalent numerical values

Linguistic variables	Number
Very low	0.00
Low	0.20
Fairly low	0.35
Medium	0.50
Fairly high	0.65
High	0.80
Very high	1.00

Table 3.9 Numerical decision matrix

Criteria	Weights	Alternatives			
		A_1	A_2	A_3	A_4
C_1	0.80	0.80	1.00	1.00	0.80
C_2	0.65	0	0	200	4,000
C_3	0.20	0.65	0.50	0.80	1.00
C_4	0.50	1.5	1.4	1.1	1.6
C_5	0.50	5	0	3	4
C_6	1.00	0.65	0.80	0.80	0.80
C_7	0.35	0.20	0.50	0.35	0.65

Table 3.10 Normalized decision matrix

Criteria	Weights	Alternatives			
		A_1	A_2	A_3	A_4
C_1	0.80	0.00	1.00	1.00	0.00
C_2	0.65	1.00	1.00	0.05	0.00
C_3	0.20	0.30	0.00	0.60	1.00
C_4	0.50	0.80	0.60	0.00	1.00
C_5	0.50	1.00	0.00	0.60	0.80
C_6	1.00	0.00	1.00	1.00	1.00
C_7	0.35	1.00	0.33	0.67	0.00

After the linguistic variables are replaced by their equivalent numerical values, a new decision matrix is obtained with numerical values in all positions. It is shown in Table 3.9.

Step 2. The evaluation numbers of the alternatives are normalized into the unit interval [0, 1] by using the linear transformation $\frac{a_{ij}-m_i}{M_i-m_i}$ for positive criteria and $\frac{M_i-a_{ij}}{M_i-m_i}$ for negative criteria, where M_i and m_i are the largest and smallest actual values, respectively, of criterion i. The normalized decision matrix is given in Table 3.10.

Table 3.11 The weighted normalized inputs

Criteria	Alternatives			
	A_1	A_2	A_3	A_4
C_1	0.00	0.80	0.80	0.00
C_2	0.65	0.65	0.03	0.00
C_3	0.06	0.00	0.12	0.20
C_4	0.40	0.30	0.00	0.50
C_5	0.50	0.00	0.30	0.40
C_6	0.00	1.00	1.00	1.00
C_7	0.35	0.12	0.23	0.00

Table 3.12 Result of rankings of the IBWT projects

Alternatives	Ranks			
	SAW and CP, $p = 1$	CP, $p = 2$	CP, $p = 10$	TOPSIS
A_1	4	4	4	4
A_2	1	1	1	2
A_3	2	2	2	1
A_4	3	3	3	3

Step 3. The normalized evaluations of the alternatives are multiplied by the criteria weights. The final evaluation matrix for this case is shown in Table 3.11.

The rankings of the alternatives are presented in Table 3.12 by using various methods.

3.9.2 Urban Water Management

City of Zahedan is the capital of the Sistan and Baluchestan state in South-eastern Iran. The countries of Afghanistan and Pakistan are the neighbors of this province. Zahedan's urban water system faces major challenges. The city had a resident population of about 450,000 in 2000. In addition, uncontrolled immigration of Afghans to Zahedan increased the population of the city by 200,000 in recent years. As a result of the population growth, Zahedan's urban water demand became 46×10^6 m^3/year, but only 24×10^6 m^3/year was supplied in 2001. The mean annual rainfall of Iran is 250 mm but it is less than 80 mm in this city.

There are no permanent rivers near the city except seasonal floods, with the average of 3×10^6 m^3/year. In addition, the groundwater resources have been extracted more than the yield capacity during the last 15 years.

The water distribution network is not in a standard condition since its 47% is older than 30 years, and it has a total water leakage of more than 30%. Due to this inadequate network, most of the people buy potable water from handy carts or trucks. To decrease the water shortage and to meet the water demand in the coming years an MCDA model is developed to find the most appropriate water distribution method. Eight alternatives are considered (Abrishamchi et al. 2005):

Alternative 1. Building a new water supply system for the whole city

Alternative 2. Building a new network for the new part of the city, renewing the existing sanitary water distribution system, extending the small drinking water distribution system with a 30 km long public standpipes within the old part of city, with water vendors, and with water kiosks

Alternative 3. Similar to A_2, but the small drinking water distribution network with public standpipes is extended to over a length of 60 km within the old part of the city

Alternative 4. Building a new drinking water supply system for the whole city and rehabilitation and extension of the existing sanitary water system

Alternative 5. Building a new drinking water distribution system for the new part of the city, rehabilitation and extension of the existing sanitary water distribution network, as well as keeping the existing small drinking water standpipes, water vendors and water kiosks

Alternative 6. Extension of the small drinking water distribution network with 30 km long public standpipes within the city, rehabilitation and extension of the existing sanitary water distribution network with water vendors and water kiosks

Alternative 7. Similar to A_6 but the small drinking water distribution network with public standpipes is extended over a length of 60 km

Table 3.13 Evaluation matrix of the urban water problem (adopted form Abrishamchi et al. 2005)

Criteria		Weights	Alternatives							
			A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
C_1	Total cost (negative)	3	116	53	50	138	76	49	52	52
C_2	Public appraisal	2	VH	H	M	H	M	VL	L	L
C_3	Political impact	2	VH	H	M	H	M	VL	L	M
C_4	Quality of water	1.8	VH	H	M	H	M	M	M	M
C_5	Health impact	2.2	VL	L	M	L	M	VH	H	H
C_6	Flexibility	2.3	VL	M	M	L	VH	M	M	H
C_7	Water demand control	1.7	VL	L	M	M	H	VH	VH	M
C_8	Time of water shortage	1.5	4	11	11	2	5	6	5	5
C_9	Population impact	1.5	VH	H	H	H	M	L	L	M

Alternative 8. Similar to A_7 with the possibility of private service connections

The best water distribution alternative should be decided in an IWRM scope. The first stage was defining the criteria and then some questionnaires were distributed among the various stakeholders of the Zahedan's urban water system and based on the answers nine criteria were adopted to be used in the evaluation of the alternatives (Abrishamchi et al. 2005). The evaluation matrix of the eight alternatives with respect to the nine criteria is presented in Table 3.13. The criteria weights were obtained from the local DM. The five-level linguistic scale (VL, L, M, H and VH) is transformed into the numerical values using the scales (1, 2, 3, 4, 5).

3.9.2.1 Dominance Method

In this case study, there is no alternative that dominates all other projects.

3.9.2.2 Sequential Optimization Method

The criterion C_1 is the most important one. By using the sequential optimization method, A_6 has the lowest cost and will be the most important alternative. The remaining alternatives are ranked based on their evaluation values with respect to C_1. There is a tie

Table 3.14 Ranks of the alternatives with various methods

Alternatives	Ranks			
	SAW and CP, $p = 1$	CP, $p = 2$	CP, $p = 5$	TOPSIS
A_1	7	8	7	7
A_2	2	2	1	2
A_3	6	4	4	5
A_4	8	7	8	8
A_5	4	3	3	3
A_6	5	6	6	6
A_7	3	5	5	4
A_8	1	1	2	1

between A_7 and A_8 therefore their evaluations with respect to the second most important criterion should be used. The final ranking of the alternatives is $A_6 \succ A_3 \succ A_8 \succ A_7 \succ A_2 \succ A_5 \succ A_1 \succ A_4$.

3.9.2.3 ε-Constraint Method

Since C_1 is the most important criterion, the DM should give acceptable levels for all remaining criteria. If he/she selects $\{\varepsilon_2$ to $\varepsilon_7 =$ Medium, $\varepsilon_8 = 3$, and $\varepsilon_9 =$ Medium$\}$ then all alternatives except A_5 and A_8 will be eliminated. Comparing their evaluations with respect to the most important criteria, C_1, alternative A_8 is selected to be the best.

Applying different methods to this problem, the results are shown in Table 3.14. In all cases either alternative A_8 or A_2 is selected as the best choice.

3.10 Discussions

This chapter introduced the most popular methods for solving discrete MCDA problems. Each method is based on a particular way how the DM expresses his/her preferences. Ordinal preferences are used in sequential optimization. The most important criterion is specified with acceptable levels for all other criteria in the case of the ε-constraint method. Importance weights are given for applying simple additive weighting. In the case of distance based methods the DM wants to get criteria values as close as possible to the ideal point or as far as possible from the nadir. The analytic hierarchy process can supply importance weights for the criteria based on pair-wise comparisons and it is also able to provide final decisions. The case studies of inter-basin water transfer and urban water management illustrate the methodology.

Different methods usually give different results. The main reason of this discrepancy is the fact that the different methods are based on different ways of expressing the preferences of the DM, and these preference information are not consistent. The best way of solving this problem is to present all results to the DM, who can then think over his/her preference information and be able to modify them appropriately. Then the computations have to be repeated with the revised preference information, and the results presented again to the DM. This iterative process continues until a satisfactory final decision can be made.

Chapter 4
Solution of Continuous MCDA Problems

4.1 Introduction

In this chapter, we introduce the most popular and most frequently used solution methods for continuous MCDA problems. The goal of these methods is to generate the set of technologically feasible and efficient alternatives and then choosing the most favorable ones.

4.2 Dominance Method

In this method we find the strongly nondominated solutions. And then any method, discussed later in this chapter, can be applied on the set of the strongly nondominated solutions to get the final choice. Let X be the decision space and let $f_i(x), i = 1, 2, \cdots, n$, denote the criteria. A solution $x^* \in X$ is weakly nondominated, if there is no $x \in X$ such that

$$f_i(x) > f_i(x^*)$$

for all i, that is, we cannot increase all criteria values simultaneously. A solution $x^* \in X$ is (strongly) nondominated, if there is no $x \in X$ such that $f_i(x) \geqq f_i(x^*)$ for all i, and with strict inequality for at least one i. That is, no criterion can be improved without worsening another one.

M. Zarghami and F. Szidarovszky, *Multicriteria Analysis*,
DOI 10.1007/978-3-642-17937-2_4,

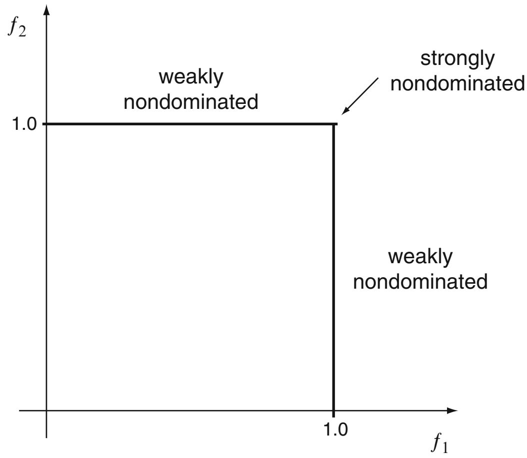

Fig. 4.1 Weakly and strongly nondominated solutions

In Fig. 4.1 the set of weakly nondominated solutions and the unique strongly nondominated solution are shown for the unit square objective space.

Example 4.1. Assume that a combination of three wastewater treatment technologies can be used. If x_1 and x_2 denote the proportion (in percent) of applying technologies 1 and 2, then $1 - x_1 - x_2$ is the proportion of the third technology. These variables clearly must satsify constraints

$$x_1, x_2 \geqslant 0$$

$$x_1 + x_2 \leqslant 1.$$

The feasible decision space is shown in Fig. 4.2.

Assume that there are two major pollutants to be removed from the wastewater. The three technology variants remove 3, 2, 1 mg/m^3 respectively from the first kind of pollutant and 2, 3, 1 mg/m^3 from the second. So the amount of removed pollutant of the first kind is

$$3x_1 + 2x_2 + 1(1 - x_1 - x_2) = 2x_1 + x_2 + 1$$

and that of the second kind is

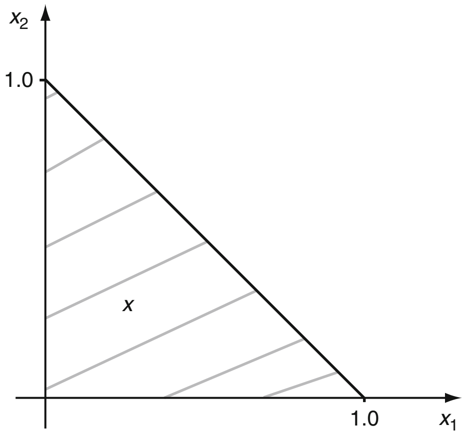

Fig. 4.2 Decision space for Example 4.1

$$2x_1 + 3x_2 + 1(1 - x_1 - x_2) = x_1 + 2x_2 + 1$$

from each unit amount of treated water.

If the DM wants to maximize the total amount of the removed material, then he/she has to solve the following problem with two criteria:

$$\begin{array}{ll} \text{Maximize} & 2x_1 + x_2, x_1 + 2x_2 \\ \text{subject to} & x_1, x_2 \geqslant 0 \\ & x_1 + x_2 \leqslant 1. \end{array}$$

The objective space can be obtained by solving equations

$$2x_1 + x_2 = f_1 \quad \text{and} \quad x_1 + 2x_2 = f_2$$

for x_1 and x_2, and then substituiting the resulting expressions into the constraints of the problem. From the first equation we have

$$x_2 = f_1 - 2x_1$$

and the second equation implies that

$$x_1 + 2f_1 - 4x_1 = f_2$$

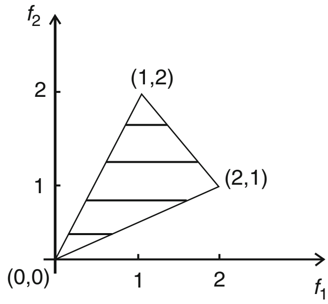

Fig. 4.3 Objective space of Example 4.1

so

$$x_1 = \frac{2f_1 - f_2}{3} \quad \text{and} \quad x_2 = f_1 - \frac{4f_1 - 2f_2}{3} = \frac{2f_2 - f_1}{3}.$$

The constraint $x_1 \geq 0$ gives $f_2 \leq 2f_1$, the constraint $x_2 \geq 0$ implies $f_2 \geq f_1/2$ and the constraint $x_1 + x_2 \leq 1$ can be rewritten as

$$\frac{2f_1 - f_2}{3} + \frac{2f_2 - f_1}{3} \leqslant 1,$$

that is,

$$f_1 + f_2 \leqslant 3.$$

The objective space is shown in Fig. 4.3.
It is the closed triangle with vertices (0, 0), (2, 1) and (1, 2).

4.3 Sequential Optimization

In this method the order of criteria is given as $(f_1 \succ f_2 \succ \cdots \succ f_n)$, where n is the number of the criteria, f_1 is the most important criterion, f_2 is the second most important, etc. and finally, f_n is the least important criterion. In this method the most important criterion is maximized first, then the second most important is maximized under the additional constraint which keeps the first criterion at optimal level. Then the third criterion is optimized with keeping the first two at their optimal levels, and so on. Therefore we solve the MCDA problem by using the following steps:

Step 1.

$$\left.\begin{array}{rr} \text{Maximize} & f_1(x) \\ \text{subject to} & x \in X \end{array}\right\} \text{ optimum value is } f_1^*$$

Step 2.

$$\left.\begin{array}{rr} \text{Maximize} & f_2(x) \\ \text{subject to} & x \in X \\ & f_1(x) = f_1^* \end{array}\right\} \text{ optimum value is } f_2^*$$

$$\vdots$$

Step k.

$$\begin{array}{rc} \text{Maximize} & f_k(x) \\ \text{subject to} & x \in X \\ & f_1(x) = f_1^* \\ & \vdots \\ & f_{k-1}(x) = f_{k-1}^*. \end{array}$$

The process terminates if either a unique optimal solution is found in any of the steps, or we proceeded n steps.

Example 4.2. The problem of Example 4.1 can be solved by sequential optimization. If f_1 is more important than f_2, then the best value of f_1 occurs at the vertex (2, 1) with decision variables

$$x_1 = \frac{2f_1 - f_2}{3} = 1 \quad \text{and} \quad x_2 = \frac{2f_2 - f_1}{3} = 0.$$

If f_2 is more important than f_1, then the best value of f_2 occurs at the vertex (1, 2) with the corresponding decision variables

$$x_1 = \frac{2f_1 - f_2}{3} = 0 \quad \text{and} \quad x_2 = \frac{2f_2 - f_1}{3} = 1.$$

This method always leads to strongly nondominated solutions, but it has a drawback since most points on the nondominated curve can be lost as potential solutions. If there is a unique solution in any earlier step, then the less important objectives are not considered at all. For example, if Step 1 provides a unique optimal solution, then the procedure terminates, so only the most important criterion is considered in the selection of the best alternative. In order to overcome this difficulty we can relax the optimality conditions in each step, so we can get the solution by a modified method, where Step k is the following:

$$\begin{array}{ll} \text{Maximize} & f_k(x) \\ \text{subject to} & x \in X \\ & f_1(x) \geqslant f_1^* - \varepsilon_1 \\ & \vdots \\ & f_{k-1}(x) \geqslant f_{k-1}^* - \varepsilon_{k-1}, \end{array}$$

where $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{k-1}$ are given, DM selected positive constants, and $f_1^*, \ldots, f_{k-1}^*$ are the optimal objective function values obtained in the earlier steps. Any solution obtained by this method is necessarily weakly nondominated, there is no guarantee for strong nondominance. Another approach is when the decision maker assigns minimal acceptable levels for all criteria and at each step of the method these values are used in additional constraints as required lower bounds for the criteria. This is the basic idea of the method introduced in the next subsection.

4.4 The ε-Constraint Method

In this method the most important criterion is selected and minimal acceptable levels are given for all other criteria. If the first criterion is the most important, then the MCDA model is formulated as follows:

$$\begin{array}{ll} \text{Maximize} & f_1(x) \\ \text{subject to} & x \in X \\ & f_2(x) \geqslant \varepsilon_2 \\ & \vdots \\ & f_n(x) \geqslant \varepsilon_n. \end{array}$$

Example 4.3. Consider again the problem of Example 4.1 and assume that $f_1 \succ f_2$ and the minimum acceptable level for f_2 is 3/2. Then the ε-constraint method solves problem

$$\begin{array}{ll} \text{Maximize} & 2x_1 + x_2 \\ \text{subject to} & x_1, x_2 \geqslant 0 \\ & x_1 + x_2 \leqslant 1 \\ & x_1 + 2x_2 \geqslant \frac{3}{2}. \end{array}$$

We can obtain the solution from the decision space and also from the objective space. Figure 4.4 shows the graphical solution in the decision space.

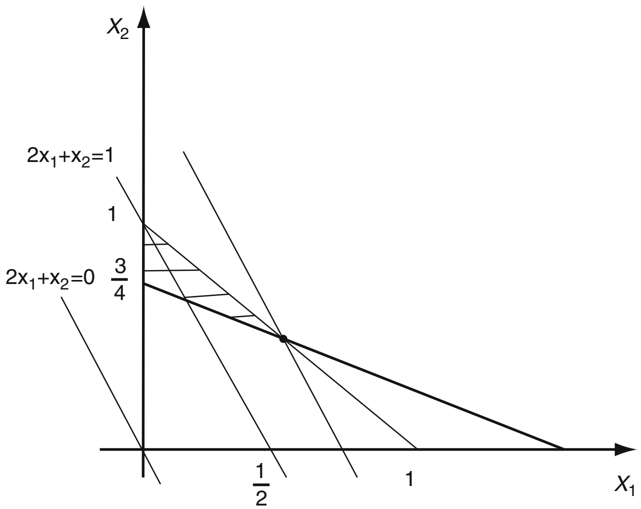

Fig. 4.4 Reduced decision space for Example 4.3

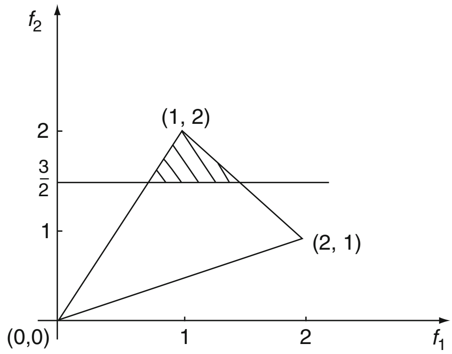

Fig. 4.5 Reduced objective space for Example 4.3

Clearly the largest value of $2x_1 + x_2$ occurs at the intercept of the lines $x_1 + x_2 = 1$ and $x_1 + 2x_2 = 3/2$ which is $x_1 = x_2 = 1/2$.

We can solve the problem by using the objective space. The reduced objective space is shown in Fig. 4.5.

The largest f_1 value occurs at the intercept of the horizontal line $f_2 = 3/2$ and linear segment connecting vertices (1, 2) and (2, 1) which is $f_1 = f_2 = 3/2$. The corresponding decision variables are

$$x_1 = \frac{2f_1 - f_2}{3} = \frac{1}{2} \quad \text{and} \quad x_2 = \frac{2f_2 - f_1}{3} = \frac{1}{2}.$$

4.5 Simple Additive Weighting

In this method the relative preferences of the DM on the set of the criteria are presented by the positive weights, $w_1, w_2, \ldots, w_n$, where we assume that $\sum_{i=1}^{n} w_i = 1$. If the complete set of the criteria has 100% interest of the DM, then these weights show how it is divided among the different criteria. In this case we have to solve the following optimization problem:

$$\begin{array}{ll} \text{Maximize} & w_1 f_1(x) + \cdots + w_n f_n(x) \\ \text{subject to} & x \in X. \end{array}$$

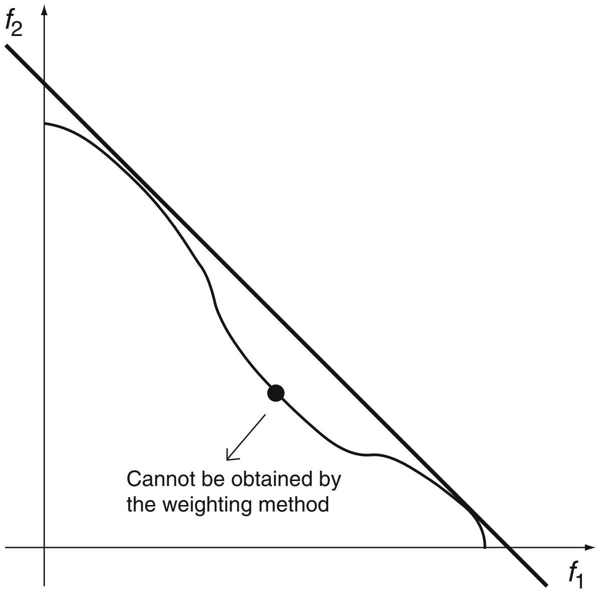

Fig. 4.6 Drawback of the SAW method

The solution of this problem is always strongly nondominated. However, by this method selection, we might lose nondominated solutions as it is shown in Fig. 4.6. We mention here that in the case of a linear problem all strongly nondominated solutions can be obtained as optimal solutions of the above problem with positive weights (see, for example Szidarovszky et al. 1986).

Example 4.4. In our earlier Example 4.1, assume that $w_1 = w_2 = \frac{1}{2}$. Then we have to solve the single-objective optimization problem

$$\begin{array}{ll} \text{Maximize} & \frac{3}{2}x_1 + \frac{3}{2}x_2 \\ \text{subject to} & x_1, x_2 \geqslant 0 \\ & x_1 + x_2 \leqslant 1. \end{array}$$

From Fig. 4.2 it is easy to see that there are infinitely many optimal solutions, all points on the linear segment connecting vertices (1, 0) and (0, 1). Assume next that the first criterion is twice more important than the second one. Then we may select

$w_1 = 2/3$ and $w_2 = 1/3$, so the problem has to be modified as follows:

$$\begin{array}{ll} \text{Maximize} & \frac{2}{3}(2x_1 + x_2) + \frac{1}{3}(x_1 + 2x_2) = \frac{5}{3}x_1 + \frac{4}{3}x_2 \\ \text{subject to} & x_1, x_2 \geqslant 0 \\ & x_1 + x_2 \leqslant 1. \end{array}$$

It is easy to see that the optimal solution is $x_1 = 1$ and $x_2 = 0$.

This method has some difficulties. First, the composite goodness measure, $\sum_{i=1}^{n} w_i f_i(x)$, usually has no meaning by itself, since different criteria mean different things. Second, the solution changes if we change the units in which the criteria are measured. In order to overcome these difficulties we have to transform the criteria to a common measure. If the DM can assign a utility function u_i which characterizes the goodness of the values of criterion i, then this common measure can be selected as the satisfaction level $u_i(f_i)$ of this criterion. If there is no such utility function, then simple normalization can be used. Define for each criterion,

$$M_i = \max\{f_i(x) | x \in X\}$$

and

$$m_i = \min\{f_i(x) | x \in X\},$$

which are the individual maximum and minimum values of f_i. We can then assume that the value M_i gives 100% satisfaction while m_i, as the worst possible outcome, gives 0%. By assuming linear scale we can normalize f_i as follows:

$$\bar{f}_i(x) = \frac{f_i(x) - m_i}{M_i - m_i}.$$

Notice in addition that $\bar{f}_i(x)$ is unitless, so the composite criterion $w_1\bar{f}_1(x) + w_2\bar{f}_2(x) + \cdots + w_n\bar{f}_n(x)$ is also unitless and represents an average satisfaction of the outcomes by selecting x as the decision.

4.6 Distance Based Methods

In this method the DM first selects an ideal point, what he/she considers the best but usually unachievable outcome. In the lack of such information we can compute an ideal point having the individual maximum values of the criteria in its components. Then the DM selects a distance measure ρ of n-dimensional vectors. The properties of the different distances and selection guidelines are given in Chap. 3. So if $f(x)$ is the criteria vector at decision x and f^* is the ideal point, then the DM wants to minimize the distance between f^* and $f(x)$ by solving the following problem:

$$\begin{array}{ll} \text{Minimize} & \rho(f^*, f(x)) \\ \text{subject to} & x \in X \end{array}$$

This concept is shown in Fig. 4.7.

Example 4.5. Consider again the problem of the previous example. The objective space is shown in Fig. 4.3. The minimum and maximum values of both criteria are 0 and 2, respectively. Therefore the ideal point can be selected as (2, 2) and the nadir is (0, 0). Similarly to relations (3.5) and (3.6) the distances of any alternative $x = (x_1, x_2)$ from the ideal point and from the nadir can be given as

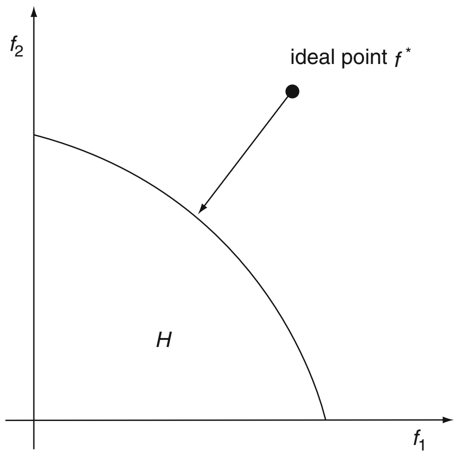

Fig. 4.7 Distance based methods

$$D_x^p = \left\{ \left(w_1 \frac{2 - 2x_1 - x_2}{2} \right)^p + \left(w_2 \frac{2 - x_1 - 2x_2}{2} \right)^p \right\}^{\frac{1}{p}},$$

and

$$d_x^p = \left\{ \left(w_1 \frac{2x_1 + x_2}{2} \right)^p + \left(w_2 \frac{x_1 + 2x_2}{2} \right)^p \right\}^{\frac{1}{p}}.$$

If $p = 1$, then the results coincide with those obtained by using the weighting method. Assume that $w_1 = w_2 = 0.5$, then minimizing D_x^p with $p = 2$ and $p = \infty$ results in the optimal solution $x_1 = x_2 = 0.5$, and maximizing d_x^p with $p = 2$ and $p = \infty$ gives the optimal solutions $x_1 = 1$ and $x_2 = 0$, and $x_1 = 0$ and $x_2 = 1$.

If the ideal point is obtained from the maximum values of the criteria, then distance-based methods with distance minimization are also called Compromise Programming.

Assume that all criteria and all constraints are linear, so the problem has the specific form:

$$\begin{array}{lll} \text{Maximize} & c_i^T x & (i = 1, 2, \ldots, n) \\ \text{subject to} & x \geqslant 0 & \\ & Ax \leqslant b. & \end{array}$$

Let a_i^* and a_{i*} denote the ith component of the ideal point and the nadir, respectively. In the case of $p = 1$ the solution is the same as the one obtained by using the weighting method with normalized criteria,

$$\begin{aligned} \sum_{i=1}^n w_i \frac{a_i^* - c_i^T x}{a_i^* - a_{i*}} &= \sum_{i=1}^n w_i \frac{a_i^* - a_{i*}}{a_i^* - a_{i*}} - \sum_{i=1}^n w_i \frac{c_i^T x - a_{i*}}{a_i^* - a_{i*}} \\ &= 1 - \sum_{i=1}^n w_i \frac{c_i^T x - a_{i*}}{a_i^* - a_{i*}}. \end{aligned}$$

In the case of $p = 2$ a quadratic programming problem has to be solved. We can easily show that the case of $p = \infty$ with distance minimization can also be solved by linear programming. The distance minimizing problem can be written as follows:

$$\begin{array}{ll} \text{Minimize} & \max\left\{ w_i \frac{a_i^* - c_i^T x}{a_i^* - a_{i*}} \right\} \\ \text{subject to} & x \geqslant 0 \\ & Ax \leqslant b. \end{array}$$

Let z denote the objective function, then for all i,

$$w_i \frac{a_i^* - c_i^T x}{a_i^* - a_{i*}} \leqslant z,$$

that is,

$$w_i c_i^T x + z(a_i^* - a_{i*}) \geqslant w_i a_i^*.$$

Therefore by adding this constraint for all values of i to the original set of constraints and minimizing z results in a linear optimization problem for unknowns $\boldsymbol{x}$ and z.

4.7 Other Methods

For a complete review and analysis on the different MCDA methods, the reader is referred to for example, Szidarovszky et al. (1986) and Ehrgott (2005). User-friendly software packages are also available to solve optimization problems. For example, the reader may consider GAMS (Brooke et al. 1996), which can be downloaded from http://www.gams.com/.

4.8 Case Studies

4.8.1 Water Allocation Problem

The Mexican Valley is a part of the watershed of the same name. It includes the metropolitan area of Mexico City, with an area of 9,674 km^2. This region has a variety of economic activities that contribute to about one third of the National Gross Product and concentrates about 20% of the population (19.9 millions) in only 0.5% of the national territory. Water availability in the Mexican Valley was 573 m^3/hab/year by 1950, and it has decreased dramatically to 85 m^3/hab/year by 2003. The distribution of the water in the State of Mexico is managed by the National Water Commission (NWC) that delivers water in bulk to the state water utility. The state water utility has to accept the water, treat it, and distribute it to various counties in the state.

Water supply is produced from six aquifers with 1,681 million cubic meter (MCM)/year, from which 234 MCM/year are provided by surface water, 353 MCM/year are from treated water supply and finally, 623 MCM/year is from water imported from other places. The gap between the growing use of water and the supply becomes larger each year, making water supply and distribution more and more difficult, so a comprehensive modeling effort is needed to find best water distribution strategies.

There are also environmental problems: the groundwater resources are overexploited beyond their capacity. Deforestation has reduced the infiltration rate and recharge of the aquifers. Wastewater generation is another problem since the capacity of the 160 wastewater treatment facilities with total capacity of 3.2 (m^3/s) is not enough to treat the current water discharge (CNA 2003). There is also a mean average precipitation in the region equivalent to 1,800 mm/year, but due to the short rainfall period from June to September, flooding becomes an additional problem due to the lack of infrastructure and management.

There is a need to decide how this limited resource should be allocated among competing users, how to restore or at least to stop the environmental damage caused by both wastewater and aquifer overexploitation and how to increase water supply in the region for future generations (Salazar et al. 2007).

4.8.1.1 Mathematical Model

In order to find an optimal water distribution strategy a linear model is developed and proposed with three water users: agriculture, industry and domestic. Each of them uses surface, ground and treated water.

Let $k = 1, 2, 3$ denote the agriculture, industry and domestic users, respectively. The decision variables for each user are as follows:s_k = surface water available for user k, g_k = groundwater available for user k, t_k = treated water available for user k, s_k^* = imported surface water available for user k, g_k^* = imported groundwater available for user k.

Variables s_k, g_k, t_k refer to local water supplies, and s_k^*,g_k^* refer to water supplies imported from other regions at the Mexican Valley. The objective of each user is to maximize water supply:

$$\text{Maximize } (s_k + g_k + t_k + s_k^* + g_k^*), \tag{4.1}$$

so we have an optimization problem with 3 criteria and 15 decision variables. Each user has its constraints, they are presented next.

Agricultural Use

The irrigated area is 50,000 ha. The main crops are corn, lucerne, oats, barley, and vegetables. Water demand for one season of each year is 474.6 MCM. The farmers have the option to use two seasons, then water demand will become 950 MCM approximately. (Water Utility by 2003 delivered 594 MCM/year for agricultural use.) The supplied water amount must not exceed demand, since we do not want the users to waste water:

$$s_1 + g_1 + t_1 + s_1^* + g_1^* \leqslant 950. \tag{4.2}$$

The minimum water amount required is approximately 475 MCM, therefore we require that

$$s_1 + g_1 + t_1 + s_1^* + g_1^* \geqslant 475. \tag{4.3}$$

Let G = set of crops that can use only groundwater, because of quality requirements, a_i = ratio of crop i in agriculture area, v_i = water need of crop i per ha.

The overall groundwater percentage must not be less than needed by crops which can be irrigated by only groundwater:

$$\frac{g_1 + g_1^*}{s_1 + g_1 + t_1 + s_1^* + g_1^*} \geqslant \frac{\sum\limits_{i \in G} a_i v_i}{V} = \frac{0.00026}{0.04468} = 0.00586,$$

where $V = \sum\limits_{\text{all } i} a_i v_i$ is the total water need per ha. These data were obtained from the annual statistics on the agriculture of Mexico. This constraint can be rewritten in a linear form as

$$\begin{aligned} &0.00586 s_1 - 0.99414 g_1 + 0.00586 t_1 + 0.00586 s_1^* \\ &\quad - 0.99414 g_1^* \leqslant 0. \end{aligned} \tag{4.4}$$

We also require that the treated water percentage cannot be larger than that in the case when all crops in T are irrigated only by treated water:

$$\frac{t_1}{s_1 + g_1 + t_1 + s_1^* + g_1^*} \leqslant \frac{\sum\limits_{i \in T} a_i v_i}{V} = \frac{0.00743}{0.04468} = 0.16629,$$

where $T =$ set of crops which can use treated water. This is equivalent to the linear form

$$\begin{aligned} &- 0.16629 s_1 - 0.16629 g_1 + 0.83371 t_1 - 0.16629 s_1^* \\ &\quad - 0.16629 g_1^* \leqslant 0. \end{aligned} \tag{4.5}$$

The rest of the crops can be irrigated by surface water.

Industrial Use

Since we do not want to supply more water than the industrial needs,

$$s_2 + g_2 + t_2 + s_2^* + g_2^* \leqslant 460. \tag{4.6}$$

According to WNC the minimum required water amount for industry is 177 MCM, so we also require that

$$s_2 + g_2 + t_2 + s_2^* + g_2^* \geqslant 177. \tag{4.7}$$

A minimum proportion of 0.7 of groundwater usage for industry was considered in an agreement with the WNC delivery in 2003. For the same period, industry can also use a maximum amount of 30 MCM/year of treated water which represents a proportion of 0.17. These water quality constraints require that

$$\frac{g_2 + g_2^*}{s_2 + g_2 + t_2 + s_2^* + g_2^*} \geqslant 0.7$$

and

$$\frac{t_2}{s_2 + g_2 + t_2 + s_2^* + g_2^*} \leqslant 0.17.$$

These constraints reflect that more groundwater usage provides better water quality but more treated water makes it worse. These constraints are equivalent to the linear forms

$$0.7s_2 - 0.3g_2 + 0.7t_2 + 0.7s_2^* - 0.3g_2^* \leqslant 0 \tag{4.8}$$

and

$$-0.17s_2 - 0.17g_2 + 0.83t_2 - 0.17s_2^* - 0.17g_2^* \leqslant 0. \tag{4.9}$$

Domestic Use

The average water usage in Mexico City and in the State of Mexico is 360 l/day/person, so for a population of 19.9 million people it gives a maximum water demand of 2,615 MCM. The water availability in the European Countries of 200 l/person/day as a lower bound produces a minimum water demand of 1,452.7 MCM. Hence we have the following constraints:

$$s_3 + g_3 + t_3 + s_3^* + g_3^* \leqslant 2,615 \tag{4.10}$$

and

$$s_3 + g_3 + t_3 + s_3^* + g_3^* \geqslant 1,452.7. \tag{4.11}$$

According to CNA (2003), 127 MCM are used each year for public services which gives a proportion of 0.06. Since treated water can be used for only this purpose, we have the corresponding constraint as

$$\frac{t_3}{s_3 + g_3 + t_3 + s_3^* + g_3^*} \leqslant 0.06,$$

or in linear form

$$-0.06s_3 - 0.06g_3 + 0.94t_3 - 0.06s_3^* - 0.06g_3^* \leqslant 0. \tag{4.12}$$

The additional constraints are

$$s_1 + s_2 + s_3 = 234 \tag{4.13}$$

and

$$g_1 + g_2 + g_3 = 1,684 \tag{4.14}$$

where 234 MCM and 1,684 MCM are the total surface and groundwater supplies in the Mexican Valley respectively. These constraints require that all local water supplies must be used before imported water can be considered. In addition, maximum 453 MCM of surface water can be imported from Rio Cutzamala (127 km away from MV) and also maximum 170 MCM of groundwater from Lerma (15 km away from MV). The corresponding availability constraints are

$$s_1^* + s_2^* + s_3^* \leqslant 453 \tag{4.15}$$

and

$$g_1^* + g_2^* + g_3^* \leqslant 170. \tag{4.16}$$

So we have a linear MCDA problem with 15 variables and 3 criteria. It can be solved by any method outlined earlier in this chapter, for example by using the simple additive weighting method with normalized criteria.

4.8.1.2 Numerical Results

The results of the MCDA approach provide optimal ground, surface and treated water distribution between the three users. We considered three scenarios and repeated the computations for each of them. First we assumed the same aquifer overexploitation, the same bound for treated water percentage for agriculture and the same water supply for domestic use 200–360 l/person/day. When agriculture is the priority, then the domestic users can get a minimum water supply of 200 l/person/day, and at the same time farmers satisfy their water demand for both seasons. On the other hand, when domestic users have the priority with water supply 292.5 l/person/day, then the farmers have available water for only one season.

In Scenario II we also wanted to reduce water shortages in all three sectors, however with decreased water extraction from the aquifer to 1,000 MCM (59%), and increased use of treated water in agriculture by 50%.

In Scenario III we also wanted to optimize water distribution of the three sectors with maintaining aquifer sustainability by extraction of maximum 788 MCM/year and by increasing the use of surface water to 857 MCM/year. This scenario is the best because in the worst case, farmers can get at least 635 MCM to cover one season and also some crops in the second season. In addition, domestic users can get slightly more than their minimum demand. The main environmental advantage of this case is the aquifer sustainability which stops aquifer overexploitation and the consequent subsidence and salinity intrusion.

Table 4.1 displays a summary of the results of the three scenarios where w_1, w_2, w_3 are the weights of the three criteria. It can be observed that a trade off is needed between government investment and the usage of ground and treated water.

Table 4.1 Optimal water distribution results in MCM for the three scenarios and two different priorities

Environmental benefits	Same situation as now Aquifers overexploited more than 100%		Reduction of aquifer overexploitation by 41%, 50% increase in treated water for agriculture		Aquifer sustainability 50% increase in treated water for agriculture, more use of surface water	
Sector/priority	Scenario 1		Scenario 2		Scenario 3	
	Domestic $w_1 = 0.01$, $w_2 = 0.09$, $w_3 = 0.9$	Agriculture $w_1 = 0.9$, $w_2 = 0.09$, $w_3 = 0.01$	Domestic $w_1 = 0.01$, $w_2 = 0.09$, $w_3 = 0.9$	Agriculture $w_1 = 0.9$, $w_2 = 0.09$, $w_3 = 0.01$	Domestic $w_1 = 0.01$, $w_2 = 0.09$, $w_3 = 0.9$	Agriculture $w_1 = 0.9$, $w_2 = 0.09$, $w_3 = 0.01$
Agriculture	474.90	950.00	635.00	689.10	635.00	950.00
Industrial	177.00	460.00	177.00	177.00	177.00	460.00
Domestic	2,125.50	1,454.33	1,481.48	1,452.70	1,918.71	1,501.28
Surface water use	233.90	233.99	234	233.99	857	857
Groundwater	1,684	1,684	999.99	1,000	788	788
Treated water	236.5	323.33	436.47	461.80	462.71	643.27
External sources	623.00	623.00	623.00	623.00	623.00	623.00
Total supply	2,777.40	2,864.33	2,293.47	2,318.80	2,730.71	2,911.27
Government investment	Same as now		Medium		High	

4.8.2 Groundwater Quality Management

Although many of the groundwater optimization problems are solved with single objective optimization, most real-world groundwater management problems constitute a MCDA problem. In this case, an unconfined, heterogeneous aquifer is pumped by three public supply wells located in close proximity to a groundwater contaminant plume (Coppola and Szidarovszky 2001). The problem of maximizing water supply via pumping while minimizing risk of well contamination was considered for a 12-month planning horizon. The basis for this continuous MCDA model was a computational neural network (CNN).

Using simulation results from MODFLOW, the finite-difference groundwater flow model developed by the United States Geological Survey, a CNN was trained to estimate the evolution of groundwater head at points of interest in the aquifer in response to monthly changes in pumping and aerial recharge rates. For the theory and applications of CNN see for example, Wasserman (1989). The CNN architecture was embedded into LINGO, a commercial linear optimization program, to generate the entire set of nondominated solutions. The CNN is linear, so a linear optimization problem with two objectives was obtained. It is well known that all nondominated solutions can be obtained by using the weighting method with appropriate weights. Therefore by systematically varying the weights and solving the resulting linear programming problems, we could generate the entire nondominated solution set.

A hypothetical but realistic isotropic, heterogeneous unconfined aquifer was modeled using MODFLOW. There are three pumping wells, P_1, P_2, P_3, where P_2 is closest to the contaminated area. The governing groundwater flow equation, that represents transient horizontal flow (Dupuit assumption) in an anisotropic, heterogeneous unconfined aquifer system, can be written as partial differential equation

$$\nabla(kh \cdot \nabla h) + Q\delta[(x - x^*)(y - y^*)] - R(x, y) = S_y \partial h/\partial t \tag{4.17}$$

where k is the hydraulic conductivity vector, h is the hydraulic head, δ is the Dirac delta function, Q is a pumpage vector, x^*, y^* is a set of well locations, R is a vector of natural recharge, and S_y is the specific yield. Solving (4.17) with a large set of pumping rates and aereal recharge rates for the head values at the points of interest the requested data set was obtained to train a CNN, which then was used to replace the physical model (4.17).

The MCDA problem of maximizing water supply via pumping while minimizing risk of well contamination was considered for a 12-month planning horizon discretized into monthly stress periods. The two criteria are conflicting because increased pumping rates induce hydraulic gradient changes along the contaminant boundary that increase the risk of well contamination. The problem is further complicated by the introduction of multiple time periods.

Supply was defined as the actual amount of groundwater pumped out by the three wells over the 12-month period. Risk was measured as the annual sum of the head differences between the downgradient and upgradient nodes (under non-pumping conditions) at the three risk pairs. Positive values indicate some risk since “upgradient” nodes would overall have a lower groundwater elevation than the “downgradient” nodes, resulting in a general gradient reversal.

Because supply and risk were quantified with different physical dimensions (volume/time versus length), they had to be normalized, which was done in such a way that in the normalized objectives, unit value corresponds to the best and zero value to the worst possibility. The nondominated set was determined by using the weighting method with varying weights:

$$\text{Maximize } [\alpha \cdot \text{Risk Normalized} + (1-\alpha) \cdot \text{Supply Normalized}] \tag{4.18}$$

subject to

$$Risk = \sum_{i=1}^{12} \left\{ (h_{6,i} - h_{5,i}) + (h_{8,i} - h_{7,i}) + (h_{10,i} - h_{9,i}) \right\}, \tag{4.19}$$

$$\text{Supply} = \sum_{k=1}^{3} \sum_{i=1}^{12} P_{k,i}. \tag{4.20}$$

State variables $h_{5,i}$ through $h_{10,i}$ are the head values at the three risk pairs (six nodes), with subscript i corresponding to months 1–12. There are 36 decision variables, denoted by $P_{k,i}$ corresponding to the monthly pumping rates, in m^3/min, for each of the three wells (e.g., $P_{1,2}$ is the monthly pumping rate of well 1 in February).

In the objective function, $\alpha = 1$ considers only risk, $\alpha = 0$ only supply, and values between 0 and 1 represents some tradeoff between risk and supply. For example, the case of $\alpha = 0.5$ weights both criteria equally. By systematically varying the value of α, the set of all non-dominated solutions was identified. In this two-criterion case, the nondominated set is a 2-dimensional graphical representation of the trade-off between supply and risk which serves as a basis for applying any MCDA method. In cases where the decision makers are unable to supply importance weights, we have to repeat the computations for a large set of systematically selected weights and present all answers to the decision makers who can then assess their priorities based on the results. The minimum acceptable weight for risk should be 0.5, that is, minimizing risk of contamination is at least as important as supply.

Figure 4.8 depicts the non-normalized nondominated set derived by the CNN and verified with MODFLOW for risk versus supply, where annual risk is reported in meters (m) and annual supply in cubic meters (m^3). Clearly the risk of well contamination increases as supply increases because larger groundwater withdrawals dictate higher pumping rates, so it further extends the hydraulic influence into the contamination zone.

Tables 4.2 and 4.3 display the results of the MCDA based on the two nondominated frontiers obtained from the use of CNN

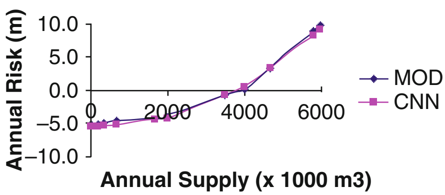

Fig. 4.8 Non-normalized nondominated set

Table 4.2 Results from CNN-derived non-dominated frontier

Supply weight $1 - \alpha$	Risk weight α	Normalized supply	Normalized risk	P_1 m^3/year	P_2 m^3/year	P_3 m^3/year	Total supply m^3/year	Total risk *m*
0	1	0	0.998	0	0	0	0	−5.5
0.025	0.975	0	0.998	0	0	0	0	−5.5
0.05	0.95	0	0.998	0	0	0	0	−5.5
0.075	0.925	0	0.998	0	0	0	0	−5.5
0.1	0.9	0	0.998	0	0	0	0	−5.5
0.125	0.875	0	0.998	0	0	0	0	−5.5
0.15	0.85	0	0.998	0	0	0	0	−5.5
0.175	0.825	0	0.998	0	0	0	0	−5.5
0.2	0.8	0.023	0.992	0	0	136,145	136,145	−5.4
0.225	0.775	0.023	0.992	0	0	136,145	136,145	−5.4
0.25	0.75	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.275	0.725	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.3	0.7	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.325	0.675	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.35	0.65	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.375	0.625	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.4	0.6	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.425	0.575	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.45	0.55	0.367	0.879	202,875	0	1,988,966	2,191,841	−3.7
0.475	0.525	0.367	0.679	202,875	0	1,988,966	2,191,841	−3.7
0.5	0.5	0.586	0.679	1,509,483	0	1,988,966	3,498,449	−0.8

Table 4.3 Results from MODFLOW-derived nondominated frontier

Supply weight $1-\alpha$	Risk weight α	Normalized supply	Normalized risk	P_1 m^3/year	P_2 m^3/year	P_3 m^3/year	Total supply m^3/year	Total risk m
0	1	0	0.991	0	0	0	0	−5.2
0.025	0.975	0	0.991	0	0	0	0	−5.2
0.05	0.95	0	0.991	0	0	0	0	−5.2
0.075	0.925	0	0.991	0	0	0	0	−5.2
0.1	0.9	0	0.991	0	0	0	0	−5.2
0.125	0.875	0	0.991	0	0	0	0	−5.2
0.15	0.85	0	0.991	0	0	0	0	−5.2
0.175	0.825	0.112	0.969	0	0	668,944	668,944	−4.7
0.2	0.8	0.112	0.969	0	0	668,944	668,944	−4.7
0.225	0.775	0.112	0.969	0	0	668,944	668,944	−4.7
0.25	0.75	0.338	0.902	29,834	0	1,988,966	2,018,800	−3.7
0.275	0.725	0.338	0.902	29,834	0	1,988,966	2,018,800	−3.7
0.3	0.7	0.338	0.902	29,834	0	1,988,966	2,018,800	−3.7
0.325	0.675	0.338	0.902	29,834	0	1,988,966	2,018,800	−3.7
0.35	0.65	0.338	0.902	29,834	0	1,988,966	2,018,800	−3.7
0.375	0.625	0.338	0.902	29,834	0	1,988,966	2,018,800	−3.7
0.4	0.6	0.338	0.902	29,834	0	1,988,966	2,018,800	−3.7
0.425	0.575	0.338	0.902	29,834	0	1,988,966	2,018,800	−3.7
0.45	0.55	0.568	0.721	1,402,221	0	1,988,966	3,391,187	−1.0
0.475	0.525	0.665	0.639	1,977,127	0	1,988,966	3,966,093	0.2
0.5	0.5	0.665	0.639	1,977,127	0	1,988,966	3,966,093	0.2

and the MODFLOW software. These results are explained in more detail in Coppola and Szidarovszky (2001). The trade-off policies identified from the CNN derived frontier usually result in a smaller supply than the MODFLOW verified frontier, but this is not always the case as it is demonstrated by supply weights ranging from 0.25 to 0.425. Overall, the results generated from the two frontiers compare favorably. The results, regardless of frontier, show pumping policies that excluded the use of well P_2. This pumping well, located closest to the contamination zone, is the most vulnerable to be contaminated. In addition, because of its close proximity to the hypothetical boundary, it has the greatest potential for increasing risk to the other wells as quantified by the three risk pairs. Keeping well P_2 turned off is consistent with the decision to select risk weight at least equal to supply weight. By comparison, if wells P_1 and P_3 pump at their maximum rates each month, the annual risk is close to 0 m. As such, the head differences on average at the three risk pairs over the 12 months are close to zero. At this value, the two wells are around the upper limit of withdrawing groundwater without reversing the hydraulic gradient and potentially spreading pollution.

4.9 Discussions

This chapter introduced the most frequently applied methods for solving continuous MCDA problems. As in the case of discrete problems each method is based on a particular way of expressing the preferences of the DM. A water allocation problem and a groundwater quality management problem illustrated the methodology. Very often different results are obtained by using different methods. The discrepancy can be eliminated by the same interactive repeated process that was explained in the last section of the previous chapter.

Chapter 5
Social Choice Methods

5.1 Introduction

Decision making in water resources and environmental management is usually based on the preferences of more than one DM. If the evaluation of the alternatives with respect to all DMs can be quantified, then the suitable MCDA formulation of the problem can be used as shown in the former chapters. However, in many cases the human preferences are difficult to be quantified. In most applications such opinions are quantified by using some subjective measures, say for example, the worst possibility is indicated by 0, the best by 10, and all values are characterized by a number on the scale [0, 10]. Even in the case of quantifiable criteria, the uncertainty in the objective functions makes any approach questionable. In most cases, many researchers therefore raise serious questions about the objectivity of these methods.

In this chapter, an alternative methodology is introduced, in which the particular preference values do not need to be precisely known. The method is based on only the ranking of the alternatives with respect to each DM. This methodology is known as social choice. In this chapter, a brief overview of the most popular methods in finding the best alternative will be given. Two interesting case studies will be then presented in water and environmental management.

M. Zarghami and F. Szidarovszky, *Multicriteria Analysis*,
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5.2 Social Choice Methods

Let n be the number of the DMs and m the number of alternatives. The exact values of the evaluations of the alternatives by the DMs are assumed to be unknown. The most appropriate method is social choice, which needs only the relative rankings of the alternatives instead of using particular values for the evaluations. In the decision matrix of these problems, the a_{ij} element represents the ranking of the jth alternative from the viewpoint of the ith DM. If $a_{ij} = 1$, then the best alternative from the viewpoint of the ith DM is the jth alternative, and if it is 2, then the jth alternative is the second best, and so on. We will next introduce the most popular social choice methods.

5.2.1 Plurality Voting

This method selects an alternative that is considered the best by most DMs. Define:

$$f(a_{ij}) = \begin{cases} 1 & \text{if } a_{ij} = 1 \\ 0 & \text{if otherwise,} \end{cases} \tag{5.1}$$

and for all alternatives compute

$$A_j = \sum_{i=1}^{n} f(a_{ij}). \tag{5.2}$$

Notice that A_j show that how many times the jth alternative is selected as the first choice by the DMs. Then alternative $j*$ with the maximum value of A_j is selected as the social choice:

$$A_{j^*} = \max\{A_j\}. \tag{5.3}$$

Table 5.1 Ranking of the different scenarios with respect to the preferences of the five DMs

Decision makers	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
DM_1	1	3	2	5	4
DM_2	1	4	5	3	2
DM_3	5	3	1	2	4
DM_4	5	2	4	1	3
DM_5	4	2	5	1	3

Example 5.1. The Caspian Sea is shared by five neighboring countries. There are also five different scenarios to divide the benefits of the sea among the stakeholders. The participant countries have ranked the scenarios as shown in Table 5.1. For the complete review on the problem, the reader may refer to Sheikhmohammady et al. (2010).

By using the plurality voting method we have $A_1 = 2$, $A_2 = 0$, $A_3 = 1$, $A_4 = 2$ and $A_5 = 0$, so both Scenarios 1 and 4 are considered the best.

If the DMs have different relative powers, then in (5.2) the quantity $f(a_{ij})$ has to be multiplied by the power of DM_i.

5.2.2 Borda Count

This method selects the decision alternative according to the total score of each alternative with respect to the preferences of the various DMs. If a_{ij} denotes the elements of the decision matrix, then the score of each alternative B_j is given as

$$B_j = \sum_{i=1}^{n} g(a_{ij}), \tag{5.4}$$

with

$$g(a_{ij}) = m - a_{ij}. \tag{5.5}$$

Notice that n is again the number of DMs and m is the number of alternatives. The alternative j^* with the maximum value of B_j is the superior choice:

$$B_{j^*} = \max\{B_j\}. \tag{5.6}$$

Note that for each alternative we have

$$B_j = \sum_{i=1}^{n} g(a_{ij}) = \sum_{i=1}^{n} (m - a_{ij}) = nm - \sum_{i=1}^{n} a_{ij}. \tag{5.7}$$

Therefore, a modified version of the Borda count is to add the a_{ij} values for $i = 1, 2, \ldots, n$ with each fixed value of j and then select the alternative with the lowest sum.

Example 5.2. The problem of Example 5.1 can also be solved by the Borda count. Based on (5.7), $B_1 = 9$, $B_2 = 11$, $B_3 = 8$, $B_4 = 13$ and $B_5 = 9$, so Scenario 4 is the most preferred choice for the group of DMs.

If the DMs have different relative powers, then each a_{ij} matrix element has to be multiplied by the power of DM_i.

5.2.3 Hare System (Successive Deletion)

This method is based on the successive deletion of less attractive alternatives until the most preferred alternative is found. In this method, the alternative j_{min} with the smallest value of A_j is deleted from the decision matrix. The values of A_j were already defined in (5.2). Then the decision matrix is modified by using the following formula:

$$a_{ij}^{\text{new}} = \begin{cases} a_{ij} & \text{if } a_{ij} < a_{ij_{\min}} \\ a_{ij} - 1 & \text{otherwise.} \end{cases} \tag{5.8}$$

This process must be repeated until the superior alternative is recognized. If at any stage, the score of one alternative is more

Table 5.2 Revised decision matrix in Step 1 of Example 5.3

	Scenario 1	Scenario 3	Scenario 4
DM_1	1	2	3
DM_2	1	3	2
DM_3	3	1	2
DM_4	3	2	1
DM_5	2	3	1

Table 5.3 Revised decision matrix in Step 2 of Example 5.3

	Scenario 1	Scenario 4
DM_1	1	2
DM_2	1	2
DM_3	2	1
DM_4	2	1
DM_5	2	1

than half of the number of DMs (i.e. $A_j > \frac{n}{2}$), then the process terminates and this alternative becomes the social choice.

Example 5.3. In the case of Example 5.1, we had $A_1 = 2, A_2 = 0$, $A_3 = 1$, $A_4 = 2$ and $A_5 = 0$. Then both of Scenarios 2 and 5 should be eliminated from the decision matrix. Since we used two eliminations in one step, we use relation (5.8) twice, first in eliminating Scenario 2 and then in eliminating Scenario 5. The revised matrix is shown in Table 5.2.

The new A_j values, based on Table 5.2, are $A_1 = 2$, $A_3 = 1$, and $A_4 = 2$. Since Scenario 3 has the lowest score, in Step 2 it has to be eliminated. The new decision matrix is shown in Table 5.3.

The new scores are $A_1 = 2$ and $A_4 = 3$. Then Scenario 4 becomes the social choice since it satisfies the relation $A_4 > \frac{5}{2}$

If the DMs have different relative powers, then in computing the A_j values by relation (5.2), each $f(a_{ij})$ value has to be multiplied by the power of DM_i.

5.2.4 *Dictatorship*

In this method one of the DMs is selected to be the dictator and then the most preferred alternative by him/her would be considered the superior one. In other words, if the i^*th DM is the dictator, then alternative j^* will be the choice if and only if

$$a_{i^*j^*} = 1. \tag{5.9}$$

Example 5.4. In the problem of Example 5.1, assume DM_2 has the major political authority at the region then he/she is the dictator in this problem. Then Scenario 1 becomes the choice of the group.

5.2.5 Pairwise Comparisons

In this method, we first calculate for each pair (a, b) of alternatives, the number of DMs who prefer a to b. Let $N(a, b)$ denote this number, then we consider alternative a to be overall more preferred than b if

$$N(a,b)>N(b,a), \tag{5.10}$$

or alternatively,

$$a \succ b \Leftrightarrow N(a,b)>\frac{n}{2}. \tag{5.11}$$

Notice that in the case, when $N(a, b) = N(b, a)$ with some alternatives a and b, then we cannot consider any of them to be more preferred than the other. Based on these values we have one of the following options:

1. We compute the $N(a, b)$ values for all pairs of alternatives. An alternative is non-dominated if there is no better alternative. In the social choice literature this alternative is called the Condorcet winner.
2. We suppose that a comparison agenda is given, in which the alternatives are ordered as $a_1, a_2, \ldots$. First, we compare a_1 with a_2 based on this schedule, and then we compare the former

winner (in the case of tie, both) with a_3 and so on. At the end of each comparison, the losing alternative is eliminated. This process continues until the last alternatives are compared, when the social choice is found as the last winner. Since the comparisons are sequentially performed according to the schedule, we do not need to compare all alternatives with all others.

Example 5.5. We calculate the $N(a, b)$ values for the problem introduced in Example 5.1. The results are shown in Table 5.4. Notice that we have to compute these values only for $a < b$ and then may use the relation $N(b, a) = n - N(a, b)$ to complete the lower part of the matrix.

First we have $N(\text{Sc.}1, \text{Sc.}2) = 2 < \frac{5}{2}$, so the second one is better and we should next compare the winner (Scenario 2) to Scenario 3. Since $N(\text{Sc.}2, \text{Sc.}3) = 3 > \frac{5}{2}$, Scenario 2 is the winner again and it should be compared to the fourth one. Notice that $N(\text{Sc.}2, \text{Sc.}4) = 1 < \frac{5}{2}$. Therefore the fourth scenario is the winner and it should be compared finally to the fifth one: $N(\text{Sc.}4, \text{Sc.}5) = 3 > \frac{5}{2}$ therefore Scenario 4 is the final winner and it is the social choice.

If the relative powers of the DMs are different then we should revise the $N(a, b)$ values as follows:

$N(a, b) =$ Sum of the relative powers of the DMs, who prefer alternative a to alternative b.

By using these modified $N(a, b)$ values, the superior alternative can be selected in the same way as before. The consideration of the different powers of the DMs is an essential feature in solving water resources and environmental management problems.

Table 5.4 $N(a, b)$ values for Example 5.1

	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Scenario 1	2	3	2	2
Scenario 2	–	3	1	4
Scenario 3	–	–	2	2
Scenario 4	–	–	–	3

5.3 Case Studies

5.3.1 Forest Treatment Problem

This case study finds the best alternative among four different forest treatment options in Northern Arizona, USA. There are six stakholders which presented their ordinal preferences on the alternatives. The study of d'Angelo et al. (1998) describes the details of the alternatives and also the stakeholders who are involved in the problem. The decision matrix is shown in Table 5.5. The alternatives can be briefly described as follows. Each of them was applied on a designated area in order to assess and compare their consequences:

A_1: On a completely clear cut watershed all merchantable poles and saw commercial wood felled. All slash and debris were machine windrowed to conserve the watershed. The woodland tree species were allowed to sprout and grow after the clearing treatment.

A_2: The watershed treated by uniform thinning and some parts of the basal were removed, leaving even-aged groups of trees. All slash was windrowed.

A_3: An irregular strip cut was applied to the third watershed. All merchantable wood was removed within irregular strips and the remaining non-merchantable trees felled. The overall treatment resulted in reduction in the basal area on the watershed. Slash was piled and burned in the cleared strips.

A_4: A part of the watershed served as a control against which the other three treatments were evaluated. The characteristics and

Table 5.5 The decision matrix of Case Study 5.3.1 (revised after d'Angelo et al. 1998)

Decision makers	A_1: clear cut	A_2: uniform thinning	A_3: strip cut and thinning	A_4: control
DM_1: water users	1	2	3	4
DM_2: wildlife advocates	4	2	1	3
DM_3: livestock producers	1	2	3	4
DM_4: wood producers	3	4	2	1
DM_5: environmentalists	4	3	2	1
DM_6: managers	4	2	3	1

resources of this watershed are investigative of what might be gained in the future if only custodial management is applied.

In this case, the power weights of the stakeholders are assumed to be identical.

5.3.1.1 Plurality Voting

Based on (5.2) we have $A_1 = 2, A_2 = 0, A_3 = 1, A_4 = 3$ implying that alternative A_4 (control) with the highest score is the social choice.

5.3.1.2 Borda Count

The Borda counts of the alternatives, based on (5.7) are $B_1 = 7, B_2 = 9, B_3 = 10, B_4 = 10$. Therefore both strip cut and thinning and control with the highest Borda scores are the choices.

5.3.1.3 Hare System

In this case, alternative 2 has the lowest A_j value, so it is eliminated first from the decision table. The resulting matrix is shown in Table 5.6.

Here the third alternative has the smallest A_j value among $A_1 = 2, A_3 = 1, A_4 = 3$, so it is next eliminated. The reduced decision matrix is shown in Table 5.7.

Since the fourth alternative, control has $A_4 = 4 > \frac{6}{2} = 3$, it is the social choice.

Table 5.6 Revised decision matrix in Step 1 of Case Study 5.3.1

Decision makers	A_1	A_3	A_4
DM_1	1	2	3
DM_2	3	1	2
DM_3	1	2	3
DM_4	3	2	1
DM_5	3	2	1
DM_6	3	2	1

Table 5.7 Revised decision matrix in Step 2 of Case Study 5.3.1

Decision makers	A_1	A_4
DM_1	1	2
DM_2	2	1
DM_3	1	2
DM_4	2	1
DM_5	2	1
DM_6	2	1

Table 5.8 The $N(a, b)$ values in the pairwise comparison of the alternatives

$N(a, b)$	A_1: clear cut	A_2: uniform thinning	A_3: strip cut and thinning	A_4: control
A_1: clear cut	–	3	2	2
A_2: uniform thinning	3	–	3	3
A_3: strip cut and thinning	4	3	–	3
A_4: control	4	3	3	–

5.3.1.4 Dictatorship

In this case the best alternative depends on the preference of only the dictator. For example if DM_6 (Managers) is the dictator then based on Table 5.5, the fourth alternative (Control) becomes the social choice.

5.3.1.5 Pairwise Comparisons

Pairwise comparisons will be now applied for the problem given in Table 5.5. First, the quantities $N(a, b)$ are determined for each ordered pair (a, b) of alternatives. For example, for the pair (1, 2) the water users, livestock producers and wood producers prefer alternative A_1 to A_2 and others prefer A_2 to A_1, therefore we have $N(1, 2) = 3$. That is, there is a tie between these alternatives, which means that no overall preference among them is found. The $N(a, b)$ values for all pairs of alternatives are summarized in Table 5.8.

Based on the values shown in Table 5.8, the preference graph is sketched in Fig. 5.1. Based on this graph, we can only conclude that clear cut should be eliminated and no comparison of the other three alternatives is found.

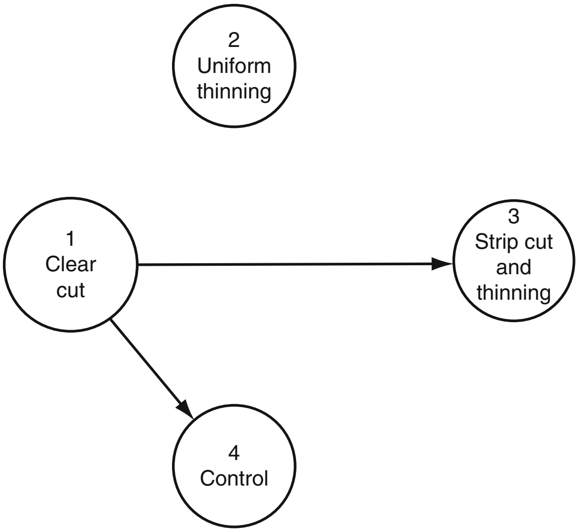

Fig. 5.1 Preference graph illustrating pairwise comparisons in Case Study 5.3.1

5.3.2 Ranking Water Resources Projects

In this case study, we will examine ten water resources development projects which are proposed to be constructed in a central watershed of Iran. The decision committee includes six representatives from the stakeholder organizations and a moderator from the government. In order to use MCDM methodology the stakeholders should be questioned concerning their preferences on these alternatives. However, there are several difficulties in obtaining objective function values for the different alternatives:

- The numerical evaluations of the alternatives with respect to some criteria are not available.
- Even in cases when they are available, they are very subjective and uncertain and the decision making process will become very complicated and the results unreliable.
- The stakeholders do not want to reveal their precise evaluation values especially in conflict situations.

Therefore it will be easier and the only possibility to obtain the relative rankings of the alternatives. The results are shown in

Table 5.9 Ranking of the alternatives with different stakeholders

Decision makers	Power of DMs	Alternatives									
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
DM_1	Medium	4	8	3	10	2	6	7	9	5	1
DM_2	High	5	9	3	8	4	2	1	10	6	7
DM_3	Slightly high	4	10	2	9	3	7	6	8	5	1
DM_4	Slightly low	1	7	2	9	3	8	6	10	5	4
DM_5	Low	7	8	5	10	3	4	6	9	2	1
DM_6	High	4	9	2	10	3	8	6	7	5	1

Table 5.10 Qualitative values and their numerical equivalents

Qualitative value	Numerical value
Very low	0.00
Low	0.20
Slightly low	0.35
Medium	0.50
Slightly high	0.65
High	0.80
Very high	1.00

Table 5.9. The relative powers of the six DMs are different in the viewpoint of the group moderator. However due to the intensive conflict among the stakeholder, the moderator did not reveal these relative power values. The linguistic weights shown in Table 5.9 are obtained from an independent senior expert who is familiar with this case.

Now, we can find the most appropriate alternative by using the various social choice methods. Notice that instead of verbal descriptions of the power values we had to use the scale shown in Table 5.10 to quantify the qualitative values.

5.3.2.1 Plurality Voting

The plurality voting method is applied for the data of Table 5.9 and the scores of the different alternatives are shown in Table 5.11. Based on these results, the last alternative has the largest A_j value. Notice that in the case of using power index p_i for DM_i, (5.2) should be revised as

Table 5.11 Scores of alternatives by using plurality voting in Case Study 5.3.2

Alternatives	A_j
A_1	0.35
A_2	0.00
A_3	0.00
A_4	0.00
A_5	0.00
A_6	0.00
A_7	0.80
A_8	0.00
A_9	0.00
$\mathbf{A_{10}}$	**2.15**

Table 5.12 Results of using Borda count in Case Study 5.3.2

Alternatives	B_j
A_1	19.45
A_2	4.05
$\mathbf{A_3}$	**24.50**
A_4	2.60
A_5	22.80
A_6	13.85
A_7	16.70
A_8	4.40
A_9	16.30
A_{10}	23.85

$$A_j = \sum_{i=1}^{n} f(a_{ij})p_i. \tag{5.12}$$

So the last alternative is the social choice, which is indicated with bold value in the table.

5.3.2.2 Borda Count

The Borda counts for this case study are shown in Table 5.12. Based on these results, the third alternative has the highest Borda count, which is indicated with bold value in the table. Therefore it is the social choice. Notice again that due to the usage of different powers, (5.7) is revised as

$$B_j = \sum_{i=1}^{n} p_i(m - a_{ij}) = m\sum_{i=1}^{n} p_i - \sum_{i=1}^{n} p_i a_{ij}. \tag{5.13}$$

5.3.2.3 Hare System

Notice that the total weights of the DMs is 3.3, and A_{10} has more than half of this value (2.15 > 1.65). Therefore it is the social choice.

5.3.2.4 Dictatorship

The result of using dictatorship depends on the choice of the dictator. Since DM_2 and DM_6 have the highest powers in the committee, one of them has to be selected as the dictator. For example if the dictator is DM_2, then alternative A_7 is the choice. If DM_6 is considered as the dictator, then alternative A_{10} will be the choice.

5.3.2.5 Pairwise Comparisons

First, we calculate the $N(a, b)$ values. In the case when the DMs have different powers, instead of using $n/2$ as a winning threshold, we should use $\frac{1}{2}\sum_{i=1}^{n} p_i$. In this case the sum of the DMs' powers is 3.3 and therefore the winning alternative should have larger score than $3.3/2 = 1.65$, and if the score is 1.65, then there is a tie between the two alternatives. Assume that the pairwise comparisons are made in the order A_1, A_2,... and the consecutive comparisons are the followings:

$N(A_1,A_2) = 0.5 + 0.8 + 0.65 + 0.35 + 0.2 + 0.8 = 3.3 > 1.65$, so A_1 is the winner;

$N(A_1,A_3) = 0.35 < 1.65$, so A_3 is the winner;

$N(A_3,A_4) = 0.5 + 0.8 + 0.65 + 0.35 + 0.2 + 0.8 = 3.3 > 1.65$, so A_3 is the winner;

$N(A_3,A_5) = 0.8 + 0.65 + 0.35 + 0.8 = 2.6 > 1.65$, so A_3 is the winner;

$N(A_3,A_6) = 0.5 + 0.65 + 0.35 + 0.8 = 2.3 > 1.65$, so A_3 is the winner;

$N(A_3,A_7) = 0.5 + 0.65 + 0.35 + 0.2 + 0.8 = 2.5 > 1.65$, so A_3 is the winner;

$N(\text{A}_3, \text{A}_8) =$
$0.5 + 0.8 + 0.65 + 0.35 + 0.2 + 0.8 = 3.3 > 1.65$, so A_3 is the winner;

$N(\text{A}_3, \text{A}_9) = 0.5 + 0.8 + 0.65 + 0.35 + 0.8 = 3.1 > 1.65$, so A_3 is still the winner;

$N(\text{A}_3, \text{A}_{10}) = 0.8 + 0.35 = 1.15 < 1.65$, so A_{10} is the final winner.

Thus, alternative A_{10} is finally the social choice.

5.4 Consensus on the Results

In a group decision making problem it is very important to evaluate the consensus measure among the DMs. Consensus is a measure to quantify the group agreement on the final social choice. A popular way to calculate the consensus measure is by using the Spearman rank correlation coefficient. It is a non-parametric method, which does not depend on the distribution of the uncertain preferences. It calculates the correlation coefficient among the ranking of alternatives declared by DM_i in comparison to the ranking obtained by the group as

$$r_i = 1 - \frac{6 \sum_{j=1}^{m} d_{ij}^2}{m(m^2 - 1)}, \tag{5.14}$$

where d_{ij} is the distance between the rank of alternative j declared by DM_i and its rank calculated by a social choice method, and m is again the number of alternatives. As an illustration we use the results of the previous subsection and compute the correlation coefficients between the DM's rankings (Table 5.9) and the results of applying the Borda count (Table 5.12). The reason of using the Borda counts is that it represents a complete ranking of the alternatives in comparison with the other four methods. The detail of calculating the d_{1j} values for DM_1 is shown in Table 5.13.

Table 5.13 Distances between the ranking obtained by DM_1 and the Borda counts in Case Study 5.3.2

Ranks	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
DM_1	4	8	3	10	2	6	7	9	5	1
Borda count	4	9	1	10	3	7	5	8	6	2
d_{1j}	0	−1	2	0	−1	−1	2	1	−1	−1
d_{1j}^2	0	1	4	0	1	1	4	1	1	1

Table 5.14 Consensus measures of the DMs in Case Study 5.3.2

Decision makers	Correlation coefficients	Significance (two-tailed)
DM_1	0.9152	0.0015
DM_2	0.5152	0.0306
DM_3	0.9636	0.0010
DM_4	0.8424	0.0029
DM_5	0.6727	0.0109
DM_6	0.9636	0.0010

Now based on (5.14) the r_1 value is obtained as

$$r_1 = 1 - \frac{6(0+1+4+0+1+1+4+1+1+1)}{10(100-1)} = 0.9152.$$

Assuming Normal distribution for r_1, its standardized value, z, is

$$z = \frac{r-\mu}{\frac{1}{\sqrt{m-1}}} = \frac{0.9152-0}{\frac{1}{\sqrt{10-1}}} = 2.7456.$$

Using the table of the normal distribution (Appendix), the cumulative probability value is 0.9970, which results in the two-tailed error of 0.0015. The correlation coefficient and the significance levels (p-value) for the consensus measures of other DMs are determined and all are shown in Table 5.14. A larger value of the Spearman's coefficient indicates larger level of consensus.

The consensus of a group is usually under criticism, if its measure is less than ~0.60 (Messier 1983; Ashton 1992). Fortunately the results of the consensus levels in Table 5.14 are greater than ~0.60 except in the case of DM_2. Therefore the moderator should have discussions and additional interview with DM_2 to revise his/her rankings and then the consensus measure should be calculated again until all of the DMs are satisfied with the results obtained by using Borda counts.

5.5 Discussions

This chapter introduced four social choice methods. They do not require the knowledge of objective function values, they only need the rankings of the alternatives by the DMs. Two case studies were used to illustrate the methodology. The first study determined the best treatment alternative of a forest region in Northern Arizona, and the second study helped to find the best project for water resources development of a watershed in Iran.

Plurality voting is a simple procedure with the disadvantage that it considers only the best rankings by the DMs. Borda count is able to provide a complete ordering of the alternatives in most cases (not in Case Study 5.3.1 but in Case Study 5.3.2). The decisions in Hare system are based on only first rankings, so it has the same disadvantage as plurality voting. Pairwise comparison results largely depend on the order in which comparisons are performed. Dictatorship considers only the ranking of only one DM. Based on the general properties of these methods as well as on the actual case studies we can recommend Borda count for solving these types of water resources problems.

The goodness of the consensus (social choice solution) can be measured by the Spearman coefficient. Its application was illustrated for Case Study 5.3.2.

Chapter 6
Conflict Resolution Methods

6.1 Introduction

Water and environmental management problems usually face conflicts among the stakeholders because of limited resources and different preferences. These problems become more complicated when stakeholders have conflicting criteria. In such cases we should find appropriate trade-offs between them. When criteria are in conflict, then any improvement in one criterion can be achieved in the expense of worsening the condition of others. For example, protecting natural resources is in conflict with the economic benefits of their utilization. If we would allocate the limited water resources just for the domestic or for the industrial usages, then it would threaten the sustainability of the environment.

In any trade-off, the DM should select a single choice from the set of nondominated solutions. A very popular approach to deal with such problems is known as conflict resolution, which is a special chapter of cooperative game theory. Any two-person conflict is mathematically defined by a pair (H, f^*), where $H \subseteq \mathbf{R}^2$ is a convex, closed, bounded and comprehensive feasible criteria space, $f^* = (f_1^*, f_2^*)$ is the disagreement or the status quo point. H is called comprehensive if $f \in H$ and $\bar{f} \leqslant f$ imply that $\bar{f} \in H$. Figure 6.1 shows this situation. Let f_i denote the criterion value of DM_i, and since no rational DM would accept

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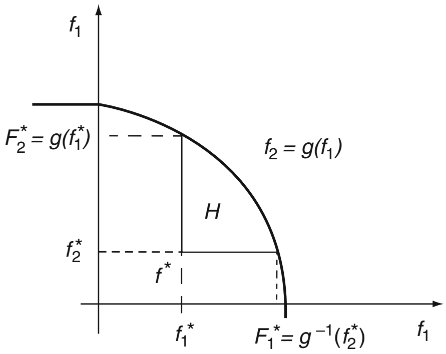


Fig. 6.1 The feasible set of ***H*** in a conflict with two DMs

a value less than that would be obtained in the case of disagreement, we have to assume $f_i \geqslant f_i^*$ for both DMs. Therefore the feasible criteria space H is reduced to its subset

$$H^* = \left\{(f_1,f_2)|(f_1,f_2) \geqslant (f_1^*,f_2^*),(f_1,f_2) \in H\right\}.$$

It is also assumed that the Pareto frontier of H^* is a strictly decreasing concave function of f_1. Notice that F_1^* and F_2^* are the maximal values of the criteria and so point $F^* = (F_1^*, F_2^*)$ is the ideal point.

6.2 The Nash Bargaining Solution

Consider this problem from the point of view of a single DM, say DM_1. He/she has no idea of the choice of the other DM, so this choice is considered to be random on the feasible set $[f_2^*, F_2^*]$. There are two possibilities. If the simultaneous decision vector (f_1, f_2) is feasible, then both DMs obtain their choices, otherwise they get the disagreement (or status quo) values. Assuming uniform distribution for f_2, the expected criterion value of DM_1 is

$$f_1 \frac{g(f_1) - f_2^*}{F_2^* - f_2^*} + f_1^* \left(1 - \frac{g(f_1) - f_2^*}{F_2^* - f_2^*}\right), \tag{6.1}$$

since vector $(f_1,f_2) \in H$ if f_2 belongs to the interval $[f_2^*, g(f_1)]$. This expected value can be rewritten as

$$f_1^* + \frac{(f_1 - f_1^*)(g(f_1) - f_2^*)}{F_2^* - f_2^*},$$

where the first term is a constant, and the denominator of the second term is a given positive value. Therefore this expected value is maximal if the product $(f_1 - f_1^*)(g(f_1) - f_2^*)$ is the largest. It gives a point (f_1, f_2) with $f_2 = g(f_1)$ on the Pareto frontier such that the product of the differences of the actual and the disagreement criterion values is as large as possible. This point is symmetric for the two DMs, so this is a common optimal solution for both of them, therefore it is a reasonable solution for the conflict situation. Notice that it can be obtained by solving the following optimization problem:

$$\begin{aligned} \text{Maximize } & (f_1 - f_1^*)(f_2 - f_2^*) \\ \text{subject to } & (f_1,f_2) \in H \\ & f_1 \geqslant f_1^* \\ & f_2 \geqslant f_2^*. \end{aligned} \tag{6.2}$$

This objective function is usually called the Nash product. Since the logarithm of the objective function is strictly concave, this problem has a unique solution, which can be obtained very easily. Observe that at the endpoints f_1^* and F_1^* of the feasible interval of f_1 the objective function has zero values, so the optimum is interior. If g is differentiable, then the first order condition implies that

$$g(f_1) - f_2^* + g'(f_1)(f_1 - f_1^*) = 0. \tag{6.3}$$

The left hand side is strictly decreasing in f_1, furthermore its value is $F_2^* - f_2^* > 0$ at $f_1 = f_1^*$ and is $g'(F_1^*)(F_1^* - f_1^*) < 0$ at $f_1 = F_1^*$. Therefore there is a unique solution in interval (f_1^*, F_1^*) which can be obtained by using any of the well known methods for solving single-variable equations (like bisection, regular falsi, secant, or Newton methods). A comprehensive summary of these procedures is given for example, in Szidarovszky and Yakowitz (1978). In his classical paper, Nash (1950) introduced six requirements which have to be satisfied by a fair solution. He also proved that there is a unique solution satisfying these axioms and it is the optimal solution of problem (6.2). Therefore it is usually called the (Symmetric) Nash Bargaining Solution.

Many authors have criticized the Nash axioms, and modified them. Any such modification has resulted in a new bargaining solution method. In the following subsections the most popular such concepts and solutions will be introduced.

6.3 The Non-symmetric Nash Solution

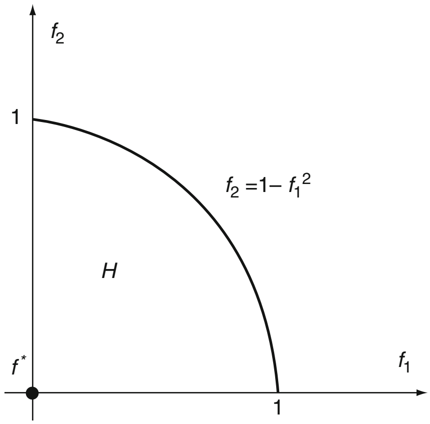
The non-symmetric Nash solution is the generalization of the Nash solution for non-symmetric DMs. It is the unique solution of the following problem:

$$\begin{aligned} \text{Maximize} \quad & (f_1 - f_1^*)^{\alpha}(f_2 - f_2^*)^{1-\alpha} \\ \text{subject to} \quad & (f_1, f_2) \in H \\ & f_1 \geqslant f_1^* \\ & f_2 \geqslant f_2^*, \end{aligned} \tag{6.4}$$

where $\alpha \in [0, 1]$ and $1 - \alpha$ are the relative powers of the two DMs or the weights of their objectives.

Example 6.1. A limited water supply should be allocated between an agricultural district and a rural area. Each of them wants more and more water. The feasible criteria space is assumed to be $H = \{(f_1, f_2) | f_1, f_2 \geqslant 0, f_2 \leqslant 1 - f_1{}^2\}$ with the disagreement payoff vector (0, 0) as shown in Fig. 6.2. The zero disagreement point shows that without an agreement none of the users can receive water, and therefore they can receive only zero benefits.

Fig. 6.2 The feasible objective set of Example 6.1



If we assume equal power weights as $\alpha = 1 - \alpha = 0.5$, then the non-symmetric Nash problem becomes the Nash bargaining solution which can be written as follows:

$$\begin{aligned} \text{Maximize} \quad & (f_1 - 0)^{0.5}(f_2 - 0)^{0.5} \\ \text{subject to} \quad & 0 \leqslant f_1 \leqslant 1 \\ & f_2 = 1 - f_1^2, \end{aligned}$$

or

$$\begin{aligned} \text{Maximize} \quad & (f_1 - 0)(1 - f_1^2 - 0) \\ \text{subject to} \quad & 0 \leqslant f_1 \leqslant 1. \end{aligned}$$

The objective function is $\left(f_1 - f_1^3\right)$ which is 0 at the endpoints $f_1 = 0$ and $f_1 = 1$ so the optimal solution is interior. The first order condition implies that

$$1 - 3f_1^2 = 0$$

implying that

$$f_1 = \frac{\sqrt{3}}{3} \approx 0.5774, \text{ and so } f_2 = g(f_1) = \frac{2}{3} \approx 0.6667.$$

So the optimal point is approximately (0.58, 0.67).

6.4 Area Monotonic Solution

The area monotonic solution is based on selecting a linear array starting from the disagreement point and dividing H into two subsets of equal area, as shown in Fig. 6.3. Hence, the first coordinate of the solution, f_1, is the root of the nonlinear equation $A_1 = A_2$ where

$$A_1 = \int_{f_1^*}^{f_1} g(x)dx - \frac{1}{2}(f_1 - f_1^*)\big(f_2^* + g(f_1)\big), \tag{6.5}$$

and

$$A_2 = \frac{1}{2}(f_1 - f_1^*)(f_2^* + g(f_1)) + \int_{f_1}^{F_1^*} g(x)dx - (F_1^* - f_1^*)f_2^*. \tag{6.6}$$

Notice that A_1 increases in f_1 and A_2 decreases in f_1. So equation $A_1 - A_2 = 0$ has a unique solution. If the conflict is

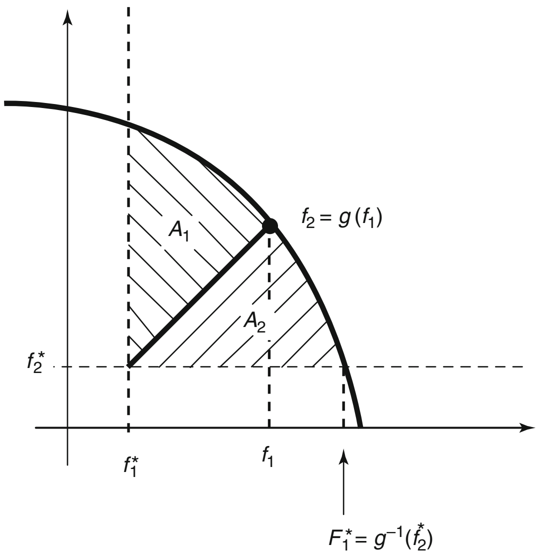


Fig. 6.3 Conflict resolution by dividing H into equal areas

not symmetric, that is $\alpha \neq 0.5$, then we might define the non-symmetric area monotonic solution by requiring that the ratio of the areas of the two regions be $\alpha/(1-\alpha)$. We should therefore solve the problem of $(1-\alpha)A_1 = \alpha A_2$.

Example 6.2. In the problem of Example 6.1 the total feasible area of H is calculated as

$$\int_0^1 (1-x^2)dx = \left[x - \frac{x^3}{3}\right]_0^1 = \frac{2}{3},$$

so both A_1 and A_2 have to be 1/3. For A_2 we have equation

$$\frac{1}{3} = \frac{f_1(1-f_1{}^2)}{2} + \int_{f_1}^1 (1-x^2)dx = \frac{f_1 - f_1{}^3}{2} + \left[x - \frac{x^3}{3}\right]_{f_1}^1$$
$$= \frac{f_1 - f_1{}^3}{2} + \frac{2}{3} - f_1 + \frac{f_1{}^3}{3},$$

that is,

$$3f_1 - 3f_1{}^3 + 4 - 6f_1 + 2f_1{}^3 = 2$$

or

$$f_1{}^3 + 3f_1 - 2 = 0.$$

Using the Newton's method in three iteration steps, the optimal point is obtained for two correct decimals: (0.60, 0.64).

6.5 Equal Loss Solution

The equal loss solution was also originally developed for the symmetric case, when the values of both criteria were relaxed simultaneously from their largest values with equal speed until an

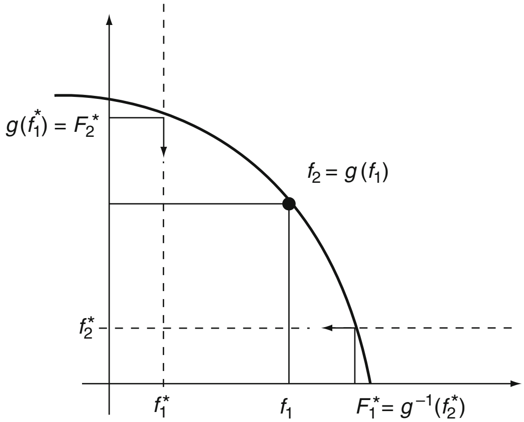


Fig. 6.4 Equal loss solution

agreement was achieved. That is, both players should decrease their payoff values from the ideal point with equal speed until feasible solution is found (see Fig. 6.4). Therefore, we determine the point $(f_1, g(f_1))$ on the Pareto frontier such that

$$F_1^* - f_1 = F_2^* - g(f_1), \tag{6.7}$$

which is a monotonic nonlinear equation for f_1.

In the case of different powers of the players, $\alpha \neq 0.5$, we may generalize this concept by requiring that the more important objective is relaxed slower than the other one by assuming that the ratio of the relaxation speeds equals $(1 - \alpha)/\alpha$. That is, f_1 is the solution of equation

$$\alpha(F_1^* - f_1) = (1 - \alpha)(F_2^* - g(f_1)). \tag{6.8}$$

Similarly to Example 6.2, the nonlinear equation for the single unknown f_1, can be easily solved by using standard methodology.

Example 6.3. Assuming equal power weights and using relation (6.7) with $F_1^* = F_2^* = 1$, we have

$$1 - f_1 = 1 - (1 - {f_1}^2),$$

that is,

$$f_1{}^2 + f_1 - 1 = 0$$

with the solution

$$f_1 = \frac{-1 + \sqrt{1+4}}{2} = \frac{-1 + \sqrt{5}}{2} \approx 0.62.$$

Then the optimal point is (0.62, 0.62).

6.6 Kalai–Smorodinsky Solution

In the case of the Kalai–Smorodinsky solution as shown in Fig. 6.5, an arrow starts from the status quo point and moves toward the ideal point. The last feasible point is then accepted as the solution. That is, we move the payoff vector from the status quo point toward the ideal point as far as possible. Hence, we have to compute the unique solution of the following equation:

$$g(f_1) - f_2^* = \frac{F_2^* - f_2^*}{F_1^* - f_1^*}(f_1 - f_1^*), \tag{6.9}$$

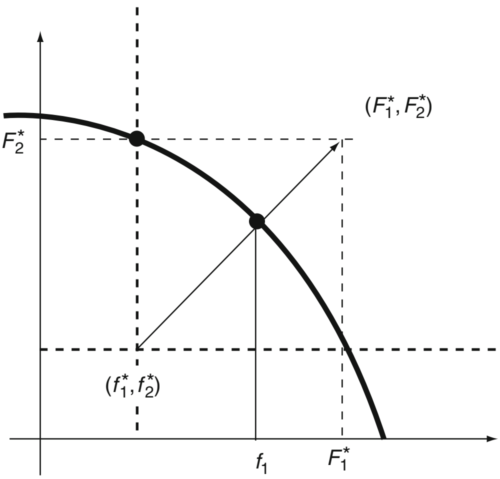


Fig. 6.5 The Kalai–Smordinsky solution

where the two-point formula is used for the equation of the arc.

If the criteria of the DMs have different importance weights, then the slope of the arc changes in such a way that the more important criterion is improved more rapidly. Therefore in this case (6.8) should be revised as follows

$$g(f_1) - f_2^* = \frac{(1-\alpha)(F_2^* - f_2^*)}{\alpha(F_1^* - f_1^*)}(f_1 - f_1^*). \tag{6.10}$$

Example 6.4. In Example 6.1 with equal power weights, (6.9) has the special form

$$(1 - {f_1}^2) - 0 = \frac{(1-0.5)(1-0)}{0.5(1-0)}(f_1 - 0),$$

that is,

$${f_1}^2 + f_1 - 1 = 0.$$

Therefore the optimal solution is again (0.62, 0.62).

6.7 Case Studies

6.7.1 Multi-reservoir Bi-objective Planning

The four-reservoir system of the Cauvery River basin in India is modelled using a bi-objective mathematical programming problem to find optimum cropping patterns, subject to land, water and downstream release constraints (Vedula and Rogers 1981). The monthly deterministic model is applied to maximize both the net economic benefit and the irrigated cropped area. The tradeoff curve between these two conflicting criteria is developed and then four conflict resolution methods are used to find the optimum design.

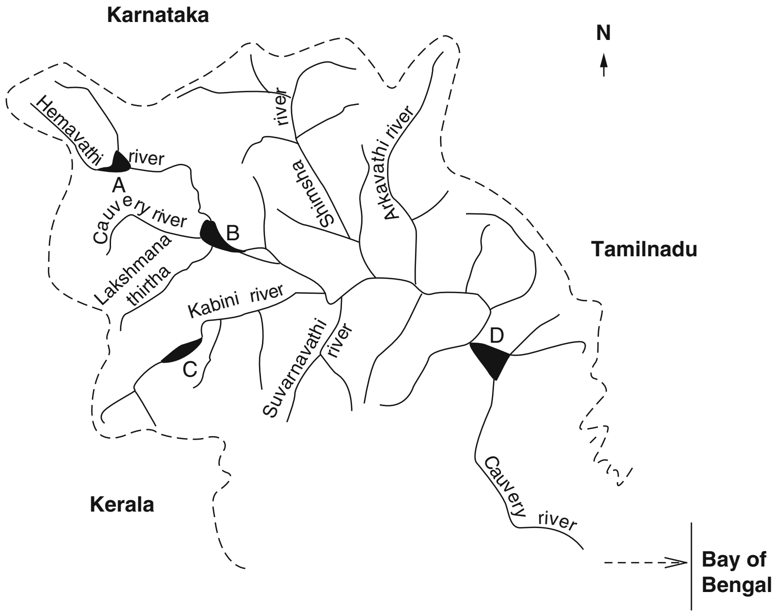


Fig. 6.6 Reservoir systems in part of the Cauvery River basin, India

In Fig. 6.6, the Hemavathy, Krishnarajasagara, Kabini and Mettur reservoirs are denoted respectively by A, B, C and D. Reservoir D mainly regulates the releases to the existing irrigation in the downstream. The groundwater supply is not considered as a usable water resource because a large part of it is in the hard rock area.

The first criterion maximizes the total economic benefit from all crops in the upper basin, where the model considers the option of irrigating or not irrigating any or all of the cropped areas. So the first criterion is the following:

$$\text{Maximize} f_1 = \sum_{j=1}^{3} \sum_{i=1}^{M_j} (\alpha_{ij} I_{ij} + \beta_{ij} U_{ij}), \tag{6.11}$$

where α_{ij} = net benefits at site j per unit area of irrigated crop i ($j = 1$, reservoir A; $j = 2$, reservoir B; and $j = 3$, reservoir C); β_{ij} = net benefits at site j per unit area of unirrigated crop i; I_{ij} = irrigated area at site j with crop i; U_{ij} = unirrigated area at site j with crop i; and M_j = total number of crops considered at site j.

The second criterion maximizes the total irrigated area from all crops in the upper basin:

$$\text{Maximize} f_2 = \sum_{j=1}^{3} \sum_{i=1}^{M_j} I_{ij}. \tag{6.12}$$

The decision variables are the cropped areas with each type of crop at each reservoir with or without irrigation and the monthly storages in each of the reservoirs. The constraints for each month at each of the reservoir sites are as follows:

- The land allocation should be such that the same area for a given crop is available throughout its growing season.
- The storage continuity constraints include all inflows, outflows (diversions and downstream releases) and evaporation losses.
- The total irrigated and unirrigated land areas in each month are limited by the physically available values.
- The downstream release from reservoir D should not be less than the estimated water requirements for existing downstream irrigation.

Evaporation losses, crop water requirements for existing and proposed irrigation, net benefits from irrigated and unirrigated cultivation per unit area of the crops were estimated from available information and summarized in Vedula and Rogers (1981). The maximum net benefit (first criterion) was found to be $2{,}085 \times 10^6$ rupees (Rs.) (at 1970–1972 price level). The maximum irrigated cropped area (second criterion) was found to be 755.6×10^3 ha. A bi-objective analysis using the ε-constraint approach (already described in Chap. 3) revealed seven Pareto frontier solutions as shown in Table 6.1. The criterion values are also normalized to be in the range of [0, 1] with 0 and 1 representing the minimum and maximum values, respectively. Based on these normalized criterion values the tradeoff curve between the net benefits and irrigated cropped areas is shown in Fig. 6.7.

Now the question is the following: which point of the tradeoff curve should be selected as the solution of the conflict. We applied

Table 6.1 Pareto frontier set and their normalized values

No.	f_1, net benefit 10^6 Rs.	f_2, area 10^3 ha	Normalized f_1	Normalized f_2
1	2,085	527.6	1.00	0.00
2	2,071	573.2	0.94	0.20
3	2,054	618.8	0.86	0.40
4	2,034	664.4	0.76	0.60
5	1,971	710.0	0.47	0.80
6	1,927	730.0	0.27	0.89
7	1,868	755.6	0.00	1.00

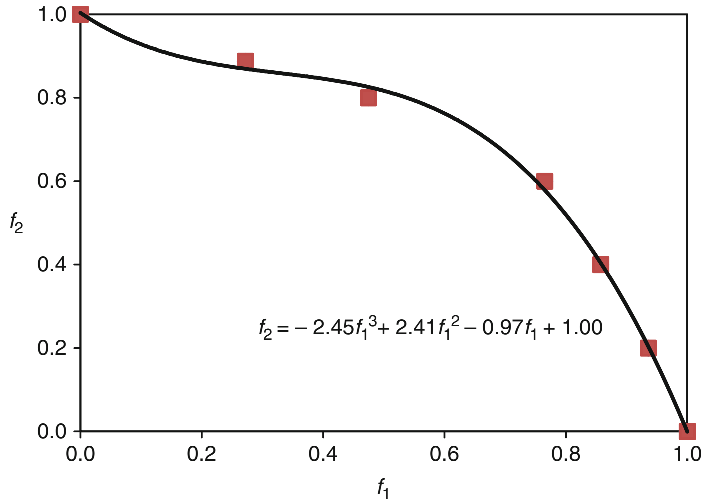


Fig. 6.7 Pareto frontier of the bi-objective model of the Cauvery River basin

the four methods discussed earlier in this chapter. We used four different weights selection for the criteria. By using least squares a cubic polynomial was fitted to the Pareto frontier and the GAMS software was used to find the numerical solutions (Table 6.2).

6.7.2 Water Distribution Network Design

The optimal operation of water distribution networks is one of the most important topics in water engineering. The existing optimization models minimize only the cost of the new or the rehabilitated

Table 6.2 Optimum decisions by varying weights for the case of the Cauvery River basin

α, weight of net benefit	1 − α, weight of area	f_1, normalized net benefit	f_2, normalized area	f_1, net benefit 10^6 Rs.	f_2, area 10^3 ha
Non-symmetric Nash solution					
1.00	0.00	1.00	0.00	2,085	527.6
0.75	0.25	0.81	0.50	2,043	641.6
0.50	0.50	0.67	0.69	2,013	686.1
0.25	0.75	0.53	0.80	1,984	709.1
0.00	1.0	0.00	1.00	1,868	755.6
Area monotonic solution					
1.00	0.00	1.00	0.00	2,085	527.6
0.75	0.25	0.88	0.35	2,059	607.4
0.50	0.50	0.69	0.67	2,018	680.4
0.25	0.75	0.37	0.85	1,948	721.4
0.00	1.0	0.00	1.00	1,868	755.6
Equal loss solution					
1.00	0.00	1.00	0.00	2,085	527.6
0.75	0.25	0.82	0.47	2,046	634.8
0.50	0.50	0.68	0.68	2,016	682.6
0.25	0.75	0.47	0.82	1,970	714.6
0.00	1.0	0.00	1.00	1,868	755.6
Kalai–Smorodinsky solution					
1.00	0.00	1.00	0.00	2,085	527.6
0.75	0.25	0.90	0.30	2,063	596.0
0.50	0.50	0.68	0.68	2,016	682.6
0.25	0.75	0.29	0.86	1,931	723.7
0.00	1.0	0.00	1.00	1,868	755.6

network. However there are other important criteria in real applications concerning the leakage, reliability, water quality, and redundancy among others. In this section, a bi-objective optimization model is developed to the benchmark problem of the Hanoi trunk network, Vietnam (Fig. 6.8). This network consists of 32 nodes and 34 pipes supplied by a fixed grade source in elevation of 100 m. The minimum required head at the junction nodes has to be 30 m. The commercially available diameters, D, for the pipes are selected from the set $A = [304.8, 406.4, 508.0, 609.6, 762.0, 1{,}016.0]$ in millimeter, and their corresponding cost ($) per unit length is assumed to be given by the function $1.1 \times D^{1.5}$.

In this case, the optimal design of the water distribution network can be formulated as follows. The first criterion is to minimize the total cost of constructing the network:

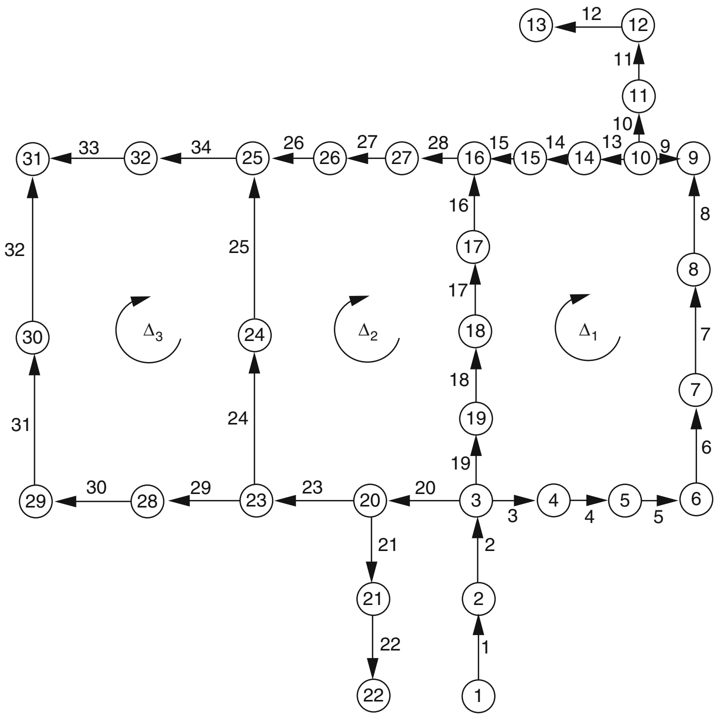


Fig. 6.8 The case of the Hanoi network (Fujiwara and Khang 1990)

$$\text{Minimize} f_1 = \sum_{i=1}^{n} C_i(D_i, L_i), \tag{6.13}$$

where n is the number of pipes in the system, C_i is the cost function for pipe i with diameter D_i and length L_i.

The second criterion is maximizing a resiliency measure. Any increase in the value of network resilience, improves the reliability of the network under failure due to the variability in different water consumption levels. That is, the second criterion is to maximize network resilience:

$$\text{Maximize} f_2 = \sum_{j=1}^{m} R(H_j - H_{j,\min}), \tag{6.14}$$

where m is the number of nodes, H_j is the head of water in node j, $H_{j,min}$ is the minimum required head in node j. In addition, R is a function of both the nodal surplus power and the uniformity in the diameters connected to that node. These criteria should be optimized subject to the energy and mass balance conditions at each node:

$$g_j(H, D) = 0, \tag{6.15}$$

$$H_j \geqslant H_{j,\min}, \tag{6.16}$$

$$D_i \in A, \tag{6.17}$$

where g_j represents the nodal mass and loops energy balance. The decision variables are the alternative design options for each pipe in the network. The hydraulic analysis on the constraints is calculated by using the EPANET software and the optimization problem is solved by using a specially developed Genetic Algorithm after converting it to a constrained, single-objective and deterministic equivalent optimization problem (Prasad and Park 2004). Thirty Pareto optimal (that is, nondominated) solutions were obtained and their normalized criteria values are shown in Fig. 6.9. Notice that the highest value of the cost criterion is normalized to 0 and the smallest value to 1. Then the new

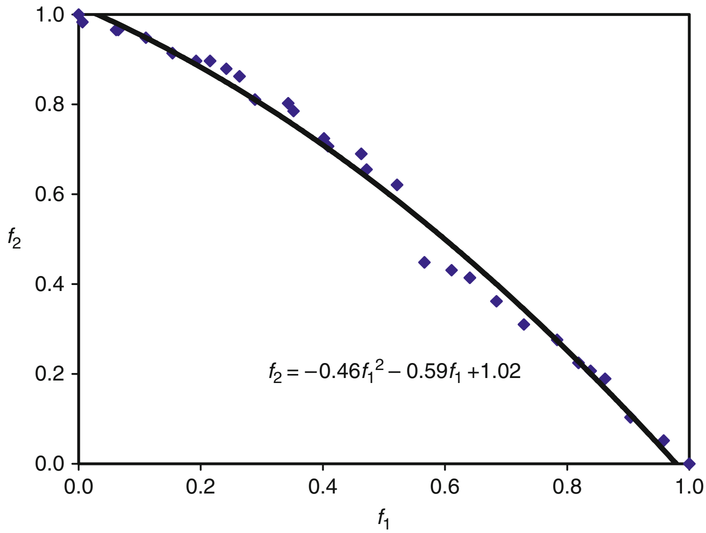


Fig. 6.9 The Pareto frontier and its continuous approximation for the water distribution problem

Table 6.3 Bi-objective optimal solutions with different weights for the Hanoi network

α, weight of normalized cost	$1-\alpha$, weight of resiliency	f_1, normalized cost	f_2, normalized resiliency	Cost (10^3 \$)	Network resilience index
Non-symmetric Nash solution					
1.00	0.00	1.00	0.00	6,349.3	0.231
0.75	0.25	0.749	0.320	6,497.1	0.250
0.50	0.50	0.533	0.575	6,624.4	0.264
0.25	0.75	0.305	0.797	6,758.7	0.277
0.00	1.0	0.00	1.00	6,938.4	0.289
Area monotonic solution					
1.00	0.00	1.00	0.00	6,349.3	0.231
0.75	0.25	0.772	0.291	6,483.6	0.248
0.50	0.50	0.537	0.570	6,622.0	0.264
0.25	0.75	0.277	0.821	6,775.2	0.279
0.00	1.0	0.00	1.00	6,938.4	0.289
Equal loss solution					
1.00	0.00	1.00	0.00	6,349.3	0.231
0.75	0.25	0.766	0.298	6,487.1	0.248
0.50	0.50	0.553	0.553	6,612.6	0.263
0.25	0.75	0.329	0.776	6,744.6	0.276
0.00	1.0	0.00	1.00	6,938.4	0.289
Kalai–Smorodinsky solution					
1.00	0.00	1.00	0.00	6,349.3	0.231
0.75	0.25	0.792	0.264	6,471.8	0.246
0.50	0.50	0.553	0.553	6,612.6	0.263
0.25	0.75	0.274	0.823	6,777.0	0.279
0.00	1.0	0.00	1.00	6,938.4	0.289

normalized cost criterion as well as the resiliency index should be maximized.

Conflict resolution methods were selected to find appropriate design from the Pareto set given in Fig. 6.9. The obtained solutions are shown in Table 6.3. Similarly to the previous case study, the results with five different weights selections are presented.

6.8 Discussions

Conflict resolution methodology was outlined in this chapter. Four different methods were introduced, each of them is based on a particular expression of the fairness of the solution. These methods can be used in both symmetric and non-symmetric cases. Since the results by applying different methods are usually different, the DM has to have a good feeling about his/her idea of the fairness of the solution and then to accept the answer which closest resembles his/her notion of fairness.

Chapter 7
MCDA Problems Under Uncertainty

7.1 Introduction

Many researchers emphasize that a real challenge in modeling MCDA problems is how to incorporate the uncertainty of the input data. MCDA models for water and environmental management, similar to many areas, face uncertainties that generally arise from two sources: random or probabilistic uncertainty related to environmental, economic or technical data, and fuzzy uncertainty related to subjective judgments and the characteristics of the DM. By considering uncertainty, the decision analysis becomes more difficult, but by ignoring it we might miss reality. This chapter discusses and illustrates the main approaches for modeling these two types of uncertainty. The studies of Sahinidis (2004) and Stewart (2005) review the literature of the different types of the uncertain MCDA models and solution procedures.

7.2 Probabilistic Methods

In water resources and environmental management, we often have quantities, which cannot be forecasted with certainty. This type of variables like water demand, energy prices, rainfall or stream flow, evaporation, wastewater pollutions and others are called random or probabilistic variables. In this section, we deal

M. Zarghami and F. Szidarovszky, *Multicriteria Analysis*,
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with random variables with regular and stationary behavior. The statistical attributes of these types of variables are not changing and they can be characterized by single probability distributions (Loucks and van Beek 2005). If there are serial correlations in the spatial or temporal chains of the random values or trends, then they have to be taken into account. Advanced statistical methods (such as factor analysis) can be used to find the independent factors and regression analysis can be applied to find the trends, which has to be added to the predicted time-related values.

7.2.1 Certain Equivalents

Assume first that the feasible set is deterministic, only the criteria depend on random parameters. Let ξ be a random vector, the components of which are the random parameters. Let x be the decision variable and X the feasible decision space. Then the problem can be formulated as

$$\begin{gathered}\text{Maximize } f_k(x,\ \xi)\\ \text{subject to } x \in X.\end{gathered}$$

In the economic literature, it has been shown that the lottery of the random outcome $f_k(x,\ \xi)$ is equivalent to a linear combination of its expectation and variance,

$$\mathrm{E}(f_k(x,\xi)) - \beta_k \mathrm{Var}(f_k(x,\xi)), \tag{7.1}$$

where β_k shows the level of risk acceptance of the DM. If $\beta_k = 0$, then he/she considers only the expected values and completely ignores risk characterized by the variance. Larger value of β_k shows that the DM is more sensitive about accepting risk and would like to reduce uncertainty. Expression (7.1) is called the certain equivalent.

Example 7.1. Consider again the problem of locating a dam, which was introduced earlier in Example 3.1. Assume that the

DM has medium sensitivity about risk in the net benefit, very low sensitivity about risk in the number of beneficiaries and he/she is very sensitive about geological stability. So he/she assigns the risk coefficients $\beta_1 = 1.0$, $\beta_2 = 0.1$ and $\beta_3 = 10$ to the three criteria. Assume in addition that the standard deviation over expected value (coefficient of variation) of the net benefit data is 5%, those of the number of beneficiaries and geological stability are 10% and 2%, respectively. The expected values are given in Table 3.1 and the variance data are shown in Table 7.1.

The certain equivalents of the random criteria values for each alternative are computed by using expression (7.1), and they are presented in Table 7.2.

Based on the new evaluation matrix any method for solving discrete MCDA problems can be used.

Example 7.2. Consider again the continuous MCDA problem introduced earlier in Example 4.1, and assume that the amount of pollutant removal by the first two technologies can be considered exact values, however the amounts estimated by using the third technology variant are uncertain with 30% standard deviations. The amount of the removed pollutant of the first kind is

$$f_1 = 3x_1 + 2x_2 + \xi_1(1 - x_1 - x_2)$$

Table 7.1 Variances of the evaluation values for Example 7.1

Criteria	Alternatives			
	A_1	A_2	A_3	A_4
C_1	24.800	18.361	25.553	22.610
C_2	0.160	3.610	16.000	25.000
C_3	1.960	1.000	0.040	0.160

Table 7.2 Evaluation table for Example 7.1 with certain equivalents

Criteria	Alternatives			
	A_1	A_2	A_3	A_4
C_1	74.800	67.339	75.547	72.490
C_2	3.984	18.639	38.400	47.500
C_3	50.400	40.000	9.600	18.400

and that of the second kind is

$$f_2 = 2x_1 + 3x_2 + \xi_2(1 - x_1 - x_2)$$

where ξ_1 and ξ_2 are random variables with

$$\mathrm{E}(\xi_1) = \mathrm{E}(\xi_2) = 1 \text{ and } \mathrm{Var}(\xi_1) = \mathrm{Var}(\xi_2) = 0.09.$$

Clearly

$$\mathrm{E}(f_1) = 3x_1 + 2x_2 + 1(1 - x_1 - x_2) = 2x_1 + x_2 + 1,$$

$$\mathrm{Var}(f_1) = (1 - x_1 - x_2)^2 0.09,$$

$$\mathrm{E}(f_2) = 2x_1 + 3x_2 + 1(1 - x_1 - x_2) = x_1 + 2x_2 + 1,$$

and

$$\mathrm{Var}(f_2) = (1 - x_1 - x_2)^2 0.09.$$

Assume that the DM is more sensitive about the risk concerning the first pollutant than the second one by selecting $\beta_1 = 2$ and $\beta_2 = 1$. So the deterministic equivalents of the two objective functions are

$$2x_1 + x_2 + 1 - 0.18(1 - x_1 - x_2)^2,$$

and

$$x_1 + 2x_2 + 1 - 0.09(1 - x_1 - x_2)^2.$$

Therefore the DM will solve the following deterministic problem:

$$\begin{aligned} \text{Maximize} \quad & 2x_1 + x_2 + 1 - 0.18(1 - x_1 - x_2)^2,\ x_1 + 2x_2 + 1 \\ & - 0.09(1 - x_1 - x_2)^2 \\ \text{subject to} \quad & x_1 + x_2 \leqslant 1 \\ & x_1, x_2 \geqslant 0. \end{aligned}$$

7.2.2 Discrete Problems

The application of certain equivalents was already introduced in the previous subsection, when each random evaluation value is replaced by its certain equivalent. Then the resulting deterministic MCDA problem can be solved by using any method introduced earlier.

Example 7.3. Returning to the problem introduced in Example 7.1, first we have to normalize the criteria. The smallest and largest values of C_1 are 67.339 and 75.547, so the normalized first criterion becomes $(C_1 - 67.339)/(75.547 - 67.339)$. Similarly, the normalized second criterion becomes $(C_2 - 3.984)/(47.500 - 3.984)$, and the normalized third criterion can be obtained as $(C_3 - 9.600)/(50.400 - 9.600)$. The normalized evaluation table is given in Table 7.3. Assume that simple additive weighting is used with weights $w_1 = 0.25$, $w_2 = 0.25$ and $w_3 = 0.50$, then the weighted averages for the four alternatives are given as follows:

$$A_1 = 0.909(0.250) + 0.000(0.250) + 1.000(0.500) = 0.727,$$

$$A_2 = 0.000(0.250) + 0.337(0.250) + 0.745(0.500) = 0.457,$$

$$A_3 = 1.000(0.250) + 0.791(0.250) + 0.000(0.500) = 0.448,$$

$$A_4 = 0.627(0.250) + 1.000(0.250) + 0.216(0.500) = 0.515.$$

Based on these results, $A_1 \succ A_4 \succ A_2 \succ A_3$.

Notice that in applying the above discussed method we used only the expectations and variances of the evaluation numbers,

Table 7.3 Normalized evaluation matrix for Example 7.3

Criteria	Alternatives			
	A_1	A_2	A_3	A_4
C_1	0.909	0.000	1.000	0.627
C_2	0.000	0.337	0.791	1.000
C_3	1.000	0.745	0.000	0.216

there was no need to know the distribution functions. In cases, when the distribution functions are available, simulation is an alternative approach. This method can be described in the following way. Select a large positive integer N as the number of generating random evaluation values. For each case, random values of the evaluation numbers are generated and they are used to form a random evaluation matrix. In the case of each evaluation matrix, the best alternative is determined by using a DM selected method. After repeating this step for N times, we have the number of cases for each alternative, when it was selected as the best choice. Let $N_1, N_2, \ldots, N_m$ denote these values, so N_k denote the number of cases when alternative A_k was the best. Then for each alternative the probability that it is optimal can be approximated by the relative frequency $p_k = N_k/N$. In this way a discrete probability distribution can be obtained on the set of the alternatives. The overall best alternative can be chosen as any central tendency of this distribution: the mean, the median or the mode. The mean is given as

$$\text{Mean} = \frac{1}{N}\sum_{k=1}^{m} kp_k, \tag{7.2}$$

which is usually a non-integer value, so we have to round it to the closest integer as the integer part of Mean + 0.5. The median is alternative k, if

$$\sum_{l=1}^{k-1} p_l < \frac{1}{2} \leqslant \sum_{l=1}^{k} p_l, \tag{7.3}$$

and the mode is the alternative which has the highest p_k probability value.

The method of generating random evaluation numbers depends on the types of the distribution functions of the elements. The random number generator of a computer supplies uniform random numbers in the unit interval [0, 1]. Let u denote such random value. Assume first an evaluation number a_{ij} has a discrete distribution with possible values $a^{(1)}<a^{(2)}<\cdots$ with occurring probabilities $\pi^{(1)},\pi^{(2)},\ldots$. The corresponding random values of a_{ij} is $a^{(k)}$, if

$$\sum_{l=1}^{k-1} p_l<u\leqslant \sum_{l=1}^{k} p_l. \tag{7.4}$$

In the case of special distribution types more simple algorithms can be used. If a_{ij} is a Bernoulli variable with parameter p, then the generated value of a_{ij} is 1 if $u \le p$, otherwise it is 0. If a_{ij} is a binomial variable with parameters n and p, then the generated value of a_{ij} is the sum of n independent Bernoulli numbers with the same parameter p. We will describe the case of Poisson variables a little later.

Assume next that a_{ij} is a continuous variable with distribution function $F(a_{ij})$, where F is assumed to be strictly increasing. If u is uniform in [0, 1], then the random number $F^{-1}(u)$ follows the given distribution function F. If a_{ij} is uniform in an interval [A, B], then the generated random value of a_{ij} is given as $u(B - A) + A$, since the distribution function of a_{ij} is $(a_{ij} - A)/(B - A)$. If a_{ij} is exponential with parameter λ, then $F(a_{ij}) = 1 - e^{-\lambda a_{ij}}$, so the random value of a_{ij} is the solution of equation

$$1 - e^{-\lambda a_{ij}} = u,$$

implying that

$$a_{ij} = -\frac{1}{\lambda}\ln(1-u) \text{ or } a_{ij} = -\frac{1}{\lambda}\ln u, \tag{7.5}$$

since if u is uniform in [0, 1] then $1 - u$ is also uniform in this interval.

Poisson variables can be easily generated by using exponential random variables. A random Poisson number with parameter λ is k, if for a sequence of independent random exponential values $e_1, e_2, \ldots$ with the same parameter value λ we have

$$\sum_{l=1}^{k-1} e_l<1\leqslant \sum_{l=1}^{k} e_l. \tag{7.6}$$

Assume next that a_{ij} is normal with parameters μ and σ^2. Using the distribution function Φ of standard normal variable, the equation for the random value of a_{ij} becomes

$$\Phi\left(\frac{a_{ij} - \mu}{\sigma}\right) = u,$$

that is,

$$a_{ij} = \Phi^{-1}(u)\sigma + \mu. \tag{7.7}$$

There is a more simple way to generate normally distributed random numbers. Let $u_1, u_2, \ldots, u_N$ be a sequence of independent uniform numbers from interval [0, 1]. If N is sufficiently large, then $\sum_{k=1}^{N} u_k$ is approximately normally distributed. The normalized value

$$\frac{\sum_{k=1}^{N} u_k - \frac{N}{2}}{\sqrt{\frac{N}{12}}}$$

is a good approximation of the standard normal variable, so

$$a_{ij} = \frac{\sum_{k=1}^{N} u_k - \frac{N}{2}}{\sqrt{\frac{N}{12}}} \cdot \sigma + \mu \tag{7.8}$$

is normally distributed with parameters μ and σ^2.

Any monograph on stochastic simulation and Monte Carlo methods gives a comprehensive summary for generating random values from the most frequently used distribution types.

7.2.3 Continuous Problems

In the case of continuous MCDA problems, we might find uncertain values in both the objective functions and in the constraints.

If only the objective function have random elements, then their certain equivalents are first determined and then the resulted deterministic problem is solved by any one of the methods discussed earlier for solving continuous MCDA problems. If there is randomness in the constraints, then there is the possibility that they become violated. In the stochastic programming literature there are two major methods to consider the violation of the constraints.

1. Depending on the levels of violations, loss functions are introduced and added to one or more objective functions or added to the problem as additional objective functions.
2. Probability levels are given for each constraint or for each group of constraints which are then replaced by the requirement that they have to be satisfied with probabilities greater than or equal to given tolerance levels (chance constraints method, see for example, Charnes and Cooper 1959).

Sometimes these two approaches are combined by introducing additional loss functions and probabilistic constraints. As an example, assume that in an allocation problem the constraint requires to receive at least amount T to be supplied by an uncertain source. Assume that X is the actually supplied amount, which is normally distributed with mean μ and variance σ^2. Then the probabilistic constraint becomes

$$P(X \geqslant T) \geqslant 1 - \varepsilon,$$

where $1 - \varepsilon$ is the given probability level. This constraint can be rewritten as

$$1 - \Phi\left(\frac{T - \mu}{\sigma}\right) \geqslant 1 - \varepsilon,$$

where Φ is the standard normal distribution function. By using its inverse, we have

$$\frac{T - \mu}{\sigma} \leqslant \Phi^{-1}(\varepsilon),$$

or

$$T \leqslant \mu + \sigma \Phi^{-1}(\varepsilon). \tag{7.9}$$

This relation shows that by selecting the target value of $\mu + \sigma \Phi^{-1}(\varepsilon)$, the received amount will be at least T with probability $1 - \varepsilon$. The table of the standard normal distribution function is given in Appendix. The function values are presented only for the nonnegative values of x. If $x < 0$, then we can use relation $\Phi(x) = 1 - \Phi(-x)$, where $-x$ is positive, so $\Phi(-x)$ can be found in the table.

Example 7.4. Consider again the water allocation problem introduced earlier in Example 6.1, where we assume that the feasible criteria space is uncertain:

$$H^* = \left\{(f_1, f_2) | f_1, f_2 \geqslant 0, f_2 \leqslant 1 - \tau f_1^2\right\},$$

where τ is a normally distributed random variable with $\mathrm{E}(\tau) = 1$ and $\mathrm{Var}(\tau) = 0.01$. If we consider this problem as a MCDA problem, then it can be formulated as

$$\begin{aligned} &\text{Maximize } f_1, f_2 \\ &\text{subject to } f_1, f_2 \geqslant 0 \\ &\qquad\qquad f_2 \leqslant 1 - \tau f_1{}^2. \end{aligned}$$

The criteria f_1, f_2 and the nonnegativity constraints are deterministic, only the last constraint is random. Let $1 - \varepsilon$ be the user selected probability level such that

$$P(f_2 \leqslant 1 - \tau f_1^2) \geqslant 1 - \varepsilon.$$

This condition can be rewritten as

$$P\left(\tau \leqslant \frac{1 - f_2}{f_1^2}\right) \geqslant 1 - \varepsilon,$$

or

$$F\left(\frac{1-f_2}{f_1^2}\right) \geqslant 1-\varepsilon,$$

where F is the distribution function of τ. Since $F(\tau) = \Phi\left(\frac{\tau-1}{0.1}\right)$, this equation can be rewritten as

$$\Phi\left(\frac{\frac{1-f_2}{f_1^2}-1}{0.1}\right) \geqslant 1-\varepsilon$$

or

$$\frac{1-f_2}{f_1^2} \geqslant 0.1\Phi^{-1}(1-\varepsilon)+1$$

which is equivalent to relation

$$f_2 \leqslant 1-f_1^2 \cdot \left(0.1\Phi^{-1}(1-\varepsilon)+1\right).$$

For example, if we take the probability level $1-\varepsilon = 97.5\%$, then $\Phi^{-1}(0.975) = 1.96$, so this last constraint becomes

$$f_2 \leqslant 1 - 1.196 f_1^2.$$

Therefore the deterministic problem has the form

$$\begin{aligned} &\text{Maximize } f_1, f_2 \\ &\text{subject to } f_1, f_2 \geqslant 0 \\ &\qquad f_2 \leqslant 1 - 1.196 f_1^2. \end{aligned}$$

The feasible set is shown in Fig. 7.1.

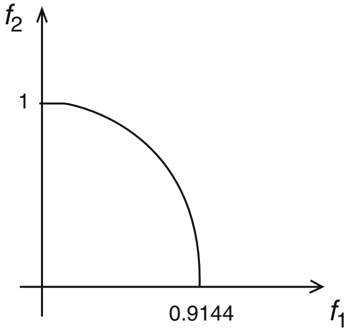

Fig. 7.1 Feasible set of Example 7.4

By assuming equal weights and using simple weighted averaging the resulting single-objective optimization problem can be written as

$$\begin{aligned} \text{Maximize } & f_1 + f_2 \\ \text{subject to } & f_1, f_2 \geqslant 0 \\ & f_2 \leqslant 1 - 1.196 f_1^{\,2}. \end{aligned}$$

It is easy to see that the optimal solution occurs when the slope of function $f_2 = 1 - 1.196 f_1^{\,2}$ is -1, that is, when

$$-2(1.196) f_1 = -1$$

implying that

$$f_1 = \frac{1}{2.392} \approx 0.418$$

and

$$f_2 = 1 - 1.196(0.418)^2 \approx 0.791.$$

Caballero et al. (2004) describe alternative methods to solve uncertain MCDA problems if randomness occurs in both the objective function and in various parameters of the constraints.

7.3 Fuzzy Methods

In the definition of most practical problems, the human judgments are often vague and therefore cannot be expressed by using exact numerical values. Fuzzy sets offer a way of representing and manipulating the data that are not precise, but rather vague. Such vague judgments are frequently used to evaluate water resources projects and therefore we have to use fuzzy set theory in tackling their uncertainty. Let X be a given nonempty set. A fuzzy set A in X is characterized by its membership function μ_A: $X \rightarrow [0, 1]$ where the value of $\mu_A(x)$ is interpreted as the degree that element x belongs to set A.

Definition 1. *The degree that a value* x *belongs to either set A or set B is the maximum of the two individual membership function values:*

$$\mu_{A \cup B}(x) = \text{Maximum}\{\mu_A(x), \mu_B(x)\}. \tag{7.10}$$

Definition 2. *The degree that a value* x *is simultaneously belongs to both sets A and B is the minimum of the two individual membership function values:*

$$\mu_{A \cap B}(x) = \text{Minimum}\{\mu_A(x), \mu_B(x)\}. \tag{7.11}$$

There is a large variety of the shapes of the different types of the membership functions applied in different applications.

Definition 3. *The trapezoidal fuzzy number is defined by the membership function with tolerance interval [a, b], left width l and right width r as shown in Fig. 7.2. If a = b, then it becomes a triangular fuzzy number.*

Let $\mu_1 = (a_1, b_1, l_1, r_1)$ and $\mu_2 = (a_2, b_2, l_2, r_2)$ be positive trapezoidal fuzzy numbers. Then the basic arithmetic operations are defined as follows (Bonissone 1982):

$$\mu_1 + \mu_2 = (a_1 + a_2\,,\ b_1 + b_2\,,\ l_1 + l_2\,,\ r_1 + r_2), \tag{7.12}$$

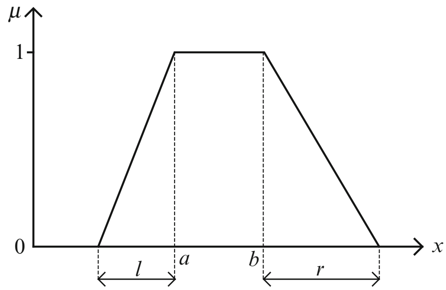

Fig. 7.2 A trapezoidal fuzzy number $\mu = (a, b, l, r)$

$$\mu_1 - \mu_2 = (a_1 - b_2\, ,\, b_1 - a_2\, ,\, l_1 + r_2\, ,\, r_1 + l_2), \tag{7.13}$$

$$\mu_1 \times \mu_2 = (a_1 a_2\, ,\, b_1 b_2\, ,\, a_1 l_2 + a_2 l_1 - l_1 l_2\, ,\, b_1 r_2 + b_2 r_1 + r_1 r_2), \tag{7.14}$$

and

$$\mu_1 / \mu_2 = \left(\frac{a_1}{b_2}\, ,\, \frac{a_2}{b_1}\, ,\, \frac{a_1 r_2 + l_1 b_2}{b_2 (b_2 + r_2)}\, ,\, \frac{b_1 l_2 + r_1 a_2}{a_2 (a_2 - l_2)} \right). \tag{7.15}$$

7.3.1 Discrete Problems

In defining water and environmental problems, we frequently use the terms of low, slightly low, medium and very high among others to describe the importance of the various criteria instead of specifying weights. When we compare alternatives without specific values, fuzzy variables are usually used. These types of variables express the vague and ambiguous values. This section illustrates how they can be modeled and used in formulating and solving discrete MCDA problems.

In applying fuzzy parameters in solving discrete MCDA problems, we may use the methods of Chap. 3 in the following steps:

– First we define suitable fuzzy membership functions for the weights of the criteria and for the evaluation values of the alternatives.

– The fuzzy arithmetic operations of (7.12–7.15) are next used to obtain the fuzzy scores of the alternatives (e.g. weighted sum of evaluation numbers).
– After obtaining the final fuzzy membership functions of the combined goodness measures, they will be compared. To compare fuzzy numbers several methods are known from the literature. The simplest way is to defuzzify them by the max-membership method, which selects the value(s) with the highest membership degree. Another approach is by using the α-cuts or the center of mass method. In the first case the membership function is cut horizontally at different α–levels between 0 and 1. For each α-level of the membership function, the minimum and maximum possible values of the variable are determined and averaged. With a given constant α value, the alternative with the higher average value is considered to have higher rank.

Chen and Hwang (1991) gives a comprehensive survey of fuzzy discrete MCDA methods.

Example 7.5. Several alternative water transfer projects are proposed and considered to conserve the drying Uremia Lake in Iran. To evaluate these projects four criteria were selected and the criteria weights were obtained by questioning three responsible DMs. However due to the uncertainty in the problem these experts presented their preferences only by linguistic terms as shown in Table 7.4.

These linguistic variables are modeled with fuzzy numbers, the membership functions of which are shown in Fig. 7.3.

Table 7.4 The evaluation of the criteria by three DMs

DMs	Power of DMs	Criteria			
		C_1: environmental impacts	C_2: construction cost	C_3: simplicity of construction	C_4: social acceptance
DM_1	High	High	Very high	Medium	Slightly low
DM_2	Medium	Slightly high	Medium	High	High
DM_3	Slightly low	Medium	Very high	Slightly low	High

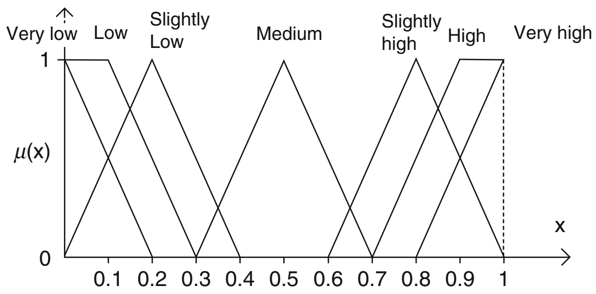

Fig. 7.3 The equivalent fuzzy numbers of the linguistic terms

By using the fuzzy simple additive weighting method and (7.12) and (7.14), we can compute the fuzzy weights of all criteria. For example, in the case of criterion 1 we have

$$\begin{aligned}F_1 = \sum_{i=1}^{3} w_i a_{i1} &= (\text{High} \times \text{High}) + (\text{Medium} \times \text{Slightly High}) \\ &\quad + (\text{Slightly low} \times \text{Medium}) \\ &= (0.9 \times 0.9, 1 \times 1, 0.9 \times 0.2 + 0.9 \times 0.2 - 0.2 \times 0.2, \\ &\quad 1.0 \times 0.0 + 1.0 \times 0.0 + 0.0 \times 0.0) + (0.5 \times 0.8, 0.5 \times 0.8, \\ &\quad 0.5 \times 0.2 + 0.8 \times 0.2 - 0.2 \times 0.2, 0.5 \times 0.2 + 0.8 \times 0.2 \\ &\quad + 0.2 \times 0.2) + (0.2 \times 0.5, 0.2 \times 0.5, 0.2 \times 0.2 + 0.5 \times 0.2 \\ &\quad - 0.2 \times 0.2, 0.2 \times 0.2 + 0.5 \times 0.2 + 0.2 \times 0.2) \\ &= (1.31, 1.50, 0.64, 0.48).\end{aligned}$$

Similar calculations show that $F_2 = (1.35, 1.45, 0.70, 0.44)$, $F_3 = (0.94, 1.04, 0.52, 0.52)$, and $F_4 = (0.81, 0.90, 0.60, 0.60)$. By using the max-membership method, the defuzzified criteria weights are 1.40, 1.40, 0.99, and 0.85 respectively concerning the criteria of environmental impacts, construction cost, simplicity of construction, and social acceptance. If we normalize the weight vector for unit sum, then it becomes {0.30, 0.30, 0.21, 0.19}.

7.3.2 Continuous Problems

There is a large variety of methods to solve continuous MCDA problems with fuzzy variables. In most of these methods there is no difference between the objectives and the constraints. One of the easiest methods is to find the intersection of the membership functions of all objectives, $\{\mu_{G1}(x), \mu_{G2}(x), \ldots\}$ and all constraints, $\{\mu_{C1}(x), \mu_{C2}(x), \ldots\}$ and then the fuzzy decision will be the value of x that maximizes the membership function $\mu_D(x)$ of the intersection. This solution concept can be mathematically formulated as

$$\begin{aligned}\text{Maximize } \mu_D(x) = \text{Maximize} \min\{&\mu_{G1}(x), \mu_{G2}(x), \ldots, \\ &\mu_{C1}(x), \mu_{C2}(x), \ldots\}.\end{aligned} \tag{7.16}$$

Example 7.6. The water level x of a reservoir varies between 0 and 10 m, which is represented by the following membership function:

$$\mu_{C1}(x) = \begin{cases} 0.25x & \text{if } 0 \leqslant x < 4 \\ 1 & \text{if } 4 \leqslant x < 6 \\ 2.5 - 0.25x & \text{if } 6 \leqslant x \leqslant 10. \end{cases} \tag{7.17}$$

The optimum level of the reservoir depends on two criteria. First, higher water level produces more recreational benefit according to the following membership function:

$$\mu_{G1}(x) = \begin{cases} 0 & \text{if } x < 6 \\ -3 + 0.5x & \text{if } 6 \leqslant x < 8 \\ 1 & \text{if } 8 \leqslant x \leqslant 10. \end{cases} \tag{7.18}$$

In contrary, higher water level is less preferred due to possible flooding. The membership function of this criterion is given as

$$\mu_{G2}(x) = 1 - 0.1x, \quad \text{if } 0 \leqslant x \leqslant 10. \tag{7.19}$$

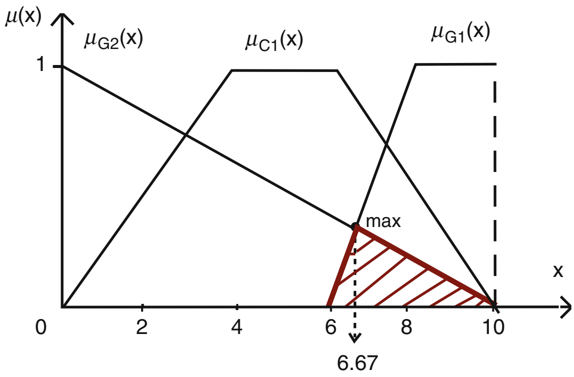

Fig. 7.4 The optimal water level in Example 7.6

To find the optimum water level the fuzzy membership functions of the constraint and the two criteria are presented in Fig. 7.4. The intersection of these membership functions which is defined by their minimal values for all x is shown by the bold broken line with equation

$$\mu_D(x) = \begin{cases} 0 & \text{if } x \leqslant 6 \\ -3 + 0.5x & \text{if } 6 < x \leqslant 20/3 \\ 1 - 0.1x & \text{if } 20/3 < x \leqslant 10. \end{cases}$$

The optimal decision is $x = 20/3 \approx 6.67$ that maximizes this function.

7.4 Probabilistic-Fuzzy MCDA Models

All methods discussed previously in this chapter did not use the combination of the different types of uncertainty in a given problem. They assumed the existence of only probabilistic or only fuzzy uncertainty. In recent studies, an increasing attention has been given to the solution of MCDA problems involving both probabilistic and fuzzy uncertainties (see for example Ben et al. 2004, among others). In this section a probabilistic-fuzzy

MCDA model in selecting optimal alternatives will be introduced. The new approach, entitled probabilistic fuzzy ordered weighted averaging (PFOWA) is based on the following method elements.

The combined goodness measure for each alternative is obtained by the OWA operator (Yager 1988). This aggregation operator has been applied in many fields including decision theory. An n-dimensional OWA operator assigns a combined goodness measure for each alternative in an MCDA problem based on an n-dimensional vector $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ of order weights, which satisfies conditions $w_i \geqslant 0$ for all i, and $\sum_{i=1}^{n} w_i = 1$. Then the combined measure is

$$F(a_1,\ a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i\, b_i = w_1\, b_1 + w_2\, b_2 + \cdots + w_n\, b_n, \tag{7.20}$$

where $F : I^n \mapsto I$ with $I = [0, 1]$, and b_i is the ith largest element in the set $\{a_1,\ a_2, \ldots, a_n\}$ of the evaluations of an alternative with respect to the n criteria. Notice that the components of the input vector have been ordered before multiplying them by the order weights. The OWA method has a large variety by the different selections of the order weights, which depend on the optimism degree of the DM. The greater the weights at the beginning of the vector are, the higher is the optimism degree. The optimism degree θ is defined as

$$\theta = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i. \tag{7.21}$$

Different methods are introduced in the literature for determining the order weights (Xu 2005). In this section fuzzy quantifiers are used to characterize aggregation imperatives, in which, the more objects are included, the higher is the satisfaction level. Some examples of these quantifiers are *most*, *half*, *few* or *at least one of them*. These linguistic inputs are modeled by regular increasing monotonic quantifiers that satisfy the following conditions:

$$Q(0) = 0, Q(1) = 1 \text{ and } Q(p_1) \geqslant Q(p_2) \text{ if } p_1 \geqslant p_2. \tag{7.22}$$

Function Q maps the unit interval $I = [0, 1]$ into itself and is really a fuzzy membership function. It can be associated to an n-dimensional OWA operator, where the components of the weighting vector are obtained as

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad i = 1, 2, \ldots, n. \tag{7.23}$$

There are many different possibilities for selecting fuzzy membership function Q. A particular form has been chosen as $Q(p) = p^{\gamma}$ with a given positive parameter γ. The corresponding optimism degree values are shown in Table 7.5.

The optimism degree can be calculated by using (7.23) and (7.21) and by introducing the new variable $p = i/n$ and computing the limit as n tends to infinity:

$$\theta \approx \int_0^1 Q(p)dp = \int_0^1 p^{\gamma} dp = \frac{1}{1+\gamma}. \tag{7.24}$$

From this equation it is clear that $\gamma = 1/\theta - 1$, and by combining (7.20), (7.23) and (7.24), we have the following form of the combined goodness measure of each alternative:

Table 7.5 Parameters of the fuzzy linguistic quantifier

Linguistic quantifier	Parameter of quantifier, γ	Optimism degree, θ
At least one of them	$\gamma \to 0.0$	0.999
Few of them	0.1	0.909
Some of them	0.5	0.667
Half of them	1.0	0.500
Many of them	2.0	0.333
Most of them	10.0	0.091
All of them	$\gamma \to \infty$	0.001

$$F = \sum_{i=1}^{n}\left[\left(\frac{i}{n}\right)^{\frac{1}{\theta}-1} - \left(\frac{i-1}{n}\right)^{\frac{1}{\theta}-1}\right]b_i. \tag{7.25}$$

The alternative which has the highest F value is selected as the most preferred one.

The evaluation numbers of the alternatives with respect to the criteria are assumed to be probabilistic. The expected value $\mathrm{E}(F_k)$ of the combined goodness measures for each alternative k, and its variance, $\mathrm{Var}(F_k)$ can be obtained from actual data, subjective assessment or by using stochastic simulation. Maximizing expectations and minimizing variances are conflicting criteria so we can use the certain equivalent introduced earlier in (7.1).

Based on the above discussed elements the procedure consists of the following steps.

Step 1. Based on the optimism degree θ of the DM, determine the value of $\gamma = 1/\theta - 1$ and the corresponding membership function $Q(p)$.

Step 2. Estimate the expectations and variances of the evaluation numbers of the alternatives with respect to the criteria.

Step 3. Obtain the risk acceptance values β_k from the DMs and use (7.1) to find the certain equivalents.

Step 4. Compute the combined goodness measure for each alternative by using (7.25) and determine the alternative with the highest such value.

The methodology discussed in this chapter will be illustrated by two case studies. The first one is a discrete problem and the second is continuous.

7.5 Case Studies

7.5.1 Long-Term Watershed Management

The application of the PFOWA approach is illustrated now to the well-known MCDA problem of the Central Tisza River in Hungary (David and Duckstein 1976). This case study consists of five distinct alternative water resources projects designed for long term planning. These decision alternatives are as follows:

A_1: The water resources of both the Tisza and the Danube rivers are used; water is transferred from Danube by a multipurpose canal-reservoir system.

A_2: A pumped reservoir system is built in the northeastern part of the region, which will be supplied from the Tisza River only.

A_3: The Tisza River water is used to supply flat-land reservoirs.

A_4: This is a mountain reservoir system in the upper Tisza River Basin, located outside Hungary.

A_5: This is a conjunctive usage scheme of the Tisza River water and groundwater of the Debrecen region based on regional water regulation.

There are 12 evaluation criteria, 8 of them are subjective. The assigned values of the subjective criteria are obtained by linguistic variables. Table 7.6 shows the alternatives, the corresponding weights and the evaluations of the alternatives with respect to the criteria. Eight criteria are positive and four are negative. The negative criteria are: costs, energy, land and forest use, and sensitivity.

In applying the PFOWA method, we first need the b_i values. The subjective evaluations were quantified by using the numerical values according to the scales given in Table 7.7. The resulted evaluation matrix with numerical values is shown in Table 7.8.

The numerical evaluations have been normalized into the unit interval [0, 1] by using the following transformation:

$$\bar{a}_{ij} = \begin{cases} \dfrac{a_{ij}}{\underset{j}{\mathrm{Max}}\,(a_{ij})} & \text{for positive criteria} \\[2ex] \dfrac{\underset{j}{\mathrm{Min}}\,(a_{ij})}{a_{ij}} & \text{for negative criteria.} \end{cases} \tag{7.26}$$

The results are summarized in Table 7.9.

The normalized evaluation values are then multiplied by the weights, which are already shown in Table 7.6. The original weights {1, 2} were replaced by {0.33, 0.66} since the outputs

Table 7.6 Evaluation matrix of the Tisza River case study

Criteria		Weights	Alternatives				
			A_1	A_2	A_3	A_4	A_5
1	Costs (billion forints/year)	2	99.6	85.7	101.1	95.1	101.8
2	Water shortage	2	4	19	50	50	50
3	Water quality	2	Very good	Good	Bad	Very good	Fair
4	Energy (reuse factor)	2	0.7	0.5	0.01	0.1	0.01
5	Recreation	2	Very good	Good	Fair	Bad	Bad
6	Flood protection	2	Good	Excellent	Fair	Excellent	Bad
7	Land and forest use (1,000 ha)	1	90	80	80	60	70
8	Manpower impact	1	Very good	Very good	Good	Fair	Fair
9	Environmental architecture	2	Very good	Good	Bad	Good	Fair
10	International cooperation	2	Very easy	Easy	Fairly difficult	Difficult	Fairly difficult
11	Development possibility	1	Very good	Good	Fair	Bad	Fair
12	Sensitivity	1	Not sensitive	Not sensitive	Very sensitive	Sensitive	Very sensitive

Table 7.7 Equivalent numerical values for linguistic evaluations

Goodness:						
Very bad	Bad	Fairly bad	Medium	Fair, fairly good	Good	Very good, excellent
Difficulty:						
Very difficult	Difficult	Fairly difficult	Medium	Fairly easy	Easy	Very easy
Sensitivity:						
Not sensitive	Low sensitive	Fairly low sensitive	Medium	Fairly sensitive	Sensitive	Very sensitive
Numeric value:						
0.05	0.2	0.35	0.5	0.65	0.8	1

Table 7.8 Evaluation matrix with numerical values

Criteria	Alternatives				
	A_1	A_2	A_3	A_4	A_5
C_1	99.6	85.7	101.1	95.1	101.8
C_2	4	19	50	50	50
C_3	1	0.8	0.2	1	0.65
C_4	0.7	0.5	0.01	0.1	0.01
C_5	1	0.8	0.65	0.2	0.2
C_6	0.8	1	0.65	1	0.2
C_7	90	80	80	60	70
C_8	1	1	0.8	0.65	0.65
C_9	1	0.8	0.2	0.8	0.65
C_{10}	1	0.8	0.35	0.2	0.35
C_{11}	1	0.8	0.65	0.2	0.65
C_{12}	0.05	0.05	1	0.8	1

Table 7.9 Normalized evaluation matrix

Criteria	Alternatives				
	A_1	A_2	A_3	A_4	A_5
C_1	0.8604	1.0000	0.8477	0.9012	0.8418
C_2	0.0800	0.3800	1.0000	1.0000	1.0000
C_3	1.0000	0.8000	0.2000	1.0000	0.6500
C_4	0.0143	0.0200	1.0000	0.1000	1.0000
C_5	1.0000	0.8000	0.6500	0.2000	0.2000
C_6	0.8000	1.0000	0.6500	1.0000	0.2000
C_7	0.6667	0.7500	0.7500	1.0000	0.8571
C_8	1.0000	1.0000	0.8000	0.6500	0.6500
C_9	1.0000	0.8000	0.2000	0.8000	0.6500
C_{10}	1.0000	0.8000	0.3500	0.2000	0.3500
C_{11}	1.0000	0.8000	0.6500	0.2000	0.6500
C_{12}	1.0000	1.0000	0.0500	0.0625	0.0500

have to be in the unit interval [0, 1]. The weighted evaluations of each alternative with respect to the criteria are then ordered in descending order. The results are shown in Table 7.10. These

Table 7.10 The ordered set of evaluations of the alternatives

Order	Alternatives				
	A_1	A_2	A_3	A_4	A_5
1	0.6600	0.6600	0.5595	0.6600	0.5556
2	0.6600	0.6600	0.4290	0.6600	0.4290
3	0.6600	0.5280	0.4290	0.5948	0.4290
4	0.6600	0.5280	0.2640	0.5280	0.2829
5	0.6600	0.5280	0.2475	0.3300	0.2310
6	0.6600	0.5280	0.2310	0.2145	0.2145
7	0.5679	0.4714	0.2145	0.1320	0.2145
8	0.5280	0.3300	0.1320	0.1320	0.1320
9	0.3300	0.3300	0.1320	0.0943	0.1320
10	0.3300	0.3300	0.0528	0.0660	0.0528
11	0.3300	0.2475	0.0165	0.0528	0.0165
12	0.2200	0.1389	0.0094	0.0206	0.0094

values are however very uncertain and therefore the optimal decision is also uncertain. Stochastic simulation is used to take the uncertainty of the data into account. The values shown in Table 7.9 were considered as expectations, and the standard deviations were computed by using υ =0.05, 0.1, 0.3, and 0.5 as the coefficient of variation. Normal distribution was assumed for all matrix elements. For each particular value of υ, 100 simulations were performed. For each run a random Table 7.9 was generated, for each case the corresponding Table 7.10 of ordered set of evaluations was computed. Based on this table the value of F_k was computed for each case. These results were obtained by using the fuzzy quantifier of *many of them* corresponding to $\theta = 0.333$ and by applying this optimism degree the order weights were calculated by using (7.23). The resulted weights vector became [0.007, 0.021, 0.035, 0.049, 0.062, 0.076, 0.090, 0.104, 0.118, 0.132, 0.146, 0.160]. Thus we generated 100 sample elements of F_k for each selection of υ and alternative k. Based on these samples the expectations and variances were estimated for each alternative and each choice of υ. The resulted expected values, $\mathrm{E}(F_k)$ and variances, $\mathrm{Var}(F_k)$ are shown in Table 7.11. The maximum expected values and the minimum variances are indicated for each scenario with boldface numbers (Zarghami and Szidarovszky 2009).

These results show that with respect to the $\mathrm{E}(F_k)$ values alone, alternative 1 is the most preferred project and alternatives 2 and

Table 7.11 Expected values and variances of the combined goodness measures

	Alternatives				
	A_1	A_2	A_3	A_4	A_5
$\upsilon = 0.05$					
$E(F_k)$	**0.4366**	0.3530	0.1330	0.1578	0.1316
$Var(F_k)$	0.0005	0.0003	**0.0000**	0.0001	**0.0000**
$\upsilon = 0.1$					
$E(F_k)$	**0.4282**	0.3462	0.1304	0.1548	0.1291
$Var(F_k)$	0.0019	0.0013	**0.0002**	0.0003	**0.0002**
$\upsilon = 0.3$					
$E(F_k)$	**0.4586**	0.3708	0.1397	0.1658	0.1382
$Var(F_k)$	0.0217	0.0142	**0.0020**	0.0028	**0.0020**
$\upsilon = 0.5$					
$E(F_k)$	**0.4427**	0.3579	0.1349	0.1600	0.1334
$Var(F_k)$	0.0573	0.0375	0.0053	0.0075	**0.0052**

The maximum expected values and the minimum variances are indicated for each scenario with boldface numbers

4 are the second and third most preferred ones. However, comparing the variances and knowing that a smaller variance represents a more robust choice, it is clear that with respect to the variances alone, alternatives 3 and 5 are the most preferred projects and the previous best decision, alternative 1 is the least preferred one. This contradiction can be addressed by using the certain equivalent including both $E(F_k)$ and $Var(F_k)$. By using relation (7.1), alternative j is considered the best if and only if for all $k \neq j$:

$$E(F_j) - \beta Var(F_j) \geqslant E(F_k) - \beta Var(F_k), \tag{7.27}$$

and with known values of the expectations and variances this is the case, when

$$\begin{aligned} &\beta \leqslant \frac{E(F_j) - E(F_k)}{Var(F_j) - Var(F_k)} \text{ if } Var(F_j) - Var(F_k) > 0 \\ &\text{and} \\ &\beta \geqslant \frac{E(F_j) - E(F_k)}{Var(F_j) - Var(F_k)} \text{ if } Var(F_j) - Var(F_k) < 0 \\ &\text{for } k = 1, 2, \ldots, j-1, j+1, \ldots, m. \end{aligned} \tag{7.28}$$

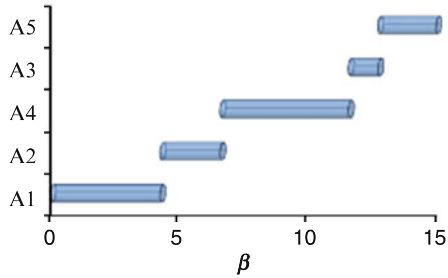

Fig. 7.5 The intervals of optimality with $\upsilon = 0.5$

These inequalities provide the range of β such that any particular decision alternative j is the best. The optimality intervals of β for each decision alternative are shown in Fig. 7.5 where the value of $\upsilon = 0.5$ and the quantifier *many of them* were selected.

7.5.2 Conservation or New Water Transfers?

7.5.2.1 Problem Description

The large city of Tabriz in Northwestern Iran is the capital of the East Azerbaijan Province (Fig. 7.6) and its urban water system faces several major challenges.

Mean annual precipitation in Tabriz is around 300 mm, which is very small compared to the worlds' average of 800 mm. The Ajichi River is the only permanent river near the city. Due to the high agricultural developments in the upper watershed and to the low water quality this supply is not enough for the city. Groundwater is extracted around its yield capacity, 40 (million cubic meter per year, MCM/Y) as shown in Fig. 7.7.

The population of the city is growing by a high rate. Equation (7.29) is used to predict the population of the city in the next years:

$$Pop_t = P_1(1 + r)^{t-1}, \tag{7.29}$$

where Pop_t is the total population of the city in year t; P_1 is the population of the city in the base year of 2009 (1.45 millions); r is

Fig. 7.6 The position of the Tabriz city in the Iran's map

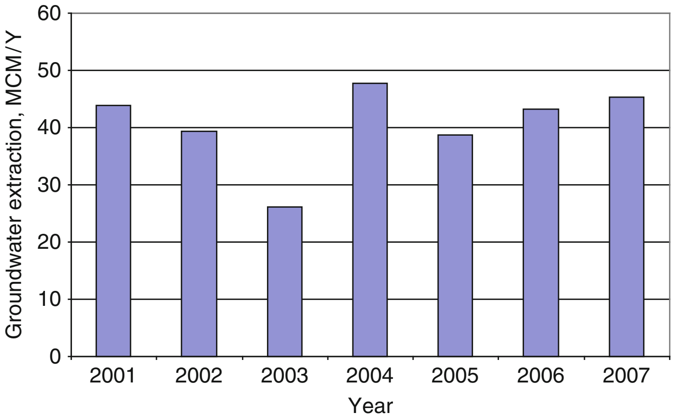

Fig. 7.7 Groundwater utilization for Tabriz city in recent years

the natural growth rate of the city which is estimated from the national statistics to be about 2%.

Another difficulty is that the water distribution pipelines are aged and the purified water leaks by the rate of 20% from the network. In this study leakage detection and the rehabilitation of the water distribution networks are considered as the main conservation measures.

Because of these shortcomings two water transfer pipelines (from Zarrinerud and Nahand reservoirs) were planned and are already in operation. To extend the water transfer from Zarrinerud, a second pipeline is under investigation. The outflow of the Zarrinerud and Nahand reservoirs is the Urmia Lake. Due to the dry condition of the lake there is a very serious problem in using its recharge resources, which would result in a limited amount of water diversions from the incoming rivers.

This study attempts to compare the conservation measures (leak detection and repair) to the new water transfer line from Zarrinerud by using a multi-criteria framework (Zarghami 2010). The uncertainty of the parameters and the constraints will be modelled by using probabilistic and fuzzy methods.

7.5.2.2 Criteria

The idea of IWRM requires that the optimal utilization of the resources has to be determined in a multi-criteria environment. According to the results of questionnaires obtained from the DMs, the main objectives of the Tabriz urban water problem are maximizing water supply, minimizing cost, and minimizing the environmental hazard in a sustainable way.

Maximizing Water Supply

The water supply per capita, S_t, from four different sources can be given as

$$S_t = ((1 - lr)\ [G_t + Z_{1t} + Z_{2t} + N_t] + L_t)/Pop_t, \tag{7.30}$$

where lr is the average water leakage rate in the network (assumed to be 20% in this case); G_t is the groundwater withdrawal rate from the aquifer (MCM/Y); Z_{1t} is the transferred water amount from the Zarrinerud reservoir by using the existing line (MCM/Y); Z_{2t} is the transferred water amount from the Zarrinerud reservoir by the new line (MCM/Y); N_t is the transferred water amount from the Nahand reservoir (MCM/Y); L_t is the conserved water amount by leakage detection and repairment (MCM/Y). All parameters refer to year t.

The water supply should satisfy the demand and satisfying the basic per capita water need is the basic priority. This basic annual need is considered about 85 m^3 per capita with zero annual growth rate because of the recent critical droughts in the region. However the satisfaction of this constraint is uncertain. If we force the optimization model to strictly follow the condition that supply has to be greater than or equal to the demand, then it will result in non-feasible solutions. Therefore the satisfaction degree of this uncertain constraint is modelled by using a hyperbolic fuzzy membership function (Fig. 7.8). The mathematical equation of the membership value, m_t, is as follows:

$$m_t = 0.5 + 0.5 \tanh(2S_t/D - 1.2), \tag{7.31}$$

where D is the water demand per capita. Our objective is to maximize this function.

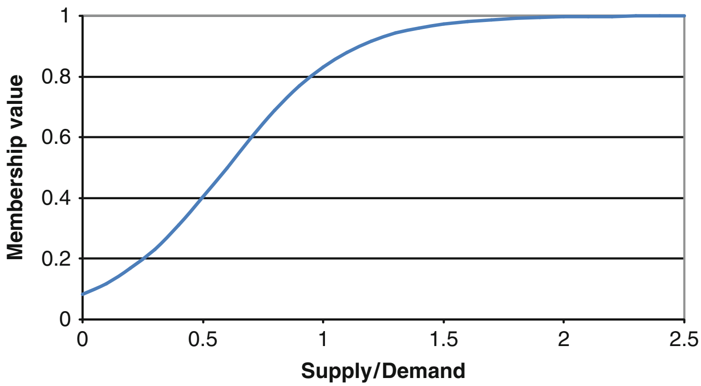

Fig. 7.8 The fuzzy membership function of water supply

Minimizing Cost

The cost function of the water supply to this city is defined as

$$C_t = 250G_t + 1,117Z_{1t} + 1,520N_t + 1,500L_t + \begin{cases} 1,117Z_{2t} + 100,000 & \text{if } Z_{2t}>0 \\ 0 & \text{if } Z_{2t} = 0, \end{cases} \tag{7.32}$$

where C_t is the total cost of water supply in the city (million IR Rials per year, 10,000 IR Rials ≈ 1 US$ in 2009). We want to minimize the value of C_t. Notice that the new water transfer from Zarrinerud requires an annual fixed cost due to the investment in pipelines and other infrastructures. This fixed cost is added to the other cost terms only if $Z_{2t} > 0$.

Minimizing Environmental Hazard

The Urmia Lake is now under great pressure, mainly due to water transfers and the diversions of its natural inflows. Figure 7.9 shows its water level shortfall in recent years.

Therefore, there is a strong conflict between environmentalists and the water supplying companies. The main objective of the environmental organization is therefore to minimize the amount of new water transfers. A simple decreasing criterion function is defined for the water transfer by the second line of Zarrinerud:

$$E_t = (37.2 - Z_{2t})/37.2, \tag{7.33}$$

which should be maximized. The term 37.2 is the maximum water amount that could be transferred with the second Zarrinerud line.

7.5.2.3 Constraints

The constraints deal with the limitations of the water resources. The available water resources are assumed to have probabilistic

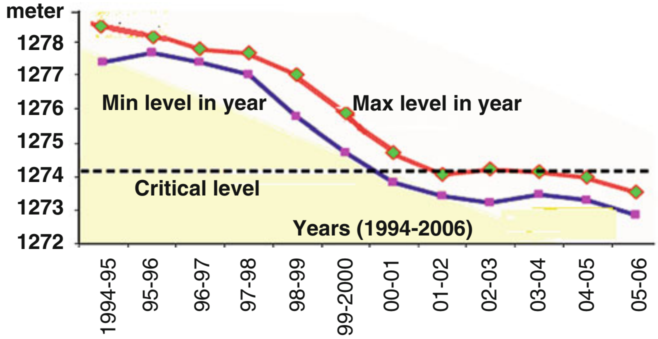

Fig. 7.9 The water level shortfalls in the Urmia Lake (modified from CIWP 2008)

Table 7.12 Characteristics of the uncertain variables

Decision variables	Average (MCM/Y)	Coefficient of variation	Probability of acceptance (%)	Deterministic limit (MCM/Y)
G_t	52	0.1	80	47.6
Z_{1t}	56	0.3	80	41.9
Z_{2t}	56	0.4	80	37.2
N_t	29	0.3	80	21.7

nature and are normally distributed. In modelling this uncertainty the chance constraint method is used. It needs the average values, the coefficients of variation, and also the probability level of accepting each constraint. The relevant parameter values are presented in Table 7.12. The last column of this table is calculated by using the chance constraints method as it was already described in Sect. 7.2.3.

Based on the data of Table 7.12, the deterministic constraint on G_t can be written as

$$0 \leqslant G_t \leqslant 47.6. \tag{7.34}$$

The maximum capacity of the water transfer from the Zarrinerud lines should be constrained as

$$0 \leqslant Z_{1t} \leqslant 41.9 \text{ and } 0 \leqslant Z_{2t} \leqslant 37.2 \tag{7.35}$$

for the existing and new lines. Maximum capacity of water transfer from Nahand should satisfy the following constraint:

$$0 \leqslant N_t \leqslant 21.7. \tag{7.36}$$

Repairing the network could conserve up to 15% of the total supplied water amount (Baumann et al. 1979) so the conserved water amount, L_t, is constrained as

$$L_t \leqslant el \times (G_t + Z_{1t} + Z_{2t} + N_t), \tag{7.37}$$

where el is the effect of leakage detection which can be selected from the range of 0–15%. However, in this study based on a questionnaire a rational rate of 8% is assumed. Notice that all variables are positive.

7.5.2.4 The Optimization Model and Results

The composite objective function of the equivalent deterministic multi-criteria optimization problem is modelled by maximizing the distance from the nadir point, as it was already described in Sect. 3.6. Because of using nonlinear terms in the criteria, the model is a nonlinear programming problem (NLP). The weights of the three main criteria $\{w_S, w_C, w_E\}$ have been also varied to create different scenarios. In Table 7.13 the desired and the worst possible values of the three criteria are presented. These values are obtained by maximizing and minimizing each criterion as a single objective problem, subject to the constraints of the model.

The distance from the nadir is as follows:

$$\begin{aligned} u_t = \{ & (w_S(m_t - 0.0)/(1.0 - 0.0))^p \\ & + (w_C(C_t - 253E9)/(0.0 - 253E9))^p \\ & + (w_E(E_t - 0.0)/(1.0 - 0.0))^p\}^{1/p}. \end{aligned} \tag{7.38}$$

Table 7.13 Ideal and worst values and weights of the objectives

Objective function	Ideal value	Worst value	Weight
Water supply	1.0	0.0	0.6
Cost (billions IR Rials)	0.0	253	0.2
Environmental protection	1.0	0.0	0.2

This utility function of the DM represents only year t. If a time period of T years is considered, then the overall objective is to maximize the sum of the discounted utility values through the entire period:

$$U_t = \sum_{t=1}^{T} \frac{u_t}{(1+d)^{t-1}}, \tag{7.39}$$

where d is the discount rate. This new objective emphasises the "dynamic efficiency" of the decision making (Griffin 2006) and gives the optimal decisions for each year t during the planning horizon $t = 1, 2,\ldots, T$.

The resulted NLP model was solved by using the Conopt solver in the GAMS (2006) software. The decision variables were G_t, Z_{1t}, Z_{2t}, N_t and L_t, for all t representing the supply terms. The optimum annual plan (for year 2009) is shown in Fig. 7.10 by using the parameter of $p = 2$, usually used in compromise programming. The discount rate is assumed to be $d = 10\%$ by the 15 years planning horizon. The results, which are described in more detail in Zarghami (2010) show that in the optimal plan, water transfer from the Zarrinerud could supply 41.9 MCM/Y with 80% reliability. In addition, groundwater can supply 40% of the water demand. The water company should not extract more than this amount due to the possible degradation of the aquifer. Surprisingly leakage detection could save about 9 MCM/Y and it can postpone the installation of the second transfer

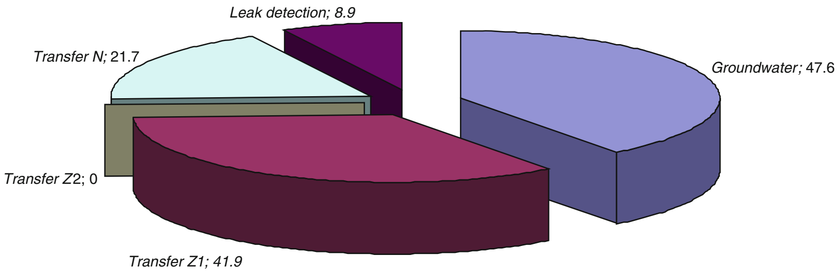

Fig. 7.10 Optimum water supply plan of Tabriz city (MCM/Y, year 2009, $p = 2$, Zarghami, 2010)

line from Zarrinerud. Therefore a more sustainable environment for the present and future generations can be guaranteed.

We also analysed the sensitivity of the results by changing the values of the weights and other scalar parameters. The strategy of the optimal solution always was the same. The results indicate that in order to solve the conflict we should use the conservation measures like leak detection rather than using new water resources. However in the following years, due to population growth and high water demand, new transfers will become mandatory.

Appendix. Cumulative Distribution of the Standard Normal Variable Function $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right) dx.$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6627	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.7	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.0	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.3	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.4	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

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