

ENGINEERING FORMULAS FOR METALCUTTING

Edmund Isakov, Ph.D.

Presented in customary U.S. and metric units of measure

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Printed in the United States of America 10 9 8 7 6 5 4 3 2 1 To my wife Yelena for inspiration and encouragement.

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Preface

Principal metalcutting operations are milling, turning, boring, and drilling. Milling is a discontinuous type of cutting, and drilling is a continuous cutting processe. Turning and boring are either continuous type of cutting (in most cases) or discontinuous cutting processes. Discontinuous turning and boring are also called interrupted cutting, which takes place when machining workpieces with keyways, splined shafts, etc. These metalcutting operations, despite their differences, have at least two common features:

- mechanical properties of work materials should be taken into consideration when selecting cutting conditions,
- 2 the workpiece must be shaped by means of chip removal to become a certain product, part of a machine, or a device.

Engineering formulas for each of the machining operations will show their differences and similarities. Formulas developed by the author are included (conversion of hardness numbers from one type into another, calculation of tensile strength of work materials, calculations of cutting forces, power consumption, and other parameters), plus conventional formulas to calculate cutting speed, feeds, metal removal rate, cutting time, and more.

Metalcutting uses an integrated system consisting of three elements: a workpiece, a cutting tool, and a machine tool (lathe, milling machine, machining center, and a drilling machine). The formulas are here described in the same sequence, so the complexity of metalcutting will be better understood.

All formulas are presented in both customary U.S. and metric units of measure. Data in metric units is based on Systeme International d'Unites (SI).

The level of difficulty of this handbook is low enough that it can be used by anyone involved in metalcutting: machinists, technicians, and engineers. Any metalworking professional will find this handbook handy and practical. The handbook can be useful for students of various manufacturing professions at trade schools, technical colleges and universities.

The original intention was to include as much information as possible. However, if some useful formulas, which you need, were not included, the editor would appreciate hearing about missing formulas as well as comments on the book. Any omissions and suggestions can be added in future edition.

Edmund Isakov, Ph.D., June 2003

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MECHANICAL PROPERTIES OF WORK MATERIALS

Work material becomes a workpiece when it is fixed in the machine tool (lathe, machining center, milling machine, etc.) and is being machined. Traditionally, guidelines for selection of cutting tools (geometry and grades) are based on tool application considerations with only limited attention to workpiece mechanical properties. The most important mechanical properties from the machining viewpoint are hardness and tensile strength. Knowledge of hardness, tensile strength and any relationship between them, adds to the guidelines for the application of cutting tools, which, in turn, increases the effectiveness of machining (estimating cutting forces, torque values, and required power).

1.1 HARDNESS

Hardness of metals is a measure of their resistance to deformation. From the machining viewpoint, hardness

is the resistance to cutting. Various static indentation and dynamic methods are used to measure hardness. Typical static indentation methods are Rockwell, Brinell, Vickers, and Knoop hardness testing, which use different loads and indenters. A typical dynamic (rebound) method is Scleroscope hardness testing, also known as the Shore method.

1.1.1 Rockwell Hardness

Rockwell hardness testing is applied to most metals and alloys, from annealed steels and copper alloys to cemented carbides. There are two types of tests: Rockwell standard and Rockwell superficial. The correct way to indicate Rockwell hardness is with three letters. The "H" indicates hardness, the "R" stands for Rockwell and the last letter indicates the scale in use. There are 15 standard hardness scales: A. B. C. D. E. F. G, H, K, L, M, P, R, S, and V. The most common are the A, B, and C scales. Superficial hardness scales include N, T, W, X, and Y. The major loads for all five of these scales are 15 kgf, 30 kgf, and 45 kgf (approximately 33 lbf, 66 lbf, and 99 lbf, respectively) where f means force. The load value stands after letter "R" and precedes the letter indicating scale. The most commonly used scales are N and T. Examples of how to indicate Rockwell superficial hardness numbers are 55 HR30N and 86 HR15T, while 84 HRA, 95 HRB, and 65 HRC, are examples of Rockwell standard hardness numbers. Indicating the scale with a subscript letter $(R_A, R_B, R_C,$ etc.) is no longer considered best practice.

Scale A is used primarily to measure the hardness of cemented carbides, thin steel, and shallow-casehardened steel. The major load is 60 kgf (132 lbf) applied to a 120° spheroconical diamond indenter. Readings between 61 HRA and 85 HRA are normal range values when the hardness of heat-treated carbon, alloy, and tool steels is measured. Typical hardness values for cemented carbides range from 83.0 to 94.5 HRA.

Scale B is used to measure the hardness of nonferrous metals and alloys, including some grades of wrought aluminum alloys and wrought copper alloys, soft grades of cast irons and unhardened steels. The major load is 100 kgf (220 lbf), which applied to a 1/16-inch-diameter (1.588 mm) ball indenter. Readings between 20 and 100, the minimum and maximum values, are in the normal range. The Rockwell B hardness numbers for carbon and alloy steels in the annealed, normalized, and quenched-and-tempered conditions are between 55 and 100.

Scale C is for heat-treated steels, cast irons, titanium alloys, and other metals harder than 100 HRB. The major load is 150 kgf (330 lbf), which is applied to a 120° spheroconical diamond indenter. Readings between approximately 20 and 68—the minimum and maximum values—are most accurate and are accepted as the normal range. The hardness of cemented carbides and PVD and CVD coatings should never be tested with Rockwell C scale.

N Scales are for materials similar to those tested with the A, C, and D scales, except they are of a thinner gauge or case depth. Each of the three major loads

mentioned above is applied to the same indenter used for scale A. The Rockwell N superficial hardness numbers measured at the 30-kgf load for carbon and alloy steels in the annealed, normalized, and quenched-andtempered conditions are between 42 (minimum) and 84 (maximum) values.

T Scales are used for materials similar to those tested with the B, F, and G scales, but they are of a thinner gauge. Each of the three major loads mentioned above is applied to the same indenter used for scale B. Readings between approximately 53 and 82—the minimum and maximum values—are accepted as the normal range at a 30-kgf load.

1.1.2 Brinell Hardness

The Brinell hardness value is a combination of a two-digit or three-digit number followed by two letters: "H" is for hardness and "B" is for Brinell. 90 HB, 250 HB and 320 HB are examples of how to indicate Brinell hardness numbers. Indicating Brinell hardness with letters BHN or Bhn is no longer considered best practice. The standard test loads are 500, 1000, 1500, 2000, 2500, and 3000 kgf. The most common loads are 500 and 3000 kgf (approximately 1,100 and 6,600 lbf), which applied to the 10-mm ball indenter. There are two types of 10-mm balls: standard, made of hardened steel, and tungsten carbide.

The 500-kgf load and hardened steel ball are used for testing unhardened steels and soft grades of wrought aluminum and copper alloys. The most accurate readings are between 16 HB, for very soft aluminum, and

100 HB, for unhardened and soft-tempered steels, gray and malleable cast irons.

The 3000-kgf load is used for testing hardened and tempered or annealed steels (except case hardened steels), cast irons, titanium alloys, and high-temperature alloys such as nickel-base and cobalt-base. Hardened steel ball indenters are used for testing materials up to 444 HB. Tungsten carbide ball indenters are recommended for similar materials having a hardness number ranging between 444 HB and 627 HB.

1.1.3 Vickers Hardness

The Vickers hardness value is a combination of a number and the letters "HV". The "H" indicates hardness and the "V" is for Vickers. For example, 360 HV represents the Vickers hardness number of 360.

The Vickers indenter is a square-based, pyramidal diamond having an included angle of 136°. The load, which is applied for 30 seconds, may be one of the following: 5, 10, 20, 30, 50, or 120 kgf.

Vickers hardness numbers for carbon and alloys steels in the annealed, normalized, and quenched-andtempered conditions are between 85 and 940—the minimum and maximum values. Vickers hardness numbers for indexable cutting inserts made of cemented carbides are between 1000 HV and 1950 HV.

1.1.4 Knoop Hardness

The Knoop hardness value is a combination of a number and the letters "HK". Knoop hardness numbers are similar to Vickers hardness numbers. The Knoop

indenter is a rhombic-based, pyramidal diamond with an included longitudinal angle of 172°30′ and an included transverse angle of 130°. The load ranges from 1 to 10000 grams (gf). The most commonly applied loads are 100, 500, 1000, and 3000 gf.

Knoop hardness numbers at the 500-gf load for carbon and alloy steels in the annealed, normalized, and quenched-and-tempered conditions, are between 97 HK (the minimum value) and 920 HK (the maximum) value.

Vickers and Knoop hardness tests are often used for measuring hardness close to the edge of a workpiece areas where Rockwell or Brinell hardness tests cannot be employed because their indenters require significantly higher loads. Vickers and Knoop hardness tests are applicable to thin metals, hard and brittle materials, and materials with surfaces that are carburized or nitrided. The Vickers and Knoop are the only methods that can be used to measure the microhardness of superhard materials such as CBN (cubic boron nitride) and PCD (polycrystalline diamond).

1.1.5 Scleroscope Hardness

The Scleroscope dynamic hardness tests are used when numerous quality checks must be performed and for measuring hardness of large forged steel pieces with a minimum weight of 2.27 kg (5 lb) or wrought alloy steel rolls. Two models of Scleroscope hardness testers are available: Model C and Model D. The major difference between these two models is that the Model D Scleroscope employs a heavier hammer and a shorter rebound height then the Model C Scleroscope. However,

falling hammers of both models develop the same striking energy. The Model C Scleroscope employs a diamondtipped hammer falling onto the surface of the metal being tested. The hammer, which weighs about 40 grains or 2.6 grams (1 grain = 0.0648 gram), is dropped from a fixed height of 10 inches (254 mm).

The Scleroscope hardness number is a measure of the height of rebound of the diamond-tipped hammer. The Scleroscope scale is graduated from 0 to 140. As the hammer strikes the metal, it rebounds, and the harder the metal, the greater the rebound. Readings between 17 (minimum value) and 97 (maximum value) are the most accurate and are accepted as normal range when testing carbon and alloy steels in the annealed, normalized, and quenched-and-tempered conditions.

More information about hardness testing equipment and testing procedures is available in popular handbooks (Ref. 1, p. 74–108; Ref. 2, p. 517–519) and, probably, can be found in some other publications.

1.2 HARDNESS-TO-HARDNESS CONVERSION

Recommendations for cutting conditions, especially, the cutting speed, are based on the work materials' hardness. It is recommended that the cutting speed be selected in relationship to the hardness numbers (preferably, in Brinell) of the workpieces being machined. If the hardness of a work material is given in Rockwell numbers (B-scale or C-scale), Vickers numbers, Knoop numbers, or Scleroscope numbers, it

is a good practice to convert given hardness numbers into Brinell hardness numbers.

Equivalent hardness numbers for Rockwell, Brinell, Vickers, Knoop, and Scleroscope can be found in Ref. 1, p. 109–114; Ref. 2, p. 520–523 and in some other publications. However, the published information is presented in such a cumbersome way that it is difficult to use. Converting the above-mentioned hardness numbers into Brinell hardness numbers is more practical and much easier. To express the relationship between Brinell hardness numbers and any of the above-mentioned hardness numbers, the author used the linear regression method to develop necessary equations.

Hardness conversion equations are listed in Tables 1.1 to 1.5. Equations to convert Rockwell B hardness numbers and Rockwell C hardness numbers into Brinell

Rockwell E Number	3 Hardness s (HRB)	Equations to Convert Rockwell B Hardness (HRB) into Brinell
From	То	Hardness (HB)
41	64.9	HB = 1.117 HRB + 33.4
65	75.9	HB = 2.279 HRB - 38.1
76	80.9	HB = 2.727 HRB - 71.3
81	85.9	HB = 3.364 HRB - 123.1
86	90.9	HB = 4.120 HRB - 187.9
91	95.9	HB = 5.431 HRB - 307.0
96	100.0	HB = 6.754 HRB - 434.2

Table 1.1. Brinell—Rockwell B Hardness Relationship

Rockwell Numbe	C Hardness rs (HRC)	Equations to Convert Rockwell C Hardness (HRC) into Brinell
From	То	Hardness (HB)
20	25.9	HB = 5.328 HRC + 119.6
26	30.9	HB = 7.077 HRC + 73.5
31	35.9	HB = 8.356 HRC + 34.5
36	40.9	HB = 8.938 HRC + 13.6
41	45.9	HB = 10.264 HRC - 41.1
46	50.9	HB = 12.610 HRC - 149.3
51	55.9	HB = 15.829 HRC - 311.9
56	60.9	HB = 19.213 HRC - 499.7
61	65.3	HB = 17.323 HRC - 386.5

Table 1.2. Brinell—Rockwell C Hardness Relationship

Vickers H Number	ardness 's (HV)	Equations to Convert Vickers Hardness (HV) into Brinell
From	То	Hardness (HB)
85	149	HB = 0.959 HV - 0.8
150	199	HB = 0.949 HV + 0.9
200	249	HB = 0.954 HV - 0.7
250	299	HB = 0.922 HV + 7.3
300	399	HB = 0.944 HV + 1.0
400	499	HB = 0.909 HV + 15.0
500	700	HB = 0.935 HV + 2.2

Knoop H Numbe	ardness rs (HK)	Equations to Convert Knoop Hardness (HK) into Brinell
From	То	Hardness (HB)
97	199	HB = 0.920 HK - 8.0
200	299	HB = 0.996 HK - 23.9
300	349	HB = 1.030 HK - 34.3
350	399	HB = 0.871 HK + 21.5
400	499	HB = 0.774 HK + 60.0
500	599	$HB = 0.840 \ HK + 26.5$
600	700	HB = 0.917 HK - 19.4

Table 1.4. Brinell—Knoop Hardness Relationship

Scleroscop	e Hardness	Equations to Convert Scleroscope
Numbe	ers (HS)	Hardness (HS) into Brinell
From	То	Hardness (HB)
17	20	HB = 5.000 HS + 26.0
21	40	HB = 7.013 HS - 11.8
41	50	HB = 8.260 HS - 62.3
51	60	HB = 8.354 HS - 68.3
61	80	HB = 10.593 HS - 207.0

hardness numbers are given in Table 1.1 and Table 1.2 respectively.

Equations in Tables 1.3, 1.4, and 1.5 were developed to convert Vickers, Knoop, and Scleroscope hardness numbers into Brinell hardness numbers respectively.

Converted Brinell hardness numbers calculated by equations in Table 1.1 through Table 1.5 may differ by less than $\pm 1.0\%$ from the actual Brinell hardness numbers measured using a 3000-kgf load.

1.3 STRENGTH-HARDNESS RELATIONSHIP OF WORK MATERIALS

When the cutting edge penetrates into a work material at a given depth of cut and feed rate, a deformed layer from the workpiece flows as a chip. The mechanism of chip formation is a complex process of elasticplastic deformation of metal removed from the workpiece. Understanding this process can be simplified by considering three phases taking place during cutting.

- 1. Material in the "cutting zone" (between the cutting edge and the undeformed layer of work material) is always under *compression*.
- 2. The deformed layer in the "cutting zone" transforms into a chip when a cutting edge generates sufficient stress, to overcome the *ultimate tensile strength* of the work material.
- 3. Separation of the chip from the parent material takes place in the "shear zone" near the cutting edge due to the *shear stress*.

Major properties of the work materials that influence the choice of a cutting insert grade, geometry, and machining conditions are the **hardness** and the **ultimate tensile strength**.

Steels that have been "fully hardened" to the same hardness when quenched will have about the same tensile strength regardless of their composition and alloying elements (Ref. 2, p. 473). The same publication recommends two formulas (p. 521) for approximate relationship between Brinell hardness (HB) and tensile strength (σ , psi) that can be applied to steels:

- $\sigma = HB \times 515$ for Brinell hardness up to 175 HB,
- $\sigma = HB \times 490$ for Brinell hardness higher than 175 HB.

These formulas provide an accuracy $\pm 6\%$ or better when used for calculating ultimate tensile strength of carbon and alloy steels, except for AISI 4340 alloy steel with the hardness range of (390–600) HB. If these formulas were used for the majority of stainless steels, the accuracy would be in the range of (10–20)%. These formulas are not recommended to calculate ultimate tensile strength of the majority of tool steels, because the Rockwell C-scale is used to measure their hardness (except for P20 mold tool steel, the hardness of which is also provided in Brinell numbers).

To increase accuracy in converting hardness numbers into ultimate tensile strength not only for steels but also for other work materials, the author applied statistical treatment of data and linear regression analysis for developing numerous formulas to calculate the ultimate tensile strength as a function of the work material hardness with a minimal error and the maximum correlation coefficient (Ref. 3, p. 20–24).

The ultimate tensile strength (in psi) of carbon, alloy, some stainless, and some tool steels should be

calculated by:

$$\sigma = \text{HB} \times 500 \tag{1.1}$$

Formula (1.1) is recommended for low-carbon, medium-carbon, and high-carbon steels, alloy steels, some stainless, and some tool steels. The range of Brinell hardness numbers for these steels is as follows:

- (90–250) HB for low-carbon steels
- (130–270) HB for medium-carbon steels
- (170–360) HB for high-carbon steels
- (140–370) HB for most alloy steels including (200–580) HB for AISI 4140 alloy steel
- (155–595) HB for martensitic 400-series stainless steels
- (250–430) HB for precipitation-hardening stainless steels (PH and AM types, and Custom 450)
- (200–250) HB for H type tool steels.

Exceptions were found for AISI 4340 alloy steel with the hardness numbers between 390 HB and 600 HB, and for precipitation-hardening stainless steels including 17-7 PH, PH 15-5 Mo (HB > 430), and Custom 455 with the hardness numbers between 370 HB and 440 HB. Formula (1.2) and formula (1.3) should be used to calculate the ultimate tensile strength of AISI 4340 alloy steel and precipitation-hardening stainless steels respectively:

 $\sigma = HB \times 540 \text{ (psi)} \tag{1.2}$

$$\sigma = \text{HB} \times 550 \text{ (psi)} \tag{1.3}$$

The ultimate tensile strength of the austenitic 300series stainless steels with the hardness range of (140–180) HB should be calculated by formula (1.4), which provides an accuracy of $\pm 7\%$ or better

$$\sigma = \text{HB} \times 560 \text{ (psi)} \tag{1.4}$$

The ultimate tensile strength of the austenitic 300series stainless steels with the hardness range of (190– 370) HB should be calculated by formula (1.5), which provides an accuracy of $\pm 7\%$ or better

$$\sigma = HB \times 470 \text{ (psi)} \tag{1.5}$$

Formula (1.5) provides an accuracy of $\pm 3\%$ or better when applied to the ferritic 400-series stainless steels with the hardness range of (140–190) HB.

To convert ultimate tensile strength from psi or lb/in^2 (customary U.S. units of measure) into N/mm² (metric units of measure), the value of σ must be divided by the conversion factor of 145.0377. For example, AISI 1045 medium-carbon steel with the hardness of 280 HB will have the ultimate tensile strength calculated by formula (1.1):

$$\sigma = 280 \cdot 500 = 140,000 \text{ psi}$$

or

$$\frac{140,000}{145.0377} = 965 \text{ N/mm}^2.$$

Statistical treatment of data and linear regression analysis resulted in development of formulas for calculating ultimate tensile strength for a variety of work materials depending on their hardness. These formulas are shown in Tables 1.6–1.13.

	ß		
Class	Designation	Hardness Range	Ultimate Tensile Strength, psi
Austenitic Austenitic Austenitic Ferritic Martensitic Precipitation-hardening Precipitation-hardening	AISI Type 201 AISI 300-series AISI 300-series AISI 400-series AISI 400-series PH, AM, Custom series PH, AM, Custom series	(253–400) HB (143–180) HB (190–371) HB (137–190) HB (156–595) HB (253–432) HB (255–46) HRC	$\sigma = 606 \text{ HB} - 31660$ $\sigma = 457 \text{ HB} + 16910$ $\sigma = 534 \text{ HB} - 16280$ $\sigma = 430 \text{ HB} + 6530$ $\sigma = 430 \text{ HB} + 6530$ $\sigma = 557 \text{ HB} - 3900$ $\sigma = 557 \text{ HB} - 20710$ $\sigma = 4725 \text{ HR} - 2370$

Table 1.6. Ultimate Tensile Strength—Hardness Relationship for Stainless Steels

Chapter 1

Mechanical Properties	s of Work Mater	rials
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Table 1.7. UI	timate Tensile Strength—	lardness Relationship f	or Tool Steels
Categories	Designation	Hardness Range	Ultimate Tensile Strength, psi
Shock-resisting	AISI Type S1	(42–58) HRC	σ = 7014 HRC – 94200
Shock-resisting	AISI Type S5	(37–59) HRC	$\sigma = 7829 \text{ HRC} - 131700$
Shock-resisting	AISI Type S7	(39–58) HRC	$\sigma = 6923 \text{ HRC} - 92700$
0il-hardening	AISI Type 01	(31–50) HRC	$\sigma = 5899 \ \text{HRC} - 60120$
0il-hardening	AISI Type 07	(31–54) HRC	$\sigma = 6332 \text{ HRC} - 78970$
Air-hardening	AISI Type A2	(30–60) HRC	$\sigma = 5920 \text{ HRC} - 53270$
Chromium hot-work	AISI Types H11, H13	(30–57) HRC	$\sigma = 6387 \text{ HRC} - 62850$
Special-purpose	AISI Type L2	(30–54) HRC	$\sigma = 6227 \text{ HRC} - 59980$
Special-purpose	AISI Type L6	(32–54) HRC	$\sigma = 6843 \text{ HRC} - 83900$
Mold	AISI Type P20	(26–54) HRC	$\sigma = 5535 \text{ HRC} - 24880$
Mold	AISI Type P20	(260–540) HB	$\sigma = 545 \text{ HB} - 11770$
Mold	AISI Type P20	(260–540) HB	$\sigma = 520 \text{ HB}$

p for Cast Irons
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e Tensilo
Ultimat
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Ultimate Tensile Strength, psi	$ \begin{aligned} \sigma &= 283 \text{ HB} - 22970 \\ \sigma &= 450 \text{ HB} \\ \sigma &= 467 \text{ HB} - 4720 \\ \sigma &= 536 \text{ HB} - 7490 \\ \sigma &= 400 \text{ HB} \\ \sigma &= 366 \text{ HB} + 6420 \end{aligned} $
Hardness Range	(160–300) HB (140–390) HB (140–390) HB (300–440) HB (150–270) HB (150–270) HB
Designation	ASTM A 48 class 20 to 60 ASTM A, SAE, GGG, etc. ASTM A, SAE, GGG, etc. ASTM A 897-90 and GGG ASTM A: 220, 602, SAE J158 ASTM A: 220, 602, SAE J158
Type of Cast Iron	Gray Ductile Ductile Ductile Malleable Malleable

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Work Material	Designation	Hardness Range	Ultimate Tensile Strength, psi
Titanium alloys Pure titanium	Alpha-beta alloys Alpha-beta alloys Alpha-beta alloys Alpha-beta alloys Alpha-beta alloys ASTM grades 1 to 4	(205–385) HB (205–385) HB (20–47) HRC (230–420) HK (230–420) HK (121–215) HB	
	ASTM grades 1 to 4	(140–240) HK	σ= 478 HK – 11730

Principal Alloying Elements	Designation	Hardness Range	Ultimate Tensile Strength, psi
Copper	2000 series	(45–130) HB	$\sigma = 580 \text{ HB}$
Copper	2000 series	(45–130) HB	$\sigma = 555 HB + 2150$
Wanganese Manganese	3000 Series 3000 Series	(28–77) HB (28–77) HB	$\sigma = 500 HB + 1595$ $\sigma = 522 HB + 1595$
Magnesium	5000 series, temper 0	(28–65) HB	$\sigma = 605 \text{ HB}$
Magnesium	5000 series, temper 0	(28–65) HB	$\sigma = 617 \text{ HB} - 557$
Magnesium	5000 series, temper H	(36–105) HB	$\sigma = 565 \text{ HB}$
Magnesium	5000 series, temper H	(36–105) HB	$\sigma = 604 \text{ HB} - 2580$
Magnesium and Silicon	6000 series, Tempers:		
	0, Т1, Т4	(25–90) HB	$\sigma = 540 \text{ HB}$
Magnesium and Silicon	6000 series, Tempers:		
Magnesium and Silicon	0, 11, 14 6000 series temners	9H (06-62)	$\sigma = 603 \text{ HB} - 3210$
	Т5, Т6, Т83, Т831, Т832	(60–120) HB	$\sigma = 460 \text{ HB}$
Magnesium and Silicon	6000 series, tempers:		
	15, 16, 183, 1831, 1832	(00-120) HB	$\sigma = 496 \text{ HB} - 2910$
Zinc	7000 series	(60–150) HB	$\sigma = 554 \text{ HB}$
Zinc	7000 series	(60–150) HB	$\sigma = 558 \text{ HB} - 409$

Table 1.10. Ultimate Tensile Strength—Hardness Relationship for Wrought Aluminum Alloys

Chapter 1

lationship for Cast Aluminum Alloys	Hardness Ultimate Range Tensile Strength, psi	$05-125$) HB $\sigma = 200$ HB + 36000 $0-110$) HB $\sigma = 446$ HB - 4465 $55-105$) HB $\sigma = 448$ HB - 645 $0-80$) HB $\sigma = 556$ HB - 6780 $0-80$) HB $\sigma = 551$ HB - 24820 $00-150$) HB $\sigma = 292$ HB + 13150
nsile Strength—Hardness Re	Designation	204.0 (1 242.0 (7 355.0 (6 356.0 (6 390.0 (1 771.0 (8
Table 1.11. Ultimate Te	Principal Alloying Elements	Copper Copper Silicon and Copper Silicon and Copper Silicon and Copper Zinc

Principal Alloying	Designation	Hardness	Ultimate
Elements		Range	Tensile Strength, psi
Zinc Zinc Zinc and Lead Zinc and Lead Zinc, Tin, and Lead Zinc, Tin, and Lead Tin Aluminum Silicon Silicon Zinc, Aluminum, Nickel Zinc, Aluminum, Nickel	C2xxxx series C2xxxx series C3xxxx series C4bxxx series C4bxxx series C4bxxx series C48xxx series C5xxxx series C5xxxx series C5xxx series C65xxx series C65000 C69000	(55-82) HRB (100-160) HB (60-85) HRB (110-165) HB (55-95) HRB (100-210) HB (55-82) HRB (100-160) HB (70-85) HRB (125-165) HB (125-200) HB (125-200) HB (110-210) HB (110-210) HB (110-240) HB	$\sigma = 933 \text{ HRB} - 1990$ $\sigma = 453 \text{ HRB} + 5150$ $\sigma = 796 \text{ HRB} + 8370$ $\sigma = 332 \text{ HB} + 21900$ $\sigma = 332 \text{ HB} + 21900$ $\sigma = 546 \text{ HRB} + 29890$ $\sigma = 546 \text{ HRB} + 27820$ $\sigma = 546 \text{ HRB} + 27820$ $\sigma = 1259 \text{ HB} + 31350$ $\sigma = 1259 \text{ HB} - 28520$ $\sigma = 474 \text{ HB} + 810$ $\sigma = 1256 \text{ HRB} - 19010$ $\sigma = 458 \text{ HB} - 4570$ $\sigma = 458 \text{ HB} - 4670$ $\sigma = 458 \text{ HB} - 6670$ $\sigma = 2137 \text{ HRB} - 91620$ $\sigma = 506 \text{ HB} + 3520$

Table 1.12. Ultimate Tensile Strength—Hardness Relationship for Wrought Copper Alloys

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Principal Alloying	Designation	Hardness	Ultimate
Elements		Range	Tensile Strength, psi
Zinc, Tin, and Lead	C8xxxx series	(50–80) HB	$ \begin{aligned} \sigma &= \ 753 \ HB - 8220 \\ \sigma &= \ 472 \ HB + 11890 \\ \sigma &= \ 265 \ HB + 35910 \\ \sigma &= \ 761 \ HB - 10930 \\ \sigma &= \ 457 \ HB + 8220 \\ \sigma &= \ 457 \ HB + 16860 \end{aligned} $
Zinc and Aluminum	C86xxx series	(110–225) HB	
Zinc and Silicon	C87xxx series	(70–120) HB	
Tin, Lead, and Zinc	C92xxx, C93xxx series	(50–75) HB	
Nickel and Tin	C94-C96-C97xxx series	(60–175) HB	
Aluminum	C95xxx series	(140–250) HB	

The majority of formulas given in Tables 1.6 to 1.13 are the linear regression equations similar to $y = ax \pm b$, where y is the ultimate tensile strength, x is the hardness number (HB, HRB, HRC, and HK), a is a slope, and b is an intercept. Other formulas are developed by statistical treatment of data and are similar to y = Ax, where A is a coefficient of proportionality based on the arithmetic average of the ultimate tensile strength to hardness number ratios.

The raw data of hardness numbers of ferrous and nonferrous alloys and respective ultimate tensile strength values can be found in various handbooks, four of which the author used to develop linear regression formulas (Ref. 4, pages 7–462; Ref. 5, pages 20, 767, 864; Ref. 6, pages 49–51, 617; and Ref. 7, pages 10–19, 211–220).

The following examples show how accurately the ultimate tensile strength can be calculated by using formulas given in Tables 1.6–1.13.

Table 1.6. The ultimate tensile strength for the AISI type 304 stainless steel with a hardness of 149 HB is calculated by the formula shown in the 2nd row. For the same stainless steel with the hardness of 240 HB, the ultimate tensile strength is calculated by the formula shown in the 3rd row.

The 2nd row: $\sigma = 457$ HB + 16910 = $457 \cdot 149$ + 16910 = 85,003 psi

The ultimate tensile strength is 85,000 psi (Ref. 4, p. 274)
Mechanical Properties of Work Materials

The formula provides an accuracy of

$$\left(\frac{85,000}{85,003}\right) \cdot 100\% \cong 100\%$$

The 3rd row: $\sigma = 534$ HB – $16280 = 534 \cdot 240 - 16280 = 111,880$ psi

The ultimate tensile strength is 110,000 psi (Ref. 4, p. 274)

The formula provides an accuracy of

$$\left(\frac{110,000}{111,880}\right) \cdot 100\% = 98.3\%$$

Table 1.7. The ultimate tensile strength for the AISI type P20 tool steel with a hardness of 286 HB is calculated by the formula shown in the next to last row. For the same tool steel with the same hardness, the ultimate tensile strength is also calculated by the formula shown in the last row to compare the accuracy of these two formulas.

Next to last row: $\sigma = 545 \text{ HB} - 11770 = 545 \cdot 286 - 11770 = 144,100 \text{ psi}$

The ultimate tensile strength is 144,000 psi (Ref. 3, p. 146)

The formula provides an accuracy of

$$\left(\frac{144,000}{144,100}\right) \cdot \ 100\% = 99.9\%$$

The last row: $\sigma = 520 \text{ HB} = 520 \cdot 286 = 148,720 \text{ psi}$

The formula provides an accuracy of

$$\left(\frac{144,000}{148,720}\right) \cdot 100\% = 96.8\%$$

The linear regression formula is more accurate.

Table 1.8. The ultimate tensile strength for the ASTM A grade 60–40–18 ductile cast iron with a hardness of 143 HB is calculated by the formulas shown in the 2nd and 3rd rows.

The 2nd row: $\sigma = 450$ HB = $450 \cdot 143 = 64,350$ psi The ultimate tensile strength is 60,000 psi (Ref. 3, p. 152)

The formula provides an accuracy of

$$\left(\frac{60,000}{64,350}\right) \cdot 100\% = 93.2\%$$

The 3rd row: $\sigma = 467 \text{ HB} - 4720 = 467 \cdot 143 - 4720 = 62,061 \text{ psi}$

The formula provides an accuracy of

$$\left(\frac{60,000}{62,061}\right) \cdot 100\% = 96.7\%$$

The linear regression formula is more accurate.

Table 1.9. The ultimate tensile strength for Ti-6Al-4V (one of alpha–beta titanium alloys) with a hardness of 32.2 HRC and 350 HK can be found by the formulas in the 3rd and 5th rows respectively.

Mechanical Properties of Work Materials

The 3rd row: $\sigma = 3300$ HRC + $35230 = 3300 \cdot 32.2 + 35230 = 141,490$ psi

The ultimate tensile strength is 145,000 psi (Ref. 6, p. 617)

The formula provides an accuracy of

$$\left(\frac{141,490}{145,000}\right) \cdot 100\% = 97.6\%$$

The 5th row: $\sigma = 440 \text{ HK} + 2910 = 440 \cdot 350 + 2910 = 156,910 \text{ psi}$

The ultimate tensile strength is 154,000 psi (Ref. 6, p. 617)

The formula provides an accuracy of

$$\left(\frac{154,000}{156,910}\right) \cdot 100\% = 98.1\%$$

Table 1.10. The ultimate tensile strength for 2024–T3 wrought aluminum alloy (one of 2000 series) with a hardness of 120 HB is calculated by the formula shown in the 2nd row. The ultimate tensile strength for 7075–T6 wrought aluminum alloy (one of 7000 series) with a hardness of 150 HB is calculated by the formula shown in the last row.

The 2nd row:
$$\sigma = 555 \text{ HB} + 2150 = 555 \cdot 120 + 2150 = 68,750 \text{ psi}$$

The ultimate tensile strength is 70,000 psi (Ref. 6, p. 49)

The formula provides an accuracy of

$$\left(\frac{68,750}{70,000}\right) \cdot 100\% = 98.2\%$$

The last row: $\sigma = 558 \text{ HB} - 409 = 558 \cdot 150 - 409 = 83,291 \text{ psi}$

The ultimate tensile strength is 83,000 psi (Ref. 6, p. 51)

The formula provides an accuracy of

$$\left(\frac{83,000}{83,291}\right) \cdot 100\% = 99.6\%$$

Table 1.11. The ultimate tensile strength for 242.0–T61 cast aluminum alloy with a hardness of 110 HB is calculated by the formula shown in the 2nd row. The ultimate tensile strength for 355.0–T62 cast aluminum alloy with a hardness of 105 HB is calculated by the formula shown in the 3rd row.

The 2nd row: $\sigma = 446 \text{ HB} - 4465 = 446 \cdot 110 - 4465 = 44,595 \text{ psi}$

The ultimate tensile strength is 47,000 psi (Ref. 6, p. 157)

The formula provides an accuracy of

$$\left(\frac{44,595}{47,000}\right) \cdot 100\% = 94.9\%$$

The 3rd row: $\sigma = 448 \text{ HB} - 645 = 448 \cdot 105 - 645 = 46,395 \text{ psi}$

The ultimate tensile strength is 45000 psi (Ref. 6, p. 164)

The formula provides an accuracy of

$$\left(\frac{45,000}{46,395}\right) \cdot 100\% = 97.0\%$$

Table 1.12. The ultimate tensile strength for C26000 wrought copper alloy (one of C2xxxx series) with a hardness of 82 HRB is calculated by the formula shown in the 1st row. The ultimate tensile strength for C48200 wrought copper alloy (one of C48xxx series) with a hardness of 75 HRB is calculated by the formula shown in the 7th row.

The first row: $\sigma = 933$ HRB – $1990 = 933 \cdot 82 - 1990 = 74,516$ psi

The ultimate tensile strength is 78,000 psi (Ref. 6, p. 301)

The formula provides an accuracy of

$$\left(\frac{74,516}{78,000}\right) \cdot 100\% = 95.5\%$$

The 7th row: $\sigma = 546$ HRB + 27820 = 546 \cdot 75 + 27820 = 68,770 psi

The ultimate tensile strength is 67,000 psi (Ref. 6, p. 320)

The formula provides an accuracy of

$$\left(\frac{67,000}{68,770}\right) \cdot 100\% = 97.4\%$$

Table 1.13. The ultimate tensile strength for C86400 cast copper alloy (one of C86xxx series) with a hardness of 108 HB is calculated by the formula shown in the 2nd row. The ultimate tensile strength for C87500 cast copper alloy (one of C87xxx series) with a hardness of 115 HB is calculated by the formula shown in the 3rd row.

The 2nd row: $\sigma = 472$ HB + 11890 = 472 · 108 + 11890 = 62,866 psi

The ultimate tensile strength is 65,000 psi (Ref. 6, p. 349)

The formula provides an accuracy of

$$\left(\frac{62,866}{65,000}\right) \cdot 100\% = 96.7\%$$

The 3rd row: $\sigma = 265 \text{ HB} + 35910 = 265 \cdot 115 + 35910 = 66,385 \text{ psi}$

The ultimate tensile strength is 68,000 psi (Ref. 6, p. 349)

The formula provides an accuracy of

$$\left(\frac{66,385}{68,000}\right) \cdot 100\% = 97.6\%$$

Concluding Remarks on Mechanical Properties of Work Materials

1. Metallic workpiece materials are ferrous and nonferrous alloys. Among **ferrous alloys**, *carbon* and *alloy steels* are by far the most frequently used workpiece

Mechanical Properties of Work Materials

materials (Ref. 5, p. 147). Stainless steels represent about 25% of the metals subjected to turning. That percentage has been rising and will continue to rise with the growth of the chemical, oil, food processing, and power industries (Ref. 8, p. 24). Machining of stainless steels, especially the austenitic type, leads to more friction, heat, and chip control problems. Among nonferrous alloys, aluminum alloys have found wide use in the automotive industry to fabricate engine blocks, cylinder heads, pistons, carburetors, wheels, etc. Aluminum alloys are machined at high speed with superhard cutting tool materials such as diamond-film-coated inserts and polycrystalline diamond tips attached to carbide substrates. Titanium alloys are preferred metals for aerospace application due to their high strength and high fatigue resistance. From a cutting viewpoint, titanium alloys are hard-to-machine work materials.

2. Hardness implies resistance to deformation, and in metalcutting this characteristic is a measure of resistance to cutting. Higher hardness means higher resistance to cutting, higher cutting forces, and higher power consumption. Ability to convert one type of hardness measurement into another is important, especially if the work materials are characterized by different methods of hardness measurements. The author developed 35 linear regression equations to convert Rockwell (B and C scales), Vickers, Knoop, and Scleroscope hardness numbers into Brinell hardness numbers (Tables 1.1–1.5). These equations provide an accuracy of 99% or better.

3. Statistical treatment of the work materials' hardness and strength data resulted in 5 formulas for calculating

ultimate tensile strength versus Brinell hardness of carbon, alloy, and some stainless steels.

4. Linear regression analysis of hardness-strength relationship for stainless steels, tool steels, titanium alloys, aluminum alloys, and copper alloys was conducted. As a result of this analysis, 73 equations for calculating ultimate tensile strength were developed (Tables 1.6–1.13).

5. Equations for calculating the ultimate tensile strength versus the work materials' hardness (Tables 1.6-1.13) provide an accuracy of 95% or better.

6. Knowledge of the mechanical properties of work materials leads to higher accuracy in calculating the cutting forces and the machining power, and results in better guidelines for cutting tool applications. This knowledge defines the first element of metalcutting as an integrated system: sufficient description of work materials.

7. The ultimate tensile strength-hardness relationship expressed by the linear regression equations, is utilized in calculating cutting force when milling, as will be shown in the next chapter.

MILLING

Milling is one of the most common metalcutting operations in which a rotating multi-edge cutting tool removes chips from a stationary workpiece to produce flat and curved surfaces in almost any direction.

Milling operation is characterized by its kinematics, productivity, and dynamics. Parameters of kinematics and productivity include:

- 1. Cutting speed
- 2. Spindle speed
- 3. Cutter diameter
- 4. Number of inserts (teeth) in the cutter
- 5. Feed per tooth (same as chip load)
- 6. Feed rate (same as table feed or machine feed)
- 7. Depth and width of cut
- 8. Number of inserts in the cut
- 9. Chip thickness and lead angle of the cutter
- 10. Feed rate adjustment
- 11. Cross-sectional area of uncut chip
- 12. Metal removal rate

13. Machining time with linear path of the cutter

14. Machining time with helical interpolation Parameters of dynamics:

- 1. Tangential cutting force
- 2. Torque at the cutter
- 3. Machining power at the cutter
- 4. Machining power at the motor

2.1 KINEMATICS AND PRODUCTIVITY

Kinematics and productivity are described by formulas showing relationships between various parameters that are required to plan milling operations using various tools: face mills, end mills, die and cavity mills, slotting cutters, thread and grooving mills. Productivity is directly proportional to cutting speed (or spindle speed), feed per tooth, feed rate, depth of cut, and width of cut.

2.1.1 Cutting Speed

Cutting speed is one of the most commonly used programmable elements for CNC milling machines and machining centers. The relationship between the cutting speed V_c (dependent variable), machine spindle speed n (independent variable), and the cutter diameter D (independent variable) is expressed by formulas,

Customary U.S. units of measure:

$$V_{\rm c} = \frac{\pi D n}{12} \; (\text{sfm or ft/min}) \tag{2.1}$$

Where D is in inches, n is in rpm or rev/min, 12 is a factor converting inches into feet.

Metric units of measure:

$$V_{\rm c} = \frac{\pi D n}{1000} \tag{2.2}$$

Where D is in millimeters (mm) and 1000 is a factor converting mm into meters (m).

For example, D = 4.0 in. (101.6 mm) and n = 470 rpm (rev/min). What is V_o ?

From formula (2.1): The cutting speed

$$V_c = \frac{\pi \cdot 4 \cdot 470}{12} = 492 \text{ sfm},$$

From formula (2.2): The cutting speed

$$V_c = \frac{\pi \cdot 101.6 \cdot 470}{1000} = 150 \text{ m/min}$$

2.1.2 Spindle Speed

If the cutting speed V_c (sfm or m/min) and the cutter diameter D (in. or mm) are independent variables, then the formula for spindle speed n (rpm or rev/min) can be derived from formulas (2.1) and (2.2),

Customary U.S. units of measure:

$$n = \frac{12V_{\rm c}}{\pi D} \ (\rm rpm) \tag{2.3}$$

For example, $V_c = 500$ sfm and D = 4 in. What is the spindle speed?

From formula (2.3):

$$n = \frac{12 \cdot 500}{\pi \cdot 4} = 477 \text{ rpm}$$

Metric units of measure:

$$n = \frac{1000V_c}{\pi D} \text{ (rev/min)}$$
(2.4)

For example, $V_c = 152.4$ m/min and D = 101.6 mm. What is the spindle speed?

From formula (2.4):

$$n = \frac{1000 \cdot 152.4}{\pi \cdot 101.6} = 477 \text{ rpm}$$

2.1.3 Cutter Diameter

If the cutting speed V_c (sfm or m/min) and spindle speed *n* (rpm or rev/min) are independent variables, then the formula for cutter diameter *D* (in. or mm) can be derived either from formulas (2.1) and (2.2) or from formulas (2.3) and (2.4),

Customary U.S. units of measure:

$$D = \frac{12V_c}{\pi n} \text{ (in.)}$$

For example, $V_c = 550$ sfm and n = 600 rpm. What is the cutter diameter?

From formula (2.5):

$$D = \frac{12 \cdot 550}{\pi \cdot 600} = 3.50 \text{ in.}$$

Metric units of measure:

$$D = \frac{1000 V_{\rm c}}{\pi n} \,\,({\rm mm})$$
 (2.6)

For example, $V_c = 170$ m/min, n = 600 rev/min. What is the cutter diameter?

From formula (2.6):

$$D = \frac{1000 \cdot 170}{\pi \cdot 600} = 90.2 \text{ mm}$$

2.1.4 Number of Inserts in the Cutter

The number of inserts or teeth in the cutter (Z) depends mainly on the cutter design, geometry, and applications. Major cutter designs can be classified as conventional and special.

Conventional die and cavity mills, or ball nose cutters, have only 1 insert (Z = 1) for various diameters, mostly from 0.250 in. (6.35 mm) to 1.250 in. (31.75 mm). Conventional end mills may have Z = 1 (diameters up to 0.500 in. or 12.70 mm), Z = 2, typically for diameters from 0.625 in. (15.9 mm) to 1.000 in. (25.4 mm), $3 \le Z \le 5$, typically for diameters from 1.250 in. (31.75 mm) to 2.000 in. (50.8 mm). Conventional end mills, which are designed for cuts up to 1.25-inch deep, have the same number of inserts as the number of flutes. If the axial depth of cut exceeds 1.25 in. (31.75 mm), each flute will have two or more inserts depending on how deep the cut must be taken. The flutes are spiral (in most designs) or straight.

Conventional face mills vary in geometries and designs and, therefore, in number of inserts. Common diameter range is 2.0 to 6.0 inches (76.2 to 152.4 mm). The number of inserts may vary from 4 to 12 depending on the cutter diameter. The number of inserts also depends on the type of pitch. A fine pitch cutter has more inserts than a coarse pitch cutter of the same diameter.

Special designs include large diameter chamfering and finishing cutters, aerospace cutters, integral shank slotting cutters, and many more.

2.1.5 Feed per Tooth

Feed per tooth f_{s} (a.k.a. chip load) is a linear distance traveled by the cutter during the engagement of one tooth with the workpiece. Feed per tooth is obtained from:

$$f_z = \frac{f}{Zn},\tag{2.7}$$

and expressed in in/tooth or mm/tooth (customary U.S. and metric units of measure respectively), where f is the feed rate of the milling machine or machining center. Formula (2.7) defines feed per tooth at the centerline of the cutter.

For example, if Z = 6, n = 475 rpm, and f = 20 ipm

(508 mm/min), the feed per tooth f_z is: $f_z = \frac{20}{(6 \cdot 475)} = 0.007$ in. (customary U.S. units of measure).

 $f_z = \frac{508}{(6 \cdot 475)} = 0.178$ mm (metric units of measure).

2.1.6 Feed Rate

Feed rate is the speed at which the table, holding a workpiece travels toward the milling cutter. Feed rate is calculated by:

$$f = f_z Z n \tag{2.8}$$

and expressed in ipm or mm/min (customary U.S. and metric units of measure respectively). Normally, feed rate would be programmed by CNC or set manually (if a milling machine does not have CNC).

- For example, if Z = 6, n = 500 rpm, and $f_z = 0.010$ in. (0.254 mm), the feed rate is:
- $f = 0.01 \cdot 6 \cdot 500 = 30$ ipm (customary U.S. units of measure)
- $f = 0.254 \cdot 6 \cdot 500 = 762$ mm/min (metric units of measure).

2.1.7 Depth of Cut and Width of Cut

Depth of cut d is the length of engagement of the cutting edge of the insert, measured perpendicular to a cut surface. This definition is applied to end milling and face milling. In slotting, the depth of cut is the distance from an uncut surface to a cut surface, measured perpendicular to the axis of rotation of a slotting cutter.

Width of cut w is the distance from the leading edge of the insert to the point of exit from a workpiece, measured perpendicular to the axis of rotation of a milling cutter. The width of cut defines position of the cutter centerline: if w > R (radius of a cutter), the cutter centerline is located on the workpiece. If w < R, the cutter centerline is not located on the workpiece. This definition applies to end milling and face milling.

In slotting, the width of cut is the distance between the two side cutting edges that are engaged with the workpiece.

Depth of cut d and width of cut w are shown in Fig. 1.

2.1.8 Number of Inserts in the Cut

The number of inserts in the cut Z_c , or in simultaneous engagement with the workpiece, depends on the



Fig. 1. Depth of cut = d and width of cut = w

number of inserts in the cutter *Z* and the engagement angle α . This relationship is expressed by:

$$Z_{\rm c} = \frac{Z\alpha}{360} \tag{2.9}$$

The engagement angle α depends on the width of cut ω and the cutter diameter *D*. This angle is shown in Fig. 2 and Fig. 3, which present the two most typical positions of the milling cutter.

Fig. 2 illustrates the condition when a cutter centerline is located on the workpiece, and the engagement angle $\alpha = 90^\circ + \alpha_1$ where α_1 is the angle between the cutter centerline and the cutter radius *R* drawn to the peripheral point *B* at which the cutter exits from the workpiece or enters it. Angle α_1 is found from right-angled triangle



Fig. 2. Centerline of the cutter is on the workpiece, w > R, 1-milling cutter, 2-workpiece



Fig. 3. Centerline of the cutter is not on the workpiece, w < R, 1-milling cutter, 2-workpiece

OAB as:

$$\sin \alpha_1 = \frac{AB}{OB} = \frac{\varpi - R}{R} = \frac{\varpi - \frac{D}{2}}{\frac{D}{2}} = \frac{2\varpi - D}{D} \qquad (2.10)$$

Thus, from formula (2.10):

$$\alpha_1 = \arcsin\left(\frac{2w - D}{D}\right)$$
(2.11)

Since the engagement angle is $\alpha = 90^{\circ} + \alpha_1$, the substitution of α_1 for its value from formula (2.11), produces:

$$\alpha = 90^{\circ} + \arcsin\left(\frac{2w - D}{D}\right)$$
 (2.12)

Substitution of α from formula (2.12) into formula (2.9) gives:

$$Z_{c} = Z \left\{ \frac{90^{\circ} + \arcsin(\frac{2w - D}{D})}{360^{\circ}} \right\}$$
(2.13)

The following calculation shows that the maximum engagement angle and the maximum number of inserts in the cut take place when the width of cut equals the cutter diameter (w = D):

$$\begin{aligned} \alpha &= 90^{\circ} + \arcsin\left(\frac{2\varpi - \varpi}{\varpi}\right) = 90^{\circ} + \arcsin 1\\ &= 90^{\circ} + 90^{\circ} = 180^{\circ}\\ Z_c &= Z\left(\frac{180^{\circ}}{360^{\circ}}\right) = 0.5 \ Z \end{aligned}$$

Thus, the maximum number of inserts in the cut, which is one half of the number of inserts in the cutter, will occur when the width of cut equals the cutter diameter.

Figure 3 illustrates the condition when a cutter centerline is not located on the workpiece, so the width of cut is less than the cutter radius. As one can see, the engagement angle is: $\alpha = 90^\circ - \alpha_1$ where α_1 is an angle found from the right-angled triangle *OAB* as:

$$\sin \alpha_1 = \frac{AB}{OB} = \frac{R - \omega}{R} = \frac{\frac{D}{2} - \omega}{\frac{D}{2}} = \frac{D - 2\omega}{D}, \quad (2.14)$$

then from formula (2.14):

$$\alpha_1 = \arcsin\left(\frac{D - 2\omega}{D}\right)$$
(2.15)

The engagement angle is $\alpha = 90^{\circ} - \alpha_1$, so the substitution of α_1 for its value from formula (2.15), produces:

$$\alpha = 90^{\circ} - \arcsin\left(\frac{D - 2w}{D}\right)$$
 (2.16)

Substitution of α from formula (2.16) into formula (2.9) gives:

$$Z_{c} = Z \left\{ \frac{90^{\circ} - \arcsin\left(\frac{D - 2w}{D}\right)}{360^{\circ}} \right\}$$
(2.17)

When the width of cut is greater than the cutter radius, use formula (2.13), when the width of cut is less than the cutter radius, use formula (2.17) to calculate the number of inserts in the cut (Z_c). The following examples show how to calculate the number of inserts in the cut when: $w > \frac{D}{2}$ (Example 1) and $w < \frac{D}{2}$ (Example 2).

Example 1 (customary U.S. units), D = 6.0 in., Z = 10, w = 4.0 in. From formula (2.13):

$$\begin{split} &Z_c = 10 \left\{ \frac{90 + \arcsin\left(\frac{2 \cdot 4 - 6}{6}\right)}{360} \right\} \\ &= 10 \left\{ \frac{90 + \arcsin\left(0.3333\right)}{360} \right\} \\ &Z_c = 10 \left\{ \frac{90 + 19.5}{360} \right\} \\ &= 10 \left\{ \frac{109.5}{360} \right\} = 10 \cdot 0.304 = 3 \text{ (3 insets in the cut).} \end{split}$$

Example 1 (metric units), D = 152.4 mm, Z = 10, w = 101.6 mm. From formula (2.13):

$$\begin{split} &Z_c = 10 \begin{cases} \frac{90 + \arcsin\left(\frac{2 \cdot 101.6 - 152.4}{152.4}\right)}{360} \\ &= 10 \left\{\frac{90 + \arcsin 0.3333}{360}\right\} \\ &Z_c = 10 \left\{\frac{90 + 19.5}{360}\right\} \\ &= 10 \left\{\frac{109.5}{360}\right\} = 10 \cdot 0.304 = 3 \text{ (3 inserts in the cut).} \end{split}$$

Example 2 (customary U.S. units), D = 6.0 in., Z = 10, w = 2.1 in. From formula (2.17):

$$\begin{split} &Z_c = 10 \begin{cases} \frac{90 - \arcsin\left(\frac{6 - 2 \cdot 2.1}{6}\right)}{360} \\ &= 10 \left\{\frac{90 - \arcsin 0.3}{360}\right\} \\ &Z_c = 10 \left\{\frac{90 - 17.5}{360}\right\} \\ &= 10 \left\{\frac{72.5}{360}\right\} = 10 \cdot 0.201 = 2 \ (2 \text{ inserts in the cut}). \end{split}$$

Example 2 (metric units), D = 152.4 mm, Z = 10, w = 53.34 mm. From formula (2.17):

$$\begin{split} Z_c &= 10 \left\{ \frac{90 - \arcsin\left(\frac{152.4 - 2 \cdot 53.34}{152.4}\right)}{360} \right\} \\ &= 10 \left\{ \frac{90 - \arcsin 0.3}{360} \right\} \\ Z_c &= 10 \left\{ \frac{90 - 17.5}{360} \right\} \\ &= 10 \left\{ \frac{72.5}{360} \right\} = 10 \cdot 0.201 = 2 \ (2 \text{ inserts in the cut}). \end{split}$$

2.1.9 Chip Thickness and Lead Angle (Cutting Edge Angle) of the Cutter

Chip thickness t_c (in. or mm) is a cutting parameter depending on feed per tooth f_s and tool lead angle λ (customary U.S. units of measure) or tool cutting edge angle γ (metric units of measure). Chip thickness is calculated from:

$$t_c = f_z \cos \lambda \tag{2.18}$$

or

$$t_c = f_s \sin \gamma \tag{2.19}$$



Fig. 4. Lead angle of the milling cutter

Formula (2.18) is applied to the lead angle λ measured between the line 1–1 parallel to the axis of rotation 0–0, and the insert cutting edge (Fig. 4). This definition is in agreement with customary U.S. units of measure. Formula (2.19) is applied to the cutting edge angle γ measured between the line 2–2 perpendicular to the axis of rotation 0–0, and the insert cutting edge (Fig. 5). This definition is in agreement with metric units of measure. As can be seen from Fig. 4 and Fig. 5, $\lambda = 90^{\circ} - \gamma$ and $\gamma = 90^{\circ} - \lambda$.

Milling cutters with 0° lead angle (customary U.S. units) and 90° tool edge angle (metric units) produce chips that have a thickness equal to the feed per tooth



Fig. 5. Cutting edge angle of the milling cutter

 $(t_c = f_x)$, since $\cos 0^\circ = \sin 90^\circ = 1$. Milling cutters with $\lambda > 0^\circ$ and $\gamma < 90^\circ$ produce chips thinner than the feed per tooth. For example, when milling with a 30° lead angle cutter (customary U.S. units), the chip thickness is: $t_c = f_x \cos 30 = 0.866 f_x$, which is about 87% of the feed per tooth at the centerline of the cutter. The same reduction in thickness occurs when milling with a 60° tool edge angle (metric units): $t_c = f_x \sin 60^\circ = 0.866 f_x$. The lead angles vary from 0° to 45° (customary U.S. units) and their values are 15°, 20°, 30°, and 45°. In metric units, the tool edge angles vary from 90° to 45° and their values are 75°, 70°, 60°, and 45°. The most common lead angle or cutting edge angle is 45°.

Lead (or tool edge) angles produce the inclination of the cutting edge. The engaged length of the cutting edge with the workpiece becomes greater than the depth of cut. The chip becomes thinner because it is distributed over a greater length of the cutting edge.

The chip-thinning phenomenon plays a negative role because more heat dissipates into the cutting edge, and a thinner chip takes less heat away. The feed per tooth should be adjusted to compensate for the chip-thinning phenomenon. The amount of adjustment is inversely proportional to the cosine of the lead angle in accordance with the customary U.S. system, and inversely proportional to the sine of the cutting edge angle in accordance with the metric system. The coefficient of adjustment, C, is found from the following formulas (respectively):

$$C = \frac{1}{\cos \lambda} \tag{2.20}$$

$$C = \frac{1}{\sin \gamma} \tag{2.21}$$

Such adjustment is needed to increase the feed per tooth f_z accordingly, in order to keep the same productivity, as would be derived from use of 0° or 90° angles. Adjusted feeds per tooth f_{za} in customary U.S. units (formula 2.22) and in metric units (formula 2.23) are

$$f_{za} = f_z C = \frac{f_z}{\cos\lambda} \tag{2.22}$$

$$f_{za} = f_z C = \frac{f_z}{\sin\gamma}$$
(2.23)

If the lead angle (or the cutting edge angle) of a cutter is 45° , the adjusted feed per tooth is:

$$f_{za} = \frac{f_z}{\cos 45} = \frac{f_z}{\sin 45} = 1.414 f_z$$

2.1.10 Feed Rate Adjustment

A feed rate adjustment (increase) should be applied in two conditions: (1) when the lead angle of the cutter $\lambda > 0^{\circ}$ (customary U.S. units) or the tool cutting edge angle $\gamma < 90^{\circ}$ (metric units), and (2) when the width of cut is less than the cutter radius (w < R). In condition (2), the cutter centerline is not located on the workpiece.

Feed rate adjustment in condition (1) requires substitution of feed per tooth f_z in formula (2.8) for adjusted feed per tooth f_{za} from formula (2.22) or formula (2.23). Therefore, adjusted feed rate f_a is calculated by (for customary U.S. and metric units of measure respectively):

$$f_a = \frac{f_z Z n}{\cos \lambda}$$
 (ipm) (2.24)

$$f_a = \frac{f_z Zn}{\sin\gamma}$$
(mm/min) (2.25)

The following examples show how to adjust the feed rate to suit the lead angle.

 $\begin{array}{ll} \mbox{Condition (1), customary U.S. units of measure,} \\ \mbox{Given:} & f_z = 0.010 \mbox{ in. (at the centerline of the cutter),} \\ \mbox{$Z = 6, n = 500 \mbox{ rpm}, \lambda = 30^\circ$} \\ \mbox{$Required: Adjusted feed rate f_a.} \\ \mbox{$Solution: Formula (2.24) is used.} \end{array}$

$$f_a = \frac{0.01 \cdot 6 \cdot 500}{\cos 30} = \frac{30}{0.866} = 34.6$$
 ipm.

$$f_a = \frac{0.254 \cdot 6 \cdot 500}{\sin 60}$$
$$= \frac{762}{0.866} = 880 \text{ mm/min} = 88 \text{ cm/min}.$$

The feed rate adjustment in condition (2) increases the feed per tooth value, to the value it would have if the cutter centerline were located on the workpiece. The adjusted (increased) feed rate value is obtained from:

$$f_a = \frac{f_s Z n}{\frac{\sqrt{(D-w)w}}{R}}$$
(ipm or mm/min) (2.26)

The following examples show how to adjust the feed rate if w < R.

Condition (2), customary U.S. units of measure, Given: Z = 6, n = 500 rpm, D = 4.0 in., w =

0.75 in., R = 2.0 in.

Required feed per tooth is: $f_{s} = 0.010$ in.

Solution: Formula (2.26) is used to calculate necessary feed rate f_a

$$f_a = \frac{0.01 \cdot 6 \cdot 500}{\sqrt{(4 - 0.75) \cdot 0.75}}$$
$$= \frac{30}{\sqrt{2.44}} = \frac{30}{0.781} = 38.4 \text{ ipm}$$

Note: if the width of cut is greater than the cutter radius (w > R), the centerline of the cutter is located on the workpiece. Therefore, no adjustment to the feed rate is needed. Feed rates should be calculated by formula (2.8). The same values given for f_s , Z, and n as in the above example produce the following feed rate:

$$f = f_z Zn = 0.01 \cdot 6 \cdot 500$$

= 30 ipm (compare with 38.4 ipm).

Condition (2), metric units of measure, Given: Z = 6, n = 500 rev/min, D = 101.6 mm,w = 19 mm, R = 50.8 mm

Required feed per tooth is: $f_z = 0.254$ mm Solution: Formula (2.26) is used to calculate the necessary feed rate f_a

$$f_{\alpha} = \frac{0.254 \cdot 6 \cdot 500}{\sqrt{(101.6 - 19) \cdot 19}}$$

$$= \frac{762}{\frac{\sqrt{1569.4}}{50.8}} = \frac{762}{\frac{39.62}{50.8}}$$

$$= \frac{762}{0.78} = 977 \text{ mm} = 97.7 \text{ cm}$$

If the feed rate adjustment is not required (w > R), then formula (2.8) should be used as in the previous example for the customary U.S. system of units:

$$f = f_z Zn = 0.254 \cdot 6 \cdot 500 = 762 \text{ mm/min}$$

= 76.2 cm/min (compare with 97.7 cm/min)

2.1.11 Cross-sectional Area of Uncut Chip

The cross-sectional area of the uncut chip A (in² or mm²) depends on the feed per tooth f_s or the adjusted feed per tooth f_{sa} and depth of cut d. The formulas for the cross-sectional area of the uncut chip are as

follows:

$$\lambda = 0^{\circ} \text{ (customary U.S. units) or}$$

$$\gamma = 90^{\circ} \text{ (metric units)}, \quad A = f_z d \qquad (2.27)$$

$$\lambda > 0^{\circ} \text{ (customary U.S. units) or}$$

$$\gamma < 90^{\circ} \text{ (metric units)}, \quad A = f_{za} d \qquad (2.28)$$

2.1.12 Metal Removal Rate

The metal removal rate (MRR, symbol Q) shows the volume of material removed in a period of time. Metal removal rate is expressed in in³/min (customary U.S. units of measure) and in cm³/min (metric units of measure). Metal removal rate is a measure of productivity and depends on the depth of cut (d), width of cut (w), and feed rate (f). MRR is calculated from:

$$Q = dwf \tag{2.29}$$

Substitution for f from formula (2.8) into formula (2.29) gives:

$$Q = dw f_z Zn \tag{2.30}$$

The following examples of MRR calculations are: (1) in customary U.S. units of measure and (2) in metric units of measure, when d = 0.2 in. (5.08 mm), w = 2 in. (50.8 mm), $f_s = 0.012$ in. (0.305 mm), Z = 6, n = 600 rpm (1), $Q = 0.2 \cdot 2 \cdot 0.012 \cdot 6 \cdot 600 = 17.28$ in³/min

(1)
$$Q = 0.2 \cdot 2 \cdot 0.012 \cdot 6 \cdot 600 = 17.28 \text{ in/min}$$

(2) $Q = \frac{5.08 \cdot 50.8 \cdot 0.305 \cdot 6 \cdot 600}{1000} = 283 \text{ cm}^3/\text{min}$

2.1.13 Machining Time with Linear Path of the Cutter

The machining time depends on the length of cut (L) and the feed rate (f). General face and end milling operations are performed by linear movements of the milling cutters along the workpiece (back and forth) at a certain depth of cut (d), width of cut (w), and feed per tooth (f_s) until the required portion of the workpiece is removed. It may take more than one pass in X-Y and Z directions. Therefore, machining time should be calculated per pass first, and then multiplied by the number of passes needed in X-Y and Z directions. Machining time per pass (T_n) is obtained from:

$$T_p = \frac{L}{f} \text{ (min)} \tag{2.31}$$

replacing f in formula (2.31) by its value from formula (2.8) gives:

$$T_p = \frac{L}{f_s Z n}$$
(min) (2.32)

The number of passes in *X*-*Y* direction (N_{xy}) is found from:

$$N_{xy} = \frac{W}{w} \tag{2.33}$$

were W is a width of the workpiece, and w is a programmed width of cut per pass.

It is a good practice to have the width of cut-to cutter diameter ratio, $\frac{w}{D} = 0.67 - 0.75$, because such ratios generate uniform chips, reduce friction at the points of entry and exit, and decrease the cutting temperature.

The number of passes in the *Z* direction (N_z) can be more than one if the height of material to be removed (*H*) exceeds the programmed depth of cut (*d*):

$$N_{z} = \frac{H}{d}$$
(2.34)

Therefore, the maximum number of passes (*N*) can be as many as:

$$N = N_{xy} + N_z = \frac{W}{w} + \frac{H}{d}$$
(2.35)

The total machining time (T) is the product of the machining time per pass (T_p) and the number of passes (N):

$$T = T_p N \tag{2.36}$$

Replacing T_p and N in formula (2.36) by their values from formula (2.32) and formula (2.35) respectively, gives:

$$T = \left(\frac{L}{f_{s}Zn}\right) \cdot \left(\frac{W}{w} + \frac{H}{d}\right) (\min) \qquad (2.37)$$

The following examples of machining time calculations are: (1) in customary U.S. units of measure and (2) in metric units of measure.

Given:

- L = 15.0 in. (381 mm), $f_z = 0.008$ in. (0.203 mm), Z = 6, n = 600 rpm,
- W = 8.0 in. (203.2 mm), w = 2.67 in. (67.8 mm), H = 0.40 in. (10.2 mm),
- d = 0.20 in. (5.08 mm).

Solution:

- (1) $T = (\frac{15}{0.008 \cdot 6 \cdot 600}) \cdot (\frac{8}{2.67} + \frac{0.40}{0.20}) = 0.521 \cdot (3+2) = 2.6 \text{ min}$
- (2) $T = (\frac{381}{0.203 \cdot 6 \cdot 600}) \cdot (\frac{203.2}{67.8} + \frac{10.16}{5.08}) = 0.521 \cdot (3+2) = 2.6 \text{ min.}$

2.1.14 Machining Time with Helical Interpolation

Another type of milling cutter performance is a helical interpolation, which is a combination of the cutter circular motion in X and Y directions, and its gradual penetration in the Z direction. Helical interpolation is a preferred method of entry into the workpiece making a cavity or a hole to its programmed diameter and depth. Helical interpolation requires only one entry and exit, where ramping requires multiple entries and exits. Other advantage is a consistent depth of cut and feed per tooth, which are maintained without dwelling on the Z-axis. Machining time calculation with helical interpolation differs from that with linear motion due to the constantly changing length of the helical path.

Advanced milling cutter geometry and modern machining centers permit programming of any kind of helical interpolation, such as cavities of various shapes

with flat and curvilinear bottoms, and holes of different diameters (straight and step, blind and through), etc.

Due to space limitations of this book, only machining time formulas for generating holes are presented, because the helical interpolation method becomes much more effective than conventional drilling, especially where a hole diameter exceeds 2 in. (50 mm). Helical interpolation generates axial force, but it is significantly lower than that in drilling, and requires significantly less machining power. Holes of any diameter can be interpolated, if geometry of the milling cutter allows plunging. Milling cutters with round and elongated indexable inserts are most commonly used for this application.

Machining time to interpolate a hole depends on its diameter (D_h) and depth (H) as shown in Fig. 6. The larger the diameter of the hole, the longer is the circular path (L_c) of the milling cutter in the X-Y direction to complete one helical revolution about the center point of the hole. The number of helical revolutions depends on the hole depth and the axial interpolation rate (f_i) . Machining time (T_h) to complete one helical revolution is calculated from:

$$T_h = \frac{L_c}{f}$$
(min) (2.38)

where L_c is circumference produced by the center point of the cutter (in., mm) f is feed rate (ipm, mm/min)



Fig. 6. Schematic of helical interpolation of a hole

As can be seen from Fig. 6:

$$L_{\rm c} = \pi D_{\rm c} = \pi (D_h - D) (\text{in., mm})$$
 (2.39)

where D = milling cutter diameter.

Replacing L_c and f values in formula (2.38) by their values from formula (2.39) and formula (2.8)
respectively, gives:

$$T_h = \frac{\pi (D_h - D)}{f_s Z n} \tag{2.40}$$

The milling cutter penetrates in the Z direction by the amount of the axial interpolation rate f_i for each helical revolution. Thus: after each helical revolution, the depth of a hole is increased by the amount f_i , so the number of required helical revolutions (N_b) is:

$$N_h = \frac{H}{f_i} \tag{2.41}$$

Total machining time for helical interpolation of a hole (T) is a product of the machining time per helical revolution (T_h) and the number of helical revolutions (N_h) :

$$T = T_h N_h \tag{2.42}$$

Replacing T_h and N_h values in formula (2.42) by their values from formula (2.40) and formula (2.41) respectively, gives:

$$T = \frac{\pi (D_h - D)}{f_s Z n} \left(\frac{H}{f_i}\right) (\min)$$
 (2.43)

The following examples of machining time calculations are: (1) in customary U.S. units of measure and (2) in metric units of measure.

Given:

 $D_{h} = 6.00 \text{ in. } (152.4 \text{ mm}), D = 2.00 \text{ in. } (50.8 \text{ mm}), f_{z} = 0.008 \text{ in. } (0.2032 \text{ mm}), Z = 4, n = 1050 \text{ rpm}, H = 2.50 \text{ in. } (63.5 \text{ mm}), f_{i} = 0.20 \text{ ipr } (5.08 \text{ mm/rev}).$ Solution: (1) $T = \frac{\pi \cdot (6-2)}{0.008 \cdot 4 \cdot 1050} \cdot \frac{2.5}{0.2} = \frac{\pi \cdot 4}{33.6} \cdot 12.5 = 4.67 \text{ min}$ (2) $T = \frac{\pi \cdot (152.4 - 50.8)}{0.2032 \cdot 4 \cdot 1050} \cdot \frac{63.5}{5.08}$ $T = \frac{\pi \cdot 101.6}{853.44} \cdot 12.5 = 4.67 \text{ min}$

2.2 DYNAMICS

Dynamics of milling is described by the cutting force, torque, and machining power formulas. These formulas are based on the relationships between the mechanical properties of work materials and the cutting conditions. The author developed some of these formulas, the other formulas are common knowledge. The formulas help to program cutting conditions within the torque and power values of the milling machines and machining centers.

2.2.1 Tangential Cutting Force

Analysis of dynamics of milling conducted by the author produced a new concept for calculating tangential cutting force. This concept has been named "Tangential Cutting Force Theory". The theory itself was "tested" in 1994 and has been used since as practical method of calculating cutting force, torque, and machining power

when applied to milling cutters with positive axial rake angles up to 20° .

This new concept for calculating tangential cutting force is based on relationships between the following parameters (Ref. 9, 10).

- (1) Ultimate tensile strength of work materials (σ) versus their hardness,
- (2) Cross-sectional area of the uncut chip (*A*) versus depth of cut and feed per tooth,
- (3) Number of inserts in the cut (Z_c) versus width of cut and number of inserts in cutter,
- (4) Machinability factor of work materials (C_m) versus cutting conditions,
- (5) Cutting tool wear factor (C_w) versus depth of cut and feed per tooth.

The author developed the tangential cutting force formula in accordance with these relationships:

$$F_t = \sigma A Z_c C_m C_w \tag{2.44}$$

(1) Ultimate tensile strength of work materials

Ultimate tensile strength is calculated by the formulas described in section 1.3 "Strength–Hardness Relationship of Work Materials". Formulas 1.1 to 1.5 are used for carbon and alloy steels. Formulas for all other materials are presented in Table 1.6 through Table 1.13.

(2) Cross-sectional area of the uncut chip

The shape of a cross section of the uncut chip depends, mainly, on the cutting insert geometry and the milling cutter lead angle (Fig. 7). Round inserts produce chips with cross sections that look like a partial crescent.



Fig. 7. Cross section of uncut chip, (a) rectangular, (b) parallelogram, (c) half-crescent

Inserts of all other shapes with straight cutting edges produce chips with either a rectangular cross section ($\lambda = 0^{\circ}$, customary U.S. units of measure, and $\gamma = 90^{\circ}$, metric units of measure) or a parallelogram cross section (milling cutters with lead angle greater than 0° and the cutting edge angle less than 90°).

The cross-sectional area of the uncut chip is calculated by formulas (2.27) and (2.28) described in paragraph 2.1.11. These formulas are easily understood when applied to rectangular and parallelogram cross-sections. The application is not so obvious when the cross section is a half-crescent (the axial depth of cut is equal to the radius of the insert) or a partial crescent (the axial depth of cut is less than the radius of the insert). The author has proven the applicability of these formulas to a half-crescent area and to a partial crescent area by integration and by algebraic-trigonometric calculations. The difference in the values calculated by formula (2.27) and by integration is less than 0.15%.

(3) Number of inserts in the cut

The number of inserts in the cut is calculated by formulas (2.13) and (2.17) described in paragraph 2.1.8. Dynamic stability of the milling cutter is very important because it affects not only the tool life and surface finish, but also the rigidity of the milling machine spindle. To obtain dynamic stability, a minimum of two cutting inserts must be in the cut (simultaneously engaged with the workpiece).

The approximate number of inserts in the cut can be calculated from:

$$Z_{\rm c} = mZ \tag{2.45}$$

where *m* is the multiplier depending on the ratio of $\frac{w}{D}$.

The following table presents *m*-values versus $\frac{w}{D}$ ratios. Formula (2.45) and Table 2.1 help to select a milling cutter with appropriate number of inserts (Ref. 11).

Table 2.1. Multiplier Values (m) vs. Width of Cut—to Cutter Diameter Ratio (w/D)

W D	1.0	0.90	0.80	0.75	0.666	0.50	0.40	0.35	0.30	0.25	0.20
m	0.50	0.40	0.35	0.33	0.304	0.25	0.22	0.20	0.185	0.167	0.148

To get $Z_c = 2$ (for dynamic stability) at $\frac{w}{D} = 0.666$ (it is a good practice to keep width of cut w equal to $\frac{2}{3}$ of the cutter diameter D to produce uniform chips), the milling cutter should have as many inserts as:

$$Z = \frac{Z_{\rm c}}{m} = \frac{2}{0.304} = 7$$

The optimal diameter of a cutter with 7 inserts is about 4 inches (101.6 mm). The width of cut should be $\frac{2}{3}$ of the cutter diameter, or $\frac{2}{3} \cdot 4 = 2.666$ in. (67.7 mm). The number of inserts in the cut is calculated by formula (2.13),

Customary U.S. units of measure:

$$Z_{c} = 7 \left\{ \frac{90 + \arcsin\left(\frac{2 \cdot 2.666 - 4}{4}\right)}{360} \right\}$$
$$= 7 \left\{ \frac{90 + \arcsin 0.333}{360} \right\}$$
$$Z_{c} = 7 \left\{ \frac{90 + 19.45}{360} \right\} = 7 \left\{ \frac{109.45}{360} \right\} = 7 \cdot 0.30 = 2.1$$

Rounding off to the nearest integer gives $Z_c = 2$. Metric units of measure:

$$Z_{c} = 7 \left\{ \frac{90 + \arcsin\left(\frac{2 \cdot 67.7 - 101.6}{101.6}\right)}{360} \right\}$$
$$= 7 \left\{ \frac{90 + \arcsin 0.3327}{360} \right\}$$
$$Z_{c} = 7 \left\{ \frac{60 + 19.43}{360} \right\} = 7 \left\{ \frac{109.43}{360} \right\} = 7 \cdot 0.30 = 2.1$$

Rounding off to the nearest integer gives $Z_c = 2$.

(4) Machinability factor of work materials

The machinability factor of work materials is a dimensionless coefficient that correlates the width of cut-to cutter diameter ratio $\left(\frac{\omega}{D}\right)$ with the tangential cutting force. Milling is more effective if sufficiently thick and uniform chips are produced. The $\frac{w}{D}$ ratio determines the uniformity of the chip thickness. When width of cut is equal to cutter diameter $(\frac{w}{D} = 1)$, the chip being formed starts off at zero thickness at the point of entry. The thickness increases to its maximum at the centerline of the cutter and thins off to zero at the point of exit. This type of cut produces a nonuniform chip and generates maximum friction at the cutting edge. As a result, the resistance to cutting increases. Effective face milling is obtained when a radial width of cut is approximately $\frac{2}{3}$ of a cutter diameter. This type of cut produces uniform and sufficiently thick chips, generates less friction at the cutting edge, and decreases resistance to cutting.

Table 2.2 presents machinability factors (C_m) for common types of workpiece materials at various $\frac{w}{D}$ ratios.

(5) Cutting tool wear factor

The cutting tool wear factor is a dimensionless coefficient that correlates tangential cutting force with tool life. When milling with new or just indexed cutting inserts for a short time, approximately 20–25% of tool life, the cutting tool wear factor $C_{w} = 1.0$. When milling is continued until the cutting inserts are indexed or replaced (100% of tool life), the following cutting tool wear factors

ומטווומטווונץ רמ	ctors $\boldsymbol{\upsilon}_m$ at vari	ous <i>W/D</i> rauos	
Type of Workpiece Material	w/D ≤ 0.5	0.5 < <i>w/D</i> < 0.7	<i>w/D</i> = 0.7–1.0
Carbon, alloy, and tool steels	1.0	1.3	1.5
Perlitic and martensitic stainless steels	1.0	1.3	1.5
Austenitic stainless steels	1.6	1.8	2.2
Gray cast iron	2.1	2.6	3.1
Ductile and malleable cast irons	1.0	1.3	1.5
Titanium alloys	1.0	1.5	1.7
Heat-resistant alloys	1.6	1.8	2.2
Aluminum alloys, $\sigma \le 70$ ksi ($\sigma \le 480$ N/mm ²)	0.7	0.8	0.9
Aluminum alloys, $\sigma > 70$ ksi ($\sigma > 480$ N/mm ²)	0.8	0.9	1.0
Copper alloys, $\sigma \leq$ 70 ksi ($\sigma \leq$ 480 N/mm ²)	0.8	0.9	1.0
Copper alloys, $\sigma >$ 70 ksi ($\sigma >$ 480 N/mm ²)	1.0	1.3	1.5

Table 2.2 Machinability Eactors Γ at Various μ/D Ratios

Chapter 2

are recommended (Ref. 2, p. 1048) for the milling categories and specified as:

- Light face milling
- Medium face milling
- Heavy-duty face milling

 $\begin{array}{l} C_w = 1.1 \\ C_w = 1.2 \\ C_w = 1.3 \end{array}$

Metalcutting related publications that were available to the author did not identify these milling categories by cutting conditions. The author, therefore, prepared a questionnaire asking for machining data that would best describe the milling categories by optimal range of depth of cut and feed per tooth data. The questionnaire was sent to more than one hundred people having practical knowledge and experience in milling applications. Statistical treatment of reported data allowed identification of each milling category by the depth of cut range and feed per tooth range.

Table 2.3 and Table 2.4 present this data in customary U.S. and metric units of measure respectively.

Example for Calculating Tangential Cutting Force

The following data is adapted from "Milling Ferrous Materials" by *Krupp Widia Tool Engineering Publication*.

Work material: DIN 1.2344 (United States similar material is AISI H13), Brinell hardness 210 HB in annealed condition;

Face milling cutter series: M40, diameter D = 160 mm (6.299 in.), tool cutting edge angle $\gamma = 75^{\circ}$ (equivalent to tool lead angle $\lambda = 15^{\circ}$ in customary U.S. units), Z = 10;

Table 2.	3. Milling Categorie	es and Cutting Tool V	Near Factors (customa	ry U.S. units)
Face Milling Category	Depth of Cut Range, in.	Feed per Tooth Range, in.	Cross-sectional Area of Chip, in ²	Cutting Tool Wear Factor, \mathcal{C}_{ω}
Light Medium	0.020-0.100 0.100-0.200	0.003-0.006 0.006-0.010	≤0.0006 ≤0.0020	1.1 1.2
Heavy-duty	0.200-0.400	0.010-0.020	>0.0020	1.3
Tab	le 2.4. Milling Cateç	jories and Cutting T	ool Wear Factors (met	ic units)
Face Milling Category	Depth of Cut Range, mm	Feed per Tooth Range, mm.	Cross-sectional Area of Chip, mm ²	Cutting Tool Wear Factor, ${\cal C}_w$
Light	0.50-2.50	0.08-0.15	≤0.040	1.1
Medium	2.50-5.00	0.15–0.25	≤0.375	1.2
Heavy-duty	5.00-10.0	0.25-0.50	>1.250	1.3

Cutting conditions: Depth of cut d = 6 mm (0.2362 in.), width of cut w = 120 mm (4.724 in.) feed per tooth 0.4 mm (0.0157 in.), cutting speed 91 m/min (298 sfm), width of cut-to-cutter diameter ratio $\frac{120}{160} = 0.75$, efficiency of the machine tool E = 70%.

Step-by-step calculations were performed using *tangential cutting force formula* and other related formulas developed by the author.

(1) Ultimate tensile strength of work material Ultimate tensile strength of DIN 1.2344 or AISI H13 is found from formula (1.1):

Customary U.S. units of measure:

$$\sigma = HB \cdot 500 = 210 \cdot 500 = 105,000 \text{ psi}$$

Metric units of measure:

$$\sigma = \frac{105,000}{145.0377} = 724 \text{ N/mm}^2$$

where 145.0377 is the conversion factor (psi into N/mm²).

(2) Cross-sectional area of the uncut chip

The cross-sectional area of the uncut chip is a function of feed per tooth and the depth of cut. The tool lead angle is 15° (in customary U.S. units of measure), so the tool cutting edge angle is 75° (in metric units of measure). Therefore, the feed per tooth should be adjusted by means of formulas (2.22 and 2.23)

respectively:

$$f_{za} = f_z C = \frac{f_z}{\cos \lambda} = \frac{0.0157}{\cos 15^\circ} = \frac{0.0157}{0.966} = 0.0163 \text{ in.}$$
$$f_{za} = f_z C = \frac{f_z}{\sin \gamma} = \frac{0.4}{\sin 75^\circ} = \frac{0.4}{0.966} = 0.4141 \text{ mm}$$

Cross-sectional area of the uncut chip is calculated by formula (2.28):

$$A = f_{sa}d = 0.0163 \cdot 0.2362 = 0.0039 \text{ in}^2$$

(customary U.S. units of measure)

$$A = f_{sa}d = 0.4141 \cdot 6 = 2.4846 \text{ mm}^2$$

(metric units of measure).

(3) Number of inserts in the cut

The number of inserts in the cut is calculated using formula (2.13) because the width of cut is greater than the cutter radius (w > R) indicating that the cutter centerline is located on the workpiece. The number of inserts in the cut can also be found from Table 2.1 using the $\frac{w}{D}$ value and the multiplier's respective value under it.

$$Z_{c} = Z \left\{ \frac{90^{\circ} + \arcsin\left(\frac{2w - D}{D}\right)}{360^{\circ}} \right\}$$

Customary U.S. units of measure:

$$\begin{split} &Z_c = 10 \begin{cases} \frac{90 + \arcsin\left(\frac{2 \cdot 4.724 - 6.299}{6.299}\right)}{360^{\circ}} \\ &= 10 \left\{\frac{90 + \arcsin 0.50}{360^{\circ}}\right\} \\ &Z_c = 10 \left\{\frac{90 + 30}{360^{\circ}}\right\} = 10 \left\{\frac{120}{360^{\circ}}\right\} = 10 \cdot 0.333 = 3.33. \end{split}$$

Rounding off to the nearest integer gives $Z_c = 3$, which is the number of inserts in the cut.

Metric units of measure:

$$Z_{c} = 10 \left\{ \frac{90 + \arcsin\left(\frac{2 \cdot 120 - 160}{160}\right)}{360^{\circ}} \right\}$$
$$= 10 \left\{ \frac{90 + \arcsin 0.50}{360^{\circ}} \right\}$$
$$Z_{c} = 10 \left\{ \frac{90 + 30}{360^{\circ}} \right\} = 10 \left\{ \frac{120}{360^{\circ}} \right\} = 10 \cdot 0.333 = 3.33.$$

Rounding off to the nearest integer gives $Z_c = 3$, which is the number of inserts in the cut.

Similar results are obtained from Table 2.1 and formula (2.45): At

$$\frac{w}{D} = \frac{120}{160} = \frac{4.724}{6.299} = 0.75,$$

the multiplier m = 0.33, $Z_c = Zm = 10 \cdot 0.33 = 3.3$.

Rounding off to the nearest integer gives $Z_c = 3$.

However, when calculating tangential cutting force it is recommended that the Z_c value be used as found (without rounding off to the nearest integer). As a matter of fact, the feed per tooth-programmed value varies during the insert's engagement with the workpiece, reaching its maximum at the centerline of the cutter. The feed per tooth minimal values are always at the points of entry and exit.

(4) Machinability factor of DIN 1.2344 (AISI H13) tool steel

The machinability factor (C_m) of this tool steel is found from Table 2.2. The C_m value is given in the right column " $\frac{w}{D} = 0.7 - 1.0$ " (since $\frac{w}{D} = 0.75$) and the first row: "Carbon, alloy, and tool steels", from which it follows that $C_m = 1.5$.

(5) Cutting tool wear factor

The cutting tool wear factors (C_w) are found in Table 2.3 for cutting conditions given in customary U.S. units of measure or in Table 2.4 for cutting conditions given in metric units of measure. For the cutting conditions given in this example, $C_w = 1.3$.

Thus, all the variables in formula (2.44) are determined, and now the tangential cutting force can be calculated.

$$F_t = \sigma A Z_c C_m C_w$$

Customary U.S. units of measure:

$$F_{t} = 105,000 \cdot 0.0039 \cdot 3.33 \cdot 1.5 \cdot 1.3 = 2659 \text{ lb}$$

Metric units of measure:

$$F_{t} = 724 \cdot 2.4846 \cdot 3.33 \cdot 1.5 \cdot 1.3 = 11681 \text{ N}$$

2.2.2 Torque at the Cutter

Torque at the cutter is generated by tangential cutting force. The amount of torque at the cutter (T_c) is directly proportional to the tangential cutting force (F_t) and the cutter radius $\frac{D}{2}$:

Customary U.S. units of measure,

$$T_{\rm c} = \frac{F_t D}{2} \text{ (in lb)}$$
 (2.46)

Metric units of measure,

$$T_{\rm c} = \frac{F_t D}{2000} \,\,({\rm N} \cdot {\rm m})$$
 (2.47)

The amount of torque generated by the tangential cutting force should not exceed the amount of a motor's output torque (T_m) . The following example of calculating

torque is based on the amount of calculated tangential force and the cutter radius.

Formula (2.46):

$$T = \frac{2659 \cdot 6.229}{2} = 8281 \text{ in } \text{lb} = \frac{8281}{12} = 690 \text{ ft } \text{lb}$$

Formula (2.47):

$$T_{\rm c} = \frac{11681 \cdot 160}{2 \cdot 1000} = 934.5 \,\mathrm{N} \cdot \mathrm{m}$$

2.2.3 Machining Power at the Cutter

Machining power at the cutter (P_c) , or required cutting power, expressed through tangential cutting force (F_t) and cutting speed (V_c) is more accurate than the currently used method of calculation based on the metal removal rate and power constants (Ref. 2, p. 1048). The following formulas are recommended to calculate machining power at the cutter:

Customary U.S. units of measure,

$$P_{\rm c} = \frac{F_t V_{\rm c}}{33,000}$$
 (hp) (2.48)

where 33,000 is a factor converting foot-pounds per minute into horsepower, since one horsepower equals 550 foot-pounds per second or $33,000(550 \cdot 60)$ foot-pounds per minute.

Metric units of measure,

$$P_{\rm c} = \frac{F_t V_{\rm c}}{60000} \ (\rm kW) \tag{2.49}$$

where 60000 is a factor converting newton-meters per minute into kilowatts, since one kilowatt equals 1000 newton-meters per second (1000 watts) or $60000(1000 \cdot 60)$ newton-meters per minute.

Machining power at the cutter is also expressed through torque at the cutter (T_c) and spindle speed (n). As can be seen from formulas (2.46) and (2.47), tangential cutting force (F_t) relates to torque at the cutter (T_c) as:

Customary U.S. units of measure,

$$F_t = \frac{2T_c}{D}$$
 (lb) (2.50)

Metric units of measure,

$$F_t = \frac{2 \cdot 1000 T_c}{D}$$
 (N) (2.51)

Relationship between spindle speed (*n*) and the cutting speed (V_c) are expressed by formulas (2.3) and (2.4) for customary U.S. and metric units of measure respectively. Replacing F_t values in formulas (2.48) and (2.49) by values from formulas (2.50) and (2.51) and V_c values in formulas (2.48) and (2.49) by values from formulas (2.3) and (2.4), results in the following equations:

Customary U.S. units,

$$P_{c} = \frac{\left(\frac{2T_{c}}{D}\right) \cdot \left(\frac{\pi Dn}{12}\right)}{33,000},$$

which, after cancellation of D, gives:

$$P_{\rm c} = \frac{2\pi T_{\rm c} n}{12 \cdot 33,000}$$

and then after dividing $(12 \cdot 33,000)$ by 2π , the equation is simplified to:

Customary U.S. units of measure,

$$P_{\rm c} = \frac{T_{\rm c} n}{63,025} \ ({\rm hp})$$
 (2.52)

Metric units,

$$P_{\rm c} = \left(\frac{2 \cdot 1000 T_{\rm c}}{D}\right) \cdot \frac{\frac{\pi D n}{1000}}{6000},$$

which after cancellation of D and 1000 gives:

$$P_{\rm c}=\frac{2\pi T_{\rm c}n}{60000},$$

and then after dividing 60000 by 2π , the equation is simplified to:

Metric units of measure,

$$P_{\rm c} = \frac{T_{\rm c} n}{9549} \ (\rm kW)$$
 (2.53)

Examples of calculating machining power at the cutter are performed using the same data as in previous examples.

Customary U.S. units of measure

Given: $F_c = 2659 \text{ lb}, T_c = 8281 \text{ in lb}, V_c = 298 \text{ sfm}, D = 6.299 \text{ in}.$

Required: Machining power at the cutter, P_c . Using formula (2.48):

$$P_c = \frac{2659 \cdot 298}{33,000} = 24.0 \text{ hp}$$

Using formula (2.52):

$$P_{c} = \frac{8281 \cdot n}{63025},$$

where n is a spindle speed, which is calculated by formula (2.3),

$$n = \frac{12 \cdot 298}{\pi \cdot 6.299} = 181$$
 rpm.

Finally, from formula (2.52),

$$P_c = \frac{8281 \cdot 181}{63025} = 23.8 \text{ hp.}$$

Metric units of measure

Given: $F_c = 11681 \text{ N}, T_c = 934.5 \text{ N} \cdot \text{m}, V_c = 91 \text{ m/min}, D = 160 \text{ mm}$ Required: Machining power at the cutter, P_c .

Using formula (2.49):

$$P_c = \frac{11681 \cdot 91}{60000} = 17.7 \text{ kW}$$

Using formula (2.53):

$$P_{c} = \frac{934.5 \cdot n}{9549},$$

where n is a spindle speed, which is calculated by formula (2.4),

$$n = \frac{1000 \cdot 91}{\pi \cdot 160} = 181$$
 rpm.

Finally, from formula (2.53),

$$P_{\rm c} = \frac{934.5 \cdot 181}{9549} = 17.7 \text{ kW}$$

2.2.4 Machining Power at the Motor

Machining power at the motor is an input power, or power consumption, which is calculated by the formula:

$$P_m = \frac{P_c}{E} \tag{2.54}$$

where *E* is a machine efficiency factor, which depends on the type of drive and expressed in percents. Type of drive is given in Table 2.5 (Ref. 2, p. 1049).

Type of Drive	<i>E</i> , %
Direct Drive (integral-motor spindle)* Direct Belt Drive Back Gear Drive Geared Head Drive Oil-Hydraulic Drive	94–95 90 75 70–80 60–90
	00 00

Table 2.5. Machine Efficiency Factors, E

* Integral-motor spindles designed by The Precise Corporation (Ref. 12).

Examples of calculating machining power at the motor (P_m) are based on machining power at the cutter (P_c) and efficiency factor (E) of oil-hydraulic drive of a milling machine. Formula (2.54) is used.

Given: $P_c = 24.0$ hp (17.7 kW), E = 70%. Required: Machining power at motor P_m Customary U.S. units of measure,

$$P_m = \frac{24.0}{0.7} = 34.3 \text{ hp}$$

Metric units of measure,

$$P_m = \frac{17.7}{0.7} = 25.3 \text{ kW}$$

Note: Power at the motor for above-mentioned cutting conditions given in "Milling Ferrous Materials" by *Krupp Widia Tool Engineering Publication*, is $P_m = 25.7$ kW (compare with $P_m = 25.3$ kW). However,

the Krupp Widia method of calculating power at the motor is more complex than that developed by the author.

Concluding Remarks on Milling

1. Kinematics of milling is expressed by numerous formulas for calculating the following parameters: cutting speed, spindle speed, feed per tooth, feed rate, and the number of inserts in the cut; feed rate adjustment in relation to lead angle of the cutter, and feed rate adjustment according to the relationship between width of cut and the cutter diameter. There are 28 formulas developed for these calculations in customary U.S. and metric units of measure.

2. Metal removal rate and machining time are major parameters that describe the productivity of milling. Machining times for the linear path of the cutter and for helical interpolation are calculated by different formulas. There are 15 formulas developed for these calculations in customary U.S. and metric units of measure.

3. Tangential cutting force, torque at the cutter, and machining power are major parameters that describe the dynamics of milling.

4. Analysis of metalcutting resulted in a new concept for calculating tangential cutting force when milling with high positive axial rake angles. This concept combines the ultimate tensile strength of work materials with the cross-sectional area of the uncut chip, the number of inserts in the cut, machinability factors, and tool wear factors.

5. To the author's best knowledge there is no other practical method that would provide accuracy of $\pm 15\%$ or better when tangential cutting force, torque, and power consumption are calculated in relationship with machining conditions (depth of cut, feed per tooth, cutting speed) and mechanical properties of work materials such as hardness and ultimate tensile strength. There are 11 formulas developed for these calculations in customary U.S. and metric units of measure.

TURNING

Turning is one of the most common metaleutting operations. In turning, a stationary cutting tool (usually a single-point) is fed into a rotating workpiece along its periphery or across its end or face for removing chips to get the required shape and dimensions of a part.

Insert identification systems, kinematics, productivity, and dynamics of turning are described in this chapter.

3.1 INSERT IDENTIFICATION SYSTEMS

Indexable inserts of various shapes, geometries, and dimensions are used in contemporary turning. Specifications for indexable inserts are given in American National Standard for Turning Tools (ANSI B212.12-1991) and in The International Standards Organization (ISO).

3.1.1 Designation System for Indexable Inserts

This system identifies indexable-type inserts by a special code of letters and numbers. Such a code consists

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of ten positions. Each position defines a characteristic of the insert in the following order: shape, clearance, tolerance class, type, size, thickness, cutting-point configuration, edge and surface preparations, hand, and manufacturer's option. Each position is described in accordance with ANSI B212.12-1991, p. 2.

The first position is a capital letter designated for the shape of an insert and its nose angle. The following sixteen letters are used (in alphabetical order):

A. parallelogram with 85°-nose angle,

B. parallelogram with 82°-nose angle,

C. diamond with 80°-nose angle,

D. diamond with 55°-nose angle,

E. diamond with 75°-nose angle,

H. hexagon, 120°-nose angle,

K. parallelogram with 55°-nose angle,

L. rectangle, 90°-nose angle,

M. diamond with 86°-nose angle,

O. octagon, 135°-nose angle,

P. pentagon, 108°-nose angle,

R. round,

S. square, 90°-nose angle,

T. triangle, 60°-nose angle,

V. diamond with 35°-nose angle,

W. trigon, 80°-nose angle.

The second position is a capital letter denoting clearance, or relief angle. The following nine letters are used (in order of increasing clearance):

N for 0°, A for 3°, B for 5°, C for 7°, P for 11°, D for 15°, E for 20°, F for 25°, and G for 30°.

The third position is a capital letter indicating the tolerance class on dimensions of gauge (column B), I.C. (diameter of inseribed circle, column A), and thickness of the insert (column T). For example, letter C indicates that tolerances are: ± 0.001 in. on both I.C. and thickness and ± 0.0005 in. on gauge (diagonal distance from I.C. to nose of an insert, a.k.a. "B" dimension); letter G indicates that tolerances are: ± 0.005 in. on thickness; letter H indicates that tolerances are: ± 0.005 in. on both gauge and I.C. and ± 0.005 in. on thickness; letter H indicates that tolerances are: ± 0.0005 in. on both gauge and I.C. and ± 0.001 in. on thickness. The M, N, and U tolerances depend on the size and shape of the insert. For example, letter M indicates that tolerances are: on gauge $\pm (0.002-0.010)$ in., on I.C. $\pm (0.002-0.004)$ in., and on thickness ± 0.005 in.

The fourth position is a capital letter indicating the insert type: with or without hole, shape of hole, with or without chip grooves. Letter A. for an insert with hole. B. insert with hole and one countersink $(70^\circ-90^\circ)$. C. with hole and two countersinks $(70^\circ-90^\circ)$. F. without hole with chip grooves on two rake faces. G. with hole and chip grooves on two rake faces. H. with hole, one countersink $(70^\circ-90^\circ)$ and chip groove on one rake face. J. with hole, two countersinks $(70^\circ-90^\circ)$ and chip groove on two rake faces. N. insert without hole. Q. with hole and two countersinks $(40^\circ-60^\circ)$. R. without hole with chip groove on one rake face. T. with hole, one countersink $(40^\circ-60^\circ)$ and chip groove on rake face. U. with hole, two countersinks $(40^\circ-60^\circ)$ and chip groove on rake face. U. with hole, two countersinks $(40^\circ-60^\circ)$ and chip groove on rake face. U. with hole, two countersinks $(40^\circ-60^\circ)$ and chip groove on rake face. U. with hole, two countersinks $(40^\circ-60^\circ)$ and chip groove on rake face. U. with hole, two countersinks $(40^\circ-60^\circ)$ and chip groove on rake face.

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on two rake faces. W. with hole and one countersink $(40^{\circ}-60^{\circ})$.

The fifth position shows the number of eights of an inch in the inscribed circle of the insert (I.C.) as follows:

 $1.2 = \frac{5}{32} \text{ inch I.C. } 1.5 = \frac{3}{16} \text{ inch I.C. } 1.8 = \frac{7}{32} \text{ inch I.C.}$ $2 = \frac{1}{4} \text{ inch I.C. } 2.5 = \frac{5}{16} \text{ inch I.C. } 3 = \frac{3}{8} \text{ inch I.C. } 4 = \frac{1}{2} \text{ inch I.C. } 5 = \frac{5}{8} \text{ inch I.C. } 6 = \frac{3}{4} \text{ inch I.C. } 8 = 1 \text{ inch I.C.}$ $10 = 1\frac{1}{4} \text{ inch I.C.}$

The sixth position shows the number of sixteenths of an inch in thickness of the insert as follows:

 $1.5 = \frac{3}{32} \text{ inch, } 2 = \frac{1}{8} \text{ inch, } 2.5 = \frac{5}{32} \text{ inch, } 3 = \frac{3}{16} \text{ inch, } 3.5 = \frac{7}{32} \text{ inch, } 4 = \frac{1}{4} \text{ inch, } 5 = \frac{5}{16} \text{ inch, } 6 = \frac{3}{8} \text{ inch, } 7 = \frac{7}{16} \text{ inch, } 8 = \frac{1}{2} \text{ inch. }$

The seventh position (cutting-point configuration) is a number designated to the nose radius of the insert.

0 = 0.004-inch radius, 0.5 = 0.008-inch radius. Each number from 1 to 8 indicates the number of sixtyfourths of an inch in the radius:

$$1 = \frac{1}{64} \text{ inch, } 2 = \frac{1}{32} \text{ inch, } 3 = \frac{3}{64} \text{ inch, } 4 = \frac{1}{16} \text{ inch,}$$

$$5 = \frac{5}{64} \text{ inch, } 6 = \frac{3}{32} \text{ inch, } 7 = \frac{7}{64} \text{ inch, } 8 = \frac{1}{8} \text{ inch.}$$

The eighth position is a capital letter designated to the cutting edge and surface preparations as follows:

A. Honed (0.0005 to less than 0.003 inch)

B. Honed (0.003 to less than 0.005 inch)

C. Honed (0.005 to less than 0.007 inch)

J. Polished to 4-microinch AA (Rake face only)

T. Chamfered—Manufacturer's standard (Negative land—Rake face only).

The ninth position is a capital letter indicating the hand of an insert: **R** is for right hand; **L** is for left hand; **N** is neutral.

The eighth and ninth positions shall only be used when required.

The tenth position is Manufacturer's option and shall only be used if desired by the manufacturer.

1st position, C. diamond with 80°-nose angle,

2nd position, N. clearance is 0°;

- 3rd position, M. tolerances are: $\pm (0.002-0.010)$ inches on gauge (B-dimension), $\pm (0.002-0.004)$ inches on I.C., and ± 0.005 inch on thickness;
- 4th position, G. with hole and chip grooves on two rake faces;
- 5th position, 4. diameter of the insert inscribed circle (I.C.) is $\frac{1}{2}$ inch;
- 6th position, **3.** thickness of the insert is $\frac{3}{16}$ inch;
- 7th position, **2**. nose radius of the insert is $\frac{1}{32}$ inch;
- 8th position, A. honed (0.0005 to less than 0.003 inch);
- 9th position, R. right hand.

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3.1.2 ISO Identification System

The ISO Standard provides a specification for indexable inserts in the metric system. The first four positions of the ISO Identification System are identical to that in the ANSI Standard. The third position indicates tolerances on B-dimension, I.C. and thickness of the insert in millimeters (mm).

The fifth position is a number indicating the diameter of the insert inscribed circle in mm, for example: 6,00; 6,35; 8,00; 10,00; 12,00; 12,70; 16,00; 20,00; 25,00; 32,00.

The sixth position is a code of letters and numbers (shown in bold) indicating the thickness of the inserts in mm as follows:

02 = 2,38 mm, 03 = 3,18 mm, T3 = 3,97 mm, 04 = 4,76 mm, 05 = 5,56 mm, 06 = 6,35 mm; 07 = 7,94 mm, 09 - 9,52 mm, 11 = 11,11 mm, and 12 = 12,70 mm.

The seventh position is a numerical code (shown in bold) indicating the size of the nose radius of the insert in mm as follows:

01 = 0,1 mm, 02 = 0,2 mm, 04 = 0,4 mm, 08 = 0,8 mm,

12=1,2 mm, 16=1,6 mm, 20=2,0 mm, 24=2,4 mm,

28 = 2,8 mm, 32 = 3,2 mm.

The eighth position is similar to that in ANSI B212.12-1991. The hone size is shown in mm and polished surface is shown in micrometers (μ m) as follows:

A. Honed (0,013 to less than 0,076 mm)

B. Honed (0,076 to less than 0,127 mm)

C. Honed (0,127 to less than 0,178 mm)

J. Polished to 0,10-micrometers $(0,10 \,\mu\text{m})$ AA (Rake face only)

T. is the same as in ANSI B212.12-1991.

The ninth and The tenth positions are similar to those described in ANSI B212.12-1991.

Comparison between ANSI and ISO Identification Systems is shown below.

Table 3.1. Identification of Indexable Insert (Example)

ANSI	С	Ν	Μ	G	4	3	2	А	R	_
ISO	С	Ν	Μ	G	12	04	08	А	R	_
Position	1	2	3	4	5	6	7	8	9	10

3.2 KINEMATICS AND PRODUCTIVITY

Kinematics and productivity are described by formulas showing relationships between various parameters, which are required to setup turning operations. Kinematics and productivity include:

- 1. Cutting speed
- 2. Spindle speed
- 3. Feed rate and feed speed
- 4. Depth of cut
- 5. Metal removal rate
- 6. Machining time when cutting along the workpiece centerline
- 7. Machining time when facing, cutting off, and deep grooving.

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3.2.1 Cutting Speed

Cutting speed is one of the most commonly used programmable parameters for CNC lathes. The relationship between cutting speed V_c (dependent variable), machine tool spindle speed *n* (independent variable), and the workpiece diameter *D* (independent variable) is expressed by formulas,

Customary U.S. units of measure:

$$V_{\rm c} = \frac{\pi Dn}{12} \text{ (sfm or ft/min)}$$
(3.1)

Where D is in inches, n is in rpm or rev/min, 12 is a factor converting inches into feet.

Metric units of measure:

$$V_{\rm c} = \frac{\pi D n}{1000}$$
 (m/min) (3.2)

Where D is in millimeters (mm) and 1000 is a factor converting mm into meters (m).

For example, D = 3.0 in. (76.2 mm) and n = 670 rpm (rev/min). What is V_0 ?

From formula (3.1): The cutting speed

$$V_{\rm c} = \frac{\pi \cdot 3 \cdot 670}{12} = 526 \, {\rm sfm},$$

From formula (3.2): The cutting speed

$$V_c = \frac{\pi \cdot 76.2 \cdot 670}{1000} = 160.3 \text{ m/min.}$$

3.2.2 Spindle Speed

If the cutting speed V_c (sfm or m/min) and the workpiece diameter D (in. or mm) are independent variables, then the formula for spindle speed n (rpm or rev/min) can be derived from formulas (3.1) and (3.2),

Customary U.S. units of measure:

$$n = \frac{12V_{\rm c}}{\pi D} \text{ (rpm or rev/min)}$$
(3.3)

For example, $V_c = 526$ sfm and D = 3 in. What is the spindle speed n?

From formula (3.3):

$$n = \frac{12 \cdot 526}{\pi \cdot 3} = 670$$
 rpm.

Metric units of measure:

$$n = \frac{1000V_{\rm c}}{\pi D} \text{ (rpm or rev/min)}$$
(3.4)

For example, $V_c = 160.3$ m/min and D = 76.2 mm. What is the spindle speed n?

From formula (3.4):

$$n = \frac{1000 \cdot 160.3}{\pi \cdot 76.2} = 670$$
 rpm.

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3.2.3 Feed Rate and Feed Speed

The feed rate is the movement of the cutting tool in relation to the revolving workpiece. Feed is a quantity expressed in a distance traveled by the cutting tool in one revolution of the workpiece. Feed rate is measured in in/rev or ipr (customary U.S. units of measure) and in mm/rev (metric units of measure). Feed rate is one of the programmable cutting parameters.

The feed speed is the speed at which the machine tool moves the cutting tool along or across the workpiece centerline, and is measured in in/min or ipm (customary U.S. units of measure) and in mm/min (metric units of measure). The feed speed is calculated by the formula:

$$V_f = f \cdot n$$
, ipm and mm/min (3.5)

where f is a feed rate (ipr or mm/rev) and n is a spindle speed (rpm)

For example, f = 0.015 ipr (0.38 mm/rev) and n = 600 rpm.

From formula (3.5):

- $V_f = 0.015 \cdot 600 = 9.0$ ipm (customary U.S. units of measure).
- $V_f = 0.38 \cdot 600 = 228$ mm/min = 22,8 cm/min (metric units of measure).

3.2.4 Depth of Cut

Depth of cut (Fig. 8) is the difference between the uncut and the cut surface of the workpiece and is a programmable parameter, as well as the feed rate. Depth of



Fig. 8. Depth of cut when turning

cut is measured perpendicular to the feed direction of the cutting tool. The relationship between the uncut workpiece diameter (D), the cut workpiece diameter (D_c) , and the depth of cut (d) is expressed by the formula:

$$d = \frac{D - D_{\rm c}}{2} \tag{3.6}$$

3.2.5 Metal Removal Rate

The metal removal rate (MRR) indicates the volume of material removed in a period of time and is also a measure of productivity expressed in in³/min (customary U.S. units of measure) and in cm³/min (metric units of measure). The metal removal rate (symbol Q) is a product of the cutting speed (V_c), depth of cut (d), and feed rate (f). MRR is calculated from:

Customary U.S. units of measure,

$$Q = 12V_c df (V_c \text{ in ft/min}, d \text{ and } f \text{ in inches})$$
 (3.7)

Turning

Metric units of measure,

$$Q = V_c df (V_c \text{ in m/min}, d \text{ and } f \text{ in mm})$$
 (3.8)

The following examples of MRR calculations are: (1) in customary U.S. units of measure and (2) in metric units of measure.

 $V_{\rm c}$ = 600 ft/min (182.9 m/min), d = 0.2 in. (5.08 mm), f = 0.02 ipr (0.508 mm/rev)

(1)
$$Q = 12 \cdot 600 \cdot 0.2 \cdot 0.02 = 28.8 \text{ in}^3/\text{min}$$

(2) $Q = 182.9 \cdot 5.08 \cdot 0.508 = 472 \text{ cm}^3/\text{min}$

3.2.6 Machining Time when Cutting Parallel to the Workpiece Centerline

The time required to cut a given length of workpiece is found by formula (3.9) in which t =time in minutes, L = length of cut in inches or millimeters, f = feed rate in inches per revolution or millimeters per revolution, and n = lathe spindle speed in revolutions per minute:

$$t = \frac{L}{fn} \tag{3.9}$$

Example of calculation is shown for the following cutting data:

L = 8 in. (203.2 mm), f = 0.02 ipr (0.508 mm/rev), n = 500 rpm.

Customary U.S. units of measure:

$$t = \frac{8}{0.02 \cdot 500} = 0.8 \text{ min},$$

Metric units of measure:

$$t = \frac{203.2}{0.508 \cdot 500} = 0.8 \text{ min},$$

3.2.7 Machining Time when Facing, Cutoff, and Deep Grooving

Facing, cutting off, and deep grooving operations deal with two types of machining: cutting to the center of the workpiece (solid cross section, Fig. 9a) and cutting to a shoulder (tubular cross section, Fig. 9b).



Fig. 9. Cross section of the workpiece, solid (a) and tubular (b) f-feed, V_c -cutting speed, n-spindle speed
Machining time when facing, cutting off, and deep grooving can be calculated by formula (3.9) only if all these operations are performed at a constant spindle speed. However, a better surface finish would be obtained if these operations were performed at a constant cutting speed. Normally, facing, cutting off, and deep grooving are performed at a constant cutting speed, and the machining time formulas have always been required to set higher productivity. The author has developed such formulas and made them available to the metalcutting industry (Ref. 13).

These formulas satisfy the following conditions:

- 1. Cutting speed remains constant for the entire facing, cutting off, or deep grooving operations.
- 2. The workpiece diameter is continuously decreasing by the value of feed per revolution.
- 3. The spindle speed rises continuously at a rate inversely proportional to the workpiece diameter until the rpm (rev/min) reaches its maximum value, since then remains constant.
- 4. Machining time is directly proportional to the unwound spiral of the cutting length and inversely proportional to the cutting speed.
- 5. The average cutting length of circumference per revolution (L_c) is found from the intermediate equations

Solid cross section (Fig. 9a),

$$L_{\rm c} = 2\pi \left[\frac{R_o + R_{\rm min}}{2} \right] \tag{3.10}$$

Tubular cross section (Fig. 9b),

$$L_c = 2\pi \left[\frac{R_o + R_i}{2}\right] \tag{3.11}$$

where: $R_o =$ radius at starting point of cut (workpiece outside radius),

- R_{\min} = workpiece minimal radius at which the lathe spindle reaches its maximum number of revolutions per minute,
- R_i = radius at end point of cut (workpiece inside radius).

Step-by-step derivation of the machining time formula for solid cross section

1. Derivation of the machining time formula is based on the general equation widely used in Mechanics:

$$t = \frac{L}{V} \tag{3.12}$$

where t = time required to cover distance *L* at the speed of *V*.

2. Total cutting length (L) equals the length of circumference per revolution (L_c) times the number of revolutions (N) required to cut the workpiece from initial radius R_o (starting point) to radius R_{\min} (radius at which the spindle reaches its maximum rpm) at given feed (f). Therefore,

$$L = L_c N \tag{3.13}$$

where

$$N = \frac{R_o - R_{\min}}{f} \tag{3.14}$$

Replacing L_c in formula (3.13) by its value from formula (3.10) and N in formula (3.13) by its value from formula (3.14) produces:

$$L = 2\pi \left[\frac{R_o + R_{\min}}{2} \right] \cdot \left[\frac{R_o - R_{\min}}{f} \right]$$
$$L = \frac{\pi (R_o^2 - R_{\min}^2)}{f}$$
(3.15)

3. Time (t_1) required for cutting the workpiece from initial radius R_o (starting point) to radius R_{\min} (radius at which the spindle reaches its maximum rpm) is calculated by formula (3.12) in which speed V is replaced by cutting speed V_c and L is replaced by its value from formula (3.15):

$$t_1 = \frac{L}{V_c} = \frac{\pi (R_o^2 - R_{\min}^2)}{V_c f}$$
(3.16)

4. Time (t_2) required for cutting the workpiece from radius R_{\min} (radius at which the spindle reaches its maximum rpm) to the centerline of the workpiece (at this point facing or cutting off is completed) is calculated

by formula (3.9) in which $L = R_{\min}$ and $n = n_{\max}$:

$$t_2 = \frac{R_{\min}}{n_{\max}f} \tag{3.17}$$

5. Total machining time (t) required when facing or cutting off the workpiece with a solid cross section (Fig. 9a) is:

$$t = t_1 + t_2 \tag{3.18}$$

6. Replacing t_1 and t_2 in formula (3.18) by their values from formula (3.16) and formula (3.17) respectively gives the final machining time formulas as follows,

Customary U.S. units of measure:

$$t = \frac{\pi (R_o^2 - R_{\min}^2)}{12V_c f} + \frac{R_{\min}}{n_{\max} f}$$
(min) (3.19)

Where: 12 is a factor converting feet into inches, because the cutting speed is expressed in feet per minute and the radii of the workpiece are given in inches;

 $R_{\rm min}$ is the variable radius of the workpiece depending on the cutting speed and the maximum spindle speed of the lathe. Since the relationship between cutting speed, spindle speed, and workpiece diameter is expressed by formula (3.1), this formula can be used to determine $R_{\rm min}$ (with these substitutions: $D=2R_{\rm min}$ and $n=n_{\rm max}$)

$$V_{\rm c} = \frac{\pi D n}{12} = \frac{\pi 2 R_{\rm min} n_{\rm max}}{12}, \quad V_{\rm c} = \frac{\pi R_{\rm min} n_{\rm max}}{6}$$
(3.20)

Solving equation (3.20) for R_{\min} we obtain,

$$R_{\min} = \frac{6V_c}{\pi n_{\max}}$$
(3.21)

Metric units of measure:

$$t = \frac{\pi (R_o^2 - R_{\min}^2)}{1000V_c f} + \frac{R_{\min}}{n_{\max} f}$$
(min) (3.22)

Where: 1000 is a factor converting meters into millimeters, because the cutting speed is expressed in meters per minute and the radii of a workpiece are given in millimeters;

 R_{\min} is the variable radius of the workpiece depending on the cutting speed and the maximum spindle speed of the lathe. The relationship between cutting speed, spindle speed, and workpiece diameter is expressed by formula (3.2), this formula can be used to determine R_{\min} (with these substitutions: $D = 2R_{\min}$ and $n = n_{\max}$)

$$V_{\rm c} = \frac{\pi D n}{1000} = \frac{\pi 2 R_{\rm min} n_{\rm max}}{1000} = \frac{\pi R_{\rm min} n_{\rm max}}{500}$$
(3.23)

Solving equation (3.23) for R_{\min} we obtain,

$$R_{\min} = \frac{500 V_c}{\pi n_{\max}} \tag{3.24}$$

Example of calculating machining time for facing and cutting off (solid cross section)

 $\begin{array}{ll} Given: & R_o = 1.5 \text{ in. (38.1 mm)}, \, V_c = 620 \,\, \mathrm{sfm} \\ & (189 \,\, \mathrm{m/min}), \,\, f = 0.005 \,\, \mathrm{ipr} \,\, (0.127 \,\,\, \mathrm{mm/min}), \, n_{\mathrm{max}} = 3500 \,\, \mathrm{rpm} \,\, (\mathrm{rev/min}). \end{array}$

Required: (1) minimal radius R_{\min} and (2) machining time t.

Customary U.S. units of measure:

(1) Formula (3.21) is used to determine R_{\min}

$$R_{\min} = \frac{6V_c}{\pi n_{\max}} = \frac{6 \cdot 620}{\pi \cdot 3500} = 0.338$$
 in.

(2) Formula (3.19) is used to calculate t

$$t = \frac{\pi (R_o^2 - R_{\min}^2)}{12V_c f} + \frac{R_{\min}}{n_{\max} f}$$

$$t = \frac{\pi (1.5^2 - 0.338^2)}{12 \cdot 620 \cdot 0.005} + \frac{0.338}{3500 \cdot 0.005}$$

$$t = \frac{\pi \cdot 2.136}{37.2} + \frac{0.338}{17.5} = \frac{6.71}{37.2} + 0.019$$

$$t = 0.180 + 0.019 = 0.199 \text{ min} = 11.9 \text{ sec.}$$

Metric units of measure:

(1) Formula (3.24) is used to determine R_{\min}

$$R_{\min} = \frac{500V_c}{\pi n_{\max}} = \frac{500 \cdot 189}{\pi \cdot 3500} = 8.59 \text{ mm.}$$

(2) Formula (3.22) is used to calculate t

$$t = \frac{\pi (R_o^2 - R_{\min}^2)}{1000V_c f} + \frac{R_{\min}}{n_{\max} f}$$

$$t = \frac{\pi \cdot (38.1^2 - 8.59^2)}{1000 \cdot 189 \cdot 0.127} + \frac{8.59}{3500 \cdot 0.127}$$

$$t = \frac{\pi \cdot 1377.8}{24003} + \frac{8.59}{444.5} = \frac{4328.5}{24003} + 0.019$$

$$t = 0.180 + 0.019 = 0.199 \text{ min} = 11.9 \text{ sec}$$

Step-by-step derivation of the machining time formula for tubular cross section

1. Derivation of the machining time formula is based on equation (3.12), the same as that used for a solid cross section of the workpiece:

2. Total cutting length (L) equals the length of circumference per revolution (L_c) times the number of revolutions (N) required to cut the workpiece from initial radius R_o (starting point) to radius R_i (radius at end point of cut, or workpiece inside radius). Therefore,

$$L = L_c \cdot N \tag{3.13}$$

where

$$N = \frac{R_o - R_i}{f} \tag{3.25}$$

Replacing L_c in formula (3.13) by its value from formula (3.11) and N in formula (3.13) by its value from formula

(3.25) produces:

$$L = 2\pi \left[\frac{R_o + R_i}{2} \right] \cdot \left[\frac{R_o - R_i}{f} \right]$$
$$L = \frac{\pi (R_o^2 - R_i^2)}{f}$$
(3.26)

3. Time (t) required to cut workpiece (Fig. 9b) from initial radius R_o (starting point) to radius R_i (radius at end point of cut, or workpiece inside radius) is found by formula (3.12) in which speed V is replaced by cutting speed V_c and L is replaced by its value from formula (3.26):

$$t = \frac{L}{V_{\rm c}} = \frac{\pi (R_o^2 - R_i^2)}{V_{\rm c} f}$$
(3.27)

Formula (3.27) is for calculating machining time when facing and cutting off a tubular cross section as well as for deep grooving in both solid and tubular cross sections.

Customary U.S. units of measure:

$$t = \frac{\pi (R_o^2 - R_i^2)}{12V_c f}$$
(3.28)

Where: 12 is a factor converting feet into inches, because the cutting speed is expressed in feet per minute and the radii of the workpiece are given in inches.

Metric units of measure:

$$t = \frac{\pi (R_o^2 - R_i^2)}{1000 V_o f}$$
(3.29)

Where: 1000 is a factor converting meters into millimeters, because the cutting speed is expressed in meters per minute and the radii of a workpiece are given in millimeters.

Formulas (3.28) and (3.29) are used when $R_i > R_{\min}$ (this is the most common condition). When $R_i < R_{\min}$, formulas (3.19), (3.21), (3.22), and (3.24) are recommended. The only change required is to substitute R_{\min} for $(R_{\min} - R_i)$.

Example of calculating machining time for facing, cutting off (tubular cross section) and deep grooving (solid and tubular cross sections)

> Given: $R_o = 2.5$ in. (63.5 mm), $R_i = 0.57$ in. (14.5 mm), $V_c = 620$ sfm (189 m/min) f = 0.005 ipr (0.127 mm/rev).

Required: Machining time *t*

Customary U.S. units of measure, formula (3.28) is used to calculate *t*:

$$t = \frac{\pi (R_o^2 - R_i^2)}{12V_c f} = \frac{\pi (2.5^2 - 0.57^2)}{12.620 \cdot 0.005}$$

$$t = \frac{\pi \cdot 5.9251}{37.2} = \frac{18.61}{37.2} = 0.50 \text{ min} = 30 \text{ sec.}$$

Metric units of measure, formula (3.29) is used to calculate *t*:

$$t = \frac{\pi (R_o^2 - R_i^2)}{1000V_c f} = \frac{\pi (63.5^2 - 14.5^2)}{1000 \cdot 189 \cdot 0.127}$$
$$t = \frac{\pi \cdot 3882}{1000 \cdot 189 \cdot 0.127} = \frac{12007}{24003} = 0.50 \text{ min} = 30 \text{ sec.}$$

3.3 DYNAMICS

Dynamics is described by formulas showing how the cutting forces, torque, and machining power depend on mechanical properties of work materials and the cutting conditions. Parameters of dynamics are:

- 1. Tangential force
- 2. Power constants
- 3. Feed and radial forces
- 4. Torque at the cutting tool
- 5. Machining power at the cutting tool
- 6. Machining power at the motor.

3.3.1 Tangential Force

Engagement of the cutting edge with the rotating workpiece at a given depth of cut and feed rate generates a cutting force. This force undercuts a layer of material and separates it from a workpiece in the form of chips. Such a cutting force (F) combines tangential (F_t), feed (F_f), and radial (F_r) forces, which could be



Fig. 10. Components of the cutting force F F_t -tangential force, F_f -feed force, F_r -radial force

measured by a three-component dynamometer (Fig. 10). Of the three cutting force components, tangential, feed, and radial, the tangential force is the greatest.

Tangential force generates torque at the spindle and accounts for more than 95% of the machining power. Therefore, it is very important to know how the tangential force can be calculated. The following is a stepby-step derivation of the formula to calculate the tangential force.

1. The relationship between tangential force (F_t) , cutting speed (V_c) , and power at the cutting tool (P_c) can be expressed in customary U.S. units of measure as:

$$P_{\rm c} = \frac{F_t V_{\rm c}}{33,000} \ (\rm hp) \tag{3.30}$$

The same relationship expressed in metric units of measure is:

$$P_{\rm c} = \frac{F_t V_{\rm c}}{60000} \ (\rm kW) \tag{3.31}$$

where F_t is the tangential force measured by dynamometer.

2. The relationship between metal removal rate (*Q*), power constant (K_p), and power at the cutting tool (P_c) is expressed by a formula currently used in the metal-cutting industry:

$$P_c = QK_p \text{ (hp or kW)} \tag{3.32}$$

3. Replacing Q in formula (3.32) by its value from formula (3.7) gives P_c in customary U.S. units of measure:

$$P_{\rm c} = 12V_{\rm c}dfK_{\rm p} \tag{3.33}$$

4. Replacing Q in formula (3.32) by its value from formula (3.8) gives P_o in *metric units of measure*:

$$P_c = V_c df K_p \tag{3.34}$$

5. By comparing formula (3.30) with formula (3.33) it becomes obvious that:

$$\frac{F_t V_c}{33,000} = 12V_c df K_p \tag{3.35}$$

After reducing formula (3.35) by V_c , we obtain:

$$\frac{F_t}{33,000} = 12 df K_p$$
 (3.36)

6. By solving equation (3.36) in respect to tangential force F_r , we get:

$$F_t = 33,000 \cdot 12 df K_p = 396,000 df K_p$$
 (3.37)

Formula (3.37) is used to calculate tangential force in *customary U.S. units of measure*.

7. By comparing formula (3.31) with formula (3.34) it becomes obvious that:

$$\frac{F_t V_c}{60000} = V_c df K_p$$
(3.38)

After reducing formula (3.38) by V_c , we obtain:

$$\frac{F_t}{60000} = df K_p$$
 (3.39)

8. By solving equation (3.39) in respect to tangential force F_r , we get:

$$F_t = 60000 df K_p$$
 (3.40)

Formula (3.40) is used for calculating the tangential force in *metric units of measure*.

3.3.2 Power Constant

As can be seen from formulas (3.37) and (3.40), tangential force is directly proportional to depth of cut (*d*), feed rate (*f*), and power constant (K_p). Depth of cut and feed rate depend on type of cutting (rough, semifinish, or finish); type of work materials and their hardness;

cutting tool geometry and cutting tool material; and machine tool power availability. Usually, machine shops select depth of cuts, feeds, and speeds according to recommendations from the cutting tool manufacturers' publications. But machine tools place limits on the cutting conditions. The magnitude of the tangential force is very important, because this force and the cutting speed determine the required machining power. In order to calculate the tangential force, knowledge of power constant values becomes necessary.

The power constant is also called the unit power, or specific power consumption. In customary U.S. units of measure the power constant is equal to the horsepower required to cut a material at a rate of one cubic inch per minute (hp/in³/min). In metric units of measure the power constant is equal to the power required to cut a material at a rate of one cubic centimeter per minute (kW/cm³/min). The power constant in customary U.S. units of measure relates to the power constant in metric units of measure as follows:

$$\frac{1 \text{ hp/in}^3/\text{min}}{1 \text{ min}} = \frac{0.7457 \text{ kW}}{(2.54 \text{ cm})^3/\text{min}}$$
$$= \frac{0.7457}{16.38706} = 0.0455 \text{ kW/cm}^3/\text{min}$$

For many years it was considered to be good practice to use the value of the power constant $K_p = 1.0$ hp/ in³/min to estimate power consumption when cutting steels (the author believes that many machine shops still use the same power constant value). Such "good

practice" leads to significant inaccuracy in estimating power consumption, because the power constants of carbon and alloy steels in annealed and normalized conditions are from 0.45 to 0.70 hp/in³/min, or from 0.0205 to 0.0318 kW/cm³/min. The same steels after heat treatment will have $K_p = (0.75 - 1.20)$ hp/in³/min (customary U.S. units of measure), or $K_p = (0.0341 - 0.0546)$ kW/cm³/min (metric units of measure).

The author's experimental study on the power constant values for most common work materials (Ref. 14, pp. 99–102, Ref. 15) revealed that feed rate, depth of cut, and hardness of the work material significantly affect the power constant values. The cutting speed also affects power constants, but to a lesser degree. Increasing each of these parameters decreases the power constant value.

Machinery's Handbook (Ref. 2, p. 1047) indicates that the hardness and microstructure of the work material, the feed rate, the rake angle of the cutting tool, and the cutting edge condition (sharp or dull) affect the value of the power constant. The value of the power constant is essentially unaffected by the cutting speed, the depth of cut, and the cutting tool material. The lack of effect of the depth of cut on the power constant value contradicts the results of the above-mentioned author's experimental study. The higher the feed rate and the depth of cut, the greater is the volume of metal removal rate. So, the depth of cut also affects the power constant, but to a lesser degree than the feed rate. Any significant increase in the cutting speed (40 to 50%) decreases the value of the power constant and increases

the rate of cutting edge wear. Cutting edge wear increases the value of the power constant. Because of this dual effect, the cutting speed is not considered as an essential contributor to the value of the power constant.

The author compared the power constant values from various publications, such as "Machining Data Handbook" (3rd Edition, Volume 2, 1980, Metcut Research Associates Inc., Cincinnati Ohio) and catalogs of major cutting tool manufacturers, and has found that the most accurate and comprehensive data on the power constants are in "Machinery's Handbook" (Ref. 2, pp. 1046–1049, 1052). The following tables contain the power constant values adapted from "Machinery's Handbook" (the author changed the metric values of K_p from kW/cm³/sec into kW/cm³/min).

As can be seen from Tables 3.2 to 3.8, the power constant values rise with increases in the work material hardness. The material microstructure also affects the power constant values. For example, free-machining medium-carbon steels listed in Table 3.2 and mediumcarbon alloy steels listed in Table 3.4 at the same hardness range of (240–260) HB have these power constants: 0.62 hp/in³/min (0.0282 kW/cm³/min) and 0.80 hp/in³/ min (0.0364 kW/cm³/min) respectively.

Other cutting parameters that affect power constants are feed rate, cutting edge condition, and cutting tool rake angle.

The relationship between feed rate values and feed factors is given in Table 3.9 (Ref. 2, p.1047, Table 25).

This following example shows how the power constant value should be adjusted in accordance with change in

Table 3.2. Power Constants, (AISI resul	$K_{p'}^{\prime}$ for Carbon Ste furized, free-mach	els, Using Sharp Cuttin iining grades)	g Tools
AISI Grades	Brinell Hardness	K _p , Customary U.S. Units of Measure, hp/in ³ /min	Kr Metric Units of Measure, kW/cm³/min
1108, 1109, 1110, 1115, 1116, 1117,	100–120	0.41	0.0187
1118, 1119, 1120, 1125, 1126, 1132.	120–140	0.42	0.0191
	140–160	0.44	0.0200
	160–180	0.48	0.0218
	180–200	0.50	0.0228
1137, 1138, 1139, 1140, 1141, 1144,	180-200	0.51	0.0232
1145, 1146, 1148, 1151.	200-220	0.55	0.0250
	220-240	0.57	0.0259
	240–260	0.62	0.0282

	,	-	
AISI Grades*	Brinell Hardness	K _p , Customary U.S. Units of Measure, hp/in ³ /min	<i>K_p</i> , Metric Units of Measure, kW/cm³/min
1005, 1006, 1008, 1010, 1012.	80-100	0.63	0.0287
1015, 1016, 1017, 1018, 1019,	100–120	0.66	0.0300
1020, 1021, 1022, 1023, 1025,	120–140	0.69	0.0314
1026, 1029, 1030, 1034, 1035,	140–160	0.74	0.0337
1037, 1038, 1039, 1040, 1042,	160–180	0.78	0.0355
1043, 1044, 1045, 1046, 1049,	180–200	0.82	0.0373
1050, 1053, 1055, 1059, 1060,	200–220	0.85	0.0387
1064, 1065, 1069, 1070, 1074,	220–240	0.89	0.0405
1078, 1080, 1084, 1085, 1086,	240–260	0.92	0.0419
1090, 1095.	260–280	0.95	0.0432
	280–300	1.00	0.0455
	300–320	1.03	0.0469
	320–340	1.06	0.0482
	340–360	1.14	0.0519

Table 3.3. Power Constants, K_{μ} for Carbon Steels, Using Sharp Cutting Tools (AISI nonresulfurized grades containing up to 1.0% manganese) *Carbon steel grades are not listed in the original table (Ref. 2, Table 30, p. 1052), so the author has identified these grades to the best of his knowledge and in accordance with AISI numerical classification (Ref. 4, pp. 3–5).

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AISI Grades	Brinell Hardness	K _p , Customary U.S. Units of Measure, hp/in³/min	<i>K_p</i> , Metric units of Measure, kW/cm ³ /min
Medium-carbon allov	140–160	0.56	0.0255
steels. Carburizing	160-180	0.59	0.0268
alloy steels:	180-200	0.62	0.0282
4130, 4320, 4615, 4620, 4626,	200–220	0.65	0.0296
5120, 8615, 8617, 8620, 8622,	220-240	0.70	0.0318
8625, 8630, 8720.	240-260	0.74	0.0337
	260-280	0.77	0.0350
	280300	0.80	0.0364
	300–320	0.83	0.0378
	320-340	0.89	0.0405
Carburizing alloy steels,	140–160	0.62	0.0282
Medium-carbon alloy steels:	160–180	0.65	0.0296
	180-200	0.69	0.0314
4023, 4024, 4027, 4028, 4032,	200-220	0.72	0.0328
4037, 4042, 4047, 4137, 4140,	220-240	0.76	0.0346
4142, 4145, 4147, 4150, 4340,	240-260	0.80	0.0364
4640, 4815, 4817, 4820, 5130,	260-280	0.84	0.0382
5132, 5135, 5140, 5145, 5150,	280300	0.87	0.0396
6118, 6150, 8637, 8640, 8642,	300-320	0.91	0.0414
8645, 8650, 8740.	320–340	0.96	0.0437
	340–360	1.00	0.0455
Manganese alloy steels:	160–180	0.79	0.0359
1330, 1335, 1340.	180-200	0.83	0.0378
Chromium high-carbon	200–220	0.87	0.0396
alloy steel E52100.	220-240	0.91	0.0414
	240–260	0.95	0.0432
	260–280	1.00	0.0455

Table 3.4. Power Constants, $K_{\rm p}$ for Alloy Steels, Using Sharp Cutting Tools

Cast Irons	Brinell Hardness	K _p , Customary U.S. Units of Measure, hp/in ³ /min	<i>K_p</i> , Metric Units of Measure, kW/cm³/min
Gray Cast Irons	100–120	0.28	0.0127
	120–140	0.35	0.0159
	140–160	0.38	0.0173
	160–180	0.52	0.0237
	180-200	0.60	0.0273
	200-220	0.71	0.0323
	220-240	0.91	0.0414
Malleable Cast Irons	150–175	0.42	0.0191
	175-200	0.57	0.0259
	200–250	0.82	0.0373
	250–300	1.18	0.0537

Table 3.5. Power Constants, K_{p} , for Cast Irons, Using Sharp Cutting Tools

Table 3.6. Power Constants, K_{ρ} , for Stainless and Tool Steels,Using Sharp Cutting Tools

Stainless and Tool Steels	Brinell Hardness	K _p , Customary U.S. Units of Measure, hp/in³/min	<i>K_p</i> , Metric Units of Measure, kW/cm³/min
Stainless steels	150–175	0.60	0.0273
	175–200	0.72	0.0328
	200–250	0.88	0.0400
Tool steels	175–200	0.75	0.0341
	200–250	0.88	0.0400
	250–300	0.98	0.0446
	300–350	1.20	0.0546
	350–400	1.30	0.0592

Table 3.7. Power (Constants, K _p , for l	High-Temperature Alloys, Usi	ng Sharp Cutting Tools
High-Temperature Alloys	Brinell Hardness	K _p , Customary U.S. Units of Measure, hp/in ³ /min	κ _p , Metric Units ⁶ p, kW/cm³/min
A286 (AISI Type 660)	165	0.82	0.0373
A286 (AISI Type 660)	285	0.93	0.0423
Chromoloy	200	0.78	0.0355
Chromoloy	310	1.18	0.0537
Inconel 700	330	1.12	0.0510
Inconel 702	230	1.10	0.0500
Hastelloy B	230	1.10	0.0500
M-252	230	1.10	0.0500
M-252	310	1.20	0.0546
Udimet 500	375	1.10	0.0500

Nonferrous Alloys	κ, Customary U.S. Units of Measure, hp/in³/min	κ _/ Metric Units of Measure, kW/cm³/min
Monel Alloys Titanium alloy (Ti-6Al-4V)	1.00 0.65	0.0455 0.0296
Copper Alloys—Brass:	000	
Hard Medium	0.50	0.0228
Soft	0.25	0.0114
Leaded	0.30	0.0136
Copper Alloys—Bronze:		
Hard	0.91	0.0414
Medium	0.50	0.0228
Copper (pure)	0.91	0.0414
Aluminum Alloys:		
Cast	0.25	0.0114
Rolled (hard)	0.33	0.0150
Magnesium Alloys	0.10	0.0045

Table 3.8. Power Constants, $K_{
m a'}$ for Nonferrous Alloys, Using Sharp Cutting Tools

Chapter 3

Customar	y U.S. Unit	ts of Measur	e	Me	etric Units	of Measure	
Feed Rate,		Feed Rate,		Feed Rate,		Feed Rate,	
ipr	С	ipr	С	mm/rev	С	mm/rev	С
0.001	1.60	0.014	0.97	0.02	1.70	0.35	0.97
0.002	1.40	0.015	0.96	0.05	1.40	0.38	0.95
0.003	1.30	0.016	0.94	0.07	1.30	0.40	0.94
0.004	1.25	0.018	0.92	0.10	1.25	0.45	0.92
0.005	1.19	0.020	06.0	0.12	1.20	0.50	0.00
0.006	1.15	0.022	0.88	0.15	1.15	0.55	0.88
0.007	1.11	0.025	0.86	0.18	1.11	09.0	0.87
0.008	1.08	0.028	0.84	0.20	1.08	0.70	0.84
0.009	1.06	0.030	0.83	0.22	1.06	0.75	0.83
0.010	1.04	0.032	0.82	0.25	1.04	0.80	0.82
0.011	1.02	0.035	0.80	0.28	1.01	0.90	0.80
0.012	1.00	0.040	0.78	0.30	1.00	1.00	0.78
0.013	0.98	0.060	0.72	0.33	0.98	1.50	0.72

Table 3.9. Feed Factors, C, for Power Constant Adjustment

Turning

the feed rate. The AISI 4340 alloy steel workpiece is to be turned at the feed rate of 0.016 ipr (0.4 mm/rev). Brinell hardness of the workpiece is 210 HB. According to Table 3.4, $K_p = 0.72$ hp/in^{3/}min (0.0328 kW/cm³/min). To adjust this power constant value for a given feed rate, an appropriate feed factor should be found from Table 3.9. Since C = 0.94, therefore, $K_p = 0.72 \cdot 0.94 = 0.68$ hp/in³/min or $K_p = 0.0328 \cdot 0.94 = 0.0308$ kW/cm³/min.

The edge of the cutting insert wears and changes from sharp condition to dull and the cutting insert is then indexed. A worn cutting edge generates higher cutting forces and requires more power to cut. Therefore, the wearing process between indexes of the insert is correlated to tool wear factors. The relationship between tool wear factors and type of operation is shown in Table 3.10 (Ref. 2, p. 1048, Table 26).

Tool wear factors presented in Table 3.10 should be used for adjusting power constants just before indexing a cutting insert. Immediately after indexing, when turning is performed by a sharp cutting edge, the tool wear factor $C_{sv} = 1.0$.

Table 3.10. Tool wear factors, G_w , for Power Constant Adjustmen	enτ
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Type of Operation	C _w
Finish turning (light cuts)	1.1
Normal rough and semifinish turning	1.3
Extra-heavy-duty rough turning*	1.6–2.0

* This category occurs only on operation where the tool is allowed to wear more than a normal amount before it is replaced, such as roll turning.

The effect of the rake angle on the power constant usually can be disregarded. Most of the data in the power constant tables are given for positive 14° rake angle inserts (Ref. 2, p. 1047).

3.3.3 Feed and Radial Forces

Normally, feed force (F_f) is the second highest after tangential force (F_t) , and radial force (F_r) is least in magnitude. It is a common practice to estimate feed and radial forces as follows: $F_f = 0.5 F_t$ and $F_r = 0.25 F_t$. Unfortunately, this method of feed and radial force estimation has acceptable accuracy only when applied to some grades of carbon steel in annealed and normalized conditions.

The author developed a new method of calculating feed and radial forces that can be applied to steels, cast irons, high-temperature alloys, and nonferrous alloys. To increase the accuracy in calculating feed and radial forces, the work materials are divided into the following groups, and the formulas are listed in Table 3.11.

- Group 1: All steels with Brinell hardness range of (80–280) HB
- Group 2: All steels (280–400) HB. Hard brass and hard bronze
- Group 3: Cray cast irons (100–240) HB. Medium brass and medium bronze
- Group 4: Ductile and malleable cast irons (150– 300) HB
- Group 5: Titanium alloys (275-340) HB
- Group 6: Nickel-base and cobalt-base hightemperature alloys (165–375) HB

Group 7: Aluminum alloys (100–150) HB. Copper alloys: soft and leaded brass.

Group of Work Materials	Feed Force Formula	Radial Force Formula
Group 1 Group 2 Group 3 Group 4 Group 5 Group 6 Group 7	$F_{f} = 0.460F_{t}$ $F_{f} = 0.683F_{t}$ $F_{f} = 0.486F_{t}$ $F_{f} = 0.532F_{t}$ $F_{f} = 0.675F_{t}$ $F_{f} = 0.609F_{t}$ $F_{f} = 0.429F_{t}$	$F_{r} = 0.221F_{t}$ $F_{r} = 0.382F_{t}$ $F_{r} = 0.269F_{t}$ $F_{r} = 0.242F_{t}$ $F_{r} = 0.411F_{t}$ $F_{r} = 0.373F_{t}$ $F_{r} = 0.211F_{t}$

Table 3.11. Formulas for Feed and Radial Forces versus Tangential Force

Example of Calculating Cutting Forces

- *Given*: A workpiece made of AISI 4140 alloy steel having hardness of 230 HB is to be machined at the depth of cut of 0.200 in. (5.08 mm) and the feed rate of 0.020 ipr (0.508 mm/rev).
 - Required: Calculation of tangential, feed, and radial forces.

Tangential force calculation

Customary U.S. units of measure Tangential force is calculated by formula (3.37):

$$F_t = 396,000 \, df K_p$$

Power constant (from Table 3.4):

$$K_p = 0.76 \text{ hp/in}^3/\text{min}$$

Feed factor (from Table 3.9):

$$C = 0.90$$

Adjusted power constant,

 $K_{pa} = K_p C = 0.76 \cdot 0.90 = 0.68 \text{ hp/in}^3/\text{min}$

Tangential force, sharp cutting tool,

 $F_t = 396,000 \cdot 0.2 \cdot 0.02 \cdot 0.68 = 1077.1$ lb

Metric units of measure

Tangential force is calculated by formula (3.40):

 $F_t = 60000 df K_p$

Power constant (from Table 3.4):

 $K_p = 0.0346 \text{ kW/cm}^3/\text{min}$

Feed factor (from Table 3.9):

C = 0.90

Adjusted power constant,

 $K_{pa} = K_p C = 0.0346 \cdot 0.90 = 0.0311 \text{ kW/cm}^3/\text{min}$

Tangential force, sharp cutting tool,

 $F_t = 60000 \cdot 5.08 \cdot 0.508 \cdot 0.0311 = 4815.5 \text{ N}$

 $\begin{aligned} & Feed \ force \ calculation \\ & Customary \ U.S. \ units \ of \ measure \\ & F_f = 0.460F_t \ (\text{Table 3.11, Group 1}). \ F_f = 0.46 \\ & 1077.1 = 495.5 \ \text{lb} \\ & \text{Metric units of measure} \\ & F_f = 0.460F_t \ (\text{Table 3.11, Group 1}). \ F_f = 0.46 \\ & 4815.5 = 2215.1 \ \text{N} \\ \hline & \text{Radial force calculation} \\ & \text{Customary U.S. units of measure} \\ & F_r = 0.221F_t \ (\text{Table 3.11, Group 1}). \ F_r = 0.221 \\ & 1077.1 = 238.0 \ \text{lb} \\ & \text{Metric units of measure} \\ & F_r = 0.221F_t \ (\text{Table 3.11, Group 1}). \ F_r = 0.221 \\ & 4815.5 = 1064.2 \ \text{N} \end{aligned}$

3.3.4 Torque at the Cutting Tool

Torque at the cutting tool (T_c) is produced by tangential force (F_t) applied to its arm. The arm of applied force is a radius (R) of the workpiece. Torque at the cutting tool is calculated by the formulas:

Customary U.S. units of measure

$$T_{\rm c} = F_t R = \frac{F_t D}{2}$$
 (in lb) (3.41)

$$T_{\rm c} = \frac{F_t D}{24}$$
 (ft lb) (3.42)

were *D* is a workpiece diameter in inches and F_t is in pounds (lb)

Metric units of measure

$$T_{\rm c} = F_t R = \frac{F_t D}{2} \,\left(\mathrm{N} \cdot \mathrm{mm}\right) \tag{3.43}$$

$$T_{\rm c} = \frac{F_t D}{2000} \,\left(\mathbf{N} \cdot \mathbf{m}\right) \tag{3.44}$$

were D is a workpiece diameter in mm and F_{t} is in Newtons (N)

Example of Calculating Torque at the Cutting Tool

This example uses the same data as in the previous example with the assumption that the workpiece diameter D = 4.0 in. (101.6 mm). Calculated tangential force value is $F_t = 1077.1$ lb (4815.5 N). From formula (3.41):

$$T_{\rm c} = \frac{1077.1 \cdot 4}{2} = 2154.2 \text{ (in lb)}$$

From formula (3.42):

$$T_c = \frac{1077.1 \cdot 4}{24} = \frac{4308.4}{24} = 179.5 \text{ (ft lb)}$$

From formula (3.44):

$$T_{\rm c} = \frac{4815.5 \cdot 101.6}{2000} = \frac{489254.8}{2000} = 244.6 \; ({\rm N} \cdot {\rm m})$$

3.3.5 Machining Power at the Cutting Tool

Machining power at the cutting tool (P_c) can be calculated by three interrelated methods: (1) tangential force and cutting speed, (2) metal removal rate and power constant, (3) torque at the cutting tool and spindle speed.

(1) Formulas (3.30) and (3.31) are recommended to calculate machining power at the cutting tool by tangential force and the cutting speed values (these formulas are expressed in customary U.S. and metric units of measure respectively).

(2) Formulas (3.33) and (3.34) are recommended to calculate machining power at the cutting tool by metal removal rate and power constant values (these formulas are expressed in customary U.S. and metric units of measure respectively).

(3) Formulas (2.52) and (2.53) can be used to calculate machining power at the cutting tool by torque and spindle speed values (these formulas are expressed in customary U.S. and metric units of measure respectively).

Example of Calculating Machining Power at the Cutting Tool

In this example, the same data are used as in the previous examples with the assumption that the cutting speed $V_c = 600$ sfm (182.9 m/min). The cutting tool is sharp, $C_{sv} = 1.0$.

Customary U.S. units of measure Method (1), formula (3.30):

$$P_{c} = \frac{F_{t}V_{c}}{33,000},$$

from previous example, tangential force, $F_t = 1077.1$ lb

$$P_{\rm c} = \frac{1077.1 \cdot 600}{33,000} = \frac{646,260}{33,000} = \underline{19.58} \text{ hp}$$

Method (2), formula (3.33):

$$P_{\rm c} = 12V_{\rm c}dfK_p,$$

from previous example, d = 0.2 in, f = 0.02 ipr, $K_p = 0.68$

$$P_{\rm c} = 12 \cdot 600 \cdot 0.2 \cdot 0.02 \cdot 0.68 = \underline{19.58} \text{ hp}$$

Method (3), formula (2.52):

$$P_{c} = \frac{T_{c}n}{63,025},$$

from previous example, $T_c = 2154.2$ in lb

Spindle speed (n) is found from formula (3.3)

$$n = \frac{12V_{c}}{\pi D} = \frac{12 \cdot 600}{\pi \cdot 4} = \frac{1800}{\pi} = 573 \text{ rpm}$$
$$P_{c} = \frac{2154.2 \cdot 573}{63.025} = \frac{1234356.6}{63025} = \underline{19.58} \text{ hp}$$

Metric units of measure Method (1), formula (3.31):

$$P_{\rm c} = \frac{F_t V_{\rm c}}{60000} \,,$$

from previous example, tangential force, $F_t = 4815.5$ N

$$P_{\rm c} = \frac{4815.5 \cdot 182.9}{60000} = \frac{880754.95}{60000} = \underline{14.68} \text{ kW}$$

Method (2), formula (3.34):

$$P_{\rm c} = V_{\rm c} df K_{\rm p},$$

from previous example, d = 5.08 mm, f = 0.508 mm/rev, $K_p = 0.0311$ kW/cm³/min

 $P_{\rm c} = 182.9 \cdot 5.08 \cdot 0.508 \cdot 0.0311 = \underline{14.68} \text{ kW}$

Method (3), formula (2.53):

$$P_c = \frac{T_c n}{9549},$$

from previous example, $T_c = 244.6 \text{ N} \cdot \text{m}$

Spindle speed (n) is found from formula (3.4)

$$n = \frac{1000V_c}{\pi D} = \frac{1000 \cdot 182.9}{\pi \cdot 101.6} = \frac{1800}{\pi} = 573 \text{ rpm}$$
$$P_c = \frac{244.6 \cdot 573}{9549} = \frac{140155.8}{9549} = \frac{14.68}{9549} \text{ kW.}$$

When the cutting insert becomes dull and must be indexed or replaced, P_c must be multiplied by the tool wear factor C_w given in Table 3.10. Cutting conditions in this example correspond to roughing, so tool wear factor $C_w = 1.3$ and machining power adjusted for a dull cutting tool P_{ca} is:

$$P_{ca} = P_c C_w \tag{3.45}$$

Customary U.S. units of measure,

$$P_{ca} = 19.58 \cdot 1.3 = 25.45$$
 hp.

Metric units of measure,

$$P_{ca} = 14.68 \cdot 1.3 = 19.08 \text{ kW}$$

3.3.6 Machining Power at the Motor

Calculation of machining power at the motor is performed similarly to that described in section 2.2.4. Formula (2.54) and Table 2.5 should be used.

Example of Calculating Machining Power at the Motor

The same machining power values at the dull cutting tool are used from the previous example:

 $P_{ca} = 25.45$ hp and $P_{ca} = 19.08$ kW. Assuming that the machine tool is an oil-hydraulic drive lathe and the machine efficiency factor E = 90%, so the calculated machining power at the motor is:

Customary U.S. units of measure,

$$P_m = \frac{25.45}{0.9} = 28.3$$
 hp.

Metric units of measure,

$$P_m = \frac{19.08}{0.9} = 21.2$$
 kW.

Concluding Remarks on Turning

1. Identification systems for indexable cutting insert geometry, dimensions, and tolerances are specified by American National Standard for Turning Tools (ANSI B212.12-1991) and International Standards Organization (ISO).

2. Kinematics of turning is expressed by six formulas in customary U.S. and metric units of measure for calculating cutting speed, spindle speed, feed speed, and depth of cut.

3. Metal removal rate and machining time are major parameters that describe productivity in turning. Machining time when turning along the workpiece centerline and across the workpiece centerline (facing, cutting off, and deep grooving) are calculated by different formulas. There are 23 formulas for such calculations in customary U.S. and metric units of measure. The author developed 20 formulas for calculating machining time when facing, cutting off, and deep grooving of workpieces with solid and tubular cross sections.

4. Cutting forces (tangential, feed, radial), torque, and machining power are major parameters related to the

dynamics of turning. Tangential force calculation is based on the depth of cut, feed rate, power constant, and tool wear factor. Calculation of machining power can be performed by: (1) tangential force and cutting speed, (2) metal removal rate and power constant, and (3) torque and spindle speed. There are 16 formulas for such calculations.

4. Analytical analysis of the cutting force components resulted in a new concept for calculating feed and radial forces in relation to tangential force. The author developed 14 formulas for these calculations (Table 3.11).

BORING

Boring is an internal turning operation performed with a boring bar to enlarge a previously drilled hole to form an internal shape of required dimensions. Boring operations range from semiroughing to finishing.

The majority of formulas described in Chapter 3 "Turning" are applicable to "Boring". Some formulas were modified and a few new formulas were developed.

Insert identification systems described in sections 3.1.1 and 3.1.2 are applicable to boring as well.

Formulas given in section 3.2 "Kinematics and Productivity" are valid for boring. To calculate cutting speed and spindle speed, formulas (3.1) through (3.4) should be used. When applied to boring, symbol D in these formulas means "internal workpiece diameter". When calculating feed speed and metal removal rate, formulas (3.5), (3.7), and (3.8) can be used with the same symbols. Depth of cut (d) can be calculated by formula (3.6) modified for boring:

$$d = \frac{D_{\rm c} - D}{2} \tag{4.1}$$


Fig. 11. Depth of cut when boring

where D_c is the workpiece cut inside diameter, D is the workpiece uncut inside diameter. When boring, $D_c > D$ (Fig. 11).

Formula (3.9) is recommended to calculate machining time in boring operations. Formula (3.27) can be used for calculating machining time when grooving. Machining time formulas when facing and cutting off are not applicable to boring, because the cutting tools (boring bars) cannot be fed perpendicular to the centerline of the workpiece.

4.1 DYNAMICS

Tangential force formulas and the power constant data (sections 3.3.1 and 3.3.2) are valid for boring. As far as tool wear factors G_w are concerned, $G_w = 1.1$ should be used because this value corresponds with finishing or light cuts (Table 3.10). Feed and radial force formulas for turning should not be used when boring.

Group of Work	Feed Force	Radial Force
Materials	Formula	Formula
Group 1 Group 2 Group 3 Group 4 Group 5 Group 6 Group 7	$\begin{split} F_{f} &= 0.537 F_{t} \\ F_{f} &= 0.869 F_{t} \\ F_{f} &= 0.579 F_{t} \\ F_{f} &= 0.596 F_{t} \\ F_{f} &= 0.725 F_{t} \\ F_{f} &= 0.658 F_{t} \\ F_{f} &= 0.486 F_{t} \end{split}$	$F_{r} = 0.308F_{t}$ $F_{r} = 0.672F_{t}$ $F_{r} = 0.422F_{t}$ $F_{r} = 0.331F_{t}$ $F_{r} = 0.592F_{t}$ $F_{r} = 0.510F_{t}$ $F_{r} = 0.311F_{t}$

Table 4.1. Formulas for Feed and Radial Forces versus Tangential Force

4.1.1 Feed and Radial Forces

As with turning, feed and radial forces are calculated proportionally to the tangential force, depending on the group number of the work materials described earlier (section 3.3.3). The author developed new formulas for calculating feed and radial forces as shown in Table 4.1.

Description of each group of work materials precedes Table 3.11.

Example of Calculating Cutting Forces

- *Given*: A workpiece is AISI 1060 high-carbon steel, 245 HB (Brinell hardness number). Boring is performed at a depth of cut of 0.080 in. (2.03 mm) and feed rate of 0.008 ipr (0.20 mm/rev).
- *Required*: Calculation of tangential, feed, and radial forces.

Tangential force calculation Customary U.S. units of measure: Tangential force is calculated by formula (3.37):

$$F_t = 396,000 \, df K_n$$

Power constant (from Table 3.3): $K_p = 0.92$ hp/in³/min Feed factor (from Table 3.9): C = 1.08

Adjusted power constant, $K_{pa} = K_p C = 0.92 \cdot 1.08 = 0.99 \text{ hp/in}^3/\text{min}$

Tangential force, sharp cutting tool, $F_t = 396,000 \cdot 0.08 \cdot 0.008 \cdot 0.99 = 250.9$ lb

Metric units of measure:

Tangential force is calculated by formula (3.40):

$$F_t = 60000 \, df K_p$$

Power constant (from Table 3.3): $K_p = 0.0419$ kW/ cm³/min

Feed factor (from Table 3.9): C = 1.08

Adjusted power constant, $K_{pa} = K_p C = 0.0419 \cdot 1.08 = 0.0452 \text{ kW/cm}^3/\text{min}$

Tangential force, sharp cutting tool, $F_t = 60000 \cdot 2.03 \cdot 0.20 \cdot 0.0452 = 1101.1 \text{ N}$

Feed force calculation Customary U.S. units of measure, Feed force, $F_f = 0.537F_t$ (Table 4.1, Group 1 of work material) $F_f = 0.537 \cdot 250.9 = 134.7$ lb.

$$\begin{split} & \textit{Metric units of measure,} \\ & \textit{Feed force, } F_{j} = 0.537F_{t} \text{ (Table 4.1, Group 1 of work material)} \\ & F_{j} = 0.537 \cdot 1101.1 = 591.3 \text{ N.} \\ & \textit{Radial force calculation} \\ & \textit{Customary U.S. units of measure,} \\ & \textit{Radial force, } F_{r} = 0.308F_{t} \text{ (Table 4.1, Group 1 of work material)} \\ & F_{r} = 0.308 \cdot 250.9 = 77.3 \text{ lb.} \\ & \textit{Metric units of measure,} \\ & \textit{Radial force, } F_{r} = 0.308F_{t} \text{ (Table 4.1, Group 1 of work material)} \\ & \textit{Metric units of measure,} \\ & \textit{Radial force, } F_{r} = 0.308F_{t} \text{ (Table 4.1, Group 1 of work material)} \\ & F_{r} = 0.308 \cdot 1101.1 = 339.1 \text{ N.} \\ \end{split}$$

4.1.2 Machining Power at the Cutting Tool

The same methods and formulas described earlier (section 3.3.5) can be used for calculating machining power at the cutting tool (P_c) . If the cutting insert is sharp, the tool wear factor $G_w = 1.0$. When the cutting insert becomes dull and must be indexed or replaced, P_c must be multiplied by the value of tool wear factor G_w given in Table 3.10. Cutting conditions in boring operation usually entail light cuts, so the tool wear factor tor used in calculations should be $G_w = 1.1$, and machining power at dull cutting tool will be:

$$P_{ca} = 1.1P_c \tag{4.2}$$

It was shown earlier that all three methods of calculating machining power at the cutting tool produced the same results, therefore only one example of calculating P_c , based on tangential force and cutting speed (method #1), is presented here.

Example of Calculating Machining Power at the Cutting Tool

- Given: Cutting speed $V_c = 600$ sfm (182.9 m/min) and tangential force (from previous example of calculation), $F_t = 250.9$ lb (1101.1 N).
- *Required:* Machining power with sharp (P_c) and dull cutting tool (P_{ca}) .
- Customary U.S. units of measure

Formula (3.30):

$$P_{\rm c} = \frac{F_t V_{\rm c}}{33,000}$$
 $P_{\rm c} = \frac{250.9 \cdot 600}{33000} = 4.56 \text{ hp}$

Formula (4.2):

$$P_{ca} = 1.1P_c$$
 $P_{ca} = 1.1 \cdot 4.56 = 5.02$ hp

Metric units of measure

Formula (3.31):

$$P_{\rm c} = \frac{F_t V_c}{60000}$$
 $P_{\rm c} = \frac{1101.1 \cdot 182.9}{60000} = 3.36 \text{ kW}$

Formula (4.2):

$$P_{ca} = 1.1P_{c}$$
 $P_{ca} = 1.1 \cdot 3.36 = 3.70 \text{ kW}$

4.1.3 Machining Power at the Motor

Calculation of machining power at the motor is performed in a manner similar to that described in section 2.2.4. Formula (2.54) and Table 2.5 should be used.

Example of Calculating Machining Power at the Motor

The same machining power values at the dull cutting tool are used from the previous example:

$$P_{ca} = 5.02 \text{ hp}$$
 and $P_{ca} = 3.70 \text{ kW}.$

Assuming that the machine tool is a gear head drive lathe and the machine efficiency factor E = 80%, so the calculated machining power at the motor is:

Customary U.S. units of measure,

$$P_m = \frac{5.02}{0.8} = 6.28$$
 hp.

Metric units of measure,

$$P_m = \frac{3.70}{0.8} = 4.62 \text{ kW}.$$

4.2 BORING BAR DEFLECTION

A boring bar has three basic elements (Fig. 12): indexable cutting insert, shank, and anchor. The anchor is the clamping portion of the shank that is held in the tool block. The minimum clamping length is approximately 3 to 4 diameters of the shank (Ref. 16). The distance the boring bar extends beyond the tool block, called overhang, determines the cutting depth. The overhang is the unsupported portion of the boring bar. Long overhang causes



Fig. 12. Elements of a boring bar, 1-indexable insert, 2-shank, 3-anchor

excessive deflection of the shank and generates vibration (chatter), which is worsening the surface finish of the bore. Eliminating chatter, especially in long-bore workpieces, is one of the greatest challenges faced by manufacturers and users of boring bars. A boring bar with the highest static and dynamic stiffness should always be chosen. Static stiffness is the ability of a boring bar to resist deflection caused by the cutting forces. Dynamic stiffness is the ability of a boring bar to damp vibration.

This chapter describes the static stiffness and deflection of a boring bar depending on the mechanical properties of the shank material, the length of the overhang, and the cutting conditions.

The following equation is used for calculating deflection (y) of a boring bar.

$$y = \frac{FL^3}{3EI} \tag{4.3}$$

- *E* is the modulus of elasticity (in tension) of a boring bar material, psi or N/mm²
- *I* is the moment of inertia of a boring bar crosssectional area, in⁴ or mm⁴.

This is the same formula that was developed to calculate deflection of a cantilever beam. A boring bar deflection is similar to that of a cantilever beam. A boring bar is a beam with a fixed support at one end (a clamping portion, or anchor) and an unsupported portion, or cantilever, at the other end (Fig. 13). Because of the similarity, the formula describing a cantilever beam's deflection is applicable for a boring bar's deflection. As can be seen from formula (4.3), there are four variables that determine a boring bar's deflection. The most sensitive variable is overhang: the amount of deflection is proportional to the increase in the length cubed. The other variables are less sensitive: deflection is directly proportional to the cutting force and inversely proportional to the modulus of elasticity and the moment of inertia.



Fig. 13. Deflection of a boring bar (y) caused by cutting force (F), 1-cantilever, 2-anchor (clamping portion)

4.2.1 Cutting Forces Acting on the Boring Bar

The cutting force generated when boring is similar to that generated when turning. Cutting force components are tangential, feed, and radial. Tangential force (F_{t}) acts perpendicular to the cutting insert rake surface and pushes the boring bar "downward", so that the cutting edge moves below the centerline of the boring bar. The feed force (F_f) acts parallel to the centerline and does not deflect the boring bar. The radial force $(F_{..})$ is perpendicular to both these forces and pushes the bar away from the side of the bore. Thus, only tangential and radial forces deflect the boring bar (Ref. 17). Since the tangential force is significantly larger than the radial force, deflection by the tangential force is also much higher than that by the radial force. Therefore, the combined deflection caused by these two cutting force components (resultant cutting force) should be calculated. The resultant cutting force (F) is found from:

$$F = \sqrt{F_t^2 + F_r^2} \tag{4.4}$$

4.2.2 Unsupported Length of a Boring Bar

The unsupported length of a boring bar plays a significant role in boring operations, since it defines the depth of a hole. With more overhang, the boring bar can machine a deeper hole. But more overhang also increases the risk of excessive deflection and vibration. For example, an increase in bar length by 1.25 times

almost doubles the amount of deflection $(1.25^3 = 1.95)$ at a given amount of the cutting force.

4.2.3 Moduli of Elasticity of Boring Bar Materials

Boring bar shanks are made either of steels, tungstenbase metals, or cemented carbides.

1. Most commonly used boring bar materials are alloy steels. Some boring bar manufacturers use AISI 1144 free machining medium-carbon steel. Regardless of grades, all carbon and alloy steels have the same modulus of elasticity:

Customary U.S. units of measure,

 $E = 30 \cdot 10^6$ psi

Metric units of measure,

 $E = 20.6 \cdot 10^4 \text{ N/mm}^2$

A common mistake is to assume that a steel shank with a higher hardness, or one made from a higher quality of steel will deflect less. As can be seen from equation (4.3), the material property that determines deflection is the modulus of elasticity. Hardness does not appear in this equation.

2. Tungsten-base metals, also known as tungsten heavy alloys, are produced by a powder metallurgy technique. High purity metal powders—tungsten, nickel, iron, and copper, are typical elements for sintering variety of alloys, some of which are used to fabricate boring bars and other tool holders (Ref. 18, p. 23; Ref. 19, pp. 1, 3). Table 4.2 shows the most popular

Mechanical Properties		Grades		
and Chemical Composition	K1700	K1800	SD175	Manufacturers*
Modulus of elasticity (x10 ⁶ psi)	45	48	48	K1700 and
Modulus of elasticity (x10 ⁴ N/mm ²)	31	33	33	K1800 grades are
Ultimate tensile strength, psi	125,000	125,000	120,000	produced by
Ultimate tensile strength, N/mm ²	860	860	830	Kulite Tungsten
Yield strength, psi	85000	00006	80000	Products,
Yield strength, N/mm ²	590	620	550	H. C. Starck.
Typical hardness, HRC	23	25	28	SD175 grade is
Tungsten content, wt.%	06	95	92.5	produced by
Nickel (Ni) content, wt.%	Total of Ni, Fe,	Total of Ni, Fe,	5.4	Droduoto An
Iron (Fe) content, wt.%	Cu are balance	Cu are balance	2.1	Allochony
Copper (Cu) content, wt.%				Technologies Co.
)

* The author obtained permission to mention these names and provide mechanical properties and chemical composition of their products.

Boring

tungsten heavy alloys used to fabricate shanks for boring bars.

Boring bars made of tungsten heavy alloys will *deflect less* than steel boring bars of the same diameter and overhang by 50 to 60 percent when cutting at the same depth of cut and feed rate.

3. Cemented carbides used for boring bars, provide *minimum deflection* because their *moduli of elasticity are higher than those of steels and tungsten heavy alloys*. Typical cemented carbides for boring bars are compositions of tungsten carbide (90 to 94%) and cobalt (10 to 6% respectively). Multi-carbide compositions of refractory metals (tungsten, titanium, tantalum, and niobium) and cobalt (as a binder) are used to fabricate boring bars.

There are many cemented carbide manufacturers in the world whose products are used for boring bars. The following makers of cemented carbides have given permission to mention their names and have provided technical information on their products.

Ultra-met[®] offers several cemented carbide grades (Ref. 20, pp. 4, 6, 7) that are used to fabricate small diameter solid boring bars (including the cutting portion) and the bar shanks of larger diameters. These grades and their properties are given in Table 4.3.

ValeniteTM offers three cemented carbide grades (Ref. 21) that are used in boring bars. Properties and chemical composition of these grades are shown in Table 4.4.

Kennametal Inc. (Ref. 22) offers two cemented carbide grades that are used in boring bars. Properties and chemical composition of these grades are shown in Table 4.5.

Mechanical Properties		Grades	
and Chemical Composition	6Z	Z23	Z50
Modulus of elasticity (x10 ⁶ psi)	82.7	89.2	75.4
Modulus of elasticity (×10 ⁴ N/mm ²)	57.0	61.5	52.0
Transverse rupture strength, psi	480,000	360,000	310,000
Transverse rupture strength, N/mm ²	3300	2480	2140
Typical hardness, HRA	91.6	93.2	91.5
Tungsten carbide, wt.%	0.06	94.0	74.3
Tantalum and Niobium carbides, wt.%		I	8.8
Titanium carbide, wt.%	Ι	Ι	8.2
Cobalt, wt.%	10.0	0.0	8.7
Diameter range of the boring bars:			
 Using 0.375-inch-diameter of I.C.* insert 	0.63–1.75 in.	0.63–1.75 in.	0.63–1.75 in.
 Using 9.52-mm-diameter of I.C. insert 	16–44 mm	16–44 mm	16–44 mm
 Using 0.500-inch-diameter of I.C. insert 	1.00–2.50 in.	1.00–2.50 in.	1.00–2.50 in.
 Using 12.7-mm-diameter of I.C. insert 	25–63 mm	25–63 mm	25–63 mm

Table 4.3. Properties and Composition of Cemented Carbides for Boring Bars

* I.C. means Inscribed Circle

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Mechanical Properties		Grades	
and Chemical Composition	VC-101	WA-10	VC-152
Modulus of elasticity (x10 ⁶ psi)	84.1	85.6	87.0
Modulus of elasticity (×10 ⁴ N/mm ²)	58.0	59.0	60.0
Transverse rupture strength, psi	590,000	575,000	474,000
Transverse rupture strength, N/mm ²	4070	3960	3270
Typical hardness, HRA	91.9	89.6	90.8
Tantalum and Niobium carbides, wt.%	0.3	0.8	0.2
Titanium carbide, wt.%	0.1	0.2	0.2
Vanadium carbide, wt.%	0.3	I	
Cobalt, wt.%	10.0	10.0	9.0
Tungsten carbide, wt.%	Balance	Balance	Balance

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Mechanical Properties	Gra	ides
and Chemical Composition	KF310	KWH
Modulus of elasticity (×10 ⁶ psi)	84.1	79.8
Modulus of elasticity ($ imes 10^4$ N/mm²)	58.0	55.0
Transverse rupture strength, psi	450,000	275,600
Transverse rupture strength,N/mm ²	3105	1900
Typical hardness, HRA	92.2	90.3
Cobalt, wt.%	10.0	8.6
Vanadium, wt.%	0.2	I
Diameter of the boring bars, in. (mm)	Under 0.5 (12.0)	0.5 (12.0) and larger

4.2.4 Moment of Inertia of a Boring Bar Cross-sectional Area

As can be seen from formula (4.3), deflection of a boring bar is inversely proportional to the moment of inertia of a bar cross-sectional area.

Moment of inertia is a property of areas. Since boring bars are available in a variety of diameters, it is important to calculate the area of a bar cross section using appropriate formulas. Boring bars are usually round with solid or tubular cross sections as shown in Fig. 14. Boring bars for dry cutting have a solid cross section, and boring bars with through-tool holes for coolant would have tubular cross section.

Moment of inertia (I) of a solid cross-sectional area is found from:

$$I = \frac{\pi (D_o)^4}{64}$$
(4.5)

where D_{α} is a bar outside diameter in inches or mm.

Moment of inertia (I) of a tubular cross-sectional area is calculated by:

$$I = \frac{\pi [(D_o)^4 - (D_i)^4]}{64}$$
(4.6)

where D_i is a bar inside diameter in inches or mm.

A typical misconception about the moment of inertia of a tubular cross-sectional area is that even a small size of through-tool hole significantly decreases moment of inertia. It is important to know that small diameter through-tool holes do not affect the moment of inertia.



Fig. 14. Cross sectional area of a boring bar (a) solid, (b) tubular

Even large holes do not reduce significantly the moment of inertia of the tubular cross-sectional area. Thus, if it is necessary, the coolant can be supplied through tool holes of large diameters, and there is no need to worry about excessive deflection of boring bars.

Examples of calculations in the following two tables illustrate how various inside diameters of a bar affect the moment of inertia of the cross-sectional area when the outside bar diameter is constant.

As can be seen from Table 4.6 and Table 4.7, the moment of inertia of a bar cross-sectional area decreases at a significantly lower rate than the rate at which the bar inside diameter increases. For example, when the boring bar inside diameter is equal to 50% of its outside diameter value, the moment of inertia of a tubular cross-sectional area is smaller than that of a solid cross-sectional area by less than 7%. Typical through-tool hole diameters are approximately 0.08 to

Bar Diamete	ers, inches	Moment	of Inertia
Outside, D _o	Inside, D _i	in ⁴	%
1 00	0.00	0 0490874	100
1.00	0.05	0.0490871	99 999
1.00	0.00	0.049082	99 990
1.00	0.15	0.049063	99.95
1.00	0.20	0.049009	99.84
1.00	0.25	0.048896	99.61
1.00	0.30	0.048690	99.19
1.00	0.35	0.048351	98.50
1.00	0.40	0.047831	97.44
1.00	0.45	0.047074	95.90
1.00	0.50	0.046019	93.75
2.00	0.00	0.785398	100
2.00	0.20	0.785320	99.99
2.00	0.30	0.785001	99.95
2.00	0.40	0.784142	99.84
2.00	0.50	0.782330	99.61
2.00	0.60	0.779036	99.19
2.00	0.70	0.773612	98.50
2.00	0.80	0.765292	97.44
2.00	0.90	0.753192	95.90
2.00	1.00	0.736311	93.75

Table 4.6. Moment of Inertia of the Cross-sectional Area at Various Boring Bar Inside Diameters (Customary U.S. units of measure)

Bar Diame	eters, mm	Moment of	of Inertia
Outside, <i>D_o</i>	Inside, D _i	mm ⁴	%
25.00	0.00	19174 8	100
25.00	1.25	19174.6	99,999
25.00	2.50	19173	99.990
25.00	3.75	19165	99.95
25.00	5.00	19144	99.84
25.00	6.25	19100	99.61
25.00	7.50	19019	99.19
25.00	8.75	18887	98.50
25.00	10.00	18684	97.44
25.00	11.25	18388	95.90
25.00	12.50	17976	93.75
50.0	0.0	306796	100
50.0	2.5	306794	99,999
50.0	5.0	306765	99.990
50.0	7.5	306641	99.95
50.0	10.0	306305	99.84
50.0	12.5	305598	99.61
50.0	15.0	304311	99.19
50.0	17.5	302192	98.50
50.0	20.0	298942	97.44
50.0	22.5	294216	95.90
50.0	25.0	287621	93.75

Table 4.7. Moment of Inertia of the Cross-sectional Area at Various Boring Bar Inside Diameters (Metric units of measure)

0.2 inches (2 to 5 mm) depending on the boring bar outside diameters.

4.2.5 Static Stiffness of Boring Bars

It was mentioned earlier that static stiffness is the ability of a boring bar to resist deflection caused by the cutting force. Static stiffness can also be described as the product of the modulus of elasticity of a boring bar material (E) and the moment of inertia of a boring bar cross-sectional area (I) and is expressed as:

$$S = EI (lb \cdot in^2 \text{ or } N \cdot mm^2)$$
(4.7)

where S is static stiffness.

Static stiffness is directly proportional to moment of inertia, so it is obvious that the largest possible diameter of boring bar should be used to minimize deflection, especially if other boring bar materials with higher moduli of elasticity are not available.

Solution:	Using formula (4.7), $S = EI = 30 \cdot 10^6$ ·
	$0.049063 = 14.7 \cdot 10^5 \text{ lb} \cdot \text{in}^2$
Metric units	of measure
Given:	modulus of elasticity $E = 20.6 \cdot 10^4$
	N/mm ² , bar outside diameter $D_0 = 25$
	mm, bar inside diameter $D_i = 3.75$ mm,
	moment of inertia $I = 19165 \text{ mm}^4$
	(from Table 4.7).
Solution:	Using formula (4.7), $S = EI = 20.6 \cdot 10^4 \cdot$
	$19165 = 39.5 \cdot 10^8 \mathrm{N} \cdot \mathrm{mm}^2$

Calculation of a tungsten heavy alloy boring bar static stiffness

Customary U.S. units of measure

- *Given:* modulus of elasticity $E = 48 \cdot 10^6$ psi, boring bar outside diameter $D_o = 1.0$ in., bar inside diameter $D_i = 0.15$ in., moment of inertia I = 0.049063 in⁴ (from Table 4.6).
- Solution: Using formula (4.7), $S = EI = 48 \cdot 10^6 \cdot 0.049063 = \underline{23.6 \cdot 10^5} \text{ lb} \cdot \text{in}^2$

Metric units of measure

- *Given:* modulus of elasticity $E = 33 \cdot 10^4$ N/mm², bar outside diameter $D_o = 25$ mm, bar inside diameter $D_i = 3.75$ mm, moment of inertia I = 19165 mm⁴ (from Table 4.7).
- Solution: Using formula (4.7), $S = EI = 33 \cdot 10^4 \cdot 19165 = 63.2 \cdot 10^8 \text{ N} \cdot \text{mm}^2$

Calculation of a cemented carbide boring bar static stiffness

Customary U.S. units of measure

- $\begin{array}{ll} Given: & \mbox{modulus of elasticity } E = 82.7 \cdot 10^6 \mbox{ psi,} \\ & \mbox{boring bar outside diameter } D_o = 1.0 \mbox{ in.,} \\ & \mbox{bar inside diameter } D_i = 0.15 \mbox{ in.,} \\ & \mbox{moment of inertia } I = 0.049063 \mbox{ in}^4 \\ & \mbox{(from Table 4.6).} \end{array}$
- Solution: Using formula (4.7), $S = EI = 82.7 \cdot 10^6 \cdot 0.049063 = 40.6 \cdot 10^5 \text{ lb} \cdot \text{in}^2$

Metric units of measure

- *Given:* modulus of elasticity $E = 57 \cdot 10^4$ N/mm², bar outside diameter $D_o = 25$ mm, bar inside diameter $D_i = 3.75$ mm, moment of inertia I = 19165 mm⁴ (from Table 4.7).
- Solution: Using formula (4.7), $S = EI = 57 \cdot 10^4 \cdot 19165 = \underline{109.2 \cdot 10^8} \text{ N} \cdot \text{mm}^2$

4.2.6 Boring Bar Overhang-to-Diameter Ratio

Analysis of equations (4.3), (4.5), and (4.6) shows what can be done to minimize boring bar deflection. The following statements are in the same sequence as the variables appear in equation (4.3):

- 1. To machine at minimum depth of cut and feed rate to reduce the cutting force.
- 2. To use a boring bar with the shortest possible overhang.

- 3. To use a boring bar material with the highest modulus of elasticity.
- 4. To use a boring bar with the largest possible diameter.

Since deflection is a function of four variables, there are several possibilities to keep a boring bar within a deflection range that will not jeopardize productivity of cutting and the quality of surface finish. The length of tool overhang and the bar diameter will determine what material should be used to fabricate the boring bar.

Boring bar overhang (L)-to-diameter (D_o) ratio is a widely used practical criterion to control deflection. The maximum overhang-to-diameter ratios ($L : D_o$) for the most common boring bar materials are:

Steel-shank bars	3.5 : 1 (Ref. 23)
Steel-shank bars	4.0:1 (Ref. 16)
Tungsten heavy alloy-shank bars	5.0:1 (Ref. 23)
Cemented carbide-shank bars	7.0:1 (Ref. 24)
Cemented carbide-shank bars	8.0:1 (Ref. 23)

When boring depth (overhang) exceeds 8 times shank diameters $(L > 8D_o)$, vibration of the boring bar occurs. A steel bar that incorporates dampening mechanism is then recommended (Ref. 23).

Example of Calculating Boring Bar Deflection

Given: AISI 1045 medium-carbon steel workpiece (250 HB) to be machined at depth of cut d = 0.100 in. (2.54 mm) and feed rate f = 0.008 ipr (0.20 mm/rev) using a boring bar with diameter $D_0 = 1.0$ in.

(25.4 mm) and unsupported length L = 4.0 in. (101.6 mm).

Required: Amount of boring bar deflection.

Calculations in Customary U.S. units of measure

Deflection of a boring bar is calculated by equation (4.3) in the following sequence:

1. Tangential Force is calculated by formula (3.37)

$$F_t = 396,000 \cdot d \cdot f \cdot K_p$$

where $K_p = 0.92$ is the power constant of 1045 steel shown in Table 3.3. The feed rate f = 0.008 ipr, so the feed factor C = 1.08 (Table 3.9). The adjusted power constant K_{pq} and tangential force F_t are equal to:

$$K_{pa} = K_p \cdot C = 0.92 \cdot 1.08 = 0.99 \text{ hp/in}^3/\text{min}$$

 $F_r = 396,000 \cdot 0.1 \cdot 0.008 \cdot 0.99 = 313.6 \text{ lb}$

2. *Radial Force* is calculated by the formula shown in the first row of Table 4.1.

$$F_r = 0.308 \cdot F_t = 0.308 \cdot 313.6 = 96.6$$
 lb

3. Resultant Force is calculated by formula (4.4)

$$F = \sqrt{F_t^2 + F_r^2} = \sqrt{313.6^2 + 96.6^2}$$
$$= \sqrt{107,676.52} = 328.1 \text{ lb}$$

4. Moment of Inertia is calculated by formula (4.5)

$$I = \frac{\pi (D_o)^4}{64} = \frac{\pi \cdot 1^4}{64} = \frac{\pi}{64} = 0.0491 \text{ in}^4$$

5. Deflection of a boring bar is calculated by equation (4.3)

$$y = \frac{FL^3}{3EI}$$

5a. Deflection of a steel boring bar, $E = 30 \cdot 10^6$ psi

$$y = \frac{328.1 \cdot 4^3}{(3 \cdot 30 \cdot 10^6 \cdot 0.0491)} = \frac{20,998.4}{4,419,000} = \underline{0.0048 \text{ in.}}$$

5b. Deflection of a tungsten heavy alloy boring bar, $E = 48 \cdot 10^6$ psi (Table 4.2)

$$y = \frac{328.1 \cdot 4^3}{(3 \cdot 48 \cdot 10^6 \cdot 0.0491)} = \frac{20,998.4}{7,070,400} = \underline{0.0030 \text{ in.}}$$

5c. Deflection of a cemented carbide boring bar, $E = 82.7 \cdot 10^6$ psi (Table 4.3)

 $y = \frac{328.1 \cdot 4^3}{(3 \cdot 82.7 \cdot 10^6 \cdot 0.0491)} = \frac{20,998.4}{12,181,710} = \underline{0.0017 \text{ in.}}$

Calculations in Metric units of measure 1. Tangential Force is calculated by formula (3.40)

$$F_t = 60000 \cdot d \cdot f \cdot K_p$$

where $K_p = 0.0419$ is the power constant of 1045 steel shown in Table 3.3. The feed rate f = 0.20 mm/rev, so the feed factor C = 1.08 (Table 3.9). The adjusted power constant K_{pa} and tangential force F_t are equal to:

$$K_{pa} = K_p \cdot C = 0.0419 \cdot 1.08 = 0.0453 \text{ kW/cm}^3/\text{min}$$

$$F_r = 60000 \cdot 2.54 \cdot 0.2 \cdot 0.0453 = 1380.7 \text{ N}$$

2. Radial Force is calculated by the formula shown in the first row of Table 4.1.

$$F_r = 0.308 \cdot F_t = 0.308 \cdot 1380.7 = 425.3 \text{ N}$$

3. Resultant Force is calculated by formula (4.4)

$$F = \sqrt{F_t^2 + F_r^2} = \sqrt{1380.7^2 + 425.3^2}$$
$$= \sqrt{2087212.58} = 1444.7 \text{ N}$$

4. Moment of Inertia is calculated by formula (4.5)

$$I = \frac{\pi (D_o)^4}{64} = \frac{\pi \cdot 25.4^4}{64} = \frac{\pi \cdot 416231}{64} = 20432 \text{ mm}^4$$

5. Deflection of a boring bar is calculated by equation (4.3)

$$y = \frac{FL^3}{3EI}$$

5*a.* Deflection of a steel boring bar, $E = 20.6 \cdot 10^4$ N/mm²

 $y = \frac{1447.7 \cdot 101.6^3}{(3 \cdot 20.6 \cdot 10^4 \cdot 20432)} = \frac{1518307363}{12626976000} = \frac{0.120 \text{ mm}}{0.120 \text{ mm}}$

5b. Deflection of a tungsten heavy alloy boring bar, $E = 33 \cdot 10^4 \text{ N/mm}^2$ (Table 4.2)

$$y = \frac{1447.7 \cdot 101.6^3}{(3 \cdot 33 \cdot 10^4 \cdot 20432)} = \frac{1518307363}{20227680000} = \underline{0.075 \text{ mm}}$$

5c. Deflection of a cemented carbide boring bar, $E = 57 \cdot 10^4 \text{ N/mm}^2$ (Table 4.3)

$$y = \frac{1447.7 \cdot 101.6^3}{(3 \cdot 57 \cdot 10^4 \cdot 20432)} = \frac{1518307363}{34638720000} = \underline{0.043 \text{ mm}}$$

4.2.7 Maximum Overhang at Permissible Deflection of a Boring Bar

When boring productivity is important, another practical criterion developed by the author, can be considered. This criterion is based on maximum overhang at permissible deflection of a boring bar made from various materials. Mathematically such criterion is expressed by equation (4.3) executed in respect to overhang (symbol "L").

$$L = \left(\frac{3EIy}{F}\right)^{1/3} \tag{4.8}$$

The following example of calculations explains the concept. Assume that the required productivity in machining AISI 4140 alloy steel with the hardness of 230 HB is satisfied at the depth of cut d = 0.100 in (2.54 mm) and at the feed rate f = 0.012 ipr (0.30 mm/rev). Permissible deflection of a boring bar y = 0.005 in (0.127 mm). The boring bar outside diameter $D_o = 1.00$ in (25.4 mm) and a through-tool hole diameter (boring bar inside diameter) $D_i = 0.156$ in (4.0 mm).

What is permissible overhang of the boring bar made from steel, tungsten heavy alloys and cemented carbides?

Prior to using equation (4.8) we have to calculate cutting force F and moment of inertia I of the cross-sectional area of the bar shank.

According to formula (4.4), cutting force F that deflects the bar shank is a resultant force of tangential F_t and radial F_r forces. These forces will be calculated first, then the moment of inertia I, and, finally, overhang L will be calculated for each of the three boring bar materials: steel, tungsten heavy alloys, and cemented carbides.

Calculations in Customary U.S. units

of measure

Tangential Force is calculated by formula (3.37)

$$F_t = 396,000 \cdot d \cdot f \cdot K_p$$

where $K_p = 0.76$ is the power constant of 4140 steel shown in Table 3.4. The feed rate f = 0.012 ipr, so the feed factor C = 1.0 (Table 3.9) and no adjustment to

power constant K_p is needed.

$$F_t = 396,000 \cdot 0.1 \cdot 0.012 \cdot 0.76 = 361.2$$
 lb

Radial Force is calculated by the formula shown in the first row of Table 4.1

$$F_r = 0.308 \cdot F_t = 0.308 \cdot 361.2 = 111.2$$
 lb

Resultant Force is calculated by formula (4.4)

$$F = \sqrt{F_t^2 + F_r^2} = \sqrt{361.2^2 + 111.2^2}$$
$$= \sqrt{142,830.88} = 377.9 \text{ lb}$$

Moment of inertia of a bar cross-sectional area is calculated by formula (4.6)

$$I = \frac{\pi [(D_o)^4 - (D_i)^4]}{64} = \frac{\pi \cdot (1^4 - 0.156^4)}{64}$$
$$= \frac{\pi \cdot 0.999408}{64} = 0.049058 \text{ in}^4$$

Permissible overhang of the boring bar is calculated by formula (4.8)

Steel boring bar, $E_s = 30 \cdot 10^6$ psi

$$\begin{split} L_S &= \left(\frac{3E_s Iy}{F}\right)^{1/3} = \left(\frac{3 \cdot 30 \cdot 10^6 \cdot 0.049058 \cdot 0.005}{377.9}\right)^{1/3} \\ L_S &= \left(\frac{4,415,220 \cdot 0.005}{377.9}\right)^{1/3} = \left(\frac{22,076.1}{377.9}\right)^{1/3} \\ L_S &= 58.42^{1/3} = 3.88 \text{ in.} \end{split}$$

Permissible overhang of the steel boring bar is: $L_s = 3.88$ in.

K1800 or SD175 tungsten heavy alloy boring bar, $E_T = 48 \cdot 10^6$ psi (Table 4.2)

$$\begin{split} L_T &= \left(\frac{3E_T Iy}{F}\right)^{1/3} = \left(\frac{3 \cdot 48 \cdot 10^6 \cdot 0.049058 \cdot 0.005}{377.9}\right)^{1/3} \\ L_T &= \left(\frac{7,064,352 \cdot 0.005}{377.9}\right)^{1/3} = \left(\frac{35,321.76}{377.9}\right)^{1/3} \\ L_T &= 93.47^{1/3} = 4.54 \text{ in.} \end{split}$$

Permissible overhang of the boring bar made from K1800 or SD175 is: $L_T = 4.54$ in.

Boring bar made from Z9 cemented carbide, $E_c = 82.7 \cdot 10^6$ psi (Table 4.3)

$$L_{C} = \left(\frac{3E_{C}Iy}{F}\right)^{1/3} = \left(\frac{3 \cdot 82.7 \cdot 10^{6} \cdot 0.049058 \cdot 0.005}{377.9}\right)^{1/3}$$
$$L_{C} = \left(\frac{12,171,289.8 \cdot 0.005}{377.9}\right)^{1/3} = \left(\frac{60,856.449}{377.9}\right)^{1/3}$$
$$L_{C} = 161.04^{1/3} = 5.44 \text{ in.}$$

Permissible overhang of the boring bar made from Z9 cemented carbide is: $L_c = 5.44$ in.

Analysis of the overhang calculations shows that the permissible overhang of a bar made from tungsten heavy alloys (L_{τ}) and cemented carbides (L_{c}) in

relation to the permissible overhang of a bar made of steel (L_S) and moduli of elasticity of the three bar materials $(E_S, E_T, \text{ and } E_C)$ can be found from the following equations:

$$L_T = L_S \left(\frac{E_T}{E_S}\right)^{1/3}$$
(4.9)

$$L_C = L_S \left(\frac{E_C}{E_S}\right)^{1/3}$$
(4.10)

Examples of calculations based on the abovementioned values

Equation (4.9):

$$L_T = 3.88 \cdot \left(\frac{48 \cdot 10^6}{30 \cdot 10^6}\right)^{1/3} = 3.88 \cdot 1.6^{1/3} = 4.54 \text{ in.}$$

Equation (4.10):

$$L_{\rm C} = 3.88 \cdot \left(\frac{82.7 \cdot 10^6}{30 \cdot 10^6}\right)^{1/3} = 3.88 \cdot 2.757^{1/3} = \underline{5.44 \text{ in.}}$$

Calculations in Metric units of measure Tangential Force is calculated by formula (4.40)

$$F_t = 60000 \cdot d \cdot f \cdot K_p$$

where $K_p = 0.0346$ is the power constant of 4140 steel shown in Table 3.4. The feed rate f = 0.30 mm/min, so the feed factor C = 1.0 (Table 3.9) and no adjustment

to power constant K_n is needed.

$$F_t = 60000 \cdot 2.54 \cdot 0.3 \cdot 0.0346 = 1581.9 \text{ N}$$

Radial Force is calculated by the formula shown in the first row of Table 4.1

$$F_r = 0.308 \cdot F_t = 0.308 \cdot 1581.9 = 487.2 \text{ N}$$

Resultant Force is calculated by formula (4.4)

$$F = \sqrt{F_t^2 + F_r^2} = \sqrt{1581.9^2 + 487.2^2}$$
$$= \sqrt{2739771.45} = 1655.2 \text{ N}$$

Moment of inertia of a bar cross-sectional area is calculated by formula (4.6)

$$I = \frac{\pi [(D_o)^4 - (D_i)^4]}{64} = \frac{\pi \cdot (25.4^4 - 4^4)}{64} = \frac{\pi \cdot 415975.4256}{64}$$
$$I = \frac{1306825.341}{64} = 20419 \text{ mm}^4$$

Permissible overhang of the boring bar is calculated by formula (4.8)

Steel boring bar, $E_s = 20.6 \cdot 10^4 \text{ N/mm}^2$

$$L_{S} = \left(\frac{3E_{S}Iy}{F}\right)^{1/3} = \left(\frac{3 \cdot 20.6 \cdot 10^{4} \cdot 20419 \cdot 0.127}{1655.2}\right)^{1/3}$$
$$L_{S} = \left(\frac{1602605634}{1655.2}\right)^{1/3} = 968225^{1/3} = 98.9 \text{ mm}$$

Permissible overhang of the boring bar made of steel is: $L_s \approx \underline{99 \text{ mm}}$.

K1800 or SD175 tungsten heavy alloy boring bar, $E_{\rm T}$ = 33 \cdot 10^4 N/mm^2 (Table 4.2)

$$\begin{split} L_T = & \left(\frac{3E_T I_y}{F}\right)^{1/3} = \left(\frac{3 \cdot 33 \cdot 10^4 \cdot 20419 \cdot 0.127}{1655.2}\right)^{1/3} \\ L_T = & \left(\frac{2567280870}{1655.2}\right)^{1/3} \\ L_T = & 1551040^{1/3} = 115.8 \text{ mm} \end{split}$$

Permissible overhang of a boring bar made from K1800 or SD175 is: $L_T \approx 116$ mm.

Boring bar made from Z9 cemented carbide, $E_c = 57 \cdot 10^4 \text{ N/mm}^2$ (Table 4.3)

$$L_{C} = \left(\frac{3E_{T}Iy}{F}\right)^{1/3} = \left(\frac{3 \cdot 57 \cdot 10^{4} \cdot 20419 \cdot 0.127}{1655.2}\right)^{1/3}$$
$$L_{C} = \left(\frac{4434394230}{1655.2}\right)^{1/3}$$
$$L_{C} = 2679069^{1/3} = 138.9 \text{ mm}$$

Permissible overhang of the boring bar made from Z9 cemented carbide is: $L_c \approx 139$ mm.

Permissible overhang of the boring bar made from K1800 or SD175 can be calculated by equation (4.9):

$$\begin{split} L_T &= 98.9 \cdot \left(\frac{33 \cdot 10^4}{20.6 \cdot 10^4}\right)^{1/3} = 98.9 \cdot 1.602^{1/3} \\ &= 115.8 \approx \underline{116 \text{ mm}} \end{split}$$

Permissible overhang of the boring bar made from Z9 cemented carbide can be calculated by equation (4.10):

$$L_C = 98.9 \cdot \left(\frac{57 \cdot 10^4}{20.6 \cdot 10^4}\right)^{1/3} = 98.9 \cdot 2.767^{1/3}$$
$$= 138.8 \approx 139 \text{ mm}$$

Overhangs of the boring bars made from tungsten heavy alloys (L_T) or cemented carbides (L_C) in comparison with overhang of a steel boring bar (L_S) can be calculated by:

$$L_T \text{ or } L_C = L_S \left[\frac{E_T \text{ or } E_C}{E_S} \right]^{1/3}$$
 (4.11)

where E_T , E_C , E_S are moduli of elasticity of tungsten heavy alloys, cemented carbides, and steels respectively.

Concluding Remarks on Boring

1. Formulas (3.1) through (3.9) and (3.27) for kinematics and productivity of turning are applicable to boring.

2. Formulas (3.30) through (3.45) for dynamics of turning are applicable to boring.

3. Analytical analysis of the cutting force components resulted in a new concept for calculating feed and radial forces in relationship to tangential force. The author developed 14 formulas for these calculations (Table 4.1).

4. Deflection of a boring bar is a function of four variables: the cutting force applied to the indexable insert, the unsupported length of the shank, the modulus of elasticity of the shank material, and the moment of inertia of the shank cross-sectional area.

5. The unsupported length of a boring bar shank or overhang is the most influential variable that affects deflection. As the overhang gets greater, the amount of deflection is cubed by the increase in the length.

6. To ensure the minimum boring bar deflection, the overhang should be as short as possible.

7. Boring bar deflection is inversely proportional to the modulus of elasticity and the moment of inertia. There are two ways to minimize deflection:

- The boring bar should be made of either tungsten heavy alloys or cemented carbides. The moduli of elasticity of tungsten heavy alloys are greater by 1.5 to 1.6 times than that of steel, and the moduli of elasticity of cemented carbides are greater by 2.5 to 3.0 times than that of steel,
- The boring bar diameter should be as large as possible to get the maximum moment of inertia.

8. The static stiffness of a boring bar is a quantity defined by the modulus of elasticity and the moment of inertia. Static stiffness is calculated by formula (4.7).

9. The boring bar overhang-to-diameter ratio $(L:D_o)$ is a practical guide to select one of the currently used materials: steel (3.5 : 1 or 4 : 1), tungsten heavy alloys (5 : 1), and cemented carbides (7 : 1 or 8 : 1).

10. When boring overhang exceeds 8 times shank diameters $(L > 8D_o)$, vibration of the boring bar occurs. A steel bar that incorporates dampening mechanism is then recommended.

11. If the maximum overhang of a steel boring bar at given cutting conditions is not long enough, equations (4.9), (4.10), and (4.11) will help to determine longer overhang by using tungsten heavy alloys and cemented carbide boring bars. The author derived these equations from equations (4.3) and (4.8).
DRILLING

Four different types of drilling tools are used in metalcutting applications: solid HSS (high speed steel) twist drills, solid carbide fluted drills, spade drills with inserted HSS and carbide blades, and carbide indexable insert drills. Each type of these drills has advantages and performance benefits. However, this chapter describes drilling only with indexable insert drills.

5.1 INDEXABLE INSERT DRILLS

Drilling tools with carbide indexable inserts can produce holes from a solid at faster rates and lower costs than HSS twist drills and spade drills in many applications. About 60% of all drilling applications in the metalcutting industry are for short holes having depths of up to four times the drill diameters (4:1), and many of these holes can be produced with indexable drills (Ref. 25, p. 1).

Indexable insert drills have been in use for over twenty years. During this time, the following major

improvements were implemented to indexable drills (Ref. 26, p. 42):

- Modifications in the holder flute design to maximize chip flushing
- Higher wear-resistant and stronger cemented carbide inserts
- New carbide insert coatings and more efficient insert geometries

A typical drill has two asymmetrically-placed indexable inserts with overlap: the inboard insert cuts through the central portion of a hole from a solid, and the outboard insert does peripheral cutting (Fig. 15). Normally, the inboard insert is made of a tougher carbide grade to better withstand the greater pressure at the center of the drill. The outboard insert is made of a more wear-resistant carbide grade to better withstand the heat generated by higher cutting speed at the periphery of the drill.

The range of drill diameters incorporating two indexable inserts depends on the manufacturer. According to



Fig. 15. Drill with two asymmetrically placed indexable inserts, 1-inboard insert, 2-outboard insert

"Modern Applications News" (Ref. 26, p. 42), indexable drills are offered in a range of diameters from 11 mm (0.433 in.) to 60 mm (2.362 in.).

One of the leading cutting tool manufacturers offers indexable drills in a range of diameters from 0.500 to 0.625 in. (from 12,5 to 16 mm) using two rectangular inserts (Ref. 27, pp. 142, 143). Indexable drills in a range of diameters from 0.625 to 3.250 in. (16 to 82 mm) have two trigon inserts (Ref. 27, pp. 144, 145, 149, 150).

A trigon insert (Fig. 16) has six sides. Two adjacent sides with included angle of 156° form the cutting edge on the top of a drill. There are three such cutting edges providing three indexes. Two adjacent cutting edges have an included angle of 84° .

The trigon geometry stabilizes the drill and provides minimal tool deflection at greater penetration rates. The trigon geometry also allows drilling into uneven surfaces without pre-drilling or pre-facing the surfaces.



Fig. 16. Trigon inserts for indexable drills

5.2 KINEMATICS AND PRODUCTIVITY

Formulas given in section 2.1 "Kinematics and Productivity" are valid for drilling. To calculate cutting speed and spindle speed, formulas (2.1) through (2.4) should be used. When applied to drilling, symbol D in these formulas is a drill diameter.

Cutting speed $V_c = 0$ at the center of a drill (if D = 0, then $V_c = 0$), and the cutting speed has its maximum value on the periphery of the outboard insert.

Penetration rate, removal rate, and drilling time are parameters of productivity.

5.2.1 Penetration Rate

The penetration rate (V_p) is a linear speed at which a drill makes a hole. The penetration rate is measured in inches per minute and millimeters per minute. To calculate the penetration rate, the following formula is used:

$$V_p = fn \text{ (ipm or mm/min)}$$
 (5.1)

where f is the feed rate (ipr or mm/rev),

n is the spindle speed (rpm or rev/min)

If the feed rate f = 0.004 ipr (0.1 mm/rev) and the spindle speed n = 500 rpm (500 rev/min), the penetration rate will be as follows,

Customary U.S. units of measure:

$$V_p = 0.004 \cdot 500 = 2$$
 ipm

Metric units of measure:

$$V_p = 0.1 \cdot 500 = 50$$
 mm/min

5.2.2 Metal Removal Rate

The metal removal rate is measured in in³/min and in cm³/min. To calculate the metal removal rate (Q), the author developed a formula based on the area of a circle (A) and penetration rate ($f \cdot n$):

$$Q = Afn \tag{5.2}$$

The area of a circle is the area of a cross section of a hole made by a drill. Such an area is found from:

$$A = \left(\frac{\pi}{4}\right) D^2 \tag{5.3}$$

where *D* is the drill diameter (in. or cm).

Replacing *A* in formula (5.2) by its value from formula (5.3) gives:

$$Q = \left(\frac{\pi}{4}\right) D^2 f n \tag{5.4}$$

For example, if drill diameter D = 1.5 in. (3.81 cm), feed rate f = 0.004 ipr (0.1 mm/rev, or 0.01 cm/rev), and spindle speed n = 500 rpm, the metal removal rate is:

Customary U.S. units of measure:

$$Q = \left(\frac{\pi}{4}\right) \cdot 1.5^2 \cdot 0.004 \cdot 500 = \left(\frac{\pi}{4}\right) \cdot 4.5 = 3.53 \text{ in}^3/\text{min}$$

Metric units of measure:

$$Q = \left(\frac{\pi}{4}\right) \cdot 3.81^2 \cdot 0.01 \cdot 500 = \left(\frac{\pi}{4}\right) \cdot 72.58$$
$$= 57.0 \text{ cm}^3/\text{min}$$

The author developed new metal removal rate formulas based on formula (5.4) and formulas (2.3) and (2.4) for spindle speed *n* described earlier.

In customary U.S. units of measure,

$$n = \frac{12V_{\rm c}}{\pi D} \tag{5.5}$$

where V_c is a cutting speed given in sfm (ft/min), and D is a drill diameter in inches.

In metric units of measure,

$$n = \frac{1000V_c}{\pi D} \tag{5.6}$$

where V_c is a cutting speed given in m/min, and D is a drill diameter given in mm

Metal removal rate formula used with customary U.S. units of measure

Replacing n in formula (5.4) by its value from formula (5.5) gives:

$$Q = \left(\frac{\pi}{4}\right) \frac{D^2 f \cdot 12V_{\rm c}}{\pi D}$$

and simplification of this formula produces

$$Q = 3V_c Df (\text{in}^3/\text{min})$$
 (5.7)

This example of a metal removal rate calculation is based on previous data. First, we calculate the cutting speed by formula (2.1):

$$V_{\rm c} = \frac{\pi Dn}{12} = \frac{\pi \cdot 1.5 \cdot 500}{12} = 196.3 \,\,{\rm sfm}$$

Then we calculate the metal removal rate using formula (5.7):

$$Q = 3 \cdot 196.3 \cdot 1.5 \cdot 0.004 = 3.53 \text{ in}^3/\text{min}$$

Metal removal rate formula used with metric units of measure

Replacing n in formula (5.4) by its value from formula (5.6) gives:

$$Q = \left(\frac{\pi}{4}\right) \frac{D^2 f \cdot 1000 V_{\rm c}}{\pi D}$$

and simplification of this formula produces

$$Q = 250V_c Df \tag{5.8}$$

To obtain Q in cm³/min, V_c is taken in m/min, D is taken in cm and f is taken in cm/rev, therefore, formula (5.8) is as follows:

$$Q = 25V_c Df \text{ (cm}^3/\text{min)}$$
 (5.9)

This example of a metal removal rate calculation is based on previous data. First, we calculate the cutting speed by formula (2.2):

$$V_{\rm c} = \frac{\pi Dn}{1000} = \frac{\pi \cdot 38.1 \cdot 500}{1000} = 59.8 \text{ m/min}$$

Then we calculate the metal removal rate using formula (5.9):

$$Q = 25 \cdot 59.8 \cdot 3.81 \cdot 0.01 = 57.0 \text{ cm}^3/\text{min}$$

5.2.3 Drilling Time

The time (t) required to drill a hole is determined by its depth (L) and the penetration rate (V_p) . The drilling time is calculated by the formula:

$$t = \frac{L}{V_p} \tag{5.10}$$

Replacing V_p in formula (5.10) by its value from formula (5.1) gives:

$$t = \frac{L}{fn} \tag{5.11}$$

For example, if L = 3.0 in (76.2 mm), f = 0.006 ipr (0.152 mm/rev), and n = 625 rev/min, then the drilling time is as follows.

Customary U.S. units of measure:

$$t = \frac{3}{0.006 \cdot 625} = 0.80 \text{ min}$$

Metric units of measure:

$$t = \frac{76.2}{0.152 \cdot 625} = 0.80 \text{ min}$$

5.3 DYNAMICS

Dynamics is described by formulas for drilling forces, torque, machining power at the drill and at the motor in relation to the mechanical properties of the work materials and the drilling conditions. The parameters of the dynamics are:

- 1. Power constant
- 2. Machining power at the drill and at the motor
- 3. Tangential cutting force
- 4. Torque at the drill
- 5. Thrust (also known as axial force).

Limited technical information on the dynamics of indexable drills can be found in a paper published by the Machining Technology Association of the Society of Manufacturing Engineers (Ref. 25, p. 5) and in some trade magazines (Ref. 26, p. 44).

Technical data on thrust, torque, and machining power for various diameters of indexable drills used in typical cutting conditions for most common work materials are given in "Metcut Drills Application Information". This technical information is shown in the following tables.

Statistical analysis of the limited data given in Tables 5.1 through 5.14 conducted by the author, allowed the following data to be obtained:

- Power constant values for the work materials mentioned in these tables
- Formula to calculate machining power based on metal removal rate and the power constant
- Formula to calculate tangential cutting force
- Formula to calculate torque
- Formula to calculate thrust

5.3.1 Power Constant

Power constant values were found through machining power (from Tables 5.1 to 5.14) and the metal removal rate calculated by formulas (5.7) and (5.9).

Drill	Cutting	Feed	Torque,	Machining	Thrust, lb
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	500	0.003	66	3.2	281
0.875	500	0.003	130	4.5	414
1.125	550	0.005	358	10.7	831
1.375	550	0.005	535	13.0	1047
1.750	600	0.006	1040	21.6	1599
2.250	600	0.006	1720	27.8	2134

Table 5.1. Carbon Steels, Brinell Hardness Range is (140–220) HB

Table 5.2. Carbon Steels, Brinell Hardness Range is (220-300) HB

Drill	Cutting	Feed	Torque,	Machining	Thrust, lb
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	500	0.003	74	3.6	313
0.875	500	0.003	145	5.0	460
1.125	550	0.005	398	11.8	925
1.375	550	0.005	595	14.4	1164
1.750	600	0.006	1157	24.0	1778
2.250	600	0.006	1912	30.9	2374

Table 5.3. Free-machining Carbon Steels, Brinell Hardness Range is (120–180) HB

Drill Diameter, in	Cutting Speed, sfm	Feed Rate, ipr	Torque, in Ib	Machining Power, hp	Thrust, lb
0.625	500	0.003	44	2.1	187
0.875	500	0.003	87	3.0	276
1.125	700	0.005	239	9.0	554
1.375	700	0.005	357	11.0	698
1.750	800	0.006	693	19.2	1066
2.250	800	0.006	1147	24.7	1423

Drill	Cutting	Feed	Torque,	Machining	Thrust, lb
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	500	0.003	52	2.5	219
0.875	500	0.003	101	3.5	322
1.125	700	0.005	279	10.5	648
1.375	700	0.005	417	12.9	816
1.750	800	0.006	811	22.5	1246
2.250	800	0.006	1340	28.9	1663

 Table 5.4. Free-machining Carbon Steels, Brinell Hardness

 Range is (180–260) HB

Table 5.5. Alloy Steels (AISI 4000, 5000, and 8000 series), B	rinell
Hardness Range is (180–260) HB	

Drill	Cutting	Feed	Torque,	Machining	Thrust, lb
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	500	0.003	66	3.2	281
0.875	500	0.003	130	4.5	414
1.125	600	0.005	358	11.6	831
1.375	600	0.005	535	14.1	1047
1.750	800	0.006	1040	28.8	1599
2.250	800	0.006	1720	37.1	2135

Table 5.6. Alloy Steels (AISI 4000, 5000, and 8000 series), Brin	ıell
Hardness Range is (260–340) HB	

Drill	Cutting	Feed	Torque,	Machining	Thrust, lb
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	500	0.003	74	3.6	312
0.875	500	0.003	144	5.0	460
1.125	600	0.005	398	12.9	924
1.375	600	0.005	595	15.7	1164
1.750	800	0.006	1156	32.0	1777
2.250	800	0.006	1911	41.2	2372

Drill	Cutting	Feed	Torque,	Machining	Thrust, lb
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	400	0.003	66	2.6	281
0.875	400	0.003	130	3.6	414
1.125	500	0.004	287	7.7	695
1.375	500	0.004	428	9.4	876
1.750	600	0.007	1214	25.2	1809
2.250	600	0.007	2006	32.4	2415

Table 5.7. Tool Steels, Brinell Hardness Range is (180-260) HB

Table 5.8. Tool Steels, Brinell Hardness Range is (260–340) HB

Drill Diameter, in	Cutting Speed, sfm	Feed Rate, ipr	Torque, in Ib	Machining Power, hp	Thrust, Ib
0.625	400	0.003	74	2.9	312
0.875	400	0.003	144	4.0	460
1.125	500	0.004	318	8.6	773
1.375	500	0.004	476	10.5	973
1.750	600	0.007	1349	28.0	2010
2.250	600	0.007	2229	36.0	2683

Table 5.9. Stainless Steels (300 & 400 series), Brinell Hardness Range is (150–200) HB

Drill	Cutting	Feed	Torque,	Machining	Thrust, lb
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	200	0.003	55	1.1	234
0.875	200	0.003	108	1.5	345
1.125	300	0.005	299	4.8	693
1.375	300		446	5.9	873
1.750	400	0.006	867	12.0	1332
2.250	400	0.006	1433	15.4	1779

Drill	Cutting	Feed	Torque,	Machining	Thrust, Ib
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	200 200	0.003	66 130	1.3 1.8	281 414
1.125	300 300	0.005	358 535	5.8 7 1	831 1047
1.750	400	0.006	1040	14.4	1599
2.250	400	0.006	1720	18.5	2135

Table 5.10. Stainless Steels (300 & 400 series), Brinell Hardness Range is (200–300) HB

Table 5.11. Cast Iron, Brinell Hardness Range is (180–250) HB

Drill	Cutting	Feed	Torque,	Machining	Thrust, Ib
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	500	0.003	52	2.5	219
0.875	500	0.003	101	3.5	322
1.125	700	0.006	334	12.6	748
1.375	700	0.006	500	15.4	942
1.750	800	0.009	1214	33.6	1720
2.250	800	0.009	2006	43.2	2296

Table 5.12. Inconel 100

Drill Diameter, in	Cutting Speed, sfm	Feed Rate, ipr	Torque, in Ib	Machining Power, hp	Thrust, lb
0.625	80	0.003	81	0.6	343
0.875	80	0.003	159	0.9	506
1.125	90	0.005	438	2.1	1016
1.375	90	0.005	654	2.6	1280
1.750	100	0.006	1272	4.4	1954
2.250	100	0.006	2102	5.7	2609

Drill Diameter, in	Cutting Speed, sfm	Feed Rate, ipr	Torque, in Ib	Machining Power, hp	Thrust, Ib
0.625	100	0.003	48	0.5	203
1.125	125	0.003	94 207	1.4	299 502
1.375 1.750	125 150	0.004 0.005	309 626	1.7 3.2	633 998
2.250	150	0.005	1035	4.2	1332

Table 5.13. Titanium Alloys

Table 5.14. Aluminum Alloys (2017T, 2042T, & 6061T Grades)

Drill	Cutting	Feed	Torque,	Machining	Thrust, lb
Diameter, in	Speed, sfm	Rate, ipr	in Ib	Power, hp	
0.625	600	0.004	30	1.7	118
0.875	600	0.004	58	2.4	174
1.125	900	0.008	191	9.3	404
1.375	900	0.008	285	11.3	508
1.750	1200	0.010	578	24.0	802
2.250	1200	0.010	995	30.9	1071

The relationship between machining power (P_c) , metal removal rate (Q), and power constant (K_p) is expressed by formula (3.32):

$$P_{\rm c} = QK_{\rm p} \ ({\rm hp \ or \ kW})$$

Rewriting this formula in respect to K_p gives:

$$K_p = \frac{P_c}{Q} \tag{5.12}$$

Replacing Q in formula (5.12) by its value from formula (5.7) gives,

Customary U.S. units of measure:

$$K_p = \frac{P_c}{3V_c Df} \text{ (hp/in^3/min)}$$
(5.13)

Replacing Q in formula (5.12) by its value from formula (5.9),

Metric units of measure:

$$K_p = \frac{P_c}{25V_c Df} \text{ (kW/cm^3/min)}$$
 (5.14)

The following example shows how to calculate the power constant by using data from the first row in Table 5.1.

Given: drill diameter D = 0.625 in., cutting speed $V_c = 500$ sfm, feed rate f = 0.003ipr, and machining power $P_c = 3.2$ hp.

Required: power constant K_p . The power constant is calculated in customary U.S.

The power constant is calculated in customary 0.8, units of measure by formula (5.13):

$$K_p = \frac{3.2}{3 \cdot 500 \cdot 0.625 \cdot 0.003} = \frac{3.2}{2.8125}$$

= 1.138 hp/in³/min.

To convert the power constant value from customary U.S. units of measure into metric units of measure, the multiplier M = 0.0455 should be used (see subdivision 3.3.2 Power Constant).

 $K_p = 1.138 \cdot 0.0455 = 0.0518$ kW/cm³/min.

Data from all rows in Table 5.1 were used to calculate power constants for carbon steels with Brinell hardness ranging from 140 to 220 HB. The results of these calculations are given in Table 5.15.

The average value of the six power constant data is calculated as the arithmetic mean.

Customary U.S. units of measure:

$$K_p = \frac{1.138 + 1.143 + 1.153 + 1.146 + 1.143 + 1.144}{6}$$
$$K_p = \frac{6.867}{6} = 1.144 \text{ hp/in}^3/\text{min.}$$

Metric units of measure:

$$K_p = \frac{0.0518 + 0.0520 + 0.0525 + 0.0522 + 0.0520 + 0.0521}{6}$$

$$K_p = \frac{0.3126}{6} = 0.0521 \text{ kW/cm}^3/\text{min.}$$

Tabl	e 5.15. Power Co	instants for Ca	arbon Steels, Ha	rdness Range is (140	–220) HB
Drill Diameter, in	Cutting Speed, sfm	Feed Rate, ipr	Machining Power, hp	Power Constant K _p , hp/in³/min	Power Constant, K _p , kW/cm ³ /min
0.625	500	0.003	3.2	1.138	0.0518
0.875	500	0.003	4.5	1.143	0.0520
1.125	550	0.005	10.7	1.153	0.0525
1.375	550	0.005	13.0	1.146	0.0522
1.750	600	0.006	21.6	1.143	0.0520
2.250	600	0.006	27.8	1.144	0.0521

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Chapter 5

Similar calculations of power constants and their average values were performed for all other work materials (Tables 5.2 through 5.14). The average value of these power constants in customary U.S. and metric units of measure are given in Table 5.16.

5.3.2 Machining Power at the Drill

Power constants given in Table 5.16 can be used to calculate machining power at the drill. These calculations are performed by formula (3.32) described in Chapter 3:

$$P_{\rm c} = QK_{\rm p}$$
 (hp or kW)

Replacing Q in this formula by its values from formulas (5.7) and (5.9) expresses machining power at the drill in customary U.S. and metric units of measure respectively.

Customary U.S. units of measure:

$$P_c = 3V_c DfK_p \text{ (hp)} \tag{5.15}$$

where V_c is in sfm, D is in inches, f is in ipr, and K_p is in hp/in³/min

Metric units of measure:

$$P_{c} = 25V_{c}DfK_{p}$$
 (kW) (5.16)

where V_c is in m/min, D is in cm, f is in cm/rev, and K_p is in kW/cm³/min.

Work Material	Brinell Hardness Range, HB	K,, Customary U.S. Units of Measure, hp/in³/min	K _p , Metric Units of Measure, kW/cm³/min
Carbon steels Carbon steels Free-machining carbon steels Free-machining carbon steels Alloy steels (AISI 4000, 5000, and 8000 series) Alloy steels (AISI 4000, 5000, and 8000 series) Tool steels Tool steels Stainless steels (AISI 300 and 400 series) Stainless steels (AISI 300 and 400 series) Cast iron Inconel 100 Trianium allows	140–220 220–300 120–180 180–260 180–260 180–260 260–340 150–200 180–250 180–250	1.144 1.272 0.759 0.891 1.142 1.144 1.274 1.274 1.274 1.274 1.276 0.956 0.889 0.889 0.892	0.0521 0.0579 0.0345 0.0405 0.0520 0.0521 0.0580 0.0405 0.0405 0.0633
Aluminum alloys (2017T, 2042T, and 6061T grades)		0.381	0.0173

The following example of calculations is based on data from Table 5.2, row 4:

 $V_c = 550 \text{ sfm} (167.6 \text{ m/min}), D = 1.375 \text{ in} (3.49 \text{ cm}), f = 0.005 \text{ ipr} (0.0127 \text{ cm/rev}), and from Table 5.16, row 2: <math>K_p = 1.272 \text{ hp/in}^3/\text{min} (0.0579 \text{ kW/ cm}^3/\text{min}).$

Customary U.S. units of measure:

 $P_c = 3 \cdot 550 \cdot 1.375 \cdot 0.005 \cdot 1.272 = 14.43$ hp

(compare with 14.4 hp shown in Table 5.2).

Metric units of measure:

 $P_{o} = 25 \cdot 167.6 \cdot 3.49 \cdot 0.0127 \cdot 0.0579 = 10.75$ kW

5.3.3 Machining Power at the Motor

Machining power at the motor is calculated in similar manner to that described in Chapter 2, "Milling". Formula (2.54) and Table 2.5 "Machine Efficiency Factors" should be used for the calculations.

5.3.4 Tangential Cutting Force

Knowledge of the tangential cutting force is very important, since this force generates torque at the drill. Using formulas (3.30), (3.31), and (3.32), described in Chapter 3, "Turning", the author developed two formulas for calculating tangential force: one formula in customary

U.S. units of measure and the other—in metric units of measure.

Customary U.S. units of measure

To develop an equation for tangential force, two formulas for machining power at the cutting tool are used: Formula (3.30)

$$P_c = \frac{F_t V_c}{33,000}$$

Formula (5.15)

$$P_{\rm c} = 3V_{\rm c}DfK_{\rm p}$$

It is obvious from comparison of these two formulas that,

$$\frac{F_t V_c}{33,000} = 3V_c D f K_p$$
(5.17)

By solving equation (5.17) in respect to tangential force F_t , we get:

$$F_t = \frac{33,000 \cdot 3V_c D f K_p}{V_c}$$
(5.18)

Cancellation by V_c and simplification, gives:

$$F_t = 99,000 Df K_p$$
 (5.19)

Tangential force calculation by formula (5.19), using the following data:

 $D = 1.375 \text{ in}, \ f = 0.005 \text{ ipr}, \text{ and } K_p = 1.272 \text{ hp/in^3/min}$ $F_t = 99,000 \cdot 1.375 \cdot 0.005 \cdot 1.272 = \underline{865.8} \text{ lb}$

Metric units of measure

To develop an equation for tangential force, two formulas for machining power at the cutting tool are used: Formula (3.31)

$$P_{\rm c} = \frac{F_t V_{\rm c}}{60,000}$$

Formula (5.16)

$$P_{\rm c} = 25 V_{\rm c} D f K_{\rm p}$$

It is obvious from comparison of these two formulas that,

$$\frac{F_t V_c}{60,000} = 25 V_c D f K_p$$
(5.20)

By solving equation (5.20) in respect to tangential force F_r , we obtain:

$$F_t = \frac{60,000 \cdot 25V_c Df K_p}{V_c}$$
(5.21)

Cancellation by V_c and simplification, gives:

$$F_t = 1500000 Df K_p$$
 (5.22)

Tangential force calculation by formula (5.22), using the following data:

- D = 3.49 cm, f = 0.0127 cm/rev, and $K_p = 0.0579$ kW/ cm³/min
- $F_t = 1500000 \cdot 3.49 \cdot 0.0127 \cdot 0.0579 = 3849.5$ N

5.3.5 Torque at the Drill

Torque at the drill (T_c) is generated by tangential force (F_t) applied to the periphery of the tool. The arm of applied force is the radius of the drill (R). Therefore, the amount of torque is found from:

Customary U.S. units of measure,

$$T_{\rm c} = F_t R = \frac{F_t D}{2}$$
 (in lb) (5.23)

where D is the drill diameter in inches

Torque calculation by formula (5.23), using the following data:

$$F_t = 865.8 \text{ lb}$$
 and $D = 1.375 \text{ in}.$
 $T_c = \frac{865.8 \cdot 1.375}{2} = \underline{595.2} \text{ in lb}$

(compare with 595 in lb, Table 5.2, row 4). *Metric units of measure*,

_ _

$$T_c = \frac{F_t D}{2000} (N \cdot m)$$
 (5.24)

where D is the drill diameter in mm.

Torque calculation by formula (5.24), using the following data:

$$F_t = 3849.5 \text{ N}$$
 and $D = 34.92 \text{ mm}$
 $T_c = \frac{3849.5 \cdot 34.92}{2 \cdot 1000} = \frac{134424.5}{2000} = 67.2 \text{ N} \cdot \text{m}$

Torque can also be calculated as a function of machining power (P_c) and spindle speed (n). This relationship is shown in Chapter 2, "Milling", equation (2.52), and equation (2.53). By solving these equations in respect to torque values, we get:

Customary U.S. units of measure:

$$T_{\rm c} = \frac{63,025P_{\rm c}}{n} \tag{5.25}$$

Metric units of measure:

$$T_{\rm c} = \frac{9549P_{\rm c}}{n}$$
 (5.26)

To calculate the torque values the following data are used:

Machining power $P_c = 14.43$ hp (10.75 kW) as previously calculated

Drill diameter D = 1.375 in. (34.92 mm) and cutting speed $V_c = 550$ sfm (167.6 m/min), as shown in Table 5.2, row 4.

Spindle speed is calculated by formulas (2.3) and (2.4):

Customary U.S. units of measure,

$$n = \frac{12V_{\rm c}}{\pi D} = \frac{12 \cdot 550}{\pi \cdot 1.375} = 1528 \text{ rpm}$$

Metric units of measure,

$$n = \frac{1000V_c}{\pi D} = \frac{1000 \cdot 167.6}{\pi \cdot 34.92} = 1528 \text{ rev/min}$$

The torque values are as follows: *Customary U.S. units of measure*, formula (5.25),

$$T_{\rm c} = \frac{63,025 \cdot 14.43}{1528} = \frac{595.2}{1528}$$
 in lb

(compare with 595 in lb, Table 5.2, row 4).

Metric units of measure, formula (5.26),

$$T_{\rm c} = \frac{9549 \cdot 10.75}{1528} = 67.2 \text{ N} \cdot \text{m}$$

5.3.6 Thrust (Axial Force)

Thrust (axial force) data when drilling into most common work materials are given in Tables 5.1 through 5.14. To develop a formula for calculating thrust, the author calculated tangential forces and compared them with thrust data given in each of six rows in all 14 tables. Statistical analysis of the relationship between thrust data and respective tangential force data (same drill

diameter, same work material, and the same drilling conditions) showed that the coefficient of proportionality between these two drilling forces varies less than $\pm 4\%$ (for all steels and Inconel 100), $\pm 5\%$ (for titanium alloys), and $\pm 6\%$ (for cast iron and aluminum alloys).

As a result of this analysis, the ratio of thrust (axial force) to tangential force was established. Table 5.17 is an example of a comparison between these forces, generated when drilling into carbon steel (140–220) HB. The drill diameter, feed rate, cutting speed, torque, and thrust data are the same as shown in Table 5.1. Tangential force values were calculated from formula (5.23):

$$T_{\rm c} = \frac{F_t D}{2},$$

from which:

$$F_t = \frac{2T_c}{D} \tag{5.27}$$

The average value of the six $\frac{F_s}{F_t}$ ratios (R_a) is calculated as an arithmetic mean.

$$R_{a} = \frac{\sum(\frac{F_{s}}{F_{t}})}{6} = \frac{1.33 + 1.39 + 1.31 + 1.35 + 1.34 + 1.40}{6}$$
$$= \frac{8.12}{6} = 1.35$$

Similar calculations of tangential forces and thrustto-tangential force ratios were performed for all other

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Drill Diameter, in	Feed Rate, ipr	Cutting Speed, sfm	Torque, $T_{c'}$ in Ib	Thrust <i>F_z,</i> Ib	Tangential Force, <i>F_t</i> , lb	F_z/F_t Ratio
0.625	0.003	500	99	281	211	1.33
0.875	0.003	500	130	414	297	1.39
1.125	0.005	550	358	831	636	1.31
1.375	0.005	550	535	1047	778	1.35
1.750	0.006	600	1040	1599	1189	1.34
2.250	0.006	600	1720	2134	1529	1.40

work materials (Tables 5.2 through 5.14). The average values of thrust-to-tangential force ratios were then calculated and the results are summarized in Table 5.18.

Table 5.18 shows that all steels and Inconel 100 have practically the same ratios of thrust to tangential force. The ratio for cast iron is only 3.7% less than that for steels, and the ratio for titanium alloys is only 3.0% greater than that for steels.

The ratio of thrust to tangential force is a coefficient of proportionality (R_a) that can be used to calculate thrust (F_s) through the tangential force (F_r) ,

$$F_{z} = R_{a}F_{t} \tag{5.28}$$

Replacing F_t in formula (5.28) by its value from formula (5.19) gives the following equation in *customary U.S. units of measure,*

$$F_{s} = 99,000R_{a}DfK_{p}$$
 (lb) (5.29)

Replacing F_t in formula (5.28) by its value from formula (5.22) gives the following equation in *metric units* of measure,

$$F_{\rm g} = 1500000 R_a D f K_p (N)$$
 (5.30)

Example of calculating thrust in customary U.S. units of measure

Using data from Table 5.6: drill diameter D = 0.625 in., feed rate f = 0.003 ipr, drilling into AISI 4000 series alloy steel with the hardness range 260 to 340 HB.

Work Material	Brinell Hardness Range, HB	Ratio of Thrust to Tangential Force, <i>R_a</i>
Carbon steels	140–220	1.35
Carbon steels	220-300	1.35
Free-machining		
carbon steels	120-180	1.35
Free-machining		
carbon steels	180-260	1.35
Alloy steels		
(AISI 4000, 5000, and		
8000 series)	180-260	1.35
Alloy steels		
(AISI 4000, 5000, and		
8000 series)	260-340	1.35
Tool steels	180-260	1.36
Tool steels	260-340	1.36
Stainless steels (AISI		
300 and 400 series)	150-200	1.35
Stainless steels (AISI		
300 and 400 series)	200-300	1.35
Cast iron	180-250	1.30
Inconel 100		1.35
Titanium alloys		1.39
Aluminum alloys (2017T, 2042T, and		
6061T grades)		1.23

Table 5.18. Thrust-to-Tangential Force Ratios for Commonly Used Work Materials

Power constant for this work material is found in Table 5.16: $K_p = 1.272$ hp/in³/min, and thrust-to-tangential force ratio for this work material is found in Table 5.18: $R_a = 1.35$.

Calculating thrust by formula (5.29):

$$F_{z} = 99,000R_{a}DfK_{p}$$

= 99,000 \cdot 1.35 \cdot 0.625 \cdot 0.003 \cdot 1.272 = 318.8 lb

Compare with $F_z = 312$ lb given in Table 5.6 row 1. The calculated thrust value exceeds the recorded one by 2.2%.

Example of calculating thrust in metric units of measure

The same data as mentioned above, but converted to metric units, are used: D = 1.59 cm, f = 0.0076 cm/rev, $K_p = 0.0579$ kW/cm³/min (Table 5.16), and $R_a = 1.35$ (Table 5.18, the same value, since it is a dimensionless coefficient of proportionality).

Calculating thrust by formula (5.30):

$$\begin{split} F_z &= 1500000 R_a D f K_p \\ &= 1500000 \cdot 1.35 \cdot 1.59 \cdot 0.0076 \cdot 0.0579 = 1416.8 \text{ N} \end{split}$$

To obtain the thrust value in lb, multiply thrust value in N by conversion factor equal to 0.2248089 lb/N:

$$F_{\sim} = 1416.8 \text{ N} \cdot 0.2248089 \text{ lb/N} = 318.5 \text{ lb}$$

(Compare with the thrust value of <u>318.8</u> lb from previous calculation).

Concluding Remarks on Drilling

1. Indexable insert drills are becoming very popular because their advanced cutting geometry and improved coated carbides allowed for drilling at much higher cutting speeds compared with other drill designs. Therefore, higher metal removal rates and productivity can be obtained with indexable drills, especially when drilling diameters are between 0.625 and 2.375 in. (16 and 60 mm).

2. To calculate cutting speeds and spindle speeds, formulas (2.1) through (2.4) for kinematics of milling can be used. Cutting speeds relate to the outside diameter of the drill. Symbol *D* in these formulas is the outside diameter of the drill.

3. The penetration rate and the metal removal rate are characteristics of productivity. The penetration rate is calculated by formula (5.1). To calculate metal removal rate, the author developed formulas (5.7) and (5.9) for customary U.S. and metric units of measure respectively.

4. As with turning and boring, power constants of the most commonly used work materials are now available for drilling. The author conducted an analytical study of the experimental data presented in Tables 5.1 through 5.14 and developed formulas (5.13) and (5.14) to calculate power constants in customary U.S. and metric units of measure respectively. Statistical analysis of data produced the arithmetic mean of power constants for the most popular work materials (Table 5.16).

5. Knowledge of power constants allows calculation of machining power at the drill through establishment of the metal removal rate using formulas (5.15) and (5.16) developed by the author for customary U.S. and metric units of measure respectively.

6. The author developed two formulas to calculate the tangential cutting force in relationship to the drill diameter, feed rate, and power constant. Formula (5.19) was developed for customary U.S. units of measure, and formula (5.22) was developed for metric units of measure.

7. The author compared recorded thrust force data in Tables 5.1 through 5.14 with tangential force values calculated for the same drilling conditions and conducted statistical analysis of thrust-to-tangential force ratios (Table 5.17). Statistical analysis of these ratios allowed establishment of coefficients of proportionality for each work material and its Brinell hardness range indicated in all 14 tables.

8. Coefficients of proportionality are summarized in Table 5.18. As can be seen from the table, all steels and Inconel 100 have practically the same coefficient of proportionality (1.35–1.36). Values for cast iron—1.30, titanium alloys—1.39, and aluminum alloys—1.23 were also established.

9. The author developed two formulas for calculating thrust (axial) force in relationship to the coefficient of proportionality, drill diameter, feed rate, and power constant. Formula (5.29) was developed for customary U.S. units of measure, and formula (5.30) was developed for metric units of measure.

Afterword

In the process of writing "Engineering Formulas for Metalcutting", the author was asked a question: "What is the unique selling feature of your book?"

"The book provides information on formulas and their applications in concise and clearly arranged form. It can serve as valuable source of knowledge in metalcutting practice and training. This book will be especially appropriate for situations in which calculations are necessary for setting maximum productivity from machine tools".

"Can you be more specific?" asked John Carleo, Director of Publications for Industrial Press.

Two interesting cases that the author has solved during two consecutive metalcutting trade shows (WESTEC-97 and WESTEC-98), explain "specifics" of required machining power versus the machine tool available power.

Case # 1, WESTEC-97

An exhibitor of bench-type milling machines complained that he could not demonstrate milling operation because his machine stalls. He believed that the milling cutter caused the problem and blamed the cutting tool manufacturer for defective face mill. The author asked the exhibitor to provide machining conditions at which he tried to machine the workpiece. The cutting conditions were as follows:

Diameter of the face mill—2.5 in., the number of indexable inserts—5,

Cutting speed—600 sfm, depth of cut—0.150 in., width of cut—1.0 in.,

Feed per tooth-0.008 in.,

Workpiece—AISI 4140 alloy steel, Rockwell hardness—48 HRC,

Available power of the milling machine—3.0 hp.

The author calculated required power at the motor of the milling machine by formulas shown in Chapter 2. "Milling", and reveled the results. The cutting conditions selected by the exhibitor, required 7.9 hp! That is why the 3—hp milling machine stalled. The author recommended the following milling conditions:

Cutting speed—400 sfm, depth of cut—0.125 in., feed per tooth—0.005 in.

The new cutting conditions required only 2.8 hp, which was within the power of the milling machine. The author witnessed an excellent performance of the milling cutter resulted in required surface finish at the maximum possible productivity of a given milling machine.

Case # 2, WESTEC-98

A visitor, the owner of a machine shop, wanted to buy a lathe, but was not sure either a 15—hp or a 30—hp machine tool he should buy. He said that the parts would be made from 4140 alloy steel with Brinell hardness ranging from 200 to 220 HB at the following cutting conditions:

Cutting speed—550 sfm, depth of cut—0.200 in., feed rate—0.012 ipr.

The author calculated required power at the motor of the lathe having an oil-hydraulic drive (machine efficiency factor is 80%) by formulas shown in Chapter 3. "Turning". The calculated value of required power at the motor was 14.3 hp. A visitor expressed his gratitude for the calculations and said that an exhibitor of machine tools recommended him a 30—hp lathe. "Now, he said, I know that a smaller lathe will do the job, and for the difference in cost I can buy the cutting tools".
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