FEDERATION
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PRECONTRAINTE

Recommendations

Practical design of structural concrete

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CEB-FIP

SEPTEMBER 1999

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FIP Commission 3 on Practical design
Working group on Recommendations for
Practical design of structural concrete

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First published by SETO, 1999

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ISBN 1 874266 48 4

Contents

Foreword

These Recommendations have been prepared by a Working Group of FIP Commission 3 on Practical Design. The work represents an update of the previous FIP Recommendations *Practical Design of Reinforced and Prestressed Concrete Structures* published in 1984.

The present document fully explains the member design and detailing by means of strut-and-tie models (STM). Reference is made to the CEB-FIP Model Code 90 and CEB Manuals as well as other FIP publications.

It is the wish of Commission 3 that this document will be of direct interest to consultants, contractors and authorities and that it will help the use of a consistent design and detailing tool like the strut-and-tie-models.

The Commission wishes to express its thanks for the work done by the Working Group and particular thanks to the convenor M. Miehlbradt, to K.-H. Reineck who edited the Recommendations and to A. J. Threlfall who did the proof reading of the final draft.

Julio Appleton

Chairman

FIP Commission 3 on Practical Design

Scope

(1) These Recommendations apply to structural concrete using normal weight aggregates for all types of structures, such as buildings and bridges. Structural concrete comprises all concrete used for structural purposes from plain concrete through to applications with ordinary non-prestressed reinforcement, pretensioned or post-tensioned tendons or their combinations.

(2) The principles of these Recommendations also apply to the assessment of existing structures.

(3) The Recommendations are intended for use by practising engineers to enable them to design in according to modern concepts. The rules are given in a concise form suitable for qualified engineers with adequate experience in design and detailing.

(4) The rules are based on the CEB/FIP Model Code 1990 (MC 90) to which frequent reference is made in the right-hand margins of this document.

1 Principles

MC 90 Section

1.1 General

1.6

(1) The design of structures should involve the following steps:

(a) check of the Ultimate Limit State (ULS)

(b) check of the Serviceability Limit State (SLS)

(c) compliance with detailing practice

(d) compliance with technological requirements

(e) compliance with durability requirements.

(2) Depending on the type of structure or the construction method employed, either the ULS or the SLS can be taken as the primary design criterion. In many cases only one of these checks will be needed if, according to experience, there is no doubt that the other one is respected *a priori*. In all cases, detailing practice and technological requirements should be carefully observed, because they are as important to the serviceability and durability of concrete structures as checks by calculation. The overall structural integrity is particularly dependent on adequate dimensioning and proper detailing, especially at geometrical or load discontinuity regions (D-regions).

(3) The primary focus of structural design should be directed towards a careful consideration of overall or global structural behaviour and the achievement of an efficient flow of forces throughout the structure. The effects of potentially damaging restraints and aggressive environmental factors should also be considered.

(4) In general, the various loadcarrying members of a structure should be interlinked so as to ensure a satisfactory overall performance with regard to structural stability and robustness. In particular, it should be ensured that the structure cannot be subjected to progressive collapse as a consequence of localized damage due to abnormal use or accident.

1.2 Ultimate Limit State (ULS)

1.6.2

(1) The ULS verifications should be based on clear and realistic models of structural behaviour that simulate the correct failure mechanisms under ultimate loads. For these calculations, the theory of plasticity (PT) provides a simple and efficient approach in many cases. Non-linear analysis methods of a more general character may also be used, particularly in cases of instability.

(2) In all cases, it must be ensured that the structure is sufficiently ductile, allowing for restraint effects to be able to reach the assumed ULS without premature brittle failure. In a sufficiently ductile structure the effects of temperature, creep, shrinkage and foundation settlements have, in general, insignificant influence on the ultimate loadcarrying capacity.

(3) The ULS condition is satisfied if the following symbolic equation is respected for all relevant combinations of actions:

$$F_{act,d}\,(\gamma_g G;\ \gamma_q \Sigma Q) \quad \leq \quad F_{res,d}\,(f_{ck}/\gamma_c;\ f_{sk}/\gamma_s) \qquad (1.1)$$

design value of load or actions $\leq$ design value of loadcarrying capacity

where the notation is as follows, with associated partial safety coefficients in parenthesis:

G = permanent actions (mean values) (γ_g)

Q = variable actions (nominal values) (γ_q)

f_{ck} = characteristic concrete strength (γ_c)

f_{sk} = characteristic strengths of reinforcing or prestressing steel (γ_s)

(4) Since it is very unlikely that the maximum values of all variable actions Q will occur at the same time, representative values as defined later may be introduced. In the case of two or more variable actions, combination values characterized by the coefficient ψ_0 may be used (see section 6.2.2). MC 90 Section

(5) The effect of prestressing may be considered as either external forces on the action side or internal forces on the resistance side.

(6) The above general criterion (3) is satisfied, if all significant sections of the structure fulfil the following condition:

$S_d(\gamma_g G; \gamma_q \Sigma Q)$	$\leq$	$R_d(f_{ck}/\gamma_c; f_{sk}/\gamma_s)$	(1.2)
design value of critical combination of action effects	$\leq$	design value of resistant action effects	

However, the overall structural integrity should also be ensured by checking arrangement and the anchorage of reinforcement, especially for discontinuity regions.

(7) The distribution of internal forces in the structure shall satisfy the conditions of equilibrium. This is always the case for elastic distributions. Thus, if the structure has already been designed for serviceability conditions (SLS), the same distribution multiplied by an appropriate load factor can often be used to check the required capacity at ULS.

(8) For structures in which equilibrium is effected by the deformations of the members, the equilibrium condition shall be formulated on the deformed structure. However, equ.(1.2) may be used if the second order effects are included in the term S_d. It should be noted that the ultimate capacity of the section is not always attained. 1.6.3

(9) Fatigue problems are normally not critical in reinforced and prestressed concrete, as long as severe cracking under the appropriate actions ($\gamma_q = 1$) is prevented (see section 6.9). 1.6.4

1.3 Serviceability Limit State (SLS) 1.6.6

(1) The SLS verifications should be based on clear and realistic models of structural behaviour including, where relevant, cracking and time-dependent effects.

(2) Normally the SLS calculations are based on the theory of elasticity (ET). In certain cases, non-linear analysis methods may be used.

(3) Depending on the particular case, the SLS check should be done by one or more of the following three methods:

(a) by limiting stresses $\sigma_d \leq \sigma_{lim}$

(b) by limiting deformations (deflections or angles) $a_d \leq a_{lim}$

(c) by limiting crack widths $w_d \leq w_{lim}$

The limit values should be established on the basis of the functional requirements of the structure. Other requirements such as watertightness, tolerances, vibrations should also be checked in appropriate cases.

(4) For the SLS calculations, the effect of prestressing may be considered either on the action side, or in the material characteristics as an imposed deformation; however, normally it is considered as an external action. The prestressing force should be considered with its mean value.

(5) The check by calculation can sometimes be omitted by respecting minimum reinforcement or detailing regulations.

1.4 Design by testing Appendix C

(1) In special cases the design of structures or structural elements may be based on testing. The test should consider all possible unfavourable conditions for the real structure, including any possible reduction of the concrete tensile strength.

(2) The following rules should be applied:

(a) The test results have to be interpreted by means of realistic analytical models from which the influence of the principal parameters involved may be estimated.

(b) The basic principles of these Recommendations, notably the criteria of all limit states, have to be applied to this experimentally derived model.

(c) The partial safety factors have to be chosen conservatively according to adequate statistical and probabilistic considerations and the level of quality control.

(d) Major deviations from accepted principles or design rules, e.g. a bearing capacity that depends considerably on the concrete tensile strength, have to be justified either by increasing the safety margins, or by a test series that is large enough to allow the estimation of the representative loadcarrying capacity to be based on the 5% fractile strength of the materials used.

(e) The undertaking of alternative procedures to those outlined in these Recommendations must be subject to the control and agreement of an appropriate authority.

2 Material characteristics

MC 90 Section

2.1 Concrete

2.1.1 Concrete strength grades

2.1.2

(1) The present document applies to concrete with normal weight aggregates. For structural concrete containing a normal amount of reinforcement a density can usually be assumed of $25kN/m^3$.

(2) The design should be based on a concrete strength class defined by the characteristic compressive strength for a cylinder f_{ck} at the age of 28 days. If the strength is determined by testing cubes ($f_{ck,cube}$), the corresponding cylinder strength can be obtained by appropriate conversion factors given in Table 2.1.1 of MC 90. The testing conditions shall be in accordance with ISO 2736/2. 2.1.3.2

(3) The preferred concrete strength classes are given in Table 2.1, and the main mechanical properties are defined for each class, where: 2.1.1.2

f_{ck} = characteristic cylinder strength

f_{ctm} = mean tensile strength (see section 2.1.4 (1))

E_{cm} = mean value of modulus of elasticity

2.1.2 Design compressive strength

(1) The uniaxial design strength of concrete is:

$$f_{1cd} = \alpha f_{ck}/\gamma_c \tag{2.1}$$

where:

α = coefficient taking account of uniaxial strength in relation to strength of control specimen and duration of loading:

$\alpha = 1.00$ for SLS; $\alpha = 0.85$ for ULS

(other values may apply for other strain rates)

γ_c = partial safety factor:

$\gamma_c = 1.00$ for SLS: $\gamma_c = 1.50$ for ULS

(2) The variation of concrete strength with age depends on many parameters (e.g. curing conditions and cement type) and a universally applicable relationship cannot be given. In the absence of more accurate data the strength increase may be estimated from Fig 2.1, which is valid for two types of Portland Cement concrete.

2.1.3 Stress-strain diagram for concrete in compression

2.1.3.1 Stress-strain diagram for structural analysis and for SLS

2.1.4.2

(1) The modulus of elasticity for linear-elastic analyses is given in Table 2.1. The range of variation may extend from $0.7E_{cm}$ to $1.3E_{cm}$.

(2) For more refined analyses reference is made to the stress-strain diagram given in section 2.1.4.4.1 of MC 90.

2.1.3.2 Stress-strain diagram for ULS

6.2.2.2

(1) For dimensioning cross-sections the parabolic-rectangular stress-strain distribution of Fig 2.2 should be used for preference.

The maximum strain is defined as follows:

$$\varepsilon_{cu} = -0.0035 \quad \text{for } f_{ck} \leq 50\text{MPa} \tag{2.2 a}$$

$$\varepsilon_{cu} = -0.0035\,(50/f_{ck}) \quad \text{for } f_{ck} > 50\text{MPa} \tag{2.2 b}$$

Table 2.1 Preferred concrete strength classes and mechanical properties

	C20	C25	C30	C35	C40	C45	C50	C60	C70	C80
f_{ck} (MPa)	20	25	30	35	40	45	50	60	70	80
f_{ctm} (MPa)	2.2	2.6	2.9	3.2	3.5	3.8	4.1	4.4	4.6	4.8
E_{cm} (GPa)	30.5	32	33.5	35	36.5	37.5	38.5	41	42.5	44.5

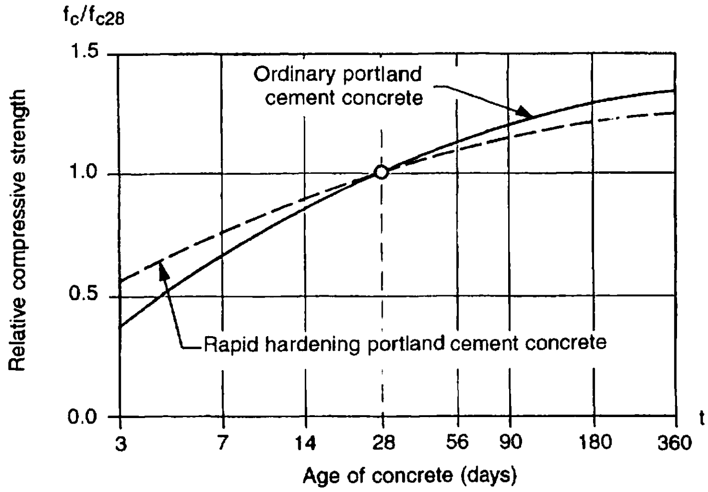


Fig 2.1 Variation of concrete strength with age

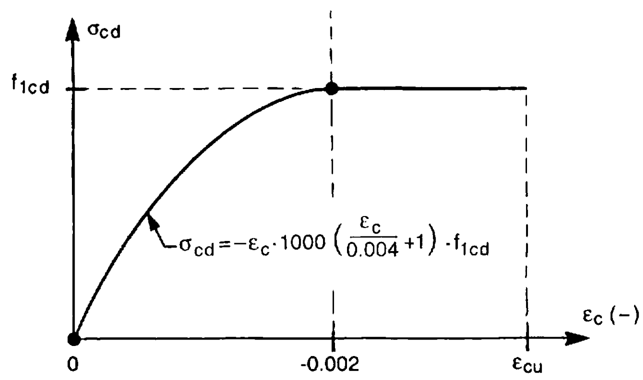


Fig 2.2 Preferred stress-strain distribution in concrete compression zones

(2) Other equivalent diagrams may be used, such as the bilinear diagram shown in Fig 2.3. A further simplification is provided by the uniform stress diagram given in section 5.3.2.

2.1.4 Tensile resistance and cracking of concrete 2.1.3.3

(1) The basic reference value for assessing the strength of concrete in tension is the uniaxial tensile strength, and its average value given in Table 2.1 is:

- for $f_{ck} \leq 50$MPa: $f_{ctm} = 0.30 f_{ck}^{2/3}$ (2.3 a)
- for $f_{ck} > 50$MPa: $f_{ctm} = 1.12 f_{ck}^{1/3}$ (2.3 b)

The lower and upper characteristic values given in Fig 2.4 are:

$f_{ctk,0.05} = 0.70 f_{ctm}$ and $f_{ctk,0.95} = 1.30 f_{ctm}$ (2.4)

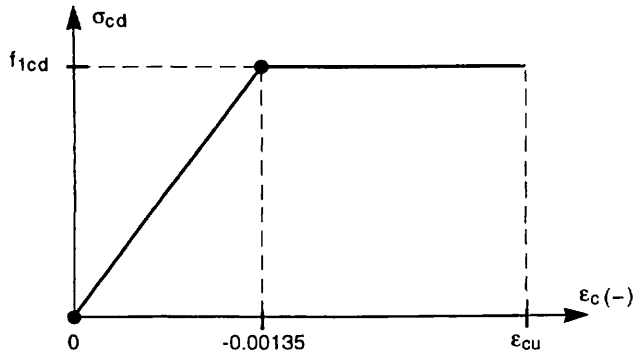


Fig. 2.3 Simplified bilinear stress-strain distribution in concrete compression zones

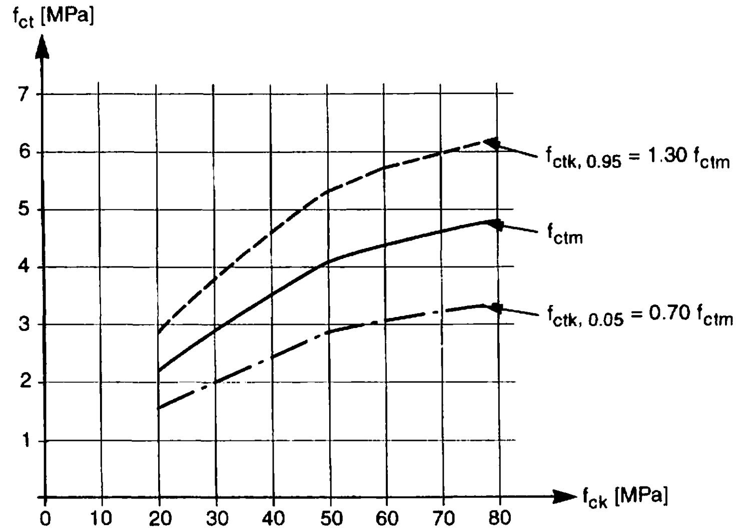


Fig 2.4 Design values for the uniaxial concrete tensile strength

(2) The design value for the uniaxial tensile strength of concrete is:

$$f_{1ct,d} = f_{ct} / \gamma_{ct} \tag{2.5}$$

where:

f_{ct}	= relevant value from equ. (2.3)
γ_{ct}	= 1.8 for ULS (or 1.0 if more unfavourable)
γ_{ct}	= 1.0 for SLS (or 1.3 in certain specific cases)

(3) The assessment of crack formation requires a realistic consideration of the fracture process in tension zones. An empirically derived practical rule, considering the size effect, is to average the tensile stress over a representative depth c_t of the stress diagram and compare it with the relevant value of the axial tensile strength. The representative depth may be taken as $c_t = 3d_g \leq 50$ mm, where d_g is the maximum aggregate size.

(4) As a practical application of the above rule, the maximum tensile stress at cracking (flexural tensile strength) may be derived for a rectangular section subjected to the cracking moment M_r combined with an axial force N_{Sd} as follows (Fig 2.5):

$$\sigma_{t,max} = f_{1ct,d} \frac{\left(1 - \frac{c_t}{h} v_t\right)}{\left(1 - \frac{c_t}{h}\right)} \leq 2 f_{1ct,d} \tag{2.6}$$

where:

v_t	$= \sigma_N / f_{1ct,d} = N_{Sd} / (b\, h\, f_{1ct,d})$ (+ for tension)
$f_{1ct,d}$	= design value for axial tensile strength according to equ. (2.5)

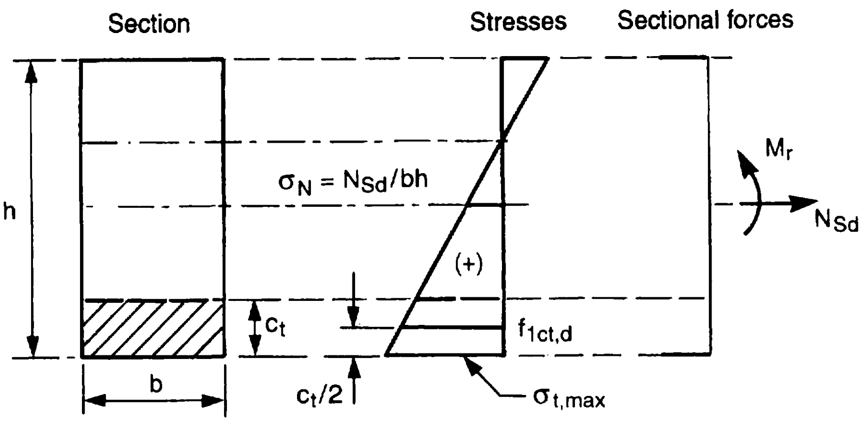


Fig 2.5 Flexural tensile strength of a rectangular section subjected to combined bending moment and axial force

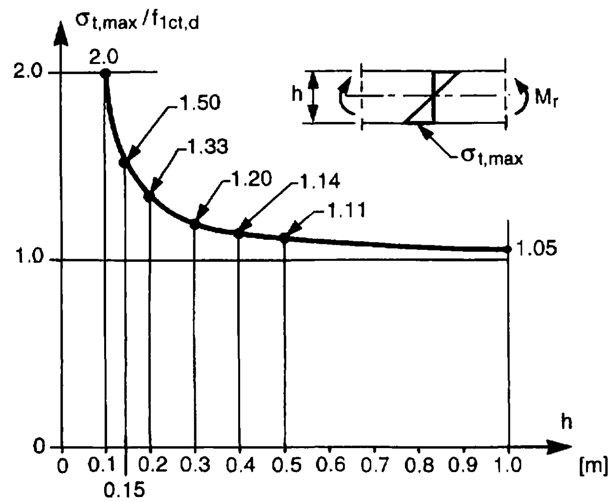


Fig 2.6 Flexural tensile strength as a function of the depth of the member (size effect) for c_t *= 50mm*

The maximum tensile stress for a section subjected to pure bending is shown in Fig. 2.6 as a function of the depth of the member.

2.1.5 Shrinkage and creep 2.1.6.4

(1) The deformations of concrete due to shrinkage and creep may vary considerably with the types of cement and aggregate, with the climate (temperature and humidity), with the member size and with the age at loading. The final values in Tables 2.2 and 2.3 are mean values, and apply to concrete of grades 20 to 50MPa subjected to a stress not exceeding $0.4f_{c,t0}$ at age t_0 of loading.

(2) The development of the shrinkage strain and of the creep coefficient with age may be estimated from Fig. 2.7.

(3) For special structures and other conditions more detailed information is required (see MC 90) or specific tests have to be carried out.

2.1.6 Coefficient of thermal expansion 2.1.8.3

The coefficient of thermal expansion may vary between 6 x 10^{-6}/°C and 15 x 10^{-6}/°C depending on the type of aggregates and the degree of saturation of concrete. For structural analysis a value of 10 x 10^{-6}/°C may be taken.

Table 2.2 Final value for shrinkage strain ε_{cs} $[10^{-3}]$ MC 90 Section

Atmospheric conditions	Effective member size $2A_c/u$ [mm]		
	50	150	600
Dry; indoors (RH = 50%)	-0.53	-0.51	-0.36
Humid; outdoors (RH = 80%)	-0.30	-0.29	-0.20

Table 2.3 Final value of the creep coefficient φ for concrete grades ≤ C50*

Age at loading t_o [days]	Atmospheric conditions					
	Dry (indoors) (RH = 50%)			Humid (outdoors) (RH = 80%)		
	Effective member size $2A_c/u$ [mm]					
	50	150	600	50	150	600
1	5.6	4.6	3.7	3.7	3.3	2.8
7	3.9	3.2	2.6	2.6	2.3	2.0
28	3.0	2.5	2.0	2.0	1.8	1.5
90	2.4	2.0	1.6	1.6	1.4	1.2
365	1.8	1.5	1.2	1.2	1.1	1.0

where: A_c = cross-sectional area of concrete
u = exposed perimeter of area A_c

*The values of the creep coefficient φ must be used in conjuction with the modulus of elasticity defined in Table 2.1. For creep-sensitive structures, characteristic values for the creep coefficient and shrinkage strain should be considered (see MC 90).

2.1.7 Fatigue strength 2.1.7

The properties of concrete in fatigue exhibit a large scatter, and the tensile strength especially should be used with caution. For ordinary buildings and bridges fatigue is rarely critical.

2.2 Reinforcing steel

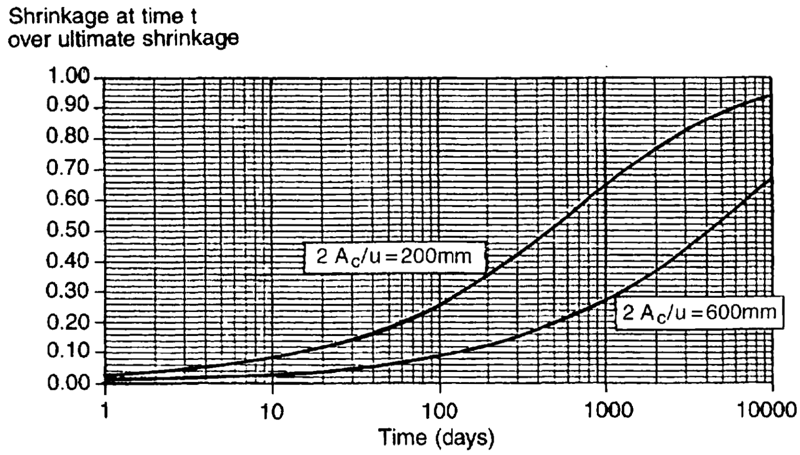
2.2.1 Steel grades 2.2.4.2

(1) The design may normally be based on a grade of ribbed steel selected from the values S400 or S500, where the numbers denote the characteristic strength f_{yk} (MPa) defined in section 2.2.2. Other values, according to national practice, may be chosen.

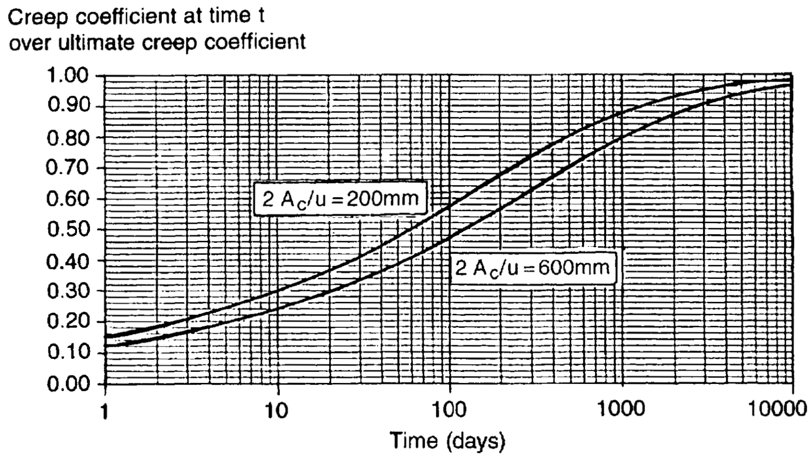
(2) As a criterion on ribbed surface the projected rib area as defined by the European Prestandard ENV 10080 may be chosen.

2.2.2 Tensile strength 2.2.4.1

(1) The characteristic strength f_{yk} is defined as the 5% fractile of the yield strength f_y or the 0.2%-proof stress (denoted as $f_{0.2}$).



(a) Development of shrinkage strain with time



(b) Average development of Φ_t with time for all environmental conditions

Fig 2.7 Development of shrinkage strain and of creep coefficient with time

(2) If the steel supplier guarantees a minimum value for f_y or $f_{0.2}$, that value may be taken as the characteristic strength.

(3) The design strength is:

$$f_{yd} = f_{yk} / \gamma_s \tag{2.7}$$

where γ_s = partial safety factor: $\gamma_s = 1.00$ for SLS; $\gamma_s = 1.15$ for ULS

2.2.3 Compressive strength

If the reinforcing steel is used in compression, normally the values f_{yk} and f_{yd} apply respectively, unless the actual value in compression (f_{yck}) is smaller than in tension (f_{ytk}). (Reference should be made to the approval document).

2.2.4 Modulus of elasticity and idealized stress-strain diagrams 2.2.4.3

(1) Due to the diversity and evolution of the manufacturing processes for bars and wires, various stress-strain diagrams may be encountered.

(2) As a simplification, actual stress-strain diagrams for all reinforcements of structural concrete may be replaced by an idealized characteristic diagram according to Fig. 2.8. The modulus of elasticity may be taken as E_s = 200GPa.

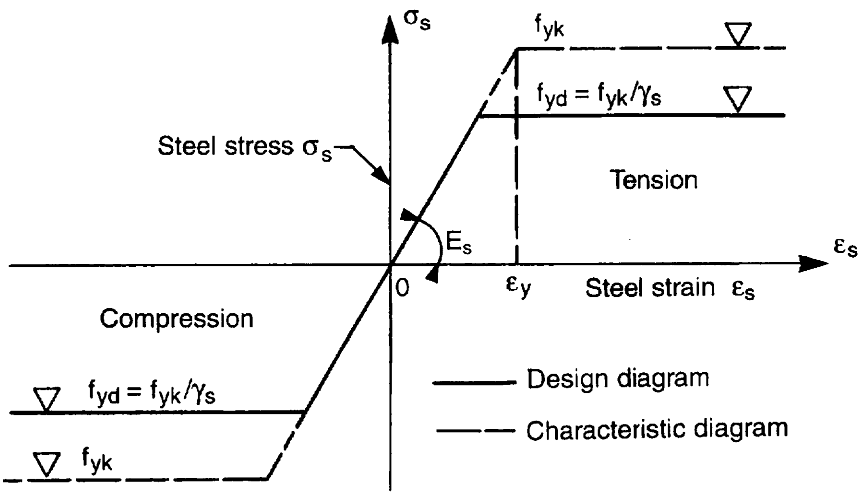


Fig 2.8 Idealized stress-strain diagram for reinforcing steel

2.2.5 High bond reinforcement

Bars and wires may be considered to be high bond reinforcement if the projected rib area f_R (defined by ENV 10080) complies with the following values:

$f_R \geq$ 0.039 for bar diameters 5 $\leq$ Ø $\leq$ 6mm

$f_R \geq$ 0.045 for bar diameters 6.5 $\leq$ Ø $\leq$ 8.5mm

$f_R \geq$ 0.052 for bar diameters 9 $\leq$ Ø $\leq$ 10.5mm

$f_R \geq$ 0.056 for bar diameters 11 $\leq$ Ø $\leq$ 40mm

2.2.6 Ductility 2.2.4.4

(1) Adequate ductility is necessary whether or not moment redistribution is taken into account in the calculations. For design purposes, this may be defined by minimum specified values for the characteristic value of the ratio f_t/f_y and the characteristic elongation ε_{uk} at maximum load as follows:

$(f_t/f_y) \geq 1.08$ and $\varepsilon_{uk} \geq 5\%$ (2.8)

The characteristic value of the ratio f_t / f_y corresponds to the 5% fractile of the relation between actual tensile strength and actual yield stress.

(2) If the above values are not respected, refer to MC 90.

2.2.7 Coefficient of thermal expansion 2.2.5.4

The coefficient of thermal expansion may be taken as 10 x 10^{-6}/°C.

2.2.8 Fatigue strength 2.2.4.5

(1) The characteristic fatigue strength $\Delta\sigma_{Rsk}$ is defined by S-N-curves.

(2) In the absence of test results the value at 10^6 cycles may be taken as 6.7.4

$\Delta\sigma_{Rsk}$ = 195MPa for straight bars.

For bent bars with a mandrel diameter d_b < 25 Ø, the value should be reduced by applying the following coefficient:

$\xi = 0.35 + 0.026 d_b/Ø$ (2.9)

(3) For welded bars, including tack welding and for butt joints, the value at 10^6 cycles may be taken as $\Delta\sigma_{Rsk}$ = 60MPa.

(4) For couplers, the manufacturer must justify the strength by test results or by means of technical approval documents.

2.3 Prestressing steel

2.3.1 Steel grades

2.3.4.1

The grade of a prestressing steel shall be specified by its characteristic strength $f_{0.1k}$ defined as the 5% fractile of the 0.1% proof stress, and its characteristic tensile strength f_{ptk} as $S(f_{0.1k}/f_{ptk})$.

2.3.2 Design strength

The design strength is defined as a simplification by:

$$f_{ptd} = 0.90 f_{ptk}/\gamma_s \tag{2.10}$$

where γ_s = partial safety factor as defined in section 2.2.2.

2.3.3 Relaxation

2.3.4.5

The relaxation values to be taken into account for the final prestress

can be obtained:

(a) from data given in the approval documents, or

(b) by using approximate values, or

(c) from the results of reliable relaxation tests.

In the absence of more accurate information, the final value relaxation loss for an initial stress $\sigma_{p0} = 0.7 f_{ptk}$ may be taken as 6% for low relaxation steels and 12% for other steels.

2.3.4 Modulus of elasticity and idealized stress-strain diagram

2.3.4.3

In the absence of more accurate information, the stress-strain-diagram in Fig 2.9 may be used. Unless more precise information is available, the modulus of elasticity of prestressing steel may be taken as:

- E_p = 205GPa for wires
- E_p = 200GPa for bars
- E_p = 195GPa for strands.

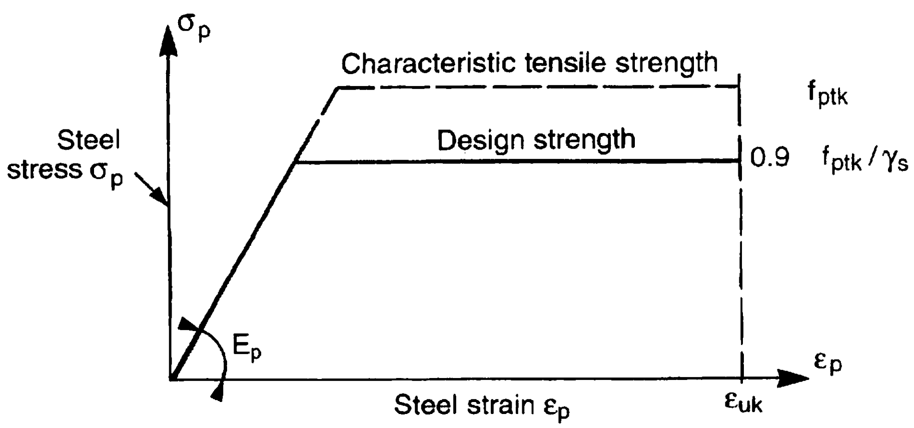


Fig 2.9 Stress-strain diagram for prestressing steel

Table 2.4 Fatigue strength $\Delta\sigma_{Rsk}$ (MPa) for steel embedded in concrete MC 90 Section

Number of cycles	$N = 10^6$	$N = 2 \times 10^6$	$N = 100 \times 10^6$
Pretensioning steel (straight):	185	175	120
Post-tensioning steel: • single layer of strand in plastic ducts (straight or curved)	185	175	120
• curved tendons in plastic ducts	160	150	95
• straight tendon in steel ducts	160	150	95
• curved tendons in steel ducts	120	110	65
• couplers	80	70	30

2.3.5 Bond properties of prestressing reinforcement

(1) The bond properties of prestressing reinforcement may be regarded as equivalent to that of high bond reinforcing bars, if the criteria in section 2.2.5 are satisfied.

(2) The bond properties of smooth wires should be determined, either on the basis of technical approval documents, or by means of tests corres-ponding to the conditions of use. Approximate values for the transmission and anchorage lengths are given in section 2.4.3.2.

2.3.6 Ductility 2.3.4.4

It should be shown that the ductility of the steel is adequate for its use in the event of a redistribution of stress. The unit elongation ε_{uk} at maximum load should be at least 3.5%.

2.3.7 Coefficient of thermal expansion 2.3.5.3

The coefficient of thermal expansion may be taken as $10 \times 10^{-6}/°C$.

2.3.8 Fatigue strength 2.3.4.6

(1) The characteristic fatigue strength $\Delta\sigma_{Rsk}$ is defined by S—N-curves.

(2) In the absence of test results, the values of $\Delta\sigma_{Rsk}$ given in Table 2.4 may be adopted for steel embedded in concrete. 6.7.4

2.4 Bond between concrete and reinforcement 6.9.3; 6.9.4

2.4.1 High bond reinforcement

(1) Bars and wires may be considered to be high bond reinforcement if the projected rib area f_R satisfies the conditions given in section 2.2.5.

(2) The bond stress τ_{bd} may be assumed to be constant over the anchorage length l_b of a straight bar of diameter Ø giving:

$$l_b = \varnothing f_{yd}/(4 f_{bd}) \qquad (2.11)$$ MC 90 Section

where:

$$f_{bd} = 1.05 f_{ctm} \qquad (2.12)$$

design value of bond strength for good bond conditions (including material safety factor)

(3) Values of f_{bd} and $l_b/\varnothing$ (for S500) for good bond conditions are given in Table 2.5 for the different concrete strength classes.

(4) The bond conditions are considered to be good for the cases shown in Fig 2.10. In all other cases the bond conditions are considered poor and the design value f_{bd} should be multiplied by 0.70.

(5) The limiting value of f_{bd} may be increased in the presence of pressure p transversely to the plane of the reinforcement, so that the anchorage length l_b may be reduced by the factor $1/(1 - 0.04p) \leq 1.5$. This value may be taken 2/3 at an end-anchorage over a support (C-C-T-node, see section 5.6.1).

(6) For bar diameters $\varnothing > 32$mm the limiting value of f_{bd} should be multiplied by $[(132 - \varnothing)/100]$.

Table 2.5 Design values of bond strength f_{bd} (MPa) and the anchorage length l_b as a multiple of the bar diameter $\varnothing$ (for S500)

	C20	C25	C30	C35	C40	C45	C50	C60	C70	C80
f_{bd}	2.3	2.7	3.0	3.4	3.7	4.0	4.3	4.6	4.8	5.0
$l_b/\varnothing$	47.3	40.3	36.2	33.0	29.4	27.2	25.3	23.6	22.6	21.6

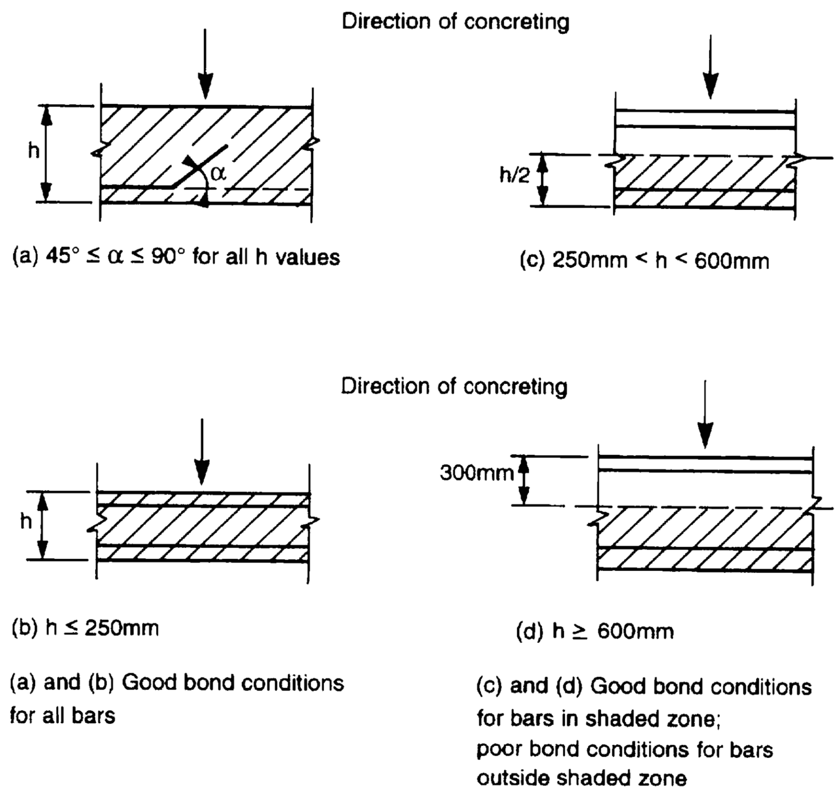


Fig 2.10 Conditions for good bond of the reinforcement

Table 2.6 Design values of bond strength f_{bpd} (MPa) for pretensioned tendons MC 90 Section

Type of tendons	C20	C25	C30	C35	C40	C45	C50	C60	C70	C80
7-wire strands	1.2	1.4	1.6	1.8	1.9	2.1	2.2	2.3	2.4	2.5
Intended or crimped wires	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	2.9	3.0

2.4.2 Bond of post-tensioned reinforcement 4.5.2

The bond properties of ribbed post-tensioned reinforcement in grouted tendons may be regarded as equivalent to that of high bond reinforcing bars if the criteria in section 2.2.5 are satisfied.

2.4.3 Bond of pretensioned reinforcement

2.4.3.1 Bond strength 6.9.11.2

The design value of the bond strength f_{bpd}, including the material safety factor, are given in Table 2.6. These values apply to good bond conditions as defined in Fig 2.10; in poor bond conditions the values should be multiplied by 0.7.

2.4.3.2 Transfer of prestress 6.9.11.4

At release of tendons, the prestress can be assumed to develop linearly from zero to its full value over the following transmission length l_{bpt}:

$$l_{bpt} = \alpha_1 \, \alpha_2 \, \alpha_3 \, Ø \, \sigma_{pi}/(4 f_{bpd}(t)) \qquad (2.13)$$

where:

α_1	= 1.00 for gradual release
	= 1.25 for sudden release
α_2	= 0.50 for verification at release of tendons
	= 1.00 for verifications at ULS
α_3	= 0.50 for 7-wire strands
	= 0.70 for intended or crimped wires with circular cross-section
$Ø$	= nominal diameter of tendon
σ_{pi}	= stress before release and before time-dependent losses
$f_{bpd}(t)$	= design value of bond strength according to Table 2.6 for concrete strength at time of release

2.4.3.3 Anchorage at ULS 6.9.11.5

The anchorage length at the ultimate limit state may be calculated as:

$$l_{bpd} = l_{bpt} + \alpha_4 \, Ø(\sigma_{pd} - \sigma_{p\infty})/(4 f_{bpd}) \qquad (2.14)$$

where:

l_{bpt}	= transmission length according to equ.(2.13)
α_4	= 0.80 for 7-wire strands
	= 1.00 for indented or cromed wires with circular cross-section
$Ø$	= nominal diameter of tendon
σ_{pd}	= stress to be anchored
$\sigma_{p\infty}$	= stress after all losses
f_{bpd}	= design value of bond strength according to Table 2.6

3 Prestressing

3.1 Definition and types of prestressing

3.1.1 Definition of prestress

(1) The prestress is applied by a construction controlled process (prestressing) by stressing tendons (prestressing reinforcement) relatively to the concrete member. Generally, the prestress is defined by the relative deformation between the prestressing steel and the concrete member.

(2) Other means of prestressing are not considered in this document.

3.1.2 Types of prestressing

(1) The prestress considered in these Recommendations is exerted by tendons made of high-strength steel (wires, strands or bars). 4.1

Tendons may be used:

(a) internal to the concrete, and

(a1) pre-tensioned, or

(a2) post-tensioned; in this case they may be bonded by grouting, or provisionally or permanently unbonded.

(b) external to the concrete; they may then be

(b1) totally within the external outline of the structure, or

(b2) partially or totally outside (except at anchorage points) the outline of the structure. However, fatigue of such structures requires special considerations and is not covered in this document.

Further reference is made to the FIP Recommendations *Acceptance of post-tensioning systems.*

(2) The prestress may be

- non-detachable and non-adjustable (which is always the case for pretensioning and internal bonded tendons),
- non-detachable but adjustable,
- detachable and adjustable.

(3) Anchorages may be active or passive or coupling.

3.2 Initial prestress 4.2

3.2.1 Prestressing steel

The tensile stress in the tendons should not exceed the following values:

(a) during tensioning: $\sigma_{pi} = 0.80 f_{ptk}$

(b) after transfer of prestress: $\sigma_{p0} = 0.75 f_{ptk}$

3.2.2 Time of prestressing

Where particular rules are not given, the time when prestressing takes place should be fixed with due regard to the following factors:

(a) Conditions for the deformation of the component

(b) Safety with respect to the actual compressive strength of concrete

(c) Safety with respect to local stresses MC 90 Section

(d) Safety with respect to the anchorages of the tendons

(e) Advantage of applying some prestress at an early stage

(f) Early creep deformation in anchorage zones.

3.3 Decrease of prestressing force ('losses of prestress')

3.3.1 General

The decrease of the prestressing force, the so-called 'losses of prestress', should be determined thoroughly, because the prestressing force is considered to be at its mean value only (see section 3.4.1).

3.3.2 Losses before releasing the tendons (pretensioning) 4.3.2

The following losses should be considered in design:

(a) losses due to friction at the deflectors; and losses due to movement in the anchoring devices (at the abutments) when anchoring on a prestressing bed,

(b) losses due to relaxation of the pretensioned tendons during the period that elapses between the tensioning of the tendons and the prestressing of the concrete.

3.3.3 Immediate losses 4.3.3

(1) The following influences should be considered in design:

(a) the instantaneous concrete deformation

(b) friction between tendon and sheathing

(c) draw-in of the anchorage

(d) steam curing, etc.

(2) The losses due to friction may be estimated as follows:

$$\Delta\sigma_{pi} = \sigma_{pi}\,[1 - \exp\{-\mu(\alpha + kx)\}] \tag{3.1}$$

where:

$\Delta\sigma_{pi}$ = loss of stress in tendon at a distance x from active end anchorage

σ_{pi} = stress in tendon at the anchorage

μ = coefficient of friction

α = sum of the angular displacements along x

k = unintentional angular displacement or wobble (per unit length)

(3) If more accurate information is not available, the following μ-values can be accepted as being representative for unlubricated tendons. The coefficient k depends essentially on the accuracy with which the theoretical shape of the tendons is attained in practice; the following are approximate values for friction and wobble effects. The following μ-values may be adopted:

(a) metal sheathing:

- for strands: $\mu = 0.18$–0.20
- for smooth wires: $\mu = 0.18$–0.20
- for wires that are not smooth: $\mu = 0.30$
- for bars: $\mu = 0.30$

(b) other sheathing:

- for plastic ducts: $\mu = 0.14$
- for unbonded mono-strands $\mu = 0.05–0.07$

The following k-values may be assumed:

- in general: $k = 0.005–0.010\text{m}^{-1}$
- in segmental construction: $k = 0.010–0.020\text{m}^{-1}$

(4) With external prestressing, the friction is concentrated at deviation devices.

3.3.4 Time-dependent losses

4.4.1

(1) The evaluation of the time-dependent losses due to shrinkage and creep of the concrete and relaxation of the steel should take into account the interdependence of these phenomena.

(2) The time-dependent losses are calculated by considering the following two reductions of stress within the steel:

(a) the reduction of stress, due to the reduction of strain caused by the deformation of the concrete due to creep and shrinkage, under quasi-permanent actions:

(a1) for bonded tendons, the local deformation at the level of the tendons has to be considered;

(a2) for unbonded tendons, the deformation at the level of the tendons averaged along the whole length between the anchorages has to be taken into account;

(b) the reduction of stress due to the relaxation of steel under tension.

(3) The relaxation of steel is modified by the reduction of strain due to creep and shrinkage of the concrete. This interaction may be taken into account in a simplified manner by reducing the value of relaxation at constant strain by 20%.

(4) An assessment of the total loss of prestress due to shrinkage, creep and relaxation can be carried out by means of the following formula where all compressive strains and stresses, as well as prestressing losses are considered as negative:

$$\Delta\sigma_p = \frac{\alpha\phi(t,t_0)(\sigma_{cg} + \sigma_{cp0}) + E_p\varepsilon_{cs} + 0.8\Delta\sigma_{pr}}{1 + \alpha\frac{A_p}{A_c}\left(1 + \alpha\frac{A_c y_p^2}{I_c}\right)\left(1 + \chi\phi(t,t_0)\right)} \tag{3.2}$$

where:

α $= E_p/E_c$

$\phi(t, t_0)$ = creep coefficient

t_0 = age of concrete at prestressing

t = age of concrete at time considered

σ_{cg} = stress in the concrete at the level of the tendons due to permanent actions (excluding prestressing)

σ_{cp0} = initial stress in the concrete at the level of the tendons due to the prestressing alone

$\Delta\sigma_{pr}$ = loss of stress in the tendon (negative) due to relaxation (at constant strain equal to value at time to) acting alone

χ = ageing coefficient, which may be taken as 0.8 for long-term calculations

3.4 Design considerations

MC 90 Section

3.4.1 Design value of prestress and requirements

4.6.3
4.6.4

(1) In general, the effect of prestress has to be considered as described in section 1.3 (4) for SLS and in section 1.2 (5) for ULS.

(2) The design value for prestress is generally taken as the mean value at SLS and ULS, given by:

$$P_m = P_i - \Delta P \tag{3.3}$$

where:

P_m = mean value at time t for section at distance x from origin

P_i = initial prestress at origin

ΔP = immediate and time-dependent losses

(3) For most cases it is sufficient to consider the values of prestress at two different times:

(a) Initial prestress ($t = 0$) after transfer of prestress:

$$P_{m0} = P_i - \Delta P_0 \tag{3.4}$$

(b) Long-term prestress ($t = \infty$):

$$P_{m\infty} = P_i - \Delta P_0 - \Delta P_\infty \tag{3.5}$$

In general, P_{m0} is critical for combination with the effects of permanent actions at transfer, whereas $P_{m\infty}$ is to be considered in combination with the total actions.

(4) At transfer of prestress, the Serviceability Limit State shall be verified for P_{m0} in combination with permanent actions. Restraining effects during transfer shall be duly considered, such as caused by deformations of the scaffolding or by longitudinal restraints, etc.

(5) The tendon anchorages shall satisfy the requirements of the FIP Recommendations *Acceptance of post-tensioning systems* for the load transfer from the anchorage into the structure (local zone around anchorage).

(6) The transfer of the tendon force from the local zone around the anchorage into the D-region of the member or structure can be verified according to section 6.5.7. At Serviceability Limit State the initial prestressing force P_i shall be used as the applied tendon force. At Ultimate Limit State the characteristic tendon force ($A_p f_{ptk}$) shall be used as the design value of the tendon force.

(7) Due consideration shall be paid to the forces in local zones caused by deviations of tendons; the verification can be carried out according to section 6.5.7.

3.4.2 Design of prestress

3.4.2.1 Definitions

(1) The following definitions are not intended as a classification of structures with regard to prestressing, but rather as an indication concerning the design criteria to be applied. As a matter of principle, the whole range from full prestress to no prestress (reinforced concrete) is allowed, and it is up to the designer to choose the most appropriate degree of prestressing for a given structure.

(2) The degree of prestress may be defined in either of the following ways described in (a) and (b).

(a) The mechanical degree of prestressing pertians to the ULS, and is defined as:

$$\lambda = \frac{A_p 0.9 f_{ptk}}{A_s f_{yk} + A_p 0.9 f_{ptk}} \tag{3.6}$$

where:

A_p = area of prestressing steel in critical sections
A_s = area of reinforcing steel in critical sections
f_{ptk} = characteristic tensile strength of prestressing steel
f_{yk} = characteristic strength of reinforcing steel

The mechanical degree of prestressing is, amongst others, particularly helpful in comparing designs based on different loading regulations and in appreciating test results.

(b) The degree of load balancing pertains primarily to the SLS, and is convenient for shallow members where deflection control for a given level of SLS loading is an important consideration. It is defined as:

$$\kappa = S_p/S_g \quad \text{or} \quad \kappa = M_p/M_g \quad \text{or} \quad \kappa = p/g \tag{3.7}$$

(general) (normal) (special)

where:

S_p = (total) action effect due to prestress
S_g = action effect due to permanent load
M_p = bending moment due to prestress
M_g = bending moment due to permanent load
p = equivalent load due to prestress
g = permanent load

Table 3.1 Different practical values for the amount of prestress

	Average prestress[(a)] [N/mm²]	Load balancing[(b)]	Tendon ratio per surface[(c)] [kg/m²]
Office floors	1.0–2.0	0.6–1.0	4–8
Raft foundations	0.75–1.50	0.6–1.0	10–20
Precast beam bridge (spans 20–35m)	–	–	15–25
In situ box girder bridge (spans 35–100m)	–	–	30–40
Precast segmental box girder bridge (spans 35–100m)	–	–	35–45

(a) Total effective prestress divided by total concrete section
(b) Total effective equivalent load due to prestress divided by total permanent loads
(c) Total weight of prestressing steel divided by surface area of structure

3.4.2.2 Design criteria for prestress

(1) The design criteria should be established in order to combine the advantages of reinforced and prestressed concrete behaviour. The prestress affects the structural behaviour favourably under service load conditions. The cracking load is increased and the steel stresses after cracking are lower, resulting in smaller crack widths. Furthermore, permanent load deflections can always be controlled by prestressing (post-tensioning).

(2) The design criteria for prestressing may be based on the deflection control requirement (common for slabs) or by limiting tensile stresses in the concrete to avoid cracking (common for beams) under frequent or permanent loads.

(3) Typical amounts of prestress in structures vary considerably with span, loading conditions, local Codes, etc. The amount of prestress can be expressed in different ways, and the values in Table 3.1 on the previous page can be often found and may serve as a starting point in a conceptual design.

4 Technological details and durability requirements

MC 90 Section

4.1 Exposure classes

1.5.2

Environmental conditions mean those chemical and physical actions to which the concrete is exposed and which result in effects that are not considered as loads or action effects in structural design. In the absence of a more specific study, these environmental conditions may be classified in the exposure classes given in Table 4.1 overleaf.

4.2 Durability design criteria

8.4.1

(1) In order to satisfy the durability requirements the following criteria should be used:

(a) An appropriate structural form should be selected at an early stage of the project, in order to avoid especially vulnerable structural arrangements and to secure adequate access to all critical parts of the structure for inspection and maintenance.

(b) An appropriate quality of concrete in the outer layer ('skin') of the structural elements should be obtained. A dense, well-compacted and well-cured, strong and low-permeability concrete is needed, which should not exhibit map cracking. Also, an adequate thickness of concrete cover should be provided.

(c) Adequate detailing of all structural concrete elements should ensure the integrity of critical surfaces, corners and edges in order to avoid any unforeseen concentration of aggressive influences.

(d) Under specified environmental conditions and/or for small diameter reinforcing bars or single prestressing wires, nominal crack widths should be controlled under specified load conditions to avoid depassivation during the specified design life.

(e) Under strongly aggressive environmental conditions, protective surface coatings may be needed.

(2) All exposed concrete surfaces should be adequately drained, so that only preplanned ponding may occur. 8.4.2

(3) Drainage of water over concrete should be limited as much as possible, and drainage over joints and seals should be avoided.

(4) In the selection of structural form, adequate care should be taken to provide robustness against deleterious liquid or gaseous substances penetrating into the structure.

(5) The geometry of exposed structural components and the form, type and placing of joints (including construction joints), connections, and supports should be chosen so as to minimize the risks of local concentrations of deleterious substances. These concentrations may develop on the surface of the structure as well as within the concrete, entering by permeation, diffusion, capillary action or similar.

(6) Care should be taken in the detailing of facades of buildings and structures in order to allow easy drainage of water and facilitate cleaning by washing.

(7) Surface areas subjected to wetting, splashing or water accumulation should be kept as small as possible.

4.3 Preferred nominal diameters for reinforcing bars

(1) The range of preferred nominal diameters ∅ [mm] for bars is as follows: 6, 8, 10, 12, 14, 16, 20, 25, 32, 40.

(2) Preferred diameters for wires used in welded mesh lie in the range 5 to 12mm in steps of 0.5mm, plus ∅14 and ∅16.

Table 4.1 Exposure classes related to environmental conditions

Exposure class	Examples of environmental conditions
1. Dry environment*	Interior of buildings for normal habitation or offices
2. Humid environment (a) without frost	 Interior of buildings where humidity is high (e.g. in commercial laundries) Exterior components Components in non-aggressive soil and/or water
(b) with frost	Exterior components exposed to frost Components in non-aggressive soil and/or water and exposed to frost Interior components when the humidity is high and exposed to frost
3. Humid environment with frost and de-icing agents	Interior and exterior components exposed to frost and de-icing agents
4. Sea-water environment (a) without frost	 Components partially immersed in sea-water or in the splash zone Components in saturated salt air (coastal area)
(b) with frost	Components partially immersed in sea-water or in the splash zone and exposed to frost Components in saturated salt air and exposed to frost
5. Aggressive chemical environment**	Refer to MC90

* This exposure class is valid only as long as during construction the structure or some of its components is not exposed to more severe conditions over a period of several months
** May occur alone or in combination with classes 1-4

4.4 Cover to reinforcement 8.4.3

(1) The minimum distance c_{min} between any concrete surface and the nearest reinforcing bar, prestressing tendon or sheathing respectively, should be obtained from Table 4.2. The values are absolute minimum values with no negative tolerances allowed.

For exposure classes 2 to 4 the minimum values may be reduced by 5mm, if the concrete strength class is at least C40.

(2) The nominal value, c_{nom}, is equal to the minimum value plus tolerance according to the rule 10.4

$$c_{nom} = c_{min} + \text{tolerance} \tag{4.1}$$

Tolerance should be taken as 10mm unless in an individual case it can be demonstrated that a lower value is obtainable (e.g. in the case of intensified quality control). The tolerance should not be less than 5mm. The relevant values of c_{nom} are given in Table 4.2.

Table 4.2 Cover to reinforcement

MC 90 Section

Exposure	Reinforcing bars		Prestressing reinforcement	
	c_{min}	c_{nom}	c_{min}	c_{nom}
1	10	15 or 20	20	25 or 30
2	25	30 or 35	35	40 or 45
3, 4	40	45 or 50	50	55 or 60
5	*	*	*	*

* Depends on the individual type of environment encountered

(3) To ensure that bond forces are safely transmitted and to prevent spalling of the concrete, the minimum cover to any bar, tendon or sheathing of diameter ∅ should be at least equal to ∅ (section 6.5.7.1).

(4) When anchoring is made by means of bends, hoops or loops, it is recommended that in the anchorage zone, the thickness of the cover should be at least equal to 3∅.

(5) Where fire resistance is necessary, other limits may apply.

4.5 Clear bar distances in the horizontal and vertical directions

4.5.1 Generally

(1) Bars in different horizontal layers should be arranged in vertical planes, leaving sufficient gaps between them to allow for internal vibrators. However, bundling is allowed, see section 5.8. 9.1.3.2

(2) Horizontal gaps between parallel single bars or vertical gaps between horizontal layers of parallel bars, should be at least equal to the largest bar diameter but not less than 20mm.

(3) The maximum size of the aggregate should be chosen to facilitate concreting and adequate compaction of the concrete surrounding the bars.

4.5.2 Members with post-tensioned prestressing reinforcement 9.1.7

(1) The sheathing should be located so that:

- the concrete can be safely placed without damaging the sheathing.
- the concrete can resist the forces from the sheathing in the curved parts before and after tensioning.
- no grout will leak into other sheathing during the grouting process.

(2) The minimum horizontal and vertical clear spacings of sheathings are given in Fig 4.1 overleaf. Rules for bundling are given in section 5.8.

4.5.3 Members with pretensioned prestressing reinforcement

The minimum horizontal and vertical clear spacings of tendons are given in Fig 4.2 overleaf. Rules for bundling are given in section 5.8.

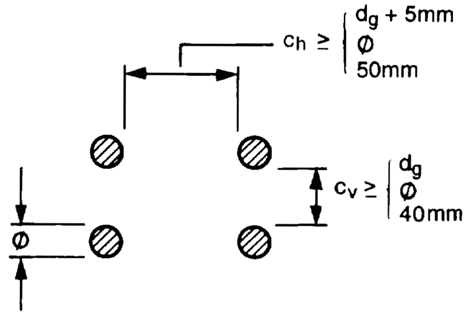


Fig 4.1 Minimum clear spacing for sheathings (where d_g *is the maximum aggregate size)*

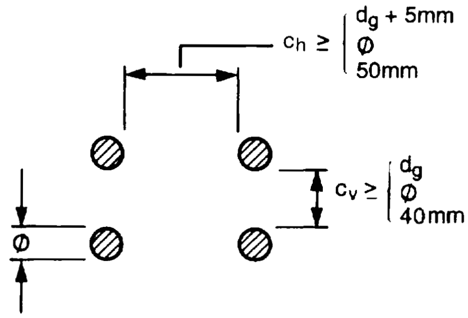


Fig 4.2 Minimum clear spacing for pretensioned tendons (where d_g *is the maximum aggregate size)*

5 Strengths of ties, struts and nodes of strut-and-tie models

MC 90 Section

5.1 General

The elements of a strut-and-tie model are ties, struts and nodes. A tie is normally the resultant of a layer of reinforcing bars or prestressing reinforcement. A strut may represent the resultant of either a parallel or a prismatic compression stress-field (e.g. a compression chord or inclined struts in webs) or a fan-shaped compression stress-field (Fig 5.1). A node is a confined volume of concrete, where struts either intersect or are deviated by ties anchored in the node. Nodes also occur where the reinforcement deviates or is spliced.

5.2 Strength of steel ties

6.2.4

(1) At the ULS the tension reinforcement normally yields, so that the stress in the tie is (see sections 2.2.2 and 2.3.2):

- for reinforcing steel: $\sigma_{sd} = f_{yd}$ (5.1)
- for bonded prestressing steel: $\sigma_{pd} = f_{ptd}$ (5.2)

In the latter case it is assumed that the steel has been prestrained to such an amount that the additional stresses due to loads lead to yielding.

(2) The resisting force of a tie is:

$$F_{Rtd} = A_s f_{yd} + A_p f_{ptd} \tag{5.3}$$

(3) In cases where the prestress is applied as an external load in the analysis only the reserve capacity at decompression, beyond the stress $\sigma_{p,Po}$ due to the prestressing force, can be utilized:

$$\Delta\sigma_{pd} = f_{ptd} - \sigma_{p,Po} \tag{5.4}$$

(4) The stress-strain diagrams are given in sections 2.2.4 and 2.3.4.

5.3 Strength of struts

5.3.1 Concrete strut in uniaxial compression

6.2.2.2

(1) The basis for any design strength is the uniaxial design strength f_{1cd} of concrete in compression as defined by equ. (2.1).

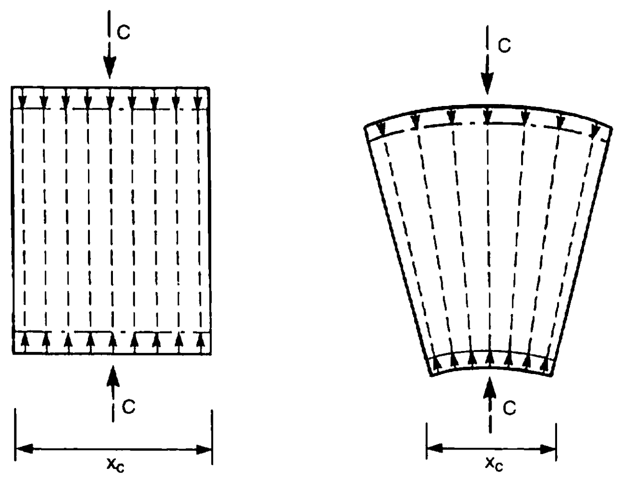


Fig 5.1 Typical compression stress-fields for struts

(2) The capacity of a strut may be determined from the compatibility of strains by using realistic stress distributions or by using constitutive laws respectively. Normally the parabolic-rectangular stress diagram (Fig 2.2) is recommended, but as a simplification the bilinear diagram (Fig 2.3) may also be used.

5.3.2 Capacity of a parallel compression field or prismatic strut

(1) The capacity of a prismatic strut or a parallel compression field of a strut-and-tie model is reduced to an effective strength $f_{cd,\ eff}$ for several reasons. This strength depends on the state of stress and strain, as well as on the crack widths and geometrical disturbances, and it may be expressed in terms of two alternative reduction-factors ν_1 and ν_2:

$$f_{cd,\ eff} = \nu_1 f_{1cd} \qquad \text{or} \qquad f_{cd,\ eff} = \nu_2 f_{1cd} \tag{5.5}$$

The resisting force of a strut is:

$$F_{Rcd} = A_c f_{cd,eff} \tag{5.6}$$

where $A_c = x_c\, b$ = area of strut (Fig 5.1)

(2) The reduction factor ν_1 applies to an uncracked strut, if a rectangular stress block is used instead of a realistic stress distribution. In the case of a compression chord of a beam with a linear strain distribution, the resultant force may be calculated with an average stress $f_{cd,eff}$ over the full depth of the compression zone (Fig 5.2) where:

$$\nu_1 = (1 - f_{ck}/250) \tag{5.7}$$

The corresponding maximum strain at the extreme concrete fibre is:

$$\varepsilon_{cu} = -\ 0.004 + 0.002\ (f_{ck}/100) \tag{5.8}$$

(3) The factor ν_2 covers several influencing effects:

(a) $\nu_2 = 1.00$ (5.9 a)

for uncracked struts with uniform strain distribution

(b) $\nu_2 = 0.80$ (5.9 b)

for struts with cracks parallel to the strut and bonded transverse reinforcement; the reduction is due to the transverse tension and to the disturbances by the reinforcement and the irregular crack surfaces.

(c) $\nu_2 = 0.60$ (5.9 c)

for struts transferring compression across cracks with normal crack widths, e.g. in webs of beams.

(d) $\nu_2 = 0.45$ (5.9 d)

for struts transferring compression across large cracks, e.g. in members with axial tension or flanges in tension.

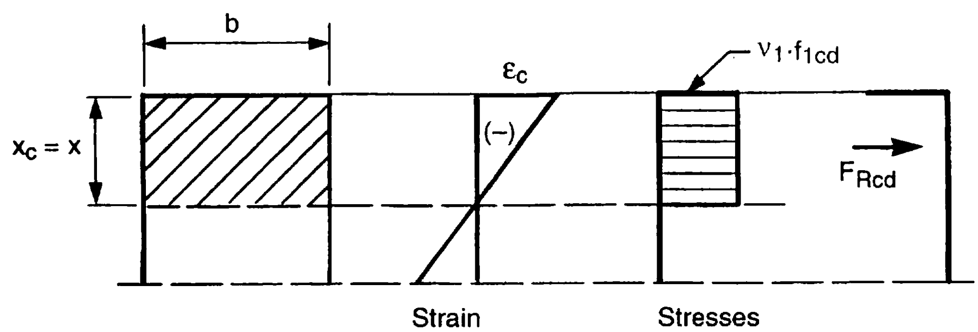


Fig 5.2 Rectangular stress-distribution (stress-block)

(4) Alternatively, for cases (c) and (d) the effective strength of the struts may be assessed by means of the constitutive laws for friction, given in section 5.5. In this case, due consideration has to be paid to realistically assessing the crack spacing and the strain condition. MC 90 Section

5.3.3 Reinforced struts 6.2.5

(1) Reinforcing steel bars should only be considered effective in compression struts if they are placed parallel to the strut, as in a compression chord or in a column.

(2) The bars must be sufficiently secured against buckling by transverse reinforcement, the amount of which is given in section 8.3.2.

(3) The resisting force of a reinforced strut is:

$$F_{Rcd} = A_c f_{cd,eff} + A_{sc}\sigma_{scd} \tag{5.10}$$

where:

A_c = area of the compression strut

$f_{cd,\,eff}$ = $\nu_1 f_{1cd}$ in chords of beams and columns

A_{sc} = area of compression reinforcement

σ_{scd} = $\varepsilon_s E_s \leq f_{ycd}$ = compressive stress in A_{sc}

5.3.4 Confined concrete struts 3.5.2

(1) The capacity of a strut may be increased by means of an appropriate amount of transverse reinforcement confining the concrete core. The increase in strength may be assessed by the following relationship shown in Fig 5.3:

$$f_{ccd} = (1 + 1.6\,\alpha\,\omega_w) f_{1cd} \tag{5.11}$$

where α and ω_w are defined in Fig. 5.3 for typical cases.

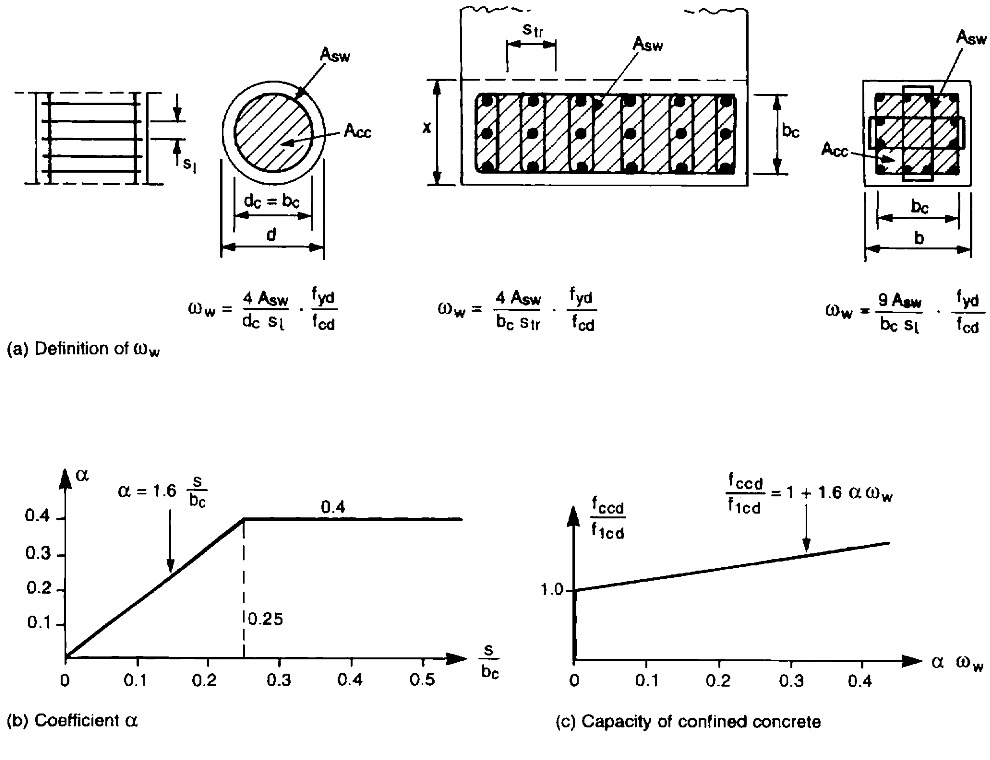


Fig 5.3 Capacity of confined concrete struts

(2) The resisting force of a confined strut is: MC 90 Section

$$F_{Rcd} = A_{cc} f_{ccd} \tag{5.12}$$

where A_{cc} = area of concrete core encompassed by the confining transverse reinforcement.

(3) The increase in ductility due to a confining reinforcement is important for the behaviour of members under reversible loading and earthquakes, and it may be assessed by the relationships given in MC 90.

5.3.5 Struts crossed by bars or ducts 6.2.2.4

(1) If a strut of width b is crossed by bars or ducts with the sum of the diameters greater than $b/6$, the compressive stresses should be calculated on the basis of a reduced width:

$$b_{red} = b - \eta \Sigma \varnothing \tag{5.13}$$

where:

$\Sigma\varnothing$ = sum of the diameters of bars or ducts at the most unfavourable level

η = coefficient depending on the stiffness of bars or ducts:

$\eta = 0.5$ for bonded bars or grouted ducts

$\eta = 1.2$ for unbonded tendons and ungrouted ducts

(2) The resisting force of the strut is given by:

$$F_{Rcd} = (x_c\, b_{red})\, f_{cd,eff} \tag{5.14}$$

where:

x_c = depth of the strut (Fig 5.1)

$f_{cd,eff}$ = effective strength according to equ. (5.5)

(3) Due consideration should be given to the provision of transverse reinforcement.

5.4 Strength of concrete ties 6.2.3

(1) Although reinforcement is normally provided to take the major tensile forces, the ultimate capacity of a member often relies on the tensile resistance of concrete, as in the case of members without transverse reinforcement or for bond and anchorage. The axial tensile strength defined in section 2.1.4 is the basic reference value for assessing the strength of concrete ties or cracking loads.

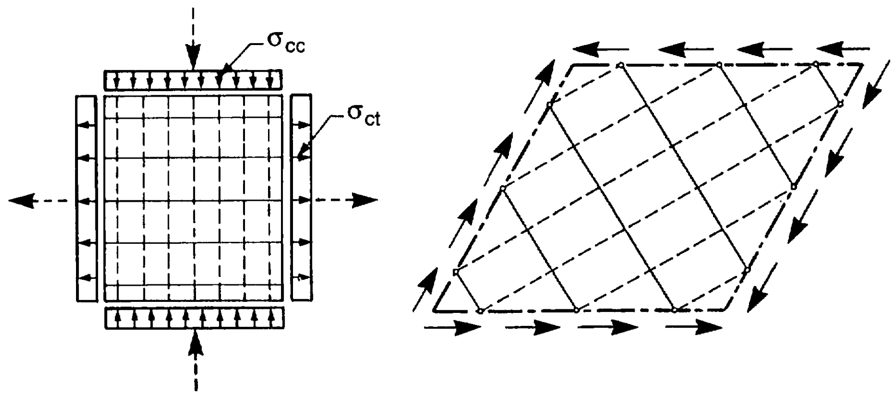
(2) For modelling uncracked regions in members the parallel biaxial stress field in Fig 5.4 may be used. It may also be used to represent the behaviour of the concrete between cracks, as in the case of webs without transverse reinforcement (e.g. slabs without shear reinforcement, see section 6.7.2), whereby the concrete tensile strength cannot be fully utilized.

(3) The biaxial bottle-shaped tension-compression stress field in Fig 5.5 may be used for modelling uncracked D-regions. Its capacity relies on transverse tensile stresses in the concrete, and these depend on the ratio of the width a of the loading plate to the total width b. For a ratio of about $a/b = 0.50$ the lowest value for the cracking load is attained with the pressure $p_a = 0.60 f_{1cd}$.

5.5 Transfer of forces by friction across interfaces 6.10.2

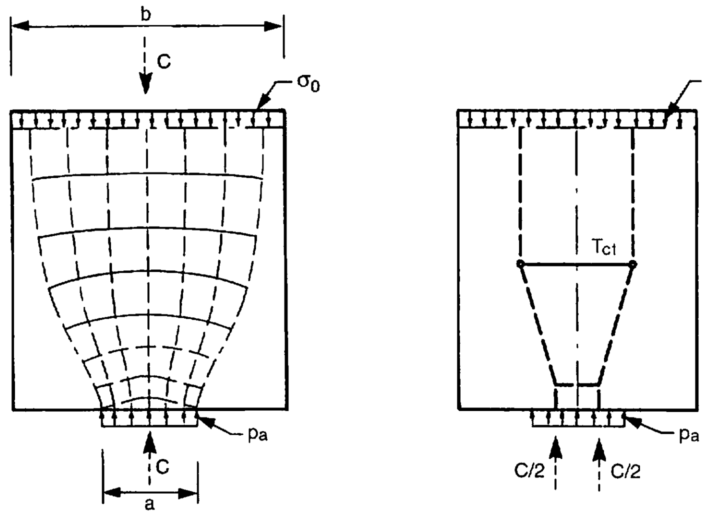
5.5.1 General

(1) The capacity for the transfer of compressive forces across an interface by means of concrete-to-concrete friction depends on the conditions of the interface and the material characteristics of the adjacent members. The capacity may generally be



(a) Rectangular plate-element (b) Skew plate-element for webs

Fig 5.4 Parallel biaxial tension-compression field in the concrete



(a) Stress trajectories (b) Strut-and-tie model

6.10.2

Fig 5.5 Bottle-shaped tension-compression field in the concrete for determining cracking loads of D-regions

assessed by a shear-friction law:

$$\tau_{fd} = \beta f_{ctd} + \mu\, \sigma_{fd} \tag{5.15}$$

where:

β	= coefficient (Table 5.1 or equ. (5.18))
σ_{fd}	= normal stress on interface (+ = compression)
μ	= friction coefficient (Table 5.1)
f_{ctd}	= design value of concrete tensile strength (see also Table 2.1)

(2) The strength of a strut transferring a parallel or prismatic compression field across an interface or a joint inclined at an angle α_f (Fig 5.6) may be derived from equ. (5.15) and be expressed in terms of an effective strength:

$$f_{cd,eff} = \nu_3\, f_{1cd} \tag{5.16 a}$$

where:

$$\nu_3 = \beta \frac{f_{ctd}}{f_{1cd}} \frac{1 + \tan^2 \alpha_f}{\tan \alpha_f - \mu} \leq 1.0 \tag{5.16 b}$$

α_f = angle as defined in Fig 5.6 a

Table 5.1 Coefficients β and μ for the friction resistance of joints

Interface condition	β	μ
Very smooth e.g cast against steel or plywood formwork	0.1	0.6
Smooth e.g. slipformed or extruded, or left without further treatment after compacting	0.2	0.6
Rough or toothed (indented) e.g with exposed aggregate, roughened by raking or brushing, or provided with shear keys (indentations)	0.4	0.9

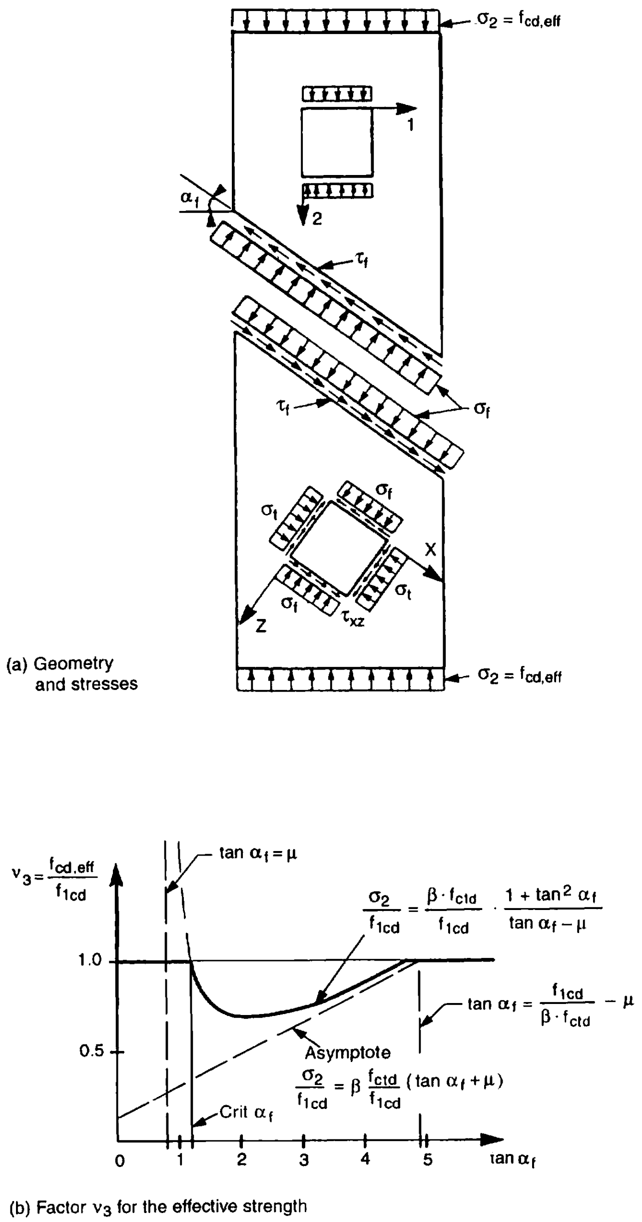


Fig 5.6 Transfer of compressive stresses of a strut across an interface or a joint by friction

5.5.2 Transfer of strut across joints

MC 90 Section

(1) The values in Table 5.1 may be assumed for the coefficients β and μ in equ. (5.15) according to the relevant interface condition.

(2) The maximum shear stress to be transferred is:

$$\tau_{fd} = 0.25 f_{1cd} \quad (5.17)$$

5.5.3 Transfer of strut over cracks (crack friction) 6.9

For crack widths not larger than 0.5mm the capacity for the transfer of forces due to concrete-to-concrete friction over cracks may be assessed for concrete classes $f_{ck} \leq 50$MPa as follows:

$$\tau_{fd} = 0.30 f_{ctd} + 1.70\sigma_{fd} \quad (5.18)$$

5.6 Strength of nodes and anchorages

5.6.1 General

(1) The nodes shall be dimensioned and detailed so that all forces are balanced and any ties are anchored or spliced securely. The concrete is bi- or triaxially stressed; either in compression only in C-C-C-nodes (C = compressive force; strut) connecting struts, or in compression and tension in C-C-T- or C-T-T-nodes (T = tension force; tie) if bonded reinforcement is anchored or spliced. The nodes must generally be verified by the following checks:

- verification of the anchorage of ties in the node.
- verification that the maximum compressive stress does not exceed the effective compressive strength.

(2) The anchorage length is defined by the beginning and end of the deviations of the compression field by the reinforcement. Any anchorage of reinforcement requires transverse tension, which should normally be taken by reinforcement (additionally or existing), but often has to be provided by concrete tensile stresses. Therefore the transfer of the forces into the struts should be thoroughly investigated 3-dimensionally, e.g. in the plane of load transfer and perpendicular to it.

(3) For C-C-T- or C-T-T-nodes the check of the compression stresses is often not critical, because either the anchorage length or the bearing pressure at the support governs the node dimensions. If in a general case such a check is required, a value of $\nu_2 = 0.85$ may be taken for the effective strength, considering any tension induced by the anchorage of bars.

5.6.2 Compression nodes

(1) In nodes connecting only compression struts the bi- or triaxial hydrostatic compressive strength of the concrete may be utilized:

- for biaxial compression: $f_{2cd} = 1.20 f_{1cd}$ (5.19 a)
- for triaxial compression: $f_{3cd} = 3.88 f_{1cd}$ (5.19 b)

When utilizing such high strengths it must be secured that the magnitude of the transverse compression is given and its value shall be critically examined. The flow of the forces in the structure must also be followed up, because transverse tension may occur requiring corresponding reinforcement.

(2) Typical compression nodes are shown in Fig 5.7. The capacity of the node may be checked by the local pressure σ_{c0} (= σ_{c1} in Fig 5.7) under the loaded area A_{c0} (see Fig 5.8): 3.3.1

$$\alpha_{c0} = F_{Sd} / A_{c0} \leq f_{2cd} \sqrt{A_{c1}/A_{c0}} \leq f_{3cd} \quad (5.20)$$

where A_{c1} = maximum area inscribed in A_c with the same shape and centroid as the loaded area A_{c0}

Special considerations should be made in the case of non-uniformly distributed pressures and additionally applied horizontal forces.

MC 90 Section

(3) In special cases such as prestressing anchorages higher local strengths may be utilized by confining reinforcement, if relevant approval documents are provided.

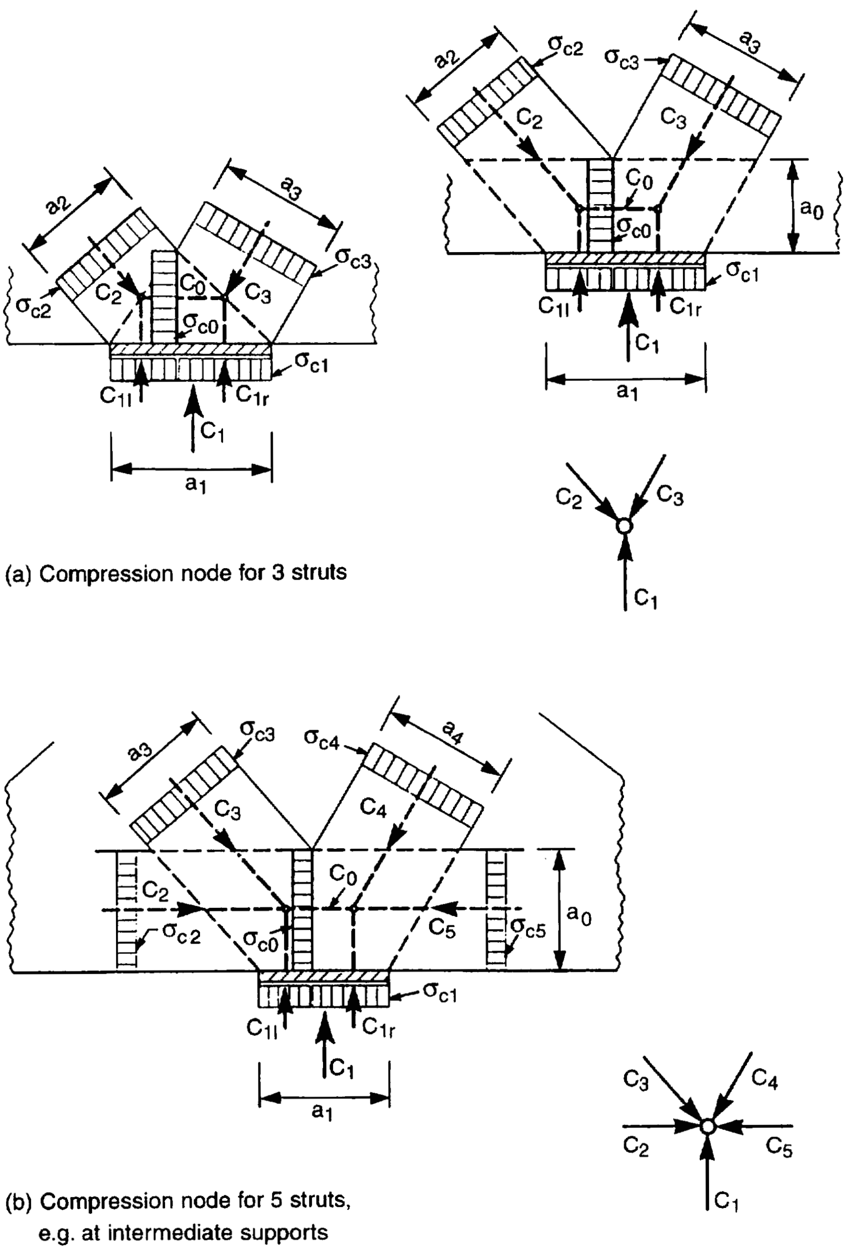


Fig.5.7 Typical compression nodes

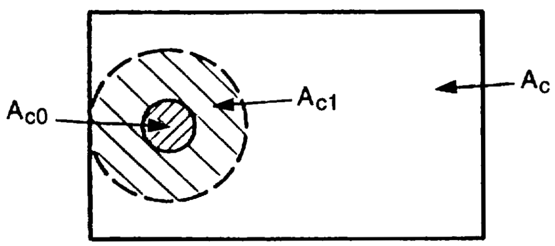


Fig 5.8 Definition of areas for local pressures

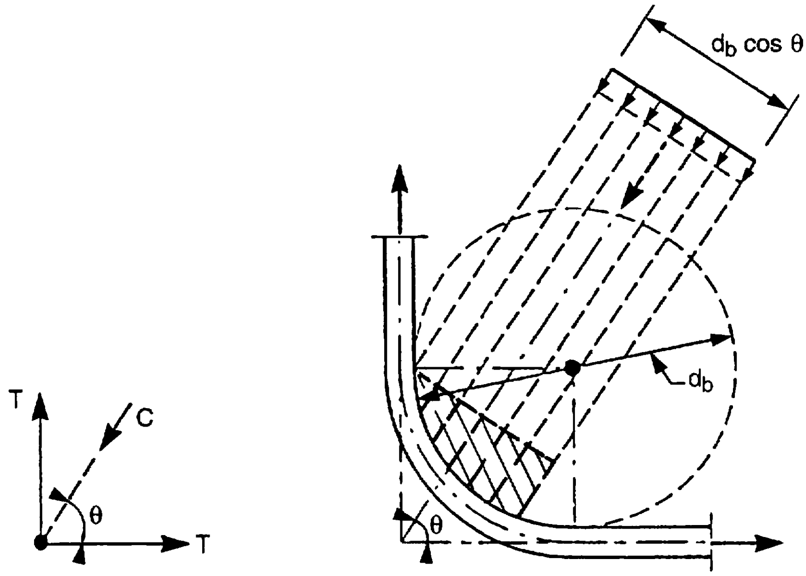


Fig 5.9 C-T-T- node at a bend in a bar

5.6.3 Bends in bars and minimum radii of curvature of tendons 6.9.2.4

5.6.3.1 Bends in bars

(1) A typical node at a bend in a bar is shown in Fig 5.9. If the angle θ is not equal to 45°, a part of the strut force is anchored at the bend. The average compressive stress at the node may be assessed by: 9.1.1

$$\sigma_c = C/(b\, d_b \cos\theta) \le \nu_2 f_{1cd} = 0.80 f_{1cd} \qquad (5.21)$$

where:

b = depth of the strut at the node

d_b = diameter of the bend

(2) The diameter of mandrel used for bending should be such as to avoid cracks in the bar and crushing or splitting of the concrete under the effect of the bearing pressure inside the bend.

These requirements are met if the minimum diameter of mandrel used for bars complies with the values given in Table 5.2.

(3) The minimum diameter of mandrel used for welded mesh fabric should comply with Fig 5.10.

Table 5.2 Minimum diameter mandrels for bars

Hooks and loops (Fig 5.11) bent-up bars and curved bars

Bar diameter ∅		Concrete cover perpendicular to plane of bend		
< 20mm	≥ 20mm	> 100mm and > 7∅	> 50mm and > 3∅	≤ 50mm and ≤ 3∅
4∅	7∅	10∅	15∅	20∅

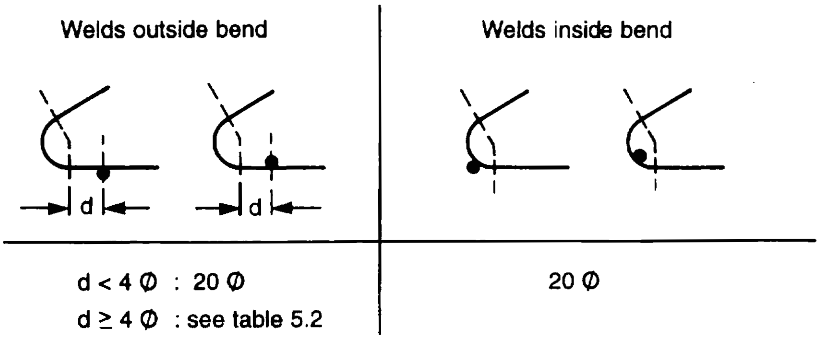


Fig 5.10 Minimum diameter of mandrel for welded mesh fabric

5.6.3.2 Minimum radii of curvature of tendons

Unless otherwise stated in technical approval documents, the following approximate values may be taken from an empirical formula for the minimum radii of curvature.

(a) internal, bonded tendons in corrugated ducts:

$$r_{min}\,[\mathrm{m}] = 3\sqrt{f_{ptk}\,A_p\,[\mathrm{MN}]} \geq 2.5\mathrm{m} \qquad (5.22\ a)$$

(b) external, unbonded multistrand tendons in smooth tube:

$$r_{min}\,[\mathrm{m}] = 1.5\sqrt{f_{ptk}\,A_p\,[\mathrm{MN}]} \geq 2\mathrm{m} \qquad (5.22\ b)$$

(c) internal, unbonded monostrand tendon (Ø 15mm):

$$r_{min} = 2.5\mathrm{m} \qquad (5.22\ c)$$

5.6.4 Nodes at anchorages of reinforcing bars 6.9.5

(1) The anchorage length $l_{b,net}$ depends on the type of anchorage as well as on the actual stress in the reinforcement, and it can be calculated from the basic value l_b as follows:

$$l_{b,net} = \alpha_a\, l_b (A_{s,req}/A_{s,prov}) \qquad (5.23)$$

where:

l_b = basic anchorage length according to equ.(2.11)

$A_{s,req}$ = area of reinforcement required

$A_{s,prov}$ = area of reinforcement provided

α_a = coefficient for type of anchorage, see Fig. 5.11

(2) A minimum anchorage length should be provided:

- for anchorages in tension: $l_{b,min} = 0.3\, l_b \geq 10\,Ø \geq 100\mathrm{mm}$
- for anchorages in compression: $l_{b,min} = 0.6\, l_b \geq 10\,Ø \geq 100\mathrm{mm}$.

(3) Transverse reinforcement with an area of 25 % of that of the main reinforcement should be provided for all anchorages, unless sufficient transverse compression exists. The transverse reinforcement should be evenly distributed over the anchorage length, with at least one bar placed near the hook, bend or loop.

(4) Bars with diameters Ø > 32mm should be anchored by bond of straight bars or by means of mechanical devices.

(5) The transverse reinforcement may be anchored in the chords using one of the anchorage types shown in Fig 5.12. Bars with diameters Ø ≥ 16mm should not be used as transverse reinforcement. A longitudinal bar should be provided inside hooks or bends. The diameter of bend for hooks and loops should comply with section 5.6.3. 9.1.1.4

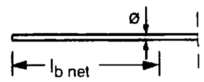
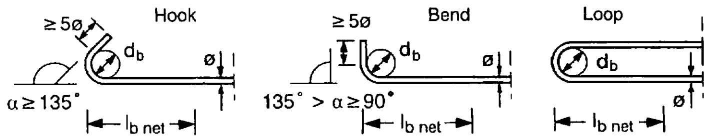
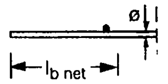
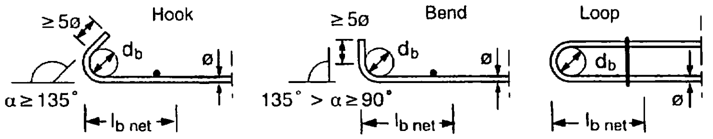
	Anchorage type	Coefficient α_a for anchorage in: Tension	Compression
1		1.0	1.0
2	Hook, Bend, Loop 	0.7	1.0
3		0.7	0.7
4	Hook, Bend, Loop 	0.5	0.7

Fig 5.11 Coefficient α_a for the type of anchorage

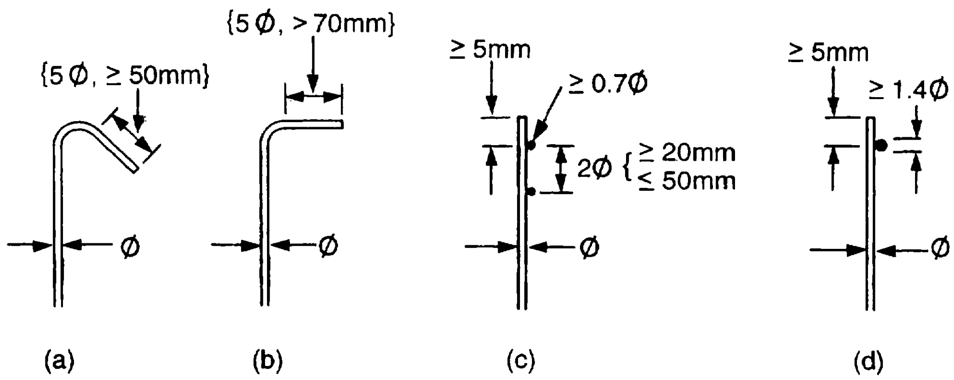


Fig 5.12 Types of anchorage for transverse reinforcement

(6) Typical cases for a C-C-T-node at end-anchorages of reinforcing bars are shown in Fig 5.13. The concrete stresses σ_{c2} in the inclined strut depend on the depth u of the node. Fig 5.13(a) shows the extreme case with $u = 0$ due to no anchorage length behind the anchor plate, whereas Fig 5.13(c) applies to nodes where several layers of bars are anchored. In the latter case the depth u should be restricted to a value $u \leq 1.5a_1$ and, in the case of very high forces T, the transfer of the strut force C_2 across the top interface by friction should be checked according to section 5.5.

For the limiting concrete stresses the value $\nu_2 = 0.85$ applies, see section 5.6.1 (3).

(7) The end-anchorage of bars with hooks or bends in a C-C-T- node is shown in Fig 5.14. This demonstrates that all three directions of a node at an anchorage should be looked at. The bearing plate should not be placed less than about $2c$ (where c = concrete cover) from the edge in order to avoid any spalling of the bottom corner.

(8) At intermediate supports of non-slender beams and deep beams the reinforcement may have to be anchored at a node as shown in Fig 5.15, which combines a C-C-C - node with a C-C-T-node.

(9) The anchorage lengths of pretensioned reinforcement are dealt with in section 2.4.3.

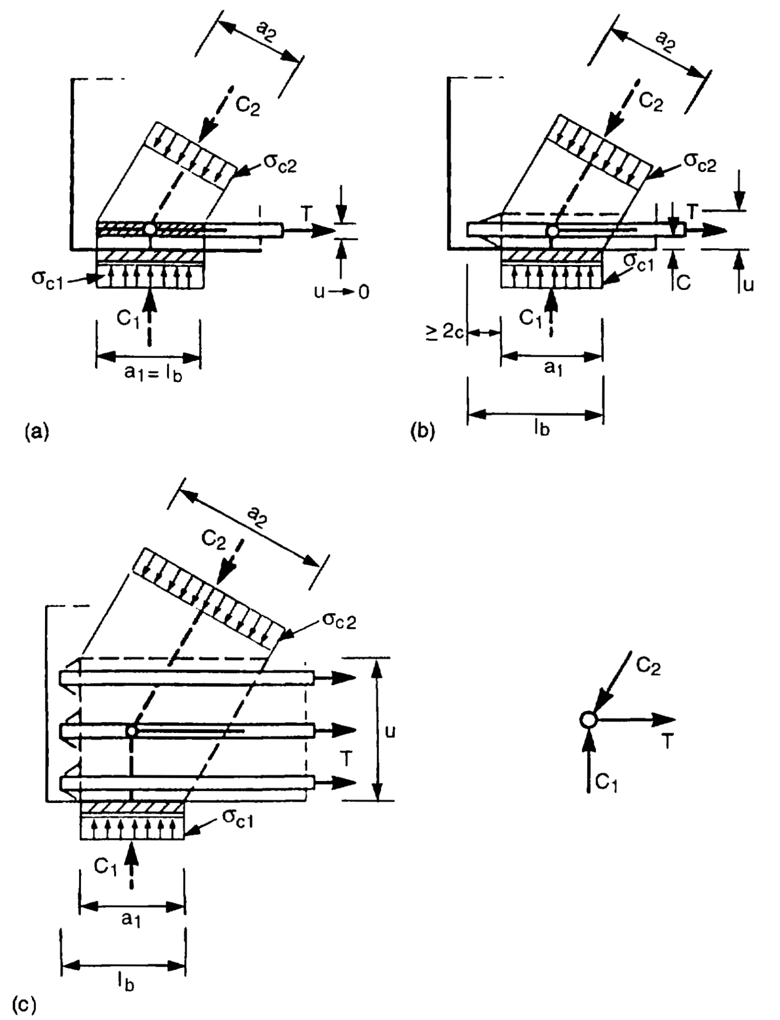


Fig 5.13 C-C-T- node at an end-anchorage of reinforcement

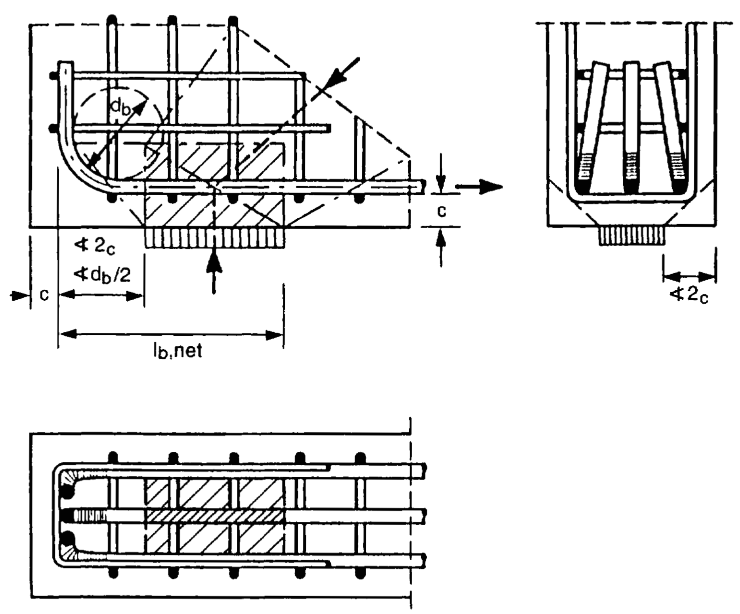


Fig 5.14 End-anchorage of bars with hooks or bends

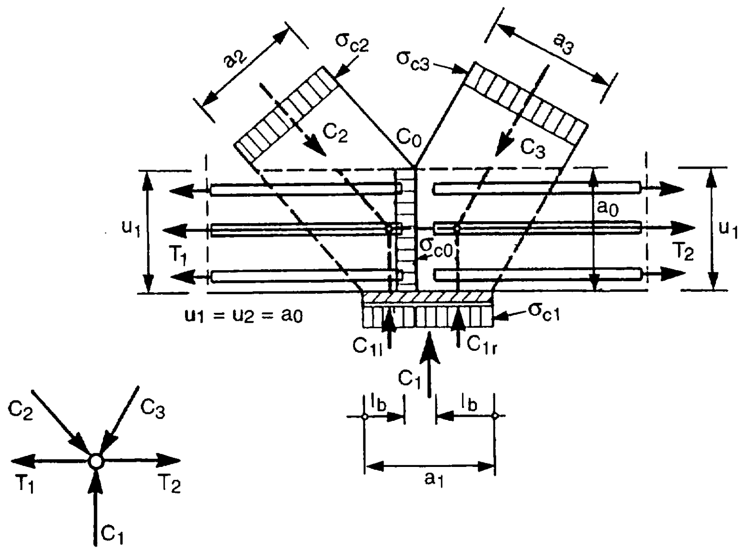


Fig 5.15 Combination of different node types at intermediate supports of deep beams and non-slender beams

5.6.5 Nodes with anchoring devices 9.1.1.3

(1) In the case of short anchorage lengths, anchor plates or anchoring devices should be provided. These should not be placed in the tension zones of members.

(2) If an anchor plate is used, then the load transfer from the tie to the struts may be regarded like a compression node (Fig 5.16). The anchor-plate must be dimensioned for the relevant stress distribution at the node face.

(3) The use of anchoring devices like studs, button heads or bolts requires appropriate technical approval documents.

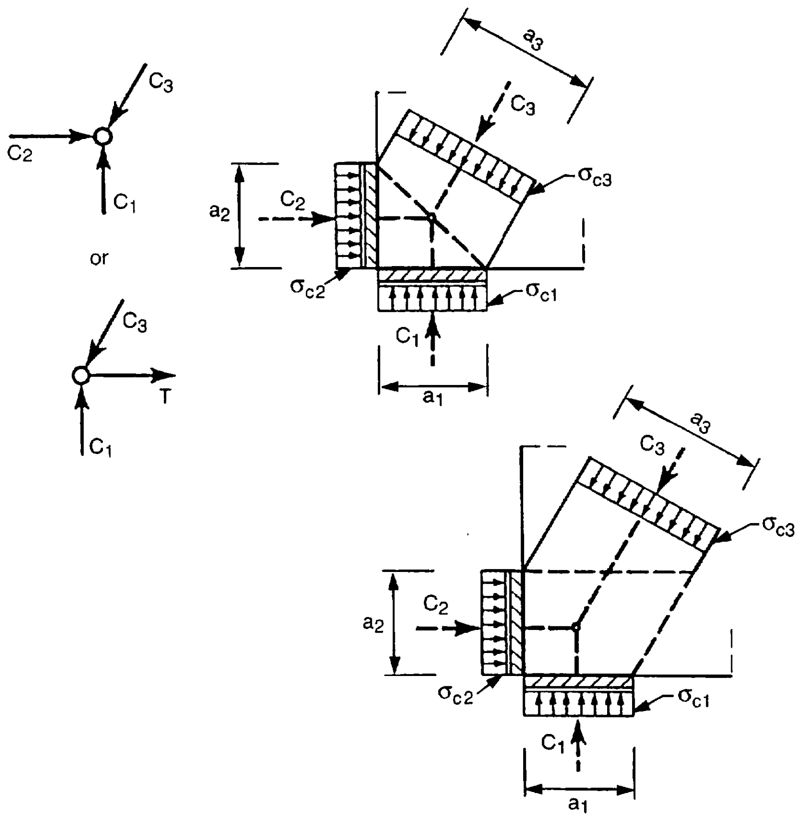


Fig 5.16 Anchor-plates for the end-anchorage of reinforcing bars result basically in a C-C-C-node

5.7 Reinforcement splices

MC 90 Section

9.1.2.1

5.7.1 General requirements

(1) Forces may be transmitted from one bar to another by:

- lapping with or without hooks, bends or loops
- welding
- mechanical devices.

(2) Any splice requires transverse tension in the plane of load transfer and perpendicular to it, which should be taken by appropriate reinforcement.

5.7.2 Splices by overlapping of bars

6.9.6

9.1.2.2.1

5.7.2.1 General requirements

(1) Laps between bars should be detailed such that the forces are fully transmitted from one bar to another without causing spalling of the concrete cover or excessive cracking. Laps should not be located at sections where the stress in the reinforcement is high, e.g $> 0.80 f_{yd}$. Laps should be placed symmetrically and parallel to the outer faces of the member.

(2) The laps between bars should be detailed and staggered in accordance with Fig 5.17.

5.7.2.2 Lap length

(1) The required lap length is given by:

$$l_0 = \alpha_s l_{b,net} = l_{0,min} \tag{5.24}$$

where:

$l_{b,net}$ = anchorage length according to equ.(5.23)

α_s = coefficient given in Table 5.3 depending on the percentage of bars lapped within the region extending $0.65 l_0$ each side of the centre of the splice and the spacing s (see Fig 5.18).

Thereby, the side cover of a splice should at least be as large as s.

(2) The required lap length should be increased by the clear spacing between the lapped bars, if this spacing exceeds $4\emptyset$.

(3) The minimum lap length is:

$$l_{0,min} = 15\,\emptyset \geq 200\text{mm} \tag{5.25}$$

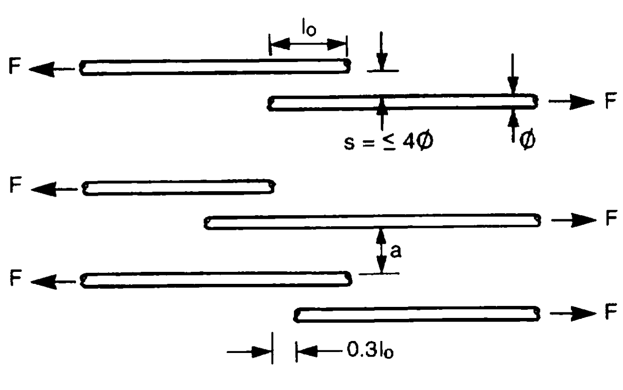


Fig 5.17 Staggering of lapped bars: $s \leq 4\,\emptyset$; $a \geq 2\emptyset$; $a \geq 20mm$

Table 5.3 Coefficient α_s for lap lengths

	bars in tension					bars in compression
% of lapped bars relative to total area of steel	≤ 20	25	33	50	> 50%	
coefficient α_s for $a \leq 10\varnothing$ and $b \leq 5\varnothing$	1.2	1.4	1.6	1.8	2.0	1.0
coefficient α_s for $a > 10\varnothing$ and $b > 5\varnothing$	1.0	1.1	1.2	1.3	1.4	1.0

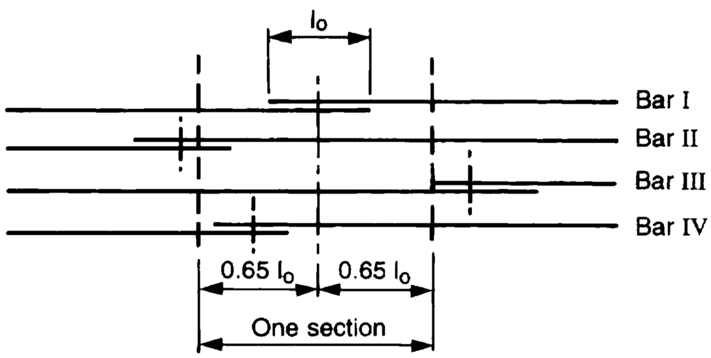


Fig 5.18 Staggering of lapped splices

5.7.2.3 Permissible percentage of lapped reinforcement 9.1.2.2.2

(1) For lapped bars in tension 100% may be lapped in one section, if they are placed only in one layer. For bars in several layers only 50% may be lapped in one section.

(2) For lapped bars in compression 100% may be lapped in any section.

5.7.2.4 Transverse reinforcement 9.1.2.2.3

(1) Transverse reinforcement provided for other reasons may also be considered to take the transverse tensile forces at a lap, for bars with diameters < Ø16mm and if less than 25% of the total reinforcement is lapped.

(2) For bars with Ø ≥ 16mm or if more than 25% of the total reinforcement is lapped, special transverse reinforcement A_{tr} should be provided. These bars should be placed between the lapped bars and the concrete surface and the amount and distribution should comply with that given in Fig 5.19. For linear elements with a clear spacing less than 10Ø between laps in the same plane the transverse reinforcement should consist of stirrups.

5.7.3 Lapping of welded mesh fabrics 6.9.8; 9.1.2.3

(1) Laps between sheets should be detailed such that the forces are fully transmitted from sheet bar to the other without causing spalling of the concrete cover or excessive cracking. In general, laps should not be located at sections where the stress in the reinforcement is high, (e.g. > $0.80 f_{yd}$).

(2) Laps may be made by either intermeshing or layering of the sheets (Fig 5.20), but intermeshing should be adopted for repeated loading.

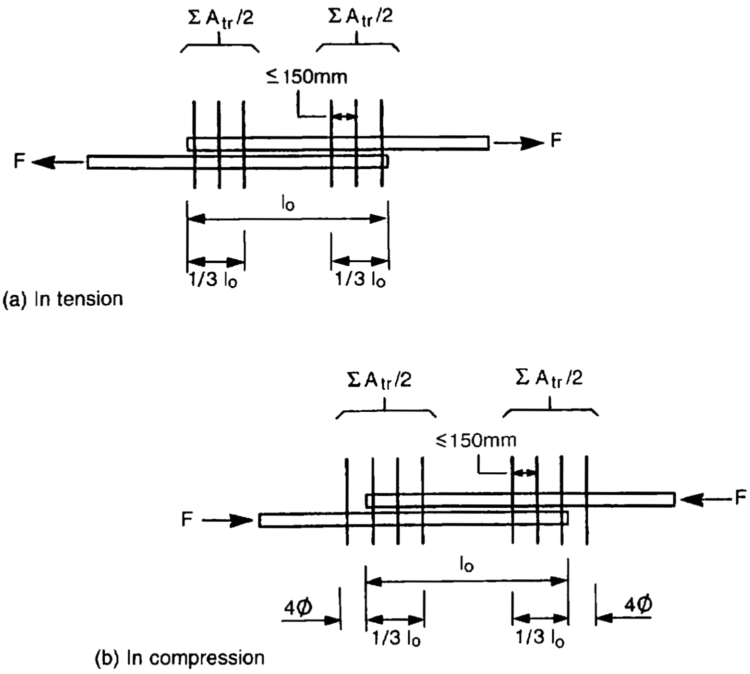


Fig 5.19 Detailing of lapped bar splices

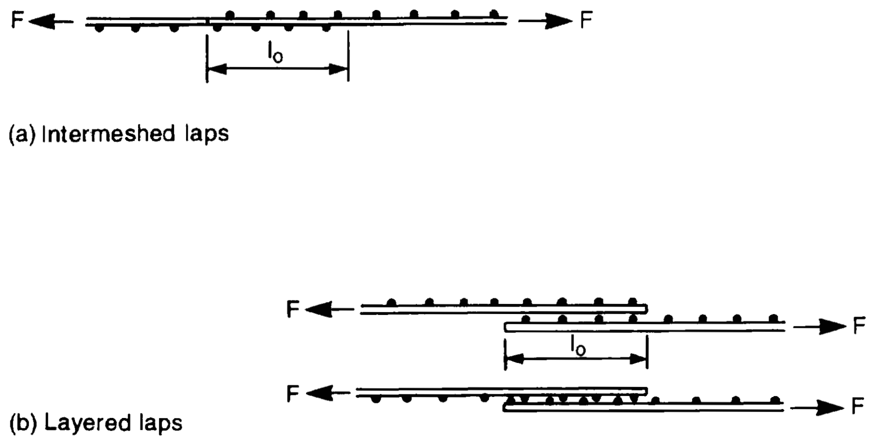


Fig 5.20 Types of laps for welded fabrics

(3) The lap length l_0 should be calculated as in section 5.7.2.2:

$$l_0 = \alpha_s \, l_{b,net} \geq l_{0,min} \tag{5.26}$$

where:

α_S $= 0.50 + \dfrac{A_{s,\,req}}{A_{s,\,prov}}$ and $1 < \alpha_S < 2$

$l_{0,min}$ = greatest of $\{0.75 l_b,\ 15\varnothing,\ s_{tr},\ 200\text{mm}\}$

$l_{b,net}$ = value given by equ. (5.23)

l_b = value according to section 5.6.4

$A_{s,req}$ = value according to section 5.6.4

$A_{s,prov}$ = value according to section 5.6.4

s_{tr} = spacing of the cross-wires

(4) Laps in several layers should be staggered by $1.3l_b$.

(5) The maximum percentage of the main reinforcement that may be lapped by layering at any one section is: MC 90 Section

- 100% if $A_s/s \leq 1200\text{mm}^2/\text{m}$;
- 60% if $A_s/s > 1200\text{mm}^2/\text{m}$, and this mesh is an interior layer of a multiple layer.

For intermeshed fabrics the requirements of section 5.7.2.3 apply.

(6) All wires acting as secondary reinforcement and the cross-wires of wire meshes may be lapped in one section.

(7) No additional transverse reinforcement is required within the lap length.

5.7.4 Splices by mechanical devices

9.1.2.4

The use of mechanical devices requires appropriate technical approval documents. These should specify the following details of the connection:

- characteristic values of yield and ultimate strength,
- deformation properties,
- fatigue characteristics.

5.8 Special rules for bundled bars and for bundled tendons

5.8.1 Bundled bars

9.1.5

(1) Bars of the same diameter may be bundled. The maximum number of bars in a bundle is limited to three, except for vertical bars in compression and laps, where 4 bars may be bundled.

(2) Arrangements of three or more bars in contact in one plane (horizontal or vertical) should not be used.

(3) For all design purposes, bundles of bars containing n bars having the same diameter should be replaced by a single notional bar having the same centroid and an equivalent diameter of:

$$\varnothing_n = \varnothing\sqrt{n} \leq 55\text{mm} \qquad (5.27)$$

(4) The cover and clear spacings of the bundles should be measured from the actual outer contour of the bundle.

(5) Bundled bars should be provided with straight anchorages and the anchorages of individual bars should be staggered. For bundles of 2, 3 and 4 bars, the staggering should be respectively 1.2, 1.3 and 1.4 times the anchorage length of the individual bar. The anchorage length for a complete bundle with $\varnothing_n < 32\text{mm}$ may be determined on the basis of the equivalent diameter.

(6) Laps can only be made with one bar of a bundle at any one section. The laps should be staggered in accordance with the above guidance.

5.8.2 Bundled tendons

(1) Up to two tendon ducts may be bundled transversly to the tendon curvature, or for straight tendons.

(2) Up to four monostrands may be bundled transversly to the tendon curvature.

(3) Tendon ducts may touch locally if they cross approximately perpendicularly, or if they touch only over a small length longitudinally.

6 Ultimate Limit State Design

MC 90 section

6.1 General requirements and definitions

6.1

(1) It shall be demonstrated that for the structure as a whole and for its members or for certain regions the probability of reaching an ultimate limit state (ULS) is small.

(2) It is generally advantageous to discern two typical regions in structures: the B- and D-regions. In a B-region the Bernoulli-hypothesis of plane sections remaining plane applies, and therefore standard dimensioning procedures may be developed (see section 6.4).

In regions with statical or geometrical discontinuities (D-regions) non-linear strain distributions occur. Examples of such D-regions are regions with concentrated load applications or member connections such as frame corners, regions with openings or abrupt changes of section.

(3) The determination of the resistance shall be based on physical models of the internal forces and the external reactions of the structure. The internal model shall represent a coherent system of internal forces, consisting of struts or compression stress fields and ties, and nodes in equilibrium with the design loads and reactions.

(4) When determining the model, compatibility should be considered at least approximately. It is generally advantageous if the model is orientated by the stress fields determined from a linear-elastic analysis. The model may be modified to account for cracking and yielding of reinforcements.

(5) The assumed nodes and ties must comply with the detailing of the reinforcements. The reinforcements must extend to the extreme fibres of the nodes or of the deviated compression stress-field. The axes of the reinforcements have to coincide with the axes of the corresponding ties in the model.

6.2 Actions and action effects

6.2.1 Definitions

(1) In general, permanent actions may be represented by a single (mean) value.

(2) The characteristic values of variable actions should be chosen along the lines of the Appendix. For live loads nominal values can be used.

(3) The prestress is defined in section 3.4.1. The partial safety factor is 1.0.

(4) Unbonded and external tendons should be treated as separate members in the analysis. The strain in the prestressing steel is equal to the strain corresponding to the forces defined in section 3.4.1, increased by the mean concrete strain between two successive points of anchorage or fixity due to load effects. This increase can be determined by a non-linear analysis of the entire structure, whereby, normally the tendons do not attain their yield strengths at ULS. For simplicity, a verification based on a linear-elastic analysis can be performed neglecting any increase in strain in the tendon (see FIP Recommendations *Design of post-tensioned slabs and foundation rafts*).

6.2.2 Combination of actions

1.6.2.5

(1) The applied loads or the acting internal forces should be determined using a partial safety factor from Table 6.1.

(2) Thus, the following combinations of actions should be considered:

(a) in the case of an unfavourable effect of G: MC 90 section

$$1.35G + 1.5Q_1 + 1.5\Sigma\, \psi_o\, Q_2 \qquad (6.1)$$

(b) in the case of a favourable effect of G:

$$1.00G + 1.5Q_1 + 1.5\Sigma\psi_o\, Q_2 \qquad (6.2)$$

where:

Q_1 = basic variable action

$\psi_o\, Q_2$ = combination value of other variable actions, see Table 6.2.

(3) Indirect actions (imposed or restrained deformations) need only to be considered if they are exceptionally large, to the extent of impairing the capacity for redistribution of internal forces, i.e. if a significant part of the plastic range in the M-κ–diagram has already been used for the redistribution of the indirect action effects (Fig 6.1). In this case, these effects are to be taken with their full value on the action side (without ψ_o).

6.2.3 Resistant action effects

The resistant internal forces should be determined on the basis of the resistances given in section 5.

Table 6.1 Partial safety factors for ULS

Actions	Unfavourable effect	favourable effect
Permanent	$\gamma_g = 1.35$	$\gamma_g = 1.00$
Variable	$\gamma_q = 1.50$	—

Table 6.2 Combination values for ULS actions

	Dwellings	Offices or retail stores	Parking areas	Highway bridges	Wind or snow
ψ_o	0.3	0.6	0.6	0.3	0.5

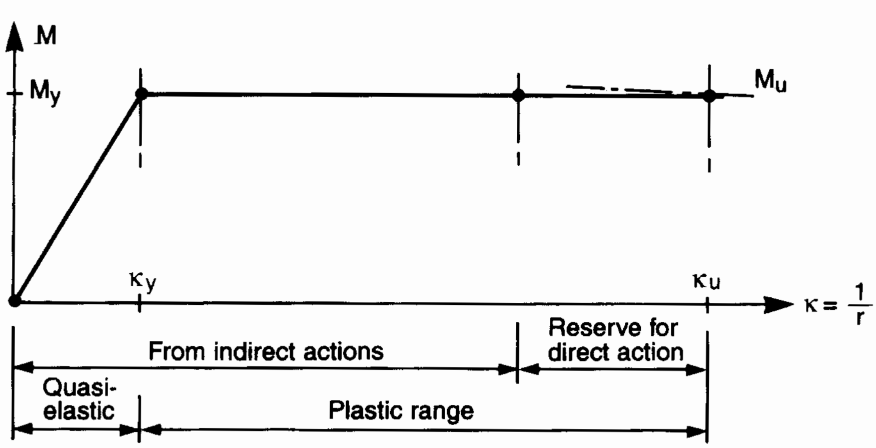


Fig 6.1 Moment-curvature diagram

6.3 Structural analysis

MC 90 section

6.3.1 General requirements

5.3.2

(1) The ULS check should be carried out according to the theory of plasticity (PT) or an appropriate non-linear method.

(2) Variable actions should be considered for the worst loading case.

(3) The effective widths of flanges may be taken as the values defined in section 7.3.2, unless proof is given for other values.

6.3.2 Static method of the theory of plasticity (PT)

5.3.2.4

(1) The use of the static method of PT is recommended whenever possible, since it yields a lower bound for the ultimate load of the structure. To this end, a plausible distribution of the internal forces is chosen, and the cross-sections or elements of the structure are dimensioned accordingly. The assumed distribution of internal forces has to satisfy the conditions of equilibrium, and should in general not differ too much from the elastic one. Otherwise, it is necessary to verify that the ductility of the structure is sufficient to allow the assumed plastic redistribution of internal forces.

(2) Linear-elastic analyses with or without redistribution are possible applications of the static method of PT.

(3) In principle, the hyperstatical effects of prestressing have no influence on the bearing capacity of the structure; however, they often give a good indication of the suitability of the assumed distribution of internal forces.

6.3.3 Kinematic method of the theory of plasticity

The kinematic method of the theory of plasticity may be used for determining the resistance of a structure, e.g the yield line theory for slabs. The designer has to make sure by experience, or trial and error, that the selected mechanism does not give an overestimated upper bound for the ultimate load.

6.3.4 Plastic rotation capacity and ductility requirement

3.7

(1) In general, the required plastic rotation capacity has to be estimated considering the non-linear behaviour of the structure, e.g. cracking and yielding.

(2) The plastic rotation capacity of a reinforced concrete flexural member may be estimated from equ. (3.7-2) in MC 90.

(3) Sufficient ductility may be presumed to exist in flexural members with a depth of the compression zone of $x \leq 0.3d$.

(4) The rotation capacity can be increased by:

(a) increasing ε_{cu}, e.g. by closely spaced stirrups confining the compression chord (see section 5.3.4).

(b) decreasing x/d (e.g. by compression reinforcement).

(5) In cases of high reinforcement ratios or high normal forces (due to actions), more detailed checks have to be carried out. Special attention should be given to cases where high strength concrete or steel with a small characteristic elongation is used.

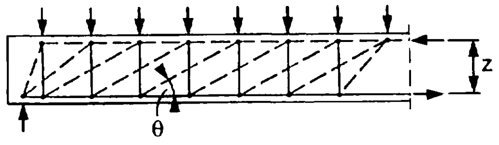
6.4 Design of B-regions

6.3.1

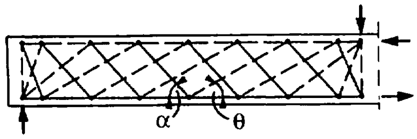
6.4. 1 Basic assumptions

(1) The design model for B-regions of linear members with a rectangular cross-section subjected to bending moments combined with an axial force and a shear force

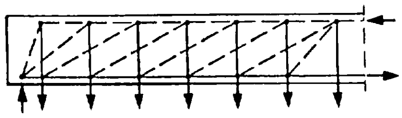
is a truss with longitudinal chords and a web (Fig 6.2). The web consists of inclined concrete struts representing a uniaxial compression stress field, and ties representing normally the distributed transverse reinforcement (Fig 6.3). For members with little or no transverse reinforcement, the ties may also represent a tension stress field in the concrete (Fig 6.4).



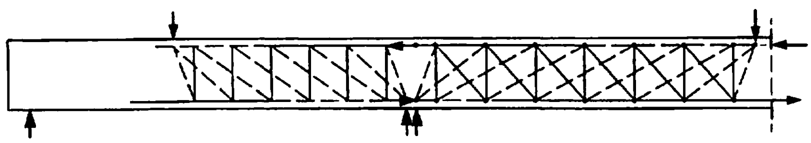
(a) End-support region with distributed loads



(b) Inclined ties



(c) Bottom loads



(d) Continuous girder with point loads

Fig 6.2 Truss models for structural concrete members

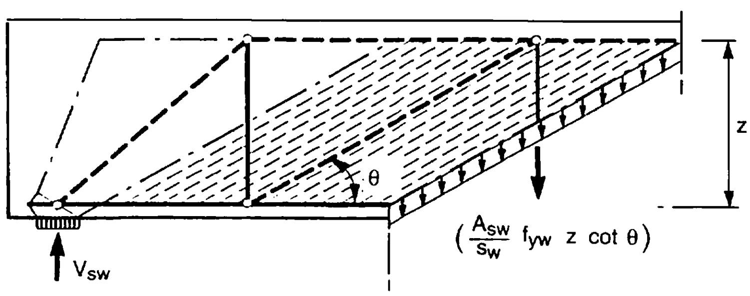


Fig 6.3 Compression stress-fields for the inclined struts

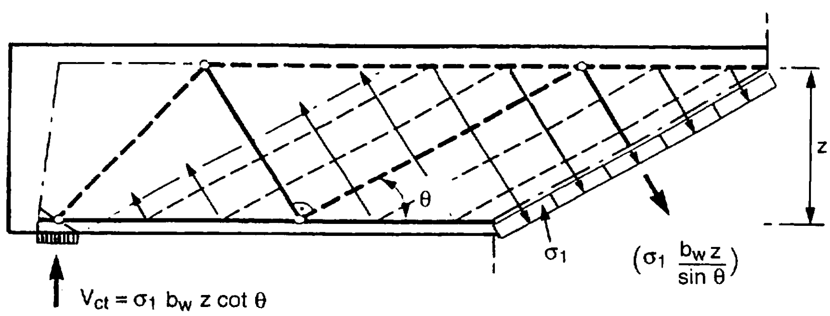


Fig 6.4 Model with concrete ties resulting from an inclined biaxial tension-compression field in the web for members with little or no transversereinforcement

(2) The geometry of the truss model is determined by the inner lever arm z between the chords and the angle θ of the inclined struts or compression stress-field in the web. The angle θ is measured relative to the tension chord. MC 90 section

The inner lever arm follows from the flexural design of section 6.4.2 for the sections with maximum moments. It may be assumed constant throughout the region in which the bending moments retain the same sign.

The angle θ of the inclined struts follows from the shear design of section 6.4.3 and varies with the magnitude of the axial force or prestress. It may be assumed constant throughout the region in which the shear force retains the same sign.

(3) A linear member with additional torsion and more complex sections, e.g. box-beams or T-beams, can be subdivided into several wall elements representing webs and flanges. These walls can then be designed for their individual action effects using the model described before.

6.4.2 Flexural design and inner lever arm of the truss 6.3.2

(1) The design moment and axial force are resisted by the chords spaced apart at distance z, and the forces in the chords should be derived on the basis of the following assumptions:

(a) the distribution of the longitudinal strain is linear over the depth of the section.

(b) the section is situated at a crack and the tensile stresses in the concrete are neglected.

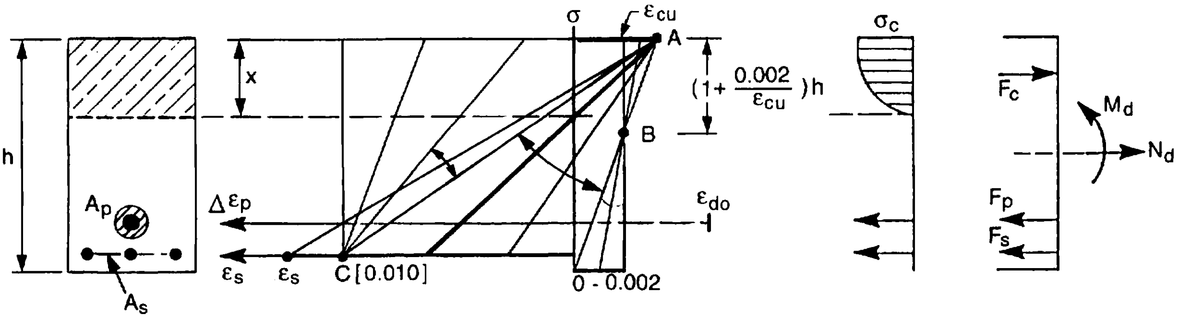
(c) bonded reinforcement is subjected to the same variations in strain as the adjacent concrete.

(d) the total strain of bonded prestressed reinforcement includes the prestrain ε_{d0} corresponding to the prestressing force after all losses.

(2) The strain diagram (Fig 6.5) should pass either through point A, defined by the maximum compressive strain ε_{cu}, or through point B in the case of uncracked members.

In cases where the steel strain may have to be limited, e.g. for low-ductility steel, the strain diagram is also restricted by point C, which is defined by a steel strain of $\varepsilon_{su} = 0.010$; other values may apply if the ductility of the steel is known.

(3) The design stress-strain diagrams for reinforcing and prestressing steel are defined in Figs 2.8 and 2.9 respectively. Alternative design stress-strain diagrams for concrete are defined in Figs 2.2 and 2.3, or, in the case of a rectangular stress distribution, in Fig 5.2.



(a) Section (b) Strain distribution (c) Stresses (d) Forces and action effects

ε_c = Compressive strain

ε_{cu} = Maximum compressive strain acc. to equ.(2.2) for a parabolic-rectangular stress-strain diagram or equ. (5.8) for a rectangular stress distribution

ε_s = Strain in reinforcing steel

$\Delta\varepsilon_p$ = Increase of strain in bonded prestressing reinforcement due to action effects

ε_{do} = Prestrain corresponding to the prestressing force after all losses

Fig 6.5 Strain distribution over the depth of the member

(4) For any combination of action effects the resistances of the chords shall not exceed the values given in sections 5.2 and 5.3. MC 90 section

(5) An increase in strength of the compression chord due to confinement may be taken into account according to section 5.3.4. The corresponding area of the compression zone is only that of the confined concrete.

6.4.3 Shear design and angle θ of the inclined struts 6.3.3

6.4.3.1 General requirements

(1) The truss models shown in Figs 6.2 and 6.3 are only valid as long as the transverse reinforcement is so closely spaced that the inclined compression stress fields can develop. Therefore, the spacing of stirrups in the longitudinal direction should not exceed the lesser of $s_w = z/5$ or 200mm. In this case the strength of the struts can be fully utilized with

$$\nu_2 = 0.80$$

(2) If larger spacings are used than those given above, then the compression field is not uniformly supported by the stirrups. In this case, the strength of the inclined strut is limited to the following values for ν_2:

- $\nu_2 = 0.60$ for $0.20 < s_w/z \leq 0.40$
- $\nu_2 = 0.45$ for $0.40 < s_w/z \leq 0.60$

(3) In no case shall the maximum spacing of the transverse reinforcement exceed the lesser of the following values:

- in the longitudinal direction: $s_{w,max} = 0.6z$ or 400mm;
- in the transverse direction: $s_{w,max} = 0.6z$ or 400mm.

(4) The angle α of any transverse reinforcement (Fig 6.2b) should not be less than 45°.

(5) The stirrups should be adequately anchored to the chords, see section 5.6.4.

(6) Bent-up bars are not recommended.

6.4.3.2 Design of the transverse reinforcement in the web

(1) The basic equation for any shear design follows directly from the vertical equilibrium of the free-body diagram in Fig 6.6:

$$V_{Rd} \geq V_{Sd} \tag{6.3}$$

where V_{Sd} = acting design shear force at a distance (z cotθ) from the face of the support (see section 6.5.2.1)

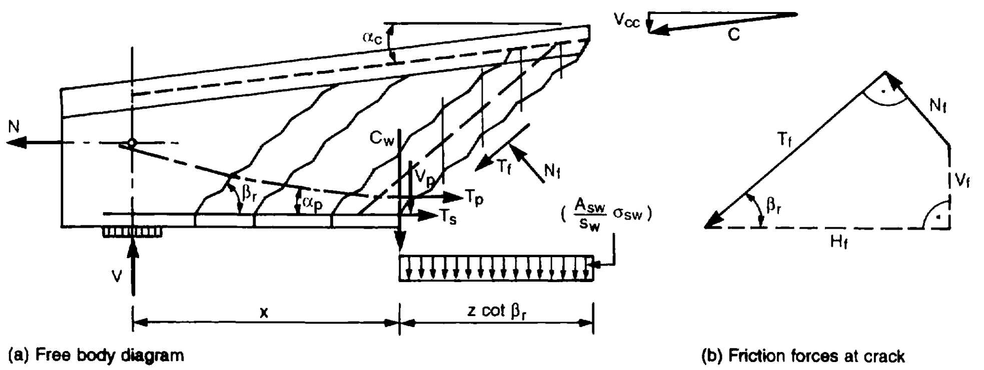


Fig 6.6 Free-body diagram for the end-support of a beam with all forces at the failure surface in the B-region

(2) The resisting shear force V_{Rd} follows from equilibrium at the inclined crack: MC 90 section

$$V_{Rd} = V_{swd} + V_{fd} + V_{pd} + V_{ccd} \tag{6.4}$$

where:

V_{swd} = shear force carried by the stirrups across the crack

V_{fd} = vertical component of friction forces at crack (Fig 6.6b)

V_{pd} = vertical component of force in prestressing tendon

V_{ccd} = vertical component of the force in an inclined compression chord.

From the equations (6.3) and (6.4) a design shear force for the web of a structural concrete member may be defined:

$$V_{Sd,web} = V_{Sd} - V_{pd} - V_{ccd} \tag{6.5}$$

and the web must exhibit the following resisting shear force:

$$V_{Rd,web} = V_{swd} + V_{fd} \geq V_{Sd,web} \tag{6.6}$$

(3) The vertical component of the force in a prestressing tendon may be taken as:

$$V_{pd} = P_{m0} \sin\alpha_p \tag{6.7}$$

where:

P_{m0} = $\sigma_{pm0} A_p$ = mean value of prestressing force (section 3.4.1)

α_p = angle of tendon at section considered

(4) The shear force component V_{swd} carried by vertical stirrups across the inclined crack is (for inclined stirrups see MC 90):

$$V_{swd} = (A_{sw}/s_w) f_{ywd}\, z \cot\beta_r \tag{6.8}$$

where:

A_{sw} = area of transverse reinforcement

s_w = stirrup spacing in the longitudinal direction

f_{ywd} = yield strength of transverse reinforcement

z = inner lever arm

β_r = crack angle

(5) The crack angle and the shear force component V_{fd} due to friction depend on the axial force as well as on the strains and crack widths in the web. As an approximation the following values may be assumed:

- for members without axial forces and prestress:

$$\cot\beta_r = 1.20 \text{ i.e. } \beta_r \approx 40° \tag{6.9 a}$$

$$V_{fd} = 0.070\ (b_w\, z\, f_{cwd}) \tag{6.10 a}$$

- for members with axial compression respectively prestress:

$$\cot\beta_r = 1.20 - 0.2\ \sigma_{xd} / f_{ctm} \tag{6.9 b}$$

$$V_{fd} = 0.10\ (1 - \cot\beta_r /4)\ (b_w\, z\, f_{cwd}) \geq 0 \tag{6.10 b}$$

- for members with axial tension:

$$\cot\beta_r = 1.20 - 0.9\ \sigma_{xd} / f_{ctm} \qquad \geq 0 \tag{6.9 c}$$

$$V_{fd} = 0.10\ (1 - 0.36/\cot\beta_r)\ (b_w\, z\, f_{cwd}) \geq 0 \tag{6.10 c}$$

where:

σ_{xd} = N_{Sd}/A_c = axial stress [(-) in compression]

f_{cwd} = $0.80\, f_{1cd}$ = compressive strength of inclined struts

6.4.3.3 Determination of the angle θ of the inclined struts

MC 90 section

The angle θ of the inclined struts of the truss model of Fig 6.3 may be determined from the above equations as follows:

$$\cot\theta = \frac{\cot\beta_r}{1 - \dfrac{V_{fd}}{V_{Sd,web}}} \qquad (6.11\ a)$$

or

$$\cot\theta = \frac{V_{Sd,web}}{\dfrac{A_{sw}}{s_w} f_{ywd}\, z} \qquad (6.11\ b)$$

6.4.3.4 Upper limit of resistant shear force

(1) The capacity of the web to transfer any shear force is limited by the compressive stresses in the struts between the cracks attaining the limiting stress $f_{cwd} = \nu_2 f_{1cd}$ (ν_2 see 6.4.3.1). The inclined compressive stresses are :

$$\sigma_{cw} = \frac{V_{Sd,web}}{b_w z} \cdot \frac{1}{\sin\theta\cos\theta} \qquad (6.12)$$

with b_w = web width, or b_{red} according to section 5.3.5

(2) The upper limit of the resistant shear force follows as:

$$V_{Rd,web,max} = b_w\, z f_{cwd} \sin\theta \cos\theta \qquad (6.13)$$

For θ = 45° and $\nu_{2,max}$ = 0.80 the highest value is reached with:

$$V_{Rd,web,max} = 0.5 b_w\, z f_{cwd} = 0.4 b_w\, z f_{1cd} \qquad (6.14)$$

6.4.4 Forces in the chords of the B-region

6.3.3.:

These chord forces may be derived from the truss model as follows:

- tension chord:

$$F_t = \frac{|M_{Sd}|}{z} + N_{Sd}\frac{z - z_s}{z} + 0.5 V_{Sd}\cot\theta \qquad (6.15)$$

- compression chord:

$$F_c = \frac{|M_{Sd}|}{z} - \frac{N_{Sd}\, z_s}{z} - 0.5 V_{Sd}\cot\theta \qquad (6.16)$$

where z_s = eccentricity of axial force relative to tension chord

6.4.5 Design of flanges of chords

6.3.4

(1) The flanges of sections act as chords for members over their effective width. The force transfer from the web into the flange may be determined by means of a truss model.

(2) In the analysis and design the effective width b_{eff} may be taken as stated in section 7.3.2. It may be kept constant over the length of the member. Larger values may be assumed if the reinforcement is correspondingly designed and detailed.

(3) The flange and the web are connected by the shear force v_{fl} per unit length:

$$v_{fl} = \Delta F_{fl}/\Delta x \qquad (6.17)$$

where:

ΔF_{fl} = change of chord force in flange over Δx

Δx = length under consideration

In the B-region the following value applies: MC 90 section

$$v_{fl} = V_{Sd} / z \tag{6.18}$$

(4) The transverse reinforcement per unit length in the flange follows from the truss model (Fig 6.7):

$$n_{sf} = (A_{sf} / s_f) f_{yd} = (b_f / b_{eff})(v_{fl} / \cot\theta_{fl}) \tag{6.19}$$

(5) The angle θ_{fl} of the struts in the flanges in equ. (6.19) may be determined for the above shear force from the rules in sections 6.4.3.2 respectively 6.4.3.3.

Alternatively, and also for simplicity, the following values may be assumed for the angle θ_{fl}:

- $\cot\theta_{fl} = 2.0$ for compression flanges; (6.20)
- $\cot\theta_{fl} = 1.0$ for tension flanges. (6.21)

The reinforcement should be distributed over the length a_{fl} (Fig 6.7).

(6) The compressive stress in the inclined struts is given by:

$$\sigma_{cf} = v_{fl} / (h_{fl} \sin\theta_{fl} \cos\theta_{fl}) \tag{6.22}$$

6.4.6 B-regions of members with torsion

6.3.5

(1) This section applies to B-regions of members where torsional resistance is required for equilibrium combined with bending and shear. Torsion due to compatibility may usually be neglected in the design, but appropriate minimum reinforcement should be provided.

Warping torsion, which may be predominant for members with open cross-sections, is not treated here.

(2) In torsion the equilibrium is maintained by a closed flow of tangential forces (circulatory torsion), which combine with the tangential forces due to the shear force in the web. For calculating the resistance an equivalent hollow section with thin walls (real or notational for members with solid cross-sections) is considered (see Fig 6.8).

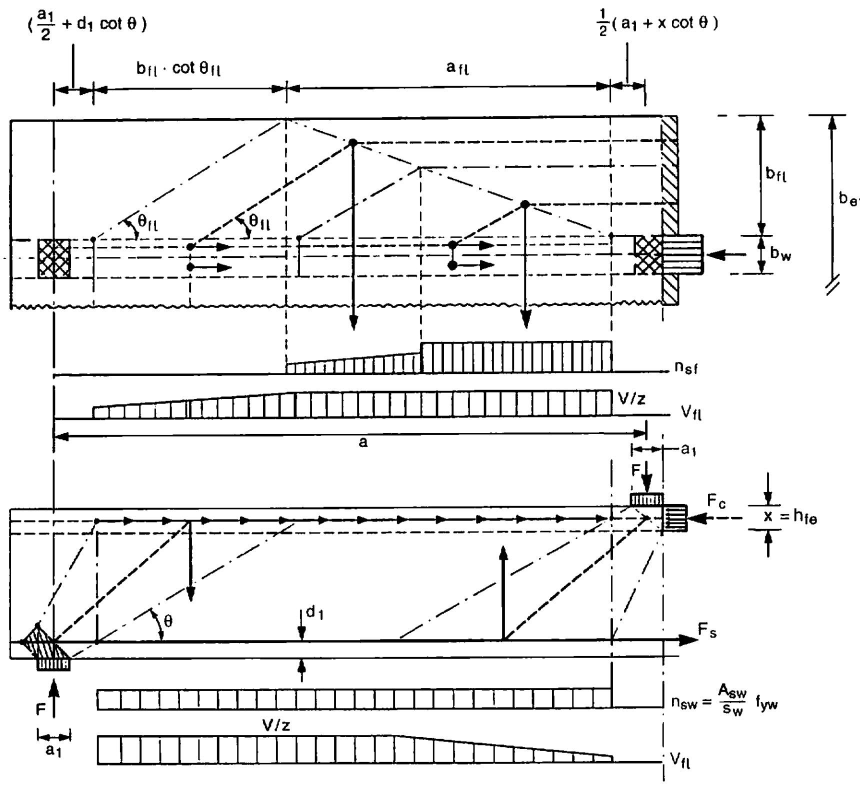


Fig 6.7 Truss model and stress fields for the transverse reinforcement of a compression flange

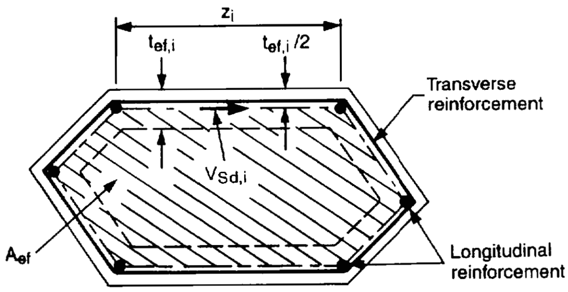


Fig 6.8 Notations for circulatory torsion

(3) The shear flow due to the torsional moment T_{Sd} alone is:

$$\tau_{t,i}\ \tau_{ef,i} = T_{Sd}/(2A_{ef}) \qquad (6.23)$$

and the tangential force $V_{Sd,i}$ in a wall is (Fig 6.9):

$$V_{Sd,i} = \tau_{t,i}\ t_{ef,i}\ z_i \qquad (6.24)$$

where:

A_{ef} = area enclosed by the centre lines of the walls

z_i = distance between the intersections of adjacent walls

$t_{ef,i}$ = the effective thickness of the wall i

The centre lines of the wall are defined by the axes of the longitudinal bars in the corners (Fig 6.9a). The effective thickness t_{ef} (= $t_{ef,i}$) of a wall is twice the distance from the centre line to the external face of the wall. In the case of hollow sections the effective thickness should not exceed the actual wall thickness.

(4) The transverse reinforcement in the wall should be designed for the combined shear flow due to torsion and shear:

$$V_{Sdi}/t_{ef,i} = V_{Sdi,T}/t_{ef,i} + V_{Sdi,V}/b_w \qquad (6.25)$$

The design may be carried out according to the rules in section 6.4.3.2, whereby the normal force of each wall may be considered.

(5) The longitudinal reinforcement should be designed for the following distributed force over the perimeter u_{ef} of the area A_{ef}:

$$n_{sl} = \frac{\Sigma A_{sl} f_{yl}}{u_{ef}} = \frac{T_{Sd}}{2A_{ef}} \cot\theta \qquad (6.26)$$

In chords these forces may be reduced by coexisting compressive forces.

The reinforcement should be distributed around the perimeter, but at least one bar should be placed in each corner of the stirrups.

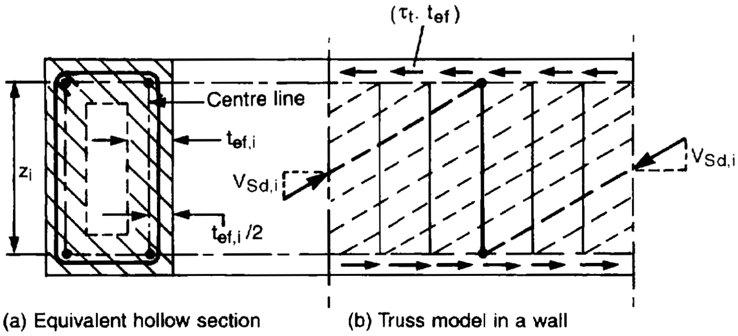


Fig 6.9 Definition of hollow section and model for torsion

(6) The upper limit of the resistance is determined by the inclined compression in the shear walls and may be regarded as sufficient, if the following condition is satisfied: MC 90 section

$$\left(\frac{T_{Sd}}{T_{Rd}}\right)^2 + \left(\frac{V_{Sd}}{V_{Rd}}\right)^2 \leq 1 \qquad (6.27)$$

where V_{Rd} = upper limit of shear force acc. to equ. (6.15).

The upper limit of torsional moment is determined for the same angle θ as used for V_{Rd} and is:

$$T_{Rd} = 2 f_{cwd} A_{ef} t_{ef} \sin\theta \cos\theta \qquad (6.28)$$

6.4.7 Shear in joints

6.10.2

(1) The capacity for the transfer of shear forces across an interface or a joint, such as a construction joint or a joint between *in situ* concrete and a precast member, depends on the capacity of the concrete-to-concrete friction limiting the transfer of the inclined compressive forces across the interface (see section 5.5). Based on that, the resistance to shear forces across a joint is given by:

$$\tau_{fRd} = \beta f_{ctd} + \mu\sigma_{cd} + (\mu \sin\alpha_j + \cos\alpha_j)\, \rho f_{yd} \qquad (6.29)$$

where:

τ_{fRd}	= design value for shear transfer by concrete friction
β	= coefficient acc. to Table 5.1
μ	= coefficient acc. to Table 5.1
f_{ctd}	= design value of concrete tensile strength acc. to section 2.1.4
ρ	= A_s/A_j = reinforcement ratio
A_s	= area of reinforcement crossing joint
A_j	= area of joint
f_{yd}	= yield strength of reinforcement
σ_{cd}	= normal stress on interface due to loading only (+ in compression)
α_j	= angle of reinforcement, see Fig 6.10

(2) The maximum shear stress to be transferred follows from the upper limit for the inclined struts acc. to section 6.4.3.4. Such a check is not required if:

$$\tau_{fRd} \leq 0.25 f_{1cd} \qquad (6.30)$$

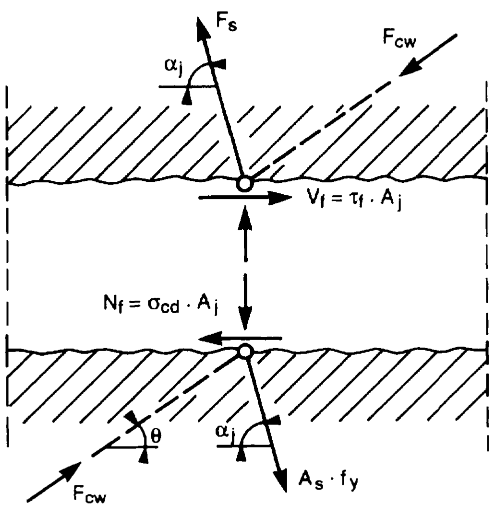


Fig 6.10 Transfer of shear forces across interfaces or joints

6.5 Design of discontinuity regions (D-regions)

6.5.1 Requirements and general criteria for modelling

(1) The determination of the resistance for discontinuity regions (D-regions) shall be based on physical models according to the requirements given in section 6.1. The model for a D-region must comply with that of the adjacent B-region(s), if any.

(2) It is to be verified that under the action of the design loads the stresses in the struts and ties do not exceed the strength criteria given in sections 5.1 to 5.4, and that the nodes and anchorages comply with sections 5.5 to 5.8.

6.5.2 Statical discontinuities: beam supports and corbels

6.5.2.1 Direct supports of beams

(1) At 'direct supports' the support force is applied by compressive stresses at the bottom face of the member. The support force $A = V_A$ is transferred into the member by an inclined strut representing a fan-shaped compression field (Fig 6.11). The geometry of the fan is defined by the flattest angle θ, which is the angle of the compression field in the B-region, intersecting the axis of the tension chord.

(2) At an end support the following force F_{sA} in the tension chord has to be anchored in the node over the support plate:

$$F_{sA} = V_A \cot\theta_A + N\,(1 - z_{s1}/z) \qquad (6.31)$$

with N (+) for tension.

The angle θ_A for the resultant of the fan-shaped compression field follows from the geometry of the fan:

$$\cot\theta_A = [0.5a_1/z + (d_1/z + 0.5)\cot\theta] \qquad (6.32)$$

This results for $\theta = 30°$ ($\cot\theta = 1.75$) in values of the order of

$$\cot\theta_A = 1.20,\ \text{i.e. } \theta_A = 40°.$$

(3) The distributed loads q over the fan are carried directly to the support, so that the transverse reinforcement near the support may be designed for the following force:

$$n_{sw,d} = A_{sw}\, f_{ywd}\, / \, s_w = \frac{V_A - q\,(0.5a_1 + (d_1 + z)\cot\theta)}{z\cot\theta} \qquad (6.33)$$

(4) At intermediate supports the design model in the web is a combination of two end-supports for the relevant shear forces (see Fig 6.12).

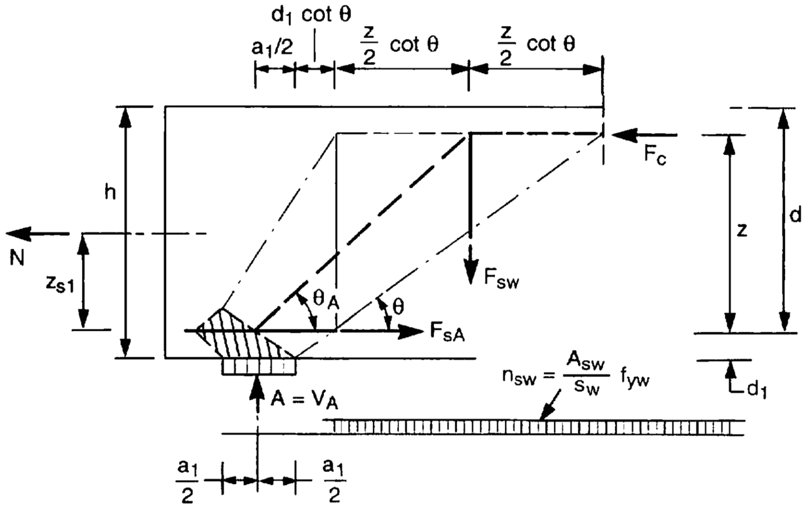


Fig 6.11 Strut-and-tie model for a 'direct' end-support

MC 90 section

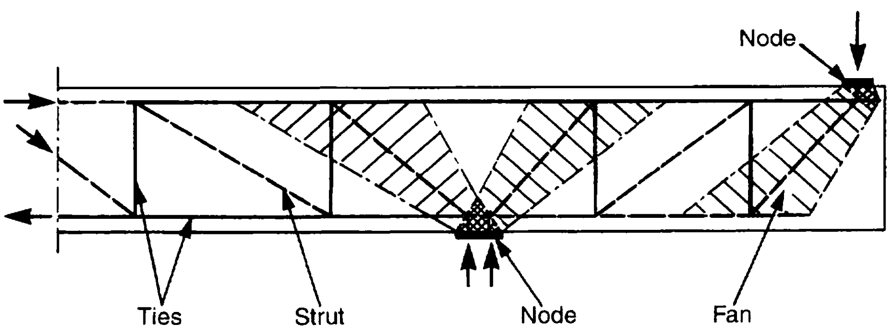


Fig 6.12 Strut-and-tie model for a 'direct' intermediate support

6.5.2.2 Indirect supports

(1) At 'indirect supports' of intersecting members the support is provided by tensile stresses over the depth of the member. The total support force has to be transferred to the top of the member by means of 'hanging-up' reinforcement within the width of the web (Fig 6.13).

(2) The load transfer into the webs of the intersecting members and the web design may be assumed to be the same as for a direct support.

(3) Careful consideration should be given to the anchorage of the main reinforcement, as there is no benefit of transverse compression for the anchorage length acc. to section 2.4.1 (5). The beginning of the anchorage length is defined by the first stirrup of the hanging-up reinforcement at the inner face of the supporting beam, which defines the deviation of the compression field.

6.5.2.3 Concentrated load near a support and corbels 6.8.4

(1) A load near a support of a beam (Fig 6.14) or a load on a corbel (Fig 6.15) may be transferred directly to the support by means of an inclined strut. The transverse reinforcement may be designed for the following part of the load:

$$F_1/F = (2a/z - 1)/3 \qquad \text{for } z/2 \le a \le 2z \tag{6.34}$$

The transverse reinforcement should be distributed over the length a_w shown in Fig 6.14. As an estimate, the value $a_w = (0.85a - z/4)$ may be taken.

(2) Unless more refined considerations are made, the strength of the inclined strut may be taken as $\sigma_{cw} \le \nu_2 f_{1cd} = 0.60 f_{1cd}$, if the width of the bearing plate fulfils the condition

$$a_F \ge \frac{x}{\sin\theta_2}\left[\frac{\nu_1}{0.60\cos\theta_2} - \cos\theta_2\right] \tag{6.35}$$

with the angle θ_2 determined from $\cot\theta_2 = a/z$.

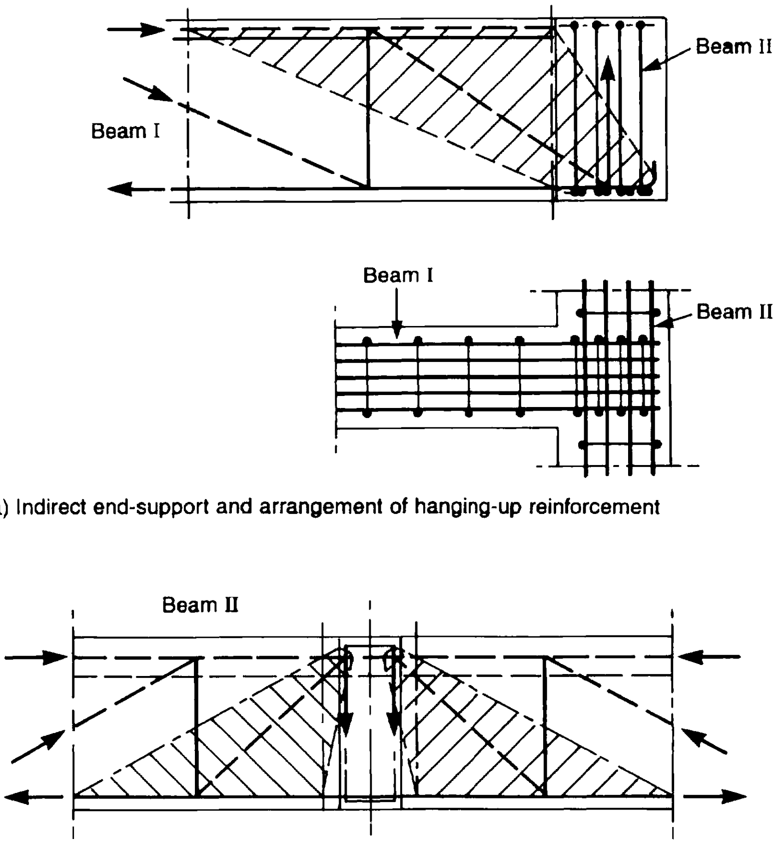
(3) Members with concentrated loads very near ($a < z/2$) or over a support require horizontal reinforcement (Fig 6.16). Unless more refined considerations are made it should be designed for a force:

$$T_3 = 0.20F \tag{6.36}$$

6.5.3 Deep beams 9.2.5

(1) Deep beams may be designed with strut-and-tie models. Special attention shall be paid to the anchorage in the nodes at the supports. Minimum reinforcement of 0.1% of the concrete section in each direction should be placed on each face.

(2) The strut-and-tie model and the distribution of the reinforcement for a deep beam on two supports is shown in Fig 6.17. The inner lever arm may be assumed at about $z = 0.6l$, so that the force in the tension chord is about $F_s = 0.2ql = 0.4A$.



(a) Indirect end-support and arrangement of hanging-up reinforcement

(b) Strut-and-tie model in supporting beam

Fig 6.13 Strut-and-tie model for an 'indirect' end-support

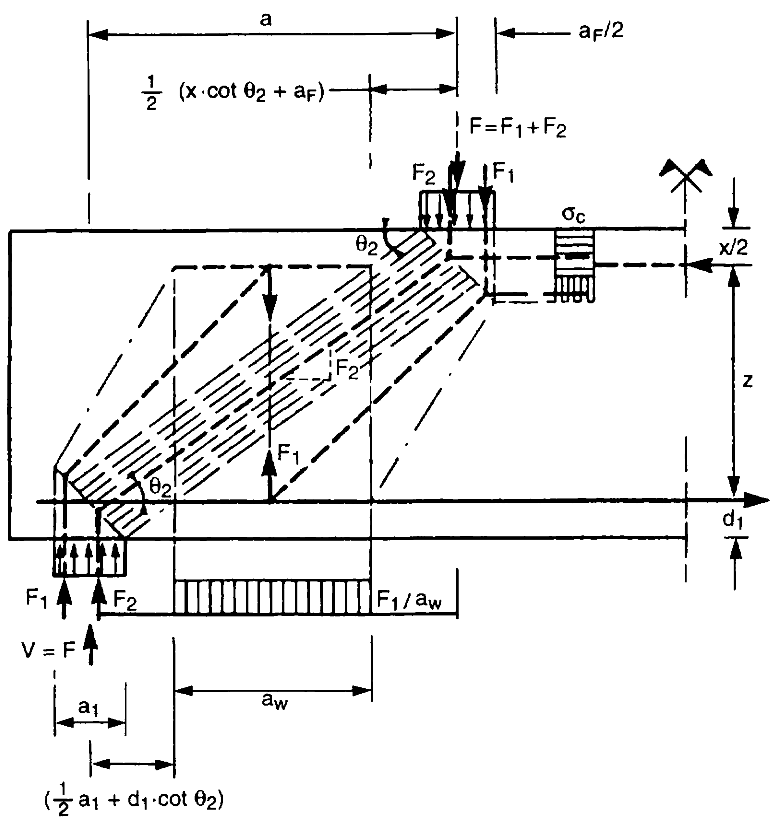
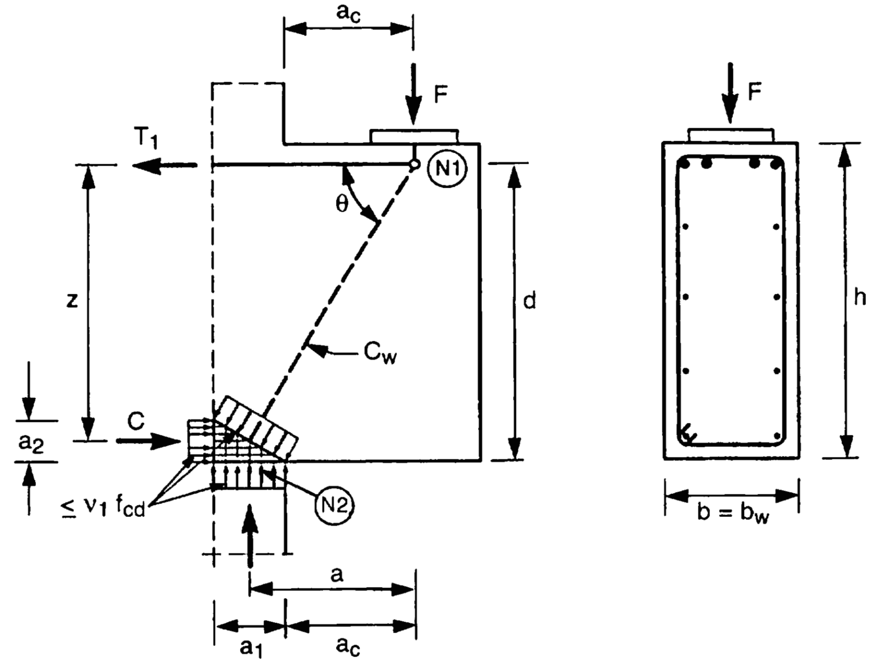


Fig 6.14 Strut-and-tie model for a concentrated load near an end-support



Step 1: $a_1 = F / b\nu_1 f_{1cd}$ with $\nu_1 = \left(1 - \frac{f_{ck}}{250}\right) \Rightarrow a = a_c + \frac{a_1}{2}$

Step 2: $a_2 d - \sqrt{d^2 - 2a\, a_1}$ $\Rightarrow z = d - \frac{a_2}{2}$

Step 3: $\cot\theta = \frac{a_2}{a_1} = \frac{a}{z}$

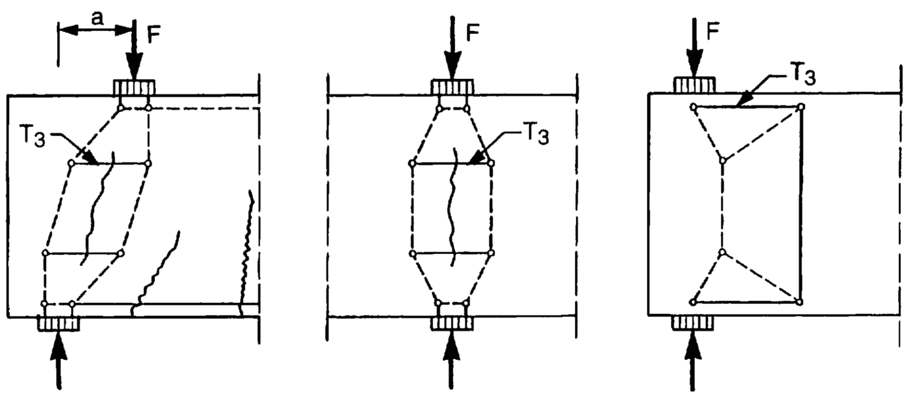
Step 4: $T_1 = F \cot\theta$ $\Rightarrow A_{s1} = T_1 / f_{syd}$

Step 5: check of node (N1): a) anchorage of tie T_1

b) bearing pressure

Step 6: check of strut C_w: not required if horizontal reinforcement acc. to Fig. 6.16 is provided

Fig 6.15 Strut-and-tie model for a corbel (with a < z/2*)*



(a) Case a < z/2 (b) Case a = 0 (c) Concentrated load at end

Fig 6.16 Models for the horizontal reinforcement for loads near or over a support

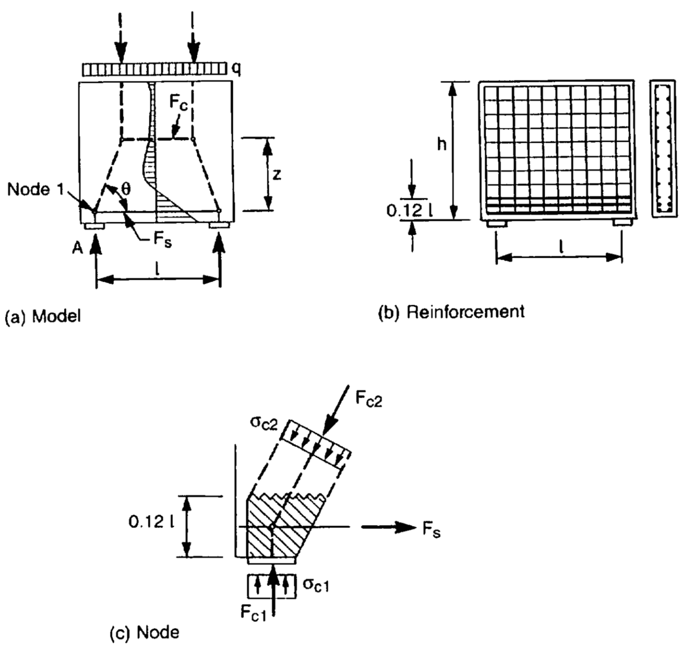


Fig 6.17 Strut-and-tie model and distribution of reinforcement for a deep beam on two supports

(3) The support zones of continuous deep beams may be designed with the model shown in Fig 6.18. Unless more refined considerations are made, the reinforcement over the support should be designed for the force $F_s = 0.2ql$ and should be distributed over a depth of $0.6l$. The force in the bottom tension chord should be assumed as $F_s = 0.16ql$ in an end-span (Fig 6.18a), and $F_s = 0.09ql$ in intermediate spans (Fig 6.18b).

6.5.4 Deviation of forces

Changes in the direction of forces may result in transverse tensile stresses or 'bursting forces' (Fig 6.19), which should be resisted by suitably anchored reinforcement.

6.5.5 Frame corners and beam-column connections

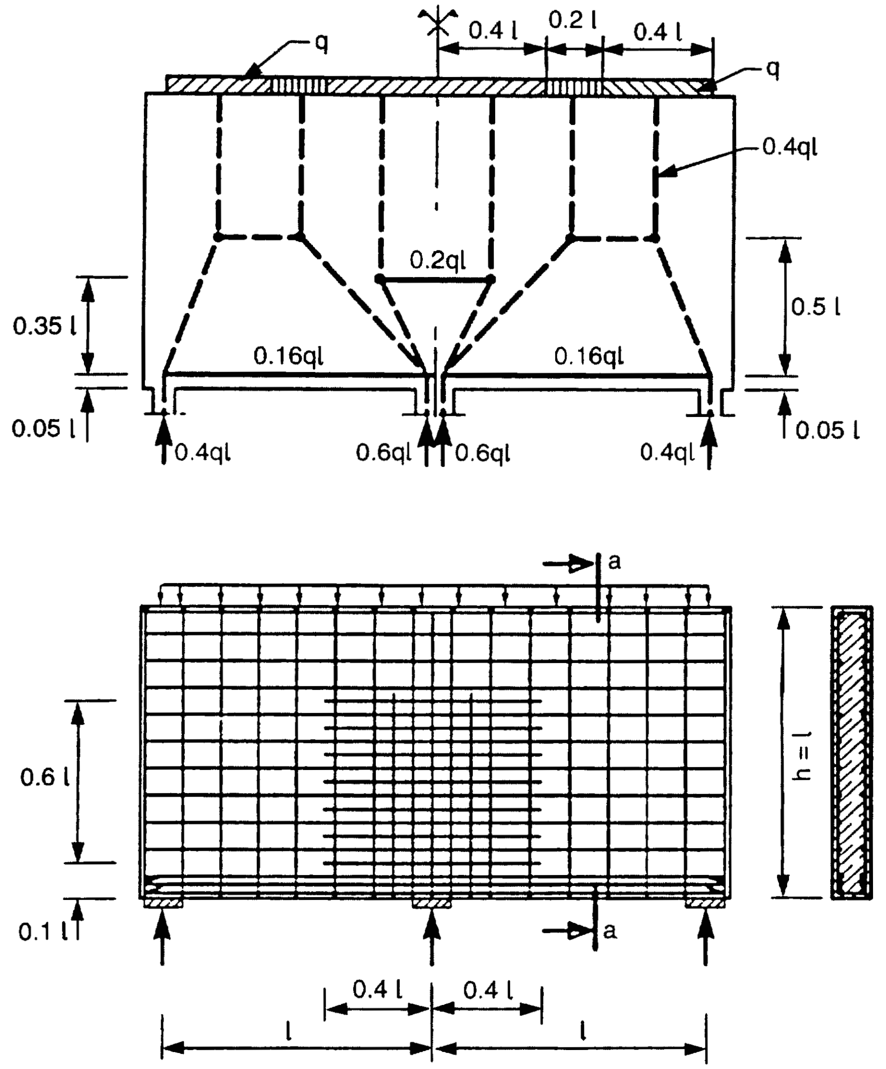
6.5.5.1 Frame corner with negative (closing) moment

(1) The basic strut-and-tie model for frame corners with negative (closing) moments 6.8.2.2.2
is shown in Fig 6.20. The critical sections 1–1 and 2–2 for determining the maximum chord forces lie within the beam-column connection at distances $x_1/2$ and $x_2/2$ respectively from the beam-column interfaces. Compression reinforcement in the chords should not be provided, because it cannot be anchored in the node (N1); however, the bi- or triaxial compressive strength may be utilized for the node (N1).

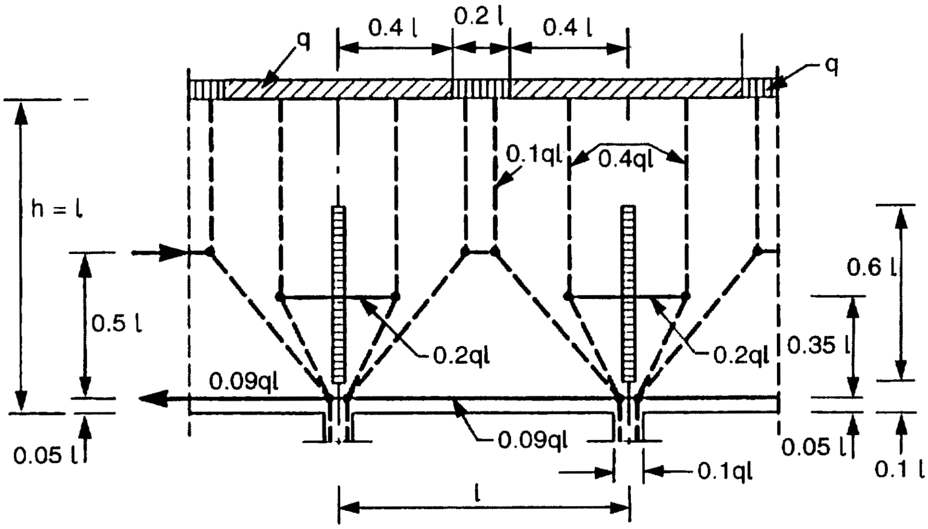
(2) The strength of the strut C3 in Fig 6.20 is normally determined by the dimensions of the node (N2) at the bend in the main bars (see section 5.6.3.1 and Fig 5.9). It should not exceed the capacity of the bottle-shaped strut of section 5.4 (3), unless the connection is reinforced in both directions.

(3) The basic strut-and-tie model of Fig 6.20 is only valid for members with approximately equal inner lever arms z_1 and z_2. For values of $z_1 > z_2$ horizontal reinforcement is required in the connection (Fig 6.21). This horizontal reinforcement may either be determined from the model given in section 6.5.2.3 (Fig 6.21a) or from the force $\Sigma T_h = T_2 - T_1$ according to the model in Fig 6.21b.

(4) Any splicing of the chord reinforcement requires additional transverse reinforcement. The side cover of the main bars should be secured by stirrups, and longitudinal bars should be provided perpendicular to the plane of the bend.

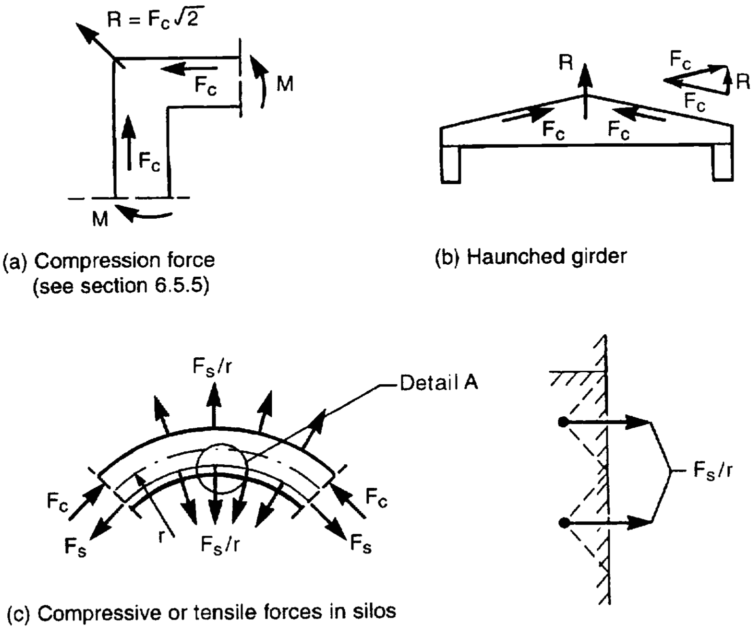


(a) Model and reinforcement for a deep beam on 3 supports



(b) Model for an intermediate span of a continuous deep beam

Fig 6.18 Strut-and-tie model and distribution of reinforcement for continuous deep beams



(a) Compression force (see section 6.5.5)

(b) Haunched girder

(c) Compressive or tensile forces in silos

Fig 6.19 Examples of transverse tension due to deviation of forces

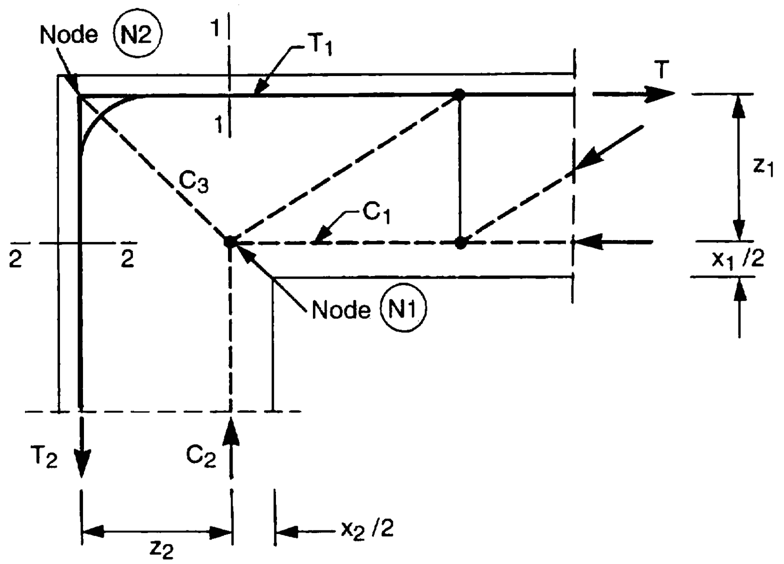
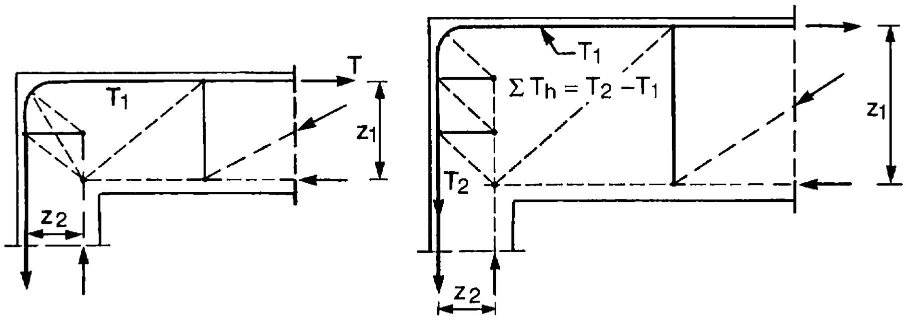


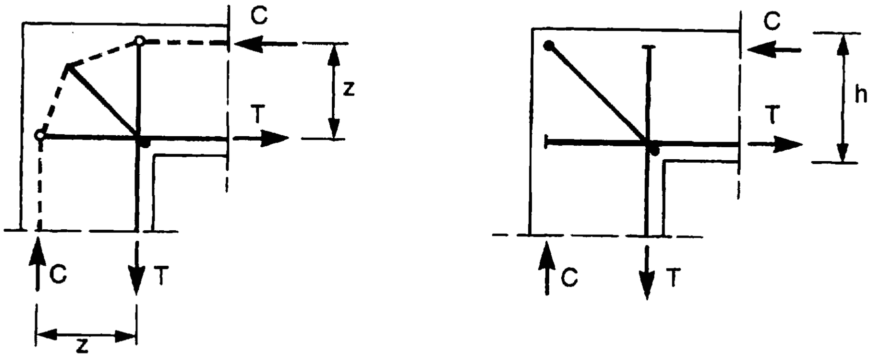
Fig 6.20 Basic strut-and-tie model for frame corners with negative moments



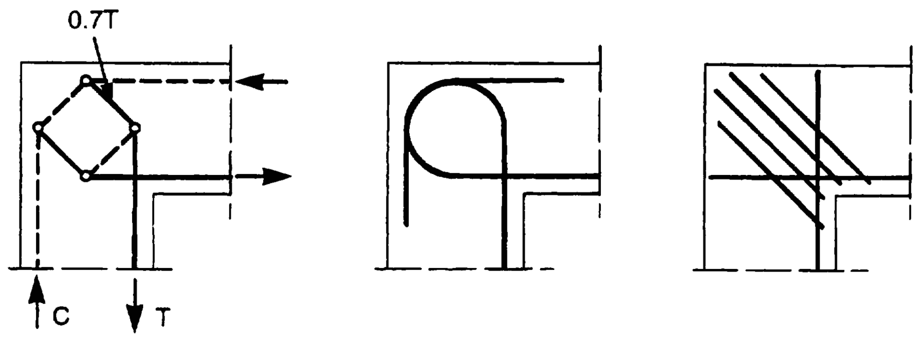
(a) Model for case $z_1 > z_2$

(b) Model for case $z_1 >> z_2$

Fig 6.21 Strut-and-tie model for frame corners with negative moments and differing depths of the members



(a) Basic model and reinforcement for small moments



(b) Model and reinforcement for moderate moments

Fig 6.22 Models for frame corners with small and moderate positive (opening) moments

6.5.5.2 Frame corners with positive ('opening') moments

(1) In frame corners with positive ('opening') moments the corner may spall off (see Fig 6.19 a) and must be secured by appropriate reinforcement. A basic strut-and-tie model (a) and a more refined one (b) are shown in Fig 6.22 along with possible reinforcement layouts.

(2) The nodes at the anchorages of the main ties of the tension chords should be thoroughly investigated. The capacity of the frame corner may be considerably reduced with increasing reinforcement ratios due to the anchorages.

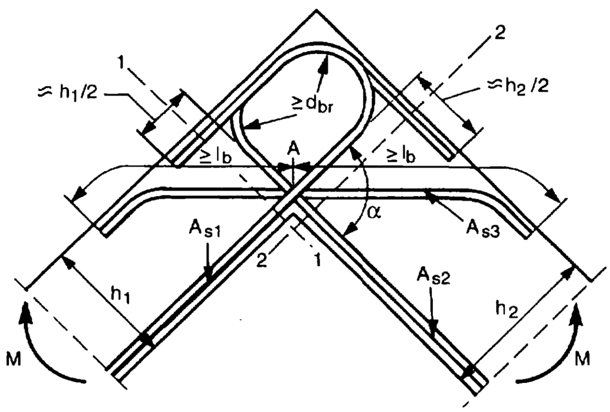
(3) Inclined bars in the corner improve the capacity and the serviceability of an opening frame corner with large reinforcement ratios, and Fig 6.23 gives two possible solutions.

6.5.5.3 Beam-column connection for an external column

(1) Simple models for a beam-column connection of members with similar depths are shown in Fig 6.24. The forces for dimensioning the connection have to be determined in the sections through the node (N1), as explained in section 6.5.2.3 (1).

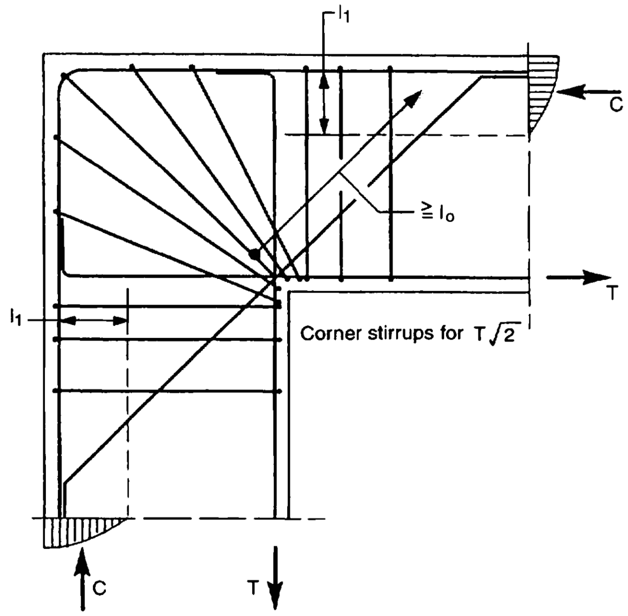
(2) The capacity of the connection depends considerably on the requirements for the diameter of the bend and the anchorage at the node (N2). In the case of insufficient anchorage length at node (N2), additional ties for a force of about $\Delta T = T_3 - T_1 = 0.3T_1$ are required over and below the reinforcement for T_1, as shown in Fig 6.25b.

(3) For connections of members with differing depths $h_1 > h_2$ (Fig 6.25) additional horizontal reinforcement is required in the connection (see also Fig 6.21), which may be designed for the force T_3.



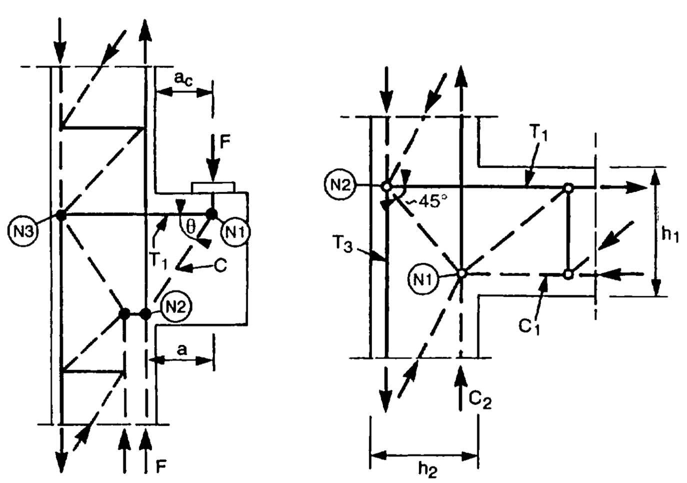
d_{br} = minimum diameter of bend h_1 or $h_2 \leq 1.0$m
Transverse reinforcement not shown

(a) Looped reinforcement and inclined bars



(b) Radial bars or spokes in combination with inclined bars

Fig 6.23 Possible reinforcement for frame corners with large opening moments



(a) Column – corbel connection (b) Connection of beam and external column

Fig 6.24 Basic strut-and-tie models for beam-column connections

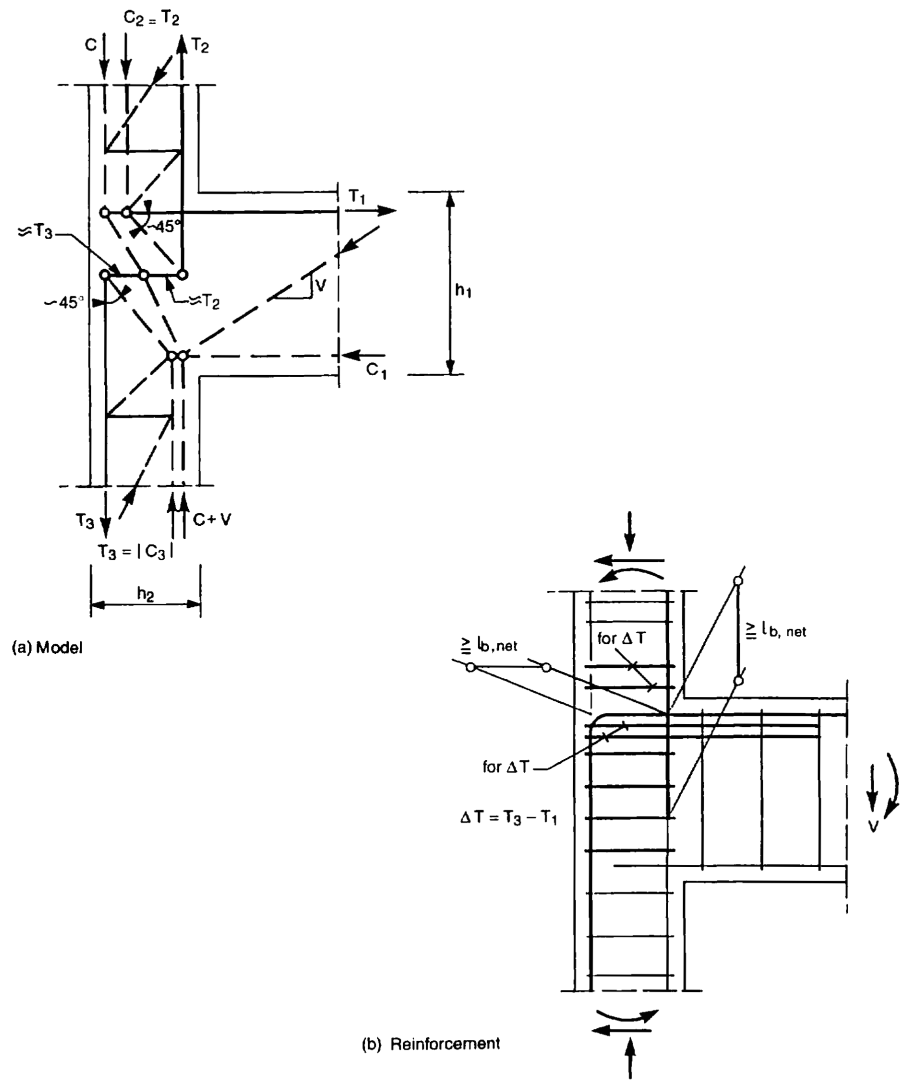


Fig 6.25 Refined model for a beam-column connection of members with different depths

6.5.6 Half joints and steps in members

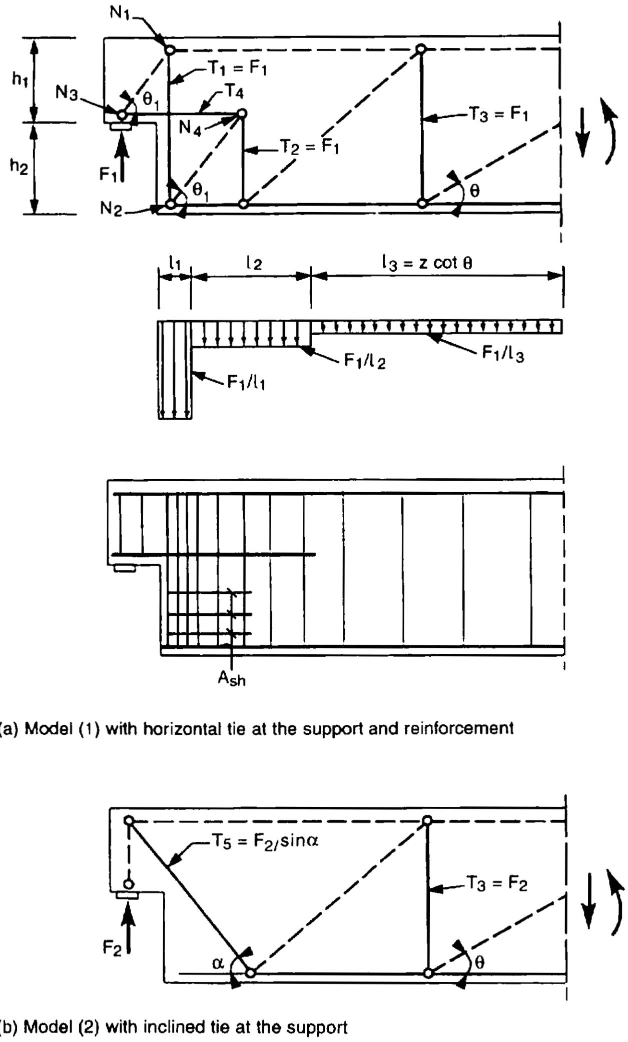
(1) Half joints or stepped-beam ends should be designed on the basis of a combination of the two strut-and-tie models in Fig 6.26a and b. Due consideration should be paid to possible horizontal forces due to friction at the support.

(2) The model in Fig 6.26 a requires transverse reinforcement for the force $(T_1 + T_2) = 2F_1$ distributed as shown. The horizontal reinforcement for the tie T_4 must be extended beyond the node (N4) by at least half of its anchorage length. Additional horizontal loops or hairpins should be provided in the bottom half of the step, if $h_2 > h_1$ or if $h_2 > 300$mm.

6.5.7 Concentrated loads in direction of member axis and anchorage zones of prestressing tendons

6.5.7.1 D-region at end-support of rectangular members

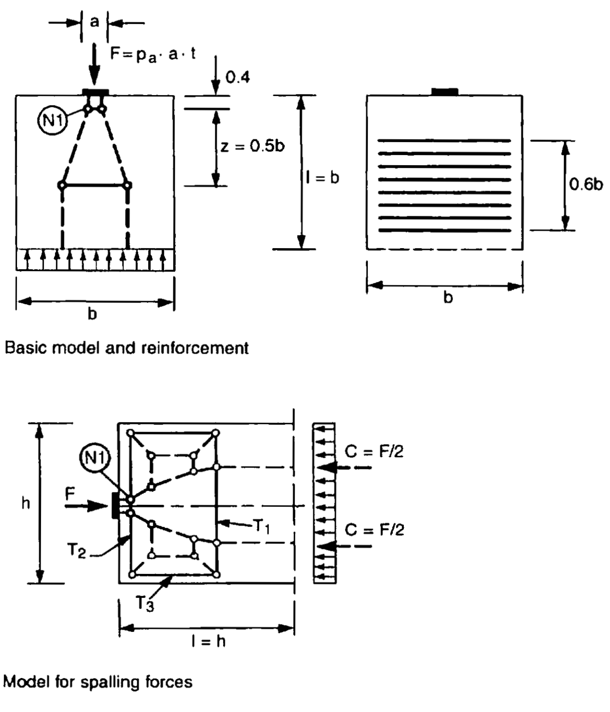
(1) The basic model in Fig 6.27 a applies to the D-region of a concentrated load on a wall or a prestressing anchor in a member of rectangular cross-section. The location of the tie T_1 was determined by the stress distribution for a linear-elastic analysis. The force T_1 may be assumed as about $T_1 = 0.25(1 - a/b)F$.



(a) Model (1) with horizontal tie at the support and reinforcement

(b) Model (2) with inclined tie at the support

Fig 6.26 Strut-and-tie models for half joints



(a) Basic model and reinforcement

(b) Model for spalling forces

Fig 6.27 Basic model for a concentrated load in direction of the member axis

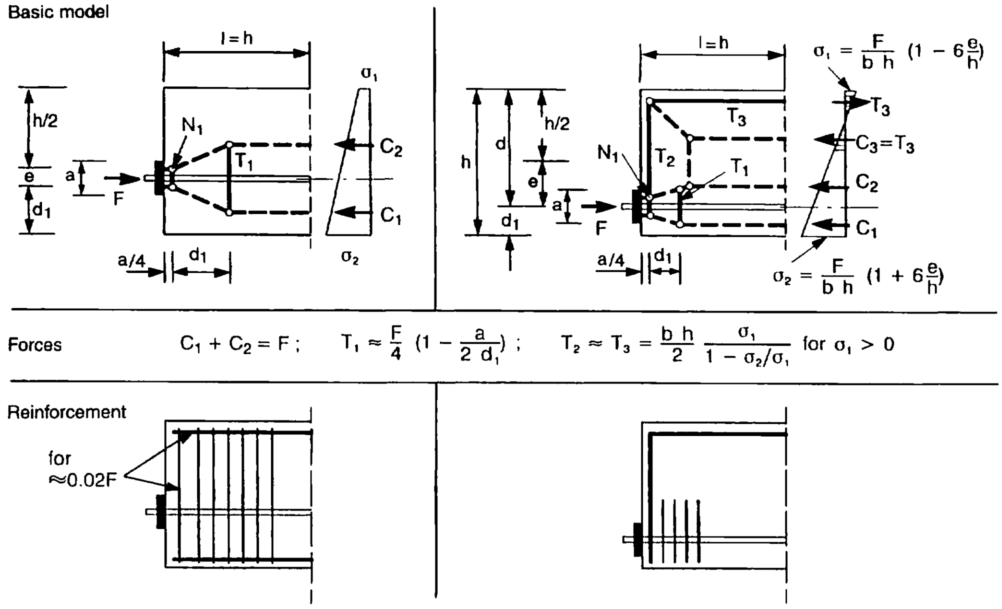


Fig 6.28 Eccentric point load in direction of the member axis

(2) The refined model in Fig 6.27b assesses the tensile forces ('spalling forces') in the concrete due to the compatibility of the inclined struts with the unstressed corners. The forces T_2 and T_3 may be assumed as $0.02F$ and are normally covered by minimum reinforcement provisions.

(3) The basic models for eccentrically applied loads follow from equilibrating the applied load with the linear-elastic stress distribution in the adjacent B-region at the opposite end of the D-region (modelling with load-path method) (Fig 6.28).

(4) Similarly, the forces may be derived for the anchorage of a post-tensioned tendon at an end-support (Fig 6.29). For simplicity the forces for the case given above may be taken.

(5) The check at node (N1) should comply with section 6.5.2.3.

(6) In the case of pre-tensioned tendons (Fig 6.30) the length of the D-region may be assumed as ($l_{bpt} + h$), where l_{bpt} is the transmission length according to section 2.4.3.2. The strut-and-tie model follows the rules given before (see Fig 6.28).

(7) The transverse force T_1 may be assumed to be taken by the concrete if the following conditions are fulfilled:

$$c/\phi > 2.5 \quad \text{and} \quad c_{eff}/\phi > 2.25 \qquad (6.37)$$

where:

c = concrete cover

c_{eff} = $[2c + 1.5(n-1)s_n\,]/2n$ = effective cover

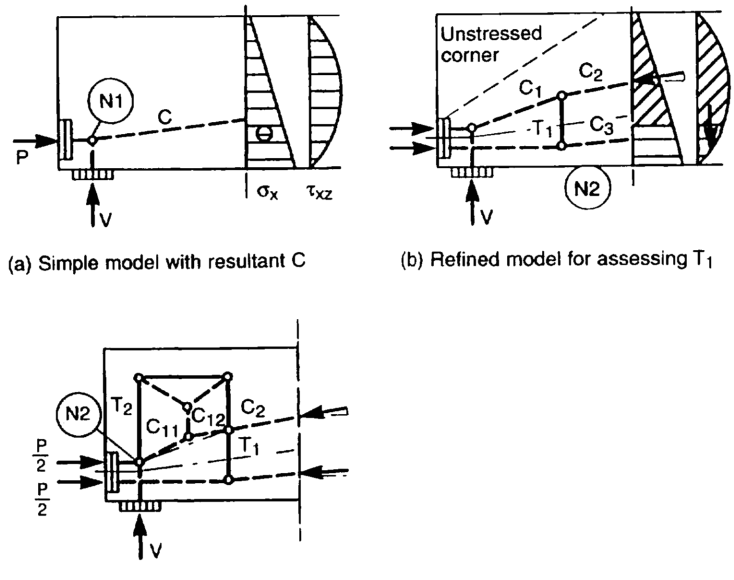
n = number of tendons

s_n = clear distance between tendons

6.5.7.2 End support of a beam with flanges

(1) The dispersion of a concentrated load into a member with flanges (Fig 6.31) requires transverse reinforcement in the flange as well as in the web.

(2) The dispersion of the prestress in a T-beam or a box-beam (Fig 6.32) follows the same principles. The transverse tie T_1 in the web of the D-region covers the forces due to the combined action of the prestress and the shear force at the support.



(a) Simple model with resultant C

(b) Refined model for assessing T_1

(c) Further refined model

Fig 6.29 Anchorage of a post-tensioned tendon at an end-support

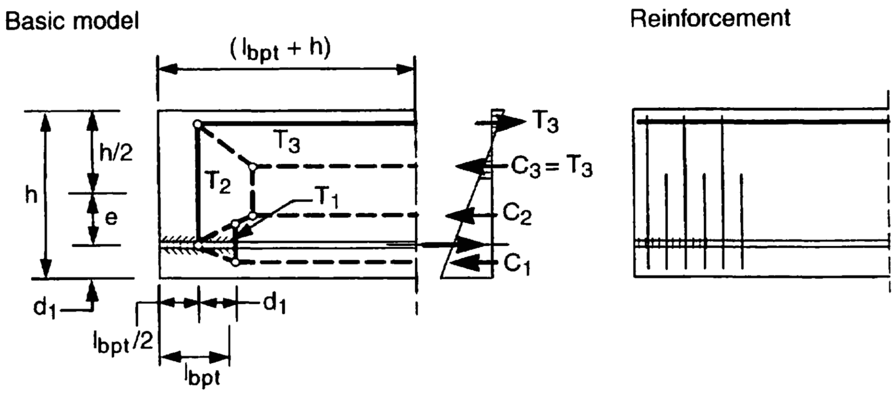


Fig 6.30 End anchorage of a pre-tensioned member

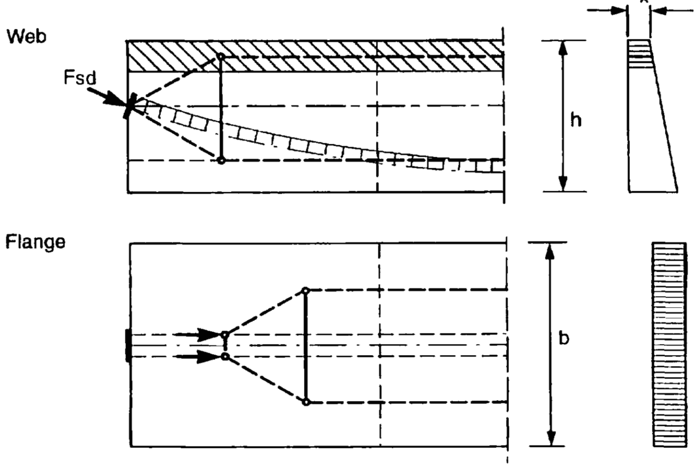


Fig 6.31 Dispersion of the prestress into a T-beam

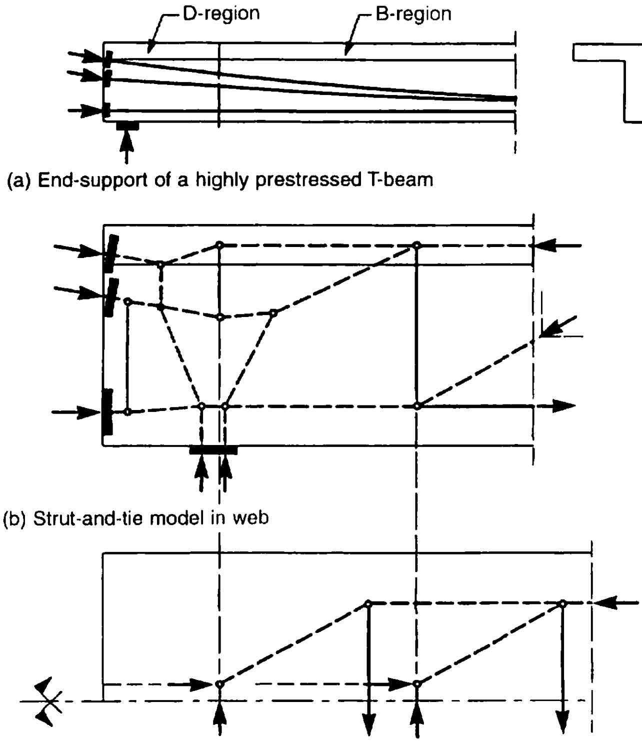


Fig 6.32 Strut-and-tie model at an end-support for a T-beam with prestressing anchorages

6.5.7.3 Interior anchorage zones and construction joints with prestressing anchorages

(1) If a load F is applied at an interior anchorage (stressing pocket) of a structural concrete member, about 25% of it should be tied back by reinforcement at the sides of the anchorage, as shown by the strut-and-tie model in Fig 6.33. This tensile force may be reduced by an amount of ($5A_1$ σ_c) in the case of compressive stresses σ_c behind the anchorage (where A_1 = area of anchor). The transverse reinforcement may be designed for the forces given in Fig 6.31, and due consideration should be given to proper anchorage length.

(2) In the case of an internal blister the prestressing force causes transverse forces due to the dispersion of the prestress as well as due to local bending, as shown by the strut-and-tie model in Fig 6.34a and b.

For the further dispersion of the prestress into the slab a simplified model is shown in Fig 6.34c; additionally longitudinal reinforcement should be provided for tying back some part of the prestressing force as shown in Fig 6.33 for an interior anchorage.

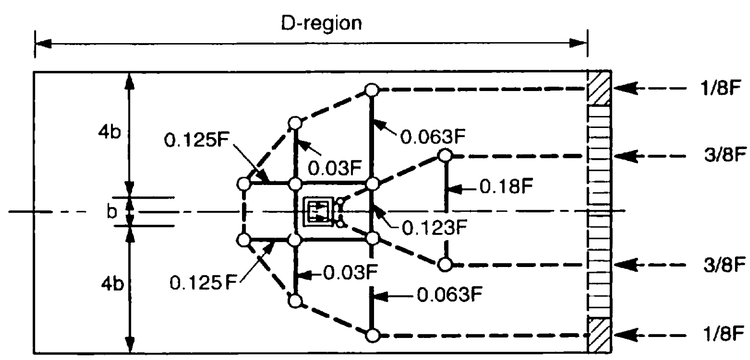


Fig 6.33 Strut-and-tie model for an interior anchorage of a tendon

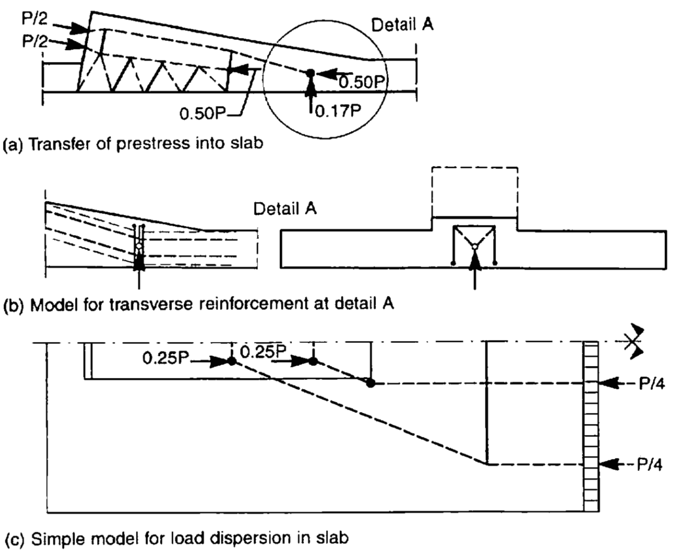


Fig 6.34 Strut-and-tie model and reinforcement for a blister

(3) A construction joint with a coupling anchorage represents a D-region where the forces concentrate at the anchorage, so that tension may occur in the section and at the edges (Fig 6.35). Therefore, an appropriate amount of minimum longitudinal reinforcement should be provided across the joint. The problem is greatly reduced if only some of the tendons are coupled and if the coupling anchorages are distributed over the depth of the web.

6.5.7.4 Deviators for external tendons

The deviation of externally applied tendons causes high concentrated forces, which have to be transferred to the webs of the beam. Fig 6.36 shows the strut-and-tie model at a deviator and demonstrates that special attention has to be paid to the transverse ties.

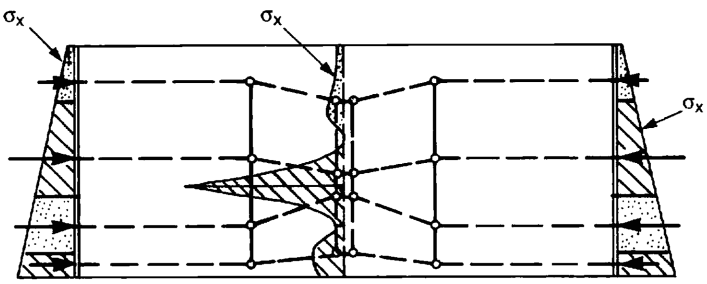


Fig 6.35 Stress distributions and strut-and-tie model for the D-region at a construction joint with a coupling anchorage

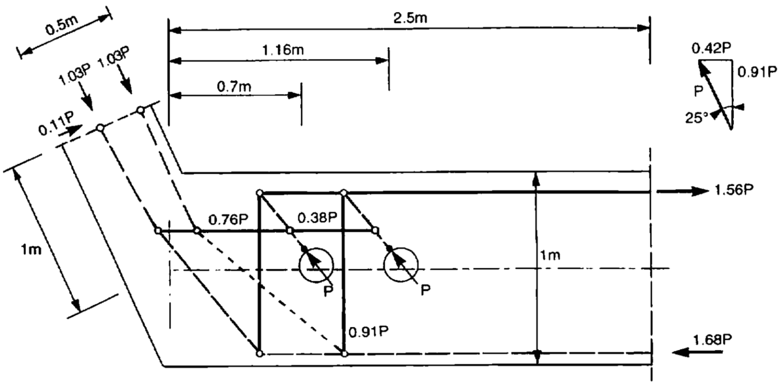


Fig 6.36 Strut-and-tie model at a deviator for external tendons

6.6 Design of slender compression members

MC 90 section 6.6

6.6.1 General

(1) This section deals with the design of slender compression members, e.g. columns, for which deformations may have a significant effect on bending moments. This so called 'second order effect' should be calculated taking into account the non-linearity of the moment-curvature relationship due to cracking, inelastic material properties and creep.

(2) Second order effects can be disregarded if the slenderness of the member is sufficiently low, see section 6.6.2. The effect of creep on second order deformations can be disregarded in some cases (see section 6.6.3).

(3) The effect of geometrical imperfections should be considered (see section 6.6.4).

(4) Simplified analyses of structures can be made by conventional methods based on second order elastic theory, if member stiffnesses are reduced to reflect the influence of cracking, material non-linearity and creep (see section 6.6.5).

(5) For isolated members various simplified methods can be used. One method is described in section 6.6.6.

(6) An accurate non-linear calculation can be made by assuming a linear strain distribution with proper stress-strain curves for the concrete (e.g. in accordance with section 2.1.3.1 (2)) and the reinforcement, by satisfying the conditions of equilibrium and deformation compatibility in selected cross-sections and by integrating the curvature to obtain the deflection. Creep can be included in the stress-strain curve for concrete. Tension stiffening can also be considered, but its effect is usually small.

A simplification of this method is to consider only one cross-section in a column and to assume a certain distribution of the curvature along the column.

6.6.2 Slenderness

(1) Second order effects can be neglected under certain circumstances, depending on slenderness, eccentricities and axial load. One simple condition is that the slenderness ratio fulfils the criterion $\lambda \leq 25$.

$$\lambda = l_o / i \qquad (6.38)$$

where:

l_o = buckling length (effective length) defined below

i = $\sqrt{\frac{I}{A}}$ = radius of gyration

I = second moment of area, normally for uncracked section

A = area of cross-section, normally for uncracked section

(2) The buckling length is defined as the length of a pin-ended column with constant axial load, having the same cross-section and buckling load as the actual column. For isolated columns with constant axial force and cross section, the buckling length can be determined directly for certain boundary conditions, and some basic cases are given in Fig 6.37.

6.6.3 Effects of creep

(1) The effects of creep can be taken into account by means of an 'effective creep ratio':

$$\varphi_{eff} = \varphi \, (M^0_{Sg} / M^0_{Sd}) \qquad (6.39)$$

where:

φ = creep coefficient according to section 2.1.5

M^0_{Sg} = first order moment under (unfactored) quasi-permanent load (see section 7.2.2)

M^0_{Sd} = first order moment under design load

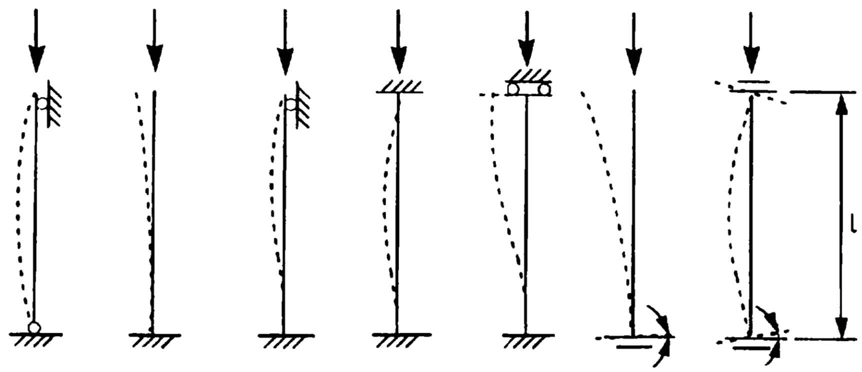


Fig 6.37 Examples of the buckling length for isolated columns

The first order moments $M^0{}_{Sg}$ and $M^0{}_{Sd}$ should include the effects of imperfections under quasi-permanent and design loads respectively (see section 6.6.4).

(2) The effects of creep may be neglected, i.e. $\varphi_{eff} = 0$, if at least two of the following conditions are fulfilled:

$$\lambda \leq 40 \qquad (6.40\ a)$$

$$e_0/h \geq 2 \qquad (6.40\ b)$$

$$N_{Sg} / N_{Sd} \leq 0.15 \qquad (6.40\ c)$$

where:

λ	= slenderness ratio according to section 6.6.2
e_0	= first order eccentricity $M^0{}_{Sd} / N_{Sd}$
h	= depth of member
$M^0{}_{Sd}$	= first order moment under design load (including imperfections)
N_{Sd}	= axial force under design load
N_{Sg}	= axial force under (unfactored) quasi-permanent load (see section 7.2.2)

6.6.4 Geometrical imperfections

(1) The effect of geometrical imperfections should be included in the analysis (unless a specific method is used which in itself includes the effect of a relevant imperfection).

(2) The effect of imperfections can be based on an inclination α, with the following design value α_a for one individual column:

$$\alpha_a = 0.01 / \sqrt{l} \qquad (6.41)$$

where l = length of the member in metres.

(3) To calculate the combined effect of imperfections from horizontally connected vertical elements, a mean value α_{am} according to equ. (6.42) can be used.

$$\alpha_{am} = \alpha_a \sqrt{0.5\left(1 + \frac{1}{m}\right)} \qquad (6.42)$$

where m = number of vertical elements contributing to the combined effect.

(4) In the design of isolated columns m is always equal to 1, i.e. $\alpha_{am} = \alpha_a$, and l is the actual length of the column, i.e. generally not the buckling length.

MC 90 section

(5) In cases other than in (4) the values m and l depend on the case considered. Two examples are shown in Fig 6.38:

(a) Case (a) in Fig 6.38a for calculating the total effect of imperfections on a 'bracing member' A. Here, l and m are defined as follows:

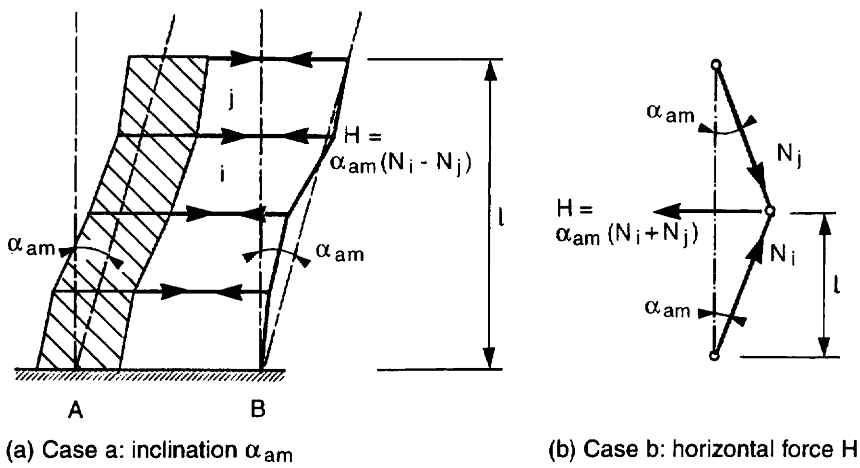
- For braced columns which are continuous throughout the building, l is the height of the building and m is the number of continuous members (including the bracing ones).
- For columns consisting of storey high elements, l is the storey height and m is the number of individual members.

(b) Case b in Fig 6.38b for calculating the total effect on a 'floor diaphragm' transferring the horizontal loads from braced to bracing members. In this case, l is the storey height and m the total number of columns in the two storeys which contribute to the total horizontal force on the floor.

(6) In the analysis of entire structures the effect of imperfections can be represented by horizontal forces, see Fig 6.38.

(7) For isolated columns the effect of imperfections can be represented by horizontal forces according to Fig 6.39 or, for non-sway columns, by an eccentricity e_a:

$$e_a = \alpha_a \, l_0 / 2 \tag{6.43}$$



(a) Case a: inclination α_{am}

(b) Case b: horizontal force H

Fig 6.38 Representation of geometrical imperfections

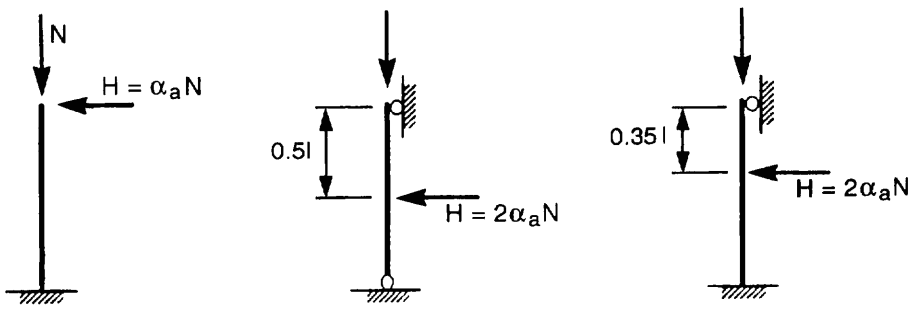


Fig 6.39 Effect of imperfections on isolated columns, expresses in terms of equivalent horizontal forces H

6.6.5 Method based on estimation of secant stiffness

MC 90 section

(1) With a reduced bending stiffness, i.e. a secant stiffness taking into account cracking, material non-linearity and creep, analysis can be made formally by second order elastic theory. In some cases amplification factors formulated on the basis of a fictitious buckling load can be used, provided the buckling load is based on such a reduced stiffness (i.e. a stiffness reflecting the conditions of the structure at ULS, not the stiffness of the structure in the fictitious state of buckling).

This method is applicable for isolated members as well as for whole structures. The effect of imperfections according to section 6.6.4 should be included in the analysis. In the following some estimates for the secant stiffness are given.

The cross-sections are designed at ULS for the normal forces and total moments resulting from the analysis.

(2) The general definition of the secant stiffness is:

$$K_B = \frac{M}{\left(1/r\right)} \tag{6.44}$$

where $1/r$ = curvature at moment M and normal force N

(3) A simple estimate of the secant stiffness is given by equ. (6.45). It can be used for symmetric cross-sections (including the reinforcement), but should not be used if the total reinforcement ratio $\rho < 0.01$.

$$K_B = \frac{0.4 E_c I_c}{1 + \varphi_{eff}} \tag{6.45}$$

where:

E_c	= design value of the modulus of elasticity of concrete
I_c	= moment of inertia (without reinforcement)
φ_{eff}	= effective creep ratio according to equ. (6.39)

(4) If the reinforcement is known (or assumed) then the following equation can be used as a better estimate:

$$K_B = \frac{0.2 E_c I_c}{\left(1 + \varphi_{eff}\right)} + E_s I_s \tag{6.46}$$

where:

E_s	= design value of modulus of elasticity of reinforcement
I_s	= moment of inertia of reinforcement (with respect to the CG of cross-section)

(5) If the cross-section can be shown to be uncracked under the design moment (including second order moment) and normal force, then the following value may be used:

$$K_B = \frac{0.8 E_c I_c}{\left(1 + \varphi_{eff}\right)} \tag{6.47}$$

(6) For symmetric cross-sections (including the reinforcement), a stiffness according to the following equation will give better agreement with the results of a refined analysis:

$$K_B = \alpha_\varphi \, \alpha_e \, E_c I_c + E_s I_s \tag{6.48}$$

where:

α_φ	$= [1 - 0.8\, \varphi_{eff} (1 - \lambda/200) \omega^{0.25}] \geq 0$
α_e	$= [0.08\, \nu f_{cd}^{0.6} e^{\lambda/100 - 2\omega}] \leq (\nu_u - \nu)$
λ	= slenderness ratio (see section 6.6.2)
ω	$= A_s f_{yd} / A_c f_{cd}$
A_s	= total area of reinforcement
ν	$= N_{Sd} / A_c f_{cd}$
ν_u	$= (1 + \omega)$

6.6.6 Simplified method for isolated columns

MC 90 section

(1) The following method is based on a simple estimate of the curvature of the critical section, giving the deformation and hence the second order moment. This is added to the first order moment to give the total design moment for which, together with the axial force, the cross-section is designed at ULS. The method should be limited to cases where the first order moment $M^0{}_{Sd}$ corresponds to an eccentricity $e_0 \geq 0.1h$.

(2) The method is best suited to individual columns with constant axial load and boundary conditions according to Fig 6.37a to e. It can also be used for other cases, provided the buckling length defined in section 6.6.2 is determined with due consideration of elastic restraints and/or variation of axial force.

(3) The total design moment is:

$$M_{Sd} = M^0{}_{Sd} + M_2 \tag{6.49a}$$

where:

$M^0{}_{Sd}$ = first order design moment, including the effect of imperfections

M_2 = second order moment

Differing first order end moments $M^0{}_1$ and $M^0{}_2$ in a non-sway column according to Fig 6.37 a, c, d and g may be replaced by an equivalent value:

$$M^0{}_e = 0.6M^0{}_1 + 0.4M^0{}_2 \tag{6.49b}$$

$M^0{}_1$ and $M^0{}_2$ should be inserted with the same sign if they give tension on the same side of the column, otherwise with opposite signs.

Further, $|M^0{}_2| \geq |M^0{}_1|$.

(4) The second order moment M_2 is determined as follows:

$$M_2 = N_{Sd}\, e_2 \tag{6.50 a}$$

$$e_2 = (1/r)\, l_0^2 / \beta \tag{6.50 b}$$

where:

N_{Sd} = axial force under design load

e_2 = deformation (eccentricity)

$1/r$ = curvature, see equ. (6.51)

l_0 = buckling length, see section 6.6.2

β = factor depending on the curvature distribution

For the factor β in equ. (6.50b), normally $\beta = 10$ ($\approx \pi^2$) can be used. However, if the first order moment is constant, then $\beta = 8$ should be used.

If the first order moment is caused by a concentrated horizontal load near midheight, or at the top of a cantilever column, then $\beta = 12$ can be used.

If the reinforcement is curtailed according to the moment diagram, then $\beta = 8$ should always be used.

(5) For cross-sections with symmetrical reinforcement, the curvature can be estimated as follows:

$$1/r = \alpha_\varphi\, \alpha_r\, (1/r_0) \tag{6.51}$$

where:

$(1/r_0)$ = $2\varepsilon_{yd}/z_s$

α_φ = correction factor for creep

α_r = correction factor for normal force, reinforcement and slenderness

ε_{yd} = f_{yd}/E_s

z_s = $2i_s$ = distance between centres of gravity of reinforcement of either side of cross-section

i_s = radius of gyration of the reinforcement area

For the correction factors α_φ and α_r two sets are given below; these values should not be mixed. MC 90 section

(6) Simplified correction factors for the curvature are as follows:

$$\alpha_\varphi = 1 + \varphi_{eff}/4 \tag{6.52 a}$$

$$\alpha_r = \frac{\nu_u - \nu}{\nu_u - \nu_{bal}} \le 1 \tag{6.52 b}$$

where:

φ_{eff}	= effective creep ratio, see section 6.6.3
ν	$= N_{Sd}/A_c f_{cd}$
ν_u	$= (1 + \omega)$
ν_{bal}	= 0.4 = normal force at maximum moment
ω	$= A_s f_{yd}/A_c f_{cd}$
A_s	= total area of reinforcement

(7) The following correction factors give better agreement with a general calculation:

$$\alpha_\varphi = 1 + (20/\lambda)\,\varphi_{eff} \tag{6.53 a}$$

$$\alpha_r = 2\,(1 - \nu/\nu_u)\, e^{-(1-\omega)\lambda/100} \tag{6.53 b}$$

where λ = slenderness ratio, see section 6.6.2.

With these factors, the limitation $e_0 < 0.1h$ is not necessary.

6.6.7 Biaxial bending

Biaxial bending of columns can be considered in the following way. Reinforcement is designed for the governing load combinations in each principal direction, disregarding biaxial bending. The following condition is then checked for combinations including biaxial bending:

$$\left(\frac{M_{Sd,x}}{M_{Rd,x}}\right)^a + \left(\frac{M_{Sd,y}}{M_{Rd,y}}\right)^b \le 1 \tag{6.54}$$

where:

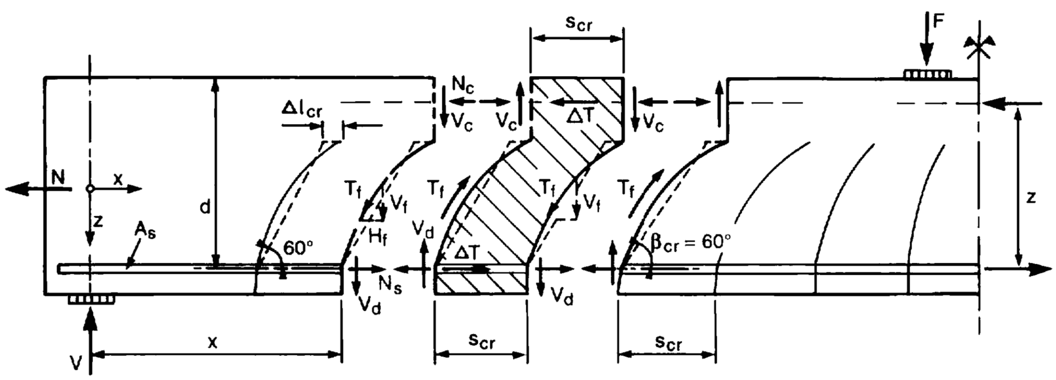
$M_{Sd,x}$	= design moment in x-dir. incl. second order moment
$M_{Sd,y}$	= design moment in y-dir. incl. second order moment
$M_{Rd,x}$	= moment capacity in x-dir. with belonging axial force
$M_{Rd,y}$	= moment capacity in y-dir. with belonging axial force
a, b	= exponents, see comment below

Exponents a and b depend on reinforcement ratio and slenderness and can be calibrated for agreement with calculations according to a general method. Such values for the exponents are given in *Betonghandbok* (1990). High values are favourable; the value 1.0 is always on the safe side and can be used as a simplification.

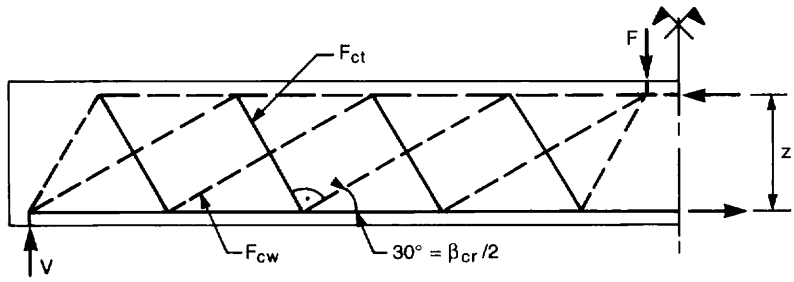
6.7 Design of slabs 6.4

6.7.1 General and design model

(1) The design model for the B-region of slabs consists of two chords or outer layers connected by a web or intermediate inner layer. In the general case of bi- or triaxially loaded slabs the chords are represented by biaxially loaded plate elements resisting the in-plane effects of the moments and axial forces and torsion, as well as additional forces due to the transfer of shear and torsion in the web due to the truss action.



(a) Cracked member and shear transfer mechanisms across cracks



(b) Truss model with biaxial tension-compression field in the concrete

Fig 6.40 Shear transfer mechanisms and truss model for members without transverse reinforcement

(2) In B-regions of slabs primarily subjected to moments parallel to the directions of the reinforcement, the flexural design to determine the inner lever arm between the chords follows section 6.4.2 and the shear design follows section 6.7.2. MC 90 section

(3) The D-region design for punching of a column through a two-way slab is dealt with in section 6.7.3.

6.7.2 Shear design of one-way spanning slabs or members

(1) The structural behaviour of slender members without transverse reinforcement (Fig 6.40a) may be represented by the truss model with a biaxial tension-compression field in the web (Fig 6.40b). According to this model the required force in the tension chord is:

$$F_s = M/z + 0.58V \qquad (6.55)$$

(2) The ultimate strength of this model is not determined by the shear transfer mechanisms across the cracks, i.e. mainly the friction of the crack surfaces and the dowel action (see Fig 6.40a). The design capacity may be assessed by the following empirical formula:

$$V_{Rd} = [0.10\kappa\,(100\rho_l f_{ck})^{1/3} - 0.12\sigma_{cd}]\, b_w d \qquad (6.56)$$

where:

κ	$= \left(1 + \sqrt{\dfrac{200}{d}}\right) \leq 2.0$ = factor for size effect with d in [mm]
f_{ck}	= characteristic cylinder strength [MPa]
ρ_l	$= A_s / b_w\, d$ = longitudinal reinforcement ratio [-]
σ_{cd}	$= N_{Sd} / (b_w\, d)$ = axial stress [MPa]; (+ for tension)

(3) If V_{Sd} exceeds V_{Rd} according to equ. (6.56), then transverse reinforcement is required which should be designed according to section 6.4.3.2. However, no minimum transverse reinforcement need be provided in regions with $V_{Sd} < V_{Rd}$.

(4) If in two-way slabs the principal shear is not in the direction of the longitudinal reinforcement the design may be carried out according to section 6.4.2.5 of MC 90. MC 90 section

6.7.3 Punching

6.7.3.1 General

(1) The punching resistance to the transverse effects, i.e. the transfer of concentrated forces (loads or reactions), acting on slabs without shear reinforcement may be verified in terms of nominal shear stresses at control perimeters around the concentrated force. This empirical approach for assessing the punching resistance does not imply any physical meaning for the nominal shear stress on the defined section.

(2) The applied shear τ_{Sd} can be determined at a critical section taken on a perimeter at a distance 2.0d from the boundary of the loaded area, as shown in Figs 6.41 and 6.42, and should satisfy the following condition:

$$\tau_{Sd} = \frac{F_{Sd}}{u_1 d} \leq \tau_{Rd} \tag{6.57}$$

where:

F_{Sd} = punching load due to the applied external loads; this may include the vertical effect of prestress acting inside a perimeter at a distance equal to half the slab depth from the periphery of the loaded area

u_1 = perimeter at a distance 2d from the boundary of the loaded area (see Figs 6.41 and 6.42)

d = effective depth (average for both directions)

(3) The applied load F_{Sd} may be reduced by the loads within the control perimeter defined above in (2), which is especially relevant for column bases.

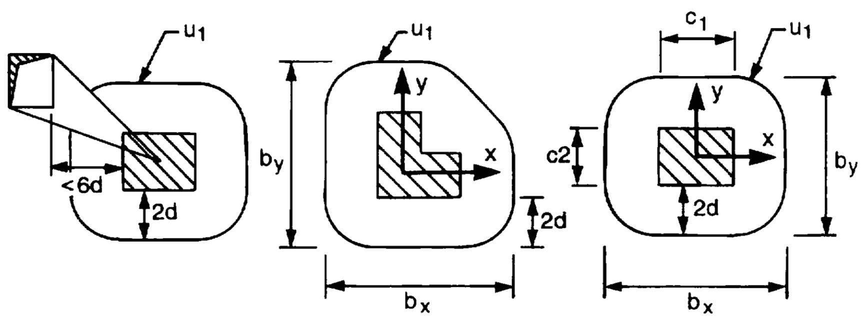


Fig 6.41 Critical perimeter u_1 *at interior columns*

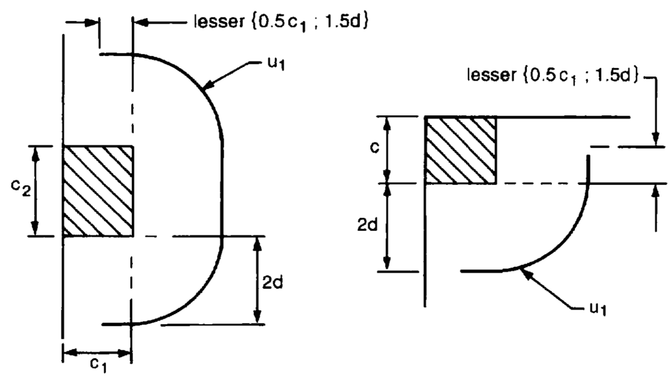


Fig 6.42 Critical perimeter u_1 *at edge and corner column*

6.7.3.2 Symmetric punching of slabs without shear reinforcement

MC 90 section

(1) For the above defined critical section, the resistant punching force for symmetrically loaded (interior) columns is given by:

$$\tau_{Rd} = F_{Rd}/u_1 d = 0.12\kappa(100\rho f_{ck})^{1/3} \quad (6.58)$$

where:

$\kappa \quad = \left(1 + \sqrt{\dfrac{200}{d}}\right) \leq 2.0$ = factor for size effect, with d (mm)

$\rho[-] \quad = \sqrt{\rho_x \rho_y}$ = flexural reinforcement ratio

In each direction ρ should be calculated for a width equal to the dimension of the loaded area plus $3d$ on each side of it (or the slab edge if it is closer).

(2) For prestressed slabs the beneficial effects of the prestress may be considered as in the FIP *Recommendations for the design of post-tensioned slabs and foundations rafts.*

6.7.3.3 Punching of slabs with transfer of moments to column

The resistant punching load is reduced, if there is a moment transfer between the slab and the column. This may be taken into account by increasing the applied shear τ_{Sd} by a factor b giving an effective design value for the punching load:

$$F_{Sd,eff} = \beta F_{Sd} \quad (6.59)$$

where $\beta = 1.15$ for edge columns and $\beta = 1.40$ for corner columns

6.7.3.4 Slabs with punching shear reinforcement

(1) When the punching capacity according to equ. (6.58) or (6.59) is insufficient, either a drop panel or shear reinforcement may be provided. If shear reinforcement is provided, the following checks have to be carried out.

(2) The maximum punching capacity is limited by the capacity of the concrete in compression at the node where the load is transferred. The maximum load transferred across the perimeter u_0 immediately adjacent to the loaded area (see Fig 6.43) is limited to:

$$F_{Sd,eff} \leq F_{Rd} = 0.5\, u_0\, d f_{1cd} \quad (6.60)$$

where u_0 = perimeter defined in Fig 6.43.

(3) Within the zone in which the punching shear reinforcement is placed, the punching capacity is given by:

$$F_{Sd,eff} \leq 0.09\,\kappa\,(100\rho f_{ck})^{1/3} u_1 d + 1.5d(A_{sw}/s_r) f_{ywd} \sin\alpha \quad (6.61)$$

where:

A_{sw} = area of shear reinforcement in each layer around the column

s_r = radial spacing of the layers of shear reinforcement (Fig 6.44)

α = angle between shear reinforcement and the plane of the slab

f_{ywd} ≤ 300MPa

(4) A minimum amount of shear reinforcement has to be provided as defined by the following equation:

$$A_{sw}/s_r = [0.03\,(100\rho f_{ck})^{1/3} u_1]/1.5 f_{ywd} \sin\alpha \quad (6.62)$$

(5) The shear reinforcement may consist of vertical or inclined bars or stirrups and must be arranged in accordance with the requirements defined in Fig 6.45.

(6) At edge- and corner-column connections, the shear reinforcement required by calculation should be placed within the regions indicated in Fig 6.44 b and c. Similar reinforcement at the same spacings should be provided in the areas between these regions and the slab edges, but should not be taken into account in the calculations.

(7) Outside the perimeter $u_{n,eff}$ as defined in Fig 6.46, within which the shear reinforcement is required, the punching capacity must comply with equ. (6.57). In this case it may be assumed that the effect of unbalanced moment transmitted by shear no longer exists.

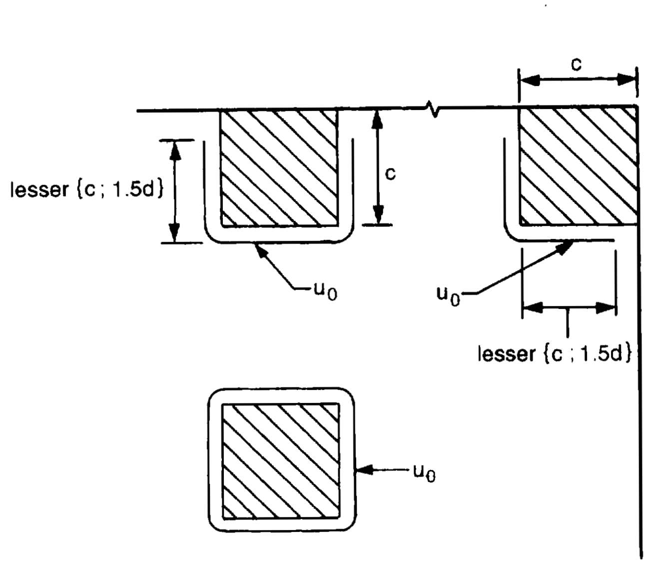


Fig 6.43 Perimeter u_0 *for maximum resistance* F_{Rd}

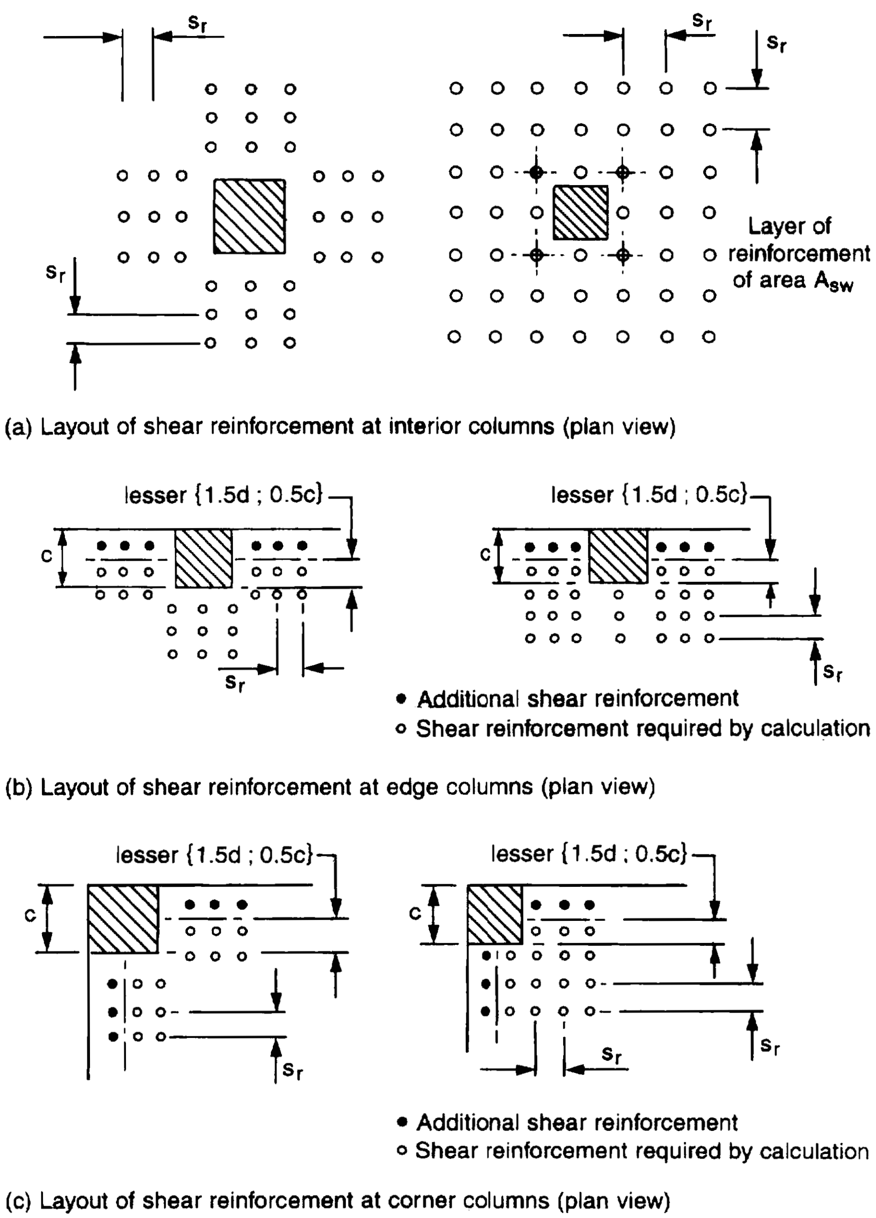


Fig 6.44 Layout of shear reinforcement for different cases

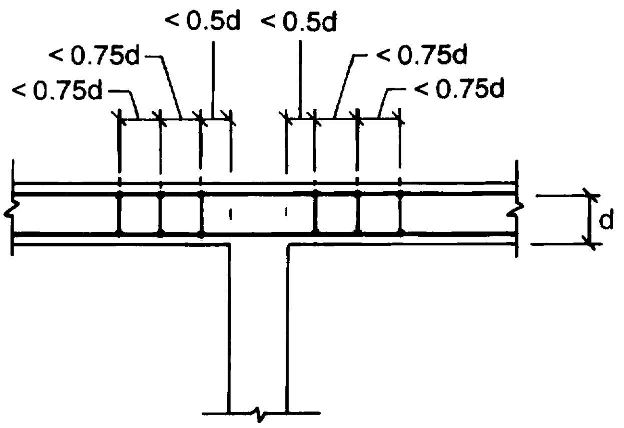


Fig 6.45 Layout of shear reinforcement (elevation)

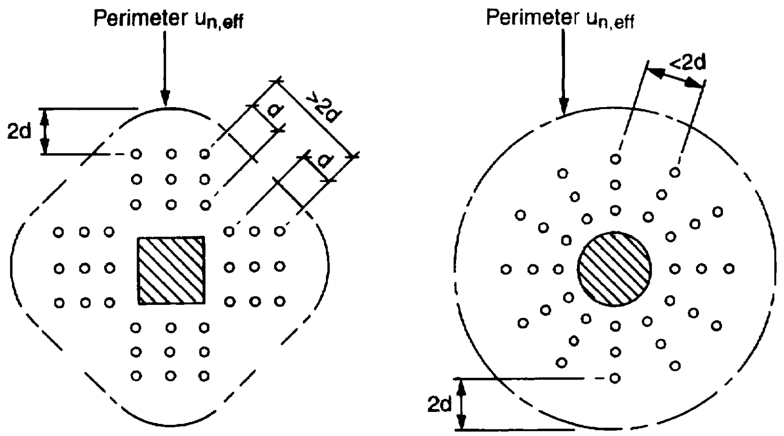


Fig 6.46 Definition of perimeter $u_{n,eff}$

6.8 Plate and shell elements

(1) A shell element is an element in the B-region of a slab, plate or shell which is subjected to combined action effects, i.e. axial forces and bending moments as well as in-plane and out-of-plane shear forces due to shear and torsion. The design model consists of two chords or outer layers connected by a web or intermediate inner layer in between. The outer layers represent plate elements subjected to in-plane normal and shear forces resulting from the combined action effects. Generally, additional forces also have to be considered, resulting from the transfer of shear and torsion in the web due to truss action.The inner layer between these outer plate elements transmit the transverse shear forces as a web between the chords.

(2) The distance between the outer layers, i.e. the inner lever arm z or the web height, is defined by the axes of the outer plate elements. The position of these axes may normally be assumed to be in the middle of the two reinforcement layers, unless large axial compressive forces act on the shell element, i.e. the compressive strength of the struts in the outer layers is exceeded. In the latter case $z = 2h/3$ may be taken as an approximation, where h = shell thickness.

(3) The design model for the plate elements under biaxial normal forces and shear forces consists of inclined struts equilibrated by ties in two orthogonal directions, normally. For biaxial tensile and compressive normal forces and for pure shear, the angle of the struts may be assumed to be at 45° with respect to the directions of the reinforcement.

(4) For more refined considerations see MC 90 as well as CEB Bulletin 141.

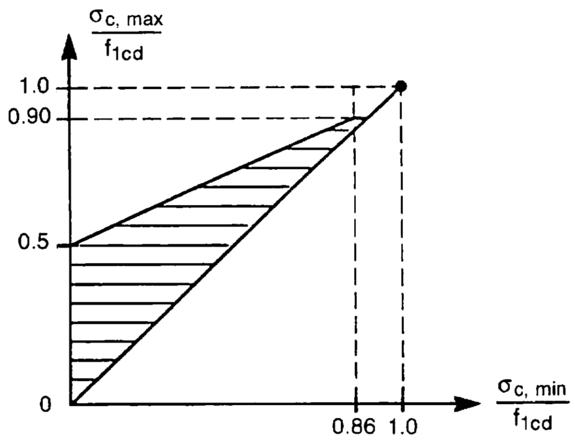


Fig 6.47 Fatigue strength of concrete in compression

6.9 Fatigue

(1) Fatigue checks should be made for the effective fatigue action based on traffic measurements and by applying $\gamma_{Sd} = 1.00$.

(2) Stresses should be determined on the basis of elastic theory, taking into account cracking of the concrete by modifying the stiffnesses accordingly.

(3) A verification of the fatigue strength under compression need not be carried out if the following condition holds (Fig 6.47):

$$\sigma_{c,max}/f_{1cd} \leq (0.50 + 0.45\sigma_{c,min}/f_{1cd}) \leq 0.90 \qquad (6.63)$$

where:

$\sigma_{c,max}$ = maximum compressive stress at fibre under frequent load combination

$\sigma_{c,min}$ = minimum compressive stress at the same fibre, i.e. where $\sigma_{c,max}$ occurs: If $\sigma_{c,min} < 0$ (tension) then the condition $\sigma_{c,max}/f_{cd} \leq 0.50$ should be fulfilled.

(4) For unwelded reinforcing bars and for prestressing steel subjected to tension, adequate fatigue resistance may be assumed if, under the frequent combination of actions, the stress variation $\Delta\sigma_s$ does not exceed 70N/mm^2.

(5) Slender and special structures may require special attention, see MC 90, section 2.1.7.

7 Serviceability Limit State

MC 90 section

7.1 Requirements

7.1

(1) It should be demonstrated that the structure and the structural elements will perform adequately in normal use. To meet these requirements the serviceability limit states should be verified.

(2) Depending on the type and function of a structure or a structural element the verification of different serviceability limit states may be relevant, such as the limitation of:

- stresses (see section 7.4)
- crack widths (see section 7.5)
- deformations (see section 7.6)
- vibrations (see section 7.7).

7.2 Actions and action effects

7.2.1 Permanent and variable actions

(1) The permanent and variable actions are defined as for the ULS (see section 6.2), but shall be applied with $\gamma_g = \gamma_q = 1.0$.

(2) Prestressing effects shall be considered using their mean value, as stated in section 3.4.1, and with a safety factor of $\gamma_p = 1.0$.

7.2.2 Load combinations

1.6.6.5

(1) The combination of loads to be considered depends on the type of SLS and on the specific problem. It is appropriate to utilize one of the combinations given in Table 7.1, i.e:

- quasi-permanent combination,
- frequent combination,
- rare combination.

(2) All direct and indirect actions such as loads and imposed or restrained deformations due to temperature effects, shrinkage, creep, changes of support conditions, etc. should be considered.

(3) The values in Table 7.1 are indicative and, wherever possible, loads and corresponding ψ-values should be taken from national or international standards.

7.2.3 Material properties

(1) The material properties shall be assumed to have their mean value or their characteristic value depending on the particular application and the relevance of the behaviour. Partial safety factor γ_c shall be as given in section 2.1.

(2) The following examples may be used as guidelines for which value to apply:

- deflections: mean value of secant modulus of elasticity, E_{cm}.
- onset of cracking for loads, crack widths: lower characteristic value of concrete tensile strength, $f_{ctk,0.05}$.
- restraint forces in uncracked structures under imposed deformation: upper characteristic value of tensile strength, $f_{ctk,0.95}$.

Table 7.1 Combination of actions for SLS

Quasi-permanent	$G + P + \Sigma\psi_2 (Q_1 + Q_2)$
Frequent	$G + P + \psi_1 Q_1 + \Sigma\psi_2 Q_2$
Rare	$G + P + Q_1 + \Sigma\psi_0 Q_2$

where:

Q_1 = basic variable action
Q_2 = other variable actions
P = prestress
ψ_1 = coefficient for frequent value of an action
ψ_2 = coefficient for quasi-permanent value of an action
ψ_0 = coefficient for combination of actions acc. to Table 6.2

Actions	ψ_1	ψ_2
Buildings		
dwellings	0.4	0.2
offices, retail store	0.6	0.3
parking areas	0.7	0.6
Highway bridges*		
l = 10m	0.7	0
l = 100m	0.5	0
Wind**	0.2 – 0.5	0
Snow**	0.2 – 0.8	0 – 0.2
Temperature	0.5	0

* For intermediate values: linear interpolation
** Depending on geographic location

7.3 Structural analysis

7.3.1 Effective span

Usually, the effective span l is equal to the distance between adjacent support axes.

In the case of a wide support width t, the support axis may be assumed at a distance of $t/3$ from the support face, but not more than $h/2$ (where h = depth of the supported member), unless more refined considerations are made.

7.3.2 Effective width of flanges

(1) In the absence of more accurate methods, the effective width b_{eff} for compression flanges of beams with solid webs, and hollow box sections, may be taken as:

$$b_{eff} = b_w + l_o/5 \quad \text{for T-beams} \qquad (7.1\text{ a})$$

$$b_{eff} = b_w + l_o/10 \quad \text{for L-beams} \qquad (7.1\text{ b})$$

where:

b_w = thickness of the web

l_o = distance between points of zero moments, see Fig 7.1

(2) The effective width b_{eff} shall not be taken as longer than the actual width b of the flange, see Fig. 7.1. These effective widths may be assumed to be constant over the entire span, including the zones near intermediate supports.

(3) In general, the effective width of a tension flange may be assumed to be the same as for a compression flange.

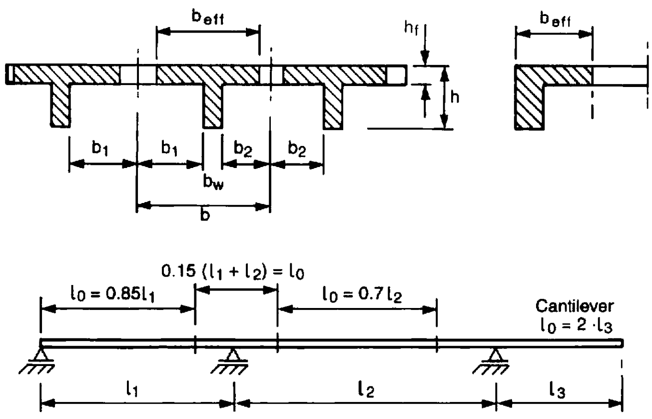


Fig 7.1 Effective widths for the flanges of T-beams

7.3.3 Distribution of internal forces

(1) For SLS checks it is, in general, assumed that the structure as a whole behaves quasi-elastically, i.e. that the distribution of internal forces of hyperstatical systems may be calculated according to the theory of elasticity. Where relevant, non-elastic effects such as cracking and, in certain cases, creep and shrinkage, are then accounted for by an appropriate reduction of stiffness.

(2) However, in many cases, it is sufficient to assume a plausible distribution of internal forces which satisfies the conditions of equilibrium and which, from experience, can be expected to differ little from the elastic one.

7.3.4 Redistribution of internal forces

(1) Relevant changes of statical systems due to different stages of construction should be considered. Due to creep and concrete relaxation the final distribution of internal forces tends towards that of the one-mass system, as if the structure had been entirely cast in one operation.

(2) Detailed step-by-step calculations could be carried out in accordance with the methods given in the CEB Bulletin 215. In most cases the changes in the distribution of action effects can be approximated by:

$$S_\infty = S_o + (S_e - S_o)\frac{\phi}{1 + 0.8\phi} \tag{7.2}$$

where:

S_∞ = final action effect after redistribution

S_o = initial action effect at the construction stage

S_e = action effect for the one-mass system

ϕ = creep coefficient.

Both S_o and S_e include the effects of prestressing forces. Fig. 7.2 illustrates the above equation.

7.4 Stress limitations 7.3

7.4.1 General and cases where stress limitations are not essential

(1) Under service load conditions stress limitations may be required for:

- tensile stresses in concrete
- compressive stresses in concrete
- tensile stresses in steel.

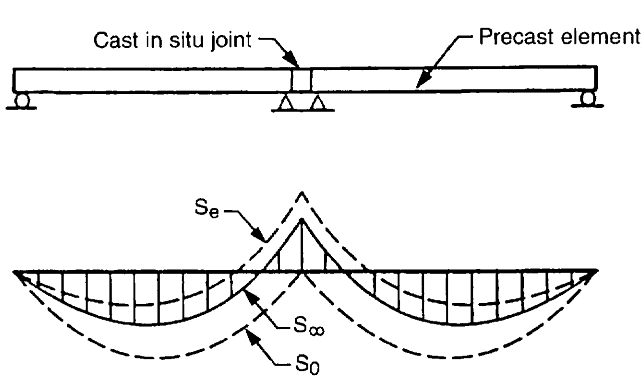


Fig 7.2 Redistribution of internal forces for a beam on three supports

(2) The stress limitations given in sections 7.4.3 and 7.4.4 below may generally be assumed to be satisfied without further calculations provided the minimum reinforcement requirements of section 7.5.5 are satisfied. 7.3.4

7.4.2 Concrete in tension

When assessing cracking according to section 2.1.4, the concrete tensile strength $f_{ctk,0.05}$ should be used. In webs with shear and torsion, the principal tensile stress should be used to assess cracking.

7.4.3 Concrete in compression 7.3.2

(1) Excessive compressive stresses in the concrete under service load may lead to longitudinal cracks and high creep that is hard to predict, with serious consequences for prestressing losses. When such effects are likely to occur, measures should be taken to limit the stresses.

(2) If under the quasi-permanent load combination the stress exceeds $0.45\,f_{cm}(t)$ a non-linear model should be used for the assessment of creep.

7.4.4 Steel 7.3.3

(1) Tensile stresses in the steel under serviceability conditions should be limited to:

$$\sigma_s \leq 0.8 f_{yk} \qquad (7.3)$$

This is to avoid inelastic deformation of the steel since this would lead to large, permanently open cracks.

(2) For more stringent crack control it may be necessary to restrict further the stress level in reinforcing steel, and the increase in the stress after decompression in prestressing steel, see section 7.5.4.

7.5 Crack control 7.4.1

7.5.1 Requirements

(1) It should be ensured, with adequate probability, that cracks will not impair the functional requirements, the durability, and the appearance of the structure.

(2) Cracks do not, *per se*, impair the serviceability or durability of a concrete structure. Cracks due to tension, bending, shear and torsion are often inevitable in reinforced concrete structures, resulting from either direct loading or restraint of imposed deformations.

(3) Thus, the designer should specify, in agreement with the client, the relevant criteria to be fulfilled for the finished structure and for intermediate construction phases. Such criteria may involve the limitation of either tensile stresses or crack widths.

Table 7.2 Limits for the characteristic crack width

MC 90 section

Exposure class according to Section 4.1	w_{lim} [mm] under the frequent load combination		
	Reinforced	Post-tensioned	Pretensioned
1	(0.30)	0.20	0.20
2	0.30	0.20	No tension within distance c_{nom} of tendons
3 and 4	0.30	a) No tension within distance c_{nom} of ducts or b) 0.20 where impermeable ducts or coating of tendons are used	

(4) Crack width limitation may be verified either by calculation of crack widths or by appropriate detailing. In cases where the ULS design leads to low reinforcement ratios, minimum reinforcement may have to be provided.

(5) Due to many uncertainties in the assumptions, actual crack widths in the structure may be larger than those assumed in the design.

7.5.2 Crack width limits

7.4.3

(1) In the absence of specific requirements (e.g. water-tightness) the limits for the characteristic crack width w_{lim} given in Table 7.2 may be applied for the exposure classes 1 to 4 defined in Table 4.1.

(2) For exposure class 1, this limit may be relaxed provided that it is not neccessary for reasons other than durability.

(3) When de-icing agents are expected to be used on the top of tension zones, appropriate limits should be specified in agreement with the client, depending on the thickness and quality of the concrete cover and the provision of additional protective layers.

(4) For corrosion protected tendons the crack width limits of reinforced concrete members shall apply. Corrosion protected tendons mean multistrand tendons encapsulated in a thick-walled plastic tube, or monostrand tendons protected with grease and extruded sheathing, or equivalent systems.

7.5.3 Calculation of crack widths

7.5.3.1 Introduction

(1) The following inequality should be observed:

$$w_k \leq w_{lim} \tag{7.4}$$

where:

w_k = characteristic crack width calculated under the appropriate combination of actions

w_{lim} = nominal crack width limit specified for cases of expected functional consequences or for cases related to durability

(2) The formation, propagation and width of cracks depend on a great number of parameters, some of which (e.g. casting and curing procedure, climatic conditions, temperature, etc.) are not known at the design stage. Elaborate crack-width calculations are thus only warranted in special cases and if the relevant parameters can be reliably predicted. MC 90 section

7.5.3.2 Basic crack width formula 7.4.3.1

(1) For all stages of cracking, the design crack width may be calculated according to:

$$w_k = 1.7\, s_r\, \varepsilon_{sm} \qquad (7.5)$$

where:

s_r = average crack spacing

ε_{sm} = average steel strain

(2) The crack spacing and average steel strain may be calculated for a concrete tie around a reinforcing ribbed bar as follows:

$$s_r = 2c + \alpha_b\, \varnothing/\rho \qquad (7.6\ a)$$

$$\varepsilon_{sm} = \varepsilon_s - 0.40\, \varepsilon_{sr1} \qquad (7.6\ b)$$

where:

c = concrete cover

ϕ = bar diameter

ρ = $A_s/A_{c,eff}$

$A_{c,eff}$ = effective concrete area defined in Fig. 7.6

α_b = coefficient for bond conditions

for high bond bars:	$\alpha_b = 0.125$ for good bond;
	$\alpha_b = 0.20$ for other cases
for pretensioned strands:	$\alpha_b = 0.19$ for good bond
	$\alpha_b = 0.3$ in other cases
for smooth bars or wires:	$\alpha_b = 0.25$ for good bond
	$\alpha_b = 0.40$ in other cases

ε_{sr1} = $f_{ct,min}/(\rho E_s)$

(3) For more details see MC 90, 7.4.3.

7.5.4 Crack control by detailing 7.4.4

(1) Applying section 7.5.3 with some simplifying assumptions, and assuming crack widths of 0.30mm for reinforced concrete members, and 0.20mm for prestressed concrete members, the following simplified rules for detailing are obtained.

(2) For cracking caused mainly by restraint, crack widths will not generally be excessive provided that the bar sizes given in Fig 7.3 are not exceeded. The σ_s-value of Fig 7.3 is that calculated at cracking of the element.

(3) For cracks caused mainly by loads, crack widths will not generally be excessive provided that either the provisions of Fig 7.3 or those of Fig 7.4 are satisfied.

7.5.5 Minimum reinforcement requirements 7.4.5

(1) In every region where under SLS conditions the tensile strength of concrete may be exceeded, a minimum amount of reinforcement should be provided to ensure predictable behaviour of the member.

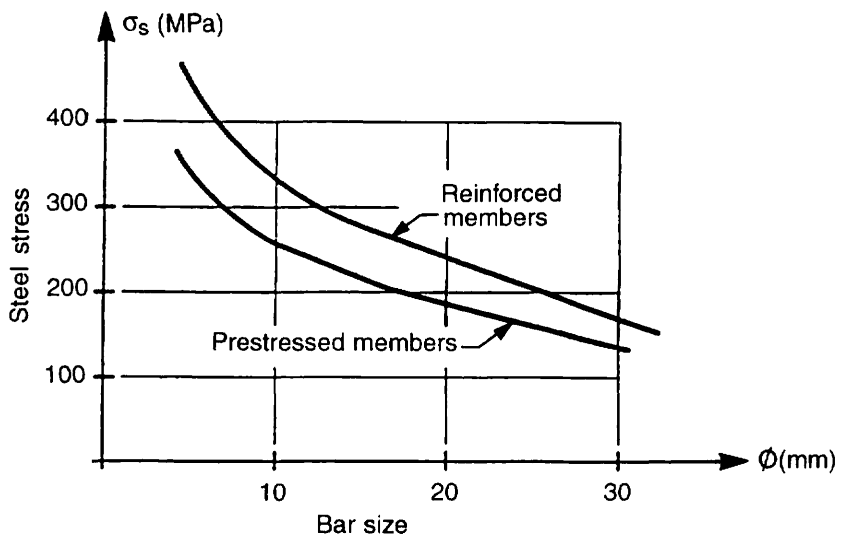


Fig 7.3 Maximum bar size

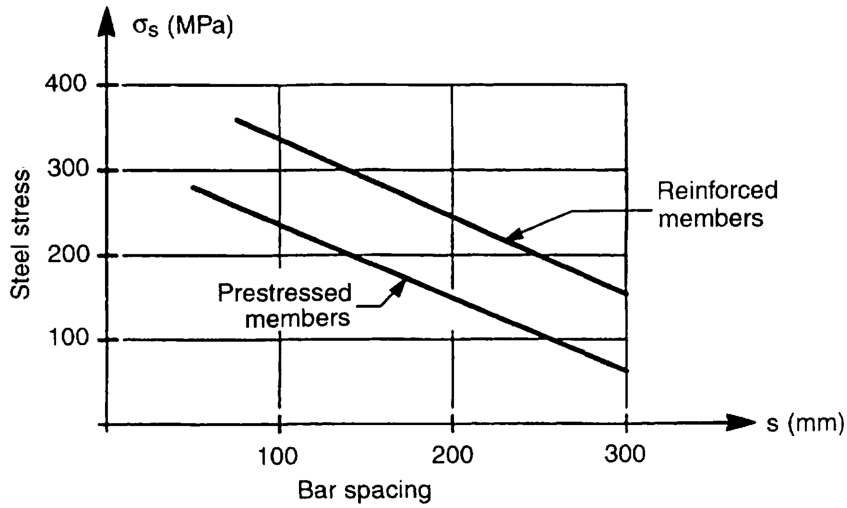


Fig 7.4 Maximum bar spacing

(2) For the combination of pure tension and flexure, and in the absence of more rigorous methods, a minimum amount of reinforcement, $A_{s,min}$, should be provided within tensioned concrete zones of all load-bearing members:

$$A_{s,min} = \rho_{r,min}\, A_{c,eff} \tag{7.7}$$

where:

$\rho_{r,min}$ = minimum reinforcing percentage acc. to Fig 7.5

$A_{c,eff}$ = efffective concrete area as defined in Fig 7.6

(3) In tension zones with large diameter bars or with bundles of bars requiring large concrete cover, a skin reinforcement in accordance with Fig 7.7 is required for adequate crack control. This reinforcement may be taken into account for the flexural and shear design, if appropriately detailed.

(4) In prestressed members or in reinforced concrete members subject to compressive normal force, the amount of minimum reinforcement may be reduced below that necessary for ordinary reinforced concrete due to the influence of:

- the increased flexural stiffness of the compression zone,
- the contribution of the prestressing tendons,
- the effect of prestress or compressive normal force contributing to crack width limitation of single cracks.

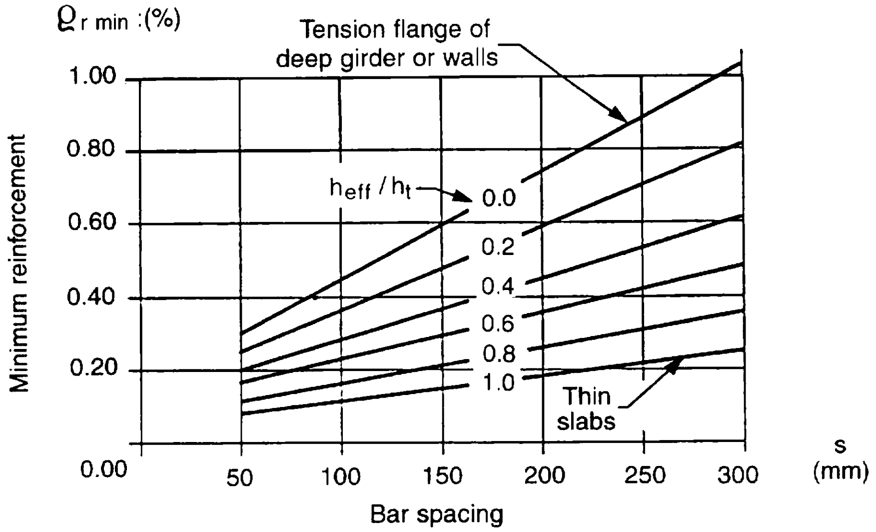


Fig 7.5 Minimum percentage $\rho_{r,min}$ (the graph is valid for f_{ctm} = 2.90MPa and f_{yk} = 460MPa; for other values extrapolate with: $\rho_{r,min}$ (f_{ctm}/2.9) (460 /f_{yk})

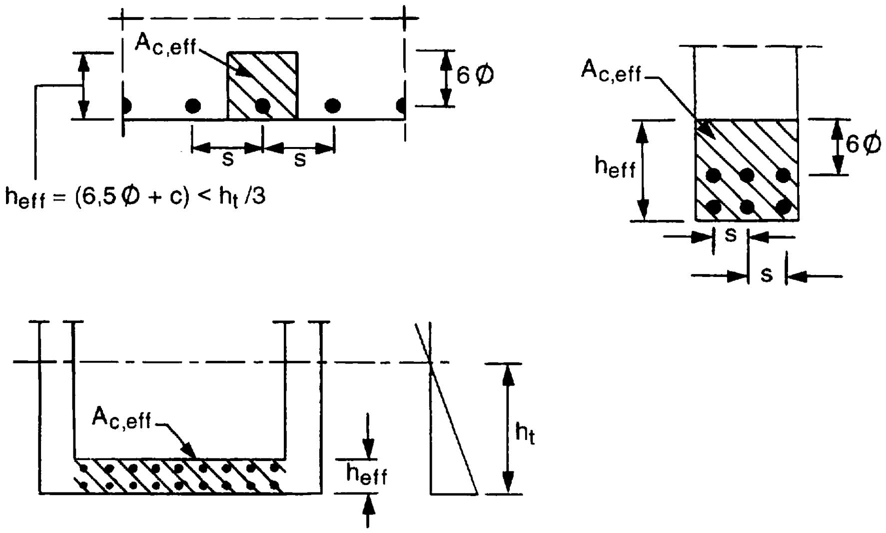


Fig 7.6 Effective concrete area for minimum reinforcements

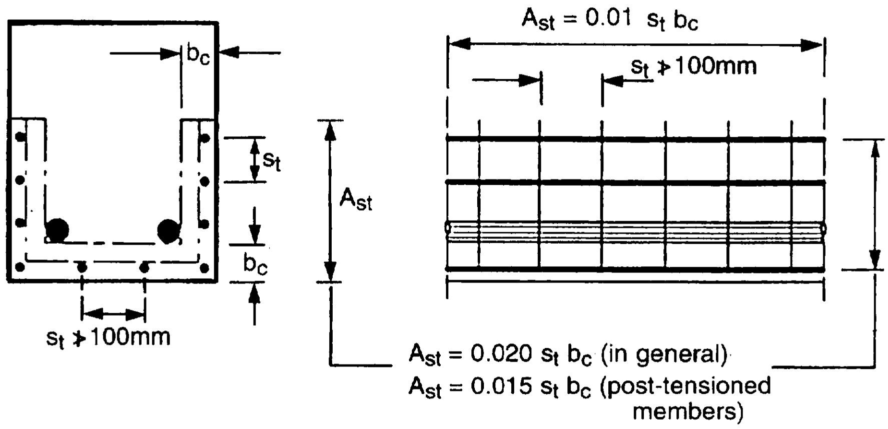


Fig 7.7 Skin reinforcement in cases of large concrete cover

(5) In prestressed members, the minimum reinforcement for crack control is not necessary in areas where, under the rare combination of actions, the concrete remains in compression. MC 90 section

(6) Prestressing tendons may be taken into account as minimum reinforcement within a 300mm square surrounding the tendon, provided that the different bond behaviour of the tendons and reinforcement is considered. In the absence of better information, the prestressing tendons may be assumed to be 50% effective.

7.5.6 Crack control for D-regions

7.5.6.1 Definition of the model

A D-region may be considered with a strut-and-tie model for the verification of cracking at the SLS. The model should be orientated by the stress fields determined from a linear-elastic analysis.

7.5.6.2 Crack control

(1) For the verification of crack control the requirements of section 7.5.1 apply.

(2) The verification of crack control is carried out by checking the crack width of the tie with the maximum force. The area of the tie is defined in Fig 7.6 according to MC 90. Based on the area and stress in the tie, the verification can be carried out in accordance with section 7.5.

7.6 Deformations 7.5

7.6.1 Requirements

(1) In-service deformations (deflections and rotations) may be harmful to:

- the appearance of the structure,
- the integrity of non-structural parts,
- the proper function of the structure or its equipment.

To avoid harmful effects of deformations appropriate limiting values should be respected.

(2) Deformation limits should normally be agreed between the designer and the client. In the absence of such agreements, the following values can be used as guidelines:

(a) total deflection below level of supports under quasi-permanent loads: span /200 to span /300

(b) deflection occuring after addition of partitions: span /500 to span /1000

More detailed guidance is given in ISO 4356.

7.6.2 Means of limiting deformations

(1) Under certain conditions, the checking of deformations by calculation may not be necessary. Conditions generally contributing to a reduction of deformations are:

(a) Prestressing, even to a low degree

(b) using high-strength concrete

(c) careful curing of the concrete

(d) removing the falsework as late as possible or else supporting the structure by temporary props

(e) proper dimensioning and detailing of the reinforcement

(f) avoiding high span/depth ratios $\alpha\, l\, /\, h$ (see Table 7.3)

(g) compensating by initial cambering.

MC 90 section

(2) For condition (f) the α-values of different structural systems which yield approximately the same deflections are given in Table 7.3. Deflections rarely become critical for spans smaller than 5m or when the $\alpha l/h$ -values do not exceed 25 for beams or 30 for slabs.

(3) Special attention should be given to the support conditions (continuity) in hyperstatic structures: a small reduction in end restraint due to cracking at the supports may lead to a considerable increase in deflection in the span.

7.6.3 Deformations due to bending

(1) For building members, long-term deflections can be evaluated by the following relations based on a bilinear relationship between load and deflection:

$$a = (1 + \phi)a_e \qquad \text{for } M_d < M_r \tag{7.8 a}$$

$$a = (h/d)^3\eta\,(1 - 20\rho_{cm})\,a_e \qquad \text{for } M_d \geq M_r \tag{7.8 b}$$

where:

M_r = cracking moment assessed according to section 2.1.4 and based on f_{ctm}

a_e = elastic deflection calculated with the rigidity $E_c\,I_c$ of the cross-section (neglecting the reinforcement)

M_d = bending moment at mid-span of a beam or a slab, or at the fixed end of a cantilever under frequent actions

ρ_{tm} = geometrical mean ratio of tensile reinforcement

ρ_{cm} = geometrical mean ratio of compressive reinforcement

η = correction factor (see Table 7.4), which includes the effects of cracking and creep

ϕ = creep coefficient

(2) The mean percentage ρ_{tm} of tensile reinforcement is determinded according to the bending moment diagram (see Fig 7.5) as:

$$\rho_{tm} = \rho_a \frac{l_a}{l} + \rho_o \frac{l_0}{l} + \rho_b \frac{l_b}{l} \tag{7.9}$$

where:

ρ_a, ρ_b = percentages of tensile/compressive reinforcement at the left and right supports, respectively

ρ_o = percentage of tensile reinforcement at the M_{max}-section

An estimate of the lengths l_a and l_b is generally sufficient.

(3) For other types of deformations, see MC 90.

Table 7.3 Values of α (ratio of notional to actual span)

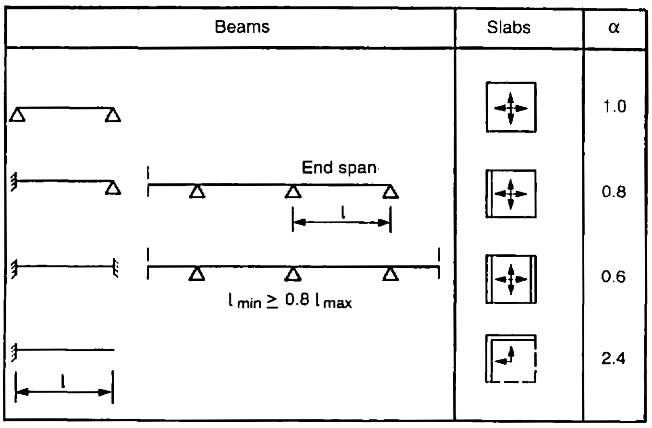


Table 7.4 Correction factor η for estimate of deflection

MC 90 section

ρ_{tm} [(%)]	0.15	0.20	0.30	0.50	0.75	1.00	1.50
η	10	8	6	4	3	2.5	2

7.6.4 Deformation control of D-regions

(1) For the verification of deformation control the requirements of section 7.6.1 apply.

(2) The deformations of the struts may be assessed with an average area over the entire length, because the struts are stiffer than the ties and therefore have only a small influence on the deformations of the D-region.

(3) The deformations of the ties should be calculated considering the tension stiffening effect provided by the concrete between the cracks. This effect may be calculated according to section 3.2 of MC 90, and in particular the expressions given in section 3.2.3.

7.7 Vibrations

7.6

7.7.1 General

Vibrations may affect the serviceability of a structure as follows:

- functional effects (discomfort to occupants, affecting operation of machines, etc.)
- structural effects (mostly on non-structural elements as cracks in partitions, loss of cladding etc.).

7.7.2 Vibrational behaviour

(1) To ensure satisfactory behaviour of a structure subject to vibrations, the fundamental natural frequency of vibration of the relevant structure should be kept sufficiently above the critical values ($f > f_{crit}$), which depend on the function of the corresponding building, see Table 7.5.

Table 7.5 Recommended lower bounds of the fundamental natural frequency f_{crit} of floors

Structures	Frequency f_{crit} [Hz]
Gymnasia and sports halls	8.0
Dance rooms and concert halls without permanent seating	7.0
Concert halls with permanent seating	3.4
Structures for pedestrians and cyclists	see below*

* Natural frequencies between 1.6 and 2.4Hz and between 3.5 and 4.5Hz are to be avoided in structures for pedestrians and cyclists. Joggers can also cause vibrations in structures with natural frequencies between 2.4 and 3.5Hz

(2) The vibrational behaviour of structures can be influenced by the following measures: MC 90 section

- changing the dynamic actions
- changing the natural frequencies by changing the rigidity of the structure or the vibrating mass
- increasing the damping features, etc.

(3) Calculations of natural frequencies should always be carried out with careful thoughts being given to the structural contribution of the floor finish, the dynamic modulus of elasticity and the extent of cracking, including the tension stiffening effect of concrete between cracks. It is advisable to carry out sensitivity analyses by varying these parameters.

8 Structural members

MC 90 Section

8.1 General

(1) Detailing of reinforcement should in general follow from the design model adopted. The axes of the reinforcement should coincide with the axes of the corresponding ties in the model. Particular care should be given to the reinforcement anchorages at the nodes. Some complementary rules for particular structural elements are given in this section.

(2) A minimum amount of reinforcement shall be provided to ensure proper behaviour of the members under the effects of all actions. Special attention should be given to restraints in respect of imposed deformations, which are not explicitly considered in the analysis.

(3) It should be remembered that prestressing is a very effective way to counteract applied loads. Very important loads (at the scale of the structure) should in principle be balanced by prestressing, improving serviceability and detailing.

8.2 Beams

8.2.1 Longitudinal reinforcement

9.2.2.1

(1) The minimum area of tension reinforcement is defined in section 7.5.5.

(2) The area of the tension reinforcement and the area of the compression reinforcement should not exceed $0.04A_c$. One-third of the maximum reinforcement needed in the span should be extended to the end supports, and one-quarter to the intermediate supports. Continuity of bottom reinforcement is recommended to resist accidental positive (sagging) moments.

(3) In flanged cross-sections, at least 50% of the longitudinal reinforcement required at the level of the flange should be located within the width of the web.

8.2.2 Transverse reinforcement

9.2.2.2

(1) Transverse reinforcement should form an angle of 45° to 90° with the axis of the beam. In most cases it consists of vertical stirrups (see Fig 8.1a), well anchored in accordance with the details given in section 5.6.4 (see Fig 5.12). It may also consist of a combination of stirrups and high bond bar in shear assemblies (Fig 8.1b).

(2) A minimum amount of transverse reinforcement should be provided, corresponding to a mechanical ratio of transverse reinforcement of 0.20, i.e.:

$$A_{sw,min} = 0.20\, b_w\, s_w \sin\alpha\, f_{ctm} / f_{yk} \tag{8.1}$$

where s_w = spacing of stirrups measured along member axis.

(3) The maximum spacing of the stirrup legs in either the longitudinal or the transverse direction shall not exceed the values given in section 6.4.3.1.

8.2.3 Torsional reinforcement

9.2.2.4

(1) The transverse reinforcement in members subjected to torsion should be detailed as shown in Fig 8.1a.

(2) At least one longitudinal bar should be placed in each corner of the stirrup. The other bars should be uniformly distributed along the internal perimeter of the stirrups, at a spacing not exceeding 350mm.

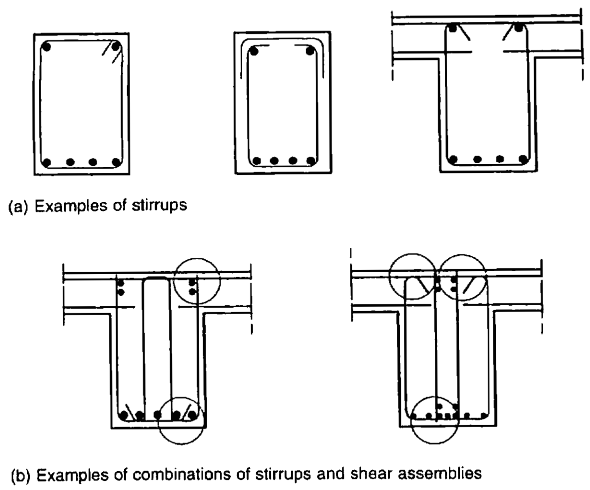


Fig 8.1 Possible layouts of transverse reinforcement in beams

8.3 Columns

9.2.3.1

8.3.1 Longitudinal reinforcement

(1) The area of longitudinal reinforcement should normally not be less than $0.008A_c$ and not be more than $0.04A_c$. In lapped joints the area of reinforcement should not exceed $0.08A_c$.

(2) The minimum number of longitudinal bars should be four for rectangular columns and six for circular columns. The diameter of the bars should not be less than 12mm.

8.3.2 Transverse reinforcement

9.2.3.2

(1) The diameter of the transverse reinforcement should not be less than 6mm or one-quarter of the diameter of the largest longitudinal bars. The spacing should not exceed the least of the following values:

- 12 times the minimum diameter of the longitudinal bars,
- the least lateral dimension of the column,
- 300mm.

(2) The transverse reinforcement should be detailed such that each bar or each group of bars placed in a corner is held by transverse reinforcement (Fig 8.2); the same principle applies to every second intermediate bars of the outer layer of reinforcement. No bar that is not held shall be located at a distance more than 150mm away from a bar that is held.

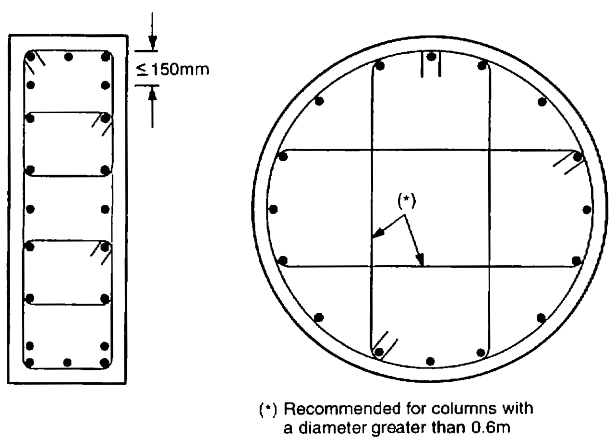


Fig 8.2 Examples of transverse reinforcement in columns

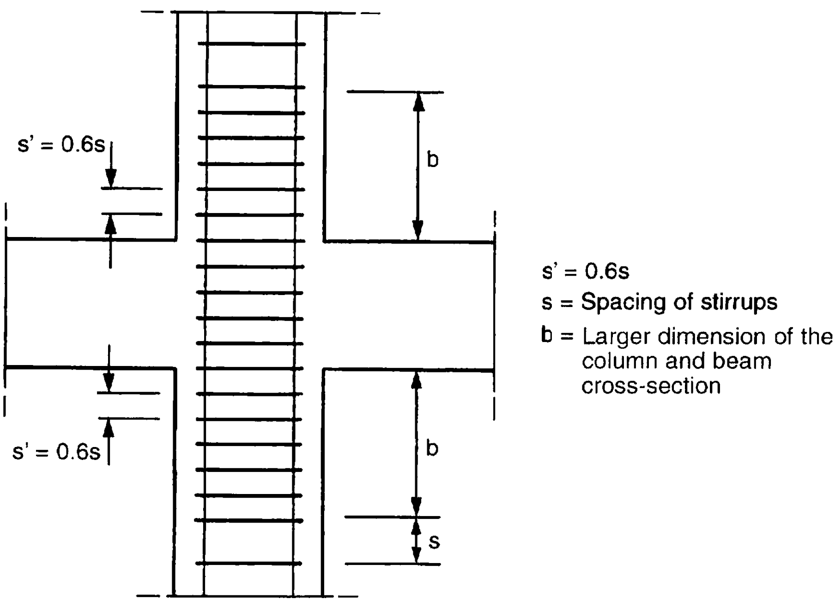


Fig 8.3 Illustration of transverse reinforcement detailing for a beam-column connection

(3) In general all transverse reinforcement should be appropriately anchored by hooks.

(4) Local effects in column D-regions (beam-column connections, cross-section variations) should be studied according to section 6.5. In general the spacing of transverse reinforcement should be decreased, e.g. by a factor of about 0.6, in the regions located above and below a beam or slab connection, as illustrated in Fig 8.3.

8.4 Slabs

9.2.1.1

8.4.1 Flexural reinforcement

(1) The minimum reinforcement is given in section 7.5.5. One-half of the reinforcement needed in the span should be extended to the end supports, one-third to intermediate supports.

(2) The ratio of secondary to main reinforcement areas should be at least equal to 0.2 at any position. For large concentrated loads this minimum ratio should be increased to 0.33.

(3) The lesser of the following values for the maximum spacing of bars is recommended:

- for main reinforcement: s_{max} = 1.2h or 350mm
- for secondary reinforcement s_{max} = 2.0h or 350mm

(4) If the corner of a slab formed by two simply supported edges is prevented from lifting and such restraint is not taken into account in the analysis, then top and bottom reinforcement capable of resisting a moment at least equal to the value of the maximum moment in the span should be provided at the corner. For a corner with one edge simply supported and the other restrained, this reinforcement should be capable of resisting a moment equal to at least one-quarter of the maximum moment in the span. The corner reinforcement should extend from the face of the support for a distance at least equal to 0.2 times the smaller span.

If the edge of a slab is partially restrained and this restraint has not been considered in the analysis, a minimum quantity of top reinforcement should be provided according to section 7.5.5. This reinforcement should extend from the face of the support for a distance at least equal to 0.2 times the corresponding span.

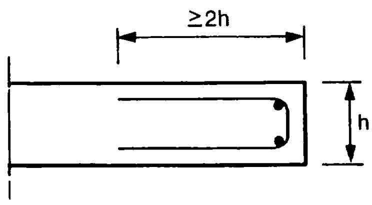


Fig 8.4 Reinforcement along a free edge of a slab

(5) Particular attention should be given to free edges (see Fig 8.4). The slab should contain:

- longitudinal reinforcement running parallel to the edge and consisting of at least two bars, one in the top corner and the other in the bottom corner;
- transverse U-shaped reinforcement running perpendicular to the edge and enclosing the longitudinal reinforcement with the legs of the U-bars extending for a distance of at least $2h$ from the edge.

8.4.2 Shear reinforcement

9.2.1.2

(1) Shear reinforcement should be provided in zones where $V_{Sd} > V_{Rd}$. This reinforcement should contain stirrups with the same minimum amount as defined for beams in section 8.2.2. The stirrups should enclose at least 50% of the longitudinal reinforcement at the bottom and the top of the slab. In general, the shear reinforcement should be inclined between 45° and 90° to the middle plane of the slab.

(2) The distance between the face of a support and the first layer of shear reinforcement should not exceed $0.5d$. The transverse spacing of bars in the same layer should not exceed $1.5d$ or 400mm, whichever is less.

(3) The requirements for punching-shear reinforcement are given in section 6.7.3.

8.5 Walls

8.5.1 Vertical reinforcement

9.2.4.1

The area of the vertical reinforcement should lie between $0.004A_c$ and $0.04A_c$; generally half of this reinforcement should be located at each face. The distance between two adjacent vertical bars should not exceed twice the wall thickness or 300mm, whichever is less.

8.5.2 Horizontal reinforcement

9.2.4.2

Horizontal reinforcement running parallel to the faces of the wall should be provided at each surface with a minimum area of 30% of that of the vertical reinforcement. The spacing should not be greater than 300mm and the diameter should not be less than one-quarter of that to the vertical bars. If the area of vertical reinforcement exceeds $0.02A_c$, then the clause in section 8.3.2 applies.

8.6 Deep beams

9.2.5

(1) The requirements and criteria for modelling of deep beams are given in section 6.5.3.

(2) The main longitudinal reinforcement corresponding to the bottom tie in the design model should be distributed over a depth of about $0.12h$ or $0.12l$ from the lower face of the beam, whichever is less (see Fig 8.5). This reinforcement should be extended from one support to the other and should be thoroughly anchored at the supports, e.g. by horizontal hooks or loops, or preferably by means of anchorage plates.

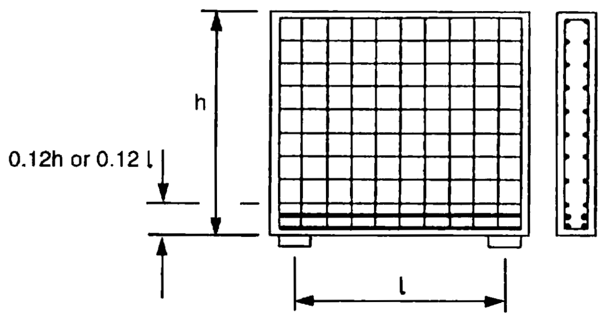


Fig 8.5 Distribution of longitudinal reinforcement in a deep beam

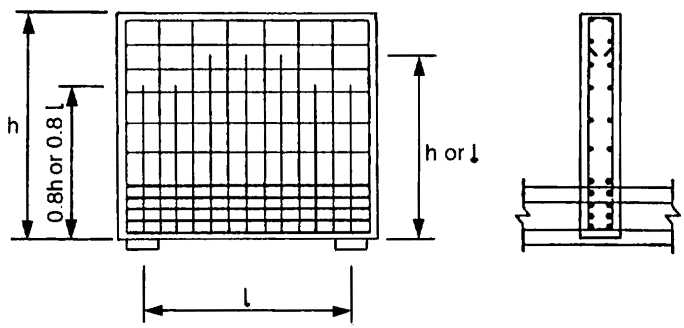


Fig 8.6 Recommended reinforcement layout for a deep beam with suspended loading (schematic)

(3) A mesh of orthogonal reinforcement with a minimum area of 0.1% of the cross-section in each direction should be provided at each face.

(4) In the case of suspended loading, additional hang-up stirrups shall be provided to transfer these loads up to a level of h or l (see Fig 8.6).

Notation

Symbols for geometry

A_c	area of a compression section or strut
A_{cc}	concrete core encompassed by confining transverse reinforcement
A_{co}	loaded area in A_c
A_{c1}	maximum area in A_c with the same centroid as the loaded area
A_{ef}	area enclosed by the centre lines of the walls
A_p	area of prestressing steel
A_s; A_{s1}	area of reinforcing steel
A_{s2}; A_{sc}	area of compression reinforcement
$A_{s,req}$	required area of reinforcement
$A_{s,prov}$	provided area of reinforcement
A_{sw}	area of transverse or web reinforcement
I	moment of inertia
I_s	moment of inertia of reinforcement (with respect to CG of cross-section)
$\varnothing$	diameter
b_{eff}	effective width of T-beam
b_w	width of web
c	concrete cover
c_{eff}	effective concrete cover
c_{nom}	nominal concrete cover
c_{min}	minimum concrete cover
d	effective depth
d_b	mandrel diameter; diameter of bend
e	eccentricity of an axial force
e_a	unavoidable eccentricity
e_o	first order eccentricity
e_2	deformation; eccentricity
h	total height of section
h_{eff}	effective height
i	radius of gyration
l	length of an element or span
l_b	anchorage length (of a straight bar)
$l_{b,min}$	minimum anchorage length
l_{bpt}	transmission length of tendon
l_{bpd}	design value of transmission length
l_o	required lap length, distance between points of zero moments; buckling length
m	number of vertical elements contributing to the combined effect (section 6.6.4)

n	number of tendons
s	spacing
s_n	clear distance between tendons
s_r	radial spacing of the layers of shear reinforcement
s_{tr}	spacing of cross wires
s_w	stirrup spacing in the longitudinal direction
t	time; thickness of thin elements
$t_{ef,i}$	effective thickness of wall i
u_1	perimeter at a distance $2d$ from the boundary of the loaded area (section 6.7.3)
v_{fl}	flange shear force per unit length
w	width of a crack
x	depth of compression zone
x_c	depth of the strut
Δx	length under consideration
z	inner lever arm
z_i	the distance between the intersections of adjacent walls
z_s	distance of reinforcement from center of cross-section
α_a	coefficient for type of anchorage
α_f	angle between direction of strut and joint
α_p	angle of tendon at considered section
α_s	coefficient for bar splices
β	angle
β_r	crack angle
κ	curvature; factor; degree of prestressing, degree of load balancing
λ	slenderness ratio; mechanical degree of prestressing
ν, ν_1, ν_2	coefficients
ρ	geometrical percentage of reinforcement (reinforcing ratio)
ξ	$= x/d$ = coefficient for depth of compression zone

Symbols relevant for safety

A	accidental load
G	permanent actions
Q	variable actions
R_d	design value of resistant action effects
S_d	design value of critical combination of action effects
S_g	action effect due to permanent load
S_o	initial action effect at the construction stage
S_p	(total) action effect due to prestress
S_∞	final action effect after redistribution
γ_g	partial safety factors of permanent distributed load
γ_q	partial safety factors of variable distributed load

γ_c	partial safety factor for concrete
γ_{ct}	partial safety factor for concrete tensile strength
γ_s	partial safety factor for steel
ψ_0	coefficient for basic combination of actions (section 6.2.2)

Symbols relevant for material

E_c	modulus of elasticity of concrete
E_{cm}	secant modulus of elasticity of concrete
E_s	modulus of elasticity of steel
E_p	modulus of elasticity of prestressing steel
f_{bd}	design value of bond strength for reinforcing steel
f_{bpd}	design value of bond strength for prestressing steel
f_c	concrete compressive strength
f_{ck}	characteristic cylinder strength of concrete
$f_{cd,eff}$	effective design value of concrete compressive strength
f_{cwd}	compressive strength of inclined struts in web
f_{1cd}	uniaxial design strength of concrete (section 2.1.2)
f_{2cd}	biaxial design strength of concrete
f_{3cd}	triaxial design strength of concrete
f_{ctm}	average concrete tensile strength
$f_{ctk,\,0.05}$	5% fractile of concrete tensile strength
$f_{ctk,\,0.95}$	95% fractile of concrete tensile strength
f_{1ct}	uniaxial tensile strength of concrete
f_y	yield strength of steel
f_{yk}	characteristic yield strength of reinforcing steel
f_{yd}	design strength of reinforcing steel
$f_{0.2k}$	0.2 % proof stress
f_t	tensile strength of steel
f_R	projected rib area (section 2.2.5)
f_{pt}	strength of prestressing steel
f_{ptk}	characteristic strength of prestressing steel
ε_c	concrete strain
ε_{cu}	maximum compressive strain
ε_{cs}	shrinkage strain
ε_{do}	prestrain corresponding to the prestressing force after creep and shrinkage
ε_{uk}	characteristic ultimate strain
μ	coefficient for friction
ϕ	creep coefficient
t_o	age of loading
τ_{bd}	bond stress
τ_{fd}	friction stress

Symbols for forces, moments, stress and strains

F_{Rt}	resisting force of a tensile chord
F_{Rcd}	resisting force of a compression chord
F_{Sd}	punching load due to the applied external loads
M_g	bending moment due to permanent load
M_p	bending moment due to prestress
M_r	cracking moment
$M^o{}_{Sg}$	first order moment under (unfactored) quasi-permanent load
$M^o{}_{Sd}$	first order moment under design load
N_{bal}	axial force with maximum moment capacity
N_{Sd}	axial force under design load
N_{Sg}	axial force under (unfactored) quasi-permanent load
N_{ud}	axial capacity of cross-section
P_0	prestressing force at time $t = 0$
P_{m0}	initial prestress ($t = 0$) after transfer of prestress
P_m	mean value of prestressing force
P_i	initial prestressing force at origin
T_{Sd}, T_{Rd}	design value of acting, resisting torsional moment
V_{Sd}, V_{Rd}	design value of acting, resisting design shear force
V_f	vertical component of friction forces at crack
V_p	vertical component of forces in prestressing tendons
V_{sw}	shear force carried by the stirrups over the cracks
g	permanent distributed load
p	equivalent load due to prestress
q	variable distributed load
ε_c	concrete compressive strain
ε_s	strain in reinforcing steel
ε_{sm}	average steel strain due to tension stiffening effect
ε_p	strain in prestressing steel
$\Delta\varepsilon_p$	increase of strain in bonded prestressing reinforcement due to action effects
ω	$= \rho f_y / f_{1c}$ = mechanical reinforcement ratio
σ_c	concrete stress
σ_{cf}	compressive stress in inclined struts
σ_{cg}	stress in concrete at level of tendons due to permanent loads
$\sigma_{cp,0}$	initial stress in concrete at level of tendons due to prestress
σ_{cw}	compressive stress of inclined struts in web
σ_f	normal stress on interface
σ_p	prestressing steel stress
$\sigma_{p\infty}$	stress after all losses
σ_{pi}	tensile stress in tendon during tensioning
σ_{p0}	tensile stress in tendons after transfer of prestress

$\Delta\sigma_{p,r}$	loss of stresses in the tendon (negative) due to relaxation (at constant length)
$\Delta\sigma_{Rsk}$	characteristic fatigue strength
τ_f	shear stress on interface
ν	dimensionless axial force

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- Part 2-1: *Densities, self weight and imposed loads*
- Part 2-3: *Snow loads*
- Part 2-4: *Wind actions*
- Part 3: *Traffic loads on bridges*
- Part 4: *Actions on silos and tanks*

ISO 2736/2: *Concrete tests — Making of test specimens* — Part 2: *Making and curing of test specimens for strength tests*

Appendix: Characteristic values of variable actions

This short appendix on variable actions certainly does not constitute a complete load specification; such an endeavour would indeed go well beyond the scope and the competence of these Recommendations. There are other national and international bodies dealing with the question of adequate load assumptions.

However, it seems somewhat futile to specify combinations and partial safety factors for actions, without also specifying the actions. Thus, the purpose of this appendix is to indicate the order of magnitude of the loads which served as guidelines for the present Recommendations.

For variable actions of natural origin, such as wind, temperature, snow, earthquake, etc., local conditions are a dominant factor; therefore no generally valid regulations can be proposed.

A1 Highway bridges: live loads

Two types of loading should be considered, as follows:

- H1 is a normal traffic load representation that includes three categories of loading to cater for different local conditions and requirements.
- H2 is an abnormal vehicular load requirement, the details of which are the prerogative of the appropriate transport authority.

A1.1 Type H1: normal traffic loading

Type H1 loading consists of a uniformly distributed load (see section A1.1.1) and an additional concentrated load (see section A1.1.2). A longitudinal force due to traction or braking of vehicles is also to be considered (see section A1.1.3).

The vertical loading includes an allowance for dynamic effects. In cases where vibrations need to be investigated explicitly, the loading will require separate consideration.

A1.1.1 Uniformly distributed load (UDL)

The UDL should be taken from Fig A1, according to the loading category. The UDL consists of joined sections of lane with constant intensity of loading.

For multilane bridges, the maximum lane loads to be considered as acting simultaneously are given in Fig A2 as percentages of the lane loading obtained from Fig A1. Longitudinal and transverse effects should be determined for the most unfavourable arrangements of lane loadings.

A1.1.2 Concentrated load

In addition to the UDL, a single concentrated load should be considered, acting in any position and distributed over either a square contact area of side 500mm or a circular contact area of diameter 550mm. The load should be taken, according to the loading category, as 400kN (heavy), 300kN (normal), or 200kN (reduced).

A1.1.3 Longitudinal force

A longitudinal force resulting from traction or braking of vehicles should be considered, acting at the road surface and parallel to it on one lane only. The force should be taken, according to the loading category, as 400kN (heavy), 300kN (normal), or 200kN (reduced).

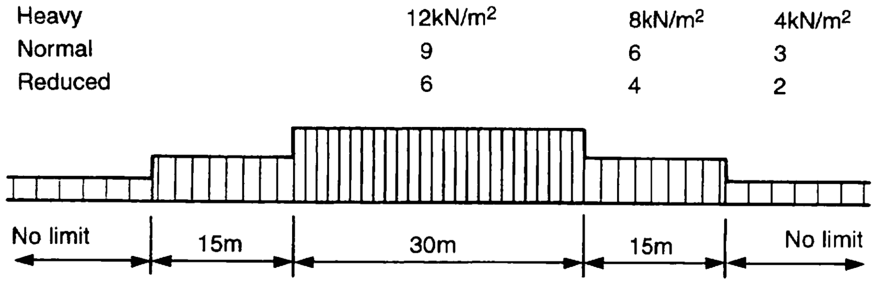

Fig A1 UDL for type H1 loading in longitudinal direction

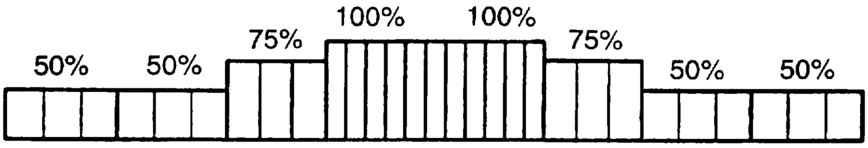

Fig A2 Multilane loading percentages in transverse direction

Table A1 Imposed floor loads in buildings

Building category or function	UDL (kN/m^2)	Concentrated load (kN)
Apartment houses, private rooms corridors	1.5 4.0	2.5 5.0
Hotels, guest rooms corridors	2.0 4.0	2.0 5.0
Schools, classrooms corridors	3.0 4.0	3.0 5.0
Office buildings, offices filing and storage areas computing, etc. areas	2.5 5.0 3.5	5.0 5.0 5.0
Hospitals, private rooms	2.0	2.0
Assembly halls, fixed seats movable seats	4.0 5.0	– 4.0
Stairs, private buildings public buildings	1.5 – 3.0 3.0	2.0 – 3.0 4.0
Restaurants, dining rooms	2.0	3.0
Libraries, stock room	9.6 min.	7.0
Garages, passenger cars only	2.5	9.0
Grandstands, stadia	5.0	4.0
Dance halls (without increase due to resonance)	5.0	4.0
Shops	4.0	4.0
Storage warehouses, light	6.0 min.	7.0
Storage warehouses, heavy	12.0 min.	9.0
Terraces, vehicular access pedestrian access	5.0 4.0	9.0 5.0

A1.1.4 Footway and cycle track load

The UDL appropriate to the reduced loading category should be taken from Fig. A1. Special consideration should be given for loaded lengths in excess of 30 m where exceptional crowds may be expected (as, for example, where a footbridge serves a sports stadium).

Where the footway or cycle track is not protected from highway traffic by an effective barrier, the concentrated load appropriate to the reduced loading category should also be considered.

A1.2 Type H2: abnormal vehicle loading

Where the controlling local authority requires the bridge to be designed for an abnormal vehicle loading, the loading specification should be obtained from the appropriate transport authority.

A2 Buildings: imposed floor loads

The uniformly distributed load on floors can be taken from Table A1, depending on the building category or function. Normal effects of impact and vibration can be assumed to be included in these values.

The values in the table are maximum values that can be assumed to occur on relatively small areas. For large areas the average load intensity can be reduced. Rules for such a reduction can be found in some codes, e.g. Eurocode 1.